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Frequency-Invariant Beamforming For Circular Arrays

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Doctor of Philosophy

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Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

14th Jan 2011

Date

张欣

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Summary

Sensor arrays have been used widely in applications including radar, sonar, seismology, biomedicine, communications, geophysical exploration, astronomy and imaging. A very popular type of sensor arrays is the circular array. It has several advantages such as the fact that it can perform 360° scan very conveniently and during the scan the gain response can be kept almost invariant. In this research study, we focused on the investigation and design of broadband beamforming techniques using circular arrays. Such techniques can perform speech acquisition and enhancement under noisy environments. Possible practical applications include video conferencing at boardrooms or hands-free communications in vehicular environments.

The design of Frequency-Invariant(FI) beampattern synthesis algorithms and adaptive broadband beamformer using circular array are addressed in this research. The first method adopted a phase mode structure and proposed a novel objective function with a quadratic constraint. The main idea of this proposed method is to have a trade off between the extent of FI characteristic in the desired direction and that in all other directions. By focusing the FI characteristics in the desired direction where the signal of interest is arriving from, and relaxing the FI requirement in other non desired directions, a better FI performance in the desired direction with fewer number of sensors used can be achieved. The principle behind this idea is that we assume a fixed number of degrees of freedom is presented in any system,

and the extra degree of freedom that are freed from the relaxed FI requirement in the non desired directions can be used to achieve a better FI performance in the desired directions, and/or reduce the number of sensors used. Moreover, the proposed constraint imposed allows the FI characteristics to be more accurately controlled over the specified bandwidth, at the expense of that over other parts of the spectrum that are of less concern to the overall performance. Mathematical transformation converts this non-convex optimization problem into a convex one. Second Order Cone Programming (SOCP) is then used to solved it with high accuracy and efficiency. Simulation results are presented to show that, using circular array, the proposed method has better performance in achieving FI in the main lobe than an existing algorithm. A conference paper and a journal paper have been published on this work.

The second proposed method extends the first idea further by incorporating multiple constraints into the optimization model. The idea is to achieve not only FI but also Spatial Invariant (SI) characteristics and/or lower Peak SideLobe (PSL) level. Specifically, one constraint is imposed in the spatial domain to ensure that the mainlobe region has a constant gain over a specified angular region across a specified wide frequency range, and another constraint is imposed on the PSL level. These modifications are simple, and yet they yield promising results at both low frequency and high frequency regions. The proposed design is formulated as a convex optimization problem. SOCP method is used to solve these problems with high accuracy and efficiency. Simulation results are presented to illustrate the performance of the proposed algorithms at different frequency ranges and for different parameters. Performance comparison between the proposed beamformer and existing beamformers is also provided. A conference paper has been published on this work and a journal paper is under revision.

The third method is the development of the above framework into an adaptive design using a multi-beam structure. In the conventional Generalized Sidelobe Canceller (GSC) structure, the received data signal is projected onto an unconstrained subspace by means of a blocking matrix and a quiescent vector. In this proposed method, the blocking matrix and the quiescent vector are replaced with the FI beampattern synthesis method proposed in this study, and the adaptive weights are calculated using the Least Mean Square (LMS) algorithm. This method effectively transforms the broadband problem into a narrowband problem, and as such only a single weight is required at the output of each FI beamformer. Simulation results show that the proposed multi-beam adaptive beamformer has better performance than that of a recently published algorithm.

Overall, this research study has provided a better understanding of the design of broadband FI beamformers using uniform circular arrays. The main contributions include the proposal of three novel algorithms, where simulation results show that they outperform some recently published algorithms. These findings have generated three conference papers (presented), and three journal papers (one in press, one under revision, and another submitted).

List of Abbreviations and Symbols

Abbreviations

ASV	array steering vector
A/D	analog to digital
BF	beamforming
DFT	discrete Fourier transform
DOA	direction of arrival
DS	delay-and-sum
FFT	fast Fourier transform
FI	frequency invariant
FIR	finite impulse response
GSC	generalized sidelobe canceller
IFFT	inverse fast fourier transform
IDFT	inverse discrete fourier transform
LCMV	linearly constrained munimum variance
LMS	least mean square
LS	least square
MSE	mean square error
MSRV	mainlobe spatial response variation

MVDR	minimum variance distortionless response
NLMS	normalized least mean square
PSL	peak sidelobe
PSC-FI	peak sidelobe constrained frequency invariant
SI	spatial invariant
SIR	signal-to-interference ratio
SINR	signal-to-interference-plus-noise ratio
SNR	signal-to-noise ratio
SOCP	Second Order Cone Programming
SOI	source of interest
SSI-FI	selective spatial invariant frequency invariant
s.t.	subject to
TD	time delay
TDOA	time difference of arrival
UCA	uniform circular array
UCCA	uniform concentric circular array
ULA	uniform linear array

Symbols

a	scalar
$\mathbf{a}$	vector
$(\cdot)$	dot product
c	sound propagation velocity
I	number of plane waves
A	amplitude of the plane wave
α_i	phase displacement of the i^{th} plane wave
K	number of microphones
τ_{ki}	time delay of the i^{th} plane wave at k^{th} sensor
$\vec{\mathbf{u}}$	the unit vector in the incident direction of the source
$\mathbf{r}$	position vector of the microphone
v	the speed of the propagating wave
$n_k(t)$	the complex random noise at k^{th} microphone
$\mathbf{x}(\mathbf{t})$	the vector of the observed signals derived at the output of the microphones
$\mathbf{a}$	the steering vector
$\mathbf{w}$	the complex weight vector
$(\cdot)^H$	the Hermitian transpose
$(\cdot)^T$	the transpose
$E\{\cdot\}$	the expectation operator
$\mathbf{R}$	the array covariance matrix
$\mathbf{g}$	the weight vector of the fixed beamformer in the GSC structure
BP	Beampattern of the array
p	output power of the beamformer
ω	angular frequency
$\star$	linear convolution

$ch_{ki}(t)$	a linear transfer function that represents propagation effects between the i^{th} source and k^{th} microphone
f_0	reference frequency
f_u	upper frequency limit
f_l	lower frequency limit
$s_k(t)$	k^{th} source in time domain
N_s	the total number of snapshot
d	inter-microphone distance
D	the number of delay elements
$\mathbf{C}_0$	the constraint matrix
$(\cdot)^*$	complex conjugate of a vector or matrix
$(\cdot)^T$	transpose of a vector or matrix
$\ \cdot\ _2$	Euclidean norm
$Re\{\cdot\}$	real operator
$\mathbf{B}$	blocking matrix of the GSC structure
Θ_{ML}	mainlobe region
Θ_{SL}	sidelobe region
$\mathbf{I}$	identity matrix
j	imaginary unit
t	time index
ϕ_k	azimuth angle of the k^{th} microphone
θ_k	elevation angle of the k^{th} microphone
λ_{min}	the wavelength corresponding to the largest frequency of the received signal
μ	step size
$\mathbf{w}$	array weight vector
$\hat{\mathbf{w}}$	estimated optimal array weight vector

r	radius of the circular array
M	total number of phases
p_m	m^{th} phase mode signal
P_m	Fourier Transform of the m^{th} phase signal
b_m	FIR coefficient at m^{th} phase
B_m	Fourier Transform of the FIR coefficients
h_m	spatial weighting at m^{th} phase
S	Fourier Transform of the desired signal $s(t)$
X	Fourier Transform of the received signal $x(t)$
$y(t)$	the output of the beamformer in time domain
Y	Fourier Transform of the beamformer output $y(t)$
G	response of the beamformer
N_m	FIR filter order at m^{th} phase

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Chapter 1

Introduction

1.1 Background and Motivation

This thesis is primarily concerned with the investigation and design of broadband beamforming techniques using circular arrays.

Beamforming, which is also known as spatial filtering, has been studied for decades as an attractive solution for signal detection and reception in harsh environments. It is a technique that exploits the use of multiple sensors to focus the reception of signals coming from one particular direction while nulling out signals coming from all other directions. In doing so, it is able to improve the quality of the desired signal in the presence of interferences based on the principle of spatial diversity. It has found many applications in radar, radio astronomy, sonar, wireless communications, seismology, speech acquisition, medical diagnosis and treatment [1, 2].

The earliest beamforming technique was developed way back during the Second World War [3, 4] and it is a mere application of Fourier-based spectral analysis. Since then, many advanced forms of beamformers have been proposed [5, 6]. In general, they can be classified into data dependent beamformer and data-independent beamformer. From the names imply, the former optimizes the weights based on the statistic of the received signal, such as the second order statistics [7–9, 13–20],

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while the later designs the beamforming weights without any knowledge of the received signal [21–24]. The spatial filtering approach, however, suffers from one fundamental limitation: its performance/beam pattern, in particular, is directly dependent upon the physical size of the array in term of frequency, regardless of the available data collection time and signal-to-noise ratio (SNR). In another words, an array with one particular physical size works well in one frequency but not the other. Undoubtedly, this will pose problem when we are dealing with signals that spanned across several octaves of frequencies.

In the literature, there are several ways to designing a beamformer that solve the above problem. One method is to use narrowband decomposition. The received broadband signal is decomposed into several narrow-band signal, and beamforming technique is applied for each frequency bin [2]. This approach requires several narrowband processing to be conducted simultaneously and is therefore computationally expensive.

Alternatively, another design approach is to employ adaptive beamforming technique. Such techniques use a bank of linear transversal filters to generate a desired beam pattern. The filter coefficients can be derived adaptively from the received signals. One classic design example is the Frost Beamformer [9], in which a number of taps are attached behind each sensor and the weights are designed such that an optimization criterion is met. However, in order to have a similar beam pattern over a broad frequency range, a large number of sensors and filter taps will be needed. This again leads to high computational complexity.

In the aim of reducing the computational complexity, frequency invariant beam-pattern synthesis is the approach for broadband signal. It designs constant spatial gain response over a wide range of frequency bands. Fundamentally, this method transforms the received signal into a frequency-independent mode, which is named

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”phase mode”. In this domain, the frequency response and the spatial response of the signal can be decoupled and adjusted independently [10–12]. A recent work produced by Chan *et al* designed a FI beamformer for all angles using an concentric uniform circular array. One limitation of Chan’s beamformer is that, a relatively large number of sensors have to be used to form the concentric circular array. Performance will be degraded when the number of sensors used is reduced.

In view of the methods above, we find that, in order to achieve frequency invariant characteristics, large number of microphones are required, either be it linear array or concentric circular array. In order to avoid spatial aliasing, the more number of microphones used, the larger is the array aperture. In the current research pool, not many articles address the array aperture issue in frequency invariant beamformer. Hence, in this PhD study, we focus on proposing techniques that achieve frequency invariant characteristic using smaller array aperture. Using the same number of microphones, linear array will result in a larger array aperture than a circular array. This can be easily conceptualized by joining the linear array end to end in a round circle. Furthermore, circular array does not have end-fire problem. It can perform 360° scan conveniently. Hence, this PhD study is more specialized in investigating the performance of a circular array in achieving frequency invariant characteristics under various conditions. Inspired by Chan’s design, phase mode is adopted in our method. We find that the phase mode structure together with circular array geometry indeed yields great flexibility in controlling the gain response in different spatial region across a wide range of frequencies. A better understanding of the relationship between phase mode structure and circular array is gained, and solutions are proposed to fill up the literature gap in this thesis.

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1.2 Objectives

The objective of this thesis is to investigate the design and performance of FI beamforming techniques using a circular array structure.

Specifically, this research aims to achieve a better understanding of the design of circular broadband beamformers in phase mode with FI characteristic across selective spatial region via computer simulation.

Secondly, this research is to study the design and performance of circular array beamformers when constraints are imposed in both frequency and spatial domains.

Lastly, this research details the study of extending the above framework into adaptive algorithm using multi-beam structure and evaluated its effectiveness.

1.3 Major Contributions of the Thesis

The main contributions of this thesis are summarized below:

- (i) This proposed algorithm aimed to minimize a function of the spatial response with a constraint on the gain being smaller than a pre-defined threshold value across a specified frequency range and at a specified angular sector for the transformed signal. The problem was formulated as a convex optimization problem using MiniMax criteria. The solution was obtained using SOCP technique. The proposed algorithm was shown to be computational effective via a computational complexity analysis. MATLAB simulations showed that the proposed method achieved better FI performance in the desired direction using fewer number of microphones.
- (ii) Two FI beampattern synthesis approaches in phase mode were proposed. Multiple-constraint were imposed to synthesize a desired beampattern. One

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constraint was imposed on spatial domain to ensure that the mainlobe region has a constant gain over an angular range across a wide spectral region. Second constraint was imposed on the peak sidelobe level. This proposed design was reformulated as a convex optimization problem using Cholesky factorization. SOCP method was used to solve these problems with high efficiency and accuracy. MATLAB simulation showed promising results at both low frequency and high frequency regions.

- (iii) An adaptive broadband beamformer using multi-beam structure was proposed. In this proposed approach, the blocking matrix and the quiescent vector in the conventional GSC structure were replaced with the beam pattern synthesis methods proposed above, and the adaptive weights were calculated using the LMS algorithm. This method transformed the broadband problem into a narrowband problem. As a result, only a single weight was required at the output of each FI beamformer. Simulation results showed that the proposed adaptive beamformer has faster convergence rate and converge to a smaller mean square error than an existing algorithm.

1.4 Organization of the Thesis

The remainder of this thesis is organized as follows. Chapter 2 provides the background materials of broadband beamforming techniques. The definition of the signal model used in array processors and the mathematical representation of different array geometries are presented. An overview of broadband beamformer with focus on adaptive beamforming techniques and frequency-invariant beam pattern synthesis techniques is demonstrated. Finally common techniques for improving robustness of a beamformer are briefly introduced.

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Frequency-invariant beamforming technique is desirable in many array applications such as speech acquisition, acoustic imaging and communication. In Chapter 3, the first method is presented in which a novel objective function with quadratic constraint was proposed. The objective function was formulated as a convex optimization problem and the solution was obtained by using the SOCP technique. An analysis of the computational complexity of the proposed algorithm will be presented as well as the performance of the proposed algorithm via computer simulation for different number of sensors and different threshold values.

Chapter 4 presents our second proposed method which further extended the first idea into frequency-spatial invariant characteristics with multiple constraints. The proposed design was then reformulated as a convex optimization problem. SOCP method was used again to solve these problems with high efficiency and accuracy. Simulation results are presented to illustrate the performance of the proposed algorithms at different frequency ranges. Performance comparison between the proposed beamformer and existing beamformers is also provided as well as the sensitivity study.

In Chapter 5, an adaptive broadband beamformer using multi-beam structure is proposed. The adaptive structure is first described where the blocking matrix and the quiescent vector in the conventional GSC structure were replaced by our previously proposed FI beamformer. The implementation of the structure is then described. Simulation results that illustrate the efficacy of the proposed system such as convergence rate and output SINR are provided. It demonstrated a better performance than an existing algorithm. A sensitivity study in evaluating the performance of the proposed system is also included.

Finally, Chapter 6 contains the conclusions for this thesis and also some suggestions for further research.

Chapter 2

An Overview of Broadband Array Signal Processing

2.1 Introduction

The aim of Chapter 2 is to provide an overview of the background materials on broadband array processing techniques available in the literature and also a foundation for thorough understanding of the proposed methods which will be presented in subsequent chapters.

Chapter 2 is organized as follows. In Section 2.2, mathematical notations and signal model used in array processing for both narrowband and broadband signal are presented. In Section 2.3, Mathematical representation for uniform linear arrays and uniform circular arrays are discussed. More emphasis is placed on circular array geometry. In Section 2.4, an overview of broadband beamforming techniques is presented with focus on adaptive broadband beamformer and frequency-invariant beamformer. Additionally, a discussion on the difference between frequency domain beamformer and phase mode beamformer is given in this section. Finally, in Section 2.5, common techniques for improving robustness of a beamformer are briefly introduced.

2.2 Signal Representation in Array Processing

It is frequently assumed in this thesis that the sensor array is located in the far field of the point source, $s(t)$. Thus, as far as the sensor array is concerned, the directional signal incident on the array for far field signal can be considered as a plane wave.

In sensor array signal processing, propagating waves convey signals from the source to the array. These signals are functions of position as well as time. They have properties governed by the laws of physics, in particular the wave equation. Discrete Fourier Transform (DFT) is commonly used to transform these signals into frequency domain. Classified by their frequency characteristic, there are two types of signals, namely, broadband signal and narrowband signal. By definition, narrowband signal has a fractional bandwidth of less than 1%, while that of a broadband signal is up to 50%. Representations for these two signals are discussed respectively in the following sections.

2.2.1 Narrowband Signal Representation

A signal environment as shown in Fig 2.1 consists of I plane waves. Each plane wave arrives at the array from a distinct direction. All the plane waves are narrowband, with the same frequency ω_0 . According to the wave equation, the noiseless signal produced at the k^{th} sensor of the array due to i^{th} plane wave can be expressed as

$$s(k, i, t) = A_i e^{j\omega_0(t - \tau_{ki}) + \alpha_i}, \quad (2.1)$$

where t is the time, A_i and α_i are the amplitude and phase displacement of the i^{th} signal produced, τ_{ki} is the time delay at k^{th} sensor for i^{th} signal.

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Figure 2.1: Array with arbitrary geometry

The expression of τ_{ki} is given by

$$\tau_{ki} = \frac{\vec{\mathbf{u}}_i \cdot \mathbf{r}_k}{v}, \quad (2.2)$$

where $\cdot$ is the dot product, $\vec{\mathbf{u}}_i$ is the unit vector in the incident direction (ϕ_i, θ_i) of the i^{th} source, $\mathbf{r}_k$ is the position vector of the k^{th} sensor, and v is the speed of the propagating wave. Clearly, $\vec{\mathbf{u}}_i$ and $\mathbf{r}_k$ can be expressed as

$$\vec{\mathbf{u}}_i = \begin{bmatrix} \sin \theta_i \cos \phi_i \\ \sin \theta_i \sin \phi_i \\ \cos \theta_i \end{bmatrix} \quad (2.3)$$

and

$$\mathbf{r}_k = \begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix} \quad (2.4)$$

respectively.

Substituting (2.3) and (2.4) into (2.2), τ_{ki} can be expressed as

$$\tau_{ki} = \frac{1}{v} [(x_k \cos \phi_i + y_k \sin \phi_i) \sin \theta_i + z_k \cos \theta_i]. \quad (2.5)$$

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The coordinate of each array element, $\mathbf{r}_k$, is assigned by taking the center of gravity of the array as the origin, such that

$$\sum_{k=0}^{K-1} \mathbf{r}_k = \mathbf{0}. \quad (2.6)$$

The selection of the center of gravity of the array as the coordinate origin will ensure that the array is equipped with the rotational invariance property [20]. This property is important as one would expect the array to have the same characteristic irrespective of whether the source is rotated in θ and/or ϕ or the array rigidly rotated by the corresponding $-\theta$ and/or $-\phi$ with a fixed source.

The sensor received signal in (2.1) can be represented by its complex amplitude, $s(k, i)$, defined by

$$\begin{aligned} s(k, i) &= A_k e^{-j\omega_0 \tau_k i + \alpha_i} \\ &= (A_i e^{j\alpha_i}) e^{-j\omega_0 \tau_k i} \\ &\triangleq a_i e^{-j\omega_0 \tau_k i}. \end{aligned} \quad (2.7)$$

The signal in (2.7) describes the noiseless signal output at the k^{th} sensor. In real life, signal often contains noise. In many array processing examples, the noise is assumed to be a white, ergodic random process. The output from each element in the array is customarily filtered to the same narrowband frequency occupied by the actual received signal.

The observed signal at the k^{th} sensor, $x_k(t)$, can be expressed as the sum of the noiseless signals produced by all the plane waves and the white noise as follows:

$$x_k(t) = \sum_{i=1}^I a_i e^{-j\omega_0 \tau_k i} + n_k(t), \quad (2.8)$$

where $n_k(t)$ is the complex random noise received at k^{th} sensor. Let $\mathbf{x}(t)$ be the vector of the observed signals derived at the output of the sensors, i.e.

$$\mathbf{x}(t) = [x_1(t) \ x_2(t) \ \cdots \ x_K(t)]^T, \quad (2.9)$$

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Figure 2.2: Narrowband beamformer with K sensors

where the superscript T denotes vector transpose.

Substitute (2.8) into (2.9), it follows that

$$\mathbf{x}(t) = \sum_{i=1}^I a_i \mathbf{a}_i(\omega_0, \phi_i, \theta_i) + \mathbf{n}(t), \quad (2.10)$$

where $\mathbf{a}_i(\omega_0, \phi_i, \theta_i)$ is the steering vector of the i^{th} source given by

$$\mathbf{a}_i(\omega_0, \phi_i, \theta_i) = \begin{bmatrix} e^{-j\omega_0 \tau_{i1}} \\ e^{-j\omega_0 \tau_{i2}} \\ \vdots \\ e^{-j\omega_0 \tau_{iK}} \end{bmatrix}, \quad (2.11)$$

and $\mathbf{n}(t)$ is the received noise vector given by

$$\mathbf{n}(t) = [n_1(t) \ n_2(t) \ \cdots \ n_K(t)]^T. \quad (2.12)$$

The structure of a narrowband beamformer comprising of K sensors is shown in Fig 2.2. The signal received at each sensor, $x_k(t)$, $k = 1, \dots, K$ are weighted by w_k , $k = 1, \dots, K$ and summed up to form the output signal $y(t)$,

$$y(t) = \sum_{k=1}^K w_k^* x_k(t) = \mathbf{w}^H \mathbf{x}(t), \quad (2.13)$$

where $\mathbf{w}$ denotes the complex weight vector and $(\cdot)^H$ denotes the Hermitian transpose.

$$\mathbf{w} = [w_1 \ \cdots \ w_k \ \cdots \ w_K]^T. \quad (2.14)$$

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For a source that can be modeled by stationary stochastic process, the mean output power from the array system is given by

$$p(\mathbf{w}) = E\{|y(t)|^2\} = \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (2.15)$$

where $E\{\cdot\}$ denotes the expectation operator, and $\mathbf{R}$ is the $K \times K$ dimensional array covariance matrix defined by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}. \quad (2.16)$$

2.2.2 Broadband Signal Representation

Speech is a broadband signal whose frequencies spanned over several octaves, i.e., $\omega \in [\omega_1, \omega_2]$, where ω_1 and ω_2 are the lower and upper frequencies of the signal, respectively.

The k^{th} array sensor received signal $x_k(t)$ due to the i^{th} source $s_i(t)$ cannot be expressed in the same form as (2.8). It should be expressed as

$$x_k(t) = s_i(t) \star ch_{ki}(t) + n_k(t) = \int ch_{ki}(t - \tau) s_i(\tau) d\tau + n_k(t), \quad (2.17)$$

where $\star$ denotes linear convolution, $n_k(t)$ is the environment noise, $ch_{ki}(t)$ is a linear transfer function that represents propagation effects between the i^{th} source and k^{th} sensor and any signal distortion in the sensor itself. In the case of ideal (non-dispersive) propagation and distortion-free omni-direction elements, $h_{ki}(t)$ corresponds to a pure time delay. In fact, this is the most often used assumption in sensor array processing. In such case, the received signal becomes

$$x_k(t) = s_i(t + \tau_{ki}(\phi_i, \theta_i)) + n_k(t), \quad (2.18)$$

where $\tau_{ki}(\phi_i, \theta_i)$ is similar to the one in (2.2).

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Figure 2.3: Broadband presteered beamformer with M sensors and J taps per sensor

With I impinging sources presented, the array received signal at k^{th} microphone is expressed as

$$x_k(t) = \sum_{i=1}^I s_i(t + \tau_{ki}(\phi_i, \theta_i)) + n_k(t). \quad (2.19)$$

To handle the broadband signals effectively, the array processor behind the sensors must be able to provide a phase shift that also varies with frequencies [3–5, 7]. This is accomplished by the use of tapped delay lines as shown in Fig 2.3.

A broadband beamformer in time domain is shown in Fig 2.3. Comparing with the narrowband beamformer in Fig 2.2, a broadband beamformer uses tapped delay line filters to replace the single weight in narrowband beamformers. Generally, the weights in narrowband beamformer are complex numbers and they are real numbers in broadband beamformer. The beamformer in Fig 2.3 consists of K sensors and J taps per channel. The array received signal is first delayed by a set

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of pure time delays $\tau_k, k = 1, \dots, K$. With these time delays, the mainbeam of the beamformer can be steered to the desired direction. Its output is expressed as

$$y(t) = \mathbf{w}^T \mathbf{x}(t), \quad (2.20)$$

where the signal vector $\mathbf{x}(t)$ is defined as

$$\mathbf{x}(t) = [x_1(t) \ \dots \ x_K(t) \ x_1(t-T_s) \ \dots \ x_K(t-T_s) \ \dots \ x_1(t-(J-1)T_s) \ \dots \ x_K(t-(J-1)T_s)]^T, \quad (2.21)$$

and the weight vector $\mathbf{w}$ is defined as

$$\mathbf{w} = [w_1 \ w_2 \ \dots \ w_{JK}]^T. \quad (2.22)$$

For the sources that can be modeled by stationary stochastic processes, the mean output power from the array system is given by

$$p(\mathbf{w}) = E\{|y(t)|^2\} = \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (2.23)$$

where $\mathbf{R}$ is the $KJ \times KJ$ dimensional array covariance matrix defined by

$$\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^H(t)\}. \quad (2.24)$$

In practice, the matrix $\mathbf{R}$ is usually estimated using sample averaging,

$$\hat{\mathbf{R}} = \frac{1}{N_s} \sum_{n=1}^{N_s} \mathbf{x}(n)\mathbf{x}^T(n), \quad (2.25)$$

where N_s is the total number of snapshot, and vector $\mathbf{x}(n)$ denotes the sampled signal received at the beamformer. $\mathbf{x}(n)$ has an expression of the following form:

$$\mathbf{x}(n) = [x_1(n) \ \dots \ x_K(n) \ x_1(n-1) \ \dots \ x_K(n-1) \ x_1(n-J+1) \ \dots \ x_K(n-J+1)]^T. \quad (2.26)$$

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Figure 2.4: Uniform linear array with K sensors

2.3 Array Geometry

The placement of array elements in the space determines the spatial sampling pattern of the signal field. It is best classified by number of dimensions that the array spans, i.e., 1-D array, 2-D array, or 3-D array. Usually, the choice of array configuration is decided by several factors, such as application requirement, physical constraint, and algorithm complexity *etc.* In the literature, many ideas developed for 1-D array are carried over to the 2-D cases such as rectangular array and circular array. Examples are given at [39–45]. In this thesis, the main focus is on circular array. Hence, in this section, we will briefly introduce conventional uniform linear array, and concentrate on circular array.

2.3.1 Uniform Linear Array

Uniform Linear Array (ULA) is widely used in many array applications due to its simple and regular physical form and its implementation efficiency in array algorithms. Fig. 2.4 shows an K -element ULA, where d is the spacing between two adjacent sensors.

Mathematically, the inter-element time delay at the k^{th} sensor can be derived

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from (2.2) by setting $y_k = z_k = 0$ and $\theta_k = 0^\circ$, i.e.,

$$\tau_k = \frac{x_k \cos \phi}{v}, \quad (2.27)$$

where ϕ is the incident angle of the directional source measured from the y-axis (broadside), and x_k is the x-coordinate of the k^{th} element.

Taking the center of the array as the reference, the inter-element time delay follows that

$$\tau_m = [(m-1) - \frac{N-1}{2}] \frac{d \cos \phi}{v}. \quad (2.28)$$

2.3.2 Uniform Circular Array

Circular arrays have been studied over the past five decades for applications at radio frequency and microwave frequencies. Comparing to other 2-D arrays, circular array has received considerable interest because it provides almost uniform beampattern for 360° azimuthal coverage. When the array elements are equally spaced, the array is entitled as Uniform Circular Array (UCA). Because of the symmetrical structure of the UCA, the steering direction can be easily changed in the azimuth angle by simply shifting weights among array elements. Recently, UCA is employed to develop frequency-invariant beamformer for broadband signal. In this thesis, only UCA is considered.

Consider a uniform circular plane array which consists of K sensors, with radius r in the azimuth plane as shown in Fig 2.5. Assume that the elements of the array have the same amplitude and phase response. The position vector of the k^{th} element is

$$\mathbf{r}_k = (r \cos \phi_k, r \sin \phi_k, 0), \quad k = 1, \dots, K, \quad (2.29)$$

where $\phi_k = 2\pi k/K$.

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Figure 2.5: Geometry of a circular array

If the plane wavefront located in the far-field of the sensor incidents from a direction as shown in Fig 2.5 to the array,

$$\vec{u}_i = \begin{bmatrix} \sin \theta_i \cos \phi_i \\ \sin \theta_i \sin \phi_i \\ \cos \theta_i \end{bmatrix}, \quad (2.30)$$

then the time delay between the signals received at the k^{th} element and the reference point (origin of circular array) is

$$\tau_k = \vec{u} \cdot \mathbf{r}_k / c = -r \sin \theta \cos(\phi_k - \phi) / c, \quad (2.31)$$

where c is the sound propagation velocity.

If the signal received at the reference point is $s(t)$, then the output of the k^{th} element of the array is

$$x_k(t) = s(t - \tau_k) + n_k(t), \quad (2.32)$$

where $s(t)$ is a broadband signal with frequency band $f \in [f_l, f_u]$, $n_k(t)$ is the additive noise on the k^{th} element. Taking Fourier transform of (2.27), we have

$$X_k(f) = e^{-j2\pi f \tau_k} S(f) + N_k(f). \quad (2.33)$$

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The frequency domain output of array denoted in matrix form is given by

$$\mathbf{X}(f) = \mathbf{a}(f, \phi, \theta) \mathbf{S}(f) + \mathbf{N}(f), \quad (2.34)$$

where $\mathbf{N}(f) = [N_0(f), \dots, N_{K-1}(f)]$ is the additive noise vector, $\mathbf{a}(f, \phi, \theta)$ is the steering vector of the circular array having the following form,

$$\mathbf{a}(f, \phi, \theta) = [e^{j2\pi f r \sin \theta \cos(\phi_0 - \phi)/c}, \dots, e^{j2\pi f r \sin \theta \cos(\phi_{K-1} - \phi)/c}]^T. \quad (2.35)$$

Without loss of generality, the incident signal is considered being in the same plane as the circular array, i.e., $\theta = \pi/2$. In this case, the steering vector is

$$\mathbf{a}(f, \phi, \theta) = [e^{j2\pi f r \cos(\phi_0 - \phi)/c}, \dots, e^{j2\pi f r \cos(\phi_{K-1} - \phi)/c}]^T. \quad (2.36)$$

The beamformer output in frequency domain is formed by applying a weight vector to the received array data,

$$Y(f_n) = \mathbf{w}_n^H \mathbf{X}(f_n), \quad (2.37)$$

where H denotes Hermitian transpose, and $\mathbf{w}_n$ is the array weight vector for n^{th} frequency bin. The spatial response of the beamformer is given by

$$BP(f_n, \phi) = \mathbf{a}(f_n, \phi)^H \mathbf{w}_n. \quad (2.38)$$

This spatial response function of a beamformer defines the transfer relation between the source and the beamformer output.

2.4 Broadband Beamforming Techniques

Array processing is a specialized branch of signal processing that focuses on signals conveyed by propagating waves. The received signals are obtained by means of an array of sensors located at different points in space in the field of interest. The

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aim of array processing is to extract useful characteristics of the received signal field (e.g., its signature, direction, speed of propagation). The collected signals at sensors are combined following certain algorithm so as to enhance the Signal-to-Noise Ratio (SNR) of the target signal, to characterize the signal wave field, or to track the signal sources as they move in space.

With distributed sensors, the array processing technique exploits not only the temporal information, but also the spatial information of signals. The temporal-spatial processor greatly extends the capability in information extraction. This technique receives great interest in past decades. In this thesis, we focus on array processor in beamforming application for broadband signal.

The inherent problem in using narrowband beamformer for broadband signal is that the spatial response of narrowband beamformer varies across frequencies. One important dimension in measuring array performance is its size in terms of operating wavelength. Thus for high frequency signal, which has a small wavelength, a fixed array will appear large and the main beam will be narrow. However, for low frequencies, which has large wavelength, the same physical array appears small and the main beam will widen. Hence, the problem in designing a broadband beamformer resolves to finding the array weights such that the resulting spatial response remains almost constant over all frequency bins of interest.

In the literature, there are many ways to design a broadband beamformer. The most commonly used method is narrowband decomposition. This approach requires several narrowband processing to be conducted simultaneously and is computationally expensive. Alternatively, we could use adaptive beamformers. Such approach employs tapped-delay lines or linear transversal filters with adaptive coefficients to generate appropriate beam pattern over a continuous range of frequencies. However, in doing so, it requires a large number of sensors and taps.

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This leads to high computational complexity. Recently a method called frequency invariant beampattern synthesis is getting more and more attention. As its name implies, frequency-invariant beamformer designs constant spatial gain response over a wide range of frequency bands.

In this section, we are going to describe firstly the conventional broadband beamforming technique represented by Frost method, followed by a frequency-invariant beampattern synthesis method for time domain processor, then we focus on frequency-invariant beampattern synthesis method for phase domain processor. Lastly, a relationship between frequency domain, time domain and phase domain array processor is discussed.

2.4.1 Adaptive Broadband Beamforming

The classical time domain broadband beamformer uses a tapped delay-line behind each element. It linearly combines the delayed $x(n)$ to get the beamformed output $y(n, \mathbf{w}(l))(l = 0, 1, \dots, D)$, where D is the number of delay elements, according to the equation

$$y(n, \mathbf{w}(l)) = \sum_{l=0}^D \mathbf{w}^H(l) \mathbf{x}(n-l), \quad \text{for } n = 1, 2, \dots, N, \quad (2.39)$$

where $\mathbf{w}(l)$ is a $(K \times 1)$ weight vector.

Assume the broadband snapshot is

$$\mathbf{x}(n) = [\mathbf{x}(n)^T, \mathbf{x}(n-1)^T, \dots, \mathbf{x}(n-D)^T]^T, \quad (2.40)$$

and the broadband weight vector is

$$\mathbf{w} = [\mathbf{w}(0)^T, \mathbf{w}(1)^T, \dots, \mathbf{w}(D)^T]^T. \quad (2.41)$$

(2.39) can be compactly written as

$$y(n) = \mathbf{w}^H \mathbf{x}(n) \quad \text{for } n = 1, 2, \dots, N. \quad (2.42)$$

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Figure 2.6: Architecture of the DS beamformer

The architecture of this classical Delay and Sum (DS) broadband beamformer is depicted in Fig. 2.6.

A typical broadband beamformer is represented by Frost beamformer. It has a structure as shown in Fig. 2.3. The optimal weight vector $\hat{\mathbf{w}}$ is obtained by minimizing the array output power while maintaining a certain frequency response in the look direction. In mathematics, it is expressed as

$$\begin{cases} \min_{\mathbf{w}} & \mathbf{w}^H \mathbf{R} \mathbf{w} \\ \text{s.t.} & \mathbf{C}_0^T \mathbf{w} = \mathbf{f}, \end{cases} \quad (2.43)$$

where $\mathbf{C}_0$ is the constraint matrix. The most commonly used one, which is known as look direction constraint matrix is defined as

$$\mathbf{C}_0 = [\mathbf{c}_1 \quad \cdots \quad \mathbf{c}_j \quad \cdots \quad \mathbf{c}_J], j = 1, 2, \dots, J, \quad (2.44)$$

where

$$\mathbf{c}_j = [0 \quad \cdots \quad 0 \quad 1 \quad \cdots \quad 1 \quad 0 \quad \cdots \quad 0]^T, j = 1, \dots, J. \quad (2.45)$$

$\mathbf{f}$ is a vector of size $J \times 1$, which determines the frequency response in the look direction. With the Lagrange multipliers methodology [62], the optimal weight vector $\hat{\mathbf{w}}$ to the problem of (2.43) is given by [9],

$$\hat{\mathbf{w}} = \mathbf{R}^{-1} \mathbf{C}_0 (\mathbf{C}_0^T \mathbf{R}^{-1} \mathbf{C}_0)^{-1} \mathbf{f}, \quad (2.46)$$

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Figure 2.7: System structure of the Generalized Sidelobe Canceller

and the optimal beamformer output power $\hat{p}$ is given by

$$\hat{p} = \mathbf{f}^T (\mathbf{C}_0^T \mathbf{R}^{-1} \mathbf{C}_0)^{-1} \mathbf{f}. \quad (2.47)$$

2.4.1.1 Generalized Sidelobe Canceler

Frost beamformer is considered as a quadratic optimization problem subject to linear constraints. In [85], Griffith *et al* proposed an alternative implementation method of the LCMV processor by splitting the processor into two paths. The upper path is a fixed beamformer which can be designed according to the requirements on the quiescent response of array processor. The lower path is a blocking matrix which blocks target signal but pass noise and interference as much as possible, then followed by a multichannel adaptive filters. The diagram of a GSC is shown in Fig. 2.7.

The weight vector $\mathbf{g}$ of the fixed beamformer in Fig 2.7 can be selected as

$$\mathbf{g} = \mathbf{C}_0(\mathbf{C}_0^T \mathbf{C}_0)^{-1} \mathbf{f}, \quad (2.48)$$

and the matrix $\mathbf{B}$ whose columns form a basis for the null space of $\mathbf{C}_0$ can be used as a blocking matrix. The adaptive filter $\mathbf{w}'$ is obtained as

$$\min_{\mathbf{w}'} (\mathbf{g} - \mathbf{B}\mathbf{w}')^T \mathbf{R} (\mathbf{g} - \mathbf{B}\mathbf{w}'). \quad (2.49)$$

The solution $\hat{\mathbf{w}}'$ is

$$\hat{\mathbf{w}}' = (\mathbf{B}^T \mathbf{R} \mathbf{B})^{-1} \mathbf{B}^T \mathbf{R} \mathbf{g}. \quad (2.50)$$

The relationship between $\hat{\mathbf{w}}$ and $\hat{\mathbf{w}}'$ is

$$\hat{\mathbf{w}} = \mathbf{g} - \mathbf{B}\hat{\mathbf{w}}'. \quad (2.51)$$

All of the LCMV beamformers can be implemented in GSC structure. The GSC implementation has some advantages: it uses unconstrained optimization, which results in simple implementation and fast convergence in adaptation of filter weights; the fixed beamformer and blocking matrix can be flexibly chosen if they satisfy the necessary conditions. Consequently, GSC is widely used in practice.

2.4.2 Frequency-Invariant Beampattern Synthesis

Frequency Invariant (FI) beampattern synthesis is a technique for choosing the weights of each sensor output in order to obtain a constant spatial response over an arbitrarily wide design bandwidth.

Many FI beampattern synthesis methods in the literature are designed in time domain. For examples, Dolph designed a beampattern that has constant sidelobe and a beamwidth that is the minimum possible for the given sidelobe level [23]. Parks and McClellan [24] formulate the Minimax problem as a polynomial approximation problem. A technique that was developed to design Finite Impulse

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Response (FIR) filter was utilized. It minimizes a maximum allowable variation in both main lobe and sidelobe region, and an optimum solution is seek to meet all these criteria.

In this section, we are going to introduce two optimal array pattern synthesis design criteria for broadband arrays. These methods will be used as comparison with one of our proposed methods.

(i) MiniMax Method

In array pattern synthesis for FI beamformer, how well a synthesized pattern approximates a desired pattern across frequencies can be measured in several ways. Typical measures are the peak synthesis error across an angular space ϕ , or the mean-squared error over ϕ . One way to design a frequency invariant beamformer is to formulate the problem as a Minimax optimization problem.

$$\begin{aligned} \min_{\mathbf{w}} \quad & \max_{k,q} \|BP_d(\phi_q) - \mathbf{a}^T(f_k, \phi_q)\mathbf{w}\| \\ \text{s.t.} \quad & \|\mathbf{a}^T(f_k, \phi_s)\mathbf{w}\| \leq \epsilon \end{aligned} \tag{2.52}$$

$$k = 1, 2, \dots, K, \quad f_k \in [f_l, f_u]$$

$$\phi_q \in \Theta_{ML}, \quad q = 1, \dots, Q,$$

$$\phi_s \in \Theta_{SL}, \quad s = 1, \dots, S,$$

where $\phi_s \in \Theta_{SL}$, $s = 1, \dots, S$ and $\phi_q \in \Theta_{ML}$, $q = 1, \dots, Q$ are the angular grids chosen that properly approximate the sidelobe region Θ_{SL} and the mainlobe region Θ_{ML} by a finite number of directions, respectively. The choice of S and Q is determined by the required accuracy of approximation. f_l and f_u are the lower and upper frequency bound respectively. $BP_d(\phi_q)$, $q = 1, \dots, Q$ is the preliminary desired beampattern within the mainlobe region Θ_{ML} . ϵ is a small positive value chosen to control the sidelobes.

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The norm constraint $\|\mathbf{w}\| \leq \Delta$ can also be added to (2.52) to control the dynamic range of the filter coefficients to avoid large noise amplification at the array output. This is similar to the so-called white noise gain constraint employed in narrowband robust adaptive beamforming. $\|\cdot\|$ here denotes the Euclidean norm, and Δ is a prescribed value to control the norm of FIR filter tap weights. However, the main shortcoming of this approach is that a desired beampattern has to be chosen preliminarily. Furthermore, it is not clear on how to choose an optimal one so that the mainlobe synthesis error are minimum.

(ii) Yan's Method

Yan [34] proposed a method which imposes constraints both on the mainlobe spatial response variation over frequency and on the sidelobes of the beamformer. A new term which is defined as the mainlobe spatial response variation (MSRV) is introduced as a measure of the quality of the approximation among the synthesized patterns at different frequencies.

The maximum MSRV of a frequency-invariant beamformer is defined as

$$\begin{aligned}\sigma_{max} &= \min_{k,q} |\mathbf{a}^T(f_0, \phi_q) \mathbf{w} - \mathbf{a}^T(f_k, \phi_q) \mathbf{w}| \\ &= \min_{k,q} |[\mathbf{a}(f_0, \phi_q) - \mathbf{a}(f_k, \phi_q)^T] \mathbf{w}| \end{aligned} \quad (2.53)$$

$$k = 1, 2, \dots, K, \quad \phi_q \in \Theta_{ML}, \quad q = 1, \dots, Q,$$

where f_0 is the reference frequency. It can be any frequency value selected within $[f_l, f_u]$.

Replacing the maximum mainlobe synthesis error in (2.52) by the maximum MSRV defined in (2.53), and imposing the distortionless constraint on the

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signal of interest, the peak sidelobes constrained Minimax MSRV beamformer design is given by,

$$\begin{aligned} \min_{\mathbf{h}} \quad & \sigma_{max} \\ s.t. \quad & \begin{cases} \mathbf{a}^T(f_0, \phi_0) \mathbf{w} = 1 \\ |\mathbf{a}^T(f_k, \phi_s) \mathbf{w}| \leq \epsilon \end{cases} \end{aligned} \quad (2.54)$$

$$f_k \in [f_l, f_u], \phi_s \in \Theta_{SL},$$

where $\mathbf{w}$ is the beamforming weights to be optimized. ϵ is the peak sidelobe constraint. Θ_{ML} and Θ_{SL} represent the mainlobe and sidelobe region respectively.

The optimal desired beampattern is chosen automatically in this method.

2.4.2.1 Frequency-Invariant Beamformer Designed in Phase Mode

Other than designing FI beamformer in time domain, some are designed in phase mode. The concept of phase mode originates from communication, where the antennae are of circular aperture. Mathematically, phase mode coefficients are the Fourier components of the aperture weighting in a circular array [54]. It was shown many years ago that, from the point of view of pattern synthesis, phase modes in circular array may be considered equivalent to elements of a linear array [35, 36]. For a long time, phase mode processing is used in communication and for narrowband application [55]. Recently, a method that exploits the concept of phase mode is used in achieving frequency invariant characteristics for concentric circular array [33].

In this method, a new digital beamformer for uniform concentric circular array having nearly frequency-invariant characteristic is proposed. The basic principle is

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Figure 2.8: Concentric circular array configuration

Figure 2.9: System structure employed by the UCCA beamformer

to transform the received signals to phase mode and remove the frequency dependency of the individual phase through the use of a digital beamforming network or frequency compensation network. As a result, the far-field pattern of the ar-

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ray, which is governed by a set of variable beamformer weight, is electronically steerable, and it is approximately invariant over a wide range of frequencies [33].

The array sensors are arranged as shown in Fig. 2.8 and the structure as shown in Fig. 2.9 is employed.

The signal received at each sensor is first transformed into phase mode via Inverse Discrete Fourier Transform (IDFT). A compensation FIR filter network is designed to remove the frequency dependency of the transformed signal. Spatial weighting coefficients are then applied before summing all the phase mode data. This makes it possible to design the compensation network filter and the beamformer weights separately. The design of the compensation network is formulated as a SOCP problem and is solved optimally for a minimax criterion.

2.4.2.2 Phase Mode Beamformer v.s. Frequency Domain Beamformer

In this section, we are going to discuss the difference between phase mode beamformer and frequency domain beamformer, as both methods involve signal transformation at the output of each sensor.

Frequency domain beamforming techniques apply Discrete Fourier Transform (DFT) to signal received at each element of the array. It has a structure as shown in Fig 2.10. At the output of each element, an A/D converter takes samples of the received signal. When K samples are available from each element, these samples are transformed using DFT. These produce K frequency domain samples of each element output. Each frequency domain sample is multiplied by a weight and then added to the corresponding frequency samples from other elements. Finally, IDFT is performed on the frequency domain samples to obtain the time-domain array output samples.

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Figure 2.10: The structure of frequency domain beamformer

Phase mode beamformer is in fact a time domain beamformer. IDFT is applied to the received signal at each sensor. It has certain advantages in processing data when use with circular array.

Let $BP(\phi, \theta)$ denote the beam pattern of a continuous circular array.

$$BP(\phi, \theta) = \int_0^{2\pi} w(\phi_1) e^{j \frac{2\pi}{\lambda} r \sin \theta \cos(\phi - \phi_1)} r d\phi_1, \quad (2.55)$$

where λ is the signal wavelength, and ϕ_1 is the angular position of the first element.

Since any weighting function $w(\phi)$ will be periodic in ϕ , the aperture weighting function is expanded in a Fourier series.

$$w(\phi) = \sum_{m=-\infty}^{\infty} w'_m e^{jm\phi}, \quad (2.56)$$

where

$$w'_m = \frac{1}{2\pi} \int_0^{2\pi} w(\phi) e^{-jm\phi} d\phi. \quad (2.57)$$

Each term is called a phase mode excitation of the aperture. Each phase mode excitation term gives rise to a spatial harmonic in the beam pattern. This correspondence means that we can take a desired beampattern in the ϕ -dimension,

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decompose it into its Fourier components, and separately excite each of the Fourier components with a phase mode term.

Hence, in conclusion, frequency domain beamforming exploits DFT at the output of each sensor to transform the signal into frequency domain, while phase mode beamforming exploits IDFT at the output of each sensor to transform the signal into ϕ dimension (a spatial dimension). The transformation into different domains for these two methods result in their different system structure and beamforming techniques.

2.5 Robustness Design of A Beamformer

In general, there are two types of robustness, one is robustness against self noise, and the other is robustness against array steering vector. There are two types of mismatches in array steering vector as well, one is mismatch as a result of calibration error, including imprecise knowledge of the element locations, and the other is mismatch due to different arrival angles from that are assumed by the array. The former is highly related to fixed beamformer, while the latter is more related to adaptive beamformer. These two types of mismatches can be referred to as “The perturbation problem”.

In the last three decades, much effort has been devoted to develop robust beamformer. In this section, we are going to examine techniques used in the literature to improve the robustness of a beamformer.

2.5.1 Diagonal Loading

This has been a popular method to improve the robustness of many beamformers. It is derived by imposing an additional quadratic constraint either on the Euclidean norm of the weight vector itself or on its difference from a desired weight vector.

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One example of it is the Gilbert and Morgan algorithm [56], where diagonal loading is added to the noise covariance matrix in the calculation of the weight vector, so as to improve the robustness of the superdirective beamformer against microphone self-noise. Sometimes diagonal loading is also proposed to alleviate various problems of using the array sample covariance matrix and to better control the peak sidelobe response as shown in the methods proposed in [57].

2.5.2 Linear Constraints

To account for the array steering vector errors, additional linear constraints, including point and derivative constraints, are imposed to improve the robustness of adaptive beamformer [20]. Point constraints fix the beamformer response at points of spatial direction and temporal frequency. The number of points at which response can be constrained is limited to number of microphones used. Derivative constraints are employed to influence response over a region of direction and/or frequency by forcing the derivatives of the beamformer response at some point of direction and frequency to be zero. If the signal arrives near the direction at which a point constraint is employed, then application of a derivative constraint at that point prevents the beamformer from synthesizing a response of zero to the desired signal. However, these constraints are not explicitly related to the uncertainty of the array steering vector. Moreover, for every additional linear constraint imposed, the beamformer loses one degree of freedom for interference suppression.

2.5.3 Subspace-based

This is also known as eigen-subspace based beamformer, which is a good candidate for robust beamforming. In this method, the presumed steering vector is projected onto the observed Signal-plus-Interference subspace. Numerical results has shown

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that Signal-to-Interference-plus-Noise Ratio (SINR) performance is increased using this method. However it possesses several limitations. Firstly, the development of the projection method is based on the assumption that the underlying covariance matrix R_{xx} is known exactly. Any imprecise knowledge of the matrix will result in performance degradation. Secondly, this method is inefficient at low input SNR due to substantial possibility of subspace swap. Moreover, if the dimension of the signal-plus-interference subspace is high or not exactly known, the performance will degrade drastically [58].

Chapter 3

Uniform Circular Broadband Array Beamformer with Selective Frequency Invariant Region

3.1 Introduction

The objective of Chapter 3 is to study the design of a circular beamformer operated in phase mode to achieve frequency invariant characteristics, and evaluate its performances under various conditions.

In the literature, there are different approaches to designing a FI beamformer. Some focus on the design based on array geometry. These include for example the 3D sensor array design reported in [59], the rectangular sensor array design reported in [60], and the design using sub-arrays in [61, 64–66]. Undoubtedly, these multi-dimensional arrays involve the use of a large number of microphones, which will contribute to an excessive computational load.

Some approaches focus on the design based on signal transformation. [33] designed a beamformer using concentric circular array by transforming the signal received at each sensor to phase mode, and outputs are summed to produce a FI beampattern. Excellent results have been produced by this algorithm (hereafter it is denoted as Chan's beamformer). In their approach, concentric circular array

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is adopted. Circles with different radii responded differently to signals of different frequencies. Hence, it is more effective in achieving a FI beampattern. One limitation is that, a relatively large number of sensors have to be used to form the concentric circular array.

Among existing FI algorithms, we find that many of them design an identical frequency response for all angles. There is a gap in the literature for which the beamformer is designed with FI characteristics across a selective spatial region. This is a trade-off between the FI performance and the FI spatial coverage. Selective FI characteristics allows the gain constraint to cover only the desired angle, hence, the FI performance of the beamformer is expected to improve in the desired angular region. Also because of this trade off, less number of microphones are required to produce results of the same quality. This will put an ease on the requirement on the computational load.

In the method presented in this chapter, an algorithm with new objective function and a quadratic constraint is proposed. Instead of achieving FI characteristics across all angles, it aims to achieve a FI beampattern for the main lobe where the signal of interest is from and relax the FI requirement at the side lobes. The basic idea is to limit the FI constraint to a specific direction so that the FI performance in that direction can be improved and a smaller number of sensors can be used to achieve the same FI beampattern. This sacrifices the FI performance in the side-lobe region. This beamformer may find its application in vehicular environment, where the desired signal and the interference are both broadband. In such case, the spectrum of interference is much broader than that of speech, this proposed beamformer is able to focus on the spectrum of the desired speech signal at the desired direction, while attenuate the broadband interference coming from other direction and the undesired portion of the spectrum.

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Figure 3.1: Uniform circular array configuration

In Chapter 3, full details of the design is provided. In addition, a computational complexity analysis of the proposed algorithm is included as well as the sensitivity performance, at different number of sensors and different constraint parameter values.

Chapter 3 is organized in the following way: in Section 3.2, problem formulation is discussed; in Section 3.3, the proposed beamforming design is described; in Section 3.4, the design of the beamforming weight using SOCP is shown; numerical results are given in Section 3.5. Finally, brief summary is given in Section 3.6.

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Figure 3.2: Relationship between inter-sensor spacing and the radius of the circular array

3.2 Problem Formulation

Consider a layer of K microphones that are distributed uniformly in a circle as shown in Fig. 3.1. Each omnidirectional sensor is located at $(r \cos \phi_k, r \sin \phi_k)$, where r is the radius of the circle, $\phi_k = 2k\pi/K$ and $k = 0, \dots, K-1$. In this configuration, the inter-sensor spacing is fixed at $\lambda/2$, where λ is the smallest wavelength of the signal of interest and is denoted as λ_{min} . Half wavelength is chosen because it is widely used in linear array to avoid spatial aliasing. The radius is calculated using the relationship as shown in Fig. 3.2, and is given by:

$$r = \lambda_{min}/4 \sin(\pi/K). \quad (3.1)$$

Assuming the circular array is on a horizontal plane, the steering vector is shown below,

$$\mathbf{a}(f, \phi) = \left[e^{\frac{j2\pi fr \cos(\phi - \phi_0)}{c}}, \dots, e^{\frac{j2\pi fr \cos(\phi - \phi_{K-1})}{c}} \right]^T, \quad (3.2)$$

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Figure 3.3: The system structure of a uniform circular array beamformer

where T denotes transpose. For the convenience of notation, let ω denote the normalized digital frequency, i.e. $\omega = \frac{2\pi f}{f_s}$, ϵ denote the ratio of the sampling frequency to the maximum frequency, i.e. $\epsilon = \frac{f_s}{f_{max}}$, and $\tilde{r}$ denote the normalized radius, i.e. $\tilde{r} = \frac{r}{\lambda_{min}}$, hence the steering vector can be represented as

$$\mathbf{a}(\omega, \phi) = [e^{j\omega\tilde{r}\epsilon \cos(\phi-\phi_0)}, \dots, e^{j\omega\tilde{r}\epsilon \cos(\phi-\phi_{K-1})}]^T. \quad (3.3)$$

Fig. 3.3 shows the system structure of the proposed uniform circular array beamformer. The sampled signal after the sensor is assumed to be given by the vector $X[n] = [x_0(n), x_1(n), \dots, x_{K-1}(n)]^T$ and is assumed to be arriving on the horizontal plane, i.e the same plane as the circular array, where n is the sampling instance and K is the number of sensors. Different from the conventional broadband beamformer, this signal received at each sensor is transformed into a set of Fourier coefficients via Inverse Discrete Fourier Transform (IDFT). Each of these coefficients is called a phase mode [67]. Let $p_m[n]$ denote the m^{th} phase signal at time instance n after the transformation,

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$$p_m[n] = \sum_{k=0}^K x_k[n] e^{\frac{j2\pi km}{K}}. \quad (3.4)$$

A compensation filter network which is characterized by the Finite Impulse Response (FIR) filter coefficient $b_m[n]$ is then designed to remove the frequency dependency of the received signal $X[n]$. h_m is the m^{th} phase spatial weighting coefficient, it governs the far field beampattern of the beamformer. Hence, the beamformer output $y[n]$ is given as follows:

$$y[n] = \sum_{m=-L}^L (p_m[n] * b_m[n]) \cdot h_m, \quad (3.5)$$

where $*$ denotes discrete-time convolution. M is defined as the total number of phases and it is assumed to be an odd number. L is defined as $(M-1)/2$. Hence in Fig. 3.3, signals received at K number of microphones are transformed into M number of phases.

In order to obtain the spatial-temporal transfer function of the beamformer, Discrete Fourier Transform (DFT) of (3.4) is taken, assuming the source signal has spectrum $S(\omega)$,

$$\begin{aligned} P_m(\omega) &= \sum_{k=0}^{K-1} X_k(\omega) e^{\frac{j2\pi km}{K}} \\ &= S(\omega) \cdot \sum_{k=0}^{K-1} e^{j\omega \tilde{r} \cos(\phi - \phi_k)} e^{\frac{j2\pi km}{K}}, \end{aligned} \quad (3.6)$$

where $X_k(\omega)$ refers to the Fourier Transform of the signal received at k^{th} microphone.

Taking DFT on both side of (3.5) and using (3.6), we have

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$$\begin{aligned} Y(\omega) &= \sum_{m=-L}^L h_m P_m(\omega) B_m(\omega) \\ &= S(\omega) \cdot \sum_{m=-L}^L h_m \left(\sum_{k=0}^{K-1} e^{j\omega\tilde{r} \cos(\phi-\phi_k)} e^{j\frac{2\pi km}{K}} \right) B_m(\omega). \end{aligned} \quad (3.7)$$

where $Y(\omega)$ is the Fourier transform of the output signal, $P_m(\omega)$ is the Fourier transform of the signal received at each phase, h_m is the spatial weighting at each phase, $B_m(\omega)$ is the Fourier transform of the FIR filter coefficients at each phase.

Hence, the response of the system is

$$G(\omega, \phi) = \sum_{m=-L}^L h_m \left(\sum_{k=0}^{K-1} e^{j\omega\tilde{r} \cos(\phi-\phi_k)} e^{j\frac{2\pi km}{K}} \right) B_m(\omega). \quad (3.8)$$

In order to obtain a FI response, terms which are functions of ω are grouped together by making use of the Jacobi-Anger expansion [78]

$$e^{j\beta \cos \gamma} = \sum_{n=-\infty}^{+\infty} j^n J_n(\beta) e^{jn\gamma}, \quad (3.9)$$

where $J_n(\beta)$ is the Bessel function of the first kind of order n .

Substituting (3.9) into (3.8),

$$\begin{aligned} G(\omega, \phi) &= \sum_{m=-L}^L h_m \times \left[\sum_{k=0}^{K-1} \sum_{n=-\infty}^{+\infty} j^n J_n(\omega\tilde{r}) \times e^{jn(\phi-2\pi k/K)} e^{j\frac{2\pi km}{K}} B_m(\omega) \right] \\ &= \sum_{m=-L}^L h_m \times \left[\sum_{n=-\infty}^{+\infty} j^n J_n(\omega\tilde{r}_p) e^{jn\phi} \times \left(\sum_{k=0}^{K-1} e^{j2\pi k \frac{m-n}{K}} \right) B_m(\omega) \right] \end{aligned} \quad (3.10)$$

The term inside the bracket is evaluated to be

$$\sum_{k=0}^{K-1} e^{j2\pi k \left(\frac{m-n}{K} \right)} = \begin{cases} K, m-n = Kx \\ 0, otherwise \end{cases}, \quad \text{where } x \in \mathbb{Z}. \quad (3.11)$$

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Substituting (3.11) into (3.10),

$$G(\omega, \phi) = K \sum_{m=-L}^L h_m \times \left[\sum_{n=m+Kx} j^n J_n(\omega \tilde{r}) e^{jn\phi} B_m(\omega) \right]. \quad (3.12)$$

From [78], the Bessel function satisfies the following property:

$$|J_{|n|}(\omega \tilde{r})| \leq \left(\frac{\omega \tilde{r}}{2|n|} \right)^{|n|}. \quad (3.13)$$

Therefore, for sufficiently large value of n , the value of the Bessel function will be negligibly small. In other words, if the number of sensors is large enough, we can truncate the infinite sum in (3.12) and retain the term with $n = m$ (*i.e.*, $x = 0$). Consequently, $G(\omega, \phi)$ can be approximated by

$$G(\omega, \phi) = \sum_{m=-L}^L h_m \cdot e^{jm\phi} \cdot K \cdot j^m \cdot J_m(\omega \tilde{r}) \cdot B_m(\omega). \quad (3.14)$$

where K is the number of sensors.

This process is reminiscent of the general approach in Parra's method [32] in which spherical harmonics are used in deriving the beampattern.

3.3 Proposed Beamformer

In some situations where the signal of interest and noise are both broadband, an FI beamformer that could discriminately enhance the broadband signal and attenuate the noise as much as possible is desirable.

In order to achieve this goal, the following objective function is proposed:

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \int_{\omega} \int_{\phi} \|G(\omega, \phi)\|^2 d\omega d\phi \\ \text{s.t.} \quad & \|G(\omega, \phi_0) - 1\| \leq \delta, \omega \in [\omega_l, \omega_u], \end{aligned} \quad (3.15)$$

where $G(\omega, \phi)$ is the spatial response of the beamformer as shown in (3.14), $\mathbf{h} = [h_L, h_{L-1}, \dots, h_{-L}]$ is the spatial weightings that are being optimized, $\mathbf{b} =$

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$[b_L[n], b_{L-1}[n], \dots, b_{-L}[n]]$ denotes the FIR filter coefficients that are being optimized. ω_l and ω_u are the lower and upper limits of the desired frequency range. ϕ_0 is the desired direction. δ is a predefined variable that controls the mainbeam ripples.

This objective function minimizes the square of the gain response across all frequencies and all angles, while constraining the gain at the desired angle to be approximately one. The main idea is to improve the FI performance in the selected region by relaxing the gain constraint to cover a desired angle. The objective function is formulated in a way that is similar to the Capon beamformer [68]. The difference is that in Capon beamformer it minimizes the data dependent array output power for a single frequency, while in our method, we minimize the data independent array output power across a wide range of frequencies with gain response constrained to one at a desired angle. By doing so, the FI characteristics of the beampattern are restricted solely along the desired direction for a range of frequency bands instead of across all directions. Hence, the resulting beamformer can enhance the desired broadband signal coming from one direction and attenuate broadband noise coming from other direction as much as possible.

In addition, this design criterion reduces the computational complexity. This is because, as compared to Chan's beamformer, our method only focuses the FI characteristic at the desired region, hence more degrees of freedom are saved. As a result, the proposed method can have better FI performance when fewer number of microphones are used or better performance on achieving FI characteristic when the same array configuration is used. This argument is further supported by the simulation results at the later section. When δ approaches zero, the objective function reduces to a linearly constrained problem.

This objective function is achieved by optimizing the coefficients of the compensation filter and the beamforming weights shown in Fig. 3.3 together. Cholesky

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factorization is used to transform the objective function further into SOCP problem. The detailed mathematical transformation will be presented in the following section.

3.4 Convex Optimization Based Implementation

In this optimization problem, Second Order Cone Programming (SOCP) is used to solve the above objective function due to the quadratic constraint [69]. SOCP was initially used in array pattern synthesis by Lebrete and Boyd [69]. Since then, it has been widely used in the optimal design of digital FIR filters and narrowband beamformer [70–72]. The global optimal solution of a SOCP problem is guaranteed if it exists. Another important advantage of SOCP is that it is very convenient to include additional linear or convex quadratic constraints, such as the norm constraint of the variable vector, to the design problem. A standard form of SOCP can be written as follows:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{b}^T \mathbf{x} \\ \text{s.t.} \quad & \mathbf{d}_i^T \mathbf{x} + q_i \geq \|\mathbf{A}_i \mathbf{x} + \mathbf{c}_i\|_2, i = 1, \dots, N, \end{aligned} \quad (3.16)$$

where $\mathbf{x} \in R^m$ is the variable vector; the problem parameters are $\mathbf{b} \in R^m$, $\mathbf{A}_i \in R^{(n_i-1) \times m}$, $\mathbf{c}_i \in R^{n_i-1}$, $\mathbf{d}_i \in R^m$ and $q_i \in R$. The norm appearing in the constraints is the standard Euclidean norm, *i.e.*, $\|u\|_2 = (u^T u)^{1/2}$

3.4.1 Convex Optimization of the Beampattern Synthesis Problem

In order to convert to the standard form, $B_m(\omega) = \sum_{n=0}^{N_m} b_m[n]e^{-jn\omega}$ is first substituted into (3.14), where N_m is the filter order for each phase,

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$$G(\omega, \phi) = \sum_{m=-L}^L h_m \cdot e^{jm\phi} \cdot K \cdot j^m \cdot J_m(\omega\tilde{r}) \cdot \left[\sum_{n=0}^{N_m} b_m[n] e^{-jn\omega} \right]. \quad (3.17)$$

Using the identity $e^{-jn\omega} = \cos(n\omega) - j \sin(n\omega)$, (3.17) becomes:

$$\begin{aligned} G(\omega, \phi) &= \sum_{m=-L}^L h_m \cdot e^{jm\phi} \cdot K \cdot j^m \cdot J_m(\omega\tilde{r}) \cdot \left[\sum_{n=0}^{N_m} b_m[n] (\cos(n\omega) - j \sin(n\omega)) \right] \\ &= K \sum_{m=-L}^L h_m \cdot e^{jm\phi} \cdot j^m \cdot J_m(\omega\tilde{r}) \cdot \left[\sum_{n=0}^{N_m} (b_m[n] \cos(n\omega) - j b_m[n] \sin(n\omega)) \right] \\ &= K \sum_{m=-L}^L h_m \cdot e^{jm\phi} \cdot j^m \cdot J_m(\omega\tilde{r}) \cdot \left[\sum_{n=0}^{N_m} b_m[n] \cos(n\omega) - j \sum_{n=0}^{N_m} b_m[n] \sin(n\omega) \right] \\ &= K \sum_{m=-L}^L h_m \cdot e^{jm\phi} \cdot j^m \cdot J_m(\omega\tilde{r}) \cdot [\mathbf{c}_m \bar{\mathbf{b}}_m - j \mathbf{s}_m \bar{\mathbf{b}}_m], \end{aligned} \quad (3.18)$$

where

$$\begin{aligned} \bar{\mathbf{b}}_m &= [b_m[0], b_m[1], \dots, b_m[N_m]]^T, \\ \mathbf{c}_m &= [\cos(0), \cos(\omega), \dots, \cos(N_m \cdot \omega)], \\ \mathbf{s}_m &= [\sin(0), \sin(\omega), \dots, \sin(N_m \cdot \omega)]. \end{aligned}$$

h_m is the spatial weighting in the system structure, and $\bar{\mathbf{b}}_m$ can be seen as the FIR filter coefficient in the frequency compensation network. The beampattern function can be further simplified by grouping h_m and $\bar{\mathbf{b}}_m$ together.

Denoting $\mathbf{u}_m = h_m \cdot j^m \cdot \bar{\mathbf{b}}_m$, we have,

$$\begin{aligned} G(\omega, \phi) &= K \sum_{m=-L}^L e^{jm\phi} \cdot J_m(\omega\tilde{r}) \cdot \mathbf{c}_m \mathbf{u}_m - j \cdot K \sum_{m=-L}^L e^{jm\phi} \cdot J_m(\omega\tilde{r}) \cdot \mathbf{s}_m \mathbf{u}_m \\ &= \bar{\mathbf{c}}(\omega, \phi) \bar{\mathbf{u}} - j \bar{\mathbf{s}}(\omega, \phi) \bar{\mathbf{u}}, \end{aligned} \quad (3.19)$$

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where

$$\bar{\mathbf{c}}(\omega, \phi) = [K e^{j(-L)\phi} J_{-L}(\omega \tilde{r}) \mathbf{c}_{-L}, \dots, K e^{j(L)\phi} J_L(\omega \tilde{r}) \mathbf{c}_L],$$

$$\bar{\mathbf{u}} = [\mathbf{u}_{-L}^T, \mathbf{u}_{-L+1}^T, \dots, \mathbf{u}_L^T]^T,$$

$$\bar{\mathbf{s}}(\omega, \phi) = [K e^{j(-L)\phi} J_{-L}(\omega \tilde{r}) \mathbf{s}_{-L}, \dots, K e^{j(L)\phi} J_L(\omega \tilde{r}) \mathbf{s}_L].$$

By representing the complex spatial response by a 2-dimensional complex vector, the above equation can be rewritten in the following form:

$$\mathbf{g}(\omega, \phi) = \begin{pmatrix} \bar{\mathbf{c}}(\omega, \phi) \\ -\bar{\mathbf{s}}(\omega, \phi) \end{pmatrix} \bar{\mathbf{u}} = \mathbf{A}(\omega, \phi)^H \bar{\mathbf{u}}. \quad (3.20)$$

$$\text{Hence, } \|G(\omega, \phi)\|^2 = \mathbf{g}^H \mathbf{g} = \left(\mathbf{A}(\omega, \phi)^H \bar{\mathbf{u}} \right)^H \left(\mathbf{A}(\omega, \phi)^H \bar{\mathbf{u}} \right) = \bar{\mathbf{u}}^H \mathbf{A}(\omega, \phi) \mathbf{A}(\omega, \phi)^H \bar{\mathbf{u}}.$$

Finally, the objective function can be written as:

$$\min_{\substack{\mathbf{h}, \mathbf{b} \\ s.t.}} \quad \bar{\mathbf{u}}^H \mathbf{R} \bar{\mathbf{u}} \quad (3.21)$$

$$\|G(\omega, \phi_0) - 1\| \leq \delta, \text{ for } \omega \in [\omega_l, \omega_u],$$

$$\text{where } \mathbf{R} = \int \int_{\omega, \phi} \mathbf{A}(\omega, \phi) \mathbf{A}(\omega, \phi)^H d\omega d\phi.$$

In order to transform (3.21) into a form defined by (3.16), the cost function must be a linear equation. By definition, Cholesky factorization decomposes symmetric positive definite matrix into the transpose of upper triangular matrix and the upper triangular matrix, i.e. $\mathbf{R} = \mathbf{D}^H \mathbf{D}$, if R is hermitian and positive definite. We substitute the above relationship into (3.21),

$$\bar{\mathbf{u}}^H \mathbf{R} \bar{\mathbf{u}} = \bar{\mathbf{u}}^H (\mathbf{D}^H \mathbf{D}) \bar{\mathbf{u}} = (\mathbf{D} \bar{\mathbf{u}})^H (\mathbf{D} \bar{\mathbf{u}}),$$

where $\mathbf{D}$ is the Cholesky factorization of $\mathbf{R}$.

As a result, (3.21) can be represented by the following form:

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$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & d^2 \\ \text{s.t.} \quad & \begin{cases} d^2 = \|\mathbf{D} \cdot \bar{\mathbf{u}}\|^2 \\ \|G(\omega, \phi_0) - 1\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u]. \end{cases} \end{aligned} \quad (3.22)$$

Denoting t as the maximum value of vector $\|\mathbf{D} \cdot \bar{\mathbf{u}}\|$ subject to various choices of $\bar{\mathbf{u}}$, we arrive at the following form for (3.22):

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & t \\ \text{s.t.} \quad & \begin{cases} \|\mathbf{D} \cdot \bar{\mathbf{u}}\| \leq t \\ \|G(\omega, \phi_0) - 1\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u]. \end{cases} \end{aligned} \quad (3.23)$$

Dividing ω uniformly within the frequency range with a sufficient number of points, says $I=64$, we yield I different constraints.

Lastly, in order to solve (3.23) by SOCP toolbox, we stack t and the coefficients of $\bar{\mathbf{u}}$ together. Let $\mathbf{c} = [1, \mathbf{0}]^T$, so that $t = \mathbf{c}^T \mathbf{y}$. The objective function is rewritten in the following form:

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \mathbf{c}^T \mathbf{y} \\ \text{s.t.} \quad & \begin{cases} \|\begin{bmatrix} \mathbf{0} & \mathbf{D} \end{bmatrix} \mathbf{y}\| \leq \mathbf{a}^T \mathbf{y} \\ \left\| \begin{bmatrix} \mathbf{0} & \mathbf{A}(\omega, \phi_0)^H \end{bmatrix} \mathbf{y} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u]. \end{cases} \end{aligned} \quad (3.24)$$

The resulting problem can now be solved using convex optimization toolbox such as SeDuMi [73] readily.

3.4.2 Computational Complexity

When interior-point methods are used to solve the SOCP problem in (3.20), the number of iterations that decrease the duality gap to a constant fraction of itself is upper bounded by $O(\sqrt{N})$ where N is the number of constraints. The amount of computational complexity per iteration is $O(n^2 \sum_i n_i)$ in the SOCP algorithm

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Table 3.1: Computational complexity of different broadband beam pattern synthesis method

Method	Number of iteration	Amount of computation per iteration
UCCA	$O\{\sqrt{I \times M}\}$	$O\{(1 + (1 + Nm))^2[2M(I + 1)]\}$
Eq (3.15)	$O\{\sqrt{1 + I}\}$	$O\{[M(Nm + 1)^2][2I + M(Nm + 1) + 1]\}$

[69]. Duality gap is the difference between the maximum value achieved by the dual problem and the minimum value of the primal problem. By reducing the gap, a feasible solution which satisfies both primal and dual problems is obtained.

The major computational demand of the broadband array pattern synthesis comes from solving the optimization problem in different objective functions. The computational complexities of different optimization problems are calculated and listed in Table 3.1.

From Table 3.1, it can be seen that the number of iterations of our proposed method is greatly reduced compared to that of Chan's method. For example, I represents the number of division within the frequency range. It takes the value of 64. M represents the total number of phase mode. Hence, according to the table, Chan's uses 33 iterations, while our proposed method uses 8 iterations. Also according to Table 3.1, Chan's has a computational load of $O(716040)$, while our proposed method has a computational load of $O(2053634)$. When these two columns are multiplied together, our proposed method has an overall computational load of (16429072) , and that of the Chan's method is $O(23629320)$. Hence, the overall computational load of the new method is less than that of Chan's method. In this complexity calculation, the number of phase is used instead of the number of microphones because the coefficients are optimized in the phase mode in this structure. Hence, for the same number of microphones, more phases are employed, better result is obtained, and higher computational load is expected.

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3.5 Numerical Results

In these numerical studies, the array performance of the proposed beamformer in terms of spatial response is compared with that of the Chan's beamformer [33] and the Yan's beamformer [34], for the specified frequency region. The evaluation metric used to quantify the FI characteristics is the mean squared error between the simulated gain response and the constraint value which is 1 in the specified direction. The sensitivity performance of the proposed algorithm will also be evaluated for different number of sensors. Different threshold values are set for magnitude control of the ripples of the main beam.

A uniform circular array of 20 microphones is considered in this numerical study. Using Fig 3.2 and applying the Cosine Rule, the radius of the microphone array is estimated to be 12 cm. The number of phase modes M is set to be 17 and thus there are 17 spatial weighting coefficients. The order of the compensation filter is set to be 16 for all the phase modes. The frequency region of interest is specified to be from $[0.3 \pi, 0.95 \pi]$ radians/sample. This normalized frequency is a fraction of the sampling frequency, which has a formula as shown below,

$$F_n = f/F_s \cdot 2\pi, \quad (3.25)$$

where F_s is taken as 8000 Hz in our simulation. The threshold value, δ , which controls the magnitude of the ripples of the main beam is set to 0.1. The specified direction is set to be 0° where the reference microphone is located. Alternatively, we can take this reference microphone being located along the x-axis.

There are several optimization criteria presented in [34]. The one that is chosen for comparison is the peak sidelobe constrained minimax mainlobe spatial response variation (MSRV) design. This algorithm was introduced in Chapter 2.

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Figure 3.4: The normalized spatial response of the proposed beamformer for $\omega=[0.3\pi, 0.95\pi]$ radians/sample

Figure 3.5: The normalized spatial response of the UCCA beamformer for $\omega=[0.3\pi, 0.95\pi]$ radians/sample

The beampattern obtained for the proposed beamformer for the frequency region of interest is shown in Figure 3.4. The spatial response of the proposed

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Figure 3.6: The normalized spatial response of Yan's beamformer for $\omega=[0.3\pi, 0.95\pi]$ radians/sample

beamformer at 10 uniformly spaced discrete frequencies are superimposed. It can be seen that, the proposed beamformer has approximately a constant gain within the frequency region of interest in the specified direction (0°). As the direction deviates from 0° , the beamformer demonstrates poor FI characteristics. We are able to achieve little amplitude variation in the mainbeam because our proposed

Table 3.2: Comparison of the peak sidelobe level at each frequency for different methods

Normalized Frequency (radians/sample)	Proposed Beamformer (dB)	Yan's beamformer (dB)	UCCA Beamformer (dB)
0.3	-15.4	-10.0	-6.2
0.4	-16.5	-15.0	-6.1
0.5	-13.1	-11.1	-6.6
0.6	-16.3	-14.1	-9.5
0.7	-14.6	-15.0	-7.2
0.8	-8.5	-12.3	-8.5
0.9	-8.9	-15.4	-6.8

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Figure 3.7: Comparison on FI characteristic between the proposed beamformer, UCCA beamformer and Yan's beamformer at 0 degree for $\omega=[0.3\pi, 0.95\pi]$ radians/sample

beamformer is designed to focus on achieving FI in the mainbeam region. The trade-off is that the sidelobe are now less controlled. The peak sidelobe level has a value of -8 dB.

The beampattern of Chan's beamformer is shown in Figure 3.5. It is noted that, the beampattern displays a larger variation in gain response across the mainlobe region than our proposed beamformer. The peak sidelobe level has a value of -6 dB. It is higher than the proposed beamformer too. The reason for this observation is that originally, Chan's beamformer was designed using concentrate circular array. Circular array with different radius will correspond to different portion of the frequency spectrum, hence achieving frequency invariant characteristics over a large frequency range and over a wide angular region. However, when Chan's beamformer used single layer of circular array, under the same optimization crite-

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Figure 3.8: Directivity versus frequency for the broadband beam pattern shown in Fig. 3.4

Figure 3.9: White noise gain versus frequency for the broadband beam pattern shown in Fig. 3.4

ria, the capability of achieving FI characteristics drops dramatically. As a result, the beampattern in Fig 3.5 is obtained.

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Table 3.3: Comparison of gain response at each frequency along the desired direction for different methods

Normalized Frequency (radians/sample)	Proposed Beamformer (dB)	Yan's beamformer (dB)	UCCA Beamformer (dB)
0.3	-0.0007	0	0.6761
0.4	-0.0248	-0.8230	0.1760
0.5	0.0044	-1.3292	-0.022
0.6	-0.0097	-1.6253	-0.2186
0.7	-0.0046	-1.8789	-0.6301
0.8	0.0085	-2.9498	-0.1291
0.9	-0.0033	-6.2886	0.1477

Table 3.4: Comparison of the percentage error of gain response at each frequency along the desired direction for different methods

Normalized Frequency (radians/sample)	Proposed Beamformer (% error)	Yan's beamformer (% error)	UCCA Beamformer (% error)
0.3	27.8	100	209.3
0.4	155.6	49.36	19.5
0.5	54.6	18.2	89.9
0.6	0.0	0.0	0.0
0.7	52.5	15.6	188.2
0.8	12.3	81.5	40.9
0.9	66.0	286.9	32.4

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The beampattern of Yan's beamformer is shown in Figure 3.6. This figure displays a variation in gain response that is smaller than Chan's beamformer but bigger than our proposed method. It has a sidelobe level of -10 dB. The reason for this observation is that the objective function in Yan's beamformer has the closest assembly to our objective function. The difference lies in our system structure and the designed frequency range. Yan's algorithm was designed for a normalized frequency range of $[0.16\pi, 0.32\pi]$ radians/sample. When it is applied to our frequency range, the FI characteristics is poorer at the desired direction. The better performance in achieving FI characteristics by our method implies that the phase mode structure together with circular array geometry indeed yields great flexibility in controlling the gain response across a wide range of frequencies. However, Yan's beamformer has the lowest sidelobe level among all.

In summary, the peak sidelobe levels at each frequency for different methods are shown in Table 3.2. At low frequency region from 0.3π radians/sample to 0.7π radians/sample, the proposed method produced the lowest peak sidelobe level among the three methods. At high frequency region from 0.8π radians/sample to 0.9π radians/sample, Yan's beamformer has the lowest peak sidelobe level. In average, Yan's beamformer has low peak sidelobe level for all frequencies. This table shows an implicit trade-off relationship between the FI characteristics in the mainlobe region and the peak sidelobe level. The better the FI characteristics in the mainlobe region, the higher the peak sidelobe level.

Table 3.3 tabulates the gain response at each frequency along the desired direction for these three methods. Table 3.4 tabulates the percentage error of the gain response at each frequency along the desired direction. Both table are to show fluctuations on gain response along the desired direction.

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Table 3.5: Comparison of the mean squared error between the simulated spatial response and the constraint value along the desired direction at each frequency for different methods

Normalized Frequency (radians/sample)	Proposed Beamformer (dB)	Yan's beamformer (dB)	UCCA Beamformer (dB)
0.3	0.028×10^{-3}	4.52	0.46
0.4	0.230×10^{-3}	1.70	0.0060
0.5	0.063×10^{-3}	0.63	0.19
0.6	0.025×10^{-3}	0.25	0.45
0.7	0.003×10^{-3}	0.06	0.17
0.8	0.040×10^{-3}	0.67	0.0015
0.9	0.001×10^{-3}	17.31	0.64

The percentage error is calculated using the following formula,

$$\% \text{ error} = \left| \frac{\text{Array gain at each frequency} - \text{Array gain at median frequency}}{\text{Array gain at median frequency}} \right| \times 100\%.$$

gain response at median frequency is taken as reference, and that for each frequency is compared with the reference gain response. Hence, the percentage error at 0.6 radians/sample are 0%. Large percentage error indicates large frequency variance along the desired direction. From here, one can clearly see that the proposed method has the smallest variation in gain response along the desired direction.

Alternatively, the mean squared errors between the simulated spatial response and the constraint value in the specified direction and across different frequencies for different methods are shown in Figure 3.7 and Table 3.5 respectively. It is also seen that, the proposed beamformer outperforms both Chan's beamformer and Yan's beamformer on achieving FI characteristic at the desired direction.

The performance of the frequency invariant beam pattern obtained by the proposed method can also be assessed by evaluating the directivity, the white-noise gain over the entire frequency band considered, as shown in Figure 3.8 and 3.9 respectively. Directivity describes the ability of the array to suppress a diffuse noise

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field, while White Noise Gain (WNG) shows the ability of the array to suppress spatially uncorrelated noise, which can be caused by self-noise of the microphones. Put it in simple words, directivity measures the capability of the beamformer suppressing noise coming from the surroundings, while WNG measures the capability of the beamformer suppressing noise coming from the microphone. The directivity $D(\omega)$ for circular array is calculated using the following equation,

$$D(\omega) = \frac{\left| \sum_{m=-L}^L B_m(\omega) \right|^2}{\sum_{m=-L}^L \sum_{n=-L}^L B_m(\omega) B_n(\omega)' \text{sinc}[(m-n)\omega r/c]}, \quad (3.26)$$

where $B_m(\omega)$ is the frequency response of the FIR filter at m^{th} phase mode, r is the radius of the circle.

The White Noise Gain (WNG) is calculated using the following equation,

$$WNG = \frac{|\mathbf{a}^H \mathbf{w}|^2}{\mathbf{w}^H \mathbf{w}}. \quad (3.27)$$

As shown in the figure, the directivity has a constant enough profile, with an average value equal to 13.1755 dB. This implies that with a directivity value equalling to 13.1755 dB, the beamformer can suppress a diffused noise field with noise power up to 20 Watt. The higher the value, the better the suppression. The white noise gain ranges from 5.5 dB to 11.3 dB. These positive values represent an attenuation of self-noise of the microphones from 3.5 Watt to 13.5 Watt. As expected, the lower the frequency, the smaller the white noise gain, and the higher the sensitivity to array imperfections. Hence, the proposed beamformer is more sensitive to array imperfection at low frequency and is the most robust to array imperfection at normalized frequency 0.75π .

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Figure 3.10: The normalized spatial response of the proposed FI beamformer for 10 microphones

Figure 3.11: The normalized spatial response of the UCCA beamformer for 10 microphones

3.6 Sensitivity Study

In this section, we are going to evaluate the array performance of the system in terms of spatial response across a range of frequency for the following aspects: the number of microphones used, the number of phase mode, different threshold values

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Figure 3.12: The normalized spatial response of the Yan's beamformer for 10 microphones

Figure 3.13: The normalized spatial response of the proposed FI beamformer for 8 microphones

of δ and different frequency ranges.

3.6.1 Effect of Number of Microphones

The number of microphone used in a beamformer is an important considering factor in industrial manufacture. The fewer number of microphones used, the less complicated is the circuit behind, the more favorable is the hardware implemen-

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Figure 3.14: The normalized spatial response of the UCCA beamformer for 8 microphones

Figure 3.15: The normalized spatial response of the Yan's beamformer for 8 microphones

tation. Most FI beamformers reported in the literature employ a large number of microphones. In this study, we are going to investigate the impact of the number of microphones used on the array performance. The number are varied from 10 to 8. The performances of the proposed FI beamformer, Chan's beamformer and Yan's beamformer are compared. Results are shown in Figures 3.10 to 3.15.

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As seen from the simulations, when 10 microphones are employed, the proposed algorithm achieves the best FI performance in the mainlobe region, with a sidelobe level of -6 dB. Chan's beamformer displays a slightly larger variation in gain response in the mainlobe region and the highest peak sidelobe level. Yan's beamformer shows the largest variation in gain response in the mainlobe region with the lowest peak sidelobe level. This is consistent with the trade-off relationship between FI characteristics in the mainlobe region and the peak sidelobe level.

When the number of microphones is further reduced to 8 in Fig 3.13, our proposed method is able to produce good FI beampattern in the mainlobe region with a peak sidelobe level of -6 dB. Whereas, the FI characteristic in the mainlobe region produced by Chan's algorithm displays a larger variation than the previous simulation. Yan's beamformer displays a smaller variation in gain response with lower peak sidelobe level than the previous simulation.

Overall, the simulations presented in this section are consistent with the trade-off relationship between FI characteristics in the mainlobe region and peak sidelobe level. The study shows that our proposed design is robust to the number of microphones used. With fewer number of microphones used, the FI characteristics in the mainlobe region of the beampattern is still well maintained.

3.6.2 Effect of Number of Phase Mode

In this section, we are going to evaluate the effect of phase mode on the performance of the proposed beamformer. The number of phase mode M is going to be varied from 15, 11, 9 and 7. Simulation results are shown from Figure 3.16 to 3.19.

From the simulation results, we can see that for all the numbers of phases tested, the FI characteristics are maintained relatively well along the desired direction. However, when the number of phase modes are reduced, the peak sidelobe level is getting higher and higher.

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Figure 3.16: The normalized spatial response of the proposed FI beamformer using 20 microphones and 15 phases

Figure 3.17: The normalized spatial response of the proposed FI beamformer using 20 microphones and 11 phases

This is because the number of phase mode is limited by the number of microphones ($M < K$). More number of phase mode employed will result in less signal lost during the transformation, hence better beamforming result is obtained. In order to have good performance, number of phase mode should be chosen as close

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Figure 3.18: The normalized spatial response of the proposed FI beamformer using 20 microphones and 9 phases

Figure 3.19: The normalized spatial response of the proposed FI beamformer using 20 microphones and 7 phases

to number of sensor as possible.

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Figure 3.20: The normalized spatial response of the proposed beamformer for $\delta = 0.001$

3.6.3 Effect of Threshold Value δ

In this proposed algorithm, δ is a parameter that defines the allowed ripples in the magnitude of the main beam spatial gain response. In this section, different values of δ are used to study the sensitivity of the performance of the proposed algorithm to this parameter value. Three values namely $\delta = [0.001, 0.01, 0.1]$ are selected and the results obtained are shown in Figures 3.20 to 3.22 respectively. The specified frequency region of interest remains the same. Figure 3.23 shows the mean squared error between the simulated gain response and the constraint value at the specified direction (0°) for the three different δ values studied.

As shown in the figures, as the value of δ decreases, the FI performance at the specified direction improves. These can be illustrated from the zoom-in plots at the mainlobe region for different values of δ in Figures 3.24 to 3.27. Also, as the value of δ decreases, the peak sidelobe level increases. This is again consistent with the trade-off relationship between the FI characteristics in the mainlobe region and the sidelobe level.

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Figure 3.21: The normalized spatial response of the proposed beamformer for $\delta = 0.01$

Figure 3.22: The normalized spatial response of the proposed beamformer for $\delta = 0.1$

Table 3.6 summarizes the peak sidelobe levels for different values of δ . This trade-off relationship can be explained by the concept of degree of freedom. If δ is set to a small value, the mainlobe region is designed with strict FI characteristics. More degree of freedom are used to ensure the strict condition, hence resulting in

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Figure 3.23: Comparison on FI characteristic of the proposed beamformer for $\delta=0.001, 0.01$ and 0.1 at 0 degree for $\omega=[0.3\pi, 0.95\pi]$ radians/sample

Table 3.6: Peak Sidelobe Level for different values of δ

δ	Peak Sidelobe Level (dB)
0.1	-10.3
0.01	-7.832
0.001	-5.976
0	-5.2

higher sidelobe. If δ is set to a relatively large value, the FI characteristics in the mainlobe region is relaxed, hence lower peak sidelobe level is obtained.

From the observation of our simulations, in order to have a balance between good FI characteristics in the mainlobe region and relatively low peak sidelobe level, 0.01 is a suitable value for δ .

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Figure 3.24: Zoom in at mainlobe region of the proposed beamformer for $\delta = 0$

Figure 3.25: Zoom in at mainlobe region of the proposed beamformer for $\delta = 0.1$

3.6.4 Effect of Frequency Range

We are going to examine the performance of our proposed method for different frequency ranges in this section.

Frequency range is sensitive in many broadband beamformer designs. This is because the fundamental limit of a beamformer is the aperture of the array in terms of wavelength. Low frequency component which has larger wavelength requires larger array aperture, while high frequency component which has shorter wavelength requires smaller array aperture. The goal of a broadband beamformer

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Figure 3.26: Zoom in at mainlobe region of the proposed beamformer for $\delta = 0.01$

Figure 3.27: Zoom in at mainlobe region of the proposed beamformer for $\delta = 0.001$

design is to trick the high frequency component into seeing the array the same way as the low frequency component does, or vice versa.

Other than the frequency range $[0.3\pi, 0.95\pi]$ covered in our earlier sections, this section focuses on frequency ranges $[0.1\pi, 0.2\pi]$ and $[0.2\pi, 0.3\pi]$.

In Fig 3.28, the beampattern of the proposed beamformer for frequency range $[0.1\pi, 0.2\pi]$ is shown. In this frequency range, the FI characteristics of the beampattern at the desired direction are achieved. However, the peak sidelobe level is extremely high.

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Figure 3.28: Spatial response of the proposed beamformer for frequency range $[0.1\pi, 0.2\pi]$ radians/sample

Figure 3.29: Spatial response of the proposed beamformer for frequency range $[0.2\pi, 0.3\pi]$ radians/sample

Fig 3.29 shows the beampattern of the proposed beamformer for frequency range $[0.2\pi, 0.3\pi]$. In this frequency range, the FI characteristics at the desired

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direction are achieved, however some degree of frequency variance are observed at the mainlobe region. The peak sidelobe level is lower than the previous simulation.

From this section of study, we can see that, our proposed method fail to perform well in low frequency ranges. The reason will be discussed in Chapter 4 Section 4.4.4.

3.7 Summary

A method which exploits uniform circular array to achieve selective frequency-invariant characteristic region is proposed in this chapter. The main idea is to have a trade-off between the extent of FIs characteristics and number of microphone used. By focusing the FI characteristics at the desired direction where the signal of interest is from, and relaxing the FI requirement on other non desired region, we achieve better FI performance in the desired direction with fewer number of microphones.

Simulation results show that:

- Our proposed algorithm is computationally effective.
- The positive values of directivity and white noise gain shows our proposed algorithm is able to attenuate diffused noise and self-microphone noise. It is the most sensitive to array imperfection at low frequency.
- Our proposed algorithm has the best FI performance in the frequency range $[0.3\pi, 0.95\pi]$.
- Different number of phase modes used has different effects on the performance of the proposed beamformer. We observe that more number of phase modes used, better beamforming performance is obtained.

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- δ is a variable that controls the balance between good FI characteristics at the desired direction and peak sidelobe level. From our observation, 0.01 is a suitable value for δ .

Chapter 4

Beampattern Synthesis For Circular Sensor Array Using Multiple-Constraint

4.1 Introduction

Based on the method derived in Chapter 3, two design optimization criteria are proposed in this chapter. The objective is to study the design and performance of circular array beamformers when multiple constraints are imposed in the optimization model.

In most of the designs for beamformers, the target DOA is assumed to be stable. When it changes with the source, the performance obtained by the beamforming technique may be degraded. In this chapter, an approach in designing an FI beamformer with frequency-spatial invariant characteristic is proposed and investigated. Similar to the concept of frequency invariant characteristic which has constant mainlobe response over the frequencies of interest, the concept of spatial invariant allows the mainlobe response being constant over the angles of interest. This is particularly useful when the exact position of the source is not known or the source is moving in a small confined area. Multiple constraints are imposed to synthesize a beampattern with frequency-spatial invariant characteristics. One

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constraint is imposed on spatial direction to ensure that the mainlobe region has constant gain over an angular space across a wide frequency range. The second proposed multiple-constraint is imposed on the peak sidelobe level and the spatial direction to achieve a spatial invariant characteristics. Cholesky factorization and minimax criteria are used to convert the non-convex problem into a convex one. Second Order Cone Programming (SOCP) method is then used to solve it with high efficiency and accuracy. In these proposed approaches, no desired beampattern is required.

Simulation results show that the beamforming designs proposed in this chapter have promising performance at both low frequency range such as from 0.1π to 0.2π and high frequency range 0.3π to 0.95π . Performance comparisons between the proposed beamformer and existing beamformers are also provided. Insightful discussions that explained the difference in performance are included at the end.

Due to the fact that the method presented in this chapter shares the same background and derivation procedure as Chapter 3, hence, the background material and SOCP based implementations are skipped in Chapter 4. The remaining text is organized as follows: in Section 4.2, the proposed pattern synthesis methods with multiple constraints is described. Section 4.3 presents the simulation results where the performances of the proposed methods are compared with the existing algorithms. Finally, a brief summary is drawn in Section 4.4.

4.2 Proposed Designs

4.2.1 Selective Spatial Invariant FI Beamformer (SSI-FI Beamformer)

The proposed selective spatial invariant FI beamformer is designed with the following objective function:

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$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \int_{\omega} \int_{\phi} \|G(\omega, \phi)\|^2 d\omega d\phi \\ \text{s.t.} \quad & \begin{cases} \|G(\omega, \phi) - 1\| \leq \delta \\ \omega \in [\omega_l, \omega_u] \\ \phi \in [\phi_L, \phi_R], \end{cases} \end{aligned} \quad (4.1)$$

where $G(\omega, \phi)$ is the spatial response of the beamformer as defined in equation (3.14), $\mathbf{h} = [h_L, h_{L-1}, \dots, h_{-L}]$ is the spatial weightings that are being optimized, $\mathbf{b} = [b_L[n], b_{L-1}[n], \dots, b_{-L}[n]]$ denotes the FIR filter coefficients that are being optimized, ω_l and ω_u are the lower and upper limits of the desired frequency range. ϕ_L and ϕ_R are the left and right limits of the spatial constraint.

This objective function minimizes the square of the amplitude response across all frequencies and all angles, and approximating the amplitude responses pertaining to a selective spatial region for a selective frequency range to be one. δ is a predefined variable that controls the allowed ripples in the desired region. This two-dimensional constraint jointly optimizes the gain response in both the frequency and spatial region. It imposes the FI characteristic requirement in the desired spatial and frequency region and relax the FI characteristic requirement in other regions.

4.2.2 Peak Sidelobe Constrained FI Beamformer (PSC-FI beamformer)

The proposed Peak Sidelobe Constrained FI beamformer is designed with the following objective function:

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \int_{\omega} \int_{\phi} \|G(\omega, \phi)\|^2 d\omega d\phi \\ \text{s.t.} \quad & \begin{cases} \|G(\omega, \phi_p) - 1\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u], \phi_p \in \Theta_{ML} \\ \|G(\omega, \phi_s)\| \leq \epsilon \text{ for } \omega \in [\omega_l, \omega_u], \phi_s \in \Theta_{SL}, \end{cases} \end{aligned} \quad (4.2)$$

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where $\phi_s \in \Theta_{SL}, s = 1, \dots, S$ and $\phi_p \in \Theta_{ML}, p = 1, \dots, P$. These angular grids approximate the sidelobe region Θ_{SL} and mainlobe region Θ_{ML} by a finite number of directions. The choices of S and P determine the smoothness of the beampattern synthesis.

In addition to the spatial constraint to the FI beamformer, this objective function imposes a constraint on the peak sidelobe level. The norm constraint $\|\mathbf{h}\| \leq \Delta$ is also added to the above two objective functions. It serves to control the dynamic range of the filter coefficients to avoid large noise amplification at the array output. This is similar to the white noise gain constraint employed in narrow-band beamforming [56]. Here $\|\cdot\|$ denotes the Euclidean norm, and Δ is a prescribed value to control the norm of filter coefficients.

4.3 Convex Optimization

Lately, Second Order Cone Programming (SOCP) has been widely used in solving practical signal processing problem [70, 72, 79]. The standard primal form of SOCP is defined as [73]:

$$\begin{aligned} \min_x \quad & c^T x \\ \text{s.t.} \quad & \begin{cases} Ax = b \\ x \in \tilde{h}, \end{cases} \end{aligned} \quad (4.3)$$

where $\tilde{h}$ is a symmetric cone. A symmetric cone is a Cartesian product of a nonnegative orthant, quadratic cones and cones of positive semi-definite matrices. In mathematical term, a quadratic cone is defined in the following form:

$$Q_{\text{cone}} := (x_1, x_2) \in \mathbb{R} \times \mathbb{R}^{N-1} | x_1 \geq \|x_2\|.$$

This quadratic cone is also known as the second order cone or Lorentz cone.

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Next, we formulate our design problems to the standard primal form of SOCP, which can then be solved efficiently using the well-established interior-point methods [73].

Following the same derivation procedure in Chapter 3, we reach at the form shown below for the objective function.

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \bar{\mathbf{u}}^H \mathbf{R} \bar{\mathbf{u}} \\ \text{s.t.} \quad & \begin{cases} \|G(\omega, \phi_p) - 1\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u], \phi_p \in \Theta_{ML} \\ \|G(\omega, \phi_s)\| \leq \epsilon \text{ for } \omega \in [\omega_l, \omega_u], \phi_s \in \Theta_{SL}, \end{cases} \end{aligned} \quad (4.4)$$

where $\mathbf{R} = \int \int_{\omega, \phi} \mathbf{A}(\omega, \phi) \mathbf{A}(\omega, \phi)^H d\omega d\phi$.

To further transform the objective function into a linear equation, Cholesky factorization is applied. By definition, $\mathbf{R} = \mathbf{D}^H \mathbf{D}$, if R is hermitian and positive definite. The above relationship is substituted into (4.4), $\bar{\mathbf{u}}^H \mathbf{R} \bar{\mathbf{u}} = \bar{\mathbf{u}}^H (\mathbf{D}^H \mathbf{D}) \bar{\mathbf{u}} = (\mathbf{D} \bar{\mathbf{u}})^H (\mathbf{D} \bar{\mathbf{u}})$, where $\mathbf{D}$ is the Cholesky factorization of matrix $\mathbf{R}$.

As a result, (4.4) can be represented by the following form alternatively:

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & d^2 \\ \text{s.t.} \quad & \begin{cases} d^2 = \|\mathbf{D} \bar{\mathbf{u}}\|^2 \\ \|G(\omega, \phi_p) - 1\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u], \phi_p \in \Theta_{ML} \\ \|G(\omega, \phi_s)\| \leq \epsilon \text{ for } \omega \in [\omega_l, \omega_u], \phi_s \in \Theta_{SL}. \end{cases} \end{aligned} \quad (4.5)$$

Denoting t as the maximum value of vector $\|\mathbf{D} \bar{\mathbf{u}}\|$, we arrive at the following form:

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & t \\ \text{s.t.} \quad & \begin{cases} \|\mathbf{D} \bar{\mathbf{u}}\| \leq t \\ \|G(\omega, \phi_p) - 1\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u], \phi_p \in \Theta_{ML} \\ \|G(\omega, \phi_s)\| \leq \epsilon \text{ for } \omega \in [\omega_l, \omega_u], \phi_s \in \Theta_{SL}. \end{cases} \end{aligned} \quad (4.6)$$

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ω and ϕ are divided uniformly within the frequency range and spatial region with a sufficient number of points, says $I=64$. Each angular constraint above will yield I different spatial constraints.

According to (4.3), we stack t and the coefficients of $\bar{\mathbf{u}}$, and define $\mathbf{y} = [t; \bar{\mathbf{u}}]$. Let $\mathbf{c} = [1, \mathbf{0}]^T$, so that $t = \mathbf{c}^T \mathbf{y}$. Hence (4.6) becomes:

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \mathbf{a}^T \mathbf{y} \\ \text{s.t.} \quad & \begin{cases} \|\mathbf{0} \ \mathbf{D}\mathbf{y}\| \leq \mathbf{c}^T \mathbf{y} \\ \left\| \begin{bmatrix} \mathbf{0} & \mathbf{A}'(\omega, \phi_p)^T \end{bmatrix} \mathbf{y} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\| \leq \delta \text{ for } \omega \in [\omega_l, \omega_u], \phi_p \in \Theta_{ML} \\ \left\| \begin{bmatrix} \mathbf{0} & \mathbf{A}''(\omega, \phi_s)^T \end{bmatrix} \mathbf{y} \right\| \leq \epsilon \text{ for } \omega \in [\omega_l, \omega_u], \phi_p \in \Theta_{SL}, \end{cases} \end{aligned}$$

where $\mathbf{0}$ is the zero matrix with its dimension determined from the context.

Similarly, the reformulation for the proposed SSI-FI beamformer can be done in the same way and the final objective function and constraint equation are as shown below:

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \mathbf{a}^T \mathbf{y} \\ \text{s.t.} \quad & \begin{cases} \|\mathbf{0} \ \mathbf{D}\mathbf{y}\| \leq \mathbf{a}^T \mathbf{y} \\ \left\| \begin{bmatrix} \mathbf{0} & \mathbf{A}'(\omega, \phi_p)^T \end{bmatrix} \mathbf{y} - \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\| \leq \delta \\ \omega \in [\omega_l, \omega_u] \\ \phi \in [\phi_L, \phi_R]. \end{cases} \end{aligned}$$

4.4 Numerical Results

Several design examples are presented to compare the array performances in terms of spatial response across a range of frequency. Among them are the Selective Spatial Invariant FI beamformer (SSI-FI beamformer), Peak Sidelobe Constrained FI Beamformer (PSC-FI beamformer), Min-max circular broadband beamformer

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and Yan's beamformer [34]. In all of the examples considered, a circular array of 20 elements that are arranged as shown in Fig 3.1 is used. The number of phase mode M is 17. Therefore we have 17 spatial weighting coefficients. The angle constraints are ± 10 degrees from direction of interest. We assume that the orders of the compensation filter are the same for all phases. It is set to be 16 in this experiment. δ which is a predefined variable that controls the allowed ripple at the mainlobe is set to be 0.01. A uniform grid of 1° is used to discretize the field of view $[-90^\circ, 90^\circ]$. Unless otherwise stated, we assume the mainlobe region is $\Theta_{ML} = [-10^\circ : 1^\circ : 10^\circ]$ and the sidelobe region are $\Theta_{SL} = [-90^\circ : 1^\circ : -30^\circ] \cup [30^\circ : 1^\circ : 90^\circ]$. The desired angle of direction of arrival is targeted at 0 degree where the reference sensor is located.

The performances are mainly evaluated by varying the length of the interested frequency region while keeping the number of sensors unchanged. The interested frequency regions are $[0.1\pi, 0.2\pi]$, $[0.2\pi, 0.3\pi]$ and $[0.3\pi, 0.95\pi]$. These three regions are chosen because they cover different portions of the frequency spectrum and the regions are of different width.

In order to have fair comparison, we designed Yan's beamformer and Minmax beamformer [34] in such a way that their FI beampatterns achieve constant gain over an angular region.

4.4.1 Example 1 (Frequency range= 0.1π to 0.2π)

In this example, we consider the beampatterns of various beamformers at low frequency. Using the formula $\omega = \frac{2\pi f}{f_s}$ ($f_s = 8kHz$), this frequency range $[0.1\pi, 0.2\pi]$ is equivalent to $[400, 800]$ Hz for $8kHz$ sampling frequency. The spatial array response of different FI beamformers are shown from Figures 4.1-4.4. These figures are plotted as gain response versus angle. Each curve corresponds to the beampattern at one frequency bin.

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It is seen that at the low frequency range $[0.1\pi, 0.2\pi]$ radians/sample, the proposed SSI-FI beamformer has constant gain at the desired direction over an angular region, however there are certain degrees of frequency variance along the mainlobe as indicated by the non-overlapping curves. The sidelobe level is -10 dB. For the proposed PSC-FI beamformer, it also forms a constant-gain spatial response region at the desired direction. Comparatively, the spatial response around the mainlobe region for PSC-FI beamformer is more frequency invariant than the proposed SSI-FI beamformer. But the spatial invariant characteristics is not as obvious as SSI-FI beamformer as indicated by the narrower mainlobe. The sidelobe level is -18 dB. A reason for this observation is mainly due to the difference in their design criterion. Without the sidelobe constraint, the design of SSI-FI beamformer focuses on the frequency invariant characteristics and the spatial invariant characteristics at the desired angular region. Once outside the constraint region, the frequency invariant characteristics of the beampattern are less observed. On the other hand, the design of PSC-FI beamformer focuses on the constraint of peak sidelobe level, hence, the spatial invariant characteristics of the beampattern are less observed.

On the contrary, Yan's beamformer [34] as shown in Fig 4.3 is unable to achieve frequency invariant characteristic at the desired direction. This is simply due to the reason that this algorithm was not designed at this frequency range. In Fig 4.4, Minmax beamformer [34] obtains a frequency invariant beampattern with sidelobe level lower than -20 dB.

4.4.2 Example 2 (Frequency range= 0.2π to 0.3π)

In this example, we again compare the beampatterns of various methods at frequency range $[0.2\pi, 0.3\pi]$ radians/sample. As shown in Fig. 4.5, the proposed

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Figure 4.1: Beampattern of the proposed Selective Spatial Invariant FI Beamformer for frequency= $[0.1\pi, 0.2\pi]$ radians/sample

Figure 4.2: Beampattern of the proposed Peak Sidelobe Constraints FI Beamformer for frequency= $[0.1\pi, 0.2\pi]$ radians/sample

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Figure 4.3: Beampattern of Yan's beamformer for frequency= $[0.1\pi, 0.2\pi]$ radians/sample

Figure 4.4: Beampattern of the MinMax beamformer for frequency= $[0.1\pi, 0.2\pi]$ radians/sample

SSI-FI beamformer has constant spatial response for ± 10 degrees away from the desired direction. However, its spatial response is rather frequency variant at the mainlobe region as indicated by the non-overlapping curves. It has a sidelobe level of -10 dB.

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Figure 4.5: Beampattern of the proposed Selective Spatial Invariant FI beamformer for frequency= $[0.2\pi, 0.3\pi]$ radians/sample

Figure 4.6: Beampattern of the proposed Peak Sidelobe constraints FI beamformer for frequency= $[0.2\pi, 0.3\pi]$ radians/sample

In Fig. 4.6, the proposed PSC-FI beamformer achieves good FI characteristic at the mainlobe region. The spatial invariant characteristics is not as obvious as SSI-FI beamformer as indicated by a narrower mainlobe. However the sidelobe level is much lower than the proposed SSI-FI beamformer.

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Figure 4.7: Beampattern of Yan's beamformer for frequency= $[0.2\pi, 0.3\pi]$ radians/sample

Figure 4.8: Beampattern of the MinMax FI beamformer for frequency= $[0.2\pi, 0.3\pi]$ radians/sample

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Figure 4.9: Beampattern of the proposed Selective Spatial Invariant FI beamformer for frequency= $[0.3\pi, 0.95\pi]$ radians/sample

Figure 4.10: Beampattern of the proposed Peak Sidelobe Constraints FI beamformer for frequency= $[0.3\pi, 0.95\pi]$ radians/sample

In Fig 4.7, Yan's beamformer [34] is unable to form frequency invariant beam-pattern at the desired direction. In Fig 4.8, MinMax beamformer [34] forms a small ripple at the mainlobe. Though they both have a sidelobe level lower than -20 dB, their mainlobe responses display some degrees of variation in gain response.

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Figure 4.11: Beampattern of Yan's FI beamformer for frequency= $[0.3\pi, 0.95\pi]$ radians/sample

Figure 4.12: Beampattern of the MinMax FI beamformer for frequency= $[0.3\pi, 0.95\pi]$ radians/sample

4.4.3 Example 3 (Frequency range= 0.3π to 0.95π)

In this example, we further enlarge the width of the interested frequency region. The beampatterns are shown from Figure 4.9 to 4.12. It is seen that Yan's beamformer and MinMax beamformer [34] fail to perform when the frequency bandwidth is large.

In Fig 4.9, the proposed SSI-FI beamformer has a constant gain spatial response at the mainlobe. Certain degree of frequency variance is observed at the mainlobe region, it is much improved than the previous two frequency ranges. The mainlobe response is flat at ± 10 degrees from the desired direction. The sidelobe level is high which has -5 dB.

In Fig 4.10, the proposed PSC-FI beamformer achieves a very frequency invariant mainlobe response at the desired direction. The sidelobe level is -20 dB.

From the above comparisons, we can see that the beampatterns obtained for these three frequency ranges are generally consistent with each other. SSI-FI beamformer has obtained beampatterns with good spatial invariant characteristics at the mainlobe region but with higher sidelobe and certain degrees of frequency invariance. PSC-FI beamformer has obtained beampatterns with better frequency invariant characteristics but with lower sidelobe and less obvious spatial invariant characteristics.

These are mainly due to the different design criterion between them. Comparing their objective functions, an additional peak sidelobe constraint is imposed in the objective function of the proposed PSC-FI beamformer. As a result, more degrees of freedom are consumed to ensure the peak sidelobe level, and less degrees of freedom are employed to ensure the frequency and spatial invariant response at the mainlobe region. Hence the proposed PSC-FI beamformer has a lower sidelobe level but comparatively not so flat mainlobe response.

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4.4.4 Discussion

Frequency range is sensitive in many broadband beamformer designs. This is because the fundamental limit of a beamformer is the aperture of the array in terms of wavelength. Low frequency component which has larger wavelength requires larger array aperture, while high frequency component which has shorter wavelength requires smaller array aperture. The goal of a broadband beamformer design is to trick the high frequency component into seeing the array the same way as the low frequency component does, or vice versa.

Comparing the PSC-FI beamformer and SSI-FI beamformer with the method presented in the previous chapter, we find that the PSC-FI beamformer produces the best beamforming result in both low frequency regions. This is because in the low frequency region, the wavelength of the signal is large. This will render the inter-sensor distance close to the spatial aliasing limit. Hence, the array is more prone to sidelobe problem at low frequency. The PSC-FI beamformer which has a peak sidelobe constraint therefore produces the best performance in both low frequency regions. This also explained the high sidelobe level appeared in the UCCB beamformer at frequency range $[0.1\pi, 0.2\pi]$ radians/sample.

At frequency range $[0.2\pi, 0.3\pi]$ radians/sample, both UCCB and SSI-FI beamformer display certain degrees of frequency variance at the mainlobe region. This is because at this frequency range, the available degrees of freedom are used to solve the sidelobe problem, hence they are deficient in ensuring the frequency invariant characteristics.

4.5 Sensitivity Study

In this section, some sensitivity tests are carried out to evaluate the array performance of the proposed design criteria in terms of spatial response across a range

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Figure 4.13: The comparison between spatial response of SSI-FI beamformer for different number of sensors for frequency range $[0.3\pi, 0.95\pi]$ radians/sample

Figure 4.14: The comparison between spatial response of PSC-FI beamformer for different number of sensors for frequency range $[0.3\pi, 0.95\pi]$ radians/sample

of frequency. Three variables are focused. They are the number of sensors used, the number of phase mode, and the value of δ .

4.5.1 Effect of Number of Sensors

The number of sensors used for three different frequency ranges is varied from 16, 8 to 4. These three numbers are chosen because they cover a circle in a symmetric way. Results are shown from Fig 4.13 to 4.18.

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Figure 4.15: The comparison between spatial response of SSI-FI beamformer for different number of sensors for frequency range $[0.2\pi, 0.3\pi]$ radians/sample

For frequency range of $[0.3\pi, 0.95\pi]$ radians/sample, the performance of SSI-FI beamformer and that of PSC-FI beamformer are compared and shown in Fig 4.13 and 4.14.

We can see that PSC-FI beamformer has better FI achievement in the mainlobe region than SSI-FI beamformer when the number of sensors used are large, in this example, it is 16. When the number of sensors are reduced significantly, PSC-FI beamformer fails to perform immediately, while SSI-FI beamformer still achieves FI along the desired direction with wider mainlobe. When the number of sensors reduces to 4, SSI-FI beamformer forms a relatively flat beampattern.

For frequency range $[0.2\pi, 0.3\pi]$ radians/sample, the performance of SSI-FI beamformer and that of PSC-FI beamformer are compared and shown in Fig 4.15 and 4.16.

SSI-FI beamformer has similar FI behavior as that in the frequency range $[0.3\pi, 0.95\pi]$ radians/sample. However, for PSC-FI beamformer, the more sensors used, the narrower is the mainlobe. The FI characteristics is more pronounced in the mainlobe region.

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Figure 4.16: The comparison between spatial response of PSC-FI beamformer for different number of sensors for frequency range $[0.2\pi, 0.3\pi]$ radians/sample

Figure 4.17: The comparison between spatial response of SSI-FI beamformer for different number of sensors for frequency range $[0.1\pi, 0.2\pi]$ radians/sample

For frequency range $[0.1\pi, 0.2\pi]$ radians/sample, the performance of SSI-FI beamformer and that of PSC-FI beamformer are compared and shown in Fig 4.17 and 4.18.

At this low frequency range, SSI-FI beamformer has relatively flat spatial response for three different numbers of sensors used. For PSC-FI beamformer, the more sensors are used, the narrower is mainlobe. The FI characteristics are more

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Figure 4.18: The comparison between spatial response of PSC-FI beamformer for different number of sensors for frequency range $[0.1\pi, 0.2\pi]$ radians/sample

pronounced in the mainlobe region.

From the above analysis, it is found that the performances of the proposed beamformers depend highly on the available degrees of freedom present in the system. Observe from the objective function (4.1) and (4.2), PSC-FI beamformer has more constraints than the SSI-FI beamformer. As a result, when the number of sensors used in the system is sufficient, PSC-FI beamformer has better performance in terms of FI characteristics and lower sidelobe level. However, when the number of sensors is not sufficient, SSI-FI beamformer will outperform PSC-FI beamformer.

4.5.2 Effect of Number of Phase Mode

To evaluate the effect of phase modes on the performance of the proposed design criteria, we are going to keep the setting of our sensor array the same as that in Section 4.4, while vary the phase mode number M .

As Fig 4.19 and 4.20 illustrated, we find that PSC-FI beamformer is very sensitive to the number of phase modes. The performance is almost identical when the numbers of phase modes are 15 and 17. When the number of phase

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Figure 4.19: The comparison between spatial response of SSI-FI beamformer for different number of phase modes

Figure 4.20: The comparison between spatial response of PSC-FI beamformer for different number of phase modes

mode is reduced to 13, it fails to perform immediately. On the other hand, SSI-FI beamformer has rather stable performance for different numbers of phase mode. The narrowest FI mainlobe and lowest sidelobe level are obtained when a large number of phase modes is employed.

This observation can be explained by the concept of degree of freedom too. Due to the additional constraint in the objective function, PSC-FI is more sensitive to

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the number of phase modes than SSI-FI beamformer.

When the signal is transformed from time domain to phase mode processing, more phase mode used will result in less signal loss during the transformation. Looking in the perspective of phase mode processing, number of phase mode determines the number of degree of freedom. Hence, when the degree of freedom is sufficient in the system, PSC-FI beamformer has better performance than SSI-FI beamformer. When the degree of freedom is not sufficient in the system, SSI-FI will stand out in terms of performance.

4.5.3 Effect of Values of δ

Lastly, in this section, we vary the value of δ and ϵ while keeping the remaining settings the same as that in section 4.4.

Fig 4.21 shows the beampattern of the SSI-FI beamformer for different δ values. According to (4.1), δ is a variable that controls the mainlobe ripples. We can see that when δ is small, it produces a wide mainlobe region with very fine FI characteristics and higher sidelobe. As δ gets larger, the mainlobe region narrows down, and sidelobe level is lower. When δ is 0.1, it has the lowest sidelobe level. However, the response at the mainlobe region is not as flat as that for other δ values.

Fig 4.22 shows the beampattern of the PSC-FI beamformer for different δ values with the same peak sidelobe constraint. As shown in the figure, lower δ value produces narrower mainlobe response.

Fig 4.23 shows the beampattern of the PSC-FI beamformer for different peak sidelobe constraint while keeping the δ value at 0.01. In this case, the observation is opposite. Higher peak sidelobe constraint value produces narrower mainlobe response.

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Figure 4.21: The comparison between spatial response of SSI-FI beamformer for different δ values

Figure 4.22: The comparison between spatial response of PSC-FI beamformer for different δ values

The analysis presented in this section shows a demanding level on the degrees of freedom for variables δ and ϵ . For SSI-FI beamformer, lower δ value ensures very fine FI characteristic in the mainlobe region, however, it is on the expense of wider mainlobe. For PSC-FI beamformer, given one variable constant, the change in the other variable will result in different beamwidths. This is because when the system resource is constant, the high demand of one variable will result in the trade-off of another variable.

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Figure 4.23: The comparison between spatial response of PSC-FI beamformer for different sidelobe level

4.6 Summary

In this chapter, array pattern synthesis approaches to designing broadband FI beamformer with multiple constraints on frequency-spatial invariant characteristic are proposed. Under this framework, two optimization criteria are presented and the corresponding convex second-order cone programming implementations are given.

For our new approaches, no desired beampattern is required and the frequency-spatial invariant characteristic is controlled over a frequency region and a desired angular region and constraint is imposed on peak sidelobe level as well.

Extensive analysis on the effect of different parameters on the performance of proposed beamformer were carried out. The simulation results show that:

- The proposed methods outperform two other existing methods on achieving frequency-invariant characteristic in both low frequency and high frequency range.

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- The performance of the proposed beamformers depend highly on the available degrees of freedom present in the system. And the degrees of freedom present in the system are mainly determined by the number of sensors and number of phase modes used. The more number of sensors used, the better performance is observed. The more number of phase modes used, the better performance is observed.
- In order to have a balance between the FI characteristics in the mainlobe region and its width, a suitable value for δ for SSI-FI beamformer is 0.01.
- In order to have a balance between peak sidelobe level and mainbeam width, a suitable value for δ for PSC-FI beamformer is 0.01 and 0.05 for ϵ .

Chapter 5

Adaptive Circular Frequency Invariant Array Beamformer Using Multi-Beam Structure

5.1 Introduction

In Chapter 5, an adaptive broadband beamformer using circular array with multi-beam structure is proposed. This is an extension of our previous beam pattern synthesis methods with application in an adaptive beamformer structure.

Beam pattern synthesis is a fixed beamforming technique. It is more robust against reverberation which can be modeled by a diffused noise field than adaptive beamformer. The measuring metric for its capability in suppressing the noise field is termed directivity. On the other hand, the advantage of an adaptive beamformer over a fixed beamformer is the ability to suppress interference. This is particularly useful when there is more than one signal present in the surrounding.

The interest in using adaptive beamforming began in the last century to solve the new problems emerged in the satellite communication. In [89], an adaptive beamformer was viewed as a system which selectively receives a signal which was contaminated with noise and interference. The performance index was a generalized SNR at the output of a beamformer. The system was designed to maximize

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the output SNR. In [90], Widrows *et al* proposed another performance index which measured the similarity of the received signal with the reference signal in LMS sense. Since these two bench-marked works, the performance indexes for adaptive control algorithms used in adaptive systems has been considerably diversified. The usual performance indexes are the mean squared error (MSE), the maximum SNR, the maximum likelihood, the minimum noise variance, and the minimum output power. In this thesis, the mean squared error is our performance index.

Alternatively, an adaptive system is also viewed as an intelligence system, because it learns from the past. For every snapshot of samples that are passed into the array, the system will automatically adjust its weights so that the system output works towards the objective function. When there are a large number of samples, the measuring index which is the mean squared error in our case will approach to a steady value. The rate at which the MSE approaches to a steady value is called the rate of convergence. For a good adaptive system, the rate of convergence should be fast [91–95].

In Chapter 5, the proposed method are devised based on the structure of General Sidelobe Canceler (GSC). GSC was introduced in Chapter 2. Conventionally, GSC performs a projection of the data onto an unconstrained subspace by means of a blocking matrix and a quiescent vector. Our method replaces the blocking matrix and the quiescent vector with the proposed beampattern synthesis methods with nulling formed at the blocking direction. Several FI beams are formed pointing to different directions. The outputs of these beams are then combined adaptively by a single weight for each of them. Since both the number of beams and the number of selected beams are small, the total number of adaptive weights is greatly reduced. A similar idea was proposed in [38]. The adaptive weights are then calculated using adaptive algorithm such as Least Mean Square (LMS).

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Simulation results prove that our proposed multi-beam adaptive beamformer has a faster rate of convergence than an existing algorithm. Sensitivity studies are also carried out to evaluate the performance of the proposed system.

Chapter 5 is organized in the following way: in Section 5.2, the adaptive multi-beam structure is introduced in great detail; in Section 5.3, the implementation of the proposed beamforming design is described; numerical results are given in Section 5.4 and sensitivity study is presented in Section 5.5. Finally, a brief summary is given in Section 5.6.

5.2 Adaptive Beamforming Using Multi-Beam Structure

5.2.1 GSC

A Linearly Constrained Minimum Variance (LCMV) beamformer with K number of microphones and J number of taps performs the minimization of the output signal's variance with respect to some given spatial and spectral constraints [9]. It can be formulated as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \quad \text{subject to} \quad \mathbf{C}^H \mathbf{w} = \mathbf{f}, \quad (5.1)$$

where $\mathbf{R}_{xx}$ is the covariance matrix of the received signal $\mathbf{x} = [x_0[n] \ x_1[n] \ \cdots \ x_{K-1}[n]]$, $\mathbf{w}$ is the weight vector, $\mathbf{C} \in \mathcal{K}^{\mathcal{J} \times \mathcal{J}}$ is the constraint matrix and $\mathbf{f}$ is the response vector.

The constrained optimization of the LCMV problem in (5.1) can be conveniently solved using a GSC structure as shown in Fig 5.1. The GSC performs a projection of the data onto an unconstrained subspace by means of a blocking

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Figure 5.1: System structure of the Generalized Sidelobe Canceler

matrix $\mathbf{B}$ and a quiescent vector $\mathbf{w}_q$. Thereafter, the standard unconstrained optimization algorithm such as the least mean square (LMS) or recursive least square (RLS) algorithm can be applied [80].

Since $\mathbf{w}_q$ is designed to satisfy the specified constraints, the signal of interest will pass through the beamformer having a desired response independent of $\mathbf{w}_a$, where $\mathbf{w}_a$ is the adaptive weighting vector. In the lower branch, the blocking matrix is required to block the signal of interest so that only interference and noise exist. When adapting $\mathbf{w}_a$, the structure will tend to cancel the interference and noise component from $d[n]$, while minimizing the variance of the output signal $e[n]$.

5.2.2 Proposed Adaptive Structure

The conventional GSC structure converts the constrained adaptive beamformer to an unconstrained beamformer, however, it has its shortcomings. If the blocking matrix is not perfect, signal will leak into the lower branch, which in turn affect

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Figure 5.2: The proposed system structure

Figure 5.3: Adaptive array structure using frequency invariant beamformer

the cancellation of noise and interference from the desired signal. In this section, we are going to propose an adaptive beamformer structure whose blocking matrix and quiescent vector are replaced by a frequency invariant beamformer.

As shown in Fig 5.2, signal received at each sensor $\mathbf{x}(n) = [x_0(n) \ x_1(n) \ \cdots \ x_{K-1}(n)]$ is first transformed into phase mode via IDFT, which is denoted by $\mathbf{V}(n) = [v_{-L}(n) \ v_{-L+1} \ \cdots \ v_L(n)]$. Assuming M is the total number of phases and it is an odd number, hence, $L = (M - 1)/2$. These phase mode data are then passed into an adaptive beamformer whose structure is shown in Fig 5.3.

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In this structure, the frequency-invariant mainbeam is pointing towards ϕ_0 which is the direction of signal of interest. The auxiliary beam is pointing towards $\phi_1, \phi_2, \dots, \phi_P$ and having a zero response at ϕ_0 at the same time. This null formed at ϕ_0 acts as a blocking matrix in the GSC structure. By this arrangement, the broadband beamforming problem is transformed into a narrowband beamforming problem. For adaptive narrowband problem, single adaptive weight is required at each frequency invariant beamformer (FIB) output. Thus, the total number of adaptive weights is reduced significantly.

The multi-beam beamforming network is constructed by the weight vectors computed using the beampattern synthesis method (5.2), and (5.3). The output of the main beam is $d(n)$. A group of adaptive weights h_i are multiplied by the outputs of the auxiliary beams $z_i(n)$. The output of the beamspace adaptive array is $e(n) = d(n) - \mathbf{h}^T \mathbf{z}(n)$, where $\mathbf{h} = [h_1 \ h_2 \ \dots \ h_P]^T$ and $\mathbf{z}(n) = [z_1(n) \ z_2(n) \ \dots \ z_P(n)]^T$. The real-valued weight vector $\mathbf{h}$ is adjusted by the error signal $e(n)$ with adaptive algorithms.

Without loss of generality, assuming that the signal of interest comes from broadside, the mainbeam should point to the direction of $\phi = 0$. The blocking matrix is arranged such that its column vectors form P FIBs with their main beam directions equally distributed over $[-\pi/2 : \pi/2]$. Since the first FIB and the P^{th} FIB actually point to the same direction, therefore we only need $P - 1$ FIBs. All of the $P - 1$ FIBs should have zero response at broadside. Fig 5.4 shows the beampatterns of the blocking matrix. As seen from Fig 5.4, a deep null is formed at the direction of signal of interest.

5.2.3 Design of Frequency Invariant Beamformer

The following beam synthesis methods are used to form the multi-beam beamforming network.

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Figure 5.4: Beampatterns of the blocking matrix with null forming at broadside direction

5.2.3.1 Selective Frequency-Invariant Beamformer

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \int_{\omega} \int_{\phi} \|G(\omega, \phi)\|^2 d\omega d\phi \\ \text{s.t.} \quad & \|G(\omega, \phi_0) - 1\| \leq \delta, \omega \in [\omega_l, \omega_u], \end{aligned} \quad (5.2)$$

where $G(\omega, \phi)$ is the spatial response of the beamformer and is given in [88], $\mathbf{h} = [h_L, h_{L-1}, \dots, h_{-L}]$ is the spatial weightings that are being optimized, $\mathbf{b} = [b_L[n], b_{L-1}[n], \dots, b_{-L}[n]]$ denotes the FIR filter coefficients that are being optimized. ω_l and ω_u are the lower and upper limit of the desired frequency range. ϕ_0 is the desired direction. δ is a predefined variable that controls the mainbeam ripples. The purpose is to minimize the square of the gain response across all frequencies and all angles, while constraining the gain at the desired angle to be approximately one.

In the design of beampattern for the auxiliary beam, an additional constraint will be included. This is because the blocking matrix in the GSC structure is

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replaced by FI Beamformers with zero response imposed at the direction of signal of interest. The design for auxiliary beam is shown below:

5.2.3.2 Selective Frequency-Invariant Beamformer for auxiliary beam design

$$\begin{aligned} \min_{\mathbf{h}, \mathbf{b}} \quad & \int_{\omega} \int_{\phi} \|G(\omega, \phi)\|^2 d\omega d\phi \\ \text{s.t.} \quad & \begin{cases} \|G(\omega, \phi_i) - 1\| \leq \delta, & \omega \in [\omega_l, \omega_u] \\ G(\omega, \phi_0) = 0, & \omega \in [\omega_l, \omega_u]. \end{cases} \end{aligned} \quad (5.3)$$

This objective function is similar to (5.2) with an additional constraint on the null formation at broadside direction.

5.3 Implementation of the Proposed System

In this section, we will describe the implementation of the proposed system from a mathematical point of view.

Consider a circular array with K omnidirectional microphones located $\{r \cos \phi_k, r \sin \phi_k\}$ as shown in Fig 5.5 where r is the radius and $\phi_k = 2\pi k/K$ ($k = 0 \dots K-1$) is the angular location of k^{th} microphone with respect to the reference axis. The microphones are uniformly distributed along the circle, and the inter-microphone spacing is fixed at $\lambda_{min}/2$, where λ_{min} denotes the smallest wavelength of the broadband signal of interest. For a plane wave (i.e., far field) arriving at azimuth angle ϕ and elevation angle θ ($\theta = 90^\circ$), the steering vector of a circular array is

$$\mathbf{a}(\omega, \phi) = [e^{j\omega \tilde{r} \epsilon \cos(\phi - \phi_0)}, \dots, e^{j\omega \tilde{r} \epsilon \cos(\phi - \phi_{K-1})}]^T, \quad (5.4)$$

where $\tilde{r}$ is the normalized radius with respect to minimum wavelength and ϵ is the ratio between sampling frequency and maximum frequency.

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Fig 5.2 and 5.3 show the beamforming structure for the circular array. The received signals are first transformed into phase mode via Inverse Discrete Fourier Transform (IDFT). The transformed data are then feed into the proposed adaptive beamformer structure. Assuming $\mathbf{x}[n] = [x_0(n), x_1(n), \dots, x_{K-1}(n)]^T$ is the signal received at instance n , the transformed data is then denoted by $\mathbf{V}[n] = \mathbf{W}_{M,K} \cdot \mathbf{x}[n]$, where $\mathbf{W}_{M,K}$ is an $M \times K$ IDFT matrix whose mk^{th} element is $e^{\frac{j2\pi km}{K}}$ and

$$\{\mathbf{V}[n]\}_m = v_m(n) = \sum_{k=0}^{K-1} x_k[n] e^{\frac{j2\pi km}{K}}. \quad (5.5)$$

Here, M is the total number of phases, and assumed to be an odd number. $L = (M - 1)/2$, and $m = -L, \dots, L$.

Assuming there is only one source signal $s(n)$ with spectrum $S(\omega)$, taking the Discrete Fourier Transform (DFT) of (5.5), we have,

$$\begin{aligned} V_m(\omega) &= \sum_{k=0}^{K-1} X_k(\omega) e^{\frac{j2\pi km}{K}} \\ &= S(\omega) \cdot \sum_{k=0}^{K-1} e^{j\omega \tilde{r}_k \cos(\phi - \phi_k)} e^{\frac{j2\pi mk}{K}} \\ &= S(\omega) \cdot a_m(\omega, \phi), \end{aligned} \quad (5.6)$$

where $X_k(\omega)$ is the Fourier Transform of $x_k[n]$.

Consider I broadband signals $\mathbf{s}[n] = [s_1[n], \dots, s_I[n]]^T$ which impinge the circular array at azimuth angles $\phi_i, i = 1, \dots, I$ respectively. Let $\mathbf{S}(\omega) = [S_1(\omega), \dots, S_I(\omega)]$ denote the spectrum of the impinging signal $\mathbf{s}(n)$, hence,

$$V_m(\omega) = \mathbf{a}_m^T(\omega, \Phi) \mathbf{S}(\omega) + N_m(\omega), \quad (5.7)$$

where $\mathbf{a}_m^T(\omega, \Phi) = [a_m(\omega, \phi_1), \dots, a_m(\omega, \phi_I)]^T$.

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The transformed data $V_m(\omega)$ are then fed into a structure as shown in Fig 5.3. The upper path is a fixed FI beamformer, while the lower path consists of $P - 1$ FI beamformers followed by a single-tap adaptive beamformer. The upper FI beamformer forms a main beam that is steered towards the signal of interest. The $P - 1$ FI beamformers in the lower path form zero response in the direction of signal of interest, which acts as a blocking matrix that prevents the desired signal at the look direction from entering the adaptive part. Consequently, the input to the adaptive part mainly consists of the undesirable signals. The adaptive coefficient vector $\mathbf{h}$ is then continuously updated in order to remove any undesired signals coming from non-desired direction. This is achieved by minimizing the output energy of the beamformer using either LMS or NLMS adaptive filtering algorithms.

Let $b_m^0[n]$ be the coefficient of the upper FI beamformer $B_m^0(\omega)$, i.e., $B_m^0(\omega) = \sum_{n=0}^N b_m^0[n]e^{-jn\omega}$, where N is the filter order, similarly, the coefficients of the $P - 1$ FI beamformers in the lower path are represented as $b_m^1[n], b_m^2[n], \dots, b_m^{P-1}[n]$. The array output of the system in spectral domain can be expressed as

$$E(\omega) = \sum_{m=-L}^L B_m^0(\omega)V_m(\omega) - \sum_{i=1}^{P-1} \left\{ \sum_{m=-L}^L B_m^i(\omega)V_m(\omega) \right\} h_i. \quad (5.8)$$

where $B_m(\omega)$ is the Fourier transform of the filter coefficients, $V_m(\omega)$ is the Fourier transform of the signal at each phase.

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Substitute (5.7) into (5.8),

$$\begin{aligned}
 E(\omega) &= \sum_{m=-L}^L B_m^0(\omega)(\mathbf{a}_m^T \mathbf{S}(\omega) + N_m(\omega)) \\
 &\quad - \sum_{i=1}^{P-1} \left\{ \sum_{m=-L}^L B_m^i(\omega)(\mathbf{a}_m^T \mathbf{S}(\omega) + N_m(\omega)) \right\} \cdot h_i \\
 &= \sum_{m=-L}^L B_m^0(\omega) \mathbf{a}_m^T(\omega, \Phi) \mathbf{S}(\omega) + B_m^0(\omega) N_m(\omega) - \\
 &\quad \sum_{i=1}^{P-1} \left\{ \sum_{m=-L}^L B_m^i(\omega) \mathbf{a}_m^T(\omega, \Phi) \mathbf{S}(\omega) + B_m^i(\omega) N_m(\omega) \right\} \cdot h_i.
 \end{aligned} \tag{5.9}$$

where $\mathbf{a}_m$ is the steering vector at each phase, $N_m(\omega)$ is the noise spectrum at each phase.

Since $B_m^0(\omega) \mathbf{a}_m^T(\omega, \Phi)$ is designed to be frequency-invariant at the direction of signal of interest from previous section, hence,

$$B_m^0(\omega) \mathbf{a}_m^T(\omega, \Phi) \approx \mathbf{a}_m^T(\Phi). \tag{5.10}$$

Also $B_m^i(\omega) \mathbf{a}_m^T(\omega, \Phi)$ are designed to have zero responses at the direction of signal of interest, hence, taking IDFT of (5.9), one gets the time-domain expression as,

$$\begin{aligned}
 e(n) &= \sum_{m=-L}^L \mathbf{a}_m^T(\Phi) s[n] + \eta_m^0[n] \\
 &\quad - \sum_{i=1}^P \sum_{m=-L}^L \eta_m^i[n] h_i,
 \end{aligned} \tag{5.11}$$

where $\eta_m[n]$ is the noise at the output of the FI beamformer.

The adaptive weights are updated using NLMS algorithm:

$$\mathbf{h}[n] = \mathbf{h}[n-1] + \mu \cdot \mathbf{z} \cdot e(n) / |\mathbf{z}|^2, \tag{5.12}$$

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where μ is a stepsize parameter and $\mathbf{z} = [z_1, \dots, z_p]^T$ are the inputs to the adaptive part and $\mathbf{h}[n] = [h_1[n], \dots, h_P[n]]^T$.

5.4 Numerical Results

In our simulations, 20 microphones are arranged as shown in Fig 3.1. For each FI beamformer, the number of phase mode M is 17, the filter order for the frequency compensation network is 16, and the interested frequency bandwidth is $[0.3\pi, 0.95\pi]$. The blocking matrix of the proposed GSC is formed by 9 FI beamformers over $[-\pi, \pi]$. One adaptive weight is used for each of the FI beamformers. Assume there are two non-coherent signals impinging on the array at angles 0° and 30° , respectively. Here, we assume the desired signal arrives at the array at angle 0° and the interference signal impinges on the array at angle 30° . The desired signal is composed of 30 sinusoidal signals with frequencies ranging from 800 Hz to 3800 Hz at an interval of 100 Hz. The interfering signal is also composed of 30 sinusoidal signals with frequencies ranging from 500 Hz to 3500 Hz at an interval of 100 Hz. The additive white Gaussian noise at each sensor is assumed to have the same power. The Signal-to-Interference-Plus-Noise-Ratio (SINR) is -20 dB, and the number of snapshots is 1450.

The array performances of the proposed method using (5.2) to (5.5) in terms of convergence rate is first evaluated in this section. And it is compared with the method proposed in [38]. As shown in Fig 5.5, in the stated frequency range, our method has faster convergence rate, and it converges to smaller mean squared error. In Fig 5.6, our proposed method obtains better output SINR in both the transient state and the steady state. It has a 20 dB improvement.

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Figure 5.5: Learning curve of the proposed method averaged over 200 trials. The step size is 0.0001 in the NLMS algorithm

Figure 5.6: Output SINR convergence curves for both methods averaged over 200 trials.

5.5 Sensitivity Study

In this section, we are going to examine the sensitivity of the proposed adaptive system in the following aspects: step-size, adaptive algorithm, SINR, the value of

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Figure 5.7: Learning curves of the proposed method for different step-sizes over 200 trials.

δ , number of microphones and angle of arrival for interference. The performances are analyzed in terms of convergence rate.

5.5.1 Effect of Step-size

From Fig 5.7, we find that as the step sizes gets smaller, the convergence rate becomes slower. For example, when the step size is 0.01, the system converge at number of sample is around 30. When the step size is 0.001, the system converge at number of sample is around 200. However, as step size gets smaller, the error fluctuation in the steady state gets bigger. This observation is consistent with the performance of a typical adaptive system.

The choice of the step size is critical, because it determines the convergence speed. When the value is chosen to be too big, the system will converge without showing the transient state. When the value is chosen to be too small, the system will take a long time to converge, which is not desirable either.

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Figure 5.8: Learning curves of the proposed method for different SINR values over 200 trials.

In all the rest simulations, the step size is set to be 0.01.

5.5.2 Effect of SINR

In Fig 5.8, the learning curves of the proposed adaptive system for SINR values of 10 dB, -10dB and -20dB are presented. We can see that as SINR increases, our proposed adaptive system converges to a stable value faster.

5.5.3 Effect of Values of δ

Fig 5.9 shows the learning curves of the proposed adaptive system for different values of δ . As analyzed in Chapter 3, the smaller value of δ , the better FI performance in the desired direction. This is reflected on the learning curve of the proposed system too. When $\delta = 0.1$, the learning curve has the slowest convergence rate. When $\delta = 0.01$, the convergence rate is significantly improved. However, when $\delta = 0.001$, the convergence rate is reduced. This is because when the δ value is small enough, the FI performance in the desired direction is good but at the

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Figure 5.9: Learning curves of the proposed method for different δ over 200 trials.

expense of sidelobe level. Hence the suppression ability is compromised, resulting in slower convergence rate.

5.5.4 Effect of Numbers of Microphones

Fig 5.10 shows the learning curves of the proposed adaptive system for different number of microphones. The number of microphone used is decreased from 20 to 18 to 16.

In this figure, we find that the proposed system is sensitive to the number of microphones used. The more microphones used the faster is the convergence rate. The reason for it is simply because the number of microphones used determine the degree of frequency invariant characteristics at the desired direction as well as the depth of the null formed at the specified direction. The deeper the null formed, the better blocking effect, hence result in faster convergence rate for the adaptive system.

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Figure 5.10: Learning curves of the proposed method for different numbers of microphones over 200 trials.

5.5.5 Effect of Angle of Arrival for Interference

Fig 5.11 shows the learning curves of the proposed adaptive system when the angle of arrival for interference is varied. We can see that regardless of how close or far between the desired signal and the interference, our proposed adaptive system has the same convergence rate. The reason for this observation is mainly due to the circular array geometry, which has no discrimination against any angle. This is so because we assume that the desired signal and the interference lie on the same plane.

5.6 Summary

In this chapter, we proposed a novel method for broadband adaptive beamforming based on the GSC structure, where the quiescent vector and blocking matrix are

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Figure 5.11: Learning curves of the proposed method for different angle of arrival for interference over 200 trials.

replaced by a series of 2-D FI beamformers. These FI beamformers transform the broadband beamforming problem into a narrow band one and ideally only one adaptive weight is required for each of the FI beamformer outputs. Due to this transformation and the resultant low adaptive filter length, a considerably increased convergence speed is achieved, as demonstrated in our simulations.

In our sensitivity study, we find that:

- Our proposed adaptive system has faster convergence rate and converge to a smaller mean squared error than an existing algorithm.
- Our proposed adaptive system responds the fastest when the SINR is the highest.
- Our proposed adaptive system has the fastest convergence rate when δ takes up the value of 0.01.

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- Our proposed adaptive system is very sensitive the number of microphones used.
- Our proposed adaptive system resolved interference coming from different angles at the same convergence rate.

Chapter 6

Conclusion and Recommendations

6.1 Conclusion

In this thesis, we have focused on investigating the design of beamforming techniques for broadband signal. Several approaches were proposed to achieve frequency invariant beampattern for a wide range of frequency spectrum.

Our first proposed method exploit uniform circular array to achieve selective frequency-invariant characteristic region. The main idea is to have a trade off between the degree of frequency-invariant (FI) characteristics and number of sensors used. By focusing the FI characteristics at the desired direction where the signal of interest is from, and relaxing the FI requirement on other non desired region, we achieve better FI performance in the desired direction with fewer number of sensors. Simulation results show that under the same array configuration, the proposed method has better performance in achieving FI at the main lobe than existing algorithms. The computational complexity analysis shows our proposed method is computational effective. The white noise gain analysis shows our proposed method is able to attenuate self-sensor noise, and it is the most sensitive to array imperfection at low frequency. The sensitivity performance evaluation at different number of sensors and different constraint parameter values conclude that

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0.01 is a suitable value for δ and more phase mode used, better FI beamforming result is obtained.

Our second proposed method extends the first idea further by incorporating multiple constraints. Two beampattern synthesis approaches were proposed. One constraint is imposed on spatial direction to ensure that the mainlobe region has constant gain over an angular region across a wide frequency range, the second constraint is imposed on the peak sidelobe level. These modifications are simple, and yet they yield promising results at both low frequency and high frequency regions. The performance of our method is compared with three other existing design criteria for different frequency ranges. In our proposed approach, no desired beampattern is required. Various performance evaluations show that our proposed design obtains better simulation result at low frequency region, and for wide frequency range. A suitable value for SSI-FI beamformer to achieve a balance between the FI characteristics in the mainlobe region and its width is 0.01. Suitable values for PSC-FI beamformer to achieve a balance between peak sidelobe level and mainbeam width are 0.01 for δ and 0.05 for ϵ .

Lastly, we look into developing an adaptive broadband beamformer using multi-beam structure with our previously proposed FI beamforming criteria. Conventional GSC beamformer is designed mainly for narrowband scenario. We extend it into broadband beamforming by replacing the blocking matrix and the quiescent vector with our proposed FI beampattern synthesis method. In the upper path of the proposed GSC structure, an FI beampattern with peak pointing at the desired direction is formed. In the lower path, a number of FI beampatterns with null formed at the desired direction are formed. Single weight is attached at the output of each FI beamformer in the lower path, and it is adaptively updated using Normalized Least Square algorithm. In the simulations, we compare the

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Figure 6.1: sensor with position error

performance with a method that uses similar GSC structure but with different FI beamforming design criteria. The result shows that we have faster convergence rate, and it converges to a smaller mean square error. The sensitivity study shows that our proposed adaptive system responds the fastest when SINR is high. It is sensitive to the number of sensors used. And it resolves interference coming from all angles at the same convergence rate.

6.2 Recommendations for Further Research

Based on the techniques and schemes developed in this thesis, the scope of the current work may be further extended in the following directions.

6.2.1 Robustness Against Sensor Mismatch

In many practical application, it is common to have sensor array characteristics error, i.e., some uncertainties in sensor phase, and position as discussed in [87]. Robustness against sensor mismatch could be taken into consideration in the future design.

Sensor position error arises when the actual position of the sensor is different from the assumed one. As illustrated in Fig 6.1, any microphone in the array that is misplaced along the circle could lead to microphone position error. As a result,

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the estimated radius will be different for each sensor. The array steering vector for circular array with position error is:

$$\mathbf{a}(\omega, \phi) = \left[e^{j\omega(\tilde{r} + \Delta_{pos_0})\epsilon \cos(\phi - \phi_0)}, \dots, e^{j\omega(\tilde{r} + \Delta_{pos_{K-1}})\epsilon \cos(\phi - \phi_{K-1})} \right]^T \quad (6.1)$$

where Δ_{pos_k} denotes the position error at k^{th} microphone.

Phase error arises when microphone is electronically steered to an incorrect direction/phase. In this case, the steering vector is:

$$\mathbf{a}(\omega, \phi) = \left[e^{j\omega\tilde{r}\epsilon \cos(\phi - \phi_0 + \Delta_{ph_0})}, \dots, e^{j\omega\tilde{r}\epsilon \cos(\phi - \phi_{K-1} + \Delta_{ph_{K-1}})} \right]^T \quad (6.2)$$

where Δ_{ph_k} denotes the phase error at k^{th} microphone.

Methods that deal with the above errors can be explored and studied further.

6.2.2 DOA Estimation

In this thesis, all the beampattern synthesis methods assume the DOAs of the desirable signals are known. However, in practise, very often the DOAs are not known and needed to be estimated. Furthermore, circular array is favored for direction finding application as it provides almost invariant azimuth angle coverage. Hence, method that estimates the DOAs of desired signal using circular array can be explored and studied further as well.

Author's Publications

Journals

- (i) X. Zhang, W. Ser, Z. Zhang and A. K. Krishna, "Selective Frequency Invariant Uniform Circular Broadband Beamformer", *EURASIP Journal on Advances in Signal Processing* (in press).
- (ii) X. Zhang, W. Ser and H. Chen, "Beampattern Synthesis for Circular Array with Frequency-Spatial Invariant Characteristics", *EURASIP Journal on Signal Processing* (revision).
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Conferences

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CHAPTER 6. CONCLUSION AND RECOMMENDATIONS

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