Essays in financial crises

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2011

Zheng, H. (2011). Essays in financial crises. Doctoral thesis, Nanyang Technological University, Singapore.

https://hdl.handle.net/10356/46229

https://doi.org/10.32657/10356/46229

Essays in Financial Crises

by

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Submitted to the Division of Economics, School of Humanities and Social Sciences
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

NANYANG TECHNOLOGICAL UNIVERSITY

April 2011

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Division of Economics, School of Humanities and Social Sciences
June 8, 2011

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Abstract

This thesis explores the power of deterministic dynamic model in capturing the qualitative attributes of financial crises and statistical features of the financial time series. It is shown that, even without any random processes/variables, the deterministic dynamic model performs well in generating crises of different patterns which are distinguished by their durations and accumulative depth and in reproducing a wide range of stylized facts that are common across financial markets. Such good performance is ascribed to the innovative modeling of investors' beliefs. Specifically, investors' beliefs are regime-dependent, they are updated according to various psychological trading regimes enclosed by different support and resistance prices.

Chapter 1 describes three types of financial crises, namely sudden crisis, smooth crisis and disturbing crisis and addresses the motivation of this thesis.

Chapter 2 examines various types of financial crises and conjectures their underlying mechanisms using a deterministic heterogeneous agent model (HAM). In a market-maker framework, forward-looking investors update their price expectations according to psychological trading windows and cluster themselves strategically to optimize their expected profits. The switches between trading strategies lead to price dynamics in market that subsequently move price up and down, and in the extreme case, cause financial crises. The model suggests that both fundamentalists and chartists could potentially contribute to the financial crises.

Chapter 3 applies the nonlinear deterministic heterogeneous agent model proposed in Chapter 2 to simultaneously reproduce a wide range of stylized facts that are common across financial markets, namely, fat tails, volatility clustering, long range dependence and leverage effect. It is inferred from the model that (i) the magnitude of the most negative return is greater than that of the most positive return as the net buying force of fundamentalists is not comparable with the collective selling force of chartists and (ii) gradual bubbles and sudden crashes could be

originated from the fact that upward price movements are counterbalanced while downward price movements are always enhanced by fundamentalists.

Chapter 4 develops a model based on Chapter 2 and Day and Huang (1990). It focuses on the regime-dependent belief, in the attempt to explain how crises differ from each other. By introducing the regime-dependent belief into a simple deterministic HAM, different types of crises can be accommodated simultaneously. Besides, the performance of HAM in capturing the salient qualitative and statistical properties in the real financial time series is improved. Specifically, the simulated results exhibit technical price patterns including head-and-shoulder and double-dip and reproduce various stylized facts.

Chapter 5 concludes by summarizing the findings and contribution of this thesis. It points out the caveats and outline the promising aspects for future research.

Chapter 1

Introduction

Market capitalization evaporates dramatically and profoundly at the time of financial crisis, making the understanding of the complexity of dynamical market behavior during the crisis particularly important. Different crises exhibit themselves differently in terms of depth and length, which are key determinants for many financial decisions. Unfortunately, forecasting the depth and length of the crisis is hard, if not impossible, as they are usually the outcome of complicated combination of internal dynamics and various external shocks. However, identifying fundamental factors, which are necessary but not sufficient conditions of the crisis, is possible. Turning to an endogenous deterministic heterogeneous agent model (HAM) we aim to investigate such possibility as we believe that the crisis has fundamentally an endogenous or internal origin and that exogenous or external shocks only serve as triggering factors[1]. If a model is capable of generating different types of crises that match with real historical scenarios, it should capture correctly some, if not all, fundamental factors of the crisis. The fundamental factors identified may not be able to explain completely why a market collapse but they are certainly indispensable in providing an understanding on the origins of

[1] See for example Brock and Hommes (1998), Hommes (2001), Sornette (2004) for theoretical discussions and Boswijk et al. (2007) for empirical evidence.

the crisis.

Qualitatively, the crisis happens when market capitalization drops by a certain scale or more cumulatively over a consecutive period of time (for an ex-ante definition from the perspective of mathematics see Watanabe et al. , 2007). We investigate crises documented by Kindleberger and Aliber (2005) whenever price data is available in Bloomberg. Following Rosser (2000), we group them into three types according to their depth and length. We name them as sudden crisis, smooth crisis and disturbing crisis, respectively. In the sudden crisis, price falls from the peak or very close to the peak precipitately down to the bottom or very close to the bottom. The length of such crisis is typically short. The price starts to rebound consecutively immediately after touching the bottom. A typical example of the sudden crisis is the Dow Jones Industrial Average Index (DJIA) in October 1987 as shown in Fig. 1-1. In the smooth crisis, the price lands smoothly from the peak to the bottom with the succession of a moderate but persistent decline. No visible crash occurs in between. The drop of DJIA during the Great Depression as shown in Fig. 1-2 is one of this kind. Somewhere between the sudden crisis and the smooth crisis is the disturbing crisis, during which the price fluctuates disturbingly with a downward trend before it plummets suddenly. Moreover, the price continue to decline after the plunge. Most historical crises are of this type. One example is the fall of DJIA during the Wall Street crash in October 1929 as shown in Fig. 1-3.

While various studies have been found focusing on the detection of the crisis, the prediction of its end, and even the schemes of avoiding it (Hart et al., 2002), they generally concentrate only on pre and post crisis market behaviors, rarely between them. Day and Huang (1990), Chiarella et al. (2003) and He and Westerhoff (2005) are a few exceptions. Day and Huang (1990) and He and Westerhoff (2005) uses chaotic HAM to examine the sudden crisis, Chiarella et al. (2003) analyzes the smooth crisis, and Gallegati et al. (2010) investigates the disturbing crisis. None of them, however, has presented all three patterns of crises within the same model. Given that crises exhibit themselves differently across markets at the same time and over time in the same market, if a model captures all key fundamental factors of crises correctly, it is expected to demonstrate multi types of crises simultaneously. Putting it loosely, the better a model captures the essential properties of crises, the more types of crises it should be able to demonstrate.

In contrast to the past literature, we model the three types of crises within the same deterministic HAM in this thesis. Chapter 2 proposes an HAM that consists of interacting heterogeneous agents, namely fundamentalists and chartists[2]. Fundamentalists believe that the price will eventually converge to its long-term fundamental value, which is determined by the real economic growth. They therefore buy in (sell out) the risky asset when it is under-valued (over-valued). Their trading activities consistently drive the price towards its long-term fundamental value. Chartists, on the other hand, extrapolate their price expectation from the price trend as well as their trading experience. Specifically, they form a series of psychological trading regimes, the thresholds of which correspond to the support and resistance price levels in technical analysis. Each period, the price expectation is updated according to the psychological trading regime that current price falls into. Their trading activities could either drive the price far away from its long-term fundamental value, which may lead to bubbles and crashes, or push the price towards its value. Neither fundamentalists nor chartists will stick to the same trading strategies over time. Instead, investors interact with each other and cluster themselves to the strategy that is expected to yield superior profit. The evolutionary switches between different trading strategies lead to dynamic market weights of fundamentalists and chartists, which embed market complexity.

For each individual, the demand function depends on his own adaptive belief. For the market as a whole, the aggregate demand function relies on both the adaptive beliefs and the dynamic market weights. According to the collective

[2]Literature of HAM that consists of fundamentalists and chartists can be traced back to Frankel and Froot (1986). This framework is then carried forward by Lux (1995), Brock and Hommes (1998), Farmer and Joshi (2002), He and Li (2007), to list a few.

order submitted by all fundamentalists and chartists, the market-maker constantly quotes the price according to the aggregate demand in order to provide liquidity and to balance inventory. When there is excess demand (supply), the market-maker supplies from (absorbs into) his inventory to balance the collective trading order and subsequently adjust the price up (down). As the price changes, investors update their price expectations and cluster themselves strategically to optimize their expected profits. Such process repeats itself, which moves the price up and down, and in the extreme case, cause financial crises of various types.

While Chapter 2 focuses on the theoretical modeling of financial crises, Chapter 3 takes a step further to evaluate the model's solidity from the statistical and qualitative perspectives. Applying the nonlinear deterministic HAM proposed in Chapter 2, a wide range of stylized facts that are common across financial markets, namely, fat tails, volatility clustering, long range dependence and leverage effect, can be simultaneously reproduced. Some of these stylized facts, such as long-range dependence, are quite hard to be duplicated in current HAMs, especially within a deterministic framework. The fact that this model is capable of generating so many stylized facts without relying on any random process, suggests that the model has well captured some of the essential factors underlying the complex financial market. Asides from its power of matching with various stylized facts, the model represents the phenomena of asymmetric returns, gradual bubbles and sudden crashes. It is inferred from the model that (i) the magnitude of the most negative return is greater than that of the most positive return as the net buying force of fundamentalists is not comparable with the collective selling force of chartists; (ii) the gradual bubbles and sudden crashes could origin from the fact that the upward price movements are counterbalanced while the downward movements are always enhanced by fundamentalists.

The model in Chapter 2 covers many aspects of investment behavior, such as adaptive learning within and across different trading groups, continuous interaction among heterogeneous agents, evolutionary switching among different strategies. The wide coverage enriches the model's performance in various dimensions. However, it also complicates the dynamic system, rending it hard to single out what contributes essentially to the financial crisis and what distinguish the crisis from each other. In order to complement this issue, Chapter 4 focuses on discussing one single factor that is innovative in this thesis, the regime-dependent belief, with investors' price expectation relying on their psychological trading regimes. To figure out the value of such a factor, we introduce the regime-dependent belief into a simple deterministic HAM established in Day and Huang (1990) and evaluate its marginal contribution to the model's performance. It is found that after accounting for the regime-dependent belief, different types of crises can be accommodated simultaneously, which cannot be fulfilled in Day and Huang (1990). It suggests that the regime-dependent belief is crucial in modeling financial crises. Perhaps more importantly, the simple set-up enables the model to shade light on why crisis differs from each other from a technical perspective. Besides, the performance of HAM in capturing the salient qualitative and statistical properties in the real financial time series is improved. Specifically, the simulated results exhibit technical price patterns including head-and-shoulder and double-dip, and reproduce various stylized facts. The model in this chapter is essentially a simplified version of that in Chapter 2, which also based on Day and Huang (1990). Despite being much more simple, its performance is not any less powerful. Like the model in Chapter 2, it is capable of replicating different types of financial crises and various stylized facts. As a result of its simplicity, it presents a better view on why crisis differs from each other compared to the model in Chapter 2.

Chapter 5 summarizes the findings of the thesis, points the caveats and extensions in methodology and outlines the promising aspects for the future research.

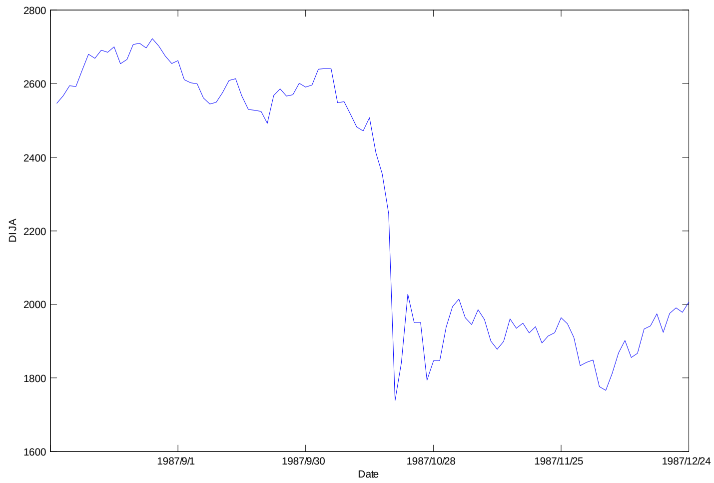


Figure 1-1: Sudden crises example: DJIA around October 19, 1987 crash.

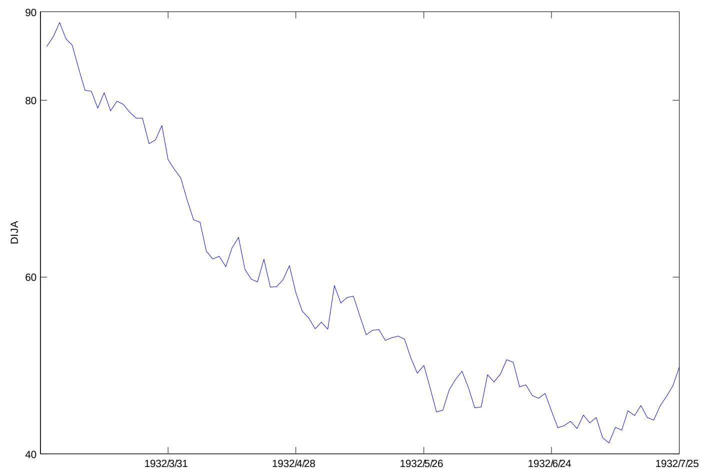


Figure 1-2: Smooth crises example: DJIA during the great depression.

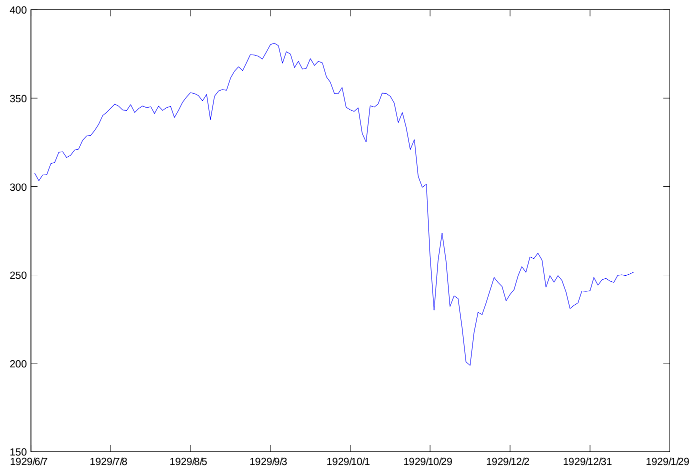


Figure 1-3: Disturbing crises example: DJIA around October 1929 Crash.

Chapter 2

Financial Crisis and Interacting Heterogeneous Agents

2.1 Introduction

The model in this chapter builds upon the heterogeneous beliefs and the market-maker framework of Day and Huang (1990), the evolutionary framework of Brock and Hommes (1998), the discount mechanism of Lux and Marchesi (1999, 2000) and the excess demand formation of Gennotte and Leland (1990). The main differences lie in the introduction of multi-phase belief system and the discounted expected profit arisen from investors' forward looking behavior. Following the work of Day and Huang (1990), Lux(1995,1998), Lux and Marchesi (1999, 2000), Farmer and Joshi (2002), and Levy (2008) and others, our model is purely deterministic so as to single out fundamental factors and avoid potential systematic pitfalls caused by random disturbances. It is found that the internal price dynamics resulted from the interactions among fundamentalists and chartists are sufficient to generate all the three types of financial crises.

The asset pricing model introduced in Brock and Hommes (1998) is the first to build a microfoundation within HAM. In the model, heterogeneous agents decide their demand or supply function through maximizing expected utility of wealth. At the end of every period, market is cleared and agents update their expected price according to a fitness measure, which is a function of past realized profits. The price dynamics driven by heterogeneous expectations are capable of explaining a range of complex financial behavior. Chiarella and He (2003) releases the assumption that market clearing price is a Walrasian equilibrium in every period and syncretizes it with the market-maker framework[1]. Such synthetical framework is widely applied in later literature such as Chiarella et al. (2009), He and Westerhoff (2005) and He and Li (2007, 2008).

If the market is cleared each period, profits from different trading strategies are liquidated immediately and can be compared directly, no discounting is necessary. In contrast, if the market is not forced to clear each period, investors are allowed to hold the risky asset for as long as they believe to be optimal. Long-term strategies, such as what are adopted by fundamental investors - buy in when price is below fundamental value and sell out when it is above, therefore become practical once a market-maker is brought in to balance aggregate demand. Since the asset holding time is no longer fixed at a single period, the agents need to discount the expected profits from various trading strategies. Lux and Marchesi (2000) raises attention on this issue by using a fixed discount factor. We extend it to account for the variance in expected holding periods. In our model, traders are assumed to be forward looking (see Keswani and Stolin, 2008; Sapp and Tiwari, 2004; Zheng, 1999), the fitness measure is thus assumed to be a function of expected paper gain instead of realized profit[2]. Investors interact with each other, evaluate discounted expected profit of various trading strategies, and evolutionarily cluster to what

[1]The two mechanisms differ mainly in the the information and and formation of expectation (Chiarella et al., 2009), which affect the market clearing mechanism. As pointed out by LeBaron (2006), the assumption of Walrasian equilibrium clears market in every period but is some what unrealistic and difficult to implement; the market maker framework does not rely on the assumption but has to deal with issues of price setting and inventory control.

[2]Various fitness measures have been proposed by e.g. Brock and Hommes (1998), Lux and Marchesi (2000), Dieci et al. (2006), Chang (2007) and Alfarano and Milakovic (2008). One thing in common for these fitness measures is that they are functions of past realized profits.

are expected to outperform the others. As investors switch trading strategies, the fractions of agents in a particular group fluctuate and subsequently lead to the dynamics in excess demand and price, and in extreme, causes crises.

All the expatiation above works for the aggregate excess demand, which directly relates to the price dynamics. Following Gennotte and Leland (1990), we normalize the excess demand with aggregate risk tolerance so that price and large trading volumes are comparable in terms of their quantities. Compared with traditional market-maker framework that uses direct excess demand, such normalization essentially captures most of the market feedback effect and leaves fewer factors to be parameterized. It makes the interpretation of dynamic in financial language easy and direct.

The belief system of representative agents is consistent with current literature in general. Fundamentalists and chartists update their price expectation according to the latest price information. Some new elements are added to the belief of chartists, who psychologically form multi trading windows for price movements and update the short-term fundamental forecast every period according to the window where the latest price locates[3].

The rest of this chapter is organized as follows. Section 2 describes the model. Section 3 focuses on the theoretical implications - mainly the relations between price fluctuation and different factors. Section 4 simulates various types of crises and offers possible economic interpretation. Section 5 concludes.

2.2 Deterministic Dynamic Model

In the market-maker based framework, investors trade risky assets with the market-maker or dealer, who buys in when investors place sell orders and sells out when investors place buy orders. The net buy orders, defined as the number of shares

[3]Similar piecewise set ups are proposed by Day (1994) and Venier (2007), with different definition and application though. Venier (2007) assigns each group with a fixed trading regimes and the investors trade only when the price falls within their affordable regimes.

demanded less the number of shares supplied from the perspective of investors, is the aggregate demand from the market-maker's point of view. The market-maker constantly quotes price according to the aggregate demand in order to provide liquidity and balance inventory. In this section, we derive the price impact function that relates the net aggregate demand at any time to prices. We first construct the general individual demand function for each investor in subsection 2.2.1, and the types of belief in subsection 2.2.2. To better proxy the actual trading behavior, we allow the investors to change their beliefs and evolutionarily cluster to trading strategy to maximize their discounted expected profit. This leads to the dynamics of market structure measured by the market fraction index in subsection 2.2.3. We then derive the price impact function in subsection 2.2.4.

2.2.1 Individual Demand Function

Consider a market with N investors investing in two types of assets - one risky and one riskfree. Let r be the *interest rate* of riskfree asset which is assumed to be completely elastic and p_t be the *price* per share of risky asset at period t[4]. $n_{i,t}$, the *number of shares* purchased by investor i ($i = 1, 2, ..., N$) with a mean variance preference in expected return, is given by:

$$n_{i,t} = a_i \frac{E_{i,t}\left(p_{t+1}|\Omega_t\right) - (1+r)p_t}{V_{i,t}(p_{t+1}|\Omega_t)}. \tag{2.1}$$

where a_i is the degree of *risk tolerance* and $a_i > 0$, $\Omega_t = \{p_t, p_{t-1}, ...p_0\}$ is the public information set available at period t, $E_i\left(p_{t+1}|\Omega_t\right)$ is the expected future price and $V_i(p_{t+1}|\Omega_t)$ is the expected price variance.

[4] Note that this price is essentially cum-price, the price that has accounted for the payment of dividend, since dividend is not explicitly considered in this model.

2.2.2 Multi-phase Heterogeneous Beliefs

Following Day and Huang (1990), N investors are classified into two broad categories according to their investment strategies - fundamentalists (α-investor) and chartists (β-investor) as they call them. For simplicity, investors taking the same strategy are assumed to be identical in terms of price-expectation, expected price variance and degree of risk tolerance. Such assumptions simplify N investors to two representative groups, distinguished by subscripts α and β, respectively.

Fundamentalists

There is x_α fundamentalists, who are assumed to hold both price expectation and variance expectation constant over time[5], that is:

$$E_\alpha\left(p_{t+1}|\Omega_t\right) \equiv \bar{p}_\alpha, \text{ and } V_\alpha(p_{t+1}|\Omega_t) = \sigma_\alpha^2,$$

where the price expectation $\bar{p}_\alpha$ is the fundamental value of the risky asset, to which they believe the price would eventually converge to. If the interest rate is zero, they buy in when the price is below $\bar{p}_\alpha$ and sell out when it is above (see Eq.(2.1)). As it takes time for price to return to its fundamental, impatient fundamentalists may lose confidence on their beliefs and switch to other group or exit market before the price reverses.We shall discuss later in subsequent section.

Chartists

There are $x_\beta\,(= N - x_\alpha)$ chartists who hold constant expected price variance:

$$V_\beta(p_{t+1}|\Omega_t) = \sigma_\beta^2.$$

[5] Chiarella and He (2003) shows that the conclusions are not significantly different between constant and floating expected variance assumptions. The same argument applies for β investors. Allowing the expected variance to float with price provides more dimensions in examining market behavior, it is however out of the concern of this chapter. We therefore reserve it for our future research.

Chartists update their price expectation constantly according to the following function:

$$E_\beta \left(p_{t+1}|\Omega_t\right) = p_t + \tau \cdot \left(p_t - \bar{p}_{\beta,t}\right),$$

where $\bar{p}_{\beta,t}$ is the short-term estimate of the 'fundamental value' and $\tau \in (0,1)$ is the adjustment speed. Greater τ indicates that chartists reacts more sensitively to past estimation bias, $p_t - \bar{p}_{\beta,t}$. Chartists who believe in the persistence of the estimation bias (trend) in the subsequent period only[6], buy in when the price goes beyond the previous short-term fundamentals and sell out when the price falls below.

In contrast to the past literature where $\bar{p}_{\beta,t}$ is assumed to be constant across time or to be governed by certain stochastic process (see Day and Huang, 1990; He and Li, 2007, 2008), the short-term 'fundamental value' is assumed to take different value when the price falls in different trading windows pre-specified according to historical prices as well as trading experience. More specifically, assume that the price domain $\mathbb{P} = [p_{\min}, p_{\max}]$ can be divided into n mutually exclusive regimes, that is:

$$\mathbb{P} = \cup_{j=1}^{n} \mathbb{P}_j = [\bar{p}_0, \bar{p}_1) \cup [\bar{p}_1, \bar{p}_2) \cup \cdots \cup [\bar{p}_{n-1}, \bar{p}_n],$$

where $\bar{p}_j$ $(j = 1, 2..., n.)$ represents the psychological thresholds corresponding to different support (resistance) levels in the chartist analysis. When price falls into a particular window, chartists extrapolate the short-term fundamental value to be in the middle of the window, which equals to the average of the top and the bottom threshold of prices that enclose that regime. Specifically,

$$\bar{p}_{\beta,t} = \left(\bar{p}_{j-1} + \bar{p}_j\right)/2 \text{ if } p_t \in [\bar{p}_{j-1}, \bar{p}_j),\ j = 1, 2..., n. \tag{2.2}$$

[6] Chartists are assumed to be myopic in the sense that they have no information about the persistence in periods farther than the subsequent period.

When $p_t \in [\bar{p}_{j-1}, \bar{p}_j)$ is observed, chartists extrapolate that the price could have been anywhere in the range of $[\bar{p}_{j-1}, \bar{p}_j)$ with a short-term fundamental value - $\bar{p}_{\beta,t}$[7]. If the price p_t deviates from its short-term fundamental value, $\bar{p}_{\beta,t}$, such deviation is expected to enlarge further in the short run. $\bar{p}_{\beta,t}$ is just like a breakpoint: when the observed p_t equals to $\bar{p}_{\beta,t}$, the price is expected to be stable; when it goes beyond $\bar{p}_{\beta,t}$, the price is expected to be farther beyond $\bar{p}_{\beta,t}$ and vice versa. In other words, chartists expect the observed trend (deviation) to persist (or the momentum to continue) in the next period.

Without lost of generality, we assume that $\bar{p}_j - \bar{p}_{j-1} \equiv \lambda$ for $j = 1, 2..., n$ and $\bar{p}_0 = 0$. Given that $\mathbb{P} = \cup_{j=1}^{n} \mathbb{P}_j = [0, \lambda) \cup [\lambda, 2\lambda) \cup \cdots \cup [(n-1)\lambda, n\lambda]$, $p_t \in [\bar{p}_{j-1}, \bar{p}_j)$ is identical to $p_t \in [(j-1)\lambda, j\lambda]$. As $j - 1 = \lfloor p_t/\lambda \rfloor$ and $j = \lceil p_t/\lambda \rceil$, which mean the largest integer not greater than p_t/λ and the smallest integer not less than p_t/λ, Eq.(4.1) can be simplified to:

$$\bar{p}_{\beta,t} = \left(\lfloor p_t/\lambda \rfloor + \lceil p_t/\lambda \rceil \right) \lambda/2 \text{ if } p_t \in [\bar{p}_{j-1}, \bar{p}_j), \quad j = 1, 2..., n. \tag{2.3}$$

2.2.3 Evolutionary Strategy Switch

Aside from updating expected price, investors also interact with each other and reshuffle the market structure, reflected by the composition of various investor groups, in every period. While some investors stick to their original strategy, others abandon less attractive strategy for what is expected to outperform the others. Specifically, the number of investors in group i, x_i ($i = \alpha, \beta$), updates according to a fitness measure - the current value of expected profit per share, or

[7] Chartists essentially adjust their price expectation based on a time-varying reference level. Similar idea could be found in Westerhoff (2003), which assumes investors anchor the expected fundamental value of the foreign exchange rate to the nearest round number and Day (1997), in which price adjustment is bounded and will eventually lie in a trapping set given by an interval.

discounted expected profit, π, depicted by:

$$\pi_{i,t} = s_{i,t} \cdot \theta_{i,t},\ i = \alpha, \beta, \tag{2.4}$$

where θ is the expected profit per share before discounting, and s is the discount factor that varies across various investor groups.

Within the framework of market-maker, market is not forced to clear every period. Investors are allowed to hold the risky asset for as long as they believe to be optimal. Learned from the belief specifications that while chartists adjust price expectation every period, fundamentalists hold their expectation stable at the fundamental value. Consequently, chartists capitalize asset return every period regardless of gain or loss (see Lux and Marchesi, 1999). The same may not happen to fundamentalists. In fact fundamentalists hold undervalued risky asset until the price goes above the fundamental and wait to refill their inventory when the price slides below the fundamental. Namely, their expected gains take time to be realized. Their discount factor thus depends on their estimation of the time interval for the price to return to its fundamental. The longer it takes, the smaller the discount factor is.

Given that chartists capitalize capital gains or loss immediately, their discount factor, $s_{\beta,t}$ is set equal to 1. For fundamentalists, the discount factor is a function of the contemporaneous price, which is assumed to be $s(p_t) = (\bar{p}_\alpha - p_t)^2 / 3\bar{p}_\alpha^2$. It indicates that the farther the price deviates from its value, the more likely it is going to reverse, and therefore the greater the discount factor is.

Before discounting, the expected profit per share, θ, for fundamentalists and chartists are given by:

$$\begin{aligned} \theta_{\alpha,t} &= |\bar{p}_\alpha - (1+r)\, p_t| - C/s(p_t) \ \text{ and} \\ \theta_{\beta,t} &= |\tau \cdot (p_t - \bar{p}_{\beta,t}) - r p_t|\,, \end{aligned} \tag{2.5}$$

where C is the *information cost.* Imposing information cost implies that it costs to become a fundamentalist. Notice that the expected profit here is non-negative and measured in per share, which is different from the realized profit applied in earlier literature (see Brock and Hommes, 1998 and Lux and Marchesi, 1999, 2000). Such definition originates from the finding that investors are forward-looking (Keswani and Stolin, 2008; Sapp and Tiwari, 2004; Zheng, 1999). As they make buy or sell decision to maximize their discounted expected profit, there is no reason that they intentionally trade to lose money.

Substituting Eq.(2.5) into Eq.(2.4) leads to the simplified function of discounted expected profit per share:

$$\begin{aligned} \pi_{\alpha,t}(p_t) &= s(p_t)\left|\bar{p}_\alpha - (1+r)\,p_t\right| - C \text{ and} \\ \pi_{\beta,t}(p_t) &= \left|\tau \cdot (p_t - \bar{p}_{\beta,t}) - rp_t\right|. \end{aligned} \tag{2.6}$$

Let $\eta_{i,t}$ be the market fraction of investors in group i, $i = \alpha, \beta$. The change in the number of investors in group i is thus reflected by its fractions $\eta_{i,t}$, whose update follows the discrete choice probability (Brock and Hommes,1998):

$$\eta_{i,t}(p_t) = x_i(p_t) \big/ \sum\nolimits_k x_k(p_t) = \exp(\rho\pi_{i,t}(p_t)) / \sum\nolimits_k \exp(\rho\pi_{k,t}(p_t)), \tag{2.7}$$

where the parameter ρ measures the speed of switching to a different trading strategy and is referred to as the intensity of choice in Brock and Hommes (1998). The $\eta_{i,t}$ is positively related to the the discounted expected profit, suggesting that investors cluster to strategy that produces higher discounted expected profit. The switch is mutual with fundamentalists and chartists switching to each other simultaneously. The switch to group with high discounted expected profit takes the lead. Note however that not all investors do so. Some stick to their original strategy even though the other strategy appears to be more attractive. This can be observed from Eq.(2.7) where $\eta_{\alpha,t}$ ($\eta_{\beta,t}$) is always greater than 0 even if $\pi_{\alpha,t} < \pi_{\beta,t}$ ($\pi_{\alpha,t} > \pi_{\beta,t}$).

As a measure of the market structure, we define the *market fraction index* m_t as the difference in market fraction between fundamentalists and chartists, that is:

$$m_t = \eta_{\alpha,t}(p_t) - \eta_{\beta,t}(p_t) = \tanh\left[\rho/2 \cdot (\pi_{\alpha,t} - \pi_{\beta,t})\right], \tag{2.8}$$

As a function of the price, the market fraction index m_t takes the value in the range of $[-1, 1]$. It is positive (negative) when $\pi_{\alpha,t} > \pi_{\beta,t}$ ($\pi_{\alpha,t} > \pi_{\beta,t}$), indicating that the group with super discounted expected profit dominates the market in terms of the number of investors.

2.2.4 Price Dynamics

Following Gennotte and Leland (1990), we define the *relative market power of group i*, ω_i, as the ratio of their weighted risk tolerance to the sum of the whole market's weighted tolerances:

$$\omega_i(p_t) \doteq a_i x_i(p_t) / \sum\nolimits_k a_k x_k(p_t) = a_i \eta_i(p_t) / \sum\nolimits_k a_k \eta_k(p_t). \tag{2.9}$$

Write $a_\beta = \delta a_\alpha$, where $\delta > 0$ is a parameter that measures the *relative risk tolerance* of chartists against fundamentalists. One of the benefits of using relative instead of absolute risk attitude is that, it allows us to account for the change in risk attitude. This is important especially during the period of crises, when investors of different types universally become less risk tolerant (or more risk averse). Simplifying the demand function is another consideration.

Substituting Eq.(2.7) into Eq.(2.9) produces the following:

$$
\begin{aligned}
\omega_{\alpha,t} &= \frac{1+m_t}{1+m_t+\delta(1-m_t)} \text{ and} \\
\omega_{\beta,t} &= \frac{\delta\,(1-m_t)}{1+m_t+\delta(1-m_t)}.
\end{aligned}
\tag{2.10}
$$

The relative market power is determined by the market fraction index and the relative risk tolerance. The group with higher relative market power dominates trading activities. Note that larger investor population does not imply higher relative market power, which is a measure of total trading volumes, the product of the number of investor and the size of individual trading volume. The relative market power could be small even though the population of that group is large if the trading volume per investor is small.

Without the loss of generality, we normalize the aggregate excess demand with the weighted sum of all investor groups' risk tolerance following Gennotte and Leland (1990), which yields:

$$
\begin{aligned}
D_t &= \sum\nolimits_i n_{i,t}x_{i,t} / \sum\nolimits_k a_k x_k = \sum\nolimits_i n_{i,t}\omega_{i,t}/a_i \\
&= \frac{\sigma_\beta^2(1+m_t)\left[\bar{p}_\alpha-(1+r)p_t\right]+\sigma_\alpha^2\delta(1-m_t)\left[\tau\cdot(p_t-\bar{p}_{\beta,t})-rp_t\right]}{\sigma_\alpha^2\sigma_\beta^2\left[(1+\delta)+(1-\delta)\,m_t\right]},
\end{aligned}
\tag{2.11}
$$

The second line is obtained by substituting Eqs.(2.1) and (2.10) into Eq.(2.11). If $D_t = 0$, the market is in equilibrium and all orders are cleared; if $D_t > 0$, demand exceeds supply and some bid orders cannot be filled in; finally, if $D_t < 0$, there is excess supply where part of ask orders fails to be executed. In the market-maker framework, all orders would be filled up as market-maker balances market orders by supplying from (taking into) inventories when there is excess demand (supply). They then adjust the price up (down) according to the excess demand in the next period. The price impact function is defined by:

$$
p_{t+1} = p_t + \gamma D_t, \tag{2.12}
$$

where γ measures the *adjustment speed of the price*. The price dynamics is essentially a one-dimensional deterministic process, as $\bar{p}_{\beta,t}$, m_t and hence D_t are nonlinear functions of p_t.

2.3 Comparative Dynamics

Due to the complexity of the nonlinear chaotic model, it is important to analyze various theoretical implications comparatively[8]. We first illustrate the equilibria and their stabilities when the market fraction index m_t is floating with the price, and then derive the precise steady states when m_t is fixed at certain value that is of interest to previous literature. To address how various factors (external shocks) affect the magnitude of price fluctuations, we proceed to evaluate their impacts on bull and bear market independently. For the discussion to be meaningful, without loss of generality, it is assumed that $\sigma_\alpha^2/\sigma_\beta^2 \in (rp_\alpha/rp_\alpha + \lambda\tau/2, \min\left[rp_\alpha/\left(rp_\alpha - \lambda\tau/2\right), (r+1)/r\right])$, which ensures the positiveness of the price and facilitates the definition of bull and bear market in subsection 2.3.2.

2.3.1 Equilibrium and Stability

In the nonlinear chaotic model, very rich and complex dynamic behaviors such as a unique stable or unstable equilibrium, multiple stable or unstable equilibria, periodic cycles, non-periodic fluctuations can occur theoretically. In particular, chaotic multi-phase switches can be observed. A typical phase-diagram is illustrated in Fig. 2-1 with $\bar{p}_\alpha = 50$, $\gamma = 2.1587$, $\delta = 10.1198$, $\tau = 0.5$, $r = 10^{-5}$, $\rho = 0.9$, $C = 3$, $\lambda = 13.1787$ and $\sigma_\alpha^2 = \sigma_\beta^2 = 1$ (All numbers are round into 4 decimal places for the clarity of presentation. This set of paramors will be referred

[8] Unstable finite cycle and quasi-periodic cycle can also arise from this model. As in the real financial market, the price appears to be random, it is best captured by the chaotic aspect of this model. We thank Volker Bohm for pointing this out.

to as *standard parameter set*). It shows that when the market fraction index is floating with the changing price, there could be multiple equilibria. But what is the characteristics of these equilibria?

Fig. 2-2 illustrates the interactions between the market fraction index m_t and the price p_t. One can observe neither monotonic nor other simple relationship between m and p. It is therefore impossible to get a closed-form solution for any equilibrium, let alone to discuss its stability and to evaluate the comparative statics. Nevertheless, at each equilibrium, the following identity must hold:

$$\overline{p} = \frac{\sigma_\beta^2 \cdot (1+\bar{m})\bar{p}_\alpha - \delta\tau\sigma_\alpha^2 \cdot (1-\bar{m})\bar{p}_\beta}{\sigma_\beta^2 \cdot (1+\bar{m})(1+r) - \sigma_\alpha^2\delta \cdot (1-\bar{m})(\tau - r)}, \tag{2.13}$$

where $\bar{p}_\beta$ and $\bar{m}$ are functions of the equilibrium price $\overline{p}$ (see Eqs.(2.3) and (2.8)). It is straightforward to verify that such an equilibrium is stable if and only if

$$0 < \gamma \frac{\sigma_\beta^2 \cdot (1+\bar{m})(1+r) - \sigma_\alpha^2\delta \cdot (1-\bar{m})(\tau - r)}{\sigma_\alpha^2\sigma_\beta^2 \cdot [(1+\delta) + (1-\delta)\bar{m}]} < 2.$$

It is interesting to examine several special cases with extreme $\bar{m}$ values.

(I) $\bar{m} = 1$, that is, *there are only fundamentalists in the market.*

A unique equilibrium exists: $\overline{p} = \bar{p}_\alpha/(1+r)$, which is stable if

$$\gamma(1+r)/\sigma_\alpha^2 < 2.$$

The phase-diagram is characterized by a unique downward straight line, as illustrated in Fig. 2-3. The typical dynamics is cyclical fluctuations, either converging or diverging.

(II) $\bar{m} = -1$, that is, *there are only chartists in the market.*

The phase-diagram is characterized by multiple straight lines with the same slope. There are multiple equilibria with the same stability. Each equilibrium must satisfy $\overline{p} = \tau\overline{p}_{\beta}/\left(\tau - r\right)$, which is stable if and only if

$$0 < \gamma\left(r - \tau\right)/\sigma_{\beta}^{2} < 2.$$

(III) $\bar{m} = 0$, that is, *the market consists of equal proportion of fundamentalists and chartists.*

The price dynamic diagram is again characterized with multiple straight lines of the same slope. In contrast to (II), there exists at most one equilibrium[9]. The equilibrium, if exists, is stable if and only if

$$0 < \gamma\left[\sigma_{\beta}^{2}(1+r) - \sigma_{\alpha}^{2}\delta\left(\tau - r\right)\right]/\left[\sigma_{\alpha}^{2}\sigma_{\beta}^{2}\left(1+\delta\right)\right] < 2.$$

The uniqueness of equilibrium can be proved by contradictory. Suppose there are two equilibria $\overline{p}_1$ and $\overline{p}_2$, with $\overline{p}_1 > \overline{p}_2$. Given Eq.(4.1), we have $\overline{p}_{\beta,1} \geq \overline{p}_{\beta,2}$. Substituting $\overline{p}_{\beta,1}$ and $\overline{p}_{\beta,2}$ into Eq.(4.6) leads to a contradictory with $\overline{p}_1 < \overline{p}_2$.

2.3.2 The Impact of Various Factors on Price Fluctuations

This subsection examines how various factors affect the *magnitude of price fluctuations* denoted by $\Delta_t p = p_{t+1} - p_t$, which subsequently shapes crises of different

[9] We exclude the singular case of infinitely many equilibria, i.e., the case in which one of the straight-line overlaps with the 45 degree line. Such case occurs when the denominator of (4.6) vanishes.

types. Before moving to the impacts, we first define

$$Y \doteq \left[\tau \cdot (p_t - \bar{p}_{\beta,t}) - rp_t\right]/\sigma_\beta^2 - \left[\bar{p}_\alpha - (1+r)p_t\right]/\sigma_\alpha^2,$$

which represents *the scaled difference between the number of shares demanded by a chartist and a fundamentalist when they bear the same degree of risk tolerance.* To ensure $Y > 0$ (< 0) is always true, it requires that $\min Y > 0$ ($\max Y(p_t) < 0$), which results in $p_t > p_{bull}$ ($p_t < p_{bear}$) given that $|p_t - \bar{p}_{\beta,t}| < \lambda/2$[10], where

$$\begin{aligned} p_{bull} &= \frac{\sigma_\beta^2 \bar{p}_\alpha}{\sigma_\beta^2 (r+1) - \sigma_\alpha^2 r} + \frac{\sigma_\alpha^2 \tau \lambda/2}{\sigma_\beta^2 (r+1) - \sigma_\alpha^2 r} \text{ and} \\ p_{bear} &= \frac{\sigma_\beta^2 \bar{p}_\alpha}{\sigma_\beta^2 (r+1) - \sigma_\alpha^2 r} - \frac{\sigma_\alpha^2 \tau \lambda/2}{\sigma_\beta^2 (r+1) - \sigma_\alpha^2 r}. \end{aligned}$$

Under the assumption that

$$\sigma_\alpha^2/\sigma_\beta^2 \in \left(rp_\alpha/rp_\alpha + \lambda\tau/2, \min\left[rp_\alpha/\left(rp_\alpha - \lambda\tau/2\right), (r+1)/r\right]\right),$$

we have $p_{bull} > \bar{p}_\alpha > p_{bear}$. Define the bull market by $p_t \in [p_{bull}, p_{\max}]$ when the price is overestimated. Similarly define the bear market by $p_t \in [p_{\min}, p_{bear}]$ when price is underestimated[11].

Since $\Delta_t p = \gamma D_t$ (see Eq.(2.12)), differentiating γD_t directly with respect to various factors gives rise to results summarized in Table 2.1. With interest in how the price bursts (bounces) in bull (bear) market, we analyze the impacts of factors on $-\Delta_t p$ ($\Delta_t p$), the magnitude of price drops (jumps), based on Table

---

[10] Y is minimized (maximized) when $p_t - \bar{p}_{\beta,t} = -\lambda/2$ ($\lambda/2$).

[11] The price movement within regime (p_{bear}, p_{bull}) is harmless and tolerable, which is left out of our discussion.

2.1. Specifically, we focus on how various factors affects the price drops (jumps) in bull (bear) market by looking into the sign of $-d\Delta_t p/dk$ ($d\Delta_t p/dk$) for $k \in \{m_t, \delta, C, \rho\}$.

Table 2.1: The impact of various factors on the price fluctuation.

	p_t	Y	$d\Delta_t p/dm_t$	$d\Delta_t p/d\delta$	$d\Delta_t p/dC$	$d\Delta_t p/d\rho$
Bull Market	$p_t \in [p_{bull}, p_{\max}]$	+	-	+	+	- if $m_t > 0$ + if $m_t < 0$
Bear Market	$p_t \in [p_{\min}, p_{bear}]$	-	+	-	-	+ if $m_t > 0$ - if $m_t < 0$
Remarks	$d\Delta_t p/dm_t = -2\gamma\delta Y/[(1+\delta)+(1-\delta)m_t]^2$ $d\Delta_t p/d\delta = \gamma(1-m_t^2)Y/[(1+\delta)+(1-\delta)m_t]^2$ $dm_t/dC = -\rho(1-m_t^2)/2$ $dm_t/d\rho = (\pi_{\alpha,t} - \pi_{\beta,t})(1-m_t^2)/2$					

Market fraction index, m

$-d\Delta_t p/dm_t$ is positive in bull market, suggesting that the larger the proportion of fundamentalists is (as indicated by a larger m_t), the greater the price drops. Similarly, $d\Delta_t p/dm_t$ is positive in bear market, suggesting that, a larger fraction of fundamentalists results in a greater price rebound. Overall, *fundamentalists play the role of bringing the price back to its fundamental.*

Relative risk tolerance, δ

The relative risk tolerance, which is rarely discussed in previous literature, is of interest especially at the time of market turbulence. When the market glooms as the panic mood spreads, investors of different types become less risk tolerant. It is likely that investors of certain types are more sensitive that they reduce their risk tolerance more than the others during crises, which may lead to a shrink in their relative market power that changes aggregate demand and subsequently price. The fact that $-d\Delta_t p/d\delta$ ($d\Delta_t p/d\delta$) is negative in bull market (bear market) indicates that the less sensitive fundamentalists are to the news compared to chartists therefore the smaller δ is, the greater the price drops (jumps). Overall, the relative risk tolerance moves the price in the direction opposite with the market fraction index in both bull and bear market. While a large proportion of fundamentalists acts to bring the price back to its fundamental, high relative risk tolerance of chartists against fundamentalists undermines such pulling back effect. In other words, the pulling back force of the market fraction index is enhance had the relative risk tolerance been low.

Information cost, C

$-d\Delta_t p/dC$ ($d\Delta_t p/dC$) is negative in bull market (bear market), meaning that high information cost smooths the price drops (jumps) when the price is overestimated (underestimated). Intuitively, if information is costly, then investors would opt to be chartists, trading on limited (nearsighted) information with moderate volumes (see Eq.(2.1)), which would not cause much price pressure, even when the price is extremely high above (or far below) its fundamental.

Intensity of choice, ρ

When $m > 0$ ($m < 0$), $-d\Delta_t p/d\rho$ is positive (negative) in bull market and $d\Delta_t p/d\rho$ is positive (negative) in bear market. It suggest that high intensity of choice enhances the herding behavior and hastens the price jumps and drops if fundamentalists dominate the market. The opposite is true if chartists govern the market. Intuitively, when fundamentalists dominate the market, high intensity of choice accelerates the accumulation of the number of fundamentalists ($dm_t/d\rho > 0$ for $m_t > 0$), which quickly assemble great selling (buying) force that imposes great pressure on price. If the price happens to be highly overestimated (underestimated), the selling (buying) force is going to be so tremendous that it could trigger catastrophic (euphoric) price drops (jumps). On the other hand, if chartists dominate, then high intensity of choice acts to smooth the price drops (jumps) when the price is highly overestimated (underestimated).

2.4 Financial Crises Demonstrations

The impacts of various factors on the magnitude of price fluctuation explored in subsection 2.3.2 help to shape price crises of various depth and length. The key determinant, however, is the internal price dynamics. In fact, different initial price could result in different types of crises ceteris paribus[12]. To show that the differences in the types of crises could be endogenous, we simulate sudden crises and smooth crises with the same set of parameters but different initial price. Disturbing crises could have been simulated using the same set of parameters. However, in order to better match the simulation with the real crises and to better contrast the disturbing crises with the smooth crises (the former exhibits a visible crash consisted of running falls), several parameters are adjusted while simulate the disturbing crises. The underlying dynamic mechanism however is the same. We further test our model in capturing the essential factors of financial markets by comparing simulated price series with real historical scenarios. If they match with a variety of real crises, the model should capture some, if not all, fundamental determinants of financial crises.

2.4.1 Sudden Crises

In a sudden crises, the price plunges from the peak (or very close to peak) precipitately down to (or very close to) the bottom. Normally for such crises, the price decline is dramatic and the length is short. The price usually starts to rebound consecutively and persistently immediately after touching the bottom. A typical example is the Oct. 19, 1987 crash in the US stock market, when DJIA lost more than 20% of its value in one day, as illustrated in Fig. 1-1. Other examples include

[12] Given the same set of parameters, different initial values alone could lead to different price levels before the crises, and subsequently lead to different magnitude of price drops that characterize the types of crises. In other words, the types of crises is path dependent. This part of results is available upon request.

the tulip mania of 1630s and the silver price bubble of 1980[13].

The sudden crises is simulated using the standard parameter set described in Section 2.3.1 with the initial price set to be $p_o = 62.71$. The upper panel of Fig. 2-4 shows the simulated price series (asterisk marked line) and the DJIA (solid line) from 1987/8/4 to 1987/12/24. The corresponding track of market fraction index m is shown in the lower panel. It is interesting to note that when the simulated price deviates high above its fundamental, investors cluster to fundamental trading strategy dramatically, which subsequently leads to a crash as fundamentalists execute their selling order collectively. This phenomenon is consistent with the real historical scenario before the Oct. 19 crash, when institutional investors, the representative of fundamentalists, sold so enormously that the price was driven down excessively (Presidential Task Force on Market Mechanisms, 1988).

As shown in Section 2.3.1, the increase in fundamentalists' share (increase in m) enlarges the magnitude of price drop in bull market when the price is high above the fundamental. However why should the drop be so dramatic? The main reason is that m increases rapidly and abruptly in a large scale. Given that the selling force of each individual fundamentalist is large at high price level, the large number of fundamentalists accumulate such great selling forces that they trigger price to plummet.

Why m increases so dramatically? Fig. 2-5 graphs the price dynamics around the crash to outline the underlying mechanism. When the price is at the peak, fundamentalists improve their discount factor as they become more certain of price reversal. Moreover, they upgrade their expected profit before discounting. All these add up to boost the discounted expected profit π_α to an extent that significantly outweighs π_β, even after accounting for the information cost. As soon as the latest price is observed, investors interact with each other and update their

[13] See Rosser (2000, Chapter 5). While Rosser ascribes the 1987 crash to be of the same type of 1929 crash, we think it as a sudden crises. It is because the price right before the plunge is quite close to the global peak. It is in fact a local peak. We allow such approximation within a standard deviation given the market volatility.

beliefs, after which many of them are so attracted by the outstanding π_α that they switch to be fundamentalists all of a sudden, which fortifies the market fraction index m significantly.

Also worthy of attention is the price movement after the crash. After the price crash, the market quickly rebound as fundamentalists step in to buy back. As the price starts going up, π_α becomes less attractive, which leads to a shrink in m. When chartists dominate, their arbitrage continues to push the price upwards.

2.4.2 Smooth Crises

In a smooth crises, the price subsides from the peak persistently in a succession of descending waves with no visible crash. The price decline is moderate but the lasting period is relatively long. The fall of DJIA during the great depression as shown in Fig. 1-2 is an example. The drop of NKY (Nikkei 225 Index) in the Japanese recession is another.

The smooth crises is simulated using the standard parameter set described in Section 2.3.1 with the initial price set to $p_o = 77.41$. Fig. 2-6 shows the simulated price series (asterisk marked line) and the DJIA (solid line) from 1932/3/4 to 1932/7/26, a small episode of the great depression. When the price is high, fundamentalists dominate slightly and pull the price down to a level that chartists find it optimal to sell. After observing the declined price, investors update their beliefs and cluster to chartists, whose arbitrage behavior contribute to the further fall of price (see Fig. 2-7).

The increase in m causes a decline but not a crash. It is because the change of m is small in magnitude as a result of the moderate difference between π_α and π_β. The slight increase in the proportion of fundamentalists accumulate some selling force, but not enough to trigger a crash.

2.4.3 Disturbing Crises

The scale of price drop and the lasting period of the disturbing crises is somewhere between the smooth and sudden crises. The crash in this case is visible but not as dramatic as that in the sudden crises. It lasts longer than the sudden crises but shorter than the smooth crises. Between the general price peak and the visible crash, there is a period when the price exhibits large clustering volatility with the general trend of falling, the phase of which is called disturbing period, or period of financial distress (Gallegati et al., 2005). Most crises are of this kind[14]. Perhaps more importantly, the crash is not the end of the crises but followed by further decline. A typical example is the fall of DJIA during the Oct of 1929.

We simulate the disturbing crises with $p_o = 71.4171$ and $\gamma = 3.0187$, $\delta = 19.3591$, $\rho = 0.6$ keeping the others in line with the standard parameter set described in Section 2.3.1. Fig. 2-8 overlaps the simulated price series (asterisk marked line) with the DJIA (solid line) between 1929/7/23 to 1929/12/17. DJIA fluctuated disturbingly with a downward trend before it eventually crashes. The simulated price characterizes such disturbance from $t = 1$ to $t = 61$. At the beginning of the disturbing period, m fluctuates moderately with investors switching between fundamental and trend following strategies disturbingly. While fundamentalists try to pull the price down, chartists does the opposite. The battle continues until fundamentalists eventually pull the price down to a level that chartists find it optimal to sell (see the dynamics when price is high as in Fig. 1-3). Later as the price falls, chartists dominate the market and lead the price up and down with a downward trend.

Unlike the 1987 crash that was an event of one day, the 1929 crash stretched from Oct 22 through Oct 29, during which the DJIA lost from 326.51 to 230.07 consecutively (see Fig. 1-3). The simulated price reproduce such crash, character-

[14] The subprime crises exhibits similar characteristics of those observed in the disturbing crises. Since the length of subprime crises remains controversial when the paper is submitted, it is not appropriate to use as an example here.

ized by running falls (step 62 to 69), with chartists continue to place selling orders of different magnitudes. Fig. 2-9 plots the price dynamic around the crash to illustrate the underlying mechanism. It shows that investors update their beliefs and stay cluster to trend following strategy. At some price level, chartists observe downward estimation bias (underestimation) and decide to sell their shares, which drives the price down. After observing the falling price, chartists revaluate their short-term fundamental and start another round of selling if the downward estimation bias still exists. This process continues until the estimation bias turns out to be non-negative.

Note that the crash characterized by running falls does not bring the crises to an end. Instead, it is followed by a bounce back as chartists start to buy in after realizing positive estimation bias, and then another round of decline (step 70 onwards). This scenario is exactly what happened after the Oct 1929 crash.

While it is controversial on the causes of 1929 crash, our simulation suggests that chartists could take the full responsibility. This argument is supported by the evidence that many pools (the bank pools for example) organized like syndicates to follow the market trend are blamed for the 1929 crash (Wigmore, 1985). As the pools were not long-term investors, they did not wait until the price recovered but cut losses even though the price dropped further, in the same way as chartists behave.

2.4.4 Discussion

While the difference in the types of crises is mainly endogenous, it is also subject to external shocks such as the change of the relative risk attitude. In fact, given the initial price, different parameter sets could lead to different types of crises. It suggests that external shocks do matter. Their impacts are however subject to the price level and the internal price dynamics.

From the analysis of crises together with Table 2.1, at the peak of the price, the smaller δ, the relative risk tolerance of chartists against fundamentalists is, the more likely the occurrence of the sudden crises, ceteris paribus. The information cost C plays similar role. An increase in the intensity of choice ρ improves the chance of sudden crises if $m > 0$, ceteris paribus.

If the price falls gradually but not crashes from the peak, the crises could be either smooth or disturbing. Recall that after chartists win the battle against fundamentalists, they pull the price down and dominate the market. Extremely, when the market is occupied by chartists only, that is

$$-\Delta p_{t+1} = -\gamma \left(\tau \cdot \left(p_t - \bar{p}_{\beta,t}\right) - r p_t\right) / \sigma_\beta^2 \leq \gamma \left(\tau \cdot \lambda + r p_t\right) / \left(2\sigma_\beta^2\right),$$

the larger λ, γ and τ are, the greater the maximum price drop is, and the more likely the occurrence of the disturbing crises characterized by a visible crash.

2.5 Conclusion

The interacting heterogeneous agent model is capable of generating three typical types of financial crises that fit into real financial series. It suggests that the model captures correctly some, if not all, key factors of financial crises. While most of our findings are consistent with the previous HAM literature, based on the model we provide new insights. First, not only crises but also the differences of crises could be endogenous, arising from the internal price dynamics. Second, market structure, measured by market fraction index, is the key for price dynamics. Factors related to market fraction index are likely to play important roles in determining crises. Third, although we do show that fundamentalists play an important role in triggering crises, they do not take on the full responsibility. Chartists could be responsible in some scenarios such as smooth and disturbing crises.

While the impact of market structure is directly observable, its origin is however complicated depending on a variety of combinations and interactions. In our framework, the market fraction index is not constant but a dynamic function of fluctuating price. It is because investors consistently cluster to strategy that would optimize their discounted expected profit. Perhaps more importantly, the model exhibits rich complex market behavior because beliefs are not constrained to be the same within a group: investors not only update their expected price but also expected short-term fundamental value according to a series of psychological windows. Such design of deterministic dynamics produces more close-to-real price series, which, in some extent, avoid the unrealistic sudden and frequent switch between bull and bear market, as shown in the earlier deterministic HAM. The strategy switch between groups and short-term fundamental value update within a group contribute to the complex dynamics of market fraction index. The intensity of choice and information cost play an important role in determining the speed and the threshold of switch. The direct relation between these two determinants add some insight to the price movements. Their impacts, however, are durable, depending on different complex scenarios.

The model focuses on the crises and leaves many dimensions unexplored. First, we do not impose constraint on the short/margin sell in this model. By constraining such behaviors, one might be able to evaluate their impact on the depth and length of crises. Second, the interest rate, which is constant in this chapter, can be released to be a variable. By doing so, one might be able to address the impact of interest policies and see whether such policies would affect the depth and length of crises. Third, the range of switching regimes in the piecewise beliefs can be released to account for different expected fluctuation intervals. Finally, although the relation between price fluctuations and intensity of choice as well as the information cost, is clear on specified scenarios, due to the complex dynamics of market structure, it is hard to describe a direct ex-ante relation between these two factors and crises. It is hard to address until after the crises, that is until after we know the market structure.

Appendix. Demonstration of path dependence in price movements

Although parameters do affect the types of crises, they are not the key. In fact, a set of parameters could generate all three types of crises, suggesting that the differences in crises could be endogenous depending on the internal price dynamics (see Fig 2-10, Fig 2-11 and Fig 2-12. In fact, given the same initial price and the same parameter set up, different types of crises can recur in different time frames. That is, for a long enough sample period, different types of crises can occur in the same simulation[15].

Common parameters: $\bar{p}_\alpha = 50$, $\gamma = 2.1585022 + 0.0001 * 3\hat{}0.5$, $\delta = 10.12 - 0.0001 * 3\hat{}0.5$, $\tau = 0.5$, $r = 0.00001$, $\rho = 0.9$, $\lambda = 13.17871$, $\sigma_\alpha^2 = \sigma_\beta^2 = 1$ and $s(p_t) = (\bar{p}_\alpha - p_t)^2 / 3\bar{p}_\alpha^2$.

[15] We thank Volker Bohm for pointing this out.

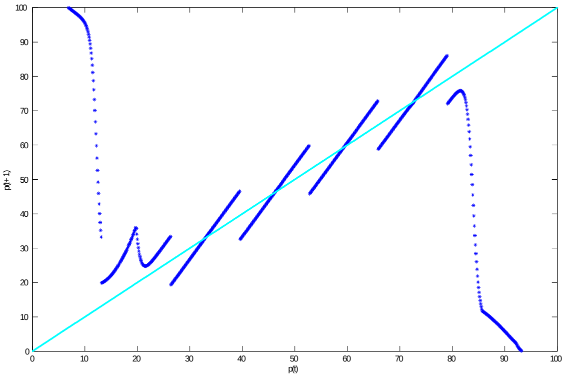


Figure 2-1: Price dynamics: floating m_t.

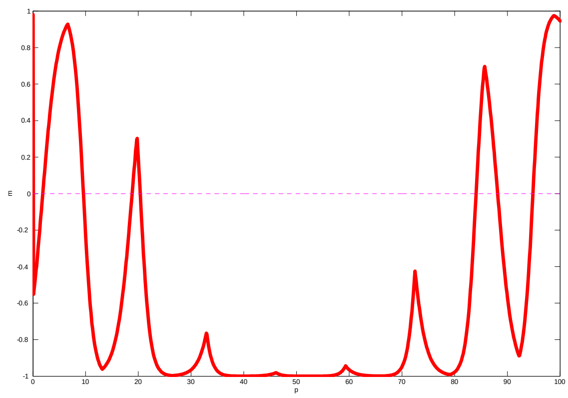


Figure 2-2: The interaction between m_t and p_t.

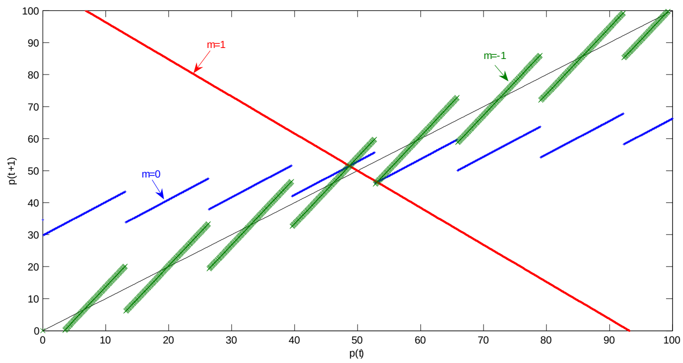


Figure 2-3: Price dynamics: fixed m.

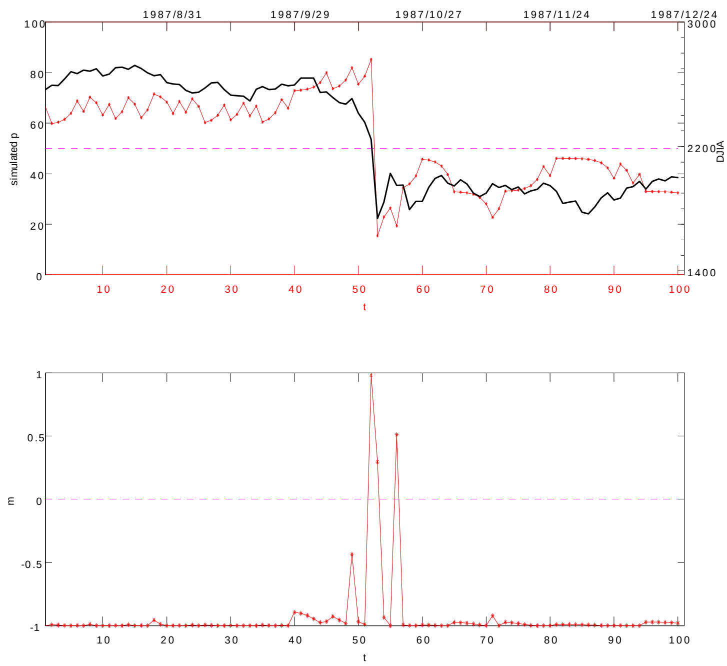


Figure 2-4: Sudden crises. The first panel compares the simulated price series (asterisk marked line) with the DJIA (solid line) from 1987/8/4 to 1987/12/24. The second panel shows the track of market fraction index m.

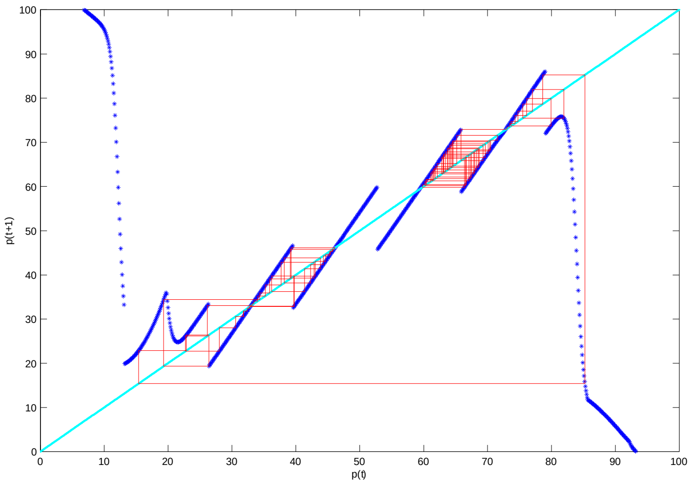


Figure 2-5: Sudden crises: The price dynamics.

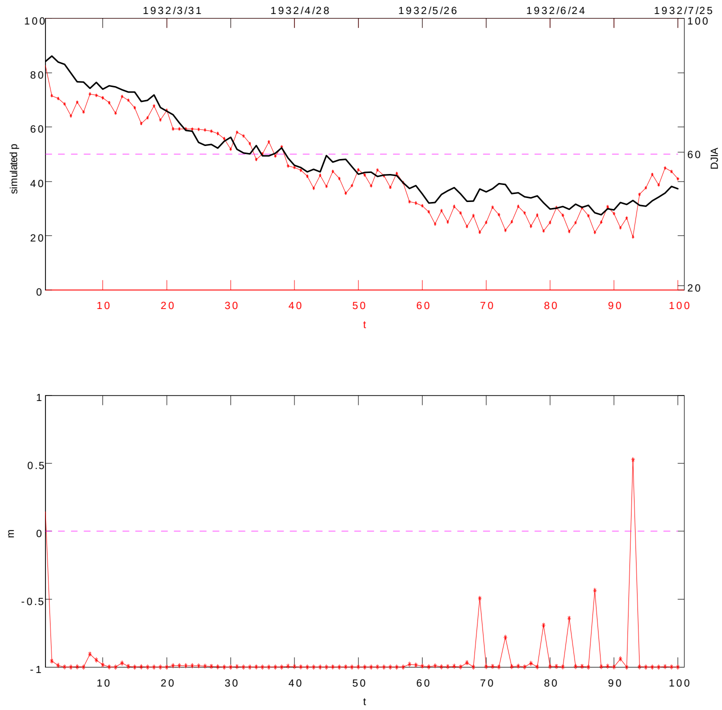


Figure 2-6: Smooth crises. The first panel compares the simulated price series (asterisk marked line) with the DJIA (solid line) from 1932/3/4 to 1932/7/26. The second panel shows the track of market fraction index m.

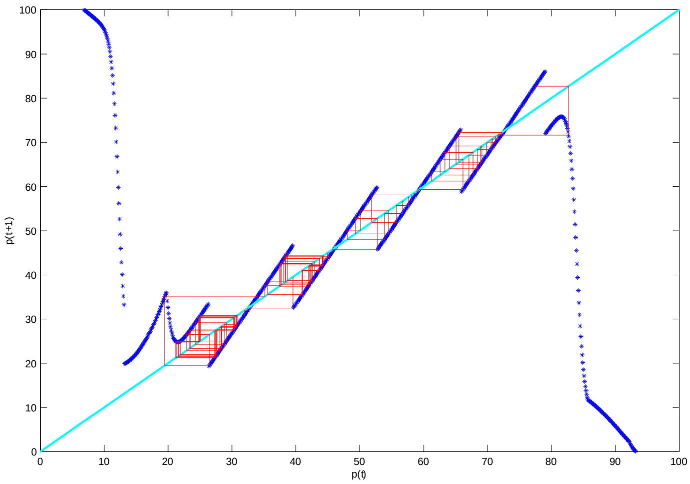


Figure 2-7: Smooth crises: The price dynamics.

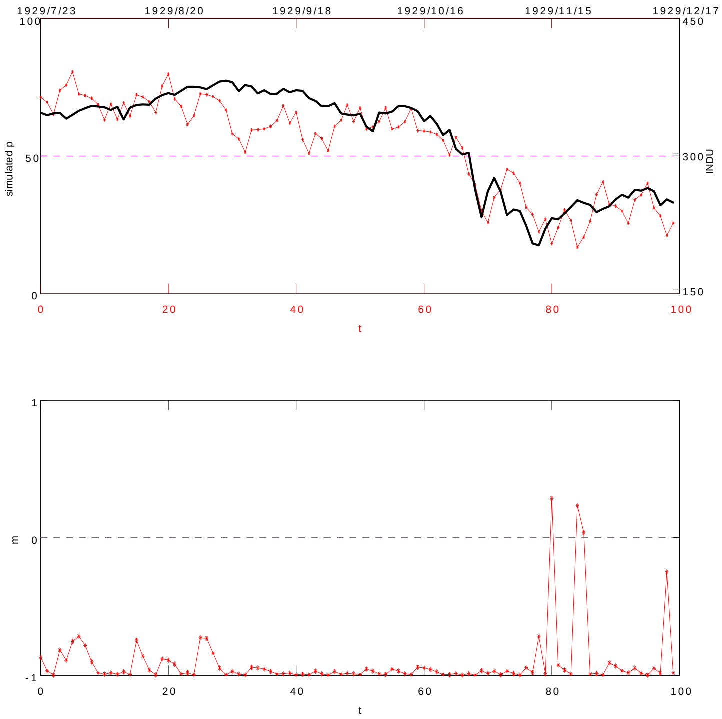


Figure 2-8: Disturbing crises. The first panel compares the simulated price series (asterisk marked line) with the DJIA (straight line) from 1929/8/15 to 1930/1/12. The second panel shows the track of market fraction index m.

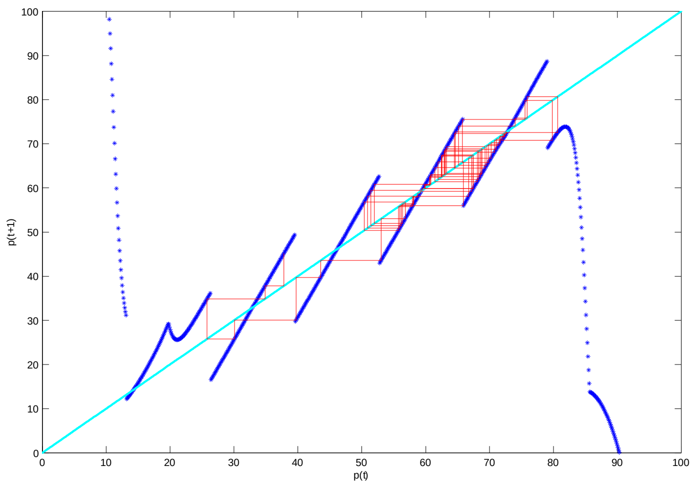


Figure 2-9: Disturbing crises: The price dynamics.

Figure 2-10: Sudden crisis ($p_0 = 76.41$): The left panel plots the price series, the right upper panel records the market fracition index and the right bottom panel describes the price dynamics.

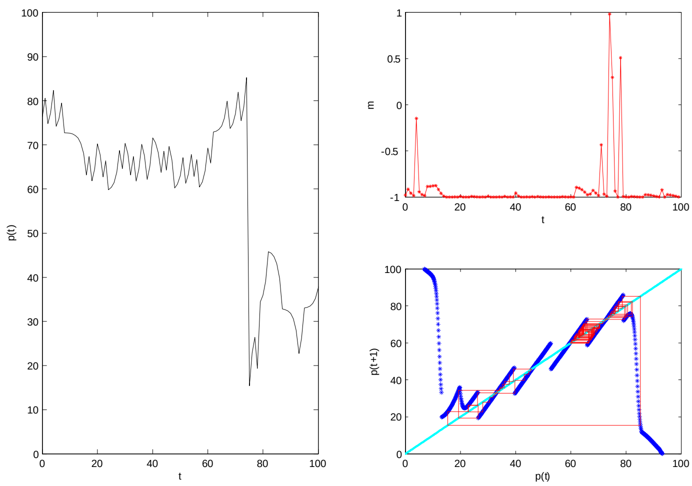


Figure 2-11: Smooth crisis ($p_0 = 77.41$): The left panel plots the price series, the right upper panel records the market fracition index and the right bottom panel describes the price dynamics.

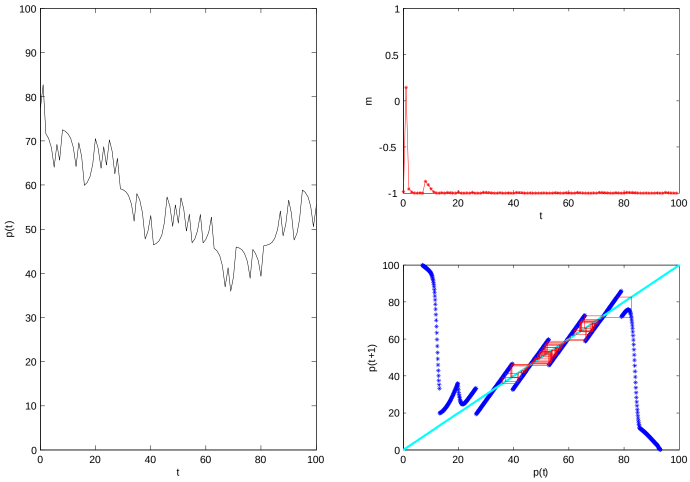
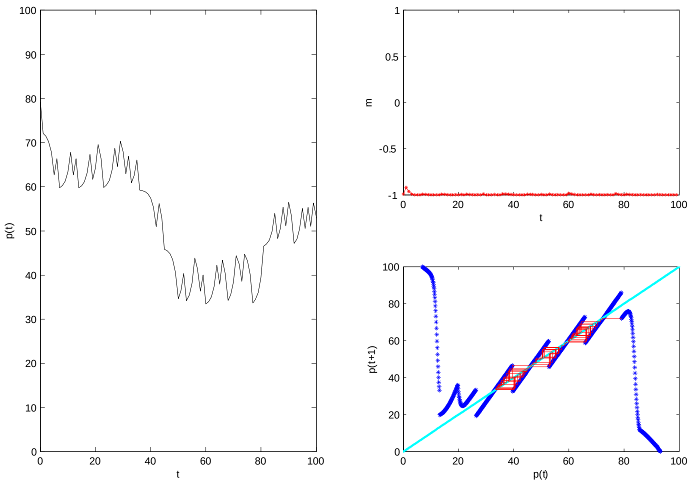


Figure 2-12: Disturbing crisis($p_0 = 79.14$): The left panel plots the price series, the right upper panel records the market fracition index and the right bottom panel describes the price dynamics.



Chapter 3

Asymmetric Returns, Gradual Bubbles and Sudden Crashes

3.1 Introduction

It has been documented that the aggregate financial market returns, such as returns of the stock market index, are asymmetric. In other words, the largest possible upward return movement is smaller than the absolute magnitude of the largest drawdown, that is, $\max(r_t, 0) < |\min(r_t, 0)|$ (Cont, 2001, Hong and Stein, 2003). For example, during the past decades (January 1900 to January 2010), the most positive monthly return of the Dow Jones Industry Average Indices (DJIA) was 34% in April 1933, the scale which was not comparable with the most negative return of -37% in September 1931. Even though the understanding on return asymmetry is important for risk management, a satisfactory economic explanation is still lacking. Among many agent-based models as surveyed by Chen et al. (2008), only a few are capable of duplicating such characteristic[1]. Another interesting observation of the financial time series is that, price bubbles appears

[1] See Johansen and Sornette (2002), Westerhoff (2003), and Hong and Stein (2003) for models that can demonstrate asymmetric returns.

gradually while price crashes suddenly (Veldkamp, 2005). For example, it took five years for the recent price bubble to reach the top (the DJIA rise from 7,591.93 in September 2002 to the peak 13,930.01 in October 2007), but it took less than two years for the bubble to burse bring the DJIA down to a level (7,062.93 in February 2009) which was much lower than the bottom of the bubble. Simple modeling of human learning and adaption enables agent-based models to reproduce bubbles and crashes[2], which are generally ascribed to the interaction of heterogeneous investors[3]. Nevertheless, only a few studies are found to address the ubiquitous feature of 'gradual' bubbles and 'sudden' crashes.

The goal of this chapter is to generate the phenomena of asymmetric returns, gradual bubbles and sudden crashes, and to explain these central qualitative attributes. Ideally, if one were to build a mathematical model based on economic forces that can duplicate such facts, it should have shade some light on these issues. However, as these phenomena are sporadic, the chance of duplicating such facts without capturing the essential fundamental factors is not impossible. For precaution, we first verify the solidity of the model by checking whether it could duplicate the most documented and frequently observed stylized facts that are common across financial markets. If the model succeeds in doing so, rely on it for origins of these sporadic phenomena shall be more convincing.

What are the most intensively discussed stylized facts that characterize the commonality of financial markets? Empirical studies consistently show that the financial time series of prices are random and nonstationary (or possess a unit root), which is in line with the efficient market hypothesis, under which prices instantly and correctly adjust to reflect new information, making arbitrage impossible. However, recent empirical evidence suggests that the financial time series of returns are predictable to some extent (Carhart, 1997, Avramov, 2002), which

[2] See, e.g. Day and Huang (1990), Frankel (2008), Gennotte and Leland (1990), Lux (1995), He and Westerhoff (2005).

[3] Kyrtsou and Terraza (2002) verify that interactions between heterogeneous investors contribute to the chaotic behaviour and excess volatility of financial time series.

challenges the idea that returns are random as implied by the market efficiency hypothesis. Some argue that a certain degree of predicability maybe necessary to reward investors for bearing certain dynamic risks and for gathering and processing information. Others suggest that returns may not fully reflect rational evaluations and investors could be boundedly rational. Motivated by these considerations, various models and techniques have been proposed to address return predicability related issues, in an attempt to beat the market. Despite the controversy on how future returns relate to historical returns, there are several well-recognized findings. First, the distribution of returns is not Gaussian. Instead, it is characterized by fat tails, with extreme returns appearing more frequently than what are predicted by Gaussian distribution. Second, there are positive autocorrelations in absolute and square returns, resulting from the volatility clustering, with periods of quiescence and turbulence cluster together. It means that large (small) return fluctuations are followed by large (small) fluctuations. Even though there is a lack of serial autocorrelation in raw returns, the volatility clustering suggests that past returns are informative for future returns and such relationship is nonlinear. Third, returns exhibit an unusually high degree of persistence or long-range dependence - their autocorrelation functions decay slowly as a power of lags. Fourth, most measures of the volatility are negatively correlated with past returns, which is called leverage effect.

Rich literature has contributed to the identifications (Cont, 2001, Pagan, 1996) and duplications of these stylized facts, in particular the characteristics of fat tails and volatility clustering in returns (Challet et al., 2001, He and Li, 2007, 2008, Kirchler and Huber, 2007, Lux and Marchesi, 1999, 2000, Shimokawa et al., 2007). Agent-based models have been quite successful in the latter. Some of them even shade light on the origins of the stylized facts (Kyrtsou and Terraza, 2002, Alfarano et al., 2008). While for early models, a close fit to some of the statistical features is developed at the expense of a bad fit to others (Cont, 2001), some recent models do demonstrate the capability of reproducing many stylized facts simultaneously (Shimokawa et al., 2007). Nevertheless, most models that fit into the real data rely crucially on a somewhat unrealistic modeling of the noise term[4] ($p_t = f(p_{t-1}) + \varepsilon_t$, $\varepsilon_t \backsim N(0, \sigma^2)$, $f(p_{t-1})$ is a function of p_{t-1}). Amilon (2008) shows that if the normally distributed noise is replaced with a more proper stochastic process ($p_t = f(p_{t-1}) + p_{t-1}\varepsilon_t$), even though the model could generate some stylized facts, the fit is generally poor. Removing the noise term ($p_t = f(p_{t-1})$) would render the model too simple to capture the dynamics of real markets and fail to demonstrate stylized facts. One may infer from these findings that, stylized facts essentially arise from exogenous shocks. There is no doubt that these models provide excellent dynamic mechanisms that convert random shocks into price patterns. They, however, leave the question that, why some properties arise in the absence of external shocks, unexplored. How do random shocks generate consistent stylized facts across markets and over time?

If the internal dynamics play the fundamental role in determining the financial market characteristics while randomness only serves as a trigger (see e.g. Farmer and Joshi, 2002 and Sornette, 2004), stylized facts that are common across various financial markets over decades ought to be endogenous, arising from the internal price dynamics. In other words, exogenous shocks shall be able to shape stylized facts of different magnitudes but not their fundamental characteristics. To show this, one shall disentangle the impact of exogenous shocks in replicating stylized facts. The safest choice is to exclude external noise terms and stochastic processes of any forms. This is however at the cost of fitness into real data as demonstrated by Amilon (2008). Nevertheless if a purely deterministic model are capable of reproducing a variety of stylized facts, one shall extrapolate it to capture some, if not all, essential of the complex market dynamics.

The structure of the chapter is organized as follows. Section 2 examines the statistical properties of the data simulated based on the model developed in Chap-

[4] The dynamic modeling of the normally distributed noise term could exhibits the features of fat tails even if the fundamental does change.

ter 2. It shows that even without the noise term and stochastic process of any forms, the simulated data matches with the well-documented statistical features that characterize the financial time series. Specifically, we find that the simulated returns demonstrate features such as fat tails, volatility clustering, long range dependence and leverage effect. In particular, we find that large returns follow a power low distribution with an exponent of three. After verifying the solidity of the model, Section 3 analyzes the underlying mechanisms of asymmetric returns as well as gradual bubbles and sudden crashes. Section 4 concludes.

3.2 Statistical Properties of Simulated Data

In this section, we verify the solidity of the model in capturing the essential of the complex dynamics by showing its capacity to generate the well-documented stylized facts described in Section 1. Asides from testing the non-stationarity of prices that supports the market efficiency[5], we also verify various forms of predicability in returns that cannot be explained by the efficient market hypothesis, such as volatility clustering, long range dependence and leverage effect. Without loss of generality, we simulate the price series using a set of parameters: $p_0 = 85 - 0.01\sqrt{2}$, $\bar{p}_\alpha = 50$, $\gamma = 2.16 + 0.01\sqrt{2}$, $\delta = 10.12 - 0.01\sqrt{2}$, $\tau = 0.5$, $r = 10^{-5}$, $\rho = 0.9$, $C = 5$, $\lambda = 11.17871$ and $\sigma_\alpha^2 = \sigma_\beta^2 = 1$. The compounded single-period return of the asset from $t-1$ to t is defined as $r_t = \log(p_t) - \log(p_{t-1})$. We could have changed certain parameters to better capture some stylized facts, however, to maintain the consistency of analysis and further support the solidity of the model, we keep to the same set of simulated data throughout the chapter.

Before moving to financial econometric tests, we present an overview of the simulated prices and returns in Fig.3-1. It is observed that (i) the price goes up

[5] The non-stationarity in price series does not necessarily imply that price is unpredictable. The stationarity test is designed to evaluate whether shocks are permant/temporatory (Campbell et al., 1997). Under the efficient market, shocks are expected to be permanent, which corresponding to a non-stationary process.

Table 3.1: Summary statistics of prices and returns.

Sample	Mean	S.D	Min	Max	N
p_t	47.53	19.097	16.445	84.986	2001
r_t	0.000	0.099	-0.311	0.279	2000

and down with occasional bubbles and crashes but without any trend or pattern, and (ii) the return moves around zero and appears to cluster. The summary statistics in Table 3.1 confirm that the average return is zero. Note that the most positive return is 0.279, the magnitude of which is smaller than the most negative return of -0.311.

3.2.1 Nonstationary Prices and Stationary Returns

Literature has consistently shown that the time series of asset prices are not stationary but that of returns are. Despite the robust evidence in finance literature, heterogeneous agent models tend to ignore these properties in prices and returns with a few exceptions, such as Lux and Marchesi (2000) and Alfarano et al. (2008). We test the existence of a unit root (non-stationarity) using the Augmented Dickey-Fuller (ADF):

$$\Delta y_t = a_0 + b y_{t-1} + \sum_{i=1}^{i=q} c_i \Delta y_{t-i} + \epsilon_t,$$

where $y_t \in \{p_t,\ \log(p_t),\ r_t\}$ and q is the number of lags, which is set to be 6 according to the information criteria and the law of parsimony.

Table 4.3 presents the estimation results. The ADF for price and log price series are -2.304 and -2.498 respectively, which are significantly greater than the critical value at 10% significance level. The corresponding MacKinnon p-values for price and log price series are 0.171 and 0.116 individually, suggesting that one cannot reject the hypothesis of a unit root process for either series even at a significance level of 10%. In the return series, the ADF test significantly rejects the hypothesis of non-stationarity at 1% significance level. The simulated data matches with the stylized facts. There is a unit root process in prices but not in returns.

Table 3.2: Unit root test.

Series	ADF(6)	MacKinnon p-value	R-squared	N
p_t	-2.304	.171	.116	1994
$\ln(p_t)$	-2.498	.116	.146	1994
r_t	-20.37	.000	.664	1993
Notes:				
Significance level		1%	5%	10%
ADF critical value		-3.43	-2.86	-2.57

3.2.2 Fat Tails

The summary statistics of returns in Panel A of Table 4.2 further support the presence of fat tails: kurtosis (the fourth moments) is 3.328, which is greater than the benchmark value when returns are normally distributed. It suggests that returns are not normally distributed but exhibit fat tails, with extreme returns appearing more frequently than what are predicted by the normal distribution. The skewness (the third moment) of returns is negative, meaning that price falls in greater scales than it rises on average.

Table 3.3: Fat tails.

Panel A: Skewness and kurtosis					
Sample	Skewness	Kurtosis	Mean	S.d	N
r_t	$-.082$	3.328	0	.099	2000
Panel B: Hill estimator of the tail index					
$\varphi\%$	10%	5%			
h	3.054	4.533			

Skewness and kurtosis are however ambiguous in measuring the characteristics of fat tails. As a complement to see how heavy the tail is, we estimate the tail index. First, the return series are sorted into descending order so that $r^{(1)} > r^{(2)} ... > r^{(N)}$. Assume Pareto-type tail in the density, we have $\Pr(R \geqslant r) = kr^{-h}$, where k is a parameter and h is the tail index, with a lower value corresponding to a fatter tail. Conditioning upon the $\varphi\%$ upper tail that includes the j $(= \lfloor \varphi\% * N \rfloor)$ largest observations, the Hill (1975) estimator is given by:

$$\hat{h} = 1/[\sum_{i=1}^{i=j} \ln(r^{(i)})/j - \ln r^{(j)}].$$

For the upper 10% and 5% tails, the Hill estimator shown in Panel B of Table 4.2 falls into $[2, 6]$, a range reported consistently in current literature (see Cont, 2001).

Since the accuracy of Hill estimator largely depends on the number of observations fall within the specified tail, we have to be cautious about the selection of percentiles. To see whether 10% or 5% is a better choice in calculating the tail index, we run a robustness check using the method of Mandelbrot (1997), by representing the sample moments as a function of the sample size. If the tail index h is larger than the moment of order c, the moment of order c exists and settles down around a finite value. Fig. 3-2 illustrates the behavior of skewness (moment of order three) and kurtosis (moment of order four) in correspondence to the sample size. As observed, while the skewness or third moment stabilizes at a negative value as the sample size increases, the kurtosis or the fourth moment shows no such stability. The results are in favor of a choice of 10% tail and a Hill estimator of around 3. This is in line with the cubic-law distribution of returns (see Gabaix et al., 2003).

3.2.3 Volatility Clustering

Except for very small trading time scales, autocorrelations function (ACF) of returns are insignificant. The absence of linear correlation does not imply independence of returns. In fact, nonlinear functions of returns, such as absolute or square returns, are significantly autocorrelated. Such correlation is positive and persistent, which is a quantitative feature of volatility clustering, with periods of

Table 3.4: NLS estimation of ACF.

	coefficient	t-statistics	R-squared	Root MSE
ς	.604	14.35	.9134	.065
d	.325	14.38		

quiescence and turbulence cluster together. Panel A of Fig. 3-3 demonstrates such properties: while the ACF of raw returns decays quickly to 0, the ACF of absolute returns is relatively large and persistent even after 100 lags. Note that the ACF of raw returns at the lag 1 and 2 is -0.28 and -0.13 respectively, the similar statistics at lag 3 diminishes to 0.01 and then fluctuates around 0 with small magnitude. The magnitude of the ACF of absolute returns remains significantly different from 0 over the 100 lags. The slow decay in the ACF of absolute returns is a characteristic (but not sufficient condition) of a long memory process.

The plot of the ACF against the number of lags is however ambiguous in measuring the decaying characteristics of volatility clustering. As a quantitative complement, exponent of power laws is proposed to measure how fast the ACF of absolute returns decays:

$$corr(|r_{t+q}|, |r_t|) \simeq \varsigma / q^d,$$

where q is the number of lags, ς is a parameter, and d is the exponent, with a smaller value corresponding to slower decay.

Nonlinear least square (NLS) estimation (see Table 3.4) shows that the ACF of absolute returns decays with an exponent $d = .325$, which matches with the real markets that typically have an exponent falling in the range of $(0.2, 0.4)$. It loosely suggests the presence of 'long-range dependence', which is to be discussed in detail later.

3.2.4 Long Range Dependence

r_t is said to have long range dependence if its ACF fades away as a power of the lag:

$$corr(r_t, r_{t+q}) \backsim L(q)/q^{1-2\tilde{d}},\ 0 < \tilde{d} < 1/2, \text{ as } q \to \infty$$

where $L(q)$ is any slowly varying function at infinity, which satisfied that, as the number of lags $q \to \infty$, it is true that $\forall a > 0$, $L(aq)/L(q) \to 1$. Long-range dependent time series exhibits an unusually high degree of persistence even at the lowest frequencies, which is thought to be a common phenomenon in financial variables.

To complement a comprehensive analysis of long-range dependence, we test the hypothesis of no long-range dependence in r_t using Lo modified range over standard deviation or R/S statistic (also called re-scaled range), which is robust to short-range dependence (Lo, 1991)[6]. The Lo modified R/S statistic over n observations, denoted by Q_n, is defined by:

$$Q_n \equiv \frac{1}{\hat{\sigma}_n(q)} \left[\max_{1 \le k \le n} \sum_{j=1}^{j=k} (r_j - \bar{r}_n) - \min_{1 \le k \le n} \sum_{j=1}^{j=k} (r_j - \bar{r}_n) \right], \tag{3.1}$$

where $\hat{\sigma}_n(q)$ is the usual sample variance:

$$\hat{\sigma}_n(q) = \frac{1}{n} \sum_{j=1}^{j=n} (r_j - \bar{r}_n) + \frac{2}{n} \sum_{j=1}^{j=q} \left(1 - \frac{j}{q+1}\right) \sum_{i=j+1}^{i=n} (r_i - \bar{r}_n)(r_{i-j} - \bar{r}_n).$$

The first and second terms in the bracket of Eq.(4.7) are the maximum of the

[6] Any incompatibility between the data and the predicted behavior of the R/S static under the null hypothesis of no long-range dependence may merely be a symptom of short-term memory, if conventional R/S is applied. That is, conventional R/S static is sensitive to short-range dependence. Lo modified R/S static accounts for the effects of short-range dependence and is considerably more accurate.

partial sums of the first k deviations of r_j from the sample mean $\bar{r}_n$ and the minimum of the same sequence of partial sums, respectively.

We show that the estimated Lo modified R/S statistic is increasing with the lag q in the Panel B of Fig. 3-3. Despite the sensitivity of the statistics to the selection of the number of lags, the overall results support the presence of long-range dependence in general. At 10% significance level, with the confidence interval of $[0.861, 1.747]$, we reject the null hypothesis of no long-range dependence through 100 lags. At 5% significance level with the confidence interval of $[0.809, 1.862]$, we find evidence of long-range dependence as long as $q \leqslant 16$.

3.2.5 Leverage Effect

Leverage effect states that most measure of volatility of an asset are negatively correlated with the past return of that asset. It is also called volatility asymmetry, with the amplitude of price fluctuations or volatility having a tendency to increase when the price drops. Represent the volatility with subsequent squared returns, the leverage effect could be measured by $corr(|r_{t+q}|^2, r_t)$ (Cont, 2001). We graph $corr(|r_{t+q}|^2, r_t)$ as a function of the lag q in the Panel C of Fig. 3-3. It turns out that the $corr(|r_{t+q}|^2, r_t)$ starts from a negative value which implies that negative returns lead to a rise in volatility. The simulated returns exhibit leverage effect as documented in many theoretical and empirical literature[7].

3.3 Numerical Analysis

After showing that the purely deterministic model are capable of generating a wide range of well-documented stylized facts, one shall extrapolate it to capture at least in part, if not all, fundamental factors of the complexity in financial market dynamics. At this point, we are more confident to take a step further to dig into such

[7] See Bouchaud et al. (2001) and the survey by Pagan(1996).

a model for the origins of return asymmetry, gradual bubbles and sudden crashes. In this section, we reproduce qualitative features and shade light on the origins of the phenomena of asymmetric returns as well as gradual bubbles and sudden crashes. Our analysis suggests that (i) returns are asymmetric because the most positive returns initiated by fundamentalist is weaken by chartists while the most negative return initiated by chartists is hardly affected by fundamentalists; and (ii) price bubbles come up gradually but crashes happen suddenly, because when the price is above the fundamental, the upward price movements are counterbalanced while the downward movements are enhanced by fundamentalists.

3.3.1 Asymmetric Returns

Returns are asymmetric in the sense that the largest upward return movement is not comparable with the absolute magnitude of the largest drawdown, that is, $\max(r_t, 0) < |\min(r_t, 0)|$. As shown in Table 3.1, the most positive return in our simulated sample is 27.8% while the most negative return is -31.1%, a match with the documented asymmetry in returns. As the statistical properties of returns have been studied in the previous section, we focus on the underlying mechanism of return asymmetry in this part. In line with Johansen and Sornette (2002) and Westerhoff (2003), we support the idea that such asymmetry in returns are endogenous.

Fig.3-4 plots the absolute returns against the price level, with the hollow circle denoting the negative return and the solid circle denoting the absolute value of the negative return. It shows that extreme returns concentrate on low price levels.

What is the driving force behind the extreme returns? When price falls below its value, fundamentalists always buy in to shore up the asset price; chartists, however, either buy or sell, depending on their forecast of piece-wise short-term fundamental (see Eq. (2.3)). At low price levels, extremely negative returns arise when the selling force of chartists outweighs the buying force of fundamentalists, which brings the price down sharply. Extremely positive returns could be generated if (i) chartists move in the same direction with fundamentalist to buy in the asset; or (ii) the buying force of fundamentalists strongly exceeds the selling force of chartists. To have a clear picture of the internal mechanism, Fig. 3-5 plots steps of price dynamics that generate returns below 10% percentile (the extremely negative returns) and above 90% percentile (the extremely positive returns). The dynamic steps associated with the extremely negative returns (top panel) all locate on the right hand side of $p_{t-1} = 19.477$ (the dash vertical line), when the market fraction index m (dash dot line) is trivial[8]. It is equivalent to say that the collective selling of chartists gives rise to the extremely negative returns. The dynamic steps associated with the extremely positive returns (bottom panel) are associated with both trivial and non-trivial m. When m is not trivial that is $p_{t-1} < 19.477$, chartists generally find it optimal to sell[9], extremely positive returns are generated because the buying force of the fundamentalists exceed that of the selling force of chartists. When m is not trivial, extremely positive returns arise because chartists who dominate the market buy in the asset in large volumes.

Why is the most negative return greater than the most positive return in terms of absolute magnitude? The most negative return (-31.1%) arises when the price falls from 22.435 to 16.445. When the price is 22.435, the market fraction of fundamentalists is trivial, the collective selling actions of chartists are hardly affected by fundamentalists. It is relative easy to move the price down for a large amplitude. The most positive return (27.8%) arises when the price jumps from 16.705 to 22.071. At the price level of 16.705, fundamentalists that are not trivial buy in the asset to shore up the price. Their actions are, however, counterbalanced in some extent by the selling behavior of chartists[10] that accounts for more than half of the market fraction (see the dash-dot line), which limits the upward movement

[8] A trivial m ($m \to 0$) indicates a large proportion of chartists and almost no fundamentalists.

[9] If there is no fundamentalists, one of the equilibria is that $p(t-1) = p(t) \approx 19$, below which chartists find it optimal to sell (see Eq.(2.11) and (2.3)). As long as the price stays within the same trading window, the lower the price is below the equilibrium price, the greater the selling force of chartists.

[10] Note that at such price level, chartists' optimal strategy is to sell.

of the subsequent return. In sum, returns are asymmetric because the most positive returns initiated by fundamentalist is weaken by the chartists while the most negative return initiated by chartists are hardly affected by fundamentalists.

3.3.2 Gradual Bubbles and Sudden Crashes

Asset price moves up and down, accompanied with sporadic bubbles and crashes. The bull-run of equity price in 2006 and its collapse during credit crunch in 2007 are representative episodes of the bubble and the crash. Current heterogeneous agent models ascribe bubbles and crashes to the interaction among heterogeneous investors. Our simulation further support this argument. Moreover, it captures the ubiquitous feature of gradual bubbles and sudden crashes that are rarely addressed in agent-based models. We argue that such characteristics could root in the facts that upward price movements are counterbalanced while the downward movements are always enhanced by fundamentalists, when the price is above the fundamental.

We first deviate from the conventional concept of bubble[11] slightly, by focusing on a more general phenomenon - price booms characterized by an upward trend regardless of whether the price is above or below the fundamental. The upper left panel of Fig. 3-6 shows that the simulated prices fluctuate randomly with booms and crashes. The typical episode of crash is reflected between period 174 and 306 (32 periods), when price falls from the peak, 84.184, to the bottom, 16.878 (declined by 67.306). The representative episode of price boom could be found from period 1346 to 1493 (47 periods), when the price bounces from the lowest point, 16.445, up to the regional peak, 83.394 (increased by 66.949). It seems to take longer for price to rise than to decline by a similar magnitude. In other words, the price crashes more quickly than its boom. In order to shade light on the phenomena of slow boom and sudden crash, the right panels of Fig. 3-6 plots the step-wise price dynamics corresponding to these episodes of crash and boom. The

[11] In the following context, we refer 'boom' to general upward trend and 'bubbles' to the upward trend when price is above the fundamental.

crash starts when the price is very high above its value. At that high price level, fundamentalists sell asset to drag the price down towards its fundamental while chartists could either buy in or sell out, depending on their expected short-term fundamental. If the selling force of fundamentalists dominates the market, price goes down for sure. As the price declines, m decreases, which weakens the selling force of fundamentalists. The price stops falling if chartists find it optimal to buy in and their buying force is greater than the selling force of fundamentalists. As soon as the price is up, m increases, the collective selling force of fundamentalists is enhanced. The rising trend reverse when the selling force of fundamentalists exceeds the buying force of chartists. The battle between fundamentalists and chartists continues until the price is brought down to a level that chartists find it optimal to sell. That is the moment when the price starts to plunge. As the price move closer to the fundamental, chartists dominate the market (m is trivial) and continue to explore the trend of price, leading to a series of moderate and disturbing declines. Similarly, the boom starts with the battle between fundamentalists and chartists. When the price is up to a certain level, chartist dominate the market and continue to explore the trend, which eventually push the price far beyond its fundamental and forms a bubble[12].

We now focus on the specific episode of the boom - the bubble, defined by the price span above the fundamental. Compared the upper right and bottom panels of Fig. 3-6, it is found from the density of the dynamic steps that, price falls from the peak less disturbingly than it approaches the peak. Intuitively, when the price is above the fundamental, fundamentalists always buy in while chartists switch between buying and selling according to their latest update of the short-term fundamental. The upward price movements initiated by chartists are impaired while the downward movements are enhanced by fundamentalists. Therefore, the price owes to drop much easier than it climbs. Fig.3-7 illustrates such asymmetry by plotting the absolute price movements against the price level.

[12] See Chapter 2 for a technical analysis.

It shows that, when the price is very high above the fundamental (eg, $p > 70$), the downward price movements (denoted by hollow circles) are greater than the upward movements (denoted by solid circles) in terms of their absolute magnitudes. The former is however sparser than the latter. It implies that price drops in large scale with low frequency while rises in relatively small amplitude with high frequency.

For a technical explanation, we turn to the dynamics of the market fraction index, m. The bottom left panel of Fig. 3-6 states that market index, m, fluctuates more disturbingly when the price falls from the peak than when the price approaches the peak. It suggests that the battle between fundamentalists and chartists last longer in forming bubbles than in generating crashes. When the price is at the peak, m is large. In this case, it is easy for fundamentalists to win the battle against chartists quickly and to drag the price down by a large scale. Whereas when the price is on the way towards the peak, as chartists push the price higher, m that is originally trivial increases gradually. The increased m fortifies the selling force of fundamentalists that counterbalances and even reverse chartists' effort to push the price up further. In sum, around the peak, it is easier for fundamentalists to win against chartists than vice versa. Therefore price goes up easier than it falls. That is why bubbles tend to be gradual while crash tends to be sudden.

3.4 Conclusion

We show that it is possible for a deterministic heterogenous agent model, involving no random elements of any forms, to generate a wide range of stylized facts simultaneously. Specifically, the chaotic return series, simulated under the deterministic model in Chapter 2, exhibits characteristics of fat tails and volatility clustering, the two stylized facts that receive the most intensive discussions in agent-based models. The tail index is around 3, in line with the cubic power low distribution of large returns. The volatility clustering decays with an exponent of 0.325, the value of which falls within the range of the empirical findings, suggesting a good match with the real data. Even without the inclusion of any normally distributed noise term or stochastic process, the simulated data reflect the properties of long range dependence as well as leverage effect. The good performance of the chaotic model in reproducing stylized facts can be attributed to the nonlinear dynamics, which is found to be important in capturing the salient feature in financial time series (Kyrtsou and Terraza, 2002, Kyrtsou and Vorlow, 2009). Accounting for the chaotic behavior through the non-linear deterministic dynamic is important in modeling the complexity in the financial market.

After verifying the solidity of the model by showing its capability to duplicate many stylized facts that are common across financial markets, we believe that the model has captured some, if not all, essential features of the complex market dynamics. So we take a step further to explore the phenomena of asymmetric returns as well as gradual bubbles and sudden crashes. Based on the same model and the same set of parameters, our numerical analysis suggests the followings: (i) returns are asymmetric because the most positive return initiated by fundamentalists are counterbalanced by chartists who trade in the opposite direction against fundamentalists, while the the most negative returns initiated by chartists are hardly affected by fundamentalists. In other words, the net buying force of fundamentalists are not comparable with the collective selling forces of chartists, rendering the magnitude of the most positive return to be smaller than that of the most negative return, and (ii) bubbles form gradually while crashes happen suddenly because, when the price is above its value, fundamentalists always sell assets to bring the price towards its fundamental, which strengthen the downward price movement but attenuate the upward movements driven by the collective buying force of chartists. Note again that the model is purely deterministic, our findings suggest that bubbles and crashes can arise even without external shocks. It indicates that regulations tend to be ineffective in preventing such phenomena from happening. However, policy makers can still monitor the market to some extent by changing the market structure or by influencing behavior of market participants. For example, they can step into the market and play the role of fundamentalists to boost the price up when the market is depressed and to stop the price from going too high. By doing so, they mitigate the size of bubbles and crashes.

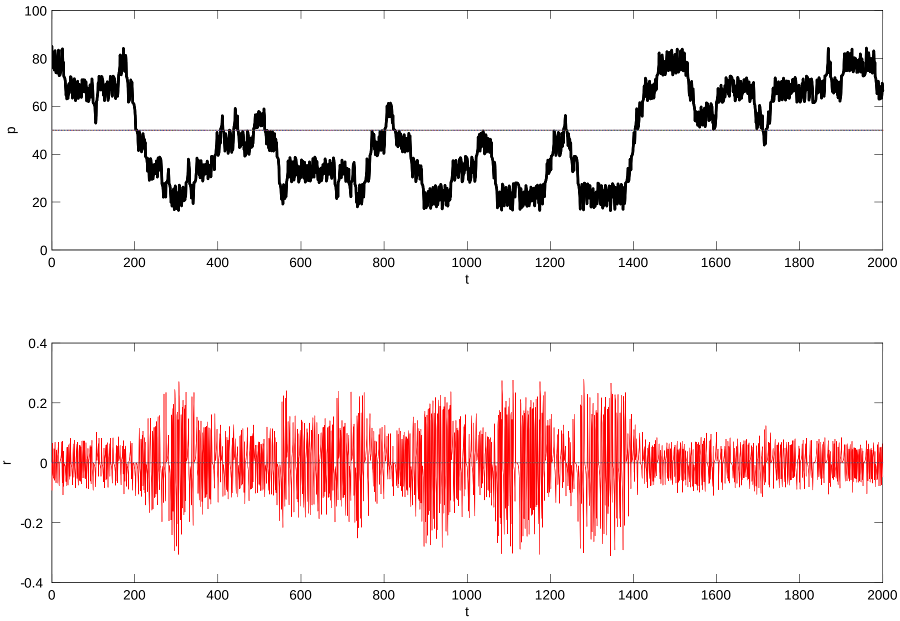


Figure 3-1: The time series of prices (top) and returns (bottom).

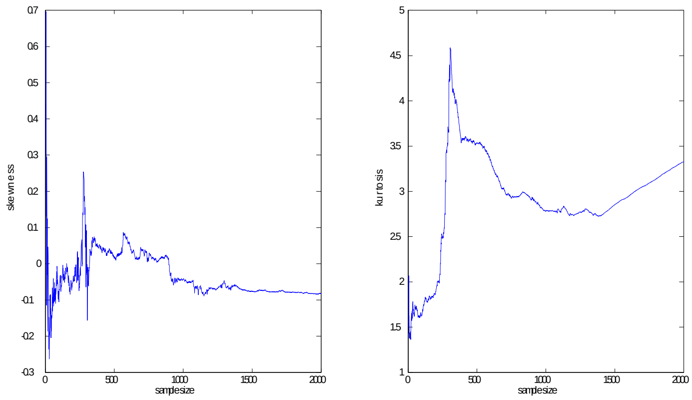
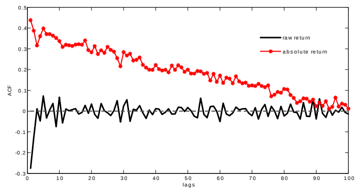


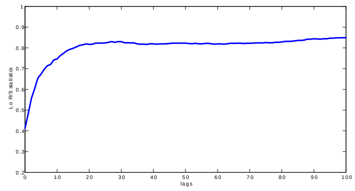
Figure 3-2: Skewness and kurtosis as a function of sample size.

Panel A



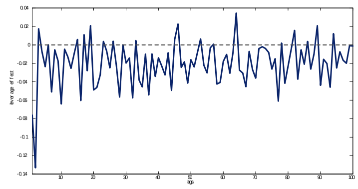
ACFs of raw and absolute returns.

Panel B



Lo modified R/S test or long-range dependence.

Panel C



Leverage effect.

Figure 3-3: ACF, Lo modified R/S and levrage effect.

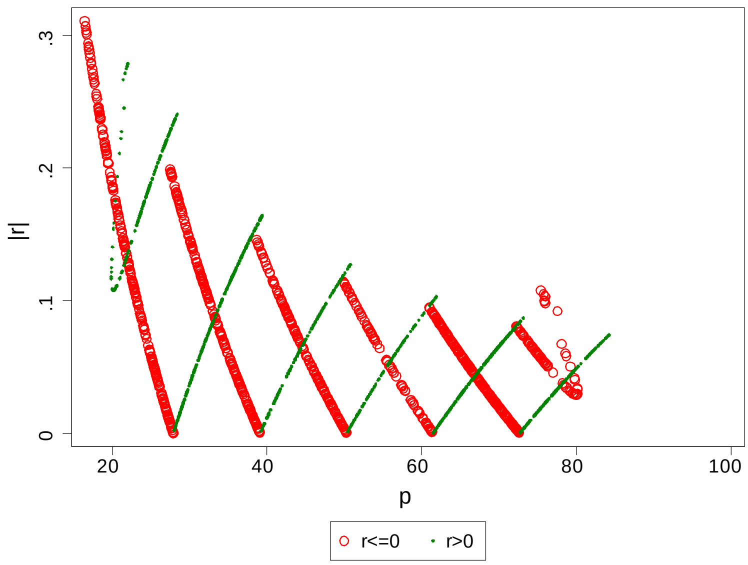
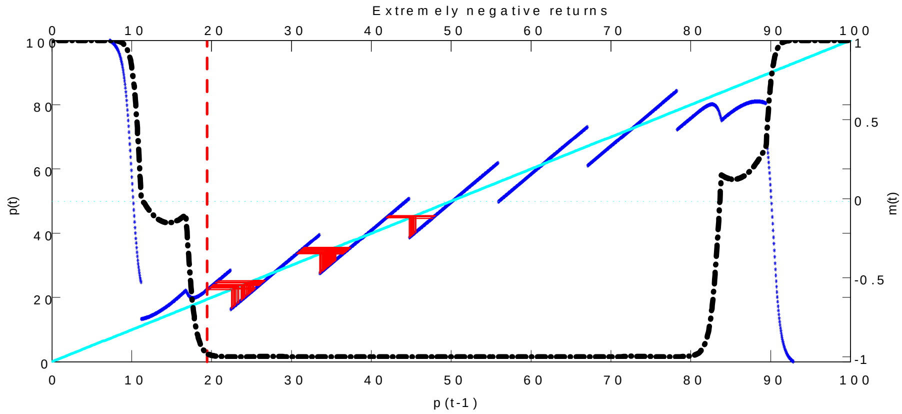


Figure 3-4: The distribution of absolute returns across different price levels. The positive returns are marked with hollow circles and the negative returns are marked with solid circles.



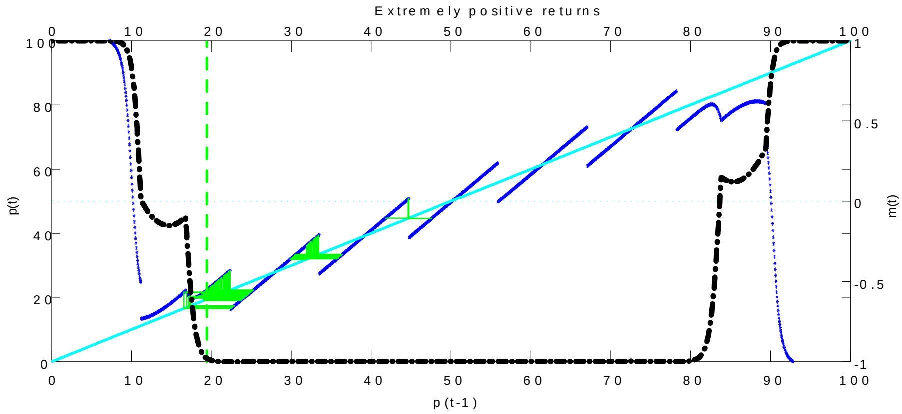


Figure 3-5: Steps of price dynamics for extreme returns. The dot line graphs the phase diagram of the price dynamics. The bold dash dot line is the market fraction index m. The solid lines plot the steps of price dynamics that generate the extremely negative returns (top) the extremely positive returns (bottom). The dash line represents $p_t = 19.477$, corresponding to the minimum price level associate with the extreme negative returns.

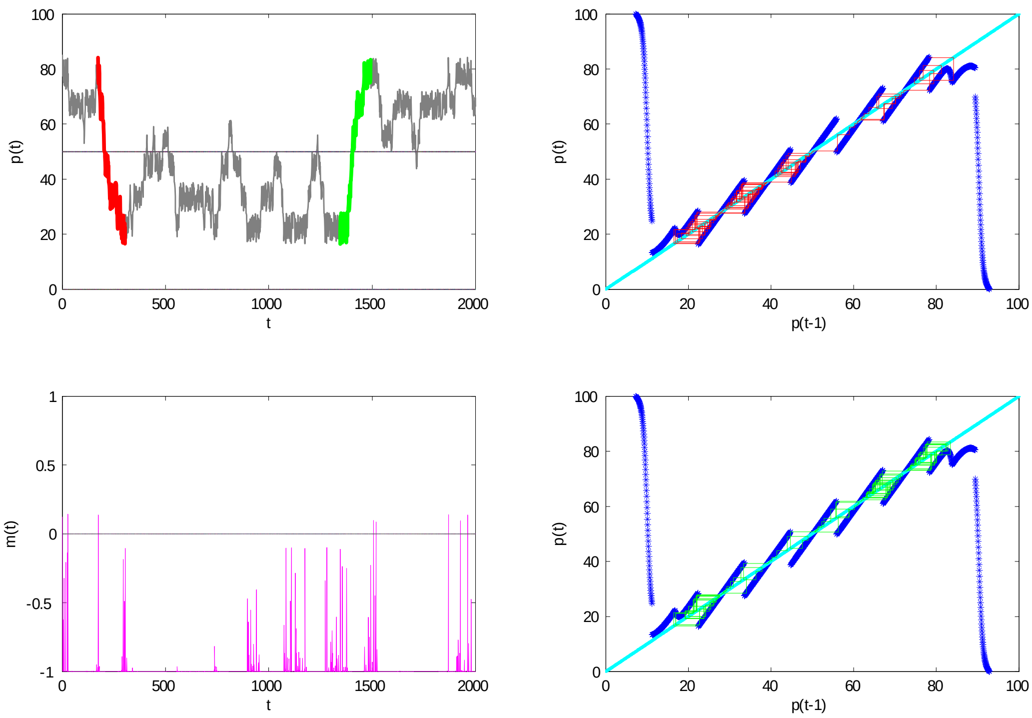


Figure 3-6: Price trajectory (upper left), market fraction index (bottom left), step-wise price dynamics of the bubble (upper right) and crash (bottom right).

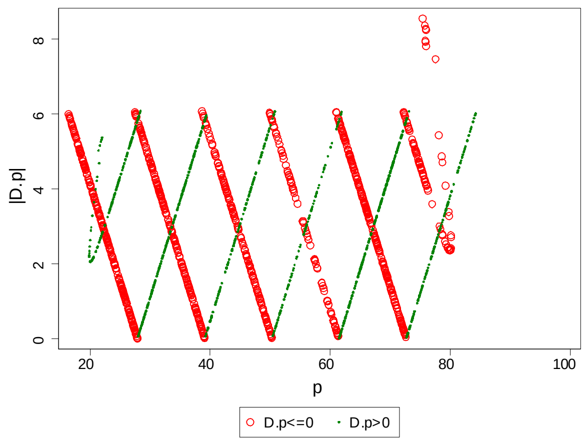


Figure 3-7: Asymmetric price movements. The solid circle denotes the upward movement and the hollow circle denotes absolute magnitude of the downward movement.

Chapter 4

Modeling Financial Crises - The Role of Regime-Dependent Beliefs

4.1 Introduction

In the technical analysis, chartists identify the support price level, at which buying force is believed to be strong enough to prevent the price from dropping further, and the resistance price level, at which selling force is thought to be large enough to curb the price from rising further. If the price moves within the regime enclosed by current support and resistance price levels, chartists stick to their original beliefs that the price will not exceed the thresholds of the regime. However, when the price breaks through the boundaries of current regime, new support and resistance price levels will be established, and chartists will shift their beliefs accordingly. According to their trading experience and analysis, chartists form a series of psychological trading regimes enclosed by different support and resistance price levels, based on which they develop their beliefs of future price movements. In order to decide optimally whether to maintain their original beliefs or shift to others, it is important for chartists to continuously update the support and resistance price level and to extrapolate the contemporaneous psychological trading regime from the latest market information. To formalize the different behavior of chartists at distinct regimes, it is therefore valuable to account for the regime-dependent belief, with the price expectation depending on the corresponding psychological trading regime.

Modeling the belief as regime-dependent is supported by the empirical evidence that the price follows a complicated process with multiple regimes and that such non-linear process affects investment decisions (Ang and Bekaert 2002; Guidolin and Timmermann, 2007, 2008). Such concept is also backed up by the literature of heterogeneous agent model (HAM). It is observed that, HAMs that explicitly incorporate the regime-dependent properties into price expectations exhibit better performance than those without, in one way or another (Manzan and Westerhoff, 2007; Tramontana et al. 2010). Manzan and Westerhoff, 2007) model the chartist sentiment (or the degree to which chartist acts on his belief) as a two-state process and show that their model exhibits out-of-sample forecasting power for some currencies. The model in Chapter 2 defines chartist's belief to depend on the pre-specified price regime and show that such a model is capable of duplicating crises of various types as well as a wide range of stylized facts, including gradual booms and sudden crash, asymmetric return, fat tails, volatility clustering and long-range dependence. Tramontana et al. (2010) describe two types of chartists and fundamentalist, whose sentiment follows either a two-state or three-state process. Their model is capable of generating a relatively large set of dynamic behaviour.

Nonetheless, due to the complexity of these HAMs and the many factors that they account for, it is difficult to analyze concretely how the regime-dependent belief contributes to the model performance. Can the regime-dependent belief alone improve a model's capability to capture the qualitative and statistical properties in the real financial time series? If so, what is the underlying mechanism? The answers to these questions remain unknown in current literature. In this chapter, we address these questions by proposing a simple HAM that exams the impact of the regime-dependent belief on the performance of the model. We first introduce the concept of regime-dependent belief into a basic deterministic HAM proposed by Day and Huang (1990). Second, we test the model's capability to produce different types of financial crises simultaneously, which cannot be fulfilled by Day and Huang (1990). As documented in Rosser (2000) and Kindleberger and Aliber (2005), crises can be classified into three categories, which are named as sudden crisis, smooth crisis and disturbing crisis in Chapter 2. The model by Day and Huang (1990) can duplicate the sudden crisis but not the other two. It suggests that this model has missed some of the key factors that are essential for the formation of general crises. The regime-dependent belief seem to compensate this missing part, as it enables the model in Day and Huang (1990) to generate all patterns of financial crises simultaneously. Asides from enhancing the model's capability to generate various financial crises, such regime-dependent belief also improves the model's performance in duplicating the salient qualitative and statistical properties of the financial time series. Specifically, the simulated price series exhibits technical trading patterns such as head-and-shoulders, double top, double bottom, V top. Moreover, the simulated time-series reproduce a wide range of stylized facts such as fat tails, volatility clustering, and long-range dependence, that are common across financial markets.

This chapter contributes to the literature by showing that the regime-dependent belief is crucial in modeling crises of different types. Although current HAMs that focus on the internal factors of the market dynamics are able to capture one type of crisis or another at a time, few are able to model the three patterns of financial crises simultaneously within the same framework[1]. The model in Chapter 2 outperforms in terms of its capacity to duplicate all crisis patterns within the deterministic HAM. This model accounts for many interlaced dynamic factors,

[1]He and Westerhoff (2005) captures the sudden crisis, Chiarella et al. (2003) the smooth crisis, and Gallegati et al. (2010) the disturbing crisis. To the best of our knowledge, the model in Chapter 2 is the only deterministic HAM that can generate all types of crises simultaneously. Introducing the jump diffusion into the model may help to capture these three patterns of crisis, but it deviates from our beliefs that crises are essentially determined by the internal dynamics rather than external shocks.

such as the regime-dependent belief, interaction among heterogeneous investors, the switch among different strategy and discounting factors used to evaluate the current value of the expected profit. Although this model addresses many aspects of investment behaviour, it is too complicated to shed light on what contributes essentially to the good performance of such a deterministic HAM. This chapter fills this gap by exploring a relatively simple model that focus only on the role of regime-dependent belief. That is, it intentionally drop out factors discussed in Chapter 2 other than the regime-dependent belief. The main driving force of the market dynamics is investors' belief for the future price. It shows that enriching the very basic HAM with only regime-dependent belief can easily improve the model's performance in capturing different types of financial crises.

More importantly, this chapter takes a step further to explain why crisis differs from each other. It shows that, the price dynamics is not only responsive to different psychological trading regimes but also very sensitive to the relative location of the latest price in the regime. Falling into different zones of the regime leads to distinct dynamic patterns that characterize crises of various types. For mean-variance maximizers, given the same risk attitude, the greater the price is expected to decline, the stronger the selling force. If the latest price falls into the zone that triggers little selling force, the price will decline a little bit and remain in the same regime. If on the other hand, the latest price falls into the zone that drives investors to sell the risky asset in a large volume, the price may decline by such a large magnitude that it breaks through the initial support price level. In this case, two scenarios can happen after investors extrapolate the regime enclosed by the newly established support and resistance price levels. If the price declines to such an extent that overall investors find it optimal to sell according to their regime-dependent belief, the price will decline further. If, on the other hand, the overall investors find it optimal to buy, the price will go up. While the first scenario creates the impression of sudden decline, with price drooping dramatically within a short period, the second scenario paints the episode with disturbing ups and downs that characterize the period of financial distress. This is especially true if such a process repeats itself over time. The different price dynamic patterns that characterize the different types of financial crises arise as the price falls into different zones of the regime. By constraining the price to fall into the specific zones in a series of regimes over time, various types of financial crises can be generated. As the chaotic model proposed in this chapter is also deterministic, one can always set the parameters and initial value in such a way that the price series satisfy the conditions for different crisis patterns.

The rest of this chapter is organized as follows. Section 2 describes the model. Section 3 discusses its theoretical implications. Section 4 simulates the sudden crisis, smooth crisis and disturbing crisis and analyze their underlying mechanism that differentiates these crises. It then shows how the model performs in capturing qualitative and statistical properties of the financial time series. Section 5 concludes.

4.2 Model

We consider a market with one risky asset and two types of investors distinguished by their trading strategies - fundamentalists (α-investor) and chartists (β-investor). A market maker steps in to balance the excessive market aggregate demand from these investors and subsequently adjusts the price up or down.

4.2.1 Fundamentalists

Fundamentalists expect the asset price to converge towards its long term fundamental value $\bar{p}_{\alpha,t}$, with a time-varying convergence speed ϑ_t. They buy in the asset when the price is below $\bar{p}_{\alpha,t}$ and sell out vice versa. Given the asset price p_t, the number of risky assets demanded by fundamentalists at period t, $D_{\alpha,t}$, is given by:

$$D_{\alpha,t} = \vartheta_t(\bar{p}_{\alpha,t} - p_t).$$

The convergence speed $\vartheta_t \in (0,1]$ is a nonlinear function of p_t following the definition of Day and Huang (1990), which is to be specified soon. The long term fundamental value $\bar{p}_{\alpha,t}$ is determined by the real economic growth. In a stable economy with zero growth rate and no permanent exogenous shock, the fundamental value $\bar{p}_{\alpha,t}$ can be treated as a constant parameter. In the presence of economic growth, the fundamental value $\bar{p}_{\alpha,t}$ is assumed to fluctuate with the business cycle.

4.2.2 Chartists

Chartists are aware of the conditional time variation in the formation process of the price time series. They update their short-term market value, $\bar{p}_{\beta,t}$, every period according to a simple process with multiple regimes. They infer the current regime from the historical price as well as their trading experience. The short-term fundamental value $\bar{p}_{\beta,t}$ is then extrapolated to be the midpoint of the regime. Following what is in Chapter 2, we assume that chartists divide the price domain $P = [p_{\min}, p_{\max}]$ into n regimes so that:

$$\mathbb{P} = \cup_{j=1}^{n} \mathbb{P}_j = [\bar{p}_0, \bar{p}_1] \cup [\bar{p}_1, \bar{p}_2] \cup \cdots \cup [\bar{p}_{n-1}, \bar{p}_n],$$

where $\bar{p}_j$ $(j = 1, 2..., n)$ represents the threshold of regime j. These threshold price can be interpreted as different support (resistance) price levels in the technical analysis. When the regime threshold is broken, the psychology behind chartists' expectation for stock movements is thought to have shifted. In this case, the psychological trading regime shifts, which is encompassed by the newly established support and resistance price levels.

The short-term fundamental value $\bar{p}_{\beta,t}$ equals to the average of the top and the bottom threshold prices that enclose the regime, into which the current price falls:

$$\bar{p}_{\beta,t} = \bar{p}_{\beta}^{j} = (\bar{p}_{j-1} + \bar{p}_j)/2 \text{ if } p_t \in [\bar{p}_{j-1}, \bar{p}_j),\ j = 1, 2..., n. \tag{4.1}$$

If the market consists of chartists only, the price in the next period p_{t+1} will decline if $p_t \in [\bar{p}_{j-1}, \bar{p}_\beta^j)$ and increase if $p_t \in [\bar{p}_\beta^j, \bar{p}_j)$.

Chartists do not expect the price to converge to $\bar{p}_{\beta,t}$. Instead, they believe that the difference between the actual price and $\bar{p}_{\beta,t}$, or the estimation bias, is going to persist to some degree. So they update their price expectations according to the latest price information and the most recent estimation bias:

$$E_{\beta,t}\left(p_{t+1}\right) = p_t + \tau \cdot \left(p_t - \bar{p}_{\beta,t}\right), \tag{4.2}$$

where $\tau \in (0,1)$ measures the sensitivity of price expectation to the most recent estimation bias $p_t - \bar{p}_{\beta,t}$. The larger τ is, the more persistent the estimation bias is expected to be. The value of τ is non-negative, suggesting that chartists expect the estimation bias to persist rather than reverse in the subsequent period.

The number of risky asset demanded by chartists is proportional to the expected price change:

$$D_{\beta,t} = \sigma\left[E_{\beta,t}\left(p_{t+1}\right) - p_t\right] = \sigma\tau\left(p_t - \bar{p}_{\beta,t}\right), \tag{4.3}$$

where σ measures the extent to which chartists act on their belief. For simplicity, the parameter σ is treated as a constant. The expression of demand function suggests that chartists buy in the risky asset when the price goes beyond the latest estimation of short-term fundamentals $\bar{p}_{\beta,t}$, and sell out when the price falls below.

4.2.3 Market Maker

The order imbalance arising from the trading activities of fundamentalists and chartists is offset by the market maker, who supplies from (absorbs into) his inventory when there is positive (negative) excess demand. In the subsequent period, the market maker adjusts the price up or down according to the weighted excess demand[2] so that

$$
\begin{aligned}
p_{t+1} = f(p_t) &= p_t + \gamma \left(D_{\alpha,t} + \phi D_{\beta,t} \right) \\
&= p_t + \gamma \vartheta_t (\bar{p}_{\alpha,t} - p_t) + \gamma \eta \left(p_t - \bar{p}_{\beta,t} \right),
\end{aligned}
\tag{4.4}
$$

where γ measures the sensitivity of price adjustment to the excess demand and ϕ is the market weight of chartists relative to that of fundamentalists, which is normalized to 1. The second line is obtained by substituting $D_{\alpha,t}$ and $D_{\beta,t}$ with Eq.(4.2) and (4.3) and by letting $\eta = \phi\sigma\tau$. When the market gets more volatile, the market maker may increase γ to compensate the risk he is bearing. Here for simplicity, we will assume γ to be a constant parameter. Given that both ϑ_t and $\bar{p}_{\beta,t}$ is a nonlinear function of p_t, the final price dynamics in Eq.(4.4) is essentially governed by a deterministic one dimensional mapping.

4.3 Theoretical Implications

In the presence of the regime-dependent belief, there are multiple equilibria, which satisfy the condition that $f(p_t) = 0$. This set up can result in periodic cycles or chaos. As the price movements in the real financial activities show no trackable trajectory, we focus on how the dynamic system leads to chaotic phenomena. Specifically, we analyze how the price dynamics leads to the ups and downs within the same regime and the switch among regimes. We address these properties within the multiple-phase dynamic system in the following discussion.

[2] For empirical evidence on how asset prices adjust with respect to market order imbalances, see Bouchaud, Farmer et al. (2009).

4.3.1 Equilibrium

It is straightforward from Eq.(4.4) that Δp_t, the price change from period $t-1$ to t, can be written as

$$\Delta p_t = p_{t+1} - p_t = \gamma\vartheta_t(\bar{p}_{\alpha,t} - p_t) + \gamma\eta\left(p_t - \bar{p}_{\beta,t}\right). \tag{4.5}$$

When the buying and selling forces balance out, the market is in equilibrium and $\Delta p_t = 0$. In the presence of the regime-dependent belief, there are more than one equilibrium price. For the equilibrium price exiting in regime j, p_j^*, it must satisfy the following condition:

$$\vartheta_t(\bar{p}_{\alpha,t} - p_j^*) + \eta\left(p_j^* - \bar{p}_\beta\right) = 0, \tag{4.6}$$

where $\bar{p}_\beta$ corresponds to the mid-point of the psychological trading regime that the equilibrium price falls into. Given that $p_j^* \in [\bar{p}_{j-1}, \bar{p}_j]$, we have $\bar{p}_\beta = \left(\bar{p}_{j-1} + \bar{p}_j\right)/2$. If the market consists of chartists only, it is always true that $p_j^* = \bar{p}_\beta$. In the presence of fundamentalists, the equilibrium price would deviates from chartists estimation of the market value.

When the price is overvalued (undervalued) such that $p_t \in (\bar{p}_{\alpha,t}, p_{\max}]$ ($p_t \in [p_{\min}, \bar{p}_{\alpha,t})$), the market is said to be bull (bear). Note from Eq.(4.6) that in bull (bear) market, it is always true that $p_j^* > \bar{p}_\beta$ ($p_j^* < \bar{p}_\beta$). This is because fundamentalists always sell out (buy in) in the bull (bear) market so as to push the price towards its long-term fundamental value $\bar{p}_{\alpha,t}$. Compared with the scenario when the market consists of chartists only, the presence of fundamentalists drives the price lower (higher) across all regimes in the bull (bear) market because they always sell out (buy in) the risky asset.

Table 4.1: Comparing declining and rising zones.

	p_t	p_j^*	$Z_{R,j}$	$Z_{D,j}$	Asymmetry
Bull Market	$p_t \in (\bar{p}_{\alpha,t}, p_{\max}]$	$p_j^* < \bar{p}_\beta$	$Z_{R,j} \subseteq (p_j^*, \bar{p}_j]$	$[\bar{p}_{j-1}, p_j^*) \subseteq Z_{D,j}$	$\Delta p_{\max} < \lvert\Delta p_{\min}\rvert$
Bear Market	$p_t \in [\mathbf{p}_{\min}, \bar{\mathbf{p}}_{\alpha,t})$	$p_j^* > \bar{p}_\beta$	$(p_j^*, \bar{p}_j] \subseteq Z_{R,j}$	$Z_{D,j} \subseteq [\bar{p}_{j-1}, p_j^*)$	$\Delta p_{\max} > \lvert\Delta p_{\min}\rvert$

4.3.2 Rising and Declining Zones

Let $Z_{R,j}$ ($Z_{D,j}$) denote the rising (declining) zone in regime j, then we have $p_t \in Z_{R,j}$ ($p_t \in Z_{D,j}$) if $\Delta p_t > 0$ ($\Delta p_t < 0$). We first analyze the properties of the rising and declining zones when the market consists of both fundamentalists and chartists. In the bull market, it is straightforward from Eq.(4.5) and Fig. 4-1 that the rising zone $Z_{R,j}$ is at most equivalent to $\left(p_j^*, \bar{p}_j\right)$ and the declining zone $Z_{D,j}$ is at least equivalent to $\left(\bar{p}_{j-1}, p_j^*\right)$[3]. Note that $p_j^* > \bar{p}_\beta$, which results in $\bar{p}_j - p_j^* < \bar{p}_j - \bar{p}_\beta < \left(\bar{p}_j - \bar{p}_{j-1}\right)/2$ and $p_j^* - \bar{p}_{j-1} > \bar{p}_\beta - \bar{p}_{j-1} > \left(\bar{p}_j - \bar{p}_{j-1}\right)/2$, the width of the rising zone $Z_{R,j}$ is smaller than that of the declining zone $Z_{D,j}$. It means that if p_t falls uniformly in regime j, the probability of the price increase is less than that of the price decrease. It suggests that the price is more likely to fall than to rise in the bull market. Similarly, it is easy to derive that, in the bear market, the price is more inclined to rise than to fall. The magnitude of price change is also asymmetric in similar manners. It is easy to derive from Eq.(4.5) that the maximum price increase $\Delta p_{\max}$ is smaller (greater) than maximum price decrease or the most negative price change, $|\Delta p_{\min}|$, in bull (bear) market. These results can be summarized in Table 4.1.

When the market consists of chartists only, following from Eq.(4.4) it is always true that $\Delta p_t > 0$ ($\Delta p_t < 0$) if $p_t > \bar{p}_\beta$ ($p_t < \bar{p}_\beta$) and $p_t \in [\bar{p}_{j-1}, \bar{p}_j]$. In this case, the rising zone in regime j is $Z_{R,j} = [\bar{p}_{j-1}, \bar{p}_\beta)$ and declining zone in regime j is $Z_{D,j} = (\bar{p}_\beta, \bar{p}_j]$ (see Fig.4-1). Given that $\bar{p}_\beta = \left(\bar{p}_{j-1} + \bar{p}_j\right)/2$, the width of rising zone equals to that of the declining zone. If p_t falls into the regime $[\bar{p}_{j-1}, \bar{p}_j]$

[3]Extremely, when the selling forces by fundamentalists in the bull market dominate any buying forces by chartists, at all price levels in regime j, the whole regime becomes the declining zone. In this case, there is not equilibrium price in regime j.

uniformly, the price increases and decreases with the same probability. Moreover, the maximum price increment is equal to the maximum decrement. Therefore, one might argue that the trading behaviour of fundamentalists contribute to the asymmetry in the rising and the declining zones and in the magnitude of price changes. Such finding sheds light on why return is asymmetric in the sense that the magnitude of the most negative return is greater than that of the most positive return. Moreover, it also helps to explain why bubbles form gradually while bust suddenly.

4.3.3 Regime Evolution

Within the multiple-phase dynamic system, price can either stay in the same regime or escape from one regime to another. As the price dynamics is deterministic, it is possible to derive both conditions and probability statements of within-regime dynamics and regime switching, which helps one to understand the price moving patterns in the next section.

Within-Regime Dynamics

Proposition 1 *If p_t falls into the regime $[\bar{p}_{j-1}, \bar{p}_j)$ uniformly and $\gamma\vartheta_t < 1$, the price in the next period stays in the same regime ($f(p_t) \in [\bar{p}_{j-1}, \bar{p}_j)$) with a probability of $(\widehat{p}^j - \widehat{p}^{j-1}) / (\bar{p}_j - \bar{p}_{j-1})$, where $\widehat{p}^{j-1} \in [\bar{p}_{j-1}, \bar{p}_j)$ and $\widehat{p}^j \in [\bar{p}_{j-1}, \bar{p}_j)$ are solutions to $f(p_t) = \bar{p}_{j-1}$ and $f(p_t) = \bar{p}_j$, respectively.*

The dynamics within the same regime reflects a relatively stable market environment, where some investors, in our example, chartists, maintain their original beliefs for support and resistance price levels and therefore $\bar{p}_\beta$. Within regime j with $p_t \in [\bar{p}_{j-1}, \bar{p}_j)$, the function $f(p_t)$ is monotonically increasing with p_t if $\gamma\vartheta_t < 1$ (sufficient but not necessary condition). Solving for $f(p_t) = \bar{p}_{j-1}$ and $f(p_t) = \bar{p}_j$ yields two boundary solutions $\widehat{p}^{j-1}$ and $\widehat{p}^j = p^j$, which enclose the price domain whose one-step mapping will remain in the same regime j. As long as $p_t \in$

$[\widehat{p}^{j-1}, \widehat{p}^{j})$, the price in the next period will sustain in the regime j and chartists will carry the psychological trading regime and the estimation of the short-term fundamental value $\bar{p}_\beta$ from period t to $t+1$. Such within-regime dynamics leads to relative moderate price movements, especially when the width of the regime is small. In order for the price to evolve in the same regime for n consecutive periods, the conditions $f^i(p_t) \in [\widehat{p}^{j-1}, \widehat{p}^{j})$ must be satisfied for $i = 1, 2, ...n$. Such scenario is more likely to happen when the market is relatively tranquil and chartists hold the same psychological trading regime in response to relative mild price changes. If $p_t = p_j^*$, the within-regime dynamics will perpetuate. Other than that, the value of n cannot be infinitely large. For any $p_t \in [\widehat{p}^{j-1}, \widehat{p}^{j}) \cap Z_{R,j}$, it is always true that $f^{i+1}(p_t) > f^i(p_t)$ for $i = 1, 2, ...n$. As $f^n(p_t)$ is monotonically increasing with n, the iterated dynamics will eventually lead the price to break through the upper bound $\widehat{p}^{j-1}$ of the self-sustaining regime as n increases, rending the price to escape regime j. The same conclusion is obtained if $p_t \in [\widehat{p}^{j-1}, \widehat{p}^{j}) \cap Z_{D,j}$.

Regime Switching

The fast growing financial market, is subject to various innovations, reversible or irreversible, that cannot be encompassed by the self-sustaining within-regime dynamics. To better understand the market, it is important to address not only the within-regime dynamics but also the regime switching. If the price escapes regime j, it must switch to another regime in the price domain. From Proposition 1, it is easy to derive that the probability for the price to escape regime j is $1 - (\widehat{p}^{j} - \widehat{p}^{j-1}) / (\bar{p}_j - \bar{p}_{j-1})$. If $p_t \in [\bar{p}_{j-1}, \widehat{p}^{j-1})$ ($p_t \in [\widehat{p}^{j}, \bar{p}_j)$), the price in the next period p_{t+1} will switch down (up) to another lower (higher) regime. Under the nonlinear dynamics, the price could switch not only to its nearby regimes but every possible regime if the dynamic system is well defined. The price in regime j can switch to regime k, where the maximum or minimum value of k is determined by the value of parameters as well as the current price level. Assuming the price dynamics leads the price to switch from regime j to regime k, the probability of falling into different regimes can be summarized in the following proposition.

Proposition 2 *Let $\widehat{p}^{k-1} \in [\bar{p}_{k-1}, \bar{p}_k)$ and $\widehat{p}^k \in [\bar{p}_{k-1}, \bar{p}_k)$ be solutions to $f(p_t) = \bar{p}_{k-1}$ and $f(p_t) = \bar{p}_k$. If p_t falls into the regime $[\bar{p}_{j-1}, \bar{p}_j)$ uniformly and $\gamma\vartheta_t < 1$, the following statements are true:*

(i) for $p_t \in [\bar{p}_{j-1}, \widehat{p}^{j-1})$, the price in the next period p_{t+1} switches downward to the regime $k < j$, that is $f(p_t) \in [\bar{p}_{k-1}, \bar{p}_k)$, with a probability of $\dfrac{\widehat{p}^k - \max\left(\bar{p}_{j-1}, \widehat{p}^{k-1}\right)}{\bar{p}_j - \bar{p}_{j-1}}$;

(ii) for $p_t \in [\widehat{p}^j, \bar{p}_j)$, the price in the next period p_{t+1} switches upward to the regime $k > j$, with a probability of $\dfrac{\min(\bar{p}_j, \widehat{p}^k) - \widehat{p}^{k-1}}{\bar{p}_j - \bar{p}_{j-1}}$.

When $p_t \in [\bar{p}_{j-1}, \widehat{p}^{j-1})$ ($p_t \in [\widehat{p}^j, \bar{p}_j)$), the selling (buying) is so strong that it drives down (up) the price so much so that the price breaks through the lower (upper) bound of regime j. As the price trajectory escapes regime j, new support and resistance price levels are established, and chartists' psychological trading regime shifts accordingly. If the mechanism is designed in such a way that the price can only switch to the nearby regime, then the probability for the price to switch from j to $k = j - 1$ ($k = j + 1$) is $\dfrac{\widehat{p}^k - \bar{p}_{j-1}}{\bar{p}_j - \bar{p}_{j-1}}$ ($\dfrac{\bar{p}_j - \widehat{p}^{k-1}}{\bar{p}_j - \bar{p}_{j-1}}$). If the price cannot only switch to its nearby regime but also to regimes far apart, the situation becomes relatively complicated. The probability is derived in two steps. First, if regime k is the lowest (highest) regime that the one-step dynamics could reach, then the probability for the price to switch from regime j to k is $\dfrac{\widehat{p}^k - \bar{p}_{j-1}}{\bar{p}_j - \bar{p}_{j-1}}$ ($\dfrac{\bar{p}_j - \widehat{p}^{k-1}}{\bar{p}_j - \bar{p}_{j-1}}$), where $\widehat{p}^k \in [\bar{p}_{k-1}, \bar{p}_k)$ and $\widehat{p}^{k-1} \in [\bar{p}_{k-1}, \bar{p}_k)$ are solutions to $f(p_t) = \bar{p}_k$ and $f(p_t) = \bar{p}_{k-1}$. Second, if regime k is not the lowest (highest) regime that the one-step dynamics could reach, given that $f(p_t)$ is a monotonic increasing function of p_t for $p_t \in [\bar{p}_{j-1}, \bar{p}_j)$, the one step dynamics governed by $f(p_t)$ can be at any points in the regime k. Therefore, the probability for the price to switch from regime j up or down to k equals to $\dfrac{\widehat{p}^k - \widehat{p}^{k-1}}{\bar{p}_j - \bar{p}_{j-1}}$. Note that $\bar{p}_j \geqslant \widehat{p}^{k-1}$ and $\widehat{p}^k \geqslant \bar{p}_{j-1}$, the results from the two steps can be summarized as the probability functions in the Proposition 2.

Transition Probability

Let the probability for the price to switch from regime i to regime j to be

$$T_{i,j} = \Pr(p_{t+1} \in [\bar{p}_{j-1}, \bar{p}_j) | p_t \in [\bar{p}_{i-1}, \bar{p}_i)).$$

Both the highest and the lowest regimes that the current price dynamics can reach depend on the current price level as well as the value of parameters. There are infinite combinations that could arise from this general set-up without specifying the value of parameters. For simplicity, the dynamic system is restricted in such a way that the price can only switch to its nearby regime so that $T_{i,j} = 0$ for $i \in (j+1, +\infty) \cup (-\infty, j-1)$. Under the assumption that p_t falls into the regime $[\bar{p}_{j-1}, \bar{p}_j)$ uniformly and $\gamma\vartheta_t < 1$, following from Propositions 1 and 2, the transition probability following from the one-step price dynamics can be summarized as below:

$$\begin{bmatrix} \frac{\widehat{p}^1 - \widehat{p}^0}{\bar{p}_1 - \bar{p}_0} & \frac{\bar{p}_1 - \widehat{p}^1}{\bar{p}_1 - \bar{p}_0} & 0 & 0 & \cdots & 0 \\ \frac{\widehat{p}^1 - \bar{p}_1}{\bar{p}_2 - \bar{p}_1} & \frac{\widehat{p}^2 - \widehat{p}^1}{\bar{p}_2 - \bar{p}_1} & \frac{\bar{p}_2 - \widehat{p}^2}{\bar{p}_2 - \bar{p}_1} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & \frac{\widehat{p}^j - \widehat{p}^{j-1}}{\bar{p}_j - \bar{p}_{j-1}} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & \cdots & & \frac{\widehat{p}^n - \widehat{p}^{n-1}}{\bar{p}_n - \bar{p}_{n-1}} \end{bmatrix}.$$

Specifically, for a price in regime j, the probability of staying in the same regime is $T_{j,j} = \frac{\widehat{p}^j - \widehat{p}^{j-1}}{\bar{p}_j - \bar{p}_{j-1}}$, the probability of switching to regime $j+1$ is $T_{j,j+1} = \frac{\bar{p}_j - \widehat{p}^j}{\bar{p}_j - \bar{p}_{j-1}}$ and the probability of switching to regime $j-1$ is $T_{j,j-1} = \frac{\widehat{p}^{j-1} - \bar{p}_{j-1}}{\bar{p}_j - \bar{p}_{j-1}}$. Note that these transition probabilities are essentially a dynamic function of p_t, their value vary with the regime j that p_t falls into and the complex dynamic system that governs values of $\widehat{p}^j$ and $\widehat{p}^{j-1}$.

4.3.4 Price Dynamic Patterns

The previous section analyzes how the one-step dynamics leads to the regime evolution. In this section, we focus on the the dynamic patterns of price movements over multiple periods. We analyze the two-step dynamics. This result can be easily extended to multiple-step dynamics by iterations. To study the price declining patterns during the financial crisis, we focus on how the price drops in the bull market. Similarly, the alternative scenario of how the price bounces up in the bear market can also be analyzed. For simplicity, we assume that dynamic system is designed in such a way that the price can only switch to its nearby regime. The scenario starts with p_t falling in the declining zone of regime j, such that $p_t \in [\bar{p}_{j-1}, p_j^*)$. Let $\widetilde{p}^{j-1} \in [\bar{p}_{j-1}, p_j^*)$ be the solution to $f(p_t) = p_{j-1}^*$, where p_{j-1}^* is the equilibrium price in regime $j-1$.

Proposition 3 *In the bull market, under the condition that $\gamma\vartheta_t < 1$, we have*

(i) $f(p_t) \in [\bar{p}_{j-2}, p_{j-1}^*) \subseteq Z_{D,j-1}$ if $p_t \in [\bar{p}_{j-1}, \widetilde{p}^{j-1})$;

(ii) $f(p_t) \in [\bar{p}_{j-1}, p_j^*) \subseteq Z_{D,j}$ if $p_t \in [\widehat{p}^{j-1}, p_j^*)$; and

(iii) $f(p_t) \in [p_{j-1}^*, \bar{p}_{j-1}) \subseteq Z_{R,j-1}$ if $p_t \in [\widetilde{p}^{j-1}, \widehat{p}^{j-1})$.

As illustrated in 4-2, if $p_t \in [\bar{p}_{j-1}, \widetilde{p}^{j-1})$, the price will switch downwards to the declining zone of a lower regime in $t+1$ so that $p_{t+1} \in [\bar{p}_{j-2}, p_{j-1}^*)$. The location of p_{t+1} in regime $j-1$ leads to further price decline in period $t+2$. The magnitude of the price decline cumulated over the two periods is the largest among the three scenarios analyzed. If the process continues for n consecutive periods so that $f^k(p_t) \in [\bar{p}_{j-k}, \widetilde{p}^{j-k})$ for every value of $k = 1, ..., n$, the magnitude of cumulative decline can be quite dramatic. We therefore refer to the regime $[\bar{p}_{j-1}, \widetilde{p}^{j-1})$ as the 'sudden declining zone'.

If $p_t \in [\widehat{p}^{j-1}, p_j^*)$, the price will drop and remain in the declining zone of the same regime so that $p_{t+1} \in [\widehat{p}^{j-1}, p_j^*)$, which triggers further price decline in period $t+2$ (see Fig.4-3). If such process repeats for n period such that $f^n(p_t) \in [\widehat{p}^{j-1}, p_j^*)$, there will be a succession of moderate price decline. We refer to the regime

$[\widehat{p}^{j-1}, p_j^*)$ as 'smooth declining zone'. Note that n cannot be infinitely large. As the price continue to decline, it will eventually drop below $\widehat{p}^{j-1}$ and escape the smooth declining zone. Therefore such smooth decline is not sustainable.

If $p_t \in [\widetilde{p}^{j-1}, \widehat{p}^{j-1})$, the price in period $t+1$, p_{t+1}, will fall into the rising zone of a lower regime $j-1$, which brings the price up in the subsequent period (see Fig.4-4). If such process repeats itself, we will observe a series of disturbing ups and downs. We therefore refer to the regime $[\widetilde{p}^{j-1}, \widehat{p}^{j-1})$ as 'disturbing declining zone'. Under certain circumstances, the price will fluctuate up and down with a downward trend, which characterized the period of financial distress that precede the disturbing crisis.

Following the results in Proposition 3, if the price dynamics frequently falls into smooth and disturbing declining zones in a series of regime, the price tends to decline smoothly. If the price falls into the 'sudden declining zone' frequently, the price shall fall dramatically in a short time. Finally, if the price first switches between smooth and disturbing declining zones, and then step into the 'sudden declining zone', the price would fluctuate disturbingly with a downward trend at the beginning and then falls precipitately afterwards.

4.4 Numerical Experiments

The previous section discusses general theoretical implications that apply for any parameter specifications. In this section, we first specify the function of ϑ_t and $\bar{p}_{\beta,t}$ to complete the price dynamics. We then specify parameters and conduct numerical simulation.

Speed of Convergence ϑ_t

Let the function of convergence speed be given by:

$$\vartheta_t = f(\bar{p}_{\alpha,t}, p_t) := (p_t - \mu_1 \bar{p}_{\alpha,t})^d (\mu_2 \bar{p}_{\alpha,t} - p_t)^d,$$

where $d < 0$, $\mu_1 \bar{p}_{\alpha,t} = \min(p_t)$, and $\mu_2 \bar{p}_{\alpha,t} = \max(p_t)$. This is essentially a generalized version of the chance function in Day and Huang (1990). ϑ_t is decreasing (increasing) with the price when $p_t \in ((\mu_1 + \mu_2) \bar{p}_{\alpha,t}/2, \mu_2 \bar{p}_{\alpha,t})$ ($p_t \in (\mu_1 \bar{p}_{\alpha,t}, (\mu_1 + \mu_2) \bar{p}_{\alpha,t}/2)$). If $\mu_2 + \mu_1 = 1$, such function indicates that the more the price deviates from its fundamental value $\bar{p}_{\alpha,t}$, the more quickly it is going to reverse to its value. If $\mu_1 + \mu_2 > 1$, when the price is below its fundamental value the convergence speed ϑ_t always decreases with p_t; when the price is above its fundamental value, it first decreases with p_t if $p_t \in (\bar{p}_{\alpha,t}, (\mu_1 + \mu_2) \bar{p}_{\alpha,t}/2)$ and then start to increase with p_t as the price exceeds the threshold $(\mu_1 + \mu_2) \bar{p}_{\alpha,t}/2$. Similarly, if $\mu_1 + \mu_2 < 1$, when the price is above its fundamental, the convergence speed always increases with the price p_t; and when the price is below its fundamental, it decreases with p_t if $p_t \in ((\mu_1 + \mu_2) \bar{p}_{\alpha,t}/2, \bar{p}_{\alpha,t})$ and decreases with p_t if the price falls below $(\mu_1 + \mu_2) \bar{p}_{\alpha,t}/2$. It means that investors believe that, if the price does not deviate much from the fundamental value, it is very likely to reverse towards fundamental in the very short-term. However, if price breaks the point $(\mu_1 + \mu_2) \bar{p}_{\alpha,t}/2$, it is more likely for the price to deviate than to reverse to the fundamental value. Such definition helps to address the asymmetric booms and burst. If $\mu_1 + \mu_2 > 1$ ($\mu_1 + \mu_2 < 1$), given the same magnitude of deviation $|p_t - \bar{p}_{\alpha,t}|$, the convergence speed is greater (smaller) when the price is above its value than when the price is below its value. Given that price declines faster than it rebounds, we opt for the identification of $\mu_1 + \mu_2 > 1$, which is in line with Day and Huang (1990).

Regime Specification

We assume that the width of the regime to be constant such that $\bar{p}_j - \bar{p}_{j-1} \equiv C$ and that $\bar{p}_0 = 0$. Under these assumptions, Eq.(4.1) can be simplified to:

$$\bar{p}_{\beta,t} = (\lfloor p_t/C \rfloor + \lceil p_t/C \rceil) \cdot C/2 \text{ if } p_t \in [\bar{p}_{j-1}, \bar{p}_j),\ j = 1, 2..., n.$$

In such a set up, the price deviation from $\bar{p}_{\beta,t}$ cannot exceed the limit $C/2$. Alternative specification of regime and its impact on our result will be discussed in the next section.

4.4.1 Basic Simulation

Once ϑ_t and $\bar{p}_{\beta,t}$ are defined, the price dynamics in Eq.(4.4) is determined. It is a one-dimensional deterministic mapping. We first consider a stable economy with zero-growth, which results in a constant fundamental value. Let the standard parameter set be the following:

$\bar{p}_{a,t} \equiv \bar{p}_\alpha = 50$, $\bar{p}_0 = 55 * 1.2^{-10}$, $d = -0.3$, $\mu_1 = 0$, $\mu_2 = 2.4$, $\gamma = 0.5$, $C = 10.02$, $\eta = 4.5$[4].

Given an initial price $p_0 = 81.6$, we simulate a price trajectory in Fig. 4-5. There are several points that worth our attention. The price series experience occasional booms and bursts. Moreover, the price series exhibits technical trading patterns such as head-and-shoulders, double top, double bottom, V top (see Fig. 4-6)[5].

4.4.2 Three Types of Financial Crises

Much literature has devoted to model the crisis (Day and Huang 1990; Rosser 2000; Chiarella et al. 2003; Gallegati et al. 2010). However only few are able to present a clear mechanism of the crisis, either due to the inclusion of a random process or the complexity of the model. The design of this simple deterministic model aim to fill this gap and shed light on why crisis differs from each other. We first demonstrate the crises of different patterns and then plot their corresponding phase diagram to illustrate the underlying dynamic mechanism.

[4] As the model is essentially chaotic, the simulate price path are very sensitive to the values of parameters as well as the initial price. The selection of these parameters is based on trial and error. Aside from this standard parameter set, we also use different parameter values to simulate different price series (as long as these series do not diverge) and find that the results discussed below are generally robust.

[5] See, for example, Bulkowski (2000) and Friesen, Weller et al. (2009) for the definition of head-and-shoulders pattern.

Sudden Crisis

Let the initial price be $p_0 = 71.6$ while keeping the other parameters the same as in the standard set. The sudden crisis exhibits itself between $t = 356$ (peak) to $t = 365$ (bottom), which is shown in the left panel of Fig. 4-7. Within 10 periods, the price drops from 80.6 to 14.7, losing more than 80% of its value. The right panel of Fig. 4-7 demonstrate the underlying mechanism of such dramatic decline. As observed, the price dynamics consecutively ends up in the declining zone of a series of regimes during the crisis. Moreover, at the beginning of the crisis (between $t = 356$ to $t = 361$), the price continuously falls into the sudden declining zone of a succession of regimes that drift lower in every period, which push the price down to precipitately. Such downward shift from one regime to another regime lower leads to a large price decline in every single period, which results in a dramatic accumulative decrement over a short time.

Smooth Crisis

Let the initial price be $p_0 = 71.6$ while keeping the other parameters the same as in the standard set. The smooth crisis is found between $t = 912$ to $t = 962$ (Fig. 4-8). During this period, the price declines from 95.5 to 5.6, shrinking by more than 90%. The overall price decline is dramatic, but the single-period drop is relatively moderate compared to that in the sudden crisis, and is frequently followed by price reversal. The phase diagram on the right panel of Fig. 4-8 shows that, the price falls frequently in the 'smooth declining zone' and 'disturbing declining zone', with occasional and inconsecutive visit to the 'sudden declining zone'. Such dynamic leads to ups and downs, with occasional steep decline. As draw-downs tend to dominate draw-ups over time, the price drifts lower after several periods' dynamics. When the price goes down to an extent that it breaks through the 'disturbing declining zone' and enters the 'sudden declining zone', the dynamic system shifts to a lower regime and triggers another round of ups and downs. As such process repeats itself, the price fluctuates with a persistent downward trend, which eventually forms smooth crisis over a relatively long time.

Disturbing Crisis

Let the initial price be $p_0 = 81.6$ while keeping the other parameters the same as in the standard set. The episode of disturbing crisis is captured from $t = 901$ to $t = 941$ (Fig. 4-9). The right panel of Fig. 4-9 plots the step-wise price dynamics for the corresponding periods. It shows that, initially, the price switches between the 'smooth declining zone' and 'disturbing declining zone' within the same regime quite disturbingly. Such dynamics leads to disturbing downward fluctuations, which characterize the period of financial distress that precede the disturbing crisis. As the price moves up and down with a downward trend, it eventually falls into the 'sudden declining zone' and shifts to a lower regime. As the subsequent price movement happens to fall into the 'sudden declining zone' continuously with occasional visit of 'smooth declining zone', the price declines successively. The dramatic price drop within a short-time characterizes the crash of the disturbing crisis. The duration of such type of crisis is shorter than the smooth crisis but longer than the sudden crisis. It is essentially a combination of smooth and sudden crisis.

4.5 Evaluating Model Fitness

The simulated price series visually match with phenomena observed from real price movements. Do they matches with the real data quantitatively and statistically? To find out, we examine their fitness to various stylized facts that are common across financial markets. Various behavioral asset pricing models have been proposed to match with stylized facts, in particular the characteristics of fat tails and volatility clustering in returns (see for example, Lux and Marchesi 1999; Lux and Marchesi 2000; Challet et al. 2001; He and Li 2007; Kirchler and Huber 2007; Shimokawa et al. 2007; He and Li 2008; Lux 2009). The degree of fitness and the range of match (how many stylized facts can the model duplicate) are ways of measuring the model performance. In addition to matching the stylized facts such as fat tails in return, unit root process in price and volatility clustering, which are frequently discussed by the HAM, we also test the model's ability to duplicate the property of long-range dependence, which can only be generated by a few models such as the HAM in Chapter 2. Without lost of generality, we study the statistical properties of price series simulated with $p_0 = 81.6$ and parameters specified in the standard set. It is worth pointing out that changing some of the parameters will increase the model's fitness in matching certain type of stylized fact. However, in order to show that the model is able to match a wide range of stylized fact simultaneously, we stick to the same price series simulated in this parameter set. The trial tests show that simulations using different parameter sets are capable of duplicating all these stylized facts simultaneously. This part of tests are not reported due to space concern but sample results are available upon request.

4.5.1 Fat Tails

The statistical analysis of financial time series suggests that return is not normally distributed but exhibits fat tails, with the extreme (positive and negative) returns happen more frequently than what is predicted by normal distribution. Such statistical properties are characterized by a kurtosis that is greater than 3. Table 4.2 presents the summary statistics of the simulated return series. The return series in the simulation has a kurtosis of 6.

Table 4.2: Summary statistics.

Series	Skewness	Kurtosis	Min	Max	Mean	S.d	N
r_t	.63	6.08	$-.56$	.77	0	.17	1000

4.5.2 Unit Root

The time series of price is universally found to be non-stationary (or has a unit root) while that of (compound) return, defined as the log difference of the price, is shown to be stationary. Table 4.3 reports the Augmented Dicky-Fuller (ADF) test for the null hypothesis of no unit root. These results are in line with the stylized fact of random price and stationary return. ADF tests of the simulated price cannot reject the hypothesis of no unit root in p_t and $\ln(p_t)$ even at a significance level of 10%. The test on simulated returns rejects the hypothesis of a unit root process in r_t at a statistical significance level of 1%.

Table 4.3: Unit root test.

	p_t	$\ln(p_t)$	r_t
ADF	-1.54	$-.56$	-13.73
Note: ADF critical values			
	1%	5%	10%
	-3.43	-2.86	-2.57

4.5.3 Volatility Clustering

Empirical evidence suggests that, even though there is little evidence of autocorrelation of raw return, volatility clustering suggests that the autocorrelation of absolute returns and squared returns decay slowly, with periods of quiescence and turbulence clustering together. The autocorrelation functions (ACFs) of r_t, $|r_t|$ and r_t^2 in Fig. 4-10 confirm such characteristics - trivial ACF in r_t and relatively persistent ACF in $|r_t|$ and r_t^2.

4.5.4 Long Range Dependence

Return r_t is said to exhibit long range dependence or possess long memory process if its ACF fades away as a power of the lag:

$$corr(r_t, r_{t+q}) \backsim L(q)/q^{1-2\tilde{d}},\ 0 < \tilde{d} < 1/2, \text{ as } q \to \infty$$

where $L(q)$ is any slowly varying function at infinity, i.e. verifies $\forall a > 0$, $L(aq)/L(q) \to 1$ as the number of lags $q \to \infty$. Long-range dependent time series exhibits an unusually high degree of persistence even at the lowest frequencies. Long-range dependence data process the feature of volatility clustering but not necessary vice versa.

We test the hypothesis of no long-range dependence in the simulated time series of price using Lo modified range over standard deviation or R/S statistic (also called re-scaled range), which is robust to short-range dependence (Lo, 1991)[6]. The Lo modified R/S statistic over n observations, denoted by Q_n, is defined by:

$$Q_n \equiv \frac{1}{\hat{\sigma}_n(q)}\left[\max_{1\leq k\leq n}\sum_{j=1}^{j=k}(r_j-\bar{r}_n)-\min_{1\leq k\leq n}\sum_{j=1}^{j=k}(r_j-\bar{r}_n)\right], \tag{4.7}$$

where $\hat{\sigma}_n(q)$ is the usual sample variance:

$$\hat{\sigma}_n(q)=\frac{1}{n}\sum_{j=1}^{j=n}(r_j-\bar{r}_n)+\frac{2}{n}\sum_{j=1}^{j=q}\left(1-\frac{j}{q+1}\right)\sum_{i=j+1}^{i=n}(r_i-\bar{r}_n)(r_{i-j}-\bar{r}_n).$$

The first and second terms in the bracket of Eq.(4.7) are the maximum of the partial sums of the first k deviations of r_j from the sample mean $\bar{r}_n$ and the minimum of the same sequence of partial sums, respectively. The tests reject the null hypothesis of no long-range dependence in simulated price at 1% significance level.

[6] Any incompatibility between the data and the predicted behavior of the R/S statistic under the null hypothesis of no long-range dependence may merely be a symptom of short-term memory, if conventional R/S is applied. That is, conventional R/S statistic is sensitive to short-range dependence. Lo modified R/S statistic accounts for the effects of short-range dependence and is considerably more accurate.

Table 4.4: Long-range dependence

Lo modified R/S	Lag	
0.52	1	
Critical values for H_0: r is not long-range dependent		
90%	95%	99%
$[0.86, 1.75]$	$[0.81, 1.86]$	$[0.72, 2.10]$

4.6 Policy Implications

The model is capable of accommodating various types of financial crises and stylized facts. It suggests the model has captured some, if not all, of the essential factors of financial markets. From the purely deterministic nonlinear model that excludes random process of any forms, we find that the crisis can arise endogenously through the internal dynamics, even without any external shocks. The external shocks, which serve as triggering or amplifying factors, can affect the duration and depth of the crisis and therefore the crisis patterns. This is because the crisis patterns are path-dependent, in the sense that the price movement are sensitive to previous price trajectory, external shocks can drive the crisis away from its original path and therefore change the crisis patterns.

The fact that the crisis can arise endogenously also implies that financial regulation and supervision can reshape crisis pattern but cannot stop the crisis from happening. One might argue that the some policies are able to influence investors' beliefs and therefore restrain the irrational exuberance that subsequently leads to financial crash. This model suggests that even though regulators can shape investors' beliefs for the time being, they cannot insulate the financial market from crises unless they can restrict the investors from shifting their beliefs to a series of downward psychological regimes. Otherwise, when the price is highly overvalued, the dynamics accompanied with regime switching will eventually lead to crisis of various. To further support the argument, we illustrate how the price movements react to 'circuit breaker' or 'collar', which stops the equity from trading for a period of time if the price drops (rises) more than certain percentage. As a robustness check, we also check whether the results are robust after accounting for the business cycle, which affects the real economic growth and therefore fundamentalists' expectation for the fundamental value.

4.6.1 Circuit Breaker

In China Shanghai Stock Exchange, the equity is banned from trading if its one-day return reach $\pm 10\%$ limit. In the New York Stock Exchange, for example, if the Dow Jones Industrial Average falls by 10%, the market might halt trading for one hour. There are other circuit breakers for 20% and 30% falls. The circuit breaker enables the market makers and traders to evaluate their trading strategies and reconsider their transactions by giving them more time. It is designed to reduce market volatility and massive panic sell-offs, which often results in less trading liquidity and market efficiency.

Take into account of the circuit breaker, chartists update their price trading regimes proportionally such that their maximum expectation for the price returns would stay within the limits[7]. Specifically, chartists assume that $\bar{p}_j \equiv \lambda \bar{p}_{j-1}$ for $j = 1, 2..., n$, where $\lambda > 1$. Under such set up, chartists' psychological return limit is consistent. Given

$$\mathbb{P} = \cup_{j=1}^{n} \mathbb{P}_j = [\bar{p}_0, \lambda \bar{p}_0] \cup [\lambda \bar{p}_0, \lambda^2 \bar{p}_0] \cup \cdots \cup [\lambda^{n-1} \bar{p}_0, \lambda^n \bar{p}_0],$$

$p_t \in [\bar{p}_{j-1}, \bar{p}_j)$ is identical to $p_t \in \left[\lambda^{j-1} \bar{p}_0, \lambda^j \bar{p}_0\right]$. Once p_t is observed, the regime that the price falls into can be identified by $j - 1 = \lfloor T \rfloor$ and $j = \lceil T \rceil$, where $T = \log \left(p_t / \bar{p}_0\right) / \log \lambda$. In this case, Eq.(4.1) can be simplified to:

$$\bar{p}_{\beta,t} = \bar{p}_0 \left(\lambda^{\lfloor T \rfloor} + \lambda^{\lceil T \rceil} \right) / 2.$$

[7] For a survey on how HAMs can be used to test the effectiveness of regulatory policies, see Westerhoff (2008).

Under proportional regimes, if the market consists of chartists only, the return is given by the following:

$$
\begin{aligned}
& \frac{\tau \cdot (p_t - \bar{p}_{\beta,t})}{p_t} \\
\geqslant\ & \frac{\tau \cdot \left(\lambda^{j-1}\bar{p}_0 - \bar{p}_0 \left(\lambda^{j-1} + \lambda^j\right)/2\right)}{\lambda^{j-1}\bar{p}_0} \\
=\ & \tau \left(1 - \lambda\right)/2
\end{aligned}
$$

The minimum return, as expected by chartists as if there are no other types of traders in the market, is $-\tau\left(\lambda - 1\right)/2$ (< 0). Similarly, the expected maximum return is $\tau\left(\lambda - 1\right)/2\lambda$ (> 0). Note that $\left|\tau\left(1-\lambda\right)/2\right| > \left|\tau\left(\lambda - 1\right)/2\lambda\right|$, the proportional regimes imply that chartists expect the return to be asymmetric in the sense that the scale of the most negative return is greater than that of the most positive return.

Let $\lambda = 1.2$ and the parameter set be the following: $\bar{p}_{a,t} \equiv \bar{p}_\alpha = 50$, $\bar{p}_0 = 55 * 1.2^{-10}$, $d = -0.3$, $\mu_1 = 0$, $\mu_2 = 2.4$, $\gamma = 0.5$, $\eta = 4.5$. Given an initial price $p_0 = 61.6$, the price trajectory and its phase diagram could be shown in Fig. 4-11. As expected, the price experiences occasional booms and busts of various forms, even though chartists' belief are reshuffled by the imposition of circuit breaker. Moreover, the price series continue to exhibit technical trading patterns such as head-and-shoulders, double top, double bottom, V top (see Fig. 4-12).

4.6.2 Business Cycles

So far we have assumed the economy to be steady, with zero growth, so that the long-term fundamental value of the risky asset is constant. If instead of being constant over time, the long term fundamental value $\bar{p}_{\alpha,t}$ follows a deterministic cycle:

$$\bar{p}_{\alpha,t} = a + b\sin(ct)$$

where a is the long term average fundamental value, b and c are parameters that guarantee the shape of the business cycle. The larger b is, the greater the swing. The greater c is, the longer the business cycle lasts.

We check the robustness of previous finding that financial crisis are endogenous by evaluating the impact of the business cycle. Under the proportional regime, let the parameters be the following: $\bar{p}_0 = 55$, $\lambda = 1.2$, $\gamma = 1$, $d = -0.3$, $\mu_1 = 0.001$, $\mu_2 = 2.4$, $\eta = 2.25$, $a = 50$, $b = 20$, $c = 0.01$ and $p_0 = 61.6$, we obtain the price trajectory (the left panel of Fig. 4-13). There are three points that worth our attention. First, the simulated price is much more volatile than the fundamental value, suggesting the existence of excessive volatility. Second, the simulated price movements trace the business cycle closely - price is relatively low when the real economy is in recession (e.g. $\bar{p}_{\alpha,t}$ is at the trough of the business cycle) and high when the real economy is booming. Third, the price appears to crash ahead of the turn-around of business cycle. Under the constant regime, let $C = 10.2$ and keep all the other parameters the same with above, we obtain a price trajectory that exhibits similar features (right panel of Fig. 4-13).

4.7 Conclusion

This chapter develops a simple heterogeneous agent model that consists of fundamentalists and chartists. Fundamentalists forecast long-term fundamental value based on the economic growth, while chartists extrapolate their regime-dependent beliefs from the historical prices as well as their trading experience. As a result of the regime-dependent belief, the deterministic model can accommodate various types of financial crises within the same market-maker framework. Falling into different parts of the same regime will result in different dynamics patterns. If the price falls into the 'sudden declining zone' consecutively (or with occasional visits to the 'smooth declining zone' only), we expect to observe the sudden crisis, characterized by dramatic decline over a short period of time. If the price falls into the 'smooth declining zone' and 'disturbing declining zone' in a succession of regimes, drifting lower and lower each period, the price trajectory will be relative smooth and exhibit a persistent declining trend. Such movements characterize the smooth crisis. Finally, if the price falls into the 'smooth declining zone' and 'disturbing declining zone' disturbingly at the beginning, and then switches to the track consists of a succession of 'sudden declining zone', we will observe the price trajectory to move up and down disturbingly at first and then fall precipitately. Such price movements is a notable manifestation of the disturbing crisis.

The fact that various types of crises can arise even without the presence of any external shocks suggests that, financial crises has their internal origins. It indicates that any financial regulation and supervision designed to curb the financial crisis will eventually be captured. Although policy makers could still restrain the irrational exuberance by imposing trading constraint, for example circuit breaker, they cannot stop investors from updating their beliefs according to their psychological trading regime. As long as the price dynamics can switch from one regime to another, investors' adaptive behavior will fail policy makers' attempt to rule out the financial crisis. Such results are robust even after accounting for the business cycle that affects fundamentalists' expectation of the long-term fundamental values.

The regime-dependent belief, which takes into account of gradual change as well as sudden movements in the price series, improves the model performance both qualitatively and statistically, compared to those that do not account for such property, such as that in Day and Huang (1990). Compared with the model in Chapter 2, this model is much simpler but not less powerful. Like the model in Chapter 2, the model in this chapter is capable of generating different types of financial crises and of reproducing a wide range of stylized facts that are common across financial markets simultaneously, including the property of long-range dependence that are considered to be the most difficult stylized fact to be duplicated. Last but not least, it is also able to replicate price trends and patterns that are frequently observed in technical analysis, such as bubbles, crashes and head-and-shoulder. Perhaps what makes this simple model outperforms that in Chapter 2 is its capability to explain why crises arise in different patterns in a more solid way.

There are several caveats in this model. First, the model is purely deterministic. It aims to improve the basic understanding of what drives prices but ignores the process of incorporating new-arrival news/information into the price movements. Second, despite our effort to simulate price series with various sets of parameters, it is impossible to cover every possible outcomes of the chaotic model, which is sensitive to the initial value and the parameter sets. Therefore, the analysis based on simulation results is only partially representative. Third, to emphasize the role of regime-dependent beliefs, we have fixed the market weight of fundamentalists and chartists, which may be not the case in real life. It would be interesting to proxy their market weights with real data, say from estimating a Markov switching heterogeneous model.

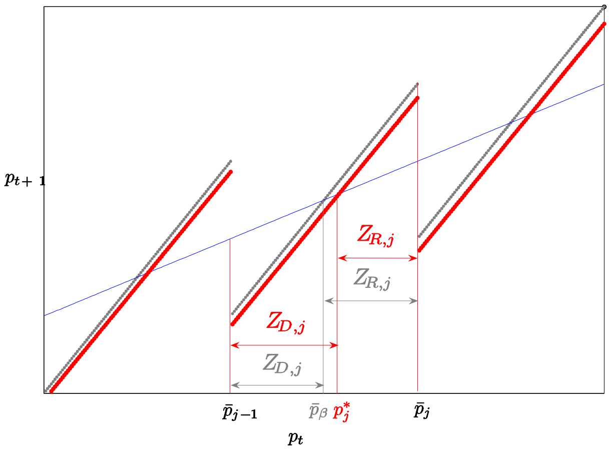


Figure 4-1: The impact of fundamentalists in bull market. The shaded thin line plots the phase diagram for the price dynamics when the market consists of chartists only. The solid bold line plots the phase diagram when there are both fundamentalists and chartists. In the bull market, the presence of fundamentalists shift the equilibrium to the right. It enlarges the declining zone $Z_{D,j}$ and compresses the rising zone $Z_{R,j}$.

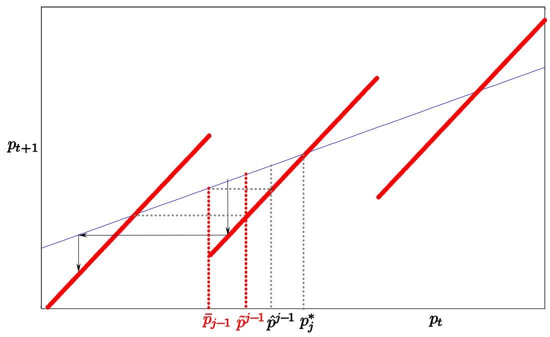


Figure 4-2: Sudden declining zone.

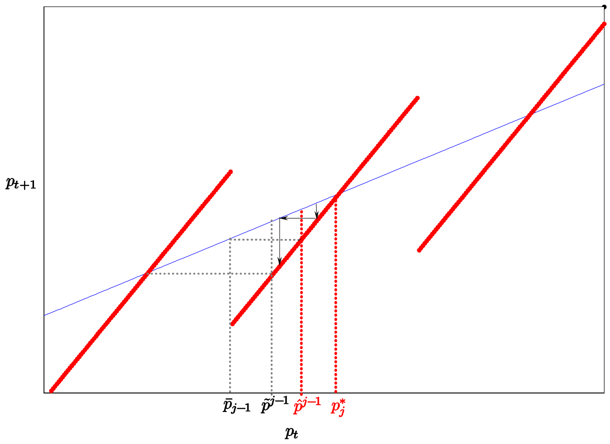


Figure 4-3: Smooth declining zone.

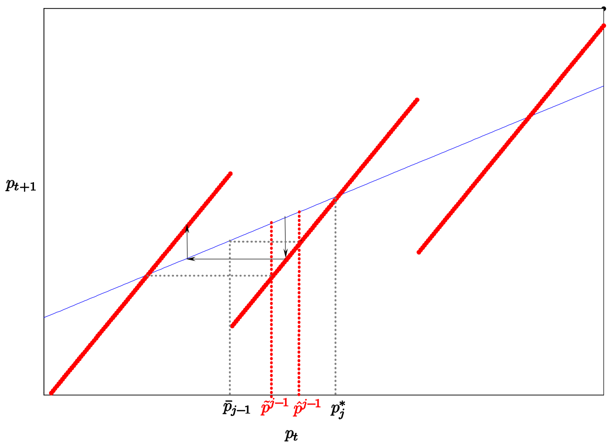


Figure 4-4: Disturbing declining zone.

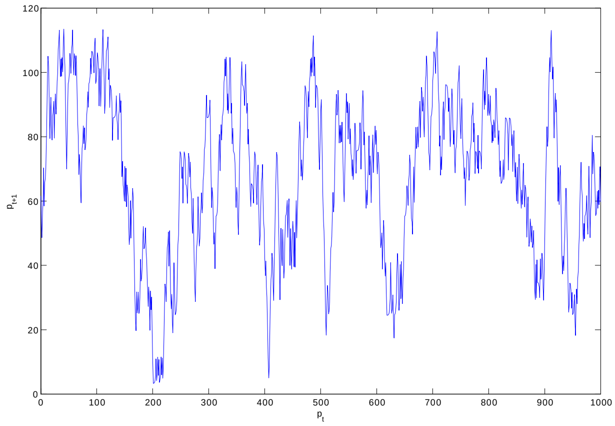


Figure 4-5: Price trajectory under constant regime switching.

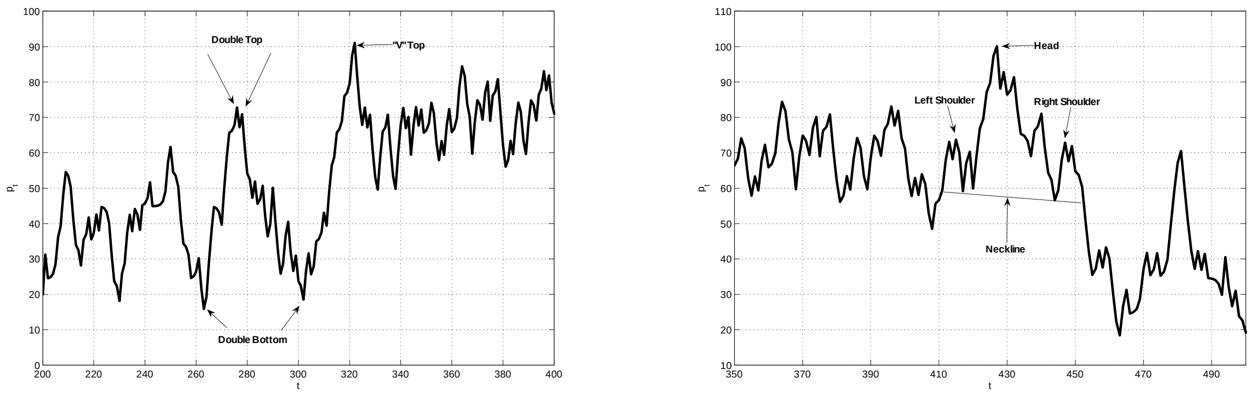


Figure 4-6: Technical price patterns.

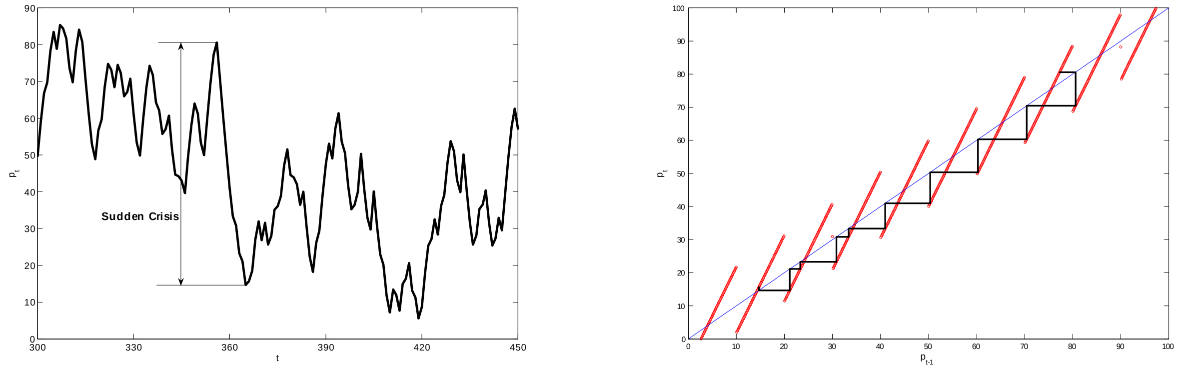


Figure 4-7: Sudden crisis.

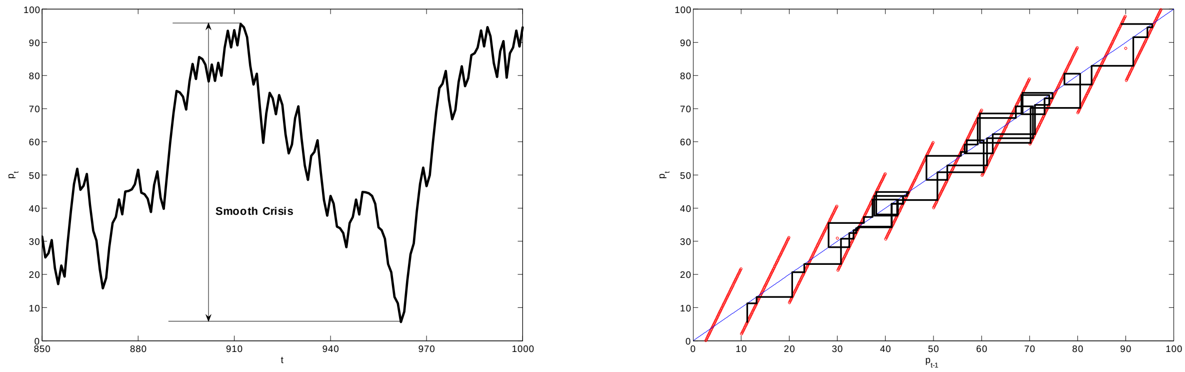


Figure 4-8: Smooth crisis.

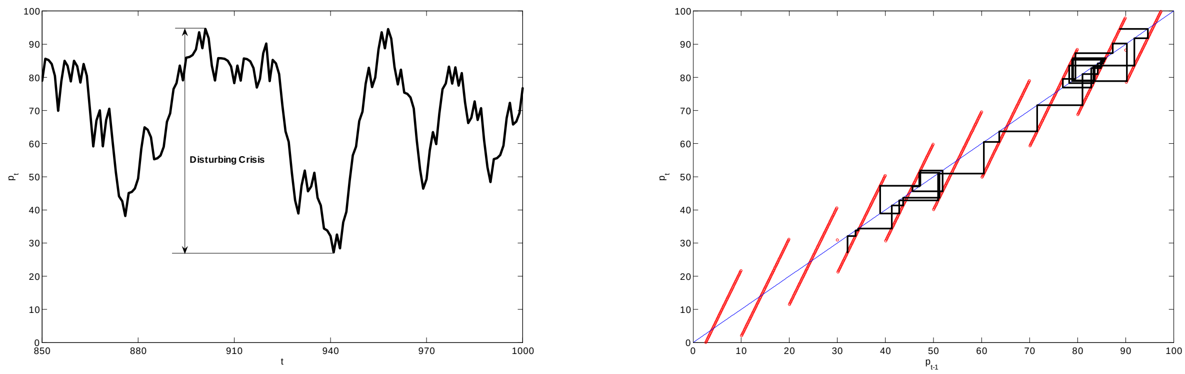


Figure 4-9: Disturbing crisis.

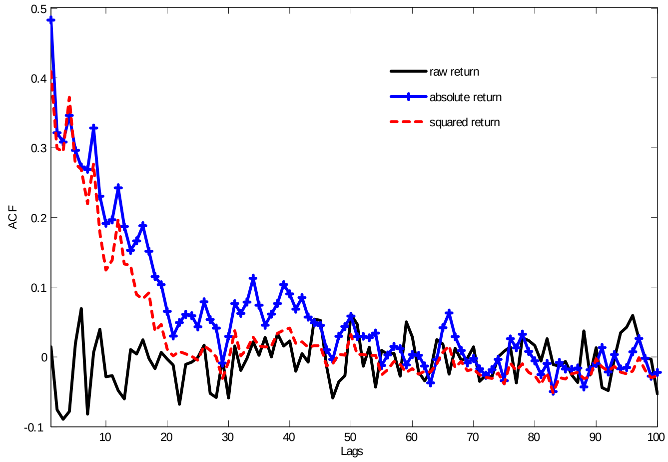


Figure 4-10: Volatility clustering. The solid, + marked, and dashed line plots the ACFs of raw, absolute and squared returns.

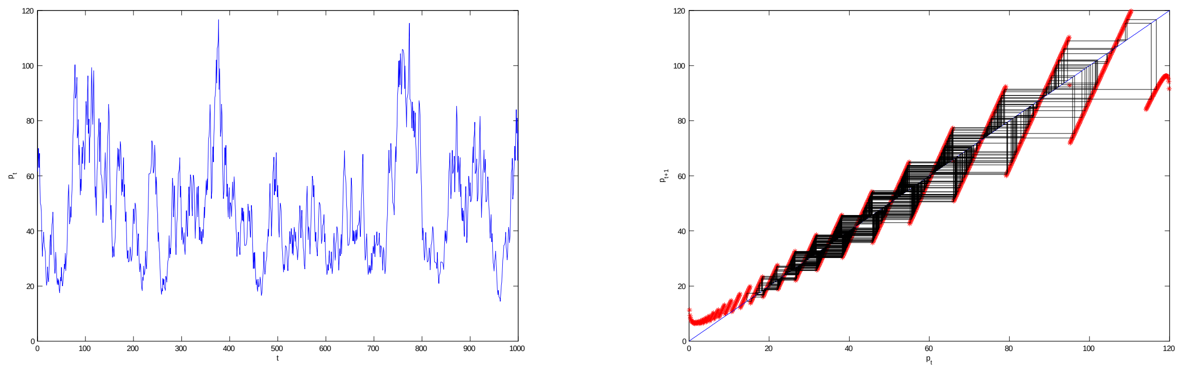


Figure 4-11: Price trajectory and phase diagram under proportional regimes.

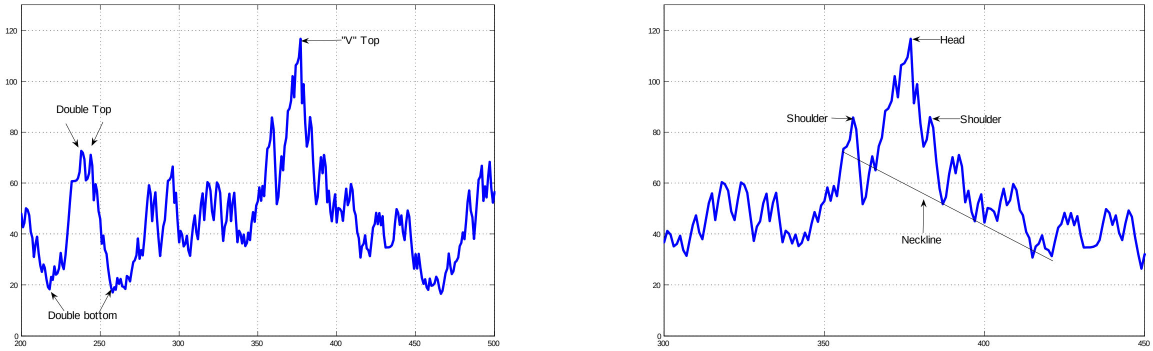


Figure 4-12: Technical price patterns under proportional regimes.

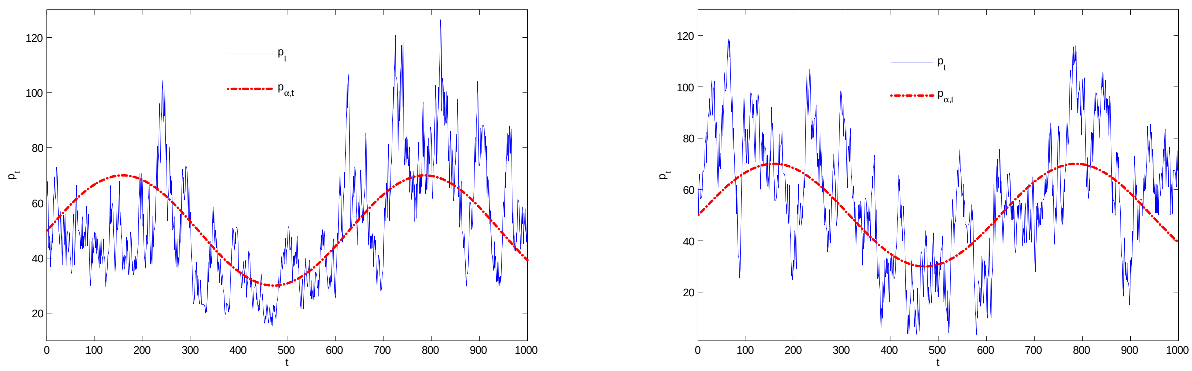


Figure 4-13: Price trajectory with business cycle. The left panel plots price trajectory for proportional regimes and the right panel graphs the price trajectory for constant regimes.

Chapter 5

Conclusion

Historically, the three types of financial crises, namely sudden crisis, smooth crisis and disturbing crisis, arise within the same market over time and across markets at the same time. Most often, financial crises happen without any obvious external shocks, at least not comparable to the depth of the crisis. It suggests that financial crises have some sort of internal origins embedded in the complex market dynamic system. The thesis supports such argument by showing that deterministic dynamic models, that exclude random process of any forms, are able to generate different types of financial crises simultaneously within the same model. Moreover, these models can duplicate a wide range of statistical and qualitative features of the real financial time series, which, in some aspects, outperform models that only account for well-defined stochastic process. It suggests that these models have done something right in capturing essential factors underlying financial crises.

The deterministic models developed in Chapter 2 and Chapter 4 are both able to generate financial crises of various types. Moreover, they convey the same dynamic mechanism for the gradual bubbles and sudden crashes. Bubbles form gradually while crashes happen suddenly because, when the price is above its value, fundamentalists always sell assets to bring the price towards its fundamental, which strengthen the downward price movement but attenuate the upward movements driven by the collective buying force of chartists. Both models can successfully reproduce a wide range of stylized facts, namely, unit root process, fat tails, volatility clustering, long-range dependence and leverage effect.

The two models, however, have different implications for the financial crises. The model in Chapter 2 ascribes the financial crisis to the interaction between fundamentalists and chartists as well as regime-dependent belief. When the price is over-valued so much so that the expected profit of fundamental trading strategy significantly dominates that of chartists, most investors would cluster to be fundamentalists. The strong selling force by the large number of fundamentalists pushes the price down dramatically, characterizing the sudden crash. In this case, the herding behavior towards fundamental strategy leads to the sudden crisis. In the disturbing crisis, however, the crisis starts with the battle between fundamentalists and chartists, which characterizes the period of financial distress. During this period, investors switch disturbingly between the fundamentalists, who sell to drag the highly over-valued price towards its long-term fundamental value, and chartists, who can either buy or sell depending on their psychological trading regime and the latest price. When the price is dragged down to such a low level that chartists dominate the market and find it optimal to sell in large volumes continuously, the price crashes dramatically over a short period of time. Therefore, both fundamentalists and chartists contribute to the disturbing crisis. Finally in the case of smooth crisis, chartists are believed to hold most responsibility. When the price is not so highly over-valued, most investors find it optimal to cluster towards chartists' strategy. It is their continuous exploration of the price trend with respect to their regime-dependent beliefs that lead to the moderate price decline over a long period of time.

The model in Chapter 4 relies solely on the regime-dependent belief to explain different types of financial crises. The price movements are path-dependent, within and across the psychological trading regimes. Falling into different part of a psychological trading regime will result in totally different price trajectory.

When the price falls into the 'sudden declining zone', it will fall with such a large magnitude that the subsequent price falls into another sudden declining zone of a lower regime. If such process continues, the price will drop dramatically within only several periods, characterizing the sudden crisis. When the price falls into a 'disturbing declining zone', it will drop with a moderate magnitude and fall into the rising zone of a lower regime, which drives the price up subsequently. If such process continues, the price will move up and down disturbingly, characterizing the period of financial distress. When the price finally breaks through the 'disturbing declining zone' and enters into a succession of sudden declining zone, the price will fall suddenly, which completes the second episode of the disturbing crisis. On the other hand, when the price falls into a 'smooth declining zone', it will decline moderately and stay within the same regime. If such process repeats itself frequently in a series of continuous regimes, the price will decline moderately and persistently over a long period of time, characterizing the smooth crisis.

5.1 Caveats and Extensions

In this thesis, we focus on the internal dynamics that is believed to be crucial for financial crises by building two purely deterministic HAMs. These models though useful in duplicating various stylized facts that are commonly observed in financial time series, they have their own limitations.

First, they may not well incorporate the newly-arrived news into the price movements. If the market is efficient in the sense that the price reflects all relevant information, such problem is trivial. However, given that the market is often perceived to be inefficient, it is also important to account for the latest information, especially those that take time to be incorporated into the price by including some random processes.

Second, due to the chaotic properties of these models, the price trajectories are very sensitive to the initial price, it is therefore impossible to simulate every possible outcomes. The large number of parameters further complicate this problem. The analysis that based on simulation results is only partially representative. However, it does serve as a good starting point to study the complexity of the financial market. Including a random process and conducting Monte-carlo simulation is unlikely to solve this problem, because the price trajectories are not just affected by the random process only. It may be useful to conduct bifurcation analysis to exam the general features of the price dynamics in response to the initial price as well as different parameter values.

Third, the assumptions that market weights of fundamentalists and chartists are either determined by the expected profit (Chapter 2) or fixed (Chapter 4) are not quite realistic. It would be interesting to calibrate time-varying market weights with real financial time series, say by estimating a Markov switching heterogeneous model.

Fourth, the definition of regime-dependent belief can be more flexible. Currently, the psychological trading regimes are mutual exclusive. When the psychology underlying chartists perception for the market shifts, new support and resistance price levels will be established. However, the regime enclosed by the new support and resistance price levels tend to overlap with the previous regime, to some extent. It may be useful to account for such overlaps when the regime shifts.

Finally, the psychological trading regimes are pre-specified. They either constant or proportional, the patterns of which are so well-defined that they may fail to capture the inconsistent behavior in the dynamic trading environment. Allowing these regimes to be defined by the technical methods, such as moving average would be useful.

5.2 Future Research

Three main strands of this model can be further explored in the near future.

First, we can polish the model and estimate it using various stock/bond/commodity market indices. This could allow one to test the out-of-sample forecasting ability of such type of model. If the out-of-sample forecasting power is promising, then we can proceed to check whether such a model is informative in predicting crisis and in identifying crisis patterns. Eventually, we can evaluate policies implemented during the recent global credit crisis, such as short-sale constraint and price limit. Ideally, as financial crises are endogenous, we will expect these policies to affect the timing and magnitude of the crisis, but not fundamentally, that is they will not stop the crisis from happening.

Second, we would like to calibrate the psychological trading regimes, which are found to be crucial in generating crises of different patterns. This could be done in three steps. we will simplify the model and estimate it under Markov-regime switching. Using the estimation results, we will specify the psychological trading regimes by utilizing the transition matrix as well as the mean value. After calibrating the model, we can proceed to explore short-term price movement and identify the regimes that are relatively fragile to market crash. Finally, we can test these results by comparing the findings to the real financial episodes.

Third, parameters embedded in the complex dynamic system can be estimated using method of simulated moments (MSM) (Westerhoff, 2011), a structural estimation technique that extends the generalized method of moments to estimate a model without direct theoretical moment functions[1].

[1] We thank Laura Guiying Wu for pointing out this aspect of future research.

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