Resource allocation in multiuser OFDM based cognitive radio systems

Dong, Huang

2010

Dong, H. (2010). Resource allocation in multiuser OFDM based cognitive radio systems.
Doctoral thesis, Nanyang Technological University, Singapore.

<https://hdl.handle.net/10356/46234>

<https://doi.org/10.32657/10356/46234>

Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems

by

DONG HUANG

A Thesis Submitted to the Nanyang Technological University

in Fulfillment of the

Requirements for the Degree of

DOCTOR OF PHILOSOPHY

School of Computer Engineering

Nanyang Technological University

Singapore

July, 2010

Abstract

Over the last decade, the number of wireless applications and subscribers has been growing at an explosive pace due to the distinct characteristics of wireless links. Higher spectrum utilization is needed in order to meet the growing demand for spectrums allocated to wireless devices with a high data rate and diverse quality of service (QoS) requirements. Cognitive radio (CR) is a novel concept for improving the efficiency of spectrum utilization by allowing secondary users (also referred to as CRUs) to access those frequency bands currently unutilized by the primary users (PUs). All CRUs compete to access the limited unlicensed spectrum in a distributed fashion. In order to achieve a good performance, a CRU is required to have the ability to adapt its transmit parameters (e.g. transmit power and channel) dynamically.

In this thesis, we address the resource allocation problems in enabling CR networks (CRNs) with the limited available resources to achieve a good performance, which include optimization, dynamical control and game theory.

Firstly, we consider the resource allocation problem in a multiuser orthogonal frequency division multiplexing (MU-OFDM) based CR system. OFDM is a good modulation candidate for a CR system due to its flexibility in allocating resources among CRUs. The scheme can achieve high spectrum efficiency and robust performance over heavily impaired wireless links due to the existence of parallel subchannels in the frequency domain.

The design of a fast and efficient method for dynamically allocating subchannels, transmit powers, and bits to CRUs in an MU-OFDM based CR system belongs to a combinatorial optimization problem. It has been shown that evolutionary based algorithms outperform traditional algorithms for many combinatorial optimization problems. We investigate the performance of memetic algorithms (MAs) in the given problem. Fitness landscape is a powerful technique for analyzing the behavior of combinatorial optimization problems. By analyzing some important properties of the fitness landscape of the given optimization problem, the guide showing how to choose appropriate genetic operators and local search methods for proposed MAs is used in order to obtain a better performance.

Secondly, since the assumption that the transmitter has perfect channel state information (CSI) is often unreasonable due to feedback delays, estimation errors, and quantization

errors, we study the resource allocation under partial CSI. We first propose a scheme to allocate the transmit power, subject to a prescribed bit error rate (BER) requirement for the case of only the partial CSI available at the transmitter, and then a novel scheme, which is robust to the changes in the correlation coefficient, is developed from the viewpoint of control theory.

Thirdly, we apply game theory to model the self-coexistence problems in IEEE 802.22 networks, which is a novel standard based on CR for wireless regional area networks (WRANs). The self-coexistence problem is an important issue, since all CRUs compete to access available TV channels independently. The best strategy to minimize the interference among the networks is developed and the Pareto efficiency at the Nash equilibrium is analyzed.

Finally, we summarize the main findings of our body of work.

Acknowledgments

First, I wish to express my sincere appreciation to my Ph.D. supervisor Prof. Chunyan Miao, for her excellent guidance and support during my research work as well as with my life during the academic years. Prof. Miao provided me with great freedom for my research, and was always willing to listen and share her experiences with me. Moreover, she spends a lot of time teaching and discussing so many research topics with me that they benefitted me immensely. Without her guidance and inspiration, this thesis could not have been successfully completed.

I would also like to thank my co-supervisor, Prof. Cyril Leung, for his invaluable support and diligent review of my work. His deep insight into theory and strong intuition on engineering always inspires and encourages me in my research. His fast and clear thinking always helps to cut a shorter path to a conclusion. Moreover, special thanks go to Prof. Zhiqi Shen, for offering his patience and encouragement when we discussed various research ideas.

I am deeply indebted to Prof. Yuan Miao, who gave me the opportunity to work with him in the School of Engineering and Science at Victoria University. During my visit to Victoria University, his constructive comments had a remarkable influence on this study. My warm thanks are due to Prof. Zhihong Man, Head of Robotics and Mechatronics, Faculty of Engineering and Industrial Sciences, Swinburne University of Technology, for his kind support and excellent advice throughout this work.

Thanks also go to the Emerging Research Lab, School of Computer Engineering, Nanyang Technological University, for the financial support and computing facilities. I would also thank my labmates and friends, Yundong Cai, Xiaogang Han, Boyang Li, Tao Qin, Hengjie Song, Jianshu Weng, Han Yu, and Guopeng Zhao etc. Their comments and advice were instrumental in the development of this thesis.

Last but not least, I would like to thank my parents for their unconditional love and support.

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Chapter 1

Introduction

In this chapter, we first provide a brief background on cognitive radio (CR) and then highlight the research motivation. The research objectives and corresponding scope and limitations follow. Then we discuss our proposed approaches. Finally, we present our contributions and organization of the thesis.

1.1 Background and Motivation

Today, wireless services and applications are becoming more affordable for most people due to the fast development of wireless communications. The number of wireless communication subscribers, applications, and services is increasing at an explosive pace. The main advantage of wireless communication is that the communication link is through the air instead of cable. It can help us to achieve highly reliable communications nearly anywhere. The objective of forthcoming wireless communication systems is to not only provide a voice calling service, but also to provide a wide variety of wireless multimedia services with diverse quality of service requirements wherever and whenever needed.

This requires more spectrums, and a higher efficiency of spectrum utilization to support these high data rate applications and services. On the other hand, the current spectrum allocation policy is fixed and the limited licensed frequency spectrums cannot support a tremendous spectrum demands unless advanced technologies are developed to achieve better efficiency of spectrum utilization.

An obvious example is that the current licensed spectrums for cellular systems are becoming scarce due to the explosive growth in cellular phone subscribers. Especially when it comes to high density population places such as airports and shopping malls, the situation of spectrum scarcity is becoming more and more obvious.

Though many different technologies for more efficient spectrum utilization have been proposed recently, they cannot change the situation much as the licensed spectrums are too small when compared to the increasing number of subscribers. On the other hand, the Federal Communication Commission (FCC) has reported that most of the other licensed spectrums are currently under-utilized [5].

A snapshot of spectrum utilization is illustrated in Fig. 1.1 [6], where the signal strength distribution over a large portion of spectrums is shown. Only a small amount of the

Figure 1.1: Spectrum utilization

spectrum is being utilized by licensed users, while most of the spectrum remains unutilized.

That is, most users compete to use a small spectrum band, but other bands are under utilized and thus wasted. Thus, spectrum scarcity has been the bottleneck for further

developing wireless communication systems. In order to improve the spectrum efficiency, the old spectrum allocation policy should be revised, and a new spectrum allocation policy that allows unlicensed users to access the frequency bands currently unutilized by the licensed users is needed.

Cognitive radio (CR), which was proposed by Joseph Mitola III in 1999, is a novel concept for improving the utilization of the current spectrum by permitting secondary (unlicensed) users to access those frequency bands which are not currently being used by the primary (licensed) users. That is, a CR system is an intelligent system that can improve the spectrums by changing its operation parameters according to the environment it senses.

A simple basic cognitive cycle of a cognitive radio system is shown in Fig. 1.2.

Figure 1.2: Basic cognitive cycle [1]

Generally, CR includes three fundamental cognitive tasks [1]:

- (1) Radio-scene analysis.

- (2) Channel-state estimation and predictive modeling.
- (3) Transmit-power control and dynamic spectrum management.

For a transmitter, its cognitive tasks are transmit power control and dynamic spectrum management. For a receiver, its cognitive tasks include radio scene analysis, which includes an estimation of the interference temperature of the radio environment, the detection of spectrum holes, and channel identification, which includes the estimation of channel state information (CSI) and the prediction of channel capacity for use by the transmitter.

Thus, by allowing a cognitive radio to access those frequency bands that are not currently used by the primary users (PUs), a cognitive radio system will mitigate the embarrassment of spectrum scarcity significantly. Based on these cognitive tasks, many authors proposed different methods for a CR system [6], [7], [8], [9], [10], [11].

However, cognitive radio is just a concept. There are many important issues for implementing a CR system [1], [12], [13], [14]. For example, when a base station (BS) wants to use unlicensed frequency bands, it needs to sense the PUs' signal before using so as to not interfere with the PUs. In this case, traditional signal detection methods may not be efficient enough to satisfy the strict requirements. Moreover, mutual interference between PUs and cognitive radio users (CRUs) would also be an important issue for the CR system to overcome when allocating the licensed spectrum for CRUs [11].

In order to make the mutual interference between PUs and CRUs satisfy a given requirement, a CR system needs to have the ability to change its transmit parameters quickly enough according to the environment it senses. Orthogonal frequency division multiplexing (OFDM) is a good candidate for a CR system due to its flexibility in allocating resources among CRUs. This important characteristic makes it seem like a good choice for CR systems.

This motivates us to study efficient algorithms to help CRUs adapt their transmit parameters in an OFDM based cognitive radio system. In addition, the transmission quality will be degraded when only partial channel state information (CSI) is available at the

transmitter. For the IEEE 802.22 networks, a good channel access strategy can help a CRU to maintain good performance with the least amount of interference.

Solutions to these research challenges will enable CRUs to achieve a good performance in different scenarios.

1.2 Research Objectives

In this section, we first present some research challenges in cognitive radio networks (CRNs) and then our research objectives are formulated.

1.2.1 Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems

OFDM is a digital multi-carrier modulation scheme, which spreads the data to be transmitted over a large number of orthogonal subcarriers. That is, in OFDM systems the available bandwidth B Hz is split into N subcarriers. Instead of transmitting digital symbols sequentially through one channel (of bandwidth B Hz), the bit stream is split into N parallel streams. A simple OFDM transmission structure is shown in Fig. 1.3.

Figure 1.3: A Simple OFDM Transmission Structure

More concretely, the generation of the OFDM symbol works as follows. Firstly, by splitting the transmitted bit stream into parallel data streams, then groups of bits of each stream are mapped to the frequency-domain representations of the corresponding

digital symbols of some modulation alphabet. Note that the modulation of each subcarrier might be different. These N symbol representations are passed to an inverse Fast Fourier transformation (IFFT), which generates a time sequence of N values. This time sequence represents one OFDM symbol. It relates to the duration of an OFDM symbol. The sequence is then transmitted at a certain center frequency f_c with a certain transmit power P_{tx} .

At the receiver the signal is passed to a Fast Fourier transformation (FFT). After applying this transformation, the frequency domain representations of the digital symbols for each sub-carrier are obtained. They are converted individually into bits, which ultimately yields the bit groups of each stream.

In contrast to conventional frequency division multiplexing, the spectral overlapping among sub-carriers is allowed in OFDM due to orthogonality. FFT will ensure the sub-carrier separation at the receiver, providing better spectral efficiency. Therefore, one of advantages in an OFDM system is that it increases the data rate by transmitting a series of bits in parallel.

For example, if an equalized single carrier modulation (SCM) system has a symbol length of T_s , the symbol length of an equivalent OFDM system is N times longer due to the fact that during each symbol duration, N symbols are transmitted in parallel.

Moreover, the increase of the symbol duration is a significant advantage of the OFDM systems when facing frequency-selective channels. In a wireless environment, there is a different delay when a digital symbol arrives at the receiver due to the effect of multi-path signal propagation. If the delay is rather large compared to the symbol duration, sequentially transmitted symbols might interfere at the receiver. This effect is called Intersymbol Interference (ISI).

Since the symbol duration can be increased by transmitting signals in parallel, ISI can be obviously mitigated.

Future wireless communication systems are generally expected to have the ability to convey much higher data rates and thus require larger bandwidth. On the other hand, ISI is still an important issue for broadband communications. In this case, OFDM systems

can solve this problem by dividing the bandwidth into a number of orthogonal sub-carriers while increasing the symbol duration. Thus, OFDM systems not only increase the symbol duration but also maintain a high overall symbol rate.

Moreover, OFDM systems are excellent transmission systems for frequency selective channels. In order to mitigate the effect of ISI completely, a cyclic extension of the OFDM symbol time sequence is added. This extension is usually larger than the delay spread and is called the guard period. It is discarded at the receiver prior to applying the FFT and removes any interference with previous OFDM symbols.

As its distinct advantages, OFDM is a promising candidate for achieving high data rate transmissions in a mobile environment and has been accepted as a mature technology for wireless broadband communication links.

We consider the resource allocation in a downlink transmission of a multiuser OFDM based cognitive radio system in Chapter 3. The resource allocation problem belongs to combinatorial optimization problems and is computationally complex. We solve the resource allocation problem by dividing it into two steps:

- (1) Determine subcarrier allocation to CRUs;
- (2) Determine bit and power allocation to CRUs.

After subcarrier allocation, the optimized problem still belongs to a combinatorial optimization problem. The steps to find the optimal solutions grow exponentially with respect to the number of subcarriers. It has been shown that evolutionary algorithms (EAs) achieve a good performance in combinatorial optimization problems. We investigate the performance of memetic algorithms (MAs) for the bit allocation for CRUs.

The allocation scheme is studied under different scenarios and with different parameter settings. Experiment results compared with existing algorithms are presented.

1.2.2 Fitness Landscape Analysis

MAs have been confirmed to outperform other traditional algorithms with many combinatorial optimization problems. Due to the complicate structure of MAs, there is little

progress obtained in investigating the behavior of them. Most of the work published on MAs only shows that they can achieve good performances for the given problems, but does not provide further analysis of how and why based on mathematical techniques.

Fitness landscape is a useful technique for understanding the behavior of combinatorial optimization algorithms and predicting their performance. For an optimization problem, we view the set of solutions as a landscape. The highness of a point in the search space represents the fitness of the solution associated with the point. A heuristic algorithm can be thought of as searching through it in order to find the highest peak of the landscape. It has been discovered that a number of properties of fitness landscapes have a great impact on the performance of heuristic optimization algorithms.

By measuring these properties, we can find appropriate local search methods and genetic operators.

In Chapter 4, we analyze the behavior of MAs for the resource allocation problem in MU-OFDM based CR systems. Based on fitness landscape analysis, appropriate local search and evolutionary operators are derived for the proposed MA.

1.2.3 Transmission Performance Analysis under Partial Channel State Information

For practical wireless transmissions, a transmitter cannot always receive the channel state information (CSI) perfectly due to feedback delays, estimation errors, and quantization errors. The quality of service (QoS) requirement in wireless systems is highly dependent on the accuracy of the CSI obtained by the transmitter. Most authors assume that the transmitter receives the CSI perfectly when considering the resource allocation in wireless communications [15], [16].

The assumption is reasonable for wireline systems, but impractical for wireless systems. In wireless communication systems, the transmitter only obtains partial CSI due to channel estimation errors and feedback delays. It will cause the system performance degradation because the QoS requirements cannot be guaranteed. For example, the bit error rate (BER) will increase when the real channel gain is smaller than the received channel gain.

In Chapter 5, we investigate the resource allocation for wireless transmissions with the Doppler effect. We then apply the mean feedback to model the transmission. Based on the given system model, we propose two transmission schemes. Firstly, we study the transmission scheme under an average BER requirement. Based on the partial CSI obtained at the transmitter, the average BER should satisfy the given BER target during transmission.

As the function of average BER is too complex, we apply a Nakagami- m distribution to approximate the original function. A simple function, which is close to the original function, is then derived.

Secondly, we analyze the system dynamics under different parameter values. Since the derived difference equations are nonlinear, we linearize them and analyze the stability of the equilibrium point. We find that the system is locally asymptotically stable in the case of the appropriate parameter values. We also find that the equilibrium point changes according to the introduced parameters. Simulation results show that the proposed allocation scheme not only suppresses the effect of the correlation coefficient, but also improves the efficiency of the resource allocation by selecting appropriate parameters.

1.2.4 Self-Coexistence Problems in IEEE 802.22 Networks

IEEE 802.22 is a novel standard based on CR for wireless regional area networks (WRANs). The objective of WRANs is to provide broadband access in rural and remote areas. WRANs operate in the TV bands between 54 MHz and 862 MHz. Multiple overlapped WRAN service providers compete to use these unlicensed channels. Since the advantage of better propagation characteristics at TV channels, WRANs have a much larger coverage range than existing networks.

Normally, the coverage range can go up to 33 Km at 4 Watts EIRP [17]. The networks operate in a point to multiple point basis (P-MP), where a base station (BS) services a number of consumer premise equipments (CPEs). Before allocating TV channels to CPEs, a BS must sense that the channels are currently not being utilized by the licensed incumbents (i.e. TV receivers and microphones). When the WRANs sense that the TV

channels they are using are being accessed by the licensed incumbents, they must vacate the channel within the channel move time, (2 seconds), and switch to a different unutilized channel [18].

Since the spectrum management among competing WRANs is freely distributed and the co-ordination amongst WRANs of different service providers does not exist, several overlapping networks may switch to the same channel when sensing an incumbent's existence. When that happens, interference among these networks occurs. The networks suffering interference have a binary action: stay in the same channel or switch to a different channel, causing some quality of service (QoS) requirements among the unsatisfied networks. Finding a way to minimize the interference among IEEE 802.22 networks and ensure the given QoS requirements are met is an issue for the IEEE 802.22 standard. Each network will usually seek their own benefits or utilities independently.

In Chapter 6, we consider the system model with multiple overlapping WRANs operated by multiple wireless service providers competing to seek available channels for their individual CPEs. When interference occurs, each network does not have any information on the what the next action taken by the other networks will be: will they stay where they are or will they switch to another channel. Each network makes decisions independently. Therefore, one of the biggest issues is the lack of co-operation between the networks. We analyze equilibrium points under different scenarios.

1.3 Scope and Limitations

In this thesis, we focus on the following three research challenges:

- Efficient resource allocation in multiuser OFDM based cognitive radio systems
- Power allocation under partial channel state information
- Optimal channel access strategy in IEEE 802.22 networks

The solutions to these three research topics will enable CRUs to achieve a good performance with the least cost. However, there are still some limitations in our research work. For the resource allocation in multiuser OFDM based cognitive radio systems, we only consider the downlink transmission model, where the base station (BS) determines the transmit power and subchannels to the CRUs.

The resource allocation is conducted by a central computing system, while there are many resource allocation problems in CRNs which are required to be conducted in a distributed fashion. Each CRU selects their transmit parameters independently. Moreover, for the channel access strategy in IEEE 802.22 networks, we only consider the cost caused by competitive CRUs. The interference cost caused by primary users has not been considered. When that factor is also included, the model will be more complex.

1.4 Approach and Methodology

This thesis addresses several research issues in achieving efficient resource allocation in cognitive radio networks, where the methods include memetic algorithms, adaptive control methods, game theory, and optimization.

For the resource allocation in multiuser OFDM based cognitive radio systems, we proposed a simple algorithm based on a greedy approach which performs well in the simulation for the subcarrier allocation and MAs for the bit allocation. The objective is to maximize the total data rate subject to given power and interference constraints.

Suboptimal algorithms with low complexity are applicable. To further improve the performance of the proposed MAs, we apply fitness landscape to investigate some important statistical properties of the given problem. This tool helps us to choose appropriate local search methods and genetic operators for the proposed MAs.

For the problem of power allocation with partial channel state information, we propose two methods to allocate the transmit power. We wish to design a transmission scheme supporting QoS. We first derive a new relationship between power and modulation while

satisfying given QoS requirements based on the statistical method. With further investigation, we find that the new allocation scheme is sensitive to changes of the correlation coefficient. We further develop a novel allocation scheme based on adaptive control methods.

The novel method is robust to the change of the correlation coefficient. With regards the self-coexistence problem in IEEE 802.22 networks, we model the problem as a non-cooperative game under the assumption that all players are rational enough to compete to access the available TV bands.

1.5 Contributions

In this thesis, we first proposed memetic algorithms to determine the power allocation and a channel allocation algorithm to assign the subcarriers to CRUs in multiuser OFDM based cognitive radio systems. By allocating the transmit power and subchannel adaptively, we can achieve maximum data rate. Then we investigated the performance degradation due to only partial channel state information being available at the transmitter, and proposed a novel power allocation. Finally, we proposed a channel access strategy for the IEEE 802.22 networks. The strategy is based on game theory and enables cognitive radio users (CRUs) to access an available unlicensed channel with the least cost.

The key contributions of this thesis for such CRNs model settings are summarized in this section.

- We proposed memetic algorithms to determine the transmit power and a channel allocation algorithm, based on a greedy approach, to assign subcarriers to CRUs. The proposed algorithms achieve better solutions than existing algorithms. And we further improved the performance of the proposed memetic algorithms based on fitness landscape analysis.
- We proposed a novel power allocation scheme, based on adaptive control methods, to maintain the transmission performance when only partial CSI is available at the transmitter.

- We proposed a channel access strategy for the CRUs in IEEE 802.22 networks. We modeled the self-coexistence problem in IEEE 802.22 networks as a game. The strategy is derived from the Nash equilibrium point. It can help CRUs to achieve given QoS requirements while minimizing the interference costs.

1.6 Organization

The thesis is organized in seven chapters. We present some related work in Chapter 2. In Chapter 3, we consider the downlink transmission in a multiuser (MU) OFDM based CR system. We propose a novel algorithm, which is based on a greedy approach, to determine the allocation of subcarriers to CRUs and MAs and to determine the power allocation to CRUs. In order to improve the performance of MAs for the bit and power allocation problem, we use fitness landscape to analyze some important properties of the given problem in Chapter 4.

A disadvantage of MAs is premature convergence. Finding a way to prevent premature convergence effectively is still an open issue. In order to achieve better performance, a hybrid local search method based MA is proposed. By analyzing some important properties of the fitness landscape of the given optimization problem, an idea of how to choose appropriate genetic operators and local search methods for proposed MAs is achieved.

In Chapter 5, we propose a new scheme for determining the transmit power under the prescribed BER requirement when only partial CSI is available at the transmitter. When a transmitter cannot received the CSI perfectly, the prescribed QoS requirements such as bit error rate (BER) cannot be guaranteed, and the system performance will then degenerate. We investigate the impact of imperfect CSI on the BER.

In Chapter 6, we consider self-coexistence problem in IEEE 802.22 networks. We use game theory to discover the best strategies for each service provider. The Pareto efficiency at the Nash equilibrium is also analyzed. Simulation results show the proposed schemes achieve a better performance than the existing schemes. Finally, Chapter 7 concludes the thesis and some future work is discussed.

Chapter 2

Related Work

In this chapter, we present some related work, which focuses on three research topics.

2.1 Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems

In this section, we consider the problem of allocating resources on the downlink of an MU-OFDM based CR system in which a base station (BS) serves one PU and K CRUs. The basic system model is illustrated in Fig. 2.1. Where solid lines denote OFDM subcarriers available for CRUs and dotted lines represent the subbands occupied by PUs and guard bands. For the bits, power and subcarrier allocation in multiuser OFDM based cognitive radio systems can be formulated as a constrained optimization problem.

It can be seen that when a channel is allocated to a CRU exclusively, the obtained signal-to-noise ratio (SNR) is at its best [15].

Therefore, the optimal subcarrier allocation solutions should satisfy the necessary condition: each channel is allocated to a maximum of one user. Then the constrained optimization problem becomes a combinatorial optimization problem. Our objective is to design an appropriate algorithm to search the global optimal solution efficiently.

Ideally, when the variables are independent, a simple greedy algorithm can find the global optimal solution with low complexity. However, the variables in this type of optimization problem are interdependent, and greedy based algorithms can get local optimal

Figure 2.1: A model of frequency occupation distribution.

solutions. In this case, the steps to discover the global optimal solution grow exponentially with respect to the number of frequency bands.

More importantly, the time for the resource allocation schedule is very limited. Therefore, only suboptimal algorithms with a low complexity are acceptable. Early works solve the problem by dividing it into two steps [19], [20], [21], [22], [23], [24]. The first step is to determine the allocation of channels to users and the second step is to determine the allocation of bits to users. Most of these algorithms are based on the greedy approach.

These two optimization problems are classes as combinatorial optimization problems. The variables in these two optimization problems are interdependent, and greedy based algorithms cannot solve the problems effectively. They often find a local optimal solution far away from the global solution. In order to achieve better solutions for the bit allocation problem, more efficient algorithms are needed.

In addition, most of these algorithms are designed for MU-OFDM systems in which there are no PUs. When determining the allocation of channels to users, only channel gain needs to be considered. In an MU-OFDM based CR system, mutual interference between

PUs and CRUs also needs to be considered.

The problem of the optimal allocation of subcarriers, bits, and transmit powers among users in an MU-OFDM CR system is more complex. It is commonly assumed that perfect CSI is available at the transmitter [24], [25].

Evolutionary algoirhtms (EAs) have been shown to solve a lot of NP-hard problems effectively. In particular, memetic algorithms (MAs) based on the genetic operators are good algorithms to use for combinatorial optimization problems [26]. Many versions of MAs have evolved because they have to face different problems. Generally speaking, a genetic algorithm (GA) combined with local search (LS) methods is called a memetic algorithm (MA). MAs have been shown to require fewer computations and produce better solutions than standard GAs for many optimization problems [26]. MAs have been successfully applied to problems such as the travelling salesman problem (TSP) and the quadratic assignment problem (QAP). They outperform traditional algorithms for many combinatorial optimization problems [27], [26], [28], [2], [29], [30].

For a given optimization problem, LS methods have an important impact on the results [31]. An appropriate LS method will generate a much better result. However, there is still little work done to advise people on the correct choice of a good local search method for a given problem.

In order to make MAs more effective, more powerful techniques for analyzing the behavior of MAs are required. Previous works on ways to improve the performance of MAs have been published in [32], [33], [34], [35]. The notion of fitness landscaping was first proposed in [36] for analyzing the gene interaction in biological evolution. It was also an important technique for analyzing the behavior of combinatorial optimization problems and predicting their behaviors [37], [38], [39].

Furthermore, it has been extended to analyze evolutionary algorithms. From [36], [38], each genotype has a “fitness” and the distribution of fitness values over the space of genotypes constitutes a fitness landscape. In our problem, we consider the set of solutions as a search space, the height of a point denotes the fitness of the solution associated with

the point. Therefore, a heuristic algorithm can be considered as searching through the search space to find the highest peak of the landscape.

Our objective is to analyze the structure of the fitness landscape of the given problem and then derive appropriate LS methods and genetic operators, so that the proposed MA can find a local optimal solution which is close to the global optimal solution of the problem more efficiently.

2.2 Adaptive Transmission with Partial Channel State Information

In performance analyses of wireless communication systems, it is often assumed that perfect channel state information (CSI) is available at the transmitter. This assumption is often not valid due to channel estimation errors and/or feedback delays. The transmission schedule model is illustrated in Fig. 2.2. To ensure that the system can satisfy the target quality

Figure 2.2: Transmission Model

of service (QoS) requirements, a careful analysis which takes into account imperfect CSI is required [40]. In wireless communication systems, the transmitter only obtains partial CSI due to channel estimation errors and feedback delays. It will cause performance degradation because the QoS requirements cannot be guaranteed. For example, the bit error rate (BER) will increase when the real channel modulus is smaller than that of the received

one. This motivates us to study the effect of imperfect CSI on the performance of the resource allocation scheme in wireless communication systems.

Generally speaking, when analyzing the partial CSI, there are two kinds of feedback which are considered [41]: mean feedback and covariance feedback. In the case of mean feedback, the channel distribution is modeled at the transmitter as $h \sim \mathcal{CN}(\mu, \alpha)$, where the mean μ denotes an estimate of channel based on the feedback, and α represents the covariance of the estimation error. In the case of covariance feedback, the channel distribution is modeled as $h \sim \mathcal{CN}(0, \Sigma)$, which denotes that the channel h varies too rapidly so that the transmitter cannot track its mean.

Recently, many authors have investigated different transmission models under partial CSI [41], [41], [42], [43], [44], [45], [3], [4]. From the viewpoint of information theory, the problems of maximizing the information transfer rate with imperfect channel feedback for both cases are presented in [41]. Based on the channel mean feedback, the strategy for optimal transmitter design for multiple antenna systems is presented in [42]. An adaptive MIMO-OFDM based on channel mean feedback is studied by [43]. Optimal transmission strategies for maximizing mutual information in MIMO systems with covariance feedback is investigated in [44].

The effects of imperfect CSI on the BER have been studied in [3] and [4]. New resource allocation schemes based on average BER requirements are derived. By further studying their performance, we found that the algorithms proposed in [3], [4] are sensitive to the change of correlation coefficient with high feedback channel gain, while more conservative in the case of low feedback channel gain.

When the correlation coefficient decreases, the resource allocation schemes are conservative and thus deteriorate spectral efficiency. This motivates us to further study a means to suppress the impact of the correlation coefficient.

2.3 Game Theory for IEEE 802.22 Networks

IEEE 802.22 networks is the first standard based on cognitive radio. The purpose of IEEE networks is to provide broadband access for rural areas and so it's also known as wireless

regional area networks (WRANs). IEEE 802.22 networks do not have licensed spectrums and are required to compete to access the unutilized spectrums in the TV bands between 54MHz and 862 MHz [18].

Due to the good propagation characteristics of TV bands, the coverage radius of a WRAN, which is around 33 Km and can go up to 100 Km, is larger than that for other existing wireless PANs/LANs/MANs. These BSs belong to competitive service providers. On the other hand, before accessing a TV channel, a BS must sense that the channel is currently not utilized by licensed incumbents (i.e., TV receivers and wireless microphones). Since all overlapping WRANs share the unutilized TV bands, it's easy to cause interference among overlapping WRANs due to the spectrum management in a distributed fashion. Overlapping WRANs compete to access the limited spectrums. The problem of managing the frequency hopping among them effectively is not a trivial one.

In order to decrease the interference among overlapping WRANs and ensure prescribed QoS requirements can be satisfied (i.e., self-coexistence problem), they have to manage their frequency hopping behavior in a coordinated fashion. On the other hand, game theory deals primarily with distributed optimization. It can be seen that the self-coexistence problem can be formulated as an uncooperative game [46]. In particular, game theory has been applied to solve different kinds of efficient resource allocation problems in wireless communications [47], [48], [49], [50], [51]. The uplink power control of a multiuser MIMO system is formulated as an uncooperative game in [47]. [48] formulates the radio resource management (RRM) in a heterogenous wireless access environment from the viewpoint of game theory. In [51], the equilibrium point between the base station and a connection for IEEE 802.16 broadband wireless networks is studied.

Recently, there are a few publications regarding the issues of the self-coexistence problem. Dynamical frequency hopping community(DFHC) was proposed in [52]. DFHC can improve the efficiency of spectrum utilization and achieve higher system throughput. However, DFHC operation requires a community leader, which is a strong constraint since the

spectrum sharing protocol among all overlapping WRANs is distributed. In [53], the authors proposed several variants of the DFH schemes that aim at reducing the coexistence problem effect.

Since both non-exclusive and exclusive spectrum sharing schemes have their individual advantages and disadvantages, an inter-BS Coexistence-Aware Spectrum Sharing (CASS) protocol was proposed in [54], which has the ability to dynamically switch between the two spectrum sharing mechanisms to minimize self-interference while keeping control overhead under control.

In [55], a network controlled spectrum access mechanism, where IEEE 802.22 BSs behave collaboratively to minimize the interference and maximize the utility obtained from the system, was proposed. A modified minority game (MMG) is proposed to model the self-coexistence problem in [56]. A special case of spectrum sharing based on game theory was proposed in [57].

Chapter 3

Memetic Algorithm for Resource Allocation in Cognitive Radio Systems

With the explosive growth of wireless applications and services, the licensed frequency bands are becoming scarce. Ways in which to use the limited resources efficiently has become very important as a result. Compared to traditional wireless networks, the key advantage of CR is that CRUs are eligible to access those frequency bands currently not utilized by PUs. Thus, it improves the efficiency of spectrum utilization and achieves reliable wireless communications anywhere and anytime [1]. Due to its distinct advantages over other wireless networks, CR is becoming the main future development direction of wireless communications.

On the other hand, when a CRU accesses the unlicensed spectrum bands, it must guarantee the interference to PUs to be acceptable for PUs. This requires the CRU to transmit signals with low power levels. In order to maintain a high speed data rate, the unlicensed channels and other relevant resources such as power must be allocated to CRUs appropriately.

Thus, the resource allocation problem can be formulated as a constrained optimization problem. According to [15], when a channel is allocated exclusively to a CRU, the signal-to-noise ratio (SNR) obtained is the best. In this case, the constrained optimization problem becomes a combinatorial optimization problem.

Ideally, when the variables are independent, a simple greedy algorithm can find the global optimal solution with low complexity. However, the variables in this type of optimization problem are interdependent, and greedy based algorithms can get local optimal solutions. In this case, the steps to discovering the global optimal solution grow exponentially with respect to the number of frequency bands.

More importantly, the time for the resource allocation schedule to take place is very limited. Therefore, only suboptimal algorithms with low complexity are acceptable. Early works [19], [21], [24], [22], [20], [23], divided the optimization problem into two steps. The first step is to determine the allocation of channels to users and the second step is to determine the allocation of bits to users. Most of these algorithms are based on a greedy approach.

On the other hand, these two optimization problems belong to the combinatorial optimization problem group. The variables in these two optimization problems are interdependent, and greedy based algorithms cannot solve the problems effectively. They often find a local optimal solution that is far away from the global solution. In order to achieve better solutions for the bit allocation problem, more efficient algorithms are needed. In addition, most of these algorithms are designed for MU-OFDM systems in which there are no PUs. When determining the allocation of channels to users, only channel gain needs to be considered. In an MU-OFDM based CR system, mutual interference between PUs and CRUs also needs to be considered.

Memetic algorithms (MAs), which are similar to genetic algorithms (GAs), are good algorithms to use for combinatorial optimization problems [26]. Normally, a genetic algorithm (GA) combined with local search (LS) methods is called a memetic algorithm (MA). MAs have been shown to require fewer computations and produce better solutions than standard GAs for many optimization problems [26]. MAs have been successfully applied to problems such as the travelling salesman problem (TSP) and the quadratic assignment problem (QAP). They outperform traditional algorithms for many combinatorial optimization problems [27], [26], [28], [2], [29], [30].

For a given optimization problem, LS methods have an important impact on the results [31]. An appropriate LS method will generate a much better result. However, there is still little work completed on the choice of a good local search method for a given problem.

In this chapter, we consider the downlink transmission in a multiuser OFDM based cognitive radio system, where a CR base station (BS) services M CRUs and one PU. Our objective is to maximize the total data rate among CRUs. It is a combinatorial optimization problem. The steps required to discover the global optimal solutions grow exponentially with respect to the number of subcarriers. Moreover, the permitted time for the resource allocation schedule is very limited. For this model, we first focus on the performance analysis of memetic algorithms (MAs) for the bit allocation problem. Its advantage over genetic algorithms (GAs) is that it introduces local search (LS) methods. The improvement is mainly dependent on the selection of LS. This requires an appropriate selection of LS for the proposed problem. Secondly, we study the performance improvement by introducing multiple LS methods. This strategy has been shown to achieve better solutions than a single LS.

The rest of this chapter is organized as follows: In Chapter 3.1, the downlink transmission in a multiuser OFDM based cognitive radio system model is presented. The subcarrier allocation problem is discussed in Chapter 3.2. In Chapter 3.3, MA is introduced and the idea of using MAs for bit allocation in a multiuser OFDM based CR system is presented. Numerical results are presented in Chapter 3.4. Finally, in Chapter 3.5, we conclude the section.

3.1 Basic Model of Resource Allocation in the Downlink Transmission in a Multiuser OFDM Based Cognitive Radio System

In this section, we study the downlink transmission in a multiuser OFDM based CR system. Without loss of generality, we consider a cognitive BS services one PU and K CRUs. The PU and CRUs occupy neighboring frequency bands as illustrated in Fig. 3.1.

Figure 3.1: Primary user band of width W_p and cognitive user sub-bands, each of width W_s .

The PU band has a width of W_p Hz and has $N/2$ subcarriers, each occupying a band of width W_s Hz, on either side. The BS allocates subcarriers, subcarrier powers and bits to the CRUs dynamically. The channels from the BS to all users are modelled as slowly time-varying, i.e. they do not change appreciably between successive allocations. The BS is assumed to have perfect channel state information (CSI) for all users and subcarriers.

The power spectral density (PSD) of the n^{th} subcarrier signal is assumed to have the form [11]

$$\Phi_n(f) = P_n T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \quad (3.1)$$

where P_n denotes the subcarrier n transmit signal power and T_s is the symbol duration. The resulting interference power spilling into the PU band is given by

$$I_n(d_n, P_n) = \int_{d_n - W_p/2}^{d_n + W_p/2} |g_n|^2 \Phi_n(f) df = P_n IF_n, \quad (3.2)$$

where g_n is the subcarrier n channel gain from the BS to the PU, d_n is the spectral distance between subcarrier n and the center frequency of the PU band, and IF_n is the interference factor for subcarrier n .

The interference power introduced by the signal destined for the PU, hereafter referred to as the PU signal, into the band of subcarrier n at user k is

$$S_{nk}(d_n) = \int_{d_n - W_s/2}^{d_n + W_s/2} |h_{nk}|^2 \Phi_{RR}(e^{jw}) dw, \quad (3.3)$$

where h_{nk} is the subcarrier n gain from the BS to user k , and $\Phi_{RR}(e^{jw})$ is the PSD of the PU signal.

Let P_{nk} denote the transmit power allocated to subcarrier n of user k . As discussed in [15] and [58], the maximum number of bits per symbol that can be transmitted on this subcarrier is

$$b_{nk} = \left\lfloor \log_2 \left(1 + \frac{|h_{nk}|^2 P_{nk}}{\Gamma(N_0 W_s + S_{nk})} \right) \right\rfloor, \quad (3.4)$$

where $\lfloor \cdot \rfloor$ denotes the floor function, N_0 is the one-sided noise PSD and S_{nk} is given by (3.3). The term Γ indicates how close the system is operating to capacity and is set to 1 for convenience. For different modulation schemes, Γ has different values.

From (3.4), the additional signal power needed to transmit one extra bit to user k on subcarrier n can be expressed as:

$$\Delta P_{nk} = \frac{N_0 W_s + S_{nk}}{|h_{nk}|^2} 2^{b_{nk}}. \quad (3.5)$$

Using (3.2), we deduce that the additional interference power generated by such an additional signal power to the PU is

$$\Delta I_{nk} = \Delta P_{nk} I F_n. \quad (3.6)$$

Let $a_{nk} \in \{0, 1\}$ be a subcarrier allocation indicator function, i.e. $a_{nk} = 1$ if and only if subcarrier n is allocated to user k . To avoid excessive interference among CRUs, it is assumed that each subcarrier can be used for transmission to at most one CRU at any given time.

The objective is to maximize the total CRU bit rate, R_s , subject to limits on the total CRU transmit power and PU tolerable interference power. More specifically, the optimization problem of interest is

$$\max R_s \triangleq W_s \sum_{k=1}^K \sum_{n=1}^N a_{nk} b_{nk} \quad (3.7)$$

subject to

$$a_{nk} \in \{0, 1\}, \forall n, k \quad (3.8)$$

$$\sum_{k=1}^K a_{nk} \leq 1, \forall n \quad (3.9)$$

$$P_{nk} \geq 0, \forall n, k \quad (3.10)$$

$$\sum_{k=1}^K \sum_{n=1}^N a_{nk} P_{nk} \leq P_{total}, \quad (3.11)$$

$$\sum_{k=1}^K \sum_{n=1}^N a_{nk} P_{nk} I F_n \leq I_{total}, \quad (3.12)$$

where P_{total} denotes the total CRU power limit and I_{total} is the maximum PU tolerable interference power. Inequality (3.9) reflects the condition that any given subcarrier can be allocated to a maximum of one user. Inequalities (3.11) and (3.12) correspond to the power and interference constraints, respectively. When inequality (3.11) is changed to $\sum_{n=1}^N a_{nk} P_{nk} \leq P_k, \forall k$, it becomes an uplink transmission [59], [60]. In the case of multiple primary users sharing the same licensed channel, all of the CRUs' transmission power should try to end the interference to each primary user at the same time. The optimization problem will then become more complex. We also analyze the optimization problem for the case of throughput requirement.

3.2 Subcarrier Allocation

The optimization problem in (3.7) is an integer programming problem whose solution has a high computational complexity. When a_{nk} is defined as the time share of the transmission slot, the optimization problem will become a liner programming problem and can be solved easily [59]. If we ignore this constraint (3.12), the algorithms in [22], [20], [23] can be used to solve the problem. These algorithms solve the problem in two steps. In the first step, subcarriers are assigned to CRUs in a greedy fashion based on the CRU subcarrier channel gains.

Two greedy algorithms proposed for multiuser OFDM based CR systems in [24] are applicable if (3.12) is included. The basic algorithm (BA) has complexity $O(4^{num_bits})$

Algorithm 3.1 Subcarrier Allocation

```

for k from 1 to N do
    find  $m_P = \arg \min_m \Delta p_{mk}$ , and  $m_I = \arg \min_m \Delta I_{mk}$ .
    if  $m_P = m_I$  then
        set  $m^* = m_P$ .
    else
        Compute  $\Delta P_{m_P^*k}$ ,  $\Delta I_{m_P^*k}$ ,  $\Delta P_{m_I^*k}$ ,  $\Delta I_{m_I^*k}$ , the total transmit power  $P$  and total
        interference  $I$ ;
         $VP = \frac{P - P_{total}}{P_{total}}$ ,  $VI = \frac{I - I_{th}}{I_{th}}$ ,
         $X = \frac{VI(\Delta P_{m_P^*k} IF_k - \Delta I_{m_I^*k})}{\Delta I_{m_I^*k}}$ , and  $Y = \frac{VP(\Delta I_{m_I^*k} / IF_k - \Delta P_{m_P^*k})}{\Delta P_{m_P^*k}}$ .
        if  $X \geq Y$  then
            set  $m^* = m_I$ .
        else
            set  $m^* = m_P$ .
        end if
    end if
    set  $a_{mk} = \begin{cases} 1 & \text{for } m = m^* \\ 0 & \text{otherwise} \end{cases}$ 
end for
    
```

Figure 3.2: Pseudo-code for subcarrier allocation algorithm

[24], where *num_bits* denotes the total number of bits loaded, and is not practical for wireless communications systems. The reduced complexity (RC) algorithm has complexity $O(\text{num_bits} \times K)$, where K is the number of subcarriers. It generally provides a solution which is very close to optimal. To reduce the complexity of the MA, we first use a method proposed in [24] to allocate the subcarriers to the CRUs. A pseudo-code listing for the subcarrier allocation algorithm is shown in Fig. 3.2. After the subcarrier allocation, the optimization problem becomes one of optimal bit and power allocation among subcarriers. This simpler problem can be formulated as follows:

$$\max R_s = \sum_{k=1}^K b_k, \quad (3.13)$$

subject to

$$P_k \geq 0, \forall k \quad (3.14)$$

$$\sum_{k=1}^K P_k \leq P_{total}, \quad (3.15)$$

$$\sum_{k=1}^K P_k I F_k \leq I_{th} \quad (3.16)$$

where P_k and b_k are the power and number of bits for subcarrier k , respectively.

For the optimization problem in (3.13), we propose to use a memetic algorithm (MA) which is a genetic algorithm (GA) combined with local search methods. MAs are evolutionary algorithms which have been shown to be more efficient than standard GAs for many combinatorial optimization problems [2], [29], [30]. As a result, MAs have been widely used with these applications.

3.3 Memetic Algorithms for Bit Allocation in a Multiuser OFDM Based Cognitive Radio System

3.3.1 Memetic Algorithm Operations

According to Darwin's natural evolution theory, evolution is mainly based on three principles: replication, variation and natural selection. Replication means the produced offspring still maintain most of the same properties as their parents. Variation causes the offspring to be slightly different from their parents. Since the resources available for organisms are finite, only the fittest organisms can survive after natural selection. Thus, natural evolution can be considered as an optimization process in which the fitness of the organisms is maximized.

Natural evolution has not only been used by biologists to study the evolution of complex organisms with changes in the environment, but it has also been used by scientists to study the applicability of simulated evolution for various optimization problems. It has been shown that it can solve various optimization problems in the fields of engineering and chemistry. Inspired by the power of natural evolution, several evolutionary algorithms

(EAs): evolutionary strategies, evolutionary programming and genetic algorithms have been proposed since the early 1960's.

When applying these EAs to solve optimization problems, the solutions evolve subject to the three operators: replication, variation and selection. From the view point of evolution, arbitrary initial solutions can evolve to find the optimal solutions given infinite time. In practice, we want to find the optimal solutions in the least amount of time possible. For certain varieties of optimization problems, the solutions quickly converge to a local optimum and it is hard to escape the local optimum within the given running time. This is called premature convergence.

As a result, we only obtain a local optimum. Generally, the final solutions are mainly dependent on the parameter settings such as replication probability, variation probability and selection strategy, and population size. A high replication probability may lead to premature convergence, while a low variation probability may lead to genetic drift. Better solutions require more appropriate parameter settings. The selection of these parameters should be based on the given problem. However, we still don't have a powerful technique for parameter selection. Working out the best way to select appropriate parameters for EAs remains an important issue.

Genetic algorithms (GAs) are powerful algorithms for combinatorial optimization problems. Compared with other heuristic algorithms such as greedy algorithms and tabu search algorithms, GAs are global algorithms. Ideally, they can search the global optimal solutions. Therefore, GAs have been widely applied to combinatorial optimization problems such as the travelling salesman problem (TSP) and the quadratic assignment problem (QAP).

GAs evolve solutions based on the same three evolutionary principles: replication (also called cross over), variation (also called mutation) and natural selection (also called selection). In practice, there are several kinds of cross over and selection operators. Generally, a typical genetic algorithm requires two things to be defined:

- (1) a simple representation of the solution domain,

- (2) an efficient fitness function to evaluate the solution domain.

A standard representation of the solution is expressed as an array of bits. For example, a solution for a TSP problem with 8 cities can be expressed as a vector:

$$\mathbf{x} = [2 \ 1 \ 3 \ 7 \ 5 \ 8 \ 6 \ 4]. \quad (3.17)$$

Therefore, the representation of a solution is fairly simple. But for the cross over operator, it is more complex. There are several kinds of cross over operators. Now we move on to the introduction of cross over and mutation operators.

Genetic Operators

Generally, a crossover operator comes in one of three forms: one point crossover, two point crossover, and uniform crossover. Without a loss of generality, the following genetic operators illustrations are based on binary vectors. Suppose $X, Y \in \{0, 1\}^n$ is a bit string which represents a candidate solution.

One point crossover

First, given the parents' bit strings, the cutting point p is randomly selected [61]. Then all data beyond the point in either bit string is swapped between the two parent bit strings.

Thus, one point crossover produces two solutions with X' and Y' with

$$X'_i = \begin{cases} X_i & \text{if } i \leq p \\ Y_i & \text{if } i > p \end{cases} \quad \text{and} \quad Y'_i = \begin{cases} X_i & \text{if } i > p \\ Y_i & \text{if } i \leq p \end{cases}.$$

More concretely, the following example illustrates the operation:

$$\begin{cases} X = 0110|010 \\ Y = 1011|001 \end{cases} \rightarrow \begin{cases} X' = 0110|001 \\ Y' = 1011|010 \end{cases}.$$

Two point crossover

Compared to the one point crossover, the main difference between them is two point crossovers cut two chromosomes into three parts by generating two cutting points p_1 and p_2 ($p_1 \leq p_2$) randomly [61]. Thus, two new solutions are constructed as:

$$X'_i = \begin{cases} X_i & \text{if } p_1 \leq i \leq p_2 \\ Y_i & \text{otherwise} \end{cases} \quad \text{and} \quad Y'_i = \begin{cases} Y_i & \text{if } p_1 \leq i \leq p_2 \\ X_i & \text{otherwise} \end{cases}.$$

An example is illustrated as follows:

$$\begin{cases} X = 01|10|010 \\ Y = 10|11|001 \end{cases} \rightarrow \begin{cases} X' = 10|10|001 \\ Y' = 01|11|010 \end{cases}.$$

Generally, a crossover operator is called k -crossover, when the solution bit arrays are cut at k randomly chosen points.

Uniform crossover

Different from one-point and two-point crossovers, uniform crossover is more complex [62]. In a uniform crossover, the bits in the two new offspring are copied by two parents, which are then swapped with a fixed probability, typically 0.5. Firstly, it randomly generates a bit string V , which has composites of 0 and 1 of the same length as the solution vector. Then, the new offspring are constructed by the following strategy:

$$X'_i = \begin{cases} X_i & \text{if } V_i = 1 \\ Y_i & \text{otherwise} \end{cases} \quad \text{and} \quad Y'_i = \begin{cases} Y_i & \text{if } V_i = 1 \\ X_i & \text{otherwise} \end{cases}.$$

The following example illustrates a uniform crossover:

$$\begin{cases} X = 0110010 \\ Y = 1011001 \end{cases} \rightarrow V = 1101010 \rightarrow \begin{cases} X' = 0110011 \\ Y' = 1011000 \end{cases}.$$

Mutation

There are two kinds of mutation operators [61]. The first one is a bit flip operator, and the second one is an inversion operator. For the bit flip operator, firstly it randomly generates a bit string of the same length as the solution vector and then selects a small number of genes to flip according to a predefined rate:

$$X = 0110\underline{0}10 \rightarrow X' = 0110\underline{1}10.$$

For an inversion operator, it first generates two cutting points randomly and then cuts the the solution vector into three parts. The bit string between the two cutting points is reversed:

$$X = 01|100|10 \rightarrow X' = 01|001|10.$$

Generally, crossover and selection operators improve the candidate solutions in genetic algorithms. Crossover operators allow GAs to find a local optimal solution quickly. Therefore, crossover operators play an important role in GAs. On the other hand, crossover operators shrink the search space. They allow GAs to easily converge on a local optimal solution, and then make it difficult for them to escape from it.

Contrary to crossover operators, mutation operators help to enlarge the total search space. The main properties of mutation operators are that they can help the GAs escape the current search space. In order to allow the GAs to achieve better solutions, we first need the mutation operators to find better solutions. In practice, the mutation rate is very low due to its diversity property.

Selection for variation:

There are two kinds of selection which can be found in GAs. The first kind is selection for variation, where individuals are chosen for crossover and mutation. The second kind is selection for survival, where individuals are selected for the next generation. Sometimes this kind of selection is also known as replacement. For the selection for variation, there are three kinds of selection strategies:

Fitness-proportionate selection

This kind of selection strategy is very simple and widely used. Firstly, we define the probability of selecting solution $\mathbf{x}_i$ as:

$$p(\mathbf{x}_i) = \frac{f(\mathbf{x}_i)}{\sum_{\mathbf{x}_j \in P} f(\mathbf{x}_j)}. \quad (3.18)$$

It can be realized by roulette wheel sampling [63]. By this selection scheme, candidate solutions with a higher fitness will be less likely to be eliminated. In addition, some weaker solutions still have a chance to survive the selection process. Sometimes weaker solutions may include an important component for the recombination process. Therefore, this selection scheme is relatively fair. However, when the variance of the fitness among candidate solutions is small, fitness proportionate selection becomes a random selection. In this case, the selection scheme can not help to select good solutions for recombination.

Rank-based selection

Compared to the fitness-proportionate selection, rank-based selection is based on the positions in the individuals rank as opposed to the actual fitness [64]. In this way, rank-based selection overcomes the scaling problems of fitness-proportionate selection. Generally, rank-based selection includes linear ranking and non-linear ranking. In the linear ranking model, the probability of selecting individual $\mathbf{x}_i$ is computed by [65]:

$$p(\mathbf{x}_i) = p_{\max} - (p_{\max} - p_{\min}) \frac{i - 1}{N - 1}, \quad (3.19)$$

where i represents the position of an individual in this population and N is the number of individuals in the population. $p_{\max}$ and $p_{\min}$ denote the maximum and minimum selection probability respectively.

Tournament selection

In tournament selection [66], k individuals are selected randomly and the best is chosen as the parent. k take values ranging from 2 to the number of individuals in the population. This process is repeated until the required number of individuals are chosen as parents.

Selection for Survival:

Since the objectives of selection for variation and survival are different, their selection strategies are distinct. In the case of selection for survival, several strategies exist.

Steady state selection

In steady state selection [67], the number of offspring is smaller than the number of parents. A good strategy for determining which parents are replaced is required. These strategies are worst replacement and oldest replacement [68].

 (μ, λ) selection

In (μ, λ) selection, μ parents are replaced by the best of the λ offspring ($\lambda \geq \mu$). The selection pressure can be raised by increasing the number of offspring λ .

Algorithm 3.2 Genetic Algorithm

```

Set  $n = 0$ ;
Initialize population  $P(0)$ ;
Evaluate  $P(0)$ ;
repeat
     $S = \text{selectforvariation}(P(t))$ ;
     $\text{crossover}(S)$ ;
     $\text{mutation}(S)$ ;
     $\text{evaluate}(S)$ ;
     $P(t + 1) = \text{selectionforsurvival}(P(t), S)$ ;
     $t = t + 1$ ;
until terminate=true

```

Figure 3.3: The GA pseudo code [2]

 $(\mu + \lambda)$ selection

In $(\mu + \lambda)$ selection, the best μ individuals are chosen from a population which includes μ parents and λ offspring. In this selection strategy, the selection scheme does not distinguish between parents and offspring.

After discussing different genetic operators and selection strategies, we are giving the genetic algorithm a pseudo code. The GA pseudo code is fairly simple. For many applications, it can find comparable good solutions within the given running time.

Therefore, GAs have been widely applied to many optimization problems, and in particular the NP-hard problems such as the travelling salesman problem (TSP) and quadratic assignment problems (QAP). On the other hand, the premature convergence in GAs leads to an increased difficulty in finding the global optimal solutions.

This property is caused by the characteristics of genetic operators. For some applications, even the best genetic operator setting cannot alleviate the premature convergence significantly. In order to make GAs more efficient, a more powerful technique is required.

Memetic algorithms (MAs) were first proposed in [69]. GAs are inspired by trying to emulate biological evolution. On the other hand, MAs are inspired by trying to mimic cultural evolution [70]. MAs are also a population based algorithm for a heuristic search in optimization problems. Basically, they combine local search methods with genetic op-

Algorithm 3.3 Memetic Algorithm

```

Set  $n = 0$ ;
Initialize population  $P(0)$ ;
Evaluate  $P(0)$ ;
localsearch( $P(0)$ );
repeat
   $S = \text{selectforvariation}(P(t))$ ;
  crossover( $S$ );
  mutation( $S$ );
  evaluate( $S$ );
  localsearch( $S$ );
   $P(t + 1) = \text{selectionforsurvival}(P(t), S)$ ;
   $t = t + 1$ ;
until terminate=true

```

Figure 3.4: The MA pseudo code [2]

erators. They are also known as hybrid genetic algorithms. The mechanism to do a local search is required to reach the local optimal solutions. Thus, each individual can achieve a certain improvement. They have been shown to be more efficient and effective than GAs for many optimization problems [26]. Fig. 3.4 shows the pseudo-code of MAs. Since they have distinct advantages over GAs, they have been widely applied to many optimization problems.

3.3.2 Memetic Algorithm with Single Local Search for Bit Allocation

Now we study the performance of an MA with a single LS method for the optimization problem in (3.13). Pseudo-code listings of the proposed MA and the local search method are shown in Figs. 3.5 and 3.6 respectively.

The local search method uses the $1 - \text{opt}$ algorithm. That is, only a gene's value of a chromosome changes once. The location of a gene to be changed is randomly generated.

Let $\mathbf{x}_i$ be the chromosome of member i in a population

$$\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iN}], \quad i = 1, 2, \dots, \text{pop_size} \quad (3.20)$$

Algorithm 3.4 MA

```

Initialize Population  $P$ ;
Set  $P = Local\_Search(P)$ .
for  $i = 1$  to  $Number\_of\_Generation$  do
     $S = SelectForVariation(P)$ ;
    for  $j = 1$  to  $\#crossover$  do
        Select  $\mathbf{x}_a, \mathbf{x}_b$  from  $S$  for crossover;
         $\mathbf{x}_c = crossover(\mathbf{x}_a, \mathbf{x}_b)$ ;
         $\mathbf{x}_c = Local\_Search(\mathbf{x}_c)$ ;
    end for
    Add individual  $\mathbf{x}_c$  to  $P$ ;
    for  $k = 1$  to  $\#mutation$  do
        Select  $\mathbf{x}_m$  from  $S$  for mutation;
         $\mathbf{x}_m = mutation(\mathbf{x}_m)$ ;
         $\mathbf{x}_m = Local\_Search(\mathbf{x}_m)$ ;
    end for
    Add individuals  $\mathbf{x}_m$  to  $P$ ;
     $P = SelectForSurvival(P)$ .
end for
    
```

Figure 3.5: Pseudo-code for the memetic algorithm [2]

Algorithm 3.5 Local Search (x)

```

Define a neighborhood  $n_x$ ;
while  $x$  is not locally optimal do
    find a new solution  $x_{new}$  in  $n_x$ ;
    if  $x_{new}$  is better than  $x$  then
         $x = x_{new}$ .
    end if
end while
    
```

Figure 3.6: Pseudo-code for the local search method

where pop_size denotes the population size. The initial integer solution vectors are randomly created within the region of admissible solutions. The original objective function of the optimization problem is evaluated as

$$eval(\mathbf{x}_i) = \begin{cases} f(\mathbf{x}_i), & \text{if } \sum_{k=1}^K P_k \leq P_{total} \\ & \text{and } \sum_{k=1}^K P_k IF_k \leq I_{th} \\ -M, & \text{otherwise} \end{cases} \quad (3.21)$$

where M is a positive large integer representing a penalty if the constraints are violated, and $f(\mathbf{x}_i)$ is the total bits achieved in (3.13).

Genetic Operations:

(1) Crossover: For a pair of parents $\mathbf{x}_1$ and $\mathbf{x}_2$,

$$\mathbf{x}_1 = [x_{11}, x_{12}, \dots, x_{1p}, x_{1(p+1)}, \dots, x_{1N}],$$

$$\mathbf{x}_2 = [x_{21}, x_{22}, \dots, x_{2p}, x_{2(p+1)}, \dots, x_{2N}],$$

we first generate a random integer $p = 1, 2, \dots, N - 1$. Then we obtain the chromosomes of two children as follows

$$\mathbf{x}'_1 = [x_{11}, x_{12}, \dots, x_{1p}, x_{2(p+1)}, \dots, x_{2N}],$$

$$\mathbf{x}'_2 = [x_{21}, x_{22}, \dots, x_{2p}, x_{1(p+1)}, \dots, x_{1N}],$$

(2) Mutation: We substitute one component of the chromosome of an individual randomly by an admissible integer for the selected position.

(3) Selection: We select the better chromosomes among parent and offspring based on their fitness values. The number to be selected is *pop_size* and we let these chromosomes enter the next generation.

3.3.3 Memetic Algorithm with Multi-Local Search for Bit Allocation

For a given problem, distinct local search methods get different solutions. A good local search method may get solutions that approximate the global optimal solution closely. An unsuitable local search method may get solutions that are far from the global optimal solution. In this case, choosing a way to select an appropriate local search method for a given problem becomes important. However, there is still little work that has been completed on the best way to make the choice of a good local search method for a given problem.

One issue is that the performance of a local search method for different problems can vary. It is very difficult to choose the best LS methods for a given problem. This motivates us to study MAs combined with multi-local-search methods (MLS) [71].

Algorithm 3.6 MSL-MA

```

Initialize population  $P$ ;
Set  $P = \text{Multi\_Local\_Search}(P)$ ;
for  $i = 1$  to  $\text{Number\_of\_Generation}$  do
    Set  $S = \text{SelectForVariation}(P)$ ;
    for  $j = 1$  to  $\#crossover$  do
        Select  $\mathbf{x}_a, \mathbf{x}_b$  from  $S$  for crossover
         $\mathbf{x}_c = \text{crossover}(\mathbf{x}_a, \mathbf{x}_b)$ ;
        Set  $\mathbf{x}_c = \text{Multi\_Local\_Search}(\mathbf{x}_c)$ ;
    end for
    Add individual  $\mathbf{x}_c$  to  $P$ ;
    for  $k = 1$  to  $\#mutation$  do
        Select  $\mathbf{x}_m$  from  $S$  for mutation
         $\mathbf{x}_m = \text{mutation}(\mathbf{x}_m)$ ;
        Set  $\mathbf{x}_m = \text{Multi\_Local\_Search}(\mathbf{x}_m)$ ;
    end for
    Add individual  $\mathbf{x}_m$  to  $P$ ;
    Set  $P = \text{select}(P)$ ;
end for

```

Figure 3.7: Pseudo-code for the MSL-MA

Pseudo-code listings of the proposed MLS-MA and the multi local search methods are shown in Figs. 3.7 and 3.8 respectively. Here, we use the $(1 - opt)$ algorithm for Local_Search1.

The location of a gene to be changed is randomly generated. Local_Search2 is based on the water filling algorithm. The parameter k in Fig. 3.8 is set to $k = 0.8$.

Algorithm 3.7 Multi_Local_Search ($\mathbf{x}$)

```

Generate a random number within in  $(0, 1)$ :
 $ran = \text{random}(0, 1)$ ;
if  $ran < k$  then
     $\mathbf{x} = \text{Local\_Search1}(\mathbf{x})$ ;
else
     $\mathbf{x} = \text{Local\_Search2}(\mathbf{x})$ ;
end if
return  $\mathbf{x}$ ;

```

Figure 3.8: Pseudo-code for the multi-local-search methods

3.4 Numerical Results

In our simulation, we consider a system consisting of one PU and $K = 4$ and 6 CRUs. The CRU band is 5 MHz wide and supports 16 subcarriers, each with a bandwidth, W_s , of 0.3125 MHz. The PU bandwidth is $W_p = W_s$ and the OFDM symbol duration is $T_s = 4 \mu s$.

It is assumed that the subcarrier gains h_{kn} and g_n , for $k \in \{1, 2, \dots, K\}$, $n \in \{1, 2, \dots, N\}$ are outcomes of independent, identically distributed (i.i.d.) Rayleigh distributed random variables (rv's) with means equal to 1. The additive white Gaussian noise (AWGN) PSD, N_0 , is set to 10^{-8} W/Hz. The PSD, $\Phi_{RR}(e^{jw})$, of the PU signal is assumed to be that of an elliptically filtered white noise process. The total CRU bit rate results are obtained by averaging over 1000 channel realizations. For our simulations, we used Matlab.

First, we compare the performance of the proposed MA in Fig. 3.5 to that of the RC algorithm in [24] for a number of different scenarios.

The parameters employed in the proposed MA are: number of generations=20, population size=25, probability of crossover=0.7 and probability of mutation=0.05.

Fig. 3.9 shows the average total CRU bit rate, R_s as a function of the maximum tolerable interference power, I_{th} , with a PU signal power $P_p = 5$ W, $P_{total} = 1$ W and $M = 4$. It can be seen that the MA provides a higher R_s than the RC algorithm. The difference is largest for small values of I_{th} . For $I_{th} = 10^{-3}$ W, the MA provides a 15% improvement in R_s . As is to be expected, for both algorithms, R_s increases with I_{th} .

Fig. 3.10 shows the average total CRU bit rate, R_s , as a function of the maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1.5$ W and $M = 4$. A comparison with Fig. 3.9 shows that R_s increases with P_{total} .

Fig. 3.11 shows the average total bit rate, R_s , as a function of the maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1$ W and $M = 6$. A comparison with Fig. 3.9 shows that R_s is larger for $M = 6$ than for $M = 4$. The improvement in R_s is due to the increased multiuser diversity that results from a larger number of CRUs.

Fig. 3.12 shows the average total bit rate, R_s , as a function of the maximum tolerable interference power, I_{th} , with $P_p = 3$ W, $P_{total} = 1$ W and $M = 4$. From Figs. 3.9 and 3.12,

Figure 3.9: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1$ W and $M = 4$.

we see that both algorithms have larger R_s values with $P_p = 3$ W compared to $P_p = 5$ W. This is due to the reduced interference power from the PU signal.

For all the four scenarios in Figs. 3.9 to 3.12, we see that the MA provides a higher average total CRU bit rate than the RC algorithm. The improvement is greatest for small values of I_{th} and large values of M and P_{total} .

Next, we study the improvement of proposed MLS-MA in Fig. 3.7.

Fig. 3.13 shows the average total CRU bit rate, R_s as a function of the maximum tolerable interference power, I_{th} , with a PU signal power $P_p = 5$ W, $P_{total} = 1$ W and $M = 4$. It can be seen that the MLS-MA provides a higher R_s than the RC algorithm. The difference is largest for small values of I_{th} . For $I_{th} = 10^{-3}$ W, the MLS-MA provides a 15% improvement in R_s . As is to be expected, for both algorithms, R_s increases with I_{th} .

Fig. 3.14 shows the average total CRU bit rate, R_s , as a function of the maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1.5$ W and $M = 4$. A comparison with Fig. 3.13 shows that R_s increases with P_{total} . Fig. 3.15 shows the average total bit rate, R_s , as a function of the maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1$ W and $M = 6$. A comparison with Fig. 3.13 shows that R_s is larger for $M = 6$

Figure 3.10: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1.5$ W and $M = 4$.

than for $M = 4$. The improvement in R_s is due to the increased multiuser diversity that results from a larger number of CRUs.

Fig. 3.16 shows the average total bit rate, R_s , as a function of the maximum tolerable interference power, I_{th} , with $P_p = 3$ W, $P_{total} = 1$ W and $M = 4$. From Figs. 3.13 and 3.16, we see that both algorithms have larger R_s values with $P_p = 3$ W compared to $P_p = 5$ W. This is due to the reduced interference power from the PU signal.

For all four of the scenarios in Figs. 3.13 to 3.16, we see that the MLS-MA provides a higher average total CRU bit rate than the RC algorithm. The improvement is greatest for small values of I_{th} and large values of M and P_{total} .

3.5 Chapter Summary

In this chapter, the resource allocation problem in an MU-OFDM based cognitive radio system is investigated. The resource allocation problem is computationally complex. To directly search the global optimal solutions would be impossible for the BS. In order to reduce the complexity, the resource allocation can be solved in two steps: (1) to determine the allocation of subcarriers to users; (2) to determine the allocation of bits to users.

Figure 3.11: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1$ W and $M = 6$.

Subcarrier allocation to users can be determined based on greedy methods. After subcarrier allocation, the bit allocation is still computationally complex.

MAs belong to evolutionary algorithms. They achieve a good performance for most combinatorial optimization problems. We propose an efficient MA to determine the bit allocation. On the other hand, premature convergence is a disadvantage of EAs. MAs with a single local search method tend to converge prematurely. Premature convergence causes the final solutions to be local optimal, and are often some way from the global optimal solutions.

In order to prevent premature convergence, MAs with a multi-local-search method can suppress the effect of premature convergence. We propose an MA with a multi-local-search method to solve the bit allocation problem and achieve a better performance than existing algorithms.

Figure 3.12: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 3$ W, $P_{total} = 1$ W and $M = 4$.

Figure 3.13: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1$ W and $M = 4$.

Figure 3.14: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1.5$ W and $M = 4$.

Figure 3.15: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 5$ W, $P_{total} = 1$ W and $M = 6$.

Figure 3.16: Average total CRU bit rate, R_s , versus maximum tolerable interference power, I_{th} , with $P_p = 3$ W, $P_{total} = 1$ W and $M = 4$.

Chapter 4

Fitness Landscape Analysis for Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems

It has been shown that MAs are suitable for solving combinatorial optimization problems. MAs belong to evolutionary algorithms. Since they are too complex, there are still very few publications analyzing how and why they can outperform other traditional algorithms for combinatorial optimization problems. When applying MAs to solve combinatorial optimization problems, most authors only show they can work well, but they do not show why.

On the other hand, it will be more valuable if we can find an efficient way to select appropriate parameters for MAs under given problems so that MAs can achieve the best performance possible. Moreover, premature convergence easily occurs in MAs (i.e. a population of solutions converge too early so that the final solutions are suboptimal). In order to make MAs more effective, more powerful techniques for analyzing the behavior of MAs are required. Previous works on ways to improve the performance of MAs have been published in [32], [33], [34], [35].

The notion of the fitness landscape was first proposed in [36] for analyzing the gene interaction in biological evolution. It was also an important technique for analyzing the behavior of combinatorial optimization problems and predict their behaviors [37], [38], [39].

Furthermore, it has been extended to analyze evolutionary algorithms. From [36], [38], each genotype has a “fitness” and the distribution of fitness values over the space of genotypes constitutes a fitness landscape.

In our problem, we consider the set of solutions as a search space, and the height of a point denotes the fitness of the solution associated with the point. Therefore, a heuristic algorithm can be considered as one that is searching through the search space to find the highest peak of the landscape. Our objective is to analyze the structure of the fitness landscape of the given problem and derive appropriate LS methods and genetic operators, so that the proposed MA finds a local optimal solution which is close to the global optimal solution of the problem more efficiently.

In this chapter we consider the resource allocation in an MU-OFDM based CR system. The application of MAs to the bit allocation problem in MU-OFDM based CR systems is studied in [72]. It mainly examines the improvement achieved by a memetic algorithm. In this chapter, we not only propose a new algorithm for subcarrier allocation, but also study how to choose appropriate genetic operators and LS methods for the proposed memetic algorithm.

We find that appropriate LS methods and genetic operators can lead to solutions that are close to the global optimal solution.

There are few papers which discuss the choice of a good LS and genetic operators for an MA. According to the No-Free-Lunch-Theorem (NFL-theorem) proposed in [73], all black box optimization techniques have the same average behavior over all optimization problems.

Therefore, the LS method and genetic operators should be specially designed for the problem at hand. We need to know how to choose an appropriate LS method and genetic operators. We apply fitness landscape to analyze the given optimization problem. By analyzing the distribution of local optima and the degree of roughness of the landscape of a given problem, we can select a good LS method as well as appropriate genetic operators for the proposed MA.

The rest of the chapter is organized as follows: In Chapter 4.1, the resource allocation model in an MU-OFDM based cognitive radio system is introduced. In Chapter 4.2, a novel subcarrier allocation algorithm is presented. For the bit allocation problem, we apply fitness landscape to analyze the performance of local search methods and genetic operators in Chapter 4.3. Numerical results are presented in Chapter 4.4. Finally, the chapter summary is presented in Chapter 4.5.

4.1 Subcarrier and Bit Allocation Model in an MU-OFDM Based Cognitive Radio System

The system model used in this chapter is the same as that in [24]. A brief description is provided below for the convenience of the reader. Consider a base station (BS) which services PUs and CRUs [6]. We focus on the forward link in a multiuser OFDM CR system in which the BS transmits to one PU and K CRUs. The PU has a bandwidth of W_p Hz and has $N/2$ subcarriers, each occupying a bandwidth of W_s Hz, on the both sides.

The baseband power spectral density (PSD) of the n^{th} subcarrier signal is assumed to have the form [11]

$$\Phi_n(f) = P_n T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2, \quad (4.1)$$

where P_n denotes the subcarrier n transmit signal power and T_s is the symbol duration. The resulting interference power spilling into the PU band is given by

$$I_n(d_n, P_n) = \int_{d_n - W_p/2}^{d_n + W_p/2} |g_n|^2 \Phi_n(f) df = P_n I F_n \quad (4.2)$$

where g_n is the subcarrier n channel gain from the BS to the PU, d_n is the spectral distance between subcarrier n and the center frequency of the PU band and $I F_n$ is the interference factor for subcarrier n .

The interference power introduced by the signal destined for the PU, hereafter referred to as the PU signal, into the band of subcarrier n at user k is

$$S_{nk}(d_n) = \int_{d_n - W_s/2}^{d_n + W_s/2} |h_{nk}|^2 \Phi_{RR}(e^{jw}) dw, \quad (4.3)$$

where h_{nk} is the subcarrier n gain from the BS to user k and $\Phi_{RR}(e^{jw})$ is the PSD of the PU signal.

Let P_{nk} denote the transmit power allocated to subcarrier n of user k . As discussed in [15], the maximum number of bits per symbol that can be transmitted on this subcarrier is

$$b_{nk} = \left\lfloor \log_2 \left(1 + \frac{|h_{nk}|^2 P_{nk}}{\Gamma(N_0 W_s + S_{nk})} \right) \right\rfloor, \quad (4.4)$$

where $\lfloor \cdot \rfloor$ denotes the floor function, N_0 is the one-sided noise PSD and S_{nk} is given by (4.3). The term Γ indicates how close the system is operating to capacity and is set to 1 for convenience.

From (4.4), the additional signal power needed to transmit one extra bit to user k on subcarrier n can be expressed as:

$$\Delta P_{nk} = \frac{N_0 W_s + S_{nk}}{|h_{nk}|^2} 2^{b_{nk}}, \quad (4.5)$$

Using (4.2), we deduce that the additional interference power generated by such an additional signal power to the PU is

$$\Delta I_{nk} = \Delta P_{nk} I F_n. \quad (4.6)$$

Let $a_{nk} \in \{0, 1\}$ be a subcarrier allocation indicator function, i.e. $a_{nk} = 1$ if and only if subcarrier n is allocated to user k . To avoid excessive interference among CRUs, it is assumed that each subcarrier can be used for transmission to a maximum of one CRU at any given time.

The objective is to maximize the total CRU bit rate, R_s , subject to limits on the total CRU transmit power and PU tolerable interference power. More specifically, the optimization problem of interest is

$$\max R_s \triangleq W_s \sum_{k=1}^K \sum_{n=1}^N a_{nk} b_{nk} \quad (4.7)$$

subject to

$$a_{nk} \in \{0, 1\}, \forall n, k \quad (4.8)$$

$$\sum_{k=1}^K a_{nk} \leq 1, \forall n \quad (4.9)$$

$$P_{nk} \geq 0, \forall n, k \quad (4.10)$$

$$\sum_{k=1}^K \sum_{n=1}^N a_{nk} P_{nk} \leq P_{total}, \quad (4.11)$$

$$\sum_{k=1}^K \sum_{n=1}^N a_{nk} P_{nk} IF_n \leq I_{total}, \quad (4.12)$$

$$R_1 : R_2 : \dots : R_K = \lambda_1 : \lambda_2 : \dots : \lambda_K, \quad (4.13)$$

where P_{total} denotes the total CRU power limit, and I_{total} is the maximum PU tolerable interference power, and

$$R_k = W_s \sum_{n=1}^N a_{nk} b_{nk}, \forall k = 1, 2, \dots, K \quad (4.14)$$

represents the total bit rate of k^{th} CRU. Inequality (4.9) reflects the condition that any given subcarrier can be allocated to one user at the most. Inequalities (4.11) and (4.12) correspond to the power and interference constraints, respectively. Equation (4.13) reflects the proportional fairness among CRUs.

4.2 Subcarrier Allocation Algorithm

The objective function in (4.7) is a combinatorial optimization problem with two levels, (i.e., determine the subcarrier allocation indicator a_{nk} and transmit bits b_{nk}). The algorithm complexity of searching the optimal solution grows exponentially with the number of subcarriers. In order to reduce the algorithm complexity, we propose a simple algorithm. Which is known as a subcarrier allocation algorithm (SA), to determine the subcarrier allocation. The pseudo-code of SA is shown in Fig. 4.1. The algorithm has complexity $O(KN)$, where K denotes the number of CRUs and N represents the number of subcarriers. Firstly, we set a threshold to delete some of the worst subcarriers for all users. For the remaining

Algorithm 4.8 SA

```

for  $n = 1$  to number of subcarriers do
    find  $k^* = \arg_k \max \frac{|h_{nk}|^2}{\Gamma(N_0 W_s + S_{nk})}$ ;
    Using (4.4), calculate the number of bits loaded on subcarrier  $n$  as  $b_{nk^*}$  with  $P_{nk^*} = \frac{P_{total}}{N}$ ;
    Initialize  $\hat{N}$  to 0;
    if  $b_{nk^*} > 2$  then
        subcarrier  $n$  is available;
        increment  $\hat{N}$  by 1;
    else
        subcarrier  $n$  is not available;
    end if;
end for;
For each  $k \in \{1, 2, \dots, K\}$ , let the number,  $m_k$ , of subcarriers allocated to user  $k$ ;
Calculate  $b_k$  by (4.17);
for  $n = 1$  to  $\hat{N}$  do
    Find  $\eta = \arg \min \frac{m_k b_k}{\lambda_k}, \forall k = 1, 2, \dots, K$ 
    Allocate subcarrier  $n$  to user  $\eta$ ;
    Increment  $m_\eta$  by 1.
end for;

```

Figure 4.1: Pseudo-code for Subcarrier Allocation Algorithm

$\hat{N}$ subcarriers we assume that each user experiences a channel factor of

$$\Omega_k = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} \frac{|h_{nk}|^2}{\Gamma(N_0 W_s + S_{nk})}, \quad \forall k = 1, 2, \dots, K \quad (4.15)$$

$$\overline{IF} = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} IF_n, \quad (4.16)$$

on each channel, equal interference to PU and equal transmit power on each channel for all users. Therefore, the available bits loaded for k^{th} CRU on each channel can be expressed as

$$b_k = \min(\lfloor \log_2(1 + \frac{\Omega_k P_{total}}{\hat{N}}) \rfloor, \lfloor \log_2(1 + \frac{\Omega_k I_{total}}{\hat{N} \overline{IF}}) \rfloor). \quad (4.17)$$

$\forall k = 1, 2, \dots, K$

Let m_k be the number of subcarriers allocated to CRU k . Then the objective is to find a set of m_k subcarriers $k = 1, 2, \dots, K$ which satisfy

$$\max R_s = W_s \sum_{k=1}^K m_k b_k, \quad (4.18)$$

subject to

$$m_1 b_1 : m_2 b_2 : \dots : m_K b_K = \lambda_1 : \lambda_2 : \dots : \lambda_K, \quad (4.19)$$

$$P \leq P_{total}, \quad (4.20)$$

$$I \leq I_{total}, \quad (4.21)$$

where P is the total transmit power allocated to all subcarriers and I represents the total interference power to the PU. After subcarrier allocation, a bit allocation solution can be expressed as

$$\mathbf{x} = [x_1 \ x_2 \ \dots \ x_N]. \quad (4.22)$$

4.3 Fitness Landscape Analysis for Bits Allocation

4.3.1 Bit Allocation Analysis

After applying the SA algorithm to determine the subcarrier allocation to CRUs, we need to determine the bits allocation to the CRUs. From the bits allocation solution expression in (4.22), the bits allocation problem is a combinatorial optimization problem. Let $\hat{b}_n$ be the number of possible bits allocated to n^{th} subcarrier, then the steps to find the optimal bits allocation is $\mathcal{O}(\prod_{n=1}^N \hat{b}_n)$. Since $\hat{b}_n \geq 2$ for real systems, $\prod_{n=1}^N \hat{b}_n \geq 2^N$. It is computationally complex. In order to make the problem tractable, we need to find a simple yet efficient algorithm to determine the bits allocation to CRUs.

4.3.2 Representation of Fitness Landscape

Though the notion of fitness landscape was first proposed in [36] for analyzing the gene interaction in biological evolution, it was also an important technique for analyzing the behavior of combinatorial optimization problems. Furthermore, it has been extended to analyze evolutionary algorithms. From [36], [38], each genotype has a “fitness” and the distribution of fitness values over the space of genotypes constitutes a fitness landscape. In our problem, we consider the set of solutions as a search space, where the height of a point denotes the fitness of the solution associated with the point.

Thus, a heuristic algorithm can be considered to be searching through the target space to find the highest peak of the landscape. Our objective is to analyze the structure of the fitness landscape of the given problem and derive appropriate LS methods and genetic operators, so that the proposed MA can find a local peak which is close to the optimal peak of the landscape more efficiently.

Generally, for a given combinatorial optimization problem, we can define a fitness landscape as $\Omega = (X, f, d)$, where the X is the set of solutions, f denotes the objective function $f : X \rightarrow \mathbb{R}$ and d represents the hamming distance of two solutions. Based on the measurement d , we also can construct the fitness landscape as a graph set: $G = (V, E)$, where V represents the set of solution (i.e. $V = X$), E denotes the set $E = \{(x, y) \in X \times X | d(x, y) = d_{\min}\}$ where $d_{\min}$ is the minimum distance between two points in the set X . The minimum distance in our problem is $d_{\min} = 1$ and the maximum distance is $d_{\max} = N$. Accordingly, we can construct the neighborhood of a point x as $\mathcal{N}_k(x) = \{y \in X | d(x, y) \leq k\}$.

For different combinatorial optimization problems, the structures of their fitness landscapes are also different. For a fitness landscape, there are several important properties which have been proven to have significant effects on the performance of a memetic algorithm. According to the “No Free Lunch” Theorem, an algorithm will perform in a different way on different problems. In order to obtain better results, we need to select an appropriate algorithm and parameters for a given problem based on domain knowledge. Therefore, analyzing the structure of a fitness landscape of a given problem is necessary.

In this chapter, we focus on the following properties of a fitness landscape:

- the landscape ruggedness
- the number of local optimal solutions in the landscape
- the distribution of the peaks in the search space
- the number of iterations to reach a local optimum
- the structure of the basins of attraction of the local optimal solutions

For these properties, some are easily determined by statistical methods, but others are difficult to identify. For example, it is very difficult to understand the properties of the number of local optimal solutions and the number of iteration to reach a local optimum by statistical methods. The NK model proposed in [38] is an important technique for analyzing these properties. In the NK model, N refers to the number of parts of a system. Each part makes a fitness contribution which depends upon that part, and upon K other parts among the N . For example, let a solution vector $\mathbf{x} = [x_1, x_2, \dots, x_N]^T$ be a binary vector of length N , the fitness function can be expressed as

$$f(\mathbf{x}) = \frac{1}{N} \sum_{i=1}^N f_i(x_i, x_{i1}, \dots, x_{ik}), \quad (4.23)$$

where the fitness contribution f_i of locus i depends on the value of gene x_i and the values of K other genes $x_{i1}, x_{i2}, \dots, x_{ik}$. The function $f_i : \{0, 1\}^{K+1} \rightarrow \mathbb{R}$ assigns a uniformly distributed random number between 0 and 1 to each of its 2^{K+1} inputs.

In the following, we introduce some related statistical methods that have been proposed to measure the properties of a fitness landscape.

Autocorrelation functions and random walk correlation functions have been proposed to measure the ruggedness of a fitness landscape in [39]. The autocorrelation function reflects the correlation of solutions with distance d in the search space. A fitness landscape is rugged if there is a low correlation between neighboring points of the landscape, and a landscape is smooth if there is a high correlation between neighboring points [38]. Therefore, the more ruggedness in a landscape, the harder the problem for an algorithm. Let $X^2(d)$ be the set of all pairs of solutions in the search space with distance d . $X^2(d)$ can be expressed as

$$X^2(d) = \{(x, y) \in X \times X | d(x, y) = d\}, \quad (4.24)$$

and let $|X^2(d)|$ be the number of pairs in the set $X^2(d)$. The autocorrelation function can be expressed as

$$\rho(d) = \frac{\mathbb{E}(f(x)f(y))_{d(x,y)=d} - \mathbb{E}^2(f)}{\mathbb{E}(f^2) - \mathbb{E}^2(f)}, \quad (4.25)$$

where $E(\cdot)$ denotes the expectation function. In addition, the random walk function can be defined as

$$r(s) = \frac{E(f(x_t)f(x_{t+s})) - E^2(f)}{E(f^2) - E^2(f)}. \quad (4.26)$$

Based on the autocorrelation function and the random walk correlation function, the correlation length ℓ of the landscape is defined as

$$\ell = -\frac{1}{\ln(|r(1)|)} = -\frac{1}{\ln(|\rho(1)|)}, \quad (4.27)$$

for $r(1), \rho(1) \neq 0$. The correlation length directly reflects the ruggedness of a landscape: the lower the value for ℓ , the more ruggedness in the landscape.

Fitness distance correlation (FDC), which was proposed in [37] as a measure for problem difficulties for genetic algorithms, is an important method. The FDC coefficient is expressed as:

$$\begin{aligned} \varrho(f, d_{opt}) &= \frac{cov(f, d_{opt})}{\sigma(f)\sigma(d_{opt})} \\ &= \frac{E(f d_{opt}) - E(f) \cdot E(d_{opt})}{\sqrt{(E(f^2) - E^2(f)) \cdot (E(d_{opt}^2) - E^2(d_{opt}))}}, \end{aligned} \quad (4.28)$$

where d_{opt} represents the distance of a point to the nearest optimum. ϱ denotes the correlation of the fitness and the distance of solutions to the nearest optimum in the search space. When $\varrho = -1.0$, it represents that the fitness and distance to the optimum are perfectly related. In this case, crossover based memetic algorithms can find a local optimum close to the global optimum. When $\varrho = 1.0$, it indicates that the fitness and distance to the optimum are not related at all. In this case, mutation based memetic algorithms are more preferable. However, there is a shortcoming for FDC as we need to know the global optimum before applying FDC. For most optimization problems, it is impossible to know the global optimum due to high complexity. We use a local optimum obtained by a simple memetic algorithm to approximate the global optimum while calculating the FDC of a fitness landscape. In addition, fitness distance analysis (FDA) is an important technique for analyzing the correlation between the fitness and distance to the nearest optimum. FDA has been

applied to analyze the fitness landscapes of combinatorial optimization problems [74], [38]. Based on FDA, we get the distribution of local optimal solutions in the search space and then decide on the appropriate search methods for the proposed memetic algorithm.

4.3.3 Local Search Analysis

We assume that subcarriers have been allocated to CRUs and we need to allocate bits among subcarriers. A bit allocation solution can be expressed as in (4.22). Accordingly, the fitness function is defined as

$$f(\mathbf{x}) = \exp(M(\frac{\min R_k/\lambda_k}{\max R_k/\lambda_k} - 1)) \sum_{n=1}^N x_n f(I) f(P), \quad (4.29)$$

where

$$f(I) = \begin{cases} 1 & \text{when } I \leq I_{total} \\ \exp(-M(I/I_{total} - 1)) & \text{otherwise} \end{cases}$$

$$f(P) = \begin{cases} 1 & \text{when } P \leq P_{total} \\ \exp(-M(P/P_{total} - 1)) & \text{otherwise.} \end{cases}$$

with I and P denoting the total interference to PU and total transmit powers among CRUs, respectively. M is a large positive number which is sensitive to the fitness value.

Let ∇f_i be the fitness gain when adding or subtracting one bit to i^{th} subcarrier. Due to the distinct characteristics of wireless communication, the resource allocation algorithms cannot tolerate a high complexity. In order to find an optimum, even a simple greedy local search needs to compare N fitness gain ∇f_i . Suppose Θ_k to be the set of the subcarrier index corresponding to the subcarriers assigned to the k^{th} CRU. Based on the fitness definition in (4.29), we propose a simple yet efficient local search method for the proposed MA. According to [38], the algorithm complexity for this local search method is $\mathcal{O}(\ln(D - 1))$, where D is the dimensionality of the genotype space. In each search loop, when the total transmit power and interference satisfy the relevant constraints, it is not necessary for the algorithm to compare every subcarrier's fitness gain, and thus it is more efficient. The algorithm is based on the (1-opt) method and the relevant pseudo-code of the algorithm

Algorithm 4.9 Local-Search Procedure

```

( $\mathbf{x} = [x_1 \ x_2 \ \dots \ x_N] \in X$ )
repeat
  if  $f(I) < 1$  or  $f(P) < 1$  then
    find  $n = \arg \max \nabla f_n \ \forall n = 1, 2, \dots, N$ ;
  else
    find  $k = \arg \max R_k / \lambda_k$ ;
    find  $n = \arg \max \nabla f_n$ , where  $n \in \Theta_k$ ;
  end if;
  update  $x_n$ ;
until  $\nabla f_i \leq 0$ ;
return  $\mathbf{x}$ ;

```

Figure 4.2: Pseudo Code of Local Search Method

Instance	P_m	P_{total}	I_{total}	K
Instance 1	3	0.8	0.006	4
Instance 2	5	1.2	0.009	4
Instance 3	7	1.6	0.012	4
Instance 4	9	2.0	0.015	4
Instance 5	11	2.4	0.018	4
Instance 6	13	2.8	0.021	4

Table 4.1: Instances Construction

is shown in Fig. 4.2. It is simpler than the original (1-opt) algorithm because there is no complete search needed for subcarriers in each loop when $f(I) \geq 1$ and $f(P) \geq 1$.

In order to get more insight into the resource allocation problem, we consider 6 instances, which are constructed as Table 4.1. When designing a memetic algorithm for a combinatorial problem, i.e. determining which local search and which genetic operators are optimal, we need to analyze its fitness landscape. Based on these instances, we use the fitness distance correlation to analyze the distribution of local optimal solutions in the search space.

Initially, we produce 2000 local optimal solutions based on the greedy local search algorithm. In addition, we also estimate the correlation length based on the equation (4.27). The local optimal solutions are obtained by the proposed local search method. The results are shown in Table 4.2, where $\min d_{opt}$ denotes the minimum distance of the

locally optimal solutions to the expected global optimum, $\bar{d}_{opt}$ is the average distance of the locally optimal solutions to the expected global optimum, $\bar{d}_{loc}$ represents the average distance between the local optima, N_d denotes the number of distinct local optimal solutions out of 2000 and ϱ is the fitness distance correlation coefficient.

According to the NK-landscape theory in [38], $K = N - 1$ for the bits allocation problem. There is a low correlation between neighboring points of the landscape. Therefore, the fitness landscape will be rugged and the number of iteration to reach a local optimum will be small. From the table, since the $\varrho \gg -1$, there is a low correlation between fitness and distance. Compared with the number of total subcarriers, ℓ is too small. Therefore, the fitness landscape is rugged. According to the statistical property of $\bar{d}_{opt}$, the local optimal solutions are distributed in a large range. The experiment results are consistent with the NK-landscape theory in [38].

The fitness distance plots for the six instances are shown in Figs. 4.3 and 4.4. Note that the structures of the fitness landscape of the given problem under different parameters are similar since the plots for these instances are similar. Thus, we do not need to analyze the structure of the fitness landscape of a given problem on-line. We can determine LS methods and genetic operators for the proposed MA off-line, which means there is no complexity introduced into the proposed MA.

Moreover, the local optimal solutions of each instance scatter in a large range and the variation of fitness difference ∇f does not show a strong relationship with the distance to the optimum d_{opt} . These properties are consistent with the fitness distance correlation analysis in Table 4.2. Since the average distance of the population converges rapidly towards zero when the crossover operator is exclusively used in an MA. In this case, mutation based MAs will generate a better performance than that of crossover based MAs.

4.3.4 The Choice of Genetic Operators

For a memetic algorithm, we need to determine not only a good local search method, but also good genetic operators (crossover and mutation). Different experiments have shown

Instance	$\min d_{opt}$	d_{opt}	d_{loc}	N_d	ϱ	ℓ
I1	5	11.0145	10.8887	2000	-0.1447	2.3979
I2	4	11.1970	11.1198	2000	-0.0992	2.0019
I3	5	11.1620	10.5577	2000	-0.0112	2.2957
I4	5	10.9370	10.7220	2000	-0.1026	2.2358
I5	5	11.5070	11.6361	2000	-0.0746	2.7233
I6	7	11.7155	10.9887	2000	-0.0820	2.8034

Table 4.2: Average Distance and Fitness Distance Correlation of Local Search

Figure 4.3: Fitness Distance Plots for Local Search Method (Instances 1-4)

Figure 4.4: Fitness Distance Plots for Local Search Method (Instances 5-6)

that the effectiveness of these evolutionary operators strongly depends on the distribution of local optimal solutions in the search space. For the choice of the local search method, we have applied FDC and FDA to analyze the problem. From the analysis above, we note that the fitness and distance to the global optimums are highly uncorrelated. Moreover, the distribution of local optimal solutions scatters over a large range. In this case, a crossover operator may have a negligible impact on the performance of MAs. Therefore, mutation based MAs will perform better with our problem.

Based on the fitness landscape analysis on the bits allocation problem, we propose an efficient memetic algorithm for the bits allocation of the optimization problem in equation (4.7). The pseudo-code of our algorithm is shown in Fig. 4.5.

Algorithm 4.10 MA

```

Initialize Population  $P$ ; {Input:  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iN}], \forall i = 1, 2, \dots, pop\_size$ }
 $P = Local\_Search(P)$ ;
for  $i = 1$  to  $Number\_of\_Generation$  do
     $S = selectForVariation(P)$ ;
     $S' = crossover(S)$ ;
     $S' = Local\_Search(S')$ ;
    add  $S'$  to  $P$ ;
     $S'' = mutation(S)$ ;
     $S'' = Local\_Search(S'')$ ;
    add  $S''$  to  $P$ ;
     $P = selectForSurvival(P)$ ;
end for
return  $P$ . {Output:  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iN}], \forall i = 1, 2, \dots, pop\_size$ }

```

Figure 4.5: Pseudo-code for the memetic algorithm

Let $\mathbf{x}_i$ be the chromosome of member i in a population.

$$\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iN}], \quad i = 1, 2, \dots, pop_size \quad (4.30)$$

where pop_size denotes the population size. The initial integer solution vectors are randomly created within the region of admissible solutions. The original objective function of the optimization problem is evaluated as in (4.29).

4.4 Numerical Results

In this section, simulation results for the proposed MA algorithm described in Fig. 4.5 are presented. Its performance is compared to that of the RC algorithm in [24] for a number of different scenarios. For our simulations, we used Matlab.

Based on the discussion above, we proposed a mutation based memetic algorithm for our problem. The parameters values were: population size=40; generations= 20; probability of crossover=0.05; probability of mutation=0.7.

The simulated system consists of one PU and $K = 4$ CRUs. The CRU band is 5 MHz wide and supports 16 subcarriers, each with a bandwidth, W_s , of 0.3125 MHz. The PU bandwidth is $W_p = W_s$ and the OFDM symbol duration is $T_s = 4\mu s$. Three cases of the bit rate requirements for users with $\lambda = [1 \ 1 \ 1 \ 1]$, $[1 \ 1 \ 1 \ 4]$ and $[1 \ 1 \ 1 \ 8]$ were considered. It is assumed that the subcarrier gains h_{nk} and g_k , for $n \in \{1, 2, \dots, N\}$, $k \in \{1, 2, \dots, K\}$ are outcomes of independent, identically distributed (i.i.d.) Rayleigh distributed random variables (rvs) with means equal to 1. The additive white Gaussian noise (AWGN) PSD, N_0 , is set to 10^{-8} W/Hz. The PSD, $\Phi_{RR}(\exp^{jw})$, of the PU signal was assumed to be that of an elliptically filtered white noise process. The total CRU bit rate results were obtained by averaging over 1000 channel realizations.

Fig. 4.6 shows the average total CRU bit rate, R_s , as a function of the maximum tolerable interference power, I_{total} , with a PU signal power $P_m = 5$ W, $P_{total} = 1$ W and $K = 4$ for the three cases. As expected, R_s increases with I_{total} . Moreover, when the bit rate requirements for users are more uniform, the total bit rate R_s is higher because of the user diversity.

Fig. 4.7 shows the average total CRU bit rate, R_s , for the RC and MA algorithms as a function of the maximum tolerable interference power, I_{total} , with $P_{total} = 1$ W, $P_m = 5$ W with $\lambda = [1 \ 1 \ 1 \ 1]$. The bit rate obtained by the proposed MA is larger than that of the RC algorithm in [24]. In the case of $I_{total} = 0.0003$ W, the MA provides a 30% improvement in R_s .

Figs. 4.8 and 4.9 show the average total CRU bit rate, R_s , as a function of the maximum tolerable interference power, I_{total} , with $P_{total} = 1$ W, $P_m = 5$ W in the case of bit rate

Figure 4.6: Average total CRU bit rate, R_s , versus maximum tolerable interference, I_{total} , with $P_{total} = 1$ W and $P_m = 5$ W.

Figure 4.7: Average total CRU bit rate, R_s , versus maximum tolerable interference, I_{total} , with $P_{total} = 1$ W and $P_m = 5$ W for $\lambda = [1 \ 1 \ 1 \ 1]$.

Figure 4.8: Average total CRU bit rate, R_s , versus maximum tolerable interference, I_{total} , with $P_{total} = 1$ W and $P_m = 5$ W for $\lambda = [1 \ 1 \ 1 \ 4]$.

requirements $\lambda = [1 \ 1 \ 1 \ 4]$ and $\lambda = [1 \ 1 \ 1 \ 8]$, respectively. The bit rates obtained by the proposed MA in these two cases are higher than that of the RC algorithm.

4.5 Chapter Summary

Cognitive radio is a promising technology that can significantly enhance the utilization of radio spectrums. Efficient resource allocation among CRUs in an MU-OFDM based CR system can provide high throughput. However, the steps required to search for the global optimal solution increase exponentially with the number of subcarriers. In this chapter, we proposed a new approach to reduce the complexity of the dynamic resource allocation of OFDM based cognitive radio systems. Firstly, we proposed a new algorithm with a low complexity to determine the subcarrier allocation. Secondly, we proposed a memetic algorithm to determine the bit allocation since MAs outperform other traditional algorithms for many combinatorial optimization problems. As the performance of MAs for a given problem is highly dependent on the selection of local search methods and the genetic operators (e.g. crossover, mutation), to further improve the performance of the proposed MA, we propose using the fitness landscape to analyze the bit allocation problem. It has

Figure 4.9: Average total CRU bit rate, R_s , versus maximum tolerable interference, I_{total} , of the primary user with $P_{total} = 1$ W and $P_m = 5$ W in the case of $\lambda = [1 \ 1 \ 1 \ 8]$.

been shown that fitness landscape is a powerful technique for analyzing a combinatorial problem. Simulation results show that it is difficult for traditional suboptimal algorithms to find solutions which are close to the global optimal solutions and the proposed MAs are more appropriate for solving the bit allocation problem. Compared to the existing algorithm, the proposed subcarrier algorithm and MA are able to obtain a better performance.

Chapter 5

Resource Allocation with Partial Channel State Information

For wireless transmissions, a successful transmission between a transmitter and a receiver requires perfect channel state information (CSI) feedback from the receiver to the transmitter. Most authors assume that the transmitter receives the CSI perfectly when considering resource allocation in wireless communications [15], [16]. This assumption is reasonable for wireline systems, yet impractical for wireless systems. In practice, the received CSI at the transmitter is different from the current CSI due to the distinct characteristics of the wireless channel.

The power allocation for signal transmission is determined by the received CSI. When the CSI obtained by the transmitter is imperfect, some given quality of service (QoS) requirements, such as bit error rate (BER), may not be satisfied. This leads to system performance degradation. Imperfect CSI may be caused by feedback delays, estimation errors, and quantization errors.

In particular, when a mobile user is located on a high speed train, there is a difference between the transmitted frequency and the received frequency due to the Doppler effect. It will cause the real situation and the feedback CSI to be highly uncorrelated. In this case, it is difficult to maintain the given QoS. Therefore, studying the transmission under imperfect CSI becomes very important.

Related work on the transmission under imperfect CSI has appeared in [75], [43], [3], [76], [41], [77]. [77] investigates an adaptive modulation schedule and the problem of maximizing

the information transfer rate in both cases is presented in [41]. An adaptive MIMO-OFDM based on channel mean feedback is studied by [43]. Here we consider a scenario similar to [76], where the imperfect CSI is caused by Doppler effects.

In this chapter, we study resource allocation in an MU-OFDM based cognitive system under imperfect CSI and affected by the Doppler effect. In this case, the received CSI will be outdated due to the Doppler effect. The system performance will degrade. An interesting side effect is that the BER will increase. We study the effects of both perfect and imperfect CSI on the data rate while satisfying the target BER requirement. Since the relationship between the transmit power and bit rate under a given BER requirement is too complex, a simpler relationship between these two variables is needed.

We apply a statistical method and Nakagami- m distribution to further analyze the power allocation function. We derive a simpler function by approximating it.

We also study other methods to solve resource allocation for this model. An existing method, based on mean feedback, can achieve the mean BER and satisfies the given requirement under imperfect CSI. We find that this method is very sensitive to the change of the correlation coefficient. To overcome this problem, we develop a system dynamics analysis method to analyze the effect of imperfect CSI on the given BER.

We find that the effect of the correlation coefficient on the allocation scheme can be suppressed by introducing appropriate parameters. Moreover, the efficiency of the allocation scheme can be further improved. Secondly, we study how to select appropriate parameters for the proposed allocation scheme to improve its efficiency while still satisfying the given QoS requirements. Simulation results show that not only is our proposed method not sensitive to the change of the correlation coefficient, but it also achieves a better performance than the existing method.

The rest of the chapter is organized as follows: In Chapter 5.1, the model of transmission under imperfect channel state information is formulated. In Chapter 5.2, a new allocation scheme based on the approximation method is presented. Then the resource allocation problem with imperfect CSI in an MU-OFDM based CR system is discussed

in Chapter 5.3. Corresponding numerical results and the main findings are presented in Chapter 5.4 and Chapter 5.5, respectively. A novel resource allocation scheme based on control theory is derived in Chapter 5.6. Numerical results and the sub-summary are presented in Chapter 5.7 and Chapter 5.8, respectively. The chapter summary is presented in Chapter 5.9.

5.1 Transmission under Imperfect Channel State Information Formulation

Generally, when analyzing the partial CSI, there are two kinds of feedback that are considered [41]: Mean feedback and Covariance feedback. In the case of mean feedback, the channel distribution is modeled at the transmitter as $H \sim \mathcal{CN}(\mu, \alpha)$, where the mean μ denotes an estimate of channel based on the feedback, and α represents the covariance of the estimation error. In the case of covariance feedback, the channel distribution is modeled as $H \sim \mathcal{CN}(0, \Sigma)$, which denotes that the channel H varies too rapidly so that the transmitter can't track its mean.

In this section, we present the transmission schedule model. The downlink of a base station (BS) with a receiver (Rx) is considered. The transmission schedule model is illustrated in Fig. 5.1. Under the condition that the channel delay is ignored, for QAM modulation,

Figure 5.1: Transmission Model

an approximation for the bit error rate (BER) can be expressed as [77]:

$$\text{BER}(t) \approx 0.2 \exp \left(\frac{-1.6P(t)|H(t)|^2}{(2^{r(t)} - 1)\sigma^2} \right), \quad (5.1)$$

where $P(t)$ represents the transmitted power at time t , $H(t)$ denotes the channel gain, and $r(t)$ is the number of bits to be transmitted and σ^2 denotes the noise power. Re-arranging (5.1), the maximum number of bits per OFDM symbol period that can be transmitted on this channel is given by

$$r(t) = \left\lfloor \log_2 \left(1 + \frac{P(t)|H(t)|^2}{\Gamma\sigma^2} \right) \right\rfloor \quad (5.2)$$

where $\Gamma \triangleq -\ln(5\text{BER})/1.5$ and $\lfloor \cdot \rfloor$ denotes the floor function.

Equation (5.1) shows the relationship between the transmit power and the number of bits loaded on the channel for a given BER requirement when perfect CSI is available at the transmitter. We now establish an analogous relationship when only partial CSI is available.

The imperfect CSI that is available to the BS is modeled as follows. We assume that perfect CSI is available at the receiver. The channel gain, $h(t)$, is the outcome of an independent complex Gaussian random variable, i.e. $H(t) \sim \mathcal{CN}(0, \sigma_h^2)$ [76], corresponding to Rayleigh fading. For clarity, we will denote random variables and their outcomes by uppercase and lowercase letters respectively.

For notational simplicity, we will use h to denote an arbitrary channel gain. The BS receives the CSI after a feedback delay $\tau_d = dT$, where T_s is the OFDM symbol duration. We assume that the noise on the feedback link is negligible. Suppose that h_f is the channel gain information that is received at the BS. Then $h_f(t) = h(t - \tau_d)$. From [78], the correlation between H and H_f is given by

$$\text{E}\{HH_f^H\} = \rho\sigma_h^2, \quad (5.3)$$

where the correlation coefficient, ρ , is given by

$$\rho = J_0(2\pi f_d dT_s). \quad (5.4)$$

In (5.3) and (5.4), $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind, f_d is the Doppler frequency, $E\{\cdot\}$ is the expectation operator and H_f^H denotes the complex conjugate of H_f . The minimum mean square error (MMSE) estimator of H based on $H_f = h_f$ is given by [79]

$$\bar{H} = E\{H|H_f = h_f\} = \rho h_f. \quad (5.5)$$

From (5.3), the actual channel gain can be written as [42]

$$h = \bar{H} + \epsilon, \quad (5.6)$$

where $\sigma \sim \mathcal{CN}(0, \sigma_\epsilon^2)$ and $\sigma_\epsilon^2 = \sigma_h^2(1 - |\rho|^2)$. Thus, $h \sim \mathcal{CN}(\rho h_f, \sigma_\epsilon^2)$. According to [80], for arbitrary vector $\alpha \sim \mathcal{CN}(\mu, \Sigma)$, the following equation holds:

$$E(\exp(-\alpha^H \alpha)) = \frac{\exp(-\mu^H (\mathbf{I} + \Sigma)^{-1} \mu)}{\det(\mathbf{I} + \Sigma)}, \quad (5.7)$$

Apply (5.7) to (5.1), we get

$$E(\text{BER}(t)) = 0.2 \frac{1}{1 + \Psi \sigma_\epsilon^2} \exp\left(-\frac{\Psi |\bar{H}(t)|^2}{1 + \Psi \sigma_\epsilon^2}\right), \quad (5.8)$$

where $\Psi = 1.5P(t)/\{(2^{r(t)} - 1)\sigma^2\}$. Based on this equation, we would like to derive the allocation scheme which is expressed as:

$$r(t) = f(E(\text{BER}(t)), \bar{H}(t), \sigma^2, \sigma_\epsilon^2). \quad (5.9)$$

Obviously, when $\rho = 1$, $\sigma_\epsilon = 0$. The right-hand side (RHS) of (5.8) turns to be the case of (5.1). In the case of $|\rho| < 1$, the equation in (5.8) is not an explicit function of $r(t)$. We need to derive an explicit function of $r(t)$ first. Note that $E(\text{BER}(t))$ is a decreasing function of Ψ . When we derive the allocation scheme as in equation (5.9), we just apply the allocation scheme

$$r'(t) = f(\text{BER}_{\text{target}}, \bar{H}(t), \sigma^2, \sigma_\epsilon^2) \quad (5.10)$$

to guarantee the requirement

$$E(\text{BER}(t)) \leq \text{BER}_{\text{target}}, \quad (5.11)$$

where $\text{BER}_{\text{target}}$ denotes the target BER.

5.2 Resource Allocation Scheme Based on Approximation

In this section, we consider the problem of allocating resources on the downlink of an MU-OFDM based CR system in which a base station (BS) serves one PU and K CRUs. The basic system model is the same as that described in [24] and is summarized here for the convenience of the reader.

The PU and CRUs occupy neighboring frequency bands as shown in Fig. 2.1. Where solid lines denote OFDM subcarriers available for CRUs and dotted lines represent the subbands occupied by PUs and guard bands. The PU channel is W_p Hz wide and the bandwidth of each OFDM subchannel is W_s Hz. On either side of the PU channel, there are $N/2$ OFDM subchannels. The BS has only partial CSI and allocates subcarriers, transmit powers and bits to the CRUs once every OFDM symbol period. The channel gain of each subcarrier is assumed to be constant during an OFDM symbol duration.

Let $\Phi_n(f)$ be the power spectral density (PSD) of the n^{th} subcarrier signal, $I_n(d_n, P_n)$ be the interference power spilling into the PU band and IF_n be the interference factor for subcarrier n . Then we have

$$I_n(d_n, P_n) = P_n \cdot IF_n. \quad (5.12)$$

Let $S_{nk}(d_n)$ be the interference power introduced by the signal destined for the PU into the band of subcarrier n at user k . Let P_{nk} denote the transmit power allocated to CRU k on subcarrier n . For QAM modulation, an approximation for the BER on subcarrier n of CRU k is [15]

$$BER[n] \approx 0.2 \exp \left(\frac{-1.5|h_{nk}|^2 P_{nk}}{(2^{b_{nk}} - 1)(N_0 W_s + S_{nk})} \right), \quad (5.13)$$

where N_0 is the one-sided noise PSD and S_{nk} is given by (4.3). Re-arranging (5.13), the maximum number of bits per OFDM symbol period that can be transmitted on this subcarrier is given by

$$b_{nk} = \left\lfloor \log_2 \left(1 + \frac{|h_{nk}|^2 P_{nk}}{\Gamma(N_0 W_s + S_{nk})} \right) \right\rfloor, \quad (5.14)$$

where $\Gamma \triangleq -\ln(5BER[n])/1.5$ and $\lfloor \cdot \rfloor$ denotes the floor function. From (5.14), note that b_{nk} is an integer variable and we have

$$b_{nk} \leq \log_2 \left(1 + \frac{|h_{nk}|^2 P_{nk}}{\Gamma(N_0 W_s + S_{nk})} \right). \quad (5.15)$$

Obviously, the minimum transmit power requirement for transmit b_{nk} bits for CRU k on subcarrier n can be expressed as

$$P_{nk} = \frac{(2^{b_{nk}} - 1)\Gamma(N_0 W_s + S_{nk})}{|h_{nk}|^2}. \quad (5.16)$$

The imperfect channel state estimation process is modeled as Chapter 5.1. Based on the partial CSI available at the BS, we wish to maximize the total CRU transmission rate, while maintaining a target BER performance on each subcarrier and satisfying PU interference and total BS CRU transmit power constraints. Let $\overline{BER}[n]$ denote the average BER on subcarrier n and $\overline{BER}_0$ represent the prescribed target BER. The optimization problem can be expressed as follows:

$$\max R_s \triangleq W_s \sum_{n=1}^N \sum_{k=1}^K a_{nk} b_{nk}, \quad (5.17)$$

subject to

$$\overline{BER}[n] \leq \overline{BER}_0, \quad \forall n \quad (5.18)$$

$$\sum_{k=1}^K \sum_{n=1}^N a_{nk} P_{nk} \leq P_{total}, \quad (5.19)$$

$$P_{nk} \geq 0, \quad \forall n, k \quad (5.20)$$

$$\sum_{k=1}^K \sum_{n=1}^N a_{nk} P_{nk} I F_n \leq I_{total}, \quad (5.21)$$

$$\sum_{k=1}^K a_{nk} \leq 1, \quad \forall n \quad (5.22)$$

$$a_{nk} \in \{0, 1\}, \quad \forall n, k \quad (5.23)$$

$$R_1 : R_2 : \dots : R_K = \lambda_1 : \lambda_2 : \dots : \lambda_K, \quad (5.24)$$

where P_{total} is the total power budget for all CRUs, I_{total} is the maximum interference power that can be tolerated by the PU and $a_{nk} \in \{0, 1\}$ is a subcarrier assignment indicator, i.e.

$a_{nk} = 1$ if and only if subcarrier n is allocated to CRU k . The term λ_k represents the nominal bit rate weight (NBRW) for CRU k , and

$$R_k = W_s \sum_{n=1}^N a_{nk} b_{nk}, \quad \forall k = 1, 2, \dots, K$$

denotes the total bit rate achieved by CRU k . Constraint (5.18) ensures that the average BER for each subcarrier is below the given BER target. When a stronger constraint (e.g., $\text{Prob}(\text{BER}(n) \geq \text{BER}_0) < \alpha$, where α is the tolerable threshold, is defined, the problem becomes more complex. Constraint (5.19) states the total power allocated to all CRUs cannot exceed P_{total} , while constraint (5.21) ensures that the interference power to the PU is maintained below an acceptable level I_{total} . Constraint (5.22) results from the assumption that each subcarrier can be assigned to a maximum of one CRU. Constraint (5.24) ensures that the bit rate achieved by a CRU satisfies a proportional fairness condition.

Based on (5.6), we calculate the average of the right-hand side (RHS) of (5.13), treating h_{nk} as an outcome of an independent complex Gaussian variable. Applying (5.7) to (5.13), we obtain

$$\overline{\text{BER}}[n] \approx 0.2 \frac{1}{1 + \Psi \sigma_\epsilon^2} \exp \left(-\frac{\Psi |\bar{H}_{nk}|^2}{1 + \Psi \sigma_\epsilon^2} \right), \quad (5.25)$$

where $\bar{H}_{nk} = \rho h_{nk}^f$ and $\Psi = 1.5 P_{nk} / \{(2^{b_{nk}} - 1)(N_0 W_s + S_{nk})\}$. h_{nk}^f denotes the channel gain that is fed back to the BS.

From (5.25), an explicit relationship between the minimum transmit power and the number of loaded bits cannot be easily derived. However, since $\overline{\text{BER}}[n]$ in (5.25) is a monotonically decreasing function of P_{nk} , we obtain the minimum power requirement while satisfying the constraint in (5.18) by setting $\overline{\text{BER}}[n] = \overline{\text{BER}}_0$.

Ideally, when $\rho = 1$, note that $\sigma_\epsilon = 0$ and the RHS of equation (5.25) turns to be the form in the RHS of equation (5.13). It indicates that the relationship between the minimum transmit power requirement and its corresponding bits does not change. Now we consider the case of $|\rho| < 1$ (i.e. $\sigma_\epsilon \neq 0$). From the equation in (5.25), we cannot derive an explicit relationship between minimum transmit power and its corresponding bits directly.

We also need to ensure the constraint in (5.18) holds. By analyzing the RHS of equation (5.25), we note that $\overline{BER}[n]$ is a monotonically decreasing function of P_{nk} . Therefore, we obtain the minimum power requirement while satisfying the constraint in (5.18) by setting $\overline{BER}[n] = \overline{BER}_0$ in equation (5.25).

However, deriving the minimum required transmit power P_{nk} as a function of $\overline{BER}[n]$ and b_{nk} from equation (5.25) is not trivial. In this case, the minimum required transmit power P_{nk} can be obtained by iterative algorithm. This will be a burden for the allocation schedule. In this chapter, we propose a method for approximating the RHS of equation (5.25).

We now derive a simpler, albeit approximate, relationship between the required transmit power, $\overline{BER}$ and the number of loaded bits.

When setting $\mathcal{K}_\mu = |\bar{H}_{nk}|^2/\sigma_\epsilon^2$, $l = 1.5P_{nk}/(N_0W_s + S_{nk})$, $g = 1/(2^{b_{nk}} - 1)$ and $\bar{\gamma} = (1 + \mathcal{K}_\mu)\sigma_\epsilon^2 l$, the RHS of equation (5.1) has the form

$$I_\mu(\bar{\gamma}, g, \theta) = \frac{(1 + \mathcal{K}_\mu) \sin^2 \theta}{(1 + \mathcal{K}_\mu) \sin^2 \theta + g\bar{\gamma}} \exp \left(-\frac{\mathcal{K}_\mu g \bar{\gamma}}{(1 + \mathcal{K}_\mu) \sin^2 \theta + g\bar{\gamma}} \right) \quad (5.26)$$

with $\theta = \pi/2$. The function $I_\mu(\bar{\gamma}, g, \theta)$ is Rician distributed with Rician factor $\mathcal{K}_\mu$ [42]. Clearly, for a Rician distribution with $\mathcal{K}_\mu$, we can approximate it by a Nakagami- m distribution [81]

$$\tilde{I}_\mu(\bar{\gamma}, g, \theta) = \left(1 + \frac{g\bar{\gamma}}{m_\mu \sin^2 \theta} \right)^{-m_\mu} \quad (5.27)$$

with $\theta = \pi/2$, where $m_\mu = \frac{(1+\mathcal{K}_\mu)^2}{1+2\mathcal{K}_\mu}$. Therefore, we can approximate the RHS of (5.25) by

$$\text{E}(\text{BER}[n]) \approx 0.2 \left(1 + \frac{(\sigma_\epsilon^2 + |\bar{H}_{nk}|^2)\Psi}{m_\mu} \right)^{-m_\mu}. \quad (5.28)$$

Then from (5.28), we obtain

$$P_{nk} \approx \frac{((5 \text{E}(\text{BER}[n]))^{-1/m_\mu} - 1)m_\mu}{\sigma_\epsilon^2 + |\bar{H}_{nk}|^2} \cdot \Upsilon, \quad (5.29)$$

where $\Upsilon = (2^{b_{nk}} - 1)(N_0W_s + S_{nk})/1.5$. From (5.29), we obtain

$$b_{nk} = \left\lfloor \log_2 \left(1 + \frac{P_{nk}(\sigma_\epsilon^2 + |\bar{H}_{nk}|^2)}{\Gamma' \sigma^2} \right) \right\rfloor, \quad (5.30)$$

where $\Gamma' = m_\mu((5\overline{BER}_0)^{-1/m_\mu} - 1)/1.5$.

5.3 Resource Allocation with Partial CSI

Note that the joint subcarrier, bit and power allocation problem in (5.17)-(5.24) belongs to the mixed integer nonlinear programming (MINP) class [82]. For brevity, we use the term “bit allocation” to denote both bit and power allocation. Since the optimization problem in (5.17)-(5.24) is generally computationally complex, we first use a suboptimal algorithm, which is based on a greedy approach, to solve the subcarrier allocation problem in Chapter 5.3.1. After subcarriers are allocated to CRUs, we apply a memetic algorithm (MA) to solve the bit allocation problem in Chapter 5.3.2.

5.3.1 Subcarrier Allocation

From (5.24), it can be seen that the subcarrier allocation depends not only on the channel gains, but also on the number of bits allocated to each subcarrier. Moreover, allocation of subcarriers close to the PU band should be avoided in order to reduce the interference power to the PU to a tolerable level. Therefore, we use a threshold scheme to select subcarriers for CRUs.

Suppose that $\hat{N}$ subcarriers are available for allocating to CRUs. We assume equal transmit power for each subcarrier. Let

$$\Omega_k = \frac{1}{\hat{N}} \sum_{n=1}^N \frac{|\bar{H}_{nk}|^2 + \sigma_\epsilon^2}{\Gamma(N_0 W_s + S_{nk})}, \quad \forall k = 1, 2, \dots, K \quad (5.31)$$

$$\overline{IF} = \frac{1}{\hat{N}} \sum_{n=1}^{\hat{N}} IF_n. \quad (5.32)$$

If a subcarrier is assigned to CRU k , the maximum number of bits which can be loaded on the subcarrier is given by

$$b_k = \min \left(\left\lfloor \log_2 \left(1 + \frac{\Omega_k P_{total}}{\hat{N}} \right) \right\rfloor, \left\lfloor \log_2 \left(1 + \frac{\Omega_k I_{total}}{\hat{N} \overline{IF}} \right) \right\rfloor \right), \quad \forall k = 1, 2, \dots, K \quad (5.33)$$

Using (5.31)-(5.33), we can determine the number of subcarriers assigned to each CRU as follows. Let m_k be the number of subcarriers allocated to CRU k . Assuming that the

same number of bits are loaded on every subcarrier assigned to a given CRU, the objective in (5.17) is equivalent to finding a set of $\{m_1, m_2, \dots, m_K\}$ subcarriers to maximize

$$R_s \triangleq W_s \sum_{k=1}^K m_k b_k \quad (5.34)$$

subject to

$$m_1 b_1 : m_2 b_2 : \dots : m_K b_K = \lambda_1 : \lambda_2 : \dots : \lambda_K, \quad (5.35)$$

$$P \leq P_{total}, \quad (5.36)$$

$$I \leq I_{total}, \quad (5.37)$$

where P is the total transmit power allocated to all subcarriers and I is the total interference power experienced by the PU due to CRU signals. The subcarrier allocation problem in (5.34)-(5.37) can be solved using the SA algorithm proposed in [83]. Note that we need to make use of (5.29) in the SA algorithm if only partial CSI is available. A pseudo code listing for the SA algorithm is shown in Fig 5.2. The algorithm has a relatively low computational complexity $\mathcal{O}(KN)$. After subcarriers are allocated to CRUs, we then determine the number, b_n , of bits allocated to subcarrier n .

5.3.2 Bit Allocation

Memetic algorithms (MAs) are evolutionary algorithms which have been shown to be more efficient than standard genetic algorithms (GAs) for many combinatorial optimization problems [2], [29], [30]. Using (5.29), the bit allocation problem can be solved using the MA algorithm proposed in [83]. It should be noted that the chosen genetic operators and local search methods greatly influence the performance of MAs. The selection of these parameters for the given optimization problem is based on the results in [83]. A pseudo code listing of the proposed memetic algorithm is shown in Fig. 5.3.

Let $\mathbf{x}_i$ be the chromosome of member i in a population, expressed as

$$\mathbf{x}_i = [x_{i1} \ x_{i2} \ \dots \ x_{iN}], \forall i = 1, 2, \dots, pop_size \quad (5.38)$$

where, pop_size denotes the population size. A brief description of the MA in [83] is now provided.

Algorithm: SA

```

for  $n = 1$  to number of subcarriers do
  find  $k^* \in \{1, 2, \dots, K\}$  which maximizes  $\arg \max \frac{|\bar{H}_{nk}|^2 + \sigma_e^2}{\Gamma'(N_0 W_s + S_{nk})}$ ;
  Using (5.30), calculate the number of bits loaded on subcarrier  $n$  as  $b_{nk^*}$  with  $P_{nk^*} = \frac{P_{total}}{N}$ ;
  initialize  $\hat{N}$  to 0;
  if  $b_{nk^*} > 2$  then
    subcarrier  $n$  is available;
    increment  $\hat{N}$  by 1;
  else
    subcarrier  $n$  is not available;
  end if
end for
For each  $k \in \{1, 2, \dots, K\}$ , set the number,  $m_k$ , of subcarriers allocated to CRU  $k$  to 0;
calculate  $b_k$  using (5.33);
for  $n = 1$  to  $\hat{N}$  do
  find the value,  $\eta$ , of  $k \in \{1, 2, \dots, K\}$  which minimizes  $\frac{m_k b_k}{\lambda_k}$ ;
  allocate subcarrier  $n$  to CRU  $\eta$ ;
  increment  $m_\eta$  by one.
end for

```

Figure 5.2: Pseudo-code for Subcarrier Allocation Algorithm

Algorithm: MA

```

initialize Population  $P$ ; {Input:  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iN}], \forall i = 1, 2, \dots, pop\_size\}$ 
 $P = Local\_Search(P)$ ;
for  $i = 1$  to Number_of_Generation do
   $S = selectForVariation(P)$ ;
   $S' = crossover(S)$ ;
   $S' = Local\_Search(S')$ ;
  add  $S'$  to  $P$ ;
   $S'' = mutation(S)$ ;
   $S'' = Local\_Search(S'')$ ;
  add  $S''$  to  $P$ ;
   $P = selectForSurvival(P)$ ;
end for
return  $P$ . {Output:  $\mathbf{x}_i = [x_{i1}, x_{i2}, \dots, x_{iN}], \forall i = 1, 2, \dots, pop\_size\}$ 

```

Figure 5.3: Pseudo-code for the Memetic Algorithm

- (1) The *selectForVariation* function selects a set, $S = \{s_1, s_2, \dots, s_{pop_size}\}$, of chromosomes from P in a roulette wheel fashion, i.e. selection with replacement.
- (2) Crossover: Suppose that $S = \{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_{pop_size}\}$. Let P_{cross} denote the crossover probability and $u_i, i = 1, 2, \dots, pop_size$ denote the outcome of an independent random variable which is uniformly distributed in $[0, 1]$. Then, $\mathbf{y}_i$ is selected as a candidate for crossover if and only if $u_i \leq P_{cross}, i = 1, 2, \dots, pop_size$. Suppose that we have n_c such candidates. We then form $\lfloor n_c/2 \rfloor$ disjointed pairs of candidates (parents).

For a pair of parents $\mathbf{y}_i$ and $\mathbf{y}_j$,

$$\mathbf{y}_i = [y_{i1} \ y_{i2} \ \dots \ y_{ip} \ y_{i(p+1)} \ \dots \ y_{iN}],$$

$$\mathbf{y}_j = [y_{j1} \ y_{j2} \ \dots \ y_{jp} \ y_{j(p+1)} \ \dots \ y_{jN}],$$

we first generate a random integer $p \in [1, N-1]$. Then obtain the (possibly identical) chromosomes of two children as follows:

$$\mathbf{y}'_i = [y_{i1} \ y_{i2} \ \dots \ y_{ip} \ y_{j(p+1)} \ \dots \ y_{jN}],$$

$$\mathbf{y}'_j = [y_{j1} \ y_{j2} \ \dots \ y_{jp} \ y_{i(p+1)} \ \dots \ y_{iN}].$$

- (3) Mutation: Let $P_{mutation}$ denote the mutation probability. For each chromosome in S , we generate $u_i, i = 1, 2, \dots, N$, where u_i denotes the outcome of an independent random variable which is uniformly distributed in $[0, 1]$. Then for each component i for which $u_i \leq P_{mutation}$, we substitute the value with a randomly chosen admissible value.
- (4) Selection of surviving chromosomes: We select the *pop_size* chromosomes of parents and offspring with the best fitness values as input for the next generation.

5.4 Numerical Results

In this section, performance results for the proposed algorithms described in Chapter 5.3 are presented. In the simulation, the parameters of the MA algorithm were chosen as follows: population size, $pop_size = 40$; number of generations = 20; crossover probability, $P_{cross} = 0.05$; mutation probability, $P_{mutation} = 0.7$. For our simulations, we used Matlab.

We consider a system with one PU and $K = 4$ CRUs. The total available bandwidth for CRUs is 5 MHz and supports 16 subcarriers with $W_s = 0.3125$ MHz. We assume $W_p = W_s$ and an OFDM symbol duration, T_s of $4\mu s$. In order to understand the impact of the fair bit rate constraint in (5.24) on the total bit rate, three cases of user bit rate requirements with $\lambda = [1 \ 1 \ 1 \ 1], [1 \ 1 \ 1 \ 4], [1 \ 1 \ 1 \ 8]$ were considered. In addition, three cases of partial CSI with $\rho = 1, 0.9$ and 0.7 were studied. It is assumed that the subcarrier gains h_{nk} and g_k , for $n \in \{1, 2, \dots, N\}, k \in \{1, 2, \dots, K\}$ are outcomes of independent, identically distributed (i.i.d.) Rayleigh distributed random variables (rvs) with mean square value $E(|H_{nk}|^2) = E(|G_k|^2) = 1$. The additive white Gaussian noise (AWGN) PSD, N_0 , was set to 10^{-8} W/Hz. The PSD, $\Phi_{RR}(f)$, of the PU signal was assumed to be that of an elliptically filtered white noise process. The total CRU bit rate, R_s , results were obtained by averaging over 10,000 channel realizations. The 95% confidence intervals for the simulated R_s results are within $\pm 1\%$ of the average values shown.

Fig. 5.4 shows the average total bit rate, R_s , as a function of the total CRU transmit power, P_{total} , for $\rho = 0.7, 0.9$ and 1 with $\lambda = [1 \ 1 \ 1 \ 1]$, $I_{total} = 0.02$ W and a PU transmit power, P_m , of 5W. As expected, the average total bit rate increases with the maximum transmit power budget P_{total} . It can be seen that the average total bit rate, R_s , varies greatly with ρ . For example, at $P_{total} = 5$ W, R_s increases by a factor of 2 as ρ increases from 0.7 to 0.9. This illustrates the big impact that inaccurate CSI may have on system performance. The R_s curves level off as P_{total} increases due to the fixed value of the maximum interference power that can be tolerated by the PU.

Corresponding results for $\lambda = [1 \ 1 \ 1 \ 4]$ and $\lambda = [1 \ 1 \ 1 \ 8]$ are plotted in Figs. 5.5 and 5.6, respectively. The average total bit rate, R_s , decreases as the NBRW distribution becomes

Figure 5.4: Average total CRU bit rate, R_s , versus total CRU transmit power, P_{total} , with $I_{total} = 0.02\text{W}$ and $P_m = 5\text{W}$ in the case of $\lambda = [1 \ 1 \ 1 \ 1]$.

Figure 5.5: Average total CRU bit rate, R_s , versus maximum transmit power budget, P_{total} , with $I_{total} = 0.02\text{W}$ and $P_m = 5\text{W}$ in the case of $\lambda = [1 \ 1 \ 1 \ 4]$.

less uniform; the reduction tends to increase with P_{total} .

Fig. 5.7 shows R_s as a function of P_{total} for three different cases of λ with $\rho = 0.9$, $I_{total} = 0.02\text{W}$ and $P_m = 5\text{W}$. As to be expected, R_s increases with P_{total} . It can be seen that R_s for $\lambda = [1 \ 1 \ 1 \ 1]$ is larger than for $\lambda = [1 \ 1 \ 1 \ 4]$ and R_s for $\lambda = [1 \ 1 \ 1 \ 4]$ is larger

Figure 5.6: Average total CRU bit rate, R_s , versus maximum transmit power budget, P_{total} , with $I_{total} = 0.02\text{W}$ and $P_m = 5\text{W}$ in the case of $\lambda = [1 \ 1 \ 1 \ 8]$.

than for $\lambda = [1 \ 1 \ 1 \ 8]$. When the bit rate requirements for CRUs become less uniform, R_s decreases due to a lowering in the benefits of user diversity. With $P_{total} = 15\text{W}$, R_s increases by about 30% when λ changes from $[1 \ 1 \ 1 \ 8]$ to $[1 \ 1 \ 1 \ 1]$. Results for $\rho = 0.7$ are shown in Fig. 5.8 and are qualitatively similar to those in Fig. 5.7.

Figure 5.7: Average total CRU bit rate, R_s , versus maximum transmit power budget, P_{total} , with $I_{total} = 0.02\text{W}$ and $P_m = 5\text{W}$ in the case of $\rho = 0.9$.

The average total bit rate, R_s , is plotted as a function of the maximum PU tolerable interference power, I_{total} , with $P_{total} = 25\text{W}$ and $P_m = 5\text{W}$, for $\rho = 0.9$ and 0.7 in Figs. 5.9 and 5.10 respectively. As expected, R_s increases with I_{total} and decreases as the CRU bit rate requirements become less uniform. The R_s curves level off as I_{total} increases due to the fixed value of the total CRU transmit power, P_{total} .

Figure 5.8: Average total CRU bit rate, R_s , versus maximum transmit power budget, P_{total} , with $I_{total} = 0.02\text{W}$ and $P_m = 5\text{W}$ in the case of $\rho = 0.7$.

5.5 Sub-Summary

The assumption of perfect CSI being available at the transmitter is often unreasonable in a wireless communication system. In Chapter 5.2, we studied an MU-OFDM CR system in which the available partial CSI is due to a delay in the feedback channel. The effect of partial CSI on the BER was investigated and a relationship between transmit power, number of bits loaded and BER was derived. This relationship was used to study the performance of a resource allocation scheme when only partial CSI is available. It is found that the performance varies greatly with the quality of the partial CSI.

Figure 5.9: Average total CRU bit rate, R_s , versus maximum interference power, I_{total} , with $P_{total} = 25\text{W}$ and $P_m = 5\text{W}$ in the case of $\rho = 0.9$.

Figure 5.10: Average total CRU bit rate, R_s , versus maximum interference power, P_{total} , with $P_{total} = 25\text{W}$ and $P_m = 5\text{W}$ in the case of $\rho = 0.7$.

5.6 Dynamical Control Based Resource Allocation Model

The assumption of perfect CSI being available at the transmitter is often unreasonable in a wireless communication system due to feedback delays, estimation errors, and quantization errors. In Chapter 5.3, we investigated the effects of partial CSI on the BER and found

that the correlation coefficient has an important impact on the system performance, which is sensitive to the changes of the correlation coefficient. When the correlation coefficient decreases, the resource allocation schemes are conservative, leading to a drastic decrease in the corresponding total data rate and thus deteriorated spectral efficiency. This motivates us to further study how to suppress the impact of the correlation coefficient.

Some basic introduction to discrete control systems is described in Chapter 5.6.1. In Chapter 5.6, a novel resource allocation scheme is developed by introducing a control parameter to the original one from the viewpoint of control theory. We first analyze the effects of partial CSI on the corresponding BER. More concretely, the channel is affected by the Doppler effect, leading the channel gain to keep changing. We then apply the mean feedback to model the feedback channel. We consider the allocation scheme as a discrete system and introduce a control parameter to the allocation scheme. Since the derived difference equations are nonlinear, we linearize it and analyze the stability of the equilibrium point. We find that the system is locally asymptotically stable in case of appropriate parameter values. We also find that the equilibrium point changes according to the introduced parameters. Simulation results show that that the proposed allocation scheme not only suppresses the effect of the correlation coefficient, but also improves the spectral efficiency by selecting appropriate parameters.

5.6.1 Discrete Control Systems

Before discussing the proposed resource allocation scheme, we first introduce dynamic systems. Generally, dynamic systems, described by difference equations, are referred to as discrete-time systems. Fig. 5.11 shows a general discrete-time control system.

From Fig. 5.11, note that a discrete-time control system can be described as

$$\mathbf{x}(k+1) = \mathbf{A}(k)\mathbf{x}(k) + \mathbf{B}(k)\mathbf{u}(k), \quad (5.39)$$

$$\mathbf{y}(k) = \mathbf{C}(k)\mathbf{x}(k) + \mathbf{D}(k)\mathbf{u}(k). \quad (5.40)$$

Where $\mathbf{x}(k)$ and $\mathbf{y}(k)$ is state vector and output vector, respectively. $\mathbf{A}(k)$, $\mathbf{B}(k)$, $\mathbf{C}(k)$ and $\mathbf{D}(k)$ are system matrices.

Figure 5.11: Discrete-time Control System Diagram.

For the simplicity of representation, we assume that the system matrix $\mathbf{A}(k)$ is constant. Thus, we can omit the index k . Now we consider the homogeneous case first:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k). \quad (5.41)$$

We assume the initial conditions $\mathbf{x}(0)$ are known, so that $\mathbf{x}(1) = \mathbf{A}\mathbf{x}(0)$. Then we get $\mathbf{x}(2) = \mathbf{A}\mathbf{x}(1) = \mathbf{A}^2\mathbf{x}(0)$. Continuing this process, we get $\mathbf{x}(k)$ at time slot k in terms of $\mathbf{x}(0)$ as

$$\mathbf{x}(k) = \mathbf{A}^k\mathbf{x}(0). \quad (5.42)$$

For the nonhomogeneous case, assume a sequence of input vectors $\mathbf{u}(0), \mathbf{u}(1), \mathbf{u}(2), \dots$

and initial condition $\mathbf{x}(0)$ are given. Then from equation (5.39), we obtain

$$\mathbf{x}(1) = \mathbf{A}\mathbf{x}(0) + \mathbf{B}(0)\mathbf{u}(0) \quad (5.43)$$

$$\mathbf{x}(2) = \mathbf{A}\mathbf{x}(1) + \mathbf{B}(1)\mathbf{u}(1) = \mathbf{A}^2\mathbf{x}(0) + \mathbf{A}\mathbf{B}(0)\mathbf{u}(0) + \mathbf{B}(1)\mathbf{u}(1) \quad (5.44)$$

$$\begin{aligned} \mathbf{x}(3) &= \mathbf{A}\mathbf{x}(2) + \mathbf{B}(2)\mathbf{u}(2) \\ &= \mathbf{A}^3\mathbf{x}(0) + \mathbf{A}^2\mathbf{B}(0)\mathbf{u}(0) + \mathbf{A}\mathbf{B}(1)\mathbf{u}(1) + \mathbf{B}(2)\mathbf{u}(2) \end{aligned} \quad (5.45)$$

$$\begin{aligned} &\vdots \\ \mathbf{x}(k) &= \mathbf{A}^k\mathbf{x}(0) + \sum_{j=0}^{k-1} \mathbf{A}^{k-1-j}\mathbf{B}(j)\mathbf{u}(j). \end{aligned} \quad (5.46)$$

A change in the dummy summation index in equation (5.46) leads to

$$\mathbf{x}(k) = \mathbf{A}^k\mathbf{x}(0) + \sum_{j=1}^k \mathbf{A}^{k-j}\mathbf{B}(j-1)\mathbf{u}(j-1). \quad (5.47)$$

Let the discrete transition matrix

$$\Phi(k, j) = \mathbf{A}^{k-j}, \quad (5.48)$$

then we get the solution

$$\mathbf{x}(k) = \Phi(k, 0)\mathbf{x}(0) + \sum_{j=1}^k \Phi(k, j)\mathbf{B}(j-1)\mathbf{u}(j-1). \quad (5.49)$$

Now we analyze the stability of the system (5.39)-(5.40).

Consider a ball rolling on the smooth surface shown in Fig. 5.12. The ball can rest at points A, B, C, D and E. Each of these points is an equilibrium point of the system. From the figure, we note that an infinite small perturbation from A and D will cause the ball to diverge from these two points. Thus A and D are unstable equilibrium points. On the other hand, the ball will eventually return to these points after small perturbation away from points B, C and E.

Definition 5.1 *The origin is a stable equilibrium point if for any given value $\epsilon > 0$ there exists a number $\delta(\epsilon, k_0) > 0$ such that if $\|\mathbf{x}(k_0)\| < \delta$, then the resulting motion $\mathbf{x}(k)$ satisfies $\|\mathbf{x}(k)\| < \epsilon$ for all $k > k_0$ [84].*

Figure 5.12: Equilibrium point of the system.

Definition 5.2 *The origin is an asymptotically stable equilibrium point if*

- (a) *it is stable, and if in addition,*
- (b) *there exists a number $\delta'(k_0) > 0$ so that whenever $\|\mathbf{x}(k_0)\| < \delta'(k_0)$ the resulting motion satisfies $\lim_{k \rightarrow \infty} \|\mathbf{x}(k)\| = 0$.*

When the input $\mathbf{u}(k)$ is nonzero, one additional type of stability is defined.

Definition 5.3 *(Bounded input, bounded output stability.) Let $\mathbf{u}$ be a bounded input with K_m as the least square bound. If there exists a scalar α so that for every k , the output satisfies $\|\mathbf{y}\| \leq \alpha K_m$, then the system is bounded input, bounded output stable, abbreviated BIBO stable.*

If the characteristic polynomial for matrix $\mathbf{A}$ is written as

$$|\mathbf{A} - \mathbf{I}\lambda| = (-\lambda)^n + c_{n-1}\lambda^{n-1} + c_{n-2}\lambda^{n-2} + \cdots + c_1\lambda + c_0 = \Delta(\lambda), \quad (5.50)$$

then the corresponding matrix polynomial is

$$\Delta(\mathbf{A}) = (-1)^n \mathbf{A}^n + c_{n-1} \mathbf{A}^{n-1} + c_{n-2} \mathbf{A}^{n-2} + \cdots + c_1 \mathbf{A} + c_0 \mathbf{I}. \quad (5.51)$$

Theorem 5.1 *Cayley-Hamilton Theorem: Every matrix satisfies its own characteristic polynomial equation; that is, $\Delta(\mathbf{A}) = \mathbf{0}$.*

Proof: Ideally, when $\mathbf{A}$ is a diagonal matrix, it is easy to derive the equation. When $\mathbf{A}$ is not a diagonal matrix, we can always find a nonsingular matrix $\mathbf{M}$ so that

$$\mathbf{A} = \mathbf{M} \mathbf{\Lambda} \mathbf{M}^{-1} \quad (5.52)$$

where $\mathbf{\Lambda}$ is a diagonal matrix. Then we get

$$\mathbf{A}^2 = \mathbf{M} \mathbf{\Lambda}^2 \mathbf{M}^{-1}, \dots, \mathbf{A}^k = \mathbf{M} \mathbf{\Lambda}^k \mathbf{M}^{-1}$$

and

$$\Delta(\mathbf{A}) = \mathbf{M} [(-1)^n \mathbf{\Lambda}^n + c_{n-1} \mathbf{\Lambda}^{n-1} + \cdots + c_1 \mathbf{\Lambda} + c_0 \mathbf{I}] \mathbf{M}^{-1}. \quad (5.53)$$

Since each term inside the brackets is a diagonal matrix, the summation is still a diagonal matrix. The diagonal element has the form

$$(-\lambda_i)^n + c_{n-1} \lambda_i^{n-1} + c_{n-2} \lambda_i^{n-2} + \cdots + c_1 \lambda_i + c_0,$$

where λ_i is a root of the characteristic equation in (5.50). Therefore, the summation is equal to zero and

$$\Delta(\mathbf{A}) = \mathbf{M} [\mathbf{0}] \mathbf{M}^{-1} = \mathbf{0}. \quad (5.54)$$

According to the Cayley-Hamilton Theorem, the eigenvalues of transition matrix $\Phi(k, 0)$ are related to the eigenvalues of $\mathbf{A}$ by

$$\alpha_i = \lambda_i^k. \quad (5.55)$$

We can easily derive the stability conditions in terms of the eigenvalues of the system matrix $\mathbf{A}$.

Theorem 5.2 *If $|\lambda_i| \leq 1$ for all simple roots and $|\lambda_i| < 1$ for all repeated roots, then system (5.41) is stable.*

Theorem 5.3 *If $|\lambda_i| < 1$ for all roots, then system (5.41) is asymptotically stable.*

Consider a time-invariant Gaussian system

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) + \mathbf{M}\mathbf{W}(k), \quad \mathbf{X}(k_0) = \mathbf{X}_0 \quad (5.56)$$

where $\mathbf{X}(k) \in \mathbb{R}^n$ is state vector and $\mathbf{A} \in \mathbb{R}^{n \times n}$ denotes state matrix. $\mathbf{W}(k) \in \mathbb{R}^n$ and $\mathbf{W}(k) \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I}_n)$ is Gaussian white noise with $\mathbf{I}_n$ denoting $n \times n$ identity matrix. Let $\boldsymbol{\mu}(k) = \mathbb{E}\{\mathbf{X}(k)\}$ and $\mathbf{Q}_x(k) = \mathbb{E}\{(\mathbf{X}(k) - \boldsymbol{\mu})(\mathbf{X}(k) - \boldsymbol{\mu})^T\}$. Clearly, $\mathbf{Q}_x(k)$ represents the variance of $\mathbf{X}(k)$. Then we have

$$\mathbb{E}\{\mathbf{X}(k+1)\} = \mathbf{A}\boldsymbol{\mu}(k) \quad (5.57)$$

and

$$\mathbf{Q}_x(k+1) = \mathbf{A}\mathbf{Q}_x(k)\mathbf{A}^T + \sigma^2\mathbf{M}\mathbf{M}^T, \quad \mathbf{Q}_x(k_0) = \mathbf{Q}_0 \quad (5.58)$$

Consider the Lyapunov equation for $\mathbf{Q} \in \mathbb{R}^{n \times n}$,

$$\mathbf{Q} = \mathbf{A}\mathbf{Q}\mathbf{A}^T + \sigma^2\mathbf{M}\mathbf{M}^T. \quad (5.59)$$

Suppose the characteristic polynomial for matrix $\mathbf{A}$ is written as

$$\begin{aligned} |\mathbf{A} - \mathbf{I}\lambda| \\ = (-\lambda)^n + c_{n-1}\lambda^{n-1} + c_{n-2}\lambda^{n-2} + \cdots + c_1\lambda + c_0. \end{aligned} \quad (5.60)$$

Let λ_i be a root of the characteristic equation in (5.60). We have the following conclusions:

Theorem 5.4 *If $|\lambda_i| < 1$, then*

(a) $\lim_{k \rightarrow \infty} \mathbf{Q}_x(k) = \mathbf{Q}$, and $\mathbf{Q}$ is a solution of the Lyapunov equation.

(b) the matrix Lyapunov equation has a unique solution which moreover satisfies $\mathbf{Q} = \mathbf{Q}^T \geq 0$.

Clearly, the variance of $\mathbf{X}(k+1)$ is determined by system matrices.

5.6.2 Further Analysis of the Resource Allocation Model

Suppose the feedback delay cannot be ignored. In order to satisfy the constraint in (5.11), a new resource allocation scheme is proposed in [4]

$$r(t) = \log_2 \left(1 + \frac{P(\sigma_\epsilon^2 + |\bar{H}(t)|^2)}{\Gamma' \sigma^2} \right), \quad (5.61)$$

where $\Gamma' = m_\mu((5 \cdot \text{BER}_{\text{target}})^{-1/m_\mu} - 1)/1.6$ with $m_\mu = \frac{(1+|\bar{H}(t)|^2/\sigma_\epsilon^2)^2}{1+2|\bar{H}(t)|^2/\sigma_\epsilon^2}$. In order to further study the allocation scheme, we first study how it can be derived. For the sake of brevity, we mainly focus on how (5.1) can be transferred. According to [80], for an arbitrary vector variable $\boldsymbol{\alpha} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, the expectation of $\boldsymbol{\alpha}$ is given by

$$\mathbb{E}\{\exp(-\boldsymbol{\alpha}^H \boldsymbol{\alpha})\} = \frac{\exp(-\boldsymbol{\mu}^H (\mathbf{I} + \boldsymbol{\Sigma})^{-1} \boldsymbol{\mu})}{\det(\mathbf{I} + \boldsymbol{\Sigma})}, \quad (5.62)$$

where $\mathbf{I}$ denotes the identity matrix. From (5.1), $\text{BER}(t)$ is a function of $h(t)$ and its expectation is given by

$$\mathbb{E}\{\text{BER}(t)\} \approx 0.2 \frac{1}{1 + \Psi \sigma_\epsilon^2} \exp \left(-\frac{\Psi |\bar{H}(t)|^2}{1 + \Psi \sigma_\epsilon^2} \right), \quad (5.63)$$

where $\bar{H}(t) = \rho h_f(t)$ and $\Psi = 1.6P(t)/\{(2^{r(t)} - 1)\sigma^2\}$. $h_f(t)$ denotes the channel gain that is fed back to the transmitter. From (5.63), note that it's difficult to find a closed-form expression for $r(t)$. We derive a simpler, albeit approximate function, which is shown in (5.61), based on Nakagami- m distribution function. For the sake of brevity, the derivation procedures of the allocation scheme (5.61) are omitted. More details are given in [4].

According to (5.63), we study the relationship between expected BER and corresponding required SNR. Figs. 5.13 and 5.14 show the the comparisons among different correlation coefficient values under a 4 QAM modulation scheme. Suppose expected BER is 10^{-3} , note that the required SNR increases around 20% for $\rho = 0.9$ in the case of $|H_f(t)|^2 = 5$, while increasing around 100% for $\rho = 0.5$. These results indicate that the allocation scheme is more conservative in the case of low channel gain feed back. From Fig. 5.14, when $|H_f(t)|^2 = 0.6$, the required SNR for all cases of ρ is almost equal. From these two figures, it can be seen that the correlation coefficient is very sensitive to the change of the channel

Figure 5.13: Expected BER versus required SNR under the case of $|H_f(t)|^2 = 5$ with 4 QAM modulation scheme.

gain. Figs. 5.15 and 5.16 show the expected BER versus required SNR in the case of a 64 QAM modulation scheme. These two figures also show similar results. In order to suppress the sensitivity of the correlation coefficient, we reconsider the allocation scheme from the viewpoint of control theory.

5.6.3 Dynamical Control for Resource Allocation Model

According to the above discussion, the resource allocation scheme in (5.8) is sensitive to the change of ρ . In order to suppress the sensitivity of ρ , we propose a new resource allocation scheme from the viewpoint of control theory. First, we reformulate the resource allocation scheme (5.13) as a discrete-time system, which is given by

$$\text{BER}(k+1) = 0.2 \exp \left(\frac{-1.6P|H(k+1)|^2}{(2^{r(k+1)} - 1)\sigma^2} \right). \quad (5.64)$$

Based on the discrete-time system in (5.64), we propose a controller

$$r(k+1) = \log_2 \left(1 + \frac{g(\text{BER}(k))P|H(k)|^2}{\Gamma(\text{BER})\sigma^2} \right), \quad (5.65)$$

where $g(\text{BER}(k))$ is a control parameter and $\Gamma(\text{BER}) = -\ln(5 \cdot \text{BER})/1.6$ with $\text{BER} = \alpha \cdot \text{BER}_{\text{target}}$. Without loss of generality, we set $\alpha = 1$ in this chapter. The equations in

Figure 5.14: Expected BER versus required SNR under the case of $|H_f(t)|^2 = 0.6$ with 4 QAM modulation scheme.

Figure 5.15: Expected BER versus required SNR under the case of $|H_f(t)|^2 = 5$ with 64 QAM modulation scheme.

Figure 5.16: Expected BER versus required SNR under the case of $|H_f(t)|^2 = 0.6$ with 64 QAM modulation scheme.

(5.64) and (5.65) formulate a closed-loop system, which is given by

$$\text{BER}(k+1) = 0.2 \exp \left(\frac{|H(k+1)|^2 \cdot \ln(5 \cdot \text{BER})}{g(\text{BER}(k))|H(k)|^2} \right). \quad (5.66)$$

Note that the closed-loop system is nonlinear and $g(\text{BER}(k))$ affects not only whether the system in (5.66) is stable, but also the corresponding equilibrium point. Before further analyzing the system (5.66), we first consider a linear discrete system

$$\mathbf{X}(k+1) = \mathbf{A}\mathbf{X}(k) \quad (5.67)$$

where $\mathbf{X}(k) \in \mathbb{R}^n$ is state vector and $\mathbf{A} \in \mathbb{R}^{n \times n}$ denotes state matrix. For the discrete system in (5.67), we have the following results:

Definition 5.4 $\mathbf{X} = \mathbf{0}$ is a stable equilibrium point if for any given value $\epsilon > 0$ there exists a number $\delta(\epsilon, k_0) > 0$ such that if $\|\mathbf{X}(k_0)\| < \delta$, then the resulting motion $\mathbf{X}(k)$ satisfies $\|\mathbf{X}(k)\| < \epsilon$ for all $k > k_0$. Where $\|\cdot\|$ denotes a norm in Euclidean space [84].

Definition 5.5 The origin is an asymptotically stable equilibrium point if

(a) it is stable, and if in addition,

(b) there exists a number $\delta'(k_0) > 0$ such that whenever $\|\mathbf{X}(k_0)\| < \delta'(k_0)$ the resulting motion satisfies $\lim_{k \rightarrow \infty} \|\mathbf{X}(k)\| = 0$.

Let λ_i be a root of the characteristic equation in (5.60). We can easily derive the stability conditions in terms of the eigenvalues of the system matrix $\mathbf{A}$.

Theorem 5.5 *If $|\lambda_i| \leq 1$ for all simple roots and $|\lambda_i| < 1$ for all repeated roots, then system (5.67) is stable.*

Theorem 5.6 *If $|\lambda_i| < 1$ for all roots, then system (5.67) is asymptotically stable.*

For the system in (5.66), suppose the system equilibrium point is at $\{H^*, \text{BER}^*, r^*\}$. Then we have

$$\lim_{k \rightarrow \infty} |H(k)| = H^*, \quad (5.68)$$

$$\lim_{k \rightarrow \infty} \text{BER}(k) = \text{BER}^*, \quad (5.69)$$

and

$$\lim_{k \rightarrow \infty} r(k) = r^*. \quad (5.70)$$

For further analysis, we linearize the system in (5.66) about the equilibrium point. The linearized system is given by

$$\delta \text{BER}(k+1) = \Phi \delta \text{BER}(k) + \Xi_1 \delta |H(k+1)|^2 + \Xi_2 \delta |H(k)|^2, \quad (5.71)$$

where

$$\delta \text{BER}(k) \triangleq \text{BER}(k) - \text{BER}^*, \quad (5.72)$$

$$\delta |H(k)|^2 \triangleq |H(k)|^2 - |H^*|^2, \quad (5.73)$$

$$\begin{aligned} \Phi = & \frac{-0.2 |H(k+1)|^2 g'(\text{BER}(k)) \ln(5 \cdot \text{BER})}{|H(k)|^2 g^2(\text{BER}(k))} \\ & \exp \left(\frac{|H(k+1)|^2 \ln(5 \cdot \text{BER})}{|H(k)|^2 g(\text{BER}(k))} \right) \Big|_{H^*, r^*, \text{BER}^*} \end{aligned} \quad (5.74)$$

$$\Xi_1 = \frac{0.2 \ln(5 \cdot \text{BER})}{g(\text{BER}(k)) |H(k)|^2} \exp \left(\frac{|H(k+1)|^2 \ln(5 \cdot \text{BER})}{|H(k)|^2 g(\text{BER}(k))} \right) |_{H^*, r^*, \text{BER}^*} \quad (5.75)$$

$$\Xi_2 = \frac{-0.2 |H(k+1)|^2 \ln(5 \cdot \text{BER})}{|H(k)|^4 g(\text{BER}(k))} \exp \left(\frac{|H(k+1)|^2 \ln(5 \cdot \text{BER})}{|H(k)|^2 g(\text{BER}(k))} \right) |_{H^*, r^*, \text{BER}^*} \quad (5.76)$$

From Theorem 5.6, when

$$|\Phi| < 1, \quad (5.77)$$

the linearized system (5.71) is asymptotically stable near the equilibrium point. Therefore, the selection of control parameter $g(\text{BER}(k))$ should be based on this stability condition in (5.77). In order to satisfy the constraint in (5.77), we choose $g(\text{BER}(k)) = \frac{\ln(\text{BER}(k))}{\ln(K \cdot \text{BER})}$ with $K \in (0, 1]$. Accordingly, we get

$$\text{BER}^* = 0.2 \exp \left(\frac{M}{\ln(\text{BER}^*)} \right), \quad (5.78)$$

where $M = \ln(5 \cdot \text{BER}) \cdot \ln(K \cdot \text{BER})$.

It is seen that, the parameter K affects the location of the equilibrium point. When setting $K = 1$, $\text{BER}^* = \text{BER}$ is an optimal solution. On the other hand, because $f(x) = x \exp(-m/\ln(x))$ is a non-decreasing function when $m > 0$, system (5.64) has a unique equilibrium point. Therefore, $\text{BER}^* = \text{BER}$. When $K \in (0, 1)$, we get

$$\text{BER}^* = 0.2 \exp \left(\frac{\ln(5 \cdot \text{BER}) \cdot \ln(K \cdot \text{BER})}{\ln(5 \cdot \text{BER}^*) \cdot \ln(\text{BER}^*)} \cdot \ln(5 \cdot \text{BER}^*) \right). \quad (5.79)$$

Since $K \in (0, 1)$, we get

$$\begin{aligned} \ln(5 \cdot \text{BER}) \cdot \ln(\text{BER}) &< \ln(5 \cdot \text{BER}) \cdot \ln(K \cdot \text{BER}) \\ &= \ln(\text{BER}^*) \cdot \ln(5 \cdot \text{BER}^*) \end{aligned} \quad (5.80)$$

Therefore, we derive

$$K \cdot \text{BER} < \text{BER}^* < \text{BER}. \quad (5.81)$$

Based on the above discussion, it is easy to prove that

$$|\Phi| = \left| \frac{\ln(5 \cdot \text{BER}^*)}{\ln(\text{BER}^*)} \right| < 1, \quad (5.82)$$

$$\Xi_1 = \frac{\text{BER}^* \cdot \ln(5 \cdot \text{BER}^*)}{|H^*|^2} \quad (5.83)$$

and

$$\Xi_2 = -\frac{\text{BER}^* \cdot \ln(5 \cdot \text{BER}^*)}{|H^*|^2}. \quad (5.84)$$

Therefore, the system is locally asymptotically stable.

From the above discussion, it can be seen that the parameter K affects the equilibrium point. When $K = 1$, the equilibrium point is the target BER, and the resource allocation scheme in (5.65) makes the state $\text{BER}(k)$ fluctuate around the target BER. When $K \in (0, 1)$, the resource allocation scheme makes the equilibrium point $\text{BER}^* \in (K \cdot \text{BER}, \text{BER})$.

Theorem 5.7 *For any given BER, there always exists a number $K \in (0, 1)$ so that the resource allocation scheme in (5.65) satisfies the constraint*

$$\mathbb{E}\{\text{BER}(k)\} \leq \text{BER}. \quad (5.85)$$

Proof: Note that the equilibrium point is an increasing function of K . We always find a $K \in (0, 1)$ so that the constraint in (5.85) holds.

Based on Theorem 5.7, we can choose an appropriate K based on adaptive algorithms so that the resource allocation scheme in (5.65) satisfies the given BER requirements.

However, when calculating $\text{BER}(k+1)$, $H(k+1)$ is not available at the transmitter in practice. Since $H(k+1) = \rho H(k) + w$ with $w \sim \mathcal{CN}(0, \sigma_\epsilon^2)$, we use

$$\widetilde{\text{BER}}(k+1) = 0.2 \frac{1}{1 + \Sigma} \exp\left(\frac{-\mu^2}{1 + \Sigma}\right) \quad (5.86)$$

to approximate the true $\text{BER}(k+1)$, where

$$\mu = \sqrt{\frac{-\rho^2 \ln(5 \cdot \text{BER})}{g(\widetilde{\text{BER}}(k))}}, \quad (5.87)$$

and

$$\Sigma = \frac{-\sigma_\epsilon^2 \ln(5 \cdot \text{BER})}{g(\widetilde{\text{BER}}(k))|H(k)|^2}. \quad (5.88)$$

In this case,

$$\text{E}(\text{BER}(k+1)) = \widetilde{\text{BER}}(k+1). \quad (5.89)$$

Therefore,

$$\text{BER}(k+1) = 0.2 \exp \left(\frac{|H(k+1)|^2 \ln(5 \cdot \text{BER})}{|H(k)|^2 g(\widetilde{\text{BER}}(k))} \right), \quad (5.90)$$

$$r(k+1) = \log_2 \left(1 + \frac{g(\widetilde{\text{BER}}(k))P|H(k)|^2}{\Gamma(\text{BER})\sigma^2} \right). \quad (5.91)$$

5.7 Numerical Results

Based on the resource allocation scheme in (5.91), we study its performance in the cases of $\rho = 0.9, 0.7, 0.5$, and compare it with the resource allocation schemes in [3], [4] in the case of $\text{BER}_{\text{target}} = 10^{-3}$. For our simulations, we used Matlab. We first used an adaptive algorithm to choose K for different cases of ρ so that the resource allocation scheme satisfies the constraint in (5.11). We set $K = 10^{-10}$ for the case of $\rho = 0.9$. $K = 10^{-18}$ for $\rho = 0.7$ and $K = 10^{-38}$ for $\rho = 0.5$.

Fig. 5.17 shows the average spectral efficiency comparisons for different values of ρ obtained by the algorithm proposed in [3]. The spectral efficiency increases with ρ and P/σ^2 . Note that the average spectral efficiency in the case of $\rho = 0.9$ is higher than that in the cases of $\rho = 0.7$ and $\rho = 0.5$. It can be seen that the average spectral efficiency decreases drastically when ρ changes from 0.9 to 0.7. This result agrees with the curves shown in Figs. 5.13 $\sim$ 5.16.

Similarly, Fig. 5.18 shows average spectral efficiency comparisons for different values of ρ obtained by the algorithm proposed in [4]. Comparing Fig. 5.17 with Fig. 5.18, shows that these two algorithms have almost the same performance.

Figure 5.17: Average spectral efficiency comparisons with $\text{BER}_{\text{target}} = 10^{-3}$ for different cases of ρ based on the algorithm proposed in [3].

Figure 5.18: Average spectral efficiency comparisons with $\text{BER}_{\text{target}} = 10^{-3}$ for different cases of ρ based on the algorithm proposed in [4].

Fig. 5.19 shows the average spectral efficiency versus P/σ^2 for different values of ρ based on our proposed resource allocation scheme in (5.91). When compared with Figs. 5.17 and 5.18, the average spectral efficiency obtained by our proposed resource allocation scheme

Figure 5.19: Average spectral efficiency comparisons with $\text{BER}_{\text{target}} = 10^{-3}$ for different cases of ρ .

is not more sensitive than that of [3] and [4]. In particular, the corresponding average spectral efficiency achieved by our proposed scheme with $\rho = 0.9$ is lower than that of the two aforementioned schemes, but is higher in the cases of both $\rho = 0.7$ and $\rho = 0.5$. Thus, our proposed resource allocation scheme is more robust to the change of ρ and achieves a better performance when ρ decreases.

5.8 Sub-Summary

In Chapter 5.6, we have studied the dynamics of a wireless resource allocation model with the Doppler effect, and developed a new resource allocation scheme. It has been seen that the newly developed resource allocation scheme is robust with respect to the change of correlation coefficient. Also, the efficiency of the resource allocation can be further improved by adjusting the control parameters. The simulation results have confirmed the efficiency of the new allocation scheme under different channel conditions.

5.9 Chapter Summary

The assumption that the transmitter always receives the channel state information perfectly is impractical for wireless systems. The system performance will degrade when the transmitter only has partial CSI. In order to maintain the system performance, an appropriate transmission schedule based on partial CSI is needed.

In this chapter, we have studied a wireless allocation scheme model affected by the Doppler effect. Firstly, we analyzed the effects of partial channel state information on the resource allocation problem in MU-OFDM based cognitive radio systems. Based on obtained partial CSI at the transmitter, the average BER should satisfy the given BER target during transmission. As the function of average BER is too complex, we apply a Nakagami- m distribution to approximate the original function. A simple function, which is close to the original function, is derived. Different cases of partial CSI and bit rate requirements are studied. Simulations show that partial CSI has a great impact on the wireless transmission. In addition, due to the lack of user diversity, the total bit rate decreases when the data rate requirements become less uniform.

Secondly, we have studied the dynamics of a wireless resource allocation model with the Doppler effect. We find that the traditional allocation scheme for this model is sensitive to the change of correlation coefficients. When $|\rho| \ll 1$, the allocation scheme varies in a small range and consumes more power. In this case, it becomes very conservative. On the other hand, the allocation scheme is very sensitive to the change of channel state when $\rho \rightarrow 1$. It approximates the real one in a good channel state while consuming more power in a bad channel state. We developed a new resource allocation scheme. It has been seen that the newly developed allocation scheme is robust with respect to the change of channel state. Also, the efficiency of the resource allocation can be further improved by adjusting the control parameters. The simulation results have confirmed the efficiency of the new allocation scheme under different channel conditions.

According to the above mentioned, partial CSI has a significant impact on data transmissions. While satisfying the QoS requirement of PUs is an important criteria for allowing

CRUs to access unlicensed frequency bands, in our future works, we would like to continue to study the ways in which to choose appropriate algorithms to approximate the real CSI for data transmission.

Chapter 6

Game Theory for Self-Coexistence Problems among IEEE 802.22 Networks

Cognitive radio (CR) is a promising technology that can significantly enhance the utilization of radio spectrums for future wireless communications by allowing cognitive radio users (CRUs) to access unlicensed radio spectrums [6]. CR is a novel concept for improving the utilization of scarce radio frequency spectrums. The mechanism of using unlicensed radio spectrums for CR is sensing before accessing. The level of interference to primary users (PUs) must be below an acceptable level.

IEEE 802.22 is a standard, which is based on CR, for wireless regional area networks (WRANs). The objective of WRANs is to provide broadband access in rural and remote areas. WRANs operate in the TV bands between 54 MHz and 862 MHz. When compared to other existing networks, WRANs have a larger coverage range and provide broadband access in rural and remote areas with performance comparable to DSL and cable modems. The networks operate in a point to multiple point basis (P-MP), where a base station (BS) services a number of consumer premise equipments (CPEs).

Before allocating TV channels to CPEs, a BS must sense that the channels are currently not being utilized by licensed incumbents (i.e. TV receivers and microphones). When the WRANs sense that the current TV channels they are using are accessed by the licensed incumbents, they must vacate the channel within a certain time (2 seconds) and switch to

other unutilized channels [4]. Since the spectrum management among competing WRANs are distributed and the coordination amongst WRANs of different service providers does not exist, several networks overlaying each other may switch to the same channel when sensing incumbents' existence. At this point, interference among these networks occurs. The competing networks have a binary action: stay in the same channel or switch to a different channel, causing some loss of quality of service (QoS) requirements among the networks. Finding ways to minimize the interference among IEEE 802.22 networks, and thus ensure that the given QoS requirements are met is a challenge issued to the IEEE 802.22 standard. These independent networks can be expected to seek their own benefits or utilities without cooperating with one another. A game theory framework for solving the self-coexistence problem is first proposed in [56]. The paper focuses on ways to select a strategy so that the expected cost of staying in the same channel is equal to the expected cost of switching to a different channel. However, the paper does not further analyze the expected costs in the game.

Game theory is a powerful tool developed to model the interactions of agents with conflicting interests. When applying game theory, there is an assumption: each agent in the game is rational. Assuming that there is more than one agent and each agent's payoff possibly depends on the other agents' actions, game theory can be applied to analyze the decision making process. Since many practical problems can be formulated in such model, game theory has been widely applied to economics, biology, engineering and political science [85]. Moreover, game theory also is an appropriate tool for analyzing some interesting problems in wireless communication systems for two reasons [51]:

- (1) Wireless communication systems are often built on standards, (e.g. CDMA system).

Devices for accessing these systems are built by a number of different manufactures. Sometimes, certain manufacturers may have an incentive to develop products with selfish behavior so that they have a better performance than products developed by other manufacturers. In order to maintain a stable system with predefined performance as designed, an appropriate strategy for making the selfish behavior unprofitable is needed;

- (2) In many cases, users in wireless communications are distributed and make their decisions independently. An individual's payoff is based on not only its action but also other users' actions. Game theory can help the users to make a good decision.

In particular, game theory has been applied to solve different kinds of efficient resource allocation problems in wireless communications [47], [48], [49], [50], [51]. The uplink power control of a multiuser MIMO system is formulated as a non-cooperative game in [47]. [48] formulates the radio resource management (RRM) in a heterogeneous wireless access environment from the viewpoint of game theory. In [51], the equilibrium point between the base station and a connection for IEEE 802.16 broadband wireless networks is studied. In addition, a special case of spectrum sharing strategy for IEEE 802.22 networks based on game theory was proposed in [57].

In this chapter, we consider the system model with multiple overlapping WRANs operated by multiple wireless service providers competing to seek available channels for their individual CPEs. When interference occurs, none of the networks know what the other networks will do: will they stay or switch? Each network makes decisions independently. Therefore, the self-coexistence problem can be formulated as a noncooperative game. We consider two cases of spectrum management: 1) common channel set case; 2) independent channel set case. We define different utility functions for these two cases. We propose a simple algorithm to solve the Nash equilibrium and its Pareto efficiency is analyzed. We also compare the proposed strategy with other strategies and find that the proposed strategy achieves better performance.

The rest of the chapter is organized as follows: In Chapter 6.1, we first discuss the IEEE 802.22 networks operation model. In Chapter 6.2, the types of game theory models are presented. The proposed strategies for the self-coexistence problem in IEEE 802.22 networks are discussed in Chapter 6.3. Numerical results and the chapter summary are presented in Chapter 6.4 and Chapter 6.5, respectively.

Figure 6.1: IEEE 802.22 networks operation topology.

6.1 IEEE 802.22 Networks Operation Model

In this section, we consider there are N available TV channels and M competing IEEE 802.22 networks (players) operated by different wireless service providers. These IEEE 802.22 networks overlap each other. The WRANs are illustrated in Fig. 6.1. For notation simplicity, we use Θ_i to represent the set that channels are occupied by incumbents sensed by player i . Φ_i denotes the channel set that are being operated by player i , Σ_i denotes the total number of available channels for player i , Ω_i is the set of channels that are claimed for backup by player i . Thus, we have $\Sigma_i \supseteq \Phi_i$ and $\Sigma_i \supseteq \Omega_i$. For the sake of convenience, we assume $\Theta_1 = \Theta_2 = \dots = \Theta_M$. In this paper, we consider two cases of spectrum management:

- (1) Common Channel Set for Backup (i.e. $\Omega_1 = \Omega_2 = \dots = \Omega_M$).
- (2) Independent Channel Set for Backup (i.e. each WRAN chooses its Ω independently).

Clearly, the first case is the most simple. All players share a common channel set for backup. The disadvantage is that the interference among WRANs will increase and causes

the given QoS requirements to be missed. The second one is more complex. But when

$$\cap_{i=1}^M \Omega_i = \emptyset, \quad (6.1)$$

it will significantly decrease the interference among WRANs. On the other hand, the spectrum management among WRANs is distributed. Different IEEE 802.22 networks operate independently instead of being controlled by a central authority. It is not easy to make (6.1) hold. Ways to minimize the interference among WRANs and maintain the predefined QoS among IEEE 802.22 networks are an important issue on the proposed IEEE 802.22 standard. The objective of each IEEE 802.22 networks is to find a strategy so that the expected cost of finding an available channel is minimized, (i.e., maintaining self-coexistence). On the other hand, game theory deals primarily with distributed optimization.

6.2 Representation of Game Theory

We assume there are N players in a game. Each player has its action set and utility function. The objective of each player in the game is to choose an appropriate strategy so that its utility function is maximized. More concretely, an N players game can be expressed in normal form: $G = (A_1, \dots, A_N; u_1, \dots, u_N)$, where A_i denotes the set of actions available to i^{th} player and $u_i : A_1 \times A_2 \times \dots \times A_N \rightarrow \mathbb{R}$ represents the i^{th} player's utility function. Let $\mathbf{N} = \{1, 2, \dots, N\}$ represent the set of players and $\mathbf{a} = (a_1, \dots, a_N) = \{a_i\}_{i \in \mathbf{N}}$ denote a profile of the actions of all the players. Therefore, $\mathbf{a} \in \mathbf{A} = A_1 \times \dots \times A_N$, where $\mathbf{A}$ represents the set of all possible actions of all the players. Let $A_{-i} = \mathbf{A} \setminus A_i$ be the set of all possible actions of all the players except player i and $a_{-i} = \mathbf{a} \setminus \{a_i\}$ be the particular actions of all the other players. Then we have $\mathbf{a} = (a_i, a_{-i})$. Note that a player's utility depends not only on his own actions, but also on the actions of the other players. Moreover, each player i tries to maximize their individual utility function u_i . In game theory, there is an important concept called the Nash equilibrium.

Definition 6.6 *Nash equilibrium:* An action vector $\mathbf{a} = (a_i, a_{-i})$ is said to be a Nash equilibrium if and only if

$$u_i(a_i, a_{-i}) \geq u_i(\tilde{a}_i, a_{-i}). \quad \forall i \in \mathbf{N} \quad (6.2)$$

		Player 2		
		Left	Middle	Right
Player 1	Up	3,2	2,1	1,1
	Down	2,0	1,0	3,1

Figure 6.2: An example game in matrix form.

Note that the Nash equilibrium is also a solution of a game. When a solution is the Nash equilibrium, it represents that no player can achieve more gain by changing its strategy based on other players' strategies. Therefore, a Nash equilibrium point corresponds to a steady state of a game. When determining strategies for a game, we need to find the Nash equilibrium points. On the other hand, Pareto efficiency is another important concept for the application of game theory.

Definition 6.7 *Pareto Efficiency:* An action $\mathbf{a} \in \mathbf{A}$ is said to be Pareto efficient if there is no action $\tilde{\mathbf{a}} \in \mathbf{A}$ so that

$$u_i(\tilde{\mathbf{a}}) \geq u_i(\mathbf{a}) \quad \forall i \in \mathbf{N} \quad (6.3)$$

with strict inequality for at least one i . That is to say, there no improvement if an action $\mathbf{a} \in A$ is Pareto efficient.

Clearly, when an action $\mathbf{a} \in \mathbf{A}$ is Pareto efficient, it must be a Nash equilibrium. On the contrary, if an action is a Nash equilibrium it does not mean that it is also Pareto efficient. Therefore, when an action is Nash equilibrium, it does not mean that all u_i is maximized. Therefore, a Nash equilibrium with Pareto efficient will be preferred. However, many Nash equilibria may be Pareto inefficient in noncooperative games [86].

We use an example game matrix in Fig. 6.2 to illustrate the above two definitions. It's a two-player game. Player 1 has two choices: Up and Down; Player 2 has three

choices: Left, Middle and Right. In this case, we assume the two players choose their actions simultaneously. The ordered pair in each box denotes the payoff corresponding to each player's action. Player 1's payoff is listed first in the ordered pair. From Fig. 6.2, note that there are two Nash equilibria (Up,Left) and (Down,Right). But only (Up,Left) is Pareto efficient. Therefore, a Nash equilibrium is not necessarily Pareto efficient. Moreover, sometimes none of the Nash equilibria are Pareto efficient in a game.

In practice, most games are more complex than the example in Fig. 6.2. It is very difficult to determine whether a Nash equilibrium is Pareto efficient from the definition of Pareto efficiency due to high algorithm complexity. Assuming a strategy profile $\mathbf{a} \in \mathbf{A}$ has discrete values, then the steps to determine whether a Nash equilibrium is Pareto efficient grow exponentially with the number of players. It becomes an NP-hard problem. On the other hand, it's easier to determine whether a Nash equilibrium is Pareto inefficient.

There are many criteria for Pareto inefficiency [87]. In this section, we use the following criteria.

For the convenience of representation, let $\mathbf{C}$ be the set of feasible $\mathbf{a}$. Suppose $\mathbf{a}^* = (a_1^*, \dots, a_N^*) \in \mathbf{A}$ is a Nash equilibrium, then let $\mathbf{N}^* = \{i | a_i^* \text{ is not a boundary value of } \mathbf{A}\}$ and $N(\mathbf{a}) \in \mathbf{A}$ be the neighborhood of $\mathbf{a}$.

Assumption 6.1 *For a Nash equilibrium $\mathbf{a}^*$, the partial derivatives of $(u_i(\mathbf{a}^*), \forall i \in \mathbf{N})$ with respect to all variables $(a_j, \forall j \in \mathbf{N}^*)$ exist and are continuous in $\mathbf{a}^* \in N(\mathbf{a}^*)$, and either of the following two cases holds [88]:*

- (1) *The utility of player i , $(u_i, \forall i \in \mathbf{N})$ is a decreasing function of $(a_j, \forall j \in \mathbf{N}^* \text{ and } j \neq i)$, that is,*

$$\left. \frac{\partial u_i}{\partial a_j} \right|_{\mathbf{a}=\mathbf{a}^*} < 0, \quad \forall j \in \mathbf{N}^* (j \neq i)$$

- (2) *The utility of player i , $(u_i, \forall i \in \mathbf{N})$ is an increasing function of $(a_j, \forall j \in \mathbf{N}^* \text{ and } j \neq i)$, that is,*

$$\left. \frac{\partial u_i}{\partial a_j} \right|_{\mathbf{a}=\mathbf{a}^*} > 0, \quad \forall j \in \mathbf{N}^* (j \neq i)$$

Assumption 6.2 *For a Nash equilibrium $\mathbf{a}^*$, there exist more than one element of $\mathbf{a}^*$ are interior values, i.e., $|\mathbf{N}^*| \geq 2$.*

Theorem 6.8 *If Assumptions 6.1 and 6.2 hold for a Nash equilibrium in $\mathbf{C}$, it is strongly Pareto inefficient.*

Note that Theorem 6.8 is an efficient condition but not an efficient and necessary condition. Since traditional algorithms to find a Nash equilibrium are based on gradient methods, some Nash equilibria may still be Pareto inefficient even though the assumptions 6.1 and 6.2 do not hold.

Game theory is a powerful technique for studying systems with dynamic decision making. Several kinds of game theory models have been successfully applied to the study of the resource management in cognitive radio networks (CRNs) [51] [89] [89] [90] [91] [92]. These game theory models are represented as follows.

6.2.1 Repeated Games

When a game only has one stage, it is called a non-repeated game. A repeated game consists of a number of sequence stages where each stage is the same normal form game. It includes two kinds of repeated games. When the number of stages in a repeated game is finite it is called finitely repeated game. Otherwise, it is called an infinitely repeated game. Players choose actions at each stage based on their past actions, current observations and future expectations.

For a given game $G = (A_1, \dots, A_N; u_1, \dots, u_N)$, we can define a finitely repeated game G^K , $K \geq 1$ as follows. For every stage $k = 1, 2, \dots, K$, player i chooses an action $a_i^k \in A_i$. Let $\mathbf{a}^k$ be a profile of actions of all the players at stage k . Player i 's utility is calculated as an average of his utilities over K stages.

$$U_i = \frac{1}{K} \sum_{k=1}^K u_i(\mathbf{a}^k). \quad (6.4)$$

Given the same game G as above, we define an infinitely repeated game G^∞ as follows. Naturally, we can define player i 's utility as

$$U_i = \lim_{K \rightarrow \infty} \frac{1}{K} \sum_{k=1}^K u_i(\mathbf{a}^k). \quad (6.5)$$

However, this limit might not exist. In order to guarantee the limit exists, we can introduce a discount parameter $\lambda \in (0, 1)$ and the utility function can be changed to

$$U_i = (1 - \lambda) \sum_{k=1}^{\infty} u_i(\mathbf{a}^k) \lambda^{k-1}. \quad (6.6)$$

Note that

$$\sum_{k=1}^{\infty} \lambda^{k-1} = \frac{1}{1 - \lambda}, \quad (6.7)$$

U_i is the weight sum of $u_i(\mathbf{a}^k)$ in each stage.

6.2.2 S-modular Games

Note that noncooperative game theory deals with optimization problems where a number of players/agents seek to achieve individual optimal utility. The problem of distributed power control in wireless networks can be considered as an S-modular game.

For a given game $G = (A_1, \dots, A_N; u_1, \dots, u_N)$, let $\mathbf{a}$, $\mathbf{b}$ be profiles of actions of all the players.

Definition 6.8 *The utility u_i for player i is supermodular if and only if for all $\mathbf{a}, \mathbf{b} \in \mathbf{A}$*

$$u_i(\mathbf{a} \wedge \mathbf{b}) + u_i(\mathbf{a} \vee \mathbf{b}) \geq u_i(\mathbf{a}) + u_i(\mathbf{b}), \quad (6.8)$$

where $\mathbf{a} \wedge \mathbf{b}$ represents the componentwise minimum and $\mathbf{a} \vee \mathbf{b}$ denotes the componentwise maximum of $\mathbf{a}$ and $\mathbf{b}$.

If $-u_i$ is supermodular then u_i is called submodular. A game that is either supermodular or submodular is called an S-modular game.

When the utility function u_i is twice differentiable, from (6.8), supermodularity is equivalent to the condition [93]

$$\frac{\partial^2 u_i}{\partial a_i \partial a_j} \geq 0. \quad \forall i \neq j \quad (6.9)$$

6.3 Proposed Strategies for Self-Coexistence Problems among IEEE 802.22 Networks

Note that the self-coexistence problem can be modeled as a non-cooperative game. In such a game, a player's utility is based on not only its actions, but also the other players' actions. Each player wishes to find an appropriate strategy to maximize its own utilities. We first analyze a simple model for the self-coexistence problem in Chapter 6.3.1. The switch probability is derived at the Nash equilibrium point. Then we study a more complex model with a candidate channel set for the self-coexistence problem. The Nash equilibrium point is analyzed and its Pareto efficiency is also discussed in Chapter 6.3.2.

6.3.1 Common Channel Set Case

In this case, all players share a common channel set for backup. When more than two players access the same channel in the common channel set, the interference occurs. The advantage of sharing a common candidate channel set among all players is to improve the efficiency of spectrum utilization. The disadvantage is that the interference increases.

Without loss of generality, we focus on a particular player $i \in \mathbf{N}$. The same strategy applies to all other players due to the homogeneity of all players. We consider the probability of any player switching to each channel to be equal. The binary action set for player i can be represented as

$$A_i = \{\text{switch}, \text{stay}\}. \quad (6.10)$$

Since all players have the same action set, we use α_1 and α_2 to denote the action stay and switch, respectively. There are two results corresponding to these actions, noninterference and interference. Let β_1 and β_2 be noninterference and interference, respectively. Suppose the switch probability for player i in step k is $P_k^i(\alpha_2)$. There are M_k remnant players and N_k available channels in step k . Let $\mathbf{m}_k = \{1, 2, \dots, M_k\}$ denote the set of players. For each $i \in \mathbf{m}_k$, we use $P_k^i(\beta_1|\alpha_1)$ and $P_k^i(\beta_1|\alpha_2)$ to represent the noninterference probability in step k under the action of stay and switch, respectively. $\bar{P}_k = \frac{1}{M_k} \sum_{j=1}^{M_k} P_k^j(\alpha_2)$ is the mean switch probability in step k . Let $C_{t,k}^i$ and $C_{w,k}^i$ be the cost of noninterference in step k for

player i under the condition of stay and switch, respectively. If player i finds an available channel in step k , the expected cost is

$$C_k^i = C_{t,k}^i P_k^i(\beta_1|\alpha_1) + C_{w,k}^i P_k^i(\beta_1|\alpha_2). \quad (6.11)$$

In addition, the interference probability in step k under the action of stay and switch shown as follows:

$$P_k^i(\beta_2|\alpha_1) = 1 - P_k^i(\beta_1|\alpha_1), \quad \forall i \in \mathbf{m}_k \quad (6.12)$$

$$P_k^i(\beta_2|\alpha_2) = 1 - P_k^i(\beta_1|\alpha_2). \quad \forall i \in \mathbf{m}_k \quad (6.13)$$

Let $\tilde{C}_{t,k}^i$ and $\tilde{C}_{w,k}^i$ be the cost of interference in step k for player i under the action of stay and switch, respectively. If player i does not find an available channel in step k , the expected cost is

$$\tilde{C}_k^i = \tilde{C}_{t,k}^i P_k^i(\beta_2|\alpha_1) + \tilde{C}_{w,k}^i P_k^i(\beta_2|\alpha_2). \quad (6.14)$$

Accordingly, our objective is to minimize the expected cost

$$\min \tilde{J}(\mathbf{p}) = \frac{1}{KM} \sum_{k=1}^K ((C_k^i + \sum_{j=0}^{k-1} \tilde{C}_j^i) * (M_{k-1} - M_k)). \quad (6.15)$$

where $\tilde{C}_0 = 0$ and $M_0 = M$. $\mathbf{p} = [\mathbf{p}_1, \dots, \mathbf{p}_M]$ is the switch probability matrix and $\mathbf{p}'_i = [p_1^i, \dots, p_K^i]$ represents the strategy selected by player i . K denotes the number of steps to find available channels for all players. According to the equation (6.15), our objective is to minimize the expected cost. While the objective of a game theory based model is to maximize its utility function. For the convenience of discussion, the utility function for player i can be defined as

$$\tilde{u}_i(\mathbf{p}_i, \mathbf{p}_{-i}) = \frac{1}{\tilde{J}(\mathbf{p})}. \quad i = 1, 2, \dots, M \quad (6.16)$$

When equation (6.16) is maximized, the objective function in equation (6.15) will be minimized. Player i wants to select appropriate $(p_1^i, \dots, p_K^i)$ to maximize its utility.

Based on the definition in (6.16), we need to find the equilibrium point $\mathbf{p}^* = (\mathbf{p}_1^*, \dots, \mathbf{p}_M^*)$ so that

$$\tilde{u}_i(\mathbf{p}_i^*, \mathbf{p}_{-i}^*) \geq \tilde{u}_i(\mathbf{p}_i, \mathbf{p}_{-i}^*). \quad i = 1, 2, \dots, M \quad (6.17)$$

Note that the utility function for player i is dependent on $\mathbf{p}_i$ and $\mathbf{p}_{-i}$. Moreover, the strategy in step $k + 1$ is dependent on the strategies in step $1, 2, \dots, k$. It belongs to a sequential decision making problem. Since none of the players know other players' strategies, it is difficult to calculate K directly. The problem is intractable due to the exponential computation complexity.

In order to cope with the problem, we consider myopic policies. Generally, the objective of a myopic policy is to minimize the immediate cost based on the current action, while ignoring the impact of the current action on the future cost. Thus, the complexity is significantly reduced. In the proposed myopic policy, the expected cost in step k is formulated as

$$\begin{aligned} J_i(\mathbf{p}_k) = & C_{t,k}^i P_k^i(\beta_1 | \alpha_1) + \tilde{C}_{t,k}^i P_k^i(\beta_2 | \alpha_1) \\ & + C_{w,k}^i P_k^i(\beta_1 | \alpha_2) + \tilde{C}_{w,k}^i P_k^i(\beta_2 | \alpha_2). \quad \forall i \in \mathbf{m}_k \end{aligned} \quad (6.18)$$

In each step, player i wants to minimize its step utility function. Then the utility function is redefined as

$$u_i(p_{k,i}, p_{k,-i}) = \frac{1}{J_i(\mathbf{p}_k)}. \quad \forall i \in \mathbf{m}_k \quad (6.19)$$

Clearly, the utility function defined in (6.19) is simpler than that of (6.16). For the convenience of representation, we use $\mathbf{p}$ to denote $\mathbf{p}_k$. In this paper, our objective is to minimize the expected cost defined in (6.18) in each step. Without loss of generality, we assume the action costs for all players are equal in each step. In particular, we consider

$$C_{t,k}^i = 1 - p_i, \forall i \in \mathbf{m}_k \quad (6.20)$$

$$\tilde{C}_{t,k}^i = 0, \forall i \in \mathbf{m}_k \quad (6.21)$$

$$C_{w,k}^i = p_i, \forall i \in \mathbf{m}_k \quad (6.22)$$

$$\tilde{C}_{w,k}^i = 0, \forall i \in \mathbf{m}_k \quad (6.23)$$

hold in each step. In this case, we get

$$\begin{aligned} J_i(\mathbf{p}) &= P^i(\beta_2 | \alpha_1)(1 - p_i) + P^i(\beta_2 | \alpha_2)p_i \\ &= P^i(\beta_2), \forall i \in \mathbf{m}_k \end{aligned} \quad (6.24)$$

and thus the objective is to minimize the interference probability. For the sake of simplicity, we first study whether the Nash equilibrium at (6.24) is Pareto efficient for $M_k = 2$. The result is given in Theorem 6.9.

Theorem 6.9 *For $M_k = 2$, the Nash equilibrium at (6.24) is Pareto efficient.*

Proof: Clearly, when $M_k = 2$, the two players' expected cost functions are given by

$$J_1(\mathbf{p}) = (1 - p_1) \cdot (1 - p_2) + p_1 \cdot \frac{p_2}{N_k} \quad (6.25)$$

$$J_2(\mathbf{p}) = (1 - p_2) \cdot (1 - p_1) + p_2 \cdot \frac{p_1}{N_k} \quad (6.26)$$

We get

$$p_1^* = p_2^* = \frac{N_k}{N_k + 1} \quad (6.27)$$

by solving

$$\left. \frac{\partial J_1(\mathbf{p})}{\partial p_1} \right|_{\mathbf{p}=\mathbf{p}^*} = 0 \quad (6.28)$$

and

$$\left. \frac{\partial J_2(\mathbf{p})}{\partial p_2} \right|_{\mathbf{p}=\mathbf{p}^*} = 0. \quad (6.29)$$

From the Nash equilibrium at (6.27), note that all players have the same switch probability.

Consider all players have the same switch probability (i.e., $p_1 = p_2 = p$), then

$$J_i(p) = (1 - p)^2 + \frac{p^2}{N_k}, \quad i = 1, 2 \quad (6.30)$$

It can be seen that $J_i(p)$ has only a local minimum on $p \in [0, 1]$ and thus the local minimum is also the global minimum. Therefore, the Nash equilibrium solution achieved at (6.27) is Pareto efficient. Based on the proof for Theorem 6.9, we conjecture that all players have the same switch probability value at Nash equilibrium point can be generalized to $M_k > 2$.

Consider the switch probability for player i is p_i . We assume all players have the same switch probability, then equation (6.24) is given by

$$J_i(p_i) = (1 - p_i)(1 - p_i^{M_k-1}) + p_i(1 - (1 - \frac{p_i}{N_k})^{M_k-1}). \forall i \in \mathbf{m}_k \quad (6.31)$$

Clearly, in order to minimize $J_i(p_i)$, player i needs to know other players' strategies before making a decision. When $M_k > 1$, we can show that $J_i(p_i)$ has a single local minimum on $p_i \in [0, 1]$ and this local minimum is also the global minimum. Thus, the same value of p_i will minimize $J_i(p_i)$. We use p^* to denote value of p_i which minimizes the expected cost function in (6.31). From Theorem 6.9, we believe p^* is Pareto efficient.

6.3.2 Independent Channel Set Case

Obviously, when all players share a common candidate channel set, the interference will occur frequently and causes prescribed QoS requirements to be missed. In this case, we assume each player has an individual candidate channel set. By dividing the common candidate channel set into separate candidate channel sets, we can decrease the interference among WRANs. For proportional fairness, the number of available TV bands allocated to each neighboring WRAN should be predefined. Let α_i be the the number of available TV bands allocated to BS i , where $\alpha_i = |\Phi_i| + |\Omega_i|$ and $\Phi_i \cap \Omega_i = \emptyset$. For M neighbor WRANs, we assume the number of available TV bands allocated to each BS should satisfy the following constraint

$$\alpha_1 : \alpha_2 : \dots : \alpha_M = n_1 : n_2 : \dots : n_M. \quad (6.32)$$

Where the total number of available TV channels $N = \sum_{i=1}^M |\Phi_i| + |\cup_{i=1}^M \Omega_i|$. Compared with the case of common candidate channel set, we not only need to find appropriate candidate channel sets for each player, but also need to punish those selfish players for the candidate channel set case. Our objective is to minimize the interference among the WRANs. We use

$$\begin{aligned} U_i &= f(\Phi, \Omega) \\ &= \exp(-\eta|\alpha_i - \frac{n_i}{\sum_{\forall j \neq i} n_j}|) \exp(-|\cap_{i=1}^M \Omega_i|) \end{aligned} \quad (6.33)$$

to denote the i th player's utility function, where $\Phi = [\Phi_1, \Phi_2, \dots, \Phi_M]$, $\Omega = [\Omega_1, \Omega_2, \dots, \Omega_M]$ and $\eta \geq 1$ is a punishment factor. Note that U_i is a discrete function since α_i and $|\cap_{i=1}^M \Omega_i|$ are discrete variables. Clearly, U_i is a decreasing function of $|\alpha_i - \frac{n_i}{\sum_{j \neq i} n_j}|$ and $|\cap_{i=1}^M \Omega_i|$. We get

$$\alpha_i = \frac{n_i}{\sum_{j \neq i} n_j}, \forall i = 1, 2, \dots, M \quad (6.34)$$

and

$$|\cap_{i=1}^M \Omega_i| = 0 \quad (6.35)$$

at the Nash equilibrium point. That is, the allocation satisfies the constraint in (6.32) and the intersection of all candidate channel set is null at the Nash equilibrium point. Therefore, before claiming an available TV channel for backup, a BS should check whether its current state satisfies the constraints in (6.34) and (6.35). In this section, we propose an algorithm to manage the spectrum among WRANs which is illustrated in Fig. 6.3.

6.4 Numerical Results

In this section, we first consider the strategy derived from (6.24). We compare the game model for common channel set case with the MMGMS scheme proposed in [56]. Given the number of competing WRANs, $M = 30$, we study how the expected cost changes when the number of available channels increases. For our simulations, we used Matlab. The simulation result is illustrated in Fig. 6.4, where the expected cost decreases with the number of available channels. The expected cost achieved by our proposed scheme is around 10% smaller than that of the MMGMS scheme. By further studying MMGMS, we find that the switch probability of MMGMS is derived by the equation $E[C^{switch}] = E[C^{stay}]$. It's objective is not to minimize the interference, while our proposed game theoretic approach is to minimize the interference. In this case, the switch probability of our proposed method causes less interference cost. Fig. 6.5 shows the expected cost as a function of the number of competing WRANs given the number of available channels, $N = 50$. As is to be expected,

```

update  $\Theta_i, \Sigma_i, \alpha_i$ ;
initialize  $\Omega_i = \emptyset, \forall i = 1, 2, \dots, M$ .
while  $\alpha_i \neq \frac{n_i}{\sum_{j \neq i} n_j}$  or  $|\Omega_i \cap \tilde{\Omega}_j, \forall j \neq i| \neq 0$  do

    if  $\alpha_i < \frac{n_i}{\sum_{j \neq i} n_j}$  then
        add  $\frac{n_i}{\sum_{j \neq i} n_j} - \alpha_i$  channels to  $\Omega_i$  from  $\Sigma_i$  such that  $U_i$  is maximized;
    else if  $\alpha_i > \frac{n_i}{\sum_{j \neq i} n_j}$  then
        delete  $\alpha_i - \frac{n_i}{\sum_{j \neq i} n_j}$  channels from  $\Omega_i$  such that  $U_i$  is maximized;
    else
        go to next step.
    end if
    if there exists a channel  $s_i \in \Omega_i$  and  $s_i \in \Omega_j, \forall j \neq i$  then
        use the strategy derived from equation (6.24) to find channels such that  $U_i$  is maximized.
    end if
    update  $\Theta_i, \Sigma_i, \alpha_i$ , and  $\Omega_i, \forall i = 1, 2, \dots, M$ .
end while
    
```

Figure 6.3: Nash Equilibrium Algorithm of Competing IEEE 802.22

Figure 6.4: Expected cost comparison between our proposed algorithm and MMGMS for different numbers of available channels.

Figure 6.5: Expected cost comparison between our proposed algorithm and MMGMS for different numbers of competing WRANs.

the expected cost increases with the number of competing WRANs. It can be seen that the our proposed scheme provides a better performance than the MMGMS scheme.

For the candidate channel set case, we consider there are $M = 5$ competing WRANs.

$$\Theta = \{s_3, s_4, s_8, s_{22}, s_{25}\}. \Phi_1 = \{s_2, s_6, s_{26}\}, \Phi_2 = \{s_1, s_7, s_9\}, \Phi_3 = \{s_{11}, s_{18}\}, \Phi_4 = \{s_5, s_{17}, s_{27}\},$$

$$\Phi_5 = \{s_{23}\}. \Sigma_1 = \{s_2, s_6, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{19}, s_{24}, s_{26}, s_{28}, s_{29}, s_{30}\},$$

$$\Sigma_2 = \{s_1, s_7, s_9, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{19}, s_{24}, s_{28}, s_{29}, s_{30}\},$$

$$\Sigma_3 = \{s_{11}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{18}, s_{19}, s_{24}, s_{28}, s_{29}, s_{30}\},$$

$$\Sigma_4 = \{s_5, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{17}, s_{19}, s_{24}, s_{27}, s_{28}, s_{29}, s_{30}\},$$

$$\Sigma_5 = \{s_{23}, s_{12}, s_{13}, s_{14}, s_{15}, s_{16}, s_{19}, s_{24}, s_{28}, s_{29}, s_{30}\}. \text{ The proportional ratio is set by } n_1 :$$

$$n_2 : n_3 : n_4 : n_5 = 7 : 6 : 5 : 4 : 3. \eta = 10 \text{ and initial } \Omega_i = \emptyset, \forall i = 1, 2, \dots, M. \text{ Each WRAN}$$

would like to maximize its utility function in (6.33). For the convenience of representation,

we assume $\Theta, \Phi_i, \forall i = 1, 2, \dots, M$ do not change when applying the proposed algorithm

in Fig. 6.3. First, we study how the expected cost changes when the switch probability

increases. From Fig. 6.6, note that the expected cost has a single minimum on $p \in [0, 1]$ and

p^* varies with the number of available channels. Then we apply the proposed algorithm to

choose appropriate channels for $\Omega_i, \forall i = 1, 2, \dots, M$. Finally, we get $\Omega_1 = \{s_{12}, s_{16}, s_{19}, s_{28}\},$

Figure 6.6: The expected cost versus switch probability.

$$\Omega_2 = \{s_{13}, s_{24}, s_{30}\}, \Omega_3 = \{s_{10}, s_{14}, s_{29}\}, \Omega_4 = \{s_{15}\}, \Omega_5 = \{s_{20}, s_{21}\}.$$

6.5 Chapter Summary

IEEE 802.22 is an important standard for WRANs for broadband access in rural and remote areas. Multiple overlapping IEEE 802.22 networks compete to access the same spectrum bands and will interfere each other, i.e., the self-coexistence problem. Improper strategy for the self-coexistence problem will increase the interference among different WRANs. Some prescribed QoS requirements will not be met. We formulate the self-coexistence problem as a noncooperative game and derive the Nash equilibrium. Both theoretical and experimental analysis are conducted. Simulation results show that experimental costs approximate the theoretical expected cost. Moreover, the proposed strategy obtains a better performance than other strategies.

Chapter 7

Conclusions and Future Work

In this chapter, we summarize the contributions of the thesis and discuss some related future research issues.

7.1 Summary of Contributions

Dynamical resource allocation schemes in CRNs can help to improve the efficiency of spectrum utilization effectively. In order to successfully compete to access the limited unlicensed spectrums at minimum cost and achieve a good performance, a CRU is required to be equipped with the ability to adapt its transmit parameters dynamically according to the wireless environment it senses. Efficient allocation of the transmit parameters among CRUs is an important research challenge in CRNs. In this thesis, we propose algorithms and strategies to adapt the transmit parameters of CRUs under different scenarios. Simulation results demonstrate how our proposed methods significantly improve the performance of CRNs.

The first contribution of this thesis is the establishment of a suboptimal algorithm for the subcarrier allocation in multiuser OFDM based cognitive radio systems and the total data rate improvement achieved by the proposed MAs. These two help to increase the total data rate of CRUs. High throughput requires efficient algorithms to allocate the resources among CRUs. In OFDM systems, the spectrum is divided into many parallel subcarriers. Each user can be assigned from one to many subcarriers. The power allocated to each subcarrier is related to which user the subcarrier belongs to. The design of a fast and

efficient method for dynamically allocating subcarriers, transmit powers and bits to CRUs in an MU-OFDM based CR system belongs to a combinatorial optimization problem. In order to reduce the complexity, the resource allocation is solved in two steps: (1) to determine the allocation of subcarriers to users; (2) to determine the allocation of bits to users. We propose a suboptimal algorithm, which is based on a greedy approach, with low complexity yet comparable performance for the subcarrier allocation. After subcarrier allocation, the bit allocation is still computationally complex. We propose an efficient MA to determine the bit allocation. On the other hand, premature convergence is a disadvantage of EAs. In order to prevent premature convergence, MAs with multi-local-search methods can suppress the effect of premature convergence. We propose an MA with multi-local-search method to solve the bit allocation problem and achieve higher throughput than existing algorithms.

In addition, we develop a policy, based on fitness landscape analysis, to select appropriate genetic operators and local methods for the proposed MAs. Simulation results have shown that the performance of MAs depends on the selection of genetic operators and local search methods. This requires further analysis of the problem at hand. It has been shown that fitness landscape analysis is a powerful technique for analyzing a combinatorial problem. The selection of genetic operators and local search methods can be conducted based on some important fitness landscape properties. When applying MAs for solving the bit allocation problem, we choose local search methods based on fitness landscape analysis. Simulation results show that it is difficult for traditional suboptimal algorithms to find solutions which are close to the global optimal solutions and the proposed MAs are more appropriate for solving the bit allocation problem. Compared to the existing algorithms, the proposed subcarrier algorithm and MA are able to obtain a better performance.

Secondly, we develop two schemes to determine the transmit power when only partial CSI is available at the transmitter. These two schemes enable the transmission with partial CSI to satisfy the given BER requirements. The assumption that the transmitter always receives the CSI perfectly is impractical for wireless systems. In order to maintain the system performance, an appropriate resource allocation scheme based on partial CSI is

needed. Firstly, we analyze the effects of partial CSI on the resource allocation problem in MU-OFDM based cognitive radio systems. Based on the obtained partial CSI at the transmitter, the average BER should satisfy the given BER target requirement during transmission. As the function of the average BER is too complex and difficult to find a closed-form of expression for transmit power, we apply a Nakagami- m distribution to approximate the original function. A simple function, which is close to the original function, is derived. Different cases of partial CSI and bit rate requirements are studied. Simulation results show that partial CSI has a great impact on the resource allocation scheme. In addition, due to the lack of user diversity, the total bit rate decreases when the data rate requirements become less uniform. Secondly, we investigate the dynamics of a wireless resource allocation model with the Doppler effect. We find that traditional allocation schemes for this model are sensitive to the changes in the correlation coefficient. When $|\rho| \ll 1$, the allocation scheme is very conservative and consumes more power to transmit the same information. In order to suppress the sensitivity of ρ , we develop a new resource allocation scheme by introducing a control parameter from the viewpoint of control theory. It can be seen that the newly developed allocation scheme is robust with respect to the changes in correlation coefficients. Also, the efficiency of the resource allocation can be further improved by adjusting the control parameters. The simulation results have confirmed the efficiency of the new allocation scheme under different channel conditions.

Finally, we propose a strategy, based on game theory, to resolve the self-coexistence problem in IEEE 802.22 networks. The proposed strategy helps to significantly decrease the interference among overlapped WRANs. IEEE 802.22 is an important standard for WRANs for broadband access in rural and remote areas. The self-coexistence problem is still a research issue in CRNs. Since the spectrum management among competing WRANs are distributed and the coordination amongst WRANs of different service providers does not exist, several networks overlaying each other may switch to the same channel when sensing an incumbents' existence. Improper strategy for the self-coexistence problem will increase the interference among different WRANs and some prescribed QoS requirements may not

be met. These different networks can be considered to seek their own benefits or utilities independently. We formulate the self-coexistence problem as a noncooperative game and derive the Nash equilibrium. Both theoretical and experimental analysis are conducted. Simulation results show that the proposed strategy achieves a better performance than other strategies.

7.2 Future Work

There are still many research challenges to be studied in the area of resource allocation in CRNs. As we discussed before, CRUs compete to access the limited unlicensed spectrums. They interfere with each other and power control is a complex issue in CRNs. On one hand, transmit power can not be too low in order to achieve good QoS. On the other hand, high transmit power will increase the interference to other CRUs. We need to find a good strategy to conduct the trade-off between transmit power and interference level.

Another possible future work is to study how the CRUs behave collaboratively to optimize the spectrum opportunity sharing where distributed CRUs compete to access the available spectrums. In such a network, each CRU is able to access a certain number of spectrums regardless of its location. We need to design a set of rules so that each CRU can opportunistically utilize its available spectrums while minimizing interference with its neighbors. Obviously, which spectrum a CRU should access not only depends on its location and environment, it also depends on the number of available spectrums. The problem of spectrum opportunity sharing is equivalent to graph coloring. Specifically, CRUs form vertices in a graph, and an edge between two vertices indicates two interfering users. Considering each frequency band as a color, we can formulate it as a graph coloring problem: Color each vertex using a number of colors from its color list under the constraint that two vertices linked by an edge cannot share the same color. The objective is to obtain a color assignment that maximizes a given utility function. However, the graph coloring problem belongs to combinatorial optimization problems and is NP-hard.

Publications

Journal

- (1) **Yuan Miao**, Dong Huang, Zhiqi Shen, Chunyan Miao, Cyril Leung, “A Game Theory Approach for the Self-Coexistence Problem among IEEE 802.22 Networks.” *To be submitted to IEEE Transaction on Vehicular Technology.*
- (2) **Dong Huang**, Chunyan Miao, Zhiqi Shen, Zhihong Man, “Further Analysis on Resource Allocation in Wireless Communications under Partial Channel State Information.” *To be submitted to IEEE Communications Letters.*
- (3) **Dong Huang**, Zhiqi Shen, Chunyan Miao, Cyril Leung, “Resource Allocation in MU-OFDM Based Cognitive Radio Systems with Partial Channel State Information.” *EURASIP Journal on Wireless Communications and Networking, Volume 2010, Article ID: 189157, 8 pages.*
- (4) **Dong Huang**, Chunyan Miao, Zhiqi Shen, Cyril Leung, “Fitness Landscape Analysis for Dynamic Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems.” *ACM Mobile Computing and Communications Review (MC2R), 13(2), pp: 26 - 36, 2009.*

Conference

- (1) **Dong Huang**, Chunyan Miao, Yuan Miao, Zhiqi Shen, “A Game Theory Approach for Self-Coexistence Analysis in IEEE 802.22 Networks,” *7th International Conference on Information, Communications and Signal Processing, Macau, pp: 1 - 5, 2009.*

- (2) **Dong Huang**, Chunyan Miao, Cyril Leung, Zhiqi Shen, “Memetic Algorithms with Multi-Local-Search for Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems,” *Third International Conference on Communications and Networking (ChinaCom’08), Hangzhou, pp: 269 - 274, 2008.*
- (3) **Dong Huang**, Cyril Leung, Chunyan Miao, “Memetic Algorithm for Dynamic Resource Allocation in Multiuser OFDM Based Cognitive Radio Systems,” *IEEE World Congress on Computational Intelligence (WCCI 2008), Hong Kong, pp: 3860 - 3865, 2008.*

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