Numerical modeling of nanophotonic crystal waveguides and waveguide devices

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NUMERICAL MODELING OF NANOPHOTONIC CRYSTAL WAVEGUIDES AND WAVEGUIDE DEVICES

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A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of the original research and has not been submitted for a higher degree to any other university or institution.

Date

Khoo Eng Huat

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Signature: _____

SUMMARY

The objective of this thesis is to construct numerical models for various characteristics of nanophotonic crystals (PCs) waveguide based on its geometrical variation and surface perturbation.

The first major work of this thesis is to improve the coupling efficiency to the narrow single mode PC waveguide (PCWG) by using PC tapered waveguide (PCTW). The tapered curvature is designed to have either linear or nonuniform shape. The curvature of the PCTW is varied by a parameter, α . For $\alpha > 1$ (and $\alpha < 1$), concave (and convex) PCTW is solved respectively whereas $\alpha = 1$ gives the linear structure. A numerical model based on the modified step-theory is developed to calculate the transmission efficiency of the various PCTW curvatures. This numerical model is computationally efficiency and sophisticated enough to include the essential field physics for the simulation of field propagation in PCTW. The numerical results show that an average coupling efficiency of greater than 97 % at $\alpha = 0.5$ is obtained for a short taper length of $36a$, where a is the lattice constant. The coupling efficiency is higher and the shorter length is much shorter compared to the previous PCTW designs. The coupling mechanisms and loss characteristics in various tapered curvatures are also discussed.

To have an in-depth understanding of the coupling mechanism and the behavior of the interaction between the modes in the tapering section, a set of coupling equations is derived from the Maxwell's equation based on the step theory. The newly developed theory, called the exact step-coupling-theory can describe the coupling mechanism and modal interaction behavior between the propagating modes occurred in waveguide structures with geometrical variation. The exact step-coupling-theory is better than the

modified step-theory and other previous semi-analytical model because it allows flexibility in the mode characterization as well as in-depth understanding of the modal behavior in the tapering section. The coupling equations are analyzed in detail to obtain a comprehensive knowledge of the mode coupling and interaction behavior in the tapering section. The coupling equations are applied to the different case studies such as a 3D PC slab tapered waveguide and lossy PCWG to investigate the loss characteristic and transmission efficiency. A transmission efficiency of approximately 90% is achieved for the 3D slab taper and the high loss is due to mismatch of mode size in the vertical plane.

In the final contribution of the thesis, the physics of surface waveguiding at the interface between PC and air is investigated. Surface waveguiding is better than the current PCWG because increases device compactness, offers higher degree of freedom in design and manipulation in the air side and make characterization and measurement easier. The properties of surface modes in PBG are discussed to obtain a useful range of frequencies for waveguiding purpose. An investigation on the transmission efficiency of light waves along the surface bend is performed and transmission efficiency of more than 90% is achieved by perturbing the corner cell of the surface bending. An add/drop circulator is designed on the circular photonic crystal using a curve surface waveguide. Light waves are coupled out by the matching resonator system at designated frequencies to redistribute it to other locations. By placing two surface waveguide together, a dual surface waveguide is also designed. By adding dipole resonator rods and other structures, a power splitter is developed to split light waves with different power ratio varying from 10 to 90 % using index modulation. The modulation range of the power splitter by using the dual surface waveguide is larger than other power splitter designs.

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CHAPTER 1

INTRODUCTION

1.1 Motivation

In the last 50 years, semiconductor industry was one of the most important research areas and industry to all researchers and scientists. However, the speed of electronic integrated circuits has been restricted [1, 2] when data are transmitted through electrons. Various new research areas have been developed to establish the possibility of transmitting data through photons rather than electrons as photons travel at much faster speed than electrons. Motivated by this idea, photonic integrated circuits have been proposed by many researchers and immense research efforts in design, fabrication packaging and characterization of the photonic integrated circuits have flourished over the last two decades.

The photonic integrated circuit is a device that contains many individual optical components which are integrated onto a single chip [3]. It can be designed to perform single or multiple functions. The photonic integrated circuit is fabricated by using silicon wafer technology rather than silica or nonlinear crystal material. The dominant technical innovation is mainly driven by the booming communication industry [4] in which the photonic integrated circuit plays a very important role. The advantages of the microscale integrated chip are high speed, small size and low cost. The innovation of the photonic integrated circuit has revolutionized the field of communication, computer system and

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biotechnology. The potential market for the photonic integrated circuit seems promising with the increasing demand of high speed information technology.

Small scale-interconnects between individual devices are important in the photonic integrated circuit in order to effectively produce a high density integrated circuit and to guide light around a tight corner [5]. The high power leakage at the tight corner has been a major challenge for scientists and researchers in the design of conventional optical silica waveguides. The ability to intersect waveguides is crucial in the construction of a complex system involving multiple waveguides for the commercialization of the photonic integrated circuit. Low crosstalk and high throughput is desired but such properties cannot be fully achieved with present optical waveguides structures. Another challenge is to reduce the size of the photonic devices to wavelength/sub-wavelength dimension for on-chip applications [6]. The objective of the thesis is therefore aimed to address these challenges by using photonic crystal technology [7].

Photonic crystal (PC) is a new class of material which has attracted the attention of many researchers and scientists. It is currently one of the latest innovations in the field of optics and photonics has been proposed to be applied to photonic system integrated onto a single chip. The control of photons in the periodic structure of PC is similar to the way one control electrons in semiconductor circuit. Greater flexibility in altering and manipulating the optical properties [8] of photons allows a vast potential application of PC in fiber optics communication and next-generation optical computer etc.

PC technology is expected to bring similar success and impact to human life as the same case for semiconductor technology in the present and future. This is indeed true

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as many miniaturized photonics devices have emerged since the discovery of PC. The simplest device formed PC is an omni-directional mirror [14], which has low absorption at almost all wavelength. Point defects are introduced to the lattice structure to trap lightwaves in suitable sites to form a small-volume photonic microcavity [15-17]. By combining these point defects, a line defect is formed so that light waves travels along the line defects with very low dispersion [18], similar to an optical waveguide medium [19-21]. PC based waveguide (PCWG) is able to address the problem of transmitting lightwaves around tight bend with low loss and high transmission efficiency.

The manipulation of the periodic lattice structure, has led to the discovery of more advanced and sophisticated optical phenomenon such as superprism [22-24], directional emission [25-27] and surface localization [28-30]. Negative refractive index [31-33] and self focusing property [34, 35] using the PC lattice structure are other attractive research areas. Another interesting optical phenomenon using PC is to stop light waves propagation or slowing down the speed of light waves [36-39]. This phenomenon may be used for package synchronization, dispersion compensation and enhanced light-matter interactions [40-42]. All these special properties enable the innovation of new integrated circuit. Beside the bandgap and defect engineering of PC, the band edge properties of PBG [43-46] are also investigated to locate the standing waves, which resembles a cavity mode lightwaves and enables coherent oscillation to be realized over a large area [44, 45]. In the recent years, a new design of PC heterostructures, which is expected to improve the performance of the photonic devices have emerged [47-50]. Using the presence of different dispersion properties of the heterostructures, a number of novel

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photonic devices have been demonstrated by manipulating the physical geometries [49, 50].

All these interesting and amazing optical properties explain the potentials of PCs for the invention of next-generation optoelectronics devices. PC technology is expected to sprung into an endless flow of innovations from the optical communication industry to transmission high speed information using photonic integrated circuit.

While PC devices are expected to contribute their potential in maneuvering the communication and computer industry, many challenges from the technology need to be address before actual realization. These challenges are coupling efficiently to small size PC devices, more sophisticated numerical model and simulation method to address the complex PC device structure and new transmission technique. They serve as a great source of motivation for this PhD research. The thesis is motivated by the need to establish and develop more efficient coupling design tool, more comprehensive and informative waveguide modeling tool and novel and sophisticated PC integrated devices and circuit.

The problem of coupling to PCs and PBG devices poses the greatest challenge due to its small size (which is about only a few optical wavelengths). Traditional methods of coupling do not work due mode mismatch and different modal shapes between PC and light sources. While grating couplers introduce excessive loss, the reliability of other coupling methods are also subjected to arguments. Slab conventional tapers for efficient mode conversion are available but requires very long (up to several millimeters) dimension. Tapers built from PC structure have the advantages of efficient mode conversion, short length, low loss and compact. There are some reported works [20-21]

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on PC waveguide for this coupler design but only a few coupling issues are clearly addressed and discussed. Most of the works address on simple coupling structures and more useful designs are not available. This is an unbalanced situation because the coupling efficient devices strongly determine the functionality, reliability and usability of the PBG device in integrated photonic devices. As a result, based on the above argument, there is a strong motivation for this PhD research to design an effective and useful PC coupling to enhance the coupling efficiency between light source to PC devices.

Mathematical models and simulation tool are necessary for the optimization and analysis of the PC waveguide and waveguide devices. The current mathematical models [5, 16-17] for PCWG are too general and often used to calculate the transmission efficiency and other simple characteristics of waveguide. These mathematical models are only applicable for straight PCWG. However, for more complex PCWG and devices, these mathematical models are either not suitable or inadequate to provide complete explanations for many optical phenomena that occur in tapering section. Therefore, the detailed analysis and investigation on the coupling mechanism in the tapering section using mathematical modeling and simulation tools are significant in this PhD research. The mathematical model must include the periodic function for more accurate modeling of the periodic boundary at PC tapers. At the same time, a thorough description and analysis of the intermodal coupling and power exchange between various modes along the tapering section is crucial for the design of the PC tapers. The analysis results serve as a useful guideline for the design and optimization of PC tapers. Therefore, it is necessary to develop new mathematical models and tools to fully describe, demonstrate and analyze the intermodal coupling mechanism of the PC coupling structures

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For this PhD project, a different kind of waveguide, known as the PC surface waveguide is studied. The motivation for this thesis is to explore new possibility for feeding and redistribution light waves using PC, to realize the photonic integrated circuit. So far, only few studies are found focusing on PC surface waveguide as compared to the conventional PCWG [5, 18]. This has further motivated the need to do a thorough investigation and design new photonic applications based on this phenomenon. The PC surface waveguide demonstrate high potential in feeding and distributing light waves. By using the higher degree of freedom in the design due to the presence of an empty area opposite to the PC lattice, the potential of the PC surface waveguide can be unleashed and a whole new group of photonic devices can be innovated. Therefore, this greatly motivates the need to study and design new photonic device for photonic integrated circuit using PC surface waveguide and waveguide devices in this PhD research.

With the strong motivations displayed in the above discussion, specific objectives are needed to be set so that a more thorough discussion and investigation can be conducted for this PhD project in the next section.

1.2 Objectives

The main objective of this thesis is to study nanophotonic crystal waveguide, which includes numerical modeling and simulations of the various coupling mechanisms for the PC waveguide and waveguide designs. In this thesis, two different types of the PC waveguide are included: (i) tapered waveguide and (ii) surface waveguides. From the physics point of view, the tapering section of the PC tapered waveguide (PCTW) offers an opportunity to investigate the coupling mechanism between various propagating modes and the design of the taper curvature for higher coupling efficiency. The

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derivation of the complete mathematical model based on the coupling equations provides a simulation tool for the design and optimization of PCTW.

The tapered waveguide structures based on the periodic lattice structures are constructed by shifting the lattice of a normal PCWG in the vertical direction to form the tapering section. The input width of the taper is set to be around the modal diameter of a single mode fiber. The output waveguide is a single row defect PCWG. The lattice parameters are chosen such that a PBG exist for TM polarization in the optical wavelength. The curvature of the PC tapered waveguide structure will be studied to give different curvature designs, depending on the curvature parameter specified in this thesis. By varying the curvature and the length of the tapering section, the maximum possible coupling efficiency with the shortest possible length for the PC tapered waveguide is achieved.

A theoretical formulation on the mode coupling equation is derived to describe the mode coupling and interaction mechanism in the tapering section. In this thesis, step-theory is used to describe the coupling behavior in the tapering section. To apply the step-theory to PCTW, some modification on the field expression is made and the modified step-theory is derived. The modified step-theory presents a simple description for the mode behavior and coupling mechanism in the geometrical varied tapering sections. The Bloch's theorem for periodic PC structure is included in the derivation of the coupling equation of the modified step-theory. The transmission coupling efficiency and the simulation results for the different taper curvature designs are compared and discussed.

An exact step-coupling-theory is derived to provide a more comprehensive and informative model for the coupling mechanism and modal interaction in the geometrical

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variation waveguide. The exact step-coupling-theory has a more complex and general set of coupling equations to model and investigate of general geometrical variation waveguide structures. The exact step-coupling-theory is derived from the Maxwell's equations with the principles of the step-theory. It can be applied to lossy waveguide design and 3D PC slab tapered waveguide studied in this PhD project and the numerical results are discussed in the later sections.

In this thesis, a new surface waveguide is studied. The location of the waveguide lies at the interface between the periodic boundary of PC lattice and air medium. The surface mode must be in the PBG and below the lightline so light waves can be localized on the PC surface. In addition, light waves must also travel with moderate group velocity so that it can be transmitted along the surface. Two case studies are investigated for surface waveguide: First, the transmission efficiency of lightwaves around perpendicular surface bend is studied and analyzed. Second, the transmission efficiency of the curve surface waveguide on a circular photonic crystal (CPC) array is investigated. Through additional resonator coupling systems, the curved surface waveguide is developed for on-chip photonic add/drop circulator filter device.

The next design is dual surface waveguide that involves two surface waveguides. The band characteristic and field distribution of the dual surface waveguides are studied to understand the conditions of transmitting light waves on both waveguides simultaneously. Based on the multipole physics, light waves are divided to two parts coupled to each of the dual surface waveguides, which provide the physical properties of the power splitter. Therefore, the new design of the power splitter is studied in this thesis.

By varying the refractive index of cell near the dipole splitting system, the splitting ratio can be tuned to different power splitting ratio.

1.3 Major contributions of the thesis

The theoretical and numerical modeling contribution of this PhD project on the different PCWG and waveguide devices designs are highlighted.

A new method known as the modified step-theory is developed to modeling the mode coupling and interaction in the PCTW. A new type of the PCTW is designed by changing the taper curvature to either linear or nonuniform (convex or concave). By simulation and optimization, the convex curvature tapered waveguide has higher coupling efficiency at shorter taper length compared to the linear and concave PCTW. The modal characteristics of PCWG such as transverse mode profile and mode number are also investigated. The coupling properties between the modes with different symmetries are also simulated using the modified step-theory and the simulation results are discussed. (see chapter 3)

Subsequently, the loss trend of the different PCTW curvature designs are analyzed to show the transmission characteristic along the tapering section. The linear and convex taper have similar loss trend while the concave taper has different loss trend due to the different taper curvature designs. The degree of power fluctuation in the transmission spectra of different taper curvatures and lengths are discussed to highlight the presence of multiple reflection and backward coupling in the tapering region. (see chapter 3)

The other theoretical model, known as the exact step-coupling-theory is derived to model the modal interaction in complete details and with greater accuracy for the

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general geometrical variation waveguide. The coupling of different field modes between forward directions, backward directions and forward/backward directions are modeled using the exact step-coupling-theory. The convergent rate of the exact step-coupling-theory is verified based on case study. The radiation modes expression are included in the coupling equations through the calculation of the transmission efficiency of a lossy “turnon” tapered waveguide. (see chapter 4)

The coupling equations of the exact step-coupling-theory are used to model the PC slab tapered waveguide. The two field components are handled separately to differentiate the guiding mechanisms. The transmission efficiencies of different PC slab tapered waveguide curvature designs are simulated. The convex PC slab tapered waveguide has the highest transmission efficiency but it is lower than the 2D counterpart due to some radiation loss in the vertical direction. (see chapter 4)

A new type of waveguiding mechanism known as the surface waveguiding is demonstrated for 2D PC structures. First, it demonstrates that the light waves are guided around the surface efficiently by modification of the size or the refractive index of the corner cell structure at the surface layer of the PC lattice. The simulation results have transmission efficiency of more 90 %. The transmission of light waves on the surface of a circular photonic crystal (CPC) structure is also investigated. By reducing the concentric period at the CPC surface, high transmission efficiency of more than 95 % is achieved. An add/drop circulator is designed by using resonator systems based on the curve surface waveguide of CPC. The simulated results shows that the add/drop circulator has a moderately-high Q value and high filtering efficiency of approximately 95 %. (see chapter 5)

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A dual surface waveguide is also designed by having two surface waveguides brought close together from two lattices. The modal characteristics of the dual surface waveguide with different separation distance are investigated. It is observed that the separation distance of the dual surface waveguide affects the modal separation of the between the two surface modes present in the PBG. Based on this observation, a power splitting device is developed. The power splitter uses a large dipole cell for splitting the lightwaves into 2 portions and a reflector cell is added to enhance splitting efficiency. The simulation results show good splitting efficiency when the reflector cell is added next to the dipole cell. The power transmission to each of the dual surface waveguide is nearly half of the incident power. The power ratio at each dual waveguide is also varied by changing the refractive index of the neighboring cells. (see chapter 5)

1.4 Organization of thesis

This thesis is divided into six chapters. An introduction of the thesis is given in Chapter 1, which covers the motivation, objective and major contributions of the thesis. The section on motivation explains the research challenge of the PC technology. The objective is to focus on the development of different PCTW curvature designs, surface waveguide and waveguide devices and mathematical model for describing the modal interaction and coupling in the forthcoming chapters. The contribution part lists all the significant innovations of both the mathematical model and design methodology.

In Chapter 2, the works that are being done previously for improving coupling efficiency to PC are reviewed. It provides a background for the PC and PCWG research and supports the current design of implementing tapered waveguide and waveguide

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devices. The past works of surface waveguide are reviewed and it can be further extended as a transmission and light distribution device.

In chapter 3, the modified step-theory is derived for the modeling of high coupling efficiency of providing high coupling efficiency between light source and the single mode PCWG. The theoretical studies include deriving the theoretical model for the explanation of the coupling mechanism in tapering region. The numerical results and conditions required for efficient coupling in tapered waveguide structures are established with the different type of tapered curvature designs.

In Chapter 4, an exact step-coupling-theory is derived for modeling the mode interaction and coupling mechanism in PC waveguide structures with geometrical variation. The assumptions and approximations is made in the modified step-theory are discard and several simplification methodology are introduced to reduce computational time. This is followed by the theoretical analysis of the exact step-coupling-theory using the coupling equations for the simulation of PC geometrical variation waveguide.

In Chapter 5, theoretical study and analysis of the PC surface waveguide is conducted and the conditions for surface waveguiding are set. The waveguiding of light waves at surface bend is investigated. Subsequently, the waveguiding efficiency at the surface of circular photonic crystal is investigated. Then an add/drop circulator based on circular PC is developed using resonator system. A dual surface waveguide structure is studied and a power splitting device, which to divided light waves power is designed in this chapter. By optimizing the physical properties of the cells in the splitting system, very good splitting efficiency can be achieved.

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Finally, the last chapter concludes the major results of this PhD project and recommends the further works.

CHAPTER 2

LITERATURE REVIEW

In this chapter, the literature survey of photonic crystal waveguide (PCWG) and coupling theory is presented. The basic knowledge of photonic crystal (PC), which includes the photonic bandgap (PBG), PC microcavities, PCWG and PC surface mode, is introduced. Other aspects of PCWG such as design principles and requirements, advantages and disadvantages of different coupling metrologies for high efficiency light transmission and semi-analytical numerical models are also discussed. PC surface modes and surface waveguide with its potential applications are also discussed.

2.1 Basic knowledge of photonic crystals (PCs)

Photonic crystals (PC) are artificial periodic structures which can manipulate electromagnetic lightwaves in one, two or three dimensions as shown in Fig. 2.1. The simplest structure is in one dimension. A good example of one dimensional (1D) PC is a multilayer film [51, 52] with alternating dielectric constant. Mathematical and physical origins of the multilayer film are derived from Bragg's multilayer reflector. With suitable contrast in dielectric constant and layer thickness, electromagnetic light waves at certain frequencies are reflected. In expanding to two dimensional (2D) PC, the structures consist of nano-pillars or cylindrical holes [53, 54] on plane and extend infinitely in the third

direction. The most complex structures are three dimensional (3D) PC lattice consisting of spherical balls [55] or “sticks” stacked in wood pile array [56].

1-dimensional (1D)

2-dimensional (2D)

3-dimensional (3D)

Fig. 2.1 Types of photonic crystal structures.

Although there are a number of ways to define the content of PC layout, the two most common periodic layouts are; (1) periodic array of high refractive index rods in vacuum and (2) air cylinder in dielectric medium. For dielectric rods layout, PBG exist for TM polarization while for air cylinder, PBG exist in the TE polarization. TM polarization is defined to have electric field in the direction parallel to the axis of the cylindrical dielectric rods while the TE mode polarization have magnetic field propagating in the direction parallel to the rod's axis.

2.1.1 Basic physics of one dimensional photonic crystal

The case of the multilayer film [51-52] is considered to understand the physical origin of PBG. This structure is periodic in one direction as shown in Fig. 2.2 and is known as 1D PC. It consists of alternating layers of materials with different dielectric constants (ϵ_{low} and ϵ_{high}) in the z direction and is stretched infinitely along the x - y plane.

Fig. 2.2 A multilayer film with lattice constant a and infinitely stretched in the x - y direction.

A light beam propagating in the z direction and crossing the layers at normal incident has a wave vector, k_z abbreviated by k and zero planar wave vector, $k_{//}$. The frequency spectrum is the light line given by $\omega(k) = \frac{ck}{\sqrt{\epsilon}}$. Based on Bloch's theorem [57], the wavevector k repeats itself after mb , where b is the reciprocal lattice vector given by $2\pi/a$ and m is an interger. For periodic lattice, only the *Brilliouin zone* is considered, restricting the wavevector k such that, $-\frac{\pi}{a} \leq k \leq \frac{\pi}{a}$.

From this restriction, k repeats itself outside the *Brilliouin zones*, folding back into the zone when they reach the edges. For the appearance of the photonic gap, the electromagnetic variation theorem is used. Due to the concentration of low and high frequency modes energies in the high ϵ_h and low ϵ_l regions respectively, the modes under the PBG have their energy concentrates in the high ϵ_h region while the modes above the gap have most of the energy concentrates in the lower ϵ_l region. The bands above the bandgap are conveniently referred as the “air band” while the band below as “dielectric band”. The band diagrams of the multilayer film with different combinations of dielectric constant are shown in Figs. 2.3(a) and (b).

(a)

(b)

Fig. 2.3 Bandgap structure of the 1D PC structure. (a) $\varepsilon_h = \varepsilon_l = 13$ (b) $\varepsilon_h = 1, \varepsilon_l = 13$

Figure 2.3(a) shows the band diagram for the film layers with the same index contrast without PBG. On the other hand, PBG appears between two bands in Fig. 2.3 (b) when there is index contrast between two film layers. The presence of index contrast causes lower frequency mode to concentrate in high ε_h while higher frequency is found in lower ε_l . The larger the indexes contrast between the two layers, the larger the PBG. It is important to have a large PBG to enable more flexibility in manipulating the photons properties in the PC structures. Also, unwanted losses such as band edge coupling can be neglected for larger PBG. Another observation is that for the one dimensional structures,

the bandgap always occurs between every set of bands, both at the Brillouin zone's edge and the centre of the lattice cell.

2.1.2 Mathematical model of photonic bandgap (PBG) structure

The electromagnetic spectrum consists of a large range of frequencies from very short Cosmic ray to long radio waves. Since visible light and infrared wavelength are parts of the spectrum, electromagnetic radiation using Maxwell's equations can be expressed as [58-59]

$$\nabla \cdot \mathbf{D} = \rho \quad (2.1a)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.1b)$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \quad (2.1c)$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} \quad (2.1d)$$

where $\mathbf{D}$ represent the electric displacement field, $\mathbf{B}$ is the magnetic flux density, $\mathbf{E}$ is the electric field, $\mathbf{H}$ is the magnetic field, $\mathbf{J}$ and ρ is the current and charge density of the electromagnetic radiation in the material media, respectively. Assuming that the medium is isotopic and low loss, the electric displacement and magnetic flux are expressed as

$$\mathbf{D}(\mathbf{r}) = \varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r}) \quad (2.2a)$$

$$\mathbf{B}(\mathbf{r}) = \mu_0 \mathbf{H}(\mathbf{r}) \quad (2.2b)$$

Expressing the Maxwell's equations in terms of magnetic field only using Eqs (2.2a) and (2.2b) gives

$$\nabla \times \left(\frac{1}{\varepsilon(\mathbf{r})} \nabla \times \mathbf{H}(\mathbf{r}) \right) = \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r}) \quad (2.3a)$$

$$\nabla \cdot \mathbf{H}(\mathbf{r}) = \nabla \cdot \mathbf{D}(\mathbf{r}) = 0 \quad (2.3b)$$

$$\mathbf{E}(\mathbf{r}) = \left(\frac{-ic}{\omega \varepsilon(\mathbf{r})} \right) \nabla \times \mathbf{H}(\mathbf{r}) \quad (2.3c)$$

Eq. (2.3a) is also called the *master* equation and solving it gives all the magnetic field modes for a given frequency. For Eq. (2.3a), a linear eigen-problem using Hermitian operator can be expressed as

$$\frac{1}{\varepsilon(\mathbf{r})} (\Theta \mathbf{H}(\mathbf{r})) = \left(\frac{\omega}{c} \right)^2 \mathbf{H}(\mathbf{r}) \quad (2.4a)$$

where $\Theta = (\nabla \times \nabla \times)$ (2.4b)

Using the Hermitian operator to solve for the magnetic field, the solution in discrete set of eigen-frequencies can be obtained. This forms a dispersion relation as a function of wavevectors giving the possible EM states of the system.

2.1.3 Different types of photonic crystal (PC) structures and applications

Some basic photonic devices developed in the study of PC are perfect *omni-directional mirror*, *resonant cavity*, and *transmission waveguide* which are used to transmit or trap light as shown in Figs. 2.4(a) – (c). These basic PC structures are created as perfect lattice, point defect (or single perturbation defects) and row defects (a cluster

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of point defects) respectively. The fourth type of PC structure, which is the surface defects with surface modes in the PBG is also discussed.

Fig. 2.4 (a) 1D omni-directional reflecting mirror (b) A 2D resonant cavity (c) A transmission waveguide (d) Surface localization at truncated boundary.

The first basic application is an omni-directional mirror [60] shown in Fig. 2.4(a). This is better than the metallic mirror because of low absorption rate for all frequencies. For the 2D PC structure, the presence of a PBG reflects all in-plane light in the range of bandgap at low absorption that can be used as a band-stop filter [61]. It can be further

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employed as a polarizer because different lattice layouts give different PBG for TM or TE polarization [62].

In the field of PC, defects break the symmetry of the crystal lattice. The point defect results in the formation of resonant cavity, which are crucial components in filtering devices [63-65]. Much work on PC devices has been reported from narrow band resonator system with extremely low loss and high Q -factor [17, 66], bistable switching device [67] and low threshold, high power and ultra narrow band membrane laser [68]. For nonlinear PC material, a wide variety of tunable PC filters and modulators devices are innovated and engineered [69, 70].

When a group of point defect structures are gathered to form a row defect, light can transmit from one location to another. This phenomenon is commonly known as PC based waveguide (PCWG), which is different as compared to the conventional optics fibres or high index waveguide. The ability to transmit lightwaves around PC bend waveguide is one of the most important applications for photonic integrated circuit and has been demonstration theoretically and experimentally [19, 20, 71]. The transmission efficiency is further improved using additional defects to minimize device loss [72-74]. It can be used for bio-sensing purposes [41] and photonic crystal fibre [75, 76]. Through the study of the defect bands in the PCWG, it can be used to slow down, stop or even change the direction of lightwaves in PCWG. These properties can be used to create different photonic devices such as delay lines [77] for light transmissions, intersecting waveguides [78], narrow bandpass waveguide filters [65], directional couplers [79] and self imaging light extraction [80]. Hetero-structures PCWG with different lattice constants also demonstrate superior cavity properties, which can provide novel functional photonic

devices such as high efficiency filters [81] and high-Q nanocavities [50]. Other combinations of PCWG with defects or PBG structures can provide various photonic devices such as LED and tunable wavelength splitter [82-88].

After discussing about the wide applications of PC waveguide devices, this thesis will focus on the coupling methodologies of PC waveguides. In the next section, the different coupling methods of PCWG are discussed.

2.2 Methodology for coupling to PCWG

PC is believed to be an important element for the building of micro-scale size photonic integrated circuit. Various PC based optical devices such as narrow band filter [17], active PC laser [68] and optical switches [87] have been developed. However, for the applications of PC devices in the photonic integrated circuit, the dimension of photonic crystal devices is at sub-micro/nano range. This gives rise to many challenges in the terms of design, fabrication and characterization measurement of the photonic integrated circuits. However, there is another problem of coupling external light sources (whether is laser, LED or other light generating sources) efficiently to a narrow single mode PCWG. Direct butt coupling from light sources to the narrow photonic crystal waveguide is not possible as the differences in modal cross-sectional size, modal lightwaves shapes and polarization orientation result in very high coupling loss. Therefore, low coupling loss is an important factor for the photonic integrated circuits to determine the commercial success. Since 3D PC fabrication is complex and difficult, researchers have redirected their research interest on 2D PC structure because both structures share many similar properties. However, coupling in 2D PBG structures is not

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really a simple task due to incompatible modal size, shapes and polarization loss between the light sources and PCWG.

Based on the PCWG study, light coupling methodologies can be categorized into vertical and lateral coupling methods. In the next section, the vertical coupling method is reviewed and discussed.

2.2.1 Vertical and lateral coupling methods for PCWG

The task to realize an all-photonics integrated circuit is limited by the poor coupling efficiency of “squeezing” enough light into the PC devices. Since PCWG is an important and fundamental element for the photonic integrated circuits, it is considered as part of the coupling system. There are two types of coupling methods: vertical coupling method which couples light from the vertical direction to the planar 2D PC and horizontal coupling method which light is transmitted from the source to PCWG at the same plane of transmission.

One of the vertical coupling methods is the use of grating couplers (GCs) for coupling light to PCWG [89]. Optical fibre is inserted to a vertical coupler and light is transmitted from the vertical to the lateral direction by grating. A small spot size converter is placed after the GC to match the mode size with the PCWG. Single wavelength channel polarization GCs has approximately 1dB insertion loss with 3 dB bandwidth in C band. The other type of the vertical coupling design is to use tapered fibre for transmission to PCWG [90]. In this system, the middle portion of the outer cladding is removed from the fibre and stretched to form an elongated tapered structure. The fibre is

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aligned with the stretched portion placed directly on top of the PCWG. The light propagating in the fibre *leaks* out at the tapered sections and coupled to the PCWG.

GCs achieve relatively high insertion loss greater than 8 dB in multiple channel devices. The couplers also generate very high reflection loss due to the different medium indices at the interface between optical fibre and high index semiconductor chip. In addition, the spot size converter can also result in more possible loss locations. For the tapered fibre coupling method, one disadvantage is difficulty to remove the cladding from the middle parts of the fibre. In practice, it is not easy to control and estimate the stretching strength. Both vertical coupling methods encounter the problem of occupying relatively large coupling space, making them difficult in meeting the requirements for the micro-scale device.

For the lateral coupling (in plane coupling), conventional rib waveguide coupling method [91] is used as a form of transmission link to bring light to PCWG. An appropriate width is selected for the waveguide so that only one waveguide mode with similar mode profile as PCWG is supported. This simple design utilizes the conventional waveguide to bring light to PCWG. For photonic crystal, the lattice is very small and hence the width of waveguide is also small. The narrow rib waveguide may not allow light to be index guided through it. Light may be trapped within the rid waveguide and fails to reach the PCWG. It is usually not easy to achieve sufficiently high coupling efficiency in practice as additional losses among different coupling components may occur.

The other lateral coupling method is to use a parabolic mirror, which is known as a J-coupler to focus light onto the entrance on the PBG waveguide [92]. The parabolic

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mirror is designed such that the focus point of the mirror is positioned exactly at the entrance of the PCWG. Collimated light from the light source propagates through a wide multimode waveguide and then incidents onto the parabolic mirror. This method is simple and useful as it neglects the mode characteristic of the wide waveguide and focuses only on the ray optics properties of the parabolic mirror. However, the curvature of the parabolic mirror requires careful design. This is because the focusing effect of the J-coupler depends on the geometry of the design and the surface roughness of the mirror. If the focusing effect is not performed accurately, the amount of light coupling of the PCWG may be reduced significantly. In addition, the metallic mirror suffers high power dissipation losses at high frequency and hence, reduces the coupling efficiency.

Another coupling method is to join an adiabatic external taper with an arrow width to the entrance of PCWG so that light from larger aperture can be brought to the narrow waveguide [93]. The external taper coupler is the conventional waveguide using homogeneous material. The taper has input width similar to the aperture size of the input light source and then slowly transmit light into the PCWG. Taper structures have been commonly used in transmitting between waveguides with different width [94]. A large amount of light can be transmitted from one waveguide to another through slow tapering. However, a problem arises due to different structural definition of the taper coupler and the PCWG; the PCWG is a periodic structure while the tapered waveguide is a homogeneous material structure. Hence, the mode matching conditions between both types of structures are different and additional loss can occur at the interface.

In order to achieve matching conditions between the lateral coupler and PCWG, it is advisable to design the tapered waveguide with structure similar to the PCWG. The taper structure based on PC periodic structure will be discussed in the next part.

2.2.2 Photonic crystal tapered waveguide

The tapering structures based on the periodic layout of PC have been designed for achieving high coupling efficiency since year 2001 [96]. The first PC based tapered waveguide (PCTW) was created by simply removing cell structures from the bulk lattice to mimic the tapering design with decreasing cross-sectional width [96]. It achieved a maximum coupling efficiency of 75% using an optical fibre light source. The taper device provides mode conversion and can be used for direct mode conversion between the light source and the PCWG. It allowed for the design of compact coupling structure but has moderate radiation loss and back reflection due to the uneven sidewall of the taper. The design is not a complex lattice layout but higher coupling efficiency for the PCTW is desired.

Another suggestion in designing PCTW is to vary the size of the unit lattice to provide adiabatic mode conversion [97, 98]. The structure is created by progressively varying the radius of the cell structures at the side of the main PCWG. This structure has similar function as the conventional width variation tapered waveguide for coupling light to PCWG. The advantage of this design is high degree of freedom in designing the taper with different radius for different wavelengths. In addition, this taper design structures can give adiabatic mode size conversion which promises high coupling efficiency. A few limitations have been identified for this tapered waveguide design. First, it is difficult to

fabricate the structure where high precise on fabrication process is required. Second, the design is very much dependent on the geometry of the holes for adiabatic mode conversion of incident mode. Third, this design cannot be used for very large light input aperture.

For the PCTW structures, light must be coupled directly from the light source directly from the aperture to the narrow PCWG so that high coupling efficiency can be achieved with small coupler size. In addition, it is necessary to have a smoother side wall without rugged steps or sharp corner to reduce the scattering and reflection loss and hence, improve the overall coupling efficiency. Therefore, the linear photonic crystal tapered waveguide with smooth sidewall has been designed to reduce the coupling losses and achieve optimum coupling efficiency [99 - 101]. It has been reported that the overall coupling efficiency of the PC based on smooth tapered waveguide is higher than the very first step tapered waveguide design [96]. In this thesis, the design of a smooth PCTW is explored with different curvature designs. The transmission characteristic of the tapering section is studied and optimum coupling efficiency of the PCTW is obtained.

2.3 Analytical numerical model for waveguides with geometrical variation

A number of semi-analytical methods [102-110] have been established to investigate the modes coupling and interaction mechanism of the waveguide structures with geometrical variations. Through the analysis of the mode coupling results obtained from these methods, the different taper curvatures are designed to achieve high coupling efficiency at the shortest possible length.

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In general, the width of the geometrical variation waveguide can either be strongly or slightly perturbed. The tapered waveguide is a special case of geometrical variation waveguide, where its width changes smoothly along the tapering section. In determining the mode coupling mechanism and the propagation characteristic, a representation of electromagnetic field in terms of the coupling modes of the tapering section is required [108]. With a series expansion of the electromagnetic fields in terms of waveguide modes, Maxwell's equation and boundary conditions of the waveguide structure, the coupling coefficient can be expressed in terms of wave equations or a generalized transmission line equations.

Various semi-analytical methods are available to analyze the mode coupling and propagation behavior in non-uniform tapered waveguides. They can be classified into two categories: time domain and frequency domain semi-analytical methods. The next section discusses the time and frequency domain methods.

2.3.1 Time domain and frequency domain semi-analytical methods

The time and frequency domain analytical methods are often to model electromagnetic light waves in the geometrical variation waveguides such as the tapered waveguide. The frequency domain semi-analytical methods are more common because of its simplicity and convenience. In this section, the advantages and disadvantages of these time and frequency domain semi-analytical method are discussed.

The most commonly used time domain semi-analytical method is the beam propagation method [102,103]. The beam propagation method is a widely used propagation techniques for waveguide with arbitrary geometries. It is because it is

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straightforward and efficient method because computational complexity can be done in optimal with adequate amount of effort. Its flexibility allows it to be easily extended and applied to complex geometries using the basic method in the areas of polarization and nonlinearity. However, the disadvantages are that it uses the parabolic approximation, which implies a paraxiality conditions only in the primary direction of propagation. Additionally, it approaches results from the assumption of scalar waves and prevents polarization effects. It also cannot account for backward reflections directly. These disadvantages can be solved by including vectorial beam propagation techniques and bidirectional algorithm, which requires the development of specialized program to calculate different geometrical variation waveguide structures.

There are a large variety of frequency domain semi-analytical methods. The most common frequency domain semi-analytical methods is the spatial coupled mode theory [104, 105], which involved expanding the electromagnetic field components in the tapering region into the eigenmodes of the waveguide and solve for the coupling amplitudes in the geometrical variation waveguide. This method is formed by transforming the usual coupled mode theory in time domain to spatial domain in the propagating direction. When combining this method with the perturbation theory, it can be used to model and simulate geometrical variation waveguide by considering geometrical variation. Then solving coupling equations, the field amplitude of the propagating modes traveling along the taper direction is obtained. The combination of coupled mode theory and perturbation theory provide very fast convergence for non-uniform waveguide with moderate or slow varying waveguide slope. By making further approximations of the field conditions [105], it can be used to improve the calculations

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speed and accuracy. However, the taper slope that varies fast needs higher order corrections in order to achieve convergence. As a result, it lengthens the computational time.

The transfer matrix method [106, 107] is another type of frequency domain method. The transfer matrix method is used to solve for the electromagnetic wave solution in multiple layer structures. A 2D PCWG can be represented as a multi-layers structure by using a transfer matrix and the electromagnetic field solution for each layer is solved. This method allows the calculation of transmission and reflection spectra from the beginning to the end of the PCWG. However, the PCTW with large number of modal interaction and coupling requires a large transfer matrix. This in turn affects the convergence because it is more difficult to solve large transfer matrix. In addition, a large transfer matrix requires longer computation time due to increase complexity of the matrix and hence, lengthens the computational time. Many versions of the transfer matrix method are also available and each version has their advantages and disadvantages. Therefore, different specialize version of the transfer matrix is necessary for different geometrical variation structures. As a result, no unified semi-analytical method is available for geometrical variation waveguides.

From these discussions, the time and frequency domain semi-analytical method for modeling the geometry variation waveguide has been reviewed. In the next section, the frequency step-theory will be discussed.

2.3.2 Frequency domain step-theory

A frequency domain called the step-theory [108-110] is also used for the analysis of waveguide with geometrical variation. This method of analysis generally requires the mode matching of the electromagnetic field components along the direction of light waves propagation in the waveguide. It is assumed that the geometrical variation waveguide can be divided into many strips. Each strip is mostly rectangular shape. The rectangular strip has longer width across the transverse profile of the field and shorter length parallel to the propagating direction. The size of each strip is dependent on the geometrical variation. The strips are known as “steps” and the mode matching method is commonly known as “step-theory”.

The step-theory has many advantages: First, it can be used for the study of the coupling and interaction mechanism between different propagating waveguide modes. It calculates the coupling efficiency of forward or backward propagating modes along the tapering section and can determine the optimal transmission in the tapered waveguide. Second, it is simple to use especially for the tapered waveguide. The length of each step is assumed to be constant for the periodic PCTW and only the step width changes according to the taper curvatures and shapes [109, 110]. Third, the step-theory can calculate the transmission and reflection for the slow and fast varying taper slope with high convergence rate and high accuracy. This is important because the coupling efficiency of the taper can be calculated to obtain the transmission and reflection spectra at various taper lengths. The transmission and reflection spectra at various lengths show the minimal length required to achieve the optimal transmission efficiency. However, a drawback for the step theory is that when the taper length increases, the number of steps

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also increases. As a result, a longer computation time is required to simulate longer tapered waveguide. Nevertheless, the step-theory is still useful for investigating the mode coupling and interaction in geometrical variation waveguide.

In this thesis, the step-theory method is used for the study of coupling and interaction mechanism in the tapering section. It gives concise understanding of the mode coupling and interaction in the taper. However, when the step-theory is applied to periodic PCTW, some modifications must be made due to different structure definitions and boundary conditions. In the next chapter, the principles of the step-theory are integrated to the general solution of wave equation to obtain a generalized set of coupling equations for the description of the mode coupling and interaction.

2.4 Photonic crystal surface mode and waveguide

For most PC systems, the devices are designed in the interior of lattice. The lattice is infinitely periodic in the planar direction and invariable in the vertical direction. The boundary effect is often neglected or assumed to have no effect on the coupling of light waves in PCWG. However, real PC structures has finite boundary. Therefore, it is important to consider the boundary effect on the PBG structure. As many concept of PC is an analogy form solid state crystal [111-113], the surface modes of PC can also be correlated with the surface states in solid state crystal. The details will be discussed in the Chapter 5 of this thesis.

2.4.1 Photonic crystal surface mode

Photonic crystal surface modes [114] are light modes on the boundary of the finite PC structure. In almost all cases the localized surface modes can trap light waves for useful applications. In the finite PC structure, the surface modes are formed by introducing defects to the surface boundary. The defects change the effective index at the surface and a defect surface mode is pulled (pushed) down (up) from the bulk modes into the PBG. The surface defects are introduced by various methods such as termination [29, 114], shape alteration [115] or size change [30] at the surface layer of lattice cells. For surface modes to be localized on the surface boundary, it must be in the PBG and below the homogenous medium lightline so that light waves decay exponentially into the PC lattice and the medium.

2.4.2 PC surface waveguide and applications

In the recent years, the PC surface modes and surface waveguide research have been studied and several applications have been proposed. One application of PC surface modes is an optical sensor for thin film characterization [28]. The ability to localize light on the PC surface at certain frequency range enables the characterization of film properties in 1D PC multilayer thin film device. The other research is the effects of surface termination on the quality of image using a PC slab [116]. When a point source is launched in a PC slab that has negative index, an image is formed at the opposite end of the PC slab. Through varying the position of the surface termination, the quality of the imaging are improved due to the matching of the PC slab effective index with surrounding index. Unique applications of PC surface mode for beam divergence control

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have also been reported [25-27]. The phenomenon of large electromagnetic field divergence angle from a subwavelength aperture is defied by crafting the intermediate surface. The surface cells act as secondary scattering points such that the central part of the output beam interferes constructively while other part interferes destructively to obtain the resulted directional emission field pattern. Another application of photonic crystal surface mode is the formation of microcavities [30] for the localization of light at the PC-air interface. It is shown that high quality factor can also be obtained through surface microcavity of the PC slab. The surface microcavity can be designed as an optical filter [117] for adding/dropping different wavelengths of light waves. It is also demonstrated that light can also be internally reflected in different direction by altering the surface angle [118]. This is due to the change in the lightwaves direction as indicated in the constant frequency constant (CFC). The direction of the transmitted waves can be predicted by studying the CFC of the truncated surface. The existence of surface mode in slab edges of 2D PC slab edges can turn into a lasing mode [119] under pulsed optical pumping conditions. The shape and termination of the slab edges is important for the Q -factor and resonant frequencies.

Another important application of PC surface mode is the transmission of light waves along the boundary [120-123]. The row defect at the interface of the PC-air can be used for the transmission of light waves by operating at an appropriate frequency range and new photonic devices can be innovated the PC surface waveguide. One application is to place a high index dielectric waveguide [120-121] sandwiched between two finite PC slab. Light waves can propagate either in the PC slab surface or the high index dielectric waveguide depending on the operating frequency. Its function is similar to a switching

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device. The surface slab waveguide can be used for side coupling to other photonic components for more sophisticated device architectures [122] such as a directional coupler. It is suggested that other photonic devices can be incorporated to the surface waveguide to realize further functionalities such as dispersion control, buffering and biochemical sensing. However, the presence of higher order modes affects the efficiency of power distribution and coupling efficiency of the directional coupler.

The potential applications for surface waveguide with high index rods structures [123] have been studied by changing the size of surface rods to create surface mode instead of terminating at certain position. The surface rods serve as a transmission path for light waves propagation. The field transmission properties at the surface waveguide have been investigated and some applications have been proposed. One application for the surface waveguide is a coupler, where incoming lightwaves are coupled to the surface waveguide and transmitted to the narrow PCWG. This method does not require very sensitive surface parameters and accurate positioning of the coupler to the PCWG. Another potential application is the directional emitter. The angular spread of a directional emitter from the surface waveguide are controlled and reduced to an even smaller angle than the emission angle of the conventional waveguide.

The PC surface waveguide will be the focus in this thesis. It is easy to perform characterization and measurement for the surface waveguide devices. The optical properties of the PC surface waveguide are investigated and new design of photonic devices based on PC surface waveguide is developed in this thesis.

2.5 Summary

This chapter provides a comprehensive literature survey on various coupling methods. The main focus is to utilize PCTW for coupling light source to the narrow PC waveguide to achieve optimum coupling efficiency at short taper length. Various frequency and time domain semi-analytical mathematical models for coupling and modal interaction in geometrical variation waveguide are reviewed in this chapter. The step-theory is used to investigate the coupling characteristic in different PCTW designs. The PC surface modes are briefly introduced and its applications are discussed. The scope of this thesis is to study the PC surface waveguide based on slab and rods structures and to develop new design of PC surface waveguide devices.

CHAPTER 3

TAPERED WAVEGUIDE AND MODIFIED STEP-THEORY

For photonic integrated circuit, the coupling efficiency between the light source and the photonic device is a very important performance indicator to measure its functionality and reliability. The objective of this chapter is to study the theory of mode coupling mechanism of the photonic crystal tapered waveguide (PCTW) for high coupling efficiency and structure optimization on a planar 2D PC.

This chapter begins with a brief introduction of the conventional dielectric optical tapered waveguide. Then, the coupling mechanisms of different electromagnetic field modes in tapering section are discussed and analyzed. Subsequently, a numerical model of the step-theory [108-110] is introduced to describe the intermodal coupling behavior in classical homogeneous tapered waveguide. Based on Bloch-Floquet theorem, a modified step-theory is derived for the PCTW. The modified step-theory is applied to obtain the transmission efficiency of a simple PC step waveguide and a step tapered waveguide structures. Finally, the modified step-theory is applied to a complex PC taper structure design with linear and non-uniform curvature. The transmission efficiency and loss characteristics in the tapering section of the PCTW are calculated and discussed in this chapter.

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3.1 Theory of optical tapered waveguide

A tapered waveguide is defined as an elongated guiding structure for electromagnetic waves that gradually becomes narrower (or broader) toward one end [124]. Such a waveguide may undergo a change in the cross sectional size and shape profile along the direction of propagation. In the field of optical integrated circuits and systems, most tapered waveguide are used for coupling electromagnetic light waves efficiently from one device to another device. When the cross section of the tapered waveguide changes, the mode profiles of the propagating light waves are transformed from one waveguide cross section to some other mode profiles in another waveguide cross section. The aim of the tapered waveguide is to provide this transition with little reflection loss and power conversion.

All tapered waveguides have in common several kind of modal interaction such as the higher and lower order guided modes, forward and backward (reflected) modes, and guided and radiation modes. These modes can propagate independently but also interact with each other along the tapered waveguide. The analysis of the mode interaction or coupling is discussed in this chapter.

3.1.1 Design of tapered waveguide

In this section, factors affecting the design of the tapered waveguide are discussed. The tapered waveguide serves as light transmission device of the photonic integrated circuits between the input light source and the output of the device. The most important factor for the dielectric optical tapered waveguide is the coupling of guided mode to the radiation modes surrounding the waveguide structure [124]. For the open

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tapered waveguide structures, the incident light waves can be reflected and radiates into the surrounding space. The reflected and radiation modes in an optical dielectric waveguide can result in an extensive loss of electromagnetic energy. The analysis and design of optical waveguide transition must take these reflected and radiated modes into consideration to obtain maximum power transmission efficiency.

The function of the optical connector is to provide power incident from one waveguide into the other waveguide. The most common case is the multimode to single mode transition. This is because most of the available light source has a large aperture while the photonic devices require operation in a single mode to prevent any intermodal coupling losses. The multimode to single mode transition is more important for optical coupling between commercial light sources and sub-micron dimension integrated circuits.

The taper length of the dielectric optical tapered waveguide is dependent on several factors such as the wavelength of the propagating modes, size reduction ratio and others. The wavelength of different modes gives different propagation phase constant, which affects the length of the tapering section. The tapered waveguide is long enough to avoid too much intermodal interaction to optimize coupling efficiency. The size reduction ratio between the input and output end of the taper is also important. A larger size reduction ratio is requires a longer taper while a smaller size reduction only needs a shorter taper.

The external factors refer to the different curvature designs of the tapered waveguide which is used for connecting waveguides with different core sizes. The different curvature designs of the tapered waveguide affects the coupling efficiency significantly. The tapered waveguide with gradual curvature reduces the possibility of

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coupling to the higher order modes and increases the coupling efficiency. On the other hand, the tapered waveguide with steep curvatures has high reflection losses due to the intermodal coupling. Hence, it is important to find a tapered waveguide curvature design that gives maximum coupling efficiency. The other consideration is the sidewall of the tapers, which is smooth enough to reduce scattering loss to the minimum.

In the next part, a numerical model and analysis of the mode coupling mechanism using the step-theory are introduced. The principles and the methodology of the step-theory are explained. The parameters affecting the step-theory are also identified.

3.1.2 Step-theory for optical tapered waveguide

The step-theory [108-110] is used to calculate the transmission efficiency of the tapered waveguide. The tapered waveguide is divided into vertical strips that are known as “steps”. By matching the field components at the boundary between each step and solving using Maxwell’s equations, the transmitted field amplitude of the different electromagnetic modes is obtained. Therefore, it provides a numerical model for the taper structure with small or moderate variation in the cross sectional size with very high convergence [108]. The main shortcoming of this method is the requirement of known propagation constants for all the guided modes in each step for adequate accuracy. This is particularly troublesome for structures with very large number of modes. In the PCTW, the number of steps required for modeling the transmission efficiency is dependent on the periodicity, the curvature and the length of the tapered waveguide.

Based on the step-theory, the numerical modeling of the transmission efficiency for a uniform homogeneous optical tapered waveguide is shown in Fig. 3.1(a). The linear

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tapered waveguide is placed between two optical waveguides with widths of d_{in} and d_{out} . Each step has a length of Δx , and the total length of taper is l . The width of each step $d(z)$ is given by

$$d(z) = d_{out} + (d_{in} - d_{out}) \cdot \left(1 - \frac{x}{l}\right) \quad (3.1)$$

The width of the steps dealing with the effective group index can be expressed as

$$d(z) = \frac{m\lambda}{\sqrt{n_c^2 - n_s^2}} \quad (3.2)$$

where m is the mode number. n_c and n_s is the refractive index of the waveguide and the background, respectively. The wavelength of the propagating modes is given by λ . Fig. 3.1(b) shows the detail field analysis of the boundary matching between two steps for electric field. For simplicity, the fields of the higher order modes are ignored. Both steps support only one mode. The step on the left-hand side has a width of $d_1(x)$, whereas the step on the right-hand side has the width of $d_2(x)$. Due to the difference in the width of the waveguide, the incident light field E_{in} experiences a change in the propagation constant where most of the light is transmitted to the step on the right. The remaining electric field is either reflected or coupled to the radiation mode outside the waveguide and radiates out. By minimizing the reflection radiation loss, higher coupling efficiency can be achieved in the tapered waveguide.

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(a)

(b)

Fig. 3.1 (a) Schematic layout of a conventional dielectric tapered waveguide. The tapered waveguide is divided into steps with width of $d(z)$. (b) Field matching analysis at the boundary between steps. It shows that the presence of electric field is reflected or transmitted when it propagates from one to another step.

By assuming only one propagating electric mode field on both steps, the fields at the interface between both steps are solved by mode matching and the reflection coefficient is approximately given as [108]

$$r \approx \left(\frac{\beta_1 - \beta_2}{\beta_1 + \beta_2} \right) \exp(-i2\beta_1) \quad (3.3)$$

and the transmission coefficient is given as

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$$t = 2 \frac{\sqrt{\beta_1 \beta_2}}{(\beta_1 + \beta_2)} \frac{I_{1,2}}{\sqrt{I_{1,1} I_{2,2}}} \exp[-i(\beta_1 + \beta_2)] \quad (3.4a)$$

where

$$I_{\alpha,\gamma} = \int E_\alpha E_\gamma dz \quad \alpha, \gamma = 1, 2 \quad (3.4b)$$

β_1 and β_2 are the propagation constants on the left and right steps, respectively. $I_{\alpha,\gamma}$ is the integral of the field overlapped between the incident and the transmitted field distributions at the step boundary. Eqs. (3.3) and (3.4) are derived based on the simplification and approximation of a single-mode transmission. In general, there are multiple modes in a tapered waveguide and they also contribute to the mode coupling mechanism along the tapering section. Therefore, the general expressions for the reflection and transmission coefficients of the multimode tapered waveguide are more complex than the single-mode example given in Eqs. (3.3) and (3.4).

For the multimode waveguide, the guided modes are classified as fundamental and higher order modes. The higher-order modes are cut-off as the taper gets narrower. This is due to the reduction in the number of modes supported in the narrower section of the tapered waveguide. The higher order modes are reflected and this causes reflection loss [124] to be the main loss mechanism in the tapered waveguide. The reflection loss can be reduced by modifying the curvature of the taper waveguide, which is discussed in the later section of this chapter.

Having introduced the principles of the step-theory with some introduction on the loss mechanism in the tapered waveguide, the next section shows the derivation of the step-theory for periodic PC structures.

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3.2 Modified step-theory

The modified step-theory is used to model the field transmission of the periodic boundary structures such as the PCTW. The modified step-theory can be used to investigate of the modal interaction and light waves transmission behavior in PCTW. It gives a relatively simple explanation of light loss, transmission and transfer of light waves power in the PCTW.

Some modifications are required before applying the step-theory for the periodic PC structures. The step-theory that has been modified is called the modified step-theory. The periodic structure of the PC is described by using including the Bloch theorem into the step-theory to define periodic wave vector in the field equations. The PCTW is divided into vertical strips known as steps and the modified step-theory calculates the transmission efficiency of light waves field from one step to another along the PCTW. Before deriving the coupling equations for the modified step-theory, the waveguide properties of the PCWG are discussed in the next section.

3.2.1 Photonic crystal waveguide (PCWG) band diagram

In this chapter, a two dimensional (2D) PC structure is considered. Electromagnetic light waves with frequencies within the bandgap are not allowed to propagate in the PC for all in-plane directions [53]. The number of possible modes and the density of states per unit frequency in the PBG is zero. However, defects introduced to the PC break the symmetry properties. As a result, a single mode or set of closely spaced modes is localized in the PBG. These mode(s) are known as the defect mode(s) in the PBG.

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The defect(s) used for the PC depends on its size, refractive index, geometrical position or the shape of a single lattice cell (rods or holes), which is known as a point defect. For a series of PC defects in the lattice structure, a row defect(s) is created. The electromagnetic light waves are localized in the row defects site. It only propagates along the row defects at the frequencies spanned by the defect mode in the PBG. As a result, a transmission-like path is formed in the row defect site. This transmission path is known as PC waveguide (PCWG).

To illustrate this concept, a case study which consist of a square array of high refractive index rods with a radius to lattice constant ratio of $r/a = 0.2$ is considered. The rod has a refractive index of 3.45 surrounded by air medium. The structural layout of the crystal is shown in Fig. 3.2(a). The simplest way to create a PCWG is to remove a row of rods in the crystal. The missing row forms a path that the electromagnetic light waves can propagate. Since the light waves cannot propagate into the bulk crystal at the side of the defect, a waveguide is formed in an empty channel.

The row of rods that has been removed is shown as dotted line in Fig. 3.2(a). It is observed that the row defect breaks the translational symmetry of the PC in the z direction. There is discrete translational symmetry in the x direction and continuous translational symmetry in the y direction. The wave vector in the x and y directions is bounded by the inequality $-\frac{\pi}{a} \leq k_x \leq \frac{\pi}{a}$ and $-\infty \leq k_y \leq \infty$. For 2D PC structures, the value of k_y is set to zero, because only the in-plane wave vector in the x and z directions are considered.

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(a)

(b)

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Fig. 3.2 (a) Schematic layout of the PCWG. The missing row of rods is shown in dotted line. (b) Projected band diagram of the PCWG, showing the defect waveguide mode in the PBG. (c) Field pattern of the light waves in the PCWG at $0.365 (a/\lambda)$.

For the wave vector k_x , it lies in the range of the first Brillouin zone with wave vector values of $\pm \frac{\pi}{a}$. The PC array in Fig. 3.2(a) has a PBG at the normalized frequency of $0.285-0.412 (a/\lambda)$ as shown in Fig. 3.2(b). The PBG is present for light waves with TM polarization where the electric field is along the y direction. However, there is no PBG in the TE mode for the electric field in the x - z plane because the field penetrates through the air region from one rod to another rod.

After defining the wave vectors of the PC square array, the band diagram of the PCWG is drawn. Since the defect modes cannot propagate into the crystal, the modes behavior is considered as a function of both the frequency, ω and the wave vector k_x . However, only certain combination of $(\omega, k_y = 0)$ and k_x gives a defect mode in the PBG. To find out which set of $(\omega, k_y = 0)$ and k_x gives a defect mode, either the radius of the

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rods or the lattice constant is varied. This process of searching for all the possible k_x for each combination of $(\omega, k_y = 0)$ is called the projecting the band structure. To calculate the band structure, the plane wave method [125] is used with the supercell of 15 rows $\times$ 1 column. The band diagram of the waveguide is shown in Fig. 3.2(b).

Figure 3.2(b) shows that there is one defect mode in the PCWG, which is known as the waveguide mode and it extends for the frequency range of 0.3-0.412 (a/λ). Fig. 3.2(c) shows the electric field distribution of light waves propagating in the waveguide at 0.365 (a/λ). The electric field distribution is calculated by using the finite different time domain (FDTD) simulation [161]. Matlab [162] is used to obtain the contour plot of the electric field distribution. It can be seen that the light waves are localized in the linear row defect site and decay into the PC. There is no field leakage from the PCWG. The number of defect modes supported by the PCWG can be increases by removing additional rows of rods. This forms a waveguide with a larger width, which supports additional defect modes in the PBG and this allows the presence of higher order modes. This implies that it is possible for the field power to be transferred from one order mode to another different order mode(s). The coupling effect and intermodal interaction mechanism between different order modes is also discussed in this chapter.

3.2.2 Electromagnetic field theory for PCWG

For the mathematical model of the light waves transmission in PCWG with periodic boundary, the field profile can be described in term of the eigenfields of the normal waveguide modes. The solution of the normal electric field modes is given as [126]

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$$E = E_0 \xi(z) \times e^{i\beta x} \quad (3.5)$$

where E_0 is the real amplitude of the propagating E -field at the position x . $\xi(z)$ is the z dependent field distribution of the guided mode. ξ is dependent on the width and the mode number of the waveguide. β is defined as the propagation constant of the guided mode and the electromagnetic wave is propagating in the x direction.

For light waves guiding in PCWG, there are two conditions to be satisfied. First, the guided mode must be in the bandgap of the PC. Second, the mode in operation is truly guided without any radiation to the PC lattice. For the PCWG, the waveguide solution is more complex because of the periodicity profile in the crystal lattice. Based on Bloch's theorem, for the 2D PC structure in the x and z directions, the field mode of a periodic function is expressed as [53]

$$E_{k_x, k_z}(r) \propto \exp(ik_x x) \times \exp(ik_z z) \times u_{k_x, k_z}(x, z) \quad (3.6)$$

where $u_{k_x, k_z}(x, z)$ is a periodic function in the x and z directions. The periodicity in the x and z directions leads to a xz dependent on the E -field, which is proportional to the xz direction plane wave and the xz -periodic function.

The transverse field profile of the PC guided mode resembles modes in the conventional waveguide [127, 128]. This is because the dispersion band diagram for the conventional waveguide is similar to the rod lattice PCWG. The equation for the mode field of the guided modes is given as [127]

$$E = E_0 \sin\left(m\pi\left(\frac{z}{b} + \frac{1}{2}\right)\right) \exp[-i\psi(x)] \quad (3.7)$$

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The modal field distribution is based on a sine function, which is dependent on the mode number, m and the width of the waveguide, b . For $m = 1, 3, 5, \dots, (2v+1), \dots$, this refers to the even mode with even field distribution. It has a cosine mode profile that is symmetrical at the x direction. The larger values imply higher order modes. When $m = 2, 4, 6, \dots, (2v), \dots$, it corresponds to odd modes with a sine profile that is asymmetrical at the x direction. The constant b is dependent on the structure of the lattice arrangement. It corresponds to the width of the periodicity imposed waveguide, which is usually in multiples of the lattice constant. ψ is the phase coefficient, which is dependent on the artificial periodicity of d expressed as

$$\psi(x) = \left(\beta + \frac{2\pi\eta}{d} \right) x \quad \eta = 0, \pm 1, \pm 2, \dots, \quad (3.8)$$

where β is the wave vector of the wave and η is the propagation direction. For periodic structure, the modes are also characterized by the Bloch modes. The integer η is related to the folding of the bands at wave vector, $k = 2\pi\eta/d$, which is at the first Brillouin zone of the photonic crystal. The phase coefficient is conserved in the propagation direction. The mode field for the guided mode in Eq. (3.7) is used to derive the modified step-theory, which is discussed in the next section.

3.2.3 Modified step-theory for periodic PCWG

The modified step-theory [129] is used to model the amplitude of the transmitted modes in the periodic PCWG. In the modified step-theory, Eq. (3.7) is used instead to account for the periodicity and discontinuous dielectric boundary in the photonic crystal structures. The equations to model the transmission amplitudes and coupling constant are

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presented in this section. Subsequently, the modified step-theory is used to calculate the transmission and reflection loss in an iterative manner from one step to another until the end of the taper.

Before deriving the modified step-theory, a few assumptions are made: First, the coupling between two reflected modes is negligible. This means that the interaction between the reflected modes are neglected and only the interaction between the transmitted modes are considered. Second, the mode coupling of the reflected and transmitted radiation with the guided modes is small and negligible. This is especially true for the case of gradual slow taper where there the reflection and radiation loss is very small. Third, the intermodal loss for a short taper can be neglected because the higher order modes are cut-off and reflected backward. The reflection loss of the higher order modes contribute to the main loss.

A slightly more complex case is compared to that in the previous section. Two guided modes, i and j traveling in a 2D PCWG with a width of b and an artificial periodicity of d . Both modes travel to a narrower waveguide of width b' with the same periodicity. The wider waveguide is labeled as the n th step with subscript n , while the narrower waveguide is labeled as the $(n+1)$ th step with subscript $n+1$. Based on Eq. (3.7) for the description of the mode field for i and j , the incident, reflected and transmitted components for the E and H field at the boundary between each step are matched and the amplitude of transmitted mode j in the $(n+1)$ th step is given by

$$A_{jn+1} \exp(-i\psi_{jn+1}) = c_{ij} A_{in} \exp(-i\psi_{in}) + c_{ji} A_{jn} \exp(-i\psi_{jn}) \quad (3.9)$$

where A_{in} refers to the ratio of the modal field amplitude of the i th mode at the n th step, in the presence of mode conversion to the initial incident amplitude that corresponds to

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the fundamental mode and has a unit power. Similar assignments are given for A_{jn+1} and A_{jn} . The coupling constant c_{ij} between mode i and j is given as

$$c_{ij} = \sqrt{\frac{\psi_{jn}}{\psi_{in}}} G \frac{I_{in,jn+1}}{\sqrt{I_{in,in} I_{jn+1,jn+1}}} \quad (3.10)$$

where

$$G = \frac{2\beta_{in}\beta_{jn} + 2\beta_{in}\beta_{jn+1} + \gamma^2 + 2\gamma\beta_{in} + \gamma\beta_{jn} + \gamma\beta_{jn+1}}{(\beta_{jn} + \beta_{jn+1} + \gamma)(\beta_{in} + \beta_{jn+1} + \gamma)} \quad (3.11)$$

and

$$\gamma = \frac{4\pi\eta}{d} \quad (3.12)$$

The purpose of γ is to account for the Bloch modes in periodic structure for convergence in high index contrast material.

The field overlap integral $I_{in,jn+1}$ is given by

$$I_{in,jn+1} = \int \sin\left[m_{in}\pi\left(\frac{z}{b} + \frac{1}{2}\right)\right] \sin\left[m_{jn+1}\pi\left(\frac{z}{b'} + \frac{1}{2}\right)\right] dz \quad (3.13)$$

The details of the equations derivation for the modeling of the fields coupling between steps is found in Appendix A. The field overlap integral, $I_{in,jn+1}$ accounts for the common field distribution between the incident and the transmitted field profile at different steps. As the width of the PCTW varies along the direction of propagation, the values of b change. Therefore, the integral varies and is evaluated for every transmission between the steps in the tapered waveguide. For the multiple modes propagating in the waveguide, the general expression for the transmitted j th mode for the sum of all the field profile of the incident mode can be expressed as

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$$A_{jn+1} \exp(-i\psi_{jn+1}) = \sum_i c_{ij} A_{in} \exp(-i\psi_{in}) \quad (3.14)$$

Based on the real and imaginary part of Eq. (3.13), the transmitted amplitude for the j^{th} mode is given by

$$A_{jn+1} = \left[\left(\sum_i c_{ij} A_{in} \cos(\psi_{in}) \right)^2 + \left(\sum_i c_{ij} A_{in} \sin(\psi_{in}) \right)^2 \right] \quad (3.15)$$

where ψ is the phase coefficient that is defined as

$$\psi_{jn+1} = \tan^{-1} \left(\frac{\sum_i c_{ij} A_{in} \sin(\psi_{in})}{\sum_i c_{ij} A_{in} \cos(\psi_{in})} \right) \quad (3.16)$$

According to Eqs. (3.15) and (3.16), the transmission and the reflected amplitude of the various modes due to mode coupling in the PCTW are obtained using the modified step-theory. In this thesis, a symmetrical PCTW is discussed. It is sufficient to consider only even propagating modes. Based on Eqs. (3.15) and (3.16), the coupling efficiency can be calculated using the instantaneous modes transmission that is propagating in any part of the taper. The modified step-theory for the 2D PCWG can be extended to 3D structures. For the 3D simulations, Eq. (3.7) is modified to include the mode field description in the third dimension. The structure is divided into finite planes, which is similar to the vertical steps of 2D PCTW. The coupling constant, c_{ij} and transmission amplitudes, A_{jn+1} are again derived for the three-dimensional photonic crystal tapered waveguide structures.

In this chapter, the simulation results are based on the 2D PCWG structures. However, it is noted that actual experimental structures [100, 130] are 3D due to

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fabrication limitations. Hence, the vertical radiation losses due to the finite height structure are taken into consideration. Actual experimental devices are subjected to fabrication process control, where the lattice dimensions across the whole crystal can varies. This results in additional scattering losses and a shift in the transmission spectrum [100]. The coupling losses between the tapered devices and the source cannot be avoided. These differences between the simulated structures and the actual experimental structures cause some disagreement in the simulation and experimental results.

It is noted that, up to this point, the derivation of the step-theory for the application in the conventional homogeneous tapered waveguide has not been discussed. This is because the modified step-theory is developed based on some modification of the original step-theory. Except for the periodicity consideration in PCWG, the steps for deriving the original step theory can be followed in similar order as shown in Appendix A. The coupling coefficient of between two different modes is expressed as

$$G = \frac{2\beta_{in}(\beta_{jn} + \beta_{in+1})}{(\beta_{jn} + \beta_{jn+1})(\beta_{in} + \beta_{in+1})} \quad (3.17)$$

where the G term is obtained by setting γ to zero in Eq. (3.11). This means that the waveguide is a uniform homogeneous structure.

Having derived the equations for the coupling efficiency between the different modes, the coupling equations of the modified step-theory are verified using the numerical results obtained from some known examples.

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3.2.4 Numerical case study

In this section, the modified step-theory is applied to the numerical modeling of the normalized frequency spectra of the taper structure. Eqs. (3.9) - (3.16) are verified by comparing with the numerical case study of the taper structure [131] as shown in Fig. 3.3(a). The PCTW consists of a smooth input linear taper structure at the entrance on the left end, which is just shifting the position of the rods accordingly in the z direction. There is also a linear row defects in the input tapering area which acts as a chain of waveguide resonators for the propagation of light waves. The input taper is then joined by an identical taper but with increasing width towards the other end. Therefore, a standard case of twin head input/output PCTW is then formed.

For this case study, the modified step-theory is applied to deal with the decreasing number of modes at the input taper coupler. The number of modes increases as the taper opens up at the other end. When applying the coupling equations to the PCTW, the centre part of the waveguide can be left out because the width remains unchanged. The propagating loss is assumed to be negligible and the coupling losses are due mainly to the variation in the width.

With the addition of line defects for guiding, the modified step-theory gives an approximation by obtaining beta values of the defect waveguide at each step. In calculating the β values, the taper are divided into steps, which corresponds to a unit cell of the waveguide. By solving the master wave equation and plotting the band diagram, the beta values at each frequency for each mode can be obtained. These beta values are then substitute into Eq. (3.10) and run iteratively to obtain the transmittance for each frequency using Eq. (3.11) as shown in Fig. 3.3(b).

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In Fig. 3.3(b), the solid line refers to the transmission spectrum that is calculated using the scattering method [131]. It should be noted that the location of incident wave and detector can influence the transmittance. The incident light wave is launched at the opening of the input PCTW while the transmittance is obtained from the exit of the output PCTW [131]. The circle dots are the simulated results that are calculated using the modified step-theory. The simulation results generally show good agreement between the modified step-theory and the scattering method. From Fig. 3.3(b), the spectrum at the band edge (< 0.23 and > 0.257) fluctuates due to low group velocity propagation that results from the presence of Fabry-Perot (FP) oscillation between the side-walls of the taper. This FP oscillation is not considered in modified step-theory. This causes the different in simulation results at the spectrum band edge (particularly < 0.23). This problem is resolved at the next the chapter when a more generalized theory is developed.

Overall, the modified step-theory provides a good estimation of the transmission spectrum with moderately high convergence and computational efficiency. The modified step-theory can also be used to analyze the mode coupling mechanism along the propagation direction in the PCTW. However, the disadvantage of the modified step-theory is the additional information required for the wave vector, β . This makes it rather tedious if there are many modes and steps with a large range of frequency. On the other hand, the scattering matrix cannot be used for analyzing the mode coupling mechanism in the tapering section and this may result in additional loss of data. The scattering matrix, however, does not require the knowledge of the wave vector for each mode.

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The average transmission at the middle of the bandgap (between 0.23-0.257) is greater than 90 %, which implies that adiabatic transmission is nearly achieved. This means that it has very little coupling to the higher order modes and maximum power transfer is maintained in the fundamental mode throughout the whole taper structure. Hence, this shows that the modified step-theory can be used to calculate both the input and output coupling tapers. Additionally, it can also be used to calculate tapered waveguide linear defects with high accuracy. In other words, the modified step-theory is capable of calculating the transmission spectra of more complex PCTW structures with high accuracy.

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(a)

(b)

Fig. 3.3 (a) Structure of the linear defect PCWG with input taper and output taper. (b) Comparison of transmission spectra using the scattering method [131] and the modified step-theory.

3.3 Step tapered waveguide

In this section, the modified step-theory is applied to investigate the effect of different taper waveguide designs on the coupling efficiency. Since the main concern for the tapered waveguide is to couple light waves efficiently into miniature PC devices, the multimode to single mode coupling behavior of the tapering section is studied in this section. The study begins with a simple case of an abrupt step waveguide which consists of two PCWGs with different widths joined together. Then more PCWGS with different width is cascaded together to form a PC step tapered waveguide.

3.3.1 Abrupt photonic crystal step waveguide

The modified step-theory is used to explain the coupling mechanism at the interface between two waveguides with different widths known as step waveguide. Then, an advanced structure of tapered waveguide is considered by cascading all the step waveguides together. The structure is known as PC step tapered waveguide. The modified step-theory determines the transmission of the step waveguide by iteratively calculating the coupling efficiency between the different modes from one step to another, until the last step which is a single mode PCWG.

The structure of the photonic crystal described is a 2D PC with square lattice arrangement. It consists of high refractive rods that are infinitely long in the third dimension surrounded by air medium. The material of the rods is silicon with the refractive index of 3.45 at optical communication wavelength in the infra-red region. a is the lattice constant, r is the radius of the rods and the r/a ratio is 0.2. Based on the plane wave supercell [132] method, the band structure for the above mentioned PC parameters

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is simulated and a PBG exists in a normalized frequency range of 0.285 - 0.412 (a/λ) for TM polarization, but there is no bandgap for TE polarization. The transverse magnetic (TM) polarization is defined to have magnetic field in the x - z plane and the electric field is perpendicular to the plane while vice versa for the transverse electric (TE) polarization.

Fig. 3.4 Schematic layout of an abrupt step waveguide with different widths. The width of the wider waveguide is $4a$ while the narrower one is $2a$.

The mode coupling characteristic of the abrupt step waveguide is analyzed using the modified step-theory. A linear row of the rods is removed from the crystal so as to push up a mode from the continuous band below the PBG. This defect mode lies in the PBG and light waves cannot propagate to the semi-crystalline structure on both sides. On the left half of the waveguide, two additional rows are removed and the resultant structure is shown in Fig. 3.4.

The wider waveguide is defined as the first step with a width of $4a$ while the narrower waveguide is the second step with a width of $2a$. From the knowledge of conventional waveguide with uniform refractive index, it can be deduced for each of the

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waveguides, the transverse mode field profiles and the number of modes are different. In general, the number of empty rows indicates the number of PCWG modes in the PBG [127].

For the narrower waveguide, the band diagram is shown in Fig. 3.5(a). There is a single localized defect mode in the bandgap. Since the waveguide has even symmetry about the x axis, the mode field distribution of the fundamental guided mode is an even mode as shown in Fig. 3.5(a). For the wider waveguide, it is expected to support three different modes with different transverse field profiles. The band diagram of the wider waveguide has three guided modes in the PBG as shown in Fig. 3.5(b). The three modes are characterized into even and odd modes depending on the field distribution profile of each modes. The inset below Fig. 3.5(b) shows the field characteristic of the different modes. The mode at the lowest frequency is has the simplest field distribution with a line-shape similar to the Gaussian line-shape. This mode is called the fundamental mode (or the lowest order mode) and has an even symmetry with respect to the x axis. For higher frequency guided modes, the field pattern is slightly more complex.

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(a)

(b)

Fig. 3.5 (a) Band diagram of the waveguide with single line defects (b) Band diagram of the waveguide with three line defects. The dotted line is odd mode while the solid lines are even modes.

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After the fundamental even mode, subsequent modes are labeled as the second order odd mode and third order even mode in increasing order. The higher order modes have higher cut-off frequencies. Hence, the number of modes supported in the PCWG may not be the same for all frequencies. For example, when the frequency is $0.368 (a/\lambda)$, the PCWG can only support two modes. This is indicated in Fig. 3.5(b), where the frequency line at $0.368 (a/\lambda)$ intersects at only two of the guided modes. For the higher frequency of $0.4 (a/\lambda)$, the wider waveguide can support all the three guided modes.

Based on the band diagrams in Fig. 3.5, an important point is demonstrated to simplify the use of the modified step-theory. The coupling between even and odd mode is not possible due to the difference in the mode field distribution. To illustrate this, the larger waveguide is excited with two different modes distributions, an even symmetry fundamental mode and an odd symmetry second order mode. Figure 3.6(a) and (b) demonstrate the case where the incident field is excited with even and odd mode respectively. From Fig. 3.6(a), it is observed that the even fundamental mode is in the larger waveguide propagates to single mode waveguide while the odd mode excited in the larger waveguide is reflected and does not transmit to the single-mode PCWG as shown in Fig. 3.6(b). The even mode propagates with nearly perfect transmission from the multimode to the single mode PCWG because both waveguides support mode of same symmetry. On the other hand, the odd symmetry excited electric field is not supported by the single-mode PCWG. Since the odd symmetry field cannot propagate into the PC, it is reflected back. Therefore, it shows that the coupling is only possible between two guided modes that have the same symmetry.

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This coupling phenomenon can also be demonstrated mathematically using the overall integral in Eq. (3.13). Using the approximate equation for the PCWG with artificial periodicity, the even and odd symmetry can be assigned as having sine and cosine field distribution respectively. For the field excited with odd symmetry and launched into the single mode PCWG with even symmetry, the overlap integral is expressed as

$$I_{in,jn+1} = \int_{-\frac{b'}{2}}^{\frac{b}{2}} \cos \left[m_{in} \pi \left(\frac{z}{b} + \frac{1}{2} \right) \right] \sin \left[m_{jn+1} \pi \left(\frac{z}{b'} + \frac{1}{2} \right) \right] dz \quad (3.18)$$

By evaluating the mathematical expression in Eq. (3.18), the overlap integral $I_{in,jn+1}$ is equal to zero. This implies that there is no coupling between modes of different symmetries (between odd and even modes). On the other hand, the even excited field gives a finite answer to the overlay integral.

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Fig. 3.6 (a) Electric field transmission of the even fundamental mode to the narrower waveguide (b) Transmission of the odd second order mode to the narrow waveguide. There is no transmission because the odd mode cannot couple to even mode.

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Based on the discussion of $I_{in,jn+l}$, the coupling coefficient of the different mode symmetry is calculated using the modified step-theory. The beta values of the odd and even modes can be calculated from the dispersion diagram for both waveguides. In this case, the beta values are taken from the dispersion diagram for the frequency of 0.368. For the wider waveguide, the beta values are $0.940k_0$ for the fundamental mode and $0.707k_0$ for the odd mode where k_0 is the free space wave vector from Fig. 3.6(a) and (b). The narrower waveguide has a propagation constant of $0.7334k_0$. According to the modified step-theory, the power transmitted from the input power of unity with the even mode is 0.6559 while it is 0.638 using the exact scattering method. This shows a good match between the modified step-theory and the exact scattering method. When the input is excited by the odd mode, the transmission is zero. The scattering method gives a value of 3×10^{-5} . The reason for this small value is due to scattered stray light that is calculated using the scattering method. However, the value detected by the scattering matrix is close to zero. Therefore, it can be concluded that the odd and even mode do not couple to each other.

In the next section, the multiple PCWGs with different width are cascaded together to form a PC step tapered waveguide. The coupling efficiency of this waveguide is calculated using the modified step-theory. Subsequently, the numerical simulation results are discussed in the next section.

3.3.2 Photonic crystal step tapered waveguide

Based on the concept of the step waveguide, the concept of the step tapered waveguide can be easily understood. The step tapered waveguide is created by cascading

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step waveguide of different width and fixed length together as shown in Fig. 3.7 [96]. This taper design is simple and can be an approximation of the classical taper waveguide. The input width is $18a$ which gives a width of approximately $10\text{ }\mu\text{m}$. This corresponds to the modal diameter of single mode optical fibre source. The output width of the step taper is $2a$. The taper is known as a “step” taper because the side-wall of taper resembles that of a step staircase as shown in Fig. 3.7. Each step is fixed at a length of $3a$ with the largest width of $18a$ at the input and $2a$ at the output of the taper. By varying the length of the step, the total length of the taper is changed, which can result in a more gradual or abrupt mode transition that affects the coupling efficiency of the taper.

For the step tapered waveguide modeling using the modified step-theory, the taper is divided into more steps to calculate the power transmission. An iterative calculation of the transmitted power from the first step to the last step gives the coupling efficiency of the taper. The first step is the one with the largest width. The subsequent steps have smaller width along the propagating direction. The 9th or the last step has only one guided mode and is the single mode PCWG. The difference from the previous case study is that to calculate the transmission from the wider to the narrower end of the taper, the value of b changes as the width changes. The modeling of the PC step tapered waveguide is more complex and tedious due to the greater number of modes involved in the coupling mechanism compared to the step waveguide. The input field is launched by a single mode optical fibre with Gaussian like even distribution. This reduced the calculation time significantly as the odd modes can be neglected.

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Fig. 3.7 Lattice layout of the “step” taper waveguide. Each step has a fixed width of $3a$. The dotted box indicates the whole step that is used by the modified step-theory.

Another point to note is that the fields within each step are assumed to be constant. This implies that the coupling coefficient only calculate the interface between PCWG of different widths.

By using the supercell and plane wave calculation method, the band diagram for each of the step are plotted. The propagation constants of the localized mode in each step are taken from the band diagram at the frequency of consideration. As the number of steps are fixed, the modified step-theory is only applied at each boundary where the width changes. When the width at the boundary between steps changes, the mode amplitude and phase are affected because the beta values changes. The power transmission for the step taper is calculated as shown in Fig. 3.8. The coupling efficiency

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increases as the taper length increases. This is because the taper structure becomes more gradual with slightly less abrupt mode conversion.

For the single step length of $6a$ (a total taper length of $48a$), the transmitted power is 74 % only. This coupling efficiency is higher than the butt coupled power of 23 %. However, this is not sufficient for waveguide device and integrated waveguide circuit requirement. From Fig. 3.8, the power transmission is observed to increase at a slower rate at longer taper length. This is because the step tapered waveguide has uneven side-wall. The uneven side-wall increases the amount of reflected and scattered light waves.

Fig. 3.8 Comparison of the transmitted power by modified step-theory and the scattering method.

Although the abrupt change in the mode size occurs at only a few interface, this has resulted in substantial amount of reflection and scattering loss. The overall transmitted power at the end of the taper is reduced considerably by the substantial amount of reflection loss. Simultaneously, the scattering loss at the corners of the abrupt

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steps also reduces the amount of the light coupling for the forward mode. This can be observed in the field distribution of the taper at length of $48a$ as shown in Fig. 3.9.

From Fig. 3.9, there are a lot of intermodal coupling due to the scattering and reflection from the abrupt corner, which excites the higher order modes. Hence, adiabatic behavior is not achieved and the higher order (even) modes are not supported and reflected at the narrower part of the taper. When the taper gets narrower, the lower order modes below the cut-off frequency propagate through the taper. Ultimately, the electromagnetic energy is only carried and transmitted by the lowest order fundamental even modes at the end of the taper. Therefore, it can be concluded that the coupling for the higher order modes needs to be minimized as much as possible for high transmission efficiency in the step tapered waveguide.

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Fig. 3.9 Field distribution for non-adiabatic behaviour of step taper waveguide with step length of $6a$ (or total length of $48a$). It can be seen that there are a lot of higher order modes existing in the tapered waveguide. This indicates that adiabatic condition is not achieved.

Fig. 3.10 Transmitted power vs relative position for the step taper with step length of $3a$ and $5a$.

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Figure 3.10 shows the transmitted power for the fundamental mode at each position of the taper for length of $24a$ and $40a$ using the modified step-theory. The transmitted power has a discrete step function because of the step structure. The power coupling only occurs at the boundary between each step. When the light waves propagate through the taper, the transmitted power begins to drop because of the uneven side-wall and scattering at the corner of the steps. The higher order even modes are excited and some power is transferred to them. Due to the abrupt change in the width of the taper, the power carried by the higher order mode is reflected rather than transferred to the lower order mode. The lower power is received in the end of the tapered section. For the shorter taper length, the transverse geometrical width changes more abruptly and can be strongly felt by the electromagnetic light waves propagating through the tapering region. Hence, the fast tapering slope of the tapered waveguide causes higher degree of intermodal interaction and coupling. When the taper length increases, the transmitted power increases because of more gradual change in the width of the taper.

The design of PC step tapered waveguide design is a simple implementation of the taper structure on the 2D PC. The crystal structure of the PC is not affected and rods are simply removed without introducing any modification to the lattice arrangement. However, this design has a major disadvantage because it cannot provide sufficient coupling transmission efficiency due to the abrupt step discontinuity. The uneven side-wall leads to a substantial amount of reflection and scattering loss, which results in poor coupling and mode size conversion efficiency. To overcome the problem given by the abrupt step discontinuity, the lattice structure of the PC is perturbed such that the design of the tapered structure has a smooth side wall in order to obtain higher power coupling.

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The modification of the taper design is introduced by shifting of the crystal lattice to produce the smooth tapered waveguide side-wall. In the next section, the modified step-theory will be used to explain the coupling characteristics of the different taper curvature designs.

3.4 Nonuniform tapered waveguide

In this section, nonuniform tapered waveguide curvature is studied. Unlike the previous section where the tapered waveguide design has uneven side and abrupt discontinuity, the new tapered waveguide structure does not have these short comings. The technique is to perturb the crystal lattice of the periodic PC by shearing so that the tapered structure with smooth side-wall is formed. With this new design, the additional loss resulted from the abrupt step tapered structure can be eliminate and higher coupling efficiency is expected. However, it is more tedious to use the modified step-theory because the more steps are required to calculate the transmission efficiency for the smooth side-wall tapered waveguide. The simulation results of the nonuniform smooth side-wall PCTW has higher coupling efficiency more than 95 %.

3.4.1 Linear and nonuniform curvature tapered waveguide

The curvature of the taper is obtained by lattice distortion of the regular crystal arrangement as shown in Fig. 3.11. The curvature of the taper is obtained by lattice distortion of the regular crystal arrangement as shown in Fig. 3.2. The equation of tapered waveguide with different curvatures are expressed as [101]

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$$z = d_i + (d_i - d_o) \left[\left(1 - \frac{x}{l} \right)^\alpha - 1 \right] \quad (3.19)$$

where the parameters, d_i and d_o are the input and output width of the taper respectively. In this paper, d_i is $18a$, which is approximately the mode diameter of light pulse from single mode optical fibre. The variable z and x are the coordinates of the rods to be placed to obtain different taper shapes.

The width of PCWG is created by simply removing a row of rods from PC lattice. The variables x and $z(x)$ are defined as the x -position and z -position of the rods with l as the total length of the taper. For square array arrangement, the x - and z -position of the rods are multiples of lattice constant. Based on Eq. (3.19), the designs of the nonuniform taper curve depend on the parameter, α . It can be deduced that Eq. (3.19) and Eq. (3.1) are identical and only differ by the parameter, α . When $\alpha = 1$, the width of the taper decrease linearly, forming a linear PCTW structure as shown in Fig. 3.11. When other values of α are substituted, a nonuniform PCTW is obtained. The nonuniform taper curvature can be divided into two groups: One group with $\alpha > 1$ and the other with $\alpha < 1$. For illustration in this thesis, consider the case when the values of $\alpha = 0.5$ and $\alpha = 2$ is investigated first to obtain two nonuniform taper. Fig. 3.11 shows the curvature of the taper with α values of 2 and 0.5. When $\alpha = 0.5$, a convex PCTW is obtained with the slope increasing toward the end of the tapered waveguide as shown in the dash-dot line. On the other hand, when $\alpha = 2$, the tapered waveguide has a concave curvature with the slope decreasing toward the end of the tapered waveguide with dash-dot-dot line.

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Fig. 3.11. Tapered waveguide shapes for different values of α . For $\alpha = 1$, the taper shape corresponds to the linear taper. For $\alpha = 2$ and $\alpha = 0.5$, the taper shape corresponds to concave and convex taper respectively.

All $\alpha > 1$ give a concave like curvature while for $\alpha < 1$, the tapered waveguide has a convex curvature. The length of taper l , is measured from the input to the output end of the taper. By varying the length of the taper, the mode conversion efficiency can be improved to increase the coupling efficiency.

Based on Eqs. (3.19), the taper shape is created by positioning the inner most row of rods that are defined by coordinates x and $z(x)$. The crystal is built in such a way that the vertical and horizontal lattice vectors are maintained at the lattice constant of a as shown in Fig. 3.12(a) - (c).

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(a)

(b)

(c)

Fig. 3.12 Schematic lattice layout of the different PCTW curvature. (a) Linear (b) Convex (c) Concave.

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In this chapter, the light source is incident from normal single-mode fibre and has a Gaussian beam mode distribution. The collected field is then converted to power using the Poynting vector theorem. This output power is normalized with the incident power to obtain the normalized transmittance for different PCTW. The fields are calculated by the modified step-theory method. Figure 3.13 shows the field distribution when an incident light waves at frequency of $0.368 (a/\lambda)$ is launched through the different taper designs.

It is observed from Fig. 3.13 that there are multimode excitations in the tapering region, implying that adiabatic condition is not achieved. In the context of electromagnetic theory, the adiabatic theorem means that the electromagnetic light waves remains in the lowest order mode and the higher order modes are not excited. In this way, the reflection loss in the tapering region is reduced to the minimum because higher order modes are not excited. Before ending this part, the changes in the transverse field profile for the different taper curvature designs are discussed. Figure 3.14 shows normalized field intensity of the propagating wave at various positions of the linear and nonuniform PCTW calculated by using the modified step-theory.

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(a)

(b)

(c)

Fig. 3.13 Field distribution for different curvature of PCTW. (a) Linear (b) Convex (c) Concave.

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Fig. 3.14 Normalize field intensity of the propagating wave as a function of taper position for different taper curvature designs.

The propagating field intensity of concave PCTW is increasing at a faster rate compared to the linear and convex PCTW. This is due to the inward curvature of the front end of concave PCTW. For the linear PCTW, the field intensity changes linearly due to the fix gradient of the taper. For the convex PCTW, greater change in the slope of the taper at the end causes some transmission loss. However, the coupling efficiency is not affected significantly.

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3.4.2 Simulation results and discussions

In this part, the length of the smooth side-wall PCTW is varied to demonstrate the relationship between taper length and power transmission for different curvature (or different values of α). The modified step-theory derived in section 3.2 is used for the calculation of the transmission efficiency. The difference of using the modified step-theory in the PCTW is the increase in number of steps for iterative calculation. In the previous section, the step tapered waveguide is considered as a discrete form with fixed step width along the tapering section. On the other hand, the smooth side-wall PCTW has width that varied continuously in the tapering region. Hence, more steps are needed for accurate transmission calculation. For this case study, two taper lengths at $l = 17a$ μm and $l = 32a$ are selected. Without the tapered waveguide, the average normalized power transmitted to the single mode PCWG is 0.2 approximately. This is because of the large difference in the mode size between the launched power from an optical fibre with modal diameter of $18a$ to the PCWG with diameter of $2a$. This coupling efficiency between the fibre light source and the single mode PCWG is insufficient because the collected power from the output of the PCTW is too weak to be used for application for photonic devices. Therefore, it is necessary for the coupling tapered waveguide device to provide an efficient coupling path between the input light sources and the narrow PCWG. Based on the modified step-theory, the transmission spectra for a linear PC taper at $l = 17a$ and $l = 32a$ are shown in Figs. 3.15(a) and (b) respectively. The linear taper has an average transmittance of 0.51 as shown in Fig. 3.15(a), while the average transmittance is 0.862 over the frequency range of 0.32 - 0.4 as shown in Fig. 3.15(b).

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(a)

(b)

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(c)

(d)

Fig. 3.15 Transmission spectrum for the various taper shapes (a) & (b) Linear PCTW for $l = 17a$ and $32a$. (c) & (d) Nonuniform convex and concave PCTW $l = 17a$ and $32a$.

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According to these observations, the modal coupling is more effective at longer taper length because losses are reduced. At longer taper length, the slope of taper is gentler. This means that the fundamental even mode does not couple with the higher order modes when it travels through the tapered waveguide. Therefore, the reflection loss is reduced and coupling efficiency increases.

The other interesting observation is the stability of the power transmitted by the tapering section at different frequencies as shown in Figs. 3.15(a) and (b). The bandpass of the spectrum at longer length have less fluctuation compared with the spectrum at shorter length. There are two reasons to explain these fluctuations. First, the fluctuations are caused by the multiple reflections from the Fabry-Perot oscillations in the cavity mirror forms by the side-wall of the taper. Second, the fluctuations are observed due to the “multi-path” interference caused by coupling between back-and-forth modes. For longer tapered waveguide as shown in Fig. 3.15(b), the interference and multiple reflections are reduced and the fluctuations are also reduced. As a result, in the coupling efficiency improved and more stable transmission spectrum. However, it is important to note that the accuracy of the fluctuations in the transmission spectra are neglected because of some assumptions made in deriving modified step-theory. In the next chapter, a more general mathematical model for describing the coupling coefficient between different modes (either differs in order or direction) is derived and the fluctuations in the transmission spectra will be compared again.

For the concave and convex PCTW, the transmission spectra with length $l = 17a$ and $l = 32a$ is shown in Figs. 3.15(c) and 3.15(d) respectively. In Fig. 3.15(c), the average transmittance of the concave PCTW is 0.4 approximately, while the average

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transmittance of the convex PCTW is slightly higher at 0.6. When the length of the PC tapered waveguide increases to $32a$, the average transmittance of both concave and convex PCTW increases to 0.8 and 0.96 respectively with a beam reduction ratio of 10:1. The mode coupling efficiency of the convex PCTW is more effective than the concave PCTW. This is because the convex PCTW has a larger receiving curvature, which means that taper slope variation is more gradual than the concave PCTW. At the wider end of the tapered waveguide, intermodal coupling is weaker and this reduces the coupling to higher order modes. Therefore, reflected light reduces as the taper become narrower. At the narrow end of the convex PCTW, there is little more reflection due to sharper change in the curvature that only affects the overall transmittance performance slightly. Therefore, the multi-path interference with input light sources for the case of convex PCTW is smaller and the band spectrum is more stable.

For the concave PCTW, the average transmittance also increases when the taper length increases. However, the improvement is less significant compared to the convex PCTW. This is due to the inward curvature of the taper. The taper slope varies steeply near the front part of the taper. This causes the large changes in the propagation constant of the modes as the wave travel through the concave PCTW. Therefore, the higher order intermodal coupling increases and there are more reflected light waves. There are also more multi-path interference and multiple reflections and, the transmission spectrum of the concave PCTW has more ripples than the convex PCTW. As the taper length increases, the inward curvature of the concave PCTW becomes gentler. However, its coupling efficiency is still more inferior compared to the convex PCTW because the reflected waves are still relatively significant.

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After comparing at taper length of $17a$ and $32a$, the transmission efficiency of the linear, concave and convex PCTW designs at other lengths are also compared. Different α values are also substituted to Eq. (3.19) to investigate the trend of mode coupling efficiency at frequency of $0.368 (a/\lambda)$. The values of α are substituted at 1.5 and 3 as well as its reciprocal to investigate the effect of the curvature of the nonuniform taper on coupling efficiency.

Figure 3.16 shows the transmittance of the linear, concave and convex PCTW curvature designs at different α values. It is observed that the convex PCTW generally gives higher coupling efficiency than the linear and the concave PCTW. From Fig. 3.16(a), the convex PCTW with α values of 0.5 and 0.667 has higher transmittance than the linear taper. When $\alpha = 0.333$, the transmittance is the lowest. It is because the slope of the convex taper for $\alpha = 0.333$ to change even more steeply at the end of the taper. Therefore, there are more reflection and scattering losses, which reduces the coupling efficiency to a substantial amount of more than 10%.

From Fig. 3.16(a), the transmittance of the tapers “saturated” after a certain taper length. This is because the tapered waveguide is beginning to experience adiabatic-like behavior. For the convex taper with $\alpha = 0.5$, the increment of the transmitted power is relatively slow after $l = 36a$. For instance, it is observed to have less than 0.01% increment for every $5a$ increase in length. Therefore, $36a$ is identified as the length where the transmittance of the convex taper “saturates”. At this point, the taper is experiencing adiabatic mode coupling and the loss is mainly from the field pattern mismatch between the Gaussian profile input and the fundamental mode of the wide PCWG.

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(a)

(b)

Fig. 3.16 Transmittance vs taper length plots for different values of α . (a) Convex PCTW for values of $\alpha = 0.333, 0.5, 0.667$. (b) Concave PCTW for values of $\alpha = 1, 1.5, 2, 3$.

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Table 3.1: Saturation transmittance and length of different α values

α values	Saturated Normalized Transmittance	Saturated Length (in lattice constant, a)
1.000	0.912	44
0.333	0.862	35
0.500	0.975	36
0.667	0.934	40
1.500*	0.910	50
2.000*	0.905	50
3.000*	0.811	50

*Note: For concave taper, the transmittance is recorded at taper length of $50a$. This is the maximum taper length considered in this thesis.

Table 1 shows the saturation lengths and the transmittance for the different taper designs at different lengths with different values of α . From Table 1, the saturated transmittance is 0.975 for the convex PCTW with $\alpha = 0.5$ and is 6% higher than the linear PCTW. It also shows that for $\alpha = 0.333$ saturation length is the shortest but the transmittance is much lower than other values of α . Based on this comparison, it is more advisable to substitute for $\alpha = 0.5$ because the coupling efficiency is higher. As a result, the curvature of the tapered waveguide is an important factor for the determination of the coupling efficiency of the nonuniform taper.

For the case of concave PCTW, the value of α are greater than 1. The mode coupling efficiency of the concave PCTW is more inferior to the convex and linear PCTW due to higher reflection losses. The reason is due to the steeper taper slope at the front part of the concave PCTW, which causes large variation in the β values of the guided modes. Therefore, there are more intermodal coupling to higher order guided modes in the concave PCTW and the reflection losses increases. When $\alpha = 3$, the inward

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curvature of the taper is very steep, which leads to high back reflection. When α decreases to 2 and then to 1.5, the transmittance increases due to the decrease in the inward curvature of the concave taper. Hence, the mode conversion efficiency of the taper can be altered with different values of α . The concave PCTW can also reach adiabatic condition at longer taper length as compared to the convex PCTW because the change in the inward curvature is becoming more gentle, which in turn reduces the back reflection and multi-path interference with the incident wave. In this chapter, taper length is restricted at $50a$, as it is not persuasive to increase mode coupling efficiency at the expense of increasing the taper length.

Fig. 3.17 Spectrum distortion for concave PCTW of length $10a$.

From Figs. 3.16(a) and (b), it is observed that there are few oscillations at shorter taper length before it increases steadily at longer length. This is due to the large amount of electromagnetic light waves reflection in the PCTW. The multiple reflections and interference of the back-and-forth modes are due to the large change in phase constant and this result in significant amount of coupling between the fundamental and higher

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order modes. The higher order modes are not supported in the narrower end. Interference of the reflected waves with incident waves causes many fluctuations in the transmission spectra as shown in Fig. 3.17 for concave PC taper at length of $10a$. For different curvature, the amount of interference is different. For example, when $\alpha = 3$, there is more interference and higher back reflection. As a result, there are more fluctuations in shorter taper with concave curvature.

In the above discussion, it can be seen that the curvature of the smooth PCTW structure is an important for obtaining maximum coupling efficiency for the single mode PCWG. The effects of taper curvature can be further investigated through the modeling of the power loss along tapered waveguide. The power loss characteristic of the taper at different length is studied in the next section.

3.4.3 Loss characteristics of different PCTW designs

The modified step-theory is used to evaluate the power loss of tapers with different curvatures at each relative position of the tapered waveguide. Figures 3.18 (a), (b) and (c) show the power loss versus the relative position for the linear, convex and concave PCTW at length of $17a$ and $32a$. From Figs. 3.18(a) and (b), the power loss at the relative position of the tapered waveguide show similar trend. The reason is due to the more abrupt change in the tapered waveguide slope between the tapered waveguide and PCWG. The abrupt change results in the scattering of light between the tapered waveguide and PCWG. Therefore, the power loss is increased as shown in Fig. 3.18(a) and (b). However, there is a sharper increase in the power loss for the convex PCTW due to a more abrupt change in β as observed from Fig. 3.18(a) and (b).

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For the concave PCTW, the loss trend is different from the linear and convex PCTW. From Fig. 3.18 (c), the loss is higher at the front part of the taper that is due to the fast reduction of taper width. There is high inter-modal coupling between the fundamental even mode and the higher order modes. Therefore, the large reflection of the higher order modes contributes to the high power loss. However, towards the narrower end of the taper, the power loss begins to drop at position of 0.7. This is because the slope of concave taper is more gentle, which reduces inter-modal coupling. Moreover, the back part of the tapered waveguide fits smoothly with the PCWG. This explains the drop in the loss at the back part of the concave PCTW as shown in Fig. 3.18(c). Even though the concave taper fits smoothly to the waveguide, the transmission is still lower than the convex or linear taper due to higher power reflection at the front part of the taper. The simulation results of the transmitted power for the fundamental mode using the modified step-theory are shown in Fig. 3.19(a), (b) and (c).

(a)

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(b)

(c)

Fig. 3.18 (a) Power loss vs relative position for (a) linear (b) convex and (c) concave PCTW at $17a$ and $32a$.

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(a)

(b)

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(c)

Fig. 3.19 Power transmission of the fundamental mode for (a) linear (b) convex (c) concave PCTW at length $17a$ and $32a$.

In Fig. 3.19(a), the power transmission of the fundamental mode decreases along to the taper position. This coincides with Fig. 3.18(a) where the power loss slowly increases until the end of the taper. The power of the shorter tapered waveguide decreases faster than the longer taper because the width of taper changes more abruptly, which causes more coupling to higher order modes. Therefore, more power is reflected as the light waves travel along the narrow part of the taper.

For the case of the convex and concave PCTW, the same observation is concluded at the longer taper length in Figs. 3.19(b) and 3.19(c) respectively. For the convex PCTW, the fundamental mode power decreases very slowly at the front part due to the slow tapered waveguide slope. The coupling of the higher order modes are very small and provide high power transmission. At the end of the taper, due the abrupt increase in

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the slope of the taper, the power transmission drops more significantly. This is because faster tapering causes excitation of the higher order modes and back reflection. For longer taper length of $32a$, the power loss remains constant through out the whole tapered waveguide. It is because the whole convex PCTW has very gradual taper slope and coupling of higher order mode is almost negligible. For the concave PCTW, the power of the fundamental mode drops fast in the front part of the tapered waveguide due to the fast change in the curvature. The power drop is slowed down at the relative position of 0.7 approximately. This is due to the gentle curvature and good fit of the structure with the PCWG.

From Fig. 3.19, the power fluctuations along the relative position of the taper are due to the intermodal coupling between the fundamental mode and the higher order modes. The upward “bumps” indicate that more power is transferred to the fundamental mode while the downward “bumps” indicate that more power is coupled to the higher order modes. In Fig. 3.19(a), when the taper length is $17a$, more fluctuations are observed and scattered at different parts of the tapered waveguide. When the taper length is $32a$, the power transmission is more stable, indicating less intermodal coupling between the fundamental mode and higher order modes.

For the linear PCTW, the fluctuations are scattered along the different parts of the taper. For the convex PCTW, the fluctuations occur towards the end of the taper when the slope of the taper becomes steeper. The intermodal coupling mechanism between the fundamental mode and higher order modes is shown in Fig. 3.19(b). When the taper length is $17a$, the fluctuations are more vicious due to steeper slope. When taper length is $32a$, the fluctuations are very small and the major fluctuations have almost disappeared.

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This is because the slow tapering of convex PCTW reduces coupling of higher order modes. Therefore, the coupling efficiency of the convex PCTW is more than 90 %. For the concave PCTW, the fluctuations are observed in the front part of the taper due to the fast reduction of the taper width. The presence of the intermodal coupling reduces the amount of power coupling to PCWG. When the taper length is $32a$, the power transmitted is only 80 % approximately.

For the convex taper with $\alpha = 0.5$, saturated transmission is achieved at the length of $36a$. Figure 3.20 shows the relationship between the power change and the relative position of the three types of taper at the length of $36a$. For the linear taper, the transmitted power is more stable. The field distribution of the linear taper is shown in Fig. 3.21(a), which display some adiabatic characteristic. It is because there is some mixture of intermodal modes and single mode in different part of the linear PCTW with transmission of 89 %. By extending the length of the taper to $44a$, the linear taper can achieve adiabatic condition with the transmission efficiency of 91.2%. The remaining losses are caused by the mode field mismatch between input Gaussian mode input and the waveguide mode with scattering loss at the end of the taper. For the convex PCTW, the power is almost constant with virtually no ripples, which shows the coupling to higher order mode is absence. The field distribution of the convex PCTW is also plotted in Fig. 3.21(b). The electric fields are in single mode throughout the whole convex PCTW. This means that the adiabatic condition is achieved for the convex PCTW at the length of $36a$ and the transmitted power is 97.5 %. For the concave PCTW, some coupling losses are observed due to fast changing curvature. Fig. 3.21(c) shows that there is a large amount of higher order modes being excited in the field distribution.

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Fig. 3.20 Power transmission of the fundamental modes for linear, convex and concave PCTW at length $36a$.

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(b)

(c)

Fig. 3.21 Field distribution for the different tapers at $36a$. (a) linear (b) convex (c) concave PCTW.

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Based on the discussions, the modified step-theory has rather high convergence for waveguide with moderate cross sectional variation. Even for short and abrupt tapered waveguide, it gives moderate convergence with reasonable accuracy as shown in Fig. 3.8. The modified step-theory can analyze the intermodal coupling mechanism and describe the modes behavior in the tapering section while the other semi analytical methods such as the transfer matrix method and the perturbation-coupled mode theory method is unable to do so. The modified step-theory can provide a design tool for PCTW structure design and optimization.

3.5 Summary

In this chapter, the technique of using tapered waveguide for coupling between input light source and output devices is introduced. Based on the step-theory and the mathematical mode of the PCWG with Bloch's theorem, the modified step-theory is developed. A numerical case study with known examples is conducted to verify its convergence and consistency. The numerical simulation results from the modified step-theory and the exact scattering method match well with only small error. Next, the modified step-theory is applied using two examples of step waveguide designs, namely the PC step waveguide and the step tapered waveguide. The objective of the step waveguide designs is to investigate the properties of the guided modes in the PCWG to perform simple numerical model using the modified step-theory. The numerical simulation results of the step tapered waveguide show a transmission efficiency of 74 %, which is not adequate for an effective coupling in the photonic integrated circuits. A smooth PCTW with linear and nonuniform curvature designs is proposed and studied.

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Using the modified step-theory, the coupling efficiency of the three tapered waveguide design is modeled. It is observed that for the convex PCTW with parameter of $\alpha = 0.5$ at taper length of $36a$ gives a maximum coupling efficiency of 97.5% with adiabatic behavior of the light waves in the tapering section. The concave and linear PCTWs have lower coupling efficiency. In addition, the loss characteristics of the three different curvature waveguides are also discussed. It is concluded that the loss trend is related to the curvature designs. The field distribution for each curvature design is simulated to demonstrate whether the adiabatic condition can be achieved. The convex PCTW achieves adiabatic condition at taper length of $36a$. The concave and linear PCTWs require longer taper length to achieve the adiabatic condition.

CHAPTER 4

EXACT STEP-COUPLING-THEORY FOR GEOMETRICAL VARIATION WAVEGUIDE

The modified step-theory provides an approximate model for investigating the coupling and intermodal interaction of waveguide modes in PCTW. The assumptions and approximations [108] are made in the derivation of the modified step-theory [129] to allow a simple and quick understanding of the coupling and mode interaction mechanism in the PCTW structures. However, the modified step-theory is limited to modeling of PCTW. In this chapter, a new exact step-coupling-theory is derived to provide a larger scope of simulation and numerical results from the coupling and modal interaction mechanism in the tapering section.

In this chapter, the assumptions and approximation of the modified step-theory are discussed. The coupling equations of the exact-step-coupling theory are derived by expanding the waveguide modes in terms of local normal modes using Maxwell's equation. The convergence characteristic of the exact step-coupling-theory is verified and light waves in different propagating directions of various PCTW design are discussed. Finally, the exact step-coupling-theory is applied to the PC slab tapered waveguide and lossy waveguide to calculate the transmission spectra and mode coupling characteristic.

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4.1 Problems of the modified step-theory

In the derivation of the modified step-theory, three simple assumptions are made to provide a more straight forward derivation process. Similarly, two approximations are applied to standardize the mathematical model of the coupling equations and field description to be standardized. However, the assumptions and approximations have to be reviewed before the derivation of a generalized set of equations to describe the mode coupling mechanism of PCTW structures with geometrical variation.

The assumptions of the modified step-theory include: (i) intermodal coupling between reflected modes is small and negligible; (ii) coupling between radiation modes and guided modes in the tapering section is very small and neglected; (iii) power loss from the coupling between the two guided modes can be neglected because reflection loss is the main loss component for the narrow part of the tapered waveguide.

In the first assumption, the inter-modal coupling between the reflected modes is considered negligible for tapers structures with gradual slope, where the change in propagating wave vector, $\Delta\beta$ is very slow. However, this is not valid for short length tapers, as well as taper structures with fast changing slope along the tapering direction because $\Delta\beta$ is relatively large. There is substantial amount of back reflected power in the tapered waveguide [110].

In the second assumption, the coupling between radiation modes and the guided modes in the tapered waveguide is very small and negligible for the guided mode with operating frequencies within the PBG. However, the guided modes operating near the edge of the PBG are subjected to possible mode coupling with the radiation mode in the bulk continuum. This coupling is very effective especially at the moderate or low group

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velocity regime of the guided mode [124]. Therefore, the modified step-theory fails to provide an accurate modeling of the transmitted power for the PCTW near the edge of the PBG.

In the third assumption, power loss coupling resulted from the coupling between the two guided modes can be neglected because the reflection loss is assumed to be the main loss component for the tapered waveguide [109, 124]. However, for longer taper, the reflection loss is reduced significantly due to slower change in the propagating wave vector, β . In this case, the intermodal coupling loss between the two guided modes dominates. The intermodal coupling occurs between one guided mode and another guided mode or radiation mode near the edge of the PBG. Therefore, power loss known as intermodal loss is observed. Beside the assumptions, two approximations are made to simplify the derivation process of the modified step-theory. More intermodal coupling characteristics can be obtained using the complete field equations from Maxwell equations.

The first approximation is the mathematical model of the transverse guided mode. It has been shown in several works that the guided mode in PBG resembles the dispersion band diagram for a metallic waveguide of width b imposed with an artificial periodicity d . Therefore, each of the transverse guided mode field profile of the PBG can be expressed as [127, 129]

$$E = E_0 \sin\left(m\pi\left(\frac{z}{b} + \frac{1}{2}\right)\right) \exp(-i\psi x) \quad (4.1a)$$

where
$$\psi = \beta + \frac{2\pi\eta}{d} \quad (4.1b)$$

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The phase coefficient shown in Eq. (4.1a) contains the wave vector, β to describe the characteristic of the light waves with a periodic term to account for Bloch modes in the periodic PCWG structures. However, it is not a complete description for the field mode near the edge of the PBG. It is appropriate for the operating frequency of the light waves far from the PBG edges. For the modes nearer to the edges of the PBG, the mathematical model of Eq. (4.1a) must include the radiation field of the bulk continuum outside the PBG. There is coupling between the guided modes and radiation modes at the band edge.

The other approximation of the reflected field of the guided mode is given by

$$E_r \approx \left(\frac{\psi_{in} - \psi_{in+1}}{\psi_{in} + \psi_{in+1}} \right) E_{in} \exp[-i(\psi_{in} - \psi_{in}^r)] \quad (4.2)$$

This approximation is not suitable for the case of short tapered waveguide. The coupling mechanism of the back reflected waves are also not covered in Eq. (4.2). Hence, the reflection coefficient of the electromagnetic light waves is redefined to describe the reflected field amplitude more accurately.

After reviewing the assumptions and approximations of the modified step-theory, it is found necessary to derive a complete set of coupling equations in the exact step-coupling-theory. This is discussed in the next section.

4.2 Exact step-coupling-theory

A set of coupling equations for modeling the coupling mechanism of the waveguide is derived in order to obtain the exact solution for the mathematical model of PCWG with geometrical variation. The coupling equations are derived by integrating the step-theory technique that divides the waveguide structure into many parts to obtain the

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general mode field solution. Then an field mode transformation is applied to the field vector equation to obtain the coupling coefficients of the exact step-coupling-theory in scalar product form.

4.2.1 Field components in Maxwell equations

In this section, Maxwell's equations are expressed in terms of electromagnetic field vectors. The curls equations (Faraday's and Ampere's law) from Maxwell's equations are used. Two other divergence equations from Gauss's law vanish in the absence of charges and current sources in electromagnetic waves. The electric field has the dielectric permittivity function $\varepsilon(x, y, z)$ and the magnetic field is characterized by the magnetic permeability function $\mu(x, y, z)$ in three dimensional space. For waveguide structures with varying width, the electric and magnetic fields are separated into longitudinal and transverse components [110]. The ∇ operator in the curl equations can be separated into the longitudinal and transverse components. However, only the transverse field components are considered as the transverse field components are more affected for geometrical varying waveguide. The Maxwell's curls equations for the transverse field can be expressed as

$$\left(\frac{1}{i\omega\mu} \right) \nabla_t \times (\nabla_t \times E_t) + e_x \times \frac{\partial H_t}{\partial x} = -i\omega\varepsilon E_t \quad (4.3a)$$

$$-\left(\frac{1}{i\omega\mu} \right) \nabla_t \times (\nabla_t \times H_t) + e_x \times \frac{\partial E_t}{\partial x} = -i\omega\mu H_t \quad (4.3b)$$

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Equations (4.3a) and (4.3b) are used to describe any fields components with arbitrary dielectric and permeability function $\varepsilon(x, y, z)$ and $\mu(x, y, z)$. Although it may be difficult to solve the Maxwell's equations in Eqs. (4.3a) and (4.3b), the curl can be removed and be replaced by scalar product of the vector fields. This is shown in the later part of this section. The field components of each propagating mode in the waveguide satisfy Eqs. (4.3a) and (4.3b) because they are solutions of Maxwell's equations. The guided modes propagating in the waveguide depends on the phase coefficient, $e^{-i\psi_v x}$. For a guided mode in the conventional waveguide, Eq. (4.3a) is expressed as

$$\left(\frac{1}{i\omega\mu}\right)\nabla_t \times (\nabla_t \times \mathcal{E}_{vt}) - i\psi_v (e_x \times \mathcal{H}_{vt}) = -i\omega\varepsilon\mathcal{E}_{vt} \quad (4.4a)$$

$$-\left(\frac{1}{i\omega\varepsilon}\right)\nabla_t \times (\nabla_t \times \mathcal{H}_{vt}) - i\psi_v (e_x \times \mathcal{E}_{vt}) = i\omega\mu\mathcal{H}_{vt} \quad (4.4b)$$

where E and H for the electric and magnetic field in Eqs. (4.4a) and (4.4b) indicate that the field vectors belong to the normal modes of the waveguide. The subscript v is used to label the different modes in the waveguide. The phase coefficient of PCWG is expressed as [129]

$$\psi_v = \beta_v + \frac{2\pi\eta}{a} \quad (4.5)$$

where β_v is the wave vector of the mode v propagating in the waveguide. Based on the Bloch-Floquet theorem, the solutions of the field in the periodic structure is in the form of Bloch waves with periodic function $f(z) = f(z+r_i)$, where r_i is the primitive lattice vector in the crystal lattice in the real space. A corollary of the theorem in the k -space required the wave vector satisfies the condition $f(k) = f(k + 2\pi/r_i)$, where $2\pi/r_i$ is in term of the

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reciprocal lattice vector. Therefore, the phase coefficient of the propagating modes in periodic waveguide also satisfies the condition $\beta_v = \beta_v + (2\pi\eta/a)$. The integer, η is related to the folding of bands at wave vector, $k = 2\pi/a$, which defines the edge of the first Brillouin zone of the PC. Since the PC structure has a uniform periodic, it is sufficient to consider the wave vector of the propagating modes in the first Brillouin zone that is defined in the k -space range, $(-\pi/a, \pi/a)$. The power carried by each propagating mode in the waveguide is expressed as

$$P_\theta = \frac{1}{2} e_x \cdot \iint (\mathcal{E}_{vt} \times \mathcal{H}_{vt}^*) dy dz \quad (4.6)$$

where the power P_θ is a real and positive quantity. In this chapter, the power transmission for different propagating modes is calculated to demonstrate the coupling mechanism that exists along the geometrical variation waveguide.

4.2.2 Local normal modes field expansion

The description of the modes coupling in waveguides is not unique [110]. Several ways are developed to derive the set of the coupling equations for the mode field components of geometrical variation waveguides, which include the effective sources method [134], transmission line representation [135] and electromagnetic field modes of waveguide structures [126]. In this chapter, the coupling equations are described using the electromagnetic fields of the normal modes of the dielectric waveguide.

To describe the fields in nonuniform geometrical variation waveguide using the normal modes, the first step requires the selection of the type of reference waveguide structure. There are two ways of setting the waveguide reference for the field description

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as shown in Fig. 4.1. The solid line shows the nonuniform dielectric waveguide with varying core width. The dash line shows the normal field components of the varying width waveguide expanded in term of modes of the ideal waveguide.

An alternative way to describe the field components is to expand the fields in terms of the normal modes belonging to a hypothesis waveguide, which is defined locally at all the points of the geometrical variation waveguide. The fields for the geometrical variation waveguide using the normal modes of the hypothesis waveguide are represented by the dash-dot line in Fig. 4.1. The light waves fields are expressed in terms of the hypothesis waveguide formed by the two dash-dot lines as a function of x .

Fig. 4.1 Definition of ideal waveguide reference and local waveguide reference.

The two field expansions have advantages for solving different types of waveguide variations. The field expansion in term of the ideal waveguide modes is more preferred for waveguide with perfect geometries but the refractive index is varied along the propagation direction. The coupling coefficients of the ideal mode expansion are simple and can be computed easily. On the other hand, the coupling coefficients of the electromagnetic field for local mode expansion are more complex than the ideal mode waveguide because it considers the waveguide variation locally. The local mode

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expansion is more useful for waveguide with geometrical variation as compared with the ideal mode expansion. It describes and analyzes the intermodal coupling behavior in nonuniform waveguide, especially the tapered waveguide. Besides, the local normal modes expansion has an additional advantage for the PCWG with geometrical variation. The geometrical variation PCWG has periodic waveguide boundaries, which can be easily applied for the local modes expansion. Therefore, the electromagnetic field is expanded in term of the local normal modes.

4.2.3 Coupling equations

In this chapter, the coupling equations for the exact step-coupling-theory are derived and the waveguide structure is divided into many steps based on the principles of the step-theory [109]. The main concept behind step-theory is mode matching of field components. When the electromagnetic light waves travel along different media, the field components change in terms of field profile distribution, amplitude, phase coefficient or velocity. This chapter deals with the mode coupling and intermodal interaction in the geometrical variation waveguide, specifically the tapered waveguide. According to field theory, the tangential component of the total electric and magnetic field is continuous across an interface and can be expressed as

$$E_{1t} = E_{2t} \quad ; \quad H_{1t} = H_{2t} \quad (4.7)$$

The boundary condition of Eq. (4.7) is general for all cases except when the interface is an ideal perfect conductor or superconductor medium. For local modes field traveling in geometrically nonuniform waveguide, Eq. (4.7) is satisfied or the law of conservation of

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field is violated. The transverse component of the electromagnetic field is considered for the derivation because the geometrical variation waveguide only changes in the transverse profile. Expanding the total field series for all the guided mode, the total electric and magnetic can be expressed as

$$E_t = \sum_v p_{vt} \mathcal{E}_{vt} \exp(-i\psi_v x) \quad (4.8a)$$

$$H_t = \sum_v q_{vt} \mathcal{H}_{vt} \exp(-i\psi_v x) \quad (4.8b)$$

The expansion of the total field components Eqs. (4.8a) and (4.8b) are used for all modal polarizations (even/odd modes and TM/TE modes) in the waveguide. The summation covers the entire guided modes in the waveguide. From Eqs. (4.8a) and (4.8b), p_{vt} is used to account for the transverse component of the electric field. A different amplitude coefficient, q_{vt} is used to account for the magnetic field. Applying Eqs. (4.8a) and (4.8b), to Eq. (4.3a) and (4.3b), the following expressions are obtained

$$\begin{aligned} & \sum_v \left[\left(\frac{1}{i\omega\mu} \right) \nabla_{vt} \times (\nabla_{vt} \times p_{vt} \mathcal{E}_{vt} \exp(-i\psi_v x)) + e_x \times \frac{\partial(q_{vt} \mathcal{H}_{vt} \exp(-i\psi_v x))}{\partial x} \right] \\ &= \sum_v -i\omega\epsilon p_{vt} \mathcal{E}_{vt} \exp(-i\psi_v x) \end{aligned} \quad (4.9a)$$

$$\begin{aligned} & \sum_v \left[-\left(\frac{1}{i\omega\epsilon} \right) \nabla_{vt} \times (\nabla_{vt} \times q_{vt} \mathcal{H}_{vt} \exp(-i\psi_v x)) + e_x \times \frac{\partial(p_{vt} \mathcal{E}_{vt} \exp(-i\psi_v x))}{\partial x} \right] \\ &= \sum_v i\omega\mu q_{vt} \mathcal{H}_{vt} \exp(-i\psi_v x) \end{aligned} \quad (4.9b)$$

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In order to simplify the notation, the exponential expression is dropped. However, this exponential expression is always present in the process of the derivation. By using Eqs. (4.4a) and (4.4b), Eqs. (4.9a) and (4.9b) can be further simplified as

$$\sum_v \left\{ \left[ip_{vt}\psi_v + \frac{\partial q_{vt}}{\partial x} \right] (e_x \times \mathcal{K}_{vt}) + (q_{vt}) \left(e_x \times \frac{\partial \mathcal{K}_{vt}}{\partial x} \right) \right\} = 0 \quad (4.10a)$$

$$\sum_v \left\{ \left[iq_{vt}\psi_v + \frac{\partial p_{vt}}{\partial x} \right] (e_x \times \mathcal{E}_{vt}) + (p_{vt}) \left(e_x \times \frac{\partial \mathcal{E}_{vt}}{\partial x} \right) \right\} = 0 \quad (4.10b)$$

Equations. (4.10a) and (4.10b) show the field expression amplitude coefficients for guided mode v . In order to obtain the coupling equation relating the modes of two waveguides with different cross sectional width, the orthogonal condition is applied. The expression relating the electric and magnetic field for the orthogonal condition is expressed as

$$\iint e_x \cdot (\mathcal{E}_{vt} \times \mathcal{K}_{\mu t}^*) dy dz = 2 \delta_{v\mu} P \frac{\beta_{\mu t}^*}{|\beta_{\mu t}|} \quad (4.11)$$

where the subscript μ is the modes in another waveguide of different cross sectional width. P is the normalized power factor the propagating light waves at operating frequency. The double integral implies that the transverse electromagnetic field span over the y and z transverse directions. The symbol $\delta_{v\mu}$ indicates Kronecker's delta expression for discrete guided modes in PBG. For the coupling between the guided modes and the bulk continuum of radiation modes outside PBG, it refers to Dirac delta function. The ratio $\beta_{\mu t}^*/|\beta_{\mu t}|$ is used to indicate whether the modes coupling involves radiation modes. If the guided mode of the PBG is considered, the ratio is given as +1

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because the β value is a real positive quantity. When there is coupling of the radiation and guided mode, β value is an imaginary quantity. The coupling equations remain complex.

According to the orthogonal relationship, the scalar product of Eq. (4.10a) is taken with the complex conjugate of the mode electric field, E_μ^* . Applying the triple product vector identities and taking the double integral over the transverse axis, this is obtained as

$$\sum_v \left[ip_v \psi_v + \frac{\partial q_v}{\partial x} \right] \iint e_x \cdot (\mathcal{K}_v \times \mathcal{E}_\mu^*) dydz = \sum_v (q_v) \iint -e_x \cdot \left(\frac{\partial \mathcal{K}_v}{\partial x} \times \mathcal{E}_\mu^* \right) dydz \quad (4.12)$$

where the subscript t is dropped for simplicity. Substituting the orthogonal relationship Eq. (4.11) into Eq. (4.12), it is simplified as

$$i\psi_\mu p_\mu + \frac{\partial q_\mu}{\partial x} = \sum_v R_{v\mu} q_v \quad (4.13)$$

where the coupling coefficient $R_{v\mu}$ is expressed as

$$R_{v\mu} = \frac{-|\psi_\mu|}{2P\psi_\mu} \iint e_z \cdot \left(\mathcal{E}_\mu^* \times \frac{\partial \mathcal{K}_v}{\partial x} \right) dydz \quad (4.14)$$

The coupling coefficient $R_{v\mu}$ is the coupling mode label from v to μ . Performing similar steps for Eq. (4.10b) by taking scalar product with H_μ^* , the following expression is obtained as

$$\sum_v \left[iq_v \psi_v + \frac{\partial p_v}{\partial x} \right] \iint e_z \cdot (\mathcal{E}_v \times \mathcal{K}_\mu^*) dydz = \sum_v (p_v) \iint -e_z \cdot \left(\frac{\partial \mathcal{E}_v}{\partial x} \times \mathcal{K}_\mu^* \right) dydz \quad (4.15)$$

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Substitute the orthogonal relationship Eq. (4.11) into Eq. (4.15) gives

$$i\psi_\mu q_\mu + \frac{\partial p_\mu}{\partial x} = \sum_v S_{v\mu} p_v \quad (4.16)$$

where the coupling coefficient $S_{v\mu}$ is given as

$$S_{v\mu} = \frac{-|\psi_\mu|}{2P\psi_\mu^*} \iint e_x \cdot \left(\frac{\partial \mathcal{E}_v}{\partial x} \times \mathcal{H}_\mu^* \right) dydz \quad (4.17)$$

Based on Maxwell's equation, the series expansion of the electromagnetic field has given a pair of coupling equations, Eqs. (4.13) and (4.16). In order to solve for the amplitude coefficient p and q , $S_{v\mu}$ and $R_{v\mu}$ are set as zero. By differentiating Eqs. (4.13) and (4.16) with respect to x and through internal substitution, a pair of second order differential equations is obtained. Solving the differential equations, the field amplitude coefficients, p_μ and q_μ , can be obtained as

$$p_\mu = \begin{cases} p_\mu^{(+)} = c_\mu^{(+)} \exp(-i\psi_\mu x) & (4.18a) \\ p_\mu^{(-)} = c_\mu^{(-)} \exp(i\psi_\mu x) & (4.18b) \end{cases}$$

$$q_\mu = \begin{cases} q_\mu^{(+)} = c_\mu^{(+)} \exp(-i\psi_\mu x) = p_\mu^{(+)} & (4.19a) \\ q_\mu^{(-)} = -c_\mu^{(-)} \exp(i\psi_\mu x) = -p_\mu^{(-)} & (4.19b) \end{cases}$$

where c_μ is the constant of the amplitude coefficients. There exist two possible solutions for each of the amplitude coefficient, p_μ and q_μ . The first solution with a plus superscript (+) indicate a wave traveling in the positive x direction and the second solution with a

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minus superscript (-) indicates a backward traveling wave. From Eqs. (4.19) and (4.20), the first set of solutions for the amplitude coefficients, q_μ and p_μ are identical. However, the second set of solutions is differentiated by the negative sign. This explains why a different amplitude coefficient for the magnetic field expansion is used because the sign for backward traveling transverse magnetic field is reversed. On the other hand, the single amplitude coefficient is used to describe the transverse (z and y) field components for the electric field in both forward and backward direction. According to field theory, it is justified that each mode must behave as an entity [110] and all field components of each mode change at the same rate.

From Eqs. (4.18) and (4.19), the following field and phase coefficient relation can be deduced as

$$\beta_\mu^{(+)} = \beta_\mu \quad (4.20a)$$

$$\beta_\mu^{(-)} = -\beta_\mu \quad (4.20b)$$

$$\mathcal{E}_\mu^{(+)} = \mathcal{E}_\mu^{(-)} = \mathcal{E}_\mu \quad (4.20c)$$

$$\mathcal{H}_\mu^{(-)} = -\mathcal{H}_\mu^{(+)} = -\mathcal{H}_\mu \quad (4.20d)$$

The common amplitude coefficient for the forward and backward propagating waves are expressed as

$$p_\mu = p_\mu^{(+)} + p_\mu^{(-)} \quad (4.21a)$$

$$q_\mu = p_\mu^{(+)} - p_\mu^{(-)} \quad (4.21b)$$

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Substituting Eq. (4.21) into Eqs. (4.13) and (4.16), the coupling equations in terms of forward and backward traveling waves can be expressed as

$$\frac{\partial p_{\mu}^{(+)}}{\partial x} + i\psi_{\mu} p_{\mu}^{(+)} - \left[\frac{\partial p_{\mu}^{(-)}}{\partial x} - i\varphi_{\mu} p_{\mu}^{(-)} \right] = \sum_{\nu} R_{\nu\mu} (p_{\nu}^{(+)} - p_{\nu}^{(-)}) \quad (4.22a)$$

$$\frac{\partial p_{\mu}^{(+)}}{\partial x} + i\psi_{\mu} p_{\mu}^{(+)} + \left[\frac{\partial p_{\mu}^{(-)}}{\partial x} - i\varphi_{\mu} p_{\mu}^{(-)} \right] = \sum_{\nu} S_{\nu\mu} (p_{\nu}^{(+)} + p_{\nu}^{(-)}) \quad (4.22b)$$

Adding and subtracting Eqs. (4.22a) and (4.22b), the coupling equations for the forward and backward waves are expressed as

$$\frac{\partial p_{\mu}^{(+)}}{\partial x} = -i\psi_{\mu} p_{\mu}^{(+)} + \sum_{\nu} \mathcal{G}_{\nu\mu}^{(+,+)} p_{\nu}^{(+)} + \mathcal{G}_{\nu\mu}^{(+,-)} p_{\nu}^{(-)} \quad (4.23a)$$

$$\frac{\partial p_{\mu}^{(-)}}{\partial x} = i\psi_{\mu} p_{\mu}^{(-)} + \sum_{\nu} \mathcal{G}_{\nu\mu}^{(-,+)} p_{\nu}^{(+)} + \mathcal{G}_{\nu\mu}^{(-,-)} p_{\nu}^{(-)} \quad (4.23b)$$

where $\mathcal{G}_{\nu\mu}$ is the overall coupling coefficient between the modes ν and μ . The coupling coefficients $R_{\nu\mu}$ and $S_{\nu\mu}$ can be expressed as

$$\begin{aligned} \mathcal{G}_{\nu\mu}^{(\alpha,\gamma)} &= \alpha\gamma R_{\nu\mu} + S_{\nu\mu} \\ &= \left[\frac{-|\psi_{\mu}|}{2P\psi_{\mu}^{(\alpha)*}} \iint e_x \cdot \left(\mathcal{E}_{\mu}^{(\alpha)*} \times \frac{\partial \mathcal{K}_{\nu}^{(\gamma)}}{\partial x} \right) dydz \right] \\ &\quad - \frac{|\psi_{\mu}|}{2P\psi_{\mu}^{(\alpha)*}} \iint e_x \cdot \left(\frac{\partial \mathcal{E}_{\nu}^{(\gamma)}}{\partial x} \times \mathcal{K}_{\mu}^{(\alpha)*} \right) dydz \end{aligned} \quad (4.24)$$

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where the parameters α and γ are used to indicate the direction of the light waves in the waveguide. When it appears as a coefficient of $R_{\nu\mu}$, it has a value of +1 or -1. Otherwise, it is a positive or a negative sign, indicating the direction of coupling coefficient. There are four different possible combinations of α and γ , which account for different intermodal coupling in the geometrical variation waveguide. From Eqs. (4.20) and (4.21), it is noted that the amplitude coefficients of $p_{\mu}^{(+)}$ and $p_{\mu}^{(-)}$ are actually rapidly varying functions of z in the absence of coupling. Hence, the parameters $c_{\mu}^{(+)}$ and $c_{\mu}^{(-)}$ are constant in the absence of coupling. However, for the coupling between modes, $c_{\mu}^{(+)}$ and $c_{\mu}^{(-)}$ are functions of x and their derivatives are present. Substituting Eqs. (4.18) and (4.19) into Eq. (4.23), the general case of the coupling equations are obtained as

$$\frac{\partial c_{\mu}^{(+)}}{\partial x} = \sum_{\nu} g_{\nu\mu}^{(+,+)} c_{\nu}^{(+)} \exp(ix(\psi_{\mu} - \psi_{\nu})) + g_{\nu\mu}^{(+,-)} c_{\nu}^{(-)} \exp(ix(\psi_{\mu} + \psi_{\nu})) \quad (4.25a)$$

$$\frac{\partial c_{\mu}^{(-)}}{\partial x} = \sum_{\nu} g_{\nu\mu}^{(-,+)} c_{\nu}^{(+)} \exp(-ix(\psi_{\mu} + \psi_{\nu})) - g_{\nu\mu}^{(-,-)} c_{\nu}^{(-)} \exp(ix(\psi_{\nu} - \psi_{\mu})) \quad (4.25b)$$

It is important to highlight that Eq. (4.25) does not have integral terms at the exponential phase coefficient, which is different from the other coupling equations [110, 131] that have an integral term due to continuous evaluation of the phase coefficient. Therefore, the transmission efficiency of the light waves can be obtained by successively applying the coupling equations at the interfaces where the cross sectional width changes.

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4.2.4 Removing the curl

The coupling equations as shown in Eq. (4.24) are derived and used to model the various inter-modal coupling mechanisms of PCTW. However, the overall coupling coefficient, $\mathcal{G}_{\nu\mu}$ has curls terms. The curls terms make it difficult and inconvenient to evaluate the coupling equations. It would also be challenging to implement Eq. (4.24) on numerical software for calculation and simulation. Hence, it is necessary to find a transformation way to remove the curl terms for the overall coupling coefficient, $\mathcal{G}_{\nu\mu}$. In this section, the longitudinal field component is used to derive the coupling equations. From Eq. (4.3), the simplified field equations are expressed

$$\nabla_t \times \mathcal{K}_\mu - i\psi_\mu (e_x \times \mathcal{K}_\mu) = -i\omega \epsilon \mathcal{E}_\mu \quad (4.26a)$$

and
$$\nabla_t \times \mathcal{E}_\mu - i\psi_\mu (e_x \times \mathcal{E}_\mu) = i\omega \mu \mathcal{K}_\mu \quad (4.26b)$$

Eqs. (4.26a) and (4.26b) are expressed in local mode field form for individual mode. By summing up the all the field mode, a complete solution similar to the form in Eq. (4.24) can be obtained. Eqs. (4.26a) and (4.26b) are superscripted with α parameter to indicate the propagation direction which can be expressed as

$$\nabla_t \times \mathcal{K}_\mu^{(\alpha)*} - i\psi_\mu^{(\alpha)*} (e_x \times \mathcal{K}_\mu^{(\alpha)*}) = -i\omega \epsilon \mathcal{E}_\mu^{(\alpha)*} \quad (4.27a)$$

and
$$\nabla_t \times \mathcal{E}_\mu^{(\alpha)*} - i\psi_\mu^{(\alpha)*} (e_x \times \mathcal{E}_\mu^{(\alpha)*}) = i\omega \mu \mathcal{K}_\mu^{(\alpha)*} \quad (4.27b)$$

To obtain the same form as the right side of Eq. (4.24), the scalar product of Eq. (4.27a) with the derivative $-dE_v^{(\gamma)}/dz$ is performed. Similarly, the scalar product of Eq. (4.27b)

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with $dH_v^{(n)}/dz$ is performed. Then by applying triple product vector identities and adding up the two equations, it is obtained as

$$\begin{aligned}
 & -i\psi_\mu^{(\alpha)*} \left\{ e_x \cdot \left[\left(\frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \times \mathcal{H}_\mu^{(\alpha)*} \right) + \left(\frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \times \mathcal{E}_\mu^{(\alpha)*} \right) \right] \right\} \\
 & = i\omega \varepsilon \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \cdot \mathcal{E}_\mu^{(\alpha)*} + i\omega \mu \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \cdot \mathcal{H}_\mu^{(\alpha)*} \\
 & + \nabla_t \cdot \left[\left(\mathcal{H}_\mu^{(\alpha)*} \times \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \right) + \left(\frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \times \mathcal{E}_\mu^{(\alpha)*} \right) \right] \quad (4.28)
 \end{aligned}$$

It is noted that the left side of Eq. (4.28) and right hand side of Eq. (4.24) are similar. On the right hand side of Eq. (4.28), the del operator ∇_t , operates on both the field terms and its associated derivative so that divergence properties can be applied to remove the curl terms. By taking the derivative of Eqs. (4.27) with respect to x , the following is obtained

$$\begin{aligned}
 & \nabla_t \times \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} - i\psi_v^{(\gamma)} \left(e_x \times \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \right) - i(e_x \times \mathcal{H}_v^{(\gamma)}) \frac{\partial \psi_v^{(\gamma)}}{\partial x} \\
 & = -i\omega \varepsilon \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} - i\omega \mathcal{E}_v^{(\gamma)} \frac{\partial \varepsilon}{\partial x} \quad (4.29a)
 \end{aligned}$$

$$\begin{aligned}
 & \nabla_t \times \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} - i\psi_v^{(\gamma)} \left(e_x \times \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \right) - i(e_x \times \mathcal{E}_v^{(\gamma)}) \frac{\partial \psi_v^{(\gamma)}}{\partial x} \\
 & = i\omega \mu \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \quad (4.29b)
 \end{aligned}$$

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From Eq. (4.29a), it is shown that the dielectric constant, ε is a function of x . The periodic boundaries of PC structures are satisfied by the ε derivative function, which accounts for the dielectric discontinuities in the periodic PC structures. The scalar product of Eq. (4.29a) with the $E_\mu^{(\alpha)*}$ and Eq. (4.29b) with $-H_\mu^{(\alpha)*}$ are taken to express in the exact form as Eq. (4.24). Summing the equations together, it is obtained as

$$\begin{aligned}
 & i\psi_v^{(\gamma)} \left\{ e_x \cdot \left[\left(\frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \times \mathcal{H}_\mu^{(\alpha)*} \right) + \left(\mathcal{E}_\mu^{(\alpha)*} \times \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \right) \right] \right\} \\
 & = -i\omega \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \cdot \mathcal{E}_\mu^{(\alpha)*} - i\omega \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \cdot \mathcal{H}_\mu^{(\alpha)*} - i\omega \mathcal{E}_v^{(\gamma)} \cdot \mathcal{E}_\mu^{(\alpha)*} \frac{\partial \varepsilon}{\partial x} \\
 & + \nabla_x \cdot \left[\left(\mathcal{E}_\mu^{(\alpha)*} \times \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \right) + \left(\frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \times \mathcal{H}_\mu^{(\alpha)*} \right) \right] \\
 & + i \frac{\partial \psi_v^{(\gamma)}}{\partial x} e_x \cdot \left[\left(\mathcal{H}_\mu^{(\alpha)*} \times \mathcal{E}_v^{(\gamma)} \right) + \left(\mathcal{H}_v^{(\gamma)} \times \mathcal{E}_\mu^{(\alpha)*} \right) \right] \quad (4.30)
 \end{aligned}$$

It can be seen that the left hand side of Eq. (4.30) has the same form as that of the right hand side of Eq. (4.24). Adding Eq. (4.28) and (4.30) and taking double integral over the y - z plane gives

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$$\begin{aligned}
& \iint i(\psi_v^{(\gamma)} - \psi_\mu^{(\alpha)*}) e_x \cdot \left[\left(\frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \times \mathcal{H}_\mu^{(\alpha)*} \right) + \left(\mathcal{E}_\mu^{(\alpha)*} \times \frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \right) \right] dydz \\
&= \iint \nabla_t \cdot \left[\left(\mathcal{H}_\mu^{(\alpha)*} \times \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \right) + \left(\frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \times \mathcal{E}_\mu^{(\alpha)*} \right) \right] dydz \\
&+ \iint i \frac{\partial \psi_v^{(\gamma)}}{\partial x} e_x \cdot \left[\left(\mathcal{H}_\mu^{(\alpha)*} \times \mathcal{E}_v^{(\gamma)} \right) + \left(\mathcal{H}_v^{(\gamma)} \times \mathcal{E}_\mu^{(\alpha)*} \right) \right] dydz \\
&- \iint i \omega \mathcal{E}_v^{(\gamma)} \cdot \mathcal{E}_\mu^{(\alpha)*} \frac{\partial \varepsilon}{\partial x} dydz \tag{4.31}
\end{aligned}$$

The left hand side of Eq. (4.31) is identical to the overall coupling coefficient $\mathcal{G}$ multiply by a factor of the phase difference between modes v and μ . Out of the three terms on the right hand side of Eq. (4.31), two of them vanish when orthogonal relation and divergence theorem are applied.

The first term is given as,

$$\iint \nabla_t \cdot \left[\left(\mathcal{H}_\mu^{(\alpha)*} \times \frac{\partial \mathcal{E}_v^{(\gamma)}}{\partial x} \right) + \left(\frac{\partial \mathcal{H}_v^{(\gamma)}}{\partial x} \times \mathcal{E}_\mu^{(\alpha)*} \right) \right] dydz \tag{4.32a}$$

Based on Strokes' theorem, this divergence term is converted from surface integral to line integral along the waveguide. The line integral around the waveguide vanishes for all the guided modes. It also vanishes for all the radiation modes as well as the guided-radiation modes for $v \neq \mu$. Hence this term does not affect the coupling coefficient.

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The second term is given as

$$\iint i \frac{\partial \psi_v^{(\gamma)}}{\partial x} e_x \cdot \left[\left(\mathcal{H}_\mu^{(\alpha)*} \times \mathcal{E}_v^{(\gamma)} \right) + \left(\mathcal{H}_v^{(\gamma)} \times \mathcal{E}_\mu^{(\alpha)*} \right) \right] dydz \quad (4.32b)$$

It is obvious that this term does not affect the coupling coefficient because it vanishes when the orthogonal relation shown in Eq. (4.11) is applied for the guided modes and the radiation modes when $v \neq \mu$.

The third term is given as

$$\iint i \omega \mathcal{E}_v^{(\gamma)} \cdot \mathcal{E}_\mu^{(\alpha)*} \frac{\partial \varepsilon}{\partial x} dydz \quad (4.33)$$

This term represents the scalar product of the field between modes v and μ . The presence of the curl terms in $\mathcal{G}_{v\mu}$, as shown in Eq. (4.24) is removed so that the coupling equations can be expressed in actual form for computational purposes. The derivative of the dielectric constant, $\partial \varepsilon / \partial x$ accounts for the periodic dielectric discontinuities of the PC lattice. The absence of the derivative for the magnetic permeability, $\partial \mu / \partial x$ is because of the nonmagnetic properties of waveguiding material. Therefore, the overall coupling coefficient is given by

$$\mathcal{G}_{v\mu}^{(\alpha\gamma)} = \frac{\omega \alpha |\psi_\mu^{(\alpha)*}|}{2 P \psi_\mu^{(\alpha)*} (\psi_\mu^{(\alpha)*} - \psi_v^{(\gamma)})} \iint \mathcal{E}_v^{(\gamma)} \cdot \mathcal{E}_\mu^{(\alpha)*} \frac{\partial \varepsilon}{\partial x} dydz \quad (4.34)$$

When the superscripts of α and γ are of different signs, there is coupling between a mode traveling in forward direction and a reflected mode traveling in the backward direction. For the gradual geometrical variation waveguide, small phase difference, $\psi_\mu - \psi_v$ gives a higher overall coupling coefficient as deduced from Eq. (4.34). This indicates strong coupling and high power are transferred from mode v to mode μ .

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With the derivation of the equation of the coupling coefficient, the next part section will discuss the backward propagation waves with the exact step-coupling-theory.

4.2.5 Backward propagation

In the whole derivation process, the backward traveling wave is not specifically discussed. Backward traveling waves are expected to be found in any general wave problem. However, the backward propagating waves are weak for the gradual geometrical variation waveguides. The amplitude of the backward propagation waves can be considered to be zero. However, this is not true for the derivative of the backward propagating waves. To investigate this issue, the coupling equation in Eq. (4.25) is separated into the forward and backward components are given as

$$\frac{\partial c_{\mu}^{(+)}}{\partial x} \exp(-i\psi_{\mu}x) - \frac{\partial c_{\mu}^{(-)}}{\partial x} \exp(i\psi_{\mu}x) = \sum_{\nu} R_{\nu\mu} (c_{\nu}^{(+)} \exp(-i\psi_{\nu}x) - c_{\nu}^{(-)} \exp(i\psi_{\nu}x)) \quad (4.35a)$$

$$\frac{\partial c_{\mu}^{(+)}}{\partial x} \exp(-i\psi_{\mu}x) + \frac{\partial c_{\mu}^{(-)}}{\partial x} \exp(i\psi_{\mu}x) = \sum_{\nu} S_{\nu\mu} (c_{\nu}^{(+)} \exp(-i\psi_{\nu}x) + c_{\nu}^{(-)} \exp(i\psi_{\nu}x)) \quad (4.35b)$$

where the amplitude coefficient $c_{\mu}^{(+)}$ and $c_{\mu}^{(-)}$ is the forward and backward propagation waves, respectively. For the case of the gradual geometrical variation waveguide, all the coupling power is concentrated in the forward direction while the back reflected power is very small. Hence, the backward amplitude coefficient is approximately zero, $c_{\mu}^{(-)} \approx 0$. However, contradiction arises when this condition is substituted into in Eq. (4.35). This is because the derivative of $c_{\mu}^{(-)}$ is considered to be zero. Substituting $c_{\mu}^{(-)} \approx 0$ and $\partial c_{\mu}^{(-)} / \partial z \approx 0$ into Eq. (4.35), the equation is obtained as

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$$R_{v\mu} = S_{v\mu} \quad (4.36)$$

The above equation shows that the coupling coefficients, $R_{v\mu}$ and $S_{v\mu}$ are equal. If Eq. (4.36) is true, then either the coupling equations in Eqs. (4.35a) or (4.35b) is redundant. This is because $\partial c_{\mu}^{(-)} / \partial z$ is also assumed to be zero. Even though the amplitude coefficient $c_{\mu}^{(-)}$ is small and negligible, it is not true to directly conclude that its derivative is also small. To further support the above statement, $S_{v\mu}$ is set to be zero in Eq. (4.35b). The backward amplitude coefficient, $c_{\mu}^{(-)}$ can be expressed as

$$c_{\mu}^{(-)}(x) = - \int_{x_0}^{x_1} \frac{\partial c_{\mu}^{(+)}}{\partial x} \exp(-2i\psi_{\mu} x) dx \quad (4.37)$$

From Eq. (4.37), the amplitude coefficient of the backward traveling waves $c_{\mu}^{(-)}$, is dependent on the derivatives of the forward propagating amplitude coefficient, $\partial c_{\mu}^{(+)} / \partial z$ and the exponential factor, $\exp(-2i\psi_{\mu} z)$. For the gradual slope tapered waveguide, the exponential factor, $\exp(-2i\psi_{\mu} z)$ cancels the backward waves by rapid oscillations. Therefore, the integral clearly shows that the amplitude coefficient of the backward traveling waves can be neglected. However, it is not sufficient to support the assumption that the derivative of $c_{\mu}^{(-)}$ is also very small. This can be explained by the following argument. At a particular point x along the tapered waveguide, light waves are coupled to the forward and backward directions. The power of the backward propagating waves along the tapered waveguide (which is $\partial c_{\mu}^{(-)} / \partial z$) will either build up or remain insignificant, depending on the phase relationship along the tapered waveguide. It adds up if all points along the waveguide are in phase with each other. Otherwise, it remains

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small and negligible. Hence, even though the amplitude coefficient $c_{\mu}^{(-)}$ is very low for gradual curvature waveguide, the same implication may not be apply to the derivative.

In the next section, the numerical model for the exact step-coupling-theory will be discussed.

4.2.6 Simplifications of the exact step-coupling-theory

The exact step-coupling-theory is simplified from two prospectives to reduce computational time: First, only modes with even field distributions are considered in the coupling process. This is because the geometrical waveguide that is considered is symmetrical at the z axis. Hence, the coupling between transverse modes with even field distributions is possible. Additionally, it is also considered that the coupling between the guided modes with different symmetries is zero [129]. This is particularly useful for large width waveguides, where there is a large number of possible intermodal couplings.

Second, only the coupling between the lowest order fundamental mode and intermediate higher order modes (up to the fifth order even mode) are considered. This is because the possibility of the coupling between the lowest fundamental even mode and the much higher order modes (usually from fifth order even mode and above) is virtually absence. This simplification further reduces the simulation time when only the coupling to the first and second even modes of the waveguide is considered. This simplification is very effective for long PC tapered waveguide (i.e. for the taper length greater then $28a$).

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4.3 Numerical results and discussions

In this section, the exact step-coupling-theory is verified using a numerical case study. The analysis of the mode coupling for different propagation directions are discussed so that the mode interaction mechanisms in the waveguide structure with geometrical variation can be understood. Subsequently, the transmission spectra for different nonuniform PC waveguide structures, especially the tapered waveguides are compared with the modified step-theory.

4.3.1 Numerical case study

For the verification of the exact step-coupling-theory, a numerical case study is performed. Fig. 4.2(a) shows the structure of a row defect PCTW structure [129, 131], consisting of a linear PCTW with a row of defects in the middle of the tapering section. The cladding rods have a r/a ratio of 0.3 with refractive index of 3.37 and the defect rods have r/a ratio of 0.2. The structure is known as a twin input/output PCTW because the waveguide decreases in width and then opens up in the opposite side. A row of defects is inserted into the tapering section to reduce the group velocity of propagating light waves and increase the coupling efficiency. The linear PCTW is divided into steps using similar method as in the previous chapter. Then, the β values at each step are obtained. For this twin input/output PCTW structure, the simulation process is less demanding due to shorter tapering section. Besides, the calculation of the uniform PCWG section in the middle can be ignored because transmission loss is assumed to be negligible. The transmitted power is taken from the open end of the twin head PCTW design. The

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simulation results of the transmission spectrum of the twin PCTW design using the exact step-coupling-theory is shown in Fig. 4.2(b).

From the transmission spectrum, it is shown that the power transmission using the exact step-coupling-theory is in very good agreement with the results using the scattering method [131]. The transmission fluctuations at the band edge (< 0.23 and > 0.257) is due to slow speed propagation from the presence of Fabry-Perot oscillation in the tapered waveguide section. The average transmission efficiency is more than 90 %. Compared to the modified step-theory, the simulation results from exact step-coupling-theory matches better with the scattering method. As observed from Fig. 4.2(b), the difference between exact step-coupling-theory and scattering method even near the edge of the spectrum where there are fluctuations is smaller than that of modified step-theory and scattering method.

To illustrate the difference numerically, the percentage error of the simulation results for the modified step-theory (from previous chapter in section 3.2.4) and exact step-coupling-theory with respect to the scattering method is plotted in Fig. 4.2 (c). It is observed that the computation error for the modified step-theory is higher than the exact step-coupling-theory, especially nearer to the band edge. The accuracy of the simulation results is improved by approximately 5 % using the exact step-coupling-theory. The exact step-coupling-theory shows very high accuracy with error less than $\pm 1\%$ through out the whole spectrum. Only a small percentage of error is found at the band edge on the left. However, the simulation results obtained by using the modified step-theory have higher percentage error and can be found over the entire transmission spectrum.

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(a)

(b)

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(c)

Fig. 4.2 (a) Structure of the twin input/output linear defect PCTW. (b) Transmission spectrum of the twin input/output linear defect PCTW with the scattering method [131] and the exact step-coupling-theory. (c) Percentage error for the transmission spectrum.

After discussing the issue for the backward propagation waves, the verification of the exact step-coupling-theory using the PC twin input/output linear defect PCTW is studied. The exact step-coupling-theory has better convergence than the modified step-theory especially for short taper length.

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4.3.2 Lossy PC waveguide with varying radii

The transmission spectrum of a nonadiabatic and lossy PCTW will be discussed using the exact step-coupling-theory. The structure of the PCTW is shown in Fig. 4.3 (a). The radius of the rods of the cladding of the PCTW is gradually increased to form a transition structure, which is called the “turnon” structure. The unperturbed rods have a r/a ratio of 0.3 with the refractive index of 3.37. The line defects rods have r/a ratio of 0.2. The transmission coupling efficiency of the “turnon” PCTW is obtained from the middle of the structure using the exact step-coupling theory. This “turnon” PCTW structure has very low coupling efficiency in the transmission spectrum shown in Fig. 4.3 (b). The low transmission coupling efficiency is due to the absence of the guided modes in the waveguide section.

One guided mode in every intermediate section of the “turnon” structure is necessary to design an effective PCTW. For this “turnon” PCTW design, this condition is not met and a PBG is found in one of the section. When light waves travels to the section that has a bandgap, it is reflected. After reflection, it radiates to the waveguide cladding. The radiation of the waveguide cladding is due to the absence of PBG in the “turnon” waveguide. This gives rise to the low power transmission across the whole spectrum as shown in Fig. 4.3(b).

In this chapter, the exact step-coupling-theory is applied to analyze the low power transmission as shown in Fig. 4.3 (b). In this case, the radiation modes are included in the exact step-coupling-theory to avoid any miscalculation. From Eq. (4.8), the series expansion of the electromagnetic field is re-expressed as

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$$E_{vt} = \sum_v p_{vt} \mathcal{E}_{vt} \exp(-i\psi_v z) + \sum_\rho \int_0^\infty p_\rho \mathcal{E}_{\rho t} \exp(-i\psi_\rho z) d\rho \quad (4.38a)$$

$$H_{vt} = \sum_v q_{vt} \mathcal{H}_{vt} \exp(-i\psi_v z) + \sum_\rho \int_0^\infty q_\rho \mathcal{H}_{\rho t} \exp(-i\psi_\rho z) d\rho \quad (4.38b)$$

Eqs. (4.38a) and (4.38b) include the radiation modes with the full series expansion of the electromagnetic field, where the field expansion was simplified for the real guided mode in the tapered waveguide section previously. Therefore, the radiation modes expression is hidden and only considered for the coupling efficiency calculation at the band edge. From Eqs. (4.38a) and (4.38b), the power flowing through the “turnon” PCTW is given as

$$P_t = P \left[\sum_v \left(|c_v^{(+)}|^2 - |c_v^{(-)}|^2 \right) + \sum_\rho \int_0^\infty \left(|c_\rho^{(+)}|^2 - |c_\rho^{(-)}|^2 \right) d\rho \right] \quad (4.39)$$

where $c_v^{(-)}$ and $c_v^{(+)}$ are the amplitude coefficients of the guided modes, and $c_\rho^{(-)}$ and $c_\rho^{(+)}$ are the amplitude coefficients of the radiation modes. At the reflection section (the section which contains a PBG), the coupling amplitude of the backward propagating light waves is much large than the forward amplitude: $c_v^{(-)} \gg c_v^{(+)}$. The integral extends over the whole propagating radiation modes. Since most light is reflected, it is expected that the radiation loss is in the backward direction. Since the amplitude $c_v^{(-)}$ and $c_\rho^{(-)}$ are larger than $c_v^{(+)}$ and $c_\rho^{(+)}$, the power P_t is negative.

Figure 4.3(c) shows the field distribution of the back reflected modes radiating out in the backward direction from the reflection section. It shows the coupling of the reflected field to the radiation field in the “turnon” section.

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(a)

(b)

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(c)

Fig. 4.3 (a) Schematic of the “turnon” PCTW structure. (b) Transmission spectrum of the “turnon” PCTW structure with the scattering method [131] and the exact step-coupling-theory. (c) Field distribution of the coupling between the reflected light and radiation modes in the cladding.

It is clearly shown that the radiated electromagnetic field is present in the cladding of the “turnon” PCTW. Therefore, the reflected field coupled to the bulk radiated field in the crystal and known as radiation loss.

4.3.3 Convergence effectiveness

In the previous part, the coupling equations of the exact step-coupling-theory are used to calculate the transmission spectrum of the lossy “turnon” PCTW. The field series expansion is manipulated to accommodate the radiation modes so that it can be used to account for the radiation loss in the lossy “turnon” PCTW. In this section, the convergence of the exact step-coupling-theory is studied. The linear and nonuniform PCTW will be used to check for the convergence of the exact step-coupling-theory with

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the scattering method. The convergence of the exact step-coupling-theory will also be compared to the modified step-theory to highlight its effectiveness.

The structure of the PCTW is identical to the previous chapter. Based on the coupling coefficient of Eq. (4.34), the transmission spectrum of the linear taper curvature design is calculated as shown in Fig. 4.4(a). The simulation results are calculated by the scattering method, the modified step-theory and the exact step-coupling-theory. The transmission efficiency is obtained for positive values of α and γ parameters, which implied forward coupling at frequency of $0.368 (a/\lambda)$. From Fig. 4.4(a), it is noted that both the modified step-theory and the exact step-coupling-theory matches well at longer taper length. The percentage error of both methods with respect to the scattering method is shown in Fig. 4.4(b). The percentage error using modified step-theory has much larger error than the error using the exact step-coupling-theory. The error using modified step-theory is 15 % while the error using the exact step-coupling-theory is only 3 % at short taper length.

(a)

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(b)

Fig. 4.4 (a) Normalized transmission vs taper length for linear PCTW. (b) Percentage error vs taper length for linear PCTW.

For the short tapered waveguide, some inter-modal coupling and backward reflection of the light waves in the short tapering section is observed. The backward power coupling among the modes cannot be neglected. This power adds up coherently along the backward direction and also affects the simulation results of the transmission coupling efficiency.

Figures 4.5(a) and 4.6(a) shows the coupling efficiency for the concave and convex PCTW, respectively with σ values of 2 and 0.5. For both cases, the modified step-theory has poorer convergence at the shorter taper length. According to the percentage error for both the PCTW designs as shown in Figs. 4.5(b) and 4.6(b), it is observed that the modified step-theory has relatively large computation percentage error.

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The exact step-coupling-theory provides better convergence and more accurate simulation results with average deviation of $\pm 1.5\%$.

In Figs. 4.4(a), 4.5(a) and 4.6(a), some fluctuations in the coupling efficiency that only exist at the region of shorter taper. This is because of the multiple reflections in the tapering section where the back reflected, and the forward traveling waves mix and interfere each other. As a result, it forms resonance standing waves oscillating in the tapered waveguide section of the PCTW.

The degree of the fluctuations in Figs. 4.4(b), 4.5(b) and 4.6(b) also indicate which PCTW curvatures designs achieved adiabatic condition at the shorter taper length. Based on the adiabatic theorem, the maximum transmission is achieved when the light waves are coupled to the lowest order fundamental mode [129] through the whole tapering region. Therefore, it is important to achieve adiabatic behavior in the tapered waveguide as it optimized the coupling efficiency tremendously.

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(a)

(b)

Fig. 4.5 (a) Normalized transmission vs taper length for concave PCTW. (b) Percentage error vs taper length for concave PCTW.

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(a)

(b)

Fig. 4.6 (a) Normalized transmission vs taper length for convex PCTW. (b) Percentage error vs taper length for convex PCTW.

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From Figs. 4.4(b), 4.5(b) and 4.6(b), it is noted that the concave PCTW has larger fluctuations than the convex and linear curvatures. This is due to the inward curvature of the concave PCTW, which changes fast at the front part of the tapered waveguide. The fast change in the slope of the concave PCTW increases the amount of inter-modal coupling in the tapering section. Therefore, the fluctuations last for a longer taper length (up to $38a$) as compared to the convex and linear PCTW. For the linear PCTW, the fluctuations vanish at a taper length of $34a$. For the convex PCTW, the fluctuations vanish at a shorter length since its fluctuations stop at the shorter taper length of $30a$ and the adiabatic condition is nearly achieved [101, 129, 131].

4.3.4 Coupling coefficients for positive α and γ

In this section, the physics of the mode coupling for the different PCTW curvature designs that affect the coupling efficiency is discussed. The parameters α and γ can appear as positive or negative in the superscripts or as ± 1 numeric factor in the coupling equations. There are a total of four possible combinations for α and γ parameters. Each of these combinations represents different mode coupling mechanism along the tapering section.

First, consider the case where α and γ is positive. For positive α and γ , the coupling between the ν and μ modes occurs in the forward direction. The phase coefficient, ψ for the modes are obtained at normalized frequency of 0.368, which is around the mid-gap of the PBG. Therefore, the phase coefficients, ψ for the ν and μ modes are real and the conjugate sign can be ignored. The coupling coefficient is simplified as

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$$g_{\nu\mu}^{(+,+)} = \frac{\omega(+1)}{2P(\psi_{\mu}^{(+)} - \psi_{\nu}^{(+)})} \iint \mathcal{E}_{\nu}^{(+)} \cdot \mathcal{E}_{\mu}^{(+)} \frac{\partial \varepsilon}{\partial x} dydz \quad (4.40)$$

For the case of forward coupling, the sign for the phase coefficient and electric field E as shown in Eq. (4.21) are positive. The PCTW is divided into steps where each steps has a length of a . This means that the derivative of the dielectric constant is treated as a constant. Therefore, the coupling coefficient is dependent on the difference of ψ values at each steps between ν and μ modes. It can be deduced for a gradual PCTW curvature design that the phase difference, $\Delta\psi$ for the lowest order fundamental mode, $\mu = \nu$ is very small which results in a very large $g_{\nu\mu}$. Therefore, high transmission efficiency is achieved in the gradual tapered waveguide. The coupling coefficient is also dependent on the electric fields amplitudes of the μ and ν modes. When the amplitude of one mode field is small or the fields are propagating at polarized right angle, the mode coupling is weak and the coefficient $g_{\nu\mu}$ is small.

For the multi-mode propagation in the PCTW, the ψ of the guided modes in the taper increases when the taper gets narrower. The velocity of the guided mode also decreases when the taper get narrower. Hence, a longer interaction time between various waveguide modes is present to reduce the transmission efficiency. The coupling coefficients at each interface of the different taper positions for the linear curvature PCTW at taper length of $17a$ and $32a$ are shown in Figs. 4.7(a) and (b). The legend of Figs. 4.7(a) and 4.7(b) shows the coupling between various guided modes. The bracket $[1, 1]$ refers to the coupling of the lowest order even mode in one step to the lowest order mode in another step, while $[1, 3]$ refers to the coupling to the coupling of the lowest

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order even mode in one step to the third order even mode in another step. From Figs. 4.7(a) and (b), the coupling amplitudes of the $[1, 1]$ mode coupling decrease gradually toward the end of the linear PCTW. For the linear PCTW with the tapered length of $17a$, the coupling amplitude drops even more sharply, especially after the middle of the tapering section. This is because the coupling of the lowest order mode power to the higher order modes is more significant. For the $[1, 3]$ and $[1, 5]$ mode coupling shown in Figs. 4.7(a) and (b), the coupling amplitude increases after the middle of the tapering section.

Besides coupling of the lower order mode to the higher order mode, reverse coupling of the higher order mode to the lower order mode is also possible. Fig. 4.7 shows this weak coupling process (for the case of $[3, 1]$). This is because the field amplitude of the higher order mode is generally smaller than the lower order mode. Therefore, the reverse coupling from the higher order modes to the lower order modes is more difficult. These higher order coupling is also terminated before the end of the taper due to the cut-off of the higher order modes at the narrow part of the taper.

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(a)

(b)

Fig. 4.7 (a) Coupling coefficient vs relative taper position for linear PCTW at length of $17a$ with positive α and γ . (b) Coupling coefficient vs relative taper position for linear PCTW at length of $32a$ with positive α and γ .

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For the concave curvature the PCTW, the taper slope decreases fast in the front part and then becomes gradual from the middle to the end of the tapered waveguide. Therefore, the coupling coefficient increases in the middle of the tapering section as shown in Figs. 4.8(a) and (b) for the $[1, 1]$ mode coupling. The coupling coefficient remains constant in the later part of the concave curvature PCTW. The coupling of the higher order modes decrease along the tapering section due to the gradual slope at the later part of the concave PCTW as shown in Fig 4.8 for the $[1, 3]$ and $[1, 5]$ mode coupling. The coupling amplitude of the lower-to-higher order modes and the higher-to-lower order modes is much dependent on the slope of the tapered waveguide.

For the convex curvature, the rate of change of the PCTW curvature is very important for the values of coupling amplitudes as shown in Figs. 4.9(a) and (b). The ψ values of the convex taper change slowly in most part of the convex PCTW. However, the slope changes abruptly at the joint between the convex PCTW and PCWG. The amplitudes of the $[1, 1]$ mode coupling drop sharply around the position of $z/l = 0.8$. After this, the curvature of the convex PCTW changes at a faster rate. Therefore, the lowest fundamental mode is coupled to the higher order modes as shown in Fig. 4.9 for the $[1, 3]$ and $[1, 5]$ mode coupling. When the convex taper is longer, the coupling of the higher modes is reduced.

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(a)

(b)

Fig. 4.8 (a) Coupling coefficient vs relative taper position for concave PCTW at length of $17a$ with positive α and γ . (b) Coupling coefficient vs relative taper position for concave PCTW at length of $32a$ with positive α and γ .

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(a)

(b)

Fig. 4.9 (a) Coupling coefficient vs relative taper position for convex PCTW at length of $17a$ with positive α and γ . (b) Coupling coefficient vs relative taper position for convex PCTW at length of $32a$ with positive α and γ .

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From Figs. 4.7, 4.8 and 4.9, it is observed that the change in the coupling amplitude of the [1, 1] is opposite to the coupling amplitude of the [1, 3] and [1, 5]. According to the conservation of energy and power, the coupling amplitudes between the different mode coupling mechanisms must satisfy the expression given by

$$\sum_v \sum_\mu |g_{v\mu}^{(\alpha,\gamma)}|^2 = 1 \quad (4.41)$$

For the gradual tapered waveguide slope, positive α and γ contributes to a larger fraction of the total energy, while negative α and γ and opposite signs of α and γ have smaller contribution. For the gradual tapered waveguide, Eq. (4.41) can be simplified to $\sum_{v=1} \sum_{\mu=1} |g_{v\mu}^{(+,+)}|^2 \approx 1$ where there is only forward coupling and all the higher orders modes and reflected modes can be neglected. However, the negative values of α and γ contribute to a larger fraction of the total energy due to higher reflection losses for the shorter tapered waveguide.

4.3.5 Coupling coefficients for negative α and γ

For negative α and γ , the coupling occurs in the backward direction (reflected modes) for the v and μ modes. The coupling coefficient is expressed as

$$g_{v\mu}^{(-,-)} = \frac{\omega}{2P(\psi_\mu - \psi_v)} \iint \mathcal{E}_v^{(-)} \cdot \mathcal{E}_\mu^{(-)} \frac{\partial \epsilon}{\partial x} dydz \quad (4.42)$$

where the notations are used to convert the sign of ψ and the electric field. Figs. 4.10(a) and (b) show the simulation results of the coupling coefficients of the linear tapered waveguide. For negative α and γ , the coupling occurs from the higher order modes to the

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lowest order fundamental mode. This is because the higher order modes are reflected rather than the lowest order fundamental mode.

From Figs. 4.10(a) and (b), it is observed that the coupling coefficient of the [1, 1] mode coupling is lower than the coupling coefficient of the [3, 1] and [5, 1] mode coupling for the linear PCTW. This is because the higher order modes are cut off when the linear PCTW becomes narrower and then propagates backward. The higher order modes would have higher power than the lower order mode in the backward direction. Hence, the coupling between the higher and the lowest order modes is more likely to occur than that between the two lowest order backward modes.

For the simulation results of the concave PCTW shown in Figs. 4.11(a) and (b), it is observed that the coupling coefficient increases to the middle of the tapered waveguide section and then decreases afterward. This is due to change in the taper slope from abrupt to gradual. The front part of the concave PCTW has an abrupt slope which leads to high reflection loss. The back part has a gradual curvature and the reflection loss is reduced significantly. The longer taper length reduces the abruptness of the concave tapered waveguide and this in turn reduces the coupling in the backward direction.

For the convex PCTW, the simulation results in Figs. 4.12(a) and (b) demonstrates that its coupling coefficient follows similar trend with the linear PCTW. However, the ψ values changes much slowly in the front part of the tapered waveguide, giving lower numerical results compared to the linear PCTW in Fig. 4.12(a) and (b). In the later part of the convex PCTW, the abrupt change in the taper slope has sharply increased the backward coupling due to higher amplitude of the backward traveling electromagnetic field.

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(a)

(b)

Fig. 4.10 (a) Coupling coefficient vs relative taper position for linear PCTW at length of $17a$ with negative α and γ . (b) Coupling coefficient vs relative taper position for linear PCTW at length of $32a$ with negative α and γ .

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(a)

(b)

Fig. 4.11 (a) Coupling coefficient vs relative taper position for concave PCTW at length of $17a$ with negative α and γ . (b) Coupling coefficient vs relative taper position for concave PCTW at length of $32a$ with negative α and γ .

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(a)

(b)

Fig. 4.12 (a) Coupling coefficient vs relative taper position for convex PCTW at length of $17a$ with negative α and γ . (b) Coupling coefficient vs relative taper position for convex PCTW at length of $32a$ with negative α and γ .

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In Figs. 4.10, 4.11 and 4.12, the coupling coefficient between the higher order modes, i.e. [3, 3] is always very close to zero. There are two reasons to explain this observation. First, the amplitudes of the higher order modes are often very low. Second, the ψ values of the higher order modes are often smaller than the fundamental modes. Therefore, the coupling between the two higher modes is small and can be neglected.

4.3.6 Coupling coefficients for opposite sign of α and γ

This combination is different from the two previous cases where the light waves are either pure forward propagating or backward reflection. The coupling coefficient for negative α and positive γ is expressed as

$$\mathcal{G}_{\nu\mu}^{(-,+)} = \frac{\omega}{2P(\psi_{\mu}^{(+)} + \psi_{\nu}^{(+)})} \iint \mathcal{E}_{\nu}^{(+)} \cdot \mathcal{E}_{\mu}^{(-)} \frac{\partial \varepsilon}{\partial x} dydz \quad (4.43)$$

while for positive α and negative γ , this is expressed as

$$\mathcal{G}_{\nu\mu}^{(+,-)} = \frac{\omega}{2P(\psi_{\mu}^{(+)} + \psi_{\nu}^{(+)})} \iint \mathcal{E}_{\nu}^{(-)} \cdot \mathcal{E}_{\mu}^{(+)} \frac{\partial \varepsilon}{\partial x} dydz \quad (4.44)$$

It can be seen that the fraction describing the phase coefficient between the ν and μ modes in Eqs. (4.43) and (4.44) are identical. This implies that the coupling coefficient is only affected by the scalar product of the electric fields in both the ν and μ modes. Figs. 4.13(a), (b) and (c) show the coupling coefficient at the taper length of $17a$ for the linear, the concave and the convex PCTW, respectively.

Since the phase coefficients add up for both the ν and μ modes in the denominator, the overall coupling coefficient, $\mathcal{G}_{\nu\mu}$ is quite small. Additionally, the

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backward traveling has lower field amplitude than the forward propagating waves. Hence, the $\mathcal{G}_{\nu\mu}$ values for the case where α and γ have the different signs are lower than the case where both α and γ have the same signs (either positive or negative) as shown in Figs. 4.13. This means that the coupling between the forward and the backward traveling waves is very weak and does not contribute much loss to the overall transmission efficiency. Therefore, for the case of the taper length of $32a$, the coupling coefficients, $\mathcal{G}_{\nu\mu}^{(+,-)}$ is even lower. For negative α and positive γ , the coupling coefficient is also expected to be very small, because of the same reasons as explained for the case where α is positive and γ is negative. According to Eqs. (4.43) and (4.44), it is expected that the coupling coefficients show similar trends for the various PCTW curvatures at each part of the tapering section.

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(a)

(b)

Chapter 4 Exact Step-Coupling-Theory for Geometrical Variation Waveguide

(c)

Fig. 4.13 (a) Coupling coefficient vs relative taper position for linear PCTW at length of $17a$ with positive α and negative γ . (b) Coupling coefficient vs relative taper position for concave PCTW at length of $17a$ with positive α and negative γ . (c) Coupling coefficient vs relative taper position for convex PCTW at length of $17a$ with positive α and negative γ .

4.3.7 Transmission spectra characteristic

The transmission spectra of the three PCTW curvature designs are studied using the scattering method, the modified step-theory and the exact step-coupling-theory in this section. The simulation results for the range of the normalized frequency between 0.32 and 0.39 using the three numerical methods are compared. The dispersion diagram of the single mode PCWG only exists for the normalized frequency between 0.32 and 0.392. For the exact step-coupling-theory, the transmission spectra are obtained by the coupling coefficients in the forward direction, i.e. α and γ are positive for all the modal interactions in the tapering section. The phase coefficient at each frequency for each guided mode is

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obtained from the dispersion band diagram at each step. The final power transmission is plotted as shown in Fig. 4.14. Fig. 4.14(a) and (b) shows the transmission spectra for the linear PCTW at the taper length of $17a$ and $32a$, respectively.

In Fig. 4.14(a), the simulation results of the modified step-theory shows a relatively large deviation with respect to the scattering method. The exact step-coupling-theory has smaller differences and is able to follow the fluctuations closely in the spectrum for nearly all points. For the longer taper length, it can be seen that both theories show accurate transmission spectra as shown in Fig. 4.14(b). This is particularly true for the exact step-coupling-theory. An approximately 15 % and 6 % accuracy improvement have been made by using the exact step-coupling-theory for the linear PCTW at taper length of $17a$ and $32a$ respectively when compared with the modified step-theory.

One important point to highlight is the convergence of the simulation results calculated using both theories near the band edge. In previous case studies, the simulation frequency is chosen to be in the middle of the PBG, where the possibility of coupling between the guided modes and the radiative modes at the band edge is very small. Therefore, it is difficult to disqualify some of the assumptions made in the modified step-theory for the short taper length. However, Figs. 4.14(a) and (b) it is show that the exact step-coupling-theory agrees very well with the scattering method even near the edge of the PBG because the exact step-coupling-theory includes all the possible interactions between the different modes coupling (including the radiation modes). At the PBG edge, there is a higher possibility of coupling to the radiation mode due to closer contact with the bulk continuum of modes. On the other hand, the modified step-theory neglects the

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coupling between the radiation and the guided modes. This assumption is invalid near the edge of the PBG because the guided mode can interact with the bulk continuum modes at low velocity. As a result, this increases the possibility of coupling between the guided and the radiation modes. Hence, this provides an explanation for the disagreement in simulation results at the edge of the band.

Figs. 4.15 and 4.16 show the transmission spectra for the case of the concave and convex PCTW, respectively. The coupling efficiency at the edge of the PBG is less accurate using the modified step-theory for the convex and concave tapers because the coupling between the guided and the radiation modes is not considered. The simulation results of the exact step-coupling-theory show better convergence and higher accuracy compared to the simulation results using the modified step-theory. The accuracy of the simulation results using the exact step-coupling-theory improve by approximately 17 % and 7 % for the concave PCTW at taper length of $17a$ and $32a$ respectively. The accuracy of the simulation results improve by approximately 12 % and 6 % for the convex PCTW at taper length of $17a$ and $32a$ respectively.

It has been proven that the convergence of the exact step-coupling-theory is better than the modified step-theory. The different combinations of the α and γ parameters are investigated and analyzed to gain more insight into the coupling mechanism in the tapered waveguide section. Subsequently, the transmission spectra of the different taper curvature designs show that the light waves coupling between the radiation and the guided modes near edge of the PBG is important for PCTW designs especially for short tapers. The PCTW slab structure using the exact step-coupling-theory will also be discussed in this chapter.

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(a)

(b)

Fig. 4.14 Transmission spectrum of linear PCTW for (a) length of $17a$. (b) length of $32a$.

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(a)

(b)

Fig. 4.15 Transmission spectra of concave PCTW for (a) length of $17a$. (b) length of $32a$.

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(a)

(b)

Fig. 4.16 Transmission spectra of convex PCTW for (a) length of $17a$. (b) length of $32a$.

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4.4 Three dimensional (3D) exact step-coupling-theory

The exact step-coupling-theory is extended to model the transmission coupling efficiency in the PC slab waveguide structure with geometrical variation. Formerly, all mathematical model and simulation results are derived based on the 2D PC structures. In this section, a PC slab waveguide structure with geometrical variations will be investigated.

4.4.1 3D PC slab waveguide

For practical devices and integrated circuits, periodic 3D PC structures are considered and 3D PBG is required [53]. However, the fabrication of the 3D structures has been a challenge; it is difficult for complex 3D inter-connectivity and strict alignment requirements [136-138]. However to obtain full confinement in all three directions, a semi-3D PC slab [139, 140] is proposed to promise fabrication technology. The PC slab structure is designed as the dielectric structure that combines homogeneous and periodic structures to guide light waves along the waveguide. The in-plane PC slab structure consists of a regular planar PC holes lattice, which guides light waves through the defect modes in the PC. In the vertical direction, the light waves are guided by index confinement (or total internal reflection). Therefore, the PC slab provides the 3D guiding mechanisms on chip.

In this chapter, the model of the PC slab consists of a high refractive index material. The slab thickness is set at $0.3\ \mu\text{m}$ approximately so that the propagating field remains as a single mode and strongly guided in the vertical direction [139, 140]. The lattice structures are made up of air-holes arranged in a triangular array. The radius to

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lattice constant ratio is set to $r/a = 0.3$. For these lattice parameters, a PBG exists in the range of the normalized frequencies 0.22 - 0.275 for the TE polarization. The linear PC slab tapered waveguide (PCST) with taper length of $24a$ is shown in Fig. 4.17. The input width of the PCST is $3\text{ }\mu\text{m}$, which corresponds to the mode size of a tapered optical fiber. The output width is the PCWG with one missing rows of holes structures (W1). Figs. 4.18(a) and (b) show mode distribution in the vertical direction and the percentage of the power coupled into the width of a $0.3\text{ }\mu\text{m}$ -thick slab. 20% of the total power is coupled into the thin slab waveguide. The remaining power is reflected backward to the source or radiates out of the slab.

Fig. 4.17 3D PC slab tapered waveguide.

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(a)

(b)

Fig. 4.18 (a) Vertical mode profile of electromagnetic field in PC slab (back view). (b) Vertical mode profile of electromagnetic field in PC slab (side view).

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The modeling for the PCST using the exact step-coupling-theory is different compared to the 2D PCTW in two aspects. First, PCST has different band diagram compared to the 2D rods structure due to the presence of lightcone. The presence of lightcone is due to the presence of vertical guiding mechanism in the PC slab which regions suitable for index confinement in the dispersion band diagram is shown. All the modes above the lightcone are bulk radiation modes, which are extended infinitely in the region above and below the slab. Therefore, the number of guided modes waveguiding in the PC slab is further reduced compared to the 2D structures. Fewer modes are considered and hence, reduces the computational time. Second, there are two different guiding mechanisms in the PC slab for the electric field. To account for the vertical confinement in the vertical direction, the field expression needs to be re-defined. However, since the thickness of the slab is chosen to support one single mode, the modeling and calculation process is simplified. The electromagnetic fields are redefined using separation of variable as

$$E_v = \left[\sum_v p_{vz} (\mathcal{E}_{vz}) \exp(-i\psi_{vz}x) \right] + p_{vy} \mathcal{E}_y \exp(-i\beta_y x) \quad (4.45a)$$

$$H_v = \left[\sum_v q_{vz} \mathcal{H}_{vz} \exp(-i\psi_{vz}x) \right] + q_{vy} \mathcal{H}_y \exp(-i\beta_y x) \quad (4.45b)$$

where the subscripts x and y refer to the electromagnetic field in the transverse direction. The electromagnetic field components in the x direction stand for the guiding mechanism by defects modes in the PBG while the fields in the y direction represent the index guiding mechanism in the PCST. Eq. (4.45) shows the expression where the summation in the x direction includes all the guided and the radiation modes in the PCST while in the y direction includes the radiation mode in air.

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Substituting Eq. (4.45) into Eq. (4.26), the new overall coupling coefficient is obtained as

$$\mathfrak{g}_{\nu\mu}^{(\alpha,\gamma)} = \frac{\omega\alpha |\psi_{\mu z}^{(\alpha)*} + \beta_y|}{2 P(\psi_{\mu z}^{(\alpha)*} + \beta_y)(\psi_{\mu z}^{(\alpha)*} - \psi_{\nu z}^{(\gamma)})} \iint \frac{\partial \varepsilon}{\partial x} [\mathcal{E}_{\nu z}^{(\gamma)} \cdot \mathcal{E}_{\mu z}^{(\alpha)*} + |\mathcal{E}_y|^2] dy dz \quad (4.46)$$

The overall coupling coefficient in Eq. (4.46) is slightly different from Eq. (4.34) due to the separation of variables into the x and y directions. However, the modes cannot be polarized in the direction of TM or TE polarized due to the lack of symmetry in the vertical direction. The guided mode can still be classified into even or odd modes with respect to the reflections through the horizontal plane of the PC slab. It is important to note that the even and odd modes fields are very similar to the TE and TM polarization. Therefore, the even modes are still referred as TE modes and the odd modes are referred as TM modes.

Fig. 4.19 Transmission spectrum of PCST for linear, concave and convex curvature designs at length of $18a$.

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For the coupling coefficients of the PCST, the simulations results of the transmission spectra for the taper length of $18a$ for the linear, the convex and the concave curvature designs are shown in Fig. 4.19. The transmission spectra are calculated for the case where α and μ are positive (in the forward direction). The coupling coefficients is used to calculate the transmission spectra and then normalized with the incident power coupled into the slab at the frequency range of 0.24 - 0.27. The convex PCST gives the highest overall coupling efficiency with its gradual curvature and average transmission efficiency of 87%. The concave PCST is the lowest due to its very fast tapering rate in the front part. The fluctuations in the spectrum are due to the multiple reflections from Fabry-Perot oscillation or multipath interference [101]. The concave PCST has more fluctuations due to the inward curvature. This results in much higher reflections in concave PCST. The high reflection causes interference in the incoming waves and as a result, there are fluctuations in the transmission spectrum. When the taper length increases, the fluctuations gradually reduce and stabilized the transmission spectrum.

Figure 4.20 shows the transmission efficiency of the three taper curvature designs at various lengths for the normalized frequency of 0.258. The maximum taper length is capped at $28a$. From Fig. 4.20, oscillations are observed at shorter length of the taper because of the large number of interferences and reflections at the shorter length of the PCST. The coupling efficiency of the convex PCST reaches 90 % at the taper length of $20a$ while the coupling efficiency of the linear PCST reaches 90 % at the taper length of $26a$. The maximum coupling efficiency of the concave PCST reaches 87 % at $28a$.

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Fig. 4.20 Transmission efficiency vs taper length of PCST waveguide for linear, concave and convex curvature designs.

For the 3D exact step-coupling-theory, there is one more point to highlight. The numerical results of the coupling efficiency of the 2D convex PCTW are higher than the convex PCST. This is because for 2D PC structures, the vertical direction is assumed to be infinitely extruded and the loss in the vertical direction is neglected. However, for PC slab structures, the vertical loss is taken into account during the modeling process while using the exact step-coupling-theory. There are some light waves radiating out of the PCST in the vertical direction and results in slight reduction in the coupling efficiency.

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4.5 Summary

In this chapter, the exact step-coupling-theory is derived to describe the coupling and intermodal interaction of the geometrical variation waveguide. The coupling equations of the exact step-coupling-theory are more complete and general to describe all of the modal interactions. The characteristics of the backward propagating waves are also discussed using the coupling equations. Two methods are applied to simplify the numerical calculation and shorten the computational time. The coupling equations are verified using known case study. The transmission simulation results agree well with those obtained from the scattering method with the percentage error lower than $\pm 1\%$. The transmission simulation results of the exact step-coupling-theory and the modified step-theory are compared with reference to the scattering method for the different PCTW curvature designs. It is concluded that the exact step-coupling-theory has improved accuracy up to 17 % and 7 % for PCTW at length of $17a$ and $32a$ when compared with the modified step-theory. This is due to the complete description of the coupling mechanism in the geometrical variation of the PCTW. Additionally, the coupling coefficients between the different combinations of α and γ are discussed. The coupling coefficients between the forward propagating modes are higher than the backward propagating modes and the coupling between the modes propagating in the different directions. In the last section, the exact step-coupling-theory is applied to the PCST and the coupling efficiency of the convex PCST is the highest, approximately 90 % for the taper length of $20a$ at frequency of 0.258. The concave and linear PCST have lower coupling efficiency at the same taper length.

CHAPTER 5

SURFACE WAVEGUIDE FOR LIGHT COUPLING AND POWER SPLITTING

The objective of this chapter is to study PC surface waveguide and photonic devices between the interface of finite lattice and air medium. The study of PC surface modes opens up new possibilities for feeding and redistribution of light waves. The surface modes at PC-air interface can be used for waveguiding purposes [123]. The transmission characteristics of the PC surface waveguide are investigated and then developed into photonic devices for applications in photonic integrated circuit.

In this chapter, the basic physics of the PC surface modes is studied through the dispersion band diagram to select the suitable ranges of frequency for waveguiding. Subsequently, the transmission efficiency for straight and bend PC surface waveguides (PCSWGs) are simulated and discussed using suitable wave vector matching model. Next, a circular waveguide is designed using circular photonic crystal (CPC) structure. Through the addition of other structures, the circular PCSWG is developed into an add-drop filtering photonic device with good add/drop properties. Lastly, another dual surface waveguide is also designed. The band properties of the dual surface waveguide are discussed. A power splitter using multipolar emission physics is developed and the splitting ratio is tuned by incorporating suitable modulating mechanism.

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5.1 Theory of photonic crystal surface mode

Majority of the research works on PC focus on the interior of lattice structure with infinite extent [53, 8]. However, there is a finite boundary in real crystal structures. This implies that surface mode may exist on the surface of PC. In this thesis, the surface mode located at the interface between the PC lattice and air medium is studied. Before studying the properties of the surface modes of the PC – air interface, it is necessary to review the surface states of electronic bandgap [113]. This is because many properties of the PC are based on the analogy of the electronic crystals structures.

For the modeling of the electronics bandgap surface states, one-dimensional (1D) crystal lattice structures are used. The case of a finite 1D atomic lattice in the z direction with the lattice period of a at the interface with the air medium is considered. Figure 5.1 shows the schematic layout at the interface of the electronics crystal lattice and the air medium. The electrons in the crystal are assumed as one-electron potential which is a periodic function of z . The air medium has a potential barrier of constant height V_0 .

Based on the Schrödinger equation, the wave function for the finite crystal and air medium is used in this case study. The adiabatic and free electron approximations are used to model the wave function of the crystal lattice. The adiabatic approximation [113] assumes that the states of an electronic system are the nuclei of atoms fixed at their own lattice positions. On the other hand, the free electron approximation is applied to treat the electrons at the surface with small perturbation. In this way, only the outermost rows of the surface electrons are affected while the rest remain unperturbed.

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Fig. 5.1 Schematic layout of the crystal lattice and air interface

Based on the Schrödinger equation, the wave functions of the finite crystal and the air medium is expressed as

$$\varphi_{crystal}(z) = \sum_n c_n \exp(i\tilde{k}z) \quad (5.1)$$

and

$$\varphi_{air}(z) = \gamma \exp\left[-\sqrt{\frac{2m_e}{h^2}(V_0 - E)}z\right] \quad (5.2)$$

where m_e and E is the rest mass and energy of the electrons, respectively. γ , and c_n is the constant for air medium wave function and finite crystal wave function, respectively. V_0 and h refers to the potential of the air medium and Planck constant, respectively. $\tilde{k}$ is the periodic wave vector in the finite crystal which is expressed as $\frac{2\pi n}{a} + k$, where k is the wave vector. $n = 1, 2, 3, \dots$, relates to the n th folding Brillouin zone in the crystal lattice.

At the interface of the crystal and the air medium, the continuity criterion is satisfied. This requires the matching of the wave function in Eq. (5.1) and its derivative with the wave function in Eq. (5.2). The equation energy eigenvalue is given as

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$$E = \frac{h^2}{2m_e} \left(k + \frac{\pi n}{a} \right)^2 \pm |V_n| \sqrt{\left[\frac{\pi h^2 n}{m_e a} \left(k + \frac{\pi n}{a} \right) \right]^2 + 1} \quad (5.3)$$

where V_n and a is the periodic potential of the n th folding Brillouin zone and the lattice constant of the crystal, respectively. From Eq. (5.3), the energy eigenvalues are divided into two different values for the finite crystal due to the periodic potential, V_n . Fig. 5.2 shows the energy band diagram with respect to the wave vector, k in the presence of V_n . Two band regions are formed as the allowed energy bands (AEBs) and the forbidden energy gap (FEG). The AEBs are the energy band at the n th Brillouin zones folds back into the first Brillouin zone with wave vector $\frac{2\pi n}{a} + k$. The FEG is an energy range that is not able to support any energy eigenvalues for all real values of wave vector, k .

The wave vector, k is a real quantity in the interior of the infinite crystal to ensure the finiteness of the wave functions in Eqs. (5.1) and (5.2). However, the wave vector, k can also be complex at the interface between the electronic crystal and the air medium. This also implies that the FEG supports some energy eigenvalues and forms complex surface states. The complex surface states have wave vector in the form of $k = q + i\kappa$. The imaginary portion of the wave vector is assumed positive, i.e $\kappa > 0$. For such case, the wave function of the electron decays exponentially on both sides at $z = 0$. Therefore, the electron is a surface state localized at the crystal – air interface.

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Fig. 5.2 Energy band diagram at the surface of the periodic crystal lattice and air medium. The dash dotted line is the surface states arising from complex k values.

In order to localize the electrons on the crystal surface, the real and imaginary part of the complex wave vector are obtained as

$$q = \pm \frac{\pi n}{a} \quad (5.4a)$$

and

$$0 \leq \kappa \leq \frac{m_e a |V_n|}{\pi \hbar^2 n} \quad n = 1, 2, 3 \dots \quad (5.4b)$$

Considering the forbidden gap between the two lowest allowed energy band, the real part of the wave vector is given as $q = \pm \frac{\pi}{a}$. For the energy of the surface state, it is directly proportional to κ .

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The surface states can either be as extrinsic or intrinsic surface states [111,113]. For the intrinsic surface states, the atoms are arranged in a well-ordered manner whereas for the extrinsic surface states, there are defects or impurities.

5.1.1 Characteristic of dispersion band diagram

Based on the study of electronic bandgap at atomic crystal-air interface, this section discusses the existent of the surface modes at PC-air interface. The dispersion band diagram for the PC-air interface is described. The band structure and characteristic at the interface of the PC-air and the conditions of the surface modes are also discussed followed by the field distributions in various parts of the surface mode and setting the range.

The surface boundary of PC is characterized by its inclination and surface termination [114]. The surface inclination is defined as the angles between the normal PC surface and the crystal orientation axis. The surface termination refers exactly to the surface which cuts across the outmost cells in the PC–air interface. The surface states at the PC-air interface are terminated with the entire or at some fractions of the unit cells [53, 114].

In this section, the lattice structure is a 2D square array of dielectric rods in air background. The circular rod is assumed to be infinite in the third direction. The dielectric rod has a refractive index of 3.45. The ratio of rod radii to lattice spacing is set as $r/a = 0.2$. Using plane wave calculation method, a PBG exists at the normalized frequency range of $0.285 - 0.412 (a/\lambda)$ for transverse magnetic (TM) polarization. The surface inclination of the array of rods is parallel to the (10) surface plane. Two different

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types of the surface termination are considered at the outermost row of PC; one with termination at the entire cell structure and the other one with the surface cells cut into half semicircle as shown in Figs. 5.3(a) and (b) respectively.

(a)

(b)

Fig. 5.3 Schematic layout for finite PC lattice structure, (a) Entire outermost cell. (b) semicircle termination.

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The discrete translational symmetry in the z direction is broken because of the finite crystal structure. Hence, the electromagnetic modes cannot be described by the wave vector k_z , in the z direction. However, there is still discrete translational symmetry in the x direction and continuous translational symmetry in the y direction. The wave vector in the x direction is bounded by the inequality $-\frac{\pi}{a} < k_x \leq \frac{\pi}{a}$ which has discrete translational symmetry [57]. In the y direction, the wave vector satisfies the inequality $-\infty < k_y \leq \infty$. For discussion in this section, the wave vector in y direction is limited to in-plane propagation for $k_y = 0$.

Similar to the defects in the interior of the PC, the surface mode is not allowed to extent into the PC lattice but decays into both the PC and the air medium. Hence, the surface mode is also below the lightline, which is a special line described by, $\omega_0 = \frac{c}{n} k$. For all frequency and wave vector ($\omega, k_x, k_y = 0$) that satisfy the inequality $\omega_0 \leq \omega = \frac{c}{n} k_x$, the electromagnetic mode is decaying exponentially and not radiating out into the air medium. Otherwise, the surface modes are coupled by the air modes and radiating out instead of localizing on the PC surface. The band diagram at the PC surface can be divided into four main regions as shown in Fig. 5.4.

Each region supports a different kind of surface modes which are labeled by two letters, E or D. The E letters refers to extended states and the D letter refers to decaying states. The first letter on the left implies that the surface mode is extending or decaying into the air while the letter on the right refers to mode in the PC.

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Fig. 5.4 Schematic band diagram showing the different surface mode regions.

In the three regions, the surface mode can be defined as surface light waves that are transmitted (EE), surface light waves that are internally reflected (DE) and surface light waves that are externally reflected (ED). The region mark EE above the lightline is the union of the photonic continuous band. The surface states in the region EE extend out into the PC and air medium, so it is possible to transmit light waves. In the DE region, the surface mode propagates in the PC lattice but exponentially decays into the air medium above the crystal. This is similar to the phenomenon of total internal reflection. In the ED region, the modes propagate in air but decays exponentially into the crystal.

The fourth region DD shows the frequency and wave vector combination in which the surface mode is only localized on the PC-air interface. In this region, the surface mode decays exponentially into both the PC and the air medium. By exciting the surface mode, light waves can be localized on the surface of PC. Hence, the surface mode can be

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truly localized on the interface of PC-air under the two conditions: (1) the mode is in the PBG and (2) the mode lies below the lightline.

The supercell plane waves method [125] is used to solve for the master equation of each k_x value. The supercell consists of a vertical column with finite cell structures. Since the band structure is projected in the air and the PC medium, the supercell is partially filled with rods and the air medium above the periodic structure. For the numerical modeling, the supercell size is set to 15 rows $\times$ 1 column, with half extending to the crystal and the other half into the air. The plane waves are expanded at a finite k_x with 300 planes waves per polarization per supercell. The dispersion band diagrams for the two different terminations are shown in Fig. 5.5(a) and 5.5(b).

Figure 5.5(a) show that no surface mode is found in PBG for the whole rods but the half-terminated rods in Fig. 5.5(b). This can be explained by the inverse relationship between the refractive index and the frequency given by $n_{eff} \propto \frac{1}{\omega}$ [8, 53]. When the effective index at the surface reduces, which mean that the rods are terminated at certain fraction and less dielectric, the frequency of the surface mode increases. This pushes up the surface mode from the bulk continuous band to the PBG. The surface mode continues to shift up in the PBG as the surface rods are terminated at larger fractions. When the row of the surface rods is terminated entirely, the surface mode disappears into the upper band of the PBG. Hence, light waves can only be localized on the surface when the surface rods are terminated at certain fraction. This is a similar situation to finite metallic structure where only surface with periodic modulation can support localized light waves to form surface plasmon polariton [141].

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Fig. 5.5(a) Band diagram for surface termination with entire cell structure. (b) Band diagram for half surface termination.

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After discussing the characteristic of the dispersion band diagram at the PC-air interface, the next section focuses on the waveguiding characteristic at the PC-air interface. The field pattern distribution at various parts of the surface mode is plotted. A discussion on the condition and the range of frequency for surface waveguiding of the PC is also provided.

5.1.2 Field distributions at different parts of the surface mode

Not all set of wave vectors and frequency in the DD region is suitable for efficient transmission of light waves for the surface boundary. As a result, the minimum wave vector/frequency that is used for designing PC surface waveguide is determined in this section. In this section, the size of the cell is reduced by half so that $r = 0.1a$ [123]. The structural layout and the band diagram onto k_x are shown in Fig. 5.6. The band diagrams of the outmost cells with smaller radii and semicircular termination are similar. A localized surface mode is found in the PBG below the lightline.

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Fig. 5.6 Band structure of the surface mode with smaller outermost cells. Thicken line bounded by the dashed line is the region 2 that is suitable for surface waveguiding.

In Fig. 5.6, the surface mode is divided into three different parts labeled as 1, 2 and 3. Each part of the surface mode refers to different electric field pattern distributions as shown in Fig. 5.7(a), (b) and (c). The surface mode is very close to the lightline at part 1. The electric field distribution of the surface mode is shown in Fig. 5.7(a). The close proximity of the surface mode with the lightline implies that its group velocity, $v_g = \frac{d\omega}{dk}$ is very similar to the speed of light, c . Hence, there is very strong coupling of the surface mode to the air medium. The surface mode field on the PC surface radiates to the air and is not suitable to transmit light waves.

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(a)

(b)

(c)

Fig.5.7 Field distribution for different parts of the surface mode (a) Part 1, near the lightline. (b) Part 2, further away from the lightline, useful for waveguiding with moderate group velocity. (c) Part 3, with very low velocity, resonating of surface mode.

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For part 2 of the surface mode, the dispersion curve moves away from the lightline, and the electric field is well localized on the surface. The group velocity of the surface mode at this part is moderate enough to satisfy the condition for waveguiding of the light waves. In this thesis, the frequency range for waveguiding with moderate velocity is at 0.34-0.38 (a/λ). The part 3 of the surface dispersion curve has relatively low group velocity and is known as “slow mode”, which can attract interest in using surface cavities to stop or slow down light waves [30].

The electric field distribution for the outermost cells which is twice as large as the lattice cells, i.e. $r = 0.4a$ is also investigated. The band diagram and the electric field distribution are shown in Figs. 5.8(a) and (b), respectively. The band diagram in Fig. 5.8(a) shows that the surface mode in the PBG is shifted to lower frequency because the large surface cell has higher dielectric factor. Based on the inverse relationship between the refractive index and the frequency, $n_{eff} \propto \frac{1}{\omega}$, the surface mode is shifted to lower frequency values. The other observation of the surface mode is that the field distribution of the larger surface cells shows a dipole emission with node in the vertical direction. A larger surface cell allows the presence of dipole emission pattern. The dipole is aligned to the vertical direction due to the presence of discrete translational symmetry in the x axis. Also, the small range of frequency is covered by the surface mode indicated that the group velocity of the propagating light waves is very low. The slow surface mode does not satisfy the condition for efficient waveguiding on the surface.

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Fig. 5.8 (a) Dispersion band diagram of the PC surface with $r = 0.4a$. (b) Field distribution of the larger surface cell showing dipole localization.

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In the next section, guiding light waves around the sharp surface bend is investigated and the solutions to transmitting light waves efficiently at the sharp surface bend are discussed.

5.2 Photonic crystal surface waveguide bend

In the theoretical study of photonic crystal surface modes [123], several advantages are found in PCSWG over the classical PCWG in the interior of the PC lattice. First, it has a single-sided PC lattice structure, which increases the compactness of waveguiding components and design of high device density photonic integrated circuit. Second, the homogeneous medium at the other side of the interface offers a high degree of freedom in design and manipulation, which opens up new possibilities for feeding and redistribution of light waves. The integration of optical/mechanical/electrical components at the outer homogeneous medium can allow a diversified evolution of new applications for sensing and integrated photonic circuits. Third, the characterization and measurement of transmission properties on the surface of the PC is made easier and more convenient when the opposite medium is air. However, there are also some research challenges for the surface waveguiding. The bandwidth supported by the PCSWG is smaller than the classical PCWG. The other problem is possibility of high radiation loss at the photonic crystal surface. For the surface mode(s), there is very strong coupling and radiation of surface light waves to the outer medium. Therefore, the selection of the frequencies range is important for the design and optimization of the PCSWG.

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5.2.1 “Classical” photonic crystal waveguide bend

A discussion on the difference between the surface wave bend and the “classical” PC waveguide bend is necessary to investigate the transmission efficiency of the PCSWG bend. Many studies on PCWG bend were reported over the last few years [19, 20, 39]. The earliest case was reported by A. Mekis *et. al* [20] in creating a “classical” PC waveguide bend in the interior of the bulk crystal lattice, which bend waveguide is created in the lattice by removing a row of rods in the vertical and horizontal directions. When light waves propagate in the waveguide bend, there are no extended modes in which the propagating mode can couple. In the other research work, the transmission efficiency around the classical PC waveguide bend was reported by perturbing the cell structures at the bend corner [72-74].

The main difference between both the “classical” PC waveguide bend and the PC surface waveguide bend is the absence of PC lattice at both sides of the bend section. For the classical PCWG, light waves is “re-directed” from one PCWG direction to the other PCWG direction at the bend due to the PC lattice. For the PCSWG bend, the absence of the PC lattice at the outer region of the bend implies light waves can easily leak out to the surrounding air background. Hence, the transmission of light waves around the surface bend is a challenge. In this chapter, the problem and solutions to transmit light waves around the surface bend is discussed.

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5.2.2 Transmission efficiency of surface waveguide bend

The PCSWG bend is shown in Fig. 5.9(a). The radius of the surface rods in both (10) and $(0\bar{1})$ directions are reduced by half, i.e. $r = 0.1a$. This creates a sharp right angle bend on the PC-air interface in the top and right outermost cells. The band structure for the surface rods at both (10) and $(0\bar{1})$ directions are identical as shown in Fig. 5.6.

A TM polarized light source with frequency, ω is launched on the left side of the surface rods along the (10) direction. The measurement of the transmitted power is carried out by a monitor toward the end of the surface waveguide at the $(0\bar{1})$ direction. The monitor is wide enough to collect the power transmitted into the $(0\bar{1})$ direction. The measurement frequency at $0.34 - 0.38 (a/\lambda)$ is considered because it is sufficiently far from the lightline and is in the PBG. The propagating electric field distribution along the PC surface waveguide bend is shown in Fig. 5.9(b) with $\omega = 0.368 (a/\lambda)$.

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(a)

(b)

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Fig. 5.9 (a) Schematic layout of the PCSWG bend. (b) Field distribution at the PCSWG bend. (c) Transmission spectrum measured.

It is observed that light waves are not transmitted to the surface waveguide in the $(0\bar{1})$ direction but radiates out into the air. The power collected in the $(0\bar{1})$ direction is plotted in the transmission spectrum as shown in Fig. 5.9(c). The low light power is transmitted to the $(0\bar{1})$ direction due to poor coupling between the different parts of the surface bend. The little pikes shown in the transmission spectrum is due to some scattered light.

The PCSWG bend is divided into three separated sections. It consists of two long waveguide sections in the (10) and $(0\bar{1})$ directions with a short bend section (the dotted square) in the $(1\bar{1})$ direction as illustrated in Fig. 5.10(a). The dispersion band diagram of the PCSWG bend in the (10) and $(0\bar{1})$ directions has the same wave vector value $k_1(\omega)$ due to identical band diagram. The dispersion band diagram of the waveguide section in the $(1\bar{1})$ direction with a wave vector k_2 is shown in Fig. 5.10(b).

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From the dispersion band diagram in Fig. 5.10(b), the band structure in the $(1\bar{1})$ and (10) (or $(0\bar{1})$) directions does not overlapped. The surface mode in the $(1\bar{1})$ direction is found at higher frequencies due to lower overall effective index. The transmission coefficient between different directions is given by [20]

$$\eta \propto \left[1 + \left(\frac{(k_1^2 - k_2^2) \sin(k_2 L)}{2k_1 k_2} \right)^2 \right]^{-1} \quad (5.5)$$

where L is the length of the bend section, which corresponds to $0.33a$. This phase mismatch between the wave vector $k_1(\omega)$ and $k_2(\omega)$ has resulted in coupling loss between the different waveguide sections. Based on Eq. (5.5), when the phase mismatch is very small, the transmission efficiency around the bending section is very high. On the contrary, the large difference between $k_1(\omega)$ and $k_2(\omega)$ as shown in Fig. 5.10(b) leads to poor coupling between the different sections of the waveguide. To achieve high transmission efficiency, the effective index along the $(1\bar{1})$ direction needs to be increased so that the surface mode is shifted to lower frequencies to match with the wave vectors k in the (10) or $(0\bar{1})$ direction. Two approaches of increasing either the radius (larger radius method) or the refractive index (higher index method) of the corner unit cell are proposed and discussed in the next section.

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(a)

(b)

Fig. 5.10 (a) Layout of the PCSWG, showing the different sections. (b) Band structure along the straight part and bend part.

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5.2.3 Larger radius method

In order to shift the mode in the $(1 \bar{1})$ direction to match with the dispersion curve with other direction, the overall index in that direction is increased. One way is by increasing the size of the corner cell as shown in Fig. 5.11(a). The size of the corner cell is first increased to a radius of $1.5a$ so that the effective index along the $(1 \bar{1})$ direction is increased. The dispersion curve along the $(1 \bar{1})$ direction is shifted down. Additionally, by increasing the corner cell size, higher order multipole field pattern [142] appears. To further enhance the transmission efficiency, the effective length of the waveguide bend [20] is increased to minimize the phase mismatch between the different waveguide sections. The larger corner cell is shifted inward along the $(1 \bar{1})$ direction with a diagonal shift of Δd . The diagonal shift, Δd is set to $0.31a$ to provide phase matching condition at the central frequency of 0.368. It is also known that when the length of the bending section is increased, the abruptness in the change of the wave vector (or to increase the bending radius) is reduced.

Using the plane wave supercell method, the band structure for larger corner cell with diagonal shift is shown in Fig. 5.11(b). It can be observed that the dispersion curve along the $(1 \bar{1})$ direction has shifted down to meet with the dispersion curve in the (10) (or $(0 \bar{1})$) direction. However, it also shows that the frequencies range is now reduced (or the group velocity reduces). The group velocity of the propagating light waves is reduced, and the interaction time between the waveguide sections in different directions is increased and improves the transmission efficiency. This can be explained by the presence of high order multipoles which exist only at some frequency range.

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(b)

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Fig. 5.11 (a) Schematic layout of the PCSWG bend with larger corner cell to improve the transmission efficiency. (b) Band structure showing the dispersion along $(1\bar{1})$ direction and (10) (or $(0\bar{1})$) direction. (c) Electric field distribution of the larger corner cell, showing dipole emission pattern in an angle of 45° with respect the vertical axis.

The electric field distributions of the PC surface waveguide bend with larger corner cell is shown in Fig. 5.11(c). From the rotational symmetry along the $(1\bar{1})$ direction, the emission direction of the dipole is rotated at an angle of 45° with respect to the vertical axis so that the light waves are transmitted from (10) to $(0\bar{1})$ direction. The dipole electric field pattern emission couples the light waves from the (10) to $(1\bar{1})$ then to $(0\bar{1})$ directions along the surface waveguide bend. Fig. 5.11(c) shows that the light waves propagate around the surface waveguide bend with little radiation loss at the bend section. In the next part, the approach by increasing the refractive index of the corner cell will be applied to transmission efficiency investigation.

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5.2.4 Higher index method

Similarly, increasing the refractive index of the corner cell can also shift the surface mode at the PCSWG bend to match with the surface modes on both the waveguide sections. The lattice layout is shown in Fig. 5.12(a). The refractive index of the corner cell is increased by $\Delta n = 0.68$. Additionally, the position of the corner cell is shifted diagonally with $\Delta d = 0.403a$. In Fig. 5.12(b), the surface band diagram along the $(1\bar{1})$ directions fits in well with the band diagram in (10) or $(0\bar{1})$ direction. The surface modes at the waveguide and bending section have a larger frequency overlapped. The surface mode at the bending section has a higher group velocity due to the absence of dipole field pattern. As a result, phase matching occurs over a broader range of frequency.

Figure 5.12(c) shows the propagating field distribution of the increased index bend section. In Fig. 5.12(c), the coupling efficiency of the surface waveguide bend is slightly lower than the larger cell method as some radiated fields can be observed in the free space even though the dispersion band along the different directions fits better. This is because light waves travel faster around the bend by the higher index method and reduces the transmission efficiency to the $(0\bar{1})$ direction. Having discussed the two methods of shifting the dispersion curve along the $(1\bar{1})$ direction to match with the dispersion curve in the other direction, the transmission spectrum for both the larger radius method and the higher index method is shown in Fig. 5.13.

Chapter 5 Surface Waveguiding for Light Coupling and Power Splitting

(a)

(b)

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Fig. 5.12 (a) Schematic layout of the PCSWG bend with increased index at corner cell. (b) Band structure showing the dispersion curves along $(1\bar{1})$ direction and (10) (or $(0\bar{1})$) direction. (c) Electric field distribution of the increase index bend.

Fig. 5.13 Transmission spectrum of light waves for different waveguides structure. Normal straight surface waveguide (solid). Bend surface waveguide with larger corner cell (dashed). Bend surface waveguide with increase index corner cell (dashed-dot).

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It is shown that both methods give different transmission bandwidths due to the different range of frequencies. The transmittance for the larger radius and higher index methods is approximately 94 % and 90.8 %, respectively as shown in Fig. 3(c). The higher index method is useful to transmit a decent range of frequencies because the dispersion band covers a larger frequency range. The higher index method has slightly lower transmission efficiency because of shorter interaction time. Hence, the coupling efficiency is lower and some light leaks to the surrounding of the corner cell. On the other hand, the larger radius method can be used to filter selective frequency due to its narrow bandwidth.

5.2.5 Photonic crystal slab with surface waveguide bend

This section focuses on the discussion of PCSWG bend around the surface of the PC slab [139, 140]. The PC slab structure consists of a triangular array of air holes extending through a silicon slab. The silicon slab has a refractive index of $n_s = 3.45$ and the holes have an index of $n_h = 1.0$. The holes have a radius of $0.3a$ and slab thickness of $0.6a$. This PC slab has a PBG at $0.208 - 0.78 (a/\lambda)$. Unlike the previous case where the radius of the surface rods is reduced to introduce surface mode, the PC slab surface waveguide is created by having an additional homogeneous slab section with width of d_1 and d_2 at the top and right boundary as shown in Fig. 5.14(a). The dispersion band diagram along the (10) and $(0\bar{1})$ directions are different due to different supercell definitions. Hence to match surface modes in these two directions, the widths are set to $d_1 = 1.138a$ and $d_2 = 1.26a$. The dispersion band diagram is shown in Fig. 5.14(b).

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(a)

(b)

Fig. 5.14 (a) Structural layout for PC slab surface bend waveguide. (b) Band diagram along the Γ -M and M-K direction with designated d_1 and d_2 .

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From this structure, an input light with frequency of $0.25(a/\lambda)$ is launched in the slab. Fig. 5.15(b) shows the field distribution for the PC slab surface waveguide bend. It is observed that light waves are localized in the corner of the slab with very little transmission of light waves to the $(0 \bar{1})$ direction. This is because of Fabry-Perot resonance where the phase condition at the corner is given by $\Phi = 2kL_l$. L_l is the length of the cavity form at the corner and k is the wave vector in unit of $2\pi/a$ along the bend corner. A two-step solution is used to solve this problem to ensure that light waves can flow around the slab surface waveguide bend with high transmission efficiency.

The first step requires the removal of a single hole cell from the top right corner of the slab waveguide bend. Then, the sharp right angle slab bend is truncated at an angle of 30° with respect to the vertical plane so that the resultant surface is parallel to the holes array axis along the $(1 \bar{1})$ direction. The perpendicular distance from the newly cut angled surface to the centre of the holes along the $(1 \bar{1})$ direction is $d_3 = 1.05a$. The final modified bend structure is shown in Fig. 5.15(a). The solution serves two purposes: First, it eliminates the possibility of cavity resonance forming at the waveguide bend by truncating the slab with a surface parallel to the holes array at the bend. Second, it increases the effective length of the waveguide bend for optimum transmission efficiency. Fig. 5.15(c) demonstrates the magnetic field travels around the surface waveguide bend without being localized at the bend at a normalized frequency of 0.25.

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From Fig. 5.15(c), it is observed that the magnetic field distribution of the truncated PC slab surface waveguide bend travels around the surface waveguide bend without being trapped at the bend at frequency of 0.25. The transmission spectrum of this PC slab surface waveguide is shown in Fig. 5.15(d). The transmission efficiency is more than 93 % for the final slab bend structure for the frequency range of 0.247-0.254 (a/λ). Hence, it shows that the surface modes can be used to transmit light waves around a perpendicular bend for rods and holes slab structures.

The transmission efficiency of the different PCSWG structures in rods and holes arrays has been discussed. In the next section of this chapter, the PC surface waveguide is designed on the outermost row of a circular photonic crystal (CPC). It is developed as an add/drop filter through the coupling resonator to select different frequency.

(a)

Chapter 5 Surface Waveguiding for Light Coupling and Power Splitting

(b)

(c)

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Fig. 5.15 (a) Structural layout of the truncated slab at the bend section. (b) Magnetic field distribution showing mode localization at bend. (c) Magnetic field distribution showing light waves propagation with truncated bend. (d) Transmission spectrum peaking at frequency of 0.25 (a/λ).

5.3 Circular surface waveguide

Most PC waveguiding structures are designed on straight, right angle bend or 60°/30° bend pattern to transmit electromagnetic light waves. This is due to limitation of crystal structures either to square or triangular array which the orders of the lattice are perfect. In this section, a circular photonic crystal (CPC) [143] is utilized to design a circular surface waveguide. Unlike the square or triangular lattice arrangement, the circular lattice is a non-periodic structure. However, it is demonstrated electromagnetic light waves can also be confined in the circular structure [144]. There are two possible explanations: (1) the circular photonic crystal lattice is taken to be a combination of square and triangular structures [145] which have very similar PBG range. (2) the CPC

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lattice arrangement is identical to the ring structures in microstructures optical fibers [146].

The CPC structure has radial periodicity, where the distance between each concentric layer is constant. When one concentric layer in the interior of the CPC is perturbed, the curve waveguide is formed. High power transmission can be achieved for curve waveguide formed [144, 145] because of its smoother bend with smaller discontinuity. The other key factor resulting in the high power transmission is the symmetry about the centre of the curve waveguide. This expands the versatility of PC structures for having a wide range of regular periodic and non periodic structures for designing photonic devices.

In this section, the transmission efficiency of guiding light waves on the surface of the CPC is investigated. The dispersion band diagram of the CPC is first plotted to find out the bandgap range. This is followed by reducing the size of the rods on the outermost row of CPC to create the surface waveguide and adding some structures to keep the light waves from leaking to the background when it propagates along the surface. Finally, the circular surface waveguide is developed into an add-drop multiplexing device using the waveguide-resonator coupling mechanism.

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5.3.1 CPC structure and dispersion band diagram

Figure 5.16(a) shows a quarter portion of a two-dimensional CPC structure. The CPC structure consists of circular cylindrical rods with radius of $0.196a$ and refractive index of 3.42 . The distance between each concentric layer, radial period is a . The CPC structure shown in Fig. 5.16(a) has a six fold symmetry and the number of rods in each layer is given by the formula $6(N - 1)$, where N denotes the number of layer with $N \geq 2$. The concentric period, d_c between the nearest-neighbor rods in each layer is $1.05a$. The calculations are based on TM-like mode polarization with electric field along the axis of the cylindrical rods for the normalized frequency range of $0.30 - 0.41$ (a/λ) as shown in Fig. 5.16(b).

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Fig. 5.16 (a) Lattice layout of the quarter – CPC structure. (b) Band diagram of the CPC showing the PBG at normalized frequency of 0.3 - 0.41 (a/λ).

5.3.2 Curve surface waveguide

A curve surface waveguide (CSWG) is formed at the outermost layer (i.e. $N = 8$) by reducing the radius of the surface rods to $0.1a$ as shown in Fig. 5.17(a). The CPC lattice has a radius of $7a$ so that it is not too large for application in integrated circuit. An input Gaussian beam light source with a width of a is launch on the left end as shown in Fig. 5.17(a). The transmission spectrum of the CSWG is calculated using finite-difference time-domain (FDTD) method.

The surface modes along k_x and k_z direction are shown in Fig. 5.17(b). The frequency range that is considered suitable for surface waveguiding purposes is 0.33 – 0.37 (a/λ). The reason is because the surface mode at this frequency range is sufficiently

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far from the lightline and electric field concentrates mainly on the surface of the CSWG. The frequency range also has higher group velocity which is suitable for waveguiding purposes. The transmission spectrum of this surface waveguide is calculated by FDTD method [147] and shown as the dash line in Fig. 5.18(a). The average transmission is around 37 %, which is less than half of the power transmission in the “interior” curve waveguide [145]. The curve waveguide lattice structure on both sides, forming Bragg reflecting like curve mirror waveguide for high transmission efficiency. On the other hand, the CSWG has only PBG on one side. Electromagnetic light waves decays exponentially into air on the outer side by operating below the lightline.

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(b)

(c)

Fig. 5.17 (a) Lattice layout of the CSWG. (b) Band diagram showing the surface modes along the x and z direction. (b) Improving the surface waveguiding efficiency by introducing additional rods shown in dash box

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(a)

(b)

Fig. 5.18 (a) Transmission spectra for the CSWG surface concentric period of $1.05a$ (dash line) and $0.525a$ (solid line). Smaller surface concentric period gives higher power transmission due to smoother changes in magnitude and direction vector of the propagating light wave. (b) Constant frequency contour at frequency of $0.365 (a/\lambda)$ for different x and z directions.

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The light waves traveling along the CSWG have to change the magnitude and direction from k_x to k_z or vice versa. The constant frequency contour (CFC) in Fig. 5.18(b) demonstrate the variation in wave vector magnitude and direction at $0.365 (a/\lambda)$ as electromagnetic light waves travel along the CSWG. With large surface period, the wave vector magnitude and direction changes rather abruptly. This results in coupling loss as light waves travel along the CSWG. A solution for this problem is to increase the radius of the CPC structure, which is equivalent to increasing the bending radius. The other solution is to reduce the surface concentric distance, d_c by half as shown in Fig. 5.17(b). This “closed-up” CSWG has higher overall surface effective index and higher confinement factor. The average transmission efficiency of the “closed-up” CSWG is calculated using FDTD method and has an average of more than 95 % in the simulated range as shown by the solid line in Fig. 5.18(a). This improvement is significant compared to the “non-closed-up” CSWG. The transmission efficiency of the CSWG without the bulk CPC (known as the periodic curve WG) structure is shown in Fig. 2(a). The transmission efficiency of the “closed-up” CSWG is higher than the periodic curve WG by approximately 10 %. This is because of more power leakage to the inner side of the periodic curve WG. The presence of the bulk CPC lattice is essential for higher transmission efficiency. With high transmission achieved through reducing the surface rods period, the next part of this section will discuss about developing the CPC structure to add/drop circulator for application in optical integrated circuit.

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5.3.3 Coupling between CPC surface waveguide and resonator modes

The quarter CPC lattice is expanded to a full disk structure shown in Fig. 5.19(a). The CSWG extends for three-quarter of the CPC. For device development, additional rods structures are either inserted or removed above the CSWG. The main requirement for designing an add/drop circulator filter is to insert a resonating structure to coupled light waves with appropriate frequency out from the circular surface waveguide. There are numerous reports and publications [63-65] for the design of optical add/drop filter based on the PC structure for photonic integrated circuit [63-65] applications. This can be achieve by adding rods or creating microcavity hole structures to function as a resonator. The resonating structure only resonates for a certain frequency (or some small frequency range depending on the Q factor). The light waves with frequency that matches the resonance frequency of the resonator system is coupled out from the CSWG and transmitted elsewhere.

For the add/drop filter design, the resonator systems are placed tangential to the CSWG at the perpendicular axes of the CPC as shown in Fig. 5.19(b). Three frequencies are dropped at different resonator systems. The resonator systems consist of rod(s) with the same index as the bulk CPC structure. The number of rods and their physical geometries in each resonator system depends on the selected frequencies.

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(a)

(b)

Fig. 5.19 (a) Schematic layout of add/drop CPC circulator. The arrows show the dropping mechanism for different frequencies at designated dropping locations. (b) Layout showing the distance between the resonator coupling system and the tangential surface rod where $n = 1, 2, 3$.

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Table 5.1: Optimized values of different parameters for the circulator

Parameters	Values
$\omega_1, \omega_2, \omega_3 (a/\lambda)$	0.340, 0.365, 0.355
d_{1a}, d_{1b}	$0.252a, 0.0136a$
d_2	$0.256a$
d_{3a}, d_{3b}	$0.272a, 0.2115a$
s_1, s_2, s_3	$0.554a, 0.52a, 0.541a$

The targeted coupling efficiency is to achieve at least 95 % dropping efficiency from the CSWG to the resonator system. The brute force method is used to obtain the optimized physical parameters of the rods as shown in Table 5.1. First, one rod is placed above each perpendicular axis. Its size and transverse distance s_n , (where $n = 1, 2$ or 3) are optimized to achieve highest dropping efficiency. When the dropping efficiency is below 95 %, a second rod is added either to the left or right of the first rod to enhance its efficiency. In this thesis, the second rod is added to the right. Its x position is set as a lattice constant from the first rod and its z position is identical to the first rod.

In order to develop the add/drop mechanism, the side coupling structure is adopted. Additional rod cells are placed on top of the surface waveguide to function as a resonator coupling system which couples electromagnetic light waves from the surface waveguide. Three frequencies ω_1, ω_2 and ω_3 are fed to the circular surface waveguide by an input feeding waveguide. The frequencies ω_1, ω_2 and ω_3 are selected to be $0.34 (a/\lambda)$, $0.365 (a/\lambda)$ and $0.355 (a/\lambda)$ respectively, which lie in frequency range suitable for surface waveguiding. For simplicity, the resonator coupling systems are placed on each right angle axis of the CPC structure, so that the frequencies ω_1, ω_2 and ω_3 are dropped at different locations along the circular surface waveguide.

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The working mechanism of the frequency-selective circulator is shown in Fig. 5.19(a). Three frequencies ω_1 , ω_2 and ω_3 are launched into the circulator on the left. When the frequency of the light waves that propagates along the CSWG matches with the resonance frequency of the resonator coupling system, it is dropped and transmitted out by a periodic waveguide. The output periodic transmission waveguide consists of a finite periodic dielectric waveguide with a spacing of $0.116a$ [148]. The dielectric rods have same refractive index as the circulator where the waveguiding range is within $0.32 - 0.4$ (a/λ).

The physics of the dropping efficiency can be explained using the simple coupled mode theory for the waveguide-resonator system. The resonator systems and the CSWG formed a side coupled waveguide-resonator system. Based on the coupled mode theory [65], the reflection coefficient, R and transmission coefficient, T of the side coupled system can be express as:

$$R = -\frac{\frac{1}{2Q_e}}{j\left(\frac{\omega}{\omega_0} - 1\right) + \frac{1}{Q_0} + \frac{1}{2Q_e}} \quad (5.6a)$$

$$T = \frac{j\left(\frac{\omega}{\omega_0} - 1\right) + \frac{1}{Q_0}}{j\left(\frac{\omega}{\omega_0} - 1\right) + \frac{1}{Q_0} + \frac{1}{2Q_e}} \quad (5.6b)$$

where Q_0 and Q_e are the quality factors due to intrinsic loss and rate of decay into the waveguide. The parameters ω and ω_0 are the waveguide frequency and the central frequency of the resonator coupling system respectively. In order to have strong coupling

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of the light waves between the CSWG and resonator coupling system, the condition requires the quality factor for the decay rate waveguide, $Q_e \gg Q_0$. To achieve this condition, the distance, s_n plays an important role. By brute force optimization, this condition can be met and light waves couples to the resonator coupling system with near 100 % efficiency and negligible power leakage.

The three different frequencies are launched in the input of the add/drop circulator and the field distribution is shown in Fig. 5.20(a) – (c) below. It is observed that if the propagating light waves frequency matches the resonance frequency of the resonator coupling system, it is “drop” out of the surface waveguide and transmitted out in the finite periodic dielectric waveguide. Otherwise, the electromagnetic light waves will continue traveling along the surface waveguide until it is filtered out. The transmission spectrum is shown in Fig. 5.20(d). At the resonance frequency, the targeted frequency is dropped from the circulator and the dropping efficiency is more than 95 % with narrow bandwidth. This indicates that the filtering response is very good. The quality factor, Q for the different frequencies are calculated to be around 2364, 1950 and 2536 for frequencies of $0.34 (a/\lambda)$, $0.365 (a/\lambda)$ and $0.355 (a/\lambda)$ respectively. This high value of Q factor shows the excellent filtering efficiency of the add/drop circulator based on circular surface waveguide.

Based on the discussion of the waveguiding characteristic of the light waves on the surface of CPC, the power splitting device is develop using the mechanism of PC surface waveguiding for power splitting will be designed in the next section. The power ratio using the large cell dipole and using index variation will be discussed.

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(a)

(b)

(c)

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(d)

Fig. 5.20 Field distribution showing the dropping of (a) 0.34 (a/λ), (b) 0.365 (a/λ), (c) 0.355 (a/λ) It is shown that the dropping efficiency is near 100 % with negligible leakage or reflection of light waves. (d) Transmission spectrum for different frequencies.

5.4 Dual photonic crystal surface waveguiding

The surface waveguiding characteristics are discussed only for a single-sided PC-air interface in the last two sections. This structure has only one surface waveguide for the transmission of light waves. Based on the single-sided surface waveguide, the optical properties of a dual surface waveguides structure is investigated. The new power splitter is designed on the dual PCSWG structure using dipole field emission to split light waves to the dual surface waveguides. The power splitting ratio is varied by modulating the refractive index of the cell structures so that the incoming power is divided into two portions with different power ratio along the two PCSWGs.

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5.4.1 Dual PC surface waveguide

The structure of the dual surface waveguide is designed based on the 2D surface waveguide with high refractive index cylindrical rods surrounded by air background. The rods are assumed to have length which is infinitely long in the third dimension. The rod has a radius of $r = 0.2a$ and a refractive index of $n = 3.45$. The PBG of the array is 0.285 - 0.412 (a/λ) for TM polarization. A row of rods is removed from the bulk crystal and the radius of the intermediate rows of rods on top and below are reduced to $r = 0.1a$. The design of the dual PCSWG is shown in Fig. 5.21(a). It can be considered as two individual surface waveguides facing each other in proximity. In the initial structure, the distance between the individual surface waveguides is set at $2a$. The band structure for the dual surface waveguide along the k_x direction using the plane wave supercell method [132] is shown in Fig. 5.21(b).

From Fig. 5.21(b), it is observed that two surface modes, $\mathbf{m}_1$ and $\mathbf{m}_2$ in the PBG form a pair of double degenerated modes. The pair of the double degenerated modes is a result of the inverted and rotational symmetry properties of the dual surface waveguides. It can be deduced from the structure layout of the dual PCSWG in Fig. 5.21(a) which has 180° rotation symmetry.

The double degenerated modes form an even and odd inverted field amplitude distribution symmetry as shown in Fig. 5.21(c). The lower frequency surface mode has even field symmetry while the higher mode has odd field symmetry. According to the energy electromagnetic variation theorem, the even field has higher energy concentration on the surface that forces the mode to lower frequency. The other surface mode has lower energy and is found at higher frequency.

Chapter 5 Surface Waveguiding for Light Coupling and Power Splitting

(a)

(b)

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Fig. 5.21 (a) Structural layout of the dual PC surface waveguide. (b) Band diagram of the dual PC surface waveguide. There is a pair of doubly degenerated surface modes, forming even and odd symmetry. (c) Field pattern of the even (left) and odd (right) symmetry for the doubly degenerated modes

An interesting phenomenon is observed when the distance between the dual surface waveguide is varied. When the distance between the surface waveguides is increased, for example $3a$, the double surface modes increase in degeneracy. When the distance between the surface waveguides is decreased, the double surface modes are separated and two non-degenerated modes are formed. The band structures are shown in Figs. 5.22(a) and (b).

The change in degeneracy, it can be explained as follow. When the distance between the surface waveguides is large, the surface modes are not able to couple and interfere with each other. Hence, the two surface modes can co-exist together with same wave vector and frequency. When the distance between the PCSWG is small, the surface modes couple and interfere with each other. As a result, the two modes have two different wave vectors at two different frequencies. This explains for the separation of the surface modes $\mathbf{m}_1$ and $\mathbf{m}_2$.

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(a)

(b)

Chapter 5 Surface Waveguiding for Light Coupling and Power Splitting

(c)

Fig. 5.22 (a) Surface modes split at near distance. (b) Surface modes co-exist together due to large separation. (c) Graph showing the change in frequencies when the distance between surface waveguide decreases.

For the mathematical model, the amplitude of each surface waveguide in the dual surface waveguide are expressed as [149]

$$\frac{da_1}{dt} = -j\beta_1 b_1 + \kappa_{12} b_2 \quad (5.7a)$$

$$\frac{da_2}{dt} = -j\beta_2 b_2 + \kappa_{21} b_1 \quad (5.7b)$$

where β_1 and β_2 are the propagation constant in either surface waveguide. The parameters a and b are the field amplitude of the waveguides. Converting Eqs. (5.7a) and (5.7b) in the form of matrix and using the relationship between coupling coefficients given by $\kappa_{12} + \kappa_{21}^* = 0$, the overall propagation is obtained as

$$\beta_{\pm} = \frac{\beta_1 + \beta_2}{2} \pm \sqrt{\left(\frac{\beta_1 - \beta_2}{2}\right)^2 + |\kappa_{12}|^2} \quad (5.8)$$

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Based on the phase constant obtained from Eq. (5.8), when s_d decreases, the coupling coefficient, κ_{12} increases due to stronger coupling between the two surface waveguides as shown in Fig 5.22(c). Therefore, the square root term in Eq. (5.8) increases and gives rise to a wider difference between β_+ and β_- . This explains why the surface modes are moving away from each other. This indicates the lost of degeneracy of both $\mathbf{m}_1$ and $\mathbf{m}_2$ surface modes in the dual surface waveguides structures. On the other hand, when s_d decreases, the coupling strength between both the surface waveguides is weakened and the coupling coefficient κ_{12} decreases. The phase constants β_+ and β_- are closer and the surface modes, $\mathbf{m}_1$ and $\mathbf{m}_2$ become nearer to each other. Then, the degeneracy of $\mathbf{m}_1$ and $\mathbf{m}_2$ is preserved. Therefore, the degeneracy of $\mathbf{m}_1$ and $\mathbf{m}_2$ surface modes is dependent on the coupling coefficient, κ_{12} , which is in turn dependent on the surface waveguide separation, s_d .

5.4.2 Power splitter using dual PC surface waveguide

A photonic power splitter is designed using the dual PC surface waveguide structure. The power splitting mechanism is designed using a large rod cell dipolar with dipolar field emission [142]. A 2D PC structure with identical lattice parameters as the previous discussion is considered. From the PC lattice, a row of rods is removed to create a PCWG. A large rod structure with radius of $r = 1.98a$ is placed in the centre of the waveguide as shown in Fig. 5.23(a). The large rod structure supports the dipolar emission field pattern at a frequency of $0.366 (a/\lambda)$ as shown in Fig. 5.23(a). In Fig. 5.23(b), the emission of the power distribution for a dipole resonator in the PCWG shows that most fields are concentrated on the z direction with only a small amount in the x direction.

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With this field distribution, the splitting mechanisms of the power splitter are shown in Figs. 5.24(a), (b) and (c).

The design of the power splitter shown in Fig. 5.24(a) consists of two surface waveguides and a dipole rod. According to the previous discussion, the surface waveguide is designed by reducing the rods radius by half. Although Fig. 5.23 (b) shows that most electric field power emits out at the z direction, some power leaks out from the dipolar emission rod in the x direction. This leaky power affects the power splitting efficiency at the splitting system. An additional rod cell is placed at the right side and labeled as a reflector to minimize the power leaking. This helps to reflect the leaky power back to the dipole rod so that it can re-split again to the surface waveguide in the z direction. The mechanisms of the forward and reflected transmissions are shown in Figs. 5.24(b) and (c). The position of the reflector is important to achieve maximum splitting efficiency. The position of the reflector is optimized at the distance of $1.05a$ to the right of the dipole rod and has the same vertical position as the dipole resonator. At the optimized position, the reflected power does not cancel the incident power in the dipole cell. The power from both the incident and reflected directions enter into the dipolar rod resonator and further split into the two output surface waveguides.

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(a)

(b)

Fig. 5.23 (a) Field emission pattern of a large dipolar rod structure placed in the centre of the missing row. (b) Field intensity measurement of the distribution pattern showing power leakage in the side.

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Fig. 5.24 (a) Structure of the power splitter with dual PC surface waveguide and dipole emission. There is leaky power from the dipolar cell. (b) Addition of a reflector cell for forward transmission. (c) Reflect back the power and re-split it again.

Figures 5.25(a) and (b) show the transmission spectra of the power splitter with and without the reflector respectively. The Figs. 5.25(a) and (b) shows that the optimized reflector improves the splitting efficiency by 10 % for each port. Each output port has transmission power near 50 %.

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(a)

(b)

Fig. 5.25 (a) Transmission spectrum of the optimized reflector. (b) Transmission spectrum of unoptimized reflector.

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Fig. 5.26 (a) Schematic layout of the concave power splitter. (b) Electric field distribution of the power splitter. (c) Identical transmission spectrum at each output port of the splitter

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The distance between each output surface waveguide in the structure is designed to create a concave curvature output [101] to achieve a high power splitting distribution system. The purpose of this design is to collect light waves more efficiently after the splitting system and conveniently distributes it to other channels. The structural layout of the concave power splitter and the field distribution pattern is shown in Fig. 5.26(a) and (b), respectively. From Fig. 5.26(b), the electric field distribution of the concave splitter shows that light waves travel along each surface waveguide independently. The transmission spectra of each output port are shown in Fig. 5.26(c). The peak frequencies is $0.366 (a/\lambda)$.

5.4.3 Tuning the power splitting ratio by index modulation

Modulation of the refractive index in PC lattice can be used through mechanical and thermal-optical tuning to more sophisticated tuning method such as induced free carrier optical absorption and nonlinearity induced material index changes [156-158]. The power splitting ratio of the dual surface power splitter is varied by the modulation of the refractive index of the rod structures. Only the linear variation of the refractive index is investigated in this section.

For the power splitter, the refractive indices of the two rods structures on one side of the surface waveguide are varied. The rods are chosen near the splitting dipolar rod cell and reflector so that the effect of the modulation of the refractive index is stronger as shown in Fig. 5.27 (a).

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Fig. 5.27 (a) Schematic layout of the power splitter through the refractive index modulation with the affected rods. (b) Field distribution of the power splitter with refractive index change.

For the modulation location, the refractive index of both the rods affects the wave vector and group velocity of light waves. The coupling coefficients of each surface waveguide are changed through increasing the index of both the rods. This reduces the group velocity at output port 2 and increases the coupling coefficient. As a result, higher light waves power is coupled to the bottom surface waveguide. The transmission spectrum of each port at $\Delta n = +0.05$ is shown in Fig. 5.28(a).

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Fig. 5.28 Transmission spectrum of the different ports with index modulation at $\Delta n = +0.05$. (b) Graph shows the power modulation at each port with different index variation.

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It is observed from Fig. 5.28(a) that the output power of the surface waveguide output port 2 is increased linearly while the index of the two rods is being modulated with positive Δn . The output power transmission at the surface waveguide output port 1 decreases linearly. The refractive indices of the two rods are varied within the range of $\Delta n = \pm 0.1$ together. Fig. 5.28(b) shows the power ratio variation from a ratio of 10 % - 90 % to 90 % - 10 % with the overall power adding up to 100 %. The large change in the splitting ratio can be used for the applications of ultrafast photonic switching devices, power splitters and, multiplexing and demultiplexing devices.

5.5 Summary

In this chapter, the optical properties of the surface waveguide are discussed and the photonic surface waveguide devices are designed. The band structure of the PC-air interface is modeled by using the plane wave supercell method. Based on the band structure, two conditions of the surface mode for localization on the PC-air interface are identified. First, the surface mode has to be in the PBG. Second, the mode has to be below the lightline so that it can decay exponentially into the air medium. Various sections of the surface mode are analyzed. For waveguiding along the surface, the frequency is set within the range of 0.33-0.37 (a/λ). The transmission efficiency of the surface waveguide bend is discussed. It is found out that the problem arises because of mode mismatch between the different waveguide sections. It is important to design the corner cell so that it matches with the other two waveguide sections. By designing suitable corner cell, the transmission efficiency of the surface waveguide bend is improved to more than 94 %. The surface waveguiding at the surface of CPC is

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investigated and an add/drop photonic filtering device is designed and simulated for communication system and integrated circuit applications. Finally, a novel dual surface waveguide is designed. The band properties of the dual surface waveguide are discussed. The effect of the separation distance between the surface waveguides is discussed. The dual surface waveguide is developed to a power splitter using dipole emission. The splitting ratio can be tuned from 10 % - 90 % to 90 % - 10 % when the refractive index of the selected rods are modulated.

CHAPTER 6

CONCLUSIONS

6.1 Conclusions

In this thesis, two types of PCWG includes: (i) tapered waveguides and (ii) surface waveguides are studied. In chapter three, PCTW with different curvatures are designed and a mathematical model called the modified step-theory is derived to describe some basic coupling mechanism in the tapering section. The next chapter, an exact step-coupling-theory is derived to give a more comprehensive description of the mode coupling and interaction mechanism in tapering section. In the last chapter, surface waveguide is investigated and photonic devices are developed. From the contributing chapters, the major conclusions of this thesis are listed below.

- (i) In the theoretical study of PCTW, a mathematical model called the modified step-coupling theory is derived. The modified step-theory is derived from the step-theory with the inclusion of Floquet-Bloch periodic wave vector in the waveguide model to account for the periodicity in PCTW boundary. The PCTW are divided into vertical steps and the transmission amplitude of the field is calculated from one step to another step iteratively. The convergence of modified step-theory is verified by comparing the transmission efficiency with a reference case study using the scattering method. The simulation results from modified step-theory and

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scattering method agree well with moderately small error. This concluded that modified step-theory can be used for calculating the transmission efficiency of the PCTW.

- (ii) The modified step-theory is applied to a case study of PC abrupt step waveguide and step tapered waveguide. The band diagrams of the different PCWG widths are studied. The wider waveguide has more modes with alternate even and odd symmetry while the narrower waveguide has one even symmetry guided mode. A Gaussian mode field with even and odd symmetry transverse field profile is launched at the wider waveguide. It shows that only modes with the same transverse symmetry profile can coupled with each other while different modal symmetry cannot couple. Subsequently, the coupling efficiency of step tapered waveguide is simulated. The step tapered waveguide has a maximum transmission coupling efficiency of 74% for a taper length of $48a$. It is concluded that this amount of power transmission coupled to the photonic integrated circuit so as to achieve reliable device performance.
- (iii) A PCTW with smooth side wall is designed with linear, convex and concave taper curvature. The PCTW has a size reduction ratio of 10:1 and the coupling efficiencies of the different PCTW curvature designs are calculated using the modified step-theory. For the linear taper with $\alpha = 1$, it has a transmittance of 0.912 at a saturated length of $44a$. For the convex taper with $\alpha = 0.5$, it has a transmittance of 0.975 at a saturated length of $36a$. For the concave taper with $\alpha = 2$, it has a transmittance of 0.918 at $50a$. The convex taper has a higher transmittance of than the concave and linear due to slow change in taper width.

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Additionally, different PCTW curvature designs are simulated with various α values and the convex taper with $\alpha = 0.5$ has the highest transmittance and shortest length.

- (iv) The loss characteristics of PCTW are investigated at different taper lengths and curvature designs. The linear and convex PCTW shares similar loss trend, which both has a sharp loss increment toward the output end. On the other hand, the concave curvature PCTW has a different loss trend. The reflection loss increases due to fast reduction of the tapered slope. The reflection loss then decrease at the later section of the concave PCTW due to very gradual taper slope. The power carried by the lowest order mode is simulated for different PCTW curvatures and taper lengths using the modified step-theory. It is concluded that longer taper has higher transmission efficiency because gradual change in the PCTW slope reduces reflection loss and intermodal coupling loss. When the field distribution in the tapering section remains uniquely single mode in the tapered waveguide session, adiabatic condition is achieved.
- (v) An exact step-coupling-theory is derived in order to have a complete picture of the coupling mechanism in the tapering section. The approximations and assumptions are discarded and the step-theory is included in the derivation to calculate the transmission efficiency in the tapered waveguide section. Based on the Maxwells' equation and step-theory, a set of coupling equations which describe all the coupling and interaction mechanism are obtained. The characteristic of the backward propagating waves are discussed using the coupling equations. Then two methodologies are introduced to simplified the numerical

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calculation and shorten the computational time. The exact step-coupling-theory is verified using numerical case study and the transmission simulation results agrees very well with the scattering method with error percentage is lower than $\pm 1\%$.

- (vi) The transmission spectra for linear, convex and concave PCTW are plotted using the modified step-theory and exact step-coupling-theory. The simulation results obtained by the exact step-coupling-theory are able to follow the transmission spectra pattern closely while the modified step theory cannot. This is observed at the band edge of the transmission spectra where the simulation results obtained from modified step-theory deviate largely from the scattering method. The error percentage of the exact step-coupling-theory with reference to the scattering method has a tolerant less than 3 % compared to the modified step-theory, which has a tolerant of 20 %. The coupling coefficients between the different combinations of α and γ are discussed and it is found that the coupling coefficients for forward-forward propagation are higher than the backward-backward propagation and forward-backward propagation.
- (vii) The exact step-coupling-theory is applied to the “turnon” waveguide, which has varying cladding radii and, the 3D structure of PCST. For the “turnon” waveguide, the mathematical model must include the radiation modes expression due to the coupling between the guided modes and the radiation modes. The simulation results of the exact step-coupling-theory agrees well with the results obtain from the scattering method. The low transmission efficiency is due to the absence of guided mode in one section of the “turnon” waveguide. The exact step-coupling-theory is applied to calculate the transmission efficiency of a PCST with

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vertical index confinement. The fields are separated into the vertical index and planar PBG components in the coupling equations. The simulation results show that the coupling efficiency of the convex PCST is the highest, with approximately 90 % for taper length of $20a$ at frequency of 0.258. The concave and linear PCST has lower coupling efficiency than the convex PCST.

- (viii) Finally, the optical properties of a surface waveguide at the PC-air interface are investigated. The band structure characteristics at the surface of a finite PC are studied. By perturbing the surface structures, a surface mode exists in the PBG and the conditions for localized surface mode on the surface are discussed and laid. The surface mode must be in the PBG and below lightline in order to be localized at the PC-air interface. For surface waveguiding purposes, it has moderate group velocity and this is in the frequency range of $0.33 - 0.37 (a/\lambda)$. Subsequently, the transmission efficiency around a perpendicular bend is investigated. Initial simulation results show poor transmission efficiency due to mode mismatch between the different waveguide sections. By increasing the index or size of the corner cell, the mode matching conditions are satisfied and light waves are transmitted around a perpendicular bend with efficiency more than 94 %.
- (ix) Another waveguiding structure is the transmission of light waves around the surface of circular photonic crystal (CPC). The radii of the rods at outermost layer of the CPC are perturbed to form a CSWG. The surface concentric period is reduced to $0.525a$ to improve confinement factor at the surface. The transmission efficiency improves from 37 % to more than 90 %. Subsequently, an add/drop

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circulator via coupling between CSWG mode and resonator mode is developed from the CSWG. The resonator coupling system is added above the CSWG and consists of rods cells with selected physical parameters to filter out frequencies at 0.34, 0.365 and 0.355 (a/λ). The transmission spectrum of the add/drop circular has filtering efficiency of more than 95 %. The Q factor of circulator the resonator coupling system is approximately 2364, 1950 and 2536 for the frequency of 0.34, 0.365 and 0.355 (a/λ) respectively.

- (x) The last section discusses about using dual surface waveguide for transmission of light waves in two directions. The dual surface waveguide has two degenerated surface modes. Each surface mode represents different field patterns. The degeneracy of the two surface modes is related to the distance between the surface waveguides. When the distance between surface waveguide increases, the degeneracy increases. When the distance decreases, the modes move away from each other and the degeneracy decreases. The separation between both surface waveguides is set at $2a$ so that the surface waveguide are far enough to have degeneracy. A power splitting device is developed by inserting a large dipole lattice cell in between the dual surface waveguide. Initial simulation results that the splitting efficiency is only 40 % at each surface waveguide. A reflector cell is added to enhance the splitting efficiency and a power splitting ratio of 50 % - 50 % on both surface waveguides. By varying the refractive index of selected cells near the dipole system, the power splitting ratio can be tuned from 10 % - 90 % to 90 % - 10 %.

6.2 Recommendations

The recommendations for future researches are summarized as below.

- (i) In the design of PC tapered waveguide, it is only limited to the case of linear, convex and concave curvature designs. To obtain an even more compact and higher efficiency coupling device, it is important to combine different curvatures together. A good suggestion is to combine the advantages of both concave and convex curvature to form a hybrid curvature taper. The hybrid curvature taper consists of a large receiving tapering section in the front part while slowly reduced the transverse beam size at the end to match with the narrow PCWG. This hybrid design of the tapered waveguide is predicted to achieve even higher coupling efficiency with shorter taper length as it combine the essence of both convex and concave curvatures. However, more complicated simulation and optimization model is needed as there are two parts in the hybrid curvature now.
- (ii) Another future research possibility is to integrate additional cells (holes or rods) structures in the tapering section to double up as a photonic device. It has been reported that addition of cells in the empty section of the tapering section can improved the coupling efficiency of taper [159-160]. By further manipulation of the cells in the tapering region, a photonic filter device can be designed and developed. This is an interesting suggestion because multi-functional coupling system allows a higher device density and reduces the size of the photonic integrated circuit. With the availability of a large empty space in the tapering section, there are unlimited possibilities in adding lattice cell structures to innovate different types of PC based device.

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- (iii) Another future research work is to extend the numerical calculation of the exact step-coupling-theory to other waveguide structures. It is recommended that the general set of coupling equations from the exact step-coupling-theory can be applied to study the mode coupling behavior in asymmetrical waveguides, tapered optical fiber and waveguide structure with sinusoidal deformation. The applications of the exact step-coupling-theory can be used for a vast variety of waveguide with small or large geometrical variations. Other possible characteristics that can be obtained from the theory include power fluctuations investigation and waveguide with spurious mode degeneracies. However some modifications to the coupling equations of the exact step-coupling-theory is required.
- (iv) It is also possible to build a large scale integrated add/drop circulator system for optical communication network system using CPC. There are three add/drop locations in the PC surface circulator. To expand the number of add/drop locations, one can insert addition resonator coupling systems along the surface of the quasi-crystal. Another possible recommendation is to cascaded several PC circulator to form a large scale integrated add/drop multiplexer which can be used for manipulating transmission paths and directions. Both of the recommendations suggested above have their own advantages and disadvantages. Adding more resonator coupling system allows a wider add/drop bandwidth capacity but may result in congestion of the single circulator. Cascading a number of the add/drop circulator together results in larger device but has greater number of dropping frequencies bandwidth.

REFERENCES

- [1] G. E. Moore, "Cramming more components onto integrated circuits". Electronics, vol. **38**, No. 8, pp. 114-117, 1965.
- [2] C. DeCusatis, "Fiber Optic Data Communication: Technological Trends and Advances", Academic Press, 2002.
- [3] http://www.rp-photonics.com/photonic_integrated_circuits.html
- [4] I. Andondovic and D. Uttamchandani, "Principles of Modern Optical Systems", Artech House, 1989.
- [5] C. Manolatau, S. G. Johnson, S. Fan, P. R. Villeneuve, H. A. Haus and J.D.Joannopoulos, "High-Density Integrated Circuits", J. Lightwaves Technol., vol. **17**, No. 9, pp. 1682-1692, 1999.
- [6] http://www.tue.nl/fileadmin/e/e/TTE/OED/Files/Pubs_2005/Smit_PIC_IPRA_05_IBWI.pdf
- [7] E. Yablonovitch, "How to be truly photonic", Science, vol. **289**, pp. 557-559, 2000.
- [8] K.Sakoda, "Optical Properties of Photonic Crystals", Springer Series in Optical Sciences, 2001.
- [9] K.Ohtaka, "Energy Band of photons and low-energy photons diffraction", Phys. Rev. B, vol. **19**, pp. 5057-5067, 1979.
- [10] E.Yablonovitch, "Inhibit spontaneous emission in solid-state physics and electronics", Phys. Rev. Lett., vol. **58**, No. 20, pp. 2059-2061, 1987.
- [11] S.John, "Strong Localization of photons in certain disorder dielectric superlattices", Phys. Rev. Lett., vol. **58**, No. 23, pp. 2486-2489, 1987.

References

-
- [12] M-C.M.Lee, D.Hah, E.K.Lau, H.Toshiyoshi, M.C.Wu, "Nano-Electro-Mechanical photonic crystal switch", OFC Proceedings, pp. 94-95, 2002.
 - [13] M.Loncar, T.Yoshie, K.Okamoto, Y.Qiu, Jelena Vuckovic and A. Scherer, "Planar Photonic Crystal Nanolasers: Porous Cavity Laser", IEICE Trans. of Electron., vol. **87**, No. 3, pp. 291-299, 2004.
 - [14] J. D. Joannopoulos, P. R. Villeneuve and S. Fan, "Photonic Crystals," *Solid State Commun.* **102**, pp. 165 (1997).
 - [15] K.Okamoto, M.Loncar, T.Yoshie and A.Scherer, "Near-field scanning optical microscopy of photonic crystal nanocavities", Appl. Phys. Lett., vol. **82**, No. 11, pp. 1676-1678, 2003.
 - [16] H. J. Kimble, "Strong interactions of single atoms and photons in cavity QED", *Physica Scripta*, **T26**, pp. 127-137, 1998.
 - [17] Y. Akahane, T. Asano, B. S. Song, and S. Noda, "High-Q, photonic nanocavity in a two-dimensional photonic crystal", *Nature*, vol. **425**, pp. 944-947, 2003.
 - [18] A.Yariv, Y.Xu, K.Lee and A.Scherer, "Coupled-resonator optical waveguide: a proposal and analysis", *Opt. Lett.*, vol. **24**, No. 11, pp. 711-713, 1999.
 - [19] S. Y. Lin, E. Chow, V. Hietch, J. D. Joannopoulos, "Experimental Demonstration of Guiding and Bending of EM Waves in Photonic Crystals", *Science*, vol. **282**, pp. 274, 1998.
 - [20] A.Mekis, J.C.Chen, I.Kurland, S.Fan, P.R.Villeneuve and J.D.Joannopoulos, "High Transmission through sharp bends in photonic crystal waveguide", *Phys. Rev. Lett.*, vol. **77**, No. 18, pp. 3787-3790, 1996.

References

- [21] T. Baba, N. Fukaya, and J. Yonekura, "Observation of light propagation in photonic crystal optical waveguides with bends", *Electron. Lett.*, vol. **35**, pp. 654-655, 1999.
- [22] D.Falco, C.Conti and G.Assanto, "Three-dimensional superprism effect in photonic crystals", *Journal of Lightwave Technology*, vol. **21**, No. 2, pp. 561-566, 2003.
- [23] A. Lupu, E. Cassan, S. Laval, L. El Melhaoui, P. Lyan, and J. M. Fedeli, "Experimental evident for superprism phenomena in SOI photonic crystals", *Opt. Express*, vol. **12**, pp. 5690-5696, 2004.
- [24] C. Luo, M. Soljacia, and J. D. Joannopoulos, "Superprism effect based on phase velocities", *Opt. Lett.*, vol. **29**, pp. 745-747, 2004.
- [25] P. Kramper, M. Agio, C. M. Soukoulis, A. Birner, F. Müller, R. B. Wehrspohn, U. Gösele, and V. Sandoghdar, "Highly directional emission from photonic crystal waveguides of subwavelength width", *Phys. Rev. Lett.*, vol. **92**, No. 11, pp. 113903, 2004.
- [26] E. Moreno, F. J. Garcia-Vidal, and L. Martin-Moreno, "Enhanced transmission and beaming of light via photonic crystal surface modes", *Phys. Rev. B* vol. **69**, pp. 121402, 2004.
- [27] W. R. Frei, D. A. Tortorelli, and H. T. Johnson, "Engineering of directional emission from photonic crystal waveguide", *Appl. Phys. Lett.* vol. **86**, pp. 111114, 2005.
- [28] F. Villa, L. E. Regalado, F. Ramos-Mendieta, J. Gaspar-Armenta, and T. Lopez-Rios, "Photonic crystal sensor based on surface waves for thin-film characterization", *Opt. Lett.*, vol. **27**, pp. 646-648, 2002.

References

-
- [29] J. M. Elson, and K. Halterman, “Local density of states analysis of surface wave modes on truncated photonic crystal surfaces with nonlinear material”, *Opt. Express*, vol. **12**, pp. 4855-4863, 2004.
 - [30] S. Xiao, and Min Qiu, “Surface mode microcavity”, *Appl. Phys. Lett.* vol. **87**, pp. 111102, 2005.
 - [31] C. Luo, S. G. Johnson, J. D. Joannopoulos, and J. B. Pendry, “All angle negative refraction without negative refractive index”, *Phys. Rev. B*, vol. **62**, pp. 10696-10705, 2000.
 - [32] E. Cubukcu, K. Aydin, E. Ozbay, S. Foteinopoulou, and S. C. M., “Negative refraction by photonic crystal”, *Nature*, vol. **423**, pp. 604-605, 2003.
 - [33] A. Berrier, M. Mulot, M. Swillo, M. Qiu, L. Thyle’n, A. Talneau, and S. Anand, “Negative refraction at infrared wavelengths in a two-dimensional photonic crystal”, *Phys. Rev. Lett.*, vol. **93**, pp. 073902, 2004.
 - [34] X. Yu, X. Jiang, and S. Fan, “Enhancing or suppressing self-focusing in nonlinear photonic crystals”, *Appl. Phys. Lett.* vol. **90**, pp. 161124, 2007.
 - [35] A. Martínez, H. Míguez, A. Griol, J. Martí, “Experimental and theoretical analysis of the self-focusing of light by a photonic crystal lens”, *Phys. Rev. B*, vol. **69**, pp. 165119, 2004.
 - [36] M. Notomi, K. Yamada, A. shinya, J. Takahashi, C. Takahashi, and I. Yokohama, “Extremely large group velocity dispersion of line defect waveguides in photonic crystal slab”, *Phys. Rev. Lett.* vol. **87**, pp. 253902, 2001.

References

- [37] H. Gersen, T. J. Karle, R. J. P. Engelen, W. Bogaerts, J. P. Korterik, N. F. van Hulst, T. F. Krauss, and L. Kuipers, “Real space observations of ultraslow light in photonic crystal waveguides”, *Phys. Rev. Lett.* vol. **94**, pp. 073903, 2005.
- [38] H. Altug and J. Vuckovic, “Experimental demonstration of the slow group velocity of light in two-dimensional coupled photonic crystal microcavity arrays”, *Appl. Phys. Lett.* vol. **86**, pp. 111102, 2005.
- [39] S. Assefa, S. J. McNab, and Y. A. Vlasov, “Transmission of slow light through photonic crystal waveguide bends”, *Opt. Lett.* vol. **31**, pp. 745-747, 2006.
- [40] M. Soljacic, S. G. Johnson, S. Fan, M. Ibanescu, E. Ippen, and J. D. Joannopoulos, “Photonic crystal slow light enhancement of nonlinear phase sensitivity”, *J. Opt. Soc. Am. B*, vol. **19**, pp. 2052-2059, 2002.
- [41] R. S. Jacobsen, K. Andersen, P. I. Borel, J. Fage-pedersen, L. H. Frandsen, O. Hansen, M. Kristensen, A. Lavrinenko, G. Moulin, H. Ou, C. Peucheret, B. Zsigri, and A. Bjarklev, “Strained silicon as a new electro-optic material”, *Nature*, vol. **441**, pp. 199-202, 2006.
- [42] S. Nishikawa, S. Lan, N. Ikeda, Y. Sugimoto, I. Hiroshi, and A. Kiyoshi, “Optical characterization of photonic crystal delay lines based on one-dimensional coupled defects”, *Opt. Lett.*, vol. **27**, pp. 2079-2081, 2002.
- [43] M. Imada, S. Noda, A. Chutinan, T. Tokuda, M. Murata, and G. Sasaki, “Coherent two-dimensional lasing action in surface-emitting laser with triangular lattice photonic crystal structure”, *Appl. Phys. Lett.* vol. **75**, pp. 316-318, 1999.

References

-
- [44] D. Ohnishi, K. Sakai, M. Imada, and S. Noda, "Continuous wave operation of surface emitting two dimensional photonic crystal laser", *Electron. Lett.*, vol. **39**, pp. 612-614, 2003.
 - [45] M. Yokoyama, and S. Noda, "Finite difference time domain simulation of two-dimensional photonic crystal surface emitting laser having a square lattice slab structure", *IEICE Trans. Electron.*, vol. **E87C**, pp. 386-392, 2004.
 - [46] S. Noda, M. Yokoyama, M. Imada, A. Chutinana, and M. Mochizuki, "Polarization mode control of two dimensional photonic crystal laser by unit cell structure design", *Science*, vol. **293**, pp. 1123-1125, 2001.
 - [47] B. S. Song, S. Noda, and T. Asano, "Photonic devices based on in plane heterophotonic crystal", *Science*, vol. **300**, pp. 1537, 2003.
 - [48] C. Zhang, F. Qiao, J. Wan, and J. Zi, "Enlargement of nontransmission frequency range in photonic crystals by using multiple heterostructures", *J. Appl. Phys.*, vol. **87**, pp. 3174-3176, 2000.
 - [49] E. Istrate, M. C. Lefort, and E. H. Sargent, "Theory of photonic crystal heterostructures", *Phys. Rev. B*, vol. **66**, pp. 075121, 2002.
 - [50] B. S. Song, S. Noda, T. Asano, and Y. Akahane, "Ultra-high- Q photonic double-heterostructure nanocavity", *Nature Mater.*, vol. **4**, pp. 207-210, 2005.
 - [51] P. Yeh, "Optical Waves in Layered Media", Wiley Publishers, New York, 1988
 - [52] Lord Rayleigh, "On the reflection of light from a regularly stratified medium", *Proceedings of the Royal Society of London*, vol. **93**, pp. 565-577, 1917.
 - [53] J. D. Joannopoulos, R. D. Meade and J. N. Winn, "Photonic crystals: Molding the Flow of Light", Princeton University Press, Princeton, 1995

References

- [54] R. D. Meade, A. M. Rappe, K. D. Brommer, J. D. Joannopoulos, and O. L. Alerhand, "Accurate theoretical analysis of photonic band-gap material", *Phys. Rev. B*, vol. **48**, pp. 8434-8437, 1993.
- [55] B. T. Holland, C. F. Blanford, and A. Stein, "Synthesis of macroporous minerals with highly ordered three-dimensional arrays of spheroidal voids", *Science*, vol. **281**, pp. 538, 1998.
- [56] S. Y. Lin, J. G. Fleming, D. L. Hetherington, B. K. Smith, R. Biswas, K. M. Ho, M. M. Sigalas, W. Zubrzycki, S. R. Kurtz, and J. Bur, "A three-dimensional photonic crystal operating at infrared wavelengths", *Nature*, vol. **394**, pp. 251-252, 1998.
- [57] F. Bloch, "Über die quantenmechanik der electronen in kristallgitten", *Physik Z*, vol. **52**, pp. 555-600, 1928.
- [58] James Clerk Maxwell, "A Dynamical Theory of the Electromagnetic Field", *Philosophical Transactions of the Royal Society of London*, vol. **155**, pp. 459-512, 1865.
- [59] James Clerk Maxwell, "A Treatise on Electricity and Magnetism", Dover, 1954.
- [60] J. Arriaga, "Omni-directional gap of 1-D photonic crystals based on porous silicon with a Gaussian profile refractive index", *Physica Status Solidi*, vol. **4**, pp. 534-536, 2007.
- [61] M. Plihal, and A. A. Maradudin, "Photonic band structure of two-dimensional systems: The triangular lattice", *Phys. Rev. B*, vol. **44**, pp. 8565-8571, 1991.

References

-
- [62] X Ao, L. Liu, L. Wosinski, and S. He, "Polarization beam splitter based on a two-dimensional photonic crystal of pillar type", *Appl. Phys. Lett.*, vol. **89**, pp. 171115, 2006.
 - [63] S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, "Channel Add-Drop filters in photonic crystals", *Opt. Express*, vol. **3**, pp. 4-11, 1998.
 - [64] D. Park, S. Kim, I. Park, and H. Lim, "Higher order optical resonant filters based on coupled defect resonators in photonic crystals", *J. Lightwaves Technol.*, vol. **23**, No. 5, pp. 1923-1928, 2005.
 - [65] H. Ren, C. Jiang, W. Hu, M. Gao, and J. Wang, "Design and analysis of two-dimensional photonic crystals channel filter", *Opt. Commun.*, vol. **266**, pp. 342-348, 2006.
 - [66] P. Yao, X. Chen, B. Chen, Y. Lu, P. Wang, X. Jiao, H. Ming, and J. Xie, "Optical reflector and high Q filter based on two-dimensional photonic-crystal waveguide", *Opt. Commun.*, vol. **236**, pp. 101-107, 2004.
 - [67] M. Notomi, A. Shinya, S. Mitsugi, G. Kira, E. Kuramochi, and T. Tanabe, "Optical bistable switching action of Si high-Q photonic crystal nanocavities", *Opt. Express*, vol. **13**, pp. 2678-2687, 2005.
 - [68] J. R. Cao, P. T. Lee, S. J. Choi, R. Shafiiha, S. J. Choi, J. D. O. Brien, P. D. Dapkus, "Nanofabrication of photonic crystal membrane lasers," *J. Vacuum Science B*, vol. **20**, pp. 618-621, 2002.
 - [69] M. Notomi, M. Taniyama, S. Mitsugi, and E. Kuramochi, "Optomechanical wavelength and energy conversion in high-Q double layer cavities of photonic crystal slab", *Phys. Rev. Lett.*, vol. **97**, pp. 023903, 2006.

References

- [70] H. Nemec, P. Kuzel, L. Duvillaret, A. Pashkin, M. Dressel, and M. T. Sebastian, “Highly tunable photonic crystal filter for the terahertz range”, *Opt. Lett.*, vol. 30, pp. 549-551, 2005.
- [71] J.D.Joannopoulos, P.R.Villeneuve and S.Fan, “Photonic crystals: Putting a New Twist on Light”, *Nature*, vol. **386**, pp. 143, 1997.
- [72] J. Moosburger, M. Kamp, A. Forchel, S. Olivier, H. Benisty, C. Weisbuch, U. Oesterle, “Enhanced transmission through photonic crystal based bend waveguides and bend engineering”, *Appl. Phys. Lett.*, vol. **79**, pp. 3579-3581, 2001.
- [73] A. Chutinan, M. Okano, and S. Noda, “Wider bandwidth with high transmission through waveguide bends in two dimensional photonic crystal slabs”, *Appl. Phys. Lett.*, vol. **80**, pp. 1698-1700, 2002.
- [74] J. S. Jensen, and O. Sigmund, “Systematic design of photonic crystal structures using topology optimization: Low-loss waveguide bends”, *Appl. Phys. Lett.*, vol. **84**, pp. 2022-2024, 2004.
- [75] T. A. Birks, J. C. Knight, and P. S. J. Russell, “Endlessly single-mode photonic crystal fiber,” *Opt. Lett.*, vol. **22**, pp. 961- 963, 1997.
- [76] J. C. Knight, “Photonic crystal fibres,” *Nature*, vol. **424**, pp. 847-851, 2003.
- [77] S. Combrie, A. De Rossi, L. Morvan, S. Tonda, S. Cassette, D. Dolfi, and A. Talneau, “Time-delay measurement in singlemode, low-loss photonic crystal waveguides”, *Electron. Lett.*, Vol. **42**, pp. 86-88, 2006.
- [78] S. G. Johnson, C. Manolatou, S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and H. A. Haus, “Elimination of cross talk in waveguide intersections”, *Opt. Lett.*, vol. **23**, pp. 1855-1857, 1998.

References

- [79] S.Boscolo, M.Midrio and Carlo G.Someda, "Coupling and Decoupling of Electromagnetic Waves in Parallel 2-D Photonic Crystal Waveguides", *J. Quant. Electron.*, vol. **38**, pp. 47-53, 2002.
- [80] H. Kim, I. Park, B. Hoan O, S. Park, E. Lee, and S. Lee, "Self-imaging phenomena in multimode photonic crystal line defect waveguides: application to wavelength demultiplexing", *Opt. Express*, vol. **12**, pp. 5625-5633, 2004.
- [81] H. Takano, B. S. Song, T. Asano, and S. Noda, "Highly efficient multi-channel drop filter in a two-dimensional hetero photonic crystal", *Opt. Express*, vol. **14**, pp. 3491-3496, 2006.
- [82] K. Ishihara, M. Fujita, I. Matsubara, T. Asano, S. Noda, H. Ohata, A. Hirasawa, H. Nakada, and N. Shimoji, "Organic light-emitting diodes with photonic crystals on glass substrate fabricated by nanoimprint lithography", *Appl. Phys. Lett.*, vol. **90**, pp. 111114, 2007.
- [83] M. Boroditsky, T. F. Krauss, R. Coccioli, R. Vrijen, R. Bhat, and E. Yablonovitch, "Light extraction from optically pumped light emitting diode by thin slab photonic crystals", *Appl. Phys. Lett.*, vol. **75**, pp. 1036-1038, 1999.
- [84] C. N. Panoiu, M. Bahl, and M. R. Osgood, "Optically tunable superprism effect in nonlinear photonic crystal", *Opt. Lett.*, vol. **28**, pp. 2503-2505, 2003.
- [85] A. Lambrecht, S. Hartwig, S. L. Schweizer, and R. B. Wehrspohn, "Miniature infrared gas sensors using photonic crystals", *Proceedings of the SPIE*, vol. 6480, pp. 64800D, 2007.

References

- [86] B. Miao, C. Chen, A. Sharkway, S. Shi, and D. W. Prather, "Two bit optical analog-to-digital converter based on photonic crystals", *Opt. Express*, vol. 14, pp. 7966-7973, 2006.
- [87] M. Richard, A. Sharkway, C. Chen, B. Miao, Eric J. Kelmelis, and D. W. Prather, "A reconfigurable self-collimation-based photonic crystal switch in silicon", *Proceedings of the SPIE*, vol. 6475, pp. 64750B, 2007.
- [88] T. Decoopman, G. Tayeb, S. Enoch, D. Maystre, and B. Gralak, "Photonic crystal lens: From negative refraction and negative index to negative permittivity and permeability", *Phys. Rev. Lett.*, vol. 97, pp. 073905, 2006.
- [89] D. Tailaert, W. Bogaerts, P. Bienstman, T. Krauss, P. V. Daele, I. Moerman, S. Verstuyft, K. D. Mesel, and R. Baets, "An out-of plane grating coupler for efficient butt coupling between compact planar waveguides and single mode fibres," *IEEE J. Quantum Electron.*, vol. 38, pp. 949–954, 2002.
- [90] P. E. Barclay, K. Srinivasan, M. Borselli, and O. Painter, "Efficient input and output fiber coupling to a photonic crystal waveguide," *Opt. Lett.*, vol. 29, pp. 697–699, 2004.
- [91] E. Miyai, M. Okano, M. Mochizuki, and S. Noda, "Analysis of coupling between 2-D photonic crystal waveguide external waveguide", *Appl. Phys. Lett.*, vol. 81, pp. 3729-3731, 2002.
- [92] D. W. Prather, J. Murakowski, S. Shi, S. Venkataraman, C. Chen, and D. Pustai, "High-efficiency coupling structure for a single-line-defect photonic crystal waveguide," *Opt. Lett.*, vol. 27, pp. 1601–1603, 2002.

References

- [93] A. Mekis and J. D. Joannopoulos, "Tapered couplers for efficient interfacing between dielectric and photonic crystal waveguides," *J. Lightwave Technol.*, vol. **19**, pp. 861–865, 2001.
- [94] R. B. Smith, "Coupling efficiency of tapered coupler", *Electron. Lett.*, vol. **11**, pp. 204-206, 1975.
- [95] R. P. Hecken, "A near optimum matching section without discontinuities", *Transaction on Microwave Theory and Techniques*, vol. 20, pp. 734-739, 1972.
- [96] T.D.Happ, M.Kamp and A.Forchel, "Photonic Crystal taper for ultracompact mode conversion", *Optics Letters*, vol. **26**, pp. 1102-1104, 2001.
- [97] P.Lalanne and A.Talneau, "Modal conversion with artificial materials for photonic crystals waveguide", *Optics Express*, vol. **10**, pp. 354-359, 2002.
- [98] M. Palamaru and Ph. Lalanne, "Photonic crystal waveguides: out-of-plane losses and adiabatic mode conversion," *Appl. Phys. Lett.*, vol. **78**, pp. 1466–1468, 2001.
- [99] P. Bienstman, S. Assefa, S. G. Johnson, J. D. Joannopoulos, G. S. Petrich, and L. A. Kolodziejski, "Taper structures for coupling into photonic crystal slab waveguide," *J. Opt. Soc. Am. B*, vol. **20**, pp. 1817–1821, 2003.
- [100] P. Pottier, I. Ntakis, and R. M. De La Rue, "Photonic crystal continuous taper for low-loss direct coupling into photonic crystal channel waveguides and further device functionality," *Opt. Commun.*, vol. **223**, pp. 339–347, 2003.
- [101] E. H. Khoo, A. Q. Liu, and J. H. Wu, "Nonuniform photonic crystal taper for high efficient mode coupling," *Opt. Express*, vol. **13**, pp. 7748-7759, 2005.

References

- [102] M. Koshiba, Y. Tsuji, and M. Hikari, "Time-domain beam propagation method and its application to photonic crystal circuits," *J. Lightwave Technol.*, vol. **18**, pp. 102-110, 2000.
- [103] T. Fujisawa, and M. Koshiba, "Time-domain beam propagation method for nonlinear optical propagation analysis and its application to photonic crystal circuits", *J. Lightwaves Technol.*, vol. **22**, pp. 684-691, 2004.
- [104] M. Skorobogatiy, S. G. Johnson, S. A. Jacobs, and Y. Fink, "Dielectric profile variations in high-indexcontrast waveguides, coupled mode theory, and perturbation expansions," *Phys. Rev. E*, vol. **67**, pp. 046613, 2003.
- [105] M. Skorobogatiy, "Modeling the impact of imperfections in high-index-contrast photonic crystal waveguides," *Phys. Rev. E*, vol. **70**, pp. 046609, 2004.
- [106] G. -W. Chern, L. A. Wang, and C.-Y. Lin, "Transfer-Matrix Approach Based on Modal Analysis for Modeling Corrugated Long-Period Fiber Gratings," *Appl. Opt.*, vol. **40**, pp. 4476-4486, 2001.
- [107] Z. Y. Li, and K. M. Ho, "Light propagation in semi-infinite photonic crystals and related waveguide structures," *Phys. Rev. B*, vol. **68**, pp. 155101, 2003.
- [108] A. F. Milton, and W. K. Burns, "Mode Conversion in planar dielectric separating waveguides," *IEEE J. Quantum Electron.*, vol. **11**, pp. 32-39, 1975.
- [109] A. R. Nelson, "Coupling optical waveguides by tapers," *Appl. Opt.*, vol. **14**, pp. 3012-3015, 1975.
- [110] D. Marcuse, *Theory of Dielectric Optical Waveguide*, Academic Press, San Diego, 1991.

References

-
- [111] R. de L. Kronig, and W. G. Penney, "Quantum Mechanics of Electrons in Crystal Lattices", Proc. R. Soc. London, vol. **130**, pp. 499-513, 1930.
 - [112] P. Yeh, and A. Yariv, "Optical surface waves in periodic layered media", Appl. Phys. Lett., vol. **32**, pp. 104-105, 1978.
 - [113] U. Rossler, Solid state theory: an introduction, Berlin, Springer, 2004.
 - [114] R. D. Meade, K. D. Brommer, A. M. Rappe, and J. D. Joannopoulos, "Electromagnetic Bloch waves at the surface of a photonic crystal", Phys. Rev. B, vol. **44**, pp. 10961, 1991.
 - [115] B. Wang, W. dai, A. Fang, L. Zhang, G. Tuttle, Th. Koschny, and C. M. Soukoulis, "Surface waves in photonic crystal slabs", Phys. Rev. B, vol. **74**, pp. 195104, 2006.
 - [116] S. Xiao, M. Qiu, Z. Ruan, and S. He, "Influence of the surface termination to the point imaging by a photonic crystal slab with negative refraction", Appl. Phys. Lett., vol. **85**, pp. 4269-4271, 2004.
 - [117] Z. Zhang, M. Dainese, L. Wosinski, S. Xiao, M. Qiu, "Optical filter based on two-dimensional photonic crystal surface mode cavity in amorphous silicon on silica structure", Appl. Phys. Lett., vol. **90**, pp. 041108, 2007.
 - [118] X. Yu, and S. Fan, "Anomalous reflection at photonic crystal surfaces", Phys. Rev. E, vol. **70**, pp. 055601, 2004.
 - [119] J. Yang, S. Kim, G. Kim, H. Park, Y. Lee, and S. Kim, "Slab-edge modes in two dimensional photonic crystals", Appl. Phys. Lett., vol. **84**, pp. 3016-3018, 2004.
 - [120] W. Lau, and S. Fan, "Creating large bandwidth line defects by embedding dielectric waveguides into photonic crystal slabs", Appl. Phys. Lett., vol. **81**, pp. 3915-3917, 2002.

References

- [121] Y. A. Vlasov, N. Moll, and S. J. McNab, "Observation of surface states in a truncated photonic crystal slab", *Opt. Lett.*, vol. **29**, pp. 2175-2177, 2004.
- [122] H. Chen, Kevin K. Tsia, and Andrew W. Poon, "Surface modes in two-dimensional photonic crystal slabs with a flat dielectric margin", *Opt. Express*, vol. **14**, pp. 7368-7377, 2006.
- [123] A. I. Rahachou, and I. V. Zozoulenko, "Waveguiding properties of surface states in photonic crystals", *J. Opt. Soc. Am. B*, vol. **23**, 1679-1682 (2006).
- [124] F. Sporleder, and H. G. Unger, "Waveguide Tapers, Transitions and Couplers", Peter Peregrinus, New York, 1974.
- [125] S. Guo, and S. Albin, "A simple plane wave implementation method for photonic crystal calculation", *Opt. Express*, vol. **11**, pp. 167-181, 2003.
- [126] J.D.Jackson, "Classical Electrodynamics", Wiley, 1999.
- [127] A. Mekis, S. Fan, and J. D. Joannopoulos, "Bound states in photonic crystal waveguides and waveguide bends", *Phys. Rev. B*, vol. **58**, pp.4809-4817, 1998.
- [128] M. Qiu, K. Azizi, A. Karlsson, M. Swillo, and B. Jaskorzynska, "Numerical studies of mode gaps and coupling efficiency for line-defect waveguides in two-dimensional photonic crystals", *Phys. Rev. B*, vol. **64**, 155113, 2001.
- [129] E. H. Khoo, A. Q. Liu, J. H. Wu, J. Li, and D. Pinjala, "Modified step-theory for investigating mode coupling mechanism in photonic crystal waveguide taper", *Opt. Express*, vol. **14**, pp. 6035-6054, 2006.
- [130] S. Assefa, P. T. Rakich, P. Bienstman, S. G. Johnson, G. S. Petrich, J. D. Joannopoulos, L. A. Kolodziejski, E. P. Ippen, and H. I. Smith, "Guiding 1.5m light

References

- in photonic crystals based on dielectric rods,” *Appl. Phys. Lett.*, vol. **85**, pp. 6110-6112, 2004.
- [131] S. G. Johnson, P. Bienstman, M. A. Skorobogatiy, M. Ibanescu, E. Lidorikis, and J. D. Joannopoulos “Adiabatic theorem and continuous coupled mode theory for efficient taper transition,” *Phys. Rev. E*, vol. **66**, pp. 066608, 2002.
- [132] H. Benisty, “Modal analysis of optical guides with two-dimensional photonic band-gap boundaries,” *J. Appl. Phys.* **79**, 7483–7492 (1996).
- [133] L. Borruel, I. Esquivias, P. Moreno, M. Krakowski, S. C. Auzanneau, M. Calligaro, O. Parillaud, M. Lecomte, S. Sujecki, J. Wykes, E. C. Larkins, “Clarinet laser: Semiconductor laser design for high-brightness applications”, *Appl. Phys. Lett.*, vol. **87**, pp. 101104, 2005.
- [134] S. A. Schelkunoff, “Conversion of Maxwell equations into generalized telegraphist’s equations”, *J. Bell, Syst Tech*, vol. **34**, pp. 995-1043, 1955.
- [135] G. Reiter, “Generalized telegraphist’s equation for waveguides of varying cross-section”, *Proc. IEE*, vol. **106B**, pp. 54-57, 1959.
- [136] E. Yablonovitch, T. J. Gmitter, and K. M Leung, “Photonic band structure: The face-centered-cubic case employing nonspherical atoms,” *Phys. Rev. Lett.*, vol. **67**, pp. 2295-2298, 1991.
- [137] S. Fan, P. R. Villeneuve, R. Meade, and J. D. Joannopoulos, “Design of three-dimensional photonic crystals at submicron lengthscales,” *Appl. Phys. Lett.*, vol. **65**, pp. 1466-1468, 1994.
- [138] H. S. Sözüer, and J. P Dowling, “Photonic Band Calculations for Woodpile Structures,” *J. Mod. Opt.*, vol. **41**, pp. 231-235, 1994.

References

- [139] S. G. Johnson, P. R. Villeneuve, S. H. Fan, and J. D. Joannopoulos, "Linear waveguide in photonic crystal slab," *Phys Rev. B*, vol. **62**, pp. 8212-8222, 2000.
- [140] S. G. Johnson, S. Fan, P. R. Villeneuve, J. D. Joannopoulos, and L. A. Kolodziejski, "Guided modes in photonic crystal slabs," *Phys. Rev. B*, vol. **60**, pp. 5751-5758, 1999.
- [141] X. Luo, J. Shi, H. Wang, and G. Yu, "Surface Plasmon polariton from metallic photonic crystal slab breaking the diffraction limit", *Modern Phys. Lett. B*, vol. **18**, pp. 945-953, 2004.
- [142] P. R. Villeneuve, S. Fan, and J. D. Joannopoulos, "Microcavities in Photonic Crystals: Mode Symmetry, Tunability and Coupling Efficiency," *Phys. Rev. B*, vol. **54**, pp. 7837, 1996.
- [143] N. Horiuchi, Y. Segawa, T. Nozokido, K. Mizuno, and H. Miyazaki, "Isotopic photonic gaps in a circular photonic crystal", *Opt. Lett.*, vol. **29**, pp. 1084-1086, 2004.
- [144] N. Horiuchi, Y. Segawa, T. Nozokido, K. Mizuno, and H. Miyazaki, "High transmission waveguide with a small radius of curvature at a bend fabricated by the use of a circular photonic crystal", *Opt. Lett.*, vol. **30**, pp. 973-975, 2005.
- [145] J. Zarbakhsh, F. Hagmann, S. F. Minaleev, K. Busch, and K. Hingerl, "Arbitrary angle waveguiding applications of two-dimensional curvilinear-lattice photonic crystal", *Appl. Phys. Lett.*, vol. **84**, pp. 4687-4689, 2004.
- [146] J. Xu, J. Song, C. Li, K. Ueda, "Cylindrical symmetrical hollow fibre", *Opt. Commun.*, vol. **182**, pp. 343-348, 2000.

References

-
- [147] A. Tavlove, "Computational electrodynamics: The finite different time domain method", Artech House, Norwood, MA, 1995.
 - [148] P. Luan, and K. Chang, "Transmission characteristics of finite periodic dielectric waveguides", *Opt. Express*, vol. **14**, pp. 3263-3272, 2006.
 - [149] A. Yariv, "Coupled mode theory for guided wave optics", *J. Quantum Electron.* **9**, pp. 919-933, 1973.
 - [150] S. Y. Lin, E. Chow, J. Bur, S. G. Johnson, and J. D. Joannopoulos, "Low-loss wide-angle, Y splitter at $\sim 1.6 \mu\text{m}$ wavelengths built with a two-dimensional photonic crystal", *Opt. Lett.*, vol. **27**, pp. 1400-1402, 2002.
 - [151] S. Boscolo, M. Midrio, and T. F. Krauss, "Y junction in photonic crystal channel waveguides: High transmission and impedance matching", *Opt. Lett.*, vol. **27**, pp. 1001-1003, 2002.
 - [152] L. H. Frandsen, P. I. Borel, Y. X. Zhuang, A. Harpoth, M. Thorhauge, M. Kristensen, W. Bogaerts, P. Dumon, R. Baets, V. Wiaux, J. Wouters, and S. Beckx, "Ultralow-loss 3-dB photonic crystal waveguide splitter", *Opt. Lett.*, vol. **29**, pp. 1623-1625, 2004.
 - [153] V. F. Rodriguez-Esquerre, M. Koshiba, H. E. Hernandez-Figueroa, and C. E. Rubio-Mercedes, "Power splitters for waveguides composed by ultralow refractive index metallic nanostructures", *Appl. Phys. Lett.*, vol. **87**, pp. 091101, 2005.
 - [154] T. Liu, A. R. Zakharian, M. Fallahi, J. V. Moloney, and M. Mansuripur, "Multimode interference-based photonic crystal waveguide power splitter", *J. Lightwave Technol.*, vol. **22**, pp. 2842-2846, 2004.

References

- [155] J. Yonekura, M. Ikeda, and T. Baba, "Analysis of finite 2-D photonic crystals of columns and lightwave devices using the scattering matrix method," *J. Lightwave Technol.*, vol. **17**, pp. 1500–1508, 1999.
- [156] Selin H. G. Teo, A. Q. Liu, J. B. Zhang, and M. H. Hong, "Induced free carriers modulation of photonic crystal optical intersection via localized optical absorption effect", *Appl. Phys. Lett.*, vol. **89**, 091910, 2006.
- [157] M. T. Tinker, and J. B. Lee, "Thermo-optic photonic crystal light modulator", *Appl. Phys. Lett.*, vol. **86**, pp.221111, 2005.
- [158] J. Manzanares-Martinez, F. Ramos-Mendieta, and P. Halevi, "Superresolution and corrections to the diffusion approximation in optical tomography", *Appl. Phys. Lett.*, vol. **87**, pp.101110, 2005.
- [159] P. Sanchis, J. Marti, J. Blasco, A. Martinez, and A. Garcia, "Mode matching technique for highly efficient coupling between dielectric waveguides and planar photonic crystal circuits", *Opt. Express*, vol. **10**, pp. 1391-1397, 2002.
- [160] P. Sanchis, J. Marti, P. Bienstman, and R. Baets, "Semianalytic approach for analyzing coupling issues in photonic crystal structures", *Appl. Phys. Lett.*, vol. **87**, pp.203107, 2005.
- [161] A. J. Ward, and J. B. Pendry, "A program for calculating photonic band structures, Green's functions and transmission/reflection coefficient using a non –orthogonal FDTD method", *Comput. Phys. Comm.* **128**, pp. 590-621, 2000.
- [162] <http://www.mathworks.com/>

AUTHOR'S PUBLICATIONS

Journal papers

1. A. Q. Liu, E. H. Khoo, T. H. Cheng, E. P. Li, and J. Li, "A frequency selective circulator via mode coupling between surface waveguide and resonators," Appl. Phys. Lett. **92**, 021119 (2008).
2. E. H. Khoo, A. Q. Liu, T. H. Cheng, J. Li, and D. Pinjala, "Light focusing via concave surface of photonic crystal," Appl. Phys. Lett. **91**, 221105 (2007).
3. E. H. Khoo, T. H. Cheng, A. Q. Liu, J. Li, and D. Pinjala, "Transmitting light efficiently on photonic crystal surface waveguide bend," Appl. Phys. Lett. **91**, 171109 (2007).
4. E. H. Khoo, A. Q. Liu, J. H. Wu, J. Li, and D. Pinjala "Modified step-theory for investigating mode coupling mechanism in photonic crystal waveguide taper," Opt. Express **14**, 6035 (2006).
5. E. H. Khoo, A. Q. Liu, and J. H. Wu, "Nonuniform photonic crystal taper for high efficient mode coupling," Opt. Express **13**, 7748 (2005).

Conference papers

1. E. H. Khoo, J. Li, H. Cai, D. Pinjala and A. Q. Liu, "An Integrated Photonic MEMS Switch System with Fast Switching Speed and Low Power Demand," Transducer 2007, 10-14 June 2007, Lyon, France
2. E.H. Khoo, A. Q. Liu, "Photonic quasi-crystal add-drop filter via coupling between surface waveguide modes and resonator modes," Photonics and Electromagnetic Crystal Symposium (PECS VII), 8-11 April 2007, Monterey, CA
3. E. H. Khoo, A. Q. Liu, "Light focusing via concave corrugated surface of photonic crystal," Photonics and Electromagnetic Crystal Symposium (PECS VII), 8-11 April 2007, Monterey, CA
4. E. H. Khoo, A. Q. Liu, "Mode splitting phenomenon in dual row defect nanophotonic waveguide," Photonics and Electromagnetic Crystal Symposium (PECS VII), 8-11 April 2007, Monterey, CA
5. E. H. Khoo, J. H. Wu and A. Q. Liu, "Investigation of Distortion Effect for Photonic Crystal Taper," Photonics and Electromagnetic Crystal Symposium (PECS VI), 19-24 June 2005, Crete, Greece
6. E.H. Khoo, J. H. Wu and A. Q. Liu, "Multipole Physics for Mode Enhancement in Photonic Crystal Taper," Photonics and Electromagnetic Crystal Symposium (PECS VI), 19-24 June 2005, Crete, Greece

Appendix A

In this appendix, the derivation of the modified step-theory is shown in details. Starting from classical Maxwell's equation, in the absence of field current and source, the electric and magnetic field in electromagnetic waves are related in the equation given as

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} \quad (\text{A1})$$

$$\nabla \times \mathbf{H} = \varepsilon \frac{\partial \mathbf{E}}{\partial t} \quad (\text{A2})$$

where μ and ε are the permeability and permittivity of the material in which the field exist. Assuming linear and lossless material, the complex field $\mathbf{E}$ and $\mathbf{H}$ can be expanded to a set of harmonic modes with complex time exponential $e^{i\omega t}$. Rearranging Eqs. (A1) and (A2) and substituting the field harmonics, the equation for the electric field is given as

$$\nabla \times (\nabla \times \mathbf{E}) = \left(\frac{\omega}{v} \right)^2 \mathbf{E} \quad (\text{A3})$$

where v is the speed of electromagnetic waves in the material with refractive index, n given by

$$v = \frac{1}{\sqrt{\varepsilon\mu}} = \frac{c}{n} \quad (\text{A4})$$

For $\mathbf{H}$ field, similar expression can be obtained. By applying vector properties for the $\mathbf{E}$ field and Coulomb's law, the curl of the curl symbol can be removed and be replaced by ∇^2 to form the scalar wave equation given as

Appendix A

$$\nabla^2 E = \left(\frac{\omega}{v}\right)^2 E \quad (\text{A5})$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Considering the electromagnetic waves propagating along the x direction in a planar channel waveguide extending the x and z direction. The wave solution for the electric field is described by the normal modes given as [25]

$$E = E_0 \xi(z) \times e^{i\beta x} \quad (\text{A6})$$

where β is the propagating constant of the waves traveling in the waveguide and $\xi(z)$ is the z dependent field distribution of the guided mode. E_0 is the real amplitude of the propagating electric field. As mentioned in section 2, photonic crystal waveguide mode field resembles modes in conventional waveguide [26-27]. A close approximation is to consider it to be a case of a metallic waveguide of width b , on which an artificial periodicity of d is imposed. The equation for the mode field of the guided modes is given as

$$E = E_0 \sin\left(m\pi\left(\frac{z}{b} + \frac{1}{2}\right)\right) \exp[-i\psi(x)] \quad (\text{A7})$$

where the phase coefficient, ψ is given as

$$\psi(x) = \left(\beta + \frac{2\pi\eta}{d}\right)x \quad \eta = 0, \pm 1, \pm 2, \dots, \quad (\text{A8})$$

Before deriving the equations, the field distribution in Eq. (A7) is replaced by the symbol ζ so as to give a clearer deriving process. Consider there are two modes, i and j are propagating at the x direction. Both modes travel to a narrower waveguide of width b' with the same periodicity. The wider waveguide is labeled as the n th step with subscript

Appendix A

n , while the narrower waveguide is labeled as the $(n+1)$ th step with subscript $n+1$. Matching the boundary conditions between the n th and $(n+1)$ th steps, the incident, reflected and transmitted field components are given as

$$\begin{aligned} & E_{in} \zeta_{in} \exp(-i\beta_{in}) + E_{jn} \zeta_{jn} \exp(-i\beta_{jn}) \\ & + E_{in}^R \zeta_{in} \exp(i\beta_{in}^R) + E_{jn}^R \zeta_{jn} \exp(i\beta_{jn}^R) + E^{ref} \\ & = E_{in+1} \zeta_{in+1} \exp(-i\beta_{in+1}) + E_{jn+1} \zeta_{jn+1} \exp(-i\beta_{jn+1}) + e^{trans} \end{aligned} \quad (A9)$$

Beside the main propagating modes, there also exist other modes such as the transmitted radiation modes, E^{trans} and the reflected radiation modes, E^{ref} . These two modes will vanish during the course of derivation by applying orthogonal condition. For H -field in z direction, similar expressions can be obtained as

$$\begin{aligned} & \beta_{in} E_{in} \zeta_{in} \exp(-i\beta_{in}) + \beta_{jn} E_{jn} \zeta_{jn} \exp(-i\beta_{jn}) \\ & - \beta_{in}^R E_{in}^R \zeta_{in} \exp(i\beta_{in}^R) - \beta_{jn}^R E_{jn}^R \zeta_{jn} \exp(i\beta_{jn}^R) \\ & + H^{ref} = \beta_{in+1} E_{in+1} \zeta_{in+1} \exp(-i\beta_{in+1}) + \beta_{jn+1} E_{jn+1} \zeta_{jn+1} \\ & \exp(-i\beta_{jn+1}) + H^{trans} \end{aligned} \quad (A10)$$

where E^R refer to the real amplitude of the reflected guided mode. By multiplying Eqs. (A9) and (A10) by a transmitted mode field distribution and integrating over x , the unguided radiation modes are eliminated because of the orthogonality. The aim now is to obtain an iterative equation for field amplitude and phase of j th mode, E_{jn+1} and ψ_{jn+1} . To do this, the expression for the reflected field of the i th mode must be obtained. Together with the assumption made in section 3 and multiplying Eqs. (A9) and (A10) by

Appendix A

transmitted mode field of the ζ_{in+1} and integrating over x , the expression for the reflected field of mode is given as

$$E_{in}^R \exp(i\psi_{in}^R) = \left(\frac{\psi_{in} - \psi_{in+1}}{\psi_{in} + \psi_{in+1}} \right) E_{in} \exp(-i\psi_{in}) \quad (A11)$$

Eqs. (A9) and (A10) are multiplied by the transmitted field ζ_{jn+1} and integrating over x . After some algebraic manipulation and substituting the reflected field for i th mode with Eq. (A11), the transmitted field expression for the j th mode is obtained as

$$E_{jn+1} I_{jn+1,jn+1} \exp(-i\beta_{jn+1}) = \left(\frac{2\beta_{in}}{\beta_{jn} + \beta_{jn+1}} \right) \frac{\beta_{jn} + \beta_{in+1}}{\beta_{in} + \beta_{in+1}} * \\ E_{in} I_{in,jn+1} \exp(-i\beta_{in}) + 2 \left(\frac{\beta_{jn}}{\beta_{jn} + \beta_{jn+1}} \right) E_{jn} I_{jn,jn+1} \exp(-i\beta_{jn}) \quad (A12a)$$

where
$$I_{in,jn+1} = \int \zeta_{in} \zeta_{jn+1} dx \quad (A12b)$$

$I_{in,jn+1}$ is refer as the field overlap integral of the respective modes at each side of the steps. The field amplitude in Eq. (A12a) is normalized by the amplitude of the incident field of the respective mode, E_m which has a mode power, P_m of unity. The mode power, P_m is related to the unity mode field E_m for the transmitted j^{th} mode at the $(n+1)$ th step can be expressed as

$$P_{mj} = (E_{mj})^2 \frac{\psi_{jn+1}}{2k_0} I_{jn+1,jn+1} = 1 \quad (A13)$$

where k_0 is the free space wave vector. All the terms of Eq. (A12a) is divided by appropriate factor of the unity mode field and simplifying, the amplitude and phase of the j^{th} mode at $(n+1)$ th step is given by

Appendix A

$$A_{jn+1} \exp(-i\psi_{jn+1}) = c_{ij} A_{in} \exp(-i\psi_{in}) + c_{jj} A_{jn} \exp(-i\psi_{jn}) \quad (\text{A14})$$

where the coupling constant c_{ij} is given by

$$\nabla^2 E = \left(\frac{\omega}{v}\right)^2 E \quad c_{ij} = \sqrt{\frac{\psi_{jn}}{\psi_{in}}} G \frac{I_{in,jn+1}}{\sqrt{I_{in,in} I_{jn+1,jn+1}}} \quad (\text{A15a})$$

$$\text{where} \quad G = \frac{2\beta_{in}\beta_{jn} + 2\beta_{in}\beta_{jn+1} + \gamma^2 + 2\gamma\beta_{in} + \gamma\beta_{jn} + \gamma\beta_{jn+1}}{(\beta_{jn} + \beta_{jn+1} + \gamma)(\beta_{in} + \beta_{jn+1} + \gamma)} \quad (\text{A15b})$$

$$\text{and} \quad \gamma = \frac{4\pi\eta}{d} \quad (\text{A15c})$$

It can be seen that the equation depends on the propagation constant, β and the Bloch mode of the periodicity. The coefficient c_{ij} can be obtained by substituting i by j . For multiple modes coupling, the general expression is given as

$$A_{jn+1} \exp(-i\alpha_{jn+1}) = \sum_i c_{ij} A_{in} \exp(-i\alpha_{in}) \quad (\text{A18})$$

By equating the real and imaginary parts of the left and right hand side, the transmitted amplitude is derived as

$$A_{jn+1} = \left[\left(\sum_i c_{ij} A_{in} \cos \alpha_{in} \right)^2 + \left(\sum_i c_{ij} A_{in} \sin \alpha_{in} \right)^2 \right]^{\frac{1}{2}} \quad (\text{A19})$$

and the phase is derived as

$$\alpha_{jn+1} = \tan^{-1} \left(\frac{\sum_i c_{ij} A_{in} \sin \alpha_{in}}{\sum_i c_{ij} A_{in} \cos \alpha_{in}} \right) \quad (\text{A20})$$