On network coding in wireless cooperative and cognitive communications

Li, Qiang

2011

Li, Q. (2011). On network coding in wireless cooperative and cognitive communications.
Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/45495>

<https://doi.org/10.32657/10356/45495>

On Network Coding in Wireless Cooperative and Cognitive Communications

Li, Qiang

School of Electrical and Electronic Engineering

Nanyang Technological University

Doctoral Dissertation

2011

To my parents

Acknowledgements

I have many to thank for their help and support during the development of this thesis. Foremost among them is Professor Ting See Ho. To me, he has been an exemplary personal and professional mentor. As my supervisor, Professor Ting always had good ideas to share, and provided easy ways to start. He was always energetic and full of passion. Whenever I met with problems in my research or daily life, he was always there ready to provide valuable comments and suggestions. He taught me how to read and write in English, how to embody research problems and solve them, everything from zero with his great patience and expertise. I would like to express my deepest gratitude and appreciation to Professor Ting See Ho for what he has done for me. His trust and support have been the main source of inspiration and impetus for me.

The other important person I would like to thank is Ho Chin Keong. Throughout the past four years, he has given me valuable lessons covering wide ranges in cooperative communications, information theory, network coding and etc. This thesis would not be in its current form without his constant guidance and constructive comments.

Ashish Pandharipande has always provided valuable comments and amendments from different perspectives when I came up with some new results. I have learnt a lot from him about how to improve the finer points of technical writing and how to come up with a decent response letter to solve the reviewers' questions. I would also like to thank Professor Guan Yong Liang, the director of Positioning and Wireless Technology Center, his suggestions and comments have certainly improved my understanding of network coding.

I would also like to thank the wonderful study group that worked with me on cooperative communications, information theory, network coding, and cognitive radios, they are Han Yang, Vivek Bohara, Harya Wicaksana, and Ping Jingjing. Without the meaningful discussions with them, I cannot have a comprehensive understanding of the current research development.

There are also many individuals who have directly or indirectly contributed to this thesis. First, I am indebted to Wang Junyan who provided many useful suggestions when I was stuck in some mathematical formulations. I am also thankful to Liu Liang, Ren Tian Peng, Fu Kai, Zhao Zhi, and Che Yueling for their friendship and encouragement in the past years. I would like also to thank all the anonymous reviewers who gave generously of their time and expertise.

Last but not least, I wish to dedicate this thesis to my parents who have supported me throughout all these years. Although it is a pity that I had little time to stay with them, their unconditional love and patience gave me great power to move on.

Li Qiang

Singapore, May 2011

Abstract

Network coding, first proposed in the seminal work by Ahlswede, Cai, Li, and Yeung, has attracted significant interest in the last decade in both academia and the industry due to its versatility and promising potential in improving the performance of communication networks. In contrast to conventional routing architecture which is typical in most computer networks where intermediate nodes of a network can only store-and-forward the received packets, the intermediate nodes are allowed to perform random mapping of the received packets in network coding. With independent information flows intended for different receivers combining at the intermediate nodes, the bottleneck of data communications has shifted from point-to-point links to the whole network topology.

Although network coding was originally designed for wireline networks with error-free links, wireless networks provide a fertile ground for various applications based on network coding. Due to the broadcast nature and the opportunistic listening among the neighbouring nodes, it is possible that some nodes in the network have overheard a subset of the packets to be transmitted from previous transmissions. This will introduce more network coding opportunities and significantly increase the network throughput. Besides, since messages are combined at the intermediate nodes which are then broadcasted, multiple copies of the same message are flooded in the network, which is able to provide diversity gain and improve the robustness of the wireless network. With these promises, much attention has been given to the applications of network coding in wireless environment in recent years.

In this thesis, network coding will be investigated and evaluated in several typical network scenarios in wireless cooperative and cognitive communications, through which we hope to provide a glance into the potential applications of network coding in wireless networks. A brief introduction of

network coding will be presented in Chapter 1. A cooperative two-phase two-way relay channel will be discussed in Chapter 2 where the use of network coding at the intermediate relay is able to significantly improve the spectral efficiency. By exploiting the existing results of broadcast channels and multiple access channels, a joint network and channel coding strategy will be proposed in Chapter 3, which shows an example where joint coding is better than separate coding in wireless networks. A broadcast channel where each receiver may have some side information will be considered in Chapter 4, where we will analyze how different amount of available receiver side information affects the overall throughput when network coding is performed. In Chapter 5, network coding will be considered with a two-way relay channel where a cognitive user gains spectrum sharing by serving as a cooperative relay for the bi-directional primary transmissions. The main results presented in this thesis will be concluded in Chapter 6, and future works will be discussed.

Contents

List of Figures	xi
List of Tables	xv
Abbreviations, Notations, and Symbols	xvii
1 Introduction	1
1.1 Network Coding	2
1.1.1 Definition	2
1.1.2 A Butterfly Network	2
1.2 Cooperation and Cognition	5
1.2.1 Cooperative Communications	5
1.2.2 Cognitive Communications	8
1.3 Motivations	10
1.3.1 A two-phase two-way relay channel	10
1.3.2 Separate/Joint Network and Channel Coding	11
1.3.3 The effect of Receiver Side Information	13
1.3.4 Cooperative Spectrum Sharing with A Two-way Relay Channel .	14
1.4 Contributions and Outline of Thesis	15
2 Adaptive Cooperative Protocol for Two-Way Relay Channels	19
2.1 Introduction	20
2.1.1 A Two-Way Relay Channel	20
2.1.2 Adaptive Relay Protocols	22

CONTENTS

2.2	System Model and Protocol Descriptions	23
2.2.1	Amplify-and-Forward	24
2.2.2	Decode-and-Forward	24
2.3	Outage Analysis of Conventional Relay Protocols	26
2.3.1	Amplify-and-Forward	27
2.3.2	Decode-and-Forward	28
2.4	An Adaptive Two-Way Relay Protocol	34
2.4.1	Adaptive AF and DF Two-Way Relay Protocol	34
2.4.2	Outage Analysis	35
2.4.3	Hybrid Case	40
2.5	Simulation Results	44
2.6	Summary	48
3	Joint Network and Channel Coding	51
3.1	Introduction	52
3.1.1	Background	52
3.1.2	Our Contributions	54
3.2	System Model	57
3.3	Joint Network and Channel Coding Strategy in a Transmission Phase .	59
3.3.1	Proposed Joint Network and Channel Coding Strategy	61
3.3.2	Achievable Rate Region	63
3.3.3	Proof for Theorem 3.3.1	64
3.3.4	Discussions	69
3.4	Network Throughput Analysis	69
3.4.1	Proposed Joint Network and Channel Coding Strategy	71
3.4.2	Separate Network and Channel Coding Strategy with Random Linear Network Coding	71
3.4.3	Comparisons	73
3.4.3.1	Efficiency	73
3.4.3.2	Memory Request	73
3.4.3.3	Complexity	74
3.5	A Toy Example	74
3.5.1	Network Topology	75

3.5.2	Proposed Joint Network and Channel Coding Strategy	75
3.5.3	Separate Network and Channel Coding Strategy with Random Linear Network Coding	79
3.5.4	Simulation Results	80
3.6	A Random Network Topology	82
3.7	Summary	83
4	Wireless Network Coding with Asymmetric Receiver Side Informa- tion	85
4.1	Introduction	86
4.2	System Model	88
4.3	Network Throughput Analysis	90
4.3.1	Random Linear Network Coding	91
4.3.2	Round Robin Scheduling	92
4.4	Asymptotic Throughput at High SNR	94
4.5	Simulation Results	96
4.6	Summary	98
5	Cognitive Spectrum Sharing with Cooperative Relay Systems	101
5.1	Introduction	102
5.2	System Model	105
5.3	Protocol Descriptions and Performance Analysis	107
5.3.1	Outage Performance of Primary System	108
5.3.2	Spectrum Sharing Requirement and Critical Region	111
5.3.3	Outage Performance of Secondary System	116
5.4	Simulation Results	117
5.5	Summary	122
6	Conclusion	123
6.1	Main Contributions and Results	123
6.2	Future Works	126
6.3	Other Applications of Network Coding	127

CONTENTS

A Sample Programs	129
A.1 Matlab Codes for the Simulation of $\Pr(O_S^{\text{af}})$ in (2.15), Figure 2.7	129
A.2 Matlab Codes for the Simulation of η in (3.43), Figure 3.7	131
A.3 Matlab Codes for the Simulation of η^{rlnc} in (4.10), Figure 4.3	136
A.4 Matlab Codes for the Simulation of $\Pr(O_A^p)$ in (5.20), Figure 5.9	139
Author's Publications	143
Bibliography	145

List of Figures

1.1	Routing and network coding.	2
1.2	The butterfly network (multicast).	3
1.3	Modified butterfly network (unicast).	4
1.4	Point-to-point, multi-hop, and relay communications.	6
1.5	Network coded cooperation with a common relay.	7
1.6	Network coded cooperation without a dedicated relay.	8
1.7	Examples of cognitive radios.	9
1.8	A two-way relay channel where S and D wish to communicate with each other with the assistance of a relay R.	10
1.9	Two-way relay channels.	11
1.10	Separate network and channel coding.	12
1.11	Joint network and channel coding.	13
1.12	A broadcast channel with receiver side information.	13
1.13	A cooperative spectrum sharing scheme.	14
2.1	Two-way relay channels.	20
2.2	A two-phase two-way relay channel.	21
2.3	Illustration of the success regions for the case $P_r = P$ and $R_D = R_S$ (i.e. $\phi = \theta$ and $n = m$) when $m \geq 1$	27
2.4	Success regions for the transmissions of x_S for the case $R_D = R_S = R_0$ and $P_r \neq P$ (i.e. $n = m$ and $\phi \neq \theta$) when $m \geq 1$	31

LIST OF FIGURES

2.5	Success regions for the case $P_r = P = \frac{P_T}{3}$ and $R_D \neq R_S$ (i.e. $\phi = \theta$ and $n \neq m$) when $mn \geq 1$ and $m < n$	33
2.6	Flow chart for the proposed adaptive AF and DF two-way relay protocol.	34
2.7	Outage probability for the transmission of x_S for the case $P_r = P = \frac{P_T}{3}$ and $R_D = R_S = 1$ (i.e. $\phi = \theta$ and $n = m$).	45
2.8	Outage probability for the transmission of x_S for the case $R_D = R_S = 1$ and $P_r = \alpha_T P_T$ (i.e. $n = m$ and $\phi \neq \theta$) where $\alpha_T \in (0, 1)$ and $P = (\frac{1-\alpha_T}{2})P_T$	47
2.9	Outage probability for the transmission of x_S for the case $P_r = P = \frac{P_T}{3}$, $R_D = 1$, and $R_S \in [0.5, 3.5]$ (i.e. $\phi = \theta$ and $n \neq m$).	49
3.1	System models for SNCC and the proposed JNCC strategies.	53
3.2	Examples for the two canonical subnetworks in wireless networks.	55
3.3	A toy example for the considered multicast network.	60
3.4	The proposed JNCC strategy where message index $w_u = 1$ is transmitted, i.e. codeword $\mathbf{x}_{u,t}(1)$ is transmitted for $t = 1, 2, \dots, T$	61
3.5	An orthogonal MAC-CS-RSI at receiver v with two intended transmitters $\mathcal{T}_v = \{u_1, u_2\}$	65
3.6	Probability region for event $\mathcal{A}_{T_2}$ in (3.39).	77
3.7	Network throughput of the toy example with $M = 10$ and $R_0 = 1$	81
3.8	A random multi-hop multicast network.	83
3.9	Network throughput of the random multi-hop multicast network.	84
4.1	A wireless broadcast channel where each receiver may have some RSI about the packets to be transmitted.	86
4.2	A wireless broadcast channel where N packets are transmitted to M receivers where each receiver may have some RSI.	89
4.3	Network throughput with $\Lambda = 20$	97
4.4	Network throughput with $\Lambda = 40$	98
5.1	An interference channel.	103
5.2	A cooperative and cognitive relay channel.	105
5.3	System model of the proposed cognitive two-way relay protocol.	106
5.4	Different scenarios in the proposed CTR protocol.	108

5.5	Critical region when $R_{pt} = 1$ and $\nu = 4$	118
5.6	Network topology for simulation.	119
5.7	Outage probability for primary and secondary systems when $\rho_P = \rho_S =$ 20dB in symmetric channels where $d_{AC} = d_{BC} = 0.5$	119
5.8	Outage probability for the primary system when $\rho_P = \rho_S = 20$ dB in asymmetric channels.	120
5.9	Outage probability for primary and secondary systems under various values of ρ_S when $\alpha = 0.5$, $\rho_P = 20$ dB, and $R_{st} = R_{pt} = 1$	121

LIST OF FIGURES

List of Tables

1.1	Transmitted/Received bits in different time slots with routing.	4
1.2	Transmitted/Received bits in different time slots with network coding. .	4
2.1	Our results under various conditions.	48
4.1	Various arrival rates of RSI with the constraint $\sum_m \lambda_m = \Lambda$	96
5.1	Comparison between interweave and overlay cognitive radio models. . .	102

Abbreviations, Notations, and Symbols

ACK	acknowledgement message
AF	amplify-and-forward
ARQ	automatic repeat request
AWGN	additive white Gaussian noise
BC	broadcast channel
BC-RSI	broadcast channel with receiver side information
CCTH	cooperative clear-to-help
CCTS	cooperative clear-to-send
CDF	cumulative distribution function
CSI	channel state information
CTR	cognitive two-way relay
DF	decode-and-forward
DMC	discrete memoryless channel
DoF	degree of freedom

ABBREVIATIONS, NOTATIONS, AND SYMBOLS

DPC	dirty-paper coding
FDMA	frequency-division multiple access
JNCC	joint network and channel coding
LLR	log-likelihood ratio
MAC	multiple access channel
MAC-CS-RSI	multiple access channel with correlated sources and receiver side information
MRC	maximal-ratio combining
NACK	negative acknowledgement message
NC	network coding
PDF	probability density function
PMF	probability mass function
PT/PR	primary transmitter/primary receiver
RLNC	random linear network coding
RRS	round robin scheduling
RSI	receiver side information
SIC	successive interference cancelation
SNCC	separate network and channel coding
SNR	signal-to-noise ratio
ST/SR	secondary transmitter/secondary receiver
$\mathbf{E}$	an event $\mathbf{E}$
$\bar{\mathbf{E}}$	complementary event of event $\mathbf{E}$
$\mathcal{CN}(\cdot, \cdot)$	a circularly symmetric complex Gaussian distribution

ABBREVIATIONS, NOTATIONS, AND SYMBOLS

$\exp(\cdot)$	an exponential distribution
$K_1(\cdot)$	modified first order Bessel function of the second kind
$\mathcal{B}(\cdot, \cdot)$	a binomial distribution
$\oplus$	bit-wise XOR
$E[\cdot]$	expectation operator
h	channel coefficient
γ	channel gain
δ	average channel gain
P	transmit power
n	zero-mean additive white Gaussian noise
σ^2	variance of the additive white Gaussian noise
ρ	transmit signal-to-noise ratio
$\Pr(\cdot)$	probability
$Q(\cdot)$	Q-function
T/R _{<i>m</i>}	transmitter/receiver <i>m</i>
S/R/D	source/relay/destination
W_i	message <i>i</i>
$\mathcal{W}$	message set
P_i	packet <i>i</i>
R_i	rate of message/packet <i>i</i>
$\mathcal{S}$	receiver side information
$ \mathcal{S} $	cardinality of $\mathcal{S}$

ABBREVIATIONS, NOTATIONS, AND SYMBOLS

$\emptyset$	null set
$\mathcal{C}$	power set
$\mathcal{G}$	finite directed graph
$\mathcal{V}$	set of vertices in the finite directed graph
$\mathcal{E}$	set of edges in the finite directed graph
e_{uv}	an edge from u to v
$\mathcal{T}_v$	intended transmitters of receiver v
$\mathcal{R}_u$	intended receivers of transmitter u
I	mutual information
$I^{\text{sum}}(T)$	mutual information accumulated over T retransmissions
L_k	k th level set, transmitters in transmission phase k
V_k	intended receivers in transmission phase k
T_k	number of retransmissions required in transmission phase k
$\mathcal{A}_{T_k}$	the event that all receivers in transmission phase k finish decoding within T_k retransmissions
T_d	overall download time
η	network throughput
$A_\epsilon^{(n)}(\cdot, \cdot)$	joint typical sequences
ξ	error event in typical-set decoding
λ_m	innovative packet arrival rate at receiver R_m
$F_g(\cdot; \cdot, \cdot)$	the cumulative distribution function of a Gamma distribution
α	power allocation factor
$f_t(\cdot; \cdot)$	the probability mass function of a truncated Poisson process

ABBREVIATIONS, NOTATIONS, AND SYMBOLS

$F_t(\cdot; \cdot)$	the cumulative distribution function of a truncated Poisson process
ν	path-loss exponent
R_{pt}	target rate of primary transmission
R_{st}	target rate of secondary transmission

Introduction

As an elegant and novel technique, network coding is a relatively young technique which was first introduced at the turn of the millennium in [1]. Ever since its birth, network coding has been widely anticipated by the research community to be a revolutionary technology for future communication networks.

In this chapter, we will present a brief introduction to network coding and its applications in different wireless cooperative and cognitive communication scenarios. Currently, most computer networks based on the store-and-forward, or routing, protocol have routers deployed at intermediate nodes to switch a data packet from an input channel to an output channel without processing the data content. On the contrary, in network coding, intermediate nodes are allowed to perform arbitrary mappings of the received packets. In other words, multiple packets that are independently produced can be combined at the intermediate nodes and extracted later at the respective receivers. Combining independent data packets allows us to better tailor information flows at the network level and accommodate the demands of specific traffic patterns [87] such as multicast. With network coding, the bottleneck of data communications, used to be on the point-to-point links, has shifted to the whole network.

Since we will discuss the applications of network coding in several typical network scenarios rather than a comprehensive review of network coding from an information-theoretic perspective, the literature referred to in this thesis is by no means complete nor exhaustive. For a detailed bibliography about network coding, please refer to [12].

Figure 1.1: Routing and network coding.

1.1 Network Coding

1.1.1 Definition

The basic concept behind network coding is in fact rather straightforward. It amounts to no more than performing some arbitrary mapping on the packets passing through the intermediate nodes. As shown in Figure 1.1, to forward two packets stored at an intermediate node, a conventional routing scheme requires two sequential transmissions. However, in network coding, the operation is more flexible. The intermediate node can transmit a linear function of p_1 and p_2 , e.g. $f(p_1, p_2) = c_1 p_1 + c_2 p_2$ where c_1 and c_2 are coefficients chosen from a finite field [2], or even simpler, a bit-wise XOR where $f(p_1, p_2) = p_1 \oplus p_2$. Thus, the conventional routing scheme can be interpreted as a trivial case of network coding where $f(p_1, p_2) = p_1$ in transmission 1 and $f(p_1, p_2) = p_2$ in transmission 2.

1.1.2 A Butterfly Network

Various theoretical and empirical studies [18]-[24] suggest that significant gains can be obtained by using network coding in multi-hop networks and for serving multicast sessions.

Consider the finite directed graph shown in Figure 1.2. In this butterfly network, which was first considered in [1], two bits p_1 and p_2 originate from the source node S, and they are to be multicast to two sink nodes D_1 and D_2 via several relay nodes. Each edge is assumed to have unit capacity. Then we face a dilemma for the use of edge $R_1 \rightarrow R_2$, arising from the fact that through this edge we can only send one bit per

Figure 1.2: The butterfly network (multicast).

time slot. In conventional routing scheme, a decision has to be made: either use edge $R_1 \rightarrow R_2$ to send bit p_1 , or use it to send bit p_2 . If for example p_1 is transmitted from R_1 to R_2 , then receiver D_1 will only receive p_1 , while receiver D_2 will receive both p_1 and p_2 .

However, we would like to send bits p_1 and p_2 to both receivers D_1 and D_2 simultaneously. The simple but important observation made in the seminal work [1] is that we can allow intermediate nodes in the network to process their incoming packets, and not just forward them. In particular, node R_1 can take a bit-wise XOR of p_1 and p_2 , i.e. $p_1 \oplus p_2$, and then send $p_1 \oplus p_2$ through $R_1 \rightarrow R_2$. Then with only one transmission from R_1 to R_2 , as shown in Figure 1.2, D_1 receives $\{p_1, p_1 \oplus p_2\}$ and D_2 receives $\{p_2, p_1 \oplus p_2\}$, and thus both destinations are able to retrieve both p_1 and p_2 .

We assume that at each time slot t , $t = 1, 2, \dots$, two innovative bits p_{2t-1} and p_{2t} will be transmitted from the source node S . Table 1.1 and 1.2 show the transmitted bits from S , and the received bits at D_1 and D_2 in each time slot for routing scheme and network coding, respectively. When routing is performed, as shown in Table 1.1, asymptotically three innovative bits will be received at D_1 and D_2 in each time slot, which achieves an average rate of 1.5 bits/second for the multicast to each destination. In contrast, when network coding is performed as shown in Table 1.2, asymptotically

1. INTRODUCTION

Table 1.1: Transmitted/Received bits in different time slots with routing.

	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
S	p_1, p_2	p_3, p_4	p_5, p_6	p_7, p_8	p_9, p_{10}	p_{11}, p_{12}	p_{13}, p_{14}	p_{15}, p_{16}	p_{17}, p_{18}
D ₁		p_1	p_3	p_5 p_1	p_7	p_9 p_3	p_{11}	p_{13} p_5	p_{15}
D ₂		p_2	p_4	p_6	p_8 p_2	p_{10}	p_{12} p_4	p_{14}	p_{16} p_6

Table 1.2: Transmitted/Received bits in different time slots with network coding.

	t=1	t=2	t=3	t=4	t=5	t=6	t=7	t=8	t=9
S	p_1, p_2	p_3, p_4	p_5, p_6	p_7, p_8	p_9, p_{10}	p_{11}, p_{12}	p_{13}, p_{14}	p_{15}, p_{16}	p_{17}, p_{18}
D ₁		p_1	p_3	p_5 $p_1 \oplus p_2$	p_7 $p_3 \oplus p_4$	p_9 $p_5 \oplus p_6$	p_{11} $p_7 \oplus p_8$	p_{13} $p_9 \oplus p_{10}$	p_{15} $p_{11} \oplus p_{12}$
D ₂		p_2	p_4	p_6 $p_1 \oplus p_2$	p_8 $p_3 \oplus p_4$	p_{10} $p_5 \oplus p_6$	p_{12} $p_7 \oplus p_8$	p_{14} $p_9 \oplus p_{10}$	p_{16} $p_{11} \oplus p_{12}$

four innovative bits will be received at the two destinations in each time slot, which achieves the min-cut capacity [91] of 2 bits/second for the multicast to each destination [18]-[24].

Figure 1.3: Modified butterfly network (unicast).

Figure 1.3 shows a modified butterfly network where p_1 and p_2 originate from two sources S_1 and S_2 respectively. Instead of multicast, we consider two unicast transmissions where p_1 is desired at D_2 and p_2 is desired at D_1 respectively. Again, through network coding, only one transmission is required to forward p_1 and p_2 from R_1 to R_2 , thus enhancing the achievable rate for unicast.

1.2 Cooperation and Cognition

For the examples presented in Section 1.1, network coding shows benefits in serving multicast/unicast where different information flows can be combined at the intermediate nodes such that the limit of communication is shifted from point-to-point links to the whole network. This provides insights to the wireless networks where the inherent broadcast provides a fertile ground for network coding. In wireless communications, all users co-exist in the same frequency band will affect each other. Rather than interfering with each other, different users may choose to cooperate or transmit in a cognitive manner. As shown later in Figures 1.5-1.7, several typical wireless network scenarios are discussed where the ideas of cooperation and cognition arise naturally in solving the coexistence problem among different users or communication systems. With network coding, different information flows from different users can be combined in a cooperative manner rather than otherwise interfere with each other, which is able to provide significant benefits in throughput, reliability, power efficiency, network latency, and is likely to be a key enabling technology for efficient utilization of the precious yet limited wireless spectrum.

1.2.1 Cooperative Communications

Figure 1.4 shows the communication from a single source to a single destination, i.e. a typical point-to-point communication. Cooperation is possible whenever there is at least one additional node willing to assist in the communication [51]. As shown in Figure 1.4, the simplest and oldest form of cooperation is perhaps multi-hop communications, which dates back to ancient times of smoke and fire. Provided the inherent attenuation of transmission with distance, replacing a single long-range link with a chain of short-range links is able to overcome the attenuation and improve the signal quality.

Figure 1.4: Point-to-point, multi-hop, and relay communications.

More recently, the three-terminal relay channel was introduced in [41]. As shown in Figure 1.4, in a relay channel, a relay node assists in the communications between the source and the destination. This captures the essence of user cooperation and serves as one of the primary building blocks for cooperation on a larger scale. Although the capacity of a general relay channel is not known even today, the upper bound to the capacity has been given in [43] which has a neat max flow-min cut interpretation. For more restrictive scenarios, the capacity of the degraded relay channel, where the relay always has a better observation of the source information than the destination, has been derived in [43].

Based on the relay channel, various cooperative scenarios have been studied in [13]-[16]. As shown in Figure 1.5, a network coded cooperation scenario was considered in [13], [14]. Instead of using two dedicated relays R_1 and R_2 , the two sources S_1 and S_2 can communicate to a common destination D with the assistance of a single relay R . Rather than forwarding the individual source information a and b , the relay R is able to help both sources simultaneously by forwarding a bit-wise XOR of the two sources' information, i.e. $a \oplus b$. It has been verified that with a single relay, each source

Figure 1.5: Network coded cooperation with a common relay.

continues to enjoy a diversity of two, and retrieving any two of the three transmissions $\{a, b, a \oplus b\}$ at the destination will recover both sources' information.

As shown in Figure 1.6, another coded cooperation scheme was considered in [15], [16]. Again, instead of using two dedicated relays R_1 and R_2 , the two sources S_1 and S_2 can cooperate with each other to communicate to a common destination D . Each source has its own information to transmit and at the same time serves as a relay for its partner. If the partner's information is successfully decoded, network coding is employed to combine its own information and the partner's information which is then broadcasted. In contrast to the scenario without cooperation, the deployment of network coding together with cooperation is able to provide better outage performance for both S_1 and S_2 [16].

Both examples above show the scenario where two communication links $S_1 \rightarrow D$ and $S_2 \rightarrow D$ co-exist in the same frequency band. Instead of a non-cooperative scheme where one link operates only when the other is idle, through cooperation where network coding plays a central role in combining the independent information flows, the two communication links can operate simultaneously in the same frequency band. This will provide significant benefits in improving the spectral efficiency.

Figure 1.6: Network coded cooperation without a dedicated relay.

1.2.2 Cognitive Communications

Wireless spectrum is a broadcast medium shared by all wireless devices co-exist in the same frequency band. A glance at the National Telecommunications and information administrations frequency allocation chart [150] reveals that almost all frequency bands below 3 GHz have been preassigned for specific uses to specific groups or companies. Then how to accommodate exponentially growing wireless services and devices and meanwhile maintain a sustainable usage of the wireless spectrum become a critical problem for the development of wireless communications. Cognitive radios, first proposed in [153], aims at efficiently exploiting the precious and limited wireless spectrum resources to overcome this spectrum shortage.

In conventional interweave cognitive radio models [153], the cognitive users will constantly sense the targeted frequency bands and transmit only when the licensed primary users are idle from the frequency bands. However, this scheme is limited to areas where the spectrum is not too crowded, and it always faces the problems of miss detections and collisions with primary transmissions.

In contrast to the conventional interweave model, much attention has been given recently to an overlay cognitive radio model [157] where the cognitive users are equipped with more sophisticated signal processing and coding technologies. Rather than transmitting only when the frequency band is idle, in overlay models, the cognitive users

Figure 1.7: Examples of cognitive radios.

are allowed to transmit simultaneously with the primary users in the same frequency band.

Consider the cognitive radio networks shown in Figure 1.7 where a primary communication system, i.e. primary transmitter (PT)→primary receiver (PR), and a secondary communication system, i.e. secondary transmitter (ST)→secondary receiver (SR), co-exist in the same frequency band. In conventional interweave model, to avoid causing interference to the primary system, ST transmits only when PT is not transmitting as shown in Figure 1.7(a).

In contrast, an overlay scheme was proposed in [159] where the two-sender, two-receiver cognitive radio network is defined as an interference channel [158] as shown in Figure 1.7(b) and ST is assumed to have perfect knowledge of the codewords that PT plans to transmit. This constitutes an asymmetric or unidirectional cooperation scenario where the primary codeword and the secondary codeword are combined at ST which is then transmitted simultaneously with the primary transmission. Since ST knows perfectly the primary codewords and the possible interference of the primary transmissions caused at SR, through the use of dirty-paper coding (DPC) [161], SR

1. INTRODUCTION

Figure 1.8: A two-way relay channel where S and D wish to communicate with each other with the assistance of a relay R.

sees no interference from PT. On the other hand, since ST allocates a fraction of its transmit power to help forward the primary codewords, the interference of the secondary transmissions caused at PR can be properly compensated by the cooperation from ST [157]. Thus spectrum sharing is achieved between primary and secondary users without degrading the primary performance.

1.3 Motivations

1.3.1 A two-phase two-way relay channel

From Figure 1.3, for the modified butterfly network, since both S_1 and D_1 know p_1 , both S_2 and D_2 know p_2 , and both R_1 and R_2 know p_1 and p_2 , we can merge S_1 and D_1 as a new node S, merge S_2 and D_2 as a new node D, and merge R_1 and R_2 as a new node R [86]. The modified butterfly network in Figure 1.3 thus becomes a two-way relay channel [54]-[56] as shown in Figure 1.8 where S and D communicate with each other with the assistance of a relay R in between. This setup constitutes a common scenario in satellite communications where two ground stations communicate with each other with the assistance of a satellite.

As shown in Figure 1.9(a), if a simple routing scheme is adopted by the relay, it requires altogether four time slots for an information exchange between S and D. In contrast, if network coding [54]-[56] is performed at R to combine the successfully decoded bi-directional data streams as shown in Figure 1.9(b), an information exchange requires only three time slots by taking advantage of the broadcast nature and the receiver side information. This improves the achievable throughput and helps recover the spectral efficiency loss due to half-duplex transmissions [46].

Can we do better with network coding? Owing to the broadcast nature of wireless communications, actually we can also squeeze the first two transmissions from $S \rightarrow R$

Figure 1.9: Two-way relay channels.

and $D \rightarrow R$ into the same slot which constitutes a multiple-access channel as shown in Figure 1.9(c). Then through network coding where the bi-directional transmissions are combined at the relay and then broadcasted [58]–[66], with only two slots, an information exchange can be achieved between S and D. This further improves the spectral efficiency and reduces the delay experienced by the end users S and D.

Motivated by the promising benefits in improving the spectral efficiency, we will consider a two-phase two-way relay channel in Chapter 2 where network coding is performed at the relay. Different relay protocols, i.e. amplify-and-forward (AF) and decode-and-forward (DF), are evaluated and compared. An adaptive AF and DF protocol is proposed which adaptively switches between AF and DF, thus achieving a better performance than both AF and DF over all SNR values.

1.3.2 Separate/Joint Network and Channel Coding

Through random mappings at the intermediate nodes, linear network coding [1], [2] has been proven to be able to achieve the min-cut capacity [91] for a class of wireline multicast networks with error-free links, e.g. the butterfly network shown in Figure 1.2.

When it comes to the wireless domain, most current works [9]–[11], [54]–[56], [102], [103] adopt the original mechanism of network coding in wireline networks. That is, for each link as shown in Figure 1.10, channel codes are used at the physical layer for

Figure 1.10: Separate network and channel coding.

each transmission to transform the noisy channels to erasure channels such that the network coding operations at the network layer remains the same.

However, network coding for wireless applications is not merely an extension or simple modification of the wireline case [83]. As demonstrated in [117]-[120], the separate network and channel coding in Figure 1.10 is in general suboptimal in wireless networks. In contrast, a joint design of network and physical layers as shown in Figure 1.11, which takes into account the inherent broadcast, fading, noise, and interference in wireless networks, is able to achieve better performance and at the same time maintain an efficient use of limited resources such as battery life and wireless spectrum [123], [125]-[131].

Motivated by the benefits of joint coding in wireless communications, we will propose a joint network and channel coding (JNCC) strategy for a wireless multicast network with multiple sources, relays, and destinations. To achieve tractable analysis and provide insights, we adopt a decode-and-forward relay protocol [46] where the end-to-end information flow across the network is divided into multi-hop transmissions without interfering with each other. Within each hop, the existing results of some well-studied canonical multi-user networks, namely broadcast channel (BC) [95]-[98] and multiple access channel (MAC) [106]-[110], are exploited. Network throughput will be

Figure 1.11: Joint network and channel coding.

analyzed and compared between the proposed JNCC strategy and a separate network and channel coding strategy.

1.3.3 The effect of Receiver Side Information

For the two-way relay channel shown in Figure 1.9, we can see that the desired bit at D (i.e. p_1) originates from S and the desired bit at S (i.e. p_2) originates from D. Thus instead of a routing scheme where the two bits are separately forwarded in two transmissions, relay R simply transmits a network coded bit, i.e. $p_1 \oplus p_2$. Since

Figure 1.12: A broadcast channel with receiver side information.

1. INTRODUCTION

S already knows p_1 and D already knows p_2 as receiver side information (RSI), one transmission from the relay R would be sufficient for the two receivers to extract their desired bits as shown in Figure 1.12(a).

This network coding scenario with RSI was extended to a broadcast channel where a group of M packets is broadcasted to M receivers where each receiver has already known the $M - 1$ packets that are desired at other receivers as RSI [102]. Then instead of forwarding M packets separately in M transmissions, all M packets can be combined through network coding to a single packet that is broadcasted such that every receiver can extract their desired packet in just one transmission. This achieves a significant benefit in network throughput.

However, if a receiver has less RSI than the others or even worse no RSI, e.g. if S knows p_1 as RSI but D knows nothing as shown in Figure 1.12(b), is it still beneficial to perform network coding? In Chapter 4, we will consider a broadcast channel with asymmetric RSI and analyze how different amount of RSI available at the receivers will affect the network throughput when network coding is performed. Network throughput will be analyzed and compared between network coding and a round robin scheduling where the packets are forwarded separately.

1.3.4 Cooperative Spectrum Sharing with A Two-way Relay Channel

For the overlay cognitive radio network shown in Figure 1.7(b), the assumption that ST knows perfectly the codewords that PT plans to transmit even before PT starts transmitting is unrealistic. In view of this, a causal overlay cognitive radio network was considered in [167], [168] as shown in Figure 1.13. Instead of assuming that ST has

Figure 1.13: A cooperative spectrum sharing scheme.

non-causal knowledge of the primary codewords, ST first listens to PT and attempts to decode the transmitted primary codewords. If PR cannot successfully decode the desired messages and sends back a NACK, ST comes in to serve as a relay for the primary system.

To achieve this cooperative spectrum sharing without degrading the performance of the primary system, ST allocates a fraction of its transmit power to relay the primary signals with the remaining for its own signal, such that the interference caused to the primary system due to the secondary transmission can be properly compensated by the cooperation from ST and at the same time the secondary spectrum sharing is achieved.

As a natural extension of the one-way relay channels considered in [167], [168], we will consider the scenario where two primary users wish to communicate with each other in Chapter 5. A cognitive spectrum sharing scheme with two-way relay channels will be proposed such that upon transmission failure between the primary users, a cognitive user serves as a cooperative relay to assist the bi-directional primary transmissions and at the same time gains an opportunity to transmit its own signals.

1.4 Contributions and Outline of Thesis

In the following chapters of this thesis, network coding will be investigated and evaluated in exploring the above mentioned problems in Section 1.3, which consist of several typical network scenarios in wireless cooperative and cognitive communications.

• Chapter 2

In Chapter 2, we will consider a basic building block in wireless multi-hop networks, namely a two-way relay channel where network coding is performed at the intermediate relay to assist the bi-directional communications of two terminals. The outage performance of various cooperative protocols, namely amplify-and-forward (AF), decode-and-forward (DF), and an adaptive AF and DF two-way relay protocol, will be analyzed.

As shown in Figure 1.2, in a two-way relay channel, the use of network coding allows us to effectively exploit the inherent broadcast nature of wireless channels and the receiver side information to reduce the number of transmissions required, thus enhancing the spectral efficiency and network throughput [54]-[57].

1. INTRODUCTION

Closed-form expressions for the outage probabilities of AF, DF, and the proposed adaptive AF and DF two-way relay protocol had been published in

[80] Q. Li, S. H. Ting, A. Pandharipande, and Y. Han, “Outage analysis for adaptive two-way relaying,” *Proc. IEEE PIMRC 2008*, Cannes, France, Sept. 2008.

[81] Q. Li, S. H. Ting, A. Pandharipande, Y. Han, “Adaptive two-way relaying and outage analysis,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3288-3299, Jun. 2009.

• Chapter 3

Motivated by the promise of joint design of network and physical layers, we will propose a joint network and channel coding strategy for a wireless multicast network with multiple sources, relays, and destinations in Chapter 3. To exploit the information-theoretic results of some canonical subnetworks, namely BC and MAC, we adopt a multi-hop DF relay protocol where the end-to-end information flow across the network can be divided into separate transmission phases and ARQ is adopted in each transmission phase to ensure reliable message delivery. To quantify the performance of the proposed joint network and channel coding strategy, network throughput will be obtained in closed-form expression. Our contributions have been published in

[134] Q. Li, S. H. Ting, and C. K. Ho, “Nonlinear network code for high throughput broadcasting with retransmissions,” *Proc. IEEE ISIT 2009*, Seoul, Korea, pp. 2853-2857, Jun. 2009.

[135] Q. Li, S. H. Ting, and C. K. Ho, “Joint network and channel coding for wireless networks,” *SECON Workshops 09*, Rome, Italy, Jun. 2009.

[136] Q. Li, S. H. Ting, C. K. Ho, “Joint network and channel coding for wireless networks,” *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 181-193, Jan. 2011.

The proposed joint network and channel coding strategy provides examples where joint coding outperforms separate coding in wireless networks in terms of network throughput.

- **Chapter 4**

A natural question arising from the example shown in Figure 1.2 is what if the receivers have different amount of RSI? That is, as a more general scenario, if a receiver has less RSI than the other receivers or even no RSI at all, then how to perform network coding? Is it still beneficial to perform network coding? We will attempt to answer these questions in Chapter 4 where a random linear network coding [3]-[6] will be considered.

In particular, we will consider different amount of receiver side information (RSI) available at the receivers of a broadcast channel and investigate how this will affect the overall network throughput when network coding is performed. In contrast to most current works [99]-[103], [130], [131] where the amount of RSI available at each receiver is assumed in an arbitrary manner, we model the accumulation of RSI at each receiver by a Poisson process where an event occurrence is analogous to an innovative packet arrival. With different packet arrival rates, we can thus model different amount of RSI available at each receiver. Taking into account the differing RSI available at each receiver, closed-form expressions of network throughput and asymptotic throughput at high SNR for a broadcast channel were reported in

[149] Q. Li, S. H. Ting, and C. K. Ho, “On the available receiver side information in wireless network coding,” *Proc. IEEE ICC 2011*, Kyoto, Japan, Jun. 2011.

Our results show that in the high SNR region, the random linear network coding always achieves a higher (or equal) asymptotic throughput than the round robin scheduling, no matter how much RSI is available at each receiver.

- **Chapter 5**

In Chapter 5, network coding will be applied to a wireless cooperative and cognitive communication scenario where two primary users wish to communicate with each other with the assistance of a secondary transceiver which acts as a relay, i.e. spectrum sharing is achieved in a cognitive two-way relay channel.

1. INTRODUCTION

To achieve spectrum sharing without degrading the performance of the primary system, the secondary transceiver serves as a primary relay to help forward the primary signals and at the same time transmits its own signal. Through a careful power allocation, the interference to the primary system due to the secondary transmission can be properly compensated by the cooperation from the secondary transceiver [157].

The outage probabilities for both the primary and secondary systems with the cognitive two-way relay channel were published in

[169] Q. Li, S. H. Ting, A. Pandharipande, Y. Han, “Cognitive spectrum sharing with two-way relaying systems,” *IEEE Trans. Veh. Tech.*, vol. 60, no. 3, pp. 1233-1240, Mar. 2011.

- **Chapter 6**

Conclusions will be drawn in Chapter 6. Main results and contributions of this thesis will be summarized and Future works will be discussed.

Adaptive Cooperative Protocol for Two-Way Relay Channels

In this chapter, we will consider a basic building block in wireless multi-hop networks, namely a two-way relay channel [54]-[56] where a single relay assists in the bi-directional communication of two end users. We assume that all nodes operate in half-duplex mode such that a node can either transmit or receive at an instant. In contrast to a simple arrangement of two relay channels [41]-[43] in parallel where an information exchange between the two end users requires four sequential time slots, we will consider a two-phase two-way relay channel with network coding [1] being used at the intermediate relay. This protocol is able to significantly recover the spectral efficiency loss due to the half-duplex transmissions [46].

The outage performance of various cooperative protocols will be analyzed. We will first derive closed-form expressions for the outage probabilities of conventional Amplify-and-Forward (AF) and Decode-and-Forward (DF) [46]-[48] two-way relay protocols. To tap the advantages of both AF and DF, We will propose a simple adaptive AF and DF two-way relay protocol which switches between AF and DF depending on the decodability of the two bi-directional data streams at the intermediate relay. Closed-form expressions for the outage probability will also be derived.

Our results show that the proposed adaptive AF and DF two-way relay protocol is able to adaptively switch between AF and DF, thus achieving a better outage performance than both AF and DF over all SNR values.

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Figure 2.1: Two-way relay channels.

2.1 Introduction

2.1.1 A Two-Way Relay Channel

In order to improve the performance of wireless networks, various cooperative communication models and protocols [43]-[45] have been proposed since the seminal works in [41]-[43]. As described in Chapter 1, the main principle behind cooperative communications is to have multiple users cooperate in transmitting and receiving information. Among the various proposed models and protocols, the two-way relay channel [54]-[57] has attracted significant research interest as it represents a canonical element in wireless multi-hop networks.

As shown in Figure 2.1(a), to achieve an information exchange between two end users, the two-way relay channel, which is a simple arrangement of two relay channels in parallel, requires four sequential time slots. In view of the spectral efficiency loss due to the half-duplex transmissions, a coded bi-directional relay scheme consists of three time slots was proposed in [54]-[56], where the bi-directional relaying transmissions, i.e. $R \rightarrow S$ and $R \rightarrow D$, are combined as a broadcast as shown in Figure 2.1(b), and the idea of network coding [1] was first introduced to wireless relaying.

To further improve the spectral efficiency and increase the achievable throughput, a two-phase AF scheme with over-the-air combining of signals¹ at the intermediate

¹This inherent combining of the bi-directional transmissions at the physical layer without decoding

Figure 2.2: A two-phase two-way relay channel.

relay was proposed in [60]-[63], where the signals of the first two transmissions are combined as a multiple access at the relay, and the bi-directional relaying transmissions are combined as a broadcast as shown in Figure 2.1(c). A general two-phase two-way relay scheme was proposed in [57] where spectral efficiency and sum rate of AF and DF were analyzed. It has been proven that this two-phase two-way relay channel equipped with network coding is capable of significantly recovering the spectral efficiency loss due to the pre-log factor of $\frac{1}{2}$ in the corresponding capacity equation for half-duplex relaying [46].

Based on the two-way relay channel, achievable rate regions of various relay protocols with network coding have been obtained in [67]-[75] independently. However, there is little work talking about the outage performance² of two-way relay protocols, which is a very important performance metric characterizing the reliability of the system under general channel conditions over all SNR values. Although outage performance for AF and DF relay protocols has been analyzed in [52], [53], the analyses were limited to conventional one-way relay channels.

In this chapter, we will focus on the two-phase³ two-way relay channel shown in Figure 2.2. We first derive closed-form expressions for the outage probabilities of conventional AF and DF two-way relay protocols. For AF, we adopt a different approach from [52], [53] in deriving the outage probabilities and our results are applicable to a more general scenario where the transmit powers at S, D, and R are different.

them is also known as analog network coding [58]-[66].

²In a recently published work [76], the outage performance for a two-way relay channel with two, three, and four time slots was analyzed where the sum-rate was also characterized.

³Due to the half-duplex operation, the direct link between the two end users $S \leftrightarrow D$ is naturally ignored in the two-phase two-way relay channel shown in Figure 2.2.

2.1.2 Adaptive Relay Protocols

There is a large amount of work on AF and DF [43], [46], [52], [53], [57], [77]. Basically, AF is simpler which requires lower implementation complexity at the relay node, but it is more vulnerable to noise amplification and error propagation. Whereas DF avoids noise amplification but requires more sophisticated signal processing and is limited to hard-decision output. Thus a natural question would be whether an adaptive switching between these two relay protocols achieves the best performance?

In [77], a decode-amplify-forward (DAF) strategy was proposed which combines the error correction in DF and the soft representation of data in AF. A concatenation of a soft-output decoder (DF) and an amplifier (AF) was built at the relay to forward the decoder log-likelihood ratio (LLR). The destination jointly decodes the messages from the source and the relay. Since this DAF strategy is essentially an “enhanced” AF where soft decisions are forwarded by the relay, it performs better than AF at all times but worse than DF when the channel condition is good enough. That is why a combination of DAF and DF was also proposed in [77] to achieve the best performance.

Outage analysis of AF, DF, and a hybrid AF/DF relay protocol was presented in [78]. In the hybrid AF/DF protocol, if the source message is successfully decoded at the relay, the relay will re-encode and forward it to the destination. Otherwise, the relay amplifies and forwards the messages as in the AF protocol. Only expressions for the outage probability of AF was obtained.

An adaptive forwarding strategy was introduced in [79] for wireless networks with relays. Depending on the reliability information of the received codeword through a soft-output detector, the relay is allowed to switch between AF and DF relay protocols. That is, if the received codeword only contains a few errors, it will be simply forwarded through AF, otherwise if the received codeword is severely corrupted, it will be forwarded through DF. To keep things simpler, direct link between the source and destination was not considered.

The above analyses for the adaptive relay protocols in [77]-[79] only considered one-way relay channels where closed-form expression was not obtained. This motivates us to propose an adaptive AF and DF protocol in two-way relay channel and derive the corresponding outage probabilities in closed-form expressions.

In this chapter, we extend the idea of adaptive AF and DF [77]-[79] to the two-way relay channel which switches between AF and DF to maximize the outage performance of the system. Our results show that the adaptive AF and DF two-way relay protocol, which is able to tap the advantages of AF and DF, always achieves a better outage performance than AF and DF over all SNR values.

The remainder of this chapter is organized as follows. Section 2.2 presents a brief introduction of conventional AF and DF two-way relay protocols. Then we derive their respective outage probabilities in Section 2.3. We propose a simple adaptive AF and DF protocol in Section 2.4, where its outage probabilities are also derived. Simulation results and comparisons are provided in Section 2.5. Finally, Section 2.6 summarizes the main results of this chapter.

2.2 System Model and Protocol Descriptions

As shown in Figure 2.2, with all nodes operating in half-duplex mode, a relay R assists in the bi-directional communication of two end users S and D. We divide the entire transmission process into two successive phases: transmission phase and relaying phase. In the transmission phase, both end users S and D transmit their messages to the relay R simultaneously. Then in the relaying phase, R broadcasts the received signals to both S and D after certain signal processing operations.

In Figure 2.2, x_S , x_D are the transmitted messages and R_S , R_D are the corresponding data rates from end users S and D respectively. The transmit powers at S and D are assumed to be equal and is denoted as P , the transmit power at relay R is denoted as P_r , and P_T is defined as the total transmit power such that $2P + P_r = P_T$. We assume the use of Gaussian codewords⁴ where $E[|x_S|^2] \leq P$ and $E[|x_D|^2] \leq P$. Denoting $h_{s,r}$ and $h_{d,r}$ as the channel coefficients for links S→R and D→R respectively, we assume independent block Rayleigh fading channels where $h_{s,r} \sim \mathcal{CN}(0, \delta_1^{-1})$ and $h_{d,r} \sim \mathcal{CN}(0, \delta_2^{-1})$, and δ_1^{-1} and δ_2^{-1} are the respective average channel gains. We assume that the channels are reciprocal, and we let $\gamma_{s,r} = |h_{s,r}|^2$ and $\gamma_{d,r} = |h_{d,r}|^2$, thus $\gamma_{s,r} \sim \exp(\delta_1)$ and $\gamma_{d,r} \sim \exp(\delta_2)$. The variance of zero-mean additive white Gaussian

⁴By the use of Gaussian codewords, the outage probability can be easily and clearly defined, which provides an upper bound for the practical performance.

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

noise (AWGN) n_0 is given by σ^2 , which is assumed to be equal at all receivers. Then at the end of the transmission phase, the received signal at relay R is given as

$$y_R = \sqrt{P}h_{s,r}x_S + \sqrt{P}h_{d,r}x_D + n_0.$$

Next, we will consider conventional AF and DF two-way relay protocols and the corresponding operations at different nodes.

2.2.1 Amplify-and-Forward

Upon receiving both messages from S and D, relay R simply broadcasts the received signals under the constraint of its transmit power P_r in the successive relaying phase. Assuming perfect channel state information (CSI) is available at S and D, from [75], the end-to-end achievable rates for AF two-way relay protocol are respectively given as

$$R_{s \rightarrow d}^{\text{af}} \approx \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{r,d} \gamma_{s,r}}{\beta \gamma_{r,d} + \gamma_{s,r}} \frac{P_r}{\sigma^2} \right), \quad (2.1)$$

$$R_{d \rightarrow s}^{\text{af}} \approx \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{r,d} \gamma_{s,r}}{\beta \gamma_{s,r} + \gamma_{r,d}} \frac{P_r}{\sigma^2} \right), \quad (2.2)$$

where $\beta = \frac{P+P_r}{P}$. The above approximations are obtained assuming $P \gg \sigma^2$, which have been shown to be a tight upper bound for the exact expressions [53]. Thus, the events that x_S and x_D are successfully received at D and S are respectively defined as

$$\mathbf{E}_{s \rightarrow d}^{\text{af}} = \{R_S \leq R_{s \rightarrow d}^{\text{af}}\}, \quad (2.3)$$

$$\mathbf{E}_{d \rightarrow s}^{\text{af}} = \{R_D \leq R_{d \rightarrow s}^{\text{af}}\}. \quad (2.4)$$

2.2.2 Decode-and-Forward

Upon receiving both messages from S and D, Relay R attempts to decode both x_S and x_D . If both are successfully decoded, then R will broadcast the decoded signals through network coding⁵. Otherwise, an outage will be declared.

⁵In contrast to a superposition coding [57] at the relay where the transmit power P_r is split to forward the two independent messages, in network coding [1] where the two decoded messages are combined through bit-wise XOR, both messages are forwarded with full transmit power.

2.2 System Model and Protocol Descriptions

In the transmission phase, we can first derive the achievable rates for the multiple access channels S→R and D→R as

$$R_{s \rightarrow r}^{\text{df}} = \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{s,r} P}{\gamma_{d,r} P + \sigma^2} \right), \quad (2.5)$$

$$R_{d \rightarrow r}^{\text{df}} = \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{d,r} P}{\gamma_{s,r} P + \sigma^2} \right), \quad (2.6)$$

where the bi-directional transmissions will interfere with each other. Thus when both events

$$\mathbf{E}_{s \rightarrow r}^{\text{df}} = \left\{ R_S \leq R_{s \rightarrow r}^{\text{df}} \right\}, \quad (2.7)$$

$$\mathbf{E}_{d \rightarrow r}^{\text{df}} = \left\{ R_D \leq R_{d \rightarrow r}^{\text{df}} \right\}, \quad (2.8)$$

are satisfied, both messages x_S and x_D can be directly decoded at relay R.

If only one of x_S and x_D is successfully decoded at R, R will attempt successive interference cancellation (SIC) such that the successfully decoded message can be peeled off [91] to decode the remaining message. That is, if only (2.8) (or (2.7)) is true, then the remaining message can be successfully decoded if the event

$$\mathbf{E}_{s \rightarrow r}^{\text{SIC}} = \left\{ R_S \leq R_{s \rightarrow r}^{\text{SIC}} = \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{s,r} P}{\sigma^2} \right) \right\} \quad (2.9)$$

or

$$\mathbf{E}_{d \rightarrow r}^{\text{SIC}} = \left\{ R_D \leq R_{d \rightarrow r}^{\text{SIC}} = \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{d,r} P}{\sigma^2} \right) \right\} \quad (2.10)$$

is satisfied⁶.

With both messages successfully decoded at R, in the successive relaying phase, the achievable rates for the relay channels R→D and R→S are respectively given as

$$R_{r \rightarrow d}^{\text{df}} = \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{d,r} P_r}{\sigma^2} \right),$$

$$R_{r \rightarrow s}^{\text{df}} = \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{s,r} P_r}{\sigma^2} \right).$$

⁶Here $R_{s \rightarrow r}^{\text{SIC}}$ ($R_{d \rightarrow r}^{\text{SIC}}$) is the achievable rate of channel S→R (R→S) after the cancellation of the interference from x_D (x_S).

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Therefore, the events that x_S, x_D are successfully received at D and S are respectively given as

$$\mathbf{E}_{r \rightarrow d}^{\text{df}} = \{R_S \leq R_{r \rightarrow d}^{\text{df}}\}, \quad (2.11)$$

$$\mathbf{E}_{r \rightarrow s}^{\text{df}} = \{R_D \leq R_{r \rightarrow s}^{\text{df}}\}. \quad (2.12)$$

2.3 Outage Analysis of Conventional Relay Protocols

In this section, we characterize the performance of two-way relay protocols discussed in Section 2.2 by outage events and probabilities. For ease of exposition, we let $m = 2^{2R_S} - 1$, $n = 2^{2R_D} - 1$, $\theta = \frac{\sigma^2}{P}$, $\phi = \frac{\sigma^2}{P_r}$, and denote $\gamma_{s,r} = x$ and $\gamma_{d,r} = y$. Thus the respective probabilities for the above success events in (2.1)-(2.12) are given as

$$\begin{aligned} \Pr(\mathbf{E}_{s \rightarrow d}^{\text{af}}) &\approx \Pr\left(y \geq f_1(x)\right), & \Pr(\mathbf{E}_{d \rightarrow s}^{\text{af}}) &\approx \Pr\left(y \geq f_2(x)\right), \\ \Pr(\mathbf{E}_{s \rightarrow r}^{\text{df}}) &= \Pr\left(y \leq f_3(x)\right), & \Pr(\mathbf{E}_{d \rightarrow r}^{\text{df}}) &= \Pr\left(y \geq f_4(x)\right), \\ \Pr(\mathbf{E}_{s \rightarrow r}^{\text{SIC}}) &= \Pr\left(x \geq m\theta\right), & \Pr(\mathbf{E}_{d \rightarrow r}^{\text{SIC}}) &= \Pr\left(y \geq n\theta\right), \\ \Pr(\mathbf{E}_{r \rightarrow d}^{\text{df}}) &= \Pr\left(y \geq m\phi\right), & \Pr(\mathbf{E}_{r \rightarrow s}^{\text{df}}) &= \Pr\left(x \geq n\phi\right), \end{aligned} \quad (2.13)$$

where $f_1(x) = \frac{m\phi x}{x - m\phi\beta}$, $f_2(x) = \frac{n\phi\beta x}{x - n\phi}$, $f_3(x) = \frac{x}{m} - \theta$, and $f_4(x) = n(x + \theta)$. From the expressions in (2.13), we are able to draw the corresponding graphs such that different events are represented by their corresponding regions [89]. After determining the success regions $\mathbf{E} = \{x \in Z_X, y \in Z_Y\}$ for the respective relay protocols as shown in Figure 2.3, the corresponding outage probabilities can be derived by

$$\Pr(\bar{\mathbf{E}}) = 1 - \Pr(\mathbf{E}) = 1 - \int_{Z_Y} \int_{Z_X} f_{X,Y}(x,y) dx dy, \quad (2.14)$$

where $f_{X,Y}(x,y) = \delta_1 \delta_2 e^{-(\delta_1 x + \delta_2 y)}$ denotes the joint probability density function (PDF) of x and y .

In this chapter, we will only consider the high-rate scenario where $m, n \geq 1$ (i.e. $R_S, R_D \geq \frac{1}{2}$) for the conventional DF and the adaptive AF and DF two-way relay protocols. For the low-rate scenario where $m, n < 1$ (i.e. $R_S, R_D < \frac{1}{2}$), there is a high probability of achieving successful decoding of both messages at the relay, thus no significant improvement can be achieved by the adaptive protocol [80, Figure 6]. Thus, we will omit this low-rate scenario due to space constraints.

Figure 2.3: Illustration of the success regions for the case $P_r = P$ and $R_D = R_S$ (i.e. $\phi = \theta$ and $n = m$) when $m \geq 1$.

2.3.1 Amplify-and-Forward

Theorem 2.3.1. *The outage probabilities for the transmissions of x_S and x_D for AF two-way relay protocol, assuming $P \gg \sigma^2$, are respectively given as*

$$\Pr(O_S^{\text{af}}) = \Pr(\overline{\mathbf{E}_{s \rightarrow d}^{\text{af}}}) \approx 1 - \delta_1 e^{-(\delta_2 + \delta_1 \beta)m\phi} \sqrt{\frac{4\delta_2 m^2 \phi^2 \beta}{\delta_1}} K_1\left(\sqrt{4\delta_1 \delta_2 m^2 \phi^2 \beta}\right), \quad (2.15)$$

$$\Pr(O_D^{\text{af}}) = \Pr(\overline{\mathbf{E}_{d \rightarrow s}^{\text{af}}}) \approx 1 - \delta_1 e^{-(\delta_1 + \delta_2 \beta)n\phi} \sqrt{\frac{4\delta_2 n^2 \phi^2 \beta}{\delta_1}} K_1\left(\sqrt{4\delta_1 \delta_2 n^2 \phi^2 \beta}\right), \quad (2.16)$$

where $K_1(\cdot)$ is the modified first order Bessel function [90, Eq.(8.407)] of the second kind and the approximations come from (2.1) and (2.2).

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Proof. From (2.13), after determining the success region that lies above $y = f_1(x) = \frac{m\phi x}{x-m\phi\beta}$, we have

$$\begin{aligned}
\Pr(O_S^{\text{af}}) &\approx 1 - \int_{m\phi\beta}^{\infty} \int_{f_1(x)}^{\infty} f_{X,Y}(x,y) dy dx \\
&= 1 - \int_{m\phi\beta}^{\infty} \int_{\frac{m\phi x}{x-m\phi\beta}}^{\infty} \delta_1 \delta_2 e^{-(\delta_1 x + \delta_2 y)} dy dx \\
&= 1 - \delta_1 \delta_2 \int_{m\phi\beta}^{\infty} e^{-\delta_1 x} \left[\int_{\frac{m\phi x}{x-m\phi\beta}}^{\infty} e^{-\delta_2 y} dy \right] dx \\
&= 1 - \delta_1 \int_{m\phi\beta}^{\infty} e^{-\delta_1 x} e^{\frac{-\delta_2 m\phi x}{x-m\phi\beta}} dx.
\end{aligned}$$

We let $x = t + m\phi\beta$, then we have

$$\begin{aligned}
&1 - \delta_1 \int_{m\phi\beta}^{\infty} e^{-\delta_1 x} e^{\frac{-\delta_2 m\phi x}{x-m\phi\beta}} dx \\
&= 1 - \delta_1 \int_0^{\infty} e^{-\delta_1(t+m\phi\beta)} e^{\frac{-\delta_2 m\phi(t+m\phi\beta)}{t}} dt \\
&= 1 - \delta_1 e^{-(\delta_2 + \delta_1\beta)m\phi} \int_0^{\infty} e^{-(\delta_1 t + \frac{\delta_2 m^2 \phi^2 \beta}{t})} dt \\
&= 1 - \delta_1 e^{-(\delta_2 + \delta_1\beta)m\phi} \sqrt{\frac{4\delta_2 m^2 \phi^2 \beta}{\delta_1}} K_1 \left(\sqrt{4\delta_1 \delta_2 m^2 \phi^2 \beta} \right)
\end{aligned}$$

from [90, Eq.(3.324)]. Thus we can obtain (2.15).

Similarly, $\Pr(O_D^{\text{af}})$ in (2.16) can be derived through

$$\Pr(O_D^{\text{af}}) \approx 1 - \int_{n\phi}^{\infty} \int_{f_2(x)}^{\infty} f_{X,Y}(x,y) dy dx.$$

□

Please note that the above closed-form expressions (2.15), (2.16) for the outage probabilities of AF two-way relay protocol are applicable to the case $P_r \neq P$ and $R_D \neq R_S$ (i.e. $\phi \neq \theta$ and $n \neq m$), which is more general than the results obtained in [52], [53].

2.3.2 Decode-and-Forward

In DF two-way relay channel, both messages x_S and x_D have to be successfully decoded at the relay R at the end of the transmission phase, otherwise an outage will be declared.

Then the corresponding success event is given as

$$\begin{aligned}\mathbf{E}_{trans}^{df} &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{df}) \cup (\mathbf{E}_{s \rightarrow r}^{df} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}} \cap \mathbf{E}_{d \rightarrow r}^{SIC}) \cup (\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC}) \\ &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{SIC}) \cup (\mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC}).\end{aligned}\quad (2.17)$$

Then from an end-to-end point of view, the events that x_S and x_D are successfully received at D and S are respectively given as

$$\begin{aligned}\mathbf{E}_S^{df} &= \mathbf{E}_{trans}^{df} \cap \mathbf{E}_{r \rightarrow d}^{df} \\ &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow d}^{df}) \cup (\mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow d}^{df}),\end{aligned}\quad (2.18)$$

$$\begin{aligned}\mathbf{E}_D^{df} &= \mathbf{E}_{trans}^{df} \cap \mathbf{E}_{r \rightarrow s}^{df} \\ &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow s}^{df}) \cup (\mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow s}^{df}).\end{aligned}\quad (2.19)$$

Therefore we can express the respective outage probabilities for the transmissions of x_S and x_D as

$$\begin{aligned}\Pr(O_S^{df}) &= \Pr(\overline{\mathbf{E}_S^{df}}), \\ \Pr(O_D^{df}) &= \Pr(\overline{\mathbf{E}_D^{df}}).\end{aligned}$$

For the most general case where $P_r \neq P$ and $R_D \neq R_S$, a closed-form solution is intractable. However, we are still able to observe some interesting phenomena and have insights into the DF two-way relay protocol by considering the following two special cases where $R_D = R_S = R_0$, $P_r \neq P$, and $P_r = P = \frac{P_T}{3}$, $R_D \neq R_S$.

Proposition 2.3.1. *For the case where $R_D = R_S = R_0$ and $P_r \neq P$ as shown in Figure 2.4, we have $n = m = 2^{2R_0} - 1$ and $\phi \neq \theta$. The outage probabilities for the transmissions of x_S and x_D when $m \geq 1$ (i.e. $R_0 \geq \frac{1}{2}$) are respectively given as follows:*

For $m\phi < m\theta$ as shown in Figure 2.4(a), that is $P_r \in (\frac{P_T}{3}, P_T)$, we have

$$\begin{aligned}\Pr(O_S^{df}) &= 1 - \frac{\delta_2 e^{-(\delta_1 + \delta_2 + \delta_1 m)m\theta}}{\delta_2 + \delta_1 m} - \frac{\delta_1 e^{-(\delta_1 + \delta_2 + \delta_2 m)m\theta}}{\delta_1 + \delta_2 m}, \\ \Pr(O_D^{df}) &= \Pr(O_S^{df});\end{aligned}\quad (2.20)$$

For $m\theta < m\phi < m(m+1)\theta$ as shown in Figure 2.4(b), that is $P_r \in (\frac{P_T}{2m+3}, \frac{P_T}{3})$, we

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

have

$$\begin{aligned}\Pr(O_S^{\text{df}}) &= 1 - \frac{\delta_2 e^{-\delta_1 m \theta - (\delta_2 + \delta_1 m) m \phi}}{\delta_2 + \delta_1 m} - \frac{\delta_1 e^{-(\delta_1 + \delta_2 + \delta_2 m) m \theta}}{\delta_1 + \delta_2 m}, \\ \Pr(O_D^{\text{df}}) &= 1 - \frac{\delta_1 e^{-\delta_2 m \theta - (\delta_1 + \delta_2 m) m \phi}}{\delta_1 + \delta_2 m} - \frac{\delta_2 e^{-(\delta_1 + \delta_2 + \delta_1 m) m \theta}}{\delta_2 + \delta_1 m};\end{aligned}\tag{2.21}$$

For $m\phi > m(m+1)\theta$ as shown in Figure 2.4(c), that is $P_r \in (0, \frac{P_r}{2m+3})$, we have

$$\begin{aligned}\Pr(O_S^{\text{df}}) &= 1 - \frac{\delta_2 + \delta_1 m + \delta_2 e^{-\delta_1 m^2 \phi}}{\delta_2 + \delta_1 m} e^{-(\delta_1 \theta + \delta_2 \phi) m} \\ &\quad + \frac{\delta_2 m e^{\delta_1 \theta - (\delta_1 + \delta_2 m) \phi}}{\delta_1 + \delta_2 m}, \\ \Pr(O_D^{\text{df}}) &= 1 - \frac{\delta_1 + \delta_2 m + \delta_1 e^{-\delta_2 m^2 \phi}}{\delta_1 + \delta_2 m} e^{-(\delta_1 \phi + \delta_2 \theta) m} \\ &\quad + \frac{\delta_1 m e^{\delta_2 \theta - (\delta_2 + \delta_1 m) \phi}}{\delta_2 + \delta_1 m}.\end{aligned}\tag{2.22}$$

Proof. Since we only consider the case for $m \geq 1$, $f_3(x)$ and $f_4(x)$ will not have an intersection point ($m \geq 1 \Rightarrow m \geq \frac{1}{m}$) for $x, y > 0$. From (2.18), we can determine the success regions of x_S when DF is performed, which are all the shaded areas shown in Figure 2.4. The success regions for x_D are the corresponding areas reflected about the $y = x$ line. Due to space constraints, we will not show the success regions for x_D . The respective outage probabilities of x_S and x_D can thus be derived as follows⁷:

For the case $m\phi < m\theta$ as shown in Figure 2.4(a),

$$\begin{aligned}\Pr(O_S^{\text{df}}) &= 1 - \overbrace{\int_{m\theta}^{\infty} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy}^{\Omega_{3a}} - \overbrace{\int_{m\theta}^{\infty} \int_{m(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx}^{\Gamma_{3a}}, \\ \Pr(O_D^{\text{df}}) &= 1 - \int_{m\theta}^{\infty} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy - \int_{m\theta}^{\infty} \int_{m(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx;\end{aligned}$$

⁷With a slight abuse in notation, we have indicated in Figure 2.4 the regions corresponding to the respective integrals with the variables above the overbraces. This also applies to Propositions 2.3.2, 2.4.1, and 2.4.2.

 (a) $m\phi < m\theta$.

 (b) $m\theta < m\phi < m(m+1)\theta$.

 (c) $m\phi > m(m+1)\theta$.

Figure 2.4: Success regions for the transmissions of x_S for the case $R_D = R_S = R_0$ and $P_r \neq P$ (i.e. $n = m$ and $\phi \neq \theta$) when $m \geq 1$.

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

For the case $m\theta < m\phi < m(m+1)\theta$ as shown in Figure 2.4(b),

$$\begin{aligned}\Pr(O_S^{\text{df}}) = 1 & - \overbrace{\int_{m\phi}^{\infty} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy}^{\Omega_{3b}} - \overbrace{\int_{m\theta}^{\infty} \int_{m(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx}^{\Gamma_{3b}}, \\ \Pr(O_D^{\text{df}}) = 1 & - \int_{m\theta}^{\infty} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy - \int_{m\phi}^{\infty} \int_{m(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx;\end{aligned}$$

For the case $m\phi > m(m+1)\theta$ as shown in Figure 2.4(c),

$$\begin{aligned}\Pr(O_S^{\text{df}}) = 1 & - \overbrace{\int_{m\phi}^{\infty} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy}^{\Omega_{3c}} - \overbrace{\int_{m\phi}^{\infty} \int_{\frac{y}{m}-\theta}^{\infty} f_{X,Y}(x,y) dx dy}^{\Gamma_{3c}}, \\ \Pr(O_D^{\text{df}}) = 1 & - \int_{m\phi}^{\infty} \int_{\frac{x}{m}-\theta}^{\infty} f_{X,Y}(x,y) dy dx - \int_{m\phi}^{\infty} \int_{m(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx.\end{aligned}$$

Thus we can obtain the results in Proposition 2.3.1. $\square$

Proposition 2.3.2. *For the case where $P_r = P = \frac{P_T}{3}$ and $R_D \neq R_S$ as shown in Figure 2.5, we have $\beta = 2$, $\phi = \theta$, and $n \neq m$. The outage probabilities for the transmissions of x_S and x_D when $m < n$ and $mn \geq 1$ are respectively given as*

$$\Pr(O_S^{\text{df}}) = 1 - \frac{\delta_2 e^{-(\delta_1 m + \delta_2 n + \delta_1 mn)\theta}}{\delta_2 + \delta_1 m} - \frac{\delta_1 e^{-(\delta_1 m + \delta_2 n + \delta_2 mn)\theta}}{\delta_1 + \delta_2 n}, \quad (2.23)$$

$$\Pr(O_D^{\text{df}}) = 1 - \frac{\delta_2 e^{-(\delta_1 m + \delta_2 n + \delta_1 mn)\theta}}{\delta_2 + \delta_1 m} - \frac{\delta_1 e^{-(\delta_1 + \delta_2 + \delta_2 n)n\theta}}{\delta_1 + \delta_2 n} + p_0, \quad (2.24)$$

where

$$p_0 = \frac{\delta_2 e^{-(\delta_1 m + \delta_2 n + \delta_1 mn)\theta} + \delta_1 m e^{-(\delta_1 + \delta_2 (\frac{n-m}{mn}))n\theta}}{\delta_2 + \delta_1 m} - e^{-(\delta_1 + \delta_2)n\theta}$$

exists only when the line $x = n\theta$ lies to the right of the point $G(m(n+1)\theta, n\theta)$, that is $(n-m) > mn$.

Proof. As shown in Figure 2.5, we only have to focus on the analysis when $f_3(x)$ and $f_4(x)$ do not have an intersection point ($mn \geq 1 \Rightarrow \frac{1}{m} \leq n$) for $x, y > 0$. From (2.18) and (2.19), again we can determine the respective success regions for the transmissions of x_S and x_D , which are all the shaded areas shown in Figure 2.5. Thus the respective

 (a) Success regions for the transmission of x_S .

 (b) Success regions for the transmission of x_D .

Figure 2.5: Success regions for the case $P_r = P = \frac{P_T}{3}$ and $R_D \neq R_S$ (i.e. $\phi = \theta$ and $n \neq m$) when $mn \geq 1$ and $m < n$.

outage probabilities can be derived as

$$\begin{aligned} \Pr(O_S^{\text{df}}) &= 1 - \overbrace{\int_{n\theta}^{\infty} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy}^{\Omega_{4a}} - \overbrace{\int_{m\theta}^{\infty} \int_{n(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx}^{\Gamma_{4a}}, \\ \Pr(O_D^{\text{df}}) &= 1 - \overbrace{\int_{n\theta}^{\infty} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy}^{\Omega_{4b}} - \overbrace{\int_{n\theta}^{\infty} \int_{n(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx}^{\Gamma_{4b}} + p_0, \end{aligned}$$

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Figure 2.6: Flow chart for the proposed adaptive AF and DF two-way relay protocol.

where

$$p_0 = \int_{m(n+1)\theta}^{n\theta} \int_{n\theta}^{\frac{x}{m}-\theta} f_{X,Y}(x,y) dy dx.$$

Then we can obtain the results in Proposition 2.3.2. The outage probabilities for the case where $m > n$ can be derived similarly. $\square$

2.4 An Adaptive Two-Way Relay Protocol

2.4.1 Adaptive AF and DF Two-Way Relay Protocol

We consider a natural but non-trivial extension of the adaptive switching between AF and DF [77]-[79] to the two-way relay channel in this section. The flow chart of the proposed adaptive AF and DF two-way relay protocol is shown in Figure 2.6. The adaptive selection of AF or DF does not require the explicit knowledge of channel coefficients. In the adaptive scheme, the relay R always attempts to decode both messages

x_S and x_D . When both messages are successfully decoded, with or without SIC, R will proceed to conventional DF operations. If neither of the messages is successfully decoded, R will switch to conventional AF operations. For the hybrid case where only one message is successfully decoded while the other cannot be decoded even through SIC, R will proceed to an intermediate state where the successfully decoded message is relayed through DF and the other is relayed through AF. However, for the hybrid case, we will prove that even for the most general case where $P_r \neq P$, and $R_S \neq R_D$, the AF relay of the undecoded message always falls into an outage.

2.4.2 Outage Analysis

Here we will first analyze all the other possible cases besides the hybrid case in the adaptive protocol. From Figure 2.6, the set of non-outage events at the end of the transmission phase can be expressed as

$$\begin{aligned} \mathbf{E}_{trans}^{ad} &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{df}) \cup (\mathbf{E}_{s \rightarrow r}^{df} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}} \cap \mathbf{E}_{d \rightarrow r}^{SIC}) \\ &\quad \cup (\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC}) \cup (\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}}) \\ &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{SIC}) \cup (\mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC}) \cup (\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}}). \end{aligned} \quad (2.25)$$

Then from an end-to-end point of view, the events that x_S and x_D are successfully received at D and S are respectively expressed as

$$\begin{aligned} \mathbf{E}_S^{ad} &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow d}^{df}) \cup (\mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow d}^{df}) \cup (\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}} \cap \mathbf{E}_{s \rightarrow d}^{af}), \\ \mathbf{E}_D^{ad} &= (\mathbf{E}_{s \rightarrow r}^{df} \cap \mathbf{E}_{d \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow s}^{df}) \cup (\mathbf{E}_{d \rightarrow r}^{df} \cap \mathbf{E}_{s \rightarrow r}^{SIC} \cap \mathbf{E}_{r \rightarrow s}^{df}) \cup (\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}} \cap \mathbf{E}_{d \rightarrow s}^{af}). \end{aligned}$$

Compared with the success events for conventional DF in (2.18) and (2.19), even when neither of x_S and x_D is successfully decoded at the end of the transmission phase at the relay R, R can still attempt AF operations. Therefore, the outage probabilities for the transmissions of x_S and x_D in the adaptive protocol are respectively given by

$$\begin{aligned} \Pr(O_S^{ad}) &= \Pr(\overline{\mathbf{E}_S^{ad}}) \\ &= \Pr(O_S^{df}) - \Pr(\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}} \cap \mathbf{E}_{s \rightarrow d}^{af}), \end{aligned} \quad (2.26)$$

$$\begin{aligned} \Pr(O_D^{ad}) &= \Pr(\overline{\mathbf{E}_D^{ad}}) \\ &= \Pr(O_D^{df}) - \Pr(\overline{\mathbf{E}_{s \rightarrow r}^{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{df}} \cap \mathbf{E}_{d \rightarrow s}^{af}). \end{aligned} \quad (2.27)$$

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Proposition 2.4.1. *We consider the case where $R_D = R_S = R_0$, $P_r \neq P$ and we first exclude the hybrid case. As shown in Figure 2.4, the outage probabilities for the transmissions of x_S and x_D in the adaptive two-way relay protocol are respectively given in (2.28) and (2.29) as*

$$\begin{aligned} \Pr(O_S^{\text{ad}}) &\approx \Pr(O_S^{\text{df}}) - \frac{\delta_1 m e^{\delta_2 \theta}}{\delta_2 + \delta_1 m} e^{-(\delta_1 + \frac{\delta_2}{m})X_B} + \frac{\delta_1 e^{-\delta_2 m \theta}}{\delta_1 + \delta_2 m} e^{-(\delta_1 + \delta_2 m)X_A} \\ &\quad - \delta_1 e^{-(\delta_2 + \delta_1 \beta)m\phi} \left(G(X_B - m\phi\beta) - G(X_A - m\phi\beta) \right), \end{aligned} \quad (2.28)$$

where $G(x)$ is given as

$$\begin{aligned} G(x) &= e^{-(\delta_1 a + \frac{\delta_2 m^2 \phi^2 \beta}{a})x} + \left(-\delta_1 + \frac{\delta_2 m^2 \phi^2 \beta}{a^2} \right) e^{-(\delta_1 a + \frac{\delta_2 m^2 \phi^2 \beta}{a})x} \frac{x^2 - 2ax}{2} \\ &\quad + \left\{ \left(\delta_1 - \frac{\delta_2 m^2 \phi^2 \beta}{a^2} \right)^2 - \frac{2\delta_2 m^2 \phi^2 \beta}{a^3} \right\} + e^{-(\delta_1 a + \frac{\delta_2 m^2 \phi^2 \beta}{a})x} \frac{x^3 - 3ax^2 + 3a^2x}{6}, \end{aligned}$$

the convergent point for the Taylor series expansion is chosen as $a = \frac{X_A + X_B}{2} - m\phi\beta$, and

$$\begin{aligned} X_A &= \frac{1}{2} \left((m\phi\beta + \phi - \theta) + \sqrt{(m\phi\beta + \phi - \theta)^2 + 4m\theta\phi\beta} \right), \\ X_B &= \frac{m}{2} \left((m\phi + \phi\beta + \theta) + \sqrt{(m\phi + \phi\beta + \theta)^2 - 4\theta\phi\beta} \right), \end{aligned}$$

are the abscissae of the intersection points A and B in Figure 2.4 respectively.

$$\begin{aligned} \Pr(O_D^{\text{ad}}) &\approx \Pr(O_D^{\text{df}}) - \frac{\delta_1 m e^{\delta_2 \theta}}{\delta_2 + \delta_1 m} e^{-(\delta_1 + \frac{\delta_2}{m})X_D} + \frac{\delta_1 e^{-\delta_2 m \theta}}{\delta_1 + \delta_2 m} e^{-(\delta_1 + \delta_2 m)X_C} \\ &\quad - \delta_1 e^{-(\delta_1 + \delta_2 \beta)m\phi} \left(G(X_D - m\phi) - G(X_C - m\phi) \right), \end{aligned} \quad (2.29)$$

where a in $G(x)$ is replaced by $a = \frac{X_C + X_D}{2} - m\phi$, and

$$\begin{aligned} X_C &= \frac{1}{2} \left((m\phi + \phi\beta - \theta) + \sqrt{(m\phi + \phi\beta - \theta)^2 + 4m\theta\phi} \right), \\ X_D &= \frac{m}{2} \left((m\phi\beta + \phi + \theta) + \sqrt{(m\phi\beta + \phi + \theta)^2 - 4\theta\phi} \right), \end{aligned}$$

are the abscissae of the intersection points C and D in Figure 2.4 respectively.

Proof. The difference of the success regions between conventional DF and the adaptive relay of x_S corresponds to the event $\overline{\mathbf{E}_{s \rightarrow r}^{\text{df}}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{df}}} \cap \mathbf{E}_{s \rightarrow d}^{\text{af}}$ given in (2.26), which is represented by the area enclosed by $f_3(x)$, $f_4(x)$, and arc AB as shown in Figure 2.4. The corresponding probability for this region can thus be derived as

$$\begin{aligned}
 & \Pr(\overline{\mathbf{E}_{s \rightarrow r}^{\text{df}}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{df}}} \cap \mathbf{E}_{s \rightarrow d}^{\text{af}}) \\
 &= \overbrace{\int_{X_A}^{X_B} \int_{f_1(x)}^{m(x+\theta)} f_{X,Y}(x,y) dy dx}^{\Delta} + \overbrace{\int_{X_B}^{\infty} \int_{\frac{x}{m}-\theta}^{m(x+\theta)} f_{X,Y}(x,y) dy dx}^{\Lambda}, \\
 &= \frac{\delta_1 m e^{\delta_2 \theta}}{\delta_2 + \delta_1 m} e^{-(\delta_1 + \frac{\delta_2}{m}) X_B} - \frac{\delta_1 e^{-\delta_2 m \theta}}{\delta_1 + \delta_2 m} e^{-(\delta_1 + \delta_2 m) X_A} \\
 &+ \delta_1 e^{-(\delta_2 + \delta_1 \beta) m \phi} \int_{X_A - m \phi \beta}^{X_B - m \phi \beta} e^{-(\delta_1 x + \frac{\delta_2 m^2 \phi^2 \beta}{x})} dx.
 \end{aligned} \tag{2.30}$$

We denote $f(x) = e^{-(\delta_1 x + \frac{\delta_2 m^2 \phi^2 \beta}{x})}$. Since $f(x)$ is infinitely differentiable in $[X_A - m \phi \beta, X_B - m \phi \beta]$ where both $X_A - m \phi \beta$ and $X_B - m \phi \beta$ are positive finite values, we can make use of the Taylor series expansion of $f(x)$ to obtain a closed-form expression for the integration in (2.30). Using second-order⁸ Taylor series expansion, we have

$$\begin{aligned}
 f(x) &\approx \sum_{n=0}^2 \frac{f^{(n)}(a)}{n!} (x-a)^n \\
 &= e^{-(\delta_1 a + \frac{\delta_2 m^2 \phi^2 \beta}{a})} + \left(-\delta_1 + \frac{\delta_2 m^2 \phi^2 \beta}{a^2} \right) e^{-(\delta_1 a + \frac{\delta_2 m^2 \phi^2 \beta}{a})} (x-a) \\
 &+ \left\{ \left(\delta_1 - \frac{\delta_2 m^2 \phi^2 \beta}{a^2} \right)^2 - \frac{2 \delta_2 m^2 \phi^2 \beta}{a^3} \right\} e^{-(\delta_1 a + \frac{\delta_2 m^2 \phi^2 \beta}{a})} \frac{(x-a)^2}{2}.
 \end{aligned} \tag{2.31}$$

Here $a = \frac{X_A + X_B}{2} - m \phi \beta$ is the convergent point for the Taylor series at which we have $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n = f(x)$. Substituting (2.31) into (2.30), we can thus obtain the results in (2.28).

The derivations for the outage probability $\Pr(O_D^{\text{ad}})$ can be derived in the same way. $\square$

Proposition 2.4.2. *We consider the case where $P_r = P = \frac{P_T}{3}$, $R_D \neq R_S$ and we first exclude the hybrid case. As shown in Figure 2.5, the outage probabilities for the*

⁸We may use an arbitrary order for the Taylor series approximation and still obtain a closed-form expression. However, for ease of exposition, we will limit ourselves to a second-order approximation here. This also applies to Proposition 2.4.2.

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

transmissions of x_S and x_D in the adaptive two-way relay protocol when $m < n$ and $mn \geq 1$ are respectively given in (2.32) and (2.33) as

$$\begin{aligned} \Pr(O_S^{\text{ad}}) &\approx \Pr(O_S^{\text{df}}) - \frac{\delta_1 m e^{\delta_2 \theta}}{\delta_2 + \delta_1 m} e^{-(\delta_1 + \frac{\delta_2}{m})X_B} + \frac{\delta_1 e^{-\delta_2 n \theta}}{\delta_1 + \delta_2 n} e^{-(\delta_1 + \delta_2 n)X_A} \\ &\quad - \delta_1 e^{-(2\delta_1 + \delta_2)m\theta} \left(H(X_B - 2m\theta) - H(X_A - 2m\theta) \right), \end{aligned} \quad (2.32)$$

where $H(x)$ is given as

$$\begin{aligned} H(x) &= e^{-(\delta_1 a + \frac{2\delta_2 m^2 \theta^2}{a})} x + \left(-\delta_1 + \frac{2\delta_2 m^2 \theta^2}{a^2} \right) e^{-(\delta_1 a + \frac{2\delta_2 m^2 \theta^2}{a})} \frac{x^2 - 2ax}{2} \\ &\quad + \left\{ \left(\delta_1 - \frac{2\delta_2 m^2 \theta^2}{a^2} \right)^2 - \frac{4\delta_2 m^2 \theta^2}{a^3} \right\} e^{-(\delta_1 a + \frac{2\delta_2 m^2 \theta^2}{a})} \frac{x^3 - 3ax^2 + 3a^2 x}{6}, \end{aligned}$$

the convergent point for the Taylor series expansion is chosen as $a = \frac{X_A + X_B}{2} - 2m\theta$, and

$$\begin{aligned} X_A &= \frac{\theta}{2n} \left((2mn + m - n) + \sqrt{(2mn + m - n)^2 + 8mn^2} \right), \\ X_B &= \frac{m\theta}{2} \left((m + 3) + \sqrt{m^2 + 6m + 1} \right), \end{aligned}$$

are the abscissae of the intersection points A and B in Figure 2.5 respectively.

$$\begin{aligned} \Pr(O_D^{\text{ad}}) &\approx \Pr(O_D^{\text{df}}) - \frac{\delta_1 m e^{\delta_2 \theta}}{\delta_2 + \delta_1 m} e^{-(\delta_1 + \frac{\delta_2}{m})X_D} + \frac{\delta_1 e^{-\delta_2 n \theta}}{\delta_1 + \delta_2 n} e^{-(\delta_1 + \delta_2 n)X_C} \\ &\quad - \delta_1 e^{-(\delta_1 + 2\delta_2)n\theta} \left(I(X_D - n\theta) - I(X_C - n\theta) \right), \end{aligned} \quad (2.33)$$

where $I(x)$ is given as

$$\begin{aligned} I(x) &= e^{-(\delta_1 a + \frac{2\delta_2 n^2 \theta^2}{a})} x + \left(-\delta_1 + \frac{2\delta_2 n^2 \theta^2}{a^2} \right) e^{-(\delta_1 a + \frac{2\delta_2 n^2 \theta^2}{a})} \frac{x^2 - 2ax}{2} \\ &\quad + \left\{ \left(\delta_1 - \frac{2\delta_2 n^2 \theta^2}{a^2} \right)^2 - \frac{4\delta_2 n^2 \theta^2}{a^3} \right\} e^{-(\delta_1 a + \frac{2\delta_2 n^2 \theta^2}{a})} \frac{x^3 - 3ax^2 + 3a^2 x}{6}, \end{aligned}$$

the convergent point a here is replaced by $a = \frac{X_C + X_D}{2} - n\theta$, and

$$\begin{aligned} X_C &= \frac{\theta}{2} \left((n + 1) + \sqrt{n^2 + 6n + 1} \right), \\ X_D &= \frac{\theta}{2} \left((2mn + m + n) + \sqrt{(2mn + m + n)^2 - 4mn} \right). \end{aligned}$$

are the abscissae of the intersection points C and D in Figure 2.5 respectively.

Proof. The difference of the success regions between conventional DF and the adaptive relay of x_S corresponds to the event $\overline{\mathbf{E}_{s \rightarrow r}^{\text{df}}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{df}}} \cap \mathbf{E}_{s \rightarrow d}^{\text{af}}$ given in (2.26), which is represented by the area enclosed by $f_3(x)$, $f_4(x)$, and arc AB as shown in Figure 2.5. The corresponding probability for this region can thus be derived as

$$\begin{aligned}
 & \Pr(\overline{\mathbf{E}_{s \rightarrow r}^{\text{df}}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{df}}} \cap \mathbf{E}_{s \rightarrow d}^{\text{af}}) \\
 &= \overbrace{\int_{X_A}^{X_B} \int_{f_1(x)}^{n(x+\theta)} f_{X,Y}(x,y) dy dx}^{\Sigma} + \overbrace{\int_{X_B}^{\infty} \int_{\frac{x}{m}-\theta}^{n(x+\theta)} f_{X,Y}(x,y) dy dx}^{\Xi}, \\
 &= \frac{\delta_1 m e^{\delta_2 \theta}}{\delta_2 + \delta_1 m} e^{-(\delta_1 + \frac{\delta_2}{m}) X_B} - \frac{\delta_1 e^{-\delta_2 n \theta}}{\delta_1 + \delta_2 n} e^{-(\delta_1 + \delta_2 n) X_A} \\
 &+ \delta_1 e^{-(2\delta_1 + \delta_2)m\theta} \int_{X_A - 2m\theta}^{X_B - 2m\theta} e^{-(\delta_1 x + \frac{2\delta_2 m^2 \theta^2}{x})} dx.
 \end{aligned} \tag{2.34}$$

We denote $f(x) = e^{-(\delta_1 x + \frac{2\delta_2 m^2 \theta^2}{x})}$. Since $f(x)$ is infinitely differentiable in $[X_A - 2m\theta, X_B - 2m\theta]$ where both $X_A - 2m\theta$ and $X_B - 2m\theta$ are positive finite values, again we can make use of the Taylor series expansion of $f(x)$ to obtain a closed-form expression for the integration in (2.34). Using second-order Taylor series expansion, we have

$$\begin{aligned}
 f(x) &\approx \sum_{n=0}^2 \frac{f^{(n)}(a)}{n!} (x-a)^n \\
 &= e^{-(\delta_1 a + \frac{2\delta_2 m^2 \theta^2}{a})} + \left(-\delta_1 + \frac{2\delta_2 m^2 \theta^2}{a^2} \right) e^{-(\delta_1 a + \frac{2\delta_2 m^2 \theta^2}{a})} (x-a) \\
 &+ \left\{ \left(\delta_1 - \frac{2\delta_2 m^2 \theta^2}{a^2} \right)^2 - \frac{4\delta_2 m^2 \theta^2}{a^3} \right\} e^{-(\delta_1 a + \frac{2\delta_2 m^2 \theta^2}{a})} \frac{(x-a)^2}{2},
 \end{aligned} \tag{2.35}$$

Here $a = \frac{X_A + X_B}{2} - 2m\theta$ is the convergent point for the Taylor series at which we have $\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x-a)^n = f(x)$. Substituting (2.35) into (2.34), we can thus obtain the results in (2.32).

The derivations for the outage probability $\Pr(O_D^{\text{ad}})$ can be derived in the same way. $\square$

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

2.4.3 Hybrid Case

Proposition 2.4.3. *For the hybrid case which occurs at R where only one of x_S and x_D is successfully decoded and the other cannot be decoded even through SIC, the AF relay of the undecoded message always falls into an outage.*

Proof. Since the signal received at relay R at the end of the transmission phase is $y_R = \sqrt{P}h_{s,r}x_S + \sqrt{P}h_{d,r}x_D + n_0$. Assuming x_S is successfully decoded at R, the hybrid case corresponds to the event $\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{df}}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}}$ (or equivalently $\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}}$). Then before power normalization, the following signal

$$x_R = x_S + \sqrt{P}h_{d,r}x_D + n_0 \quad (2.36)$$

will be broadcast by R in the successive relaying phase.

From the structure of (2.36), we can actually deal with x_S and the remaining components separately. Denoting $\alpha_r \in (0, 1)$ as the power allocation factor, we presume $\alpha_r P_r$ is used to forward x_S and $(1 - \alpha_r)P_r$ is used to forward $\sqrt{P}h_{d,r}x_D + n_0$. Then assuming $P_r \gg \sigma^2$, the power normalization factors can be derived as

$$\begin{aligned} g_1 &= \sqrt{\frac{\alpha_r P_r}{1}} = \sqrt{\alpha_r P_r}, \\ g_2 &= \sqrt{\frac{(1 - \alpha_r)P_r}{P\gamma_{d,r} + \sigma^2}} \approx \sqrt{\frac{(1 - \alpha_r)P_r}{P\gamma_{d,r}}}. \end{aligned} \quad (2.37)$$

Thus the signals received at end users S and D at the end of the relaying phase are respectively given as

$$\begin{aligned} y_S &= g_1 h_{r,s} x_S + g_2 \sqrt{P} h_{d,r} h_{r,s} x_D + g_2 h_{r,s} n_0 + n_0, \\ y_D &= g_1 h_{r,d} x_S + g_2 \sqrt{P} h_{d,r} h_{r,d} x_D + g_2 h_{r,d} n_0 + n_0. \end{aligned}$$

At S and D, the interference due to its own transmitted message can be canceled away to obtain the effectively received signal as

$$y'_S = g_2 \sqrt{P} h_{d,r} h_{r,s} x_D + g_2 h_{r,s} n_0 + n_0, \quad (2.38)$$

$$y'_D = g_1 h_{r,d} x_S + g_2 h_{r,d} n_0 + n_0. \quad (2.39)$$

Using (2.37), the achievable rates for links R→D and D→S can be respectively derived

as

$$\begin{aligned} R_{r \rightarrow d}^{\text{df}'} &= \frac{1}{2} \log_2 \left(1 + \frac{g_1^2 \gamma_{d,r}}{(g_2^2 \gamma_{d,r} + 1) \sigma^2} \right) \\ &\approx \frac{1}{2} \log_2 \left(1 + \frac{\alpha_r P_r P \gamma_{d,r}}{(1 - \alpha_r) P_r + P} \frac{1}{\sigma^2} \right), \end{aligned} \quad (2.40)$$

$$\begin{aligned} R_{d \rightarrow s}^{\text{af}'} &= \frac{1}{2} \log_2 \left(1 + \frac{g_2^2 P \gamma_{s,r} \gamma_{d,r}}{(g_2^2 \gamma_{s,r} + 1) \sigma^2} \right) \\ &\approx \frac{1}{2} \log_2 \left(1 + \frac{(1 - \alpha_r) P_r \gamma_{s,r} P \gamma_{d,r}}{(1 - \alpha_r) P_r \gamma_{s,r} + P \gamma_{d,r}} \frac{1}{\sigma^2} \right). \end{aligned} \quad (2.41)$$

Then the events that x_S and x_D are successfully received in the relaying phase are respectively given as

$$\begin{aligned} \mathbf{E}_{r \rightarrow d}^{\text{df}'} &= \left\{ R_S \leq R_{r \rightarrow d}^{\text{df}'} \right\}, \\ \mathbf{E}_{d \rightarrow s}^{\text{af}'} &= \left\{ R_D \leq R_{d \rightarrow s}^{\text{af}'} \right\}. \end{aligned}$$

When considering the AF relay of the undecoded data stream x_D , which corresponds to event

$$\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{d \rightarrow s}^{\text{af}'},$$

because

$$\begin{aligned} R_{d \rightarrow s}^{\text{af}'} &\approx \frac{1}{2} \log_2 \left(1 + \frac{(1 - \alpha_r) P_r \gamma_{s,r} P \gamma_{d,r}}{(1 - \alpha_r) P_r \gamma_{s,r} + P \gamma_{d,r}} \frac{1}{\sigma^2} \right) \\ &< \frac{1}{2} \log_2 \left(1 + \frac{\gamma_{d,r} P}{\sigma^2} \right) = R_{d \rightarrow r}^{\text{SIC}}, \end{aligned} \quad (2.42)$$

it is clear that events $\mathbf{E}_{d \rightarrow s}^{\text{af}'} = \left\{ R_D \leq R_{d \rightarrow s}^{\text{af}'} \right\}$ and $\overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} = \left\{ R_D > R_{d \rightarrow r}^{\text{SIC}} \right\}$ are mutually exclusive events. Thus we have $\overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{d \rightarrow s}^{\text{af}'} = \emptyset$ and $\Pr(\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{d \rightarrow s}^{\text{af}'}) = 0$.

The above derivations hold for the most general case where $P_r \neq P$, and $R_S \neq R_D$. Therefore, the AF relay of the undecoded message always falls into an outage in the hybrid case. $\square$

Remark 2.4.1. From the analysis above, regardless of the value for α_r , the undecoded message at R always fails to be successfully relayed through AF. Thus we consider the scenario where all transmit power P_r is assigned to forward the successfully decoded message in the successive relaying phase.

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Proposition 2.4.4. *Whenever the hybrid case occurs, the corresponding success probability for the uni-directional DF relay of the successfully decoded message at R should be taken into account in the overall outage probability.*

As shown in Figure 2.4(a), for the case where $R_D = R_S = R_0$ and $P_r \neq P$, when $P_r > P$, the success probabilities for the corresponding uni-directional DF relay of the successfully decoded message (x_S or x_D) are respectively given as

$$P_{H_S} = \frac{\delta_2 e^{-\delta_1 m \theta}}{\delta_2 + \delta_1 m} \left(e^{-(\delta_2 + \delta_1 m) m \phi} - e^{-(\delta_2 + \delta_1 m) m \theta} \right), \quad (2.43)$$

$$P_{H_D} = \frac{\delta_1 e^{-\delta_2 m \theta}}{\delta_1 + \delta_2 m} \left(e^{-(\delta_1 + \delta_2 m) m \phi} - e^{-(\delta_1 + \delta_2 m) m \theta} \right). \quad (2.44)$$

As shown in Figure 2.5(a), for the case where $P_r = P = \frac{P_T}{3}$ and $R_D \neq R_S$, when $R_S < R_D$ ($m < n$), the success probability for the uni-directional DF relay of x_S is given as

$$P_{H_S} = \frac{\delta_2 e^{-\delta_1 m \theta}}{\delta_2 + \delta_1 m} \left(e^{-(\delta_2 + \delta_1 m) m \theta} - e^{-(\delta_2 + \delta_1 m) n \theta} \right). \quad (2.45)$$

When $R_S > R_D$ ($m > n$), the success probability for the uni-directional DF relay of x_D is given as

$$P_{H_D} = \frac{\delta_1 e^{-\delta_2 n \theta}}{\delta_1 + \delta_2 n} \left(e^{-(\delta_1 + \delta_2 n) n \theta} - e^{-(\delta_1 + \delta_2 n) m \theta} \right). \quad (2.46)$$

The overall outage probabilities for the transmissions of x_S and x_D in the proposed adaptive AF and DF two-way relay protocol can thus be obtained as

$$\Pr(O_S^{\text{ad}'}) = \Pr(O_S^{\text{ad}}) - P_{H_S}, \quad (2.47)$$

$$\Pr(O_D^{\text{ad}'}) = \Pr(O_D^{\text{ad}}) - P_{H_D}, \quad (2.48)$$

where $\Pr(O_S^{\text{ad}})$ and $\Pr(O_D^{\text{ad}})$ can be obtained from (2.26) and (2.27) respectively.

Proof. Since x_S is successfully decoded, from (2.13), the corresponding event for this hybrid case can be expressed as $\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{df}}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}}$ (or equivalently $\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}}$). Then for x_S to be successfully received in the successive relay phase, we need to ensure that event⁹

$$\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{r \rightarrow d}^{\text{df}'} \neq \emptyset,$$

where

$$\begin{aligned} \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} &= \{y < n\theta\}, \\ \mathbf{E}_{r \rightarrow d}^{\text{df}'} &= \{y \geq m\phi\}. \end{aligned}$$

⁹We have $\mathbf{E}_{r \rightarrow d}^{\text{df}'} = \mathbf{E}_{r \rightarrow d}^{\text{df}} = \{R_S \leq R_{r \rightarrow d}^{\text{df}}\}$ because in both cases x_S is forwarded with full power P_r .

Therefore, the uni-directional DF relay of the successfully decoded message x_S succeeds with a non-zero probability only when $m\phi < n\theta$.

For the case $R_D = R_S = R_0$ and $P_r \neq P$, to ensure $m\phi < n\theta$, we have $\phi < \theta$, or equivalently $P_r > P$. Therefore, only when $P_r > P$, the uni-directional DF relay of x_S succeeds with a non-zero probability. The corresponding success region is shown in Figure 2.4(a), and the corresponding probability can be derived as

$$\begin{aligned} P_{H_S} &= \Pr(\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{r \rightarrow d}^{\text{df}'}) \\ &= \int_{m\phi}^{m\theta} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy \\ &= \frac{\delta_2 e^{-\delta_1 m\theta}}{\delta_2 + \delta_1 m} \left(e^{-(\delta_2 + \delta_1 m)m\phi} - e^{-(\delta_2 + \delta_1 m)m\theta} \right). \end{aligned} \quad (2.49)$$

For the case $P_r = P = \frac{P_T}{3}$ and $R_D \neq R_S$, to ensure $m\phi < n\theta$, we have $m < n$, or $R_S < R_D$. Therefore, only when $R_S < R_D$, the uni-directional DF relay of x_S succeeds with a non-zero probability¹⁰. The corresponding success region is shown in Figure 2.5(a), and the corresponding probability can be derived as

$$\begin{aligned} P_{H_S} &= \Pr(\mathbf{E}_{s \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{d \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{r \rightarrow d}^{\text{df}'}) \\ &= \int_{m\theta}^{n\theta} \int_{m(y+\theta)}^{\infty} f_{X,Y}(x,y) dx dy \\ &= \frac{\delta_2 e^{-\delta_1 m\theta}}{\delta_2 + \delta_1 m} \left(e^{-(\delta_2 + \delta_1 m)m\theta} - e^{-(\delta_2 + \delta_1 m)n\theta} \right). \end{aligned} \quad (2.50)$$

For the uni-directional DF relay of x_D in the hybrid case where only x_D is successfully decoded at R, which corresponds to event $\overline{\mathbf{E}_{s \rightarrow r}^{\text{df}}} \cap \mathbf{E}_{d \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{s \rightarrow r}^{\text{SIC}}}$ (or equivalently $\mathbf{E}_{d \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{s \rightarrow r}^{\text{SIC}}}$), again we need to ensure that event¹¹

$$\mathbf{E}_{d \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{s \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{r \rightarrow s}^{\text{df}'} \neq \emptyset,$$

where

$$\begin{aligned} \overline{\mathbf{E}_{s \rightarrow r}^{\text{SIC}}} &= \{x < m\theta\}, \\ \mathbf{E}_{r \rightarrow s}^{\text{df}'} &= \{x \geq n\phi\}. \end{aligned}$$

Therefore, the uni-directional DF relay of x_D succeeds with a non-zero probability only when $n\phi < m\theta$.

¹⁰This means that when x_S is successfully decoded at R in a hybrid case, if $R_S > R_D$, an outage always occurs in the successive relaying phase.

¹¹We have $\mathbf{E}_{r \rightarrow s}^{\text{df}'} = \mathbf{E}_{r \rightarrow s}^{\text{df}} = \{R_D \leq R_{r \rightarrow s}^{\text{df}}\}$ because in both cases x_D is forwarded with full power P_r .

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

For the case $R_D = R_S = R_0$ and $P_r \neq P$, to ensure $n\phi < m\theta$, we have $\phi < \theta$, or equivalently $P_r > P$. Therefore, only when $P_r > P$, the uni-directional DF relay of x_D succeeds with a non-zero probability. The corresponding success region is shown in Figure 2.4(a), and the corresponding probability can be derived as

$$\begin{aligned}
 P_{H_D} &= \Pr(\mathbf{E}_{d \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{s \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{r \rightarrow s}^{\text{df}'}) \\
 &= \int_{m\phi}^{m\theta} \int_{m(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx \\
 &= \frac{\delta_1 e^{-\delta_2 m\theta}}{\delta_1 + \delta_2 m} \left(e^{-(\delta_1 + \delta_2 m)m\phi} - e^{-(\delta_1 + \delta_2 m)m\theta} \right). \tag{2.51}
 \end{aligned}$$

For the case $P_r = P = \frac{P_T}{3}$ and $R_D \neq R_S$, to ensure $n\phi < m\theta$, we have $n < m$, or equivalently $R_D < R_S$. Therefore, only when $R_D < R_S$, the uni-directional DF relay of x_D succeeds with a non-zero probability¹². The corresponding success probability can be derived as

$$\begin{aligned}
 P_{H_D} &= \Pr(\mathbf{E}_{d \rightarrow r}^{\text{df}} \cap \overline{\mathbf{E}_{s \rightarrow r}^{\text{SIC}}} \cap \mathbf{E}_{r \rightarrow s}^{\text{df}'}) \\
 &= \int_{n\theta}^{m\theta} \int_{n(x+\theta)}^{\infty} f_{X,Y}(x,y) dy dx \\
 &= \frac{\delta_1 e^{-\delta_2 n\theta}}{\delta_1 + \delta_2 n} \left(e^{-(\delta_1 + \delta_2 n)n\theta} - e^{-(\delta_1 + \delta_2 n)m\theta} \right). \tag{2.52}
 \end{aligned}$$

□

2.5 Simulation Results

In this section, we will show some simulation results to compare the outage performance between the proposed adaptive AF and DF two-way relay protocol and the conventional non-adaptive protocols. Matlab is used in deriving all simulation results in this chapter. Given the channel parameters, i.e. δ_1 and δ_2 , each channel is randomly generated through Monte Carlo method for 50,000 times. Then we count the ratio that the channel fails to support the data rate (i.e. R_S and R_D), which is the corresponding outage probability for this channel. We will only present the outage probabilities for x_S (i.e. AF_S , DF_S , and Adapt_S), the results for x_D are analogous. A sample program for the simulation of $\Pr(O_S^{\text{af}})$ in (2.15) is provided in Appendix A.1 and the corresponding results are shown in Figure 2.7.

¹²This means that when x_D is successfully decoded at R in a hybrid case, if $R_D > R_S$, an outage always occurs in the successive relay phase.

Figure 2.7: Outage probability for the transmission of x_S for the case $P_r = P = \frac{P_T}{3}$ and $R_D = R_S = 1$ (i.e. $\phi = \theta$ and $n = m$).

For the case $P_r = P = \frac{P_T}{3}$ and $R_D = R_S = 1$ as shown in Figure 2.7, we consider symmetric channels where $\delta_1 = \delta_2 = 1$ and asymmetric channels¹³ where $\delta_1 = 10$, $\delta_2 = 1$ and $\delta_1 = 100$, $\delta_2 = 1$. It is clear that the analytical results agree well with the simulation results and the adaptive protocol outperforms both AF and DF protocols with both symmetric and asymmetric channels.

For the comparisons between symmetric and asymmetric channels, we can see that the outage performance of the adaptive and AF protocols becomes worse for the asymmetric channels where the channel $S \rightarrow R$ has a higher attenuation (from $\delta_1 = 1$ to $\delta_1 = 100$). This is reasonable as the outage performance is degraded by the poor channel $S \rightarrow R$. However, for DF, the result is actually opposite. We can see that the outage performance of DF with asymmetric channels is better than that with symmetric chan-

¹³The scenario of asymmetric channels is equivalent to the scenario where the receivers have different noise variances.

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

nels, although both will experience a floor as SNR increases. This phenomenon can be deduced from (2.5) and (2.6) as both equations indicate that the performance of DF is interference-limited. In the asymmetric channels, it is easier to first decode the message from the stronger channel and then use SIC to decode the remaining one. Thus we can observe a relatively lower floor in asymmetric channels than in symmetric channels. However, due to this error floor, DF performs worse than AF in the high SNR region. This result indicates that a simple network coding scheme in two-phase two-way relay channel where the decoded bi-directional transmissions are combined is not necessarily the best choice. The AF protocol, which corresponds to an inherent combining of the bi-directional transmissions at the physical layer without decoding them, or in other words an analog network coding scheme [58]-[66], achieves a better performance in the high SNR region. This is because in AF, the relay does not attempt to decode the source messages and thus the performance will not be limited by the capacity of the relay channels [82].

For the case $R_D = R_S = R_0$ and $P_r \neq P$ as shown in Figure 2.8, by letting $R_0 = 1$, $P_r = \alpha_T P_T$ where $\alpha_T \in (0, 1)$ and $P = (\frac{1-\alpha_T}{2})P_T$, we first consider the symmetric channels where $\delta_1 = \delta_2 = 1$ shown in Figure 2.8(a). We have drawn all the outage probabilities at different SNR values, and it is clear that the adaptive protocol always outperforms conventional non-adaptive protocols. We can also observe that the optimal power allocation at R is approximately at $\alpha_T = 0.5$ where $P_r = \frac{P_T}{2}$ and $P = \frac{P_T}{4}$. This indicates that for symmetric channels, to relay the bi-directional transmissions, relay R should be allocated with twice the power as S and D.

For the comparisons between asymmetric channels shown in Figure 2.8(b), we have drawn all the outage probabilities at $P_T/\sigma^2 = 40\text{dB}$. It is clear that the overall outage performance is degraded by the poorer channel with increasingly higher attenuation (i.e. $\delta_1 = 10$, $\delta_1 = 100$, and $\delta_1 = 1000$). However, the outage performance of DF actually improves when the channel attenuation increases from $\delta_1 = 10$ to $\delta_1 = 100$. Again this is attributed to the interference-limited characteristics of DF due to the multiple access in the transmission phase. Asymmetric channels improve the probability of successful decoding through SIC and this fact can also be observed from Figure 2.8(b) where the outage performance of DF approaches that for the adaptive protocol.

For the case $P_r = P = \frac{P_T}{3}$ and $R_D \neq R_S$ as shown in Figure 2.9, by letting $R_D = 1$ and $R_S \in [0.5, 3.5]$, we will mainly consider the symmetric channels where $\delta_1 = \delta_2 = 1$.

 (a) Comparison between symmetric channels $\delta_1 = \delta_2 = 1$.

 (b) Comparison between asymmetric channels at $P_T/\sigma^2 = 40\text{dB}$.

Figure 2.8: Outage probability for the transmission of x_S for the case $R_D = R_S = 1$ and $P_r = \alpha_T P_T$ (i.e. $n = m$ and $\phi \neq \theta$) where $\alpha_T \in (0, 1)$ and $P = (\frac{1-\alpha_T}{2})P_T$.

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Again all the outage probabilities at different SNR values are given. We can see that the overall outage performance is degraded by an increase in R_S . Due to the second-order Taylor series approximation, there is slight discrepancy between the analytical and simulation results of the adaptive scheme at 40dB. But still we can see that the adaptive protocol outperforms conventional non-adaptive protocols.

2.6 Summary

In this chapter, we have analyzed the outage performance of various cooperative protocols for a two-phase two-way relay channel where network coding is performed.

We first derived closed-form expressions for the outage probabilities of conventional AF and DF two-way relay protocols. Compared with AF, DF has poor performance in symmetric channels but good performance in asymmetric channels. This is mainly due to the interference-limited characteristics of the DF protocol.

Then we proposed an adaptive AF and DF protocol in the two-way relay channel where the relay R adaptively selects AF or DF depending on the decodability of the bi-directional transmissions. Closed-form expressions of the outage probability were also obtained. Our results show that the adaptive protocol is able to effectively tap the advantages of both AF and DF, thus achieving a better performance over all SNR values.

Table 2.1 summarizes the main results in this chapter for AF, DF, and the proposed adaptive AF and DF protocols under various conditions.

Table 2.1: Our results under various conditions.

AF	DF		Adaptive		
$P_r \neq P$ $R_S \neq R_D$	$P_r \neq P$ $R_S = R_D$	$P_r = P$ $R_S \neq R_D$	$P_r \neq P$ $R_S = R_D$	$P_r = P$ $R_S \neq R_D$	Hybrid Case
Theorem 2.3.1	Proposition 2.3.1	Proposition 2.3.2	Proposition 2.4.1	Proposition 2.4.2	Proposition 2.4.3 & 2.4.4

Figure 2.9: Outage probability for the transmission of x_S for the case $P_r = P = \frac{P_T}{3}$, $R_D = 1$, and $R_S \in [0.5, 3.5]$ (i.e. $\phi = \theta$ and $n \neq m$).

2. ADAPTIVE COOPERATIVE PROTOCOL FOR TWO-WAY RELAY CHANNELS

Joint Network and Channel Coding

In this chapter, we will propose a joint network and channel coding (JNCC) strategy [134]-[136] for a wireless multicast network with multiple sources, relays, and destinations. To exploit the existing results of some well-studied canonical multi-user networks, namely broadcast channel (BC) [95]-[98] and multiple access channel (MAC) [106]-[110], we adopt a multi-hop decode-and-forward relay [46] protocol and the end-to-end information flow across the network is divided into separate transmission phases.

As we shall show, the proposed JNCC strategy does not require the knowledge of receiver side information (RSI) nor any transmit channel state information. A simple ARQ scheme [144] will be adopted where each transmitter simply performs retransmissions from independent and identically generated codebooks. On the other hand, the intended receivers perform joint typical-set decoding [91] on the received codewords until enough mutual information is accumulated to successfully decode all transmitted messages.

To evaluate the performance of the proposed JNCC strategy with ARQ, we will derive closed-form expressions of network throughput by applying the renewal-reward theorem [142], [143]. Analytical results show that the proposed JNCC strategy outperforms, in terms of network throughput, the conventional separate network and channel coding (SNCC) strategy with random linear network coding (RLNC) [3]-[6], thus providing an example where joint coding is better than separate coding in wireless networks.

3.1 Introduction

3.1.1 Background

In the seminal work by Shannon [114], it was proven that information can be transmitted with an arbitrarily small probability of error if and only if the entropy rate of the information source is less than the capacity of the channel, and this can always be achieved asymptotically by designing source and channel codes separately. As an extension, a separation principle for network and channel coding [115] was proven in a single-source communication network with discrete memoryless channels (DMC). Originally proposed in wireline networks with error-free links, (linear) network coding [1]-[2] has been proven to be capable of achieving the min-cut capacity [91] for a class of multicast networks with a separation of network and channel coding.

As introduced in Chapter 1, wireless networks provide a fertile ground for various applications based on network coding [83] where opportunistic listening [9]-[11] between neighboring nodes brings in many network coding opportunities. However, most of the existing works directly transplanted the separation of network and channel coding from wireline networks to the wireless realm [9]-[11], [54]-[56], [102], [103]. That is, as shown in Figure 3.1(a), channel coding is performed at the physical layer for each transmission to transform the noisy channels into erasure channels [140]. Then at the network layer, network coding is performed on top of these erasure channels to combine the messages as if the transmissions are error-free. A natural question would be whether such a separation is still optimal in wireless networks.

In contrast to the optimality of separate network and channel coding in networks with a single source and multiple sinks [116] and in networks with point-to-point DMC [115], this separation was shown to be suboptimal in [118], [119] for wireless deterministic relay networks (i.e. Aref networks [117]) and discrete memoryless broadcast channels. In [124], the separation of channel and network coding was demonstrated to be suboptimal in general for wireless networks. An end-to-end algorithm was presented for Gaussian and erasure wireless networks where instead of forcing each intermediate nodes to perform decoding, better performance can be achieved by flexibly allowing the nodes to choose between decode-and-forward (DF) and amplify-and forward (AF) operations. Besides, the suboptimality of a separation of source and network codes was

Figure 3.1: System models for SNCC and the proposed JNCC strategies.

demonstrated for non-multicast networks [123], multicast networks [124], and multiple access channels [113] respectively.

From the perspective of network topologies, it was shown in [121], [122] that achieving capacity on the subnetworks of a wireless network does not guarantee achieving the capacity globally. That is, instead of separation-based approaches, optimal design of a wireless network should take into account the global structure of the network. To be specific, for a wireless multi-hop network, it is not necessarily the best choice that each relay node decodes the message it receives.

An unified framework for source, channel, and network coding based on linear codes was proposed in [123], where the suboptimality of separate coding strategies across canonical wireless subnetworks [121], [122], i.e. BC and MAC, was proven. That is, for general wireless networks, although codes optimal for canonical subnetworks can be separately implemented, the resulting solution is in general not optimal for the network as a whole, which makes the case for the need to perform end-to-end joint coding across the network.

However, for a general wireless multi-terminal network [91], a comprehensive end-to-end coding strategy is beyond current research achievements. Thus, most current works [125]-[131] mainly focus on the canonical wireless subnetworks, i.e. BC and MAC [99]-[105], [111]-[113], instead.

A network-channel coding scheme, based on Turbo codes, was proposed in [125]-

3. JOINT NETWORK AND CHANNEL CODING

[127] for a two-way relay channel and a multiple-access relay channel, respectively. The main idea is that the redundancy in the network code can be used to augment the channel code for better error protection at the destination. However, to recover the source messages, separate network and channel coding was performed at the intermediate relay. An alternative network-channel coding design was proposed for the binary symmetric channel and additive white Gaussian noise (AWGN) channel in [129]. By exploiting the linear property of both the channel and network codes, network coding is performed prior to channel decoding by directly combining the hard or soft decisions of each received packet.

A nested coding scheme [130], where multiple information words are first separately channel encoded and then XORed together (i.e. network encoding), was implemented in [131] as a way of achieving joint channel and network coding. A two-hop multi-source multi-destination network scenario was considered. To ensure reliable message delivery, retransmissions and feedbacks were performed based on erasure channels where only correctly received codewords are stored for final decoding.

A BC where each receiver may possess some RSI was analyzed in [99]-[101] where the capacity regions were derived for a degraded broadcast channel with two receivers. To achieve the respective capacity regions, different coding strategies were proposed independently in [99]-[101], where explicit code designs consisting of superposition coding, nested coding, and linear network coding were performed, depending on the exact RSI available at each receiver.

3.1.2 Our Contributions

In order to implement the end-to-end joint coding across a general network, there needs to exist a “genie” who has knowledge of the whole network topology as well as the global channel state information (CSI). These pieces of information are difficult to obtain in practice, especially for large and dynamic networks. Thus, instead of an ambitious end-to-end joint coding scheme, it will be pragmatic to focus on some canonical multi-user networks [92]-[94], i.e. BC and MAC, and see what are the gains that can be achieved if joint coding is applied, as compared to a simplistic separate network and channel coding (SNCC) strategy.

Owing to the broadcast nature of wireless networks, every transmission can be categorized as a BC from the perspective of a transmitter, while every reception can be

(a) Broadcast channel with receiver side information (BC-RSI).

(b) Multiple access channel with correlated sources and receiver side information (MAC-CS-RSI).

Figure 3.2: Examples for the two canonical subnetworks in wireless networks.

3. JOINT NETWORK AND CHANNEL CODING

categorized as a MAC from the perspective of a receiver if multiple nodes transmit simultaneously. As an example, in Figure 3.2(a), due to opportunistic listening, Rx1 and Rx2 have already received message W_1 and W_2 respectively from Tx1 and Tx2. Suppose Tx3 now broadcasts messages $\{W_1, W_2, W_3\}$ to Rx1 and Rx2, this will constitute a BC with receiver side information (BC-RSI) [99]-[105]. Similarly, as shown in Figure 3.2(b), Rx has already received message W_1 from Tx1. Suppose both Tx2 and Tx2' now transmit simultaneously to Rx, this will constitute a MAC with correlated sources (with $\{W_1, W_3\}$ as the common messages between Tx2 and Tx2') and receiver side information (MAC-CS-RSI) [109]-[113].

In this chapter, we consider a wireless multicast network with discrete memoryless fading channels where a group of sources wish to transmit independent messages to a group of destinations via some intermediate nodes and each destination desires the messages from all sources. To exploit the existing information-theoretic results for BC-RSI and MAC-CS-RSI, we adopt a decode-and-forward (DF) relay protocol [46] at each intermediate node. The end-to-end information flow across the network is divided into separate transmission phases for which a joint network and channel coding (JNCC) strategy is proposed. Our contributions are as follows.

- When a transmitter broadcasts to a group of receivers, owing to the independent wireless fading channels between the transmitter and multiple receivers, it is highly probable that some receivers are not able to decode all desired messages after a single transmission. To ensure reliable message delivery, we consider an automatic repeat request (ARQ) scheme [144] where the transmitter simply performs retransmissions until an acknowledgement¹ (ACK) is sent back from each of the intended receivers indicating successful decoding of all desired messages.
- In [99]-[103], either RSI or transmit CSI is assumed to be available at the transmitter such that explicit code designs² and rate adaptation can be performed. In contrast, for the proposed JNCC strategy, without any knowledge of the available RSI at each receiver, the transmitter simply uses independent and identically generated codebooks for each transmission and at the same time, optimal rate

¹For simplicity, we assume that the ACK feedback channel is delay-free and error-free [143].

²Although RSI and CSI can be obtained via feedback, explicit code design is a NP-hard problem and is usually very complex [99]-[101], [131] in general.

adaptation is achieved without the need for transmit CSI or any explicit rate adaptation algorithm.

- In contrast to [99]-[101], [131] where the generalization to a broadcast channel with more than two receivers is highly non-trivial, our proposed JNCC strategy is applicable to a BC with an arbitrary number of receivers. As will be explained later, the encoder for the proposed JNCC strategy is independent of the number of receivers and the RSI they possess. The receiver that experiences a better channel and with more RSI available is able to decode the transmitted messages faster.
- Taking into account the ARQ protocol, we quantify the performance of the proposed JNCC strategy in terms of network throughput by applying the renewal-reward theorem [142], [143].

Analytical and simulation results show that the proposed JNCC strategy substantially outperforms the conventional SNCC strategy with random linear network coding (RLNC) [3]-[6] in terms of network throughput, thus proving that joint coding is better than separate coding in wireless networks. The organization of this chapter is as follows. In Section 3.2, we present the network model for the considered wireless multicast network. In Section 3.3, a JNCC strategy is proposed and the achievable rate region is presented. In Section 3.4, we derive the network throughput for both the proposed JNCC strategy and SNCC with RLNC. In Section 3.5, closed-form expressions of network throughput and simulation results are obtained for a toy example. To further illustrate the improvement of the proposed JNCC, a toy example of random networks will be analyzed in Section 3.6. Finally, Section 3.7 summarizes the main results of this chapter.

3.2 System Model

We consider a wireless multicast network with multiple sources, relays, and destinations where all nodes are assumed to operate in half-duplex mode. Each destination desires the messages from all sources. A DF relay protocol is adopted such that an intermediate relay node will not transmit until it has successfully decoded all desired messages. To

3. JOINT NETWORK AND CHANNEL CODING

avoid interference among the different incoming links to a common receiver, we assume an orthogonal MAC where FDMA is employed.

- The considered wireless multicast network is represented by a finite directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ is the set of vertices representing the communication nodes, and $\mathcal{E}$ is the set of edges representing the wireless channels. A communication channel exists between node u and node v if $e_{uv} \in \mathcal{E}$, where e_{uv} is an edge from u to v . To model a BC, we assume that all outgoing edges from a transmitter form a hyperarc carrying the same transmit information. To model an orthogonal MAC, we assume that the receiving node receives non-interfering signals from each of the incoming edges. We define the set of sources as $\mathcal{V}_s \subset \mathcal{V}$, and the set of destinations as $\mathcal{V}_d \subset \mathcal{V}$. We assume that each source has a set of independent messages to transmit and each destination desires the messages from all sources.
- All nodes in $\mathcal{V}$ are grouped into disjoint level sets $L_1, \dots, L_{K+1}$. Level sets $L_1 \triangleq \mathcal{V}_s$ and $L_{K+1} \triangleq \mathcal{V}_d$. Nodes in L_k receive messages only from nodes in levels $L_{k'}$ where $k' < k$. Then nodes in L_k will send messages only to nodes in levels $L_{k''}$ where $k'' > k$, after they have successfully decoded all desired messages from $L_{k'}$. This model constitutes a feedforward flowgraph [141] which has edges of the form $\{e_{uv} | u \in L_k, v \in L_{k''}\}$, $\forall k \in \{1, \dots, K\}$, $k'' \in \{2, \dots, K+1\}$, and $k'' > k$.
- All nodes in $L_k, k \in \{1, \dots, K\}$ are assumed to be time synchronized and they transmit simultaneously. Transmission phase k is defined as the period when nodes in L_k start transmitting until all intended receivers, denoted as V_k where $V_k \subseteq \{L_{k''} | k'' > k\}$, have successfully decoded all desired messages from L_k . A simple ARQ scheme is adopted within transmission phase k where each transmitter performs retransmissions until all intended receivers have successfully decoded all desired messages and sent back an ACK. Immediately after, transmission phase $k+1$ proceeds where nodes in level L_{k+1} start transmitting³. Thus, information flow across the network can be visualized as a series of time-orthogonal multi-hop transmissions where each hop corresponds to a transmission phase.

³In a typical wireless fading channel, although some receivers will be able to finish decoding earlier, they will have to wait for all receivers in V_k to finish decoding before transmission phase $k+1$ proceeds.

3.3 Joint Network and Channel Coding Strategy in a Transmission Phase

- In transmission phase k , a transmitter $u \in L_k$ transmits to a group of intended receivers denoted as $\mathcal{R}_u \subseteq V_k$, such that $\{e_{uv}|v \in \mathcal{R}_u\} \subseteq \mathcal{E}$. The intended receivers in $\mathcal{R}_u$ may have some RSI. Thus from the perspective of the transmitter u , we have a BC-RSI. On the other hand, a receiver $v \in V_k$ receives from a group of intended transmitters denoted as $\mathcal{T}_v \subseteq L_k$, such that $\{e_{uv}|u \in \mathcal{T}_v\} \subseteq \mathcal{E}$. Since all transmitters $u \in L_k$ operate under FDMA, we have an orthogonal MAC-CS-RSI from the perspective of a receiver v .

In Figure 3.3, we show a toy example where all nodes in the network are grouped into four disjoint level sets and the end-to-end information flow can thus be characterized by three separate transmission phases. In transmission phase 1, three sources $L_1 = \{S_1, S_2, S_3\}$ initialize the transmission to the intended receivers $V_1 = \{R_1, R_2, D_1, D_2\}$. After all nodes in V_1 have finished decoding their desired messages from L_1 , transmission phase 2 begins where $L_2 = \{R_1, R_2\}$ transmit to the intended receiver $V_2 = \{R_3\}$, which forms an orthogonal MAC for R_3 . After R_3 has finished decoding, transmission phase 3 begins where $L_3 = \{R_3\}$ transmits to the intended receivers $V_3 = L_4 = \{D_1, D_2\}$, which forms a BC-RSI for R_3 . Thus, the end-to-end transmission from multiple sources $\{S_1, S_2, S_3\}$ to multiple destinations $\{D_1, D_2\}$ takes place in terms of multi-hop transmissions across the disjoint level sets $\{L_1, L_2, L_3, L_4\}$, where each hop corresponds to a transmission phase.

3.3 Joint Network and Channel Coding Strategy in a Transmission Phase

As described in Section 3.2, since end-to-end information flow across the network is divided into separate transmission phases, we can focus on each transmission phase separately. In this section, we will present the mathematical formulations and propose a JNCC strategy for the network model described in Section 3.2 and then derive the achievable rate region for a transmission phase k .

We first define the global message set as $\mathcal{W}_G = \{W_i|i \in \mathcal{I}_G\}$, which contains all source messages to be transmitted across the network $\mathcal{G}$, and $\mathcal{I}_G$ is the corresponding set of message identifiers which is unique for each message. Each message W_i is drawn

3. JOINT NETWORK AND CHANNEL CODING

Figure 3.3: A toy example for the considered multicast network.

3.3 Joint Network and Channel Coding Strategy in a Transmission Phase

from an index set $\{1, 2, \dots, 2^{nR_i}\}$ where R_i is the information rate⁴ of message W_i .

Without loss of generality, we assume that a transmitter $u \in L_k$ has a set of independent messages $\mathcal{W}_u = \{W_i | i \in \mathcal{I}_u\} = \{1, 2, \dots, 2^{nR_u^{\text{sum}}}\}$ to transmit, where $\mathcal{I}_u \subseteq \mathcal{I}_G$ is the corresponding set of message identifiers at u and $R_u^{\text{sum}} = \sum_{i \in \mathcal{I}_u} R_i$. Thus for a receiver $v \in V_k$, a set of messages $\mathcal{W}_v = \cup_{u \in \mathcal{T}_v} \mathcal{W}_u$ from all intended transmitters in $\mathcal{T}_v$ is desired. Receiver v attempts to decode for $\mathcal{W}_v$ with the assistance of the available⁵ RSI $\mathcal{S}_v \subseteq \mathcal{W}_v$ and the possible correlation between the intended transmitters in $\mathcal{T}_v$.

3.3.1 Proposed Joint Network and Channel Coding Strategy

Figure 3.4: The proposed JNCC strategy where message index $w_u = 1$ is transmitted, i.e. codeword $\mathbf{x}_{u,t}(1)$ is transmitted for $t = 1, 2, \dots, T$.

- *Encoding:* For a transmitter $u \in L_k$, consider a n -length code of rate R_u^{sum} with

⁴Here $R_i = k_i/n$ is defined as the ratio of the number of information bits k_i to the number of channel uses n per ARQ transmission [144].

⁵Note that the proposed JNCC strategy includes the special case of $\mathcal{S}_v = \emptyset$ when no RSI is available at v .

3. JOINT NETWORK AND CHANNEL CODING

an encoder

$$X_u : (\{1, 2, \dots, 2^{nR_u^{\text{sum}}}\}) \longrightarrow \mathcal{X}_u^n, \quad (3.1)$$

where $\mathcal{X}_u$ denotes the finite input alphabet set of transmitter u . Equation (3.1) indicates that the encoder of u performs joint encoding⁶ of all messages to be transmitted. This is where the notion of network coding comes in as “mixing” of messages is performed at the intermediate nodes. It is obvious from (3.1) that the proposed JNCC strategy is an arbitrary mapping of messages to a joint network channel codeword. Thus in general, this joint network and channel code is non-linear in nature. For codebook generation, $2^{nR_u^{\text{sum}}}$ codewords are independently and identically generated according to some distribution $p(x_u)$ on $\mathcal{X}_u$ and indexed as $\mathbf{x}_u(w_u)$, where $w_u \in \{1, 2, \dots, 2^{nR_u^{\text{sum}}}\}$. This joint network channel codeword is then broadcast by transmitter u . It is clear that each index w_u corresponds to a message index in $\mathcal{W}_u$.

- *ARQ*: As shown in Figure 3.4, to ensure reliable message delivery, a transmitter u performs retransmissions such that for each retransmission $t = 1, 2, \dots, T$, we have an encoder

$$X_{u,t} : (\{1, 2, \dots, 2^{nR_u^{\text{sum}}}\}) \longrightarrow \mathcal{X}_u^n, \quad (3.2)$$

and codeword $\mathbf{x}_{u,t}(w_u)$ is broadcast from independent and identically generated codebooks⁷ until an ACK has been received from each intended receiver $v \in \mathcal{R}_u$ indicating successful decoding of all desired messages.

- *Decoding*: For receiver $v \in V_k$, as shown in Figure 3.4, joint typical-set decoding [91] is performed using all received codewords $\mathbf{y}_{uv,t} \forall t = 1, 2, \dots, T$ such that with each retransmission a decreasing set of possible transmitted codeword candidates is produced. The correct transmitted codeword can then be recovered with a high probability after the T th transmission when only one unique codeword is found in the intersection set across all retransmissions. This decoding can be further assisted by exploiting the available RSI $\mathcal{S}_v$ and the possible correlation

⁶A similar idea was proposed in [131] where the encoder jointly encodes all its messages and the overall code rate increases with the number of messages that are jointly encoded, unlike in RLNC [3]-[6] where the overall code rate remains constant.

⁷Without loss of generality, in Figure 3.4, we assume that message index $w_u = 1$ is being transmitted, i.e. codeword $\mathbf{x}_{u,t}(1)$ is sent for $t = 1, 2, \dots, T$.

3.3 Joint Network and Channel Coding Strategy in a Transmission Phase

between the intended transmitters $\mathcal{T}_v$. We will defer the details to the proof for Theorem 3.3.1.

3.3.2 Achievable Rate Region

Theorem 3.3.1. *For a receiver v , suppose that each intended transmitter $u \in \mathcal{T}_v$ has a set of messages, $\mathcal{W}_u = \{W_i | i \in \mathcal{I}_u\}$, to transmit to v . Thus, in total, a set of messages $\mathcal{W}_v = \cup_{u \in \mathcal{T}_v} \mathcal{W}_u$ from all intended transmitters in $\mathcal{T}_v$ is desired at v . Assuming that receiver v possesses some RSI $\mathcal{S}_v \subseteq \mathcal{W}_v$, the set of messages that is effectively transmitted to v becomes $\mathcal{W}_v \setminus \mathcal{S}_v$. Then $\{R_i | W_i \in \mathcal{W}_v \setminus \mathcal{S}_v\}$ satisfying*

$$\sum_{i \in \{i | W_i \in \mathcal{C}\}} R_i < \sum_{u \in \{u | \mathcal{C} \cap \mathcal{W}_u \neq \emptyset\}} \sum_{t=1}^T I(X_{u,t}; Y_{uv,t}) \quad \forall \mathcal{C} \subseteq \mathcal{W}_v \setminus \mathcal{S}_v \quad (3.3)$$

is achievable over T retransmissions for some product distribution $\prod_{u \in \mathcal{T}_v} p(x_u)$. In (3.3), $Y_{uv,t}$ is the corresponding received signal at v from u in the t th transmission where $t = 1, 2, \dots, T$, and $\sum_{t=1}^T I(X_{u,t}; Y_{uv,t}) \triangleq I_{u \rightarrow v}^{\text{sum}}(T)$ is the sum of mutual information offered by the channel $e_{uv} \in \mathcal{E}$ over T retransmissions.

Remark 3.3.1. *Theorem 3.3.1 is applicable to all receivers in V_k . Since the encoding at all transmitters and decoding at all receivers are independent, the overall achievable rate region for transmission phase k is thus given by the intersection of (3.3) for all receivers in V_k .*

Next we give some examples to provide insights to Theorem 3.3.1.

Example 1 (Point-to-point transmission): Suppose that a message W_1 is transmitted from a transmitter u_1 to a receiver v . From Theorem 3.3.1, the achievable rate region of R_1 is

$$R_1 < I_{u_1 \rightarrow v}^{\text{sum}}(T), \quad (3.4)$$

which indicates that R_1 is upper bounded by the sum of mutual information offered by the channel $e_{u_1 v}$ over T retransmissions. From Section 3.3.1, the code rate is $R_{u_1}^{\text{sum}} = R_1$ and the corresponding joint network channel codeword is $\mathbf{x}_{u_1,t}(w_{u_1})$ for the t th transmission where $w_{u_1} \in \{1, 2, \dots, 2^{nR_{u_1}^{\text{sum}}}\}$. Joint typical-set decoding is performed over T retransmissions such that the unique codeword that is present in the intersection set of all codeword candidates is chosen as the transmitted codeword. The first observation that can be made is that the instantaneous mutual information offered by the channel $e_{u_1 v}$ is accumulated for each retransmission. This effectively means that rate adaptation

3. JOINT NETWORK AND CHANNEL CODING

is achieved passively without the need for transmit CSI nor any explicit rate adaptation algorithms. In fact, this is an optimal rate adaptation as we are able to achieve the mutual information of the instantaneous channel across every retransmission.

Example 2 (Orthogonal MAC-CS): We extend Example 1 with another transmitter u_2 which also transmits the same message W_1 to v , i.e. $\mathcal{T}_v = \{u_1, u_2\}$. From Theorem 3.3.1, the achievable rate region of R_1 is

$$R_1 < I_{u_1 \rightarrow v}^{\text{sum}}(T) + I_{u_2 \rightarrow v}^{\text{sum}}(T). \quad (3.5)$$

Owing to the correlation between u_1 and u_2 , a higher rate of R_1 can be achieved in (3.5) than in (3.4) since more mutual information is accumulated from the two independent input channels. Conversely, the time taken T for successful decoding by v is reduced, compared to Example 1, since two sets of codeword candidates are obtained in every retransmission.

Example 3 (BC-RSI): Suppose that u wants to transmit a set of messages $\{W_1, W_2, W_3\}$ to two intended receivers v_1 and v_2 , which have $\mathcal{S}_{v_1} = \{W_1\}$ and $\mathcal{S}_{v_2} = \{W_1, W_2\}$ as RSI respectively. From Theorem 3.3.1, the achievable rate region of $\{R_2, R_3\}$ is

$$R_2 + R_3 < I_{u \rightarrow v_1}^{\text{sum}}(T), \quad (3.6)$$

$$R_3 < I_{u \rightarrow v_2}^{\text{sum}}(T). \quad (3.7)$$

Owing to the available RSI, receiver v_1 desires $\{W_2, W_3\}$ and receiver v_2 desires W_3 only. When the channel e_{uv_2} is no worse than e_{uv_1} , i.e. $I_{u \rightarrow v_2}^{\text{sum}}(T) \geq I_{u \rightarrow v_1}^{\text{sum}}(T) \forall T$, from (3.6) and (3.7), it is obvious that v_2 will finish decoding faster than v_1 for a fixed value of R_3 . However, as described in Section 3.2, v_2 has to wait for v_1 to finish decoding before the next transmission phase can proceed.

3.3.3 Proof for Theorem 3.3.1

To prove Theorem 3.3.1, for ease of exposition, we consider an orthogonal MAC-CS-RSI with two intended transmitters. The results can then be easily generalized to an orthogonal MAC-CS-RSI for any receiver with an arbitrary set of intended transmitters.

Figure 3.5: An orthogonal MAC-CS-RSI at receiver v with two intended transmitters $\mathcal{T}_v = \{u_1, u_2\}$.

As shown in Figure 3.5, we consider an orthogonal MAC-CS-RSI with DMC for a receiver v with two intended transmitters $\mathcal{T}_v = \{u_1, u_2\}$:

$$\left(\mathcal{X}_{u_1} \times \mathcal{X}_{u_2}, \prod_{u \in \{u_1, u_2\}} p_{Y_{uv}|X_u}(y_{uv}|x_u), \mathcal{Y}_{u_1v} \times \mathcal{Y}_{u_2v} \right), \quad (3.8)$$

where $\mathcal{X}_{u_1}$ and $\mathcal{X}_{u_2}$ denote the input alphabet sets of transmitters u_1 and u_2 , $\mathcal{Y}_{u_1v}$ and $\mathcal{Y}_{u_2v}$ denote the corresponding output alphabet sets at v , and $\prod_{u \in \{u_1, u_2\}} p_{Y_{uv}|X_u}(y_{uv}|x_u)$ is the channel transition probability. All alphabets are assumed to be finite.

We seek to derive the achievable rate region and the corresponding coding strategy for a scenario where transmitters u_1, u_2 each has a set of independent messages $\mathcal{W}_{u_1} = \{W_1, W_2, W_3, W_4\}$, and $\mathcal{W}_{u_2} = \{W_3, W_4, W_5, W_6\}$ to transmit. The correlation between the two intended transmitters, via W_3 and W_4 , may come from an earlier transmission phase from other links. Thus, in total a set of independent messages $\mathcal{W}_v = \cup_{u \in \{u_1, u_2\}} \mathcal{W}_u = \{W_1, W_2, W_3, W_4, W_5, W_6\}$ is desired at v . Without loss of gen-

3. JOINT NETWORK AND CHANNEL CODING

erality, we assume that $\mathcal{S}_v = \{W_2, W_4, W_6\}$ is known *a priori* at v as RSI. Thus the set of messages that is effectively transmitted to v becomes $\mathcal{W}_v \setminus \mathcal{S}_v = \{W_1, W_3, W_5\}$.

Lemma 3.3.1. *We consider the orthogonal MAC-CS-RSI in (3.8) where $\mathcal{W}_{u_1} = \{W_1, W_2, W_3, W_4\}$, $\mathcal{W}_{u_2} = \{W_3, W_4, W_5, W_6\}$, and $\mathcal{S}_v = \{W_2, W_4, W_6\}$. Retransmissions are performed over T time slots, with each time slot consisting of $n \rightarrow \infty$ channel uses. Then $\{R_1, R_3, R_5\}$ satisfying*

$$\begin{aligned} R_1 &< I_1^{\text{sum}}(T), & R_5 &< I_2^{\text{sum}}(T), \\ R_1 + R_3 + R_5 &< I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T), \end{aligned} \tag{3.9}$$

is achievable for some product distribution $p(x_{u_1})p(x_{u_2})$ on $\mathcal{X}_{u_1} \times \mathcal{X}_{u_2}$, where $I_1^{\text{sum}}(T)$ ($I_2^{\text{sum}}(T)$) is the sum mutual information offered by the channel $u_1 \rightarrow v$ ($u_2 \rightarrow v$) across T retransmissions.

Remark 3.3.2. *If the two transmitters u_1, u_2 know $\mathcal{S}_v$ and simply transmit $\{W_1, W_3\}$ and $\{W_3, W_5\}$ respectively, then clearly (3.9) is the achievable rate region. However, we will show in the following proof that the transmitters do not need to explicitly know the RSI in order to achieve (3.9).*

Achievability Proof: Employing the proposed JNCC strategy in Section 3.3.1, the codebooks for encoding by u_1 and u_2 are independently and identically generated for every transmission. We denote the transmitted and received codewords in the t th transmission as $\mathbf{x}_{u,t}$ and $\mathbf{y}_{uv,t}$ respectively, where $u \in \{u_1, u_2\}$, $t = 1, 2, \dots, T$.

Let $A_\epsilon^{(n)}(X_{u,t}, Y_{uv,t})$ be the set of joint typical $(\mathbf{x}_{u,t}, \mathbf{y}_{uv,t})$ sequences. The decoder of v chooses the unique $\{\hat{w}_1, \hat{w}_3, \hat{w}_5\}$ satisfying

$$\begin{aligned} &\bigcap_{t=1}^T \left\{ \left\{ (\mathbf{x}_{u_1,t}(\hat{w}_1, w_2, \hat{w}_3, w_4), \mathbf{y}_{u_1v,t}) \in A_\epsilon^{(n)}(X_{u_1,t}, Y_{u_1v,t}) \right\} \right. \\ &\quad \cap \left. \left\{ (\mathbf{x}_{u_2,t}(\hat{w}_3, w_4, \hat{w}_5, w_6), \mathbf{y}_{u_2v,t}) \in A_\epsilon^{(n)}(X_{u_2,t}, Y_{u_2v,t}) \right\} \right\}. \end{aligned} \tag{3.10}$$

as the transmitted codewords $\{w_1, w_3, w_5\}$. That is, for every transmission, joint typical-set decoding [91] is employed to produce a set of possible codewords, then we choose the unique $\{\hat{w}_1, \hat{w}_3, \hat{w}_5\}$ that are present in all sets across T retransmissions. If none or more than one $\{\hat{w}_1, \hat{w}_3, \hat{w}_5\}$ is found, an error is declared.

3.3 Joint Network and Channel Coding Strategy in a Transmission Phase

Analysis of Probability of Error: By the symmetry of the code construction, the probability of error does not depend on the particular codeword that was sent. Thus, without loss of generality, we assume that codewords $\mathbf{x}_{u_1,t}(1, w_2, 1, w_4)$ and $\mathbf{x}_{u_2,t}(1, w_4, 1, w_6)$ are transmitted for $t = 1, 2, \dots, T$.

We note that due to the RSI $\mathcal{S}_v = \{W_2, W_4, W_6\}$, the decoder of v is able to narrow down the number of incorrect codewords for $\mathbf{x}_{u_1,t}$ from $(2^{n(R_1+R_2+R_3+R_4)} - 1)$ to $(2^{n(R_1+R_3)} - 1)$, and for $\mathbf{x}_{u_2,t}$ from $(2^{n(R_3+R_4+R_5+R_6)} - 1)$ to $(2^{n(R_3+R_5)} - 1)$ [99]-[103], [130], [131]. For ease of exposition, we define $\mathcal{W}_{e1}$, $\mathcal{W}_{e3}$, and $\mathcal{W}_{e5}$ as the sets of incorrect codewords for messages W_1 , W_3 , and W_5 respectively. An error occurs when the incorrect codewords are jointly typical with the received sequences.

We define the events $\mathbf{E}_t^1(\hat{w}_1, w_2, \hat{w}_3, w_4) = \{(\mathbf{x}_{u_1,t}(\hat{w}_1, w_2, \hat{w}_3, w_4), \mathbf{y}_{u_1v,t}) \in A_\epsilon^{(n)}(X_{u_1,t}, Y_{u_1v,t})\}$, $\mathbf{E}_t^2(\hat{w}_3, w_4, \hat{w}_5, w_6) = \{(\mathbf{x}_{u_2,t}(\hat{w}_3, w_4, \hat{w}_5, w_6), \mathbf{y}_{u_2v,t}) \in A_\epsilon^{(n)}(X_{u_2,t}, Y_{u_2v,t})\}$, and $\overline{\mathbf{E}}_t^1(\hat{w}_1, w_2, \hat{w}_3, w_4)$, $\overline{\mathbf{E}}_t^2(\hat{w}_3, w_4, \hat{w}_5, w_6)$ as the corresponding complementary events. In total, we have the following four cases for the error events.

1: *Correct codewords are not jointly typical with received sequences.*

$$\xi_1 = \bigcup_{t=1}^T \overline{\mathbf{E}}_t^1(1, w_2, 1, w_4) \cup \bigcup_{t=1}^T \overline{\mathbf{E}}_t^2(1, w_4, 1, w_6). \quad (3.11)$$

2: *For codewords that satisfy the typicality test, one of $\{\hat{w}_1, \hat{w}_3, \hat{w}_5\}$ is wrong.*

$$\xi_2 = \bigcup_{\hat{w}_5 \in \mathcal{W}_{e5}} \left(\bigcap_{t=1}^T \mathbf{E}_t^1(1, w_2, 1, w_4) \cap \bigcap_{t=1}^T \mathbf{E}_t^2(1, w_4, \hat{w}_5, w_6) \right), \quad (3.12)$$

$$\xi_3 = \bigcup_{\hat{w}_1 \in \mathcal{W}_{e1}} \left(\bigcap_{t=1}^T \mathbf{E}_t^1(\hat{w}_1, w_2, 1, w_4) \cap \bigcap_{t=1}^T \mathbf{E}_t^2(1, w_4, 1, w_6) \right), \quad (3.13)$$

$$\xi_4 = \bigcup_{\hat{w}_3 \in \mathcal{W}_{e3}} \left(\bigcap_{t=1}^T \mathbf{E}_t^1(1, w_2, \hat{w}_3, w_4) \cap \bigcap_{t=1}^T \mathbf{E}_t^2(\hat{w}_3, w_4, 1, w_6) \right). \quad (3.14)$$

3: *For codewords that satisfy the typicality test, two of $\{\hat{w}_1, \hat{w}_3, \hat{w}_5\}$ are wrong.*

$$\xi_5 = \bigcup_{\hat{w}_3 \in \mathcal{W}_{e3}, \hat{w}_5 \in \mathcal{W}_{e5}} \left(\bigcap_{t=1}^T \mathbf{E}_t^1(1, w_2, \hat{w}_3, w_4) \cap \bigcap_{t=1}^T \mathbf{E}_t^2(\hat{w}_3, w_4, \hat{w}_5, w_6) \right), \quad (3.15)$$

$$\xi_6 = \bigcup_{\hat{w}_1 \in \mathcal{W}_{e1}, \hat{w}_5 \in \mathcal{W}_{e5}} \left(\bigcap_{t=1}^T \mathbf{E}_t^1(\hat{w}_1, w_2, 1, w_4) \cap \bigcap_{t=1}^T \mathbf{E}_t^2(1, w_4, \hat{w}_5, w_6) \right), \quad (3.16)$$

$$\xi_7 = \bigcup_{\hat{w}_1 \in \mathcal{W}_{e1}, \hat{w}_3 \in \mathcal{W}_{e3}} \left(\bigcap_{t=1}^T \mathbf{E}_t^1(\hat{w}_1, w_2, \hat{w}_3, w_4) \cap \bigcap_{t=1}^T \mathbf{E}_t^2(\hat{w}_3, w_4, 1, w_6) \right). \quad (3.17)$$

3. JOINT NETWORK AND CHANNEL CODING

4: For codewords that satisfy the typicality test, none of $\{\hat{w}_1, \hat{w}_3, \hat{w}_5\}$ is correct.

$$\xi_8 = \bigcup_{\hat{w}_1 \in \mathcal{W}_{e1}, \hat{w}_3 \in \mathcal{W}_{e3}, \hat{w}_5 \in \mathcal{W}_{e5}} \left(\bigcap_{t=1}^T \mathbf{E}_t^1(\hat{w}_1, w_2, \hat{w}_3, w_4) \cap \bigcap_{t=1}^T \mathbf{E}_t^2(\hat{w}_3, w_4, \hat{w}_5, w_6) \right). \quad (3.18)$$

From [91], by the union bound and the use of i.i.d. codebooks, the error probability is given as $p_e = \sum_{i=1}^8 \Pr(\xi_i)$, where

$$\begin{aligned} \Pr(\xi_1) &< \epsilon \\ \Pr(\xi_2) &< 2^{nR_5} 2^{-n(I_2^{\text{sum}}(T) - 3\epsilon)}, \\ \Pr(\xi_3) &< 2^{nR_1} 2^{-n(I_1^{\text{sum}}(T) - 3\epsilon)}, \\ \Pr(\xi_4) &< 2^{nR_3} 2^{-n(I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T) - 3\epsilon)}, \\ \Pr(\xi_5) &< 2^{n(R_3 + R_5)} 2^{-n(I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T) - 3\epsilon)}, \\ \Pr(\xi_6) &< 2^{n(R_1 + R_5)} 2^{-n(I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T) - 3\epsilon)}, \\ \Pr(\xi_7) &< 2^{n(R_1 + R_3)} 2^{-n(I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T) - 3\epsilon)}, \\ \Pr(\xi_8) &< 2^{n(R_1 + R_3 + R_5)} 2^{-n(I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T) - 3\epsilon)}. \end{aligned}$$

Therefore, we have $p_e \rightarrow 0$ for sufficiently large n and

$$\begin{aligned} R_5 &< I_2^{\text{sum}}(T), \\ R_1 &< I_1^{\text{sum}}(T), \\ R_3 &< I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T), (\text{redundant}). \\ R_3 + R_5 &< I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T), (\text{redundant}), \\ R_1 + R_5 &< I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T), (\text{redundant}), \\ R_1 + R_3 &< I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T), (\text{redundant}), \\ R_1 + R_3 + R_5 &< I_1^{\text{sum}}(T) + I_2^{\text{sum}}(T), \end{aligned} \quad (3.19)$$

Hence, with the proposed JNCC strategy, we can choose ϵ and n such that the rates in (3.19) can be achieved asymptotically.

The results of Lemma 3.3.1 can be readily generalized to any receiver with arbitrary RSI and an arbitrary set of intended transmitters and messages to obtain Theorem 3.3.1. Since the encoding at all transmitters $u \in L_k$ and the decoding at all receivers $v \in V_k$ are independent, it is clear that Remark 3.3.2 holds true.

3.3.4 Discussions

The work by Avestimehr et al. [132] presented an achievable rate for general deterministic relay networks with broadcast at the transmitters and interference at the receivers. With a single source serving unicast or multicast information flows, linear finite-field deterministic network was discussed where the deterministic functions at the intermediate nodes are linear over a finite field. In contrast, as described in Section 3.3.1, the proposed JNCC is nonlinear in nature.

A similar idea was presented recently in [133] by S. H. Lim et al. This work was published after our conference paper [134], [135]. The basic idea is the same, i.e., the same message is sent multiple times using independent codebooks and the decoder performs joint typicality decoding on the received signals from all the blocks. An inner bound for the noisy network coding for a multi-source multicast network was presented and proved theoretically, however no closed-form expression was given.

For analog network coding (i.e. AF) or network coding at the physical layer [58]-[66], usually a two-hop network, e.g. a two-way relay channel or a multiple-access relay channel, is considered where the signals are combined at the relay without decoding them. With an inherent combining of the signals or the soft information of the signals rather than the decoded bits at the relay, joint decoding can be performed at the destination. However, as far as I know, there is no published work talking about analog network coding for general networks with more than two hops.

3.4 Network Throughput Analysis

We first define the event $\mathcal{A}_{v,t} = \{\text{receiver } v \text{ has recovered all desired messages } \mathcal{W}_v \text{ within } t \text{ retransmissions}\}$. Then $\mathcal{A}_t = \bigcap_{v \in V_k} \mathcal{A}_{v,t}$ corresponds to the event that transmission phase k finishes within t retransmissions. From Remark 3.3.1, since the encoding at all transmitters and decoding at all receivers are independent, event $\mathcal{A}_t$ occurs with probability

$$\Pr(\mathcal{A}_t) = \prod_{v \in V_k} \Pr(\mathcal{A}_{v,t}). \quad (3.20)$$

Then the probability that transmission phase k finishes at exactly the T_k th retransmission (and not earlier) is given by

$$\Pr(C_{T_k}) = \Pr(\mathcal{A}_{T_k}) - \Pr(\mathcal{A}_{T_k-1}). \quad (3.21)$$

3. JOINT NETWORK AND CHANNEL CODING

From an end-to-end perspective, in order to obtain the normalized⁸ network throughput which quantifies the average rate of successful message delivery, we assume that $\mathcal{V}_s$ will initiate the transmission of a new set of messages right after $\mathcal{V}_d$ have successfully recovered the previous message set $\mathcal{W}_G$, and this transmission cycle will repeat itself indefinitely. For each transmission cycle, the total time-frequency resource that is being used can be quantified by

$$\Gamma = \sum_{k=1}^K f_k T_k \quad (3.22)$$

where f_k denotes the number of orthogonal frequencies used in transmission phase k for FDMA and T_k denotes the number of retransmissions required for all receivers in transmission phase k to achieve successful decoding. The reward $\mathfrak{R}$ of the network for a single transmission cycle can be quantified by the sum rate of the messages in $\mathcal{W}_G$, i.e. $\mathfrak{R} = \sum_{i \in \mathcal{I}_G} R_i$. It is evident that Γ is a random variable as T_k , $k = 1, 2, \dots, K$ is a function of the fading channels.

We assume that reward $\mathfrak{R}$ is the same for each transmission cycle and each channel is independent and identically distributed (i.i.d.) for every transmission. With such an end-to-end traffic model, we can apply the renewal-reward theorem [142], [143] to obtain the normalized network throughput as

$$\eta = \frac{\mathfrak{R}}{E[\Gamma]} = \frac{\mathfrak{R}}{\sum_{k=1}^K f_k E[T_k]} \quad [\text{bits} / \text{time-frequency resource}] \quad (3.23)$$

where

$$E[T_k] = \sum_{T_k=1}^{\infty} T_k \Pr(C_{T_k}) \quad (3.24)$$

and $\Pr(C_{T_k})$ is given in (3.21). For the rest of the chapter, we will simply refer to the normalized network throughput as network throughput.

From Theorem 3.3.1, as long as $\sum_{i \in \{i | W_i \in \mathcal{C}\}} R_i$ is finite, a receiver v is able to finish decoding within a finite number of retransmissions because $I_{u \rightarrow v}^{\text{sum}}(t)$ is a strictly increasing function of t . Since the number of retransmissions required for each receiver to do successful decoding is finite, T_k is also finite and thus $E[T_k]$ in (3.24) converges.

⁸The network throughput is normalized with respect to time and frequency resource.

3.4.1 Proposed Joint Network and Channel Coding Strategy

For a receiver $v \in V_k$ to successfully receive all desired messages within t retransmissions, from the proof for Theorem 3.3.1, the event

$$\mathcal{A}_{v,t} = \left\{ \bigcap_{\mathcal{C} \subseteq \{\mathcal{W}_v \setminus \mathcal{S}_v\}} \left\{ \sum_{i \in \{i | W_i \in \mathcal{C}\}} R_i < \sum_{u \in \{u | \mathcal{C} \cap \mathcal{W}_u \neq \emptyset\}} I_{u \rightarrow v}^{\text{sum}}(t) \right\} \right\}$$

has to be satisfied. Thus from (3.20), the probability that all receivers in V_k retrieve all desired messages within T_k retransmissions is given by

$$\Pr(\mathcal{A}_{T_k}) = \Pr \left(\bigcap_{v \in V_k} \mathcal{A}_{v,T_k} \right) = \prod_{v \in V_k} \Pr(\mathcal{A}_{v,T_k}). \quad (3.25)$$

Substituting (3.25) into (3.21) and (3.24), we are able to obtain

$$E[T_k] = \sum_{T_k=1}^{\infty} T_k [\Pr(\mathcal{A}_{T_k}) - \Pr(\mathcal{A}_{T_k-1})] \quad (3.26)$$

and thus network throughput η can be calculated easily from (3.23). The analytical result for η will be derived in closed-form expressions for a toy example in the next section.

3.4.2 Separate Network and Channel Coding Strategy with Random Linear Network Coding

For comparison purposes, we derive similar network throughput expressions for the conventional SNCC strategy with RLNC. We adopt an algebraic approach [3]-[6] and consider whether each receiver $v \in V_k$ has collected sufficient degrees of freedom⁹ (DoF) to decode all desired messages $\mathcal{W}_v$. Essentially, the DoF of $\mathcal{W}_v$ is equal to the total number of independent messages $|\mathcal{W}_v|$. We may equivalently associate a vector space Ω to $\mathcal{W}_v$ where $\text{rank}(\Omega) = |\mathcal{W}_v|$. The available RSI $\mathcal{S}_v \subseteq \mathcal{W}_v$ can be viewed as a vector subspace $\Omega_S \subseteq \Omega$ with DoF $|\mathcal{S}_v| = \text{rank}(\Omega_S)$. Thus in order to retrieve all desired messages in $\mathcal{W}_v$, receiver v with RSI $\mathcal{S}_v$ has to collect an additional $|\mathcal{W}_v| - |\mathcal{S}_v|$ DoF¹⁰.

⁹This is similar to rateless codes [137], [138] where successful decoding does not depend on receiving a specific codeword, as long as any subset of the transmitted codewords of size equal to or only slightly larger than the number of source messages (DoF) is collected, all original source messages can be ideally recovered.

¹⁰An ACK will be sent back to each of the intended transmitters after all $|\mathcal{W}_v| - |\mathcal{S}_v|$ DoF have been collected.

3. JOINT NETWORK AND CHANNEL CODING

For simplicity, we assume that $R_i = R_0 \forall i$. Instead of transmitting a codeword from $\mathcal{X}_u^n$ with rate R_0 , transmitter u may choose to transmit at a higher rate¹¹ of $R_{\text{rlnc}} = bR_0$ where $b \in \mathbb{Z}^+$ is fixed due to the lack of transmit CSI. In other words, for each transmission that lasts n channel uses, u is able to send b codewords from $\mathcal{X}_u^{n/b}$. Thus as a best-case lower bound¹², receiver v with $\mathcal{S}_v$ only has to receive $N_v = \left\lceil \frac{|\mathcal{W}_v| - |\mathcal{S}_v|}{b} \right\rceil$ transmissions correctly in order to retrieve all messages in $\mathcal{W}_v$. It is a reasonable assumption that for v to receive N_v transmissions correctly in total, it has to receive on average $\bar{N} = \left\lceil \frac{|\mathcal{W}_v| - |\mathcal{S}_v|}{b|\mathcal{T}_v|} \right\rceil$ transmissions from each intended transmitter $u \in \mathcal{T}_v$.

Let us first consider a receiver $v \in V_k$ and that T_k transmissions have occurred. Denoting s_{uv, T_k} as the number of correctly received transmissions at v from u out of T_k transmissions, thus $\mathcal{A}_{v, T_k}^{\text{rlnc}} = \cap_{u \in \mathcal{T}_v} \{s_{uv, T_k} \geq \bar{N}\}$ corresponds to the event that receiver v has collected sufficient DoF to retrieve all desired messages. It is clear that $\Pr(\mathcal{A}_{v, T_k}^{\text{rlnc}}) = 0$ when $T_k < \bar{N}$.

Since the outage probability for channel $e_{uv} \in \mathcal{E}$ at the t th transmission is given by $p_{uv, t} = \Pr(I(X_{u, t}; Y_{uv, t}) < R_{\text{rlnc}}) = p_{uv}$, the random variable s_{uv, T_k} follows a binomial distribution $\mathcal{B}(T_k, 1 - p_{uv})$ with T_k trials and success probability of $(1 - p_{uv})$. Hence, event $\mathcal{A}_{v, T_k}^{\text{rlnc}}$ occurs with probability

$$\Pr(\mathcal{A}_{v, T_k}^{\text{rlnc}}) = \prod_{u \in \mathcal{T}_v} \sum_{t=\bar{N}}^{T_k} \binom{T_k}{t} (1 - p_{uv})^t p_{uv}^{(T_k-t)}. \quad (3.27)$$

Thus the probability that all receivers $v \in V_k$ retrieve their desired messages within T_k retransmissions is given by

$$\begin{aligned} \Pr(\mathcal{A}_{T_k}^{\text{rlnc}}) &= \prod_{v \in V_k} \Pr(\mathcal{A}_{v, T_k}^{\text{rlnc}}) \\ &= \prod_{v \in V_k} \prod_{u \in \mathcal{T}_v} \sum_{t=\bar{N}}^{T_k} \binom{T_k}{t} (1 - p_{uv})^t p_{uv}^{(T_k-t)}. \end{aligned} \quad (3.28)$$

Substituting (3.28) into (3.21) and (3.24), we are able to derive $E[T_k]$ for RLNC as

$$E[T_k] = \sum_{T_k=\bar{N}}^{\infty} T_k \left[\Pr(\mathcal{A}_{T_k}^{\text{rlnc}}) - \Pr(\mathcal{A}_{T_k-1}^{\text{rlnc}}) \right]. \quad (3.29)$$

¹¹This can be achieved through the use of higher order modulation.

¹²We assume that the finite field size used for RLNC is sufficiently large.

The network throughput for SNCC with RLNC will be derived in closed-form expressions for a toy example in the next section.

3.4.3 Comparisons

3.4.3.1 Efficiency

Rather than collecting DoF over the erasure channels where hard-decision decoding is performed and the erased codewords are simply discarded in RLNC and rateless codes, the proposed JNCC strategy performs joint decoding of all received codewords which is similar to a soft-decision decoding performed across retransmissions. That is, even when a codeword is not successfully decoded, some information can be obtained to narrow down the possible candidates for the source messages through joint typical-set decoding until successful decoding can be done.

In RLNC or rateless codes, each codeword is equally important which carries information about all combined original messages, however, just by itself it does not allow to recover any part of the original messages. In contrast, all messages can be recovered from a single codeword in JNCC, as long as the channel is good enough to support the encoded data rate.

3.4.3.2 Memory Request

To perform network coding, there is a memory requirement at the intermediate nodes for both the proposed JNCC strategy and SNCC with RLNC. Intuitively, the storage requirement for the proposed JNCC is higher, since a receiver needs to store all received codewords to accumulate mutual information until successful decoding can be done. In contrast, for RLNC, only correctly decoded codewords are stored for final decoding.

However, it is not necessarily true that the proposed JNCC strategy always requires a higher memory requirement than RLNC. Our results, which will be shown in the next section, indicate that the proposed JNCC strategy always finishes decoding with a less number of transmissions than RLNC thus achieving a higher network throughput. To be specific, to collect N DoF in RLNC, a receiver has to store at least $\lceil \frac{N}{b} \rceil$, $b \in \mathbb{Z}^+$, correctly decoded codewords. However, for the proposed JNCC strategy, to accumulate sufficient mutual information for successful decoding, fewer retransmissions are required

3. JOINT NETWORK AND CHANNEL CODING

for a better channel. Thus fewer codewords are stored at the receiver and a lower memory storage is required for the proposed JNCC strategy.

3.4.3.3 Complexity

In terms of complexity, RLNC is relatively simple as both encoding and decoding are achieved through linear operations¹³. Although the proposed JNCC strategy is inherently more complex due to the nonlinear operations, this strategy is still an attractive alternative given the substantial performance improvements which will be illustrated in Section 3.5.

Recently, rateless codes with soft-decision decoding have been discussed in noisy channels [139] where the receiver uses all received codewords to perform decoding. This rateless code could serve as a potential candidate for the implementation of JNCC to achieve the promising results.

3.5 A Toy Example

In this section, we apply the proposed JNCC strategy to a toy example shown in Figure 3.3. Network throughput is obtained in closed-form expressions and compared with SNCC with RLNC. We assume independent block Rayleigh fading channels such that at the t th transmission, the received signal at v from u is given by

$$\mathbf{y}_{uv,t} = h_{uv,t}\mathbf{x}_{u,t} + \mathbf{n}_{v,t}, \quad \forall e_{uv} \in \mathcal{E} \quad (3.30)$$

where $\mathbf{n}_{v,t}$ is i.i.d. circularly symmetric complex Gaussian noise vector of zero mean and variance $\sigma^2 I$, and $h_{uv,t}$ is the channel coefficient. We assume the use of Gaussian codewords such that $\mathbf{x}_{u,t}$ is subjected to a power constraint $E[|\mathbf{x}_{u,t}|^2]/n \leq P$ where P is the average transmit power at $u \forall u \in \mathcal{V}$. From (3.30), assuming receiver v has perfect knowledge of $h_{uv,t}$, the mutual information offered by the channel e_{uv} for the t th transmission can be expressed as

$$I(X_{u,t}; Y_{uv,t}) = \log_2(1 + \rho\gamma_{uv,t}), \quad (3.31)$$

where $\gamma_{uv,t} = |h_{uv,t}|^2$ and $\rho = P/\sigma^2$.

¹³However, network and channel coding has to be done separately as shown in Figure 3.1(a).

3.5.1 Network Topology

We consider the toy example shown in Figure 3.3(a) where three sources S_1, S_2 , and S_3 transmit independent sets of messages $\mathcal{W}_{S_1} = \{W_1, \dots, W_M\}$, $\mathcal{W}_{S_2} = \{W_{M+1}, \dots, W_{2M}\}$, and $\mathcal{W}_{S_3} = \{W_{2M+1}, \dots, W_{3M}\}$ respectively, to two destinations D_1 and D_2 through a group of intermediate nodes R_1, R_2 , and R_3 . Without loss of generality, we assume that the rates of all messages are equal where $R_i = R_0 \forall i$. We can then derive the corresponding feedforward flowgraph in Figure 3.3(b) where the end-to-end information flow can be divided into three transmission phases.

In transmission phase 1, S_1, S_2, S_3 initiate the transmission. The corresponding intended receivers R_1, R_2, D_1 , and D_2 attempt to decode for $\{\mathcal{W}_{S_1}, \mathcal{W}_{S_2}\}$, $\{\mathcal{W}_{S_2}, \mathcal{W}_{S_3}\}$, $\mathcal{W}_{S_1}$, and $\mathcal{W}_{S_3}$ respectively. In transmission phase 2, both R_1 (knowing $\mathcal{W}_{S_1}, \mathcal{W}_{S_2}$) and R_2 (knowing $\mathcal{W}_{S_2}, \mathcal{W}_{S_3}$) transmit what they have received in the previous transmission phase to R_3 , thus forming an orthogonal MAC-CS. In transmission phase 3, R_3 (knowing $\mathcal{W}_{S_1}, \mathcal{W}_{S_2}, \mathcal{W}_{S_3}$) broadcasts to D_1 and D_2 , which attempt to decode all source messages with the assistance of available RSI $\mathcal{S}_{D_1} = \{\mathcal{W}_{S_1}\}$ and $\mathcal{S}_{D_2} = \{\mathcal{W}_{S_3}\}$ obtained during transmission phase 1, thus forming a BC-RSI.

3.5.2 Proposed Joint Network and Channel Coding Strategy

In transmission phase 1, we assume that all intended receivers R_1, R_2, D_1, D_2 finish decoding within T_1 retransmissions. Thus we have for each intended receiver

$$\begin{aligned} \Pr(\mathcal{A}_{R_1, T_1}) &= \prod_{u \in \{S_1, S_2\}, v=R_1} \Pr \left(\sum_{t=1}^{T_1} \log_2(1 + \rho \gamma_{uv,t}) > R_u^{\text{sum}} \right) \\ &\approx Q \left(\frac{MR_0 - T_1\mu}{\sqrt{T_1\tau^2}} \right)^2, \end{aligned} \quad (3.32)$$

$$\begin{aligned} \Pr(\mathcal{A}_{R_2, T_1}) &= \prod_{u \in \{S_2, S_3\}, v=R_2} \Pr \left(\sum_{t=1}^{T_1} \log_2(1 + \rho \gamma_{uv,t}) > R_u^{\text{sum}} \right) \\ &\approx Q \left(\frac{MR_0 - T_1\mu}{\sqrt{T_1\tau^2}} \right)^2, \end{aligned} \quad (3.33)$$

3. JOINT NETWORK AND CHANNEL CODING

$$\begin{aligned}\Pr(\mathcal{A}_{D_1, T_1}) &= \Pr\left(\sum_{t=1}^{T_1} \log_2(1 + \rho\gamma_{S_1 D_1, t}) > R_{S_1}^{\text{sum}}\right) \\ &\approx Q\left(\frac{MR_0 - T_1\mu}{\sqrt{T_1}\tau^2}\right),\end{aligned}\tag{3.34}$$

$$\begin{aligned}\Pr(\mathcal{A}_{D_2, T_1}) &= \Pr\left(\sum_{t=1}^{T_1} \log_2(1 + \rho\gamma_{S_3 D_2, t}) > R_{S_3}^{\text{sum}}\right) \\ &\approx Q\left(\frac{MR_0 - T_1\mu}{\sqrt{T_1}\tau^2}\right),\end{aligned}\tag{3.35}$$

where $Q(\cdot)$ is the Q-function [89], and $R_u^{\text{sum}} = MR_0 \forall u \in \{S_1, S_2, S_3\}$ is the sum rate at each source. Since $\gamma_{uv, t}$ is i.i.d. over t , from the Central Limit Theorem, we can approximate $\sum_{t=1}^{T_1} \log_2(1 + \rho\gamma_{uv, t})$ as a Gaussian distribution $\mathcal{N}(T_1\mu, T_1\tau^2)$, where $\mu = E[\log_2(1 + \rho\gamma_{uv, t})]$ and $\tau^2 = E[|\log_2(1 + \rho\gamma_{uv, t}) - \mu|^2]$, thus obtaining the Q-function approximations¹⁴ in (3.32)-(3.35). From (3.25), event $\mathcal{A}_{T_1}$ occurs with probability

$$\Pr(\mathcal{A}_{T_1}) = \Pr(\mathcal{A}_{R_1, T_1}) \Pr(\mathcal{A}_{R_2, T_1}) \Pr(\mathcal{A}_{D_1, T_1}) \Pr(\mathcal{A}_{D_2, T_1}).\tag{3.36}$$

Similarly, we denote $\mathcal{A}_{T_2}$ as the event that R_3 finishes decoding within T_2 retransmissions in transmission phase 2. From (3.25), we have

$$\begin{aligned}\Pr(\mathcal{A}_{T_2}) = \Pr\left\{ \bigcap_{u \in \{R_1, R_2\}, v=R_3} \left(\sum_{t=1}^{T_2} \log_2(1 + \rho\gamma_{uv, t}) > R_u^{\text{sum}} \right) \right. \\ \left. \cap \left(\sum_{u \in \{R_1, R_2\}, v=R_3} \sum_{t=1}^{T_2} \log_2(1 + \rho\gamma_{uv, t}) > 3R_u^{\text{sum}} \right) \right\}.\end{aligned}\tag{3.37}$$

From Figure 3.3, for the second transmission phase, since the message set $\mathcal{W}_{S_1}$ is transmitted solely through channel $R_1 \rightarrow R_3$, $\mathcal{W}_{S_3}$ is transmitted solely through channel

¹⁴Simulation results shown later in Figure 3.7 validate the accuracy of the approximations used in this section.

Figure 3.6: Probability region for event $\mathcal{A}_{T_2}$ in (3.39).

$R_2 \rightarrow R_3$, and $\mathcal{W}_{S_2}$ is transmitted through both channels, we have for $\Pr(\mathcal{A}_{T_2})$

$$\begin{aligned} \Pr(\mathcal{A}_{T_2}) = \Pr \bigg(& \left\{ \sum_{t=1}^{T_2} \log_2(1 + \rho\gamma_{uv,t}) > MR_0 \right\}_{u=R_1, v=R_3} \\ & \cap \left\{ \sum_{t=1}^{T_2} \log_2(1 + \rho\gamma_{uv,t}) > MR_0 \right\}_{u=R_2, v=R_3} \\ & \cap \left\{ \sum_{u \in \{R_1, R_2\}, v=R_3} \sum_{t=1}^{T_2} \log_2(1 + \rho\gamma_{uv,t}) > 3MR_0 \right\} \bigg). \end{aligned} \quad (3.38)$$

For ease of exposition, we denote $x = \sum_{t=1}^{T_2} \log_2(1 + \rho\gamma_{uv,t})$ where $u = R_1$ and $v = R_3$, and denote¹⁵ $y = \sum_{t=1}^{T_2} \log_2(1 + \rho\gamma'_{uv,t})$ where $u = R_2$ and $v = R_3$. Thus,

$$\Pr(\mathcal{A}_{T_2}) = \Pr(\{x > MR_0\} \cap \{y > MR_0\} \cap \{x + y > 3MR_0\}), \quad (3.39)$$

¹⁵Note that $\gamma_{uv,t}$ and $\gamma'_{uv,t}$ are i.i.d. random variables.

3. JOINT NETWORK AND CHANNEL CODING

which is denoted by the shaded region $\widehat{DBCE}$ shown in Figure 3.6. Then we have $\Pr(\mathcal{A}_{T_2}) = P_{\widehat{DBCE}} = P_{\widehat{DAE}} - P_{\widehat{BAC}}$, where

$$\begin{aligned} P_{\widehat{DAE}} &= \int_{MR_0}^{\infty} \int_{MR_0}^{\infty} f_{X,Y}(x, y) dy dx \\ &= \int_{MR_0}^{\infty} f_X(x) dx \int_{MR_0}^{\infty} f_Y(y) dy \\ &\approx Q\left(\frac{MR_0 - T_2\mu}{\sqrt{T_2\sigma^2}}\right)^2, \end{aligned} \quad (3.40)$$

$f_X(x)$ and $f_Y(y)$ are the probability density functions for x and y respectively. The approximation in (3.40) comes from the Central Limit Theorem where $x \sim \mathcal{N}(T_2\mu, T_2\tau^2)$ and $y \sim \mathcal{N}(T_2\mu, T_2\tau^2)$ approximately for large T_2 , where $\mu = E[\log_2(1 + \rho\gamma_{uv,t})]$ and $\tau^2 = E[|\log_2(1 + \rho\gamma_{uv,t}) - \mu|^2]$. Since the integration for the triangular area $\widehat{BAC}$ is intractable, we can approximate $P_{\widehat{BAC}}$ by calculating the sum probability of a set of smaller non-overlapping square areas within $\widehat{BAC}$. From (3.40), we can see that for a square area defined by (x, y) and $(x + c, y + c)$ where $x, y, c \geq 0$, the corresponding probability can be derived as

$$\left[Q\left(\frac{x - T_2\mu}{\sqrt{T_2\tau^2}}\right) - Q\left(\frac{x + c - T_2\mu}{\sqrt{T_2\tau^2}}\right) \right] \cdot \left[Q\left(\frac{y - T_2\mu}{\sqrt{T_2\tau^2}}\right) - Q\left(\frac{y + c - T_2\mu}{\sqrt{T_2\tau^2}}\right) \right]. \quad (3.41)$$

Thus the probability of $P_{\widehat{BAC}}$ can be derived in terms of a summation of (3.41) for different x , y , and c . In our derivations, we use 7 smaller squares to approximate the triangular area $\widehat{BAC}$ as shown in Figure 3.6, and simulation results show that it is a good approximation. We omit the details here due to space constraints.

We denote $\mathcal{A}_{T_3}$ as the event that both D_1 and D_2 finish decoding within T_3 retransmissions in transmission phase 3. Due to the respective available RSI $\mathcal{S}_{D_1} = \{\mathcal{W}_{S_1}\}$ and $\mathcal{S}_{D_2} = \{\mathcal{W}_{S_3}\}$ obtained in transmission phase 1, we have from (3.25)

$$\begin{aligned} \Pr(\mathcal{A}_{T_3}) &= \prod_{u=R_3, v \in \{D_1, D_2\}} \Pr\left(\sum_{t=1}^{T_3} \log_2(1 + \rho\gamma_{uv,t}) > 2R_u^{\text{sum}}\right) \\ &\approx Q\left(\frac{2MR_0 - T_3\mu}{\sqrt{T_3\tau^2}}\right)^2. \end{aligned} \quad (3.42)$$

Substituting (3.36), (3.37), and (3.42) into (3.21) and (3.23), we can thus obtain

the network throughput for the proposed JNCC strategy as

$$\eta = \Re \left/ \left\{ \begin{aligned} & 3 \sum_{T_1=1}^{\infty} T_1 [\Pr(\mathcal{A}_{T_1}) - \Pr(\mathcal{A}_{T_1-1})] \\ & + 2 \sum_{T_2=1}^{\infty} T_2 [\Pr(\mathcal{A}_{T_2}) - \Pr(\mathcal{A}_{T_2-1})] \\ & + \sum_{T_3=1}^{\infty} T_3 [\Pr(\mathcal{A}_{T_3}) - \Pr(\mathcal{A}_{T_3-1})] \end{aligned} \right\} \right., \quad (3.43)$$

where $\Re = \sum_{u \in \{S_1, S_2, S_3\}} R_u^{\text{sum}} = 3MR_0$.

3.5.3 Separate Network and Channel Coding Strategy with Random Linear Network Coding

In transmission phase 1, for R_1, R_2, D_1, D_2 to successfully decode $\{\mathcal{W}_{S_1}, \mathcal{W}_{S_2}\}$, $\{\mathcal{W}_{S_2}, \mathcal{W}_{S_3}\}$, $\mathcal{W}_{S_1}$, and $\mathcal{W}_{S_3}$ respectively, as a best-case scenario, they each have to receive $\bar{N} = \lceil \frac{M}{b} \rceil$ transmissions correctly from every incoming link. Again we assume that all receivers finish decoding within T_1 retransmissions. From (3.27), for large T_1 , the binomial distribution $\mathcal{B}(T_1, 1 - p_{uv})$ can be approximated [89] as a Gaussian distribution $\mathcal{N}(T_1(1 - p_{uv}), T_1 p_{uv}(1 - p_{uv}))$. Thus we have for each intended receiver

$$\Pr(\mathcal{A}_{R_1, T_1}^{\text{rlnc}}) = \Pr(\mathcal{A}_{R_2, T_1}^{\text{rlnc}}) \approx Q \left(\frac{\lceil \frac{M}{b} \rceil - T_1(1 - p_{uv})}{\sqrt{T_1 p_{uv}(1 - p_{uv})}} \right)^2, \quad (3.44)$$

$$\Pr(\mathcal{A}_{D_1, T_1}^{\text{rlnc}}) = \Pr(\mathcal{A}_{D_2, T_1}^{\text{rlnc}}) \approx Q \left(\frac{\lceil \frac{M}{b} \rceil - T_1(1 - p_{uv})}{\sqrt{T_1 p_{uv}(1 - p_{uv})}} \right), \quad (3.45)$$

where $p_{uv} = \Pr(\log_2(1 + \rho\gamma_{uv,t}) < R_{\text{rlnc}} = bR_0)$. From (3.28), we have

$$\Pr(\mathcal{A}_{T_1}^{\text{rlnc}}) = \Pr(\mathcal{A}_{R_1, T_1}^{\text{rlnc}}) \Pr(\mathcal{A}_{R_2, T_1}^{\text{rlnc}}) \Pr(\mathcal{A}_{D_1, T_1}^{\text{rlnc}}) \Pr(\mathcal{A}_{D_2, T_1}^{\text{rlnc}}). \quad (3.46)$$

In transmission phase 2, for R_3 to successfully decode $\{\mathcal{W}_{S_1}, \mathcal{W}_{S_2}, \mathcal{W}_{S_3}\}$, it has to receive $\bar{N} = \lceil \frac{3M}{2b} \rceil$ transmissions correctly (best-case scenario) from each of the two incoming links $R_1 \rightarrow R_3$ and $R_2 \rightarrow R_3$. From (3.28) we have event $\mathcal{A}_{T_2}^{\text{rlnc}} = \mathcal{A}_{R_3, T_2}^{\text{rlnc}} = \{s_{R_1 R_3, T_2} \geq \lceil \frac{3M}{2b} \rceil\} \cap \{s_{R_2 R_3, T_2} \geq \lceil \frac{3M}{2b} \rceil\}$, which occurs with probability

$$\Pr(\mathcal{A}_{T_2}^{\text{rlnc}}) \approx Q \left(\frac{\lceil \frac{3M}{2b} \rceil - T_2(1 - p_{uv})}{\sqrt{T_2 p_{uv}(1 - p_{uv})}} \right)^2. \quad (3.47)$$

3. JOINT NETWORK AND CHANNEL CODING

In transmission phase 3, for D_1, D_2 to successfully decode $\{\mathcal{W}_{S_1}, \mathcal{W}_{S_2}, \mathcal{W}_{S_3}\}$, they both only have to receive $\bar{N} = \lceil \frac{3M-M}{b} \rceil$ transmissions correctly (best-case scenario) due to the respective RSI $\mathcal{S}_{D_1} = \{\mathcal{W}_{S_1}\}$ and $\mathcal{S}_{D_2} = \{\mathcal{W}_{S_3}\}$ obtained in transmission phase 1. From (3.28), we have event $\mathcal{A}_{T_3}^{\text{rlnc}} = \{s_{R_3 D_1, T_3} \geq \lceil \frac{2M}{b} \rceil\} \cap \{s_{R_3 D_2, T_3} \geq \lceil \frac{2M}{b} \rceil\}$, which occurs with probability

$$\Pr(\mathcal{A}_{T_3}^{\text{rlnc}}) \approx Q \left(\frac{\lceil \frac{2M}{b} \rceil - T_3(1 - p_{uv})}{\sqrt{T_3 p_{uv}(1 - p_{uv})}} \right)^2. \quad (3.48)$$

Substituting (3.46), (3.47), and (3.48) into (3.21) and (3.23), we can also obtain the network throughput for SNCC with RLNC as

$$\begin{aligned} \eta^{\text{rlnc}} = \Re \Bigg/ & \left\{ \begin{aligned} & 3 \sum_{T_1=\lceil \frac{M}{b} \rceil}^{\infty} T_1 \left[\Pr(\mathcal{A}_{T_1}^{\text{rlnc}}) - \Pr(\mathcal{A}_{T_1-1}^{\text{rlnc}}) \right] \\ & + 2 \sum_{T_2=\lceil \frac{3M}{2b} \rceil}^{\infty} T_2 \left[\Pr(\mathcal{A}_{T_2}^{\text{rlnc}}) - \Pr(\mathcal{A}_{T_2-1}^{\text{rlnc}}) \right] \\ & + \sum_{T_3=\lceil \frac{2M}{b} \rceil}^{\infty} T_3 \left[\Pr(\mathcal{A}_{T_3}^{\text{rlnc}}) - \Pr(\mathcal{A}_{T_3-1}^{\text{rlnc}}) \right] \end{aligned} \right\}. \end{aligned} \quad (3.49)$$

3.5.4 Simulation Results

We assume that each source has a set of $M = 10$ independent messages to transmit, $R_0 = 1$ and that all the channels in the network experience i.i.d. Rayleigh fading such that $E[\gamma_{uv,t}] = 1 \forall e_{uv} \in \mathcal{E}$. Thus for RLNC, the average outage probability of e_{uv} is given by

$$\begin{aligned} p_{uv,t} &= \Pr(\log_2(1 + \rho\gamma_{uv,t}) < R_{\text{rlnc}}) \\ &= 1 - e^{-\frac{2^{R_{\text{rlnc}}}-1}{\rho}} \quad \forall e_{uv} \in \mathcal{E}, \forall t. \end{aligned} \quad (3.50)$$

Matlab is used in deriving all simulation results in this chapter. Given the channel parameters, i.e. $E[\gamma_{uv,t}] = 1$, each channel is randomly generated through Monte Carlo method. An outage occurs if the instantaneous channel cannot support the data rate. Then the throughput across a channel can be obtained as the ratio between the number of packets successfully delivered and the number of transmissions used. A

Figure 3.7: Network throughput of the toy example with $M = 10$ and $R_0 = 1$.

sample program for the simulation of η in (3.43) is provided in Appendix A.2 and the corresponding results are shown in Figure 3.7.

In Figure 3.7, the network throughput η is shown for both the proposed JNCC and SNCC with RLNC under different values of ρ . It can be observed that the simulation results denoted by markers agree well with the analytical results which are denoted by lines, and the proposed JNCC strategy always outperforms SNCC with RLNC.

This result is mainly due to the fact that in the proposed JNCC, we can take advantage of the accumulation of mutual information and this means that no received codeword is wasted, regardless of whether it can be correctly decoded or not. In contrast, SNCC with RLNC is originally designed for erasure channels and thus only codewords that are successfully decoded will contribute to the final decoding for source messages $\{\mathcal{W}_{S_1}, \mathcal{W}_{S_2}, \mathcal{W}_{S_3}\}$.

For SNCC with RLNC, different rates of $R_{\text{rlnc}} = bR_0$, $b = 1, 2, 5$ are adopted at the

transmitters¹⁶. From Figure 3.7, it can be observed that the throughput performance of SNCC is limited by the choice of b . Although a higher throughput can be achieved in the high SNR region with a larger b , the corresponding throughput performance is severely reduced in the low SNR region due to the high outage probability. Conversely, for a smaller b , although a better performance is achieved in the low SNR region, the corresponding throughput in the high SNR region is limited. This is because without transmit CSI at the transmitters, SNCC with RLNC is not able to perform any meaningful rate adaptation to take advantage of the fluctuating channel conditions. However for the proposed JNCC strategy, as demonstrated in Theorem 3.3.1 and Example 1-3, optimal rate adaptation is passively achieved without the requirement for transmit CSI and this translates into good throughput performance throughout the whole SNR range.

3.6 A Random Network Topology

In addition to the toy example shown in Figure 3.3, the proposed JNCC strategy is also applied to a random multi-hop multicast network shown in Figure 3.8. In L_1 , each of the three sources S_1, S_2, S_3 (with the same transmit power P) has $M = 10$ independent messages to transmit and each message is of the same rate $R_0 = 1$. In L_4 , three destinations D_1, D_2, D_3 desire the messages from all sources. 10 relay nodes are then randomly distributed among two level sets L_2 and L_3 . Communication links between neighbouring levels are randomly chosen under the constraint that there is at least a path from each source to each destination. Under the same channel conditions as the toy example above, we obtain the simulation results for the corresponding network throughput averaged over 1000 network realizations in Figure 3.9. To better illustrate the spread of the network throughput with respect to the average value, we also plot the standard deviation and the 10th percentile throughput. To reduce clutter in the figure, we have just shown one representative standard deviation bar for each graph. Again we can see that the proposed JNCC strategy substantially outperforms SNCC with RLNC (different rates of $R_{\text{rlnc}} = bR_0$, $b = 1, 2, 5$ are adopted at the transmitters) in terms of network throughput.

¹⁶We assume that all transmitters in the network (i.e. sources and the intermediate nodes) adopt the same rate of bR_0 .

Figure 3.8: A random multi-hop multicast network.

3.7 Summary

In this chapter, we have considered a wireless multicast network with multiple sources, relays, and destinations. For tractable analysis, we adopted a multi-hop DF relay protocol and considered a feedforward flowgraph where all nodes in the network were divided into disjoint level sets such that the end-to-end information flow can be characterized by separate transmission phases consisting of BC and MAC.

A joint network and channel coding strategy was proposed by exploiting ARQ, receiver side information, and the correlated sources. Without requiring receiver side information for explicit code design, nor transmit channel state information for rate adaptation, a transmitter simply performs retransmissions until each of the intended receivers has successfully decoded all desired messages and sent back an ACK.

To measure the performance of the proposed joint network and channel coding strategy, we quantified the network throughput over retransmissions by applying the renewal-reward theorem. Our results show that the proposed joint network and channel coding strategy substantially outperforms the conventional separate network and channel coding strategy with random linear network coding.

3. JOINT NETWORK AND CHANNEL CODING

Figure 3.9: Network throughput of the random multi-hop multicast network.

Wireless Network Coding with Asymmetric Receiver Side Information

As discussed in Chapter 2 and Chapter 3, due to the available receiver side information (RSI), a higher achievable rate or throughput is achieved [99], [130], [131] with network coding where more information is communicated with fewer transmissions. In this chapter, we will focus on a broadcast channel and analyze how different amount of RSI available at the receivers will affect the overall network throughput when network coding is performed. Two schemes, namely random linear network coding (RLNC) [3]-[6] and round robin scheduling (RRS) [148], will be considered and compared.

Effectively, the accumulation of RSI at each receiver is an innovative packet arrival process from the neighbouring nodes in the previous transmissions. Thus by modeling the arrival of innovative packets at each receiver as a Poisson process [89] with different packet arrival rate, we can model different amount of RSI available at the receivers.

Taking into account the differing amount of RSI available at each receiver, closed-form expressions of network throughput and asymptotic throughput at high SNR will be obtained. Our results show that the asymptotic throughput of RLNC is always higher (or equal) than RRS no matter how much RSI is available at each receiver. With a sum constraint for the total amount of receiver side information in the broadcast channel, the maximum throughput is achieved for RLNC if each receiver accumulates the same

4. WIRELESS NETWORK CODING WITH ASYMMETRIC RECEIVER SIDE INFORMATION

Figure 4.1: A wireless broadcast channel where each receiver may have some RSI about the packets to be transmitted.

amount of RSI.

4.1 Introduction

The broadcast channel [95]-[98] with network coding [1] has recently attracted great interest in research communities [99]-[105], [130], [131]. Most current works assume that each receiver knows *a priori* a certain subset of the transmitted packets as RSI such that with network coding, fewer transmissions are required for the receivers to decode all transmitted packets, thus enhancing the network throughput.

For example, as shown in Figure 4.1(a), a transmitter T has $N = 3$ packets to

broadcast to $M = 3$ receivers. Each receiver has RSI of the packets intended to the other $M - 1 = 2$ receivers. Then through network coding [1], where all 3 packets are combined (e.g. XORed) to a single packet which is then broadcasted, all receivers are able to extract their desired packet in only one transmission. In contrast to a RRS scheme [148] where the 3 packets are separately transmitted from T, this simple network coding operation brings about a 3-fold increase in the overall throughput.

Besides the ideal case described in Figure 4.1(a), the scenario where each receiver has different amount of RSI was considered in [103]. To maximize the throughput for one transmission, an opportunistic scheduling scheme was proposed where only the packets destined for a subset of the receivers are combined and transmitted according to the RSI they have and the instantaneous channel conditions. That is, for the scenario shown in Figure 4.1(b), only packets 1 and 3 are combined and transmitted from T such that receivers R_2 and R_3 can decode both packets 1 and 3 in one transmission. However, this is not a broadcast strictly and most of the time the receivers with less RSI (e.g. receiver R_1) or poor channel conditions will not be served.

To the best of our knowledge, there is no published work discussing about how different amount of RSI available at the receivers will affect the overall network throughput of a broadcast channel when network coding is performed. As shown in Figure 4.1(b), if some receivers have RSI of only one packet or even no side information, is it still beneficial to perform network coding? Specifically, what is the throughput advantage of network coding over RRS? In this chapter, we will analyze how different amount of RSI available at the receivers affects the network throughput of a broadcast channel when network coding is performed.

Basically, the accumulation of RSI at each receiver comes from the correctly decoded packets in the previous transmissions. Due to the channel fluctuations and the dynamic network topology, the accumulation of innovative packets at each receiver resembles a random process where the collection of an innovative packet is analogous to the occurrence of an “event”. Thus instead of arbitrarily assuming certain packets as available RSI in a broadcast channel [99]-[103], [130], [131], we model the accumulation of RSI at each receiver R_m by a Poisson process with innovative packet arrival rate λ_m . With different packet arrival rates, we can thus model different amount of RSI available at the receivers.

4. WIRELESS NETWORK CODING WITH ASYMMETRIC RECEIVER SIDE INFORMATION

We consider two transmission schemes for a broadcast channel, namely random linear network coding (RLNC) [3]-[6] and RRS [148]. To ensure reliable packet delivery, we consider an ARQ scheme where the transmitter T performs retransmissions until an acknowledgement (ACK) is sent back from every receiver indicating successful decoding of the transmitted packet. Taking into account the differing amount of RSI available at each receiver, closed-form expressions of network throughput for both RLNC and RRS are obtained by applying the renewal-reward theorem [142], [143]. The asymptotic throughput at high SNR is also characterized.

Our results show that the RLNC always achieves a higher (or equal) asymptotic throughput than RRS no matter how much RSI is available at each receiver. It will also be shown that with a sum constraint of the innovative packet arrival rate λ_m over all receivers in the broadcast channel, the network throughput of RLNC is maximized if all λ_m are equal.

4.2 System Model

We consider the wireless broadcast channel shown in Figure 4.2. A set of N packets $\mathcal{W}_T = \{P_1, P_2, \dots, P_N\}$, where P_i is of rate R_i , $i \in \{1, 2, \dots, N\}$, is broadcast by the transmitter T to a group of M receivers R_m , $m \in \{1, 2, \dots, M\}$. The transmit power at T is given by P_T . We assume independent block Rayleigh fading channels where $h_{m,t} \sim \mathcal{CN}(0, \delta_m^{-1})$ denotes the channel coefficient of link $T \rightarrow R_m$ at the t th transmission. Thus, we have $\gamma_{m,t} \triangleq |h_{m,t}|^2 \sim \exp(\delta_m)$. The additive white Gaussian noise (AWGN) at each receiver is assumed to be zero mean with the same variance σ^2 . Assuming receiver R_m has perfect knowledge of $h_{m,t}$, the mutual information of the channel $T \rightarrow R_m$ at the t th transmission is given by

$$I_{m,t} = \log_2(1 + \rho\gamma_{m,t})$$

where $\rho = \frac{P_T}{\sigma^2}$ denotes the transmit SNR.

Owing to the independent fading channels between the transmitter and multiple receivers, some receivers may not successfully decode the transmitted packet after a single transmission. To ensure reliable packet delivery, we consider a simple ARQ

Figure 4.2: A wireless broadcast channel where N packets are transmitted to M receivers where each receiver may have some RSI.

scheme where the transmitter performs retransmissions until an ACK¹ is sent back from every receiver indicating successful decoding of the transmitted packet².

With the assistance of the available RSI, denoted by $\mathcal{S}_m \subseteq \mathcal{W}_T$, each receiver R_m attempts to decode all N packets in $\mathcal{W}_T$. The accumulation of $|\mathcal{S}_m| = k$, $k \in \{0, 1, \dots, N\}$, innovative packets as RSI is modeled as a truncated Poisson process

¹For simplicity, we assume that the ACK feedback channel is delay-free and error-free [143].

²As explained later in Section 4.3, for RLNC where all packets are combined to a single packet that is broadcast, an ACK will be sent back from a receiver if it has collected sufficient DoF to decode *all* N packets. In contrast, for RRS where the packets are broadcast sequentially, an ACK will be sent back from a receiver if it has decoded *the transmitted packet*.

4. WIRELESS NETWORK CODING WITH ASYMMETRIC RECEIVER SIDE INFORMATION

with probability mass function (PMF) [89]

$$f_t(k; \lambda_m) = \frac{\lambda_m^k e^{-\lambda_m}}{k!} \frac{1}{\sum_{k=0}^N \frac{\lambda_m^k e^{-\lambda_m}}{k!}} = \frac{\lambda_m^k}{k!} \frac{1}{\sum_{k=0}^N \frac{\lambda_m^k}{k!}}, \quad (4.1)$$

and cumulative distribution function (CDF)

$$F_t(k; \lambda_m) = \sum_{i=0}^k \frac{\lambda_m^i}{i!} \frac{1}{\sum_{i=0}^N \frac{\lambda_m^i}{i!}}. \quad (4.2)$$

With different arrival rates λ_m of innovative packets, receiver R_m is able to accumulate different amount of RSI. The sum over all arrival rates $\sum_m \lambda_m$ characterizes the total amount of RSI available in the broadcast channel on average. By modeling the arrival of RSI at each receiver as an independent truncated Poisson process, we next investigate how different amount of RSI available at the receivers of a broadcast channel will affect the overall network throughput.

4.3 Network Throughput Analysis

Here we adopt the same network throughput analysis as in Chapter 3. We first define an event $\mathcal{A}_{m,t} = \{\text{receiver } R_m \text{ has decoded all } N \text{ packets within } t \text{ retransmissions}\}$. Thus $\mathcal{A}_t \triangleq \bigcap_{m \in \{1,2,\dots,M\}} \mathcal{A}_{m,t}$ corresponds to the event that all M receivers finish decoding all N packets within t retransmissions. Since the receivers experience independent channels, event $\mathcal{A}_t$ occurs with probability

$$\Pr(\mathcal{A}_t) = \prod_{m \in \{1,2,\dots,M\}} \Pr(\mathcal{A}_{m,t}). \quad (4.3)$$

Then the probability that the broadcasting finishes at exactly the T_d th retransmission (and not earlier) is given by [134]-[136]

$$\Pr(C_{T_d}) = \Pr(\mathcal{A}_{T_d}) - \Pr(\mathcal{A}_{T_d-1}). \quad (4.4)$$

In order to obtain the network throughput which quantifies the average rate of successful packet delivery over a network [143], we assume that the transmitter T will initiate the transmission of a new set of N packets right after all M receivers have successfully decoded the previous message set, and this transmission cycle will repeat itself indefinitely. For each transmission cycle, the number of retransmissions required

for all M receivers to finish decoding, i.e. the download time, is denoted by T_d . The reward $\mathfrak{R}$ of the broadcast channel for a single transmission cycle can be quantified by the sum rate of the packets in $\mathcal{W}_T$, i.e. $\mathfrak{R} = \sum_{i=1}^N R_i$.

We assume that the reward $\mathfrak{R}$ is identical for every transmission cycle and each channel is independent and identically distributed (i.i.d.) for every transmission. Thus we can apply the renewal-reward theorem [142], [143] to obtain the network throughput as

$$\eta = \frac{\mathfrak{R}}{E[T_d]} \quad [\text{bits/s/Hz}] \quad (4.5)$$

where

$$E[T_d] = \sum_{T_d=1}^{\infty} T_d \Pr(C_{T_d}). \quad (4.6)$$

Remark 4.3.1. *As long as the reward $\mathfrak{R}$ is finite, through RLNC or RRS, receiver $R_m \forall m$ is able to finish decoding all N packets within a finite number of retransmissions, thus the download time T_d is finite and (4.6) converges.*

4.3.1 Random Linear Network Coding

For RLNC [3]-[6], we adopt an algebraic approach and consider whether each receiver has collected sufficient DoF to decode all transmitted packets. Essentially, the DoF of $\mathcal{W}_T$ is equal to the total number of innovative packets N . We may equivalently associate a vector space Ω to $\mathcal{W}_T$ where $\text{rank}(\Omega) = |\mathcal{W}_T| = N$. The available RSI $\mathcal{S}_m \subseteq \mathcal{W}_T$ can be viewed as a vector subspace $\Omega_m \subseteq \Omega$ with DoF $|\mathcal{S}_m| = \text{rank}(\Omega_m)$. Thus in order to decode all N packets, R_m with $|\mathcal{S}_m| = k$ has to collect an additional $N - k$ DoF.

We define $\mathcal{A}_{m,T_d}^{\text{rlnc}}$ as the event that R_m has collected $N - k$ DoF to decode all N packets within T_d retransmissions. Denoting s_{m,T_d} as the number of correctly received packets out of T_d retransmissions, as a best-case lower bound³, we have $\mathcal{A}_{m,T_d}^{\text{rlnc}} = \{s_{m,T_d} \geq N - k\}$. For simplicity, we assume that $R_i = R_0 \forall i$, then the outage probability for link $T \rightarrow R_m$ is given by

$$p_{m,t} = \Pr(I_{m,t} < R_0) \triangleq p_m \quad (4.7)$$

where $p_{m,t}$ is independent of t . Thus the random variable s_{m,T_d} follows a binomial distribution $\mathcal{B}(T_d, (1 - p_m))$ with T_d trials and success probability of $(1 - p_m)$. It is

³We assume that the finite field size used for RLNC is sufficiently large.

4. WIRELESS NETWORK CODING WITH ASYMMETRIC RECEIVER SIDE INFORMATION

clear that $\Pr(\mathcal{A}_{m,T_d}^{\text{rlnc}}) = 0$ if $T_d < N - k$. Intuitively, this is because to collect $N - k$ DoF, at least $N - k$ transmissions are needed. Hence, event $\mathcal{A}_{m,T_d}^{\text{rlnc}}$ occurs with probability

$$\Pr(\mathcal{A}_{m,T_d}^{\text{rlnc}}) = \sum_{k=0}^N \left[\sum_{t=N-k}^{T_d} \binom{T_d}{t} (1 - p_m)^t p_m^{(T_d-t)} \right] \cdot f_t(k; \lambda_m). \quad (4.8)$$

Then event $\mathcal{A}_{T_d}^{\text{rlnc}} = \cap_{m \in \{1, 2, \dots, M\}} \mathcal{A}_{m,T_d}^{\text{rlnc}}$ occurs with probability

$$\Pr(\mathcal{A}_{T_d}^{\text{rlnc}}) = \prod_{m \in \{1, 2, \dots, M\}} \Pr(\mathcal{A}_{m,T_d}^{\text{rlnc}}). \quad (4.9)$$

Substituting (4.9) into (4.4)-(4.6), we can thus derive the network throughput for RLNC as

$$\eta^{\text{rlnc}} = \frac{\Re}{\sum_{T_d=1}^{\infty} T_d \left[\Pr(\mathcal{A}_{T_d}^{\text{rlnc}}) - \Pr(\mathcal{A}_{T_d-1}^{\text{rlnc}}) \right]}. \quad (4.10)$$

4.3.2 Round Robin Scheduling

For comparison purposes, we derive similar network throughput expression for RRS. In RRS [148], retransmissions are carried out for a packet P_i , $i \in \{1, 2, \dots, N-1\}$, until it has been decoded by all M receivers, then packet P_{i+1} is broadcasted by T. We assume that the innovative packets known *a priori* at a receiver are uniformly distributed, i.e., if receiver R_m has $|\mathcal{S}_m| = k$ innovative packets as RSI, then the probability that one of these packets is P_i is given by $\Pr(R_m \text{ has } P_i \mid |\mathcal{S}_m| = k) = \frac{k}{N}$. Thus from (4.1), the probability that R_m has P_i as RSI is given by

$$\begin{aligned} \Pr(R_m \text{ has } P_i) &= \sum_{k=0}^N \Pr(R_m \text{ has } P_i \mid |\mathcal{S}_m| = k) \cdot \Pr(|\mathcal{S}_m| = k) \\ &= \frac{1}{N} \sum_{k=0}^N k \cdot f_t(k; \lambda_m) = \frac{E[|\mathcal{S}_m|]}{N}, \end{aligned} \quad (4.11)$$

where $E[|\mathcal{S}_m|] = \sum_{k=0}^N k \cdot f_t(k; \lambda_m)$ denotes the average number of innovative packets known *a priori* at receiver R_m .

During the broadcasting of packet P_i , for those receivers that already have P_i , they will feedback to T immediately after the first transmission. For those receivers that do not have P_i , they will attempt to decode P_i through maximal ratio combining (MRC) of all packets received over the retransmissions.

We define $\mathcal{A}_{m,T_i}^{\text{rrs}}$ as the event that R_m finishes decoding P_i within T_i retransmissions. The probability that R_m finishes decoding P_i within the first transmission is given by

$$\begin{aligned} \Pr(\mathcal{A}_{m,T_i=1}^{\text{rrs}}) &= \Pr(R_m \text{ has } P_i) \\ &+ \Pr(R_m \text{ does not have } P_i) \Pr(\log_2(1 + \rho\gamma_{m,1}) > R_i) \\ &= \frac{E[|\mathcal{S}_m|]}{N} + \left(1 - \frac{E[|\mathcal{S}_m|]}{N}\right) \Pr\left(\gamma_{m,1} > \frac{2^{R_i} - 1}{\rho}\right). \end{aligned}$$

Similarly, the probability that R_m finishes decoding for P_i within $T_i \geq 2$ retransmissions is

$$\begin{aligned} \Pr(\mathcal{A}_{m,T_i}^{\text{rrs}}) &= \Pr(R_m \text{ has } P_i) \\ &+ \Pr(R_m \text{ does not have } P_i) \Pr\left(\log_2\left(1 + \rho \sum_{t=1}^{T_i} \gamma_{m,t}\right) > R_i\right) \\ &= \frac{E[|\mathcal{S}_m|]}{N} + \left(1 - \frac{E[|\mathcal{S}_m|]}{N}\right) \Pr\left(\sum_{t=1}^{T_i} \gamma_{m,t} > \frac{2^{R_i} - 1}{\rho}\right). \end{aligned}$$

Since $\gamma_{m,t} \sim \exp(\delta_m)$, we have from [89] $\Pr\left(\sum_{t=1}^{T_i} \gamma_{m,t} > \frac{2^{R_i}-1}{\rho}\right) = 1 - F_g(G_i; T_i, \delta_m^{-1})$, where $F_g(\cdot; \cdot, \cdot)$ is the CDF of a Gamma distribution and $G_i = \frac{2^{R_i}-1}{\rho}$. Thus event $\mathcal{A}_{m,T_i}^{\text{rrs}}$, $T_i \geq 1$, occurs with probability

$$\Pr(\mathcal{A}_{m,T_i}^{\text{rrs}}) = \frac{E[|\mathcal{S}_m|]}{N} + \left(1 - \frac{E[|\mathcal{S}_m|]}{N}\right) [1 - F_g(G_i; T_i, \delta_m^{-1})]. \quad (4.12)$$

Next, we define $\mathcal{A}_{T_i}^{\text{rrs}} = \cap_{m \in \{1,2,\dots,M\}} \mathcal{A}_{m,T_i}^{\text{rrs}}$ as the event that all M receivers finish decoding P_i within T_i retransmissions, which occurs with probability

$$\Pr(\mathcal{A}_{T_i}^{\text{rrs}}) = \prod_{m \in \{1,2,\dots,M\}} \Pr(\mathcal{A}_{m,T_i}^{\text{rrs}}). \quad (4.13)$$

Substituting (4.13) into (4.4)-(4.6), we obtain

$$E[T_i] = \sum_{T_i=1}^{\infty} T_i [\Pr(\mathcal{A}_{T_i}^{\text{rrs}}) - \Pr(\mathcal{A}_{T_i-1}^{\text{rrs}})].$$

Then the corresponding average download time required for all M receivers to decode all N packets in $\mathcal{W}_T$ is given by

$$E[T_d] = \sum_{i=1}^N E[T_i]. \quad (4.14)$$

4. WIRELESS NETWORK CODING WITH ASYMMETRIC RECEIVER SIDE INFORMATION

Thus we can derive the network throughput for RRS as

$$\eta^{\text{rrs}} = \frac{\Re}{\sum_{i=1}^N \sum_{T_i=1}^{\infty} T_i \left[\Pr(\mathcal{A}_{T_i}^{\text{rrs}}) - \Pr(\mathcal{A}_{T_i-1}^{\text{rrs}}) \right]}. \quad (4.15)$$

Remark 4.3.2. For RRS, transmitter T can also broadcast packets $P_1, P_2, \dots, P_N$ one by one in a circular manner, i.e. in the t th transmission where $t = 1, 2, \dots$, packet $P_{t \bmod N}$ is broadcast unless it has already been successfully decoded by all M receivers. This will end up with the same analysis and results as above.

4.4 Asymptotic Throughput at High SNR

Theorem 4.4.1. We consider the broadcast channel shown in Figure 4.2, where a set of N packets $\mathcal{W}_T = \{P_1, P_2, \dots, P_N\}$ is broadcasted by the transmitter T to a group of M receivers R_m , $m = 1, 2, \dots, M$, and each of them may have some receiver side information of the packets to be transmitted. Then in the high SNR region, random linear network coding always achieves a higher (or equal) asymptotic throughput than round robin scheduling.

Proof. 1. Asymptotic throughput of RLNC

We denote T_m as the download time for receiver R_m . From (4.7), $p_m \rightarrow 0 \forall m$ when $\rho \rightarrow \infty$. Thus it requires only $\lim_{\rho \rightarrow \infty} T_m = N - |\mathcal{S}_m|$ transmissions for R_m to decode all N packets. Since the overall download time T_d is limited by the worst receiver that finishes decoding last, we have

$$\lim_{\rho \rightarrow \infty} T_d = \max_m \left(\lim_{\rho \rightarrow \infty} T_m \right) = N - \min_m (|\mathcal{S}_m|). \quad (4.16)$$

We let $Z = \min_m (|\mathcal{S}_m|)$, $Z = 0, 1, \dots, N$. At each receiver R_m , since the arrival of $|\mathcal{S}_m|$ innovative packets is modeled as an independent truncated Poisson process given in (4.1) and (4.2), we have the CDF of Z as

$$\begin{aligned} F_Z(z) &= 1 - \Pr(Z > z) \\ &= 1 - \Pr(|\mathcal{S}_1| > z) \cdot \Pr(|\mathcal{S}_2| > z) \cdots \Pr(|\mathcal{S}_M| > z) \\ &= 1 - [1 - F_t(z, \lambda_1)] \cdot [1 - F_t(z, \lambda_2)] \cdots [1 - F_t(z, \lambda_M)]. \end{aligned} \quad (4.17)$$

Then the corresponding PMF of Z can be obtained by $f_Z(z) = F_Z(z) - F_Z(z-1)$. Thus we have the average download time when $\rho \rightarrow \infty$

$$E[\lim_{\rho \rightarrow \infty} T_d] = \sum_z (N - z) f_Z(z) = N - \sum_z z \cdot f_Z(z), \quad (4.18)$$

and the corresponding asymptotic throughput

$$\lim_{\rho \rightarrow \infty} \eta^{\text{rlnc}} = \frac{\Re}{N - \sum_z z \cdot f_Z(z)}. \quad (4.19)$$

2. Asymptotic throughput of RRS

When $\rho \rightarrow \infty$, we have from (4.12) $\lim_{\rho \rightarrow \infty} \Pr(\mathcal{A}_{m,T_i=1}^{\text{rrs}}) = 1, \forall m, i$, thus

$$\lim_{\rho \rightarrow \infty} \Pr(\mathcal{A}_{T_i=1}^{\text{rrs}}) = \prod_{m \in \{1,2,\dots,M\}} \lim_{\rho \rightarrow \infty} \Pr(\mathcal{A}_{m,T_i=1}^{\text{rrs}}) = 1, \forall i \quad (4.20)$$

which means that it requires only $\lim_{\rho \rightarrow \infty} T_i = 1$ transmission for all M receivers to successfully decode P_i . Then we can obtain the corresponding average download time as

$$E[\lim_{\rho \rightarrow \infty} T_d] = E[\lim_{\rho \rightarrow \infty} \sum_{i=1}^N T_i] = E[\sum_{i=1}^N \lim_{\rho \rightarrow \infty} T_i] = N, \quad (4.21)$$

and the asymptotic throughput is thus

$$\lim_{\rho \rightarrow \infty} \eta^{\text{rrs}} = \frac{\Re}{N}. \quad (4.22)$$

Thus from (4.19) and (4.22), the random linear network coding always achieves a higher (or equal) throughput than round robin scheduling in the high SNR region. $\square$

Rather than investigating the effect of the available RSI on the overall network throughput, there are some existing works talking about network coding and scheduling schemes where the advantages of network coding were demonstrated in terms of the delay performance or throughput.

Scaling laws governing the delay performance of network coding and scheduling were studied and compared in a cellular downlink channel [145]. Simulation results showed that with different traffic modes, network coding achieves significant gains over scheduling. The asymptotic delay performance of network coding and round robin scheduling was analyzed and compared in [146], the results showed that the performance of both network coding and round robin scheduling in a noisy channel can be substantially worse than that in a noiseless channel, and network coding achieves a better performance than round robin scheduling. A network with a source serving multiple multicast flows was considered in [147], where scheduling and network coding schemes were compared. The authors showed that there are instances where the random linear network coding outperforms any scheduling strategy in terms of sum stable throughput.

4. WIRELESS NETWORK CODING WITH ASYMMETRIC RECEIVER SIDE INFORMATION

Table 4.1: Various arrival rates of RSI with the constraint $\sum_m \lambda_m = \Lambda$

Arrival rate	λ_1	λ_2	λ_3	λ_4	λ_5	$\min_m (\lambda_m)$
Scenario 1	0.2Λ	0.2Λ	0.2Λ	0.2Λ	0.2Λ	0.2Λ
Scenario 2	0.6Λ	0.1Λ	0.1Λ	0.1Λ	0.1Λ	0.1Λ
Scenario 3	Λ	0	0	0	0	0

4.5 Simulation Results

For the broadcast channel shown in Figure 4.2, we assume that the transmitter T has a set of $N = 10$ innovative packets where $R_i = R_0 = 1 \forall i$ to broadcast to a group of $M = 5$ receivers. We let $\delta_m = 1 \forall m$. Thus for RLNC, the outage probability is given by

$$p_{m,t} = 1 - e^{-\frac{2^{R_0}-1}{\rho}}, \forall m, t. \quad (4.23)$$

To evaluate how different amount of RSI available at the receivers will affect the network throughput of a broadcast channel, we consider a sum constraint where $\sum_m \lambda_m = \Lambda$. In Table 4.1, Scenario 1 represents the case where the arrival rate of innovative packets is the same for all receivers. Scenario 2 represents the case where the arrival rate is skewed towards a single receiver (i.e. R_1). Scenario 3 is the extreme case where all RSI is concentrated in just one receiver (i.e. R_1).

Matlab is used in deriving all simulation results in this chapter. Given the channel parameters, i.e. δ_m , each channel $T \rightarrow R_m$ is randomly generated through Monte Carlo method. For the transmission of a packet, it is successfully delivered if the target rate R_0 can be supported by the instantaneous channel, otherwise an outage occurs. A sample program for the simulation of η^{rlnc} in (4.10) is provided in Appendix A.3 and the corresponding results are shown in Figure 4.3.

In Figure 4.3 and 4.4, the network throughput is shown for both RLNC and RRS under different scenarios with the constraint of $\Lambda = 20$ and 40 respectively. We can see that the analytical results agree well with the simulation results. The network throughput of RLNC is bounded tightly by the corresponding asymptotic throughput given in (4.19). And for RRS, its network throughput is always upper bounded by $\lim_{\rho \rightarrow \infty} \eta^{\text{rrs}} = \frac{\mathfrak{R}}{N} = 1$ as given in (4.22).

An important observation from Figure 4.3 and 4.4 is that the case where each receiver is able to accumulate the same amount of RSI (Scenario 1) achieves the maxi-

Figure 4.3: Network throughput with $\Lambda = 20$.

imum throughput⁴ for RLNC. Similar observation holds for RRS, though the difference in throughput is less significant. When some receiver is able to accumulate more RSI and others have little or even no RSI available (Scenarios 2 and 3), the network throughput of RLNC could be severely reduced since the download time is limited by the worst receiver with the least amount of RSI. However, as proved in Theorem 4.4.1, we can see that in Figure 4.3 and 4.4 the asymptotic throughput of RLNC is always higher (or equal) than RRS no matter how much RSI is available at each receiver. This is because in RLNC, all packets are combined to a single packet which is broadcast such that the available RSI can be utilized to effectively reduce the number of transmissions required for all receivers to finish decoding, as shown in (4.18). In contrast, for RRS where the

⁴An extension where the receivers have different channel conditions and how this affects the optimal distribution of RSI such that the maximum throughput is achieved will be discussed in Chapter 6.

Figure 4.4: Network throughput with $\Lambda = 40$.

packets are transmitted separately, it requires at least N transmissions for all receivers to decode all N packets no matter how much RSI is available at the receivers, as shown in (4.21). Similar results can be observed in Figure 4.4 where higher network throughput can be achieved for RLNC since more innovative packets have been accumulated as RSI at the receivers.

4.6 Summary

In this chapter, by modeling the arrival of innovative packets at each receiver as an independent Poisson process, we have investigated how different amount of RSI available at the receivers affects the overall network throughput of a broadcast channel when network coding is performed.

Two schemes, namely random linear network coding and round robin scheduling, were considered. Taking into account the differing amount of RSI available at each receiver, the corresponding network throughput was derived in closed-form expressions and the asymptotic throughput at high SNR was also characterized.

Our results showed that the asymptotic throughput of RLNC is always no worse than RRS no matter how much RSI is available at each receiver. With a sum constraint for the overall arrival rate of innovative packets in the broadcast channel, i.e. $\sum_m \lambda_m = \Lambda$, the case where each receiver is able to accumulate the same amount of RSI achieves the maximum throughput for RLNC.

4. WIRELESS NETWORK CODING WITH ASYMMETRIC RECEIVER SIDE INFORMATION

Cognitive Spectrum Sharing with Cooperative Relay Systems

To accommodate the exponentially emerging wireless devices and services, various spectrum sharing models, protocols, and standards [152]-[168] have been proposed to improve the spectral efficiency. In this chapter, network coding will be applied to a wireless cooperative and cognitive communication scenario [166] to achieve spectrum sharing between the primary and secondary systems.

We consider a scenario where two primary users communicate with each other, upon transmission failure between the primary users, a cognitive user is willing to serve as a primary relay to cooperate with the primary system. As an incentive, the cognitive user gains spectrum access by superimposing the secondary transmission on network coded [1] primary signals. Through a careful power allocation, it is possible that a better performance can be achieved by the primary system than the case without spectrum sharing [157].

We will analytically derive the outage probabilities for both primary and secondary systems with the proposed cognitive two-way relay protocol. Our results show that a critical region exists such that as long as the secondary transceiver is located within this region, there always exists a power allocation threshold above which the proposed protocol is able to provide a better (or equal) outage performance for the primary system, and at the same time achieve secondary spectrum sharing.

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

Table 5.1: Comparison between interweave and overlay cognitive radio models.

Interweave	Overlay
The secondary system operates under the constraint that it causes minimal performance degradation to the primary system	
Secondary user constantly senses the frequency bands and it knows the spectrum holes in space, time, or frequency when the primary user is not transmitting	Secondary user knows the channel gains, codewords, and codebook of the primary system
Secondary user transmits simultaneously with the primary user only in the event of a false spectral hole detection and this will cause severe interference to the primary system	Secondary user is able to transmit simultaneously with the primary user in the same frequency band. The interference to primary system can be offset by using part of the secondary user's power to relay the primary signals

5.1 Introduction

With exponentially increasing wireless devices and services squeezing into the already crowded wireless spectrum, almost all frequency bands below 3 GHz have been preassigned as shown in the National Telecommunications and Information Administrations frequency allocation chart [150]. On the other hand, spectrum usage measurement [151] indicates that at any given time or location, a large proportion of the licensed spectrum lies idle. This contradiction has lead to great research interest in different models for spectrum sharing [153]-[168] in the last decade. With the increasing efforts being injected in this promising field for continuable reuse and development of frequency resources, the first cognitive radio wireless regional area network standard [152], IEEE 802.22, is on its way to put to good use in practice.

Being aware of the environment, cognitive radio [154], [157] is an intelligent wireless communication system which is able to learn from the environment and make corresponding changes in certain operating parameters to adapt to the outside variations. To ensure reliable communications as well as improve efficient utilization of the radio spectrum, cognitive radios operate under the constraint that the secondary transmissions result in limited harmful interference to the primary system [155]-[157].

In contrast to the conventional interweave model [153] where the secondary users constantly sense the licensed frequency bands for spectrum holes, much attention has

Figure 5.1: An interference channel.

been given recently to an overlay model [157] where through sophisticated signal processing, the secondary users are able to operate simultaneously with the primary system in the same frequency band [159]-[165]. For a general comparison between the interweave and overlay cognitive radio models, please refer to Table 5.1. Although the overlay model requires more sophisticated signal processing at the cognitive radios, it provides a feasible technique to achieve spectrum sharing in urban areas where the spectrum is too crowded for spectrum holes to exist for the cognitive users in interweave models.

The scenario where primary and the cognitive radios wish to communicate simultaneously with different receivers and subject to mutual interference falls in the scope of an interference channel [158]. As shown in Figure 5.1, in an interference channel, two independent sources transmit separate symbols X_1 and X_2 simultaneously to two independent destinations respectively through a common channel where Y_1 and Y_2 denote the corresponding received symbols. The solid lines denote the transmission to desired destinations and the dashed lines denote the mutual interference. Each receiver attempts to retrieve their respective source symbol which is subjected to the interference from the other source and the additive noise denoted by N_1 and N_2 .

In [159], the cognitive radio channel is defined as a two-sender, two-receiver interference channel as shown in Figure 5.1, where sender 2 (secondary transmitter) is assumed to have perfect knowledge of the codewords sender 1 (primary transmitter)

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

plans to transmit. Borrowing ideas from Gel'fand and Pinsker's coding for channels with known interference at the transmitter [160], Costa's dirty-paper coding [161], and the achievable rate region of the interference channel described by Han and Kobayashi [162], an achievable rate region was developed which falls between the capacities of a conventional interference channel (no cooperation between two transmitters) and a 2x2 MIMO channel (full cooperation between two transmitters). Based on the same interference channel model, different achievable rate regions for cognitive radios have been obtained in [163], [164] where the secondary transmitter has knowledge, full or partial, of the messages transmitted by the primary transmitter.

Motivated by the same model as in [159], an information-theoretic perspective of overlay cognitive radios was presented in [165]. The fundamental limits of cognitive radio networks were studied under the constraint that no rate degradation is created for the primary system and the primary receiver uses a single-user decoder, as if there is no cognitive radio. With the assumption that the cognitive transmitter has non-causal knowledge of the codewords of the primary transmitter [161], achievable rates of the cognitive user were characterized under different constraints.

However, the assumption that secondary transmitter knows perfectly the codewords that primary transmitter plans to transmit even before primary transmitter starts transmitting is unrealistic. By considering a cooperative relay channel as shown in Figure 5.2, a causal overlay cognitive radio network was discussed in [167] and [168]. A cognitive spectrum sharing scheme was proposed where the secondary transmitter (ST) obtains the packets of the primary transmitter (PT) in a causal manner by acting as a relay for the primary system in phase 1. When ST helps relay the primary packet to the primary receiver (PR) in phase 2, it superimposes its own secondary packet on the primary packet which is desired at the secondary receiver (SR). The outage performance of the primary system can be improved due to the cooperation from the secondary system and at the same time secondary spectrum sharing is achieved.

In this chapter, we will consider the overlay cognitive radio model and extend the cooperative and cognitive relay protocols [166]-[168] to a two-way relay scenario where two primary users A and B communicate with each other with the assistance of a secondary transceiver C as shown in Figure 5.3. The primary system is ignorant of the presence of the secondary system, and the onus is on C to "disguise" itself as a primary relay. To achieve spectrum sharing without degrading the performance of the primary

Figure 5.2: A cooperative and cognitive relay channel.

system, C allocates a fraction of its transmit power to help relay the primary signals, such that the interference to the primary system due to the secondary transmission can be properly compensated by the cooperation from C [157]. A similar mechanism was presented in [166] where the primary system explicitly decides whether to lease the spectrum to secondary users in exchange for cooperation (a fraction of the leased time is used to relay the primary transmission), such that its quality of service (e.g. rate or outage probability) can be maximized.

We will analytically derive the outage probabilities for both the primary and secondary systems with the proposed cognitive two-way relay (CTR) protocol. A critical region is obtained such that as long as the secondary transceiver C is located within this region, it can always choose a suitable power allocation to relay the primary signals such that a better (or equal) outage performance can be achieved by the primary system than in the case without spectrum sharing, and at the same time the secondary system gains an opportunity for spectrum sharing.

5.2 System Model

Consider a bi-directional communication scenario where two primary users A and B wish to communicate with each other. Whenever the channel condition between A and B drops below a particular threshold, they will seek cooperation from neighboring nodes. Using a practical handshake mechanism [171], [172], primary user A first broadcasts a cooperative right-to-send (CRTS) message, and B responds to CRTS by broadcasting a cooperative clear-to-send (CCTS) message. We assume that there exists a secondary user C in between who is willing and also capable to help the primary

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

Figure 5.3: System model of the proposed cognitive two-way relay protocol.

transmissions. Upon overhearing CRTS and CCTS, C broadcasts a cooperative clear to help (CCTH) message, and upon receiving this confirmation, the primary system correspondingly switches to a relay transmission mode¹, with C acting as the relay.

As shown in Figure 5.3, we consider a three-phase CTR scenario where a DF relay protocol is adopted at the the secondary transceiver C. In phase 1, primary user A transmits x_A , followed by B transmitting x_B in phase 2, and C attempts to decode both

¹For the proposed CTR protocol, we assume that the primary system supports relay functionality and network coding [33] such that the switch to relay mode is within the operating capability of the primary users.

x_A and x_B . If both x_A and x_B are correctly decoded², a network-coded [1] primary signal, i.e. $x_A \oplus x_B$, will be broadcast by C in phase 3 together with the secondary signal x_S intended for the secondary receiver D. Superposition coding is utilized at C where a fraction α , $0 < \alpha < 1$, of its transmit power is assigned to relay $x_A \oplus x_B$, and the remaining power for x_S as given in (5.5) in the next section. Through a careful choice of α , a better (or equal) outage performance can be achieved for the primary system than in the case without spectrum sharing, and at the same time secondary spectrum sharing can be achieved.

We assume that all nodes in the proposed CTR protocol operate in half-duplex mode and the channels are reciprocal and experience independent block Rayleigh fading with channel coefficients denoted in Figure 5.3. Thus, we have $h_{ij} \sim \mathcal{CN}(0, d_{ij}^{-\nu})$, $i, j \in \{A, B, C, D\}$, $i \neq j$, where ν is the path-loss exponent and d_{ij} is the normalized distance between transmitter i and receiver j . This normalization is done with respect to the distance between A and B, i.e. $d_{AB} = 1$. We denote $\gamma_{ij} = |h_{ij}|^2$, thus $\gamma_{ij} \sim \exp(d_{ij}^{-\nu})$. Let x_A , x_B , and x_S denote the primary signal from A, B, and the secondary signal from C respectively, with zero mean and $E[|x_A|^2] = E[|x_B|^2] = E[|x_S|^2] = 1$. The transmit powers at A and B are equal and is denoted as P , and the transmit power at C is denoted as P_S . The additive white Gaussian noise (AWGN) at receiver j is denoted as $n_j \sim \mathcal{CN}(0, \sigma^2)$, $\forall j \in \{A, B, C, D\}$.

5.3 Protocol Descriptions and Performance Analysis

In phase 1, x_A is transmitted by primary user A, the corresponding received signal at B, C, and D is

$$y_{j,1} = \sqrt{P}h_{Aj}x_A + n_j, \quad \text{where } j \in \{B, C, D\}. \quad (5.1)$$

The achievable rate for link $A \rightarrow j$ is thus

$$R_{Aj} = \frac{1}{3} \log_2 \left(1 + \frac{P\gamma_{Aj}}{\sigma^2} \right), \quad j \in \{B, C, D\} \quad (5.2)$$

where $\frac{1}{3}$ accounts for the fact that the overall transmission is being split into three phases.

²The other scenarios where incorrect decoding occurs at the secondary transceiver C will be discussed in detail in the next section.

Figure 5.4: Different scenarios in the proposed CTR protocol.

Then in phase 2, x_B is transmitted by primary user B. The corresponding received signal at A, C, and D is

$$y_{j,2} = \sqrt{P}h_{Bj}x_B + n_j, \quad \text{where } j \in \{A, C, D\}. \quad (5.3)$$

The achievable rate for link $B \rightarrow j$ is similarly

$$R_{Bj} = \frac{1}{3} \log_2 \left(1 + \frac{P\gamma_{Bj}}{\sigma^2} \right), \quad j \in \{A, C, D\}. \quad (5.4)$$

5.3.1 Outage Performance of Primary System

Upon receiving $y_{C,1}$ and $y_{C,2}$, secondary transceiver C attempts to decode both x_A and x_B , giving rise to the following three scenarios³ as illustrated in Figure 5.4.

³We assume that the occurrence of a particular scenario will be conveyed to A, B and D through simple MAC control messages inserted in the header of each data frame. Such control messages will be

Scenario 1: Both x_A and x_B are successfully decoded. In this case, a network-coded [1] signal $x_A \oplus x_B$ (bit-wise XOR) is superimposed with the secondary signal x_S to obtain a composite signal

$$x_C = \sqrt{\alpha P_S}(x_A \oplus x_B) + \sqrt{(1-\alpha)P_S}x_S. \quad (5.5)$$

Then in phase 3, x_C is broadcast by C and the corresponding received signal at A, B, and D is

$$y_{j,3} = \sqrt{\alpha P_S}h_{Cj}(x_A \oplus x_B) + \sqrt{(1-\alpha)P_S}h_{Cj}x_S + n_j, \quad j \in \{A, B, D\}. \quad (5.6)$$

Primary user B attempts to decode x_A from $y_{B,3}$. The achievable rate for link C→B, conditioned on the successful decoding of both x_A and x_B at C, is thus given by

$$R_{CB}^{\text{NC}} = \frac{1}{3} \log_2 \left(1 + \frac{\alpha P_S \gamma_{CB}}{(1-\alpha)P_S \gamma_{CB} + \sigma^2} \right). \quad (5.7)$$

As long as $(x_A \oplus x_B)$ is successfully decoded at B, x_A can be recovered since x_B is known *a priori* at B. Otherwise, an outage will be declared. Do note that the network-coded signal $x_A \oplus x_B$ is different from x_A and thus maximal-ratio combining (MRC) cannot be used to combine the two signals. The same argument applies for the decoding of x_B at primary user A.

Scenario 2: Only one of x_A and x_B is successfully decoded, e.g. x_A . Then x_A is superimposed with x_S to obtain a composite signal

$$x_C = \sqrt{\alpha P_S}x_A + \sqrt{(1-\alpha)P_S}x_S. \quad (5.8)$$

Then in phase 3, x_C is broadcast by C and the corresponding received signal at A, B, and D is

$$y_{j,3} = \sqrt{\alpha P_S}h_{Cj}x_A + \sqrt{(1-\alpha)P_S}h_{Cj}x_S + n_j, \quad j \in \{A, B, D\}. \quad (5.9)$$

Since the broadcast signal x_C does not contain x_B , it is of no interest to primary user A and thus A will simply ignore the received signal in phase 3. For this scenario, $y_{B,1}$ and $y_{B,3}$ are combined at B using maximal-ratio combining (MRC) to decode for x_A . The achievable rate for link C→B, conditioned on the successful decoding of only x_A at C, is thus

$$R_{CB}^{\text{MRC}} = \frac{1}{3} \log_2 \left(1 + \frac{\alpha P_S \gamma_{CB}}{(1-\alpha)P_S \gamma_{CB} + \sigma^2} + \frac{P \gamma_{AB}}{\sigma^2} \right). \quad (5.10)$$

necessary in any system that supports two-way relay.

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

On the other hand, since x_B fails to be decoded at C, A will attempt to decode x_B from the direct link B→A (using $y_{A,2}$) only. The same argument applies for the case where only x_B is successfully decoded at C.

Scenario 3: Neither x_A nor x_B is successfully decoded. Then C simply broadcasts a negative acknowledgement (NACK) message in phase 3. A and B will then attempt to decode for x_B and x_A using $y_{A,2}$ and $y_{B,1}$ respectively.

From the above three mutually exclusive scenarios, an outage is declared if x_A (or x_B) cannot be successfully decoded at B (or A) at the end of phase 3. The overall outage probability for the transmission of x_A with target rate R_{pt} is thus given by

$$\Pr(O_A^p) = 1 - \Pr(R_{AC} > R_{pt}) \Pr(R_{BC} > R_{pt}) \Pr(R_{CB}^{\text{NC}} > R_{pt}) \quad (5.11)$$

$$- \Pr(R_{AC} > R_{pt}) \Pr(R_{BC} < R_{pt}) \Pr(R_{CB}^{\text{MRC}} > R_{pt}) \quad (5.12)$$

$$- \Pr(R_{AC} < R_{pt}) \Pr(R_{BC} > R_{pt}) \Pr(R_{AB} > R_{pt}) \quad (5.13)$$

$$- \Pr(R_{AC} < R_{pt}) \Pr(R_{BC} < R_{pt}) \Pr(R_{AB} > R_{pt}). \quad (5.14)$$

For (5.2), (5.4), and (5.7), we have from [89]

$$\Pr(R_{ij} > R_{pt}) = e^{-d_{ij}^{\nu} \frac{R_P}{\rho_P}}, \quad (5.15)$$

and

$$\Pr(R_{CB}^{\text{NC}} > R_{pt}) = \begin{cases} 0, & 0 < \alpha < \frac{R_P}{1+R_P}, \\ e^{-d_{CB}^{\nu} \frac{R_P}{[\alpha(1+R_P)-R_P]\rho_S}}, & \frac{R_P}{1+R_P} < \alpha < 1, \end{cases} \quad (5.16)$$

where $R_P = 2^{3R_{pt}} - 1$, $\rho_P = \frac{P}{\sigma^2}$, $\rho_S = \frac{P_S}{\sigma^2}$. Assuming $P_S \gg \sigma^2$, we have from (5.10)

$$\begin{aligned} \Pr(R_{CB}^{\text{MRC}} > R_{pt}) &\approx \Pr\left\{\frac{1}{3} \log_2 \left(1 + \frac{\alpha}{(1-\alpha)} + \frac{P\gamma_{AB}}{\sigma^2}\right) > R_{pt}\right\} \\ &= \begin{cases} e^{-\frac{R_P - \alpha(1+R_P)}{(1-\alpha)\rho_P}}, & 0 < \alpha < \frac{R_P}{1+R_P}, \\ 1, & \frac{R_P}{1+R_P} < \alpha < 1. \end{cases} \end{aligned} \quad (5.17)$$

Substituting (5.15)-(5.17) into (5.11)-(5.14), we are able to derive $\Pr(O_A^p)$ as

$$\Pr(O_A^p) \approx \begin{cases} \Pr(O_A^{p,1}), & 0 < \alpha < \frac{R_P}{1+R_P}, \\ \Pr(O_A^{p,2}), & \frac{R_P}{1+R_P} < \alpha < 1, \end{cases} \quad (5.18)$$

where

$$\begin{aligned} \Pr(O_A^{p,1}) &= 1 - e^{-d_{AC}^{\nu} \frac{R_P}{\rho_P}} \left(1 - e^{-d_{BC}^{\nu} \frac{R_P}{\rho_P}}\right) e^{-\frac{R_P - \alpha(1+R_P)}{(1-\alpha)\rho_P}} \\ &\quad - \left(1 - e^{-d_{AC}^{\nu} \frac{R_P}{\rho_P}}\right) e^{-\frac{R_P}{\rho_P}}, \end{aligned} \quad (5.19)$$

$$\begin{aligned}
 \Pr(O_A^{p,2}) = 1 & - e^{-(d_{AC}^\nu + d_{BC}^\nu) \frac{R_P}{\rho_P}} e^{-d_{CB}^\nu \frac{R_P}{[\alpha(1+R_P)-R_P]\rho_S}} \\
 & - e^{-d_{AC}^\nu \frac{R_P}{\rho_P}} \left(1 - e^{-d_{BC}^\nu \frac{R_P}{\rho_P}} \right) \\
 & - \left(1 - e^{-d_{AC}^\nu \frac{R_P}{\rho_P}} \right) e^{-\frac{R_P}{\rho_P}}.
 \end{aligned} \tag{5.20}$$

The outage probability for the transmission of x_B , i.e. $\Pr(O_B^p)$, can be derived in the same way as $\Pr(O_A^p)$.

5.3.2 Spectrum Sharing Requirement and Critical Region

For the original primary system without spectrum sharing, the achievable rate for link $A \leftrightarrow B$ is $R'_{AB} = \frac{1}{2} \log_2 \left(1 + \frac{P\gamma_{AB}}{\sigma^2} \right)$. Thus the outage probability for x_A (x_B) with target rate R_{pt} is

$$\Pr(O_A) = \Pr(O_B) = \Pr(R'_{AB} < R_{pt}) = 1 - e^{-\frac{R'_P}{\rho_P}}, \tag{5.21}$$

where $R'_P = 2^{2R_{pt}} - 1$.

Cognitive radio operates under the constraint that it causes minimal degradation to the performance of the primary system. That is, in the proposed CTR protocol, the outage performance of the primary system is better than (or equal to) the case without spectrum sharing, i.e.

$$\Pr(O_A^p) \leq \Pr(O_A) \text{ and } \Pr(O_B^p) \leq \Pr(O_B). \tag{5.22}$$

To ensure (5.22), from (5.18), we have the following two cases for the power allocation factor α .

Case 1: $0 < \alpha < \frac{R_P}{1+R_P}$

We first consider the primary signal x_A . Substituting $\Pr(O_A^{p,1})$ and (5.21) into (5.22), after some manipulations we obtain

$$\alpha \geq \alpha_{1,A}^* = \frac{R_P - \rho_P \ln(f_{1,A}^{-1})}{R_P - \rho_P \ln(f_{1,A}^{-1}) + 1} \tag{5.23}$$

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

where $f_{1,A} = \frac{c_1 - (1-b_1)c_2}{b_1(1-b_2)} > 0$, $c_1 = e^{-\frac{R'_P}{\rho_P}}$, $c_2 = e^{-\frac{R_P}{\rho_P}}$, $b_1 = e^{-d_{AC}^\nu \frac{R_P}{\rho_P}}$, $b_2 = e^{-d_{BC}^\nu \frac{R_P}{\rho_P}}$, and $\alpha_{1,A}^*$ is the power allocation threshold such that when $\alpha \geq \alpha_{1,A}^*$, the spectrum sharing requirement $\Pr(O_A^P) \leq \Pr(O_A)$ is satisfied. For x_B , we similarly have

$$\alpha \geq \alpha_{1,B}^* = \frac{R_P - \rho_P \ln(f_{1,B}^{-1})}{R_P - \rho_P \ln(f_{1,B}^{-1}) + 1} \quad (5.24)$$

where $f_{1,B} = \frac{c_1 - (1-b_2)c_2}{b_2(1-b_1)} > 0$. Since $0 < \alpha_{1,A}^*, \alpha_{1,B}^* < \frac{R_P}{1+R_P}$ under Case 1, from (5.23) and (5.24), after some manipulations we obtain $0 < f_{1,A}, f_{1,B} < 1$. Thus, the spectrum sharing region, or critical region, can be derived as

$$d_{AC}, d_{BC} < d_1^* = \left[\frac{\rho_P}{R_P} \ln(\Omega) \right]^{\frac{1}{\nu}},$$

where $\Omega = \frac{(1-c_2)^2 - c_1 + c_2}{(1-c_2)(c_1 - c_2)}$ and $\rho_P < \frac{R'_P}{\ln(2)}$.

(5.25)

From (5.25), we can see that as long as the secondary transceiver C is located within a radius of d_1^* from both primary users A and B and $\rho_P < \frac{R'_P}{\ln(2)}$, there exists a power allocation threshold $\alpha_1^* = \{\alpha_{1,A}^*, \alpha_{1,B}^*\}$ above which the spectrum sharing requirement in (5.22) can be achieved.

Proof. From (5.23), to ensure that $0 < \alpha_{1,A}^* < \frac{R_P}{1+R_P}$, we have

$$0 < \frac{R_P - \rho_P \ln(f_{1,A}^{-1})}{R_P - \rho_P \ln(f_{1,A}^{-1}) + 1} < \frac{R_P}{R_P + 1}. \quad (5.26)$$

Taking note that $c_1 = e^{-\frac{R'_P}{\rho_P}}$, $c_2 = e^{-\frac{R_P}{\rho_P}}$, $b_1 = e^{-d_{AC}^\nu \frac{R_P}{\rho_P}}$, $b_2 = e^{-d_{BC}^\nu \frac{R_P}{\rho_P}}$, we can see that $f_{1,A} = \frac{c_1 - (1-b_1)c_2}{b_1(1-b_2)} > e^{-\frac{R_P}{\rho_P}}$ always holds true, i.e. $R_P - \rho_P \ln(f_{1,A}^{-1}) > 0$, thus we have from (5.26)

$$\begin{aligned} (R_P - \rho_P \ln(f_{1,A}^{-1})) (R_P + 1) &< (R_P - \rho_P \ln(f_{1,A}^{-1}) + 1) R_P, \\ R_P + R_P^2 - \rho_P \ln(f_{1,A}^{-1}) - R_P \rho_P \ln(f_{1,A}^{-1}) &< R_P^2 - R_P \rho_P \ln(f_{1,A}^{-1}) + R_P, \\ -\rho_P \ln(f_{1,A}^{-1}) &< 0. \end{aligned}$$

Hence $f_{1,A} < 1$, which can be further expressed as

$$\begin{aligned} \frac{c_1 - (1-b_1)c_2}{b_1(1-b_2)} &< 1, \\ c_1 - (1-b_1)c_2 - b_1(1-b_2) &< 0, \\ b_2 &< 1 - c_2 - \frac{c_1 - c_2}{b_1}. \end{aligned} \quad (5.27)$$

Hence,

$$d_{CB} > \delta_2 = \left[\frac{\rho_P}{R_P} \ln \left(\frac{1}{1 - c_2 - \frac{c_1 - c_2}{b_1}} \right) \right]^{\frac{1}{\nu}}. \quad (5.28)$$

In (5.27), since $b_2 > 0$, to ensure that $1 - c_2 - \frac{c_1 - c_2}{b_1} > 0$, we have

$$\begin{aligned} b_1 &> \frac{c_1 - c_2}{1 - c_2}, \\ \therefore d_{AC} &< \delta_1 = \left[\frac{\rho_P}{R_P} \ln \left(\frac{1 - c_2}{c_1 - c_2} \right) \right]^{\frac{1}{\nu}}. \end{aligned} \quad (5.29)$$

Similarly, to ensure that $0 < \alpha_{1,B}^* < \frac{R_P}{1 + R_P}$ for x_B , we have the following symmetric conditions where

$$d_{BC} < \delta_1 \text{ and } d_{CA} > \delta_3 = \left[\frac{\rho_P}{R_P} \ln \left(\frac{1}{1 - c_2 - \frac{c_1 - c_2}{b_2}} \right) \right]^{\frac{1}{\nu}}. \quad (5.30)$$

Combining (5.28), (5.29), and (5.30), spectrum sharing requirement in (5.22) can be achieved when $\delta_1 > d_{CB} > \delta_2$ and $\delta_1 > d_{AC} > \delta_3$. In other words, $\delta_1 > \delta_2$ and $\delta_1 > \delta_3$ have to be satisfied. To ensure that $\delta_1 > \delta_2$, we have

$$\begin{aligned} \left[\frac{\rho_P}{R_P} \ln \left(\frac{1 - c_2}{c_1 - c_2} \right) \right]^{\frac{1}{\nu}} &> \left[\frac{\rho_P}{R_P} \ln \left(\frac{1}{1 - c_2 - \frac{c_1 - c_2}{b_1}} \right) \right]^{\frac{1}{\nu}}, \\ \frac{(1 - c_2)^2 - c_1 + c_2}{(1 - c_2)(c_1 - c_2)} &> \frac{1}{b_1}, \end{aligned}$$

hence

$$d_{AC} < d_1^* = \left[\frac{\rho_P}{R_P} \ln(\Omega) \right]^{\frac{1}{\nu}}, \quad (5.31)$$

where $\Omega = \frac{(1 - c_2)^2 - c_1 + c_2}{(1 - c_2)(c_1 - c_2)}$. To ensure that $\Omega > 1$, we have

$$\begin{aligned} (1 - c_2)^2 - c_1 + c_2 &> (1 - c_2)(c_1 - c_2), \\ 2c_1 - c_1 c_2 &< 1, \\ 2c_1 &< 1, \end{aligned}$$

hence

$$\rho_P < \frac{R'_P}{\ln(2)}. \quad (5.32)$$

To ensure that $\delta_1 > \delta_3$, we similarly have

$$d_{BC} < d_1^* = \left[\frac{\rho_P}{R_P} \ln(\Omega) \right]^{\frac{1}{\nu}}, \text{ and } \rho_P < \frac{R'_P}{\ln(2)}. \quad (5.33)$$

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

Combining (5.31) and (5.33), we can thus obtain the critical region given in (5.25). $\square$

Remark 5.3.1. Under practical scenarios, e.g. $R_{pt} = 1$, secondary spectrum sharing is feasible only when the transmit SNR $\rho_P = \frac{P}{\sigma^2}$ is very small, i.e. $\rho_P < \frac{R_P'}{\ln(2)} \approx 6\text{dB}$, which is not realistic and thus for Case 1 where $0 < \alpha < \frac{R_P}{1+R_P}$, we can reasonably assume that secondary spectrum sharing is not possible, i.e. $f_{1,A}, f_{1,B} > 1$.

Case 2: $\frac{R_P}{1+R_P} < \alpha < 1$

Substituting $\Pr(O_A^{p,2})$ and (5.21) into (5.22), we obtain

$$f_{2,A} \leq e^{-d_{CB}^v \frac{R_P}{[\alpha(1+R_P)-R_P]\rho_S}} \quad (5.34)$$

where $f_{2,A} = \frac{c_1-(1-b_1)c_2-b_1(1-b_2)}{b_1b_2}$. When $f_{1,A} > 1$, we have $f_{2,A} > 0$, and thus from (5.34),

$$\alpha \geq \alpha_{2,A}^* = \frac{\frac{d_{CB}^v R_P}{\rho_S \ln(f_{2,A}^{-1})} + R_P}{1 + R_P} \quad (5.35)$$

where $\alpha_{2,A}^*$ is the corresponding power allocation threshold in Case 2. For x_B , we similarly have

$$\alpha \geq \alpha_{2,B}^* = \frac{\frac{d_{CB}^v R_P}{\rho_S \ln(f_{2,B}^{-1})} + R_P}{1 + R_P} \quad (5.36)$$

where $f_{2,B} = \frac{c_1-(1-b_2)c_2-b_2(1-b_1)}{b_1b_2} > 0$. Since $\frac{R_P}{1+R_P} < \alpha_{2,A}^*, \alpha_{2,B}^* < 1$, after some manipulations we can obtain the critical region under Case 2 as

$$d_{AC}, d_{BC} < d_2^* = \min \left\{ \left[\frac{\rho_P}{R_P} \ln \left(\frac{1-c_2}{c_1-c_2} \right) \right]^{\frac{1}{\nu}}, \left[\frac{\rho_S}{R_P} (1-c_1) \right]^{\frac{1}{\nu}} \right\} \quad (5.37)$$

such that as long as the secondary transceiver C is located within a radius of d_2^* from both primary users A and B, there exists a power allocation threshold $\alpha_2^* = \{\alpha_{2,A}^*, \alpha_{2,B}^*\}$ above which the spectrum sharing requirement in (5.22) can be achieved.

Proof. From (5.35), to ensure that $\frac{R_P}{1+R_P} < \alpha_{2,A}^* < 1$, we have

$$\frac{R_P}{1+R_P} < \frac{\frac{d_{CB}^v R_P}{\rho_S \ln(f_{2,A}^{-1})} + R_P}{1 + R_P} < 1, \quad (5.38)$$

$$0 < \frac{d_{CB}^v R_P}{\rho_S \ln(f_{2,A}^{-1})} < 1. \quad (5.39)$$

Rewriting (5.39) in terms of $f_{2,A}$ and taking note that $f_{2,A} > 0$ holds true, we obtain

$$f_{2,A} = \frac{c_1 - (1 - b_1)c_2 - b_1(1 - b_2)}{b_1 b_2} < e^{-d_{CB}^\nu \frac{R_P}{\rho_S}}. \quad (5.40)$$

From 1st-order Taylor's series approximation, $e^{-d_{CB}^\nu \frac{R_P}{\rho_S}} > 1 - d_{CB}^\nu \frac{R_P}{\rho_S}$. Thus we have from (5.40)

$$f_{2,A} < 1 - d_{CB}^\nu \frac{R_P}{\rho_S}. \quad (5.41)$$

To ensure that $1 - d_{CB}^\nu \frac{R_P}{\rho_S} > 0$, we have

$$d_{CB} < \left(\frac{\rho_S}{R_P} \right)^{\frac{1}{\nu}}. \quad (5.42)$$

Next, (5.41) can be expanded as

$$\begin{aligned} \frac{c_1 - (1 - b_1)c_2 - b_1(1 - b_2)}{b_1 b_2} &< 1 - d_{CB}^\nu \frac{R_P}{\rho_S}, \\ d_{CB}^\nu \frac{R_P}{\rho_S} b_1 b_2 &< (1 - c_2)b_1 - c_1 + c_2, \\ \ln \left(d_{CB}^\nu \frac{R_P}{\rho_S} \right) - d_{AC}^\nu \frac{R_P}{\rho_P} - d_{BC}^\nu \frac{R_P}{\rho_P} &< \ln((1 - c_2)b_1 - c_1 + c_2), \\ \ln \left(d_{CB}^\nu \frac{R_P}{\rho_S} \right) &< \ln((1 - c_2)b_1 - c_1 + c_2), \\ d_{CB}^\nu \frac{R_P}{\rho_S} &< (1 - c_2)b_1 - c_1 + c_2, \\ d_{CB}^\nu \frac{R_P}{\rho_S} &< (1 - c_2) - c_1 + c_2. \end{aligned} \quad (5.43)$$

Hence,

$$d_{CB} < d_{CB}^* = \left[\frac{\rho_S}{R_P} (1 - c_1) \right]^{\frac{1}{\nu}}, \quad (5.44)$$

which is tighter than the condition in (5.42).

On the right hand side of (5.43), to ensure that the argument of the $\ln(\cdot)$ function is positive, i.e.

$$\begin{aligned} (1 - c_2)b_1 - c_1 + c_2 &> 0, \\ \therefore d_{AC} &< d_{AC}^* = \left[\frac{\rho_P}{R_P} \ln \left(\frac{1 - c_2}{c_1 - c_2} \right) \right]^{\frac{1}{\nu}}. \end{aligned} \quad (5.45)$$

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

To ensure that $\frac{R_P}{1+R_P} < \alpha_{2,B}^* < 1$, we can similarly derive for x_B

$$d_{BC} < d_{BC}^* = \left[\frac{\rho_P}{R_P} \ln \left(\frac{1-c_2}{c_1-c_2} \right) \right]^{\frac{1}{\nu}}, \quad (5.46)$$

$$d_{CA} < d_{CA}^* = \left[\frac{\rho_S}{R_P} (1-c_1) \right]^{\frac{1}{\nu}}. \quad (5.47)$$

Combining (5.44)-(5.47), we can thus obtain the critical region given in (5.37). $\square$

5.3.3 Outage Performance of Secondary System

For the secondary system, upon receiving $y_{D,1}$ and $y_{D,2}$, the secondary receiver D attempts to decode both x_A and x_B , and stores the decoding results if it succeeds.

Scenario 1: C and D have successfully decoded the same copy of primary signals. For example, both C and D have successfully decoded both x_A and x_B , or x_A only. Then in phase 3, from (5.6) and (5.9), the interference from the primary signals can be perfectly canceled out at D to obtain an effective received signal

$$y'_{D,3} = \sqrt{(1-\alpha)P_S} h_{CD} x_S + n_D. \quad (5.48)$$

The achievable rate for link C→D is thus given by

$$R_{CD} = \frac{1}{3} \log_2 \left(1 + \frac{(1-\alpha)P_S \gamma_{CD}}{\sigma^2} \right). \quad (5.49)$$

For all other scenarios where interference cancellation is not possible, e.g. both x_A and x_B are successfully decoded at C but only x_A is successfully decoded at D, an outage will be declared for the secondary system. The overall outage probability for the transmission of x_S with target rate R_{st} is thus given by

$$\begin{aligned} \Pr(O_S^p) = 1 & - \Pr(R_{AC} > R_{pt}) \Pr(R_{BC} > R_{pt}) \\ & \Pr(R_{AD} > R_{pt}) \Pr(R_{BD} > R_{pt}) \Pr(R_{CD} > R_{st}) \\ & - \Pr(R_{AC} > R_{pt}) \Pr(R_{BC} < R_{pt}) \Pr(R_{AD} > R_{pt}) \Pr(R_{CD} > R_{st}) \\ & - \Pr(R_{AC} < R_{pt}) \Pr(R_{BC} > R_{pt}) \Pr(R_{BD} > R_{pt}) \Pr(R_{CD} > R_{st}). \end{aligned} \quad (5.50)$$

For (5.49), we have from [89]

$$\Pr(R_{CD} > R_{st}) = e^{-d_{CD}^{\nu} \frac{R_S}{(1-\alpha)\rho_S}} \quad (5.51)$$

where $R_S = 2^{3R_{st}} - 1$. Thus we can obtain the closed-form expression for the outage probability of the secondary system as

$$\begin{aligned} \Pr(O_S^p) = 1 & - e^{-(d_{AC}^\nu + d_{BC}^\nu + d_{AD}^\nu + d_{BD}^\nu) \frac{R_P}{\rho_P}} e^{-d_{CD}^\nu \frac{R_S}{(1-\alpha)\rho_S}} \\ & - e^{-(d_{AC}^\nu + d_{AD}^\nu) \frac{R_P}{\rho_P}} \left(1 - e^{-d_{BC}^\nu \frac{R_P}{\rho_P}}\right) e^{-d_{CD}^\nu \frac{R_S}{(1-\alpha)\rho_S}} \\ & - e^{-(d_{BC}^\nu + d_{BD}^\nu) \frac{R_P}{\rho_P}} \left(1 - e^{-d_{AC}^\nu \frac{R_P}{\rho_P}}\right) e^{-d_{CD}^\nu \frac{R_S}{(1-\alpha)\rho_S}}. \end{aligned} \quad (5.52)$$

5.4 Simulation Results

To evaluate the outage performance of the primary and secondary systems under different settings, for ease of exposition, we consider a system topology in a two-dimensional X-Y plane where A and B are located at points (0,0) and (1,0) respectively, i.e. $d_{AB} = 1$. Matlab is used in deriving all simulation results in this chapter. Given the normalized distance of each channel and the corresponding path-loss exponent, the channels are randomly generated through Monte Carlo method for 50,000 times. Then we count the ratio that the intended receiver fails to receive the desired packet, which is the corresponding outage probability. All simulation results in this chapter are obtained for $R_{pt} = 1$ and path-loss exponent $\nu = 4$. Only the outage probability $\Pr(O_A^p)$ for x_A will be shown as the results for x_B are similar. A sample program for the the Simulation of $\Pr(O_A^p)$ in Equation (5.20) is provided in Appendix A.4 and the corresponding results are shown in Figure 5.9.

In Figure 5.5, we show the critical region under various values of ρ_P and ρ_S . As long as secondary transceiver C is located within this region, it can always choose a suitable power allocation such that the spectrum sharing requirement in (5.22) is satisfied. The simulation results are represented by dots and the analytical results are represented by the intersection region of the two circles defined in (5.37) for Case 2 where $\frac{R_P}{1+R_P} < \alpha < 1$. A simple observation is that secondary spectrum sharing is generally not possible when $\rho_S < \rho_P$. This indicates that the secondary user has to pay a price for a spectrum sharing opportunity by adopting a higher transmit SNR than the primary users (i.e. $\rho_S > \rho_P$). Although not shown in Figure 5.5, the simulation results also concurred with Remark 5.3.1 that secondary spectrum sharing is not possible for Case 1 where $0 < \alpha < \frac{R_P}{1+R_P}$ when $\rho_P > 6\text{dB}$.

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

Figure 5.5: Critical region when $R_{pt} = 1$ and $\nu = 4$.

Next, we focus on the network topology shown in Figure 5.6 where C moves between A and B, i.e. $d_{AC} + d_{CB} = d_{AB} = 1$, and D moves along $x = d_{AC}$ in the vertical direction. We let $\rho_P = \rho_S = 20\text{dB}$ and $d_{CD} = 0.5$. In Figure 5.7, we show the outage probability for both primary and secondary systems in symmetric relay channels where $d_{AC} = d_{BC} = 0.5$. We can see that there exists a power allocation threshold $\alpha^* \approx 0.9$ such that when $\alpha^* < \alpha < 1$, the proposed CTR protocol is able to achieve a better outage performance for the primary system than in the case without spectrum sharing, i.e. $\Pr(O_A^P) < \Pr(O_A)$. Two cases are considered for the secondary system where

Figure 5.6: Network topology for simulation.

 Figure 5.7: Outage probability for primary and secondary systems when $\rho_P = \rho_S = 20\text{dB}$ in symmetric channels where $d_{AC} = d_{BC} = 0.5$.

$R_{st} = 1$ and $R_{st} = 0.5$ respectively. It can be seen that in both cases, a reasonable outage performance of below 10^{-1} can be achieved by the secondary system.

In Figure 5.8, we also show the outage performance for the primary system in

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

Figure 5.8: Outage probability for the primary system when $\rho_P = \rho_S = 20\text{dB}$ in asymmetric channels.

asymmetric channels where C is located closer to one primary user (A or B). Although not shown here due to space constraints, among the three scenarios described in Section 5.3.1 ((5.11)-(5.14)), Scenario 1 where both x_A and x_B are successfully decoded at C has the highest occurrence probability. Thus the relay transmission C $\rightarrow$ B in phase 3 is dominant in the overall outage probability $\Pr(O_A^p)$. This is also evident in Figure 5.8 where a lower outage probability $\Pr(O_A^p)$ can be achieved for when d_{BC} decreases⁴. Since there is only a slight difference for the outage performance of the secondary system in asymmetric channels, we have omitted it from Figure 5.8.

In Figure 5.9, we show the effects of ρ_S and d_{CD} on the outage performance of the primary and secondary systems. We let $R_{pt} = R_{st} = 1$, $d_{AC} = d_{BC} = 0.5$, and

⁴Conversely, the outage performance for x_B , i.e. $\Pr(O_B^p)$, will worsen as d_{BC} decreases.

Figure 5.9: Outage probability for primary and secondary systems under various values of ρ_S when $\alpha = 0.5$, $\rho_P = 20\text{dB}$, and $R_{st} = R_{pt} = 1$.

$\alpha = 0.9$. From (5.16), we have $\Pr(R_{CB}^{\text{NC}} > R_{pt}) \rightarrow 1$ when ρ_S increases. Thus the link A→C becomes the limiting factor for $\Pr(O_A^p)$ in (5.20) and it is lower bounded by $\left(1 - e^{-d_{AC}^\nu \frac{R_P}{\rho_P}}\right) \left(1 - e^{-\frac{R_P}{\rho_P}}\right)$, which is independent of P_S . This is why we can observe that $\Pr(O_A^p)$ experiences a floor in the high ρ_S region. Similarly, from (5.51), we have $\Pr(R_{CD} > R_{st}) \rightarrow 1$ when ρ_S increases. Thus the links A→C, A→D, B→C, B→D in (5.52) become the limiting factors for $\Pr(O_S^p)$ and they are independent of ρ_S . That is why an outage floor is also evident for the secondary system. Two scenarios are considered for the secondary system where $d_{CD} = 0.5$ and $d_{CD} = 0.1$. When C is located closer to D than to the primary users A and B (i.e. $d_{CD} = 0.1$), a better outage performance can be achieved for the secondary system even with a small value of ρ_S , without affecting the outage performance of the primary system.

5. COGNITIVE SPECTRUM SHARING WITH COOPERATIVE RELAY SYSTEMS

From the above observations and analyses, we can see that when the secondary transceiver C is located within the critical region, in order to achieve a better outage performance for the primary system than in the case without spectrum sharing, C can allocate more transmit power (i.e., with a higher ρ_S or a larger α) to help forward the primary signals. On the other hand, to maintain an acceptable outage performance for the secondary transmission without affecting the primary system, the secondary system can transmit at a lower target rate, or the secondary transceiver C is situated closer to the secondary receiver D than to the primary users [165].

5.5 Summary

In this chapter, we considered cognitive spectrum sharing with cooperative relay systems. A cognitive two-way relay protocol was proposed where two primary users A and B communicate with each other with the assistance of a secondary transceiver C which acts as a relay for the primary system.

By superimposing the secondary transmission on the network-coded primary signals, we showed that it is possible to find a suitable power allocation at the secondary transceiver C to forward the primary signals such that the outage performance of the primary system with spectrum sharing is at least as good as that without spectrum sharing, as long as C is located within a critical region. And at the same time, secondary spectrum sharing can be achieved.

Conclusion

Network coding, first proposed in 2000, has come a long way in the last decade. It has been widely extended to various domains ranging from packet switching networks to a variety of wireless networks, e.g. cellular networks, mesh networks, cognitive radio networks, and etc. While the original focus is on the throughput improvement in wireline networks, an increasing number of applications and research directions have been inspired by this promising technique in wireless networks.

6.1 Main Contributions and Results

In this thesis, the fundamental concept of network coding was first introduced and explained in Chapter 1. In contrast to the conventional routing architectures where the intermediate nodes of a network can only store-and-forward the received packets without processing them, intermediate nodes are allowed to perform random mappings of the received packets in network coding. The inherent advantages of network coding were illustrated through a butterfly network shown in Figure 1.1 which was first considered in [1].

Motivated by the significant benefits promised by network coding, we proceeded to investigate and evaluate network coding in several typical wireless network scenarios. In Chapter 2, we considered a basic building block in wireless multi-hop networks, namely a two-way relay channel where two end users communicate with each other with the assistance of a common relay in between. To take advantage of the broadcast nature of wireless channels and the available receiver side information, we considered

6. CONCLUSION

network coding at the relay to combine the bi-directional transmissions. An information exchange between the two end users thus requires only two time slots, which is able to significantly enhance the network throughput and spectral efficiency [54]-[57]. The outage performance of amplify-and-forward (AF) and decode-and-forward (DF) relay protocols was analyzed. As far as I know, the obtained closed-form expression for the outage probability of AF applies to more general network scenarios than the existing works.

To effectively tap the advantages of both AF and DF, an adaptive AF and DF two-way relay protocol was proposed. Depending on the decodability of the bi-directional transmissions received at the relay, the proposed protocol is able to adaptively switch between AF and DF, thus achieving a better outage performance over all SNR values.

In Chapter 3, we discussed the problem of separate/joint network and channel coding in wireless networks. It is well known that network coding was originally designed for wireline networks with error-free links. When it comes to the wireless realm, most current works directly transplant the mechanism in wireline networks and adopt a separate network and channel coding strategy where channel coding is performed at the physical layer to combat the noisy channel, and then network coding is performed at the network layer to combine the channel-decoded messages. This separate coding strategy, however, was proven to be suboptimal in general for wireless networks [118], [119].

Motivated by the benefits of joint design of network and physical layers for wireless networks, we proposed a joint network and channel coding strategy for a wireless multicast network with multiple sources, relays, and destinations. To achieve tractable analysis and provide insights, we adopted a decode-and-forward relay protocol and the end-to-end information flow across the network was divided into multi-hop transmissions. Then within each hop, a ARQ scheme was employed to ensure reliable message delivery and the information-theoretic results of some canonical subnetworks, namely broadcast channel and multiple access channel, were exploited. Closed-form expressions of the network throughput were obtained and compared to a separate network and channel coding strategy. In contrast to the optimality of separate network and channel coding in wired multicast networks with error-free links, our proposed strategy provides an example where joint coding outperforms separate coding in wireless networks.

Both Chapter 2 and Chapter 3 demonstrated the throughput enhancement achieved by network coding together with the inherent receiver side information (RSI) in wireless networks. Due to the opportunistic listening among the neighbouring nodes, it is possible that some nodes in the network have overheard a subset of the packets to be transmitted from the previous transmissions. For an ideal case as considered in most current works, each receiver of a broadcast channel knows *a priori* all packets to be transmitted except one, then through network coding where all packets are combined at the transmitter as a single packet which is broadcasted, each receiver is able to extract the desired packet by XORing the received packet with the packets it already knew. Thus a less number of transmissions is required for reliable packet delivery to all receivers of a broadcast channel, which significantly enhances the network throughput [99]-[103], [130], [131].

However, this ideal case is generally not true in wireless networks. Instead of the symmetric RSI available at all receivers of a broadcast channel, it is highly probable that some receiver has less RSI than the others, or even worse, no RSI at all. To account for different amount of receiver side information available at the receivers of a broadcast channel, we modeled the accumulation of RSI at each receiver as an independent Poisson process where the collection of an innovative packet is analogous to the occurrence of an “event”. With different innovative packet arrival rates, we can thus model different amount of RSI available at the receivers of a broadcast channel and investigate how this affects the overall network throughput when network coding is performed. Our results showed the advantage of random linear network coding over round robin scheduling. In the high SNR region, the random linear network coding always achieves a higher (or equal) throughput than the round robin scheduling, no matter how much RSI is available at each receiver.

In Chapter 5, we go back to the two-way relay channel where a relay assists the bi-directional transmissions of two end users. The difference is that the relay here belongs to a different communication system (with lower priority) and it needs incentives to cooperate with the transmissions between the two end users. This scenario results in a natural combination of cooperation and cognition where two primary users and a cognitive user co-exist. Rather than operating in a non-cooperative manner and interfere with each other, it is more beneficial that the cognitive user serves as a relay for the primary system and at the same time gains an opportunity to transmit its own

6. CONCLUSION

packets. Again network coding is applied at the cognitive relay to combine the decoded primary packets. We name this network scenario as cognitive spectrum sharing with two-way relay channels in Chapter 5.

To achieve spectrum sharing without degrading the performance of the primary system, the cognitive relay allocates a fraction of its transmit power to help forward the primary packets, with the remaining for its own packet. Closed-form expressions for the outage probabilities of both primary and secondary systems were derived. Our results showed that a critical region exists and as long as the cognitive relay is located in this region, it can always choose a suitable power allocation to forward the primary packets such that the interference to the primary system due to the secondary transmission can be properly compensated by the cooperation from the cognitive relay [157], and at the same time spectrum sharing is achieved.

In exploring the above problems discussed in each chapter of this thesis, network coding has been extensively investigated and evaluated in several typical wireless network scenarios. We can see that to solve the co-existence problem between different users or communication systems and at the same time maintain spectral efficiency, cooperation and cognition techniques have demonstrated significant benefits. With network coding, especially a joint network and channel coding which jointly considers the broadcast nature, interference, noise, and unreliability of wireless networks, more benefits can be achieved for the wireless communication networks.

I believe that the technical and analytical methods adopted in this thesis would be helpful to a wide range of readers who will benefit from an understanding of wireless cooperative and cognitive communications together with the utilization of network coding.

6.2 Future Works

The joint network and channel coding strategy proposed in Chapter 3 demonstrated significant benefits in improving the network throughput. And of course it can be readily adopted in two-way relay channels and broadcast channels discussed in other chapters of this thesis to achieve even better performance. However, the results presented in Chapter 3 are limited to information-theoretic analysis and there is no such constructive code in practice yet. So in the future, it would be interesting to find out

a good candidate code that can be deployed to achieve the benefits promised by joint network and channel coding.

For the broadcast channel considered in Chapter 4, assuming that the receivers experience i.i.d. channels (i.e. $p_m = p_0 \forall m$), I have shown that with a sum constraint of all arrival rates of innovative packets, i.e. $\sum_m \lambda_m = \Lambda$, the scenario where each receiver is able to collect the same amount of RSI, i.e. $\lambda_m = \frac{\Lambda}{M} \forall m$, achieves the maximum throughput for random linear network coding. A natural and interesting extension is to find out if the receivers have different channel conditions, then what assignment of the arrival rate λ_m at each receiver achieves the maximum throughput when random linear network coding is performed?

Since the download time for the broadcast channel is determined by the worst receiver that finishes decoding last, intuitively the receiver with a worse channel should be assigned with a higher packet arrival rate, i.e. if $p_m > p_{m'}$, then $\lambda_m > \lambda_{m'}$, where $m \neq m'$ and $m, m' \in \{1, 2, \dots, M\}$. Thus as a special case where $p_m = p_0 \forall m$, it is reasonable that the scenario where $\lambda_m = \frac{\Lambda}{M} \forall m$ achieves the maximum throughput, as demonstrated in Figure 4.3 and Figure 4.4 in the thesis.

A hardware implementation of the cooperative relay communications at the MAC layer, i.e. CoopMAC, was presented in [171]. Being backward compatible to the existing communication standards, e.g. IEEE 802.11, the cooperative relay communications demonstrated a significant advantage over the direct transmissions without relay. In view of this, it would be meaningful to implement a prototype for the cognitive spectrum sharing scenario discussed in Chapter 5 to provide insights to the practical applications. For the cognitive spectrum sharing with two-way relay channels, besides the establishment of the cooperation as discussed in [171], other practical issues need to be considered, e.g. security, authentication, timing issues, and etc.

6.3 Other Applications of Network Coding

As illustrated in the first few chapters, network coding is expected to offer benefits along diverse dimensions of wireless communications such as throughput, power efficiency, spectral efficiency, latency, and robustness. The interest in network coding continues to increase with the emergence of new applications in both theory and practice.

6. CONCLUSION

Due to the broadcast nature of wireless transmissions and random mappings performed at the intermediate nodes, multiple copies of the same message are being flooded in the network, thus network coding can be adopted as an efficient tool in wireless cooperative communications to provide diversity [13]-[17] so as to improve the robustness of the system.

Since the transmitted packets are widely flooded in the network, security is an important issue that should be considered. In contrast to the conventional error correction done on a link-by-link basis, network coding has been applied to control error from an end-to-end aspect [34]-[40]. Due to the random mappings of packets at the intermediate nodes without necessarily decoding them, network coding provides inherent benefits in security. For example, as shown in the butterfly network in Figure 1.2, if R_2 is a malicious node or it is being wiretapped, even if it obtains $p_1 \oplus p_2$, it can recover neither p_1 nor p_2 .

The problem of designing secure network codes for wiretap networks was investigated in [34]-[37]. The source combines the original data with random information and designs a network code such that only the receivers are able to decode the original packets. From an information-theoretic point of view, the mutual information between the original packets and the packets obtained by the eavesdroppers is zero. A weaker form of network security was investigated in [38] where nodes can only decode packets if they have received a sufficient number of independent encoded packets. Under this condition, although the eavesdropper obtains some encoded packets of the source information, it is almost impossible to get any meaningful information.

The great amount of theoretical results have also encouraged the implementation of network coding. Microsoft has adopted network coding as a core technology in its Avalanche project [25]-[28], which aims at developing a peer-to-peer file distribution system by exploiting the significant improvement provided by network coding. Besides its promise in peer-to-peer file distribution systems [29], network coding has also been widely considered in cellular networks and wireless mesh networks [30]-[32].

With the great efforts put in and the significant improvements achieved in various aspects of communications in the last decade, the era of network coding may finally be here [83].

Appendix A

Sample Programs

A.1 Matlab Codes for the Simulation of $\Pr(O_S^{\text{af}})$ in (2.15), Figure 2.7

```
1 clear all
2
3 %Number of iterations
4 iter=50000;
5
6 %Same input data rate from S and D
7 RS=1; RD=1;
8
9 m=2^(2*RS) - 1;
10
11 %Parameters for the relay channels
12 delta_1=1;
13 delta_2=1;
14
15 %Equal power allocation at S, D, and R assuming 2P+Pr=1
16 P=1/3; Pr=1/3; b=(P+Pr)/P;
17
18 SNR = 0:5:60;
19 snr = 10.^(SNR/10);
20 noise = 1./snr;
21
22 for isnr = 1:length(SNR)
23
24 %Analytical results for the outage probability of AF
```

A. SAMPLE PROGRAMS

```

25     u(isnr)=noise(isnr)/P;
26     AF1(isnr)=1-delta_1*exp(-(delta_2+2*delta_1)*m*u(isnr)
    .*sqrt(8*m^2*u(isnr).^2*delta_2/delta_1)).*besselk
    (1,sqrt(8*m^2*u(isnr).^2*delta_2*delta_1));
27
28 %Monte Carlo simulation
29     N2=0;
30
31     for k=1:iter
32
33         isnr
34         k
35
36 %Each channel is randomly generated
37         gamma_s=abs(sqrt(1/delta_1)*(randn(1)+1i*randn(1)))/
    sqrt(2))^2;
38         gamma_d=abs(sqrt(1/delta_2)*(randn(1)+1i*randn(1)))/
    sqrt(2))^2;
39
40 %End-to-end achievable rate of AF
41         R_sd(isnr,k)=0.5*log2(1+gamma_d*gamma_s/(b*gamma_d+
    gamma_s))./u(isnr));
42
43         if R_sd(isnr,k)>=RS
44             N2=N2+1;
45         end
46
47     end
48
49 %Simulation results for the outage probability of AF
50     AF2(isnr)=1-N2/iter;
51
52 end
53
54 figure
55 semilogy(SNR,AF1,'k—','LineWidth',1)
56 hold on
57 semilogy(SNR,AF2,'ko','LineWidth',1,'MarkerSize',8)
58 legend('AF_S_(analytical)','AF_S_(simulation)')
59 xlabel('P_T/\sigma^2[dB]')
60 ylabel('Outage_Probability')

```

A.2 Matlab Codes for the Simulation of η in (3.43), Figure 3.7

```

1  clear all
2
3  %Number of messages at each source
4  M = 10;
5
6  %Rate of a message, assumed to be equal for all messages
7  R0 = 1;
8
9  %Average channel gain
10 gamma =1;
11
12 %Sum rate of all source messages
13 R_sum = 3 * M * R0;
14
15 rho_dB = 0:5:30;
16 rho = 10.^(rho_dB/10);
17
18 iterations = 50000;
19 T_max = 100;
20
21 %Monte Carlo simulation of E[T1]
22 for irho = 1:length(rho)
23
24 %The probability that all receivers in phase 1 finish decoding
   after the 1st transmission
25     h_1 = sqrt(gamma)*(randn(iterations,6)+1i*randn(
       iterations,6))/sqrt(2);
26     rate_1 = log2(1+rho(irho).*abs(h_1).^2);
27
28     N_1=0;
29     for i=1:iterations
30         if (rate_1(i,1)>=10) && (rate_1(i,2)>=10) && (
           rate_1(i,3)>=10) && (rate_1(i,4)>=10) && (rate_1
           (i,5)>=10) && (rate_1(i,6)>=10)
31             N_1=N_1+1;
32         end
33     end
34
35     PrRT1(1) = N_1/iterations;
36     ET1_tmp = PrRT1(1);

```

A. SAMPLE PROGRAMS

```

37
38 %The probability that all receivers in phase 1 finish decoding
   within iT_1 retransmissions
39 for iT_1 = 2:T_max
40
41     h1 = sqrt(gamma)*(randn(iterations,iT_1,6)+1i*randn(
         iterations,iT_1,6))/sqrt(2);
42     rate1 = sum( log2(1+rho(irho).*abs(h1).^2), 2 );
43
44     N1=0;
45     for i=1:iterations
46         if (rate1(i,1)>=10) && (rate1(i,2)>=10) && (rate1(i
         ,3)>=10) && (rate1(i,4)>=10) && (rate1(i,5)>=10)
         && (rate1(i,6)>=10)
47             N1=N1+1;
48         end
49     end
50
51     PrRT1(iT_1) = N1/iterations;
52
53 %The probability that all receivers in phase 1 finish decoding
   at exactly the iT_1th transmission
54     PcT1(iT_1) = PrRT1(iT_1) - PrRT1(iT_1-1);
55     ET1_tmp = ET1_tmp + iT_1*PcT1(iT_1);
56
57 end
58
59     ET_1(irho) = ET1_tmp;
60
61 end
62
63 %Monte Carlo simulation of E[T2]
64 for irho = 1:length(rho);
65
66 %The probability that all receivers in phase 2 finish decoding
   after the 1st transmission
67     h_2 = sqrt(gamma)*(randn(iterations,2)+1i*randn(
         iterations,2))/sqrt(2);
68     rate_2 = log2(1+rho(irho).*abs(h_2).^2);
69
70     N_2=0;
71     for i=1:iterations
72

```

```

73         if (rate_2(i,1)>=10) && (rate_2(i,2)>=10) && (
74             rate_2(i,1)+rate_2(i,2)>=30)
75             N_2=N_2+1;
76         end
77     end
78
79     PrRT2(1) = N_2/iterations;
80     ET2_tmp = PrRT2(1);
81
82     %The probability that all receivers in phase 2 finish decoding
83     %within iT_2 retransmissions
84     for iT_2 = 2:T_max
85         h2 = sqrt(gamma)*(randn(iterations,iT_2,2)+1i*randn(
86             iterations,iT_2,2))/sqrt(2);
87         rate2 = sum( log2(1+rho(irho).*abs(h2).^2), 2 );
88         N2=0;
89         for iter=1:iterations
90             if (rate2(i,1)>=10) && (rate2(i,2)>=10) && (rate2(i
91                 ,1) + rate2(i,2)>=30)
92                 N2=N2+1;
93             end
94         end
95     end
96
97     PrRT2(iT_2) = N2/iterations;
98
99     %The probability that all receivers in phase 2 finish decoding
100     %at exactly the iT_2th transmission
101     PcT2(iT_2) = PrRT2(iT_2) - PrRT2(iT_2-1);
102     ET2_tmp = ET2_tmp + iT_2*PcT2(iT_2);
103
104     end
105
106     ET_2(irho) = ET2_tmp;
107
108 end
109
110 %%Monte Carlo simulation of E[T3]
111 for irho = 1:length(rho)

```

A. SAMPLE PROGRAMS

```
111
112 %The probability that all receivers in phase 3 finish decoding
    after the 1st transmission
113     h_3 = sqrt(gamma)*(randn(iterations,2)+1i*randn(
        iterations,2))/sqrt(2);
114     rate_3 = log2(1+rho(irho).*abs(h_3).^2);
115
116     N_3=0;
117     for i=1:iterations
118         if (rate_3(i,1)>=20) && (rate_3(i,2)>=20)
119             N_3=N_3+1;
120         end
121     end
122
123     PrRT3(1) = N_3/iterations;
124     ET3_tmp=PrRT3(1);
125
126 %The probability that all receivers in phase 3 finish decoding
    within the iT_3 retransmissions
127     for iT_3 = 2:T_max
128
129         h3 = sqrt(gamma)*(randn(iterations,iT_3,2)+1i*randn(
            iterations,iT_3,2))/sqrt(2);
130         rate3 = sum( log2(1+rho(irho).*abs(h3).^2), 2 );
131
132         N3=0;
133         for iter=1:iterations
134
135             if (rate3(i,1)>=20) && (rate3(i,2)>=20)
136                 N3=N3+1;
137             end
138
139         end
140
141         PrRT3(iT_3) = N3/iterations;
142
143 %The probability that all receivers in phase 3 finish decoding
    at exactly the iT_3th transmission
144     PcT3(iT_3) = PrRT3(iT_3) - PrRT3(iT_3-1);
145     ET3_tmp = ET3_tmp + iT_3*PcT3(iT_3);
146     ET_count3(irho,iT_3) = ET3_tmp;
147
148 end
```

```

149         ET_3(irho) = ET3_tmp;
150
151
152     end
153
154     %Network throughput for the proposed JNCC strategy
155     ET = 3*ET_1 + 2*ET_2 + ET_3;
156     eta = R_sum./ET;
157
158     figure
159     plot(rho_dB, eta, 'ro', 'Linewidth', 1, 'MarkerSize', 8)
160     legend('JNCC_(simulation)')
161     xlabel('\rho_{dB} ')
162     ylabel('Network_Throughput_[bits/s/Hz] ')

```


A.3 Matlab Codes for the Simulation of η^{rlnc} in (4.10), Figure 4.3

```

1  clear all
2
3  %Number of packets to be broadcasted
4  N = 10;
5
6  %Number of receivers
7  M = 5;
8
9  %Rate of a packet, assumed to be equal for all packets
10 R0 = 1;
11
12 %Sum rate of all packets to be broadcasted
13 R_sum = N * R0;
14
15 %Equal packet arrival rate at each receiver
16 lambda=4;
17
18 %Average channel gain
19 gamma = 1;
20
21 rho_dB = 0:5:40;
22 rho = 10.^(rho_dB/10);
23
24 iter = 1000;
25 T_max = 100;
26
27 %Monte Carlo simulation for the throughput of RLNC
28 %The probability that a receiver collects k packets as RSI
29 R=poissrnd(lambda, 1,50000);
30     N10=length( find (R<=10));
31     N9=length( find (R<=9));
32     N8=length( find (R<=8));
33     N7=length( find (R<=7));
34     N6=length( find (R<=6));
35     N5=length( find (R<=5));
36     N4=length( find (R<=4));
37     N3=length( find (R<=3));
38     N2=length( find (R<=2));
39     N1=length( find (R<=1));
40     N0=length( find (R<=0));

```

```

41
42     p(1)=N0/N10;
43     p(2)=(N1-N0)/N10;
44     p(3)=(N2-N1)/N10;
45     p(4)=(N3-N2)/N10;
46     p(5)=(N4-N3)/N10;
47     p(6)=(N5-N4)/N10;
48     p(7)=(N6-N5)/N10;
49     p(8)=(N7-N6)/N10;
50     p(9)=(N8-N7)/N10;
51     p(10)=(N9-N8)/N10;
52     p(11)=(N10-N9)/N10;
53
54     for irho = 1:length(rho)
55
56         %Success probability of a channel
57         h1 = sqrt(gamma)*(randn(1,iter)+1i*randn(1,iter))/sqrt
            (2);
58         s1 = log2(1+rho(irho).*abs(h1).^2);
59         n1 = length(find(s1>=R0));
60         Ps(irho)=n1/iter;
61
62         %The probability that all M receivers finish decoding after
            the 1st transmission
63         ET_tmp(irho)=(p(11)+p(10).*Ps(irho)).^M;
64         ET_1 = ET_tmp(irho);
65
66         %The probability that all M receivers finish decoding within
            the iTth transmission
67         for iT=2:T_max
68
69             EP=0;
70             for k=0:N
71
72                 h_1=sqrt(gamma)*(randn(iter,iT)+1i*randn(iter,iT))/
                    sqrt(2);
73                 s_1 = log2(1+rho(irho).*abs(h_1).^2);
74
75                 N_1=0;
76                 for i=1:iter
77                     if length(find(s_1(i,:)>=R0))>=N-k
78                         N_1=N_1+1;
79                     end

```

A. SAMPLE PROGRAMS

```
80         end
81         p_1=N_1/iter;
82         PK=p(k+1)*p_1;
83         EP=EP+PK;
84
85     end
86
87     PrRT(irho,iT) = EP^M;
88     PrRT(irho,1) = ET_tmp(irho);
89
90     %The probability that all M receivers finish decoding at
91     %exactly the iTth transmission
92     PcT(irho,iT) = PrRT(irho,iT) - PrRT(irho,iT-1);
93     ET_1 = ET_1 + iT*CpT(irho,iT);
94
95 end
96
97     ET(irho) = ET_1;
98
99 end
100 T_rlnc = R_sum./ET;
101
102 figure
103 plot(rho_dB,T_rlnc,'r*','Linewidth',1,'MarkerSize',8)
104 legend('\eta^{rlnc}(simulation)')
105 xlabel('\rho [dB]')
106 ylabel('Network Throughput [bits/s/Hz]')
```

A.4 Matlab Codes for the Simulation of $\Pr(O_A^p)$ in (5.20), Figure 5.9

```

1  clear all
2
3  %Number of iterations
4  iter=50000;
5
6  %Target rates of primary and secondary systems
7  Rpt=1; Rst=1;
8
9  %Distance between the primary users and secondary users
10 d_AB=1; d_BA=1;
11 d_AC=0.5; d_BC=0.5; d_CB=0.5;
12
13 %Path-loss exponent
14 v=4;
15
16 R_P=2^(3*Rpt) - 1;
17
18 SNR = 0:5:60;
19 snr = 10.^(SNR/10);
20 noise_s = 1./snr;
21
22 %Primary transmit SNR=20dB
23 noise_p=0.01;
24
25 %Power allocation at the secondary transceiver to forward the
    primary packet
26 alpha = 0.9;
27
28 %Analytical results for the outage probability of the primary
    system
29 P_1=exp(-d_AC^v*R_P*noise_p);
30 P_2=exp(-d_AB^v*R_P*noise_p);
31 P_3=exp(-d_BC^v*R_P*noise_p);
32
33 for i = 1:length(SNR)
34
35     P_4=exp(-d_CB^v*(R_P.*noise_s(i)/(alpha*(1+R_P)-R_P)));
36     PrA1(i)=1-P_1*P_3*P_4-P_1*(1-P_3)-(1-P_1)*P_2;
37
38 end

```

A. SAMPLE PROGRAMS

```

39
40 %Monte Carlo simulation for the outage probability of the
    primary system
41 for i=1:length(SNR)
42
43     N_1=0;
44     for k=1:iter
45
46         h_AC=sqrt(d_AC^(-v))*(randn(1)+1i*randn(1))/sqrt(2);
47         h_AB=sqrt(d_AB^(-v))*(randn(1)+1i*randn(1))/sqrt(2);
48         h_BC=sqrt(d_BC^(-v))*(randn(1)+1i*randn(1))/sqrt(2);
49         h_CB=sqrt(d_CB^(-v))*(randn(1)+1i*randn(1))/sqrt(2);
50         g_AC=abs(h_AC)^2;
51         g_AB=abs(h_AB)^2;
52         g_BC=abs(h_BC)^2;
53         g_CB=abs(h_CB)^2;
54
55         R_AC(i,k)=1/3*log2(1 + g_AC/noise_p);
56         R_BC(i,k)=1/3*log2(1 + g_BC/noise_p);
57         R_AB(i,k)=1/3*log2(1 + g_AB/noise_p);
58         R_CB_1(i,k)=1/3*log2(1 + alpha*g_CB./((1-alpha)*g_CB+
            noise_s(i)));
59         R_CB_2(i,k)=1/3*log2(1 + g_AB/noise_p + alpha*g_CB
            ./((1-alpha)*g_CB+noise_s(i)));
60
61         if ( (R_AC(i,k)>Rpt) && (R_BC(i,k)>Rpt) && (R_CB_1
            (i,k)>Rpt) ) || ( (R_AC(i,k)>Rpt) && (R_BC(i,k)
            <Rpt) && (R_CB_2(i,k)>Rpt) ) || ((R_AC(i,k)<Rpt)
            && (R_AB(i,k)>Rpt) )
62
63             N_1=N_1+1;
64
65         end
66
67     end
68
69     PrA2(i)=(1-N_1/iter);
70
71 end
72
73 figure
74 semilogy(SNR,PrA1,'r:','LineWidth',1,'MarkerSize',8)
75 hold on

```

A.4 Matlab Codes for the Simulation of $\Pr(O_A^p)$ in (5.20), Figure 5.9

```
76 semilogy(SNR, PrA2, 'rd', 'LineWidth', 1, 'MarkerSize', 8)
77 legend('P(O_A^p)_analytical', 'P(O_A^p)_simulation')
78 xlabel('\rho [dB]')
79 ylabel('Outage_Probability')
```


Author's Publications

Conference Papers

- Q. Li, S. H. Ting, and C. K. Ho, "On the available receiver side information in wireless network coding," *Proc. IEEE ICC 2011*, Kyoto, Japan, Jun. 2011.
- Q. Li, S. H. Ting, and C. K. Ho, "Joint network and channel coding for wireless networks," *SECON Workshops 09*, Rome, Italy, Jun. 2009.
- Q. Li, S. H. Ting, and C. K. Ho, "Nonlinear network code for high throughput broadcasting with retransmissions," *Proc. IEEE ISIT 2009*, Seoul, Korea, pp. 2853-2857, Jun. 2009.
- Q. Li, S. H. Ting, A. Pandharipande, and Y. Han, "Outage analysis for adaptive two-way relaying," *Proc. IEEE PIMRC 2008*, Cannes, France, Sept. 2008.

Journal Papers

- Q. Li, S. H. Ting, A. Pandharipande, and Y. Han, "Cognitive spectrum sharing with two-way relaying systems," *IEEE Trans. Veh. Tech.*, vol. 60, no. 3, pp. 1233-1240, Mar. 2011.

AUTHOR'S PUBLICATIONS

- Q. Li, S. H. Ting, C. K. Ho, “Joint network and channel coding for wireless networks,” *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 181-193, Jan. 2011.
- Q. Li, S. H. Ting, A. Pandharipande, Y. Han, “Adaptive two-way relaying and outage analysis,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3288-3299, Jun. 2009.

Bibliography

- [1] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inf. Theory*, vol. IT-46, pp. 1204-1216, 2000.
- [2] S.-Y. R. Li, R. W. Yeung, and N. Cai, "Linear network coding," *IEEE Trans. Inf. Theory*, vol. 49, no. 2, pp. 371-381, Feb. 2003.
- [3] R. Köetter and M. Médard, "Beyond routing: an algebraic approach to network coding," *Proc. IEEE INFOCOM 2002*, pp. 122-130, Jun. 2002.
- [4] T. Ho, R. Köetter, M. Médard, D. R. Karger, and M. Effros, "The benefits of coding over routing in a randomized setting," *Proc. IEEE ISIT 2003*, Yokohama, Japan, p. 442, Jun. 2003.
- [5] R. Köetter and M. Médard, "An algebraic approach to network coding," *IEEE/ACM Trans. Networking*, vol. 11, no. 5, pp. 782-795, Oct. 2003.
- [6] T. Ho, M. Médard, J. Shi, M. Effros, and D. R. Karger, "On randomized network coding," *Proc. 41st Annual Allerton Conference on Communication Control and Computing*, Monticello, U.S.A., Oct. 2003.
- [7] P. A. Chou, Y. Wu, and K. Jain, "Practical network coding," *51st Allerton Conf. Communication, Control and Computing*, Oct. 2003.
- [8] C. Fragouli, J. Y. L. Boudec, and J. Widmer, "Network coding: an instant primer," *SIGCOMM Comput. Commun. Rev.*, vol. 36, no. 1, pp. 63-68, Jan. 2006.

BIBLIOGRAPHY

- [9] S. Katti, D. Katabi, W. Hu, H. Rahul, and M. Médard, “The importance of being opportunistic: practical network coding for wireless environments,” *Proc. of Allerton Conference on Communication, Control, and Computing*, 2005.
- [10] S. Katti, H. Rahul, W. Hu, D. Katabi, M. Médard, and J. Crowcroft, “XORs in the air: Practical wireless network coding,” *IEEE/ACM Trans. Networking*, vol. 16, no. 3, pp. 497-510, Jun. 2008.
- [11] A. Argyriou, “Wireless network coding with improved opportunistic listening,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 2014-2023, Apr. 2009.
- [12] The Network Coding Home Page. Available: <http://www.networkcoding.info>.
- [13] Y. Chen, S. Kishore, and J. Li, “Wireless diversity through network coding,” *Proc. IEEE WCNC 2006*, Las Vegas, USA, pp. 1681-1686, Apr. 2006.
- [14] X. Bao and J. Li, “Adaptive network coded cooperation (ANCC) for wireless relay networks: matching code-on-graph with network-on-graph,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 2, pp. 574-583, Feb. 2008.
- [15] T. E. Hunter, S. Sanayei, and A. Nosratinia, “Outage analysis of coded cooperation,” *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 375-391, Feb. 2006.
- [16] L. Xiao, T. Fuja, J. Klierer, and D. Costello, “A network coding approach to cooperative diversity,” *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3714-3722, Oct. 2007.
- [17] Z. Ding, K. K. Leung, D. L. Goeckel, and D. Towsley, “On the study of network coding with diversity,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1247-1259, Mar. 2009.
- [18] D. S. Lun, M. Médard, R. Köetter, “Network coding for efficient wireless unicast,” *IEEE International Zurich Seminar on Communications*, pp. 74-77, Feb. 2006.
- [19] A. Lee and M. Médard, “Simplified random network codes for multicast networks,” *Proc. IEEE ISIT 2005*, Adelaide, Australia, pp. 1725-1729, Sept. 2005.

- [20] X. Yan, Z. Zhang, and J. Yang, "Explicit inner and outer bounds for multi-source multi-sink network coding," *Proc. IEEE ISIT 2006*, Seattle, USA, pp. 64-58, Jul. 2006.
- [21] L.-L. Xie, "Network coding and random binning for multi-user channels," *Proc. of 10th Canadian Workshop on Inf. Theory*, Edmonton, Canada, Jun. 2007.
- [22] A. T. Campo and A. Grant, "On random network coding for multicast," *Proc. IEEE ISIT 2007*, Nice, France, pp. 1591-1595, Jun. 2007.
- [23] X. Yan, R. W. Yeung, and Z. Zhang, "The capacity region for multi-source multi-sink network coding," *Proc. IEEE ISIT 2007*, Nice, France, pp.116-120, Jun. 2007.
- [24] T. Ho, M. Médard, R. Köetter, D. R. Karger, M. Effros, J. Shi, and B. Leong, "A random linear network coding approach to multicast," *IEEE Trans. Inf. Theory*, vol. 52, no. 10, pp. 4413-4430, Oct. 2006.
- [25] Avalanche: File swarming with network coding.
<http://research.microsoft.com/en-us/projects/avalanche/default.aspx>.
- [26] C. Gkantsidis and P. Rodriguez, "Network coding for large scale content distribution," *Proc. IEEE INFOCOM 2005*, Mar. 2005.
- [27] C. Gkantsidis and P. Rodriguez, "Cooperative security for network coding file distribution," *Proc. IEEE INFOCOM 2006*, Apr. 2006.
- [28] C. Gkantsidis, J. Miller, and P. Rodriguez, "Anatomy of a P2P content distribution system with network coding," *Proc. 5th International Workshop on Peer-to-Peer Systems (IPTPS 2006)*, 2006.
- [29] G. Ma, Y. Xu, K. Ou, and W. Luo, "How can network coding help P2P content distribution?" *Proc. IEEE ICC 2009*, Dresden, Germany, Jun. 2009.
- [30] J. Sundararajan and H. Viswanathan, "Comparison of schemes for streaming multicast in cellular networks with relays," *Proc. IEEE WCNC 2006*, Las Vegas, USA, pp. 1178-1183, Apr. 2006.

BIBLIOGRAPHY

- [31] S. Jeon and S. Lee, "A relay-assisted handover technique with network coding over multihop cellular networks," *IEEE Commun. Lett.*, vol. 11, no. 3, pp. 252-254, Mar. 2007.
- [32] A. Al Hamra, C. Barakat, and T. Turletti, "Network coding for wireless mesh networks: A case study," *7th IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks (WoWMoM)*, Niagara-Falls, Buffalo-NY, USA, Jun. 2006.
- [33] A. Argyriou, "Network coding in IEEE 802.11 wireless LANs with an enhanced channel access scheme," *Proc. IEEE GLOBECOM 2008*, New Orleans, LO, Nov. 2008.
- [34] N. Cai and R. W. Yeung, "Network coding and error correction," *Proc. IEEE ITW 2002*, Bangalore, India, pp. 119-122, Oct. 2002.
- [35] N. Cai and R. W. Yeung, "Network error correction," *Proc. IEEE ISIT 2003*, Yokohama, Japan, p. 101, Jun. 2003.
- [36] R. W. Yeung and N. Cai, "Network error correction, Part I: Basic concepts and upper bounds," *Communications in Information and Systems*, vol. 6, no. 1, pp. 19-36, 2006.
- [37] N. Cai and R. W. Yeung, "Network error correction, Part II: Lower bounds," *Communications in Information and Systems*, vol. 6, no. 1, pp. 37-54, 2006.
- [38] K. Bhattad and K. R. Narayanan, "Weakly secure network coding," *NetCod*, Apr. 2005.
- [39] D. Silva, F. R. Kschischang, and R. Köetter, "A rank-metric approach to error control in random network coding," *IEEE Trans. Inf. Theory*, vol. 54, no. 9, pp. 3951-3967, Sept. 2008.
- [40] R. Köetter and F. R. Kschischang, "Coding for errors and erasures in random network coding," *IEEE Trans. Inf. Theory*, vol. 54, no. 8, pp. 3579-3591, Aug. 2008.

- [41] E. C. van der Meulen, "Three-terminal communication channels," *Adv. Appl. Probab.*, vol. 3, pp. 120-154, 1971.
- [42] E. C. van der Meulen, "A survey of multi-way channels in information theory: 1961-1976," *IEEE Trans. Inf. Theory*, vol. IT-23, no. 1, pp. 1-37, Jan. 1977.
- [43] T. M. Cover and A. El Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inf. Theory*, vol. 25, pp. 572-584, Sep. 1979.
- [44] G. Kramer and M. Gapster, "Capacity theorems for wireless relay channels," *Presented at the 41st Allerton Conf. on Commun., Control, and Comp.*, Oct. 2003.
- [45] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inf. Theory*, vol. 51, pp. 3037-3063, Sept. 2005.
- [46] N. J. Laneman, D. N. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.
- [47] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, pp. 1927-1938, Nov. 2003.
- [48] —, "User cooperation diversity-Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, pp. 1939-1948, Nov. 2003.
- [49] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. Commun.*, vol. 52, no. 10, pp. 1820-1830, Oct. 2004.
- [50] A. Nosratinia and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Commun. Mag.*, vol. 42, no. 10, pp. 74-80, Oct. 2004.
- [51] Frank H. P. Fitzek and Marcos D. Katz (eds.), *Cooperation in Wireless Networks: Principles and Applications*. 2006 Springer.

BIBLIOGRAPHY

- [52] M. O. Hasna and M.-S. Alouini, "Performance analysis of two-hop relayed transmissions over Rayleigh fading channels," *Proc. IEEE VTC 2002-Fall*, pp. 1992-1996, Sept. 2002.
- [53] M. O. Hasna and M.-S. Alouini, "Harmonic mean and end-to-end performance of transmission systems with relays," *IEEE Trans. Commun.*, vol. 52, no. 1, pp. 130-135, Jan. 2004.
- [54] Y. Wu, P. A. Chou, and S.-Y. Kung, "Information exchange in wireless networks with network coding and physical-layer broadcast," *Proc. CISS*, Mar. 2005.
- [55] P. Larsson, N. Johansson, and K.-E. Sunell, "Coded bi-directional relaying," *AD-HOC05*, May 2005.
- [56] P. Larsson, N. Johansson, and K.-E. Sunell, "Coded bi-directional relaying," *Proc. 63rd IEEE Vehicular Technology Conference (VTC Spring 2006)*, Melbourne, Australia, May 2006.
- [57] B. Rankov and A. Wittneben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Selected Areas Commun.*, vol. 25, pp. 379-389, Feb. 2007.
- [58] S. Zhang, S. Liew, and P. Lam, "Physical layer network coding," *Proc. 12th Annual International Conference on Mobile Computing and Networking (ACM MobiCom 2006)*, Sept. 2006.
- [59] S. Katti, Shyamnath Gollakota, and Dina Katabi, "Embracing wireless interference: Analog network coding," *SIGCOMM 2007*, Kyoto, Japan, Aug. 2007.
- [60] P. Popovski and H. Yomo, "Bi-directional amplification of throughput in a wireless multihop network," *Proc. IEEE VTC 2006-Spring*, Melbourne, pp. 588-593, May 2006.
- [61] —, "The anti-packets can increase the achievable throughput of a wireless multi-hop network," *Proc. IEEE ICC 2006*, Istanbul pp. 3885-3860, Jun. 2006.
- [62] —, "Wireless network coding by amplify-and-forward for bi-directional traffic flows," *IEEE Commun. Lett.*, vol. 11, no. 1, pp. 16-18, Jan. 2007.

- [63] —, “Physical network coding in two-way wireless relay channels,” *Proc. ICC 2007*, Glasgow, Scotland, pp.707-712, Jun. 2007.
- [64] S. Zhang and S.-C. Liew, “Channel coding and decoding in a relay system operated with physical-layer network coding,” *IEEE J. Select areas in Commun.*, vol. 27, no. 5, pp. 788-796, Jun. 2009.
- [65] H. Gacanin and F. Adachi, “Performance of physical layer network coding in a frequency-selective fading channel,” *Proc. IEEE PIMRC 2009*, Tokyo, pp. 928-932, Sept. 2009.
- [66] P. Hu and M. Ibnkahla, “A survey of physical-layer network coding in wireless networks,” *25th Biennial Symposium on Communications*, pp. 311-314, May 2010.
- [67] B. Rankov and A. Wittneben, “Achievable rate regions for the two-way relay channel,” *Proc. IEEE ISIT 2006*, Seattle, USA, pp. 1668-1672, Jul. 2006.
- [68] T. J. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, “Achievable rate region of a two phase bidirectional relay channel,” *Proc. CISS*, pp. 408-413, Mar. 2007.
- [69] —, “Broadcast capacity region of two-phase bidirectional relaying,” *IEEE Trans. Inf. Theory*, vol. 54, no. 1, pp. 454-458, Jan. 2008.
- [70] R. F. Wyrembelski, T. J. Oechtering, and H. Boche, “Decode-and-forward strategies for bidirectional relaying,” *Proc. IEEE PIMRC 2008*, Cannes, France, Sept. 2008.
- [71] T. J. Oechtering and H. Boche, “Piggyback a common message on half-duplex bidirectional relaying,” *IEEE Trans. Wireless Commun.*, vol. 7, no. 9, pp. 3397-3406, Sept. 2008.
- [72] C.-H. Liu and F. Xue, “Network coding for two-way relaying: rate region, sum rate and opportunistic scheduling,” *Proc. IEEE ICC 2008*, Beijing, China, pp. 1044-1049, May 2008.
- [73] S. J. Kim, P. Mitran, and V. Tarokh, “Performance bounds for bi-directional coded cooperation protocols,” *IEEE Trans. Inf. Theory*, vol. 54, no. 11, pp.5235-5241, Nov. 2008.

BIBLIOGRAPHY

- [74] Y. Han, S. H. Ting, C. K. Ho, and W. H. Chin, "High rate two-way amplify-and-forward half-duplex relaying with OSTBC," *Proc. IEEE VTC 2008-Spring*, Singapore, pp. 2426-2430, May 2008.
- [75] —, "Performance bounds for two-way amplify-and-forward relaying," *IEEE Trans. Wireless Commun.*, vol. 8, no. 1, pp. 432-439, Jan. 2009.
- [76] R. H. Y. Louie, Y. Li, and B. Vucetic, "Practical physical layer network coding for two-way relay channels: performance analysis and comparison," *IEEE Trans. Wireless Commun.*, vol. 9, no. 2, pp. 764-777, Feb. 2010.
- [77] X. Bao and J. Li, "Decode-amplify-forward (DAF): A new class of forwarding strategy for wireless relay channels," *Proc. IEEE Workshop on SPAWC*, pp. 816-820, Jun. 2005.
- [78] M. R. Souryal and B. R. Vojcic, "Performance of amplify-and-forward and decode-and-forward relaying in Rayleigh fading with turbo codes," *Proc. IEEE ICASSP*, pp. 681-684, May 2006.
- [79] J. Ch. Fricke, M. M. Butt, and P. A. Hoeher, "Quality-oriented adaptive forwarding for wireless relaying," *IEEE Commun. Lett.*, vol. 12, pp. 200-202, Mar. 2008.
- [80] Q. Li, S. H. Ting, A. Pandharipande, and Y. Han, "Outage analysis for adaptive two-way relaying," *Proc. IEEE PIMRC 2008*, Cannes, France, Sept. 2008.
- [81] —, "Adaptive two-way relaying and outage analysis," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 3288-3299, Jun. 2009.
- [82] H. Wicaksana, S. H. Ting, M. Motani, Y. L. Guan, "On the diversity-multiplexing tradeoff of amplify-and-forward half-duplex relaying," *IEEE Trans. Commun.*, vol. 58, no. 12, pp. 3621-3630, Dec. 2010.
- [83] T. Ho and D. S. Lun, *Network Coding: An Introduction*. Cambridge University Press 2008.
- [84] R. W. Yeung, *A First Course in Information Theory*. New York, NY: Kluwer Academic/Plenum, 2002.

- [85] R. W. Yeung, S.-Y. R. Li, N. Cai, and Z. Zhang, “Network coding theory.” *Foundations and Trends in Communications and Information Theory*, vol. 2, 2005.
- [86] R. W. Yeung, *Information Theory and Network Coding*. Springer 2008.
- [87] C. Fragouli and E. Soljanin, “Network coding fundamentals.” *Foundations and Trends in Networking*, vol. 2, no. 1, 2007.
- [88] —, “Network coding application.” *Foundations and Trends in Networking*, vol. 2, no. 2, 2007.
- [89] A. Papoulis and S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, 4th Edition. McGraw Hill, 2002.
- [90] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*, 7th Edition. Academic, 2007.
- [91] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. 2nd edition. Wiley, 2006.
- [92] A. Wyner, “Recent results in the Shannon theory,” *IEEE Trans. Inf. Theory*, vol. 20, no. 1, pp. 2-10, Jan. 1974.
- [93] A. El Gamal and T. M. Cover, “Multiple user information theory,” *Proc. of the the IEEE*, vol. 68, no. 12, pp. 1466-1483, Dec. 1980.
- [94] G. Kramer, “Topics in multi-user information thoery,” *Foundations and Trends in Networking*, vol. 4, no. 4-5, 2008.
- [95] T. M. Cover, “Broadcast channels,” *IEEE Trans. Inf. Theory*, vol. 18, pp. 2-14, Jan. 1972.
- [96] P. P. Bergmans, “A simple converse for broadcast channels with additive white Gaussian noise,” *IEEE Trans. Inf. Theory*, vol. 20, pp. 279-280, Mar. 1974.
- [97] R. G. Gallager, “Capacity and coding for degraded broadcast channels,” *Probl. Peredachi Inf.*, vol. 10, no. 3, pp. 3-14, 1974.
- [98] T. M. Cover, “Comments on broadcast channels,” *IEEE Trans. Inf. Theory*, vol. 44, no. 6, pp. 2524-2530, Oct. 1998.

BIBLIOGRAPHY

- [99] Y. Wu, "Broadcasting when receivers know some messages a priori," *Proc. IEEE ISIT 2007*, Nice, France, Jun. 2007.
- [100] F. Xue and S. Sandhu, "PHY-layer network coding for broadcast channel with side information," *Proc. IEEE ITW 2007*, Tahoe City, CA, pp. 108-113, Sept. 2007.
- [101] G. Kramer and S. Shamai, "Capacity for classes of broadcast channels with receiver side information," *IEEE ITW 2007*, Tahoe City, CA, Sept. 2007.
- [102] H. Yomo and P. Popovski, "Opportunistic scheduling for wireless network coding," *Proc. IEEE ICC 2007*, Glasgow, Scotland, pp. 5610-5615, Jun. 2007.
- [103] —, "Opportunistic scheduling for wireless network coding," *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2766-2770, Jun. 2009.
- [104] E. Tuncel, "Slepian-Wolf coding over broadcast channels," *IEEE Trans. Inf. Theory*, vol. 52, no. 4, pp. 1469-1482, Apr. 2006.
- [105] C. Fragouli, J. Widmer, and J.-Y. L. Boudec, "Efficient broadcasting using network coding," *IEEE/ACM Tran. Networking*, vol. 16, no. 2, pp. 450-463, Apr. 2008.
- [106] H. Liao, "Multiple access channels," Ph.D. Dissertation, Univesity of Hawaii, Honolulu, 1972.
- [107] D. Slepian and J. K. Wolf, "Noiseless coding of correlated information sources," *IEEE Trans. Inf. Theory*, vol. IT-19, no. 4, pp. 471-480, Jul. 1973.
- [108] —, "A coding theorem for multiple access channels with correlated sources," *Bell Syst. Tech. J.*, vol. 52, pp. 1037-1076, Sept. 1973.
- [109] T. S. Han, "The capacity region of general multiple-access channel with certain correlated sources," *Information and Control* 40(1), pp. 37-60, 1979.
- [110] T. M. Cover, A. El Gammal, and M. Salehi, "Multiple access channels with arbitrarily correlated sources," *IEEE Trans. Inf. Theory*, vol. IT-26, no. 6, pp. 648-657, Nov. 1980.

- [111] T. Ho, M. Médard, M. Effros, and R. Köetter, “Network coding for correlated sources,” *Proc. CISS 2004*.
- [112] J. Barros and S. D. Servetto, “Network information flow with correlated sources,” *IEEE Trans. Inf. Theory*, vol. 52, no. 1, pp. 155-170, Jan. 2006.
- [113] B. Nazer and M. Gapster, “Computation over multiple-access channels,” *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3498-3516, Oct. 2007.
- [114] C. E. Shannon, “A mathematical theory of communication,” *Bell Syst. Tech. J.*, vol. 27, pp. 379-423, 1948.
- [115] L. Song, R. W. Yeung, and N. Cai, “A separation theorem for single-source network coding,” *IEEE Trans. Inf. Theory*, vol. 52, no. 5, pp. 1861-1871, May 2006.
- [116] S. P. Borade, “Network information flow: limits and achievability,” *Proc. IEEE ISIT 2002*, Lausanne, Switzerland, p. 139, Jun. 2002.
- [117] M. R. Aref, “Information flow in relay networks,” Ph.D. dissertation, Stanford Univ., Stanford, CA, 1980.
- [118] N. Ratnakar and G. Kramer, “On the separation of channel and network coding in Aref networks,” *Proc. IEEE ISIT 2005*, Adelaide, Australia, pp. 1716-1719, Sept. 2005.
- [119] N. Ratnakar and G. Kramer, “The multicast capacity of deterministic relay networks with no interference,” *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2425-2432, Jun. 2006.
- [120] R. Gowaikar, A. F. Dana, B. Hassibi, and M. Effros, “A practical scheme for wireless network operation,” *IEEE Trans. Commun.*, vol. 55, pp. 463-476, Mar. 2007.
- [121] A. F. Dana, R. Gowaikar, B. Hassibi, M. Effros, and M. Médard, “Should we break a wireless network into sub-networks?” *Proc. Annu. Allerton Conf. Commun., Control, Comput.*, 2003.

BIBLIOGRAPHY

- [122] A. F. Dana, M. Sharif, B. Hassibi, and M. Effros, “Is broadcast plus multiaccess optimal for Gaussian wireless networks with fading?” *Proc. 37th Asilomar Conf. Signals, Syst. Comput.*, pp. 1748-1752, Nov. 2003.
- [123] M. Effros, M. Médard, T. Ho, S. Ray, D. Karger, R. Köetter, and B. Hassibi, “Linear network codes: A unified framework for source, channel, and network coding,” *DIMACS workshop on network information theory, invited paper*, 2003.
- [124] A. Ramamoorthy, K. Jain, P. A. Chou, and M. Effros, “Separating distributed source coding from network coding,” *IEEE Trans. Inf. Theory*, vol. 52, no. 6, pp. 2785-2795, Jun. 2006.
- [125] C. Hausl and J. Hagenauer, “Iterative network and channel decoding for the two-way relay channel,” *Proc. IEEE ICC 2006*, Istanbul, Turkey, vol. 4, pp. 1568-1573, Jun. 2006.
- [126] C. Hausl and P. Dupraz, “Joint network-channel coding for the multiple-access relay channel,” *3rd Annual IEEE Communications Society on Sensor and Ad Hoc Communications and Networks*, Virginia, U.S.A., vol. 3, pp. 817-822, Sept. 2006.
- [127] C. Hausl, “Joint network-channel coding for the multiple-access relay channel based on turbo codes,” *European Transactions on Telecommunications*, vol. 20, pp. 175-181, Jan. 2009.
- [128] S. Yang and R. Köetter, “Network coding over a noisy relay: a belief propagation approach,” *Proc. IEEE ISIT 2007*, Nice, France, pp. 801-804, Jun. 2007.
- [129] S. Zhang, Y. Zhu, S. C. Liew, and K. B. Letaief, “Joint design of network coding and channel decoding for wireless networks,” *Proc. IEEE WCNC 2007*, Hongkong, pp. 779-784, Mar. 2007.
- [130] L. Xiao, T. E. Fuja, J. Klierer, and D. J. Costello, Jr., “Nested codes with multiple interpretations,” *Proc. CISS*, Princeton, U.S.A., Mar. 2006.
- [131] J. Klierer, T. Dikaliotis, and T. Ho, “On the performance of joint and separate channel and network coding in wireless fading networks,” *Proc. IEEE ITW 2007*, California, U.S.A., pp. 1-5, Jul. 2007.

- [132] S. Avestimehr, S. Diggavi, and D. Tse, "Wireless network information flow: A deterministic approach," submitted to *IEEE Trans. Inf. Theory*, 2009. [Online]. Available: <http://arxiv.org/abs/0906.5394>
- [133] S. H. Lim, Y. Kim, A. El Gamal, and S. Chung, "Noisy network coding," *Proc. IEEE ITW 2010*, Cairo, Egypt, Jan. 2010.
- [134] Q. Li, S. H. Ting, and C. K. Ho, "Nonlinear network code for high throughput broadcasting with retransmissions," *Proc. IEEE ISIT 2009*, Seoul, Korea, pp. 2853-2857, Jun. 2009.
- [135] —, "Joint network and channel coding for wireless networks," *SECON Workshops 09*, Rome, Italy, Jun. 2009.
- [136] —, "A joint network and channel coding strategy for wireless decode-and-forward relay networks," *IEEE Trans. Commun.*, vol. 59, no. 1, pp. 181-193, Jan. 2011.
- [137] Michael Luby, "LT-Codes," *Proceedings of 43rd Annual IEEE Symposium on Foundations of Computer Science (FOCS)*, Vancouver, BC, Canada, pp.271-280, Nov. 2002.
- [138] A. Shokrollahi, "Raptor codes," *IEEE Trans. Inf. Theory*, vol.52, no.6, pp.2551-2567, Jun. 2006.
- [139] D. Park and S.-Y. Chung, "Performance-complexity tradeoffs of rateless codes," *Proc. IEEE ISIT 2008*, Toronto, ON, pp. 2056-2060, Jul. 2008.
- [140] A. F. Dana, R. Gowaikar, R. Palanki, B. Hassibi, and M. Effros, "Capacity of wireless erasure networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 3, pp. 789-804, Mar. 2006.
- [141] P. Gupta and P. R. Kumar, "Towards an information theory of large networks: an achievable rate region," *IEEE Trans. Inf. Theory*, vol. 49, no. 8, pp. 1877-1894, Aug. 2003.
- [142] R. Wolff, *Stochastic Modeling and the Theory of Queues*. Upper Saddle River, NJ: Prentice-Hall, 1989.

BIBLIOGRAPHY

- [143] G. Caire and D. Tuninetti, "The throughput of hybrid-ARQ protocols for the Gaussian collision channel," *IEEE Trans. Inf. Theory*, vol. 47, pp. 1971-1988, Jul. 2001.
- [144] R. Comroe and D. Costello, "ARQ schemes for data transmission in mobile radio systems," *IEEE J. Select Areas Commun.*, vol. 2, no. 4, pp. 472-481, Jul. 1984.
- [145] E. Ahmed, A. Eryilmaz, M. Medard, and A. Ozdaglar, "On the scaling law of network coding gains in wireless networks," *Proc. MILCOM 2007*, Orlando, FL, Oct. 2007.
- [146] I. Stojanovic, Z. Wu, M. Sharif, and D. Starobinski, "Data dissemination in wireless broadcast channels: Network coding versus cooperation," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1726-1732, Apr. 2009.
- [147] R. Cogill, B. Shrader, and A. Ephremides, "Stable throughput for multicast with random linear coding," *IEEE Trans. Inf. Theory*, vol. 57, no. 1, pp. 267-281, Jan. 2011.
- [148] J. Golestani, "A self-clocked fair queueing scheme for broadband applications," *Proc. IEEE INFOCOM 1994*, pp. 636-646, Jun. 1994.
- [149] Q. Li, S. H. Ting, and C. K. Ho, "On the available receiver side information in wireless network coding," *Proc. IEEE ICC 2011*, Kyoto, Japan, Jun, 2011.
- [150] National Telecommunications and information administration (NTIA), "FCC frequency allocation chart," 2003. [Online]. Available: <http://www.ntia.doc.gov/osmhome/allochrt.pdf>.
- [151] FCC Spectrum Policy Task Force, "Report of the spectrum efficiency working group," Nov. 2002. [Online]. Available: <http://www.fcc.gov/sptf/reports.html>.
- [152] C. Stevenson, G. Chouinard, Z. Lei, W. Hu, S. Shellhammer, and W. Caldwell, "IEEE 802.22: the first cognitive radio wireless regional area network standard," *IEEE Communnications Magazine*, vol. 47, no. 1, pp. 130-138, Jan. 2009.
- [153] J. Mitola, "Cognitive radio: an integrated agent architecture for software defined radio," Ph.D. dissertation, KTH, Stockholm, Sweden, Dec. 2000.

- [154] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select Areas Commun.*, vol. 23, pp. 201-220, Feb. 2005.
- [155] Y. Xing, C. N. Mathur, M. A. Haleem, R. Chandramouli, and K. P. Subbalakshmi, "Dynamic spectrum access with QoS and interference temperature constraints," *IEEE Trans. Mobile Comput.*, vol. 6, pp. 423-433, Apr. 2007.
- [156] Q. Zhao and B. M. Sadler, "A survey of dynamic spectrum access," *IEEE Signal Process. Mag.*, pp. 79-89, May 2007.
- [157] A. Goldsmith, S. A. Jafar, I. Marić, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. of the IEEE*, vol. 97, no. 5, pp. 894-914, May 2009.
- [158] A. B. Carleial, "Interference channels," *IEEE Trans. Inf. Theory*, vol. IT-24, no. 1, pp. 60-70, Jan. 1978.
- [159] N. Devroye, P. Mitran, and V. Tarokh, "Achievable rates in cognitive radio channels," *IEEE Trans. Inf. Theory*, vol. 52, pp. 1813-1827, May 2006.
- [160] S. Gel'fand and M. Pinsker, "Coding for channels with random parameters," *Probl. Contr. and Inf. Theory*, vol. 9, no. 1, pp. 19-31, 1980.
- [161] H. M. Costa, "Writing on dirty paper," *IEEE Trans. Inf. Theory*, vol. IT-29, no. 3, pp. 439-441, May 1983.
- [162] T. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *IEEE Trans. Inf. Theory*, vol. IT-27, no. 1, pp. 49-60, Jan. 1981.
- [163] I. Marić, A. Goldsmith, G. Kramer, and S. Shamai, "On the capacity of interference channels with a cognitive transmitter," *Information Theory and Applications Workshop*, pp. 268-273, Jan. 2007.
- [164] I. Marić, R. D. Yates, and G. Kramer, "Capacity of interference channels with partial transmitter cooperation," *IEEE Trans. Inf. Theory*, vol. 53, no. 10, pp. 3536-3548, Oct. 2007.
- [165] A. Jovićić and P. Viswanath, "Cognitive radio: an information-theoretic perspective," *IEEE Trans. Inf. Theory*, vol. 55, no. 9, pp. 3945-3958, Sept. 2009.

BIBLIOGRAPHY

- [166] O. Simeone, J. Gambini, Y. Bar-Ness, and U. Spagnolini, "Cooperation and cognitive radio," *Proc. IEEE ICC 2007*, Glasgow, pp. 6511-6515, Jun. 2007.
- [167] Y. Han, A. Pandharipande, and S. H. Ting, "Cooperative spectrum sharing via controlled amplify-and-forward relaying," *Proc. IEEE PIMRC 2008*, Cannes, France, Sept. 2008.
- [168] —, "Cooperative decode-and-forward relaying for secondary spectrum access," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 4945-4950, Oct. 2009.
- [169] Q. Li, S. H. Ting, A. Pandharipande, Y. Han, "Cognitive spectrum sharing with two-way relaying systems," *IEEE Trans. Veh. Tech.*, vol. 60, no. 3, pp. 1233-1240, Mar. 2011.
- [170] O. Simeone, I. Stanojev, S. Savazzi, Y. Bar-Ness, U. Spagnolini, and R. Pickholtz, "Spectrum leasing to cooperating secondary ad hoc networks," *IEEE J. Select Areas Commun.*, vol. 26, no. 1, pp. 203-213, Jan. 2008.
- [171] P. Liu, Z. Tao, S. Narayanan, T. Korakis, and S. Panwar, "A cooperative MAC protocol for wireless LANs," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 340-354, Feb. 2007.
- [172] S. Valentin, H. S. Lichte, H. Karl, S. Simoens, G. Vivier, J. Vidal, and A. Agustin, "Implementing cooperative wireless networks," *Cognitive Wireless Networks*, F. H. P. Fitzek and M.D. Katz, Ed. Netherlands: Springer, 2007, pp. 155-178.