Waveform diversity for HFSW and MIMO radars

Wang, Guohua

2010

Wang, G. (2010). Waveform diversity for HFSW and MIMO radars. Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/42364>

<https://doi.org/10.32657/10356/42364>

WAVEFORM DIVERSITY FOR HFSW AND MIMO RADARS

WANG GUOHUA

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2010

Acknowledgments

Firstly, I would like to express my deepest gratitude and thanks to my supervisor, Professor LU Yilong, for his valuable guidance, concurrent encouragement, and continuous support throughout the past several years. His understanding and patience provided me with large degrees of freedom in research while his constructive comments and criticism kept me from straying far away from the main points. It's really my great pleasure to be associated with Professor LU Yilong.

I am also indebted to Professor Alexander Haimovich, for his guidance during my visiting study at New Jersey Institute of Technology. Professor Haimovich taught me to look at the big picture and to catch the physical essence before digging deeply into mathematics. I gained much from him. Thanks are also due to Dr. LIU Weixian, for guiding me at the beginning of my Ph.D work. His rich experience always made hard things easy and funny.

Thanks also go to all students and staffs associated with Radar Center, Dr. FENG Hongchuan, Dr. SUN Hongbo, Mr. LI Wei, Mr. TANG Yue, Mr. DUAN Ran, who stretched another colorful dimension, with knowledge and friendship, to my life of graduate study.

I thank deeply my family, in particular, my parents. Their simple belief in knowledge and education runs me on a way from primary school to college. Their selfless love and support encourage me to complete this studying journey.

Finally, I wish to express my deep appreciation to my wife, SUN Mei, for her long time patience and encouragement during the course of my Ph. D study. It would be impossible to complete this work without her continuous support from all aspects.

Table of Contents

Acknowledgments	i
Table of Contents	ii
Summary	vi
List of Figures	vii
List of Tables	xi
Abbreviations	xii
Chapter 1 Introduction	1
1.1 Radar Basics	1
1.1.1 Detection and Ranging	2
1.1.2 Velocity Estimation	3
1.2 Waveform Diversity	4
1.3 MIMO Radar and HFSW Radar	7
1.3.1 MIMO Radar	7
1.3.2 Waveform Diversity in MIMO Radar	10
1.3.3 Waveform Diversity in HFSW Radar	11
1.4 Contributions	12
1.5 Dissertation Outline	14
Chapter 2 Radar Ambiguity Function	17
2.1 Review of Classic Ambiguity Function	18
2.1.1 Definition of Ambiguity Function	18
2.1.2 Properties of Ambiguity Function	21

2.2	Composite Ambiguity Function	24
2.2.1	Volume Clearance.....	25
2.2.2	η Clear	29
2.2.3	Minimum Mean Sidelobe Level.....	30
2.3	Ambiguity functions for MIMO radar.....	30
2.3.1	Coherent MIMO Radar Ambiguity Function	31
2.3.2	Non-coherent MIMO Radar Ambiguity Function.....	32
2.4	Concluding Remarks	33
Chapter 3	Sparse Frequency Waveform Design	34
3.1	Introduction	34
3.2	Phase-Only Design: Optimization Model and PSO	37
3.2.1	Optimization Model.....	38
3.2.2	PSO Based Optimization Engine.....	42
3.3	Phase-Amplitude Joint Design	45
3.4	Design Examples and Discussions	47
3.4.1	Case 1: Implementation of Standard PSO	47
3.4.2	Case 2: Implementation of Constrained PSO	51
3.4.3	Case 3: Joint Design	55
3.4.4	Further Discussions	58
3.5	Multiple Waveforms Design.....	61
3.5.1	Optimization Model.....	62
3.5.2	Design Example.....	64
3.5.3	Further Discussions	68
3.6	Concluding Remarks	71
Chapter 4	Iterative Method of Designing Sparse Frequency Waveform	74
4.1	Introduction	74
4.2	Problem Formulation and Iterative Algorithm	76

4.2.1	Problem Formulation.....	76
4.2.2	Iterative Algorithm	78
4.3	Extension to MIMO Radar	84
4.4	Designing Examples	87
4.4.1	Example 1: HF Band Application.....	87
4.4.2	Example 2: UHF Band Application.....	92
4.4.3	MIMO Radar Application.....	94
4.5	Concluding Remarks	99
Chapter 5	Waveform Diversity in MIMO STAP Radar	101
5.1	Introduction	101
5.2	System Configuration and Signal Model	105
5.3	Sensitivity	113
5.3.1	Waveform Effect on TR Gain.....	114
5.3.2	Waveform Effect on SINR.....	119
5.4	Clutter Rank.....	122
5.4.1	Full Rank R_S	127
5.4.2	Rank Deficient R_S	127
5.4.3	Other Cases.....	132
5.4.4	Numerical Studies and Discussions.....	132
5.5	Waveform Effect on Optimum Adapted Pattern.....	137
5.6	Concluding Remarks	141
Chapter 6	High Resolution MIMO-HFSW Radar	143
6.1	Introduction	143
6.2	Ambiguity Function of the Proposed System.....	145
6.3	Resolution Capacity and PSL Performance.....	150
6.4	Doppler and Geometry Factor	158
6.5	Concluding Remarks	161

Chapter 7 Conclusions and Future Work	163
7.1 Conclusions	163
7.2 Future Work	168
Author's Publications	171
Bibliography	172
Appendix A: Proof of (2.17)	181

Summary

This thesis presents a study on the waveform diversity and optimal waveform design approaches for high frequency surface wave (HFSW) radar and multiple-input multiple-output (MIMO) radar. Main contributions of this study are as follows.

First, methods for designing single/multiple sparse frequency waveforms with sidelobe constraint are presented. The proposed methods can design digital waveforms that satisfy simultaneously the requirement on Power Spectral Density (PSD) and the requirement on sidelobe in correlation function, i.e., autocorrelation function for single waveform design and both autocorrelation functions and cross-correlation functions for multiple waveforms design. Objective functions are constructed based on the PSD and sidelobe requirements and are minimized by two optimization approaches. These two approaches together can provide much flexibility in designing sparse frequency waveforms for different applications. Some implementation issues such as quantization effect, property of Low Probability of Intercept, and Doppler Tolerance are discussed as well.

Second, some basic issues regarding to space-time adaptive processing (STAP) in MIMO radar with waveform diversity are investigated in order to predict the performance of MIMO STAP. Specifically, this thesis focuses on the clutter rank of STAP in MIMO radar. For a special class of rank deficient waveforms, three rules are proposed to predict their clutter ranks. A unified rule for full rank waveforms and rank-1 waveforms is proposed, which can be the generalized form of current rank estimation rules for conventional STAP and MIMO STAP with orthogonal waveforms.

Third, a high resolution HFSW radar with MIMO radar configuration is proposed. Ambiguity function is developed for the proposed radar system. Associated range sidelobe suppression method is proposed by using sparse frequency waveforms. In addition, bandwidth effect on the ambiguity function is also investigated.

List of Figures

Figure 1.1: Basic radar scenario.	2
Figure 1.2: Basic radar scenario with a moving target.	4
Figure 1.3: Collocated MIMO radar (N transmitters and M receivers).	7
Figure 1.4: Distributed MIMO radar (N transmitters and M receivers).	10
Figure 2.1: Ambiguity function of LFM waveform of pulse width T	21
Figure 2.2: Partial ambiguity functions of two waveforms.	23
Figure 2.3: Contour plots of partial ambiguity functions.	24
Figure 2.4: Autocorrelation functions of two waveforms.	24
Figure 3.1: PSD performance comparison (Case 1).	48
Figure 3.2: ACF comparison (Case 1).	50
Figure 3.3: PSD comparison (Case 2).	53
Figure 3.4: ACF Comparison (Case 2).	54
Figure 3.5: PSD of two methods (Case 3).	56
Figure 3.6: ACF comparison of two methods (Case 3).	56
Figure 3.7: Amplitude comparison (Case 3).	57
Figure 3.8: Comparison of short-time PSD.	58
Figure 3.9: Partial ambiguity function of the designed waveform.	60
Figure 3.10: Matched filter output peaks of the waveform with the echo Doppler shifted.	60

Figure 3.11: ACFs (Example 1).	66
Figure 3.12: CCFs (Example 1).	66
Figure 3.13: PSD performance (Example 1).	67
Figure 3.14: ACF performance (Example 2).	69
Figure 3.15: CCF performance (Example 2).	70
Figure 3.16: PSD performance (Example 2).	70
Figure 3.17: Resolution performance comparison.	71
Figure 4.1: Performance comparison, weight coefficient $\lambda=0.9$ (HF band application).	89
Figure 4.2: Performance of phase-amplitude method.	90
Figure 4.3: Performance comparison, weight coefficient $\lambda=0.5$ (HF band application).	91
Figure 4.4: ACF comparison (UHF band application).	93
Figure 4.5: PSD comparison (UHF band application).	94
Figure 4.6: (a) ACF and (b) CCF performance of 3 waveforms in MIMO-Case 1.	96
Figure 4.7: PSD of 3 waveforms in MIMO-Case 1.	96
Figure 4.8: (a) ACF and (b) CCF performance of 3 waveforms.	98
Figure 4.9: PSD of 3 waveforms.	99
Figure 5.1: MIMO STAP system geometry.	105
Figure 5.2: Patterns for different waveforms.	117

Figure 5.3: Patterns when azimuth of target is different from that of transmitting and receiving.....	118
Figure 5.4: Illustration of detectability of minimum velocity of different waveforms ($N=M=5$, $L=10$).	120
Figure 5.5: Output SINR comparison of MIMO and conventional SIMO STAP processor ($N=M=10$ for MIMO, $N=10$ for SIMO, $L=10$ for both).	121
Figure 5.6: Example of full rank $\mathbf{R}_s$	133
Figure 5.7: Illustration of <i>Rule 1</i> with coherent waveforms.	134
Figure 5.8: The transmitting beampattern of waveforms designed by method of maximum power criterion.....	135
Figure 5.9: Illustration of <i>Rule 2</i>	136
Figure 5.10: Adapted patterns of different waveforms.	139
Figure 5.11: Doppler slice of 2-D adaptive pattern of MIMO STAP with waveform diversity.....	140
Figure 6.1: MIMO radar configuration.....	146
Figure 6.2: AFs of the proposed system with different waveform sets	154
Figure 6.3: Resolution and sidelobe performances with different waveforms.....	155
Figure 6.4: High resolution ability of 3 point targets case.....	156
Figure 6.5: Bandwidth effect on resolution and sidelobe performances.	157
Figure 6.6: Illustration of geometry factor in Case 1.....	158
Figure 6.7: Resolution and sidelobe performances of different configurations in	

Case 1.....160

Figure 6.8: Ambiguity functions of different velocity case with Configuration 1.

.....161

List of Tables

Table 1: Passbands Distribution.....	51
Table 2: Resolution and Sidelobe Characters of Different Waveforms	151

Abbreviations

AF	Ambiguity Function
ACF	Autocorrelation Function
CA	Cyclic Algorithm
CCF	Cross-correlation Function
CCM	Clutter Covariance Matrix
CF	Correlation Function
CW	Continuous Wave
FFT	Fast Fourier Transform
FMICW	Frequency Modulated Interrupted Continuous Wave
GA	Genetic Algorithm
GMTI	Ground Moving Target Indication
HF	High Frequency
HFSW	High Frequency Surface Wave
HFSWR	High Frequency Surface Wave Radar
ISL	Integrated Sidelobe Level
LFM	Linear Frequency Modulation
LPI	Low Probability of Intercept
MIMO	Multiple-Input Multiple-Output
MTI	Moving Target Indication
OTHR	Over-The-Horizon radar
PISR	Peak-To-Integrated-Sidelobe-Ratio
PSD	Power Spectral Density
PSL	Peak Sidelobe Level
PSO	Particle Swarm Optimization
SFW	Sparse Frequency Waveform
SIMO	Single-Input Multiple-Output
SINR	Signal-to-Interference-plus-Noise-Ratio
STAP	Space-Time Adaptive Processing

UHF	Ultra High Frequency
WCM	Waveform Covariance Matrix

Chapter 1 Introduction

1.1 Radar Basics

Radar is a system that can detect a target and locate its position [1]. It operates by radiating electromagnetic signals from a transmitter and detecting the presence and character of the echo returned from the target with a receiver. Detection of echo energy reveals the presence of the target. Comparison of the echo with the transmitted signal produces such information about the target as its location, its size, and its velocity. According to the configuration of transmitter and receiver, radar systems can be categorized into monostatic radar (with transmitter and receiver collocated), bistatic radar (with transmitter and receiver distributed far away from each other relative to the wavelength occupied by the radar), and multistatic radar (with multiple transmitters and receivers sparsely distributed from each other). According to the continuity of the transmitted signals, radar systems can be categorized into continuous waveform radar and pulse radar. Continuous waveform radars transmit a continuous waveform while pulse radars transmit multiple pulses. Most radar systems are monostatic pulse radar. Various functions can be seen in radar applications. However, to a certain extent the most basic radar functions are detection, ranging, and velocity estimation.

1.1.1 Detection and Ranging

Figure 1.1: Basic radar scenario.

In a basic radar application scenario as illustrated in Fig. 1.1, a monostatic radar emits a waveform $u(t)$ into space from its transmitter antenna. A target located at range R_0 intercepts and reradiates a portion of the radar signal. Then, the echo signal at the receiver after demodulation is $\alpha \cdot u\left(t - \frac{2R_0}{c}\right)$, where α is the target reflection coefficient determined by the radar cross section of the target, the distance between radar and target, the beampattern of the antenna, and the azimuth of the target, and c is the velocity of light. To detect the presence of the target, a matched filter is usually employed to process the received signals. By comparing the output of the matched filter at different time delay point with a threshold, the radar can determine whether a target is present or not.

After claiming the presence of the target, the target's range can then be

calculated. When the target is present, the corresponding delay point, say t_0 , is associated with an output that surpasses the threshold. Then, the range is determined by

$$R_0 = \frac{c \cdot t_0}{2}. \quad (1.1)$$

1.1.2 Velocity Estimation

In many radar applications, the potential target is moving towards or away from the radar with certain velocity v_0 . Thus it is a natural requirement for radar to extract the information of target velocity. Consider another radar scenario illustrated in Fig. 1.2. The transmitted signal is $u(t)e^{j2\pi f_0 t}$, where f_0 is the carrier frequency of the radar system. This signal meets the narrowband requirement, which means that the bandwidth of the signal is much smaller than the carrier frequency. The echo signal after demodulation is $\alpha \cdot u\left(t - \frac{2R_0}{c}\right)e^{j2\pi f_d t}$, where f_d is the Doppler frequency caused by the relative movement between radar and target. Mathematically, the Doppler frequency is expressed as:

$$f_d = \frac{2v_0}{c} f_0. \quad (1.2)$$

As the Doppler frequency is always much smaller compared to the carrier frequency, a coherent pulse train is always used to measure it in order to capture its full information. In the case that a target is present, after matched filtering, a sequence of matched filter output is obtained. Each element of this sequence is

just like a sample of a single tone with frequency f_d . Then, many methods can be adopted to estimate the frequency. Simply, one can use Fourier transform to get it. With the Doppler frequency on hand, one can calculate the target velocity with the assistance of its azimuth.

Figure 1.2: Basic radar scenario with a moving target.

1.2 Waveform Diversity

As mentioned above, the basic function of radar is to determine the presence, the range and the velocity of a target by transmitting a waveform and then exploring information contained in the received waveform modulated by the target and polluted by noise. From the very beginning of radar development, conflicting requirements have always been laid on radar design [2-4]. In the earliest stages, requirement of range resolution always competed with that of maximum detection range. That was because good range resolution needs small

pulse width while small pulse width could not provide as much energy as needed for a larger detection range. This problem was latterly theoretically resolved by Woodward [5]. In his legendary research article, Woodward pointed out that the range resolution is a function of bandwidth [5]. Thus, waveforms with large time-bandwidth product can satisfy simultaneously the requirements of range resolution and maximum detection range. Large time-bandwidth product can be achieved through maintaining the pulse as wide as possible while modulating the pulse in such a manner that can increase its bandwidth. This is the basis for the technique of pulse compression [110]. However, Woodward at the same time also pointed out that the overall ability of any waveform with fixed energy to distinguish targets in range-Doppler domain is same, which means if one wants to increase the ability in range domain, one must sacrifice that in Doppler domain, or vice versa [2, 8]. These fundamental limitations on radar performance laid by waveforms, together with large variety of radar applications, makes waveform analysis and design always an important and challenging field to radar engineers.

During the course of radar development, much work has been dedicated to designing good waveforms to reach a compromise between different constraints. One valuable concept to do this is to utilize diversity in waveform design. One long-being used diversity technique is the multiple pulse repetition frequency technique in pulse-Doppler radar [6, 7]. Though other methods of diversity are

also straightforward in concept, they have not seen wide application because of processing complexity and hardware limitations [8].

Nowadays, advances in digital signal processing algorithms and hardware allow different diversity waveforms to be adopted in a single radar system to improve radar performance [9-11]. This opens up a new and rich research direction, which is called waveform diversity. With waveform diversity, a single radar system can perform multiple functions automatically and adaptively tuned for a dynamic environment. A general definition of waveform diversity is the exploitation of different transmitted waveforms from a single system to improve performance [11]. The differences, i.e., diversity dimensions, of waveforms with diversity commonly used are: (i) Spatial – different waveforms transmitted at different spatial locations; (ii) Temporal – different waveforms transmitted at different times; (iii) Frequency – waveforms differ in frequency; (iv) Polarization – waveforms differ in polarization; (v) Modulation – waveforms differ in modulation. Obviously, multiple combinations are possible, and sometimes at least two forms of diversity are selected in real applications.

Selection of proper waveforms for a particular radar application is a problem of waveform design. Basic functions of radar require its waveform to obtain enough energy and bandwidth, which determines the probability of detection and the resolution of ranging, respectively. High level applications may require radar to distinguish the type of targets, i.e., target classification or recognition,

which needs more powerful waveforms besides advanced methods of signal and information processing. More advanced applications may further require a single system to adapt its waveforms for various functions coping with a dynamically changing environment. These kinds of increasing needs for advanced radar functions ask for high flexibility in waveform design. By exploiting waveform diversity, more freedoms in waveform design can be benefited in order to meet the specific requirements from powerful radar functions. More importantly, supports from hardware technology lead waveform diversity and design from an ideal concept to be a real rich research topic today.

1.3 MIMO Radar and HFSW Radar

1.3.1 MIMO Radar

Figure 1.3: Collocated MIMO radar (N transmitters and M receivers).

Multiple-input multiple-output (MIMO) radar [12-20] is new radar architecture that has been developing fast in recent years. MIMO radar, though no standard definition is yet available for it, in general is a kind of radar that can both transmit and receive multiple waveforms in order to improve radar performance.

There are two basic architectures of MIMO radar considered in current articles. One is like the conventional phased array radar, say, with transmit array elements and receive array elements collocated; but it can transmit different waveforms from different antennas. This architecture is illustrated in Fig. 1.3, where there are N transmitting elements spaced at half-wavelength and M receiving elements spaced at N times half-wavelength; each transmitting element emits its own waveform $u_n(t)$, $n=1, \dots, N$ and each receiving element has a filter bank consisting of N matched filters. With this architecture, MIMO radar observes a target with azimuth θ in its far field so that it can perform coherent processing easily like conventional phased array radar. The advantages of this MIMO radar over conventional phased array radar may include the following points [14-18]:

- 1) improved parameter identifiability
- 2) improved detection and estimation performance
- 3) flexibility in beampattern design

Generally, these attributes come from the increased degrees of freedom enabled

by multiple transmitting waveforms. In particular, multiple probing signals make each channel separable on receive. Thus, there are more independent observations than its phased array counterpart and a virtual array of much larger aperture can be formed. Thereby come the improved performances. The other, as illustrated in Fig. 1.4, is like traditional multistatic radar, say, with both transmit elements and receive elements widely distributed; but it employs multiple waveforms on transmit site and jointly processes all returns on receive site. This kind of configuration allows MIMO radar to exploit the advantages from both waveform diversity and spatial diversity. Through transmitting and receiving orthogonal waveforms from distributed sensors, each bistatic pair (channel) can observe a particular aspect of the target and obtain an independent scattering response. Then, by centrally processing all signals from all channels, the magnitude of target fluctuation can be reduced, and consequently the radar detection performance can be improved, especially for high signal-to-noise ratio cases [12, 19]. High resolution capability can also be exploited through coherent processing in this MIMO radar architecture [20] though the complexity of this kind of processing may be large.

MIMO radar has already been proposed for many applications. For example, MIMO space-time adaptive processing (STAP) was proposed in [14] and further studied in [21-27], MIMO Ground Moving Target Indication (GMTI) was further discussed in [28], MIMO Over-The-Horizon radar (OTHR) was

proposed in [29, 30], MIMO Synthetic Aperture Radar (SAR) was proposed in [31]. It seems to be that almost every conventional radar application can be upgraded to MIMO radar architecture.

Figure 1.4: Distributed MIMO radar (N transmitters and M receivers).

1.3.2 Waveform Diversity in MIMO Radar

MIMO radar is an emerging radar technology that inherently takes advantage of waveform diversity. Theoretically, arbitrary waveforms that meet certain application requirement can be adopted by MIMO radar. With orthogonal waveforms, signal processing can enjoy more degrees of freedom by separating them on receive site. With coherent waveforms, MIMO radar reduces to conventional radar that has maximum power in illumination. With partially correlated waveforms, flexibility in beampattern design can be

achieved. In a word, waveform diversity can bring much superiority to MIMO radar [12-21].

Current methods of waveform design for MIMO radar can be classified into three types. The first type is to design waveforms with good sidelobe performance. Good sidelobe performance means that sidelobe levels in both the autocorrelation functions (ACFs) and cross-correlation functions (CCFs) should be approximately zero. This kind of waveform can approximately possess the property of orthogonality. The method of designing this kind of waveform has two streams. One is to minimize the penalty function based on the sidelobe energy in both the ACFs and the CCFs using stochastic evolutionary methods [32, 33]. The other is direct waveform design method via cyclic algorithms [34-35]. The second type is to design waveforms with good performance in terms of such metric figures as maximum mutual information, maximum signal-to-interference-plus-noise-ratio (SINR), and so on [36-39]. The last kind is to utilize partially correlated waveforms to design certain illuminating beampattern for colocated MIMO radars [40-42].

1.3.3 Waveform Diversity in HFSW Radar

High frequency surface wave (HFSW) radar, or for brief, HFSWR, is a long range radar system that operates in HF band (3-30 MHz). HF electromagnetic signals in vertical polarization can propagate along ocean surface and easily go beyond the line-of-sight due to the high conductivity of the ocean surface. Thus,

HFSWR can detect targets in a large oceanic area beyond the horizon. As a result, HFSWR is widely used in maritime surveillance for both military and civil purposes. Many countries are highly involved in the research and development of practical HFSWR systems [43-46, and references therein].

Waveform diversity has long been used in HFSWR systems for various objectives. Frequency Modulated Interrupted Continuous Wave (FMICW) was proposed and analyzed in [46, 47]. This FMICW technique is mainly to obtain isolation between transmitter and receiver while keeping the transmitting power on an acceptable level. Dual frequency waveforms were discussed in [48]. This technique is theoretically based on the fact that the separation between targets and clutter is through the difference in their Doppler frequencies, which are different functions of frequency of operation [49, 50]. A more general form of techniques in [48], i.e, frequency agility, was further discussed in [51]. A sparse frequency waveform design problem was introduced in [52]. Sparse frequency waveform is highly desirable in order to avoid interference from assigned spectrum in HF band.

1.4 Contributions

The main contributions of this thesis are as follows.

- 1) First, re-investigation of composite ambiguity function. New bounds on the volume and height distribution for composite ambiguity function are developed. It is found that the volume clearance area is no smaller than

the conventional ambiguity function.

- 2) Development of two flexible approaches of designing sparse frequency waveform with sidelobe constraint for both conventional radar and MIMO radar. With the proposed methods, the designed waveforms can simultaneously meet the requirements of power spectral density (PSD) and sidelobe level in ACF (and CCF in MIMO radar case). With different variations of our method, we can either design waveforms with sidelobe reduced or maintain all sidelobe level below certain level. This makes our methods much more flexible than others for different application occasions.
- 3) Construction of unified framework of signal model for STAP in MIMO radar with waveform diversity and the sensitivity evaluation of MIMO STAP with waveform diversity with the eigenvalue of waveform covariance matrix.
- 4) Investigation of the clutter property of STAP in MIMO radar and development of several new rules for estimating the clutter rank of STAP in MIMO radar with waveform diversity. These rules can serve as the generalized forms of Brennan's rule and current rule for clutter rank estimation in MIMO STAP with orthogonal waveforms.
- 5) Proposal of high resolution MIMO-HFSW radar system. Ambiguity function for the proposed system is constructed. Meanwhile, to reduce

the incurred high sidelobe level, sparse frequency waveform scheme is proposed.

1.5 Dissertation Outline

The dissertation focuses on the waveform diversity and design in HFSWR and MIMO radar. First, the waveform design issue discussed in the dissertation is called sparse frequency waveform design. Sparse frequency waveform is a kind of waveform that has certain stopbands sparsely distributed over the entire spectrum occupied. Sparse frequency waveform has good resolution capacity and good anti-interference performance. However, high sidelobe level from ACF always occurs in implementation due to sparsity in spectrum. The problem to be addressed in this thesis is to design waveforms that can simultaneously meet the requirements of sidelobe performance and PSD performance. In essence, this problem is concerning sidelobe reduction. However, the difference compared to conventional sidelobe reduction problem is distinct as it also takes into account the PSD. Second, with regard to waveform diversity, this thesis will first evaluate several basic performance of STAP in MIMO radar with waveform diversity. That includes the sensitivity of MIMO STAP radar and the clutter property of MIMO STAP radar. Then, a concept of high resolution MIMO-HFSWR will be introduced. The organization of the dissertation is as following.

In Chapter 2, the development of radar ambiguity function is extensively

reviewed as ambiguity function is the basic tool for waveform design and evaluation. Three types of radar ambiguity function are reviewed: conventional radar ambiguity function, composite radar ambiguity function and MIMO radar ambiguity function. Some properties of composite ambiguity function are investigated.

Chapter 3 presents a new method for optimal sparse frequency waveform design. Single waveform design case is discussed first. A new penalty function encompassing the sidelobe requirement and the PSD requirement is first constructed. Particle Swarm Optimization (PSO) algorithm is then utilized to minimize this penalty function. Besides the implementation of standard PSO algorithm in this optimization, constrained PSO algorithm is also exploited for precisely controlling the Peak Sidelobe Level (PSL). Then, the method is extended for multiple waveforms design. In multiple waveforms design case, the penalty function also includes the sidelobe energy in the CCFs.

Chapter 4 presents a different method for designing single/multiple sparse frequency waveforms. This new method is to optimize an almost equivalent form of the penalty function derived in chapter 3 via iterative algorithm. Compared to the method proposed in Chapter 3, this method is more efficient in computation. However, it cannot control precisely the peak sidelobe level though the total sidelobe energy can be suppressed.

Chapter 5 discusses the basic STAP performance in MIMO radar with

waveform diversity. Specifically, the clutter rank of MIMO STAP with waveform diversity is investigated. New rules for estimating the clutter rank is proposed. The sensitivity of different waveforms is also investigated. To evaluate these performances, a unified signal model incorporating waveforms for STAP in MIMO radar with waveform diversity is developed.

Chapter 6 proposes a HFSWR configuration in the mode of widely spaced MIMO radar. The system configuration, signal model are constructed. The ambiguity function of the proposed system is derived. Meanwhile, sparse frequency waveforms are proposed to significantly reduce the sidelobes caused by the sparsely located antennas.

Finally, Chapter 7 concludes the dissertation and outlines some topics to be investigated in the future.

Chapter 2 Radar Ambiguity Function

In radar applications there are two basic functions. One is to detect the presence of a target; the other is to estimate parameters of a target, such as range, velocity, etc. A problem associated with these functions is to identify two or more targets closely located in a range-velocity, or delay-Doppler space. To evaluate waveform's ability of identification, Woodward proposed the tool of ambiguity function [5, 53]. From then on, the radar ambiguity function as well as its properties has been extensively studied and playing an important role in radar signal design and analysis [54-60]. Radar ambiguity functions have a peak at the origin and a constant volume under the ambiguity surface, which is called property of volume invariance or radar uncertainty principle. The property of volume invariance makes it not achievable to have an ideal ambiguity function to be zero everywhere except at the origin. As a result, radar signal design is always a tradeoff between the sharpness of the central spike and the spreading of the ambiguity volume in the ambiguity surface [8].

Since the first radar ambiguity function was defined, some other kinds of ambiguity function have also been proposed by other researchers for different radar applications. Guery and Bell proposed a composite ambiguity function for diversity waveform set for delay-Doppler imaging [61]. Different MIMO radar ambiguity functions were proposed and investigated [20, 62-65]. In this chapter,

we will focus on the review as well as re-investigation of radar ambiguity function because it is a major tool for radar signal analysis and design and it will serve the following chapters extensively. We will introduce the classic radar ambiguity function first. Then, we will study some properties of the composite ambiguity function in Section 2.2. After that, we will review the MIMO radar ambiguity function in Section 2.3. And this chapter is finally concluded in Section 2.4.

2.1 Review of Classic Ambiguity Function

2.1.1 Definition of Ambiguity Function

We first introduce the definition of radar ambiguity function by following [2]. It is derived from a physical perspective based on a common methodology that we distinguish different objects according to their likeness or difference. For a basic radar scenario as illustrated in Fig. 1.2, there is a radar transmitting and receiving a waveform $u(t)$, whose energy is normalized to be 1. There is a target moving from a distance R_0 towards the radar at a velocity v_0 . The range between the radar and the target is a function of time t :

$$R(t) = R_0 - v_0 t \quad (2.1)$$

Then the echo delay t_d is a function of t :

$$t_d = \frac{2R(t)}{c + v_0} \quad (2.2)$$

If the transmitting waveform is expressed in a complex analytical form:

$$s(t) = u(t) \exp(j2\pi f_0 t) \quad (2.3)$$

where f_0 is the radar centre frequency, then the echo signal can be modeled as:

$$\begin{aligned} s(t - t_d) &= u(t - t_d) \exp(j2\pi f_0 (t - t_d)) \\ &= u\left(\alpha t - \frac{2R_0}{c + v_0}\right) \exp\left(j2\pi f_0 \left(\alpha t - \frac{2R_0}{c + v_0}\right)\right) \end{aligned} \quad (2.4)$$

where $\alpha = 1 + 2v_0/c$, the reflection coefficient of target is assumed to be 1 for brevity. To this point, we can see that, the target velocity has two effects [109]:

- 1) A compression or stretching of the time scale of the complex envelope.
- 2) A shift of the carrier frequency.

In most cases, for narrow band waveforms, the first effect can be ignored. Thus, the echo signal can be modeled as:

$$s(t - t_d) = u\left(t - \frac{2R_0}{c}\right) \exp(j2\pi (f_0 + f_d)t) \quad (2.5)$$

by ignoring the phase shift, and setting

$$\frac{2R_0}{c + v_0} \approx \frac{2R_0}{c} \triangleq \tau \quad \text{and} \quad f_d = f_0 \frac{2v_0}{c}.$$

From (2.5), we can see that, the echo signal will be shifted by a time delay and frequency shift besides a scaling in amplitude. Thus, from a general point of view, the return signal can be expressed as $s(t + \tau, f + f_d)$. To distinguish the transmitting signal and the received echo, a common idea is to evaluate the ‘mean square departure’, i.e., the difference between them, as:

$$\Delta(\tau, f_d) = \int_{-\infty}^{\infty} |s(t) - s(t + \tau, f + f_d)|^2 dt \quad (2.6)$$

Expanding (2.6), one can get:

$$\begin{aligned}
\Delta(\tau, f_d) &= \int_{-\infty}^{\infty} |s(t) - s(t + \tau, f + f_d)|^2 dt \\
&= \int_{-\infty}^{\infty} |s(t)|^2 dt + \int_{-\infty}^{\infty} |s(t + \tau, f + f_d)|^2 dt \\
&\quad - 2 \operatorname{Re} \left\{ \int_{-\infty}^{\infty} s(t) s^*(t + \tau, f + f_d) dt \right\} \\
&= k_e - 2 \operatorname{Re} \left\{ \int_{-\infty}^{\infty} s(t) s^*(t + \tau, f + f_d) dt \right\}
\end{aligned} \tag{2.7}$$

The first item k_e stands for the energy of signals, and the integral term means the projection of its shifted version on waveform itself, demonstrating the alikeness between them. Then, we define the ambiguity function based on the integral term as:

$$\chi(\tau, f_d) = \int s(t) s^*(t + \tau) e^{-j2\pi f_d t} dt \tag{2.8}$$

A typical ambiguity function of a Linear Frequency Modulation (LFM) signal is shown in Fig. 2.1 [8]. In some applications, one may be interested in the ambiguity function along the zero-Doppler axis. Set the Doppler shift as zero, then, the ambiguity function becomes:

$$\chi(\tau, 0) = \int s(t) s^*(t + \tau) dt \tag{2.9}$$

This function is also called as autocorrelation function.

By expressing the time delay and Doppler frequency as functions of other variables, for example, x , y , and z , we can get the ambiguity function in a more general form as:

$$\chi(\tau, f_d) = \chi(\tau(x, y, z), f_d(x, y, z)) \tag{2.10}$$

With this form, we can evaluate properties of the ambiguity function along different dimensions. For example, express the delay as a function of range and

azimuth, and then we can directly investigate the resolution along range and azimuth dimensions. Meanwhile, the ambiguity function can be plotted either with a linear vertical axis or in dB. The latter may be useful to show the level of sidelobes.

Figure 2.1: Ambiguity function of LFM waveform of pulse width T .

2.1.2 Properties of Ambiguity Function

Several main properties of the ambiguity function are listed as follows. The property of ambiguity function is always evaluated through its squared form or absolute value, i.e., $|\chi(\tau, f_d)|^2$ or $|\chi(\tau, f_d)|$. In this subsection, we assume that the energy of the transmitting waveform is normalized to unity.

Property 1: Symmetry about the origin $(0, 0)$

$$|\chi(\tau, f_d)| = |\chi(-\tau, -f_d)| \quad (2.11)$$

This property allows us to obtain the whole picture of the ambiguity function by

studying and plotting only two adjacent quadrants.

Property 2: Maximum at (0, 0)

$$|\chi(\tau, f_d)| \leq |\chi(0, 0)| = 1 \quad (2.12)$$

This property says that the ambiguity function has a greatest value equal to the signal energy at the origin. This can be easily interpreted from a physical perspective that because the ambiguity function reveals the likeness between a signal and its shifted version, the version without any shift will be most like the waveform itself.

Property 3: Volume invariance

$$\int_{-\infty}^{\infty} |\chi(\tau, f_d)|^2 d\tau df_d = 1 \quad (2.13)$$

It leads to the conclusion that the total potential radar ambiguity is the same for all signals that possess the same energy and that all the signals are equally good as long as they are not compared against a specific application.

Property 2 and 3 tell us that if we squeeze the ambiguity function to a narrow peak at the origin, the peak cannot exceed a value of 1, and the energy squeezed from the peak must appear somewhere else.

As the total potential ambiguity is constant, there must be the trade-off between the time ambiguity and the Doppler ambiguity. In other words, the time resolution and Doppler resolution cannot be simultaneously as good as possible. In this sense, this volume invariance property is also called the uncertainty property.

Property 4: Parabolic phase

If a complex envelope $u(t)$ has ambiguity function $\chi(\tau, f_d)$, then adding quadratic-phase modulation, or Linear Frequency Modulation to $u(t)$, the resulting ambiguity function becomes:

$$u(t) \exp(j2\pi kt^2) \Leftrightarrow |\chi(\tau, f_d - k\tau)| \quad (2.14)$$

This property means adding LFM shears the resulting ambiguity function. This property is illustrated in Fig. 2.2 and Fig. 2.3. In Fig. 2.2, the ambiguity function of a single tone rectangular signal vanishes quickly along the Doppler axis and spreads along the delay axis. After adding LFM, the resulting ambiguity function vanishes quickly along delay axis. With contour plots, these effects are clearly shown in Fig.2.3. As a consequence, the corresponding matched filter output displays a pulse compression effect on the autocorrelation function, which is shown in Fig. 2.4. Therefore, this property is a basis for pulse compression technique.

Figure 2.2: Partial ambiguity functions of two waveforms.

(a) Constant frequency waveform.

(b) LFM waveform.

Figure 2.3: Contour plots of partial ambiguity functions.

Figure 2.4: Autocorrelation functions of two waveforms.

Property 5: Volume Free Area and Average Sidelobe Level

This property says that the maximum possible volume free area that can surround a spike located at the origin is 4. This implies that the maximum range-Doppler area that can be unambiguously explored is unity [2, 54].

2.2 Composite Ambiguity Function

The composite ambiguity function is a kind of radar ambiguity function

proposed by Guery and Bell in [61] to measure the discrimination characteristics of a set of multiple waveforms in a delay-Doppler plane. In an idealized situation, the radar system is capable of operating simultaneously and independently on several independent channels without crosstalk between them. Thus, by coherently summing the images from each channel up, one can get a composite ambiguity function. The composite ambiguity function has less volume of ambiguity than traditional ambiguity function, which makes it possible to incur less ambiguity for methods of waveform-diverse multiple-measurements technique [61]. Recently, the concept of composite ambiguity function has been employed in some works for designing Doppler resilient Golay complementary pairs for radar [66-67] as it is actually a more general tool extending the concept of Golay code to a 2D delay-Doppler context. Like in conventional radar ambiguity function, the maximum volume-free area in composite ambiguity function is an interesting property to be studied since it will set an upper bound on the delay-Doppler discrimination ability of any waveform set adopted in such a manner, which in turn will determine the best waveform set possible for delay-Doppler discrimination. In this section, we will investigate the bounds on the maximum volume clearance area of composite ambiguity function.

2.2.1 Volume Clearance

The composite ambiguity function is defined for a set of K waveforms

$\{s_0(t), \dots, s_{K-1}(t)\}$ as [61]:

$$C_{AF}(\tau, \nu) = \sum_{i=0}^{K-1} \chi_i(\tau, \nu) \quad (2.15)$$

where χ_i is the ambiguity function of $s_i(t)$, $i=0, 1, \dots, K-1$. To evaluate the bound on volume clear area, we define the volume under the composite ambiguity function as a function of region A like:

$$V(A) = \iint_A |C_{AF}(\tau, \nu)|^2 d\tau d\nu \quad (2.16)$$

We can obtain the Fourier Transform property of the proposed ambiguity function as:

$$\iint \left| \sum_{i=0}^{K-1} \chi_i(\tau, f) \right|^2 e^{-j2\pi\nu\tau + j2\pi\sigma f} d\tau df = \sum_{n=0}^{K-1} \sum_{m=0}^{K-1} |\chi_{nm}(\sigma, \nu)|^2 \quad (2.17)$$

where χ_{nm} is the cross-ambiguity function between $s_n(t)$ and $s_m(t)$, $n, m=0, 1, \dots, K-1$, and when $n=m$, χ_{nm} becomes the ambiguity function of $s_n(t)$. More details on proof of (2.17) can be found in Appendix A at the end of the thesis. Note that, we can easily derive the Theorem 1 in paper [61] from (2.17). It follows from (2.17) and Parseval's theorem that if $\varphi(\tau, f)$ is any quadratically integrable function having the two-dimensional Fourier transform [54]:

$$\Phi(\tau, f) = \iint_{-\infty}^{\infty} \varphi(\sigma, \nu) e^{-j2\pi\nu\tau + j2\pi\sigma f} d\sigma d\nu \quad (2.18)$$

Then

$$\iint_{-\infty}^{\infty} \left| \sum_{i=0}^{N-1} \chi_i(\tau, f) \right|^2 \varphi(\tau, f) d\tau df = \sum_{n=0}^{K-1} \sum_{m=0}^{K-1} \iint_{-\infty}^{\infty} |\chi_{nm}(\tau, f)|^2 \Phi(\tau, f) d\tau df \quad (2.19)$$

Following the methodology in [54], we restrict $\varphi(\tau, f)$ to have a non-negative transform $\Phi(\tau, f)$ so that $\varphi(\tau, f)$ is a two-dimensional autocorrelation

function and may therefore be written as:

$$\varphi(\tau, f) = \int \int_{-\infty}^{\infty} \varphi_1(x, y) \varphi_1(x + \tau, y + f) dx dy \quad (2.20)$$

for some $\varphi_1(x, y)$. Then applying the Schwarz inequality to (2.20) yields the condition:

$$|\varphi(\tau, f)| \leq \varphi(0, 0) \quad (2.21)$$

It is further required that $\varphi(\tau, f)$ be real and non-positive outside the τ - f region A in (2.16). Thus, the following inequalities are obtained:

$$\begin{aligned} \varphi(0, 0) V(A) &\geq \int \int_{-\infty}^{\infty} \left| \sum_{i=0}^{K-1} \chi_i(\tau, f) \right|^2 \varphi(\tau, f) d\tau df \\ &= \sum_{n=0}^{K-1} \sum_{m=0}^{K-1} \int \int_{-\infty}^{\infty} |\chi_{nm}(\tau, f)|^2 \Phi(\tau, f) d\tau df \\ &\geq \beta \sum_{n=0}^{K-1} \sum_{m=0}^{K-1} \int \int_{A'} |\chi_{nm}(\tau, f)|^2 d\tau df \\ &= \beta V(A') \end{aligned} \quad (2.22)$$

where A' is another region in the τ - f plane associated with ambiguity volume $V(A')$, and β denotes the greatest lower bound on $\Phi(\tau, f) \in A'$. If the area A' shrinks to origin $(0, 0)$, then β approaches $\Phi(0, 0)$, and

$$\begin{aligned} V(0, 0) &= \sum_{n=0}^{K-1} \sum_{m=0}^{K-1} \int \int_{A' \rightarrow (0,0)} |\chi_{nm}(\tau, f)|^2 d\tau df \\ &= \sum_{n=0}^{K-1} \int \int_{A' \rightarrow (0,0)} |\chi_{nn}(\tau, f)|^2 d\tau df + \sum_{\substack{n=0 \\ n \neq m}}^{K-1} \sum_{m=0}^{K-1} \int \int_{A' \rightarrow (0,0)} |\chi_{nm}(\tau, f)|^2 d\tau df \end{aligned} \quad (2.23)$$

When $\{s_0(t), \dots, s_{K-1}(t)\}$ is a set of equal-energy orthogonal waveforms, we can easily find $V(0, 0)$ approaches its minimum as:

$$V(0, 0) = K V_0 \quad (2.24)$$

where V_0 is the volume of peak impulse from each waveform. For

non-orthogonal waveform set, $V(0,0) > KV_o$ must hold. In particular, $V(0,0)$ has a maximum of K^2V_o when all waveforms in the set are same. Now, like [54], we restrict the region A of (2.16) to be any set of points in the delay-Doppler (τ, f) plane satisfying:

(a) A is convex

(b) $(\tau, f) \in A \rightarrow (-\tau, -f) \in A$

Like [54], we then construct the weight function $\varphi(\tau, f)$ in (2.18)-(2.23) as the correlation function

$$\varphi(\tau, f) = \int \int_{-\infty}^{\infty} I_A(2x, 2y) I_A(2x + 2\tau, 2y + 2f) dx dy \quad (2.25)$$

where the characteristic function of the set A is

$$I_A(\tau, f) = \begin{cases} 1; (\tau, f) \in A \\ 0; (\tau, f) \notin A \end{cases}$$

Then following the discussion in [54], from (2.22)-(2.25), for orthogonal waveforms of equal energy we can have:

$$\begin{aligned} V(A) &\geq \frac{\Phi(0,0)}{\varphi(0,0)} KV_o \\ &= \frac{\int \int_{-\infty}^{\infty} \varphi(\sigma, \nu) d\sigma d\nu}{\varphi(0,0)} KV_o = \frac{\left[\int \int_{-\infty}^{\infty} I_A(2\sigma, 2\nu) d\sigma d\nu \right]^2}{\int \int_{-\infty}^{\infty} I_A^2(2\sigma, 2\nu) d\sigma d\nu} KV_o \quad (2.26) \\ &= \frac{1}{4} \frac{\left[\int \int_{-\infty}^{\infty} I_A(\sigma, \nu) d\sigma d\nu \right]^2}{\int \int_{-\infty}^{\infty} I_A(\sigma, \nu) d\sigma d\nu} KV_o = \frac{1}{4} C(A) KV_o \end{aligned}$$

Meanwhile, under the assumption that all waveforms are constrained with the same time-bandwidth product and contain equal energy, the volume at the origin of the composite ambiguity function is [62]

$$\begin{aligned}
V_{c0} &= \iint_{A' \rightarrow (0,0)} \left| \sum_{i=0}^{K-1} \chi_i(\tau, f) \right|^2 d\tau df \\
&= K^2 V_0
\end{aligned} \tag{2.27}$$

In this way, from both (2.26) and (2.27), we can get

$$V(A) \geq \frac{1}{4} C(A) K V_o = \frac{1}{4} C(A) \frac{V_{c0}}{K} \tag{2.28}$$

From (2.28) the volume clearance corollary is that if $C(A) > 4K$, then the composite ambiguity function must contain some volume in A in addition to those of the impulse at the origin. Thus, in this case, the composite ambiguity function obtains K times as large volume clearance area as conventional ambiguity function. Similarly, for non-orthogonal waveforms, the maximum clear area of composite ambiguity function must decrease as compared with orthogonal waveforms and approaches to that of conventional ambiguity function when all waveforms are same. In this way, the maximum clear area of composite ambiguity function is no smaller than that of conventional ambiguity function.

2.2.2 η Clear

Similar to the discussions in [54], we can investigate the “ η clear” area, for which $|C_{AF}(\tau, \nu)|^2 \leq \eta$. For orthogonal waveform case, the “ η clear” over A is satisfying

$$V_{c0} + \eta C(A) \geq V(A) \geq \frac{1}{4K} C(A) V_{c0} \tag{2.29}$$

For $\eta < V_o/4K$, this inequality can hold if and only if

$$C(A) \leq \frac{4KV_{c0}}{V_{c0} - 4K\eta} \quad (2.30)$$

which gives the largest possible area that could be clear to level η for all kind of waveform set.

2.2.3 Minimum Mean Sidelobe Level

For orthogonal waveforms, the average sidelobe level s over the region A for composite ambiguity function is:

$$\begin{aligned} s^2 &= \frac{V(A) - V_{c0}}{C(A)} \geq \frac{\frac{1}{4K}C(A)V_{c0} - V_{c0}}{C(A)} \\ &= \frac{1}{4K}V_{c0} \left(1 - \frac{4K}{C(A)} \right) \end{aligned} \quad (2.31)$$

For $C(A) \gg 4K$, the minimum sidelobe level must exceed $\sqrt{V_{c0}/K}/2$. This is also the minimum value achievable for all kinds of waveform set. Obviously, minimum sidelobe level from non-orthogonal waveforms must increase but can not be higher than that from a single waveform.

2.3 Ambiguity functions for MIMO radar

The concept of ambiguity function analysis was firstly introduced to MIMO radar by Santonio, Fuhrmann, and Rocky in [63, 64]. A general form of ambiguity function of MIMO radars was proposed by them [63, 64]. Specific forms with respect to particular applications were studied in [20, 62, 65]. In particular, [20] proposed an ambiguity function for high resolution MIMO radar imaging. Paper [62] proposed an ambiguity function based on the radar

performance in clutter. Paper [65] proposed another ambiguity function for MIMO radar with array antenna. These ambiguity functions can be categorized into two kinds. One is the coherent ambiguity function. The other is the non-coherent ambiguity function. The coherent ambiguity function focuses on the target response of a MIMO radar system, i.e., system ability of distinguish different targets. The non-coherent ambiguity function mainly concerns about the clutter issue incurred by MIMO radar.

2.3.1 Coherent MIMO Radar Ambiguity Function

For a general MIMO radar configuration, suppose there are N transmitters and M receivers. The n -th transmitter has a baseband transmitting waveform $s_n(t)$ and a central frequency f_0 . We assume the target of interest to be a point target with reflection coefficient 1. Target 1 has position vector $\mathbf{P}_1$ and velocity vector $\mathbf{V}_1$. Let $\Theta_1 = [\mathbf{P}_1; \mathbf{V}_1]$. For target 2, let $\Theta_2 = [\mathbf{P}_2; \mathbf{V}_2]$. Then, we define the delay from the n -th transmitter to the m -th receiver via target 1 as $\tau_{nm}(\mathbf{P}_1)$, target 2 as $\tau_{nm}(\mathbf{P}_2)$; define the Doppler frequency observed by the m -th receiver due to the n -th transmitter via target 1 as $f_{nm}(\Theta_1)$, target 2 as $f_{nm}(\Theta_2)$.

The MIMO radar ambiguity function can then be expressed as [64]:

$$AF(\Theta_1, \Theta_2) = \left| \sum_{m=1}^M \sum_{n=1}^N \sum_{n'=1}^N e^{-j2\pi f_0(\tau_{nm}(\mathbf{P}_1) - \tau_{n'm}(\mathbf{P}_2))} \chi_{nn'}(\Theta_1, \Theta_2, m) \right|^2 \quad (2.32)$$

Where

$$\chi_{nn'}(\Theta_1, \Theta_2, m) = \int_{-\infty}^{\infty} s_n(t - \tau_{nm}(\mathbf{P}_1)) s_{n'}^*(t - \tau_{n'm}(\mathbf{P}_2)) e^{j2\pi t(f_{nm}(\Theta_1) - f_{n'm}(\Theta_2))} dt \quad (2.33)$$

From (2.32) and (2.33) we can see that the MIMO radar ambiguity function is dependent on both the sensor distribution and the waveforms. This is different from the conventional ambiguity function. The sensor distribution mainly affects the phase shift and the Doppler shift. While the waveform set determines the autocorrelation and crosscorrelation terms. Thus, joint sensor-configuration and waveform optimization may be applicable to MIMO radar performance optimization.

2.3.2 Non-coherent MIMO Radar Ambiguity Function

For a typical MIMO radar architecture, the clutter power at the output of the MIMO radar is:

$$P_c = \sigma_c^2 \sum_{j=1}^N \sum_{k=1}^N \iint_A |\chi_{jk}(\tau, f)|^2 \phi(\tau, f) d\tau df \quad (2.34)$$

where $\sigma_c^2 \phi(\tau, f)$ is the clutter distribution function over the area A .

To evaluate the MIMO radar performance in a clutter environment, the non-coherent MIMO radar ambiguity function is defined as [62]

$$\chi_N(\tau, f) = \sum_{n=1}^N \sum_{n'=1}^N |\chi_{nn'}(\tau, f)|^2 \quad (2.35)$$

Compared with (2.32), the non-coherent ambiguity function discards the relative phase information between channels and utilizes the power of correlations of waveforms. If there is a sidelobe area, say, A , free of ambiguity, then, the clutter power could be greatly decreased. Paper [62] studies the volume clearance condition for this MIMO radar ambiguity function and finds

that the maximum volume clearance area of MIMO radar is only $1/N$ of the conventional signal waveform ambiguity function, which significantly limits the performance of MIMO radar operated in a clutter environment.

2.4 Concluding Remarks

In this chapter, we reviewed different kinds of ambiguity functions. After introducing the conventional radar ambiguity function, we introduced and studied the composite ambiguity function first and then introduced the concept of MIMO radar ambiguity function. Specifically, we investigated the bounds on the volume and height distributions of the composite ambiguity function. It is found that the volume clearance area of composite ambiguity function is no smaller than the conventional ambiguity function. In the case of orthogonal waveform set, the volume clearance area of composite ambiguity function is K times as large as that of conventional ambiguity function. Both the conventional radar ambiguity function and composite ambiguity function focus on waveform's inherent ability of discrimination. However, MIMO radar ambiguity function may involve the effect of radar geometry configuration.

Chapter 3 Sparse Frequency Waveform Design

3.1 Introduction

Waveform design is of fundamental importance in designing high performance radar systems. With ever more congested electromagnetic spectrum, a sparse frequency waveform (SFW) design employing a number of discontinuous bands can be used to solve some spectrum congestion problems. However, conventional SFW design by frequency filtering results in high range sidelobes. There are two kinds of approaches to solve high range sidelobe problems. The first kind of approaches is to employ post processing in the receiver to achieve better sidelobe performance without significant loss in target detectability and target resolution. The second kind of approaches is to design an optimal SFW with better power spectral density (PSD) and sidelobe performance. There are many papers on the first kind of approach and paper [68] is a detailed review on this topic.

There are several papers on the second kind of approach to solve the sidelobe problem. A method of stepped frequency pulse train for high frequency surface wave radar (HFSWR) application is reported in [52]. In this method, each pulse in the train is divided into a number of segments whose frequency modulation rate is optimized. If the length of each segment is too small, frequency spreading would occur. On the other hand, if the length of each

segment is too large, there would not be enough segments in each pulse to get an optimal waveform. In summary, the method in [52] requires long pulse duration for optimal results but such long pulse duration may not be achievable for practical radar applications.

Another algorithm to design SFWs for radar and communication applications is reported in [68]. This method is like the optimum phase-only adaptive nulling presented in [69]. This method is efficient in computation and can potentially achieve stopband suppression for certain sparse frequency assignments in both transmit waveform and receive waveform. However, there are also several constraints for practical applications. Firstly, although the receive waveform design can benefit from amplitude variation and reduces sidelobe levels in the range response, the weighting method used may cause mismatch problems [8] or reduce the range resolution. Secondly, even after achieving sidelobe reductions in the receive waveform design, a high peak sidelobe level (PSL) still occurs in range response. For example, Fig. 4 in [68] reported a PSL of -7 dB, which may be too high for some detection techniques, and may deteriorate the image property in imaging application by reducing the contrast or dynamic range in the range response so that post-processing would still be required. Thirdly, for the soft power constraint method, though it could benefit from amplitude diversity by allowing the transmitter power to be varied, it would cause power loss in transmitter efficiency, leading to the loss of radar

sensitivity. Furthermore, the linear dynamic range of the power amplifier has to be checked to ensure that the amplitude variation is achievable. In summary, the soft power constraint method may not be as robust as the phase-only method.

This chapter presents an optimal SFW design method. The transmit waveform designed using the proposed method can simultaneously meet the requirements of both sparse frequency spectrum and low sidelobe levels in the autocorrelation function (ACF) by optimizing the total penalty function incorporating these two requirements for the same transmit waveform. As the proposed method reduces sidelobe levels in ACF, it will not cause mismatch loss. This is one basic difference between the proposed method and the method in [68]. In addition, since an optimal SFW with acceptable ACF and sparse frequency performance is achieved with the proposed method, the post-processing effort is minimized. The proposed approach is based on a phase-only method with a constant signal magnitude, allowing the transmitter to constantly transmit full power across the pulse. For completeness, the proposed approach is also extended to phase-amplitude joint optimization by introducing a further constraint on power distribution. Besides designing single SFW, this chapter also extends the proposed method to design multiple SFWs.

This chapter is organized as follows. Section 3.2 presents the proposed methodology for phase-only design, including the optimization model and a brief introduction to implementation of the particle swarm optimization (PSO)

algorithm. Section 3.3 presents the phase-amplitude joint design method. Section 3.4 shows some case studies and discussions with regard to these two methods. Section 3.5 presents the method for designing multiple SFWs. Section 3.6 finally concludes this chapter.

3.2 Phase-Only Design: Optimization Model and PSO

When the requirements for both sparse frequency and range sidelobe level are considered in the transmit waveform design, it is very difficult to apply directly an adaptive method like [68]. This is because when the requirement for an ACF constraint in the transmit waveform is considered, the corresponding penalty function should be formed from the ACF of the desired waveform. However, the derivative of the ACF on the phase values does not have a concise form. Meanwhile, the minimum value of $\mathbf{s}^H \tilde{\mathbf{R}} \mathbf{s}$, where $\mathbf{s}$ is the desired transmitting waveform, the superscript H stands for the conjugate transpose, and $\tilde{\mathbf{R}}$ is a matrix with unknown $\mathbf{s}$, subject to linear constraints cannot be easily obtained through the way solving a linear constrained minimization problem [68]. Therefore this chapter employs a flexible optimization approach to solve this problem. The total penalty function consists of two parts. The first part is from the constraint on the PSD, namely, the power difference between the PSD of a SFW candidate and the desired PSD, which mainly reduces the energy in the stop bands. The second part comes from the constraint on ACF sidelobes. Such evolutionary optimization algorithms as genetic algorithms (GAs) [70-72]

or PSO algorithms [73-75] could be used as the optimization engines. Based on our extensive survey, simulation and comparison studies, we have found that PSO is more efficient than GA for this problem. Besides, as an increasingly popular optimization method, a PSO algorithm is easier to implement than a GA. Thus, the PSO algorithm is selected as the optimization engine for this study.

3.2.1 Optimization Model

Based on the constraints on PSD and ACF, there are two sub-penalty functions to be satisfied:

- 1) PSD sub-penalty function, denoted by P_{PSD} , is determined by the PSD constraints and is derived in Sub-Section A.
- 2) ACF sub-penalty function, termed by P_{ACF} , is determined by the ACF constraints and is derived in Sub-Section B.

The total penalty function can be constructed based on these two sub-penalty functions as below:

$$P_{Total} = \lambda \cdot P_{PSD} + (1 - \lambda) \cdot P_{ACF} \quad (3.1)$$

where λ , $0 < \lambda < 1$, is a real weight parameter to balance the contributions from these two sub-objective functions.

A) PSD Constraints

A candidate transmit waveform, which is a time domain signal, is a set of N samples from a continuous complex waveform. The number of N is derived

based on the pulse duration and the sampling rate satisfying the Nyquist Sampling Theorem. For the phase-only transmit waveform design, the digital waveform is assumed to be phase-controlled with a constant amplitude. The set of waveform samples is defined in the form of a discrete complex vector as $\mathbf{s} = [\exp(j\phi_1); \exp(j\phi_2); \dots; \exp(j\phi_N)]$, where ϕ_n , $n=1, \dots, N$ are phases. We also define the phase vector $\mathbf{\Phi} = [\phi_1; \phi_2; \dots; \phi_N]$. The PSD of the candidate signal can be expressed as $|x(f)|^2$, where $x(f)$ is the discrete Fourier transform of the waveform. The penalty function with the stop band PSD constraint is:

$$P_{PSD} = \sum_{k=1}^{N_s} \int_{f_k^1}^{f_k^2} w_k(f) |x(f)|^2 df \quad (3.2)$$

where f_k^1 and f_k^2 are the starting and ending frequencies of the k -th stop band, N_s is the number of stop bands, $w_k(f)$ is a weight for frequency f . Different weights may be assigned to different stop bands. P_{PSD} expresses the total energy in all stop bands and the integral in (3.2) is evaluated in discrete form. Because $x(f)$ is a function of the phase vector, the PSD penalty function is also a function of the phase vector. Clearly speaking, the PSD in stopbands should be as low as possible. Although no constraints are placed in the pass bands, the range resolution requirement in the ACF objective function (Sub-Section B) has the equivalent effect.

In practical implementation, (3.2) may not be resolved for some severe cases in which a required sub-channel bandwidth is less than the frequency resolution given the sampling rate. That is because (3.2) is evaluated by a numerical

method in discrete form. Therefore, when the stop band has an extremely small bandwidth, the number of Fast Fourier Transform (FFT) points should be large enough to meet the frequency resolution requirement. Thus the computation effort would be considerably increased. Therefore, an alternate penalty function is regained such as one based on the methods in [68]. Referring to [68], the objective function for minimizing the energy of stopbands is $P_{stopband} = \mathbf{s}^H \mathbf{R}_s \mathbf{s}$, where $\mathbf{R}_s$ is the stopband penalty matrix and is defined by

$$\mathbf{R}_s(m, n) = \sum_{k=1}^{N_s} w_k \begin{cases} \frac{\exp[2\pi j(m-n)f_k^2 \Delta t] - \exp[2\pi j(m-n)f_k^1 \Delta t]}{2\pi j(m-n)\Delta t} & m \neq n \\ f_k^2 - f_k^1 & m = n \end{cases} \quad (3.3)$$

where w_k is the constant weight for the k -th stopband, m and n are the indices of matrix element, Δt is the sampling interval.

In addition to constructing the $\mathbf{R}_s$ matrix for stopbands, a matrix for passbands is also constructed in the proposed method. By substituting corresponding parameters of passbands into (3.3), the penalty matrix for passbands could be constructed, named as $\mathbf{R}_p$. The penalty function for passbands is

$$\begin{aligned} P_{passband} &= \sum_{l=1}^{N_p} w_l \int_{f_l^1}^{f_l^2} |x(f)|^2 df \\ &= \mathbf{s}^H \mathbf{R}_p \mathbf{s} \end{aligned} \quad (3.4)$$

where w_l is the weight for the l -th passband, f_l^1 and f_l^2 the starting and ending frequency of the l -th passband, N_p the pass band number. Thus, $P_{passband}$ can also be computed efficiently by matrix multiplication.

The alternate penalty function for PSD now becomes

$$P_{PSD} = P_{stopband} / P_{passband} \quad (3.5)$$

Because the ratio in (3.5) is determined by both the overall energy in stopbands and passbands, the stopbands can be suppressed and the passbands can be enhanced by appropriate choice of weights to certain stopbands and passbands.

Extensive experimentation with both PSD objective functions has shown, in general, that (3.2) has better computation efficiency and better PSD shaping capacity than (3.5). Only in a severe case as mentioned above, (3.5) may be used for an alternative solution.

B) ACF Constraints

For waveform design, the range sidelobe levels of the ACF are very important. In the phase-only waveform design, the ACF is a function of the phase vector. The ACF constraint aims to minimize sidelobes in the ACF to obtain acceptable sidelobe levels. For a pulse waveform, the ACF penalty function can be expressed as

$$P_{ACF} = \sum_{n=1}^{N-NI/2} |ACF(\Phi, n)|^2 + \sum_{N+NI/2}^{2N-1} |ACF(\Phi, n)|^2 \quad (3.6)$$

where $NI+1$ is the mainlobe width and N is the number of samples. The mainlobe width is defined by $NI+1 = F_s/BW1$, where F_s is the sampling rate for complex data, $BW1$ is the effective bandwidth of all passbands. F_s should be no less than the total bandwidth, based on Nyquist Sampling Theorem. If the

waveform is a continuous wave (CW), then a circular or periodic ACF [76] is used in (3.6).

3.2.2 PSO Based Optimization Engine

PSO is a powerful optimization algorithm. It has been widely used for a wide range of applications [77-80] to find global optimal solutions. In addition, PSO is efficient in computing time and memory requirements. For this waveform optimization, the main computational load for each iteration comes from two parts; one is the update of particle position and velocity, the other is the computation of the objective function. The first part involves only element multiplication and addition, and the second part can be efficiently computed by FFT. Thus, the computation effort and memory requirements are approximately linear in the dimension of problem space N . In this chapter, the PSO algorithm is used to search for the optimal phase vectors that can simultaneously meet the requirements of sparse frequency channel assignments and main-to-sidelobe ratio. We can either use the standard PSO or the constrained PSO to accomplish this work.

A) Standard PSO

When the standard PSO algorithm is used to optimize (3.1), the optimizer initially generates a group of, say M , random phase vectors as potential solutions. Each phase vector is called a particle with its cost value evaluated from the specified objective function. Each particle will keep track of its

position in the problem space and store the position with the smallest cost value it has achieved so far as the population best, say, $\mathbf{P}_{best}$. The optimizer will also keep track of the particle position that has the smallest cost value obtained so far by any particle in the population. This position is taken as the global best, say, $\mathbf{G}_{best}$. The population best and the global best are initialized as the following:

$$\mathbf{P}_{best}^j \leftarrow \Phi^j \text{ and } \mathbf{G}_{best} \leftarrow \arg \min \text{Obj}_{Total}(\Phi^j), j = 1, \dots, M$$

After initialization, the iteration progress will begin. In each iteration, particles are employed to evaluate their cost values based on the objective function, and the cost values are taken to update the $\mathbf{G}_{best}$ and $\mathbf{P}_{best}$. Afterwards each particle is updated according to (3.7) and (3.8):

$$\mathbf{v}(i+1) = \mu \cdot \mathbf{v}(i) + c_1 \cdot \mathbf{u}(0,1) \odot (\mathbf{P}_{best}(i) - \Phi(i)) + c_2 \cdot \mathbf{u}(0,1) \odot (\mathbf{G}_{best}(i) - \Phi(i)) \quad (3.7)$$

$$\Phi(i+1) = \Phi(i) + \mathbf{v}(i) \quad (3.8)$$

where $\Phi(i)$ is the particle position of the i -th iteration and $\Phi(i+1)$ the particle position of the following iteration, $\mathbf{v}(i)$ is the velocity of the i -th iteration and $\mathbf{v}(i+1)$ the velocity of the next iteration, μ is the inertia weight, c_1 and c_2 are the learning factors, $\mathbf{u}(0,1)$ is a random vector with elements uniformly distributed in the region of $[0,1]$, and the operator $\odot$ stands for the Hadamard matrix operator. The selection of parameters μ , c_1 , and c_2 can be referred to [81, 82], that is, $c_1=c_2=2$ and μ has an initial value around 1 and gradually declines towards 0. PSO will iterate until the desired cost value is achieved or a given

maximum number of iterations is reached.

B) Constrained PSO

The method of constrained PSO was discussed in detail in [83]. For brevity, the details will not be repeated here. In constrained PSO the penalty function is the PSD objective function and the constraint is to confine all sidelobes in ACF below a given level. There are two main differences between the standard PSO and the constrained PSO. Firstly, in the initialization step, the particles in constrained PSO should all be verified to satisfy the ACF constraint. Secondly, during the iteration process, only those particles satisfying the ACF constraint will be used to update $\mathbf{G}_{best}$ and $\mathbf{P}_{best}$.

The constrained PSO algorithm is not as efficient as the standard PSO algorithm in computation because it needs extra effort to check the constraint and there will be fewer effective particles due to the ACF constraint. But it has several advantages. One advantage is that the constrained PSO does not need the weights adjustment for the two parts of penalty function in (3.1). The other advantage is that any kind of ACF constraint could be easily implemented. Moreover, the constrained PSO can keep the PSL under certain level while optimizing the PSD penalty function, which is highly desirable when the spectrum of interest is extremely sparse, leading to unacceptable PSL by using the standard PSO implementation. Usually, these two PSO algorithms could be adopted alternatively.

3.3 Phase-Amplitude Joint Design

In fact, the waveform can also be modulated in amplitude. By introducing diversity of amplitude, more design freedom can be utilized to achieve better PSD and/or ACF performance, certainly, with loss of transmitting power. In this section, we investigate the phase-amplitude joint design method.

In this case, the set of waveform samples is defined in the form of a discrete complex vector as $\mathbf{s} = [a_1 \exp(j\phi_1); a_2 \exp(j\phi_2); \dots; a_N \exp(j\phi_N)]$, where ϕ_n , and a_n , $n=1, \dots, N$ are phases and magnitudes of samples, respectively. We also define the phase vector $\mathbf{\Phi} = [\phi_1; \phi_2; \dots; \phi_N]$ and an amplitude vector $\mathbf{a} = (a_1; a_2; \dots; a_N)$. Then the PSD penalty function can be constructed similarly as that in the phase-only design method. The difference now only lies in that the PSD penalty function is a function of both phase vector and amplitude vector. Similarly, the ACF penalty function can be constructed based on (3.6). Then, the total penalty function is now:

$$P_{Total} = \lambda \cdot P_{PSD}(\mathbf{\Phi}, \mathbf{a}) + (1 - \lambda) \cdot P_{ACF}(\mathbf{\Phi}, \mathbf{a}) \quad (3.9)$$

The parameters to be optimized are now twice as long as those of phase-only method. Meanwhile, the implementation of PSO algorithm will be different. As illustrated in (3.9), the total penalty function is a function of phase vector and amplitude vector. For normal PSO optimization problem, the particles should consist of both of them. However, the phase vector and amplitude vector in this chapter have not the same effect on the penalty

function and have not the same range of variety and therefore should not be computed and updated in the same way. Thus we take two kinds of particles for this problem. And the particle evolutionary step in PSO algorithm should also be revised to be apt to this optimization problem. Two groups of particles, phases and amplitudes, are randomly initialized and then the iteration begins. In each iteration, two groups of particles are employed to evaluate the penalty function, and the fitness value is taken to update the population best ($\mathbf{P}_{best}$) and global best ($\mathbf{G}_{best}$). Unlike normal PSO algorithm, there are two kinds of population-best: one, $\mathbf{P}_{best}$, is for phase particles and the other, $\mathbf{P}_{a_{best}}$, is for amplitude particles. And there are two kinds of global best for phase particles and amplitude particles, $\mathbf{G}_{best}$ and $\mathbf{G}_{a_{best}}$, respectively. After that each particle would be updated according to (3.10)-(3.13):

$$\mathbf{v}(i+1) = \mu \cdot \mathbf{v}(i) + c_1 \cdot \mathbf{u}(0,1) \odot (\mathbf{P}_{best}(i) - \Phi(i)) + c_2 \cdot \mathbf{u}(0,1) \odot (\mathbf{G}_{best}(i) - \Phi(i)) \quad (3.10)$$

$$\Phi(i+1) = \Phi(i) + \mathbf{v}(i) \quad (3.11)$$

$$\mathbf{v}_a(i+1) = \mu \cdot \mathbf{v}_a(i) + c_1 \cdot \mathbf{u}(0,1) \odot (\mathbf{P}_{a_{best}}(i) - \mathbf{a}(i)) + c_2 \cdot \mathbf{u}(0,1) \odot (\mathbf{G}_{a_{best}}(i) - \mathbf{a}(i)) \quad (3.12)$$

$$\mathbf{a}(i+1) = \mathbf{a}(i) + \mathbf{v}_a(i) \quad (3.13)$$

where $\Phi(i)$ and $\mathbf{a}(i)$ are the phase vector and amplitude vector of current iteration, $\Phi(i+1)$ and $\mathbf{a}(i+1)$ are the phase vector and amplitude vector of the following iteration, $\mathbf{v}(i)$ and $\mathbf{v}_a(i)$ are the velocity vectors of phase and amplitude in the present iteration, $\mathbf{v}(i+1)$ and $\mathbf{v}_a(i+1)$ are the next iteration. Because the power of the transmitting signal should be in an acceptable range,

the value of the amplitude vector should be restricted in a corresponding range too. This can be done by giving limits to $\mathbf{a}$. PSO will keep iterating until the desired fitness is achieved or an assumed number of iterations have been reached.

3.4 Design Examples and Discussions

Examples are presented here to show that the proposed approach is highly flexible and can be used in various SFW design applications.

3.4.1 Case 1: Implementation of Standard PSO

Case 1 illustrates the capability of the proposed approach by comparing it with the phase-only transmit design of [68]. The desired waveform has 3 passbands and 2 stopbands distributed in a total bandwidth of 2 MHz. The stopbands are [0.8, 0.9] MHz and [1.1, 1.2] MHz, respectively. In practical design stopbands should be inserted at both ends of the spectrum to minimize spectral leakage. In the case studies of this chapter, we just want to know the performance of the SFW design method, so we did not insert stopbands at both ends of the spectrum, which would not affect the discussion.

Assume that the pulse duration is 200 μs , and the sampling rate is 2 MHz, there are 401 complex samples in each pulse. We choose the weighting coefficient $\lambda=0.01$ for (3.1), and $NI=9$ for (3.6). We take the same stopbands weights for our method and for the method in [68]. For other parameters such

as diagonal loading, we use the same value as described in [68]. We also compare these two methods with a linear frequency modulation (LFM) pulse train using 3 sub-pulses to represent 3 pass bands. The sub-pulse duration is 200 μ s.

Figure 3.1: PSD performance comparison (Case 1).

Fig. 3.1 and Fig. 3.2 show comparisons of PSD and ACF results from the three methods mentioned above. Fig. 3.1 shows that the ratio of mean passband power to stopband power are more than 90 dB using the proposed method, about 53 dB using the method in [68], and about 13 dB using the LFM train. It indicates that the proposed PSO approach outperforms the other two methods in the PSD performance based on the stopbands PSD levels. From Fig. 3.2, the ACF from the method in [68] has the PSL of -17.4 dB, and the Integrated Sidelobe Level (ISL) of about -4.6 dB. The ACF from the proposed PSO approach has a PSL of -19.7 dB and ISL of about -6.0 dB. The ACF of LFM

train has a PSL of -13.2 dB and ISL of -9.8 dB. Moreover, the ACF result obtained by the proposed method has a narrower mainlobe width than that in [68]. Thus the proposed method has the best overall performance considering the requirements for both PSD and ACF.

Iteration time is also considered here for computation load comparison between our PSO approach and the method in [68]. For 3000 iterations, the average computing time is 64.6 seconds for PSO with (3.2), 491.9 seconds for PSO with (3.5), and 91.5 second for 1000 iterations by the method in [68]. Usually 3000 and 1000 iterations are good enough for the two algorithms to converge, respectively. The method in [68] usually converges much faster but can easily fall into a local minimum while PSO can always find the global minimum. In other words, method in [68] may likely take much more trials than PSO before a global minimum is found, especially when available channel distribution is complicated. The optimization was performed by a PC with 3GB RAM and Intel 1.86GHz dual core CPU.

In this case, we can easily obtain an optimal result by either our PSO approach or the method in [68]. The PSO optimization method with (3.2) outperforms the adaptive algorithm in both PSD and range response performance. Moreover, from our extensive simulations trials, the final ACF performance by using (3.5) is still better than the adaptive algorithm in [68] because of the global optimization property of PSO, but it takes more CPU time

than the adaptive algorithm. Because we need to use (3.5) only when the bandwidth of certain passband is smaller than the frequency resolution, for brevity, we do not show the results of this method.

(a)

(b) Zoom-in of (a).

Figure 3.2: ACF comparison (Case 1).

3.4.2 Case 2: Implementation of Constrained PSO

Table 1: Passbands Distribution

Channel	Frequency Range(MHz)	Bandwidth (kHz)
1	5.11–5.14	30
2	5.26–5.27	10
3	5.29–5.31	20
4	5.37–5.39	20
5	5.46–5.48	20
6	5.51–5.52	10
7	5.54–5.59	50
8	5.64–5.66	20
9	5.72–5.73	10
10	5.77–5.80	30

This set of simulation parameters is based on the HF channel information from a Canadian coast high frequency surface wave radar system application report [84]. The channel availability is listed in Table 1 with a total of 10 stopbands covering remaining frequencies in the frequency span from 5.11 MHz to 5.80 MHz. The pulse duration is 200 μ s. We set the mainlobe width as 3 for (3.6). We take same weights for stopbands in our method and the method of [68]. We use diagonal loading of 10^{-5} relative to the matrix diagonal for the method in [68], the same value as described in [68]. This example shows how the constrained PSO algorithm performs in the highly congested HF bands. The direct implementation of the conventional LFM trains causes high sidelobes

[52]. Unlike the standard PSO, the constrained PSO treats the ACF objective function as a constraint and optimize only the PSD objective function. Thus we do not need to balance the weights between the two parts of the objective function.

Fig. 3.3 and Fig. 3.4 show comparisons of PSD and ACF results from the phase-only adaptive algorithm in [68], conventional LFM train and our PSO optimization approach. The average CPU time is about 158.7 seconds, tested in the same PC as mentioned above. That computation efficiency makes this method applicable for real HFSWR operation. In Fig. 3.3, the ratio of mean passband power to stopband power is 17 dB for paper [68], and about 13 dB for this chapter using the constrained PSO, which is still acceptable compared with about 14 dB from LFM pulse train. Our result seems to have the worst PSD performance. The reason is that we constrain heavily on our ACF performance to make the waveform have an optimal performance in view of both PSD and ACF. For the waveform of the proposed method, the first sidelobe is -13.2 dB and the PSL is -12.9 dB, and the ISL is about $+3.2$ dB. The waveform from the method in [68] has a high PSL of -2.5 dB and an ISL of $+6.5$ dB. The reasons for the improved sidelobe performance in our method as compared to the adaptive method may consist of two points. One is that our ACF constraint is only on the sidelobes beyond the mainlobe, which is superior to the adaptive method with all range responses included. The other is that the PSO algorithm

has better global optimal searching capability than the adaptive method in [68].

Compared to both the ACF performance and the PSD performance by the method in [68], our approach is more flexible because it can control both the PSD and the ACF performance simultaneously. Simulation study also shows that PSD performance will be sacrificed if better sidelobe level is required or vice versa. In practical operation, one has to make a compromise to obtain the best achievable solution if the desired solution is not available. For example, one may take the solution with better PSD (ACF) performance at the cost of ACF (PSD) performance.

Figure 3.3: PSD comparison (Case 2).

(a)

(b) Zoom in of (a).

Figure 3.4: ACF Comparison (Case 2).

3.4.3 Case 3: Joint Design

This set of simulation parameters is also based on the HF channel information from Canadian report [84]. The 10 clear channels are illustrated in Table 1. And 10 stopbands cover remaining frequencies in the region of 5.11-5.81MHz. The pulse duration is 200 μ s. 141 samples in time domain were tested with the sampling rate of 700 kHz. Both of the method of this chapter and the method of [68] are tested and the performances are compared as stated in the following paragraphs.

The performance of PSD shaping is illustrated in Fig. 3.5. By the method of this chapter, the pass-to-stop band power ratio for the PSD is about 18 dB. By the adaptive method of [68], the pass-stop-band power ratio for the PSD is about 36 dB. However, the sparse waveform performance can not be judged only by the PSD shaping. We have to consider the ACF and the power dynamic range. Meanwhile, based on our simulation studies, our approach can also achieve the comparable and even better PSD performance if it did not take into account the ACF constraint and power dynamic constraint.

The ACF from our approach has a low PSL of -12.2 dB demonstrated as the solid line in Fig. 3.6. This is acceptable for the high frequency surface wave radar system. As a comparison, the PSL of the waveform from [68] is -6.1 dB, which is too high for the radar system. Considering the ACF and PSD performance together, we have to choose the optimal waveform by our

approach.

Figure 3.5: PSD of two methods (Case 3).

Figure 3.6: ACF comparison of two methods (Case 3).

Figure 3.7: Amplitude comparison (Case 3).

Finally, we investigate the power controlling performance of our method. As shown in solid line in Fig. 3.7, the power from this method varies from -3 dB to 0 dB , which causes the power loss of 1.2 dB after averaging. Obviously, the power variation is within the dynamic range of the radar power amplifier. In another word, the power variation of the signal could be well controlled for any practical requirements by this method. While for method of [68], as shown in the dotted line in Fig. 3.7, the power varies sharply from -30 dB to 0 dB , with the average power loss about 10 dB . It is not easy to realize such a large dynamic power change in practical systems, especially for the radar power amplifier.

Comparing the results in Case 2 and Case 3 based on both the PSD and the

ACF performances, we can see that by introducing amplitude variation, we can get better PSD performance with relative same ACF performance. Thus, in some cases, we can sacrifice the power performance for better PSD or ACF.

3.4.4 Further Discussions

With regard to practical applications, several points, say, Low Probability of Intercept (LPI), Doppler resilience, and Quantization effect, should be further claimed. We in this section investigate these properties based on the designed waveform in Case 2, Section 3.4.2.

A) LPI property

Figure 3.8: Comparison of short-time PSD.

In a real application environment, the transmit pulse may be detected and intercepted by others by examining the instantaneous spectral energy. Thus, instantaneous spectral energy concentration is of interest to measure the waveform's property for LPI [85]. We use the short-time Fourier transform of our waveform to evaluate the LPI property. The short-time PSD of our SFW is

given in Fig. 3.8 (a). As a comparison, we also show the short-time PSD of the LFM pulse train in (b) of Fig. 3.8. From Fig. 3.8, we can see that the power of our waveform is sparsely distributed over the passbands during the whole pulse while the power of the LFM train jumps from pulse to pulse. Therefore, our SFW has good ability of LPI.

B) Doppler Resilience

In practical operations there will be Doppler caused mismatch in the matched filter when the signal is reflected back by moving targets [8, 86]. Doppler effect can be illustrated by ambiguity function in general. Partial ambiguity function of the resulting waveform is illustrated in Fig. 3.9, from which we can see that, as the Doppler frequency increases, the performance degradation due to mismatch will increase. More specifically, we see the matched filter output peaks of this waveform in Fig. 3.10. As illustrated in Fig. 3.10, as long as the Doppler-time product of waveform is smaller than 0.5, the output signal amplitude will not be significantly reduced (signal loss < 3 dB). However, for larger Doppler-time product, the performance will degrade significantly. That is mainly because the designed waveform has random phases, and thus, it is noise-like and very sensitive to the Doppler frequency when filtered by matched filter.

Figure 3.9: Partial ambiguity function of the designed waveform.

Figure 3.10: Matched filter output peaks of the waveform with the echo
Doppler shifted.

A basic solution for this problem is to use a Doppler-matched filter bank in the radar processing end to filter the echo signal [1, ch. 10]. Each

Doppler-matched filter is designed to match a signal with certain Doppler-frequency shift. Then, based on the maximum output from the filter bank, detection decision is made.

C) Quantization Effects

For practical operations, the optimal phase values may be quantized before implementation in order to simplify the hardware configuration. In simulation, we can simply round down the phase values to the nearest angle bin. The loss in stopband suppression is 4.4 dB, 0.4 dB, 0.02 dB, and 0 dB for 2, 4, 8, and 16 bit quantization. Meanwhile, the PSLs of the quantized waveforms are no higher than their original counterpart. Thus, in this case, even with 4-bit quantization, we can get comparable PSD and ACF performance.

3.5 Multiple Waveforms Design

Multiple-input multiple-output (MIMO) radar is a new concept of radar that is attracting extensive studies nowadays [12–22]. In many literatures, MIMO radar waveforms are required to be orthogonal with each other. Orthogonal waveform design is therefore an important topic in MIMO radar study [32–35]. The main idea in these papers is to reduce the sidelobe levels in both the autocorrelation functions and the cross-correlation functions (CCFs) of different waveforms by various optimization methods.

As MIMO radar enters many fields of conventional radar application, it would also face the problem of congested spectrum. In this section we extend

our works on designing sparse frequency transmitting waveform to MIMO radar. The designed waveforms can simultaneously satisfy the requirements of desired sparse frequency property and low sidelobe levels in both the ACFs and the CCFs. This is done by minimizing a penalty function constructed based on the power spectral density and correlation requirements. To achieve an optimal solution to this problem, we employ PSO algorithm as the searching engine. The waveforms designed by this part are phase-controlled with constant modulus. Several design examples shows the performance of the proposed method along with discussions on the properties of designed waveforms.

3.5.1 Optimization Model

The basic concept of designing multiple sparse frequency transmit waveforms for MIMO radar is similar to designing single waveform. The difference lies in that there is a group of waveforms to be optimized rather than only one waveform. Consequently, there come larger problem space dimension, somehow new PSD penalty function, new ACF penalty function, and more importantly, totally new CCF penalty function, which is not considered in single waveform design. The total penalty function in the multiple waveforms design is constructed in a way like that in the single waveform design, but with different PSD and sidelobe penalty functions.

A) PSD Penalty Function

We assume that there are M (set size) waveforms to be designed. Each is

represented by a sequence of N (sequence length) samples. Thus, the waveform for the m -th transmitter is represented by a complex number sequence as

$$\mathbf{x}_m = \left[e^{j\phi_{m,1}}, e^{j\phi_{m,2}}, \dots, e^{j\phi_{m,N}} \right]^T \in C^{N \times 1} \quad (3.14)$$

where, $n=1, \dots, N$, and the superscript T denotes the vector transpose, and $\phi_{m,n}$, $m=1, \dots, M$ are phases. The waveform set, i.e., waveform matrix, assembling all M waveforms is expressed as

$$\mathbf{X}(\Phi) = [\mathbf{x}_1 \quad \dots \quad \mathbf{x}_M]_{N \times M} \quad (3.15)$$

where Φ is the phase vector defined by

$$\Phi = [\phi_{1,1}; \phi_{1,2}; \dots; \phi_{m,n}; \dots; \phi_{M,N}]_{NM \times 1}, 1 \leq n \leq N, 1 \leq m \leq M \quad (3.16)$$

Thus, the PSD penalty function can be expressed as:

$$P_{PSD} = \sum_{m=1}^M \sum_{k=1}^{N_s} \int_{f_k^1}^{f_k^2} w_k(f) |x_m(f)|^2 df \quad (3.17)$$

where f_k^1 and f_k^2 are the starting and ending frequencies of the k -th stop band, N_s is the number of stop bands, $w_k(f)$ is a weight for frequency f , $x_m(f)$ is Fourier transform of m -th waveform. The computation of (3.17) can follow the discussion for (3.2).

B) ACF & CCF Penalty Function

For MIMO radar waveform design, we are concerned about not only the sidelobe levels in ACFs but those in CCFs. That is the main difference compared with single waveform design. The mathematical expression of them is developed in below.

Firstly, for phase-only digital waveforms, the autocorrelations and

crosscorrelations in discrete forms are defined by [32]:

$$ACF(\mathbf{x}_m, k) = \frac{1}{M} \sum_{n=0}^{N-1-|k|} x_m(n) x_m^*(n+k), \quad -N < k < N, m=1, \dots, M \quad (3.18)$$

$$CCF(\mathbf{x}_p, \mathbf{x}_q, k) = \frac{1}{M} \sum_{n=0}^{N-1-|k|} x_p(n) x_q^*(n+k), \quad -N < k < N, p \neq q, p, q=1, \dots, M \quad (3.19)$$

where $ACF(x_m, k)$ and $CCF(x_p, x_q, k)$ are the aperiodic autocorrelation function of digital sequence x_m and the crosscorrelation function of digital sequences x_p and x_q , respectively, the asterisk denotes the complex conjugate, and k is the discrete time index.

Base on the ACF and CCF, we construct a total correlation function (CF) penalty function as below:

$$P_{CF} = \sum_{m=1}^M |ACF(\mathbf{x}_m, k)|^2 + \sum_{p=1}^M \sum_{\substack{q=1 \\ p \neq q}}^M |CCF(\mathbf{x}_p, \mathbf{x}_q, k)|^2 \quad (3.20)$$

3.5.2 Design Example

A) Example 1: Three Stopbands

In this case study, there are three stopbands located at 200-220 kHz, 320-340 kHz, 400-430 kHz over a total bandwidth 0-600 kHz. The pulse duration is 100 μ s. Fig. 3.11 shows the results of autocorrelation, Fig. 3.12 crosscorrelation, and Fig. 3.13 power spectral density. For the ACFs, the PSLs of three waveforms are -13.5 dB, -14.2 dB, -14.6 dB, respectively. The ISLs are 3.8 dB, 3.2 dB, and 3.5 dB, respectively. For the CCFs, the PSLs of three

waveforms are -12.3 dB, -10.5 dB, -12.7 dB, respectively. The ISLs are 3.1 dB, 3.2 dB, and 2.9 dB, respectively. As the MIMO radar performance can in some sense be constrained by the ambiguity function [62], the averaged performance is also a metric for evaluating the set performance. We evaluate the averaged performance based on the MIMO radar ambiguity function given in [62]. Referring to ambiguity function discussion in Chapter 2, the ambiguity function of MIMO radar is given by:

$$\chi_{MIMO}(\tau, \nu) = \sum_{p=1}^M \sum_{q=1}^M |\chi_{pq}(\tau, \nu)|^2 \quad (3.21)$$

By setting the Doppler shift to be zero, we obtain

$$\begin{aligned} \chi_{MIMO}(\tau, 0) &= \sum_{m=1}^M |\chi_{mm}(\tau, 0)|^2 + \sum_{p=1}^M \sum_{\substack{q=1 \\ p \neq q}}^M |\chi_{pq}(\tau, 0)|^2 \\ &\triangleq M \chi_{ACF} + (M^2 - M) \chi_{CCF} \end{aligned} \quad (3.22)$$

where χ_{ACF} and χ_{CCF} are the averaged ACFs and the averaged CCFs of MIMO radar, respectively. For ACF, the averaged PSL is -16.5 dB, and the averaged ISL is 3.5 dB. For CCF, the averaged PSL is -16.9 dB, and the averaged ISL is 3.1 dB. Thus, collectively, this waveform set has better PSL performance in both ACF and CCF compared to each individual waveform. The stopband suppression in each waveform is 22.1 dB, 23.0 dB, and 25.0 dB.

Figure 3.11: ACFs (Example 1).

Figure 3.12: CCFs (Example 1).

Figure 3.13: PSD performance (Example 1).

B) Example 2: Five Stopbands

Then, let's study a relative complicated case. There are five stopbands at 90-160 kHz, 200-240 kHz, 320-350 kHz, 400-440 kHz, and 500-520 kHz over a total band of 0-600 kHz. The sparseness, the ratio of bandwidth in all stopbands to the total bandwidth, of this case is $1/3$. The pulse duration is 200 μ s. In this example, there are 2 times as many samples as in three-stopbands case. Fig. 3.14 shows the results of autocorrelation, Fig. 3.15 the result of crosscorrelation, and Fig. 3.16 the result of power spectral density. For ACF, the PSLs of three waveforms are -9.7 dB, -9.7 dB, -8.3 dB, respectively. And the averaged PSL is -9.7 dB. The ISLs are 0.35 dB, -0.11 dB, and 0.29 dB, respectively. And the averaged ISL is 0.17 dB. Compared to the results in Example 1, the ACF performance of each waveform in this case decreases

largely. That is mainly because the sparseness of each waveform increases largely. For CCF, the PSLs of three waveforms are -14.0 dB, -13.6 dB, -13.9 dB, respectively. And the averaged PSL is -16.0 dB. The ISLs are 1.90 dB, 1.44 dB, and 1.15 dB, respectively. And the averaged ISL is 1.49 dB. Compared with Example 1, though the PSLs of CCFs are relatively lower in this case, the ISL performance in this case is impaired. Based on both the ACF performance and the CCF performance, we can see that the sparseness has more effect on the performance of ACF than that of CCF. The stopband suppression for three waveforms is 18.3 dB, 19.8 dB, 19.2 dB, respectively.

Comparing these two cases, we can see that due to the increase in sparseness, the total performance in Example 2 is decreased. Based on these two examples, one important point is that for MIMO radar application, we can have a good chance to reduce the averaged PSL while still satisfying the other performances.

3.5.3 Further Discussions

The LPI property, Doppler resilience, and Quantization effect for each waveform in this multiple waveforms design are all similar to the discussions in single waveform design. For brevity, we will not give separate discussions here. A utilization example is given to show the high resolution property of SFWs. In this example, we have two point target closely located at range gate No.90, and No. 95 with respect to the resolution determined by the SFWs. A uniform linear array with 3 elements distributed in half wavelength serves as both transmitter

and receiver. One waveform set is the sparse frequency waveform set with 5 stopbands designed in Example 2. The other is a LFM signal with 100 kHz bandwidth and 100 μ s pulse duration. The interference is assumed to be White Gaussian noise with a Signal-to-Noise-Ratio of 10 dB. Figure 3.17 shows the simulation result. We can clearly see the superiority of sparse frequency waveform, with which the two point target can be absolutely separated.

Figure 3.14: ACF performance (Example 2).

Figure 3.15: CCF performance (Example 2).

Figure 3.16: PSD performance (Example 2).

Figure 3.17: Resolution performance comparison.

3.6 Concluding Remarks

In this chapter, we studied the method of designing sparse frequency waveform. For sparse frequency waveform design, one big concern is the sidelobe level in ACF, the other is the power distribution in PSD. We firstly designed single SFW that can simultaneously satisfy the property of low sidelobe level in ACF and the requirement on PSD. To achieve this objective, we constructed a penalty function based on these two requirements. Then, PSO algorithm was employed to search for the global minimum of this penalty function in order to get the optimum waveform. Two forms of PSO algorithm were implemented for this optimization. One is the standard PSO algorithm, which can minimize the total penalty function. The other is the constrained PSO, which minimize a sub-penalty function of PSD while controlling the PSL in the

ACF under certain value. The former approach is efficient in computation while without exact control on PSL. The later can control precisely the PSL level but is much more time consuming as in the iteration step of PSO, many of the particles that minimize the PSD penalty function would be rejected as they do not meet the requirement of PSL. With these two implementation methods, our proposed approach is very flexible. Meanwhile, PSO is easy to implement and efficient in computation and memory effort, which makes the proposed approach even applicable to some real-time applications.

Both phase-only design method and the phase-amplitude-joint design method were discussed. The phase-only method can generate waveforms with constant modulus. The phase-amplitude method can have better performance in nulling the stopbands and reducing the sidelobe level but would incur power loss.

Several design examples were provided to illustrate the effectiveness of the proposed methods. For relative simple PSD structure, both standard PSO and constrained PSO can produce good results. While for waveforms with complicated PSD structure, though standard PSO can still reduce the sidelobe, it can not guarantee the PSL to be below certain level. Thus, in this case, the constrained PSO may be preferred. Meanwhile, as the PSD and the ACF are a Fourier transform pair, the better performance of one comes at the price of the other. Thus, for practical application, one must trade off these two performances

in light of a total performance criterion. Many application issues were also discussed, for example, the LPI property, the Doppler effect, the Quantization effect.

We also extended this method to design multiple SFWs for MIMO radar. Though the basic concept for the multiple waveforms design is similar with the single waveform design, there are indeed some differences. First, the sidelobe in CCFs should be considered. Second, the increase in waveform number leads to a much larger problem space, which challenges the implementation of PSO algorithm. Nevertheless, as shown in this chapter, for problems with small dimension and relative simple PSD structure, the proposed method can perform well. Moreover, one point of great interest is that for MIMO radar application, we can have a good chance to reduce the averaged PSL while maintaining the other performances satisfying.

Chapter 4 Iterative Method of Designing Sparse Frequency Waveform

4.1 Introduction

Sparse frequency waveform is a kind of waveform that has some stopbands sparsely distributed over a wide spectrum band. It is a desirable option for radars operating in such highly congested spectrum regions as HF, VHF, and UHF bands. With sparse frequency waveform, radar systems can avoid spectrum interference from and/or to other users. Meanwhile, combining several clear bands together can also improve the range resolution of range compression systems as the total bandwidth is increased. In addition, sparse frequency waveform can also be used in cognitive radios to optimize the efficiency of spectrum usage. However, it could lead to the problem of high range sidelobes in autocorrelation function (ACF). There are many papers on designing sparse frequency waveforms. Most of the work was reviewed in [68]. In [68], a sparse frequency transmit-receive waveform design was introduced. However, the range sidelobe property of the transmitting waveform is not considered. Mismatch processing in the receiving end would also incur loss of processing gain. Papers [52] and [87] then proposed improved sparse frequency waveform design method using stochastic evolutionary algorithms such as Genetic Algorithm and Particle Swarm Optimization Algorithm. These methods

bring the sidelobe constraint into the transmit waveform design, thus the spectrum performance and the sidelobe performance can be balanced in such a way that the total performance is acceptable. Though these methods are highly flexible in concept, they are not efficient in designing waveforms with large dimension. That is because the convergence of such algorithms will become slower as the number of samples in the waveform increases.

In this chapter, we propose a new iterative algorithm to design sparse frequency transmit waveform with the integrated sidelobe level (ISL) minimized or, more practically, suppressed. This method is derived based on the Cyclic Algorithm (CA) introduced in [34, 35, 88]. A new optimization model as well as the computation model is constructed based on the property of sparse frequency spectrum and requirement on low range sidelobe level. The new penalty function is inherently different from that in [88], thus the cyclic algorithm in [88] is no longer suitable for this problem. Therefore, a new iterative algorithm is proposed for local minimization of the penalty function. Extension to design multiple sparse frequency waveforms is also presented. As the proposed methods are based on solving Toeplitz systems and fast Fourier transform, it is computationally efficient and thereby can be used to design waveforms with large number of samples. The designed waveform can be either phase-amplitude modulated or phase-only modulated. Phase-only modulation possesses constant modulus and is preferable for transmitter to consistently

deliver maximum power.

This chapter is organized as follows. Section 4.2 introduces the signal model, the new optimization method. Section 4.3 presents the extension to multiple sparse frequency waveforms design. Section 4.4 shows several design examples to validate the proposed algorithm. Section 4.5 concludes this chapter.

4.2 Problem Formulation and Iterative Algorithm

4.2.1 Problem Formulation

The waveform designed in this chapter is a set of digital samples of complex waveform. Let the bandwidth of the desired waveform be B , pulse duration be T . Then, based on Nyquist Theorem, there will be $N=T\Delta$ samples, where Δ ($\Delta \geq B$) is the sampling rate. For brevity, the waveform is assumed to be phase-only modulated and it is described in a discrete form by stacking the samples in a vector form as:

$$\mathbf{x}=[x_1; \dots; x_N]=[e^{j\phi_1}; \dots; e^{j\phi_N}] \in C^{N \times 1}, \quad (4.1)$$

where $\phi_n, n=1, 2, \dots, N$, is the phase value of the n -th sample.

The goal for this waveform design problem is to find a proper waveform $\mathbf{x}$ that satisfies both the requirements for power spectrum density (PSD) and ACF. With regard to the PSD requirement, the power distributed in the N_s stopbands should be as low as possible. Thus, we can construct a penalty function P_{PSD} based on the method in [68] such as

$$P_{PSD} = \mathbf{x}^H \mathbf{R} \mathbf{x}$$

$$R_{pq} = \sum_{m=1}^{N_s} w_m \begin{cases} \frac{\exp(2\pi j(p-q)f_{m2}\Delta) - \exp(2\pi j(p-q)f_{m1}\Delta)}{2\pi j(p-q)\Delta}, & p \neq q \\ f_{m2} - f_{m1}, & p = q \end{cases} \quad (4.2)$$

where the superscript H denotes the complex transpose, w_m is the weight factor for the m -th stopband, f_{m1} and f_{m2} are the starting frequency and the ending frequency of the m -th stopband, respectively.

With regard to the ACF requirement, the sidelobe level in the ACF should be as low as possible. Thus, we can construct a penalty function P_{ACF} based on the integrated sidelobe level in the ACF as:

$$P_{ACF} = \sum_{k=1}^{N-1} |r_k|^2 = \mathbf{x}^H \mathbf{G}^H \mathbf{G} \mathbf{x}, \quad \mathbf{G} = \begin{bmatrix} 0 & 0 & \cdots & x_1^* \\ 0 & \cdots & x_1^* & x_2^* \\ \vdots & \vdots & \vdots & \vdots \\ 0 & x_1^* & \cdots & x_{N-1}^* \end{bmatrix} \quad (4.3)$$

with

$$r_k = \sum_{n=k+1}^N x_n x_{n-k}^* = r_{-k}^*, k = 0, \dots, N-1 \quad (4.4)$$

where r_k is the autocorrelation function of waveform $\mathbf{x}$, and $(\bullet)^*$ denotes the complex conjugate.

The problem as mentioned above is to simultaneously minimize the P_{PSD} in (4.2) and P_{ACF} (4.3). Obviously, to minimize these two functions, we can construct a total penalty function as:

$$P_{Total} = \lambda P_{PSD} + (1 - \lambda) P_{ACF} \quad (4.5)$$

where λ , $0 < \lambda < 1$, is a weight coefficient to balance P_{PSD} and P_{ACF} .

4.2.2 Iterative Algorithm

Though the PSD penalty function is a quadratic function of x_n , the ACF penalty function is a quartic function of x_n , which makes the problem difficult to tackle by an iterative procedure proposed in [68]. So, we try to look for an equivalent form of ACF penalty function with lower order. Then, the minimization problem may be solved easily. Papers [88-91] suggest the possibility of solving the problem through an equivalent form.

It is well known that the ACF and the PSD are Fourier transform pair. Particularly, in a discrete form, the PSD can be expressed, for any $\omega \in [0, 2\pi]$, as

$$\Phi(\omega) = \left| \sum_{n=1}^N x_n e^{-j\omega n} \right|^2 = \sum_{k=-(N-1)}^{N-1} r_k e^{-j\omega k} \quad (4.6)$$

We set a group of Fourier frequencies as:

$$\omega_p = \frac{2\pi}{2N} p, \quad p = 1, 2, \dots, 2N \quad (4.7)$$

Then, we obtain a relationship between the PSD and the total energy in the ACF like

$$\begin{aligned} \sum_{p=1}^{2N} \Phi(\omega_p)^2 &= \sum_{p=1}^{2N} \left(\sum_{k=1-N}^{N-1} r_k e^{-jk\omega_p} \right)^2 \\ &= \sum_{k=1-N}^{N-1} \sum_{\tilde{k}=1-N}^{N-1} r_k r_{\tilde{k}}^* \sum_{l=1}^{2N} e^{-j\omega_p(k-\tilde{k})} \\ &= \sum_{k=1-N}^{N-1} \sum_{\tilde{k}=1-N}^{N-1} r_k r_{\tilde{k}}^* 2N \delta(k-\tilde{k}) \\ &= 2N \sum_{k=1-N}^{N-1} |r_k|^2 \end{aligned} \quad (4.8)$$

where

$$\delta_k = \begin{cases} 1, k = 0 \\ 0, k \neq 0 \end{cases} \quad (4.9)$$

$$\begin{aligned} \sum_{p=1}^{2N} e^{-j\omega_p(k-\tilde{k})} &= e^{-j\frac{2\pi}{2N}(k-\tilde{k})} \frac{e^{-j2\pi(k-\tilde{k})} - 1}{e^{-j\frac{2\pi}{2N}(k-\tilde{k})} - 1} \\ &= 2N\delta_{(k-\tilde{k})} \end{aligned} \quad (4.10)$$

From this relationship, we can see that the total energy in the ACF is determined by the squared sum of the PSD. Then, from this relationship we can also easily derive the generalized ISE expression in [88] as

$$\begin{aligned} \sum_{p=1}^{2N} [\Phi(\omega_p) - N]^2 &= 2N \sum_{k=-(N-1)}^{N-1} |(r_k - N\delta_k)|^2 \\ &= 4N \sum_{k=1}^{N-1} |r_k|^2 \end{aligned} \quad (4.11)$$

Therefore, the ISL metric is the same as [88]:

$$\text{ISL} = \frac{1}{4N} \sum_{p=1}^{2N} [\Phi(\omega_p) - N]^2 \quad (4.12)$$

The ISL metric in (4.12) is quartic function of $\{x_n\}$. Following the discussion in [88], the minimization of ISL is equivalent to minimizing a frequency domain metric like:

$$\sum_{p=1}^{2N} \left[\left| \sum_{n=1}^N x_n e^{-j\omega_p n} \right|^2 - N \right]^2 \quad (4.13)$$

Therefore, based on the discussion in [88], we know that this metric is ‘almost equivalent’ to the one in a compact form:

$$\|\mathbf{A}^* \mathbf{z} - \mathbf{v}\|^2 \quad (4.14)$$

where

$$\mathbf{A} = \frac{1}{\sqrt{2N}} [\mathbf{a}_1; \dots; \mathbf{a}_{2N}]_{2N \times 2N} \text{ with } \mathbf{a}_p = [e^{j\omega_p} \dots e^{j2N\omega_p}], \quad p = 1, 2, \dots, 2N$$

$$\mathbf{v} = \frac{1}{\sqrt{2}} [e^{j\phi_1}; \dots; e^{j\phi_{2N}}]_{2N \times 1}$$

$$\mathbf{z} = \begin{bmatrix} \mathbf{x} \\ \mathbf{0} \end{bmatrix}_{2N \times 1}$$

To this point, we can get an almost equivalent form of the penalty function in (4.5) as

$$\begin{aligned} P_{Total} &= \lambda \mathbf{x}^H \mathbf{R} \mathbf{x} + (1 - \lambda) \|\mathbf{A}^* \mathbf{z} - \mathbf{v}\|^2 \text{ with } \mathbf{R}_1 = \begin{bmatrix} \mathbf{R} & \mathbf{R}_{12} \\ \mathbf{R}_{21} & \mathbf{R}_{22} \end{bmatrix}_{2N \times 2N} \\ &= \lambda \mathbf{z}^H \mathbf{R}_1 \mathbf{z} + (1 - \lambda) \|\mathbf{A}^* \mathbf{z} - \mathbf{v}\|^2 \end{aligned} \quad (4.15)$$

where, $\mathbf{R}_{12}$, $\mathbf{R}_{21}$, $\mathbf{R}_{22}$ can be arbitrarily chosen as they do not have any effect on the PSD penalty function at all. However, in all the following discussion, we chose them in such a way that $\mathbf{R}_1$ is Toeplitz matrix with the elements of first row in $\mathbf{R}_{12}$ to be 0. In addition, we may diagonally load $\mathbf{R}_1$ in order to make it to be positive definite.

The new penalty function in (4.15) cannot be directly minimized through the cyclic algorithm proposed in [88] as there is no direct closed form solution of $\mathbf{z}$ for a given $\mathbf{v}$. Thus, we alternatively choose a more general optimization method, iterative algorithm for the minimization of (4.15). That is, we first minimize the total penalty function with respect to $\mathbf{v}$ for fixed $\mathbf{z}$, and then minimize the penalty function with respect to $\mathbf{z}$ for fixed $\mathbf{v}$.

Firstly, for given $\mathbf{z}$, we find the minimization of P_{Total} with respect to $\mathbf{v}$. In this case, the minimization of (4.15) is equivalent to the minimization of the second part of the right hand side of (4.15). The solution is direct. Let

$$\mathbf{f} = \mathbf{A}^* \mathbf{z} \quad (4.16)$$

then

$$\varphi_p = \arg(f_p), \quad p=1, \dots, 2N \quad (4.17)$$

Secondly, for given $\mathbf{v}$, we look for $\mathbf{x}$. Then, we can change the total penalty function as follows:

$$\begin{aligned} P_{Total} &= \lambda \mathbf{z}^H \mathbf{R}_1 \mathbf{z} + (1-\lambda) \|\mathbf{A}^* \mathbf{z} - \mathbf{v}\|^2 \\ &= \lambda \mathbf{z}^H \mathbf{R}_1 \mathbf{z} + (1-\lambda) (\mathbf{z}^H \mathbf{A}^T \mathbf{A}^* \mathbf{z} - \mathbf{z}^H \mathbf{A}^T \mathbf{v} - \mathbf{v}^H \mathbf{A}^* \mathbf{z}) + (1-\lambda) \mathbf{v}^H \mathbf{v} \quad (4.18) \\ &\triangleq \mathbf{z}^H \mathbf{R}_2 \mathbf{z} - (1-\lambda) (\mathbf{z}^H \mathbf{A}^T \mathbf{v} + \mathbf{v}^H \mathbf{A}^* \mathbf{z}) + (1-\lambda) \mathbf{v}^H \mathbf{v} \end{aligned}$$

where $\mathbf{R}_2 = \lambda \mathbf{R}_1 + (1-\lambda) \mathbf{A}^T \mathbf{A}^* = \lambda \mathbf{R}_1 + (1-\lambda) \mathbf{I}$, is a positive definite Toeplitz matrix as $\mathbf{R}_1$ is a positive definite Toeplitz matrix. Now that $\mathbf{R}_2$ is positive definite, we have a transform matrix, unitary matrix $\mathbf{T}$ satisfying

$$\begin{cases} \mathbf{T}^H \mathbf{T} = \mathbf{I} \\ \mathbf{T}^H \mathbf{R}_1 \mathbf{T} = \mathbf{\Lambda} = \text{diag}(e_1, \dots, e_N) \end{cases}$$

Then, the total penalty function can be written as:

$$\begin{aligned} P_{Total} &= \mathbf{z}^H \mathbf{T}^H \mathbf{T} \mathbf{R}_2 \mathbf{T}^H \mathbf{T} \mathbf{z} - (1-\lambda) (\mathbf{z}^H \mathbf{T}^H \mathbf{T} \mathbf{A}^T \mathbf{v} + \mathbf{v}^H \mathbf{A}^* \mathbf{T}^H \mathbf{T} \mathbf{z}) + (1-\lambda) \mathbf{v}^H \mathbf{v} \\ &= (\mathbf{T} \mathbf{z})^H \mathbf{\Lambda} (\mathbf{T} \mathbf{z}) - (1-\lambda) ((\mathbf{T} \mathbf{z})^H \mathbf{T} \mathbf{A}^T \mathbf{v} + \mathbf{v}^H \mathbf{A}^* \mathbf{T}^H \mathbf{T} \mathbf{z}) + 2(1-\lambda) N \\ &\triangleq \mathbf{w}^H \mathbf{\Lambda} \mathbf{w} - (1-\lambda) (\mathbf{w}^H \mathbf{b} + \mathbf{b}^H \mathbf{w}) + 2(1-\lambda) N \quad (4.19) \\ &\triangleq \mathbf{w}^H \mathbf{\Lambda}_1^H \mathbf{\Lambda}_1 \mathbf{w} - (1-\lambda) (\mathbf{w}^H \mathbf{\Lambda}_1^H \mathbf{b}_1 + \mathbf{b}_1^H \mathbf{\Lambda}_1 \mathbf{w}) + 2(1-\lambda) N \\ &= \|\mathbf{\Lambda}_1 \mathbf{w} - (1-\lambda) \mathbf{b}_1\|^2 \\ &= \|\mathbf{\Lambda}_1 \mathbf{T} \mathbf{z} - (1-\lambda) \mathbf{\Lambda}_1^{-1} \mathbf{T} \mathbf{A}^T \mathbf{v}\|^2 \end{aligned}$$

where

$$\mathbf{w} \triangleq \mathbf{T} \mathbf{z}$$

$$\mathbf{b} \triangleq \mathbf{T} \mathbf{A}^T \mathbf{v}$$

$$\Lambda_1^H \mathbf{b}_1 = \mathbf{b}, \text{ i.e., } \mathbf{b}_1 = \Lambda_1^{-1} \mathbf{b}.$$

Through (4.19), we observe that we can only get an exact solution for $\mathbf{z}$ as:

$$\mathbf{z} = (1 - \lambda) \mathbf{T}^H \Lambda_1^{-1} \Lambda_1^{-1} \mathbf{T} \mathbf{A}^T \mathbf{v} = (1 - \lambda) (\mathbf{R}_2)^{-1} \mathbf{A}^T \mathbf{v} \quad (4.20)$$

An alternative approach to get this solution is that the solution can be found when the derivative of (4.15) with respect to $\mathbf{z}$ equals zero. That is, to solve the system as follows:

$$\begin{aligned} \frac{\partial p_{Total}}{\partial \mathbf{z}} &= \lambda 2 \mathbf{R}_1 \mathbf{z} + 2(1 - \lambda) (\mathbf{A}^T \mathbf{A}^* \mathbf{z} - \mathbf{A}^T \mathbf{v}) \\ &= 2(\lambda \mathbf{R}_1 + (1 - \lambda) \mathbf{A}^T \mathbf{A}^*) \mathbf{z} - 2(1 - \lambda) \mathbf{A}^T \mathbf{v} \\ &= \mathbf{0} \end{aligned}$$

The interesting point indicated by (4.20) is that, the PSD constraint leads to an operation of matrix inversion on $\mathbf{R}_1$, which is dedicated to suppress the PSD energy of stopbands. Meanwhile, the solution (4.20) requires a Toeplitz matrix inversion, or solution of simultaneous Toeplitz equations, which can be efficiently computed offline prior to the iteration [68]. However, through (4.20), we can not guarantee that the resulting waveform has constant modulus as $\mathbf{z}$ is indeed phase-amplitude modulated. Thus, through (4.20), we can get a phase-amplitude modulated waveform.

In order to get a solution with constant modulus, we attempt to exploit a different approach, say, the steepest descent method. This method is as follows.

We rewrite the total penalty function as:

$$\begin{aligned}
P_{Total} &= \lambda \mathbf{z}^H \mathbf{R}_1 \mathbf{z} + (1 - \lambda) (\mathbf{z}^H \mathbf{A}^* \mathbf{A} \mathbf{z} - \mathbf{z}^H \mathbf{A}^* \mathbf{v} - \mathbf{v}^H \mathbf{A} \mathbf{z} + \mathbf{v}^H \mathbf{v}) \\
&= \lambda \mathbf{x}^H \mathbf{R} \mathbf{x} + (1 - \lambda) (\mathbf{z}^H \mathbf{A}^* \mathbf{A} \mathbf{z} - \mathbf{z}^H \mathbf{A}^* \mathbf{v} - \mathbf{v}^H \mathbf{A} \mathbf{z} + \mathbf{v}^H \mathbf{v}) \\
&\triangleq \mathbf{x}^H (\lambda \mathbf{R} + (1 - \lambda) \mathbf{A}_1) \mathbf{x} - (1 - \lambda) (\mathbf{x}^H \mathbf{b} + \mathbf{b}^H \mathbf{x} - 2N) \\
&= \mathbf{x}^H \mathbf{R}_3 \mathbf{x} - (1 - \lambda) (\mathbf{x}^H \mathbf{b} + \mathbf{b}^H \mathbf{x} - 2N)
\end{aligned} \tag{4.21}$$

where

$$\begin{aligned}
\mathbf{b}_1 &\triangleq \mathbf{A}^* \mathbf{v} = [b_1; \dots; b_N; \dots; b_{2N}] \\
\mathbf{b} &\triangleq [b_1; \dots; b_N] \\
\mathbf{A}^* \mathbf{A} &\triangleq \begin{bmatrix} \mathbf{A}_1 & \mathbf{A}_0 \\ \mathbf{A}_0^* & \mathbf{A}_2 \end{bmatrix}
\end{aligned}$$

An iterative solution can be found as:

$$\begin{aligned}
\phi_n^{(p+1)} &= \phi_n^{(p)} - \mu \frac{\partial p_{Total}}{\partial \phi_n^{(p)}} \\
\frac{\partial p_{Total}}{\partial \phi_n} &= 2 \operatorname{Im} \left(\exp(-j\phi_n) \sum_{m=1}^N \mathbf{R}_3(n, m) \exp(j\phi_m) \right) \\
&\quad - 2(1 - \lambda) \operatorname{Im}(\exp(-j\phi_n) \mathbf{b}(n))
\end{aligned} \tag{4.22}$$

The p -th iteration of the n -th phase sample is $\phi_n^{(p)}$. Initial phases can be random values between 0 and 2π . We can also choose the solutions in previous steps to accelerate the searching process. Hence, through (4.22), we can get $\mathbf{x}$ with constant modulus that minimize the total penalty function with given $\mathbf{v}$.

Then, based on the phase values obtained from (4.22) the waveform sequence $\{x_n\}$ is given by

$$x_n = e^{j\phi_n} \tag{4.23}$$

For phase-only design method, the iterative algorithm to minimize both the stopbands power in PSD and sidelobe power in ACF can be summarized as follows.

Step 1) Set the waveform $\mathbf{x}$ to be some initial values and construct $\mathbf{R}_3$ with

given λ .

Step 2) With given $\mathbf{x}$, compute $\varphi_p, p=1, \dots, 2N$, and get $\mathbf{v}$.

Step 3) With given $\mathbf{v}$, compute phase values with (4.22).

Step 4) Compute waveform sequence with (4.23)

Step 5) Repeat Steps 2) -4) until a prespecified stop criterion is satisfied.

4.3 Extension to MIMO Radar

As mentioned earlier, MIMO radar is an emerging radar technology. In many literatures, MIMO radar is supposed to transmit multiple independent waveforms on transmit ends so that receiver ends can separate them and thereby achieve more degrees of freedom in signal processing than conventional radar. As MIMO radar would also face the highly congested spectrum environment, it may need sparse frequency waveforms for some applications. In this section, we extend the above method to design multiple sparse frequency transmit waveforms for MIMO radar.

Let M be the waveform number. All waveforms have the same bandwidth B and pulse duration T . Then, for a given sampling rate Δ , there are $N=T\Delta$ samples in each waveform. Thus, the total waveform set can be expressed as:

$$\mathbf{X}=[\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M]^T, \mathbf{x}_m=[e^{j\phi_{m,1}}; \dots; e^{j\phi_{m,N}}], m=1, 2, \dots, M. \quad (4.24)$$

Like that in [35], the correlation penalty function is “almost equivalent” to

$$\begin{aligned}
P_{CF}^{MIMO} &= \|\mathbf{A}^H \tilde{\mathbf{X}} - \mathbf{V}\|^2 \\
\tilde{\mathbf{X}} &= \begin{bmatrix} \mathbf{X} \\ 0 \end{bmatrix}_{2N \times M} = [\mathbf{z}_1 \quad \cdots \quad \mathbf{z}_M] \\
\mathbf{V} &= [\mathbf{a}_1 \cdots \mathbf{a}_{2N}]^T = [\mathbf{v}_1 \quad \cdots \quad \mathbf{v}_M]_{2N \times M}, \|\mathbf{a}_p\|^2 = \frac{1}{2}, p = 1, \dots, 2N
\end{aligned} \tag{4.25}$$

The PSD penalty function is

$$P_{PSD}^{MIMO} = \sum_{m=1}^M \mathbf{z}_m^H \mathbf{R}_1 \mathbf{z}_m \tag{4.26}$$

The total penalty function for designing waveform set with good correlation and PSD property is:

$$P_{Total}^{MIMO} = \lambda P_{PSD}^{MIMO} + (1 - \lambda) P_{CF}^{MIMO} \tag{4.27}$$

To solve this minimization, we also adopt an iterative algorithm. In the first step, for given $\mathbf{X}$, we can find that the minimization of the total penalty function is equivalent to the minimization of the correlation penalty function. We can easily get the solution for $\mathbf{V}$ as [35]

$$\mathbf{a}_p = \frac{1}{\sqrt{2}} \frac{\mathbf{c}_p}{\|\mathbf{c}_p\|}, p = 1, 2, \dots, 2N \tag{4.28}$$

where $\mathbf{c}_p^T$ is the p -th row of $\mathbf{A}^* \tilde{\mathbf{X}}$.

In the second step, for given $\mathbf{V}$, we solve $\mathbf{X}$. Because $\mathbf{V}$ is fixed now we can have

$$P_{CF}^{MIMO} = \|\mathbf{A}^H \tilde{\mathbf{X}} - \mathbf{V}\|^2 = \|\tilde{\mathbf{X}} - \mathbf{A} \mathbf{V}\|^2 = \sum_{m=1}^M \|\mathbf{z}_m - \mathbf{A} \mathbf{v}_m\|^2 \tag{4.29}$$

Then, in this step, the minimization of (4.27) with respect to $\mathbf{z}_m$ can be obtained by solving

$$\begin{aligned}
\frac{\partial p_{Total}^{MIMO}}{\partial \mathbf{z}_m} &= \lambda 2 \mathbf{R}_1 \mathbf{z}_m + 2(1-\lambda)(\mathbf{z}_m - \mathbf{A} \mathbf{v}_m) \\
&= 2(\lambda \mathbf{R}_1 + (1-\lambda) \mathbf{I}) \mathbf{z}_m - 2(1-\lambda) \mathbf{A}^T \mathbf{v}_m \\
&= \mathbf{0}
\end{aligned} \tag{4.30}$$

We can get

$$\mathbf{z}_m = (\lambda \mathbf{R}_1 + (1-\lambda) \mathbf{I})^{-1} (1-\lambda) \mathbf{A}^T \mathbf{v}, \quad m = 1, \dots, M \tag{4.31}$$

This result, as in single waveform case, leads to a set of phase-amplitude jointly modulated waveforms.

For phase-only design method, we look for another approach for the second step. Based on (4.26) and (4.29), we can rewrite the total penalty function for MIMO radar case as:

$$\begin{aligned}
P_{Total}^{MIMO} &= \lambda P_{PSD}^{MIMO} + (1-\lambda) P_{CF}^{MIMO} \\
&= \sum_{m=1}^M \left(\lambda \mathbf{z}_m^H \mathbf{R}_1 \mathbf{z}_m + (1-\lambda) \|\mathbf{z}_m - \mathbf{A} \mathbf{v}_m\|^2 \right)
\end{aligned} \tag{4.32}$$

Thus, in the second step, the problem of minimization of the total penalty function is divided into M independent problems, each of which is corresponding to one single waveform. Then, following the discussion for single waveform case, we can obtain the phase values for each waveform and then construct the waveform $\mathbf{x}_m$ based on the phase-only solution in $\mathbf{z}_m$. Then, the solution for $\mathbf{x}_m$ is:

$$\mathbf{x}_m(n) = \exp(j \phi_{m,n}) \tag{4.33}$$

In summary, for the MIMO radar case, the stopband suppression objective is mainly achieved in the second step while the correlation reduction is achieved through these two steps, which is similar to the correlation reduction in CAN

algorithm in [35].

The algorithm to minimize (4.27) can be summarized as follows.

Step a) Set the waveform $\mathbf{X}$ to be some initial values and construct $\mathbf{R}_3$ with given λ .

Step b) With given $\mathbf{X}$, compute $\mathbf{V}$ using (4.28).

Step c) With given $\mathbf{V}$, compute $\mathbf{X}$ with (33).

Step d) Repeat Steps b) and c) until a prespecified stop criterion is satisfied.

4.4 Designing Examples

4.4.1 Example 1: HF Band Application

This set of simulation parameters is based on the HF channel information from Canadian report [84]. The 6 equivalent baseband clear channels are: 0.11-0.14, 0.26-0.27, 0.29-0.31, 0.37-0.39, 0.46-0.48, 0.51-0.82 MHz. And 5 stopbands cover remaining frequencies in the region of 0.11-0.82 MHz. In practical design, it is desirable to place stopbands on both edges of the spectrum to avoid frequency leakage. However, in the case studies of this chapter, we focus on the performance of the proposed method, so we did not insert stopbands at both ends of the spectrum, which would not affect the discussion. The pulse duration is 200 μs . 143 samples in time domain were obtained with the sampling rate of 710 kHz. We also compare our results with the method in [68]. We set all w_m in (4.2) to be 10 for both methods for all discussion that

follows and $\mu=0.01$.

Firstly, we set $\lambda=0.9$. The ACF and PSD performances are shown in Fig. 4.1. In Fig. 4.1 (b), the frequency region is shifted to the region of 0.0-0.71 MHz. The Peak-to-Integrated-Sidelobe-Ratio (PISR) is 1.6 dB and -2.8 dB for our method and method in [68], respectively. The PSL is -15.7 dB for our method and -9 dB for method in [68]. The average power spectrum level in stopbands is suppressed to be 25.9 dB below that of passbands with our method. The method in [68] suppresses the average power spectrum level in stopbands to -25.8 dB. Therefore, our method can better balance the total performance. Meanwhile, the result of phase-amplitude method using (4.20), as illustrated in Fig. 4.2, can achieve 38 dB stopband-suppression and improved PISR of 3.2 dB. The performance improvement attributes to varying amplitude.

To see the effect of λ in balancing these two requirement, we further set $\lambda=0.5$. The ACF and PSD performances are shown in Fig. 4.3. The PISR is now 3.8 dB for our method, which is better than 1.6 dB in last example due to a larger weight on ISL. The PSL is -15 dB for our method. The average power spectrum level in stopbands is suppressed to be about 13.6 dB below that of passbands with our method, which is not as good as the 25.9 dB suppression in last example due to a smaller weight on PSD. Thus, as the PSD and the ACF are a Fourier transform pair, the better PSD performance always comes at the price of ACF, or vice versa.

(a) ACF comparison.

(b) PSD comparison.

Figure 4.1: Performance comparison, weight coefficient $\lambda=0.9$ (HF band application).

(a) ACF.

(b) PSD.

Figure 4.2: Performance of phase-amplitude method.

(a) ACF comparison.

(b) PSD comparison.

Figure 4.3: Performance comparison, weight coefficient $\lambda=0.5$ (HF band application).

4.4.2 Example 2: UHF Band Application

The simulation scenario is taken from [68]. There is one stopband at 450-475 MHz in 420-580 MHz. The equivalent baseband case is one stopband at 30-55 MHz in 0-160 MHz. The pulse duration is 10 μ s. We take the sampling rate as the total bandwidth. Then, there are 1601 samples. We take $\lambda=0.8$. We take the adaptive method in [68] to compare. We set w_m in (4.2) to be 10 for both methods.

In, Fig. 4.4 we compare the ACF performance of two methods. Fig. 4.4 (b) is the zoomed in of Fig. 4.4 (a). And Fig. 4.5 shows the comparison of PSD. The PISR for the proposed method in this chapter is 8.5 dB, much better than 1.5 dB of the method in [68]. Meanwhile, the PSL for this chapter is -28.2 dB and -24.0 dB for the method in [68]. With regard to the PSD, our method suppresses the average power in stopband to be 29.2 dB below the average power in passbands. The averaged passband-to-stopband power ratio for method in [68] is 26.0 dB. This suppression is not as good as the one showed in [68]. That's because we take an example with relatively low PISR and PSL so that the total performance in terms of both PSD and ACF is balanced. Meanwhile, it can be easily observed from the PSD distribution that the passbands power variation of our PSD is much less than that from [68], which makes our waveform much closer to be white noise, i.e., with lower ISL [88].

(a) Full range index.

(b) Zoom in range index.

Figure 4.4: ACF comparison (UHF band application).

Figure 4.5: PSD comparison (UHF band application).

Based on numerical studies above, we can see that our method is much more flexible in balancing the requirements of PSD and ACF. The comparison with the method proposed in Chapter 3 will be shown in the following example.

4.4.3 MIMO Radar Application

Finally, we investigate a multiple waveforms design case. The simulation scenario is based on the HF channel information from Canadian report [84]. We set $M=3$. We choose $\lambda=0.99$ to suppress stopband heavily. For each waveform, there are 5 stopbands located in 0.03-0.15, 0.16-0.18, 0.20-0.26, 0.28-0.35, 0.37-0.40 MHz. And 6 passbands cover remaining frequencies in the region of 0.00-0.71 MHz. The pulse duration is 200 μ s. 143 samples for each waveform in time domain are obtained with the sampling rate of 710 kHz. We name this numerical study as MIMO-Case 1. The ACF results are illustrated in Fig. 4.6 (a),

Cross-correlation Function (CCF) in Fig. 4.6 (b) and PSD in Fig. 4.7. The PSLs in ACFs are -12.8 dB, -10.8 dB, and -12.4 dB for 3 waveforms, respectively. The average PSL is -14.5 dB. The PISRs are -0.6 dB, -2.0 dB, -1.9 dB. The PSLs in CCFs are -13.2 dB, -13.3 dB, and -14.0 dB, with an average value of -15.9 dB. The average power levels in stopbands are suppressed to be 27.6 dB, 29.3 dB, and 27.8 dB below those of passbands. If we want a waveform set with better ACF and CCF performance, the PSD performance would be sacrificed. For brevity, we did not discuss this part further. Thus, even for multiple waveform cases, our method can also perform well in simultaneous ISL and stopbands suppression.

(a)

(b)

Figure 4.6: (a) ACF and (b) CCF performance of 3 waveforms in MIMO-Case

1.

Figure 4.7: PSD of 3 waveforms in MIMO-Case 1.

Meanwhile, we compare this method with the method proposed in Chapter 3.

We take spectrum parameters from Example 2 in Section 3.5.2. We name this numerical study as MIMO-Case 2. Fig. 4.8 shows the ACF and CCF performances. Fig. 4.9 shows the PSD performance. For ACF, the PSLs of three waveforms are -9.4 dB, -9.3 dB, -8.0 dB, respectively. And the averaged PSL is -9.2 dB. The ISL are 0.15 dB, -0.69 dB, and -0.11 dB, respectively. And the averaged ISL is -0.23 dB. For CCF, the PSLs of three waveforms are -11.3 dB, -13.4 dB, -12.2 dB, respectively. And the averaged PSL is -14.9 dB. The ISL are 0.50 dB, 1.68 dB, and 0.50 dB, respectively. And the averaged ISL is 0.93 dB. The stopbands suppressions for three waveforms are 24.0 dB, 25.1 dB, and 25.0 dB, respectively. For this case, the method proposed in this chapter has better total PSD performance and worse performance of correlations than the method proposed in last chapter. Thus, these two methods seem to be comparable in terms of the overall performance of the waveform set. But the new method in the chapter obtains computation advantage over the method of Chapter 3. Based on our experiment study, these conclusions also apply to the case for single waveform design.

(a)

(b)

Figure 4.8: (a) ACF and (b) CCF performance of 3 waveforms.

Figure 4.9: PSD of 3 waveforms.

4.5 Concluding Remarks

In this chapter, we proposed a new method for designing sparse frequency transmit waveform with good correlation property. The new method is an efficient iterative algorithm that can simultaneously optimize the PSD function with sidelobe constraint. Thus, the PSD performance and the ACF performance can be balanced in the transmit waveform. The proposed algorithm involves both fast Fourier transform and solving Toeplitz systems, thus it is very efficient in computation and memory requirements. Extension to design multiple sparse frequency transmitting waveforms was also exploited. Numerical examples in different application contexts were shown to demonstrate the effectiveness of the proposed method. Compared with the method proposed in Chapter 3, the new method can be used for designing sparse frequency waveforms with large

number of samples. However, this method can not control precisely the peak sidelobe to be below certain level, which is an advantage of the approach proposed in Chapter 3. These two methods together make it more flexible to design sparse frequency waveforms for various conditions.

Chapter 5 Waveform Diversity in MIMO STAP

Radar

5.1 Introduction

In the recent years MIMO radar has been shown with some advantages over conventional radar due to its ability of transmitting multiple correlated or uncorrelated waveforms via its antennas (see, for example, [12-22] and references therein). More recently, studies of MIMO radar have been extended to space-time adaptive processing (STAP) for moving target indication (MTI) [14, 23-28]. STAP technique can effectively suppress the ground clutter of inherent two-dimensional nature and therefore it is an effective technique for target detection in a combined clutter and jamming environment [92-99]. By applying STAP in a MIMO radar system, one can potentially increase the STAP ability to discriminate clutter as well as improve the target detection and estimation performances. Paper [14] firstly analyzed the performance of MIMO radar with application to STAP in terms of degrees of freedom and resolution. Paper [23] proposed a subspace method for MIMO STAP processing by using spherical wave functions. The clutter rank of MIMO STAP was studied in [23-25]. The proposed clutter model is data independent while is limited to the assumption of ideal orthogonal transmitting waveforms. Papers [26] and [27] studied the multipath clutter mitigation by slow-time STAP in MIMO radar.

This proposed slow-time MIMO technique can judiciously allocate the available degrees of freedom between Doppler and transmit beam pattern synthesis. However, the total degree of freedom is the same as that of conventional STAP. Paper [28] investigated the GMTI application of MIMO radar. In these studies, the waveforms transmitted by separate transmitters are assumed to be orthogonal with each other. Thus it is possible for receiver to separate the signals from different transmitting antennas and to utilize full degrees of freedom.

From a general point of view, one distinct property of MIMO radar is waveform diversity. In many applications, more general waveforms, i.e., correlated waveforms may be occupied by MIMO radar other than orthogonal waveforms [36-42, 103]. For example, by using partially correlated waveforms, the beam-shape of transmitting waveform can be optimized according to different purpose of application. Many papers are related to design these kinds of waveforms for MIMO radar based on different criteria [40-42]. To the best of our knowledge, when it comes to the partially correlated waveforms, the performance of MIMO STAP radar is not analyzed yet. Problems such as the optimum filter design, clutter subspace characteristics, and system sensitivity, etc., when considering waveform diversity are still open to be investigated now. Further more, to analyze the performance of MIMO STAP with waveform diversity can provide us more insight into MIMO radar, and can show both the

advantages and the disadvantages of certain waveforms and thus can guide us to select proper waveforms for the application of MIMO radar in certain practical operation, and can show us how the radar performance is changing from the conventional coherent waveforms to orthogonal waveforms. In a word, the distinctive role that waveform diversity plays in MIMO STAP radar may be worthy of exploration. Therefore, this chapter will focus on the evaluation of the performance of MIMO STAP systems with waveform diversity. Through this work, we want to show that what and how much price we should pay for the advantages of certain waveforms, and how should we deal with the waveform-induced performance degradation.

This study develops a unified framework for STAP in MIMO radar with waveform diversity. Several key performance metrics related to MIMO STAP radars are analyzed according to waveform diversity. We start by constructing a signal model based on waveform diversity, i.e., transmitted waveforms are not constrained to be orthogonal. Performance metrics are then formulated and expressed as functions of the waveforms. With that, waveform effects on STAP performance can be studied from several perspectives. Firstly, this chapter studies the sensitivity, i.e., the maximum range detectable, a metric of great importance for radar applications. Though some papers pointed out that the MIMO radar will suffer from loss of transmitting power compared with single-input multiple-output (SIMO) radar [23], this chapter provides a general

approach to evaluate the sensitivity of MIMO STAP radar with waveform diversity. It is achieved by investigating the effective transmitting-and-receiving (TR) gain and the processing gain according to different waveforms. The sensitivity of MIMO STAP radar with waveform diversity is shown to be determined by eigenvalue distribution of the waveform covariance matrix (WCM), or waveform correlation matrix called in some papers (for example[40-42]). Along with it, we also compare the sensitivity of MIMO STAP applying orthogonal waveforms with conventional SIMO STAP. Secondly, waveform diversity, like geometry configuration, impacts the clutter subspace observed by MIMO radar. Clutter rank is shown to be determined by the rank and structure of WCM. Several rules to predict the clutter rank of MIMO STAP radar with waveform diversity are developed. Thirdly, the optimum space-time filter is shown to be determined not only by the array manifold but also by the WCM, which is unlike the conventional STAP or STAP in MIMO radar with orthogonal waveforms. Waveform effect on the optimum adapted pattern is studied through some numerical simulations together with theoretical analysis.

This chapter is organized as follows. The configuration and signal model is presented in Section 5.2. In Section 5.3, the waveform effect on sensitivity of MIMO radar is analyzed both theoretically and by simulations. Section 5.4 investigates the waveform effects on the clutter subspace discrimination.

Section 5.5 studies the waveform effect on the optimum space-time filter. And section 5.6 finally concludes this chapter.

5.2 System Configuration and Signal Model

Figure 5.1: MIMO STAP system geometry.

The MIMO radar system geometry is shown in Fig. 5.1. In this chapter we use the same conventions as in [92]. The radar platform travels at velocity v_P in the positive direction of the x axis. The altitude of the array phase center is h over the x - y plane. The surface of the earth is assumed to be flat for the interested area of observation. There are N omnidirectional transmitting elements uniformly spaced at distance d_{Tx} , and M receiving elements uniformly spaced at distance d_{Rx} . As illustrated in Fig.5.1, the transmitting antennas are

collocated with the receiving antennas meaning that this system has a monostatic MIMO radar configuration. Thus, all the elements in the transmitting and receiving arrays view the target with the same azimuth and elevation angles. The elevation angle φ and the azimuth angle θ are illustrated in Fig. 5.1. At each transmitter, a coherent processing interval (CPI) consists of a sequence of L waveforms. Let $\mathbf{s}_n \in \mathbb{C}^{N_s}$ be the discrete baseband waveform transmitted at the n -th transmitting element in each pulse repetition interval (PRI). Then, in each PRI, the transmitting waveform set is denoted by $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \dots, \mathbf{s}_N]' \in \mathbb{C}^{N \times N_s}$, where the prime means matrix transpose. We also define the covariance matrix of transmitting waveforms as $\mathbf{R}_s = \mathbf{S}\mathbf{S}^H/N_s \in \mathbb{C}^{N \times N}$, where the superscript H means the conjugate transpose. Orthogonal waveforms are defined as those with $\mathbf{R}_s = \mathbf{I}_N$, where $\mathbf{I}_N$ is an identity matrix of size N , coherent waveforms defined as those with $\mathbf{R}_s = \mathbf{1}_{N \times N}$, where $\mathbf{1}_{N \times N}$ is an N -dimension matrix with each element equal to one, while waveforms other than orthogonal waveforms and coherent waveforms defined as partially correlated waveforms. We also use rank-1 waveform matrix to denote a waveform matrix whose rank is 1. We assume that the transmitting waveforms meet the narrowband assumption. Thus the Doppler Effect within the waveforms can be neglected. The pulse repetition interval is T_r and the waveform duration is T_p .

After down-conversion, the echo for the l -th PRI at the m -th receiver from

possible moving target and all clutter patches at the range ring of interest are:

$$\begin{aligned}
\mathbf{y}_m &= \sum_{n=1}^N \gamma \mathbf{s}_n e^{j \frac{2\pi}{\lambda_o} \sin(\theta_t) \cos(\varphi_t) ((n-1)d_{Tx} + (m-1)d_{Rx})} e^{j \frac{2\pi}{\lambda_o} (2v_t T_r (l-1))} + \\
&\quad \int_{\theta=0}^{2\pi} \sum_{n=1}^N \xi(\theta) \mathbf{s}_n e^{j \frac{2\pi}{\lambda_o} \sin(\theta) \cos(\varphi) ((n-1)d_{Tx} + (m-1)d_{Rx})} e^{j \frac{2\pi}{\lambda_o} (2v_p T_r (l-1) \sin(\theta) \cos(\varphi))} d\theta \\
&= \sum_{n=1}^N \gamma \mathbf{s}_n e^{j 2\pi f_{s,t} ((n-1)\alpha + (m-1))} e^{j 2\pi f_{d,t} (l-1)} \\
&\quad + \int_{\theta=0}^{2\pi} \sum_{n=1}^N \xi(\theta) \mathbf{s}_n e^{j 2\pi f_{s,c} ((n-1)\alpha + (m-1))} e^{j 2\pi f_{d,c} (l-1)} d\theta
\end{aligned} \tag{5.1}$$

where λ_o is the wavelength corresponding to the carrier frequency; γ is the target reflection coefficient; θ_t is the target azimuth angle and φ_t the target elevation angle; v_t is the relative velocity between the target and the platform; $f_{s,t} = d_{Rx} \sin(\theta_t) \cos(\varphi_t) / \lambda_o$ and $f_{d,t} = (2v_t T_r) / \lambda_o$ are normalized target spatial and temporal frequencies, respectively; $\alpha = d_{Tx} / d_{Rx}$; $f_{s,c} = d_{Rx} \sin(\theta) \cos(\varphi) / \lambda_o$ and $f_{d,c} = \sin(\theta) \cos(\varphi) (2v_p T_r) / \lambda_o$ are normalized spatial and temporal frequencies associated with clutter of azimuth θ and elevation φ , respectively; and $\xi(\theta)$ is the reflection coefficient of clutter associated with angle θ . We also set $\beta = 2v_p T_r / d_{Rx}$.

We can define a general transmitting steering vector and a receiving steering vector by:

$$\mathbf{v}_{Tx}(\theta) = [1; e^{j 2\pi f_{s,t} \alpha}; \dots; e^{j 2\pi f_{s,t} \alpha (N-1)}] \in C^{N \times 1} \tag{5.2}$$

$$\mathbf{v}_{Rx}(\theta) = [1; e^{j 2\pi f_{s,c} \alpha}; \dots; e^{j 2\pi f_{s,c} \alpha (M-1)}] \in C^{M \times 1} \tag{5.3}$$

Define a Doppler vector as

$$\mathbf{u}(\theta) = [1; e^{j 2\pi f_{d,t}}; \dots; e^{j 2\pi (L-1) f_{d,t}}] \in C^{L \times 1} \tag{5.4}$$

To obtain the sufficient statistics for STAP signal processing, we can employ $\Phi = \mathbf{S}^H / N_s$ as the filter bank in the receivers. Thus the target echoes, the jamming signals and the clutter echoes are compressed by Φ after filtering. Then the stacked data of target from angle θ_l is

$$\boldsymbol{\chi}_l = \mathbf{u}(\theta_l) \otimes \text{stack}(\gamma \mathbf{v}_{Rx}(\theta_l) \mathbf{v}_{Tx}'(\theta_l) (\mathbf{R}_s)) = \gamma \mathbf{v}_l \in C^{NML \times 1} \quad (5.5)$$

where $\mathbf{v}_l$ is the target space-time steering vector, $\otimes$ stands for the Kronecker Product. Through (5.5) the two-dimensional space-time sampling data are mapped into an effective one-dimensional data sequence.

If the iso-range ring is divided in the cross-range dimension into N_C ($N_C \gg NML$) clutter patches, then the discrete form of the clutter data for the l -th PRI is

$$\mathbf{x}_l = \text{stack} \left(\sum_{i=1}^{N_C} \xi(\theta_{c,i}) e^{j2\pi(l-1)f_{c,i}} \mathbf{v}_{Rx}(\theta_{c,i}) \mathbf{v}_{Tx}^T(\theta_{c,i}) \mathbf{R}_s \right) \in C^{NM \times 1} \quad (5.6)$$

where $\theta_{c,i}$ is the azimuth of the i -th clutter patch; $\mathbf{v}_{Rx}(\theta_{c,i})$ and $\mathbf{v}_{Tx}(\theta_{c,i})$ are obtained from (5.2) and (5.3) by substituting f_s with $f_{s,c}$, respectively; $\xi(\theta_{c,i})$ can be modeled as a zero-mean independent complex Gaussian random variable with the variance σ_i^2 . In the content following, we will write $\xi(\theta_{c,i})$ as ξ_i . The clutter component of the space-time snapshot is then given by

$$\boldsymbol{\chi}_c = [\mathbf{x}_0; \mathbf{x}_1; \dots; \mathbf{x}_{L-1}] \quad (5.7)$$

The i -th clutter echoes from $\theta_{c,i}$ can be stacked in the form

$$\begin{aligned} \boldsymbol{\chi}_{c,i} &= [\mathbf{x}_{0,i}; \mathbf{x}_{1,i}; \dots; \mathbf{x}_{L-1,i}] = \xi_i \mathbf{u}(\theta_{c,i}) \otimes \text{stack}(\mathbf{v}_{Rx}(\theta_{c,i}) \mathbf{v}_{Tx}'(\theta_{c,i}) \mathbf{R}_s) \\ &= \xi_i \mathbf{u}(\theta_{c,i}) \otimes ((\mathbf{R}_s)' \otimes \mathbf{v}_{Rx}(\theta_{c,i})) \mathbf{v}_{Tx}(\theta_{c,i}) \triangleq \xi_i \mathbf{v}_i \in C^{NML \times 1} \end{aligned} \quad (5.8)$$

Then we define

$$\mathbf{V}_C = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_{N_c}] \quad (5.9)$$

$$\mathbf{\Xi} = \text{diag}(\sigma_1^2, \sigma_2^2, \dots, \sigma_{N_c}^2) \quad (5.10)$$

The clutter covariance matrix (CCM) can be written as

$$\mathbf{R}_C = E(\boldsymbol{\chi}_C \boldsymbol{\chi}_C^H) = \mathbf{V}_C \mathbf{\Xi} \mathbf{V}_C^H \quad (5.11)$$

where $E[\cdot]$ is the expected value operator.

For the noise component, we assume the noise snapshot, $\boldsymbol{\chi}_n$, are both temporally and spatially de-correlated after compression, then the noise covariance matrix is

$$\mathbf{R}_n = E[\boldsymbol{\chi}_n \boldsymbol{\chi}_n^H] = \sigma^2 \mathbf{I}_{NML} \quad (5.12)$$

where σ^2 is the noise power at each virtual receive element.

For a jamming signal at θ_j, φ_j , we assume its temporal behavior as thermal noise, but with the characteristics of a point target in the spatial domain. We define $f_{s,j} = d_{Rx} \sin(\theta_j) \cos(\varphi_j) / \lambda_o$ as the normalized spatial frequency. The spatial steering vector associated with the jammer is

$$\mathbf{v}_{Rx,j} = [1; e^{j2\pi f_{s,j}}; \dots; e^{j2\pi f_{s,j}(M-1)}] \in C^{M \times 1} \quad (5.13)$$

The jammer samples at the l -th pulse interval have the form of $\mathbf{j}_l \in C^{N_s}$. Based on the assumption that the jammer is uncorrelated temporally, we can have that $E[\mathbf{j}_l (\mathbf{j}_{l+\Delta})^H] = \varsigma \sigma^2 \delta(\Delta) \mathbf{I}$, where ς is the jamming-to-noise ratio and $\delta(\Delta)$ means the value of Dirac delta function at Δ . Therefore, after filtering, there would be NM jamming outputs for PRI. The consequent jamming component of spatial

snapshot is in the form of

$$\mathbf{x}_{j,l} = \text{stack} \left(\mathbf{v}_{Rx,j}(\theta_j, \varphi_j) \mathbf{j}_l ((l-1)T_r + \tau)^T \boldsymbol{\Phi} \right) \quad (5.14)$$

Then the jammer space-time snapshot is

$$\boldsymbol{\chi}_j = [\mathbf{x}_1; \mathbf{x}_2; \cdots; \mathbf{x}_L] \quad (5.15)$$

For each pulse, the jamming outputs covariance matrix can be

$$\begin{aligned} & E[\mathbf{x}_l \mathbf{x}_l^H] \\ &= (\boldsymbol{\Phi}^T \otimes \mathbf{v}_{Rx,j}(\theta_j, \varphi_j)) E[\mathbf{j}_l ((l-1)T_r + \tau) \mathbf{j}_l ((l-1)T_r + \tau)^H] (\boldsymbol{\Phi} \otimes \mathbf{v}_{Rx,j}(\theta_j, \varphi_j)^T) \\ &= (\boldsymbol{\Phi}^T \otimes \mathbf{v}_{Rx,j}(\theta_j, \varphi_j)) \mathbf{I} \zeta \sigma^2 (\boldsymbol{\Phi} \otimes \mathbf{v}_{Rx,j}(\theta_j, \varphi_j)^T) \\ &= \zeta \sigma^2 (\boldsymbol{\Phi}^T \boldsymbol{\Phi} \otimes \mathbf{v}_{Rx,j}(\theta_j, \varphi_j) \mathbf{v}_{Rx,j}(\theta_j, \varphi_j)^T) \\ &\triangleq \zeta \sigma^2 \mathbf{R}_j \end{aligned} \quad (5.16)$$

If we redefine $\boldsymbol{\Phi} = \mathbf{S}^H(\mathbf{R}_S)^{-1/2}/N_s$ in (5.16), it is clear that that correlation property of jamming signal will not be affected by the waveform covariance matrix according to the filter bank definition [36]. Because the jamming is temporally uncorrelated between pulses, thus

$$E[\mathbf{x}_{l_1} \mathbf{x}_{l_2}^H] = \mathbf{0} \quad (5.17)$$

Thus, the covariance matrix of jamming is

$$\mathbf{R}_j = E[\boldsymbol{\chi}_j \boldsymbol{\chi}_j^H] = \zeta \sigma^2 \mathbf{I}_L \otimes \mathbf{R}_j \quad (5.18)$$

Thus, the total space-time snapshot including target, clutter, jamming and noise can be

$$\begin{aligned} \boldsymbol{\chi} &= \boldsymbol{\chi}_t + \boldsymbol{\chi}_j + \boldsymbol{\chi}_c + \mathbf{n} \\ &\triangleq \boldsymbol{\chi}_t + \boldsymbol{\chi}_u \end{aligned} \quad (5.19)$$

where χ_u , the undesired component, consists of clutter, jamming, and noise. For the problem of detecting the target in a complex environment with clutter, jamming, and noise, the optimum space-time filter under the criterion of maximum Signal to Interference plus Noise Ratio (SINR) is found from [92]:

$$\max \frac{|\mathbf{w}^H \mathbf{v}_t|^2}{\mathbf{w}^H \mathbf{R}_u \mathbf{w}} \text{ subject to } \mathbf{w}^H \mathbf{w} = 1 \quad (5.20)$$

By solving (5.20), we get

$$\mathbf{w} = \kappa \mathbf{R}_u^{-1} \mathbf{v}_t \quad (5.21)$$

with

$$\mathbf{R}_u = E \{ \chi_u \chi_u^H \} \quad (5.22)$$

A useful equivalent representation of (5.21) is always expressed in eigen form.

We derive it based on the discussion in [100] as following.

We start by considering the eigen-decomposition of covariance matrix $\mathbf{R}_u$. It is

$$\mathbf{R}_u = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^H = \sum_{k=1}^{NML} \lambda_k \mathbf{u}_k \mathbf{u}_k^H \quad (5.23)$$

where

$$\mathbf{U} = [\mathbf{u}_1 \quad \cdots \quad \mathbf{u}_{NML}] \in C^{NML \times NML} \quad (5.24)$$

is the $NML \times NML$ matrix whose columns are the orthonormal eigenvectors of

$\mathbf{R}_u$, and

$$\mathbf{\Lambda} = \text{diag} \{ \lambda_1 \quad \cdots \quad \lambda_{NML} \} \in R^{NML \times NML} \quad (5.25)$$

is the $NML \times NML$ diagonal matrix of corresponding strictly positive

eigenvalues.

From this matrix decomposition, we have

$$\mathbf{R}_u^{-1} = \mathbf{U}\mathbf{\Lambda}^{-1}\mathbf{U}^H = \sum_{k=1}^{NML} \frac{\mathbf{u}_k \mathbf{u}_k^H}{\lambda_k} \quad (5.26)$$

Submitting (5.26) into (5.21) and rearranging terms yields:

$$\begin{aligned} \mathbf{w} &= \kappa \mathbf{R}_u^{-1} \mathbf{v}_t = \tilde{\kappa} \lambda_{\min} \mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^H \mathbf{v}_t \\ &= \tilde{\kappa} \left(\mathbf{v}_t - \mathbf{v}_t + \lambda_{\min} \mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^H \mathbf{v}_t \right) \\ &= \tilde{\kappa} \left[\mathbf{v}_t - \left(\mathbf{v}_t - \lambda_{\min} \mathbf{U} \mathbf{\Lambda}^{-1} \mathbf{U}^H \mathbf{v}_t \right) \right] \quad \text{with } \tilde{\kappa} = \kappa / \lambda_{\min} \\ &= \tilde{\kappa} \left[\mathbf{v}_t - \mathbf{U} \left(\mathbf{I} - \lambda_{\min} \mathbf{\Lambda}^{-1} \right) \mathbf{U}^H \mathbf{v}_t \right] \end{aligned} \quad (5.27)$$

Finally, the eigen form of the optimum filter is

$$\begin{aligned} \mathbf{w} &= \tilde{\kappa} \left[\mathbf{v}_t - \sum_{k=1}^{NML} \left(1 - \frac{\lambda_{\min}}{\lambda_k} \right) \left(\mathbf{u}_k^H \mathbf{v}_t \right) \mathbf{u}_k \right] \\ &= \tilde{\kappa} \left[\mathbf{v}_t - \sum_{k=1}^{NML} \left(\frac{\lambda_k - \lambda_{\min}}{\lambda_k} \right) \left(\mathbf{u}_k^H \mathbf{v}_t \right) \mathbf{u}_k \right] \end{aligned} \quad (5.28)$$

Equation (5.28) shows that the optimum filter is formed by subtracting a weighted sum of the interference eigenvectors from the target steering vector and that only the eigenvectors with corresponding eigenvalues greater than the minimum one may contribute to the sum. The exact amount they contribute is proportional to the difference between their eigenvalues and the minimum eigenvalue, as well as the amount of colinearity relationship between their eigenvectors and the target steering vector. That is the underlying support for some reduced-rank methods of STAP.

Based on the above signal model, it can be seen that unlike conventional SIMO radar, the space-time snapshots of target signal, clutter signal are both

affected not only by the array manifold, but by the waveforms, i. e., covariance matrix of waveforms. MIMO radars with both orthogonal waveforms and coherent waveforms can be taken as a special form of the MIMO radar with waveform diversity. More importantly, many practical waveforms for orthogonal MIMO radar are not ideally orthogonal at all. Through the formulas derived above, the effects of non-ideal orthogonal waveforms can also be studied.

5.3 Sensitivity

The sensitivity of the radar is of great importance because it will determine the functional range of the radar. Radar sensitivity is conventionally evaluated by the radar equation [1]. We here rewrite the basic free-space radar equation without considering pattern propagation factor as:

$$R_{\max}^4 = \frac{E_t G_T G_R \sigma_t \lambda_o^2}{(4\pi)^3 k T_o F_n (E / N_0)} = \frac{E_t G_{TR} \sigma_t \lambda_o^2}{(4\pi)^3 k T_o F_n (E / N_0)} \quad (5.29)$$

where $R_{\max}$ is the maximum detection range for a given target, G_{TR} is the total transmitting-receiving gain, E_t is the total energy contained in one waveform, σ_t is the target cross-section, λ_o is the wavelength of radar central frequency, k is Boltzmann's constant, T_o is a reference temperature, F_n is the receiver noise figure, E signal energy, and N_0 white noise energy. From (5.29), we can see that the sensitivity of radar is proportional to the transmitting-receiving gain given other factors fixed. Meanwhile, in STAP radar, the maximum detection range is

proportional to the gain of spatial and temporal coherent integration over all elements and all pulses [92]. The gain of spatial coherent integration is just the G_{TR} from the transmitting-receiving beamforming. The gain of temporal coherent integration over all pulses is determined by the number of pulses. Obviously, the temporal coherent integration gain would be same for different kind of waveforms. Thus, we will focus on the gain from spatial coherent integration in order to investigate the sensitivity.

Therefore, in this section we will study the sensitivity of MIMO STAP radar with diverse waveforms from the perspective of waveform effects on TR gain. We also compare the sensitivity of MIMO STAP and that of conventional SIMO STAP.

5.3.1 Waveform Effect on TR Gain

For a given system with waveform diversity, all parameters are fixed other than waveforms. Thus, the G_{TR} will be varying according to different waveform sets. The following will show this consideration. For MIMO radar, the TR gain can be written as [18]:

$$\begin{aligned}
 G_{TR}(\theta) &= \frac{\left| \text{stack} \left(\mathbf{v}_{Rx}(\theta) \mathbf{v}_{Tx}^T(\theta) \mathbf{R}_S^{\frac{1}{2}} \right)^H \text{stack} \left(\mathbf{v}_{Rx}(\theta_t) \mathbf{v}_{Tx}^T(\theta_t) (\mathbf{R}_S)^{\frac{1}{2}} \right) \right|^2}{\left\| \text{stack} \left(\mathbf{v}_{Rx}(\theta) \mathbf{v}_{Tx}^T(\theta) (\mathbf{R}_S)^{\frac{1}{2}} \right)^H \right\|^2} \quad (5.30) \\
 &= \frac{|\mathbf{v}_{Tx}^H(\theta) \mathbf{R}_S \mathbf{v}_{Tx}(\theta_t)|^2 |\mathbf{v}_{Rx}^H(\theta) \mathbf{v}_{Rx}(\theta_t)|^2}{\mathbf{v}_{Tx}^H(\theta) \mathbf{R}_S \mathbf{v}_{Tx}(\theta)}
 \end{aligned}$$

where $|x|^2$ is the magnitude of the scalar x , and $\|\mathbf{x}\|^2$ is the 2-norms of vector $\mathbf{x}$.

From (5.30) we can see that the transmitting-receiving gain of MIMO radar is the product of transmitting gain and the receiving gain. For STAP radar systems with given antenna configuration, the G_{TR} can be optimized with respect to $\mathbf{R}_S$. Because $\mathbf{R}_S$ is positive semi-definite, it can be decomposed by single value decomposition as $\mathbf{R}_S = \mathbf{U}^H \mathbf{\Lambda} \mathbf{U}$. Then we can have

$$\begin{aligned}
G_{TR}(\theta) &= \frac{\left| \mathbf{v}_{Tx}^H(\theta) \mathbf{R}_S \mathbf{v}_{Tx}(\theta_t) \right|^2 \left| \mathbf{v}_{Rx}^H(\theta) \mathbf{v}_{Rx}(\theta_t) \right|^2}{\mathbf{v}_{Tx}^H(\theta) \mathbf{R}_S \mathbf{v}_{Tx}(\theta)} \\
&= \frac{\left| \mathbf{v}_{Tx}^H(\theta) \mathbf{U}^H \mathbf{\Lambda} \mathbf{U} \mathbf{v}_{Tx}(\theta_t) \right|^2 \left| \mathbf{v}_{Rx}^H(\theta) \mathbf{v}_{Rx}(\theta_t) \right|^2}{\mathbf{v}_{Tx}^H(\theta) \mathbf{R}_S \mathbf{v}_{Tx}(\theta)} \\
&\leq \frac{\left\| \mathbf{v}_{Tx}^H(\theta) \mathbf{U}^H \mathbf{\Lambda} \right\|^2 \left\| \mathbf{U} \mathbf{v}_{Tx}(\theta_t) \right\|^2 \left| \mathbf{v}_{Rx}^H(\theta) \mathbf{v}_{Rx}(\theta_t) \right|^2}{\mathbf{v}_{Tx}^H(\theta) \mathbf{R}_S \mathbf{v}_{Tx}(\theta)} \\
&= \mathbf{v}_{Tx}^H(\theta_t) \mathbf{R}_S \mathbf{v}_{Tx}(\theta_t) \left| \mathbf{v}_{Rx}^H(\theta) \mathbf{v}_{Rx}(\theta_t) \right|^2
\end{aligned} \tag{5.31}$$

where inequality comes from rule of Cauchy–Schwarz inequality. As the transmit steering vector has constant Euclidean norm, by applying property of Rayleigh quotient we can get

$$\mathbf{v}_{Tx}^H(\theta_t) \mathbf{R}_S \mathbf{v}_{Tx}(\theta_t) \leq \lambda_{\max}(\mathbf{R}_S) \tag{5.32}$$

From (5.31) and (5.32), we can see that the maximum TR gain is proportional to the largest eigenvalue of WCM. Thus, waveforms that obtain the largest eigenvalue will have the maximum TR gain. The maximum value is obtained in the case of coherent waveforms, i. e., where the total energy of $\mathbf{R}_S$ is concentrated in one eigenvalue. In other words, in the case of coherent waveforms, MIMO radar reduces to conventional phased array radar that is

optimal in terms of TR gain. While for orthogonal signals, the G_{TR} will be decreased by a factor of $10\log N$, because all eigenvalues are the same. Thus, the TR gain of orthogonal waveform is only $1/N$ of coherent waveforms. Meanwhile, the TR gain of partially correlated waveforms may be better than that of orthogonal waveform but worse than that of coherent waveform. Fig. 5.2 illustrates the above analysis, where we have 10 antenna elements as both transmitters and receivers spaced by half wavelength. Target azimuth is zero degree. Coherent waveforms, orthogonal waveforms and partially correlated waveforms are used in this example. The partially correlated waveforms are designed by the method of *Minimum Sidelobe Design* described in [13]. The mainbeam is centered at 0 degree, and 3dB mainbeam area is set to $[-12, 12]$ degrees.

However, as the transmitting gain of the orthogonal waveform is uniform over all azimuth angles, the maximum TR gain of orthogonal waveforms will keep the same no matter what the target azimuth is, which is illustrated in Fig. 5.3 (a). For coherent or partially correlated waveform, the maximum TR gain will be affected by the difference between the azimuth of transmitting beam and target azimuth, which is illustrated in Fig. 5.3 (b). In this way, though orthogonal waveforms can provide much more degrees of freedom than correlated waveforms, it will have lower TR gain. Meanwhile, though orthogonal waveform set have lower TR gain, it has better ubiquitous searching

performance than other waveform sets because we only need to do the digital beam-forming at the receiver end to cover all azimuth regions of interest in orthogonal waveform case.

(a) Transmitting gain.

(b) TR Gain.

Figure 5.2: Patterns for different waveforms.

(a) Orthogonal waveforms.

(b) Partially correlated waveforms.

Figure 5.3: Patterns when azimuth of target is different from that of transmitting and receiving.

5.3.2 Waveform Effect on SINR

In this section, we will study the waveform effects on the optimal SINR.

The optimal SINR is defined as:

$$SINR = \frac{|\mathbf{w}^H \mathbf{v}_t|^2}{\mathbf{w}^H \mathbf{R}_u \mathbf{w}} \quad (5.33)$$

From (5.33), the waveform effect can be investigated by holding the target angle fixed and varying the target Doppler frequency. Meanwhile, by definition, the input SNR (the SNR at a single receive element due to a single transmitting pulse from a single transmit element) is taken to be 0 dB. Three kinds of waveforms are considered here, i.e., ideally orthogonal waveform, coherent waveform, and partially correlated waveform designed by the method as mentioned in the previous subsection. The system configuration is like that $N=M=5$, $L=10$, θ_t is zero, $d_{Rx}=\lambda_o/2$, $\alpha=5$, $v_P=100$ m/s, $v_t=70$ m/s, $\beta=1$, $f_o=500$ MHz the clutter-to-noise ratio is 30 dB and the jamming-to-noise ratio is 30 dB. Two jamming signals come from the directions of -40 degree and 30 degree, respectively. This configuration will be employed throughout the following content unless there are specific notations. The real value of SINR for MIMO radar with orthogonal waveforms is about $10\log_{10}NML=24.0$. The gain of NML represents coherent spatial and temporal integration over NM virtual elements and L pulses. While this metric has a larger value for coherent and partially coherent waveforms because there are larger TR gains for these two cases. To roughly compare the detectability of minimum velocity, we simply normalize

these three SINRs in Fig. 5.4. From Fig. 5.4, the orthogonal waveform set has the best velocity detectability as it has the narrowest clutter notch, while the coherent waveform set has the worst.

Based on abovementioned studies, we can conclude that sensitivity of MIMO radar with waveform diversity is proportional to the maximum eigenvalue of the WCM. Specifically, the sensitivity of orthogonal waveform is only $1/N$ of the sensitivity of coherent waveform; the sensitivity of partially correlated waveforms may be better than that of orthogonal waveform but worse than that of coherent waveform. Thus MIMO radar may sacrifices the sensitivity to better velocity detectability and better spatial resolution capability owing to more degrees of freedom.

Figure 5.4: Illustration of detectability of minimum velocity of different waveforms ($N=M=5$, $L=10$).

We can also compare the sensitivity of MIMO STAP with orthogonal waveform and conventional SIMO STAP. As for the conventional STAP, beamforming is conducted during transmitting. While for MIMO STAP with orthogonal waveform, “beamforming” will be performed after receiving. Thus, if SIMO STAP has the same number of transmitters as that of MIMO STAP, i.e., the same total transmit power, then SNR_{in} of SIMO STAP is $N \times N$ times of SNR_{in} of MIMO radar. If the pulse number is L for MIMO radar and SIMO radar, then the comparison of sensitivity can be illustrated by the ratio between the output SINRs of MIMO radar and SIMO radar like that

$$\frac{\text{SINR}_{\text{opt}}^{\text{MIMO}}}{\text{SINR}_{\text{opt}}^{\text{SIMO}}} = \frac{\text{SNR}_{\text{in}}^{\text{MIMO}}}{\text{SNR}_{\text{in}}^{\text{SIMO}}} \frac{\text{SINR}_{\text{MIMO}}}{\text{SINR}_{\text{SIMO}}} = \frac{1}{N^2} \frac{NML}{NL} = \frac{M}{N^2} \quad (5.34)$$

Figure 5.5: Output SINR comparison of MIMO and conventional SIMO STAP

processor ($N=M=10$ for MIMO, $N=10$ for SIMO, $L=10$ for both).

Thus, compared with SIMO STAP, the total sensitivity of MIMO STAP with orthogonal waveform will decrease as long as $M < N^2$. Fig. 5.5 shows the output SINRs of MIMO and SIMO when $L=10$, and $N=M=10$. In this example, the output SINR of MIMO is approximately $10\log_{10}10=10$ dB less than the output SINR of SIMO.

Therefore, in practical operation, the sensitivity loss of MIMO radar with orthogonal waveforms must be compensated by improving the integration time for some applications in order to achieve equivalent performance as SIMO radar. The final sensitivity of MIMO radar after integration can be calculated based on the available optimum integration time [104]. As described in [104], the optimum integrating time can be seriously limited by transmitter spurious-frequency spread and target modulation; the advantage of integration may not be fully realized. Thus, the sensitivity of MIMO radar should be carefully treated before real application.

5.4 Clutter Rank

STAP is an effective technique for target detection in a combined clutter and jamming environment. By applying STAP in a MIMO radar system, one can have more degrees of freedom to improve the performance of target detection. However, the severity of clutter will also increase due to the same reason, which is determined by the rank of clutter covariance matrix. The knowledge of clutter rank is important for reducing the computation complexity while

improving the efficiency of signal processing. Therefore, characterization of the clutter rank is very important for predicting the performance of MIMO STAP radar systems.

So far, the clutter rank estimation of CCM has been intensively studied for conventional adaptive radars with uniform linear arrays aligned along with the radar platform's motion direction and with pulse repetition frequency matched to the antenna element spacing. Several rules [92, 101] have been developed for these STAP systems under ideal conditions, where such effects as the internal clutter motion, velocity misalignment, array manifold mismatch, and channel mismatch are not taken into account. Specifically, paper [92] introduced the well known Brennan's rule and proved a special case of it. Paper [101] extended Brennan's rule to estimate the clutter rank in the case of subarraying. Paper [102] generalized these rules for analyzing the rank and the eigenspectrum of the CCM observed by space-time radar systems with arbitrarily configured arrays and varying look geometry. Based on abovementioned rules, paper [23] derived the rule for estimating the clutter rank of MIMO radar with orthogonal waveforms. All of these studies considered only conventional transmitting waveforms, i. e., coherent or orthogonal waveforms. However, as illustrated in [34, 36-42, 103], MIMO radar can utilize correlated waveforms in many applications. In these cases, the transmitting waveforms may be partially correlated other than orthogonal or

coherent with each other. When it comes to the partially correlated waveforms, the clutter rank estimation will be different from those in the current studies. Hence, it is worthy of studying what the severity of clutter subspace would be when using different waveforms, in other words, what the exact relationship would be between the clutter rank and the transmitting waveforms.

In this section, we extend the current rules to estimate the rank of CCM observed by MIMO radar that operates with waveform diversity under ideal conditions. We have constructed the signal model of clutter echoes based on transmitting waveforms and the configuration of MIMO STAP system in Section 5.2. The CCM is expressed as a function of waveform covariance matrix (WCM). With that, the clutter rank is estimated by investigating the properties of CCM associated with different WCMs. It is found that the structure and rank of a WCM have large effect on the clutter rank. As a WCM from different waveforms may be full rank or rank deficient, the clutter rank is investigated for different rank-cases. We firstly study the full-rank WCM case and show that the clutter rank of this case is the same as that of orthogonal waveform cases. Secondly, we study the case with rank deficient WCM. This case is much more complicated than the full-rank WCM case. For this case, we propose three close-form rules for a class of special WCMs based on current clutter rank estimation rule, i.e., the rule of time-bandwidth product. For other WCMs not included in these cases, we also provide some experience-based

conclusions to predict approximately the clutter rank based on our intensive numerical experiments. Simulation results are provided to demonstrate the proposed rank estimation rules.

In this section we develop the rank estimation rules for MIMO radar with waveform diversity. Referring to (5.11), because Ξ is a positive-definite matrix, the rank of $\mathbf{R}_C$ is equal to the rank of $\mathbf{V}_C$ [92]. Therefore, we can just study the rank of $\mathbf{V}_C$. Furthermore, because each column of $\mathbf{V}_C$ is a function of $\mathbf{R}_S$, the rank of $\mathbf{V}_C$ is dependent on the rank of $\mathbf{R}_S$, let us say, r ($r \leq N$). For simplification, we set $Z_i = \exp[j2\pi d_{Rx} \sin(\theta_i) \cos(\varphi) / \lambda_0] = \exp[j2\pi f_{s,i}]$, $\beta = 2vT_r/d_{Rx}$, and $(\mathbf{R}_S)_{k,j} = R_{k,j}$. Then we can rearrange the elements of $\mathbf{v}_i$ in (5.8) by assembling together the elements with same indexes m (receiver index) and l (pulse index) and obtain

$$\begin{aligned} \mathbf{v}_{i,m,l} &= z_i^{(m-1)+(l-1)\beta} \mathbf{R}_S^T \mathbf{v}_{Tx,i} \\ &= z_i^{(m-1)+\beta(l-1)} \begin{bmatrix} R_{1,1} + R_{2,1}z_i^\alpha + R_{3,1}z_i^{2\alpha} + \dots + R_{N,1}z_i^{(N-1)\alpha} \\ R_{1,2} + R_{2,2}z_i^\alpha + R_{3,2}z_i^{2\alpha} + \dots + R_{N,2}z_i^{(N-1)\alpha} \\ \vdots \\ R_{1,N} + R_{2,N}z_i^\alpha + R_{3,N}z_i^{2\alpha} + \dots + R_{N,N}z_i^{(N-1)\alpha} \end{bmatrix} \end{aligned} \quad (5.35)$$

where $\mathbf{v}_{i,m,l} \in \mathbb{C}^{N \times 1}$ and $\mathbf{v}_{Tx,i} = \mathbf{v}_{Tx}(\theta_i)$. Based on (5.35), we define

$\mathbf{Z}_{i,m,l} = z_i^{(m-1)+\beta(l-1)}$ and rewrite $\mathbf{V}_C$ as

$$\begin{aligned} \bar{\mathbf{V}}_C &= \begin{bmatrix} \mathbf{v}_{1,1,l} & \mathbf{v}_{2,1,l} & \dots & \mathbf{v}_{N_C,1,l} \\ \mathbf{v}_{1,2,l} & \mathbf{v}_{2,2,l} & \dots & \mathbf{v}_{N_C,2,l} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{v}_{1,M,L} & \mathbf{v}_{2,M,L} & \dots & \mathbf{v}_{N_C,M,L} \end{bmatrix} \\ &= (\mathbf{I}_{ML} \otimes \mathbf{R}_S^T) \begin{bmatrix} \mathbf{v}_{Tx,1} \mathbf{Z}_{1,1,1} & \mathbf{v}_{Tx,2} \mathbf{Z}_{2,1,1} & \dots & \mathbf{v}_{Tx,N_C} \mathbf{Z}_{N_C,1,1} \\ \mathbf{v}_{Tx,1} \mathbf{Z}_{1,2,1} & \mathbf{v}_{Tx,2} \mathbf{Z}_{2,2,1} & \dots & \mathbf{v}_{Tx,N_C} \mathbf{Z}_{N_C,2,1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{v}_{Tx,1} \mathbf{Z}_{1,M,L} & \mathbf{v}_{Tx,2} \mathbf{Z}_{2,M,L} & \dots & \mathbf{v}_{Tx,N_C} \mathbf{Z}_{N_C,M,L} \end{bmatrix} \\ &\triangleq (\mathbf{I}_{ML} \otimes \mathbf{R}_S^T) \mathbf{V}_0 \end{aligned} \quad (5.36)$$

As above operations, i.e., reordering the rows, will not change the rank of

original clutter matrix $\mathbf{V}_C$, the rank of $\bar{\mathbf{V}}_C$ is equal to the rank of $\mathbf{V}_C$. Thus, we can just study the rank of $\bar{\mathbf{V}}_C$ instead. From (5.36) we can find that the new clutter matrix $\bar{\mathbf{V}}_C$ is a product of two matrices. One is from the Kronecker product of the identity matrix and the waveform covariance matrix. The other, denoted by $\mathbf{V}_O$, is a clutter matrix associated with the ideally orthogonal waveforms. Please note that through (5.36) we can see that in each column of $\mathbf{V}_O$, $\mathbf{v}_{Tx}$ will be compressed, or filtered, by $\mathbf{R}_S$. This filtering process is in effect a beamforming process. The dependence of clutter rank on $\mathbf{R}_S$ is related to this beamforming process, which is determined by $\mathbf{R}_S$. Different waveform matrices may have different $\mathbf{R}_S$, thereby, different clutter property. As $\text{rank}(\mathbf{AB}) \leq \min(\text{rank}(\mathbf{A}), \text{rank}(\mathbf{B}))$ [105], the rank of $\bar{\mathbf{V}}_C$ is no larger than the rank of $\mathbf{V}_O$. Thus, the rank of CCM from orthogonal waveforms behaves as an upper bound for the rank of CCM from any other waveform matrix. As the coherent waveforms have the most concentrated illuminating energy, the rank of CCM from coherent waveforms is the lower bound for the rank of CCM from arbitrary waveform matrix. In the following sections, we will investigate the filtering effect according to different $\mathbf{R}_S$ and develop rules to predict the corresponding clutter rank. As the WCM can be full rank or rank deficient in different applications, we will discuss the rank estimation problem according to the rank property of the WCM.

5.4.1 Full Rank $\mathbf{R}_S$

In many proposed MIMO radar applications, the waveform matrix has full rank, leading to a full rank $\mathbf{R}_S$. When $\mathbf{R}_S$ is full-rank, $(\mathbf{I}_{ML} \otimes \mathbf{R}_S^T)$ is also full-rank. We also have that if $\mathbf{A} \in C^{M \times M}$ and $\mathbf{C} \in C^{N \times N}$ are both full rank, then [105]:

$$\text{rank}(\mathbf{B}) = \text{rank}(\mathbf{AB}) = \text{rank}(\mathbf{BC}) = \text{rank}(\mathbf{ABC}) \quad (5.37)$$

So that in this case, the rank of $\bar{\mathbf{V}}_C$ ($\mathbf{V}_C$) is the same as the rank of $\mathbf{V}_O$ from orthogonal waveforms. Therefore, the rule to predict the clutter rank for waveforms with full rank WCM is [23]:

$$r_c = \text{rank}(\mathbf{V}_C) = \lfloor M + \beta(L-1) + \alpha(N-1) \rfloor \quad (5.38)$$

where the brackets $\lfloor \rfloor$ indicate rounding to the nearest integer.

This conclusion is useful in many applications where full rank WCM from approximately orthogonal waveforms or judiciously designed full rank waveform matrix (see [38, 101], for example) is available. When applying (5.38), we should calculate the rank of WCM if the WCM is not judiciously designed to be full rank. To calculate the rank of $\mathbf{R}_S$ (of dimension N by N) is significantly efficient than to calculate the rank of CCM (of dimension NML by NML).

5.4.2 Rank Deficient $\mathbf{R}_S$

As one can expect, in many cases, the WCM may be rank deficient, for example, the WCM that is designed to obtain a certain beampattern [40-42]. We

in this section discuss the rank estimation for a rank deficient WCM. The basic concept we use is the rule of time-bandwidth product [23, 101-102]. As the ‘bandwidth’ of spatial frequency f_s is known for given system configuration, the main problem is to calculate the ‘time width’.

Because $\mathbf{R}_S$ is normal and positive semi-definite it can be decomposed as $\mathbf{R}_S = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{U} \in C^{N \times N}$ is a unitary matrix, and $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_r, 0, \dots, 0)$ is a diagonal matrix with only r positive diagonal elements. We also denote $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_N]$. Thus we can obtain $\mathbf{R}_S \mathbf{U} = \mathbf{U} \mathbf{\Lambda}$, which indicates that we can obtain the below equation:

$$\mathbf{U}^T \mathbf{R}_S^T \mathbf{v}_{Tx,i} = \mathbf{\Lambda} \mathbf{U}^T \mathbf{v}_{Tx,i} = [f_1(z_i); \dots; f_r(z_i); 0; \dots; 0] \in C^{N \times 1} \quad (5.39)$$

where, $f_q(z_i) = \lambda_q (\mathbf{u}_q)^T \mathbf{v}_{Tx,i}$, $q=1, \dots, r$, is a polynomial function of argument z_i . If we can express $f_q(z_i)$ as $f_q(z_i) = a_q z_i^{b_q}$, where a_q and b_q are real numbers, the rank estimation problem will be perfectly solved as discussed in the following.

The condition for $f_q(z_i) = a_q z_i^{b_q}$ is equivalent to

$$\begin{cases} \text{Re}(f_q(z_i)) = a_q \cos[b_q(2\pi f_{s,i})] \\ \text{Im}(f_q(z_i)) = a_q \sin[b_q(2\pi f_{s,i})] \end{cases}, q=1, \dots, r, \quad (5.40)$$

where $\text{Re}(x)$ and $\text{Im}(x)$ denote the real and imaginary components of x , respectively. The unknowns in (5.40) can be easily resolved analytically or by numerical method when the condition holds. With the unknowns in (5.40) resolved, we can obtain from $\mathbf{v}_{i,m,l}$

$$\begin{aligned} \tilde{\mathbf{v}}_{i,m,l} &= \mathbf{U}^T \mathbf{v}_{i,m,l} \\ &= z_i^{(m-1)+\beta(l-1)} [f_1(z_i); \dots; f_r(z_i); 0; \dots; 0] \\ &= z_i^{(m-1)+\beta(l-1)} [a_1 z_i^{b_1}; \dots; a_r z_i^{b_r}; 0; \dots; 0] \in C^{N \times 1} \end{aligned} \quad (5.41)$$

Similarly, the same left-multiplication by $\mathbf{U}^T$ (like row manipulations) can be conducted over all blocks of $\bar{\mathbf{V}}_C$ and finally leads to another clutter matrix

$$\tilde{\mathbf{V}}_C = (\mathbf{I}_{ML} \otimes \mathbf{U}^T) \bar{\mathbf{V}}_C = \begin{bmatrix} \tilde{\mathbf{v}}_{1,1,l} & \tilde{\mathbf{v}}_{2,1,l} & \cdots & \tilde{\mathbf{v}}_{N_C,1,l} \\ \tilde{\mathbf{v}}_{1,2,l} & \tilde{\mathbf{v}}_{2,2,l} & \cdots & \tilde{\mathbf{v}}_{N_C,2,l} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{\mathbf{v}}_{1,M,L} & \tilde{\mathbf{v}}_{2,M,L} & \cdots & \tilde{\mathbf{v}}_{N_C,M,L} \end{bmatrix} \in C^{NML \times N_C} \quad (5.42)$$

Note that, the rank of $\tilde{\mathbf{V}}_C$ is also equal to the rank of $\bar{\mathbf{V}}_C$, i.e., the rank of $\mathbf{V}_C$.

Because there are some all-zero rows, we can rearrange the rows of $\tilde{\mathbf{V}}_C$ and obtain

$$\hat{\mathbf{V}}_C = \begin{bmatrix} \hat{\mathbf{v}}_{1,l} & \hat{\mathbf{v}}_{2,l} & \cdots & \hat{\mathbf{v}}_{N_C,l} \\ \hat{\mathbf{v}}_{1,2} & \hat{\mathbf{v}}_{2,2} & \cdots & \hat{\mathbf{v}}_{N_C,2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{v}}_{1,r} & \hat{\mathbf{v}}_{2,r} & \cdots & \hat{\mathbf{v}}_{N_C,r} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{bmatrix} \triangleq [\hat{\mathbf{v}}_{C,1} \quad \hat{\mathbf{v}}_{C,2} \quad \cdots \quad \hat{\mathbf{v}}_{C,N_C}] \in C^{NML \times N_C} \quad (5.43)$$

with

$$\begin{aligned} \hat{\mathbf{v}}_{i,q} &= f_q(z_i) \begin{bmatrix} z_i^0; \cdots; z_i^{(M-1)+\beta(L-1)} \end{bmatrix} \\ &= a_q \begin{bmatrix} z_i^{b_q+0}; \cdots; z_i^{b_q+(M-1)+\beta(L-1)} \end{bmatrix}_{ML \times 1}, i=1,2,\dots,N_C, \quad q=1,2,\dots,r \end{aligned} \quad (5.44)$$

where $\hat{\mathbf{v}}_{i,q}$ is obtained by putting together the matrix elements with same b_q from all pairs of m and l (referring to (5.41)) in each column. Without loss of generality, we assume $b_1 \leq \dots \leq b_q \leq \dots \leq b_r$ in the following discussion for convenience. Because the above row operation will not change the rank of matrix, thus the rank of new clutter matrix $\hat{\mathbf{V}}_C$ is the same as the rank of original clutter matrix $\mathbf{V}_C$. Then we can just estimate the clutter rank by investigating $\hat{\mathbf{V}}_C$ in stead of the original clutter matrix.

As pointed out in [23], the column vector $\hat{\mathbf{v}}_{C,i}$ in $\hat{\mathbf{V}}_C$ can be viewed as a

sampled version of a time-and-band-limited sinusoidal function

$$\hat{V}(f_{s,i}, x) = \begin{cases} \sum_{q=1}^r a_q e^{j2\pi f_{s,i} x}, & x \in [b_q, b_q + X] \\ 0, & \text{otherwise} \end{cases} \quad (5.45)$$

where $X = M - l + \beta(L - l)$, $f_{s,i} \in [-0.5, 0.5]$ is the frequency value, and x is the time vale. We also define the one-side bandwidth of the sinusoidal function as $W = 0.5$ based on $-0.5 \leq f_{s,i} \leq 0.5$. The dimension of this kind of function can be well approximated by the rule of time-bandwidth product [106]. Therefore, the function in (5.45) has the same dimension as the one below

$$V(f_{s,i}, x) = \begin{cases} e^{j2\pi f_{s,i} x}, & x \in \{[b_1, b_1 + X], \dots, [b_r, b_r + X]\} \\ 0, & \text{otherwise} \end{cases} \quad (5.46)$$

since they have the same time-bandwidth product. From (5.46), we can see that the time domain of this sinusoidal function may consist of r sub-time-domains, and each sub-domain has a time-width of X . However, the total time width may be different for different group of $(b_1, \dots, b_r)$, which is more complicated than the situation faced by coherent waveforms and orthogonal waveforms. Thus, we can no longer apply the rule of time-bandwidth product directly without considering the function's time domain property. Meanwhile, as pointed in [102], the average sampling rate should satisfy the Nyquist rate for the application of the rule of time-bandwidth product. Thus, for the problem considered in this chapter, we assume $\beta \leq M$ for each sub-time-domain in order to make the average sampling rate satisfy the Nyquist rate, which is not pointed out in [23]. Therefore, each sub-time-domain has approximately $2WX + 1 = X + 1$

eigenvalues according to the rule of time-bandwidth product. Considering the time domain property of the function in (5.46), we apply the rule of time-bandwidth product to estimate the rank of CCM and obtain the following three rules.

Rule 1: If $b_{q-l}+X \geq b_q$, $q=2, \dots, r$, which means that the succeeding sub-time-domain will overlap the preceding sub-time-domain and the total time domain is one continuous interval with total time width $X_{\text{total}} = M-l+\beta(L-l)+b_r-b_l$, then the rank of CCM is

$$r_c = \text{rank}(\mathbf{V}_c) \approx \lfloor 2WX_{\text{total}} + 1 \rfloor = \lfloor M + \beta(L-1) + b_r - b_l \rfloor \quad (5.47)$$

Rule 2: If $b_q+X < b_{q+l}$, $q=1, \dots, r-l$, which means that the succeeding sub-domain will not overlap the preceding sub-domain and the total time domain has r independent intervals, then the rank of CCM is

$$r_c = \text{rank}(\mathbf{V}_c) \approx \left\lfloor \sum_{q=1}^r (2WX + 1) \right\rfloor = \left\lfloor \sum_{q=1}^r (\beta(L-1) + M) \right\rfloor \quad (5.48)$$

Rule 3: Otherwise, the rank of CCM should be separately computed by applying *Rule 1* for overlapping sub-domains and *Rule 2* for independent sub-domains.

Though these three rules are constrained by the condition of (5.40), they are applicable to many waveforms adopted in MIMO radar analysis so far. For example, orthogonal waveforms (though the corresponding WCM is full rank) and coherent waveforms satisfy the condition of (5.40). a_q and b_q can be easily resolved analytically and *Rule 1* can be found to be proper to these two kinds of

waveforms. Then, by simplifying *Rule 1*, the clutter ranks of coherent waveforms and orthogonal waveforms can be predicted by a unified rule.

$$r_c = \text{rank}(\mathbf{V}_c) = \lfloor M + \beta(L-1) + \alpha(r-1) \rfloor \quad (5.49)$$

Reminding that a full rank waveform matrix leads to same clutter rank as orthogonal waveforms, we can arrive at the conclusion that clutter rank from any full rank WCM or rank-1 WCM can be estimated by using (5.49). When applying (5.49), we should also calculate the rank of WCM if the WCM is not judiciously designed to be full rank or rank-1. However, to calculate the rank of $\mathbf{R}_s$ is significantly more efficient than to calculate directly the rank of CCM.

5.4.3 Other Cases

As WCMs may have large variety and complicated structure, many waveforms may not be included in the scope of the above discussions. However, based on our extensive simulation study, we have found that when a WCM is close to be full rank, the clutter rank will close to the maximum rank predicted by orthogonal waveforms. More clearly, when the rank of a WCM is larger than half of its dimension, this experience conclusion always holds. Nevertheless, further study should be dedicated to the waveforms not well investigated in this chapter.

5.4.4 Numerical Studies and Discussions

In this section we verify the rank prediction rules derived in this chapter by

simulated clutter data. The clutter simulation assumptions are given such as those in [102]. Briefly, the simulation divides a constant range ring into many clutter patches. The ground clutter is assumed to be homogeneous within the range bin of interest. Each element pattern is assumed to be isotropic. The system is operating under ideal condition, i.e., no velocity misalignment, intrinsic clutter motion, etc. The clutter to noise ratio is 50 dB. The basic parameters used in the simulations are $v_t=200$ m/s, $h=10$ km, $\lambda_0=0.6666$ m, $d_{Rx}=0.3333$ m, $\varphi=0$ degree.

Figure 5.6: Example of full rank $\mathbf{R}_s$.

Firstly, we will demonstrate the rule for full rank WCMs. As shown in (5.38), any full rank WCM leads to the same clutter rank as WCM of orthogonal waveforms. We take a special Toeplitz matrix as the WCM [42]. The WCM can be written as:

$$\mathbf{R}_s = \begin{bmatrix} 1, \rho, \dots, \rho^{N-1}; & \rho, 1, \dots, \rho^{N-2}; & \dots; & \rho^{N-1}, \rho^{N-2}, \dots, 1 \end{bmatrix}.$$

When we take ρ to be 0.1 in this $\mathbf{R}_S$, the corresponding WCM is full rank. We also set $N=5$, $M=5$, $L=10$, $\beta=1$, $\alpha=5$. The simulation result is presented in Fig. 5.6. One can see that the predicted rank for this full rank WCM is exactly the same as that from the eigen-value decomposition, the same as the rank of ideal orthogonal waveforms.

Figure 5.7: Illustration of *Rule 1* with coherent waveforms.

After that, we illustrate the rules for rank deficient cases. Since *Rule 3* is the combination of *Rule 1* and *Rule 2*, we only need to illustrate *Rule 1* and *Rule 2* with different waveforms. We take two WCMs proposed for MIMO radar applications in [40]. One is from coherent waveforms; the other is from the method of maximum power design for known target location. Fig. 5.7 shows eigenspectrum of the CCM with coherent waveform to verify *Rule 1* of this chapter. We here set $N=10$, $M=10$, $L=16$, $\alpha=10$. For coherent waveform, rank $r=1$, when $\beta=1$, $r_C=M+\beta(L-1)+\alpha(r-1)=25$, when $\beta=2.4$,

$r_c = M + \beta(L-1) + \alpha(r-1) = 46$. As illustrated Fig. 5.7, when β is an integer, the eigenspectrum exhibits a sharp cutoff along the predicted rank. When $\beta = 2.4$, the eigenspectrum has only a few significant eigenvalues beyond a knee near the predicted rank. The proposed rule well predicts the rank of the simulated clutter data.

(a) Half wavelength element spacing.

(b) 5 wavelength element spacing.

Figure 5.8: The transmitting beam pattern of waveforms designed by method of

maximum power criterion.

Figure 5.9: Illustration of *Rule 2*.

Fig. 5.9 shows eigenspectrum of CCM to verify *Rule 2* of this chapter. The WCM is from the method of maximum power design for known target location. Here we set $N=10$, $M=10$, $L=16$, $\alpha=10$. The focus region is set to be $[-20, 20]$ degree and 3 dB points are -12 degree and 12 degree. The transmit beam-pattern of this waveform set is illustrated in Fig. 5.8. The rank of the WCM is 2. Through matrix operation, we can get $\mathbf{U}^H \mathbf{R}_S = \mathbf{U} \mathbf{\Lambda} = [0, \dots, 0; \dots; 0, \dots, 0; 0, \dots, 0, 1; 3, \dots, 3, 0]$. From (18), we can get $b_2 = (N-1)\alpha = 90$ and $b_1 = (N-2)\alpha/2 = 40$. Thus $\Delta = b_2 - b_1 = 90 - 40 = 50 > X$ for $\beta=1, 2.4$. Therefore, *Rule 2* is suitable to this case. When $\beta=1$, $r_C = r(M + \beta(L-1)) = 50$; when $\beta=2.4$, $r_C = r(M + \beta(L-1)) = 92$. From Fig. 5.9 we can see that *Rule 2* well predicts the clutter rank of the simulated data. However, if the rule of time-bandwidth product is applied without considering the waveform effect, the rank will be

significantly overestimated. As pointed above, the rank from orthogonal waveforms is the upper bound as illustrated by the red solid line in Fig. 5.9.

In this section we have extended Brennan's rule for estimating the rank of CCM in MIMO radar with more flexible waveform diversity. It is found that the clutter rank is dependent on the rank and the structure of the covariance matrix of the transmitting waveforms, and that clutter rank of orthogonal waveforms acts as the upper bound for all kinds of waveforms and clutter rank of coherent waveforms the lower bound. A unified rule is proposed for full rank WCM and rank-1 WCM. In addition, we have presented three rules to estimate the clutter rank from a special class of rank deficient WCM. Based on the proposed rules in this chapter, the clutter rank of MIMO radar with common waveforms can be estimated directly. Simulation results verified the proposed rules.

5.5 Waveform Effect on Optimum Adapted Pattern

The optimum space-time filter response as a function of angle and Doppler is one indicator of the STAP processor performance, and this response is called the adapted pattern, defined by

$$P_w(\theta, f_d) = \left| \mathbf{w}^H \mathbf{v}(\theta, f_d) \right|^2 \quad (5.50)$$

The adapted pattern is a two-dimensional angle-Doppler frequency response. We can see from (5.5) and (5.20) that waveform property is involved in the optimum filter. Therefore, in MIMO STAP processing, the adapted pattern will

be changing corresponding to different waveforms. Fig. 5.10 shows the adapted pattern from coherent waveform and ideal orthogonal waveform, where $N=M=5$, $L=10$, θ_t is zero, $d_{Rx}=\lambda_o/2$, $\alpha=5$, $v_p=100$ m/s, $v_t=70$ m/s, $\beta=1$, $f_o = 500$ MHz, the clutter-to-noise ratio is 30 dB and the jamming-to-noise ratio is 30 dB. Two jamming signals come from the direction of -40 degree and 30 degree, respectively. Fig. 5.11 is the target Doppler slice from the patterns in Fig. 5.10. Fig. 5.11 also show the fact that even with the same antenna configuration, the spatial resolution of the MIMO beamformer is different for different waveform sets because the effective virtual array will be different from different waveform sets. This can be expressed by (5.5). For example, the orthogonal waveform will extend the receiving array by N times to form an $N \times M$ elements array with half wavelength spacing. While the coherent waveform will not extend the receiving array because it will concentrate the transmitting energy on the receiving element as $(\mathbf{R}_s)^{1/2}$ has only the last column to be $\mathbf{1}_{N \times 1}$. Thus, the waveform decorrelation in transmitting may provide more applicable degrees of freedom in the optimum processing and thereby improves the spatial resolution.

(a) Coherent waveform case.

(b) Orthogonal waveform case.

Figure 5.10: Adapted patterns of different waveforms.

(a) Noise only.

(b) Noise plus Clutter plus Jammers (30 and -40 degree).

Figure 5.11: Doppler slice of 2-D adaptive pattern of MIMO STAP with waveform diversity.

5.6 Concluding Remarks

In this chapter, we have discussed several basic issues of STAP in MIMO radar with waveform diversity. We have developed a unified framework for signal model incorporating the transmitting waveforms. The performance figures of metric have consequently been expressed as functions of waveforms and have been evaluated as well.

Waveform effect on sensitivity of MIMO STAP has been investigated first. As shown in this chapter, the sensitivity of MIMO STAP is proportional to the largest eigenvalue of waveform covariance matrix. With this on hand, we can precisely evaluate the sensitivity of certain waveforms. This is important for practical applications as it can provide information about how much compensation from integration should be performed in order to meet the total sensitivity requirement.

Waveform effect on the property of clutter has also been investigated. As clutter rank is a critical parameter for STAP, we have mainly studied the clutter rank estimation for STAP in MIMO radar with waveform diversity. As shown in this chapter, the clutter rank of MIMO radar depends on not only the array manifold but also the waveform covariance matrix. It is found that the clutter rank of orthogonal waveforms acts as the upper bound for all kinds of waveforms and clutter rank of coherent waveforms the lower bound. A unified rule has been proposed for full rank WCM and rank-1 WCM. In addition, we

have presented three rules to estimate the clutter rank from a special class of rank deficient WCM. The proposed rules can serve as an extension of the Brennan's rule.

Finally, the effect of waveform on the optimum adapted pattern has been evaluated.

Chapter 6 High Resolution MIMO-HFSW Radar

6.1 Introduction

High frequency surface wave (HFSW) radar is a long range radar system that adopts vertically polarized high frequency (3-30 MHz) electromagnetic signals which propagate along the ocean surface. A preferable property of HFSW radar is that it can detect and track ship and aircraft targets beyond the horizon. Due to this reason, HFSW radar has a wide range of applications in both civil and military fields. For conventional HFSW radar systems, its range resolution is highly restricted by the bandwidth of available clear channels in a congested spectrum environment [45], [84], while the azimuth resolution is also constrained by the physical dimension of the radar antenna aperture.

Multiple-input multiple-output (MIMO) radar is a new radar concept that may lead to improved performances in many radar applications. MIMO radar can transmit multiple waveforms that are separable at the receivers so that it can obtain more degrees of freedom compared with conventional radars which transmit a single waveform [12-20]. With widely distributed antennas, angular diversity can be fully achieved to reduce the effect of target scintillations [12-13, 19-20]. Meanwhile, MIMO radars with widely distributed antennas can gain high resolution by coherent processing [20]. Like distributed MIMO radar, a single-input multiple-output (SIMO) radar system with sparse coherent

receiving aperture can also achieve high resolution on the order of one wavelength with limited bandwidth as reported in [107].

The sparse frequency waveform problem has been studied in [68, 72] and literatures therein. For HFSW radar, sparse frequency waveform can provide large flexibility to choose clear channels and thereby reduce interferences from occupied channels. Motivated by the potential benefits from sparse frequency waveforms and high resolution capacity of coherent multistatic radar and MIMO radar systems, we in this chapter propose a new MIMO-HFSW radar using sparse frequency waveforms to break down the limitation on range and azimuth resolutions of conventional HFSW radar. The ambiguity function (AF) is derived in detail and fully investigated in this chapter to analyze the performance of the proposed system. Unlike that of paper [20], we take Doppler effect in the AF for analysis. Through AF analysis it is demonstrated that this system has high flexibility in operation and attractive improvement on resolution in the restricted geographical condition as well as the congested spectrum environment. In particular, by using the widely separated antennas, it abates the aperture limitation as well as the rigorous land requirement successfully. In addition, by using sparse frequency waveform it not only brings more clear channels into use to compete with co-channel interference but also reduces the peak sidelobe level (PSL) in range responses.

The remainder of this chapter is organized as follows. The AF of the

proposed MIMO-HFSW radar using sparse frequency waveforms is derived in Section 6.2. Based on AF analysis and simulations the system is evaluated in terms of resolution capacity and PSL performance, with zero Doppler frequency in Section 6.3 and under the factors of geometric configurations and Doppler effects in Section 6.4. Finally, conclusions and future work are outlined in Section 6.5.

6.2 Ambiguity Function of the Proposed System

The ambiguity function is an important tool in conventional radar analysis as it shows radar's inherent capacity of discriminating targets associated with different time delay and Doppler frequency. Thus, in this chapter, we also employ AF to evaluate the performance of the proposed system.

The proposed MIMO-HFSW radar system consists of M transmitters transmitting M waveforms and N receivers. Each transmitter is assigned a distinct channel with starting frequency $f_m, m=1, 2, \dots, M$. Thus, collectively, the transmitting waveforms will have a sparse spectrum, because which we call the transmitting waveforms sparse frequency waveform. All antennas are arbitrarily located with mutual separation distance larger than several wavelengths in a 3-dimensional space. Fig. 6.1 shows the system configuration, where R_n and T_m refer to the n -th receiver and the m -th transmitter, respectively. Each transmitter and each receiver are located at a point represented by a 3-dimensional vector in the Cartesian coordinate system. For example, the m -th

transmitter is associated with a vector $\mathbf{c}_{t,m}=[x_m, y_m, z_m]$, and the n -th receiver $\mathbf{c}_{r,n}=[x_n, y_n, z_n]$. For simplification, this chapter considers only single point target case. And the target is assumed to be located at a general point $\mathbf{x} = [x, y, z]$ with constant velocity of $\mathbf{v} = [v_x, v_y, v_z]$. As the antennas are widely distributed, each of them will view the target with a different angle. Angle variables θ and φ refer to the true elevation and azimuth as illustrated in Fig. 6.1. We also assume that the phases and time at the transmitters and receivers are synchronized in advance. Meanwhile, the signal attenuation in different path is assumed to be the same.

Figure 6.1: MIMO radar configuration.

Let $x_m(t)$ be the signal transmitted by the m -th transmitter that meets the requirement of narrow band assumption. It is expressed as

$$x_m(t) = \exp(j2\pi f_m t) s_m(t) \quad (6.1)$$

where $s_m(t)$ is the baseband waveform of the m -th transmitter. After the signal impinged back from the target to the n -th receiver, the echo is:

$$e_n(t) = \sum_{m=1}^M \gamma x_m(t - \tau_{nm}) \exp(-j2\pi f d_{nm} t) \quad (6.2)$$

where γ is the complex reflection coefficient of the target, τ_{nm} is the round-trip delay, and $f d_{nm}$ is the Doppler frequency of the echo at the n -th receiver due to the m -th transmitter. We take the assumption that the target stops during the pulse transmission and reception. Then τ_{nm} is the round-trip delay at the start of observation time, and has the form

$$\tau_{nm} = r_T^m(t)/c + r_R^n(t)/c \Big|_{t=0} = \sqrt{(x-x_m)^2 + (y-y_m)^2 + (z-z_m)^2}/c + \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}/c \quad (6.3)$$

where c is the velocity of light in the media that the transmitters, receivers and targets are located in. In the case that the transmitters or the receivers are mounted on moving platform, the platform velocity can also be easily included in (6.3). For different scatter, τ_{nm} is a function of variables x , y , and z . Thus, through Taylor-series analysis at a reference point $\mathbf{x}_0 = [x_0, y_0, z_0]$, (6.3) can be changed to

$$\begin{aligned} \tau_{nm} &= r_T^m(0)/c + r_R^n(0)/c \Big|_{[x,y,z]} \\ &\doteq \left(r_T^m(0)/c + r_R^n(0)/c \Big|_{[x_0,y_0,z_0]} \right) + \\ &\quad \left(\frac{(x-x_0)(\cos\theta_m \cos\varphi_m + \cos\theta_n \cos\varphi_n)}{c} + \right. \\ &\quad \left. \frac{(y-y_0)(\cos\theta_m \sin\varphi_m + \cos\theta_n \sin\varphi_n)}{c} + \frac{(z-z_0)(\sin\theta_m + \sin\theta_n)}{c} \right) \\ &= \tau_{nm}^0 + \tau_{nm}' \end{aligned} \quad (6.4)$$

where

$$\tau_{nm}^0 = r_T^m(0)/c + r_R^n(0)/c \Big|_{[x_0, y_0, z_0]} \quad (6.5)$$

and

$$\tau'_{nm} = \left(\frac{(x-x_0)(\cos\theta_m \cos\varphi_m + \cos\theta_n \cos\varphi_n)}{c} + \frac{(y-y_0)(\cos\theta_m \sin\varphi_m + \cos\theta_n \sin\varphi_n)}{c} + \frac{(z-z_0)(\sin\theta_m + \sin\theta_n)}{c} \right) \quad (6.6)$$

fd_{nm} has the form

$$fd_{nm} = \frac{1}{\lambda_m} \frac{d}{dt} (r_T^m(t) + r_R^n(t)) \quad (6.7)$$

Again, through Taylor-series analysis (6.7) can be changed to

$$\begin{aligned} f_{dnm} &= \frac{v_x(x-x_m) + v_y(y-y_m) + v_z(z-z_m)}{\lambda_m \sqrt{(x-x_m)^2 + (y-y_m)^2 + (z-z_m)^2}} \\ &+ \frac{v_x(x-x_n) + v_y(y-y_n) + v_z(z-z_n)}{\lambda_m \sqrt{(x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2}} \\ &= \frac{v_x(\cos\theta_m \cos\varphi_m + \cos\theta_n \cos\varphi_n)}{\lambda_m} \\ &+ \frac{v_y(\cos\theta_m \sin\varphi_m + \cos\theta_n \sin\varphi_n)}{\lambda_m} + \frac{v_z(\sin\theta_m + \sin\theta_n)}{\lambda_m} \end{aligned} \quad (6.8)$$

It can be easily proved that (6.8) is equivalent to the expression for the conventional Doppler frequency of bistatic radar.

As each transmitting waveform is assigned to a distinct channel, orthogonality holds for all the transmitting signals. Thus, at each receiver, signals from M transmitters can be firstly separated by down-converting into M channels. Then, for each channel, a matched filter of corresponding transmitting waveform is employed at the interested range cell centered at $\mathbf{x}_0 = [x_0, y_0, z_0]$. Thus, the m -th

filter output at the n -th receiver can be expressed as

$$\begin{aligned}
& y_{nm}(\mathbf{x}, \mathbf{x}_0, \mathbf{v}) \\
&= \gamma \exp(-j2\pi f_m(\tau_{nm} - \tau_{nm}^0)) \\
&\times \int_{-\infty}^{+\infty} s_m(t - \tau_{nm}^0) \exp(j2\pi f d_{nm} t) s_m^*(t - \tau_{nm}) dt \\
&+ n_{nm}(t) \\
&= \gamma \exp(-j2\pi f_m \tau'_{nm}) \mathbf{A}_{mm}(\tau'_{nm}, f d_{nm}) + n_{nm}(t)
\end{aligned} \tag{6.9}$$

where $\mathbf{A}_{mm}$ is the correlation between the m -th transmitting waveform and its delay-Doppler shifted version, and $n_{nm}(t)$ is the noise component of the output.

Collectively, there are NM outputs after matched filtering. By coherently summing all these outputs we can get

$$\mathbf{A}(\mathbf{x}, \mathbf{x}_0, \mathbf{v}) = \left| \sum_{n=1}^N \sum_{m=1}^M y_{nm}(\mathbf{x}, \mathbf{x}_0, \mathbf{v}) \right|^2 \tag{6.10}$$

Because both τ'_{nm} and $f d_{nm}$ in (6.9) are affected by azimuth angles and elevation angels, the geometry configuration represented by a matrix $\mathbf{C}$ consisting of all the azimuth and elevation angels should be included in the ambiguity function. Ignoring the noise-based component and discarding the target reflection coefficient in (6.10), we can define the normalized ambiguity function for the proposed system as

$$\chi(\mathbf{x}, \mathbf{x}_0, \mathbf{v}, \mathbf{C}) = \frac{1}{M^2 N^2} \left| \sum_{n=1}^N \sum_{m=1}^M \exp(-j2\pi f_m \tau'_{nm}) \mathbf{A}_{mm}(\tau'_{nm}, f d_{nm}) \right|^2 \tag{6.11}$$

As τ'_{nm} and $f d_{nm}$ are related to $x-x_0$, $y-y_0$, $z-z_0$, v_x , v_y , v_z , azimuth angles and elevation angels, the range and azimuth resolution as well as the effects of velocity components and geometry configuration can be assessed through the

AF analysis.

Besides, different waveforms may obtain different $\mathbf{A}_{mm}$, thus waveforms also play a key role in the MIMO radar ambiguity function. We take Linear Frequency Modulation (LFM) waveforms as an example. A conventional LFM waveform defined by $u(t)=\text{rect}(t/T)\exp(j\pi kt^2)$ has an correlation function like [8]:

$$A(\tau, \nu)_{LFM} = \exp(-j\pi k\tau^2) \left(1 - \frac{|\tau|}{T}\right) \text{sinc} \left((\nu T - B\tau) \left(1 - \frac{|\tau|}{T}\right) \right) \quad (6.12)$$

where ν is the Doppler frequency, τ is the time-delay, k is the chirp rate, T is the pulse width, B is the bandwidth. From (6.12), it can be easily inferred that the bandwidth of single waveform adopted would also impact the performance of the MIMO-HFSW radar system.

6.3 Resolution Capacity and PSL Performance

In this section, the resolution capacity and PSL of the MIMO-HFSW radar system using sparse frequency waveform are assessed by setting the Doppler frequency to zero in AF analysis. Sparse frequency waveforms consisting of stepped frequency linear frequency modulation signals are investigated. For simplification we just study a simplified 2-dimensional configuration.

As we can see from above analysis, the ambiguity function of the proposed MIMO-HFSW radar system depends on the system geometry configuration confined by all azimuth and elevation angles. Thus, we can ignore the true

position of transmitters and receivers. We here take nine transmitters and nine receivers located evenly over spatial region of $(-\pi/4, \pi/4)$ for φ . Each transmitter will emit one LFM with an assigned start frequency. The pulse width is $100 \mu\text{s}$ for all transmitters. The bandwidth of each LFM pulse is 500 kHz. The nine start frequencies of LFM waveforms are defined as the sequence of $\{5, 6, 7, 8, 9, 8, 7, 6, 5\}$ MHz. Orthogonality can be achieved by sequentially transmitting at transmitters or by setting the first five LFM waveforms to be up-chirps and the left four down-chirps [8]. We in this chapter utilize the first mechanism. As a comparison, we also take an ambiguity function from a single LFM waveform with the same bandwidth and pulse width as well as the start frequency of 9 MHz. We also suppose to transmit it sequentially in time domain so that we can separate at each receiver the returns from different transmitters. By central coherent processing we can also get the results of AF as showed in Fig. 6.2 (b), which seems the same as that in [20].

Table 2: Resolution and Sidelobe Characters of Different Waveforms

Item	Sparse frequency waveform	LFM
Resolution along x -axis	0.31 lamda	0.58 lamda
PSL along x -axis	-14.9 dB	-3.2 dB
Resolution along y -axis	0.9 lamda	1.6 lamda
PSL along y -axis	-13.5 dB	-16.5 dB

The mesh plots of the AF are showed in Fig. 6.2 and more details on resolutions and sidelobe characteristics are given in Fig.6.3 and Table 2. As is obvious from both Fig. 6.3 and Table 2, the resolutions of MIMO-HFSW radar are at the level of one wavelength for both sparse frequency waveforms and single LFM waveform. And the resolutions of sparse frequency waveforms are even better than those of common LFM signals. This is a great improvement for azimuth resolution and even for range resolution from conventional several kilometers to several tens meters. Meanwhile, as HFSW radar always works in a highly congested spectrum environment, the sparse frequency waveform approach can provide better flexibility on choosing available channels than waveforms confined in only one channel. The sidelobes along x -axis and y -axis are well below -13 and -14 dB for sparse frequency waveforms, respectively, which is a significant improvement compared with the side lobe level from the single LFM waveform within the same channel. It demonstrates that the sidelobe levels can be suppressed by frequency diversity in random arrays [108]. This is another advantage of sparse frequency waveform.

Multiple targets case are illustrated in Fig. 6.4, where three targets are located in $[0, 0]$, $[0, 10]$, and $[-10, -10]$. The coordinate system is expressed in multiples of wavelength. As we can see, by using sparse frequency waveform set, the system can better distinguish different targets than by using waveform set in the same channel. Bandwidth effect is illustrated in Fig. 6.5. We take a set

of sparse waveforms like that mentioned above. The difference is that the bandwidth is 50 kHz for each waveform. From Fig. 6.5 we can see that even with smaller bandwidth, the resolution capacity is not much impacted. However, the PSL performance is deteriorated. The PSL in x -axis is about -12.9 dB and is about -10 dB in y -axis for this waveform set. Thus, we can see that the larger the bandwidth adopted, the lower the sidelobes in both x -axis and y -axis.

In this case study, the simulation results demonstrate that MIMO-HFSW radar with sparse frequency waveform has superior resolution than conventional HFSW radar in both range and downrange domain. Sidelobe levels can be suppressed by using sparse frequency waveforms. Further work will be focused on the suppression of sidelobe levels by waveforms with effective frequency diversity scheme.

(a) Sparse frequency waveform.

(b) LFM waveform.

Figure 6.2: AFs of the proposed system with different waveform sets

(a) x -axis.

(b) y -axis.

Figure 6.3: Resolution and sidelobe performances with different waveforms.

(a) Sparse frequency waveform.

(b) LFM waveform.

Figure 6.4: High resolution ability of 3 point targets case.

(a) x -axis.

(b) y -axis.

Figure 6.5: Bandwidth effect on resolution and sidelobe performances.

6.4 Doppler and Geometry Factor

(a) Configuration 1, (9×9).

(b) Configuration 2, (5×5).

Figure 6.6: Illustration of geometry factor in Case 1.

As HFSW radar is always operated in an environment where the echoes are subject to Doppler effect, the AF with Doppler effects should be further investigated. Meanwhile, unlike monostatic radar the distributed MIMO radar is confined by the geometry configuration. Thus the geometry factor should also be investigated. Two cases are given below to investigate the Doppler and

configuration effect.

In Case 1, we study the configuration effect. Target of this case is with velocity of $[v_x, v_y] = [500, 500]$ m/s. This is a high velocity target case corresponding to air targets. There are two configurations. For the Configuration 1, in the region of $(-\pi/4, \pi/4)$ there are nine transmitters and nine receivers, evenly distributed. The transmitting waveforms are defined as those in section III except that each one has 10 kHz bandwidth. For practical HFSW radar application, only a limited number of continuous clear channels with bandwidth of a few kilo-Hertz in the 3-30 MHz high frequency band can be found and used at a time when interference is considered [45], [84]. Thus, 10 kHz bandwidth is used for a much more similitude in real condition of the HFSW radar system. For the Configuration 2, in the same region of $(-\pi/4, \pi/4)$ there are five transmitters and five receivers, evenly distributed. The waveforms are the first five used in Configuration 1 of Case 1. Thus, the total spectra employed by these two configurations are the same. As illustrated in Fig. 6.6, Fig. 6.7, both configurations in case 1 show high resolution capabilities. However, Configuration 1 with more transmit-receive pairs shows better sidelobe performance in both x -axis and y -axis. The PSL in x -axis is about -12 dB and is about -10.5 dB in y -axis for Configuration 1. Based on our numerous simulation experiments, it is found that as more pairs of transmitter and receiver are set in a much wider spatial region, the resolutions can be slightly improved

and the PSLs of y -axis and x -axis can be further reduced. However, systematic study on the PSLs reduction through geometry optimization will be explored in the future.

(a) x -axis.

(b) y -axis.

Figure 6.7: Resolution and sidelobe performances of different configurations in

Case 1.

In Case 2: we have four velocity settings like $[0, 0]$ m/s, $[100, -100]$ m/s, $[-10, 5]$ m/s, and $[500, 500]$ m/s. The geometry configuration in Case 2 is the same as Configuration 1 in Case 1. We also take the waveform set of Configuration 1 of Case 1 in this case study. Fig. 6.8 shows the results of Case 2. We can see from Fig. 6.8 that the proposed system shows similar characteristics in different Doppler context, which means the resolution and PSL performance are both insensitive to Doppler frequency. Thus, for both high speed air targets and low velocity surface targets, the proposed system can also have high resolution performance.

Figure 6.8: Ambiguity functions of different velocity case with Configuration 1.

6.5 Concluding Remarks

In this chapter, the concept of distributed MIMO-HFSW radar using sparse

frequency waveforms is proposed. The AF of this proposed system is derived in detail. Potential advantages of the proposed system on resolution capacity and PSL performance are assessed through AF analysis and simulations. The impacts of Doppler effects and the geometry configuration factor are also studied. It has been found that the system has several distinguished characteristics. Firstly, the range resolution and the azimuth resolution can be improved to the level of one wavelength, namely, only tens meters and the PSL is reduced to a much lower level with sparse frequency waveforms. Meanwhile, the resolutions are not restricted by individual bandwidth while the PSL can benefit from large bandwidth. Secondly, the performance of fine resolution and low PSL are insensitive to the Doppler effects. Thus, for both high speed air and low velocity surface targets, the proposed system also has high performance. Thirdly, the resolution capacity and PSL performance can be optimized through geometry configuration optimization. In addition, multistatic configuration provides large flexibility to find a proper place to locate the radar transmitters and receivers; by using sparse frequency waveforms, it is much easier to find more available channels in different locations thus the co-channel interference can be avoided and the performance can be further improved. Further studies will be conducted on the surveillance strategy and high quality waveforms with better AF performance. Meanwhile, synchronization problem should also be paid special attention to so that coherent processing can be conducted perfectly.

Chapter 7 Conclusions and Future Work

7.1 Conclusions

As the sensing medium of a radar, waveforms play a key role in radar functioning. Since properties of a waveform are constricted by its ambiguity function, a fixed waveform can only be optimum to some certain applications. A method to compensate the inherent constraint on radar performance posed by waveform is to adapt the receiving processing to application objectives. However, this approach is still not complete in the sense of “global optimum”. Then, to further improve the radar ability, adaptive transmitting is required to tune the radar waveform to certain operation with optimum performance. Therefore, there comes the topic of waveform diversity. With waveform diversity, radar can choose a proper waveform for a given application and can even adapt itself with optimum waveforms for a series of commands in a dynamic operation environment in a real-time manner. Although the idea of waveform diversity is not very new and indeed has long been utilized in some applications, it has not seen so much attention from both the industry and academic until recently.

The concept of MIMO, once extensively exploited in the wireless communication field, is now attracting much attention in radar society. MIMO concept can bring traditional radar some significant benefits and gives birth to a

new radar technology called MIMO radar. When transmitting multiple independent waveforms, MIMO radar can separate different transmitting channels on each receiver such that the available degrees of freedom are significantly increased compared to traditional radar. Meanwhile, with MIMO configuration, waveform diversity can be better exploited than in a conventional radar configuration.

To exploit the advantages of these abovementioned new technologies, this thesis is focused on waveform diversity and design for radar applications, especially for MIMO radar and HFSW radar. There are five contributions. First, radar ambiguity function is extensively reviewed. Composite ambiguity function is re-investigated by studying the bounds on the volume and height distribution. Second, two flexible approaches have been developed for designing sparse frequency waveform with sidelobe constraint for both conventional radar and MIMO radar. Third, a unified framework of signal model for STAP in MIMO radar with waveform diversity is constructed and the sensitivity of MIMO STAP with waveform diversity is evaluated by the eigenvalue of waveform covariance matrix. Fourth, the clutter property of STAP in MIMO radar is investigated and several rules are developed for estimating the clutter rank of STAP in MIMO radar with waveform diversity. These rules can serve as the generalized rules for Brennan's rule and current rule for clutter rank estimation in MIMO STAP with orthogonal waveforms.

Fifth, a high resolution MIMO-HFSW radar system is proposed. Meanwhile, to reduce the incurred high sidelobe level, sparse frequency waveform scheme is proposed.

In Chapter 2, ambiguity function was extensively reviewed and re-investigated since it is the basic tool for waveform analysis and design. Till to today, there are three kinds of radar ambiguity functions: the ambiguity function for conventional radar, the composite ambiguity function, and the ambiguity function for MIMO radar. New bounds on the volume and height distribution for the composite ambiguity function were developed. It is found in this chapter that the volume clearance area is no smaller than the conventional ambiguity function. In the case of orthogonal waveform set, the volume clearance area of composite ambiguity function is K times as large as that of conventional ambiguity function. This feature reveals a fundamental limit on delay-Doppler imaging ability of a set of waveforms adopted in a manner of composite ambiguity function. The η clear area as well as the average sidelobe level of composite ambiguity function was also investigated.

In Chapter 3, the method for designing sparse frequency waveform with sidelobe constraint was presented. The single waveform design case was first studied. Unlike previous approach for designing sparse frequency waveform, the proposed method designs phase-only or phase-amplitude modulated digital waveform that satisfies simultaneously the requirement of PSD and the

requirement of sidelobe level in ACF. Two kinds of implementations were proposed. One is that can minimize the penalty function consisting of the PSD and ACF sub-penalty functions. With this method, the total energy in sidelobe area can be reduced. The other is one that minimizes the PSD sub-penalty function while keeping the sidelobe in the ACF under certain level. This method can be used to control the PSL in some challenging cases with very high PSL. Then the method was extended for designing multiple sparse frequency waveforms. The difference for these two cases lies in the construction of penalty function. For single waveform case, the sidelobe sub-penalty function is constructed based on the total power in the sidelobe area of ACF. For the multiple waveforms case, the sidelobe sub-penalty function is constructed based on both the total sidelobe power in all ACFs and that in all CCFs. That is because in MIMO radar, cross-filtering will result in cross-correlations, which have large effect on the radar performance. Some implementation issues such as quantization effect, LPI property, Doppler Tolerance, and high resolution effect were discussed as well.

In Chapter 4, a new iterative algorithm for designing single/multiple sparse frequency waveforms was developed. The basic idea of this method is to use an equivalent form of sidelobe sub-penalty function with lower order than the original one. Original sidelobe sub-penalty function is quartic, which makes it difficult to tackle by conventional methods. Equivalent form of it is available in

quadratic form. Then, combining this equivalent form with the PSD sub-penalty function, we got a total equivalent form of quadratic form. In this way, iterative algorithm was employed to find the local minimum of the function. The extension of this method to multiple waveforms case was also investigated. Compared to the method developed in the previous chapter, this method is more efficient in computation and memory but can not control precisely the PSL to below certain value.

In Chapter 5, some basic issues regarding to STAP in MIMO radar with waveform diversity were investigated to predict the performance of MIMO STAP. We constructed a unified framework of signal model with general form of waveform. The sensitivity of MIMO radar with waveform diversity was studied. It is found to be proportional to the maximum eigenvalue of waveform covariance matrix. The SINR with respect to different waveforms was also studied. A comparison with conventional STAP was performed. Then, the chapter focused on the clutter rank of STAP in MIMO radar with waveform diversity. Based on the investigation, it is found that clutter rank of orthogonal waveforms performs as the upper bound of rank of all waveforms, while coherent waveforms the lower bound. It is also found that the clutter rank of full rank waveforms is the same as orthogonal waveforms. For a special class of rank deficient waveforms, we presented three rules to predict their clutter ranks based on the rule of time-bandwidth product. Finally, a unified rule for full rank

waveforms and rank-1 waveforms was proposed, which can serve as the generalized form of current rank estimation rules for conventional STAP and MIMO STAP with orthogonal waveforms.

In Chapter 6, high resolution HFSW radar with MIMO radar configuration was proposed. The ambiguity function for the proposed radar system was constructed. Different from its counterpart of conventional radar, this ambiguity function is indeed reflecting the effect of both waveform and system configuration. Through ambiguity function analysis this MIMO-HFSW radar shows the high resolution capability which is not constrained by the bandwidth of waveforms. Meanwhile, by using sparse frequency waveforms, the sidelobe in the ambiguity function can be significantly decreased. In addition, bandwidth effect on the ambiguity function was also investigated.

7.2 Future Work

Waveform diversity and design are rich research topics nowadays. This thesis only studied sparse frequency waveform design and some basic performance evaluation of waveform diversity in MIMO radar. In the future, several points could be further exploited.

Firstly, as mentioned in Chapter 1, there are several branches for MIMO radar waveform design. The first is to design waveforms with good correlation property, i.e., good sidelobe performance. The second is to design waveforms that can meet such criterion like maximum SINR, maximum mutual

information. The third is to design waveforms that can shape a certain beam pattern. As can be seen, the study in this thesis targets the first topic. Advanced topics concerning the performance of detection, performance of target classification require waveforms that belong to the second branch. Requirement on time-energy management may also ask for waveforms that generate a desired pattern. With these requirements, modern convex optimization theory could be further exploited to design sparse frequency waveform with more complicated constraints. In addition, composite ambiguity function can obtain less ambiguity than conventional waveform ambiguity function. Can we operate sparse frequency waveform in such a manner that the total performance is evaluated with composite ambiguity function in order to gain further reduction in sidelobe? Combination of these topics with sparse frequency requirement may further lead to other research activities.

Secondly, the ultimate purpose of the sparse frequency waveform study is the implementation of the resulting waveform. Before implementation, performance evaluation with respect to practical application should be studied further. What is the criterion for acceptable stopband suppression for real application? To what extent we can sacrifice one performance to another? Meanwhile, as cognitive radar is attracting more and more attentions, dynamic spectrum sensing and access may also be further linked to static sparse frequency waveform design to better utilize the spectrum resource.

Thirdly, waveform diversity brings many advantages to MIMO STAP radar. But these advantages do not come without a price. The significant increase in the problem dimension leads to huge effort in computation of the clutter matrix inversion. Thus, effective reduced rank method that carries out sub-optimal processing may be highly wanted. Conventional pre-Doppler and post-Doppler STAP approach in both element space and beamspace should firstly be investigated in a MIMO context before new algorithms be developed. In addition, the clutter property for MIMO radar could be further exploited due to the large variety of waveforms applicable.

Fourthly, as shown in Chapter 6, MIMO radar can improve the resolution capability of HFSW Radar. Flexible configurations as well as signal processing scheme that are suitable for practical operation could be further exploited. Meanwhile, systematic method for reducing the sidelobe level in the filter output could also be studied in the future. STAP processing in this configuration could also be studied for MTI application.

Author's Publications

International Journal Papers

1. **G. H. Wang** and Y. L. Lu, "High resolution HFSWR-MIMO radar using sparse frequency waveform," *Wireless Sensor Network*, Oct. 2009.
2. **G. H. Wang**, Y. L. Lu, "Clutter rank of STAP in MIMO radar with waveform diversity," *IEEE Trans. on Signal Processing*, Feb. 2010.
3. **G. H. Wang**, Y. L. Lu, "Designing of single/multiple sparse frequency transmitting waveforms with sidelobe constraint," accepted by *IET RSN*, Oct. 2009.
4. **G. H. Wang**, Y. L. Lu, "Sparse frequency waveform design for MIMO radar," *Progress In Electromagnetics Research-B*, Feb. 2010.
5. **G. H. Wang**, Y. L. Lu, and W.X. Liu, "Sparse frequency waveform design by using PSO algorithm," under 3rd review, accepted due to revision for publication by *IEEE Trans. on Aerospace and Electronic Systems*.
6. **G. H. Wang**, Y. L. Lu, "Bounds on the generalized integrated sidelobe level in waveforms with stopbands," accepted by *Electronic Letters*.

International Conferences

1. **G. H. Wang** and Y. L. Lu, "Sparse frequency waveform design with soft power constrain using PSO algorithm," *Proc. of 2008 IEEE Radar Conference*, Rome, Italy, 26-29, May 2008.
2. **G. H. Wang** and Y. L. Lu, "High resolution MIMO-HFSWR using sparse frequency waveform," *Proc. of 9th International Conference on Signal Processing*, Beijing, China, 26-29, Oct 2008.
3. **G. H. Wang** and Y. L. Lu, "Clutter rank of MIMO STAP with a special class of waveforms," *Proc. of 2009 International Waveform Diversity & Design Conference*, Orlando, FL, USA, 8-13, Feb 2009.
4. **G. H. Wang**, Y. L. Lu, and J. P. Sun, "STAP performance analysis of MIMO radar with waveform diversity," *Proc. of 2009 IEEE Radar Conference*, Pasadena, CA, USA, 4-8, May 2009. (**Best Student Paper nominee.**)
5. Y. L. Lu, W. X. Liu and **G. H. Wang**, "Optimal Sparse Frequency Waveform Design for HF Surface Wave Radar Applications," *Proc. of International Radiowave Oceanography Workshop*, Split, Croatia, 19-22, May 2009.
6. **G. H. Wang**, Y. L. Lu, "Designing sparse frequency waveform using iterative algorithms," *Proc. International Radar Symposium, IRS-2010*, Vilnius, Lithuania, Jun 2010.

Bibliography

- [1] M. I. Skolnik, *Radar Handbook*, New York: McGraw-Hill, 1970.
- [2] C. E. Cook, and M. Bernfeld, *Radar Signals – An Introduction to Theory and Application*, New York: Academic Press, 1967.
- [3] A. W. Rihaczek, *Principles of High Resolution Radar*, New York: McGraw-Hill, 1969.
- [4] F. E. Nathanson, *Radar Design Principles: Signal Processing and the Environment*. New York: McGraw-Hill, 1969.
- [5] P. M. Woodward, *Probability and Information Theory, with Applications to Radar*, Elmsford, NY: Pergamon Press, 1953.
- [6] L. E. Brennan, “Optimum processing of unequally spaced radar pulse trains for clutter rejection,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 4, no. 3, pp.474-477, May 1968.
- [7] O. J. Jacomini, “Weighting function and transmission time optimization in video MTI systems,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 8, no. 4, pp.517-527, Jul. 1972.
- [8] N. Levanon, E. Mosezen, *Radar Signals*. New Jersey: John Wiley & Sons, 2004.
- [9] C. J. Baker and H. D. Griffiths, “Radar systems and waveforms,” *Proc. of 2004 International Waveform Diversity and Design Conference*, Edinburgh, UK, Nov. 2004.
- [10] M. C. Wicks and P. Antonik, “Waveform diversity in intelligent sensor systems,” *Proc. of the IET Forum on Waveform Diversity & Design in Communications, Radar, and Sonar*, pp.1-6, London, UK, Nov. 2006.
- [11] J. W. Garnham and J. Roman, “Why and what is waveform diversity and how does it affect electromagnetics,” *Proc. of 2007 IEEE International Symposium on Electromagnetic Compatibility*, Honolulu, HI, USA, pp.1-5, Jul. 2007.
- [12] E. Fisher, A. Haimovich, R. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, “MIMO radar: An idea whose time has come,” *Proc. of 2004 IEEE Radar Conference*, pp.71-78, Philadelphia, PA, USA, Apr., 2004.
- [13] E. Fisher, A. Haimovich, R. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, “Spatial Diversity in Radars—Models and Detection Performance,” *IEEE Trans. on Signal Processing*, vol. 20, no. 3,

pp.823-838, Mar. 2006.

- [14] D. W. Bliss, and K. W. Forsythe, "Multiple-input multiple-output (MIMO) radar and imaging: degrees of freedom and resolution," *Proc. of 37th Asilomar Conference on Signals, System, and Computers*, pp.54-59, Nov. 2003.
- [15] J. Li, "MIMO radar: diversity means superiority," *Proc. of 2006 Adaptive Sensor Array Processing*, pp.305-309, Jun. 2006.
- [16] J. Li, P. Stoica, L. Xu, and W. Roberts, "On parameter identifiability of MIMO radar," *IEEE Signal Processing Letters*, vol. 14, no.12, pp. 968-971, Dec. 2007.
- [17] P. F. Sammartino, C. J. Baker, and H. D. Griffiths, "MIMO radar performance in clutter environment," *Proc. of 2006 CIE Radar Conference*, Shanghai, China, pp. 1-4, Oct. 2006.
- [18] I. Bekkerman and J. Tabrikian, "Target detection and localization using MIMO radars and sonars," *IEEE Trans. on Signal Processing*, vol. 54, no. 10, pp.3873-3883, Oct. 2006.
- [19] E. Fisher, A. Haimovich, R. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, "Performance of MIMO radar systems: advantages of angular diversity," *Proc. of 38th Asilomar Conference on Signal, Systems, and Computers*, pp.305-309, Nov. 2004.
- [20] N. Lehmann, A. Haimovich, R. Blum, L. Cimini, "High resolution capabilities of MIMO radar," *Proc. of 40th Asilomar Conference on Signal, Systems, and Computers*, pp.25-30, Nov. 2006.
- [21] D. J. Rabideau, and P. A. Parker, *Ubiquitous MIMO Multifunction Digital Array Radar ... and the Role of Time-Energy Management in Radar*, Project Report DAR-4, Lincoln Laboratory, MIT, Dec. 2003.
- [22] D. J. Rabideau, and P. A. Parker, "Ubiquitous MIMO Multifunction Digital Array Radar," *Proc. of 37th Asilomar Conference on Signals, System, and Computers*, pp. 1057-1064, Nov. 2003.
- [23] C. Y. Chen, and P. P. Vaidyanathan, "A subspace method for MIMO radar space-time adaptive processing," *Proc. of 2007 IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp.925-928, Hawaii, HI, USA, Apr., 2007.
- [24] C. Y. Chen, and P. P. Vaidyanathan, "Beamforming issues in modern MIMO radars with Doppler," *Proc. of 40th Asilomar Conference on Signal, Systems, and Computers*, pp.41-45, Oct. 2006.
- [25] C. Y. Chen, and P. P. Vaidyanathan, "MIMO radar space time adaptive processing using prolate spheroidal wavefunctions," *IEEE Trans. on*

Signal Processing, pp. 623-635, vol. 56 , no. 2, Feb. 2008.

- [26] V. Mecca, D. Ramakrishnan, and J. Krolik, "MIMO Radar Space-Time Adaptive Processing for Multipath Clutter Mitigation," *Proc. of IEEE Workshop on Sensor Array and Multichannel Processing*, pp.249-253, Jul., 2006.
- [27] V. Mecca, J. Krolik, and F. Robey, "Beamspace slow-time MIMO radar for multipath clutter mitigation," *Proceedings of 2008 IEEE International Conference on Acoustics, Speech, and Signal Processing*, pp. 2313-2316, Apr., 2008.
- [28] D. W. Bliss, and K. W. Forsythe, S.K. Davis, G. S. Fawcett, etc., "GMTI MIMO radar," *Proc. of 2007 International Waveform Diversity and Design Conference*, pp.118-122, Feb. 2009.
- [29] G. J. Frazer, Y. I. Abramovich, B. A. Johnson, F. C. Robey, "Recent results in MIMO over-the-horizon radar," *Proc. of 2008 IEEE Radar Conference*, pp. 1-6, Roma, Italy, May 2008.
- [30] G. J. Frazer, Y. I. Abramovich, B. A. Johnson, "Multiple-input multiple-output over-the-horizon radar: experiment results," *IET Proc. on Radar Sonar and Navigation*, vol. 3, no. 4, pp. 290-303, Oct. 2009.
- [31] G. Krieger, N. Gebert, and A. Morira, "Multidimensional waveform encoding: A new digital beamforming technique for synthetic aperture radar remote sensing," *IEEE Trans. on Geoscience and Remote Sensing*, vol. 46, no. 1, pp.31-45, Jan. 2008.
- [32] H. Deng, "Polyphase code design for orthogonal netted radar systems," *IEEE Trans. on Signal Processing*, vol.52, no.11, Nov. 2004, pp.3126-3135.
- [33] H. A. Khan, Y. Y. Zhang, C. Ji, C. J. Stevens, D. J. Edwards, and D. O'Brien, "Optimizing Polyphase Sequences for Orthogonal Netted Radar," *IEEE Signal Processing Letters*, vol. 13, no.10, Oct. 2006, pp.589-592.
- [34] J. Li, P. Stoica, and X. Zhu, "MIMO radar waveform synthesis," *Proc. of 2008 IEEE Radar Conference*, pp.1-6, Roma, Italy, May, 2008.
- [35] H. He, P. Stoica, and J. Li, "Designing unimodular sequence sets with good correlations -- Including an application to MIMO radar," *To appear in IEEE Trans. on Signal Processing*.
- [36] K. W. Forsythe, and D. W. Bliss, "Waveform correlation and optimization for MIMO radar," *Proc. of 39th Asilomar Conference on Signal, Systems, and Computers*, pp.1306-1310, Oct, 2005.
- [37] B. Friedlander, "Waveform design for MIMO radars," *IEEE Trans. on*

- Aerospace and Electronic Systems*, vol. 43, no. 3, pp. 1227-1238, July 2007.
- [38] J. Li, L. Xu, P. Stoica, K. W. Forsythe, and D. W. Bliss, "Range compression and waveform optimization for MIMO radar: A Cramer-Rao bound based study," *IEEE Trans. on Signal Processing*, vol. 55, no. 8, pp. 4151-4161, Aug. 2007.
 - [39] Y. Yang and R. S. Blum, "MIMO radar waveform design based on mutual information and minimum mean-square error estimation," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 43, Jan. 2007, pp.330-343.
 - [40] P. Stoica, J. Li, and Y. Xie, "On probing signal design for MIMO radar," *IEEE Trans. on Signal Processing*, vol. 55, no. 8, pp. 4151-4161, Aug. 2007.
 - [41] G. S. Antonio, D. R. Fuhrmann, and F. C. Robey, "Transmit beamforming for MIMO radar systems using partial signal correlation," *Proc. of Asilomar Conference on Signal, Systems, and Computers*, pp.295-299, Nov, 2004.
 - [42] D. R. Fuhrmann, and G. S. Antonio, "Transmit beamforming for MIMO radar systems using signal cross-correlation," *IEEE Trans. on Aerospace and Electronic Systems*, pp. 171-186, vol. 44, no. 1, Jan. 2008.
 - [43] K. W. Gurgel, H. H. Essen, and T. Schlick, " HF surface wave radar for oceanography- a review of activities in Germany," *Proc. of 2003 International Radar Conference*, pp. 700-705, Adelaide, Australia, Sep. 2003.
 - [44] D. Barrick, "History, present status, and future directions of HF surface wave radars in the U.S.," *Proc. of 2003 International Radar Conference*, pp. 652-655, Adelaide, Australia, Sep. 2003.
 - [45] R. J. Riddolls, *A Canadian Perspective on High Frequency Over-The-Horizon Radar*, Technical Memorandum, DRDC OTTAWA TM-2006-285, Defense R&D Canada-Ottawa, 2006.
 - [46] H. Su, H. Liu, P. Shui, S. Zhang, and Z. Bao, "HFSWR based on synthetic impulse and aperture processing," *IET Proc. on Radar Sonar and Navigation*, vol. 153, no. 5, pp. 445-453, Oct. 2006.
 - [47] R. H. Khan and D. K. Mitchell, "Waveform analysis for high-frequency FMICW radar," *IET Proc. on Radar Sonar and Navigation*, vol. 138, no. 5, pp. 411-419, Oct. 1991.
 - [48] H. Leong and A. Ponsford, "The advantage of dual-frequency operation in ship tracking by HF surface wave radar," *Proc. of 2004 International Radar Conference*, Toulouse, France, pp. 652-655, Sep. 2003.

- [49] D. D. Crombie, "Doppler spectrum of sea echo at 13.56 Mc./s.," *Nature*, vol. 175, pp. 681-682, April, 1955.
- [50] E. D. Shearman, "Propagation and scattering in MF/HF Groundwave radar," *IEE Proc.-F Communications, Radar, and Signal Processing*, vol. 130, no. 7, pp. 579-591, Dec. 1983.
- [51] H. Chan, *Coherent Processing with Frequency Agility*, Technical Memorandum, DRDC OTTAWA TM-2006-263, Defence R&D Canada-Ottawa, 2006.
- [52] W. X. Liu, Y. L. Lu, and M. Leisturgie, "Optimal sparse waveform design for HFSWR system," *Proc. 2007 International Waveform Diversity and Design Conference*, pp. 127-130, May 2007.
- [53] P. M. Woodward, "Theory of radar information," *Trans. of the IRE Professional Group on Information Theory*, vol. 1, no. 1, pp. 108-113, 1953.
- [54] R., Price, and E. M., Hofstetter, "Bounds on the volume and height distributions of the ambiguity functions," *IEEE Trans. on Information Theory*, vol. 11, pp. 207-214, Apr. 1965.
- [55] C. A. Stutt, "A note on invariant relations for ambiguity and distance functions," *IRE Trans. on Information Theory*, vol.4, no. 5, pp.164-167, Dec., 1959.
- [56] C. A. Stutt, "Some results on real-part/imaginary -part and magnitude-phase relations in ambiguity functions," *IEEE Trans. on Information Theory*, vol. 10, no. 4, pp.321-327, Oct. 1964.
- [57] L. Auslander, and R. Tolimieri, "Characterizing the radar ambiguity functions," *IEEE Trans. on Information Theory*, vol. 30, no. 6, pp.246-254, Apr. 1962.
- [58] S. M. Sussman, "Least-square synthesis of radar ambiguity functions," *IEEE Trans. on Information Theory*, vol. 8, pp.246-254, Apr. 1962.
- [59] A. W. Rihaczek, "Radar waveform selection – a simplified approach," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 7, pp. 1078-1086, Nov. 1971.
- [60] D. Delong, and E. Hofstetter, "On the design of optimum radar waveforms for clutter rejection," *IEEE Trans. on Information Theory*, vol. 13, no. 3, pp. 454-463, Jul. 1967.
- [61] J. C. Guery and M. R. Bell, "Diversity waveform sets for delay-Doppler imaging," *IEEE Trans. on Information Theory*, vol. 44, no. 4, pp. 1504-1522, Jul. 1967.
- [62] Y. I. Abramovich, "Bounds on the volume and height distributions for the

- MIMO radar ambiguity function,” *IEEE Signal Processing Letters*, vol. 15, no. 3, pp. 505-508, 2008.
- [63] G. S. Antonio, D. R. Fuhrmann, and F. C. Robey, “MIMO radar ambiguity function,” *IEEE Journal of Selected Topic in Signal Processing*, vol. 1, no.1, pp. 167-177, Jun. 2007.
 - [64] G. S. Antonio, D. R. Fuhrmann, and F. C. Robey, “MIMO radar ambiguity functions,” *Proc. of 40th Asilomar Conference on Signal, Systems, and Computers*, pp.36-40, Oct. 2006.
 - [65] C. Y. Chen and P. P. Vaidyanathan, “MIMO radar ambiguity properties and optimization using frequency-hopping waveforms,” *IEEE Trans. on Signal Processing*, vol. 56, no. 12, pp.5926-5936, Dec. 2008.
 - [66] A. Pezeshki, A. R. Calderbank, S. D. Howard, and W. Moran, “Doppler resilient Golay complementary pairs for radar,” *IEEE Workshop on Statistical Signal Processing*, Madison, WI, USA, Aug. 26-29, 2007.
 - [67] A. Pezeshki, A. R. Calderbank, S. D. Howard, and W. Moran, “Doppler resilient Golay complementary waveforms,” *IEEE Trans. on Information Theory*, vol. 54, no. 9, pp. 4254-4266, Sep. 2008.
 - [68] M. J. Lindenfeld, “Sparse frequency transmit and receive waveform design,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 40, pp. 851-861, Jul. 2004.
 - [69] S.T. Smith, “Optimum phase-only adaptive nulling,” *IEEE Trans. on Signal Processing*, vol. 47, no.2, pp. 1835-1843, Feb. 1999.
 - [70] K. K. Yan, and Y. L. Lu, “Sidelobe reduction in array pattern synthesis using genetic algorithm,” *IEEE Trans. on Antennas and Propagation*, vol. 45, no. 7, pp.1117-1122, Jul. 1997.
 - [71] B. K. Yeo, and Y. L. Lu, “Array failure correction by using the genetic algorithm,” *IEEE Trans. on Antennas and Propagation*, vol. 47, no. 5, pp. 823-828, May 1999.
 - [72] W. X. Liu, Y. L. Lu, and J. S. Fu, “Data fusion of multiradar system by using genetic algorithm,” *IEEE Trans. on Aerospace and Electronic System*, vol. 38, no. 2, pp. 601-612, 2002.
 - [73] J. Kennedy, and R. Eberhart, “Particle swarm optimization,” *Proc. of IEEE Conference on Neural Networks IV*, pp.1942-1948, 1995.
 - [74] R. Eberhart, and J. Kennedy, “A new optimizer using particle swarm theory,” *Proc. of the 6th International Symposium on Micro Machine and Human Science*, pp. 39-43, Apr. 1995.
 - [75] R. C. Eberhart, and Y. Shi, “Particle swarm optimization: development, applications and resources,” *Proc. of 2001 Congress on Evolutionary*

Computation, vol. 1, pp. 81-86, 2001.

- [76] N. Levanon, and A. Freedman, "Periodic ambiguity function of CW signals with perfect periodic autocorrelation," *IEEE Trans. on Aerospace and Electronic System*, vol. 28, no.2, pp. 387-395, 1992.
- [77] D. W. Boeringer, and D. H. Werner, "Particle swarm optimization versus genetic algorithms for phased array synthesis," *IEEE Trans. on Antennas and Propagation*, vol. 52, no. 3, pp. 771-779, Mar. 2004.
- [78] J. Robinson, and Y. Rahmat-Samii, "Particle swarm optimization in electromagnetics," *IEEE Trans. on Antennas and Propagation*, vol. 52, no. 2, pp.397-407, Feb. 2004.
- [79] W. Wang, and Y. L. Lu, "Particle swarm optimization and finite-element based approach formicrowave filter design," *IEEE Trans. on Maganetics*, vol. 41, no. 5, pp. 1800-1803, May 2005.
- [80] M. M. Khodier, and C. G. Christodolou, "Linear array geometry synthesis with minimum sidelobe level and null control using Particle Swarm Optimization," *IEEE Trans. on Antenna and Propagation*, vol. 53, no. 8, pp. 2674-2679, 2005.
- [81] I. C. Trelea, "The particle swarm optimization algorithm: convergence analysis and parameter selection," *Information Processing Letters*, vol. 85, pp. 317-325, 2003.
- [82] K. E. Parsopoulos, and M. N. Vrahatis, "Recent approaches to global optimization problems through particle swarm optimization," *Natural Computing*, vol. 1, pp. 235-306, 2001.
- [83] X. H. Hu, and R. Eberhart, "Solving constrained nonlinear optimization problems with particle swarm optimization," *Proc. of the 6th World Multiconference on Systemics, Cybernetics and Informatics*, pp. 1-4, 2002.
- [84] H. W. Leong, and B. Dawe, *Channel Availability for East Coast High Frequency Surface Wave Radar Systems*, Defence R&D Canada, Technical report, DREO TR 2001-104, Nov. 2001.
- [85] P. E. Pace, *Detecting and Classifying Low Probability of Intercept Radar*, MA: Artech House, 2004.
- [86] M. A. Richards, *Fundamentals of Radar Signal Processing*, McGraw-Hill, 2005.
- [87] G. H. Wang, and Y. L. Lu, "Sparse Frequency transmit waveform design with soft-power constraint by using PSO algorithm," *Proc. of 2008 IEEE Radar Conference*, Rome, Italy, pp. 127-130, May 2008.
- [88] P. Stoica, H. He, and J. Li, "New algorithm for designing unimodular

- sequences with good correlation properties,” *IEEE Trans. on Signal Processing*, vol. 57, no. 4, pp.1415-1425, Apr. 2009.
- [89] P. Stoica, J. Li, and L. Xu “Waveform synthesis for diversity based transmit beampattern design,” *IEEE Trans. on Signal Processing*, vol. 56, no. 6, pp.2593-2598, Jun. 2008.
 - [90] J. Li, P. Stoica, and X. Zhen, “Signal synthesis and receiver design for MIMO radar imaging,” *IEEE Trans. on Signal Processing*, vol. 56, no. 8, pp.3959-3968, Aug. 2008.
 - [91] J. Tropp, I. Dhillon, R. Heath, and T. Strohmer, “Design structured tight frames via an alternating projection method,” *IEEE Trans. on Information Theory*, vol.51, no. 1, pp.188-209, Jan. 2005.
 - [92] J. Ward, *Space-Time Adaptive Processing for Airborne Radar*. MIT Lincoln Laboratory, Lexington, MA, Tech. Rep. 1015, Dec. 1994.
 - [93] R. Klemm, *Space-Time Adaptive Processing: Principles and Applications*. IEEE Press, 1988.
 - [94] L. E. Brennan, and I. S. Reed, “Theory of adaptive radar,” *IEEE Trans. on Aerospace and Electronic System*, vol. 9, no.1, pp. 237-251, 1973.
 - [95] I. S. Reed, J. D. Mallet, and L.E. Brennan, “Rapid convergence rate in adaptive arrays,” *IEEE Trans. on Aerospace and Electronic System*, vol. 10, pp. 853-863, Oct. 1974.
 - [96] H. Wang, “An overview of space-time adaptive processing for airborne radars,” *Proc. of 1996 CIE international Radar Conference*, Beijing, China, pp. 789-794, Oct. 1996.
 - [97] L. E. Brennan, J. D. Mallet, and I. S. Reed, “Adaptive arrays in airborne MTI radar,” *IEEE Trans. on Antennas and Propagation*, vol. 24, no. 5, pp. 607-615, Sep. 1976.
 - [98] R. Klemm, and J. Ender, “New aspects of airborne MTI,” *Proc. of 1990 International Radar Conference*, Arlington, VA, USA, pp. 335-340, May. 1990.
 - [99] R. Klemm, “New airborne MTI techniques,” *Proc. of 1990 International Radar Conference*, pp. 380-384, London, UK, Oct. 1987.
 - [100] J. R. Guerci, *Space-Time Adaptive Processing for Radar*, Artech House, 2003.
 - [101] Q. Zhang and W. B. Mikhael, “Estimation of the clutter rank in the case of subarraying for space-time adaptive processing,” *Electronic Letters*, vol. 33, no. 5, pp. 419-420, Feb. 1997.
 - [102] N. A. Goodman and J. M. Stiles, “ On clutter rank observed by arbitrary

- Arrays,” *IEEE Trans. on Signal Processing*, vol. 55 , no. 1, pp. 178-186, Jan. 2007.
- [103] A. D. Maio and M. Lops, “Design principles of MIMO radar detectors,” *IEEE Trans. on Aerospace and Electronic Systems*, vol. 43, no. 3, pp. 886-898, Jul. 2007.
 - [104] J. M. Flaherty, and E. Kadak, “Optimum radar integration time,” *IRE Trans. on Antenna and Propagation* , vol. 8, no. 2, pp. 183-185, Mar. 1960.
 - [105] R. Horn, and C. Johnson, *Matrix Analysis*, Cambridge University Press, 1990.
 - [106] D. Slepian, and H. O. Pollak, “Prolate Spheroidal Wave Function, Fourier Analysis and Uncertainty-III: the dimension of the space of essentially time-and-band-limited signals,” *Bell System Technical Journal*, pp. 1295-1336, July 1962.
 - [107] D. R. kirk , J. S. Bergin, P. M. Techau, and J. E. Don Carlos, “Multi-static coherent sparse aperture approach to precision target detection and estimation,” *Proc. of 2005 IEEE Radar Conference*, pp. 579-584, Washington DC, USA, May 2005.
 - [108] B. D. Steinberg and E. H. Attia, “Sidelobe reduction of random arrays by element position and frequency diversity,” *IEEE Trans. on Signal Processing*, vol. 31, no. 6, pp. 922-930, Nov. 1983.
 - [109] H. L. Van Trees, *Detection, Estimation and Modulation Theory, Part III: Radar-Sonar Signal Processing and Gaussian Signals in Noise*, John Wiley & Sons, 2001.
 - [110] J.R. Klauder, A.C. Price, S. Arlington, and W. J. Albershim, “The theory and design of chirp radars,” *Bell System Technical Journal*, vol. XXXIX, no.4, pp745-809, July, 1960.

Appendix A: Proof of (2.17)

We first show the development of (2.17) step by step as below and then give some explanation on critical steps.

$$\begin{aligned}
& \iint_{\tau f} \left| \sum_{n=0}^{N-1} \chi_{nn}(\tau, f) \right|^2 e^{-j2\pi v\tau + j2\pi\sigma f} d\tau df \\
&= \iint_{\tau f} \sum_{n=0}^{N-1} \chi_{nn}(\tau, f) \sum_{m=0}^{N-1} \chi_{mm}^*(\tau, f) e^{-j2\pi v\tau + j2\pi\sigma f} d\tau df \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \iint_{\tau f} \chi_{nn}(\tau, f) \chi_{mm}^*(\tau, f) e^{-j2\pi v\tau + j2\pi\sigma f} d\tau df \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \iint_{\tau f} \int s_n(t) s_n^*(t+\tau) e^{-j2\pi ft} dt \int s_m^*(t_1) s_m(t_1+\tau) e^{j2\pi ft_1} dt_1 e^{-j2\pi v\tau + j2\pi\sigma f} d\tau df \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \iint_{\tau f} \int s_n(t) s_n^*(t+\tau) e^{-j2\pi ft} dt \int u_m(\eta) u_m^*(\eta+f) e^{j2\pi\eta\tau} d\eta e^{-j2\pi v\tau + j2\pi\sigma f} d\tau df \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \iint_{t\eta} \int u_m^*(\eta+f) e^{-j2\pi ft} e^{j2\pi\sigma f} df \int s_n^*(t+\tau) e^{j2\pi\eta\tau} e^{-j2\pi v\tau} d\tau s_n(t) u_m(\eta) dt d\eta \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \iint_{t\eta} s_n(t) s_m^*(t-\sigma) e^{j2\pi(t-\sigma)\eta} u_m(\eta) u_n^*(\eta-v) e^{-j2\pi t(\eta-v)} dt d\eta \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \iint_{t_1\eta_1}^{t-\sigma=t_1, \eta-v=\eta_1} s_n(t_1+\sigma) s_m^*(t_1) e^{j2\pi t_1(\eta_1+v)} u_m(\eta_1+v) u_n^*(\eta_1) e^{-j2\pi(t_1+\sigma)\eta_1} dt_1 d\eta_1 \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \iint_{t_1\eta_1} s_n(t_1+\sigma) s_m^*(t_1) e^{j2\pi t_1 v} u_m(\eta_1+v) u_n^*(\eta_1) e^{-j2\pi\eta_1\sigma} dt_1 d\eta_1 \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \int s_n(t_1+\sigma) s_m^*(t_1) e^{j2\pi t_1 v} dt_1 \int u_m(\eta_1+v) u_n^*(\eta_1) e^{-j2\pi\eta_1\sigma} d\eta_1 \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \chi_{mn}^*(\sigma, v) \chi_{mn}(\sigma, v) \\
&= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \left| \chi_{mn}(\sigma, v) \right|^2
\end{aligned}$$

The development from Step 3 to Step 4 is by expressing

$\int s_m^*(t_1) s_m(t_1+\tau) e^{j2\pi ft_1}$ in terms of the corresponding frequency functions $u_m(\eta)$ and $u_m^*(\eta)$. More specifically, the derivation from Step 3 to Step 4 is as

follows:

$$\begin{aligned}
& \int s_m^*(t_1) s_m(t_1 + \tau) e^{j2\pi f t_1} dt_1 \\
&= \int \int u_m^*(\eta) e^{-j2\pi \eta t_1} d\eta \int u_m(\eta_1) e^{j2\pi \eta_1(t_1 + \tau)} d\eta_1 e^{j2\pi f t_1} dt_1 \\
&= \int \int u_m^*(\eta) u_m(\eta_1) e^{j2\pi \eta \tau} \int e^{-j2\pi \eta t_1} e^{j2\pi \eta_1 t_1} e^{j2\pi f t_1} dt_1 d\eta d\eta_1 \\
&= \int \int u_m^*(\eta) u_m(\eta_1) e^{j2\pi \eta \tau} \delta(\eta - \eta_1 - f) d\eta d\eta_1 \\
&= \int u_m^*(\eta_1 + f) u_m(\eta_1) e^{j2\pi \eta_1 \tau} d\eta_1
\end{aligned}$$

The development from Step 5 to Step 6 uses the following two Fourier

Transform based derivations:

$$\begin{aligned}
& \int u_m^*(\eta + f) e^{-j2\pi f t} e^{j2\pi \sigma f} df \\
&= \left(\int u_m(\eta + f) e^{j2\pi f(t - \sigma)} df \right)^* \\
&= \left(\int u_m(f_1) e^{j2\pi(f_1 - \eta)(t - \sigma)} df \right)^* \\
&= s_m^*(t - \sigma) e^{j2\pi(t - \sigma)\eta}
\end{aligned}$$

$$\begin{aligned}
& \int s_n^*(t + \tau) e^{j2\pi \eta \tau} e^{-j2\pi v \tau} d\tau \\
&= \left(\int s_n(t + \tau) e^{-j2\pi \tau(\eta - v)} d\tau \right)^* \\
&= \left(\int s_n(t_1) e^{-j2\pi t_1(\eta - v)} e^{j2\pi t(\eta - v)} dt_1 \right)^* \\
&= u_n^*(\eta - v) e^{-j2\pi t(\eta - v)}
\end{aligned}$$

The development from Step 9 to Step 10 is based the expression of

cross-ambiguity function in frequency domain:

$$\begin{aligned}
& \int u_m(\eta_1 + v) u_n^*(\eta_1) e^{-j2\pi \eta_1 \sigma} d\eta_1 \\
&= \int \int s_m(t) e^{-j2\pi t(\eta_1 + v)} dt \int s_n^*(t_1) e^{j2\pi t_1 \eta_1} dt_1 e^{-j2\pi \eta_1 \sigma} d\eta_1 \\
&= \int \int s_m(t) s_n^*(t_1) e^{-j2\pi t v} dt dt_1 \int e^{-j2\pi t \eta_1} e^{j2\pi t_1 \eta_1} dt_1 e^{-j2\pi \eta_1 \sigma} d\eta_1 \\
&= \int \int s_m(t) s_n^*(t_1) e^{-j2\pi t v} \delta(t - t_1 + \sigma) dt dt_1 \\
&= \int s_m(t) s_n^*(t + \sigma) e^{-j2\pi t v} dt \\
&= \chi_{mn}(\sigma, v)
\end{aligned}$$