Printed periodic waveguide structures : full-wave characterization, guided-wave characteristics and applications

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**PRINTED PERIODIC WAVEGUIDE STRUCTURES:
FULL-WAVE CHARACTERIZATION,
GUIDED-WAVE CHARACTERISTICS AND APPLICATIONS**

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A thesis submitted to the Nanyang Technological University
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Summary

Printed periodic waveguide structures are investigated for applications in waveguide based band pass filters, electromagnetic band gap (EBG) structures and double negative (DNG) metamaterials. Such waveguide structures are characterized using the *Hybrid Mom-Immittance Approach* in which Immittance Approach is employed for calculation of spectral dyadic Green's functions for multilayered waveguide structures. Method-of-moments (MoM) is utilized for solving unknown current densities on printed strips/slots and Mode-matching Method has been used for obtaining reflection and transmission coefficients for various waveguide discontinuity elements. The impedance-type Hybrid MoM-Immittance Approach is formulated here for the transversely printed strip obstacle whereas the admittance-type Hybrid MoM-Immittance Approach is formulated for the transversely printed slot obstacle in shielded waveguide environment.

From full-wave characterization of a single unit/cell of the printed periodic waveguide structure, *guided-wave per-unit length transmission parameters* of frequency selective surface (FSS) printed periodic waveguide structures are numerically obtained. Guided-wave characteristics consists of complex propagation constant ($\gamma=\alpha+j\beta$) and complex wave impedance ($Z=\text{Re}\{Z\}+j\text{Im}\{Z\}$) unlike the usually used Brillouin diagram for periodic structures which shows only the imaginary part of the complex propagation constant, i.e., phase constant. It has been observed that different FSS loading configurations exhibit distinctive guided-wave characteristics such as slow-wave, fast-wave, backward wave, forward wave, passband and stopband. The possible applications of such periodic waveguide structures are in the design of waveguide based EBG structures with the stopband and slow-wave, band pass filters with the passband, leaky wave antennas with the fast-wave, metamaterials with the backward wave, etc.

Printed periodic waveguide structures are investigated further in depth for some specific applications. Firstly, we have introduced a new type of waveguide band pass filter in which printed *stepped impedance slot (SIS) resonators* are linked by $\lambda/4$

waveguide sections or impedance transformers. This novel waveguide filter shows a considerable improvement in stop band filter performance: an increased and steeper out-of-band rejection. Secondly, FSS printed periodic waveguide structures have been investigated for the design of *EBG transmission media*. A printed FSS element with widest stopband and highest slow-wave factor in the guided-wave characteristics has been employed for constructing a wideband waveguide based EBG circuits. Various FSS elements inside waveguide have also been investigated to improve the roll-off characteristics of the EBG structures in the passband. Finally, a novel and simple architecture for *waveguide based double-negative (DNG) metamaterials* is proposed. It has been observed that periodic waveguide structures loaded with double strips printed on two sides of dielectric substrate exhibits the fundamental properties of DNG metamaterials viz., negative permeability, negative permittivity, negative refractive index and backward wave propagation.

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List of Acronyms

MoM	Method-of-moments
EFIE	Electric field integral equation
MFIE	Magnetic field integral equation
EM	Electromagnetic
SDA	Spectral Domain Approach
FDTD	Finite-Difference Time-Domain
FEM	Finite Element Method
GSM	Generalized Scattering Matrix
PWS	Piece wise sinusoidal
TE	Transverse electric
TM	Transverse magnetic
FSS	Frequency selective surface
CPW	Coplanar waveguide
BW	Bandwidth
EBG	Electromagnetic band gap
PBG	Photonic band gap
HFSS	High-frequency structure simulator
SIR	Stepped impedance resonator
UIR	Uniform impedance resonator
SIS	Stepped impedance slot
UIS	Uniform impedance slot
DNG	Double negative
SRR	Split ring resonator

Chapter 1

Introduction

1.1 Objective and Outline

An increasing interest has been generated recently to design various periodic waveguide structures with varied frequency-dependent propagation characteristics for applications in electromagnetic band gap (EBG) structure with band stop feature, band pass filters with pass band characteristics and double negative (DNG) metamaterials with backward wave behavior and so on. In this thesis, we will investigate printed periodic waveguide structures for such applications. Printed strip/slots are basic building blocks for many waveguide based structures and they are printed on transverse dielectric layers inside waveguide hence the name printed strip and slot.

First step in the study of printed periodic waveguide structure is to develop an efficient and accurate numerical approach for full-wave characterization of such waveguide based structures. We can use a general purpose numerical method like Finite Element Method (FEM) and Finite-Difference Time-Domain (FDTD) technique, which employs volumetric discretization so that any arbitrary shaped structure can be modeled. However, it will lead to large matrix size, increased CPU time and memory. Instead, we can employ surface discretization such as in Method-of-moments (MoM) which requires less CPU time and memory for full-wave analysis. One of the important steps in MoM is calculation of dyadic Green's functions. Immittance Approach is widely accepted as a simple, efficient and

accurate technique for calculation of spectral domain dyadic Green's functions for multilayered structures. Hence, we will employ a hybrid approach of these two numerical methods. The proposed technique for efficient and accurate full-wave analysis of printed periodic waveguide structures is termed as Hybrid MoM-Immittance Approach. Mode-matching Method is often applied for finding reflection and transmission coefficients for various waveguide discontinuities and is employed here too. In fact, the Hybrid MoM-Immittance Approach is hybrid of the above three basic methods. Equivalent circuits of printed strips and slots in layered waveguide are extracted using the Hybrid MoM-Immittance Approach. Complex waveguide structures are designed using the simple and efficient cascaded network based synthesis.

Next, we study the guided-wave characteristics of printed periodic waveguide structures. Guided-wave characteristics comprise complex propagation constant and complex wave impedance unlike the usually used Brillouin diagram for study of periodic structures which shows only the imaginary part of complex propagation constant, i.e., phase constant. From the guided-wave characteristics, we can infer various possible applications of such printed periodic waveguide structures. Here, we have originated a novel periodic waveguide structure loaded with various FSS elements. Different FSS loading configurations show varied propagation behaviors and correspondingly different applications. In particular, we will investigate such printed periodic waveguide structure for applications in design of waveguide based EBG structures, waveguide band pass filters and waveguide based DNG metamaterials.

A novel printed stepped impedance slot (SIS) resonator is proposed here. One of the basic geometrical configurations of SIS resonator shows an increased frequency bandstop between the fundamental and spurious harmonic resonant frequencies. Such a large frequency bandstop can't be achieved with the conventional uniform impedance slot (UIS) resonators. Three-stage waveguide band pass filters are designed using such SIS resonators linked by $\lambda/4$ waveguide sections or impedance transformers. In addition to size reduction than that of the conventional waveguide band pass filters with $\lambda/2$ waveguide section resonators, such waveguide filters have

the flexibility of improving filter stopband performance by adjusting the geometrical configurations of SIS resonators. An increased and steeper stopband filter performance has been observed. Second application under consideration is in design of waveguide based EBG structures. FSS element with the widest stopband and highest slow-wave factor in the guided-wave characteristics has been utilized in design of waveguide based wideband EBG structure. Third application under investigation of printed periodic waveguide structures are in waveguide based DNG metamaterials. A novel and simple architecture for waveguide based DNG metamaterials is proposed. Periodic waveguide structures loaded with double strips printed on dielectric exhibits the fundamental properties of metamaterials like negative electric permittivity, negative magnetic permeability, negative refractive index and backward wave propagation.

A detailed outline of chapters in this thesis is presented below. *Chapters containing original and significant contributions are indicated by a *.*

Chapter 1 presents a brief outline of the thesis followed by original contributions related to this research work. It also provides a brief introduction to various terms used in this thesis.

Chapter 2* contains formulation of impedance-type Hybrid MoM-Immittance Approach for printed strip and admittance-type Hybrid MoM-Immittance Approach for printed slot in multilayered waveguide environment. The Hybrid MoM-Immittance Approach is hybrid of three basic methods: MoM, Immittance Approach and Mode-matching Methods. Here, MoM has been employed for determining unknown current densities on the printed strip/slot. Immittance Approach is utilized for calculation of spectral dyadic Green's functions for multilayered waveguide structures. Mode-matching Method is used for determining reflection and transmission coefficients of various waveguide discontinuities.

Chapter 3 presents numerical verifications of the developed electromagnetic (EM) program. Firstly, the EM program is checked for numerical convergence. Then it is verified for accuracy by comparing with High-frequency Structure Simulator (HFSS)

and experimental results. The equivalent circuits for printed strip and slot in X-band waveguide has also been extracted so that waveguide based cascaded synthesis design of various passive microwave devices can be executed easily. The EM program developed is found to be accurate and faster than available commercial software like HFSS.

Chapter 4* investigates the per-unit length transmission parameters of such printed periodic waveguide structures loaded with FSS strip/slot layers. The per-unit length transmission parameter constitutes complex propagation constant ($\gamma=\alpha+j\beta$) and complex wave impedance $\{Z=\text{Re}(Z)+j\text{Im}(Z)\}$. It has been observed that different FSS elements exhibit varied guided-wave characteristics. The possible applications of such printed periodic waveguide structures are in design of band pass filters with the passband, EBG structures with the stopband, leaky wave antennas with the fast wave, DNG metamaterials with the backward wave, etc.

Chapter 5* provides an extensive study on stepped impedance slot (SIS) resonators for the design of waveguide band pass filters. Two basic geometries of SIS resonators are studied here in comparison to uniform impedance slot (UIS) resonator in maximizing the frequency bandstop between fundamental and spurious harmonic resonant frequencies. The SIS resonator with the maximum frequency bandstop has been used for the design of a novel class of waveguide band pass filter. This new filter comprises SIS resonators linked by $\lambda/4$ impedance transformers or waveguide sections. This novel waveguide filter shows considerable improvement in the filter out-of-band rejection performance with good in-band filter characteristics.

Chapter 6* studies a waveguide based EBG structure constructed by periodically loading rectangular waveguide with strip layers printed on dielectric. The effect of various parameters such as periodicity and number of periodic cells/units on rejection band or bandgap has also been investigated. Finally, a wideband waveguide based 1-D EBG structure is designed using FSS square loop strip loaded periodic waveguide structure.

Chapter 7* proposes a simple architecture for waveguide based DNG metamaterials. It has been observed that periodic waveguide structure loaded with double strips printed on dielectric exhibits fundamental properties of metamaterials

like negative permeability, negative permittivity, negative refractive index and backward wave propagation.

Chapter 8 provides summary of the works presented in this thesis and suggest directions for future research works.

1.2 Original Contributions

The original contributions presented in this thesis are:

- Formulation and implementation of Hybrid MoM-Immittance Approach for full-wave characterization of multiple printed strip/slot layers inside waveguide.
- Investigation on guided-wave characteristics of printed periodic waveguide structures loaded with various FSS strip/slot layers.
- Extensive studies on printed SIS resonators for designing waveguide band pass filters with improved filter performances.
- EBG structures design using FSS elements loaded printed periodic waveguide structures.
- Proposition and demonstration of a simple architecture for waveguide based DNG metamaterials.

1.3 Introduction to Various Terminologies

1.3.1 Numerical Methods

Printed strip is defined as an interface where metallic portion is small with respect to dielectric portion as shown in Figure 1.1 (a). Hence it is more efficient to analyze metallic portion than the remaining non-metallic or dielectric portion. It is also called as electric layer or interface. It can be numerically characterized using full-wave impedance type MoM formulation. Printed slot is defined as an interface where dielectric portions is small with respect to metallic portion as depicted in Figure 1.1 (b). Hence, it is more efficient to analyze dielectric portion than the metallic portion.

It is also called as magnetic layer or interface. It can be numerically characterized using full-wave admittance type MoM formulation.

Figure 1.1: (a) Printed strip and (b) Printed slot

- **Method-of-moments (MoM)**

MoM is a numerical technique for solving complex integral equations. In electromagnetics, many problems are formulated as integral equations in which analytical solutions do not exist and therefore must be solved numerically. The MoM basically transforms integral equations into matrix systems of linear equations which can be solved using computers. A simple mathematical outline of MoM is given next for a brief introduction to this numerical method.

Consider the following inhomogeneous equation

$$Lu = f \quad (1.1)$$

$$\text{or } Lu - f = 0 \quad (1.2)$$

where L is a linear operator, u is an unknown function and f is a known function. In order to solve for u , let us approximate it by a set of basis functions or expansion functions as given below.

$$u_n = \sum_{k=1}^n \alpha_k \varphi_k, \quad n = 1, 2, \dots \quad (1.3)$$

where ϕ_k is the expansion function, α_k is its unknown amplitude, k and n are order and total number of expansion functions. Replacing u by u_n and taking inner product with a set of weighting or testing functions w_m gives

$$\langle (Lu_n - f), w_m \rangle = 0, \quad m = 1, 2, \dots, n \quad (1.4)$$

$$\text{or } \sum_{k=1}^n \alpha_k \langle L\phi_k, w_m \rangle = \langle f, w_m \rangle, \quad m = 1, 2, \dots, n \quad (1.5)$$

which can be written in matrix form as

$$Ax = b \quad (1.6)$$

with each matrix and vector defined by

$$x = (\alpha_1 \quad \alpha_2 \quad \dots \quad \alpha_n)$$

$$b = (\langle f, w_1 \rangle \quad \langle f, w_2 \rangle \quad \dots \quad \langle f, w_n \rangle)^T$$

and

$$A = [a_{mk}]$$

where T denotes the transpose and a_{mk} are individual matrix elements given by

$$a_{mk} = \langle L\phi_k, w_m \rangle$$

Hence, we can solve the unknown matrix x of amplitudes of basis function from equation (1.6) as

$$x = A^{-1}b \quad (1.7)$$

- **Immittance Approach**

It is a simple method for formulating spectral domain dyadic Green's functions in terms of generalized printed transmission lines which may contain several dielectric layers and strips/slots appearing at dielectric interfaces.

Figure 1.2: (a) 1-D inhomogeneous multilayer structures excited by transverse electric/magnetic point sources (b) Equivalent TE/TM circuit models for electric point source excitation and (c) Equivalent TE/TM circuit models for magnetic point source excitation

Here is a brief summary of the approach.

- First draw equivalent TE and TM circuit models of the multilayered structure. Each dielectric layer is represented by a particular transmission line. Whenever strips are present at a particular interface, place electric current sources at the junctions between transmission lines. Whenever slots are present, place voltage source at the interface.
- For printed strip case, equivalent impedance at particular observation point can be calculated using simple impedance transformations of transmission line. In case of slot, equivalent admittance at any location can be calculated similarly from admittance transformations of transmission line in the equivalent TE and TM circuit models.
- Hence, we can obtain the voltage due to current sources of printed strips from the equivalent TE and TM circuit models. Similarly, we can also calculate the current at the observation point due to voltage sources of printed slots from the equivalent TE and TM circuit models. From voltage values, we can get electric dyadic Green's functions in spectral domain

whereas from current values, we can derive magnetic dyadic Green's functions in spectral domain. Using dyadic analysis, various components of dyadic Green's functions can be obtained.

This technique is quite versatile and simple formulation of dyadic Green's function in spectral domain may be done by almost inspection for many multilayered structures.

- **Mode-matching Methods**

Mode-matching is often applied to find amplitude of guided modes in waveguide with a complicated cross-sectional geometry. To illustrate this method, let us choose a simple waveguide step discontinuity as depicted in Figure 1.3 with a TE_{n0} excitation.

Figure 1.3: Waveguide discontinuities

The first step is to expand E_y and H_x in terms of modal functions $\zeta_{an}(x)$ and $\zeta_{bn}(x)$, $n=1,2,\dots$. Next, continuity of E_y and H_x is applied at the discontinuity $z=0$.

$$E_y : \sum_{n=1}^{\infty} (A_n^+ + A_n^-) \zeta_{an} = \begin{cases} \sum_{n=1}^{\infty} (B_n^+ + B_n^-) \zeta_{bn} & 0 < x < b \\ 0 & b < x < a \end{cases} \quad (1.8)$$

$$H_x : \sum_{n=1}^{\infty} (A_n^+ - A_n^-) Y_{an} \zeta_{an} = \sum_{n=1}^{\infty} (B_n^+ - B_n^-) Y_{bn} \zeta_{bn} \quad 0 < x \leq b \quad (1.9)$$

where superscripts + and – indicate amplitudes of the modal waves propagating in positive and negative directions of z respectively. A_n and B_n are incident terms and

usually only one of them is nonzero for a single-mode analysis. Using orthogonality of $\zeta_{bn}(x)$ in region $0 \leq x \leq b$, we get,

$$\sum_{n=1}^{\infty} H_{nm} (A_n^+ + A_n^-) = B_m^+ + B_m^- \quad (1.10)$$

$$\sum_{n=1}^{\infty} H_{nm} Y_{an} (A_n^+ - A_n^-) = Y_{bm} (B_m^+ - B_m^-) \quad (1.11)$$

Applying orthogonality of $\zeta_{an}(x)$ for $0 \leq x \leq a$, we have

$$A_m^+ + A_m^- = \sum_{n=1}^{\infty} H_{mn} (B_n^+ + B_n^-) \quad (1.12)$$

$$\text{where } H_{mn} = \int_0^b \zeta_{an}(x) \zeta_{bn}(x) dx$$

Eliminating the unknown B_m , we get,

$$\sum_{n=1}^{\infty} (Y_{an} + Y_{bm}) H_{nm} A_n^- = \sum_{n=1}^{\infty} (Y_{an} - Y_{bm}) H_{nm} A_n^+ + 2Y_{nm} B_n^- \quad m=1,2,\dots \quad (1.13)$$

This set of linear simultaneous equations for unknown A_n can be solved numerically. While solving it, matrix number must to be truncated to a finite size say $n, m=1,2,\dots,N$.

In chapter 2, we will introduce a new hybrid method in which all the above three methods is utilized to efficiently and accurately characterize printed strips and slots in multilayered waveguide.

1.3.2 Periodic Structures

Waveguides and transmission lines loaded at periodic intervals with reactive elements such as diaphragms and stubs are referred to as periodic structures. The interest in waveguiding structures of this type arises from some basic properties common to all periodic structures, namely, (a) passband-stopband characteristics (b)

propagation characteristics like fast wave, slow wave, forward wave, backward wave and (c) applications for design of filters, EBG structures, leaky-wave antennas, traveling-wave tubes, DNG metamaterials, etc.

- **Phase and group velocity**

Phase velocity of a wave is the rate at which phase of wave propagates in space. This is the velocity at which phase of any one frequency component of the wave will propagate. In mathematical terms, ω be the angular frequency and k the wavenumber. Then phase velocity v_p is defined as $v_p = \frac{\omega}{k}$. The group velocity of a wave is the velocity with which the overall shape of wave amplitude (known as the envelope of the wave) propagates through space. The group velocity is the velocity with which energy propagates and is defined by $v_g = \frac{\partial \omega}{\partial k}$. The phase velocity is also given by the slope of a line from origin to a point on dispersion curve, while the group velocity is given by slope of a tangent to the dispersion curve.

Figure 1.4: Dispersion curve

- **Forward and backward wave**

Dispersive propagation occurs when phase velocity depends on frequency. Most of our acquaintance with the waves is non dispersive propagation, in which phase velocity is independent of frequency. For forward wave, phase and group velocity are constant and equal to one another. This simple relation does not hold in general. Backward wave, in which phase velocity is opposite in direction of propagation to group velocity is very counter-intuitive concept. But, they are simply a normal phenomenon that occurs with dispersive propagation.

- **Forward wave transmission line**

Let us consider the simplified distributed equivalent circuit of conventional transmission line as illustrated in Figure 1.5. It consists of series inductance and shunt capacitance in the LC ladder network. For such line, $k = \omega\sqrt{LC}$ is the wavenumber. Here, $\frac{\omega}{k} = \frac{1}{\sqrt{LC}}$ and $\frac{\partial\omega}{\partial k} = \frac{\omega}{k} = \frac{1}{\sqrt{LC}}$. Hence, wave packet travels at the phase velocity without change of shape.

Figure 1.5: Forward wave transmission line

- **Backward wave transmission line**

Consider the backward wave transmission line as shown in Figure 1.6. It consists of series capacitor and shunt inductor in the LC ladder network. Hence, it is a high-pass line unlike the forward-wave line we considered before, which is a low-pass line.

The dispersion relation is given by $1/k = -\omega\sqrt{LC}$ or $v_p = \frac{\omega}{k} = -\frac{1}{k^2\sqrt{LC}}$. This line

is strongly dispersive. $v_g = \frac{\partial\omega}{\partial k} = \frac{1}{k^2\sqrt{LC}}$, is negative of the phase velocity. The

wavelets in a wave packet appear to travel in a direction opposite to that of the packet itself.

Figure 1.6: Backward wave transmission line

Figure 1.7: (a) Geometry of a generalized frequency selective surface (FSS)
(b) Geometrical configurations of some typical FSS elements

• **Frequency Selective Surfaces**

FSS surface comprise periodically arranged metallic patch or aperture elements on a metallic screen in 1-D or 2-D and exhibit total reflection (patches) or transmission (apertures) at the resonance. They are frequently used as filters for microwaves and

optics. Various FSS geometries of patch or aperture elements give different transmission or reflection characteristics. Figure 1.7 (a) shows geometry of a 2-D FSS and Figure 1.7 (b) depicts some typical FSS element geometries.

1.3.3 Applications

- **Microwave filters**

The ideal filter is a device which provides perfect transmission for all frequencies in certain passband and infinite attenuation in stopband region. For practical filters, this can not be achieved so the goal of filter design is to imitate the ideal case to some acceptable limits. Filters fall into three categories, namely, (a) low-pass filters that transmits all signals between zero and some upper limit ω_c and attenuate all signals above cutoff frequency ω_c (b) high-pass filters that pass all frequencies above a lower cutoff value ω_c and reject all frequencies below ω_c and (c) band pass filters that pass all frequencies in a range ω_1 and ω_2 and reject frequencies outside this range. Some of the commonly used waveguide band pass filters are direct-coupled, band pass filters with $\lambda/4$ and $\lambda/2$ resonators. Advantages of filters with $\lambda/4$ resonators over filters with $\lambda/2$ resonators are given below.

- The second passband center appears at 3ω instead of 2ω as in case of $\lambda/2$ resonators.
- The overall length of resonators is cut to half.
- $\lambda/4$ resonator filters usually have considerably more mid-stop-band attenuation.

Both the above filters uses the transmission line or waveguide sections as resonator, there is another kind of microwave filters in which chains of resonant elements coupled by $\lambda/4$ sections of transmission line or waveguides. Such filters have advantages of $\lambda/4$ resonator filter plus additional flexibility of improving filter performance by choosing proper resonators like SIS resonators which has been introduced in chapter 5.

- **EBG Structures**

By introducing periodic perturbations like dielectric rods, holes and patterns in waveguides and microstrip substrates, it is possible to achieve complete forbidden of electromagnetic waves propagation in some specific frequency region. Such periodic structures are called Photonic Band Gap (PBG) structures in optics and preferably termed as EBG structures in microwave community. EM waves behave in such crystals similar to that of electronic behavior in semiconductors hence it is named as EBG structures. The periodic perturbations can be introduced in ground plane or over the substrate which considerably improves performance of filters, antennas and other microwave devices without increasing the device dimensions. On basis of dimensions in which periodic perturbations are introduced, EBG structures can be categorized as 1-D, 2-D or 3-D.

- **DNG metamaterials**

It is a new class of periodic structures that can act as a medium that possesses an effective negative refractive index. These structures have also been termed as negative-refractive-index (NRI) medium, backward wave materials, left-handed materials, etc. Such periodic structure is composed of pairs of elements, in which an array of one element produces an effective negative dielectric constant and an array of other elements contributes to an effective negative permeability. A proper combination of the two arrays yields a backward wave medium with negative refraction. A simple novel architecture for waveguide based DNG metamaterials is proposed in chapter 7.

Chapter 2

Hybrid MoM-Immittance Approach

2.1 Introduction

A full-wave Hybrid MoM-Immittance Approach for efficient and accurate analysis of printed slots and strips of arbitrary shape and size in layered waveguide for various applications has been formulated. An impedance-type Hybrid MoM-Immittance Approach is formulated from the electric field integral equation (EFIE) for printed strip case and an admittance-type Hybrid MoM-Immittance Approach is formulated from the magnetic field integral equation (MFIE) for printed slot case. This Hybrid MoM-Immittance Approach is initially established as an EFIE/MFIE with unknown electric current density over the printed strip/slot and then numerically implemented using Galerkin's technique. Spectral domain dyadic Green's functions for the layered waveguide are deduced using the Immittance Approach and finally the Mode-matching Method is used for solving reflection and transmission coefficients due to the waveguide discontinuity element or obstacle for various waveguide modes: dominant and higher order modes. For efficient analysis of large and complex structures, equivalent circuit parameter of a block is first extracted and complete structure is analyzed through cascaded ABCD matrices. The EM program developed is checked for its numerical accuracy and efficiency with HFSS, analytical and experimental results in the subsequent chapters. It shows good accuracy and high efficiency. The proposed Hybrid MoM-Immittance Approach will

be a very useful tool for full-wave analysis of periodic waveguide structures, waveguide filter and synthesis design.

2.2 Literature Review

Waveguide discontinuity has been extensively studied as basic building blocks in past decades for development of low-loss and high-power waveguide-based components, devices and systems [1]-[7]. Very recently, research interest in this topic continues to be quickly shifted to full-wave characterization of waveguide discontinuities with printed strip and/or slot layers on dielectric substrate [8] for waveguide-based power combiners and for harmonic rejection waveguide filters [9]. There are many numerical techniques for full-wave characterization of such waveguide discontinuities as described in [10]. Spectral Domain Approach (SDA) is sometimes easier, faster, accurate and efficient method for analyzing planar circuits [11]-[14]. Complicated boundary value problems in spatial domain are transformed into spectral domain using the Fourier transformation to obtain a much more simplified boundary value problem in spectral domain. Immittance Approach is an efficient method for analyzing multi-layered EM structures [15] and numerical analysis is done in Fourier-transformed domain. The available commercial software like HFSS and XFDTD uses volumetric meshing so that arbitrarily shaped structures can be modeled. HFSS employs Finite Element Method (FEM) [16]-[17] and XFDTD uses Finite-Difference Time Domain (FDTD) method [18]-[20]. Hence, these commercial softwares need a large CPU memory and computation time. MoM [21]-[24] on the other hand, utilizes surface discretization thereby requires less memory and CPU time. Mode-matching Method has been also widely used for analyzing waveguide discontinuities [25]-[30] such as waveguide steps and junctions. The first step in Mode-matching Method is expansion of unknown fields in individual regions in terms of normal modes. And modal coefficients can be determined using orthogonal property of normal modes which leads to a finite set of linear simultaneous equations for the modal coefficients. Generalized Scattering Matrix (GSM) approach has also been used to study waveguide discontinuities [31]-

[33]. It is employed for analyzing complicated discontinuity problems, mainly cascaded ones. It considers the effect of dominant modes as well as higher order modes in numerical characterization. Here, we will formulate a hybrid approach of MoM, Immittance Approach and Mode-matching Method for efficient and accurate full-wave characterization of printed waveguide structures.

2.3 Impedance-type Hybrid MoM-Immittance Approach

Figure 2.1 shows geometry of printed strip transversely placed in multi-layered waveguide. There are three regions inside waveguide, region I of relative permittivity ϵ_1 , region II of relative permittivity ϵ_2 and region III of relative permittivity ϵ_3 . A planar strip of any arbitrary shape and size is printed at interface #1 on dielectric substrate layer, region II, of thickness h . The incident electric field from region I induce a surface current on printed metallic strip which then produces a scattered electric field at the interface #1 and a certain portion of it is reflected back to region I and remaining electric field is transmitted through region II to region III.

Figure 2.1: Geometry of printed metallic strip in multi-layered waveguide

2.3.1 Electric Field Integral Equation

Electric field integral equation (EFIE) can be obtained from boundary condition of transverse electric field, i.e., total incident and scattered electric field at the interface is zero.

$$\hat{z} \times (\bar{E}^{inc}(\bar{r}) + \bar{E}^{scatt}(\bar{r})) = 0 \quad (2.1)$$

The scattered electric field can be expressed in terms of electric dyadic Green's function for the multi-layered waveguide and unknown electric current density on printed strip. Hence,

$$\hat{z} \times (\bar{E}^{inc}(\bar{r}) + \int_{strip} \bar{\bar{G}}_{EJ}(\bar{r}, \bar{r}') \cdot \bar{J}(\bar{r}') d\bar{S}') = 0 \quad (2.2)$$

The unprimed coordinates represent observation point whereas source location is represented by primed coordinates.

The electric current density on printed strip can be expressed as

$$\bar{J}(x', y') = J_x(x', y')\hat{x} + J_y(x', y')\hat{y} \quad (2.3)$$

where $J_x(x', y')$ and $J_y(x', y')$ are unknown current density along x-axis and y-axis respectively.

The transverse components of electric dyadic Green's functions are

$$\begin{aligned} & \bar{\bar{G}}_{EJ}(x, y; x', y') \\ &= G_{EJ}^{xx}(x, y; x', y')\hat{x}\hat{x} + G_{EJ}^{xy}(x, y; x', y')\hat{x}\hat{y} \\ &+ G_{EJ}^{yx}(x, y; x', y')\hat{y}\hat{x} + G_{EJ}^{yy}(x, y; x', y')\hat{y}\hat{y} \end{aligned} \quad (2.4)$$

Since we have considered planar strip case and assumed that printed strip is infinitely thin, we are only concerned with transverse components of electric dyadic Green's functions. Using dyadic analysis, we can get the following scalar EFIE from the above vector EFIE.

$$\begin{aligned} & E_x^{inc}(x, y) \\ &= - \iint_{strip} G_{EJ}^{xx}(x, y; x', y') J_x(x', y') dx' dy' - \iint_{strip} G_{EJ}^{xy}(x, y; x', y') J_y(x', y') dx' dy' \end{aligned} \quad (2.5a)$$

$$E_y^{inc}(x, y) = - \iint_{strip} G_{EJ}^{yx}(x, y; x', y') J_x(x', y') dx' dy' - \iint_{strip} G_{EJ}^{yy}(x, y; x', y') J_y(x', y') dx' dy' \quad (2.5b)$$

2.3.2 Impedance-type MoM-Immittance Approach Matrix

Let us first transform the multilayered space domain Green's functions into their spectral domain counterparts using the well known modal series expansion in waveguide [34].

$$G_{EJ}^{xx}(x, y; x', y') = \sum_{m=0}^{M+1N+1} \sum_{n=0} \tilde{G}_{EJ}^{xx}(k_{xm}, k_{yn}) \phi_{mn}^x(x, y) \phi_{mn}^x(x', y') \quad (2.6a)$$

$$G_{EJ}^{xy}(x, y; x', y') = \sum_{m=0}^{M+1N+1} \sum_{n=0} \tilde{G}_{EJ}^{xy}(k_{xm}, k_{yn}) \phi_{mn}^x(x, y) \phi_{mn}^y(x', y') \quad (2.6b)$$

$$G_{EJ}^{yx}(x, y; x', y') = \sum_{m=0}^{M+1N+1} \sum_{n=0} \tilde{G}_{EJ}^{yx}(k_{xm}, k_{yn}) \phi_{mn}^y(x, y) \phi_{mn}^x(x', y') \quad (2.6c)$$

$$G_{EJ}^{yy}(x, y; x', y') = \sum_{m=0}^{M+1N+1} \sum_{n=0} \tilde{G}_{EJ}^{yy}(k_{xm}, k_{yn}) \phi_{mn}^y(x, y) \phi_{mn}^y(x', y') \quad (2.6d)$$

Here, $\phi_{mn}^x(x, y)$ and $\phi_{mn}^y(x, y)$ are orthonormal eigenfunctions satisfying boundary conditions on the metallic waveguide.

$$\phi_{mn}^x(x, y) = \sqrt{\frac{\epsilon_{0m}}{a}} \sqrt{\frac{\epsilon_{0n}}{b}} \cos(k_{xm}x) \sin(k_{yn}y) \quad (2.7a)$$

$$\phi_{mn}^y(x, y) = \sqrt{\frac{\epsilon_{0m}}{a}} \sqrt{\frac{\epsilon_{0n}}{b}} \sin(k_{xm}x) \cos(k_{yn}y) \quad (2.7b)$$

where ϵ_{0m} , ϵ_{0n} are Neuman indices; $k_{xm} = \frac{m\pi}{a}$; $k_{yn} = \frac{n\pi}{b}$ and m, n indicate integer number in conjunction with various waveguide modes.

The spectral domain multilayered Green's functions can be analytically deduced as the response of voltages in equivalent transverse TE/TM transmission line networks representing the layered medium as described in [35] and will be discussed in detail in section 2.3.3. Substituting equations (2.6a) and (2.6b) into equation (2.5a), we get,

$$\begin{aligned}
 E_x^{inc}(x, y) &= - \iint_{strip} \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{G}_{EJ}^{xx}(k_{xm}, k_{yn}) \varphi_{mn}^x(x, y) \varphi_{mn}^x(x', y') J_x(x', y') dx' dy' \\
 &\quad - \iint_{strip} \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{G}_{EJ}^{xy}(k_{xm}, k_{yn}) \varphi_{mn}^x(x, y) \varphi_{mn}^y(x', y') J_y(x', y') dx' dy'
 \end{aligned} \tag{2.8}$$

Separating source (primed) and observation (unprimed) variables and bringing in double integration over the source variables, we get,

$$\begin{aligned}
 E_x^{inc}(x, y) &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{G}_{EJ}^{xx}(k_{xm}, k_{yn}) \varphi_{mn}^x(x, y) \iint_{strip} J_x(x', y') \varphi_{mn}^x(x', y') dx' dy' \\
 &\quad - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{G}_{EJ}^{xy}(k_{xm}, k_{yn}) \varphi_{mn}^x(x, y) \iint_{strip} J_y(x', y') \varphi_{mn}^y(x', y') dx' dy'
 \end{aligned} \tag{2.9}$$

Let us define a new transformation

$$\tilde{J}_x(k_{xm}, k_{yn}) = \iint_{strip} J_x(x', y') \varphi_{mn}^x(x', y') dx' dy' \tag{2.10a}$$

$$\tilde{J}_y(k_{xm}, k_{yn}) = \iint_{strip} J_y(x', y') \varphi_{mn}^y(x', y') dx' dy' \tag{2.10b}$$

$$\tilde{T}_x^*(k_{xm}, k_{yn}) = \iint_{strip} T_x^*(x, y) \varphi_{mn}^x(x, y) dx dy \tag{2.10c}$$

$$\tilde{T}_y^*(k_{xm}, k_{yn}) = \iint_{strip} T_y^*(x, y) \varphi_{mn}^y(x, y) dx dy \tag{2.10d}$$

Using the above transformation we can further simplify equation (2.9) as below.

$$\begin{aligned}
 E_x^{inc}(x, y) &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{G}_{EJ}^{xx}(k_{xm}, k_{yn}) \varphi_{mn}^x(x, y) \tilde{J}_x(k_{xm}, k_{yn}) \\
 &\quad - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{G}_{EJ}^{xy}(k_{xm}, k_{yn}) \varphi_{mn}^x(x, y) \tilde{J}_y(k_{xm}, k_{yn})
 \end{aligned} \tag{2.11}$$

After a set of testing functions are weighted on both sides of the above integral equation, we get,

$$\begin{aligned}
 &\iint_{strip} T_x^*(x, y) E_x^{inc}(x, y) dx dy \\
 &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \iint_{strip} T_x^*(x, y) \varphi_{mn}^x(x, y) dx dy \tilde{G}_{EJ}^{xx}(k_{xm}, k_{yn}) \tilde{J}_x(k_{xm}, k_{yn}) \\
 &\quad - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \iint_{strip} T_x^*(x, y) \varphi_{mn}^x(x, y) dx dy \tilde{G}_{EJ}^{xy}(k_{xm}, k_{yn}) \tilde{J}_y(k_{xm}, k_{yn}) \\
 &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{T}_x^*(k_{xm}, k_{yn}) \tilde{G}_{EJ}^{xx}(k_{xm}, k_{yn}) \tilde{J}_x(k_{xm}, k_{yn}) \\
 &\quad - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{T}_x^*(k_{xm}, k_{yn}) \tilde{G}_{EJ}^{xy}(k_{xm}, k_{yn}) \tilde{J}_y(k_{xm}, k_{yn})
 \end{aligned} \tag{2.12}$$

Similarly, we can simplify EFIE in equation (2.5b) into the following equation.

$$\begin{aligned}
 &\iint_{strip} T_x^*(x, y) E_y^{inc}(x, y) dx dy \\
 &= - \sum_{m=0}^{M+1} \sum_{n=0}^{N+1} \tilde{T}_y^*(k_{xm}, k_{yn}) \tilde{G}_{EJ}^{yx}(k_{xm}, k_{yn}) \tilde{J}_x(k_{xm}, k_{yn}) \\
 &\quad - \sum_{m=0}^{M+1} \sum_{n=0}^{N+1} \tilde{T}_y^*(k_{xm}, k_{yn}) \tilde{G}_{EJ}^{yy}(k_{xm}, k_{yn}) \tilde{J}_y(k_{xm}, k_{yn})
 \end{aligned} \tag{2.13}$$

In order to solve EFIE numerically, unknown current density can be expressed in terms of superposition of basis functions, composed of x-directed piecewise sinusoidal (PWS) functions, $B_x^{ij}(x, y)$ and y-directed PWS functions, $B_y^{ij}(x, y)$ as follows:

$$J_x(x, y) = \sum_{i=1}^{I+1} \sum_{j=1}^{J+1} I_x^{ij} B_x^{ij}(x, y) \quad (2.14)$$

$$\text{where } B_x^{ij}(x, y) = \frac{\sin[k_s(l_x - |x - x_i|)]}{l_y \sin(k_s l_x)} \quad |x - x_i| \leq l_x; |y - y_j| \leq l_y / 2$$

$$J_y(x, y) = \sum_{i=1}^{I+1} \sum_{j=1}^{J+1} I_y^{ij} B_y^{ij}(x, y) \quad (2.15)$$

$$\text{where } B_y^{ij}(x, y) = \frac{\sin[k_s(l_y - |y - y_j|)]}{l_x \sin(k_s l_y)} \quad |x - x_i| \leq l_x / 2; |y - y_j| \leq l_y$$

In above equation, $l_x = x_{i+1} - x_i$, $l_y = y_{j+1} - y_j$ are steps of discretization along x-axis and y-axis respectively and k_s is a scaling parameter which can be chosen as k in dielectric layer.

Figure 2.2: (a) x-directed and (b) y-directed PWS functions

Figure 2.2 (a) shows x-directed PWS basis function whereas Figure 2.2(b) depicts y-directed PWS basis function. For instance, consider the discretization of a rectangular metallic strip as shown in Figure 2.3. The x-directed, y-directed basis and testing functions are shown by the direction of arrow head. A basis function spanned over two rectangular cells. I_x^{ij} and I_y^{ij} are respectively the $(i, j)^{th}$ unknown current expansion coefficients associated with each x-directed and y-directed basis function on the rectangular strip. They are calculated at the peak of each basis

function. We have chosen Galerkin's method in which both basis and testing functions are the same.

Figure 2.3: Assignment of x-directed and y-directed electric currents on a rectangular strip

The expressions for unknown current densities in transformed domain of the new transformation defined in equation (2.10) are as follows.

$$\tilde{J}_x(k_{xm}, k_{yn}) = \sum_{i=1}^{I+1} \sum_{j=1}^{J+1} I_x^{ij} \tilde{B}_x^{ij}(k_{xm}, k_{yn}) \quad (2.16)$$

$$\text{where } \tilde{B}_x^{ij}(k_{xm}, k_{yn}) = \iint_{\text{strip}} B_x^{ij}(x, y) \varphi_{mn}^x(x, y) dx dy$$

$$= \frac{4 \sqrt{\frac{\epsilon_{0m}}{a}} \sqrt{\frac{\epsilon_{0n}}{b}} \sin(k_{yn} y_j) \sin(k_{yn} \frac{l_y}{2}) k_s \cos(k_{xm} x_i) [\cos(k_s l_x) - \cos(k_{xm} l_x)]}{k_{yn} l_y \sin(k_s l_x) (k_{xm}^2 - k_s^2)}$$

$$\tilde{J}_y(k_{xm}, k_{yn}) = \sum_{i=1}^{I+1} \sum_{j=1}^{J+1} I_y^{ij} \tilde{B}_y^{ij}(k_{xm}, k_{yn}) \quad (2.17)$$

$$\text{where } \tilde{B}_y^{ij}(k_{xm}, k_{yn}) = \iint_{\text{strip}} B_y^{ij}(x, y) \varphi_{mn}^y(x, y) dx dy$$

$$= \frac{4\sqrt{\frac{\epsilon_{0m}}{a}}\sqrt{\frac{\epsilon_{0n}}{b}}\sin(k_{xm}x_i)\sin(k_{xm}\frac{l_x}{2})k_s\cos(k_{yn}y_j)[\cos(k_sl_y)-\cos(k_{yn}l_y)]}{k_{xm}l_x\sin(k_sl_y)(k_{yn}^2-k_s^2)}$$

Hence, from equations (2.12), (2.13), (2.16) and (2.17), we can obtain a matrix system of linear equations for the EFIE of equation (2.2) which can be solved numerically.

$$\begin{bmatrix} [Z_{xx}] & [Z_{xy}] \\ [Z_{yx}] & [Z_{yy}] \end{bmatrix} \begin{bmatrix} [I_x] \\ [I_y] \end{bmatrix} = \begin{bmatrix} [V_x] \\ [V_y] \end{bmatrix} \quad (2.18)$$

where the elements of each sub-matrices are expressed as

$$z_{xx}(m,n) = -\sum_{m=0}^{M+1N+1}\sum_{n=0}^{M+1N+1}\tilde{T}_x^*(k_{xm},k_{yn})\tilde{G}_{EJ}^{xx}(k_{xm},k_{yn})\tilde{B}_x(k_{xm},k_{yn})$$

$$z_{xy}(m,n) = -\sum_{m=0}^{M+1N+1}\sum_{n=0}^{M+1N+1}\tilde{T}_x^*(k_{xm},k_{yn})\tilde{G}_{EJ}^{xy}(k_{xm},k_{yn})\tilde{B}_y(k_{xm},k_{yn})$$

$$z_{yx}(m,n) = -\sum_{m=0}^{M+1N+1}\sum_{n=0}^{M+1N+1}\tilde{T}_y^*(k_{xm},k_{yn})\tilde{G}_{EJ}^{yx}(k_{xm},k_{yn})\tilde{B}_x(k_{xm},k_{yn})$$

$$z_{yy}(m,n) = -\sum_{m=0}^{M+1N+1}\sum_{n=0}^{M+1N+1}\tilde{T}_y^*(k_{xm},k_{yn})\tilde{G}_{EJ}^{yy}(k_{xm},k_{yn})\tilde{B}_y(k_{xm},k_{yn})$$

$$v_x = \iint_{strip} \tilde{T}_x^*(x,y)E_x^{inc}(x,y)dxdy$$

$$v_y = \iint_{strip} \tilde{T}_y^*(x,y)E_y^{inc}(x,y)dxdy$$

The impedance sub-matrix $[z_{xy}]$ denotes the x-directed testing of electric field produced by y-directed current basis elements (refer to figure 2.3). The sub-matrices $[V_x]$ and $[V_y]$ refer to x- and y-directed testing of incident electric field. $[I_x]$ and $[I_y]$ sub-matrices are respectively the unknown electric current expansion coefficients associated with x-directed and y-directed basis function and they can be numerically calculated as follows:

$$[I] = [Z]^{-1} [V] \quad (2.19)$$

2.3.3 Spectral Domain Electric Dyadic Green's Functions

Figure 2.4: Transverse electric point source excitation $\bar{J}(\bar{r}')$ inside rectangular waveguide

The printed strip inside layered waveguide of geometry depicted in Figure 2.1 can be represented by transverse electric point source excitation as shown in Figure 2.4. For transverse electric point source excitation, equivalent TE and TM circuit models [15, 35] are shown in Figure 2.5. In Figure 2.5(a), Z_1^{TE} , Z_2^{TE} and Z_3^{TE} are respectively the wave impedances of TE mode in region I, region II and region III of the layered waveguide. Similarly, Z_1^{TM} , Z_2^{TM} and Z_3^{TM} in Figure 2.5(b) are respectively the wave impedances of TM mode in region I, region II and region III of the layered waveguide. The equivalent impedance looking up from interface #1 for TE and TM mode cases are

$$Z_u^{TE} = Z_1^{TE} \quad (2.20a)$$

$$Z_u^{TM} = Z_1^{TM} \quad (2.20b)$$

where $Z_i^{TE} = \frac{j\omega\mu_i}{\gamma_i}$; $Z_i^{TM} = \frac{\gamma_i}{j\omega\epsilon_i}$; $\gamma_i = \sqrt{\frac{m\pi^2}{a} + \frac{n\pi^2}{b} - k_0^2 \mu_i \epsilon_i}$, where $i=1,2,3$ are propagation constant of TE/TM waves in region I, II and III respectively.

The equivalent impedance looking down from interface #1 for TE and TM mode cases are

$$Z_d^{TE} = Z_2^{TE} \frac{Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h)}{Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)} \quad (2.21a)$$

$$Z_d^{TM} = Z_2^{TM} \frac{Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h)}{Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)} \quad (2.21b)$$

Let us denote the equivalent impedance by Z^{TE} for TE mode then

$$Z^{TE} = \frac{Z_1^{TE} Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))}{Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)) + Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))} \quad (2.22)$$

due to shunt combination of Z_u^{TE} and Z_d^{TE} . Similarly, let us denote the equivalent impedance for TM mode as Z^{TM} then

$$Z^{TM} = \frac{Z_1^{TM} Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))}{Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)) + Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))} \quad (2.23)$$

The electric dyadic Green's function when source and observation points are both at interface #1 is

$$\begin{aligned} \tilde{\mathbf{G}}_{EJ}^{11}(\bar{k}_t, z=0, z'=0) \\ = -V_1^{TE}(\bar{k}_t, 0, 0)(\hat{k}_t \times \hat{z})(\hat{k}_t \times \hat{z}) - V_1^{TM}(\bar{k}_t, 0, 0)\hat{k}_t \hat{k}_t \\ = -(Z^{TE})(\hat{k}_t \times \hat{z})(\hat{k}_t \times \hat{z}) - (Z^{TM})\hat{k}_t \hat{k}_t \end{aligned} \quad (2.24)$$

The primed z coordinates represent source locations whereas unprimed z coordinates represent observation point. V_1^{TE} and V_1^{TM} are voltages across interface #1 for TE and TM mode respectively.

(a) TE mode

(b) TM mode

Figure 2.5: Equivalent TE and TM circuit models for transverse electric point source excitation $\bar{J}(\bar{r}')$ inside rectangular waveguide

$\bar{k}_t$ is the transverse vector wave number and can be expressed as

$$\bar{k}_t = |\bar{k}_t| \hat{k} = |\bar{k}_t| \left\{ \frac{k_{xm}}{\sqrt{k_{xm}^2 + k_{yn}^2}} \hat{x} + \frac{k_{yn}}{\sqrt{k_{xm}^2 + k_{yn}^2}} \hat{y} \right\} \quad (2.25)$$

Putting this expression of $\hat{k}$ from equation (2.25) into equation (2.24) and using dyadic analysis,

$$\begin{aligned} & \tilde{\mathbf{G}}_{EJ}^{11}(\bar{k}_t, z=0, z'=0) \\ &= -\frac{Z^{TE} k_{yn}^2 + Z^{TM} k_{xm}^2}{k_{xm}^2 + k_{yn}^2} \hat{x}\hat{x} - \frac{k_{xm} k_{yn} (Z^{TM} - Z^{TE})}{k_{xm}^2 + k_{yn}^2} \hat{x}\hat{y} \\ & \quad - \frac{k_{xm} k_{yn} (Z^{TM} - Z^{TE})}{k_{xm}^2 + k_{yn}^2} \hat{y}\hat{x} - \frac{Z^{TE} k_{xm}^2 + Z^{TM} k_{yn}^2}{k_{xm}^2 + k_{yn}^2} \hat{y}\hat{y} \\ &= \tilde{G}_{EJ}^{xx11}(k_{xm}, k_{yn}) \hat{x}\hat{x} + \tilde{G}_{EJ}^{xy11}(k_{xm}, k_{yn}) \hat{x}\hat{y} \\ & \quad + \tilde{G}_{EJ}^{yx11}(k_{xm}, k_{yn}) \hat{y}\hat{x} + \tilde{G}_{EJ}^{yy11}(k_{xm}, k_{yn}) \hat{y}\hat{y} \end{aligned} \quad (2.26)$$

Hence,

$$\begin{aligned} & \tilde{G}_{EJ}^{xx11}(k_{xm}, k_{yn}) \\ &= -\frac{k_{yn}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_1^{TE} Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))}{Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)) + Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))} \right\} \\ & \quad - \frac{k_{xm}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_1^{TM} Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))}{Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)) + Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))} \right\} \end{aligned} \quad (2.27a)$$

$$\begin{aligned} & \tilde{G}_{EJ}^{yy11}(k_{xm}, k_{yn}) \\ &= -\frac{k_{xm}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_1^{TE} Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))}{Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)) + Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))} \right\} \\ & \quad - \frac{k_{yn}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_1^{TM} Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))}{Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)) + Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))} \right\} \end{aligned} \quad (2.27b)$$

$$\begin{aligned}
 & \tilde{G}_{EJ}^{xy11}(k_{xm}, k_{yn}) \\
 &= \tilde{G}_{EJ}^{yx11}(k_{xm}, k_{yn}) \\
 &= \frac{k_{xm}k_{yn}}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_1^{TE} Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))}{Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)) + Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))} \right\} \\
 &- \frac{k_{xm}k_{yn}}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_1^{TM} Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))}{Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)) + Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))} \right\} \quad (2.27c)
 \end{aligned}$$

Similarly, when source is at interface #1 and observation point is at interface #2, electric dyadic Green's function can be derived as

$$\begin{aligned}
 & \tilde{\mathbf{G}}_{EJ}^{21}(\bar{k}_t, z = h, z' = 0) \\
 &= -V_2^{TE}(\bar{k}_t, h, 0)(\hat{k}_t \times \hat{z})(\hat{k}_t \times \hat{z}) - V_2^{TM}(\bar{k}_t, h, 0)\hat{k}_t \hat{k}_t \\
 &= -\left(\frac{Z_3^{TE} \times Z^{TE}}{Z_d^{TE}}\right)(\hat{k}_t \times \hat{z})(\hat{k}_t \times \hat{z}) - \left(\frac{Z_3^{TM} \times Z^{TM}}{Z_d^{TM}}\right)\hat{k}_t \hat{k}_t \\
 &= -\left\{ \frac{\frac{Z_3^{TE} \times Z^{TE}}{Z_d^{TE}} k_{yn}^2 + \frac{Z_3^{TM} \times Z^{TM}}{Z_d^{TM}} k_{xm}^2}{k_{xm}^2 + k_{yn}^2} \right\} \hat{x} \hat{x} \\
 &- \left\{ \frac{k_{xm} k_{yn} \left(\frac{Z_3^{TM} \times Z^{TM}}{Z_d^{TM}} - \frac{Z_3^{TE} \times Z^{TE}}{Z_d^{TE}} \right)}{k_{xm}^2 + k_{yn}^2} \right\} \hat{x} \hat{y} \\
 &- \left\{ \frac{k_{xm} k_{yn} \left(\frac{Z_3^{TM} \times Z^{TM}}{Z_d^{TM}} - \frac{Z_3^{TE} \times Z^{TE}}{Z_d^{TE}} \right)}{k_{xm}^2 + k_{yn}^2} \right\} \hat{y} \hat{x} \\
 &- \left\{ \frac{\frac{Z_3^{TE} \times Z^{TE}}{Z_d^{TE}} k_{xm}^2 + \frac{Z_3^{TM} \times Z^{TM}}{Z_d^{TM}} k_{yn}^2}{k_{xm}^2 + k_{yn}^2} \right\} \hat{y} \hat{y} \\
 &= \tilde{G}_{EJ}^{xx21}(k_{xm}, k_{yn}) \hat{x} \hat{x} + \tilde{G}_{EJ}^{xy21}(k_{xm}, k_{yn}) \hat{x} \hat{y} \\
 &+ \tilde{G}_{EJ}^{yx21}(k_{xm}, k_{yn}) \hat{y} \hat{x} + \tilde{G}_{EJ}^{yy21}(k_{xm}, k_{yn}) \hat{y} \hat{y} \quad (2.28)
 \end{aligned}$$

Hence, we can get the four components of electric dyadic Green function $\tilde{G}_{EJ}^{21}(\bar{k}_t, z=h, z'=0)$ as

$$\begin{aligned} & \tilde{G}_{EJ}^{xx\ 21}(k_{xm}, k_{yn}) \\ &= -\frac{k_{yn}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_3^{TE} Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h))}{Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)) + Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))} \right\} \\ & - \frac{k_{xm}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_3^{TM} Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h))}{Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)) + Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))} \right\} \end{aligned} \quad (2.29a)$$

$$\begin{aligned} & \tilde{G}_{EJ}^{yy\ 21}(k_{xm}, k_{yn}) \\ &= -\frac{k_{xm}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_3^{TE} Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h))}{Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)) + Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))} \right\} \\ & - \frac{k_{yn}^2}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_3^{TM} Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h))}{Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)) + Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))} \right\} \end{aligned} \quad (2.29b)$$

$$\begin{aligned} & \tilde{G}_{EJ}^{xy\ 21}(k_{xm}, k_{yn}) = \tilde{G}_{EJ}^{yx\ 21}(k_{xm}, k_{yn}) \\ &= \frac{k_{xm} k_{yn}}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_3^{TE} Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h))}{Z_1^{TE} (Z_2^{TE} + Z_3^{TE} \tanh(\gamma_2 h)) + Z_2^{TE} (Z_3^{TE} + Z_2^{TE} \tanh(\gamma_2 h))} \right\} \\ & - \frac{k_{xm} k_{yn}}{k_{xm}^2 + k_{yn}^2} \left\{ \frac{Z_3^{TM} Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h))}{Z_1^{TM} (Z_2^{TM} + Z_3^{TM} \tanh(\gamma_2 h)) + Z_2^{TM} (Z_3^{TM} + Z_2^{TM} \tanh(\gamma_2 h))} \right\} \end{aligned} \quad (2.29c)$$

The same method can be used to calculate spectral dyadic Green's functions for multiple numbers of printed strips in multi-layered waveguide.

2.3.4 Reflection and Transmission Coefficients

The incident field at $z=0$ in region I of the three-layered waveguide whose geometry is shown in Figure 2.1 consists of various waveguide TE and TM modes [11].

$$\bar{E}_{\tan}^{inc} = \begin{Bmatrix} \bar{\psi}_{m_i n_i}^{TE} \\ \bar{\psi}_{m_i n_i}^{TM} \end{Bmatrix} \quad (2.30a)$$

$$\bar{H}_{\tan}^{inc} = \begin{Bmatrix} Y_{m_i n_i}^{TE,1} & \bar{\chi}_{m_i n_i}^{TE} \\ Y_{m_i n_i}^{TM,1} & \bar{\chi}_{m_i n_i}^{TM} \end{Bmatrix} \quad (2.30b)$$

where $\bar{\psi}_{mn}^{TE}$ and $\bar{\psi}_{mn}^{TM}$ are TE and TM modal functions for rectangular waveguide and $\bar{\chi} = \hat{z} \times \bar{\psi}$.

The reflected field at $z=0^-$ in region I of the three-layered waveguide can be expressed as

$$\bar{E}_{\tan}^{refl} = \sum_{m,n} a_{mn} \bar{\psi}_{mn}^{TE} + b_{mn} \bar{\psi}_{mn}^{TM} \quad (2.31a)$$

$$\bar{H}_{\tan}^{refl} = -\sum_{m,n} a_{mn} Y_{mn}^{TE} \bar{\chi}_{mn}^{TE} + b_{mn} Y_{mn}^{TM} \bar{\chi}_{mn}^{TM} \quad (2.31b)$$

The transmitted field at $z=0^+$ in region II of the three-layered waveguide can also be expressed as

$$\bar{E}_{\tan}^{trans} = \sum_{m,n} a'_{mn} A_{mn}^{TE} \bar{\psi}_{mn}^{TE} + b'_{mn} A_{mn}^{TM} \bar{\psi}_{mn}^{TM} \quad (2.32a)$$

$$\bar{H}_{\tan}^{trans} = \sum_{m,n} Y_{mn}^{TE,2} a'_{mn} B_{mn}^{TE} \bar{\chi}_{mn}^{TE} + Y_{mn}^{TM,2} b'_{mn} B_{mn}^{TM} \bar{\chi}_{mn}^{TM} \quad (2.32b)$$

where $a'_{mn} = a_{mn} + \delta_{m-m_i}$ for $TE_{m_i n_i}$ incidence;

$b'_{mn} = b_{mn} + \delta_{m-m_i}$ for $TM_{m_i n_i}$ incidence;

δ_{m-m_i} is the Kronecker delta function;

$$A_{mn}^{TE/TM} = 1 + R_{mn}^{TE/TM};$$

$$B_{mn}^{TE/TM} = 1 - R_{mn}^{TE/TM} \text{ and}$$

$$R_{mn}^{TE/TM} = \frac{Y_{mn}^{TE/TM,2} - Y_{mn}^{TE/TM,3}}{Y_{mn}^{TE/TM,2} + Y_{mn}^{TE/TM,3}} e^{-2\gamma_{mn}^2 h}.$$

From boundary condition, we have

$$\bar{J} = \hat{z} \times (\bar{H}^+ - \bar{H}^-) = \hat{z} \times (\bar{H}^{trans} - \bar{H}^{inc} - \bar{H}^{refl}) \quad (2.33)$$

Substituting for magnetic fields from equations (2.30b), (2.31b) and (2.32b) into equation (2.33) and using the vector identity $\hat{z} \times (\hat{z} \times \bar{\chi}) = -\bar{\chi}$ gives

$$\begin{aligned} \bar{J} = & -2 \sum_{m,n} a_{mn} \left(\frac{Y_{mn}^{TE,1} + B_{mn}^{TE} Y_{mn}^{TE,2}}{2} \right) \bar{\psi}_{mn}^{TE} + b_{mn} \left(\frac{Y_{mn}^{TM,1} + B_{mn}^{TM} Y_{mn}^{TM,2}}{2} \right) \bar{\psi}_{mn}^{TM} \\ & + \left\{ \begin{array}{cc} Y_{m_i n_i}^{TE,1} - B_{mn}^{TE} Y_{m_i n_i}^{TE,2} & \bar{\psi}_{m_i n_i}^{TE} \\ Y_{m_i n_i}^{TM,1} - B_{mn}^{TM} Y_{m_i n_i}^{TM,2} & \bar{\psi}_{m_i n_i}^{TM} \end{array} \right\} \end{aligned} \quad (2.34)$$

By applying orthogonal properties of waveguide normal modes [36], we can solve coefficients a_{mn}, b_{mn} of the reflected and transmitted fields for TE and TM modes in terms of induced current $\bar{J}$ on printed strip.

$$\left\{ \begin{array}{c} a_{mn} \\ b_{mn} \end{array} \right\} = \delta_{m-m_i} \left\{ \begin{array}{c} R_{mn}^{TE'} \\ R_{mn}^{TM'} \end{array} \right\} - \frac{\iint \bar{J}(x, y) \cdot \left\{ \begin{array}{c} \bar{\psi}_{mn}^{TE} \\ \bar{\psi}_{mn}^{TM} \end{array} \right\} dx dy}{2 \left\{ \begin{array}{c} Y_{mn}^{TE'} \\ Y_{mn}^{TM'} \end{array} \right\} \left(\frac{ab}{\epsilon_{0m} \epsilon_{0n}} \right) [k_{xm}^2 + k_{yn}^2]} \quad (2.35)$$

$$\text{where } R_{mn}^{TE'} = \frac{Y_{mn}^{TE,1} - B_{mn}^{TE} Y_{mn}^{TE,2}}{2Y_{mn}^{TE'}};$$

$$R_{mn}^{TM'} = \frac{Y_{mn}^{TM,1} - B_{mn}^{TM} Y_{mn}^{TM,2}}{2Y_{mn}^{TM'}};$$

$$Y_{mn}^{TE'} = \frac{Y_{mn}^{TE,1} + B_{mn}^{TE} Y_{mn}^{TE,2}}{2}$$

$$\text{and } Y_{mn}^{TM'} = \frac{Y_{mn}^{TM,1} + B_{mn}^{TM} Y_{mn}^{TM,2}}{2}$$

2.4 Admittance-type Hybrid MoM-Immittance Approach

Figure 2.6 shows three-dimensional geometry of waveguide discontinuity under consideration. There are three regions inside waveguide, region I of relative permittivity ϵ_1 , region II of relative permittivity ϵ_2 and region III of relative permittivity ϵ_3 . A printed slot of any arbitrary shape and size is printed at interface #2.

2.4.1 Magnetic Field Integral Equation

Magnetic field integral equation (MFIE) is obtained by enforcing boundary condition of tangential continuity of the total magnetic field, which consists of incident and scattered magnetic field, on printed slot.

$$\hat{z} \times (\bar{H}^{inc}(\bar{r}) + \bar{H}^{scatt}(\bar{r})) \Big|_{z=0^-} = \hat{z} \times (\bar{H}^{scatt}(\bar{r})) \Big|_{z=0^+} \quad (2.36)$$

In above equation, scattered magnetic field can be expressed in terms of magnetic dyadic Green's functions, $\bar{\bar{G}}_{HM}(\bar{r}, \bar{r}')$ and magnetic current density, $\bar{M}(\bar{r}')$ over printed slot as given below.

$$\hat{z} \times \bar{H}^{inc}(\bar{r}) + \int_{slot} \bar{\bar{G}}_{HM}^-(\bar{r}, \bar{r}') \cdot \bar{M}(\bar{r}') d\bar{S}' = - \int_{slot} \bar{\bar{G}}_{HM}^+(\bar{r}, \bar{r}') \cdot \bar{M}(\bar{r}') d\bar{S}' \quad (2.37a)$$

$$- \hat{z} \times \bar{H}^{inc}(\bar{r}) = \int_{slot} \bar{\bar{G}}_{HM}^-(\bar{r}, \bar{r}') \cdot \bar{M}(\bar{r}') d\bar{S}' + \int_{slot} \bar{\bar{G}}_{HM}^+(\bar{r}, \bar{r}') \cdot \bar{M}(\bar{r}') d\bar{S}' \quad (2.37b)$$

$$- \hat{z} \times \bar{H}^{inc}(\bar{r}) = \int_{slot} \bar{\bar{G}}_{HM}(\bar{r}, \bar{r}') \cdot \bar{M}(\bar{r}') d\bar{S}' \quad (2.37c)$$

$$\text{where } \bar{\bar{G}}_{HM} = \bar{\bar{G}}_{HM}^- + \bar{\bar{G}}_{HM}^+$$

Figure 2.6: Geometry of printed slot in multi-layered waveguide

2.4.2 Admittance-type MoM-Immittance Approach Matrix

Following similar formulation as in section 2.3.2, from MFIE, we can get the following admittance-type MoM matrix equations.

$$\begin{bmatrix} [Y_{xx}] & [Y_{xy}] \\ [Y_{yx}] & [Y_{yy}] \end{bmatrix} \begin{bmatrix} [V_x] \\ [V_y] \end{bmatrix} = \begin{bmatrix} [I_x] \\ [I_y] \end{bmatrix} \quad (2.38)$$

where the elements of each sub-matrices are expressed as

$$\begin{aligned} y_{xx}(m, n) &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{T}_x^*(k_{xm}, k_{yn}) \tilde{G}_{HM}^{xx}(k_{xm}, k_{yn}) \tilde{B}_x(k_{xm}, k_{yn}) \\ y_{xy}(m, n) &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{T}_x^*(k_{xm}, k_{yn}) \tilde{G}_{HM}^{xy}(k_{xm}, k_{yn}) \tilde{B}_y(k_{xm}, k_{yn}) \\ y_{yx}(m, n) &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{T}_y^*(k_{xm}, k_{yn}) \tilde{G}_{HM}^{yx}(k_{xm}, k_{yn}) \tilde{B}_x(k_{xm}, k_{yn}) \\ y_{yy}(m, n) &= - \sum_{m=0}^{M+1N+1} \sum_{n=0}^{M+1N+1} \tilde{T}_y^*(k_{xm}, k_{yn}) \tilde{G}_{HM}^{yy}(k_{xm}, k_{yn}) \tilde{B}_y(k_{xm}, k_{yn}) \\ i_x &= \iint_{slot} T_x^*(x, y) H_x^{inc}(x, y) dx dy \\ i_y &= \iint_{slot} T_y^*(x, y) H_y^{inc}(x, y) dx dy. \end{aligned}$$

The admittance sub-matrix $[y_{xy}]$ denotes x-directed testing of incident magnetic field produced by y-directed magnetic current basis elements. The sub-matrices $[I_x]$ and $[I_y]$ refer to x- and y-directed testing of incident magnetic field. $[V_x]$ and $[V_y]$ are respectively the unknown magnetic voltage expansion coefficients associated with x-directed and y-directed basis functions which can be expressed as below:

$$[V] = [Y]^{-1} [I] \quad (2.39)$$

2.4.3 Spectral Domain Magnetic Dyadic Green's Functions

Printed slot inside the layered waveguide of geometry depicted in Figure 2.6 can be represented by transverse magnetic point source excitation as shown in Figure 2.7. For transverse magnetic point source excitation, equivalent TE and TM circuit models [15, 35] are shown in Figure 2.8. In Figure 2.8(a), Y_1^{TE} , Y_2^{TE} and Y_3^{TE} are respectively the wave admittances of TE mode in region I, region II and region III of waveguide. Similarly, Y_1^{TM} , Y_2^{TM} and Y_3^{TM} in Figure 2.8(b) are respectively the wave admittances of TM mode in region I, region II and region III of waveguide. The equivalent admittances looking down from interface #2 for TE and TM mode cases are

$$Y_d^{TE} = Y_3^{TE}; Y_d^{TM} = Y_3^{TM} \quad (2.40)$$

where $Y_i^{TE} = \frac{\gamma_i}{j\omega\mu_i}$; $Y_i^{TM} = \frac{j\omega\epsilon_i}{\gamma_i}$; $\gamma_i = \sqrt{\frac{m\pi^2}{a} + \frac{n\pi^2}{b} - k_0^2\mu_i\epsilon_i}$ where $i=1,2,3$ are propagation constant of EM wave in region I, II and III respectively.

Figure 2.7: Transverse magnetic point source excitation $\bar{M}^+(\bar{r}')$ and $\bar{M}^-(\bar{r}')$ inside waveguide

Whereas equivalent admittances looking up from interface #2 for TE and TM mode cases are

$$Y_u^{TE} = Y_2^{TE} \frac{Y_2^{TE} + Y_1^{TE} \coth(\gamma_2 h)}{Y_2^{TE} \coth(\gamma_2 h) + Y_1^{TE}} \quad (2.41a)$$

$$Y_u^{TM} = Y_2^{TM} \frac{Y_2^{TM} + Y_1^{TM} \coth(\gamma_2 h)}{Y_2^{TM} \coth(\gamma_2 h) + Y_1^{TM}} \quad (2.41b)$$

Figure 2.8: Equivalent TE and TM circuit models for transverse magnetic point source excitation in waveguide

The magnetic dyadic Green's function when source and observation points are both at interface #2 is

$$\begin{aligned}
 & \tilde{\tilde{G}}_{HM}^{22}(\bar{k}_t, z=h, z'=h) \\
 &= -I_d^{TE}(\bar{k}_t, h, h)(\hat{k}_t \times \hat{k}_t)(\hat{k}_t \times \hat{k}_t) - I_d^{TM}(\bar{k}_t, h, h)\hat{k}_t\hat{k}_t \\
 &= -Y_d^{TE}(\hat{k}_t \times \hat{k}_t)(\hat{k}_t \times \hat{k}_t) - Y_d^{TM}\hat{k}_t\hat{k}_t \\
 &= -Y_3^{TE}(\hat{k}_t \times \hat{k}_t)(\hat{k}_t \times \hat{k}_t) - Y_3^{TM}\hat{k}_t\hat{k}_t
 \end{aligned} \tag{2.42}$$

Similarly, magnetic dyadic Green's function when source is at interface #2 and observation point is at interface #1 is

$$\begin{aligned}
 & \tilde{\tilde{G}}_{HM}^{12}(\bar{k}_t, z=0, z'=h) \\
 &= -I_d^{TE}(\bar{k}_t, 0, h)(\hat{k}_t \times \hat{k}_t)(\hat{k}_t \times \hat{k}_t) - I_d^{TM}(\bar{k}_t, 0, h)\hat{k}_t\hat{k}_t \\
 &= -Y_u^{TE}(\hat{k}_t \times \hat{k}_t)(\hat{k}_t \times \hat{k}_t) - Y_u^{TM}\hat{k}_t\hat{k}_t \\
 &= -Y_2^{TE} \frac{Y_2^{TE} + Y_1^{TE} \coth(\gamma_2 h)}{Y_2^{TE} \coth(\gamma_2 h) + Y_1^{TE}} (\hat{k}_t \times \hat{k}_t)(\hat{k}_t \times \hat{k}_t) \\
 &\quad - Y_2^{TM} \frac{Y_2^{TM} + Y_1^{TM} \coth(\gamma_2 h)}{Y_2^{TM} \coth(\gamma_2 h) + Y_1^{TM}} \hat{k}_t\hat{k}_t
 \end{aligned} \tag{2.43}$$

The four components of magnetic dyadic Green's function $\tilde{\tilde{G}}_{HM}^{22}(\bar{k}_t, z=h, z'=h)$ are as follows:

$$\tilde{G}_{EJ}^{xx22}(k_{xm}, k_{yn}) = -\frac{k_{xm}^2 Y_3^{TE} + k_{yn}^2 Y_3^{TM}}{k_{xm}^2 + k_{yn}^2} \quad (2.44a)$$

$$\tilde{G}_{EJ}^{yy22}(k_{xm}, k_{yn}) = -\frac{k_{xm}^2 Y_3^{TE} + k_{yn}^2 Y_3^{TM}}{k_{xm}^2 + k_{yn}^2} \quad (2.44b)$$

$$\begin{aligned} &\tilde{G}_{EJ}^{xy22}(k_{xm}, k_{yn}) \\ &= \tilde{G}_{EJ}^{yx22}(k_{xm}, k_{yn}) = -\frac{k_{xm} k_{yn} (Y_3^{TM} - Y_3^{TE})}{k_{xm}^2 + k_{yn}^2} \end{aligned} \quad (2.44c)$$

Similarly, four components of magnetic dyadic Green's function $\tilde{\tilde{G}}_{HM}^{12}(\bar{k}_t, z=0, z'=h)$ are

$$\begin{aligned} &\tilde{G}_{EJ}^{xx12}(k_{xm}, k_{yn}) = \\ &= -\frac{k_{xm}^2 Y_2^{TE} \frac{Y_2^{TE} + Y_1^{TE} \coth(\gamma_2 h)}{Y_2^{TE} \coth(\gamma_2 h) + Y_1^{TE}} + k_{yn}^2 Y_2^{TM} \frac{Y_2^{TM} + Y_1^{TM} \coth(\gamma_2 h)}{Y_2^{TM} \coth(\gamma_2 h) + Y_1^{TM}}}{k_{xm}^2 + k_{yn}^2} \end{aligned} \quad (2.45a)$$

$$\begin{aligned} &\tilde{G}_{EJ}^{yy12}(k_{xm}, k_{yn}) \\ &= -\frac{k_{xm}^2 Y_2^{TE} \frac{Y_2^{TE} + Y_1^{TE} \coth(\gamma_2 h)}{Y_2^{TE} \coth(\gamma_2 h) + Y_1^{TE}} + k_{yn}^2 Y_2^{TM} \frac{Y_2^{TM} + Y_1^{TM} \coth(\gamma_2 h)}{Y_2^{TM} \coth(\gamma_2 h) + Y_1^{TM}}}{k_{xm}^2 + k_{yn}^2} \end{aligned} \quad (2.45b)$$

$$\begin{aligned} &\tilde{G}_{EJ}^{xy12}(k_{xm}, k_{yn}) \\ &= \tilde{G}_{EJ}^{yx12}(k_{xm}, k_{yn}) \\ &= -\frac{k_{xm} k_{yn} \left(Y_2^{TM} \frac{Y_2^{TM} + Y_1^{TM} \coth(\gamma_2 h)}{Y_2^{TM} \coth(\gamma_2 h) + Y_1^{TM}} - Y_2^{TM} \frac{Y_2^{TM} + Y_1^{TM} \coth(\gamma_2 h)}{Y_2^{TM} \coth(\gamma_2 h) + Y_1^{TM}} \right)}{k_{xm}^2 + k_{yn}^2} \end{aligned} \quad (2.45c)$$

The above method can be easily extended for any number of layers inside the waveguide and hence the Hybrid MoM-Immittance Approach developed can be used for analysis of multiple numbers of printed strips and slots in multi-layered waveguide environment.

2.4.4 Reflection and Transmission Coefficients

The scattered magnetic field vectors in region III ($z>h$) of waveguide discontinuity as depicted in Figure 2.6 can be expressed as

$$\bar{H}_{scatt}^3 = \iint_{slot} \bar{G}_{HM}^{22}(\bar{r}, \bar{r}') \cdot \bar{M}(\bar{r}, \bar{r}') d\bar{S}' \quad (2.46)$$

The transverse magnetic field vectors in region I ($z<0$) of waveguide is given by

$$\bar{H}_{scatt}^1 = \iint_{slot} \bar{G}_{HM}^{12}(\bar{r}, \bar{r}') \cdot \bar{M}(\bar{r}, \bar{r}') d\bar{S}' \quad (2.47)$$

The incident magnetic field coming from region III ($z=h^+$) can be expressed in terms of TE and TM modal functions.

$$\bar{H}_{tan}^{inc} = \begin{Bmatrix} Y_{m,n_i}^{TE,3} & \bar{\chi}_{m,n_i}^{TE} \\ Y_{m,n_i}^{TM,3} & \bar{\chi}_{m,n_i}^{TM} \end{Bmatrix} \quad (2.48)$$

The electromagnetic fields in each region can be represented in terms of the respective waveguide modes (with the z-component suppressed). The scattered magnetic field at $z=h^+$ is

$$\bar{H}_{scatt}^3 = \sum_{m,n} a_{mn}^3 Y_{mn}^{TE,3} \bar{\chi}_{mn}^{TE} + b_{mn}^3 Y_{mn}^{TM,3} \bar{\chi}_{mn}^{TM} \quad (2.49)$$

The scattered magnetic field at $z=0^-$ is

$$\bar{H}_{scatt}^1 = \sum_{m,n} a_{mn}^1 Y_{mn}^{TE,1} \bar{\chi}_{mn}^{TE} + b_{mn}^1 Y_{mn}^{TM,1} \bar{\chi}_{mn}^{TM} \quad (2.50)$$

The mode reflection and transmission coefficient can be obtained from orthogonal property of modal vectors.

$$\begin{Bmatrix} a_{mn}^1 \\ b_{mn}^1 \end{Bmatrix} = \frac{\iint_{\text{slot}} \iint \left\{ \overline{\overline{G}}_{HM}^{22}(\bar{r}, \bar{r}') \cdot \overline{M}(\bar{r}, \bar{r}') \right\} \begin{Bmatrix} \bar{\chi}_{mn}^{TE} \\ \bar{\chi}_{mn}^{TM} \end{Bmatrix} d\bar{S}' d\bar{S}}{\begin{Bmatrix} Y_{mn}^{TE,1} \\ Y_{mn}^{TM,1} \end{Bmatrix} \iint \begin{Bmatrix} \bar{\chi}_{mn}^{TE} \\ \bar{\chi}_{mn}^{TM} \end{Bmatrix} \cdot \begin{Bmatrix} \bar{\chi}_{mn}^{TE} \\ \bar{\chi}_{mn}^{TM} \end{Bmatrix} d\bar{S}} \quad (2.51a)$$

$$\begin{Bmatrix} a_{mn}^3 \\ b_{mn}^3 \end{Bmatrix} = \frac{\iint_{\text{slot}} \iint \left\{ \overline{\overline{G}}_{HM}^{12}(\bar{r}, \bar{r}') \cdot \overline{M}(\bar{r}, \bar{r}') \right\} \begin{Bmatrix} \bar{\chi}_{mn}^{TE} \\ \bar{\chi}_{mn}^{TM} \end{Bmatrix} d\bar{S}' d\bar{S}}{\begin{Bmatrix} Y_{mn}^{TE,1} \\ Y_{mn}^{TM,1} \end{Bmatrix} \iint \begin{Bmatrix} \bar{\chi}_{mn}^{TE} \\ \bar{\chi}_{mn}^{TM} \end{Bmatrix} \cdot \begin{Bmatrix} \bar{\chi}_{mn}^{TE} \\ \bar{\chi}_{mn}^{TM} \end{Bmatrix} d\bar{S}} \quad (2.51b)$$

2.5 Conclusion

In this chapter, we have formulated a unified approach of MoM, Mode-matching Method and Immittance Approach for full-wave characterization of printed periodic waveguide structures. MoM is employed for transforming the integral equations into a matrix system of linear equations which can be solved using computer codes. Impedance-type Hybrid MoM-Immittance Approach is developed for printed strip inside waveguide whereas the Admittance-type Hybrid MoM-Immittance Approach is developed for printed slot layers inside waveguide. Immittance Approach is employed for calculating the spectral Dyadic Green's functions for the multilayered waveguide structure. Mode-matching method is used for obtaining the transmission and reflection coefficients of various waveguide modes. In the next chapter, we will do the numerical convergence study and validation our electromagnetic code developed using this unified approach.

Chapter 3

Numerical Verifications and Equivalent Circuit Extraction

3.1 Numerical Convergence

The Hybrid MoM-Immittance Approach formulated in the previous chapter has been implemented in a computer program using MATLAB, C and C++. For all EM simulators, it is mandatory to check its numerical convergence, accuracy and efficiency before utilizing it for numerical characterization of microwave devices. Let us check the convergence of our developed EM program. In the following sections, we will also check its numerical accuracy and efficiency by comparing with experimental results and results generated from other available commercial software. It has been observed that for obtaining convergent results printed slot or strip surface must be divided into at least 10 numbers of meshes. For accurate scattering parameter results, number of meshes should be chosen around 15 for the dimension of slot or strip equal to wavelength at frequency under consideration. Figure 3.2 (a) and 3.2 (b) illustrate convergence plot of S-parameter amplitude and phase for a printed slot of dimension $w=17.0\text{mm}$ and $l=3.0\text{mm}$ placed inside a X-band rectangular waveguide ($a=22.86\text{mm}$ and $b=10.16\text{mm}$) at 17GHz. The three regions of waveguide in Figure 2.6 have relative permittivity of $\epsilon_1 = 1.0$, $\epsilon_2 = 1.7$, $\epsilon_3 = 1.0$ and h is taken as 1.0mm. It can be seen that amplitude of S_{11} starts to converge around 10 numbers of meshes. For phase of S_{11} parameter, convergent result is obtained when number of meshes is about 15. Figure 3.2(c) illustrates the mode

convergence plot for the same printed slot. It can be observed that for this specific printed slot with the increase in number of modes we get a convergent result for the printed slot magnitude of S_{11} parameter around 12 modes. The number of modes for obtaining a convergent result also depends on the reference plane where we calculate the scattering parameters. Here we have chosen the reference plane at a distance of 3mm from the waveguide discontinuity. If we calculate the scattering parameters at a distance far away from the waveguide discontinuity (greater than $\lambda_g/2$), we will get the mode convergence very fast. For calculation of scattering parameters near a waveguide discontinuity, we will have to consider more number of modes.

Figure 3.1: Geometry of (a) printed rectangular strip (b) printed rectangular slot

(b)

Figure 3.2: Rate of convergence of numerical characterization for a printed slot inside a rectangular waveguide (a) amplitude and (b) phase of S_{11} (c) Mode convergence

3.2 HFSS Validation of Printed Slot and Strip

Let us consider the case of printed strip in a layered waveguide as shown in Figure 3.1(a). A rectangular strip of $l=9.0\text{mm}$ and $w=3.0\text{mm}$ is placed at the center of an X-band rectangular waveguide ($a=22.86\text{mm}$ and $b=10.16\text{mm}$). The three regions of

waveguide as illustrated in Figure 2.1 have relative permittivity of $\epsilon_1 = 1.0$, $\epsilon_2 = 2.25$, $\epsilon_3 = 1.0$ and h is taken as 1.0mm. The scattering parameters for a resonant printed strip inside an X-band rectangular waveguide is derived using the developed EM program and shows good agreement with HFSS results as illustrated in Figure 3.3(a). It has been observed that printed strip totally reflects incident wave at resonance. Next, let us consider the case of printed slot in a layered waveguide as shown in Figure 3.1 (b). A rectangular slot of $l=2.7\text{mm}$ and $w=15.5\text{mm}$ is placed at center of an X-band rectangular waveguide. The three regions of waveguide as shown in Figure 2.6 have relative permittivity of $\epsilon_1 = 1.0$, $\epsilon_2 = 2.25$, $\epsilon_3 = 1.0$ and h is taken as 1mm. The scattering parameters for a resonant printed slot inside an X-band rectangular waveguide is obtained using the developed EM program and shows good agreement with HFSS results as shown in Figure 3.3(b). It has been observed that printed slot totally transmits incident wave at the resonance.

(a)

(b)

Figure 3.3: Scattering parameter for a resonant printed (a) strip and (b) slot in layered waveguide

3.3 Efficiency Comparison with FEM

Let us consider a waveguide filter composed of resonant iris, i.e., printed slot with no dielectric layer ($h=0$). A compact and broadband iris filter has been designed based on simple equivalent transmission line network, in which each parameter is derived using the proposed hybrid method. The geometry and equivalent circuit topology of the waveguide filter are shown in Figure 3.4(a) and 3.4(b) respectively. The distance d between resonant irises is taken as 9.0mm. The dimension of iris is $w=15.5$ mm and $l=2.7$ mm. The overall ABCD parameter of the filter is obtained by cascading ABCD parameters of every block. The ABCD parameter is transformed back to scattering parameters to observe the filter performance. The predicted result is compared with those of HFSS as shown in Figure 3.4(c) and they are in good agreement. The waveguide filter hence designed shows the broadband characteristics with bandwidth $\approx 28.0\%$ for S_{11} below -15 dB at centre frequency of 10.7GHz. The EM program achieved convergence with 15 basis functions and 10 modes. The overall CPU time

for calculation of 1000 frequency points was 43.7 minutes on a PC Pentium 4 @ 1.7 GHz (40 minutes for simulation of a single iris and 3.7 minutes to get the performance of the overall filter).

Figure 3.4: (a) Geometry (b) Equivalent circuit and (c) Scattering parameters of a waveguide band pass filter

FEM based HFSS version 8.025 has been used for simulation of the same waveguide band pass filter. With a λ -refinement based upon a target frequency of 11 GHz, discrete sweeps were run between 7 and 17 GHz. The number of tetrahedra used was 37901 and it gave a variation in the S parameters of $\Delta S=0.0074774$. The overall CPU time for the calculation of the frequency response of the filter on the same PC configuration was 13.7 hours. HFSS usually takes several hours for such waveguide structures simulations whereas the proposed method takes several minutes to simulate the same device.

3.4 Equivalent Circuit Parameter Extraction

A reciprocal two-port network can be represented by either a T- or π - network as shown in Figure 3.5.

Figure 3.5: (a) π -network (b) T-network

The elements of a two-port network can be calculated from transmission matrix by a simple transformation as below:

$$Z_a = \frac{A-1}{C} \quad Z_b = \frac{D-1}{C} \quad Z_c = \frac{1}{C} \quad (3.1)$$

$$Y_a = \frac{1}{B} \quad Y_b = \frac{A-1}{B} \quad Y_c = \frac{D-1}{B} \quad (3.2)$$

The transmission matrix can be obtained from scattering matrix as follows:

$$A = \frac{(1 + s_{11})(1 - s_{22}) + s_{12}s_{21}}{2s_{21}} \quad (3.3a)$$

$$B = \frac{Z_0(1 + s_{11})(1 + s_{22}) - s_{12}s_{21}}{2s_{21}} \quad (3.3b)$$

$$C = \frac{(1 - s_{11})(1 - s_{22}) - s_{12}s_{21}}{2Z_0s_{21}} \quad (3.3c)$$

$$D = \frac{(1 - s_{11})(1 + s_{22}) + s_{12}s_{21}}{2s_{21}} \quad (3.3d)$$

where Z_0 is wave impedance of the waveguide for TE or TM mode under consideration. By considering equivalent circuit network for various TE and TM waveguide modes, we can get multimode equivalent circuit network of the waveguide discontinuity under consideration. Usually equivalent circuits are derived for dominant mode under the assumption that other higher waveguide modes has been fully attenuated at the reference plane under consideration which is generally taken at a distance beyond $\lambda_g/2$. For lossless networks, impedance or admittance elements are purely imaginary. Hence, we can get equivalent circuit parameters from the sign of impedance or admittance elements in the two-port equivalent network.

The equivalent circuit of printed strip is shown in Figure 3.6(a) and its equivalent circuit parameters in layered waveguide have been extracted following the method described above. Figure 3.6(b) and 3.6(c) show variation of normalized X_b and X_c with frequency in equivalent T-network of Figure 3.5(b) for various strip dimensions. From Figure 3.6(b) we can infer that normalized X_b is positive and quasi-linearly increasing with frequency thereby it acts as an inductive element. It can be observed from Figure 3.6(c) that normalized X_c is negative at lower frequency region then crosses zero axis and becomes positive at higher frequency region, showing series resonant behavior. It can be further observed that as we increase length of the resonant printed strip, resonant frequency is lowered. Normalized X_a is very small and can be neglected in the equivalent T-network. The equivalent circuit of printed slot is shown in Figure 3.7(a). Figures 3.7(b) and 3.7(c) show variation of

normalized X_b and X_c with frequency in the equivalent T-network for various slot dimensions. Normalized X_b is positive and increases linearly with frequency as illustrated in Figure 3.7(b) hence it also acts as an inductive element. From Figure 3.7(c), we can infer that normalized X_c is positive in the lower frequency range then goes towards $+\infty$ and suddenly comes down to $-\infty$ around resonant frequency and becomes negative at higher frequency range which shows parallel resonant behavior. We can also observe that resonant frequency is lowered when width of slot is increased. Normalized X_a is very small and can be neglected in equivalent T-network.

(c)

Figure 3.6: (a) Equivalent circuit (b) Normalized X_b versus frequency and (c) Normalized X_c versus frequency for resonant printed strip in layered waveguide

(a)

(b)

(c)

Figure 3.7: (a) Equivalent circuit (b) Normalized X_b versus frequency and (c) Normalized X_c versus frequency for printed slot in layered waveguide

3.5 Experimental Validation of a Wide Resonant Strip

K. Chang et al [7] has developed an accurate numerical approach to calculate the susceptance due to a wide resonant strip on the transverse plane of a rectangular waveguide. K. Chang et al observed that their new method has good agreement with the experimental results for a wide resonant strip unlike their previous analysis which doesn't show good agreement near the resonance. Figure 3.8(a) shows a X-band waveguide with a centered strip of width=0.280 inch and depth=0.360 inch. The equivalent circuit of the waveguide structure under investigation can be represented by a shunt susceptance as shown in Figure 3.8(b). Figure 3.8(c) illustrates the normalized susceptance versus frequency. Note the choice of basis functions: half-basis functions are employed at the edge where the strip touches the waveguide walls. It can be observed that there is close agreement between the Hybrid MoM-Immittance Approach [42] and experimental results [7].

Figure 3.8: A rectangular waveguide with a wide resonant strip (a) Cross section (b) Equivalent circuit and (c) Normalized susceptance of a centered strip of $w=0.280$ inch and $d=0.360$ inch

Chapter 4

Guided-wave Characteristics of Periodic Waveguide Structures

4.1 Literature Review

Frequency Selective Surface (FSS) is widely used in microwave and optical engineering as spatial and frequency filters [37]-[39]. In this chapter, a new class of periodic waveguide structure loaded with various FSS strip/slot layers is originated and studied in terms of their per-unit-length guided-wave parameters, i.e., complex propagation constant ($\gamma=\alpha+j\beta$) and complex wave impedance ($Z=\text{Re}(Z)+j\text{Im}(Z)$). In past, propagation characteristics of a rectangular waveguide, periodically loaded with simple rectangular iris has been partially studied in form of Brillouin diagram [40]-[41] which limitedly illustrates frequency-dependent imaginary part of the complex propagation constant, i.e., phase constant. No proper analysis has been reported today to investigate the complete guided-wave characteristic of periodic waveguide structures with different FSS loading configurations with resort to the above per-unit-length parameters that are parameters of one-dimensional (1-D) periodically loaded medium. The Hybrid MoM-Immittance Approach [42] is utilized here for investigation on guided-wave characteristics of the novel periodic waveguide structures loaded with various FSS strip/slot layers. Derived results are at first compared with available analytical results [36] with resort to the imaginary part of complex propagation constant, i.e., phase constant, of a rectangular iris-loaded

periodic waveguide structure, giving an evident validation of our approach in this guided-wave problem. Next, we move to study periodic waveguide structure loaded with FSS elements, i.e., cross, square loop, square, ring and Jerusalem cross strip/slot layers, in order to demonstrate their distinctive guided-wave propagation characteristics with the passband and/or stopband for 1-D periodically loaded medium. It is observed that different FSS strip/slot layers loading lead to varied propagation behaviors such as slow wave, fast wave, backward wave, forward wave, passband, stopband, etc. The possible applications of such periodic waveguide structures are in design of waveguide based EBG structures with stopband, bandpass filters with the passband, leaky wave antennas with the fast-wave, metamaterials with the backward wave, etc.

4.2 Theory and Validation

4.2.1 Theory

Consider the periodic waveguide structure of periodicity p as illustrated in Figure 4.1(b). It consists of cascade connection of an infinite number of lossless waveguide discontinuities whose unit cells are depicted in Figure 4.1(a). Each waveguide discontinuity can be represented by unit cell ABCD parameters which can be obtained using Hybrid MoM-Immittance Approach [42]. Such a periodic waveguide structure with discontinuity can be equivalently perceived as uniform waveguide section of length p as illustrated in Figure 4.1(c) and its performance can be modeled in terms of equivalent per-unit-length propagation constant ($\gamma=\alpha+j\beta$) and complex wave impedance ($Z=\text{Re}(Z)+j\text{Im}(Z)$) similar to previous studies [43] for periodic CPW structure or medium with inductive loading. Let us assume that only the dominant TE_{10} mode is propagating. If any other higher order modes are propagating at frequency under consideration then guided-wave parameters for that mode can also be studied in addition to dominant mode similar to what have been investigated for corrugated waveguides [44]. Since the Hybrid MoM-Immittance Approach can

be extended for multimode full-wave characterization of such kind of periodic waveguide structures hence guided-wave characteristics of first few propagating modes can also be obtained. But we will consider guided-wave parameters for the dominant TE_{10} mode only for the sake of simplicity. It is worth mentioning the concept of “accessible” and “localized” modes which has been introduced in [40]. Accessible modes are waveguide modes that are being excited at the location of one waveguide discontinuity and are also “seen” by adjacent waveguide discontinuities. This includes all propagating modes of original (unloaded) waveguide, plus, possibly, first few evanescent ones, depending on separation between the two adjacent discontinuities. All remaining modes, purely evanescent, are called localized, as they remain localized to neighborhood of the discontinuity that excites them. The periodicity p of the periodic structures is chosen such that all higher order evanescent modes excited by the waveguide discontinuity have decayed to a negligible value at the next consecutive discontinuities, i.e., all higher order modes are localized. β_0 and Z_0 in Figure 4.1 (a) and Figure 4.1 (b) are respectively the phase constant and wave impedance of dominant mode of the original waveguide (no loading). Whereas, γ and Z are respectively the propagation constant and wave impedance of the corresponding equivalent uniform waveguide of printed periodic waveguide structure (with FSS loading), as illustrated in Figure 4.1(c). If a wave is to propagate down the periodic waveguide structure, it is necessary for voltage and current at the $(n+1)^{th}$ terminal to be equal to voltage and current at the n^{th} terminal, apart from the phase delay due to finite propagation time.

$$\begin{aligned} V_{n+1} &= e^{-\gamma p} V_n \\ I_{n+1} &= e^{-\gamma p} I_n \end{aligned} \quad (4.1)$$

where $\gamma = \alpha + j\beta$ is propagation constant of the periodic waveguide structure with FSS loading.

In terms of transmission matrix for a unit cell, we now have,

$$\begin{aligned} \begin{bmatrix} V_n \\ I_n \end{bmatrix} &= \begin{bmatrix} A_u & B_u \\ C_u & D_u \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = e^{\gamma p} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} \\ \Rightarrow \begin{bmatrix} A_u & B_u \\ C_u & D_u \end{bmatrix} - \begin{bmatrix} e^{\gamma p} & 0 \\ 0 & e^{\gamma p} \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} &= 0 \end{aligned} \quad (4.2)$$

This equation is a matrix eigenvalue equation for γ . Here, the subscript u in ABCD parameters denotes unit/cell.

Figure 4.1: (a) General waveguide discontinuity as unit/cell for the periodic waveguide structure (b) Infinite periodic waveguide structure composed of cascade connection of units/cells (c) Equivalent uniform waveguide of the infinite periodic waveguide structure

A nontrivial solution for V_{n+1} , I_{n+1} exists only if the determinant vanishes. Hence, considering a reciprocal network in which $A_u D_u - B_u C_u = 1$, we get,

$$\det \left\{ \begin{bmatrix} A_u - e^{\gamma p} & B_u \\ C_u & D_u - e^{\gamma p} \end{bmatrix} \right\} = A_u D_u - B_u C_u + 2e^{\gamma p} - e^{\gamma p} (A_u + D_u) = 1 + 2e^{\gamma p} - e^{\gamma p} (A_u + D_u) = 0 \quad (4.3)$$

So we obtain,

- 1) Propagation: When $\left| \frac{A_u + D_u}{2} \right| < 1$, we must have $\gamma = j\beta$ and $\alpha = 0$; so that

$$\cos \beta p = \frac{A_u + D_u}{2} \quad (4.4)$$

- 2) Attenuation: When $\frac{A_u + D_u}{2} > 1$, we must have $\gamma = \alpha$ and $\beta = 0$; so that

$$\cosh \alpha p = \frac{A_u + D_u}{2} \quad (4.5)$$

- 3) Complex mode: When $\frac{A_u + D_u}{2} < -1$, we must have $\gamma p = \alpha + j\pi$, so that

$$\cosh \gamma p = \cosh(\alpha + j\pi) = -\cosh \alpha p = \frac{A_u + D_u}{2} \quad (4.6)$$

We will not consider complex modes in this discussion. Let us write down the expression for normalized characteristic wave impedance which can be derived similarly for the periodic waveguide structure (for complete derivation, refer to [36]).

$$\frac{Z}{Z_0} = \pm \sqrt{\frac{B}{C}} \quad (4.7)$$

Figure 4.3 and Figure 4.13 show the three-dimensional (3-D) geometry of an infinitely extended waveguide based periodic structure loaded with any arbitrary FSS strip and slot layers respectively. The FSS strip/slot layers are transversely put at a periodicity of p inside a uniform waveguide with width of a and height of b . The FSS strips/slots are placed at the symmetrical center of each transverse strip layers inside waveguide. The infinitely extended periodic waveguide structure can be perceived as a 1-D guided-wave medium, thus its per-unit-length medium parameters can be simply analyzed by characterizing a single unit/cell of periodicity p for the periodic waveguide structure. Using the Hybrid MoM-Immittance Approach [42], two-port ABCD parameters of such FSS strip/slot layers driven by two waveguide sections of length $p/2$ can be numerically derived and hence the guided-wave parameters using equations (4.4), (4.5) and (4.7).

4.2.2 Brillouin Diagram

Figure 4.2: Numerically calculated k_0 - β diagram for a capacitive strip loaded waveguide based periodic structure

Firstly, a rectangular iris loaded periodic waveguide structure is analyzed. Figure 4.2 shows numerically calculated k_0 - β diagram for a rectangular waveguide (22.86mm $\times$ 5.08mm), loaded with capacitive strips of height $d=3.81$ mm, placed at a period of $p=10$ mm. The brillouin diagram for the periodic waveguide structure is calculated using the Hybrid MoM-Immittance Approach and compares well with the analytical results given in [36], thereby verifying the accuracy of the developed approach.

4.3 FSS Strips Printed Periodic Waveguide Structures

Firstly, let us consider first printed FSS strip layers loaded periodic waveguide structures and study their guided wave characteristics in terms of per-unit-length guided-wave parameters. Figure 4.3 illustrates three-dimensional (3-D) geometry of an infinitely extended waveguide based periodic structure loaded with any arbitrary FSS strip layers. The periodicity p of the printed periodic waveguide structure is chosen as 4.00mm for all simulations. The rectangular waveguide under consideration is an X-band waveguide whose dimensions are $a=22.86$ mm and $b=10.16$ mm. Three typical FSS strip elements have been chosen for our investigation on the guided wave study: Square loop, Cross and Jerusalem cross strips. Let us study them one by one.

Figure 4.3: Geometry of an infinite periodic waveguide structure loaded with any arbitrary FSS strip layer

4.3.1 Printed Square Loop Strip

Figure 4.4 depicts the front view of a periodic waveguide structure loaded with square loop strip layers and dimensions of the strips are specified by l_1 , l_2 and w_1 . They are chosen as $l_1=7.0\text{mm}$, $l_2=7.00\text{mm}$ and $w_1=2.00\text{mm}$ for our study. Figure 4.5 and Figure 4.6 show respectively the normalized propagation constant ($\alpha/k_0+j\beta/k_0$) and characteristics wave impedance $\{\text{Re}(Z/Z_0)+j\text{Im}(Z/Z_0)\}$ of the square loop strip loaded periodic waveguide structure. From Figure 4.5, we can observe that there is visible slow-wave behavior ($\beta/k_0>1$) in the lower passband ($\alpha=0$) whereas fast-wave ($\beta/k_0<1$) behavior in the upper passband. Besides, it can be observed that guided-wave propagates in form of forward wave, in which phase velocity ($\omega/\beta>0$) is in-phase with group velocity ($\partial\omega/\partial\beta>0$) in both passbands. It can be observed that within stopband ($\beta=0$), the attenuation constant increases slowly first from zero, then increases sharply to 3.5 across the resonant frequency of the FSS strip and decreases to zero again at higher frequency. The stopband width is 6.7GHz at the mid-stopband frequency of 16.7GHz. It can be further observed that the real part of normalized wave impedance $\{\text{Re}(Z/Z_0)\}$ is less than 1 and almost constant within the whole lower passband whereas in the upper passband it goes to $+\infty$ and decreases sharply to a value around 1.5 as illustrated in Figure 4.6. It can also be seen here that $\text{Im}(Z/Z_0)$ becomes non-zero within whole stopband and its value starts to increase slowly at first and then rapidly toward $+\infty$ at higher end of this stopband. This completes the guided-wave passband/stopband characteristic of waveguide based periodic structure loaded with square loop strip layers.

Figure 4.4: Front view of a periodic waveguide structure loaded with square loop strip layers

Figure 4.5: Normalized propagation constant ($\gamma/k_0=\alpha/k_0+j\beta/k_0$) of square loop strip loaded periodic waveguide structure

Figure 4.6: Normalized wave impedance (Z/Z_0) of square loop strip loaded periodic waveguide structure

4.3.2 Printed Cross Strip

Figure 4.7 depicts the front view of a periodic waveguide structure loaded with cross strip layers and dimensions of the strips are specified by l_1 , l_2 , w_1 and w_2 . They are chosen as $l_1=7.0\text{mm}$, $l_2=7.00\text{mm}$, $w_1=2.00\text{mm}$ and $w_2=2.00\text{mm}$. Figure 4.8 and Figure 4.9 show respectively the normalized propagation constant and wave impedance of cross strip loaded periodic waveguide structure. From Figure 4.8, we can infer that there is visible slow-wave behavior in lower passband and fast-wave behavior in upper passband similar to the previous case of square loop strip. Besides, it can be observed that guided-wave propagates in the form of forward wave.

Figure 4.7: Front view of a periodic waveguide structure loaded with cross strip layers

Figure 4.8: Normalized propagation constant ($\gamma/k_0=\alpha/k_0+j\beta/k_0$) of cross strip loaded periodic waveguide structure

It can be observed that within the stopband region, the attenuation constant increases slowly first from zero, then increases sharply to 3.45 across the resonant frequency of the FSS strip and decreases to zero again as frequency increases. The stopband width has been considerably reduced to 4.5GHz at the mid-stopband frequency of 20.75GHz. It can be further observed that the real part of normalized wave impedance $\{Re(Z/Z_0)\}$ is less than 1 and remains almost constant within the whole lower passband whereas in the upper passband this quantity goes to $+\infty$ and decreases sharply to a value around 1.3, as illustrated in Figure 4.9. It can also be seen here that the $Im(Z/Z_0)$ becomes non-zero within the whole stopband and its value starts to increase slowly at first and then rapidly toward $+\infty$ at the higher end of this stopband. This completes the guided-wave passband/stopband characteristics of waveguide based periodic structure loaded with cross strip layers.

Figure 4.9: Normalized wave impedance (Z/Z_0) of cross strip loaded periodic waveguide structure

4.3.3 Printed Jerusalem Cross Strip

Figure 4.10 depicts the front view of a periodic waveguide structure loaded with Jerusalem cross strip layers and dimensions of the strips are specified by l_1 , l_2 , l_3 , w_1 ,

w_2 and w_3 . The dimensions of strip are chosen as $l_1=7.0\text{mm}$, $l_2=7.00\text{mm}$, $l_3=3.50\text{mm}$, $w_1=2.00\text{mm}$, $w_2=2.00\text{mm}$ and $w_3=1.00\text{mm}$. Figure 4.11 and Figure 4.12 show respectively the normalized propagation constant and wave impedance of Jerusalem cross strip loaded periodic waveguide structure. From Figure 4.11, we can infer that there is visible slow-wave and fast-wave behaviors in both the lower and upper passbands unlike the previous two strip cases. Besides, it can be observed that guided-wave propagates in form of backward wave [45] in lower passband where group velocity ($\partial\omega/\partial\beta < 0$) is anti-phase with phase velocity ($\omega/\beta > 0$). It can be observed that within the stopband region, the attenuation constant increases slowly first from zero, then increases sharply to 4.7 across the resonant frequency of the FSS strip and decreases to zero again as frequency increases. The stopband width is approximately 6.5GHz at the mid-stopband frequency of 14GHz. It can be further observed that the real part of normalized wave impedance $\{\text{Re}(Z/Z_0)\}$ is less than 1 and remains almost constant within the whole lower passband whereas in the upper passband this quantity goes to $+\infty$ and decreases sharply to a value around 1, then to zero as illustrated in Figure 4.12. It can also be seen here that $\text{Im}(Z/Z_0)$ becomes non-zero within whole stopband and its value starts to increase slowly at first and then rapidly toward $+\infty$ at the higher end of stopband. This completes the guided-wave passband/stopband characteristic of the waveguide based periodic structure loaded with Jerusalem cross strip layers which shows distinctive behavior from the previous two FSS strips loaded periodic waveguide structures.

Figure 4.10: Front view of a periodic waveguide structure loaded with Jerusalem cross strip layers

Figure 4.11: Normalized propagation constant ($\gamma/k_0=\alpha/k_0+j\beta/k_0$) of Jerusalem cross strip loaded periodic waveguide structure

Figure 4.12: Normalized wave impedance (Z/Z_0) of Jerusalem cross strip loaded periodic waveguide structure

4.4 FSS Slots Printed Periodic Waveguide Structures

Let us now consider printed FSS slot layers loaded periodic waveguide structures and study their guided-wave characteristics in terms of per-unit-length guided-wave parameters. Figure 4.13 illustrates the three-dimensional (3-D) geometry of an infinitely extended waveguide based periodic structure loaded with any arbitrary FSS slot layers. A typical FSS ring slot element has been chosen for our investigation on the guided wave study.

Figure 4.13: Geometry of an infinite periodic waveguide structure loaded with any arbitrary FSS slot layer

4.4.1 Printed Ring Slot

Figure 4.14 depicts the front view of a periodic waveguide structure loaded with ring slot layers and dimensions of the ring slot are specified by r_1 and r_2 . The dimensions of printed slot are chosen as $r_1=4.5\text{mm}$ and $r_2=1.5\text{mm}$. Figure 4.15 and Figure 4.16 show respectively the normalized propagation constant $(\alpha/k_0 + j\beta/k_0)$ and characteristics wave impedance $\{\text{Re}(Z/Z_0) + j\text{Im}(Z/Z_0)\}$ of ring slot loaded periodic waveguide. From Figure 4.15, we can observe that there is visible fast-wave behavior in the passband and guided-wave propagates in the form of forward wave. It can also be observed that within stopband region, the attenuation constant decreases gradually towards zero.

Figure 4.14: Front view of a periodic waveguide structure loaded with ring slot layers

Figure 4.15: Normalized propagation constant ($\gamma/k_0=\alpha/k_0+j\beta/k_0$) of ring slot loaded periodic waveguide structure

It can be further observed that the real part of normalized wave impedance $\{\text{Re}(Z/Z_0)\}$ decreases suddenly from $+\infty$ at the start of passband then gradually towards a value less than 1 at the higher end of passband as shown in Figure 4.16. It can also be seen here that the $\text{Im}(Z/Z_0)$ becomes non-zero within the whole stopband and its value starts to increase slowly at first and then rapidly toward $+\infty$ at the higher end of stopband. This completes the guided-wave passband/stopband characteristic of waveguide based periodic structure loaded with ring slot layers.

Figure 4.16: Normalized wave impedance (Z/Z_0) of ring slot loaded periodic waveguide structure

4.5 Effect of Dielectric Filling on Periodic Waveguide Structures

As usual, dielectric layer may be required to support various FSS elements in practical implementation of various waveguide based periodic structures. Dielectric profile is an important parameter for designing FSS for spatial and frequency filters as mentioned in various literatures like [46]. Let us study the effect of dielectric filling on guided wave characteristics of such waveguide based periodic structures loaded with FSS elements. Let us choose the cross strip loaded infinite periodic waveguide structure whose front view is shown in Figure 4.7. The dimensions of the cross strip and periodicity p are kept the same as in previous study except that the infinitely extended waveguide of Figure 4.3 is filled with dielectric permittivity of ϵ equal to 1.7.

Figure 4.17: Effect of dielectric filling on the normalized propagation constant ($\gamma/k_0=\alpha/k_0+j\beta/k_0$) of cross strip loaded periodic waveguide structure

Figure 4.18: Effect of dielectric filling on the normalized wave impedance (Z/Z_0) of cross strip loaded periodic waveguide structure

The normalized propagation constant and wave impedance are plotted in Figure 4.17 and Figure 4.18 respectively. In comparison to previous plots of normalized

propagation constant and wave impedance for cross strip loaded periodic waveguide structure (Figure 4.8 and Figure 4.9), we can infer that there is visible change and shift in the width of passband and stopband. Hence, it would be wise to choose low-permittivity and electrically thin substrate for the printed strip/slots so as to minimize the undesired influence of dielectric substrate on guided-wave characteristics of printed periodic waveguide structure.

Chapter 5

Stepped Impedance Slot Resonator Waveguide Filters

5.1 Literature Review

Advanced microwave filters are usually designed for size reduction and/or improving in-band and out-of-band filter performances [47]-[48]. Parallel coupled stripline stepped-impedance resonator (SIR) has been extensively used for controlling the spurious response by changing the aspect ratio of the two different characteristic impedances [49]. Size miniaturization of microstrip band pass filters has been achieved using the folded SIR in [50]. SIR has also been investigated for improving the stopband performance of filters in rectangular waveguide configuration [51]. In this chapter, we have investigated stepped-impedance slot (SIS) resonators for increasing the rejection bandwidth for application in the design of harmonic-suppressed waveguide band pass filters. First, two basic SIS resonators have been studied in comparison to the conventional uniform impedance slot (UIS) resonator in maximizing the frequency bandstop between the fundamental (f_r) and spurious harmonic (f_s) resonant frequencies. Among the two basic SIS resonators, one reduces the frequency bandstop ($f_s - f_r$) whereas the other increases ($f_s - f_r$) frequency bandstop with respect to conventional UIS resonator. SIS resonator with the maximum ($f_s - f_r$) frequency bandstop has been used for designing a waveguide band pass filter with $\lambda/4$ couplings or impedance transformers [52]-[53]. Unlike the conventional waveguide filters with $\lambda/2$ or $\lambda/4$ waveguide section resonators coupled by iris or other waveguide discontinuities [48], the proposed waveguide band pass

filter has the additional flexibility of controlling the frequency of operation, spurious response and insertion loss by changing the geometrical configuration of SIS resonators. A three-stage waveguide filter is optimally designed using SIS resonators linked by $\lambda/4$ waveguide sections or impedance transformers. The performance of this new filter has been characterized using the accurate and efficient Hybrid MoM-Immittance Approach [42]. It has been observed that at the passband, printed slot has an equivalent circuit of shunt LC resonator. Finally, a three-stage waveguide filter is optimally designed using SIS resonators and its predicted filtering performances have been well confirmed by independent HFSS simulated results.

5.2. SIS and UIS Resonators

Let us investigate UIS and two basic SIS resonators whose geometrical layouts are shown in Figure 5.1(a), Figure 5.1(b) and Figure 5.1(c), respectively. These slot resonators are studied here to increase the $(f_s - f_r)$ frequency bandstop. The various parameters which affect this performance are: (a) w for UIS resonator (b) w_1 (w_2 kept minimum to increase the aspect ratio of the two slot widths) and l_1 ($2l_2 + l_1 = l$ overall length kept fixed for a specific w_1/w_2 ratio) for SIS resonator I (c) w_2 (w_1 kept minimum to increase the aspect ratio of the two slot widths) and l_1 ($2l_2 + l_1 = l$ overall length kept fixed for a specific w_1/w_2 ratio) for SIS resonator II. In our work, these slot resonators are assumed to be shielded in X-band waveguide ($a=22.86\text{mm}$ and $b=10.16\text{mm}$). Front view of a UIS resonator is depicted in Figure 5.1(a). The length of this UIS resonator is kept fixed ($l=10.0\text{mm}$) whereas the width is varied from $w=1.0\text{mm}$, 3.0mm and 5.0mm . It can be inferred from Figure 5.2(a) that the fundamental resonant frequency f_r increases as the width of UIS resonator increases whereas there is only slight change in spurious harmonic resonant frequency f_s thereby decreasing the $(f_s - f_r)$ frequency bandstop.

Figure 5.1: Front view of (a) UIS resonator (b) SIS resonator I and (c) SIS resonator II

(a)

Figure 5.2: (a) Return loss of UIS resonator versus frequency for different values of w and (b) Return loss comparison of UIS resonator, SIS resonator I and SIS resonator II versus frequency for increasing the rejection bandwidth

The maximum fractional stopband width occurs when $w=1.00\text{mm}$ $[(f_s-f_r)/f_r=(32.2-15.3)\text{GHz}/15.3\text{GHz}=1.10]$. It means that the spurious harmonic resonant frequency f_s is 2.10 times the fundamental resonant frequency f_r and $(f_s-f_r=1.10\times f_r)$ is the widest possible frequency bandstop between the two resonant frequencies possible for the conventional UIS resonator. Figure 5.2(b) illustrates the return loss of the three types of slot resonators versus frequency. The dimensions of these three slot resonators are chosen such that the first resonant frequency (f_r) for all three resonators is the same at 9 GHz. It can be observed that the spurious harmonic frequency f_s is highest for SIS resonator II (31.5 GHz), lowest for SIS resonator I (22.4 GHz) and that of UIS resonator is 27.5 GHz. The performance of SIS resonator I in increasing the rejection bandwidth with respect to that of UIS resonator deteriorates; hence no further investigation of such resonators will be done. Instead we will study SIS resonator II in the next section to increase the rejection bandwidth by optimizing the various parameters.

5.3 SIS Resonator II

Let us consider SIS resonator II whose geometrical layout is shown in Figure 5.1(c). The main parameter which affects the performance for such slot resonator is w_2 and the other three parameters are kept fixed ($w_1=1.00\text{mm}$ kept minimum, $l_1=3.00\text{mm}$ and $l_2=3.50\text{mm}$). Figure 5.3(a) illustrates the return loss of these slot resonators with respect to w_2 and it can be inferred that as w_2 increases from 3.00mm, 5.00mm and 7.00mm, the fundamental resonant frequency f_r further decreases whereas there is no visible change in the spurious harmonic resonant frequency f_s and consequently increases (f_s-f_r) frequency bandstop. The maximum fractional stopband width is $[(f_s-f_r)/f_r=(31.8-9.5)\text{GHz}/9.5\text{GHz}=2.34]$ when $w_2=7.00\text{mm}$. This means that the spurious harmonic resonant frequency f_s is 3.34 times the fundamental resonant frequency f_r and such a large frequency bandstop (f_s-f_r) between the two resonant frequencies is not possible for the conventional UIS resonators.

(a)

(b)

(c)

Figure 5.3: (a) Return loss of SIS resonator II versus frequency for different values of w_2 (b) Return loss of SIS resonator II versus frequency for different values of l_1 and (c) Relationship between w_2/w_1 ratio and normalized spurious resonance frequency (f_s/f_r) for different values of l_1

The other parameter which affects performance of the SIS resonator II is l_1 (keeping

$w_1=1.00\text{mm}$, $w_2=7.00\text{mm}$ and $2l_2=10.0-l_1$ fixed). Figure 5.3(b) illustrates the return loss of these slot resonators with respect to l_1 and it can be observed that as l_1 increases from 3.00mm , 5.00mm and 7.00mm , the fundamental resonant frequency f_r decreases greatly whereas there is not much change in spurious harmonic resonant frequency f_s thereby increasing the (f_s-f_r) frequency bandstop. When $l_1=7.00\text{mm}$, maximum fractional stopband width is achieved $[(f_s-f_r)/f_r=(32.0-8.3)\text{GHz}/8.3\text{GHz}=2.85]$. This means that the spurious harmonic resonant frequency f_s is 3.85 times the fundamental resonant frequency f_r hence the (f_s-f_r) frequency bandstop between the two resonant frequencies is further enhanced. Further optimization is also possible for enhancing the maximum fractional stopband width by properly adjusting the aspect ratio of w_2 , w_1 and the dimension of l_1 . Figure 5.3(c) shows a complete full-wave characterization of SIS resonator II with respect to various parameters viz. w_2 , w_1 and l_1 in terms of normalized spurious resonant frequency (f_s/f_r) . By changing the aspect ratio of two slot widths (w_2 , w_1) and l_1 , we can also control fundamental resonant frequency, f_r , of the SIS resonator, thereby the frequency of operation of the waveguide band pass filter.

5.4 SIS Resonators Waveguide Band Pass Filters

A simplified algorithm for designing SIS resonator waveguide band pass filter is given in Figure 5.4. Using those steps outlined, a three-stage waveguide band pass filter has been optimally designed by linking three SIS resonators II with two $\lambda/4$ waveguide sections or impedance transformers as depicted in Figure 5.5(a). The periodicity p between the SIS resonators inside X-band waveguide ($a=22.86\text{mm}$, $a=10.16\text{mm}$) has been set to about $\lambda/4$ at the operation frequency. Its optimized value is 8.23mm for the chosen dimensions of the SIS resonators ($w_1=1.0\text{mm}$, $w_2=9.0\text{mm}$, $l_1=3.0\text{mm}$ and $l_2=3.5\text{mm}$). It is a hollow rectangular waveguide and finite conductivity has been neglected in our present analysis.

Figure 5.4: Simple flowchart for designing SIS resonator waveguide filters

Figure 5.5(b) is our optimized results that are obtained using the Hybrid MoM-Immittance approach [42] and are in good agreement with that of HFSS simulation. This designed waveguide band pass filter achieves a bandwidth of 23.8% for return loss $< -10\text{dB}$ at the mid-band frequency of 9.3GHz. The out-of-band filter performance has been considerably improved than conventional waveguide band pass filters in many respects. In particular, the out-of-band rejection bandwidth is increased extensively ($\sim 23\text{GHz}$) and mid-stop band attenuation is significantly reduced to less than -100dB at 19.4 GHz. Furthermore, frequency response at two sides of passband becomes much steeper than the conventional band pass filters. No finite metal thickness has been considered in the current version of this numerical technique As usual, finite metal thickness usually bring out some harmful effect on the filtering performance of narrow band pass filter with a slight shift (less than 0.5%) in the central frequency passband. But for the filter with 23.8% bandwidth that is considered here, there is no significant effect in the filtering performance from the finite metal thickness.

(c)

Figure 5.5: (a) 3-D Geometry (b) Equivalent Circuit Network and (b) Scattering parameters of the three-stage SIS resonator waveguide band pass filters

5.5 Conclusion

SIS resonators have been investigated for increasing the stopband width between the fundamental and spurious harmonic resonant frequencies. Among the two basic SIS resonators, one of them increases quantitatively the frequency stopband width whereas the other decreases the stopband width. The SIS resonator with the widened stopband has been employed for designing a novel waveguide band pass filter with good harmonic suppression. It comprises SIS resonators linked by $\lambda/4$ waveguide sections or impedance transformers. The newly proposed waveguide filter has good in-band filter response and its out-of-band characteristic has been improved considerably. In particular, a steeper and increased out-of-band rejection is observed which has been well confirmed by independent HFSS simulated results.

Chapter 6

Waveguide Based EBG Structures

6.1 Literature Review

Photonic bandgap (PBG) structures are artificially made structures with periodically loaded obstacles in 1-, 2-, or 3-D. They are capable of forbidding electromagnetic propagation in either all or selected directions to realize spatial filters in microwave and optical domains [54]-[55]. Although periodic structure has been investigated in microwave community, new ideas and concepts developed in optical domain [56]-[57] have renewed interest in microwave area. EM waves behave in such crystals similar to that of electronic behavior in semiconductors hence it is also named as Electromagnetic bandgap (EBG) structure. In microwave community, preferable nomenclature is EBG structure [58]. As in Photonic crystals (PC), photon propagation is impeded by periodic discontinuity, EM waves in EBG materials are hindered by periodic discontinuity making it a slow wave structure. The slow wave behavior in passband characteristics of EBG structures can be used as a slow wave medium for size miniaturized microwave devices and circuits. The EBG materials exhibit wide passband and stopband characteristics at microwave and millimeter-wave frequencies. Due to passband - stopband characteristics of PBG structures, they find potential applications in filters, antennas, waveguides, phased arrays and many other microwave devices. Surface waves propagating in high dielectric constant slabs carry substantial energy in unwanted directions and create unnecessary coupling between the devices. EBG structures can be used to alleviate these problems by suppressing higher order modes and surface waves [59]. In filters, EBG structures

are employed for harmonic suppression and improving filter performance without increasing dimension of the device. Since PBG structures are etched on ground plane of filter it is also called as defected ground structure [60]. In antenna design, EBG structures can be used to enhance antenna broadside gain, to suppress surface waves and to reduce cross-polarization levels [61]. On the basis of dimensions in which periodic perturbations like dielectric rods, holes and patterns in waveguides and microstrip substrates are introduced, EBG structures can be categorized as 1-D, 2-D or 3-D. Conventional PBG structures are 1-D/2-D/3-D periodic structures that satisfy Bragg's conditions, i.e, inter-cell separation (period) is close to half guided wavelength. In this chapter, an alternative waveguide based EBG structure is constructed by periodically loading rectangular waveguide with FSS strip layers printed on dielectric substrate. Efficient and accurate Hybrid MoM-Immittance Approach [42] has been used for all the simulation works.

First, a wideband waveguide based EBG structure is validated with HFSS simulation results. There is good agreement between the FEM and Hybrid MoM-Immittance results for finite periodic waveguide structure loaded with square loops in X-band rectangular waveguide. Next, we study the effect of various parameters on the EBG width for simple square FSS strip loaded periodic waveguide structures. From the parametric study of effect of various factors on the EBG width, we have designed an optimized wideband EBG structure using the FSS square strip. It has been observed that although square FSS strip loaded periodic waveguide structure gives a very broad EBG width, the roll-off characteristics in the passband of the periodic waveguide structure is not good. Hence we try to improve this performance by considering various existing FSS and novel FSS strip elements loaded waveguide structure. Seven FSS strip structures viz., square (FSS1), square loop (FSS2), ring (FSS3), cross (FSS4) and other novel FSS structures: double square loop (FSS5), FSS6 (FSS2+FSS4) and FSS7 (FSS1+FSS4) loaded waveguide has been investigated. Double square loop loaded periodic waveguide structure shows a promising candidate for improving the roll-off factors in the waveguide based EBG structures. The scattering performance for a periodic waveguide based EBG structure loaded with different number of double square loop unit/cells have been investigated.

It is observed that with the increase of number of the unit/cells, the insertion loss goes into deep rejection band and the roll-off factor in the passband improves. This study gives us a novel waveguide based EBG structure with the enhanced EBG performance. It can be used for various applications like design of harmonic suppressed waveguide filters, band rejection filters and harmonic suppression of slot antennas in the waveguide.

6.2 Waveguide Based EBG Structure Validation

A wideband waveguide based EBG structure is constructed by periodically loading square loop in waveguide at a periodicity of $p=10.2\text{ mm}$. The periodic waveguide structure is finitely extended and number of unit cells n is chosen as five. The 3-D geometry and equivalent circuit topology of the waveguide based EBG structure is shown in Figure 6.1(a) and 6.1(b) respectively. The period p is chosen approximately $\lambda_g/2$ (the Bragg's frequency). The scattering performance of the designed waveguide based EBG structure is plotted in Figure 6.1(c). It can be seen that the Hybrid MoM-Immittance Approach results compares well with HFSS generated results. There exists a wide EBG or forbidden band with $|S_{11}|$ of about 0dB in frequency region from 10.5GHz to 21.5GHz.

Figure 6.1: (a) Geometry of waveguide based finite periodic structure acting as EBG structure (b) Equivalent circuit topology and (c) Scattering parameters

6.3 Effect of various parameters on the EBG width

Figure 6.2 shows a rectangular waveguide loaded with a FSS square strip printed on a dielectric layer. The waveguide extends infinitely in both $+y$ and $-y$ directions. It has been shown truncated in both directions for illustration purpose only. In order to understand the EBG performances properly, let us investigate the effect of various parameters on the EBG. The actual waveguide based EBG structures composed of periodic waveguide structures. Here, we have considered a single unit/cell for investigation on the -10dB insertion loss EBG width so that its performance on the

overall finite periodic waveguide structures can be estimated from the single unit/cell performance. The various parameters which may control the EBG performances are:

- 1) Dimension of FSS square strip
- 2) Dielectric constant of dielectric layer on which the FSS elements are printed
- 3) Thickness of the dielectric layer
- 4) Period of the periodic waveguide structure
- 5) Number of unit/cells

The square FSS strip elements are printed on a dielectric layer of $\epsilon_r=3$ and thickness $h=1\text{mm}$. The waveguide dimension are $a=22.86\text{mm}$ and $b=10.16\text{mm}$ and the dimensions of FSS strip elements are chosen as $l=7\text{mm}$.

Figure 6.2: 3-D geometry of a rectangular waveguide ($a \times b$) loaded with a square strip (l) printed on a dielectric layer of thickness h

Dimensions of the FSS strip elements (l)

It has been observed that with the increase of dimensions for the FSS square strip from $l=7\text{mm}$, 8mm and 9mm (other parameters kept the same), there is downward shift in the resonant frequency of the FSS square strip element as illustrated in Figure

6.3. There is also significant increase in the -10dB insertion loss EBG width with the increase in the dimension of the FSS square strip (refer to Table I).

Figure 6.3: Insertion loss versus frequency for different dimension of square strip (l)

Figure 6.4: Insertion loss versus frequency for different values of relative electrical permittivity (ϵ_r)

Dielectric constant of dielectric layer on which the FSS elements are printed (ϵ_r)

As the relative permittivity (ϵ_r) of the dielectric layer on which FSS square strip are printed increases from 3, 4 and 5.7 (other parameters kept constant for all the three cases), the resonant frequency of the FSS square printed waveguide structure decreases as depicted in Figure 6.4 and the fractional EBG width increases from 24%, 27.2% and 31.25%, respectively.

a	9mm	8mm	7mm
EBG width	(18.4-8.0)/13.2	(18.2-12.2)/15.2	(18.2-14.3)/16.2
	78.78%	39.47%	24%
Relative permittivity	3	4	7
EBG width	(18.2-14.3)/16.25	(16.7-12.7)/14.7	(14.8-10.8)/12.8
	24%	27.2%	31.25%
h	1mm	2mm	3mm
EBG width	(18.2-14.3)/16.25	(16.4-13.8)/15.1	(15.8-13.8)/14.8
	24%	17.21%	13.51%
p	4.8mm	5.8mm	6.8mm
EBG width	(15.5-9.6)/12.55	(15.3-10.1)/12.7	(15.2-10.8)/13.0
	47.01%	40.94%	33.84%

Table I: -10 dB insertion loss EBG width versus various parameters

Thickness of the dielectric layer (h)

An interesting observation is that when we increase the thickness of the dielectric layer, the fractional EBG width decreases. When there is a dielectric layer of $\epsilon_r=3$ and thickness of the dielectric layer $h=1\text{mm}$, the fractional EBG width is 24% (refer to Figure 6.5 and Table I). It reduces to 13.51% as we increase the dielectric layer thickness h to 3mm, keeping same the other parameters of the waveguide structure of Figure 6.2.

Figure 6.5: Insertion loss versus frequency for different thickness of the dielectric layer (h)

Figure 6.6: Insertion loss versus frequency for different period (p)

Periodicity (p)

It is a good idea to investigate the role of periodicity p in increasing the EBG width. Figure 6.6 illustrates the bandstop of square FSS printed waveguide structure of

Figure 6.2 for various periodicity p . It can be observed that there is a downward frequency shift in bandstop as periodicity p is increased from $p=4.8\text{mm}$, 5.8mm , and 6.8mm . Besides, there is visible enhanced bandwidth for periodicity of $p=4.8\text{mm}$ in comparison to other values of p . It is because the connecting waveguide section approaches half guided-wavelength, the frequency at which each square loop resonates, thereby further widening the EBG width.

Number of Unit/cells (n)

Figure 6.7 shows insertion loss (S_{21}) for the finitely extended periodic structure with number of unit cells varying from $n=2, 3$ and 5 for a fixed periodicity $p=4.8\text{mm}$. It can be observed that as number of unit cells (n) increases, insertion loss goes into deep rejection band as mentioned in many literatures [43]. There is also slight decrease in the EBG width as the number of unit/cells increases as shown in Figure 6.7.

Figure 6.7: Insertion loss versus frequency for different number of units/cells (n)

6.4 Compact Waveguide Based EBG Structure

Figure 6.8: Geometry of the waveguide based EBG structure (a) Front view and (b) Side view (c) Scattering performance of the waveguide based EBG structure

A waveguide based EBG is constructed by periodically loading FSS square strips in rectangular waveguide whose geometry (side view) is shown in Figure 6.7(b) and its

front view is depicted in Figure 6.7(a). The thickness of the dielectric layer h is chosen as 1mm and the relative permittivity of the dielectric layer is 3. The period p is chosen approximately $\lambda_g/2$ (at the Bragg's frequency) which is equal to 4.8mm. The scattering performance of the designed EBG structure is plotted in Figure 6.7(c). It can be observed that there exists a bandgap or forbidden band with $|S_{11}|$ of about 0dB in the frequency region from 10.5GHz to 15.5GHz. There is significant reduction in the EBG width in comparison to the case of square loop FSS loaded EBG structure of Figure 6.1, it is due to the dielectric layer of thickness 1mm and relative permittivity 3 which is utilized for printing the FSS strip in the rectangular waveguide (as we have observed in Figure 6.5 that with increase of dielectric layer thickness h the EBG bandwidth decreases). But this EBG structure is more compact than the previous one due the increased slow-wave factor due to the dielectric layer. For $n=5$, the overall length of the 1-D EBG structure of Figure 6.1 is ($5 \times 10.2\text{mm} = 51\text{mm}$) whereas for the present EBG structure of Figure 6.8 the overall length along the longitudinal direction of the waveguide is just ($5 \times 4.8\text{mm} = 24\text{mm}$). But this comes at the expense of reduced EBG width.

6.5 Improving the Roll-off Characteristics

Roll-off characteristics of the previous EBG structures are not good. Let us consider various FSS strip loaded waveguide structure to improve the roll-off factor. Several FSS strip structures viz., square (FSS1), square loop (FSS2), ring (FSS3), cross (FSS4) and other novel FSS structures: double square loop (FSS5), FSS6 (FSS2+FSS4) and FSS7 (FSS1+FSS4) loaded waveguide has been investigated and their front view is depicted in Figure 6.9 (a). The insertion loss in dB for all these 7 FSS structures loaded waveguide structure is shown in Figure 6.9(b). It can be observed that the insertion loss characteristics after resonance go upward towards zero as the frequency increases whereas the insertion loss before resonance usually touches around -10dB and goes downward further as the frequency decreases. To improve the roll-off characteristics, it should go to 0dB and then should go down as

the frequency increases. The best insertion loss characteristics for improving the roll-off characteristics is for FSS 5 i.e., double square loop FSS structure.

Figure 6.9: (a) Front view of the waveguide loaded with various FSS strips and (b) Magnitude of scattering parameters

(a)

(b)

Figure 6.10: (a) Geometry of double square loop FSS printed finite periodic waveguide structure acting as EBG structures and (b) Magnitude of scattering parameters

Since the double square loop FSS strip loaded waveguide unit/cell gives the best roll-off characteristics. Let us study the effect of number of unit/cells on the EBG performance specially the roll-off characteristics. From Figure 6.10 (b), we can see that as the number of unit/cells of the finite periodic waveguide structure whose

geometry depicted in Figure 6.10 (a), the roll-off factor increases. The dimensions of the double square loops are chosen as $l_1=9\text{mm}$ and $l_2=5\text{mm}$. Both the square loops have thickness 1mm. They are transversely put in a X-band waveguide. The period p is chosen as 5.58mm and thickness of the dielectric layer of dielectric constant 3 is taken as $h=1\text{mm}$. Due to increased roll-off factors in the passband characteristics, such EBG structures are more suitable for use in harmonic suppression of filters, antennas and in design of band rejects waveguide filters.

Chapter 7

Simple Architecture for Waveguide Based DNG Metamaterials

7.1 Literature Review

Metamaterials (meta means ‘beyond’ in Greek) are new artificial materials with unusual electromagnetic properties that are not found in naturally occurring materials. All ‘natural’ materials have positive electrical permittivity, magnetic permeability and index of refraction. In these new artificially fabricated materials, all these material parameters are negative. These unusual material parameters can be harnessed for creating new generation of size-miniaturized antennas and microwave components/devices with the enhanced performance for applications in wireless communications and defense industries. In optics, every lens made until now has been limited by the wavelength of light it focuses. Using the so-called ‘beyond’ materials it is possible to construct ‘super’ lens which can focus features smaller than the wavelength of light itself. It opens up a new regime of sub-wavelength magnetic resonance imaging (MRI) for medical applications.

7.1.1 Properties

Electrical permittivity ϵ (response to electric field) and magnetic permeability μ (response to magnetic field) are the main determinants of a material’s response to electromagnetic (EM) waves. In metamaterials, both these material parameters are

negative. Correspondingly, refractive index of the metamaterials is also negative since refractive index $n = \sqrt{\epsilon\mu}$. Hence, light bends the “wrong way” when it enters from vacuum to metamaterials as illustrated in Figure 7.1. Usually, light bend towards the normal (normal to the interface shown by dotted line in Figure 7.1) when it enters from a rarer medium to a denser medium. But for metamaterials, light bends away from the normal.

Figure 7.1: Light bends the “wrong way” when it enters from vacuum to metamaterials

An immediate outcome of negative ϵ and μ in metamaterials is that when we change the sign of ϵ and μ in four Maxwell’s equations, we get a left-handed triplet of electric field **E**, magnetic field **H** and phase vector **k**. For normal materials, it is a right-handed triplet as shown in Figure 7.2. This implies that phase vector **k** and group velocity or Poynting vector **S** are in opposite direction for metamaterials. Thereby, backward waves flow in such artificial materials (i.e. wave phase appears to undulate backwards while energy propagates along the incident forward direction). The direction of phase velocity is specified by phase vector **k** whereas direction of propagation of energy or group velocity is indicated by Poynting vector **S**.

Figure 7.2: **E**, **H** and **k** form left-handed triplet in metamaterials unlike normal materials

Among other strange properties of metamaterials is reverse Doppler effect which makes a train-whistle sound higher in pitch as it approaches and lower in pitch as it recedes. Possible application for such phenomenon is in mobile wireless communications. Cerenkov radiation occurs when a charged particle moves in a material with a speed faster than that of light. Reverse Cerenkov effects are hypothesized in metamaterials which means a charged particle passing through a medium emits light from a cone behind the particle rather than in front. One of the possible applications of such unique property is in the construction of metamaterials based Cerenkov detector which could be used in particle physics to identify charged particles of various velocities [62].

7.1.2 Applications

Metamaterials were theoretically proposed by Russian physicist V. Veselago in 1968 [63]. However, experimental verifications of such materials with simultaneously negative ϵ and μ did not occur until recently. In 2001 April issue of Science, D. Smith and his colleagues [64] at the University of California at San Diego (UCSD) announced experimental verification of negative refraction from a 2-D array of repeated unit cells of copper strips and split ring resonators (SRR) on the interlocking strips of standard circuit board materials. The scattering angle measured from the transmitted beam through a prism fabricated from these metamaterial structures shows a frequency band where index of refraction is negative. Since then, researchers throughout world have been trying to demystify these novel artificial materials with unusual properties and to explore their possible applications. The first possible applications of such artificially composed metamaterials are in optics due to their negative refractive index. J. B. Pendry, a physicist at the Imperial College London, has claimed that a perfect flat lens made up of such metamaterials could focus light for objects of size lesser than the wavelength λ to a geometric point [65]. All lens made of naturally occurring materials known today cannot focus light onto an area smaller than square wavelength of the light used to examine it (the diffraction limit). For instance, atoms are smaller than wavelength of visible light

and hence can't be seen using optical microscopes. Normally evanescent waves are too weak to pass through 'normal' lenses. But with metamaterial made lenses, evanescent waves which carry object's sub-wavelength details grow their amplitude inside such lenses. At the same time, the lens corrects phase of the waves by focusing diverging waves into a beam, thereby enhancing lens resolution to a great extent. Sub-wavelength resolution or focusing beyond diffraction limit is possible with perfect flat lens made up of metamaterials. That's why such lens is also called as 'super' lens. G. V. Eleftheriades from University of Toronto has reported in March 19, 2004 that it is possible to achieve $\lambda/2$ resolution using such kind of 'super' lens [66]. Anything smaller than the wavelength of light such as atoms and molecules are out of reach for even the best available optical microscopes. But for the metamaterials based lenses, resolution is lower than the wavelength of light. Hence, such 'super' lens would let one literally see previously invisible objects like atoms in near future.

Figure 7.3: Perfect flat lens focusing light to a geometric point for objects of size less than wavelength

Another possible application is in the area of communications. It is possible to construct leaky-wave antennas with backward to forward scanning capability using such metamaterials [67]. Conventional leaky-wave antennas have drawbacks of scanning only the half-space i.e. forward scanning capability. Utilizing metamaterials, we can have the backward scanning capability. If we combine the conventional and metamaterials based leaky-wave antennas, it is possible to construct backfire-to-endfire leaky-wave antennas with capability of scanning in all

directions. Beside these examples, many researchers are trying to improve the performance of microwave, wireless communications, microelectronics and optical devices using these new metamaterials. For instance, several novel microwave devices and structures using metamaterials like a coupled-line coupler with arbitrary coupling, a compact increased-bandwidth hybrid ring, a dual-band inharmonic coupler, etc. with the enhanced performance has been proposed by T. Itoh and others [68] at the University of California, Los Angeles (UCLA).

Figure 7.4: Photograph of the first fabricated metamaterial (Courtesy of David Smith, UCSD)

7.1.3 Construction

Now, let us consider how such new materials with these unusual properties are created. The simplest way is to construct a periodic structure composed of pairs of elements, in which one element array produces a negative ϵ and the other element array produces a negative μ . In 1996, J. B. Pendry et al propounded that a periodic array of copper wires with a specific radius and spacing produces an electromagnetic response of negative ϵ materials [69] and three years later he further proposed that a periodic array of SRRs would have a frequency band where μ is negative [70]. D. Smith et al envisage that if they could somehow combine these two J. B. Pendry's proposed artificial materials, then the composite material will be metamaterials with both negative ϵ and μ , which V. Veselago hypothesized 30 years ago. So they started constructing such composite artificial materials and observe their material properties. As an outcome, the first metamaterial was fabricated by interleaving split

ring resonators (SRR) and copper wires. D. Smith and his colleagues at UCSD reported experimental verification of negative refraction in this metamaterial structure in April 2001. In such metamaterials, an array of SRR contributes to negative μ ; copper wire array contributes to negative ϵ and the combined array results in a negative permeability and permittivity material. Another metamaterial was reported in July 2003 by R. W. Ziolkowski from the University of Arizona [71]. This metamaterial comprise of a substrate with embedded capacitively loaded strips (CLSs) and square split ring resonators (SRRs). CLSs produce strong dielectric-like response thereby contributes to negative ϵ and square SRRs give strong magnetic material-like response which contributes to negative μ . And the combined composite structure of CLSs array and square SRRs array exhibits a frequency band where both ϵ , μ are negative and consequently negative index of refraction. It has been reported that metamaterials can be designed, fabricated and tested with normal microwave engineering tools. These available fabrication techniques like clean room etching, shadow mask/etching, etc. could be applied in many frequency bands. But right now, attention has been focused on Radio Frequency (RF) to 100GHz frequency range because fabrication becomes more difficult and costly as the wavelength involved gets shorter at higher frequencies. Every new discovery or invention has a controversy surrounding it. Metamaterial is no exception. P. Valanju et al at the University of Texas at Austin claims that negative refraction would violate the fundamental speed limit of light [72]. But there have been quite a few experimental results to support negative refraction as mentioned above. With such mounting evidences, researchers around the world have started accepting this new artificial material with unusual properties.

7.2 Proposed Architecture for Metamaterials

Available architectures/configurations/geometries for DNG metamaterials [73]-[77] are quite complex. Here, we propose a new much more simplified structure/architecture for DNG metamaterials in shielded waveguide environment.

X-band waveguide structure loaded with double strip layers printed on dielectric substrate shows a passband characteristic where electric permittivity, magnetic permeability and refractive index are negative. Full-wave Hybrid MoM-immittance Approach [42] has been used for extraction of material parameters like effective electric permittivity, magnetic permeability and refractive index. The scattering parameters obtained for a unit cell of the proposed DNG metamaterial architecture from the Hybrid MoM-Immittance Approach are transformed to ABCD parameters and then transformed to effective electric permittivity, magnetic permeability and refractive index as described in [78]. Figure 7.5(a) illustrates 3-D geometry of rectangular waveguide loaded with double strips printed on a dielectric layer. The dimensions for the periodic waveguide structure depicted in Figure 7.5(a) are chosen as $w=8.00\text{mm}$, $l=6.00\text{mm}$, $h=1.00\text{mm}$, $p=3.00\text{mm}$ and $\epsilon_r=7.00$. We will consider the two-port network representation of a lossless transmission line (waveguide) of length p as depicted in Figure 7.5(b) to extract the various material parameters. Figure 7.5(c) illustrates the DNG passband which is centered at the frequency of around 8.7GHz. The -5dB insertion loss bandwidth is 8.3-8.7GHz.

(c)

Figure 7.5: (a) Proposed architecture for waveguide based DNG Metamaterials (b) Two-port network representation of a lossless transmission line (waveguide) of length p of a periodic cell (c) Scattering parameters showing the DNG passband

7.3 Extraction of Material Parameters

7.3.1 Extracted Relative Permittivity

Assuming that p/λ is small, we may obtain relative permittivity from the ABCD parameters of a unit cell using the following equations [78].

$$\epsilon = \frac{C}{j\omega\epsilon_0 pA} \quad (7.1)$$

The material parameters are extracted from the ABCD-parameters using the full-wave Hybrid MoM-Immittance Approach (HMIA) and HFSS. The real parts of extracted relative permittivity vs frequency are plotted in Figure 7.6. The amplitude of real part of relative permittivity ϵ_r reaches its most positive value, i.e., 500 at 8.2 GHz, passes through zero at 8.2GHz and then reaches its most negative value, i.e., -

230 at 8.3GHz. The extremely large values of extracted material parameters are due to resonant nature of the response. There is also slight shift in the HMIA and HFSS extracted material parameters. This may be due to HFSS usually do single mode analysis. Although, only dominant TE₁₀ mode is propagating in the X-band waveguide in this frequency range. But the effect of higher order evanescent modes is also more pronounced for such kinds of periodic waveguide structures.

Figure 7.6: Extracted Effective Relative Permittivity

Here, the double strips printed on two sides of dielectric layer for a unit cell of the periodic waveguide structure mainly produce effective permeability whereas dielectric layer inside multilayered waveguide environment gives main contribution to effective permittivity.

7.3.2 Extracted Relative Permeability

For small p/λ , we may obtain the relative permeability from ABCD parameters the following equations [78].

$$\mu = \frac{B}{j\omega\mu_0 pA} \tag{7.2}$$

The amplitude of real part of the relative permeability μ_r reaches its most positive value, i.e., 11 at 8.2 GHz, passes through zero at 8.27GHz and then reaches its most negative value, i.e., -7 at 8.3GHz as shown in Figure 7.7.

Figure 7.7: Extracted Effective Relative Permeability

7.3.3 Extracted Index of Refraction

The index of refraction of the DNG metamaterials can be numerically calculated from ϵ and μ using the following formula:

$$n = \sqrt{\epsilon} \sqrt{\mu} \quad (7.3)$$

n is complex variable here. We show only real parts of extracted index of refraction for the sake of brevity. The real part of the amplitude of n reaches its most positive value, i.e., 70 at 8.2GHz, passes through zero at 8.27GHz and then reaches its most negative value, i.e., -30 at 8.3GHz as illustrated in Figure 7.8.

Figure 7.8: Extracted Index of refraction

The material parameters of a periodic waveguide structure loaded with double printed strips on dielectric substrate has been extracted and it exhibits the properties of metamaterials viz., negative electric permittivity, negative magnetic permeability, and negative index of refraction in DNG passband region.

Chapter 8

Conclusions and Future Research

8.1 Conclusions

Printed periodic waveguide structures have been explored for various applications. This thesis can be divided into three main sections: full-wave characterization, guided-wave characteristics and applications for printed periodic waveguide structures. In order to efficiently and accurately characterize such waveguide structures, a hybrid numerical method has been developed. This method is hybrid of three basic numerical methods viz., MoM, Immittance Approach and Mode-matching Method. The unknown current densities on printed strips/slots are numerically calculated using Galerkin's method. The sub-domain basis and testing functions are chosen as piecewise sinusoidal (PWS) functions. Immittance Approach has been utilized for calculating spectral dyadic Green's functions for multilayered waveguide structures from the equivalent TE/TM transmission line equivalent circuit models. The Mode-matching Method has been employed for calculation of reflection and transmission coefficients for various waveguide discontinuities. The EM program developed has been checked for numerical convergence. It has been observed that it is necessary to discretize the printed strip/slot at least 15-20 per wavelength to achieve numerical convergence and accuracy. The accuracy of the EM program has been checked in comparison to HFSS, analytical and experimental results. It shows good agreement. Besides, the present method is much faster than available commercial software like HFSS due to surface discretization. HFSS employs FEM which utilizes volumetric meshing. For instance, a waveguide filter whose simulation takes several hours using HFSS, the Hybrid MoM-Immittance Approach takes few minutes for full-wave characterization. Equivalent circuits of

printed strips and slots in shielded waveguide environment have also been extracted so that network based synthesis design can be implemented.

From the full-wave characterization of a single unit/cell, per-unit length transmission parameters or guided-wave characteristics for printed periodic waveguide structures loaded with various FSS elements has been numerically calculated. The guided-wave characteristics comprise complex propagation constant ($\gamma=\alpha+j\beta$) and complex wave impedance $\{Z=\text{Re}(Z)+j\text{Im}(Z)\}$. Periodic waveguide structures loaded with various FSS elements exhibit varied guided-wave characteristics. The possible applications of such printed periodic waveguide structures are in design of waveguide band pass filters, EBG structures, leaky wave antennas, DNG metamaterials, etc. Some specific applications of such printed periodic waveguide structures are investigated further.

For many decades, SIR filters have been used for improving the performance of waveguide and stripline band pass filters. We have extended this idea to build a new class of stepped impedance slot (SIS) resonator waveguide band pass filters. SIS resonators have been extensively investigated for maximizing the frequency bandstop between the fundamental and spurious harmonic resonant frequencies (f_s-f_r). By changing the various geometrical configurations of such SIS resonators, it is possible to increase the above-said frequency bandstop. The SIS resonator with the maximum frequency bandstop has been utilized for designing a three-stage waveguide band pass filter with the $\lambda/4$ waveguide sections or impedance transformers linking them. It has been observed that there is considerable improvement in filter out-of-band rejection performance without compromising the in-band filter performance. From the guided-wave characteristics, printed FSS strip element with the widest stopband and highest slow-wave factor has been used for designing a wideband EBG. Finally, periodic waveguide structures loaded with double printed strips on dielectric has been investigated for possible applications in design of waveguide based DNG metamaterials. It has been observed that indeed such periodic waveguide structures exhibit the properties of metamaterials like negative permeability, negative permittivity, negative refractive index and backward wave propagation in certain frequency region.

8.2 Future Research Directions

- Losses due to dielectrics and metallic portions have been neglected. This can be taken into account in the numerical analysis by considering lossy effect into the formulation of dyadic Green's functions.
- We can further investigate the printed periodic waveguide structures with fast-wave and utilize them as a new feeding line for applications in design of leaky-wave antennas.
- We have designed a wideband EBG using printed FSS elements, but such EBG is only one-dimensional. It should be better if the present analysis can be extended to 2-D or 3-D case.
- Here, we have come up with a simple architecture for waveguide based DNG metamaterials. In future, we can further investigate such kinds of printed periodic waveguide structures for widening the DNG passband with good impedance matching conditions.
- Before using our EM program for designing various microwave devices, we have validated it by comparing with HFSS, analytical and experimental results. But it would be good to fabricate the actual microwave device and see its performance.

Author's Publications

Journal Papers

1. R. S. Kshetrimayum and L. Zhu, "Novel SIS Resonators Waveguide Filters" accepted for publication in *International Journal of RF and Microwave Computer-Aided Engineering*.
2. R. S. Kshetrimayum and L. Zhu, "Guided-wave Characteristics of Waveguide Based Periodic Structures Loaded with Various FSS Strip Layers," *IEEE Transactions on Antennas and Propagation: Special Issue on Artificial Magnetic Conductors, Soft/Hard Surfaces and other Complex Surfaces*, vol. 53, no. 1, pp. 120-124, Jan. 2005.
3. R. S. Kshetrimayum, "A Brief Intro to Metamaterials", *IEEE Potentials*, vol. 23, no. 5, pp. 44-46, Dec. 2004-Jan. 2005.
4. R. S. Kshetrimayum and L. Zhu, "Hybrid MoM-Immittance approach for full-wave characterization of printed strips and slots in layered waveguide and its applications," *IEICE Transactions on Electronics: Special Section on Measurement Technologies for Microwave Materials, Devices and Circuits*, vol. E87-C, no. 5, pp. 700-707, May 2004.
5. R. S. Kshetrimayum, K. J. Vinoy and L. Zhu, "Simple Architecture for Waveguide Based DNG Metamaterials," submitting to *Applied Physics Letters*.
6. R. S. Kshetrimayum, K. J. Vinoy and L. Zhu, "Equivalent parameter extraction of double strip printed periodic waveguide structure," submitting to *IEE Electronics Letters*.

7. R. S. Kshetrimayum and L. Zhu, "EBG design using FSS elements in rectangular waveguide" submitted to *Applied Electromagnetics Society Journal*.

Conference Papers

1. R. S. Kshetrimayum and L. Zhu, "A Novel Waveguide Based Metamaterials," in *Proc. International Symposium on Antennas and Propagation*, Sendai, Japan, pp. 469-472, Aug.17-21, 2004.
2. R. S. Kshetrimayum and L. Zhu, "Equivalent Circuit Model of Planar Strips in Layered Waveguide for Synthesis Design of Printed Periodic Waveguide Structures," in *Proc. Asia Pacific Microwave Conference*, Seoul, South Korea, pp. 471-474, Nov. 4-7, 2003.
3. R. S. Kshetrimayum and L. Zhu, "Multimode Network Equivalence of Waveguide Discontinuities using full-wave Method-of-moments for Spatial Power Combining System," in *Proc. Progress in Electromagnetics Research Symposium*, Singapore, pp. 72, January 7-10, 2003.

Exhibitions/Talks

1. R. S. Kshetrimayum, "Printed Periodic Waveguide Structures," Talk at Microwave Lab, Electrical Communication Engineering, Indian Institute of Science, Bangalore, October 20, 2004.
2. R. S. Kshetrimayum, "Metamaterials – Extending Materials Properties," Lecture: Institute of Materials (East Asia), March 17, 2004.
3. R. S. Kshetrimayum, S. Sun, H Shi, R. Fu, H. Wang, J. Gao and L. Zhu, "Metamaterials – Beyond the Laws of Nature," COE Technology Week, March 8-13, 2004, Singapore.

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