Novel frequency selective structures based on a two-dimensional periodic array of vertical microstrip lines

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**NOVEL FREQUENCY SELECTIVE STRUCTURES BASED ON A
TWO-DIMENSIONAL PERIODIC ARRAY OF VERTICAL
MICROSTRIP LINES**

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Dedicated to my parents

Malik Abdur Rasheed and Seema Hamid

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SUMMARY

Frequency selective structures (FSSs) are widely used in antenna radomes, antenna sub-reflectors, spatial filtering, and many other applications. Traditional FSS consists of a two-dimensional (2-D) periodic array of unit-cells, which are either printed on a dielectric layer or etched out of a conductive surface. For practical applications, it is desirable to realize a thin FSS with high selectivity of filtering performance, and stable response under different angles of incidence. However, all these features are difficult to obtain through traditional designs. Recently, an innovative concept of three-dimensional (3-D) FSS was proposed, which is very promising for the realization of compact high-performance FSSs. These FSSs basically consist of a 2-D array of cavities whose modes and coupling with air can be controlled to obtain a desirable frequency response. This thesis presents analysis and design of a new 3-D FSS, which consists of a 2-D periodic array of vertical microstrip lines.

The proposed array of vertical microstrip lines supports two quasi-TEM modes, and a thick array basically forms a dual-mode resonator. By controlling the resonant frequencies of those modes and their couplings with air, it is possible to obtain a desirable pseudo-elliptic filtering response. With a suitable selection of the substrate material and geometrical dimensions, it is possible to include more resonances, and resultantly the thick array may also form a multi-mode resonator. Because of the quasi-TEM modes, it is also possible to use an arbitrarily smaller unit-cell of this geometry, which leads to stable filtering response even when the structure is subjected to a large variation of the angle of incidence.

The proposed FSS has been analyzed using an efficient full-wave mode-matching method. The analysis comprises two parts. In first part, the propagation characteristics of the array have been analyzed by considering a unit-cell of this geometry based on the Floquet theorem. In the second part, scattering from the proposed array has been investigated by solving the air-to-array discontinuity. FSS

is modeled as a cascaded junction of two air-to-array discontinuities, and its S-parameters are obtained by cascading the relevant generalized scattering matrices.

Both the propagation and scattering characteristics of the proposed array have been elaborated through a number of parametric studies, which include variation of geometrical dimensions, polarization, and the angle of incidence. Based on these parametric studies, and suitable modeling of the proposed FSS, a list of design guidelines has been evolved. A few design examples for both narrow-band and wideband FSSs are presented. These structures have also been fabricated and measurement results are in an agreement with the predicted results.

Since circuit analog microwave absorbers are traditionally considered as closely related or a sub-set of FSS, the proposed array has also been demonstrated to realize a wideband microwave absorber. It has been noted that when a sub-wavelength unit-cell is used, the air-to-array discontinuity offers a wideband natural transition between air and the two quasi-TEM modes of the array. Hence, the proposed array is actually a very suitable candidate for wideband applications. A scheme of absorbing the energy carried by the two quasi-TEM modes has been proposed. Based on this scheme, the absorber problem is modeled to derive simple design formulas. Complete design procedure has been illustrated through a design example. The same example has also been fabricated, and measurement results validate the modeling and procedure presented in this work. It is seen that an absorption bandwidth in excess of 110% is easily achievable with the proposed 2-D periodic array of vertical microstrip lines.

The thesis has been concluded by pointing out a few limitations of the proposed array. Since the performance of the proposed array is mainly based on the two quasi-TEM modes of microstrip lines, it is relatively polarization sensitive and operates under those polarizations which favor the excitation of quasi-TEM modes. With further future research on the proposed array, it is expected that the present limitations may be overcome, and this structure may find applications in more areas of microwave engineering.

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CHAPTER 1

INTRODUCTION

Frequency selective structures (FSS) are widely used in antenna radomes, sub-reflectors, spatial filters, and related applications. Traditional FSSs are a cascaded combination of planar geometries, which suffer from a number of performance limitations. Recently, a concept of three-dimensional (3-D) FSS has been proposed, whereby an FSS consists of a two-dimensional (2-D) array of ‘monolithic’ cavities. The number of propagating modes in those cavities, and their couplings with air are controlled to obtain a desirable filtering performance. Resultantly, it is possible to excite multi-mode propagation, which may also lead to an elliptical filtering response.

This thesis presents a new type of 3-D FSS based on a two-dimensional (2-D) periodic array of vertical microstrip lines. The proposed FSS exhibits quasi-elliptical filtering response, which remains stable under even a large variation of the angle of incidence.

1.1 Motivation

As mentioned above, the proposed FSS built from a 2-D periodic array of vertical microstrip lines exhibits superior performance over many traditional FSS designs. This forms the prime motivation behind the research presented here. In the following paragraphs, it is shown that the performance of traditional FSSs is limited by a number of factors, and it is difficult to overcome those limitations with traditional techniques. The limitations of traditional FSSs are discussed through a number of simple examples. The concept of 3-D FSS appears to be an attractive alternative, which, in principle, rectifies most of the limitations present in traditional structures. Based on the principle of 3-D FSS, motivation for the proposed research is more clearly established, and brief structural details of the proposed FSS are also given under the present section.

1.1.1 Basics of Frequency Selective Surfaces

Frequency selective surface is a periodic geometry which is basically a spatial filter when exposed to an incidence of electromagnetic waves [1]. Part of the incident energy is reflected back while the remaining portion of it passes through the FSS. Frequency response of this scattered energy follows a prescribed filtering characteristics whereby the reflection/transmission of certain frequency band is achieved.

Figure 1.1: A single-layer traditional 2-D bandstop FSS consisting of ring resonators printed on a thin dielectric layer.

Traditional frequency selective surface is a two-dimensional periodic arrangement of unit-cell shapes which may be perforated through a conducting screen or they may be printed on a dielectric layer. Based on that, it forms either a capacitive or an inductive screen leading to bandpass or bandstop frequency response, respectively. Since the problem of a periodic array is reduced to that of a unit-cell as required by the Floquet theorem [2], performance of an FSS can be completely determined from its unit-cell shape and size. Unit-cell size mainly

determines the center frequency of operation, and generally its dimensions are comparable with the operating wavelength. On the other hand, the unit-cell shape primarily controls the rejection skirt and bandwidth of a single-layer FSS.

Figure 1.2: Unit-cell of the FSS shown in Figure 1.1, (a) Perspective view, (b) Front View, (c) Equivalent Circuit.

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is an inductive type of FSS, and its unit-cell can be characterized by a series LC resonator, which is in shunt connection with the input and output ports, as shown in Figure 1.2. S-parameters of this FSS are given in Figure 1.3, which clearly show a bandstop behavior centered at 10 GHz.

Figure 1.4: Unit-cell of a simple bandpass FSS, (a) Perspective view, (b) Front View, (c) Equivalent Circuit.

Figure 1.5: S-parameters of the bandpass FSS ($w = 0.2 \text{ mm}$, $r = 4.2 \text{ mm}$, $p = 12 \text{ mm}$).

A simple capacitive surface can be realized by making similar periodic ring slots into a conductive layer. In the example presented here, the conductive layer is also backed by a thin dielectric layer of the same thickness and material as that used in

the case of Figure 1.3. Figure 1.4 shows the unit-cell in this case. Equivalently, it can be represented by a parallel LC resonator which is in shunt connection with the input/output ports. S-parameters of this FSS are given in Figure 1.5, which show bandpass frequency response around a center frequency of 10 GHz.

The above two examples clearly demonstrate the bandstop and bandpass characteristics of inductive and capacitive FSSs based on ring resonators. Center frequency of these FSSs is related with the circumference of the ring and the backing dielectric layer. Basically, this circumference equals a guided wavelength at the center frequency where resonance occurs. Bandwidth and the skirt of the frequency response can be slightly improved by introducing certain modifications to the present unit-cell or by employing different type of unit-cell geometries [1]. On the other hand, a cascaded combination of multiple single-layer surfaces is an effective technique for significant improvement of filtering response. However, it results in an increased size, while a sharp rejection skirt of the filtering response is rarely possible to obtain. This and other limitations of the traditional FSSs are discussed as follows.

1.1.2 Limitations of Traditional Frequency Selective Surfaces

Traditional frequency selective surfaces suffer from the following major limitations:

- 1) Poor filtering response
- 2) Sensitivity to the angle of incidence
- 3) Effect of finite size

The following paragraphs discuss these problems in more detail. Though there exist a number of traditional techniques, which theoretically solve these problems to some extent. However, those solutions often results in multi-layer configurations, which may not be practically feasible for size and weight constraints.

Figure 1.6: Unit-cell and equivalent circuit of a two-layer FSS formed by cascaded single-layer surfaces.

Figure 1.7: S-parameters of a cascaded combination of two single-layer bandpass FSSs of Figure 1.4 ($w = 0.2 \text{ mm}$, $r = 4.2 \text{ mm}$, $p = 12 \text{ mm}$, $t = 5 \text{ mm}$, $\epsilon_r = 2.2$).

1.1.2.1 Poor Filtering Response

As shown in the previous section, frequency response of a single-layer frequency selective surface is equivalent to that of an LC resonator. For practical applications, two or more single-layer surfaces are cascaded to form multi-layer configuration. A dielectric spacer may be placed between these layers. Thickness of the dielectric

spacers is generally kept close to a quarter wavelength whereby they act as impedance inverters. Equivalently, a simple cascaded combination of 2-D frequency selective surfaces results in a periodic arrangement of resonators and impedance inverters.

Figure 1.6 and Figure 1.7 show an example where two copies of the surface discussed in Figure 1.4 have been cascaded through a dielectric layer. Two resonances may be noted in this case, which basically correspond to the two resonators separated through an impedance inverter. Adjusting the separation width t and resonant frequencies of the resonators, position of these two resonances can be controlled to obtain a maximally flat or an equally ripple response.

Based on the above equivalent circuit, it is easy to add more transmission poles with an inclusion of more layers. However, it is very difficult to obtain transmission zeros with this traditional approach. It requires either cross couplings [3] among different layers or provision of parallel coupling paths achieved through two slots as done in [4]. On the other hand, a desired number of transmission zeros at finite frequencies can be easily realized through three-dimensional (3-D) FSSs consisting of multi-mode cavities. An FSS built from a two dimensional periodic array of multimode cavities actually offers greater flexibility in terms of controlling the number and positions of desired transmission poles and zeros. This recent development of 3-D FSS is discussed in Section 1.1.3.

1.1.2.2 Sensitivity to the Angle of Incidence

Most of traditional frequency selective surfaces are relatively sensitive to the angle of incidence, and their response generally undergoes a frequency shift as the angle of incidence is changed. Effect of oblique incidence can be understood from a simple illustration given in Figure 1.8 where a one-dimensional (1-D) periodic array of certain shapes is subjected to an illumination of plane wave. It is easy to see that signal arriving at the adjacent unit-cells undergoes a phase shift Ψ which is a function of the angle of incidence θ and the period p , as follows:

$$\psi = kp \sin \theta. \quad (1.1)$$

Since the current distribution at the surface of these shapes, induced by the plane wave, is affected accordingly, a higher value of Ψ results in a more pronounced change in the performance of the array. To obtain stable angular performance at a given frequency, one needs to reduce Ψ , which can be realized in two ways. These traditionally well-known possible solutions and their respective limitations are elaborated as follows.

Figure 1.8: A one-dimensional periodic structure under the illumination of plane wave arriving at an oblique angle.

Figure 1.9: A one-dimensional periodic structure coated with a dielectric layer, and subjected to an illumination of plane wave arriving at an oblique angle.

- 1) It is noted from (1.1) that a reduction in the period p directly reduces the induced phase shift. However in case of a frequency selective surface, since the size of a unit-cell is a design parameter affecting mainly the center frequency of operation, it may not be always possible to reduce the period significantly. On the other hand, there may exist certain unit-cell shapes which, unlike the previous example of ring resonator, may be more compact and can be overlapped together to reduce the period p . Ref. [1] includes a comprehensive discussion on such unit-cell shapes.
- 2) Traditionally, sandwiching a frequency selective surface between two additional dielectric layers is the most common technique of obtaining an angular stable response. As illustrated in Figure 1.9, the incident plane wave is refracted when it travels through the dielectric material. Resultantly, the current distribution on the FSS induced by this plane wave remains relatively stable under a considerable variation of the angle of incidence. Choosing an appropriate dielectric constant as well as the thickness of these additional dielectric layers is an important design consideration. However, the design calculations often result in the values which may not be commercially available [1]. Moreover, it also increases the size and weight of an FSS.

1.1.2.3 Effect of Finite Size

Since the problem of an infinite frequency selective surface is reduced to that of a unit-cell by the Floquet theorem [2], therefore, the existing design techniques are generally based on the analysis of a unit-cell only. On the other hand, practical designs are always finite in size, and the actual dimensions depend upon the application under consideration. This leads to a serious discrepancy between the measured performance of a finite FSS and its expected theoretical results, especially when the size of the FSS is only a few wavelengths. Use of FSS as antenna sub-reflector and many other applications actually involve a small size of FSS, which is difficult to accurately analyze using traditional techniques.

1.1.3 Operating Principle of Three-Dimensional Frequency Selective Structures

Three-dimensional frequency selective structure is a recent development, which holds significant promise for solving the limitations of traditional designs more effectively. With this new concept, in principle, it is possible to obtain a pseudo-elliptic filtering response exhibiting a desired number of transmission zeros and poles.

Figure 1.10: Illustration of a 3-D FSS consisting of a 2-D periodic array of multimode cavities (a) FSS (b) Unit-cell.

Figure 1.10 shows a generalized view of 3-D FSS consisting of a periodic array of multimode cavities/resonators. Its unit-cell is shown in Figure 1.10(b). This multimode cavity may be an SIW structure [5], a shielded microstrip line [6], or any other suitable geometry. One may obtain a desired frequency response by controlling the number of propagating modes and their couplings with air. As indicated in Figure 1.11, an air-to-multimode resonator discontinuity can be

represented by a K-inverter and transmission line sections of electrical length Φ_0 , $\Phi_1, \dots, \Phi_N$, where N denotes the number of propagating modes [7]. These electrical lengths are merely a representation of the parasitic effect of the discontinuity and their values can be either positive or negative. Since a 3-D FSS can be treated as a cascaded junction of two air-to-multimode resonator discontinuities, its complete equivalent circuit is also shown in Figure 1.11. $\Psi_0, \Psi_1, \dots, \Psi_N$ represent the electrical lengths of the multi-mode resonator corresponding to N propagating modes.

Figure 1.11: Equivalent circuit of a multi-mode cavity subjected to an incidence from air.

The outer sections of length Φ_0 can be absorbed into the source/load feed lines (representing air), and the inner circuit can be solved to obtain the reflection

coefficient of an n th mode as follows,

$$S_{11}^{(n)} = \frac{j(\bar{K}_n^4 - 1)\tan\varphi_n}{2\bar{K}_n^2 + j(\bar{K}_n^4 + 1)\tan\varphi_n}, \quad (1.2)$$

where

$$\bar{K}_n = \frac{K_n}{\sqrt{Z_0 Z_n}}, \quad \varphi_n = 2\Phi_n + \psi_n.$$

From Figure 1.11, it is easy to see that a transmission zero occurs when

$$\left| \sum_{n=1}^N S_{11}^{(n)} \right| = 1. \quad (1.3)$$

Similarly, the condition for a reflection zero (or transmission pole) can be given by

$$\left| \sum_{n=1}^N S_{11}^{(n)} \right| = 0. \quad (1.4)$$

Equations (1.2)-(1.4) show that a 3-D FSS can exhibit a desired pseudo-elliptic performance when these conditions for the number and locations of transmission zeros/poles are satisfied.

1.1.4 The Proposed Frequency Selective Structure

The proposed FSS is illustrated in Figure 1.12, which shows a two-dimensional periodic array of microstrip lines. It can be constructed by stacking up a number of identical printed circuit boards (PCBs) on a common plane. Each PCB is printed with a one-dimensional periodic array of microstrip lines of width t . Thickness of the printed strips is kept negligibly small. Periods along the x - and y - axes are denoted by b and h , respectively. Thickness of the substrate is denoted by d . As indicated in Figure 1.12, a unit cell of this periodic array forms a microstrip line

shielded with side walls characterized by a periodic boundary condition (PBC). Following are the salient features of the structure studied in this thesis:

Figure 1.12: Geometry of the proposed FSS. (a) Perspective view, (b) Top view, (c) Front view, (d) Enlarged view of the cross-section of a unit-cell.

- 1) Since its unit-cell consists of three isolated conductors, it supports two quasi-TEM modes [2].
- 2) Inline with the general discussion of 3-D FSSs as given in the previous sub-section, it is possible to obtain a desired pseudo-elliptic performance from this proposed structure by controlling the number and locations of transmissions zeros/poles.
- 3) Based on the existence of quasi-TEM modes, it is possible to realize any arbitrarily smaller size of a unit-cell, which is desirable for an angular stable performance of the FSS.

- 4) For a sub-wavelength size of the unit-cell, a practical FSS comprises large number of unit-cells. This reduces the effect of the finite size, and leads to an excellent agreement between the measured results and those expected from the theoretical analysis.

It should be pointed out that the above features are based on the existence of two quasi-TEM modes in the proposed array, which are excited when the polarization of incident plane wave is mainly perpendicular to the underlying microstrip lines. For this reason, the performance of the proposed FSS is not independent of the polarization of incidence. However, it is expected that future studies and extensions of the present work may overcome this limitation, and resultantly it may be possible to realize a compact, angular stable and polarization-insensitive FSS exhibiting desired elliptic performance.

1.2 Objectives

It has been shown in the previous paragraphs that traditional frequency selective structures suffer from a number of serious limitations, which may not be effectively addressed by the existing techniques of FSS design. In this context, the idea of three-dimensional FSS appears very attractive, and the present research aims at investigating a novel 3-D FSS based on a 2-D periodic array of vertical microstrip lines. The objective of this thesis is to present a complete study of the proposed FSS, based on which, an interested reader may easily design a working structure for his/her application. Also, design guidelines and demonstration of a related circuit analog absorber is part of the objectives. It includes the following goals:

1. A comprehensive review of the literature on FSS and microwave absorbers is the first step for the proposed research. The primary target of this step is to understand the fundamental operating principles of both the classic and recent designs. This review also aims to identify the limitations of traditional approaches, methods for improvements, and the future direction of research.

Resultantly, the motivation for the proposed research may be established more clearly.

2. As the next objective, propagation characteristics of the proposed 2-D periodic array of vertical microstrip lines need to be investigated using an efficient full-wave method. Understanding these propagation characteristics is important to later formulate some useful guidelines for the design of FSS or microwave absorber. Since the modes of the proposed array are a function of its geometry and excitation, effect of the both needs to be examined through a number of parametric studies.
3. The proposed array is an asymmetric structure, and those polarizations and angles of incidence need to be identified which may lead to the excitation of a desired set of modes. Based on that, a full-wave scattering analysis of the array shall be formulated and implemented. Scattering by single air-to-array discontinuity, and a cascaded junction of two air-to-array discontinuities shall be investigated.
4. Using the above studies, it is aimed to outline useful guidelines and procedures to design working FSSs and microwave absorbers. To achieve this, circuit models of the proposed FSS and microwave absorber need to be considered and analyzed.
5. For a demonstration of the proposed concept, it is targeted to fabricate a number of prototype structures, and measurement results shall be compared with the predicted performance of those prototypes. This exercise is also expected to show if the response of a finite-sized practical structure is close to the theoretical results obtained above whose modeling is primarily based on the assumption of an infinite array as required by the Floquet theorem.

1.3 Major Contributions of the Thesis

This thesis introduces a novel frequency selective structure with pseudo-elliptic filtering performance, and a wideband circuit analog microwave absorber. Both of them are based on a 2-D periodic array of vertical microstrip lines, whose analysis and design procedures are presented in this work. Major contributions of the thesis may be identified as the following:

1. Propagation characteristics of the proposed 2-D periodic array of vertical microstrip lines have been investigated by analyzing a unit-cell of this structure. This unit-cell consists of a microstrip line shielded with periodic side walls, which are characterized by a periodic boundary condition (PBC). For comparative studies, a microstrip line shielded with perfect electric conductor (PEC), or perfect magnetic conductor (PMC) was also considered. It has been found that, unlike a microstrip line shielded with PEC side walls, the microstrip line with PBC or PMC side walls supports two quasi-TEM modes. This dual-mode phenomenon of a shielded microstrip line has not been previously reported in the open literature, or exploited for a useful application. Interestingly, the dual-mode existence was already noted for an open microstrip line grating [8].
2. An LSE/LSM modes based efficient mode-matching method has been proposed for the analysis of a microstrip line shielded with PBC side walls. By invoking the Floquet theorem [2], this method is also suitable for studying the problems of shielded strip gratings. Previously, the mode-matching method was often hybridized with the point-matching method when studying the grating problems [9, 10], which forms a relatively inefficient approach.
3. A discontinuity between air and the proposed array has been analyzed using full-wave mode-matching method. It has been found

that under suitable polarizations of incident plane wave, this discontinuity offers an excellent broadband transition from air to the two quasi-TEM modes of the array of vertical microstrip lines. The relevant suitable polarizations and the range of angles of incidence have been identified. Based on this natural air-to-array transition, the proposed array appears attractive for wideband applications in microwave absorbers [11] or spatial power combiners [12], etc. This is unlike the discontinuity between air and for example, the widely studied array of rectangular waveguides [13, 14], or even an array of shielded microstrip lines [15].

4. Based on the above findings, a novel high-performance FSS has been introduced, which overcomes many limitations of the traditional FSSs. It exhibits pseudo-elliptic filtering response, whereby the number and locations of the transmission poles/zeros can be controlled for a desired frequency response. Its unit-cell size is sub-wavelength, which leads to stable performance under a large variation of the angle of incidence. Because of the sub-wavelength size of a unit-cell, a physically finite structure comprises many unit-cells, which reduces the effect of finite-truncation as observed in the case of traditional FSSs. This is further verified by a number of fabricated prototype structures, whose measurement results are in an excellent agreement with those predicted by a Floquet theorem based numerical analysis.
5. The thesis also introduces the design procedure and modeling of a novel wideband microwave absorber based on the 2-D periodic array of vertical microstrip lines. Design of the proposed absorber is based on closed-form or simple formulas derived in Chapter 5. This is unlike most of the traditional RLC absorbers, where simple design formulas are not available, and instead the absorber problem becomes a problem of numerical optimization.

1.4 Organization of the Thesis

The thesis has been organized into six chapters. The first chapter primarily explains the motivation behind this work by identifying limitations of previous designs, and advantages of the proposed novel structure. It establishes that the proposed solution, in principle, can overcome most of those problems of traditional structures.

Chapter 2 presents a comprehensive literature review of FSSs and microwave absorbers. Traditional as well as recent designs have been considered. Operating principle and the main design techniques are discussed in sufficient details. Some design examples of traditional structures have also been presented to compare certain classes.

Chapter 3 covers the propagation characteristics of a 2-D periodic array of vertical microstrip lines, which have been studied using an efficient full-wave mode-matching method. Using Floquet theorem [2], a unit-cell of the array is analyzed, which basically consists of a microstrip line shielded with PBC side walls. A number of parametric studies have been presented to understand the effect of geometrical dimensions and excitation on the propagation characteristics of the proposed array. A similar mode-matching method based formulation has also been presented for the case of a microstrip line shielded with PMC or PEC side walls. These cases are compared with the unit-cell of the proposed array, and some interesting similarities and differences have been noted.

Chapter 4 formulates the scattering problem where an air-to-array discontinuity has been analyzed using full-wave mode-matching method. Modes in the air region are expressed in terms of TE and TM Floquet modes, while the modes in the array region have already been calculated in Chapter 3. Similar to the previous chapter, a number of parametric studies have been presented to develop a better understanding and physical significance of this scattering problem. Effect of structural dimensions, polarization, and angle of incidence has been studied in sufficient details.

Based on the above two major studies, Chapter 5 demonstrate some useful applications of the proposed array. Circuit model and design guidelines of the proposed FSS have been presented to more clearly explain its operating principle, and lay down a faster design procedure. Similarly, the problem of a wideband microwave absorber based on this array has been suitably modeled to derive simple design formulas. A number of prototype structures have been fabricated and the measurement results validate the design procedures presented in this chapter.

Chapter 6 concludes this thesis by summarizing the findings and results of this research. It also points out certain limitations of the proposed structure. Some suggestions for future explorations leading to possible improvements/extensions of the proposed structure have also been listed in this last chapter.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Frequency selective structure (FSS) is one of the widely researched fields, and a large number of geometries and techniques have been proposed since over the past four decades. Munk [1] authored a comprehensive account of design techniques of classical FSSs. The understanding and principles of classical FSSs presented in this chapter are mainly based on the work of [1].

For practical applications, it is desirable to realize a thin FSS with high selectivity of filtering performance, and stable response under different angles of incidence. However, all these features are difficult to obtain through traditional designs, and many new techniques are being investigated recently. These new methods and design have been also reviewed in this chapter.

The last section of this chapter explores design techniques, and recent trends of wideband microwave absorbers. Traditionally, the subject of circuit analog (CA) absorber is treated as a sub-set of FSS since these absorbers consist of either a lossy FSS or an FSS backed by resistive layer [1]. Based on this convention, the proposed FSS has also been extended to realize a wideband CA absorber. Therefore, a comprehensive review of previously known microwave absorbers becomes relevant to more clearly distinguish this research from the previous studies on microwave absorbers.

2.2 Frequency Selective Structures

Traditional FSS consists of a periodic array of patches, apertures, or complex shapes that are designed to control the transmission/reflection of the incident

electromagnetic energy. Reflection/transmission properties of FSS are a function of frequency and the angle of incidence. Although initially, FSS was proposed and used as a spatial filter with potential applications in antenna radomes, yet later it found applications in various other areas. Principles of many recent developments such as electronic bandgap (EBG) structure [16-20], artificial magnetic conductor (AMC) [20-22], and metamaterials are largely based on the theory of classical FSSs [1]. EBG structures are mainly used for guided-wave applications, and they can be derived from an FSS with addition of a substrate and a conducting plane. EBG structures exhibit a bandgap that can be explained from the phenomenon of Bragg reflections. Bragg reflections play a dominant role if the distance between unit-cells is comparable with a half wavelength at the operating frequency. If, however, the unit-cell distance is much smaller than the wavelength, Bragg reflections condition breaks down, and ‘effective medium’ theory is applied [21]. The structure is now treated as a homogeneous material characterized by ‘effective’ permittivity and permeability parameters [23]. The periodic structure in that limit is preferably termed as a metamaterial [21].

2.2.1 Applications

Frequency selective structures have been used for a large number of applications in antenna radomes, reflector antennas, spatial filters, grid arrays [24], ground planes for surface wave suppression, photonic crystals, artificial magnetic conductors [20-22], and absorbers [25] etc. Historically, FSS was initially used to realize band pass radomes. Radomes are an important part of antenna in many applications, and their performance directly affects the overall performance of an antenna. FSS provides flexibility and improvement in radome designs. FSS application in dual-reflector antenna systems is also well established whereby different frequency feeds are placed at real and virtual foci of a sub-reflector. One of the feeds is covered with an FSS that either transmits the incident signal to one feed or reflects it to the other feed, discriminating on the basis of signal frequency. FSS can be used as spatial filter, and it is possible to discriminate the signal on the basis of angle of incidence.

Recently, frequency selective surfaces combined with a grounded substrate have been demonstrated to act as a band-gap structure. EBG structures, as they are called, have been used for realizations of novel filters. They have also been used to suppress surface wave propagation within a narrow band that improves antenna efficiency. The band-gap property has been also used in reduction of mutual coupling in antenna arrays. High impedance surfaces and artificial magnetic conductors also use FSS patterns. Further, FSS is useful in size reduction and bandwidth enhancement of a Salisbury screen, leading to the realization of circuit-analog absorbers. This technique has been discussed under Section 2.3.1.

Figure 2.1: Unit-cell shapes of traditional FSSs.

2.2.2 Design Principles of Traditional Structures

Traditional FSS consists of a two-dimensional (2-D) periodic array of unit-cells, which are either printed on a dielectric layer or etched out of a conductive surface. Shape and size of these unit-cells are important design parameters which control the operating frequency, bandwidth, stability with variation of the angle of incidence, etc. Performance of traditional designs can be further improved through a cascaded

combination of these 2-D surfaces and thick dielectric spacers [1]. Effect of the above various design parameters is discussed as follow:

2.2.2.1 Elements Shape

The shape of a unit-cell element is important in controlling bandwidth, filtering selectivity and stability with the angle of incidence, of an FSS. There exists a large number of possible shapes, and each carries certain advantages and limitations. Some shapes are more broadband than others. Performance of some shapes is more sensitive to the angle of incidence. Similarly, the filtering selectivity of different shapes also varies significantly. Selection of a suitable shape mainly depends upon the application under consideration. Munk [1] divided these traditional unit-cell shapes into four distinct groups, as given below:

- 1) The center connected or N-poles (dipole, tripole, square spiral, the Jerusalem crosses etc).
- 2) The loop types (three or four legged loaded elements, circular, square and hexagonal loops, etc.).
- 3) Solid interior or plate type of various shapes.
- 4) Others miscellaneous shapes, which are generally a combinations of the above types. They also includes fractal FSSs, and complex shapes designed through numerical optimizations.

Figure 2.1 illustrates some representative shapes of each group. Wu [24] compared these various shapes with respect to angular stability of resonant frequency, cross polarization, bandwidth, and band separation (of transmit and reflect frequencies). The relevant table is reproduced here as Table I. It concludes that the loop elements possess best overall features, while the solid interior elements exhibit poor-most performance. A similar conclusion and discussion may also be found in [1]. It should be noted that this conclusion has been reached only for the case of free standing single-layer 2-D frequency selective surfaces, which are not backed by any additional dielectric layer.

Table I: Comparison of various shapes [24], where a lower value refers to better performance.

Type of element	Stability with angle of incidence	Cross-polarization level (perpendicular polarization)	Larger bandwidth	Higher Filtering Selectivity
Loaded dipole	1	2	1	1
Jerusalem cross	2	3	2	2
Rings	1	2	1	1
Tripole	3	3	3	2
Crossed dipole	3	3	3	3
Square loop	1	1	1	1
Dipole	4	1	4	1

2.2.2.2 Element Size

The size of a unit cell element determines the resonant frequency of an FSS, which is generally kept to be the center frequency of an operating band. The exact relation between this center frequency and dimensions of an element depends on the element shape. For example, a dipole element resonates at a frequency where its length equals half of the operating wavelength. On the other hand, a ring element exhibits resonance when its circumference becomes equal to one wavelength at the operating frequency. In presence of an additional dielectric sheet, the effect of dielectric layer is also considered, which lowers the operating frequency and resultantly the size of a dipole and a ring element becomes less than a half and a full wavelength, respectively. For more complex shapes, an analytical close-form relation between its dimensions and resonant frequency is difficult to determine, and numerical simulations may be used for designing these FSSs.

2.2.2.3 Inter-Elements Spacing

Similar to the case of elements shape, the spacing between elements (the size of a unit-cell) also affects bandwidth, filtering selectivity, and angular stability of an FSS. Both the bandwidth and stability with the angle of incidence increase with a denser packing of the FSS elements. However, it is not always possible to reduce the inter-elements spacing arbitrarily, especially for the case of loop and solid interior elements where elements size is comparable with the operating wavelength, and hence the unit-cell size may not be reduced significantly. On the other hand, N-pole elements offer reasonably good flexibility in packing these elements closer. Ref. [1] shows a number of configurations for close packing of N-pole elements.

2.2.2.4 Effect of Additional Dielectric Layer

The placement of an additional dielectric layer is a very common technique to improve the performance of traditional frequency selective surfaces. Basically, this layer leads to two important effects. It improves the stability of response when the angle of incidence varies, as discussed in Section 1.1.2.2. Secondly, it lowers the resonant frequency of otherwise free-standing FSS. For example, a ring-based FSS will still resonate when the circumference of the ring is one wavelength, but its guided wavelength is lower than the free space wavelength due to the effect of the dielectric layer. Equivalently, this change in guided wavelength can be related to the reduction in resonant frequency.

2.2.2.5 Multi-layer Structures

Traditional 2-D single-layer frequency selective surface basically forms a resonator as explained in Section 1.1.1. Its filtering response can be improved through a cascade of multiple layers. In this technique, 2-D single-layer FSSs are combined together through dielectric spacer layers between them. Figure 2.2 presents an

equivalent circuit of such combination where three single-layer FSSs are combined with the help of dielectric spacers of thickness l_1 and l_2 , respectively. A multi-layer structure is equivalent to a higher order filter whereby filtering selectivity improves with an increasing filter order. However, this improvement in filtering response costs an increase in size and weight of the resultant multi-layer structure.

Figure 2.2: Equivalent circuit of a three-layer FSS.

2.2.3 Recent Trends

As shown in Figure 2.2, traditional cascade of single-layer FSSs forms direct-coupled resonators, which leads to a Butterworth filtering response [3], where out-of-band rejection increases monotonically. Recently, a number of designs have been proposed for improvement of filtering function to realize Chebyshev and even elliptic filtering response. This is achieved through increased coupling between the layers as indicated by C_{12} , C_{23} , C_{13} in Figure 2.3. Denoting the couplings among different layers of FSS by capacitors is merely a representation, while in actual case, these may also be inductive or mixed couplings [3].

Ref. [26] presents a technique of introducing strong coupling between adjacent layers by using aperture-coupled microstrip patches, which leads to an improved Chebyshev like filtering response. Ref. [4] introduces a novel configuration to realize these couplings whereby both Chebyshev and quasi-elliptic filtering responses have been demonstrated. A resonant coupling structure was inserted between two layers of circular patches, and the resulting antenna-filter-antenna configuration exhibits Chebyshev response [4]. Using two coupling structures in a

single unit-cell provides two parallel paths leading to a quasi-elliptic response where one transmission zero is realized [4]. Further recently, FSS designs using substrate integrated waveguide (SIW) and vertical microstrip lines have also been used to obtain quasi-elliptic performance [5, 6]. These designs are fundamentally different from previous approaches. Unlike the traditional cascaded combinations of 2-D FSSs, these structures consist of a 2-D periodic array of ‘monolithic’ multi-mode cavities whose coupling with air is controlled to obtain a desired quasi-elliptic filtering response.

Figure 2.3: An equivalent circuit of a three-layer FSS with Chebyshev or elliptic filtering response.

As discussed previously, a reduced separation between unit-cells of an FSS is desirable to obtain stable filtering response when it is subjected to plane wave incident from different angles. There has been a trend of employing lumped elements to realize compact unit-cell geometries. Ref. [27] presents a useful design method of lumped elements loaded FSS. It is also possible to obtain a tunable filtering performance by using active lumped elements [28].

2.3 Microwave Absorbers

Microwave absorbers are important primarily for defense applications. Research on RF/Microwave absorbers had started even before the Second World War, but major

developments came up during the war period. Progress on radars triggers further research on the absorbers so as to stealth against the new radar capabilities. Recently, ultra wide band radars are getting popular, and subsequently there is a greater need for the development of wideband absorbers.

Broadband microwave absorbers also carry significant commercial potential as they are widely used in anechoic chambers and electromagnetic interference (EMI)/electromagnetic compatibility (EMC) test ranges. The technology is also used in fabrication of some RF/microwave components, with an aim to control EMI. Antenna systems also use absorber coatings to avert undesired diffraction.

Microwave absorbers are broadly categorized into resonant and non-resonant structures. Resonant absorbers are inherently narrow-band, yet an improved bandwidth can be obtained through multi-layer multi-resonant designs. Non-resonant absorbers exhibit wideband operation. However, their applications are often restricted by size and weight limitations. Recently, new findings of high-impedance surface [29], artificial magnetic conductor, and artificial dielectric slabs [30], have been applied to absorber design. Most of these metamaterial absorbers exhibit highly narrow-band performance, and it may take more research and time before the practical feasibility of these recent trends is fully established.

In the following paragraphs, a review of the principles of traditional absorbers consisting of resonant and non-resonant structures is presented. Application of metamaterial leads to novel geometries, which are also reviewed from the viewpoint of their absorption performance, and limitations.

2.3.1 Traditional Structures

Traditional design of microwave absorbers is primarily based on two fundamental techniques namely Salisbury screen [31], and Dallenbach layer [32]. There have evolved many variants of these techniques, and they are often treated as an

independent class of absorbers. For instance, the design consisting of a cascaded combination of Salisbury screens is used for bandwidth enhancement, and it is termed as the Jaumann absorber. A method to reduce the thickness of Salisbury screen relies on capacitive loading of the resistive film, and this class of absorbers has been named as circuit analog (CA) absorbers. In the following paragraphs, design principles of both the fundamental and derived forms are discussed under the broad categories of Salisbury screen and Dallenbach layer.

Figure 2.4: Salisbury screen and its equivalent transmission line model.

2.3.1.1 Salisbury Screen and Its Variants

A classic Salisbury screen is shown in Figure 2.4. It comprises three parts:

- 1) A perfect electric conductor (PEC) plane or any conducting surface for practical purpose. Basically, it is the target object whose radar cross section (RCS) is reduced with the placement of an absorbing screen.
- 2) A lossless spacer layer (for instance, foam or a dielectric material) which is placed on top of the target or the PEC plane indicated by Region 1 in Figure 2.4.

- 3) A lossy top layer, which is purely resistive for classic Salisbury screen [31]. However, circuit analog absorbers [1] may include a reactive component for performance improvement. The lossy top layer has been shown as Region 2 in Figure 2.4.

From Figure 2.4, it is easy to see that impedance at the interface of lossy and lossless layers (while looking towards the PEC side) is a parallel combination of surface resistance of the top layer and the reactance offered by a conductor-backed dielectric spacer, as follows.

$$Z_{IN} = Z_R \parallel j\eta_1 \tan(k_1 d), \quad (2.1)$$

where

$$\eta_1 = \sqrt{\frac{\mu_r \mu_0}{\epsilon_r \epsilon_0}}, \quad k_1 = \omega \sqrt{\mu_r \mu_0 \epsilon_r \epsilon_0}.$$

Substituting $d = \lambda/4$ in (2.1) leads to

$$Z_{IN} = Z_R, \quad (2.2)$$

where Z_R refers to surface impedance of the top lossy layer, and λ denotes the wavelength at a frequency where absorption is desired. For a thin resistive layer, this surface impedance Z_R can be calculated as

$$Z_s = \frac{1}{\sigma t}, \quad (2.3)$$

where σ and t represent the conductivity and the thickness of a resistive layer.

Based on the well known transmission line theory [33], reflection coefficient Γ seen by incident plane wave at the interface of the top lossy layer and free space, is given by

$$\Gamma = \frac{Z_{IN} - Z_0}{Z_{IN} + Z_0} = \frac{Z_{IN} - \eta_0}{Z_{IN} + \eta_0}. \quad (2.4)$$

Since the goal of an absorber is to minimize this reflection coefficient, Z_R is kept equal to the intrinsic impedance of free space, and resultantly one can obtain zero reflection as implied by (2.4). Thus, in principle, it is possible to obtain a zero-reflection interface by using a lossy top layer of surface resistance $Z_R = \eta_0 = 377 \Omega$, and thickness $d = \lambda / 4$, where λ is the wavelength at a desired frequency of absorption. However, this method results in a very narrow-band structure. For a wideband design, Chambers [34] concluded the following important design guidelines of an optimally thin Salisbury screen:

- 1) For a known ϵ_r , thickness d is straightforwardly related to the desired center frequency of operation as follows:

$$d = \frac{c}{4\sqrt{\epsilon_r} f_c}, \quad (2.5)$$

where f_c and c refer to center frequency of absorber and speed of light in free space, respectively.

- 2) For known d and Z_R , maximum achievable bandwidth decreases when ϵ_r is larger than unity.
- 3) Given ϵ_r , d , maximum bandwidth is obtained when

$$Z_R = Z_0 \frac{1 - |\rho|^2}{1 + |\rho|^2} \quad (2.6)$$

where ρ is the prescribed level of reflection coefficient Γ based on which the absorber bandwidth has been defined. For example, if one is interested in 10-

dB bandwidth, it gives $|\rho| = 0.3162$. Considering free space intrinsic impedance $Z_0 = 377 \Omega$, (2.6) leads to a selection of $Z_R = 308.45 \Omega$.

Figure 2.5 shows an example of an optimally designed wideband Salisbury screen using the above steps. For a center frequency of 3.4 GHz, this procedure leads to an absorber thickness $d = 22.1 \text{ mm}$. This thickness can be reduced by using a dielectric material with higher dielectric constant ϵ_r . However, a Salisbury screen with dielectric material has narrow bandwidth compared with the one filled with foam or air [35], as illustrated in Figure 2.6.

Figure 2.5: Frequency response of an optimally designed Salisbury screen with $f_c = 3.4 \text{ GHz}$, ($\epsilon_r = 1$, $d = 22.1 \text{ mm}$, $Z_R = 308.45 \Omega$).

It is easy to see from Figure 2.4 that if d is decreased, Z_{IN} becomes inductive, and hence it is not possible to perfectly match it with free space impedance (377Ω) unless Z_R is made capacitive. Adding capacitive component to Z_R , in principle, compensates for the inductive component of Z_{IN} , and resultantly, it is possible to reduce the thickness of Salisbury screen while still obtaining good bandwidth of absorption [36]. This new class of absorbers is termed as **Circuit Analog (CA)**

absorber. Figure 2.7 presents an example where thickness of the Salisbury screen of Figure 2.5 has been reduced through a capacitive loading of the top lossy layer. The optimum design of this shunt RC absorbers has been discussed in [36]. Recently, Shen [37] showed that a series RC configuration is superior to a shunt RC network when wideband absorption is targeted.

Figure 2.6: Frequency response of a reduced thickness, optimally designed Salisbury screen with $f_c = 3.4$ GHz, ($\epsilon_r = 1.2$, $d = 20$ mm, $Z_R = 308.45 \Omega$).

Design procedure of a practical CA absorber may be found in [1, 38, 39]. It is based on a lossy FSS, which can be modeled as an RLC resonator. Following the extraction of R , L , C parameters of an FSS, the absorber design is iteratively optimized around those circuit component values which are realizable through the FSS under consideration [40].

In general, simultaneous improvement in size and bandwidth is not possible with electric type Salisbury screen and CA absorbers. A **magnetic Salisbury screen** has been reported in [41], but it is rather closer to a *magnetic Dallenbach layer* as the width of Region 1 (Figure 2.4) is assumed to be negligibly small, and Region 2 is characterized by its magnetic properties. This structure is one of the best traditional single layer broadband absorbers, and bandwidth as high as three octaves for a

single thin-layer has been reported [42]. More discussion on magnetic absorbers is given in the following section.

Figure 2.7: Frequency response of an optimally designed shunt RC absorber with $f_c = 3.4$ GHz, ($\epsilon_r = 1$, $d = 20$ mm, $R = 308.45 \Omega$, $C = 24.4$ fF).

2.3.1.2 Dallenbach Layer and Its Variants

Dallenbach layer consists of a homogeneous lossy layer backed by PEC as shown in Figure 2.8. The layer is characterized by complex constitutive parameters, μ_r and ϵ_r . Imaginary parts of these parameters represent magnetic and electric losses, respectively.

$$\begin{aligned}\mu_r &= \mu_r' - j\mu_r'' \\ \epsilon_r &= \epsilon_r' - j\epsilon_r''\end{aligned}\quad (2.7)$$

For the case of normal incidence, the equivalent transmission line model of the Dallenbach layer is given in Figure 2.8. This leads to

$$Z_{IN} = j\eta_1 \tan(k_1 d) . \quad (2.8)$$

Enforcing the condition of zero reflection $Z_{IN} = \eta_0$ results in

$$\eta_0 = j\eta_1 \tan(k_1 d) ,$$

or

$$j \tan(k_1 d) = \sqrt{\frac{\epsilon_r}{\mu_r}} , \quad (2.9)$$

where

$$k_1 = \omega \sqrt{\epsilon_0 \epsilon_r \mu_0 \mu_r} .$$

Equation (2.9) is regarded as the fundamental design equation for this absorber. Based on it, the absorber problem is reduced to that of finding a suitable material whose constitutive parameters and thickness fulfill (2.9) at a desired frequency of absorption.

Figure 2.8: The Dallenbach layer and its equivalent transmission line model.

If $\mu_r = 1$, only electric losses contribute to microwave absorption, and the absorber is called an *electric Dallenbach layer*, or simply the Dallenbach layer.

For a particular case, Knott [35] showed that the electric Dallenbach layer has a fractional bandwidth of around 20% for thickness close to $\lambda/4$. This bandwidth is less than that of a Salisbury screen.

On the other hand, the bandwidth can be greatly improved by introducing magnetic losses. In fact, with a **magnetic Dallenbach layer**, it is possible to improve both the bandwidth and the thickness, simultaneously. There have been significant investigations on this class of absorber, and it is named as **broadband magnetic absorber** [43]. Reduction in layer thickness improves actually its bandwidth. However, it imposes certain constraints on the material properties. When d is very small, (2.8) can be written as,

$$\begin{aligned} Z_{IN} &= j\eta_1 \tan(k_1 d) \approx j\eta_1 k_1 d \\ &= j \sqrt{\frac{\mu_1}{\varepsilon_1}} \omega \sqrt{\varepsilon_1 \mu_1} d \\ &= j\omega \mu_1 d. \end{aligned} \quad (2.10)$$

Enforcing $Z_{IN} = \eta_0$ (to obtain zero reflection) gives

$$\eta_0 = j\omega \mu_1 d = j\omega \mu_0 (\mu_r' - j\mu_r'') \quad (2.11)$$

Using an assumption that $\mu_r'' \gg \mu_r'$, (2.11) reduces to

$$\eta_0 = \omega \mu_0 \mu_r'' d. \quad (2.12)$$

This is the primary design guideline for magnetic thin broadband absorbers. Wallace [42] presented a number of examples where 10 dB reflection coefficient bandwidth of a 0.6 mm single layer exceeded two octaves (160-870 MHz). In a different design consisting of two layers, bandwidth exceeded even three octaves (190-1530 MHz). Magnetic broadband absorbers have traditionally been an

excellent solution for VHF/UHF radar cross section reduction [43]. They have also been demonstrated at lower microwave frequencies. However, they suffer from the following limitations [42]:

- 1) They generally consist of ferrite particles, which are heavy in weight. Thus, multi-layer structures might not be suitable for air-borne applications [44].
- 2) Magnetic Dallenbach layer does not offer a general solution for a desired frequency band, since it is not always possible for every frequency range to find a suitable material satisfying both (2.12) and the assumption $\mu_r'' \gg \mu_r'$.
- 3) μ_r changes rapidly with frequency. At microwave frequencies, magnetization suffers significantly, and thus it is difficult to find the suitable material operating in microwave range.

Recently, there has been progress on synthesizing nano-particles based magnetic materials [45, 46]. These novel techniques almost compensate for the above mentioned limitations, and they might revive interest in this class of absorbers especially for frequencies in the range of microwave and beyond.

2.3.1.3 Multilayer Structures

Cascading two or more above mentioned structures results in an improved bandwidth at the expense of size. Therefore, the number of layers in practical applications is constrained by the size and weight limitations. Figure 2.9 illustrates a cascaded combination of two Salisbury screens, which is also referred to as *Jaumann absorber*. A detailed design procedure for optimum Jaumann absorber may be found in [47-51].

Cascaded Dallenbach layers are also commonly used for broadband applications. Dallenbach layers provide more flexibility and ease of fabrication. Inhomogeneous layered absorbers are built as combination of thin homogeneous layers [52]. Hybrid absorbers have been used for increased bandwidth by combining the electric and magnetic Dallenbach layers. Magnetic materials are known to suit lower frequency applications whereas the electric layers are more practical for higher frequency range. A combination of the both has been reported as a promising candidate for multi-layer broadband absorbers [35].

Figure 2.9: A 2-layer Jaumann sbsorber.

From a designer point of view, the addition of more layers gives more parameters to control. Simple algorithms have found to be slow and inefficient for optimization of multi-layer structures, and many research groups have developed specialized algorithms for such problems as given in [48, 53].

2.3.2 Recent Designs

Recently, a number of metamaterial based absorber designs have been presented [30, 54, 55], which appear to carry certain advantages over traditional absorbers. In general, metamaterial techniques are attractive since they offer a control over the electromagnetic properties (electric permittivity and magnetic permeability) of a material and it is, in principle, possible to construct a material of any arbitrary ϵ , μ ,

be it positive or negative. Further, it is also realizable to get desired anisotropic variation in the material parameters. Anisotropy combined with periodicity and frequency dependent ϵ , μ can be used to design certain interesting structures that could exhibit useful peculiar properties. The recently proposed mushroom high impedance structure [18], and other artificial magnetic conductors (AMCs) lead to a significant size reduction of the traditional Salisbury screen, and a number of thin absorbers have been recently proposed. The following paragraphs briefly review the application of both the metamaterial slabs and AMCs in the development of microwave absorbers.

2.3.2.1 Absorbers Based On Metamaterial Slabs/Substrates

Similar to the previously discussed traditional Dallenbach layer, it is possible to design a lossy metamaterial slab to fulfill (2.12), and hence form an absorber. Since natural materials offer a limited range of ϵ , μ , metamaterials present more flexibility as they are theoretically suitable for realizing any arbitrary ‘effective’ values of the permittivity and permeability. However, the practical metamaterial slabs are not very popular for absorber applications at microwave frequencies, and only very few experiments have been performed so far. Bilotti [56] reported a metamaterial slab based absorber, which consists of an array of SRR structures backed by a thin resistive layer. It does not require a metal backing, and it can be used for RCS reduction of non-metallic objects. Wang [57] presented a three-dimensional polarization-independent absorber, which is also based on a well-packed SRR structure.

At microwave frequencies, metamaterial slabs do not offer drastic size reduction. Moreover, they are very dispersive in comparison with natural materials. At higher frequencies especially in terahertz range, metamaterial slab based absorbers might find more useful role to play. Devices at microwave frequencies work by flow of electrons, while at UV and higher frequencies photons are the center of focus. There is an in-between region where natural materials do not respond well to either electrons or photons. This has been named as ‘terahertz gap’,

and it ranges from 0.1-10 THz (λ varies from 3mm- 30 μ m). Metamaterial seems good choice for devices in this gap, and recently an absorbers based on metamaterial slab (wire and SRR combination) has been reported in [30]. It is expected to be useful for absorption of thermally THz imaging.

2.3.2.2 Absorbers Based on High Impedance Surfaces

High impedance surfaces have been shown to possess ‘in-phase reflection’ of normal incidence, opposed to the traditionally known ‘out of phase reflection’ from a perfect electric conductor (PEC). These structures are also termed as artificial magnetic conductors (AMCs), and they have been studied by a number of research groups [19, 58]. Application of AMC to microwave absorber has been mainly focused on exploitation of the ‘in-phase reflection’ feature for size reduction of a Salisbury screen. The primitive Salisbury screen is known to be a quarter wavelength thick in order to cancel the reflection from the bottom PEC and present an ‘open circuit’ in parallel to the lossy (resistive) layer. Using an AMC, ‘open circuit’ condition is available with even zero thickness, as it offers very high surface impedance. Engheta [54] has explained the design of very thin absorbers based on this principle. Similar approach was used by [55, 59-64] and an impressive size reduction has been reported.

2.3.2.3 Limitations

Practical applications of the above mentioned work are greatly limited by the highly dispersive response of the metamaterial slabs and high impedance surfaces. Though it can be shown theoretically that a material with (for instance) negative ϵ , μ can demonstrate an exceptional bandwidth [65], yet realization of such a material with constant ϵ , μ , in the desired practical frequency band of absorption, is not possible with the current state of understanding and research on left-handed metamaterials. In

fact the slope of permittivity and permeability (ϵ, μ) with frequency is so steep that only single frequency or very narrowband absorbers are realizable.

2.4 Conclusions

Frequency selective surfaces and microwave absorbers have been comprehensively reviewed with a focus on the design principles and recent trends. It has been noted that there exists a clear direction of future explorations on FSSs, as 3-D FSS offers many desirable features. There is a room for significant further research on 3-D FSSs in order to examine different 3-D unit-cell geometries for their effect on the elliptic filtering response, angular stability, and bandwidth of an FSS. Based on that trend, this thesis marks a significant contribution, since it presents a complete study of a new 3-D FSS.

Following the review of microwave absorbers, it is seen that the recent trend of using metamaterials has shown great reduction in the size of traditional microwave absorbers. However, these designs are very narrow-band solutions. Based on the review of open literature, it is difficult to establish a clear direction for future explorations of wideband microwave absorbers.

The theoretical work by Shen [37] shows that series RC absorber is a superior choice for wideband absorption. With that motivation, this thesis proposes that the 2-D periodic array of microstrip lines is a suitable structure to implement a series RC network based absorber proposed in [37]. Although the structure proposed in this thesis carries certain limitations, it may lead to a preferred direction for future explorations whereby further improvements may overcome the limitations, and may render an enhanced performance.

CHAPTER 3

PROPAGATION CHARACTERISTICS OF A 2-D PERIODIC ARRAY OF VERTICAL MICROSTRIP LINES

3.1 Introduction

Understanding the propagation characteristics of a two-dimensional (2-D) periodic array of vertical microstrip lines is important for its application as frequency selective structure (FSS) and microwave absorber, to be discussed later in Chapter 5. Accurate calculation of modes and the corresponding modal coefficients of this array is necessary for improved convergence and accuracy of the scattering problem when air-to-array discontinuity is analyzed using the mode-matching method in Chapter 4. Similarly, a detailed parametric study of the propagation characteristics is also important for establishing useful design guidelines of FSS and microwave absorbers to be discussed in Chapter 5. Based on that, both the calculation of modes and the understanding of propagation characteristics are the prime focus of the study presented in this chapter.

A closely related study of guided waves along an open strip grating is a classic topic with wide range of applications in leaky-wave antennas, traveling wave antennas, traveling wave amplifiers, and bandpass filters, etc. Open strip grating has been investigated under various assumptions by a number of authors [8, 9, 66-69] with a focus on mainly the leaky wave and surface wave modes. Shielded grating was also studied by a few authors [10, 70]. The classification of distinct regions and the use of point-matching technique in Ref. [10] forms an inefficient approach, whereby a large matrix needs to be solved. Moreover, the results reported in [10] are incomplete and it is difficult to develop a useful understanding of the shielded grating. Ref. [70] considers a grating placed inside a rectangular waveguide.

The mode-matching method is well known to be an efficient approach for guided-wave problems [71]. However, its hybridization with point-matching technique to solve the strip grating problem results in an over-all inefficient approach [9, 10] especially when a large number of modes need to be accurately computed. For this reason, the present investigation is based solely on an efficient mode-matching technique whereby a unit-cell is divided into three distinct regions, and the tangential components of fields are enforced to be continuous at the discontinuities. The periodic boundary is characterized by a periodic phase shift [2], and the same phase shift is then enforced on the tangential field components.

The detailed formulation of this efficient mode-matching method to calculate the propagation characteristics of a 2-D periodic array of vertical microstrip lines is given in the following paragraphs. Results have been compared with those obtained from CST microwave studio (MWS). It is interesting to note that an array of vertical microstrip lines supports two quasi-TEM modes. A comprehensive parametric study of the propagation characteristics is also presented. Further, this problem is compared with the cases when a microstrip line is shielded with magnetic or electric side walls. A few interesting similarities are noted between these cases and the problem under consideration where a microstrip line is basically shielded with periodic side walls. These comparisons and the discussions of the parametric study aim to build an in-depth understanding, which shall be later useful in evolving some intuitive guidelines for designing an FSS or microwave absorber based on the proposed periodic array of vertically placed microstrip lines.

3.2 Analysis Model

Figure 3.1 shows a 2-D periodic array of vertically placed microstrip lines. Basically, it is a periodic stack of a number of printed circuit boards (PCBs) where each PCB consists of a one-dimensional (1-D) periodic array of microstrip lines. Ground plane of each PCB is assumed be a perfect electric conductor (PEC). It is seen that this geometry can be divided into many identical unit-cells, as indicated in

Figure 3.1(b). Since the problem of a 2-D array can be reduced to that of a unit-cell based on the Floquet theorem [2], it is therefore appropriate to study the propagation characteristics of the unit-cell using an efficient full-wave mode-matching method, as formulated in this chapter.

Figure 3.1: Illustration of the 2-D periodic array of vertical microstrip lines, (a) Perspective view, (b) Top view.

Figure 3.2: A unit-cell of the 2-D periodic array of microstrip lines.

3.2.1 Geometry of the Problem

Figure 3.1 also shows the geometrical dimensions of the array. Periods along x - and y - axes are denoted by b and h , respectively. Width of the microstrip lines is

represented by t , and they are printed on a substrate material of dielectric constant ϵ_r and height d . Thickness of the strips is assumed to be negligibly small. A unit-cell of this geometry is given in Figure 3.2. Basically, it comprises a microstrip line shielded with periodic side-walls. Under an oblique incidence, the periodic boundary can be characterized by a phase shift Φ_x [2] which is a function of the angle of incidence. The top and bottom PEC planes refer to the ground of a printed circuit board.

3.2.2 Analysis Method

The unit-cell can be divided into three distinct regions, as shown in Figure 3.2. Region I consists of an inhomogeneous waveguide that supports LSE and LSM modes [2]. Region II and III form homogeneous regions and their modes are known by closed-form expressions. For this problem, y -directed Hertzian potentials for all the three regions are assumed. These lead to LSE/LSM field components in each region. The continuity of tangential field components at the interface of these regions is enforced. This discontinuity/interface is shown through a dotted line in Figure 3.2. Similarly the condition for periodic boundary is also satisfied, and the resulting set of linear equations leads to the propagation constant of this geometry.

3.3 Formulation

Following the method outlined above, expressions for fields in the three regions are derived in this section as follows.

3.3.1 Modes in Region I

Since Region I consists of an inhomogeneous waveguide, transverse electric (TE) and transverse magnetic (TM) modes are not supported [2, 72, 73], and instead it

supports longitudinal section electric (LSE) and longitudinal section magnetic (LSM) modes [2, 74]. Based on the coordinate-axis shown in Figure 3.2, LSE and LSM modal fields can be derived from y -directed Hertzian potentials as [2, 75]:

$$\begin{aligned}\vec{H} &= \nabla \times \nabla \times \vec{\Pi}^h + j\omega\epsilon \nabla \times \vec{\Pi}^e, \\ \vec{E} &= \nabla \times \nabla \times \vec{\Pi}^e - j\omega\mu \nabla \times \vec{\Pi}^h,\end{aligned}\tag{3.1}$$

where

$$\vec{\Pi}^h = \phi^h \hat{a}_y, \quad \vec{\Pi}^e = \phi^e \hat{a}_y.$$

ϕ^h and ϕ^e refer to magnetic- and electric-type scalar potentials, respectively. These potentials can be derived from the boundary conditions of the unit-cell [2, 75] as follows:

3.3.1.1 Potentials of Region I

As shown in Figure 3.2, there exists PEC boundary condition at $y = 0$ and $y = h$. Since the normal magnetic field vanishes near an electric wall, while the electric field can be normal to an electric wall [2, 33], the y -directed scalar magnetic and electric potentials of an n -th mode are required to fulfill the following conditions:

$$\phi_n^{lh} \Big|_{y=0,h} = 0, \quad \phi_n^{le} \Big|_{y=0,h} = 1.$$

This leads to,

$$\phi_n^{lh} = e^{-\gamma z} f_n^{lh}(x) \begin{cases} \frac{\sin(k_{yn}^{lh1} y)}{\sin(k_{yn}^{lh1} d)} & 0 < y < d \\ \frac{\sin[k_{yn}^{lh2} (h-y)]}{\sin[k_{yn}^{lh2} (h-d)]} & d < y < h \end{cases}, \tag{3.2}$$

$$\phi_n^{Ie} = e^{-\gamma z} f_n^{Ie}(x) \begin{cases} \frac{\cos(k_{yn}^{Ie1} y)}{\varepsilon_r \cos(k_{yn}^{Ie1} d)} & 0 < y < d \\ \frac{\cos[k_{yn}^{Ie2} (h-y)]}{\cos[k_{yn}^{Ie2} (h-d)]} & d < y < h \end{cases}, \quad (3.3)$$

where $\sin(k_{yn}^{Ih1} d)$, $\sin[k_{yn}^{Ih2} (h-d)]$, $\cos(k_{yn}^{Ie1} d)$, and $\cos[k_{yn}^{Ie2} (h-d)]$ are the normalization constants. γ represents the propagation constant of a microstrip line with periodic side walls shown in Figure 3.2. $f_n^{Ih}(x)$ and $f_n^{Ie}(x)$ denote the horizontal variation of these modal potentials. Region I also obeys the following relations [2, 75, 76]:

$$\frac{\tan(k_{yn}^{Ih1} d)}{k_{yn}^{Ih1}} + \frac{\tan[k_{yn}^{Ih2} (h-d)]}{k_{yn}^{Ih2}} = 0,$$

$$\frac{k_{yn}^{Ie1} \tan(k_{yn}^{Ie1} d)}{\varepsilon_r} + k_{yn}^{Ie2} \tan[k_{yn}^{Ie2} (h-d)] = 0,$$

$$k_{xn}^{Ip} = +\sqrt{\gamma^2 + k_0^2 - (k_{yn}^{Ip2})^2}, \quad p = h, e.$$

From the continuity of fields at the interface of the two sub-regions within Region I, one can write

$$k_0^2 - (k_{yn}^{Ip2})^2 = \varepsilon_r k_0^2 - (k_{yn}^{Ip1})^2$$

where $p = h, e$.

The horizontal variation of the modal potentials or fields cannot be expressed completely by a single sine or cosine function, and instead it may be written in a more general form as:

$$f_n^{lh}(x) = A_n^{lh} \frac{\sin\{k_{xn}^{lh}(x-x_1)\}}{\sin(k_{xn}^{lh}x_1)} + B_n^{lh} \frac{\cos\{k_{xn}^{lh}(x-x_1)\}}{\cos(k_{xn}^{lh}x_1)},$$

$$f_n^{le}(x) = A_n^{le} \frac{\cos\{k_{xn}^{le}(x-x_1)\}}{\cos(k_{xn}^{le}x_1)} + B_n^{le} \frac{\sin\{k_{xn}^{le}(x-x_1)\}}{\sin(k_{xn}^{le}x_1)},$$

where $x_1 = \frac{x_0}{2}$. A_n^{lh} , B_n^{lh} , A_n^{le} , and B_n^{le} are the modal coefficients of n -th mode of the dielectric-loaded waveguide formed by Region I.

3.3.1.2 Fields in Region I

Fields of an LSE mode can be obtained from (3.1) by substituting only the magnetic-type Hertzian potential [2] given by (3.2). It leads to the following expressions of the tangential fields of n -th mode. e and h denote the modal electric and magnetic field components, respectively.

$$h_{yn}^{lh} = \left[(k_{xn}^{lh})^2 - \gamma^2 \right] e^{-\gamma z} f_n^{lh}(x) \begin{cases} \frac{\sin(k_{yn}^{lh1}y)}{\sin(k_{yn}^{lh1}d)} & 0 < y < d \\ \frac{\sin[k_{yn}^{lh2}(h-y)]}{\sin[k_{yn}^{lh2}(h-d)]} & d < y < h \end{cases}, \quad (3.4)$$

$$h_{zn}^{lh} = \gamma e^{-\gamma z} f_n^{lh}(x) \begin{cases} \frac{-k_{yn}^{lh1} \cos(k_{yn}^{lh1}y)}{\sin(k_{yn}^{lh1}d)} & 0 < y < d \\ \frac{k_{yn}^{lh2} \cos[k_{yn}^{lh2}(h-y)]}{\sin[k_{yn}^{lh2}(h-d)]} & d < y < h \end{cases}, \quad (3.5)$$

$$e_{zn}^{lh} = -j\omega\mu k_{xn}^{lh} e^{-\gamma z} \bar{f}_n^{lh}(x) \begin{cases} \frac{\sin(k_{yn}^{lh1} y)}{\sin(k_{yn}^{l1} d)} & 0 < y < d \\ \frac{\sin[k_{yn}^{lh2}(h-y)]}{\sin[k_{yn}^{l2}(h-d)]} & d < y < h \end{cases}, \quad (3.6)$$

where

$$\bar{f}_n^{lh}(x) = A_n^{lh} \frac{\cos\{k_{xn}^{lh}(x-x_1)\}}{\sin(k_{xn}^{lh} x_1)} - B_n^{lh} \frac{\sin\{k_{xn}^{lh}(x-x_1)\}}{\cos(k_{xn}^{lh} x_1)}.$$

Similarly, the tangential field components of an LSM mode of Region I can be obtained by substituting the electric type Hertzian potential given by (3.3) into (3.1), which results in

$$e_{yn}^{le} = \left[(k_{xn}^{le})^2 - \gamma^2 \right] e^{-\gamma z} f_n^{le}(x) \begin{cases} \frac{\cos(k_{yn}^{le1} y)}{\varepsilon_r \cos(k_{yn}^{l1} d)} & 0 < y < d \\ \frac{\cos[k_{yn}^{le2}(h-y)]}{\cos[k_{yn}^{le2}(h-d)]} & d < y < h \end{cases}, \quad (3.7)$$

$$e_{zn}^{le} = \gamma e^{-\gamma z} f_n^{le}(x) \begin{cases} \frac{k_{yn}^{le1} \sin(k_{yn}^{le1} y)}{\varepsilon_r \cos(k_{yn}^{l1} d)} & 0 < y < d \\ \frac{-k_{yn}^{le2} \sin[k_{yn}^{le2}(h-y)]}{\cos[k_{yn}^{le2}(h-d)]} & d < y < h \end{cases}, \quad (3.8)$$

$$h_{zn}^{le} = -j\omega\epsilon_0 k_{xn}^{le} e^{-\gamma z} \bar{f}_n^{le}(x) \begin{cases} \frac{\cos(k_{yn}^{le1} y)}{\cos(k_{yn}^{le1} d)} & 0 < y < d \\ \frac{\cos[k_{yn}^{le2} (h-y)]}{\cos[k_{yn}^{le2} (h-d)]} & d < y < h \end{cases}, \quad (3.9)$$

where

$$\bar{f}_n^{le}(x) = A_n^{le} \frac{\sin\{k_{xn}^{le}(x-x_1)\}}{\cos(k_{xn}^{le} x_1)} - B_n^{le} \frac{\cos\{k_{xn}^{le}(x-x_1)\}}{\sin(k_{xn}^{le} x_1)}.$$

3.3.2 Modes in Region II

Unlike Region I, this region forms a homogenous waveguide and supports TE and TM modes, which can be derived from z -directed Hertzian potentials [33]. However, for a possibly faster convergence when fields are matched at the discontinuity of Region II with Region I, it may be useful to follow a similar procedure as done for Region I, and consider y -directed Hertzian potentials [75]. It leads to LSE and LSM modes whose field expressions are derived in the following paragraphs.

3.3.2.1 Potentials of Region II

Similar to the previous case of Region I, the y -directed Hertzian potentials for an n -th mode of Region II are required to fulfill the following conditions:

$$\phi_n^{IIh} \Big|_{y=d,h} = 0, \quad \phi_n^{IIe} \Big|_{y=d,h} = 1.$$

This leads to the following assumptions for the two types of potentials.

$$\phi_n^{IIh} = e^{-\gamma z} f_n^{IIh}(x) \sin\{k_{yn}^{IIh}(y-d)\}, \quad (3.10)$$

$$\phi_n^{Ile} = e^{-\gamma z} f_n^{Ile}(x) \cos\{k_{yn}^{Ile}(y-d)\}, \quad (3.11)$$

where

$$f_n^{IIh}(x) = A_n^{IIh} \frac{\sin\{k_{xn}^{IIh}(x-x_2)\}}{\sin(k_{xn}^{IIh}x_2)} + B_n^{IIh} \frac{\cos\{k_{xn}^{IIh}(x-x_2)\}}{\cos(k_{xn}^{IIh}x_2)},$$

$$f_n^{Ile}(x) = A_n^{Ile} \frac{\cos\{k_{xn}^{Ile}(x-x_2)\}}{\cos(k_{xn}^{Ile}x_2)} + B_n^{Ile} \frac{\sin\{k_{xn}^{Ile}(x-x_2)\}}{\sin(k_{xn}^{Ile}x_2)},$$

$$k_{yn}^{IIh} = \frac{n_h \pi}{h-d}, \quad k_{yn}^{Ile} = \frac{n_e \pi}{h-d}, \quad x_2 = x_0 + \frac{t}{2},$$

$$k_{xn}^{Ilp} = +\sqrt{k_0^2 + \gamma^2 - (k_{yn}^{Ilp})^2}, \quad p = h, e.$$

3.3.2.2 Fields in Region II

Substituting the expressions for the y -directed Hertzian potentials given by (3.10) and (3.11) into (3.1) results in the following tangential field components in Region II. $\vec{e}_n^{Ilp}$ and $\vec{h}_n^{Ilp}$ denote the tangential electric and magnetic field vectors supported by Region II. The superscript $p = e$ or h , which refers to an LSM or an LSE mode, respectively.

$$\vec{e}_n^{Ile} = e^{-\gamma z} f_n^{Ile}(x) \left\{ \begin{aligned} &\hat{a}_y \left[(k_{xn}^{Ile})^2 - \gamma^2 \right] \cos\{k_{yn}^{Ile}(y-d)\} \\ &+ \hat{a}_z \gamma k_{yn}^{Ile} \sin\{k_{yn}^{Ile}(y-d)\} \end{aligned} \right\}, \quad (3.12)$$

$$\vec{e}_n^{IIh} = -\hat{a}_z j e^{-\gamma z} \omega \mu k_{xn}^{IIh} \bar{f}_n^{IIh}(x) \sin\{k_{yn}^{IIh}(y-d)\}, \quad (3.13)$$

$$\vec{h}_n^{IIh} = e^{-\gamma z} f_n^{IIh}(x) \left\{ \begin{aligned} &\hat{a}_y \left[(k_{xn}^{IIh})^2 - \gamma^2 \right] \sin\{k_{yn}^{IIh}(y-d)\} \\ &- \hat{a}_z \gamma k_{yn}^{IIh} \cos\{k_{yn}^{IIh}(y-d)\} \end{aligned} \right\}, \quad (3.14)$$

$$\vec{h}_n^{IIe} = -\hat{a}_z j e^{-\gamma z} \omega \epsilon_0 \epsilon_r k_{xn}^{IIe} \vec{f}_n^{IIe}(x) \cos\{k_{yn}^{IIe}(y-d)\}, \quad (3.15)$$

where

$$\vec{f}_n^{IIe}(x) = A_n^{IIe} \frac{\sin\{k_{xn}^{IIe}(x-x_2)\}}{\cos(k_{xn}^{IIe}x_2)} - B_n^{IIe} \frac{\cos\{k_{xn}^{IIe}(x-x_2)\}}{\sin(k_{xn}^{IIe}x_2)},$$

$$\vec{f}_n^{IIIh}(x) = A_n^{IIIh} \frac{\cos\{k_{xn}^{IIIh}(x-x_2)\}}{\sin(k_{xn}^{IIIh}x_2)} - B_n^{IIIh} \frac{\sin\{k_{xn}^{IIIh}(x-x_2)\}}{\cos(k_{xn}^{IIIh}x_2)}.$$

3.3.3 Modes in Region III

Expressions for the modal fields of Region III are very similar to those of Region II since Region III also forms a homogenous waveguide. Although, it can support TE and TM modes, yet for the sake of consistency with the other regions to obtain an improved convergence, LSE and LSM modes are considered in this region. These modes are derived from y -directed Hertzian potentials as done for the cases of Region I and Region II.

3.3.3.1 Potentials of Region III

Based on the configuration shown in Figure 3.2, the following conditions are enforced on y -directed scalar Hertzian potentials of this region.

$$\phi_n^{IIIh} \Big|_{y=0,d} = 0, \quad \phi_n^{IIIe} \Big|_{y=0,d} = 1.$$

The resulting magnetic and electric types of Hertzian potentials can be written as

$$\phi_n^{IIIh} = e^{-\gamma z} f_n^{IIIh}(x) \sin(k_{yn}^{IIIh}y), \quad (3.16)$$

$$\phi_n^{IIIe} = e^{-\gamma z} f_n^{IIIe}(x) \cos(k_{yn}^{IIIe} y), \quad (3.17)$$

where

$$f_n^{IIIh}(x) = A_n^{IIIh} \frac{\sin\{k_{xn}^{IIIh}(x-x_2)\}}{\sin(k_{xn}^{IIIh} x_2)} + B_n^{IIIh} \frac{\cos\{k_{xn}^{IIIh}(x-x_2)\}}{\cos(k_{xn}^{IIIh} x_2)},$$

$$f_n^{IIIe} = A_n^{IIIe} \frac{\cos\{k_{xn}^{IIIe}(x-x_2)\}}{\cos(k_{xn}^{IIIe} x_2)} + B_n^{IIIe} \frac{\sin\{k_{xn}^{IIIe}(x-x_2)\}}{\sin(k_{xn}^{IIIe} x_2)},$$

$$k_{yn}^{IIIh} = \frac{n_h \pi}{d}, \quad k_{yn}^{IIIe} = \frac{n_e \pi}{d}, \quad x_2 = x_0 + \frac{t}{2},$$

$$k_{xn}^{IIIp} = +\sqrt{\varepsilon_r k_0^2 + \gamma^2 - (k_{yn}^{IIIp})^2}, \quad p = h, e.$$

3.3.3.2 Fields in Region III

Similar to the previous cases of Region I and Region II, tangential field components of an n -th mode in Region III are obtained by substituting (3.16) and (3.17) into (3.1). It gives the following field expressions. $\vec{e}_n^{IIIp}$ and $\vec{h}_n^{IIIp}$ denote the tangential electric and magnetic field vectors in Region III, while the superscript $p = e$ or h , refers to an LSM or an LSE mode, respectively.

$$\vec{e}_n^{IIIe} = e^{-\gamma z} f_n^{IIIe}(x) \left\{ \hat{a}_y \left[(k_{xn}^{IIIe})^2 - \gamma^2 \right] \cos(k_{yn}^{IIIe} y) + \hat{a}_z \gamma k_{yn}^{IIIe} \sin(k_{yn}^{IIIe} y) \right\}, \quad (3.18)$$

$$\vec{e}_n^{IIIh} = -\hat{a}_z j e^{-\gamma z} \omega \mu k_{xn}^{IIIh} \bar{f}_n^{IIIh}(x) \sin(k_{yn}^{IIIh} y), \quad (3.19)$$

$$\vec{h}_n^{IIIh} = e^{-\gamma z} f_n^{IIIh}(x) \left\{ \hat{a}_y \left[(k_{xn}^{IIIh})^2 - \gamma^2 \right] \sin(k_{yn}^{IIIh} y) - \hat{a}_z \gamma k_{yn}^{IIIh} \cos(k_{yn}^{IIIh} y) \right\}, \quad (3.20)$$

$$\vec{h}_n^{IIIe} = -\hat{a}_z j e^{-\gamma z} \omega \varepsilon_0 \varepsilon_r k_{xn}^{IIIe} \vec{f}_n^{IIIe}(x) \cos(k_{yn}^{IIIe} y), \quad (3.21)$$

where

$$\begin{aligned} \vec{f}_n^{IIIh} &= A_n^{IIIh} \frac{\cos\{k_{xn}^{IIIh}(x-x_2)\}}{\sin(k_{xn}^{IIIh} x_2)} - B_n^{IIIh} \frac{\sin\{k_{xn}^{IIIh}(x-x_2)\}}{\cos(k_{xn}^{IIIh} x_2)}, \\ \vec{f}_n^{IIIe} &= A_n^{IIIe} \frac{\sin\{k_{xn}^{IIIe}(x-x_2)\}}{\cos(k_{xn}^{IIIe} x_2)} - B_n^{IIIe} \frac{\cos\{k_{xn}^{IIIe}(x-x_2)\}}{\sin(k_{xn}^{IIIe} x_2)}. \end{aligned}$$

3.3.4 Boundary Conditions

The tangential electric and magnetic field components in any of the three regions can be represented by a general form as,

$$\begin{aligned} \vec{e}_n^{ip} &= e^{-\gamma z} \left[A_{nE}^{ip} f_{nE}^{ipA}(x) + B_{nE}^{ip} f_{nE}^{ipB}(x) \right] \vec{g}_{nE}^{ip}(y), \\ \vec{h}_n^{ip} &= e^{-\gamma z} \left[A_{nH}^{ip} f_{nH}^{ipA}(x) + B_{nH}^{ip} f_{nH}^{ipB}(x) \right] \vec{g}_{nH}^{ip}(y), \end{aligned} \quad (3.22)$$

where $p = e, h$, and $i = I, II, III$. f and g denote the variation of fields with respect to x - and y - axes, respectively. From the orthogonality of field components, it is known that [2]

$$\int_0^h \hat{a}_x \cdot (\vec{e}_m^{ip} \times \vec{h}_n^{ip*}) dy = 0, \quad m \neq n. \quad (3.23)$$

where $(\cdot)^*$ represents complex conjugate of a vector.

For a fixed $x = a$, substituting (3.22) into (3.23) leads to

$$\int_0^h \hat{a}_x \cdot (\vec{g}_{nE}^{ip} \times \vec{g}_{nH}^{ip}) dy = 0, \quad m \neq n. \quad (3.24)$$

It appears that (3.24) is a simpler relation which is easier to implement than (3.23). Based on that, in the following procedure, inner products are taken by multiplying the two sides of an equation with only the y -variation of electric and magnetic fields, and therefore invoking (3.24).

For the problem under consideration, basically a total of four boundary conditions need to be fulfilled, as follows:

1) The tangential components of electric and magnetic fields are known to be continuous at the interface between Region I and Regions II, III. This can be written as,

$$E_t^I \Big|_{x=x_0} = \begin{cases} E_t^{II} \Big|_{x=x_0} & y > d \\ E_t^{III} \Big|_{x=x_0} & y < d \end{cases},$$

or

$$\sum_{n=1}^{N_I} [\vec{e}_n^{Ih} + \vec{e}_n^{Ie}]_{x=x_0} = \begin{cases} \sum_{l=0}^{N_{II}} [\vec{e}_l^{IIIh} + \vec{e}_l^{IIe}]_{x=x_0} & y > d \\ \sum_{m=0}^{N_{III}} [\vec{e}_m^{IIIh} + \vec{e}_m^{IIIe}]_{x=x_0} & y < d \end{cases}, \quad (3.25)$$

where

$$\vec{e}_n^{Ih} = \hat{a}_z e_{zn}^{Ih}, \quad \vec{e}_n^{Ie} = \hat{a}_y e_{yn}^{Ie} + \hat{a}_z e_{zn}^{Ie}.$$

Taking a cross product on both sides of (3.25) with $\vec{g}_{nH}^{Ih}(y)$ and $\vec{g}_{nH}^{Ie}(y)$, and then using (3.24) leads to a set of equations that can be represented in a matrix form as follows,

$$\mathbf{U}_A^I \begin{bmatrix} \mathbf{A}^{Ih} \\ \mathbf{A}^{Ie} \end{bmatrix} + \mathbf{U}_B^I \begin{bmatrix} \mathbf{B}^{Ih} \\ \mathbf{B}^{Ie} \end{bmatrix} = \mathbf{P}_A \begin{bmatrix} \mathbf{A}^{IIIh} \\ \mathbf{A}^{IIIe} \end{bmatrix} + \mathbf{P}_B \begin{bmatrix} \mathbf{B}^{IIIh} \\ \mathbf{B}^{IIIe} \end{bmatrix} + \mathbf{Q}_A \begin{bmatrix} \mathbf{A}^{IIIh} \\ \mathbf{A}^{IIIe} \end{bmatrix} + \mathbf{Q}_B \begin{bmatrix} \mathbf{B}^{IIIh} \\ \mathbf{B}^{IIIe} \end{bmatrix}, \quad (3.26)$$

where

$$\begin{bmatrix} \mathbf{U}_X^I \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{Xhh}^I & \mathbf{U}_{Xeh}^I \\ \mathbf{0} & \mathbf{U}_{Xee}^I \end{bmatrix}, \quad [\mathbf{P}_X] = \begin{bmatrix} \mathbf{P}_{Xhh} & \mathbf{P}_{Xeh} \\ \mathbf{0} & \mathbf{P}_{Xee} \end{bmatrix}, \quad [\mathbf{Q}_X] = \begin{bmatrix} \mathbf{Q}_{Xhh} & \mathbf{Q}_{Xeh} \\ \mathbf{0} & \mathbf{Q}_{Xee} \end{bmatrix},$$

The subscript $X = A \text{ or } B$.

$\mathbf{U}_{Xhh}^I$ and $\mathbf{U}_{Xee}^I$ are diagonal matrices of size $N_I \times N_I$ each, where the diagonal elements are given by

$$\mathbf{U}_{Xhh}^I(n, n) = \int_0^h f_{nE}^{IhX}(x) \left[\bar{g}_{nE}^{Ih}(y) \times \bar{g}_{nH}^{Ih}(y) \right] dy,$$

$$\mathbf{U}_{Xee}^I(n, n) = \int_0^h f_{nE}^{IeX}(x) \left[\bar{g}_{nE}^{Ie}(y) \times \bar{g}_{nH}^{Ie}(y) \right] dy.$$

Elements of all the above matrices are given in the Appendix.

2) Continuity of electric field at the periodic boundary can be given by

$$E_t^I \Big|_{x=0} = \begin{cases} e^{-j\Phi_x} E_t^{II} \Big|_{x=b} & y > d \\ e^{-j\Phi_x} E_t^{III} \Big|_{x=b} & y < d' \end{cases},$$

or

$$\sum_{n=1}^{N_I} \left[\bar{e}_n^{Ih} + \bar{e}_n^{Ie} \right]_{x=0} = \begin{cases} e^{-j\Phi_x} \sum_{l=0}^{N_{II}} \left[\bar{e}_l^{IIIh} + \bar{e}_l^{IIIe} \right]_{x=b} & y > d \\ e^{-j\Phi_x} \sum_{m=0}^{N_{III}} \left[\bar{e}_m^{IIIh} + \bar{e}_m^{IIIe} \right]_{x=b} & y < d' \end{cases},$$

where Φ_x represents the periodic boundary phase shift, which is a function of the angle of incidence.

Similar to Step 1, taking a cross product with $\bar{g}_{nH}^{Ih}(y)$, $\bar{g}_{nH}^{Ie}(y)$ in this case leads to

$$\bar{\mathbf{U}}_A^I \begin{bmatrix} \mathbf{A}^{Ih} \\ \mathbf{A}^{Ie} \end{bmatrix} + \bar{\mathbf{U}}_B^I \begin{bmatrix} \mathbf{B}^{Ih} \\ \mathbf{B}^{Ie} \end{bmatrix} = \bar{\mathbf{P}}_A \begin{bmatrix} \mathbf{A}^{IIh} \\ \mathbf{A}^{IIe} \end{bmatrix} + \bar{\mathbf{P}}_B \begin{bmatrix} \mathbf{B}^{IIh} \\ \mathbf{B}^{IIe} \end{bmatrix} + \bar{\mathbf{Q}}_A \begin{bmatrix} \mathbf{A}^{IIIh} \\ \mathbf{A}^{IIIe} \end{bmatrix} + \bar{\mathbf{Q}}_B \begin{bmatrix} \mathbf{B}^{IIIh} \\ \mathbf{B}^{IIIe} \end{bmatrix}. \quad (3.27)$$

These matrices are very similar to those in (3.26), and their elements are also given in the Appendix.

3) Since tangential magnetic field is also continuous at the interface of the three regions, it can be expressed as

$$H_t^I \Big|_{x=x_0} = \begin{cases} H_t^{II} \Big|_{x=x_0} & y > d \\ H_t^{III} \Big|_{x=x_0} & y < d \end{cases},$$

or

$$\sum_{n=1}^{N_I} [\vec{h}_n^{Ih} + \vec{h}_n^{Ie}]_{x=x_0} = \begin{cases} \sum_{l=0}^{N_{II}} [\vec{h}_l^{IIh} + \vec{h}_l^{IIe}]_{x=x_0} & y > d \\ \sum_{m=0}^{N_{III}} [\vec{h}_m^{IIIh} + \vec{h}_m^{IIIe}]_{x=x_0} & y < d \end{cases}.$$

A cross product with $\vec{g}_{nE}^{IIh}(y)$, $\vec{g}_{nE}^{IIe}(y)$, $\vec{g}_{nE}^{IIIh}(y)$, and $\vec{g}_{nE}^{IIIe}(y)$, leads to two sets of equations as follows,

$$\mathbf{R}_A \begin{bmatrix} \mathbf{A}^{Ih} \\ \mathbf{A}^{Ie} \end{bmatrix} + \mathbf{R}_B \begin{bmatrix} \mathbf{B}^{Ih} \\ \mathbf{B}^{Ie} \end{bmatrix} = \mathbf{U}_A^{II} \begin{bmatrix} \mathbf{A}^{IIh} \\ \mathbf{A}^{IIe} \end{bmatrix} + \mathbf{U}_B^{II} \begin{bmatrix} \mathbf{B}^{IIh} \\ \mathbf{B}^{IIe} \end{bmatrix}, \quad (3.28)$$

$$\mathbf{S}_A \begin{bmatrix} \mathbf{A}^{Ih} \\ \mathbf{A}^{Ie} \end{bmatrix} + \mathbf{S}_B \begin{bmatrix} \mathbf{B}^{Ih} \\ \mathbf{B}^{Ie} \end{bmatrix} = \mathbf{U}_A^{III} \begin{bmatrix} \mathbf{A}^{IIIh} \\ \mathbf{A}^{IIIe} \end{bmatrix} + \mathbf{U}_B^{III} \begin{bmatrix} \mathbf{B}^{IIIh} \\ \mathbf{B}^{IIIe} \end{bmatrix}, \quad (3.29)$$

where

$$\mathbf{U}_X^q = \begin{bmatrix} \mathbf{U}_{Xhh}^q & \mathbf{0} \\ \mathbf{U}_{Xhe}^q & \mathbf{U}_{Xee}^q \end{bmatrix}, \quad \mathbf{R}_X = \begin{bmatrix} \mathbf{R}_{Xhh} & \mathbf{0} \\ \mathbf{R}_{Xhe} & \mathbf{R}_{Xee} \end{bmatrix}, \quad \mathbf{S}_X = \begin{bmatrix} \mathbf{S}_{Xhh} & \mathbf{0} \\ \mathbf{S}_{Xhe} & \mathbf{S}_{Xee} \end{bmatrix},$$

and

$$q = II, III. \quad X = A, B.$$

$\mathbf{U}_{Xhh}^q$ and $\mathbf{U}_{Xee}^q$ are diagonal matrices of size $N_q \times N_q$ each, and their diagonal elements are given by,

$$\begin{aligned} \mathbf{U}_{Xhh}^q(n, n) &= \int_0^h f_{nH}^{qhX}(x) \left[\bar{g}_{nH}^{qh}(y) \times \bar{g}_{nE}^{qh}(y) \right] dy, \\ \mathbf{U}_{Xee}^q(n, n) &= \int_0^h f_{nH}^{qeX}(x) \left[\bar{g}_{nH}^{qe}(y) \times \bar{g}_{nE}^{qe}(y) \right] dy. \end{aligned}$$

The expressions of all these matrices elements are given in the Appendix.

4) Enforcing the periodic boundary condition for magnetic field results in

$$H_t^I \Big|_{x=0} = \begin{cases} e^{-j\Phi_x} H_t^{II} \Big|_{x=b} & y > d \\ e^{-j\Phi_x} H_t^{III} \Big|_{x=b} & y < d \end{cases},$$

or

$$\sum_{n=1}^N \left[\tilde{h}_n^{lh} + \tilde{h}_n^{le} \right]_{x=0} = \begin{cases} e^{-j\Phi_x} \sum_{l=0}^{N_{II}} \left[\tilde{h}_l^{IIIh} + \tilde{h}_l^{IIIe} \right]_{x=b} & y > d \\ e^{-j\Phi_x} \sum_{m=0}^{N_{III}} \left[\tilde{h}_m^{IIIh} + \tilde{h}_m^{IIIe} \right]_{x=b} & y < d \end{cases}.$$

Similar to the previous case, taking a cross product with $\bar{g}_{nE}^{IIh}(y)$, $\bar{g}_{nE}^{IIe}(y)$, $\bar{g}_{nE}^{IIIh}(y)$, $\bar{g}_{nE}^{IIIe}(y)$, and simplifying the resulting equations leads to the following sets of matrices.

$$\bar{\mathbf{R}}_A \begin{bmatrix} \mathbf{A}^{lh} \\ \mathbf{A}^{le} \end{bmatrix} + \bar{\mathbf{R}}_B \begin{bmatrix} \mathbf{B}^{lh} \\ \mathbf{B}^{le} \end{bmatrix} = \bar{\mathbf{U}}_A^{II} \begin{bmatrix} \mathbf{A}^{IIh} \\ \mathbf{A}^{IIe} \end{bmatrix} + \bar{\mathbf{U}}_B^{II} \begin{bmatrix} \mathbf{B}^{IIh} \\ \mathbf{B}^{IIe} \end{bmatrix}, \quad (3.30)$$

$$\bar{\mathbf{S}}_A \begin{bmatrix} \mathbf{A}^{lh} \\ \mathbf{A}^{le} \end{bmatrix} + \bar{\mathbf{S}}_B \begin{bmatrix} \mathbf{B}^{lh} \\ \mathbf{B}^{le} \end{bmatrix} = \bar{\mathbf{U}}_A^{III} \begin{bmatrix} \mathbf{A}^{IIIh} \\ \mathbf{A}^{IIIe} \end{bmatrix} + \bar{\mathbf{U}}_B^{III} \begin{bmatrix} \mathbf{B}^{IIIh} \\ \mathbf{B}^{IIIe} \end{bmatrix}. \quad (3.31)$$

These matrices are also very similar to those in (3.28)-(3.29), and their elements are also given in the Appendix.

3.3.5 Solution

Solving (3.26)-(3.31) simultaneously gives,

$$[\mathbf{Y}] \begin{bmatrix} \mathbf{A}^{Ih} \\ \mathbf{A}^{Ie} \\ \mathbf{B}^{Ih} \\ \mathbf{B}^{Ie} \end{bmatrix} = \mathbf{0}, \quad (3.32)$$

where

$$[\mathbf{Y}] = [\mathbf{U}^I] - [\mathbf{P}][\mathbf{U}^{II}]^{-1}[\mathbf{R}] - [\mathbf{Q}][\mathbf{U}^{III}]^{-1}[\mathbf{S}],$$

and

$$\mathbf{U}^i = \begin{bmatrix} \mathbf{U}_A^i & \mathbf{U}_B^i \\ \bar{\mathbf{U}}_A^i & \bar{\mathbf{U}}_B^i \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} \mathbf{P}_A & \mathbf{P}_B \\ \bar{\mathbf{P}}_A & \bar{\mathbf{P}}_B \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} \mathbf{Q}_A & \mathbf{Q}_B \\ \bar{\mathbf{Q}}_A & \bar{\mathbf{Q}}_B \end{bmatrix},$$

$$\mathbf{R} = \begin{bmatrix} \mathbf{R}_A & \mathbf{R}_B \\ \bar{\mathbf{R}}_A & \bar{\mathbf{R}}_B \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} \mathbf{S}_A & \mathbf{S}_B \\ \bar{\mathbf{S}}_A & \bar{\mathbf{S}}_B \end{bmatrix}, \quad i = I, II, III.$$

Determinant of the Matrix $\mathbf{Y}$ is tested against a sweep of possible values of the unknown propagation constant γ . Only those γ values are noted as valid modes of this structure, for whom determinant of the matrix $\mathbf{Y}$ vanishes [77].

3.4 Results and Discussions

To ensure a fast relative convergence of the above formulation, the number of modes in each region is dynamically chosen for a simulation frequency point. For an operating point k_o , a cut-off wave number k_c is defined, and the number of propagating modes N_{II} and N_{III} in Regions II and III are calculated for k_c . This cut-off wave number is kept much higher than the operating wave number k_o and its purpose is to incorporate the effect of both the heights ratio d/h and dielectric constant ϵ_r in order to obtain the number of modes in Region III relative to those in Region II. The number of modes in Region I is taken as an algebraic sum of N_{II} and N_{III} . Based on this scheme, convergent results may be obtained when k_c is chosen ten times the operating wave number k_o , as indicated in Table 3.1.

Table 3.1: Convergence behavior of the first two modes

($t = 4 \text{ mm}$, $b = 10 \text{ mm}$, $h = 3 \text{ mm}$, $d = 1.27 \text{ mm}$, $\epsilon_r = 2.2$, $f = 10 \text{ GHz}$, $\Phi_x = 0$).

Mode #	Cut-off Wave Number (rad/m)		
	$k_c = 5 k_o$ (16 x 16 Matrix)	$k_c = 10 k_o$ (24 x 24 Matrix)	$k_c = 20 k_o$ (32 x 32 Matrix)
1	298	297	297
2	220	220	220

Figure 3.3: Dispersion diagram of the first six modes of a periodic array of microstrip lines ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $\Phi_x = 0$).

Figure 3.3 and Figure 3.4 show a comparison of the dispersion diagram generated by the full-wave mode-matching method presented above with the one obtained from CST Microwave Studio (MWS). An excellent agreement of results can be observed. It takes 15 seconds for our program routine to generate this diagram, while CST MWS takes 6 minutes for the same problem. The existence of two quasi-TEM modes at lower phase shifts may also be noted. Most of the power carried by the first mode is concentrated in the substrate region, while the second mode propagates dominantly in the air region above the substrate. Based on that, these may be termed as ‘substrate-mode’, and ‘air-mode’ [6]. Figure 3.5 verifies this point whereby the substrate mode is strongly affected by a variation of the substrate dielectric constant. Interestingly, the existence of dual-mode propagation was previously also observed for an open strip grating, and these were named as grating and surface wave modes [8].

Figure 3.4: Dispersion diagram of the first six modes of a periodic array of microstrip lines
($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $\Phi_x = \pi$).

3.4.1 Effect of Periodic Boundary Condition

Figure 3.7 presents the variation of propagation constants of the first three modes when Φ_x is varied from 0 to 120° . In the region of lower frequencies, propagation constants of these modes decrease with an increase in the periodic phase shift. The substrate-mode remains almost stable while the air-mode becomes evanescent at lower frequencies when the periodic phase shift is increased, as shown in Figure 3.7. Basically, when the periodic phase shift $\Phi_x = 0$, virtual magnetic walls may be considered between the strips of the periodic array of microstrip lines. The unit-cell of this geometry then takes the form of a microstrip line shielded with PMC sidewalls as shown in Figure 3.6(b). Hence, the existence of two quasi-TEM modes can be attributed to the presence of three isolated conductors in this case. On the other extreme, when $\Phi_x = \pi$, a virtual PEC wall may be considered between the strips and the unit-cell of the resulting periodic array is given in Figure 3.6(a). Since this geometry is known to support the propagation of single quasi-TEM mode, it may explain the variation of the air-mode in Figure 3.7, which becomes evanescent

with an increase of the periodic boundary phase shift.

Figure 3.5: Effect of dielectric constant ϵ_r on the propagation constants of the first two modes of a microstrip line shielded with periodic side walls ($t = 2 \text{ mm}$, $b = 10 \text{ mm}$, $d = 3.175 \text{ mm}$, $h = 4.7 \text{ mm}$, $f = 5 \text{ GHz}$, $\Phi_x = 0$).

Alternatively, the trend of Figure 3.7 can also be understood by considering the fact that there exists a preferred direction of the group velocity for every propagating mode of the periodic array of microstrip lines [8], as indicated in Figure 3.8. If b is the periodicity of this array, then the periodic boundary phase shift Φ_x can be written as:

$$\Phi_x = b\Gamma \sin(\psi). \quad (3.33)$$

where Γ is the propagation constant along the direction of propagation making an angle ψ with the y -axis, as shown in Figure 3.8. Since β is the z -directed projection of Γ [8], it leads to

$$\beta = \Gamma \cos(\psi). \quad (3.34)$$

Figure 3.6: Cross-section of a microstrip line shielded with (a) PEC (b) PMC side walls.

Figure 3.7: Variation of the propagation constant β as a function of periodic phase shift Φ_x for first three modes of a periodic array of microstrip lines ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$).

Figure 3.8: A schematic diagram of 1-D periodic array of microstrip lines, indicating the direction of propagation of a mode inside the periodic structure.

For the region of lower frequencies when the periodicity of the array is much smaller than the operating wavelength, Γ can be assumed to be independent of Ψ , which means that the group velocity of a mode remains the same in every direction since the layer of strips acts as a homogenous surface. This assumption seems reasonable especially for the quasi-TEM modes at lower frequencies where the field distribution is quite uniform within the substrate or the air-region. Differentiating both sides of (3.33) and (3.34) with respect to Ψ results in

$$\frac{\partial \Phi_x}{\partial \psi} = b\Gamma \cos(\psi) = b\beta,$$

and

$$\frac{\partial \beta}{\partial \psi} = -\Gamma \sin(\psi) = -\frac{\Phi_x}{b}.$$

This leads to

$$\frac{\partial \beta}{\partial \Phi_x} = -\frac{\Phi_x}{b^2 \beta}. \quad (3.35)$$

The above result is clearly in agreement with the trend of Figure 3.7 at lower frequencies, where reduction in the propagation constant of a mode is noted with an

increase in the periodic boundary phase shift. Also, the air-mode changes more rapidly than the substrate-mode, as expected from (3.35), since the rate of variation is inversely proportional to the propagation constant of a mode. However, this relation breaks down at higher frequencies, which implies that the magnitude of the group velocity of a mode is not independent of its direction of propagation at higher frequencies, as assumed in the derivation of (3.35). This is understandable because the periodicity of the array becomes comparable with its operating guided wavelength at higher frequencies.

3.4.2 Parametric Study

Figure 3.9: Variation of the propagation constants of first two modes of the array as a function of dielectric constant ϵ_r and Φ_x ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $f = 10 \text{ GHz}$).

As established in Chapter 1, a 2-D periodic array is attractive primarily for it supports two quasi-TEM modes. These modes lead to the realization of frequency selective structures with elliptical filtering response, and microwave absorbers with wideband absorption, to be discussed in Chapter 5. Due to the quasi-TEM nature of these modes, a unit-cell size can be sub-wavelength, which results in stable

performance with different angles of incidence. Based on that, design of the structures proposed in this thesis is mainly concerned with the first two quasi-TEM modes of the 2-D periodic array of microstrip lines. Therefore, in the following paragraphs the effect of various structural parameters is investigated on these first two modes only.

Figure 3.10: Variation of the propagation constants of first two modes of the array as a function of unit-cell height h and Φ_x ($t = 2$ mm, $b = 5$ mm, $d = 1.524$ mm, $\epsilon_r = 3.38$, $f = 10$ GHz).

Figure 3.9 illustrates the effect of the substrate's dielectric constant on the two quasi-TEM modes. As expected, the substrate-mode is strongly affected by the change of dielectric constant. Also, with an increase in the periodic boundary phase shift, the air-mode changes more rapidly than the substrate-mode, as expected by (3.35).

Figure 3.10 presents the effect of the height of a unit-cell, which shows that the air-mode is more strongly affected by this parameter. Since the substrate-mode is mainly concentrated in the substrate region, it is relatively less affected by the variation of height.

Figure 3.11: Variation of the propagation constants of first two modes of the array as a function of strip width t and Φ_x ($b = 5$ mm, $h = 5$ mm, $d = 1.524$ mm, $\epsilon_r = 3.38$, $f = 10$ GHz).

Figure 3.11 shows the variation of propagation constant as a function of the strip width t . An increased strip width basically isolates the substrate and the air regions. More Field lines of the two modes are restricted to their respective regions, and hence the propagation constant of the substrate-mode increases, while that of the air-mode decreases with an increased strip width.

3.5 Comparison of Periodic Boundary with PMC and PEC Walls

In many electromagnetic problems of periodic structures, especially those involving microstrip line arrays, a periodic boundary is often replaced by a PEC or a PMC wall [6, 66]. Considering a one-dimensional (1-D) periodic array of microstrip lines, it can be intuitively argued that when the strips are excited in phase, virtual PMC walls shall be formed between the adjacent strips. Similarly, if the two

adjacent strips are excited out of phase, a virtual PEC wall may be considered between them. Based on that, it may be an interesting investigation to seek a comparison of PMC and PEC walls with PBC when the periodic boundary phase shift is set to be $\Phi_x = 0^\circ$ and 180° , respectively.

3.5.1 Formulation - Microstrip Line Shielded with PEC Side Walls

Figure 3.12 shows the cross-sectional view of a microstrip line shielded with PEC side walls. It also illustrates a classification of four distinct regions. Unlike most of the traditional formulations of shielded microstrip line where a symmetry plane is considered [75, 78], the given classification is also valid for asymmetrically placed shielded microstrip line. The mode-matching procedure for this case is similar to the one used in the previous section, since this configuration also enforces four boundary conditions once the tangential field components of each region are derived from their respective Hertzian potentials. Also, the y -directed Hertzian potentials are considered, which lead to LSM/LSE modes in each region.

Figure 3.12: Cross-sectional view of a microstrip line shielded with PEC side walls.

Region I forms an inhomogeneous waveguide, which supports the following y -directed Hertzian potentials:

$$\phi_m^{lh} = D_m^{lh} e^{-\gamma z} \frac{\cos(k_{xm}^{lh} x)}{\cos(k_{xm}^{lh} x_1)} \begin{cases} \frac{\sin(k_{ym}^{lhb} y)}{\sin(k_{ym}^{lhb} d)} & 0 < y < d \\ \frac{\sin[k_{ym}^{lha} (h-y)]}{\sin[k_{ym}^{lha} (h-d)]} & d < y < h \end{cases}, \quad (3.36)$$

$$\phi_m^{le} = D_m^{le} e^{-\gamma z} \frac{\sin(k_{xm}^{le} x)}{\sin(k_{xm}^{le} x_1)} \begin{cases} \frac{\cos(k_{ym}^{leb} y)}{\varepsilon_r \cos(k_{ym}^{leb} d)} & 0 < y < d \\ \frac{\cos[k_{ym}^{lea} (h-y)]}{\cos[k_{ym}^{lea} (h-d)]} & d < y < h \end{cases}, \quad (3.37)$$

where D_m^{lh} and D_m^{le} represent the modal coefficients of m -th mode. γ is the propagation constant of the microstrip line shielded with PEC side walls. $\cos(k_{xm}^{lp} x_1)$, $\sin(k_{ym}^{lpb} d)$, and $\sin[k_{ym}^{lpa} (h-d)]$ are the normalization constants where $p = e$ or h . The normalization basically prevents possible ill-conditioned matrices when this formulation is implemented. The following relations also hold for Region I.

$$\frac{\tan(k_{ym}^{lhb} d)}{k_{ym}^{lhb}} + \frac{\tan[k_{ym}^{lha} (h-d)]}{k_{ym}^{lha}} = 0,$$

$$\frac{k_{ym}^{leb} \tan(k_{ym}^{leb} d)}{\varepsilon_r} + k_{ym}^{lea} \tan[k_{ym}^{lea} (h-d)] = 0,$$

$$k_{xm}^{lp} = +\sqrt{\gamma^2 + k_0^2 - (k_{ym}^{lpa})^2}, \quad p = h, e.$$

Region II forms an air-filled homogenous waveguide, and the following potentials are supported in this case.

$$\phi_m^{IIh} = e^{-\gamma z} \sin[k_{ym}^{IIh}(y-d)] \left\{ D_m^{IIh} \frac{\sin[k_{xm}^{IIh}(L-x)]}{\sin(k_{xm}^{IIh}t)} + F_m^{IIh} \frac{\cos[k_{xm}^{IIh}(L-x)]}{\cos(k_{xm}^{IIh}t)} \right\}, \quad (3.38)$$

$$\phi_m^{IIe} = e^{-\gamma z} \cos[k_{ym}^{IIe}(y-d)] \left\{ D_m^{IIe} \frac{\cos[k_{xm}^{IIe}(L-x)]}{\cos(k_{xm}^{IIe} \frac{t}{2})} + F_m^{IIe} \frac{\sin[k_{xm}^{IIe}(L-x)]}{\sin(k_{xm}^{IIe} \frac{t}{2})} \right\}. \quad (3.39)$$

where D_m^{IIp} and F_m^{IIp} are the modal coefficients. Also,

$$k_{ym}^{IIh} = \frac{m_h \pi}{h-d}, \quad k_{ym}^{IIe} = \frac{m_e \pi}{h-d},$$

$$k_{xm}^{IIp} = +\sqrt{\gamma^2 + k_0^2 - (k_{ym}^{IIp})^2}, \quad p = h, e.$$

Region III is also a homogenous region filled with substrate material of dielectric constant ϵ_r . Very similar to those for Region II, the following potentials may be assumed for Region III.

$$\phi_m^{IIIh} = e^{-\gamma z} \sin(k_{ym}^{IIIh} y) \left\{ D_m^{IIIh} \frac{\sin[k_{xm}^{IIIh}(L-x)]}{\sin(k_{xm}^{IIIh} \frac{t}{2})} + F_m^{IIIh} \frac{\cos[k_{xm}^{IIIh}(L-x)]}{\cos(k_{xm}^{IIIh} \frac{t}{2})} \right\}, \quad (3.40)$$

$$\phi_m^{IIIe} = e^{-\gamma z} \cos(k_{ym}^{IIIe} y) \left\{ D_m^{IIIe} \frac{\cos[k_{xm}^{IIIe}(L-x)]}{\cos(k_{xm}^{IIIe} \frac{t}{2})} + F_m^{IIIe} \frac{\sin[k_{xm}^{IIIe}(L-x)]}{\sin(k_{xm}^{IIIe} \frac{t}{2})} \right\}, \quad (3.41)$$

where D_m^{IIIp} , F_m^{IIIp} are the modal coefficients, and

$$k_{ym}^{IIIh} = \frac{m_h \pi}{d}, \quad k_{ym}^{IIIe} = \frac{m_e \pi}{d},$$

$$k_{xm}^{IIIp} = +\sqrt{\gamma^2 + \varepsilon_r k_0^2 - \left(k_{ym}^{IIIp}\right)^2}, \quad p = h, e.$$

Region IV is closely related to Region I, and the potential functions of this region may be assumed as,

$$\phi_m^{IVh} = D_m^{IVh} e^{-\gamma z} \frac{\cos\left[k_{xm}^{IVh}(b-x)\right]}{\cos\left[k_{xm}^{IVh}(b-x_2)\right]} \begin{cases} \frac{\sin\left(k_{ym}^{IVhb} y\right)}{\sin\left(k_{ym}^{IVhb} d\right)} & 0 < y < d \\ \frac{\sin\left[k_{ym}^{IVha}(h-y)\right]}{\sin\left[k_{ym}^{IVha}(h-d)\right]} & d < y < h \end{cases}, \quad (3.42)$$

$$\phi_m^{IVe} = D_m^{IVe} e^{-\gamma z} \frac{\sin\left[k_{xm}^{IVe}(b-x)\right]}{\sin\left[k_{xm}^{IVe}(b-x_2)\right]} \begin{cases} \frac{\cos\left(k_{ym}^{IVeb} y\right)}{\varepsilon_r \cos\left(k_{ym}^{IVeb} d\right)} & 0 < y < d \\ \frac{\cos\left[k_{ym}^{IVea}(h-y)\right]}{\cos\left[k_{ym}^{IVea}(h-d)\right]} & d < y < h \end{cases}, \quad (3.43)$$

where

$$\frac{\tan\left(k_{ym}^{IVhb} d\right)}{k_{ym}^{IVhb}} + \frac{\tan\left[k_{ym}^{IVha}(h-d)\right]}{k_{ym}^{IVha}} = 0,$$

$$\frac{k_{ym}^{IVeb} \tan\left(k_{ym}^{IVeb} d\right)}{\varepsilon_r} + k_{ym}^{IVea} \tan\left[k_{ym}^{IVea}(h-d)\right] = 0,$$

$$k_{xm}^{IVp} = +\sqrt{\gamma^2 + k_0^2 - \left(k_{ym}^{IVpa}\right)^2}, \quad p = h, e.$$

As discussed in the previous section, tangential field expression for each region are obtained from (3.1). Based on the variables used in Figure 3.12, the following boundary conditions are enforced on the tangential field components of the four regions.

- 1) Continuity of tangential electric and magnetic fields at the discontinuity plane $x = x_1$:

$$E_t^I \Big|_{x=x_1} = \begin{cases} E_t^{II} \Big|_{x=x_1} & y > d \\ E_t^{III} \Big|_{x=x_1} & y < d \end{cases}, \quad H_t^I \Big|_{x=x_1} = \begin{cases} H_t^{II} \Big|_{x=x_1} & y > d \\ H_t^{III} \Big|_{x=x_1} & y < d \end{cases}.$$

- 2) Continuity of the tangential electric and magnetic fields at $x = x_2$:

$$E_t^{IV} \Big|_{x=x_2} = \begin{cases} E_t^{II} \Big|_{x=x_2} & y > d \\ E_t^{III} \Big|_{x=x_2} & y < d \end{cases}, \quad H_t^{IV} \Big|_{x=x_2} = \begin{cases} H_t^{II} \Big|_{x=x_2} & y > d \\ H_t^{III} \Big|_{x=x_2} & y < d \end{cases}.$$

The procedure of taking cross-product with orthogonal functions is very similar to that discussed in Section 3.2, and the same orthogonality relations are applicable here. Resulting matrices are solved to evolve a single matrix $\mathbf{Y}$ as in Section 3.2, whose determinant is tested against a sweep of possible solutions of γ , which leads to the modes of this structure.

3.5.2 Formulation - Microstrip Line Shielded with PMC Side Walls

Side Walls

Figure 3.13: Cross-sectional view of a microstrip line shielded with PMC side walls.

A microstrip line shielded with PMC side walls is shown in Figure 3.13, whose cross-section has been classified into four regions. This classification and the geometry are identical to the one discussed previously and given in Figure 3.12. The only difference is a boundary condition whereby the electric side walls have been replaced with magnetic side walls in this new structure. Based on that, the same formulation as discussed in the previous section is still applicable here, except that the Hertzian potentials for Region I and IV are modified to suit the PMC side walls.

Region I now supports the following y -directed Hertzian potentials:

$$\phi_m^{Ih} = D_m^{Ih} e^{-\gamma z} \frac{\sin(k_{xm}^{Ih} x)}{\sin(k_{xm}^{Ih} x_1)} \begin{cases} \frac{\sin(k_{ym}^{Ihb} y)}{\sin(k_{ym}^{Ihb} d)} & 0 < y < d \\ \frac{\sin[k_{ym}^{Iha}(h-y)]}{\sin[k_{ym}^{Iha}(h-d)]} & d < y < h \end{cases}, \quad (3.44)$$

$$\phi_m^{Ie} = D_m^{Ie} e^{-\gamma z} \frac{\cos(k_{xm}^{Ie} x)}{\cos(k_{xm}^{Ie} x_1)} \begin{cases} \frac{\cos(k_{ym}^{Ieb} y)}{\varepsilon_r \cos(k_{ym}^{Ieb} d)} & 0 < y < d \\ \frac{\cos[k_{ym}^{Iea}(h-y)]}{\cos[k_{ym}^{Iea}(h-d)]} & d < y < h \end{cases}, \quad (3.45)$$

where the interpretation of various notations is the same as discussed previously for the case of a microstrip line shielded with PEC side walls in Section 3.5.1.

Similarly, the modified form of Region IV potential function is the following:

$$\phi_m^{IVh} = D_m^{IVh} e^{-\gamma z} \frac{\sin[k_{xm}^{IVh}(b-x)]}{\sin[k_{xm}^{IVh}(b-x_2)]} \begin{cases} \frac{\sin(k_{ym}^{IVhb} y)}{\sin(k_{ym}^{IVhb} d)} & 0 < y < d \\ \frac{\sin[k_{ym}^{IVha}(h-y)]}{\sin[k_{ym}^{IVha}(h-d)]} & d < y < h \end{cases}, \quad (3.46)$$

$$\phi_m^{IVe} = D_m^{IVe} e^{-\gamma z} \frac{\cos[k_{xm}^{IVe}(b-x)]}{\cos[k_{xm}^{IVe}(b-x_2)]} \begin{cases} \frac{\cos(k_{ym}^{IVeb} y)}{\varepsilon_r \cos(k_{ym}^{IVeb} d)} & 0 < y < d \\ \frac{\cos[k_{ym}^{IVea}(h-y)]}{\cos[k_{ym}^{IVea}(h-d)]} & d < y < h \end{cases} \quad (3.47)$$

3.5.3 Results and Discussions

Figure 3.14 shows dispersion diagrams of a microstrip line shielded with PEC sidewalls, and that of a periodic array of microstrip lines when its adjacent strips are fed out of phase. As expected, the dominant modes of the two cases are identical. However, it is interesting to note that certain higher order modes are also similar, for instance, the third and fifth modes in Figure 3.14.

Figure 3.14: Comparison of dispersion diagrams of a microstrip line with PEC side walls, and PBC side walls when the periodic phase shift $\Phi_x = 180^\circ$ ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\varepsilon_r = 3.38$).

Figure 3.15: Comparison of dispersion diagrams of a microstrip line with PEC side walls, and PBC side walls when the periodic phase shift $\Phi_x = 0^\circ$ ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$).

Figure 3.16: Dispersion diagram of the first two modes of a microstrip line shielded with periodic side walls ($t = 2 \text{ mm}$, $b = 4 \text{ mm}$, $\epsilon_r = 3.38$, $d = 1.52 \text{ mm}$, $h = 7.52 \text{ mm}$, $\Phi_x = 0$).

Figure 3.15 presents the dispersion diagram of a microstrip line shielded with PMC side walls compared with that of a periodic array of microstrip lines when the strips are excited in phase. The first two modes of these cases are identical, as expected. However, the higher order modes are significantly different in this case.

Based on the above, it is seen that a periodic boundary is significantly different from a PEC or a PMC wall. A PEC or PMC wall requires the tangential electric or magnetic field components to vanish on the wall surface, while a periodic boundary condition requires the phase shifted replicas of field pattern based on Floquet theorem. However, under certain excitation conditions, some modes of a microstrip line with PBC become identical to those of a microstrip line with PEC/PMC side-wall. For those modes, PEC/PMC side-wall may be treated as special cases of periodic boundaries.

Figure 3.17: Effect of the separation between two side walls on the propagation constants of the first two modes of a microstrip line with periodic side walls ($t = 2 \text{ mm}$, $\epsilon_r = 3.27$, $d = 5.08 \text{ mm}$, $h = 11.08 \text{ mm}$, $f = 5 \text{ GHz}$, $\Phi_x = 0$).

Figure 3.16 compares the dispersion diagram of the two modes of a microstrip line with periodic boundary when $\Phi_x = 0$, with that of the single mode of a microstrip line shielded with PEC side walls. It is seen that the substrate-mode is

more dispersive than the dominant mode of a microstrip line shielded with PEC side walls, while the air-mode is the least dispersive of all. The substrate-mode is mainly concentrated in the substrate region, and the field pattern between the strip and the bottom PEC is very similar to that of a dielectric-filled parallel-plate waveguide. There penetrates only a small portion of fields into the air region, and hence the dispersion diagram of this mode is close to the $\beta = \sqrt{\epsilon_r} k_0$ line. For a microstrip line with PEC side walls, the electric field pattern is known to be significantly different as the tangential field has to vanish near the side walls, and it results in more fields into the air region. Effective dielectric constant of the microstrip line shielded with PEC side walls is hence lower than that of the substrate-mode of a microstrip line with periodic side walls ($\Phi_x = 0$). Based on an identical argument, since the field pattern of an air-mode is similar to that of a parallel plate waveguide, there is less penetration of fields into the substrate region, and resultantly the propagation constant of this mode is close to the $\beta = k_0$ line.

Figure 3.18: Effect of the vertical period on the propagation constants of the first two modes of a microstrip line with periodic side walls ($t = 2 \text{ mm}$, $\epsilon_r = 3.27$, $d = 5.08 \text{ mm}$, $b = 10 \text{ mm}$, $f = 5 \text{ GHz}$, $\Phi_x = 0$).

Figure 3.17 shows the effect of the horizontal period on the propagation constants of the two modes when $\Phi_x = 0$, or alternatively when the microstrip line is shielded with PMC side walls. For $t/b = 1$, the configuration of Figure 3.13 reduces to a stack of two parallel-plate waveguides supporting two TEM modes. The top parallel-plate waveguide is air-filled, while the lower is completely filled with dielectric, and the reduced structure supports $\beta = k_0$ and $\beta = \sqrt{\epsilon_r} k_0$ modes as expected. When the gap between the side wall and the strip is increased, the propagation constants of the first two modes come close to each other and tend to merge together. When t/b approaches zero, the microstrip line in fact becomes a partially dielectric-filled parallel-plate waveguide, which is known to support single mode propagation.

Figure 3.18 illustrates the effect of vertical period h on the propagation constant of the first two modes of the shielded microstrip line with periodic side walls. As it has been previously noted, the substrate-mode is mainly concentrated within the substrate region, and hence it is negligibly affected by the height of the shielding enclosure or the unit cell. However, the air-mode is significantly affected by the variation of this parameter.

3.6 Summary

Propagation characteristics of a 2-D periodic array of vertical microstrip lines have been investigated in sufficient details. By invoking Floquet theorem, a unit-cell of the array has been analyzed using an efficient mode-matching method. Results of this method are in excellent agreement with those obtained from CST Microwave Studio. A number of studies have been presented to build an understanding, and to point out the physical significances of various parameters which affect the propagation characteristics of this array.

It has been noted that the proposed array supports two quasi-TEM modes when the periodic boundary phase shift is small. These quasi-TEM modes are actually the

prime motivation behind the research presented in this thesis. Based on their importance in the design of proposed frequency selective structure and microwave absorber, propagation characteristics of these two modes have been investigated in sufficient detail and the effect of various geometrical parameters on these modes has been understood through a number of parametric studies.

A unit-cell of the proposed array consists of a microstrip line whose side walls are characterized by a periodic boundary condition. This unit-cell is compared with a microstrip line shielded with either PEC or PMC side walls. It is seen that the under a periodic phase shift $\Phi_x = 0$, the first two modes of this array are identical to that of a microstrip line shielded with PMC side walls. With an increase of the periodic phase shift, one of these modes becomes evanescent and resultantly this structure supports single quasi-TEM mode when $\Phi_x = \pi$. Interestingly, the dispersion diagram of this single mode is identical to that a microstrip line shielded with PEC side walls. Further, a number of comparison were made between the propagation characteristics of the first two quasi-TEM modes of the array and the single quasi-TEM mode of a microstrip line shielded with PEC side walls. These comparisons complement the previous parametric studies to build better understanding of the proposed array.

CHAPTER 4

SCATTERING BY A 2-D PERIODIC ARRAY OF MICROSTRIP LINES

4.1 Introduction

Scattering by a 2-D periodic array of vertical microstrip lines refers to the actual frequency response of the structure when it is subjected to plane wave incidence arriving from certain angle. A detailed investigation of scattering properties of this array is important because it is directly applicable for the design of frequency selective structures proposed in this thesis. The mathematical formulation of scattering problem actually completes the analysis of the proposed FSS. However, the primary objective of this work is to go beyond this full-wave analysis, and develop a useful understanding of the scattering problem, which later leads to the intuitive design guidelines of a practical FSS based on this array. Previously, the propagation characteristics of the array have also been discussed with the same objective. However, knowing only the propagation characteristics is not fully sufficient to design an FSS, since these modes may or may not be excited under certain incidence of plane wave. The excitation of desired modes under given polarizations and angles of incidence is effectively covered by the scattering problem discussed in this chapter.

Scattering by periodic surfaces has been widely investigated for their applications in frequency selective surfaces, leaky-wave antennas, shielding effectiveness, etc. Chen's work [79, 80] forms one of the most useful classical references on this subject, where scattering by a two-dimensional periodic array of conducting plates or perforated conducting screen was considered. Modes in the air region were formulated as a combination of Floquet TE and TM modes [81]. Based on a similar method, a large number of more complicated geometries were investigated later [1, 24, 82, 83], and most of this work has been compiled in [1]. It

uses method of moments whereby currents on a periodic surface are assumed to be a sum of orthogonal functions, and continuity of tangential fields at its interface with air is enforced. Scattering from conducting strips and gratings has been especially of interest [66, 84-88] for leaky-wave antenna and other application. A representation of these geometries is shown in Figure 4.1 where a plane wave illuminates one-dimensional or two-dimensional array of horizontally placed patches/strips.

Figure 4.1: Traditional scattering problems (a) a 2-D array of dipoles printed on a dielectric layer, (b) a 2-D array of printed patches with a dielectric sheet, (c) a strip grating.

Scattering by a simple 2-D array of rectangular waveguides has been of interest for a few decades. It was initially investigated as a thick FSS [89], later as a technique to reduce radar cross-section of a cavity [13], and recently its application as a shielding structure has also been studied [14]. This later study [14] involves the mode-matching method, and the major steps are similar to the mode-matching based formulation of the scattering problem discussed in this chapter. The air region is modeled in terms of Floquet modes, while the modes in rectangular waveguide are written as a combination of TE and TM modes. Continuity of tangential field components is enforced at both the front and back interfaces with air, and resultantly scattering from an array of finite width is obtained.

Unlike [14], the mode-matching formulation presented in this chapter calculates the scattering by single interface between air and the array, which is equivalent to considering the scattering from an array of infinite extent. The problem of an array with finite thickness L is then treated as a cascaded junction of two air-to-array discontinuities, whose S-parameters can be straightforwardly calculated from the simple relations given in [71]. This approach results in reduced-size matrices, and the overall approach is more efficient than the case of solving simultaneously the both front and back discontinuities of an array with finite-thickness L . Further, it is possible to retrieve more understanding and physical insight of the problem by studying the S-parameters of both the single air-to-array discontinuity, and the cascaded junction of two air-to-array discontinuities.

A full-wave mode-matching method based formulation of scattering by a 2-D periodic array of vertical microstrip lines is presented in the following paragraphs. A number of parametric studies are presented and discussed to evolve a better understanding of this problem. The proposed array is an asymmetrical structure, and its response is anticipated to be polarization-dependent. Since the primary motivation of the proposed array lies in its support for two quasi-TEM modes, this chapter identifies those polarizations and angles of incidence, which favor the excitation of quasi-TEM modes of the microstrip lines. Further, the effect of angle of incidence is also investigated with an objective to obtain an angular stability of frequency response. The investigations and conclusions made in this chapter are

useful in forming the intuitive design guidelines of the proposed array, when it is used as an FSS or a microwave absorber, to be discussed in Chapter 6.

Figure 4.2: An array of shielded microstrip lines subjected to an oblique incidence.

4.1.1 Analysis Model

A 2-D periodic array of vertical microstrip lines under plane wave incidence is shown in Figure 4.2. Based on Floquet theorem [2], the problem of air-to-array discontinuity is reduced to that of an air-to-microstrip line discontinuity, where the microstrip line is shielded with periodic boundary condition as shown in Figure 4.3. The periodic boundary phase shift is actually related with the angle of incidence. It should be noted that this approach assumes an infinite array of vertical microstrip lines.

4.1.2 Geometry of the Problem

Figure 4.3 shows geometrical details of the reduced problem, which basically consists of an air-to-microstrip line discontinuity. Cross-sectional view of the microstrip line section is also shown, where horizontal and vertical periods have

been denoted by b and h , respectively. Floquet modes in air are then written in terms of these periods. The periodic boundaries of microstrip line region have been characterized in terms of a periodic phase shift Φ_x , which can be related to the incident angles θ and φ as follows:

$$\Phi_x = -k_0 b \cos(\theta) \sin(\varphi), \quad (4.1)$$

where k_0 denotes the propagation constant of incident plane wave in the air region. Definitions of angles θ and φ are given in Figure 4.2. Figure 4.4 shows a unit-cell of the proposed FSS of thickness L , which is under illumination of plane wave incident from air. Regions 1 and 2 denote the air region and the microstrip line region, respectively.

Figure 4.3: (a) Side view of the air-to-microstrip line discontinuity, (b) Cross-section of Region 2.

4.1.3 Analysis Method

Figure 4.3 illustrates the classification of two distinct regions used in the full-wave mode-matching method to be presented in the following section. Region 1 represents air, while Region 2 denotes the microstrip line section. Fields and modes in Region 2 are already known from Chapter 3, where the mode-matching method has been

used to study the propagation characteristics of this region. Region 1 supports TE and TM Floquet modes [81], whose field expressions are available in [14, 79, 80].

Figure 4.4: Side view of the unit-cell of a finite-thickness array illustrating a cascaded junction of two air-to-microstrip line discontinuities.

Once the expressions for electric and magnetic fields in the two regions are known, their tangential components are forced to be continuous at the interface between Regions 1 and 2. Using suitable orthogonality relations leads to a set of independent equations, which are written in matrices form. S-parameters of the interface are then obtained from these matrices using simple algebraic manipulations [90]. Since an array of finite thickness L can be modeled as a cascaded junction of air-to-microstrip line discontinuities as shown in Figure 4.4, its S-parameters are also straightforwardly calculated by combining three generalized scattering matrices [71]. The detailed steps of this method are elaborated as follows.

4.2 Formulation

Based on the method summarized above, the following field expressions are assumed for Region 1 and 2 classified in Figure 4.3.

4.2.1 Modes in Region 1

Region 1 supports the following transverse fields [14, 79, 80].

$$\begin{aligned}
\vec{E}_t^{(1)} &= \sum_{p=-P_1}^{P_1} \sum_{q=-Q_1}^{Q_1} \sum_{r=1}^2 \left[A_{pqr}^+ e^{j\beta_{pqr}^{(1)} z} + A_{pqr}^- e^{-j\beta_{pqr}^{(1)} z} \right] \vec{e}_{pqr}^{(1)}, \\
\vec{H}_t^{(1)} &= \sum_{p=-P_1}^{P_1} \sum_{q=-Q_1}^{Q_1} \sum_{r=1}^2 \left[A_{pqr}^+ e^{j\beta_{pqr}^{(1)} z} - A_{pqr}^- e^{-j\beta_{pqr}^{(1)} z} \right] Y_{pqr}^{(1)} \left(-\hat{a}_z \times \vec{e}_{pqr}^{(1)} \right),
\end{aligned} \tag{4.2}$$

where

$$\begin{aligned}
\vec{e}_{pqr}^{(1)} &= \frac{1}{\sqrt{U_{pq}^{(1)}}} \begin{cases} \left(k_{yq}^{(1)} \hat{a}_x - k_{xp}^{(1)} \hat{a}_y \right) e^{-jk_{xp}^{(1)} x} e^{-jk_{yq}^{(1)} y} & r=1 \\ \left(k_{xp}^{(1)} \hat{a}_x + k_{yq}^{(1)} \hat{a}_y \right) e^{-jk_{xp}^{(1)} x} e^{-jk_{yq}^{(1)} y} & r=2 \end{cases}, \\
Y_{pqr}^{(1)} &= \begin{cases} \frac{\beta_{pqr}^{(1)}}{\omega\mu} & r=1 \\ \frac{\omega\epsilon_0}{\beta_{pqr}^{(1)}} & r=2 \end{cases}, \quad U_{pq}^{(1)} = \begin{cases} \frac{\beta_{pqr}^{(1)} \left(k_{xp}^{(1)2} + k_{yq}^{(1)2} \right) bh}{\omega\mu} & r=1 \\ \frac{\omega\epsilon_0 \left(k_{xp}^{(1)2} + k_{yq}^{(1)2} \right) bh}{\beta_{pqr}^{(1)}} & r=2. \end{cases}, \\
k_{xp}^{(1)} &= -k_0 \sin \theta \cos \phi + \frac{2\pi p}{b}, \quad k_{yq}^{(1)} = -k_0 \sin \theta \sin \phi + \frac{2\pi q}{h}, \\
\beta_{pqr}^{(1)} &= -j \sqrt{k_{xp}^{(1)2} + k_{yq}^{(1)2} - k_0^2}.
\end{aligned}$$

Indices p and q refer to the horizontal and vertical variations of the fields in Region 1. Two values of index r represent TE and TM modes. A^+ and A^- are the modal coefficients of forward and backward traveling waves. $\beta^{(1)}$ denotes the propagation constant of a mode. $Y^{(1)}$ is the modal admittance, whose expressions are different for TE and TE modes. $U^{(1)}$ is a normalization factor. b and h are the horizontal and vertical periods of the array, respectively. Definition of the incident of angles θ and ϕ has been indicated in Figure 4.2.

4.2.2 Modes in Region 2

Region 2 supports hybrid modes which are combination of LSE and LSM modes in the three sub-regions I, II and III. Propagation characteristics of this region have

already been investigated in Chapter 3. Transverse field components in Region 2 can be represented as

$$\begin{aligned}\vec{E}_t^{(2)} &= \sum_{n=1}^N \left[B_n^+ e^{-j\beta_n^{(2)}z} + B_n^- e^{j\beta_n^{(2)}z} \right] \vec{e}_n^{(2)}, \\ \vec{H}_t^{(2)} &= \sum_{n=1}^N \left[B_n^+ e^{-j\beta_n^{(2)}z} - B_n^- e^{j\beta_n^{(2)}z} \right] \vec{h}_n^{(2)},\end{aligned}\tag{4.3}$$

where

$$\begin{aligned}\vec{e}_n^{(2)} &= \frac{1}{\sqrt{U_n^{(2)}}} \begin{bmatrix} \sum_{m=1}^{M_I} \left\{ (e_{mnx}^{Ie} + e_{mnx}^{Ih}) \hat{a}_x + e_{mny}^{Ie} \hat{a}_y \right\}, & R^I \\ \sum_{m=0}^{M_{II}} \left\{ (e_{mnx}^{IIe} + e_{mnx}^{IIh}) \hat{a}_x + e_{mny}^{IIe} \hat{a}_y \right\}, & R^{II} \\ \sum_{m=0}^{M_{III}} \left\{ (e_{mnx}^{IIIe} + e_{mnx}^{IIIh}) \hat{a}_x + e_{mny}^{IIIe} \hat{a}_y \right\}, & R^{III} \end{bmatrix}, \\ \vec{h}_n^{(2)} &= \frac{1}{\sqrt{U_n^{(2)}}} \begin{bmatrix} \sum_{m=1}^{M_I} \left\{ (h_{mnx}^{Ie} + h_{mnx}^{Ih}) \hat{a}_x + h_{mny}^{Ie} \hat{a}_y \right\}, & R^I \\ \sum_{m=0}^{M_{II}} \left\{ (h_{mnx}^{IIe} + h_{mnx}^{IIh}) \hat{a}_x + h_{mny}^{IIe} \hat{a}_y \right\}, & R^{II} \\ \sum_{m=0}^{M_{III}} \left\{ (h_{mnx}^{IIIe} + h_{mnx}^{IIIh}) \hat{a}_x + h_{mny}^{IIIe} \hat{a}_y \right\}, & R^{III} \end{bmatrix},\end{aligned}$$

$$\{(0 \leq x \leq x_0) \wedge (0 \leq y \leq h)\} \in R^I,$$

$$\{(x_0 \leq x \leq b) \wedge (d \leq y \leq h)\} \in R^{II},$$

$$\{(x_0 \leq x \leq b) \wedge (0 \leq y \leq d)\} \in R^{III},$$

and

$$\begin{aligned}U_n^{(2)} &= \int_0^b \int_0^h \hat{a}_z \cdot \left(\vec{e}_n^{(2)*} \times \vec{h}_n^{(2)} \right) dx dy \\ &= \begin{bmatrix} \sum_{m=1}^{M_I} \sum_{m'=1}^{M_I} \int_0^{x_0} \int_0^h \left(e_{mnx}^{Ie*} h_{m'ny}^{Ih} + e_{mnx}^{Ih*} h_{m'ny}^{Ie} - e_{mny}^{Ie*} h_{m'nx}^{Ie} - e_{mny}^{Ih*} h_{m'nx}^{Ih} \right) dx dy + \\ \sum_{m=0}^{M_{II}} \sum_{m'=0}^{M_{II}} \int_{x_0}^b \int_d^h \left(e_{mnx}^{IIe*} h_{m'ny}^{IIh} + e_{mnx}^{IIh*} h_{m'ny}^{IIe} - e_{mny}^{IIe*} h_{m'nx}^{IIe} - e_{mny}^{IIh*} h_{m'nx}^{IIh} \right) dx dy + \\ \sum_{m=0}^{M_{III}} \sum_{m'=0}^{M_{III}} \int_{x_0}^b \int_0^d \left(e_{mnx}^{IIIe*} h_{m'ny}^{IIIh} + e_{mnx}^{IIIh*} h_{m'ny}^{IIIe} - e_{mny}^{IIIe*} h_{m'nx}^{IIIe} - e_{mny}^{IIIh*} h_{m'nx}^{IIIh} \right) dx dy \end{bmatrix}.\end{aligned}$$

B^+ and B^- represent the modal coefficients of forward and backward traveling waves. $\beta^{(2)}$ is the propagation constant of a mode, which has already been calculated in Chapter 3. R^I , R^{II} , and R^{III} denote the three distinct regions of the microstrip line

cross-section as shown in Figure 4.3(b). $U^{(2)}$ is a normalization factor. e and h refer to the electric and magnetic fields, which have already been calculated in Chapter 3.

4.2.3 Air-to-Array Discontinuity

Since the tangential components of electric field are known to be continuous at the interface between Region 1 and 2, it gives

$$E_t^{(1)} \Big|_{z=0} = E_t^{(2)} \Big|_{z=0} \Rightarrow \sum_{s=1}^S [A_s^+ + A_s^-] \vec{e}_s^{(1)} = \sum_{n=1}^N [B_n^+ + B_n^-] \vec{e}_n^{(2)} \quad (4.4)$$

where index ‘ s ’ is a unified mapping of index ‘ pqr ’ for Region 1, and S represents the total number of modes considered in Region 1. It is easy to see that these Floquet modes obey the following orthogonality relation:

$$\int_0^b \int_0^h (\vec{e}_s^{(1)} \cdot \vec{e}_u^{(1)*}) dx dy = 0, \quad s \neq u. \quad (4.5)$$

Taking a dot product on both sides of (4.4) with $\vec{e}_u^{(1)*}$, and using (4.5) leads to a set of equations given by

$$[A_u^+ + A_u^-] = \sum_{n=1}^N [B_n^+ + B_n^-] G_{un}, \quad (4.6)$$

where

$$G_{un} = G(u, n) = \int_0^b \int_0^h (\vec{e}_n^{(2)} \cdot \vec{e}_u^{(1)*}) dx dy$$

$$= \begin{bmatrix} \sum_{m=1}^{M_I} \left\{ \int_0^h \int_0^{x_0} (e_{mnx}^{Ie} e_{ux}^{(1)*} + e_{mnx}^{Ih} e_{ux}^{(1)*} + e_{mny}^{Ie} e_{uy}^{(1)*}) dx dy \right\} + \\ \sum_{m=0}^{M_{II}} \left\{ \int_0^h \int_{x_0}^b (e_{mnx}^{IIe} e_{ux}^{(1)*} + e_{mnx}^{IIh} e_{ux}^{(1)*} + e_{mny}^{IIe} e_{uy}^{(1)*}) dx dy \right\} + \\ \sum_{m=0}^{M_{III}} \left\{ \int_0^d \int_{x_0}^b (e_{mnx}^{IIIe} e_{ux}^{(1)*} + e_{mnx}^{IIIh} e_{ux}^{(1)*} + e_{mny}^{IIIe} e_{uy}^{(1)*}) dx dy \right\} \end{bmatrix} \quad \begin{cases} u = 1, 2, \dots, S \\ n = 1, 2, \dots, N \end{cases}$$

The above equations can also be written in a matrix form as

$$\left[\mathbf{A}^+ + \mathbf{A}^- \right] = \mathbf{G} \left[\mathbf{B}^+ + \mathbf{B}^- \right], \quad (4.7)$$

where $\mathbf{A}$ and $\mathbf{B}$ are column vectors consisting of S and N elements respectively. $\mathbf{G}$ is an $S \times N$ matrix.

Enforcing the continuity of tangential magnetic field at the interface of Region 1 and 2 leads to

$$H_t^{(1)} \Big|_{z=0} = H_t^{(2)} \Big|_{z=0} \Rightarrow \sum_{s=1}^S \left[A_s^+ - A_s^- \right] Y_s^{(1)} \left(-\hat{a}_z \times \vec{e}_s^{(1)} \right) = \sum_{n=1}^N \left[B_n^+ - B_n^- \right] \vec{h}_n^{(2)}. \quad (4.8)$$

The following orthogonality relation [2] is useful in this case.

$$\int_0^b \int_0^h \hat{a}_z \cdot \left(\vec{e}_u^{(2)*} \times \vec{h}_n^{(2)} \right) dx dy = 0, \quad u \neq n. \quad (4.9)$$

Taking a cross product on both sides of (4.8) with $\vec{e}_u^{(2)*}$, and integrating over the interface area ($0 \leq x \leq b, 0 \leq y \leq h$) results in

$$\sum_{s=1}^S \left[A_s^+ - A_s^- \right] Y_s^{(1)} \bar{G}_{us} = \left[B_u^+ - B_u^- \right], \quad (4.10)$$

where (4.9) has been used to retain the single term at the right hand side of (4.10).

Also,

$$\begin{aligned} \bar{G}_{us} &= \int_0^b \int_0^h \hat{a}_z \cdot \left[\vec{e}_u^{(2)*} \times \left(-\hat{a}_z \times \vec{e}_s^{(1)} \right) \right] dx dy \\ &= - \left[\begin{aligned} &\sum_{m=1}^{M_I} \left\{ \int_0^h \int_0^{x_0} \left(e_{mux}^{Ie*} e_{sx}^{(1)} + e_{mux}^{Ih*} e_{sx}^{(1)} + e_{muy}^{Ie*} e_{sy}^{(1)} \right) dx dy \right\} + \\ &\sum_{m=0}^{M_{II}} \left\{ \int_d^h \int_{x_0}^b \left(e_{mux}^{IIe*} e_{sx}^{(1)} + e_{mux}^{IIh*} e_{sx}^{(1)} + e_{muy}^{IIe*} e_{sy}^{(1)} \right) dx dy \right\} + \\ &\sum_{m=0}^{M_{III}} \left\{ \int_0^d \int_{x_0}^b \left(e_{mux}^{IIIe*} e_{sx}^{(1)} + e_{mux}^{IIIh*} e_{sx}^{(1)} + e_{muy}^{IIIe*} e_{sy}^{(1)} \right) dx dy \right\} \end{aligned} \right] \begin{cases} u = 1, 2, \dots, N \\ s = 1, 2, \dots, S \end{cases}. \end{aligned}$$

Similar to (4.7), the above result can also be expressed in a matrix form as follows,

$$\begin{bmatrix} \mathbf{A}^+ - \mathbf{A}^- \end{bmatrix} \mathbf{Y}^{(1)} \bar{\mathbf{G}} = \begin{bmatrix} \mathbf{B}^+ - \mathbf{B}^- \end{bmatrix}. \quad (4.11)$$

Since $\mathbf{A}$ and $\mathbf{B}$ are column vectors of length S and N , respectively, (4.7) and (4.11) can be re-written as,

$$\begin{bmatrix} \mathbf{I}^{(1)} & -\mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{A}^- \\ \mathbf{B}^- \end{bmatrix} = \begin{bmatrix} -\mathbf{I}^{(1)} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{A}^+ \\ \mathbf{B}^+ \end{bmatrix}, \quad (4.12)$$

$$\begin{bmatrix} -\mathbf{Y}^{(1)} \bar{\mathbf{G}} & \mathbf{I}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{A}^- \\ \mathbf{B}^- \end{bmatrix} = \begin{bmatrix} -\mathbf{Y}^{(1)} \bar{\mathbf{G}} & \mathbf{I}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{A}^+ \\ \mathbf{B}^+ \end{bmatrix}, \quad (4.13)$$

where $\mathbf{I}^{(1)}$ and $\mathbf{I}^{(2)}$ are identity matrices of size $S \times S$ and $N \times N$, respectively. Combining (4.12) and (4.13) gives

$$\begin{bmatrix} \mathbf{I}^{(1)} & -\mathbf{G} \\ -\mathbf{Y}^{(1)} \bar{\mathbf{G}} & \mathbf{I}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{A}^- \\ \mathbf{B}^- \end{bmatrix} = \begin{bmatrix} -\mathbf{I}^{(1)} & \mathbf{G} \\ -\mathbf{Y}^{(1)} \bar{\mathbf{G}} & \mathbf{I}^{(2)} \end{bmatrix} \begin{bmatrix} \mathbf{A}^+ \\ \mathbf{B}^+ \end{bmatrix}. \quad (4.14)$$

The generalized scattering matrix of the air-to-array discontinuity is calculated from (4.14) as:

$$\begin{bmatrix} \mathbf{S}_{11}^s & \mathbf{S}_{12}^s \\ \mathbf{S}_{21}^s & \mathbf{S}_{22}^s \end{bmatrix} = \begin{bmatrix} \mathbf{I}^{(1)} & -\mathbf{G} \\ -\mathbf{Y}^{(1)} \bar{\mathbf{G}} & \mathbf{I}^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} -\mathbf{I}^{(1)} & \mathbf{G} \\ -\mathbf{Y}^{(1)} \bar{\mathbf{G}} & \mathbf{I}^{(2)} \end{bmatrix}, \quad (4.15)$$

where the superscript s refers to a single air-to-array discontinuity. It is to differentiate these S-parameters from those of a cascaded junction of air-to-microstrip line to be discussed in the following paragraphs.

4.2.4 Cascaded Junction of Air-to-Array Discontinuities

An array of finite thickness can be modeled as a cascaded junction of two air-to-array discontinuities as indicated in Figure 4.4. Its S-parameters can be calculated as

[71],

$$\begin{aligned}\mathbf{S}_{11} &= \mathbf{S}_{11}^s + \mathbf{S}_{12}^s \mathbf{D} \left[\mathbf{I}^{(2)} - \mathbf{S}_{22}^s \mathbf{D} \mathbf{S}_{22}^s \mathbf{D} \right]^{-1} \mathbf{S}_{22}^s \mathbf{D} \mathbf{S}_{21}^s \\ \mathbf{S}_{21} &= \mathbf{S}_{12}^s \mathbf{D} \left[\mathbf{I}^{(2)} - \mathbf{S}_{22}^s \mathbf{D} \mathbf{S}_{22}^s \mathbf{D} \right]^{-1} \mathbf{S}_{21}^s\end{aligned}\quad (4.16)$$

where $\mathbf{D}$ is an $N \times N$ diagonal matrix based on propagation constants of N modes in Region 2, as calculated in the Chapter 3 of this thesis.

$$\mathbf{D} = \begin{bmatrix} e^{-j\beta_1^{(2)}L} & 0 & 0 & 0 \\ 0 & e^{-j\beta_2^{(2)}L} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & e^{-j\beta_N^{(2)}L} \end{bmatrix}. \quad (4.17)$$

$\beta_n^{(2)}$ refers to the propagation constant of an n -th mode in Region 2. L denotes thickness of the array, which is basically the separation between the two air-to-array discontinuities, shown in Figure 4.4.

4.3 Numerical Results

For a given angle of incidence θ and φ , the periodic boundary shift induced in the 2-D array of microstrip lines can be calculated as,

$$\Phi_x = -(k_0 \sin \theta \cos \varphi) b. \quad (4.18)$$

Based on this Φ_x , modes in Region 2 are calculated from the procedure discussed in Chapter 3. Modal field coefficients are obtained from the null space of matrix $\mathbf{Y}$ in (3.32), using the singular value decomposition method [91]. Knowing the propagation constants and fields in Region 2, the above formulation is used to obtain scattering parameters of the air-to-array discontinuity.

The number of modes considered in Region 1 is a function of the frequency, angle of incidence, and the geometrical parameters of the array. Similar to the scheme discussed in Chapter 3, a cut-off wave number k_c is defined, which is much higher than the operating wave number k_o . The number of propagating Floquet modes in the air region corresponding to k_c are calculated for a known angle of incidence, and geometrical dimensions of the array. These numbers for P and Q are then related to the modes considered in Region 1 for an operating point k_o .

Table 4.1: Convergence behavior of S-parameters with number of modes in Region 1 ($t = 1 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $f = 10 \text{ GHz}$, $\Phi_x = 0$, $N=10$).

	Cut-off Wave Number (rad/m)		
	$k_c = 5 k_0$ ($P = Q = 1$)	$k_c = 10 k_0$ ($P = Q = 2$)	$k_c = 20 k_0$ ($P = Q = 4$)
S_{11}	$0.44 \angle 153^\circ$	$0.42 \angle 152^\circ$	$0.42 \angle 152^\circ$
S_{21}	$0.90 \angle -117^\circ$	$0.91 \angle -118^\circ$	$0.91 \angle -118^\circ$

Table 4.2: Convergence behavior of S-parameters with number of modes in Region 2 ($t = 1 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $f = 10 \text{ GHz}$, $\Phi_x = 0$, $P = Q=2$).

	Number of Modes in Region 2		
	$N = 2$	$N = 4$	$N = 8$
S_{11}	$0.42 \angle 152^\circ$	$0.42 \angle 152^\circ$	$0.42 \angle 152^\circ$
S_{21}	$0.91 \angle -118^\circ$	$0.91 \angle -118^\circ$	$0.91 \angle -118^\circ$

The number of propagating modes in Region 2 cannot be found analytically for a hypothetical test point k_c . For this reason, the number of modes in Region 2 is fixed to a small finite value N . It is seen that the mode-matching approach actually gives rapidly converging results and hence a few modes are sufficient for Region 2. Table 4.1 shows the convergence behavior of S-parameters when the cut-off wave number k_c is varied to change the number of modes in Region 1. It is noted that $k_c = 10k_0$ leads to sufficiently convergent results. Table 4.2 shows convergence behavior of S-

parameters when the number of modes in Region 2 is varied. Figure 4.5 compares results of the above formulation with those obtained from CST MWS, whereby a very good agreement is noted.

Figure 4.5: S- parameters of a cascaded junction of two air-to-array discontinuities under TE incidence ($t = 0.2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 3.124 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3$, $L = 9.5 \text{ mm}$, $\phi = 0$, $\theta = 40^\circ$).

4.3.1 Excitation of Quasi-TEM modes

As highlighted previously, the most salient feature of the proposed array is its support for two quasi-TEM modes, which in turn leads to wide bandwidth, elliptic filtering response, and stability of performance under large variation of the angle of incidence. Since it is an asymmetric structure, the excitation of these quasi-TEM modes is a function of the polarization and the angle of incidence. Further, unlike many common scattering problems [14] where modes are an intrinsic property of the array and the angle of incidence affects only the excitation of those modes, the propagation characteristics of the proposed array are significantly affected by the incidence. As given by (4.18) and discussed in Chapter 3, the incident plane wave

induces a phase shift on the unit-cells of this array, which in turn determines the propagation characteristics. Thus, the polarization and the arriving angle of the incident plane wave are important for both the excitation and the propagation characteristics of the array.

(a) TM incidence

(b) TE incidence

Figure 4.6: Reflection coefficient of air-to-microstrip line discontinuity, and transmission coefficients of the first two propagating modes under TE and TM incidence ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $\varphi = 0$, $\theta = 0$).

Figure 4.7: Effect of angle θ on the reflection coefficient of the air-to-array discontinuity, and transmission coefficients of the first two modes of the array, for TE incidence ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $f = 10 \text{ GHz}$, $\phi = 0$).

Figure 4.8: Effect of angle ϕ on the reflection coefficient of the air-to-array discontinuity, and transmission coefficients of the first two modes of the array, for TE incidence ($t = 2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $f = 10 \text{ GHz}$, $\theta = 0$).

(a) TE Incidence

(a) TM Incidence

Figure 4.9: S-parameters of a cascaded junction of air-to-array discontinuities ($t = 0.2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 2.5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $L = 9.5 \text{ mm}$, $\varphi = 0$, $\theta = 20^\circ$).

(a) TE Incidence

(a) TM Incidence

Figure 4.10: S-paramters of a cascaded junction of air-to-array discontinuities ($t = 0.2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 2.5 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $L = 9.5 \text{ mm}$, $\varphi = 90^\circ$, $\theta = 20^\circ$).

Figure 4.6 shows the reflection coefficient of a single air-to-array discontinuity for TE and TM incident waves. It also presents the transmission coefficients of the first two modes of the array, where $S_{21}(1)$ and $S_{21}(2)$ refer to the substrate-mode and the air-mode, respectively. It may be noted that most of the incident energy carried by TE wave is coupled to the air-mode (second mode) of the array. On the other hand, the first two modes of the array are not excited by the TM incidence. Therefore, it results in strong reflection, as indicated by Figure 4.6 (a). A case of normal incidence has been considered for these two results.

Under off-normal incidence, the variation of S-parameters is shown in Figure 4.7. Here, TE incidence is considered, and θ is varied while $\varphi = 0$. Basically, electric field has only one component E_y , which remains perpendicular to the strips when the angle of incidence is changed. A reasonably stable scattering is observed with even a significantly larger variation of the angle of incidence. On the other hand, the proposed array is relatively more sensitive to the incident polarization as indicated by Figure 4.8. The same case of TE wave is considered, and φ is varied while fixing $\theta = 0$. This actually refers to normal incidence when polarization is rotated from being perpendicular to the strips ($\varphi = 0$) to becoming parallel with them ($\varphi = 90^\circ$). The variation of S-parameters is sharper than the trend in Figure 4.7, when the polarization was fixed and instead the angle of incidence was varied. Based on these results, one may note that the polarization of incident wave is more important consideration than the angle of incidence, when stable performance of this array is desired.

Figure 4.9 shows S-parameters of a cascaded junction of air-to-array discontinuities, which is subjected to an incidence of TE and TM waves ($\varphi = 0$). In the case of TE incidence, the two modes of microstrip line are excited, which leads to two reflection zeros and one transmission zero of filtering response. Basically, the two modes may be treated as two parallel paths with different phase shifts, and a transmission zero is observed when the signal traveling through the finite thickness L is combined out of phase. As expected, strong reflection is observed for the case of TM incidence since no component of electric field is perpendicular to the microstrip line, and the dominant quasi-TEM modes of the array are not excited

under this incidence. Table 4.3 lists the field components supported by TE and TM waves, based on the co-ordinates and notations defined in Figure 4.2. Since any arbitrary plane wave can be represented in terms of TE and TM components, it may be sufficient to consider the response of the proposed array under the possibilities listed in Table 4.3.

Figure 4.10 shows the S-parameters of a cascaded junction when $\varphi = 90^\circ$. Since no component of the incident electric field is perpendicular to the strips, strong reflection is observed for the case of TE incidence. However, in case of TM incidence, the y -component of electric field E_y is present, which excites the quasi-TEM modes of this array, and resultantly an elliptical filtering response is observed. Using Table 4.3, it is easy to explain and predict the excitation of quasi-TEM modes under an arbitrary polarization and angle of incidence.

Table 4.3: Field components supported by TE and TM waves

	$ \theta $	$ \varphi $	E_x	E_y	E_z	H_x	H_y	H_z
TE	≥ 0	0		✓		✓		✓
		90°	✓				✓	✓
TM	≥ 0	0	✓		✓		✓	
		90°		✓	✓	✓		

4.3.2 Parametric Study

Since this array supports two quasi-TEM modes, a cascaded junction of two air-to-array discontinuities basically forms two resonators when the unit cell size is sub-wavelength. Based on that, it is possible to obtain a transmission zero at a finite frequency as seen in the results of a cascaded junction of air-to-array discontinuities discussed above. Corresponding to the two modes, two reflection zeros are also noted whose position is a function of the propagation constants of these modes.

Figure 4.11: Effect of dielectric constant ϵ_r on the filtering response of a cascaded junction of air-to-array discontinuities ($t = 0.2 \text{ mm}$, $b = 5 \text{ mm}$, $h = 2.5 \text{ mm}$, $d = 1.524 \text{ mm}$, $L = 9.5 \text{ mm}$, $\varphi = 0$, $\theta = 0$).

Figure 4.12: Effect of vertical period h on the filtering response of a cascaded junction of air-to-array discontinuities ($t = 0.2 \text{ mm}$, $b = 5 \text{ mm}$, $\epsilon_r = 3.38$, $d = 1.524 \text{ mm}$, $L = 9.5 \text{ mm}$, $\varphi = 0$, $\theta = 0$).

Figure 4.13: Effect of strip width t on the filtering response of a cascaded junction of air-to-array discontinuities ($b = 5 \text{ mm}$, $h = 2 \text{ mm}$, $\epsilon_r = 3.38$, $d = 1.524 \text{ mm}$, $L = 9.5 \text{ mm}$, $\varphi = 0$, $\theta = 0$).

Figure 4.11 considers the effect of dielectric constant ϵ_r on the positions of these transmission/reflection zeros. Since the air-mode is less affected by a change in the substrate material, the second reflection zero is seen to be relatively stable, while the first reflection zero undergoes a large shift of frequency. On the other hand, since this substrate-mode mainly propagates in the substrate region, the first reflection zero is less affected by a change in the vertical period of the array illustrated in Figure 4.12. The effect of strip width is depicted in Figure 4.13 whereby both the reflection zeros are affected. This is also understandable from the discussion in Chapter 3. Basically, a strip isolates the air and the substrate regions of microstrip line, and propagation constants of both the modes are affected with this parameter.

4.4 Summary

A full-wave mode-matching method has been formulated to study the scattering by a 2-D periodic array of vertical microstrip lines. An already well-known Floquet modes based approach has been used for the air region. Modes in the array have been calculated earlier in Chapter 3. Based on those modes and fields, the detailed steps of the formulation have been explained in this chapter. Results of this method have also been compared with those obtained from CST MWS, and an excellent agreement has been noted.

The proposed array is an asymmetrical structure whose scattering is a function of both the polarization and the angle of the incident plane wave. This effect has been discussed in sufficient details through a number of simulation experiments. It has been illustrated that the desired quasi-TEM modes of the array are excited when any component of the incident electric field is perpendicular to the strips. In principle, the performance of this array can be made very stable under large variation of the angle of incidence by choosing a small size of its unit-cell. However, the sensitivity of this array to polarization of incidence remains a critical point for its practical applications.

The effect of various geometrical parameters on the filtering response of this array has also been studied. It has been noted that the positions of transmission/reflections zeros exhibited by a finite-width array can be straightforwardly explained from its propagation characteristics established earlier in Chapter 3. The air-to-array discontinuity itself is a very good transition especially when the incident electric is perpendicular to the strips. In this context, the array may be easily matched with air, and the resulting filtering performance becomes a function the propagation constants of the modes in array whose variation with geometrical dimensions have already been well-understood in Chapter 3 of this thesis.

CHAPTER 5

DESIGN GUIDELINES AND EXAMPLES

5.1 Introduction

The proposed 2-D periodic array of vertical microstrip lines is suitable for a number of interesting and practical applications such as frequency selective structure (FSS) [6], microwave absorber [11], spatial power combiner [12], etc. Its support for two quasi-TEM modes is the primary attractive feature, which leads to wideband, and angular stable performance for the above and other possible applications. This chapter considers the application of the proposed array in FSS and microwave absorbers, and lays down useful design guidelines. For the purpose of demonstration, the designed examples of frequency selective structures and microwave absorber have also been fabricated, and the measurement results are in agreement with the expected performance, which validate the guidelines and formulae presented in this chapter.

5.2 Frequency Selective Structures

The proposed array may be treated as a multi-mode resonator whose resonances, and coupling with air is controlled to realize a desirable filtering response. In principle, it is possible to obtain a required number of transmission zeros and poles at desired frequency locations by choosing appropriate dimensions of a unit-cell. The operating principle of an FSS based on this array is comprehensively explained through its circuit model. Based on the understanding developed in the previous chapters, useful design guidelines are also listed. Examples of narrow-band and wideband FSSs have been designed and the measurement results are also presented in this section.

5.2.1 Circuit Model

It has been already shown in Chapter 3 that the proposed array supports two quasi-TEM modes because its unit-cell consists of three isolated conductors, and the top and bottom conductors of this unit-cell can have different electric potentials. It was also observed that one of these two modes is primarily concentrated in the substrate region and it was termed as the substrate-mode. The second quasi-TEM mode is concentrated mainly in the air region and it was named as the air-mode. A discontinuity of air-to-array thus results in a three-port network since the incident plane wave is coupled to these two quasi-TEM modes. A cascaded junction of two such discontinuities then leads to the circuit model shown in Figure 5.1 where the two resonators are formed by the first two resonances of the array with finite thickness L . In order to relate this network with already well-established microwave filter theory [3, 92], the representation has been retained in a general form where the air region on the two sides of the network model is termed as source and load, respectively. In the actual case of the proposed FSS, source and load are identical i.e.,

$$M_{S1} = M_{L1}, \quad M_{S2} = M_{L2},$$

where M_{S1} , M_{L1} , and M_{S2} , M_{L2} denote the couplings between the air region and the two fundamental modes of the array. The source-load direct coupling is given with a dotted line because it may be observed only at very low frequency. At higher frequencies, however, this direct source-load coupling becomes negligible and instead the incident wave is coupled to the two modes of the proposed array.

Based on the theory of cross-coupled resonators [92-96], the topology given in Figure 5.1 is known to generate two reflection zeros and one transmission zero at finite frequencies. Each resonator contributes one reflection zero at a frequency where its electrical length becomes equal to π [97, 98]. The topology is applicable for both band-pass and the band-reject filtering functions [99]. The coupling matrix elements needed for the band-reject response are, however, easier to realize with this structure. Also due to the presence of a strong direct source-load coupling at lower

frequencies, the proposed FSS is inherently suitable for band-reject applications, and the design examples presented in this chapter focus on the band-reject behavior, although other filtering responses may also be possible. Figure 5.2 gives an example to show the two reflection zeros and one transmission zero associated with the block diagram in Figure 5.1.

Figure 5.1: Equivalent network model of the proposed FSS.

Figure 5.2: S-parameters of the proposed FSS. ($t = 0.2 \text{ mm}$, $b = 10 \text{ mm}$, $h = 2.524 \text{ mm}$, $d = 1.524 \text{ mm}$, $\epsilon_r = 3.38$, $L = 9.5 \text{ mm}$).

Figure 5.3: A generalized equivalent block representation of the proposed FSS with N resonators.

For a wideband FSS, more transmission zeros may be needed and they can be realized if higher-order resonances are also considered. Reflection zeros corresponding to these resonances occur at frequencies where the electrical length of the resonators becomes a multiple of π . In that case, the proposed FSS can be generalized as the block diagram shown in Figure 5.3. The resulting coupling matrix M is an $(N+2) \times (N+2)$ matrix, where N refers to the number of resonances. This configuration can generate N reflection zeros at finite frequencies [93, 94, 100, 101].

5.2.2 Design Guidelines

The frequency response of the proposed FSS is mainly controlled by three design parameters:

- 1) Dielectric constant ϵ_r ,
- 2) Strip width to unit-cell width ratio t/b ,
- 3) Substrate height to unit-cell height ratio d/h .

Effect of these parameters on S-parameters of the FSS is related to the propagation characteristics of the proposed array. Based on that, certain useful guidelines may be deduced for designing a high-performance FSS. These guidelines

are intuitive, and lead to a good estimate of the required FSS dimensions. Those dimensions may then be verified and fine-tuned using the efficient full-wave mode-matching method presented in Chapter 4. Alternatively, this refining process can also be performed using a suitable commercial software package if available.

For a given center frequency f_0 , bandwidth Δf , and minimum out-of-band return loss $RL_{\min}$, the following steps may be considered.

- 1) Based on the given bandwidth requirement, one has to determine the number of transmission zeros needed in the reject-band. For narrow-band cases, single transmission zero may serve the purpose. However, two or more transmission zeros are desirable for wideband designs. The number of transmission zeros is related to the number of resonances required for the FSS. If a single transmission zero is required, a lower value of ε_r may be selected. For two or more transmission zeros, a higher ε_r is more suitable. A higher value of ε_r reduces the thickness of the structure, and the pass-band (out-of-band response) may not be very large as further higher-order modes may deteriorate the upper end of the out-of-band performance.
- 2) As a first estimate, the thickness L of the array may be chosen as $\lambda_e/2$ where $\lambda_e = \lambda_0 / \sqrt{(\varepsilon_r + 1)/2}$ and λ_0 is the free space wavelength at the center frequency f_0 . The final value of L may slightly be different from the first estimate due to the parasitic effects of the air-to-microstrip line discontinuity.
- 3) The required $RL_{\min}$ is obtained by setting a proper ratio d/h between the substrate width and the unit-cell height. For a given ε_r , the minimum return loss $RL_{\min}$ varies between $\infty < RL_{\min} < -10 \log \left(\frac{\sqrt{\varepsilon_r} - 1}{\sqrt{\varepsilon_r} + 1} \right)$ as $0 < \frac{d}{h} < 1$. The parameter d/h also affects the bandwidth of the transmission zero. For an extremely narrow-band or a notch-band response, d/h is a relatively independent design parameter, and a higher value of it may be preferred for

an improved out-of-band rejection. For wideband designs, it is required to choose a highest possible value of d/h that can be allowed by the given $RL_{\min}$ specification. Higher value of d/h increases coupling to the higher-order modes of the resonators. The principle here is similar to that of the known ultra wideband filters based on microstrip line multi-mode resonators [97, 98], where this coupling is increased by creating a ground-plane aperture. With inclusion of the higher-order modes of the resonating section, more transmission zeros are obtained at finite frequencies and the response curve is flattened through an increased coupling to the higher-order resonances.

- 4) Since the reflection zeros corresponding to the two modes of a microstrip line are placed close to each other for narrow-band design, a lower value of t/b is selected. For wideband FSS, a higher t/b ratio is desired to increase the separation between the reflections zeros so that the second reflection zero does not deteriorate the in-band response.

Table 5.1: Comparison of design parameters for narrow-band and wideband FSSs.

Design Parameter	Narrow-band FSS	Wideband FSS
ϵ_r	Lower	Higher
t/b	Lower	Higher
d/h	Arbitrary or lower	Higher

Table 5.1 gives a summary of the above discussions of the design parameters, and compares them for the two cases of narrow-band and wideband FSSs.

5.2.3 Designed Examples

In the following sub-sections, a total of four designed examples are presented. They include both the narrow-band and wideband structures. These FSSs have been fabricated based on the following steps:

- 1) A large double-layer PCB is used to fabricate one-dimensional (1-D) periodic array of long microstrip lines on its top layer.
- 2) The board is then cut into a number of small parts containing 1-D periodic array of short microstrip lines.
- 3) The individual small PCBs of equal length are then stacked vertically up to form the proposed FSS through two plastic rods at two ends.

Figures 5.4 and 5.5 show the scheme of measurement for reflection and transmission of the designed FSS examples. As illustrated in Figure 5.4, two antennas are placed to one side of an FSS at a distance R , which should be more than the far-field range of these antennas. One of them acts as a transmitting antenna while the second antenna picks the power reflected by the FSS. For accurate measurement and calibration of this set-up, the FSS is removed and response of an empty chamber is measured. After that, a rectangular PEC plate whose radar cross-section is already known is tested in place of the FSS. Based on these calibration measurements, one is able to get an accurate reflection coefficient of the FSS under test.

Similar calibration steps are also taken for the measurement of transmission coefficient whose experimental set-up is schematically shown in Fig. 5.5. Since the space requirement for this set-up is almost twice larger than that for the reflection measurement, an open lab hall was used for this purpose. However, the size of our available anechoic chamber was sufficient for the experimental set-up of Figure 5.4, and hence reflection coefficients of all the designed examples were measured in the anechoic chamber.

Figure 5.4: Experimental set-up for the measurement of reflection coefficient.

Figure 5.5: Experimental set-up for the measurement of transmission coefficient.

5.2.3.1 Narrow-Band FSS

The equivalent network of the basic configuration shown in Figure 5.1 is inherently narrow-band, and it is thus easy to obtain a notch-like response. The aim is to place the two reflection zeros close to each other with a transmission zero in between. A lower ϵ_r can effectively achieve this goal. The separation between the two reflection zeros can be controlled by changing the t/b ratio. Though the ratio d/h also affects the separation between these reflection zeros, its effect is negligibly small in comparison with its impact on the out-of-band return loss performance of the structure.

(a) FSS-A, 250 mm x 220 mm.

(b) FSS-B, 280 mm x 280mm.

Figure 5.6: Photos of the fabricated prototype narrow-band FSSs.

Figure 5.6 shows photos of the two design examples, which have been fabricated and tested. Since only one transmission zero is desired in this case, a lower ϵ_r values of 3.38 and 3 have been selected for the design A and B, respectively. These FSSs are fabricated using RO-4003, and RO-4230 substrate materials. Reflection coefficient measurements are performed in an anechoic chamber. Due to the small size of the anechoic chamber available, its transmission coefficient is measured in an open lab hall. Figure 5.7 shows the simulated and measured reflection and transmission coefficients of the narrow-band FSS-A when it is subjected to normal TE incidence, whereby the incident electric field is perpendicular to the strips. It is seen that the measured results are in good agreement with those obtained from the full-wave mode-matching method. The overall size of FSS-A is 250 mm x 220mm, which contains 41 x 62 unit-cells.

Narrow-band FSS-B is larger in size, while its unit-cell is smaller than that of FSS-A. It was constructed to evaluate the performance of the proposed array under oblique incidence. Figure 5.8 shows its results under TE incidence when $\varphi = 0$, and θ is varied. As seen in this figure, the FSS exhibits very stable response under large variation of the angle of incidence. Similarly, a stable performance is also possible under TM incidence when $\varphi = 90^\circ$, as shown in Figure 5.9.

(a) $|S_{11}|$

(b) $|S_{21}|$

Figure 5.7: S-parameters of the narrow-band FSS – A, under TE incidence. ($t = 0.2$ mm, $b = 6$ mm, $h=3.524$ mm, $d = 1.524$ mm, $\epsilon_r = 3.38$, $L = 9.5$ mm, $\varphi = 0$, $\theta = 0$).

(a) $|S_{11}|$

(b) $|S_{21}|$

Figure 5.8: S-parameters of the narrow-band FSS-B, under TE incidence. ($t = 0.2$ mm, $b = 5$ mm, $h = 3.124$ mm, $d = 1.524$ mm, $\epsilon_r = 3$, $L = 9.5$ mm, $\varphi = 0$).

(a) $|S_{11}|$

(b) $|S_{21}|$

Figure 5.9: S-parameters of the narrow-band FSS-B, under TM incidence. ($t = 0.2$ mm, $b = 5$ mm, $h = 3.124$ mm, $d = 1.524$ mm, $\epsilon_r = 3$, $L = 9.5$ mm, $\varphi = 90^\circ$).

(a) FSS-C, 180 mm x 180 mm.

(b) FSS-D, 260 mm x 230 mm.

Figure 5.10: Photos of wideband FSS prototypes.

5.2.3.2 Wideband FSS

As discussed earlier, wideband response can be obtained with an inclusion of more transmission zeros through higher-order resonances. A higher ϵ_r of 10.2 is used in

this example, and one higher order resonance is included in the reject-band. Moreover, t/b is chosen to be high so that the reflection zero corresponding to the second quasi-TEM mode of the microstrip line may not occur in the frequency band of interest. Within that stop-band, one more transmission zero is generated due to the first higher-order resonance of the substrate-mode, and the overall stop-band is then flattened through increased coupling to substrate-mode by decreasing the d/h ratio. This FSS is fabricated using Rogers RT Duroid 6010 substrate material. Similar to the previous case, the reflection and transmission coefficients are measured in an anechoic chamber and an open lab hall, respectively. As shown in Figure 5.11, measured results are in good agreement with those calculated by the full-wave mode-matching method. There are small differences in the location of the reflection zeros and the insertion loss, which may be attributed to the fabrication tolerances and the measurement errors. The measurement error at the upper end of frequencies is expected as it is not possible to fulfill the far-field criterion at higher frequencies, due to the small size of the anechoic chamber that was available for these measurements.

Purpose of FSS-C was to validate the concept, and hence this structure was constructed with an objective of normal incidence only. Based on the encouraging results of FSS-C, a new wideband design FSS-D was then constructed to test the performance under oblique incidence. FSS-D is larger in over-all size so that the diffraction could be minimized even under oblique incidence when the effective intercepting area of a structure reduces. Further, its unit-cell size is smaller than that of FSS-C, which leads to stable response even under large variation of the angle of incidence as demonstrated for the cases of TE and TM incidence, in Figure 5.12 and Figure 5.13, respectively.

An excellent agreement of results is observed between measurement and the full-wave mode-matching analysis. Though the analysis is based on the assumption of an infinite array, yet its results are in agreement with the measured results of the finite array. This may be related to the sub-wavelength size of the proposed FSS. A physically small FSS actually consists of a large number of unit-cell, which reduce

the edge effect of a finite array on its performance. Based on this feature, the proposed FSS is suitable for sub-reflector antenna or similar applications.

(a) $|S_{11}|$

(b) $|S_{21}|$

Figure 5.11: S-parameters of the wideband design FSS-C, under TE incidence ($t = 4$ mm, $b = 4.2$ mm, $h = 2$ mm, $d = 1.27$ mm, $\epsilon_r = 10.2$, $L = 6.8$ mm, $\varphi = 0$, $\theta = 0$).

(a) $|S_{11}|$

(b) $|S_{21}|$

Figure 5.12: S-parameters of the wideband design FSS-D, under TE incidence ($t = 1.3$ mm, $b = 1.5$ mm, $h = 2$ mm, $d = 1.27$ mm, $\varepsilon_r = 10.2$, $L = 7$ mm, $\varphi = 0$).

(a) $|S_{11}|$

(b) $|S_{21}|$

Figure 5.13: S-parameters of the wideband design FSS-D, under TM incidence ($t = 1.3$ mm, $b = 1.5$ mm, $h = 2$ mm, $d = 1.27$ mm, $\varepsilon_r = 10.2$, $L = 7$ mm, $\phi = 90^\circ$).

5.3 Wideband Microwave Absorber

It has been observed from the dispersion diagram of the proposed array in Chapter 3 that the dual-mode propagation can be obtained over significantly wider range. This is achieved by choosing a smaller size of the unit-cell, and a lower value of the dielectric constant ϵ_r , which delays the onset of higher order modes of the array. Further, it was also noted in Chapter 4 that the air-to-array single discontinuity exhibits an almost frequency independent response (Figure 4.6) when sub-wavelength unit-cells are used. Based on these findings, it readily appears attractive to explore if the proposed array can be used to realize a wideband microwave absorber.

Figure 5.14: A schematic illustration of the proposed circuit analogue absorber.

Recently a few microwave absorbers have been reported [102, 103], which are actually based on the proposed array or a similar geometry. As explained in [102, 103], the incident electromagnetic waves are basically intercepted by a transmission line grid, which converts the wave energy into electric currents. These currents are then controlled and absorbed through suitable lumped circuit elements. Since this grid can be divided into many identical unit-cells and the absorber design is basically to match a unit-cell to the free space through lumped elements. The results reported in [102, 103] are very encouraging, yet they consist of an experimental

work only. In this section, the absorber problem has been formulated formally to lay down the basic design foundations for this new class of microwave absorbers using 2-D periodic array of vertical microstrip lines.

Figure 5.15: Illustrative details of a unit-cell of the proposed wideband microwave absorber.

The conceptual details of the proposed absorber are shown in Figure 5.14, where an incidence of plane wave is considered whose polarization is perpendicular to the strips. Similar to the previous approach of analyzing the 2-D periodic array of vertical microstrip lines, this absorber can also be divided into many identical unit-cells, and the problem of absorber is reduced to that of a unit-cell shown in Figure 5.15. For a sub-wavelength size of the unit-cell, since this array supports only two quasi-TEM modes, it is easy to absorb them through a three lumped elements based scheme illustrated in Figure 5.15. Since the dominant mode of the array is mainly concentrated in the substrate region of the microstrip line, it can be absorbed by terminating the microstrip line with resistor R_1 whose value is equal or close to the characteristics impedance of the substrate-mode. The second quasi-TEM mode of the microstrip line may be absorbed through a series RC network shown in Figure 5.15.

As previously noted in Chapter 4 (Figure 4.6), most of the incident energy is coupled to the second mode (air-mode) of the microstrip line. For this reason, it is

important to effectively absorb this mode over a wide frequency range. A series RC network appears to be an easy and superior choice for this application. Shen [37] has recently shown that a series RC network may even lead to an infinite bandwidth if the thin absorber is modeled as an inductor. This idea of Shen [37] is revisited and extended in the following section. Based on that, a design procedure of the proposed absorber is evolved in Section 5.3.2. Further, a prototype absorber has been fabricated and tested to validate the concept and the design procedure.

5.3.1 Modeling of a Thin Absorber with series RC Matching Network

Figure 5.16: Broadband impedance matching of a thin absorber represented by an inductor.

It is well known that a short-circuited microstrip line of small length l can be represented by an inductor whose inductance L is approximately equal to

$$L = \frac{Z_o \tan(\beta l)}{\omega}, \quad (5.1)$$

where Z_o and β are the characteristic impedance and propagation constant of the line, respectively. The absorber design problem is basically reduced to the one of matching this inductance with free space through a lossy network over a large bandwidth. Traditionally, the matching circuits of a general inductive load have

been assumed to be lossless, and resultantly only limited bandwidth can be obtained [104]. Recently, it has been shown [37] that with the inclusion of loss, an inductive load can be matched for an infinite bandwidth under the ideal situation. In the formulation given in [37], both the shunt and series RC networks were considered as the potential lossy matching networks, and it was shown that the series RC network performs better in terms of the obtained bandwidth. In the following paragraphs, this interesting finding is revisited to evolve useful guidelines for an optimum design through a series RC network. Certain limitations of this technique have also been identified.

Figure 5.16 shows the circuit representation of a thin-layer absorber connected in parallel with a series RC network. The inductance L is assumed to be known, and it can be calculated from (5.1). Y_0 represents the characteristic admittance of free space. R and C are the required component values that need to be calculated for an optimally thin and broadband absorber. From Figure 5.16, the reflection coefficient Γ can be derived as

$$\Gamma = \frac{Y_{in} - Y_0}{Y_{in} + Y_0} = \frac{1 - \omega^2 LC + Y_0 \omega^2 RLC + j(\omega RC - Y_0 \omega L)}{1 - \omega^2 LC - Y_0 \omega^2 RLC + j(\omega RC + Y_0 \omega L)}, \quad (5.2)$$

or

$$|\Gamma|^2 = \Gamma \Gamma^* = \frac{(1 - \omega^2 LC + Y_0 \omega^2 RLC)^2 + (\omega RC - Y_0 \omega L)^2}{(1 - \omega^2 LC - Y_0 \omega^2 RLC)^2 + (\omega RC + Y_0 \omega L)^2}, \quad (5.3)$$

where Γ^* represents the complex conjugate of Γ .

A required level of absorption is commonly specified in terms of the magnitude M of the reflection coefficient Γ . Basically, this specified level is used to define the reflection coefficient bandwidth [105] of the absorber, which refers to the range of frequencies for which the magnitude of Γ remains less than M . Letting $|\Gamma|^2 = M^2$ leads to

$$a^2 \omega^2 - p = -\frac{1}{\omega^2}, \quad (5.4)$$

where

$$a^2 = FC^2, \quad p = 2LC - R^2C^2 - Y_0^2L^2,$$

$$F = L^2 + Y_0^2R^2L^2 + 2\frac{(M^2+1)}{(M^2-1)}Y_0RL^2.$$

Figure 5.17: Reflection coefficient of the circuit shown in Figure 5.16 for different selections of R . ($L = 25 \text{ nH}$, $C = 0.7 \text{ pF}$, $Y_0 = 0.0026 \text{ mho}$).

It may be noted that the left side of (5.4) is a parabolic equation in ω , which intersects the $-1/\omega^2$ curve at two points of the positive ω -axis when $a^2 > 0$ and $p > 0$. However, for the case when $a^2 < 0$, there exists only a single positive root of ω , which implies the existence of an infinite bandwidth [37]. Solving for $a^2 < 0$, hence, leads to the following condition of an infinite bandwidth.

$$R > \frac{1-M}{1+M} \frac{1}{Y_0}. \quad (5.5)$$

Figure 5.17 illustrates this trend whereby an increase in R results in an increase of the level M for which the reflection coefficient bandwidth becomes infinite. This root of ω beyond which the magnitude of reflection coefficient remains below M can be given by

$$\omega_i = \frac{p - \sqrt{p^2 - 4a^2}}{2a^2}. \quad (5.6)$$

It may be noticed from Figure 5.17 that ω_i increases if R is increased while L and C are kept constant.

Figure 5.18: Reflection coefficient of the circuit shown in Figure 5.16 for different selections of L ($R = 200 \Omega$, $Y_0 = 0.0026 \text{ mho}$).

For the design of a thin absorber, one should seek to minimize ω_i through a suitable selection of the capacitance C . Differentiating (5.6) with respect to C , and

enforcing $\frac{d\omega_i}{dC} = 0$ leads to the following condition for an optimum capacitance

C_{opt} .

$$-\frac{2L}{C_{opt}^2} + \frac{2Y_0^2 L^2}{C_{opt}^3} - \frac{\left(-\frac{2L}{C_{opt}^2} + \frac{2Y_0^2 L^2}{C_{opt}^3}\right) \left(\frac{2L}{C_{opt}} - R^2 - \frac{Y_0^2 L^2}{C_{opt}^2}\right) + \frac{4F}{C_{opt}^3}}{\sqrt{\left(\frac{2L}{C_{opt}} - R^2 - \frac{Y_0^2 L^2}{C_{opt}^2}\right)^2 - \frac{4F}{C_{opt}^2}}} = 0. \quad (5.7)$$

Although it is difficult to obtain a closed-form expression for the optimum C_{opt} satisfying (5.7), yet it is easy to obtain a numerical solution of (5.7) through the bisection method [77]. The optimum selection of R is the minimum value allowed by (5.5).

5.3.1.1 Limitation

Unlike the commonly known shunt or series RLC circuits, it is not possible to tune the circuit of Figure 5.16 for an arbitrarily lower frequency. For a given inductance L , the lowest operating frequency ω_i can be obtained by using C_{opt} given in (5.7). Selection of R is a function of the prescribed level of $|\Gamma|$ and hence it is largely independent of L and C as given by (5.5). Based on these conditions (5.5), (5.7) on R and C , respectively, the inductance L turns out to be the only independent parameter for obtaining a desired lower operating frequency. Figure 5.18 illustrates this limitation in more detail. In this case, the value of R has been fixed to $200 \, \Omega$ as required by (5.5) for a 10 dB bandwidth of $|\Gamma|$ when Z_0 is $377 \, \Omega$. Inductance L is varied from $10 \, nH$ to $50 \, nH$, and for each case, an optimum value of C is calculated from (5.7).

Figure 5.18 and other investigations show that ω_i is a monotonic function of L whereby a lower ω_i can be obtained only by using a higher value of L . As

discussed later, this consideration is important for obtaining a thinner design of the proposed microwave absorber.

Figure 5.19: Reflection coefficient of the circuit shown in Figure 5.16 when the inductive load L is replaced by a capacitance C_L ($R = 300 \Omega$, $C = 0.7 \text{ pF}$, $Y_0 = 0.0026 \text{ mho}$).

5.3.1.2 The Case of a Capacitive Load

Since a short-circuited transmission line may become capacitive for certain frequencies when its electrical length is more than a quarter-wavelength, it appears relevant to consider the effect of the proposed RC circuit in this frequency range. The load inductance L in Figure 5.16 may be replaced with a capacitance C_L , and following similar steps as those in the previous formulation leads to

$$\bar{a}^2 \omega^2 - \bar{p} = -\frac{1}{\omega^2}, \quad (5.8)$$

where

$$\bar{a}^2 = \frac{R^2 C^2 C_L^2}{Y_0^2}, \quad \bar{p} = - \left\{ \frac{(C + C_L)^2}{Y_0^2} + R^2 C^2 + \frac{2RC^2}{Y_0} \left(\frac{M^2 + 1}{M^2 - 1} \right) \right\}.$$

Since $\bar{a}^2 > 0$, it is not possible to obtain an infinite bandwidth in this case. However, the capacitive load can still be matched over a finite bandwidth using the proposed RC circuit. Figure 5.19 presents the frequency response of the circuit shown in Figure 5.16 when inductance L is replaced by a capacitance C_L . A dip can be noted in this case. Based on this finding, it is later easier to explain the two dips observed in the frequency response of the proposed microwave absorber. Since the load offered by a short-circuited shielded microstrip line varies with the operating frequency, the two dips are observed at the frequencies where this load becomes a suitable inductance and capacitance as required by the above formulation.

5.3.2 Design Procedure

The two-dimensional periodic array of vertical microstrip lines supports two quasi-TEM modes whose discontinuity with air results in a three-port equivalent circuit shown in Figure 5.20. Figures 5.20 and 5.21 also illustrate the proposed scheme for wideband absorption of these two modes. Since the substrate-mode is concentrated mainly in the substrate region, it can be easily terminated with a resistor soldered in the microstrip line, while the energy carried by the air-mode is absorbed through a series RC network discussed in the previous section.

The case of an infinite bandwidth is based on the assumption of a fixed inductance, which is not strictly true for the present case since the inductance offered by a short-circuited microstrip line is known to vary with frequency. However, the expressions and the procedure derived for an infinite bandwidth provide a very good estimate of the optimum lumped component values required for a broadband absorption. Moreover, the concept of matching a capacitive load with the RC network is useful in this case, and it provides a substantial increase in the

over-all bandwidth of the proposed absorber. Based on that, the following steps may be pursued for designing the proposed wideband microwave absorber:

Figure 5.20: Equivalent circuit of the proposed absorber illustrating the placement of lumped elements.

Figure 5.21: Illustration of the placement of the matching lumped circuit elements in a unit-cell of the two-dimensional periodic array of microstrip lines, (a) front view, (b) side view.

- 1) A substrate material of nominal dielectric constant may be selected.
- 2) As an initial guess, the thickness l may be chosen as $\lambda/10$, where λ refers to the free space wavelength at the lowest operating frequency.
- 3) Since the substrate-mode can be easily terminated through a resistor placed in the microstrip line, one should mainly focus on the propagation characteristics of the air-mode while selecting the unit-cell dimensions. It is desired to choose the dimensions that maximize the product of the propagation constant and the characteristic impedance of the air-mode. This in turn leads to a higher level of inductance given by (5.1). As shown in the previous section, a higher inductance is desirable for lowering ω_i and hence obtaining a thinner design.
- 4) Once the geometrical dimensions of the unit-cell are finalized, the characteristic impedance of the substrate-mode Z_{01} is calculated, which leads to the initial guess of the lumped resistor R_1 , i.e., $R_1 = Z_{01}$.
- 5) The initial value of R_2 is also straightforwardly obtained from (5.5).
- 6) The propagation constant and the characteristic impedance of the air-mode is then calculated based on the formulation of Chapter 3. Using them, the equivalent inductance of the air-mode is extracted from (5.1). Its variation as a function of frequency is analyzed, and an optimum C_2 is found from (5.7), which corresponds to the lowest possible value of ω_i .
- 7) A small tuning of the component values may improve the results slightly because the frequency dependence of the line inductance was not included in the simple circuit model of the absorber. Also in the above procedure, the parasitic effect of the air-to-microstrip line discontinuity has been assumed to be negligibly small, which may not be strictly true.

5.3.3 Designed Example

The design procedure is demonstrated here through an example. For the absorber to be designed, a start frequency of 1.5 GHz is considered for the wide operating band.

A substrate material RO 4230 ($\epsilon_r = 3$) is used for the purpose of this design example, and following Step 2, the thickness l is fixed to 20 mm. With the understanding of the propagation characteristics of the array discussed in Chapter 3, the following nominal dimensions of the unit-cell are selected, as given in Table 5.2.

Table 5.2: Cross-sectional dimensions of a unit-cell of the absorber design-example.

Dimension	Value (mm)
d	1.524
h	11.524
t	2
b	10

Figure 5.22: Inductance of a short-circuit array, considering the air-mode only. ($t = 2$ mm, $\epsilon_r = 3$, $d = 1.524$ mm, $h = 11.524$ mm, $b = 10$ mm, $l = 20$ mm).

The characteristic impedances of the first two modes of the microstrip line are calculated to be 52 Ω and 386 Ω , respectively. This gives $R_1 = 52 \Omega$. The lower limit allowed by (5.5) for 10 dB reflection coefficient bandwidth is calculated to be 226 Ω .

It leads to the initial selection of $R_2 = 230\Omega$. Knowing the characteristic impedance of the air-mode, and calculating the propagation constant, one can generate a plot of the inductance of the air-mode as a function of frequency, as shown in Figure 5.22. With an already fixed R_2 , the lowest ω_i becomes a function of the inductance. Since inductance varies with frequency, C_{opt} for each frequency point is calculated, and the corresponding ω_i^{opt} is observed as given by (5.6). For a frequency point f , if the corresponding f_i^{opt} equals f , that frequency point is taken as the realizable start frequency of the absorber, and the corresponding C_{opt} is noted as the initial selection for the lumped component C_2 . Figure 5.23 shows a plot of f_i^{opt} calculated using (5.6) and (5.7) based on the inductance corresponding to every frequency point. It is seen from this plot that the frequency point $f = 1.4$ GHz equals to its corresponding f_i^{opt} , and hence the same is selected as the realizable start frequency. Since the corresponding C_{opt} is 0.6 pF, it leads to the selection of $C_2 = 0.6$ pF.

Figure 5.23: Realizable start frequency for the design example corresponding to the inductance obtained at every frequency point.

Figure 5.24 presents the frequency response of the resulting absorber. The final design obtained after a little tuning of the lumped elements is also given. It exhibits

112.5% bandwidth (1.49 GHz – 5.32 GHz) while its thickness is less than $\lambda/10$ of the lowest frequency. The two dips of the frequency response correspond to the matching of the series RC network when the load in Figure 5.16 becomes a suitable inductance or capacitance.

Figure 5.24: Reflection coefficient of the designed example. (Before tuning: $C_2 = 0.6 \text{ pF}$, $R_2 = 230 \Omega$, $R_1 = 52 \Omega$, After tuning: $C_2 = 0.5 \text{ pF}$, $R_2 = 180 \Omega$, $R_1 = 120 \Omega$).

Figure 5.25: Performance of the designed absorber under oblique TE incidence.

Figure 5.25 shows the absorbing performance of the designed absorber under different angles of incidence. A stable frequency response is observed, which is primarily due to the relatively smaller dimensions of a unit-cell. Since the proposed absorber is based on quasi-TEM modes of a microstrip line, therefore in principle, it is possible to choose an arbitrarily small dimension of a unit-cell, which leads to a stable angular performance and an effective prevention of grating lobes.

Figure 5.26: Photo of the fabricated wideband microwave absorber.

The absorber with tuned component values has been fabricated and its photo is given in Figure 5.26. The fabricated structure is 20 cm x 20 cm in size, and comprises 20 x 18 unit cells. Measurement results of this absorber under normal incidence are shown in Figure 5.27. The measured 10 dB bandwidth is 113% (1.40 GHz – 5.06 GHz), and the absorber thickness is less than $\lambda/10$ of the lowest frequency. There are some differences in simulated and measured results, which may be attributed to the fabrication tolerances, variation of lumped element values with frequency, and the parasitic effects of the lumped elements. It should also be pointed out that simulation results have been obtained using CST MWS for an infinite array of the absorber unit-cells.

Figure 5.27: Simulated and measured reflection coefficient of the fabricated wideband absorber. ($t = 2 \text{ mm}$, $\epsilon_r = 3$, $d = 1.524 \text{ mm}$, $h = 11.524 \text{ mm}$, $b = 10 \text{ mm}$, $l = 20 \text{ mm}$, $C_2 = 0.5 \text{ pF}$, $R_2 = 180 \Omega$, $R_1 = 120 \Omega$).

5.4 Summary

The two-dimensional periodic array of vertical microstrip lines, presented in this thesis, is useful for a number of applications. Its use as a frequency selective structure and a microwave absorber has been demonstrated in this chapter. Design guidelines and the procedures have been explained through a number of examples, which have also been fabricated, and the measurement results clearly validate the methods presented here.

An array of finite-thickness forms a dual-mode resonator, which is attractive for application as FSS. By controlling the propagation constants of the two modes, and their coupling with air, a suitable quasi-elliptic filtering performance is obtained. With a reduced size of the unit-cell, it is also possible to stabilize its performance under a large variation of the angle of incidence. The bandwidth of the proposed

FSS can also be controlled, and the array is very suitable for both narrow-band and wideband applications.

A scheme for wideband absorption of the two modes of this array has been presented, which requires three lumped elements for each unit-cell. Unlike many other known circuit analog absorbers, the proposed scheme is based on closed-form or simple formulas. It has been demonstrated through a design example that more than 110% bandwidth is achievable with the proposed absorber.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

A new class of three-dimensional frequency selective structure (FSS) has been presented, which overcomes many of the limitations of traditional designs previously used for frequency selective applications. Traditional structures suffer from poor selectivity of filtering response, and their performance is sensitive to the angle of incidence. Some improvements are made by adding dielectric layers and cascading a number of two-dimensional surfaces to realize higher order filtering function. However, these improvements lead to an increase in size and weight of the resulting design, while the difference in performance may not be substantial. On the other hand, the structure proposed in this thesis offers a remarkable performance improvement while maintaining a practically reasonable size and weight of the physical product.

The proposed FSS consists of a simple geometry based on a two-dimensional periodic array of vertical microstrip lines. The array supports two quasi-TEM modes which lead to the formation of a dual-mode resonator when a structure of finite-thickness is considered. Using a substrate material of higher dielectric constant, it is possible to include higher order resonances of these two modes, and in general, this array may be termed as a multi-mode resonator. Resonant frequencies of these modes and their coupling with air can be controlled to obtain a desirable elliptic filtering response, whereby a required number of transmission/reflection zeros can be realized at finite frequency points. Further, the support of quasi-TEM modes allows an arbitrarily small size of the unit-cell, which is an attractive feature to achieve stable performance under a large variation of the angle of incidence. A sub-wavelength size of the unit-cell leads to a large number of cells in a physically small structure, which minimizes the effect of finite truncation of a theoretically infinite

periodic array as assumed in the Floquet theorem based analysis of the proposed structure.

The proposed array has been analyzed using an efficient full-wave mode-matching method. The analysis has been divided into two parts, as discussed in Chapter 3 and 4, respectively. First, the propagation characteristics of this array are investigated by invoking Floquet theorem, and analyzing a unit-cell of this geometry. Basically, it is a shielded microstrip line with periodic side walls. Its periodic phase shift is related to the angle of incidence, and resultantly the propagation characteristics of the proposed array become a function of its excitation. This is unlike many classic problems, for example, a thick 2-D array of rectangular waveguides, where the modes inside the array are an intrinsic property of the geometry and do not depend upon the excitation conditions. Because of their significance in the design of FSS and microwave absorber, the two quasi-TEM modes of the array have been studied in detail. Effect of the periodic phase shift and geometrical dimensions on the propagation characteristics of these modes has been discussed and understood. Further, the propagation characteristics of the proposed array have also been compared with those of the microstrip lines shielded with PEC or PMC side walls. A few similarities and differences among them have been noted, which basically lead to better understanding of the modes in a 2-D periodic array of vertical microstrip lines.

In the second part of the analysis, scattering by this array is studied using a full-wave mode-matching method. Modes in the air region have been expressed in terms of TE and TM Floquet modes. Modes and fields in the array were calculated using the mode-matching method as discussed previously. Enforcing the continuity of fields at the interface between air and the array leads to scattering parameters of this discontinuity. Since FSS can be modeled as a cascaded junction of two air-to-array discontinuities, S-parameters of FSS are efficiently computed through a cascaded combination of generalized scattering matrices. Effect of the angle and polarization of incidence has been investigated, and conditions for the excitation of quasi-TEM modes have been identified. It is seen that the two quasi-TEM modes are excited only when an electric field component of the incident plane wave is perpendicular to

the underlying strips. For a sub-wavelength size of the unit-cell, scattering performance becomes stable even with a significant variation of the incident angle. However, the performance is sensitive to the polarization of incidence even for a sub-wavelength size of the unit-cell. It is also noted that under the favorable polarization of incidence, the air-to-array discontinuity itself offers a good transition, and most of the incident power is easily coupled to the air-mode of the array. S-parameters of air-to-array discontinuity are almost frequency independent when a sub-wavelength unit-cell is used, and the onset of higher order modes is delayed.

A frequency selective surface can be modeled as a cascaded junction of two air-to-array discontinuities, and it has been analyzed based on the above mentioned mode-matching method. Since scattering by an air-to-array discontinuity does not rapidly change with frequency when a sub-wavelength unit-cell is selected, it may be appropriate to employ coupling matrix based synthesis for the proposed array. For this reason, equivalent circuit of the proposed FSS is related with the known coupling matrix topology of microwave filters. Further, based on the understanding of scattering problem and the propagation characteristics of the proposed structure, some useful guidelines have been suggested. A number of design examples have been fabricated, and the measurement results validate the concepts and methods presented in this thesis.

Traditionally, circuit analog microwave absorber is treated as a closely related topic, and sometimes a sub-set of frequency selective structures. For completeness of this study on a new FSS, the same structure has been explored for its application in microwave absorbers. With proper termination of the two quasi-TEM modes, the proposed array has been demonstrated as an attractive candidate for wideband microwave absorbers. Problem of a circuit analog absorber with series RC network has been modeled in sufficient details to derive closed-form or simple design formulas. This is unlike many traditional RLC circuit analog absorbers where explicit design formulas for an optimum design are not available, and instead numerical optimizations are commonly used. A design example was fabricated and the measurement results show that an absorption bandwidth in excess of 110% is achievable with the proposed 2-D periodic array of vertical microstrip lines.

It should also be mentioned that the proposed array suffers from a few limitations, and hence it may not be a suitable selection for every application. Based on its asymmetric geometry, it is sensitive to the polarization of incidence. Its salient features rely on the excitation of quasi-TEM modes, which happens only when a component of electric field is perpendicular to the strips. Also, the structure is relatively difficult to assemble since it consists of a three-dimensional geometry. For application as a microwave absorber, the proposed design technique is based on lumped elements. With the state of present technology, it may not be possible to find the lumped elements for very high frequencies, especially for the upper band of microwaves.

6.2 Recommendations for Future Work

This thesis has presented design of novel frequency selective structures and microwave absorber. Complete analysis and guidelines of a primitive form of this concept have been presented. However, like any useful new idea, it offers opportunities for many future investigations on the same or similar structures consisting of microstrip line arrays, which support quasi-TEM modes. Some of these possible studies are suggested as follows:

1. A 2-D periodic array of vertical microstrip lines is suitable for band-reject applications since the incident plane wave is easily coupled to the air-mode of the array for a wide range of frequencies. Based on this, the FSS design examples presented in Chapter 6 focus on the band-reject applications. In future, an investigation for bandpass FSS may be carried out using the same array, or instead a similar concept of 2-D periodic array of shielded vertical microstrip lines as suggested in [15].
2. The proposed structure suffers from the limitation of polarization whereby a component of the incident electric field is required to

fall perpendicular to the underlying strips. Some modifications or extensions on the proposed structure may be studied to overcome this shortcoming of the present geometry.

3. The use of microstrip line offers an easy placement of lumped elements for FSS applications. By soldering suitable active or passive lumped elements in the microstrip lines, it may be possible to reduce the thickness of the proposed FSS. It may also lead to active and tunable structures.
4. More than one microstrip line may be used in a unit-cell as shown in Figure 6.1. This technique may lead to more design flexibility, and it may also be possible to realize novel multi-band frequency selective structures.
5. The absorber presented in this thesis is based on a non-magnetic lossless substrate material. Inclusion of loss and magnetic properties of the substrate layer may lead to an improved design, since recently, materials with lossy electric and magnetic properties have been developed for absorber applications.

Figure 6.1: Cross-sectional view of the unit-cell of a potential multi-band FSS.

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Journal Papers

1. A. K. Rashid and Z. Shen, "Scattering by a two-dimensional periodic array of vertical microstrip lines," *Accepted for Publication in IEEE Trans. on Antennas and Propag.*, 2010.
2. A. K. Rashid, Z. Shen, and S. Aditya, "Wideband microwave absorber based on a two-dimensional periodic array of microstrip lines," *IEEE Trans. on Antennas and Propag.*, vol. 58, no. 12, pp. 3913-3922, 2010.
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1. A. K. Rashid and Z. Shen, "Three-dimensional monolithic frequency selective structure with dielectric loading," *Asia Pacific Microwave Conference*, Japan, 2010.
2. A. K. Rashid and Z. Shen, "Three-dimensional frequency selective surfaces," *International Conference on Communications, Circuits, and Systems (ICCCAS)*, China, July 2010.
3. A. K. Rashid and Z. Shen, "Bandpass frequency selective surface based on a two-dimensional periodic array of shielded microstrip lines," *IEEE International Symposium on Antennas and Propagation*, Toronto, Canada, July 2010.
4. A. K. Rashid and Z. Shen, "Scattering by a two-dimensional periodic array of vertical microstrip lines," *International Symp. on Antenna Tech. and Applied Electromag. (ANTEM)*, Canada, July 2010.

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APPENDIX – MATRICES ELEMENTS FOR THE TWO-DIMENSIONAL PROBLEM

The mode-matching method based formulation of the two-dimensional problem of the array is given in Chapter 3. Various matrices involved in that formulation have also been explained in the same chapter. Following are the complete expansions of the elements of those matrices.

Matrices $\mathbf{U}_A^I$ and $\mathbf{U}_B^I$ can be written as,

$$\mathbf{U}_A^I = \begin{bmatrix} \mathbf{U}_{Ahh}^I & \mathbf{U}_{Aeh}^I \\ \mathbf{0} & \mathbf{U}_{Aee}^I \end{bmatrix}, \quad \mathbf{U}_B^I = \begin{bmatrix} \mathbf{U}_{Bhh}^I & \mathbf{U}_{Beh}^I \\ \mathbf{0} & \mathbf{U}_{Bee}^I \end{bmatrix},$$

where $\mathbf{U}_{Ahh}^I$ and $\mathbf{U}_{Aee}^I$ are diagonal matrices of size $N_I \times N_I$ each, and their diagonal elements are given by

$$U_{Ahh}^I(r, r) = j\omega\mu k_{xr}^{lh} \cot(k_{xs}^{lh} x_1) \left\{ \frac{SS^{(i)}(k_{yr}^{lh1}, k_{yr}^{lh1})}{\sin^2(k_{yr}^{lh1} d)} + \frac{SS^{(ii)}(k_{yr}^{lh2}, k_{yr}^{lh2})}{\sin^2[k_{yr}^{lh2} (h-d)]} \right\},$$

$$U_{Aee}^I(r, r) = \left[(k_{xr}^{le})^2 - \gamma^2 \right] \left\{ \frac{CC^{(i)}(k_{yr}^{lh1}, k_{yr}^{lh1})}{\varepsilon_r \cos^2(k_{yr}^{le1} d)} + \frac{CC^{(ii)}(k_{yr}^{lh2}, k_{yr}^{lh2})}{\cos^2[k_{yr}^{le2} (h-d)]} \right\},$$

$\mathbf{U}_{Ahh}^I$ is also an $N_I \times N_I$ matrix, and its element can be expressed as

$$U_{Aeh}^I(r, s) = -\gamma \left\{ \frac{k_{ys}^{le1} SS^{(i)}(k_{ys}^{le1}, k_{yr}^{lh1})}{\varepsilon_r \cos(k_{ys}^{le1} d) \sin(k_{yr}^{lh1} d)} - \frac{k_{ys}^{le2} SS^{(ii)}(k_{ys}^{le2}, k_{yr}^{lh2})}{\cos[k_{ys}^{le2} (h-d)] \sin[k_{yr}^{lh2} (h-d)]} \right\},$$

$\mathbf{U}_B^I$ is very similar to $\mathbf{U}_A^I$, and their individual elements are related by the following equations.

$$U_{Bhh}^I(r, r) = -\tan^2(k_{xs}^{Ih} x_1) U_{Ahh}^I(r, r),$$

$$U_{Bee}^I(r, r) = U_{Aee}^I(r, r), \quad U_{Beh}^I(r, s) = U_{Aeh}^I(r, s).$$

The following definitions have been used in the above expressions:

$$CC^{(i)}(k_i, k_j) = \int_0^d \cos(k_i y) \cos(k_j y) dy = \frac{1}{2} \left[\frac{\sin\{(k_i + k_j)d\}}{k_i + k_j} + \frac{\sin\{(k_i - k_j)d\}}{k_i - k_j} \right],$$

$$SS^{(i)}(k_i, k_j) = \int_0^d \sin(k_i y) \sin(k_j y) dy = -\frac{1}{2} \left[\frac{\sin\{(k_i + k_j)d\}}{k_i + k_j} - \frac{\sin\{(k_i - k_j)d\}}{k_i - k_j} \right],$$

$$\begin{aligned} CC^{(ii)}(k_i, k_j) &= \int_d^h \cos\{k_i(h-d)\} \cos\{k_j(h-d)\} dy \\ &= \frac{1}{2} \left[\frac{\sin\{(k_i + k_j)(h-d)\}}{k_i + k_j} + \frac{\sin\{(k_i - k_j)(h-d)\}}{k_i - k_j} \right], \end{aligned}$$

$$\begin{aligned} SS^{(ii)}(k_i, k_j) &= \int_d^h \sin\{k_i(h-d)\} \sin\{k_j(h-d)\} dy \\ &= -\frac{1}{2} \left[\frac{\sin\{(k_i + k_j)(h-d)\}}{k_i + k_j} - \frac{\sin\{(k_i - k_j)(h-d)\}}{k_i - k_j} \right]. \end{aligned}$$

Similarly, the matrices $\mathbf{P}_A$, $\mathbf{P}_B$, $\mathbf{Q}_A$ and $\mathbf{Q}_B$ can be written as,

$$\begin{aligned} \mathbf{P}_A &= \begin{bmatrix} \mathbf{P}_{Ahh} & \mathbf{P}_{Aeh} \\ \mathbf{0} & \mathbf{P}_{Aee} \end{bmatrix}, & \mathbf{P}_B &= \begin{bmatrix} \mathbf{P}_{Bhh} & \mathbf{P}_{Beh} \\ \mathbf{0} & \mathbf{P}_{Bee} \end{bmatrix}, \\ \mathbf{Q}_A &= \begin{bmatrix} \mathbf{Q}_{Ahh} & \mathbf{Q}_{Aeh} \\ \mathbf{0} & \mathbf{Q}_{Aee} \end{bmatrix}, & \mathbf{Q}_B &= \begin{bmatrix} \mathbf{Q}_{Bhh} & \mathbf{Q}_{Beh} \\ \mathbf{0} & \mathbf{Q}_{Bee} \end{bmatrix}, \end{aligned}$$

where each of the sub-matrices $\mathbf{P}_{Ahh}$, $\mathbf{P}_{Aeh}$ and $\mathbf{P}_{Aee}$ is an $N_I \times N_{II}$ matrix, whose elements are

$$P_{Ahh}(r, s) = j\omega\mu k_{xs}^{IIh} \cot(k_{xs}^{IIh} x_2) \frac{SS^{y(ii)}(k_{ys}^{IIh}, k_{yr}^{IIh2})}{\sin[k_{yr}^{IIh2}(h-d)]},$$

$$P_{Aee}(r, s) = \left[(k_{xs}^{IIe})^2 - \gamma^2 \right] \frac{CC^{y(ii)}(k_{ys}^{IIe}, k_{yr}^{IIe2})}{\cos[k_{yr}^{IIe2}(h-d)]},$$

$$P_{Aeh}(r, s) = -\gamma k_{ys}^{IIe} \frac{SS^{y(ii)}(k_{ys}^{IIe}, k_{yr}^{IIh2})}{\sin[k_{yr}^{IIh2}(h-d)]}.$$

Each of the sub-matrices $\mathbf{Q}_{Ahh}$, $\mathbf{Q}_{Aeh}$ and $\mathbf{Q}_{Aee}$ is an $N_I \times N_{III}$ matrix, and the elements are given as follows:

$$Q_{Ahh}(r, s) = j\omega\mu k_{xs}^{IIIh} \cot(k_{xs}^{IIIh} x_2) \frac{SS^{y(i)}(k_{ys}^{IIIh}, k_{yr}^{IIIh1})}{\sin(k_{yr}^{IIIh1} d)},$$

$$Q_{Aee}(r, s) = \left[(k_{xs}^{IIIe})^2 - \gamma^2 \right] \frac{CC^{y(i)}(k_{ys}^{IIIe}, k_{yr}^{IIIe1})}{\cos(k_{yr}^{IIIe1} d)},$$

$$Q_{Aeh}(r, s) = -\gamma k_{ys}^{IIIe} \frac{SS^{y(i)}(k_{ys}^{IIIe}, k_{yr}^{IIIh1})}{\sin(k_{yr}^{IIIh1} d)}.$$

Elements of $\mathbf{P}_B$ and $\mathbf{Q}_B$ can be derived from the previously calculated elements as,

$$P_{Bhh}(r, s) = -\tan^2(k_{xs}^{IIh} x_2) P_{Ahh}(r, s),$$

$$P_{Bee}(r, s) = P_{Aee}(r, s), \quad P_{Beh}(r, s) = P_{Aeh}(r, s),$$

$$Q_{Bhh}(r, s) = -\tan^2(k_{xs}^{IIh} x_2) Q_{Ahh}(r, s),$$

$$Q_{Bee}(r, s) = Q_{Aee}(r, s), \quad Q_{Beh}(r, s) = Q_{Aeh}(r, s).$$

As shown in Chapter 3, Matrices $\mathbf{U}_A^I$, $\mathbf{U}_B^I$, $\mathbf{U}_A^{II}$ and $\mathbf{U}_B^{II}$ are based on the following form:

$$\mathbf{U}_A^I = \begin{bmatrix} \mathbf{U}_{Ahh}^I & \mathbf{0} \\ \mathbf{U}_{Ahe}^I & \mathbf{U}_{Aee}^I \end{bmatrix}, \quad \mathbf{U}_B^I = \begin{bmatrix} \mathbf{U}_{Bhh}^I & \mathbf{0} \\ \mathbf{U}_{Bhe}^I & \mathbf{U}_{Bee}^I \end{bmatrix},$$

$$\mathbf{U}_A^{II} = \begin{bmatrix} \mathbf{U}_{Ahh}^{II} & \mathbf{0} \\ \mathbf{U}_{Ahe}^{II} & \mathbf{U}_{Aee}^{II} \end{bmatrix}, \quad \mathbf{U}_B^{II} = \begin{bmatrix} \mathbf{U}_{Bhh}^{II} & \mathbf{0} \\ \mathbf{U}_{Bhe}^{II} & \mathbf{U}_{Bee}^{II} \end{bmatrix},$$

where $\mathbf{U}_{Ahh}^I$ and $\mathbf{U}_{Aee}^I$ are diagonal matrices of size $N_I \times N_I$, and their diagonal elements are given by

$$U_{Ahh}^I(r, r) = \left[\left(k_{xr}^{Ih} \right)^2 - \gamma^2 \right] SS^{(ii)}(k_{yr}^{Ih}, k_{yr}^{Ih}),$$

$$U_{Aee}^I(r, r) = j\omega\epsilon_0 k_{xr}^{Ie} \tan(k_{xr}^{Ie} x_2) CC^{(ii)}(k_{yr}^{Ie}, k_{yr}^{Ie}).$$

$\mathbf{U}_{Ahe}^I$ is also an $N_I \times N_I$ matrix with the elements

$$U_{Ahe}^I(r, s) = \gamma k_{ys}^{Ih} CC^{(ii)}(k_{ys}^{Ih}, k_{yr}^{Ie}).$$

Similarly, $\mathbf{U}_{Ahh}^{II}$ and $\mathbf{U}_{Aee}^{II}$ are diagonal matrices of size $N_{II} \times N_{II}$ whose diagonal elements are given as follows:

$$U_{Ahh}^{II}(r, r) = \left[\left(k_{xr}^{IIh} \right)^2 - \gamma^2 \right] SS^{(i)}(k_{yr}^{IIh}, k_{yr}^{IIh}),$$

$$U_{Aee}^{II}(r, r) = j\omega\epsilon_0 \epsilon_r k_{xr}^{IIe} \tan(k_{xr}^{IIe} x_2) CC^{(i)}(k_{yr}^{IIe}, k_{yr}^{IIe}).$$

The elements of $\mathbf{U}_{Ahe}^{II}$ can be expressed as the following. It is also an $N_{II} \times N_{II}$ matrix.

$$U_{Ahe}^{II}(r, s) = \gamma k_{ys}^{IIh} CC^{(i)}(k_{ys}^{IIh}, k_{yr}^{IIe}).$$

Similar to the previous cases, the elements of $\mathbf{U}_B^{II}$ and $\mathbf{U}_B^{III}$ can be written in terms of $\mathbf{U}_A^{II}$ and $\mathbf{U}_A^{III}$ as follows,

$$\begin{aligned} U_{Bhh}^{II}(r, r) &= U_{Ahh}^{II}(r, r), & U_{Bhe}^{II}(r, s) &= U_{Ahe}^{II}(r, s), \\ U_{Bee}^{II}(r, r) &= -\cot^2(k_{xr}^{Ile} x_2) U_{Aee}^{II}(r, r), \\ U_{Bhh}^{III}(r, r) &= U_{Ahh}^{III}(r, r), & U_{Bhe}^{III}(r, s) &= U_{Ahe}^{III}(r, s), \\ U_{Bee}^{III}(r, r) &= -\cot^2(k_{xr}^{IIIe} x_2) U_{Aee}^{III}(r, r). \end{aligned}$$

The matrices $\mathbf{R}_A$, $\mathbf{R}_B$, $\mathbf{S}_A$ and $\mathbf{S}_B$ have been previously expressed in the following form as given in Chapter 3.

$$\begin{aligned} \mathbf{R}_A &= \begin{bmatrix} \mathbf{R}_{Ahh} & \mathbf{0} \\ \mathbf{R}_{Ahe} & \mathbf{R}_{Aee} \end{bmatrix}, & \mathbf{R}_B &= \begin{bmatrix} \mathbf{R}_{Bhh} & \mathbf{0} \\ \mathbf{R}_{Bhe} & \mathbf{R}_{Bee} \end{bmatrix}, \\ \mathbf{S}_A &= \begin{bmatrix} \mathbf{S}_{Ahh} & \mathbf{0} \\ \mathbf{S}_{Ahe} & \mathbf{S}_{Aee} \end{bmatrix}, & \mathbf{S}_B &= \begin{bmatrix} \mathbf{S}_{Bhh} & \mathbf{0} \\ \mathbf{S}_{Bhe} & \mathbf{S}_{Bee} \end{bmatrix}, \end{aligned}$$

where all the sub-matrices of $\mathbf{R}_A$ and $\mathbf{S}_A$ are of size $N_I \times N_{II}$ and $N_I \times N_{III}$, respectively.

They can be expanded as

$$\begin{aligned} R_{Ahh}(r, s) &= \left[\left(k_{xs}^{Ih} \right)^2 - \gamma^2 \right] \frac{SS^{(ii)}(k_{ys}^{Ih2}, k_{yr}^{IIIh})}{\sin[k_{ys}^{Ih2}(h-d)]}, \\ R_{Aee}(r, s) &= j\omega\epsilon_0 k_{xs}^{Ie} \tan(k_{xs}^{Ie} x_1) \frac{CC^{(ii)}(k_{ys}^{Ie2}, k_{yr}^{IIe})}{\cos[k_{ys}^{Ie2}(h-d)]}, \\ R_{Ahe}(r, s) &= -\gamma k_{ys}^{Ih2} \frac{CC^{(ii)}(k_{ys}^{Ih2}, k_{yr}^{IIe})}{\sin[k_{ys}^{Ih2}(h-d)]}, \end{aligned}$$

$$S_{Ahh}(r, s) = \left[\left(k_{xs}^{Ih} \right)^2 - \gamma^2 \right] \frac{SS^{(i)} \left(k_{ys}^{Ih1}, k_{yr}^{IIIh} \right)}{\sin \left(k_{ys}^{Ih1} d \right)} \int_0^d \frac{\sin \left(k_{ys}^{Ih1} y \right) \sin \left(k_{yr}^{IIIh} y \right)}{dy},$$

$$S_{Aee}(r, s) = j\omega\epsilon_0 k_{xs}^{Ie} \tan \left(k_{xs}^{Ie} x_1 \right) \frac{CC^{(i)} \left(k_{ys}^{Ie1}, k_{yr}^{IIIe} \right)}{\cos \left(k_{ys}^{Ie1} d \right)},$$

$$S_{Ahe}(r, s) = \gamma k_{ys}^{Ih1} \frac{CC^{(i)} \left(k_{ys}^{Ih1}, k_{yr}^{IIIe} \right)}{\sin \left(k_{ys}^{Ih1} d \right)}.$$

Since $\mathbf{R}_B$ and $\mathbf{S}_B$ are very similar to $\mathbf{R}_A$ and $\mathbf{S}_A$, their elements are given by the following relations.

$$R_{Bhh}(r, s) = R_{Ahh}(r, s), \quad R_{Bhe}(r, s) = R_{Ahe}(r, s),$$

$$R_{Bee}(r, s) = -\cot^2 \left(k_{xs}^{Ie} x_1 \right) R_{Aee}(r, s),$$

$$S_{Bhh}(r, s) = S_{Ahh}(r, s), \quad S_{Bhe}(r, s) = S_{Ahe}(r, s),$$

$$S_{Bee}(r, s) = -\cot^2 \left(k_{xs}^{Ie} x_1 \right) S_{Aee}(r, s).$$

All the remaining matrices are simply related with the above matrices, and these relations may be given as follows.

$$\bar{\mathbf{U}}_A^I = \mathbf{U}_A^I, \quad \bar{\mathbf{U}}_B^I = -\mathbf{U}_B^I,$$

$$\bar{\mathbf{P}}_A = e^{-j\Phi_x} \mathbf{P}_A, \quad \bar{\mathbf{P}}_B = -e^{-j\Phi_x} \mathbf{P}_B,$$

$$\bar{\mathbf{Q}}_A = e^{-j\Phi_x} \mathbf{Q}_A, \quad \bar{\mathbf{Q}}_B = -e^{-j\Phi_x} \mathbf{Q}_B,$$

$$\bar{\mathbf{U}}_A^{II} = -e^{-j\Phi_x} \mathbf{U}_A^{II}, \quad \bar{\mathbf{U}}_B^{II} = e^{-j\Phi_x} \mathbf{U}_B^{II},$$

$$\bar{\mathbf{U}}_A^{III} = -e^{-j\Phi_x} \mathbf{U}_A^{III}, \quad \bar{\mathbf{U}}_B^{III} = e^{-j\Phi_x} \mathbf{U}_B^{III},$$

$$\bar{\mathbf{R}}_A = -\mathbf{R}_A, \quad \bar{\mathbf{R}}_B = \mathbf{R}_B,$$

$$\bar{\mathbf{S}}_A = -\mathbf{S}_A, \quad \bar{\mathbf{S}}_B = \mathbf{S}_B.$$