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ACTIVELY MODE-LOCKED FIBER LASERS FOR HIGH SPEED TELECOMMUNICATION SYSTEMS

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A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Date

Lam Quoc Huy

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Summary

We propose and develop a novel mathematical series method for analyzing actively mode-locked lasers. Classical analysis methods (self-consistence methods) are very useful in determining the steady-state pulse parameters but don't show the transient evolution of the pulse and the role of amplified stimulated emission (ASE) noise in the process of pulse formation. To overcome this, we propose and derive a set of mathematical series, which are easily calculated from the laser cavity parameters, to trace the evolution of the signal in the laser cavity. Using this approach, we reveal the transient shaping of the pulse from ASE noise under the filtering and modulation effects. We also achieve the steady-state pulse directly from the limit of this transient shaping evolution. Furthermore, we investigate the detuning of actively mode-locked lasers using this method and show that mode-locked pulses are still obtained when the modulation frequency is detuned.

The other advantage of our proposed method in comparison with self-consistence analysis methods is that it can be applied to analyze not only laser model without noise but also laser model with ASE noise. We found that minimizing cavity loss and amplifier noise would increase the laser's Signal to Noise Ratio (SNR). On the other hand, the laser's SNR rapidly decreases as the laser is detuned since the pulse experiences higher loss when passing the modulator with position shift. As a result, the locking range is found to be limited by the noise.

We also develop a simulation model to investigate the dispersion and nonlinear effects on the output pulses of the lasers. Combining the abnormal dispersion effect with the nonlinear effect leads to soliton formation in the actively mode-locked fiber

laser. Pulse width shortening by this soliton forming is achieved. However, the locking range decreases as the pulse becomes shorter and hence optimization of the laser parameters is required in order to obtain stable short pulse over a large locking range.

We then investigate techniques to design an actively mode-locked fiber laser with fast wavelength tuning ability. We propose the use of a phase modulator to shift the optical field inside a Lyot birefringence fiber filter to tune the central wavelength of the filter. Electrical ultra fast tunable optical filter has been achieved using this technique. We demonstrate picoseconds-order tuning speed of the filter by using it to build a 10 Gbps frequency shift keying (FSK) modulation transmitter. The proposed ultra-fast filter is then applied to our design to obtain an ultra fast wavelength tunable actively mode-locked fiber laser.

Furthermore, by employing a birefringent ring cavity, we generate a dual amplitude pulse train from an actively mode-locked fiber laser. We also report for the first time the switching of the polarization state of the output pulse train when the modulation frequency is tuned. This is conducted by exploiting the existence of two different fundamental frequencies in the birefringent cavity. The slow-axis modes are excited if the modulation frequency is equal to a harmonic of their fundamental frequency and vice versa.

List of Publications

1. H. Q. Lam, P. Shum, L. N. Binh, and Y. D. Gong, "Dual locking modes in a weakly birefringent ring active mode locked fiber laser," *Passive Components and Fiber-Based Devices Iii, Pts 1 and 2*, vol. 6351, pp. U707-U714, 2006.
2. H. Q. Lam, P. Shum, L.N. Binh, Y.D. Gong, "8 ps Pulse From a Dual Locking Range Active Mode-Locked Fiber Laser," *The Fifth International Conference on Optical Communications and Networks (ICOON2006)*, ChengDu, China, 2006.
3. H. Q. Lam, P. Shum, L. N. Binh, and Y. D. Gong, "Polarization-Dependent Locking in SOA Harmonic Mode-Locked Fiber Laser," *IEEE Photonics Technology Letters*, vol. 18, pp. 2404-2406, 2006.
4. H. Q. Lam, P. Shum, L. N. Binh, Y. D. Gong, and M. Tang, "Dual Orthogonal Polarization States in an Active Mode-Locked Birefringent Fiber Ring Laser," *IEEE Photonics Technology Letters*, vol. 19, pp. 635-637, 2007.
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6. H. Q. Lam, P. Shum, L. N. Binh, Y. D. Gong, and S. Fu, "Series analysis of Signal to Noise Ratio (SNR) in active mode-locked lasers," *IEEE and IFIP International Conference on Wireless and Optical Communications Networks (WOCN) 2007*, Grand Hyatt, Singapore, 2007.

7. H. Q. Lam, P. Shum, L. N. Binh, Y. D. Gong, and S. Fu, "Pulse dropout and subharmonic locking in an active mode-locked birefringent fiber laser," *The 12th Optoelectronics and Communications Conference / The 16th International Conference on Integrated Optics and Optical Fiber Communication OECC/IOOC 2007*, Yokohama, Japan, 2007.
8. H. Q. Lam, P. Shum, L. N. Binh, Y. D. Gong, and S. N. Fu, "Mathematical series for analysis of detuned active mode locked," *Passive Components and Fiber-Based Devices Iv, Pts 1 and 2*, vol. 6781, pp. D7813-D7813, 2007.
9. H. Q. Lam, P. Shum, L. N. Binh, Y. D. Gong, S. Fu, and M. Tang, "Electrically Wavelength Tunable Active Mode-Locked Fiber Laser Using a Phase Modulator as Both Mode Locker and Wavelength Selector," *The Optical Fiber Communication Conference & Exposition and the National Fiber Optic Engineers Conference (OFC/NFOEC)*, 2008. OFC 2008, San Diego, CA, 2008.
10. H. Q. Lam, P. Shum, L. N. Binh, Y. D. Gong, and S. Fu, "Series Analysis of Active Mode-Locked Laser Under the Influence of ASE Noise," *IEEE/OSA Journal of Lightwave Technology*, vol. 26, pp. 1671-1680, 2008
11. H. Q. Lam, P. Shum, L. N. Binh, Y. D. Gong, S. Fu, and M. Tang " Dispersion Effects in Active Mode-locked Fiber Lasers," *The International Conference on Advanced Infocomm Technology 2008 (ICAIT 2008)*, Shenzhen, China, 2008.

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List of Abbreviations

AM	Amplitude Modulation
AOM	Acousto-optic modulator
APL	Additive pulse limiting
ASE	Amplified spontaneous emission
AWG	Arrayed waveguide grating
BPF	Bandpass filter
CW	Continuous-wave
DC	Direct current
DSF	Dispersion shifted fiber
DWDM	Dense-wavelength-division multiplexing
EDF	Erbium-doped fiber
EDFA	Erbium-doped fiber amplifier
E/O	Electrical-optical conversion
FBG	Fiber Bragg grating
FPF	Fabry-Perot Filter
FHWM	Full width at half maximum
FM	Frequency modulation
FSK	Frequency shift keying
FSR	Free spectral range
FTTH	Fiber to the home
FTTx	Fiber to the x
GVD	Group velocity dispersion
HDTV	High definition television
HiBi	Hibirefringence
HMLFL	Harmonic mode-locked fiber laser
HNLF	Highly nonlinear fiber
IPTV	Internet Protocol Television
LD	Laser diode
LHS	Left hand side
MLL	Mode-locked laser
MLFL	Mode-locked fiber laser
MZIM	Mach-Zehnder intensity modulator
NF	Noise figure
NLSE	Nonlinear Schrödinger equation
NOLM	Nonlinear optical loop mirror

NPR	Nonlinear polarization rotation
O/E	Optical-electrical conversion
OEO	Opto-electrical oscillator
OSC	Oscilloscope
OSA	Optical spectrum analyzer
OTDM	Optical time division multiplexing
PC	Polarization controller
PD	Photodetector
PM	Phase modulation
PMF	Polarization maintaining fiber
Pol	Polarizer
PZT	Piezo-electric transducer
RF	Radio frequency
RHMLFL	Rational harmonic mode-locked fiber laser
RHS	Right hand side
RMS	Root mean square
SAW	Surface acoustic wave
SMF	Single-mode fiber
SNR	Signal-to-noise ratio
SOA	Semiconductor optical amplifier
SPM	Self phase modulation
TBP	Time bandwidth product
WDM	Wavelength-division multiplexing

1 Introduction

1.1. BACKGROUND AND MOTIVATIONS

Four decades ago, G. R. Moore made a prediction, which is now the famous Moore's law, that the number of integrated transistors on a chip will be doubled for every 18 months. This is still true now and is expected to be valid for another decade at least. The Moore's law is applied not only in the growth of semiconductor industry but also in the growth of data traffic. The data traffic is projected to grow exponentially as shown in Figure 1.1 [1]. This is mainly driven by the emerging of high bandwidth consuming applications and services such as Internet Protocol Television (IPTV), file sharing, and high definition television (HDTV). Besides that, the expansions of the major telecommunications infrastructure in the Far East (China, India) and in developing countries also bring up huge data traffic.

Figure 1.1. Metro internet-traffic data growth over time and in the future [1]

Rapid deployment of Fiber To The x (FTTx) (see Figure 1.2) further pushes the demand for network capacity for delivering the data traffic. Recently, Infocomm Development Authority (IDA) of Singapore has kicked off a project to design, build and operate the active infrastructure of Singapore's Next Generation National Broadband Network [2, 3]. The project aims at providing broadband access links, which can scale up to 1 Gbps, to 50% of Singapore residents by 2012.

Figure 1.2. Global FTTx deployment [4]

The development of optical communications after the invention of optical fiber has fulfilled the demand in the past. But in order to keep up with the exponential growths of the data traffic in the future, more and more hardware components and transmission technologies have to be developed. Wavelength-division multiplexing (WDM), the technology of combining a number of wavelengths into the same fiber, successfully increases the fiber capacity by 32 times or even hundreds times if its advanced version, dense wavelength-division multiplexing (DWDM), is employed. However, the bottleneck is at the routers and switches where hundreds of channels must be demultiplexed for O/E (optical electrical conversion), routing, E/O (electrical optical conversion) and then multiplexing [5, 6]. The whole network speed will be increased if the optical signal can be directly routed/switched in optical domain without the need of O/E. All-optical switching, signal processing and optical time division multiplexing (OTDM) are promising solutions [7-12].

In OTDM systems, data is encoded using ultra-short optical pulses occupying N time slots in the OTDM time frame. Each channel is assigned a time slot and its data can be accessed with the aid of an optical clock pulse train corresponding to that time slot. Hence, generation of ultra-short optical pulses with multiple gigabits repetition rate is critical for ultra-high bit rate optical communications, particularly for the next generation of terabits/sec, optical fiber systems. The mode locked fiber laser offers a potential source of such pulse train. Furthermore, the field of optical packet switching has gained recognition in recent years and requires ultra-short and high peak power pulse generators to provide all optical switching [13-19].

Although the generation of ultra-short pulses by mode locking of a multi-modal ring laser is well known, the applications of such short pulse trains in multi-gigabits/sec optical communications challenge designers on their stability and spectral properties

as well as the multiplication of the repetition rate. Recent reports on the generation of short pulse trains at repetition rates in the order of 40 Gb/s [20, 21], and possibly higher in the near future, has motivated us to design, experiment and model these sources in order to evaluate whether they can be employed in practical optical communications and all optical switching systems. Furthermore, the development of multiplexed transmission at 160 Gb/s and higher in the foreseeable future, requires us to experiment with an optical pulse source with a short pulse duration and high repetition rates.

1.2. OBJECTIVES

The main objective of this research work is to investigate new devices and design methods for actively mode-locked fiber lasers to be used in the future high speed optical communication systems. Analytical study on the influence of the cavity design on the performance of the lasers is conducted to serve as design guideline for the laser systems. Besides that, simulation algorithm and program are to be developed to investigate in detail the behaviors of actively mode-locked fiber lasers under different configurations and conditions so that optimized design parameters can be obtained. In addition, method for fast tuning the laser's operation wavelength is also investigated to assist the ability of fast recovery or reconfiguration of the future optical networks. Moreover, when the whole usable spectra of the fiber are fully filled up by the huge data traffic in the future, polarization division multiplexing (PDM) may be a solution to double the system capacity. Therefore, orthogonal polarization mode-locked pulses generation is also investigated by introducing birefringence cavity in the laser design.

1.3. OUTLINE OF THE THESIS

After the introduction in Chapter 1, the fundamentals of actively mode-locked fiber lasers are presented in Chapter 2. Review on the development of mode-locked lasers is also discussed. Mathematical description of principle of mode locking is derived. Two techniques for mode locking, passive mode locking and active mode locking, are discussed. Passive mode locking is suitable for generation of ultra-short pulses and high peak power while active mode locking is preferred for generation of high repetition rate pulses for high speed telecommunication systems. The repetition rate of actively mode-locked fiber lasers can even be increased by using rational harmonic mode locking technique where the modulation frequency is deliberately detuned by f_R/N . Alternately, an intra-cavity Fabry-Perot etalon filter can be used to increase the pulses' repetition rate by suppressing the unwanted modes. In addition, supermode noise causes fluctuation in the pulse amplitude and can be suppressed by filtering or self phase modulation (SPM) effect. The change of temperature and environment can make the modulating frequency differ from the harmonic of the laser fundamental frequency and hence causes the laser unstable. Monitoring the repetition rate and using a PZT for dynamical adjustment of the cavity length can stabilize the lasers.

In Chapter 3, we propose and develop a novel analysis method to study the actively mode-locked lasers under the influence of Amplified Spontaneous Emission (ASE) noise. Instead of assuming a Gaussian pulse solution for the mode locking equation and applying invariant condition to obtain the pulse's steady-state parameters, we propose using mathematical series to trace the pulse's evolution in the cavity and derive the pulse's parameters directly from the series. Detuning of the laser is also investigated and locking range is derived with the consideration of ASE noise. We

show that ASE noise reduces the locking range of the laser and hence the optical amplifier noise should be minimized in order to improve the laser performance.

Dispersion and nonlinear effects on the performance of actively mode-locked fiber lasers are presented in Chapter 4. A comprehensive laser model which includes the dispersion and nonlinear effects is presented. We demonstrate that nonlinearity assists in shortening the pulse and plays an important role in soliton shaping of the laser pulse. Moreover, dispersion improves the stability of the laser against the modulation detuning through shifting the lasing wavelength to compensate for delay/advance caused by the detuning. We show that there is a trade-off between pulse shortening and stability of the laser. Cavity dispersion and nonlinearity should be optimized to obtain the shortest pulse while the laser is still stable within a certain amount of detuning.

In Chapter 5, we propose and demonstrate an ultra-fast electrically wavelength tunable actively mode-locked fiber laser. Based on a Lyot birefringence filter, we introduce a phase modulator to shift the phase of the optical field so that the transmission peak wavelength of the filter can be tuned by an external electrical voltage. The filter is then applied to the laser cavity design. Moreover, the phase modulator of the filter is also used for mode locking and thus reduces the cost.

Polarization dynamics of actively mode-locked fiber lasers are presented in Chapter 6. We investigate the behaviors of the lasers when the birefringence is introduced into the cavity. Pulses with dual polarization states are generated and polarization switching of the mode-locked pulses is observed. Pulse drop out and sub-harmonic mode locking due to mode competition are also reported.

Chapter 7 summarizes the main achievements and draws the conclusions of the thesis. Recommendations and discussion of possible future works are presented.

1.4. ORIGINAL CONTRIBUTIONS

The original contributions of this thesis are as follows:

- Proposal and demonstration of a novel analysis method using mathematical series to analyze actively mode-locked lasers under the influence of ASE noise.
- Applications of the series analysis method in analyzing the laser noise, deriving the laser locking range and studying the transition evolution of mode-locked pulse.
- Numerical investigation of dispersion and nonlinear effects on the performance of actively mode-locked fiber lasers.
- Design and demonstration of ultra fast electrical tunable filter.
- Design and demonstration of ultra fast wavelength tunable actively mode-locked fiber lasers.
- Generation of pulse train with dual polarization states by introducing birefringence into the cavity of an actively mode-locked fiber laser.
- Demonstration of switching of mode-locked pulses' polarization by tuning modulation frequency in an actively mode-locked fiber laser with birefringence cavity.

2 Principles of Actively Mode-locked Fiber Lasers

2.1. INTRODUCTION AND LITERATURE REVIEW

In 1964, DiDomenico predicted that small signal modulating of cavity loss in a laser at a frequency equal to multiples of axial mode spacing causes mode coupling with well defined amplitude and phase [22]. This phenomenon was later called mode locking of the laser.

In the same year, the first mode-locked laser was demonstrated by Hargrove et al. [23]. The authors placed an acousto-optic modulator inside a He-Ne laser to periodically alter the laser loss and obtained mode locking. The mode-locked amplitude was five-fold over the average unlocked one and the optical spectrum changed from rapidly fluctuation to stationary spectrum. The repetition rate and pulsewidth are *56 MHz* and *2.5 ns* respectively.

In 1970, the analytic theory of active mode locking was firmly established by Kuizenga and Siegman [24]. The authors studied the process in time domain and derived mode locking equations for both amplitude modulation (AM) and frequency

modulation (FM). However, the intra-cavity filter which is normally used to stabilize the laser was not included in the analysis.

In 1975, Haus presented a frequency domain approach for mode locking theory [25]. Using a circuit model, which is similar to that of [26], the author analyzed the evolution of the pulse in frequency domain and derived differential equations for forced mode locking. The equations had a Hermite-Gaussian solution and the mode-locked pulse characteristic was obtained. The author also derived differential equations in time domain which follows the form of Schrödinger equation. It is noted that a cosinusoidal modulation was used to acquire injection locking. Furthermore, continuum spectrum approximation of discrete frequency spectrum and assumption of dense spectrum were made to derive the equations.

In 1989, the first report of mode locked operation in an YbEr-doped fiber laser [27]. The authors used AM modulator to achieve mode locking. Pulses with repetition rate of *100 MHz* and pulsewidth of *70 ps* were generated in a linear cavity.

One year later, Tadaka et al. applied harmonic mode locking with a ring configuration to generate pulses with repetition rate as high as 30 GHz by using high speed LiNO₃ modulator [28]. Instead of modulating the signal at the cavity fundamental frequency, the authors applied a high frequency microwave signal at high order harmonic of the fundamental frequency to the modulator and obtained the pulse train at the modulating rate.

The major problem in active mode locked lasers is the stability. Many researches have been done to overcome this problem. In 1993, Harvey et al. used a sub cavity with a free spectral range that matches the modulation frequency to stabilize the pulse energy [29]. Coupling of portions of each pulse energy to successive pulses causes injection

locking the pulse train optically and hence equalizes the pulse energy. The disadvantage of this method is that it requires strictly matching between the fundamental frequency, the free spectral range of the sub cavity and the modulation frequency.

Another approach is the use of Additive Pulse Limiting (APL) technique [30]. In this technique, the high energy pulse is rotated more than the low energy and experiences more loss. By proper adjusting the polarization bias, the pulse energy can be clamped to a common value.

Self phase modulation effect was also used to stabilize the pulse energy in 1996[31]. SPM broadens the spectrum of high intense pulses. With a spectral filter placed inside the cavity, the high intense pulses experience more loss and hence energy stability is obtained.

Variation of the fiber length due to temperature fluctuation may cause the pulse to lose synchronism with the modulation. Several techniques have been proposed to stabilize the laser against temperature. A common technique is to adjust the cavity length to compensate for the variation such as in [32, 33]. The authors used a piezoelectric drum driven by an error signal to adjust the fiber length. The phase mismatch between the output pulse and the modulation signal was used as the error signal. Alternatively, the modulation frequency can be adjusted so that it is kept in synchronization with the fiber. This technique is called regenerative feedback since the modulation frequency is derived directly from the pulse change[21, 34, 35].

However, maximum pulse repetition rate is limited by the bandwidth of the LiNbO_3 modulator. The repetition rate can be increased by using rational harmonic mode-locking technique [36-38]. The rational harmonic mode-locked fiber laser (RHMLFL)

configuration is the same as that of harmonic mode-locked fiber laser (HMLFL) except that the modulating frequency f_m is not an integer number of the fundamental frequency. The modulating frequency f_m is now detuned by a fraction of fundamental frequency f_R , that means $f_m = (p+1/k)f_R$. Given that detuning, the output pulse train has a repetition rate of $(kp+1)f_R$, k times the modulating frequency.

The disadvantage of RHMLFL is that the pulse amplitudes vary from pulse to pulse due to the different losses which the pulses experience when passing the modulator. In 2002, Gupta et al. proposed the use of a Fabry Perot filter (FPF) inserted into the ring to equalize the pulse amplitudes in the RHMLFL[39]. However, this method requires the FPF's free spectral range (FSR) must be equal to the repetition rate and additional circuit to stabilize the FPF.

2.2. AMPLIFYING MEDIUM AND LASER SYSTEM

There are certain discrete energy levels in a particle that electrons can occupy as shown in Figure 2.1. The electrons can jump between those levels depend on whether they receive or release energy. When receiving energy from other sources such as electronic or optical source they jump from a lower level to higher levels. The electrons at high level states are excited electrons and can randomly jump back to a lower energy state without any stimulating source. When changing from high energy level E_i to lower energy E_j , the electrons release energy in form of a photon whose frequency can be calculated from:

$$\nu_{ij} = \frac{E_i - E_j}{h} \quad (2.1)$$

Chapter 2

where ν_{ij} is the released photon's frequency, h is Plank constant, E_i and E_j are the energies of the electron at level i and j respectively.

The emission of photons through this random jumping process is called spontaneous emission and the phases of the released photons are random. By contrast, when a photon enters a medium with excited electrons as shown in Figure 2.2, it will cause a stimulated emission where the electron transits to lower level and releases a new photon. The photons released through this emission have the same energy and phase as those of the stimulating photon. They then stimulate new emission and hence more and more photons are generated. Therefore, the optical signal is amplified and the medium with electrons pumped into excited states is called amplifying medium.

Figure 2.1. Energy level diagram for a particle

Figure 2.2. Stimulated emission

The amplifying medium is now put inside a cavity formed by two mirrors to form a typical laser system as shown in Figure 2.3. One of the mirrors totally reflects the

light while the other only reflects a part of the light and allows the rest to be transmitted to the output. The photons, which are initially emitted from the spontaneous emission process, are amplified when traveling through the amplifying medium via the stimulated emission. The photons are reflected back and forth between the mirrors and grow exponentially if their frequencies or wavelengths satisfy the phase condition:

$$\omega = \frac{N\pi c}{nL} \quad (2.2)$$

where L is the cavity length, n is the refractive index of the medium or the effective refractive index of the guided mode if the medium is an optical waveguide, N is an integer number, and $c = 3 \times 10^8 \text{ m/s}$ is the speed of light in vacuum.

Figure 2.3. Schematic diagram of a basic laser system

It can be seen from equation (2.2) that the laser normally generates lightwaves at a large number of wavelengths, which are called longitudinal modes. Several research works have been conducted to force the laser resonate at only one particular mode so as to narrow the laser linewidth [40-42].

Alternately, the laser is designed to oscillate at multiple longitudinal modes but those modes are controlled so that train of very short pulses can be generated. Such pulses find important applications in photonic switching and photonic communication systems[27, 43-45]. One of the typical techniques to generate short pulse laser is Q-switching [46-50]. In this technique, the quality factor Q of the optical resonator is

lowered to prevent the laser from oscillating during pumping period. The gain/population inversion therefore can build up to value higher than normal pumping. When the population reaches its peak, the quality factor Q is switched to a high value. Therefore the threshold for oscillating is restored to a value that is much below the gain. This causes an exponential grow of the intra-cavity intensity and the laser delivers a pulse of high intensity and short duration. However Q-switching technique can just generate pulses no shorter than a few ns . For generating ps pulses, mode-locking techniques are commonly employed [51, 52].

2.3. PRINCIPLE OF MODE-LOCKING

It can be seen from (2.2) that the laser can oscillate at a number of resonant frequencies whose spacing is equal to the fundamental frequency ω_R of the cavity:

$$\omega_i - \omega_{i-1} = \pi c / L = \omega_R \quad (2.3)$$

The output electric field of the generated lightwave in the temporal domain is the summation of all the oscillating modes given as:

$$e(t) = \sum_n E_n e^{j[(\omega_0 + n\omega_R)t + \Phi_n]} \quad (2.4)$$

where ω_0 is referenced center oscillating frequency, and E_n and Φ_n are amplitude and phase of the n^{th} mode, respectively, $j = (-1)^{1/2}$

When the laser is in the free oscillating state, E_n and Φ_n can take any value without any bound. This results in continuous wave (CW) laser. An example of this is shown in Figure 2.4, where electrical fields of five modes with random phase and the power of the total signal are plotted.

On the other hand, when the modes are forced to lock together, that means all the modes are either in phase or different in a multiple number of 2π phase, E_n and Φ_n are fixed, resulting in the pulse shape of $e(t)$. The simplest case is when $E_n = 1$, and $\Phi_n = 0$, using (2.4) leads to

$$e(t) = \cos \omega_0 t \frac{\sin(N\omega_R t / 2)}{\sin(\omega_R t / 2)} \quad (2.5)$$

This is an oscillation at frequency ω_0 modulated with the *sinc* envelope function

$$f(t) = \frac{\sin(N\omega_R t / 2)}{\sin(\omega_R t / 2)} \quad (2.6)$$

The average power is thus given by

$$P(t) \propto \frac{\sin^2(N\omega_R t / 2)}{\sin^2(\omega_R t / 2)} \quad (2.7)$$

This represents a periodic train of pulses that have following properties:

- The pulse period is $T = 2\pi/\omega_R$
- The peak power is N times the average power
- The peak field amplitude is N times the amplitude of a single mode.
- The pulse width, defined as the time from the peak to the first zero, is $\tau = T/N$, and shortens as N increased.

An example of such pulse is plotted in Figure 2.5.

Figure 2.4. Electrical fields of 5 individual modes with random phases and the power of the total signal.

Figure 2.5. Electrical fields of 5 in-phase individual modes and the power of the total signal.

2.4. MODE-LOCKING TECHNIQUES

There are several techniques for mode locking and they are divided into two classifications: passive mode locking and active mode locking.

2.4.1. Passive mode locking

Passive mode locking technique refers to those locking without any external RF signal for seeding. The pulses are formed passively through internal structure of the laser that gives more advantage (less loss, high gain) to the signal if it travels in pulse form

rather than in continuous wave (CW) [53-58]. The simplest method is to insert a saturable absorber into the cavity as shown in Figure 2.6. Saturable absorber is a nonlinear optical component whose absorption coefficient decreases when the optical intensity increases. The pulse train with high peak intensity will pass through the absorber with less loss than a CW laser with many modes. It results that the pulse train is formed in the cavity.

Figure 2.6. Schematic of a passively mode-locked laser with a saturable absorber in the cavity

Instead of using saturable absorber, nonlinear polarization rotation (NPR) can be used for passive mode locking as plotted in Figure 2.7 [59]. In this system, a nonlinear phase shift is imposed on signal with high peak power and thus rotates its polarization state to align with the analyzer axis. The signal thus passes through the analyzer with minimum loss. On the other hand, CW signal with low average power experiences zero nonlinear phase shift and thus its polarization state is not rotated to align with the analyzer axis. It is then blocked by the analyzer. Therefore, pulses with high peak power are formed in the cavity instead of continuous wave (CW) with low average power.

Figure 2.7. Schematic of a nonlinear polarization rotation passive mode-locked fiber laser

Passively mode-locked laser has a potential to produce very short laser pulse, down to femto-seconds. The shortest pulse generated in the communication wavelength window to date is 5 fs [60, 61]. Moreover, passive mode-locking is a self mode-locking technique; it means that the pulse is formed without any external modulating signal. However, high speed passively mode-locked lasers normally require harmonic-mode locking. In this regime, the spacing between two pulses is varied from pulse to pulse since there is no control mechanism to force the pulses to be equally spaced. This is the disadvantage which makes passively mode-locked lasers not suitable for high speed optical transmission systems, in which precise timing from pulse to pulse is required. The accuracy of data recovery at the receiver side of a transmission system critically depends on how close to the optimum decision time the data pulse is sampled. The higher data rate the more accuracy is required. A drop from 10^{-12} to 10^{-8} of the Bit Error rate (BER) of a 20 Gbps system when the timing jitter increased by 500 fs was reported in [62]. Therefore, actively mode-locked lasers are preferred in high speed transmission systems. Since the pulses pass through a modulator for every round trip in an actively mode-locked laser, any drift of the pulse timing is corrected by the modulator. By adding a bit complexity to the design with stabilization circuit, actively mode-locked lasers can generate stable pulses with timing jitter as low as tens

of femtoseconds. The pulse width covers the range from hundreds of femtoseconds to several tens of picoseconds while the repetition rate can be tuned from MHz to sub-THz.

2.4.2. Active mode locking

In actively mode-locked lasers, mode locking is induced through modulation. When modulated by an external RF signal with frequency of f_m , the oscillating mode at optical frequency f_0 shares its energy to the two side bands located at $f_0 + f_m$ and $f_0 - f_m$ as shown in Figure 2.8. If the modulating frequency f_m is chosen so that $f_m = f_R$, the side bands are coincident with the adjacent modes of the lasers and hence the energy from mode i^{th} is transferred to its adjacent modes $(i + 1)^{th}$ and $(i - 1)^{th}$. In another word, the energies of the modes in the laser are transferred from one to another. This causes the phase of the modes locked together and hence mode-locked pulse train is formed.

In actively mode-locked lasers, mode-locking is induced by modulating the gain or loss of the cavity with an external signal at the fundamental frequency f_R .

Figure 2.8. Active mode locking by modulation; (a) Energy transferred to two sidebands when modulating; (b) Oscillating modes in a cavity with fundamental frequency of f_R ; (c) Energy distribution when modulating the oscillating modes with frequency of f_m ; (d) Energy transferring between the modes when $f_m = f_R$

A common method for active mode-locking is inserting an acousto-optic modulator into the cavity as shown in Figure 2.9. This is the first method used in the experiment for observation of mode-locked pulse in a helium-neon laser [23]. The acousto-optic modulator (AOM) is used to deflect light energy from the cavity by varying the refractive index of the medium in a standing wave profile. Thus the undeflected beam is amplitude modulated. Acousto-optic modulator is widely used in several mode-locked lasers as it offers a very low insertion loss [63-66] [67]. However, the operating frequency of the acousto-optic modulator is limited and far below the pulse rate required for modern optical communication, typically from a few hundreds MHz to a few GHz depending on the acoustic velocity of the acousto-optic material.

Figure 2.9. Mode-locked laser with an intra-cavity acousto-optic modulator ([67])

Due to limited bandwidth of acousto-optic modulators [68-70], they are normally replaced by LiNbO_3 modulators whose bandwidth may reach 40 GHz or wider in high repetition rate actively mode-locked lasers. In addition, all-fiber cavity is also used in those lasers to take advantages of fiber lasers such as low threshold, single mode, high beam quality. Moreover, high optical intensity in small cross section of the fiber make it possible to obtain nonlinear effect for shortening pulses with even low pump power.

Another method of mode-locking is to modulate the gain of the amplifying medium as in [71, 72]. In those experiments, the gain of the semiconductor amplifier is modulated by an external modulating signal. The lightwaves therefore experience a periodical gain and form the mode-locked pulses in the cavity. Semiconductor laser has a potential to generate very short pulse due to its wide spectral gain region.

2.5. ACTIVELY MODE-LOCKED FIBER LASERS

2.5.1. Actively mode-locked fiber lasers

Figure 2.10 shows the schematic diagram of an actively mode-locked fiber laser (MLFL). The laser has a ring configuration incorporating an isolator for unidirectional lasing. An erbium doped fiber amplifier (EDFA) is used to provide the gain for the lightwave traveling in the ring. The lasing wavelength is selected by using an optical bandpass filter (BPF). Mode locking is obtained by inserting into the ring a Mach-

Zehnder intensity modulator (MZIM) that periodically modulates the loss of the lightwaves traveling around the ring. A polarization controller (PC) is also employed to maximize the coupling of lightwaves from fiber to the diffused waveguides of the MZIM. Output pulse trains are extracted via a 90:10 coupler.

The fundamental frequency of the ring is:

$$f_R = c/nL \quad (2.8)$$

where n is effective refractive index of the single mode fiber, $c=3 \times 10^8 \text{ m/s}$ speed of light in vacuum, L is the total length of the ring.

Since the total length of the ring L is usually very long, from a few meters to hundreds of meters, the fundamental frequency f_R is low and hence the repetition rate of the output pulse train is also low. To increase the repetition rate, harmonic mode locking is used. In harmonic mode-locked fiber laser (HMLFL), the modulator is driven by a signal with frequency equal to a high harmonic order of the fundamental frequency. With this arrangement, the laser is mode-locked to the modulating frequency, which can be up to few tens of GHz, instead of the low fundamental frequency f_R , depending on the bandwidth of the optical modulator.

Figure 2.10. Schematic of an active harmonic mode-locked fiber laser

2.5.2. Multiplication of repetition rate

Although high repetition rate pulse train can be obtained by driving actively mode-locked fiber laser at a harmonic of fundamental frequency of the ring cavity, the maximum pulse repetition rate is still limited by the bandwidth of the modulator. To increase the repetition rate rational harmonic mode-locking technique can be used [35-38, 72-75].

The rational harmonic mode-locked fiber laser (RHMLFL) configuration is the same as that of harmonic mode-locked fiber laser except that the modulation frequency f_m is not an integer number of the fundamental frequency. The modulation frequency f_m is now detuned by a fraction of fundamental frequency f_R , that means $f_m = (p+1/k)f_R$. With that detuning, the output pulse train has a repetition rate of $(kp+1)f_R$, k times the modulation frequency.

A rational harmonic mode-locked laser with a repetition rate of 200 GHz when modulating at 40 GHz has been report in [38] and its setup is shown in Figure 2.11. A multiplication of 5 has been demonstrated. The disadvantage of RHMLFL is that the pulse amplitude is varied from pulse to pulse as seen in Figure 2.12 due to the different losses the pulses experience when passing the modulator.

Figure 2.11. Rational harmonic mode-locked fiber laser ([38])

Figure 2.12. Output pulse train of the RHMLFL (a) Auto correlation waveform at 160GHz, (b) Spectral profile at 160GHz, (c) Auto correlation waveform at 200GHz, (d) Spectral profile at 200GHz ([38])

Alternatively, multiplication of the repetition rate can be obtained by using an intra-cavity optical filter [76, 77]. Figure 2.13 shows the schematic of a HMLFL with pulse repetition rate multiplied by using a Fabry-Perot filter (FPF). The schematic is similar to that of the HMLFL with an FPF inserted into the ring. The free spectral range (FSR) of the FPF is K times the modulation signal frequency, where K is the

multiplication factor. The FPF is used to select higher order longitudinal modes that correspond to its transmission peaks, while filter out the intermediate longitudinal modes. The condition for this laser is that the modulation frequency f_m is not only a harmonic multiple of the fundamental frequency f_R but also a sub-harmonic multiple of FSR of the FPF

$$f_m = Nf_R \quad (2.9)$$

and
$$f_m = \text{FSR}/k \quad (2.10)$$

where N and k are integer numbers

When those conditions are satisfied, only the longitudinal modes equal to the peak frequencies of the FPF are selected while the other intermediate longitudinal modes are suppressed as shown in Figure 2.14. This results in a pulse train whose repetition rate is equal to the FSR. The pulse train generated using this technique has equal amplitude but the repetition rate is fixed to the FSR.

Figure 2.13. Rational harmonic mode-locked fiber laser

Figure 2.14. Frequency domain depiction of pulse repetition frequency multiplication in HMLFL via intra-cavity optical filtering; (a) Spectrum structure of the pulse train without the FPF, (b) The FPF transmission characteristic, (c) Spectrum structure of the pulse train filtered by the FPF

2.5.3. Equalizing pulses in RHMLFL

The Fabry Perot filter (FPF) can also be used to equalize the pulse amplitudes in rational harmonic mode-locked fiber laser (RHMLFL) [39, 78-80]. As mentioned above, RHMLFL can generate pulse train with high repetition rate but the pulse train suffers with pulse amplitude instability that includes both amplitude noise and unequal pulse amplitudes. The amplitude noise is mainly due to super-mode noise caused by the unequal energy distribution amongst the cavity modes. The unequal pulse amplitudes in optical pulse train are mainly due to asymmetric cavity loss modulation within the cavity caused by the present of randomly oscillating intermediate modes that are frequency spaced at f_m and become predominant at a pulse repetition rate greater than $2f_m$.

The RHMLFL is setup as shown in Figure 2.15. When the modulation frequency is detuned by f_R/k (k is an integer), $f_m = (p + 1/k)f_R$, the output pulse train repetition

frequency is $(kp+1)f_R$, k times the modulation frequency. However the modulation signal also generates intermediate longitudinal modes at f_m , $2f_m$, and $3f_m$... Those intermediate modes cause the pulse amplitude instability. If the FPF is chosen so that its FSR is equal to kf_m then those intermediate modes are filtered out, leaving the modes at kf_m as shown in Figure 2.16. Therefore the pulse becomes stable at the repetition rate of kf_m .

Figure 2.15. Schematic of a RHMLFL with FPF for equalizing pulse amplitudes

Figure 2.16. Frequency domain depiction of intermediate frequency filtering of FPF;

(a) Spectrum struture of the pulse train without the FPF, (b)The FPF transmission characteristic, (c) Spectrum structure of the pulse train filtered by the FPF

2.5.4. Laser stabilization

HMLFL has the ability to generate very short pulse at high repetition rate but a long term stable output pulse train is hard to obtained due to its long cavity, which is easily affected by the change of the environment such as mechanical vibration or temperature variation [32, 81-83]. Therefore, there is a need to stabilize the laser so that the output pulse train can be used as a practical source for transmission of ultra-high speed data.

The length of the ring in the HMLFL can be varied when the temperature changes, which in turn change the fundamental frequency of the ring and hence the oscillating longitudinal modes are out of synchronous with the modulating signal. The length of the ring can be control by wounding the fiber on a piezo-electric transducer (PZT) and using a feedback circuit to control the PZT.

Figure 2.17 shows a schematic of HMLFL using PZT to stabilize the laser. The fiber is wound on a PZT so that the length of the fiber can be controlled by applying a DC voltage into the PZT. The output of the laser is converted to electrical pulse by a photodiode (PD) and filtered by a bandpass filter (BPF). The output is then compared with the modulation signal which is extracted from a signal generator. The error is used for controlling the PZT through a control circuit.

Figure 2.17. Schematic of a HMLFL using PZT to control the length of the cavity

Alternately, laser stabilizing can also be achieved by using a feedback scheme as shown in Figure 2.18. This configuration is normally referred as regenerative harmonic mode-locked fiber laser [81, 84]. The electrical output pulses after being detected by the photodiode (PD) are fed into the clock recovery to extract the repetition frequency of the pulses. The output sinusoidal signal is used to control the modulator after amplified and phase shifted. The phase between the pulses and the modulation signal is adjusted by the phase shifter so that the pulses always experience the minimum loss when they pass through the modulator. Long term stability using this stabilization technique has been demonstrated by Nakazawa et al [81].

Figure 2.18. Schematic of a regenerative harmonic mode-locked fiber laser

The instability of the fiber laser is also caused by the birefringence of the fiber. Random change of the polarization state of the light traveling in the fiber causes fluctuates of the pulse amplitudes. Polarization maintaining (PM) fiber can be used to preserve the polarization of the light and hence help to stabilize the laser[34, 85-88]. Lasers using PM fiber have been reported to produce stable pulse train with repetition rate up to 40 GHz and at wavelengths tunable over the whole C-band (from 1530 nm to 1565 nm) [21, 89].

3 Series Analysis of Actively Mode-locked Lasers

3.1. INTRODUCTION

As discussed in Chapter 2, actively mode-locked laser (MLL) has potential applications in the future high speed optical networks due to its ability to generate very short pulse at high repetition rate with low timing jitter. Since its first demonstration in 1964 by Hargrove et al [23], actively mode-locked laser has been improved from year to year with numerous proposed designs [23, 33, 90-92]. Besides that, theories on mode locking were also extensively developed to study the mode-locked laser and to serve as guide line for the laser design. In 1970, the analytic theory of active mode locking was firmly established by Kuizenga and Siegman [24]. They studied the process in time domain and derived mode locking equations for both amplitude modulation (AM) and frequency modulation (FM). In their paper, Gaussian pulse solution was assumed and approximations of the line shape and modulation function to keep the pulse Gaussian were applied.

In 1975, Haus presented a frequency domain approach for mode locking theory [25]. Using a circuit model, which is similar to that of [26], the author analyzed the

evolution of the pulse in frequency domain and derived differential equations for forced mode locking. The equations had a Hermite-Gaussian solution and the mode-locked pulse characteristics were obtained. In addition, the analysis of the detuning effects in actively mode-locked laser was reported by Li *et al.* with the use of consistence condition and the assumption that the pulse has a Gaussian shape [93]. This analysis was based on the invariant condition of the Gaussian pulse shape after traveling a round trip. Pulse parameters in steady state and locking range were derived. The dispersion effect on the detuning properties of actively mode-locked fiber lasers was recently reported by Zhu and Dutta [94]. The authors found that under strong cavity dispersion effect the lasing wavelength is shifted from the central wavelength of the filter when the laser was detuned.

However, the analyses were based on the assumption that the pulse shape is Gaussian in the steady state or the consistence condition is held to obtain the mode-locked pulse parameters. Moreover, Amplified Spontaneous Emission (ASE) noise, which cannot be avoided in the optical gain medium of the laser, has not been considered in the models. Thus there are needs to study the evolution of the pulse shape in mode-locked lasers under the existence of optically amplified noises. In this chapter, we employ a mathematical series instead of the self-consistence condition to trace the evolution of a white noise traveling inside the cavity. The shape of the noise is featured by eight series S_n , Q_n , R_n , a_n , b_n , c_n , r_n and P_m which can be easily calculated from the laser parameters. Using this method, we observe that the noise is gradually shaped to steady-state pulse under the shaping effect of the filter and modulator for every round trip. We obtain the steady-state pulse through transient process without the need of presumed Gaussian pulse shape. Equations for determining the steady-state pulse parameters and signal to noise ratio (SNR) are also derived using these mathematical

series. These equations can be used to study the effects of the laser parameters and detuning condition on the output pulse characteristics and to determine the locking range for a specific requirement of the laser SNR.

The analysis of actively mode-locked lasers using self-consistence condition with assumed Gaussian pulse shape is presented in Section 3.2. Following is the description of our series analysis method for actively mode-locked lasers. Applications of our method in analysis of the mode-locked lasers with and without detuning are presented in Sections 3.4 and 3.5 respectively. The results are then confirmed with simulation results in Section 3.6. And finally, conclusions are given in Section 3.7.

3.2. ANALYSIS OF ACTIVELY MODE-LOCKED LASERS USING SELF-CONSISTENCE CONDITION WITH ASSUMED GAUSSIAN PULSE SHAPE

Figure 3.1. Actively mode-locked laser model

In this section, we present the analysis of actively mode-locked laser by using the self-consistence condition method with assumed Gaussian pulse shape [93, 95]. Figure 3.1 shows the laser model which consists of an in-line optical amplifier, a bandpass optical filter and an intensity modulator. Since the gain medium linewidth is normally much larger than the filter bandwidth, for lasers with long gain-recovery time

constant, e.g. Erbium doped gain medium, the gain can be considered flat over the filter spectrum with a saturated value of G_0 . The filter transmittance function $T(\omega)$ and the Mach-Zehnder modulator transmission function $M(t)$ can be written as

$$T(\omega) = \exp(-\beta\omega^2), \quad (3.1)$$

$$M(t) = \cos\left[\frac{\pi}{2}(1-b) - \frac{\pi}{4}m \cos(\omega_m t)\right], \quad (3.2)$$

where $\beta = 2\ln 2/\Delta\omega^2$, $\Delta\omega$ is the filter 3 dB bandwidth, b accounts for the bias point of the modulator, $b = 0.5$ for the quadrature bias point, m is modulation index, ω_m is modulation frequency. If the pulses pass the modulator at the time near the peak of transmission function, $t = 0$, we expand (3.2) using the Taylor series, and hence $M(t)$ can be approximated by

$$M(t) = \Gamma \exp(-\delta_m t^2) \quad (3.3)$$

in which $\delta_m = m\omega_m^2/4$, Γ accounts for the total loss of the laser cavity including insertion loss, modulator bias loss and coupler loss. Suppose the pulse entering the gain medium after the n^{th} round trip is

$$A_n(t) = A \exp\left[-\alpha(t-t_s)^2\right], \quad (3.4)$$

where A is the pulse amplitude, α is the Gaussian pulse parameter, and t_s is the position of the pulse with reference to the transmission peak of the modulator. Hence the signal after the optical filter, in the time domain, is

$$A_{n'}(t) = F^{-1}\left\{G_0 T(\omega) F[A_n(t)]\right\}, \quad (3.5)$$

where F and F^{-1} denote the Fourier and its inverse transform respectively. Substitute (3.1) and (3.4) into (3.5) we obtain

$$A_{nl}(t) = \frac{G_0 A}{\sqrt{1+4\alpha\beta}} \exp\left[-(t-t_s)^2 / (1+4\alpha\beta)\right] \quad (3.6)$$

If the modulation frequency is detuned by Δf_m from its ideal N^{th} order harmonic mode locking frequency f_{m0} : $\Delta f_m = f_m - f_{m0}$, the pulse arrives at the modulator with the delay of

$$t_d = N \left(\frac{1}{f_{m0}} - \frac{1}{f_m} \right) \approx \frac{N \Delta f_m}{f_{m0}^2}. \quad (3.7)$$

After passing through the modulator the pulse completes one round trip and becomes

$$A_{n+1}(t) = A_{nl}(t) M(t) = \frac{AG_0\Gamma}{\sqrt{1+4\alpha\beta}} \exp\left\{-\left[\frac{4\alpha}{1+4\alpha\beta} + \delta_m\right](t-t_s-t_d)^2\right\} \quad (3.8)$$

Using the self consistence condition in steady state $A_{n+1}(t) = A_n(t)$, and balancing the terms in both sides we obtain

$$\alpha = \frac{1}{2} \sqrt{\delta_m^2 + \frac{\delta_m}{\beta}} + \frac{\delta_m}{2} \quad (3.9)$$

$$t_s = \frac{t_d}{4\alpha\beta} \quad (3.10)$$

$$G_0 = \frac{\sqrt{1+4\alpha\beta}}{\Gamma} \exp\left(\frac{t_d^2}{4\beta}\right). \quad (3.11)$$

From these analytical results, we observe that:

- The pulse width is independent of the detuning and identical to that obtained with exact tuning.
- Detuning does not change the pulsewidth of the steady pulse but shifts the pulse position away from the transmission peak. The shift of the pulse position from the transmission peak is linearly proportional to the detuning Δf_m .

- Due to additional loss caused by position shift, the required gain is increased by a factor of $\exp(t_d^2/4\beta)$.

When exact tuning happens, that is $\Delta f_m = 0$, $t_d = 0$, we get

$$\alpha = \frac{1}{2} \sqrt{\delta_m^2 + \frac{\delta_m}{\beta}} + \frac{\delta_m}{2} \quad (3.12)$$

$$t_s = 0 \quad (3.13)$$

$$G_0 = \frac{\sqrt{1+4\alpha\beta}}{\Gamma}. \quad (3.14)$$

This confirms the results obtained in [24] if we substitute β and δ_m in the above definition.

3.3. SERIES APPROACH FOR ACTIVELY MODE-LOCKED LASER ANALYSIS

Figure 3.2. Actively mode-locked laser model with ASE noise included

The ASE noise of the optical amplifier is included in the laser model shown in Figure 3.2. The lasing process is started by the ASE noise which is modeled as a white noise source ρ_{ASE} with a variance of σ_N^2 . For every round trip, the ASE noise is filtered, shaped, and amplified when passing the optical band pass filter, modulator and amplifier in the loop. In frequency domain, the ASE signal after passing the filter is

$$\rho_{1A} = \sigma_N e^{-\beta\omega^2} \quad (3.15)$$

Due to detuning, after one round trip the pulse arrives at the modulator with the delay of t_d . Delay t_d in time domain is equivalent to multiplying by $\exp(-j\omega t_d)$ in frequency domain. Hence, the noise after the first round trip is

$$\begin{aligned} \rho_1(\omega) &= G \times F \left[F^{-1} \left(\sigma_N e^{-\beta\omega^2} e^{-j\omega t_d} \right) \times \Gamma e^{-\delta_m t^2} \right] \\ &= \Gamma G \sigma_N \times F \left[\frac{1}{2\sqrt{\pi\beta}} e^{-(t-t_d)^2/4\beta} \times e^{-t^2 x/4\beta} \right] \\ &= \Gamma G \sigma_N \times F \left[\frac{1}{2\sqrt{\pi\beta}} e^{-A_1(t)} \right] \end{aligned} \quad (3.16)$$

where F and F^{-1} denotes Fourier transform and its inverse transform, respectively, $x = 4\beta\delta_m$ and

$$\begin{aligned} A_1(t) &= \frac{(t-t_d)^2}{4\beta} + \frac{xt^2}{4\beta} \\ &= \frac{1+x}{4\beta} \left(t - \frac{t_d}{1+x} \right)^2 + \frac{xt_d^2}{(1+x)} \\ &= \frac{Q_1}{4\beta S_1} \left(t - \frac{S_1}{Q_1} t_d \right)^2 + \frac{R_1}{4\beta Q_1} t_d^2 \end{aligned} \quad (3.17)$$

Therefore

$$\begin{aligned} \rho_1(\omega) &= \sigma_N \Gamma G \times F \left[\frac{1}{2\sqrt{\pi\beta S_1}} e^{-\frac{Q_1}{4\beta S_1} \left(t - \frac{S_1}{Q_1} t_d \right)^2} e^{-\frac{R_1}{4\beta Q_1} t_d^2} \right] \\ &= \sigma_N \Gamma G \frac{1}{2\sqrt{\pi\beta S_1}} \sqrt{\frac{\pi 4\beta S_1}{Q_1}} e^{-\frac{S_1}{Q_1} \beta \omega^2} e^{-j\omega \frac{S_1}{Q_1} t_d} e^{-\frac{R_1}{4\beta Q_1} t_d^2} \\ &= \sigma_N \Gamma G \frac{1}{\sqrt{Q_1}} e^{-\frac{R_1}{4\beta Q_1} t_d^2} e^{-\frac{S_1}{Q_1} \beta \omega^2} e^{-j\omega \frac{S_1}{Q_1} t_d} \end{aligned} \quad (3.18)$$

in which $S_1 = 1, Q_1 = x + 1, R_1 = x$.

Similarly the signal after the second round trip is

$$\begin{aligned}
\rho_2(\omega) &= G \times F \left\{ F^{-1} \left[\rho_1(\omega) e^{-\beta\omega^2} e^{-j\omega t_d} \right] \times \Gamma e^{-\delta_m t^2} \right\} \\
&= \frac{\sigma_N (\Gamma G)^2}{\sqrt{Q_1}} \times F \left\{ F^{-1} \left[e^{-\frac{R_1}{4\beta Q_1} t_d^2} e^{-\frac{S_1+Q_1}{Q_1} \beta\omega^2} e^{-j\omega \frac{S_1+Q_1}{Q_1} t_d} \right] e^{-\frac{x}{4\beta} t^2} \right\} \\
&= \sigma_N (\Gamma G)^2 \frac{e^{-\frac{R_1}{4\beta Q_1} t_d^2}}{\sqrt{Q_1}} \times F \left\{ \frac{e^{-\frac{Q_1}{4\beta(S_1+Q_1)} \left(t - \frac{S_1+Q_1}{Q_1} t_d \right)^2} e^{-\frac{x}{4\beta} t^2}}{2\sqrt{\pi\beta(S_1+Q_1)/Q_1}} \right\} \\
&= \sigma_N (\Gamma G)^2 \frac{e^{-\frac{R_1}{4\beta Q_1} t_d^2}}{2\sqrt{\pi\beta(S_1+Q_1)}} \times F \left\{ e^{-A_2(t)} \right\}
\end{aligned} \tag{3.19}$$

where

$$\begin{aligned}
A_2(t) &= \frac{Q_1}{4\beta(S_1+Q_1)} \left(t - \frac{S_1+Q_1}{Q_1} t_d \right)^2 + \frac{x}{4\beta} t^2 \\
&= \frac{Q_1+x(S_1+Q_1)}{4\beta(S_1+Q_1)} \left(t - \frac{S_1+Q_1}{Q_1+x(S_1+Q_1)} t_d \right)^2 \\
&\quad + \frac{2xS_1+R_1}{4\beta[Q_1+x(S_1+Q_1)]} t_d^2 - \frac{R_1}{4\beta Q_1} t_d^2 \\
&= \frac{Q_2}{4\beta S_2} \left(t - \frac{S_2}{Q_2} t_d \right)^2 + \frac{R_2}{4\beta Q_2} t_d^2 - \frac{R_1}{4\beta Q_1} t_d^2
\end{aligned} \tag{3.20}$$

in which $S_2 = S_1 + Q_1$, $Q_2 = xS_2 + Q_1$, $R_2 = 2xS_2 + R_1$. Hence

$$\begin{aligned}
\rho_2(\omega) &= \frac{\sigma_N (\Gamma G)^2}{2\sqrt{\pi\beta S_2}} e^{-\frac{R_1}{4\beta Q_1} t_d^2} \times F \left\{ e^{-\frac{Q_2}{4\beta S_2} \left(t - \frac{S_2}{Q_2} t_d \right)^2} e^{-\frac{R_2}{4\beta Q_2} t_d^2} e^{-\frac{R_1}{4\beta Q_1} t_d^2} \right\} \\
&= \sigma_N (\Gamma G)^2 \frac{1}{2\sqrt{\pi\beta S_2}} \sqrt{\frac{\pi 4\beta S_2}{Q_2}} e^{-\frac{S_2}{Q_2} \beta\omega^2} e^{-j\omega \frac{S_2}{Q_2} t_d} e^{-\frac{R_2}{4\beta Q_2} t_d^2} \\
&= \sigma_N (\Gamma G)^2 \frac{1}{\sqrt{Q_2}} e^{-\frac{R_2}{4\beta Q_2} t_d^2} e^{-\frac{S_2}{Q_2} \beta\omega^2} e^{-j\omega \frac{S_2}{Q_2} t_d}
\end{aligned} \tag{3.21}$$

Thus in general, the signal after the n^{th} round trip is

$$\rho_n(\omega) = \sigma_N (\Gamma G)^n \frac{1}{\sqrt{Q_n}} e^{-\frac{R_n}{4\beta Q_n} t_d^2} e^{-\frac{S_n}{Q_n} \beta\omega^2} e^{-j\omega \frac{S_n}{Q_n} t_d} \tag{3.22}$$

where

$$\begin{cases} S_1 = 1, Q_1 = x + 1, R_1 = x \\ S_n = S_{n-1} + Q_{n-1}, Q_n = xS_n + Q_{n-1}, R_n = nxS_n + R_{n-1} \end{cases} \quad (3.23)$$

The total signal in the ring is the sum of the 1st, 2nd, ..., ∞^{th} round trip signal and is given as

$$\eta(\omega) = \sum_{n=1}^{\infty} \rho_n(\omega) \quad (3.24)$$

3.4. MODE LOCKING WITH ZERO DETUNING

First of all, we consider the case of zero detuning in which the modulation frequency is exactly equal to a harmonic of the fundamental frequency of the laser ring. Without detuning, there is no additional delay per round trip, thus we can substitute $t_d = 0$ into (3.22) and then analyze the result in two conditions: noiseless amplifier and amplifier with ASE noise.

3.4.1. Noiseless amplifier

To compare our results with those obtained by using classical approaches in which the gain medium is modeled as a noiseless amplifier[24], we assume that the amplifier used here is also noiseless. The signal is formed from a white noise traveling the loop infinitely without any ASE added for every round trip. Therefore, the steady-state signal is

$$\eta(\omega) = \lim_{n \rightarrow \infty} \rho_n(\omega) \propto e^{-\lim_{n \rightarrow \infty} \left(\frac{S_n}{Q_n} \right) \beta \omega^2} = e^{-\frac{2}{x + \sqrt{4x + x^2}} \beta \omega^2} \quad (3.25)$$

Using inverse Fourier transform, we obtain the Gaussian pulse shape in time domain as

$$\eta(t) \propto e^{-\frac{x+\sqrt{4x+x^2}}{8\beta}t^2} = e^{-\alpha t^2} \quad (3.26)$$

where $\alpha = \frac{x+\sqrt{4x+x^2}}{8\beta}$ is the Gaussian pulse parameter. This is exactly the same result as in (3.9).

To calculate the gain required for lasing, we consider the power of the signal after n round trips

$$P_n = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\rho_n(\omega)|^2 d\omega = \frac{\sigma_N^2}{\sqrt{8\pi\beta}} \frac{(\Gamma G)^{2n}}{\sqrt{S_n Q_n}} \quad (3.27)$$

The requirement is that the power is not reduced to zero or increased to infinite but approach a constant as $n \rightarrow \infty$. It is found that $\lim_{n \rightarrow \infty} P_n = \text{const}$ when

$$G = G_{0l} = \frac{\sqrt{1+4\alpha\beta}}{\Gamma} \quad (3.28)$$

This is the required gain for lasing and is the same as (3.14), but here we obtain by using different approach.

3.4.2. Amplifier with ASE noise

In practice, the noise is continuously added to the signal for every round trip passing the gain medium. The newly added noise is then shaped closer to the steady-state Gaussian pulse in (3.26) for every round traveling in the loop. The total signal is hence the sum of all shaped noise as presented in (3.24). Since the individual shaped noise converges to the Gaussian pulse in (3.26), their sum also converges to that steady-state Gaussian pulse. It should be noted that if the gain is kept the same as that in the ideal amplifier case, the power of individual Gaussian pulse is unchanged but

the total power is increased to infinity due to the contribution of the added noise in every round trip. This cannot happen in practice thanks to the saturation of the gain medium[96-99]. The increase of power will reduce the gain and thus the net gain in one round trip is less than one which makes the power decrease. The result is that the total power is kept unchanged and the gain is reduced by a factor of r compared to the noiseless amplifier case

$$G = G_{0l}r = \frac{\sqrt{1+4\alpha\beta}}{\Gamma}r \quad (3.29)$$

Since the gain is reduced, the power of individual signal P_n is no longer conserved but exponential decreased to zero as $n \rightarrow \infty$. The power of the signal after n round trips now becomes

$$P_n = \frac{\sigma_N^2}{\sqrt{8\pi\beta}} \frac{(1+4\alpha\beta)^n}{\sqrt{S_n Q_n}} r^{2n} \quad (3.30)$$

From the above analysis, the evolution of the steady-state pulse can be described as following. The spontaneous noise of the amplifying medium is shaped to the steady-state pulse under the shaping effect of the filter and modulator for every round trip. It converges to the steady-state pulse as n approach infinite. By examining (3.22) and (3.30) we see that there are three series that control the dynamic evolution of the pulse: $a_n = S_n/Q_n$, $b_n = (1+4\alpha\beta)^n/\sqrt{S_n Q_n}$ and $r_n = r^{2n}$. a_n series controls the evolution of the pulse width, b_n series controls the evolution of the power, and r_n series controls the decaying process of the individual signal.

The value of r is determined from balancing the total power

$$P = \sum_{n=1}^{\infty} P_n = \sum_{n=1}^{\infty} \frac{\sigma_N^2}{\sqrt{8\pi\beta}} \frac{(1+4\alpha\beta)^n}{\sqrt{S_n Q_n}} r^{2n} \approx \sum_{n=1}^{\infty} P_l r^{2n} = \frac{P_l}{1-r^2} \quad (3.31)$$

$$\text{where } P_l = \lim_{n \rightarrow \infty} \frac{\sigma_N^2}{\sqrt{8\pi\beta}} \frac{(1+4\alpha\beta)^n}{\sqrt{S_n Q_n}}$$

with the power related to the saturated gain [52]

$$P = \frac{G^2}{G^2 - 1} P_{sat} \ln \frac{G_{ss}}{G^2} \approx \frac{r^2}{r^2 - 1/G_{0l}^2} P_{sat} \ln \frac{G_{ss}}{G_{0l}^2} \quad (3.32)$$

Figure 3.3 plots the evolution of a_n , b_n , r_n , P_n series for a mode-locked laser with parameters as following: central wavelength $\lambda = 1550 \text{ nm}$, filter bandwidth $BW = 1.2 \text{ nm}$, modulation frequency $f_m = 10 \text{ GHz}$, modulation index $m = 0.5$, small signal gain $G_{ss} = 20 \text{ dB}$, noise figure $NF = 3 \text{ dB}$, total cavity loss $\Gamma = 6 \text{ dB}$. Figure 3.3a shows the evolution of the shaped noise for the first 200 round trips. During the first round trips (shaping phase) the shaping effects of filter and modulator are dominant and change the shaped noise after every round trip. The pulse width, which is determined by a_n , quickly converges to the steady-state value and reaches to 99% of the value after about 50 round trips. Similarly, the power parameter, b_n , rapidly converges to steady state and obtains less than 1% difference from the steady-state value after about 20 round trips. The gain reduced factor r is determined to be 0.99999969 and hence has negligible effect compared to the filtering and modulating effects during the first round trips. However, it has a long term effect on the power of the shaped noise when a_n and b_n already settle to their steady-state values. Figure 3.3b shows the long term evolution of the signal. After obtaining shaping phase steady state, the signal power does not remain constant but decreases for every round trip since net gain is negative due to gain reduced factor r . The power decreases with a very slow rate since $r \approx 1$. It can be seen that the power is just reduced by about 6 times after 5000000 round trips.

Figure 3.3. Evolution of a_n , b_n , r_n , P_n series in an actively mode-locked laser; (a) First 200 round trip evolution; (b) Long term evolution; the a_n series controlling the evolution of the pulse width and b_n series controlling the evolution of the power rapidly converge to steady state after about 50 round trips; the gain reduced factor series r_n plays negligible role during the first round trips (shaping phase), but is the main factor causing the power series P_n to decrease in long term (decaying phase).

Given the steady-state pulse signal of $\exp(-\omega^2/4\alpha)$, we can determine the noise contained in $\eta_n(\omega)$ as

$$\eta_n^{noise}(\omega) = \frac{\sigma_n (G_o \Gamma)^n}{\sqrt{Q_n}} \left[e^{-\omega^2 \beta a_n} - e^{-\omega^2 / 4\alpha} \right] \quad (3.33)$$

The noise power is

$$\begin{aligned} P_n^{noise} &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} |\rho_n^{noise}(\omega)|^2 d\omega \\ &= P_n \left[1 + 2\sqrt{\alpha\beta a_n} - 4\sqrt{\frac{2\alpha\beta a_n}{1 + 4\alpha\beta a_n}} \right] \end{aligned} \quad (3.34)$$

Therefore, the Signal to Noise Ratio (SNR) of the output pulse is

$$\begin{aligned}
 SNR &= \frac{\sum_{n=1}^{\infty} (P_n - P_n^{noise})}{\sum_{n=1}^{\infty} (P_n^{noise})} \\
 &= \frac{\sum_{n=1}^{\infty} P_n \left(4 \sqrt{\frac{2\alpha\beta a_n}{1+4\alpha\beta a_n}} - 2\sqrt{\alpha\beta a_n} \right)}{\sum_{n=1}^{\infty} P_n \left(1 + 2\sqrt{\alpha\beta a_n} - 4 \sqrt{\frac{2\alpha\beta a_n}{1+4\alpha\beta a_n}} \right)} \quad (3.35)
 \end{aligned}$$

Using (3.35) we can now evaluate the effect of the laser parameters such as small signal gain, noise figure and cavity loss on the laser quality. Figure 3.4 shows the dependence of the laser SNR on noise figure and cavity loss. The SNR decreases as the cavity loss increases. This can be understood that the required gain for lasing increases as the cavity loss increases. Thus the optical amplifier must work in high gain region which generates more ASE noise than in saturated region. Higher noise figure also adds more noise to the signal for every round trip and hence also reduce the signal quality. For the given laser parameters, the SNR is about 53.7 dB. However, it is noted that SNR as high as more than 60 dB can be obtained if the cavity loss and the noise figure of the amplifier are kept at low values. This is consistent with the low noise feature of reported mode-locked lasers [100, 101].

Figure 3.4. Dependence of Signal to Noise Ratio (SNR) on the cavity loss and amplifier Noise Figure (NF); small cavity loss and small NF help to increase the SNR.

3.5. MODE LOCKING WITH DETUNING

3.5.1. Noiseless amplifier

The steady-state signal is the limit of $\rho_n(\omega)$ when n approach infinite and is determined as

$$\eta(\omega) = \lim_{n \rightarrow \infty} \rho_n(\omega) \propto e^{-\lim_{n \rightarrow \infty} \left(\frac{S_n}{Q_n} \right) \beta \omega^2} e^{-j \omega \lim_{n \rightarrow \infty} \left(\frac{S_n}{Q_n} \right) t_d} = e^{-\frac{2}{x + \sqrt{4x + x^2}} \beta \omega^2} e^{-j \omega \frac{2}{x + \sqrt{4x + x^2}} t_d} \quad (3.36)$$

Therefore, the pulse shape in time domain is

$$\eta(t) \propto e^{-\frac{x + \sqrt{4x + x^2}}{8\beta} \left(t - \frac{2}{x + \sqrt{4x + x^2}} t_d \right)^2} = e^{-\alpha \left(t - \frac{t_d}{4\alpha\beta} \right)^2} \quad (3.37)$$

where $\alpha = \frac{x + \sqrt{4x + x^2}}{8\beta}$ is the Gaussian pulse parameter. The pulse width when

detuning is the same as that of exact tuning. However, the pulse position does not pass the modulator at the peak of the modulation transfer function but shifted by an amount

of $t_s = t_d/4\alpha\beta$, which is proportional to the detuning Δf_m . This is exactly the same result as in section 2.

The power of the signal after n round trips

$$P_n = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \rho_n^2(\omega) d\omega = \frac{\sigma_N^2}{\sqrt{8\pi\beta}} \frac{(\Gamma G)^{2n}}{\sqrt{S_n Q_n}} e^{-\frac{R_n}{2\beta Q_n} t_d^2} \quad (3.38)$$

The gain required for $\lim_{n \rightarrow \infty} P_n = \text{const}$ is

$$G = G_l = \frac{\sqrt{1+4\alpha\beta}}{\Gamma} e^{\frac{t_d^2}{4\beta}} \quad (3.39)$$

This is the same as that obtained using the classical approach. Compared to the zero-detuning case, the required gain increases by $\exp(t_d^2/4\beta)$ due to the addition loss caused by position shifting.

Locking range

Detuning causes the pulse to pass the modulator not at the transmission peak but at a shifted position t_s . This induces more loss to the pulse and hence requires more gain to compensate for the loss. The larger the detuning the higher the gain is required. However, the maximum gain provided by the amplifying medium is limited to G_{ss} , the small signal gain. If the required gain exceed G_{ss} mode locking is not obtained. Using (3.39), we got

$$\frac{\sqrt{1+4\alpha\beta}}{\Gamma} \exp\left(\frac{t_d^2}{4\beta}\right) < G_{ss} \quad (3.40)$$

$$\Rightarrow |\Delta f_m| < \frac{f_m^2}{N} \sqrt{4\beta \ln\left(\frac{G_{ss}\Gamma}{\sqrt{1+4\alpha\beta}}\right)} \quad (3.41)$$

The normalized locking range is

$$|\delta f_m| \equiv \frac{|\Delta f_m|}{f_m} < \frac{f_m}{N} \sqrt{4\beta \ln \left(\frac{G_{ss}\Gamma}{\sqrt{1+4\alpha\beta}} \right)} \quad (3.42)$$

In addition to the limitation impinged by the required gain, the locking range is also limited by the maximum allowable value of pulse position shift when detuned. In order to reshape and retime the pulse to keep the pulse unchanged for every round trip, a negative second derivative in the modulation transmission function is required [102]. In the case of Mach-Zehnder Intensity modulator, the second derivative change sign at about $T_m/4$. Therefore the pulse must arrive at the modulator no later than $T_m/4$. Equation (3.37) shows that the steady-state pulse position when detuned is t_s . When it arrives the modulator in the next round its position becomes $t_s + t_d$ and must satisfy the condition $|t_s + t_d| < T_m/4$. Using this condition, we obtain the locking range as

$$|\delta f_m| \equiv \frac{|\Delta f_m|}{f_m} < \frac{\alpha\beta}{N(1+4\alpha\beta)} \quad (3.43)$$

Combining (3.42) and (3.43) the locking range is determined as

$$|\delta f_m| < \min \left\{ \frac{f_m}{N} \sqrt{4\beta \ln \left(\frac{G_{ss}\Gamma}{\sqrt{1+4\alpha\beta}} \right)}, \frac{\alpha\beta}{N(1+4\alpha\beta)} \right\} \quad (3.44)$$

In most of the cases, the loss of the cavity is kept at very low while the amplifying medium gain is large enough that the first term in the RHS of (3.44) is larger than the later one. Hence the locking range is usually limited by the later term. Using the given laser parameters, we obtain $|\delta f_m| < \min\{2.64 \times 10^{-5}, 1.3 \times 10^{-5}\} = 1.3 \times 10^{-5}$, which will be verified with simulation result in section 3.6.

3.5.2. Amplifier with ASE noise

Following procedure similar to that of zero detuning, we obtain that the required gain for lasing and the power of the signal after n round trips as

$$G = G_l r = \frac{\sqrt{1+4\alpha\beta}}{\Gamma} e^{\frac{t_d^2}{4\beta}} r \quad (3.45)$$

$$P_n = \frac{\sigma_N^2}{\sqrt{8\pi\beta}} \frac{(1+4\alpha\beta)^n}{\sqrt{S_n Q_n}} e^{\frac{t_d^2}{2\beta} \left(n - \frac{R_n}{Q_n}\right)} r^{2n} \quad (3.46)$$

In which the noise is

$$\begin{aligned} P_n^{noise} &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| \rho_n^{noise}(\omega) \right|^2 d\omega \\ &= P_n \left[1 + 2\sqrt{\alpha\beta a_n} - 4\sqrt{\frac{2\alpha\beta a_n}{1+4\alpha\beta a_n}} e^{-4\alpha(t_d a_n - t_s)^2 / (1+4\alpha\beta a_n)} \right] \end{aligned} \quad (3.47)$$

Therefore, the SNR of the output pulse is

$$\begin{aligned} SNR &= \frac{\sum_{n=1}^{\infty} (P_n - P_n^{noise})}{\sum_{n=1}^{\infty} (P_n^{noise})} \\ &= \frac{\sum_{n=1}^{\infty} P_n \left(4\sqrt{\frac{2\alpha\beta a_n}{1+4\alpha\beta a_n}} e^{-4\alpha(t_d a_n - t_s)^2 / (1+4\alpha\beta a_n)} - 2\sqrt{\alpha\beta a_n} \right)}{\sum_{n=1}^{\infty} P_n \left(1 + 2\sqrt{\alpha\beta a_n} - 4\sqrt{\frac{2\alpha\beta a_n}{1+4\alpha\beta a_n}} e^{-4\alpha(t_d a_n - t_s)^2 / (1+4\alpha\beta a_n)} \right)} \end{aligned} \quad (3.48)$$

The evolution of the signal also consists of two phases: shaping phase and fading phase. In the shaping phase, filtering and modulating have strong effect and shape the noise to steady-state Gaussian pulse. This phase is characterized by the developing of three series: $a_n = S_n/Q_n$, $b_n = (1+4\alpha\beta)^n/\sqrt{S_n Q_n}$ and $c_n = \exp((n-R_n/Q_n)t_d^2/2\beta)$. a_n series controls the evolution of the pulse width and pulse position, while b_n and c_n series control the evolution of the power. When a_n , b_n and c_n obtain their steady states,

the shaping phase is complete and the signal gets the shape of the steady-state pulse as determined in the noiseless amplifier case. In the fading phase, the $r_n = r^{2n}$ series plays dominant role and the signal power decreases exponentially with r^2 . The value of r is determined by solving the equation

$$P = \sum_{n=1}^{\infty} P_n = \frac{G_l^2 r^2}{G_l^2 r^2 - 1} P_{sat} \ln \frac{G_{ss}}{G_l^2 r^2} \quad (3.49)$$

Figure 3.5 plots the series of a_n , b_n , c_n , r_n , P_n for the mode-locked laser with frequency detune of $\delta f_m = 10^{-5}$ ($\Delta f_m = 100$ kHz). During the first round trips (shaping phase), the noise is strongly shaped to steady-state pulse due to filtering, modulating and delaying effects. This reflects on the rapid evolution and converging of a_n , b_n and c_n . They reach 99% of their steady-state values after 47, 19 and 63 round trips, respectively. Compared to exact tuning, the shaping phase takes more round trips since it involves an additional series evolution c_n . In contrast to exact tuning, the value of gain reduced factor r in detuning is rather small, especially when the detuning is large. This causes the signal to rapidly decrease in the fading phase. It can be seen from Figure 3.5b that the signal power falls to nearly zero after just 20000 round trips. As the noise is stronger in the shaping phase than in the fading phase, the long shaping phase and short fading phase in detuning makes the laser's signal to noise ratio reduced. For the given laser, the SNR is only 28.4 dB, largely reduced from the value of 53.7 dB when zero detuning.

Figure 3.5. Evolution of pulse width and pulse position controlling series a_n , power controlling series b_n , c_n , gain reduced factor series r_n , and power series P_n in a detuned actively mode-locked laser with frequency detune of $\delta f_m = 10^{-5}$; (a) First 200 round trip evolution; (b) Long term evolution; the additional power controlling series c_n when detuned make the pulse need more round trips to reach the steady state; small gain reduced factor r causes the power series P_n to decrease faster; longer shaping phase and shorter decaying phase reduce the SNR of the output pulse when detuned.

The dependence of SNR on frequency detune is shown in Figure 3.6. As detuned frequency increases, SNR rapidly decreases. SNR drops to 18.6 dB with just a normalized detune of 1.2×10^{-5} . In practice, the laser SNR must satisfy a minimum value to be useful in applications. A minimum value of 25 dB is required for telecommunication application. Using this requirement, we can determine the locking range of the laser in the presence of ASE noise. Figure 3.7 shows the locking range of a

mode-locked laser with different filter bandwidth and noise figures. The position limited locking range, which is specified in (3.44), is also plotted for comparison. It is shown that with the present of ASE noise and a requirement for minimum laser SNR, the locking range of the laser is limited to a narrower range compared to that limited by the maximum shifted position.

Figure 3.6. Degrading of the mode-locked laser's SNR with increase of frequency
detune

Figure 3.7. Maximum locking range of the mode-locked laser with SNR threshold of 25 dB; locking range when the ASE noise is considered is narrower than that in noiseless situation.

Figure 3.8. SNR contours of the actively mode-locked laser.

For different requirement of SNR of the laser, we can use the SNR contour plotted in Figure 3.8 to determine the locking range. For a given SNR quality, the filter bandwidth and detuning frequency should be kept in the region under the specific SNR contour line. As the filter bandwidth increases, not only that the maximum detuning frequency is decreased, but also the spacing between the contour line is decreased. Hence, the laser quality is more sensitive to the detuning; a small detuning amount can degrade the laser SNR from high quality to very low quality.

3.6. SIMULATION RESULTS

The simulation model consists of a gain saturated amplifying medium with white Gaussian noise, a bandpass filter and a modulator. The build up process of the pulse in the simulation is similar to that in the real mode-locked laser which is a self-started process from amplifying spontaneous emission (ASE) noise. The initial signal is a set of random complex number which represents the ASE white noise. The signal is then amplified in a saturated gain medium with white Gaussian noise. Filtering is implemented in frequency domain using Fast Fourier Transform (FFT) algorithm. The signal is first converted to frequency domain and then multiplied with the filter transfer function. The signal is then converted back to time domain using Inverted

Fast Fourier Transform (IFFT) algorithm and passed through the modulator. The Mach-Zehnder modulator transmission function expressed in (3.2) is used in the simulation except clearly specified in the text.

After passing the modulator, the signal completes one round trip and is fed back to the amplifier for the next round. The process is continuing until equilibrium is obtained.

Figure 3.9. Pulse evolution in an actively mode-locked laser with a normalized detuning frequency of 5×10^{-6} ($\Delta f_m = 50$ kHz); (a) Time domain; (b) Frequency domain

Figure 3.10. Evolution of pulse peak value (a) and pulse position (b) during pulse build up process in the detuned mode-locked laser

First of all the laser is simulated with a noiseless amplifier in which the ASE noise source is turned off. The theoretical Gaussian modulation transmission function is

used for easy comparing with analytical results. Figures 3.9 and 3.10 show the simulation build up process of the pulse inside the cavity when the modulation frequency is detuned. Initially, the pulse position is shifted away from the transmission peak for every roundtrip due to detuning. After about 100 round trips, the pulse position inclines to a constant which is the steady-state value t_s predicted by the theory. At this position, the retiming effect of the modulation completely compensates for the delay due to detuning and hence the pulse position is fixed or every round trip afterward.

The simulation also confirms that the pulse width is independent of detuning and has the same value as that when exact tuning. Variation of the pulse position and the gain are shown in Figure 3.11. Within the calculated locking range, $\pm\delta f_{max} = \pm 1.3 \times 10^{-5}$, the simulation results are in good agreement with the analytical results. As predicted in theory, the pulse position linearly increases while the required gain exponentially increases with detuning. When the detune frequency exceeds δf_{max} , the theoretical calculation based on the Gaussian approximation is no longer valid. The pulse position does not increase linearly with detuning but inclines to a constant of about $T_m/4$.

Figure 3.11. Normalized pulse position (a) and gain (b) of the mode-locked laser when detuned; the pulse position linearly increases with detuning within the locking

$$\text{range } \pm \delta f_{\text{max}} = \pm 1.3 \times 10^{-5}.$$

Figure 3.12. Normalized pulse position (a) and gain (b) of the mode-locked laser with MZIM when detuned; the results are nearly the same as that with Gaussian modulation function.

Simulation with the MZIM transmission function is shown in Figure 3.12. The results are nearly the same as that of approximated Gaussian transmission function. The small difference in the required gain is due to the difference between the two transmission functions.

Figure 3.13 shows the pulse building up process, its peak power and position evolution over 10000 round trips of the laser with ASE noise when the frequency

detune is zero. Compared to noiseless case, the steady-state peak power and position have slight fluctuation due to the noise generated by the amplifier. The standard deviations of the power and position are 8.8×10^{-5} and 4.3×10^{-7} , respectively. Figure 3.14 -Figure 3.16 show the evolution of the pulse with different detuned frequency. As the detune increases, the fluctuation of the pulse increases. This is also observed in Figure 3.17 which shows pulse position when frequency detuning changing from -2×10^{-5} to 2×10^{-5} . The simulation is run over 10000 round trips for every frequency detuning value. The pulse positions after a few hundred transient round trips are recorded and used to calculate the means and standard deviations. The red dots are the means of the pulse position in steady state while the bars are the standard deviations. It can be seen from the simulation results that the pulse quality has been critically decreased before the detuned frequency reaches 1.3×10^{-5} , the limit for noiseless case. This confirms that the locking range when considering ASE noise is narrower than that derived from the pulse position limiting.

Figure 3.13. Simulation of zero-detuned mode-locked laser under the influence of ASE noise; (a) Pulse build up process, (b) pulse peak power, and (c) normalized pulse position

Figure 3.14. Simulation of mode-locked laser under the influence of ASE noise when normalized detuning frequency $\delta f_m = 8 \times 10^{-6}$; (a) Pulse build up process, (b) pulse peak power, and (c) normalized pulse position

Figure 3.15. Simulation of mode-locked laser under the influence of ASE noise when normalized detuning frequency $\delta f_m = 10 \times 10^{-6}$; (a) Pulse build up process, (b) pulse peak power, and (c) normalized pulse position

Figure 3.16. Simulation of mode-locked laser under the influence of ASE noise when normalized detuning frequency $\delta f_m = 11 \times 10^{-6}$; (a) Pulse build up process, (b) pulse peak power, and (c) normalized pulse position

Figure 3.17. Normalized pulse position of the mode-locked laser simulated with ASE noise included, the error bar shows the standard variation of the simulated position.

3.7. CONCLUSIONS

In this chapter, we have presented an analysis of actively mode-locked laser using mathematical series to trace the evolution of the signal in the laser cavity. Using this approach, we can obtain not only the steady state but also the transient evolution processes of the mode-locked pulse. The noiseless amplifier laser model has been analyzed and the results are consistent with those obtained using classical analysis.

Steady-state pulse parameters in either exact tuning or detuning are derived. It is shown that the pulse width when detuned is the same as that when exactly tuned. The pulse position is linearly shifted with detune and this limits the locking range of the laser.

The advantage of our proposed series method is its ability of analyzing the laser under the influence of the ASE noise that is normally ignored in classical analyses. Using the series method, we have calculated the Signal to Noise Ratio (SNR) of the output pulse and studied the dependence of SNR on the cavity design parameters such as cavity loss and amplifier noise. The decrease of SNR when the laser is detuned has also been studied. It is found that the presence of ASE noise narrows down the locking range compared to the case in which an ideal noiseless amplifier is employed

4 Dispersion and Nonlinearity Effects in Actively Mode-locked Fiber Lasers

4.1. INTRODUCTION

Fiber lasers have several interesting characteristics that distinguish them from other solid state or gas lasers. The low loss of the fiber (0.2 dB/km attenuation) makes it possible to form a low loss cavity and thus low threshold laser. Since the intensity in the fiber is very high due to its small core area, nonlinear cavity can be easily obtained. Moreover, the cavity dispersion can also be easily controlled by using fibers with different group velocity dispersion. With the introduction of nonlinearity and dispersion into the cavity, lasers with advance quality and performance can be obtained by optimizing the laser parameters.

In fact, the effects of dispersion and nonlinearity on an optical pulse when it propagates through a fiber have been studied extensively using Nonlinear Schrödinger

equation (NLS) [52, 103-109]. By solving the NLS equation using inverse scattering method [110], the authors obtained the solution in form of soliton whose shape is kept unchanged if the dispersion and nonlinear effects are balanced. Although soliton in optical fiber was predicted in 1973 [107], it was not experimentally observed until 1980 [111] due to the lack of short optical pulse source [106]. Numerical solving the NLS equation using Split Step Fourier [112] or Finite-difference methods shows that the pulse can be compressed by nonlinear effect or be broadened by dispersion effect.

Since pulses are formed and travel in actively mode-locked fiber lasers, dispersion and nonlinearity of the fiber cavity would play an important role in this mode locking process. By introducing the master equation for mode locking and numerical simulation, Haus et al. showed that self phase modulation (SPM) can shorten of the mode-locked pulse by a factor of 2 [113]. Later, Kartner et al. added that further shortening factor could be obtained when abnormal dispersion is introduced into the cavity [114]. The pulse shortening is the result of soliton pulse formation in the cavity and has been reported in many papers [20, 34, 91, 114-120]. However, the results were obtained with the assumption that mode locking conditions are satisfied and pulse propagation in the cavity can be approximately described by the master equation. The performance of the laser and characteristic of the pulse when the laser is detuned were not discussed.

In this chapter, we present a detail study on the dispersion and nonlinear effects on the performance of actively mode-locked fiber laser. Characteristics of the mode-locked pulses are discussed not only when the laser is exactly tuned but also when the laser is detuned. We show that there is a trade-off between pulse shortening and stability of

the laser. Cavity dispersion and nonlinearity should be optimized to obtain the shortest pulse while the laser is still stable within a certain amount of detuning.

The structure of this chapter is as follows. Propagation of optical pulses in a fiber is given in Section 4.2 where the effects of dispersion and nonlinearity on the propagating pulse are discussed. The laser model is then described in Section 4.3. Dispersion and nonlinear effects on properties of the mode-locked pulse and locking range of the laser are discussed in Section 4.4 and 4.5 respectively. Soliton formation under the combination effects of dispersion and nonlinearity is examined in Section 4.6. The numerical results are confirmed with experimental results presented in Section 4.7. And finally, conclusions are given in Section 4.8.

4.2. PROPAGATION OF OPTICAL PULSES IN A FIBER

4.2.1. Dispersion effect

When propagating through free space, the lightwave travels at a constant speed c . However, when traveling through a material rather than free space, the light speed is slowed down due to the interaction between the light's electromagnetic field and the material's electron cloud. The degree to which the light is slowed down is given by the material's refractive index n , which relates to the speed of the light in the material by:

$$v = c/n \quad (4.1)$$

Since the interaction of the electromagnetic field with the material depends on the frequency of the light, the refractive index also depends on the light frequency. This

property is referred to as chromatic dispersion. Chromatic dispersion is also called material dispersion and is given by

$$D_M(\lambda) = -\frac{\lambda}{n} \frac{d^2 n}{d\lambda^2} \quad (4.2)$$

In which the refractive index n is a function of wavelength λ

$$n^2(\lambda) = 1 + \sum_{j=1}^m \frac{G_j \lambda^2}{\lambda^2 - \lambda_j^2} \quad (4.3)$$

where G_j are the Sellmeier constants.

A plot of the refractive index over the wavelength region for optical communication for a pure silica fiber is shown in Figure 4.1.

Figure 4.1. Variation in the silica refractive index as a function of optical wavelength

Chromatic dispersion plays a critical role in the propagation of optical pulse. If the optical pulse contained only one frequency component, i.e. zero bandwidth, it would

travel through the material without any distortion. In reality, an optical pulse actually includes a range of different frequencies. The shorter the pulse the wider the range of frequency it covers. Different frequency components will travel at different speeds, some of them travel faster, while others lag behind. This results in broadening of the pulse.

Besides chromatic dispersion, when traveling in an optical fiber, the light also experiences an additional dispersion caused by the structure of the optical fiber/waveguide. This dispersion is called waveguide dispersion and given by

$$D_w(\lambda) = -\frac{n_2(\lambda)\Delta}{c\lambda} V \frac{d^2(Vb)}{dV^2} \quad (4.4)$$

where $n_2(\lambda)$ is the refractive index of the core, Δ is the relative refractive index difference, b is the normalized propagation constant, and V is the normalized frequency.

The total dispersion of the fiber is the combination of chromatic dispersion and waveguide dispersion

$$D(\lambda) = D_M(\lambda) + D_w(\lambda) \quad (4.5)$$

A plot of the fiber dispersion over the wavelength region for optical communication is shown in Figure 4.2

Figure 4.2. Fiber dispersion as a function of optical wavelength

The propagation of an optical pulse in a fiber is characterized by a mode propagation constant β . The mode propagation constant β can be expanded in a Taylor series about the center frequency ω_0

$$\beta(\omega) = \frac{n(\omega)\omega}{c} = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}\beta_2(\omega - \omega_0)^2 + \dots \quad (4.6)$$

where

$$\beta_m = \left. \frac{d^m \beta}{d\omega^m} \right|_{\omega=\omega_0} \quad (m = 0, 1, 2, \dots) \quad (4.7)$$

The pulse envelope moves at the group velocity ($v_g = 1/\beta_1$) while the parameter β_2 is responsible for pulse broadening. β_2 is generally referred to as the group velocity dispersion (GVD) parameter and relates to the fiber dispersion by

$$\beta_2 = -\frac{\lambda^2 D}{2\pi c} \quad (4.8)$$

The fiber is said to exhibit normal dispersion for $\beta_2 > 0$, and to exhibit abnormal dispersion for $\beta_2 < 0$. In normal dispersion regime, high frequency (blue-shifted) components of an optical pulse travel slower than the low frequency (red-shifted) components. The opposite occurs in abnormal dispersion regime.

Dispersion can be a major problem in fiber optic communication systems due to its pulse broadening effect. Broadened pulse has lower peak intensity than the initial pulse, making it more difficult to detect. Worse still, pulse broadening can cause two adjacent pulses to overlap, leading to errors at the receiver. However, dispersion is not always a harmful effect. When combined with self phase modulation (SPM) effect, it can lead to a formation of optical soliton, the wave that keep its shape unchanged during propagating through the fiber.

4.2.2. Nonlinear effect

The fiber refractive index not only varies with wavelength but also varies with the intensity of the light traveling inside the fiber. The interaction between the light and the material electron cloud increases nonlinearly with light intensity, so the high intensity light travels slower than the low intensity one. The result is that the refractive index of the fiber increases with the light intensity. This is known as fiber nonlinearity.

The fiber refractive index can be written as a function of wavelength and light intensity

$$n(\lambda, |E|^2) = n(\lambda) + n_2 |E|^2 \quad (4.9)$$

Where $n(\lambda)$ is the linear part given by Eq. (4.3), $|E|^2$ is the optical intensity inside the fiber, and n_2 is the nonlinear-index coefficient.

The fiber nonlinearity causes are several effects on the optical pulse but the most concern to the pulse propagation theory is self phase modulation (SPM) effect. Under the effect of self phase modulation, the optical pulse experiences a phase shift induced by the intensity-dependent refractive index while the pulse shape remains unchanged. The nonlinear phase shift induced to the optical pulse traveling a distance L is given by

$$\phi_{NL}(L, T) = |U(0, T)|^2 (L/L_{NL}) \quad (4.10)$$

where $U(0, T)$ is the normalized pulse envelope at $z = 0$, L_{NL} is the nonlinear length defined as

$$L_{NL} = (\gamma P_0)^{-1} \quad (4.11)$$

with P_0 is the peak power of the pulse, and γ is the nonlinear coefficient.

γ can be calculated from the nonlinear-index coefficient n_2 as

$$\gamma = n_2 \omega_0 / c A_{eff} \quad (4.12)$$

where A_{eff} is the effective area of the fiber.

The temporally varying phase ϕ_{NL} implies that the instantaneous optical frequency differs from its central frequency ω_0 . The frequency difference $\delta\omega$ is given by

$$\delta\omega(T) = -\frac{\partial\phi_{NL}}{\partial T} = -\left(\frac{L}{L_{NL}}\right) \frac{\partial|U(0, T)|^2}{\partial T} \quad (4.13)$$

It can be seen from Eq. (4.13) that new frequency components are continuously generated as the pulse propagates down the fiber. Those new frequencies broaden the spectrum over its initial value at $z = 0$. The time dependence of $\delta\omega$ is referred to as frequency chirping.

4.2.3. Soliton

Consider the case that an optical pulse is launched into a fiber with $\beta_2 < 0$, anomalous dispersion. Self phase modulation (SPM) leads to lower frequencies at the leading edge of the pulse and higher frequencies at the trailing edge. In anomalous dispersion regime, lower frequencies travel slower than higher frequencies. Therefore, anomalous dispersion causes the leading edge of the pulse to slow down while the trailing edge travels faster. Effectively, anomalous dispersion compresses the pulse and undoes the frequency chirp induced by SPM. If the pulse has the shape so that the effects of SPM and GVD cancel each other then the pulse remains its initial width along the entire length of the fiber. This pulse is called soliton.

The pulse which has the above property is the *sech* profile pulse

$$U(T) = P_0 \text{sech}(T/T_0) \quad (4.14)$$

Where P_0 is the peak power, T_0 is the pulse width related to the full width at half maximum (FWHM) T_{FWHM} as

$$T_0 = T_{FWHM}/1.665 \quad (4.15)$$

The peak power P_0 required to support the fundamental soliton is given by

$$P_0 = |\beta_2|/\gamma T_0^2 \quad (4.16)$$

In a real system, it is difficult to generate pulses with exact parameters specified above for soliton formation. Theoretical and experimental results show that if the pulse does not have hyperbolic-secant shape and exact power as specified in (4.16), soliton pulse can still be formed with the condition that the initial values must be close enough to the final soliton values [52, 106]. The interplay between GVD and SPM can mold the pulse into a hyperbolic-secant shape with appropriate peak power.

4.2.4. Propagation equation in optical fibers

Propagation of optical pulses in an optical fiber, including all linear and nonlinear effects, can be described by the following equation

$$\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial t^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial t^3} + \frac{\alpha}{2} A = j\gamma |A|^2 A - k_1 \frac{\partial(|A|^2 A)}{\partial t} - k_2 A \frac{\partial |A|^2}{\partial t} \quad (4.17)$$

where A is the pulse envelope, z is the axial distance, t is the time, β_m is the m^{th} order derivative of the mode-propagation constant β and defined in Eq. (4.7), α is the fiber loss, γ is the nonlinear coefficient, k_1 is given by $k_1 = \gamma/\omega_0$, and k_2 is given by $k_2 = i\gamma T_R$. T_R is related to the slope of Raman gain spectrum and can be approximated to be about 3 fs.

Making the transformation

$$T = t - z/v_g = t - \beta_1 z \quad (4.18)$$

Eq. (4.17) becomes

$$\frac{\partial A}{\partial z} + \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} - \frac{1}{6} \beta_3 \frac{\partial^3 A}{\partial T^3} + \frac{\alpha}{2} A = j\gamma |A|^2 A - k_1 \frac{\partial(|A|^2 A)}{\partial T} - k_2 A \frac{\partial |A|^2}{\partial T} \quad (4.19)$$

On the left-hand side of the equation, the second term is responsible for the GVD, the third term governs the effects of third-order dispersion, and the last term accounts for the loss. On the right hand side, the first term governs the effects of SPM and plays important role in the formation of soliton. The term proportional to k_1 is responsible for the self-steepening and shock formation. The last term is related to Raman scattering and accounts for the self-frequency shift.

For pico-second pulses, the effects of self-steepening and self-frequency shift are so small that the last two terms on the right-hand side can be neglected. Since the

contribution of the third-order dispersion term is also quite small, the third term on the left-hand side can also be neglected. The propagation equation can be simplified

$$\frac{\partial A}{\partial z} + \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \frac{\alpha}{2} A = j\gamma |A|^2 A \quad (4.20)$$

In the special case where $\alpha = 0$, Eq (4.20) becomes the well known nonlinear Schrödinger equation (NLSE) describing the propagation of moderately short pulses.

The fundamental soliton solution for NLS equation is given by

$$A(z, T) = \sqrt{P_0} \operatorname{sech}(T/T_0) \exp(iz/2L_D) \quad (4.21)$$

where T_0 is the pulse width, P_0 is the peak power satisfied the Eq. (4.16), and L_D is the dispersion length defined as

$$L_D = T_0^2 / |\beta_2| \quad (4.22)$$

4.3. THE LASER MODEL

Figure 4.3 shows the laser model which consists of an optical amplifier, a bandpass optical filter (BPF), a fiber and an intensity modulator. The optical wave is amplified in the optical amplifier and then filtered by the BPF. After filtering, the lightwave propagates through the fiber, where it experiences the dispersion and nonlinear effects, and finally is modulated by the optical modulator. The output of the modulator is fed back into the amplifier to create a loop.

Figure 4.3. General schematic model of the actively mode-locked fiber laser

4.3.1. Modeling of the fiber

The fiber can be modeled by the propagation equation [52]

$$\frac{\partial A}{\partial z} + \frac{j}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \frac{\alpha}{2} A = j\gamma |A|^2 A \quad (4.23)$$

where A is the signal envelope, z is the axial distance, T is the delayed time ($T = t - z/v_g$), v_g is group velocity, α is the linear attenuation factor of the fiber and accounts for the loss, γ is the nonlinear coefficient which accounts for the self phase modulation effect, β_2 is the second order derivative of the propagation constant β and can be calculated from the fiber dispersion parameter as

$$\beta_2 = -\frac{\lambda^2 D}{2\pi c} \quad (4.24)$$

in which λ is the operating wavelength in vacuum, D is the dispersion factor of the fiber, c is the speed of light.

4.3.2. Modeling of the EDFA

Since the bandwidth the optical amplifier is much larger than the filter bandwidth, it can be modeled by a wavelength independent gain with saturated equation:

$$g_0 = \frac{g_{ss}}{1 + P_{av}/P^{sat}} \quad (4.25)$$

where g_{ss} is small signal gain factor, P_{av} is the signal average power, P^{sat} is the saturation power level.

4.3.3. Modeling of the optical modulator

The modulator can be modeled from the transmission function determined in (4.20)

$$T = \alpha_m \cos^2 \left(\frac{\pi(v_m - V_{sh})}{2V_\pi} \right) \quad (4.26)$$

where V_π is the voltage applied into the modulator causes π phase shift in one arm of the integrated optical interferometer, V_{sh} accounts for the DC drift of the modulator, α_m is the insertion loss, v_m is the modulating voltage signal and can be given by

$$v_m = V_m \cos(\omega_m t) + V_b \quad (4.27)$$

in which V_m is the amplitude of the modulating signal, ω_m is the modulating frequency, V_b is the bias voltage.

Substituting (4.27) into (4.26) and note that V_{sh} can be assumed to be zero without any affect to the final result, the transmission function of the modulator can be written as

$$T = \alpha_m \cos^2 \left(\frac{\pi}{4} \Delta_m \cos(\omega_m t) + \frac{\pi}{2} \frac{V_b}{V_\pi} \right) \quad (4.28)$$

where $\Delta_m = 2V_m/V_\pi$ is the modulation depth. When the modulation is biased at the quadrature point $V_b = V_\pi/2$, (4.28) becomes

$$T = \alpha_m \cos^2 \left[\frac{\pi}{4} (\Delta_m \cos(\omega_m t) + 1) \right] \quad (4.29)$$

4.3.4. Modeling of the optical filter

The optical filter BPF can be described by the following transfer function following a Gaussian profile as

$$H(f) = \alpha_F \exp\left(-\frac{1}{2}\left(\frac{f}{B_0}\right)^2\right) \quad (4.30)$$

where α_F is the insertion loss, B_0 is half of the $(1/e)$ bandwidth of the filter

4.4. DISPERSION EFFECTS IN ACTIVELY MODE-LOCKED FIBER LASERS

4.4.1. Zero detuning

To study the dispersion effects in actively mode-locked fiber lasers, we set the nonlinearity parameter in the fiber model to zero so that the laser cavity exhibits only the dispersion property. The cavity dispersion is determine as

$$D_{cav} = D_f L \quad (4.31)$$

where D_f is the fiber dispersion, L is the fiber length.

For convenience, we normalize it using

$$D_n = \frac{D_{cav}}{D_{ref}} \quad (4.32)$$

in which

$$D_{ref} = \frac{2\pi c(\tau_G / 1.703)^2}{\lambda_0^2} \quad (4.33)$$

where τ_G is the pulsewidth of the Gaussian pulse generated when fiber effect is ignored. For given values of modulation frequency f_m , modulation index m , and filter bandwidth Ω , from (3.37) we have

$$\tau_G = 2(2 \ln 2)^{3/4} \sqrt{\frac{1}{m\omega_m^2 \Omega^2}} \quad (4.34)$$

The simulation parameters are as following: central wavelength $\lambda = 1550 \text{ nm}$, filter bandwidth $BW = 1.2 \text{ nm}$, modulation frequency $f_m = 10 \text{ GHz}$, modulation index $m = 0.5$, small signal gain $G_{ss} = 20 \text{ dB}$, noise figure $NF = 3 \text{ dB}$, total cavity loss $\Gamma = 6 \text{ dB}$.

Figure 4.4a shows the buildup process of the pulse in an actively mode-locked fiber laser with a normalized cavity dispersion of 0.2. Similarly to the case of zero dispersion cavity shown in Figure 4.4b, the pulse is rapidly obtain its steady state after about 100 round trips. However the pulse width for the dispersive cavity is much larger than that of zero dispersion. Steady-state pulses with different cavity dispersion values are plotted in Figure 4.5. It is shown that the higher the dispersion value the wider the pulse obtained.

Figure 4.4. Pulse buildup in an actively mode-locked fiber laser with (a) normalized cavity dispersion of 0.2 and (b) zero dispersion

Figure 4.5. Steady-state pulses with different cavity dispersion; increasing of the cavity dispersion broadens the mode-locked pulse due to velocity mismatch of different frequency components in the pulse

The dependence of pulse width on the cavity dispersion is presented in Figure 4.6. It shows a symmetric dependence of the pulsewidth on the cavity dispersion over the zero dispersion value. Either normal dispersion or abnormal dispersion has the same broadening effect on the pulse. The pulsewidth may increase from 13.6 ps for zero dispersion to up to 32.8 ps for an absolute normalized cavity dispersion value of 1 .

Figure 4.6. Symmetric dependence of the pulsewidth on the cavity dispersion over the zero dispersion value

The pulse broadening effect of dispersion on the mode-locked pulse can be explained due to the velocity difference between different frequency components in the mode-locked pulse. In an abnormal dispersion cavity ($D > 0$), low frequency (red-shifted) components of the pulse travel slower than the high frequency (blue-shifted) ones. This results in broadening of the pulse with high frequencies at the leading edge and low frequencies at the trailing edge. The pulse is called down-chirp [105]. In opposite, low frequency components of the pulse travel faster than the high frequency components in a normal dispersion cavity ($D < 0$). This also causes the pulse broadened but the low frequencies are at the leading edge while the high frequencies are at the trailing edge. And hence up-chirp pulse is obtained. This is clearly confirmed in Figure 4.7 which shows that up-chirp pulses are generated in a normal dispersion cavity mode-locked laser while down-chirp pulses are generated in an abnormal dispersion cavity

Figure 4.7. Up-chirped pulses are generated in a normal dispersion cavity mode-locked laser while down-chirp pulses are generated in an abnormal dispersion cavity

4.4.2. Dispersion effects in detuned actively mode-locked fiber lasers

As discussed in previous chapter, when the modulation frequency is slightly detuned from the harmonic of the fundamental frequency the laser is still under mode-locking state. However, the pulse position is no longer at the transmission peak of the modulation function but shifted linearly with the detuning frequency. The position shift causes a pushing (pulling) effect on the pulse when it passes the modulator and hence compensates for the detuning. In a dispersive cavity, the shifting of the pulse when detuned is also observed as shown in Figure 4.8. The figure shows the pulse buildup processes of 10 GHz actively mode-locked fiber lasers when the modulation frequency is detune by an amount of $\delta f_m = 8 \times 10^{-6}$ ($\Delta f_m = 80$ kHz) for the case of zero dispersion and dispersion equal to 0.1. Compared to the zero dispersion cavity, the position shifting in a dispersive cavity is less severe.

Figure 4.8. Pulse buildup in an actively mode-locked fiber laser with detuning frequency of $\delta f_m = 8 \times 10^{-6}$; (a) normalized cavity dispersion $D = 0.1$; (b) zero dispersion

Figure 4.9. Steady-state pulses of a detuned mode-locked fiber laser ($\delta f_m = 8 \times 10^{-6}$) with different cavity dispersion values; large dispersion causes pulses broadening but reduces the pulse position shifting effect when detuned.

Figure 4.10. Increasing of cavity dispersion (absolute value) reduces pulse shifting effect when the laser is detuned

Figure 4.9 and Figure 4.10 show the effect of dispersion on the steady-state pulse and on its position when the laser is detuned. Both normal dispersion and abnormal dispersion cause the same effects on the detuned laser. The dispersion broadens the pulse but keeps it from shifting far away from the transmission peak. It can be understood that as the pulse becomes wider, it will experience stronger shaping effect by the modulation which pushes (or pulls) it back to the transmission peak. Therefore, the pulse is shifted less in the dispersive cavity than in the zero dispersion cavity although the detuning frequency is the same.

Besides the pulse position shifting effect on the detuned mode-locked fiber lasers, the dispersion also shifts the optical carrier towards faster/slower wavelength to compensate for the delay/advance caused by detuning. In the zero dispersion cavity, all wavelengths travel at the same speed. Therefore, the only way to keep the pulse synchronized with the modulation frequency is to shift the pulse position so that the pushing/pulling effect of the modulator transmission function compensates for the detuning. However, in a dispersive cavity, different wavelengths travel at different

velocities. Instead of lasing at the central wavelength, the laser can shift the wavelength to slower or faster one so that the change of propagation speed cancels the frequency detuning. Figure 4.11 shows the shifting of the lasing wavelength in a positive detuned mode-locked fiber laser when the dispersion varies from normal dispersion to abnormal dispersion. It can be seen that the lasing wavelength is shifted to lower frequency for normal dispersion ($D < 0$). This is because lower frequency travels faster in normal dispersion cavity and hence it compensates for the delay caused by positive detuning. On the other hand, the lasing wavelength is shifted to higher frequency when the cavity dispersion is abnormal ($D > 0$).

Figure 4.11. Change of carrier frequency in a detuned actively mode-locked fiber laser ($\Delta f_m = 80$ kHz) when the cavity dispersion increases.

4.4.3. Locking range

Since the dispersion compensates for the detuning by shifting the lasing wavelength, the locking range of the laser is increased with increasing of dispersion as shown in Figure 4.12. The locking range has been enhanced from only 9×10^{-6} for zero dispersion to 2.2×10^{-4} for the dispersion of one. This improvement is achieved with the degradation (broadening) of the pulsewidth. The pulsewidth has been widened to

28.5 ps when the normalized dispersion increases to 1. However, it is noted that the pulse is heavily chirped and hence it can be compressed by propagating through an appropriate length of dispersion fiber outside the cavity [106].

Figure 4.12. Locking range vs. cavity dispersion

4.5. NONLINEAR EFFECTS IN ACTIVELY MODE-LOCKED FIBER LASERS

4.5.1. Zero detuning

The cavity nonlinearity is determined as

$$\gamma = \frac{2\pi L n_2}{\lambda_0 A_{eff}} \quad (4.35)$$

where L is the fiber length, n_2 is the fiber nonlinear-index coefficient, λ_0 is the operating wavelength, and A_{eff} is the effective area of the fiber. For convenience, we normalize the cavity nonlinearity using

$$\gamma_n = \frac{\gamma P_{sat}}{1.5 \tau_G f_m k_{sat}} \quad (4.36)$$

in which P_{sat} is saturation power of the Erbium-doped fiber amplifier, f_m is the modulation frequency, k_{sat} is the saturation coefficient, and τ_G is the pulsewidth given in (4.34)

Figure 4.13 shows the pulse buildup process in an actively mode-locked laser with cavity nonlinearity of 0.2. Steady-state pulse is obtained after about 200 round trips and its pulsewidth is shorter than that without nonlinearity effect. Comparison between steady-state pulses of different nonlinearity values and the dependence of pulsewidth on the cavity nonlinearity are given in Figure 4.14 and Figure 4.15 respectively. As the nonlinearity increases the pulse becomes shorter and shorter. Since the nonlinearity introduces nonlinear phase variation to the optical pulse, which in turn generates new frequencies, the spectrum of the optical pulse is broadened. When the spectrum becomes wider, more modes can be locked and hence the pulse becomes shorter. This is verified in Figure 4.16 which shows that the spectrum of the pulse with high nonlinearity is wider than that without nonlinearity effect.

Figure 4.13. Pulse buildup in an actively mode-locked fiber laser with (a) normalized cavity nonlinearity of 0.2 and (b) zero nonlinearity

Figure 4.14. Pulses with different cavity nonlinearity values

Figure 4.15. Pulse shortening when nonlinearity increases

Figure 4.16. Pulse spectra with different cavity nonlinearity values

It can be seen from Figure 4.14 and Figure 4.16 that when the nonlinearity increases from zero to 0.2, the pulsewidth is shortened by a half while the pulse bandwidth is increased four times. This indicates that the pulse is chirped when the nonlinearity increases.

The dependence of the pulse chirp on the cavity nonlinearity is shown in Figure 4.17. The pulse is up chirp with low frequency at the leading edge and high frequency at the trailing edge. Recalled from (4.13) for the new frequency generated due to the nonlinearity effect of the fiber

$$\delta\omega(T) = -\frac{\partial\phi_{NL}}{\partial T} = -\left(\frac{L}{L_{NL}}\right)\frac{\partial|U(0,T)|^2}{\partial T} \quad (4.37)$$

For the leading edge $dU^2/dT > 0 \rightarrow d\omega < 0$, low frequency is generated. While for the trailing edge, $d\omega > 0$, high frequency is generated. Therefore, the pulse is up-chirped.

Figure 4.17. The nonlinearity causes the pulse up-chirped and the chirp increases as nonlinearity increases; the chirp is measured by taking the second deviation of the phase of the optical field and then multiplying with square of the pulse width

4.5.2. Detuning in actively mode-locked fiber laser with nonlinearity effect

Figure 4.18a shows the steady-state pulses of an actively mode-locked fiber laser having a cavity nonlinearity of 0.1 with different frequency detuning values. Similar to the case of linear cavity, the pulse position is shifted away from the transmission peak (the centre) of the modulation function. For positive detune ($f_m > 0$), the pulse is delayed and the leading edge experiences less loss than the trailing edge when passing the modulator. This pushes the pulse towards the centre and hence compensates for the delay caused by the detune. And hence steady-state pulse is formed. The position shift also increases linearly with the detune as shown in Figure 4.19a.

The spectra of the pulses are plotted in Figure 4.18b. It is seen that the lasing wavelength is shifted toward the lower frequency for positive detuning as in the case of normal dispersion. Although the pulse is also up-chirped as the case of normal dispersion but the principle of wavelength shifting is different. The wavelength shifting in nonlinear case is the result of filtering effect of the modulator. Since the pulse is up-chirped, the low frequency is at the leading edge and hence it experiences lower loss than the high frequency when passing the modulator. Therefore, the spectrum is shifted towards low frequency as shown in Figure 4.19b. On the other hand, when detuning is negative, the pulse is lag behind. Its trailing edge, where the high frequency is distributed, passes the modulator at the transmission peak. The modulator now acts as a filter which is in favor of the high frequency and hence shifts the spectrum towards high frequency.

While the wavelength shifting in dispersive cavity helps to compensate and balance for the detune, the wavelength shifting in nonlinear cavity does not give any help in

compensating for the detuning. The wavelength shifting is just the result of modulator filtering effect on the up-chirp distribution of the frequency components of the pulse. The only effect that reacts and compensates for the detuning is the pushing or pulling effect when the pulse position is shifted. However, these pushing/pulling effects are weaker for the narrower pulse under nonlinear effect. Therefore, the laser is more sensitive to the detuning. Figure 4.20 shows the locking range of the laser when the nonlinearity is changed. The locking range starts dropping dramatically as the nonlinearity reaches 0.04 where the nonlinearity start taking its shortening effect on the pulse as shown in Figure 4.15.

Figure 4.18. Steady-state pulses and spectra of an actively mode-locked fiber laser having cavity nonlinearity of 0.1 with different detuning values

Figure 4.19. Shifting of pulse position and lasing wavelength in detuned actively mode-locked fiber laser with nonlinearity effect

Figure 4.20. Detuning range vs. cavity nonlinearity

4.5.3. Pulse amplitude equalization in harmonic mode-locked fiber laser.

In harmonic mode-locked lasers, the modulation frequency is not equal to the fundamental frequency of the cavity but is a harmonic of it. There is not only one pulse but N pulses traveling together in the cavity. This results in N times increasing of repetition rate. However, the pulse amplitude may be different from one to another due to the super-mode noise [121-123] which causes the laser modes jump from one group to the other groups.

Chapter 4

The amplitude difference between two pulses in a harmonic mode-locked laser without nonlinearity is clearly seen in Figure 4.21a but their amplitudes become equalized when nonlinearity is added in the cavity as shown in Figure 4.21b. In harmonic mode-locking, there is not only one pulse but many pulses simultaneously traveling in the loop. Without the nonlinearity effect, the pulses experience independent shaping processes and hence may result in pulses with different amplitudes. When the nonlinearity is introduced to the cavity, the pulses will experience SPM effect which broadens their spectra. The higher energy pulse experiences higher SPM effect and hence its spectrum becomes wider. Therefore it will experience more loss when passing through the filter. This results in equalization of the pulse energy.

Figure 4.21. Harmonic mode-locked pulses with and without nonlinearity effect

4.6. SOLITON FORMING IN ACTIVELY MODE-LOCKED FIBER LASERS WITH COMBINED EFFECTS OF DISPERSION AND NONLINEARITY

4.6.1. Zero detuning

As discussed in previous sections that nonlinearity introduces up-chirp to the pulse while abnormal dispersion causes the pulse down-chirped. The combination of nonlinearity and dispersion would balances those effects and generates soliton pulse as what happens in the propagation of pulse in single mode fiber [105]. Figure 4.22 shows the dependence of pulse width on the nonlinearity in the laser with cavity dispersion of 0.1 . The pulse width is broadened from 13.5 ps to 16.8 ps with the presence of 0.1 dispersion. However, the pulse width decreases dramatically as the nonlinearity increases in the laser and obtains 5.8 ps , less than half of that of the linear cavity, when the nonlinearity is 0.2 . Further increase of nonlinearity causes the laser unstable due to the strong nonlinear phase shift of the pulse. When the dispersion is double, the required nonlinearity for shortening the pulse is also nearly double as shown in Figure 4.23.

Figure 4.22. Pulse width shortened with increase of nonlinearity, $D = 0.1$

Figure 4.23. Pulse width shortened with increase of nonlinearity, $D = 0.2$

The buildup process and steady-state pulse of the laser with cavity dispersion of 0.1 and nonlinearity of 0.2 are shown in Figure 4.24. The pulse shape is well fitted with a sech^2 profile. This confirms the theoretical prediction that soliton is formed in the laser when abnormal dispersion and nonlinearity are taken effects together.

Figure 4.24. Soliton pulse formed in the active MLL when nonlinearity and abnormal dispersion effects are introduced into the cavity; (a) pulse buildup; (b) steady-state pulse with profile fitting; (c) plot of (b) in log scale

Figure 4.25 gives a detail view on the effects of dispersion and nonlinearity on the pulse width of the mode-locked pulse. A pulse width contour is plotted on a plan of dispersion and nonlinearity. The unstable area is also presented. In the area near the origin, where the dispersion and nonlinearity are small, the modulator and filter play the dominant role in shaping the pulse as presented in Chapter 3. The pulse is compressed when passing the modulator while it is widened when passing the filter due to narrower spectrum after filtering. Stable pulse is obtained when the modulation and filtering effects are balance.

The left side of the figure is the area of normal dispersion. In addition to the modulation and filtering effects, the pulse in this area is also broadened by the dispersion effect. Due to normal dispersion, the low frequency components of the pulse travel faster than the high frequency ones. This causes the pulse up-chirp with low frequencies at the leading edge while the high frequencies at the trailing edge. The increase of nonlinearity in this area causes the pulse highly up-chirp and hence enhances the normal dispersion effect. Therefore, the pulse is more broadened.

Figure 4.25. Pulse width contour with cavity dispersion and cavity nonlinearity as the parameters; * area is unstable area where the steady-state pulse is not obtained

In opposite, on the right side where the cavity dispersion is anomalous, the pulse is compressed when the nonlinearity is increased. In this area, the anomalous dispersion causes the pulse down-chirp while the nonlinearity introduces up-chirp to the pulse. This results in the balance between the anomalous dispersion and nonlinearity effects and hence the soliton is formed. The soliton formation is clearly indicated as the pulse width contour in this area is almost linear.

However, there is an unstable regime in the anomalous dispersion area. During the soliton formation process, part of the pulse energy is shed away as the dispersive wave. The dispersion moves this dispersive wave toward the wings of the pulse where it is attenuated when passing the modulator. If the dispersion is not large enough, compared to the nonlinearity, the dispersive wave will interact with the soliton pulse

and cause the pulse unstable. Therefore, in the large nonlinearity and small anomalous dispersion area, the pulse is continuously formed and then destroyed. The steady-state pulse cannot be obtained.

4.6.2. Detuning and locking range in mode-locked fiber laser with nonlinearity and dispersion effects

Similar to the other cases, when the modulation frequency is detuned the pulse position is also shifted away from the centre as shown in Figure 4.26. The pushing/pulling effect resulted from this shifting compensates for the detuning and hence keeps the pulse stable. However, as the pulse becomes shorter and shorter under the soliton effect, the pushing/pulling effect is weaker and hence the amount of detuning that it can compensate for is reduced. This results in smaller locking range for shorter pulse.

Figure 4.26. Pulse buildup in a detuned actively mode-locked fiber laser with both dispersion and nonlinear effects.

Figure 4.27. Locking range contour with cavity dispersion and cavity nonlinearity as the parameters

Figure 4.27 plots the locking range contour on the plane of cavity nonlinearity and dispersion. Combining with the pulse width contour in Figure 4.25, we see that the large locking range contours correspond to the wide pulse width contours while the small locking range contours correspond to narrow pulse width contours. There is a trade-off between the short pulse and large locking range. If the laser is designed to give a short pulse then its locking range would be sacrificed and its stability against the change of environment is reduced. On the other hand, if the laser is designed to have a large locking range then the shortest pulse which can be obtained is limited and depends on the cavity dispersion. Examples are given in Figure 4.28 and Figure 4.29 where the dependence of shortest pulse width on the cavity dispersion is plotted for locking ranges of $\delta f_m = 10^{-6}$ ($\Delta f_m = 10 \text{ kHz}$) and $\delta f_m = 10^{-7}$ ($\Delta f_m = 1 \text{ kHz}$). It is

shown that shorter pulse is obtained if the cavity dispersion is increased but the required nonlinearity is also increased.

Figure 4.28. Shortest pulse width and the required cavity nonlinearity vs. cavity

dispersion for a given locking range of $\delta f_m = 10^{-6}$ ($\Delta f_m = 10$ kHz)

Figure 4.29. Shortest pulse width and the required cavity nonlinearity vs. cavity

dispersion for a given locking range of $\delta f_m = 10^{-7}$ ($\Delta f_m = 1$ kHz)

4.7. EXPERIMENTAL RESULTS

4.7.1. Experimental setup

Figure 4.30. Actively mode-locked fiber laser setup; EDF: Erbium doped fiber; BPF: bandpass filter; PC: polarization controller; MZIM: Mach-Zehnder intensity modulator; PD: photodiode; OSC: oscilloscope; OSA: optical spectrum analyzer; Sig Gen: signal generator

Figure 4.30 shows the experimental setup of an actively mode-locked fiber laser using Erbium-doped fiber (EDF) as a gain medium. The EDF is 5.6 m long and pumped by a 980 nm laser diode through a WDM coupler. An isolator integrated in the WDM ensures unidirectional lasing. A LiNbO₃ Mach-Zehnder intensity modulator (MZIM) is inserted in the ring for mode locking. Mode locking is obtained by introducing a periodical loss to the signal when driving the modulator by a sinusoidal signal extracted from an RF signal generator. The polarization controller (PC) is used to adjust the polarization state of the lightwave signal traveling in the ring. The lasing wavelength can be tuned by adjusting the central wavelength of the thin-film tunable

optical bandpass filter with a 3 dB bandwidth of 1.2 nm. The optical signal in the ring is coupled to the output port through a 70:30 coupler.

4.7.2. Mode-locked pulse train with 10 GHz repetition rate

The total length of the cavity is about 20.7 m which corresponds to a fundamental frequency of 9.859 MHz. When the modulation frequency is set at 10.006885 GHz, which corresponds to the 1015th harmonic of the fundamental frequency, a stable mode-locked pulse train is obtained as shown in Figure 4.31. The total dispersion of the cavity is about 0.352 ps/nm which corresponds to a normalized dispersion of 0.0064 while the normalized nonlinearity is about 0.011. The laser hence operates near the regime of zero dispersion and nonlinearity. In this regime, the pulse shaping mechanism is dominated by the modulation and filtering effect which results in the Gaussian pulse shape. The full width at half maximum (FWHM) of the pulse measured from the oscilloscope is about 21 ps. It is noted that this value includes the risetime of the photodiode and the oscilloscope. Using an autocorrelator for measuring the pulsewidth to get rid of those electrical components' risetimes, we obtain the pulsewidth of 14.3 ps.

Figure 4.31. 10 GHz repetition rate pulse train generated from the actively mode-locked fiber laser when the modulator is driven by a sinusoidal RF signal.

The optical spectrum of the pulse train is shown in Figure 4.32. It can be seen from Figure 4.32 that the pulse train spectrum has sidelobes with a separation of 0.08 nm , which corresponds to a longitudinal mode separation of 10 GHz . This spectrum profile is typical for the mode-locked laser and usually referred as the mode-locked structure spectrum [21, 86, 89].

The above spectrum profile can be explained by taking the Fourier transform of the periodic Gaussian pulse train:

$$p(t) = e^{-t^2/2T_0^2} * \text{III}\left(\frac{t}{T}\right) \quad (4.38)$$

where T_0 is the width of the Gaussian pulse, T is the pulse repetition period and $\text{III}(t/T)$ is the comb function given by

$$\text{III}\left(\frac{t}{T}\right) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \quad (4.39)$$

$$P(\omega) = F(p(t)) = T_0 \sqrt{2\pi} e^{-T_0^2 \omega^2 / 2} \sum_{n=-\infty}^{\infty} \delta(\omega - n\Omega) \quad (4.40)$$

where $\Omega = 2\pi/T$. $P(\omega)$ has the structure of a train of Dirac function pulses separated by the repetition frequency with a Gaussian envelope.

The laser wavelength is tunable over the whole C-band by tuning the central wavelength of the thin film bandpass filter inserted in the loop.

Figure 4.32. Spectrum of the mode-locked pulse train; the 10 GHz (0.08 nm) spacing modes are clearly observed.

4.7.3. Wavelength shifting in a detuned actively mode-locked fiber laser with dispersive cavity

To study the dispersion effect in the actively mode-locked fiber, we inserted 200 m of SMF fiber into the cavity in the experimental setup shown in Figure 4.33. The total cavity dispersion is about 3.302 ps/nm or equivalent to a normalized dispersion of 0.07 . As low as 40 mW of 980 nm signal was pumped to the EDF so that the laser had enough gain for lasing but the mode-locked pulse power was kept at low level. Therefore, the nonlinear effect can be negligible.

Figure 4.33. Actively mode-locked fiber laser with dispersive cavity enhanced by 200 m of SMF; EDF: Erbium doped fiber; BPF: bandpass filter; PC: polarization controller; MZIM: Mach-Zehnder intensity modulator; PD: photodiode; OSC: oscilloscope; OSA: optical spectrum analyzer; Sig Gen: signal generator

Figure 4.34 shows the mode-locked pulse train of the laser when the modulation frequency was tuned to 10.006900 GHz corresponding to the 1083^{rd} harmonic of the fundamental frequency of 924 kHz (the total length of the cavity is 220.9 m). The pulse was slightly broadened due to the dispersion of the cavity. The pulse width was measured to be about 16.2 ps . The pulse spectrum in Figure 4.35 also exhibits the mode locking structure with 0.08 nm spacing peaks. The bandwidth of the signal is about 0.26 nm which gives a time bandwidth product (TBP) of 0.526 . This is slightly higher than 0.44 , the TBP of an unchirp Gaussian pulse. Therefore, the pulse is slightly chirped.

Figure 4.34. The mode-locked pulse is broadened due to the dispersion effect when the dispersion of the cavity increases.

Figure 4.35. Optical spectrum of the pulse train when the modulation frequency is set at 10.006900 GHz, the 1083 harmonic of the f_R .

When the modulation frequency is detuned by 10 kHz to 10.006910 GHz, the spectrum is shifted toward shorter wavelength as shown in Figure 4.36. The central wavelength is now moved to 1549.9 nm. This confirms the above simulation results where the lasing wavelength is shifted to higher frequency (shorter wavelength) when the abnormal dispersion cavity mode-locked laser is positive detuned ($\Delta f_m > 0$). On

the other hand, as seen in Figure 4.37, the lasing wavelength is shifted to longer wavelength when the modulation frequency is decreased by 10 kHz.

Figure 4.36. The spectrum is shifted to shorter wavelength when the modulation frequency is positive detuned by 10 kHz from 10.006900 GHz.

Figure 4.37. The spectrum is shifted to longer wavelength when the modulation frequency is negative detuned by 10 kHz from 10.006900 GHz.

4.7.4. Pulse shortening and spectrum broadening under nonlinearity effect

The nonlinearity effect is explored by increasing the optical power pumped into the EDF. As the power is increased the nonlinear effect is enhanced and thus contributes to the formation of the mode-locked pulse. Figure 4.38 shows the oscilloscope trace of the pulse train when the pumped power is increased to 190 mW . The pulse width has been shortened to 12.3 ps , 24 % decreased compared to that of dispersive cavity without nonlinear effect. The pulse will be compressed more if higher optical power is pumped into the EDF. The pulse shape is close to hyperbolic secant due to the combination effects of anomalous dispersion and nonlinearity. Putting the experimental values of normalized dispersion and nonlinearity, 0.07 and 0.075 respectively, into the theoretical prediction of the pulse width contour in Figure 4.25, we find that the laser operated in the soliton shaping regime. The predicted pulse width is about 11 ps which is within the reasonable agreement with the experimental measurement of 12.3 ps .

Figure 4.38. Oscilloscope trace of the mode-locked pulse train when the pump power increases to 190 mW ; pulse shortened when the pumped power increases.

The optical spectrum of the pulse is shown in Figure 4.39. As discussed in previous section that the nonlinear effect generates new frequencies when it introduces nonlinear phase change to the pulse and thus broadens its spectrum. The 3 dB bandwidth of the pulse has been increased from 0.26 nm to 0.5 nm when the pump power increases from 40 mW to 190 mW.

Figure 4.39. Nonlinearity causes the pulse spectrum broaden when the pump power increases to 190 mW

4.8. CONCLUSIONS

We have presented the dispersion and nonlinearity effects on the actively mode-locked fiber lasers. A laser model which includes optical amplifier, bandpass filter, modulator and optical fiber has been developed for studying the dispersion and nonlinear effects. It is found that dispersion causes the pulse broadened and chirped. The sign of the chirp depends on the cavity dispersion: positive chirp (down chirp) for normal dispersion while negative chirp for abnormal dispersion. Similarly to the zero dispersion case, the pulse in the dispersion MLFL is also shifted from the transmission peak of the modulation function when the laser is detuned. However, the

shift is reduced as the dispersion increases. The reduction of position shift is the result of moving lasing wavelength towards faster/slower wavelength to compensate for the delay/advance of the pulse when detuning. Therefore, the cavity dispersion enhances the locking range and thus improves the stability of the laser against the environmental perturbation.

In contrast to dispersion effect, nonlinearity compresses the pulse and reduces the locking range. The mode-locked pulse is positive chirped due to the generation of new frequencies through SPM effect and thus the pulse bandwidth is increased. When combining the nonlinear effect with the abnormal dispersion effect, we are able to generate soliton pulse directly from actively mode-locked fiber laser. The pulsewidth of the soliton pulse is shorter than that of Gaussian mode-locked pulse. However, there is a trade of between short pulsewidth and large locking range. If the laser is designed to have shorter pulsewidth, the locking range of the laser is narrower and vice verse. Therefore, our laser model can be used to optimize the laser parameters to obtain the required pulse width while maintaining the laser stability over a certain locking range.

5 Ultra-fast Tunable Actively Mode-locked Fiber Lasers

5.1. INTRODUCTION

Tunable optical filters are essential elements for selecting the lasing wavelength of actively mode-locked fiber laser. Numerous techniques have been developed for tuning the filter's central wavelength. The simplest tuning technique is mechanical rotating the angular position of the substrate in the commonly used tunable thin film filter. This rotation changes the light path length and hence sets different resonant condition at a particular wavelength at which the maximum transmission occurs. Another technique is mechanical or piezo-electrical stretching of the resonant cavity in a Fabry Perot filter [124]. Thermal tuning has also been introduced in thin film filters [125], arrayed waveguide grating [126] or fiber Bragg grating [127]. In addition, the redirection property of the liquid crystal under an electric field has been employed to make electrically tunable filter [128]. In this technique, the physical etalon filter thickness is unchanged but the reflective index is altered under the

redirection of the liquid crystals inside the cavity hence leading to the change of effective thickness of the etalon filter. Technique involving the interaction between lightwave and surface acoustic waves (SAW) in an anisotropic LiNbO_3 to convert the transverse electric guided mode to transverse magnetic mode has also been applied to implement acousto-optic tunable filters[129]. The central wavelength is tuned by changing the frequency of the SAW for satisfying the phase matching condition of different guided modes.

However, besides the acousto-optic filters, all those tuning techniques involve the moving or rotation of material: moving of the substrate, lengthening of the cavity by stretching or heating, rotating of the liquid crystals. Such mechanical tuning causes the slow response time in the order of μs or ms . The tuning time of acoustic filter is also in order of μsec .

In this chapter, we present an electrically tunable filter with picosecond-scale response time. By introducing a phase modulator into a Lyot birefringence fiber filter, we are able to control the phase shift of the lightwave inside the retarder section and hence the resonant wavelength can be easily tuned. Since only the phase of the signal is changed and no mass moving is required, our proposed scheme can obtain an ultra-fast tuning speed, in the order of 10 ps . The fast tuning speed of the proposed filter is demonstrated by the implementation of a frequency shift keying (FSK) modulation at 10 Gb/s .

We then apply the filter to design an ultra fast wavelength tunable actively mode-locked fiber laser. The lasing wavelength is easily tuned with a DC voltage applied to a LiNbO_3 phase modulator. Moreover, the modulator in our scheme is also used to modulate the signal thus reducing the requirement of additional modulator for mode

locking. This helps to simplify the setup and reduce the cost as well. Since the tuning mechanism is based on the electro-optic effect of the LiNbO₃ crystal with response time of tens of picoseconds, the tuning speed is much faster than that based on liquid crystal or mechanical rotating of the angle tuned etalon filter [20, 29, 38, 119, 130]. Therefore, our scheme is promising for applications that require fast tuning of the wavelength such as wavelength switching, remote sensor, and optical spread spectrum communications.

5.2. BIREFRINGENCE FILTER

Figure 5.1. Setup of a Lyot filter with HiBi fiber serving as a retarder. Pol: Polarizer.

Figure 5.1 shows the setup of a Lyot filter using HiBirefringence (HiBi) fiber as a retarder. The filter consists of a piece of HiBi fiber sandwiched between two polarizers. The HiBi fiber is spliced with a 45° angle rotated of its slow axis to the polarizer. Using Jone matrix calculation, we obtain the transmission function of the filter as

$$T(\lambda) = \cos^2\left(\frac{\pi L \Delta n}{\lambda}\right) \quad (5.1)$$

where L is the length of the fiber, $\Delta n = \lambda_0/L_B$ is the birefringence of the fiber, and L_B is the beat length at the reference wavelength λ_0 .

This is a comb filter with free spectral range (FSR) of $\frac{\lambda_0 L_B}{L}$. The calculated transmission function of a Lyot filter with 0.7 m length of Panda fiber is shown in

Figure 5.2. The FSR of the filter is 7.75 nm . The FSR of the filter can be increased while keeping the same filter bandwidth by cascading several single Lyot filter to form a higher order filter. The lengths of individual filters' HiBi fiber are $L, L/2, L/4, \dots, L/2^N$. Figure 5.3 shows transmission function of a second-order Lyot filter. The filter FSR has been double to 15.5 nm while the bandwidth is kept unchanged.

Figure 5.2. Transmittance of a Lyot filter with a 0.7 m of HiBi fiber serving as a retarder. The Free Spectral Range: $\text{FSR} = 7.75 \text{ nm}$; bandwidth: $\text{BW} = 3.875 \text{ nm}$

Figure 5.3. The FSR of the filter is enlarged by cascading two first-order Lyot filters to form a second-order Lyot filter. The FSR is twice that of first-order Lyot filter while keeping the same bandwidth.

5.3. ULTRA-FAST ELECTRICALLY TUNABLE FILTER BASED ON ELECTRO-OPTIC EFFECT OF LiNbO_3

5.3.1. Lyot filter and wavelength tuning by a phase shifter

The central wavelength of the filter can be tuned by varying the length of the HiBi fiber through stretching or heating the fiber. However, tuning speed of those methods is slow and response time is limited to *ms*. We thus propose to insert a phase shifter into the HiBi fiber section to introduce an ultra-fast phase shift to the lightwave signal. The transmission function now becomes

$$T(\lambda) = \cos^2\left(\frac{\pi L \Delta n}{\lambda} + \varphi\right) \quad (5.2)$$

The central wavelength can now be easily tuned by varying the shifted phase φ . Figure 5.4 shows the shifting of central wavelength when the phase is shifted by $\pi/2$.

Figure 5.4. Central wavelength of the filter can be tuned by a phase shifter; solid: zero phase shift; dash-dot: $\pi/2$ phase shift

5.3.2. Experimental results

Figure 5.5. Setup of an electrically tunable HiBi-fiber Lyot filter with a phase modulator acting as a phase shifter. Pol: Polarizer; Phase Mod: Phase modulator

Experimental setup of an electrically tunable filter is shown in Figure 5.5. The filter consists of two polarizers sandwiching an optical phase modulator. All the fibers in the setup are HiBi Panda fibers. The modulator pigtails with the length of 7.6 m and 2.8 m, respectively, are spliced with a rotated angle of 45° to the slow axes of the polarizers' pigtails. By applying an electrical voltage to the phase modulator, we can easily shift the phase of the signal and hence tune the central wavelength of the filter.

The transmission function of the filter is given as

$$T(\lambda) = \cos^2 \left(\frac{\pi B}{\lambda} + \frac{\pi V}{V_\pi} \right) \quad (5.3)$$

Where V is the applied voltage, V_π is the π -phase-shift voltage of the modulator, $B = \Delta n_{PMF} (L_1 + L_2) \pm \Delta n_c L_c$, Δn_{PMF} is the birefringence of the PM fiber, L_c and Δn_c is the length and birefringence of the phase modulator crystal, respectively. The shifted wavelength is given by a linear function of applied voltage:

$$\Delta\lambda = \frac{FSR}{2V_\pi} V \quad (5.4)$$

Figure 5.6. Central wavelength of the filter is shifted when a DC voltage is applied to the phase modulator.

Figure 5.6 shows the transmission function of the setup filter. The FSR of the filter is about 16.06 nm . The central wavelength has been tuned from 1550.14 nm to 1543.52 nm when the applied voltage increases from 0 V to 0.9 V , respectively. It is noticed that using (5.4), we obtain a shift of $FSR/2$ when the applied voltage is equal to V_π . However, in the experiment, the wavelength has been tuned by $FSR/2$ while the applied voltage is less than V_π (5 V). This enhancement of wavelength tuning is due to the increase of the temperature through heating effect. As the phase modulator used in the experiment is a traveling waveguide type, it consumes the electrical power of the applied voltage and heats the modulator. This causes the thermal expansion and pyro-optic effect on the LiNbO_3 crystal [131-134], hence contributing to the larger shift of the central wavelength.

It is suggested that the modulator's temperature should be stabilized by using a temperature controller. Another possible solution would be the use of isolated electrodes instead of the traveling waveguide electrodes for controlling the phase

modulator. With isolated electrodes, the driving current is nullified and thus the heating effect can be avoided.

To eliminate the thermal effect on the measurement of the wavelength shift, we applied a square wave to the phase modulator and measure the wavelength shift when the voltage is switching from one peak to the other. The results are shown in Figure 5.7. The wavelength shift increases linearly with the increase of applied voltage. This is in consistent with the theoretical estimation using (5.4).

Figure 5.7. Shifting of the central wavelength as a function of applied square wave voltage (peak-to-peak).

The fast tuning response of the filter has been tested with the frequency shift keying modulation setup shown in Figure 5.8. Signals from two lasers at 1543 nm and 1550 nm are combined by a 50/50 coupler and fed into the filter. The phase modulator is driven by a bit stream of 1100001011000000 from a bit pattern generator at 10 Gb/s . The bandpass filter is tuned to either 1543 nm or 1550 nm to monitor the signals at the correspondent wavelength. The results in Figure 5.9 show that the filter alternately passes the signals at the two wavelengths when driven by the bit stream. The rise time of about 40 ps has been measured.

Figure 5.8. Experimental setup of a 10Gb/s frequency shift keying (FSK) modulation using the high speed tunable filter; BPG: bit pattern generator; BPF: bandpass filter;

PD: photo diode; OSC: oscilloscope

Figure 5.9. The lights at 1543 nm and 1550 nm alternately pass through the filter when the phase modulator is driven by a bit stream of 1100001011000000; switching up to 10 Gb/s has been obtained.

Since the tuning mechanism is based on the electro-optic effect of the LiNbO_3 crystal, we can obtain a very fast tuning with sub-picosecond scale time response. The tuning speed is only limited due to the slow rise time of the modulator's electrodes. Given this limitation, we still achieve the tuning speed in the order of 10 ps using current 40 GHz phase modulator technology [135]. This is faster by the order of 2 compared to ns- or ms-scale tuning speed of the liquid crystal or thermal based tunable filters [125, 128].

5.4. ULTRA-FAST ELECTRICALLY TUNABLE MLL

5.4.1. Experimental setup

Figure 5.10. Setup of an actively mode-locked fiber laser using the electrically tunable Lyot filter; the phase modulator serves as a mode-locker as well as a wavelength selector; Pol: Polarizer, Phase Mod: phase modulator; EDF: Erbium doped fiber; DSF: Dispersion shifted fiber; PC: polarization controller; PD: photodiode; OSA: Optical Spectrum analyzer; OSC: Oscilloscope; T: bias Tee; Sig Gen: Signal generator.

The electrically tunable filter is then used in an actively mode-locked fiber laser as shown in Figure 5.10. The laser also includes 5 m of EDF pumped by a 980 nm laser diode through an isolator integrated WDM coupler, 50 m of dispersion shifted fiber (DSF), two polarization controllers, and a 90/10 output coupler. A bias DC voltage and a microwave signal are combined together by a bias tee to drive the phase modulator. The modulator here plays dual roles: mode-locker and wavelength tuner. The lasing wavelength can be tuned by controlling the bias DC voltage. As the bias DC voltage changes, the phase modulator shifts the phase of the signal by an amount proportional to the DC voltage and hence changes the central wavelength of the filter.

When the modulator is driven by a microwave signal, it introduces phase modulation to the optical signal passing through it. If the microwave frequency is equal to a harmonic of the fundamental frequency of the laser cavity, the laser is mode locked and generates pulse train with the repetition rate at the microwave frequency.

The signal is coupled out through a $90/10$ coupler and detected by a 45 GHz bandwidth photodiode. The detected signal is then monitored using a high speed oscilloscope with 70 GHz sampling head. The optical signal tapped out from the WDM coupler is monitored using an optical spectrum analyzer with a resolution of 0.02 nm .

5.4.2. Experimental results

Figure 5.11a shows the oscilloscope trace of the mode-locked pulse train when a zero DC voltage and a 10.002223 GHz microwave signal are applied to the modulator. The pulse width measured from the oscilloscope is about 21 ps . It is noticed that this includes the risetime of the photodiode and the sampling head. The pulse train is stable for more than 2 hours without any adjustment of the polarization controllers or modulation frequency. The optical spectrum of the pulse train in Figure 5.11b shows clearly the modes with separation of 0.08 nm which is correspond to 10 GHz repetition rate of the pulses. The pulse has a 0.41 nm bandwidth spectrum centered at 1558.85 nm . The time bandwidth product (TBP) of 1.07 indicates that the pulse is highly chirped. This is due to the chirp caused by the phase modulator and the dispersion of the fiber. It is noticed that the dispersion shifted fiber (DSF) and the right hand side polarizer in the setup can be removed to simplify the laser design. However, employing these components helps to reduce the super-mode noise and equalize the pulse amplitude through additive pulse limiting (APL) effect [30].

Figure 5.11. Pulse train generated from the electrically wavelength tunable actively mode-locked laser (a) and its optical spectrum (b).

Using the phase noise power density of the pulse train plotted in Figure 5.12b, we obtained the RMS timing jitter of about 630 fs with the integrated frequency range from 100 Hz to 1 MHz . On the other hand, the timing jitter can also be calculated from the electrical spectrum of the pulse train shown in Figure 5.12a. By applying the Vonderlinde method [136], we obtained the timing jitter of 530 fs . This is in consistent with the value measured using the phase noise power density. The measured timing jitter is larger than that reported in [33, 117], which is 200 fs and 120 fs respectively. The increase of timing jitter in our experiment is due to high cavity loss caused by the insertion of two polarizers and cross-splices of polarization maintaining fibers. It is also noted that both lasers in [33, 117] include stabilization feedback loops which contribute to the low timing jitters of the lasers.

Figure 5.12. Electrical intensity spectrum (a) and phase noise spectrum density (b) of the mode-locked pulse train.

The tuning of wavelength by applying electrical bias voltage to the modulator is shown in Figure 5.13. The lasing wavelengths are 1558.85 nm , 1558.45 nm and 1554.98 nm for the DC voltage of 0 V , 0.7 V , and 1.2 V respectively. The lasing wavelength does not shift linearly with the DC voltage. This is due to the thermal effect as explained above. Using a phase modulator with isolated electrical electrodes for the DC bias would avoid such thermal induced shifting effect.

Figure 5.13. Shifting of the wavelength when changing the bias voltage applied to the modulator; (a) oscilloscope traces; (b) optical spectrums.

5.5. CONCLUSIONS

We have proposed and demonstrated an electronically tunable optical filter with fast tuning characteristic. The filter consists of a PM-pigtail phase modulator spliced between two polarizers with a 45° rotating angle of the principle axes. Tuning of 6.62 nm for a 16.06 nm FSR filter has been obtained with an applied voltage of 0.9 V . Fast tuning speed with a rise time of 40 ps has been demonstrated.

We have also proposed and demonstrated an actively mode-locked fiber laser with a fast electrically wavelength tuning ability. A phase modulator inserted in a Lyot birefringence filter has been used as both mode-locker and wavelength selector. Since the wavelength tuning is based on the electro-optic effect of the LiNbO_3 crystal with the response time of about tens of picoseconds, our scheme is promising for a very fast electrically tunable pulse laser source. The tuning speed is only limited by the cavity loop length. Possible applications of such laser include wavelength switching, remote sensing, and optical spread spectrum.

6 **Actively Mode-locked Fiber Lasers with Birefringent Cavity**

6.1. INTRODUCTION

In theory, a single mode fiber (SMF) can be perfect cylindrical symmetry and indiscriminately propagate the light polarized along any direction. Light emitted from fiber lasers thus can have any polarization. With this property, fiber lasers are usually modeled using scalar equations in which the polarizations of the fields are ignored [24, 25, 52, 118, 137]. These models have successfully explained and predicted several characteristics of the lasers such as lasing threshold, lasing wavelength, lasing modes, mode locking, pulse width, steady state... In addition, they can also be applied to analyze lasers which employ polarization maintaining fiber to keep the light polarization unchanged along the laser cavity [21, 52]. In such lasers, since the light polarization is fixed along an eigenstate of the fiber, normally the X-axis, they can be analyzed using the scalar models.

However, in practice, the laser cavity is not ideal isotropic. Different polarized components may undergo different processes due to the cavity anisotropies. In fact, several phenomena related to polarization dynamic of the lasers have been reported [138-140]. In 1992, Bielawski et al. presented an antiphase behavior in the Nd-doped fiber laser [139]. By simultaneously measuring the two orthogonal polarizations, the authors observed that the maximum output intensity in one polarization corresponds to a minimum peak in the other one. Variation of threshold pump power due to gain and loss anisotropies in Nd-doped fiber laser was also observed in 1994 [140]. The authors showed that the pump power threshold can be varied by controlling the anisotropy of the gain medium. Those phenomena were supposed to relate to the pump anisotropies or the asymmetries of the lasing medium.

Besides the pump and lasing medium anisotropies, the birefringence of the cavity may also be the cause of polarization dynamics in the lasers. Laser frequency splitting due to cavity birefringence was reported [141, 142]. In [141] two quarter-wave plates were inserted into the cavity to separate the longitudinal modes. By adjusting the rotation angle of the two wave plates, dual-frequency lasing was observed. Similarly, Zhang S. et al. used a KD*P crystal to induce the birefringence in the cavity and also obtained similar result.

Although polarization dynamics of the CW lasers have been researched for long time, it is hard to find any work related to those dynamic behaviors in mode locked fiber lasers. Perhaps passive mode locked fiber laser based on the Nonlinear Polarization Rotation (NPR) effect is one of the rare mode locked fiber models that considers the polarization of the light inside the cavity [52, 95]. The mode locking mechanism is that pulse peak polarization is rotated owing to the nonlinear phase change so that it

experiences less loss than the wings with low intensity. The result is that the pulse becomes shorter and shorter for every round trip in the cavity until equilibrium. The mode locked pulse is formed. However this model just considers the nonlinear phase change and ignores the velocity mismatch between two orthogonal components which is caused by the birefringence of the fiber.

Therefore, in this chapter, we present the study on a mode-locked fiber laser with birefringence cavity in which the polarization of the optical field inside the cavity is considered. By introducing birefringence into the laser cavity, we reveal several lasers' dynamic behaviors including polarization switching and dual locking range operation.

6.2. BIREFRINGENCE CAVITY ACTIVELY MODE-LOCKED FIBER LASER.

As presented in previous sections, actively mode-locked fiber laser is normally characterized by its fundamental frequency f_R which is inverse to the round trip traveling time. Mode locking condition is obtained when the modulation frequency is equal to a harmonic of f_R [102, 118]. Slight detuning of the modulation frequency has also been studied and it is shown that the laser is still mode-locked if the detuning is within the locking range defined in (3.44).

However, this condition is no longer valid when the birefringence of the cavity is large enough that it cannot be ignored. Since the lightwaves traveling in the X-axis sees an effective refractive index n_x different from n_y when traveling in the Y-axis, the traveling round trip time of the ring is thus different from X-axis to Y-axis. The laser can thus resonate at two different fundamental frequencies, f_{Rx} for the X-axis and f_{Ry}

for the Y-axis, instead of a single fundamental frequency as reported in [52, 76, 86, 90]. When the modulation frequency is equal to an integer number multiple of f_{Rx} the laser is excited and mode-locked in the X-axis, resulting in the output pulse has X polarized state. On the other hand, when the modulation frequency is equal to an integer number multiple of f_{Ry} , harmonic mode locking in the Y-axis is obtained. The locking ranges can be estimated using (3.44) while the difference between two modulation frequencies is derived as following.

In fiber laser, the birefringence is induced by inserting a piece of HiBi fiber into the cavity. The refraction index difference between the fast and slow axes of the fiber can be calculated as [105]

$$\Delta n = \frac{\lambda}{L_b} \quad (6.1)$$

Where λ is the operating wavelength and L_b is the beat length of the fiber. The round trip traveling time difference between the two locking regime is thus given by

$$\Delta T = L_{PM} \times \frac{\Delta n}{c} \quad (6.2)$$

where L_{PM} is the length of the PM fiber and c is the speed of light. Therefore, the difference between the two fundamental frequencies f_{Rx} and f_{Ry} is

$$\Delta f_R = f_{Ry} - f_{Rx} = \frac{1}{T_{Ry}} - \frac{1}{T_{Rx}} = \frac{\Delta T}{T_{Ry} T_{Rx}} = \Delta T \times f_R^2 \quad (6.3)$$

Where T_{Rx} , T_{Ry} is the round trip time of the light traveling in the X- and Y- axes, respectively, and f_R is the average fundamental frequency

$$f_R = \sqrt{f_{Rx} f_{Ry}} \quad (6.4)$$

Substituting (6.1), (6.2) and (6.3), we obtain

$$\Delta f_R = \frac{L_{PM} \lambda f_R^2}{L_b c} \quad (6.5)$$

When the laser is locked to an N^{th} order harmonic mode of the fundamental frequency, the difference between the two modulation frequencies, f_{mx} and f_{my} , in order to mode lock the laser ring into the two orthogonal regimes is

$$\Delta f_m = f_{mx} - f_{my} = N \Delta f_R = N \frac{L_{PM} \lambda f_R^2}{L_b c} \quad (6.6)$$

6.3. SIMULATION MODEL

Figure 6.1 shows the model of a birefringence cavity mode-locked fiber laser. The model consists of an amplifier, a bandpass filter, a piece of birefringence fiber and an intensity modulator.

Figure 6.1. Model of the actively mode-locked fiber laser with a birefringence cavity

The optical signal is modeled by a vector (E_x, E_y) , where E_x and E_y is the envelope of the electric field on the X-axis and Y-axis respectively. Initially, E_x and E_y is one set of random complex numbers representing random noise. After passing the amplifier, the signal becomes

$$\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}_B = \begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}_A \times \sqrt{G} + \begin{pmatrix} E_{ASE,x} \\ E_{ASE,y} \end{pmatrix} \quad (6.7)$$

where $E_{ASE,x}$ $E_{ASE,y}$ are the ASE noises of the amplifier and their spectral densities are given by

$$\frac{dP_{ASE,x}}{df} = \frac{dP_{ASE,y}}{df} = \frac{10^{NF/10} (G-1) hf}{2} \quad (6.8)$$

in which NF is the noise figure, $h = 6.626 \times 10^{-34}$ is the plank constant, f is the optical frequency, and $G = P_{out}/P_{in}$ is the gain of the amplifier.

The saturation of the amplifier is also taken into account and its effect on the gain factor g is given as

$$g = \frac{g_{ss}}{1 + P / P_{sat}} \quad (6.9)$$

where g_{ss} is the small signal gain factor, P is the signal power, and P_{sat} is the saturation power.

The signal is then passed through a bandpass filter. The filter is modeled as a first order Gaussian transmission transfer function

$$T(f) = \exp\left(-2 \ln 2 \times \frac{f^2}{\Delta f^2}\right) \quad (6.10)$$

where Δf is the 3 dB bandwidth

The signal in frequency domain at the output of the filter is

$$\begin{pmatrix} E_x(f) \\ E_y(f) \end{pmatrix}_C = \begin{pmatrix} E_x(f) \\ E_y(f) \end{pmatrix}_B \times \exp\left(-2 \ln 2 \times \frac{f^2}{\Delta f^2}\right) \quad (6.11)$$

After filtered, the signal propagates in a HiBi fiber which can be modeled by a nonlinear Schrödinger (NLS) equation [106].

$$\begin{aligned}\frac{\partial A_x}{\partial z} + \beta_{1x} \frac{\partial A_x}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_x}{\partial t^2} + \frac{\alpha}{2} A_x &= i\gamma \left(|A_x|^2 + B|A_y|^2 \right) A_x \\ \frac{\partial A_y}{\partial z} + \beta_{1y} \frac{\partial A_y}{\partial t} + \frac{i\beta_2}{2} \frac{\partial^2 A_y}{\partial t^2} + \frac{\alpha}{2} A_y &= i\gamma \left(|A_y|^2 + B|A_x|^2 \right) A_y\end{aligned}\quad (6.12)$$

For simplification only the birefringence term in the NLS equation is kept while the loss, nonlinear and the dispersive terms are ignored. A simple equation governing the propagation of the signal in a birefringence fiber is given as

$$\left(\frac{\partial}{\partial z} + \beta_{1x,y} \frac{\partial}{\partial t} \right) E_{x,y}(z, t) = 0 \quad (6.13)$$

where $\beta_{1x,y}$ is the propagation constant on the X-axis and Y-axis respectively. Hence the X-axis and Y-axis fields after passing through the birefringence fiber experience different delays due to the birefringence and the output signal is

$$\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}_D = \begin{pmatrix} E_x(t - \beta_{1x}L) \\ E_y(t - \beta_{1y}L) \end{pmatrix}_C \quad (6.14)$$

where L is the length of the birefringence fiber. This birefringence delay leads to two different fundamental frequencies f_{Rx} and f_{Ry} for the two orthogonal fields E_x and E_y respectively. The signal is then modulated by the MZI modulator. After one-round trip, the signal returns to point A as

$$\begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}_{A'} = \begin{pmatrix} E_x(t) \\ E_y(t) \end{pmatrix}_D \times \Gamma \cos \left\{ \frac{\pi}{4} [1 - m \cos(\omega_m t)] \right\} \quad (6.15)$$

where m is the modulation index, ω_m is the modulation frequency, and Γ represents the total loss of the cavity.

The signal is fed in the amplifier again and completes another round trip. The process is repeated until equilibrium.

6.4. SIMULATION RESULTS

The parameters used in the simulation are as follow: small signal gain $G_{ss} = 18$ dB, $NF = 8$ dB, output saturation power $P_{sat} = 5$ dBm, filter bandwidth $BW = 1.2$ nm, fundamental frequencies of the X-axis and Y-axis $f_{Rx} = 12.00059$ MHz, $f_{Ry} = 12.00127$ MHz, harmonic order $N = 834$, modulation index $m = 0.5$, and total cavity loss $\Gamma = 10$ dB.

Figure 6.2 shows the simulation results when the modulation frequency is set to an integer multiple of the X-axis fundamental frequency ($f_m = f_{mx} = Nf_{Rx} = Nf_{Ry} - N\Delta f_R = 10.008490$ GHz). It is clearly shown that the X-axis component gradually forms the mode-locked pulse while the Y-axis component is suppressed to zero after several round trips. This results in the X-polarized mode-locked pulse. The laser is thus locked in the X-axis.

On the other hand, when the modulation frequency is increased by $\Delta f_m = 570$ kHz to be equal to a multiple of f_{Ry} ($f_m = f_{my} = Nf_{Ry} = Nf_{Rx} + N\Delta f_R = 10.009060$ GHz), the X-axis component is suppressed and the Y-axis component is built up as shown in Figure 6.3. Moreover, in Figure 6.4, there is no mode-locked pulse formed in either X-axis or Y-axis when the modulation frequency is set in the middle of f_{mx} and f_{my} .

Figure 6.2. Forming of the pulse in the cavity when the modulation frequency is set to
10.00849 GHz

Figure 6.3. Forming of the pulse in the cavity when the modulation frequency is set to
10.00906 GHz

Figure 6.4. Forming of the pulse in the cavity when the modulation frequency is set to
10.00876 GHz

Figure 6.5 plots the output power of the X-axis and Y-axis components when the modulation frequency is detuned. It can be seen that there are two locking ranges. One is located at f_{mx} and the other at f_{my} . When the modulation frequency is in the locking range of f_{mx} the laser is mode locked and the output pulse is polarized on the X-axis. We call this H-mode regime. In contrast, the laser is mode locked in V-mode regime, in which the output pulse is Y polarized if the modulation is in the locking range of f_{my} . Outside the locking ranges mode locking is not obtained.

It is noted that when the locking range of H-mode and V-mode is overlapped the two locking ranges are merged. This occurs when Δf_R is small and the simulation result is plotted in Figure 6.6. It can be seen that there is a transition point at which the output pulse polarization will be switched from X-axis to Y-axis and vice versa.

Figure 6.5. Dependence of the output power of the x-axis and y-axis components on the detuning of modulation frequency f_m ; F_m is the offset to 10 GHz: $F_m = f_m - 10$ GHz

Figure 6.6. Dependence of the output power of the x-axis and y-axis components on the detuning of modulation frequency f_m when Δf_m is reduced to 100 kHz; F_m is the offset to 10 GHz: $F_m = f_m - 10$ GHz

6.5. POLARIZATION SWITCHING IN AN ACTIVELY MODE-LOCKED FIBER LASER WITH BIREFRINGENCE CAVITY

6.5.1. Experimental setup

Figure 6.7. SOA based harmonic mode-locked fiber laser; SOA: semiconductor optical amplifier; BPF: bandpass filter; pc: polarization controller; MZIM: Mach-Zehnder intensity modulator; PMF: polarization maintaining fiber; PD: photodiode; OSC: oscilloscope; OSA: optical spectrum analyzer

Figure 6.7 shows the experimental setup of the semiconductor optical amplifier (SOA) based actively mode-locked fiber laser. The laser has a ring configuration incorporating an isolator to ensure unidirectional lasing. The optical gain is provided by an SOA with a small signal gain of 18.2 dB and a saturation power of 11.3 dBm while a thin-film 1.2 nm tunable optical bandpass filter is used to tune the lasing wavelength. Mode locking of the lasing lightwave is obtained through amplitude modulation process by incorporating in the ring a LiNbO_3 Mach-Zehnder intensity

modulator (MZIM) with the insertion loss of 3 dB. As large as 30% of the optical power in the ring can be coupled to the output port through a 70:30 coupler.

The output signal is split into two paths for simultaneously monitoring and characterization. One of them is amplified and fed into a high speed photodiode (45 GHz 3 dB bandwidth). The electrical signal output from the photodiode is then monitored by a high speed 50 GHz sampling oscilloscope. The other path is analyzed by a polarizer followed by an autocorrelator and/or an optical spectrum analyzer.

6.5.2. Results and discussion

The fundamental frequency of the ring, estimated as the inverse of the round trip period of the lightwave traveling in the ring, is about 12 MHz. Figure 6.8a shows the harmonic mode-locked pulse sequence at a repetition rate of about 10 GHz when the modulation frequency is set at 10.008426 GHz, according to the 834th harmonic of the fundamental frequency. The pulse sequence optical spectrum in Figure 6.8b clearly shows lasing longitudinal mode spacing of 0.08 nm. The 3 dB bandwidth of the spectrum envelop has been measured as large as 0.77 nm. The locking range is about 260 kHz. However, when the modulation frequency is tuned to 10.009106 GHz, or 680 kHz away from 10.008426 GHz, a stable well-defined mode-locked pulse train appears again as shown in Figure 6.9. The 3 dB bandwidth is estimated to be 0.36 nm under this condition. It is interesting that the polarization state of the pulse sequence in this case is orthogonal to that of previous one, which is mode locked at 10.008426 GHz. Hence we denote the two polarization locking states as the V- and H- mode regimes respectively.

Figure 6.8. H-mode pulse trace (a) and its optical spectrum (b)

Figure 6.9. V-mode pulse trace (a) and its optical spectrum (b)

H-mode regime.

Figure 6.10a and b show the autocorrelation traces of the X and Y polarization component, respectively, of the H-mode pulse sequence. Higher power on the X-axis demonstrates that the pulse has the major polarization state aligning along this axis. This is confirmed by the optical spectra of the two orthogonal polarization components shown in Figure 6.11. The central wavelength modes of the pulse polarize along the X-axis and hence pass through the X-axis polarizer without attenuation whereas suffering as much as 10 dB loss when passing through the Y-axis polarizer. It is noted that the sideband modes far from the central wavelength don't have strong X polarized state as the central wavelength. They have equal powers on both axes.

The pulse width of the Y-axis component is slightly larger than that of the X-axis. The pulse full width at half maximum (FWHM) of 24.8 ps and 19.8 ps have been achieved, assuming Gaussian shape, for the Y and X components, respectively.

Figure 6.10. Autocorrelation traces of H-mode pulses: (a) X-axis component, (b) Y-axis component

Figure 6.11. Optical spectra of H-mode pulses: (a) X-axis component, (b) Y-axis component

V-mode regime.

The polarization state of the mode-locked pulse is now switched to the other orthogonal state, the V-mode regime, by tuning the modulation frequency to 10.009106 GHz . The autocorrelation traces shown in Figure 6.12 indicates that the pulse power is more distributed on the Y-axis than on the X-axis, hence contrasting to that observed in the H-mode regime. In addition, the optical spectra shown in Figure

6.13 also show that the central wavelength is polarized along the Y-axis and has not been affected by the Y-axis polarizer. However, there is a slight difference of the polarization state of the sideband modes from the H-mode regime. The sideband modes are now polarized along the axis orthogonal to the polarization axis of the central wavelength and significantly attenuated when passing through the Y-axis polarizer.

Similar to the H-mode regime, the pulse width of the major axis (the higher power axis) component is slightly narrower than that of the minor axis. The pulse width is 29.4 ps and 19.1 ps for the X- and Y- axis components respectively. This can be easily understood that the minor axis component is detuned while the major axis component is tuned to its locking frequency.

It is also noted that the time bandwidth products (TBP) are very large in both locking regimes. This indicates that the pulses are heavily chirped and can be linearly compressed externally [105].

Figure 6.12. Autocorrelation traces of V-mode pulses: (a) X-axis component, (b) Y-axis component

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Figure 6.13. Optical spectra of V-mode pulses: (a) X-axis component, (b) Y-axis component

In our experiment, $L_{PM} = 3.2 \text{ m}$, $L_b = 3.5 \text{ mm}$, $\lambda = 1550 \text{ nm}$, $f_R = 12 \text{ MHz}$, $N = 834$, the fundamental frequency difference and the modulation frequency difference is thus estimated as $\Delta f_R = 680 \text{ Hz}$ and $\Delta f_m = 567 \text{ kHz}$. These values agree well with the experimental result where the modulation frequencies for X- and Y- polarized mode locking states are $f_{mx} = 10.008426 \text{ GHz}$ and $f_{my} = 10.009106 \text{ GHz}$, respectively. The difference is $\Delta f_m = 680 \text{ kHz}$.

6.6. DUAL ORTHOGONAL POLARIZATION STATES IN AN ACTIVELY MODE-LOCKED BIREFRINGENT FIBER RING LASER

6.6.1. Experimental setup

Figure 6.14. Actively mode-locked fiber laser with birefringence cavity; PM EDF: Polarization maintaining Erbium doped fiber; BPF: bandpass filter; PC: polarization controller; MZIM: Mach-Zehnder intensity modulator; Pol: Polarizer; PD: photodiode; OSC: oscilloscope; OSA: optical spectrum analyzer; Sig Gen: signal generator

Figure 6.14 shows the experimental setup of the actively mode-locked fiber laser incorporating a birefringent cavity. The laser has a ring configuration with 15 m of polarization maintaining Erbium-doped fiber (PM-EDF) serving as a gain medium. This PM-EDF also causes the laser cavity becoming birefringent. The PM-EDF is pumped by a 980 nm laser diode through a WDM coupler. An isolator integrated in the WDM ensures unidirectional lasing. A thin-film 1.2 nm tunable optical bandpass filter is used to tune the lasing wavelength. A LiNbO₃ Mach-Zehnder intensity

modulator (MZIM) is incorporated in the cavity to stimulate mode locking through amplitude modulation process. The modulator is driven by a microwave signal extracted from a signal generator. The polarization controller (pc) is used to adjust the polarization state of the lightwave signal traveling in the ring. The optical signal in the ring is coupled to the output port through a 70:30 coupler.

The output signal is monitored by a high speed sampling oscilloscope (50 GHz 3 dB bandwidth) preceded by a photo diode (45 GHz 3 dB bandwidth). Polarizer is also inserted in front of the photo diode when polarization state of the output signal is analyzed. An optical spectrum analyzer with resolution of 0.01 nm is used to record the pulse spectrum.

6.6.2. Results and discussion

The total length of the cavity is about 29.5 m which corresponds to a fundamental frequency f_R of 6.923 MHz. The modulator is biased at the quadrature point. The central wavelength of the filter is 1550 nm. The modulation frequency is 2.997668 GHz which corresponds to the resonance of the 433rd harmonic. When the polarization controller is adjusted at an appropriate position, we observed on the oscilloscope a well defined mode-locked pulse train as shown in Figure 6.15a. It is interesting to note that the pulse sequence have two amplitude levels. This indicates that the pulse train actually consists of low and high amplitude pulses which can be denoted as the X- and Y- pulse sequences, respectively. It is noticed that the X-pulse sequence appears brighter than the Y pulse. This means that there are more X pulses than that of the Y- in the pulse train. The sampling oscilloscope samples the pulse train for every sampling interval and each sample gives a dot on the screen. Therefore,

more samples obtained for the high population of X pulses result in brighter trace of the X pulse on the screen.

Figure 6.15. Oscilloscope trace of the output pulses (a) shows dual-amplitude property of the pulse train; the pulse spectrum (b) also shows dual wavelengths

Figure 6.15b shows the optical spectrum of the pulse train. The spectrum is totally different from a normal mode-locked pulse train spectrum. It does not follow a Gaussian shape with peak at the filter centre wavelength; instead there exists two peaks located on two sides of the filter centre wavelength. The separation is about 0.25 nm . The longer wavelength peaks at 1550.12 nm and has the mode-locked structure of several longitudinal modes of spacing about 0.02 nm that corresponds to the modulation frequency of 2.998 GHz . The shorter wavelength peaks at 1549.87 nm and has smoother structure. It thus demonstrates that the mode-locked pulses are not only dual amplitudes in time domain but also dual wavelengths in frequency domain. It is questionable that whether the two wavelengths are from the two types of pulses. One corresponds to X pulses and the other corresponds to Y pulses.

Figure 6.16. Pulse trace (a) and its spectrum (b) when the polarization controllers are adjusted so that only low amplitude pulses exist in the loop; clear underfoot shows no pulse dropping

Figure 6.17. Pulse trace (a) and its spectrum (b) when the polarization controllers are adjusted so that only high amplitude pulses exist in the loop; underfoot shows that there are some pulses dropped in the loop

To answer this question, we operated the laser in two different modes by adjusting the polarization controllers. Figure 6.16 shows the oscilloscope trace of the pulse train and its spectrum when the polarization controller is adjusted to support only X pulse. There is now only the low amplitude component in the pulse trace. Clear underfoot pulse trace indicates that there is no pulse dropping. Every time slot in the pulse train is filled with the X pulse. This results in the mode locking as observed in its spectrum in Figure 6.16b. The spectrum shows that the X pulse is corresponding to the long wavelength component of the dual pulse spectrum.

Furthermore the polarization controllers are also adjusted to lock the laser to the high amplitude level pulse sequence. The low amplitude pulses no longer exist as shown in Figure 6.17a. In contrast to the X pulse locking case, the pulse trace here has an underfoot. This means that not every time slot in the pulse train is filled. It is easily understood that the pulse dropping is caused by the energy limitation of the laser. High energy of the Y pulse requires high power from the pump to make all time slots filled. Failure to this requirement results in pulse dropping [118]. It is noted that the high amplitude pulse is brighter than that of the dual pulses case. This means there are more Y pulses in the loop than before. The energies of the X pulses transfer to the Y pulses and hence more Y pulses are formed but not all slots of the pulse train circulating in the loop is filled. Pulse dropping causes unequal spacing of the pulses. Therefore, the Fourier transformed pulse sequence no longer has a smooth mode locking structure as the spectrum shown in Figure 6.17b. This spectrum corresponds to the short wavelength component of the dual pulse spectrum.

Figure 6.18. X-axis component (a) and Y-axis component (b) of the dual pulse train; high amplitude pulses and low amplitude pulses are orthogonally polarized.

Dual-pulses locking can be explained as following. With proper setup of polarization controller in the cavity, the laser can support two orthogonal modes which can be considered as the two eigen-modal solutions of the coupled nonlinear Schrödinger equations.

In the N^{th} order harmonic mode-locked fiber laser, there is not only one pulse but there are N pulses simultaneously traveling in the loop. Pulse of any solution can occupy one time slot in the pulse train. Therefore, there is a mixture of pulses from the two modes in the pulse train which appears as dual amplitude pulses overlapping on the oscilloscope screen.

Moreover, the slow and fast axis modes are under different detuning conditions due to birefringence of the cavity. Slow axis mode is negatively detuned while fast axis mode is positively detuned. It should be noted that the laser cavity has a normal dispersion with average value of -28.5 ps/nm/km . In order to keep synchronization, the wavelength of the slow axis mode is shifted to longer wavelength so that it travels

faster than the fast axis mode in the normal dispersion cavity. In contrast, the fast axis mode is shifted to shorter wavelength and hence is slowed down by the normal dispersion of the fiber cavity [94, 120]. The wavelength shifting of the two modes is clearly observed in Figure 6.15 – Figure 6.17. The X-pulse spectrum is at longer wavelength while the Y-pulse spectrum is at shorter wavelength.

To verify that the pulse train consists of pulses on two orthogonal axes, we use a polarizer to analyze the output pulse train. The polarizer is rotated to two orthogonal positions. Figure 6.18a is the pulse trace when the polarizer is at position 1. The high amplitude pulses are attenuated and appear as the underfoot while the low amplitude pulses reach the maximum value. In contrast, as shown in Figure 6.18b, the low amplitude pulses are highly attenuated and the high amplitude pulses reach their maxima when the polarizer is set at position 2.

It is noted that although no stabilization technique was applied, stable pulses were obtained for several hours when the laser was operated to generate either X pulse or Y pulse alone; dual-pulse operation was stable for more than one hour. Besides, the phenomenon was also obtained when the modulation frequency increased to 10 GHz but the stability was an issue. Moreover, adjustment need to be carefully handled since the higher the frequency, the larger the bandwidth of the mode-locked laser hence the PMD effect will cause some polarization fading and coupling.

6.7. PULSE DROP OUT AND SUB-HARMONIC LOCKING

In addition to the dual polarization state pulses generation, we have also recorded the pulse drop out and sub-harmonic locking when the polarization controllers are set to obtain coupling between the two orthogonal modes. The experimental setup is the

same as in section 6.6. When the polarization controllers are adjusted at an appropriate position, we observed on the oscilloscope a well-defined mode-locked pulse train as shown in Fig 2a. Starting from this setting, we recorded the evolution of the pulse train while rotating the middle plate of the polarization controller (PC).

Remarkable frames extracted from the video file are presented in Figure 6.19 a – d. It can be seen from Figure 6.19b that underfoot appears at every alternative pulse position as the polarization controller is rotated. The pulses in the pulse train are no longer unified but divided into two groups, denoted as group A and group B, respectively. The pulses in group B are gradually dropped out when the PC is rotated. Finally, all pulses of group B are dropped and the pulse train consists of pulses from group A only (see Figure 6.19c). The repetition rate of the pulse train is half of the modulation frequency, sub-harmonic locking occurs. It is noted that the amplitude of the pulse in this case is nearly double that of the normal locking case since the energy of the dropped pulses are transferred to the remained pulses. Further rotating of the PC results in dropping of pulses in group A as shown in Figure 6.19d.

Figure 6.19. Oscilloscope trace of the output pulses (a) full rate pulse train with clear pedestal; (b) pulses of group B are dropped in some slots; (c) All group-B pulses are dropped leaving a subharmonic mode-locked pulse with repetition rate at half of modulation frequency; (d) pulses of group A are dropped as further rotating of the polarization controller.

Pulse dropout and sub-harmonic locking were also observed when the modulation frequency increased to 6.009125 GHz. However, the pulses in this case are classified into 4 groups and have the pattern of ABCDABCD as shown in Figure 6.20. The pulses are also gradually dropped as the PC is rotated. The evolution of the pulse train is as follow: ABCDABCD, _BCD_BCD, __CD__CD, ___D___D. This results in sub-harmonic locking of the laser to a quarter of the modulation frequency.

Figure 6.20. Oscilloscope trace of the output pulses (a) full rate pulse train with clear pedestal; (b) group-A pulses are dropped; (c) group-B pulses are also dropped; (d) pulses from groups A, B and C are dropped leaving a subharmonic mode-locked pulse with repetition rate at a quarter of the modulation frequency.

The phenomenon can be explained due to the beating between different modes simultaneously oscillating in the laser cavity. In a birefringence cavity, there are two cavities, one with a fundamental frequency of f_{RX} , the other with f_{RY} . Thus, there are two sets of modes oscillating inside the laser cavity. With proper setting of the PC, those two sets may independently oscillate or mutually couple energy from one to another and beat with each other inside the cavity. The independent oscillating of the two sets causes the dual polarization states locking, which has been reported above. In contrast, the beating of the two sets may change the pattern of the pulse train through

pulse dropping and causes the laser to operate at repetition rate lower than the modulation frequency, i.e. sub-harmonic locking.

6.8. CONCLUSIONS

By employing a birefringent ring cavity, we have generated a dual amplitude pulse train from an actively mode-locked fiber laser. The pulse train consists of pulses polarized on both X and Y axes. The X-polarized pulses have lower amplitude and lase at longer wavelength than that of the Y axis ones. Therefore, we observed the dual amplitude pulse trace and dual wavelengths on the optical spectrum analyzer. Besides that, locking to only X axis pulse or Y axis pulse has also been obtained by controlling the coupled polarization state of the generated lightwave.

In addition, switching of the polarization state of the output pulse sequence of an actively mode-locked fiber laser due to birefringence in the ring has been reported. Since there are two different fundamental frequencies according to the resonance of the two orthogonal polarized regimes, the polarization state of the laser can be switched between two orthogonal states by tuning the modulation frequency.

Moreover, sub-harmonic locking where the output pulse train repetition rate is half or a quarter of the modulation frequency has been obtained by proper controlling the polarization state of the light insides the birefringent cavity laser. This sub-harmonic locking may be exploited to develop optical frequency dividers or optical clock recovery circuits for fully optical signal processing systems.

7 Conclusions and Recommendations for Future Works

7.1. CONCLUSIONS

We have proposed and demonstrated a novel analysis method for studying the characteristics and performance of actively mode-locked lasers. Instead of using self-consistence condition to obtain the steady-state pulse parameters as in classical analysis methods, we derived a set of mathematical series to trace the evolution of the signal in the laser cavity. Using this approach, we not only obtained the steady-state pulse parameters but also revealed the transient evolution of the pulse. Round trip by round trip shaping of the pulse from ASE noise under the modulation and filtering effects has been observed. In addition, detuning of actively mode-locked lasers was also investigated by using this method. It is shown that mode locking still occurs when the modulation frequency is detuned within a certain locking range. This locking range is limited due to the shifting of the pulse position when the laser is detuned. Except the shift of pulse position, the other steady-state pulse parameters when detuned are the same as those when exactly tuned.

Another advantage of our proposed series method is that it can be used for analyzing the laser model with ASE noise which is normally ignored in classical analyses. Signal to Noise Ratio (SNR) of the output pulses can be calculated using the series method. Its dependence on the cavity design parameters such as cavity loss and amplifier noise has been discussed. It is found that the output pulse SNR can be improved by minimizing cavity loss and amplifier noise. On the other hand, detuning would decrease the laser's SNR and the presence of ASE noise limits the detuning frequency to a narrower locking range compared to the case in which an ideal noiseless amplifier is employed.

Besides the theoretical analysis method, we have also developed a laser model for numerical simulation of the actively mode-locked fiber lasers. Dispersion and nonlinear effects play very important roles in the pulse formation and they have been studied in details. Chirped and broadened pulses are attributed to the dispersion of the laser cavity. The sign of the chirp depends on the cavity dispersion: positive chirp (down chirp) for normal dispersion while negative chirp for abnormal dispersion. It is interesting that the lasing wavelength is no longer at the central of the intra cavity filter but shifted away due to the dispersion when the laser is detuned. The wavelength shifting towards faster/slower wavelength compensates for the delay/advance caused by detuning and hence enhances the locking range of the laser. This improves the stability of the laser against environmental perturbations.

In contrast, the laser is more sensitive to detuning when large nonlinearity is introduced into the cavity. Although pulse shortening is obtained with nonlinear cavity, the locking range is dramatically reduced. The pulse bandwidth is wider due to the new frequencies generated through the SPM effect. Moreover the mode-locked

pulse is positively chirped. By combining the abnormal dispersion effect with the nonlinear effect, soliton pulses can be directly generated from the laser and shorter pulse width is obtained from this soliton forming effect. However, the pulse is shortened with the trade-off for the locking range. Therefore, the laser parameters must be optimized to obtain the required pulse width while the laser stability is still maintained over a certain locking range.

Furthermore, we have also investigated techniques to improve the tunability of the lasers. The lasers are normally tuned by using mechanically tunable optical filters whose tuning speed is rather slow, in the order of μs or ms . We have proposed and designed an ultra-fast electrical tunable optical filter. The filter is based on a Lyot birefringence filter setup where the birefringence waveplates replaced by polarization maintaining fibers. We introduced a phase modulator to shift the phase of the signal and thus tune the filter wavelength. Fast tuning speed with a rise time in the order of picoseconds has been demonstrated.

The proposed ultra-fast filter has been applied to our design to obtain an actively mode-locked fiber laser with fast electrically wavelength tuning ability. Moreover, the phase modulator, which is used to tune the wavelength in the filter, was also used to mode lock the laser. This simplifies the design and thus lowers the cost of the laser. Since our scheme uses the LiNbO_3 crystal's electro-optic effect with the response time of about tens of picoseconds for wavelength tuning, it is very promising for ultra-fast electrically tunable pulse laser source.

Actively mode-locked fiber lasers with birefringent cavity have also been investigated. By introducing birefringence into the cavity, we have generated a pulse train with dual amplitudes from an active mode-locked fiber laser. Different to normal

mode-locked pulses with identical amplitude and polarization state, pulses polarized on both X and Y axes simultaneously exist in the output pulse train. The two orthogonal pulse sequences have different amplitudes and lase at different wavelengths. Dual wavelengths are the results of red shift and blue shift of the X and Y polarization states of the generated pulses, respectively, due to the detuning phenomena. Locking to individual X-axis or Y-axis polarized pulse is also obtained by adjusting polarization controllers.

In addition, switching of the polarization state of the output pulse train generated from an active harmonic mode-locked fiber laser by exploiting the birefringence of the fiber ring have been reported for the first time. Two polarization modes have different fundamental frequencies and hence the laser can be forced to mode locking states of the two orthogonal polarization regimes by tuning or detuning the modulation frequency.

Moreover, by proper controlling the polarization state of the light insides the birefringent cavity laser, we have generated a pulse train with a repetition rate at half or a quarter of the modulation frequency. This sub-harmonic locking may find applications in design and development of optical frequency dividers and optical clock recoveries which are very important components in fully optical signal processing systems.

7.2. RECOMMENDATIONS FOR FUTURE WORKS

Generation of ultra-short high repetition rate optical pulses has been studied in this thesis. This pulse source is an import clock for OTDM system. Delivery and then

recovery of the clock is also very important in OTDM system. Therefore, opto-electronic oscillator is recommended for studying in the future work.

Figure 7.1. Schematic of an opto-electrical oscillator

Figure 7.1 shows the setup of the proposed opto-electrical oscillator (OEO). The oscillator is based on an actively mode-locked fiber laser which can be either electrically or optically mode-locked. Mode locking to an electrical signal input can be done by using an electro-optic modulator as shown in this thesis. On the other hand, mode locking to an optical signal may be done through optical injection locking with a nonlinear medium such as SOA or Nonlinear loop optical mirror (NOLM). There must be some ways to synchronize these two mode locking mechanisms so that the laser is mode-locked. The output pulses of the laser are detected and filtered to obtain the microwave signal at the repetition rate of the pulse f_m . The microwave signal is then combined and mixed with the input electrical signal to generate the output signal and control signals for the actively mode-locked fiber laser.

The oscillator has an optical and an electrical input ports, an optical and an electrical output ports. This feature makes the oscillator easy and flexible in interacting with complex fiber-optic communication systems. Moreover, the oscillator can be run with

different configurations for different applications. Following are some examples of applications of the oscillator.

- **Reference optical and electrical clocks**

The oscillator can be free run without any inputs and simultaneously generate both optical and electrical reference clocks for optical communication systems as seen in Figure 7.2.

Figure 7.2. Free running opto-electrical oscillator used as reference clock signal

- **Clock recovery**

If the OEO's optical input is fed with an optical data, it is locked to the clock embedded in the data. Therefore, we obtain the recovered clock at the output ports optically as well as electrically. The recovered electrical clock can be used for triggering a data recovery unit while the optical clock can be distributed with very low loss through optical fiber to other units or systems.

Figure 7.3. Opto-electrical oscillator used as clock recovery

- **Atomic clock reference and distribution system**

The OEO can be locked to an atomic clock reference through the electrical input as shown in Figure 7.4. The highly accurate clock signal is delivered through the optical network and regenerated at each station by using the same OEO.

Figure 7.4. Atomic clock reference and distribution system

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