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ACOUSTICAL TIME-REVERSAL SIGNAL PROCESSING: NEW DEVELOPMENTS AND APPLICATIONS

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SUMMARY

This dissertation focuses on new developments and applications of time-reversal (TR) signal processing techniques for acoustic waves in the case of multiple input channels. It is well known that the acoustic wave equation in a non-dissipative medium contains a second-order time-derivative operator, which in turn is invariant under a TR operation. Therefore, the TR is a novel technique that can employ back-propagation of the wave to focus energy onto a specific location in space and time. This is because back-propagated waves can retrace all of the complex paths such as reflection, refraction and multiple scattering that they underwent during the forward propagation. Thus, it results in highly focusing onto the initial source location. Based on this property, there has been a strong interest in the application of TR techniques in a broader topic of the acoustic field. Thus, the motivation of our dissertation is to develop new algorithms based on the TR technique for applying in some important topics of the acoustic field, including acoustic focusing, acoustic detection and localization, acoustic inverse scattering as well as stereophonic acoustic echo cancellation.

In particular, one of the most well-known methods using TR technique is DORT (French acronym for Décomposition de l'Opérateur de Retournement Temporel) method. This method decomposes the invariant of the time-reversal operator (TRO) obtained by measurement of the scattering data in a pulse-echo mode. Thus, the DORT method has been known as an active approach in detection and selective focusing on the well-resolved scatterers. At the beginning of this dissertation, the

parameters of the DORT method are studied to investigate the quantitative focusing performance in the well-resolved scatterer case. In the following parameter investigation, a weighted least-squares (WLS) algorithm is also introduced together with the DORT algorithm to achieve selective focusing in the non-well-resolved scatterer case. This method is called the WLS-DORT, which can achieve better spatial focusing resolution compared to the conventional DORT method, and also results in simpler computational complexity than minimum variance beamforming method under practical implementation.

Subsequently, another TR technique called time-reversal multiple-signal-classification (TR-MUSIC) is comprehensively studied and investigated in acoustic detection and localization. Specifically, the theory of TR-MUSIC algorithm is studied and compared between single and multiple scattering cases. Besides, the TR-MUSIC algorithms in both cases are investigated in different scenarios via several numerical simulations. Following that, the TR-MUSIC algorithm is also compared with the conventional MUSIC algorithm in terms of data measurement and computational complexity. In overall, the TR-MUSIC can achieve high accuracy in detecting the number of targets and super-resolution quality in localizing the position of the targets in the medium.

Furthermore, a new approach based on the conventional DORT method is developed to solve the acoustic inverse scattering problem in a small elastic scatterer case. The proposed approach not only estimates the position of an unknown scatterer but also determines the physical properties of the scatterer such as the shape (cylinder or sphere), the material (density), and the size (radius) in an anisotropic scattering

case. The numerical technique of this approach is also developed to provide a feasible solution in a far-field and small aperture condition. The advantage of this proposed approach is to avoid the complex iterative scheme in solving the direct scattering problem and results in smaller computational load as well as faster implementation.

The TR operation can also be applied to the stereophonic acoustic echo cancellation (SAEC) due to its inherent decorrelating property. A novel method of applying TR transformation on selective block signal of only one channel is presented in this thesis. Thus, the proposed method can significantly increase the convergence performance of adaptive filters and at the same time, efficiently preserve the stereophonic perception and quality. Besides, the proposed algorithms are studied in both quantitative and subjective performance in the SAEC system. Numerical simulation results are also investigated to show that the proposed method achieves faster convergence rate of misalignment and better stereophonic perception with less audio distortion compared to some other conventional approaches in the SAEC system.

Hence, in this dissertation, we have proposed and developed several new algorithms based on the TR technique to apply in different topics of the acoustic field. These algorithms can be directly implemented in some promising applications such as highly selective focusing, super-resolution target localization, high-accuracy target characterization as well as fast-convergence stereophonic acoustic echo cancellation. Besides, the proposed algorithms can be extended and adapted to some other important applications, such as nondestructive evaluation, nondestructive testing, remote sensing, and medical imaging.

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List of Abbreviations and Acronyms

AEC	Acoustic Echo Cancellation
AP	Affine Projection
AR	Autogressive
DOA	Direction-of-Arrival
DORT	Décomposition de l'Opérateur de Retournement Temporel
etc.	et cetera
i.e.	id est (that is)
ERLE	Echo-Return-Loss Enhancement
FDTD	Finite-Difference Time-Domain
FT	Fourier Transform
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
HWR	Half-Wave Rectifier
IEIR	Inter-element Impulse Response
LTI	Linear Time-Invariant
MDM	Multistatic Data Matrix
MSE	Mean-Square Error
MUSIC	MUltiple SIgnal Classification
MVBF	Minimum Variance Beamforming
NDE	Nondestructive Evaluation
NDT	Nondestructive Testing

NLMS	Normalized Least-Mean-Square
RLS	Recursive Least-Squares
RNA	Random Noise Addition
RT60	Reverberation Times
SAEC	Stereophonic Acoustic Echo Cancellation
SNR	Signal-to-Noise Ratio
SRR	Signal-to-Ripple Ratio
STRB	Selective Time-Reversal Block
SVD	Singular Value Decomposition
TMDM	Truncated Multistatic Data Matrix
TR	Time-Reversal
TRC	Time-Reversal Cavity
TRM	Time-Reversal Mirror
TR-MUSIC	Time-Reversal MULTiple Signal Classification
TRO	Time-Reversal Operator
TRP	Time-Reversal Process
TRSP	Time-Reversal Signal Processing
ULA	Uniform Linear Array
WLS	Weighted Least-Squares
WLS-DORT	Weighted Least-Squares DORT
XM	Exclusive-maximum

List of Symbols

$\mathbf{A}^{-1}$	Inverse of matrix $\mathbf{A}$
$\mathbf{A}^T$	Transpose of matrix $\mathbf{A}$
$\mathbf{A}^H$	Hermitian transpose of matrix $\mathbf{A}$
$\mathbf{I}_{N \times N}$	The $N \times N$ identity matrix
$\mathbf{0}$	Zero vector or matrix
$\mathbf{K}$	Multistatic data matrix
$\mathbf{K}^H \mathbf{K}$	Time-reversal operator
$\hat{\mathbf{R}}$	Array covariance matrix
$\mathbf{S}_\theta$	Steering vector
$\mathbf{V}$	Eigenvector
$\mathbf{w}$	Weight matrix
$\mathbf{\Lambda} = \text{diag}(\cdot)$	Diagonal matrix
$\sum$	Summation function
$\lim_{x \rightarrow \infty}$	Limitation function
$J = E\{e^2(n)\}$	MSE function
$E\{\cdot\}$	Statistical expectation
$G(\mathbf{r}, \mathbf{r}_0)$	Green's function
$O(\cdot)$	Order of complexity

$\ \cdot\ $	Euclidean norm
$ \cdot $	Absolute value
$\otimes$	Convolution
$*$	Complex conjugate
$\square$	Gradient
$p(\mathbf{r},t)$	Acoustic pressure
$\approx$	Approximately equal to
$\triangleq$	Defined as
$\gg$	Much larger than
$\ll$	Much smaller than
∞	Infinity
α	Density contrast
β	Compressibility contrast
ω	Angular frequency
λ	Wavelength
ρ	Density
σ	Singular value
Δ	Spacing
ε	Threshold
μ	Step-size parameter
δ	Regularization parameter
$\delta(n)$	Impulse response

D	Aperture size
M	Number of targets (scatterers)
N	Number of transducers
a	Radius
c	Wave speed
c^c	Scattering coefficient
f	Frequency
j	$\sqrt{-1}$
k_0	Wavenumber
t_R	Recording time
t_L	Time length

Chapter 1

INTRODUCTION

1.1 Literature Review

Time-reversal acoustic (TRA) has been shown to be a novel and efficient technique in focusing acoustic waves on targets inside a medium [1]-[3]. In the early 1990s, several TRAs were successfully implemented in time-domain experiments by Fink and his group in Laboratoire Ondes et Acoustique (LOA) [4], [5]. In a time-reversal process (TRP), the wave field is recorded simultaneously at each transducer, and then this wave field is subsequently time-reversed and retransmitted to the medium in a “last in first out” (LIFO) procedure. These time-reversed waves tend to refocus on the initial source position at a specific time. These results are obtained when the propagation medium satisfies a non-dissipative and linear time-invariant (LTI) system [6]. This condition is valid due to the fact that the acoustic wave equation in a linear and lossless medium is invariant for the time inversion transformation [7]. Besides, a perfect TRP requires the transducers to completely surround the propagation medium with a transducer spacing of $\lambda/2$, which is called time-reversal cavity (TRC) [8]. In practice, a complete TRC is impossible to build, thus a time-reversal mirror (TRM) with linear array of transducers and limited angular aperture is commonly used [9]. A plane TRM is also shown to be an efficient alternative to a TRC in a theoretical way [10]. Continuously, there are several devices built to illustrate the feasibility of the

TRM [4], [11]-[15]. These experiments employ an array of transducers, which can be operated as either transmitters or receivers. Each transducer is connected to its own electronic devices including receiving amplifier, A/D converter, storage memory, and programmable transmitter [4], [15], as shown in Fig. 1.1. These devices are able to synthesize a time-reversed version of the stored signal [2].

Figure 1.1 Electronic devices of a channel in a TRM (adapted and modified from [81]).

The TRMs have also been conducted for various experiments in different complex media, as shown in Fig. 1.2. One of the first TRM experiments was carried out in multiple scattering medium with 2000 steel rods randomly distributed and immersed in the water [16]. Despite of high-order multiple scattering paths involved, the observed focusing resolution was one-sixth of the theoretical limit for the TRM aperture in free space medium. The improvement in spatial resolution comes from the fact that the array of random rods (scatterers) acts as a “pseudo-lens” with an angular aperture much wider than the size of the TRM alone. Following this experiment, some other theoretical and experimental works have also been developed and

investigated in different multiple scattering media [17]-[19], as well as subsequently employed in telecommunication applications [20], [21].

Figure 1.2 Time-reversal signal processing developed in different media.

Moreover, the TRMs were also conducted in ultrasonic waveguides [22]. This experiment showed how a TRM was able to compensate for the distortions introduced by multiple paths in an acoustic transmission. Specifically, multiple reflections on the waveguide interfaces significantly improved the effective virtual aperture of the TRM [22], [23]. This observation was also investigated in phase conjugation for the monochromatic waves [24]-[26], as well as TRM for broadband time-domain signals [27], [28]. Thus, the TRM procedure is an efficient technique to perform focusing or nulling applications in underwater acoustic communication [29]-[32]. It is also useful to note that the main difference between a multiple scattering medium and a waveguide is that the arrival time of each wavefront in a waveguide is easily predictable based on the length and the depth of the waveguide [2], [22].

In other experiments, the TRMs have also been investigated in chaotic cavities such as elastic waves propagating in a silicon wafer. Draeger *et al.* [33]-[35] showed experimentally and theoretically that in this particular case a refocusing signal on the initial position can be obtained using only one time-reversal (TR) channel operating in a closed cavity. Moreover, the underlying TR principles in random media have been developed through wave propagation analysis and mathematical statistics [36]-[39]. Following these theoretical works, there are several numerical simulations conducted to verify the results of the TR techniques in random media [40]-[44].

Based on these intensive studies, it was shown that the TR technique can achieve super resolution and high stability in complex and random media. Thus, the TR technique can be applied to many important fields, such as medical therapy [45]-[53], underwater acoustics [29]-[32], [54]-[59], acoustic communication [60]-[63], nondestructive testing [64]-[67], geophysics [68]-[70], electromagnetism [71]-[76], biomedical engineering [77] and structural health monitoring [78], [79]. For example, in medical therapy, the TR technique has been used for the destruction of kidney or gallbladder stones [45]-[47], high-precision brain hyperthermia for local destruction of malignant brain tumors [48]-[50] and microwave breast cancer detection [51]-[53]. Besides, the TR is a promising technique for the detection and focusing applications in pulse-echo mode. Specifically, in this mode, a TRP operates in three steps [80]. In the first step, one transducer in a linear array of transducers transmits acoustic waves through the medium containing scatterers. In the second step, the waves reflecting from the scatterers are recorded by all transducers in a linear array. In the third step, the recorded waves are time-reversed and back-

propagated to the medium. The second and third steps are iterated until the waves can converge on the most reflective scatterer in the medium. This process is called the iterative TRP [13], [80]. Based on the iterative TRP, DORT (French acronym for Décomposition de l'Opérateur de Retournement Temporel) method was developed to perform detection and selective focusing on weaker targets in a multiple target medium [14], [15], [81]. The DORT method has been derived from the mathematical analysis of the ITR process and consists of the signal patterns that are invariants under a TRP. Those invariants appear as the eigenvectors of a matrix called the time-reversal operator (TRO). Thus, back-propagation of these eigenvectors can refocus signal on the corresponding scatterers in the medium. Other variant DORT methods include the beam-space DORT method which was employed in underwater environment [82], and the focused DORT method used in medical imaging [83]. Subsequently, the DORT method was also developed in time-domain for ultra-wideband electromagnetic fields [84]. Hence, due to the invariance of acoustic wave equation during a TR operation, the iterative TRP and DORT method have been developed to detect, focus and characterize an unknown target in the complex and lossless media.

In this thesis, we also exploit other intrinsic beneficial properties of the TR operation in acoustic signal processing field. This beneficial property is found from the fact that time-reversed signal is highly uncorrelated to the original signal. Thus, the TR technique can be applied to decorrelate any highly correlated signals. For example, in the case of two highly correlated signals $x_1(t)$ and $x_2(t)$, if the TR operation is only applied on signal $x_1(t)$, then the cross-correlation between $x_1(-t)$ and

$x_2(t)$ are usually much smaller compared to the original cross-correlation between $x_1(t)$ and $x_2(t)$ [85], [86]. Hence, the TR operation can efficiently reduce the high interchannel coherence among acoustic signals in multiple channels.

1.2 Motivation

Based on literature reviews conducted in the previous section, the TR signal processing is shown as a novel and robust approach to achieve detection, focusing, as well as target characterization in the acoustic fields. Specifically, in the case of multiple targets, detection and selective focusing on the desired targets is possible via the DORT method [14], [15], [81]-[84]. Initially, the DORT method was developed in the case of well-resolved and point-like scatterers embedded in the medium [14], [15]. In this case, it is known that the number of significant eigenvalues is equal to the number of the scatterers, and each eigenvector is associated to one scatterer [14]. Generally, the situation is more complex and depends on several parameters such as the number of targets, their reflectivities, their positions with respect to the array, the properties of the propagation medium, and the spatial resolution of the system. These parameters of the DORT method were investigated and implemented successfully in real-time experiments [15]. Thus, this work gave us the motivation to further investigate the parameters of the DORT method in a numerical technique and study their performances via simulation results. These works have also been conducted in a well-resolved scatterer case, which requires the distance between two nearest scatterers is larger than the classical diffraction limit [87].

However, there are still challenges in performing selective focusing via DORT method in a non-well-resolved scatterer case. The underlying reason is the coupling between two scatterers becomes significant in the non-well-resolved case. Subsequently, the selective focusing on desired scatterers can no longer be obtained with back-propagating the corresponding eigenvectors of TRO. This problem was clearly stated and analyzed in the well-known work by Prada *et al.* [83].

Furthermore, in a recent research, a time-reversal multiple signal classification (TR-MUSIC) algorithm has also been developed for acoustic detection and localization of the unknown target in the medium [88]-[91]. This algorithm uses noise subspace data, in contrast to the DORT algorithm employing signal subspace data [92], [93] to localize the target as the peak of the pseudo-spectrum. Thus, in the thesis, the TR-MUSIC algorithm has also been studied and investigated in different scenarios for both single and multiple scattering cases.

In addition, the TR-MUSIC algorithm has been applied to solve an inverse scattering problem, which is a well-known problem of determining the shape and/or the physical properties of unknown scatterers. Specifically, these solutions can localize the position and determine either the reflectivity or the polarization tensors of unknown point-like scatterers in an electromagnetic inverse scattering problem [94], [95]. However, these solutions are limited to the isotropic scattering condition where the scatterer is assumed to be point-like [94] or a very small elastic object with $k_0 a \ll 1$ (where k_0 is the wavenumber and a is the size of the scatterer) [95]. In other words, these solutions are not appropriate to solve inverse scattering problem in the case of larger scatterers, which belongs to an anisotropic scattering condition. Therefore, this

limitation provides us the motivation to develop a new approach based on the DORT method to solve an acoustic inverse scattering problem in the anisotropic scattering condition.

Besides applying TR techniques to acoustic focusing, acoustic detection and localization, as well as acoustic inverse scattering problems, we have found new application of the TR technique in stereophonic acoustic echo canceller (SAEC). The SAEC suffers from poor convergence of the adaptive filters due to the high interchannel coherence between the stereophonic (two-channel) input signals, which results in an ill-conditioning problem [96], [97]. To reduce the interchannel coherence between stereophonic signals before adaptation, several decorrelating techniques have been proposed in the past [96]-[104]. However, these decorrelation preprocessing techniques often introduce additional signals in the form of either noises [98]-[101] or distortions [96], [102]-[104] which in turn degrade either the audio quality or the stereophonic image. In particular, a survey of these techniques in the SAEC system can be found in [105]. In order to solve the SAEC problem, a TR approach is proposed in this thesis to improve the convergence rate of the adaptive filters and at the same time reduce the distortions in audio quality as well as stereophonic image.

1.3 Major Contributions of the Thesis

In this thesis, several TR techniques have been studied, analyzed and applied to solve some acoustics-related problems. Several newly proposed algorithms are investigated in different scenarios and their performances are evaluated. Here, we summarize the contributions of this thesis as follows:

- *Investigation of the DORT method under different scenarios:* The parameters of the DORT method have been studied and selective focusing performance has been investigated in different scenarios [106]. Besides, truncated multistatic data matrix (TMDM) approach has been presented to achieve faster implementation with smaller computational load requirement [107].
- *Numerical technique of the DORT method:* The DORT method has been studied and developed via numerical approach in a homogenous medium [108]. Subsequently, the numerical form of singular values and singular vectors are expressed based on the reflectivity of the scatterers and the known geometry of transducers as well as scatterers.
- *Weighted Least-Squares DORT (WLS-DORT) method for selective focusing in non-well-resolved scatterer problem:* The weighted least-square algorithm has been combined with the DORT algorithm to perform selective focusing on point-like scatterers in the non-well-resolved scatterer case [109], [110]. The proposed WLS-DORT algorithm is also compared with the minimum variance beamforming algorithm under practical implementation.
- *Time-reversal in acoustic detection and localization:* a review and study of the MUSIC and TR-MUSIC algorithms in both single and multiple scattering cases. This is followed by several comparisons between MUSIC and TR-MUSIC algorithms. Besides, the TR-MUSIC algorithm has been analyzed and investigated in different settings of signal-to-noise ratio [200].

- *The DORT solution in an acoustic inverse scattering problem:* A new approach based on the DORT method has been developed to solve an acoustic inverse scattering problem in an anisotropic scattering condition [111], [112]. The proposed approach can determine the position and the physical properties of an unknown small elastic scatterer without requiring any iterative techniques.
- *TR approach in stereophonic acoustic echo cancellation problem:* A novel algorithm of selective time-reversal block (STRB) is proposed to solve the SAEC problem with significantly increase in the convergence rate of adaptive filters and preserve the stereophonic image as well as quality [113]. Theoretical and numerical simulation results have also been studied and investigated to show that the proposed algorithm achieves faster convergence rate and better stereophonic perception with less audio distortion compared to other well-known algorithms in the SAEC system [114].

Finally, the major contributions of this thesis are highlighted with the author's publications in these areas, as shown in Fig. 1.3.

Figure 1.3 Organization of major contributions in the thesis.

1.4 Organization of the Thesis

The organization of the thesis is as follows. In Chapter 2, the TR invariance in acoustic wave equation is introduced. Subsequently, we present and review the iterative TRP, as well as the DORT method. In Chapter 3, the DORT method is

investigated in the well-resolved scatterer case with different scenarios to study their performances in selective focusing. The numerical technique of the DORT method is also developed to build a feasible solution in homogeneous medium and known array geometry system. Moreover, we propose the WLS-DORT algorithm to perform selective focusing on desired scatterers in non-well-resolved scatterer case. In Chapter 4, the MUSIC and TR-MUSIC algorithms are reviewed and investigated in different scenarios to study their performances in acoustic detection and localization. Besides, there are some comparisons and discussions between TR-MUSIC and conventional MUSIC algorithms represented in the theoretical view of subspace signal processing. In Chapter 5, a new approach based on the DORT method is presented to solve an acoustic inverse scattering problem in the anisotropic scattering condition. The proposed solution can determine the location, shape and physical properties (i.e., density and radius) of an unknown scatterer. Moreover, this chapter discusses and compares the proposed approach with some conventional solutions in acoustic inverse scattering problem. Using decorrelating property of the TR operation, the STRB algorithm is proposed to solve the SAEC problem in Chapter 6. The proposed algorithm combines the TR operation with magnitude detector to achieve fast convergence rate of adaptive filters and excellent audio quality, as well as undistorted stereophonic image. Besides, this algorithm is compared with other conventional decorrelation algorithms and evaluated under different scenarios. Finally, Chapter 7 concludes our thesis and discusses some future works related to our research topics and contributions.

CHAPTER 2

TIME-REVERSAL ACOUSTICS

For nearly twenty years, time-reversal (TR) techniques have been developed and applied in many different fields, including ultrasound medical imaging and therapy [45]-[53], underwater acoustics [29]-[32], [54]-[59], acoustic communication [60]-[63], detection of defects in solids [64]-[67], geophysics [68]-[70], electromagnetism [71]-[76], biomedical engineering [77] and structural health monitoring [78], [79]. The TR technique can be applied in the acoustics field due to the underlying physical process of acoustic wave propagation is unchanged if time is reversed. Specifically, in a non-dissipative medium, the acoustic wave equation guarantees that for every burst of wave diverging from a source, there always exists a set of waves that can precisely retrace the path of the wave back to its initial source.

Therefore, in this chapter, the basic principles of time-reversal acoustics (TRAs) are described in Section 2.1. Using TRAs principles, there are some theoretical derivations and analyses in the development of two important TR techniques, including iterative TRP and the DORT method. In particular, the theory of the iterative TRP is described in Section 2.2, and the development of the DORT method based on the iterative TRP is discussed and explained in Section 2.3. Finally, Section 2.4 summarizes and concludes this chapter.

2.1 Time-Reversal Acoustics

In a non-dissipative medium, the pressure waves satisfy the acoustic wave equation [7]:

$$\nabla \left(\frac{\nabla p(\mathbf{r}, t)}{\rho(\mathbf{r})} \right) - \frac{1}{\rho(\mathbf{r})c^2(\mathbf{r})} \frac{\partial^2 p(\mathbf{r}, t)}{\partial t^2} = 0 \quad (2.1)$$

where $p(\mathbf{r}, t)$ is the acoustic pressure field (scalar), $\rho(\mathbf{r})$ is the density, $c(\mathbf{r})$ is the wave speed, $\mathbf{r}$ is the spatial vector, t is the time and

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}, \quad (2.2)$$

is the gradient expressed in the spatial coordinates (x, y, z) . The acoustic wave equation (2.1) consists of only a second-order time-derivative operator. This property shows that the TR principle can be applied in the acoustic wave equation. In particular, if $p(\mathbf{r}, t)$ is a pressure field solution of the acoustic wave equation, then $p(\mathbf{r}, -t)$ is also a solution of this problem. However, the second-order time-derivative property is only valid in a non-dissipative medium. If acoustic wave propagation is affected by dissipation effects, an odd-order time-derivative appears in the wave equation and the TR invariance is lost. Nevertheless, in a common case where the absorption coefficient is sufficiently small in the frequency bandwidth of the waves used for the experiments, the TR invariance still remains valid. This beautiful symmetry has been illustrated by Stokes in the classical case of plane wave reflection and transmission through a plane interface between two layers of different wave speeds [117]. Hence, in theory, there always exists a dual solution $p(\mathbf{r}, t)$ (forward direction) and $p(\mathbf{r}, -t)$ (backward direction) to satisfy the acoustic wave equation, as

described in Fig. 2.1. This means that the acoustic wave equation is invariant under a TR operation.

Figure 2.1 Description of a dual solution in the acoustic wave equation (adapted and modified from [118]).

Considering an impulse response $\delta(t)$ at source $\mathbf{r}_0$ with initial time $t_0 = 0$, (2.1) can be transformed as [118]

$$\left\{ \nabla \left(\frac{\nabla}{\rho(\mathbf{r})} \right) - \frac{1}{\rho(\mathbf{r})c^2(\mathbf{r})} \frac{\partial^2}{\partial t^2} \right\} G(\mathbf{r}, \mathbf{r}_0; t) = -\delta(\mathbf{r} - \mathbf{r}_0) \delta(t). \quad (2.3)$$

Thus, the causal solution to (2.1) is the causal Green's function $G(\mathbf{r}, \mathbf{r}_0; t)$ that takes into account of the heterogeneities and boundaries of the medium [119]. It is useful to note that the causal Green's function is only selected while the anti-causal Green's function $G(\mathbf{r}, \mathbf{r}_0; -t)$ is neglected in respect to the causality in physics. Thus, the goal of TR experiment is to generate the anti-causal Green's function by modifying the

initial conditions. In the case of an acoustic wave, the TR experiment can be achieved with large array of transducers, which can perform as either transmitter or receiver in a wave propagation process [4], [5], [13]-[15].

2.2 Iterative Time-Reversal Process

In this section, the TRP and iterative TRP have been introduced and analyzed in a theoretical way. The TRP can be applied to focus signal on one scatterer in the case of single scatterer in the medium. On the other hand, the iterative TRP can be employed to focus signal on the most reflective scatterer in the case of multiple scatterers in the medium.

2.2.1 Time-reversal process

Based on the TR invariance of the acoustic wave equation in non-dissipative medium, the TRP is developed to achieve self-focusing on a reflective target [80]. The idea is to convert a divergent wave issue from the reflective target into a convergent wave focused on the target. In particular, the TRP includes three steps that are listed below

- (1)*Transmission step* – A first incident wave is transmitted into the medium by the array of transducers;
- (2)*Reception step* – The waves reflected by the target are measured with the same array of transducers and the corresponding signals are recorded in a storage memory;
- (3)*Retransmission step* – These signals are time-reversed and transmitted to the programmable generators before retransmitting a second wave through the medium, and consequently this wave focuses onto the target.

Hence, the TRP can perform self-focusing on a reflective target without requiring a *priori* knowledge of the medium. The description of TRP can be illustrated in the Fig. 2.2.

Figure 2.2 Description of the TRP (adapted and modified from [80]).

2.2.2 Iterative time-reversal process

In the case of the medium containing several reflective targets, the TRP may not lead to a one point focused wave but this process can efficiently be iterated in order to focus on the most reflective target. The underlying reason is the wave transmitted after one TRP leads to a second reflected wave that can also be measured and time-reversed [80]. Thus, in the iterative TRP, the second and third steps of the TRP are iterated until the waves can highly converge on the most reflective scatterer in the medium, as shown in Fig. 2.3.

In order to describe the concept clearly, let us analyze the iterative TRP in the case of two point-like targets A and B with their reflectivity a and b (where $a > b$), respectively. These targets are placed symmetric with respect to the array of transducers and also the same distance from the array in linear time-invariant (LTI) medium. Besides, these targets are well-resolved and satisfy single scattering conditions. In the first iteration, the time-reversed wave can be expressed as the sum of a wave focused on the target A of relative amplitude a and a wave focused on the target B of relative amplitude b . At the second iteration, those two waves are reflected by the targets A and B that produce two spherical waves of relative amplitude a^2 and b^2 , respectively. Thus, after n iterations of the TRP, the relative amplitudes of reflected signals from the targets A and B are a^n and b^n , respectively. Hence, if the number of iteration n is large enough, the reflected signals from the target B becomes insignificantly compared to the reflected signals from the target A due to their different reflectivity, i.e., $b < a$. Hence, the iterative TRP automatically learns how to focus on the target A, which is the most reflective target in the medium.

Figure 2.3 Description of iterative TRP in the case of two point-like targets A and B in the LTI medium.

2.2.3 Time-reversal operator

Following the description of the principle behind the iterative TRP, we next explain the process of iterative TRP in mathematical framework that formulates transmit and receive operation with an array of transducers. In particular, the received signals are expressed as a function of the transmitted ones in a scattering medium. The whole system is considered as a LTI system. Besides, there are some assumptions made including linear operation of the transducers and linear acoustic properties in wave propagation.

2.2.3.1 Inter-element impulse response

In order to express the received signals as a general function of the transmitted ones, an inter-element impulse response (IEIR) is defined for each pair of transducers [80]. The IEIR includes all the propagation effects through the medium and the acousto-electric responses of the two elements. In particular, the IEIR $k_{lm}(t)$ is defined as the signal received on the l^{th} channel after a temporal delta function $\delta(t)$ is applied to the m^{th} channel, as shown in Fig. 2.4.

Figure 2.4 IEIR measurements in the pulse-echo mode.

Let $e_m(t)$, where $1 \leq m \leq N$ with N is the number of transducers, be the N input signals. Then, the output signals $r_l(t)$, $1 \leq l \leq N$, are given by the following equation

$$r_l(t) = \sum_{m=1}^N k_{lm}(t) * e_m(t), \quad (2.4)$$

where $*$ denotes the convolution in time.

A temporal Fourier transforms leads to the relation:

$$R_l(\omega) = \sum_{m=1}^N \int_{-\infty}^{+\infty} k_{lm}(t) * e_m(t) e^{-j\omega t} dt = \sum_{m=1}^N K_{lm}(\omega) E_m(\omega). \quad (2.5)$$

Equation (2.5) can be expressed in a matrix form:

$$\mathbf{R}(\omega) = \mathbf{K}(\omega) \mathbf{E}(\omega), \quad (2.6)$$

where $\mathbf{E}(\omega)$ and $\mathbf{R}(\omega)$ are the $N \times 1$ vectors of the transmitted and received signals at array of transducers and $\mathbf{K}(\omega) = (K_{lm}(\omega))_{1 \leq m, l \leq N}$ is known as a multistatic data matrix (MDM) with a dimension of $N \times N$.

The reciprocity theorem indicates that the point-like source and the observer can be reversed without altering the acoustic field [3]. A consequence of this theorem is that the IEIR of the m^{th} transducer to the l^{th} transducer is equal to the one of the l^{th} transducer to the m^{th} transducer, so that $K_{lm} = K_{ml}$. In other words, the matrix $\mathbf{K}$ is symmetrical. It is useful to note that the spatial reciprocity is considered in the mathematical analysis of the iterative TRP and DORT method in the following parts.

2.2.3.2 Mathematical formulations of the iterative TRP

The TRP can be described at a given frequency. Thus, for simplicity, the term ω is omitted in the following explanation. Let $\mathbf{E}_0$ be the initial input vector signal. The resulting output signal is $\mathbf{R}_0 = \mathbf{K}\mathbf{E}_0$. It is interesting to note that the TR operation in the time domain is equivalent to a phase conjugation in the frequency domain [1]. Thus, in the iterative TRP, the new input signal $\mathbf{E}_1$ at array of transducers is the phase conjugate of the preceding output signal $\mathbf{R}_0$, and $\mathbf{E}_1$ can be expressed as

$$\mathbf{E}_1 = \mathbf{K}^* \mathbf{E}_0^* . \quad (2.7)$$

This results in the new output signal as

$$\mathbf{R}_1 = \mathbf{K}\mathbf{K}^* \mathbf{E}_0^* . \quad (2.8)$$

In the same way, the input signals at the second and third iterations are

$$\mathbf{E}_2 = \mathbf{R}_1^* = \mathbf{K}^* \mathbf{K} \mathbf{E}_0 , \quad (2.9)$$

$$\mathbf{E}_3 = \mathbf{K}^* \mathbf{E}_2^* = \mathbf{K}^* \mathbf{K} \mathbf{K}^* \mathbf{E}_0^* . \quad (2.10)$$

Therefore, these formulae can be generalized by the induction technique. Specifically, the input signal at the iteration number $2n$ (even iteration) and $2n+1$ (odd iteration) are given by

$$\mathbf{E}_{2n} = \left[\mathbf{K}^* \mathbf{K} \right]^n \mathbf{E}_0 , \quad (2.11)$$

$$\mathbf{E}_{2n+1} = \left[\mathbf{K}^* \mathbf{K} \right]^n \mathbf{K}^* \mathbf{E}_0^* . \quad (2.12)$$

Hence, the iterative TRP depends on the behavior of the invariant operator $\mathbf{K}^* \mathbf{K}$, which is called the time-reversal operator (TRO).

The symmetric property of MDM $\mathbf{K}$ implies that the TRO $\mathbf{K}^* \mathbf{K}$ is a Hermitian matrix since

$$[\mathbf{K}^* \mathbf{K}]^H = [\mathbf{K} \mathbf{K}^*]^T = [\mathbf{K}^*]^T [\mathbf{K}]^T = \mathbf{K}^* \mathbf{K}, \quad (2.13)$$

where $\mathbf{K}^T$ is the transpose of MDM $\mathbf{K}$, $\mathbf{K}^*$ is the complex conjugate of MDM $\mathbf{K}$, and $\mathbf{K}^H$ is the transpose conjugate of MDM $\mathbf{K}$. According to the Hermitian theory, the TRO $\mathbf{K}^* \mathbf{K}$ can be diagonalized. Its eigenvectors are orthogonal and its eigenvalues are real [120]. Besides, using the eigen-decomposition technique, we obtain

$$\|\mathbf{K}\mathbf{V}\|^2 = [\mathbf{K}\mathbf{V}]^H [\mathbf{K}\mathbf{V}] = \mathbf{V}^H \mathbf{K}^* \mathbf{K} \mathbf{V} = \mathbf{V}^H \boldsymbol{\sigma} \mathbf{V} = \sigma \|\mathbf{V}\|^2, \quad (2.14)$$

where σ is the eigenvalue of TRO $\mathbf{K}^* \mathbf{K}$ and $\mathbf{V}$ is the associated eigenvector. Equation (2.14) implies that the eigenvalues of TRO are positive.

Assume that the TRO $\mathbf{K}^* \mathbf{K}$ has p ($p \leq N$) distinct eigenvalues arranged as $\sigma_1 > \sigma_2 > \dots > \sigma_p$ with their associated eigenspaces of $\Omega_1, \Omega_2, \dots, \Omega_p$ [15]. In fact, an eigenspace is the vector space of eigenvectors associated to the same eigenvalue and any vector can be written as the sum of vectors of the eigenspaces. Thus, writing the first input vector $\mathbf{E}_0$ as the sum of vectors of the eigenspaces, one obtains

$$\mathbf{E}_0 = \mathbf{V}_1 + \mathbf{V}_2 + \dots + \mathbf{V}_p, \quad (2.15)$$

where $\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_p$ are different eigenmodes of the iterative TRP and $\mathbf{V}_j \in \Omega_j, 1 \leq j \leq p$.

After $2n$ (even) iterations of the iterative TRP, the new input signal can be expressed as

$$\begin{aligned} \mathbf{E}_{2n} &= [\mathbf{K}^* \mathbf{K}]^n \mathbf{E}_0 \\ &= \sigma_1^n \mathbf{V}_1 + \sigma_2^n \mathbf{V}_2 + \dots + \sigma_p^n \mathbf{V}_p. \end{aligned} \quad (2.16)$$

If $\mathbf{V}_1 \neq 0$ and for large number of iteration n with σ_1 being the largest eigenvalue, (2.16) can be approximated as

$$\mathbf{E}_{2n} \approx \sigma_1^n \mathbf{V}_1. \quad (2.17)$$

On the other hand, after $2n+1$ (odd) iterations of the iterative TRP, the new input signal can be expressed as

$$\begin{aligned} \mathbf{E}_{2n+1} &= [\mathbf{K}^* \mathbf{K}]^n \mathbf{K}^* \mathbf{E}_0 \\ &= \sigma_1^n \mathbf{K}^* \mathbf{V}_1 + \sigma_2^n \mathbf{K}^* \mathbf{V}_2 + \dots + \sigma_p^n \mathbf{K}^* \mathbf{V}_p. \end{aligned} \quad (2.18)$$

Thus, for large values of n , (2.18) can be approximated as

$$\mathbf{E}_{2n+1} \approx \sigma_1^n \mathbf{K}^* \mathbf{V}_1. \quad (2.19)$$

Hence, these results in (2.17) and (2.19) ensure the convergence of the iterative TRP towards odd and even limit. Besides, this analysis shows that the iterative TRP has eigenmodes that are eigenvectors of the TRO $\mathbf{K}^* \mathbf{K}$. Specifically, the

eigenvector $\mathbf{V}_1$ associated to the largest eigenvalue among all scatterers can be selected to focus on the most reflective target in the medium [80].

2.3 Decomposition of Time-Reversal Operator Method

In the previous section, the iterative TRP was presented and its ability in focusing on the most reflective scatterer in the medium was illustrated. However, in multiple scatterers' situations, the iterative TRP cannot perform focusing on weaker scatterers. Thus, in this section, the DORT method is introduced to provide a solution to this problem. In particular, the DORT method was derived from the theoretical study of iterative TRP and offline calculation of an array response measurement in a pulse-echo mode [15].

2.3.1 Analysis of time-reversal operator for point-like scatterers

In the case of point-like scatterers, the MDM $\mathbf{K}$ can be derived as follows. We assume that the array of transducers is made of N similar elements with uniformly linear configuration. The medium consists of M point-like (Rayleigh) scatterers with complex frequency-dependent reflectivity coefficients of $C_1(\omega)$, $C_2(\omega)$, ..., $C_M(\omega)$ [120]. Thus, the MDM $\mathbf{K}$ can be written as the product of three matrices including

- (a) a propagation matrix $\mathbf{H}$ that describes the transmission and the propagation from the array of transducers to the scatterers,

- (b) a scattering matrix $\mathbf{C}$ which is diagonal in the case of single scattering process; and
- (c) a back-propagation matrix $\mathbf{H}^T$ that describes the propagation from the scatterers to the array of transducers.

2.3.1.1 Time-reversal operator

Let $h_{il}(t)$ be the diffraction impulse response of the l^{th} transducer to the i^{th} scatterer. Additionally, let $a_e(t)$ and $a_r(t)$ be the transducer acousto-electrical response in the emission and reception, with Fourier transforms $A_e(\omega)$ and $A_r(\omega)$, respectively. If the input signal at the l^{th} transducer is $e_l(t)$, then the pressure at the i^{th} scatterer is

$$p_i(t) = \sum_{l=1}^N h_{il}(t) * a_e(t) * e_l(t). \quad (2.20)$$

Equation (2.20) can be written in the frequency domain as

$$P_i = A_e \sum_{l=1}^N H_{il} E_l, \quad (2.21)$$

where $1 \leq i \leq M$.

Thus, with M point-like scatterers, we can express the pressure in matrix notation as

$$\mathbf{P} = A_e \mathbf{H} \mathbf{E}, \quad (2.22)$$

where $\mathbf{E}$ is the input signal vector of dimension $N \times 1$ at array of transducers, $\mathbf{P}$ is a vector of dimension $M \times 1$ representing the pressure received by M point-like scatterers, and $\mathbf{H}$ is a matrix of dimension $M \times N$, which is called the propagation matrix. Hence, the condition of the propagation matrix $\mathbf{H}$ depends on number of the scatterers M and number of the transducers N in the medium.

In the case of a single scattering, the pressure reflected by the i^{th} scatterer is $Q_i = C_i P_i$. Thus, the vectors $\mathbf{P}$ and $\mathbf{Q}$ are linked together by a diagonal matrix $\mathbf{C}$ of dimension $M \times M$ as

$$\mathbf{Q} = \mathbf{C}\mathbf{P}, \quad (2.23)$$

where $C_{ij} = \delta_{ij} C_i$ for all $i, j \in 1, \dots, M$ and δ is a delta function.

According to the spatial reciprocity principle, when the target emits a delta signal $\delta(t)$, the l^{th} transducer receives the signal $h_{il}(t)$. Thus, the back-propagation matrix is the transpose of the propagation matrix $\mathbf{H}$ so that the vector of the pressure reflected by M point-like scatterers, $\mathbf{Q}$, is linked to the output signal $\mathbf{R}$ by the relation

$$\mathbf{R} = A_r \mathbf{H}^T \mathbf{Q}. \quad (2.24)$$

By combining (2.22), (2.23) and (2.24), the MDM $\mathbf{K}$ can be expressed as

$$\mathbf{K} = A_e A_r \mathbf{H}^T \mathbf{C} \mathbf{H}. \quad (2.25)$$

In the following sections, the acousto-electrical responses will be omitted as they are factorized into the operator. Thus, the simple formulation of a MDM $\mathbf{K}$ in point-like scatterers' case is

$$\mathbf{K} = \mathbf{H}^T \mathbf{C} \mathbf{H}. \quad (2.26)$$

2.3.1.2 Decomposition of time-reversal operator

If the input signals at array of transducers are chosen as $\mathbf{H}_i^* = [H_{i1}^* \ H_{i2}^* \ \dots \ H_{iN}^*]^T$, then after reflecting on the scatterers, the received signals at array of transducers are

$$\mathbf{R} = \mathbf{K} \mathbf{H}_i^*. \quad (2.27)$$

In particular, the l^{th} element of the vector $\mathbf{R}$ of dimension $N \times 1$ can be expressed as

$$R_l = \sum_{m=1}^N K_{lm} H_{im}^* . \quad (2.28)$$

From (2.26), we have

$$K_{lm} = \sum_{k=1}^d H_{kl} C_k H_{km} . \quad (2.29)$$

Substituting (2.29) into (2.28), we can obtain

$$\begin{aligned} R_l &= \sum_{m=1}^N K_{lm} H_{im}^* = \sum_{m=1}^N \sum_{k=1}^d H_{kl} C_k H_{km} H_{im}^* \\ &= \sum_{k=1}^d H_{kl} C_k \sum_{m=1}^N H_{km} H_{im}^* . \end{aligned} \quad (2.30)$$

The summation over m in (2.30) can also be expressed as

$$\sum_{m=1}^N H_{km} H_{im}^* = \mathbf{H}_k^T \mathbf{H}_i^* . \quad (2.31)$$

It is interesting to note that the scatterers are well-resolved when it is possible to focus on one of the scatterers without sending energy to the others [14]. This condition is satisfied when the distance between two nearest scatterers is larger than classical diffraction limit [15]. In this case, the vectors $\mathbf{H}_k$ and $\mathbf{H}_i$ are orthogonal when $k \neq i$. Thus, in the well-resolved scatterers' case, (2.31) becomes

$$\sum_{m=1}^N H_{km} H_{im}^* = \begin{cases} 0 & k \neq i \\ \sum_{m=1}^N |H_{im}|^2 & k = i \end{cases} . \quad (2.32)$$

The proof of this above orthogonality property can be found in Appendix D.

Substituting (2.32) into (2.30), we obtain

$$R_l = \sum_{m=1}^N K_{lm} H_{im}^* = H_{il} C_i \sum_{m=1}^N |H_{im}|^2 . \quad (2.33)$$

Equation (2.33) can be written in matrix form as

$$\mathbf{R} = \mathbf{K}\mathbf{H}_i^* = C_i \sum_{m=1}^N |H_{im}|^2 \mathbf{H}_i. \quad (2.34)$$

Performing conjugate of matrix $\mathbf{R}$, we have

$$\mathbf{R}^* = \mathbf{K}^* \mathbf{H}_i = C_i^* \sum_{m=1}^N |H_{im}|^2 \mathbf{H}_i^*. \quad (2.35)$$

From (2.35), we deduce

$$\mathbf{K}^* = C_i^* \sum_{m=1}^N |H_{im}|^2 \mathbf{H}_i^* \mathbf{H}_i^{-1}. \quad (2.36)$$

Multiplying the left and right terms of (2.36) with the left and right terms of (2.34), respectively, which lead to the following equation

$$\mathbf{K}^* \mathbf{K} \mathbf{H}_i^* = C_i^2 \left(\sum_{m=1}^N |H_{im}|^2 \right)^2 \mathbf{H}_i^*. \quad (2.37)$$

Thus, the eigenvalues of TRO $\mathbf{K}^* \mathbf{K}$ corresponding to the i^{th} scatterer can be deduced as

$$\sigma_i = |C_i|^2 \left(\sum_{m=1}^N |H_{im}|^2 \right)^2. \quad (2.38)$$

As can be seen in (2.38), the eigenvalue depends on the reflectivity of the scatterer and the propagating effects. Furthermore, the eigenvector associated with the eigenvalue σ_i is $\mathbf{H}_i^* = [H_{i1}^* \ H_{i2}^* \ \dots \ H_{iN}^*]^T$.

Hence, in the well-resolved and point-like scatterers' case, the number of significant eigenvalues corresponds to number of scatterers in the medium. Specifically, each eigenvector of the TRO $\mathbf{K}^* \mathbf{K}$ is associated with one of the scatterer. It is also exactly the vector signal that would be emitted after one TRP when only one scatterer exists in the medium.

2.3.2 DORT method

In this section, the DORT method can be deduced from the preceding theoretical results. The DORT method is an experimental technique that includes a mathematical processing of the measured data [14], [15]. This method consists of three steps to perform selective focusing signal on desired scatterers, as shown in Fig. 2.5. The main aims of the first and second steps are to detect the scatterers in the medium while the third step focuses the eigenvector signal onto the desired scatterer [15].

Figure 2.5 Description of the DORT method.

In the first step, the IEIR is measured. This measurement requires N transmit-receive operations, as shown in Fig 2.6. The first transducer of the array is excited with a signal $\delta(t)$. The signals received by all N transducers are stored. This operation is repeated for all the transducers of the array with the same transmitted signal $\delta(t)$. The components of MDM $\mathbf{K}$ of dimension $N \times N$ are obtained by Fourier transforms

of each signal. This measurement can also be carried out with any multiplexed system using N transmit-receive operations.

Figure 2.6 Description of the IEIR process.

In the second step, the TRO $\mathbf{K}^*\mathbf{K}$ is diagonalized at a chosen frequency. For this computation, the matrix has to be symmetrical; however, this requirement is normally not achieved in the experiments. In fact, the reciprocity principle is exactly unsatisfied due to noise and also the differences between the acousto-electrical responses of the transducers. Thus, in the DORT experiments, it is useful to make the matrix $\mathbf{K}$ symmetrical, i.e., by replacing each element K_{lm} with $1 \leq l, m \leq N$ by $\frac{1}{2}(K_{lm} + K_{ml})$ [15]. Thus, the TRO $\mathbf{K}^*\mathbf{K}$ can be calculated and diagonalized to obtain eigenvalues and eigenvectors. In the case of well-resolved, isotropic and point-like scatterers, the number of significant eigenvalues is exactly equal to the number of scatterers in the medium. Besides, each eigenvector associated to eigenvalue of the TRO corresponds to a scatterer in the media [14].

In the third step, the signal at array of transducers is back-propagated using the phase and amplitude provided by the eigenvector found in the second step. The signals focus only on the corresponding scatterer in the medium. For example, the phase and amplitude of the eigenvectors $\mathbf{V}_1, \mathbf{V}_2$ or $\mathbf{V}_3$ can be selected and applied to array of transducer to focus on the scatterer 1, scatterer 2 or scatterer 3, respectively, as shown in Fig. 2.7. This step can be done either experimentally [14], [15], [121] or numerically [93], [108], [122]. Specifically, the experimental back-propagation requires programmable generators controlled in parallel in the TRM. Pulsed signals are built from the monochromatic data given by the eigenvectors. Thus, the experimental technique allows one to transmit a focused wave on one of the scatterers

without requiring the well-known geometry of the array of transducers or the properties of the propagating medium.

Figure 2.7 Selective focusing via the DORT method on (a) scatterer 1, (b) scatterer 2 and (c) scatterer 3.

2.4 Conclusions

This chapter focused on the introduction and review of TR principle in the acoustic field. The acoustic wave equation is invariant under a TR operation. Thus, the TR technique is applied to retrace the multiple paths to refocus acoustic waves on the initial position in complex media. Besides, the TR method can be proven as an optimal focusing method in inhomogeneous media [206]. Specifically, two well-known methods have been studied and developed from TR technique including iterative TRP and DORT method.

The iterative TRP is an experimental technique with the ability of focusing wave on the most reflective scatterer in the case of multiple scatterers. This technique is a promising solution in realistic applications since it does not require a *priori* knowledge of the medium. Thus, the iterative TRP can be adapted with complex media such as human body, underwater environments, etc. However, the iterative TRP requires the use of programmable generators in the array of transducers to perform the TR operation and iterative processes.

On the other hand, the DORT method provides an elegant way of focusing on weaker reflective scatterers in multiple scatterers' media. This method was derived from a theoretical study of the iterative TRP. It was shown that the TRO can be diagonalized into eigenmodes. Specifically, in the case of well-resolved scatterers, each eigenvector associated to a significant eigenvalue provides the phase law to be applied to the array of transducers in order to focus on the corresponding scatterer. Hence, the DORT method consists of the scattering data measurement and

computational processing to determine the eigenvectors, as well as employs them for detection and selective focusing applications.

Hence, the iterative TRP and DORT method are powerful tools for detection and focusing in multiple targets' media [13]-[15], [80], [81]. Thus, these methods create several axes of research interests and applications in the scattering environment. Specifically, the iterative TRP has been employed in medical treatment to detect and destroy kidney or gallbladder stones [45]-[47]. On the other hand, the DORT method has been used in medical imaging to detect the microcalcification in the breast [123]. The DORT method is also applied to nondestructive testing for finding defects that are close to the interfaces in solid waveguides [121] and underwater applications for mine countermeasures [124]. Subsequently, the DORT method is also investigated and developed to perform selective focusing signal on the desired targets in the following chapter.

CHAPTER 3

TIME-REVERSAL TECHNIQUE IN ACOUSTIC FOCUSING PROBLEM

With the introduction of the DORT method in Chapter 2, active detection and selective focusing on the desired target using array of transducers are now becoming possible. The DORT method is an analysis technique using the IEIRs in the pulse-echo measurements (or reflection data). From these responses, the MDM and TRO can be deduced and diagonalized. Each pair of eigenvalue and eigenvector of the TRO is associated to a corresponding point-like scatterer in the medium. Subsequently, back-propagation of these eigenvectors performs selective focusing signal on different scatterers.

In this chapter, we study the focusing performance of the DORT method by investigating various parameters, such as array size (or number of transducers), array aperture, recording time, etc. A graphical simulation environment is being created to evaluate the performance of the DORT method in different scenarios based on Finite-Difference Time-Domain (FDTD). In order to save the recording time and memory size as well as computational load, a truncated version of MDM (TMDM) approach is proposed. The appropriate parameters' selection in the TMDM approach can provide significant set-up conditions to implement the DORT method in practical cases.

Moreover, the DORT method has been studied in two configurations of point-like scatterers in the medium, which are well-resolved and non-well-resolved scatterer cases. Specifically, in the well-resolved scatterer case, the numerical technique of the DORT method has been developed to provide a feasible solution in numerical implementation. Besides, a combination method between weighted least-squares and DORT algorithms (WLS-DORT) has been proposed to achieve better focusing in non-well-resolved scatterer case.

This chapter is organized as follows. Firstly, the key parameters of the DORT method are discussed in Section 3.1. Numerical simulations are carried out to evaluate the contributions of these parameters under different scenarios. In Section 3.2, the TMDM approach is proposed to reduce computational time and memory size in its implementation. In Section 3.3, the numerical technique of the DORT method is developed to detect and focus on the scatterers in well-resolved scatterer case. Subsequently, the WLS-DORT method is proposed to perform focusing on one scatterer in non-well-resolved scatterer case in Section 3.4. Finally, Section 3.5 summarizes and concludes this chapter.

3.1 Investigation of the DORT method

In this section, some important parameters of the DORT method are studied and their performances are investigated. This investigation is conducted by using Matlab simulation. Subsequently, these simulation results are analyzed to verify the effects of the DORT parameters on the acoustic focusing performance.

3.1.1 DORT method

The DORT method is an experimental technique that includes a mathematical processing of the measured data [14], [15]. This method comprises of three steps to selectively focus signal on each scatterer. The first and second steps deal with the detection of scatterers, while the aim of the third step is to focus the signal on the targeted scatterer.

Specifically, in the first step, the IEIRs are measured. This measurement requires N transmit-receive operations. The components of MDM $\mathbf{K}$ in frequency domain are obtained by Fourier transforms of time-domain signals. In the second step, the TRO $\mathbf{K}^*\mathbf{K}$ is diagonalized at a chosen frequency. In the case of well-resolved, isotropic and point-like scatterers, each eigenvector of the TRO is associated to a scatterer in the scattering medium [14], [80]. In the third step, the signals at the array of transducers are back-propagated using the phase and amplitude provided by the eigenvector found in the second step. These signals focus on the corresponding scatterer in the medium. This step can be done experimentally [14], [15], [80]-[83] and numerically [84], [108]. Moreover, the detail formulations of the above steps in the DORT method are described and analyzed in Section 2.3.

3.1.2 DORT simulation set-up

To allow easy alteration of various DORT parameters in different virtual experiments, we have developed a simulation platform with graphical user interface (GUI) in Matlab. The simulation platform consists of 5 main portions, namely “Control Panel”, “Feedback Bar”, “Signal Plot”, “Simulation Set-up”, and “Simulation”, as shown in

Fig. 3.1. In particular, the “Control Panel” is used to set-up, run, and playback the simulation. The “Feedback Bar” is used to show the simulation status, and the “Signal Plot” is used to plot the received time-domain signals at the chosen scatterer in the medium. The “Simulation Set-up” is used to indicate the positions of the transducers and scatterers in the medium. Finally, in the “Simulation” setting, the two-dimensional (2D) Finite-Difference Time-Domain (FDTD) is used to model the wave propagation in the simulated medium [84], [126], [137]. Hence, from the simulation platform, we can alter the input parameters of the simulation, including the geometry of simulation set-up, the signal model and the environment effects.

Figure 3.1 Matlab GUI simulation of the DORT method.

The DORT simulation is set up as shown in Fig. 3.2. The dimension of the field is fixed at 200 cells $\times$ 200 cells. Each cell is equal to $\lambda/10 \times \lambda/10$ in dimension. The

simulation platform allows users to randomly place the scatterers and the array of transducers under different scenarios. The target is commonly chosen as the point-like scatterer that has the largest reflectivity in the case of multiple scatterers. In all simulations reported in this chapter, the scatterer A with a reflectivity of 0.5 is chosen as the target. On the other hand, clutters are chosen as the point-like scatterers that have a small reflectivity of 0.01. Moreover, the target is placed furthest away from the array of transducers, as shown in the Fig. 3.2. This geometry set-up is to verify whether the array of transducers can focus a distant target with significant reflectivity. The input signal is chosen as Gaussian modulated sine wave, which is a typical signal emitted from a piezoelectric source [6]. This input signal has been widely applied in ultrasonic TR in focusing and localization [125], as well as non-destructive characterization of defects [148]. The central frequency is chosen as 0.5 MHz.

Figure 3.2 Simulation set-up of the DORT method.

Some important parameters used in the DORT method are defined, as shown in Fig. 3.2. Parameter t_L is defined as the time length that signal travels from the first (or last) element of the array of transducers to the furthest corner of the field of interest. Parameter t_R is defined as the recording time of the received signal at the array of transducers in the IEIR measurements. Due to multiple reflections among scatterers, t_R is usually a multiple of t_L , i.e., $t_R \geq 2t_L$ [16], [17], [20], [89]. The array size (or number of transducers) and the aperture size are denoted as N and D , respectively. Moreover, the elevation angle θ is the angle between the first (or last element) of the array of transducers and the highest (or lowest) scatterer.

To study the effects of the parameters into focusing performance, the DORT method is conducted in 12 different simulation set-ups, as shown in Table 3.1. In particular, set-ups #1 to #4 are used to obtain the efficient range of recording time to achieve focusing on the target, as tabulated in Table 3.2. Subsequently, set-ups #1 to #4 and #9 are used to determine the minimum array size. Its results are tabulated in Table 3.3. Set-ups #4 to #8 are used to determine a suitable aperture size in these specific scenarios, as shown in Table 3.4. Lastly, set-ups #10 to #12 are used to compare the focusing performance in two environments, including free space (without waveguide) and waveguide medium with the same geometry set-up. Its results are tabulated in Table 3.5.

Table 3.1 The FDTD simulation set-ups.

Set-up No.	Scatterer A and B				Array of Transducers			
	X	Y	Space		X	Y	Size	Spacing
1	20	30	100		185	3	40	5
2	20	30	100		185	6	20	10
3	20	30	100		185	3	14	15
4	20	30	100		185	10	10	20
5	20	30	100		185	15	10	15
6	20	30	100		185	35	10	10
7	20	30	100		185	57	10	5
8	20	30	100		185	67	10	3
9	20	30	100		185	12	8	28
10	20	30	100		185	70	10	5
11	20	30	100		185	90	10	5
12	20	30	100		185	110	10	5

3.1.3 Investigation of the DORT parameters

This section investigates the performance of the DORT method with variations of its four main parameters. These parameters include the recording time (t_R), the array size (N), the aperture size (D), and the waveguide effect [106]. The MDM is decomposed in order to compute eigenvalues and eigenvectors. The target has the largest reflectivity coefficient than the other scatterers. Thus, the eigenvector corresponding to the largest eigenvalue is chosen to back-propagate and focus signal on the target. The focusing performance criterion is evaluated by the signal-to-ripple ratio (SRR), which is defined as the ratio between peak mainlobe signal and peak sidelobe signal, is calculated in decibel (dB). The higher SRR indicates better focusing performance on the desired target.

3.1.3.1 Recording time

In the first experiment, the recording time t_R varies within the range of $[2t_L, 4t_L]$. Besides, to give more comprehensive results, these experiments are conducted over the array size that varies from 10 to 40, corresponding to set-ups #1 to #4. From Table 3.2, when $t_R = 3t_L$ and $t_R = 4t_L$, the same SRR results are obtained. The main reason for this similar result is due to the fact that no reflected signals from the scatterers embedded in the medium for $t_R \geq 3t_L$. Thus, we can estimate exactly the eigenvector and back-propagate this eigenvector to focus on the corresponding target in the case of $t_R = 3t_L$. Moreover, we observe that the SRR performance is degraded when using smaller recording time. The selection of the threshold $t_R = 2.5t_L$ as the minimum recording time to focus on the target is based on the largest decrease in SRR magnitude when decreasing the recording time, as shown in Table 3.2. Therefore, to achieve focusing on the desired target, the conventional range of recording time t_R must be within $[2.5t_L, 3t_L]$.

Table 3.2 Focusing performance in different recording time.

t_R	Set-up #1	Set-up #2	Set-up #3	Set-up #4
4.0	12.0 dB	12.0 dB	12.0 dB	12.0 dB
3.0	12.0 dB	12.0 dB	12.0 dB	12.0 dB
2.9	12.0 dB	11.5 dB	11.5 dB	11.5 dB
2.8	11.5 dB	11.0 dB	10.5 dB	10.5 dB
2.7	11.5 dB	10.5 dB	10.0 dB	9.5 dB
2.5	10.5 dB	10.0 dB	9.5 dB	9.0 dB
2.4	8.5 dB	8.5 dB	8.0 dB	7.5 dB
2.0	6.0 dB	6.0 dB	6.0 dB	5.5 dB

3.1.3.2 Array size

In this subsection, the recording time t_R is chosen as $3t_L$, and the number of the scatterers is chosen as 10. Besides, the array size (number of transducer) N varies from 8 to 40, corresponding to set-ups #1 to #4 and #9, as shown in Table 3.3.

Table 3.3 Focusing performance in different array sizes.

Set-up No	t_R	Array Size	Aperture Size	SRR
1	3.0	40	19.5λ	12.0 dB
2	3.0	20	19.0λ	12.0 dB
3	3.0	14	19.5λ	12.0 dB
4	3.0	10	18.0λ	12.0 dB
9	3.0	8	19.6λ	NA**

NA** represents unable to focus.

From Table 3.3, it is observed that in order to focus on target in a scattering environment of 10 random scatterers, a minimum of 10 transducers are required. This implies that the DORT method requires more number of transducers than number of scatterers to achieve selective focusing. These simulation results are in agreement with the theoretical DORT analysis that is mentioned in previous papers [15], [84].

3.1.3.3 Aperture size

In this subsection, the recording time and the array size are fixed at $t_R = 3t_L$ and $N = 10$. The aperture size is calculated as

$$D = (N - 1)\Delta, \quad (3.1)$$

where Δ is the array spacing between two neighboring transducers. Thus, when the array spacing Δ decreases, the aperture size D also decreases, which in turn, increases

the elevation angles θ . From Table 3.4, we can see that when the array aperture (D) decreases from 18λ to 4.5λ , the SRR decreases from 12 dB to 10 dB. In set-up #8, the focusing on the target is not achieved due to the aperture size is too small. Hence, smaller aperture size causes poorer focusing performance on the target. The underlying reason is that array of transducers with smaller aperture size receive less reflected signals in IEIR measurement. As a result, decomposition of TRO $\mathbf{K}^* \mathbf{K}$ does not provide accurate eigenvectors associated to the scatterers.

To evaluate the effect of the aperture size on the focusing performance of SRR, we consider the ratio between the aperture size D and the observation distance F as

$$r_{DF} = D / F, \quad (3.2)$$

where F is the observation distance between the target and array of transducers.

Table 3.4 Focusing performance in different array apertures.

Set-up No	t_R	Aperture Size	SRR	Elev [^]
4	3.0	18.0λ	12.0 dB	-6.91°
5	3.0	13.5λ	12.0 dB	-5.19°
6	3.0	9.0λ	11.5 dB	1.74°
7	3.0	4.5λ	10.0 dB	9.29°
8	3.0	2.7λ	NA**	12.64°

Elev[^] is the elevation angle from the array of transducers to the lowest or highest scatterer.

As shown in Table 3.4, when the observation distance is fixed at 16.5λ , we observe that the minimum aperture size to achieve focusing on target is 4.5λ with an elevation angle of 9.29° . Thus, in the configuration ($t_R = 3t_L$ and $N = 10$), the ratio r is required to be larger than $3/11$ to achieve good focusing on the desired target. This

ratio r is also employed and verified in the experimental DORT method on an elastic scatterer [127].

3.1.3.4 Waveguide effect

In this subsection, the DORT simulation is carried out in free space (without waveguide) and in a waveguide medium. The waveguide has two boundaries with a vertical distance of 100λ . The simulation set-ups in both cases are similar with fixed aperture size and the array of transducers moving vertically to the bottom of the FDTD field, which corresponds to set-ups #10 - #12, as shown in Table 3.5.

Table 3.5 Focusing performance in free space and waveguide media.

Set-up No	t_R	Free Space		Waveguide	
		SRR	Elev [^]	SRR	Elev [^]
10	3.0	11.0 dB	-6.91°	12.0 dB	-6.91°
11	3.0	NA**	-5.19°	12.0 dB	-5.19°
12	3.0	NA**	1.74°	11.5 dB	1.74°

From Table 3.5, we can see that the DORT method in free space cannot focus signal when the array of transducers is near the bottom of the FDTD field as in set-ups #11 and #12. On the other hand, the DORT method still focuses signal on target in waveguide medium although the array of transducers is near the waveguide boundary that is placed at the FDTD boundary. The underlying reason is that the effective aperture size is larger than the actual aperture size due to virtual images of array of transducers in waveguide medium [22]. Thus, the larger effective aperture

size improves focusing performance on the target in waveguide medium as compared to the free space (without waveguide) case.

3.1.4 Discussions

In the previous subsections, we investigated four parameters in the DORT method including the recording time, array size, aperture size and waveguide effect. Firstly, the optimal range of the recording time to focus on the farthest scatterer in the medium must be within the range of $[2.5t_L, 3t_L]$. Secondly, to fully achieve focusing on all scatterers in the scattering environment, the number of transducers must be larger than number of scatterers. Besides, the aperture size can significantly affects the focusing performance. Specifically, to achieve better focusing performance, it requires bigger aperture size compared to the observation distance in free space. On the other hand, in waveguide medium, the effective aperture size is larger due to virtual images of the array of transducers. Besides, it is observed that if the scatterers are very close to the array of transducers (less than $0.05t_L$), the overall focusing would be affected. Thus, aperture size and array size are also dependent on the geometry set-up and the types of environment.

From the above investigations, we can arrange the DORT parameter effects in two types that are due to environment effects and channel effects. Environment effects include parameters that affect the wave propagation in the medium, such as waveguide effect [22], [31], [106], [147] and/or scattering effect [16], [17], [89], [137]. On the other hand, the channel effects include parameters related to the array of transducers, such as aperture size [15], array size [90], [137] and recording time

[106], [107]. In the DORT experiments and simulations, the focusing performance is more sensitive to environment effects than channel effects. Therefore, the contributions from these effects determine how the DORT method should be designed in practical cases. These observations are summarized in Table 3.6.

Table 3.6 Statistics of the parameter effects in focusing performance via DORT method.

Parameter effects	Parameters	Observations
Environment effects	Waveguide medium	9-times smaller lateral focusing resolution compared with free space [22]
	Scattering medium	6-times smaller spatial focusing resolution compared with free space [16]
Channel effects	Recording time	Longer recording time gives better focusing performance
	Array size	Larger array size gives better focusing performance
	Aperture size	Larger aperture size gives better focusing performance

3.2 Truncated Multistatic Data Matrix

In the DORT method, the measurement of the MDM in the conventional DORT method requires large memory storage and long recording time [14], [15], [84]. Thus, this limitation provides us a motivation to propose a truncated MDM (TMDM) to achieve shorter recording time, smaller computational load and memory size requirement [107].

3.2.1 Multistatic data matrix (MDM)

The MDM is obtained from N measurements of inter-element impulse response (IEIR) $k_{ij}(t)$, the signal received by the i^{th} transducer from emitted signal $\delta(t)$ at the j^{th} transducer, as shown in Fig. 3.3. In the first measurement, the first transducer sends a pilot signal $\delta(t)$ and the reflected signals from the scatterers are recorded in the array of transducers. The recorded signals are stored in the first column of matrix $\mathbf{K}(t)$. Subsequently, after N measurements, an $N \times N$ matrix $\mathbf{K}(t)$ is formed as:

$$\mathbf{K}(t) = \begin{pmatrix} k_{11}(t) & k_{12}(t) & \dots & k_{1N}(t) \\ k_{21}(t) & k_{22}(t) & \dots & k_{2N}(t) \\ \vdots & \vdots & & \vdots \\ k_{N1}(t) & k_{N2}(t) & \dots & k_{NN}(t) \end{pmatrix}. \quad (3.3)$$

Figure 3.3 Description of the IEIR measurements in the DORT method.

The matrix $\mathbf{K}(t)$, which relates the transmitted and received signals provides a mathematical description of the TR process. A TR operation $t \rightarrow -t$ is equivalent to a

phase conjugation in the frequency domain [14], [15]. In the case of a narrowband signal, the DORT method is commonly carried out in central frequency ω [14], [15], [80]-[83]. The main reason is that the frequency domain gives a simpler implementation than the time-domain in this case. Thus, the DORT method is also implemented in the frequency domain in the following parts.

In the frequency domain, $K_{ij}(\omega)$ is obtained by taking the Fourier transforms of the signal $k_{ij}(t)$ in the time-domain. For simplicity, we drop the frequency variable symbol ω from all the remaining expressions in this chapter. Thus, the MDM $\mathbf{K}$ of dimension $N \times N$ can be expressed in the central frequency as

$$\mathbf{K} = \begin{pmatrix} K_{11} & K_{12} & \dots & K_{1N} \\ K_{21} & K_{22} & \dots & K_{2N} \\ \vdots & \vdots & & \vdots \\ K_{N1} & K_{N2} & \dots & K_{NN} \end{pmatrix}. \quad (3.4)$$

3.2.2 Truncated multistatic data matrix (TMDM)

In the DORT method, the measurement of the IEIR is the first and most important step in achieving good focusing. In fact, larger data measurement in IEIR in the time-domain produces a more accurate MDM and eigenvectors in the frequency domain, and thus results in better focusing on the target. From Table 3.2, the full MDM is defined as the MDM with $t_{RO} = 3t_L$, which results in the best focusing performance (highest SRR). However, the full MDM also requires long recording time, large memory size and high computational load in the DORT implementation [126].

Figure 3.4 A full IEIR measurement.

In a multiple scattering environment [16], [17], [20], [89], the IEIR has an experimentally decaying characteristic, and the tail end of a full IEIR has negligible amplitudes that does not contribute significantly to construct the MDM in the frequency domain, as shown in Fig. 3.4. Thus, a truncated multistatic data matrix (TMDM) is proposed to achieve smaller recording time and memory size over the full MDM in the DORT method. The proposed approach comprises of three steps, as shown in Fig. 3.5:

- (i) The first step is to truncate the rear end of full IEIR with the truncation length $(3-x)$, where x is the recording time factor in TMDM. The recoding time factor x can be chosen in the range $[2.5, 3)$.
- (ii) The second step is to zero-pad the truncated IEIR with the length $(3-x)$.
- (iii) Lastly, we apply fast Fourier transformation (FFT) on the zero-padding signal to construct the TMDM $\mathbf{K}_T$ in frequency domain.

Figure 3.5 Illustration of full MDM and TMDM approaches.

3.2.3 Comparison between MDM and TMDM approaches

In the following subsections, the TMDM approach is compared to the full MDM approach in terms of recording time saving, memory size saving, signal-to-ripple ratio (SRR) and spatial focusing.

3.2.3.1 Recording time saving and memory size saving

In the full MDM approach, each IEIR data requires a recording time of $t_{RO} = 3t_L$. Thus, the total recording time after N transmit-receive operation in the IEIR measurement is

$$T_{RO} = 3t_L N. \quad (3.5)$$

In the TMDM approach, each IEIR data requires a recording time of $t_{RT} = xt_L$, where x is the recording time factor in the range $[2.5, 3)$, as shown in Fig. 3.5. Thus, the total recording time of the TMDM approach is

$$T_{RT} = xt_L N. \quad (3.6)$$

Therefore, the saving in recording time using the TMDM approach is

$$T_S = T_{RO} - T_{RT} = (3 - x)t_L N. \quad (3.7)$$

Moreover, the saving in recoding time reduces the memory size requirement in the IEIR measurement. Define S as the memory size needed to save a frame of data, per IEIR element, then the total memory size, S_T , needed to store the truncated IEIR using TMDM approach in $N \times N$ data is

$$S_T = xt_L N^2 S. \quad (3.8)$$

The total memory size needed using full MDM approach in $N \times N$ data is

$$S_O = 3t_L N^2 S. \quad (3.9)$$

Thus, the memory size saving using the TMDM approach is

$$S_S = S_O - S_T = (3 - x)t_L N^2 S. \quad (3.10)$$

From (3.5), (3.7), (3.9) and (3.10), we can see that the proposed TMDM approach can achieve recording time and memory size saving ratio of

$$R = (3 - x) / x. \quad (3.11)$$

Figure 3.6 Saving ratio R using TMDM approach.

The maximum saving ratio R using the proposed TMDM approach can achieve 20% when $x = 2.5$, as seen in Fig. 3.6. Besides, the recording time saving T_S and memory size saving S_S are proportional to the number of transducers (N) and the square of number of transducers (N^2), respectively. In particular, the number of transducers used in the DORT experiments is very large (around 100 transducers) [14], [15]. Thus, the proposed TMDM approach can be an appropriate solution to the DORT implementations with limited time and memory size.

3.2.3.2 Signal-to-ripple ratio (SRR)

In this subsection, we evaluate the quantitative performance of the proposed TMDM and full MDM approaches in term of signal-to-ripple ratio (SRR), which indicates the focusing performance on target at the focusing time. In particular, Fig. 3.7 shows the received signals in the target of full MDM, TMDM with $2.5t_L + 0.5t_L$ zeros, and TMDM with $2.8t_L + 0.2t_L$ zeros. The SRRs using full MDM approach are 0 - 0.8 dB

(depending on the value of x in the range $[2.5, 3)$) larger compared to the proposed TMDM approach. The poorer SRRs using TMDM approach is mainly due to the received signals on the target using TMDM, which has higher sidelobe compared to the full MDM, as shown in Fig. 3.7.

Figure 3.7 Received signals at target in full MDM, TMDM with $2.5t_L + 0.5t_L$ zeros and TMDM with $2.8t_L + 0.2t_L$ zeros.

3.2.3.3 Spatial focusing

Figures 3.8, 3.9 and 3.10 show the spatial focusing of full MDM and TMDM approaches with $2.8t_L + 0.2t_L$ zeros and $2.5t_L + 0.5t_L$ zeros in FDTD simulations [84], [126]. The focusing spots are defined as the darker red spots and marked in circles. From these figures, the spatial focusing spots of both full MDM and TMDM are also approximated as 20×1 cells (vertical $\times$ horizontal cells). Besides, we observe that the focusing spots estimated by using full MDM and TMDM approaches are close to the actual position of the target (the square marker). Hence, both full MDM and TMDM approaches can perform good focusing on the target at the same focusing time.

Figure 3.8 Spatial focusing in full MDM.

Figure 3.9 Spatial focusing in TMDM with $2.5t_L + 0.5t_L$ zeros.

Figure 3.10 Spatial focusing in TMDM with $2.8t_L+0.2t_L$ zeros.

3.2.4 Discussions

The advantage of the proposed TMDM approach is to reduce recording time and memory size requirement, as well as computational load compared to the full MDM approach. The drawback of the TMDM approach is poorer focusing performance (SRR) on the target. However, the TMDM approach still maintains spatial focusing as full MDM approach does. Thus, the proposed TMDM approach can be an alternative solution in the cases of the DORT implementations with limited time and resources. In summary, the comparison between TMDM and full MDM approaches is illustrated in Table 3.7.

Table 3.7 Comparison of performance between TMDM and full MDM approaches.

	Recording Time	Memory Size	SRR	Spatial Focusing
TMDM $2.5t_L + 0.5t_L$ zeros	$T_{RT} = 2.5t_L N^2$	$S_T = 2.5t_L N^2 S$	11.2 dB	20×1
Full MDM	$T_{RO} = 3t_L N^2$	$S_O = 3t_L N^2 S$	12.0 dB	20×1
Improvement	Saving 20%	Saving 20%	-0.8 dB reduction	Same

3.3 DORT Method in Well-Resolved Problem

In this section, we consider and analyze the DORT method in a well-resolved scatterer case. A well-resolved scatterer case is only applicable to case where energy is solely focused on only one scatterer, and zero energy is sent to other scatterers. The scatterers are also considered as point-like and isotropic, as shown in Fig. 3.11. Thus, the number of significant eigenvalues of the TRO corresponds to the number of the scatterers and each eigenvector of the TRO is associated to only one scatterer in the medium. For example, the eigenvector associated to the i^{th} scatterer is $\mathbf{H}_i^*$, where $\mathbf{H}_i^* = [H_{i1}^* \dots H_{iL}^* \dots H_{iN}^*]$, consists of the propagating paths between the i^{th} scatterer and all the transducers in the array of transducers. The eigenvalue corresponding to the i^{th} scatterer depends on the reflectivity of this scatterer and the propagating paths as

$$\sigma_i = |C_i|^2 \left(\sum_{i=1}^N |H_{il}|^2 \right)^2, \quad (3.12)$$

where C_i is the reflectivity coefficient of the i^{th} scatterer.

Figure 3.11 Propagating paths in the well-resolved scatterer case.

3.3.1 DORT method: Experimental technique

The DORT method is a self-adaptive approach to complex environments due to the measurements of the IEIRs' data are conducted in real-time experiments [13]-[15]. However, the DORT experiment requires long duration to implement due to its large number of IEIR measurements in transmit-receive operation. Besides, these measurements require large number of transducers and a multi-channel acquisition system. In order to conduct the DORT experiment, it requires high resources and high computational complexity [14], [15], [80]-[82].

After performing IEIR measurements, the MDM $\mathbf{K}$ can be obtained by using Fourier transforms. The TRO $\mathbf{K}^H \mathbf{K}$ is also constructed and diagonalized to find its eigenvalues and corresponding eigenvectors. The eigenvalue decomposition (EVD) of the TRO can be replaced by the singular value decomposition (SVD) of the MDM by squaring the singular values of the MDM to obtain the eigenvalues of TRO before extracting singular vectors of MDM as the eigenvectors of TRO. Thus, this offline

calculation requires intensive computational load because of large matrix dimension in the MDM and TRO.

3.3.2 DORT method: Numerical technique

The experimental technique of the DORT method is a powerful tool to detect and focus signal on the target. However, it requires high resources and high computational complexity. Thus, these requirements motivate us to develop the numerical technique of the DORT method with the aim of performing selective focusing on the desired scatterer in a common scenario (i.e., homogeneous medium) [108]. In particular, the proposed technique provides a set of algorithm to estimate the eigenvalues and eigenvectors of the TRO without requiring large resources and experimental measurements.

In the numerical technique, the scenario is considered as a homogeneous medium with embedded scatterers. The array of transducers is linear and consists of N point-like transducers with equal spacing Δ . There are M point-like scatterers that have associated reflectivity coefficients of $C_1, C_2, \dots, C_M$. The origin of Cartesian system (2D) is at the $(m+1)^{\text{th}}$ transducer where $m = \lfloor N / 2 \rfloor$.

In a homogeneous medium, the Green's function estimation only changes the time shift and amplitude reduction of the initial source wave [128]. The diffraction impulse response $h_{ik}(t)$ between the k^{th} transducer and the i^{th} scatterer is the Green's function estimation matrix, which emits signal $\delta(t)$ from the array of transducers:

$$h_{ik}(t) = g(r_k; t) = \frac{1}{S_k} \delta\left(t - \frac{S_k}{c}\right), \quad (3.13)$$

where $S_k = |r_k - r_i|$, c is wave speed (m/s), and $k = 1, 2, \dots, N$. In the frequency domain, the Green's function estimation can be expressed as

$$H_{ik}(\omega) = \frac{1}{S_k} e^{-j\omega S_k / c}. \quad (3.14)$$

To develop the numerical technique of the DORT method, we need to perform estimations of the eigenvalues in terms of the parameters of a well-known geometry and the reflectivity of the scatterers. The parameters of a well-known geometry include the number of transducers N , the array spacing Δ , and the position of the scatterers in the medium. In particular, to estimate the eigenvalue in numerical form, we need to consider the number of transducers N in two different cases, including odd case ($N = 2m + 1$) and even case ($N = 2m$).

In the odd case, the distance between the k^{th} transducer ($0, (m + 1 - k)\Delta$) and the i^{th} scatterer (x_i, y_i) is calculated as

$$S_k = \sqrt{x_i^2 + (y_i - (m + 1 - k)\Delta)^2}, \quad (3.15)$$

where $k = 1, 2, \dots, 2m + 1$. Combining (3.14) and (3.15), we obtain

$$\sum_{k=1}^N |H_{ik}|^2 = \sum_{k=1}^{2m+1} \frac{1}{S_k^2}, \quad (3.16)$$

where

$$\begin{aligned} \sum_{k=1}^{2m+1} \frac{1}{S_k^2} &= \frac{1}{S_{i1}^2} + \frac{1}{S_{i2}^2} + \dots + \frac{1}{S_{i2m}^2} + \frac{1}{S_{i2m+1}^2} \\ &= \left(\frac{1}{S_1^2} + \frac{1}{S_{2m+1}^2} \right) + \left(\frac{1}{S_2^2} + \frac{1}{S_{2m}^2} \right) + \dots + \frac{1}{S_{m+1}^2}. \end{aligned} \quad (3.17)$$

Transform the first term in the above summation (3.17) to

$$\begin{aligned} \left(\frac{1}{S_1^2} + \frac{1}{S_{2m+1}^2} \right) &= \frac{1}{x_i^2 + (y_i - m\Delta)^2} + \frac{1}{x_i^2 + (y_i + m\Delta)^2} \\ &= \frac{2(x_i^2 + y_i^2 + m^2\Delta^2)}{(x_i^2 + y_i^2 + m^2\Delta^2 - 2y_i m\Delta)(x_i^2 + y_i^2 + m^2\Delta^2 + 2y_i m\Delta)} \end{aligned} \quad (3.18)$$

Set $a = x_i^2 + y_i^2 + m^2\Delta^2$ and (3.18) becomes

$$\frac{2a}{(a - 2y_i m\Delta)(a + 2y_i m\Delta)} = \frac{2a}{a^2 - 4y_i^2 m^2 \Delta^2} = \frac{2}{a - 4y_i^2 m^2 \Delta^2 / a} \quad (3.19)$$

Because Δ is very small, we can approximate (3.18) as

$$\left(\frac{1}{S_1^2} + \frac{1}{S_{2m+1}^2} \right) = \frac{2}{x_i^2 + y_i^2 - m^2\Delta^2} \quad (3.20)$$

Using the same transformation, (3.16) can be expressed as

$$\sum_{k=1}^{2m+1} \frac{1}{S_k^2} = \frac{2}{x_i^2 + y_i^2 - m^2\Delta^2} + \dots + \frac{2}{x_i^2 + y_i^2 - \Delta^2} + \frac{1}{x_i^2 + y_i^2} \quad (3.21)$$

Additionally,

$$\frac{1}{x_i^2 + y_i^2 - m^2\Delta^2} + \frac{1}{x_i^2 + y_i^2 - \Delta^2} \approx \frac{2}{x_i^2 + y_i^2 - ((m+1)\Delta/2)^2} \quad (3.22)$$

Combining (3.21), (3.22) and (3.16), we obtain

$$\sum_{k=1}^N |H_{ik}|^2 = \frac{2m}{x_i^2 + y_i^2 - ((m+1)\Delta/2)^2} + \frac{1}{x_i^2 + y_i^2} \quad (3.23)$$

Hence, the eigenvalue of the TRO associated with the i^{th} scatterer is estimated as:

$$\sigma_i = C_i^2 \left(\frac{2m}{x_i^2 + y_i^2 - ((m+1)\Delta/2)^2} + \frac{1}{x_i^2 + y_i^2} \right)^2 \quad (3.24)$$

Following to the mathematical process of the odd case, the eigenvalue of TRO associated with the i^{th} scatterer in the even case can also be estimated as

$$\sigma_i = C_i^2 \left(\frac{2(m-1)}{x_i^2 + y_i^2 - (m\Delta/2)^2} + \frac{1}{x_i^2 + y_i^2} + \frac{1}{x_i^2 + (y_i - m\Delta)^2} \right)^2. \quad (3.25)$$

In the well-resolved scatterer case, the eigenvector of the TRO associated to the i^{th} scatterer can be expressed as

$$\mathbf{H}_i^* = [H_{i1}^* \dots H_{ik}^* \dots H_{iN}^*], \quad (3.26)$$

where $H_{ik}^* = \frac{e^{j\omega S_k/c}}{S_k}$ with $k = 1, 2, \dots, N$.

Hence, the numerical technique provides a feasible solution of the DORT method to estimate the eigenvalue and the eigenvector associated to each scatterer in a well-known geometry. These values can be employed to perform back-propagation and selective focusing on the desired scatterer.

3.3.3 Discussions

As stated in the preceding section, the eigenvalue and eigenvector of the TRO associated with a specific scatterer can be estimated by the proposed numerical technique. The numerical form of the eigenvalue is dependent on the array spacing Δ . This spacing is significant in the near-field geometry, in which the scatterer is close to the array of transducers. However, in the far-field geometry, the spacing becomes trivial. Thus, we can verify the convergence of the eigenvalue in the far-field condition by testing its limit in odd and even cases.

Consider b_i as the ratio of the half aperture size to the distance between the scatterer and the array of transducers. We can obtain

$$b_i \triangleq \frac{D/2}{F_i} = \begin{cases} \frac{(m+1)\Delta}{\sqrt{x_i^2 + y_i^2}} & N = 2m+1 \\ \frac{m\Delta}{\sqrt{x_i^2 + y_i^2}} & N = 2m \end{cases} . \quad (3.27)$$

The far-field condition occurs when $b_i \leq 0.01$. In the odd case ($N = 2m + 1$), the limit of the eigenvalue can be calculated as

$$\begin{aligned} & \lim_{b_i \rightarrow 0} C_i^2 \left(\frac{2m}{x_i^2 + y_i^2 - ((m+1)\Delta/2)^2} + \frac{1}{x_i^2 + y_i^2} \right)^2 \\ &= \lim_{b_i \rightarrow 0} C_i^2 \left(\frac{2m}{1 - b_i^2/4} + \frac{1}{x_i^2 + y_i^2} \right)^2 \\ &= C_i^2 \left(\frac{2m+1}{x_i^2 + y_i^2} \right)^2 = C_i^2 \left(\frac{N}{x_i^2 + y_i^2} \right)^2 \end{aligned} . \quad (3.28)$$

Similarly, in the even case ($N = 2m$), the limit of the eigenvalue can be calculated as

$$\begin{aligned} & \lim_{b_i \rightarrow 0} C_i^2 \left(\frac{2(m-1)}{x_i^2 + y_i^2 - (m\Delta/2)^2} + \frac{1}{x_i^2 + y_i^2} + \frac{1}{x_i^2 + (y_i - m\Delta)^2} \right)^2 \\ &= \lim_{b_i \rightarrow 0} C_i^2 \left(\frac{2(m-1)}{1 - b_i^2/4} + \frac{1}{x_i^2 + y_i^2} + \frac{1}{x_i^2 + y_i^2 - b_i^2(2y_i/m\Delta - 1)} \right)^2 \\ &= C_i^2 \left(\frac{2m}{x_i^2 + y_i^2} \right)^2 = C_i^2 \left(\frac{N}{x_i^2 + y_i^2} \right)^2 \end{aligned} . \quad (3.29)$$

These results (3.28) and (3.29) ensure that the convergence of the eigenvalue towards the same value in either odd or even limits. Therefore, in far-field condition, the eigenvalue associated to the i^{th} scatterer can be expressed as

$$\sigma_i^f = \left(\frac{C_i N}{x_i^2 + y_i^2} \right)^2. \quad (3.30)$$

Hence, in far-field geometry, the estimated eigenvalue associated to each scatterer depends only on the reflectivity coefficient and its own position. This result derived from the numerical technique is also in agreement with that in the experimental technique [15].

The numerical technique of the DORT method has some advantages compared to the experimental technique of the DORT method to perform selective focusing in homogeneous medium. Specifically, the numerical technique does not require the measurement of the IEIRs and diagonalization of the TRO, which are the first and second steps in the experimental technique. Therefore, the numerical technique can save $N \times N$ memory size of the MDM $\mathbf{K}$. In addition, the numerical technique requires much smaller computational complexity of $O(N)$ for estimating eigenvalues and eigenvectors, compared to the much complex SVD of TRO $\mathbf{K}^* \mathbf{K}$ of $O(N^3)$ in the experimental technique.

3.3.4 Numerical simulation results

In this subsection, we conduct some computer simulations of two approaches including virtual experiment and numerical technique under two-dimensional (2D) FDTD simulations. The simulation is set up with 200 cells $\times$ 200 cells. Each cell is equal to $\lambda/10 \times \lambda/10$ in dimension. The array of transducers is linear, and consists of 10 transducers with a spacing of $\Delta = \lambda$, as shown in Fig. 3.12. The target is chosen as the upper scatterer in the left corner of FDTD with the position (20, 30). The

reflectivity of the target is chosen as 0.5. Besides, there are different point-like scatterers placed randomly between array of transducers and the target. These scatterers have the reflectivity of only 0.01.

Figure 3.12 Simulation set-up in the well-resolved scatterer case.

In the virtual experiment, we measure the IEIRs by using wave propagation in the FDTD simulation. Each transducer transmits an impulse response to the scatterers embedded in the medium. These signals are reflected from the scatterers and recorded in all transducer elements of the array. This measurement is also repeated with other elements of array of transducers. Subsequently, the IEIRs are Fourier transformed to construct the MDM $\mathbf{K}$. The SVD of $\mathbf{K}$ is used to calculate the eigenvalues and eigenvectors. The largest eigenvalue is associated to the target due to the largest reflectivity of the target compared to other scatterers. The back-propagation of the eigenvector corresponding to the largest eigenvalue focuses on the target. In the numerical technique, we can directly perform back-propagation using the largest

eigenvalue and the corresponding eigenvector estimated in numerical forms (3.24) – (3.26) in order to focus signal on the target.

Figures 3.13 and 3.14 show the focusing on the target using virtual experiment and numerical technique in FDTD, respectively. In these figures, the darker red patch defines the focusing positions when performing back-propagation of the eigenvector associated to the largest eigenvalue. In both cases of the virtual experiment and numerical technique, the focusing positions are close to the actual position of the target. The spatial focusing of the virtual experiment is slightly smaller compared to that of the numerical technique. The main reason is that the eigenvalues and eigenvectors obtained from virtual experiment are more accurate when performing measurement of IEIRs compared to our approximations in the numerical technique.

Figure 3.13 Focusing on the target using virtual experimental technique of the DORT method.

Figure 3.14 Focusing on the target using numerical technique of the DORT method.

3.4 DORT Method in Non-Well-Resolved Problem

In the well-resolved scatterers case, it is possible to focus on one of the scatterers without sending energy to the other scatterers [15]. From a mathematical point of view, it means that the columns of the propagation matrix $\mathbf{H}$ are orthogonal. Suppose that a point-like source is placed at the position of the i^{th} scatterer and emits a temporal signal $\delta(t)$. Following that, the signal received on the l^{th} transducer is the impulse diffraction response $h_{il}(t)$. The best way to focus on the same scatterer is to use the input signal as the time-reversed signal $h_{il}(-t)$ [15]. However, the pressure signal received by the j^{th} scatterer is given as

$$P_{ij} = \sum_{l=1}^N H_{jl} H_{il}^* . \quad (3.31)$$

In the well-resolved scatterer case, P_{ij} is almost zero for $j \neq i$. Thus, the pressure is significant only at the same transmitted and received scatterer. Since each eigenvector

of the TRO is associated to one point-like scatterer, selective focusing on the scatterer can be achieved by back-propagating signals with the phase of the corresponding eigenvector.

However, in the non-well-resolved scatterer case, there is interference among scatterers due to close proximity among scatterers. Thus, the pressure signal P_{ij} is still significant for $j \neq i$. Subsequently, the selective focusing performance on each scatterer via the DORT method is degraded. This problem motivates us to propose a new solution for handling non-well-resolved scatterer case [109].

3.4.1 Weighted Least-Square DORT Method

In the non-well-resolved scatterer case, the columns of the propagation matrix $\mathbf{H}$ are not necessary orthogonal. Thus, the scatterers can be chosen randomly in the medium. In this case, the scatterers are defined as either the target region or the null region. The target includes the focused scatterer and the null region includes the remaining scatterers, as shown in Fig. 3.15.

Figure 3.15 Signal propagation from the target to the array of transducers.

Suppose that a source is placed at the position of the i^{th} scatterer (target) and emits a temporal signal $\delta(t)$. The received signal at array of transducers consisting of N transducers in the frequency domain is

$$\mathbf{H}_i = [H_{i1} \dots H_{il} \dots H_{iN}]. \quad (3.32)$$

To focus signal via the DORT method, the eigenvector associated target used in input signal is the phase conjugation of the received signal vector at the array of transducers [14]. In back-propagation, the input signal at the array of transducers is

$$\mathbf{H}_i^* = [H_{i1}^* \dots H_{il}^* \dots H_{iN}^*]. \quad (3.33)$$

To reduce interference among non-well-resolved scatterers and reinforce signal at target, a weight matrix $\mathbf{w}$, which is also a diagonal matrix, is proposed in the array of transducers [109]. The weighted matrix is applied before back-propagating signals to focus on the target, as shown in Fig. 3.16.

Figure 3.16 Signal back-propagation from array of transducers to the target.

The pressure signal received by a null region in the frequency domain is

$$P_n = \sum_{l=1}^N H_{nl} w_l H_{il}^*, \quad (3.34)$$

where $\mathbf{H}_n = [H_{n1}, \dots, H_{nl}, \dots, H_{nN}]$ is the propagation matrix of the null region, and

$\mathbf{H}_i = [H_{i1}, \dots, H_{il}, \dots, H_{iN}]$ is the propagation matrix of the i^{th} scatterer (target).

Expanding (3.34), we have

$$P_n = \mathbf{H}_n \mathbf{w} \mathbf{H}_i^H = (H_{n1} \dots H_{nN}) \begin{pmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_N \end{pmatrix} \begin{pmatrix} H_{i1}^* \\ \vdots \\ H_{iN}^* \end{pmatrix}. \quad (3.35)$$

Denote $\mathbf{w}_\diamond = (w_1^* H_{i1} \dots w_N^* H_{iN})$, we obtain

$$\mathbf{w}_\diamond^H = \mathbf{w} \mathbf{H}_i^H = \begin{pmatrix} w_1 & & 0 \\ & \ddots & \\ 0 & & w_N \end{pmatrix} \begin{pmatrix} H_{i1}^* \\ \vdots \\ H_{iN}^* \end{pmatrix}. \quad (3.36)$$

Substituting (3.36) into (3.35), the pressure signal received by a null region is

$$P_n = \mathbf{H}_n \mathbf{w}_\diamond^H. \quad (3.37)$$

Similarly, the total pressure signal received by all scatterers is

$$P = \mathbf{H} \mathbf{w}_\diamond^H, \quad (3.38)$$

where $\mathbf{H} = (\mathbf{H}_i \ \mathbf{H}_n)^T$ is the propagation matrix of the scattering environment.

Therefore, selective focusing on target is to find a weight matrix $\mathbf{w}$ which results in distortionless response at the focal point and minimizes the intensity coming from elsewhere in signal back-propagation [129]. This problem reduces to the optimization problem of the following objective function:

$$\min_{\mathbf{w}_\diamond} \mathbf{w}_\diamond^H \mathbf{R} \mathbf{w}_\diamond \text{ subject to } \mathbf{H}_i \mathbf{w}_\diamond^H = 1, \quad (3.39)$$

where $\mathbf{R} = \mathbf{H}^H \mathbf{H}$ is the spatial correlation matrix.

To solve the above constrained optimization problem, the classical Lagrange multiplier method can be used. Let:

$$\mathbf{F} = \mathbf{w}_\diamond \mathbf{R} \mathbf{w}_\diamond^H - \nu_1 (\mathbf{w}_\diamond \mathbf{H}_i^H - 1) - \nu_2 (\mathbf{H}_i \mathbf{w}_\diamond^H - 1). \quad (3.40)$$

Taking the derivative of (3.40) with respect to the weight matrix $\mathbf{w}_\diamond$ and minimizing $\mathbf{F}$, we have

$$\frac{\partial \mathbf{F}}{\partial \mathbf{w}_\diamond} = \mathbf{R} \mathbf{w}_\diamond^H - \nu_1 \mathbf{H}_i^H = 0, \quad (3.41)$$

The solution of (3.41) can be found as

$$\mathbf{w}_\diamond = \frac{\mathbf{H}_i (\mathbf{R}^{-1})^H}{\mathbf{H}_i (\mathbf{R}^{-1})^H \mathbf{H}_i^H}. \quad (3.42)$$

From (3.36), we have

$$\mathbf{w}_\diamond = \mathbf{H}_i \mathbf{w}^H. \quad (3.43)$$

Combining (3.42) and (3.43), we can obtain

$$\mathbf{w} = \frac{\mathbf{R}^{-1}}{\mathbf{H}_i \mathbf{R}^{-1} \mathbf{H}_i^H}, \quad (3.44)$$

where $\mathbf{H}_i$ is the propagation matrix or Green's function estimation between the i^{th} scatterer (target) and the array of transducers.

Hence, the weight matrix $\mathbf{w}$ corresponding to the target (the i^{th} scatterer) depends on the appropriate eigenvector and the spatial correlation matrix. The eigenvector is measured by the experimental technique of the DORT method. In a homogeneous medium, the Green's function connecting to the scatterers is known and the eigenvectors can be estimated by the numerical technique [108]. Thus, the WLS-DORT method is flexibly adapted in both numerical and experimental technique to focus signal on the target. Besides, the WLS-DORT method can be applied in both well and non-well-resolved scatterer cases. Hence, the WLS-DORT method is a more general and more robust technique than conventional DORT method for practical implementation. However, the proposed WLS-DORT method also requires more computational load than the DORT method.

3.4.2 Numerical simulation results

A computer simulation is set up to verify the WLS-DORT method and compare with the DORT method in the non-well-resolved scatterers case. We analyze their performance under 2D FDTD simulations. The simulation is set up with $200 \text{ cells} \times 200 \text{ cells}$ in dimension. Each cell is equal to $\lambda/10 \times \lambda/10$. The array of transducers is linear, and made up of 20 transducers with equal array spacing of 0.5λ . Two isotropic point-like scatterers A and B are placed very close with each other and the vertical spacing only 0.2λ , as shown in Fig. 3.17. Thus, the two scatterers, A and B satisfy the non-well-resolved scatterer condition. Both scatterers also have a reflectivity of 0.5. Besides, there are 10 other point-like scatterers placed randomly between TRA and scatterers A and B. These scatterers have the reflectivity of 0.01, which can provide scattering effects in a homogeneous medium.

Figure 3.17 Simulation set-up in the non-well-resolved scatterer case.

In Figs. 3.18 and 3.19, the solid line represents the received signal at scatterer A and the dot line represents the received signal at scatterer B. In the WLS-DORT method, the scatterer A is chosen as the target that we want to focus on and the scatterer B is considered as null region. From Figs. 3.18 and 3.19, we can see that the amplitude of received signal at scatterer A in the WLS-DORT method is slightly bigger than that in the DORT method. On the other hand, the amplitude of received signal at scatterer B in the WLS-DORT method is smaller than that in the DORT method. These simulation results are also in good agreement with the theoretical analysis of the DORT and WLS-DORT methods.

In the DORT method, the peak amplitudes of received signals at scatterer A are only 1.3 times bigger than that in scatterer B. Thus, we cannot separately focus signal on one scatterer. However, in the WLS-DORT method, the peak amplitudes of

received signals at scatterer A are about 6 times bigger than that in scatterer B. Thus, we can focus signal on scatterer A (target) and neglect the interference effects of scatterer B. Hence, the WLS-DORT method achieves better spatial focusing performance in selective focusing on the target compared to the DORT method in the non-well-resolved scatterer case.

Figure 3.18 Received signals at scatterer A and scatterer B when using the DORT method.

Figure 3.19 Received signals at scatterer A and scatterer B when using the WLS-DORT method.

3.4.3 Discussions

In the adaptive beamforming method, the minimum variance beamforming (MVBF) method is used to emphasize the signal from propagation direction $\mathbf{e}$ and suppress the

signal from other directions [130]. The weight vector of the minimum variance beamformer is the solution to the problem:

$$\min_{\mathbf{w}} E \left[|\mathbf{w}^H \mathbf{y}|^2 \right] \text{ subject to } Re[\mathbf{e}^H \mathbf{w}] = 1. \quad (3.45)$$

And, the weight matrix is calculated by:

$$\mathbf{w} = \frac{\mathbf{R}^{-1} \mathbf{e}}{\mathbf{e}^H \mathbf{R}^{-1} \mathbf{e}}, \quad (3.46)$$

where $\mathbf{e}$ is steering vector and $\mathbf{R}$ is spatial correlation matrix.

In the focusing field, both MVBF and WLS-DORT methods try to minimize the power with unity response constraint. The weight matrix is found to enhance signal on desired direction and minimize signal in other directions. However, the MVBF method requires estimation of the steering vector by using array calibration techniques [130]. On the other hand, the WLS-DORT method estimates the eigenvector by performing SVD of the MDM [14], [15].

However, the weight matrix in the WLS-DORT method (3.44) has simpler computational complexity than the MVBF method (3.46) because the WLS-DORT uses eigenvector of the TRO as input signal. Thus, the WLS-DORT algorithm requires only $O(N^2)$ operations, whereas the MVBF algorithm requires $O(N^3)$ operations, resulting in significantly faster computation.

Besides, both MVBF and WLS-DORT methods can be used to focus signal on the target in both well-resolved and non-well-resolved scatterer cases. However, the MVBF performance is degraded in environmental distortions such as local scattering

medium and environmental non-stationarities [131]. On the other hand, the WLS-DORT method can perform selective focusing better in more complex environments, especially in multiple scattering or waveguide medium. The underlying reason is that the time-reversed signal can converge and focus wave on the target in spite of the sound speed fluctuations in the complex medium [14].

Hence, the WLS-DORT method achieves more significant computational saving and more robust performance compared to the MVBF method. The comparison results between WLS-DORT and MVBF are summarized in Table 3.8. The WLS-DORT method becomes a powerful tool of analysis of the eigenstructure for selective focusing in complex medium.

Table 3.8 Comparisons of WLS-DORT and MVBF methods.

	WLS-DORT	MVBF
Signal Vector	Eigenvector	Steering vector
Data	Multistatic Data Matrix	Covariance Data Matrix
Computational Complexity	Weight algorithm $O(N^2)$	Weight algorithm $O(N^3)$
Environmental Application	Complex medium (scattering, waveguide)	Stationary and no distortions medium
Case	Both well-resolved and non-well-resolved scatterer	Both well-resolved and non-well-resolved scatterer

3.5 Conclusions

This chapter focused on the study and development of the DORT method in acoustic focusing field. In particular, the DORT method was investigated to disclose the

standard requirement of these parameters to achieve good focusing with minimum cost under different configurations. These results and observations provided useful set-up conditions to achieve focusing in practical implementations. Subsequently, the TMDM approach was proposed to achieve saving of up to 20% in terms of time and memory size requirement without much degradation of focusing performances in signal-to-ripple ratio (SRR) and spatial focusing compared to the full MDM approach. Thus, the TMDM approach is a suitable approach in the DORT implementations that require limited time, memory size and computation. These findings provided a practical mean in implementing TMDM in some applications such as nondestructive testing [66], medical imaging [49], underwater acoustics [55], electromagnetism [72] etc.

Moreover, a part of this chapter is on the numerical technique of the DORT method to perform selective focusing on target in the well-resolved scatterer condition. In this condition, the eigenvectors of the TRO are linked to the targets. The eigenvalue depends on the reflectivity of the target and the geometry of the system. Besides, the eigenvector provides the phase and amplitude to focus on the target. In numerical technique, the eigenvalues and eigenvectors were estimated in geometry-based algorithms. The theoretical study shows that the numerical technique can achieve smaller computational load compared to experimental technique to perform selective focusing. However, the limitation of the numerical DORT technique is only applied in homogeneous medium. For heterogeneous or complex medium, the experimental DORT technique is a more appropriate approach with super-resolution focusing on the target.

Finally, the final part of this chapter outlined the WLS-DORT method in selective focusing on target in the non-well-resolved scatterer condition. The WLS-DORT method has been developed based on the DORT and least-squares algorithms. In this method, the back-propagation of corresponding eigenvector of the TRO and the weight matrix linked to the target can select focusing on target. Specifically, the weight matrix depends on the spatial correlation matrix and the corresponding eigenvector. The theoretical results showed that the proposed technique can greatly reduce computational complexity compared to the minimum variance beamforming technique in focusing field. Besides, the simulation results also showed that the WLS-DORT method can achieve better spatial focusing performance than the conventional DORT method in the non-well-resolved scatterer condition. Therefore, the proposed WLS-DORT method can overcome the present technical challenges of the DORT and MVBF methods, and can be applied in acoustic communications [60], non-destructive evaluation [37], as well as ultrasound focusing in medical therapy [83].

CHAPTER 4

TIME-REVERSAL TECHNIQUE IN ACOUSTIC DETECTION AND LOCALIZATION PROBLEM

Detection and localization of embedded sources (or targets) in the medium finds wide range of applications in medical imaging, nondestructive defect testing, as well as underwater object detection and classification. There are different techniques to detect and localize unknown sources (or targets) in the acoustic field [130], [133], [134]. In this chapter, we will conduct comprehensive reviews and intensive investigation of two commonly used and well-known methods in acoustic detection and localization.

The first method is the MULTiple-Signal-Classification (MUSIC) algorithm, which is a passive source detection method [120]. This method can be used to detect and localize the incoming sources in the medium [130], [132]-[134]. In particular, the MUSIC algorithm utilizes the eigenvectors associated with small eigenvalues of the array covariance matrix to localize the direction of the source. The MUSIC algorithm can achieve super-resolution results, but it requires a large number of snapshots to estimate the array covariance matrix [138]. Thus, the MUSIC algorithm requires high computational load and large memory size to form a large dimension of array covariance matrix. The MUSIC algorithm has a limitation of resolving highly correlated and closely spaced sources (i.e., coherent sources) in the medium.

The second method uses the time-reversal technique with multiple signal classification (TR-MUSIC), which is an active detection and localization method [120]. Specifically, this method can detect the number of targets and localize the position of the targets in the medium [135]-[138]. The TR-MUSIC method employs multistatic data matrix (MDM), which requires only a single snapshot in measurement of reflected data [91]. Thus, the TR-MUSIC method can greatly reduce computational complexity in data measurement compared to the MUSIC method [138]. Moreover, the TR-MUSIC method can be applied in multiple scattering case [17], [20], [22]. Thus, the TR-MUSIC method can achieve larger effective aperture size compared to the MUSIC method, which is only limited in physical aperture size.

This chapter is organized as follows. Firstly, the fundamental theory of the MUSIC method is introduced in Section 4.1. In Section 4.2, the TR-MUSIC method is studied and analyzed in both single and multiple scattering cases. Subsequently, the MUSIC and TR-MUSIC algorithms are simulated and investigated in different scenarios in Section 4.3. Besides, there are some discussions and comparisons between MUSIC and TR-MUSIC methods conducted in Section 4.4. Finally, Section 4.5 summarizes and concludes this chapter.

4.1 MUSIC Method

MUSIC is a statistical method used to estimate the source parameters. It is an efficient method based on the principle of eigenvector decomposition techniques [130], [132]-[134]. Thus, the MUSIC algorithm can be applied in direction-of-arrival (DOA) estimation or source direction finding in communications [132].

4.1.1 Array geometry

A source direction finding method depends very much on the geometry condition, which can be chosen as either near-field or far-field. In the near-field condition, the azimuth angles, which are the angles between the source and array of transducers, are varied. On the other hand, in the far-field condition, the azimuth angles are fixed angle and the type of the propagating signal is considered as plane wave.

The configuration of the array of transducers depends on the shape, the size and the array spacing. In particular, the conventional configuration of the array of transducers is the uniform linear array (ULA). This array configuration consists of N point-like transducers of equal array spacing Δ along one-dimension (1D) axis. A source signal, which is a plane wave, arrives at a fixed azimuth angle of θ with respect to the array of transducers, as shown in Fig. 4.1. Hence, in MUSIC algorithm, the array geometry is commonly chosen to satisfy far-field condition with the ULA configuration.

Figure 4.1 Array geometry with ULA configuration.

To implement the array of transducers with the aim of estimating the source direction, there are also two requirements related to the array geometry must be fulfilled. The first requirement is the determination of an appropriate aperture size to estimate the source directions with using a source angle resolution of δ_0 . The second requirement is the appropriate array spacing to determine exactly the main source propagating from the direction $\vec{\zeta}$.

Firstly, in the ULA configuration with finite number of transducers N and array spacing Δ , the physical aperture size D is stated in (3.1) [107]. The array angle resolution determined in the array of transducers can be expressed as [130]:

$$\delta_a = \lambda / D, \quad (4.1)$$

where $\lambda = c / f$ is a wavelength with the wave speed c and frequency f . Thus, to estimate two source directions with angle resolution δ_0 , it requires that the array angle resolution is larger than the source angle resolution, i.e., $\delta_a \geq \delta_0$ [130]. Therefore, the physical aperture size must satisfy the condition of $D \geq \lambda / \delta_0$.

Secondly, in array signal processing, each element in the array of transducers receives the wavefield at discrete spatial positions $\vec{r}_m$, where $m = 1, 2, \dots, N$. Thus, the array pattern can be expressed as

$$W(\vec{k}) = \sum_{m=1}^N \exp(j\vec{k} \cdot \vec{r}_m), \quad (4.2)$$

where $\vec{k} = k\vec{\zeta}$ and $k = \omega / c$ is the wavenumber. To avoid aliasing in the array pattern due to any periodic scheme, the array spacing must satisfy the following condition:

$$\Delta \leq \pi / k = \lambda / 2. \quad (4.3)$$

Therefore, the array spacing larger than half wavelength leads to spatial aliasing and grating lobes in the array pattern. For example, in the case of $\Delta = 1.5\lambda$, there are two grating lobes presents together with the mainlobe in the array pattern, as shown in Fig. 4.2. Signals propagating from grating lobe directions would be indistinguishable with the signals propagating from the mainlobe direction. Hence, to uniquely determine the direction of the source, the array spacing is to be equal or smaller than the half wavelength at a chosen frequency, i.e., $\Delta \leq \lambda/2$.

Figure 4.2 Array pattern in the case of $\Delta = 1.5\lambda$.

4.1.2 MUSIC algorithm

The MUSIC algorithm relies on the property of the array covariance matrix. This matrix can be decomposed and partitioned eigenvalues and eigenvectors into two subspaces, including the signal subspace and the noise subspace. In fact, the steering

vectors corresponding to the direction of the source is orthogonal to the noise subspace [132]. Besides, the noise subspace is orthogonal to the signal subspace in the MUSIC algorithm [130], [132]. Thus, these observations imply that the steering vectors corresponding to the source directions belong to the signal subspace.

There are two main steps in the MUSIC algorithm, as shown in Fig. 4.3. The first step is to estimate the array covariance matrix from the available data samples (snapshots). The array covariance matrix is computed as

$$\hat{\mathbf{R}} = \frac{1}{N_d} \sum_{i=1}^{N_d} \mathbf{X}_i \mathbf{X}_i^H, \quad (4.4)$$

where N_d is number of snapshots and $\mathbf{X}_i = [X_{i1} \ X_{i2} \ \dots \ X_{iN}]^T$ is the i^{th} snapshot vector received at the array of transducers with N elements. To get a desired accurate estimation of the array covariance matrix, it requires large number of snapshots N_d . Thus, the MUSIC algorithm requires large memory size to store the data samples.

The second step of the MUSIC algorithm is to perform the eigen-decomposition of the array covariance matrix $\hat{\mathbf{R}}$ so as to compute the eigenvalues and the corresponding eigenvectors. Given that the number of significant eigenvalues is denoted as N_s , and these eigenvalues are arranged in decreasing value order, i.e., $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_{N_s} > 0$, while $\sigma_i = 0$ for $N \geq i \geq N_s + 1$. Besides, the corresponding eigenvectors are separated into two subspaces as follows:

- (i) The eigenvectors corresponding to first N_s eigenvalues are selected as the signal subspace eigenvectors.
- (ii) The remaining $(N - N_s)$ eigenvectors are classified as the noise subspace eigenvectors.

Figure 4.3 Flowchart of the MUSIC algorithm.

Thus, the MUSIC algorithm can detect the number of unknown sources presence in the medium as the number of signal subspace eigenvalues N_s . Subsequently, the noise subspace eigenvectors are used to compute the directions of unknown sources by calculating the pseudo-spectrum as

$$P = \frac{1}{\|\mathbf{E}_n^H \mathbf{S}_\theta\|^2}, \quad (4.5)$$

where $\|\cdot\|$ denotes the Euclidean norm of a vector, $\mathbf{E}_n$ are the noise subspace eigenvectors of dimension $N \times 1$ and $\mathbf{S}_\theta$ is the test steering vector of dimension $N \times 1$ at a testing arrival angle θ where $-180^\circ \leq \theta \leq 180^\circ$, which can be defined as

$$\mathbf{S}_\theta = \left[e^{j\omega\Delta\cos\theta/c}, \dots, e^{j\omega m\Delta\cos\theta/c}, \dots, e^{j\omega N\Delta\cos\theta/c} \right]^T, \quad m=1,2,\dots,N. \quad (4.6)$$

In the MUSIC algorithm, the peak values of the pseudo-spectrum P occur at the points when $\|\mathbf{E}_n^H \mathbf{S}_\theta\|$ are close to zero, i.e., $\|\mathbf{E}_n^H \mathbf{S}_\theta\| \approx 0$. This condition implies that the steering vectors $\mathbf{S}_\theta$ are orthogonal to the noise subspace eigenvectors $\mathbf{E}_n$. Thus, based on the peak values of the pseudo-spectrum, we can determine the direction of unknown sources in the medium. The flowchart of the MUSIC algorithm is also summarized in Fig. 4.3.

Furthermore, when the number of transducer is much larger than the number of sources, i.e., $N \gg M$, the signal subspace eigenvectors can be used instead of the noise subspace eigenvectors to compute the pseudo-spectrum as

$$P = \frac{1}{\|\mathbf{I} - \mathbf{E}_s^H \mathbf{S}_\theta\|^2}, \quad (4.7)$$

where $\mathbf{E}_s$ is the signal subspace eigenvectors of dimension $N \times 1$ and $\mathbf{I}$ is an identity matrix of dimension $N \times N$. Compared to (4.5), the computational complexity of (4.7) is much reduced since the number of the signal subspace eigenvectors is smaller than the number of the noise subspace eigenvectors.

The MUSIC algorithm can obtain good resolution in source direction finding and high accuracy in source detection applications [132]-[134], [138]. However, the MUSIC algorithm also has some limitations. Firstly, in order to estimate exactly the number and direction of unknown sources existing in the medium, the MUSIC algorithm requires that number of the transducers should be larger than the number of the sources, i.e., $N > M$ [130], [132], [133]. Secondly, the accuracy of the MUSIC algorithm also depends very much on the estimation of array covariance matrix. This estimation requires large number of snapshots measurement in monostatic configuration [138]. In order to achieve a smaller angle resolution, larger aperture size is required. These requirements imply large number of transducers spaced at an array spacing of $\lambda/2$. This latter restriction also leads to more computational load in performing the eigen-decomposition of an array covariance matrix of large dimension. Besides, for the multiple source detection, the MUSIC algorithm only performs well for high signal-to-noise ratio (SNR) cases [133]. Finally, the estimation of array covariance matrix is also not accurate in the case of some highly correlated sources. Although there are some algorithms proposed to solve this problem, but they are quite difficult to apply and require wideband data [137].

4.2 TR-MUSIC Method

The MUSIC algorithm is an efficient tool to detect the number of sources and direction of sources existing in the medium. However, to achieve good detection performance, the MUSIC algorithm requires large number of snapshots and/or large number of transducers due to its monostatic configuration [138]. Besides, the conventional MUSIC algorithm is usually considered in the homogeneous medium.

On the other hand, the TR technique can be used to locate point-like targets that exist in a complex medium [13]-[15]. Due to the multistatic configuration, where each transducer transmits alternatively, and all the transducers receive the reflected data, the TR technique can achieve a large effective array aperture size [11], [12]. Thus, the TR technique can be applied to any array configurations and/or array geometry conditions. However, the performance of the TR technique degrades seriously when the targets are in the non-well-resolved case [13]-[15], [138]. In this case, the separation between two targets is much smaller compared to the wavelength, and the conventional TR technique fails to detect and localize the targets.

In this section, the conventional MUSIC algorithm is applied into the MDM used in the TR technique. The time-reversal MUSIC (TR-MUSIC) method, which is the combination of the TR technique and the MUSIC algorithm [88]-[90], allows super-resolution detection and localization of unknown point-like targets in both well-resolved and non-well-resolved cases [88], [89]. In particular, as long as the number of the transducer exceeds the number of the existing targets, the MDM has a rank equals to the number of the targets and the pseudo-spectrum of the TR-MUSIC algorithm localizes the position of the targets.

4.2.1 Mathematical model

In this subsection, the scalar wave formulation is used to model the wavefield. Specifically, the Green's function is used to model the wave propagation in the scattering medium [136], [137]. Following that, the MDM is formulated and analyzed in both single and multiple scattering cases. In the frequency domain, the TR

operation in time-domain becomes phase conjugation operation, and the TR-MUSIC algorithm is developed at a central frequency ω [88]-[90], [135]-[138]. Therefore, for simplicity, we will omit the frequency index ω in all of the following developments with the understanding that all equations are expressed in the frequency domain at the frequency ω .

4.2.1.1 Green's function model

In the case of a homogeneous medium such as free space, the Green's function is given by [89]

$$G(\mathbf{r}, \mathbf{r}_0) = \frac{1}{4\pi R} e^{-jkR}, \quad (4.8)$$

where $R = |\mathbf{r} - \mathbf{r}_0|$ is the distance between the positions $\mathbf{r}$ and $\mathbf{r}_0$ in the medium, $k = \omega / c$ is the wavenumber and c is the acoustic (or electromagnetic) wave velocity.

According to (4.8), the Green's function depends on the distance between two positions $\mathbf{r}$ and $\mathbf{r}_0$. The reciprocity theorem indicates that the point-like sources and the observers can be reversed without altering the acoustic field [3]. Therefore, the consequence of applying reciprocity theorem results in

$$G(\mathbf{r}, \mathbf{r}_0) = G(\mathbf{r}_0, \mathbf{r}) = \frac{e^{-jkR}}{4\pi R}. \quad (4.9)$$

The estimation of the Green's function in a homogeneous medium is also provided and discussed in detail in Appendix A.

4.2.1.2 Multistatic data matrix

The MDM is an important data inversion schemes for acoustic localization and subsurface imaging. The array of the transducers consists of N elements with each transducer located at $\mathbf{r}_j, j = 1, 2, \dots, N$. Each transducer is assumed to radiate a scalar field $\psi_j(\mathbf{r})$ into the medium containing some targets. It is useful to note that the theory is developed for ideal point-like (monopole) transducers and point-like targets (similar to the scatterers mentioned in Chapters 2 and 3), which are assumed and employed in a numerous works [13]-[15], [88]-[90], [135], [138].

In the single scattering case, the wavefield radiated by the j^{th} transducer is expressed as

$$\psi_j(\mathbf{r}) = G(\mathbf{r}, \mathbf{r}_j) e_j, \quad (4.10)$$

where e_j is the amplitude of the input signal applied at the j^{th} transducer. Thus, the resulting scattered wavefields are

$$\psi_j^{\text{scatt}}(\mathbf{r}) = \sum_{m=1}^M G(\mathbf{r}, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_j) e_j, \quad (4.11)$$

where M is the number of targets, Γ_m is the reflectivity of the m^{th} target and $\mathbf{x}_m$ is the location of the m^{th} target. For simplicity, the amplitude of the input signal e_j is set as unity. At a central frequency, the K_{ij} element of the MDM $\mathbf{K}$ is the wavefield received in the i^{th} transducer after reflecting from all the scatterers when the initial transmission comes from the j^{th} transducer. The element in the MDM is given as:

$$K_{ij} = \psi(\mathbf{r}_i, \mathbf{r}_j) = \sum_{m=1}^M G(\mathbf{r}_i, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_j), \quad (4.12)$$

where $i, j = 1, 2, \dots, N$. Define the Green's function associated to the m^{th} target as a column vector of dimension $N \times 1$ as

$$\mathbf{G}(\mathbf{x}_m) = [G(\mathbf{r}_1, \mathbf{x}_m) \ G(\mathbf{r}_2, \mathbf{x}_m) \ \dots \ G(\mathbf{r}_N, \mathbf{x}_m)]^T. \quad (4.13)$$

Then, the MDM $\mathbf{K}$ can be expressed in a detailed matrix form as

$$\begin{aligned} \mathbf{K} &= \sum_{m=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m) \\ &= \begin{bmatrix} \sum_{m=1}^M G(\mathbf{r}_1, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_1) & \sum_{m=1}^M G(\mathbf{r}_1, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_2) & \dots & \sum_{m=1}^M G(\mathbf{r}_1, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_N) \\ \sum_{m=1}^M G(\mathbf{r}_2, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_1) & \sum_{m=1}^M G(\mathbf{r}_2, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_2) & \dots & \sum_{m=1}^M G(\mathbf{r}_2, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_N) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{m=1}^M G(\mathbf{r}_N, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_1) & \sum_{m=1}^M G(\mathbf{r}_N, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_2) & \dots & \sum_{m=1}^M G(\mathbf{r}_N, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_N) \end{bmatrix}. \end{aligned} \quad (4.14)$$

In (4.14), the multiple scattering among the targets are assumed to be very small and can be ignored. This case is also called the Born-approximation [89]. Thus, in the case of the non-negligible multiple scattering existed between the targets, a general model can be described as below.

In the case of multiple scattering, the exact scattered field $\psi_j^{\text{scatt}}(\mathbf{r})$ at position $\mathbf{r}$ is due to the radiation by the j^{th} transducer, and can be expressed in the Newman series [140] as

$$\begin{aligned} \psi_j^{\text{scatt}}(\mathbf{r}) &= \sum_{m=1}^M G(\mathbf{r}, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{r}_j) \\ &\quad + \sum_{m=1}^M \sum_{m'=1}^M G(\mathbf{r}, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{x}_{m'}) \Gamma_{m'} G(\mathbf{x}_{m'}, \mathbf{r}_j) (1 - \delta_{m,m'}) \\ &\quad + \sum_{m=1}^M \sum_{m'=1}^M \sum_{m''=1}^M G(\mathbf{r}, \mathbf{x}_m) \Gamma_m G(\mathbf{x}_m, \mathbf{x}_{m'}) \Gamma_{m'} G(\mathbf{x}_{m'}, \mathbf{x}_{m''}) \Gamma_{m''} G(\mathbf{x}_{m''}, \mathbf{r}_j) (1 - \delta_{m,m'}) (1 - \delta_{m',m''}) \\ &\quad + \dots \end{aligned} \quad (4.15)$$

where δ denotes the Kronecker delta, which can be expressed as

$$\delta_{m,m'} = \begin{cases} 0 & m \neq m' \\ 1 & m = m' \end{cases}. \quad (4.16)$$

The first summation in the series (4.15) corresponds to the first-order scattering contribution. Subsequently, the second summation corresponds to the second-order scattering contribution, and so on [140]. Hence, if third and higher order scattering effect is insignificant, this condition is called the second-order scattering model [140]. In this chapter, we only consider the first- and second-order scattering contributions in the MDM formulation.

4.2.1.3 Time-reversal operator

As discussed in Chapters 2 and 3, time-reversal operator (TRO) is defined as

$$\mathbf{T} \triangleq \mathbf{K}^H \mathbf{K} = \mathbf{K}^* \mathbf{K}, \quad (4.17)$$

since the MDM $\mathbf{K}$ is a symmetric matrix [15]. The scattering model used in this analysis is based on Born-approximation. Substituting (4.12) into (4.17), we obtain

$$\begin{aligned} \mathbf{T} &= \left[\sum_{m=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m) \right]^* \left[\sum_{m'=1}^M \mathbf{G}(\mathbf{x}_{m'}) \Gamma_{m'} \mathbf{G}^T(\mathbf{x}_{m'}) \right] \\ &= \sum_{m=1}^M \sum_{m'=1}^M \Gamma_m \Gamma_{m'} \mathbf{G}_m^* \mathbf{G}_m^H \mathbf{G}_{m'} \mathbf{G}_{m'}^T, \end{aligned} \quad (4.18)$$

where $\mathbf{G}_m \triangleq \mathbf{G}(\mathbf{x}_m)$ and $\mathbf{G}_{m'} \triangleq \mathbf{G}(\mathbf{x}_{m'})$. Since $\mathbf{G}_m^H \mathbf{G}_{m'}$ is the inner product, (4.18)

can be written as

$$\mathbf{T} = \sum_{m=1}^M \sum_{m'=1}^M \Lambda_{m,m'} \mathbf{G}_m^* \mathbf{G}_{m'}^T, \quad (4.19)$$

where

$$\Lambda_{m,m'} = \Gamma_m^* \Gamma_{m'} \mathbf{G}_m^H \mathbf{G}_{m'} = \Gamma_m^* \Gamma_{m'} \sum_{i=1}^N G^*(\mathbf{r}_i, \mathbf{x}_m) G(\mathbf{r}_i, \mathbf{x}_{m'}). \quad (4.20)$$

Assume that the separation among targets is much larger than the wavelength. In this case, the targets satisfy the well-resolved condition, and the Green's function vectors $\mathbf{G}_m$ and $\mathbf{G}_{m'}$ are orthogonal when $m \neq m'$ as [89]

$$\mathbf{G}_m^H \mathbf{G}_{m'} \approx \|\mathbf{G}_m\|^2 \delta_{m,m'}, \quad (4.21)$$

where $\delta_{m,m'}$ is the Kronecker delta function and

$$\|\mathbf{G}_m\|^2 = \mathbf{G}_m^H \mathbf{G}_m, \quad (4.22)$$

is the squared norm of the Green's function vector evaluated at the target location $\mathbf{x}_m$.

Therefore, substituting (4.21) into (4.18), a simple form of the TRO can be expressed as

$$\mathbf{T} = \sum_{m=1}^M |\Gamma_m|^2 \|\mathbf{G}_m\|^2 \mathbf{G}_m^* \mathbf{G}_m^T. \quad (4.23)$$

Subsequently, we can apply the same transformation to decompose the TRO $\mathbf{T}$ as

$$\begin{aligned} \mathbf{T} \mathbf{G}_{m_0}^* &= \sum_{m=1}^M |\Gamma_m|^2 \|\mathbf{G}_m\|^2 \mathbf{G}_m^* \mathbf{G}_m^T \mathbf{G}_{m_0}^* \\ &= \sum_{m=1}^M |\Gamma_m|^2 \|\mathbf{G}_m\|^2 \mathbf{G}_m^* \|\mathbf{G}_{m_0}\|^2 \delta_{m,m_0}, \\ &= |\Gamma_{m_0}|^2 \|\mathbf{G}_{m_0}\|^4 \mathbf{G}_{m_0}^* \end{aligned} \quad (4.24)$$

where $\mathbf{G}_{m_0} \triangleq \mathbf{G}(\mathbf{x}_{m_0})$ is the Green's function of a target at the position $\mathbf{x}_{m_0}$ with $\mathbf{x}_{m_0} \in [\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_M]$. From (4.24), we can deduce that the eigenvectors $\mathbf{V}$ of the TRO

$\mathbf{T}$ as $\mathbf{G}_{m_0}^*$, which are the complex conjugate of the Green function vector at the targets' location. Besides, the associated eigenvalues can be expressed as

$$\sigma_{m_0} = |\Gamma_{m_0}|^2 \|\mathbf{G}_{m_0}\|^4. \quad (4.25)$$

As can be seen in (4.25), the eigenvalues depend on the targets' reflectivity and the Green's function associated to the target. It is useful to note again that these results are only true in the case of the number of the scatterers M equals or smaller than the number of the transducers N , i.e., $M \leq N$. Hence, for the well-resolved targets and $M < N$ case, we can determine the number of unknown targets as the number of significant eigenvalues M since eigenvectors and eigenvalues of TRO are related in a one-to-one manner to the well-resolved targets. Besides, we can obtain the corresponding eigenvectors to localize the position of the targets that exists in the medium. The target localization will be discussed clearly in the subsequent subsections.

4.2.2 TR-MUSIC Algorithm

To localize the position of the unknown targets, the TR-MUSIC algorithm is introduced to compute the pseudo-spectrum. In particular, TR-MUSIC algorithm is an improved version of the conventional MUSIC algorithm since array covariance matrix in MUSIC algorithm is replaced by the MDM in the TR algorithm, as described in Fig. 4.4.

In the localization process, the TR-MUSIC algorithm requires the knowledge of the background Green function vectors, which is similar to the steering vectors in the conventional MUSIC algorithm. The underlying reason is that the

Figure 4.4 Description of the TR-MUSIC method.

background Green function vectors of the targets must be orthogonal to the noise subspace eigenvectors. Thus, the pseudo-spectrum according to the TR-MUSIC algorithm can be expressed as

$$P(\mathbf{x}_p) = \frac{1}{\sum_{n=M+1}^N \|\mathbf{V}_n^H \mathbf{G}(\mathbf{x}_p)\|^2}, \quad (4.26)$$

where $\mathbf{G}(\mathbf{x}_p) = [G(\mathbf{r}_1, \mathbf{x}_p) \ G(\mathbf{r}_2, \mathbf{x}_p) \ \dots \ G(\mathbf{r}_N, \mathbf{x}_p)]^T$ is the Green's function (or steering vector) at the test location $\mathbf{x}_p$ and there are $(N - M)$ noise subspace eigenvectors denoted as $\mathbf{V}_n$. It is interesting to note that the targets belong to the signal subspace, which is orthogonal to the noise subspace [132], [137]. Thus, the inner product $\|\mathbf{V}_n^H \mathbf{G}(\mathbf{x}_p)\|^2$ should be zero if the test locations $\mathbf{x}_p$ coincide with the exact target locations $\mathbf{x}_m$. This observation is also true with the non-well-resolved target case. Therefore, the peaks of the pseudo-spectrum P localize the position of the

targets in the medium. Finally, the TR-MUSIC algorithm can be summarized and illustrated in Fig. 4.5.

Figure 4.5 Flowchart of the TR-MUSIC algorithm.

4.3 Numerical Simulation Results

4.3.1 MUSIC algorithm

The MUSIC algorithm is used for the direction-of-arrival or direction finding applications. In this particular simulation, three source signals are chosen to propagate and impinge on the array of transducers from three different directions 30° , 35° and 90° . In array geometry set-up, we perform the numerical simulations in two conventional cases. In the first case, the array of transducers includes 5 elements with uniform array spacing along the y-axis. In particular, the exact locations of the transducers are (0, 2.3) m, (0, 2.4) m, (0, 2.5) m, (0, 2.6) m and (0, 2.7) m in 2D geometry, as described in Fig. 4.1. This array geometry satisfies the conditions of the ULA configuration. Based on Fig. 4.6(a), we observe that there are three peak values in the signal spectrum, which correspond to the angles of 30° , 35° and 90° . Specifically, these estimated results are in good agreement with the exact sources' direction.

In the second case, five elements of the array of transducers are perturbed with the corresponding biases -0.01 m, 0 m, 0.03 m, 0.001 m and -0.01 m with respect to the original positions along the y-axis. From Fig. 4.6(b), there are only two peak values in the range of angles $[30^\circ, 35^\circ]$ and $[80^\circ, 90^\circ]$. Therefore, we cannot clearly estimate three source signals from 30° , 35° and 90° in the perturbed array case.

Hence, the MUSIC algorithm can achieve effective detection of the number of sources and accurate localization of the arrival angle of the sources in the case of exact measurement of the array geometry. However, the MUSIC algorithm is sensitive to any perturbation in the array geometry. The underlying reason is that the

noise subspace eigenvectors of the array covariance matrix in the perturbed case are not totally orthogonal to the background Green's function at the target location.

Figure 4.6 MUSIC spectrum with three different directions arriving from 30, 35 and 90 degrees in two conventional cases: (a) well-known array geometry, (b) perturbed array geometry.

4.3.2 TR-MUSIC algorithm

In this subsection, we consider two separate cases of the TR-MUSIC algorithm including the Born-approximation and the second-order scattering cases. In the Born-approximation case, the scattering among targets is small enough so that only the direct reflections are considered. On the other hand, in the second-order scattering case, the MDM includes the second-order scattering contribution besides the direct reflections from the targets. These scattering effects contribute to the formulation of different MDMs in the TR-MUSIC algorithm.

4.3.2.1 TR-MUSIC in Born-approximation case

In the Born-approximation case, we only consider the reflected signals from the targets to the array of transducers. The simulation set-up with fifty transducers in the ULA configuration and five point-like targets randomly placed in a homogeneous and stationary medium. The input signal is chosen as a Gaussian modulated sine wave [6], [125]. The simulation parameters are also listed in Table 4.1.

Table 4.1 Simulation parameters in Born-approximation case.

Number of transducers	50
Array spacing	0.1 m (along y-axis)
Number of targets	5
Target reflectivity	1
Target locations (X, Y)	(0.5, 2.4) m, (1.7, 2.4) m, (2.6, 0.6) m, (3.6, 3.4) m, (4.5, 4.2) m
Signal speed	340 m/s
Signal peak frequency	500 Hz
Wavelength	0.68 m

Figure 4.7 Simulation geometry for Born-approximation case.

The geometry simulation is considered in 2D, as illustrated in the Fig. 4.7. The transducers (indicated as asterisks) placed along the y-axis transmit the signal to the medium. These signals are scattered by the targets (indicated as circles) in the medium. The signal reflections are then recorded at the array of transducers. These time-domain signals are frequency transformed to construct the MDM. The decomposition of the MDM provides the eigenvalues and eigenvectors, which are associated to the target information. Specifically, the number of significant eigenvalues indicates the number of unknown targets in the medium. Moreover, the noise subspace eigenvectors, which are associated to the insignificant eigenvalues, are employed to compute the pseudo-spectrum and localize the exact position of the targets in the medium.

In the ideal medium, the background noise addition is neglected. Thus, the eigenvalue distribution and the TR-MUSIC pseudo-spectrum can be computed and plotted in Figs. 4.8 and 4.9. From Fig. 4.8, we observe that there are five significant eigenvalues above the threshold, which is chosen from the characteristic of the eigenvalue distribution. In the noise-free condition, the selection of the threshold between significant and insignificant eigenvalues is based on the largest decrease in the eigenvalue magnitude when plotted in logarithmic (dB) scale [136]. Thus, based on the number of significant eigenvalues, we can determine that there are five targets in the medium. Subsequently, we can localize the position of these targets as the colorful spots, as indicated in Fig. 4.9. In the noise-free setting, we can determine the exact position of the targets in the geometry set-up shown in Fig. 4.7.

Figure 4.8 Eigenvalues in Born-approximation case with noise-free.

Figure 4.9 Pseudo-spectrum results in Born-approximation case with noise-free.

The red markers indicate the estimated target locations and the white outline circles indicate the exact target location.

However, in a realistic scenario, there exists a background noise in the medium. In our simulation, a white Gaussian noise (with zero mean and variance of σ^2) is chosen and added to the input signal. With different noise power, different settings of signal-to-noise ratio (SNR) are evaluated. In particular, we consider the TR-MUSIC algorithm in four different SNR settings of SNR = 20 dB, 30 dB, 40 dB and 70 dB. In each SNR setting, the eigenvalues of TRO are calculated and plotted, as shown in Figs. 4.10 - 4.13. It is useful to note that the selection of the threshold between significant and insignificant eigenvalues is based on the corresponding level of SNR settings. The number of significant eigenvalues corresponds to the number of targets presence in the medium.

Figure 4.10 Eigenvalues in Born-approximation case with SNR = 20 dB.

Figure 4.11 Eigenvalues in Born-approximation case with SNR = 30 dB.

Figure 4.12 Eigenvalues in Born-approximation case with SNR = 40 dB.

Figure 4.13 Eigenvalues in Born-approximation case with SNR = 70 dB.

In the cases of background noise addition, it is more difficult to exactly estimate the number of unknown targets. In particular, when the SNR is smaller than 20 dB, the estimated number of targets is not correct. As shown from Table 4.2, we can efficiently determine the number of the targets when the SNR is larger than 40 dB.

Table 4.2 Number of target detection in Born-approximation case.

Settings of SNRs	Number of target presence	Number of target detection
Noise-free	5	5
SNR = 20 dB	5	1
SNR = 30 dB	5	2
SNR = 40 dB	5	4
SNR = 70 dB	5	5

To localize the position of the targets, we analyze the pseudo-spectrum results, as shown in Figs. 4.14 - 4.17. In the cases of background noise addition, it is more difficult to localize the position of the targets than the ideal condition (noise-free). For example, some of the targets (Targets 3, 4 and 5) cannot be detected in the SNR settings of 20 dB and 30 dB, as shown in Figs. 4.14 and 4.15. Thus, in low SNR settings, we can only identify some significant targets, which are close to the array of transducers, i.e., Targets 1 and 2. On the other hand, we can detect and localize the position of the targets with good resolution when the SNR is larger than 40 dB, as shown in Figs. 4.16 and 4.17. Hence, in high SNR settings, the TR-MUSIC algorithm can localize the exact position of all targets in the medium.

Figure 4.14 Pseudo-spectrum results in Born-approximation case with $\text{SNR} = 20 \text{ dB}$. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

Figure 4.15 Pseudo-spectrum results in Born-approximation case with $\text{SNR} = 30 \text{ dB}$. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

Figure 4.16 Pseudo-spectrum results in Born-approximation case with SNR = 40 dB. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

Figure 4.17 Pseudo-spectrum results in Born-approximation case with SNR = 70 dB. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

4.3.2.2 TR-MUSIC in the second-order scattering case

In the second-order scattering case, the scattering signals include single (first-order) scattering (direct reflections) and second-order scattering (scattered signals between two targets). Thus, the MDM includes both single scattering and second-order scattering contributions, which can be expressed as follows:

$$\mathbf{K} = \sum_{m=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m) + \sum_{m=1}^M \sum_{m'=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m, \mathbf{x}_{m'}) \Gamma_{m'} \mathbf{G}(\mathbf{x}_{m'}) (1 - \delta_{m,m'}) \quad (4.27)$$

From (4.27), we observe that the formulation of MDM in second-order scattering case is more complex and resembles the wave propagation in a realistic scattering medium.

Figure 4.18 Simulation geometry for second-order scattering case.

The simulation set-up of the second-order scattering case includes four point-like targets randomly placed at (1.0, 2.2) m, (1.0, 2.4) m, (2.6, 3.5) m and (2.6, 4.2) m, as shown in Fig. 4.18. Specifically, Target 1 and 2 are placed closely with spacing of 0.2 m ($\Delta \approx 0.3\lambda$) to evaluate the super-resolution performance of the TR-MUSIC algorithm in a second-order scattering case. All other simulation parameters are similar to that used in the Born-approximation case, as indicated in Table 4.5.

Firstly, we consider the second-order scattering case in ideal setting without background noise. Using the TR-MUSIC algorithm, the pseudo-spectrum results are shown in Fig. 4.19. From these results, we can detect and localize all targets presence in the medium. In particular, Target 1 and 2 are also detected distinctively although they are closely spaced. Hence, the TR-MUSIC algorithm can achieve super-resolution in target localization in the second-order scattering case.

Figure 4.19 Pseudo-spectrum results in second-order scattering case with noise-free. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

Furthermore, in a more realistic set-up with second-order scattering case, four different SNRs of 20 dB, 30 dB, 40 dB and 70 dB are considered. The pseudo-spectrum results to detect and localize the targets in the medium are evaluated as shown in the Figs. 4.20 to 4.23 for the four SNR settings.

Specifically, in the SNR setting of 20 dB, the TR-MUSIC algorithm can only detect Target 1 which is indicated as a colorful spot near the array of transducers, as shown in Fig. 4.20. Thus, the TR-MUSIC algorithm cannot separately localize two close targets (Target 1 and Target 2) in low SNR conditions. Subsequently, in the SNR setting of 30 dB, we can detect two targets including Target 1 and Target 2, which are indicated in small colorful spots shown in Fig. 4.21. However, it is difficult to detect Target 3 and Target 4, which are far away from the array of transducers, since there are some ghost patterns around these targets in the pseudo-spectrum results.

On the other hand, in the SNR settings of 40 dB and 70 dB, we can distinctively detect four targets which are indicated as four small colorful spots in the pseudo-spectrum results, as shown in Figs. 4.22 and 4.23. Especially, in the second-order scattering case, we observe that Target 1 and Target 2 can be separately detected and localized in good resolution regardless of close separation among them ($\Delta \approx 0.3\lambda$). This is because the TR-MUSIC algorithm can employ higher-order scattering contributions into MDM to achieve larger effective aperture size of the array compared to the conventional MUSIC algorithm [88]. Therefore, the TR-MUSIC algorithm can achieve super-resolution in acoustic detection and localization in the higher-order scattering case.

Figure 4.20 Pseudo-spectrum results in second-order scattering case with $\text{SNR} = 20 \text{ dB}$. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

Figure 4.21 Pseudo-spectrum results in second-order scattering case with $\text{SNR} = 30 \text{ dB}$. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

Figure 4.22 Pseudo-spectrum results in second-order scattering case with $\text{SNR} = 40 \text{ dB}$. The colorful parts indicate the estimated target locations and the white outline circles indicate the exact target location.

Figure 4.23 Pseudo-spectrum results in second-order scattering case with $\text{SNR} = 70 \text{ dB}$. The colorful parts indicate the estimated target locations and the white circles indicate the exact target location.

4.4 Discussions

In this section, we further investigate the differences between the MUSIC and TR-MUSIC algorithms in terms of data measurement and computational complexity. Subsequently, the performances of the TR-MUSIC algorithm are studied and analyzed under different SNR settings. Finally, some key observations of the TR-MUSIC algorithm conducted in single and multiple scattering cases are presented.

4.4.1 MUSIC and TR-MUSIC

In Section 4.3.1, the conventional MUSIC algorithm for acoustic detection applications was studied by analyzing the signal spectrum results. When array geometry is accurately measured, we observe that three different sources are clearly separated by the MUSIC algorithm, as shown in Fig. 4.6(a). However, in the perturbed array geometry, it is difficult to separate different sources and also hard to decide the number of sources existing in the medium, as shown in Fig. 4.6(b). Hence, the conventional MUSIC algorithm is very sensitive to array configuration and geometry due to the usage of monostatic configuration to form the array covariance matrix.

To improve the acoustic detection performance, the TR-MUSIC algorithm is developed and analyzed, as shown in Section 4.3.2. Specifically, the TR-MUSIC algorithm employs the multistatic configuration, where each transducer alternatively transmits and all the transducers receive the reflected signals. Thus, the multistatic configuration gives a richer data matrix than the monostatic configuration. Also, the TR-MUSIC algorithm is not limited to any special array configurations and/or

restricted geometry model (far-field condition), which are imposed on the conventional MUSIC algorithm [138]. Hence, the main difference between TR-MUSIC and MUSIC algorithms is the configuration of matrix data measurement.

On the other hand, the similarity of MUSIC and TR-MUSIC algorithms is mainly developed based on subspace signal processing [135]. In particular, both algorithms share the principles of eigen-decomposition technique in matrix data measurement to obtain eigenvalues and eigenvectors [120]. Besides, these eigenvalues and eigenvectors are arranged and partitioned into two subspaces including signal and noise. Subsequently, both MUSIC and TR-MUSIC algorithms determine the number of sources (MUSIC) or targets (TR-MUSIC) based on the number of signal subspace eigenvalues. Moreover, both algorithms also employ the noise subspace eigenvector to construct the pseudo-spectrum for localizing the position of the sources/targets presence in the medium. Hence, both MUSIC and TR-MUSIC algorithms achieve super-resolution performance in acoustic detection and localization.

Finally, we compare the MUSIC and TR-MUSIC algorithms in term of computational complexity, as shown in Table 4.3. It is useful to note that the two algorithms are compared with the same number of transducers N , as well as number of sources/targets M . In the MUSIC algorithm, N_d is denoted as the number of snapshots to formulate the array covariance matrix. From Table 4.3, we observe that the difference of computational complexity between MUSIC and TR-MUSIC algorithms depends on matrix data measurement. Specifically, if $N_d \geq 2M$, the computational complexity of TR-MUSIC algorithm is smaller than that of MUSIC

algorithm, and vice versa for $N_d < 2M$. In general, to achieve super-resolution in MUSIC, it requires number of snapshots N_d much larger than number of sources/targets M . Therefore, the MUSIC algorithm usually possesses larger computational load and longer processing time compared to the TR-MUSIC algorithm in implementation.

Table 4.3 Comparison of computational complexity between MUSIC and TR-MUSIC algorithms.

	MUSIC	TR-MUSIC
Data Matrix	$\hat{\mathbf{R}} = \frac{1}{N_d} \sum_{i=1}^{N_d} \mathbf{x}_i \mathbf{x}_i^H$ <div> Multiplication Addition $N_d \times N \times N$ $(N_d - 1) \times N \times N$ </div>	$\mathbf{K} = \sum_{m=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m)$ <div> Multiplication Addition $2 \times M \times N \times N$ $(M - 1) \times N \times N$ </div>
Pseudo-spectrum	$P = \frac{1}{\ \mathbf{E}_n^H \mathbf{S}_\theta\ ^2}$ <div> Multiplication Addition $(N - M) \times N$ $(N - M) \times (N - 1)$ </div>	$P(\mathbf{x}_p) = \frac{1}{\sum_{n=M+1}^N \ \mathbf{V}_n^H \mathbf{G}(\mathbf{x}_p)\ ^2}$ <div> Multiplication Addition $(N - M) \times N$ $(N - M) \times (N - 1)$ </div>
Total	<div> Multiplication Addition $N_d \times N \times N$ $(N_d - 1) \times N \times N$ $+(N - M) \times N$ $+(N - M) \times (N - 1)$ </div>	<div> Multiplication Addition $2 \times M \times N \times N$ $(M - 1) \times N \times N$ $+(N - M) \times N$ $+(N - M) \times (N - 1)$ </div>

4.4.2 Robustness of the TR-MUSIC algorithm

To verify the robustness of the TR-MUSIC algorithm based on pseudo-spectrum results, we compare the pseudo-spectrum resolution in different SNR settings, as illustrated in Table 4.4. This investigation is conducted in the Born-approximation

case. It is useful to note that the target numbering in Table 4.4 was previously shown in Fig. 4.3. In the simulation set-up, each point-like target is represented by a grid. The estimated target resolution quality can then be measured by how many colorful grids at the exact target location. For example, the number '1' represents one grid size; the number '2' represents two grid sizes and so on as shown in Table 4.4.

Table 4.4 Comparison of estimated target area in Born-approximation case with different SNR settings.

Targets	No noise	SNR = 20 dB	SNR = 30 dB	SNR = 40 dB	SNR = 70 dB
1	1	2	1	1	1
2	1	>5	2	1	1
3	1	>20	>5	1	1
4	1	>20	>10	2	1
5	1	>20	>20	>10	1

To classify the criteria performance of acoustic localization, we define the spatial resolutions as follows:

- Firstly, if the estimated target area is smaller than two grids, we can obtain the exact position of the target. Thus, the estimated target localization is defined as good resolution;
- Secondly, if the estimated target area is in the range of [5, 10] grids, we still localize the position of the target. Thus, the estimated target localization is defined as medium resolution;
- Finally, if the estimated target area is larger than 10 grids, we cannot localize the exact position of the target in the medium. Thus, the estimated target localization is defined as poor resolution.

Based on the above performance criteria, we can determine the number of targets localized in good, medium and poor resolutions under different SNR settings, as shown in Table 4.5. From these results, we can conclude that when a higher SNR is used, more targets can be detected in good resolution. Especially, when SNR setting is 70 dB, all targets are detected and localized in good resolution, which is similar to the results in the ideal setting with noise-free setting. Besides, we observe that Target 1 always results in good resolution regardless of SNR. This is because Target 1 is closest to the array of transducers.

Table 4.5 Comparison of target resolution in Born-approximation case with different scenarios of SNRs.

Scenario of SNRs	Number of Targets Detected		
	Good resolution	Medium resolution	Poor resolution
No noise addition	5	0	0
SNR = 20 dB	1	1	3
SNR = 30 dB	2	1	2
SNR = 40 dB	4	0	1
SNR = 70 dB	5	0	0

4.4.3 TR-MUSIC in Born-approximation and second-order scattering cases

In this section, we compare the performance of the TR-MUSIC algorithm in Born-approximation and second-order scattering cases. The main difference between the two cases is the construction of the MDM. The computational complexities of the formulation of the MDM in both Born-approximation and second-order scattering

cases are tabulated in Table 4.6. In particular, the computational complexity of the MDM in the second-order scattering case is 3.5 and 2 times larger in multiplication and addition, respectively, compared to the Born-approximation case. Thus, the second-order scattering case requires larger computational load and memory in MDM measurement compared to the Born-approximation case. It is useful to note that both cases use the same pseudo-spectrum formulation to localize the position of the targets in the medium.

Table 4.6 Comparison of computational complexity between Born-approximation and second-order scattering cases.

	Born-approximation case	Second-order scattering case
Multistatic Data Matrix	$\mathbf{K} = \sum_{m=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m)$	$\mathbf{K} = \sum_{m=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m) + \sum_{m=1}^M \sum_{m'=1}^M \mathbf{G}(\mathbf{x}_m) \Gamma_m \mathbf{G}^T(\mathbf{x}_m, \mathbf{x}_{m'}) \Gamma_{m'} \mathbf{G}(\mathbf{x}_{m'}) (1 - \delta_{m,m'})$
	<p>Multiplication Addition</p> <p>$2 \times M \times N \times N$ $(M - 1) \times N \times N$</p>	<p>Multiplication Addition</p> <p>$7 \times M \times N \times N$ $2 \times (M - 1) \times N \times N$</p>

Under the pseudo-spectrum performance, the TR-MUSIC algorithm in second-order scattering case can exactly localize all the targets with good resolution for the SNR of 40 dB. On the other hand, in this setting, the TR-MUSIC algorithm used in the Born-approximation case cannot localize one target, that is placed far away from the array of transducer. Moreover, the TR-MUSIC algorithm used in the second-order

scattering case can efficiently localize two closely spaced targets, as shown in Figs. 4.22 and 4.23. Thus, the second-order scattering case can improve the localization performance compared to the Born-approximation case.

In addition, we discuss the eigenvectors and the target characterization of the TR-MUSIC under both Born-approximation and second-order scattering cases. Specifically, in the Born-approximation case, the principal eigenvectors are precisely the complex conjugates of the Green function vectors, as shown in (4.25). Besides, each eigenvector is associated with a particular target in the medium [14]. On the other hand, in the second-order scattering case, the principal eigenvectors become linear combinations of the complex conjugates of the Green function vectors [140]. These eigenvector results in the second-order scattering case are similar to the results for the non-well-resolved case, which was mentioned in [15], [110]. Furthermore, once the target locations are determined from the peaks in the TR-MUSIC pseudo-spectrum, we can also compute the unknown reflectivity of the targets. This computation can be performed by substituting the values of target locations into MDM formulation and solving the resulting system of equations. Therefore, the computation of target reflectivity depends on the scattering model employed, which becomes more exact when higher-order scattering model is used. Hence, the second-order scattering case can achieve better resolution and higher accuracy in target localization and characterization than the Born-approximation case. However, the second-order scattering case also requires large computational load and memory storage, which can be a trade-off compared to the Born-approximation case. Thus, the Born-approximation case is still a common scattering model to implement in

numerical simulations, as can be found in most of the works in acoustic detection and localization [130], [88]-[90], [135]-[138].

4.5 Conclusions

In this chapter, we conducted extensive reviews and investigations of two well-known algorithms in acoustic detection and localization. The MUSIC algorithm can be applied in detecting the number of sources and localizing the direction of sources in the medium. The TR-MUSIC algorithm can be applied in detecting the number of targets and localizing the position of targets in the medium. Both algorithms are developed from subspace signal processing technique. Thus, they can also achieve super-resolution performance in acoustic detection and localization.

The theory of both MUSIC and TR-MUSIC algorithms was reviewed and comparatively studied in different array geometries and mathematical models. In particular, the performance of MUSIC algorithm depends on the formulation of array covariance matrix, which is based on monostatic configuration. Thus, the MUSIC algorithm requires larger amount of data to construct the array covariance matrix. The large dimension of this matrix results in high computational load and large memory storage requirements. On the other hand, the TR-MUSIC algorithm depends on the formulation of MDM, which is based on multistatic configuration. Thus, the TR-MUSIC algorithm does not require multiple snapshots and subsequently, it can greatly reduce the computational complexity in data measurement compared to the conventional MUSIC algorithm.

In addition, the TR-MUSIC algorithm was studied and investigated in both single and multiple scattering cases. The single scattering (Born-approximation) and second-order scattering cases were analyzed and simulated under different SNR settings. From numerical results, the TR-MUSIC algorithm can effectively detect the number of targets based on signal subspace eigenvalues and efficiently localize the position of targets based on noise subspace eigenvectors. In particular, the TR-MUSIC method was found to work well even for closely spaced targets and low SNR conditions when including second-order scattering contribution in MDM. Thus, the TR-MUSIC algorithm is an effective solution in the scattering medium with multiple scattering, which models the actual environment more closely. However, multiple scattering also contributes additional computational load, which can be a trade-off in implementation with limited resources and time.

Hence, the incorporation of the TR technique to subspace signal processing can efficiently improve the detection and localization of unknown targets in the medium. Therefore, the TR technique is employed to develop many important applications including seismo-acoustic imaging [136], [89], ocean acoustics for target location [54], [55], [93], target characterization [94], [95] and ultrasonic nondestructive evaluation of parts [65].

CHAPTER 5

TIME-REVERSAL TECHNIQUE IN ACOUSTIC INVERSE SCATTERING PROBLEM

In Chapters 2 and 3, the DORT method was developed in the isotropic scattering condition with the case of well-resolved and point-like scatterers in the medium [13]-[15]. In this case, the number of significant eigenvalues is equal to the number of the scatterers and each eigenvector is associated to one corresponding scatterer in the medium. Subsequently, the phase and amplitude of the eigenvector can be applied to an array of transducers in order to focus on that particular scatterer.

However, in realistic experiments, the scatterers are commonly elastic objects, thus the isotropic scattering condition is no longer valid. The anisotropy of the scattering was first studied for the DORT method in 1999 [139]. In this work, the invariants were expressed as the normal modes of vibration of the object in the case of a cylindrical electromagnetic scatterer, which was observed by a complete circular one-dimensional (1D) array. Subsequently, many studies in the anisotropic scattering condition has been conducted for different DORT configurations and shapes of the scatterer, such as small dielectric cylinder observed by a linear antenna array [140], small dielectric sphere [141], [142], small dielectric ellipsoid [143], large elastic sphere [144] and large elastic cylinder/sphere [127], etc. A review of these results can

be found in Chambers' paper [145]. Hence, based on a great amount of collected data and mathematical analysis, the DORT method can extract the relevant information from these data to form an image of a scattering object [119].

Recently, time-reversal multiple signal classification (TR-MUSIC) method, which was explained in the previous chapter, has also been developed to perform acoustic detection and localization [88]-[91], [135]-[138]. In particular, the TR-MUSIC method uses signal subspace data to detect the number of scatterers, as well as employs noise subspace data to localize the position of the scatterers in the medium. The TR-MUSIC method can achieve super-resolution in detection and localization although it requires large computational complexity. Following that, the TR-MUSIC method has also been applied to solve an inverse scattering problem, which is a well-known problem of determining the shape and/or the physical properties of the scatterer. These solutions can localize the position and determine the reflectivity or polarization tensors of unknown scatterers in an inverse scattering problem [94], [95]. However, these solutions are only limited to solving the inverse scattering problem in the isotropic scattering condition, where the scatterer is assumed to be a point-like [94] or very small elastic object with $k_0 a \ll 1$ (k_0 is the wavenumber and a is the size of the scatterer) [95].

This limitation motivates us to develop a new approach based on the DORT method to solve an acoustic inverse scattering problem in the anisotropic scattering condition. To achieve this objective, a new technique is proposed to distinguish the shape of a small elastic scatterer (cylinder or sphere with $k_0 a < 0.5$) based on the DORT singular values. Besides, both closed-form equation and numerical solution

are developed to estimate the density and the radius corresponding to the shape of the scatterer (cylinder or sphere). These algorithms can also deal with different material properties of a small elastic scatterer. The proposed approach can be a favorable solution to acoustic inverse scattering problem since it does not require any complex optimization step, which involves solving the direct scattering problem at each step of the iterative scheme [149]–[151]. Thus, the proposed approach can achieve smaller computational load and faster implementation compared to conventional solutions in some specific conditions of small scatterer's size and limited shape. This approach can also be used to localize the position of the scatterer in the medium. Hence, the proposed solution consists of an efficient and non-iterative technique to determine the physical properties of a small elastic scatterer in the anisotropic scattering condition.

This chapter is organized as follows. Firstly, we describe the general knowledge of DORT method in the isotropic and anisotropic scattering conditions in Section 5.1. Secondly, we develop a numerical technique to estimate the responses of the scatterer in anisotropic scattering condition in Section 5.2. Subsequently, the proposed approach to solve the acoustic inverse scattering problem is developed and analyzed in Section 5.3. Several numerical simulations are also presented for different scenarios and significant observations are discussed in Section 5.4. Finally, Section 5.5 summarizes and concludes this chapter.

5.1 DORT Method

In this section, the fundamental theory of the DORT method is studied in the isotropic and anisotropic scattering conditions with only one scatterer in the medium. In the

isotropic scattering condition, the scatterer is chosen as very small size ($k_0 a \ll 1$). On the other hand, in the anisotropic scattering condition, the shape of the scatterer is considered as a cylinder or a sphere with radius a satisfying the condition of $k_0 a < 0.5$. This analysis is performed in the frequency domain, thus for simplicity, the frequency dependence in all equations is kept implicit.

5.1.1 DORT method in isotropic scattering condition

In this subsection, the isotropic scatterer is characterized by a scattering coefficient R , which corresponds to the monopolar mode of vibration [146]. The scattering coefficient can be expressed as $R = |R|e^{i\phi}$, with $|R|$ is the modulus of the scattering coefficient and ϕ is the scattering phase shift. We define the Green's function describing the propagation from the i^{th} transducer to the scatterer as h_i . Thus, we can construct a vector $\mathbf{h} = [h_1 \ h_2 \ \dots h_i \ \dots h_N]^T$, where N is number of element in the array of transducers. Assume that spatial reciprocity is valid throughout the whole medium, the vector describing the propagation for the wave from the transducer array to the scatterer is given by the transpose of vector $\mathbf{h}$, denoted by $\mathbf{h}^T$ [147]. Thus, the MDM $\mathbf{K}$ in the isotropic scattering condition can be expressed as

$$\mathbf{K} = \mathbf{h}^T R \mathbf{h}. \quad (5.1)$$

The MDM $\mathbf{K}$ is symmetrical and its rank equals 1 due to the isotropic scattering condition. Therefore, we can perform the SVD of MDM $\mathbf{K}$ as

$$\mathbf{K} = \mathbf{U}_1 \sigma_1 \mathbf{U}_1^T, \quad (5.2)$$

where σ_1 is the unique positive singular value, $\mathbf{U}_1$ is the normalized singular vector.

By comparing (5.1) with (5.2), the singular value can be expressed as

$$\sigma_1 = |R| \|\mathbf{h}\|^2, \quad (5.3)$$

where $\|\cdot\|$ indicates the standard Euclidean vector norm. Moreover, the corresponding singular vector of dimension $N \times 1$ is expressed as

$$\mathbf{U}_1 = \frac{\mathbf{h}^T e^{i\phi/2}}{\|\mathbf{h}\|}. \quad (5.4)$$

Hence, in the isotropic scattering condition, the singular value depends only on the scattering coefficient R and the propagation effects $\|\mathbf{h}\|^2$, which can be calculated as the summation of the squared magnitude of the Green's functions between the array of transducers and the scatterer. Besides, the phase of the singular vector corresponds to the phase shift due to the propagation from the transducer array to the scatterer plus half scattering phase shift.

5.1.2 DORT method in anisotropic scattering condition

In an anisotropic scattering medium, a one-dimensional (1D) array of N transducers is considered with the same geometry set-up in [127]. The position of the j^{th} transducer of the array is denoted as y_j . A scatterer (cylinder or sphere) of radius a can be located in the medium. The scatterer, located at position dy along the (Oy) axis, is placed at a distance F from the transducer array. The center of the scatterer is placed at the origin

of a spherical coordinate ($O \theta \psi$), with θ as the angle in the (Oxy) plane and ψ as the angle with respect to the (Oz) axis, as seen in Fig. 5.1.

Based on above geometry set-up, the position between the j^{th} transducer and the center of the scatterer is marked by the distance $r_j = \sqrt{F^2 + (y_j - dy)^2}$ and the retro-diffusion angle $\varphi_j = \theta_j - \pi$. This angle is defined by $\sin \varphi_j = (y_j - dy) / r_j$, so that the terms φ_j and $(y_j - dy)$ have the same sign. The aperture function O_j of the j^{th} transducer is defined by $O_j = \text{sinc}(A \sin \varphi_j)$ [127], [154]. The emission and reception transducers are denoted as i and j , respectively, the angle φ_{ij} between incident and scattering directions in the (Oxy) plane is defined as $\varphi_{ij} = \varphi_j - \varphi_i$.

Figure 5.1 Geometry of the DORT method in an elastic scatterer (cylinder or sphere).

Note that the positive direction is downward and the positive angle is counterclockwise (adapted and modified from [127]).

In a general anisotropic scattering condition, an element K_{ij} of the MDM $\mathbf{K}$ is written as the sum of $(m + 1)$ cylindrical (or spherical) normal modes of vibrations for an elastic cylinder (superscript c) or an elastic sphere (superscript s) [127], respectively as

$$K_{ij}^c = O_i O_j \sum_{n=0}^m \varepsilon_n R_n H_n^{(1)}(k_0 r_i) H_n^{(1)}(k_0 r_j) \cos(n\varphi_{ij}), \quad (5.5a)$$

$$K_{ij}^s = O_i O_j \sum_{n=0}^m (2n+1) r_n h_n^{(1)}(k_0 r_i) h_n^{(1)}(k_0 r_j) P_n^0(\cos \varphi_{ij}), \quad (5.5b)$$

with $1 \leq i \leq N$, $1 \leq j \leq N$. The $e^{-i\omega t}$ term is omitted. The Neumann coefficient ε_n is expressed as $\varepsilon_0 = 1$ and $\varepsilon_n = 2$ for $n \geq 1$. The corresponding spherical term is $(2n + 1)$. Each normal mode is weighted by scattering coefficients R_n for a cylinder [127], [151] or r_n for a sphere [127]. The scattering coefficients depend on the elastic properties of the scatterer and the surrounding medium. Their expressions are provided in detail in Appendix B. The cylindrical or spherical Hankel functions of the first kind, $H_n^{(1)}$ or $h_n^{(1)}$, depend on the wavenumber $k_0 = 2\pi/\lambda_0 = \omega/c_0$, where λ_0 and c_0 are the wavelength and sound speed in the surrounding medium, respectively [153]. The angular dependence is $\cos(n\varphi_{ij})$ for the n^{th} cylindrical normal mode and $P_n^0(\cos \varphi_{ij})$ is the Legendre polynomial for the n^{th} spherical normal mode [152]. For the cosine term, the Legendre polynomial and the aperture function are even, the MDM $\mathbf{K}$ is symmetric as expected, i.e., $K_{ij} = K_{ji}$. The m term denotes the highest significant normal mode order and is of the order of $k_0 a$ ($0 \leq n \leq m$). Note that the superscripts c and s denote the cylindrical and spherical scatterer, respectively.

In this chapter, we consider the anisotropic scattering condition in the case of a small elastic scatterer with $k_0 a < 0.5$. In this case, the two first normal modes are significant, i.e., $m = 1$, with monopole mode ($n = 0$) and dipole mode ($n = 1$) [127]. Thus, the elements K_{ij}^c and K_{ij}^s of the MDM $\mathbf{K}$ for a small cylinder (superscript c) and a small sphere (superscript s), respectively, can be expressed as [122]

$$K_{ij}^c = O_i O_j \left(R_0 H_0^{(1)}(k_0 r_i) H_0^{(1)}(k_0 r_j) + 2 R_1 H_1^{(1)}(k_0 r_i) H_1^{(1)}(k_0 r_j) \cos(\varphi_{ij}) \right), \quad (5.6a)$$

$$K_{ij}^s = O_i O_j \left(r_0 h_0^{(1)}(k_0 r_i) h_0^{(1)}(k_0 r_j) P_0^0(\cos \alpha_{ij}) + 3 r_1 h_1^{(1)}(k_0 r_i) h_1^{(1)}(k_0 r_j) P_1^0(\cos \varphi_{ij}) \right), \quad (5.6b)$$

where r_i and r_j are the distances from the i^{th} and j^{th} transducers to the scatterer. O_i and O_j are the aperture functions of the i^{th} and j^{th} transducers, respectively. $P_0^0(\cos \varphi_{ij})$ and $P_1^0(\cos \varphi_{ij})$ are the Legendre polynomial for the spherical monopole and dipole modes, respectively. $H_0^{(1)}, H_1^{(1)}$ and $h_0^{(1)}, h_1^{(1)}$ are the cylindrical and spherical Hankel functions of the first kind, respectively. Moreover, the cylindrical and spherical scattering coefficients are approximated as $R_0 \approx \alpha^c c^c$, $R_1 \approx -\beta^c c^c / 2$ (for cylinder) and $r_0 \approx \alpha^s c^s$, $r_1 \approx -\beta^s c^s / 3$ (for sphere) [127], where α and β are the compressibility and density contrasts, respectively, and c is the scattering coefficient, expressed as

$$\alpha^c = 1 - \frac{\rho_0 c_0^2}{\rho_1 (c_L^2 - c_T^2)}, \quad (5.7a) \quad \alpha^s = 1 - \frac{\rho_0 c_0^2}{\rho_1 (c_L^2 - 4c_T^2 / 3)}, \quad (5.7b)$$

$$\beta^c = 2 \frac{\rho_1 - \rho_0}{\rho_1 + \rho_0}, \quad (5.8a) \quad \beta^s = 3 \frac{\rho_1 - \rho_0}{2\rho_1 + \rho_0}, \quad (5.8b)$$

$$c^c = -i\pi(k_0 a)^2 / 4, \quad (5.9a) \quad c^s = -i(k_0 a)^3 / 3, \quad (5.9b)$$

where ρ_0 is the density of the surrounding medium, ρ_1 is the density of the scatterer, and c_L, c_T are longitudinal and transverse wave speeds in the scatterer, respectively.

In the case of a small elastic scatterer with $k_0 a < 0.5$, the decomposition of the MDM $\mathbf{K}$ provides two significant singular values with one singular value in the monopole mode and one singular value in the anti-symmetric dipole mode [154], which are expressed as follows:

$$\sigma_0^c = (\alpha^c + \beta^c) |c^c| \|\mathbf{H}_0\|^2, \quad (5.10a) \quad \sigma_0^s = (\alpha^s + \beta^s) |c^s| \|\mathbf{h}_0\|^2, \quad (5.10b)$$

$$\sigma_1^c = \beta^c |c^c| \|\mathbf{H}_{11}\|^2, \quad (5.11a) \quad \sigma_1^s = \beta^s |c^s| \|\mathbf{h}_{11}\|^2. \quad (5.11b)$$

$\mathbf{H}_0, \mathbf{H}_{11}$ are cylindrical monopole mode and anti-symmetric dipole mode responses

expressed as $\mathbf{H}_0 = [H_{0,1} \dots H_{0,j} \dots H_{0,N}]^T$, where $H_{0,j} = O_j \sqrt{\frac{2}{i\pi k_0 r_j}} e^{ik_0 r_j}$; and

$\mathbf{H}_{11} = [H_{11,1} \dots H_{11,j} \dots H_{11,N}]^T$, where $H_{11,j} = \sin(\varphi_j) H_{0,j}$. $\mathbf{h}_0, \mathbf{h}_{11}$ are spherical monopole mode and anti-symmetric dipole mode responses expressed as

$\mathbf{h}_0 = [h_{0,1} \dots h_{0,j} \dots h_{0,N}]^T$, where $h_{0,j} = O_j \frac{-ie^{ik_0 r_j}}{k_0 r_j}$; and $\mathbf{h}_{11} = [h_{11,1} \dots h_{11,j} \dots h_{11,N}]^T$,

where $h_{11,j} = iO_j P_1^1(\cos \alpha_j) h_1^{(1)}(k_0 r_j)$. Besides, the corresponding singular vectors

($\mathbf{V}_0$ and $\mathbf{V}_1$) to two significant singular values σ_0 and σ_1 of a small elastic scatterer

(cylinder or sphere) are also verified as $\mathbf{V}_0 \propto \tilde{\mathbf{V}}_0$ and $\mathbf{V}_1 \propto \tilde{\mathbf{V}}_{11}$ [127], where

$$\tilde{\mathbf{V}}_0^c = \frac{\mathbf{H}_0}{\|\mathbf{H}_0\|}, \quad \tilde{\mathbf{V}}_{11}^c = \frac{\mathbf{H}_{11}}{\|\mathbf{H}_{11}\|}, \quad (5.12a)$$

$$\tilde{\mathbf{V}}_0^s = \frac{\mathbf{h}_0}{\|\mathbf{h}_0\|}, \quad \tilde{\mathbf{V}}_{11}^s = \frac{\mathbf{h}_{11}}{\|\mathbf{h}_{11}\|}. \quad (5.12b)$$

5.2 Numerical approximation of responses in a small scatterer

In the DORT method, the cylindrical and spherical responses are commonly measured by conducting realistic experiments in pulse-echo mode [127], [154]. However, these measurements are very complex and costly due to large array of transducers and complex hardware (i.e., an acquisition system in every channel). Thus, in this section, the numerical approximation of these responses is developed in order to reduce the computational complexity for feasible implementation.

5.2.1 The geometry and the conditions

Firstly, let us verify the geometry and the conditions of the case study in the numerical approximation. The center-position scatterer is situated at the center axis of the array of transducer, as shown in Fig. 5.2. The far-field condition is considered where the scatterer size a is small compared to the observation distance F , i.e., $a/F \ll 1$. The aperture is considered to be small as the array aperture D is much smaller

Figure 5.2 Geometry of a small aperture, far-field and center-position scatterer case (adapted and modified from [154]).

compared to the observation distance F , i.e., $D/F \ll 1$. This condition implies that the j^{th} observation angle φ_j is small compared to 1 radian [127]. Subsequently, it implies that the aperture function O_j of the j^{th} transducer is equal to 1. A single scatterer is chosen with metallic material, i.e., $\alpha \approx 1$, and small size satisfying the condition of $k_0 a < 0.5$ [154].

5.2.2 Numerical approximation of the responses in a small elastic cylinder

In a small cylinder of $k_0 a < 0.5$, there are only two significant cylindrical responses $\mathbf{H}_0$ and $\mathbf{H}_{11}$. In a small aperture condition, the element of cylindrical monopole mode response is approximated as [127]

$$H_{0,j} \approx \sqrt{\frac{2}{i\pi k_0 F}} e^{ik_0 F}, \quad 1 \leq j \leq N. \quad (5.13)$$

Thus, the squared norm of the cylindrical monopole mode response is

$$\|\mathbf{H}_0\|^2 = \sum_{j=1}^N |H_{0,j}|^2 = \frac{2N}{\pi k_0 F}. \quad (5.14)$$

The anti-symmetric response $\mathbf{H}_{11}$ in dipole mode is related to the symmetric response $\mathbf{H}_0$ in monopole mode as

$$H_{11j} = \sin(\varphi_j) H_{0j}, \quad (5.15)$$

where $\sin(\varphi_j) = \frac{y_j}{F} = \frac{p(j - N/2)}{F}$ and p is the array pitch. Thus, the squared norm of the cylindrical anti-symmetric dipole mode response is

$$\|\mathbf{H}_{11}\|^2 = \sum_{j=1}^N |H_{11,j}|^2 = |H_{0,j}|^2 \frac{p^2}{F^2} \sum_{j=1}^N (j - N/2)^2. \quad (5.16)$$

Expanding the sum of square terms in (5.16), we have

$$\sum_{j=1}^N (j - N/2)^2 = \sum_{j=1}^N (j^2 - jN + N^2/4) = \frac{N(N+1)(2N+1)}{6} - \frac{N^2(N+1)}{2} = \frac{N^3 + 2N}{12}. \quad (5.17)$$

Substituting (5.17) into (5.16), the squared norm of the cylindrical anti-symmetric dipole mode responses is

$$\|\mathbf{H}_{11}\|^2 = H_0^2 \frac{p^2}{F^2} \frac{N^3 + 2N}{12} \approx \frac{ND^2}{6\pi k_o F^3}, \quad (5.18)$$

Hence, the cylindrical ratio between the squared norm of the monopole mode and the squared norm of the anti-symmetric dipole mode responses is defined and expressed as

$$M^c \triangleq \frac{\|\mathbf{H}_0\|^2}{\|\mathbf{H}_{11}\|^2} \approx \frac{2N / \pi k_o F}{ND^2 / 6\pi k_o F^3} = 12 \left(\frac{F}{D} \right)^2. \quad (5.19)$$

5.2.3 Numerical approximation of the responses in a small elastic sphere

In a small elastic sphere, the components of the spherical monopole mode response in a small aperture can be expressed as

$$h_{0,j} \approx \frac{-ie^{ik_o F}}{k_o F}, \quad 1 \leq j \leq N. \quad (5.20)$$

Therefore, the squared norm of the spherical monopole mode responses is

$$\|\mathbf{h}_0\|^2 = \sum_{j=1}^N |h_{0,j}|^2 = \frac{N}{(k_0 F)^2}. \quad (5.21)$$

In a small aperture and far-field case, the angle φ_j is much smaller than 1 radian, i.e., $\varphi_j \ll 1$, and $\cos \varphi_j \approx 1$. Thus, the components of the spherical anti-symmetric dipole mode responses are

$$h_{11,j} = i\sqrt{\frac{1}{2}}P_1^1(\cos \varphi_j)h_1^{(1)}(k_0 F), \quad (5.22)$$

where Legendre polynomial of the first degree is $P_1^1(\cos \varphi_j) = jp/F$ and the spherical Hankel function of the first kind is $h_1^{(1)}(k_0 F) = \frac{-ie^{ik_0 F}}{k_0 F} \left(1 + \frac{i}{k_0 F}\right) \approx \frac{-ie^{ik_0 F}}{k_0 F}$,

where $k_0 F \gg 1$ [152]. Thus, the squared norm of the spherical anti-symmetric dipole mode responses is

$$\begin{aligned} \|\mathbf{h}_{11}\|^2 &= \sum_{j=1}^N |h_{11,j}|^2 = \frac{p^2}{2(k_0 F^2)^2} \sum_{j=1}^N j^2 = \frac{p^2}{2(k_0 F^2)^2} \frac{N(N+1)(2N+1)}{6} \\ &\approx \frac{p^2}{2(k_0 F^2)^2} \frac{2N^3}{6} = \frac{1}{6N(k_0 p)^2} \frac{(Np)^4}{F^4} = \frac{1}{6N(k_0 p)^2} \left(\frac{D}{F}\right)^4. \end{aligned} \quad (5.23)$$

Therefore, the spherical ratio between the squared norm of the monopole mode and the squared norm of the anti-symmetric dipole mode responses is defined and expressed as

$$M^s \triangleq \frac{\|\mathbf{h}_0\|^2}{\|\mathbf{h}_{11}\|^2} = \frac{6(k_0 Np)^2}{(k_0 F)^2} \left(\frac{F}{D}\right)^4 = 6\left(\frac{F}{D}\right)^2. \quad (5.24)$$

It is useful to note that the cylindrical and spherical ratios (M^c and M^s) are always larger than 1 in a small aperture case. This observation also implies that the squared norm of the monopole mode response is always larger than that of the anti-symmetric dipole mode response in a small cylinder and a small sphere.

5.3 DORT Solution in Acoustic Inverse Scattering Problem

The solution of an acoustic inverse scattering problem requires the determination of the shape and the physical properties of an unknown scatterer in a scattering medium. Thus, to determine the position, shape, material and the size of a small elastic scatterer, the proposed solution based on the DORT method includes seven steps, as shown in Fig. 5.3.

- (i) In the first step, the IEIRs are measured by sending pilot signal in each transducer and receiving scattered signal in all transducers to construct the MDM $\mathbf{K}$. Note that the rank of $\mathbf{K}$ is 2 in a small scatterer condition ($k_0 a < 0.5$).
- (ii) The MDM $\mathbf{K}$ is decomposed by the SVD technique to find its singular values and singular vectors.
- (iii) The two most significant singular values of $\mathbf{K}$ are obtained and classified as σ_0 and σ_1 .
- (iv) Subsequently, the position of the scatterer can be located by the TR technique. Based on the pseudo-spectrum created by the TR technique, we can localize the position of the scatterer. Thus, the observation distance F between the scatterer and the array of transducers can also be estimated.

- (v) Furthermore, the ratio of two singular values (σ_0/σ_1) is evaluated and compared with the reference data to determine the shape of the scatterer as either cylinder or sphere.
- (vi) In addition, the density ρ corresponding to the shape of the scatterer is estimated. The estimated density is compared with the reference density to determine the material of the scatterer.
- (vii) Lastly, the radius a is calculated to determine the size of the scatterer.

Figure 5.3 Description of the proposed approach to solve acoustic inverse scattering problem of a small scatterer (cylinder or sphere).

5.3.1 Classification of the singular values and singular vectors

In the DORT implementation, IEIRs are measured by sending pilot signal to one transducer at a time and recording reflected signals in all transducers in the time-domain. After that, the Fourier transform is applied on these time-domain signals to obtain the MDM $\mathbf{K}$. The SVD computation of $\mathbf{K}$ at a chosen frequency provides the distribution of singular values and singular vectors versus the number of transducers. Thus, we can assume that the singular values and singular vectors are *a priori* knowledge obtained from the DORT implementations, which are commonly conducted experimentally in [14], [15], [127] and [154].

In a metallic scatterer, the square of the norm in monopole mode responses $\|\mathbf{H}_0\|^2$ (cylinder) and $\|\mathbf{h}_0\|^2$ (sphere) is always larger than that in anti-symmetric dipole mode responses $\|\mathbf{H}_{11}\|^2$ (cylinder) and $\|\mathbf{h}_{11}\|^2$ (sphere), as has been shown in Section 5.2. Besides, the compressibility contrast α is always positive when the density of the surrounding medium is smaller than that of the scatterer. This condition is also called the Neumann (or rigid boundary) condition [127]. Thus, the singular value σ_0 corresponding to monopole mode is always larger than the singular value σ_1 corresponding to anti-symmetric dipole mode in both a small cylinder and sphere, i.e., $\sigma_0 > \sigma_1$.

Hence, based on the singular values obtained from the DORT implementation, we can classify the largest singular value as σ_0 and the second largest singular value as σ_1 . The corresponding singular vectors to singular values σ_0 , σ_1 are defined

as $\mathbf{V}_0$ and $\mathbf{V}_1$, respectively. This classification is necessary to identify the scatterer's shape and estimate the physical properties of the scatterer in later steps.

5.3.2 Estimation of the scatterer 's position and the observation distance

In general, the position of the scatterer can be localized by using TR detection and localization techniques. Specifically, two well-known TR detection and localization techniques are DORT [92] and TR-MUSIC [90], as introduced in Chapters 3 and 4, respectively. The first technique can locate the point-like scatterers in a well-resolved condition when the Green's functions associated to any two scatterers are orthogonal. On the other hand, the latter technique can work in both well-resolved and non-well resolved conditions. In an isotropic scattering case, each singular vector associated with a significant singular value corresponds to one point-like scatterer. Thus, the number of significant singular values determines the number of point-like scatterers in the medium. In an anisotropic scattering case with a small elastic scatterer ($k_0 a < 0.5$), the rank of the MDM $\mathbf{K}$ is 2 [154]. Thus, there are two significant singular values associated to signal subspace (the scatterer), and $(N-2)$ singular values are very small (close to zero), associated to the noise subspace. Since signal subspace is smaller than noise subspace and one scatterer always satisfies the well-resolved scatterer condition, it is better to apply the DORT algorithm to localize the position of the scatterer than applying the TR-MUSIC algorithm. Thus, the DORT algorithm [92] is extended to estimate the pseudo-spectrum in this anisotropic scattering case as

$$\phi(r) = \sum_{n=0}^1 \left\| \mathbf{U}_n^H g_0(r) \right\|^2, \quad (5.25)$$

where $\mathbf{U}_n$ (with $n = 0, 1$) are two singular vectors corresponding to two significant singular values in the signal subspace and $g_0(r)$ is the background Green's function associated to the scatterer [154]. The peak pseudo-spectrum image indicates the position of the scatterer in the anisotropic scattering medium. From this image, we can estimate the observation distance F_e as the horizontal distance between the transducer array and the scatterer. This estimated observation distance can be employed to develop the numerical technique of the proposed approach in later steps.

5.3.3 Determination of the shape of the scatterer

Based on the estimation of the scatterer position and observation distance, we can determine the shape of the scatterer as either cylinder or sphere. Using same material and size ($k_0 a < 0.5$), the density contrast, the compressibility contrast and the scattering coefficients of a small cylinder are always larger than that of a small sphere, i.e., $\alpha^c > \alpha^s, \beta^c > \beta^s$ and $c^c > c^s$. These observations can be proven theoretically as follows.

Due to the fact that $c_T^2 > 0$, thus the density contrast of the cylinder is always larger than that of the sphere as

$$1 - \frac{\rho_0 c_0^2}{\rho_1 (c_L^2 - c_T^2)} > 1 - \frac{\rho_0 c_0^2}{\rho_1 (c_L^2 - 4c_T^2 / 3)} \Leftrightarrow \alpha^c > \alpha^s. \quad (5.26)$$

In a Neumann condition [127], the density of the scatterer is larger than the density of the surrounding medium, i.e., $\rho_1 > \rho_0$. Thus, the compressibility contrast of the cylinder is always larger than that of the sphere, as shown:

$$2 \frac{\rho_1 - \rho_0}{\rho_1 + \rho_0} > 3 \frac{\rho_1 - \rho_0}{2\rho_1 + \rho_0} \Leftrightarrow \beta^c > \beta^s. \quad (5.27)$$

Moreover, since $k_0 a < 0.5$, the scattering coefficient of the cylinder is always larger than that of the sphere as

$$\pi(k_0 a)^2 / 4 < (k_0 a)^3 / 3 \Leftrightarrow c^c > c^s. \quad (5.28)$$

The real physical parameters (α , β) of some sample metallic scatterers (cylinder or sphere), illustrated in Table 5.1 (where c_L , c_T , ρ parameters are obtained from [155]), are also in good agreement with the above observation. Hence, we can distinguish the shape of the scatterer if there exist information (density contrast or compressibility contrast or scattering coefficient) of both cylinder and sphere.

Table 5.1 Physical parameters of the metallic scatterer (adapted and modified from [155]).

Materials	c_L	c_T	ρ	α^c	α^s	β^c	β^s	$\frac{\alpha^c + \beta^c}{\beta^c}$	$\frac{\alpha^s + \beta^s}{\beta^s}$
Aluminum	6.40	3.00	2.70	0.9739	0.9712	0.9189	0.7969	2.0599	2.2188
Steel	5.75	3.00	7.80	0.9883	0.9864	1.5455	1.2289	1.6395	1.8026
Nickel	5.60	3.00	8.80	0.9986	0.9868	1.5918	1.2581	1.6210	1.7844
Copper	5.01	2.27	8.90	0.9872	0.9860	1.5960	1.2606	1.6186	1.7821
Silver	3.60	1.60	10.50	0.9794	0.9776	1.6522	1.2955	1.5928	1.7546
Tungsten	5.20	2.90	19.30	0.9937	0.9926	1.8030	1.3864	1.5512	1.7160

In a general case, we can determine the shape of the scatterer to be either a cylinder or a sphere based on singular values that are estimated by decomposing MDM $\mathbf{K}$ in the DORT implementation. We define the singular value ratio δ as the ratio between the largest singular value σ_0 and the second largest singular value σ_1 as

$$\delta \triangleq \sigma_0 / \sigma_1 . \quad (5.29)$$

Combining (5.10), (5.11) and (5.29), the singular value ratio in a small cylinder and sphere can be expressed, respectively, as

$$\delta^c = \frac{\alpha^c + \beta^c}{\beta^c} \frac{\|\mathbf{H}_0\|^2}{\|\mathbf{H}_{11}\|^2}, \quad (5.30a)$$

$$\delta^s = \frac{\alpha^s + \beta^s}{\beta^s} \frac{\|\mathbf{h}_0\|^2}{\|\mathbf{h}_{11}\|^2}. \quad (5.30b)$$

In a small aperture case, we can estimate the cylindrical and spherical ratios between squared norm of monopole mode response and squared norm of anti-symmetric dipole mode response, as shown in Section 5.2. Thus, to determine the shape of the scatterer, we can define the Q value which is the ratio between singular value ratio (σ_0/σ_1) and the square of distance-to-aperture ratio $(F/D)^2$ as

$$Q \triangleq \frac{(\sigma_0 / \sigma_1)}{(F / D)^2} . \quad (5.31)$$

Combining (5.29) with (5.31), the Q value for a small cylinder and sphere can be expressed, respectively, as

$$Q^c \approx 12(\alpha^c + \beta^c) / \beta^c , \quad (5.32a)$$

$$Q^s \approx 6(\alpha^s + \beta^s) / \beta^s . \quad (5.32b)$$

Based on the physical properties of some sample metals in Table 5.1 and (5.32), the theoretical ranges of the value Q for a small metallic cylinder and sphere can be calculated as [18.6, 24.7] and [10.3, 13.3], respectively, corresponding to the metallic density range [2.7, 19.3] (g/cm^3). Moreover, due to some errors in the estimated observation distance F_e and the calibration of the aperture size D , the percentage error of Q value can be as high as 10%, as consideration in the upper and lower bounds of the range of the cylinder and the sphere, respectively. As shown in Fig. 5.4, we observe that the conventional Q ranges of the cylinder and sphere are separated. Thus, the Q value can be estimated and compared with these conventional ranges to determine the shape of an unknown small scatterer as

$$Q \triangleq \frac{(\sigma_0 / \sigma_1)}{(F_e / D)^2} \in \begin{cases} [16.9, 27.2] \text{ indicates cylindrical scatterer,} \\ [9.4, 14.6] \text{ indicates spherical scatterer.} \end{cases} \quad (5.33)$$

Figure 5.4 Conventional Q value ranges for a small metallic cylinder and a small metallic sphere.

5.3.4 Estimation of the material and the size of the scatterer

According to the shape of the scatterer, two unknown physical properties of the scatterer including density and radius can be estimated in closed-form equation or through numerical solution. In a small scatterer with metallic material case ($k_0 a < 0.5$ and $\alpha \approx 1$), two significant singular values of the scatterer (cylinder or sphere) can be expressed as (5.9) and (5.10). Dividing (5.10a) to (5.9a) and (5.10b) to (5.9b), we can obtain the density contrast β of the cylinder and sphere, respectively, as

$$\beta^c = \frac{\sigma_1^c \|\mathbf{H}_0\|^2}{\sigma_0^c \|\mathbf{H}_{11}\|^2 - \sigma_1^c \|\mathbf{H}_0\|^2}, \quad (5.34a)$$

$$\beta^s = \frac{\sigma_1^s \|\mathbf{h}_0\|^2}{\sigma_0^s \|\mathbf{h}_{11}\|^2 - \sigma_1^s \|\mathbf{h}_0\|^2}. \quad (5.34b)$$

Combining (5.10), (5.11) and (5.34), the closed-form formulae of the density of the cylinder and the sphere can be expressed, respectively, as

$$\rho_1^c = \frac{2\sigma_0^c \|\mathbf{H}_{11}\|^2 - \sigma_1^c \|\mathbf{H}_0\|^2}{2\sigma_0^c \|\mathbf{H}_{11}\|^2 - 3\sigma_1^c \|\mathbf{H}_0\|^2} \rho_0, \quad (5.35a)$$

$$\rho_1^s = \frac{3\sigma_0^s \|\mathbf{h}_{11}\|^2 - 2\sigma_1^s \|\mathbf{h}_0\|^2}{3\sigma_0^s \|\mathbf{h}_{11}\|^2 - 5\sigma_1^s \|\mathbf{h}_0\|^2} \rho_0. \quad (5.35b)$$

From the estimation of the scatterer's density, we can find the corresponding material of the scatterer by comparing the estimated density with the reference density of the sample materials, as shown in Table 5.1.

Continuously, substituting (5.35a) into (5.10a) and (5.35b) into (5.10b) yields

$$|c^c| = \frac{\sigma_0^c \|\mathbf{H}_{11}\|^2 - \sigma_1^c \|\mathbf{H}_0\|^2}{\|\mathbf{H}_{11}\|^2 \|\mathbf{H}_0\|^2}, \quad (5.36a)$$

$$|c^s| = \frac{\sigma_0^s \|\mathbf{h}_{11}\|^2 - \sigma_1^s \|\mathbf{h}_0\|^2}{\|\mathbf{h}_{11}\|^2 \|\mathbf{h}_0\|^2}. \quad (5.36b)$$

Substituting (5.36a) into (5.11a) and (5.36b) into (5.11b), we can derive the closed-form expressions of the radius of the cylinder and the sphere are, respectively, as

$$a^c = \frac{2}{\sqrt{\pi}k_0} \left(\frac{\sigma_0^c \|\mathbf{H}_{11}\|^2 - \sigma_1^c \|\mathbf{H}_0\|^2}{\|\mathbf{H}_{11}\|^2 \|\mathbf{H}_0\|^2} \right)^{1/2}, \quad (5.37a)$$

$$a^s = \frac{\sqrt[3]{3}}{k_0} \left(\frac{\sigma_0^s \|\mathbf{h}_{11}\|^2 - \sigma_1^s \|\mathbf{h}_0\|^2}{\|\mathbf{h}_{11}\|^2 \|\mathbf{h}_0\|^2} \right)^{1/3}. \quad (5.37b)$$

It is useful to note that the closed-form expressions of the density (5.35) and the radius (5.37) can be applied to any geometry configurations of a small scatterer (cylinder or sphere). However, this estimation requires to measure the cylindrical responses ($\|\mathbf{H}_0\|^2$ and $\|\mathbf{H}_{11}\|^2$) or spherical responses ($\|\mathbf{h}_0\|^2$ and $\|\mathbf{h}_{11}\|^2$) in the anisotropic scattering experiment. However, these measurements are complex and costly to carry out [127].

In a common case, i.e., far-field, small aperture and center-position scatterer case, the cylindrical and spherical responses can be approximated by using numerical techniques, as shown in Section 5.2. Thus, the numerical forms of the density and the radius of a small cylinder and sphere can be expressed, respectively, as

$$\rho_n^c = \frac{D^2 \sigma_0 - 6F_e^2 \sigma_1}{D^2 \sigma_0 - 18F_e^2 \sigma_1} \rho_0, \quad (5.38a)$$

$$\rho_n^s = \frac{D^2 \sigma_0 - 4F_e^2 \sigma_1}{D^2 \sigma_0 - 10F_e^2 \sigma_1} \rho_0, \quad (5.38b)$$

$$a_n^c = \sqrt{\frac{2F_e}{k_0 N D^2} (D^2 \sigma_0 - 12F_e^2 \sigma_1)}, \quad (5.39a)$$

$$a_n^s = \sqrt[3]{\frac{3F_e^2}{k_0 N D^2} (D^2 \sigma_0 - 6F_e^2 \sigma_1)}. \quad (5.39b)$$

Hence, in the numerical technique, the density and radius of a small elastic scatterer can be estimated from basic and feasible parameters including the singular values σ_0 and σ_1 , estimated observation distance F_e , the aperture size D and the number of the transducers N . Thus, the numerical technique can achieve lesser memory storage requirement and smaller computational load compared to the experimental technique due to avoid the measurement of MDM and mode responses. Therefore, it can provide faster implementation in digital processors. Finally, the proposed formulae to solve acoustic inverse scattering problem are summarized in Table 5.2.

Table 5.2 The proposed formulae to solve acoustic inverse scattering problem.

$$\phi(r) = \sum_{n=0}^1 |\mathbf{U}_n^H g_0(r)|^2 .$$

$$Q \triangleq \frac{(\sigma_0 / \sigma_1)}{(F_e / D)^2} \in \begin{cases} [16.9, 27.2] \text{ indicates cylindrical scatterer,} \\ [9.4, 14.6] \text{ indicates spherical scatterer.} \end{cases}$$

$$\rho_e^c = \frac{2\sigma_0 \|\mathbf{H}_{11}\|^2 - \sigma_1 \|\mathbf{H}_0\|^2}{2\sigma_0 \|\mathbf{H}_{11}\|^2 - 3\sigma_1 \|\mathbf{H}_0\|^2} \rho_0, \quad a_e^c = \frac{2}{\sqrt{\pi}k_0} \left(\frac{\sigma_0 \|\mathbf{H}_{11}\|^2 - \sigma_1 \|\mathbf{H}_0\|^2}{\|\mathbf{H}_{11}\|^2 \|\mathbf{H}_0\|^2} \right)^{1/2},$$

$$\rho_e^s = \frac{3\sigma_0 \|\mathbf{h}_{11}\|^2 - 2\sigma_1 \|\mathbf{h}_0\|^2}{3\sigma_0 \|\mathbf{h}_{11}\|^2 - 5\sigma_1 \|\mathbf{h}_0\|^2} \rho_0, \quad a_e^s = \frac{\sqrt[3]{3}}{k_0} \left(\frac{\sigma_0 \|\mathbf{h}_{11}\|^2 - \sigma_1 \|\mathbf{h}_0\|^2}{\|\mathbf{h}_{11}\|^2 \|\mathbf{h}_0\|^2} \right)^{1/3}.$$

$$\rho_n^c = \frac{D^2 \sigma_0 - 6F_e^2 \sigma_1}{D^2 \sigma_0 - 18F_e^2 \sigma_1} \rho_0, \quad a_n^c = \sqrt{\frac{2F_e}{k_0 N D^2} (D^2 \sigma_0 - 12F_e^2 \sigma_1)},$$

$$\rho_n^s = \frac{D^2 \sigma_0 - 4F_e^2 \sigma_1}{D^2 \sigma_0 - 10F_e^2 \sigma_1} \rho_0, \quad a_n^s = \sqrt[3]{\frac{3F_e^2}{k_0 N D^2} (D^2 \sigma_0 - 6F_e^2 \sigma_1)}.$$

5.4 Numerical Simulation Results and Discussions

The numerical simulations are conducted in a water medium with $\rho_0 = 1$ (g/cm³) and $c_0 = 1.5$ (mm/ μ s). A linear array of transducer of 96 elements with a pitch of 0.417 mm operates at a central frequency of 1.5 MHz. A single scatterer is placed at the origin of the 2D geometry set-up and the observation distance between transducer array and the scatterer is 200 mm, as shown in Fig. 5.5. The dimension of the field is fixed at 500 cells $\times$ 100 cells. Each cell is equal to 0.417 mm $\times$ 0.417 mm. The material of the scatterer can be chosen randomly in the metallic density range [2.7, 19.3] (g/cm³). The shape of the scatterer is chosen as either a cylinder or a sphere. The radius of the scatterer is 0.05 mm. Thus, to satisfy the small scatterer condition of $k_0 a < 0.5$, the maximum frequency chosen is 2.3 MHz, which corresponds to the operating frequency bandwidth of 1.6 MHz and available frequency range [0.7, 2.3]

MHz. The ratio D/F and a/F are equal to 0.2 and 2.5×10^{-4} , respectively, satisfying the small aperture and the far-field conditions. Hence, all parameters in the simulation set-up are assigned to satisfy the assumption of an anisotropic scattering and numerical technique conditions described in Sections 5.1 and 5.2.

Figure 5.5 Two-dimensional (2D) geometry set-up.

5.4.1 Numerical simulation results

In the virtual (computer) experiment, the array response matrix $\mathbf{K}$ in an elastic scatterer (cylinder or sphere) can be synthesized from an accurate model that is a sum of normal modes of vibrations [127], [154], and also expressed in (5.5) in Section 5.1. Besides, the expression of $\mathbf{K}$ in (5.5) was verified in very good agreement with the measurement data in the experiments of Minonzio *et al.* [127], [154]. Thus, the array response matrix $\mathbf{K}$ of a small steel scatterer (cylinder or sphere) was simulated in Matlab to verify the performance of our proposed solution in an acoustic inverse scattering problem. The decomposition of MDM $\mathbf{K}$ is conducted at central frequency

of 1.5 MHz to achieve the singular values and singular vectors. From Fig. 5.6, we notice that there are two dominant virtual experimental singular values over 96 singular values calculated from MDM modeled in (5.6). This result confirms that the rank of $\mathbf{K}$ is 2 in a small scatterer case ($k_0 a < 0.5$), as explained in Section 5.1. Besides, two theoretical singular values estimated in closed-form expressions (5.10) and (5.11) are in good agreement with the virtual experimental singular values, as shown in Fig. 5.6.

Figure 5.6 Singular values obtained by virtual (computer) experiment and theoretical approximation at central frequency 1.5 MHz, (a) normalized by the radius a for a steel cylinder and (b) normalized by the radius a^2 for a steel sphere.

Moreover, the experimental and theoretical singular values are investigated in the frequency range from 0.7 to 2.3 MHz, corresponds to $k_0 a$ ranging from 0.15 to 0.48. From Fig. 5.7, we observe that theoretical approximations of the singular values are

close to those obtained from virtual experiment for the cases of small cylinder and small sphere. The differences between experimental and theoretical results are mainly due to the acoustic resonances for a metallic scatterer [127]. Specifically, these singular values are linear dependence with the frequency for the anisotropic scattering condition of $k_0 a < 0.5$. These observations also coincide with those experimental results reported in [154]. Thus, we can employ the singular values and singular vectors obtained from the Matlab simulation at central frequency of 1.5 MHz to verify the proposed algorithms in solving the acoustic inverse scattering problem of an unknown scatterer.

Figure 5.7 Two significant singular values obtained by virtual experiment and theoretical approximation in (a) normalized by the radius a for a steel cylinder with the size of $50 \mu\text{m}$ and (b) normalized by the radius a^2 for a steel sphere with the size of $50 \mu\text{m}$, versus $k_0 a$.

To localize the position of the scatterer, we apply the DORT algorithm that is described in (5.25). The peak amplitude of the mesh plot illustrates the position of the scatterer in the scattering medium, as shown in Fig. 5.8. Moreover, the pseudo-spectrum image is also plotted to locate the position of the scatterer as a brighter spot, as shown in Fig. 5.9. This image of the scatterer (brighter spot) is also close to the actual position of the scatterer (indicated as circle). The observation distance between the transducer array and the scatterer by using the DORT algorithm can be estimated approximately as 196 mm. Hence, the estimated observation distance is very close (approximately 2%) to the actual observation distance defined at 200 mm, as shown in Fig. 5.5. This error is considered as acceptable in detection and localization [95].

Figure 5.8 Mesh plot of a small elastic scatterer by using DORT method.

Figure 5.9 Pseudo-spectrum image of a small elastic scatterer by using the DORT method.

Based on the estimated observation distance and obtainable singular values, we can measure the Q value and determine the shape of a metallic scatterer as shown in (5.33). The shape of the scatterer is verified with different metallic materials from low density (i.e., aluminum) to high density (i.e., tungsten), corresponding to the metallic density range $[2.7, 19.3]$ (g/cm^3), as described in Table 5.1. From Fig. 5.10, we observe that the measurement of Q values (circle) can either belong to the range of a metallic cylinder $[16.9, 27.2]$ or the range of a metallic sphere $[9.4, 14.6]$. Thus, after measuring the Q value, we can compare this value with two above shape (cylinder and sphere) ranges to determine the corresponding shape of the scatterer.

Figure 5.10 Measurement of actual Q values based on virtual experimental singular values.

After determining the observation distance and the shape of the scatterer, the density and the radius of the scatterer can be obtained from the density and radius formulae (5.35), (5.36), (5.38), (5.39) corresponding to the disclosed shape of the scatterer. The closed-form and numerical form expressions of these formulae are summarized in Table 5.2. To evaluate the accuracy of these estimations, the percentage errors between the estimated results and the actual one are considered. In particular, the density error and radius error are defined as

$$\eta_{\rho} = \frac{|\rho - \rho_e|}{\rho} \times 100\%, \quad (5.40)$$

$$\eta_a = \frac{|a - a_e|}{a} \times 100\% , \quad (5.41)$$

where ρ_e and a_e are the estimated density and radius, ρ and a are the actual density and radius.

To verify the robustness of the proposed solution in acoustic inverse scattering problem, the size and the radius of the scatterer are investigated in different materials of scatterers, as described in Table 5.1. Firstly, the density and the radius are estimated in a closed-form expression using virtual experimental data. These results are also compared with the actual density and radius of the scatterer (cylinder or sphere), as shown in Tables 5.3 and 5.4. Based on the results in Table 5.3, we can see that the density error is normally below 10% for the metallic scatterer with low and medium densities. This density error is acceptable due to the fact that there is always a considerable amount of density gap between two nearest metallic materials [155]. Thus, we can compare the estimated density with a list of reference density to determine the scatterer's material. The reference material having the smallest error with estimated one discloses the material of an unknown scatterer. It is useful to note that our proposed approach is only applied to a metallic scatterer. The underlying reason is that closed-form expressions of two singular values (5.10) and (5.11) contain three parameters including the compressibility contrast α , the density contrast β and the scattering coefficient c . In order to derive two closed-form expressions for the density and radius of an unknown scatterer based on two significant singular values, we need to reduce a parameter to satisfy the condition for solving a linear system of equations. Based on this reason, the material of the scatterer is limited to

metallic material whose compressibility contrast is approximately unity ($\alpha \approx 1$), as stated in Table 5.1. On the other hand, the compressibility contrast of a non-metallic scatterer is undetermined ($\alpha < 1$). Thus, the closed-form expressions for the density and radius of a small non-metallic scatterer cannot be derived based on two significant singular values.

Table 5.3 The estimated density of a small scatterer (cylinder or sphere).

Material	Scatterer	Cylinder		Sphere	
	Actual Density (g/cm ³) [155]	Estimated Density (g/cm ³)	Density Error (%)	Estimated Density (g/cm ³)	Density Error (%)
Aluminum	2.70	2.786	3.19	2.811	4.12
Steel	7.80	8.179	4.85	8.358	7.14
Nickel	8.80	9.263	5.26	9.496	7.90
Copper	8.90	9.429	5.94	9.652	8.45
Silver	10.50	11.777	12.16	12.372	17.83
Tungsten	19.30	20.541	6.43	21.271	10.21

Table 5.4 The estimated radius of a small scatterer (cylinder or sphere) with scatterer's size of 50 μm .

Material	Scatterer	Cylinder		Sphere	
	Actual Radius (μm)	Estimated Radius (μm)	Radius Error (%)	Estimated Radius (μm)	Radius Error (%)
Aluminum	50	49.344	1.31	49.516	0.97
Steel	50	49.699	0.60	49.771	0.46
Nickel	50	49.713	0.57	49.779	0.44
Copper	50	49.682	0.64	49.768	0.46
Silver	50	49.482	1.04	49.623	0.75
Tungsten	50	49.843	0.31	49.877	0.25

In addition, the radius of the scatterer (cylinder or sphere) is also estimated with percentage error below 1%, as shown in Table 5.4. We also observe that with the same material, the error in radius estimation of a metallic cylinder is always larger than that of a metallic sphere. A main reason is that the synthesis model of the MDM for a small cylinder, expressed in (5.6a), is only applicable for infinite length or 2D geometry. However, in a practical experiment or simulation, a finite length cylinder is used, that results in extra errors in radius estimation to characterize an unknown cylinder.

Secondly, the radius and the density can also be estimated in numerical technique by calculating the observation distance from the pseudo-spectrum image. In this case, the observation distance error (or position error) is 2%, thus it causes some residual errors in the estimation of the radius and density in numerical technique compared to virtual experimental technique. However, since these residual errors are small, the percentage errors of the density and radius between virtual experimental technique (excluding the position errors) and numerical technique (including the position errors) are very close, respectively, as shown in Figs. 5.11 and 5.12. Hence, the proposed method is quite robust in estimating the position, the shape (cylinder or sphere), the material and the size of an unknown small scatterer.

Figure 5.11 Density errors of the estimated density in different metallic materials.

Figure 5.12 Radius errors of the estimated radius with the scatterer's size of $50\text{ }\mu\text{m}$ in different metallic materials.

Furthermore, to verify the range of the scatterer's size, we have performed the investigation in two cases of the scatterer (cylinder or sphere) with the radius of 35 μm and 65 μm . It is useful to note that these scatterer's sizes were also investigated in the realistic experiments [154]. Specifically, two significant singular values computed using theoretical approximation are verified in good agreement with the singular values found from the experimental measurement. The material of the scatterer was also chosen as metallic samples in the range of density [2.7, 19.3] (g/cm^3), as illustrated in Table 5.1. Subsequently, the size of the scatterer in two different cases of 35 μm and 65 μm can be estimated and listed in Tables 5.5 and 5.6, respectively.

Table 5.5 The estimated radius of a small scatterer (cylinder or sphere) with scatterer's size of 35 μm .

Material	Scatterer	Cylinder		Sphere	
	Actual Radius (μm)	Estimated Radius (μm)	Radius Error (%)	Estimated Radius (μm)	Radius Error (%)
Aluminum	35	34.56	1.26	34.66	0.97
Steel	35	34.80	0.57	34.84	0.46
Nickel	35	34.81	0.54	34.85	0.43
Copper	35	34.78	0.63	34.84	0.46
Silver	35	34.65	1.00	34.74	0.74
Tungsten	35	34.90	0.29	34.91	0.26

Table 5.6 The estimated radius of a small scatterer (cylinder or sphere) with scatterer's size of 65 μm .

Material	Scatterer	Cylinder		Sphere	
	Actual Radius (μm)	Estimated Radius (μm)	Radius Error (%)	Estimated Radius (μm)	Radius Error (%)
Aluminum	65	64.19	1.25	64.36	0.98
Steel	65	64.63	0.57	64.69	0.48
Nickel	65	64.64	0.55	64.71	0.45
Copper	65	64.60	0.62	64.70	0.46
Silver	65	64.36	0.98	64.51	0.75
Tungsten	65	64.81	0.29	64.84	0.25

Based on results in these tables, we observe that the estimated radius of a cylinder (or sphere) using our proposed approach is very close to the actual radius (size) with the radius errors are normally smaller than 1%. Also, the radius error in estimating the size of the cylinder is always higher than that in estimating the size of the sphere due to using finite-length cylinder in simulation model. Lastly, the radius errors are also plotted versus different metallic densities in three cases of the scatterer's size (35 μm , 50 μm and 65 μm), as shown in Fig. 5.13. Based on Fig. 5.13, in the same density, we observe that the cylindrical (or spherical) radius errors in three cases of the scatterer's size are very close with the maximum deviation of radius error only 0.05%. Hence, the proposed approach based on DORT method is quite robust technique to estimate the physical properties (size and density) of a small metallic scatterer in an acoustic inverse scattering problem.

Figure 5.13 Radius errors of the estimated radius in three cases of scatterer's size (35 μm , 50 μm and 65 μm).

5.4.2 Discussions

In this subsection, we discuss and compare the proposed approach based on the DORT method with some conventional solutions in an acoustic inverse scattering problem. Firstly, the proposed solution is developed based on the DORT method, thus it can extract the physical properties of the scatterer by decomposing the MDM. On the other hand, the conventional solution usually includes optimization procedure that requires a direct scattering problem to be solved at each step of the iterative scheme in low frequency region where $k_0 a \ll 1$ [149]. Moreover, in high frequency region, the classical approach for solving inverse problem is based on the computational complex Kirchhoff approximation or geometrical optics [150], [151]. Hence, the proposed approach can avoid solving direct scattering problems, which depend heavily on the

frequency of the wave motion and intensive mathematical computations, used in classical approaches. Thus, the proposed approach requires smaller computational load and faster numerical implementation than iterative approaches to solve acoustic inverse scattering problem [149]-[151].

Moreover, the proposed solution can explore more physical properties of the scatterer compared to other non-iterative approaches in some specific conditions (i.e., small scatterer's size and limited shape) [156]-[160]. In particular, the proposed approach based on the DORT method can be applied to solve acoustic inverse scattering problem with two possible shapes (cylinder and sphere), compared to other solutions developed to explore physical properties of a fixed shape such as a cylinder [156], a sphere [157] or an ellipsoid [158]. Moreover, the proposed approach includes the estimation of the density and the size of a small elastic scatterer in both closed-form and numerical form expressions, which are not provided in two well-known non-iterative solutions in recovering the shape of the scatterer [159], [160]. Thus, the proposed approach based on the DORT method can provide a direct solution and feasible results for numerical implementation in some specific conditions mentioned above.

However, the main drawbacks of the proposed approach are its limitation in applying to the scatterer with size $k_0a < 0.5$ and/or limited shape (cylinder or sphere). For a larger size of the scatterer, the anisotropic scattering model used is highly complex and consists of larger number of modes. Thus, closed-form expressions of the physical properties of a large scatterer are not easily derived. For an unknown scatterer with large size and arbitrary shape, we can refer to three well-known

approaches in inverse obstacle problem including non-iterative techniques [159], [160], iterative techniques solving nonlinear optimization problem [161] and linearized approach [162]. The rigorous justification for these approaches has been undertaken in an intensive mathematical book of Colton and Kress in inverse scattering problem [163].

5.5 Conclusions

In this chapter, we analyzed and proposed an approach based on DORT method to solve the acoustic inverse scattering problem in the anisotropic scattering condition. The scatterer is chosen as a small metallic scatterer with a shape of either cylinder or sphere. In a small metallic scatterer condition of $k_0 a < 0.5$, there are two significant singular values corresponding to monopole and dipole modes in an anisotropic scattering medium. Based on these values, new algorithms are devised to estimate the observation distance, shape, size and the material of an unknown small scatterer. The position and the observation distance of the scatterer can be determined by using the DORT algorithm and singular vectors associated to the scatterer. As a result, we can determine the shape of the scatterer is as either cylinder or sphere by evaluating the ratio of the singular values. Besides, the algorithms of the density and the size corresponding to the shape of the scatterer (cylinder or sphere) were derived in closed-form and numerical form expressions. Thus, we can use these expressions to determine the material and the size of an unknown scatterer without requiring any iterative process. These estimations were verified with a model data in the DORT simulation results.

The proposed approach can provide a direct solution with small computational load to distinguish the shape (cylinder or sphere) and physical properties of a small elastic scatterer. The advantage of the proposed approach lies in its simplicity in computation without the needs of solving direct scattering problem or forward scattering solvers. Besides, the proposed approach can extract simultaneously standard physical information of an unknown scatterer (density and size) in an anisotropic scattering medium. It is useful to note again that the proposed approach only works in a small elastic scatterer with two common shapes (cylinder or sphere).

It is known that the proposed approach based on the DORT method requires a mathematical model of the scatterer in an anisotropic scattering medium. Thus, this proposed approach can possibly be extended to determine physical properties of a small scatterer with ellipsoid shape, which is studied and derived in closed-form acoustic scattering expressions [207]. Moreover, our proposed approach is also a potential solution in the case of two small elastic cylinders, which are studied and developed in multiple scattering model [146].

In conclusion, the proposed approach based on the DORT method is a suitable technique in nondestructive testing (NDT) [164], nondestructive evaluation (NDE) [165] and target characterization applications [127]. Besides, it can be extended to apply in other important fields such as radar [166], sonar [167], geophysical exploration [168], medical imaging [83] and biomedical applications [48].

CHAPTER 6

TIME-REVERSAL TECHNIQUE IN STEREOPHONIC ACOUSTIC ECHO CANCELLATION PROBLEM

From Chapter 3 to Chapter 5, time-reversal (TR) is an invariant property in the acoustic equation. Thus, TR can be developed and applied in some acoustic fields including acoustic focusing, acoustic detection and localization, as well as acoustic inverse scattering problem. Subsequently, in this chapter, a decorrelation property of TR is studied and developed to solve a fundamental problem in stereophonic acoustic echo cancellation (SAEC). Specifically, the SAEC system can enhance spatial information which in turn provides a more immersive experience for users. Stereophonic acoustic echo cancellers often employ a pair of adaptive filters for the estimation of acoustic impulse responses in the receiving room. The effectiveness of echo cancellation is often determined by the performance of these adaptive filters which is quantified by both their convergence rate and their steady-state normalized misalignment. One of the main challenges for a stereophonic acoustic echo canceller is that it suffers from poor convergence [96], [97]. This poor convergence has been found to be caused by the high interchannel coherence between the two-channel input signals, which as a consequence, give rise to an ill-conditioning problem [96]. To address this problem, several algorithms have since been proposed with the common

aim of achieving high rate of convergence by decorrelating the stereophonic input signals.

One of the first algorithms proposed for SAEC involves the addition of random noise to the stereophonic signals [98]. A similar approach computes the amount of noise to be added adaptively through the use of variable gain and energy detection so as to achieve sufficient decorrelation for fast convergence [99]. One of the main disadvantages of this approach is the degradation of audio quality in order to achieve sufficiently high convergence for the adaptive filters.

To address the trade-off between maintaining audio quality and decorrelation, psychoacoustic signal processing techniques have been proposed in [100], [101]. The first method introduces the addition of spectrally shaped random noise that is designed to be an auxiliary signal based on the human auditory properties [100]. Although this technique aims to maintain audio quality by reducing the effect of perceived noise, the operation of frequency masking results in an undesirable processing delay. The signal decorrelation technique proposed in [101] approximates the masking patterns by employing an autoregressive (AR) analysis for the source in the transmission room. These techniques however require high computational complexity in terms of their design and implementation.

The use of time-varying all-pass filters have also been studied in two signal decorrelation techniques [169], [170]. In particular, the first technique achieves signal decorrelation by altering the phase of the transmitted signals. This is achieved by filtering the signals with two independent time-varying first-order all-pass filters [169]. This approach can achieve significant reduction in correlation for higher

frequencies with the imposed constraints. However, lower frequencies are still fairly unaffected by the time variation. The second technique applies a periodically varying filter to either the left or right channel. The signal is either delayed by one sample or unprocessed without any delay [170]. Although these techniques can achieve high convergence for the adaptive filters, altering the phase of the transmitted signals can change the stereophonic perception of the SAEC system.

Nonlinear transformation to the original stereophonic signals has also been proposed to reduce the linear relationship between the transmitted signals [96]. In this technique, nonlinear signals are added to the original stereophonic signals. Due to the nonlinearity principle, this transformation can reduce the interchannel coherence and consequently the convergence rate of the adaptive filters can be increased. Based on similar concept, a new configuration with nonlinear pre-processing is also proposed in [102]. In this configuration, the nonlinear processed signals are used as inputs to the exclusive adaptive filters before tap estimates are copied to a fixed two-channel filter is subsequently used to perform the echo cancellation using the unprocessed signals. The advantage of this method is the ability to use the simple normalized least-mean-square (NLMS) algorithm to achieve high convergence rate, instead of the more complex adaptive algorithms such as recursive least squares (RLS) and affine projection (AP) algorithms. However, the stereophonic perception of the processed signals is somewhat degraded due to additional harmonic distortions introduced by the nonlinearities [171]. Another approach is to decorrelate the channels by means of complementary comb filters [98], [103]. This technique is based on removing the energy in certain frequency bands of the speech signal from one channel. It works

well for frequencies above 1 kHz but it requires a combination of nonlinear transformations at lower frequencies. A survey of existing solutions for decorrelation in the SAEC system can be found in [105]. Among these methods, the nonlinear transformation [96] provides an effective approach to achieve interchannel decorrelation which results in good convergence performance. The nonlinear transformation was investigated using different types of nonlinearities [104] and it has been shown that the half-wave rectifier (HWR) offers a good tradeoff in terms of stereophonic quality, as well as convergence rate.

In addition to the above decorrelation techniques, a multichannel AP algorithm was also proposed to overcome the slow convergence rate of adaptive algorithm by using additional projections [172]. Recently, the exclusive-maximum (XM) tap-selection algorithm has been developed to reduce the high interchannel coherence between stereophonic signals by selecting an exclusive set of filter coefficients to update at each sample iteration [173], [174]. It then minimizes the degradation in convergence rate due to tap-selection by jointly maximizing the energies of the selected tap-inputs. This technique can be applied to NLMS, AP and RLS algorithms with a nonlinear processor to achieve a significant improvement in convergence rate. Hence, as can be seen by the discussions above, there is often a tradeoff between convergence rate and stereophonic perception since the processed signals using such decorrelation techniques often introduce additional signals either in the form of noise or distortion which in turn degrade the stereophonic quality.

To solve the SAEC problem, we proposed to apply TR technique in one channel which in turn reduces the linear relation between the stereophonic signals [113],

[114]. As will be shown in this chapter, we exploit the intrinsic beneficial properties of TR transformation which preserves the magnitude response of the original signal so as to minimize frequency distortion. To address the undesirable phase distortion brought about by this TR process, we propose to process one of the input signals based on its average energy within a given frame. Our motivation of employing this selective time-reversal block (STRB) technique is to ensure that both the audio quality and the stereophonic image of the SAEC system are preserved as much as possible. In order to illustrate the benefits brought about by this TR process, we incorporate the STRB algorithm with the NLMS algorithm for SAEC application.

This chapter is organized as follows. Firstly, Section 6.1 reviews the common problem in the SAEC system and describes the nonlinear transformation solution. Subsequently, we will discuss how to develop the proposed STRB algorithm to become a novel solution to the SAEC problem in Section 6.2. The proposed algorithm is also studied in quantitative performance in the SAEC system in Section 6.3. Then, we conduct several numerical simulations, whose results are discussed in Section 6.4. Additionally, there are some experimental results presented in Section 6.5. Finally, Section 6.6 concludes this chapter.

6.1 Review of SAEC Problem and Nonlinear Transformation Solution

In this section, the problem of the SAEC system is shown in poor convergence rate of the adaptive filters due to high interchannel coherence between stereophonic signals. Subsequently, a nonlinear transformation is presented as a conventional solution to

reduce interchannel coherence as well as improve the convergence rate of the adaptive filters in the SAEC system.

6.1.1 Review of SAEC problem

Figure 6.1 shows an SAEC system, where two microphones in the transmission room pick up speech signals from a source $s(n)$ through two acoustic impulse responses

$\mathbf{g}_i = [g_{i,0} \ g_{i,1} \ \dots \ g_{i,M-1}]^T$ each of length M and $i = 1, 2$ is defined as the channel index.

Stereophonic signals $x_i(n)$ are transmitted to the receiving room, which in turn are coupled to both microphones in the receiving room via another set of acoustic echo paths

$\mathbf{h}_i = [h_{i,0} \ h_{i,1} \ \dots \ h_{i,L-1}]^T$, where L is defined as the length of $\mathbf{h}_i$. Similar to that of

[96], we describe the problem of SAEC for one microphone since similar discussion can be extended to the second channel.

Figure 6.1 Schematic diagram of the SAEC system. Note that only single-channel algorithm is considered, i.e., one of the two microphones in the receiving room is neglected.

In order to minimize the echo in the SAEC system, a pair of finite impulse response (FIR) adaptive filters $\hat{\mathbf{h}}_i(n) = [\hat{h}_{i,0}(n) \hat{h}_{i,1}(n) \dots \hat{h}_{i,L-1}(n)]^T$ is employed where the length of the adaptive filters is assumed to be the same as that for $\mathbf{h}_i(n)$. These adaptive filters are used to estimate the unknown acoustic echo paths $\mathbf{h}_i(n)$ in the receiving room and the output of these adaptive filters are given by

$$\hat{y}(n) = \hat{\mathbf{h}}_1^T(n) \mathbf{x}_1(n) + \hat{\mathbf{h}}_2^T(n) \mathbf{x}_2(n) , \quad (6.1)$$

where $\mathbf{x}_i(n) = [x_i(n) x_i(n-1) \dots x_i(n-L+1)]^T$ for $i=1, 2$, are the tap-input vectors of length L . The microphone signal in the receiving room is then given by

$$y(n) = \mathbf{h}_1^T(n) \mathbf{x}_1(n) + \mathbf{h}_2^T(n) \mathbf{x}_2(n) + w(n) , \quad (6.2)$$

where $w(n)$ is defined as the background noise. Employing (6.1) and (6.2), the acoustic echo is then given as

$$e(n) = [\mathbf{h}_1^T(n) - \hat{\mathbf{h}}_1^T(n)] \mathbf{x}_1(n) + [\mathbf{h}_2^T(n) - \hat{\mathbf{h}}_2^T(n)] \mathbf{x}_2(n) + w(n) . \quad (6.3)$$

It has been shown and described comprehensively in [96] that, for a realistic SAEC system where $L < M$, a unique solution exists. However, due to the high interchannel coherence between input vectors $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$, the convergence rate of the adaptive filters is reduced significantly.

The main reason for the high interchannel coherence in an SAEC system is the existence of a linear relation between tap-input vectors $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ due to them being generated from the same source in the transmission room. In particular, with a linear and time invariant system, such as occur in the transmission room, the relation between the stereophonic signals for the SAEC system [96] can be expressed as

$$\mathbf{x}_{1,M}^T(n) \mathbf{g}_{2,M}(n) = \mathbf{x}_{2,M}^T(n) \mathbf{g}_{1,M}(n), \quad (6.4)$$

where $\mathbf{x}_{i,M}(n) = [x_i(n) \ x_i(n-1) \ \dots \ x_i(n-M+1)]^T$ for channels $i = 1, 2$.

6.1.2 Nonlinear transformation solution

As discussed in Section 6.1.1, the high interchannel coherence degrades the convergence performance of adaptive filters that are employed in SAEC systems. In addition, it has been shown in [175] that reducing the interchannel coherence improves the conditioning of the SAEC problem, which in turn brings about an improvement in the convergence of the adaptive filters. Thus, to reduce the effect of high interchannel coherence brought about by the above linear relation, one possible solution is to apply nonlinear transformation to the stereophonic signals before transmitting these processed signals to the receiving room in the SAEC system [96]. The nonlinear transformation solution in the SAEC problem is to add a nonlinear function given by [96]:

$$x'_i(n) = x_i(n) + \alpha q[x_i(n)], \quad i = 1, 2, \quad (6.5)$$

where q is a nonlinear function and α is a control variable that determines the tradeoff between convergence rate and the audio quality.

Of the several types of nonlinearities that have been investigated in [104], the half-wave rectifier (HWR) function is found to achieve good convergence rate and stereophonic perception performance. This function can be expressed as

$$q[\mathbf{x}(n)] = [\mathbf{x}(n) \pm |\mathbf{x}(n)|] / 2, \quad (6.6)$$

giving

$$\mathbf{x}'_1(n) = \mathbf{x}_1(n) + \frac{\alpha}{2} [\mathbf{x}_1(n) + |\mathbf{x}_1(n)|], \quad (6.7a)$$

$$\mathbf{x}'_2(n) = \mathbf{x}_2(n) + \frac{\alpha}{2} [\mathbf{x}_2(n) - |\mathbf{x}_2(n)|], \quad (6.7b)$$

where $0 < \alpha \leq 0.5$ [96]. Further studies on the effect of the HWR on audio quality show that when α approaches 0.5, the listener experiences harmonic distortions introduced by the HWR transformation [104].

6.2 The Proposed Selective Time-Reversal Block (STRB) Solution

To address the SAEC problem, the main task is to reduce the linear relation given by (6.4) between stereophonic signals. We therefore propose to employ TR operation to only one channel of the SAEC system. However, it is expected that TR operation can severely distort the audio quality and stereophonic perception if it is being applied across every frame of this channel. Hence in order to preserve the audio quality as well as the stereophonic perception, we propose a selective time-reversal block (STRB) algorithm that only selects and time-reverses input blocks of one channel with an average magnitude less than a specified threshold. Following that, we will discuss how our proposed algorithm can be incorporated into the NLMS algorithm.

6.2.1 TR technique

In this section, we investigate and show that time-reversing a frame corresponding to one channel can reduce the linear relationship of the tap-input vectors which as a

consequence reduces the interchannel coherence. Due to the high coherence between two channels, we can choose any channel to apply the TR technique. Specifically, we perform TR processing on the first channel and leave the second channel unprocessed. It is useful to note that the stereophonic signals are first normalized before performing the STRB algorithm.

Therefore, if we apply TR processing on $\mathbf{x}_1(n)$ and assume that the linear relation among stereophonic signals after this transformation still exists, it implies that

$$\bar{\mathbf{x}}_{1,M}^T(n) \mathbf{g}_{2,M}(n) = \mathbf{x}_{2,M}^T(n) \mathbf{g}_{1,M}(n), \quad (6.8)$$

where $\bar{\mathbf{x}}_{1,M}(n) = [x_1(n-M+1) \dots x_1(n-1) x_1(n)]^T$ is defined as the time-reversed signal of $\mathbf{x}_{1,M}(n)$. Combining (6.8) with (6.4), the linear relation between $\bar{\mathbf{x}}_{1,M}(n)$ and $\mathbf{x}_{2,M}(n)$ exists if and only if

$$\mathbf{x}_{1,M}^T(n) \mathbf{g}_{2,M}(n) = \bar{\mathbf{x}}_{1,M}^T(n) \mathbf{g}_{2,M}(n). \quad (6.9)$$

For this equation to be valid, either of the two conditions listed below have to be satisfied:

- 1) $\mathbf{g}_{2,M}(n) = \mathbf{0}$, which indicates that the impulse response in the transmission room associated with the second channel has all its amplitudes equal to zero;
- 2) $\mathbf{x}_{1,M}(n) = \bar{\mathbf{x}}_{1,M}(n)$, which implies that the input signal of the first channel is symmetric in time domain.

It is foreseeable that these two conditions are violated in practice. This is because an acoustic impulse response always has finite amplitude due to the finite distance between the source and microphone as well as reverberation of the transmission room

[96]. In addition, input signals such as speech, music or noise are seldom symmetric within a time domain processing block [171]. These two conditions jointly show that (6.8) cannot be satisfied and as a result, TR processing can efficiently decorrelate stereophonic signals.

6.2.2 Magnitude detector technique

It is expected that the proposed technique to decorrelate the stereophonic signals will significantly degrade the stereophonic perception if this process is applied to all input signal blocks of the first channel. To address this concern, we propose to select and time-reverse only blocks of input signal which are relatively inaudible to the listener. To achieve this, we employ a magnitude detector which selects input blocks having an average magnitude smaller than a specified threshold ε . These blocks are in turn selected to perform TR processing, while those with an average magnitude above ε will be unprocessed.

In order to perform TR, the input signals are pre-processed in blocks of B samples where $B < L$. Defining an input block of the first channel as

$$\mathbf{x}_1(m) = [x_1(mB) \ x_1(mB-1) \ \dots \ x_1(mB-B+1)]^T, \quad (6.10)$$

where m denotes the block index, the time-reversed input signal is then expressed as

$$\bar{\mathbf{x}}_1(m) = [x_1(mB-B+1) \ \dots \ x_1(mB-1) \ x_1(mB)]^T. \quad (6.11)$$

In order to reduce perceived signal distortion, we compute the mean absolute magnitude

$$k(m) = \frac{1}{B} \sum_{i=0}^{B-1} |x_1(mB-i)|, \quad (6.12)$$

and time-reverse only blocks with $k(m) < \varepsilon$, i.e.,

$$\hat{\mathbf{x}}_1(m) = \begin{cases} \bar{\mathbf{x}}_1(m), & \text{if } k(m) < \varepsilon \\ \mathbf{x}_1(m), & \text{if } k(m) \geq \varepsilon \end{cases}. \quad (6.13)$$

Hence, by comparing the average magnitude of each block with specified threshold and performing TR operation appropriately, the input signal $\mathbf{x}_1(n)$ can be transformed as signal $\hat{\mathbf{x}}_1(n)$, as shown in Fig. 6.2.

Figure 6.2 Schematic diagram of the SAEC system with applying STRB transformation. Note that only single-channel algorithm is considered, i.e., one of the two microphones in the receiving room is neglected.

To evaluate the appropriate specified threshold ε for the TR operation, the sound pressure level (SPL) for a typical 16-bit implementation is considered. The

relationship between SPL and the amplitude of the speech input signal A can be expressed as

$$\text{SPL} = 96 \text{ dB} + 10 \log_{10}(A^2). \quad (6.14)$$

Under the absolute threshold in quiet region approximated by Terhardt [176], the typical SPL in a meeting room is 50 dB which is equivalent to the average magnitude of the speech signal in an inaudible region of 0.005. It is important to note that in a practical teleconferencing system, a comfort noise is often inserted at the network terminals in order to mask pauses provide a signal to the far end user when the transmission is interrupted [177]. Thus, a larger threshold than 0.005 can be used to sustain good audio quality in a meeting room.

6.2.3 STRB algorithm

After performing STRB transformation on the input signal of the first channel, the stereophonic signals are then transmitted to the receiving room. To reduce the echo from the receiving room, two adaptive filters employing the NLMS algorithm can be used. After performing STRB algorithm as stated in (6.13), the B data samples of the m^{th} block are used for sample-by-sample adaptation. While the filter coefficients of the NLMS algorithm are being updated using the m^{th} block, our proposed STRB algorithm is concurrently applied to $(m+1)^{\text{th}}$ block as shown in Fig. 6.3. It is therefore expected that, since STRB is applied to the first channel, there will be a delay of B samples with respect to the other channel without STRB preprocessing. In order to preserve the stereophonic image, we propose that the second channel is also delayed by B samples without STRB preprocessing. Defining δ as the regularization

parameter for the NLMS algorithm, the proposed STRB-NLMS algorithm is listed in Table 6.1.

Figure 6.3 Procedure in implementing STRB-NLMS algorithm in the SAEC system.

It is also useful to note that the proposed STRB algorithm can easily be implemented. In practice, digital signal processors commonly provide real-time block processing algorithms and the TR operation can be easily achieved by using circular buffer addressing [178]. In addition, the computational complexity of our STRB algorithm is largely dependent on the magnitude detector algorithm, which only requires $(B-1)$ additions and one division in each block signal.

Table 6.1 The STRB-NLMS algorithm.

Magnitude detector	$k(m) = \frac{1}{B} \sum_{b=0}^{B-1} x_1(mB-b) $
STRB	$\hat{\mathbf{x}}_1(m) = \begin{cases} \bar{\mathbf{x}}_1(m), & \text{if } k(m) < \varepsilon \\ \mathbf{x}_1(m), & \text{if } k(m) \geq \varepsilon \end{cases}$ $\hat{\mathbf{x}}(n) = [\hat{\mathbf{x}}_1^T(n) \ \mathbf{x}_2^T(n)]^T$ $\hat{\mathbf{h}}(n) = [\hat{\mathbf{h}}_1^T(n) \ \hat{\mathbf{h}}_2^T(n)]^T$ <p>If $n \leq B$</p> $\hat{\mathbf{h}}(n) = \mathbf{0}$ <p>else</p>
NLMS	$\hat{y}(n-B) = \hat{\mathbf{h}}^T(n) \hat{\mathbf{x}}(n-B)$ $e(n-B) = y(n-B) - \hat{y}(n-B)$ $\hat{\mathbf{h}}(n+1) = \hat{\mathbf{h}}(n) + \mu \frac{\hat{\mathbf{x}}(n-B)e(n-B)}{\ \hat{\mathbf{x}}(n-B)\ ^2 + \delta}$

6.3 Quantitative Performance of STRB Algorithm

We evaluate the quantitative and subjective performance of the STRB algorithm on the transmitted signals $\mathbf{x}_1(m)$ and $\mathbf{x}_2(m)$ in terms of their interchannel coherence and distortion measurements both in time and frequency domains. In addition, we illustrate the comparisons between our proposed STRB algorithm and the HWR processor [96] as well as the selective random noise addition (SRNA) algorithm. For fair comparison, the SRNA is modified from the work reported in [98], where random noise is only added on selected blocks with mean absolute magnitude smaller than the specified threshold ε . In numerical simulations, we generate the impulse responses using the method of images [179] with the source location at $\{2.2, 1.5, 1.6\}$ m, while

the two microphones are placed at $\{1.5, 2.5, 1.6\}$ m and $\{2.5, 2.5, 1.6\}$ m in a room having dimensions of $4 \times 5 \times 3$ m³. The reverberation time (RT60) is set to 128 ms.

6.3.1 Interchannel coherence

We first show that the STRB algorithm reduces interchannel coherence between the stereophonic signals by first defining the interchannel coherence between $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ as

$$\gamma(f) = \frac{S_{x_1 x_2}(f)}{\sqrt{S_{x_1 x_1}(f) S_{x_2 x_2}(f)}}, \quad (6.15)$$

$$\text{where } S_{x_k x_l}(f) = \sum_{\tau=-\infty}^{+\infty} E\{x_k(n)x_l(n-\tau)\}e^{-j2\pi f\tau} = \sum_{\tau=-\infty}^{+\infty} R_{x_k x_l}(\tau)e^{-j2\pi f\tau}, \quad k, l = 1, 2,$$

and f is the normalized frequency. When applying the STRB algorithm on the first channel, the interchannel coherence between the stereophonic signals is given by

$$\hat{\gamma}(f) = \frac{S_{\hat{x}_1 \hat{x}_2}(f)}{\sqrt{S_{\hat{x}_1 \hat{x}_1}(f) S_{x_2 x_2}(f)}}. \quad (6.16)$$

We note that a time-reversed signal does not change the signal amplitude since it only reverses the elemental positions of the elements in $\mathbf{x}_1(m)$. As a result of this intrinsic property, the auto-correlation function of the STRB processed signal is the same as the auto-correlation of the original signal, i.e.,

$$R_{\hat{x}_1 \hat{x}_1}(\tau) = E\{\hat{x}_1(n)\hat{x}_1(n-\tau)\} = E\{x_1(n)x_1(n-\tau)\} = R_{x_1 x_1}(\tau). \quad (6.17)$$

By taking Fourier transform of above auto-correlation functions, we obtain the auto-spectral density function in the frequency domain giving

$$S_{\hat{x}_1\hat{x}_1}(f) = S_{x_1x_1}(f). \quad (6.18)$$

On the other hand, the cross-spectral density function between STRB processed signals $\hat{\mathbf{x}}_1(m)$ and $\mathbf{x}_2(m)$ is

$$S_{\hat{x}_1x_2}(f) = \sum_{\tau=-\infty}^{+\infty} R_{\hat{x}_1x_2}(\tau) e^{-j2\pi f\tau} = \sum_{\tau=-\infty}^{+\infty} E\{\hat{x}_1(n)x_2(n-\tau)\} e^{-j2\pi f\tau}. \quad (6.19)$$

An important relation involving the magnitude of the cross-spectrum is the cross-spectrum inequality, given by [180]

$$\left| S_{\hat{x}_1x_2}(f) \right|^2 \leq S_{\hat{x}_1\hat{x}_1}(f) S_{x_2x_2}(f). \quad (6.20)$$

Substituting (6.18) into (6.20), we have

$$\left| S_{\hat{x}_1x_2}(f) \right|^2 \leq S_{x_1x_1}(f) S_{x_2x_2}(f). \quad (6.21)$$

In the SAEC system, due to the linear relation between the stereophonic signals in the time domain, the interchannel coherence function is very close to unity [96]. Thus, for the unprocessed SAEC system,

$$|\gamma(f)|^2 \approx 1 \rightarrow \left| S_{x_1x_2}(f) \right|^2 \approx S_{x_1x_1}(f) S_{x_2x_2}(f). \quad (6.22)$$

By substituting (6.22) into (6.21), we can deduce

$$\left| S_{\hat{x}_1x_2}(f) \right|^2 \leq \left| S_{x_1x_2}(f) \right|^2, \quad (6.23)$$

which as a consequence implies that

$$|\hat{\gamma}(f)|^2 \leq |\gamma(f)|^2. \quad (6.24)$$

Hence, our proposed STRB algorithm can reduce the interchannel coherence of the original stereophonic signals in the SAEC system. In addition, the TR properties are also studied and provided in detail in Appendix C for reference.

To illustrate the decorrelation performance of TR technique, we apply the TR technique on stereophonic signals generated from a short speech input of 128 data samples. Note that TR processing is applied only on the first channel of the SAEC system. Interchannel coherence results achieved by using the half-wave rectifier (HWR) method [96], the random noise addition (RNA) method with signal-to-noise ratio (SNR) of 30 dB [98] and the TR technique are 0.9002, 0.5153 and 0.1072, respectively. Thus, TR processing results in higher reduction of interchannel coherence compared to the HWR and RNA processors in the SAEC system.

Furthermore, we compare the interchannel coherence of the proposed STRB algorithm with HWR and SRNA algorithms. We then compute, using (6.15) and (6.16), the time-averaged interchannel coherence for a speech sequence of 10 s using a frame size of 128 ms at 8 kHz sampling rate. This time-averaged interchannel coherence is then plotted against frequency as shown in Fig. 6.4. As can be seen from Figs. 6.4(a) and 6.4(d), the time-averaged interchannel coherence using our proposed STRB algorithm is smaller than that of the original signals across all frequencies. More importantly, the proposed STRB algorithm achieves smaller mean interchannel coherence than the HWR algorithm with a nonlinearity control factor $\alpha = 0.5$. This observation can also be made when our proposed algorithm is compared with the SRNA algorithm using a SNR of 30 dB. The average reduction in mean interchannel coherence across all frequencies for the STRB algorithm over HWR and SRNA algorithms are 0.0278 and 0.0655 respectively.

Figure 6.4 Interchannel coherence plots for (a) no decorrelation, (b) HWR algorithm, (c) SRNA algorithm and (d) STRB algorithm.

6.3.2 Objective distortion measurement in time domain

To compare the distortions introduced by different decorrelation techniques in time domain, we evaluate the speech difference between the processed and the original speeches. For clarity of presentation, Fig. 6.5 shows a speech segment of the speech sequence with duration of 2 seconds received from the first microphone in the transmission room. We can see that the processed speech using STRB method, shown in Fig. 6.5(b), is different from the original speech in the first channel, shown in Fig. 6.5(a), in only a few segments with small magnitudes indicated by the rectangular dotted blocks.

Figure 6.5 (a) Original speech and (b) processed speech using STRB method. Note that these signals are in time domain and belong to the first channel of the SAEC system.

To compare the distortions introduced by different decorrelation techniques in time domain, we evaluate the speech difference between the processed and the original speech signal. Figure 6.6(a) shows the speech difference between the original and the HWR processed speech with $\alpha = 0.5$, while Fig. 6.6(b) shows such differences for the proposed STRB processed speech signal with $B = 128$ and $\varepsilon = 0.01$. Comparing Figs. 6.6(a) and 6.6(b), it can be seen that the STRB algorithm introduces significantly lesser distortion to the speech signal than the HWR algorithm in the first channel. On the other hand, the HWR algorithm introduces distortion proportional to the magnitude of the received signal for decorrelation in both channels

of the SAEC system. It is important to note that, unlike the HWR algorithm, our proposed STRB algorithm does not process the second channel and hence no distortion is introduced in this channel.

Figure 6.6 Speech differences between original and (a) HWR processed signals and (b) STRB processed signals.

In order to quantify the effect of the speech difference introduced by the proposed STRB algorithm, we define the peak signal-to-difference ratio (PSDR) as

$$\text{PSDR} = 20 \log_{10} (1/\sigma), \quad (6.25)$$

where σ is the mean speech difference between the original and processed speech signals. Within a block of the STRB processed signal, the mean speech difference σ is always smaller than the chosen magnitude threshold ε . Thus, we can relate the PSDR of the STRB algorithm to the specified magnitude threshold ε as

$$\text{PSDR} = 20\log_{10}(1/\sigma) \geq 20\log_{10}(1/\varepsilon). \quad (6.26)$$

Figure 6.7 The PSDR of HWR algorithm with $\alpha = 0.5$ (dashed line) and STRB algorithm with ε in the range $(0, 0.3]$ (solid line).

To evaluate the distortions of the STRB processed signal in time domain, the PSDR measurements versus the specified threshold ε is plotted as shown in Fig. 6.7. From this figure, we observe that the minimum PSDR of the STRB algorithm is always greater than 40 dB and therefore it is foreseeable that the proposed algorithm can sustain good speech quality. Specifically, the PSDR of the STRB algorithm with $\varepsilon = 0.01$ and HWR algorithm with $\alpha = 0.5$ are found to be 57.59 dB and 34.44 dB, respectively. Hence, our proposed algorithm with $\varepsilon = 0.01$ and $B = 128$ shows a PSDR improvement of 67.22% over the HWR algorithm with $\alpha = 0.5$.

6.3.3 Objective distortion measurement in frequency domain

We now evaluate the speech distortion in terms of phase and magnitude differences between the original and processed signals of the first channel. The phase and magnitude responses of the original and processed signals are analyzed using a 128-point fast Fourier transform (FFT). These values are then time-averaged across the entire speech sequence. We then compute and plot the time-averaged phase differences for signals processed using HWR preprocessing, STRB processing with no overlap and STRB processing with 50% overlap, respectively as shown in Figs. 6.8(a), 6.8(b) and 6.8(c). We included STRB processing with 50% overlap for the purpose of investigating whether any instantaneous change between a time-reversed block and a neighboring unprocessed block will bring about a large phase distortion over the first channel.

As can be seen from Fig. 6.8, the time-averaged phase difference using STRB algorithm is smaller than that using HWR algorithm across most frequencies. Specifically, the time-averaged phase difference of the STRB processing with no overlap is approximately three times smaller than that of the HWR algorithm. More importantly, we notice that the time-averaged phase difference using STRB processing with no overlap is very close to that using STRB processing with 50% overlap. In particular, the mean phase difference across all frequencies of STRB processing with 50% overlap is 0.0003 radians smaller than that of STRB processing with no overlap. This phase difference corresponds to only a $0.012 \mu\text{s}$ time delay at 8 kHz sampling rate, which can be negligible, compared to the just noticeable difference (JND) for inter-aural time difference (ITD) of $80 \mu\text{s}$ [181]. Since the STRB

processing with 50% overlap requires higher computational load and more complex implementation compared to conventional STRB processing with no overlap, the STRB processing with no overlap offers a good trade-off between computational complexity and phase distortion across frames in the first channel. In addition, as can be seen from Fig. 6.8, the STRB processing does not introduce significant phase distortions in low frequency range of 0–1 kHz where the mean phase difference across this frequency range is 0.0086 radians. This corresponds to a time delay of $2.73 \mu\text{s}$ which is insignificant in terms of acceptable JND for ITD [181]. Hence, we note that the proposed STRB algorithm preserves the stereophonic image of the SAEC system.

Figure 6.8 Time-averaged phase differences of (a) HWR processing, (b) STRB processing with no overlap and (c) STRB processing with 50% overlap.

Figure 6.9 Spectrograms of signal differences (a) between HWR processed signal and original signal, and (b) STRB processed signal and original signal. Note that these signals are belong to the first channel of the SAEC system.

The spectrograms of the signal differences between the processed signals using HWR as well as STRB algorithms and original signal are also evaluated as shown in Figs. 6.9(a) and 6.9(b), respectively. We observe that the spectrogram differences

between the original and STRB processed signals are negligible across most time instances. The spectrogram of the signal difference using the STRB algorithm as shown in Fig. 6.9(b) is much lower than that using the HWR algorithm as shown in Fig. 6.9(a) across all frequencies. Hence, we note that the STRB algorithm preserves much of the speech quality. These observations in frequency domain are also consistent with the objective distortion measurement results in time domain as explained in above section. These smaller objective distortion measurements can be achieved in the STRB algorithm since it performs TR operation on selective blocks.

Furthermore, it is important to relate the phase and magnitude responses with inter-aural time difference (ITD) and inter-aural level difference (ILD). According to the duplex theory [182], ITD is used to localize low frequency sounds, while ILD is used in the localization of high frequency sounds. As can be seen from the discussion above, the STRB processed signal of the first channel can achieve negligible phase distortions in low frequency range and small magnitude distortions in high frequency range. Since our proposed algorithm does not perform any STRB processing on the second channel, the overall joint effect shows that the proposed algorithm introduces negligible distortion to the stereophonic image of the transmitted signals.

6.4 Numerical Simulations Results and Discussions

In numerical simulations, a male speech with duration of 10 s is used to verify the effectiveness of our proposed STRB algorithm in an SAEC system. The speech and audio duration of 10 s are commonly used in telecommunication and internet communication applications that are evaluated statistically in [183] and [184]. Two

microphone signals $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ are obtained by convolving the speech signal with two impulse responses $\mathbf{g}_1(n)$ and $\mathbf{g}_2(n)$ in the transmission room. These impulse responses are generated using the method of images [179] with the source at $\{2.2, 1.5, 1.6\}$ m, while the microphones are placed at $\{1.5, 2.5, 1.6\}$ m and $\{2.5, 2.5, 1.6\}$ m. The echo was generated by further convolving $\mathbf{x}_1(n)$ and $\mathbf{x}_2(n)$ with impulse responses $\mathbf{h}_1(n)$ and $\mathbf{h}_2(n)$ in the receiving room. Impulse responses $\mathbf{h}_1(n)$ and $\mathbf{h}_2(n)$ are also generated using the method of images with the loudspeakers at $\{1, 2.5, 1.6\}$ m and $\{3, 2.5, 1.6\}$ m and microphone at $\{2.1, 1.5, 1.6\}$ m. All impulse responses are of length $L = 1024$ samples and both rooms are of dimensions $4 \times 5 \times 3$ m³. A sampling frequency of 8 kHz and the two-channel NLMS algorithm with filter length of 512 data samples are used throughout the simulation. The reverberation time (RT60) is set to 128 ms. A white noise is added to the microphone signal in the receiving room to achieve an SNR of 30 dB.

6.4.1 Comparison between STRB and HWR algorithms

In this section, we used $B = 128$ and $\varepsilon = 0.01$ for the evaluation of our proposed STRB algorithm. This block length results in a processing delay of 16 ms at 8 kHz sampling rate, which is smaller than the maximum allowable delay budget of 252 ms for a real-time implementation [185]. The threshold ε is chosen as 0.01 which corresponds to 56 dB SPL giving an acceptable audio quality in a common meeting room [176]. The STRB algorithm is benchmarked against the HWR algorithm with a commonly used value of $\alpha = 0.5$ [96]. The quantitative performance of the two algorithms are evaluated and compared in terms of the Bark Spectral Distortion

measurement, convergence rate of normalized misalignment, echo return loss enhancement and subjective listening test.

6.4.1.1 Bark spectral distortion (BSD)

In order to verify that our proposed STRB algorithm introduces less distortion compared to the HWR algorithm, we employ the BSD measurement, whereby a smaller value corresponds to a smaller distortion [186]. The BSD takes into account auditory frequency warping, critical band integration, amplitude sensitivity variations with frequency, and subjective loudness. The mean BSD of the STRB and HWR algorithms are found to be 0.0010 and 0.0274, respectively. Thus, the STRB algorithm shows an improvement of 96.35% in terms of BSD over the HWR algorithm. Hence, the proposed STRB algorithm can achieve better BSD measurement compared to the conventional HWR algorithm in the SAEC system [96].

Figure 6.10 Computation of the BSD.

6.4.1.2 Convergence rate of normalized misalignment

We now evaluate the convergence performance of our proposed STRB and the HWR algorithms in terms of normalized misalignment defined by

$$\eta(n) = 10 \log_{10} \left[\frac{\|\mathbf{h} - \hat{\mathbf{h}}(n)\|^2}{\|\mathbf{h}\|^2} \right]. \quad (6.27)$$

In the first simulation, we compare the convergence rate between STRB and HWR algorithms using a synthesized white Gaussian noise (WGN) as input signal. The purpose of using synthesized WGN as a source signal is to allow the STRB algorithm to selectively time-reverse blocks having energies that are less than a specified threshold. This also allows us to compare our algorithm with that of the HWR algorithm [96]. The synthesized WGN is created by concatenating and alternating between two WGNs whose variances are 1 with duration of 1 s and 0.01 with duration of 1.5 s, respectively. This synthesized WGN closely mimics a standard speech signal [187] where approximately 38.53% of the speech duration consisting of high speech energy information while the remaining 61.47% consisting of pauses. The total duration of synthesized WGN is 60 s as shown in top part of Fig. 6.11. In order to achieve the same steady-state normalized misalignment, we use a step-size of 0.5 and 0.7 for the STRB and HWR algorithms, respectively. As shown in Fig. 6.11, both the STRB and HWR algorithms both converge to the steady-state normalized misalignment of 8.5 dB. The proposed STRB algorithm achieves a 2.3 dB improvement in convergence rate compared to the HWR algorithm. It is also useful to note that the steady-state normalized misalignment is computed using time-averaged window of 1024 samples. In our subsequent speech simulations, we use these step-size values when comparing STRB and HWR algorithms.

Figure 6.11 Convergence rate of normalized misalignment plots for HWR algorithm with step-size $\mu = 0.7$ (dashed line) and STRB algorithm with step-size $\mu = 0.5$ (solid line). The input signal is synthesized WGN source.

Figure 6.12 (a) Normalized speech and misalignment plots for (b) no decorrelation, (c) HWR algorithm, and (d) STRB algorithm.

In the second simulation based on a male speech input with duration 10 s, we investigate the convergence performances of the STRB algorithm labeled as (d) against that using the HWR algorithm labeled as (c), as shown in Fig. 6.12. As shown in the top part of Fig. 6.12, selective TR is only applied to speech blocks marked by rectangular dotted lines. As can be seen from this result, the proposed STRB-NLMS algorithm can achieve an improvement of approximately 2.6 dB steady-state normalized misalignment compared to the HWR-NLMS algorithm [96]. The main reason for this better performance is due to the fact that STRB processing can decorrelate stereophonic signals more efficiently than the HWR processing in the SAEC system.

6.4.1.3 Echo-return-loss enhancement (ERLE)

Besides evaluating the normalized misalignment, the ERLE defined by

$$\text{ERLE} = 10 \log_{10} \left(\frac{E\{y^2(n)\}}{E\{e^2(n)\}} \right), \quad (6.28)$$

is chosen as a metric for performance evaluation [188]. Figure 6.13 shows the ERLE performance of the HWR and STRB algorithms denoted by dashed and solid lines respectively. These plots are generated using a time-averaged window of 1024 samples. As can be seen, our proposed STRB algorithm achieves higher ERLE than the HWR algorithm throughout the speech duration. In particular, the time-averaged ERLE performance using the STRB algorithm is 2 dB higher than that using the HWR algorithm.

Figure 6.13 The ERLE performances using HWR algorithm (dashed line) and STRB algorithm (solid line).

6.4.2 Robustness of the STRB algorithm in the SAEC system

In this section, the proposed STRB algorithm is simulated in two different conditions including an abrupt change in the transmission room impulse responses [189] and background noise addition to speech signals in the transmission room. This abrupt change is achieved by shifting the source from $\{2.2, 1.5, 1.6\}$ m to $\{1.9, 1.5, 1.6\}$ m, after 5 s from the start of the adaptive process. All acoustic impulse responses are generated using the method of images [179] with the microphones at $\{1.5, 2.5, 1.6\}$ m and $\{2.5, 2.5, 1.6\}$ m. As shown in Fig. 6.14, our proposed STRB algorithm can achieve an approximately 4.7 dB improvement in normalized misalignment compared to the HWR algorithm before the echo path change. After the echo path changes at 5 s, the STRB algorithm is able to track the unknown system by maintaining its high

convergence rate giving approximately 2 dB improvement in convergence over that of the HWR algorithm.

Figure 6.14 Normalized misalignment plots for HWR algorithm (dashed line) and STRB algorithm (solid line). An abrupt change is made in the transmission room at 5 s.

Secondly, the proposed STRB algorithm is verified for the case of input speeches containing a background noise in the transmission room. The background noise is chosen as a common car engine noise, which is added to the input speeches. In this experiment, the impulse responses in the transmission room are generated using the method of images [179] with the source at $\{2.2, 1.5, 1.6\}$ m, while the microphones are placed at $\{1.5, 2.5, 1.6\}$ m and $\{2.5, 2.5, 1.6\}$ m. Due to background noise added to achieve an SNR of 20 dB, the threshold ε is adjusted as 0.03 to achieve sufficient convergence of the STRB algorithm. As shown in Fig. 6.15, the STRB algorithm achieves 7 dB improvement in terms of steady-state normalized misalignment compared to the HWR algorithm. In listening tests, the undesirable

harmonic distortion of the HWR processed speech can be perceived more clearly. On the other hand, the STRB processed speech is perceived to be similar to that of the original speech. This is due to the fact that the background noise can mask time-reversed signal with small amplitude. Hence, our proposed STRB algorithm is robust under different changes in the transmission room.

Figure 6.15 (a) Normalized speech containing background noise in the transmission room and normalized misalignment plots for (b) no decorrelation (dotted line), (c) HWR algorithm with $\alpha = 0.5$ (dashed line) and (d) STRB algorithm with $\varepsilon = 0.03$ and $B = 128$ (solid line).

6.4.2 Investigation of the STRB parameters

The threshold and the block length of the STRB algorithm are studied and their effects on convergence performance are investigated in this subsection. We first evaluate the performance of the STRB algorithm for different thresholds ($\varepsilon = 0.005$, 0.01, 0.02 and 0.03) and a fixed block length $B = 128$. As shown in Fig. 6.16, the

convergence rate increases with threshold at the expense of degradation in terms of audio quality. With $\varepsilon = 0.005$, the distortions caused by blocks of time-reversed speech are less audible since the STRB processed speech is approximately the same as that for the original speech. This threshold also results in higher rate of convergence of about 1.1 dB over the HWR algorithm. In the case of no background noise added to the transmission room, we observe that the STRB algorithm using thresholds of $\varepsilon = 0.02$ and $\varepsilon = 0.03$ achieve similar steady-state normalized misalignment of 10.1 dB. Thus, in this case, the threshold can be chosen to range between 0.005 and 0.02 with a trade-off between convergence performance and speech quality. A threshold range of $[0.02, 0.03]$ can be used in the actual transmission room with environmental noise.

Figure 6.16 (a) Normalized speech and normalized misalignment plots for (b) no decorrelation (c) HWR, (d) STRB with $\varepsilon = 0.005$, (e) STRB with $\varepsilon = 0.01$, (f) STRB with $\varepsilon = 0.02$ and (g) STRB with $\varepsilon = 0.03$ (All STRBs have fixed $B = 128$.)

We further investigate the performance of the proposed STRB algorithm with different block lengths ($B = 128, 256$ and 512 corresponding to adaptive processing delay of 16 ms, 32 ms and 64 ms, respectively). These experiments are investigated using four different thresholds of $\varepsilon = 0.005, 0.01, 0.02$ and 0.03 and plotted in terms of normalized misalignment, respectively as shown in Figs. 6.17, 6.18, 6.19 and 6.20. With the same threshold, we observe that smaller block length leads to a higher rate of convergence. This is because more blocks are being time-reversed giving lower interchannel coherence between the stereophonic channels. Finally, the PSDR, BSD and normalized misalignment results under different thresholds and block lengths are summarized and listed in Table 6.2. This table is useful to appropriately select the threshold and block length of the STRB algorithm for a desired performance in an SAEC system.

Figure 6.17 (a) Normalized speech and misalignment plot for (b) no decorrelation, (c) HWR with $\alpha = 0.5$, (d) STRB with $\varepsilon = 0.005$ and $B = 128$, (e) STRB with $\varepsilon = 0.005$ and $B = 256$ and (f) STRB with $\varepsilon = 0.005$ and $B = 512$.

Figure 6.18 (a) Normalized speech and misalignment plot for (b) no decorrelation, (c) HWR with $\alpha = 0.5$, (d) STRB with $\varepsilon = 0.01$ and $B = 128$, (e) STRB with $\varepsilon = 0.01$ and $B = 256$ and (f) STRB with $\varepsilon = 0.01$ and $B = 512$.

Figure 6.19 (a) Normalized speech and normalized misalignment plots for (b) no decorrelation (c) HWR with $\alpha = 0.5$ (d) STRB with $\varepsilon = 0.02$ and $B = 128$ (e) STRB with $\varepsilon = 0.02$ and $B = 256$ and (f) STRB $\varepsilon = 0.02$ and with $B = 512$.

Figure 6.20 (a) Normalized speech and misalignment plot for (b) no decorrelation (c) HWR with $\alpha = 0.5$ (d) STRB with $\varepsilon = 0.03$ and $B = 128$ (e) STRB with $\varepsilon = 0.03$ and $B = 256$ and (f) STRB with $\varepsilon = 0.03$ and $B = 512$.

Table 6.2 Investigation of the proposed STRB algorithm in different thresholds and block lengths.

Threshold	Block Length	PSDR	BSD	Normalized Misalignment*
$\varepsilon = 0.005$	$B = 512$	62.20 dB	9.4×10^{-4}	7.2 dB
	$B = 256$	63.30 dB	9.3×10^{-4}	6.9 dB
	$B = 128$	63.51 dB	3.5×10^{-4}	6.9 dB
$\varepsilon = 0.01$	$B = 512$	62.20 dB	21×10^{-4}	8.0 dB
	$B = 256$	58.74 dB	16×10^{-4}	8.3 dB
	$B = 128$	57.59 dB	10×10^{-4}	8.3 dB
$\varepsilon = 0.02$	$B = 512$	47.68 dB	80×10^{-4}	10.2 dB
	$B = 256$	49.93 dB	41×10^{-4}	10.5 dB
	$B = 128$	49.82 dB	23×10^{-4}	10.8 dB
$\varepsilon = 0.03$	$B = 512$	45.70 dB	114×10^{-4}	10.0 dB
	$B = 256$	44.95 dB	57×10^{-4}	10.3 dB
	$B = 128$	44.11 dB	38×10^{-4}	10.8 dB

* The normalized misalignment is evaluated as the steady-state normalized misalignment with input speech duration of 10 s.

6.4.3 Comparison among different decorrelation techniques applied to selected blocks in the SAEC system

We now compare the performance of TR against other decorrelation techniques' effects on selected blocks. Based on conventional HWR and RNA algorithms, we extended two different algorithms including selective random noise addition (SRNA) and selective half-wave rectifier (SHWR). The SRNA and SHWR algorithms perform RNA [98] and HWR transformations [96] on selected blocks with mean absolute magnitude smaller than the specified threshold ε , similar to that of the proposed STRB algorithm. The SRNA algorithm adds a WGN with an SNR of 30 dB on the selected blocks, and the SHWR algorithm introduces the HWR processing, as described in (6.6), with $\alpha = 0.5$ on the selected blocks. A white noise is added to the microphone signal in the receiving room to achieve an SNR of 30 dB. We compare the convergence rate among SHWR, SRNA and STRB algorithms, respectively labeled as (c), (d) and (e) in Fig. 6.21. The specified threshold ε is selected as 0.01. As seen in Fig. 6.21, the STRB algorithm achieves a 4.4 dB improvement in terms of steady-state normalized misalignment compared to the SHWR algorithm. On the other hand, the STRB algorithm achieves similar steady-state normalized misalignment compared to the SRNA algorithm. However, the SRNA algorithm distorts the audio quality significantly due to the large variance of the random noise added to the selected blocks of original signals. This is in contrast to the STRB algorithm, which retains the audio and stereophonic quality of the SAEC system.

Figure 6.21 (a) Normalized speech and normalized misalignment plots for (b) no decorrelation, (c) SHWR algorithm, (d) SRNA algorithm and (e) STRB algorithm.

Note that the selected blocks are indicated as rectangular dotted lines in the normalized speech.

6.4.4 Subjective listening test

Based on the ITU recommendation [190], subjective listening tests are undertaken with 10 listeners. The stereophonic signals comprise of male and female speeches with a duration of 10 s. The stereophonic signals are processed using different decorrelation techniques including STRB with $B = 128$ and $\varepsilon = 0.01$, HWR with $\alpha = 0.5$, SHWR with $\alpha = 0.5$ as well as SRNA with 30 dB noise addition. Independent listeners hear and rank the quality of these processed speech signals using the Mean Opinion Score (MOS) with 5 scales of subjective quality (1 - Bad, 2 – Poor, 3 – Fair, 4 – Good and 5 – Excellent). The MOS of the processed male and female speech signals are averaged over all listeners for each of the five scales. As mentioned in

Table 6.3, the mean MOS values of the HWR, SHWR and STRB processed signals are recorded as 3.6, 4.25 and 4.15 respectively, indicating that these processed signals achieve good audio quality. On the other hand, the SRNA processed speeches are evaluated as poor quality due to the addition of 30 dB random noise on selected blocks which can be easily be perceived as speech distortions.

Table 6.3 Mean Opinion Score value of the stereophonic signals employing different decorrelation techniques.

Processed Signals	MOS (Male Speech)	MOS (Female Speech)	MOS
SRNA algorithm	1.30	1.20	1.25
SHWR algorithm	4.20	4.30	4.25
STRB algorithm	4.10	4.20	4.15
HWR algorithm	3.60	3.60	3.60

6.4.5 Summary

In summary, the comparison of all decorrelation techniques mentioned in this chapter including SRNA, SHWR, STRB and HWR algorithms are listed in Table 6.4. These techniques are evaluated in terms of PSDR, BSD, normalized misalignment and ERLE performances. From Table 6.4, we observe that the proposed STRB algorithm can achieve fastest convergence rate with excellent stereophonic quality (PSDR and BSD) compared to other existing decorrelation techniques. Thus, the proposed STRB algorithm can achieve an effective trade-off between convergence rate and stereophonic perception. In addition, the STRB algorithm is computationally efficient to implement for an SAEC system.

Table 6.4 Comparisons among different decorrelation techniques in SAEC system.

Types	PSDR	BSD	Normalized Misalignment	ERLE
SRNA algorithm	52.52 dB	460×10^{-4}	8.6 dB	18.50 dB
SHWR algorithm	72.57 dB	3.5×10^{-4}	4.2 dB	14.15 dB
STRB algorithm	57.59 dB	10×10^{-4}	8.6 dB	15.39 dB
HWR algorithm	34.44 dB	274×10^{-4}	6.0 dB	13.39 dB

6.5 Experimental Results

In this section, we investigate our proposed STRB algorithm [114] and the conventional HWR algorithm [96] in different meeting room environments. All the impulse responses in the transmission room were measured using a Techron TEF-20 measurement system via the MLS module of Techron's Sound Lab software [191]. The configuration of the experimental setup including room size, source and microphone positions in the transmission room is illustrated in Fig. 6.22. The volume of the transmission room is $14.5 \times 8.4 \times 4.6 = 560 \text{ m}^3$. A Genelec loudspeaker driven by an HTL50SPA power amplifier is used as the source. A B&K 4007 microphone and a B&K 2801 preamplifier are used to measure impulse responses at different positions. During the experiments, the room was empty and the door was closed. The loudspeaker was fixed and the microphone was first located at location "1" as illustrated in Fig. 6.22. After measuring the impulse response at this microphone location, the microphone was subsequently moved along the arc from location "2" to location "9" to obtain nine impulse response measurements. The radius of the arch was set at 1.5 m and the distance between adjacent microphone locations was

separated by 0.3 m. Each set of impulse response was measured at the sampling frequency 48 kHz, and averaged across 10 measurements at each location. After obtaining impulse responses at the sampling frequency 48 kHz, these impulse responses were decimated to a lower sampling rate of 8 kHz using a lowpass filter designed by the Digital Filter Design Package (DFDP4).

Figure 6.22 Configuration of the microphones and loudspeaker in a transmission room.

On the other hand, the impulse responses in the receiving room were measured in a smaller conference room. The configuration of the experimental setup including room size, loudspeakers and microphone positions in the receiving room is illustrated in Fig. 6.23. The volume of the receiving room is $6.3 \times 3.6 \times 2.8 = 64 \text{ m}^3$. Similar experimental measurements are carried out as in the transmission room.

Figure 6.23 Configuration of the microphones and loudspeaker in a receiving room.

In the SAEC system, realistic impulse responses measured in the transmission and receiving rooms are employed to investigate the performance of the STRB and HWR algorithms. In this investigation, we consider four different settings of a pair of microphones in the transmission room. The first setting is a pair of microphones placed at locations “2” and “6”. The second, third and fourth settings is a pair of microphones placed at locations “3” and “7”, “4” and “8”, as well as “5” and “9”, respectively. As shown in Fig. 6.22, we observe that the distance between a pair of microphones is increased when the setting changes from 1 to 4. In this experiment, we also use a male speech input with duration 10 s and a step-size of 0.5 and 0.7 for the STRB and HWR algorithms, respectively. Subsequently, the normalized misalignments of the STRB and HWR algorithms are computed based on (6.27). From that, we can investigate the convergence performances of the STRB algorithm

labeled as (c) against that using the HWR algorithm labeled as (b), as shown in Figs. 6.24 - 6.27. As can be seen from these results, the STRB algorithm can achieve an improvement of 2.7 to 3.0 dB steady-state normalized misalignment compared to the conventional HWR algorithm in all the four different settings. Specifically, the normalized misalignments of the HWR and STRB algorithms are also evaluated and tabulated in Table 6.5. From this table, we also observe the normalized misalignment of both STRB and HWR algorithms is smaller when larger spacing between two microphones is employed. Hence, in all different experimental settings, the proposed STRB algorithm [114] still achieves better convergence rate of the adaptive filters compared to the conventional HWR algorithm in the SAEC system [96].

Table 6.5 Investigation of the normalized misalignments in different experimental settings.

Settings	Spacing [^]	HWR algorithm	STRB algorithm	Improvement ^{**}
1	0.6 m	17.0 dB	20.0 dB	3.0 dB
2	1.05 m	16.3 dB	19.1 dB	2.8 dB
3	1.45 m	16.2 dB	19.0 dB	2.8 dB
4	1.85 m	16.0 dB	18.7 dB	2.7 dB

^{**} The improvement is evaluated as the difference of steady-state normalized misalignment between STRB and HWR algorithms with the input speech duration of 10 s.

[^] Spacing is evaluated as the distance among two microphones in the transmission room

Figure 6.24 (a) Normalized speech and misalignment plots for (b) HWR algorithm, and (c) STRB algorithm in setting 1 (a pair of microphones placed at “2” and “6”).

Figure 6.25 (a) Normalized speech and misalignment plots for (b) HWR algorithm, and (c) STRB algorithm in setting 2 (a pair of microphones placed at “3” and “7”).

Figure 6.26 (a) Normalized speech and misalignment plots for (b) HWR algorithm, and (c) STRB algorithm in setting 3 (a pair of microphones placed at “4” and “8”).

Figure 6.27 (a) Normalized speech and misalignment plots for (b) HWR algorithm, and (c) STRB algorithm in setting 4 (a pair of microphones placed at “5” and “9”).

We further investigate the performance of the STRB and HWR algorithms in an asymmetric setting of a pair of microphones in the transmission room. The first microphone is placed at location “5”, and the other microphone is placed at location “1”. In this experiment, we also use a male speech input with duration 10 s and a step-size of 0.5 and 0.7 for the STRB and HWR algorithms, respectively. As shown in Fig. 6.28, the STRB algorithm can achieve an improvement of 2 dB steady-state normalized misalignment compared to the conventional HWR algorithm in the asymmetric setting.

Figure 6.28 (a) Normalized speech and misalignment plots for (b) HWR algorithm, and (c) STRB algorithm in the asymmetric setting (a pair of microphones placed at “5” and “1”).

6.6 Conclusion

In this chapter, the STRB algorithm was presented to mitigate the well-known misalignment problem in the SAEC system. The STRB algorithm employs a magnitude detector to select and time-reverse appropriate blocks in order to achieve signal decorrelation. The motivation of this method is that the stereophonic speech signals are random signals with many silences or small magnitude periods that are usually inaudible. Thus, the STRB algorithm can achieve smaller interchannel coherence, faster convergence rate of normalized misalignment and better ERLE compared to that for the HWR algorithm. Additionally, the proposed STRB algorithm achieves better stereophonic image and smaller speech distortions over the HWR algorithm.

In addition, the SRNA and SHWR algorithms were developed and compared with the proposed STRB method in terms of convergence rate and stereophonic perception performances. After evaluating these standard criteria, the STRB method also performs favorably when compared with these decorrelation techniques for the SAEC system. Moreover, some parameters of the proposed STRB method were investigated to verify their effectiveness and robustness on the SAEC performance. It was shown that the STRB method still achieved good convergence rate at lower thresholds, and further enhanced on its speech quality. The proposed STRB method is also robust and performs consistently better than other methods in time-varying environment. Hence, the proposed STRB method can overcome present technical challenges of an SAEC system and provides a suitable and cost effective solution for teleconferencing and multi-participant desktop conferencing applications.

CHAPTER 7

CONCLUSIONS AND FUTURE WORKS

7.1 Conclusions

This thesis detailed the derivation and analysis of some new time-reversal (TR) algorithms in the acoustic field. Subsequently, these algorithms were employed to solve some acoustic problems including acoustic focusing, acoustic detection and localization as well as acoustic inverse scattering and stereophonic acoustic echo cancellation (SAEC). Thus, these algorithms can be applied to improve the performance in some important applications such as nondestructive evaluation, medical imaging, target characterization, multi-participant desktop conferencing and many more.

In particular, Chapter 2 introduced and reviewed the TR principle in the acoustic field. The acoustic wave equation is invariant under a TR operation. Thus, the TR technique can be applied to retrace multiple paths and refocus acoustic waves on the initial position in a complex medium. Subsequently, two well-known TR techniques including iterative time-reversal process (TRP) and decomposition of time-reversal operator (DORT) method have been studied and analyzed in a pulse-echo mode. Specifically, if there is only one target (i.e., scatterer) in the medium, a single TRP can achieve focusing on this target in the presence of different distortions of the

medium. However, if there are several targets in the medium, the focusing via a single TRP is not possible. Therefore, the iterative TRP was developed with the ability of focusing on the most reflective target. On the other hand, to achieve focusing on weaker target, the DORT method was employed by back-propagating the corresponding eigenvector of time-reversal operator (TRO). The eigenvectors are calculated offline after the measurement of the multistatic data matrix (MDM) in the presence of the scattering medium. This approach has been applied in many experimental situations such as microcalcification detection in the breast [123], mine countermeasures in underwater [124], defects detection in nondestructive testing [121] etc.

In Chapter 3, we concentrated on the development of the DORT method in acoustic focusing topic. In the first part of Chapter 3, the DORT method was investigated to disclose the requirement of standard parameters including recording time, number of transducers, aperture size as well as environment effects [106], [107]. The main purpose of this investigation is to achieve good focusing with minimum cost via using the DORT method. Besides, these simulation results and observations provided useful set-up conditions to achieve focusing on target in practical implementations. Subsequently, based on these intensive investigations, we also found that the MDM measurement requires much processing time and memory size. Thus, we proposed the truncated multistatic data matrix (TMDM) approach to achieve saving of up to 20% in terms of processing time and memory size with minimum degradation of focusing performances compared to the full MDM approach [106], [107]. Hence, the proposed TMDM is a suitable approach in the DORT

implementations that demand faster processing time and smaller memory size requirement.

The second part of Chapter 3 is to develop the numerical technique of the DORT method for selective focusing on target in the well-resolved condition [108]. In this condition, the expressions of the eigenvalues and eigenvectors were developed in numerical form based on well-known array geometry. Specifically, the eigenvalue depends on the reflectivity of the corresponding target as well as the position of the transducers and targets in the medium. The phase and amplitude of the eigenvector can also be estimated and employed to back-propagate and focus on the corresponding target. Moreover, we also compare our proposed numerical technique with the experimental technique in terms of computational complexity and focusing performances. In particular, the proposed numerical technique can achieve smaller computational load compared to the experimental technique since the numerical technique does not require the MDM measurement and eigenvalue decomposition. However, the limitation of the proposed numerical technique is only applied in homogeneous medium, where the closed-form expression of the corresponding Green's function is available. Therefore, in an inhomogeneous medium, the experimental technique of the DORT method is still a more appropriate approach with super-resolution focusing on the target.

It is well-known that the conventional DORT method cannot perform focusing performance in the non-well-resolved condition, where targets are closely spaced [13], [14]. In order to perform selective focusing on a target in the non-well-resolved condition, we proposed a weighted least-square DORT (WLS-DORT) algorithm in

the final part of Chapter 3 [109], [110]. The main objective of our proposed algorithm is to achieve high signal response on the target and minimize the signal response in other regions surrounding the target. Thus, in the WLS-DORT algorithm, the least-squares algorithm is integrated with the conventional DORT algorithm. In particular, a weight matrix, which is calculated based on the spatial correlation matrix and the corresponding eigenvector, is applied at the array of transducers before back-propagation of the corresponding eigenvector. Subsequently, the simulation results showed that the WLS-DORT algorithm can achieve 4 times better spatial focusing performance than the DORT method in the non-well-resolved scatterer condition. Moreover, the proposed WLS-DORT algorithm can also reduce $O(N)$ operations in terms of the computational complexity compared to the minimum variance beamforming algorithm in the focusing field. Hence, the proposed WLS-DORT algorithm can provide an efficient solution to apply in some non-well-resolved scenarios, which exist in acoustic communications [60], non-destructive evaluation [37], as well as ultrasound focusing in medical therapy [83].

Besides the development of TR in acoustic focusing topic, the TR technique was also employed and analyzed in acoustic detection and localization topic in Chapter 4. In particular, we conducted intensive reviews and comprehensive investigations of two well-known methods in the detection and localization topic. The first method is the multiple signal classification (MUSIC) algorithm, which can be applied in detecting number of sources and localizing the direction of sources in the medium. The second method is the time-reversal multiple signal classification (TR-MUSIC) algorithm, which can be applied in detecting the number of targets and

localizing the position of targets in the medium. The first part of Chapter 4 focused on the theoretical analysis of both MUSIC and TR-MUSIC algorithms in general as well as in some specific array geometry and mathematical models. Specifically, both methods were developed from subspace signal processing technique. Thus, they can also achieve super-resolution performance in acoustic detection and localization. The later part of Chapter 4 focused on the comparison between MUSIC and TR-MUSIC algorithms in terms of data measurement as well as computational complexity. In particular, the TR-MUSIC algorithm can achieve smaller computational complexity than the MUSIC algorithm due to the number of transducers used in the TR-MUSIC algorithm is much smaller than the number of snapshots required in the MUSIC algorithm. Subsequently, the robustness of the TR-MUSIC algorithm was also investigated in different background noise settings [200]. The simulation results showed that the TR-MUSIC algorithm can achieve good detection and accurate localization when SNR is larger than 40 dB. Moreover, the TR-MUSIC algorithm was also developed and studied in both single and multiple scattering cases in the final part of Chapter 4. The finding showed that the multiple scattering effects can provide more multistatic data in the TR-MUSIC algorithm, which as a consequence, efficiently detecting two closely spaced targets with super-resolution image. Therefore, the TR-MUSIC algorithm is shown as a robust and efficient solution in acoustic detection and localization. Hence, the TR-MUSIC algorithm is a promising approach in active detection applications, such as nondestructive testing [64], [66], nondestructive evaluation [65], target characterization [95] etc.

In Chapters 3 and 4, the TR techniques were studied and developed in the case of point-like targets presence in the medium. In this case, the scattering condition is isotropic. On the other hand, the scattering condition is anisotropic in the case of elastic targets. Therefore, in chapter 5, we proposed and analyzed an approach based on DORT method to solve the acoustic inverse scattering problem in the anisotropic scattering condition [111], [112]. The target was chosen as elastic target with small size and the shape either cylinder or sphere. In a small elastic target condition of $k_0 a < 0.5$, there are two significant singular values corresponding to monopole and dipole modes in an anisotropic scattering medium. The underlying reason for choosing this range of $k_0 a$ is that the acoustic response of a small scatterer can be modeled exactly as the experimental results [127], and the singular values can be expressed in the formulation of the compressibility contrast, density contrast and scattering coefficient. We can then obtain the singular values resulted from the DORT method to derive and determine the observation distance, the shape and the physical properties (density, size) of an unknown scatterer in the medium. The advantage of the proposed approach based on the DORT method is to avoid solving direct scattering problems used in classical approaches in an acoustic inverse scattering problem. Thus, the proposed approach requires smaller computational demands and faster numerical implementation than the iterative approaches in order to solve an acoustic inverse scattering problem. Furthermore, the proposed solution can determine more physical properties of the scatterer compared to other non-iterative approaches in some specific conditions (i.e., small scatterer's size and limited shape). In particular, the proposed approach based on the DORT method can be applied to

solve an acoustic inverse scattering problem with two possible shapes (cylinder and sphere), compared to other solutions developed to determine physical properties of a fixed shape such as a cylinder, a sphere, or an ellipsoid. Moreover, the proposed approach includes the estimation of the density and the size of a small elastic scatterer in both closed-form and numerical form expressions, which are not provided in two well-known non-iterative solutions in recovering the shape of the scatterer. Therefore, the proposed approach based on the DORT method is a suitable technique in some important applications such as flaw detection in solids by nondestructive evaluation [165] and target characterization [127].

In previous chapters, we considered the invariant property of the TR operation in acoustic wave equation. Based on this property, the recorded signals can be time-reversed and back-propagated to perform focusing, detection, localization and even characterization, which were discussed detail from Chapter 2 to Chapter 5. In Chapter 6, we exploited other intrinsic beneficial property of the TR operation in acoustic signal processing field. This property is found from the fact that time-reversed signal is highly uncorrelated to the original signal. Thus, the selective time-reversal block (STRB) algorithm was proposed and applied in one channel to decorrelate two highly correlated stereophonic signals in a SAEC system [113], [114]. It is also useful to note that the proposed STRB processes one of the stereophonic signals based on its average energy within a given frame with the aim of sustaining good audio quality and stereophonic image. Moreover, the simulation and experimental results also showed that STRB algorithm can achieve smaller interchannel coherence, faster convergence rate of normalized misalignment and better ERLE compared with the

conventional half-wave rectifier (HWR) algorithm in the SAEC system. Besides, the proposed STRB algorithm achieves better stereophonic image and smaller speech distortions over the HWR algorithm.

In addition, to verify the effectiveness of the TR property versus other decorrelation properties, some decorrelating algorithms were developed onto selective blocks and then compared with the proposed STRB method in terms of convergence rate and stereophonic perception performances. The STRB method was found to achieve better decorrelation with minimum distortion to the stereophonic signals. Moreover, the proposed STRB method is also robust and performs consistently in time-varying environment. Therefore, the proposed STRB method can provides a suitable and cost effective solution for teleconferencing and audio/video conferencing systems applications.

7.2 Future Work

Application of the TR operation using the acoustic waves is a relatively new and open topic. Thus, different applications including acoustic communication, acoustic de-reverberation, as well as acoustic focusing can be further explored using the TR techniques. Some of these applications are described in details as follows.

Recent research in the area of acoustic communications in random medium has showed that reverberation or scattering of waves in a disordered medium can greatly increase the information transfer rate [17], [19]-[21]. In particular, it was shown that a TR antenna could efficiently focus a short pulse through a multiple scattering medium

with a spatial resolution of the order of the wavelength [19]. Subsequently, Derode *et al.* [20] proposed a TR communication scheme between a base antenna and a set of users to achieve a multiple-input multiple-output multiple-user (MIMO-MU) communication. In this application, TR operation gives an elegant solution to focus a pulse in different points [12]. Besides, TR operation is considered as a matched filter because it maximizes the ratio between the energy of the pulse at the focus point and the total energy sent by the transmitting antenna. On the other hand, from a mathematical point of view, TR operation is also an approximation of the inverse filter [23]. To converge to the optimal approximation of the inverse filter, an iterative TR was proposed with the aim of reducing computational complexity compared to inverse filter method [21]. Moreover, this iterative technique can achieve lower inter-symbol interference and lower error rate than classical TR technique in communication in highly scattering medium. The lower inter-symbol interferences permit a faster error-free communication. However, in the presence of external noises in the medium, the iterative technique has lower signal-to-noise ratio (SNR) for a given amount of transmitted energy compared to classical TR technique [21]. The higher SNR can support larger receiving data information in communication. Thus, there exists a compromise between lower error rate and higher SNR when choosing either iterative TR technique or classical TR technique in communication. This compromise can be potentially solved by a new approach that combines iterative TR technique and DORT method. This approach is known as the iterative DORT method. The main rationale behind the combination is that the DORT method can efficiently perform selective focusing on different users in the medium [13]-[15], [84], while the

iterative TR technique can achieve a lower inter-symbol interference in communication [20], [21]. Thus, the iterative DORT method is a potential solution to enhance the transfer rate as well as the receiving data size in acoustic communications in the complex and highly scattering medium.

The theory of TR techniques in acoustic focusing and acoustic localization of point-like targets has been developed in the case of the array of transmitter and array of receiver is coincident [13]-[15], [89]-[91]. The background medium is also assumed to be reciprocal. This case is commonly known as the reflection measurement mode [136]. Subsequently, the theory of TR techniques was generalized to the case where the transmitters and receivers are not in coincident and the background medium can be nonreciprocal [136], [202]. This general case is known as the transmission measurement mode [136]. In the reflection measurement mode, the pilot signals are sent from the array of transducers, after that, these signals scattered from the targets are recorded in the same array. Thus, in this mode, the MDM is a symmetric and squared matrix with the dimension corresponding to the number of transducer in the array. On the other hand, in the transmission measurement mode, the pilot signals are sent from the array of transmitters and these signals scattered from the targets are then recorded in the distinct array of receivers. Thus, in the transmission mode, the MDM is a general matrix with one dimension corresponding to the number of transducer in the array of transmitters and other dimension corresponding to the number of transducer in the array of receivers. It is useful to note that the array of receivers is placed in front of the array of transmitters in the medium and the targets are embedded in the middle. In both measurements, we observe that

the scattered signals are only recorded in the array of transducer in one side (left or right) with respect to the targets presence in the medium. Hence, the existing scattering measurements (reflection or transmission) do not include total scattered signals on the targets in the medium. Thus, to obtain higher accuracy of scattering measurement, we can propose a new approach to perform measurement in both reflection and transmission modes. In this approach, the pilot signals are sent from the array of transmitters, and these signals scattered from the targets are then recorded in both array of receivers and array of transmitters. Subsequently, two MDMs corresponding to reflection and transmission modes are constructed and decomposed to explore the singular values and singular vectors. In a well-resolved condition, it can be found that one pair of singular value and singular vector in reflection mode, and another pair in transmission mode are associated to a point-like target. Two corresponding singular vectors are then selected to back-propagate and focus on the desired target. This approach can result in better focusing with higher resolution compared to conventional approaches (reflection or transmission mode).

In addition, the TR technique can also be applied in acoustic de-reverberation in underwater environment [124] and/or acoustic source localization in skull [201]. Thus, it is potential to develop TR technique to achieve higher focusing performance in the reverberant room environments, such as concert hall, meeting room, etc. To achieve this objective, it is necessary to separate between reverberation effects of the room and scattering effects from the target. Thus, the reverberation effects of the room can be possibly found based on an analysis of the eigenvectors and eigenvalues obtained from the DORT method. Subsequently, the eigenvectors of TRO

corresponding to reverberation will be eliminated before back-propagating the eigenvectors corresponding to the desired target. Thus, the signal in the desired direction is enhanced; on the other hand, the signal in other directions is suppressed in the reverberant environments. Therefore, the TR technique can reduce reverberation effects and highly focus on the desired region or target. This approach will also be directly employed in some important applications such as acoustic de-reverberation or acoustic focusing in the reverberant environments.

It is well-known that sound focusing in reverberant rooms remains a complicated problem, essentially because of the multiple reflections on obstacles and walls occurring during propagation [203]. To overcome this difficulty, the TR technique was investigated in audible range acoustics with the ability of recreating a desired sound field. As recently shown in Yon's works [203], [204], some TR experiments were also conducted in an audio room. Compared to the classical focusing techniques such as delay law focusing, the TR technique can improve the quality of both temporal and spatial focusing considerably. Thus, based on these works, the TR technique can potentially be extended to develop a new approach to perform sound steering in a reverberant room environment. To achieve this objective, it requires integrating array steering algorithms [205] into TR technique when back-propagating signals. The proposed approach will potentially provide new solution to some novel applications including directional messaging in audio billboards and directories, and multi-party teleconferencing.

Appendix A

Green's Function Estimations

This appendix demonstrates some new techniques to estimate Green's function in the free-field condition [116]. In this condition, the Green's function estimation only changes amplitude and time of initial source wave when the acoustic wave is propagated in a homogeneous medium [128]. Hence, the Green's function estimation depends only on total length estimation of the path from transmitter to the receiver through multiple scatterers.

In acoustic communication, the geometry is set up in Cartesian coordinate system in two-dimensional (2D) with an arbitrary origin, as shown in Fig. A.1 [128]. For a clearer illustration, one pair of point-like transmitter and receiver is chosen, which are located at (x_t, y_t) and (x_r, y_r) , respectively [128]. Besides, there are M point-like scatterers, which are located at $(x_1, y_1), \dots, (x_i, y_i), \dots, (x_M, y_M)$ [116]. The Green's function in a homogeneous medium is estimated in time-domain as

$$G(r; t) = \sum_{i=1}^M \frac{1}{R_i} \delta(t - \frac{R_i}{c}), \quad (\text{A.1})$$

where c is wave speed and R_i is the total length propagation, which is the summation of the distance between the transmitter and the scatterer with the distance between the scatterer and the receiver, and can be expressed as

$$R_i = R_i(x_i, y_i) = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2} + \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2}, \quad (\text{A.2})$$

$$\text{or } R_i = R_i(x_i, y_i, z_i) = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2 + (z_i - z_t)^2} + \sqrt{(x_i - x_r)^2 + (y_i - y_r)^2 + (z_i - z_r)^2}. \quad (\text{A.3})$$

Figure A.1 Conventional 2D Cartesian coordinate system in an acoustic communication.

A. Green's function in 2D geometry

It is useful to note that the estimation of the total distance propagation depends on the origin O of the Cartesian coordinate system. To reduce the complexity, it is necessary to choose the appropriate origin O in different scattering scenarios. In single scattering case, the origin of the Cartesian coordinate system in a 2D geometry can be chosen to coincide with the position of the transmitter, as shown in Fig. A.2.

Figure A.2 Proposed 2D Cartesian coordinate system in single scattering case.

The total length propagation from transmitter to receiver through the i^{th} scatterer in single scattering case is estimated as

$$L_{it}(x_i, y_i) = \sqrt{x_i^2 + y_i^2} + \sqrt{(x_i - a)^2 + y_i^2}, \quad (\text{A.4})$$

where a is the x -coordinate of the receiver.

In a single scattering case, the acoustic waves from the transmitter are propagated to the scatterers and after reflection; they are directly propagated to the receiver. It is also called the Born-approximation case [89]. Hence, the Green's function estimation in a single scattering case can be calculated as

$$\hat{g}_t^{2D}(r; t) = \sum_{i=1}^M \frac{C_i}{\hat{L}_{it}^{2D}} \delta\left(t - \frac{\hat{L}_{it}^{2D}}{c}\right), \quad (\text{A.5})$$

where C_i is the reflectivity coefficient of the i^{th} scatterer and $\hat{L}_{it}^{2D} = L_{it}(x_i, y_i)$.

In multiple scattering case, there exists the wave reflection among scatterers in the medium. Thus, the total length of wave propagation in multiple scattering is longer than that in single scattering case. The consequence is that the equation for estimating total length propagation in a multiple scattering case is also the upper bound of the total length propagation in a single scattering case. In a multiple scattering case, the origin of the Cartesian coordinate system can be chosen to be the middle point between the position of the transmitter and the position of the receiver, as shown in Fig. A.3. Thus, the total length propagation through the i^{th} scatterer in single scattering case is expressed as

$$L_i = \sqrt{(x_i + b)^2 + y_i^2} + \sqrt{(x_i - b)^2 + y_i^2}, \quad (\text{A.6})$$

where $b = a/2$. Thus, the total length propagation through the i^{th} scatterer in multiple scattering case will be a Big O notation of (A.6) [192], which can be expressed as

$$L_{ir}(x_i, y_i) = O(L_i(x_i, y_i)). \quad (\text{A.7})$$

Figure A.3 Proposed 2D Cartesian coordinate system in multiple scattering case.

To obtain the expression of the total length propagation in multiple scattering case, we apply the Cauchy–Bunyakovski–Schwarz inequality [193] in (A.7) as

$$\begin{aligned} L_i^2 &= \left(\sqrt{(x_i + b)^2 + y_i^2} + \sqrt{(x_i - b)^2 + y_i^2} \right)^2 \leq 2 \left((x_i + b)^2 + y_i^2 + (x_i - b)^2 + y_i^2 \right) \\ &\Leftrightarrow L_i^2 \leq 2 \left(2x_i^2 + 2b^2 + 2y_i^2 \right) \\ &\Leftrightarrow L_i \leq 2\sqrt{x_i^2 + b^2 + y_i^2} \end{aligned} \quad (\text{A.8})$$

Combining (A.7) and (A.8), we deduce

$$L_{ir} = 2\sqrt{x_i^2 + b^2 + y_i^2}. \quad (\text{A.9})$$

Hence, the Green's function estimation in the multiple scattering case is expressed as

$$\hat{g}_r^{2D}(r; t) \approx \sum_{i=1}^M \frac{C_i}{\hat{L}_{ir}^{2D}} \delta\left(t - \frac{\hat{L}_{ir}^{2D}}{c}\right), \quad (\text{A.10})$$

where C_i is the reflectivity coefficient of the i^{th} scatterer and $\hat{L}_{ir}^{2D} = L_{ir}(x_i, y_i)$.

B. Green's function estimation in 3D geometry

In the previous section, the Green's function estimation in single scattering and multiple scattering was developed in 2D geometry. In this section, the Green's function estimation in 3D geometry is also developed by extending the results in 2D geometry.

In particular, the Green's function estimation in 3D geometry in single scattering can be expressed as

$$\hat{g}_t^{3D}(r; t) = \sum_{i=1}^M \frac{C_i}{\hat{L}_{it}^{3D}} \delta\left(t - \frac{\hat{L}_{it}^{3D}}{c}\right), \quad (\text{A.11})$$

$$\text{where } \hat{L}_{it}^{3D} = \hat{L}_{it}(x_i, y_i, z_i) = \sqrt{(x_i^2 + y_i^2 + z_i^2)} + \sqrt{(x_i - a)^2 + y_i^2 + z_i^2}. \quad (\text{A.12})$$

Furthermore, the Green's function estimation in 3D geometry in multiple scattering case can be expressed as

$$\hat{g}_r^{3D}(r; t) \approx \sum_{i=1}^M \frac{C_i}{\hat{L}_{ir}^{3D}} \delta\left(t - \frac{\hat{L}_{ir}^{3D}}{c}\right), \quad (\text{A.13})$$

$$\text{where } \hat{L}_{ir}^{3D} = L_{ir}(x_i, y_i, z_i) \approx 2\sqrt{x_i^2 + y_i^2 + z_i^2} + b^2. \quad (\text{A.14})$$

Hence, by choosing the appropriate Cartesian coordinate system, the Green's function estimation can be simplified in 2D and 3D geometry.

C. Discussions

Subsequently, there are some comparisons among Green's functions estimation between single scattering and multiple scattering cases in both 2D and 3D geometry.

In a homogeneous medium, the Green's function estimation depends only on the total

length propagation estimation from transmitter to receiver through scatterers. Hence, the complexity of the Green's function estimation depends mainly on the complexity of total length propagation estimation. Thus, we can compare the complexity of total length propagations in conventional, single scattering as well as multiple scattering cases in both 2D and 3D geometry, as shown in Tables A.1 and A.2, respectively.

Table A.1 Comparison complexity of total length propagations in 2D geometry

Complexity	Conventional	Single scattering	Multiple scattering
Additions	7	4	3
Multiplication	4	4	3
Square root	2	2	1
Total complexity	13	10	7

Table A.2 Comparison complexity of total length propagations in 3D geometry

Complexity	Conventional	Single scattering	Multiple scattering
Additions	11	6	3
Multiplication	6	6	5
Square root	2	2	1
Total complexity	19	14	9

Based on these tables, we have found three observations in Green's function estimation. Firstly, the proposed total length estimation from transmitter to receiver in

single scattering and multiple scattering cases can reduce computational complexity (in terms of addition and multiplication) by 27.27% to 52.94% compared to that in conventional case. The Green's function estimation is also proportional to number of the scatterers. Thus, if the number of scatterers is large, the proposed Green's function estimation in single scattering and multiple scattering cases has more significant computational complexity saving compared to conventional Green's function estimation. Secondly, in embedded digital processing implementation, fewer quantization errors occur when there are fewer computational complexities in the Green's function estimation [178]. Especially in embedded signal processing system, the square root operation is highly complex to implement. Thus, by reducing a square root operation in the proposed approach, the Green's function estimation is easier and more accurate to implement in practical applications. Finally, the proposed Green's function estimation [128] can contribute to a feasibility study of time-reversal (TR) technique in acoustic communication with fast simulation and high accuracy experiment in homogeneous and scattering medium.

Appendix B

General Expression of the Scattering Coefficients

In the anisotropic scattering condition, the multistatic data matrix (MDM) measurement depends on the number of significant normal modes of vibration, which is also the order of $k_0 a$. In particular, the MDM of an elastic cylinder (or sphere) can be written as a sum of $(m + 1)$ cylindrical (or spherical) normal modes of vibrations, respectively. Each cylindrical (or spherical) normal mode of vibration has a specific cylindrical (or spherical) scattering coefficient. Thus, this appendix shows the general expressions of the cylindrical (or spherical) scattering coefficients. These scattering coefficients were also mentioned and employed for some DORT experiments in the work [127].

The cylindrical and spherical scattering coefficients, R_n and r_n , are expressed in function of the terms x , x_L and x_T equal to $\omega a / c_0$, $\omega a / c_L$ and $\omega a / c_T$ respectively, where c_L , c_T are longitudinal and transverse wave speeds in the scatterer. Specifically, the cylindrical scattering coefficient R_n depends on the cylindrical Bessel and Hankel function J_n and $H_n^{(1)}$ as [194]-[196]

$$R_n = -\frac{J_n(x)\zeta_n - xJ_n'(x)}{H_n^{(1)}(x)\zeta_n - xH_n^{(1)'}(x)}. \quad (\text{B.1})$$

where

$$\zeta_n = \frac{\rho_0}{\rho_1} x_T^2 \frac{A_n / B_n - C_n / D_n}{H_n^{(1)}(x)\zeta_n - xH_n^{(1)'}(x)}. \quad (\text{B.2})$$

In the Neumann or rigid boundary condition corresponding to $\rho_1 \gg \rho_0$, the cylindrical scattering coefficient can be expressed as

$$R_n = -\frac{J'_n(x)}{H_n^{(1)'}(x)}. \quad (\text{B.3})$$

On the other hand, in the Dirichlet or soft boundary condition corresponding to $\rho_1 \ll \rho_0$, the cylindrical scattering coefficient can be expressed as

$$R_n = -\frac{J_n(x)}{H_n^{(1)}(x)}. \quad (\text{B.4})$$

The different terms of A_n to F_n for an elastic cylindrical scatterer are written as [194]

$$A_n = x_L J'_n(x_L), \quad (\text{B.5})$$

$$B_n = 2n[J_n(x_L) - x_L J'_n(x_L)], \quad (\text{B.6})$$

$$C_n = nJ_n(x_T), \quad (\text{B.7})$$

$$D_n = (x_T^2 - 2n^2)J_n(x_T) + 2x_T J'_n(x_T), \quad (\text{B.8})$$

$$E_n = (x_T^2 - 2n^2)J_n(x_L) + 2x_L J'_n(x_L), \quad (\text{B.9})$$

$$F_n = 2n[J_n(x_T) - x_T J'_n(x_T)]. \quad (\text{B.10})$$

Similar to the cylindrical scattering, the spherical scattering coefficient r_n is also expressed as (B.1), where the cylindrical Bessel and Hankel function J_n and $H_n^{(1)}$

being replaced by the spherical Bessel and Hankel function j_n and $h_n^{(1)}$, respectively as [195], [197], [198]

$$r_n = -\frac{j_n(x)\zeta_n - x\dot{j}_n(x)}{h_n^{(1)}(x)\zeta_n - xh_n^{(1)\prime}(x)}. \quad (\text{B.11})$$

The term ζ_n is expressed similar as (B.2) with the different terms A_n to F_n for an elastic spherical scatterer are written as [198]

$$A_n = x_L \dot{j}_n(x_L), \quad (\text{B.12})$$

$$B_n = 2[j_n(x_L) - x_L \dot{j}_n(x_L)], \quad (\text{B.13})$$

$$C_n = n(n+1)j_n(x_T), \quad (\text{B.14})$$

$$D_n = 2x_T \dot{j}_n(x_T) + (x_T^2 - 2n(n+1) + 2)j_n(x_T), \quad (\text{B.15})$$

$$E_n = (x_T^2 - 2n(n+1))j_n(x_L) + 4x_L \dot{j}_n(x_L), \quad (\text{B.16})$$

$$F_n = 2n(n+1)[j_n(x_T) - x_T \dot{j}_n(x_T)]. \quad (\text{B.17})$$

Appendix C

Time-Reversal Property in Convolution and Correlation Functions

In this appendix, we investigate the time-reversal (TR) property in convolution and correlation functions. Thus, the TR property can be used to link correlation with convolution and prove that the auto-correlation and convolution of even signals are even function. These properties were verified by the experiment reported in [6].

As a starting point, an acoustic medium is considered under the following three general hypotheses:

- (i) the elastic wave propagation through the medium is completely linear,
- (ii) spatial reciprocity is valid throughout the whole medium,
- (iii) the elastic wave propagation properties of the medium are considered stationary in time, i.e., the Green's function of the medium for any pair of points is invariant under time shift transformations.

Continuously, the TR process includes two ideal point-like transducers, A and B , located in the medium at different positions $\mathbf{r}_A$ and $\mathbf{r}_B$. Note that A is the source of the forward propagation and B plays the role of the TR mirror. At the initial time ($t = 0$), the source at A emits a pulse signal $s(t)$, which is symmetric about $t = 0$. The signal is transmitted to the medium and recorded at transducer B as $s_B(t)$ during time interval $[0; t_R]$. The time-reversed version of this signal, $s_B(t_R - t)$, is re-broadcasted (assuming that $t=0$ is the start time of the backward propagation). The induced signal at A , $s_A(t)$,

is recorded during the back-propagation in order to compare with the time-reversed version of the source signal $s(t)$.

The Green's function $G(\mathbf{r}, \mathbf{r}', t, t')$ of the wave equation for a specific mode of propagation in an unbounded medium is the solution to the scalar wave equation of the type [199]

$$\left[\frac{1}{c_i^2} (\partial / \partial t) - \Delta \right] G(\mathbf{r}, \mathbf{r}', t, t') = \delta(\mathbf{r} - \mathbf{r}') \delta(t - t'), \quad (\text{C.1})$$

where c_i is the phase velocity, $\partial / \partial t$ is the first-order partial time derivative operator and Δ is the Laplacian operator in 2D space. The right hand side term in (C.1) represents a point-like source in space and in time. Due to hypothesis (iii) of a time-invariant medium, the Green's function can be expressed as

$$G(\mathbf{r}, \mathbf{r}', t, t') = G(\mathbf{r}, \mathbf{r}', t - t', 0). \quad (\text{C.2})$$

Thus, we can define $G_{A,B}(t, t') = \tilde{G}_{A,B}(t - t')$ as the Green's function of the medium from point B to point A .

After describing the TR process and denoting the Green's function in linear and time invariant (LTI) medium, the properties of TR in convolution and correlation functions can be explored as follows [6].

A. Property I: Link between time-reversed signal and convolution

Given the definition of the signal created at location $\mathbf{r}_B$ during the forward propagation by the source signal $s(t)$ at $\mathbf{r}_A$,

$$s_B(t) = \left(\tilde{G}_{A,B}(t) * s(t') \right)(t) = \int_{-\infty}^{+\infty} \tilde{G}_{A,B}(t - t') s(t') dt', \quad (\text{C.3})$$

where $(.*) (t)$ indicates the convolution-in-time integral function, evaluated at time t .

Therefore, the time-reversed of $s_B(t)$ can be written as

$$s_B(-t) = \int_{-\infty}^{+\infty} \tilde{G}_{A,B}(-t-t')s(t')dt' . \quad (C.4)$$

By changing in time variable $t' = -t''$, (C.4) can be rewritten as

$$s_B(-t) = \int_{-\infty}^{+\infty} \tilde{G}_{A,B}(-(t-t''))s(-t'')dt'' . \quad (C.5)$$

Hence, combining (C3) and (C5), the time-reversed output signal can be expressed as the convolution of the time-reversed input signal with the time-reversed Green's function

$$s_B(-t) = \left(\tilde{G}_{A,B}(-t') * s(-t') \right) (t) . \quad (C.6)$$

B. Property II: Link auto-correlation and convolution functions

The auto-correlation function of a signal represented by a real-valued function $f(t)$ can be expressed in terms of the convolution-in-time product of the function and its time-reversed version. The auto-correlation function of $f(t)$ is defined as

$$F(t) = \int_{-\infty}^{+\infty} f(t')f(t'+t)dt' . \quad (C.7)$$

By substituting in time variable $t'' = t' + t$ inside the integral in (C.7) leads to the following definition

$$F(t) = \int_{-\infty}^{+\infty} f(-(t-t''))f(t'')dt'' . \quad (C.8)$$

Equation (C.8) corresponds to

$$F(t) = (f(t) * f(-t))(t). \quad (\text{C.9})$$

Hence, the auto-correlation can be linked with the convolution, as described in (C.9).

C. Property III: Autocorrelation function is even function

To prove the autocorrelation as an even function, the autocorrelation at time $-t$, which is defined as $F(-t)$, is calculated as

$$F(-t) = \int_{-\infty}^{+\infty} f(t') f(t' - t) dt'. \quad (\text{C.10})$$

Thus, changing time variable $t'' = t - t'$ in (C.10) leads to

$$F(-t) = - \int_{-\infty}^{+\infty} f(t - t'') f(-t'') dt''. \quad (\text{C.11})$$

Another change in temporal frame of reference, $t'' = -t'''$, in (C.11) leads to

$$F(-t) = \int_{-\infty}^{+\infty} f(t + t''') f(t''') dt''' = F(t). \quad (\text{C.12})$$

Hence, the auto-correlation function of a signal $f(t)$ is an even function.

D. Property IV: Convolution of two even functions

Given two even function $a(t)$ and $b(t)$ with symmetric about $t=0$, their convolution-in-time is still an even function

$$(a(t) * b(t))(-\tau) = \int_{-\infty}^{+\infty} a(-\tau - t') b(t') dt'. \quad (\text{C.13})$$

Note that $a(-\tau - t') = a(\tau + t')$ and applying the change in variable $t' = -t''$ in the integral of (C.13)

$$(a(t) * b(t))(-\tau) = \int_{-\infty}^{+\infty} a(\tau - t'')b(t'')dt'' . \quad (\text{C.14})$$

Since $b(t)$ is an even function, i.e., $b(-t'') = b(t'')$, (C.14) becomes

$$(a(t) * b(t))(-\tau) = \int_{-\infty}^{+\infty} a(\tau - t'')b(t'')dt'' = (a(t) * b(t))(\tau) . \quad (\text{C.15})$$

Hence, convolution of two even function is also an even function.

Appendix D

Orthogonality of the Propagation Matrices in the Well-Resolved Scatterer Case

In this appendix, we provide the proof about the orthogonality of the propagation matrices in the well-resolved scatterer case [110]. It is interesting to note that the scatterers are defined in well-resolved case when it is possible to focus on one of the scatterers without sending energy to the others [14]. Suppose that a point-like source is placed at the position of the i^{th} scatterer and emits a temporal signal $\delta(t)$. Then, the signal received on the m^{th} transducer is the impulse diffraction response $h_{im}(t)$. The best way to focus on the same scatterer (i^{th} scatterer) is to use the input signal as the time-reversed signal $h_{im}(-t)$ [14]. However, the pressure signal received by the k^{th} scatterer in the frequency domain is given as:

$$P_{ik} = \sum_{m=1}^N H_{km} H_{im}^* . \quad (\text{D.1})$$

In the well-resolved scatterer case, the pressure signal P_{ik} is almost zero if $k \neq i$, i.e., $P_{ik} = 0$ [14]. Thus, in this case, we can obtain

$$\sum_{m=1}^N H_{km} H_{im}^* = \begin{cases} 0 & k \neq i \\ \sum_{m=1}^N |H_{im}|^2 & k = i \end{cases} . \quad (\text{D.2})$$

Hence, in the well-resolved scatterer case, the vectors $\mathbf{H}_k$ and $\mathbf{H}_i$ are orthogonal when $k \neq i$.

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