Performance analysis of MC-CDMA systems over frequency-selective fading channels

Hou, Zhihua

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**Performance Analysis of MC-CDMA Systems over
Frequency-Selective Fading Channels**

Hou Zhihua

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2005

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Date

Hou Zhihua

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Summary

This research is mainly concerned with the performance analysis of multicarrier code-division multiple-access (MC-CDMA) systems over frequency-selective fading channels. The performance of MC-CDMA systems with space time block codes is investigated in conjunction with a new transmit diversity scheme for MC-CDMA systems by exploiting some intrinsic properties of MC-CDMA systems. The channel estimation algorithms for multicarrier systems are also considered.

To analyze the performance of MC-CDMA systems over fading channels a Gaussian approximation of the multiple-access interference is commonly adopted with the assumption that the fading among subcarriers is independent of each other. However, the accuracy of Gaussian approximation depends on system configurations, especially on the number of users and their powers. It is well known that the Gaussian approximation is not accurate when the number of users is small or power control is inaccurate. Furthermore, the assumption of independent fading is usually not practical since the fading of the subcarrier is highly correlated due to insufficient subcarrier spacing. Therefore, an exact bit-error rate (BER) analysis is always desirable. In this thesis, we propose performance analysis methods for MC-CDMA systems over downlink transmission. The analysis assumes the two most popular fading statistics, namely Rayleigh and Rician. An exact BER formula is obtained

without making the unrealistic assumption of independent fading among subcarriers. The performance analysis is performed in the time domain rather than the usual frequency domain. A closed-form BER expression is given based on the method of characteristic function. The theoretical results reveal that the performance of MC-CDMA is a function of the correlation property in the frequency domain of the spreading codes as the spreading is performed in the frequency domain rather than the time domain for MC-CDMA systems with maximal ratio combining. The performance analysis method is further extended to include synchronous MC-CDMA systems in the uplink. A simple and accurate performance evaluation method is also provided by expressing the decision variables in Gaussian quadratic forms and using the moment generating function approach.

The performance of MC-CDMA systems with space time block codes (STBC) is investigated next. MC-CDMA systems combined with STBC can achieve both maximum transmitter diversity and frequency diversity. Among four commonly used combining schemes, minimum mean square error combining (MMSEC) performs best because of its property to maintain the orthogonality among users without the excessive noise amplification. To fully exploit the information provided by all the subcarriers, a new minimum mean square error (MMSE) per user combining scheme is proposed. The proposed combining scheme is able to achieve considerable performance improvement compared with the original MMSE per subcarrier combining scheme. For MC-CDMA system with STBC, transmit diversity is achieved at each subcarrier where the assumption of quasi-static flat fading is necessarily needed. A new transmit diversity scheme for MC-CDMA system is also

proposed by exploiting the intrinsic property of MC-CDMA system. The proposed scheme maintains comparable performance without making any assumption regarding the channel.

Finally, we consider the channel estimation for orthogonal frequency division multiplexing (OFDM), MC-CDMA and related multicarrier systems which is a crucial aspect in the design of multicarrier modulation (MCM) systems especially for multiple-input multiple-output (MIMO) systems. An improved finite-alphabet based channel estimation method is developed based on polynomial approximation of the channel in both the time and frequency domain without the knowledge of the channel statistics. The proposed estimator is more robust to the variations of channel statistics and results in considerable improvement in performance as compared to the existing methods under practical channel conditions. The mean square error (MSE) is used as the performance measure in the above mentioned analysis.

List of Abbreviations and Symbols

Abbreviations

2D	two- dimensional
4G	the fourth generation
ADSL	asymmetric digital subscriber loop
AWGN	additive white Gaussian noise
BER	bit-error rate
BPSK	binary phase-shift keying
CDMA	code-division multiple access
CF	characteristic function
CR	channel response
CP	cyclic prefix
DAB	digital audio broadcasting
DFT	discrete Fourier transform

DS-CDMA	direct-sequence code-division multiple access
DVB	digital video broadcasting
EGC	equal gain combining
ETSI	European Telecommunications Standards Institute
FDMA	frequency division multiple access
FFT	fast Fourier transform
GA	Gaussian approximation
HDSL	high rate digital subscriber line
ICI	interchannel interference
IDFT	inverse discrete Fourier transform
IEEE	Institute of Electrical and Electronics Engineers
ISI	intersymbol interference
LAN	local area network
LOS	line of sight
MAI	multiple access interference
MC-CDMA	multicarrier code-division multiple access
MCM	multicarrier modulation

MGF	moment generating function
MIMO	multiple-input multiple-output
MIP	multipath intensity profile
MMSE(C)	minimum mean square error (combining)
MRC	maximal-ratio combining
MSE	mean square error
OFDM	orthogonal frequency division multiplexing
ORC	orthogonality restoring combining
PAPR	peak-to-average power ratio
PDF	probability density function
PSD	power spectrum density
PSK	phase-shift keying
QAM	quadrature amplitude modulation
RF	radio frequency
RVs	random variables
SC	single carrier
STBC	space time block codes

TDL	tapped delay line
WCDMA	wideband code-division multiple access
WSSUS	wide-sense stationary-uncorrelated scattering

Symbols

$a_{k,0}(i)$	information sequence
$b(\tau; t)$	baseband impulse response of a multipath channel
$c_{k,i}$	spreading sequence
$E[\cdot]$	expectation operator
E_b	the bit energy
$FFT(\cdot, n)$	n -point fast Fourier Transform (FFT)
f_c	carrier frequency
f_d	Doppler spread
h_i	the channel gain for the i th subcarrier
$IFFT(\cdot, n)$	inverse n -point FFT
$\text{Im}[\cdot]$	imaginary part of a complex number

$J_0(\cdot)$	zero-order Bessel function of the first kind
K	the number of active users
$K_f(K_{fl})$	Rician K -factor (of the l th path)
L	the number of non-zero paths of the channel
L_0	the number of resolvable paths of the channel
N	the number of subcarrier
$n(t)$	AWGN
n_i	the AWGN term at the i th subcarrier
$p_b(t)$	rectangular symbol pulse waveform
$\text{Re}[\cdot]$	real part of a complex number
T_c	chip duration
T_b	bit duration
T_d	multipath spread
T_s	symbol duration
ν_l	normalized propagation delay
W	the bandwidth of the bandpass signal

$\delta(\cdot)$	Dirac delta function
σ_l^2	the variance of path l
σ_n^2	the variance of the noise in the time domain
ρ_i	amplitude of the channel gain of i th subcarrier
φ_i	phase of the channel gain of i th subcarrier
μ_l	mean of the path gain
$\eta(i)$	AGWN noise in the time domain
γ_b	the signal-to-noise ratio (SNR) per bit
$ \cdot $	norm of a complex variable or a vector
$\langle \cdot, \cdot \rangle$	point multiplication
$(\cdot)^*$	complex conjugate
$\otimes$	circular convolution
$(m)_N$	$m + pN$ (p is an integer)

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Chapter 1

Introduction

1.1 Motivation

Emerging requirements for higher rate data services and better spectrum efficiency are the main drivers identified for the next generation mobile radio systems (e. g. the fourth generation (4G)). The common feature of the next generation wireless technologies will be the convergence of multimedia services such as speech, audio, video, image and data. This implies that a future wireless terminal, by guaranteeing high-speed data, will be able to connect to different networks in order to support various services: switched traffic, IP data packets and broadband streaming services such as video. The development of wireless terminal with generic protocols and multiple-physical layers or software-defined radio interfaces is expected to allow users to seamlessly switch access between existing and future standards. Although the exact requirements for 4G have not been commonly defined, its new air interface shall fulfill at least the following requirement [43]:

- Generic architecture, enabling the integration of existing technologies.

- High spectral efficiency, offering higher data rates in a given scarce spectrum.
- High scalability, designing different cell configurations (hot spot, ad hoc), hence better coverage.
- High adaptability and reconfigurability, supporting different standards and technologies.
- Low cost, enabling a rapid market introduction.
- Future proof, opening the door for new technologies.

Therefore, besides offering new services and applications, the success of the next generation wireless systems will strongly depend on the choice of the concept and technology innovation in architecture, spectrum allocation, spectrum utilization and exploitation. New high-performance physical layer and multiple access technologies are needed to provide high speed data rates with flexible bandwidth allocation. A low-cost generic radio interface, being operational in mixed-cell and in different environments with scalable bandwidth and data rates, is expected to have better acceptance.

The technique of multi-carrier transmission has been receiving wide interest recently, especially for high data-rate broadcast applications. The main advantages of multi-carrier transmission are its robustness in frequency selective fading channels and, in particular, the reduced signal processing complexity by performing equalization in the frequency domain. Combining the techniques of multicarrier

modulation (MCM) and the spread spectrum technique, multicarrier CDMA systems allows one to benefit from several advantages of both multi-carrier modulation and spread spectrum systems. These systems are able to offer, for instance, high flexibility, high spectral efficiency, simple and robust detection techniques and narrow band interference rejection capability. Multi-carrier modulation and multicarrier CDMA systems are today considered potential candidates to fulfill the requirement of next generation high-speed wireless multimedia communication systems, where spectral efficiency and flexibility are considered the most important criteria for the choice of the air interface.

In the design and development of the most promising technique for the next generation systems, the system engineers usually have to go through many steps such as theoretical analyses, simulations, field trials, and measurements prior to practical realization of a real system. The theoretical analyses and simulations are important in the early stage for learning the qualitative nature of various components/parameters and estimating the performance and capacity of a system. Based on these results, a more accurate field trial can be carried out and the stage of redesigning and modification can be minimized. Due to its unavoidable importance, theoretical studies have been an interesting topic over the years. As multicarrier CDMA system is a relatively new technique proposed in 1993, intense research is essential to provide us valuable information both in theory and implementations.

1.2 Objective

Although multicarrier CDMA systems have attracted attention of many researchers and much progress has been made in studying the performance of the systems, many of them were based on unrealistic assumptions. The closed-form expressions are still rare due to the complex structure of the system. Therefore, the main purpose of this thesis is to focus on the analysis of the performance of multicarrier CDMA systems. Among the three kinds of proposed structure of multicarrier CDMA systems, we discuss MC-CDMA systems, since it outperforms the other two over downlink [50] and uplink [51] transmissions. We aim to provide an accurate and simple method in closed form to evaluate the performance of MC-CDMA systems without making the unrealistic assumptions.

The performance of MC-CDMA system with transmit diversity is also considered. The use of multiple antennas at the transmitter and receiver in order to improve the quality of the received signal is another hot research topic. Due to the efficiency of adding equipments to base stations rather than the remote units, transmit diversity schemes are relatively more attractive. Space time block codes (STBC) are used for transmission in conjunction with multiple transmit antennas. By applying STBC to MC-CDMA system, the system is able to achieve full spatial diversity provided by the orthogonal design of STBC, but also yield full frequency diversity by carefully combining components of each subcarrier. The in-depth study of the performance of the system with different combining schemes is also provided. A new transmit

diversity scheme based on the specific structure of MC-CDMA systems in order to achieve an improved performance is another objective of our research.

For spectrally efficient wireless communication with high data rates, coherent modulation is more appropriate than differential modulation schemes. However, coherent detection requires channel state information (CSI). In this thesis, we also investigate channel estimation algorithms for multicarrier transmission. We aim at developing a robust channel estimator which is able to provide good trade off between performance and complexity, and is also not sensitive to the channel statistics at the same time.

1.3 Major Contribution of the thesis

Though there have been a number of papers on the calculation of error probabilities for MC-CDMA systems in the open literature, we find that most of these papers [1][20][51][52] use Gaussian approximation (GA) in the calculation of error probability, assuming that the fading in different subcarriers is independent of each other. However, the assumption is not guaranteed in practice, since the fading of the subcarriers is usually correlated due to insufficient frequency separation between the subcarriers. Furthermore, its accuracy depends on system configurations, especially on the number of users and their powers. It is well known that the Gaussian approximation is not accurate when the number of users is small or when

power control is inaccurate. Therefore, emphasis of this thesis is focused on the exact analysis of MC-CDMA systems in more realistic channel conditions.

We propose a characteristic function and residue calculation based approach to investigate the error performance of downlink MC-CDMA with maximal ratio combining (MRC) in correlated Rayleigh fading channels. The error performance is obtained in the time domain instead of the usual frequency domain. The most important feature of this approach is that we can obtain an exact BER performance without making any assumptions about the distribution of multiple access interference (MAI). To generalize our method, we consider the performance of MC-CDMA systems over channels with Rician distribution. Based on the Gaussian quadratic form, an exact moment generating function (MGF) is obtained for the decision variables. A simple BER expression is obtained. The method provides a unified solution for evaluating the performance of MC-CDMA systems in both downlink and uplink scenarios. The error probability of MC-CDMA systems in the uplink is also considered for synchronous transmissions. It is not necessary to make assumption regarding the correlation property of the spreading sequence. The closed form error performance may provide helpful insights in designing the spreading sequences for the MC-CDMA systems and may lead to an improved performance for MRC as it has been shown to suffer severe MAI when the number of user is large in the downlink.

Transmit diversity is considered to be an effective method of combating detrimental effects in wireless fading channels. The application of space-time block codes (STBC) to multicarrier CDMA (MC-CDMA) systems is considered. We

investigate the performance of four commonly used combining schemes, namely orthogonality restoring combining (ORC), equal gain combining (EGC), maximal ratio combining (MRC) and minimum mean square error combining (MMSEC). In addition, we propose a MMSE per user combining scheme for the MC-CDMA system with STBC based on MMSE criterion. The above analysis is based on Gaussian approximation assuming the fading in different subcarriers is independent of each other. In order to get a more accurate result we also propose an exact evaluation method for MC-CDMA systems with STBC.

Though introducing STBC to MC-CDMA system can dramatically improve the performance, for space-time block codes, it's necessary to assume that the path gains are constant during a frame and vary from frame to frame (quasi-static flat fading). This requirement may not be satisfied in cases when Doppler spread is high, that is, the remote unit is moving at high speed or the signal is transmitted at high carrier frequency. In order to address the problem, we propose a transmit diversity scheme for MC-CDMA system, which is able to achieve the same performance as the above mentioned system without the assumption of quasi-static fading channel for space-time block codes. The adaptability of the system is thus enhanced. Taking advantage of the intrinsic structure of MCM, different frequency groups are assigned to individual antenna. The performance of MC-CDMA is greatly improved compared with the one without transmit diversity. Different combining schemes such as minimum mean square error combining (MMSEC) and maximum ratio combining (MRC) are used to detect the information bearing symbols of the desired user in a frequency-selective Rayleigh fading channel. The performance of the proposed

system is accurately evaluated based on the characteristic function (CF) method and residue calculation.

Channel estimation is crucial for coherent demodulation in multicarrier systems. We propose an improved, finite alphabet based channel estimation method for multicarrier systems. Finite alphabet based channel estimation was first introduced in [74]. However, the original estimator is applicable to slowly varying channels. We design an efficient noise reduction scheme for phase-shift keying (PSK) modulated signals based on polynomial approximation of the channel responses (CRs) in both the time and frequency domains. The noise can be greatly suppressed by this nonlinear two-dimensional (2D) regression model. With this noise reduction algorithm, the finite alphabet based channel estimator can be used to track even fast fading channels without reading any further information about the statistical property of CR.

1.4 Organization of the thesis

The organization of the remainder of the thesis is as follows. Chapter 2 provides an overview of the multicarrier CDMA systems. After brief introduction of the time- and frequency-selective fading channel model, we characterize the multicarrier signal. The basic concept of orthogonal MCM is explained, and then followed by the fundamentals of multicarrier CDMA systems. Some important concepts for the multicarrier CDMA systems are introduced there. The advantages of multicarrier

CDMA systems over its conventional single carrier (SC) counterpart are discussed. In the following section, we present several major drawbacks which are inherent in multicarrier systems. The last part of this chapter provides materials relating to the thesis, such as space-time block codes and characteristic function method.

In Chapter 3 the exact analysis of MC-CDMA systems with MRC over frequency-selective Rayleigh and Rician channels is presented in the downlink. The exact BER formula is derived by applying power series expansion to obtain a general closed-form expression and a Gauss-Chebyshev quadrature rule to provide a more computational efficient solution. An approximation method with lower complexity is also developed for the case when the number of users is large for practical implementation.

Chapter 4 discusses the performance of synchronous MC-CDMA systems over frequency-selective Rayleigh fading channels in the uplink. The proposed technique is shown to be a unified method for the performance analysis in both the downlink and the uplink.

Chapter 5 discusses MC-CDMA systems with transmit diversity. MC-CDMA system with space-time block codes is first introduced and the performance of the system with four commonly used combining schemes is presented using analytical method and verified by simulations. An improved MMSE per user combiner which performs better than MMSE per carrier combiner is proposed for the system. We also propose a new transmit diversity scheme for MC-CDMA system based on its particular system structure.

Chapter 6 proposes a channel estimation method which greatly improves the performance of finite alphabet based channel estimator for multicarrier systems. The channel estimation scheme is based on polynomial approximation of the CRs in both the time and frequency domains.

Chapter 2

Overview of Multicarrier CDMA Systems

2.1 Introduction

Multicarrier code-division multiple-access (MC-CDMA) system based on the combination of code division multiple access (CDMA) and orthogonal frequency division multiplexing (OFDM) techniques was proposed in 1993 [1]. The combination of OFDM signaling and CDMA scheme has one major advantage that it can lower the symbol rate in each subcarrier which makes it easier to quasi-synchronize the received signals. Another advantage is that these signals can be easily transmitted and received using the fast Fourier transform (FFT) device without increasing the complexity of transmitter and receiver, and have an attractive feature of high spectral efficiency due to minimum subcarrier spacing. Moreover, the technique allows the use of a guard interval in order to absorb echoes due to multipath propagation, thus, frequency-selective channel impairments are

compensated by using a simple one-tap equalizer per subcarrier. The MCM has been adopted for several applications in the past, including Digital Audio Broadcast (DAB) [44], Digital Video Broadcast (DVB) [45] and recently in the interactive terrestrial return channel (DVB-RCT) [46], wireless local area network (LAN) standards HIPERLAN/2 [47] and IEEE802.11a [48], and also Asymmetric Digital Subscriber Line (ADSL 6Mbps) [49] and very-high-speed digital subscriber lines (VDSL 100Mbps) in fixed line communications. Multicarrier CDMA systems are also considered as potential candidates of next generation (4G) high-speed wireless multimedia communication systems.

In this chapter, the fundamentals of multicarrier CDMA systems are described. First we begin with wireless channel modeling in Section 2.2. By introducing the structure of multicarrier signals in Section 2.3, we examine the reason why multicarrier signals exhibit immunity to multipath whereas single carrier systems severely suffer from multipath. Detection techniques with simulation results for multicarrier CDMA systems are presented in Section 2.4. The major advantages of multicarrier CDMA systems are explained in Section 2.5. We also show some disadvantages of multicarrier CDMA systems in Section 2.6. Section 2.7 concludes the chapter.

2.2 Channel Model

The mobile radio channel places fundamental limitation on the performance of wireless communication systems. The signal transmitted between transmitter and the receiver suffers severe impairments of multiple paths (multipath) due to atmospheric reflection, refraction, or reflection from buildings and other objects. The different paths have different time delays and attenuation which are time varying resulting from the relative motion between the base station and the mobile and the motion of the surrounding objects. If we assume wide sense stationary uncorrelated scattering (WSSUS) [2], the channel is characterized by the delay power spectrum (or multipath intensity profile) and Doppler spectrum. The different paths may consist of several *discrete* paths each with a different time delay and attenuation or they may consist of a *continuum* of paths. The latter is called a diffuse multipath channel. The impulse response is a wideband channel characterization and contains all information necessary to simulate or analyze any type of radio transmissions through the channel. This stems from the fact that a mobile radio channel may be modeled as a linear filter with a time varying impulse response, where the time variation is due to the motion of the receiver in space. The filtering nature of the channel is caused by the summation of amplitudes and delays of the multiple arriving waves at any instant of time. Let $b(\tau;t)$ denote the time-varying complex baseband impulse response of a multipath channel. When a signal $x(t) = \Re\{\tilde{x}(t)e^{j2\pi f_c t}\}$ having a complex envelope $\tilde{x}(t)$ and center frequency f_c is transmitted, the received signal has complex envelope [3]

$$\tilde{y}(t) = \int_{-\infty}^{\infty} b(\tau; t) \tilde{x}(t - \tau) d\tau. \quad (2.1)$$

We consider multipath channels that are randomly varying in time. The channel is said to exhibit uncorrelated scattering (US) if [3]

$$\frac{1}{2} E \{ b^*(\tau_a; t_1) b(\tau_b; t_2) \} = \phi_b(\tau_a; t_1, t_2) \delta(\tau_a - \tau_b). \quad (2.2)$$

where $E\{\cdot\}$ is the expectation operator, $(\cdot)^*$ means complex conjugate, $\phi_b(\tau_a; t_1, t_2)$ is the autocorrelation function of $b(\tau; t)$. Note that the channel is said to be a wide-sense stationary (WSS) channel if [3]

$$\phi_b(\tau; t_1, t_2) = \phi_b(\tau; t_2 - t_1). \quad (2.3)$$

If we set $t_2 - t_1$ in (2.3) equal to zero, the resulting autocorrelation function $\phi_b(\tau) \equiv \phi_b(\tau; 0)$, is called the multipath intensity profile or delay power spectrum (power delay profile). The range of values of τ over which $\phi_b(\tau)$ is essentially nonzero is called the multipath spread of the channel and is denoted by T_d . Clearly, the correlation as a function of frequency separation can be represented as

$$R_b(\Delta f) = \int_{-\infty}^{\infty} \phi_b(\tau) e^{-j2\pi\Delta f\tau} d\tau. \quad (2.4)$$

The reciprocal of the multipath spread is a measure of the coherence bandwidth of the channel. Therefore, the coherence bandwidth can be easily obtained as

$$(\Delta f)_c \approx \frac{1}{T_d}. \quad (2.5)$$

On the other hand, the channel is said to be frequency-selective if the bandwidth of the transmitted signal is larger in comparison with the coherence bandwidth $(\Delta f)_c$.

We now consider the time variations of the channel. By taking the Fourier transform of $b(\tau; t)$, we obtain the time-variant transfer function $B(f; t)$, where f is the frequency variable. Thus

$$B(f; t) = \int_{-\infty}^{\infty} b(\tau; t) e^{-j2\pi f\tau} d\tau. \quad (2.6)$$

Since $b(\tau; t)$ is modeled as a complex-valued Gaussian random process in the t variable, it follows that $B(f; t)$ also has the same statistics. Under the WSS assumption, the autocorrelation of $B(f; t)$ can be defined as [3]

$$\begin{aligned} \phi_B(f_1, f_2; \Delta t) &= \frac{1}{2} E \{ B^*(f_1; t) B(f_2; t + \Delta t) \} \\ &= \phi_B(\Delta f; \Delta t). \end{aligned} \quad (2.7)$$

where $\Delta f = f_2 - f_1$. The correlation as a function of time separation can be represented by setting Δf in (2.7) to zero

$$\phi_B(\Delta t) = \phi_B(0; \Delta t). \quad (2.8)$$

The Doppler power spectral density of the channel connected therewith is the Fourier transform of $\phi_B(\Delta t)$, and it is defined as

$$S_B(f) = \int_{-\infty}^{\infty} \phi_B(\Delta t) e^{-j2\pi f\Delta t} d\Delta t. \quad (2.9)$$

The range of values of f over which $S_H(f)$ is essentially nonzero is called the Doppler spread f_d of the channel. As a result, the reciprocal of f_d is a measure of the coherence time of the channel. That is

$$(\Delta t)_c \approx \frac{1}{f_d}. \quad (2.10)$$

where $(\Delta t)_c$ is the coherence time. Clearly, a channel is said to be a time-selective channel (or fast changing channel) if the symbol duration is longer than the coherence time.

If the signal is band-limited, then the time-varying diffuse multipath channel can be represented as a tapped-delay line with time-varying coefficients and fixed tap spacing. The baseband equivalent output signal becomes

$$\tilde{y}(t) = \frac{1}{W} \sum_m b(t, \frac{m}{W}) \tilde{x}(t - \frac{m}{W}). \quad (2.11)$$

where W is the bandwidth of the bandpass signal.

In discrete time model, the output signal through the time varying channel can be expressed as follows

$$\tilde{y}(n) = \sum_{l=0}^{L-1} b(n, l) \tilde{x}(n-l). \quad (2.12)$$

where L is the number of resolvable paths of the channel $L = \lceil WT_d \rceil + 1$.

The complex equivalent low-pass time-variant impulse response is given by

$$b(\tau; n) = \sum_{l=0}^{L-1} b(n, l) \delta(\tau - \tau_l). \quad (2.13)$$

where n and τ are the time and the delay normalized by the sample interval, respectively, $\delta(\cdot)$ is the Dirac delta function, $b(n, l)$ is the path gain of the l th tap at time n which is independently generated by a filtering of a white complex Gaussian process through the low pass filter whose spectrum is identically designed to the Doppler spectrum, and τ_l is the propagation delay for the l th path. Figure 2.1 shows the corresponding multipath delay profile (we assume that there is no signal whose propagation delay exceeds the symbol duration)

Figure 2.1 Multipath delay profile.

The autocorrelation function of the channel is

$$E[b(n_1, l_1) \cdot b^*(n_2, l_2)] = \phi_t(n_1 - n_2) \phi_L(l_1) \delta(l_1 - l_2). \quad (2.14)$$

where $\phi_t(n)$ is the spaced-time correlation function with properties $\phi_t(n) = \phi_t(-n)$,

$\phi_t(0) = 1$ and $\phi_L(l)$ is the multipath intensity profile with $\sum_{l=0}^{L-1} \phi_L(l) = 1$. Assuming that

the sampling frequency is N/T and that an omnidirectional monopole antenna is used at the receiver and the angular distribution of wave arrival on each path is uniformly distributed, $\phi_t(\Delta n)$ is given by

$$\phi_t(\Delta n) = J_0(2\pi f_d T \Delta n / N). \quad (2.15)$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind.

2.3 Multicarrier CDMA Systems

2.3.1 Basic Concept

The basic principle of multicarrier systems is to split a high-rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. The relative amount of dispersion in time caused by multipath delay spread reduces as the symbol duration increases for the lower rate parallel streams. Intersymbol interference (ISI) is eliminated almost completely by introducing a guard time in every multicarrier symbol. In the guard time, the multicarrier symbol is cyclically extended to avoid intercarrier interference. The simplified transmitter structures of single carrier and multicarrier system are shown in Figure 2.2. T_s denotes symbol duration. In single carrier system, as the symbol duration decreases in order to achieve fast data transmission, the normalized delay spread (T_d/T_s) increases. The delay spread might be hundreds of times larger than the symbol duration. This makes the equalizer structure complex. Current equalizer techniques

cannot recover the original transmitted signal for such cases. The MLSE can minimize the BER under perfect knowledge of the channel state information (CSI). However, its complexity exponentially increases with the delay spread and the order of the modulation. Therefore, the single carrier scheme cannot be a good candidate for high rate data transmission.

Figure 2.2 Single Carrier and Multicarrier.

Figure 2.3 Power spectrum density of FDMA and multicarrier systems.

In order to effectively reduce the influence of the multipath, the symbol duration must be greater than the delay spread. Then, only the fraction of the symbol energy is corrupted by ISI. Thus, we can ignore the ISI. To keep the data rate equal to that of the single carrier system, a parallel data transmission is required. In multicarrier systems, the high rate data stream is split to lower rate parallel data stream by the Serial-to-Parallel (S/P) block. Each data stream modulates its own carrier called a subcarrier. Thus we can achieve the flat fading channel at each subcarrier without losing the data rate. The simulation results for multicarrier CDMA system employing various detection techniques are presented in Section 2.4. The natural questions which arise are how to assign the subcarrier frequencies and what kind of pulse shape can be used to prevent the interference between subchannels. One simple example for multicarrier systems is the frequency division multiple access (FDMA) scheme shown in Figure 2.3 (a). Each subchannel spectrum does not overlap and it requires a guard band to avoid interference between adjacent subchannels. However, in OFDM and MC-CDMA systems each subchannel spectrum overlaps since the subcarrier spacing is set to $1/T_s$. General multicarrier signal can be written as follows [4]

$$s(t) = \sqrt{E_s} \sum_{m=-\infty}^{\infty} \sum_{k=0}^{N-1} a_k(m) \phi_k(t - mT). \quad (2.16)$$

where N is the number of subchannels, T is the symbol duration after S/P block, E_s is the symbol energy per subchannel, $a_k(m)$ is the data for k th subchannel at m th multicarrier symbol, and $\phi_k(t)$ should satisfy following orthogonality condition

$$\langle \phi_k(t - mT), \phi_l(t - nT) \rangle = \delta(k - l) \delta(m - n). \quad (2.17)$$

where $\langle \cdot \rangle$ denotes inner product between two functions. One of the simplest examples satisfying the above condition is the complex exponentials

$$\phi_k(t) = \frac{1}{\sqrt{T}} e^{j2\pi kt/T}, \quad 0 \leq t \leq T. \quad (2.18)$$

The waveform can be implemented through the IFFT. The complex baseband multicarrier signal as defined by (2.18) is in fact nothing more than the inverse Fourier transform of N input symbols. In practice, this transform can be implemented very efficiently by the inverse fast Fourier transform (IFFT). The IFFT drastically reduces the amount of calculations by exploiting the regularity of the operations in the inverse discrete Fourier transform (IDFT), which is a major advantage of the multicarrier system. As a result, throughout this study, we use the orthogonal waveform shown in (2.18) because it is widely used in many applications.

Figure 2.4 shows the spectra of individual subcarriers. It is clear that the multicarrier spectrum fulfills Nyquist criteria for an intersymbol interference free pulse shape. Notice that the pulse shape is present in the frequency domain and not in the time domain, for which the Nyquist criteria is usually applied. Therefore, instead of intersymbol interference (ISI), it is intercarrier interference (ICI) that is avoided by having the maximum of one subcarrier spectrum correspond to zero crossings of all the others.

Figure 2.4 Spectra of individual subcarriers.

Since the waveform of (2.18) is strictly a time-limited function, it has a large side lobe power in the frequency domain. Therefore, one can also use the raised cosine pulse or square root raised cosine pulse which is widely used in single carrier modulation.

2.3.2 Guard Time and Cyclic Extension

One of the main features of multicarrier is its effectiveness against the multipath delay spread frequently encountered in mobile communication channels. The reduction of the symbol rate by N times, results in a proportional reduction of the relative multipath delay spread, relative to the symbol time. To completely eliminate even the very small ISI that results, a guard time is introduced for each multicarrier symbol. The guard time must be chosen to be larger than the expected delay spread, such that multipath components from one symbol cannot interfere with the next symbol. If the guard time is left empty, this may lead to inter-carrier interference

(ICI), since the carriers are no longer orthogonal to each other. To avoid such a cross talk between sub-carriers, the multicarrier symbol is cyclically extended in the guard time. This ensures that the delayed replicas of the multicarrier symbols always have an integer number of cycles within the FFT interval as long as the multipath delay spread is less than the guard time.

Figure 2.5 Cyclic Prefix for a multicarrier symbol with two subcarriers.

The principle of cyclic prefix is shown in **Figure 2.5**. A guard interval (cyclic prefix) with a length greater than the delay spread is inserted before transmission, which is just replica of the tail of that symbol. The guard interval is discarded at the receiver, then the received signal excluding the guard interval is not corrupted by ISI. Each subcarrier experiences a flat fading channel even if the channel is a frequency selective channel. This property makes the equalizer structure simple because the multipath channel can be modeled by a complex number at each subcarrier, affecting the magnitude and phase of each subcarrier. Since the guard interval is not used for

detection, there is a loss in transmitted energy. However, because the symbol duration is designed much longer than the delay spread, one can neglect the energy loss. The guard interval is a good redundancy which can be exploited for timing and carrier synchronization.

2.3.3 Three Types of Multicarrier CDMA systems

In the first type of system, namely, MC-CDMA system (Figure 2.6), a spreading sequence is serial-to-parallel converted, and each chip modulates a different carrier frequency. This implies that the number of carriers N should be equal to the processing gain M , and each carrier conveys a narrowband waveform, rather than a DS waveform. In other words, the resulting signal has a PN coded structure in the frequency domain.

Figure 2.6 MC-CDMA System Structure.

In the second type of system, which is usually called MC-DS-CDMA system (Figure 2.7), the available frequency spectrum is divided into N equal width

frequency bands, typically much less than the processing gain M , and each frequency band is used to transmit a narrowband DS waveform.

Another representative proposal in combining MCM with CDMA is the multitone-CDMA (Figure 2.8) where MCM is first performed and the resulting symbols are then spread. The time-spreading scrambles the output of MCM and makes resulting spectrum random. Given a constant bandwidth, the chip duration has to be constant, as does the ratio between the number of tones and the number of chips per symbol.

Figure 2.7 MC-DS-CDMA system structure.

Figure 2.8 MT-CDMA system structure.

The main difference between MC-CDMA and MC-DS-CDMA is how they achieve spreading process. For MC-CDMA system, the spreading operation is done in the frequency domain, while the spreading operation is done in the time domain for MC-DS-CDMA system. Another major difference is the method by which both the systems separate the data from different subcarrier. In the transmitter, the spread baseband data from different users is modulated by numbers of subcarriers, which the frequency separation between each subcarrier is equal. The modulated signal is summed up to form a multicarrier signal. The complex baseband multicarrier signal is in fact nothing more than the inverse Fourier transform of the input symbols. Carrier frequencies satisfy the following condition as they are chosen to be orthogonal to each other

$$\int_0^{T_c} \cos(\omega_i t + \phi_i) \cdot \cos(\omega_j t + \phi_j) dt = 0, \quad \text{for } i \neq j. \quad (2.19)$$

Where T_c is the chip duration, ω_i and ω_j are the i th and j th carrier frequencies respectively, and ϕ_i and ϕ_j are arbitrary carrier phases, respectively. This is done so that a signal in the j th frequency band does not cause interference in the correlation receiver for the i th frequency band. Thus the signal of different subcarriers can be easily separated using FFT in the receiver. This is the case of MC-CDMA. However, in an asynchronous CDMA system, signals from other users are no longer orthogonal to the desired signal, even if (2.19) is satisfied. In addition, orthogonality might be lost because of multipath propagation or Doppler frequency shift even for a synchronous system. This implies that co-channel interference in one frequency band causes interference not only at the output of the correlator for that frequency band,

but also in the signals out of all the other correlators. To minimize such unnecessary self-interference, in multicarrier DS CDMA systems, chip matched filter for each subcarrier is used to extract data from the multicarrier signal. The extracted signal of each carrier is multiplied by carrier signal and low pass filter is used to allow us to ignore double frequency terms. Thus orthogonality among carriers is not required. Also this signaling scheme prevents narrowband waveforms from causing interference to all frequency bands. Unfortunately, the receiver structure of this scheme is rather complicated.

In fact, both the systems show a similar fading mitigation effect over a frequency selective channel. However, the latter system requires only M subcarriers, which is much smaller than the former one.

Another major advantage of the combination of OFDM signaling and CDMA scheme is that it can lower the symbol rate in each subcarrier so that a longer symbol duration makes it easier to quasi-synchronous system. In addition, it is potentially robust to channel frequency selectivity with high frequency utilization efficiency.

2.4 Transmitter, Receiver and Combining Schemes

The difference between downlink transmission and uplink transmission will be explained first. By ‘downlink’, we mean, the signals are transmitted from base station to remote subscribers. Compared with uplink, which means the signals are transmitted from remote subscribers to base station, the downlink problem is quite

different since each user only needs to extract its own signal from mixed CDMA signals broadcast from the base station. In summary, the differences between downlink and uplink transmission are:

- Downlink signals of different users are perfectly synchronized at the transmitter;
- All downlink signals go through the same multipath channel;
- The receiver only knows his own spreading codes, and has no knowledge of other users' codes. And only requires his own data from the mixed CDMA signals.

In our case, we discuss downlink transmission. In order to focus much attention on the BER variations by different combining strategies, we assume perfect subcarrier synchronization with no frequency offset and no nonlinear distortion and perfect subcarrier amplitude/phase estimation.

At the base station, signals of different users are first spread by their unique spreading codes. The spread signals are modulated by multicarrier with equal space in frequency. That is, MC-CDMA transmitter spreads the original signal using a given spreading code in the frequency domain. In other words, a fraction of the symbol corresponding to a chip of the spreading code is transmitted through a different subcarrier. For MC transmission, it is essential to have frequency nonselective fading over each subcarrier. Therefore, if the original symbol rate is high enough and subjected to frequency-selective fading, the signal needs to be first serial-to-parallel converted before spreading over the frequency domain, which

means more than one original signals are transmitted at same time. The number of signals depends on the delay spread of the channel and the processing gain of the spreading code. The basic transmitter structure of MC-CDMA scheme is similar to that of OFDM scheme used in digital audio broadcasting (DAB) system. The main difference is that the MC-CDMA scheme transmits the same symbol in parallel through a lot of subcarriers whereas the OFDM scheme transmits different symbols. The signals transmitted at each subcarrier actually are the sum of the spread signals from different users. After despreading procedure, the unique signals of the desired user are extracted from the mixed signals. And copies of user signal are carefully combined to produce the decision variable. Compared with OFDM, this kind of scheme exploits entire frequency diversity of the channel. At this case, MAI is the main cause of performance loss, especially in large SNR case.

Figure 2.9 MC-CDMA transmitter for the k th user.

Figure 2.9 shows the MC-CDMA transmitter for the k th user. The input information sequence is first converted into P parallel data sequences $(a_{k,0}(i), a_{k,1}(i), \dots, a_{k,P-1}(i))$ and then each Serial/Parallel converter output is multiplied with the spreading code with length M . All the data in total $N = P \times M$ (corresponding to the total number of subcarriers) are modulated in baseband by the IDFT and converted back into serial data. The guard interval Δ is inserted between symbols to avoid intersymbol interference caused by multipath fading, and finally the signal is transmitted after radio frequency (RF) upconversion. For simplicity, we just assume that there is only one original symbol transmitted at a time. That is, $P = 1$. Then the data sequences are just $a_k(i)$ for the k th user. The complex equivalent low-pass transmitted signal can be written as [13]

$$S^k(t) = \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} a_k(i) c_{k,m} p_s(t - iT_s') e^{i2\pi m \Delta f' (t - iT_s')} \quad (2.20)$$

$$\Delta + T_s' = T_s \quad (2.21)$$

$$\Delta f' = 1/(T_s' - \Delta) \quad (2.22)$$

where $\{c_{k,0}, c_{k,1}, \dots, c_{k,M-1}\}$ is the spreading code with length M , T_s' is the symbol duration at subcarrier, $\Delta f'$ is the minimum subcarrier separation, and $p_s(t)$ is the rectangular symbol pulse waveform defined as

$$p_s(t) = \begin{cases} 1, & (-\Delta \leq t \leq T_s' - \Delta) \\ 0, & \text{otherwise} \end{cases} \quad (2.23)$$

The bandwidth of the transmitted signal spectrum is written as

$$\begin{aligned} B &= (M-1)/(T_s' - \Delta) + 2/T_s' \\ &\approx M/T_s'/(1-\Delta) \\ &= (1-\Delta)M/T_s'. \end{aligned} \quad (2.24)$$

where Δ is the bandwidth expansion factor associated with the guard interval insertion.

Note that, in (2.20), no spreading operation is done in the time domain. We consider the IFFT signal points as the sampling signal in the time domain. We can also see that the chip duration at subcarrier level is M times as long as the original chip duration due to serial/parallel conversion. As mentioned before, when the delay spread of the channel is comparable or even longer than the symbol duration, we must choose a larger P . But this is an easy extension.

Figure 2.10 MC-CDMA receiver for the k th user.

The received signal is written as

$$\begin{aligned}
 r(t) &= \sum_{k=1}^K \int_{-\infty}^{+\infty} s^k(t-\tau) \otimes b^k(\tau; t) d\tau + n(t) \\
 &= \sum_{i=-\infty}^{+\infty} \sum_{m=0}^{M-1} \sum_{k=1}^K h_m(t) a_k(i) c_{k,m} p_s(t - iT_s') e^{j2\pi m \Delta f' t} + n(t).
 \end{aligned} \tag{2.25}$$

where $h_m(t)$ is the received complex envelope at the m th subcarrier.

MC-CDMA receiver requires coherent detection for successful despreading operation. This could give us an impression that the structure of MC-CDMA receiver is very complicated, as compared with that of normal OFDM receiver which employs differential detection to avoid complicated subcarrier recovery. Figure 2.10 shows the MC-CDMA receiver for the k th user. After downconversion, the m th subcarrier component corresponding to the received data is first coherently detected with discrete Fourier transform (DFT) and then multiplied with the designed gains to combine the energy of the received signals scattered in the frequency domain. The decision variable is the sum of the weighted baseband components given by

$$D^{k'}(t = iT_s') = \sum_{m=0}^{M-1} c_{k',m}^* g_{k'}(m) y(m). \tag{2.26}$$

$$y(m) = \sum_{k=1}^K h_m(iT_s') a_k c_{k,m} + n_m(iT_s'). \tag{2.27}$$

where $y(m)$ and $n_m(iT_s')$ are the complex baseband component of the received signal after downconversion and the complex additive Gaussian noise at the m th subcarrier at $t = iT_s'$, respectively. The combining strategies are designed to eliminate MAI and to avoid noise enhancement at the same time. One important fact is that the orthogonality among users is destroyed by the multipath character of the channel.

There're four commonly used combining strategies: Equal Gain Combining (EGC), Maximum Ratio Combining (MRC), Orthogonality Restoring Combining (ORC) and Minimum Mean Square Error Combining (MMSEC).

- EGC (Equal Gain Combining)

This is the simplest of all combining methods. The gain factor for each subcarrier is chosen to be $g(m) = \frac{h^*(m)}{|h(m)|}$. The receiver does not use any gain control to combat the fading of individual channels, only phase correction is performed.

- MRC (Maximum Ratio Combining)

This technique tries to maximize the signal to noise ratio. The gain factor for each channel is the complex conjugate of the channel gain $g(m) = h^*(m)$. Even if the signal to noise power ratio is maximized, the orthogonality among spread K data symbols is destroyed, resulting into a low SNR.

- ORC (Orthogonality Restoring Combining)

This technique attempts to restore the orthogonality of each data. The gain factor is chosen as $g(m) = \frac{1}{h(m)}$. Inter-user interference is completely eliminated. However, weak channels are multiplied with a high gain and thus amplify the noise. A variation of this scheme is the Controlled Equalization (CE). In CE, a threshold is used to prevent the noise enhancement. If the channel is too weak, it is not used for decision. The difficulty of this scheme is the optimum threshold varies with signal to noise power ratio.

- MMSEC (Minimum Mean Square Error Combining)

This technique minimizes the noise enhancement and inter-user interference. The equalizer gain factor is chosen that the error must be orthogonal to all the received baseband components at the subcarriers based on the minimum mean square estimation (MMSE)

$$E\left[\left(a_k - \tilde{a}_k\right) \cdot y(m)\right] = 0, \quad (m = 0, 1, \dots, M-1). \quad (2.28)$$

The gain factor becomes

$$g(m) = \frac{h^*(m)}{|h(m)|^2 + 1/SNR}. \quad (2.29)$$

Note that, in the downlink application, for small $|h(m)|$, the gain becomes small to avoid the excessive noise amplification, whereas for large $|h(m)|$, it becomes in proportion to the inverse of the subcarrier envelop $\frac{h^*(m)}{|h(m)|^2}$ in order to recover orthogonality among users. As a result, this method achieves the best performance in a downlink.

The performance of downlink MC-CDMA system with four combining schemes is given in Figure 2.11. It is shown that MRC performs well when the number of user is small, say fewer than 10 users. For single user case, its performance is near the theoretical lower bound for MC-CDMA system. The performance abruptly becomes worse when the number of user becomes large, because the interference resulting from distorted code orthogonality is multiplied in the combining process. The BER

performance of ORC is independent of the number of users, however its performance can't be acceptable even when the number of user is small. The EGC can perform better than the MRC with relatively large number of users in the system as the interference is not multiplied. The MMSEC performs best among the four combining strategies, although it requires information regarding the number of total active users and the noise power, in addition to the channel condition (Figure 2.12).

Figure 2.11 Downlink BER of MC-CDMA with four combining scheme (10 users).

Figure 2.12 Downlink BER of MC-CDMA system with MMSEC.

2.5 Advantages of Multicarrier CDMA

2.5.1 Frequency Efficiency

Since there is no guard band between adjacent subchannels, the overall data rate of the orthogonal multicarrier system is $\left\lfloor \frac{N}{N+1} \right\rfloor R_{\max}$, where $R_{\max}$ is the Nyquist rate and N stands for the number of subcarriers for which unrealizable rectangular filters with sharp cutoffs and linear phases are required [9]. Thus, as N increases, the data rate approaches the theoretical maximum rate.

On the other hand, the introduction of a guard interval of length ν expands the transmitted signal bandwidth by a factor of ν/N [10]. In order to limit bandwidth expansion, the length of the guard interval must be a small fraction of the FFT block

length. In practical applications, the guard interval length is chosen as a function of the expected maximum length of the channel impulse.

MC-CDMA is more bandwidth efficient than unfiltered direct-sequence code-division multiple access (DS-CDMA) and does not require filtering [11]. It was found that spreading codes also play a pulse shaping role in MC-CDMA based systems. Therefore, in designing codes for such systems one can jointly protect the transmitted signal from external interference and optimize its spectral properties. The superior bandwidth efficiency accounts for the popularity of multicarrier transmission in many recent communication systems.

2.5.2 Robust Performance in Multipath Fading Channel

As discussed earlier, the increase in the symbol duration of the multicarrier symbol by N times, leads to a corresponding increase in the effectiveness of multicarrier against the ISI caused due to multipath delay spread. Multicarrier signaling converts the frequency-selective fading channel into a series of flat fading subchannels. It can be summarized that if the number of subcarriers is sufficiently large, the channel transfer function becomes virtually nonselective within the bandwidth of a particular subcarrier. Therefore, the influence of the fading channel reduces to an attenuation and phase shift. Furthermore, using the cyclic extension process and proper design, one can completely eliminate ISI from the system. While in conventional single carrier system, time dispersion due to the delay spread introduces intersymbol interference and reduces the maximum achievable data rate.

A time domain equalizer with adequate taps can mitigate this to some extent, but at the cost of increasing noise.

In addition to delay variations in the channel, the lack of amplitude flatness in the frequency response of the channel also causes ISI in digital communication systems. A typical example would be the twister-pair used in telephone lines. These transmission lines are used to handle voice calls and have a poor frequency response when it comes to high frequency transmission. In systems that use single-carrier transmission, an equalizer might be required to mitigate the effect of channel distortion. The complexity of the equalizer depends upon the severity of the channel distortion and there are usually issues such as equalizer non-linearity and error propagation etc may result in additional impairments. In multicarrier systems on the other hand, since the bandwidth of each subcarrier is very small, the amplitude response over this narrow bandwidth will be basically flat (of course, one can safely assume that the phase response will be linear over this narrow bandwidth). Even in the case of extreme amplitude distortion, an equalizer of very simple structure will be enough to correct the distortion in each subcarrier.

2.5.3 Frequency Diversity

For multicarrier CDMA systems, frequency diversity is inherently present in the system (i.e. it is available for free). Performance benefits are achieved by combining carriers to maximize frequency diversity benefits [12]. In contrast, in DS-CDMA, performance benefits are achieved by combining various paths in the multipath communication channel (typically using a RAKE receiver). Theoretically, the path

diversity achieved via DS-CDMA should be identical to the frequency diversity available in MC-CDMA [13]. As it turns out, MC-CDMA significantly outperforms its DS-CDMA counterpart. First, MC-CDMA systems are capable of better exploiting the energy spread introduced by the channel. In DS-CDMA, the energy in the path spread out over the 3 or 4 highest-energy paths is practically completely lost at the RAKE receiver. In MC-CDMA, on the other hand, the equivalent effect to multipath (in the time domain) is frequency-selectivity (in the frequency domain), which is completely resolved by the multiple carriers and can be fully exploited by the cross-carrier combiner at the MC-CDMA receiver side. In other words, rather than to separate paths in the time domain to exploit path diversity (in DS-CDMA), there is an attempt to separate narrow-band carriers in the frequency domain to achieve frequency diversity (in MC-CDMA). Hence, each carrier branch experiences a high SIR (low interference), and as such the MC-CDMA receiver can significantly outperform its DS-CDMA counterpart.

2.5.4 Immunity to Wideband and Narrowband Impulsive Noise

Communication systems are seldom interfered by white Gaussian noise alone [14, 15]. Impulse noise is usually a burst of interference caused in channels such as the return path HFC (Hybrid-Fiber-Coaxial), twisted-pair and wireless channels affected by atmospheric phenomena such as lightning etc. It is common for the length of the interference waveform to exceed the symbol duration of a typical digital communication system, and a impulse noise waveform, lasting for a couple of micro-seconds can cause a burst of errors that cannot be corrected using normal error-correction coding. Usually, the discrete time domain model of the wide-band

impulsive noise is modeled as a Bernoulli-Gaussian process, i.e., a product of a real Bernoulli process and a complex Gaussian process. It is equivalent to the widely used continuous time domain model which considers a train of Poisson arriving delta functions with random area distributed according to a "power" Rayleigh PDF, followed by matched filtering and symbol sampling at the input to the decision device [16]-[18]. Usually complicated Reed-Solomon codes in conjunction with huge interleaves are used to correct this problem. Multicarrier systems are inherently robust against impulse noise, since the symbol duration of a multicarrier signal is much larger than that of the corresponding single-carrier system and thus, it is less likely that impulse noise might cause (even single) symbol errors. It has been shown that in most cases, multicarrier systems perform better than single carrier systems, specifically when the probability of an impulse is not too high and the impulse power is moderate [16-19]. Thus, complicated error-control coding and interleaving schemes for handling burst-type errors are not really required for multicarrier systems simplifying the transceiver design.

Moreover, multicarrier CDMA systems have a narrowband interference suppression effect [20]. In the narrowband impulsive noise environments, Middleton's Class A model has been widely used [21]-[24]. Under the assumption that the narrow-band impulsive noise only affects a limited number of subcarriers at a time, the performance for a multicarrier CDMA system definitely outperforms its single carrier counterpart.

2.6 Some Drawbacks

The performance loss of a multicarrier CDMA system is mainly due to the presence of nonlinear radio frequency (RF) power amplifiers, phase noise, and frequency offset of the RF oscillator, chip synchronization timing errors and fading. Each impairment produces loss of orthogonality between subcarriers, resulting in an increase in the interchannel interference.

2.6.1 More Vulnerable to Frequency Offset and Phase Noise

Frequency offsets for a FFT based multicarrier systems are created by differences in RF oscillators in transmitter and receiver, Doppler shifts, or phase noise introduced by non-linear channels, while phase noise is caused by imperfections in the transceiver oscillators.

In [25], a comprehensive analysis taking into consideration the exact statistics of the multiplicative phase noise is presented assuming the free-running local oscillator model used in [26]. In particular, the symbol error rate (SER) for the additive white Gaussian noise (AWGN) channel is analytically derived. In [27], Steendam study the effect of local oscillator imperfections on MC-CDMA performance based on the phase-locked oscillator model from [28] for an AWGN channel. There are two main destructive effects caused by carrier frequency offset and phase noise in multicarrier systems. One is the reduction of signal amplitude and the other is the introduction of ICI from other subcarriers. The latter is caused by the loss of orthogonality between

subcarriers. It is found that a multicarrier system is much more sensitive to frequency offset than a single carrier system [25, 26].

In order to overcome this problem, a multicarrier receiver calls for accurate compensation of the carrier frequency offset in the input signal. A frequency offset as small as a few percent of the subcarrier spacing is sufficient to impair the performance as is pointed out in [28]. Frequency offset estimators have been proposed for OFDM and related multicarrier system. Most frequency estimators proposed are data aided, insofar as they rely on the periodic transmission of known data blocks [28][29]. On the one side, this quality ensures reliable operation of the synchronizer, but on the other, it impacts spectrum efficiency. A few non-data aided, or blind, low-complexity schemes have been proposed and assessed over the additive white Gaussian noise channel [30][31]. Other blind algorithms for the selective channel were also proposed, but they bear high complexity, and they rely on the presence of virtual carriers in the multicarrier symbol [32][33].

2.6.2 High Peak-to-Average Power Ratio

For a multicarrier system, if the sinusoids have equal amplitudes the potential worst case peak-to-average power ratio (PAPR) for amplifiers in the transmission path is $10\log_{10} N$ dB. The worst case of peak occurs when equal amplitude cosine or sine carriers happen to have zero or $\pi/2$ phase at the same time. Severe intermodulation (IM) distortion may occur when an amplifier is not designed to accommodate these peaks [34, 35]. But, if the amplifier is designed to accommodate these peaks, it tends to be very inefficient [36, 37].

In [38], a predistortion compensation technique was used to combat nonlinear distortion at the transmitter. In [39], a design of a MC-MMSE transceiver to suppress IMD and MAI for coded multicarrier CDMA systems by adaptively suppressing both the MAI and IM, and also achieving a coding gain, without requiring knowledge of other user's spreading codes, timing or phase information. An accurate time-domain analysis is used to evaluate the baseband IM distortion when a number of subcarriers share a common nonlinear amplifier. A MC-CDMA system with a nonlinear amplifier is analyzed for uncoded BER and an adaptive predistorter can significantly reduce the nonlinear effect of the amplifier [40].

To reduce the PAPR, clipping the modulated signal is a relatively simple method [41]. The characteristic of the clipped signal is discussed in [42].

2.7 Other Related Materials

2.7.1 Space Time Block Codes

Antenna diversity is a practical, effective and, hence, a widely applied technique for reducing the effect of multipath fading in most scattering environments. Transmit diversity is particularly appealing because of its relative simplicity of implementation and the feasibility of multiple antennas at the base station.

The first bandwidth efficient transmit diversity scheme was proposed by Witteneben [83]. Later Foschini introduced a multi-layered space-time architecture

[84]. Space-time trellis coding (STTC) was introduced in [85] as an effective transmit diversity technique to combat fading. However its decoding complexity increases exponentially with the transmission rate. In addressing the issue of decoding complexity, Alamouti discovered a remarkable scheme for transmission using two antennas [69]. Space-time block coding, introduced in [86], generalizes the transmission scheme discovered by Alamouti to an arbitrary number of transmit antennas and is able to achieve the full diversity promised by transmit and receive antennas. These codes retain the property of having a very-simple maximum likelihood decoding algorithm based only on linear processing at the receiver and are able to achieve full spatial diversity and a high transmission rate.

2.7.2 Characteristic Function Method

The characteristic function method was first proposed by Helstrom [87] to compute the probability of error. This technique was later applied to the performance evaluation of DS-CDMA [53]. The method was first extended to MC-CDMA systems by Smida [56]. The technique avoids the exact calculation of the PDF of the interference. Hence, it is widely used in performance evaluation recently [54][55].

2.8 Summary

Based on the combination of CDMA and OFDM techniques, multicarrier CDMA technique is a promising scheme for next generation wireless communications. Inheriting from the nature of multicarrier signals, multicarrier CDMA systems have

major advantages as robust to frequency-selective channels, high bandwidth efficiency, easy to quasi-synchronize the transmission and highly computationally efficient. Besides, due to the unique structure they can achieve efficient frequency diversity compared with traditional OFDM.

However, multicarrier CDMA systems are more susceptible to time varying channel than its single carrier counterpart since the orthogonality of subcarriers is destroyed, causing ICI. Other drawbacks of multicarrier CDMA system due to the nature of the multicarrier signals are the PAPR which severely reduces the efficiency of the power amplifier and the effect of carrier frequency offset caused by the drift of the oscillator at both transmitter and receiver resulting in severe performance degradation. Furthermore, in order to restore the orthogonality between different users destroyed by multipath channels and at the same time without amplifying the noise term, combining scheme should be carefully designed to achieve the optimum performance.

Chapter 3

Exact Analysis of Downlink MC-CDMA Systems over Frequency-Selective Fading Channels

3.1 Introduction

There have been a number of papers on the calculation of error probabilities for MC-CDMA systems in the open literature. Most of these papers [1][52] use Gaussian approximation in the calculation of error probability, assuming that the fading in different subcarriers is independent of each other. Nevertheless, the assumption is not guaranteed in practice, since the fading of the subcarriers is usually correlated due to insufficient frequency separation between the subcarriers. An exact error floor without considering the noise term is obtained in [54] under the assumption of exponential multipath intensity profile. In [55], a closed-form BER expression for a synchronous MC-CDMA under the assumption of independent fading among subcarriers has been obtained. In [56], a performance evaluation method is proposed

for MC-CDMA system for uplink transmission with equal gain combining (EGC) scheme. Here we propose a characteristic function and residue calculation based approach to investigate the error performance of downlink MC-CDMA with maximal ratio combining (MRC) in correlated Rayleigh fading channels. The error performance is obtained in the time domain instead of usual frequency domain. The most important feature of this approach is that we can obtain an exact BER expression without making any assumptions about the distribution of MAI. Moreover, it is not necessary to assume that the fading of the subcarriers is independent of each other. In addition, a simple closed-form expression for the BER can be obtained.

The organization of the remainder of this chapter is as follows. The performance analysis for MC-CDMA systems over Rayleigh fading channels is presented in Section 3.2. First we introduce the system model in Section 3.2.1. Then the basic idea of the time domain approach is explained in Section 3.2.2. We derive a decision variable in the time domain for further processing. In Section 3.2.3, the moment generating function (MGF) of the decision variable is obtained. An exact BER formula is given in Section 3.2.4. In Section 3.2.5, numerical results and discussions are given. The theoretical performance evaluation of MC-CDMA systems over Rician fading channels is provided Section 3.3. The detailed derivation of the exact BER formula is addressed as for Rayleigh fading case. Finally, concluding remarks are given in Section 3.4.

3.2 BER Performance of Downlink MC-CDMA over Rayleigh Fading Channels

3.2.1 System Model

A. Transmitter

Suppose that there are K active users in a downlink. At the transmitter side, the spreading binary data stream is serial-to-parallel converted to N parallel substreams. All the data in N subcarriers are modulated in baseband by the IDFT using binary phase-shift keying (BPSK). The resultant signals are then converted back into serial data. The guard interval is inserted between symbols to avoid intersymbol interference caused by multipath fading, and finally the signal is transmitted after radio frequency (RF) upconversion. Let $a_k(q)$ and $\{c_{k,i}\}_{i=0}^{N-1}$ denote the q th data bit and spreading sequence of user k , respectively. The equivalent low-pass transmitted signal for user k can be expressed as [54]

$$S^k(t) = \sum_{q=-\infty}^{+\infty} \sum_{i=0}^{N-1} \sqrt{\frac{E_b}{N}} a_k(q) c_{k,i} p_b(t - qT_b) e^{j2\pi\Delta f(t - qT_b)}. \quad (3.1)$$

where E_b is the power of the data bit and assumed to be same for all users, $p_b(t)$ is a rectangular pulse defined in $[0, T_b]$ with T_b denoting the bit duration, and $\Delta f = 1/T_b$ is the minimum subcarrier separation.

B. Channel model

In the following, we consider a slowly varying frequency-selective Rayleigh fading channel which means that the channel parameters are considered as being unchanged over one bit duration T_s . Then the channel in (2.13) is modeled as a tapped delay line (TDL) having the following baseband equivalent impulse response [3]

$$b(t) = \sum_{l=0}^{L_0-1} b(l) \delta(t - lT_c). \quad (3.2)$$

where T_c is the chip duration which is the sampling interval for multicarrier CDMA systems, L_0 is the number of resolvable paths sampled at the rate $1/T_c$, $b(l)$ is the l th path gain which is a random process and is mutually independent for different l . Since only the non-zero paths need to be considered, we define $\{v_l\}_{l=0}^{L-1}$ as the propagation delay for the non-zero paths normalized by the sample interval, and σ_l^2 as the variance, L is the number of the non-zero paths.

In the frequency domain, all subcarriers are assumed to experience flat but correlated fading so long as the number of subcarriers N is sufficiently large. If $h_i = \rho_i e^{j\varphi_i}$ is the channel gain for the i th subcarrier, it has been shown as [57]

$$h_i = \sum_{n=0}^{L_0-1} b(n) \exp(-j2\pi n i / N), \quad (i = 0, 1, \dots, N-1). \quad (3.3)$$

Or equivalently

$$h_i = \sum_{n=0}^{L-1} b(v_n) \exp(-j2\pi i v_n / N), \quad (i = 0, 1, \dots, N-1).$$

Actually, we have $h_i = B(\frac{i}{NT_c})$, $(i = 0, 1, \dots, N-1)$.

The path gain for l th path $b(l)$ is determined by specific channel condition. For Rayleigh channels, $b(l)$ is modeled as a complex Gaussian random process with zero mean, ρ_i is Rayleigh distributed with $E[\rho_i^2] = 1$ and φ_i uniformly distributed over $[0, 2\pi)$.

The received signal can be written as

$$r(t) = \sum_{q=-\infty}^{\infty} \sqrt{\frac{E_b}{N}} \sum_{k=1}^K a_k(q) p_b(t - qT_b) \sum_{i=0}^{N-1} \rho_i c_{k,i} e^{j[2\pi i \Delta f (t - qT_b) + \varphi_i]} + n(t), \quad (3.4)$$

where $n(t)$ is additive white Gaussian noise (AWGN) having a double-sided power spectrum density (PSD) of N_0 .

C. Receiver

At the receiver, the RF signal is first converted to baseband signal. After the portion of the signal corresponding to the prefix is removed, DFT is performed on the signal samples. Then a coherent correlation receiver with MRC is used. As our focus is on the evaluation of BER, we assume perfect subcarrier synchronization with no frequency offset and no nonlinear distortion and also assume perfect subcarrier amplitude/phase estimation. Assuming user u is the desired user, the decision variable of the *zeroth* data bit is given by

$$Z_u(0) = \text{Re}\left\{\sum_{i=0}^{N-1} \rho_i c_{u,i} \frac{1}{T_b} \int_0^{T_b} r(t) e^{-j(2\pi i \Delta f t + \phi_i)} dt\right\}. \quad (3.5)$$

3.2.2 The Decision Variable in the Time Domain

The following sections will focus on BER performance evaluation. The basic idea is that first we transform the calculation of the decision variable from the frequency domain to the time domain by applying discrete Parseval's theorem, so that the MGF of the decision variable can be derived. The exact bit error probability is then obtained from the MGF by Laplace inversion integral or numerical methods.

The length of the cyclic prefix is assumed to be longer than the length of the channel impulse response, so that there is no intersymbol interference (ISI). Here we discuss real spreading codes, which are commonly used in practice (Extension to complex-valued codes is straightforward). We don't consider the power loss due to the cyclic prefix in the analysis. From (3.5), the decision variable of the zeroth data bit can be written as [54]

$$Z_u(0) = \text{Re}\left\{\sqrt{\frac{E_b}{N}} \sum_{i=0}^{N-1} \beta_{u,i} h_i h_i^* + \sum_{i=0}^{N-1} c_{u,i} n_i h_i^*\right\}. \quad (3.6)$$

where

$$\beta_{u,i} = a_u(0) + \sum_{k=1, k \neq u}^K a_k(0) c_{k,i} c_{u,i}, \quad i = 0, 1, \dots, N-1. \quad (3.7)$$

n_i corresponds the AWGN term at the i th subcarrier. Without loss of generality, a factor equal to $\sqrt{2/N_0}$ is multiplied to (3.6). Then the following equation is achieved

$$Z_u(0) = \text{Re} \left\{ \sqrt{\frac{2\gamma_b}{N}} \sum_{i=0}^{N-1} \beta_{u,i} h_i h_i^* + \sum_{i=0}^{N-1} c_{u,i} \bar{n}_i h_i^* \right\}, \quad (3.8)$$

This implies that the $\bar{n}_i$ in (3.8) is normalized AWGN noise with a variance of $\sigma_n^2 = 1$, and the ratio $\gamma_b = E_b / N_0$ is the signal-to-noise ratio (SNR) per bit.

As the noise is AWGN, the multiplication of the noise by spreading codes of the desired user will not change the distribution of the noise. The AWGN noise in the frequency domain is actually the Discrete Fourier Transform (DFT) of the normalized AWGN noise in the time domain $\eta(i)$ multiplied by $1/\sqrt{N}$

$$\bar{n}_i = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \eta(n) \exp(-j2\pi in / N), \quad i = 0, 1, \dots, N-1, \quad (3.9)$$

Our aim is to obtain a concise form of the decision variable for further processing. We make use of the sparsity constraints on the channel which means that there are usually only a few active taps compared to the number of subcarriers. Two time domain parameters are introduced in order to obtain the final result.

Assume $\mathbf{y}$ is the corresponding signal of β_u in the time domain given the value $\mathbf{y} = \text{IFFT}(\beta_u, N)$, $\mathbf{x}$ is the signal corresponding to $\langle \beta_u \cdot \mathbf{h} \rangle$ in the time domain, that is $\mathbf{x} = \text{IFFT}(\langle \beta_u \cdot \mathbf{h} \rangle, N)$. As we know, multiplication of the DFT of two sequences is

equivalent to the circular convolution of the two sequences in the time domain [58], the relationship between $\mathbf{x}$ and $\mathbf{y}$ can be expressed as

$$\begin{aligned} x(i) &= y(i) \otimes b(i) \\ &= \sum_{m=0}^{N-1} b(m) y(i-m)_N = \sum_{m=0}^{L-1} b(v_m) y(i-v_m)_N, \quad i = 0, 1, \dots, N-1 \end{aligned} \quad (3.10)$$

Applying discrete Parseval's theorem [58], the summation in the frequency domain in (3.8) can be converted to the summation in the time domain. Omitting a factor equal to $\sqrt{N}$, the decision variable in (3.8) can be rewritten in the time domain as

$$Z_u(0) = \text{Re} \left\{ \sqrt{2\gamma_b} \sum_{i=0}^{N-1} x(i) b(i)^* + \sum_{i=0}^{N-1} \eta(i) b(i)^* \right\}. \quad (3.11)$$

From (3.10) and (3.11), the decision variable in the time domain can be obtained as

$$Z_u(0) = \text{Re} \left\{ \sqrt{2\gamma_b} \sum_{i=0}^{L-1} \sum_{m=0}^{L-1} b(v_m) y(v_i - v_m)_N b(v_i)^* + \sum_{i=0}^{L-1} \eta(v_i) b(v_i)^* \right\}, \quad (3.12)$$

For $a_u(0) = 1$, an error occurs if $Z_u(0) < 0$. For $a_u(0) = -1$, and error occurs if $Z_u(0) > 0$.

3.2.3 Derivation of Moment Generating Function

The decision variable $Z_u(0)$ is conditioned on the channel coefficients $\{b(v_i)\}_{i=0}^{L-1}$ and user data $\{a_k(0)\}_{k=1}^K$. We compute the BER by first averaging over $\{b(v_i)\}_{i=0}^{L-1}$ and

then over $\{a_k(0)\}_{k=1}^K$. In order to obtain an exact error performance in closed form, we calculate the Laplace transform of $Z_u(0)$ conditioned on $\{a_k(0)\}_{k=1}^K$, $\Phi(s) = E(e^{-sZ})$, that is, the MGF of the decision variable $Z_u(0)$. Usually it's not easy to calculate MGF for a random vector because it is by definition the mean of an exponential function of random variables involved. This leads to the calculation of multi-dimensional integral and requires the knowledge of the joint probability density of the random variables. In this case since the calculation is converted to the time domain, the channel coefficients are statistically independent and the exponential of the function is a pure quadratic form, the integral can be calculated by proper variable replacement.

The decision variable in (3.12) can be written as

$$Z_u(0) = \text{Re}\left\{\sqrt{2\gamma_b}\left[\sum_{i=0}^{L-1}|b(v_i)|^2 y(0) + \sum_{i=0}^{L-1}\sum_{m=0, m \neq i}^{L-1} b(v_i)^* b(v_m) y(v_i - v_m)_N\right] + \sum_{i=0}^{L-1} \eta(v_i) b(v_i)^*\right\}. \quad (3.13)$$

Assuming the complex variables as

$$\begin{aligned} \eta(v_i) &= m_{ix} + jm_{iy}, \\ b(v_i) &= b_{ix} + jb_{iy}, \quad i = 0, 1, \dots, L-1 \end{aligned} \quad (3.14)$$

On substituting (3.14) into (3.13), we obtain

$$\begin{aligned} Z_u(0) &= \sqrt{2\gamma_b} \left\{ 2 \sum_{i=0}^{L-1} \sum_{m=i+1}^{L-1} \left[\text{Re}[y(v_i - v_m)_N] (b_{ix} b_{mx} + b_{iy} b_{my}) + \text{Im}[y(v_i - v_m)_N] (b_{iy} b_{mx} - b_{ix} b_{my}) \right] \right. \\ &\quad \left. + y(0) \sum_{i=0}^{L-1} (b_{ix}^2 + b_{iy}^2) \right\} + \sum_{i=0}^{L-1} [m_{ix} b_{ix} + m_{iy} b_{iy}] \end{aligned}$$

(3.15)

Because $b(n)$ are independent complex Gaussian random variables for Rayleigh fading channel, and the AWGN noise term is independent of the channel impulse response, the moment-generation function $\Phi(s) = E(e^{-sZ})$ can be calculated as

$$\begin{aligned}\Phi(s) &= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-sZ} f(m_{0x}, m_{0y}, \cdots, m_{(L-1)x}, m_{(L-1)y}, b_{0x}, b_{0y}, \cdots, b_{(L-1)x}, b_{(L-1)y}) dm_{0x} dm_{0y} \cdots db_{0x} db_{0y} \cdots \\ &= \alpha \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-\left[sZ + \sum_{i=0}^{L-1} \frac{(b_{ix}^2 + b_{iy}^2)}{2\sigma_i^2} + \sum_{i=0}^{L-1} \frac{(m_{ix}^2 + m_{iy}^2)}{2} \right]} dm_{0x} dm_{0y} \cdots dm_{(L-1)x} dm_{(L-1)y} db_{0x} db_{0y} \cdots db_{(L-1)x} db_{(L-1)y}\end{aligned}\quad (3.16)$$

where $\alpha = \frac{1}{(2\pi)^{2L} \prod_{i=0}^{L-1} \sigma_i^2}$, and $f(\cdot)$ is the joint probability density function (PDF) of

the random variables.

Noticing that the exponential of (3.16) is in pure quadratic form. The integral can be calculated by properly changing the variables

$$\begin{cases} m'_{ix} = m_{ix} + sb_{ix}, \\ m'_{iy} = m_{iy} + sb_{iy}, \\ b'_{ix} = b_{ix} + \frac{\sum_{k=i+1}^{L-1} P_{ikx} b_{kx} - P_{iky} b_{ky}}{R_i}, \\ b'_{iy} = b_{iy} + \frac{\sum_{k=i+1}^{L-1} P_{iky} b_{kx} + P_{ikx} b_{ky}}{R_i} \end{cases} \quad (i = 0, 1, \cdots, L-1) \quad (3.17)$$

where

$$\begin{aligned}
 M_i &= \sqrt{2\gamma_b} y(0)s + \frac{1}{2\sigma_i^2} - \frac{s^2}{2}, \\
 J_{ikx} &= \sqrt{2\gamma_b} \operatorname{Re}[y(v_i - v_k)_N]s, \quad / \quad J_{iky} = \sqrt{2\gamma_b} \operatorname{Im}[y(v_i - v_k)_N]s, \\
 P_{ikx} &= J_{ikx} - \sum_{n=0}^{i-1} \frac{P_{nix}P_{nqx} + P_{niy}P_{nky}}{R_n}, \quad / \quad P_{iky} = J_{iky} - \sum_{n=0}^{i-1} \frac{P_{nix}P_{nky} - P_{niy}P_{nqx}}{R_n}, \\
 R_i &= M_i - \sum_{n=0}^{i-1} \frac{P_{nix}^2 + P_{niy}^2}{R_n}, \quad (k=0,1,\dots,L-1) \text{ and } (k > i)
 \end{aligned}$$

Then, (3.16) becomes

$$\begin{aligned}
 \Phi(s) &= J\alpha \prod_{i=0}^{L-1} \left[\int_{-\infty}^{\infty} e^{-\frac{1}{2}m_{ix}^2} dm_{ix} \int_{-\infty}^{\infty} e^{-\frac{1}{2}m_{iy}^2} dm_{iy} \right] \cdot \prod_{i=0}^{L-1} \left[\int_{-\infty}^{\infty} e^{-R_i b_{ix}^2} db_{ix} \int_{-\infty}^{\infty} e^{-R_i b_{iy}^2} db_{iy} \right] \\
 &= \frac{1}{2^L \prod_{i=0}^{L-1} \sigma_i^2 R_i}
 \end{aligned} \tag{3.18}$$

where $J=1$ is the determinant value of the Jacobian of the transformation (3.17).

Here, the MGF of the decision variable for 2-ray channel model is given as an example

$$\Phi(s) = \frac{1}{c_4 s^4 + c_3 s^3 + c_2 s^2 + c_1 s + c_0}. \tag{3.19}$$

where the coefficients are given by

$$\begin{aligned}
 c_4 &= \sigma_0^2 \sigma_1^2, \\
 c_3 &= -4\sqrt{2\gamma_b} \sigma_0^2 \sigma_1^2 y(0), \\
 c_2 &= -(\sigma_0^2 + \sigma_1^2) + 8\gamma_b \sigma_0^2 \sigma_1^2 y(0)^2 - 8\gamma_b \sigma_0^2 \sigma_1^2 |y(v_0 - v_1)_N|^2, \\
 c_1 &= 2\sqrt{2\gamma_b} (\sigma_0^2 + \sigma_1^2) y(0), \\
 c_0 &= 1.
 \end{aligned}$$

The region of convergence is the vertical strip enclosing the $j\omega$ axis bounded by the closest poles on either side.

3.2.4 Exact BER Formula

According to [59], the BER of user u conditioned on $\{a_k(0)\}_{k=1}^K$ can be exactly calculated through residues

$$P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} = - \sum_{RHS} \text{Residue} \left[\frac{\Phi(s)}{s} \right], \quad (3.20)$$

where the residues are calculated and summed over the poles in the right-hand side (RHS) of the complex s -plane, i.e., the real part of s is positive.

For coherent detection

$$P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} = P_{e,u} \big|_{a_u(0)=-1, \{a_k(0)\}_{k=1, k \neq u}^K}. \quad (3.21)$$

Thus, the calculation of BER can be reduced to half.

By averaging (3.20) over $\{a_k(0)\}_{k=1, k \neq u}^K$ which are independent and identically distributed binary random variables (RVs), we obtain the error performance conditioned on $a_u(0)$

$$P_{e,u} \big|_{a_u(0)=1} = - \sum_{a_1(0) \in \{1, -1\}} \cdots \sum_{a_{k \neq u}(0) \in \{1, -1\}} \cdots \sum_{a_K(0) \in \{1, -1\}} \frac{1}{2^{K-1}} \sum_{RHS} \text{Residue} \left[\frac{\Phi(s)}{s} \right]. \quad (3.22)$$

For coherent detection, the BER performance of the user u can now be written as

$$P_{e,u} = \frac{1}{2}(P_{e,u} |_{a_u(0)=1} + P_{e,u} |_{a_u(0)=-1}) = P_{e,u} |_{a_u(0)=1} . \quad (3.23)$$

The average BER of all users is given by

$$P_e = \frac{1}{K} \sum_{i=1}^K P_{e,i} . \quad (3.24)$$

3.2.5 Numerical Results and Discussions

In this section, we present numerical results. In [13], it is pointed out that the lower bound on BER of MC-CDMA systems is same as that of DS-CDMA systems. This lower bound is achieved when there's only one user in the system. We first verify the results of our proposed method (MGF) described herein with the lower bound of DS-CDMA systems derived by Proakis [3]. To calculate the BER, it is assumed that the number of subcarriers N is 32, and three channel delay profiles are discussed, a simple two-ray multipath delay profile with equal energy for each path often encountered in urban and hilly areas [60], a three-path and a four-path channel with linear delay power profile [61]. We assume the transmission rate $R=3\text{Msymbols/s}$, and rms delay spread $(\tau_{rms})=70\text{ns}$. It is easy to calculate that $v_0=0, v_1=14$. For linear delay power profile, the channel paths are located at the first n (e.g. $n=3$ for three-path channel) chips. For example, $v_0=0, v_1=1, v_2=2$ for three-channel. Furthermore, Walsh Hadamard codes are used as the spreading codes. In all of the cases, the results obtained by the two methods are exactly same as shown in (Figure 3.1). In (Figure 3.2), Monte Carlo simulation results under the two-ray, three-path and four-path delay profile are provided. The simulation results agree very

well with the theoretical results obtained by MGF technique proposed. The simulation results are based on the calculation of the decision variable for each bit, and averaging the results over large number of bits.

Next, the effect of the channel delay profile and the spreading sequence on the BER performance is investigated using the analytical formula derived. It is demonstrated in [13], for MC-CDMA system with MRC, the performance is quite good when the number of users is small, while the performance degrades substantially when the number of users is large. From Figure 3.3, we can see this performance degradation when the number of users increases in a 2-ray Rayleigh fading channel by analytical and simulation results. The performance degradation depends on the channel condition and the spreading codes. We will give a simple example to illustrate the reason by using the formula we've derived (3.19). It can be shown from (3.19) that, for 2-ray channel model with equal energy for each path, that is $\sigma_0^2 = \sigma_1^2$, the BER performance is determined mainly by the value of $y(0)$ and $|y(v_0 - v_1)_N|^2$ at given γ_b . When Walsh Hadamard codes are selected in sequence, $y(0) = 1$. The Y which denotes the value of $|y(v_0 - v_1)_N|^2$ averaged over $\{a_k(0)\}_{k=1, k \neq u}^K$ as a function of the number of users is also shown in (Figure 3.3). The BER curve exactly follows the variation of Y , which indicates a close relationship between the two. Figure 3.4 shows the value of Y when v_1 varies. It is shown that the value of Y doesn't vary monotonically with v_1 which is used as the measure of delay spread. The sudden increase of Y for particular v_1 (16, 8, 12, 14) also corresponds to the value when sharp degradation in the performance occurs, as shown in Figure 3.3. The

system performance is determined by the values of Y at the non-zero taps. Figure 3.5 shows the BER comparison for users with different spreading codes for various values of SNR. Users are referenced by their spreading codes. For example, user 1 represents the one with the first row of Walsh Hadamard codes. Once again it can be seen that there is a close match between the variation of Y and the BER. The smaller values of Y for user 1, 2, 5 and 6 correspond to fairly good performance, while the larger ones for user 9 and 10 lead to high BER. Moreover, the effect of Y appears more distinct when SNR becomes larger. From the results in Figure 3.3, 4 and 5, it can be confirmed that the value of y (e.g. $y(0)$ and $y(v_0 - v_1)_N$) may provide helpful insights in analyzing the system performance. As a consequence, it should be the major consideration for selecting or designing the spreading codes for MC-CDMA system.

Figure 3.1 Theoretical results of BER for MC-CDMA using MRC, K=1.

Figure 3.2 BER of MC-CDMA for 2-ray, 3-path and 4-path channel model. $K=10$ and $K=20$.

Figure 3.3 BER of MC-CDMA system versus the number of users, $\text{SNR}=10\text{dB}$, 2-ray channel model, $v_0 = 0$.

Figure 3.4 Y versus v_1 for various number of users, 2-ray channel model, $v_0 = 0$.

Figure 3.5 $y(0)$, Y and BER of users with different spreading codes for various values of SNR, $v_0 = 0$, $v_1 = 6$, $K = 10$.

3.3 BER Performance of Downlink MC-CDMA over Rician Fading Channels

3.3.1 System Model

We consider the system discussed in Section 3.2.1.

In this section, we derive the error performance of MC-CDMA systems in frequency-selective Rician fading channel. When there is a dominant stationary (nonfading) signal component present, such as a line of sight propagation path, the fading envelope distribution is Rician. In this case the $b(l)$'s are complex Gaussian random processes with non-zero mean and are mutually independent for different l . The Rician probability density function (PDF) is obtained as the PDF of $|b(l)| = \sqrt{b_{lx}^2 + b_{ly}^2}$, where $b_{lx} \sim N(\mu_{lx}, \sigma_l^2)$, $b_{ly} \sim N(\mu_{ly}, \sigma_l^2)$, and b_{lx} , b_{ly} are independent. The Rician K -factor of l th path gain K_l is defined as the ratio of signal power in dominant component over the (local-mean) scattered power. We have

$$K_l = (u_{lx}^2 + u_{ly}^2) / (2\sigma_l^2). \quad (3.25)$$

3.3.2 The Decision Variable in the Time Domain

From the analysis in Section 3.2.2, the decision variable in the time domain can be obtained as

$$Z_u(0) = \text{Re}\left\{\sqrt{2\gamma_b} \sum_{i=0}^{L-1} \sum_{m=0}^{L-1} b(v_m) y(v_i - v_m)_N b(v_i)^* + \sum_{i=0}^{L-1} \eta(v_i) b(v_i)^*\right\}. \quad (3.26)$$

3.3.3 Derivation of Moment Generating Function

The decision variable $Z_u(0)$ is conditioned on channel impulse response $\{b(v_i)\}_{i=0}^{L-1}$ and user data $\{a_k(0)\}_{k=1}^K$. We compute the BER by first averaging over $\{b(v_i)\}_{i=0}^{L-1}$ and then over $\{a_k(0)\}_{k=1}^K$. In order to obtain an exact error performance, we calculate the Laplace transform of $Z_u(0)$ conditioned on $\{a_k(0)\}_{k=1}^K$, $\Phi(s) = E(e^{-sZ})$, that is, the MGF of the decision variable $Z_u(0)$. Usually it's not an easy task to calculate the MGF for a random vector since it is by definition the mean of an exponential function of the random variables involved. This generally requires the calculation of multi-dimensional integral and the knowledge of the joint probability density of the random variables. In this case because the calculation is converted to the time domain, the channel coefficients $\{b(v_i)\}_{i=0}^{L-1}$ for different paths are statistically independent Gaussian random variables [3] and the exponential of the function is in pure quadratic form. The integral can be calculated by a simple method.

The decision variable $Z_u(0)$ in (3.26) can be rewritten as

$$Z_u(0) = \text{Re}\{\sqrt{2\gamma_b} \left[\sum_{i=0}^{L-1} |b(v_i)|^2 y(0) + \sum_{i=0}^{L-1} \sum_{m=0, m \neq i}^{L-1} b(v_i)^* b(v_m) y_{i,m} \right] + \sum_{i=0}^{L-1} \eta(v_i) b(v_i)^* \} \quad . \quad (3.27)$$

with $y(v_i - v_m)_N$ denoted by $y_{i,m}$ for simplicity.

If the decision variable in (3.27) is written in vector form, the derivation of the MGF can be expressed more concisely. The vector form of the decision variable for the desired user is given by

$$Z_u(0) = \mathbf{v}^H(0) \mathbf{Q}(0) \mathbf{v}(0). \quad (3.28)$$

where

$$\mathbf{v}(0) = [b(v_0) \quad b(v_1) \quad \cdots \quad b(v_{L-1}) \quad \eta(v_0) \quad \eta(v_1) \quad \cdots \quad \eta(v_{L-1})]^T. \quad (3.29)$$

$\mathbf{Q}(0)$ is a $2L \times 2L$ matrix defined as

$$\mathbf{Q}(0) = \begin{bmatrix} \mathbf{Q}_1 & 0.5\mathbf{I}_L \\ 0.5\mathbf{I}_L & \mathbf{0}_L \end{bmatrix}. \quad (3.30)$$

where

$$\mathbf{Q}_1 = \sqrt{2\gamma_b} \begin{bmatrix} y(0) & y_{0,1} & y_{0,2} & \cdots & y_{0,L-1} \\ y_{0,1}^* & y(0) & y_{1,2} & \cdots & y_{1,L-1} \\ y_{0,2}^* & y_{1,2}^* & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & y_{L-2,L-1} \\ y_{0,L-1}^* & y_{1,L-1}^* & \cdots & y_{L-2,L-1}^* & y(0) \end{bmatrix}. \quad (3.31)$$

In (3.30), $\mathbf{0}_L$ and $\mathbf{I}_L$ are the $L \times L$ zero and identity matrices, respectively. Here we assume real-valued spreading codes are used as mentioned above.

With the assumption that the channel coefficients $b(l)$ for different paths are statistically independent from each other and the AWGN noise term is independent of the channel impulse response, a simple expression of the covariance matrix of $\mathbf{v}(0)$ is achieved

$$\mathbf{P}_v = E\{\mathbf{v}(0)\mathbf{v}^H(0)\} = 2diag(\sigma_0^2, \sigma_1^2, \dots, \sigma_{L-1}^2, \underbrace{1, \dots, 1}_L). \quad (3.32)$$

The mean vector of $\mathbf{v}(0)$ is given by

$$\boldsymbol{\mu}_v = E(\mathbf{v}(0)) = \left[(\mu_{0x} + j\mu_{0y}), (\mu_{1x} + j\mu_{1y}), \dots, (\mu_{(L-1)x} + j\mu_{(L-1)y}), \underbrace{0, \dots, 0}_L \right]^T. \quad (3.33)$$

Employing a result for the distribution of a noncentral Gaussian quadratic form [62][63], the MGF of $Z_u(0)$ in (3.28) can be obtained as

$$\Phi(-s) = \frac{\exp(\boldsymbol{\mu}_v^H \mathbf{F}(s) \boldsymbol{\mu}_v)}{\det(\mathbf{I} - s\mathbf{P}_v^{1/2} \mathbf{Q} \mathbf{P}_v^{1/2})}. \quad (3.34)$$

and

$$\mathbf{F}(s) = \mathbf{P}_v^{-1/2} \left((\mathbf{I} - s\mathbf{P}_v^{1/2} \mathbf{Q} \mathbf{P}_v^{1/2})^{-1} - \mathbf{I} \right) \mathbf{P}_v^{-1/2}. \quad (3.35)$$

where $\mathbf{P}_v^{-1/2}$ is the inverse of the symmetric square root of $\mathbf{P}_v$.

To represent the conditioned MGF $\Phi(-s)$ in a more compact form, we define a matrix

$$\mathbf{A} = \mathbf{P}_v^{1/2} \mathbf{Q} \mathbf{P}_v^{1/2}. \quad (3.36)$$

and represent its eigen-decomposition as $\mathbf{A} = \mathbf{U}_v \boldsymbol{\Lambda}_v \mathbf{U}_v^H$, where

$$\boldsymbol{\Lambda}_v = diag(\lambda_1, \lambda_2, \dots, \lambda_{2L}). \quad (3.37)$$

is the eigenvalue matrix of $\mathbf{A}$. Also, we define

$$\tilde{\mathbf{\mu}}_v = [\tilde{\mu}_1, \tilde{\mu}_2, \dots, \tilde{\mu}_{2L}]^T = \mathbf{U}_v^H \mathbf{P}_v^{-1/2} \mathbf{\mu}_v. \quad (3.38)$$

Equation (3.34) can now be simplified to

$$\begin{aligned} \Phi(-s) &= \frac{\exp(\tilde{\mathbf{\mu}}_v^H \mathbf{\Lambda}(s) \tilde{\mathbf{\mu}}_v)}{\det(\mathbf{I} - s \mathbf{\Lambda}_v)} \\ &= \frac{\exp\left(\sum_{k=1}^{2L} \frac{|\tilde{\mu}_k|^2 s \lambda_k}{1 - s \lambda_k}\right)}{\prod_{k=1}^{2L} (1 - s \lambda_k)} \end{aligned} \quad (3.39)$$

where $\mathbf{\Lambda}(s) = (\mathbf{I} - s \mathbf{\Lambda}_v)^{-1} - \mathbf{I}$ is a diagonal matrix.

In the case of repeated eigenvalues, the MGF of $Z_u(0)$ is given by

$$\Phi(-s) = \frac{\exp\left(\sum_{m=1}^M \frac{b_m^2 s \lambda_m}{1 - s \lambda_m}\right)}{\prod_{m=1}^M (1 - s \lambda_m)^{w_m}}. \quad (3.40)$$

where λ_m , for $m = 1, \dots, M$, are distinct eigenvalues of $\mathbf{A}$ defined in (3.36) where M is the total number of distinct eigenvalues, w_m is the multiplicity of the eigenvalue λ_m . It is clear that $(-1/\lambda_1, \dots, -1/\lambda_M)$ are the M distinct poles of the MGF $\Phi(s)$ of $Z_u(0)$ conditioned on $\{a_k(0)\}_{k=1}^K$.

In (3.40) we have

$$b_m^2 = \sum_{k \in \kappa_m} |\tilde{\mu}_k|^2. \quad (3.41)$$

where κ_m denotes the set of k indices associated with the m th distinct eigenvalue.

Here, the MGF of the decision variable for 2-ray channel model is given as an example

$$\Phi(s) = \frac{e^{-(K_{f0}+K_{f1})}}{R} e^{\frac{2(\sigma_1^2 K_{f0} M_1 + \sigma_0^2 K_{f1} M_0 - W)}{R}}. \quad (3.42)$$

where the coefficients are given by

$$R = c_4 s^4 + c_3 s^3 + c_2 s^2 + c_1 s + 1,$$

$$c_4 = \sigma_0^2 \sigma_1^2,$$

$$c_3 = -4\sqrt{2\gamma_b} \sigma_0^2 \sigma_1^2 y(0),$$

$$c_2 = -(\sigma_0^2 + \sigma_1^2) + 8\gamma_b \sigma_0^2 \sigma_1^2 y(0)^2 - 8\gamma_b \sigma_0^2 \sigma_1^2 |y_{0,1}|^2,$$

$$c_1 = 2\sqrt{2\gamma_b} (\sigma_0^2 + \sigma_1^2) y(0),$$

$$\text{and } W = \sqrt{2\gamma_b} s \left[\mu_{0x} \mu_{1x} \text{Re}(y_{0,1}) + \mu_{0y} \mu_{1x} \text{Im}(y_{0,1}) + \mu_{0y} \mu_{1y} \text{Re}(y_{0,1}) - \mu_{0x} \mu_{1y} \text{Im}(y_{0,1}) \right].$$

The region of convergence is the vertical strip enclosing the $j\omega$ axis bounded by the closest poles on either side.

3.3.4 Exact BER Formula

A. Closed-Form Expression

If an exact calculation of $P(Z_u(0) < 0)$ is sought, a common starting point is the Laplace inversion formula [3]

$$P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} = P(Z_u(0) \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} \leq 0) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \Phi(s) \frac{ds}{s}. \quad (3.43)$$

where c is a small positive constant between zero and the smallest positive pole of $\Phi(s)/s$.

This complex contour integral might be evaluated exactly by calculating the residues of $\Phi(s)/s$ [59] over the poles in the right-hand side (RHS) of the complex s -plane. Since $\Phi(s)$ contains essential singularities which makes the residue evaluation very difficult, power series expansion is applied to solve the problem. Assuming that among the M distinct poles of $\Phi(s)$ (3.40), M_1 of them are positive. Following the method described in [64], we obtain a closed-form expression for (3.43) as

$$P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} = \sum_{k=1}^{M_1} \left(\prod_{m=1, m \neq k}^M \left(\left(\frac{\lambda_k}{\lambda_k - \lambda_m} \right)^{w_m} \exp \left(\frac{b_m^2 \lambda_m}{\lambda_k - \lambda_m} \right) \right) \right) \cdot \exp(-b_k^2) \cdot \left(\sum_{n=0}^{\infty} \sum_{r_k=0}^{n+w_k-1} \frac{b_k^{2n} G_k^{(r_k)}(\lambda)}{n! r_k! (-\lambda_k)^{r_k}} \right) \quad (3.44)$$

where

$$G_k^{(r)}(\lambda) = \left[\sum_{r_1=0}^{r-1} \binom{r-1}{r_1} g_k^{(r-1-r_1)}(\lambda) \cdot \sum_{r_2=0}^{r_1-1} \binom{r_1-1}{r_2} g_k^{(r_1-1-r_2)}(\lambda) \cdots \right] \quad (3.45)$$

In (3.45), $g_k^{(n)}(\lambda)$ is given by

$$g_k^{(n)}(\lambda) = \sum_{m=1, m \neq k}^M \left[n! \left(\frac{\lambda_k \lambda_m}{\lambda_k - \lambda_m} \right)^{n+1} \left(w_m + (n+1) \frac{b_m^2 \lambda_m}{\lambda_k - \lambda_m} \right) \right] \quad (3.46)$$

B. Numerical Method

Equation (3.44) gives a general closed-form BER expression for MC-CDMA systems over arbitrary Rician multipath fading channels. Since (3.44) is derived using the fast-convergent power series expansion of the MGF about its positive poles, it is expected to converge rapidly with respect to the index n . However, as shown in [64], the series (3.44) converges more slowly as the Rician K -factor increases, therefore it is more suitable for analysis of Rician fading channels with a small K -factor. To achieve a more efficient solution to this problem, a different approach towards the exact calculation of (3.43) is used instead [65].

Since the left-hand side of (3.43) is a real-valued quantity, we can write

$$\begin{aligned} P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\Phi(c + j\omega)}{c + j\omega} d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{c \operatorname{Re}\{\Phi(c + j\omega)\} + \omega \operatorname{Im}\{\Phi(c + j\omega)\}}{c^2 + \omega^2} d\omega. \end{aligned} \quad (3.47)$$

The change of variable $\omega = c\sqrt{1-x^2}/x$ yields

$$\begin{aligned} P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} &= \frac{1}{2\pi} \int_{-1}^1 \left\{ \operatorname{Re} \left[\Phi \left(c + jc \frac{\sqrt{1-x^2}}{x} \right) \right] + \frac{\sqrt{1-x^2}}{x} \operatorname{Im} \left[\Phi \left(c + jc \frac{\sqrt{1-x^2}}{x} \right) \right] \right\} \frac{dx}{\sqrt{1-x^2}}. \end{aligned} \quad (3.48)$$

Finally, using a Gauss-Chebyshev quadrature (GCQ) rule with an even number v of nodes, we have

$$P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} = \frac{1}{v} \sum_{k=1}^{v/2} \left\{ \operatorname{Re} [\Phi(c + jc\tau_k)] + \tau_k \operatorname{Im} [\Phi(c + jc\tau_k)] \right\} + E_v. \quad (3.49)$$

where $\tau_k = \tan((2k-1)\pi/(2\nu))$ and the error term E_ν vanishes as $\nu \rightarrow \infty$. In order to achieve the desired degree of accuracy it is practical increasing values of ν and accepting the result when P does not change significantly. In general, 32 to 64 nodes are sufficient to obtain an accuracy better than 10^{-5} . The value of c affects the number of nodes necessary to achieve a preassigned accuracy. The detailed discussion of the selection of c can be found in [65]. In our numerical results we assume $\nu = 64$, and c is set equal to one half the smallest real part of the poles of $\Phi(s)$. However, even with a suboptimum choice of c the value of ν does not grow so large as to become unmanageable.

In the above section, a residue calculation method and a numerical method via the GCQ rule are given to obtain the error probability conditioned on the user data based on the MGF. For coherent detection, we have

$$P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} = P_{e,u} \big|_{a_u(0)=-1, \{a_k(0)\}_{k=1, k \neq u}^K} . \quad (3.50)$$

By averaging the conditioned BER (3.44) or (3.49) over $\{a_k(0)\}_{k=1, k \neq u}^K$ which are independent and identically distributed binary random variables, we obtain the BER performance of the user u

$$P_{e,u} = \frac{1}{2^{K-1}} \sum_{a_1(0) \in \{1, -1\}} \cdots \sum_{a_{k \neq u}(0) \in \{1, -1\}} \cdots \sum_{a_K(0) \in \{1, -1\}} P_{e,u} \big|_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} . \quad (3.51)$$

The average BER of all users is given by

$$P_e = \frac{1}{K} \sum_{i=1}^K P_{e,i} . \quad (3.52)$$

As the computational complexity to evaluate (3.51) increases exponentially with the number of users, the proposed approach is appropriate for systems with relatively small number of users, say $K < 20$. When the number of user is large, we may resort to traditional Gaussian approximation. In the following section we propose a simple evaluation method to remedy this problem.

C. Gaussian approximation method for large number of users

From (3.26), the decision variable of the desired user can be rewritten as

$$Z_u(0) = D_u + I_u + N_u. \quad (3.53)$$

where

$$D_u = \sqrt{\frac{E_b}{N}} a_u(0) \sum_{i=0}^{N-1} |h_i|^2. \quad (3.54)$$

$$I_u = \sqrt{\frac{E_b}{N}} \sum_{i=0}^{N-1} |h_i|^2 \sum_{k=1, k \neq u}^K a_k(0) c_{u,i} c_{k,i}. \quad (3.55)$$

$$N_u = \text{Re} \left[\sum_{i=0}^{N-1} c_{u,i} h_i^* n_i \right]. \quad (3.56)$$

The term D_u is the desired output. I_u is the MAI component. N_u is the noise term contributed by AWGN. Due to the equal power, equal probable, antipodal data modulation $a_k(0)$ in (3.55), we can apply the Central Limit Theorem and

approximate $g_{u,i} = \sum_{k=1, k \neq u}^K a_k(0) c_{u,i} c_{k,i}, (i = 0, 1, \dots, N-1)$ as a zero mean Gaussian

random variable. Note that $g_{u,i}$'s are correlated between different subcarriers. The covariance matrix M_u is a $N \times N$ matrix with each element given by

$$M_u(i1, i2) = \sum_{k=1, k \neq u}^K c_{u,i1} c_{k,i1} c_{u,i2} c_{k,i2}.$$

The decision variable conditioned on $\{h_i\}_{i=0}^{N-1}$ is a Gaussian random variable with

$$E(D_u) = \sqrt{\frac{E_b}{N}} a_u(0) \sum_{i=0}^{N-1} |h_i|^2 = \sqrt{\frac{E_b}{N}} a_u(0) \sum_{i=0}^{N-1} \rho_i^2. \quad (3.57)$$

$$\text{var}(I_u) \approx \frac{E_b}{N} \sum_{i1=0}^{N-1} \sum_{i2=0}^{N-1} |h_{i1}|^2 |h_{i2}|^2 M_u(i1, i2) = \frac{E_b}{N} \sum_{i1=0}^{N-1} \sum_{i2=0}^{N-1} \rho_{i1}^2 \rho_{i2}^2 M_u(i1, i2). \quad (3.58)$$

$$\text{var}(N_u) = \frac{N_0}{2} \sum_{i=0}^{N-1} |h_i|^2 = \frac{N_0}{2} \sum_{i=0}^{N-1} \rho_i^2. \quad (3.59)$$

Equation (3.58) is derived from [66] for the variance of the linear combination of correlated random variables. The probability of error conditioned on $\{\rho_i\}_{i=0}^{N-1}$ is simply given by

$$P[e | \mathbf{p}] = \frac{1}{2} \text{erfc} \left(\sqrt{\frac{E(D_u)}{\text{var}(I_u) + \text{var}(N_u)}} \right). \quad (3.60)$$

The BER for the zeroth data stream is obtained via averaging $P[e | \mathbf{p}]$ over $\mathbf{p}$

$$P_e = \int_0^\infty \int_0^\infty \cdots \int_0^\infty p[e | \mathbf{p}] p(\rho_0, \rho_1, \dots, \rho_{N-1}) d\rho_0 d\rho_1 \cdots d\rho_{N-1}. \quad (3.61)$$

The evaluation of (3.61) can be done by Monte Carlo integration [51][52].

3.3.5 Numerical Results and Discussions

In this section, we present numerical results. To calculate the BER, it is assumed that the number of subcarriers is 32, and two channel delay profiles are discussed, a simple two-ray multipath delay profile often encountered in urban and hilly areas, and a three-path channel with linear delay power profile. For the two-ray channel, we assume $v_0 = 0, v_1 = 14$. Furthermore, Walsh Hadamard codes are used as the spreading codes.

For the channels and the signals described above, we can compute the BER based on the closed-form expression given by (3.44) and the GCQ method (3.49). Figure 3.6 shows the BER curves obtained by the two methods. The results obtained by the two methods match very well as long as the value of $n_{\max}$ is sufficiently large. To check how quickly the infinite series in (3.44) converges with respect to n , we give the BER values based on the truncated series for the index n summing up from 0 to $n_{\max}$, and the results are given in **Table 3.1** and **Table 3.2** for two-ray and 3-path channels. The BER for $n_{\max} = \infty$ is obtained by setting $n_{\max}$ large enough in (3.44), so that the first nine significant digits for each BER value converge. It is also shown in the tables that the results of the closed-form expression conform exactly to those computed by the GCQ method. The results in **Table 3.1-4** also show that the series in (3.44) converges very quickly for a relatively small K -factor, and $n_{\max} = 5-10$ may be accurate enough for practical use. For large K -factors, it requires $n_{\max} = 15-30$ to get an accurate approximation. Moreover, it is relatively more difficult for the series to converge in the channels with more paths, and the number of users in the system

does not affect the convergence speed. If $n_{\max} = 15 - 30$ or more, the closed-form method is far more complex than GCQ method. Since the GCQ method appears more computationally efficient compared with the closed-form expression especially when the K -factor is large. In the Figure 3.7-Figure 3.12, the analytical results are derived using the GCQ method.

The results of the proposed time domain method described herein are verified by comparing with the Monte Carlo simulations. In Figure 3.7 and Figure 3.8 the K -factors for different taps are equal, while in Figure 3.9, the K -factors for each tap are different. In all these cases, the simulation results agree very well with the theoretical results obtained by the technique proposed in this paper. The simulation results are based on the calculation of the decision variable for each bit, and averaging the results over large number of bits (say 100,000 bits). Figure 3.10 shows the accuracy of our low-complexity method proposed in Section 3.3.4. The results indicate that our low-complexity method give quite a good approximation as compared with the accurate result, and the accuracy improves when the number of user becomes high.

Next, the effect of the Rician K -factor on the BER performance is investigated using the analytical formula derived in Section 3.3. The Rician K -factor is defined as the ratio of signal power in dominant component over the (local-mean) scattered power which corresponds a deterministic strong wave received. It's natural to expect a better performance for larger values of K -factor. From the analytical results shown in (Figure 3.11), it can be seen that the performance improvement due to the increase in K -factor is evident when the number of user is small (i.e. MAI is not dominant), while for 20 users (where MAI dominates), there's no difference between the BER

performance for different values of K -factor (that is $K_f = 0.2$, $K_f = 1$ and $K_f = 10$).

For this case, MAI trends to be so severe and it becomes the dominant factor in determining the system performance. Simulation results are also given to validate analytical results. Figure 3.12 shows the BER comparison for different values of K -factor when SNR equals 10dB for 1, 10 and 20 users. A distinct variation can be seen when the number of users is small, while for 20 users, there is hardly any variation in the BER as MAI dominates the performance in this case.

$n_{\max}$	K -factor		
	0.2	1	4.5
5	0.00539032392	0.00375964584	0.08947561189
10	0.00539032392	0.00375964646	0.00267096359
15	0.00539032392	0.00375964646	0.00161027475
∞	0.00539032392	0.00375964646	0.00163187943
GCQ	0.00539032392	0.00375964646	0.00163187943

Table 3.1: BER values with truncated series for 2-ray channel, $K=1$, SNR=10dB.

$n_{\max}$	K -factor		
	0.2	1	4.5
5	0.0155976326	0.0125381670	0.00958320166
10	0.0155976326	0.0125382808	0.00962593545
15	0.0155976326	0.0125382808	0.00962593744
∞	0.0155976326	0.0125382808	0.00962593744
GCQ	0.0155976326	0.0125382808	0.00962593744

Table 3.2: BER values with truncated series for 2-ray channel, $K=10$, SNR=10dB.

$n_{\max}$	K -factor		
	0.2	1	2.8
10	0.00286634634	0.00448217119	0.01146651024
15	0.00286634634	0.00194363052	0.00966268806
20	0.00286634634	0.00194363052	0.00103149956
∞	0.00286634634	0.00194363052	0.00131590191
GCQ	0.00286634634	0.00194363052	0.00131590191

Table 3.3: BER values with truncated series for 3-path channel, $K=1$, SNR=10dB.

$n_{\max}$	K -factor		
	0.2	1	2.8
10	0.108401585	0.00875032091	0.0646651024
15	0.108401585	0.00427313056	0.0168790431
20	0.108401585	0.00427313056	0.0117689852
∞	0.108401585	0.00427313056	0.0939229789
GCQ	0.108401585	0.00427313056	0.0939229789

Table 3.4: BER values with truncated series for 3-path channel, $K=10$, SNR=10dB.

Figure 3.6 Theoretical BER of MC-CDMA for 2-ray ($K_{f0} = K_{f1} = K_f$), 3-path channel model ($K_{f0} = K_{f1} = K_{f2} = K_f$) using MRC by the closed-form and the GCQ method.

Figure 3.7 BER of MC-CDMA for 2-ray ($K_{f0} = K_{f1} = 0.2$), 3-path channel model ($K_{f0} = K_{f1} = K_{f2} = 0.2$) using MRC.

Figure 3.8 BER of MC-CDMA for 2-ray ($K_{f0} = K_{f1} = 1$), 3-path channel model

($K_{f0} = K_{f1} = K_{f2} = 1$) using MRC.

Figure 3.9 BER of MC-CDMA using MRC, for 2-ray channel model:

($K_{f0} = 10, K_{f1} = 4$), for 3-path channel model: ($K_{f0} = 10, K_{f1} = 2.5, K_{f2} = 0$).

Figure 3.10 BER of MC-CDMA system with MRC versus SNR for 2-ray channel

model: $v_0 = 0, v_1 = 14$, and $K_{f0} = K_{f1} = 1$.

Figure 3.11 BER of MC-CDMA system versus SNR with various number of users

and K -factor, 2-ray channel model: $v_0 = 0, v_1 = 14$, $K_{f0} = K_{f1} = K_f$.

Figure 3.12 BER of MC-CDMA system versus K -factor, SNR=10dB, 2-ray channel

$$\text{model, } v_0 = 0, v_1 = 14, K_{f0} = K_{f1} = K_f.$$

3.4 Conclusion

In this chapter, we propose an accurate and simple method to calculate the performance of MC-CDMA systems with MRC combining scheme over a downlink correlated Rayleigh and Rician fading channel. The main idea of the time domain approach is as follows. First, the number of resolvable paths in the time domain is usually far less than the number of subcarriers. Second, the channel coefficients of the taps are statistically independent of each other while the channel parameters are highly correlated at each subcarrier. As a result, transforming the summation in the frequency domain to the time domain simplifies the calculation significantly. Using moment generating function method, we are able to avoid the complexity encountered in deriving exact probability density function (PDF) of the decision

variable. It is shown that the method leads to an elegant solution to the performance evaluation of MC-CDMA systems.

Till now, we have discussed T-spaced channel model which means that all the delay paths of the channel are at the sampling instances. The algorithm proposed here works quite efficiently in this scenario because of the sparsity constraints on the channel. For non T-spaced channel, we may refer to the cyclic model proposed in [73] which describes the non T-spaced channel as T-spaced channel by discrete Fourier transform. The validity of the cyclic model depends on how well the objective of the guard space is met, i.e., how well it eliminates inter-block interference.

We present two methods for deriving the MGF of the decision variables for Rayleigh and Rician cases. Actually they are all based on the Gaussian quadratic forms. However, the method proposed in Section 3.3.3 provides a more compact form. In addition, the method can achieve a unified solution for MC-CDMA systems over both the downlink and uplink transmission which will be presented in the next chapter.

Chapter 4

Exact BER for Synchronous MC-CDMA Systems in the Uplink

4.1 Introduction

In this chapter we discuss the BER performance of MC-CDMA in uplink transmission. To evaluate the performance of MC-CDMA, a common method is using Gaussian approximation assuming that the fading in different subcarriers is independent of each other [1][52]]. However, the assumption may not be satisfied in practice, since the fading of the subcarriers is usually correlated due to insufficient frequency separation between the subcarriers. Furthermore, its accuracy depends on system configurations, especially on the number of users and their powers. It is well known that the Gaussian approximation is not accurate when the number of users is small or power control is inaccurate. As mentioned in the last chapter [55], a closed-form BER expression for a synchronous MC-CDMA has been derived based on the assumption of independent fading among subcarriers and exponential distributed

multipath intensity profile (MIP). In this chapter, we propose an exact BER analysis for synchronous MC-CDMA with maximal ratio combining (MRC) in correlated Rayleigh fading channels based on Gaussian quadratic forms and characteristic function. Note that this method does not require making the unrealistic assumptions of independent fading among subcarriers and regarding MIP. Moreover, a simple closed-form expression can be obtained.

The organization of the remainder of this chapter is as follows. In Section 4.2, we describe in detail the proposed BER evaluation approach for synchronous MC-CDMA systems. In Section 4.2.1, system model is presented. The exact BER formula in closed-form for correlated Rayleigh fading channel is given in Section 4.2.2. In Section 4.2.3, numerical results and discussions are given. Finally, the conclusion is presented in Section 4.3.

4.2 BER Performance for Synchronous MC-CDMA Systems

4.2.1 System Model

A synchronous MC-CDMA system [55] with K active users and N subcarriers in the uplink is considered. Assume that binary phase-shift keying (BPSK) modulation and deterministic binary spreading sequence are employed. Let $a_k(q)$ and $\{c_{k,i}\}_{i=0}^{N-1}$

denote the q th data bit and spreading sequence of user k , respectively. The equivalent low-pass transmitted signal for user k can be expressed as

$$S^k(t) = \sum_{q=-\infty}^{\infty} \sqrt{\frac{E_b(k)}{N}} a_k(q) p_b(t - qT_b) \cdot \sum_{i=0}^{N-1} c_{k,i} \exp(j[\omega_i t + \theta_k]). \quad (4.1)$$

where $E_b(k)$ is the power of the data bit of user k , $p_b(t)$ is a rectangular pulse defined in $[0, T_b]$ with T_b denoting the bit duration, and $\omega_i = 2\pi i / T_b$ is the i th subcarrier frequency, and the random phase θ_k is uniformly distributed over $[0, 2\pi)$.

A slowly varying frequency-selective Rayleigh fading channel is considered. The channel gain for the i th subcarrier of user k can be modeled as a zero mean complex Gaussian random variable $h_{k,i} = \rho_{k,i} e^{j\varphi_{k,i}}$ with $E[\rho_{k,i}^2] = 1$. We assume that the fading processes for different users are independent and the covariance matrix is denoted by $\Sigma_{kk} = E(\mathbf{h}_k \mathbf{h}_k^H)$. In addition, we define $\{b_{k,l}\}_{l=0}^{L_k-1}$ as channel coefficient for non-zero paths in the time domain, the $\{v_{k,l}\}_{l=0}^{L_k-1}$ as the propagation delay for the non-zero paths normalized by the sample interval, and $\sigma_{k,l}^2$ as the variance of the real and imaginary parts, L_k is the number of the non-zero paths of user k . It has been shown that

$$h_{k,i} = \sum_{n=0}^{L_k-1} b_{k,n} e^{-j2\pi i v_{k,n} / N}, \quad i = 0, 1, \dots, N-1. \quad (4.2)$$

From the relationship (4.2), the covariance matrix can be calculated by

$$\Sigma_{kk}(i, n) = \sum_{l=0}^{L_k-1} 2\sigma_{k,l}^2 e^{j2\pi(i-n)v_{k,l} / N}. \quad (4.3)$$

The received signal at the base station can be written as

$$r(t) = \sum_{q=-\infty}^{\infty} \sum_{k=1}^K \sqrt{\frac{E_b(k)}{N}} a_k(q) p_b(t - qT_b) \cdot \sum_{i=0}^{N-1} h_{k,i} c_{k,i} \exp(j[\omega_i t + \theta_k]) + n(t). \quad (4.4)$$

where $n(t)$ is additive white Gaussian noise (AWGN) having a double-sided PSD of $N_0/2$ for both real and imaginary components.

A coherent correlation receiver with maximal ratio combining is used. Assuming user u is the desired user, the decision variable of the *zeroth* data bit is given by

$$Z_u(0) = \text{Re} \left\{ \sum_{i=0}^{N-1} h_{u,i}^* c_{u,i} \frac{1}{T_b} \int_0^{T_b} r(t) \exp(-j[\omega_i t + \theta_u]) dt \right\}. \quad (4.5)$$

4.2.2 Performance Analysis

To obtain the exact BER, we first derive the MGF expression for the decision variables which is defined as $\Phi_Z(s) = E(e^{-sZ})$ and then evaluate the probability of error based on the MGF. Assuming the length of the cyclic prefix is longer than that of the CR, so that there is no ISI. Assuming $a_u(0) = 1$, the decision variable of the *zeroth* data bit can be derived from (4.5)

$$Z_u(0) = \text{Re} \left\{ \sum_{k=1, k \neq u}^K \sum_{i=0}^{N-1} \left[\sqrt{\frac{E_b(k)}{N}} a_k(0) h_{u,i}^* h_{k,i} c_{u,i}^* c_{k,i} e^{j(\theta_k - \theta_u)} \right] + \sum_{i=0}^{N-1} \left[\sqrt{\frac{E_b(u)}{N}} h_{u,i} h_{u,i}^* \right] + \sum_{i=0}^{N-1} [c_{u,i}^* h_{u,i} n_i e^{-j\theta_u}] \right\}. \quad (4.6)$$

If we define

$$y_{u,i} = \sqrt{\frac{E_b(u)}{N}} h_{u,i} + \sum_{k=1, k \neq u}^K \left[\sqrt{\frac{E_b(k)}{N}} a_k(0) c_{u,i}^* c_{k,i} h_{k,i} e^{j(\theta_k - \theta_u)} \right] + c_{u,i}^* n_i e^{-j\theta_u}. \quad (4.7)$$

The decision variable for the desired user conditioned on $a_k(0), (k = 1, 2, \dots, K)$ can be rewritten as

$$Z_u(0) = \text{Re}(\mathbf{h}_u^H \mathbf{y}_u) = \mathbf{v}^H(0) \mathbf{Q}(0) \mathbf{v}(0). \quad (4.8)$$

where

$$\mathbf{v}(0) = [\mathbf{h}_u^T, \mathbf{y}_u^T]^T. \quad (4.9)$$

$\mathbf{Q}(0)$ is a $2N \times 2N$ matrix defined as

$$\mathbf{Q}(0) = \begin{bmatrix} \mathbf{0}_N & 0.5\mathbf{I}_N \\ 0.5\mathbf{I}_N & \mathbf{0}_N \end{bmatrix}. \quad (4.10)$$

In (4.10), $\mathbf{0}_N$ and $\mathbf{I}_N$ are the $N \times N$ zero and identity matrices, respectively.

Since in (4.8), $\{h_{k,i}\}_{i=0}^{N-1}$ are identical but correlated Rayleigh random variables with $E[|h_{k,i}|^2] = 1$, $\varphi_{k,n}$ and $\{\theta_k - \theta_u\}$ are identical random variables uniformly distributed over $[0, 2\pi)$, $\{\rho_{k,n} e^{j(\varphi_{k,i} + \theta_k - \theta_u)}\}$ can be viewed as correlated Gaussian random variables with zero mean and variance $1/2$. Hence, (4.8) is a central Gaussian quadratic form.

Note that $\mathbf{P}_v = E\{\mathbf{v}(0)\mathbf{v}^H(0)\}$ is rank deficient. In this case, linear transformation is used first to make it full rank

$$\mathbf{v}(0) = \mathbf{B}\tilde{\mathbf{v}}(0). \quad (4.11)$$

where

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{0}_{L_u} \\ \mathbf{0}_{N \times L_u} & \mathbf{I}_{L_u} \end{bmatrix}.$$

In (4.11), each element in $\mathbf{B}_1$ is given by

$$B_u(i, n) = \exp(j2\pi i v_{u,n} / N), \quad (i = 0, 1, \dots, N-1; \quad n = 0, 1, \dots, L_u-1). \quad (4.12)$$

Accordingly,

$$\tilde{\mathbf{Q}}(0) = \mathbf{B}^H \mathbf{Q}(0) \mathbf{B}. \quad (4.13)$$

The resulting vector $\tilde{\mathbf{v}}(0)$ after transformation is given by

$$\tilde{\mathbf{v}}(0) = [\mathbf{b}_u^T, \mathbf{y}_u^T]^T. \quad (4.14)$$

with the mean vector of $\tilde{\mathbf{v}}(0)$ shown by

$$\boldsymbol{\mu}_v = E(\tilde{\mathbf{v}}(0)) = \mathbf{0}_{(L_u+N) \times 1}.$$

With the assumption that the channel coefficients $b(l)$'s for different paths are statistically independent from each other and the AWGN noise term is independent of the channel impulse response, a simple expression of the covariance matrix of $\tilde{\mathbf{v}}(0)$ is achieved

$$\tilde{\mathbf{P}}_v = E\{\tilde{\mathbf{v}}(0)\tilde{\mathbf{v}}^H(0)\} = \begin{bmatrix} E(\mathbf{b}_u \mathbf{b}_u^H) & E(\mathbf{b}_u \mathbf{y}_u^H) \\ E(\mathbf{y}_u \mathbf{b}_u^H) & E(\mathbf{y}_u \mathbf{y}_u^H) \end{bmatrix}. \quad (4.15)$$

where

$$\mathbf{P}_1 = E(\mathbf{b}_u \mathbf{b}_u^H) = 2 \text{diag}(\sigma_{u,0}^2, \sigma_{u,1}^2, \dots, \sigma_{u,L_u-1}^2). \quad (4.16)$$

For each element in $\mathbf{P}_2 = E(\mathbf{b}_u \mathbf{y}_u^H)$

$$\mathbf{P}_2(i, n) = \sqrt{\frac{E_b(u)}{N}} (2\sigma_{u,i}^2) \exp(j2\pi n v_{u,i} / N). \quad (4.17)$$

For each element in $\mathbf{P}_3 = E(\mathbf{y}_u \mathbf{y}_u^H)$

$$\mathbf{P}_3(i, n) = \sum_{k=1}^K \left[\frac{E_b(k)}{N} c_{u,i} c_{k,i} c_{u,n} c_{k,n} \sum_{kk} (i, n) \right]. \quad (4.18)$$

Employing a result for the distribution of a noncentral Gaussian quadratic form [62][63], the MGF of $Z_u(0)$ in (4.7) can be obtained as

$$\Phi(s) = \frac{\exp(\boldsymbol{\mu}_v^H \mathbf{F}(s) \boldsymbol{\mu}_v)}{\det(\mathbf{I} + s \tilde{\mathbf{P}}_v^{1/2} \tilde{\mathbf{Q}}(0) \tilde{\mathbf{P}}_v^{1/2})} = \frac{1}{\det(\mathbf{I} + s \tilde{\mathbf{P}}_v^{1/2} \tilde{\mathbf{Q}}(0) \tilde{\mathbf{P}}_v^{1/2})}. \quad (4.19)$$

and

$$\mathbf{F}(s) = \tilde{\mathbf{P}}_v^{-1/2} \left((\mathbf{I} + s \tilde{\mathbf{P}}_v^{1/2} \tilde{\mathbf{Q}}(0) \tilde{\mathbf{P}}_v^{1/2})^{-1} - \mathbf{I} \right) \tilde{\mathbf{P}}_v^{-1/2}.$$

where $\tilde{\mathbf{P}}_v^{-1/2}$ is the inverse of the symmetric square root of $\tilde{\mathbf{P}}_v$. Note that the sign of s is different from that in [62] due to the difference in the definition of MGF.

To represent the conditioned MGF $\Phi(s)$ in a more compact form, we define a matrix

$$\mathbf{A} = \tilde{\mathbf{P}}_v^{1/2} \tilde{\mathbf{Q}}(0) \tilde{\mathbf{P}}_v^{1/2}. \quad (4.20)$$

and represent its eigen-decomposition as $\mathbf{A} = \mathbf{U}_v \mathbf{\Lambda}_v \mathbf{U}_v^H$, where

$$\mathbf{\Lambda}_v = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_{L_u+N}). \quad (4.21)$$

is the eigenvalue matrix of $\mathbf{A}$ containing the singular values $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_{L_u+N}$ on its diagonal. Equation (4.19) can now be simplified to

$$\Phi(s) = \frac{1}{\det(\mathbf{I} + s\mathbf{\Lambda}_v)} = \frac{1}{\prod_{k=1}^{L_u+N} (1 + s\lambda_k)}. \quad (4.22)$$

In the case of repeated eigenvalues, the MGF of $Z_u(0)$ is given by

$$\Phi(s) = \frac{1}{\prod_{m=1}^M (1 + s\lambda_m)^{w_m}}. \quad (4.23)$$

where M is the total number of distinct eigenvalues, w_m is the multiplicity of the eigenvalue λ_m .

From (4.16), (4.17), (4.18) and (4.19), we notice that the MGF of $Z_u(0)$ is not related to the value of $\{a_k(0)\}_{k=1, k \neq u}^K$, which lead to a simple solution to the problem without suffering the heavy computational complexity as in [54]. Furthermore, equation (4.18) clearly indicates that the MGF depends on the value of the spreading sequences for correlated fading channels among subcarriers, while the statement that the MAI is not associated with the spreading sequences [55] is only applicable to the

case when the fading channels among different subcarriers is independent from each other. In this case, $\mathbf{P}_3$ is a diagonal matrix, and no longer associates with the spreading sequence.

If an exact calculation of $P(Z_u(0) < 0)$ is sought, a common starting point is the Laplace inversion formula [3]

$$P_{e,u} = P(Z_u(0) \leq 0) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \Phi(s) \frac{ds}{s}. \quad (4.24)$$

where c is a small positive constant between zero and the smallest positive pole of $\Phi(s)/s$. This complex contour integral might be evaluated exactly by calculating the residues of $\Phi(s)/s$ [59] over the poles in the right-hand side (RHS) of the complex s -plane

$$P_{e,u} |_{\{a_k(0)\}_{k=1, k \neq u}^K} = - \sum_{RHS} \text{Re sidue} \left[\frac{\Phi(s)}{s} \right]. \quad (4.25)$$

where the residues are calculated and summed over the poles in the right-hand side (RHS) of the complex s -plane.

Assuming that among the M distinct poles of $\Phi(s)$ (4.21), the first M_1 of them are positive. Following the method described in [62], we obtain a closed-form expression for (4.25) as

$$P_e = -\sum_{m=1}^{M_1} \frac{1}{(w_m - 1)!} \left\{ \frac{d^{w_m-1}}{ds^{w_m-1}} \left[\frac{\Phi(s)}{s} (s - \lambda_m)^{w_m-1} \right] \right\} \Big|_{s=\lambda_m} \quad (4.26)$$

$$= \sum_{k=1}^{M_1} \left(\prod_{m=1, m \neq k}^M \left(\frac{\lambda_k}{\lambda_k - \lambda_m} \right)^{w_m} \right) \sum_{r_k=0}^{w_k-1} \frac{G_k^{(r_k)}(\lambda)}{r_k! (-\lambda_k)^{r_k}}.$$

where

$$G_k^{(r)}(\lambda) = \left[\sum_{r_1=0}^{r-1} \binom{r-1}{r_1} g_k^{(r-1-r_1)}(\lambda) \cdot \sum_{r_2=0}^{r_1-1} \binom{r_1-1}{r_2} g_k^{(r_1-1-r_2)}(\lambda) \cdots \right]. \quad (4.27)$$

In (4.27), $g_k^{(n)}(\lambda)$ is given by

$$g_k^{(n)}(\lambda) = \sum_{m=1, m \neq k}^M \left[n! w_m \left(\frac{\lambda_k \lambda_m}{\lambda_k - \lambda_m} \right)^{n+1} \right]. \quad (4.28)$$

If the eigenvalues of A are distinct, (4.26) is simplified to

$$P_e = \sum_{k=1}^{M_1} \prod_{\substack{m=1 \\ m \neq k}}^M \frac{\lambda_k}{\lambda_k - \lambda_m}. \quad (4.29)$$

4.2.3 Numerical Results and Discussions

It is assumed that the number of subcarriers is 32, and two channel delay profiles are discussed, a simple two-ray multipath delay profile often encountered in urban and hilly areas, a three-path channel with linear delay power profile. And the received powers at the base station for all users are identical due to perfect power control. In all of the cases, the simulation results agree very well with the theoretical results obtained by the technique proposed (Figure 4.1 and Figure 4.2).

Next, we discuss the MAI by using the theoretical formula. Taking the simplest 2-ray channel model as an example. Walsh Hadamard codes are used. The MAI is caused by the cross-talk between the independent two paths in the decision variable which is due to the imperfect combining scheme. The cross-talk is determined by the correlation property of the spreading codes in the frequency domain and the relative time delay of the two paths. Figure 4.3 shows the close relationship between the BER performance and the two key elements stated above. The correlation parameter of the spreading codes is denoted by R . It is interesting that the BER does not vary monotonously with ν_1 which is the measure of delay spread.

Figure 4.1 BER of synchronous MC-CDMA for 2-ray channel model.

lines: analytical results (A), marks: simulation results (S).

Figure 4.2 BER of synchronous MC-CDMA for 3-path channel model.

lines: analytical results (A), marks: simulation results (S).

Figure 4.3 (1)(2):BER and R versus the number of users for 2-ray channel model,.

(3):BER versus v_1 of the desired user, $K=16$. All: $SNR=10dB$, $v_0 = 0$.

4.3 Conclusion

A simple closed-form BER expression for a synchronous MC-CDMA system over correlated Rayleigh fading channels is proposed in the chapter. Based on the Gaussian quadratic form, an exact MGF is obtained for the decision variables. A simple BER expression is derived accordingly. The results obtained by the analytical formula have been verified by computer simulations.

In Chapter 3 and Chapter 4, the performance of MC-CDMA system is analyzed over frequency-selective fading channels. To improve the performance in scattering environments, transmit diversity is a practical, effective and hence, widely applied technique to reduce the effect of multipath fading. In the next chapter, we focus on MC-CDMA system with transmit diversity.

Chapter 5

MC-CDMA with Transmit Diversity

5.1 Introduction

Transmit diversity is considered to be an effective method of combating detrimental effects in wireless fading channels. In this chapter, we consider MC-CDMA systems with transmit diversity. The application of space-time block codes (STBC) to multicarrier CDMA (MC-CDMA) systems proposed in [67] and [68] is investigated. By linearly combining received data in two consecutive symbol intervals, diversity is achieved at each subcarrier. Different combining schemes are used to gather all the information at each subcarrier to make a decision. The combining scheme should be carefully designed to mitigate both multiple access interference as well as AWGN noise, and at the same time should be able to achieve high spectral efficiency. We investigate the performance of four commonly used combining schemes, namely ORC, EGC, MRC and MMSEC. In addition, we propose a MMSE per user combining scheme for MC-CDMA system with STBC based on MMSE criterion. It is shown by simulation results that the proposed

combining scheme can achieve considerable performance improvement when the system is not fully loaded. It is also shown that MC-CDMA system with STBC not only can achieve full spatial diversity provided by the orthogonal design of STBC, but also achieve full frequency diversity by carefully combining components of each subcarrier.

In [67], error performance analysis for MC-CDMA system with STBC using EGC is provided by making Gaussian approximation assuming that the fading in different subcarriers is independent of each other. Nevertheless, as mentioned earlier, the assumption is not guaranteed in practice. In this chapter, we propose a characteristic function and residue calculation based approach to investigate the error performance of the system with MRC scheme in correlated Rayleigh fading channels. As in the previous chapter, the error performance is obtained in the time domain instead of usual the frequency domain. The most important feature of this approach is that we can obtain an exact BER performance without making any assumptions about the distribution of MAI and any further assumptions with respect to the statistical or spectral nature of the channel. Moreover, it is not necessary to assume that the fading of the subcarriers is independent of each other and the desired signal and the MAI from different users are mutually independent. Another feature is that a simple closed-form expression for the BER can be obtained.

It has been shown that introducing transmit diversity in MC-CDMA system can dramatically improve the performance. By applying space-time block codes, it's sufficient to assume that the path gains are constant during a frame and vary from frame to frame (quasi-static flat fading). The requirements may not be satisfied in

cases when Doppler spread is high, that is, the remote unit is moving at high speed or the signals are transmitted at high carrier frequency. We propose a transmit diversity scheme for MC-CDMA system, which is able to achieve the same performance as MC-CDMA system with STBC without the assumption of quasi-static fading channel for space-time block codes. Taking advantage of the structure of MCM, different frequency groups are assigned to individual antenna. Simulation results show that the performance of MC-CDMA is greatly improved. An exact BER performance of the proposed system in correlated Rayleigh fading channels is obtained based on the CF method and residue calculation.

In this chapter, first we investigate the performance of MC-CDMA systems with STBC using four commonly used combining schemes in Section 5.2. A new MMSE per user combining scheme is proposed in Section 5.3. In Section 5.4 we derive the exact performance analysis for MC-CDMA systems with STBC. Finally, we present a new transmit diversity scheme for MC-CDMA systems in Section 5.5. Conclusion is given in Section 5.6.

5.2 Performance Investigation of MC-CDMA Systems with Space-time Block Codes

5.2.1 System Model

A MC-CDMA transmitter with space-time block codes is shown in Figure 5.1.

Figure 5.1 MC-CDMA transmitter with STBC.

We use the space-time block codes proposed by Alamouti with two transmitters and one receiver [69]. The processing gain N is assumed to be equal to the number of carriers. The original signal is first space-time coded and then subjected to the common process of MC-CDMA. The spread signal is modulated at different carriers. IDFT is used to implement MCM. After the cyclic prefix is added, the signal is transmitted from one of the antennas. At the receiver, the RF signal is first converted to baseband. After the portion of the signal corresponding to the prefix is removed, DFT is performed on the signal samples. Simple decoding algorithms based on linear processing are used to extract the information from the received signal. Various combining schemes can then be used to gather all the information at each subcarrier to make a decision.

5.2.2 Performance Analysis Using Gaussian Approximation

The following section will focus on BER performance evaluation. The length of the cyclic prefix is assumed to be longer than the length of the channel impulse response, so that there is no ISI. It is appropriate to use space-time block codes as the CR of each subcarrier can be considered as flat fading. It is sufficient to consider the received signal in two consecutive blocks, say block 0 and block 1. We assume the channel is slow fading, and the CRs of block 0 and block 1 are same. The receiver receives the signals from both of the transmitters. The orthogonality character of STBC allows us to extract easily the transmitted signal in different time blocks from the combined signal at the receiver. After DFT is performed, $(r_{0,i}, r_{1,i})$ which is the data received at i th carrier in block 0 and block 1, can be written as

$$r_{0,i} = \sum_{k=1}^K \sqrt{\frac{E_s}{2N}} (h_{0,i} c_{k,i} a_{0,k} + h_{1,i} c_{k,i} a_{1,k}) + n_{0,i} \quad (5.1)$$

$$r_{1,i} = \sum_{k=1}^K \sqrt{\frac{E_s}{2N}} (-h_{0,i} c_{k,i}^* a_{1,k}^* + h_{1,i} c_{k,i}^* a_{0,k}^*) + n_{1,i} \quad (5.2)$$

where K is the number of active users, $c_{k,i}$ is the i th chip of the k th user's spreading code, $(a_{0,k}, a_{1,k})$ denotes the transmitted signal from the k th user at time block 0 and 1, respectively, and E_s is the symbol energy of the signal. In order to have the same total radiated power from two transmit antennas, the energy allocated to each symbol should be halved. We assume users have the same symbol energy, and $(n_{0,i}, n_{1,i})$ are double-sided additive white Gaussian noise with variance σ_n^2 , and $h_{0,i}$ and $h_{1,i}$ are

CR from two transmit antennas to the receiver at the i th carrier. We assume the channel impulse response for all users and each subcarrier is independent and identically distributed Rayleigh random variables, and the CR from two transmit antennas is independent of each other. The power loss due to the insertion of cyclic prefix is not considered.

To demonstrate the relation between the transmitted signal and the received signal in compact form, we write (5.1) and (5.2) in vector form

$$\mathbf{R}_0 = \sqrt{\frac{E_s}{2N}}(\mathbf{H}_0 \mathbf{C} \mathbf{A}_0 + \mathbf{H}_1 \mathbf{C} \mathbf{A}_1) + \mathbf{N}_0. \quad (5.3)$$

$$\mathbf{R}_1 = \sqrt{\frac{E_s}{2N}}(-\mathbf{H}_0 \mathbf{C} \mathbf{A}_1^* + \mathbf{H}_1 \mathbf{C} \mathbf{A}_0^*) + \mathbf{N}_1. \quad (5.4)$$

Here, we denote $\mathbf{R}_0 = [r_{0,0}, r_{0,1}, \dots, r_{0,N-1}]^T$, and $\mathbf{R}_1 = [r_{1,0}, r_{1,1}, \dots, r_{1,N-1}]^T$ are received signal vector at time block 0 and 1. Let $\mathbf{N}_0 = [n_{0,0}, n_{0,1}, \dots, n_{0,N-1}]^T$ and $\mathbf{N}_1 = [n_{1,0}, n_{1,1}, \dots, n_{1,N-1}]^T$. $\mathbf{H}_0 = \text{diag}(h_{0,0}, h_{0,1}, \dots, h_{0,N-1})$ and $\mathbf{H}_1 = \text{diag}(h_{1,0}, h_{1,1}, \dots, h_{1,N-1})$ are CR matrices from transmitter 0 and 1 to the receiver. $\mathbf{C} = [\mathbf{C}_1, \mathbf{C}_2, \dots, \mathbf{C}_K]$ where each column of the matrix, namely $\mathbf{C}_k$ is the spreading code of the k th user. $\mathbf{A}_0 = [a_{0,1}, a_{0,2}, \dots, a_{0,K}]^T$, $\mathbf{A}_1 = [a_{1,1}, a_{1,2}, \dots, a_{1,K}]^T$ are input signal vectors of K users at time block 0 and 1.

A. *The BER Performance for EGC*

The gain for the equal gain combining is given by [68]

$$Z[0] = \sum_{i=0}^{N-1} \left(\frac{h_{0,i}^* r_{0,i}}{\sqrt{|h_{0,i}|^2 + |h_{1,i}|^2}} + \frac{h_{1,i} r_{1,i}^*}{\sqrt{|h_{0,i}|^2 + |h_{1,i}|^2}} \right). \quad (5.5)$$

Substituting (5.1) and (5.2) into (5.5), we get

$$Z[0] = \sum_{i=0}^{N-1} c_{1,i} \left(\sqrt{\frac{E_s}{2N}} \sqrt{|h_{0,i}|^2 + |h_{1,i}|^2} \left(\sum_{k=1}^K a_{0,k} c_{k,i} \right) + \frac{h_{0,i}^* n_{0,i} + h_{1,i} n_{1,i}^*}{\sqrt{|h_{0,i}|^2 + |h_{1,i}|^2}} \right). \quad (5.6)$$

If User 1 is the desired user, the decision variable $Z[0]$ for time block 0 can be written as

$$Z[0] = D[0] + I[0] + N[0]. \quad (5.7)$$

where

$$D[0] = \sqrt{\frac{E_s}{2N}} a_{0,1} \sum_{i=0}^{N-1} \sqrt{|h_{0,i}|^2 + |h_{1,i}|^2}, \quad (5.8)$$

$$I[0] = \sqrt{\frac{E_s}{2N}} \sum_{k=2}^K a_{0,k} \sum_{i=0}^{N-1} c_{1,i} c_{k,i} \sqrt{|h_{0,i}|^2 + |h_{1,i}|^2}, \quad (5.9)$$

$$N[0] = \sum_{i=0}^{N-1} \frac{h_{0,i}^* n_{0,i} + h_{1,i} n_{1,i}^*}{\sqrt{|h_{0,i}|^2 + |h_{1,i}|^2}}, \quad (5.10)$$

The desired signal component, MAI and the noise component for time block 0 are $D[0]$, $I[0]$ and $N[0]$ respectively. If the spreading codes and the input data symbol are random, MAI component can be approximated to a zero-mean Gaussian distributed random variable by invoking the Central Limit Theorem when the number of subcarriers is large enough. Similarly, the noise component is approximated as a

zero-mean Gaussian distributed random variable. The expectation of the desired signal, the variance of MAI and the noise component are (See Appendix A for detail)

$$E(D[0]) = N \sqrt{\frac{9\pi P_i}{8}}, \quad (5.11)$$

$$\text{Var}(I[0]) \approx (K-1)\left(4 - \frac{9\pi}{8}\right)P, \quad (5.12)$$

$$\text{Var}(N[0]) \approx NN_0, \quad (5.13)$$

where P_i for $(i=0, \dots, N-1)$ is the local mean power of each subcarrier, and P is the total local-mean power of each user.

Finally, the BER can be approximated by

$$P_e = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{\frac{9}{32}\pi P}{2\left(1 - \frac{9\pi}{32}\right)\frac{(K-1)}{N}P + \frac{N_0}{2}}}\right), \quad (5.14)$$

B. The BER Performance for MRC

For MRC case, the gain is given by

$$Z[0] = \sum_{i=0}^{N-1} h_{0,i}^* r_{0,i} + h_{1,i} r_{1,i}^*, \quad (5.15)$$

Combining with (5.1) and (5.2), we get

$$Z[0] = \sum_{i=0}^{N-1} c_{1,i} \left[\sqrt{\frac{E_s}{2N}} (|h_{0,i}|^2 + |h_{1,i}|^2) \left(\sum_{k=1}^K a_{0,k} c_{k,i} \right) + (h_{0,i}^* n_{0,i} + h_{1,i} n_{1,i}^*) \right] \quad (5.16)$$

Similarly, the signal, MAI and noise component can be approximated by

$$E(D[0]) = 4P, \quad (5.17)$$

$$\text{Var}(I[0]) \approx \frac{8(K-1)P^2}{N}, \quad (5.18)$$

$$\text{Var}(N[0]) \approx 4PN_0, \quad (5.19)$$

We derive the BER performance for MRC as

$$P_e = \frac{1}{2} \text{erfc}\left(\sqrt{\frac{P}{\frac{(K-1)}{N}P + \frac{N_0}{2}}}\right). \quad (5.20)$$

C. Orthogonality Restoring Combining (ORC)

For ORC case, the decision variable is given by

$$Z[0] = \sum_{i=0}^{N-1} c_{1,i} \left[\sqrt{\frac{E_s}{2N}} \left(\sum_{k=1}^K a_{0,k} c_{k,i} \right) + \frac{(h_{0,i}^* n_{0,i} + h_{1,i} n_{1,i}^*)}{|h_{0,i}|^2 + |h_{1,i}|^2} \right] \quad (5.21)$$

Because the orthogonality among different users is fully restored, the receiver can eliminate multiple-access interference perfectly. The signal and noise component can be approximated by

$$E(D[0]) = \sqrt{\frac{PN}{2}}, \quad \text{Var}(N[0]) = N_0 \sum_{n=0}^{N-1} \frac{1}{|h_{0,n}|^2 + |h_{1,n}|^2}. \quad (5.22)$$

D. Minimum Mean Square Error Combining (MMSEC)

The gain for MMSEC is also provided as

$$Z[0] = \sum_{i=0}^{N-1} \frac{h_{1,i}^* r_{0,i} + h_{0,i} r_{1,i}^*}{(|h_{0,i}|^2 + |h_{1,i}|^2) + 2\sigma^2 N / (E_s K)}. \quad (5.23)$$

5.2.3 Numerical Results and Discussions

In order to demonstrate the numerical results, we assume following system parameters: transmission rate $R=1\text{M}$ symbols/s (QPSK format), Walsh-Hadamard code with $N=32$, linearly delay profile with delay spread of 3 chip duration [61], Doppler power spectrum with maximum Doppler frequency $f_d=10\text{Hz}$. From Figure 5.2 and Figure 5.3, it is clear that MC-CDMA system with transmit diversity performs better than MC-CDMA system without transmit diversity with the same total radiated power. For four combining schemes, it is seen that transmit diversity can improve the BER performance irrespective of how many users are in the system. It is also seen from Figure 5.4-6 that as in MC-CDMA system without transmit diversity, the MRC for MC-CDMA system with transmit diversity performs quite well when the number of users is small ($K=10$), while the performance becomes worse when the number of users is relatively large ($K=16$ and 32). For ORC, since the orthogonality among users is fully restored, MAI is completely eliminated. The performance does not vary with the number of users. It may also be pointed out that in MC-CDMA system with ORC scheme, low-level subcarriers tend to be multiplied by the high gains, and the noise components are amplified at weaker subcarriers. This noise amplification effect degrades the BER performance. In the case of MC-CDMA

system with transmit diversity, because the probability that both the signals transmitted from two antennas experience deep fading channel is small, this noise amplification effect is greatly suppressed. The performance improvement is evident for the system with transmit diversity compared to the system without transmit diversity. MMSEC performs best among the four schemes no matter how many users are active.

In Figure 5.7 and Figure 5.8, the performance evaluated using the closed-form BER equations developed in (5.14), (5.20) for EGC and MRC is verified by simulations. The simulation results agree very well with analytical results for the case of independent fading among subcarriers (i.e. There are independent 32 subcarriers scattered in the symbol duration [13]). However, the results obtained by Gaussian approximation in (5.14), (5.20) are not applicable to the case when there're not sufficient independent paths during the symbol duration, the CR among subcarriers is highly correlated (Figure 5.8). In the following, we provide an exact performance analysis for MC-CDMA systems with STBC over correlated Rayleigh fading channels.

Figure 5.2 BER of MC-CDMA with and without transmit diversity (MRC).

Figure 5.3 BER of MC-CDMA with and without transmit diversity (MMSEC).

Figure 5.4 BER of MC-CDMA with transmit diversity (using four combining schemes), $K=10$.

Figure 5.5 BER of MC-CDMA with transmit diversity (using four combining schemes), $K=16$.

Figure 5.6 BER of MC-CDMA with transmit diversity (using four combining schemes), $K=32$.

Figure 5.7 Downlink BER for MC-CDMA with STBC (EGC).

Figure 5.8 Downlink BER for MC-CDMA with STBC (MRC).

5.3 MMSE per User Combining

In the MMSEC receiver described in [68], the equalization coefficients are based on the MMSE criterion applied independently per subcarrier. The complexity of the equalizer is low. However, this MMSE equalization per carrier method is not optimal, since it does not take into account the despreading process and thus does not minimize the mean square error at the input of the threshold detector.

We now propose MMSE per user combining for MC-CDMA with transmit diversity.

If User I is the desired user, the aim of the proposed method is to minimize the mean square error between the transmitted symbol $a_{0,I}$ and the estimated symbol $\hat{a}_{0,I}$.

Let $\mathbf{W}_1^T = [w_1^0, w_1^1, \dots, w_1^{N-1}]$ be the optimal weighting vector. The estimated symbol of User I is

$$\hat{a}_{0,I} = \mathbf{W}_1^T \cdot (\mathbf{H}_0^* \mathbf{R}_0 + \mathbf{H}_1 \mathbf{R}_1^*) = \mathbf{C}_1^T \cdot \mathbf{G} \cdot \mathbf{R} \quad (5.24)$$

where

$$\begin{aligned} \mathbf{R} &= (\mathbf{H}_0^* \mathbf{R}_0 + \mathbf{H}_1 \mathbf{R}_1^*) \\ &= (|\mathbf{H}_0|^2 + |\mathbf{H}_1|^2) \mathbf{C} \mathbf{A}_0 + (\mathbf{H}_0^* \mathbf{N}_0 + \mathbf{H}_1 \mathbf{N}_1^*) \end{aligned} \quad (5.25)$$

Let

$$\mathbf{N} = \mathbf{H}_0^* \mathbf{N}_0 + \mathbf{H}_1 \mathbf{N}_1^*. \quad (5.26)$$

According to Wiener filter, the optimal weighting vector is

$$\mathbf{W}_1 = \mathbf{\Gamma}_{\mathbf{RR}}^{-1} \cdot \mathbf{\Gamma}_{\mathbf{R}a_{0,I}} \quad (5.27)$$

where $\mathbf{\Gamma}_{\mathbf{RR}}$ is the autocorrelation matrix of the received vector $\mathbf{R}$ and $\mathbf{\Gamma}_{\mathbf{R}a_{0,I}}$ is the cross-correlation vector between the desired symbol $a_{0,I}$ and the received signal vector $\mathbf{R}$. These terms are equal to

$$\begin{aligned} \mathbf{\Gamma}_{\mathbf{RR}} &= E\{\mathbf{R}^* \cdot \mathbf{R}^T\} \\ &= (|\mathbf{H}_0|^2 + |\mathbf{H}_1|^2) \cdot \mathbf{C} \cdot E\{\mathbf{A}_0^* \cdot \mathbf{A}_0^T\} \cdot \mathbf{C}^T + (|\mathbf{H}_0|^2 + |\mathbf{H}_1|^2) + E\{\mathbf{N}^* \cdot \mathbf{N}^T\} \end{aligned} \quad (5.28)$$

$$\begin{aligned}\Gamma_{\mathbf{R}a_{0,1}} &= E\{\mathbf{R}^* a_{0,1}\} \\ &= (|\mathbf{H}_0|^2 + |\mathbf{H}_1|^2)CE\{\mathbf{A}_0^* \cdot a_{0,1}\}\end{aligned}\quad (5.29)$$

The optimal weighting vector can then be written as

$$\mathbf{W}_1^T = E\{a_{0,1} \cdot \mathbf{A}^{*T}\} \cdot \mathbf{C}^T \mathbf{H} \cdot (\mathbf{H} \cdot \mathbf{C} \cdot E\{\mathbf{A} \cdot \mathbf{A}^{*T}\} \cdot \mathbf{C}^T \cdot \mathbf{H} + E\{\mathbf{N} \cdot \mathbf{N}^{*T}\})^{-1}. \quad (5.30)$$

We assume user signals have the same power ($E(\mathbf{A}_k \cdot \mathbf{A}_k^*) = E_c = E_s / N$), and are independent to each other, then $E\{\mathbf{A} \cdot \mathbf{A}^{*T}\} = E_c \cdot \mathbf{U}$, where $\mathbf{U}$ is a diagonal matrix, with $\mathbf{U}_{kk} = 1$ when the k th user is active and $\mathbf{U}_{kk} = 0$ when the k th user is inactive.

The part of AWGN in (5.30) is given by

$$E\{\mathbf{N} \cdot \mathbf{N}^{*T}\} = 2\sigma^2 (|\mathbf{H}_0|^2 + |\mathbf{H}_1|^2) \mathbf{I}, \quad (5.31)$$

where $\mathbf{I}$ is the $(N \times N)$ identity matrix.

Finally, we get the equalization coefficient matrix as

$$\mathbf{G} = \mathbf{H} \cdot (\mathbf{H} \cdot \mathbf{C} \cdot \mathbf{U} \cdot \mathbf{C}^T \cdot \mathbf{H} + 2\sigma^2 \mathbf{H})^{-1}, \quad (5.32)$$

where $\mathbf{H} = |\mathbf{H}_0|^2 + |\mathbf{H}_1|^2$.

It is seen from Figure 5.9, that the performance of MMSEC per user scheme is better than MMSEC per carrier scheme, when the system is not fully loaded. For a fully loaded system (that is, 32 users are active for 32 subcarriers case), the quantity $\mathbf{C} \cdot \mathbf{U} \cdot \mathbf{C}^T$ is equal to the identity matrix and the equalization coefficients matrix $\mathbf{G}$ is a diagonal matrix with the k th subcarrier equalization coefficient equal to that of

MMSEC per carrier scheme. In this case (and only in this case) the performance of the two MMSEC approaches is same.

Figure 5.9 BER of MC-CDMA with transmit diversity (using two MMSEC).

5.4 Exact BER Performance Evaluation

5.4.1 Exact BER Formula

The following section will focus on BER performance evaluation for the systems described in Section 5.2.1 with MRC. With the same assumption that the length of the cyclic prefix is longer than the length of the channel impulse response, there is no ISI. From the analysis in Section 5.2.2, $(r_{0,i}, r_{1,i})$ which is the data received at i th carrier in block 0 and block 1, can be written as

$$r_{0,i} = \sum_{k=1}^K \sqrt{\frac{E_s}{2N}} (h_{0,i} c_{k,i} a_k(0) + h_{1,i} c_{k,i} a_k(1)) + n_{0,i} \quad (5.33)$$

$$r_{1,i} = \sum_{k=1}^K \sqrt{\frac{E_s}{2N}} (-h_{0,i} c_{k,i}^* a_k(1)^* + h_{1,i} c_{k,i}^* a_k(0)^*) + n_{1,i} \quad (5.34)$$

If user u is the desired user, the gain for MRC is given by

$$\begin{aligned} r_u[0] &= \sum_{i=0}^{N-1} c_{u,i} (h_{0,i}^* r_{0,i} + h_{1,i} r_{1,i}^*) \\ r_u[1] &= \sum_{i=0}^{N-1} c_{u,i} (h_{1,i}^* r_{0,i} - h_{0,i} r_{1,i}^*) \end{aligned}, \quad (5.35)$$

Substituting (5.33) and (5.34) into (5.35), we get

$$r_u(0) = \sqrt{\frac{E_s}{2N}} \sum_{i=0}^{N-1} \beta_i (h_{0,i}^* h_{0,i} + h_{1,i} h_{1,i}^*) + \sum_{i=0}^{N-1} c_{u,i} (h_{0,i}^* n_{0,i} + h_{1,i} n_{1,i}^*), \quad (5.36)$$

where

$$\beta_i = a_u(0) + \sum_{k=1, k \neq u}^K a_k(0) c_{k,i} c_{u,i}, \quad (i = 0, 1, \dots, N-1)$$

Without loss of generality, a factor equal to $\sqrt{2/N_0}$ is multiplied to (5.36). Then the following equation is achieved

$$r_u(0) = \sqrt{\frac{\gamma_b}{N}} \sum_{i=0}^{N-1} \beta_i (h_{0,i}^* h_{0,i} + h_{1,i} h_{1,i}^*) + \sum_{i=0}^{N-1} c_{u,i} (h_{0,i}^* n_{0,i}' + h_{1,i} n_{1,i}^*), \quad (5.37)$$

This implies that $n_{0,i}'$ and $n_{1,i}'$ in (5.37) are normalized AWGN noise with variance of $\sigma_n^2 = 1$, and the ratio $\gamma_b = E_b / N_0$ is the signal-to-noise ratio per bit. The decision variable with MRC is obtained

$$Z_u(0) = \text{Re}[r_u(0)]. \quad (5.38)$$

Applying discrete Parseval's theorem and the characteristic of DFT that multiplication of the DFT of two sequences is equivalent to the circular convolution of the two sequences in the time domain, the decision variable in (5.38), omitting a factor equal to $\sqrt{N}$, can be rewritten in the time domain as

$$Z_u(0) = \text{Re} \left\{ \sqrt{\gamma_b} \sum_{i=0}^{N-1} [x_{0,i} b_{0,i}^* + x_{1,i} b_{1,i}^*] + \sum_{i=0}^{N-1} [\eta_{0,i} b_{0,i}^* + \eta_{1,i} b_{1,i}^*] \right\} \quad (5.39)$$

where

$$x_{t,i} = y(i) \otimes b_{t,i} \\ = \sum_{m=0}^{N-1} b_{t,i} y(i-m)_N = \sum_{m=0}^{L-1} b_{t,v_m} y(i-v_m)_N, \quad (5.40)$$

where $(i = 0, 1, \dots, N-1)$ and $(t = 0 \text{ or } 1)$. $\eta_{0,i}$ and $\eta_{1,i}$ are the corresponding normalized noise in the time domain which is also AWGN. $\mathbf{y}$ is the corresponding signal of $\boldsymbol{\beta}$ in the time domain given the value $\mathbf{y} = \text{IFFT}(\boldsymbol{\beta}, N)$

$$Z_u(0) = \text{Re} \left\{ \sqrt{\gamma_b} \sum_{i=0}^{L-1} \sum_{m=0}^{L-1} [b_0(v_m) y(v_i - v_m)_N b_0(v_i)^* + b_1(v_m) b_1(v_i)^* \right. \\ \left. \cdot y(v_i - v_m)_N] + \sum_{i=0}^{L-1} [\eta_0(v_i) b(v_i)^* + \eta_1(v_i)^* b_1(v_i)] \right\} \quad (5.41)$$

For $a_u(0) = 1$, an error occurs if $Z_u(0) < 0$. For $a_u(0) = -1$, and error occurs if $Z_u(0) > 0$.

The decision variable $Z_u(0)$ is conditioned on the CR $\{b(v_i)\}_{i=0}^{L-1}$ and user data $\{a_k(0)\}_{k=1}^K$. We compute the BER by first averaging over $\{b(v_i)\}_{i=0}^{L-1}$ and then over

$\{a_k(0)\}_{k=1}^K$. In order to obtain an exact error performance in closed form, we calculate the MGF of $Z_u(0)$ conditioned on $\{a_k(0)\}_{k=1}^K$, shown in the next section. The BER of user u conditioned on $\{a_k(0)\}_{k=1}^K$ can be exactly calculated through residues

$$P_{e,u} |_{a_u(0)=1, \{a_k(0)\}_{k=1, k \neq u}^K} = - \sum_{RHS} \text{Residue} \left[\frac{\Phi(s)}{s} \right]. \quad (5.42)$$

where the residues are calculated and summed over the poles in the right-hand side (RHS) of the complex s -plane, i.e., the real part of s is positive.

By averaging (5.42) over $\{a_k(0)\}_{k=1, k \neq u}^K$ which are independent and identically distributed binary RVs, we obtain the error performance conditioned on $a_u(0)$

$$P_{e,u} |_{a_u(0)=1} = - \sum_{a_1(0) \in \{1, -1\}} \cdots \sum_{a_{k \neq u}(0) \in \{1, -1\}} \cdots \sum_{a_K(0) \in \{1, -1\}} \frac{1}{2^{K-1}} \sum_{RHS} \text{Residue} \left[\frac{\Phi(s)}{s} \right]. \quad (5.43)$$

The BER performance of the user u can now be written as

$$\begin{aligned} P_{e,u} &= \frac{1}{2} (P_{e,u} |_{a_u(0)=1} + P_{e,u} |_{a_u(0)=-1}) \\ &= P_{e,u} |_{a_u(0)=1} \end{aligned} \quad (5.44)$$

The average BER of all users is given by

$$P_e = \frac{1}{K} \sum_{i=1}^K P_{e,i}. \quad (5.45)$$

5.4.2 Derivation of MGF

In this section, we derive the MGF function of the decision variable for Rayleigh fading channel. Assuming the complex variables in (5.41) as

$$\begin{aligned}\eta(v_{ti}) &= m_{ti} + jm_{tiy}, \\ b(v_{ti}) &= b_{ti} + jb_{tiy}, \quad (i = 0, 1, \dots, L-1) \text{ and } (t = 0 \text{ or } 1)\end{aligned}\quad (5.46)$$

The decision variable in (5.41) can be written as (5.47)

$$\begin{aligned}Z_u(0) &= \sum_{t=0}^1 \left\{ \sqrt{\gamma_b} \left\{ 2 \sum_{i=0}^{L-1} \sum_{m=i+1}^{L-1} \left[\text{Re}[y(v_i - v_m)_N] (b_{ti} b_{tm} + b_{tiy} b_{tmy}) + \text{Im}[y(v_i - v_m)_N] (b_{ti} b_{tm} - b_{tiy} b_{tmy}) \right] \right. \right. \\ &\quad \left. \left. + y(0) \sum_{i=0}^{L-1} (b_{ti}^2 + b_{tiy}^2) \right\} \right\} + \sum_{t=0}^1 \sum_{i=0}^{L-1} [m_{ti} b_{ti} + m_{tiy} b_{tiy}] \\ &\quad .\end{aligned}\quad (5.47)$$

From (5.47), it is observed that the decision variable consists of two parts corresponding to $t=0$ and $t=1$. From the assumption that the CRs from two transmit antennas is independent and identical random process, it's easy to verify that the MGF of the two parts can be calculated separately. In the following, the index of the transmitter, that is t , is omitted from the expressions.

Because $b(n)$'s are independent complex Gaussian random variables for Rayleigh fading channel, and the AWGN noise term is independent of the channel impulse response, the MGF can be calculated as

$$\begin{aligned}\Phi_0(s) &= \alpha \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} e^{-\left[sZ + \sum_{i=0}^{L-1} \frac{(b_{ix}^2 + b_{iy}^2)}{2\sigma_i^2} + \sum_{i=0}^{L-1} \frac{(m_{ix}^2 + m_{iy}^2)}{2} \right]} dm_{0x} dm_{0y} \cdots dm_{(L-1)x} dm_{(L-1)y}, \\ &\quad db_{0x} db_{0y} \cdots db_{(L-1)x} db_{(L-1)y}\end{aligned}\quad (5.48)$$

$$\text{where } \alpha = \frac{1}{(2\pi)^{2L} \prod_{i=0}^{L-1} \sigma_i^2}.$$

Noticing that the exponential of (5.48) is a pure quadratic form. The integral can be calculated by properly changing the variables

$$\begin{cases} m'_{ix} = m_{ix} + sb_{ix}, & / & m'_{iy} = m_{iy} + sb_{iy}, \\ b'_{ix} = b_{ix} + \frac{\sum_{k=i+1}^{L-1} P_{ikx} b_{kx} - P_{iky} b_{ky}}{R_i}, & / & b'_{iy} = b_{iy} + \frac{\sum_{k=i+1}^{L-1} P_{iky} b_{kx} + P_{ikx} b_{ky}}{R_i}. \end{cases} \quad (5.49)$$

where $(i = 0, 1, \dots, L-1)$ and

$$\begin{aligned} M_i &= \sqrt{\gamma_b} y(0)s + \frac{1}{2\sigma_i^2} - \frac{s^2}{2}, \\ J_{ikx} &= \sqrt{\gamma_b} \operatorname{Re}[y(v_i - v_k)_N]s, & / & & J_{iky} &= \sqrt{\gamma_b} \operatorname{Im}[y(v_i - v_k)_N]s, \\ P_{ikx} &= J_{ikx} - \sum_{n=0}^{i-1} \frac{P_{nix} P_{nqx} + P_{niy} P_{nky}}{R_n}, & / & & P_{iky} &= J_{iky} - \sum_{n=0}^{i-1} \frac{P_{nix} P_{nky} - P_{niy} P_{nqx}}{R_n}, \\ R_i &= M_i - \sum_{n=0}^{i-1} \frac{P_{nix}^2 + P_{niy}^2}{R_n}, & & & & (k = 0, 1, \dots, L-1) \text{ and } (k > i) \end{aligned}$$

Then, (5.48) becomes (5.50), where $J=1$ denotes the Jacobian of the transformation (5.49)

$$\begin{aligned} \Phi_0(s) &= J\alpha \prod_{i=0}^{L-1} \left[\int_{-\infty}^{\infty} e^{-\frac{1}{2}m_{ix}^{'2}} dm_{ix}' \int_{-\infty}^{\infty} e^{-\frac{1}{2}m_{iy}^{'2}} dm_{iy}' \right] \cdot \prod_{i=0}^{L-1} \left[\int_{-\infty}^{\infty} e^{-R_i b_{ix}^{'2}} db_{ix}' \int_{-\infty}^{\infty} e^{-R_i b_{iy}^{'2}} db_{iy}' \right] \\ &= \frac{1}{2^L \prod_{i=0}^{L-1} \sigma_i^2 R_i}, \end{aligned} \quad (5.50)$$

Finally, the MGF of the decision variable is derived

$$\Phi(s) = \frac{1}{\left\{ 2^L \prod_{i=0}^{L-1} \sigma_i^2 R_i \right\}^2}, \quad (5.51)$$

Here, the MGF of the decision variable for 2-ray channel model is given as an example

$$\Phi(s) = \frac{1}{(\sigma_0^2 \sigma_1^2 s^4 + c_3 s^3 + c_2 s^2 + c_1 s + 1)^2}, \quad (5.52)$$

where the coefficients are given by

$$\begin{aligned} c_3 &= -4\sqrt{\gamma_b} \sigma_0^2 \sigma_1^2 y(0), \\ c_2 &= -(\sigma_0^2 + \sigma_1^2) + 4\gamma_b \sigma_0^2 \sigma_1^2 y(0)^2 - 4\gamma_b \sigma_0^2 \sigma_1^2 |y(v_0 - v_1)_N|^2, \\ c_1 &= 2\sqrt{\gamma_b} (\sigma_0^2 + \sigma_1^2) y(0), \end{aligned}$$

The region of convergence is the vertical strip enclosing the $j\omega$ axis bounded by the closest poles on either side.

5.4.3 Numerical Results and Discussions

In this section, we present numerical results. It is assumed that the number of subcarriers is 32, transmission rate is 1M symbols/s, and Walsh-Hadamard codes are used as spreading codes. Two types of channel delay profiles are discussed, a simple two-ray multipath delay profile often encountered in urban and hilly areas, a three-path and a four-path channel with linear delay power profile. In all of the cases (Figure 5.10) and (Figure 5.11), the simulation results with 5, 10 and 20 users in the system shown by marks agree very well with the theoretical results obtained by MGF technique proposed in this paper indicated by lines. It can also be confirmed from the

simulation and analytical results that MC-CDMA system with space-time block codes can achieve not only full spatial diversity provided by the orthogonal design of STBC, but also yield full frequency diversity provided by MC-CDMA structure.

Figure 5.10 BER of MC-CDMA with STBC for 2-ray channel model (MRC),

$$v_0 = 0, v_1 = 14$$

Figure 5.11 BER of MC-CDMA with STBC for 3-path channel model (MRC).

5.5 A New Transmit Diversity Scheme for MC-CDMA

5.5.1 System Model

The algorithm is proposed based on the fact that for M -ary amplitude-shift keying (MASK), the signals transmitted at subcarriers are orthogonal with the frequency separation of $1/(2T_s)$, where T_s is the symbol duration [88]. As a result, the number of subcarriers doubles compared with the number of subcarriers when the frequency separation is $1/T_s$ if the available bandwidth is given.

Assuming the number of subcarriers is equal to $2N$. We assign two groups of N frequency points to different transmit antennas, where N is the processing gain or the number of chips per symbol. The carrier frequencies assigned to the first transmitter

are, $f_i = f_1 + 2i/(2T_s)$, ($i = 0, 1, \dots, N-1$), where T_s is the symbol duration, while the carrier frequencies assigned to the second transmitter will be, $f_i = f_1 + (2i+1)/(2T_s)$.

On the transmitter side, MASK is used. The duplicated signals are spread and modulated by the two groups of frequency points. The MCM can be easily implemented by taking IDFT with $2N$ points, then selecting the signal at the frequency points which correspond to the specific transmitter. After the cyclic prefix is added, the resulting multicarrier signal is transmitted from one of the antennas. On the receiver side, the received signal is first converted to baseband signal. After the portion of the signal corresponding to the cyclic prefix is removed, discrete cosine transform (DCT) is performed on the $2N$ signal samples. We assume that the length of the cyclic prefix is longer than the length of the channel impulse response, so that no ISI is introduced. A decomposer is used to separate the signal transmitted from different transmit antennas. Different combining schemes are used to gather all the information at each subcarrier to produce a decision. We use two commonly used combining schemes, namely, MRC and MMSEC. Simulation results show that the new system outperforms MC-CDMA system without space diversity with the same total transmit power.

As far as the complexity of implementation is concerned, a simple selector matrix Φ is used to extract the desired information from the combined signal, where Φ is $2N \times N$ permutation matrix constructed from the N columns of the $2N \times 2N$ identity matrix I_{2N} corresponding to the frequency points assigned to the desired transmitter. The resulting signal at the transmitter can then undergoes IFFT operation.

As permutation matrices have orthogonal columns, it follows that the selector matrices are orthogonal to each other, i.e. $\Phi_u^T \Phi_m = \delta(\mu - m) I_N$, where $\delta(\cdot)$ denotes Kronecher's delta, u and m denotes different transmitters.

So far we proposed a new transmit diversity scheme based on the intrinsic structure of MC-CDMA systems. For frequency selective fading, the channel impulse response for MCM with cyclic prefix insertion can be deemed as flat fading per subcarrier, because a circular matrix can be diagonalized by pre- and post-multiplication with FFT and IFFT matrices [70]. The new MC-CDMA system is able to achieve full transmit diversity without increasing the total bandwidth. Furthermore, the assumption of quasi-static fading channel for space-time block codes is also not required for the new scheme.

5.5.2 Performance Analysis for the Proposed System

So far we have designed a multicarrier transceiver that is able to achieve full transmit diversity gain regardless of the underlying frequency-selective channels. In this section, we consider the performance of the proposed system. The power loss due to the introduction of the cyclic prefix isn't considered in the analysis. The received signal after FFT operation can be written as

$$r_u(0) = \sqrt{\frac{E_b}{NN_0}} \sum_{i=0}^{N-1} \beta_i (h_{1,2i} + h_{2,2i+1}) + \sum_{i=0}^{2N-1} c_{u,i} n_i, \quad (5.53)$$

where

$$\beta_i = a_u(0) + \sum_{k=1, k \neq u}^K a_k(0) c_{k,i} c_{u,i}, \quad (i = 0, 1, \dots, N-1)$$

where $a_k(q)$ and $\{c_{k,i}\}_{i=1}^N$ denote the q th data bit and spreading sequence of user k , respectively, n_i is normalized AWGN noise with a variance $\sigma_n^2 = 1$, and the ratio $\gamma_b = E_b / N_0$ is the signal-to-noise ratio per bit. If user u is the desired user, the decision variable with MRC can be obtained as

$$Z_u(0) = \text{Re} \left\{ \sqrt{\frac{\gamma_b}{N}} \left[\sum_{i=0}^{N-1} \beta_i h_{1,2i} h_{1,2i}^* + \sum_{i=0}^{N-1} \beta_i h_{2,2i+1} h_{2,2i+1}^* \right] + \sum_{i=0}^{N-1} (n_{2i} h_{1,2i}^* + n_{2i+1} h_{2,2i+1}^*) \right\}, \quad (5.54)$$

The signal terms in (5.54) can be further expressed as

$$h_{1,2i} = \sum_{n=0}^{N-1} [b_1(n) + b_1(n+N)] e^{-j2\pi n / N}$$

$$h_{2,2i+1} = \sum_{n=0}^{N-1} [b_2(n) - b_2(n+N)] e^{-j\pi i / N} e^{-j2\pi n / N}$$

$$n_{2i} = \frac{1}{\sqrt{2N}} \sum_{n=0}^{N-1} [\eta(n) + \eta(n+N)] e^{-j2\pi n / N}$$

$$n_{2i+1} = \frac{1}{\sqrt{2N}} \sum_{n=0}^{N-1} [\eta(n) - \eta(n+N)] e^{-j\pi i / N} e^{-j2\pi n / N}$$

Applying discrete Parseval's theorem and the characteristic of DFT that multiplication of the DFT of two sequences is equivalent to the circular convolution of the two sequences in the time domain, the decision variable in (5.54), omitting a factor equal to $\sqrt{N}$, can be rewritten in the time domain as

$$\begin{aligned}
 Z_u(0) = & \text{Re}\{\sqrt{\gamma_b} \sum_{i=0}^{L-1} \sum_{m=0}^{L-1} [d_1(v_m) y(v_i - v_m)_N d_1(v_i)^* + d_2(v_m) \cdot y(v_i - v_m)_N d_2(v_i)^*] \\
 & + \frac{1}{\sqrt{2}} \sum_{i=0}^{L-1} [\eta_1(v_i) d_1(v_i)^* + \eta_2(v_i) d_2(v_i)^*]\}
 \end{aligned} \tag{5.55}$$

where $\mathbf{y}$ is the corresponding signal of $\boldsymbol{\beta}$ in the time domain, where,

$\mathbf{y} = \text{IFFT}(\boldsymbol{\beta}, N)$, and

$$d_1(i) = b(i) + b(i + N),$$

$$d_2(i) = [b_2(i) - b_2(i + N)]e^{-j\pi i / N},$$

$$\eta_1(i) = \eta(i) + \eta(i + N),$$

$$\eta_2(i) = [\eta(i) - \eta(i + N)]e^{-j\pi i / N}, \quad (i = 0, 1, \dots, N-1)$$

Using the algorithm presented in Section 5.4.2, we calculate the MGF of $Z_u(0)$, $\Phi(s) = E(e^{-sZ})$ conditioned on $\{a_k(0)\}_{k=1}^K$. The method to obtain the BER for the desired user u based on MGF is also similar to the technique described in Section 5.4.1, so we will not repeat it again for conciseness.

5.5.3 Numerical Results and Discussions

In our simulation, we assume a simple two-path multipath delay profile often encountered in urban and hilly areas. The close form of BER obtained in 5.5.2 has been validated by computer simulation (Figure 5.12). It is also shown that the performance of MC-CDMA system with transmit diversity improves compared to

conventional MC-CDMA system with the same radiated power. For MRC combining scheme, it is seen that transmit diversity can improve the BER performance. The same results hold for MMSE combining scheme shown in Figure 5.13. As in MC-CDMA systems, MC-CDMA systems with transmit diversity using MRC combining perform quite well when the number of user is small, while the performance degrades when the number of user becomes large. MMSEC performs better than MRC irrespective of the number of active users because it is able to restore orthogonality among users as well as avoid excessive noise enhancement by adaptively changing the gain according to the received signal and noise energy.

Figure 5.12 BER of MC-CDMA with and without transmit diversity (MRC).

Figure 5.13 BER of MC-CDMA with and without transmit diversity (MMSEC).

5.6 Conclusion

The performance of MC-CDMA systems with transmit diversity is considered in this chapter. The BER performance of the system with four commonly used combining schemes is evaluated by analysis and computer simulations. Introducing STBC to traditional MC-CDMA can improve the performance if combining schemes are properly selected. Among the four schemes, MMSEC performs best irrespective of how many users are active. The improved MMSE per user combining scheme for MC-CDMA system with STBC performs better than MMSE per carrier combining scheme, when the system isn't fully loaded.

In order to improve the accuracy of the above performance analysis methods, we have also propose an accurate and simple method to calculate the performance of

MC-CDMA systems with space-time block codes with MRC combining scheme over correlated Rayleigh fading channel. An exact BER is obtained in the time domain based on the characteristic function and residue calculation method. The results obtained by the analytical formula have been verified by computer simulation.

Furthermore, a new transmit diversity scheme for MC-CDMA system by assigning two groups of frequency points to different antennas is proposed. Full transmit diversity can be achieved. Simulation and analytical results show that the proposed system outperforms MC-CDMA system without space diversity having the same total radiated power. MMSEC performs better than MRC for the proposed scheme.

So far we discussed the performance of MC-CDMA system over downlink and uplink transmission and MC-CDMA system with transmit diversity. In the previous analysis, perfect channel state information is assumed. However, novel parameter estimation approaches are desired for coherent detection and decoding of space-time codes. In the next chapter, we propose one such technique of channel estimation for multicarrier system.

Chapter 6

Channel Estimation for Multicarrier Systems

6.1 Introduction

In this chapter, we address channel estimation algorithms for multicarrier systems. The proposed channel estimation is developed for classical cyclic prefix (CP) based OFDM system and related MC-CDMA system. It is applicable to general multicarrier systems, since MC-CDMA system relies on multicarrier transmissions to suppress multiuser interference and easily reduces the channel estimation to the OFDM-like single-user problem treated in this chapter.

It is necessary to compensate the amplitude and phase distortions for coherent transmission in multicarrier systems. The blind channel estimation and equalization methods are appropriate as they avoid use of training sequences, save bandwidth and are capable of tracking slow channel variations. The presence of CP has been utilized to devise methods for blind or semi-blind channel estimation in [71][72] and [79]-

[82]. Among these, the statistically inspired blind estimators in [71] and [72] rest on the inherent CP-induced cyclostationarity at the transmitter explicitly or implicitly, while the estimators in [80] and [81] belong to the class of deterministic subspace approach. Specifically, Heath and Giannakis [71] proposed a blind method based on the cyclostationarity property of the time-varying correlation of the received data samples due to the CP insertion at the transmitter; however, this approach suffers from slow convergence of the estimator. Cai *et al.* [81] developed a *noise subspace method* by utilizing the structure of the filtering matrix introduced by the CP insertion that achieves faster convergence for smaller data records. In [74], a new blind (semi-blind) channel estimation method that is applicable to both cyclic prefix (CP)- and zero padding (ZP)-OFDM transmissions and relies on the finite alphabet property of information-bearing symbols is proposed. The method has the major advantage that it can reduce the size of required samples and is easy to solve the scalar ambiguity compared with other blind methods [71]. In order to reduce the enhanced noise power, simple averaging across many data blocks is used. However this noise reduction scheme would cause severe distortion in fast fading channels due to the rapid variations in the channels. To solve the problem, we propose an efficient noise reduction scheme for PSK modulated signals based on polynomial approximation of the power of the CRs in both the time and frequency domains [76][78]. The noise can be greatly suppressed by this nonlinear two-dimensional (2D) regression model. With this noise reduction algorithm, the finite alphabet based channel estimator can be used to track even fast fading channels. The main advantage of the proposed algorithm is that it does not require any further information about the

statistical properties of the CR. Furthermore, it is applicable to the channel where the delay paths are not at the sampling instances, that is non T-spaced channel [73].

The organization of the remainder of this chapter is as follows. In Section 6.2, the improved finite-alphabet based channel estimation method for OFDM systems is presented. After introducing the system model in Section 6.2.1, we describe the original finite-alphabet based channel estimation method in Section 6.2.2. In Section 6.2.3 we explain our noise reduction approach which is shown to greatly reduce the estimation error. Numerical results are presented in Section 6.2.4. In Section 6.3 we draw the conclusion.

6.2 Improved Finite-Alphabet Based Channel Estimation for OFDM Systems

6.2.1 System Model

We discuss a typical CP-OFDM baseband model. The system divides the whole bandwidth B_d into K subchannels and modulates a block of data onto a set of subcarriers of corresponding subchannels. The sub-channels are divided evenly, and the bandwidth of the subchannels is $\Delta f = B_d / K$. Input data are first buffered to blocks and then grouped into K bit streams. These bit streams are mapped to some complex constellation points $x_{i,k}$, $k = 0, \dots, K-1$ at the i th block. The modulation is implemented by a K -point IDFT. Then the modulated data are passed through a P/S

converter to form serial data $X_{i,k}$. A cyclic prefix which is the copy of the last L samples of $X_{i,k}$'s is inserted before sending them to the channel, where L is the number of resolvable paths of the channel. Now it follows that the symbol duration is K/B_d . However, due to the adding of CP, the actual block duration is $T_f = (K + L)/B_d$ with a sampling rate B_d .

At the receiver, the signal corresponding the CP is discarded. The demodulation is performed by the discrete Fourier transform (DFT) operation. Assuming that the use of CP both preserves the orthogonality of the tones and eliminates ISI between consecutive OFDM symbols. Furthermore, the channel is assumed to be constant during one OFDM symbol. We describe the OFDM system as [74]

$$y(i,k) = x(i,k)h(i,k) + n(i,k), \quad k \in [0, K-1]. \quad (6.1)$$

where $y(i,k)$ denotes the demodulated data at subcarrier k of the i th OFDM symbol. $h(i,k)$ is the channel frequency response and $n(i,k)$ is the corresponding channel noise. $n(i,k)$ is assumed to be a white Gaussian process with zero mean and variance σ_n^2 . The noise is assumed to be uncorrelated with the channel coefficients.

Equation (6.1) can also be written in vector form as

$$\mathbf{y} = \mathbf{x}\mathbf{h} + \mathbf{n}. \quad (6.2)$$

where $\mathbf{y}$ is the received vector, $\mathbf{x}$ is the diagonal matrix containing the transmitted signaling points, $\mathbf{h}$ is the channel attenuation vector, and $\mathbf{n}$ is the vector of noise.

Considering the channel model described in Section 2.2, the channel responses $h(i, k)$'s are the samples of

$$B(t, f) = \int_{-\infty}^{\infty} b(t, \tau) e^{-j2\pi\tau f} d\tau. \quad (6.3)$$

That is, $h(i, k) = B(iT_f, k\Delta f)$.

As illustrated in [74], CP-OFDM and ZP-OFDM yield the same equivalent channel model and both convert the FIR convolutional channel to parallel flat fading channels. Therefore, the method described here is also applicable to ZP-OFDM systems.

6.2.2 Finite-Alphabet Based Channel Estimation

The algorithm is developed based on the nice property of PSK constellations that

$$x^J(i, k) = \text{Const}. \quad (6.4)$$

where the Const could either be -1 or $+1$ depending on the selection of constellation points, J is the size of the finite alphabet set.

Starting from (6.1), the following equation can be obtained

$$\begin{aligned} y^J(i, k) & \quad k \in [0, K-1] \\ &= [x(i, k)h(i, k) + n(i, k)]^J \\ &= x^J(i, k)h^J(i, k) + Jn(i, k)x^{J-1}(i, k)h^{J-1}(i, k) + O(n^2(i, k)) \end{aligned} \quad (6.5)$$

As $x^J(i, k)$ is a constant, $h^J(i, k)$ can be estimated from the value of $y^J(i, k)$ with noise term (the second and the third term of (6.5)). Omitting the noise term

$O(n^2(i, k))$ at high SNR, the SNR pertinent to the channel estimation is now $|h(i, k)|^2 \rho_s^2 / (J^2 \rho_n^2)$, where $\rho_s^2 (\rho_n^2)$ stands for signal (noise) power. For training based channel estimation, we receive (6.1) with the effective SNR being $|h(i, k)|^2 \rho_s^2 / (\rho_n^2)$. Hence, the finite-alphabet method is shown $10\log_{10}(J^2)$ dB loss relative to the training-based method. In [74], this noise enhancement is reduced by averaging across many blocks in order to decrease the noise power and thus enhance the SNR. However, this simple averaging method cannot be applied to track fast channel variations. In the next section, we propose a polynomial model based algorithm to improve the performance.

From the frequency domain estimates $h^J(i, k)$ estimated from (6.5), Minimum Distance (MD) algorithms is developed. Here a brief description is given. We define $\beta_j^T = [\beta_0, \dots, \beta_{JL}] = b(i, l) \otimes_J b(i, l)$, where $\otimes_J$ is the J -fold convolution, $\tilde{\mathbf{H}}_j := [h^J(i, 0), \dots, h^J(i, K-1)]^T$. Define $\mathbf{V}_j$ to be a scaled version of the first $JL-J+1$ columns of FFT matrix. The vector β_j can be estimated by $\tilde{\beta}_j = \frac{1}{K} \mathbf{V}_j^H \tilde{\mathbf{H}}_j$. For each $k \in [0, K-1]$, we have $\tilde{h}(i, k) = \lambda_k [\tilde{h}^J(i, k)]^{1/J}$ where $\lambda_k \in \{e^{j(2\pi/J)n}\}_{n=0}^{J-1}$ is the corresponding scalar ambiguity in taking the J th root. To resolve these ambiguities, we exhaustively search over all J^K possible vectors $\tilde{\mathbf{h}}_1 := [\lambda_0 [\tilde{h}^J(i, 0)]^{1/J}, \dots, \lambda_{K-1} [\tilde{h}^J(i, K-1)]^{1/J}]^T$. Through Least Squares fitting we compute the corresponding time domain vector: $\tilde{\mathbf{b}}_1 = (1/K) \mathbf{V}_1^H \tilde{\mathbf{h}}_1$, where $\mathbf{V}_1$ is a scaled version of the first L columns of FFT matrix. Channel estimates are found by minimizing the Euclidean distance $\tilde{\mathbf{b}} = \arg \min_{\tilde{\mathbf{b}}_1} \|\tilde{\beta}_j - \tilde{\mathbf{b}}_1 \otimes_J \tilde{\mathbf{b}}_1\|$.

MD will perform considerably better than maximum likelihood (ML) channel and symbol estimation at the expense of increasing complexity exponentially. Because the channel order L is usually much smaller than the number of subcarriers N , a modified Minimum Distance (MMD) algorithm will greatly reduce the complexity of MD while still achieving fairly good performance. Detailed description of MD and MMD algorithm is provided in [74].

6.2.3 Model Based Channel Estimation Method

From the approximation theory [77], we know that a smoothly varying function can be approximated by projecting to a finite set of basis functions. On the other hand, the band-limited nature of the CR suggests that the channel variation in the physical world is smooth in both the time and frequency domains. Since the OFDM CRs are located in a time-frequency plane, it is natural to project the responses over a time–frequency window $(2M_0 + 1)\Delta f \times (2N_0 + 1)T_f$ to a small set of polynomial basis functions around a center point (i_0, k_0) , in which M_0 and N_0 are the number of subcarriers and symbols of the operating block in the time-frequency plane, i.e.,

$$h_{i,k} = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} h_{i_0, k_0} (nm)(k - k_0)^m (i - i_0)^n + R_{MN}, \quad (6.6)$$

for $k_0 - M_0 \leq k \leq k_0 + M_0; i_0 - N_0 \leq i \leq i_0 + N_0$

where M and N are the model orders for frequency and time domains, respectively.

R_{MN} is the model error.

Using least square methods, we can get the estimation of the coefficients of the polynomial basis $h_{i_0, k_0}(nm)$ from the temporary estimation. A common practice for getting the temporary estimation is to insert pilot symbols at some predetermined (pilot) locations in the time–frequency plane. One obvious CR estimate at a pilot location is the LS estimate. Since the polynomial approximation is done in the time domain and the frequency domain, the noise can be greatly suppressed because fewer coefficients need to be estimated. Comparing to the Fourier transform used in [89]–[92], the error caused by polynomial model approximation is bounded and hence is more robust to the channel statistics and system parameters.

6.2.4 Polynomial Model Based Noise Reduction Method

In this section, we propose a simple approach to reduce the noise term shown in (6.5). Taking into account the time variation property of the channel, the approach is based on the on a time–frequency polynomial model of the CR described in Section 6.2.3 instead of simply averaging across many blocks as shown in [74], the noise can be greatly suppressed because fewer coefficients need to be estimated compared with the number of elements known.

We define the power of the CR as

$$\Gamma(i, k) = h^J(i, k). \quad (6.7)$$

Shown in (6.3), the discrete CR $h^J(i, k)$ can be viewed as a sampled version of the 2-D continuous complex baseband fading process $B^J(f; t)$. The band-limited nature of the CR suggests that the channel variation $B(f; t)$ in the physical world is

smooth in both the time and frequency domain. It is easy to verify that $B^J(f;t)$ is also band limited. Similarly, such a smoothly varying function can be approximated by projecting it onto a finite set of basis functions over a time-frequency window $(M_0 \cdot \Delta f \times N_0 \cdot T_f)$. We can model the signal generated by $B^J(f;t)$ in this region with respect to the origin (i_0, k_0) as

$$\Gamma(i, k) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \Gamma_{i_0, k_0}(m, n) (k - k_0)^m (i - i_0)^n + R_{MN}, \quad (i, k) \in \mathbb{Q}. \quad (6.8)$$

where M and N are the model orders for the frequency and time domains, respectively. $\Gamma_{i_0, k_0}(m, n)$'s are the coefficients to be estimated, R_{MN} represents the modeling error, and $\mathbb{Q}$ stands for the operating block.

Actually, (6.8) is nothing but multivariate Taylor's theorem. As a result, for a given operating block $\mathbb{Q}$, as the order $M \times N \rightarrow \infty$, the mean square value of the error term $R_{MN} \rightarrow 0$ since $B^J(f;t)$ is bandlimited in t . Thus with increasing $M \times N$, the polynomial approximation becomes more and more accurate. However, there is a tradeoff between the window size and the polynomial order. As model order increases, the number of unknowns increases too. For a fixed number of observations, these unknowns cannot be estimated accurately if they are too numerous. In conclusion, for a model basis like the polynomial model, the model error increases when the model order becomes smaller or the window dimension becomes larger. The tradeoff can be reached by adjusting the window dimension and model order to the channel statistics.

Using least square methods, we can obtain the estimation of the coefficients of the polynomial basis from the value of $y^J(i, k)$ in the operating block. By carefully choosing the modeling orders and the size of the operating block, the time-frequency modeling error can be limited to a certain level. We get

$$\tilde{\Gamma}(i, k) = \mathbf{c}^H \mathbf{q}_{i,k} . \quad (6.9)$$

where

$$\begin{aligned} \mathbf{q}_{ik}^T &\triangleq (q_{M-1, N-1}^{k-k_0, i-i_0}, \dots, q_{0, N-1}^{k-k_0, i-i_0}, \dots, q_{M-1, 0}^{k-k_0, i-i_0}, \dots, q_{0, 0}^{k-k_0, i-i_0}), \\ \mathbf{c}^H &\triangleq (\Gamma_{i_0, k_0}(M-1, N-1), \dots, \Gamma_{i_0, k_0}(0, N-1), \dots, \Gamma_{i_0, k_0}(M-1, 0), \dots, \Gamma_{i_0, k_0}(0, 0)). \end{aligned} \quad (6.10)$$

with $q_{m,n}^{k,i} = k^m i^n$ for $k = -(M_0 - 1), \dots, 0, \dots, M_0 - 1$, $i = -(N_0 - 1), \dots, 0, \dots, N_0 - 1$,
 $m = 0, \dots, M - 1$ and $N = 0, \dots, N - 1$.

The coefficients are chosen such that

$$\min_{\tilde{\mathbf{c}}} \sum_{(i,k) \in \mathbb{Q}} |y^J(i, k) - \tilde{\mathbf{c}}^H \mathbf{q}_{ik}|^2 . \quad (6.11)$$

Taking the derivative of (6.11) with respect to $\tilde{\mathbf{c}}$ and invoking the definitions

$$\mathbf{Q} \triangleq \sum_{(i,k) \in \mathbb{Q}} \mathbf{q}_{ik} \mathbf{q}_{ik}^T, \quad \mathbf{P} = \mathbf{Q}^{-1} . \quad (6.12)$$

$$\tilde{\mathbf{b}} \triangleq \sum_{(i,k) \in \mathbb{Q}} \mathbf{q}_{ik} y^{J*}(i, k) . \quad (6.13)$$

The coefficients are derived

$$\tilde{\mathbf{c}} = \mathbf{P} \tilde{\mathbf{b}} . \quad (6.14)$$

Thus we obtain the estimation of $\Gamma(i, k)$ as

$$\tilde{\Gamma}(i, k) = \mathbf{q}_{ik}^T \tilde{\mathbf{c}}^* . \quad (6.15)$$

Based on the refined estimation of $\Gamma(i, k)$, the CR can be estimated by using the MD or MMD algorithms.

6.2.5 Numerical Results and Discussions

First, the performance of estimation of $\Gamma(i, k)$ defined in (6.8) which is the basis for the finite-alphabet estimation of the channel parameter is discussed. For comparison, we consider the direct estimator adopted in [74] and the linear regression method proposed in this chapter. Two normalized Doppler spread ($f_d T_f$) values are chosen as $6.9\text{e-}4$ and $6.9\text{e-}3$. The selection of appropriate model orders and the window dimension is based on the time varying nature of the channel. The estimation error of $\Gamma(i, k)$ is greatly reduced by using this linear regression method, especially when the average SNR per bit is low. The performance of our method is fairly robust to the variations of the normalized Doppler spread (Figure 6.1). When the average SNR per bit is relatively high (i.e. in the range of 20-30dB), however, the modeling error increases as well as the estimation error when the channel varies more rapidly. In this case, a higher order estimator can be used to mitigate the modeling error as shown in Figure 6.2. A higher order model may be more robust to the fast fading environment at the expense of some performance degradation at low SNR. In Figure 6.3, we discuss the effect of the size of the operating block on the performance. From Figure 6.3, we conclude that in environment where noise

dominates, that is low average SNR per bit, a larger operating block is more effective in reducing the noise by averaging across the operating block. The modeling error increases for a relatively larger block. A smaller operating block is more preferable at large SNR as a result. In conclusion, when the averaging SNR is high, the modeling error dominates the system performance. The polynomial approximation is more sensitive to rapid channel variation and a larger operating block.

Next, we'll apply our algorithm to estimate the fading channel's information. The same estimation method MMD is used as in [74]. When the channel is slowly varying, say the normalized Doppler spread equals $f_d T_f = 6.9e-4$, the performance of estimation using our noise reduction method results in much less estimation error and has achieved more than 3 dB gain over the original method (Figure 6.4). When the channel is fast fading, that is the normalized Doppler spread is $f_d T_f = 6.9e-3$. MMD estimation with noise reduction maintains considerable improvements at low SNR environment. At high SNR, proposed noise reduction method is still comparable with the original MMD estimation method proposed in [74].

Figure 6.1 Estimation error of $\Gamma(i, k)$ for system with and without noise reduction.

NR: Noise reduction method, FA: Direct estimation.

Figure 6.2 Comparison of estimation error of $\Gamma(i, k)$ with different model order.

Figure 6.3 Comparison of estimation error of $\Gamma(i, k)$ with different size of operating block.

Figure 6.4 Mean square error of MMD estimator with and without noise reduction.

MMD-NR: MMD with noise reduction, FA-MMD: finite-alphabet based MMD.

6.3 Conclusion

In this chapter, we have proposed a noise reduction method for the finite-alphabet based channel estimation for OFDM systems. The proposed scheme is to project the responses over a time–frequency window to a small set of polynomial basis functions around a center point, based on the fact that the CR with band-limited nature is a smoothly varying function on the local time-frequency domain. It has relatively low complexity and does not require the knowledge of the statistics of the multipath fading channel. The simulation results show that the proposed algorithm is able to achieve 3dB gain over the original finite alphabet based estimator if the modeling order and the operating block are properly selected. It is also robust to the rapid variations in the fading channels, especially at relatively lower average SNR.

Chapter 7

Conclusion and Recommendations

7.1 Conclusion

In this thesis, we have focused on the performance analysis of MC-CDMA systems over frequency-selective fading channels. The multicarrier CDMA systems are very attractive in combating the multipath fading and impulsive noise. The enlarged symbol duration of multicarrier systems effectively changes the frequency selective channel to the parallel flat fading channels as shown in Chapter 2. In contrast with the complex equalizer structure in the single carrier system, the equalization is performed by a one-tap multiplier at each subcarrier. By spreading the data over the frequency domain, MC-CDMA systems can exploit the frequency selective channel as a frequency diversity source.

We discussed performance analysis and capacity improvement for MC-CDMA systems. First, we derived closed-form BER expressions of MC-CDMA systems over multipath Rayleigh and Rician channels. In order to improve the capacity of MC-CDMA systems, we investigated application of transmit diversity to MC-CDMA

systems. Since channel estimation is a tough task for coherent detection, especially in MIMO MCM systems, we proposed a channel estimation algorithm for general multicarrier systems.

In detail, we have proposed a performance evaluation method in a downlink frequency selective Rayleigh and Rician fading channel for MC-CDMA system, which can achieve an exact and closed-form BER formula without making the unrealistic assumption that the fading among subcarriers is independent of each other. In order to obtain the moment generating function of the decision variable, the evaluation is performed in the time domain. An exact BER is derived through residue calculation or numerical methods from the MGF of the decision variable. Since the complexity of our proposed algorithm increases exponentially with the number of users, for practical consideration, we have also presented the simple approximate evaluation method to reduce the complexity. Based on the exact BER formula, we have found that the performance of MC-CDMA systems highly dependent on the correlation property of the spreading codes in the frequency domain and the distribution of the non-zero channel paths.

We also provide an exact BER evaluation approach for synchronous MC-CDMA systems in the uplink based on the Gaussian quadratic form. The method leads to a simple solution without encountering the heavy computational burden as in the downlink case.

Applying space time block codes to MC-CDMA systems can dramatically improve the performance with the same total radiated power. MMSE combining scheme performs best among four commonly used combining schemes as it can

retain the orthogonality between different users and avoid excessive noise amplification at the same time. The disadvantage of this combining scheme is that the detector requires the knowledge of channel state information as well as the signal to noise ratio. In the MMSE combining per user method, we propose to take into account the information provided by all the subcarriers and thus minimize the mean square error at the input of the threshold detector. The new transmit diversity scheme for MC-CDMA systems exploits the intrinsic structure of multicarrier CDMA systems and achieves comparable performance as MC-CDMA with STBC without making the quasi-static assumption.

Finally, we investigate the channel estimation over a frequency-selective fading channel for multicarrier systems. Polynomial approximation in both the time and frequency domain exploits the time and frequency correlation of the CRs and greatly suppresses the noise term. The proposed estimator is found to be more robust to the channel statistics and system parameters and achieves considerable performance improvement over the existing method under practical channel conditions. Being simple and flexible it holds high application for multicarrier transmissions.

7.2 Recommendations for Further Research

Finally, we suggest a few topics which can be explored as future work based on the thesis.

- Performance analysis for MMSEC: In Chapter 2, we derived an exact closed-form BER expression for downlink MC-CDMA systems with MRC combining scheme. This may be a starting point for further research on MC-CDMA systems with other combining schemes. Since MMSEC provides the best performance among the four commonly used combining schemes for MC-CDMA systems, the evaluation of the performance for MC-CDMA with MMSEC is of great importance.
- Performance analysis for Nakagami fading channel model: In our study, we have discussed two most popular channel models: Rayleigh and Rician distribution. Another Nakagami-m fading distribution has been widely adopted for multipath modeling in wireless communications due to its good accuracy and versatility. It is a generalized distribution which often gives the best fit to land-mobile and indoor-mobile multipath propagation environments as well as to scintillating ionospheres radio links. The derivation of the BER performance of MC-CDMA systems for Nakagami-m fading model would be desirable in the theoretical sense.
- Optimum spreading codes design: Through out this study, we assumed the Walsh-Hadamard code as the frequency domain spreading codes. From the analysis in Chapter 3, it is shown that the correlation property in the frequency domain of the spreading codes dominates the performance of MC-CDMA systems under practical channel conditions. The analysis may lead a new criterion designing the specific spreading codes for MC-CDMA systems. With prior knowledge of CR, specific spreading codes may be

designed or selected to ensure that there's no crosstalk between different users. The optimal spreading codes should be designed in order to maximize the information rate.

- Space-frequency coding for MC-CDMA systems: We have discussed two transmit diversity schemes for MC-CDMA systems. It is promising to consider space-frequency (SF) code technique to MC-CDMA systems. The SF codes where two-dimensional coding is applied to distribute channel symbols across space (transmit antenna) and frequency (subcarriers), ensure full diversity at the price of symbol rate decrease compared with the symbol rate of the underlying ST codes [75]. It is promising applying SF codes to MC-CDMA systems as well as designing optimal spreading codes for MC-CDMA systems with transmit diversity.
- Channel estimation for MIMO multicarrier systems: For MIMO multicarrier systems, channel estimation is a tougher task compared with SISO systems. Since two or more different signals are transmitted from different antennas simultaneously, the received signal is the superposition of these signals. If the channel parameters corresponding to each transmitter and receiver antenna pair are estimated by the estimator developed for SISO systems, the signal from other transmitter antenna(s) will become interference. Hence, novel parameter estimation approaches are desired for MIMO multicarrier systems.

Author's Publications

Journal Papers

1. Z. Hou and V. K. Dubey, "Exact analysis for downlink MC-CDMA in Rayleigh fading channels," *IEEE Commun. Lett.*, vol. 8, No.2, Feb. 2004.
2. Z. Hou and V. K. Dubey, "BER performance for downlink MC-CDMA systems over Rician fading channel," to appear in *EURASIP Journal on Applied Signal Processing*, 2005.
3. Z. Hou and V. K. Dubey, "Exact Analysis for Synchronous MC-CDMA Over Correlated Rayleigh Fading Channels." Submitted to *IEEE Commun. Lett.*, in Jan., 2005.

Conference Papers

4. Z. Hou and V. K. Dubey, "Application of Space-time Block Codes to MC-CDMA Systems," in *Proc. 2003 IEEE International Conference on Software, Telecommunications and Computer Networks*, Split, Croatia, Oct. 2003.
5. Z. Hou and V. K. Dubey, "A new transmit diversity scheme for MC-CDMA system," in *Proc. 2003 IEEE International Conference on Software, Telecommunications and Computer Networks*, Split, Croatia, Oct. 2003.

6. Z. Hou and V. K. Dubey, "Performance Analysis for Downlink MC-CDMA Systems with Space-time Block Codes in Frequency-selective Rayleigh Fading Channels," in *Proc. 2003 Fourth International Conference on Information, Communications & Signal Processing Fourth IEEE Pacific-Rim Conference On Multimedia*, Singapore, Dec. 2003.
7. Z. Hou and V. K. Dubey, "Bit Error Probability of MC-CDMA System over Rayleigh Fading Channel," in *Proc. 2004 IEEE International Symposium on Spread Spectrum Techniques and Applications*, Sydney, Australia, Sept. 2004.
8. Z. Hou and V. K. Dubey, "Improved Finite-Alphabet Based Channel Estimation for OFDM Systems'," in *Proc. Ninth International Conference on Communication Systems*, Singapore, Sept. 2004.

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Appendix A

BER Performance Evaluation

In this appendix, we derive (5.11), (5.12) and (5.13) from (5.8), (5.9) and (5.10).

For multicarrier systems with proper cyclic extension and sample timing, it has been shown in Section 2.2 that, with tolerable leakage, the channel frequency response can be expressed as,

$$h_i = \sum_{l=0}^{L-1} b(l) \exp(-j2\pi il / N), \quad (i = 0, 1, \dots, N-1) \quad (\text{A.1})$$

where $b(l)$ is the channel parameters of the resolvable L paths. Hence, h_i is a complex Gaussian random variable for a Rayleigh fading channel. From (5.8), the individual part $h_i = \sqrt{|h_{0,i}|^2 + |h_{1,i}|^2}$ is a random variable of generalized Rayleigh distribution with $n=4$, under the assumption that the CR from different transmit antennas $h_{0,i}$ and $h_{1,i}$ is independent and identical random process [3]. The PDF of h_i can be written in the form

$$p_{h_i}(x) = \frac{x^{n-1}}{2^{(n-2)/2} \sigma^n \Gamma(\frac{1}{2}n)} e^{-x^2/2\sigma^2}, \quad (x \geq 0) \quad (\text{A.2})$$

And the expectation of h_i is obtained as

$$E(h_i) = (2\sigma^2)^{1/2} \frac{\Gamma(5/2)}{\Gamma(2)} = \sqrt{\frac{9}{8}} \sigma^2 \pi, \quad (\text{A.3})$$

The expectation of the desired signal component can be achieved as in (5.11).

From (5.9), the variance of h_i is given by

$$\begin{aligned} \text{Var}(h_i) &= E(h_i^2) - [E(h_i)]^2 \\ &= E(|h_{0,i}|^2 + |h_{1,i}|^2) - [E(\sqrt{|h_{0,i}|^2 + |h_{1,i}|^2})]^2 \\ &= (4 - 9\pi/8)P_i \end{aligned} \quad (\text{A.4})$$

Then we get the variance of MAI as

$$\begin{aligned} \text{Var}(I[0]) &= (K-1)N(4 - 9\pi/8)P_i \\ &= (K-1)(4 - 9\pi/8)P \end{aligned} \quad (\text{A.5})$$

We can easily get the variance of the noise component shown in (5.13). The BER performance for MRC and ORC can be obtained similarly.