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On the Capacity and Diversity of Cooperative Wireless Communication Networks

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To my parents

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Summary

The explosive expansion of wireless communication applications has imposed great demand in developing wireless communication systems with high spectrum efficiency, high energy efficiency and good quality of service. Cooperative communication has been proposed to meet this demand and is becoming one of the most promising techniques for future wireless communication systems. Taking advantage of the broadcast nature of wireless communication systems, cooperative communication allows users in a network to help each other with the information transmission, which enables more efficient usage of the network resources and/or provides more robust communication links for the network users.

In this thesis, we study the effect of user cooperation in wireless communication networks from both spectrum efficiency and communication reliability perspective. In particular, information transmission rates and fundamental tradeoffs between diversity and multiplexing gains are the main focuses. Channel scenarios such as the single-user multi-hop relay channel, the cooperative multiple-access channel (CMAC), and the interference channel (IFC) with source cooperation are investigated.

For the single-user multi-hop relay channel, we consider a scenario where all the nodes are equipped with multiple antennas and the relays work in a full-duplex mode. We propose an amplify-and-forward (AF) based space-time relay scheme and show that with proper design of space-time codes at the source, the diversity-multiplexing tradeoff (DMT) upper bound of the channel can be achieved. The proposed relay scheme essentially creates an artificial multipath channel, enabling the exploitation of multipath diversity.

We then move on to the cooperative multiple-access channel and study the effect of source cooperation on the DMT performance. All the nodes in the network are assumed to be equipped with multiple antennas and the K sources can operate at different multiplexing levels (asymmetric). Based on Wyner-Ziv-type compress-and-forward (CF) cooperation, we propose a transmission scheme and show that the DMT upper bound of the channel can be achieved. We find that for CF-based cooperation to be DMT optimal, each source should properly allocate its transmission resources (power, antenna) for the transmission of its own information and the relaying of the messages from the other sources. As a by-product of the DMT analysis, an achievable rate region for the K -user MAC with source cooperation is also proposed. Based on the DMT result, we propose a way of partitioning the degree-of-freedom (DOF) region of the channel. The partition reflects the effect of user cooperation on the

diversity performance for each targeted multiplexing rate. Based on the partition, efficient cooperation schemes can be chosen for each targeted diversity order and multiplexing rate.

Following that, the rate region and the DMT behavior of the two-user interference channel (IFC) with source cooperation are studied. Based on a combined decode-and-forward (DF) and CF cooperation strategy, an achievable rate region for the channel under full-duplex cooperation constraint is derived by applying a proposed coding scheme that jointly implements rate splitting, block Markov coding, superposition coding, Gel'fand-Pinsker coding and backward decoding. The proposed rate region extends some existing rate regions of the channel.

The DMT behavior of the two-user IFC with source cooperation is first investigated by assuming full-duplex source cooperation and full channel state information at the transmitter (CSIT). We derive a DMT upper bound using a capacity upper bound and two DMT lower bounds based on two DF-based cooperation schemes. It is shown that in most channel settings, the upper bound is achievable, and thus is tight. We then assume only partial CSIT and study the DMT behavior of the channel with both full-duplex and half-duplex source cooperation in a scenario where the long-term fading of all the links are of comparable strength. Under the full-duplex source cooperation constraint, a transmission scheme which requires only one bit of feedback information for each link between the sources and the destinations is proposed. We show that the proposed transmission scheme achieves a DMT that meets the DMT upper bound, and thus is DMT optimal. Under the half-duplex constraint, a capacity upper bound and a corresponding DMT upper bound of the channel are derived. We then propose a transmission scheme to show that the DMT upper bound can indeed be achieved. Similar to the full-duplex case, only a few bits of feedback information concerning the magnitude of each link between the sources and the destinations are required to implement the proposed transmission scheme. Our DMT analysis shows that even under the half-duplex constraint, communication with source cooperation yields higher diversity gains than that without cooperation over all the multiplexing rates.

List of Abbreviations

AF	Amplify-and-forward
AWGN	Additive white Gaussian noise
BC	Broadcast channel
BER	Bit-error rate
CF	Compress-and-forward
CMAC	Cooperative multiple access channel
CSI	Channel state information
CSIT	Channel state information at transmitter
DDF	Dynamic decode-and-forward
DF	Decode-and-forward
DMT	Diversity-multiplexing tradeoff
DPC	Dirty-paper coding
FF	Flip-forward
HK	Han-Kobayashi
IFC	Interference channel
MAC	Multiple-access channel
MAF	Multiple-access amplify-and-forward
MARC	Multiple-access relay channel
NAF	Non-orthogonal amplify-and-forward
PEP	Pairwise error probability
SAF	Slotted amplify-and-forward
SF	Space-frequency
SNR	Signal-to-noise ratio
ST	Space-time
WZ	Wyner-Ziv

List of Symbols

X	Random variables
x	Realizations of random variables
x^n	Length n vector
x_t^n	The t th element of x^n
$\mathbf{A}'$	Transpose of matrix $\mathbf{A}$
$(\cdot)^\dagger$	Conjugate transpose of a matrix or conjugate of a complex scalar
$H(\cdot)$	Shannon entropy
$h(\cdot)$	Differential entropy
$I(X; Y)$	Mutual information between random variables X and Y
$\mathbb{E}\{X\}$	Expectation of random variable X
$\mathcal{CN}(0, \sigma^2)$	Complex normal distribution with zero mean and variance σ^2
$\max\{x, y\}$	The maximum of x and y
$\min\{x, y\}$	The minimum of x and y
$(x)^+$	$(x)^+ \triangleq \max\{0, x\}$
$\bar{x}$	$\bar{x} \triangleq 1 - x$ for $0 \leq x \leq 1$
$\log$	Base-2 logarithm
$\text{diag}(\mathbf{x})$	A diagonal matrix with the elements of $\mathbf{x}$ being the diagonal elements
$\text{vec}(\mathbf{X})$	A vector obtained by stacking the columns of the matrix $\mathbf{X}$
$g(x) = O(f(x))$	Indicates $\lim_{x \rightarrow \infty} \frac{g(x)}{f(x)}$ is a constant for functions $g(x)$ and $f(x)$
$f(x) = \Theta(g(x))$	Indicates $f(x) = O(g(x))$ and $g(x) = O(f(x))$
$f(x) \doteq g(x)$	Indicates $\lim_{x \rightarrow \infty} \frac{\log(f(x))}{\log x} = \lim_{x \rightarrow \infty} \frac{\log(g(x))}{\log x}$
$f(x) \dot{\leq} g(x)$	Indicates $\lim_{x \rightarrow \infty} \frac{\log(f(x))}{\log x} \leq \lim_{x \rightarrow \infty} \frac{\log(g(x))}{\log x}$
$f(x) \dot{\geq} g(x)$	Indicates $\lim_{x \rightarrow \infty} \frac{\log(f(x))}{\log x} \geq \lim_{x \rightarrow \infty} \frac{\log(g(x))}{\log x}$

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Chapter 1

Introduction

1.1 Motivation

A long-standing pursuit in communication research societies and industries is to develop communication systems with good spectrum efficiency and/or communication reliability. In modern wireless communications, the rapid advancement of broadband wireless communication applications and services makes the need of taking the best use of the limited frequency spectrum to provide a reliable high-speed data transmission even more urgent. With the potential of providing a reliable and spectrum efficient communication, cooperative communication, first proposed in the early 1970's [1], has regained great research interests nowadays.

By taking advantage of the broadcast nature of wireless communications, cooperative communication allows users in a network to hear and help the information transmission of each other. With the assistance from other users, two remote users may be able to communicate without increasing the transmit power, a user may be able to run bandwidth-consuming wireless applications without asking for more spectrum resources, and two users may be able to reliably communicate with each other even if the communication link between them suffers from severe fading or shadowing. Cooperative communication is beneficial in improving network connectivity, enhancing power and spectrum efficiency, and improving communication reliability [2]. Moreover, comparing to other emerging techniques that could achieve similar performance advantage such as the multiple-input-multiple-output (MIMO) technique, cooperative communication is superior in deployment flexibility and hardware feasibility. These merits of cooperative communication make it a promising technique for next-generation wireless communication systems.

Despite its wide recognition, the fundamental issues of cooperative wireless communication such as the data transmission capacity and the error probability for each targeted rate are still largely unresolved. Consequently, the design of cooperative schemes that could

efficiently exploit the potential of cooperative communication is left with lack of guidance. It is thus of great interest to investigate the performance limits of cooperative communication and to design practical cooperation schemes that could approach or even achieve the limits.

1.2 Objectives

The main objective of this thesis is to understand the fundamental mechanisms of cooperative communication and to design practical cooperation schemes to better exploit its potentials. Towards this objective, the two fundamental issues of communication, i.e., spectrum efficiency and communication reliability, are studied for some basic cooperative wireless communication channel models using the achievable rate region and the diversity-multiplexing tradeoff (DMT) as the main performance metrics. As will be introduced in Chapter 2, the achievable rate region measures the system performance in fast fading channels, while the DMT reflects the system performance in slow fading channels. Specifically, we aim to achieve the following targets.

- To characterize the DMT behavior of the MIMO multihop relay channel and design practical relay scheme with good DMT performance.
- To evaluate the DMT of the MIMO cooperative multiple-access channel (CMAC) and design practical cooperation schemes with good DMT behavior.
- To design cooperation schemes for the general discrete memoryless interference channel (IFC) with source cooperation in order to extend the existing achievable rate region of the channel.
- To study the DMT of the fading IFC with source cooperation and design practical transmission schemes with good DMT behavior.

1.3 Major Contributions of the Thesis

In this thesis, we evaluate the performance limits of some basic cooperative wireless communication networks from the perspectives of the achievable rate region and the diversity-multiplexing tradeoff. Based on that, we design practical cooperative transmission schemes to approach or even achieve the performance limits. The major contributions are listed as below.

- i. For the Rayleigh-faded MIMO multihop relay channel where one source-destination pair communicates through multiple layers of full-duplex relays and all the communication nodes are equipped with multiple antennas, we propose an amplify-and-forward (AF) based space-time relay scheme and show that it is DMT optimal,

i.e., the DMT lower bound of the channel derived from the proposed relay scheme achieves the DMT upper bound. Based on that, we show that the DMT behavior of the channel is fully characterized.

- ii. For the K -user MIMO CMAC, we fully characterize its DMT behavior by deriving both the DMT upper bound and a DMT lower bound and showing that the lower bound meets the upper bound. The DMT upper bound is derived from the capacity cut-set bound and the lower bound is achieved by a proposed Wyner-Ziv (WZ) type compress-and-forward (CF) based transmission scheme. As a by-product, an achievable rate region with WZ-CF based cooperation is also obtained for the general K -user discrete memoryless CMAC.
- iii. We derive a new achievable rate region for the discrete memoryless two-user IFC with source cooperation using a cooperation scheme that combines decode-and-forward (DF) and WZ-CF. The derived achievable rate region is shown to extend the existing rate region results for some channel scenarios.
- iv. We evaluate the DMT behavior of the Rayleigh-faded two-user single-antenna IFC with source cooperation. Under the assumption of full channel state information at transmitter (CSIT) and full-duplex source cooperation, we derive an DMT upper bound based on a capacity upper bound and two DMT lower bounds based on two DF-based cooperation schemes. For most of the channel settings, the upper bound is shown to be achievable. We then study the DMT behavior of the channel by assuming only partial CSIT. Under both full-duplex and half-duplex source cooperation constraints, the DMT behaviors are fully determined for a channel scenario where the long-term fading effects of all the links are comparable. Under each cooperation constraint, a transmission scheme requiring only partial CSIT is proposed and shown to be DMT optimal. The partial CSIT is provided by destination feedback. For the full-duplex case, each destination represents the magnitude of each of its incoming links using 1 bit of information and feeds this information back to both sources. For the half-duplex case, the feedback information is obtained by representing the magnitude of each of its incoming links using no more than 3 information bits.

1.4 Organization of the Thesis

In Chapter 2, we provide some background knowledge on network information theory and cooperative communication networks. The basic performance metrics used in this thesis are also introduced. In Chapter 3, we focus on the DMT evaluation of the MIMO multihop relay channel. Chapter 4 is devoted to the DMT study of the MIMO CMAC. In Chapter 5, we provide a new achievable rate region for the discrete memoryless two-user IFC with

source cooperation. The DMT evaluation of the Rayleigh-faded two-user IFC with source cooperation is provided in Chapter 6. We conclude this thesis in Chapter 7, where some possible directions for future work are also proposed.

Chapter 2

Background Review

In this chapter, we first briefly review the progresses made on cooperative communications over the past several decades. The basic concepts and approaches of user cooperation are then introduced. Following that, we provide some preliminary knowledges about the performance metrics used in the thesis. We first introduce the information-theoretical capacity of communication networks and summarize some known capacity results. The concept of DMT is then explained and the DMT results on cooperative communication networks are summarized.

2.1 Cooperative Communications

The idea of using a third communication node to assist signal transmission between an information source and a sink has a long history of practice. One of the primary applications of user cooperation is the multi-hopping based long-distance communication where repeaters are used to re-boost attenuated signals.

Recent researches on user cooperation was initiated by van der Meulen in the early 1970's [1]. In [1], van der Meulen constructed a three-terminal relay channel and derived upper and lower bounds on its channel capacity. The capacity of the relay channel is further exploited by Cover and El Gamal in the prominent paper published in 1979 [3]. It was shown that with the help of a relay, the data transmission capacity between a source and a destination can be significantly improved. The capacity results, the relay schemes and the analysis approaches proposed in [3] have a significant importance on promoting researches in information theory as a whole and in cooperative communications as a particular. Besides the relay channel, user cooperation in multiple-access channels has also attracted plenty of research interests in the 1970's and 1980's. A large number of illuminating theoretical works concerning the capacity of the multiple-access channel (MAC) with feedback [4]–[9],

the MAC with partially cooperating encoders [10] and the MAC with cribbing encoders [11] have emerged during that period. All these works have shown that the capacity of MAC can be improved by feedback or source cooperation. Some other outstanding theoretical works in that period that have imposed great influence on the contemporary researches includes [12]–[21]. In particular, the capacity region of multiple-access channel was found by Ahlswede in [12]. The Wyner-Ziv source coding with receiver side information proposed in [15] has been widely used in distributed source compression [22] and in cooperative communication networks [3]. The Han-Kobayashi (HK) coding scheme for the interference channels proposed in [18] is one of the most spectrum efficient coding schemes for the two-user IFC developed so far. The Gel'fand-Pinsker coding scheme proposed in [19] and the corresponding dirty-paper coding (DPC) for the Gaussian channels proposed in [20] have been used for arranging source transmission in the broadcast channel (BC). It has been proven that for both the scalar and vector Gaussian BCs, the DPC rate regions achieve the channel capacity regions [23], [24].

After a development slow-down in the 1990's, propelled by the development requirements of next-generation broadband wireless communications and the awareness on the potential of cooperative communications, cooperative communications was experiencing a great resurgence both in research and in application started from the early 2000's. Led by some pioneer works [25]–[29], a huge amount of research efforts have been put on understanding and exploiting the potential of communications from the perspectives of spectrum efficiency [25], [26], [30]–[79], reliability [28], [29], [80]–[101], and power efficiency [35], [102], [77], [103], etc.

From the spectrum efficiency perspective, current researches basically follow two major directions. One direction focuses on the micro scale and attempts to characterize the data transmission capacity for each user in the network. The other direction focuses on the macro scale and investigates the aggregate capacity of the entire network. Major metrics used in the first research direction are the achievable rate region, the capacity region, and the degree of freedom region. The major metric used in the second direction is the capacity scaling law which measures the scaling of the sum capacity with respect to the number of users in the network.

Following the first direction, capacities results on the relay channel [30]–[41], the cooperative multiple-access channel [42], [43], the multiple-access relay channel [32], [44], the broadcast channel with receiver cooperation [45], the relay broadcast channel [46], the interference channel with source and destination cooperation [47]–[56], and the interference relay channel [57]–[61] are obtained. All the capacity results unexceptionally showed that with user cooperation, the capacity regions could be greatly improved as compared to the ones without cooperation. More discussions on the information-theoretical capacity of communication networks will be provided in Section 2.4.

Following the second direction, capacity scaling laws have been studied under different assumptions on network setting, channel attenuation and node mobility. For extended random wireless networks, where the area of the network scales as the number of users growth while the node density remains fixed, with n random located users and a path-loss exponent greater than 3, the best achievable capacity scaling is shown to be $\Theta(\sqrt{n})$ [62]–[67]. A multihop transmission scheme, proposed by Gupta and Kumar in their seminal paper [25] is proven to be order-optimal. For the extended network with path-loss exponent in between 2 and 3 and the dense random wireless networks, where the total area of the network is fixed and the node density increases as more users come in, the capacity can at best be scaled as $O(n \log n)$. Under these channel settings, the multihop transmission, which again achieves a scaling of $\Theta(\sqrt{n})$ [25], [67] is strictly suboptimal. Better capacity scaling behavior can be obtained by user cooperation [68]–[70]. Particularly, in [70], Özgür, Lévêque and Tse proposed a hierarchical cooperation scheme based on the idea of local user cooperation and long-range distributed MIMO transmission. The hierarchical cooperation scheme was shown to achieve a capacity scaling of $\Theta(n^{1-\epsilon})$ (for any $\epsilon > 0$) and thus is near optimal. Further developments of the idea of hierarchical cooperation can be found in [71]–[74]. Besides the large random wireless networks, the capacity scaling of the wireless relay networks has been studied in [75]–[78]. For the single-source single-destination K -relay network, it was shown in [75]–[77] that the data transmission capacity between the source and the destination scales as $O(\log(K))$, i.e., the channel capacity grows in log scale with respect to the number of relays. For the multiple-source-destination-pair multiple-relay channel, it was shown in [78] that the relays help to decouple the network so that each source-destination pair can communicate as if there is no interference from other simultaneously communicated user pairs. Therefore, capacity scaling results from the single-source-destination-pair multi-relay network can be directly applied in the case with multiple source-destination pairs. The effect of node mobility on the network capacity was studied in [79]. It was shown that by allowing for each mobile node to store its packets and forward only when it moves close to its desired destination or some intermediate relay nodes, a capacity scaling of $\Theta(n)$ can be achieved.

From the communication reliability perspective, it is well understood that user cooperation introduces cooperation diversity and is effective in combating fading. Following the works in [28] and [29], a huge amount of research works focused on designing cooperation schemes to exploit the diversity advantage of cooperative communication under various network settings, such as the single-user single-relay channel [80], the single-user multiple-relay channel [81], [82], and the cooperative multiple-access channel [83]–[86]. Instead of solely focusing on exploiting the diversity advantage at a fixed transmission rate, many research works attempt to study the diversity advantage over the whole rate region. The metric diversity-multiplexing tradeoff is adopted as a major performance metric. The works in

[87]–[101] investigated the DMT of various cooperative wireless communication networks and designed cooperation schemes with optimal or near-optimal DMT behavior. A detailed summary of the DMT results is provided in Section 2.5.

Besides capacity and diversity, power efficiency of a network can also be improved by user cooperation, as shown in [35], [77], [102], [103]. The work in [102] evaluated the energy efficiency of long-range transmissions. It was shown that by asking the closely located users to share and cooperatively send information, forming a cooperative MIMO, the energy efficiency can be significantly improved over the traditional multihop based transmission. In [35], the upper and lower bounds on the minimum energy-per-bit of the Gaussian relay channel were studied. It was shown that energy consumption can be effectively improved by using a relay. The works in [77] and [103] investigated the energy scaling of the multiuser multiple-relay channel. It was shown in [103] that with respect to the number of relays, K , the per-bit energy consumption can be scaled as better as $\Theta(1/K)$, indicating the ability of relays in improving system power efficiency.

2.2 General Model of Communication Networks

A communication network is composed of a set of communication nodes and a set of communication links that connect the nodes. If a communication link exists between two nodes, then the two nodes are said to be connected. Depending on the transmission media, communication networks can be classified into wireline networks and wireless networks. Our focus in this thesis is on wireless communication networks. Each node in the network can act as a transmitter, a receiver, or both. Consider a network with a set $\mathcal{U}$ of communication nodes, the received signal $y_{d,t}$ at receiver node $d \in \mathcal{U}$ during time instance t can be expressed as

$$y_{d,t} = f_{d,t}(x_{\mathcal{S}}^t, y_d^{t-1}), \quad (2.2.1)$$

where the set $\mathcal{S} \subseteq \mathcal{U}$ is the set of transmitter nodes that are connected to node d , $x_{\mathcal{S}} = \{x_s, s \in \mathcal{S}\}$, and x_s is the signal transmitted from node s . The transmitting power of the signals satisfies $E\{|X_s|^2\} \leq P_s$. The function $f_{d,t}$ is determined by the channel between the transmitter nodes in $\mathcal{S}$ and the receiver node d . If x_s and y_d ($s, d \in \mathcal{U}$) are both taken from some finite alphabets $\mathcal{X}_s$ and $\mathcal{Y}_s$, respectively, and the function $f_{d,t}$ is determined by a probability transmission $p(y_d|x_{\mathcal{S}})$, then the channel is said to be *discrete*. From (2.2.1), it can be seen that $y_{d,t}$ depends on the signals transmitted in time instances $0, \dots, t$ and all the previous received $y_{d,0}, \dots, y_{d,t-1}$. In this sense, the channel has memory. If $y_{d,t}$ only depends on $x_{\mathcal{S},t}$, i.e.,

$$y_{d,t} = f_{d,t}(x_{\mathcal{S},t}), \quad (2.2.2)$$

then the channel is said to be *memoryless*. In this thesis, we only consider memoryless channels.

Denoting $W_{s,d}$ as the message of source s intended for destination d in a transmission block of length n , $x_{s,t}$ (for $t = 1, \dots, n$) is generated as a function of $W_{s,d}$ ($d \in \mathcal{D}$) and y_s^{t-1} , where $\mathcal{D}$ is the set of destination nodes that receives from source s . With the received block of signals, destination node d tries to decode the source message from y_d^n , obtaining $\hat{W}_{s,d}$. The probability of error is defined as $P_e = \Pr(W_{s,d} \neq \hat{W}_{s,d})$. An information transmission rate $R_{s,d}$ between s and d is said to be achievable if $P_e \rightarrow 0$ when $n \rightarrow \infty$.

In the case that the links are Rayleigh faded and suffer from additive white Gaussian noise (AWGN), (2.2.2) can be further expressed as

$$y_{d,t} = \sum_{s \in \mathcal{S}} h_{s,d}(t) x_{s,t} + z_{d,t}, \quad (2.2.3)$$

where $h_{s,d}(t) \sim \mathcal{CN}(0, 1)$ represents the fading coefficient between nodes s and d at time instance t and $z_{d,t} \sim \mathcal{CN}(0, \sigma_w^2)$ denotes the AWGN.

Some elementary channel models for the wireless communication network include the multiple-access channel, the broadcast channel, and the interference channel. In a multiple-access channel, multiple sources communicate with a common destination. In the Rayleigh-fading case, the channel model (2.2.3) becomes

$$y_{d,t} = \sum_{s \in \mathcal{U} \setminus \{d\}} h_{s,d}(t) x_{s,t} + z_{d,t}. \quad (2.2.4)$$

In a broadcast channel, there is only one source node and all the other nodes are destination nodes, receiving information sent from the source. The channel model (2.2.3) becomes

$$y_{d,t} = h_{s,d}(t) x_{s,t} + z_{d,t}, \text{ for } d \in \mathcal{U} \setminus \{s\}. \quad (2.2.5)$$

In an interference channel, according to the definition in [16], there is a one-to-one correspondence between the sources and the destinations. Each source-destination pair is distinguished. The channel model can still be expressed as (2.2.3) with half of the nodes in $\mathcal{U}$ serving as sources and the other half as destinations.

2.3 Basics of Cooperative Communication Networks

Due to the broadcast nature of wireless communications, the signal transmitted by a source node may be overheard by some nearby nodes (not necessary the desired destination). If

these non-destination nodes are allowed to help forward the source signal to its destination, then the network becomes a cooperative communication network. Some basic channel models of cooperative communication networks include:

- Single-user single-relay channel, where the communication between a source-destination pair is assisted by a relay node. The relay node does not have information of its own to transmit, and only helps to forward the source information. Despite its simplicity, the single-user single-relay channel model is the most basic cooperation communication channel model. Based on this model, Cover and El Gamal, in their prominent paper [3], proposed several illuminating ideas on ways of implementing cooperation and on approaches to analyze the achievable rates. Their research work has imposed great influences on researches of network information theory and cooperative communications.
- Single-user two-hop multiple-relay channel, where a pair of source and destination communicates with the help of a layer of multiple relay nodes. Since all the relay nodes have access to the source signal, they can help the source transmission like a distributed antenna array. Some space-time (ST) processing techniques used in the MIMO system can thus be applied [81], [82].
- Single-user multihop relay channel, where a pair of source and destination communicates through multiple hops of relays, with multiple relays for each hop. This channel model is a multihop extension of the previous two-hop channel. Like the two-hop channel, the first hop can be viewed as a broadcast channel from the single source to multiple relays, and the last hop can be viewed as a multiple-access channel. For the middle hops, however, the transmission between two layers of relay resembles a distributed MIMO channel. This gives some new features to the multihop channel as compared to the two-hop channel. Moreover, in a fading environment, since each hop of transmission will suffer from fading, the task of combating fading in a multihop network is tougher than in a two-hop network.
- Multiple-user relay channel, where the communication of several source-destination pairs are assisted by relays. This channel model is a multiuser extension of the single-user relay channels. Due to the existence of multiple source-destination pairs, each destination sees interference from the other undesired sources. The presence of interference makes the handling of the multiuser case more technically challenged than the single-user case. If we restrict all the sources communicate with one common destination, or all destinations receive from one common source, then two variations of the multiple-user relay channel can be obtained, namely, the multiple-access relay channel and the broadcast relay channel, respectively.
- Cooperative multiple-access channel, where the users (information sources) can ex-

change information and assist with each other to communicate with their common destination. From each user's perspective, all the other users function like relay nodes to assist its transmission. Unlike the relay channels, in CMAC, however, all the users have their own information to transmit, and each user is asked to help with the other user. By information exchange and collaboration, the multiple users form a distributed transmit antenna array. Space-time processing can thus be used to arrange the transmission, as proposed in [29] and [83]. For networks with a large number of randomly located source nodes, instead of asking each user to indiscriminately help all the other users, it may be better to ask the user to only help a group of users, such as the users that are located nearby. The problems of user grouping and selection for cooperation have been studied in [85] and [86].

- Broadcast channel with destination cooperation, where the destination nodes exchange their received signal to help each other in decoding. In this sense, the destination nodes form a distributed receive antenna array. Since the destination nodes only listen and are not supposed to transmit, cooperation between destinations is not that natural as source cooperation. To enable destination cooperation, separate cooperation links or time slots are usually allocated for the destinations to share information.
- Interference channel with source/destination cooperation, where the sources and/or the destinations can exchange information to collaboratively transmit and/or decode. By source cooperation, the sources could know part of each other's message. They can then collaborate in broadcasting their messages to the destination nodes. This makes the IFC with source cooperation mimic a broadcast channel with a multiple-antenna source. Some of the coding schemes for the broadcast channels, such as the dirty-paper coding, can then be applied by the sources to arrange their transmission. Similarly, by destination cooperation, the destinations can perform joint decoding. The effect of interference can thus be partially mitigated [56]. This makes the channel resemble a multiple-access channel with a multiple-antenna receiver.

Cooperative communication can be realized in a causal or a non-causal way. By causal cooperation, the cooperating nodes have no *a priori* knowledge about each other. They must first exchange their information before performing any form of cooperation. Depending on the applications and the hardware limitations, causal cooperation can be achieved in a full-duplex or half-duplex mode. With full-duplex cooperation, each node can transmit and receive at the same time. With half-duplex cooperation, each node can either transmit or receive, but not both. Full-duplex cooperation puts more stringent constraints on the hardware. By non-causal cooperation, some cooperating nodes have full knowledge about the other's information in advance. They can thus implement cooperative transmission without

prior information exchange. Non-causal cooperation is commonly assumed in cognitive radio networks and sensor networks. In this thesis, we only consider causal cooperation. Unless otherwise stated, the term cooperation indicates a causal cooperation.

To implement cooperation, each cooperating node processes the signals it received during the information exchange phase and then forwards the processed signals toward some desired destinations in the next appropriate time. This whole procedure, including the choice of the signals to be sent cooperatively, the method used in processing the signals and the ways to forward the processed cooperation signals, is governed by cooperation protocols. In the following, we use the single-user single-relay channel as an example to illustrate the three basic cooperation protocols that are widely used in the literature.

- i. Amplify-and-forward (AF): By the AF protocol, the relay simply scales its received source signal according to its transmit power constraint and forwards the scaled signal in the next transmission slot. The destination decodes the source message using the signals received from the source and the relay.
- ii. Compress-and-forward (CF): By the CF protocol, the relay first maps its received signal y_r into another signal $\hat{y}_r$, and then encodes $\hat{y}_r$ into a codeword x_r by taking the signal y_d received at destination as side information. x_r is then transmitted in the next transmission slot. Since $\hat{y}_r$ and y_d are correlated, in generating x_r , Wyner-Ziv coding is used to reduce the coding rate. With the received signals, the destination first decodes x_r , based on which $\hat{y}_r$ is recovered. Using y_d and the recovered $\hat{y}_r$, the source message is then decoded.
- iii. Decode-and-forward (DF): By the DF protocol, the relay decodes the source message from y_r , re-encodes it into a new codeword x_r and re-transmits it in the next transmission slot. To enable coherent decoding at the destination, block Markov coding is used by the source. The destination can decode the source message using successive decoding [3], sliding-window decoding [6] or backward decoding [11]. For the single-user channels, all the decoding methods have the same performance in terms of achievable rates. But for channels with multiple sources, backward decoding is superior than the other two decoding methods [39].

As compared to the other two protocols, the most prominent merit of AF is simplicity. Its implementation only requires simply signal processing with little software and hardware modifications. The major problem with the AF protocol is the accumulation of noise and channel fading, especially in the multihop multi-relay channels, as indicated in [104]. Some variations of the AF protocols include the non-orthogonal AF (NAF) protocol [80], [87] and the slotted AF (SAF) protocol [92]. The CF protocol and the DF protocol are originally proposed by Cover and El Gamal in [3]. Similar to AF, CF does not require decoding of the source message at the relay, and can be classified as an analog cooperation protocol. Unlike

AF, however, CF re-transmits a re-encoded codeword based on the received signal, and thus avoids the noise and channel fading accumulation problem encountered by AF. As a result, CF has better performance than AF in terms of achievable rate and error probability. A major drawback of CF is that it requires the relay to have full channel state information which increases the implementation complexity. Some variations of CF are hash-and-forward [36] and quantization-and-map [41]. Particularly, it was shown in [41] that the achievable rate of the quantization-and-map protocol is within a constant gap to the cut-set capacity upper bound of the relay channel. The DF protocol requires the relay to decode the source message and is a digital cooperation protocol. It was shown in [3] that for a physically degraded relay channel, DF is capacity achieving. For general relay channels, DF achieves a higher rate than CF when the relay is placed near the source, while the achievable rate of CF is higher when the relay is located near the destination [32]. The reason is that when the source-relay link is weak, requiring the relay to fully decode the source message becomes the main factor that limits the maximum achievable rate. In this case, the CF protocol, without performing decoding, could achieve a better rate. Furthermore, for cases where all nodes in the channel are equipped with multiple antennas, requiring the relay to fully decode is strictly suboptimal in terms of diversity performance, as indicated in [90]. One variation of the DF protocol is the dynamic DF (DDF) protocol proposed in [87].

2.4 Capacity of Communication Networks

The capacity of a communication network defines the maximum information transmission rate between any two communicating nodes in the network. It is important in guiding the design of practical network protocols and benchmarking the performance of network protocols. Determining the capacity of communication networks has been a long-standing research interest for many decades. Despite the tremendous efforts made, except for some specified network models, the communication network capacity is not known in general. In face of the huge difficulties in finding the exact capacity, people then resort to evaluate the capacity upper bounds, lower bounds (or the achievable rates), and, more recently, the degrees of freedom (DOF). It also constitutes an alternative way of characterizing the network capacity. For a given network, if we can find an upper bound and a lower bound that are *tight*, i.e., the upper bound meets the lower bound, then the capacity of the network can be fully determined. Or less stringently, if the gap between the upper bound and the lower bound is within a constant number of bits, then it still gives a good characterization of the network capacity with a constant number of bit gap from the exact capacity. In the following, we review the capacity results of some basic communication networks.

2.4.1 Cut-Set Upper Bound

Finding the capacity upper bounds often requires some sophisticated application of information theoretical inequalities and smart building of genie-aided channels. For different network topologies, we may have different upper bounds. Even for the same network, different upper bounding approaches could result in different upper bounds. Despite the differences, there is one common capacity upper bound applicable to all the communication networks. That is the cut-set upper bound.

A *cut* across a network divides the network into two parts. The cut-set upper bound delineates the upper bound for the maximum rate of information flow across any cut in the network. For the network model in (2.2.2), denote $\mathcal{S}$ as the set of nodes in one side of an arbitrary cut and its complementary $\mathcal{S}^c$ as the set of nodes in the other side of the cut. The achievable rate $R_{i,j}$ between any node $i \in \mathcal{S}$ and $j \in \mathcal{S}^c$ is bounded by the max-flow min-cut theorem [105]

$$\sum_{i \in \mathcal{S}, j \in \mathcal{S}^c} R_{i,j} \leq I(X_{\mathcal{S}}; Y_{\mathcal{S}^c} | X_{\mathcal{S}^c}). \quad (2.4.1)$$

The cut-set upper bound has been further developed in [31] to include practical constraints, such as the half-duplex cooperation constraint. Denote M as the number of states in the networks, and $t_m, (m = 1, \dots, M)$ as the portion of time that the network stays in state m , the cut-set upper bound for the achievable rates $R_{i,j}$ becomes

$$\sum_{i \in \mathcal{S}, j \in \mathcal{S}^c} R_{i,j} \leq \sum_{m=1}^M t_m I(X_{\mathcal{S}}^{(m)}; Y_{\mathcal{S}^c}^{(m)} | X_{\mathcal{S}^c}^{(m)}), \quad (2.4.2)$$

where $X_{\mathcal{S}}^{(m)}$ denotes the transmitted signals from sources in set $\mathcal{S}$ when the network operates in state m . $Y_{\mathcal{S}^c}^{(m)}$ and $X_{\mathcal{S}^c}^{(m)}$ are similarly defined.

2.4.2 Lower Bounds

The lower bounds are usually found based on some designed coding or transmission schemes. Therefore, the lower bounds are often termed as the achievable rates. In the following, we review the capacity lower bound results of some basic channel models in communication networks.

(i). Multiple-Access Channel

For the two-user MAC given in (2.2.4) with $\mathcal{S} = \{1, 2\}$ and $d = 3$, it has been shown that by independent encoding at each source and joint typical set decoding, the rate region can be achieved for the discrete memoryless MAC is the convex closure of the following

region: [12]

$$R_1 \leq I(X_1; Y_3 | X_2) \quad (2.4.3a)$$

$$R_2 \leq I(X_2; Y_3 | X_1) \quad (2.4.3b)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_3). \quad (2.4.3c)$$

Here, since each source only transmits to one destination (i.e., unicast), we express the rate from source i to destination node 3 as R_i , instead of $R_{i,3}$, for simplicity. In the following, for all the unicast cases, we express the rate of source i using R_i . It can be checked that the capacity lower bound (2.4.3) meets the cut-set upper bound of the MAC. Therefore, the rate region (2.4.3) is indeed the capacity region of the channel.

The two-user rate region (2.4.3) can be extended to the case with K users. The capacity of the K -user discrete memoryless MAC is shown to be

$$\sum_{i \in \mathcal{U}} R_i \leq I(X_{\mathcal{U}}; Y_{K+1} | X_{\mathcal{U}^c}), \text{ for all } \mathcal{U} \subseteq \mathcal{S} = \{1, \dots, K\}. \quad (2.4.4)$$

The capacity can still be achieved by a scheme that applies independent source coding and joint typical set decoding. The capacity region of the discrete memoryless MAC is fully determined.

(ii). Broadcast Channel

Consider a two-user broadcast channel and denote the two destinations as nodes 1 and 2, respectively. The source sends an independent message to each destination. Depending on the transmission schemes applied by the source, different achievable rate regions of the channel can be obtained. There are two well-known transmission schemes proposed in the literature. One is based on Gel'fand-Pinsker binning and the other uses superposition coding.

Using Gel'fand-Pinsker binning, the information rates R_i ($i = 1, 2$) (for brevity, we use R_i to represent $R_{s,i}$) that can be achieved is contained in the following region:

$$R_1 \leq I(U_1; Y_1) \quad (2.4.5a)$$

$$R_2 \leq I(U_2; Y_2) \quad (2.4.5b)$$

$$R_1 + R_2 \leq I(U_1; Y_1) + I(U_2; Y_2) - I(U_1; U_2), \quad (2.4.5c)$$

where U_1 and U_2 are generated from a joint probability distribution $p(u_1, u_2)$, and the source signal x is generated as a function of u_1 and u_2 . $[U_1, U_2] - X - [Y_1, Y_2]$ forms a Markov chain.

Based on the superposition coding, the achievable rate region is

$$R_2 \leq I(U; Y_2) \quad (2.4.6a)$$

$$R_1 \leq I(X; Y_1|U) \quad (2.4.6b)$$

$$R_1 + R_2 \leq I(X; Y_2), \quad (2.4.6c)$$

where U carries the message targeted for destination node 2. X carries both messages targeted for destinations 1 and 2 and is generated on top of U . $U - X - [Y_1, Y_2]$ forms a Markov chain.

A broadcast channel is said to be physically *degraded* if $p(y_1, y_2|x) = p(y_1|x)p(y_2|y_1)$, i.e., $X - Y_1 - Y_2$ forms a Markov chain. It has been shown that for degraded broadcast channels, the rate region (2.4.6) achieves the channel capacity. Since all the scalar Gaussian BCs are degraded, superposition coding is optimal in capacity for scalar Gaussian BCs. Moreover, it can be shown that for the scalar Gaussian BC, the rate region of the Gel'fand-Pinsker binning scheme also achieves the capacity. Gel'fand-Pinsker binning is also capacity optimal for scalar Gaussian BCs.

For general broadcast channels (non-degraded), the capacity is still not known. But for vector Gaussian BCs, the channel capacity has been determined in [23], [24]. It was shown that the capacity region of the vector Gaussian BC with transmitting power constraint P is equal to the capacity region of its dual vector Gaussian MAC with the same transmitting power constraint. The capacity region of the vector Gaussian BC can be achieved by the dirty-paper coding, which is an application of Gel'fand-Pinsker binning in the Gaussian channels.

(iii). Interference Channel

We consider a two-user interference channel with a set of source nodes $\mathcal{S} = \{1, 2\}$ and a set of destination nodes $\mathcal{D} = \{3, 4\}$. Source i ($i = 1, 2$) communicates with destination node $i + 2$ at rate R_i . For the Gaussian IFC, its channel model can be expressed in the following standard form.

$$y_3 = x_1 + a_{2,3}x_2 + \tilde{z}_1, \quad (2.4.7)$$

$$y_4 = a_{1,4}x_1 + x_2 + \tilde{z}_2. \quad (2.4.8)$$

The channel is said to be under strong interference if $|a_{1,4}|^2 \geq 1$ and $|a_{2,3}|^2 \geq 1$. It is said to be under very strong interference if $|a_{1,4}|^2 \geq 1 + P_2$ and $|a_{2,3}|^2 \geq 1 + P_1$.

One lower bound of the two-user IFC is achieved by a scheme that lets the two destinations to decode messages from both sources. The achievable rate region is given as

follows:

$$R_1 \leq I(X_1; Y_3 | X_2) \quad (2.4.9a)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_3) \quad (2.4.9b)$$

$$R_2 \leq I(X_2; Y_4 | X_1) \quad (2.4.9c)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_4). \quad (2.4.9d)$$

This rate region is shown to achieve the capacity upper bound for the strong and very strong IFCs.

For general interference channels, however, the capacity region is not known, even for the two-user case. The best achievable rate region known so far is the HK region proposed by Han and Kobayashi in [18]. It is achieved by a coding scheme that splits the message of each source into two parts with one part (the common message part) being decoded by both destinations and the other part (the private message part) only decoded by its intended destination. A simplified expression of the HK rate region was recently found in [106], which is given as follows:

$$R_1 \leq I(X_1; Y_1 | V_2) \quad (2.4.10a)$$

$$R_2 \leq I(X_2; Y_2 | V_1) \quad (2.4.10b)$$

$$R_1 + R_2 \leq I(X_2, V_1; Y_2) + I(X_1; Y_1 | V_1, V_2) \quad (2.4.10c)$$

$$R_1 + R_2 \leq I(X_1, V_2; Y_1) + I(X_2; Y_2 | V_1, V_2) \quad (2.4.10d)$$

$$R_1 + R_2 \leq I(X_1, V_2; Y_1 | V_1) + I(X_2, V_1; Y_2 | V_2) \quad (2.4.10e)$$

$$2R_1 + R_2 \leq I(X_1, V_2; Y_1) + I(X_1; Y_1 | V_1, V_2) + I(X_2, V_1; Y_2 | V_2) \quad (2.4.10f)$$

$$R_1 + 2R_2 \leq I(X_2, V_1; Y_2) + I(X_2; Y_2 | V_1, V_2) + I(X_1, V_2; Y_1 | V_1), \quad (2.4.10g)$$

where V_1, V_2 are generated according to the probability distribution $p(v_1, v_2) = p(v_1)p(v_2)$, X_1, X_2 are generated on top of V_1 and V_2 according to $p(x_1, x_2 | v_1, v_2) = p(x_1 | v_1)p(x_2 | v_2)$. Based on the HK-scheme, the capacity of a two-user Gaussian interference channel has recently be characterized by to within one bit [107].

Results concerning the capacity of IFCs with more than two users only emerged very recently [108]–[111]. The transmission schemes proposed are based on structured coding and interference alignment.

(iv). Relay Channel

We consider a single-user single-relay channel with nodes 1, 2 and 3 serving as the source, the relay and the destination, respectively. Based on the DF and CF relay schemes, two capacity lower bounds are derived by Cover and El Gamal in [3]. Using DF relaying, the source can achieve the rate

$$R \leq \min\{I(X_1, X_2; Y_3), I(X_1; Y_2|X_2)\}, \quad (2.4.11)$$

where X_1 and X_2 are generated from the joint probability distribution $p(x_1, x_2) = p(x_2)p(x_1|x_2)$.

Based on the CF relay scheme, the following rate is achievable:

$$R \leq I(X_1; \hat{Y}_2, Y_3|X_2), \quad (2.4.12)$$

subject to the constraint

$$I(Y_2; \hat{Y}_2|X_2, Y_3) \leq I(X_2; Y_3), \quad (2.4.13)$$

where X_1 , X_2 and $\hat{Y}_2$ are generated according to $p(x_1, x_2, \hat{y}_2, y_2, y_3) = p(x_1)p(x_2)p(y_1, y_2|x_1, x_2)p(\hat{y}_2|x_2, y_2)$.

It has been shown in [3] that the DF-based lower bound (2.4.11) achieves the capacity of the *degraded* relay channel, where $X_1 - [X_2, Y_2] - Y_3$ forms a Markov chain.

2.4.3 Degrees of Freedom Region

The degrees of freedom of a channel is defined as the dimension of the received signal space. It is reflected by the pre-log factor of the channel capacity. For a network with S source nodes and D destination nodes and capacity region $\mathcal{C}(\text{SNR}_1, \dots, \text{SNR}_S)$ at signal-to-noise ratios $\text{SNR}_1, \dots, \text{SNR}_S$, the DOF region is calculated as

$$\mathcal{K} = \lim_{\text{SNR}_i \rightarrow \infty, i \leq S} \left\{ (k_{i,j})_{\substack{i=1, \dots, S \\ j=1, \dots, D}} \in \mathbb{R}_+^{SD} : (k_{i,j} \log \text{SNR}_i)_{\substack{i=1, \dots, S \\ j=1, \dots, D}} \in \mathcal{C}(\text{SNR}_1, \dots, \text{SNR}_S) \right\}. \quad (2.4.14)$$

For example, the DOF of the two-user MIMO MAC with M_1 and M_2 antennas at the two sources, respectively, and N_D antennas at the destination is given by

$$k_1 \leq \min\{M_1, N_D\} \quad (2.4.15a)$$

$$k_2 \leq \min\{M_2, N_D\} \quad (2.4.15b)$$

$$k_1 + k_2 \leq \min\{M_1 + M_2, N_D\}. \quad (2.4.15c)$$

Compared to the capacity bounds, DOF region is less technically challenging. It is still a good approximate for the channel capacity in the high SNR region. The DOF region has been widely used in characterizing the capacity of complicated networks such as networks with MIMO nodes and the interference networks [108]–[112].

2.5 Diversity-Multiplexing Tradeoff

The ergodic channel capacity discussed in the previous section is a sufficient performance metric for channels suffering from fast fading. For slow fading channels, however, outage probability should be used to describe the system performance at each targeted rate. Due to the fundamental tradeoff between the spectrum efficiency and error performance in communication systems, there is a tradeoff between transmission rate and outage performance. The increase in transmission rate will also lead to an increase in probability of outage. To characterize this fundamental tradeoff, diversity-multiplexing tradeoff was proposed [113].

Diversity determines the slope of the log-scaled outage error probability in the high SNR regime. For a system operating at signal-to-noise ratio SNR and rate R , diversity can be calculated from the outage probability $P_{\text{out}}(\text{SNR}, R)$ as

$$d = - \lim_{\text{SNR} \rightarrow \infty} \frac{\log P_{\text{out}}(\text{SNR}, R)}{\log \text{SNR}}. \quad (2.5.1)$$

Multiplexing rate is defined as the pre-log factor of the transmission rate $R(\text{SNR})$ in the high SNR regime, i.e.,

$$r = \lim_{\text{SNR} \rightarrow \infty} \frac{R(\text{SNR})}{\log \text{SNR}}. \quad (2.5.2)$$

Diversity is thus a function of multiplexing rate. In the high SNR regime, the tradeoff between outage probability and transmission rate can be well reflected by the tradeoff between diversity and multiplexing rate.

Diversity-multiplexing tradeoff was first defined by Zheng and Tse in the seminal paper [113], where the DMT of MIMO systems were evaluated. Following that, the DMT of various channel models such as the MIMO MAC [114], [115], the single-user single-relay channel [87], [88], [90], [100], the single-user multiple-relay channel [87], [89], [92], [94], [98], the multihop relay channel [93], [96], [97], the CMAC [87], [99], the MARC [90], [101], and the IFC [116]–[120] were investigated. A common method to determine the optimal DMT of a fading channel is to find the DMT upper bound and the lower bound and show that the two bounds are tight, i.e., the two bounds are coincided. The DMT upper bound can be calculated from the capacity upper bound. The lower bound can be derived based on some designed transmission schemes. A transmission scheme is said to be DMT optimal if its

achievable DMT achieves the optimal DMT of the channel. For a fading MIMO system with M_t transmit antennas and M_r receive antennas, it was shown in [113] that the optimal DMT is a piecewise line joining the curve $d(r) = (M_t - r)(M_r - r)$ for $r = 1, \dots, \min\{M_t, M_r\}$. A transmission scheme applying Gaussian coding at the source and maximum-likelihood (ML) decoding at the destination was shown to be DMT optimal.

A system is said to be *symmetric* if all the source users transmit at a same multiplexing rate r . In contrast, an *asymmetric* system corresponds to a system whose users may operate at different multiplexing gains. For the fading K -user symmetric MIMO MAC with M_r receive antennas at the destination and M_t transmit antennas at each of the K sources, the DMT is given in [114] as

$$d_{\text{MAC}}(r) = \begin{cases} d_{M_t, M_r}(r) & \text{if } 0 \leq r \leq \min\{M_t, \frac{M_r}{K+1}\} \\ d_{KM_t, M_r}(Kr) & \text{if } \min\{M_t, \frac{M_r}{K+1}\} < r \leq \min\{M_t, \frac{M_r}{K}\}, \end{cases} \quad (2.5.3)$$

where $d_{m,n}(r)$ represents the DMT expression of an $m \times n$ MIMO channel, and is given by $d_{m,n}(r) = (m - r)(n - r)$ for $r = 0, \dots, \min\{m, n\}$.

For the fading single-antenna single-user single-relay channel, the DMT upper bound is shown to be $d(r) = 2(1 - r)^+$ under both full-duplex and half-duplex cooperation constraints [90]. With full-duplex cooperation, this optimal DMT can easily be achieved by DF-based relaying. With half-duplex constraint, due to the tighter constraint on the use of relay resources, more elaborated transmission schemes should be designed to achieve the optimal DMT. Towards this end, transmission schemes like NAF, DDF, and CF have been applied. By using NAF, the achievable DMT under the half-duplex constraint is shown to be [87]

$$d(r) \leq (1 - r)^+ + (1 - 2r)^+. \quad (2.5.4)$$

The achievable DMT for DDF is given in [87] as

$$d(r) = \begin{cases} 2(1 - r) & \text{if } 0 \leq r \leq \frac{1}{2} \\ \frac{1-r}{r} & \text{if } \frac{1}{2} \leq r \leq 1. \end{cases} \quad (2.5.5)$$

By using the CF-based relaying, the achievable DMT is shown to be $d(r) = 2(1 - r)^+$ [90], which is just the optimal DMT. The DMT optimality of the CF relaying also holds for MIMO single-user single-relay channels. Figure 2.1 shows the DMT curves achieved by NAF, DDF and CF. It can be seen that NAF is strictly DMT suboptimal over all the multiplexing rates $r > 0$. DDF is optimal for multiplexing rates $0 \leq r \leq 1/2$, but fails to achieve the optimal DMT for higher multiplexing rates. CF achieves the DMT upper bound over all the

Figure 2.1: DMT curves of NAF, DDF and CF protocols under a fading single-user single-relay network with single-antenna terminals.

multiplexing rates.

For the single-antenna single-user K -relay channel with a direct link between source and destination, a DMT upper bound derived from the capacity cut-set bound is shown to be $d(r) = (K + 1)(1 - r)^+$. In [87], two DMT lower bounds were derived using NAF and DDF relaying under half-duplex relay constraint. With NAF relaying, the achievable DMT is given by

$$d(r) = (1 - r)^+ + (K - 1)(1 - 2r)^+. \quad (2.5.6)$$

The achievable DMT of the DDF relay scheme is

$$d(r) = \begin{cases} K(1 - r) & \text{if } 0 \leq r \leq \frac{1}{K} \\ 1 + \frac{(K-1)(1-2r)}{1-r} & \text{if } \frac{1}{K} < r \leq \frac{1}{2} \\ \frac{1-r}{r} & \text{if } \frac{1}{2} < r \leq 1. \end{cases} \quad (2.5.7)$$

In Fig. 2.2, the DMT curves of the two lower bounds are compared with the upper bound for a channel with 4 relays. It can be seen that at low multiplexing rate region, DDF achieves the DMT upper bound, but as multiplexing rate increases, DDF loses its optimality. NAF achieves the DMT upper bound only when $r = 0$. For all the other multiplexing rates, NAF is strictly suboptimal. In [92], by using a slotted AF relay scheme, a better DMT lower

Figure 2.2: DMT curves of NAF, DDF, and SAF under a network with single-source, single-destination and 4 relays. All the terminals have single antenna. For the SAF protocol, systems with slot lengths $N_s = 5$ and $N_s = 10$ are presented.

bound was derived as

$$d(r) = (1 - r)^+ + K \left(1 - \frac{N_s}{N_s - 1} r \right)^+, \quad (2.5.8)$$

where N_s is the number of slots. As $N_s \rightarrow \infty$, the DMT lower bound by SAF approaches the optimal DMT. Fig. 2.2 demonstrates the DMT curves of the SAF for cases with $N_s = 5$ and $N_s = 10$.

For the MIMO single-user N -hop relay channels, the DMT upper bound is found using capacity cut-set bound as

$$d(r) = \min_{i=1, \dots, N} (K_{i-1} - r)(K_i - r), \quad (2.5.9)$$

where K_i is the total number of transmit antennas in each layer. In [93], a AF-based flip-forward (FF) relay scheme was proposed and a DMT lower bound was derived. In [97], an improved scheme with a better DMT lower bound was proposed. The DMT upper bound and the lower bounds of a two-hop relay channel with a 2-antenna source, 2-antenna destination and a layer of single/multiple-antenna relays with a total of 4 transmit antennas are shown in Fig. 2.3. It can be seen that the lower bounds are quite far away from the upper bound over all the multiplexing gains $r > 0$.

Figure 2.3: DMT curves of a two-hop relay channel with a 2-antenna source, 2-antenna destination and a layer of single/multiple-antenna relays with totally 4 transmit antennas.

For the single-antenna K -user symmetric cooperative MAC, the optimal DMT is $d(r) = K(1 - Kr)^+$. A half-duplex NAF cooperation scheme is shown to be DMT optimal [87]. For the two-user symmetric MARC, the DMT upper bound is given in [101] as

$$d(r) = \begin{cases} 2(1 - r) & \text{if } 0 \leq r \leq \frac{1}{4} \\ 3(1 - 2r) & \text{if } \frac{1}{4} < r \leq \frac{1}{2}. \end{cases} \quad (2.5.10)$$

Three DMT lower bounds were proposed using a multiple-access AF (MAF) relay scheme, a DDF relay scheme and a CF relay scheme in [91], [87] and [90], respectively. Based on MAF, the DMT lower bound was found as

$$d(r) = \begin{cases} 2 - 3r & \text{if } 0 \leq r \leq \frac{1}{3} \\ 3(1 - 2r) & \text{if } \frac{1}{3} < r \leq \frac{1}{2}. \end{cases} \quad (2.5.11)$$

Figure 2.4: DMT curves of single-antenna two-user CMAC, MARC and MAC without source cooperation.

Using the DDF scheme, the DMT is

$$d(r) = \begin{cases} 2(1-r) & \text{if } 0 \leq r \leq \frac{1}{4} \\ 3(1-2r) & \text{if } \frac{1}{4} < r \leq \frac{1}{3} \\ \frac{1-2r}{r} & \text{if } \frac{1}{3} < r \leq \frac{1}{2}. \end{cases} \quad (2.5.12)$$

For the CF scheme, the achievable DMT is

$$d(r) = \begin{cases} 2(1-2r)^+ & \text{if } 0 \leq r \leq \frac{1}{3} \\ 1-r & \text{if } \frac{1}{3} < r \leq \frac{2}{5} \\ 3(1-2r) & \text{if } \frac{2}{5} < r \leq \frac{1}{2}. \end{cases} \quad (2.5.13)$$

In Fig. 2.4, the DMT results for the CMAC and the MARC are presented. It can be observed that a relay is more helpful in boosting diversity performance of MACs than user cooperation. For the relay schemes developed for MARC, DDF achieves the DMT upper bound at low multiplexing rate region, while at high multiplexing regions, MAF and CF achieve the DMT upper bound. At the high-multiplexing-rate region, asking the relay to decode both users' information becomes the main factor that restricts the DMT behavior of DDF.

Most recently, thanks to the advances in characterizing the capacity of two-user IFCs [107], [121], results on the DMT of such a channel have been obtained [116]–[120]. It has been shown in [116], [117] and [120] that for channel scenarios where the interference signal level received by each destination is of comparable strength with the desired signal level, by assuming full CSIT, the DMT of the delay-limited two-user single-antenna fading IFC is upper bounded by

$$d(r) = \min\{(1 - r)^+, 3(1 - 2r)^+\}. \quad (2.5.14)$$

In [120], this upper bound is shown to be achievable using a transmission scheme that only requires partial CSIT.

Chapter 3

DMT of MIMO Multihop Relay Channels

For single-user fading multihop relay channels with multiple-antenna nodes, the diversity upper bound can be achieved in an ideal case where the relays work in full-duplex mode and can perfectly cooperate to perform joint decoding and re-encoding [95]. For practical scenarios, however, the current achievable DMT lower bounds are far from optimal, as can be seen from the literature review in Chapter 2. In this chapter, we propose an AF-based space-time relay scheme and show that with proper design of space-time codes at the source, the proposed relay scheme can achieve the optimal DMT for full-duplex MIMO multihop relay channels.

3.1 Channel Model

We consider a full-duplex MIMO multihop relay channel with N layers of full-duplex multiple-antenna relays assisting the communication between a pair of source and destination, both equipped with multiple antennas. The number of antennas at the source and destination are M_t and M_r , respectively. The total number of antennas equipped at the relays in the i th ($i = 1, \dots, N$) layer is K_i . We assume no direct link between the source and the destination. Signals transmitted from the source experience $N + 1$ hops of relaying before arriving at the destination. The relays in non-adjacent layers are not connected. Figure 3.1 shows such a channel scenario. The channels between any transmit and receive antennas are assumed to be independent, quasi-static and flat-Rayleigh faded. We express the fading channel between the i th layer and the $(i + 1)$ th layer as a $K_{i+1} \times K_i$ matrix $\mathbf{H}^{(i)}$. Each element in $\mathbf{H}^{(i)}$ is independent and identically complex Gaussian distributed with zero mean and unit variance. For notational unification, we denote the source as the 0th layer and the destination as the $(N + 1)$ th layer. Consequently, the channels in the first hop and the last hop can be denoted as $\mathbf{H}^{(0)}$ and $\mathbf{H}^{(N+1)}$, respectively. Based on the quasi-static assumption, the channel

Figure 3.1: Multihop relay channel model.

fading coefficients remain fixed for a block of T symbol intervals and change independently from block to block. Based on the independent assumption, in the following context we will equivalently consider a system with the i th layer containing K_i single-antenna relays, and denote such a system as a $\langle M_t, K_1, K_2, \dots, K_N, M_r \rangle$ multihop relay system.

We denote the $M_t \times T$ matrix $\mathbf{X} = [\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{T-1}]$ as the transmitted symbol matrix from M_t transmit antennas in the source during T symbol intervals, the $M_r \times T$ matrix $\mathbf{Y}$ as the received symbol matrix at the destination during T symbol intervals, the $K_i \times T$ matrix $\mathbf{R}^{(i)} = [\mathbf{r}_1^{(i)}, \mathbf{r}_2^{(i)}, \dots, \mathbf{r}_{K_i}^{(i)}]'$ as the received symbol matrix at the K_i relays in the i th layer, and the $K_i \times T$ matrix $\mathbf{S}^{(i)} = [\mathbf{s}_1^{(i)}, \mathbf{s}_2^{(i)}, \dots, \mathbf{s}_{K_i}^{(i)}]'$ as the transmitted symbol matrix from the K_i relays in the i th layer during T symbol intervals. All the transmitting symbol matrices ($\mathbf{X}$ and $\mathbf{S}^{(i)}$'s) are normalized such that $\mathbb{E}\{\|\mathbf{X}\|_F^2\} \leq T$ and $\mathbb{E}\{\|\mathbf{S}^{(i)}\|_F^2\} \leq T$. The information flow in the network can be expressed as

$$\mathbf{R}^{(1)} = \sqrt{\frac{P_0}{M_t}} \mathbf{H}^{(0)} \mathbf{X} + \mathbf{W}^{(1)}, \quad (3.1.1)$$

$$\mathbf{R}^{(i)} = \sqrt{\frac{P_{i-1}}{K_{i-1}}} \mathbf{H}^{(i-1)} \mathbf{S}^{(i-1)} + \mathbf{W}^{(i)}, \quad i = 2, 3, \dots, N, \quad (3.1.2)$$

and

$$\mathbf{Y} = \sqrt{\frac{P_N}{K_N}} \mathbf{H}^{(N)} \mathbf{S}^{(N)} + \mathbf{W}^{(N+1)}, \quad (3.1.3)$$

where P_i is the total transmitting power at the i th layer, and $\mathbf{W}^{(i)} = [\mathbf{w}_1^{(i)}, \mathbf{w}_2^{(i)}, \dots, \mathbf{w}_{k_i}^{(i)}]'$ is the additive noise matrix at the i th layer. The elements of $\mathbf{W}^{(i)}$ are independent and identically distributed complex Gaussian random variables with zero mean and unit variance. Note that here we do not consider the optimal power allocation between relay nodes in each layer, and simply assume that the total transmitting power at each layer is shared equally

among the relays.

3.2 An AF-Based Space-Time Relay Strategy

For the proposed AF-based space-time relay strategy, each relay applies a space-time linear transformation on each of its received symbol vector, scales it to meet the power constraint, and retransmits it. Specifically, for the received symbol vector $\mathbf{r}_k^{(i)}$ at the k th relay in the i th layer, a $T \times T$ linear transformation matrix $\mathbf{T}_k^{(i)}$ is applied. After power scaling, we retransmit the following symbol vector

$$\mathbf{s}_k^{(i)} = \sqrt{\frac{1/K_i}{P_{i-1} + 1}} (\mathbf{T}_k^{(i)})' \mathbf{r}_k^{(i)}. \quad (3.2.1)$$

From (3.1.1), (3.1.2) and (3.2.1), the received symbol at the k th relay in the i th layer can be found as

$$(\mathbf{r}_k^{(i)})' = P^{(i-1)} \sum_{k_{i-1}=1}^{K_{i-1}} \sum_{k_{i-2}=1}^{K_{i-2}} \cdots \sum_{k_1=1}^{K_1} \left(\mathbf{H}_{k,k_{i-1}}^{(i-1)} \left(\prod_{m=1}^{i-2} \mathbf{H}_{k_{i-m},k_{i-1-m}}^{(i-1-m)} \right) \mathbf{H}_{k_1,\cdot}^{(0)} \mathbf{X} \prod_{m=1}^{i-1} \mathbf{T}_{k_m}^{(m)} \right) + \mathbf{z}_k^{(i)}, \quad (3.2.2)$$

where $P^{(i-1)}$ is a power factor given by

$$P^{(i-1)} = \sqrt{\frac{\prod_{m=0}^{i-1} P_m}{M_t \prod_{m=1}^{i-1} K_m \prod_{m=0}^{i-2} (P_m + 1)}}, \quad (3.2.3)$$

and

$$\begin{aligned} \mathbf{z}_k^{(i)} = & \sum_{n=1}^{i-1} \left(\sqrt{\frac{\prod_{m=n}^{i-1} P_m}{\prod_{m=n}^{i-1} K_m \prod_{m=n-1}^{i-2} (P_m + 1)}} \sum_{k_{i-1}=1}^{K_{i-1}} \sum_{k_{i-2}=1}^{K_{i-2}} \cdots \sum_{k_n=1}^{K_b} \left(\mathbf{H}_{k,k_{i-1}}^{(i-1)} \right. \right. \\ & \left. \left. \prod_{m=1}^{i-1-n} \mathbf{H}_{k_{i-m},k_{i-1-m}}^{(i-1-m)} (\mathbf{w}_{k_n}^{(n)})' \prod_{m=n}^{i-1} \mathbf{T}_{k_m}^{(m)} \right) \right) + (\mathbf{w}_k^{(i)})' \end{aligned} \quad (3.2.4)$$

is the additive noise. The received symbol at the destination can thus be written as

$$\mathbf{Y} = \sum_{k_N=1}^{K_N} \mathbf{H}_{:,k_N}^{(N)} (\mathbf{s}_{k_N}^{(N)})' + \mathbf{Z}, \quad (3.2.5)$$

where $\mathbf{Z}$ is an additive noise term which includes all the noise added in the transmission between the layers of relays. Note that $\mathbf{Z}$ is not white in general. Since our focus is on the

Figure 3.2: Illustration of a $\langle 2, 2, 2, 2 \rangle$ channel with the proposed linear transformation.

system asymptotic behavior and it has been shown that in this scale of interest non-Gaussian noise does not affect the analytical results [93], [96], [104], we treat $\mathbf{Z}$ as white Gaussian noise in the following analysis.

The linear transformation matrix $\mathbf{T}_k^{(i)}$ is constructed as

$$\mathbf{T}_k^{(i)} = \mathbf{D}^{(k-1)} \prod_{m=1}^{i-1} K_m, \quad (3.2.6)$$

where $\mathbf{D} = \text{diag}_{i=0}^{T-1} \{\exp(-j2\pi i/T)\}$. By this choice of linear transformation, the end-to-end channel analogies to a multipath channel with $L = \prod_{m=1}^N K_m$ distinct paths. The linear transformations introduce different delays to each path. Figure 3.2 demonstrates a $\langle 2, 2, 2, 2 \rangle$ channel applying the proposed linear transformation. We see that the linear transformations create 4 paths with delays $\mathbf{D}^0$, $\mathbf{D}^1$, $\mathbf{D}^2$, and $\mathbf{D}^3$. By properly combining the different delayed copies of the received signal at the destination, multipath diversity as well as spatial diversity can be obtained. The introduced multipath diversity leads to the superior DMT behavior of our proposed AF relaying scheme over the simple AF relaying scheme without linear transformation.

With this linear operation and from (3.2.5), the input-output relationship between the source and the destination can be expressed as

$$\text{vec}(\mathbf{Y}) = \sqrt{\frac{\text{SNR}}{M_t L}} \begin{bmatrix} \mathbf{G}(0) & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{G}(1) & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & \mathbf{G}(T-1) \end{bmatrix}_{TM_r \times TM_t} \text{vec}(\mathbf{X}) + \text{vec}(\mathbf{Z}_w), \quad (3.2.7)$$

where the matrix $\mathbf{G}(t)$ for $t = 0, 1, \dots, T-1$ is given by

$$\mathbf{G}(t) = \sum_{k_N=1}^{K_N} \sum_{k_{N-1}=1}^{K_{N-1}} \cdots \sum_{k_1=1}^{K_1} \left(\mathbf{H}_{:,k_N}^{(N)} \left(\prod_{m=1}^{N-1} \mathbf{H}_{k_{N-m+1},k_{N-m}}^{(N-m)} \right) \mathbf{H}_{k_1,:}^{(0)} \right)$$

$$\times \exp \left(-j2\pi \frac{t \left(\sum_{m=1}^N (k_m - 1) \prod_{p=1}^{m-1} K_p \right)}{T} \right). \quad (3.2.8)$$

Note that $\mathbf{Z}_w$ denotes an additive Gaussian noise matrix whose elements are independent and identically distributed with zero mean and unit variance.

The channel model in (3.2.7) is very similar to the frequency-domain system model of a T -tone MIMO orthogonal-frequency-division-multiplexing (OFDM) system undergoing a multipath fading channel with L resolvable paths. This builds an analog between the considered system and an OFDM modulated MIMO system. The main difference is that in a MIMO-OFDM system undergoing multipath fading, the resolvable paths are often regarded as independent. In our system, however, the paths, whose fading coefficient matrices are given by $\mathbf{H}_{:,k_N}^{(N)} \left(\prod_{m=1}^{N-1} \mathbf{H}_{k_{N-m+1},k_{N-m}}^{(N-m)} \right) \mathbf{H}_{k_1,:}^{(0)}$, are not independent, i.e., several paths share common transmission links between one or more hops. Despite this, some useful ideas and methods used in the analysis and design of MIMO-OFDM systems, such as [122]–[129], can still be adopted for our proposed system.

To give a better illustration of this “multipath” nature of the system concerned, we provide an alternative expression for the system model in (3.2.7) as

$$(\mathbf{Y})' = \sqrt{\frac{\text{SNR}}{M_t L}} \mathbf{C}(\mathbf{X}) \mathcal{H}_P + (\mathbf{Z}_w)', \quad (3.2.9)$$

where

$$\mathbf{C}(\mathbf{X}) = \left[\mathbf{D}^0 \mathbf{X}', \quad \mathbf{D} \mathbf{X}', \quad \dots, \quad \mathbf{D}^{\sum_{m=1}^N (k_m - 1) \prod_{p=1}^{m-1} K_p} \mathbf{X}', \quad \dots, \quad \mathbf{D}^{L-1} \mathbf{X}' \right] \quad (3.2.10)$$

and

$$\mathcal{H}_P = \begin{bmatrix} (\mathbf{H}_{:,1}^{(N)} \mathbf{H}_{1,1}^{(N-1)} \dots \mathbf{H}_{1,:}^{(0)})' \\ (\mathbf{H}_{:,1}^{(N)} \mathbf{H}_{1,1}^{(N-1)} \dots \mathbf{H}_{2,:}^{(0)})' \\ \vdots \\ (\mathbf{H}_{:,k_N}^{(N)} \mathbf{H}_{k_N,k_{N-1}}^{(N-1)} \dots \mathbf{H}_{k_1,:}^{(0)})' \\ \vdots \\ (\mathbf{H}_{:,K_N}^{(N)} \mathbf{H}_{K_N,K_{N-1}}^{(N-1)} \dots \mathbf{H}_{K_1,:}^{(0)})' \end{bmatrix}. \quad (3.2.11)$$

Since the system model in (3.2.7) reflects the input-output relationship in time domain, we denote it as the system’s time-domain model. The system model in (3.2.11) reflects the multipath property of the system, we denote it as the system’s multipath model.

3.3 DMT Analysis

As shown in Chapter 2, the DMT upper bound of the MIMO multihop relay channel is

$$d(r) = \min_{i=1, \dots, N+1} d_i(r), \quad (3.3.1)$$

where

$$d_i(r) = \begin{cases} (M_t - r)(K_1 - r) & \text{if } i = 1 \\ (K_{i-1} - r)(K_i - r) & \text{if } i = 2, 3, \dots, N \\ (K_N - r)(M_r - r) & \text{if } i = N + 1. \end{cases}$$

Our target here is to prove that the proposed AF-based space-time relay scheme is DMT optimal, i.e., it can achieve the DMT upper bound (3.3.1). Towards this end, we analyze the DMT of the channel under the proposed relay scheme in two steps. We first derive a DMT upper bound and show that this upper bound coincides with the upper bound given in (2.5.9). We then show that by using properly designed space-time codes at the transmitter, the DMT upper bound can be achieved.

3.3.1 DMT Upper Bound

Based on the time-domain system model in (3.2.7), the mutual information of the system is

$$\begin{aligned} I_T(\text{SNR}) &= \frac{1}{T} \sum_{t=0}^{T-1} \log \det \left(\mathbf{I}_{M_r} + \frac{\text{SNR}}{M_t L} \mathbf{G}(t) (\mathbf{G}(t))^\dagger \right) \\ &= \frac{1}{T} \sum_{t=0}^{T-1} \log \det \left(\mathbf{I}_{M_t} + \frac{\text{SNR}}{M_t L} (\mathbf{G}(t))^\dagger \mathbf{G}(t) \right). \end{aligned} \quad (3.3.2)$$

Here, we assume that the input symbol vectors at different symbol intervals are uncorrelated and satisfy the condition $\mathbf{Q}_t = \mathbf{I}_{M_t}$, where $\mathbf{Q}_t$ is the covariance matrix of the input symbol vector $\mathbf{x}_t$.

To analyze the system outage behavior and consequently the DMT, the distribution pattern of $I_T(\text{SNR})$ is needed. Since $I_T(\text{SNR})$ involves a sum of correlated fading channels, direct calculation of the distribution is difficult. Therefore, we resort to an upper bound of $I_T(\text{SNR})$. Applying Jensen's inequality to (3.3.2), we have

$$I_T(\text{SNR}) \leq \log \det \left(\mathbf{I}_m + \frac{\text{SNR}}{M_t L T} \mathcal{H}_T \mathcal{H}_T^\dagger \right) \triangleq I_J(\text{SNR}), \quad (3.3.3)$$

where $m = \min\{M_t, M_r\}$, and

$$\mathcal{H}_T = \begin{cases} [\mathbf{G}(0), \mathbf{G}(1), \dots, \mathbf{G}(T-1)] & \text{if } M_r \leq M_t, \\ [(\mathbf{G}(0))^\dagger, (\mathbf{G}(1))^\dagger, \dots, (\mathbf{G}(T-1))^\dagger] & \text{if } M_r > M_t. \end{cases} \quad (3.3.4)$$

Since $I_T(\text{SNR}) \leq I_J(\text{SNR})$, we have

$$\Pr(I_T(\text{SNR}) < r \log \text{SNR}) \geq \Pr(I_J(\text{SNR}) < r \log \text{SNR}). \quad (3.3.5)$$

This implies that the outage probability for a system with channel $\mathcal{H}_T$ is a lower bound for the outage probability of our system. The DMT of our considered system will be upper bounded by the DMT obtained based on $\mathcal{H}_T$.

From (3.2.8), for $M_r \leq M_t$, we have

$$\mathcal{H}_T = \begin{bmatrix} \mathbf{H}_{:,1}^{(N)} \mathbf{H}_{1,1}^{(N-1)} \dots \mathbf{H}_{1,:}^{(0)}, & \dots, & \mathbf{H}_{:,k_N}^{(N)} \mathbf{H}_{k_N,k_{N-1}}^{(N-1)} \dots \mathbf{H}_{k_1,:}^{(0)}, & \dots, \\ \mathbf{H}_{:,K_N}^{(N)} \mathbf{H}_{K_N,K_{N-1}}^{(N-1)} \dots \mathbf{H}_{K_1,:}^{(0)} \end{bmatrix} \mathbf{F}, \quad (3.3.6)$$

where

$$\mathbf{F} = \begin{bmatrix} \exp(-j2\pi \frac{0 \cdot 0}{T}) & \dots & \exp(-j2\pi \frac{(T-1) \cdot 0}{T}) \\ \vdots & \vdots & \vdots \\ \exp(-j2\pi \frac{0 \cdot l(k_1, \dots, k_N)}{T}) & \dots & \exp(-j2\pi \frac{(T-1) \cdot l(k_1, \dots, k_N)}{T}) \\ \vdots & \vdots & \vdots \\ \exp(-j2\pi \frac{0 \cdot (L-1)}{T}) & \dots & \exp(-j2\pi \frac{(T-1) \cdot (L-1)}{T}) \end{bmatrix}, \quad (3.3.7)$$

and $l(k_1, \dots, k_N) = (\sum_{m=1}^N (k_m - 1) \prod_{p=1}^{m-1} K_p) - 1$. Since the $L \times T$ matrix $\mathbf{F}$ satisfies $\mathbf{F}\mathbf{F}^\dagger = T\mathbf{I}_L$, we have

$$\begin{aligned} \mathcal{H}_T \mathcal{H}_T^\dagger &= T \sum_{k_N=1}^{K_N} \sum_{k_{N-1}=1}^{K_N} \dots \sum_{k_1=1}^{K_1} \mathbf{H}_{:,k_N}^{(N)} |\mathbf{H}_{k_N,k_{N-1}}^{(N-1)}|^2 \dots |\mathbf{H}_{k_2,k_1}^{(1)}|^2 \|\mathbf{H}_{k_1,:}^{(0)}\|_F^2 (\mathbf{H}_{:,k_N}^{(N)})^\dagger \\ &= T \mathbf{H}^{(N)} \mathbf{\Sigma}^{(N)} (\mathbf{H}^{(N)})^\dagger, \end{aligned} \quad (3.3.8)$$

where $\mathbf{\Sigma}^{(N)} = \text{diag}_{k=1}^{K_N} \{\mu_1^{(N)}, \mu_2^{(N)}, \dots, \mu_{K_N}^{(N)}\}$ and

$$\mu_k^{(N)} = \sum_{k_{N-1}=1}^{K_{N-1}} \dots \sum_{k_1=1}^{K_1} |\mathbf{H}_{k,k_{N-1}}^{(N-1)}|^2 |\mathbf{H}_{k_{N-1},k_{N-2}}^{(N-2)}|^2 \dots |\mathbf{H}_{k_2,k_1}^{(1)}|^2 \|\mathbf{H}^{(0)}(k_1, :)\|_F^2. \quad (3.3.9)$$

Similarly, for the case $M_r > M_t$, we have

$$\mathcal{H}_T \mathcal{H}_T^\dagger = T(\mathbf{H}^{(0)})^\dagger \tilde{\Sigma}^{(N)} \mathbf{H}^{(0)}, \quad (3.3.10)$$

where $\tilde{\Sigma}^{(N)} = \text{diag}_{k=1}^{K_N} \{\tilde{\mu}_1^{(N)}, \tilde{\mu}_2^{(N)}, \dots, \tilde{\mu}_{K_1}^{(N)}\}$ and

$$\tilde{\mu}_k^{(N)} = \sum_{k_2=1}^{K_2} \dots \sum_{k_N=1}^{K_N} |\mathbf{H}_{k_2,k}^{(1)}|^2 |\mathbf{H}_{k_3,k_2}^{(2)}|^2 \dots |\mathbf{H}_{k_N,k_{N-1}}^{(N-1)}|^2 \|\mathbf{H}_{\cdot,k_N}^{(N)}\|_F^2. \quad (3.3.11)$$

Since $\mathcal{H}_T \mathcal{H}_T^\dagger$ has a same structure for both the $M_r \leq M_t$ and the $M_r > M_t$ cases, we will focus on the case with $M_r \leq M_t$ in the following analysis.

For a system with channel model $\mathcal{H}_T$, the system outage probability at rate $R = r \log \text{SNR}$ can be evaluated as

$$\begin{aligned} P_T(r \log \text{SNR}) &\doteq \Pr(\log \det(\mathbf{I}_{M_r} + \text{SNR} \mathcal{H}_T^\dagger \mathcal{H}_T) < r \log \text{SNR}) \\ &= \Pr\left(\log \prod_{v=1}^q (1 + \text{SNR} \lambda_v) < r \log \text{SNR}\right), \end{aligned} \quad (3.3.12)$$

where λ_v , for $v = 1, 2, \dots, q$, are the eigenvalues of $\mathcal{H}_T \mathcal{H}_T^\dagger$ and $q = \min\{M_r, K_N\}$.

Let $\alpha_v \doteq -\log \lambda_v / \log \text{SNR}$, (3.3.12) can be rewritten as

$$\begin{aligned} P_{\text{out}}(r \log \text{SNR}) &= \Pr\left(\prod_{v=1}^q (1 + \text{SNR}^{1-\alpha_v}) < \text{SNR}^r\right) \\ &\doteq \Pr\left(\text{SNR}^{\sum_{v=1}^q (1-\alpha_v)^+} < \text{SNR}^r\right) \\ &= \Pr\left(\sum_{v=1}^q (1 - \alpha_v)^+ < r\right). \end{aligned} \quad (3.3.13)$$

The outage probability can thus be calculated as the probability of $\boldsymbol{\alpha}$ falling in the set $\mathcal{A} = \{\boldsymbol{\alpha} : \sum_{v=1}^q (1 - \alpha_v)^+ < r\}$, i.e.,

$$P_{\text{out}}(r \log \text{SNR}) \doteq \int_{\mathcal{A}} p(\boldsymbol{\alpha}) d\boldsymbol{\alpha}, \quad (3.3.14)$$

where $\boldsymbol{\alpha}$ is denoted as the set $\{\alpha_1, \dots, \alpha_q\}$.

To find $p(\boldsymbol{\alpha})$, or alternatively $p(\boldsymbol{\lambda})$, we use the idea that conditioned on the channel realizations of the first $n - 1$ hops, the channel parameter distribution of the first n hops only depends on that of the n th hop. We evaluate $p(\boldsymbol{\lambda})$ by starting from the last hop, the

$(N + 1)$ th hop, and find $p(\boldsymbol{\lambda}|\boldsymbol{\mu}^{(N)})$, where $\boldsymbol{\mu}^{(N)}$ is a vector of parameters determined jointly by the first N hops. We then proceed to evaluate $p(\boldsymbol{\mu}^{(N)}|\boldsymbol{\mu}^{(N-1)})$, up until $p(\boldsymbol{\mu}^{(1)})$. In this way, a joint distribution of $(\boldsymbol{\lambda}, \boldsymbol{\mu}^{(N)}, \boldsymbol{\mu}^{(N-1)}, \dots, \boldsymbol{\mu}^{(1)})$ can be obtained and $p(\boldsymbol{\lambda})$ can be easily calculated by taking integral over $(\boldsymbol{\mu}^{(N)}, \boldsymbol{\mu}^{(N-1)}, \dots, \boldsymbol{\mu}^{(1)})$.

Specifically, note that

$$\frac{1}{T} \mathcal{H}_T \mathcal{H}_T^\dagger = \mathbf{H}^{(N)} \boldsymbol{\Sigma}^{(N)} (\mathbf{H}^{(N)})^\dagger = \mathbf{H}^{(N)} (\boldsymbol{\Sigma}^{(N)})^{1/2} (\boldsymbol{\Sigma}^{(N)})^{1/2} (\mathbf{H}^{(N)})^\dagger, \quad (3.3.15)$$

the eigenvalues of $\mathcal{H}_T \mathcal{H}_T^\dagger / T$ are the same as that of $\mathcal{W} = ((\boldsymbol{\Sigma}^{(N)})^{1/2} (\mathbf{H}^{(N)})^\dagger) ((\boldsymbol{\Sigma}^{(N)})^{1/2} (\mathbf{H}^{(N)})^\dagger)^\dagger$, which is a complex Wishart matrix with M_r degrees of freedom and covariance matrix $\boldsymbol{\Sigma}^{(N)}$. Conditioned on the ordered eigenvalues of $\boldsymbol{\Sigma}^{(N)}$, $\mu_1^{(N)} > \mu_2^{(N)} > \dots > \mu_{K_N}^{(N)} > 0$, the pdf of the ordered eigenvalues of $\mathcal{W}$, and consequently $\mathcal{H}_T \mathcal{H}_T^\dagger / T$, $\lambda_1 > \lambda_2 > \dots > \lambda_q > 0$ can be found using the eigenvalue distribution for Wishart matrix [93], [130]. Following that, we can obtain $p(\boldsymbol{\lambda}|\boldsymbol{\mu}^{(N)})$.

Notice that $\mu_k^{(N)}$ given by (3.3.9) can be rewritten as

$$\mu_k^{(N)} = \mathbf{H}_{k,:}^{(N-1)} \boldsymbol{\Sigma}^{(N-1)} (\mathbf{H}_{k,:}^{(N-1)})^\dagger, \quad (3.3.16)$$

where $\boldsymbol{\Sigma}^{(N-1)} = \text{diag}\{\mu_1^{(N-1)}, \mu_2^{(N-1)}, \dots, \mu_{K_{N-1}}^{(N-1)}\}$, and

$$\mu_k^{(N-1)} = \sum_{k_{N-2}=1}^{K_{N-2}} \dots \sum_{k_1=1}^{K_1} |\mathbf{H}_{k,k_{N-2}}^{(N-2)}|^2 \dots |\mathbf{H}_{k_2,k_1}^{(1)}|^2 \|\mathbf{H}_{k_1,:}^{(0)}\|_F^2, \quad k = 1, 2, \dots, K_{N-1}. \quad (3.3.17)$$

When conditioned on $\boldsymbol{\mu}^{(N-1)}$, $\mu_k^{(N)}$'s are independent and each $\mu_k^{(N)}$ is the eigenvalue of a Wishart matrix $((\boldsymbol{\Sigma}^{(N-1)})^{1/2} (\mathbf{H}_{k,:}^{(N-1)})^\dagger) ((\boldsymbol{\Sigma}^{(N-1)})^{1/2} (\mathbf{H}_{k,:}^{(N-1)})^\dagger)^\dagger$. Therefore, $p(\mu_k^{(N)}|\boldsymbol{\mu}^{(N-1)})$ can be obtained. The joint pdf of ordered $\boldsymbol{\mu}^{(N)}$ conditioned on $\boldsymbol{\mu}^{(N-1)}$ can thus be calculated as

$$p(\boldsymbol{\mu}^{(N)}|\boldsymbol{\mu}^{(N-1)}) = K_N! \prod_{k=1}^{K_N} p(\mu_k^{(N)}|\boldsymbol{\mu}^{(N-1)}). \quad (3.3.18)$$

Similar method can be used to find $p(\boldsymbol{\mu}^{(i)}|\boldsymbol{\mu}^{(i-1)})$, for $i = 2, 3, \dots, N - 1$, where the $\boldsymbol{\mu}^{(i)}$'s are defined as

$$\mu_k^{(i)} = \sum_{k_{i-1}=1}^{K_{i-1}} \dots \sum_{k_1=1}^{K_1} |\mathbf{H}_{k,k_{i-1}}^{(i-1)}|^2 \dots |\mathbf{H}_{k_2,k_1}^{(1)}|^2 \|\mathbf{H}_{k_1,:}^{(0)}\|_F^2, \quad k = 1, 2, \dots, K_i \quad (3.3.19)$$

and

$$\mu_k^{(1)} = \|\mathbf{H}_{k,:}^{(0)}\|_F^2, \quad k = 1, 2, \dots, K_1. \quad (3.3.20)$$

Each $\mu_k^{(1)}$ satisfies the chi-square distribution with $2M_t$ degrees of freedom, $p(\boldsymbol{\mu}^{(1)})$ can be easily obtained. The joint pdf of $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}^{(N)}, \boldsymbol{\mu}^{(N-1)}, \dots, \boldsymbol{\mu}^{(1)}$ can be found as

$$p(\boldsymbol{\lambda}, \boldsymbol{\mu}^{(N)}, \boldsymbol{\mu}^{(N-1)}, \dots, \boldsymbol{\mu}^{(1)}) = p(\boldsymbol{\lambda}|\boldsymbol{\mu}^{(N)})p(\boldsymbol{\mu}^{(N)}|\boldsymbol{\mu}^{(N-1)}) \cdots p(\boldsymbol{\mu}^{(2)}|\boldsymbol{\mu}^{(1)})p(\boldsymbol{\mu}^{(1)}). \quad (3.3.21)$$

After obtaining this joint distribution, the subsequent calculation can follow the standard method developed in [113]. We provide the detailed calculation in Appendix A. Note that by using this direct calculation method, we are only able to prove that the DMT upper bound of the proposed scheme achieves the DMT cut-set upper bound in a specific case where the numbers of transmit antennas at the source, receive antennas at the destination, and the total antennas at each layer of relays are the same, i.e., $M_t = K_1 = \dots = K_N = M_r = K$. For the general cases, we use the following alternative proof to show the achievement of the DMT cut-set upper bound.

From the mutual information expressions given in (3.3.3), we observe that the considered multihop system is equivalent to a two-hop system, where the first hop has a channel matrix $(\boldsymbol{\Sigma}^{(N)})^{1/2}$ and the second hop has a channel matrix $\mathbf{H}^{(N)}$. The channel in the first hop is essentially a parallel channel with K_N parallel subchannels and channel coefficients specified by the parameters $\boldsymbol{\mu}^{(N)} = [\mu_1^{(N)}, \mu_2^{(N)}, \dots, \mu_{K_N}^{(N)}]$. The channel coefficients of all the parallel subchannels satisfy the same probability distribution. The outage probability of each subchannel is the same. The second channel is a MIMO channel with the fading coefficient in each subchannel follows a Rayleigh distribution.

The outage probability of such a concatenation of a parallel channel and a MIMO channel can be evaluated as

$$P_T = P_{\text{out}}((\boldsymbol{\Sigma}^{(N)})^{1/2}) + P_{\text{out}}(\mathbf{H}^{(N)}) + P_{\text{out}}(\mathbf{H}^{(N)}(\boldsymbol{\Sigma}^{(N)})^{1/2}|\mathbf{H}^{(N)}, (\boldsymbol{\Sigma}^{(N)})^{1/2}), \quad (3.3.22)$$

where $P_{\text{out}}((\boldsymbol{\Sigma}^{(N)})^{1/2})$ is the outage probability of the channel $(\boldsymbol{\Sigma}^{(N)})^{1/2}$, $P_{\text{out}}(\mathbf{H}^{(N)})$ is the outage probability of the channel $\mathbf{H}^{(N)}$, and $P_{\text{out}}(\mathbf{H}^{(N)}(\boldsymbol{\Sigma}^{(N)})^{1/2}|\mathbf{H}^{(N)}, (\boldsymbol{\Sigma}^{(N)})^{1/2})$ denotes the outage probability of the concatenated channel given the two component channels are good.

The last term in (3.3.22) essentially denotes the probability of the outage event caused by the mismatch between the two component channels. Drawing a cut-set between the two channels, mismatch happens when the cut-set fails. Figure 3.3 gives an example of such a situation, where the subchannels crossed by the dashed curve comprise a cut-set. The outage probability caused by mismatch is the summation of the outage probabilities of all the possible cut-sets. This probability is dominated by the outage probability of the cut-set

Figure 3.3: An illustration of a cut-set.

with the highest order outage probability. In our case, since the fading coefficients of the subchannels in each component channel have the same probability distribution, the outage probability caused by channel mismatch satisfies

$$P_{\text{out}}(\mathbf{H}^{(N)}(\Sigma^{(N)})^{1/2} | \mathbf{H}^{(N)}, (\Sigma^{(N)})^{1/2}) \leq \min \left\{ P_{\text{out}}((\Sigma^{(N)})^{1/2}), P_{\text{out}}(\mathbf{H}^{(N)}) \right\}. \quad (3.3.23)$$

Therefore, the outage caused by channel mismatch is not the dominate outage event of the concatenated channel. We therefore have

$$P_T \doteq P_{\text{out}}((\Sigma^{(N)})^{1/2}) + P_{\text{out}}(\mathbf{H}^{(N)}), \quad (3.3.24)$$

which indicates that the outage probability of the concatenated channel is dominated by the outage probability of the component channels.

The outage probability of $(\Sigma^{(N)})^{1/2}$ is the probability that outage happens when joint decoding is performed at the output of the parallel channel. Since the fading coefficient of each subchannel in $(\Sigma^{(N)})^{1/2}$ can be expressed as

$$(\mu_k^{(N)})^{1/2} = \mathbf{H}^{(N-1)}(k, :) (\Sigma^{(N-1)})^{1/2}, \quad (3.3.25)$$

$(\Sigma^{(N)})^{1/2}$ is equivalent to a concatenation of a parallel channel $(\Sigma^{(N-1)})^{1/2}$ and a MIMO channel $\mathbf{H}^{(N-1)}$. Similarly, the outage probability of $(\Sigma^{(N)})^{1/2}$ in the high SNR region can be evaluated as

$$P_{\text{out}}((\Sigma^{(N)})^{1/2}) \doteq P_{\text{out}}((\Sigma^{(N-1)})^{1/2}) + P_{\text{out}}(\mathbf{H}^{(N-1)}). \quad (3.3.26)$$

Applying the above analysis successively, we conclude that the outage probability of the

multihop system at the high SNR region can be evaluated as

$$P_T \doteq \sum_{i=0}^N P_{\text{out}}(\mathbf{H}^{(i)}), \quad (3.3.27)$$

which is the same as the outage probability of a system where the relays in each layer perform the joint decode-and-forward operation. Since a clustered DF-based relay system achieves the optimal DMT [93], [95], our proposed system achieves the optimal DMT as well.

The above derivation reflects one major difference between our proposed AF-based space-time relay scheme and the conventional AF relay scheme where each relay simply scales and forwards its received signal without performing any form of space-time linear transformation. To further illustrate it, we take a two-hop system as an example. In our proposed relay scheme, as stated in the analysis, the overall channel between the source and the destination is equivalent to a concatenation of a parallel channel and a MIMO channel. The fading coefficients of the subchannels in the parallel channel follow the same distribution. For the conventional AF relay scheme, by applying singular value decomposition (SVD) on the channel matrix of the first hop together with the fact that applying unitary transformation on a isotropically distributed matrix does not change its distribution, the overall channel can also be equivalently expressed as a concatenation of a parallel channel and a MIMO channel. The fading coefficients of the subchannels in the parallel channel are the singular values of the channel matrix of the first hop. However, in the conventional AF relay scheme, unlike the case in our proposed relay system, the fading coefficients of the subchannels in the parallel channel are not identically distributed. Consequently, the outage probabilities of the subchannels are not the same, i.e., the outage probabilities of the “weak” subchannels will be higher than that of the “strong” subchannels. Therefore, outage caused by channel mismatch becomes the dominate outage event and leads to an inferior DMT behavior.

3.3.2 Achievability of the DMT Upper Bound

From previous discussions, we know that the proposed relay scheme amounts to create an artificial multipath channel, as indicated by the multipath channel model (3.2.9). To exploit the introduced multipath diversity, space-time codes could be used at the source. By analyzing the pairwise error probability (PEP), we find that if the applied space-time code satisfies certain criterion, the DMT upper bound can be achieved. We summarize the code design criterion in the following proposition. The detailed proof is given in Appendix B.

Proposition 3.3.1. *A family of space-time codes $\mathcal{C}$ with codewords $\mathbf{X} \in \mathbb{C}^{M_t \times T}$ achieve*

DMT optimal performance as long as the difference matrix $\Delta\mathbf{C} = \mathbf{C}(\mathbf{X}) - \mathbf{C}(\tilde{\mathbf{X}})$ for any pair of distinct codewords $\mathbf{X}$ and $\tilde{\mathbf{X}}$ satisfies

$$\text{rank}(\Delta\mathbf{C}) = M_t \prod_{i=1}^N K_i, \quad (3.3.28)$$

and the minimum eigenvalue $\lambda_{\min}$ of $\Delta\mathbf{C}$ among all pairs of $\mathbf{X}, \tilde{\mathbf{X}} \in \mathcal{C}$ satisfies

$$\lambda_{\min}^q \geq \text{SNR}^{-(r-\epsilon)} \quad (3.3.29)$$

for any $\epsilon > 0$.

Note that in analyzing the PEP, we use a similar method as in [122] where the DMT of selective fading MIMO channel was analyzed. There is also a similarity between the DMT-optimal code design criterion in Proposition 1 for our considered channel and the DMT-optimal space-time/frequency code design criterion proposed in [122] for the selective fading MIMO channel. This observation indicates that any space-frequency (SF) code designed to achieve DMT optimal performance in a MIMO system undergoing frequency-selective fading will also be DMT optimal when applied in our proposed MIMO relay system. In particular, the maximum diversity at $r = 0$ can be achieved using the SF codes designed to exploit the full spatial and frequency diversity of a MIMO-OFDM system [124]–[126].

Moreover, noncoherent SF codes [127] and differential SF codes [128], [129] designed for achieving full diversity of MIMO-OFDM systems with no channel state information at the receiver can also be adopted in the considered MIMO relay channel for communication in scenarios where the channel knowledge is unavailable at the destination. We conjecture that the full cooperative diversity can be achieved as well.

3.4 Numerical Results

We simulate the outage probability for the proposed relay scheme and the bit-error probability (BER) curves of a ST coded multihop MIMO relay channel based on the proposed relay scheme. In the simulation, we assume that the fading coefficients of the flat Rayleigh-fading channel remain constant over a coherence interval of $T = M_t L$. For the ST coded system with a targeted transmission rate R , the n th ($n = 1, 2, \dots, 2^{RT}$) codeword in the codebook of size 2^{RT} is constructed as

$$\mathbf{X}_n = \mathbf{V}^{n-1} \mathbf{B}, \quad (3.4.1)$$

Figure 3.4: Outage probabilities for a 2×2 MIMO channel, a $\langle 2, 2, 2, 2 \rangle$ channel with the conventional AF relay, a $\langle 2, 2, 2, 2 \rangle$ channel with the proposed relay scheme, a $\langle 2, 2, 3, 2 \rangle$ channel with the proposed relay scheme and a $\langle 2, 4, 2, 2 \rangle$ channel with the proposed relay scheme. The targeted rates are $R = 4$ and $R = 8$ bits per channel use.

where $\mathbf{V} = \text{diag}_{m=0}^{T-1}\{\exp(-j2\pi u_m/T)\}$, u_m ($m = 0, 1, \dots, T-1$) are the parameters found by random search to maximize

$$\det((\mathbf{V}^0 - \mathbf{V}^n)^\dagger(\mathbf{V}^0 - \mathbf{V}^n)), \quad \forall n = 1, 2, \dots, 2^{RT}, \quad (3.4.2)$$

and the $T \times M_t$ matrix $\mathbf{B}$ is constructed from a $T \times T$ fast Fourier transform (FFT) matrix. Specifically, the p th column of $\mathbf{B}$ is chosen as $\mathbf{f}^{L(p-1)}$, where $\mathbf{f} = [\omega_T^0, \omega_T^1, \dots, \omega_T^{T-1}]$ and $\omega_T = \exp(-j2\pi/T)$. Note that in constructing the ST code, we do not impose the criteria in Proposition 3.3.1, the constructed ST code is not necessarily DMT optimal. Despite that, by choosing the parameters u_m in a way that (3.4.2) is maximized, we can ensure that the constructed ST code exhibits a good diversity behavior, as will be demonstrated in the numerical results.

In Fig. 3.4, the outage probability curves of a $\langle 2, 2, 2, 2 \rangle$ channel, a $\langle 2, 2, 3, 2 \rangle$ channel and a $\langle 2, 4, 2, 2 \rangle$ channel operating under the proposed relay scheme are presented. Two targeted data rates, $R = 4$ and 8 bits per channel use, are considered. Since the rates are fixed, the corresponding multiplexing rate is $r = 0$. To benchmark the probability behavior of our proposed relay scheme, we also present the outage probability curves of a 2×2 MIMO channel, and a $\langle 2, 2, 2, 2 \rangle$ channel with conventional AF relaying. From the numerical results,

Figure 3.5: Outage probabilities for a 2×3 MIMO channel, a $\langle 2, 3, 3, 2 \rangle$ relay channel with conventional AF relay and a $\langle 2, 3, 3, 2 \rangle$ relay channel with the proposed relay scheme. The targeted multiplexing rate is $r = 1$.

it can be observed that the outage probability curves of the channels using our proposed relay scheme and the MIMO channel are almost parallel, both of which enjoy higher slope than the conventional AF relay scheme. This indicates that our proposed relay scheme achieves full diversity at multiplexing rate $r = 0$ and outperforms the conventional AF relay scheme in terms of outage probability. The MIMO channel exhibits the best outage probability behavior over all the simulated systems. The gap in outage probability between the multihop channel and the MIMO channel is due to the noise and fading accumulation during the AF-based multihop transmission. Moreover, it can be seen that with the proposed relay scheme, the $\langle 2, 2, 3, 2 \rangle$ channel and the $\langle 2, 4, 2, 2 \rangle$ channel achieve better outage behaviors than the $\langle 2, 2, 2, 2 \rangle$ channel. One explanation for this phenomenon is that although using more relays in one layer will introduce more noise, it also helps to create more paths, which gives a better outage performance. To show the diversity behavior of the proposed relay scheme in high multiplexing rates, we simulate the outage probabilities of a 2×3 MIMO channel, a $\langle 2, 3, 3, 2 \rangle$ relay channel with conventional AF relaying and a $\langle 2, 3, 3, 2 \rangle$ relay channel with the proposed AF-based space-time relay scheme at multiplexing rate $r = 1$, as shown in Fig. 3.5. It can be observed that the outage probability curves of the $\langle 2, 3, 3, 2 \rangle$ channel under the proposed relay scheme is in parallel with that of the 2×3 MIMO channel, indicating the diversity optimality of the proposed scheme at $r = 1$. This result further confirms the DMT optimality of the proposed relay scheme.

Figure 3.6: BER performance of a $\langle 1, 2, 1 \rangle$ channel, a $\langle 1, 2, 2, 1 \rangle$ channel, a $\langle 2, 2, 2 \rangle$ channel and a $\langle 2, 2, 2, 2 \rangle$ channel with rate 1 symbol per channel use.

Figure 3.6 shows the BER performance of the proposed space-time coded AF-based space-time relay scheme. The simulation is done under the constraint that for all channels simulated, the total transmit power of relays in each layer is the same. The coding rate is 1 symbol per channel use. From the simulation results, we can observe that at high SNR, the BER curves of the $\langle 2, 2, 2 \rangle$ channel and the $\langle 2, 2, 2, 2 \rangle$ channel become parallel, indicating a same diversity order for the two channels. A similar observation can be made for the $\langle 1, 2, 1 \rangle$ and the $\langle 1, 2, 2, 1 \rangle$ channels. It can also be seen that the two-hop relay channel has better BER performance than that of the three-hop relay channel. This observation is consistent with what we have observed in Figs. 3.4 and 3.5, where the MIMO channel shows better outage performance than the three-hop relay channel.

In Fig. 3.7, the BER curves of a $\langle 2, 2, 2 \rangle$ channel, a $\langle 2, 3, 2 \rangle$ channel and a $\langle 2, 4, 2 \rangle$ channel with ST coding and the proposed relay scheme are presented. The code rate of all the systems is 1 symbol per channel use. For the channels simulated, the total transmitting power of relays are the same. From the simulation results, we can see that the $\langle 2, 4, 2 \rangle$ channel has the highest diversity order, and the diversity order of the $\langle 2, 3, 2 \rangle$ channel is higher than that of the $\langle 2, 2, 2 \rangle$ channel. This observation is intuitively clear. With more number of relays in two-hop relay channels, more independent paths can be obtained, resulting in a better protection and a higher diversity.

Figure 3.7: BER performance of a $\langle 2, 2, 2 \rangle$ channel, a $\langle 2, 3, 2 \rangle$ channel and a $\langle 2, 4, 2 \rangle$ channel with rate 1 symbol per channel use.

3.5 Conclusion

In this chapter, the DMT of full-duplex single-user MIMO multihop relay channel has been investigated. We have proposed an AF-based space-time scheme and shown that with proper design of ST codes at the source, the DMT upper bound can be achieved. A ST code design criterion has also been proposed to achieve the optimal DMT.

The reason why the proposed scheme can achieve the DMT bound for the considered MIMO multihop relay channel is intuitively clear: From the time-domain point of view, linear transformations at relays convert the end-to-end flat fading channel into an equivalent time-varying channel with periodically varying channel fading coefficients. As can be seen from the channel model (3.2.7), at different time instances within a period of T , the effective fading experienced by each received signal at the destination is different. By applying space-time coding within the time period, time diversity as well as spatial diversity can be exploited. This gives the diversity advantage of the proposed scheme over the conventional AF relay scheme where only spatial diversity is exploited.

Viewing it from another perspective, we observe some similarities between the channel model in (3.2.7) and the frequency-domain channel model of a MIMO channel with OFDM modulation. For the MIMO-OFDM system, besides the frequency-domain channel model, the system can also be represented in time domain as a summation of signals transmitted from different paths. Likewise, our proposed system can also be expressed by a multipath

channel model given in (3.2.9) where the end-to-end channel is equivalently expressed as an L -path channel with each path experiencing different time delays. Signals transmitted through different paths can be separated and properly combined at the destination. Multipath diversity can thus be obtained. Note that unlike the genuine multipath channel, where the paths are usually regarded as independent, the artificial paths created by the proposed relay scheme are not independent. Among the L paths, the number of independent paths is restricted by the number of transmission links in the mini-cut hop. Therefore, the proposed MIMO relay network exhibits the DMT property demonstrated in this chapter.

Chapter 4

DMT of Asymmetric MIMO Multiple-Access Channels with Source Cooperation

The DMT behavior of the symmetric single-antenna multi-user fading cooperative MAC has been studied in [87] and [99], where an AF-based user cooperation scheme [87] and a DF-based user cooperation scheme [99] were proposed and both have been shown to be DMT optimal. For multiple-antenna CMACs, however, the DMT optimal scheme is still unknown. Moreover, compared to the single-antenna systems, a system with multiple-antenna users is more likely to be asymmetric. It is thus of great interest to study the DMT behavior of asymmetric MIMO fading CMACs, as pointed out in [87]. Motivated by this awareness, in this chapter, we investigate the DMT of delay-limited multi-user asymmetric CMAC under both full-duplex and half-duplex user cooperation constraints. The channel is delay-limited in the sense that each source should maintain its transmission rate unless outage happens. We propose a Wyner-Ziv-type CF-based user cooperation scheme and show that it can achieve the optimal DMT of the channel. An achievable rate region of a full-duplex discrete memoryless multi-user CMAC based on CF cooperation is also found as a by-product. Based on the DMT result, we partition the DOF region to reflect, for a given point within the DOF region, the effect of cooperation on the diversity performance of each user. This partition of DOF region facilitates the system design and gives us guidance on designing the most efficient communication scheme for each targeted diversity gain and multiplexing rate pair.

4.1 Channel Model

We consider a cooperative multiple-access channel with K sources and one destination. Besides transmitting its own information, each source could also assist the communications between the other sources and the destination. Figure 4.1 shows such a channel scenario

Figure 4.1: A three-user cooperative multiple-access channel model.

with $K = 3$. Denote $\mathcal{S} = \{1, 2, \dots, K\}$ as the set of source nodes. All nodes in the channel are equipped with multiple antennas with M_i ($i = 1, 2, \dots, K$) antennas at source i and N_D antennas at the destination. The fading channel is assumed to be quasi static and frequency-flat Rayleigh faded. The channel between sources i and j is represented by the $M_i \times M_j$ fading coefficient matrix $\mathbf{H}_{i,j}$ and the channel between source i and the destination is represented by the $M_i \times N_D$ matrix $\mathbf{H}_{i,D}$. By the Rayleigh fading assumption, the entries in $\mathbf{H}_{i,j}$ and $\mathbf{H}_{i,D}$ are both independent and identically complex Gaussian distributed with zero mean and unit variance. The $M_i \times 1$ symbol vector transmitted by user k is denoted as $\mathbf{x}_k$ subjected to a power constraint $E\{||\mathbf{x}_k||^2\} \leq P_k$. We assume the same order of transmit powers for all users, i.e., $P_i \doteq P_j$ for all $i, j \in \mathcal{S}$. The signal received at source node i and at the destination can be found, respectively, as

$$\mathbf{y}_i = \sum_{k=1, k \neq i}^K \mathbf{H}_{k,i} \mathbf{x}_k + \mathbf{z}_i, \quad (4.1.1)$$

$$\mathbf{y}_D = \sum_{k=1}^K \mathbf{H}_{k,D} \mathbf{x}_k + \mathbf{z}_D, \quad (4.1.2)$$

where $\mathbf{z}_i$ and $\mathbf{z}_D$ represent additive white Gaussian noise vectors at source i and the destination, respectively. The elements of $\mathbf{z}_i$ and $\mathbf{z}_D$ are both independent and identically complex Gaussian distributed with zero mean and unit variance.

In the following studies, we focus on delay-limited applications. Each source cannot adjust its transmission rate according to the channel variation, even if it is fully aware of the channel state information.

4.2 DMT with Full-duplex Source Cooperation

4.2.1 DMT Upper Bound

We use the capacity cut-set bound to derive a DMT upper bound for the channel. Applying the network capacity cut-set bound to the considered CMAC, the information rate $\{R_i\}$ of a set of users $\mathcal{T} \subseteq \mathcal{S}$ is upper-bounded by

$$\sum_{i \in \mathcal{T}} R_i \leq I(\mathbf{x}_{\mathcal{T}}; \mathbf{y}_{\mathcal{T}^c} \mathbf{y}_D | \mathbf{x}_{\mathcal{T}^c}), \quad (4.2.1)$$

where $\mathcal{T}^c$ is the complement set of $\mathcal{T}$, $\mathbf{x}_{\mathcal{T}}$ denotes the transmitted signals from all the users in the subset $\mathcal{T}$, and $\mathbf{y}_{\mathcal{T}^c}$ denotes the signals received by all the users in the subset $\mathcal{T}^c$.

Users in the subset $\mathcal{T}$ will be in outage when the available information transmission capacity $\sum_{i \in \mathcal{T}} R_i$ cannot support the targeted information rate $R_{\mathcal{T}}^*$. Define this outage event as

$$\mathcal{O}_{\mathcal{T}} \triangleq \{\mathbf{G} \in \mathbb{C}^{N_D \times \sum_{k=1}^K M_k} : \sum_{i \in \mathcal{T}} R_i < R_{\mathcal{T}}^*\}, \quad (4.2.2)$$

where $\mathbf{G} = [\mathbf{H}_{1D}, \mathbf{H}_{2D}, \dots, \mathbf{H}_{KD}]$ is the channel matrix between the K users and the destination. Applying the cut-set upper bound, the probability for the outage event $\mathcal{O}_{\mathcal{T}}$ to occur can be lower bounded by

$$P(\mathcal{O}_{\mathcal{T}}) \geq P(\mathcal{O}_{\mathcal{T}}^{(LB)}), \quad (4.2.3)$$

where the outage event $\mathcal{O}_{\mathcal{T}}^{(LB)}$ is defined as

$$\mathcal{O}_{\mathcal{T}}^{(LB)} \triangleq \{\mathbf{G} \in \mathbb{C}^{N_D \times \sum_{k=1}^K M_k} : I(\mathbf{x}_{\mathcal{T}}; \mathbf{y}_{\mathcal{T}^c} \mathbf{y}_D | \mathbf{x}_{\mathcal{T}^c}, \mathbf{G}) < R_{\mathcal{T}}^*\}. \quad (4.2.4)$$

We denote $\mathcal{T}_k = \{k\} \cap \tilde{\mathcal{T}}_k$, $\forall \tilde{\mathcal{T}}_k \subset \{1, 2, \dots, k-1, k+1, \dots, K\}$ as the subset of users that user k is included in. Following [114] and [115], the outage event for user k is

$$\mathcal{O}_k \triangleq \bigcup_{\mathcal{T}_k} \mathcal{O}_{\mathcal{T}_k}. \quad (4.2.5)$$

Its outage probability can be evaluated as

$$P(\mathcal{O}_k) = P\left(\bigcup_{\mathcal{T}_k} \mathcal{O}_{\mathcal{T}_k}\right) \geq \max_{\mathcal{T}_k} P(\mathcal{O}_{\mathcal{T}_k}^{(LB)}). \quad (4.2.6)$$

Since $P(\mathcal{O}_{\mathcal{T}_k}^{(LB)})$ can be equivalently evaluated as the outage probability of a MIMO channel

with $\sum_{i \in \mathcal{T}_k} M_i$ transmit antennas and $\sum_{j \in \mathcal{T}_k^c} M_j + N_D$ receive antennas, we have

$$\max_{\mathcal{T}_k} \mathbb{P}(\mathcal{O}_{\mathcal{T}_k}^{(LB)}) \doteq \text{SNR}^{-\min_{\mathcal{T}_k} d_{\sum_{i \in \mathcal{T}_k} M_i, \sum_{j \in \mathcal{T}_k^c} M_j + N_D}(\sum_{i \in \mathcal{T}_k} r_i)}, \quad (4.2.7)$$

and thus

$$\mathbb{P}(\mathcal{O}_k) \doteq \text{SNR}^{-d_k(r_1, r_2, \dots, r_K)} \geq \text{SNR}^{-\min_{\mathcal{T}_k} d_{\sum_{i \in \mathcal{T}_k} M_i, \sum_{j \in \mathcal{T}_k^c} M_j + N_D}(\sum_{i \in \mathcal{T}_k} r_i)}. \quad (4.2.8)$$

From here, we have the DMT upper bound

$$d_k(r_1, r_2, \dots, r_K) \leq \min_{\mathcal{T}_k} d_{\sum_{i \in \mathcal{T}_k} M_i, \sum_{j \in \mathcal{T}_k^c} M_j + N_D} \left(\sum_{i \in \mathcal{T}_k} r_i \right). \quad (4.2.9)$$

To achieve this upper bound, we propose to use CF-based cooperation among the users. In the following two subsections, we will show that given properly designed transmission scheme for the users' own information and the cooperation information, CF-based cooperation can achieve this DMT upper bound. We start with a simple two-user channel to demonstrate the basic idea and then proceed to give a general proof for the K -user channel.

Before going into the detailed illustration of the proposed cooperation scheme, we would like to first compare the DMT of the CMAC with that of the asymmetric MAC. In [132], DMT of the asymmetric MAC was evaluated, which is given as

$$d_k^{\text{MAC}}(r_1, r_2, \dots, r_K) \leq \min_{\mathcal{T}_k} d_{\sum_{i \in \mathcal{T}_k} M_i, N_D} \left(\sum_{i \in \mathcal{T}_k} r_i \right). \quad (4.2.10)$$

By comparing the DMT expression of the MAC and the CMAC, the improvement in diversity by source cooperation can be easily observed.

4.2.2 Achievable Scheme: Two-User Case

For a two-user CMAC, the DMT upper bound in (4.2.9) for user 1 becomes

$$d_1(r_1, r_2) \leq \min\{d_{M_1, M_2 + N_D}(r_1), d_{M_1 + M_2, N_D}(r_1 + r_2)\}. \quad (4.2.11)$$

In the following, we will show that by a properly designed CF-based cooperation scheme, the DMT upper bound can be achieved.

At each time instance, each source transmits two parts of information, namely, the user's own information, and the cooperation information for the other user. The transmitted

signal of user i ($i = 1, 2$) can thus be represented as $\mathbf{x}_i(\mathbf{x}_{i0}, \mathbf{x}_{i1})$, where $\mathbf{x}_{i0}$ represents the part containing the user's own information and $\mathbf{x}_{i1}$ represents the part that contains the cooperation information. The transmission rates for $\mathbf{x}_{i0}$ and $\mathbf{x}_{i1}$ are denoted as R_{i0} and R_{i1} , respectively.

The achievable rate region for the CF-based two-user CMAC with Wyner-Ziv-type compression, block Markov encoding and successive decoding has been found in [42]. Applying the rate region results into our considered Rayleigh-faded vector channel, the achievable rate region can be found as

$$R_{10} \leq I(\mathbf{x}_{10}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - \delta \quad (4.2.12a)$$

$$R_{20} \leq I(\mathbf{x}_{20}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}) - \delta \quad (4.2.12b)$$

$$R_{10} + R_{20} \leq I(\mathbf{x}_{10}, \mathbf{x}_{20}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - \delta \quad (4.2.12c)$$

subject to the following constraints

$$I(\hat{\mathbf{y}}_1; \mathbf{y}_1 | \mathbf{x}_{10}, \mathbf{x}_{11}) + h(\hat{\mathbf{y}}_1 | \hat{\mathbf{y}}_2, \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_1 | \mathbf{x}_{11}, \mathbf{x}_{21}) + \delta \leq I(\mathbf{x}_{11}; \mathbf{y}_D | \mathbf{x}_{21}) \quad (4.2.13a)$$

$$I(\hat{\mathbf{y}}_2; \mathbf{y}_2 | \mathbf{x}_{20}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_2 | \hat{\mathbf{y}}_1, \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}) + \delta \leq I(\mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) \quad (4.2.13b)$$

$$I(\hat{\mathbf{y}}_1; \mathbf{y}_1 | \mathbf{x}_{10}, \mathbf{x}_{11}) + I(\hat{\mathbf{y}}_2; \mathbf{y}_2 | \mathbf{x}_{20}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_1 | \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}) + \delta \leq I(\mathbf{x}_{11}, \mathbf{x}_{21}; \mathbf{y}_D), \quad (4.2.13c)$$

where $\delta > 0$ denotes an arbitrary small number, and $\hat{\mathbf{y}}_1$ and $\hat{\mathbf{y}}_2$ are the compressed signals obtained from $\mathbf{y}_1$ and $\mathbf{y}_2$, respectively. For the Gaussian channel case, $\hat{\mathbf{y}}_1$ and $\hat{\mathbf{y}}_2$ can be calculated as

$$\hat{\mathbf{y}}_1 = \mathbf{y}_1 + \mathbf{z}_{W1} \quad (4.2.14)$$

$$\hat{\mathbf{y}}_2 = \mathbf{y}_2 + \mathbf{z}_{W2}, \quad (4.2.15)$$

where $\mathbf{z}_{W1}$ and $\mathbf{z}_{W2}$ denote the compression noise with variances σ_{W1}^2 and σ_{W2}^2 , respectively.

Note that by applying the results in [42], we have made some modifications to the constraints in (4.2.13). In [42], the constraints are stricter due to the conditions on entropies $h(\hat{\mathbf{y}}_1 | \mathbf{x}_{11}, \mathbf{x}_{21})$ and $h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21})$. Although the rate region expression in [42] is more explicit, by examining the error probability analysis, we find that the rate region given in (4.2.12) and (4.2.13) is more accurate. Also note that the above rate region is derived for discrete channels, but our considered system has continuous variables. However, according to [32],

for Gaussian input distributions, we can still use the rate region results in our system. The same argument holds for the K -user channel presented in the following subsection.

User 1 will be in outage when either one of the following two events happens: user 1 is in outage while user 2 can be correctly detected and both user 1 and user 2 are in outage. The outage probability for user 1 can thus be evaluated as

$$P(\mathcal{O}_1) = P(\mathcal{O}_{\{1\}} \cup \mathcal{O}_{\{1,2\}}) \leq P(\mathcal{O}_{\{1\}}) + P(\mathcal{O}_{\{1,2\}}). \quad (4.2.16)$$

For a targeted multiplexing rate r_1 , the outage probability $P(\mathcal{O}_{\{1\}})$ can be computed as

$$P(\mathcal{O}_{\{1\}}) \doteq P(I(\mathbf{x}_{10}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) < r_1 \log \text{SNR}). \quad (4.2.17)$$

The mutual information $I(\mathbf{x}_{10}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20})$ can be evaluated as

$$\begin{aligned} I(\mathbf{x}_{10}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) &\geq I(\mathbf{x}_{10}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) \\ &\geq h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) \\ &\quad - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}). \end{aligned} \quad (4.2.18)$$

Denoting

$$h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) = \log |\pi e \boldsymbol{\Sigma}_{1D}|, \quad (4.2.19)$$

where $\boldsymbol{\Sigma}_{1D}$ is a matrix specified by the channel, and together with the fact that

$$h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) = \log |\pi e (1 + \sigma_{W2}^2) \mathbf{I}_{M_2}|, \quad (4.2.20)$$

the outage probability in (4.2.17) becomes

$$\begin{aligned} P(\mathcal{O}_{\{1\}}) &\leq P\left(\frac{|\boldsymbol{\Sigma}_{1D}|}{(1 + \sigma_{W2}^2)^{M_2}} < \text{SNR}^{r_1}\right) \\ &= P\left(\frac{1}{|\boldsymbol{\Sigma}_{1D}|^{1/M_2}} + \frac{\sigma_{W2}^2}{|\boldsymbol{\Sigma}_{1D}|^{1/M_2}} > \frac{1}{\text{SNR}^{r_1/M_2}}\right) \\ &\leq P(|\boldsymbol{\Sigma}_{1D}|^{1/M_2} < 2\text{SNR}^{r_1/M_2}) + P\left(\frac{|\boldsymbol{\Sigma}_{1D}|^{1/M_2}}{\sigma_{W2}^2} < 2\text{SNR}^{r_1/M_2}\right) \\ &\doteq P(|\boldsymbol{\Sigma}_{1D}| < \text{SNR}^{r_1}) + P\left(\frac{|\boldsymbol{\Sigma}_{1D}|}{|\sigma_{W2}^2 \mathbf{I}_{M_2}|} < \text{SNR}^{r_1}\right) \\ &\doteq P(h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) < r_1 \log \text{SNR}) \\ &\quad + P(h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20})) \end{aligned} \quad (4.2.21)$$

$$-\log |\pi e \sigma_{W_2}^2 \mathbf{I}_{M_2}| < r_1 \log \text{SNR}), \quad (4.2.22)$$

where the inequality in (4.2.21) is obtained based on

$$\mathbb{P}(A + B > c) \leq \mathbb{P}((A > c/2) \cup (B > c/2)) \leq \mathbb{P}(A > c/2) + \mathbb{P}(B > c/2),$$

where A and B are any positive random variables, and $c > 0$ is a constant.

For the first term in (4.2.22), we have

$$\begin{aligned} & h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) \\ & \geq h(\mathbf{y}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) \\ & = I(\mathbf{x}_{10}; \mathbf{y}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}). \end{aligned} \quad (4.2.23)$$

To evaluate the second term of (4.2.22), the value of $\sigma_{W_2}^2$ is needed. It can be found using the constraints in (4.2.13). Among the three constraints in (4.2.13), (4.2.13a) and (4.2.13b) impose constraints on $\sigma_{W_1}^2$ and $\sigma_{W_2}^2$, respectively, and (4.2.13c) imposes constraints on both $\sigma_{W_1}^2$ and $\sigma_{W_2}^2$. We find that in the high SNR region, the values of $\sigma_{W_1}^2$ and $\sigma_{W_2}^2$ can be equivalently characterized using (4.2.13a), (4.2.13b) or (4.2.13c) (see Appendix C for the proof). Therefore, we simply use (4.2.13b) to calculate the value of $\sigma_{W_2}^2$.

From (4.2.13b), we have

$$\begin{aligned} & h(\hat{\mathbf{y}}_2 | \mathbf{x}_{20}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_2 | \mathbf{y}_2, \mathbf{x}_{20}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_2 | \hat{\mathbf{y}}_1, \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}) \\ & \leq h(\mathbf{y}_D | \mathbf{x}_{11}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}). \end{aligned} \quad (4.2.24)$$

Together with the fact that

$$h(\hat{\mathbf{y}}_2 | \mathbf{y}_2, \mathbf{x}_{20}, \mathbf{x}_{21}) = \log |\pi e \sigma_{W_2}^2 \mathbf{I}_{M_2}|, \quad (4.2.25)$$

the second term in (4.2.22) can be found as

$$\begin{aligned} & h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) - \log |\pi e \sigma_{W_2}^2 \mathbf{I}_{M_2}| \\ & = h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{20}, \mathbf{x}_{21}) \\ & \quad - h(\hat{\mathbf{y}}_2 | \hat{\mathbf{y}}_1, \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}) + h(\mathbf{y}_D | \mathbf{x}_{11}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \\ & \geq h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) + h(\hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}, \mathbf{x}_{20}) \\ & \quad - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}) + h(\mathbf{y}_D | \mathbf{x}_{11}) - h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \\ & = I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - I(\mathbf{x}_{20}; \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}) \end{aligned}$$

$$= I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}). \quad (4.2.26)$$

Using (4.2.23) and (4.2.26), $P(\mathcal{O}_{\{1\}})$ in (4.2.22) can be evaluated as

$$\begin{aligned} P(\mathcal{O}_{\{1\}}) &\leq P(I(\mathbf{x}_{10}; \mathbf{y}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) < r_1 \log \text{SNR}) \\ &\quad + P(I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \\ &\quad - I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}) < r_1 \log \text{SNR}). \end{aligned} \quad (4.2.27)$$

To further calculate the value of (4.2.27), the ways of arranging the transmission of the two information parts $\mathbf{x}_{i0}$ and $\mathbf{x}_{i1}$ need to be specified. We first consider an uniform superposition based transmission scheme, where both parts of information are transmitted using all the transmit antennas and at tantamount power. The symbol transmitted by source i can be formulated as

$$\mathbf{x}_i = \mathbf{x}_{i0} + \mathbf{x}_{i1} = \sqrt{\alpha_i P_i} \tilde{\mathbf{x}}_{i0} + \sqrt{\bar{\alpha}_i P_i} \tilde{\mathbf{x}}_{i1}, \quad (4.2.28)$$

where $\tilde{\mathbf{x}}_{i0} \in \mathbb{C}^{M_i \times 1}$, $\tilde{\mathbf{x}}_{i1} \in \mathbb{C}^{M_i \times 1}$, $0 < \alpha_i < 1$ is a fixed power allocation factor independent of P_i , and $\bar{\alpha}_i = 1 - \alpha_i$. By the uniform superposition based scheme, the mutual information expression in the first term of (4.2.27) can be viewed as the mutual information of an $M_1 \times (M_2 + N_D)$ MIMO channel. The first term of (4.2.27) can thus be evaluated as

$$P(I(\mathbf{x}_{10}; \mathbf{y}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) < r_1 \log \text{SNR}) \doteq \text{SNR}^{-d_{M_1, (M_2 + N_D)}(r_1)}. \quad (4.2.29)$$

For the second term of (4.2.27), using the chain rule and some simple calculations, the mutual information expression can be obtained as

$$\begin{aligned} &I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}) \\ &= I(\mathbf{x}_{10}; \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) + I(\mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}) \\ &\doteq I(\mathbf{x}_{10}; \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}). \end{aligned} \quad (4.2.30)$$

The last step of the above derivation is due to $I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) = 0$, and for uniform superposition based transmission, $I(\mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) \doteq 0 \cdot \log \text{SNR}$ and $I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}) \doteq 0 \cdot \log \text{SNR}$. The mutual information in (4.2.30) can be viewed as the mutual information of an $M_1 \times N_D$ MIMO channel. Therefore

$$P(I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}) < r_1 \log \text{SNR})$$

$$\doteq \text{SNR}^{-d_{M_1, N_D}(r_1)}. \quad (4.2.31)$$

The overall outage probability is thus upper bounded by

$$\text{P}(\mathcal{O}_{\{1\}}) \leq \text{SNR}^{-d_{M_1, M_2 + N_D}(r_1)} + \text{SNR}^{-d_{M_1, N_D}(r_1)} \doteq \text{SNR}^{-d_{M_1, N_D}(r_1)}. \quad (4.2.32)$$

From the above derivation, we can see that with the uniform superposition based transmission scheme, CF-based CMAC cannot achieve the DMT upper bound. The reason is explained as follows. By the CF-based cooperation strategy, the destination should first decode $\mathbf{x}_{i1}$, based on which $\hat{\mathbf{y}}_i$ can be recovered and further used to help the decoding of $\mathbf{x}_{j0}$ (user j is the user that user i helps with). If uniform superposition is adopted for organizing the transmission of $\mathbf{x}_{i0}$ and $\mathbf{x}_{i1}$, with the uniformly presence of $\mathbf{x}_{i0}$ as an interference, the transmission rate of $\mathbf{x}_{i1}$ cannot grow with SNR. In other words, the multiplexing gain for transmitting $\mathbf{x}_{i1}$ is zero. The low transmission rate of $\mathbf{x}_{i1}$ will lead to high compression distortion at user i , which could degrade the usefulness of the cooperation information. In the worst case, the cooperation information might be totally useless and the DMT behavior of the cooperative transmission scheme would be no better than that of the non-cooperative scheme.

Let us view it from another angle. Since

$$I(\mathbf{x}_{i0}, \mathbf{x}_{i1}; \mathbf{y}_D) = I(\mathbf{x}_{i1}; \mathbf{y}_D) + I(\mathbf{x}_{i0}; \mathbf{y}_D | \mathbf{x}_{i1}), \quad (4.2.33)$$

given a good channel condition between user i and the destination, for example, $I(\mathbf{x}_{i0}, \mathbf{x}_{i1}; \mathbf{y}_D) \doteq M_i \log \text{SNR}$, $I(\mathbf{x}_{i1}; \mathbf{y}_D) \doteq 0 \cdot \log \text{SNR}$ implies $I(\mathbf{x}_{i0}; \mathbf{y}_D | \mathbf{x}_{i1}) \doteq M_i \log \text{SNR}$. If the targeted multiplexing rate of user i is $r_i < M_i$, then the transmission of $\mathbf{x}_{i0}$ through a channel that can accommodate a higher rate will lead to a waste of resources that could be otherwise used for cooperation. This provides user i with unneeded extra protection while depriving the user in need of obtaining this protection.

Therefore, to achieve the optimal DMT, the transmission of $\mathbf{x}_{i0}$ and $\mathbf{x}_{i1}$ should be properly organized to make full use of the transmission resources (power, antennas, etc.). This intuition leads to our proposed coding scheme. The basic idea is as follows. Since the CF-based cooperation scheme requires users to know all the channel state information, each user can arrange the transmission by taking the channel conditions into consideration. Instead of simply transmitting the two parts of information at fixed power and through all the antennas, each of these two information parts can be transmitted using part of the antennas, or at a transmitting power that is in the order of SNR^β ($0 \leq \beta \leq 1$). The antennas used for transmitting each part of information (in the first approach) and the value of β (in the second

approach) are chosen based on the channel condition and the required transmission rate of the user. The first approach can be viewed as a spatial-multiplexing-based scheme, and the second approach can be viewed as a rate-splitting-based scheme [131]. The target of both approaches is to restrict the resources allocated for transmitting the users' own information to be just what is needed, and consequently, to release more resources for cooperation.

In the following, we take the spatial-multiplexing-based approach as an example to demonstrate the details of the proposed transmission scheme. In the discussion, the multiplexing gains are assumed to be integers. By combining the spatial-multiplexing-based approach and the rate-splitting-based approach, any arbitrary multiplexing gains can be achieved. With the knowledge on the channel state information about all the channels, and assuming $\mathbf{x}_i/\sqrt{P_i} \in \mathbb{C}^{M_i \times 1}$, user i calculates $I(\mathbf{x}_i; \hat{\mathbf{y}}_i, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21})$ and decides the scheme for transmitting $\mathbf{x}_{i0}$ and $\mathbf{x}_{i1}$. For a targeted multiplexing rate r_i , the following transmission scheme is used.

- If $I(\mathbf{x}_i; \hat{\mathbf{y}}_i, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \leq r_i \log \text{SNR}$, uniform superposition is used to encode and transmit $\mathbf{x}_{i0}$ and $\mathbf{x}_{i1}$.
- If $I(\mathbf{x}_i; \hat{\mathbf{y}}_i, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) > r_i \log \text{SNR}$, $\mathbf{x}_{i0}$ is encoded among and transmitted through some selected antennas such that $I(\mathbf{x}_{i0}; \hat{\mathbf{y}}_i, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \doteq r_i \log \text{SNR}$. Being allocated with sufficient (but fixed and independent of SNR) transmitting power, we can ensure $I(\mathbf{x}_{i0}; \hat{\mathbf{y}}_i, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) = R_{i0}$, and $\mathbf{x}_{i0}$ can be correctly decoded at destination with high probability. $\mathbf{x}_{i1}$ is transmitted through the remaining antennas using the remaining power. This provides $\mathbf{x}_{i1}$ a non-zero multiplexing rate.

Denoting $I_i = I(\mathbf{x}_i; \hat{\mathbf{y}}_i, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21})$ with $\mathbf{x}_i/\sqrt{P_i} \in \mathbb{C}^{M_i \times 1}$, and defining

$$\mathcal{E}_i \triangleq (I_i \leq r_i \log \text{SNR}), \quad (4.2.34)$$

$$\bar{\mathcal{E}}_i \triangleq (I_i > r_i \log \text{SNR}), \quad (4.2.35)$$

we discuss the outage probability by considering the following four scenarios:

- $I_1 > r_1 \log \text{SNR}$ and $I_2 > r_2 \log \text{SNR}$.

In this case, each user can be decoded at the destination without using the cooperation information and simply treating the other user as interference. Both users can arrange its transmission using the proposed spatial-multiplexing-based scheme. Since joint decoding and cooperation can only enhance performance, we have

$$P(\mathcal{O}_{\{1\}} | \bar{\mathcal{E}}_1, \bar{\mathcal{E}}_2) = 0, \quad (4.2.36)$$

$$P(\mathcal{O}_{\{2\}} | \bar{\mathcal{E}}_1, \bar{\mathcal{E}}_2) = 0, \quad (4.2.37)$$

$$P(\mathcal{O}_{\{1,2\}} | \bar{\mathcal{E}}_1, \bar{\mathcal{E}}_2) = 0. \quad (4.2.38)$$

ii. $I_1 \leq r_1 \log \text{SNR}$ and $I_2 \leq r_2 \log \text{SNR}$.

In this case, both users might be in outage. Each user arranges its transmission using the uniform superposition scheme. The mutual information in the second term of (4.2.27) can be evaluated as

$$\begin{aligned} & I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{11}; \hat{\mathbf{y}}_2 | \mathbf{x}_{21}) \\ & \geq I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) - r_2 \log \text{SNR}, \end{aligned} \quad (4.2.39)$$

Therefore, the outage probability in (4.2.27) becomes

$$\begin{aligned} P(\mathcal{O}_{\{1\}} | \mathcal{E}_1, \mathcal{E}_2) & \leq P(I(\mathbf{x}_{10}; \mathbf{y}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) < r_1 \log \text{SNR}) \\ & \quad + P(I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) < (r_1 + r_2) \log \text{SNR}). \end{aligned} \quad (4.2.40)$$

Similarly,

$$\begin{aligned} P(\mathcal{O}_{\{2\}} | \mathcal{E}_1, \mathcal{E}_2) & \leq P(I(\mathbf{x}_{20}; \mathbf{y}_1, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}) < r_2 \log \text{SNR}) \\ & \quad + P(I(\mathbf{x}_{20}, \mathbf{x}_{10}, \mathbf{x}_{11}; \mathbf{y}_D | \mathbf{x}_{21}) < (r_1 + r_2) \log \text{SNR}). \end{aligned} \quad (4.2.41)$$

Furthermore, the probability that both users are in outage can be evaluated as

$$\begin{aligned} P(\mathcal{O}_{\{1,2\}}) & \doteq P(I(\mathbf{x}_{10}, \mathbf{x}_{20}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) < (r_1 + r_2) \log \text{SNR}) \\ & \leq P(I(\mathbf{x}_{10}, \mathbf{x}_{20}; \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) < (r_1 + r_2) \log \text{SNR}). \end{aligned} \quad (4.2.42)$$

iii. $I_1 \leq r_1 \log \text{SNR}$ and $I_2 > r_2 \log \text{SNR}$.

In this case, user 1 might be in outage. The information of user 2 can be successfully decoded at the destination with high probability by simply treating the signals from user 1 as interference. User 2 can spare part of its transmitting resources to help user 1, while user 1 must make full use of its transmission resources for its own communication. Therefore, user 1 arranges its transmission using uniform superposition, and user 2 uses spatial-multiplexing-based scheme with $I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \doteq r_2 \log \text{SNR}$. The outage probabilities are evaluated as

$$\begin{aligned} P(\mathcal{O}_{\{1\}} | \mathcal{E}_1, \bar{\mathcal{E}}_2) & \leq P(I(\mathbf{x}_{10}; \mathbf{y}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) < r_1 \log \text{SNR}) \\ & \quad + P(I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D | \mathbf{x}_{11}) < (r_1 + r_2) \log \text{SNR}), \end{aligned} \quad (4.2.43)$$

$$P(\mathcal{O}_{\{2\}} | \mathcal{E}_1, \bar{\mathcal{E}}_2) = 0, \quad (4.2.44)$$

and

$$\begin{aligned}
& P(\mathcal{O}_{\{1,2\}}|\mathcal{E}_1, \bar{\mathcal{E}}_2) \\
& \doteq P(I(\mathbf{x}_{10}, \mathbf{x}_{20}; \hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D|\mathbf{x}_{11}, \mathbf{x}_{21}) < (r_1 + r_2) \log \text{SNR}) \\
& \leq P(I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D|\mathbf{x}_{11}, \mathbf{x}_{21}) + I(\mathbf{x}_{10}; \hat{\mathbf{y}}_2, \mathbf{y}_D|\mathbf{x}_{11}, \mathbf{x}_{21}) < (r_1 + r_2) \log \text{SNR}) \\
& \doteq P(I(\mathbf{x}_{10}; \hat{\mathbf{y}}_2, \mathbf{y}_D|\mathbf{x}_{11}, \mathbf{x}_{21}) < r_1 \log \text{SNR}). \tag{4.2.45}
\end{aligned}$$

iv. $I_1 > r_1 \log \text{SNR}$ and $I_2 \leq r_2 \log \text{SNR}$.

In this case, user 1 transmits using spatial-multiplexing-based scheme while user 2 uses uniform superposition. Following the same discussion as in the previous case, we have

$$P(\mathcal{O}_{\{1\}}|\bar{\mathcal{E}}_1, \mathcal{E}_2) = 0, \tag{4.2.46}$$

$$\begin{aligned}
P(\mathcal{O}_{\{2\}}|\bar{\mathcal{E}}_1, \mathcal{E}_2) & \leq P(I(\mathbf{x}_{20}; \mathbf{y}_1, \mathbf{y}_D|\mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}) < r_2 \log \text{SNR}) \\
& + P(I(\mathbf{x}_{20}, \mathbf{x}_{10}, \mathbf{x}_{11}; \mathbf{y}_D|\mathbf{x}_{11}) < (r_1 + r_2) \log \text{SNR}), \tag{4.2.47}
\end{aligned}$$

and

$$P(\mathcal{O}_{\{1,2\}}|\bar{\mathcal{E}}_1, \mathcal{E}_2) \leq P(I(\mathbf{x}_{20}; \hat{\mathbf{y}}_1, \mathbf{y}_D|\mathbf{x}_{11}, \mathbf{x}_{21}) < r_2 \log \text{SNR}). \tag{4.2.48}$$

In summary, $P(\mathcal{O}_1)$ can be evaluated as

$$\begin{aligned}
P(\mathcal{O}_1) & = P(\mathcal{O}_{\{1\}}|\bar{\mathcal{E}}_1, \bar{\mathcal{E}}_2)P(\bar{\mathcal{E}}_1, \bar{\mathcal{E}}_2) + P(\mathcal{O}_{\{1\}}|\mathcal{E}_1, \bar{\mathcal{E}}_2)P(\mathcal{E}_1, \bar{\mathcal{E}}_2) + P(\mathcal{O}_{\{1\}}|\bar{\mathcal{E}}_1, \mathcal{E}_2)P(\bar{\mathcal{E}}_1, \mathcal{E}_2) \\
& + P(\mathcal{O}_{\{1\}}|\mathcal{E}_1, \mathcal{E}_2)P(\mathcal{E}_1, \mathcal{E}_2) \\
& \leq P(I(\mathbf{x}_{10}; \mathbf{y}_2, \mathbf{y}_D|\mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{20}) < r_1 \log \text{SNR}) \\
& + P(I(\mathbf{x}_{10}, \mathbf{x}_{20}, \mathbf{x}_{21}; \mathbf{y}_D|\mathbf{x}_{11}) < (r_1 + r_2) \log \text{SNR}), \tag{4.2.49}
\end{aligned}$$

where the mutual informations are evaluated based on a strategy where $\mathbf{x}_{10}$ and $\mathbf{x}_{11}$ are transmitted using uniform superposition and $\mathbf{x}_{20}$ and $\mathbf{x}_{21}$ are transmitted using spatial multiplexing. The mutual information in the first term is equivalent to that of an $M_1 \times (M_2 + N_D)$ MIMO channel, and the mutual information in the second probability term is equivalent to that of an $(M_1 + M_2) \times N_D$ MIMO channel. Therefore,

$$P(\mathcal{O}_{\{1\}}) \doteq \text{SNR}^{-d_{M_1, M_2 + N_D}(r_1)} + \text{SNR}^{-d_{M_1 + M_2, N_D}(r_1 + r_2)}. \tag{4.2.50}$$

Similarly, $P(\mathcal{O}_{\{2\}})$ can be evaluated as

$$\begin{aligned} P(\mathcal{O}_{\{2\}}) &\leq P(I(\mathbf{x}_{20}; \mathbf{y}_1, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{10}) < r_2 \log \text{SNR}) \\ &\quad + P(I(\mathbf{x}_{20}, \mathbf{x}_{10}, \mathbf{x}_{11}; \mathbf{y}_D | \mathbf{x}_{21}) < (r_1 + r_2) \log \text{SNR}) \\ &\doteq \text{SNR}^{-d_{M_2, M_1 + N_D}(r_2)} + \text{SNR}^{-d_{M_1 + M_2, N_D}(r_1 + r_2)}. \end{aligned} \quad (4.2.51)$$

Here, the mutual information in the probability terms is evaluated based on a strategy that user 2 arranges its transmission using uniform superposition and user 1 arranges its transmission using spatial multiplexing.

The outage probability $P(\mathcal{O}_{\{1,2\}})$ can be found as

$$\begin{aligned} P(\mathcal{O}_{\{1,2\}}) &\leq \max \left\{ P(I(\mathbf{x}_{10}, \mathbf{x}_{20}; \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) < (r_1 + r_2) \log \text{SNR}), \right. \\ &\quad \left. P(I(\mathbf{x}_{i0}; \hat{\mathbf{y}}_j, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) < r_i \log \text{SNR}) \right\} \end{aligned} \quad (4.2.52)$$

$$\doteq \max \left\{ \text{SNR}^{-d_{M_1 + M_2, N_D}(r_1 + r_2)}, \text{SNR}^{-d_{M_i, M_j + N_D}(r_i)} \right\}, \quad (4.2.53)$$

where $i = 1, 2$, $j = 1, 2$ and $i \neq j$, the mutual information expressions in (4.2.52) are evaluated by assuming that both users use uniform superposition. The values of i and j are chosen depending on the focused user. For instance, when trying to evaluate $P(\mathcal{O}_1)$, then $i = 1$ and $j = 2$ are chosen.

Applying the outage probability results, we have

$$P(\mathcal{O}_1) \leq P(\mathcal{O}_{\{1\}}) + P(\mathcal{O}_{\{1,2\}}) \leq \text{SNR}^{-d_{M_1, M_2 + N_D}(r_1)} + \text{SNR}^{-d_{M_1 + M_2, N_D}(r_1 + r_2)}, \quad (4.2.54)$$

and

$$P(\mathcal{O}_2) \leq P(\mathcal{O}_{\{2\}}) + P(\mathcal{O}_{\{1,2\}}) \leq \text{SNR}^{-d_{M_2, M_1 + N_D}(r_2)} + \text{SNR}^{-d_{M_1 + M_2, N_D}(r_1 + r_2)}. \quad (4.2.55)$$

The DMT expressions of the two users can thus be lower-bounded by

$$d_1(r_1, r_2) \geq \min\{d_{M_1, M_2 + N_D}(r_1), d_{M_1 + M_2, N_D}(r_1 + r_2)\} \quad (4.2.56)$$

and

$$d_2(r_1, r_2) \geq \min\{d_{M_2, M_1 + N_D}(r_2), d_{M_1 + M_2, N_D}(r_1 + r_2)\}, \quad (4.2.57)$$

respectively. Since this lower bound meets the upper bound given in (4.2.9), with the pro-

Figure 4.2: Outage probabilities of a two-user CMAC with $M_1 = 3$, $M_2 = 3$ and $N_D = 6$. The two users operate at multiplexing gains $r_1 = 2$ and $r_2 = 1$.

posed transmission scheme, the CF-based CMAC achieves the optimal DMT.

In Fig. 4.2, simulation results for the outage probabilities of a two-user CMAC with $M_1 = 3$, $M_2 = 3$ and $N_D = 6$ are shown. The targeted multiplexing gains for the two users are $r_1 = 2$ and $r_2 = 1$, respectively. The outage probability curves of the uniform-superposition-based transmission scheme and our proposed spatial-multiplexing-based transmission scheme are simulated and compared. As a benchmark, we also show the outage probability curves of a non-cooperative two-user MAC and a two-user CMAC with perfect user cooperation. For the perfect cooperation case, the optimal diversity orders of the two users are $d_1 = 7$ and $d_2 = 9$, respectively. For the non-cooperative case, the optimal diversity orders are $d_1 = 4$ and $d_2 = 9$, respectively. Since the difference in diversity for the cooperative system and the non-cooperative system is only reflected by the first user, we show all the outage probability curves for user 1, while for user 2 we only show the outage probability curves of our proposed transmission scheme and the system with perfect cooperation. From the simulation results, we see that for both users, the outage probability curves of our proposed transmission scheme are almost parallel with that of the perfect cooperation, both of which exhibit higher diversity order than the system using uniform superposition. One can also observe that the outage probability curve of the system using uniform superposition is almost parallel with that of non-cooperative system. This observation is consistent with our conclusion that if uniform superposition is used, the cooperative system will yield no better diversity gain than that of the non-cooperative system.

4.2.3 Achievable Scheme: K -User Case

We extend the analysis in the previous section to the K -user channel. This extension is non-trivial since in order to achieve the DMT upper bound, in addition to the requirement on the transmission arrangement of the user information and the cooperation information as shown in the two-user case, in the K -user case, each user also requires to decode the cooperation part of information received from other users before forwarding the message. In other words, the users should try not to forward unneeded message (the cooperation part of information transmitted by other users) and only focus on the useful part (other users' user information). Moreover, since the users may have different decoding abilities, the cooperation part of information should be designed such that it can be partially decoded by the other users. In the following context, we evaluate the DMT by first providing an achievable rate region for the discrete memoryless K -user CMAC.

To the best of our knowledge, the achievable rate region for a CF-based K -user CMAC is unknown. We provide such an achievable rate region in the following theorem.

Theorem 4.2.1. *An achievable rate region of a discrete memoryless K -user cooperative multiple-access channel with Wyner-Ziv-type CF cooperation, successive decoding at the destination, and (partial) decoding of the cooperation information at each user is*

$$R_{\mathcal{T}} < I(X_{\mathcal{T}0}; \hat{Y}_{\mathcal{S}}, Y_D | X_{\mathcal{T}^c0}, X_{\mathcal{S}1}), \text{ for all } \mathcal{T} \subseteq \mathcal{S}, \quad (4.2.58)$$

for some joint probability mass distribution

$$\begin{aligned} & p(x_{10}, x_{20}, \dots, x_{K0}, x_{11}, x_{21}, \dots, x_{K1}, y_1, y_2, \dots, y_K, \hat{y}_1, \hat{y}_2, \dots, \hat{y}_K, y_D) \\ = & \left(\prod_{k=1}^K p(x_{k1}) p(x_{k0} | x_{k1}) \right) \cdot \left(\prod_{k=1}^K p(\hat{y}_k | y_k, x_{k0}, x_{k1}) \right) \\ & \cdot p(y_1, y_2, \dots, y_K, y_D | x_{10}, x_{20}, \dots, x_{K0}, x_{11}, x_{21}, \dots, x_{K1}), \end{aligned} \quad (4.2.59)$$

and subject to the following constraints

$$\sum_{k \in \mathcal{T}} I(\hat{Y}_k; Y_k | X_{k0}, X_{k1}, X_{\mathcal{S}_k^{dc}}) + H(\hat{Y}_{\mathcal{T}} | X_{\mathcal{S}1}, \hat{Y}_{\mathcal{S}^c}, Y_D) - \sum_{k \in \mathcal{T}} H(\hat{Y}_k | X_{\mathcal{S}1}) < I(X_{\mathcal{T}1}; Y_D | X_{\mathcal{T}^c1}), \quad (4.2.60)$$

for all $\mathcal{T} \subseteq \mathcal{S}$. Here, $\mathcal{S}_k = \mathcal{S} / \{k\}$, and $X_{\mathcal{S}_k^{dc}}^{dc}$ denotes the part of $X_{\mathcal{S}_k1}$ that has been decoded by user k and satisfies

$$I(X_{\mathcal{S}_k^{dc}}^{dc}; Y_k) \doteq I(X_{\mathcal{S}_k1}; Y_k), \quad (4.2.61)$$

$$I(X_{\mathcal{S}_k^1}^{ndc}; Y_k | X_{\mathcal{S}_k^1}^{dc}) \doteq 0 \cdot \log \text{SNR}, \quad (4.2.62)$$

where $X_{\mathcal{S}_k^1}^{ndc}$ is the part of $X_{\mathcal{S}_k}$ that cannot be decoded by user k .

Proof. See Appendix D. □

Applying the above rate region to the Rayleigh-faded vector channel, the probability that $\mathcal{T} \subseteq \mathcal{S}$ users are in outage can be calculated using a similar approach as in the two-user case as follows:

$$\begin{aligned} P(\mathcal{O}_{\mathcal{T}}) &= P(I(\mathbf{x}_{\mathcal{T}0}; \hat{\mathbf{y}}_{\mathcal{S}}, \mathbf{y}_D | \mathbf{x}_{\mathcal{T}^c0}, \mathbf{x}_{\mathcal{S}1}) < \sum_{i \in \mathcal{T}} r_i \log \text{SNR}) \\ &\leq P(I(\mathbf{x}_{\mathcal{T}0}; \mathbf{y}_{\mathcal{T}^c}, \mathbf{y}_D | \mathbf{x}_{\mathcal{T}^c0}, \mathbf{x}_{\mathcal{S}1}) < \sum_{i \in \mathcal{T}} r_i \log \text{SNR}) \\ &\quad + P(I(\mathbf{x}_{\mathcal{S}0}, \mathbf{x}_{\mathcal{T}^c1}; \mathbf{y}_D | \mathbf{x}_{\mathcal{T}1}) - I(\mathbf{x}_{\mathcal{T}^c0}; \hat{\mathbf{y}}_{\mathcal{T}^c}, \mathbf{y}_D | \mathbf{x}_{\mathcal{S}1}) \\ &\quad - \sum_{i \in \mathcal{T}^c} I(\mathbf{x}_{\mathcal{S}_i^1}^{ndc}, \hat{\mathbf{y}}_i | \mathbf{x}_{i1}, \mathbf{x}_{\mathcal{S}_i^1}^{dc}) < \sum_{i \in \mathcal{T}} r_i \log \text{SNR}). \end{aligned} \quad (4.2.63)$$

For the CF-based K -user CMAC to be DMT optimal, we use a cooperative transmission scheme that satisfies the following two requirements:

- i. The transmission of the user information part and the cooperation part are arranged in a way that if the channel between a user and the destination has a higher multiplexing gain than what the user needs, then just use the needed transmission resources (power, antennas, etc.) to transmit the user information. The remaining transmission resources are used in transmitting the cooperation part of information. Otherwise, use uniform superposition to transmit the two parts of information.
- ii. Before performing compression and forwarding, each user first tries its best to decode the received cooperation part of information sent by the other users.

The first requirement ensures

$$I(\mathbf{x}_{\mathcal{T}^c0}; \hat{\mathbf{y}}_{\mathcal{T}^c}, \mathbf{y}_D | \mathbf{x}_{\mathcal{S}1}) \leq \sum_{i \in \mathcal{T}^c} r_i \log \text{SNR}. \quad (4.2.64)$$

The second requirement ensures

$$I(\mathbf{x}_{\mathcal{S}_i^1}^{ndc}, \hat{\mathbf{y}}_i | \mathbf{x}_{i1}, \mathbf{x}_{\mathcal{S}_i^1}^{dc}) \doteq 0 \cdot \log \text{SNR}. \quad (4.2.65)$$

Following a similar procedure as in the two-user case, the outage probability can be evaluated

as

$$\begin{aligned}
P(\mathcal{O}_{\mathcal{T}}) &\stackrel{\cdot}{\leq} P(I(\mathbf{x}_{\mathcal{T}0}; \mathbf{y}_{\mathcal{T}^c}, \mathbf{y}_D | \mathbf{x}_{\mathcal{T}^c0}, \mathbf{x}_{S1}) < \sum_{i \in \mathcal{T}} r_i \log \text{SNR}) \\
&\quad + P(I(\mathbf{x}_{S0}, \mathbf{x}_{\mathcal{T}^c1}; \mathbf{y}_D | \mathbf{x}_{\mathcal{T}1}) < \sum_{i=1}^K r_i \log \text{SNR}) \\
&\stackrel{\cdot}{=} \text{SNR}^{-d_{\sum_{i \in \mathcal{T}} M_i, \sum_{j \in \mathcal{T}^c} M_j + N_D}(\sum_{i \in \mathcal{T}} r_i)} + \text{SNR}^{-d_{\sum_{i=1}^K M_i, N_D}(\sum_{i=1}^K r_i)}. \quad (4.2.66)
\end{aligned}$$

Here, the mutual information in the probability terms is calculated based on a strategy that the users in $\mathcal{T}$ are transmitted using uniform superposition. The diversity gain can therefore be lower-bounded as

$$d_{\mathcal{T}}(r_1, r_2, \dots, r_K) \geq \min \left\{ d_{\sum_{i \in \mathcal{T}} M_i, \sum_{j \in \mathcal{T}^c} M_j + N_D} \left(\sum_{i \in \mathcal{T}} r_i \right), d_{\sum_{i=1}^K M_i, N_D} \left(\sum_{i=1}^K r_i \right) \right\}. \quad (4.2.67)$$

The DMT lower bound meets the upper bound, and therefore the optimal DMT of the K -user CMAC can be achieved by using CF cooperation and the proposed transmission scheme.

Note that the second requirement is crucial since if without partial decoding of the cooperation information, we have $\mathbf{x}_{S_{\bar{k}1}}^{dc} = 0$ and $\mathbf{x}_{S_{\bar{k}1}}^{ndc} = \mathbf{x}_{S_{\bar{k}1}}$. In this case, given the fulfillment of the first requirement, we may have

$$\sum_{i \in S^c} I(\mathbf{x}_{S_i1}^{ndc}; \hat{\mathbf{y}}_i | \mathbf{x}_{i1}, \mathbf{x}_{S_i1}^{dc}) = \sum_{i \in S^c} I(\mathbf{x}_{S_i1}; \hat{\mathbf{y}}_i | \mathbf{x}_{i1}) \stackrel{\cdot}{=} \sum_{i \in S^c} r_{i1} \log \text{SNR} > 0 \cdot \log \text{SNR}. \quad (4.2.68)$$

Substituting (4.2.68) into the outage probability expression (4.2.63), one can easily see that the resulting diversity order is suboptimal. Partial decoding of the cooperation information is not required in the two-user case. Since, for the two-user case, outage might happen only when the direct link between a user and destination cannot support the targeted rate. In this case, the user uses uniform superposition and allocates zero multiplexing gain for the cooperation part. According to the derived outage probabilities (for example, (4.2.27)), whether to decode this cooperation part or not at the other user does not affect the diversity result.

To fulfill the first requirement, we propose a simple transmission scheme based on time-sharing between different transmission arrangements at each user. In the following, a three-user system is used to illustrate the proposed scheme. We consider a channel scenario where

for $\mathbf{x}_i/\sqrt{P_i} \in \mathbb{C}^{M_i}$,

$$\begin{aligned} I(\mathbf{x}_1; \mathbf{y}_D | \mathbf{x}_{20}, \mathbf{x}_{30}, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) &\leq r_1 \log \text{SNR}, \\ I(\mathbf{x}_2; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{30}, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) &> r_2 \log \text{SNR}, \\ I(\mathbf{x}_3; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{20}, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) &> r_3 \log \text{SNR}, \\ I(\mathbf{x}_2, \mathbf{x}_3; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) &> (r_2 + r_3) \log \text{SNR}. \end{aligned}$$

Under this channel scenario, user 1 might be in outage, while the information of user 2 and 3 can be correctly detected at the destination by simply treating the signal from user 1 as interference. User 1 uses uniform superposition to transmit its user information and cooperation information. By assuming $\mathbf{x}_3/\sqrt{P_3} \in \mathbb{C}^{M_3}$, user 2 arranges its transmission as follows:

- If $I(\mathbf{x}_2; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) > r_2 \log \text{SNR}$ and $I(\mathbf{x}_3; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{20}, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) > r_3 \log \text{SNR} > I(\mathbf{x}_3; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31})$, user 2 transmits its user information through the antennas such that $I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) \doteq r_2 \log \text{SNR}$, and uses the remaining antennas to transmit the cooperation information.
- If $I(\mathbf{x}_2; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{30}, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) > r_2 \log \text{SNR} > I(\mathbf{x}_2; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31})$ and $I(\mathbf{x}_3; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) > r_3 \log \text{SNR}$, user 2 transmits its user information through the antennas such that $I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{30}, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) \doteq r_2 \log \text{SNR}$, and uses the remaining antennas to transmit the cooperation information.
- If $I(\mathbf{x}_2; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) > r_2 \log \text{SNR}$ and $I(\mathbf{x}_3; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) > r_3 \log \text{SNR}$, user 2 arranges its transmission by time-sharing between the following two modes: (i) in $t \in (0, 1)$ portion of each transmission block, transmits its user information through the antennas such that $I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) \doteq r_2 \log \text{SNR}$, and the remaining antennas for the cooperation information, and (ii) in $(1 - t)$ portion of each block, transmits its user information through the antennas such that $I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{30}, \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) \doteq r_2 \log \text{SNR}$, and the remaining antennas for the cooperation information.

User 3 arranges its transmission in the same fashion as user 2 with a time-sharing schedule that complements with that of user 2. The time-sharing schedule is predetermined and is aware by all the users in the network beforehand. Users 2 and 3 can arrange their individual transmission without coordination. The above transmission scheme applies a spatial multiplexing based approach. A similar scheme can be designed using the rate splitting based approach.

With the knowledge of the transmission scheme used by each user, the destination can arrange the decoding order to ensure that $I(\mathbf{x}_{20}, \mathbf{x}_{30}; \hat{\mathbf{y}}_2, \hat{\mathbf{y}}_3, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}, \mathbf{x}_{31}) \doteq (r_2 +$

r_3) log SNR. For the other scenarios in the three-user system and for systems with arbitrary numbers of users, by applying similar transmission scheme as the one described above, the first requirement for CF-based CMACs to be DMT optimal can also be satisfied.

4.3 DMT with Half-Duplex Source Cooperation

In this section, we focus on the two-user channel to study the DMT of CF-based CMACs with half-duplex constraint. We find that the requirements for the full-duplex systems to achieve the optimal DMT still apply in the half-duplex case. To illustrate this, we first give the DMT upper bound and then provide a transmission scheme that can achieve this upper bound.

4.3.1 DMT Upper Bound

We assume that user 1 transmits $\mathbf{x}_1^{(1)}$ in $\tau \in (0, 1)$ portion of a frame time and listens to user 2 in the remaining $(1 - \tau)$ portion of the time period. User 2 listens to user 1 during τ portion of the time and transmits $\mathbf{x}_2^{(2)}$ in the remaining $(1 - \tau)$ portion. We denote $\mathbf{y}_D^{(1)}$ and $\mathbf{y}_2^{(1)}$ as the signals received by the destination and user 2 during the τ time interval that user 1 transmits, respectively. Denote $\mathbf{y}_D^{(2)}$ and $\mathbf{y}_1^{(2)}$ as the signal received by the destination and user 1 during the $(1 - \tau)$ time interval that user 2 transmits, respectively.

The capacity cut-set bound for the half-duplex two-user CMAC is

$$R_1 < \tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_2^{(1)}, \mathbf{y}_D^{(1)}) \quad (4.3.1a)$$

$$R_2 < (1 - \tau) I(\mathbf{x}_2^{(2)}; \mathbf{y}_1^{(2)}, \mathbf{y}_D^{(2)}) \quad (4.3.1b)$$

$$R_1 + R_2 < \tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)}) + (1 - \tau) I(\mathbf{x}_2^{(2)}; \mathbf{y}_D^{(2)}). \quad (4.3.1c)$$

Based on the capacity cut-set bound, and using a similar approach used in finding the DMT upper bound in the full-duplex case, the diversity upper bound can be found as

$$d_1(r_1, r_2) \leq \min\{d_{C_1}(r_1), d_{C_D}(r_1 + r_2)\} \quad (4.3.2)$$

$$d_2(r_1, r_2) \leq \min\{d_{C_2}(r_2), d_{C_D}(r_1 + r_2)\}, \quad (4.3.3)$$

where

$$d_{C_1} = - \lim_{\text{SNR} \rightarrow \infty} \frac{\log P\left(\tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_2^{(1)}, \mathbf{y}_D^{(1)}) < r_1 \log \text{SNR}\right)}{\log \text{SNR}} \quad (4.3.4)$$

$$d_{\mathcal{C}_2} = - \lim_{\text{SNR} \rightarrow \infty} \frac{\log \mathbb{P} \left((1 - \tau) I(\mathbf{x}_2^{(2)}; \mathbf{y}_1^{(2)}, \mathbf{y}_D^{(2)}) < r_2 \log \text{SNR} \right)}{\log \text{SNR}} \quad (4.3.5)$$

$$d_{\mathcal{C}_D} = - \lim_{\text{SNR} \rightarrow \infty} \frac{\log \mathbb{P} \left(\tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)}) + (1 - \tau) I(\mathbf{x}_2^{(2)}; \mathbf{y}_D^{(2)}) < (r_1 + r_2) \log \text{SNR} \right)}{\log \text{SNR}} \quad (4.3.6)$$

In general, the optimal value of τ that maximizes the DMT upper bound depends on the multiplexing rates r_i ($i = 1, 2$) and the number of transmit antennas m_i . Here, we evaluate the optimal τ value for the symmetric case where $r_1 = r_2 = r$ and $M_1 = M_2 = M$. In the symmetric case, the diversity gains $d_{\mathcal{C}_1}(r_1)$, $d_{\mathcal{C}_2}(r_1)$ and $d_{\mathcal{C}_D}(r_1 + r_2)$ can be calculated as

$$d_{\mathcal{C}_1}(r_1) = \left(M - \frac{r_1}{\tau} \right) \left(M + N_D - \frac{r_1}{\tau} \right), \quad (4.3.7)$$

$$d_{\mathcal{C}_2}(r_2) = \left(M - \frac{r_2}{1 - \tau} \right) \left(M + N_D - \frac{r_2}{1 - \tau} \right), \quad (4.3.8)$$

and

$$d_{\mathcal{C}_D}(r_1 + r_2) = \begin{cases} \left(M - \frac{(2r - \tau \min\{M, N_D\})^+}{1 - \tau} \right) \left(N_D - \frac{(2r - \tau \min\{M, N_D\})^+}{1 - \tau} \right) & \text{if } 0 < \tau \leq \frac{1}{2} \\ \left(M - \frac{(2r - (1 - \tau) \min\{M, N_D\})^+}{\tau} \right) \left(N_D - \frac{(2r - (1 - \tau) \min\{M, N_D\})^+}{\tau} \right) & \text{if } \frac{1}{2} < \tau \leq 1. \end{cases} \quad (4.3.9)$$

It is easy to see that the DMT is maximized when $\tau = 1/2$, meaning that it is optimal to let each user transmit half of the time regardless of the channel states.

4.3.2 Achievable Scheme

To show that the half-duplex DMT upper bound is achievable, we again consider CF-based cooperation. We find that given a properly designed transmission scheme, the CF-based CMAC is still DMT optimal under the half-duplex constraint.

By the half-duplex CF-based cooperation, during the first τ time intervals of each symbol block, user 1 transmits the cooperation information $\mathbf{x}_{11}^{(1)}$ as well as its own information $\mathbf{x}_{10}^{(1)}$. User 2 listens to user 1, compresses its received signal as $\hat{\mathbf{y}}_2^{(1)}$ and re-encodes it into a codeword $\mathbf{x}_{21}^{(2)}$. During the following $(1 - \tau)$ time interval of the symbol block, user 2 transmits $\mathbf{x}_{21}^{(2)}$ together with its own information $\mathbf{x}_{20}^{(2)}$. User 1 compresses its received $\mathbf{y}_1^{(2)}$ into $\hat{\mathbf{y}}_1^{(2)}$ and re-encodes it into $\mathbf{x}_{11}^{(1)}$ which will be transmitted in the next frame. The signals transmitted from user 1 and user 2 can thus be represented as $\mathbf{x}_1^{(1)}(\mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)})$ and $\mathbf{x}_2^{(2)}(\mathbf{x}_{20}^{(2)}, \mathbf{x}_{21}^{(2)})$, respectively.

Using some standard techniques as in [105], we find that the following rate region is

achievable for the CF-based half-duplex discrete memoryless CMAC

$$R_{10} < \tau I\left(X_{10}^{(1)}; \hat{Y}_2^{(1)}, Y_D^{(1)} | X_{11}^{(1)}, X_{21}^{(2)}\right) \quad (4.3.10a)$$

$$R_{20} < (1 - \tau) I\left(X_{20}^{(2)}; \hat{Y}_1^{(2)}, Y_D^{(2)} | X_{11}^{(1)}, X_{21}^{(2)}\right), \quad (4.3.10b)$$

subject to the constraints

$$(1 - \tau) I\left(\hat{Y}_1^{(2)}; Y_1^{(2)} | X_{10}^{(1)}, X_{11}^{(2)}\right) < (1 - \tau) I\left(\hat{Y}_1^{(2)}; Y_D^{(2)} | X_{11}, X_{21}\right) + \tau I\left(X_{11}^{(1)}; \mathbf{y}_D^{(1)}\right), \quad (4.3.11a)$$

$$\tau I\left(\hat{Y}_2^{(1)}; Y_2^{(1)} | X_{20}^{(2)}, X_{21}^{(2)}\right) < \tau I\left(\hat{Y}_2^{(1)}; Y_D^{(1)} | X_{11}^{(1)}, X_{21}^{(2)}\right) + (1 - \tau) I\left(X_{21}^{(2)}; Y_D^{(2)}\right) \quad (4.3.11b)$$

Applying the rate region results to the vector channel, the outage probabilities at the targeted rates $R_{10} \doteq r_1 \log \text{SNR}$, $R_{20} \doteq r_2 \log \text{SNR}$ can be computed as

$$\begin{aligned} P(\mathcal{O}_1) &= P\left(\tau I\left(\mathbf{x}_{10}^{(1)}; \hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) < r_1 \log \text{SNR}\right) \\ &\leq P\left(\tau I\left(\mathbf{x}_{10}^{(1)}; \mathbf{y}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}\right) < r_1 \log \text{SNR}\right) \\ &\quad + P\left(\tau I\left(\mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}; \mathbf{y}_D^{(1)}\right) + (1 - \tau) I\left(\mathbf{x}_{20}^{(2)}, \mathbf{x}_{21}^{(2)}; \mathbf{y}_D^{(2)}\right) \right. \\ &\quad \left. - \tau I\left(\mathbf{x}_{11}^{(1)}; \hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)}\right) - (1 - \tau) I\left(\mathbf{x}_{20}^{(2)}; \mathbf{y}_D^{(2)} | \mathbf{x}_{21}^{(2)}\right) < r_1 \log \text{SNR}\right), \end{aligned} \quad (4.3.12)$$

and

$$\begin{aligned} P(\mathcal{O}_2) &= P\left((1 - \tau) I\left(\mathbf{x}_{20}^{(2)}; \hat{\mathbf{y}}_1^{(2)}, \mathbf{y}_D^{(2)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) < r_2 \log \text{SNR}\right) \\ &\leq P\left((1 - \tau) I\left(\mathbf{x}_{20}^{(2)}; \mathbf{y}_1^{(2)}, \mathbf{y}_D^{(2)} | \mathbf{x}_{21}^{(2)}\right) < r_2 \log \text{SNR}\right) \\ &\quad + P\left(\tau I\left(\mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}; \mathbf{y}_D^{(1)}\right) + (1 - \tau) I\left(\mathbf{x}_{20}^{(2)}, \mathbf{x}_{21}^{(2)}; \mathbf{y}_D^{(2)}\right) \right. \\ &\quad \left. - (1 - \tau) I\left(\mathbf{x}_{21}^{(2)}; \hat{\mathbf{y}}_1^{(2)}, \mathbf{y}_D^{(2)}\right) - \tau I\left(\mathbf{x}_{10}^{(1)}; \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}\right) < r_2 \log \text{SNR}\right). \end{aligned} \quad (4.3.13)$$

The derivation is provided in Appendix E.

Based on the outage probability expressions, a similar transmission strategy as in the full-duplex case can be used to ensure an optimal DMT behavior for the considered half-duplex CF-based CMAC. We first compute $\tau I\left(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)}\right)$ and $(1 - \tau) I\left(\mathbf{x}_2^{(2)}; \mathbf{y}_D^{(2)}\right)$ by assuming $\mathbf{x}_1^{(1)}/\sqrt{P_1} \in \mathbb{C}^{M_1 \times 1}$ and $\mathbf{x}_2^{(2)}/\sqrt{P_2} \in \mathbb{C}^{M_2 \times 1}$. The transmission of the two parts of information can then be arranged as follows:

- If $\tau I\left(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)}\right) \leq r_1 \log \text{SNR}$ ($(1 - \tau) I\left(\mathbf{x}_2^{(2)}; \mathbf{y}_D^{(2)}\right) \leq r_2 \log \text{SNR}$), user 1 (user 2) uses uniform superposition to transmit $\mathbf{x}_{10}^{(1)}$ and $\mathbf{x}_{11}^{(1)}$ ($\mathbf{x}_{20}^{(2)}$ and $\mathbf{x}_{21}^{(2)}$), i.e., to transmit $\mathbf{x}_{10}^{(1)}$

and $\mathbf{x}_{11}^{(1)}$ ($\mathbf{x}_{20}^{(2)}$ and $\mathbf{x}_{21}^{(2)}$) using all the antennas and use a fixed portion of transmitting power independent of SNR. Since in this case, user 1 (user 2) may be in outage and the major portion of its resources should be used for transmitting its own information, such transmission strategy ensures that $\tau I(\mathbf{x}_{10}^{(1)}; \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}) \doteq \tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)})$ ($(1 - \tau)I(\mathbf{x}_{20}^{(2)}; \mathbf{y}_D^{(2)} | \mathbf{x}_{21}^{(2)}) \doteq (1 - \tau)I(\mathbf{x}_2^{(2)}; \mathbf{y}_D^{(2)})$).

- If $\tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)}) > r_1 \log \text{SNR}$ ($(1 - \tau)I(\mathbf{x}_2^{(2)}; \mathbf{y}_D^{(2)}) > r_2 \log \text{SNR}$), user 1 (user 2) chooses the antennas such that $\tau I(\mathbf{x}_{10}^{(1)}; \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}) \doteq r_1 \log \text{SNR}$ ($(1 - \tau)I(\mathbf{x}_{20}^{(2)}; \mathbf{y}_D^{(2)} | \mathbf{x}_{21}^{(2)}) \doteq r_2 \log \text{SNR}$) to transmit $\mathbf{x}_{10}^{(1)}$ ($\mathbf{x}_{20}^{(2)}$) with some fixed transmitting power independent of SNR, and use the remaining antennas and power to transmit the cooperation part of information $\mathbf{x}_{11}^{(1)}$ ($\mathbf{x}_{21}^{(2)}$). In this case the messages sent by user 1 (user 2) can be successfully detected by the destination even without cooperation, and therefore user 1 (user 2) can spare part of its resources for helping the other user. This transmission strategy ensures that

$$\begin{aligned} \tau I(\mathbf{x}_{11}^{(1)}; \mathbf{y}_D^{(1)}) &= \tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)}) - \tau I(\mathbf{x}_{10}^{(1)}; \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}) \\ &\doteq \tau I(\mathbf{x}_1^{(1)}; \mathbf{y}_D^{(1)}) - r_1 \log \text{SNR} > 0 \cdot \log \text{SNR}, \end{aligned}$$

and therefore $\mathbf{x}_{11}^{(1)}$ can be transmitted at a higher multiplexing level and provides more help for user 2. Similarly, by the proposed transmission scheme, it is possible for user 2 to spare part of its resources to assist the transmission of user 1.

Following a similar analysis as in the two-user full-duplex case, the outage probability in (4.3.12) becomes

$$\begin{aligned} P(\mathcal{O}_1) &\leq P\left(\tau I(\mathbf{x}_{10}^{(1)}; \mathbf{y}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}) < r_1 \log \text{SNR}\right) \\ &\quad + P\left(\tau I(\mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}; \mathbf{y}_D^{(1)}) + (1 - \tau)I(\mathbf{x}_{20}^{(2)}, \mathbf{x}_{21}^{(2)}; \mathbf{y}_D^{(2)}) < (r_1 + r_2) \log \text{SNR}\right) \\ &\doteq \text{SNR}^{-\min\{d_{C_1}(r_1), d_{C_D}(r_1+r_2)\}}, \end{aligned} \tag{4.3.14}$$

where the mutual information in the probability terms are evaluated using a strategy that user 1 transmits its two parts of information using uniform superposition. The outage probability in (4.3.13) becomes

$$\begin{aligned} P(\mathcal{O}_2) &\leq P\left((1 - \tau)I(\mathbf{x}_{20}^{(2)}; \mathbf{y}_1^{(2)}, \mathbf{y}_D^{(2)} | \mathbf{x}_{21}^{(2)}) < r_2 \log \text{SNR}\right) \\ &\quad + P\left(\tau I(\mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}; \mathbf{y}_D^{(1)}) + (1 - \tau)I(\mathbf{x}_{20}^{(2)}, \mathbf{x}_{21}^{(2)}; \mathbf{y}_D^{(2)}) < (r_1 + r_2) \log \text{SNR}\right) \\ &\doteq \text{SNR}^{-\min\{d_{C_2}(r_2), d_{C_D}(r_1+r_2)\}}, \end{aligned} \tag{4.3.15}$$

where the mutual information in the probability terms are calculated based on that user 2 performs uniform superposition transmission.

The DMT of the two-user half-duplex CMAC can thus be lower bounded as

$$\begin{aligned} d_1(r_1, r_2) &\geq \min\{d_{C_1}(r_1), d_{C_D}(r_1 + r_2)\} \\ d_2(r_1, r_2) &\geq \min\{d_{C_2}(r_2), d_{C_D}(r_1 + r_2)\}. \end{aligned} \quad (4.3.16)$$

The lower bounds coincides with the DMT upper bounds in (4.3.2) and (4.3.3). Therefore, with the proposed transmission strategy, the CF-based cooperation is DMT optimal.

4.4 Reflections on the Multiplexing-Gain Region of Cooperative MAC

Since the pure relay channel and the non-cooperative multiple-access channel are two extremes of the cooperative multiple-access system, it is interesting to study under what situation the CMAC goes to one of the extremes. This gives us a way of partitioning the multiplexing-gain region which in turn could help us to decide, for a targeted multiplexing gain, the most efficient transmission scheme.

When operating at a point within the achievable multiplexing-gain region, if one user, say user 1, in the CMAC achieves the same diversity order as in a channel where the other users act as pure relay for it, we say that user 1 in the CMAC achieves a same diversity behavior as in a pure relay channel. In this case, user 1 enjoys a diversity gain as if the other users have no information of their own to transmit and devote all their resources to help the communication of user 1. On the other hand, if one user operating in a CMAC sees no improvement in diversity as compared to the case when it operates in a non-cooperative MAC, we say that for that user, the CMAC relegates to a non-cooperative MAC and the user gains no benefit from cooperation, though it may help to improve the performance of the other users in the system. When all the users in a CMAC gain no benefit from cooperation, there is no need for cooperation and a simple non-cooperative transmission scheme will do.

We consider a two-user MAC to further illustrate the idea. Assuming a fixed r_1 , and varying r_2 , the following three situations delineate, for user 1, whether cooperation is helpful or not and to what extent cooperation could help.

- i. $d_{M_1, M_2 + N_D}(r_1) \leq d_{M_1 + M_2, N_D}(r_1 + r_2)$

In this case, user 1 in the CMAC achieves a same diversity order as in a channel where user 2 act as pure relay for it. User 2 helps the transmission of user 1 as if it has no information of its own to transmit.

Figure 4.3: Illustration of partition of multiplexing gain region of a two-user CMAC.

ii. $d_{M_1, N_D}(r_1) < d_{M_1+M_2, N_D}(r_1 + r_2) < d_{M_1, M_2+N_D}(r_1)$

In this case, the diversity order of user 1 is lower than in the pure relay channel, but higher than what can be obtained in a non-cooperative MAC, indicating that cooperation is still helpful as compared to the cases where no cooperation is used.

iii. $d_{M_1+M_2, N_D}(r_1 + r_2) \leq d_{M_1, N_D}(r_1)$

In this case, the diversity order of user 1 is no better than the diversity order obtained when operating in a non-cooperative MAC. Cooperation provides no benefit for user 1 in terms of improving diversity gain.

If we fix r_2 and vary r_1 , a similar three situations can be defined which tells the degree of improvement in diversity order for user 2.

Based on the above discussion, we partition the multiplexing-gain region into 9 subregions, as illustrated in Fig. 4.3. The boundaries $r_{2,a}^*$ and $r_{2,b}^*$ can be found by solving the following equations for r_2 :

$$d_{M_1, M_2+N_D}(r_1) = d_{M_1+M_2, N_D}(r_1 + r_2) \quad (4.4.1)$$

$$d_{M_1, N_D}(r_1) = d_{M_1+M_2, N_D}(r_1 + r_2). \quad (4.4.2)$$

We obtain

$$r_{2,a}^* = \frac{(N_D + M_1 + M_2 - 2r_1) - \sqrt{\Delta_{2,a}}}{2} \quad (4.4.3)$$

$$r_{2,b}^* = \frac{(N_D + M_1 + M_2 - 2r_1) - \sqrt{\Delta_{2,b}}}{2}, \quad (4.4.4)$$

where

$$\begin{aligned} \Delta_{2,a} &= (N_D + M_1 + M_2 - 2r_1)^2 - 4M_2(N_D - M_1), \\ \Delta_{2,b} &= (N_D + M_1 + M_2 - 2r_1)^2 - 4M_2(N_D - r_1). \end{aligned}$$

The boundaries $r_{1,a}^*$ and $r_{1,b}^*$ are calculated by solving the following equation for r_1 :

$$d_{M_2, M_1 + N_D}(r_2) = d_{M_1 + M_2, N_D}(r_1 + r_2) \quad (4.4.5)$$

$$d_{M_2, N_D}(r_2) = d_{M_1 + M_2, N_D}(r_1 + r_2), \quad (4.4.6)$$

resulting in

$$r_{1,a}^* = \frac{(N_D + M_1 + M_2 - 2r_2) - \sqrt{\Delta_{1,a}}}{2} \quad (4.4.7)$$

$$r_{1,b}^* = \frac{(N_D + M_1 + M_2 - 2r_2) - \sqrt{\Delta_{1,b}}}{2}, \quad (4.4.8)$$

where

$$\begin{aligned} \Delta_{1,a} &= (N_D + M_1 + M_2 - 2r_2)^2 - 4M_1(N_D - M_2), \\ \Delta_{1,b} &= (N_D + M_1 + M_2 - 2r_2)^2 - 4M_1(N_D - r_2). \end{aligned}$$

For user 1, subregions (I), (II) and (III) correspond to the cases where user 2 acts as pure relay for it. Subregions (IV), (V) and (VI) correspond to the cases where cooperation can still help to improve the diversity order of user 1 but not as much as that in the pure relay case. Subregions (VII), (VIII), and (IX) correspond to the cases where cooperation offers no help in improving the diversity order of user 1. For user 2, the three areas delineated by the two vertical boundaries $r_{1,a}^*$ and $r_{1,b}^*$ corresponds, from left to right, to the pure relay case, the cooperation-beneficial case and the case where cooperation does not help, respectively. Note that Fig. 4.3 is plotted for illustration. In practice, $r_{1,i}^*$ ($i = a, b$) depends on r_2 (similarly $r_{2,i}^*$) and hence should not be straight lines.

By considering the two users together, we see that when operating in subregion (I), each user obtains a diversity gain as if the other user does not transmit new information and simply act as pure relay. When operating in subregions (II), (IV) and (V), each user can benefit from cooperation but can also feel the influence of the other user. When operating

Figure 4.4: Partition of multiplexing-gain region of a two-user CMAC with $M_1 = 4$, $M_2 = 5$ and $N_D = 6$.

in subregions (III) and (VI), user 1 is heavily loaded and cannot help user 2. However, user 2 can still help user 1 in improving diversity. Similarly, in subregions (VII) and (VIII), user 2 is heavily loaded and cannot help user 1 but user 1 is lightly loaded and can help to improve user 2's diversity performance. When operating in subregion (IX), both users are heavily loaded and no user benefits from cooperation, the optimal DMT can be achieved using a simple non-cooperative multiple-access communication scheme. In Fig. 4.4, we show a numerically generated partition of multiplexing-gain region of a CMAC with $M_1 = 4$, $M_2 = 5$ and $N_D = 6$. The curves that divide the partitions are calculated using (4.4.3)-(4.4.4), (4.4.7)-(4.4.8).

4.5 Conclusion

In this chapter, the DMT of delay-limited asymmetric cooperative multiple-access channel with multiple-antenna terminals was studied. Based on the network capacity cut-set bound, we found the DMT upper bound for the considered channel scenario under both full-duplex and half-duplex constraints. We showed that Wyner-Ziv-type CF-based cooperation can achieve the upper bound given that the following two requirements are satisfied:

- The transmission resources (antennas, power, etc.) of each user, should be properly allocated such that more resources can be saved for cooperation.
- Each user should try its best to decode the cooperation part of information received

from the other users so that it can concentrate on compressing and forwarding the part that contains the other users' information.

To satisfy the first requirement, we proposed to use a spatial-multiplexing-based scheme, a rate-splitting-based scheme, or their combination to arrange the transmission of the user information and the cooperation information at each user. Depending on the channel state information, each user only uses the necessary resources to transmit its own information and leaves the remaining resources for cooperation. To achieve this, channel state information is required at each user. However, since channel knowledge is a must for Wyner-Ziv-type CF cooperation, the proposed transmission scheme can be realized without extra cost.

To satisfy the second requirement, the cooperation information should be encoded such that it can be (partially) decoded by other users. In the derivation of the achievable rate region for the K -user CF-based CMAC, a coding scheme that can be partially decoded was provided. Since only the user information is the part that needs to be forwarded and cooperatively detected, requiring a user to indiscriminately forwarding all the information received from the other users would only waste the limited communication resources. Decoding, and consequently deducting, of the cooperation information received from other users could minimize this waste and focus the communication resources on the part of information that needs to be cooperatively resolved.

Although the pure relay channel and the multiple-access channel are two special cases of the CMAC, the above-mentioned two requirements are not needed for these two channels to achieve the optimal DMT. Since in the pure relay channel, the relays do not have their own information to transmit, they can simply devote all their resources for cooperation. In a non-cooperative multiple-access channel, each user only needs to transmit its own information, there is also no such issue as allocating resources for cooperation. Cooperative multiple-access channel gives rise to issues that could not be encountered in the pure relay channel and the non-cooperative multiple-access channel, thus provides us an opportunity to better understand the essence of cooperation and the ways to make best use of the cooperation resources.

Although the two requirements are set for CF-based cooperation schemes, we conjecture that similar requirements may also needed for DF-based cooperation schemes. However, due to the same reasons as in the multiple-antenna relay channel, DF-based cooperation schemes may not able to achieve the optimal DMT of the multiple-antenna CMAC. For AF-based cooperation schemes, it is still unclear whether these requirements are necessary. Since, unlike the CF-based scheme where the cooperation information should be decoded prior to the user information, by an AF-based scheme, the cooperation information and the user information can be jointly decoded. But this joint decoding may lead to high decoding complexity and large delay. Characterizing the DMT behavior for AF and DF

based asymmetric multiple-antenna CMAF is still an open problem.

Based on the DMT results, we provided a way of partitioning the multiplexing gain region of a two-user CMAF to reflect the effect of cooperation on each user. This partition reveals the interaction between users in realizing cooperation and indicates the multiplexing-gain subregions that each user can benefit from cooperation in terms of diversity. This partition also provides us a guidance on choosing the most efficient cooperation scheme for an asymmetric CAM channel.

Chapter 5

An Achievable Rate Region for Two-User Interference Channels with Source Cooperation

The achievable rate regions of two-user IFCs with source cooperation have been studied in many works based on different assumptions. The works in [51], [133]-[135] assumed that one user (the cognitive user) has non-causal (*a priori*) knowledge about the message transmitted by the other user (the non-cognitive user), while the non-cognitive user has no knowledge about the message transmitted by its counterpart. Such a channel was termed as a genie-aided cognitive radio channel in [133]-[134], an interference channel with degraded message sets in [135]-[136], and an interference channel with one cooperating transmitter in [51]. Network scenarios that allow for this non-causal cooperation includes the sensor networks and the cognitive radio networks, but for some other network scenarios, this non-causal cooperation assumption is too idealistic. A more realistic assumption is to let one user acquire the other user's message in a causal way as considered in [137]. Such a channel scenario was denoted as interference channel with causal unidirectional cooperation in [137]. With the non-causal or causal knowledge, some coding schemes can be implemented at the cognitive user to enhance the spectrum efficiency of the system as have been done in [133]-[137].

The above-mentioned works only assume one user helping the other one but not *vice versa*. A step further is to study the case where both users could help with each other. The works in [48], [49], [52], [54], [138] are based on this assumption. This channel model was termed as an interference channel with transmitter cooperation in [48] and [54], an interference channel with generalized feedback in [52], and an interference channel with conferencing in [49] and [138]. Full-duplex causal cooperation was assumed by all these works and achievable rate regions were proposed based on DF cooperation. In particular, the sum capacity of the two-user IFC with source cooperation has been studied in [54], where DF-based cooperation schemes were proposed and shown to achieve a sum rate that is within

Figure 5.1: Channel model of two-user IFC with source cooperation.

a fixed number of bits to the sum capacity upper bound.

In this chapter, we consider the same causal full-duplex channel scenario. A new achievable rate region is derived based on a cooperation scheme that combines DF and CF. To realize source cooperation, we develop a coding strategy that combines rate splitting developed by Han and Kobayashi for interference channels [18], block Markov coding developed by Cover and El Gamal for relay channels [3], Gel'fand-Pinsker binning [19] applied in broadcast channels [17], and superposition coding applied by Chong, Motani and Garg for interference channels [106]. For the decoding, we use backward decoding which has been shown to achieve higher rates than forward decoding in networks with multiple sources [39]. Based on the proposed coding scheme, an achievable rate region is derived for the discrete memoryless IFC with source cooperation. As a corollary, an achievable rate region for the channel with pure CF cooperation is also obtained. By applying to Gaussian channels, the achievable rate region is numerically demonstrated and compared with some existing rate region results. The comparisons confirm the advantage of our proposed rate region in some network settings.

5.1 Discrete Memoryless Interference Channel Model

We consider a two-user discrete memoryless IFC with source cooperation as shown in Fig. 5.1. The channel is defined by two finite channel input alphabets $\mathcal{X}_1$ and $\mathcal{X}_2$, four finite channel output alphabets $\mathcal{Y}_1, \mathcal{Y}_2, \mathcal{Y}_3$ and $\mathcal{Y}_4$, and a channel transition probability $p(y_1, y_2, y_3, y_4 | x_1, x_2)$.

Transmission is arranged into blocks of n channel uses. At the beginning of each block, transmitter i ($i = 1, 2$) maps its message into an index $w_i \in \mathcal{W}_i = \{1, \dots, 2^{nR_i}\}$, and encodes w_i into a length- n codeword vector x_i^n . The t th element in x_i^n is generated according to $x_{i,t}^n = f_{i,t}(w_i, y_i^{t-1})$. By the end of each block, with the received y_{i+2}^n , decoder i in node $i + 2$ decodes the message sent by transmitter i as $\hat{w}_i = g_i(y_{i+2}^n)$, where $g_i(\cdot)$ is the decoding function defined by decoder i . The message sets $\mathcal{W}_i$, the encoding functions $f_{i,t}$, and the decoding functions g_i define a $(2^{nR_1}, 2^{nR_2}, n)$ code for the two-user IFC with source cooperation.

5.2 An Achievable Rate Region Based on Combined DF and CF Cooperation

In this section, we establish an achievable rate region for the two-user IFC with combined DF and CF source cooperation. The rate region is derived based on a proposed coding scheme that jointly applies rate splitting, block Markov coding, superposition coding, Gel'fand-Pinsker coding and backward decoding.

To implement the combined DF and CF cooperative transmission scheme, transmitter i partially decodes the message of the other transmitter (say transmitter j , $j = 1, 2$, $j \neq i$) from y_i^n , cancels out the decoded message and compresses the remaining part of y_i^n using Wyner-Ziv-type compression. The decoded message and the compressed signal are then re-encoded and re-transmitted together with the transmitter's own information. Therefore, the message transmitted by transmitter i contains three parts: the decode-and-forward message part $\tilde{w}_i$, the compress-and-forward message part z_i and the transmitter's own message w_i . w_i is the message targeted for the intended receiver of transmitter i , while $\tilde{w}_i$ and z_i are the messages targeted for the intended receiver of transmitter j . At receiver node $j + 2$, $\tilde{w}_i$ and z_i are decoded and then used to assist the decoding of w_j .

The considered IFC with source cooperation can therefore be viewed as a combination of relay channels, multiple-access channels, broadcast channels and interference channels. Transmitter i acts as a relay for transmitter j . w_i and $(\tilde{w}_i, z_i)$ are broadcasted from transmitter i to receiver nodes $i + 2$ and $j + 2$, respectively. Decoder j at receiver node $j + 2$ decodes $(\tilde{w}_i, z_i)$ and w_j with the presence of signals carrying w_i , $\tilde{w}_j$ and z_j as interference. The coding techniques such as rate splitting and block Markov coding developed for the interference channels and the relay channels, respectively, and the Gel'fand-Pinsker coding applied in broadcast channels can thus be incorporated in the coding scheme for the considered channel.

In the proposed coding scheme, following the HK rate-splitting approach [18], we first split w_i into w_{i1} and w_{i2} , where w_{i1} is the common user information that can be decoded by both receivers and w_{i2} is the private user information that can only be decoded by receiver node $i + 2$. The transmitters decode part of each other's common user information and forward to the receivers, while the remaining part of the common user information and the private user information are sent via CF-based cooperation. To enable the partial decoding of the common user information, we further split w_{i1} into w_{i1d} and w_{i1c} , where w_{i1d} is the common user information to be sent by DF-based cooperation and w_{i1c} is the common user information to be sent by CF-based cooperation. The compress-and-forward message z_i is also split into z_{i1} and z_{i2} , where z_{i1} is the part that can be decoded by both receivers and z_{i2} is the part that can only be decoded by its intended receiver, i.e., receiver $j + 2$.

Figure 5.2: Illustration of the proposed coding scheme.

A sequence of codewords is generated to transmit the messages. Figure 5.2 illustrates the encoding procedure at transmitter i . We first generate a sequence of codewords X_0^n , X_{i1d}^n and S_{i1}^n by superposition coding, carrying the messages $\tilde{w}_i$, w_{i1d} and z_{i1} , respectively. Following that, superimposed on the generated X_0^n , X_{i1d}^n and S_{i1}^n , two sequences of codewords are generated in parallel: one contains the codewords X_{i1c}^n and U_{i2}^n and the other contains the codeword V_{i2}^n . U_{i2}^n and V_{i2}^n are then used to generate the codewords X_{i2}^n and S_{i2}^n following the Gel'fand-Pinsker coding procedure for broadcast channels. X_{i1c}^n , X_{i2}^n and S_{i2}^n carry the messages w_{i1c} , w_{i2} and z_{i2} , respectively. Since $\tilde{w}_i$ is common for both transmitters, we use the same codeword X_0^n to encode this message at both transmitters. The codeword X_i^n is then generated as a deterministic function of $(X_0^n, X_{i1d}^n, X_{i1c}^n, X_{i2}^n, S_{i1}^n, S_{i2}^n)$.

The decoding strategy used is backward decoding. To facilitate backward decoding, transmission is arranged in frames of $(B+1)$ blocks. During the first B blocks, new information as well as cooperative information is sent, while in the $(B+1)$ th block, only cooperative information is transmitted. For large B , the effect of this one-block overhead on the overall transmission rate is negligible. Upon receiving a complete frame, receivers start decoding from the $(B+1)$ th block and proceed towards the first block. For each block, the decoding process can be completed in three steps, as shown in Fig. 5.3. In the first step, X_0^n , X_{i1d}^n , X_{i1c}^n are jointly decoded at each receiver and (w_{i1d}, w_{i1c}) is recovered. In the second step, receiver node $i+2$ jointly decodes S_{i1}^n , S_{j1}^n and S_{j2}^n , and recovers $\hat{Y}_j^n$, which is the compressed version of Y_j^n . Finally, with $\hat{Y}_j^n$ and Y_{i+2}^n , receiver node $i+2$ jointly decodes X_{j1c}^n , X_{i1c}^n and X_{i2}^n , and recovers (w_{i1c}, w_{i2}) .

We denote R_{i1d} , R_{i1c} and R_{i2} as the rates for sending the messages w_{i1d} , w_{i1c} and

Figure 5.3: Illustration of the decoding scheme.

w_{i2} , respectively, and L_{i1} and L_{i2} as the rates for sending s_{i1} and s_{i2} , respectively. The information rate of transmitter i can be expressed as $R_i = R_{i1d} + R_{i1c} + R_{i2}$. We further denote $L_i = L_{i1} + L_{i2}$, $\tilde{L}_{i2} = L_{i2} + L'_{i2}$ and $\tilde{R}_{i2} = R_{i2} + R'_{i2}$. The achievable rate region for the two-user IFC with combined DF and CF cooperation and our proposed coding scheme is presented in the following theorem.

Theorem 5.2.1. *For discrete memoryless two-user IFC with source cooperation, the rates $R_i = R_{i1d} + R_{i1c} + R_{i2}$ ($i = 1, 2$) are achievable if*

$$\tilde{L}_{12} \leq I(V_{12}; Y_4 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}) \quad (5.2.1a)$$

$$L_{11} + \tilde{L}_{12} \leq I(S_{11}, V_{12}; Y_4 | X_0, X_{11d}, X_{21d}, S_{21}) \quad (5.2.1b)$$

$$L_{21} + \tilde{L}_{12} \leq I(S_{21}, V_{12}; Y_4 | X_0, X_{11d}, X_{21d}, S_{11}) \quad (5.2.1c)$$

$$L_{21} + L_{11} + \tilde{L}_{12} \leq I(S_{11}, S_{21}, V_{12}; Y_4 | X_0, X_{11d}, X_{21d}) \quad (5.2.1d)$$

$$\tilde{L}_{22} \leq I(V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}) \quad (5.2.1e)$$

$$L_{21} + \tilde{L}_{22} \leq I(S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{11}) \quad (5.2.1f)$$

$$L_{11} + \tilde{L}_{22} \leq I(S_{11}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{21}) \quad (5.2.1g)$$

$$L_{11} + L_{21} + \tilde{L}_{22} \leq I(S_{11}, S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}) \quad (5.2.1h)$$

$$R_{11d} \leq I(X_{11d}; Y_2 | X_0, X_{21d}, S_{21}, V_{22}, X_{21c}, U_{22}) \quad (5.2.1i)$$

$$R_{21d} \leq I(X_{21d}; Y_1 | X_0, X_{11d}, S_{11}, V_{12}, X_{11c}, U_{12}) \quad (5.2.1j)$$

$$R_{11d} + R_{21d} \leq I(X_0, X_{11d}, X_{21d}; Y_3) \quad (5.2.1k)$$

$$R_{11d} + R_{21d} \leq I(X_0, X_{11d}, X_{21d}; Y_4) \quad (5.2.1l)$$

$$\tilde{R}_{12} \leq I(U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}, X_{11c}, X_{21c}) \quad (5.2.1m)$$

$$R_{11c} + \tilde{R}_{12} \leq I(X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}, X_{21c}) \quad (5.2.1n)$$

$$R_{21c} + \tilde{R}_{12} \leq I(X_{21c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}, X_{11c}) \quad (5.2.1o)$$

$$R_{21c} + R_{11c} + \tilde{R}_{12} \leq I(X_{11c}, X_{21c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}) \quad (5.2.1p)$$

$$\tilde{R}_{22} \leq I(U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{12}, X_{11c}, X_{21c}) \quad (5.2.1q)$$

$$R_{21c} + \tilde{R}_{22} \leq I(X_{21c}, U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{12}, X_{11c}) \quad (5.2.1r)$$

$$R_{11c} + \tilde{R}_{22} \leq I(X_{11c}, U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{12}, X_{21c}) \quad (5.2.1s)$$

$$R_{11c} + R_{21c} + \tilde{R}_{22} \leq I(X_{11c}, X_{21c}, U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{12}) \quad (5.2.1t)$$

$$R'_{12} + L'_{12} \geq I(X_{11c}, U_{12}; V_{12} | X_0, X_{11d}, S_{11}) \quad (5.2.1u)$$

$$R'_{22} + L'_{22} \geq I(X_{21c}, U_{22}; V_{22} | X_0, X_{21d}, S_{21}) \quad (5.2.1v)$$

for some joint probability mass function

$$\begin{aligned} & p(x_0, x_{11d}, x_{11c}, u_{12}, x_{21d}, x_{21c}, u_{22}, s_{11}, s_{12}, v_{21}, v_{22}, x_1, x_2, y_3, y_4) \\ &= p(x_0)p(x_{11d}|x_0)p(x_{21d}|x_0)p(s_{11}|x_0, x_{11d})p(s_{21}|x_0, x_{21d})p(x_{11c}|x_0, x_{11d}, s_{11})p(x_{21c}|x_0, x_{21d}, s_{21}) \\ & p(u_{12}|x_0, x_{11d}, s_{11}, x_{11c})p(u_{22}|x_0, x_{21d}, s_{21}, x_{21c})p(v_{12}|x_0, x_{11d}, s_{11})p(v_{22}|x_0, x_{21d}, s_{21}) \\ & p(x_1|x_0, x_{11d}, s_{11}, x_{11c}, u_{12}, v_{12})p(x_2|x_0, x_{21d}, s_{21}, x_{21c}, u_{22}, v_{22})p(y_3, y_4|x_1, x_2), \end{aligned}$$

subject to the constraints

$$I(\hat{Y}_1; Y_1 | X_0, X_{11d}, X_{21d}, X_{11c}, U_{12}, S_{11}, V_{12}) - I(\hat{Y}_1; Y_4 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{12}) \leq L_1 \quad (5.2.2a)$$

$$I(\hat{Y}_2; Y_2 | X_0, X_{11d}, X_{21d}, X_{21c}, U_{22}, S_{21}, V_{22}) - I(\hat{Y}_2; Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}) \leq L_2. \quad (5.2.2b)$$

Proof. Please refer to Appendix F. □

For the special case with $S_{11} = \emptyset$, $S_{21} = \emptyset$, $V_{12} = V_1$, $V_{22} = V_2$, $L'_{12} = 0$, $L'_{22} = 0$, the rate region in Theorem 5.2.1 becomes

$$R_{11d} \leq I(X_{11d}; Y_2 | X_0, X_{21d}, X_{21c}, V_2, U_{22}) \quad (5.2.3a)$$

$$R_{21d} \leq I(X_{21d}; Y_1 | X_0, X_{21d}, X_{11c}, V_1, U_{12}) \quad (5.2.3b)$$

$$R_{11d} + R_{21d} \leq I(X_0, X_{11d}, X_{21d}; Y_3) \quad (5.2.3c)$$

$$R_{11d} + R_{21d} \leq I(X_0, X_{11d}, X_{21d}; Y_4) \quad (5.2.3d)$$

$$R_{12} \leq I(U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, X_{11c}, X_{21c}, V_2) - G_1 \quad (5.2.3e)$$

$$R_{11c} + R_{12} \leq I(X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, X_{21c}, V_2) - G_1 \quad (5.2.3f)$$

$$R_{21c} + R_{12} \leq I(X_{21c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, X_{11c}, V_2) - G_1 \quad (5.2.3g)$$

$$R_{21c} + R_{11c} + R_{12} \leq I(X_{21c}, X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, V_2) - G_1 \quad (5.2.3h)$$

$$R_{22} \leq I(U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, X_{11c}, X_{21c}, V_1) - G_2 \quad (5.2.3i)$$

$$R_{21c} + R_{22} \leq I(X_{21c}, U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, X_{11c}, V_1) - G_2 \quad (5.2.3j)$$

$$R_{11c} + R_{22} \leq I(X_{11c}, U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, X_{21c}, V_1) - G_2 \quad (5.2.3k)$$

$$R_{11c} + R_{21c} + R_{22} \leq I(X_{11c}, X_{21c}, U_{22}; \hat{Y}_1, Y_4 | X_0, X_{11d}, X_{21d}, V_1) - G_2 \quad (5.2.3l)$$

subject to

$$I(\hat{Y}_1; Y_1 | X_0, X_{11d}, X_{21d}, X_{11c}, U_{12}, V_1) - I(\hat{Y}_1; Y_4 | X_0, X_{11d}, X_{21d}, V_1) \leq I(V_1; Y_4 | X_0, X_{11d}, X_{21d}) \quad (5.2.4a)$$

$$I(\hat{Y}_2; Y_2 | X_0, X_{11d}, X_{21d}, X_{21c}, U_{22}, V_2) - I(\hat{Y}_2; Y_3 | X_0, X_{11d}, X_{21d}, V_2) \leq I(V_2; Y_3 | X_0, X_{11d}, X_{21d}), \quad (5.2.4b)$$

where $G_i = I(X_{i1c}, U_{i2}; V_i | X_0, X_{i1d})$.

Applying Fourier-Motzkin elimination and recalling that $R_i = R_{i1d} + R_{i1c} + R_{i2}$, we have the following theorem.

Theorem 5.2.2. *For the case with $S_{11} = \emptyset$, $S_{21} = \emptyset$, $V_{12} = V_1$, $V_{22} = V_2$, $L'_{12} = 0$, $L'_{22} = 0$, the rate region in Theorem 5.2.1 can be explicitly expressed as*

$$R_1 \leq (5.2.3a) + (5.2.3f) \quad (5.2.5a)$$

$$R_2 \leq (5.2.3b) + (5.2.3j) \quad (5.2.5b)$$

$$R_1 + R_2 \leq \min\{(5.2.3c), (5.2.3d)\} \\ + \min\{(5.2.3e) + (5.2.3l), (5.2.3g) + (5.2.3k), (5.2.3h) + (5.2.3i)\} \quad (5.2.5c)$$

$$R_1 + R_2 \leq (5.2.3a) + (5.2.3b) \\ + \min\{(5.2.3e) + (5.2.3l), (5.2.3g) + (5.2.3k), (5.2.3h) + (5.2.3i)\} \quad (5.2.5d)$$

$$R_1 + R_2 \leq (5.2.3j) + (5.2.3f) + \min\{(5.2.3c), (5.2.3d)\} \quad (5.2.5e)$$

$$R_1 + R_2 \leq \frac{1}{2}((5.2.3j) + (5.2.3e) + (5.2.3k) + (5.2.3h) + 2 \min\{(5.2.3c), (5.2.3d)\}) \quad (5.2.5f)$$

$$R_1 + R_2 \leq \frac{1}{2}((5.2.3i) + (5.2.3g) + (5.2.3f) + (5.2.3l) + 2 \min\{(5.2.3c), (5.2.3d)\}) \quad (5.2.5g)$$

$$2R_1 + R_2 \leq 2(5.2.3a) + (5.2.3e) + (5.2.3k) + (5.2.3h) + (5.2.3b) \quad (5.2.5h)$$

$$2R_1 + R_2 \leq (5.2.3a) + (5.2.3e) + (5.2.3k) + (5.2.3h) + \min\{(5.2.3c), (5.2.3d)\} \quad (5.2.5i)$$

$$R_1 + 2R_2 \leq (5.2.3a) + (5.2.3i) + (5.2.3g) + (5.2.3l) + 2(5.2.3b) \quad (5.2.5j)$$

$$R_1 + 2R_2 \leq (5.2.3i) + (5.2.3g) + (5.2.3l) + (5.2.3b) + \min\{(5.2.3c), (5.2.3d)\}. \quad (5.2.5k)$$

subject to the constraints defined in (5.2.4).

Corollary 5.2.1. *Achievable rate regions for two-user IFCs with pure CF-based source co-operation can be obtained by setting $X_0 = \emptyset$, $X_{11d} = \emptyset$ and $X_{21d} = \emptyset$ in the rate regions defined in Theorems 5.2.1 and 5.2.2.*

Setting $X_0 = \emptyset$, $X_{11d} = \emptyset$, $X_{21d} = \emptyset$, $V_1 = \emptyset$, $V_2 = \emptyset$ in the rate region in Theorem 5.2.2, we can recover the CMG region [106] for interference channels, which is shown to be a compact description of the well-known HK region [18].

5.3 Numerical Results

We apply the rate region defined in Theorem 5.2.2 to a Gaussian interference channel model given by

$$y_1 = h_{2,1}x_2 + z_1, \quad (5.3.1)$$

$$y_2 = h_{1,2}x_1 + x_2, \quad (5.3.2)$$

$$y_3 = h_{1,3}x_1 + h_{2,3}x_2 + z_3, \quad (5.3.3)$$

$$y_4 = h_{1,4}x_1 + h_{2,4}x_2 + z_4, \quad (5.3.4)$$

where $h_{i,j}$ denotes the channel coefficient from transmitter i to node j , z_j is the additive Gaussian noise with zero mean and unit variance, and the transmitted signals are subjected to the average power constraints $\mathbb{E}\{|X_i|^2\} \leq P_i$ ($i = 1, 2$). In our simulation, we let $h_{i,j} = 1/|d_{i,j}|$, where $d_{i,j}$ is the distance between nodes i and j .

For some $0 \leq \alpha_i \leq 1$, $0 \leq \beta_i \leq 1$, $0 \leq \gamma_i \leq 1$, $0 \leq \theta_i \leq 1$ ($i = 1, 2$), we choose $X_0 \sim \mathcal{CN}(0, 1)$, $J_{i1d} \sim \mathcal{CN}(0, \alpha_i \bar{\beta}_i P_i)$, $J_{i1c} \sim \mathcal{CN}(0, \bar{\alpha}_i \gamma_i \theta_i P_i)$, $J_{i2} \sim \mathcal{CN}(0, \bar{\alpha}_i \gamma_i \bar{\theta}_i P_i)$, and $Q_i \sim \mathcal{CN}(0, \bar{\alpha}_i \bar{\gamma}_i P_i)$. The codewords are constructed as

$$X_{i1d} = \sqrt{\alpha_i \beta_i P_i} X_0 + J_{i1d}, \quad (5.3.5)$$

$$X_{i1c} = \sqrt{\alpha_i \beta_i P_i} X_0 + J_{i1d} + J_{i1c}, \quad (5.3.6)$$

$$V_i = \sqrt{\alpha_i \beta_i P_i} X_0 + J_{i1d} + Q_i, \quad (5.3.7)$$

$$U_{i2} = \sqrt{\alpha_i \beta_i P_i} X_0 + J_{i1d} + J_{i1c} + J_{i2} + \lambda_i Q_i, \quad (5.3.8)$$

$$X_i = \sqrt{\alpha_i \beta_i P_i} X_0 + J_{i1d} + J_{i1c} + J_{i2} + Q_i \quad (5.3.9)$$

where the parameter $\lambda_i \geq 0$. Using the constructed signals, the achievable rate region in Theorem 5.2.2 can be easily calculated. Since both transmitters send the signal component X_0 , this allows for coherent addition at the receivers.

To illustrate the derived rate region result, we consider both static and dynamic channel settings. In the static case, we assume fixed locations for the nodes and consider four types of channel configurations, namely, the strong-cooperation strong-interference channel, the strong-cooperation weak-interference channel, the weak-cooperation strong-interference channel and the weak-cooperation weak-interference channel. Each of the four configurations represents one of the four situations specified by the relative strength of the cooperation link and the interference link with respect to that of the direct link. With the dynamic channel setting, we assume mobile nodes and consider average rates. Under each of the channel settings, the achievable rate region presented in Theorem 5.2.2 is numerically compared with the the HK region [18], the DF-based rate regions [52], [54], and the pure CF-based rate region (Corollary 5.2.1). The numerical results are presented in Figs. 5.5-5.8 and 5.10.

For the static channel setting, we place transmitters 1 and 2 at fixed points in a two-dimensional Cartesian system with coordinates $(-a/2, -b/2)$ and $(-a/2, b/2)$, respectively. By choosing different values for a and b and different locations for the receivers, the four network configurations can be constructed. Specifically, for cases with strong cooperation links, we choose $a = 2$ and $b = 1$. For cases with weak cooperation links, we choose $a = 1$ and $b = 2$. For the cases with strong interference links, we let destination nodes 3 and 4 located at points $(a/2, -b/2)$ and $(a/2, b/2)$, respectively. For the cases with weak interference links, the location of the two destination nodes are swapped. Figure 5.4 illustrates the four network configurations. The numerical results for the four channel configurations are shown in Figs. 5.5-5.8, respectively. In all the simulations, the transmit power levels are set as $P_1 = P_2 = 6$ dB.

From Figs. 5.5 and 5.6, which correspond to the cases with strong cooperation links, we observe that the rate region achieved by the proposed combined CF and DF cooperation

Figure 5.4: Simulation environment for the static case.

Figure 5.5: Rate region results for $a = 2, b = 1, P_1 = P_2 = 6$ dB and the receivers suffer from strong interference from the transmitters.

Figure 5.6: Rate region results for $a = 2$, $b = 1$, $P_1 = P_2 = 6$ dB and the receivers suffer from weak interference from the transmitters.

Figure 5.7: Rate region results for $a = 1$, $b = 2$, $P_1 = P_2 = 6$ dB the receivers suffer from strong interference from the transmitters.

scheme dominates that of the DF scheme proposed in [52] (DF-I) where the two sources decode and cooperatively send part of each others' common messages. On the other hand, by comparing with the rate region achieved by the DF scheme proposed in [54] (DF-II) where part of the private messages are sent and exchanged by the sources in advance by taking

Figure 5.8: Rate region results for $a = 1$, $b = 2$, $P_1 = P_2 = 6$ dB and the receivers suffer from weak interference from the transmitters.

advantage of the strong cooperation link, we can see that the proposed combined CF and DF scheme achieves a higher maximum rate for each source but is inferior to the DF-II in other rate points. This observation indicates that in the case with strong cooperation links, the DF-II scheme makes a better use of the cooperation resources than the proposed combined DF and CF cooperation scheme. Moreover, by comparing the rate regions achieved by systems with and without source cooperation, the advantage of source cooperation in boosting the achievable rate of the channel can be easily observed.

In Fig. 5.7, which corresponds to the weak-cooperation strong-interference case, we can observe the superiority of the proposed combined CF and DF cooperation scheme over the DF-I scheme. It can also be observed that the rate region achieved by the pure CF-based scheme overlaps with that of the combined DF and CF scheme. This result indicates that when the cooperation links are weak, CF-based cooperation scheme is better than DF-based ones in terms of achievable rates. Note that in the weak cooperation link cases, only the DF-I scheme can be implemented since the two sources are not able to decode each others' private message part due to the weak cooperation links. From Fig. 5.8, which corresponds to the weak-cooperation weak-interference case, we observe that cooperation provides little improvement in information rate over the non-cooperative case. This observation is intuitively clear: with both weak cooperation links and interference links, the users can do little in helping with each other.

For the channel with mobile nodes, we consider a setting as shown in Fig. 5.9, where the

Figure 5.9: Simulation environment for the dynamic case.

Figure 5.10: Rate region results for the system with mobile transmitters.

two destination nodes are placed 1 meter above the ground with a distance of 2 meters apart from each other and the two transmitters are moved randomly within a $6\text{m} \times 6\text{m}$ square area on the ground. Throughout the simulation, the transmitters adjust their transmission rates to ensure reliable communication (the transmission rates fall in the instantaneous achievable rate region). We take 100 samples of the random locations of the transmitters, and calculate the average achievable rate region for each of the cooperation scheme. The numerical results are shown in Fig. 5.10. It can be observed that with the combined DF and CF cooperation scheme, each user achieves a higher maximum rate. Note that in the simulation, we only consider the DF-I scheme, since this scheme can be implemented despite the condition of the cooperation links.

5.4 Conclusions and Discussions

In this chapter, we have derived an achievable rate region for the two-user interference channel with combined DF and CF source cooperation. A coding scheme that applies rate splitting, block Markov coding, superposition coding, Gel'fand-Pinsker coding and backward decoding was proposed to achieve the rate region. For Gaussian channels, numerical results have shown that for some channel settings, the derived rate region extends the rate region achieved by some pure DF-based cooperation schemes. This result indicates a better spectrum efficiency by using the combined DF and CF cooperation.

Chapter 6

On the DMT of Two-User Fading Interference Channels with Source Cooperation

Due to the lack of knowledge about the capacity of interference channels with source cooperation, the DMT of the channel is not known even for the simplest two-user case. In this chapter, we make a first attempt to evaluate the DMT of the symmetric Rayleigh-faded two-user IFC with source cooperation. The focus is still on the delay-limited applications.

We first assume full CSIT and study the DMT of the general two-user fading IFC with full-duplex source cooperation. DMT upper and lower bounds are derived for such a channel scenario. The upper bound is derived based on a channel capacity upper bound and the two lower bounds are derived using a partially DF-based cooperation scheme and a fully DF-based cooperation scheme, respectively. In most channel settings, the bounds are found to be tight. This result gives us the optimal DMT of the channel and also implies that the optimal DMT can be achieved by DF-based cooperation schemes.

We then assume only partial CSIT and study the DMT of the channel under a scenario where the interference signal and the desired signal received by each destination node are of comparable strength. Both full-duplex and half-duplex source cooperation constraints are considered in the DMT study. Under the full-duplex source cooperation constraints, we propose a transmission scheme and show that its achievable DMT meets the derived DMT upper bound. Under the half-duplex constraint, we derive a DMT upper bound using a proposed capacity upper bound and a lower bound using a proposed transmission scheme. The two bounds are shown to be tight. A full characterization of the DMT of the channel is thus established. For the DMT-optimal transmission scheme designed under the full-duplex constraint, the required partial CSIT is acquired by letting each destination represent the fading magnitude of each of its incoming links using 1 bit of information and feed back this information to the sources. For the DMT-optimal transmission scheme designed under the half-duplex constraint, no more than 3 bits of feedback information are needed to represent

Figure 6.1: Channel model for the two-user IFC with source cooperation.

the fading magnitude of each link. The DMT results show that source cooperation can notably improve the system diversity performance over the whole multiplexing gain region even with partial CSIT and half-duplex cooperation constraint.

6.1 Fading Interference Channel Model

We consider a block Rayleigh-faded two-user IFC as shown in Fig. 6.1. Source nodes 1 and 2 communicate with destination nodes 3 and 4, respectively. The two source nodes operate within the same frequency band and are allowed to transmit simultaneously. The communication links between the two sources allow source cooperation. This channel can be modeled as

$$y_1(t) = \rho_{2,1}h_{2,1}x_2(t) + z_1(t), \quad (6.1.1)$$

$$y_2(t) = \rho_{1,2}h_{1,2}x_1(t) + z_2(t), \quad (6.1.2)$$

$$y_3(t) = \rho_{1,3}h_{1,3}x_1(t) + \rho_{2,3}h_{2,3}x_2(t) + z_3(t), \quad (6.1.3)$$

$$y_4(t) = \rho_{1,4}h_{1,4}x_1(t) + \rho_{2,4}h_{2,4}x_2(t) + z_4(t). \quad (6.1.4)$$

where, $x_i(t)$ and $y_i(t)$ are the signals transmitted from and received at node i at time instance t , respectively (for $i = 1, \dots, 4$). The sources are under an average power constraint $\mathbb{E}\{X_i\} \leq P_i$. For $i = 1, \dots, 4$ and $j = 1, \dots, 4$, $h_{i,j}$ represents the short-term channel fading coefficient of the link between node i and j . $\rho_{i,j}$ represents the long-term channel fading effects, such as the path loss. For the Rayleigh fading channel, $h_{i,j}$ is modeled as a complex Gaussian distributed random variable with zero mean and unit variance. By the block fading assumption, $h_{i,j}$ holds constant within a fading block of length N and changes independently from block to block. We define such a length N fading block as a transmission frame. By assuming reciprocity in the cooperation links between the two sources, we let $\rho_{1,2} = \rho_{2,1} = \rho_c$ and $|h_{1,2}| = |h_{2,1}| = h_c$. $z_i(t) \in \mathcal{CN}(0, \sigma_w^2)$ denotes the additive white Gaussian noise at node i .

Define the signal-to-noise ratio and the interference-to-noise ratio of user i as

$$\text{SNR}_i = \frac{\mathbb{E}\{|\rho_{i,i+2}h_{i,i+2}|^2\}P_i}{\sigma_w^2} = \frac{\rho_{i,i+2}^2 P_i}{\sigma_w^2},$$

and

$$\text{INR}_i = \frac{\mathbb{E}\{|\rho_{i,j+2}h_{i,j+2}|^2\}P_i}{\sigma_w^2} = \frac{\rho_{i,j+2}^2 P_i}{\sigma_w^2}, \text{ for } j \neq i,$$

respectively. For symmetric channels, we let $P_1 = P_2 = P$, $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$, and $\text{INR}_1 = \text{INR}_2 = \text{INR}$. Following [107], the interference level α_i is defined as

$$\alpha_i = \frac{\log \text{INR}}{\log \text{SNR}}.$$

We further define the cooperation link level α_c as

$$\alpha_c = \frac{\log \text{CNR}}{\log \text{SNR}},$$

where

$$\text{CNR} = \frac{\mathbb{E}\{|\rho_c h_c|^2\}P_i}{\sigma_w^2} = \frac{\rho_c^2 P_i}{\sigma_w^2}.$$

The parameters α_c and α_i reflect the relative channel gains of the cooperation links and the interference links with respect to the direct links, respectively. Note that when $\alpha_c < 0$ and $\alpha_i < 0$, the received signal at the sources and the interference signals at the destinations are much lower than the received signal from the direct transmission link. The channel can be viewed as two parallel single-input-single-output channels without source cooperation. Therefore, in the following, we only focus on the cases with $\alpha_c \geq 0$ and $\alpha_i \geq 0$.

It has been shown in [107] that for a symmetric two-user IFC cooperation, the maximum achievable DOF of each user is given by

$$\text{DOF}_{IFC}(\alpha_i) = \begin{cases} 1 - \alpha_i, & \text{if } 0 \leq \alpha_i < \frac{1}{2} \\ \alpha_i, & \text{if } \frac{1}{2} \leq \alpha_i < \frac{2}{3} \\ 1 - \frac{\alpha_i}{2}, & \text{if } \frac{2}{3} < \alpha_i \leq 1 \\ \frac{\alpha_i}{2}, & \text{if } 1 \leq \alpha_i < 2 \\ 1, & \text{if } \alpha_i \geq 2. \end{cases} \quad (6.1.5)$$

With source cooperation, the DOF of each user is given in [140] as

$$\text{DOF}_{CIFIC}(\alpha_i, \alpha_c) = \min \left\{ \max\{(1 - \alpha_i)^+, (\alpha_i - \alpha_c)^+\} + \alpha_c, (\max\{1, \alpha_i, \alpha_c\} + (1 - \alpha_i)^+)/2, \max\{1, \alpha_c\}, \max\{1, \alpha_i\} \right\}. \quad (6.1.6)$$

In particular, if $\alpha_i = \alpha_c = \alpha$, the DOF of each user is given in [53] as

$$\text{DOF}_{CIFIC}(\alpha) = \frac{\max\{1, \alpha\} + (1 - \alpha)^+}{2}. \quad (6.1.7)$$

From (6.1.5), one can see that the DOF of IFCs without cooperation is bounded by 1. From (6.1.6) and (6.1.7), it can be seen that the DOF of IFCs with cooperation can go unbounded as α_i and α_c increase. This gives us an idea about how the channel capacity can be improved by cooperation.

6.2 DMT with Full CSIT

6.2.1 Upper Bound

To find a DMT upper bound for the channel, a capacity upper bound is required. In [54], the sum capacity of two-user IFCs with source cooperation has been characterized to within a fixed number of bits. Using this sum capacity result together with the capacity cut-set upper bound, an capacity upper bound for the channel can be found as follows:

$$0 \leq R_1 \leq \log \left(1 + \text{SNR}|h_{1,3}|^2 + \text{SNR}^{\alpha_c}|h_c|^2 \right), \quad (6.2.1a)$$

$$0 \leq R_1 \leq \log \left(1 + (\text{SNR}^{1/2}|h_{1,3}| + \text{SNR}^{\alpha_i/2}|h_{2,3}|)^2 \right), \quad (6.2.1b)$$

$$0 \leq R_2 \leq \log \left(1 + \text{SNR}|h_{2,4}|^2 + \text{SNR}^{\alpha_c}|h_c|^2 \right), \quad (6.2.1c)$$

$$0 \leq R_2 \leq \log \left(1 + (\text{SNR}^{1/2}|h_{2,4}| + \text{SNR}^{\alpha_i/2}|h_{1,4}|)^2 \right), \quad (6.2.1d)$$

$$\begin{aligned} R_1 + R_2 \leq & \log 2 \left(1 + (\text{SNR}^{1/2}|h_{1,3}| + \text{SNR}^{\alpha_i/2}|h_{2,3}|)^2 \right) \\ & + \log \left(1 + \frac{\max\{\text{SNR}|h_{2,4}|^2, \text{SNR}^{\alpha_i}|h_{2,3}|^2, \text{SNR}^{\alpha_c}|h_c|^2\}}{\max\{1, \text{SNR}^{\alpha_i}|h_{2,3}|^2\}} \right), \end{aligned} \quad (6.2.1e)$$

$$\begin{aligned} R_1 + R_2 \leq & \log 2 \left(1 + (\text{SNR}^{1/2}|h_{2,4}| + \text{SNR}^{\alpha_i/2}|h_{1,4}|)^2 \right) \\ & + \log \left(1 + \frac{\max\{\text{SNR}|h_{1,3}|^2, \text{SNR}^{\alpha_i}|h_{1,4}|^2, \text{SNR}^{\alpha_c}|h_c|^2\}}{\max\{1, \text{SNR}^{\alpha_i}|h_{1,4}|^2\}} \right), \end{aligned} \quad (6.2.1f)$$

$$\begin{aligned}
R_1 + R_2 \leq & \log \left(1 + \left(\frac{\text{SNR}^{1/2}|h_{1,3}|}{\max\{1, \text{SNR}^{\alpha_i/2}|h_{1,4}|\}} + \frac{\text{SNR}^{\alpha_i/2}|h_{2,3}|}{\max\{1, \text{SNR}^{\alpha_c/2}|h_c|\}} \right)^2 \right) (1 + \text{SNR}^{\alpha_c}|h_c|^2) \\
& + \log \left(1 + \left(\frac{\text{SNR}^{1/2}|h_{2,4}|}{\max\{1, \text{SNR}^{\alpha_i/2}|h_{2,3}|\}} + \frac{\text{SNR}^{\alpha_i/2}|h_{1,4}|}{\max\{1, \text{SNR}^{\alpha_c/2}|h_c|\}} \right)^2 \right) (1 + \text{SNR}^{\alpha_c}|h_c|^2)
\end{aligned} \tag{6.2.1g}$$

$$\begin{aligned}
R_1 + R_2 \leq & \log \left(1 + 2(\text{SNR}|h_{1,3}|^2 + \text{SNR}|h_{2,4}|^2 + \text{SNR}^{\alpha_i}|h_{1,4}|^2 + \text{SNR}^{\alpha_i}|h_{2,3}|^2) \right. \\
& \left. + 4(\text{SNR}^2|h_{1,3}h_{2,4}|^2 + \text{SNR}^{2\alpha_i}|h_{1,4}h_{2,3}|^2 - 2\text{SNR}^{1+\alpha_i}|h_{1,3}h_{2,4}h_{1,4}h_{2,3}|\cos\theta) \right).
\end{aligned} \tag{6.2.1h}$$

Here, (6.2.1a)-(6.2.1d) are obtained from the capacity cut-set bound, (6.2.1e)-(6.2.1h) are obtained from the sum capacity in [54]. The parameter θ reflects the phase difference of the direct links and the interference links.

Based on the capacity upper bound (6.2.1), we derive a DMT upper bound given in the following theorem.

Theorem 6.2.1. *The DMT of a symmetric two-user IFC with source cooperation is upper bounded by*

$$d^{ub}(r) = \min\{d_a^{ub}(r), d_b^{ub}(r), d_c^{ub}(r), d_d^{ub}(r)\}, \tag{6.2.2}$$

where

$$d_a^{ub}(r) = (1 - r)^+ + (\alpha_c - r)^+, \tag{6.2.3}$$

$$d_b^{ub}(r) = (1 - r)^+ + (\alpha_i - r)^+, \tag{6.2.4}$$

$$d_c^{ub}(r) = \begin{cases} 2(1 - 2r)^+ + (\alpha_c - 2r)^+ + (\alpha_i - 2r)^+, & \text{if } \alpha_i \geq 2r \\ (1 + \alpha_c - \alpha_i - 2r)^+, & \text{if } \alpha_i < 1, 2r > 1, \alpha_c > 1 \\ (2 - \alpha_i - 2r)^+ + (\alpha_c - 2r)^+, & \text{otherwise} \end{cases} \tag{6.2.5}$$

$$d_d^{ub}(r) = \begin{cases} \min\{d_{d,1}^{ub}(r), d_{d,3}^{ub}(r), d_{d,4}^{ub}(r), d_{d,5}^{ub}(r)\}, & \text{if } \alpha_i \leq \min\{1, \frac{1+\alpha_c}{2}, r\} \\ \min\{d_{d,2}^{ub}(r), d_{d,3}^{ub}(r), d_{d,4}^{ub}(r), d_{d,5}^{ub}(r)\}, & \text{if } \alpha_i > \min\{1, \frac{1+\alpha_c}{2}, r\}, \end{cases} \tag{6.2.6}$$

where

$$d_{d,1}^{ub}(r) = 2(1 - \alpha_i - r)^+ + (\alpha_c - (\alpha_i + r - 1)^+)^+, \tag{6.2.7}$$

$$\begin{aligned}
d_{d,2}^{ub}(r) = & 2(1 - 2r)^+ + (\alpha_c - \min\{1, (2r - 1)^+\})^+ \\
& + (2\alpha_i - \min\{1, 2r\} - \min\{\alpha_c, 1, (2r - 1)^+\})^+,
\end{aligned} \tag{6.2.8}$$

$$d_{d,3}^{ub}(r) = 2(\alpha_i - r)^+ + 2(1 - \min\{2\alpha_i, 2r\})^+ + (\alpha_c - \min\{\alpha_i, r, (\min\{2\alpha_i, 2r\} - 1)^+\})^+, \quad (6.2.9)$$

$$d_{d,4}^{ub}(r) = 2(\alpha_i - r)^+ + 2(1 - \min\{\alpha_i, r\})^+ + (\alpha_c - \min\{\alpha_i, r\})^+, \quad (6.2.10)$$

$$d_{d,5}^{ub}(r) = (\alpha_c - r)^+ + 2(\alpha_i - \min\{\alpha_c, r\})^+ + 2(1 - \min\{\alpha_i, \alpha_c, r\})^+. \quad (6.2.11)$$

Proof. Please refer to Appendix G for the proof of this theorem. $\square$

6.2.2 Lower Bounds

We provide two DMT lower bounds based on two DF cooperation schemes proposed in [52] and [54], respectively. Based on their ways of implementing cooperation, we term the DF scheme proposed in [52] as a partially DF-based cooperation scheme and the one proposed in [54] as a fully DF-based scheme. By applying the achievable rate regions given in [52] and [54], the achievable DMTs of the two DF schemes can be derived. Note that since our focus is on signal-antenna systems, DF-based cooperation schemes are able to achieve a good DMT behavior, as will be shown later. For systems with multiple antennas, CF-based cooperation schemes (as the one we proposed in Chapter 5) might be needed for a tighter DMT lower bound.

In the partially DF source cooperation scheme, the messages of a source are divided into a cooperative common message part, a non-cooperative common message part, and a private message part. The common messages (cooperative and non-cooperative) are decoded by both destinations, while the private message is only decoded by the intended destination. Moreover, the cooperative common message is also decoded by the other user and cooperatively sent to the destinations. The non-cooperative common message and the private message are all sent without cooperation. This scheme is mainly applied in scenarios where the links between the two sources are weaker than the direct links from the sources to their respective destinations, i.e., $\alpha_c \leq 1$. The achievable DMT is presented in the following theorem.

Theorem 6.2.2. *The DMT of symmetric two-user IFC with source cooperation and the partially DF cooperation scheme is*

$$d^{DF_p}(r) = \min \{d_a^{DF_p}(r), d_b^{DF_p}(r), d_c^{DF_p}(r), d_d^{DF_p}(r), d_e^{DF_p}(r), d_f^{DF_p}(r), d_g^{DF_p}(r)\}. \quad (6.2.12)$$

Here

$$d_a^{DF_p}(r) = \begin{cases} (1 + \alpha_c - \alpha_i - r)^+, & \text{if } \alpha_i \leq \min\{\alpha_c, 1, r\} \\ \min\{d_{a,1}^{DF_p}(r), d_{a,2}^{DF_p}(r)\}, & \text{if } \alpha_i > \min\{\alpha_c, 1, r\} \end{cases} \quad (6.2.13)$$

$$d_b^{DF_p}(r) = \min\{d_{b,1}^{DF_p}(r), d_{b,2}^{DF_p}(r)\}, \quad (6.2.14)$$

$$d_c^{DF_p}(r) = 2(1 - r - \min\{r, \frac{\alpha_c}{2}\})^+ + (\alpha_i - 2r)^+, \quad (6.2.15)$$

$$d_d^{DF_p}(r) = \begin{cases} (2 + \alpha_c - \alpha_i - 2r)^+, & \text{if } \alpha_i \leq \min\{1, 2r\} \\ \min\{d_{d,1}^{DF_p}(r), d_{d,2}^{DF_p}(r)\}, & \text{if } \alpha_i > \min\{1, 2r\} \end{cases} \quad (6.2.16)$$

$$d_e^{DF_p}(r) = d_d^{ub}(r), \quad (6.2.17)$$

$$d_f^{DF_p}(r) = \min\{d_{f,1}^{DF_p}(r), d_{f,2}^{DF_p}(r)\}, \quad (6.2.18)$$

$$d_g^{DF_p}(r) = \begin{cases} \min\{d_{g,1}^{DF_p}(r), d_{g,3}^{DF_p}(r)\}, & \text{if } \alpha_i \leq \min\{1, \frac{1+\alpha_c}{2}, \frac{3r}{2}\} \\ \min\{d_{g,2}^{DF_p}(r), d_{g,3}^{DF_p}(r)\}, & \text{if } \min\{\frac{1+\alpha_c}{2}, \frac{3r}{2}\} < \alpha_i \leq 1 \\ d_{g,4}^{DF_p}(r), & \text{if } 1 < \alpha_i \leq \alpha_c \\ d_{g,5}^{DF_p}(r), & \text{if } \alpha_i > \max\{1, \alpha_c\} \end{cases} \quad (6.2.19)$$

where

$$d_{a,1}^{DF_p}(r) = (1 - r)^+ + (\alpha_c - \min\{1, r\})^+, \quad (6.2.20)$$

$$d_{a,2}^{DF_p}(r) = (1 - \min\{\alpha_c, r\})^+ + (\alpha_c - r)^+, \quad (6.2.21)$$

$$d_{b,1}^{DF_p}(r) = (1 - r)^+ + (\alpha_i - \min\{1, r\})^+, \quad (6.2.22)$$

$$d_{b,2}^{DF_p}(r) = (\alpha_i - r)^+ + (1 - \min\{\alpha_i, r\})^+, \quad (6.2.23)$$

$$d_{d,1}^{DF_p}(r) = (\alpha_c - (2r - \alpha_i)^+)^+ + (\alpha_i - 2r)^+ + 2(1 - \min\{\alpha_i, 2r\})^+, \quad (6.2.24)$$

$$d_{d,2}^{DF_p}(r) = (\alpha_c - (2r - 1)^+)^+ + (\alpha_i - \min\{1, 2r\})^+ + 2(1 - 2r)^+, \quad (6.2.25)$$

$$d_{f,1}^{DF_p}(r) = \begin{cases} (1 - 3r)^+ + (1 - \alpha_i - \frac{1}{2}(3r - 1)^+)^+ \\ \quad + (\alpha_c - \min\{\alpha_i, \frac{1}{2}(3r + 2\alpha_i - 3)^+\})^+, & \text{if } \alpha_i \leq \min\{\frac{1}{2}, \frac{3r}{2}\} \\ \frac{1}{2}(1 + 2\alpha_i - 3r)^+ + (\alpha_c - \min\{\alpha_i, 2\alpha_i - 1 + \frac{1}{2}(3r - 1 - 2\alpha_i)^+\})^+, \\ \quad \text{if } 1 < 2\alpha_i \leq 1 + \min\{1, \alpha_c\}, 4\alpha_i \leq 1 + 3r \\ (1 - 3r)^+ + (\alpha_c - \frac{1}{2}(3r - 1)^+)^+ \\ \quad + (1 - \frac{1}{2} \min\{1, 3r\} - \frac{1}{2} \min\{\alpha_c, \frac{1}{2}(3r - 1)^+\})^+ \\ \quad + 2(\alpha_i - \min\{1, \frac{1}{2} \min\{1, 3r\} + \frac{1}{2} \min\{\alpha_c, \frac{1}{2}(3r - 1)^+\})^+. & \text{otherwise} \end{cases} \quad (6.2.26)$$

$$d_{f,2}^{DF_p}(r) = 2\left(\alpha_i - \frac{3r}{2}\right)^+ + (\alpha_c - \min\{\alpha_i, (3r - 2\alpha_i)^+\})^+ + \left(1 - \min\left\{\alpha_i, \frac{3r}{2}\right\}\right)^+ \\ + \left(1 - 2\min\left\{\alpha_i, \frac{3r}{2}\right\} + \min\{\alpha_c, \alpha_i, (3r - 2\alpha_i)^+\}\right)^+, \quad (6.2.27)$$

$$d_{g,1}^{DF_p}(r) = \left(1 - \alpha_i - \frac{1}{2}(3r - 1 - \alpha_c)^+\right)^+ + (1 + \alpha_c - 3r)^+, \quad (6.2.28)$$

$$d_{g,2}^{DF_p}(r) = \left(1 - \frac{3r}{2}\right)^+ + \left(\alpha_c - \frac{3r}{2} - \left(\frac{3r}{2} - 1\right)^+\right)^+ \\ + \left(1 - \min\left\{\alpha_i, \frac{3r}{2}\right\}\right)^+ + 2\left(\alpha_i - \min\left\{1, \frac{3r}{2}\right\}\right)^+, \quad (6.2.29)$$

$$d_{g,3}^{DF_p}(r) = \left(1 - \frac{3r}{2}\right)^+ + 2\left(\alpha_i - \min\left\{1, \frac{3r}{2}\right\}\right)^+ + (1 + \alpha_c - \min\{2\alpha_i, 3r\})^+, \quad (6.2.30)$$

$$d_{g,4}^{DF_p}(r) = \begin{cases} 2(\alpha_i - \frac{3r}{2})^+ + 2(1 - \frac{3r}{2})^+ + (\alpha_c - \frac{3r}{2})^+, & \text{if } 3r \leq 2 \\ (\alpha_i - 1)^+ + (1 + \alpha_i - 3r)^+ + (1 + \alpha_c - 3r)^+, & \text{if } 2 < 3r \leq 1 + \alpha_i \\ (\alpha_i + \alpha_c - 3r)^+, & \text{if } 1 + \alpha_i < 3r \leq \alpha_c + \alpha_i \end{cases} \quad (6.2.31)$$

$$d_{g,5}^{DF_p}(r) = 2\left(\alpha_i - \frac{3r}{2}\right)^+ + (1 + \alpha_c - \min\{2\alpha_i, 3r\})^+ + \left(1 - \min\left\{\alpha_i, \frac{3r}{2}\right\}\right)^+ \quad (6.2.32)$$

Proof. Please refer to Appendix H for the proof of this theorem. $\square$

The fully DF-based cooperation scheme is designed for scenarios where the cooperation links between the two sources are stronger than that of the direct links, i.e. $\alpha_c > 1$. To make full use of the strong cooperation link, at the t th transmission interval, the source i transmits not only the signal $x_i(t)$ desired for this interval but also a signal $s_i(t+1)$ that carries the message for transmission in the $(t+1)$ th interval. By assigning proper power to $s_i(t+1)$ and due to the relatively weak direct links compared to the cooperation links, $s_i(t+1)$ can be received and correctly decoded by the other source, but it can only be received by the destinations at a power level which is below noise level. Thus, transmitting $s_i(t+1)$ will help the sources to gain, in advance, some information about the messages to be transmitted in the $(t+1)$ th interval, but will not cause extra interference to the destinations. With this advanced knowledge, actions can be taken by a source to pre-cancel part of the interference caused by the other source.

The fully DF scheme in [54] is designed for analyzing the channel ergodic rate region. In our case, we focus on the effect of outage in a fading channel. Since the messages are required to be fully decoded by the sources, the cooperation links become the main factor that restricts the DMT behavior. We therefore make a modification to the fully DF-based cooperation scheme to relax the restriction of the cooperation link.

In the modified fully DF scheme, depending on the targeted multiplexing rate, we switch between two transmission strategies. When the targeted multiplexing rate is less than the maximum multiplexing rate that can be supported by the corresponding interference

channel without cooperation, the transmission is implemented as follows. If the source can successfully decode each other's message, the transmission scheme of the original fully DF scheme is used. If the sources fail to correctly decode each other's message due to, for example, deep fading in the cooperation link, the sources do not cooperate and simply transmit their own messages using the transmission schemes for the IFC without cooperation, such as the HK scheme [18]. When the targeted multiplexing rate exceeds the maximum multiplexing rate that can be supported by the corresponding IFC without cooperation, then the original fully DF scheme is used. The following theorem provides the DMT achieved by the modified fully DF-based cooperation scheme.

Theorem 6.2.3. *The DMT of the symmetric IFC with modified fully DF-based source cooperation is given as*

$$d^{DF_f}(r) = \begin{cases} \min\{d_c^{DF_f}(r), d_d^{DF_f}(r), d^{IFC}(r) + (\alpha_c - r)^+\}, & \text{if } r \leq \text{DOF}_{IFC}(\alpha_i) \\ \min\{d_a^{DF_f}(r), d_b^{DF_f}(r), d_c^{DF_f}(r), d_d^{DF_f}(r)\}. & \text{if } \text{DOF}_{IFC}(\alpha_i) < r \leq \text{DOF}_{CIFC}(\alpha_i, \alpha_c), \end{cases} \quad (6.2.33)$$

where

$$d_a^{DF_f}(r) = (\alpha_c - r)^+, \quad (6.2.34)$$

$$d_b^{DF_f}(r) = \begin{cases} (1 + \alpha_c - \alpha_i - 2r)^+, & \text{if } \alpha_i \leq \min\{1, \alpha_c, 2r\} \\ \min\{d_{b,1}^{DF_f}(r), d_{b,2}^{DF_f}(r), d_{b,3}^{DF_f}(r)\}, & \text{if } \alpha_i > \min\{1, \alpha_c, 2r\} \end{cases} \quad (6.2.35)$$

$$d_c^{DF_f}(r) = (1 - r)^+ + (\alpha_i - r)^+, \quad (6.2.36)$$

$$d_d^{DF_f}(r) = d_c^{ub}(r), \quad (6.2.37)$$

and

$$d_{b,1}^{DF_f}(r) = (1 - 2r)^+ + (\alpha_i - \min\{1, 2r\})^+ + (\alpha_c - \min\{1, \alpha_i, 2r\})^+, \quad (6.2.38)$$

$$d_{b,2}^{DF_f}(r) = (\alpha_c - 2r)^+ + (\alpha_i - \min\{\alpha_c, 2r\})^+ + (1 - \min\{\alpha_c, \alpha_i, 2r\})^+, \quad (6.2.39)$$

$$d_{b,3}^{DF_f}(r) = (\alpha_i - 2r)^+ + (1 - \min\{\alpha_i, 2r\})^+ + (\alpha_c - \min\{\alpha_i, 2r\})^+. \quad (6.2.40)$$

$d^{IFC}(r)$ is the DMT of interference channel given in [116], and is included here for completeness.

$$d^{IFC}(r) = \min\{d_a^{IFC}(r), d_b^{IFC}(r), d_c^{IFC}(r), d_d^{IFC}(r)\}, \quad (6.2.41)$$

where

$$d_a^{IFC}(r) = (1 - r)^+, \quad (6.2.42)$$

$$d_b^{IFC}(r) = 2\left(1 - r - \min\left\{r, \frac{\alpha_i}{2}\right\}\right)^+ + (\alpha_i - 2r)^+, \quad (6.2.43)$$

$$d_c^{IFC}(r) = 2(1 - r - \min\{r, \alpha_i\})^+ + 2(\alpha_i - r)^+, \quad (6.2.44)$$

$$d_d^{IFC}(r) = \begin{cases} (1 - \alpha_i - \frac{1}{2}(3r - 1))^+ + (1 - 3r)^+, & \text{if } \alpha_i \leq \min\{\frac{1}{2}, \frac{3r}{2}\} \\ (1 - \frac{3r}{2})^+ + (1 - 3r)^+ + 2(\alpha_i - \frac{3r}{2})^+, & \text{if } \alpha_i > \min\{\frac{1}{2}, \frac{3r}{2}\}. \end{cases} \quad (6.2.45)$$

Proof. Please refer to Appendix I for the proof of this theorem. $\square$

From the DMT upper bound given in Theorem 6.2.1 and the two achievable DMT results given in Theorems 6.2.2 and 6.2.3, it can be seen that at $r = 0$, $d^{ub}(0) = d^{DF_p}(0) = d^{DF_f}(0) = \min\{1 + \alpha_i, 1 + \alpha_c\}$. This indicates that the maximum diversity order of the IFC with source cooperation is given by $\min\{1 + \alpha_i, 1 + \alpha_c\}$ and can be achieved by both the partially and fully DF-based schemes. For the other multiplexing values, due to the complicated DMT expressions, it is hard to tell exactly in which scenarios the DMT upper bound is achievable. In the face of the analytical difficulties, we resort to the numerical method to gain some insights about the tightness of the derived DMT bounds.

6.2.3 Numerical Evaluation

We numerically evaluate the DMT upper and lower bounds of the channel by considering different values of α_i and α_c . The DMT curves of the channels with different values of α_c and $\alpha_i = 0.5, 0.8, 1.2$ are drawn in Figs. 6.2-6.4, respectively. The following observation can be made from the numerical results.

First of all, the DMT upper bound is tight for most values of α_i and α_c . This can be seen from Figs. 6.3 and 6.4. For all the simulated cases in Figs. 6.3 and 6.4, the DMT upper bounds can be achieved by the modified fully DF-based cooperation scheme. In general, according to our simulation, the DMT upper bound is tight for the cases with (i) $\alpha_i \leq 0.3, \forall \alpha_c$, (ii) $\alpha_i \geq 0.8, \forall \alpha_c$, and (iii) $\alpha_c \geq 1, \forall \alpha_i$. For the cases with $0.3 < \alpha_i < 0.8$, the DMT upper bound is tight for some values of α_c . Specifically, the DMT upper bound is tight for cases where $\alpha_i = 0.4, \alpha_c \geq 0.1$, or $\alpha_i = 0.5, \alpha_c \geq 0.4$, or $\alpha_i = 0.6, \alpha_c \geq 0.5$, or $\alpha_i = 0.7, \alpha_c \geq 0.5$. In these cases, the upper bound can be achieved by either the partially DF scheme or the modified fully DF scheme.

Since the DMT upper bound is derived from a capacity upper bound, which may not be tight in general, there are cases where the DMT upper bound is not tight. An example is shown in Fig. 6.2(a). For the case with $\alpha_c = 0$, the strength of the cooperation link is

(a) $\alpha_i = 0.5, \alpha_c = 0$

(b) $\alpha_i = 0.5, \alpha_c = 0.4$

(c) $\alpha_i = 0.5, \alpha_c = 0.6$

(d) $\alpha_i = 0.5, \alpha_c = 0.9$

(e) $\alpha_i = 0.5, \alpha_c = 1.0$

(f) $\alpha_i = 0.5, \alpha_c = 1.1$

Figure 6.2: DMT curves of IFCs with $\alpha_i = 0.5$ and various values of α_c .

(a) $\alpha_i = 0.8, \alpha_c = 0$

(b) $\alpha_i = 0.8, \alpha_c = 0.6$

(c) $\alpha_i = 0.8, \alpha_c = 0.7$

(d) $\alpha_i = 0.8, \alpha_c = 1.0$

(e) $\alpha_i = 0.8, \alpha_c = 1.1$

Figure 6.3: DMT curves of IFCs with $\alpha_i = 0.8$ and various values of α_c .

(a) $\alpha_i = 1.2, \alpha_c = 0$

(b) $\alpha_i = 1.2, \alpha_c = 0.8$

(c) $\alpha_i = 1.2, \alpha_c = 0.9$

(d) $\alpha_i = 1.2, \alpha_c = 1.2$

(e) $\alpha_i = 1.2, \alpha_c = 1.3$

Figure 6.4: DMT curves of IFCs with $\alpha_i = 1.2$ and various values of α_c .

very weak and almost useless. We expect that the IFCs with cooperation has the same DMT as the IFCs without cooperation. However, in Fig. 6.2(a), the DMT curves of the case with $\alpha_c = 0$ show that although the two DF-based schemes have the same DMT as the IFC without cooperation, the upper bound is higher in some multiplexing rates and therefore is not tight. Our further experiments show that as α_c increases, the DMT lower bounds achieved by the two DF schemes also increase but still below the upper bound. The DMT lower bound meets the upper bound only when $\alpha_c \geq 0.4$, as shown in Fig. 6.2(b).

To obtain a tighter DMT upper bound, a better capacity upper bound is needed. In the capacity upper bound given by (6.2.1), we only consider bounds for the single rates R_1 , R_2 and the sum rate $R_1 + R_2$. However, for a typical expression of the rate region of IFCs, there are terms that define the bounds for $2R_1 + R_2$ and $R_1 + 2R_2$. The absence of the bounds on $2R_1 + R_2$ and $R_1 + 2R_2$ makes the capacity upper bound (6.2.1) not tight, which in turn leads to the not-so-tight behavior of the derived DMT upper bound.

Secondly, for $0 \leq \alpha_i < 0.8$, the partially DF cooperation scheme has a better DMT for small values of α_c . As α_c increases, the DMT of the modified fully DF scheme improves and will eventually exceed that of the partially DF scheme. Figure 6.2 clearly reflects this trend. For the cases where $\alpha_i \geq 0.8$ and $\forall \alpha_c$, the modified fully DF scheme dominates the partially DF scheme and achieves the DMT upper bound. This can be partly observed from Figs. 6.3 and 6.4. Moreover, for all values of α_i , as long as $\alpha_c \geq 1$, the modified fully DF scheme achieves the DMT upper bound and thus is DMT optimal. The above observations imply that as long as the cooperation links are better than the direct links, no matter what the status of the interference links are, the modified fully DF scheme is DMT optimal. Since in most scenarios, cooperation happens when the cooperation link is better than the direct link, to achieve the optimal DMT behavior, we can simply use the modified fully DF-based cooperation scheme.

Thirdly, if $\alpha_i \leq 1$, the partially DF scheme achieves the maximum multiplexing rate when $\alpha_c \leq 1$. If $\alpha_i > 1$, the partially DF scheme achieves the maximum multiplexing rate when $\alpha_c \leq \alpha_i$. This can be observed from the comparisons between Figs. 6.2(e) and 6.2(f), Figs. 6.3(d) and 6.3(e), and Figs. 6.4(d) and 6.4(e). This observation is consistent with the target of the partially DF scheme, i.e., it is designed for cases where the cooperation links are relatively weak and the two sources cannot fully decode the messages from each other. The modified fully DF scheme cannot achieve the maximum multiplexing rate when $\alpha_i \leq 0.6$ and $\alpha_c < 1$. For all the other cases, our experiments show that the modified fully DF scheme can achieve the maximum multiplexing rate.

6.3 DMT with Partial CSIT: Full-Duplex Case

In this section, we focus on the case with $\alpha_i = 1$ and $\alpha_c = 1$. This corresponds to a channel scenario where the long-term fading effects of all the links are of a similar order. Without loss of generality, we assume $\rho_{1,3} = \rho_{1,4} = \rho_{2,3} = \rho_{2,4} = 1$. By applying Theorem 6.2.1 under the case $\alpha_i = 1$ and $\alpha_c = 1$ we know that the DMT of the channel can be upper bounded by

$$d^{FD}(r) = \min\{2(1-r)^+, 4(1-2r)^+\}. \quad (6.3.1)$$

Note that although this upper bound is derived under a full CSIT assumption, for delay-limited applications, it is still an effective upper bound for channels with partial or without CSIT, as suggested by [113] and [141]. In the following, we show that the DMT upper bound can indeed be achieved by a proposed transmission scheme. Therefore, the DMT bound in (6.3.1) is the optimal DMT of the considered channel.

In the achievable transmission scheme, with full knowledge on the channel state information (CSI), each destination represents the fading magnitude of each of its incoming links using one information bit. The bits for CSI are then fed back to the sources. Transmission at each source is arranged based on the feedback bits. Specifically, for $i = 1, 2$ and $j = 1, 2$ the feedback bit $b_{i,j+2}^{(0)}$ for the fading magnitude $|h_{i,j+2}|$ is defined as follows:

$$b_{i,j+2}^{(0)} = \begin{cases} 0 & \text{if } 0 \leq \beta_{i,j+2} \leq (1-2r)^+, \\ 1 & \text{if } \beta_{i,j+2} > (1-2r)^+, \end{cases} \quad (6.3.2)$$

where $\beta_{i,j+2} \doteq -\log |h_{i,j+2}|^2 / \log \text{SNR}$. To facilitate illustration, we further define a parameter L_j for $i \neq j$ as

$$L_j = \begin{cases} 0 & \text{if } b_{i,j+2}^{(0)} = 1, b_{j,j+2}^{(0)} = 1, \\ 1 & \text{if } b_{i,j+2}^{(0)} = 0, b_{j,j+2}^{(0)} = 1, \\ 2 & \text{if } b_{j,j+2}^{(0)} = 0. \end{cases} \quad (6.3.3)$$

Let $\beta_c \doteq -\log |h_c|^2 / \log \text{SNR}$, the transmission strategy is described as follows:

- i. When $\beta_c > (1-r)^+$, no source cooperation is used. The two sources simply transmit together using the transmission scheme developed in [120] for the two-user IFCs, where the fading magnitude of each link in the channel is represented by one bit which is revealed to the sources for arranging transmission.
- ii. When $(1-2r)^+ < \beta_c \leq (1-r)^+$, the following strategy is used.
 - (a) If $(L_i, L_j) = (0, 0)$ or $(2, 2)$, the two sources simply transmit simultaneously without cooperation. Each source uses a Gaussian codebook to encode its

message.

- (b) If $L_i = 1$, the two sources transmit cooperatively. Each source encodes its message using a Gaussian codeword. At each time instance, each source fully decodes the message sent by the other source, re-encodes it, and then forwards together with its own message in the next time slot. Each destination performs backward decoding to jointly decode the messages from both sources.
 - (c) If $L_i = 0$, $L_j = 2$, source i encodes its message into a Gaussian codeword x_i as in the previous cases. Source j splits its message into two parts: a common message part that can be decoded by both destinations and a private message part that can only be decoded by its intended destination. The public part is then encoded into a Gaussian codeword x_{jw} at multiplexing rate r_{jw} . The private part is encoded into a Gaussian codeword x_{ju} at multiplexing rate r_{ju} . The total multiplexing rate of user j is thus $r_j = r_{jw} + r_{ju}$. The message of source j is transmitted as a superposition of two Gaussian codewords $x_j = x_{jw} + x_{ju}$ with the public part x_{jw} and the private part x_{ju} being allocated with power $\text{SNR} - \text{SNR}^\eta$ and SNR^η , respectively. For notational simplicity, we denote the resulting superimposed codeword x_j as a two-level Gaussian code. In contrast, the Gaussian codeword x_i generated without message-splitting is denoted as a one-level Gaussian codeword. Cooperation is realized by letting source j fully decode and forward the message sent by source i . Source i solely sends its own message without helping to relay the message of source j . At the receiver side, both destinations perform backward decoding. At node $i + 2$, which is the corresponding destination for source i , x_i and x_{jw} are jointly decoded treating x_{ju} as interference. At destination node $j + 2$, x_i and x_{jw} are first jointly decoded treating x_{ju} as interference. x_{ju} is then decoded without the presence of x_i and x_{jw} .
- iii. When $\beta_c \leq (1 - 2r)^+$, for most of the channel conditions, the transmission strategy is the same as the one described in item ii, except for the case where $L_i = 0, L_j = 1$. When $L_i = 0, L_j = 1$, the coding scheme employed by the two sources is the same as the one proposed in the case ii-(c). The difference is in the way cooperation being implemented. Unlike in the case ii-(c), where only source j helps relaying the message of source i and source i solely transmits its own message, in the current considered case, both sources fully decode and forward each other's message. The re-encoded codewords still keep their original form, i.e., the message of source i is still encoded as a one-level Gaussian code at source j and the message of source j is still encoded as a two-level Gaussian code with the same rate and power allocation at source

i. Decodings at both destinations are again implemented in a backward fashion. The message of source *i* and the public message of source *j* are jointly decoded at both destinations treating the signal carrying the private message of source *j* as interference. The private message of source *j* is then decoded at destination node *j* + 2.

For the cases where message splitting is applied by source *i*, depending on the targeted multiplexing rate *r*, the rate and power are allocated as follows:

- When $r \in [0, 1/3]$, take $r_{iw} = 0$, $r_{iu} = r$, and $\eta = 1 - r$.
- When $r \in (1/3, 1/2]$, take $r_{iw} = 3r - 1$, $r_{iu} = 1 - 2r$, and $\eta = 2(1 - 2r)$.

By analyzing the DMT of the proposed transmission scheme, we have the following theorem.

Theorem 6.3.1. *The achievable DMT of the proposed transmission scheme for the symmetric two-user fading IFC with full-duplex source cooperation is*

$$d^{FD-LB}(r) = \min\{2(1 - r)^+, 4(1 - 2r)^+\}. \quad (6.3.4)$$

Proof. Please refer to Appendix J. □

The DMT lower bound achieved by the proposed scheme meets the DMT upper bound given in (6.3.1). One can thus conclude that the DMT bounds are tight and the proposed transmission scheme is DMT optimal. The underlining idea behind the proposed transmission scheme can be explained as follows.

When $\beta_c > (1 - r)^+$, according to the analysis in Appendix J, the two sources cannot fully decode each other. In this case, they can simply transmit simultaneously without cooperation using the transmission scheme designed in [120]. Outage analysis in Appendix J shows that the corresponding DMT result is above the upper bound over the whole multiplexing gain region. Therefore, this case puts no limitation on the achievement of the DMT upper bound. In the following, we can simply focus on the case where $\beta_c \leq (1 - r)^+$.

When $\beta_c \leq (1 - r)^+$, depending on the coding scheme used, the two sources may be able to decode each other. This enables user cooperation. When both users are able to fully decode each other, as in the cases where both users use one-level Gaussian code, we show in Appendix J that as long as one of the incoming links to a destination node *j* + 2, say link $h_{i,j+2}$, satisfies $\beta_{i,j+2} \leq (1 - 2r)^+$, the destination can fully decode both messages from the two sources and, therefore, no outage occurs. This corresponds to the case where $L_1 = 1, 2$ and $L_2 = 1, 2$. Outage happens when both incoming links to destination node *i* + 2 are not so strong, i.e., $\beta_{1,i+2} > (1 - 2r)^+$ and $\beta_{2,i+2} > (1 - 2r)^+$. This corresponds to the cases where $L_i = 0, L_j = 0, 1, 2$.

For the case with $L_i = 0$ and $L_j = 0$, we observe from Appendix J that the probability for this to happen is in the order of $\text{SNR}^{-4(1-2r)^+}$. Even with a decoding error probability of 1, a diversity order $4(1-2r)^+$ can still be obtained, which is well above the DMT upper bound. Therefore, under this channel scenario, we may simply use one-level coding at the sources and let them transmit together without cooperation.

The case with $L_i = 0$ and $L_j = 2$ corresponds to the scenario where all the incoming links to destination i are relatively weak, i.e., below $\text{SNR}^{-(1-2r)^+}$, the direct link between destination j and its corresponding source is strong. If destination node $i+2$ is asked to fully decode both messages from the two sources, a major constraint will be put on the achievable DMT that prevents the achievement of the upper bound. Since destination node $i+2$ is not interested in the message sent by source j , we may relax the full decoding constraint by asking destination node $i+2$ to only partially decode the message of source j . To enable this partial decoding, source j needs to split its messages into two parts and encode them independently. Outage analysis in Appendix J shows that this partial decoding can effectively improve the DMT behavior such that the upper bound can be achieved. On the other hand, since the direct link between source j and its corresponding destination node $j+2$ is strong enough to convey the two-level coded message sent from source j , there is no need for source i to help relaying, and thus no need for source i to decode the message of source j .

For the case with $L_i = 0$ and $L_j = 1$, all the incoming links to destination $i+2$ are relatively weak, the cross link between source i and destination node $j+2$ is strong while the direct link between source j and destination $j+2$ is weak. In this case, as explained above, source j should use two-level coding for destination $i+2$ to partially decode. Moreover, since the direct link between source j and destination node $j+2$ is weak, source i must help to relay the message from source j , which requires source i to decode the message from source j . From Appendix J, we know that if $\beta_c \leq (1-2r)^+$, source i can successfully decode the two-level coded message from source j . Therefore, by using two-level coding at source j and letting the two sources to fully decode and relay each other, the DMT upper bound can be achieved. However, when $(1-2r)^+ < \beta_c \leq (1-r)^+$, although the sources can decode each other if one-level Gaussian codes are used, the private message cannot be decoded if two-level codes are used. In this case, source i may only decode the public message from source j to help partial relaying, however, it can be seen that the probability for the scenario $\beta_c > (1-2r)^+, L_i = 0, L_j = 1$ to happen is in the order of $\text{SNR}^{-4(1-2r)^+}$. Due to the low occurrence probability, this scenario imposes no limitation for the achievement of the upper bound. We can simply use one-level coding at each source and conduct cooperative transmission at the sources and joint decoding at the destinations.

6.4 DMT with Partial CSIT: Half-Duplex Case

In this section, we again consider the case with $\alpha_i = 1$ and $\alpha_c = 1$. A half-duplex cooperation constraint is imposed by the sources. Under the half-duplex constraint, each source cannot transmit and receive at the same time. The system can operate either in a mode where one source transmits and the other listens or in a mode where the two sources transmit together without mutual communication. A transmission frame can thus be divided into three phases: in the first τ_1 portion of a frame time, source 1 transmits while source 2 listens. In the next τ_2 portion of the frame, source 2 transmits while source 1 listens. In the remaining $(1 - \tau_1 - \tau_2)$ portion of the frame, both sources transmit. In the following, we derive a DMT upper bound and a lower bound. The two bounds are shown to be coincided. The optimal DMT of the channel is thus established.

6.4.1 Upper Bound

To find a tight DMT upper bound, a tight capacity upper bound is needed. We find that the existing capacity upper bounds, such as the ones proposed in [142] and [143], are not tight enough to meet this end. In the following theorem, we propose a new capacity upper bound for the two-user Gaussian IFC with half-duplex source cooperation. Our analysis shows that with this capacity upper bound, we are able to find a tight DMT upper bound that can be achieved in practice.

Theorem 6.4.1. *The capacity of the two-user single-antenna Gaussian IFC with half-duplex source cooperation is contained in the following region*

$$R_1 \leq \tau_1 \log(1 + \text{SNR}(|h_{1,2}|^2 + |h_{1,3}|^2)) + (1 - \tau_1 - \tau_2) \log(1 + \text{SNR}|h_{1,3}|^2) \quad (6.4.1a)$$

$$R_1 \leq \tau_1 \log(1 + \text{SNR}|h_{1,3}|^2) + \tau_2 \log(1 + \text{SNR}|h_{2,3}|^2) \\ + (1 - \tau_1 - \tau_2) \log(1 + \text{SNR}(|h_{1,3}| + |h_{2,3}|)^2) \quad (6.4.1b)$$

$$R_2 \leq \tau_2 \log(1 + \text{SNR}(|h_{2,1}|^2 + |h_{2,4}|^2)) + (1 - \tau_1 - \tau_2) \log(1 + \text{SNR}|h_{2,4}|^2) \quad (6.4.1c)$$

$$R_2 \leq \tau_1 \log(1 + \text{SNR}|h_{1,4}|^2) + \tau_2 \log(1 + \text{SNR}|h_{2,4}|^2) \\ + (1 - \tau_1 - \tau_2) \log(1 + \text{SNR}(|h_{1,4}| + |h_{2,4}|)^2) \quad (6.4.1d)$$

$$R_1 + R_2 \leq \tau_1 \log(1 + \text{SNR}(|h_{1,3}|^2 + |h_{1,4}|^2)) + \tau_2 \log(1 + \text{SNR}(|h_{2,3}|^2 + |h_{2,4}|^2)) \\ + (1 - \tau_1 - \tau_2) \log\left(1 + \text{SNR}(|h_{1,3}|^2 + |h_{2,3}|^2 + |h_{1,4}|^2 + |h_{2,4}|^2) \right. \\ \left. + \text{SNR}^2(|h_{1,3}|^2|h_{2,4}|^2 + |h_{1,4}|^2|h_{2,3}|^2 - 2\text{Re}(h_{1,3}h_{1,4}^\dagger h_{2,4}h_{2,3}^\dagger))\right) \quad (6.4.1e)$$

$$\begin{aligned}
R_1 + R_2 \leq & \tau_1 \log \left(1 + \frac{\text{SNR}|h_{1,3}|^2}{1 + \text{SNR}|h_{1,4}|^2} \right) + \tau_2 \log \left(1 + \frac{\text{SNR}|h_{2,3}|^2}{1 + \text{SNR}|h_{2,1}|^2} \right) \\
& + (1 - \tau_1 - \tau_2) \log \left(1 + \text{SNR}|h_{2,3}|^2 + \frac{\text{SNR}|h_{1,3}|^2}{1 + \text{SNR}|h_{1,4}|^2} \right) + \tau_1 \log (1 + \text{SNR}|h_{1,2}|^2) \\
& + \tau_1 \log \left(1 + \frac{\text{SNR}|h_{1,4}|^2}{1 + \text{SNR}|h_{1,2}|^2} \right) + \tau_2 \log \left(1 + \frac{\text{SNR}|h_{2,4}|^2}{1 + \text{SNR}|h_{2,3}|^2} \right) \\
& + (1 - \tau_1 - \tau_2) \log \left(1 + \text{SNR}|h_{1,4}|^2 + \frac{\text{SNR}|h_{2,4}|^2}{1 + \text{SNR}|h_{2,3}|^2} \right) + \tau_2 \log (1 + \text{SNR}|h_{2,1}|^2)
\end{aligned} \tag{6.4.1f}$$

$$\begin{aligned}
R_1 + R_2 \leq & \tau_1 \log (1 + \text{SNR}|h_{1,3}|^2) + \tau_2 \log (1 + \text{SNR}|h_{2,3}|^2) \\
& + (1 - \tau_1 - \tau_2) \log (1 + \text{SNR}(|h_{1,3}|^2 + |h_{2,3}|^2)) + (1 - \tau_1) \log \left(1 + \frac{\text{SNR}|h_{2,4}|^2}{1 + \text{SNR}|h_{2,3}|^2} \right) \\
& + \tau_2 \log \left(1 + \frac{\text{SNR}|h_{2,1}|^2}{1 + \text{SNR}(|h_{2,4}|^2 + |h_{2,3}|^2)} \right)
\end{aligned} \tag{6.4.1g}$$

$$\begin{aligned}
R_1 + R_2 \leq & \tau_1 \log (1 + \text{SNR}|h_{1,4}|^2) + \tau_2 \log (1 + \text{SNR}|h_{2,4}|^2) \\
& + (1 - \tau_1 - \tau_2) \log (1 + \text{SNR}(|h_{1,4}|^2 + |h_{2,4}|^2)) + (1 - \tau_2) \log \left(1 + \frac{\text{SNR}|h_{1,3}|^2}{1 + \text{SNR}|h_{1,4}|^2} \right) \\
& + \tau_1 \log \left(1 + \frac{\text{SNR}|h_{1,2}|^2}{1 + \text{SNR}(|h_{1,3}|^2 + |h_{1,4}|^2)} \right)
\end{aligned} \tag{6.4.1h}$$

Proof. The single rate bounds (6.4.1a)-(6.4.1d) and the sum rate bound (6.4.1e) are based on the cut-set bounds. The sum rate bound (6.4.1f) is derived based on a genie-aided channel upper bound. The sum rate bounds (6.4.1g) and (6.4.1h) are based on the Z-channel upper bound. The detailed proof is provided in Appendix K. $\square$

The DMT upper bound is derived from the capacity upper bound in Theorem 6.4.1. We find that the single-rate bounds (6.4.1a)-(6.4.1d) and the sum rate bounds (6.4.1g)-(6.4.1h) are enough to provide a tight DMT upper bound. The resultant DMT upper bound is presented in the following theorem.

Theorem 6.4.2. *The DMT of the two-user fading IFC with static half-duplex source cooperation is upper bounded by*

$$d^{HD}(r) = \begin{cases} 2 - 3r & \text{if } 0 \leq r \leq \frac{1}{3} \\ \frac{(1-2r)^+}{\tau^*} & \text{if } \frac{1}{3} < r \leq \frac{5}{12} \\ 4(1 - 2r)^+ & \text{if } \frac{5}{12} < r \leq \frac{1}{2}, \end{cases} \tag{6.4.2}$$

where $\tau^* = \frac{3-5r-\sqrt{(5r-3)^2-4(1-2r)}}{2}$.

Figure 6.5: DMT results for the IFC with full-duplex source cooperation, static half-duplex source cooperation and dynamic half-duplex source cooperation.

Proof. Please refer to Appendix L. □

Note that the capacity upper bound (6.4.1) applies for both the static and dynamic half-duplex source cooperation, while the DMT upper bound in 6.4.2 is for the static half-duplex source cooperation. For the dynamic half-duplex source cooperation where the time-sharing coefficients τ_1 and τ_2 are adapted according to the channel conditions, a DMT upper bound also can be derived using the capacity upper bound (6.4.1). This DMT upper bound for dynamic half-duplex cooperation should lie in between the full-duplex DMT (6.3.1) and the static half-duplex DMT upper bound (6.4.2). Since under dynamic half-duplex source cooperation, τ_1 and τ_2 will be functions of the channel coefficients $h_{i,j}$ ($i = 1, 2$ and $j = 3, 4$), it is hard to derive an exact DMT upper bound expression. Instead, numerical methods can be applied. To do this, we adjust the τ_i ($i = 1, 2$) values for each set of channel realizations to prevent outage. Consequently, outage happens only when no such τ_i found. In Fig.6.5, the DMT upper bounds under the static and the dynamic half-duplex source cooperation and the DMT of the full-duplex cooperation are plotted. It can be seen that while in low multiplexing rates, dynamic half-duplex cooperation achieves a higher DMT upper bound than static half-duplex source cooperation, for higher multiplexing rates (i.e., $r > 1/3$), dynamic and static half-duplex source cooperation share the same upper bound. This observation is important since it facilitates the design of practical cooperation schemes that achieve the

DMT upper bound in (6.4.2). To be more specified, for multiplexing rate $r > 1/3$, the designed cooperation scheme could not be restricted to static half-duplex cooperation. A dynamic half-duplex cooperation scheme, which allows a more flexible cooperation, is also applicable.

In the next section, we design practical cooperation schemes to show the achievability of the DMT upper bound (6.4.2). For $r \in [0, 1/3]$, the designed cooperation scheme follows strict static half-duplex cooperation. For $r \in (1/3, 1/2]$, dynamic half-duplex cooperation is allowed. Since in the multiplexing rate region $r \in (1/3, 1/2]$, static and dynamic half-duplex cooperation have a common DMT upper bound, by showing the achievability of the DMT upper bound in this region, we prove the optimal DMT of the channel that allows dynamic half-duplex cooperation. Therefore, in the multiplexing rate region $r \in [0, 1/3]$, the DMT expression in (6.4.2) gives us the optimal DMT of the channel under static half-duplex source cooperation, while in the multiplexing rate region $r \in (1/3, 1/2]$, the optimal DMT expression (6.4.2) applies to channels with dynamic half-duplex cooperation.

6.4.2 Achievable Scheme

In the proposed scheme, depending on the targeted rate and the channel scenarios indicated by the feedback bits, different transmission schemes are selected. In the following, we first give a sketch of the transmission schemes proposed, and then specify the applications of the schemes under different targeted rates and different channel conditions. To facilitate illustration, we denote the two sources as source A and B and their respective destinations as destination A and B .

Scheme I: This transmission scheme is illustrated in Table 6.1. For each frame, transmission completes in two phases. The first phase occupies t_1 portion of the frame time and the second phase occupies t_2 portion of the frame time with $t_1 + t_2 = 1$. The values of t_1 and t_2 are chosen based on the individual channel conditions, and will be specified later. Source A divides its message into two parts and encodes into two Gaussian codewords $x_{A1}^{(1)}$ and x_{A2} at rates r_{A1} and r_{A2} , respectively. Source B encodes its message into a Gaussian codeword x_B at rate r . In the t_1 time portion, source A transmits $x_{A1}^{(1)}$. Source B simply listens to source A . In the t_2 time portion, source A re-encodes the message carried by $x_{A1}^{(1)}$ into another Gaussian codeword $x_{A1}^{(2)}$ and transmits it on top of x_{A2} . The total transmission power of source A is equally shared by the two codewords. Source B decodes $x_{A1}^{(1)}$, re-encodes it into $\tilde{x}_{A1}^{(2)}$ and transmits it together with its own codeword x_B . $\tilde{x}_{A1}^{(2)}$ and x_B equally share the transmission power of source B . Joint backward decoding is performed by each destination to decode the messages from both sources.

Scheme II: We illustrate this scheme in Table 6.2. A transmission frame is divided

Table 6.1: Illustration of the transmission scheme I

	t_1	t_2
Source A	$x_{A1}^{(1)}$	$x_{A1}^{(2)}, x_{A2}$
Source B		$\tilde{x}_{A1}^{(2)}, x_B$

Table 6.2: Illustration of the transmission scheme II

	t_1	t_2	t_3
Source A	$x_{A1}^{(1)}, \tilde{x}_{B1}^{(1)}$		$x_{A1}^{(3)}, \tilde{x}_{B1}^{(3)}, x_{A2}$
Source B		$x_{B1}^{(2)}$	$\tilde{x}_{A1}^{(3)}, x_{B2}$

into three phases, occupying t_1 , t_2 and t_3 portions of the frame time, respectively, with $t_1 + t_2 + t_3 = 1$. Source A divides its message into two parts, and encodes into Gaussian codewords $x_{A1}^{(1)}$ and x_{A2} with rates r_{A1} and r_{A2} , respectively. Source B also divides its message into two parts, and encodes into Gaussian codewords $x_{B1}^{(2)}$ and x_{B2} with rates r_{B1} and r_{B2} , respectively. The exact value for the coding rates depends on the targeted rates and the channel conditions and will be specified later. In the t_1 portion of a frame time, source A decodes the signal it received during the t_2 time portion of the previous frame, re-encodes it into $\tilde{x}_{B1}^{(1)}$ and transmits it together with its own codeword $x_{A1}^{(1)}$. The transmission power allocated for $x_{A1}^{(1)}$ and $\tilde{x}_{B1}^{(1)}$ are $\text{SNR} - \text{SNR}^\eta$ and SNR^η , respectively. The value of η is chosen based on the channel conditions and will be specified later. Source B does not transmit in the t_1 time portion. In the t_2 portion of a frame time, source B transmits $x_{B1}^{(2)}$. Source A does not transmit and simply listens. In the t_3 portion of a frame time, both sources transmit. Source A decodes $x_{B1}^{(2)}$, re-encodes it into $\tilde{x}_{B1}^{(3)}$ and transmits it together with $x_{A1}^{(3)}$ and x_{A2} , where $x_{A1}^{(3)}$ is a re-encoded Gaussian codeword carrying the same message as $x_{A1}^{(1)}$. The transmission power allocated to $x_{A1}^{(3)}$, x_{A2} and $\tilde{x}_{B1}^{(3)}$ are $(\text{SNR} - \text{SNR}^\eta)/2$, $(\text{SNR} - \text{SNR}^\eta)/2$, and SNR^η , respectively. Source B decodes $x_{A1}^{(1)}$, re-encodes it into $\tilde{x}_{A1}^{(3)}$ and re-transmits it together with its own codeword x_{B2} . The transmission power of source B is equally shared by $\tilde{x}_{A1}^{(3)}$ and x_{B2} . At the receiver side, both destinations perform backward decoding. Destination A jointly decodes its own message and the message carried by x_{B2} from the signal received during the t_1 and t_3 time portions, treating $\tilde{x}_{B1}^{(1)}$ and $\tilde{x}_{B1}^{(3)}$ as interference. Destination B first jointly decodes the message of source A and its own message carried by x_{B2} from the signal received during the t_1 and t_3 time portions. It then decodes the remaining part of its message from $x_{B1}^{(2)}$, $\tilde{x}_{B1}^{(1)}$, and $\tilde{x}_{B1}^{(3)}$. Therefore, source B decodes messages from both sources. Source A only decodes part of the message from source B. The message part carried by $x_{B1}^{(2)}$ can be viewed as a private message for destination B.

Scheme III: This scheme is similar to the two-level coding scheme proposed in [120], the difference is in the power and rate allocation. For the sake of completeness, a brief description of the scheme is presented in the following. Source A encodes its message into a Gaussian code x_A . Source B divides its message into two parts and encodes into two Gaussian codes x_{Bw} and x_{Bu} with rates r_{Bw} and r_{Bu} , respectively. The transmission power allocated for x_{Bw} and x_{Bu} are $\text{SNR} - \text{SNR}^\eta$ and SNR^η , respectively. The values for the rates and the parameter η will be specified later based on the channel conditions. Destination A jointly decodes x_A and x_{Bw} , treating x_{Bu} as interference. Destination B first jointly decodes x_A and x_{Bw} , and then decodes x_{Bu} . The message carried by x_{Bu} can be viewed as a private message for destination B .

Scheme IV: With this scheme, the two sources simply transmit together using one-level Gaussian codes x_1 and x_2 without any form of cooperation. The messages of both sources are jointly decoded by each destination.

Depending on the targeted rate and the channel conditions provided by the feedback bits, one of the schemes I-IV is applied by the sources. We divide the multiplexing gain region into the following three segments: 1) $r \in [0, 1/3]$, 2) $r \in (1/3, 5/12]$, and 3) $r \in (5/12, 1/2]$. The same feedback bits as defined in (6.3.2) are used to indicate the magnitude of the fading links. For $i = 1, 2, j = 1, 2, j \neq i$, we further define

$$J_j = \begin{cases} 0 & \text{if } b_{i,j+2}^{(0)} = 0, b_{j,j+2}^{(0)} = 0, \\ 1 & \text{if } b_{i,j+2}^{(0)} = 1, b_{j,j+2}^{(0)} = 0, \\ 2 & \text{if } b_{i,j+2}^{(0)} = 0, b_{j,j+2}^{(0)} = 1, \\ 3 & \text{if } b_{i,j+2}^{(0)} = 1, b_{j,j+2}^{(0)} = 1. \end{cases} \quad (6.4.3)$$

The transmission schemes adopted under the different rates and different channel scenarios are described as follows.

When $\beta_c > (1 - 2r)^+$, the two sources transmit together without cooperation, using the transmission scheme III. As shown in Appendix M, the DMT upper bound is not determined by this channel scenario. In the following, we focus on the cases where $0 \leq \beta_c \leq (1 - 2r)^+$.

i.) $r \in [0, 1/3]$

a) When $J_i = 2$ and $J_j = 0$, depending on the value of β_c , we have the following scheme.

When $0 \leq \beta_c \leq (1 - 3r)^+$, a variation of scheme II is adopted. We let $A = i$, $B = j$, i.e., source node i takes the role of source A and source node B takes the role of source B . The values of the time portions are chosen as $t_1 = t_2 = t_3 = 1/3$. We choose $x_{i1}^{(1)} = x_i^{(1)}$, $x_{i2} = 0$, $x_{i1}^{(3)} = x_i^{(3)}$, $x_{j1}^{(2)} = x_j^{(2)}$, $x_{j2} = 0$, $\tilde{x}_{j1}^{(1)} = 0$, $\tilde{x}_{j1}^{(3)} = \tilde{x}_j^{(3)}$, $\tilde{x}_{i1}^{(3)} = \tilde{x}_i$, $r_{i1} = r$, $r_{i2} = 0$, $r_{j1} = r$, and $r_{j2} = 0$. In another word, each source encodes their message into a single codeword. The channel condition $0 \leq \beta_c \leq (1 - 3r)^+$ allows the two sources to fully decode each other. The

messages of source j are all treated as private messages to the corresponding destination of source i , node $i + 2$. The value of η is chosen as $\eta = 1 - r$.

When $(1 - 3r)^+ < \beta_c \leq (1 - 2r)^+$, scheme II is used with the following specifications. We let $A = i$ and $B = j$, and choose $t_1 = t_2 = t_3 = 1/3$, $\tilde{x}_{j1}^{(1)} = 0$, $r_{i1} = (1 - \beta_c)^+/3$, $r_{i2} = r - r_{i1}$, $r_{j1} = (1 - \beta_c)^+/3$ and $r_{j2} = r - r_{j1}$. By this choice, source i does not help relaying the message of source j in the first t_1 time portion. η is chosen as $\eta = 2(1 - 2r)^+ - 2\beta_c$.

b) When $J_i = 3$ and $J_j = 0$, the transmission scheme adopted is very similar to the one for the channel scenario $J_i = 2$, and $J_j = 0$. The transmission is also arranged according to the value of β_c . The only difference is in the choice of the power parameter η . When $0 \leq \beta_c \leq (1 - 3r)^+$, η is chosen as $\eta = r$. When $(1 - 3r)^+ < \beta_c \leq (1 - 2r)^+$, η is chosen as $\eta = 3(1 - 2r)^+/2 - \beta_c/2$.

c) For all the other channel scenarios, scheme IV is used by the sources. Since for the scheme IV, the sources keep transmitting in the whole time block, it can be viewed as a transmission scheme with time-sharing $t_1 = t_2 = t_3 = 1/3$ and each source transmitting all the time.

ii.) $r \in (5/12, 1/2]$

a) When $J_i = 2$ and $J_j = 0$, apart from $b_{i,j+2}^{(0)}$, one more bit is needed to represent the magnitude of each link between the sources and the destinations. We define

$$b_{m,n+2}^{(1)} = \begin{cases} 0 & \text{if } 0 \leq \beta_{m,n+2} \leq \frac{1}{2}(1 - 2r)^+ \\ 1 & \text{if } \frac{1}{2}(1 - 2r)^+ < \beta_{m,n+2} \leq (1 - 2r)^+, \end{cases} \quad (6.4.4)$$

for $m = 1, 2$ and $n = 1, 2$. Depending on the values of $b_{m,n+2}^{(1)}$, the transmission scheme is depicted as follows.

When any one of $b_{i,j+2}^{(1)}$, $b_{j,i+2}^{(1)}$, $b_{j,j+2}^{(1)}$ is 1, scheme IV is applied.

When all of $b_{i,j+2}^{(1)}$, $b_{j,i+2}^{(1)}$, $b_{j,j+2}^{(1)}$ are 0, scheme I is adopted with $A = i$, $B = j$, $t_1 = 1/6$ and $t_2 = 5/6$. The rates are chosen as $r_{i1} = (1 - \beta_c)/6$ and $r_{i2} = r - r_{i1}$.

b) When $J_i = 3$ and $J_j = 0$, we still need one more bit as defined by $b_{i,j}^{(1)}$ in (6.4.4) to represent the channel fading magnitude.

If $b_{j,j+2}^{(1)} = 0$, scheme III is used with $A = i$, $B = j$, $\eta = 3(1 - 2r)^+/2 - \beta_c/2$, $r_{jw} = 1/2 - \eta$ and $r_{ju} = r - r_{jw}$.

If $b_{j,j+2}^{(1)} = 1$, scheme III is used with $A = i$, $B = j$, $\eta = (1 - r)/2$, $r_{jw} = \eta$ and $r_{ju} = r - r_{jw}$.

c) When $J_i = 3$ and $J_j = 1$, we use scheme III by setting $A = i$, $B = j$, $\eta = 3(1 - 2r)^+/2 - \beta_c/2$, $r_{jw} = 1 - \eta - r$ and $r_{ju} = 1 - r_{jw}$.

d) When $J_i = 3$ and $J_j = 2$, scheme II is applied with the setting $A = i$, $B = j$, $t_1 = t_2 = t_3 = 1/3$, $\eta = 3(1 - 2r)^+/2 - \beta_c/2$, $r_{i1} = (1 - \beta_c)^+/3$, $r_{i2} = r - r_{i1}$, $r_{j1} = (1 - \beta_c)^+/3$

and $r_{j2} = r - r_{j1}$.

e) For all the other channel scenarios, simply use scheme IV will ensure the achievement of the DMT upper bound.

iii.) $r \in (1/3, 5/12]$

In this region, the transmission scheme requires at most 3 bits to represent the fading magnitude of each link.

a) When $J_i = 2$ and $J_j = 0$, the following scheme is used.

If anyone of $\beta_c, \beta_{j,i+2}, \beta_{i,j+2}, \beta_{j,j+2}$ is no less than $\frac{\tau^*}{(1-2\tau^*)}(1-2r)^+$, then the two transmitters simply transmit together using one-level Gaussian coding without cooperation.

If $0 \leq \beta_{j,i+2} \leq \frac{1}{2t_2^*-1}(1 - \frac{t_1^*}{\tau^*})(1-2r)^+$ with $t_2^* = \frac{\tau^*(1-r)}{1-2r}$ and $t_1^* = 1 - t_2^*$, scheme I is used by choosing $A = i$, $B = j$, $t_1 = t_1^*$, $t_2 = t_2^*$, $r_{i1} = t_1(1 - \beta_c)^+$, and $r_{i2} = r - r_{i1}$.

If $\beta_{j,i+2} \in (\frac{1}{2t_2^*-1}(1 - \frac{t_1^*}{\tau^*})(1-2r), \frac{\tau^*}{1-2\tau^*}(1-2r))$, scheme II is used by setting $A = i$ and $B = j$. To determine the time partition, we further divide the interval $(\frac{1}{2t_2^*-1}(1 - \frac{t_1^*}{\tau^*})(1-2r), \frac{\tau^*}{1-2\tau^*}(1-2r))$ into the following three intervals: i) $(\frac{1}{2t_2^*-1}(1 - \frac{t_1^*}{\tau^*})(1-2r), \frac{1}{2} \frac{\tau^*}{1-2\tau^*}(1-2r)]$, ii) $(\frac{1}{2} \frac{\tau^*}{1-2\tau^*}(1-2r), \frac{2}{3} \frac{\tau^*}{1-2\tau^*}(1-2r)]$, and iii) $(\frac{2}{3} \frac{\tau^*}{1-2\tau^*}(1-2r), \frac{\tau^*}{1-2\tau^*}(1-2r))$. Feedback bits are used to indicate which interval $\beta_{j,i+2}$ falls in. Based on this information, the values of the time portions are determined accordingly. The detailed method of choosing the values of t_1, t_2 and t_3 is presented at the end of Appendix E. The value of η is chosen as $\eta = \frac{1-2r}{t_1+t_3} - \frac{t_2}{t_1+t_3}\beta_c$. The rates are $r_{i1} = t_1(1 - \beta_c)^+$, $r_{i2} = r - r_{i1}$, $r_{j1} = t_2(1 - \beta_c)^+$ and $r_{j2} = r - r_{j1}$.

b) When $J_i = 3$, and $J_j = 0$, the transmission scheme is as follows.

If $\beta_c \geq 0.4(1-2r)^+$, or $0 \leq \beta_c < 0.4(1-2r)^+$ and $\beta_{j,i+2} > 1.3(1-2r)^+$, scheme III is used by taking $A = i$, $B = j$, $\eta = \frac{1}{2}(\frac{1-2r}{\tau^*} - \beta_c)$, $r_{jw} = 1 - r - \eta$ and $r_{ju} = r - r_{jw}$.

If $0 \leq \beta_c < 0.4(1-2r)^+$ and $\beta_{j,i+2} \in ((1-2r)^+, 1.3(1-2r)^+]$, scheme II is used by setting $A = i$, $B = j$ and with a proper choice of the time portions based on a further partitioning of the interval of $\beta_{j,i+2}$ into the following three subintervals: i) $((1-2r)^+, 1.1(1-2r)^+]$, ii) $(1.1(1-2r)^+, 1.2(1-2r)^+]$ and iii) $(1.2(1-2r)^+, 1.3(1-2r)^+]$. The detailed method of choosing the values of t_1, t_2 and t_3 is provided at the end of Appendix E. The value of η is chosen as $\eta = \frac{1-2r}{t_1+t_3} - \frac{t_2}{t_1+t_3}\beta_c$. The rates are $r_{i1} = t_1(1 - \beta_c)^+$, $r_{i2} = r - r_{i1}$, $r_{j1} = t_2(1 - \beta_c)^+$ and $r_{j2} = r - r_{j1}$.

c) When $J_i = 3$ and $J_j = 1$, transmission scheme III is used with $A = i, B = j$, $\eta = \frac{1}{2}(\frac{1}{\tau^*} - 1)(1-2r)^+ - \frac{\beta_c}{2}$, $r_{2w} = 1 - r - \eta$, and $r_{2u} = r - r_{2w}$.

d) When $J_i = 3$ and $J_j = 2$, we use the same transmission scheme as in the case $r \in (5/12, 1/2]$ with the corresponding channel condition.

e) For all the other channel scenarios, scheme IV is sufficient to achieve the DMT upper bound.

Based on the DMT analysis of the proposed transmission scheme, we have the following theorem.

Theorem 6.4.3. *The proposed transmission scheme for the symmetric two-user fading IFC with half-duplex source cooperation achieves a DMT given in (6.4.2).*

Proof. Please refer to Appendix M. □

Theorem 6.4.3 states that the DMT upper bound can actually be achieved by the proposed transmission scheme. This proves the tightness of the DMT upper bound.

The basic principle in designing the cooperative transmission scheme is to ensure that the transmission abilities of the links that are in good condition are fully exploited so that the performance loss due to the poor links could be compensated. The impact of this principle is perhaps more salient under the half-duplex constraint due to the even tighter restriction on the use of the transmission resources. Our proposed transmission scheme very well reflects this point.

Scheme I is essentially to let one source spare a part of the transmission time to help relay part of the message of the other source. The main application of scheme I is for scenarios where the direct link between a source and its desired destination, say source 1 and destination node 3, is poor while all the other links are in good condition. In this case, the message from source node 1 may not be correctly received by destination node 3. Therefore, to secure a low error probability for source 1, assistance from source 2 is needed. On the other hand, since the link between source 2 and its corresponding destination node 4 is good, to achieve a low error probability, source 2 may only require part of its transmission resources. This entails source 2 the ability to help the message transmission of source 1. Moreover, since the links between source 2 and destination node 3 is also good, the information of source 2 can be decoded by destination 3. The main factor that affects the error performance is the poor direct link between source 1 and destination 3 in the sense that even without interference from source 2, a satisfactory low error probability cannot be achieved. Therefore, letting both destinations to fully decode the messages from both sources do not affect the performance. All the messages from source 2 can be made public for destination 3.

In scheme II, sources A and B both help with relaying part of each other's message. The message from source A are public to both destinations, while the message part of source B that is relayed by source A is designed as a private message and is only decodable at destination B . Scheme II is mainly designed for the cases where all the incoming links to a destination node, say node 3, are poor, while the link from the corresponding source (node 1) to the other destination (node 4) is good. In this case, since the links from both sources to destination 3 are poor, requiring destination 3 to fully decode both sources could result in a poor error performance. To achieve a better performance, we must relax the decoding requirement at destination 3 by asking only partial decoding of source 2's message. This means that part of the message from source 2 should be made private to destination

4 and cannot be decoded by destination 3. Also, since the direct link between source 1 and destination 3 is poor, it is preferable for source 2 to help the transmission of source 1. On the other hand, since the link between source 1 and destination node 4 is good, this link should be made best use to help convey the information of source 2, i.e., source 1 should also help to relay the message of source 2. Due to the poor link quality between source 1 and destination 3, we can choose a proper transmission power for the message relayed by source 1 such that it can be made blind to destination 3 or at least cause little interference. This motivates the idea of making the part of message of source 2 that is relayed by source 1 a private message that could not be decoded by destination 3.

Scheme III also lets one source divide its message into a public part and a private part, as done in scheme II, but does not conduct source cooperation. Each destination should decode its desired messages solely from the signal transmitted from its corresponding source. Scheme III can be used for scenarios where all the incoming links to a destination node, say node 3, are poor, and the link from source 1 to destination 4 is also poor. In this case, due to the poor incoming links to destination node 3, it can only decode part of the message of source 2. The remaining message part of source 2 should be kept private. Moreover, due to the poor link quality between source 1 and destination 4, source 1 could not help relaying the message of source 2 without jeopardizing the reception of its own message at destination 3. But fortunately, thanks to the good quality of the direct link between source 2 and destination 4 (which is an underlining assumption, since if not, then all the four links are in poor condition, which has a very low occurrence probability and will not affect the overall outage behavior), destination node 4 can fully decode the message from source 2 without the help of source 1 even when source 2 uses two-level Gaussian coding and transmit the private message part at a low power level.

Another feature of the proposed transmission scheme is that for the middle-to-high rate region $r \in (1/3, 5/12]$, more feedback bits are needed to achieve the DMT optimal behavior. With the one bit of feedback, we are actually partitioning the channel fading magnitude into two intervals. We find that this partition is sufficient to come up with a DMT optimal transmission scheme for most channel scenarios in the low and high rate region. However, for the middle-to-high rate region, it is very hard to design a DMT optimal transmission scheme with such little knowledge about the channel fading. We thus proceed to further partition the magnitude of some of the links into 6 intervals, as suggested in the description of the transmission scheme, which requires 3 bits to represent. A more refined partition for the fading links gives a more precise knowledge about the channel fading at the transmitter side. With these information, the DMT optimal transmission scheme can finally be designed in this rate region.

Figure 6.6: Comparison between the DMT results for the symmetric two-user fading IFC without source cooperation, with full-duplex (FD) source cooperation, with half-duplex (HD) source cooperation, with strict time-sharing (TS) half-duplex source cooperation and with the orthogonal AF scheme proposed in [28] (LTW-AF).

6.4.3 Reflections

By comparing the DMT result under the half-duplex constraint with that of under the full-duplex constraint, it can be easily observed that the half-duplex constraint does impair the DMT of the system. This is in contrast with the single-antenna single-relay channel and the two-user cooperative MAC, where for each channel model, the same DMT result holds under both full-duplex and half-duplex relaying constraints. By comparing the DMT of the channel without source cooperation, we see that even with half-duplex constraint, source cooperation can still help to improve the system performance over all multiplexing gains. The DMTs of the two-user IFC without cooperation, with full-duplex source cooperation and with half-duplex source cooperation are compared in Fig. 6.6. The advantage of source cooperation in terms of DMT can be easily observed.

An alternative cooperation strategy under the half-duplex constraint is to let the sources transmit in a strictly time-sharing way, i.e., one source transmits in τ_1 portion of a frame time, the other source transmits in the remaining $\tau_2 = 1 - \tau_1$ portion of a frame time. When transmitting, each source transmits the message it received from the other source during the listening period as well as its own message. Essentially, the cooperation scheme with the strict time-sharing source transmission is a special case for the general three-phase

transmission scheme. It can be obtained by simply omitting the last phase in the three-phase transmission scheme. Based on the capacity cut-set upper bound, we find the DMT upper bound for systems operating under such a cooperation scheme, which is given in the following theorem.

Theorem 6.4.4. *The DMT upper bound for the symmetric two-user fading IFC with the strict time-sharing half-duplex source cooperation is*

$$d^{HD-TS}(r) = 2(1 - 2r)^+. \quad (6.4.5)$$

Proof. The proof is presented in Appendix N. □

This DMT upper bound coincides with the DMT achieved by the incremental relay scheme proposed in [28]. Therefore, (6.4.5) is the optimal DMT of the channel under the strict time-sharing half-duplex source cooperation. The incremental relay scheme is DMT optimal. In Fig. 6.6, the DMT curves of the strict time-sharing cooperation and the DMT curve achieved by the AF-based orthogonal source cooperation scheme proposed by Laneman, Tse and Wornell in [28] are also plotted. The inferior performance of the strict time-sharing kind of cooperation scheme and the orthogonal AF scheme in terms of DMT is obvious. In the high multiplexing gain region, these schemes lead to DMTs that are even worse than that of without cooperation.

6.5 Conclusion

In this chapter, we have investigated the DMT behavior of the symmetric two-user interference channel with source cooperation. We first assumed full CSIT, full-duplex cooperation between the sources and a general channel scenario, and derived the DMT upper and lower bounds of the channel. It has been shown via numerical simulation that the DMT upper and lower bounds meet for most channel scenarios. We then assumed only partial CSIT and a channel scenario that the long-term fading effects of all the links in the channel are of the same order. Both the full-duplex and the half-duplex source cooperation were considered. The DMT behavior of the channel was fully determined for this channel setting. Under the full-duplex cooperation constraint, we proposed a DMT optimal transmission scheme that achieves the derived DMT upper bound. The proposed transmission scheme only requires limited feedback about the fading magnitude of the channel from the destinations. Under the half-duplex cooperation, both the DMT upper and lower bounds were derived and shown to be tight. The upper bound was derived based on a proposed capacity upper bound. The

lower bound was derived based on a proposed DMT optimal transmission scheme. The proposed transmission scheme again requires partial CSIT that is obtained from destination feedback. From the DMT results, we showed that in the whole multiplexing gain region, the diversity performance of the channel can be greatly improved by source cooperation even under the half-duplex cooperation constraint. The proposed transmission schemes offer ways of fully exploiting the potential of source cooperation.

The DMT evaluation in this chapter focused on single-antenna IFC. For the multiple-antenna IFC with source cooperation, as discussed in the previous chapters, DF-based cooperation may limit the diversity benefits introduced by user cooperation. To achieve a better DMT performance, CF-based cooperation scheme such as the one we proposed in Chapter 5 might be required. Evaluation of the optimal DMT of the multiple-antenna IFC with source cooperation and the achievable DMT of our proposed combined CF and DF cooperation scheme is an interesting future research direction.

Chapter 7

Conclusion and Future work

7.1 Conclusion

In this thesis, we focused on evaluating and exploiting the potentials of user cooperation in improving the spectrum efficiency and error performance of wireless communication networks, such as the single-user MIMO multihop relay channel, the multiple-user cooperative multiple-access channel and the two-user interference channel with source cooperation. Achievable rate region and diversity-multiplexing tradeoff were used as the main performance metrics.

For the MIMO multihop relay channel, by exploiting the multipath nature of the relay channel, we proposed an AF-based space-time relay scheme and showed that the optimal DMT of the channel can be achieved by the proposed relay scheme given proper design of space-time codes at the source. The proposed relay scheme fills in the gap between the DMT upper bound and the achievable DMTs of the channel proposed in the literature and provides a practical scheme for efficient relaying.

For the asymmetric MIMO multiple-user CMAC, we proposed a WZ-CF based relay scheme and showed that it can achieve the optimal DMT of the channel. We showed that proper allocation of transmission resources is a must for the CF-based relay scheme to be DMT optimal. This results highlighted the differences on achieving the optimal DMT of the CMAC and the relay channel. Based on the DMT results, we partitioned the DOF region of the CMAC to reflect the effect of user cooperation on the diversity performance for each targeted rate. This partition of DOF region provides some guidances for designing the most efficient communication scheme for each targeted diversity order and multiplexing rate.

For the two-user IFC with source cooperation, we derived an achievable rate region for the general discrete memoryless channel model and investigated the DMT behavior of the channel under Rayleigh fading. The derived achievable rate region was based on a

cooperation scheme that combines both DF and CF. We proposed a coding scheme that jointly applies rate splitting, block Markov coding, superposition coding, Gel'fand-Pinsker coding and backward decoding. The proposed rate region extends the existing rate regions under some channel scenarios.

The DMT behavior of the symmetric single-antenna two-user IFC with source cooperation was first studied under the full CSIT assumption. We derived the DMT upper and lower bounds of the channel under full-duplex source cooperation and showed that the upper and lower bounds are tight in most of channel scenarios. We then assumed partial CSIT and focused on a channel scenario where the long-term fading effects of all the links are in the same order. Under both full-duplex and half-duplex source cooperation constraints, DMT upper and lower bounds of the channel were derived and shown to be tight. The optimal DMT of the channel was then fully determined. The lower bounds were derived based on our proposed cooperative transmission schemes which require only limited feedback bits from the destinations representing the magnitude of the fading links. Under the full-duplex source cooperation, only one bit of feedback information is needed for each fading link. Under the half-duplex source cooperation, no more than three bits of feedback information are needed. The DMT results showed that even under half-duplex cooperation, the diversity performance of the channel can be notably improved over all the multiplexing rates. The proposed transmission schemes provide some insights on how to achieve the optimal DMT of the channel.

7.2 Future Work

Following the research direction of this thesis, it is interesting to extend the study on achievable rate and the DMT to other basic cooperative network scenarios such as the interference channel with destination cooperation and the interference relay channel. For these two channel models, only some achievable rate results were derived by some recent works [56], [57]–[61]. Further investigation is needed to characterize their capacity regions and the DMTs. Practical cooperation schemes with better achievable rates and DMT behavior also need to be designed.

As fundamental issues in wireless communications, capacity and diversity deserve great future research efforts. Besides some simply and specific networks, the capacity and the DMT behavior of more general and more complicated networks such as the interference network with more than two users are still unknown. Researches in this direction is of great interest both theoretically and practically. For the IFCs, current researches suggested that with more than two users, the capacity approaching coding schemes are quite different from that of the two user case. Techniques such as interference alignment and structured coding

[108]–[111] instead of rate splitting and random Gaussian coding might be needed to achieve the capacity of the multi-user IFCs. Research in this direction is still in its early stage, where only sum rates are investigated. There is still a long way to go to characterized the entire capacity region and the DMT behavior.

Appendix A

Proof of the DMT Upper Bound for the MIMO Multihop Relay Channel with AF-Based Space-Time Relaying

According to [93], [130], the probability distribution $p_{\lambda|\mu^{(N)}}(\lambda|\mu^{(N)})$ can be found as

$$p_{\lambda|\mu^{(N)}}(\lambda|\mu^{(N)}) = \begin{cases} L_{K_N, M_r} |\det(\mathbf{\Omega}_1)| \prod_{i=1}^{K_N} (\mu_i^{(N)})^{K_N - M_r - 1} \lambda_i^{M_r - K_N} \prod_{i < j}^{K_N} \frac{\lambda_i - \lambda_j}{\mu_i^{(N)} - \mu_j^{(N)}} & \text{if } M_r \geq K_N \\ G_{K_N, M_r} |\det(\mathbf{\Omega}_2)| \prod_{i < j}^{K_N} \frac{1}{\mu_i^{(N)} - \mu_j^{(N)}} \prod_{i < j}^{M_r} (\lambda_i - \lambda_j) & \text{if } M_r < K_N, \end{cases} \quad (\text{A.1})$$

where L_{K_N, M_r} and G_{K_N, M_r} are normalization factors, $\mathbf{\Omega}_1 = [e^{-\lambda_j/\mu_i^{(N)}}]_{i,j=1}^{K_N}$, $\mathbf{\Omega}_2 = [\mathbf{\Omega}_{2,1}, \mathbf{\Omega}_{2,2}]$, and

$$\mathbf{\Omega}_{2,1} = \begin{bmatrix} 1 & \mu_1^{(N)} & \cdots & (\mu_1^{(N)})^{K_N - M_r - 1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \mu_{K_N}^{(N)} & \cdots & (\mu_{K_N}^{(N)})^{K_N - M_r - 1} \end{bmatrix}, \quad (\text{A.2})$$

$$\mathbf{\Omega}_{2,2} = \begin{bmatrix} (\mu_1^{(N)})^{K_N - M_r - 1} e^{-\frac{\lambda_1}{\mu_1^{(N)}}} & \cdots & (\mu_1^{(N)})^{K_N - M_r - 1} e^{-\frac{\lambda_{M_r}}{\mu_1^{(N)}}} \\ \vdots & \ddots & \vdots \\ (\mu_{K_N}^{(N)})^{K_N - M_r - 1} e^{-\frac{\lambda_1}{\mu_{K_N}^{(N)}}} & \cdots & (\mu_{K_N}^{(N)})^{K_N - M_r - 1} e^{-\frac{\lambda_{M_r}}{\mu_{K_N}^{(N)}}} \end{bmatrix}. \quad (\text{A.3})$$

Based on the analysis in Section III, $p(\mu_k^{(N)}|\mu^{(N-1)})$ can also be obtained using (A.1). Substituting into (3.3.18), $p(\mu^{(N)}|\mu^{(N-1)})$ can be calculated. Using the same method successively, we can find $p(\mu^{(i)}|\mu^{(i-1)})$ for $i = 2, 3, \dots, N-1$. Since $\mu_k^{(1)}$ satisfies the chi-square

distribution with $2M_t$ degrees of freedom, $p(\boldsymbol{\mu}^{(1)})$ is given by

$$p(\boldsymbol{\mu}^{(1)}) = \frac{K_1!(1/2)^{M_t}}{\Gamma(M_t)} \prod_{k=1}^{K_1} (\mu_k^{(1)})^{M_t-1} \exp\left(-\frac{\sum_{k=1}^{K_1} \mu_k^{(1)}}{2}\right). \quad (\text{A.4})$$

With $p_{\boldsymbol{\lambda}|\boldsymbol{\mu}^{(N)}}(\boldsymbol{\lambda}|\boldsymbol{\mu}^{(N)})$, $p(\boldsymbol{\mu}^{(i)}|\boldsymbol{\mu}^{(i-1)})$ and $p(\boldsymbol{\mu}^{(1)})$, the joint probability distribution $p(\boldsymbol{\lambda}, \boldsymbol{\mu}^{(N)}, \boldsymbol{\mu}^{(N-1)}, \dots, \boldsymbol{\mu}^{(1)})$ can be calculated from (3.3.21).

Define $\beta_k^{(i)} = \log \mu_k^{(i)} / \log \text{SNR}$ for $i = 1, 2, \dots, N$, the joint pdf of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}^{(N)}, \boldsymbol{\beta}^{(N-1)}, \dots, \boldsymbol{\beta}^{(1)}$ can be easily obtained by applying change of variables in (3.3.21). Specifically, we have

$$p(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}) = p_{\boldsymbol{\lambda}|\boldsymbol{\mu}^{(N)}}(\text{SNR}^{-\boldsymbol{\alpha}}|\text{SNR}^{-\boldsymbol{\beta}^{(N)}})(\log \text{SNR})^q \prod_{v=1}^q \text{SNR}^{-\alpha_v}, \quad (\text{A.5})$$

$$p(\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)}) = (\log \text{SNR})^{K_i} p_{\boldsymbol{\mu}^{(i)}|\boldsymbol{\mu}^{(i-1)}}(\text{SNR}^{-\boldsymbol{\beta}^{(i)}}|\text{SNR}^{-\boldsymbol{\beta}^{(i-1)}}) \prod_{k=1}^{K_i} \text{SNR}^{-\beta_k^{(i)}}, \quad (\text{A.6})$$

and

$$p(\boldsymbol{\beta}^{(1)}) = \frac{(1/2)^{M_t} K_1!}{\Gamma(M_t)} (\log \text{SNR})^{K_1} \prod_{k=1}^{K_1} \text{SNR}^{-M_t \beta_k^{(1)}} \exp\left(-\frac{\sum_{k=1}^{K_1} \text{SNR}^{-\beta_k^{(1)}}}{2}\right). \quad (\text{A.7})$$

Since for $\beta_k^{(1)} < 0$, $p(\boldsymbol{\beta}^{(1)})$ decays exponentially with SNR, at the high SNR region, $p(\boldsymbol{\beta}^{(1)})$ exhibits the following asymptotic property

$$p(\boldsymbol{\beta}^{(1)}) \doteq \begin{cases} \text{SNR}^{-E(\boldsymbol{\beta}^{(1)})}, & \text{for } \boldsymbol{\beta}^{(1)} \in \mathcal{R}_{\boldsymbol{\beta}^{(1)}}, \\ \text{SNR}^{-\infty}, & \text{otherwise,} \end{cases} \quad (\text{A.8})$$

where

$$E(\boldsymbol{\beta}^{(1)}) = \sum_{k=1}^{K_1} M_t \beta_k^{(1)} \quad (\text{A.9})$$

and

$$\mathcal{R}_{\boldsymbol{\beta}^{(1)}} = \{0 \leq \beta_1^{(1)} \leq \beta_2^{(1)} \leq \dots \leq \beta_{K_1}^{(1)}\}. \quad (\text{A.10})$$

Applying the results in [93], the asymptotic pdf of $p(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N-1)})$ is given by

$$p(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}) \doteq \begin{cases} \text{SNR}^{-E(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)})}, & \text{for } (\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N)}) \in \mathcal{R}_{\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}}, \\ \text{SNR}^{-\infty}, & \text{otherwise,} \end{cases} \quad (\text{A.11})$$

where

$$E(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}) = \sum_{v=1}^q (M_r+1-v)\alpha_v + \sum_{j=1}^q (j-M_r-1)\beta_j^{(N)} + \sum_{j=1}^q \sum_{v<j} (\alpha_v - \beta_j^{(N)})^+ + \sum_{j=q+1}^{K_N} \sum_{v=1}^q (\alpha_v - \beta_j^{(N)})^+, \quad (\text{A.12})$$

and

$$\mathcal{R}_{\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}} = \{\alpha_1 \leq \dots \leq \alpha_q, \beta_1^{(N)} \leq \dots \leq \beta_{K_N}^{(N)}, \text{ and, } \beta_v^{(N)} \leq \alpha_v, \text{ for } v = 1, 2, \dots, q\}. \quad (\text{A.13})$$

Similarly, the asymptotic pdf of $p(\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)})$, for $i = 2, 3, \dots, N$, can be found as

$$p(\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)}) \doteq \begin{cases} \text{SNR}^{-E(\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)})}, & \text{for } (\boldsymbol{\beta}^{(i)}, \boldsymbol{\beta}^{(i-1)}) \in \mathcal{R}_{\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)}} \\ \text{SNR}^{-\infty}, & \text{otherwise,} \end{cases} \quad (\text{A.14})$$

where

$$E(\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)}) = \sum_{k=1}^{K_i} (\beta_k^{(i)} - \beta_1^{(i-1)}) + \sum_{k=1}^{K_i} \sum_{j=2}^{K_{i-1}} (\beta_k^{(i)} - \beta_j^{(i-1)})^+ \quad (\text{A.15})$$

and

$$\mathcal{R}_{\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)}} = \{\beta_1^{(i-1)} \leq \beta_2^{(i-1)} \leq \dots \leq \beta_{K_{i-1}}^{(i-1)}, \text{ and } \beta_1^{(i-1)} \leq \beta_k^{(i)}, \text{ for } k = 1, 2, \dots, K_i\}. \quad (\text{A.16})$$

Based on the above analysis, the asymptotic pdf of $p(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N-1)}, \dots, \boldsymbol{\beta}^{(1)})$ is

$$p(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N-1)}, \dots, \boldsymbol{\beta}^{(1)}) \doteq \begin{cases} \text{SNR}^{-E(\boldsymbol{\alpha}, \boldsymbol{\beta})}, & \text{for } (\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N-1)}, \dots, \boldsymbol{\beta}^{(1)}) \in \mathcal{R}, \\ \text{SNR}^{-\infty}, & \text{otherwise,} \end{cases} \quad (\text{A.17})$$

where

$$E(\boldsymbol{\alpha}, \boldsymbol{\beta}) = E(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}) + E(\boldsymbol{\beta}^{(N)}|\boldsymbol{\beta}^{(N-1)}) + \dots + E(\boldsymbol{\beta}^{(2)}|\boldsymbol{\beta}^{(1)}) + E(\boldsymbol{\beta}^{(1)}) \quad (\text{A.18})$$

and

$$\mathcal{R} = \mathcal{R}_{\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}} \cap \mathcal{R}_{\boldsymbol{\beta}^{(N)}|\boldsymbol{\beta}^{(N-1)}} \cap \dots \cap \mathcal{R}_{\boldsymbol{\beta}^{(2)}|\boldsymbol{\beta}^{(1)}} \cap \mathcal{R}_{\boldsymbol{\beta}^{(1)}}. \quad (\text{A.19})$$

With $p(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N-1)}, \dots, \boldsymbol{\beta}^{(1)})$, the outage probability given in (3.3.14) can be evaluated as

$$P_T(r \log \text{SNR}) \doteq \int_{\mathcal{A}} p(\boldsymbol{\alpha}) d\boldsymbol{\alpha}$$

$$\begin{aligned}
&= \int \cdots \int_{\mathcal{A} \cap \mathcal{R}} p(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N-1)}, \dots, \boldsymbol{\beta}^{(1)}) d\boldsymbol{\beta}^{(1)} \cdots d\boldsymbol{\beta}^{(N)} d\boldsymbol{\alpha} \\
&\doteq \int \cdots \int_{\mathcal{A} \cap \mathcal{R}} \text{SNR}^{-E(\boldsymbol{\alpha}, \boldsymbol{\beta})} d\boldsymbol{\beta}^{(1)} \cdots d\boldsymbol{\beta}^{(N)} d\boldsymbol{\alpha}.
\end{aligned} \tag{A.20}$$

According to the analysis in [113], at the high SNR region, the integral in (A.20) is dominated by the term with the largest SNR exponent, i.e.,

$$P_T(r \log \text{SNR}) \doteq \text{SNR}^{-d_T(r)}, \tag{A.21}$$

where

$$d_T(r) = \min_{\boldsymbol{\alpha} \in \mathcal{A} \cap \mathcal{R}} E(\boldsymbol{\alpha}, \boldsymbol{\beta}) \tag{A.22}$$

is the DMT formula we want to find. To obtain $d_T(r)$, we need to minimize $E(\boldsymbol{\alpha}, \boldsymbol{\beta})$ among all the $(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N-1)}, \dots, \boldsymbol{\beta}^{(1)}) \in \mathcal{A} \cap \mathcal{R}$. In the following, we illustrate the optimization process for the case where $M_t = K_1 = \cdots = K_N = M_r = K$. For the general cases, it is difficult to minimize $E(\boldsymbol{\alpha}, \boldsymbol{\beta})$ using the same method. An alternative proof for the achievement of the DMT cut-set upper bound is given in Section 3.3.1 for the general case.

When $M_t = K_1 = \cdots = K_N = M_r = K$, $E(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N-1)})$, $E(\boldsymbol{\beta}^{(n)}|\boldsymbol{\beta}^{(n-1)})$'s and $E(\boldsymbol{\beta}^{(1)})$ become

$$E(\boldsymbol{\alpha}|\boldsymbol{\beta}^{(N)}) = \sum_{v=1}^K (K+1-v)\alpha_v + \sum_{j=1}^K (j-K-1)\beta_j^{(N)} + \sum_{j=1}^K \sum_{v < j} (\alpha_v - \beta_j^{(N)})^+, \tag{A.23}$$

$$E(\boldsymbol{\beta}^{(i)}|\boldsymbol{\beta}^{(i-1)}) = \sum_{k=1}^K (\beta_k^{(i)} - \beta_1^{(i-1)}) + \sum_{k=1}^K \sum_{j=2}^K (\beta_k^{(i)} - \beta_j^{(i-1)})^+, \tag{A.24}$$

and

$$E(\boldsymbol{\beta}^{(1)}) = \sum_{k=1}^K K\beta_k^{(1)}. \tag{A.25}$$

Our objective is to minimize $E(\boldsymbol{\alpha}, \boldsymbol{\beta})$ subject to the constraint that $(\boldsymbol{\alpha}, \boldsymbol{\beta}^{(N)}, \dots, \boldsymbol{\beta}^{(1)}) \in \mathcal{R}$. To this end, we implement the optimization process as follows. First, we fix $\boldsymbol{\beta}^{(2)}$, and optimize $\boldsymbol{\beta}^{(1)}$ such that the last two terms of $E(\boldsymbol{\alpha}, \boldsymbol{\beta})$ are minimized. Following that, we go on to minimize the summation of the last three terms by fixing $\boldsymbol{\beta}^{(3)}$ and optimizing $\boldsymbol{\beta}^{(2)}$. During the optimization process, we keep the relationship between $\boldsymbol{\beta}^{(2)}$ and $\boldsymbol{\beta}^{(1)}$ obtained in the first optimization step unchanged, i.e., $\boldsymbol{\beta}^{(1)}$ will also be reduced or increased according to the reduction or increase in $\boldsymbol{\beta}^{(2)}$. This is to ensure that the summation of the last two

terms remains in the minimized state. When the minimized value of the summation of the last three terms is obtained, we can get a set of optimized values of $\beta^{(2)}$ and $\beta^{(1)}$. Applying the similar procedure to the other terms successively, the minimum value of $E(\alpha, \beta)$ can be obtained. In the following, we will describe the optimization process in detail.

For the last two terms of $E(\alpha, \beta)$,

$$E(\beta^{(2)}|\beta^{(1)}) + E(\beta^{(1)}) = \sum_{k=1}^K (\beta_k^{(2)} - \beta_1^{(1)}) + \sum_{k=1}^K \sum_{j=2}^K (\beta_k^{(2)} - \beta_j^{(1)})^+ + \sum_{j=1}^K K\beta_j^{(1)}, \quad (\text{A.26})$$

we start the optimization process with a set of $\beta^{(1)}$ and $\beta^{(2)}$, which satisfies

$$\beta_1^{(1)} < \beta_1^{(2)} < \beta_2^{(2)} < \dots < \beta_K^{(2)} < \beta_2^{(1)} < \dots < \beta_K^{(1)}. \quad (\text{A.27})$$

With this set of $\beta^{(1)}$ and $\beta^{(2)}$, we have

$$E(\beta^{(2)}|\beta^{(1)}) + E(\beta^{(1)}) = \sum_{k=1}^K (\beta_k^{(2)} - \beta_1^{(1)}) + \sum_{j=1}^K K\beta_j^{(1)}. \quad (\text{A.28})$$

With fixed $\beta_k^{(2)}$'s, the value of the right hand side of (A.28) can be reduced by moving each $\beta_j^{(1)}$ to the left of $\beta_k^{(2)}$, i.e., reduce $\beta_j^{(1)}$ such that it becomes smaller than the $\beta_k^{(2)}$'s. Each time $\beta_j^{(1)}$ passes through a $\beta_k^{(2)}$ from right to left, the coefficient of $\beta_j^{(1)}$ will be reduced by one and the coefficient of $\beta_k^{(2)}$ will be increased by one. The value of $E(\beta^{(2)}|\beta^{(1)}) + E(\beta^{(1)})$ in (A.28) will be reduced by $\beta_{j,\text{old}}^{(1)} - (\beta_k^{(2)} - \beta_{j,\text{new}}^{(1)})$, where $\beta_{j,\text{old}}^{(1)}$ denotes the value of $\beta_j^{(1)}$ before this step of reduction and $\beta_{j,\text{new}}^{(1)}$ denotes the value of $\beta_j^{(1)}$ after reduction. Whenever the coefficient of $\beta_j^{(1)}$ becomes non-positive, further reduction of $\beta_j^{(1)}$ will be stopped. Since in this case, any further reduction of $\beta_j^{(1)}$ will lead to an increase in the value of $E(\beta^{(2)}|\beta^{(1)}) + E(\beta^{(1)})$. With this procedure, the minimized value of $E(\beta^{(2)}|\beta^{(1)}) + E(\beta^{(1)})$ can be found as

$$E(\beta^{(2)}|\beta^{(1)}) + E(\beta^{(1)}) = \sum_{k=1}^K K\beta_k^{(2)}, \quad (\text{A.29})$$

which is achieved when

$$0 \leq \beta_1^{(1)} \leq \beta_2^{(1)} \leq \dots \leq \beta_K^{(1)} \leq \beta_1^{(2)} \leq \beta_2^{(2)} \leq \dots \leq \beta_K^{(2)}. \quad (\text{A.30})$$

For notational simplicity, we denote this relationship between $\beta_k^{(1)}$'s and $\beta_k^{(2)}$'s as

$$0 \leq \boldsymbol{\beta}^{(1)} \leq \boldsymbol{\beta}^{(2)}. \quad (\text{A.31})$$

Similar procedure can be applied to minimize the other terms. For fixed values of α_v 's, the minimized value of $E(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is found as

$$E(\boldsymbol{\alpha}, \boldsymbol{\beta}) = \sum_{v=1}^K (K + K - 2v + 1) \alpha_v. \quad (\text{A.32})$$

This optimized value is obtained when

$$0 \leq \boldsymbol{\beta}^{(1)} \leq \boldsymbol{\beta}^{(2)} \leq \dots \leq \boldsymbol{\beta}^{(N-1)} \leq \boldsymbol{\beta}^{(N)} \leq \boldsymbol{\alpha}. \quad (\text{A.33})$$

With (A.32), $d_{\text{out}}(r)$ can then be explicitly calculated as

$$d_T(r) = \min_{\boldsymbol{\alpha} \in \mathcal{A}} \sum_{v=1}^K (K + K - 2v + 1) \alpha_v, \quad (\text{A.34})$$

which is just the DMT formula of a $K \times K$ MIMO system [113]. Therefore, in the case with $M_t = K_1 = \dots = K_N = M_r = K$, the DMT upper bound of the proposed relay scheme coincides with the DMT cut-set upper bound.

Appendix B

Proof of Proposition 3.3.1

Based on the multipath system model given in (3.2.9), the pairwise error probability (PEP) of ML decoding is upper bounded by

$$P(\mathbf{X} \rightarrow \tilde{\mathbf{X}}) \leq \mathbb{E}_{\mathcal{H}_P} \left\{ \exp \left(- \frac{\text{SNR}}{M_t L} \text{tr}(\mathcal{H}_P^\dagger \Delta \mathbf{C}^\dagger \Delta \mathbf{C} \mathcal{H}_P) \right) \right\}. \quad (\text{B.1})$$

Since $\Delta \mathbf{C}^\dagger \Delta \mathbf{C}$ is a Hermitian matrix, it can be decomposed as $\Delta \mathbf{C}^\dagger \Delta \mathbf{C} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^\dagger$. We denote the minimum eigenvalue of $\Delta \mathbf{C}^\dagger \Delta \mathbf{C}$ as η . Since $\text{rank}(\Delta \mathbf{C}) = M_t L$, $\eta > 0$, we have

$$\text{tr}(\mathcal{H}_P^\dagger \mathbf{U} \mathbf{\Lambda} \mathbf{U}^\dagger \mathcal{H}_P) \geq \eta \text{tr}(\mathcal{H}_P^\dagger \mathbf{U} \mathbf{U}^\dagger \mathcal{H}_P) = \eta \text{tr}(\mathcal{H}_P^\dagger \mathcal{H}_P). \quad (\text{B.2})$$

The PEP expression in (B.1) can thus be further upper bounded as

$$P(\mathbf{X} \rightarrow \tilde{\mathbf{X}}) \leq \mathbb{E}_{\mathcal{H}_P} \left\{ \exp \left(- \frac{\eta \text{SNR}}{M_t L} \text{tr}(\mathcal{H}_P^\dagger \mathcal{H}_P) \right) \right\}. \quad (\text{B.3})$$

From (3.2.11), we know that $\mathcal{H}_P^\dagger \mathcal{H}_P$ can be evaluated as

$$\mathcal{H}_P^\dagger \mathcal{H}_P = \mathbf{H}^{(N)} \mathbf{\Sigma}^{(N)} (\mathbf{H}^{(N)})^\dagger, \quad (\text{B.4})$$

which is equal to $\mathcal{H}_T \mathcal{H}^\dagger / T$. Therefore, we have

$$\text{tr}(\mathcal{H}_P^\dagger \mathcal{H}_P) = \frac{1}{T} \sum_{v=1}^q \lambda_v \doteq \frac{1}{T} \sum_{v=1}^q \text{SNR}^{-\alpha_v}. \quad (\text{B.5})$$

Substituting (B.5) into (B.3), we get

$$P(\mathbf{X} \rightarrow \tilde{\mathbf{X}}) \leq \mathbb{E}_\alpha \left\{ \exp \left(- \frac{\eta}{T M_t L} \sum_{v=1}^q \text{SNR}^{1-\alpha_v} \right) \right\}. \quad (\text{B.6})$$

Since for $1 - \alpha_v < 0$, (B.6) decays exponentially with SNR, we only consider the α_v 's that satisfies $1 - \alpha_v > 0$. Defining $\mathcal{L} = \{v : \alpha_v < 1\}$, for a realization of the random vector α , we have

$$\begin{aligned} \sum_{v=1}^q \text{SNR}^{1-\alpha_v} &\geq \sum_{v \in \mathcal{L}} \text{SNR}^{1-\alpha_v} \\ &\geq |\mathcal{L}| \text{SNR}^{\frac{1}{|\mathcal{L}|} \sum_{v \in \mathcal{L}} (1-\alpha_v)} \\ &= |\mathcal{L}| \text{SNR}^{\frac{1}{|\mathcal{L}|} \sum_{v=1}^q (1-\alpha_v)^+}. \end{aligned} \quad (\text{B.7})$$

Applying (B.7) in (B.6), the upper bound of PEP becomes

$$\text{P}(\mathbf{X} \rightarrow \tilde{\mathbf{X}}) \leq \mathbb{E}_{\alpha} \left\{ \exp \left(- \frac{\eta |\mathcal{L}|}{T M_t L} \text{SNR}^{\frac{1}{|\mathcal{L}|} \sum_{v=1}^q (1-\alpha_v)^+} \right) \right\}. \quad (\text{B.8})$$

According to the values of α_i 's, the overall error probability can be calculated as

$$\begin{aligned} \text{P}_e(r \log \text{SNR}) &= \text{Pr}(\text{error}, \alpha \in \mathcal{A}) + \text{Pr}(\text{error}, \alpha \notin \mathcal{A}) \\ &= \text{Pr}(\alpha \in \mathcal{A}) \text{Pr}(\text{error} | \alpha \in \mathcal{A}) + \text{Pr}(\alpha \notin \mathcal{A}) \text{Pr}(\text{error} | \alpha \notin \mathcal{A}) \\ &\leq \text{Pr}(\alpha \in \mathcal{A}) + \text{Pr}(\alpha \notin \mathcal{A}) \text{Pr}(\text{error} | \alpha \notin \mathcal{A}). \end{aligned} \quad (\text{B.9})$$

For any $\alpha \notin \mathcal{A}$, $\sum_{v=1}^q (1 - \alpha_v)^+ > r$. For a code with length T and rate $R = r \log \rho$, the corresponding codebook contains SNR^{Tr} codewords. Following (B.8) and note that $1 \leq |\mathcal{L}| \leq q$, the union bound can be found as

$$\text{Pr}(\text{error} | \alpha \notin \mathcal{A}) \leq \text{SNR}^{Tr} \exp \left(- \frac{\eta}{T M_t L} \text{SNR}^{r/q} \right). \quad (\text{B.10})$$

Therefore, for any $r > 0$, as long as $\eta > \text{SNR}^{-r/q}$, $\text{Pr}(\text{error} | \alpha \notin \mathcal{A})$ decays exponentially with SNR, and we have

$$\text{Pr}(\text{error}, \alpha \notin \mathcal{A}) \leq \text{SNR}^{Tr} \exp \left(- \eta \text{SNR}^{r/q} \right), \quad (\text{B.11})$$

where we use the fact that $\text{Pr}(\alpha \notin \mathcal{A}) \leq 1$. It can be seen that $\text{Pr}(\text{error}, \alpha \notin \mathcal{A})$ decays exponentially with SNR for $r > 0$ and $\eta > \text{SNR}^{-r/q}$.

Since $\text{Pr}(\alpha \in \mathcal{A}) \doteq \text{SNR}^{-d_T(r)}$, we obtain

$$\text{P}_e(r \log \text{SNR}) \leq \text{SNR}^{-d_T(r)}. \quad (\text{B.12})$$

Let

$$P_e(r \log \text{SNR}) \doteq \text{SNR}^{d_e(r)}, \quad (\text{B.13})$$

from (B.12), it can be seen that

$$d_e(r) \geq d_T(r). \quad (\text{B.14})$$

Since $d_T(r)$ achieves the DMT upper bound, we therefore obtain

$$d_e(r) = d_T(r), \quad (\text{B.15})$$

which indicates that the DMT upper bound derived from the channel $\mathcal{H}_T$ can be achieved by applying the considered ST coding scheme at the source and the proposed AF-based space-time relaying.

Appendix C

Evaluating the Values of σ_{W1}^2 and σ_{W2}^2 in the Full-Duplex Two-User CMAc with CF-Based Cooperation

Equations (4.2.13a) and (4.2.13b) impose constraints on the values of σ_{W1}^2 and σ_{W2}^2 , respectively. The constraint (4.2.13c) imposes constraint on both σ_{W1}^2 and σ_{W2}^2 . In this section, we show that the values $|\sigma_{W1}^2 \mathbf{I}_{M1}|$ and $|\sigma_{W2}^2 \mathbf{I}_{M2}|$ calculated using (4.2.13a) and (4.2.13b), respectively, and the value $|\tilde{\sigma}_{W1}^2 \mathbf{I}_{M1}| |\tilde{\sigma}_{W2}^2 \mathbf{I}_{M2}|$ calculated using (4.2.13c) are in the same order in terms of SNR.

Using (4.2.13a), (4.2.13b) and (4.2.13c), the variances of the compression noises can be calculated as

$$\begin{aligned} \log |\pi e \sigma_{W1}^2 \mathbf{I}_{M1}| &\geq h(\hat{\mathbf{y}}_1 | \mathbf{x}_{10}, \mathbf{x}_{11}) - h(\hat{\mathbf{y}}_1 | \mathbf{x}_{11}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \\ &\quad - h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\mathbf{y}_D | \mathbf{x}_{21}) + h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}), \end{aligned} \quad (\text{C.1})$$

$$\begin{aligned} \log |\pi e \sigma_{W2}^2 \mathbf{I}_{M2}| &\geq h(\hat{\mathbf{y}}_2 | \mathbf{x}_{20}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \\ &\quad - h(\hat{\mathbf{y}}_1, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\mathbf{y}_D | \mathbf{x}_{11}) + h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}), \end{aligned} \quad (\text{C.2})$$

and

$$\begin{aligned} \log |\pi e \tilde{\sigma}_{W1}^2 \mathbf{I}_{M1}| |\pi e \tilde{\sigma}_{W2}^2 \mathbf{I}_{M2}| &\geq h(\hat{\mathbf{y}}_1 | \mathbf{x}_{10}, \mathbf{x}_{11}) + h(\hat{\mathbf{y}}_2 | \mathbf{x}_{20}, \mathbf{x}_{21}) - h(\hat{\mathbf{y}}_1 | \mathbf{x}_{11}, \mathbf{x}_{21}) \\ &\quad - h(\hat{\mathbf{y}}_2 | \mathbf{x}_{11}, \mathbf{x}_{21}) + h(\hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\mathbf{y}_D). \end{aligned} \quad (\text{C.3})$$

Based on that, we have

$$\log |\sigma_{W1}^2 \mathbf{I}_{M1}| + \log |\sigma_{W2}^2 \mathbf{I}_{M2}| - \log |\tilde{\sigma}_{W1}^2 \mathbf{I}_{M1}| |\tilde{\sigma}_{W2}^2 \mathbf{I}_{M2}|$$

$$\begin{aligned}
&= h(\hat{\mathbf{y}}_1, \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{12}) - h(\hat{\mathbf{y}}_1, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{12}) - h(\hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{12}) \\
&\quad + 2h(\mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) - h(\mathbf{y}_D | \mathbf{x}_{11}) - h(\mathbf{y}_D | \mathbf{x}_{21}) + h(\mathbf{y}_D) \\
&= -I(\hat{\mathbf{y}}_1; \hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) - I(\mathbf{x}_{11}; \mathbf{y}_D | \mathbf{x}_{21}) + I(\mathbf{x}_{11}; \mathbf{y}_D).
\end{aligned} \tag{C.4}$$

From our following discussions in section 4.2.2, the situation we are interested in is when either $I(\mathbf{x}_{10}; \hat{\mathbf{y}}_1, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \leq r_1 \log \text{SNR}$ or $I(\mathbf{x}_{20}; \hat{\mathbf{y}}_2, \mathbf{y}_D | \mathbf{x}_{11}, \mathbf{x}_{21}) \leq r_2 \log \text{SNR}$ happens. In this case, uniform superposition transmission is used by one of the users, and hence $I(\mathbf{x}_{12}; \mathbf{y}_D | \mathbf{x}_{21}) \doteq I(\mathbf{x}_{12}; \mathbf{y}_D)$. Moreover, since $I(\hat{\mathbf{y}}_1; \hat{\mathbf{y}}_2 | \mathbf{y}_D, \mathbf{x}_{11}, \mathbf{x}_{21}) = 0$, we have

$$\log |\sigma_{W1}^2 \mathbf{I}_{M1}| + \log |\sigma_{W2}^2 \mathbf{I}_{M2}| - \log |\tilde{\sigma}_{W1}^2 \mathbf{I}_{M1}| |\tilde{\sigma}_{W2}^2 \mathbf{I}_{M2}| \doteq 0 \cdot \log \text{SNR}. \tag{C.5}$$

Therefore, both σ_{W1}^2 , σ_{W2}^2 and $\tilde{\sigma}_{W1}^2$, $\tilde{\sigma}_{W2}^2$ can be used in evaluating DMT. The results will be the same.

Appendix D

Proof of Theorem 4.2.1

In this section, we provide an outline of the proof for the rate region of the K -user CMA channel with (partial) decoding performed by each user. The proof uses some basic techniques used in [3]. The error probability analysis is similar to the ones in [3], [32], and [42], and is thus omitted here.

We denote R_{k1} as the information rate of transmitting X_{k1} and $\hat{R}_k$ as the Wyner-Ziv compression rate used by user k . Let $R_{k1} \doteq r_{k1} \log \text{SNR}$, R_{k1} can then be expressed as $R_{k1} = r_{k1} \log \text{SNR} + \tilde{R}_{k1}$, where $\tilde{R}_{k1}$ denotes a constant rate offset independent of SNR. We assume that the codewords are of length n .

Random code generation:

The random codewords of user k are generated in the following way:

i. Generate the codebooks of x_{k1}^n as follows.

(a) Generate $2^{n\tilde{R}_{k1}}$ i.i.d. $x_{k1,0}^n$ according to

$$p(x_{k1,0}^n) = \prod_{t=1}^n p_{X_{k1,0}}(x_{k1,0,t}).$$

Index them as $x_{k1,0}^n(l_{k1,0})$, $l_{k1,0} \in [1, 2^{n\tilde{R}_{k1}}]$.

(b) For each $x_{k1,0}^n$, generate $2^{n \log \text{SNR}}$ i.i.d. $x_{k1,1}^n$ according to

$$p(x_{k1,1}^n | x_{k1,0}^n(l_{k1,0})) = \prod_{t=1}^n p_{X_{k1,1}|X_{k1,0}}(x_{k1,1,t} | x_{k1,0,t}(l_{k1,0})).$$

Index them as $x_{k1,1}^n(l_{k1,1} | l_{k1,0})$, $l_{k1,1} \in [1, 2^{n \log \text{SNR}}]$.

(c) For each pair of $(x_{k1,0}^n, x_{k1,1}^n)$, generate $2^{n \log \text{SNR}}$ i.i.d. $x_{k1,2}^n$ according to

$$p(x_{k1,2}^n | x_{k1,0}^n(l_{k1,0}), x_{k1,1}^n(l_{k1,1})) = \prod_{t=1}^n p_{X_{k1,2}|X_{k1,0}X_{k1,1}}(x_{k1,2,t} | x_{k1,0,t}(l_{k1,0}), x_{k1,1,t}(l_{k1,1})).$$

Index them as $x_{k1,2}^n(l_{k1,2}|l_{k1,0}, l_{k1,1})$, $l_{k1,2} \in [1, 2^{n \log \text{SNR}}]$.

- (d) Continue this process until all the $2^{r_{k1} \log \text{SNR} + \tilde{R}_{k1}}$ codewords of x_{k1}^n have been generated. Each codeword can therefore be represented as $x_{k1}^n(x_{k1,0}^n, x_{k1,1}^n, \dots, x_{k1,r_{k1}}^n)$, with $x_{k1,j}^n$ ($0 < j \leq r_{k1}$) being generated according to $p(x_{k1,j}^n | x_{k1,0}^n, \dots, x_{k1,j-1}^n)$.

Index the generated codewords of X_{k1} as $x_{k1}^n(s_{k1})$, $s_{k1} \in [1, 2^{nR_{k1}}]$.

- ii. For each $x_{k1}^n(s_{k1})$, generate $2^{nR_{k0}}$ i.i.d. x_{k0}^n according to

$$p(x_{k0}^n | x_{k1}^n(s_{k1})) = \prod_{t=1}^n p_{X_{k0}|X_{k1}}(x_{k0,t} | x_{k1,t}(s_{k1})).$$

Index them as $x_{k0}^n(s_{k0}|s_{k1})$, $s_{k0} \in [1, 2^{nR_{k0}}]$.

- iii. For each $x_{k1}^n(s_{k1})$, generate $2^{n\hat{R}_k}$ i.i.d. $\hat{y}_k^n$ according to

$$p(\hat{y}_k^n | x_{k1}^n(s_{k1})) = \prod_{t=1}^n p_{\hat{Y}_k|X_{k1}}(\hat{y}_{k,t} | x_{k1,t}(s_{k1})).$$

Index them as $\hat{Y}_k(z_k | s_{k1})$, $z_k \in [1, 2^{n\hat{R}_k}]$.

- iv. Randomly partition the set $\{1, 2, \dots, 2^{n\hat{R}_k}\}$ into $2^{nR_{k1}}$ cells $\mathcal{F}_{s_{k1}}$, $s_{k1} \in [1, 2^{nR_{k1}}]$.

Encoding:

Assuming that at the end of block $b-1$, with $y_{k,b-1}$ and $x_{k1}(s_{k1,b-1})$, source k determines that $\hat{y}_k(z_{k,b-1})$ was received using the decoding step iv described below, the codewords $x_{k0}(s_{k0,b})$ and $x_{k1}(s_{k1,b})$ for transmitting in the b th block are encoded as follows:

- i. With the knowledge of $z_{k,b-1}$, source k chooses $s_{k1,b}$ so that $z_{k,b-1}$ falls in $\mathcal{F}_{s_{k1,b}}$.
- ii. Let $s_{k0,b}$ be the new message index sent in block b , then $(X_{k0}(s_{k0,b}|s_{k1,b}), X_{k1}(s_{k1,b}))$ will be sent by user k in block b .

Decoding:

At the end of block b , with the received $y_{D,b}$ at destination and the received $y_{k,b}$ at user k , the following decoding steps are implemented.

- i. The destination decodes $\hat{s}_{S1,b}$, so that $(x_{S1}^n(\hat{s}_{S1,b}), y_{D,b}^n)$ is jointly ϵ -typical. This can be done reliably if

$$R_{S1} < I(X_{S1}, X_D | X_{S^c1}) \text{ for all } S \subseteq \mathcal{S}. \quad (\text{D.1})$$

- ii. Assuming that both $s_{S1,b-1}$ and $s_{S1,b}$ have been correctly decoded, the destination finds an ambiguity set $\mathcal{L}(y_{D,b-1})$ of $z_{S,b-1}$ such that $(\{x_{k1}^n(s_{k1,b-1}), \hat{y}_k^n(z_{k,b-1}) : k \in \mathcal{S}\}, y_{D,b-1}^n)$ is jointly ϵ -typical. The destination then decides that $\hat{z}_{S,b-1}$ was sent in

block $(b-1)$ if it finds a unique $\hat{z}_{S,b-1}$ that falls in the set $(\mathcal{F}_{s_{11,b}} \times \mathcal{F}_{s_{21,b}} \times \cdots \times \mathcal{F}_{s_{K,b}}) \cap \mathcal{L}(Y_{D,b-1})$. This can be done reliably if

$$\sum_{k \in S} \hat{R}_k < \sum_{k \in S} H(\hat{Y}_k | X_{S1}) - H(\hat{Y}_S | X_{S1}, \hat{Y}_{S^c}, Y_D) + R_{S1}. \quad (\text{D.2})$$

- iii. Assuming that $s_{S1,b-1}$, $z_{S,b-1}$ have been successfully found, the destination then declares that $x_{S0}^n(\hat{s}_{S0})$ was sent if $(\{x_{k0}^n(s_{k0,b-1}), x_{k1}^n(s_{k1,b-1}), \hat{y}_k^n(z_{k,b-1}) : k \in \mathcal{S}\}, y_{D,b-1}^n)$ is jointly ϵ -typical. This can be done reliably if

$$R_S < I(X_{S0}; \hat{Y}_S, Y_D | X_{S1}, X_{S^c0}) \text{ for all } S \subseteq \mathcal{S}. \quad (\text{D.3})$$

- iv. At source k , upon receiving $y_{k,b}^n$, it does its best to decode $s_{i1}(l_{i1,0}, l_{i1,1}, \dots, l_{i1,r_{i1}})$ for all $i \in \mathcal{S}_{\bar{k}}$ up to a point that

$$I(X_{\mathcal{S}_{\bar{k}}1}^{dc}; Y_k) \doteq I(X_{\mathcal{S}_{\bar{k}}1}; Y_k),$$

and consequently

$$I(X_{\mathcal{S}_{\bar{k}}1}^{ndc}; Y_k | X_{\mathcal{S}_{\bar{k}}1}^{dc}) \doteq 0 \log \text{SNR}, \quad (\text{D.4})$$

obtaining the partially decoded $s_{\mathcal{I}1, \mathcal{J}_{\mathcal{I}}}$, where $\mathcal{I} \subseteq \mathcal{S}_{\bar{k}}$ and for each $i \in \mathcal{I}$, $\mathcal{J}_i \subseteq \{0, 1, \dots, r_{i1}\}$. Together with $s_{k1,b}$ and $y_{k,b}^n$, user k chooses $z_{k,b}$ such that $(x_{k1}^n(s_{k1,b}), x_{k0}^n(s_{k0,b} | s_{k1,b}), x_{\mathcal{I}1, \mathcal{J}_{\mathcal{I}}}^n(l_{\mathcal{I}1, \mathcal{J}_{\mathcal{I}}, b}), \hat{y}_k^n(z_{k,b}), y_{k,b}^n)$ is jointly ϵ -typical. This can be done reliably if

$$\hat{R}_k > I(\hat{Y}_k; Y_k | X_{k0}, X_{k1}, X_{\mathcal{S}_{\bar{k}}}^{dc}). \quad (\text{D.5})$$

Summarizing (D.1)-(D.5), Theorem 4.2.1 follows.

Appendix E

Outage Probabilities of CF-based Half-Duplex Two-User CMAAC

In this section, we provide derivations for the outage probability expressions in (4.3.12) and (4.3.13). For $P(\mathcal{O}_1)$, we have

$$\begin{aligned}
P(\mathcal{O}_1) &= P\left(\tau I\left(\mathbf{x}_{10}^{(1)}; \hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) < r_1 \log \text{SNR}\right) \\
&\leq P\left(h\left(\hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) - h\left(\mathbf{y}_D^{(1)} | \mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) \right. \\
&\quad \left. - h\left(\hat{\mathbf{y}}_2^{(1)} | \mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) < \frac{r_1}{\tau} \log \text{SNR}\right) \\
&\leq P\left(h\left(\hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) - h\left(\mathbf{y}_D^{(1)} | \mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) < \log(2^{M_2} \text{SNR}^{\frac{r_1}{\tau}})\right) \\
&\quad + P\left(h\left(\hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) - h\left(\mathbf{y}_D^{(1)} | \mathbf{x}_{10}^{(1)}, \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) \right. \\
&\quad \left. - \log |\pi e \sigma_{W2}^2 \mathbf{I}_{M_2}| < \log(2^{M_2} \text{SNR}^{\frac{r_1}{\tau}})\right) \tag{E.1}
\end{aligned}$$

$$\begin{aligned}
&\leq P\left(I\left(\mathbf{x}_{10}^{(1)}; \mathbf{y}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}\right) < \frac{r_1}{\tau} \log \text{SNR}\right) \\
&\quad + P\left(\tau I\left(\mathbf{x}_{10}, \mathbf{x}_{11}; \mathbf{y}_{D|1}\right) + (1 - \tau) I\left(\mathbf{x}_{20}^{(2)}, \mathbf{x}_{21}^{(2)}; \mathbf{y}_D^{(2)}\right) \right. \\
&\quad \left. - \tau I\left(\mathbf{x}_{11}^{(1)}; \hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)}\right) - (1 - \tau) I\left(\mathbf{x}_{20}^{(2)}; \mathbf{y}_D^{(2)} | \mathbf{x}_{21}^{(2)}\right) < r_1 \log \text{SNR}\right), \tag{E.2}
\end{aligned}$$

where (E.1) is based on the similar procedure as used in calculating the full-duplex outage probability in (4.2.22). To obtain the last inequality in (E.2), we calculate σ_{W2}^2 by using the constraint in (4.3.11b). The value is

$$\begin{aligned}
\log |\pi e \sigma_{W2}^2 \mathbf{I}_{M_2}| &\geq h\left(\hat{\mathbf{y}}_2^{(1)} | \mathbf{x}_{20}^{(2)}, \mathbf{x}_{21}^{(2)}\right) - h\left(\hat{\mathbf{y}}_2^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) + h\left(\hat{\mathbf{y}}_2^{(1)}, \mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) \\
&\quad - h\left(\mathbf{y}_D^{(1)} | \mathbf{x}_{11}^{(1)}, \mathbf{x}_{21}^{(2)}\right) - \frac{1 - \tau}{\tau} I\left(\mathbf{x}_{21}^{(2)}; \mathbf{y}_D^{(2)}\right). \tag{E.3}
\end{aligned}$$

Substituting (E.3) into (E.2) and using chain rule and some properties such as conditioning reduces entropy, we obtain the outage expression given in (E.2). The outage probability

$P(\mathcal{O}_2)$ can be similarly derived.

Appendix F

Proof of Theorem 5.2.1

Random Code Generation:

In the following, we describe the codebook generation procedure at transmitter 1. The codebooks at transmitter 2 are generated in the same way.

- i. Generate $2^{(nR_{11d}+nR_{12d})}$ i.i.d. x_0^n according to

$$p(x_0^n) = \prod_{t=1}^n p(x_{0,t}).$$

Index them as $x_0^n(w_{11d}^{(p)}, w_{21d}^{(p)})$, $w_{11d}^{(p)} \in [1, 2^{nR_{11d}}]$, $w_{21d}^{(p)} \in [1, 2^{nR_{21d}}]$.

- ii. For each $x_0^n(w_{11d}^{(p)}, w_{21d}^{(p)})$, generate $2^{nR_{11d}}$ i.i.d. x_{11d}^n according to

$$p(x_{11d}^n | x_0^n) = \prod_{t=1}^n p(x_{11d,t} | x_{0,t}(w_{11d}^{(p)}, w_{21d}^{(p)})).$$

Index them as $x_{11d}^n(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$, $w_{11d} \in [1, 2^{nR_{11d}}]$.

- iii. For each pair of $(x_0^n(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d}^n(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}))$, generate $2^{nL_{11}}$ i.i.d. s_{11}^n according to

$$p(s_{11}^n | x_0^n, x_{11d}^n) = \prod_{t=1}^n p(s_{11,t} | x_{0,t}(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d,t}(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})).$$

Index them as $s_{11}^n(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$, $z_{11} \in [1, 2^{nL_{11}}]$.

- iv. For each triple of $(x_0^n(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d}^n(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), s_{11}^n(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}))$, generate $2^{nR_{11c}}$ i.i.d. x_{11c}^n according to

$$\begin{aligned}
& p(x_{11c}^n | x_0^n, x_{11d}^n, s_{11}^n) \\
&= \prod_{t=1}^n p(x_{11c,t} | x_{0,t}(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d,t}(w_{11d}^{(p)}, w_{11d}^{(p)}, w_{21d}^{(p)}), s_{11,t}(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})).
\end{aligned}$$

Index them as $x_{11c}^n(w_{11c}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$, $w_{11c} \in [1, 2^{nR_{11c}}]$.

- v. For each $(x_0^n(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d}^n(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), s_{11}^n(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11c}^n(w_{11c}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}))$ tuple, generate $2^{n(R_{12}+R'_{12})}$ i.i.d. u_{12}^n according to

$$\begin{aligned}
& p(u_{12}^n | x_0^n, x_{11d}^n, s_{11}^n, x_{11c}^n) \\
&= \prod_{t=1}^n p(u_{12,t} | x_{0,t}(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d,t}(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), s_{11,t}(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), \\
&\quad x_{11c,t}(w_{11c}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})).
\end{aligned}$$

Index as $u_{12}^n(l_{12}, w_{11c}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$, $l_{12} \in [1, 2^{n(R_{12}+R'_{12})}]$. Uniformly distribute the u_{12}^n 's into $2^{nR_{12}}$ bins. Denote as $\mathcal{B}_1(w_{12}, w_{11c}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$, $w_{12} \in [1, 2^{nR_{12}}]$.

- vi. For each triple of $(x_0^n(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d}^n(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), s_{11}^n(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}))$, generate $2^{n(L_{11}+L'_{11})}$ i.i.d. v_{12}^n according to

$$\begin{aligned}
& p(v_{12}^n | x_0^n, x_{11d}^n, s_{11}^n) \\
&= \prod_{t=1}^n p(v_{12,t} | x_{0,t}(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d,t}(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), s_{11,t}(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})).
\end{aligned}$$

Index them as $v_{12}^n(k_{12}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$, $k_{12} \in [1, 2^{n(L_{12}+L'_{12})}]$. Uniformly distribute the v_{12}^n 's into $2^{nL_{11}}$ bins. Denote as $\mathcal{C}_1(z_{12}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$, $z_{12} \in [1, 2^{nL_{12}}]$.

- vii. For each $(x_0^n(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d}^n(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), x_{21d}^n(w_{21d}, w_{11d}^{(p)}, w_{21d}^{(p)}), s_{11}^n(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), v_{12}^n(k_{12}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}))$ tuple, generate $2^{\hat{R}_1}$ i.i.d. $\hat{y}_1^n$ according to

$$\begin{aligned}
& p(\hat{y}_1^n | x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, v_{12}^n) \\
&= \prod_{t=1}^n p(\hat{y}_{1,t} | x_{0,t}(w_{11d}^{(p)}, w_{21d}^{(p)}), x_{11d,t}(w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), x_{21d,t}(w_{21d}, w_{11d}^{(p)}, w_{21d}^{(p)}), \\
&\quad s_{11,t}(z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)}), v_{12,t}(k_{12}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})).
\end{aligned}$$

Index them as $\hat{y}_1^n(g_1, k_{12}, z_{11}, w_{11d}, w_{11d}^{(p)}, w_{21d}^{(p)})$. Randomly partition the $2^{\hat{R}_1}$ $\hat{y}_1^n$'s into $2^{n(L_{11}+L_{12})}$ cells, denote the cells as $\mathcal{S}_1(z_1)$, $z_1 \in [1, 2^{n(L_{11}+L_{12})}]$.

Encoding:

We describe the encoding process at transmitter 1 for the b th block. The encoding at transmitter 2 can be similarly performed. We assume that by the end of block $b - 1$, transmitter 1 has decoded $w_{21d,b-1}$ and has found a message index $z_{1,b}$ from the decoding steps 1 and 2 described in the next section. Assuming $w_{1,b}$ is the new message to be transmitted in block b , encoding at transmitter 1 is implemented as follows.

Split the nR_1 bits of $w_{1,b}$ into nR_{11d} bits of $w_{11d,b}$, nR_{11c} bits of $w_{11c,b}$ and nR_{12} bits of $w_{12,b}$. Split the nL_1 bits of $z_{1,b}$ into nL_{11} bits of $z_{11,b}$ and nL_{12} bits of $z_{12,b}$. Using the message $(w_{11d,b-1}, w_{21d,b-1})$ obtained from the $(b - 1)$ th block, select the codeword $x_0^n(w_{11d,b-1}, w_{21d,b-1})$ from the codebook generated beforehand. Based on that, choose a sequence of codewords $x_{11d}^n(w_{11d,b}, w_{11d,b-1}, w_{21d,b-1})$, $s_{11}^n(z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1})$ and $x_{11c}^n(w_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1})$. Following that, choose the codeword pair

$$\begin{aligned} & (u_{12}^n(l_{12,b}, w_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), v_{12}^n(k_{12,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1})) \\ & \in \mathcal{A}_\epsilon \cap (\mathcal{B}_1(w_{12,b}, w_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}) \times \mathcal{C}_1(z_{12,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1})). \end{aligned}$$

As can be seen from the error probability analysis presented later, such a pair can be found with high probability if

$$R'_{12} + L'_{12} \geq I(X_{11c}, U_{12}; V_{12} | X_0, X_{11d}, S_{11}). \quad (\text{F.1})$$

A codeword x_1^n is then generated as a deterministic function of $(x_0^n, x_{11d}^n, s_{11}^n, x_{11c}^n, u_{12}^n, v_{12}^n)$.

Decoding:

The transmitters decode by the end of every received block. The receivers wait until the complete reception of all the blocks and apply backward decoding starting from the last block. In the following description, we mainly consider the decoding processes at transmitter 1 and receiver node 3. The decoding processes at transmitter 2 and receiver node 4 follow the same way. For decoding at transmitter 1 in the b th block, we assume that $w_{21d,b-1}$ has been successfully decoded from the $(b - 1)$ th block. For decoding at receiver node 3 in the b th block, we assume that $w_{11d,b}$, $w_{21d,b}$ and $z_{2,b+1}$ have been successfully decoded from the $(b + 1)$ th block. The decoding procedures can be summarized as follows:

- i. At transmitter 1, upon receiving $y_{1,b}^n$, it estimates $\hat{w}_{21d,b}$ by finding a unique $x_{21d}^n(\hat{w}_{21d,b}, w_{11d,b-1}, w_{21d,b-1})$ such that $(x_{21d}^n(\hat{w}_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), x_0^n(w_{11d,b-1}, w_{21d,b-1}), x_{11d}^n(w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{11}^n(z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{11c}^n(w_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), u_{12}^n(l_{12,b}, w_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), v_{12}^n(k_{12,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), y_{1,b}^n) \in \mathcal{A}_\epsilon$. Applying the conventional error probability analysis

method for point-to-point channels, $\hat{w}_{21d,b} = w_{21d,b}$ with high probability if

$$R_{21d} \leq I(X_{21d}; Y_1 | X_0, X_{11d}, X_{11c}, U_{12}, S_{11}, V_{12}). \quad (\text{F.2})$$

- ii. At transmitter 1, with the decoded $\hat{w}_{21d,b}$, it tries to find a $g_{1,b}$ such that $(x_0^n(w_{11d,b-1}, w_{21d,b-1}), x_{11d}^n(w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{21d}^n(w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{11}^n(z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{11c}^n(w_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), u_{12}^n(l_{12,b}, w_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), v_{12}^n(k_{12,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), \hat{y}_1^n(g_{1,b}, k_{12,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), y_{1,b}^n) \in \mathcal{A}_\epsilon$. According to [139], such a $g_{1,b}$ can be found with high probability if

$$\hat{R}_1 \geq I(\hat{Y}_1; Y_1 | X_0, X_{11d}, X_{21d}, X_{11c}, U_{12}, S_{11}, V_{12}). \quad (\text{F.3})$$

If $g_{1,b} \in \mathcal{S}_1(z_{1,b+1})$, transmitter 1 knows $z_{1,b+1}$ to be sent in the next block.

- iii. At receiver node 3, with $y_{3,b}^n$, it finds $\hat{w}_{11d,b-1}$ and $\hat{w}_{21d,b-1}$ such that $(x_0^n(\hat{w}_{11d,b-1}, \hat{w}_{21d,b-1}), x_{11d}^n(w_{11d,b}, \hat{w}_{11d,b-1}, \hat{w}_{21d,b-1}), x_{21d}^n(w_{21d,b}, \hat{w}_{11d,b-1}, \hat{w}_{21d,b-1}), y_{3,b}^n) \in \mathcal{A}_\epsilon$. $(\hat{w}_{11d,b-1}, \hat{w}_{21d,b-1}) = (w_{11d,b-1}, w_{21d,b-1})$ with high probability if

$$R_{11d} + R_{21d} \leq I(X_0, X_{11d}, X_{21d}; Y_3). \quad (\text{F.4})$$

- iv. At receiver node 3, assuming $(w_{11d,b-1}, w_{21d,b-1})$ has been successfully found, it then tries to find some $(\hat{z}_{11,b}, \hat{z}_{21,b}, \hat{k}_{22,b})$ such that $(x_0^n(w_{11d,b-1}, w_{21d,b-1}), x_{11d}^n(w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{21d}^n(w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{11}^n(\hat{z}_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{21}^n(\hat{z}_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), v_{22}^n(\hat{k}_{22,b}, \hat{z}_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), y_{3,b}^n) \in \mathcal{A}_\epsilon$. If all the $\hat{k}_{22,b}$ found fall in the bin $\mathcal{C}_2(\hat{z}_{22,b}, \hat{z}_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1})$, then the receiver declares that $(\hat{z}_{11,b}, \hat{z}_{21,b}, \hat{z}_{22,b})$ was received. By analyzing the error probability, we have $(\hat{z}_{11,b}, \hat{z}_{21,b}, \hat{z}_{22,b}) = (z_{11,b}, z_{21,b}, z_{22,b})$ with high probability if

$$L_{22} + L'_{22} \leq I(V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}) \quad (\text{F.5a})$$

$$L_{21} + L_{22} + L'_{22} \leq I(S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{11}) \quad (\text{F.5b})$$

$$L_{11} + L_{22} + L'_{22} \leq I(S_{11}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{21}) \quad (\text{F.5c})$$

$$L_{11} + L_{21} + L_{22} + L'_{22} \leq I(S_{11}, S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}). \quad (\text{F.5d})$$

With $(\hat{z}_{21,b}, \hat{z}_{22,b})$, $\hat{z}_{2,b}$ can be obtained.

- v. At receiver node 3, assuming $(z_{21,b}, z_{22,b})$ has been successfully decoded, the receiver calculates a set $\mathcal{L}(y_3^n(b))$ of $g_{2,b}$ such that $g_{2,b} \in \mathcal{L}(y_3^n(b))$ if $(x_0^n(w_{11d,b-1}, w_{21d,b-1}),$

$x_{11d}^n(w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{21d}^n(w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{11}^n(z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{21}^n(z_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), v_{22}^n(k_{22,b}, z_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), \hat{y}_2^n(g_{2,b}, k_{22,b}, z_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), y_{3,b}^n) \in \mathcal{A}_\epsilon$. With the $z_{2,b+1}$ obtained from the $(b+1)$ th block, the receiver also knows that $g_{2,b}$ falls in the cell $\mathcal{S}_2(z_{2,b+1})$. The receiver then declares that $\hat{g}_{2,b}$ was sent if there is a unique $\hat{g}_{2,b} \in \mathcal{L}(y_{3,b}^n) \cap \mathcal{S}_2(z_{2,b+1})$. $\hat{g}_{2,b} = g_{2,b}$ with high probability if

$$\hat{R}_2 \leq I(\hat{Y}_2; Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}) + L_{11} + L_{12}. \quad (\text{F.6})$$

vi. At receiver node 3, with the recovered $\hat{Y}_{2,b}$, it finds some triples of $(\hat{w}_{11c,b}, \hat{w}_{21c,b}, \hat{l}_{12,b})$ such that $(x_0^n(w_{11d,b-1}, w_{21d,b-1}), x_{11d}^n(w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{21d}^n(w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{11}^n(z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), s_{21}^n(z_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), v_{22}^n(k_{22,b}, z_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{11c}^n(\hat{w}_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), x_{21c}^n(\hat{w}_{21c,b}, z_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), u_{12}^n(\hat{l}_{12,b}, \hat{w}_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1}), \hat{y}_2^n(g_{2,b}, k_{22,b}, z_{21,b}, w_{21d,b}, w_{11d,b-1}, w_{21d,b-1}), y_{3,b}^n) \in \mathcal{A}_\epsilon$. If all the $\hat{l}_{12,b}$ found belong to the bin $\mathcal{B}_1(\hat{w}_{12}, \hat{w}_{11c,b}, z_{11,b}, w_{11d,b}, w_{11d,b-1}, w_{21d,b-1})$, then the decoder declares that $\hat{w}_{12}$ was received. Error probability analysis shows that $(\hat{w}_{11c,b}, \hat{w}_{21c,b}, \hat{w}_{22,b}) = (w_{11c,b}, w_{21c,b}, w_{22,b})$ with high probability if

$$R_{12} + R'_{12} \leq I(U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}, X_{11c}, X_{21c}) \quad (\text{F.7a})$$

$$R_{11c} + R_{12} + R'_{12} \leq I(X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}, X_{21c}) \quad (\text{F.7b})$$

$$R_{21c} + R_{12} + R'_{12} \leq I(X_{21c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}, X_{11c}) \quad (\text{F.7c})$$

$$R_{21c} + R_{11c} + R_{12} + R'_{12} \leq I(X_{11c}, X_{21c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}). \quad (\text{F.7d})$$

Error Probability Analysis:

In the subsequent error probability analysis, without loss of generality, we assume that the messages $(w_{11d,b-1}, w_{21d,b-1}, w_{11d,b}, w_{11c,b}, w_{12,b}, w_{21d,b}, w_{21c,b}, z_{11,b}, z_{12,b}, z_{21,b}, z_{22,b}, l_{12,b}, k_{12,b}, k_{22,b}) = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$ were sent. We focus on the encoding error probability at transmitter 1 and the decoding error probability at receiver node 3. The error probability for the other steps is quite standard and is thus omitted here.

We first study the error probability for encoding at transmitter 1. The encoding error occurs when the encoder cannot find a $u_{12}^n(l_{12,b}, 1, 1, 1, 1, 1)$ in bin $\mathcal{B}_1(1, 1, 1, 1, 1, 1)$ and/or a $v_{12}^n(k_{12,b}, 1, 1, 1, 1, 1)$ in bin $\mathcal{C}_1(1, 1, 1, 1, 1, 1)$ such that $(u_{12}^n(l_{12,b}, 1, 1, 1, 1, 1), v_{12}^n(k_{12,b}, 1, 1, 1, 1, 1)) \in \mathcal{A}_\epsilon$.

The probability of this error event is bounded as

$$\begin{aligned}
P_{e,enc} &\leq \Pr\left(\bigcap_{\substack{u_{12}^n(l_{12,b}, \dots) \in \mathcal{B}_1(1, \dots) \\ v_{12}^n(k_{12,b}, \dots) \in \mathcal{C}_1(1, \dots)}} (u_{12}^n(l_{12,b}), v_{12}^n(k_{12,b}), x_0^n, x_{11d}^n, s_{11}^n, x_{11c}^n) \notin \mathcal{A}_\epsilon\right) \\
&= \prod_{\substack{u_{12}^n(l_{12,b}, \dots) \in \mathcal{B}_1(1, \dots) \\ v_{12}^n(k_{12,b}, \dots) \in \mathcal{C}_1(1, \dots)}} (1 - \Pr((u_{12}^n(l_{12,b}), v_{12}^n(k_{12,b}), x_0^n, x_{11d}^n, s_{11}^n, x_{11c}^n) \in \mathcal{A}_\epsilon)) \\
&\leq (1 - (1 - \epsilon)2^{-n(I(X_{11c}, U_{12}; V_{12}|X_0, X_{11d}, S_{11}) + 4\epsilon)})^{2^{n(R'_{12} + L'_{12})}} \\
&\leq \exp(-(1 - \epsilon)2^{n(R'_{12} + L'_{12} - I(X_{11c}, U_{12}; V_{12}|X_0, X_{11d}, S_{11}) - 4\epsilon)}). \tag{F.8}
\end{aligned}$$

Here, the second inequality in (F.8) is obtained based on

$$\begin{aligned}
&\Pr((u_{12}^n(l_{12,b}), v_{12}^n(k_{12,b}), x_0^n, x_{11d}^n, s_{11}^n, x_{11c}^n) \in \mathcal{A}_\epsilon) \\
&= \sum_{(u_{12}^n, v_{12}^n, x_0^n, x_{11d}^n, s_{11}^n, x_{11c}^n) \in \mathcal{A}_\epsilon} p(x_0^n, x_{11d}^n, s_{11}^n) p(u_{12}^n, x_{11c}^n | x_0^n, x_{11d}^n, s_{11}^n) p(v_{12}^n | x_0^n, x_{11d}^n, s_{11}^n) \\
&\geq (1 - \epsilon)2^{n(H(X_0, X_{11d}, S_{11}, X_{11c}, U_{12}, V_{12}) - H(X_0, X_{11d}, S_{11}) - H(U_{12}, X_{11c} | X_0, X_{11d}, S_{11}) - H(V_{12} | X_0, X_{11d}, S_{11}) + 4\epsilon)} \\
&\geq (1 - \epsilon)2^{-n(I(X_{11c}, U_{12}; V_{12}|X_0, X_{11d}, S_{11}) + 4\epsilon)}. \tag{F.9}
\end{aligned}$$

The third inequality in (F.8) is derived based on $(1 - a)^b \leq \exp(-ab)$.

For the decoding error probability at receiver node 3, we define the following events: 1) $E_{e,f}^{(1)} = \{(x_0^n(e, f), x_{11d}^n(1, e, f), x_{21d}^n(1, e, f), y_3^n(b)) \in \mathcal{A}_\epsilon\}$, 2) $E_{e,f,g}^{(2)} = \{(x_0^n(1, 1), x_{11d}^n(1, 1, 1), x_{21d}^n(1, 1, 1), s_{11}^n(e, 1, 1, 1), s_{21}^n(f, 1, 1, 1), v_{22}^n(g, f, 1, 1, 1), y_3^n(b)) \in \mathcal{A}_\epsilon\}$, 3) $E_{e,f,g}^{(3)} = \{(x_0^n(1, 1), x_{11d}^n(1, 1, 1), x_{21d}^n(1, 1, 1), s_{11}^n(1, 1, 1, 1), s_{21}^n(1, 1, 1, 1), v_{22}^n(1, 1, 1, 1, 1), x_{21c}^n(e, 1, 1, 1, 1), x_{11c}^n(f, 1, 1, 1, 1), u_{12}^n(g, f, 1, 1, 1, 1), \hat{y}_2^n(b), y_3^n(b)) \in \mathcal{A}_\epsilon\}$. Each event corresponds to one of the three decoding steps. For the first decoding step at the receiving node 3, there are four error events: $E_{1,1}^{(1)c}$, $E_{w'_{11d,b-1},1}^{(1)}$, $E_{1,w'_{11d,b-1}}^{(1)}$, and $E_{w'_{11d,b-1},w'_{21d,b-1}}^{(1)}$ for some $(w'_{11d,b-1}, w'_{21d,b-1}) \neq (1, 1)$.

By the asymptotic equipartition property (AEP) [105], $\Pr(E_{1,1}^{(1)c}) < \epsilon$ for large n . For the event $E_{w'_{11d,b-1},1}^{(1)}$, we have

$$\begin{aligned}
\Pr(E_{w'_{11d,b-1},1}^{(1)}) &= \sum_{w'_{11d,b-1}=2}^{2^{nR_{11d}}} \sum_{(x_0^n, x_{11d}^n, x_{21d}^n, y_3^n) \in \mathcal{A}_\epsilon} p(x_0^n, x_{11d}^n, x_{21d}^n) p(y_3^n) \\
&\leq 2^{nR_{11d}} 2^{n(H(X_0, X_{11d}, X_{21d}, Y_3) - H(X_0, X_{11d}, X_{21d}) - H(Y_3) + 3\epsilon)} \\
&= 2^{n(R_{11d} - I(X_0, X_{11d}, X_{21d}; Y_3) + 3\epsilon)}. \tag{F.10}
\end{aligned}$$

Similarly, the probabilities for the other two error events can be calculated as

$$\Pr(E_{1,w'_{11d,b-1}}^{(1)}) \leq 2^{n(R_{21d}-I(X_0,X_{11d},X_{21d};Y_3)+3\epsilon)}, \quad (\text{F.11})$$

and

$$\Pr(E_{w'_{11d,b-1},w'_{21d,b-1}}^{(1)}) \leq 2^{n(R_{11d}+R_{21d}-I(X_0,X_{11d},X_{21d};Y_3)+3\epsilon)}, \quad (\text{F.12})$$

respectively. The dominant error probability is (F.12).

For the second decoding step at receiver node 3, we have the following error events: $E_{(1,1,1)}^{(2)c}$, $E_{(1,1,k'_{22,b})}^{(2)}$, $E_{(1,z'_{21,b},1)}^{(2)}$, $E_{(1,z'_{21,b},k'_{22,b})}^{(2)}$, $E_{(z'_{11,b},1,k'_{22,b})}^{(2)}$, $E_{(z'_{11,b},z'_{21,b},1)}^{(2)}$, and $E_{(z'_{11,b},z'_{21,b},k'_{22,b})}^{(2)}$ for some $(z'_{11,b}, z'_{21,b}, k'_{22,b}) \neq (1, 1, 1)$. Note that although the decoder only needs to find the index of the bin that $k_{22,b}$ falls in, by asking the decoder to decode the exact $k_{22,b}$, we can obtain an error probability upper bound.

By AEP, $\Pr(E_{(1,1,1)}^{(2)c}) < \epsilon$. For the other error events, we have

$$\begin{aligned} & \Pr(E_{(1,1,k'_{22,b})}^{(2)}) \\ &= \sum_{k'_{22,b}=2}^{2^{n(L_{22}+L'_{22})}} \sum_{(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, v_{22}^n, y_3^n) \in \mathcal{A}_\epsilon} p(v_{22}^n | x_0^n, x_{11d}^n, x_{21d}^n, s_{21}^n) p(x_0^n, x_{11d}^n, x_{21d}^n, s_{21}^n, s_{11}^n, y_3^n) \\ &\leq 2^{n(L_{22}+L'_{22})} 2^{n(H(X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}, Y_3) - H(V_{22} | X_0, X_{11d}, X_{21d}, S_{21}) - H(X_0, X_{11d}, X_{21d}, S_{21}, S_{11}, Y_3) + 3\epsilon)} \\ &\leq 2^{n(L_{22}+L'_{22}-I(V_{22};Y_3|X_0,X_{11d},X_{21d},S_{11},S_{21})+3\epsilon)}, \end{aligned} \quad (\text{F.13})$$

$$\Pr(E_{(1,z'_{21,b},1)}^{(2)}) \quad (\text{F.14})$$

$$\begin{aligned} &= \sum_{z'_{21,b}=2}^{2^{n(L_{21})}} \sum_{(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, v_{22}^n, y_3^n) \in \mathcal{A}_\epsilon} p(s_{21}^n, v_{22}^n | x_0^n, x_{11d}^n, x_{21d}^n) p(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, y_3^n) \\ &\leq 2^{n(L_{21}-I(S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{11})+3\epsilon)}, \end{aligned} \quad (\text{F.15})$$

$$\Pr(E_{(1,z'_{21,b},k'_{22,b})}^{(2)}) \quad (\text{F.16})$$

$$\begin{aligned} &= \sum_{z'_{21,b}=2}^{2^{n(L_{21})}} \sum_{k'_{22,b}}^{2^{n(L_{22}+L'_{22})}} \sum_{(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, v_{22}^n, y_3^n) \in \mathcal{A}_\epsilon} p(s_{21}^n, v_{22}^n | x_0^n, x_{11d}^n, x_{21d}^n) p(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, y_3^n) \\ &\leq 2^{n(L_{21}+L_{22}+L'_{22}-I(S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{11})+3\epsilon)}. \end{aligned} \quad (\text{F.17})$$

Similarly,

$$\Pr(E_{(z'_{11,b}, 1, k'_{22,b})}^{(2)}) \leq 2^{n(L_{11}+L_{22}+L'_{22}-I(S_{11}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d}, S_{21})+3\epsilon)}, \quad (\text{F.18})$$

$$\Pr(E_{(z'_{11,b}, z'_{21,b}, 1)}^{(2)}) \leq 2^{n(L_{11}+L_{21}-I(S_{11}, S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d})+3\epsilon)}, \quad (\text{F.19})$$

$$\Pr(E_{(z'_{11,b}, z'_{21,b}, k'_{22,b})}^{(2)}) \leq 2^{n(L_{11}+L_{21}+L_{22}+L'_{22}-I(S_{11}, S_{21}, V_{22}; Y_3 | X_0, X_{11d}, X_{21d})+3\epsilon)}. \quad (\text{F.20})$$

Defining the event $E_a^{(4)} = \{(x_0^n(1, 1), x_{11d}^n(1, 1, 1), x_{21d}^n(1, 1, 1), s_{11}^n(1, 1, 1, 1), s_{21}^n(1, 1, 1, 1), v_{22}^n(1, 1, 1, 1, 1), \hat{y}_2^n(a, 1, 1, 1, 1, 1), y_3^n(b)) \in \mathcal{A}_\epsilon\}$, the error probability of recovering $\hat{y}_2$ from the decoded $z_{2,b}$, is upper bounded by

$$\begin{aligned} & \Pr(E_{\hat{g}_{2,b}}^{(4)} \cap \{g_{2,b} \in \mathcal{S}_2(z_{2,b+1})\}) \\ & \leq \sum_{\substack{\hat{g}_{2,b} \neq g_{2,b} \\ g_{2,b} \in \mathcal{S}_2(z_{2,b+1})}} \sum_{\substack{(x_0^n, x_{11d}^n, x_{21d}^n, s_{21}^n, v_{22}^n) \\ s_{21}^n, v_{22}^n, \hat{y}_2^n, y_3^n \in \mathcal{A}_\epsilon}} p(\hat{y}_2^n | x_0^n, x_{11d}^n, x_{21d}^n, s_{21}^n, v_{22}^n) p(x_0^n, x_{11d}^n, x_{21d}^n, s_{21}^n, v_{22}^n, s_{11}^n, y_3^n) \\ & \leq 2^{n(\hat{R}_2 - L_{21} - L_{22})} \\ & \quad \times 2^{n(H(X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}; \hat{Y}_2, Y_3) - H(\hat{Y}_2 | X_0, X_{11d}, X_{21d}, S_{21}, V_{22}) - H(X_0, X_{11d}, X_{21d}, S_{21}, S_{11}, V_{22}, Y_3) + 3\epsilon)} \\ & \leq 2^{n(\hat{R}_2 - L_{21} - L_{22} - I(\hat{Y}_2; Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}) + 3\epsilon)}. \end{aligned} \quad (\text{F.21})$$

For the last decoding step at receiver node 3, error happens when the following events occur: $E_{1,1,1}^{(3)c}$, $E_{1,1,l'_{12,b}}^{(3)}$, $E_{1,w'_{11c,b},1}^{(3)}$, $E_{1,w'_{11c,b},l'_{12,b}}^{(3)}$, $E_{w'_{21c,b},1,l'_{12,b}}^{(3)}$, $E_{w'_{21c,b},w'_{11c,b},1}^{(3)}$, $E_{w'_{21c,b},w'_{11c,b},l'_{12,b}}^{(3)}$ for some $(w'_{11c,b}, w'_{21c,b}, l'_{12,b}) \neq (1, 1, 1)$. Again, asking the decoder to decode the exact $l_{12,b}$ yields an error probability upper bound.

By AEP, $\Pr(E_{1,1,1}^{(3)c}) < \epsilon$ for large n . The probabilities for the other error events can be calculated as

$$\begin{aligned} & \Pr(E_{1,1,l'_{12,b}}^{(3)}) \\ & = 2^{n(R_{12}+R'_{12})} \sum_{l'_{12,b}=2} \sum_{\substack{(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, \\ v_{22}^n, x_{11c}^n, x_{21c}^n, u_{12}^n, \hat{y}_2^n, y_3^n) \in \mathcal{A}_\epsilon}} p(u_{12}^n | x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, x_{11c}^n) \\ & \quad \times p(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, v_{22}^n, x_{11c}^n, x_{21c}^n, \hat{y}_2^n, y_3^n) \\ & \leq 2^{n(R_{12}+R'_{12})} 2^{-n(I(U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, X_{11c}, X_{21c}, V_{22}) - 3\epsilon)}, \end{aligned} \quad (\text{F.22})$$

$$\begin{aligned}
& \Pr(E_{1,w'_{11c,b},1}^{(3)}) \\
&= \sum_{w'_{11c,b}=2}^{2^{nR_{11c}}} \sum_{\substack{(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, \\ v_{22}^n, x_{11c}^n, x_{21c}^n, u_{12}^n, \hat{y}_2^n, y_3^n) \in \mathcal{A}_\epsilon}} p(x_{11c}^n, u_{12}^n | x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n) \\
&\quad \times p(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, v_{22}^n, x_{21c}^n, \hat{y}_2^n, y_3^n) \\
&\leq 2^{nR_{11c}} 2^{-n(I(X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, X_{21c}, V_{22}) - 3\epsilon)}, \tag{F.23}
\end{aligned}$$

$$\begin{aligned}
& \Pr(E_{1,w'_{11c,b},l'_{12,b}}^{(3)}) \\
&= \sum_{w'_{11c,b}=2}^{2^{nR_{11c}}} \sum_{l'_{12,b}=2}^{n(R_{12}+R'_{12})} \sum_{\substack{(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, \\ v_{22}^n, x_{11c}^n, x_{21c}^n, u_{12}^n, \hat{y}_2^n, y_3^n) \in \mathcal{A}_\epsilon}} p(x_{11c}^n, u_{12}^n | x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n) \\
&\quad \times p(x_0^n, x_{11d}^n, x_{21d}^n, s_{11}^n, s_{21}^n, v_{22}^n, x_{21c}^n, \hat{y}_2^n, y_3^n) \\
&\leq 2^{n(R_{11c}+R_{12}+R'_{12})} 2^{-n(I(X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, X_{21c}, V_{22}) - 3\epsilon)}. \tag{F.24}
\end{aligned}$$

The probabilities of the remaining three error events can be similarly calculated as

$$\Pr(E_{w'_{21c,b},1,l'_{12,b}}^{(3)}) \leq 2^{n(R_{21c}+R_{12}+R'_{12})} 2^{-n(I(X_{21c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, X_{11c}, V_{22}) - 3\epsilon)}, \tag{F.25}$$

$$\Pr(E_{w'_{21c,b},w'_{11c,b},1}^{(3)}) \leq 2^{n(R_{21c}+R_{11c})} 2^{-n(I(X_{21c}, X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}) - 3\epsilon)}, \tag{F.26}$$

$$\Pr(E_{w'_{21c,b},w'_{11c,b},l'_{12,b}}^{(3)}) \leq 2^{n(R_{21c}+R_{11c}+R_{12}+R'_{12})} 2^{-n(I(X_{21c}, X_{11c}, U_{12}; \hat{Y}_2, Y_3 | X_0, X_{11d}, X_{21d}, S_{11}, S_{21}, V_{22}) - 3\epsilon)}. \tag{F.27a}$$

Similar analyses can be made for encoding at transmitter 2 and decoding at receiver node 4. Summarizing all the results, one can see that transmitting at rates within the rate region defined in Theorem 5.2.1 ensures reliable transmission over the two-user IFC with source cooperation. This completes the proof for Theorem 5.2.1.

Appendix G

Proof of Theorem 6.2.1

Following the analysis in [113], the outage probability of the symmetric IFC with source cooperation can be upper bounded as

$$P_{\text{out}}(r) \doteq \int_{\mathcal{A}(\beta)} \text{SNR}^{-(\beta_{1,3}+\beta_{1,4}+\beta_{2,3}+\beta_{2,4}+\beta_c)} d\beta_{1,3}d\beta_{1,4}d\beta_{2,3}d\beta_{2,4}d\beta_c, \quad (\text{G.1})$$

where $\mathcal{A}(\beta) = \{\beta_{i,j} \geq 0, \beta_c \geq 0 : (r \log \text{SNR}, r \log \text{SNR}) \notin \mathcal{R}(\beta)\}$ and $\mathcal{R}(\beta)$ denotes the capacity upper bound region. In the high SNR regime, the outage probability can be further approximated as

$$P_{\text{out}}(r) \doteq \text{SNR}^{-\min_{\hat{\mathcal{A}}(\beta)} \beta_{1,3}+\beta_{1,4}+\beta_{2,3}+\beta_{2,4}+\beta_c}, \quad (\text{G.2})$$

where the set $\hat{\mathcal{A}}(\beta)$ is the approximation of the integral region $\mathcal{A}(\beta)$ under high signal-to-noise ratio condition. Using the capacity outer bound in (6.2.1) and considering the assumption of symmetric rates, $\hat{\mathcal{A}}(\beta)$ is found as

$$\hat{\mathcal{A}}(\beta) = \{\beta_{i,j} \geq 0, \beta_c \geq 0 :$$

$$\max\{(1 - \beta_{1,3})^+, (\alpha_c - \beta_c)^+\} \leq r, \quad (\text{G.3a})$$

$$\max\{(1 - \beta_{1,3})^+, (\alpha_i - \beta_{2,3})^+\} \leq r, \quad (\text{G.3b})$$

$$\begin{aligned} &\max\{(1 - \beta_{1,3})^+, (\alpha_i - \beta_{2,3})^+\} \\ &\quad + \max\{(1 - \beta_{2,4})^+, (\alpha_i - \beta_{2,3})^+, (\alpha_c - \beta_c)^+\} - (\alpha_i - \beta_{2,3})^+ \leq 2r \end{aligned} \quad (\text{G.3c})$$

$$\begin{aligned} &\max\{(1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{2,3})^+, (\alpha_c - \beta_c)^+\} \\ &\quad + \max\{(1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{1,4})^+, (\alpha_c - \beta_c)^+\} \leq 2r, \end{aligned} \quad (\text{G.3d})$$

$$\max\{(2 - \beta_{1,3} - \beta_{2,4})^+, (2\alpha_i - \beta_{1,4} - \beta_{2,3})^+\} \leq 2r. \quad (\text{G.3e})$$

Here (G.3a), (G.3b), (G.3c), (G.3d) and (G.3e) are derived from (6.2.1a), (6.2.1b), (6.2.1e), (6.2.1g) and (6.2.1h), respectively.

The diversity order at multiplexing gain r can be calculated as

$$d^{ub}(r) = \min_{\hat{\mathcal{A}}(\beta)} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c. \quad (\text{G.4})$$

By solving the optimization problem within the set $\hat{\mathcal{A}}(\beta)$, the DMT result in Theorem 6.2.1 can be obtained. Specifically,

$$\begin{aligned} d_a^{ub}(r) &= \min \beta_{1,3} + \beta_c, \text{ s.t. } (G.3a), \\ d_b^{ub}(r) &= \min \beta_{1,3} + \beta_{2,3}, \text{ s.t. } (G.3b), \\ d_c^{ub}(r) &= \min \beta_{1,3} + \beta_{2,3} + \beta_{2,4} + \beta_c, \text{ s.t. } (G.3c), \\ d_d^{ub}(r) &= \min \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \text{ s.t. } (G.3d). \end{aligned}$$

Since the constraint in (G.3e) is less stringent than that in (G.3b), we do not need to consider it in the DMT upper bound.

In the following, we take $d_d^{ub}(r)$ as an example to illustrate the derivation process. The problem can be recapped as

$$\begin{aligned} d_d^{ub}(r) &= \min \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \\ \text{s.t.} \quad &\beta_{i,j} \geq 0, \beta_c \geq 0, \\ &\max\{(1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{2,3})^+, (\alpha_c - \beta_c)^+\} \\ &+ \max\{(1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{1,4})^+, (\alpha_c - \beta_c)^+\} \leq 2r. \end{aligned}$$

We solve this problem by partitioning and optimizing the values of $\beta_{i,j}$ and β_c in the following four cases.

- (i). $(1 - \beta_{1,3})^+ \geq (\alpha_i - \beta_{1,4})^+, (1 - \beta_{2,4})^+ \geq (\alpha_i - \beta_{2,3})^+.$
- (ii). $(1 - \beta_{1,3})^+ \geq (\alpha_i - \beta_{1,4})^+, (1 - \beta_{2,4})^+ \leq (\alpha_i - \beta_{2,3})^+.$
- (iii). $(1 - \beta_{1,3})^+ \leq (\alpha_i - \beta_{1,4})^+, (1 - \beta_{2,4})^+ \geq (\alpha_i - \beta_{2,3})^+.$
- (iv). $(1 - \beta_{1,3})^+ \leq (\alpha_i - \beta_{1,4})^+, (1 - \beta_{2,4})^+ \leq (\alpha_i - \beta_{2,3})^+.$

For case (i), the constraint (G.3d) becomes

$$\begin{aligned} &\max\{(1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{2,3})^+\} \\ &+ \max\{(1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{1,4})^+\} \leq 2r. \end{aligned}$$

This case can be further divided into four sub-cases, by considering the relative values be-

tween $(1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+$ and $(\alpha_i - \beta_{2,3})^+$, and $(1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+$ and $(\alpha_i - \beta_{1,4})^+$.

For example, if $(1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+ \geq (\alpha_i - \beta_{2,3})^+$, and $(1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+ \geq (\alpha_i - \beta_{1,4})^+$, the optimization problem becomes

$$\begin{aligned}
d_d^{ub}(r) &= \min \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \\
\text{s.t.} \quad &\beta_{i,j} \geq 0, \beta_c \geq 0, \\
&(1 - \beta_{1,3})^+ + (1 - \beta_{2,4})^+ + 2(\alpha_c - \beta_c)^+ - (\alpha_i - \beta_{1,4})^+ - (\alpha_i - \beta_{2,3})^+ \leq 2r, \\
&(1 - \beta_{1,3})^+ \geq (\alpha_i - \beta_{1,4})^+, \\
&(1 - \beta_{2,4})^+ \geq (\alpha_i - \beta_{2,3})^+, \\
&(1 - \beta_{1,3})^+ + (\alpha_c - \beta_c)^+ \geq (\alpha_i - \beta_{1,4})^+ + (\alpha_i - \beta_{2,3})^+, \\
&(1 - \beta_{2,4})^+ + (\alpha_c - \beta_c)^+ \geq (\alpha_i - \beta_{1,4})^+ + (\alpha_i - \beta_{2,3})^+.
\end{aligned}$$

Solving for this linear programming problem, we have the following results:

- If $\alpha_i \leq \{1, (1 + \alpha_c)/2, r\}$, the minimum value is

$$d_{d,A1}^{ub}(r) = 2(1 - \alpha_i - r)^+ + (\alpha_c - (\alpha_i + r - 1)^+)^+.$$

This is achieved by taking $\beta_{1,4} = \beta_{2,3} = 0$, $\beta_{1,3} = \beta_{2,4} = (1 - \alpha_i - r)^+$, and $\beta_c = (\alpha_c - (\alpha_i + r - 1)^+)^+$.

- If $\alpha_i > \{1, (1 + \alpha_c)/2, r\}$, the minimum value is

$$\begin{aligned}
d_{d,A1}^{ub}(r) &= 2(1 - 2r)^+ + (\alpha_c - \min\{1, (2r - 1)^+\})^+ \\
&\quad + (2\alpha_i - \min\{1, 2r\} - \min\{1, \alpha_c, (2r - 1)^+\})^+.
\end{aligned}$$

This is achieved by taking $\beta_{1,3} = \beta_{2,4} = (1 - 2r)^+$, $\beta_c = (\alpha_c - \min\{1, (2r - 1)^+\})^+$, and $\beta_{1,4} = \beta_{2,3} = (2\alpha_i - \min\{1, 2r\} - \min\{1, \alpha_c, (2r - 1)^+\})^+/2$.

The analysis for the remaining cases follows the same procedure. The basic approach is to partition the $\beta_{i,j}$ and β_c values into several regions such that in each region, the problem can be expressed as a linear programming program which can be solved by some standard methods.

Appendix H

Proof of Theorem 6.2.2

In this proof, we use the achievable rate region of the IFC with partially DF-based source cooperation proposed in [52] and [54]. Let $R_i = R_{v_i} + R_{u_i} + R_{z_i}$ ($i = 1, 2$), for Gaussian channels, the achievable rate region at high SNR is given as

$$R_{v_1} \leq \log(1 + \text{SNR}^{\alpha_c} |h_c|^2), \quad (\text{H.1a})$$

$$R_{z_1} \leq \log\left(1 + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}}\right), \quad (\text{H.1b})$$

$$R_{u_1} + R_{z_1} \leq \log\left(1 + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}} + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}}\right), \quad (\text{H.1c})$$

$$R_{u_2} + R_{z_1} \leq \log\left(1 + \frac{\text{SNR} |h_{2,3}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}} + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}}\right), \quad (\text{H.1d})$$

$$\begin{aligned} R_{u_1} + R_{u_2} + R_{z_1} \leq \log\left(1 + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}} \right. \\ \left. + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} + \frac{\text{SNR}^{\alpha_i} |h_{2,3}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}}\right), \end{aligned} \quad (\text{H.1e})$$

$$\begin{aligned} R_1 + R_{v_2} + R_{u_2} \leq \log\left(1 + \text{SNR} |h_{1,3}|^2 + \text{SNR}^{\alpha_i} |h_{2,3}|^2 + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}} \right. \\ \left. + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} + \frac{\text{SNR}^{\alpha_i} |h_{2,3}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}}\right), \end{aligned} \quad (\text{H.1f})$$

$$R_{v_2} \leq \log(1 + \text{SNR}^{\alpha_c} |h_c|^2), \quad (\text{H.1g})$$

$$R_{z_2} \leq \log\left(1 + \frac{\text{SNR} |h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}}\right), \quad (\text{H.1h})$$

$$R_{u_2} + R_{z_2} \leq \log\left(1 + \frac{\text{SNR} |h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}} + \frac{\text{SNR} |h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}}\right), \quad (\text{H.1i})$$

$$R_{u_1} + R_{z_2} \leq \log\left(1 + \frac{\text{SNR} |h_{1,4}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}} + \frac{\text{SNR} |h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}}\right), \quad (\text{H.1j})$$

$$R_{u_1} + R_{u_2} + R_{z_2} \leq \log\left(1 + \frac{\text{SNR} |h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_c} |h_c|^2\}}\right)$$

$$+ \frac{\text{SNR}|h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_i}|h_{2,3}|^2\}} + \frac{\text{SNR}^{\alpha_i}|h_{1,4}|^2}{\max\{1, \text{SNR}^{\alpha_c}|h_c|^2\}} \Bigg), \quad (\text{H.1k})$$

$$R_2 + R_{v_1} + R_{u_1} \leq \log \left(1 + \text{SNR}|h_{2,4}|^2 + \text{SNR}^{\alpha_i}|h_{1,4}|^2 + \frac{\text{SNR}|h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_c}|h_c|^2\}} \right. \\ \left. + \frac{\text{SNR}|h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_i}|h_{2,3}|^2\}} + \frac{\text{SNR}^{\alpha_i}|h_{1,4}|^2}{\max\{1, \text{SNR}^{\alpha_c}|h_c|^2\}} \right). \quad (\text{H.1l})$$

Applying Fourier-Motzkin elimination, we express the rate region in the following compact form:

$$R_1 \leq \min \{ (H.1f), (H.1a) + (H.1c) \}, \quad (\text{H.2a})$$

$$R_2 \leq \min \{ (H.1l), (H.1g) + (H.1i) \}, \quad (\text{H.2b})$$

$$R_1 + R_2 \leq \min \{ (H.1b) + (H.1l), (H.1f) + (H.1h), (H.1a) + (H.1d) + (H.1g) + (H.1j), \\ (H.1a) + (H.1b) + (H.1g) + (H.1k), (H.1a) + (H.1e) + (H.1g) + (H.1h) \}, \quad (\text{H.2c})$$

$$R_1 + 2R_2 \leq \min \{ 2(H.1g) + (H.1h) + (H.1a) + (H.1d) + (H.1k), \\ (H.1g) + (H.1h) + (H.1l) + (H.1d) \}, \quad (\text{H.2d})$$

$$2R_1 + R_2 \leq \min \{ 2(H.1a) + (H.1b) + (H.1g) + (H.1j) + (H.1e), \\ (H.1a) + (H.1b) + (H.1f) + (H.1j) \}. \quad (\text{H.2e})$$

Using the rate region result in (H.2), following the same procedure as in Appendix G, the diversity value at a targeted multiplexing value r can be calculated as

$$d^{DF_p}(r) = \min_{\hat{\mathcal{B}}(\beta)} (\beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c), \quad (\text{H.3})$$

where

$$\hat{\mathcal{B}}(\beta) = \{ \beta_{i,j} \geq 0, \beta_c \geq 0 :$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+, (\alpha_c - \beta_c)^+\} \leq r, \quad (\text{H.4a})$$

$$\max\{(1 - \beta_{1,3})^+, (\alpha_i - \beta_{2,3})^+\} \leq r, \quad (\text{H.4b})$$

$$((1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+)^+ + \max\{(1 - \beta_{2,4})^+, (\alpha_i - \beta_{1,4})^+\} \leq 2r, \quad (\text{H.4c})$$

$$(\alpha_c - \beta_c)^+ + ((1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+)^+$$

$$\begin{aligned}
& + \max\{(1 - \beta_{2,4})^+, (\alpha_i - \beta_{1,4})^+, (\alpha_c - \beta_c)^+, \\
& (1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+\} \leq 2r,
\end{aligned} \tag{H.4d}$$

$$\begin{aligned}
& \max\{(1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{2,3})^+, (\alpha_c - \beta_c)^+\} \\
& + \max\{(1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{1,4})^+, (\alpha_c - \beta_c)^+\} \leq 2r,
\end{aligned} \tag{H.4e}$$

$$\begin{aligned}
& (\alpha_c - \beta_c)^+ + ((1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+)^+ \\
& + \max\{(1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+, (\alpha_i - \beta_{1,4})^+, (\alpha_c - \beta_c)^+\} \\
& + \max\{(1 - \beta_{1,3})^+, (\alpha_i - \beta_{2,3})^+, (\alpha_c - \beta_c)^+, \\
& (1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+ + (\alpha_c - \beta_c)^+\} \leq 3r,
\end{aligned} \tag{H.4f}$$

$$\begin{aligned}
& ((1 - \beta_{1,3})^+ - (\alpha_i - \beta_{1,4})^+)^+ + \max\{(1 - \beta_{1,3})^+, (\alpha_i - \beta_{2,3})^+\} \\
& + \max\{(\alpha_i - \beta_{1,4})^+, (\alpha_c - \beta_c)^+, (1 - \beta_{2,4})^+ - (\alpha_i - \beta_{2,3})^+ + (\alpha_c - \beta_c)^+\} \leq 3r
\end{aligned} \tag{H.4g}$$

}.

Using the same approach as in Appendix G, the DMT expressions $d_a^{DF_p}(r), \dots, d_g^{DF_p}(r)$ in Theorem 6.2.2 can be obtained from (H.4a)-(H.4g), respectively.

Appendix I

Proof of Theorem 6.2.3

For Gaussian IFCs, the achievable rate region of IFC with full DF-based source cooperation is given as [54]

$$R_{s_1} \leq \log \left(1 + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR} |h_{1,3}|^2, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} \right), \quad (\text{I.1a})$$

$$R_{z_1} + R_{s_1} \leq \log \left(1 + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2, \text{SNR} |h_{1,3}|^2\}} \right) \quad (\text{I.1b})$$

$$R_1 \leq \log \left(1 + \text{SNR}^{\alpha_c} |h_c|^2 + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2, \text{SNR} |h_{1,3}|^2\}} \right) \quad (\text{I.1c})$$

$$R_{z_1} \leq \log \left(1 + \frac{\text{SNR} |h_{1,3}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} \right) \quad (\text{I.1d})$$

$$R_{s_1} + R_{z_1} \leq \log \left(1 + \frac{\text{SNR} |h_{1,3}|^2}{\text{SNR}^{\alpha_i} |h_{1,4}|^2} + \frac{|\text{SNR} |h_{1,3}| |h_{2,4}| - \text{SNR}^{\alpha_i} |h_{1,4}| |h_{2,3}| e^{j\theta}|^2}{\max\{\text{SNR} |h_{2,4}|^2, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} + \frac{\text{SNR}^{\alpha_i} |h_{2,3}|^2}{\text{SNR} |h_{2,4}|^2} \right) \quad (\text{I.1e})$$

$$R_1 + R_{v_2} \leq \log \left(1 + \text{SNR} |h_{1,3}|^2 + \text{SNR}^{\alpha_i} |h_{2,3}|^2 + \frac{\text{SNR} |h_{1,3}|^2}{\text{SNR}^{\alpha_i} |h_{1,4}|^2} + \frac{|\text{SNR} |h_{1,3}| |h_{2,4}| - \text{SNR}^{\alpha_i} |h_{1,4}| |h_{2,3}| e^{j\theta}|^2}{\max\{\text{SNR} |h_{2,4}|^2, \text{SNR}^{\alpha_i} |h_{1,4}|^2\}} + \frac{\text{SNR}^{\alpha_i} |h_{2,3}|^2}{\text{SNR} |h_{2,4}|^2} \right) \quad (\text{I.1f})$$

$$R_{s_2} \leq \log \left(1 + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR} |h_{2,4}|^2, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}} \right) \quad (\text{I.1g})$$

$$R_{z_2} + R_{s_2} \leq \log \left(1 + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}} + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2, \text{SNR} |h_{2,4}|^2\}} \right) \quad (\text{I.1h})$$

$$R_2 \leq \log \left(1 + \text{SNR}^{\alpha_c} |h_c|^2 \right)$$

$$+ \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}} + \frac{\text{SNR}^{\alpha_c} |h_c|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2, \text{SNR} |h_{2,4}|^2\}} \Big) \quad (\text{I.1i})$$

$$R_{z_2} \leq \log \left(1 + \frac{\text{SNR} |h_{2,4}|^2}{\max\{1, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}} \right) \quad (\text{I.1j})$$

$$R_{s_2} + R_{z_2} \leq \log \left(1 + \frac{\text{SNR} |h_{2,4}|^2}{\text{SNR}^{\alpha_i} |h_{2,3}|^2} + \frac{|\text{SNR} |h_{1,3}| |h_{2,4}| - \text{SNR}^{\alpha_i} |h_{1,4}| |h_{2,3}| e^{j\theta}|^2}{\max\{\text{SNR} |h_{1,3}|^2, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}} + \frac{\text{SNR}^{\alpha_i} |h_{1,4}|^2}{\text{SNR} |h_{1,3}|^2} \right) \quad (\text{I.1k})$$

$$R_2 + R_{v_1} \leq \log \left(1 + \text{SNR} |h_{2,4}|^2 + \text{SNR}^{\alpha_i} |h_{1,4}|^2 + \frac{\text{SNR} |h_{2,4}|^2}{\text{SNR}^{\alpha_i} |h_{2,3}|^2} + \frac{|\text{SNR} |h_{1,3}| |h_{2,4}| - \text{SNR}^{\alpha_i} |h_{1,4}| |h_{2,3}| e^{j\theta}|^2}{\max\{\text{SNR} |h_{1,3}|^2, \text{SNR}^{\alpha_i} |h_{2,3}|^2\}} + \frac{\text{SNR}^{\alpha_i} |h_{1,4}|^2}{\text{SNR} |h_{1,3}|^2} \right). \quad (\text{I.1l})$$

After Fourier-Motzkin elimination, we can express the rate region (I.1) as

$$R_1 \leq \min\{(I.1c), (I.1f)\}, \quad (\text{I.2a})$$

$$R_2 \leq \min\{(I.1i), (I.1l)\}, \quad (\text{I.2b})$$

$$R_1 + R_2 \leq \min\{(I.1l) + (I.1b), (I.1l) + (I.1e), (I.1l) + (I.1a) + (I.1d), \\ (I.1f) + (I.1h), (I.1f) + (I.1k), (I.1f) + (I.1g) + (I.1j)\}. \quad (\text{I.2c})$$

We define $\mathcal{E}_{coop}$ as the event that the sources fail to decode each other. When $r \leq \text{DOF}_{IFC}(\alpha_i)$, the outage probability of the modified scheme can be calculated as

$$\begin{aligned} P_{\text{out}}(r) &= P_{\text{out}}(r|\bar{\mathcal{E}}_{coop})\Pr(\bar{\mathcal{E}}_{coop}) + P_{\text{out}}(r|\mathcal{E}_{coop})\Pr(\mathcal{E}_{coop}) \\ &\doteq P_{\text{out}}(r|\bar{\mathcal{E}}_{coop}) + P_{\text{out}}(r|\mathcal{E}_{coop})\Pr(\mathcal{E}_{coop}) \\ &\doteq \text{SNR}^{-d^{DFf,2}(r)} + \text{SNR}^{-d^{IFC}(r)} \text{SNR}^{-(\alpha_c-r)^+}. \end{aligned} \quad (\text{I.3})$$

When $\text{DOF}_{IFC}(\alpha_i) < r \leq \text{DOF}_{CIFC}(\alpha_i, \alpha_c)$, the outage probability is

$$P_{\text{out}}(r) \doteq \text{SNR}^{-d^{DFf,1}(r)}. \quad (\text{I.4})$$

The overall DMT expression can thus be evaluated as

$$d^{DFf}(r) = \begin{cases} \min\{d^{DFf,2}(r), d^{IFC}(r) + (\alpha_c - r)^+\}, & \text{if } r \leq \text{DOF}_{IFC}(\alpha_i), \\ d^{DFf,1}(r), & \text{if } \text{DOF}_{IFC}(\alpha_i) < r \leq \text{DOF}_{CIFC}(\alpha_i, \alpha_c). \end{cases} \quad (\text{I.5})$$

Using the rate region results in (I.2) and together with the assumption that the rate is

no longer restricted by the cooperation link, $d^{DF,1}(r)$ can be evaluated as

$$d^{DF,1}(r) = \min_{\hat{\mathcal{O}}_1(\beta)} (\beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c). \quad (\text{I.6})$$

The set $\hat{\mathcal{O}}_1(\beta)$ is defined as

$$\hat{\mathcal{O}}_1(\beta) = \{\beta_{i,j} \geq 0, \beta_c \geq 0 :$$

$$(\alpha_c - \beta_c)^+ \leq r \quad (\text{I.7a})$$

$$\max\{(1 - \beta_{2,4})^+, (\alpha_i - \beta_{1,4})^+\} + ((\alpha_c - \beta_c)^+ - (\alpha_i - \beta_{1,4})^+)^+ \leq 2r \quad (\text{I.7b})$$

$$\max\{(1 - \beta_{1,3})^+, (\alpha_i - \beta_{2,3})^+\} \leq r \quad (\text{I.7c})$$

$$\begin{aligned} & \max\{(1 - \beta_{2,4})^+, (\alpha_i - \beta_{1,4})^+\} \\ & + \max\{(\alpha_c - \beta_c)^+, (1 - \beta_{1,3})^+, (\alpha_i - \beta_{1,4})^+\} - (\alpha_i - \beta_{1,4})^+ \leq 2r \end{aligned} \quad (\text{I.7d})$$

$\}.$

Note that $d^{DF,2}(r)$ is calculated as

$$d^{DF,2}(r) = \min_{\hat{\mathcal{O}}_2(\beta)} (\beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c). \quad (\text{I.8})$$

The set $\hat{\mathcal{O}}_2(\beta)$ is defined by taking the constraints (I.7c) and (I.7d) in $\hat{\mathcal{O}}_1(\beta)$.

Using the same approach as in Appendix G, the DMT expressions $d_a^{DF_f}(r), \dots, d_d^{DF_f}(r)$ can be obtained by minimizing the objective function (I.6) under the constraints of (I.7a)-(I.7d), respectively.

Appendix J

Proof of Theorem 6.3.1

We first show that for the two sources to fully decode each other, with one-level Gaussian codes, we must have $\beta_c \leq (1 - r)^+$. With the proposed two-level Gaussian codes, we must have $\beta_c \leq (1 - 2r)^+$.

When one-level Gaussian codebooks are used, the two sources can fully decode each other if

$$R_i \leq I(X_i; Y_j), \quad i = 1, 2, j = 1, 2, j \neq i. \quad (\text{J.1})$$

For the considered Rayleigh fading channel, and for $R_1 = R_2 \doteq r \log \text{SNR}$, (J.1) becomes

$$r \log \text{SNR} \leq \log(1 + \text{SNR}|h_c|^2) \doteq \log(1 + \text{SNR}^{(1-\beta_c)}) \doteq (1 - \beta_c)^+ \log \text{SNR}. \quad (\text{J.2})$$

Therefore, if $\beta_c \leq (1 - r)^+$, the one-level Gaussian encoded message can be decoded.

With the two-level Gaussian codes, we use a decoding scheme that the public codeword part is first decoded treating the private codeword as interference. The private codeword is then decoded after removing the public part. For successful decoding at source j , source i should encode the public and the private message parts at rates that satisfy

$$R_{iw} \leq I(X_{iw}; Y_j), \quad (\text{J.3a})$$

$$R_{iu} \leq I(X_{iu}; Y_j | X_{iw}). \quad (\text{J.3b})$$

For Rayleigh-faded channels, we have

$$\begin{aligned} I(X_{iw}; Y_j) &= \log(1 + \text{SNR}|h_c|^2) - \log(1 + \text{SNR}^\eta |h_c|^2) \\ &\doteq ((1 - \beta_c)^+ - (\eta - \beta_c)^+) \log \text{SNR}, \end{aligned} \quad (\text{J.4})$$

and

$$I(X_{iu}; Y_j | X_{iw}) = \log(1 + \text{SNR}^\eta |h_c|^2) \doteq (\eta - \beta_c)^+ \log \text{SNR}. \quad (\text{J.5})$$

By the proposed rate and power allocation, we know that for $r \in [0, 1/3]$, $\eta = 1 - r$, $r_{iw} = 0$ and $r_{iu} = r$. In this multiplexing gain region, all the messages are encoded as private message, to decode this private message, we need

$$r \stackrel{\cdot}{\leq} (\eta - \beta_c)^+ = (1 - r - \beta_c)^+ \log \text{SNR}, \quad (\text{J.6})$$

which is equivalent to require $\beta_c \leq (1 - 2r)^+$.

When $r \in (1/3, 1/2]$, we take $\eta = 2(1 - 2r)^+$, $r_{iw} = (3r - 1)$, and $r_{iu} = (1 - 2r)$. For successful decoding of the public message, it requires

$$r_{iw} \stackrel{\cdot}{\leq} (1 - \beta_c)^+ - (\eta - \beta_c)^+. \quad (\text{J.7})$$

When $\beta_c \leq (1 - 2r)^+$, we have $\beta_c < \eta$, (J.7) becomes

$$r_{iw} \stackrel{\cdot}{\leq} (1 - \eta) = 4r - 1. \quad (\text{J.8})$$

Since $r_{iw} = 3r - 1$, the above requirement can be satisfied for sure.

For the decoding of the private message, we must have

$$r_{iu} \stackrel{\cdot}{\leq} (\eta - \beta_c)^+, \quad (\text{J.9})$$

which is equivalent to require $\beta_c \leq (\eta - r_{iu}) = (1 - 2r)^+$. Therefore, when $\beta_c \leq (1 - 2r)^+$, both the public and the private message parts can be successfully decoded with high probability.

Based on our proposed transmission scheme which classifies the channel into different categories, the probability that the outage event $\mathcal{O}$ happens can be evaluated as

$$\begin{aligned} & P(\mathcal{O}) \\ &= P(\mathcal{O} | \beta_c > (1 - r)^+) P(\beta_c > (1 - r)^+) + P(\mathcal{O} | \beta_c \leq (1 - 2r)^+) P(\beta_c \leq (1 - 2r)^+) \\ &\quad + P(\mathcal{O} | (1 - 2r)^+ < \beta_c \leq (1 - r)^+) P((1 - 2r)^+ < \beta_c \leq (1 - r)^+) \\ &= P(\mathcal{O} | \beta_c > (1 - r)^+) P(\beta_c > (1 - r)^+) \\ &\quad + P(\beta_c \leq (1 - 2r)^+) \left(\sum_{m=0}^2 \sum_{n=0}^2 P(\mathcal{O} | L_1 = m, L_2 = n, \beta_c \leq (1 - 2r)^+) P(L_1 = m, L_2 = n) \right) \\ &\quad + P((1 - 2r)^+ < \beta_c \leq (1 - r)^+) \times \end{aligned}$$

$$\left(\sum_{m=0}^2 \sum_{n=0}^2 P(\mathcal{O} | L_1 = m, L_2 = n, (1-2r)^+ < \beta_c \leq (1-r)^+) P(L_1 = m, L_2 = n) \right).$$

In the following, we calculate the outage probability by considering the different channel scenarios.

i) When $\beta_c > (1-r)^+$

In this case, the transmission scheme is to let the sources transmit together without cooperation using the transmission scheme developed in [120]. According to [120], we know that

$$P(\mathcal{O} | \beta_c > (1-r)^+) = \text{SNR}^{-\min\{(1-r)^+, 3(1-2r)^+\}}. \quad (\text{J.10})$$

For the Rayleigh-faded channel h_c , we have

$$P(\beta_c > (1-r)^+) = P(|h_c|^2 < \text{SNR}^{-(1-r)^+}) \doteq \text{SNR}^{-(1-r)}. \quad (\text{J.11})$$

Therefore,

$$P(\mathcal{O} | |h_c|^2 < \text{SNR}^{-(1-r)^+}) P(|h_c|^2 < \text{SNR}^{-(1-r)^+}) = \text{SNR}^{-d_1(r)}, \quad (\text{J.12})$$

where $d_1(r) = \min\{2(1-r)^+, (1-r)^+ + 3(1-2r)^+\}$. It can be easily seen that $d_1(r) \geq d^{\text{FD}}(r)$ over all $r \in [0, 1/2]$.

ii) When $\beta_c < (1-r)^+$, $L_1 = 0$ and $L_2 = 0$

In this case, we have $|h_{i,j+2}|^2 < \text{SNR}^{-(1-2r)^+}$ for all $i = 1, 2, j = 1, 2$. The occurrence probability for this scenario is in the order of $\text{SNR}^{-4(1-2r)^+}$. Therefore we have

$$\begin{aligned} & P(\mathcal{O} | L_1 = 0, L_2 = 0, \beta_c \leq (1-2r)^+) P(L_1 = 0, L_2 = 0, \beta_c \leq (1-2r)^+) \\ & \leq P(L_1 = 0, L_2 = 0, \beta_c \leq (1-2r)^+) \doteq \text{SNR}^{-4(1-2r)}. \end{aligned} \quad (\text{J.13})$$

iii) When $\beta_c < (1-r)^+$, $L_i = 1, 2$ and $L_j = 1, 2$

In this case, one-level Gaussian codes are adopted by the sources. Source i fully decodes the message of source j , re-encodes it into $\tilde{x}_j$ and re-transmits it on top of its own codeword. The total transmit power at each source is equally shared by codeword carrying its own message and the codeword carrying the cooperative message. The signals received by the two destinations become

$$y_3 = \frac{1}{\sqrt{2}} h_{1,3}(x_1 + \tilde{x}_2) + \frac{1}{\sqrt{2}} h_{2,3}(\tilde{x}_1 + x_2) + z_3, \quad (\text{J.14})$$

$$y_4 = \frac{1}{\sqrt{2}} h_{1,4}(x_1 + \tilde{x}_2) + \frac{1}{\sqrt{2}} h_{2,4}(\tilde{x}_1 + x_2) + z_4. \quad (\text{J.15})$$

Destinations perform backward decoding to jointly decode the two messages. For successful decoding at destination 3, the transmission rates should fall in the following region

$$R_1 \leq I(X_1, \tilde{X}_1; Y_3 | X_2, \tilde{X}_2), \quad (\text{J.16a})$$

$$R_1 + R_2 \leq I(X_1, \tilde{X}_1, X_2, \tilde{X}_2; Y_3). \quad (\text{J.16b})$$

The rate region for successful decoding at destination node 4 can be obtained by replacing Y_3 with Y_4 and swapping the subscripts 1 and 2 in (J.16).

The mutual informations in (J.16) can be calculated as

$$\begin{aligned} I(X_1, \tilde{X}_1; Y_3 | X_2, \tilde{X}_2) &= \log \left(1 + \frac{\text{SNR}}{2} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) \\ &\doteq \max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} \log \text{SNR} \\ &\geq 2r \log \text{SNR}, \end{aligned} \quad (\text{J.17})$$

where the last inequality is because for the considered channel scenarios, either $\beta_{13} \leq (1 - 2r)^+$ or $\beta_{23} \leq (1 - 2r)^+$.

$$I(X_2, \tilde{X}_2; Y_3 | X_1, \tilde{X}_1) = \log \left(1 + \frac{\text{SNR}}{2} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) \geq 2r \log \text{SNR}, \quad (\text{J.18})$$

and

$$I(X_1, \tilde{X}_1, X_2, \tilde{X}_2; Y_3) = \log \left(1 + \text{SNR} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) \geq 2r \log \text{SNR}. \quad (\text{J.19})$$

It can be seen that for all $R_1 = R_2 = r \log \text{SNR}$, the system operates within the rate region for decoding at destination 3, no outage happens. Similar analysis can be made for decoding at destination 4. The outage probability under the considered channel scenarios is thus given by

$$\text{P}(\beta_c \leq (1-r)^+) \left(\sum_{i=1}^2 \sum_{j=1}^2 \text{P}(\mathcal{O} | L_1 = i, L_2 = j, \beta_c \leq (1-r)^+) \text{P}(L_1 = i, L_2 = j) \right) \doteq \text{SNR}^{-d_2(r)}, \quad (\text{J.20})$$

where $d_2(r) \rightarrow \infty$. Therefore outage behavior under the considered channel scenarios imposes no constraint on the overall outage behavior.

iv) When $(1 - 2r)^+ < \beta_c \leq (1 - r)^+$, $L_1 = 0$ and $L_2 = 1, 2$

For the case with $L_1 = 0$ and $L_2 = 1$, the transmission scheme is to let both sources

encode using one-level Gaussian code. Both source perform decode-and-forward. Each destination jointly decodes messages from both sources. The achievable rate region of this transmission scheme is (J.16). For the mutual information expressions (J.17)-(J.19) calculated for the Gaussian channel model and applying similar method as in Appendix A, the DMT expression can be evaluated as

$$d_3(r) = \min_{\mathcal{A}_1 \cup \mathcal{B}_1} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{J.21})$$

where the sets $\mathcal{A}_1$ and $\mathcal{B}_1$ are defined as

$$\mathcal{A}_1 = \{0 \leq \beta_{i,j} \leq 1, 0 \leq \beta_c \leq 1 : \quad (\text{J.22a})$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} < r_1 \quad (\text{J.22a})$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} < r_1 + r_2 \quad (\text{J.22b})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} < r_2 \quad (\text{J.22c})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} < r_1 + r_2 \quad (\text{J.22d})$$

$$\}$$

and

$$\begin{aligned} \mathcal{B}_1 = \{ & \beta_{1,3} > (1 - 2r)^+, \beta_{2,3} > (1 - 2r)^+, \beta_{2,4} > (1 - 2r)^+, \beta_{1,4} \leq (1 - 2r)^+, \\ & (1 - 2r)^+ < \beta_c \leq (1 - r)^+ \}. \end{aligned} \quad (\text{J.23})$$

Solving (J.38), $d_3(r)$ can be easily found as

$$d_3(r) = 4(1 - 2r)^+. \quad (\text{J.24})$$

For the case $L_1 = 0$ and $L_2 = 2$, the transmission scheme is to let source 1 encode its message into a one-level Gaussian code x_1 and source 2 encode its message into a two-level Gaussian code $x_2 = x_{2w} + x_{2u}$. The power allocated for x_{2w} and x_{2u} are $\text{SNR} - \text{SNR}^\eta$ and SNR^η , respectively. Source 2 decodes and re-encodes the message from source 1 into $\tilde{x}_1$ and transmits together with its own codeword x_2 . Source 1 does not help relay source 2's message and simply transmit its own codeword x_1 . Upon reception, destination node 3 jointly decodes x_1 , $\tilde{x}_1$ and x_{2w} , treating x_{2u} as interference. Destination node 4 jointly

decode messages from both sources. The Gaussian channel model becomes

$$y_3 = h_{1,3}x_1 + \frac{1}{\sqrt{2}}h_{2,3}(\tilde{x}_1 + x_{2w} + x_{2u}) + z_3 \quad (\text{J.25})$$

$$y_4 = h_{1,4}x_1 + \frac{1}{\sqrt{2}}h_{2,4}(\tilde{x}_1 + x_{2w} + x_{2u}) + z_4. \quad (\text{J.26})$$

The following rate region can be achieved by the proposed transmission scheme.

$$R_1 \leq I(X_1, \tilde{X}_1; Y_3 | X_{2w}) \quad (\text{J.27a})$$

$$R_{2w} + R_1 \leq I(X_1, \tilde{X}_1, X_{2w}; Y_3) \quad (\text{J.27b})$$

$$R_1 \leq I(X_1, \tilde{X}_1; Y_4 | X_{2w}) \quad (\text{J.27c})$$

$$R_{2w} \leq I(X_{2w}; Y_4 | X_1, \tilde{X}_1) \quad (\text{J.27d})$$

$$R_1 + R_{2w} \leq I(X_1, \tilde{X}_1, X_{2w}; Y_4) \quad (\text{J.27e})$$

$$R_{2u} \leq I(X_{2u}; Y_4 | X_1, \tilde{X}_1, X_{2w}). \quad (\text{J.27f})$$

For the considered Gaussian channel, the rate region becomes

$$R_1 \leq \log \left(1 + \frac{\text{SNR}}{2} (2|h_{1,3}|^2 + |h_{2,3}|^2) \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} |h_{2,3}|^2 \right) \quad (\text{J.28a})$$

$$R_{2w} + R_1 \leq \log \left(1 + \text{SNR} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} |h_{2,3}|^2 \right) \quad (\text{J.28b})$$

$$R_1 \leq \log \left(1 + \text{SNR} |h_{1,4}|^2 + \frac{\text{SNR} + \text{SNR}^\eta}{2} |h_{2,4}|^2 \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} |h_{2,4}|^2 \right) \quad (\text{J.28c})$$

$$R_{2w} \leq \log \left(1 + \frac{\text{SNR}}{2} |h_{2,4}|^2 \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} |h_{2,4}|^2 \right) \quad (\text{J.28d})$$

$$R_1 + R_{2w} \leq \log \left(1 + \text{SNR} |h_{1,4}|^2 + \text{SNR} |h_{2,4}|^2 \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} |h_{2,4}|^2 \right) \quad (\text{J.28e})$$

$$R_{2u} \leq \log \left(1 + \frac{\text{SNR}^\eta}{2} |h_{2,4}|^2 \right). \quad (\text{J.28f})$$

By applying the similar method as in Appendix G, the DMT expression can be evaluated as

$$d_4(r) = \min_{\mathcal{A}_2 \cup \mathcal{B}_2} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{J.29})$$

where where the sets $\mathcal{A}_2$ and $\mathcal{B}_2$ are defined as

$$\mathcal{A}_2 = \{0 \leq \beta_{i,j} \leq 1, 0 \leq \beta_c \leq 1 :$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} - (\eta - \beta_{2,3})^+ < r_1 \quad (\text{J.30a})$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} - (\eta - \beta_{2,3})^+ < r_1 + r_{2w} \quad (\text{J.30b})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - (\eta - \beta_{2,4})^+ < r_1 \quad (\text{J.30c})$$

$$(1 - \beta_{2,4})^+ - (\eta - \beta_{2,4})^+ < r_{2w} \quad (\text{J.30d})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - (\eta - \beta_{2,4})^+ < r_1 + r_{2w} \quad (\text{J.30e})$$

$$(\eta - \beta_{2,4})^+ < r_{2u} \quad (\text{J.30f})$$

}

and

$$\begin{aligned} \mathcal{B}_2 = & \{ \beta_{1,3} > (1 - 2r)^+, \beta_{2,3} > (1 - 2r)^+, \beta_{2,4} \leq (1 - 2r)^+, \\ & (1 - 2r)^+ < \beta_c \leq (1 - r)^+ \}. \end{aligned} \quad (\text{J.31})$$

Solving (J.29), $d_3(r)$ can be easily found as

$$d_3(r) = (1 - 2r)^+ + \min\{2(1 - r), 4(1 - 2r)^+\}. \quad (\text{J.32})$$

v) When $\beta_c \leq (1 - 2r)^+$, $L_1 = 0$ and $L_2 = 0, 1, 2$

For the case $L_1 = L_2 = 0$, we have $|h_{i,j}|^2 < \text{SNR}^{-(1-2r)^+}$ for all $i = 1, 2, j = 1, 2$. The occurrence probability for this scenario is in the order of $\text{SNR}^{-4(1-2r)^+}$. Therefore we have

$$\begin{aligned} & \text{P}(\mathcal{O} | L_1 = 0, L_2 = 0, \beta_c \leq (1 - 2r)^+) \text{P}(L_1 = 0, L_2 = 0, \beta_c \leq (1 - 2r)^+) \\ & \leq \text{P}(L_1 = 0, L_2 = 0, \beta_c \leq (1 - 2r)^+) \doteq \text{SNR}^{-4(1-2r)^+}. \end{aligned} \quad (\text{J.33})$$

For the case with $L_1 = 0$ and $L_2 = 1$, the transmission scheme is to let source 1 encode using one-level Gaussian code, source 2 use two-level Gaussian code. Source 1 re-encodes the decoded message from source 2 into a two-level code $\tilde{x}_2 = \tilde{x}_{2w} + \tilde{x}_{2u}$ with $\tilde{x}_{2w}$ at a power $\text{SNR} - \text{SNR}^\eta$ and $\tilde{x}_{2u}$ at a power SNR^η , and transmits together with its own codeword x_1 . Source 2 re-encodes the message from source 1 as $\tilde{x}_1$ and retransmits it with its own codeword $x_2 = x_{2w} + x_{2u}$. The signals received at the destinations are thus

$$y_3 = \frac{1}{\sqrt{2}} h_{1,3} (x_1 + \tilde{x}_{2w} + \tilde{x}_{2u}) + \frac{1}{\sqrt{2}} h_{2,3} (\tilde{x}_1 + x_{2w} + x_{2u}) + z_3 \quad (\text{J.34})$$

$$y_4 = \frac{1}{\sqrt{2}} h_{1,4} (x_1 + \tilde{x}_{2w} + \tilde{x}_{2u}) + \frac{1}{\sqrt{2}} h_{2,4} (\tilde{x}_1 + x_{2w} + x_{2u}) + z_4. \quad (\text{J.35})$$

For successful decoding, the transmission rates should fall in the following region.

$$R_1 \leq I(X_1, \tilde{X}_1; Y_3 | X_{2w}, \tilde{X}_{2w}) \quad (\text{J.36a})$$

$$R_{2w} + R_1 \leq I(X_1, \tilde{X}_1, X_{2w}, \tilde{X}_{2w}; Y_3) \quad (\text{J.36b})$$

$$R_1 \leq I(X_1, \tilde{X}_1; Y_4 | X_{2w}, \tilde{X}_{2w}) \quad (\text{J.36c})$$

$$R_{2w} \leq I(X_{2w}, \tilde{X}_{2w}; Y_4 | X_1, \tilde{X}_1) \quad (\text{J.36d})$$

$$R_1 + R_{2w} \leq I(X_1, \tilde{X}_1, X_{2w}, \tilde{X}_{2w}; Y_4) \quad (\text{J.36e})$$

$$R_{2u} \leq I(X_{2u}, \tilde{X}_{2u}; Y_4 | X_1, \tilde{X}_1, X_{2w}, \tilde{X}_{2w}). \quad (\text{J.36f})$$

For the considered Gaussian channel, the rate region becomes

$$R_1 \leq \log \left(1 + \frac{\text{SNR} + \text{SNR}^\eta}{2} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) \quad (\text{J.37a})$$

$$R_{2w} + R_1 \leq \log \left(1 + \text{SNR} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) \quad (\text{J.37b})$$

$$R_1 \leq \log \left(1 + \frac{\text{SNR} + \text{SNR}^\eta}{2} (|h_{1,4}|^2 + |h_{2,4}|^2) \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} (|h_{1,4}|^2 + |h_{2,4}|^2) \right) \quad (\text{J.37c})$$

$$R_{2w} \leq \log \left(1 + \frac{\text{SNR}}{2} (|h_{1,4}|^2 + |h_{2,4}|^2) \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} (|h_{1,4}|^2 + |h_{2,4}|^2) \right) \quad (\text{J.37d})$$

$$R_1 + R_{2w} \leq \log \left(1 + \text{SNR} (|h_{1,4}|^2 + |h_{2,4}|^2) \right) - \log \left(1 + \frac{\text{SNR}^\eta}{2} (|h_{1,4}|^2 + |h_{2,4}|^2) \right) \quad (\text{J.37e})$$

$$R_{2u} \leq \log \left(1 + \frac{\text{SNR}^\eta}{2} (|h_{1,4}|^2 + |h_{2,4}|^2) \right). \quad (\text{J.37f})$$

Following a similar method used in Appendix G, the DMT expression can be evaluated as

$$d_5(r) = \min_{\mathcal{A}_3 \cup \mathcal{B}_3} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{J.38})$$

where the sets $\mathcal{A}_3$ and $\mathcal{B}_3$ are defined as

$$\begin{aligned} \mathcal{A}_3 = \{ & 0 \leq \beta_{i,j} \leq 1, 0 \leq \beta_c \leq 1 : \\ & \max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} - \max\{(\eta - \beta_{1,3})^+, (\eta - \beta_{2,3})^+\} < r_1 \end{aligned} \quad (\text{J.39a})$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} - \max\{(\eta - \beta_{1,3})^+, (\eta - \beta_{2,3})^+\} < r_1 + r_{2w} \quad (\text{J.39b})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - \max\{(\eta - \beta_{1,4})^+, (\eta - \beta_{2,4})^+\} < r_1 \quad (\text{J.39c})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - \max\{(\eta - \beta_{1,4})^+, (\eta - \beta_{2,4})^+\} < r_{2w} \quad (\text{J.39d})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - \max\{(\eta - \beta_{1,4})^+, (\eta - \beta_{2,4})^+\} < r_1 + r_{2w} \quad (\text{J.39e})$$

$$\max\{(\eta - \beta_{1,4})^+, (\eta - \beta_{2,4})^+\} < r_{2u} \quad (\text{J.39f})$$

$\},$

$$\begin{aligned} \mathcal{B}_3 = \{ & \beta_{1,3} > (1 - 2r)^+, \beta_{2,3} > (1 - 2r)^+, \\ & \beta_{2,4} > (1 - 2r)^+, \beta_{1,4} \leq (1 - 2r)^+, \beta_c \leq (1 - 2r)^+ \}. \end{aligned} \quad (\text{J.40})$$

By the proposed rate and power allocation, it can be easily calculated that

$$d_5(r) = \min\{2(1 - r), 4(1 - 2r)^+\}. \quad (\text{J.41})$$

For the case with $L_1 = 0$ and $L_2 = 2$, similar transmission scheme is adopted, the DMT can also be calculated from (J.38) with $\mathcal{B}_3$ being replaced by $\mathcal{B}_4$, defined as

$$\mathcal{B}_4 = \{\beta_{1,3} > (1 - 2r)^+, \beta_{2,3} > (1 - 2r)^+, \beta_{2,4} \leq (1 - 2r)^+, \beta_c \leq (1 - 2r)^+\}. \quad (\text{J.42})$$

Simple calculation leads to

$$d_6(r) = \min\{2(1 - r), 4(1 - 2r)^+\}. \quad (\text{J.43})$$

Due to the symmetry in the channel settings, the DMT for cases with $L_2 = 0$ and $L_1 = 0, 1, 2$ can be analyzed in the same fashion. Summarizing all the channel scenarios, we have

$$P(\mathcal{O}) \leq \sum_{i=1}^6 \text{SNR}^{-d_i(r)} + \text{SNR}^{-4(1-2r)} \doteq \text{SNR}^{-d_6(r)}. \quad (\text{J.44})$$

Therefore, the achievable DMT is

$$d^{FD-LB} = \min\{2(1-r)^+, 4(1-2r)^+\}. \quad (\text{J.45})$$

Appendix K

Proof of Theorem 6.4.1

The upper bounds (6.4.1a)-(6.4.1e) are obtained by using the cut-set upper bound for networks with half-duplex constraints proposed in [31].

The bound in (6.4.1f) is obtained by using a genie-aided channel, which is constructed in a same way as in [54]. We denote

$$s_1(t) = h_{1,2}x_1(t) + z_2(t) \quad (\text{K.1})$$

$$s_2(t) = h_{2,1}x_2(t) + z_1(t) \quad (\text{K.2})$$

$$s_3(t) = h_{1,4}x_1(t) + z_4(t) \quad (\text{K.3})$$

$$s_4(t) = h_{2,3}x_2(t) + z_3(t), \quad (\text{K.4})$$

the genie-aided channel is constructed by letting a genie provide s_1 , s_2 and s_3 to destination node 3 and a genie provide s_1 , s_2 and s_4 to destination node 4. Let W_i as the message transmitted from source i during one block of length N (here, we assume a channel with ergodic fading), the sum rate of the original channel can be bounded as follows:

$$\begin{aligned} R_1 + R_2 &\leq \frac{1}{N} (H(W_1) + H(W_2)) \\ &\leq \frac{1}{N} (I(W_1; Y_3^N) + I(W_2; Y_4^N)) \\ &\leq \frac{1}{N} (I(W_1; Y_3^N, S_1^N, S_2^N, S_3^N) + I(W_2; Y_4^N, S_1^N, S_2^N, S_4^N)). \end{aligned} \quad (\text{K.5})$$

Following the same procedure as in [54], the sum rate in (K.5) can be further upper bounded as

$$R_1 + R_2 \leq \frac{1}{N} \left\{ H(Y_3^N | S_3^N, S_2^N) - \sum_{t=1}^N H(S_4(t) | X_2(t)) + H(S_1^N) - \sum_{t=1}^N H(S_1(t) | X_1(t)) \right\}$$

$$+\frac{1}{N}\left\{H(Y_4^N|S_4^N, S_1^N) - \sum_{t=1}^N H(S_3(t)|X_1(t)) + H(S_2^N) - \sum_{t=1}^N H(S_2(t)|X_2(t))\right\}. \quad (\text{K.6})$$

For the first term in (K.6), we have

$$\begin{aligned} & \frac{1}{N}\left\{H(Y_3^N|S_3^N, S_2^N) - \sum_{t=1}^N H(S_4(t)|X_2(t)) + H(S_1^N) - \sum_{t=1}^N H(S_1(t)|X_1(t))\right\} \\ & \leq \frac{1}{N} \sum_{t=1}^N H(Y_3(t)|S_3(t), S_2(t)) - \sum_{t=1}^N H(S_4(t)|X_2(t)) + \sum_{t=1}^N H(S_1(t)) - \sum_{t=1}^N H(S_1(t)|X_1(t)) \\ & \leq \frac{\tau_1 N}{N} \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} H(Y_3(t)|S_3(t)) + \frac{\tau_2 N}{N} \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} H(Y_3(t)|S_2(t)) \\ & \quad + \frac{(1-\tau_1-\tau_2)N}{N} \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N H(Y_3(t)|S_3(t)) - \frac{1}{N} \sum_{t=1}^N H(Z_3(t)) \\ & \quad + \frac{\tau_1 N}{N} \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} (H(S_1(t)) - H(Z_2(t))) \\ & \leq \tau_1 \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} \left(\log(\pi e)^2 (\sigma_w^4 + P_{1t} \sigma_w^2 (|h_{1,3}|^2 + |h_{1,4}|^2)) - \log \pi e (\sigma_w^2 + P_{1t} |h_{1,4}|^2) \right) \\ & \quad + \tau_2 \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} \left(\log(\pi e)^2 (\sigma_w^4 + P_{2t} \sigma_w^2 (|h_{2,3}|^2 + |h_{2,1}|^2)) - \log \pi e (\sigma_w^2 + P_{2t} \sigma_w^2 |h_{2,1}|^2) \right) \\ & \quad + (1-\tau_1-\tau_2) \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N \left(\log(\pi e)^2 (\sigma_w^4 + P_{1t} \sigma_w^2 (|h_{1,4}|^2 + |h_{1,3}|^2) \right. \\ & \quad \left. + P_{2t} |h_{2,3}|^2 (P_{1t} |h_{1,4}|^2 + \sigma_w^2)) - \log \pi e (\sigma_w^2 + P_{1t} |h_{1,4}|^2) \right) - \log \pi e \sigma_w^2 \\ & \quad + \tau_1 \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} \left(\log \pi e (\sigma_w^2 + |h_{2,1}|^2 P_{1t}) - \log \pi e \sigma_w^2 \right) \\ & \leq \tau_1 \log \pi e \left(\frac{\sigma_w^4 + \frac{\sigma_w^2}{\tau_1 N} \sum_{t=1}^{\tau_1 N} P_{1t} (|h_{1,3}|^2 + |h_{1,4}|^2)}{\sigma_w^2 + \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} P_{1t} |h_{1,4}|^2} \right) \\ & \quad + \tau_2 \log \pi e \left(\frac{\sigma_w^4 + \frac{\sigma_w^2}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} P_{2t} (|h_{2,3}|^2 + |h_{2,1}|^2)}{\sigma_w^2 + \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} P_{2t} |h_{2,1}|^2} \right) \\ & \quad + (1-\tau_1-\tau_2) \log \pi e \left(\sigma_w^2 + \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N P_{2t} |h_{2,3}|^2 \right) \end{aligned}$$

$$\begin{aligned}
& + \frac{\frac{\sigma_w^2}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N P_{1t}|h_{1,3}|^2}{\sigma_w^2 + \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N}^N P_{1t}|h_{1,4}|^2} \Big) - \log \pi e \sigma_w^2 + \tau_1 \log \left(1 + \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} \frac{P_{1t}}{\sigma_w^2} |h_{1,2}|^2 \right) \\
\leq & \tau_1 \log \left(1 + \frac{\text{SNR}|h_{1,3}|^2}{1 + \text{SNR}|h_{1,4}|^2} \right) + \tau_2 \log \left(1 + \frac{\text{SNR}|h_{2,3}|^2}{1 + \text{SNR}|h_{2,1}|^2} \right) \\
& + (1 - \tau_1 - \tau_2) \log \left(1 + \text{SNR}|h_{2,3}|^2 + \frac{\text{SNR}|h_{1,3}|^2}{1 + \text{SNR}|h_{1,4}|^2} \right) + \tau_1 \log \left(1 + \text{SNR}|h_{1,2}|^2 \right), \quad (\text{K.7})
\end{aligned}$$

where in the first two inequalities, we use the fact that conditioning reduce entropy, in the third inequality, we use the fact that Gaussian inputs maximize entropy, and in the last two inequalities, we use Jensen's inequality.

Similarly, for the second term in (K.6), we have

$$\begin{aligned}
& \frac{1}{N} \left\{ H(Y_4^N | S_4^N, S_1^N) - \sum_{t=1}^N H(S_3(t) | X_1(t)) + H(S_2^N) - \sum_{t=1}^N H(S_2(t) | X_2(t)) \right\} \\
\leq & \tau_1 \log \left(1 + \frac{\text{SNR}|h_{1,4}|^2}{1 + \text{SNR}|h_{1,2}|^2} \right) + \tau_2 \log \left(1 + \frac{\text{SNR}|h_{2,4}|^2}{1 + \text{SNR}|h_{2,3}|^2} \right) \\
& + (1 - \tau_1 - \tau_2) \log \left(1 + \text{SNR}|h_{1,4}|^2 + \frac{\text{SNR}|h_{2,4}|^2}{1 + \text{SNR}|h_{2,3}|^2} \right) + \tau_2 \log \left(1 + \text{SNR}|h_{2,1}|^2 \right). \quad (\text{K.8})
\end{aligned}$$

The upper bound in (6.4.1f) can be obtained by adding (K.7) and (K.8).

The upper bounds in (6.4.1g) and (6.4.1h) are obtained by using the capacity upper bound for the corresponding Z channels. In particular, (6.4.1g) is obtained by giving destination node 4 full information about the message transmitted from source 1, and (6.4.1h) is obtained by providing destination node 3 full information about the message transmitted from source 2. In the following, the derivation for the upper bound (6.4.1g) is provided. From [54], we know that

$$\begin{aligned}
R_1 + R_2 & \leq \frac{1}{N} (H(W_1) + H(W_2)) \\
& \leq \frac{1}{N} (I(W_1; Y_3^N) + I(W_2; Y_4^N)) \\
& \leq \frac{1}{N} (I(W_1; Y_3^N) + I(W_2; Y_4^N, S_4^N, S_1^N, S_2^N, W_1)) \\
& = \frac{1}{N} (I(W_1; Y_3^N) + I(W_2; Y_4^N, S_4^N, S_1^N, S_2^N | W_1)) \\
& \leq \frac{1}{N} \left(\sum_{t=1}^N H(Y_3(t) | Y_3^{t-1}) - H(S_4(t) | Y_4^{t-1}, S_4^{t-1}, S_1^{t-1}, S_2^{t-1}, W_1, W_2) \right. \\
& \quad \left. + H(Y_4(t) | X_1(t), S_4(t)) - H(Y_4(t) | X_1(t), X_2(t)) \right)
\end{aligned}$$

$$\begin{aligned}
& +H(S_2(t)|Y_4(t), S_4(t), X_1(t)) - H(S_2(t)|X_2(t))) \\
\leq & \frac{1}{N} \left(\sum_{t=1}^N H(Y_3(t)) - H(Z_3(t)) + H(Y_4(t)|X_1(t), S_4(t)) - H(Z_4(t)) \right. \\
& \left. + H(S_2(t)|Y_4(t), S_4(t), X_1(t)) - H(Z_1(t)) \right). \tag{K.9}
\end{aligned}$$

The terms in (K.9) can be evaluated as

$$\begin{aligned}
& \frac{1}{N} \left(\sum_{t=1}^N H(Y_3(t)) - H(Z_3(t)) \right) \\
= & \frac{\tau_1 N}{N} \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} H(Y_3(t)) + \frac{\tau_2 T}{N} \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} H(Y_3(t)) \\
& + \frac{(1-\tau_1-\tau_2)N}{N} \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N H(Y_3(t)) - H(Z_3^N) \\
\leq & \tau_1 \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} \log \pi e(\sigma_w^2 + P_{1t}|h_{1,3}|^2) + \tau_2 \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} \log \pi e(\sigma_w^2 + P_{2t}|h_{2,3}|^2) \\
& + (1-\tau_1-\tau_2) \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N \log \pi e(\sigma_w^2 + P_{1t}|h_{1,3}|^2 + P_{2t}|h_{2,3}|^2) - \log \pi e \sigma_w^2 \\
\leq & \tau_1 \log(1 + \text{SNR}|h_{1,3}|^2) + \tau_2 \log(1 + \text{SNR}|h_{2,3}|^2) \\
& + (1-\tau_1-\tau_2) \log(1 + \text{SNR}(|h_{1,3}|^2 + |h_{2,3}|^2)), \tag{K.10}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{N} \left(\sum_{t=1}^N H(Y_4(t)|X_1(t), S_4(t)) - H(Z_4(t)) \right) \\
\leq & \frac{\tau_1 N}{N} \frac{1}{\tau_1 N} \sum_{t=1}^{\tau_1 N} H(Y_4(t)|X_1(t)) + \frac{\tau_2 N}{N} \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} H(Y_4(t)|S_4(t)) \\
& + \frac{(1-\tau_1-\tau_2)N}{N} \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N H(Y_4(t)|X_1(t), S_4(t)) - \frac{1}{N} \sum_{t=1}^N H(Z_4(t)) \\
\leq & \tau_1 \log(\pi e \sigma_w^2) \\
& + \tau_2 \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} \left(\log(\pi e)^2 (\sigma_w^4 + P_{2t} \sigma_w^2 (|h_{2,4}|^2 + |h_{2,3}|^2)) - \log \pi e (\sigma_w^2 + P_{2t} |h_{2,3}|^2) \right) \\
& + (1-\tau_1-\tau_2) \frac{1}{(1-\tau_1-\tau_2)N} \sum_{t=(\tau_1+\tau_2)N+1}^N \left(\log(\pi e)^2 (\sigma_w^4 + P_{2t} \sigma_w^2 (|h_{2,4}|^2 + |h_{2,3}|^2)) \right. \\
& \left. - \log \pi e (\sigma_w^2 + P_{2t} |h_{2,3}|^2) \right) - \log \pi e \sigma_w^2
\end{aligned}$$

$$\leq (1 - \tau_1) \log \left(1 + \frac{\text{SNR} |h_{2,4}|^2}{1 + \text{SNR} |h_{2,3}|^2} \right), \quad (\text{K.11})$$

and

$$\begin{aligned} & \frac{1}{N} \left(\sum_{t=1}^N H(S_2(t)|Y_4(t), S_4(t), X_1(t)) - H(Z_1(t)) \right) \\ &= \frac{\tau_2 T}{N} \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} H(S_2(t)|Y_4(t), S_4(t)) - H(Z_1(t)) \\ &\leq \tau_2 \frac{1}{\tau_2 N} \sum_{t=\tau_1 N+1}^{(\tau_1+\tau_2)N} \left(\log(\pi e)^3 (\sigma_w^6 + P_{2t} \sigma_w^4 (|h_{2,1}|^2 + |h_{2,4}|^2 + |h_{2,3}|^2)) \right. \\ &\quad \left. - \log(\pi e)^2 (\sigma_w^4 + P_{2t} \sigma_w^2 (|h_{2,4}|^2 + |h_{2,3}|^2)) \right) - \tau_2 \log \pi e \sigma_w^2 \\ &\leq \tau_2 \log \left(1 + \frac{\text{SNR} |h_{2,1}|^2}{1 + \text{SNR} (|h_{2,4}|^2 + |h_{2,3}|^2)} \right). \end{aligned} \quad (\text{K.12})$$

Summarizing (K.10)-(K.12), we have the upper bound (6.4.1g). The upper bound (6.4.1h) can be derived in a same fashion.

Appendix L

Proof of Theorem 6.4.2

Using the capacity upper bounds (6.4.1a)-(6.4.1d) and (6.4.1g)-(6.4.1h), the DMT upper bound of the channel can be evaluated as

$$d(r) = \min_{\mathcal{F}} \beta_{1,3} + \beta_{2,3} + \beta_{1,4} + \beta_{2,4} + \beta_c, \quad (\text{L.1})$$

where the set $\mathcal{F}$ is defined as

$$\mathcal{F} = \{\boldsymbol{\beta} : 0 \leq \beta_{i,j} \leq 1, 0 \leq \beta_c \leq 1$$

$$\tau_1 \max\{(1 - \beta_{1,2})^+, (1 - \beta_{1,3})^+\} + (1 - \tau_1 - \tau_2)(1 - \beta_{1,3})^+ < r_1 \quad (\text{L.2a})$$

$$\tau_1(1 - \beta_{1,3})^+ + \tau_2(1 - \beta_{2,3})^+ + (1 - \tau_1 - \tau_2) \max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} < r_1 \quad (\text{L.2b})$$

$$\tau_2 \max\{(1 - \beta_{2,1})^+, (1 - \beta_{2,4})^+\} + (1 - \tau_1 - \tau_2)(1 - \beta_{2,4})^+ < r_2 \quad (\text{L.2c})$$

$$\tau_1(1 - \beta_{1,4})^+ + \tau_2(1 - \beta_{2,4})^+ + (1 - \tau_1 - \tau_2) \max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} < r_2 \quad (\text{L.2d})$$

$$\begin{aligned} & \tau_1(1 - \beta_{1,3})^+ + \tau_2(1 - \beta_{2,3})^+ + (1 - \tau_1 - \tau_2) \max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} \\ & + (1 - \tau_1)((1 - \beta_{2,4}) - (1 - \beta_{2,3})^+)^+ \\ & + \tau_2((1 - \beta_{2,1}) - \max\{(1 - \beta_{2,4})^+, (1 - \beta_{2,3})^+\})^+ < r_1 + r_2 \end{aligned} \quad (\text{L.2e})$$

$$\begin{aligned} & \tau_1(1 - \beta_{1,4})^+ + \tau_2(1 - \beta_{2,4})^+ + (1 - \tau_1 - \tau_2) \max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} \\ & + (1 - \tau_2)((1 - \beta_{1,3}) - (1 - \beta_{1,4})^+)^+ \\ & + \tau_1((1 - \beta_{1,2}) - \max\{(1 - \beta_{1,3})^+, (1 - \beta_{1,4})^+\})^+ < r_1 + r_2 \end{aligned} \quad (\text{L.2f})$$

$\}$.

Due to the symmetry of the expressions in the set $\mathcal{F}$, it can be easily seen that the DMT is maximized when $\tau_1 = \tau_2 = \tau$. By some simple calculation, we have

$$d^{HD}(r) = \max_{0 \leq \tau \leq 1} \min\{d_1^{HD}(\tau, r), d_2^{HD}(\tau, r)\}, \quad (\text{L.3})$$

where

$$d_1^{HD}(\tau, r) = \begin{cases} 2 - \frac{r}{\tau} & \text{if } \tau \leq \min\{r, \frac{1}{3}\} \\ \left(1 + \frac{\tau}{1-2\tau}\right) - \frac{r}{1-2\tau} & \text{if } r < \tau \leq \frac{1}{3} \\ 2\left(1 - \frac{r}{1-2\tau}\right)^+ & \text{if } \tau > \frac{1}{3}, \end{cases} \quad (\text{L.4})$$

and

$$d_2^{HD}(\tau, r) = \begin{cases} \frac{3(1-2r)^+}{1-\tau} & \text{if } \tau \leq \min\{2r, \frac{1}{4}\} \\ 3 + \left(1 - \frac{2r}{\tau}\right)^+ & \text{if } 2r < \tau \leq \frac{1}{4} \\ 1 + 3\left(1 - \frac{2r}{1-\tau}\right)^+ & \text{if } \frac{1}{4} < \tau \leq (1-2r) \\ \frac{(1-2r)^+}{\tau} & \text{if } \tau > \max\{(1-2r), \frac{1}{4}\}. \end{cases} \quad (\text{L.5})$$

It can be verified that the maximum value of $d_1^{HD}(\tau, r)$ is $(2-3r)$, which is obtained by setting $\tau = 1/3$. The maximum value of $d_2^{HD}(\tau, r)$ is $4(1-2r)^+$, which is obtained by setting $\tau = 1/4$. If $\tau = 1/3$, we have

$$d^{HD}\left(\frac{1}{3}, r\right) = \min\left\{d_1^{HD}\left(\frac{1}{3}, r\right), d_2^{HD}\left(\frac{1}{3}, r\right)\right\} = \begin{cases} 2-3r & \text{if } r \leq \frac{1}{3} \\ 3(1-2r)^+ & \text{if } \frac{1}{3} < r \leq \frac{1}{2}. \end{cases} \quad (\text{L.6})$$

If $\tau = 1/4$, we have

$$d^{HD}\left(\frac{1}{4}, r\right) = \min\left\{d_1^{HD}\left(\frac{1}{4}, r\right), d_2^{HD}\left(\frac{1}{4}, r\right)\right\} = \begin{cases} 2(1-2r)^+ & \text{if } r \leq \frac{1}{4} \\ \frac{3}{2} - 2r & \text{if } \frac{1}{4} < r \leq \frac{5}{12} \\ 4(1-2r)^+ & \text{if } \frac{5}{12} < r \leq \frac{1}{2}. \end{cases} \quad (\text{L.7})$$

If τ can be either $1/3$ or $1/4$, we have

$$\tilde{d}^{HD}(r) = \min\left\{d^{HD}\left(\frac{1}{3}, r\right), d^{HD}\left(\frac{1}{4}, r\right)\right\} = \begin{cases} 2-3r & \text{if } r \leq \frac{1}{3} \\ 3(1-2r) & \text{if } \frac{1}{3} < r \leq \frac{3}{8} \\ \frac{3}{2} - 2r & \text{if } \frac{3}{8} < r \leq \frac{5}{12} \\ 4(1-2r)^+ & \text{if } \frac{5}{12} < r \leq \frac{1}{2}. \end{cases} \quad (\text{L.8})$$

Since for $r \leq 1/3$ and $5/12 < r \leq 1/2$, $\tilde{d}^{HD}(r)$ achieves the maximum values of $d_1^{HD}(\tau, r)$ and $d_2^{HD}(\tau, r)$, respectively, the DMT upper bound in these regions can thus be settled. What remains to be shown is DMT upper bound in the multiplexing gain region $1/3 < r \leq 5/12$.

It can be shown that for $1/3 < r \leq 5/12$, the optimal τ value should be in the range $[1/4, 1/3)$, therefore

$$d^{HD}(r) = \max_{\tau \in [1/4, 1/3)} \min \left\{ \left(1 + \frac{\tau}{1-2\tau}\right) - \frac{r}{1-2\tau}, \frac{1-2r}{\tau} \right\}. \quad (\text{L.9})$$

For any fixed $r \in (1/3, 5/12]$, $(1 + \frac{\tau}{1-2\tau}) - \frac{r}{1-2\tau}$ is an increasing function of τ and $\frac{1-2r}{\tau}$ is a decreasing function of τ . When τ increases from $1/4$ to $1/3$, $(1 + \frac{\tau}{1-2\tau}) - \frac{r}{1-2\tau}$ increases from $\frac{3}{2} - 2r$ to $4(1-2r)$ and $\frac{1-2r}{\tau}$ decreases from $4(1-2r)$ to $3(1-2r)$, the optimal τ^* value is the one that achieves

$$\left(1 + \frac{\tau^*}{1-2\tau^*}\right) - \frac{r}{1-2\tau^*} = \frac{1-2r}{\tau^*}. \quad (\text{L.10})$$

Solving for τ^* , we have

$$\tau^* = \frac{(3-5r) - \sqrt{(5r-3)^2 - 4(1-2r)}}{2}. \quad (\text{L.11})$$

The DMT upper bound for $r \in (1/3, 5/12]$ is thus

$$d^{HD}(r) = \frac{(1-2r)^+}{\tau^*} = \frac{2(1-2r)^+}{(3-5r) - \sqrt{(5r-3)^2 - 4(1-2r)}}. \quad (\text{L.12})$$

Together with the DMT results in (L.8), we have the DMT upper bound expression given in (6.4.2). This completes the proof for Theorem 6.4.1.

Appendix M

Proof of Theorem 6.4.3

We first evaluate the achievable rate regions for the schemes I-IV. We denote $Y_j^{(m)}$, ($j = 1, \dots, 4, m = 1, 2, 3$), as the received signal at destination node j in the m th transmission phase. To facilitate illustration, we further assume $A = 1$ and $B = 2$, i.e., set source A as the source node 1 and source B as the source node 2. By applying some standard methods in information theory, the achievable rate regions for the schemes I-IV can be found. Here, we omit the proof and only present the results. For scheme I, the rate region for a general discrete memoryless channel is given as

$$R_{11} \leq t_1 I(X_{11}^{(1)}; Y_2^{(1)}) \quad (\text{M.1a})$$

$$R_{12} \leq t_2 I(X_{12}; Y_3^{(2)} | X_{11}^{(2)}, \tilde{X}_{11}^{(2)}, X_2) \quad (\text{M.1b})$$

$$R_2 + R_{12} \leq t_2 I(X_{12}, X_2; Y_3^{(2)} | X_{11}^{(2)}, \tilde{X}_{11}^{(2)}) \quad (\text{M.1c})$$

$$R_1 + R_2 \leq t_1 I(X_{11}^{(1)}; Y_3^{(1)}) + t_2 I(X_{12}, X_2, X_{11}^{(2)}, \tilde{X}_{11}^{(2)}; Y_3^{(2)}) \quad (\text{M.1d})$$

$$R_2 \leq t_2 I(X_2; Y_4^{(2)} | X_{11}^{(2)}, \tilde{X}_{11}^{(2)}, X_{12}) \quad (\text{M.1e})$$

$$R_{12} + R_2 \leq t_2 I(X_2, X_{12}; Y_4^{(2)} | X_{11}^{(2)}, \tilde{X}_{11}^{(2)}) \quad (\text{M.1f})$$

$$R_1 + R_2 \leq t_1 I(X_{11}^{(1)}; Y_4^{(1)}) + t_2 I(X_2, X_{12}, X_{11}^{(2)}, \tilde{X}_{11}^{(2)}; Y_4^{(2)}). \quad (\text{M.1g})$$

For scheme II, the rate region is given as

$$R_{11} \leq t_1 I(X_{11}^{(1)}; Y_2^{(1)} | \tilde{X}_{21}^{(1)}) \quad (\text{M.2a})$$

$$R_{21} \leq t_2 I(X_{21}^{(2)}; Y_1^{(2)}) \quad (\text{M.2b})$$

$$R_{12} \leq t_3 I(X_{12}; Y_3^{(3)} | X_{11}^{(3)}, \tilde{X}_{11}^{(3)}, X_{22}) \quad (\text{M.2c})$$

$$R_{22} + R_{12} \leq t_3 I(X_{22}, X_{12}; Y_3^{(3)} | X_{11}^{(3)}, \tilde{X}_{11}^{(3)}) \quad (\text{M.2d})$$

$$R_1 + R_{22} \leq t_1 I(X_{11}^{(1)}; Y_3^{(1)}) + t_3 I(X_{11}^{(3)}, \tilde{X}_{11}^{(3)}, X_{12}, X_{22}; Y_3^{(3)}) \quad (\text{M.2e})$$

$$R_{12} \leq t_3 I(X_{12}; Y_4^{(3)} | X_{11}^{(3)}, \tilde{X}_{11}^{(3)}, X_{22}) \quad (\text{M.2f})$$

$$R_{22} \leq t_3 I(X_{22}; Y_4^{(3)} | X_{11}^{(3)}, \tilde{X}_{11}^{(3)}, X_{12}) \quad (\text{M.2g})$$

$$R_{12} + R_{22} \leq t_3 I(X_{12}, X_{22}; Y_4^{(3)} | X_{11}^{(3)}, \tilde{X}_{11}^{(3)}) \quad (\text{M.2h})$$

$$R_1 + R_{22} \leq t_1 I(X_{11}^{(1)}; Y_4^{(1)}) + t_3 I(X_{11}^{(3)}, \tilde{X}_{11}^{(3)}, X_{12}, X_{22}; Y_4^{(3)}) \quad (\text{M.2i})$$

$$\begin{aligned} R_{21} &\leq t_1 I(\tilde{X}_{21}^{(1)}; Y_4^{(1)} | X_{11}^{(1)}) + t_2 I(X_{21}^{(2)}; Y_4^{(2)}) \\ &\quad + t_3 I(\tilde{X}_{21}^{(3)}; Y_4^{(3)} | X_{11}^{(3)}, \tilde{X}_{11}^{(3)}, X_{12}, X_{22}). \end{aligned} \quad (\text{M.2j})$$

For scheme III, we have the rate region

$$R_1 \leq I(X_1; Y_3 | X_{2w}) \quad (\text{M.3a})$$

$$R_{2w} + R_1 \leq I(X_{2w}, X_1; Y_3) \quad (\text{M.3b})$$

$$R_1 \leq I(X_1; Y_4 | X_{2w}) \quad (\text{M.3c})$$

$$R_{2w} \leq I(X_{2w}; Y_4 | X_1) \quad (\text{M.3d})$$

$$R_1 + R_{2w} \leq I(X_1, X_{2w}; Y_4) \quad (\text{M.3e})$$

$$R_{2u} \leq I(X_{2u}; Y_4 | X_1, X_{2w}). \quad (\text{M.3f})$$

For scheme IV, the rate region is

$$R_1 \leq I(X_1; Y_3 | X_2) \quad (\text{M.4a})$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_3) \quad (\text{M.4b})$$

$$R_2 \leq I(X_2; Y_4 | X_1) \quad (\text{M.4c})$$

$$R_1 + R_2 \leq I(X_1, X_2; Y_4). \quad (\text{M.4d})$$

Applying the rate region results to Gaussian channels, we have that for scheme I with the channel model

$$y_2^{(1)} = h_{1,2} x_{11}^{(1)} + z_2^{(1)} \quad (\text{M.5a})$$

$$y_3^{(1)} = h_{1,3} x_{11}^{(1)} + z_3^{(1)} \quad (\text{M.5b})$$

$$y_4^{(1)} = h_{1,4}x_{11}^{(1)} + z_4^{(1)} \quad (\text{M.5c})$$

$$y_3^{(2)} = h_{1,3}(x_{11}^{(2)} + x_{12}) + h_{2,3}(\tilde{x}_{11}^{(2)} + x_2) + z_3^{(2)} \quad (\text{M.5d})$$

$$y_4^{(2)} = h_{1,4}(x_{11}^{(2)} + x_{12}) + h_{2,4}(\tilde{x}_{11}^{(2)} + x_2) + z_4^{(2)}, \quad (\text{M.5e})$$

the rate region becomes

$$R_{11} \leq t_1 \log \left(1 + \text{SNR} |h_{1,2}|^2 \right) \quad (\text{M.6a})$$

$$R_{12} \leq t_2 \log \left(1 + \frac{\text{SNR}}{2} |h_{1,3}|^2 \right) \quad (\text{M.6b})$$

$$R_2 + R_{12} \leq t_2 \log \left(1 + \frac{\text{SNR}}{2} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) \quad (\text{M.6c})$$

$$R_1 + R_2 \leq t_1 \log \left(1 + \text{SNR} |h_{1,3}|^2 \right) + t_2 \log \left(1 + \text{SNR} (|h_{1,3}|^2 + |h_{2,3}|^2) \right) \quad (\text{M.6d})$$

$$R_2 \leq t_2 \log \left(1 + \frac{\text{SNR}}{2} |h_{2,4}|^2 \right) \quad (\text{M.6e})$$

$$R_{12} + R_2 \leq t_2 \log \left(1 + \frac{\text{SNR}}{2} (|h_{1,4}|^2 + |h_{2,4}|^2) \right) \quad (\text{M.6f})$$

$$R_1 + R_2 \leq t_1 \log \left(1 + \text{SNR} |h_{1,4}|^2 \right) + t_2 \log \left(1 + \text{SNR} (|h_{1,4}|^2 + |h_{2,4}|^2) \right). \quad (\text{M.6g})$$

For scheme II, with the Gaussian channel model

$$y_2^{(1)} = h_{1,2}(x_{11}^{(1)} + \tilde{x}_{21}^{(1)}) + z_2^{(1)} \quad (\text{M.7a})$$

$$y_1^{(2)} = h_{2,1}x_{21}^{(2)} + z_1^{(2)} \quad (\text{M.7b})$$

$$y_3^{(1)} = h_{1,3}(x_{11}^{(1)} + \tilde{x}_{21}^{(1)}) + z_3^{(1)} \quad (\text{M.7c})$$

$$y_4^{(1)} = h_{1,4}(x_{11}^{(1)} + \tilde{x}_{21}^{(1)}) + z_4^{(1)} \quad (\text{M.7d})$$

$$y_3^{(2)} = h_{2,3}x_{21}^{(2)} + z_3^{(2)} \quad (\text{M.7e})$$

$$y_4^{(2)} = h_{2,4}x_{21}^{(2)} + z_4^{(2)} \quad (\text{M.7f})$$

$$y_3^{(3)} = h_{1,3}(x_{11}^{(3)} + x_{12} + \tilde{x}_{21}^{(3)}) + h_{2,3}(\tilde{x}_{11}^{(3)} + x_{22}) + z_3^{(3)} \quad (\text{M.7g})$$

$$y_4^{(3)} = h_{1,4}(x_{11}^{(3)} + x_{12} + \tilde{x}_{21}^{(3)}) + h_{2,4}(\tilde{x}_{11}^{(3)} + x_{22}) + z_4^{(3)}, \quad (\text{M.7h})$$

the achievable rate region becomes

$$R_{11} \leq t_1 \log \left(1 + (\text{SNR} - \text{SNR}^\eta) |h_{1,2}|^2 \right) \quad (\text{M.8a})$$

$$R_{21} \leq t_2 \log (1 + \text{SNR}|h_{2,1}|^2) \quad (\text{M.8b})$$

$$R_{12} \leq t_3 \left(\log \left(1 + \frac{\text{SNR} + \text{SNR}^\eta}{2} |h_{1,3}|^2 \right) - \log (1 + \text{SNR}^\eta |h_{1,3}|^2) \right) \quad (\text{M.8c})$$

$$R_{22} + R_{12} \leq t_3 \left(\log \left(1 + \frac{\text{SNR} + \text{SNR}^\eta}{2} |h_{1,3}|^2 + \frac{\text{SNR}}{2} |h_{2,3}|^2 \right) - \log (1 + \text{SNR}^\eta |h_{1,3}|^2) \right) \quad (\text{M.8d})$$

$$\begin{aligned} R_1 + R_{22} &\leq t_1 \left(\log (1 + \text{SNR}|h_{1,3}|^2) - \log (1 + \text{SNR}^\eta |h_{1,3}|^2) \right) \\ &\quad + t_3 \left(\log (1 + \text{SNR}(|h_{1,3}|^2 + |h_{2,3}|^2)) - \log (1 + \text{SNR}^\eta |h_{1,3}|^2) \right) \end{aligned} \quad (\text{M.8e})$$

$$R_{12} \leq t_3 \left(\log \left(1 + \frac{\text{SNR} + \text{SNR}^\eta}{2} |h_{1,4}|^2 \right) - \log (1 + \text{SNR}^\eta |h_{1,4}|^2) \right) \quad (\text{M.8f})$$

$$R_{22} \leq t_3 \left(\log \left(1 + \frac{\text{SNR}}{2} |h_{2,4}|^2 + \text{SNR}^\eta |h_{1,4}|^2 \right) - \log (1 + \text{SNR}^\eta |h_{1,4}|^2) \right) \quad (\text{M.8g})$$

$$R_{12} + R_{22} \leq t_3 \left(\log \left(1 + \frac{\text{SNR} + \text{SNR}^\eta}{2} |h_{1,4}|^2 + \frac{\text{SNR}}{2} |h_{2,4}|^2 \right) - \log (1 + \text{SNR}^\eta |h_{1,4}|^2) \right) \quad (\text{M.8h})$$

$$\begin{aligned} R_1 + R_{22} &\leq t_1 \left(\log (1 + \text{SNR}|h_{1,4}|^2) - \log (1 + \text{SNR}^\eta |h_{1,4}|^2) \right) \\ &\quad + t_3 \left(\log (1 + \text{SNR}(|h_{1,4}|^2 + |h_{2,4}|^2)) - \log (1 + \text{SNR}^\eta |h_{1,4}|^2) \right) \end{aligned} \quad (\text{M.8i})$$

$$R_{21} \leq t_1 \log (1 + \text{SNR}^\eta |h_{1,4}|^2) + t_2 \log (1 + \text{SNR}|h_{2,4}|^2) + t_3 \log (1 + \text{SNR}^\eta |h_{1,4}|^2). \quad (\text{M.8j})$$

For scheme III, with the Gaussian channel model

$$y_3 = h_{1,3}x_1 + h_{2,3}(x_{2,w} + x_{2,u}) + z_3 \quad (\text{M.9a})$$

$$y_4 = h_{1,4}x_1 + h_{2,4}(x_{2,w} + x_{2,u}) + z_4, \quad (\text{M.9b})$$

the rate region is

$$R_1 \leq \log (1 + \text{SNR}|h_{1,3}|^2) - \log (1 + \text{SNR}^\eta |h_{2,3}|^2) \quad (\text{M.10a})$$

$$R_{2w} + R_1 \leq \log (1 + \text{SNR}(|h_{1,3}|^2 + |h_{2,3}|^2)) - \log (1 + \text{SNR}^\eta |h_{2,3}|^2) \quad (\text{M.10b})$$

$$R_1 \leq \log (1 + \text{SNR}|h_{1,4}|^2) - \log (1 + \text{SNR}^\eta |h_{2,4}|^2) \quad (\text{M.10c})$$

$$R_{2w} \leq \log (1 + \text{SNR}|h_{2,4}|^2) - \log (1 + \text{SNR}^\eta |h_{2,4}|^2) \quad (\text{M.10d})$$

$$R_1 + R_{2w} \leq \log (1 + \text{SNR}(|h_{1,4}|^2 + |h_{2,4}|^2)) - \log (1 + \text{SNR}^\eta |h_{2,4}|^2) \quad (\text{M.10e})$$

$$R_{2u} \leq \log(1 + \text{SNR}^\eta |h_{2,4}|^2). \quad (\text{M.10f})$$

For scheme IV, with the Gaussian channel model

$$y_3 = h_{1,3}x_1 + h_{2,3}x_2 + z_3 \quad (\text{M.11a})$$

$$y_4 = h_{1,4}x_1 + h_{2,4}x_2 + z_4, \quad (\text{M.11b})$$

the rate region becomes

$$R_1 \leq \log(1 + \text{SNR}|h_{1,3}|^2) \quad (\text{M.12a})$$

$$R_1 + R_2 \leq \log(1 + \text{SNR}(|h_{1,3}|^2 + |h_{2,3}|^2)) \quad (\text{M.12b})$$

$$R_2 \leq \log(1 + \text{SNR}|h_{2,4}|^2) \quad (\text{M.12c})$$

$$R_1 + R_2 \leq \log(1 + \text{SNR}(|h_{1,4}|^2 + |h_{2,4}|^2)). \quad (\text{M.12d})$$

The outage probability can be evaluated as

$$\begin{aligned} \text{P}(\mathcal{O}) &= \text{P}(\mathcal{O}|\beta_c > (1-2r)^+) \text{P}(\beta_c > (1-2r)^+) + \text{P}(\mathcal{O}|\beta_c \leq (1-2r)^+) \text{P}(\beta_c \leq (1-2r)^+) \\ &= \text{P}(\mathcal{O}|\beta_c > (1-2r)^+) \text{P}(\beta_c > (1-2r)^+) \\ &\quad + \text{P}(\beta_c \leq (1-2r)^+) \\ &\quad \times \left(\sum_{m=0}^3 \sum_{n=0}^3 \text{P}(\mathcal{O}|J_1 = m, J_2 = n, \beta_c \leq (1-2r)^+) \text{P}(J_1 = m, J_2 = n) \right). \end{aligned} \quad (\text{M.13})$$

When $\beta_c > (1-2r)^+$, the transmission scheme is to let the two sources transmit together without cooperation, using the scheme proposed in [120]. According to [120],

$$\text{P}(\mathcal{O}|\beta_c > (1-2r)^+) \doteq \text{SNR}^{-\min\{(1-r)^+, 3(1-2r)^+\}}. \quad (\text{M.14})$$

Since

$$\text{P}(\beta_c > (1-2r)^+) \doteq \text{SNR}^{-(1-2r)^+}, \quad (\text{M.15})$$

we have

$$\text{P}(\mathcal{O}|\beta_c > (1-2r)^+) \text{P}(\beta_c > (1-2r)^+) \doteq \text{SNR}^{-\min\{(2-3r)^+, 4(1-2r)^+\}} \leq \text{SNR}^{-d^{HD}(r)}. \quad (\text{M.16})$$

Therefore, the outage probability under the channel condition $\beta_c > (1-2r)^+$ does not

dominate the overall outage probability. In the following, we concentrate on the cases where $\beta_c \leq (1 - 2r)^+$.

As shown in (M.13), for $\beta_c \leq (1 - 2r)^+$, the outage probability can be evaluated by further partitioning the channel according to the values of $\beta_{i,j}$, ($i = 1, 2, j = 3, 4$). For each kind of channel environment, one of the schemes I-IV can be applied. The overall DMT can thus be evaluated by considering the individual channel conditions with the respective transmission schemes.

When operating at scheme I, the DMT can be evaluated as

$$d_I(r) = \min_{\mathcal{S}_1 \cup \mathcal{H}} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{M.17})$$

where the set $\mathcal{S}_1$ is defined as

$$\mathcal{S}_1 = \{0 \leq \beta_{1,3} \leq 1, 0 \leq \beta_{1,4} \leq 1, 0 \leq \beta_{2,3} \leq 1, 0 \leq \beta_{2,4} \leq 1, 0 \leq \beta_c \leq 1 :$$

$$t_1(1 - \beta_c)^+ < r_{11} \quad (\text{M.18a})$$

$$t_2(1 - \beta_{1,3})^+ < r_{12} \quad (\text{M.18b})$$

$$t_2 \max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} < r_2 + r_{12} \quad (\text{M.18c})$$

$$t_1(1 - \beta_{1,3})^+ + t_2 \max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} < r_1 + r_2 \quad (\text{M.18d})$$

$$t_2(1 - \beta_{2,4})^+ < r_2 \quad (\text{M.18e})$$

$$t_2 \max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} < r_{12} + r_2 \quad (\text{M.18f})$$

$$t_1(1 - \beta_{1,4})^+ + t_2 \max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} < r_1 + r_2 \quad (\text{M.18g})$$

$\}$.

Here, (M.18a)-(M.18g) are derived from (M.6a)-(M.6g), respectively. The set $\mathcal{H}$ specifies the particular channel condition.

With scheme II, the DMT can be found as

$$d_{II}(r) = \min_{\mathcal{S}_2 \cup \mathcal{H}} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{M.19})$$

where the set $\mathcal{S}_2$ is defined as

$$\mathcal{S}_2 = \{0 \leq \beta_{1,3} \leq 1, 0 \leq \beta_{1,4} \leq 1, 0 \leq \beta_{2,3} \leq 1, 0 \leq \beta_{2,4} \leq 1, 0 \leq \beta_c \leq 1 :$$

$$t_1(1 - \beta_c)^+ < r_{11} \quad (\text{M.20a})$$

$$t_2(1 - \beta_c)^+ < r_{21} \quad (\text{M.20b})$$

$$t_3((1 - \beta_{1,3})^+ - (\eta - \beta_{1,3})^+) < r_{12} \quad (\text{M.20c})$$

$$t_3(\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} - (\eta - \beta_{1,3})^+) < r_{22} + r_{12} \quad (\text{M.20d})$$

$$t_1((1 - \beta_{1,3})^+ - (\eta - \beta_{1,3})^+) + t_3(\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} - (\eta - \beta_{1,3})^+) < r_1 + r_{22} \quad (\text{M.20e})$$

$$t_3((1 - \beta_{1,4})^+ - (\eta - \beta_{1,4})^+) < r_{12} \quad (\text{M.20f})$$

$$t_3((1 - \beta_{2,4})^+ - (\eta - \beta_{1,4})^+) < r_{22} \quad (\text{M.20g})$$

$$t_3(\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - (\eta - \beta_{1,4})^+) < r_{12} + r_{22} \quad (\text{M.20h})$$

$$t_1((1 - \beta_{1,4})^+ - (\eta - \beta_{1,4})^+) + t_3(\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - (\eta - \beta_{1,4})^+) < r_1 + r_{22} \quad (\text{M.20i})$$

$$t_1(\eta - \beta_{1,4})^+ + t_2(1 - \beta_{2,4})^+ + t_3(\eta - \beta_{1,4})^+ < r_{21} \quad (\text{M.20j})$$

$\}$.

Here, (M.20a)-(M.20j) are derived from (M.8a)-(M.8j), respectively.

For scheme III, the DMT can be calculated as

$$d_{III}(r) = \min_{\mathcal{S}_3 \cup \mathcal{H}} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{M.21})$$

where the set $\mathcal{S}_3$ is defined as

$$\mathcal{S}_3 = \{0 \leq \beta_{1,3} \leq 1, 0 \leq \beta_{1,4} \leq 1, 0 \leq \beta_{2,3} \leq 1, 0 \leq \beta_{2,4} \leq 1, 0 \leq \beta_c \leq 1 :$$

$$(1 - \beta_{1,3})^+ - (\eta - \beta_{2,3})^+ < r_1 \quad (\text{M.22a})$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} - (\eta - \beta_{2,3})^+ < r_1 + r_{2w} \quad (\text{M.22b})$$

$$(1 - \beta_{1,4})^+ - (\eta - \beta_{2,4})^+ < r_1 \quad (\text{M.22c})$$

$$(1 - \beta_{2,4})^+ - (\eta - \beta_{2,4})^+ < r_{2w} \quad (\text{M.22d})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} - (\eta - \beta_{2,4})^+ < r_1 + r_{2w} \quad (\text{M.22e})$$

$$(\eta - \beta_{2,4})^+ < r_{2u} \quad (\text{M.22f})$$

$\}$.

Here (M.22a)-(M.22f) are derived from (M.10a)-(M.10f), respectively.

For scheme IV, we have

$$d_{IV}(r) = \min_{\mathcal{S}_4 \cup \mathcal{H}} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{M.23})$$

where the set $\mathcal{S}_4$ is defined as

$$\mathcal{S}_4 = \{0 \leq \beta_{1,3} \leq 1, 0 \leq \beta_{1,4} \leq 1, 0 \leq \beta_{2,3} \leq 1, 0 \leq \beta_{2,4} \leq 1, 0 \leq \beta_c \leq 1 :$$

$$(1 - \beta_{1,3})^+ < r_1 \quad (\text{M.24a})$$

$$\max\{(1 - \beta_{1,3})^+, (1 - \beta_{2,3})^+\} < r_1 + r_2 \quad (\text{M.24b})$$

$$(1 - \beta_{2,4})^+ < r_2 \quad (\text{M.24c})$$

$$\max\{(1 - \beta_{1,4})^+, (1 - \beta_{2,4})^+\} < r_1 + r_2 \quad (\text{M.24d})$$

$\}.$

Here (M.24a)-(M.24d) are derived from (M.12a)-(M.12d), respectively.

For each rate region and channel condition, using one of (M.17), (M.19), (M.21) and (M.23) according to the selected transmission scheme, it can be shown that the proposed transmission scheme has a DMT that achieves the DMT upper bound. For example, for $r \in (5/12, 1/2]$, $J_1 = 3$ and $J_2 = 2$, the DMT can be calculated using (M.19) and by taking the set $\mathcal{H}$ as

$$\begin{aligned} \mathcal{H} = & \{0 \leq \beta_c \leq (1 - 2r)^+, \beta_{1,3} > (1 - 2r)^+, \\ & \beta_{2,3} > (1 - 2r)^+, \beta_{2,4} > (1 - 2r)^+, 0 \leq \beta_{1,4} \leq (1 - 2r)^+\}. \end{aligned}$$

We omit the detailed evaluation for the DMTs here. The basic approach is to use some linear programming methods to find the minimized value of $\sum_{i=1}^2 \sum_{j=3}^4 \beta_{i,j} + \beta_c$ subject to the constraints imposed by $\mathcal{H}$ and each of the sets $\mathcal{S}_1, \dots, \mathcal{S}_4$.

In the following, we present the method of choosing the values of t_1 , t_2 and t_3 for the case where $r \in (1/3, 5/12]$, $J_i = 2$, $J_j = 0$ and $\beta_{j,i+2} \in (\frac{1}{2t_2^*-1}(1 - \frac{t_1^*}{\tau^*})(1 - 2r), \frac{\tau^*}{1-2\tau^*}(1 - 2r))$, and the case where $r \in (1/3, 5/12]$, $J_i = 2$, $J_j = 0$, $\beta_c < 0.4(1 - 2r)^+$ and $\beta_{j,i+2} \in ((1 - 2r)^+, 1.3(1 - 2r)^+]$. For simplicity, we assume $i = 1$ and $j = 2$.

i) When $r \in (1/3, 5/12]$, $J_1 = 2$, $J_2 = 0$, and $\beta_{2,3} \in (\frac{1}{2t_2^*-1}(1 - \frac{t_1^*}{\tau^*})(1 - 2r), \frac{\tau^*}{1-2\tau^*}(1 - 2r))$, we further divide the interval of $\beta_{2,3}$ into three subintervals, as proposed in Section 6.4.2, and denote the lower bound of each subinterval as $\beta_{2,3}^{(l)}$ and the upper bound as $\beta_{2,3}^{(u)}$. The

value of t_1 , t_2 and t_3 should be chosen such that $t_1 + t_2 + t_3 = 1$,

$$\min\{d_a(r), d_b(r)\} \geq \frac{(1-2r)^+}{\tau^*}, \quad (\text{M.25})$$

and

$$1 + \frac{t_2 - r}{t_3} - \frac{1 - 2r}{t_1 + t_3} \geq \left(1 + \frac{t_2}{t_3} - \frac{t_2}{t_1 + t_3}\right) \frac{\tau^*}{1 - 2\tau^*} (1 - 2r), \quad (\text{M.26})$$

where

$$d_a(r) = \begin{cases} \frac{1}{t_1}(1-2r)^+ + (1 - \frac{t_3}{t_1})\beta_{2,3}^{(l)} - (\frac{t_2}{t_1} - 1)^+ \frac{\tau^*}{1-2\tau^*}(1-2r) & \text{if } t_1 \geq t_3 \\ \frac{1}{t_1}(1-2r)^+ + (1 - \frac{t_3}{t_1})\beta_{2,3}^{(u)} - (\frac{t_2}{t_1} - 1)^+ \frac{\tau^*}{1-2\tau^*}(1-2r) & \text{if } t_1 < t_3, \end{cases} \quad (\text{M.27})$$

$$d_b(r) = 1 + \frac{t_1 - r}{t_3} + \beta_{2,3}^{(l)} - \left(\frac{t_1}{t_3} - 1\right)^+ \frac{\tau^*}{1 - 2\tau^*} (1 - 2r). \quad (\text{M.28})$$

It can be easily verified by computer simulation that for each $r \in (1/3, 5/12]$, we can always find sets of values of (t_1, t_2, t_3) that satisfy the requirement.

ii) When $r \in (1/3, 5/12]$, $J_1 = 2$, $J_2 = 0$, $\beta_c < 0.4(1-2r)^+$ and $\beta_{2,3} \in ((1-2r)^+, 1.3(1-2r)^+]$, we further divide the interval of $\beta_{2,3}$ into three subintervals, as proposed in Section 6.4.2. The lower and upper bounds of each subinterval are denoted as $\beta_{2,3}^{(l)}$ and $\beta_{2,3}^{(u)}$, respectively. The values of t_1 , t_2 and t_3 are chosen such that $t_1 + t_2 + t_3 = 1$,

$$\min\{d_c(r), d_d(r), d_e(r)\} \geq \frac{(1-2r)^+}{\tau^*}, \quad (\text{M.29})$$

and

$$1 + \frac{t_2 - r}{t_3} - \eta - \frac{t_2}{t_3}\beta_c \geq \frac{\tau^*}{1 - 2\tau^*} (1 - 2r), \quad (\text{M.30})$$

where

$$d_c(r) = \frac{2(1-2r)^+}{t_3} + \left(1 - \frac{2(t_1 + t_2)}{t_3}\right)\beta_c, \quad (\text{M.31})$$

$$d_d(r) = 1 + \frac{t_1 - r}{t_3} + \left(1 - \frac{t_1}{t_3}\right)\beta_c + \beta_{2,3}^{(l)}, \quad (\text{M.32})$$

$$d_e(r) = \begin{cases} d_{e1}(r) & \text{if } \beta_{2,3}^{(u)} \leq \eta \\ d_{e2}(r) & \text{if } \beta_{2,3}^{(u)} > \eta, \end{cases} \quad (\text{M.33})$$

where

$$d_{e1}(r) = \begin{cases} \frac{(1-2r)^+}{t_1} + (1 - \frac{t_3}{t_1})\beta_{2,3}^{(l)} + (1 - \frac{t_2}{t_1})\beta_c & \text{if } t_3 \leq t_1 \\ \frac{(1-2r)^+}{t_1} + (1 - \frac{t_3}{t_1})\beta_{2,3}^{(u)} + (1 - \frac{t_2}{t_1})\beta_c & \text{if } t_3 > t_1, \end{cases} \quad (\text{M.34})$$

and

$$d_{e2}(r) = \min\{d_{e2,1}(r), d_{e2,2}(r)\}, \quad (\text{M.35})$$

where

$$d_{e2,1}(r) = \begin{cases} \frac{(1-2r)^+}{t_1} + \left(1 - \frac{t_3}{t_1}\right)\beta_{2,3}^{(l)} + \left(1 - \frac{t_2}{t_1}\right)\beta_c & \text{if } t_3 \leq t_1 \\ \frac{2(1-2r)^+}{t_1+t_3} + \left(1 - \frac{2t_2}{t_1+t_3}\right)\beta_c & \text{if } t_3 > t_1, \end{cases} \quad (\text{M.36})$$

and

$$d_{e2,2}(r) = \begin{cases} \frac{2(1-2r)^+}{t_1+t_3} + \left(1 - \frac{2t_2}{t_1+t_3}\right)\beta_c & \text{if } t_3 \leq t_1 \\ \frac{(1-2r)^+}{t_1} + \left(1 - \frac{t_3}{t_1}\right)\beta_{2,3}^{(u)} + \left(1 - \frac{t_2}{t_1}\right)\beta_c & \text{if } t_3 > t_1. \end{cases} \quad (\text{M.37})$$

Appendix N

Proof of Theorem 6.4.4

We assume that in the τ_1 portion of a frame time, source 1 transmits $x^{(1)}$ and in the τ_2 portion of a frame time, source 2 transmits $x^{(2)}$. The received signal at node $j = 1, \dots, 4$ during the τ_i (for $i = 1, 2$) time portion is denoted as $y_j^{(i)}$. Using the capacity cut-set upper bound for the half-duplex system [31], a capacity upper bound of the discrete memoryless two-user IFC with strict time-sharing half-duplexing source cooperation can be found as

$$R_1 \leq \tau_1 I(X_1^{(1)}; Y_2^{(1)}, Y_3^{(1)}) \quad (\text{N.1a})$$

$$R_1 \leq \tau_1 I(X_1^{(1)}; Y_3^{(1)}) + \tau_2 I(X_2^{(2)}; Y_3^{(2)}) \quad (\text{N.1b})$$

$$R_2 \leq \tau_2 I(X_2^{(2)}; Y_1^{(2)}, Y_4^{(2)}) \quad (\text{N.1c})$$

$$R_2 \leq \tau_1 I(X_1^{(1)}; Y_4^{(1)}) + \tau_2 I(X_2^{(2)}; Y_4^{(2)}) \quad (\text{N.1d})$$

$$R_1 + R_2 \leq \tau_1 I(X_1^{(1)}; Y_3^{(1)}, Y_4^{(1)}) + \tau_2 I(X_2^{(2)}; Y_3^{(2)}, Y_4^{(2)}). \quad (\text{N.1e})$$

Applying (N.1) to the Gaussian channel

$$y_2^{(1)} = h_{1,2}x_1^{(1)} + z_2^{(1)} \quad (\text{N.2a})$$

$$y_3^{(1)} = h_{1,3}x_1^{(1)} + z_3^{(1)} \quad (\text{N.2b})$$

$$y_4^{(1)} = h_{1,4}x_1^{(1)} + z_4^{(1)} \quad (\text{N.2c})$$

$$y_1^{(2)} = h_{2,1}x_2^{(2)} + z_1^{(2)} \quad (\text{N.2d})$$

$$y_3^{(2)} = h_{2,3}x_2^{(2)} + z_3^{(2)} \quad (\text{N.2e})$$

$$y_4^{(2)} = h_{2,4}x_2^{(2)} + z_4^{(2)}, \quad (\text{N.2f})$$

we have

$$R_1 \leq \tau_1 \log (1 + \text{SNR}(|h_{1,2}|^2 + |h_{1,3}|^2)) \quad (\text{N.3a})$$

$$R_1 \leq \tau_1 \log (1 + \text{SNR}|h_{1,3}|^2) + \tau_2 \log (1 + \text{SNR}|h_{2,3}|^2) \quad (\text{N.3b})$$

$$R_2 \leq \tau_2 \log (1 + \text{SNR}(|h_{2,1}|^2 + |h_{2,4}|^2)) \quad (\text{N.3c})$$

$$R_2 \leq \tau_1 \log (1 + \text{SNR}|h_{1,4}|^2) + \tau_2 \log (1 + \text{SNR}|h_{2,4}|^2) \quad (\text{N.3d})$$

$$R_1 + R_2 \leq \tau_1 \log (1 + \text{SNR}(|h_{1,3}|^2 + |h_{1,4}|^2)) + \tau_2 \log (1 + \text{SNR}(|h_{2,3}|^2 + |h_{2,4}|^2)). \quad (\text{N.3e})$$

Based on (N.3), the DMT upper bound for the channel with symmetric rates $r_1 = r_2 = r$ can be evaluated as

$$d^{HD-TS} = \max_{\tau_1, \tau_2} \min_{\mathcal{S}_{ts}} \beta_{1,3} + \beta_{1,4} + \beta_{2,3} + \beta_{2,4} + \beta_c, \quad (\text{N.4})$$

where $\mathcal{S}_{ts}$ is defined as

$$\mathcal{S}_{ts} = \{0 \leq \beta_{1,3} \leq 1, 0 \leq \beta_{1,4} \leq 1, 0 \leq \beta_{2,3} \leq 1, 0 \leq \beta_{2,4} \leq 1, 0 \leq \beta_c \leq 1 :$$

$$\tau_1 \max\{(1 - \beta_c)^+, (1 - \beta_{1,3})^+\} < r \quad (\text{N.5a})$$

$$\tau_1(1 - \beta_{1,3})^+ + \tau_2(1 - \beta_{2,3})^+ < r \quad (\text{N.5b})$$

$$\tau_2 \max\{(1 - \beta_c)^+, (1 - \beta_{2,4})^+\} < r \quad (\text{N.5c})$$

$$\tau_1(1 - \beta_{1,4})^+ + \tau_2(1 - \beta_{2,4})^+ < r \quad (\text{N.5d})$$

$$\tau_1 \max\{(1 - \beta_{1,3})^+, (1 - \beta_{1,4})^+\} + \tau_2 \max\{(1 - \beta_{2,3})^+, (1 - \beta_{2,4})^+\} < 2r \quad (\text{N.5e})$$

$$\}.$$

Some simple calculation reveals that the DMT upper bound is optimized by choosing $\tau_1 = \tau_2 = 1/2$ and is given as $d^{HD-TS}(r) = 2(1-2r)^+$. This completes the proof for Theorem 6.4.4.

Author's Publications

Journal Papers and Manuscripts

1. Q. Li, K. H. Li, K. C. Teh, "Space-frequency codes based on geometric design", *IEEE Trans. Veh. Technol.*, vol. 58, pp. 1604-1607, Mar. 2009.
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6. Q. Li, K. H. Li and K. C. Teh, "Diversity-multiplexing tradeoff of wireless communication systems with user cooperation," submitted to *IEEE Trans. Inform. Theory*, under revision.
7. Q. Li, K. H. Li and K. C. Teh, "Diversity-multiplexing tradeoff of symmetric MIMO interference channel with partial CSIT", submitted to *IEEE Trans. Wireless Commun.*, under revision.

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1. Q. Li, K. H. Li and K. C. Teh, "MIMO-OFDM systems based on space-time coding and OFDM carrier block spreading," in *Proc. ICICS2007*, Singapore, Dec. 2007.

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