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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**AN EFFICIENT METHODOLOGY FOR ANALYZING
HEAD DISK INTERFACE
WITH APPLICATION TO UNLOADING PROCESS**

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in fulfillment of the requirement for the Degree of
Doctor of Philosophy

2011

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*Dedicated to my parents,
to my wife, Shi Yin
and to my daughter, Liu Jingxuan*

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ABSTRACT

Head disk interfaces (HDI) and Load/Unload (L/UL) processes have been studied numerically for over a decade. Several simplified and comprehensive models have been developed to study the HDI in L/UL processes. All existing methods simulated the L/UL behaviors of a slider by iteratively obtaining the instantaneous attitude of a slider through repeatedly solving a modified Reynolds equation which is coupled with the dynamics of the suspension. Despite its relative accuracy, this approach requires huge amount of computational time and power. Hence, developing a simple and efficient model, which is capable of modeling the behavior of the head-disk interface during the L/UL processes, is desired and critical to aid the design process of slide/suspension systems.

In this dissertation, a simple but fairly efficient and accurate method was proposed to model and analyze the unloading behavior of a subambient pressure slider. A dual scale model for head disk interfaces was proposed and implemented by considering the incomparability between the milli-scale deformation of the suspension and the nano-scale variations of the air bearing gap.

The suspension was modeled as a 3-DOF lumped parameters model. Realistic values of the parameters in the model were obtained from a comprehensive finite element analysis and verified with experiments. Three stages in the unloading process and their transitional conditions were analyzed using the FEM and the simplified lumped parameters model.

Finite difference method and finite volume method were employed to solve the modified Reynolds equations governing the slider air bearing. Nonlinear variations of

air bearing forces and moments with flying attitude in the L/UL processes were characterized and calculated by the proposed simple performance functions, which were easily obtained by function-fitting those discrete numerical solutions from the Reynolds equations with the minimal flying height and the pitch angle chosen as independent variables. The two variables were decoupled in obtaining the performance functions.

Finally, an efficient and fairly accurate method for modeling the unload process of a slider was achieved by coupling the 3-DOF suspension lumped parameters model with the air bearing performance functions. A typical unloading process was simulated. Parametric studies and trend analyses were performed to demonstrate the efficiency of the method and its potential for practical application.

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Nomenclature of key variables and symbols

Suspension parameters

ΔF_r	the change of the force applied on the lift tab in the vertical direction
ΔF_s	the change of the force applied on the slider in the vertical direction
ΔM_θ	the change of the moments applied on the slider in the pitch direction
Δz_r	variations of lift-tab translational displacement
Δz_s	variations of slider translational displacement
$\Delta \theta$	variations of slider angular displacement
k_{xx}	coefficients of stiffness matrix
$K^{(d)}$	stiffness matrix for dimple engaged
$K^{(g)}$	stiffness matrix for dimple separated
$K^{(L)}$	stiffness matrix for limiter engaged
λ_{xx}	coefficients of flexibility matrix
$L^{(d)}$	Equivalent free length of the dimple
$L^{(g)}$	Equivalent free length of the limiter
(d)	dimple engaged stage
(g)	dimple separated stage
(L)	limiter engaged stage

Air bearing parameters

A_x, A_y	bearing number in the x and y directions, respectively
σ	squeeze number
F_a	total air bearing force on the slider;
M_θ	total air bearing moment in pitch direction on the slider.

F_p	total air bearing positive force
F_n	total air bearing negative force
P_p	positive air bearing pressure at the grid points
P_n	negative air bearing pressure at the grid points
M_p	total positive moment
M_n	total negative moment
M_{pp}	positive moment caused right-hand side positive force
M_{pn}	positive moment caused left-hand side negative force
M_{np}	negative moment caused left-hand side positive force
M_{nn}	negative moment caused right-hand side negative force
a, b, c, d	variable coefficients changing with θ
m, n	constants for a slider adjusting the curvature of the fitting
f, u, w, v	moments curve fitting coefficients

CHAPTER ONE

INTRODUCTION

1.1 HISTORY AND OVERVIEW OF HARD DISK DRIVES

Punch tapes were used as the first storage medium on computers for many years until the creation of magnetic tapes. However, the tapes also gave their way to disk drives when random-access to the data was needed for quick and efficient use of data stored. IBM announced a disk drive product that offered unprecedented random-access storage in May 1955. These early prototypes with their heads in contact with the disk surfaces lead serious tribological problems. As IBM engineers realized that read/write (R/W) heads with a proper design could be suspended above disk surfaces and read the bits as the disks passed by underneath, IBM introduced model 3340, the ancestor of the modern disk drives. Over the succeeding years, as the technology improved incrementally, bit density, capacity and performance all increased.

Hard disk drives (HDDs), which benefit from their inexpensive media and stable storage, are the most important magnetic storage devices for computers and other data processing systems now. The demand for greater capacities and higher data transfer rates have also increased dramatically and been enabled by technological advances in magnetic, mechanics, tribology, and several other areas of research [1-4].

In the most commercial HDDs, digital data are stored on the disks coated with thin magnetic layers and arranged in concentric circles or tracks. They are read or

written with R/W heads. A typical structure of hard disk drives is shown in Fig. 1.1 [5-7].

Fig. 1.1 Schematic of A Hard Disk Drive

1. **DISK.** Every hard disk drive has one or more flat rotating platters with either Al–Mg alloy substrates or mixture substrates of glass and ceramic. A thin magnetic material layer allowing data storage is coated on both sides of each platter with a protective carbon overcoat.
2. **ACTUATOR ASSEMBLY.** The actuator assembly is formed with a voice coil motor (VCM), a data flex cable, actuator arms, and crash-stops at both ends of travel. The preamplifier is often included as a part of the actuator assembly. The actuator moves the heads radically across the disks as they spin, allowing each head access to the entirety of the platter. Data are written to the disks by transmitting electromagnetic flux to magnetic materials.
3. **HEAD/SUSPENSION ASSEMBLY.** In modern HDDs, reading and writing are performed with separate heads who are positioned on a slider with an air-bearing

surface design. The slider is attached to a stainless steel suspension with a gimbal to allow pitch and roll motion, while the suspension itself is attached to the arm of the actuator with a hinge.

4. **SPINDLE AND MOTOR ASSEMBLY.** The movable heads access to the random areas on the disks when the disks rotate. All modern hard disks use servo-controlled DC motors with smooth spindle motor bearings. The disks are clamped on the spindle motor.

5. **DEVICE ENCLOSURE.** Two major parts, the base casting and the top cover, are screwed together and sealed with a gasket. The base casting supports the main parts, such as the electronics card, the spindle parts, and the actuator parts. The disk surface and the internal environment of a HDD must be kept immaculate to prevent damage from fingerprints, hair, dust, smoke particles, etc. With the help of a breather filter, the air in the cavity can be kept away from outside dust and other contamination, then reducing the possibility of head crashes.

6. **ELECTRONICS CARD.** This part provides an interface to the host computer. Read heads retrieve the information stored on the disk by sensing the electrical changes induced by the movement of the magnetic fields when the platters rotate. The associated electronic system rotates the disks and controls the movement and position of the actuator. Firmware schedules the read/write actions efficiently. The read/write head flies on a nano-scale air bearing cushion, which relies heavily on the air pressure inside the drive. If the air pressure is too small, the air will not provide enough force to push the head to the designed flying height, thus raising the risk of

head crashes and data loss [8-9].

Since reproduced magnetic signal amplitude decreases with a decrease of the recording wavelength and the track width, the signal loss is affected by the magnetic coating thickness, the read gap length, and the flying height. According to the Wallace equation, the signal loss caused by the spacing can be increased exponentially as the gap between the head and the medium increases. Thus a zero flying height is an ideal circumstance. However, the zero spacing is prevented by tribological issues [10].

Because the flying heights are the same order of magnitude of the peak-to-valley distance of head and medium surfaces, head-disk contacts may occur occasionally. On one hand, the air bearing film should be thin to give a strong read-back magnetic signal; on the other hand, the air film in operation is expected to be thick to minimize the asperity contacts. Thus, the head–disk interface should be designed with optimum conditions [11, 12].

Smoother surfaces are preferred to realize higher recording densities, whereas they lead to an increase in adhesion, friction, and wear. Friction and wear issues can be solved by selecting proper interface materials and lubricants, by optimizing the dynamics of the head and medium. A fundamental understanding of the tribology of the magnetic head–medium interface becomes crucial for the continuing growth of the magnetic storage industry [13].

As the flying height of magnetic sliders is approaching 5 nm and even lower, ultrasmooth disks are required to reduce head-disk wear caused by contacts. Since a textured landing zone is difficult to be implemented for low flying heights, the load/unload mechanism has widely replaced the conventional zone technology in a

hard disk drive. Compared with the contact start-stop (CSS) technology, load/unload mechanism can decrease the possibility of stiction and wear failure, increase the areal density, reduce the power consumption, and improve the shock robustness [14, 15].

1.2 PROJECT OBJECTIVES

Head disk interfaces (HDI) and Load/Unload (L/UL) processes have been studied numerically for over a decade. Several simplified and comprehensive models have been developed to study the HDI in L/UL processes. All existing methods simulated the L/UL behaviors of a slider by iteratively obtaining the instantaneous attitude of a slider through repeatedly solving a modified Reynolds equation which is coupled with the dynamics of the suspension. Despite its relative accuracy, this approach requires a huge amount of computational time and power. Even a minor redesign needs the whole process rerun, causing a long product development period. Therefore, developing a simple and efficient model, who can model the behavior of the head-disk interface during the L/UL processes, is desired and critical to aid the design of slider/suspension.

In this dissertation, an efficient methodology was proposed to model and analyze the unloading behavior of a subambient pressure slider.

1.3 OUTLINE OF THE DISSERTATION

In current HDD industry, the air bearing sliders and the suspensions are normally designed and fabricated separately by different companies. Assembling them efficiently and precisely is the focus of all the HDD manufacturing companies. This dissertation covers the research work on developing a novel and efficient scheme for studying the

head disk interface with its application to the unloading process. The whole structure of the dissertation can be illustrated as Fig. 1.2. It is organized as following.

In Chapter Two, an overview of the hard disk drive and the load/unload technology was presented. Two major components, the magnetic components and the electromechanical components, were discussed in detail. Two slider parking technologies, CSS technology and ramp load/unload technology, were compared. Details of the head disk interface (HDI) and the ramp load/unload technology were presented. Various air bearing models, suspension models, and some existing results on unloading process were enumerated.

In Chapter Three, the structures of a typical suspension and a ramp load/ unload mechanism were briefly introduced. A dual scale model for analyzing unloading processes was developed by considering milli-scale deformation of the suspension and nano-scale variation of the air bearing gap in a head disk interface (HDI). The suspension was modelled as the 3-DOF lumped parameter system. Three stages in the unload process and transitional conditions were discussed with the 3 DOF suspension model.

In Chapter Four, a comprehensive finite element model including all major components of a suspension was modelled. Static analyses were performed to obtain the stiffness matrices required by the 3-DOF lumped parameters model and the equivalent lengths of the suspension at different unloading stages. Quasi-static analyses were performed to study the behaviors of the suspension in unloading processes. Stages in the

Fig. 1.2 Outline of the dissertation

unloading processes were discussed. Three experiments were designed and conducted to verify the equivalent lengths and a part of parameters in the stiffness matrices.

In Chapter Five, head disk interfaces (slider air bearing) in modern HDD were modelled through the modified Reynolds equations. Finite difference method and finite volume method were employed to solve the modified Reynolds equation numerically in the chapter. Nonuniform grids were arranged for the meshing scheme to accommodate the nonlinear air bearing pressure distribution of subambient sliders with steep walls.

In Chapter Six, a novel surface fitting scheme was introduced to characterize and approximate the nonlinear variations of the air bearing forces and moments with flying attitude in unloading processes. The air bearing forces and the moments acting on a subambient slider were studied by separately calculating the contribution of positive and negative pressures on the slider. First, a family of exponential-type functions was employed to fit the air bearing forces and moments with the minimal flying height chosen as an independent variable. Second, the pitch angle was introduced as another independent variable by fitting the coefficients in the exponential-type functions. Thus, performance functions for characterizing and calculating the nonlinear air bearing forces and moments in unloading processes were obtained. Three subambient pressure sliders with simple as well as complex surface design were studied using the performance functions approach.

In Chapter Seven, the 3-DOF lumped parameter suspension model was coupled with the performance functions to achieve an efficient and fairly accurate unloading model. A typical unloading process was simulated. Parametric studies and trend

analyses were performed to demonstrate the efficiency of the method and its potential for practical applications.

In Chapter Eight, current works were concluded, and some future works were presented.

CHAPTER TWO

LITERATURE REVIEW

2.1 GENERAL COMPONENTS OF A HARD DISK DRIVE

Modern HDDs normally consist of four parts: magnetic components, mechanical components, electromechanical components, and electronic components [6]. The components related to the research work are introduced in the follows.

2.1.1 Magnetic Components

The media and the head are the kernel components used to store and retrieve binary information. Read/write heads store information bits into or retrieve them out from concentric data tracks on a rotating magnetically coated disk. Disks are clamped on a spindle motor that spins the disks. A typical layered construction of a thin magnetic film hard disk is shown in Fig. 2.1.

Fig. 2.1 A typical layered construction of a thin magnetic film hard disk

The disk consists of a flat rigid substrate, a recording layer about 20~50 nm thick, a protective overcoat about 3~5 nm, and a lubricant layer less than 4 nm. The materials

for each layer are shown in Fig. 2.1. Two kinds of substrates, aluminum-magnesium alloy and glass, are practically chosen. A lubricant layer (tetrafluoroethylene) superficially applied on the protective layer is adopted to improve tribological performance between the head and the disk [17-19].

Read and write heads fabricated together and mounted at the trailing edge of the slider are crucial components in writing data to or reading data from the disks. The interface between the magnetic physical media and the read/write elements which are electrically connected to the drive decides the overall performance of the hard disk. The write heads convert electric bits to magnetic pulses and store them on the disks while the read heads reverse the process. Sliders which accommodate the read/write heads at the trailing edge determine the flying spacing and the behavior of the head-disk interface. The slider positions the read/write heads in close proximity to the magnetized bits by flying over the surface of the spinning disk. The air bearing is formed when the air moving with the spinning disk enters the gap between the disk and the slider aerodynamic surface. The surface of the disk should be kept smooth enough to stabilize the read/write electric signals. A modern slider flying only few nanometers above the disk is cut off from a wafer. To resist shock, improve time response, and lower flying heights, the slider/suspension assemblies are carefully designed to reduce their overall mass and size [20-22]. Fig. 2.2 shows the evolution of slider/air bearing size and surface.

In the early 1990's, subambient (negative) pressure air bearings were introduced with the emergence of an innovative manufacturing method. Cavities on the surface of the slider facing the bearing create a subambient pressure area opposing to the positive pressure in the rest of the air bearing by expanding the flow. Negative pressure sliders can reduce the flying height, increase the stability, reduce the contact area between the slider and the disk, and uniform the flying height from the inner to the outer edge of the

disk. However they also develop an undesirable suction force during the dynamic load/unload processes. In the normal operational condition, the net air bearing force is positive to balance the suspension preload [23, 24].

Fig. 2.2 Evolution of slider/air bearing size and surface

2.1.2 Electromechanical Components

The spindle motor and the actuator are two principal electromechanical components in a hard disk drive.

Normally, a brushless direct current (DC) spindle motor is used to spin a disk or a stack of disks in a HDD. As the velocity of the disk influences the flying height, and further affects the bit density, the variation of the spindle speed is a major source of disturbance in the tracking servo loop. So the speed of the motors must be controlled precisely to ensure the conformity of the rate of read back bits.

Read/write heads attached to an actuator arm is driven by a voice coil motor. The coil of a rotary VCM turns sidewise and the tip of the actuator arm traces an arc. Since the actuator arms are made of significantly thick steel or aluminum, it is too strong to provide flexibility to fly the read/write heads over the disk surface. Practically, a thin and more flexible sheet made of stainless steel called suspension is attached by a hinge

to the end of the actuator arm. [6, 10]

The slider floats over the spinning disk. The suspension provides a balance force and moment on the slider to counteract the aerodynamic force and moment of the air bearing surface. The suspension also must allow the slider to rotate in the pitch and roll directions so it can stay close to the surface despite the presence of long wave fluctuation and shortwave asperities on the disk surface. An even more flexible metal sheet called gimbal is used to connect the load beam and the slider. However the HDD servomechanism uses an error signal to guide the input torque for positioning the head. The gimbal cannot be too soft since the servomechanism asks for high stiffnesses in the other direction so it can be swung back and forth rapidly during the tracking and the seeking without producing excessive vibration. The gimbal is spot-welded on the suspension load beam. The suspension is thicker than the flexure and has a flange on both sides. The back side of the gimbal tongue is propped and the slider pivots around a spherical protrusion called "dimple" in the suspension. The head/suspension assembly constraints three of the six degrees of freedom (DOF) of the slider, and other three degrees of freedom, the vertical translation, and the rotations around the x and y-axis, remain. During the normal working conditions, the vertical force and stiffness acting on the slider are mainly caused by the suspension, while the rotary moments and stiffnesses are governed by the gimbal. However during the L/UL processes, the situation becomes more complicated [25-27].

2.2 OVERVIEW OF THE LOAD/UNLOAD TECHNOLOGY

2.2.1 Overview of Contact Start-Stop Technology

When hard disk drives are powered off, the rotational speed of the disks slows down to stop, and therefore, the dynamic air bearing supporting the slider disappears. To avoid the direct slider-disk contact, the sliders should land on a non-data zone before the air bearing breaks down. Two types of technologies, contact start-stop (CSS) shown in Fig.2.3 (a) and load/unload (L/UL) shown in Fig.2.3 (b), have been adopted for slider landing [28].

Fig. 2.3 Slider parking technology

In conventional CSS type disk drives which rest the sliders on landing zones, the sliders slide on the disk while the disks slow to a stop, and the reverse processes occur when the drives are turned on. Stiction and adhesive force may occur between the surface of the slider and that of the disk during start-up. During the start-up cycle, the sliders does not take off from the disk surface until an air pressure formed by the spinning disks lift them off, and then the heads fly above the disk platter at the designated flying height.

One of the most significant drawbacks of the design is the damage caused by the adhesion and stiction of two contacted smooth surfaces, the disk surface and the slider surface in this case. To reduce the stiction, the landing zone is texturized so the area of

the contact between the two surfaces can be reduced. Besides the problems caused by the stiction, wear at the slider-disk interface can be another consideration after a large number of start/stop cycles.

Although CSS techniques have satisfied the requirements in the past, the desire to store more data into the same surface area asks for ultrasmooth surfaces. As the areal densities increase up to 1000GB/in^2 , the magnetic spacing between the read/write head and the disk has to be further reduced down to 7~8 nm. The corresponding flying height has to be reduced down to 3~4 nm. Then the thickness of the protective layers of both the head-slider and the disk media must be reduced to 2 and 1 nm, respectively. Because of the inherent limit in the CSS mechanism, it is necessary to switch the head-slider parking technology from the CSS mechanism to the ramp loading/unloading mechanism to achieve a higher storage density. Load/unload implementation provides numerous benefits, including greater durability, more efficient power utilization and superior shock resistance [29].

2.2.2 Load/Unload Mechanism

With the load/unload technology, higher storage capacities, greater durability, lower power consumption and enhanced shock tolerance become feasible. Very smooth recording media allowing for lower fly heights is usable by resting the heads off the disk surface to avoid stiction. Hard disk drives based on the load/unload mechanism can resist wear and tolerate more start/stop cycles by eliminating the direct slider-disk contact. The combination of the load/unload technology and the giant magneto resistive (GMR) heads increases the track and bit densities of the disk significantly. Advanced power management modes are enabled by the improvement of the overall drive reliability.

Ramp loading as shown in Fig. 2.4 (a) is one convenient design to implement the load/unload technology. The ramp is mounted at the out edge of the disk as shown in Fig. 2.4 (c). The idea is to load or unload the slider with a rotary actuator moving the suspension along a ramp as shown in Fig. 2.4 (b). Since the design keeps the slider away from the disk during the off period, shock-induced impacts between the slider and the disk seems unlikely. Therefore, the ramp loading system becomes a good alternative to CSS in high performance hard disk drives. Hard disk drives which adopt the ramp load/unload (L/UL) technology load/unload the heads at the outer edge of the disk while the disk is still rotating.

Fig. 2.4 Ramp load/unload mechanism

Ramp L/UL systems can be designed with several different geometries, such as end-lift and mid-lift designs. End-lift designs have a lift extension on the suspension while mid-lift designs have a feature on the suspension load beam between the suspension hinge and the head engages the ramp. End-lift designs allow the use of

backside merge ramps on the ramp structure to facilitate installation of the actuator during drive assembly. However, the drawback of the design is even a small extra mass added at the end of the suspension creates a significant increase in actuator inertia and lowers the resonant frequencies of suspension modes. Two variations, those with centered and offset lift features, exist in end-lift designs. Lift tabs which extend along the centerline of the suspension are designed with several advantages of simplicity, low cost, generally with a lower mass and least deflect under load. The main disadvantage of centered lift tabs is that ramp must overhang the disk surface to lift the head at the proper position and the debris produced during ramp-suspension contact may spread on the surface of the disk [30-32].

The main concern with the mechanisms is there must be no slider-disk contact during the entire L/UL process. Whether the phenomenon occurs depends on total system parameters such as air bearing design, stiffness of suspension, L/UL speed, ramp angle, and disk vibration. Therefore, the L/UL behaviors should be analyzed as a whole by experiments and simulations. L/UL dynamic characteristics are affected by a number of operating parameters, such as pitch static attitude (PSA), roll static attitude (RSA), L/UL velocities, disk RPM, suspension limiter, and slider air-bearing surface (ABS) design [33, 34].

Although some research has been conducted for decades [35-38], the behavior of the slider during the load/unload processes has not been well understood. A crucial question is how the slider establishes its air bearing cushion against the disk without resulting in hazardous slider-disk contacts and rests on the ramp smoothly and swiftly without damaging the surface of the disk. The conventional slider-suspension assembly with a gimbal mechanism attaching the slider to the suspension was not originally designed for dynamic load/unload. Ultra small head-disk spacing increases the risk of

slider-disk contact, whereas the reduction of the protective layer thickness puts a stricter requirement on the prevention of the slider-disk contact and the impact. The slider-disk contact during the ramp load/unload processes may lead even more severe consequences. Therefore, it is essential to investigate the slider-disk contact/interaction during the load/unload processes thoroughly.

Currently, the investigations on the L/UL are either based on computer modeling or experimental observations of surface damage and slider wear after a long-term reliability test. The L/UL processes are complicated dynamic processes, which involve the dynamic behaviors of the air bearing slider and the dynamic behaviors of suspension, gimbal, and limiter. Simulation alone is not enough to provide detailed understanding of slider–disk interaction in the loading and unloading processes. Experiments will be used to test and help modifying the simulation models.

2.3 HEAD DISK INTERFACE

2.3.1 Head Disk Interface

In the implementation of the L/UL mechanisms, an important concern is the head-disk interface reliability. Currently, the read-write element is mounted on the trailing edge of an air bearing slider, which flies over an air cushion caused by the relative motion of the slider and the rotating disk as shown in Fig. 2.5.

Fig. 2.5 Head disk interface

Air bearing force and moment are generated by the extremely thin squeezed air layer under the slider. The thickness of the air cushion called the flying height is determined by the force and moment. As mentioned earlier, the slider can be considered having three degrees of freedom, namely the vertical motion and rotational motion in the pitch and roll directions. The skew angle, which is defined as the angle between the central line of the slider along the flying direction and the flow direction at each radial position, is temporarily regarded fixed in the L/UL processes. The read-write signal is significantly affected by the distance between the read-write element and the disk surface. The fluctuation of the distance will add noise into the read-write signal. The suspension forces and moments are obtained from structural mechanics, while the air bearing forces and moments are obtained by solving a modified Reynolds equation. Conventionally, dynamic analysis of a slider flying over a rotating disk involves solving the time-dependent generalized Reynolds equation and the equation of motion of the slider and suspension simultaneously. The dynamics of a slider is completely determined by the balance of the air bearing pressure, the suspension force, the damping force, and its inertia. The suspension force can be represented using either the flexure stiffness and approximate damping coefficients or the suspension dynamics.

It is critically to maintain a minimal flying height for reliable operation. Originally, the gap was kept by pressurizing a slider in which the recording head was embedded statically. Currently, the read/write heads were mounted at the trailing edge of sliders with carefully designed air bearings that is self-pressurized by the airflow of the rapidly spinning disk. The minimal flying height down to 7 or 8 nanometers or less currently is realized by designing the air bearing and the head disk interface carefully in drives. Furthermore, durability should be increased by considering the

disk roughness and waviness, the protective overcoats on disk and slider, and the lubrication. The requirement for higher recording densities prefers to an even smaller flying spacing.

The head-disk interfaces are widely analyzed with experiments and simulations. Experiments are implemented with the help of various mechanical, optical and magnetic devices, and simulations are supported by solid dynamics, fluid dynamics, and numerical methods. The head-disk interaction and wear mechanisms are investigated under a wide spectrum of conditions. Research on designs of air bearing, suspensions, and advanced head-disk material is conducted to achieve low flying height designs with high reliability. All of these research activities are expected to boost the growth rates of the magnetic recording densities continuously.

2.3.2 Slider Air Bearing and Modeling

Air bearings in HDI utilize a thin film of pressurized air, due to the high rotation speed of the disk, to provide an exceedingly low friction load-bearing interface between surfaces. The two wedged surfaces don't touch. Being non-contact, air bearings avoid the traditional bearing-related problems of friction, wear. The design of the air bearing is such that, although the air constantly escapes from the bearing gap, the pressure between the faces of the bearing is enough to support the working loads.

Reynolds equation derived from the Navier-Stokes equations and the continuity of the hydrodynamic fluid thin film could be used to model the air bearing. The Navier-Stokes equations are the equations of the motion for a viscous fluid. These equations establish a relation between the geometry of the surfaces, relative sliding velocity, the property of the fluid and the magnitude of the normal load the bearing can support. The derivation of the Reynolds equation is referred to appendix F. From the point of view of

physical meaning, the Reynolds equation describes a combination of two types of fluid flow: Poiseuille flow and Couette flow. The Poiseuille flow terms describe the flow due to pressure gradients within the lubricated area, while the Couette flow term describes the shear flow due to the relative motion of the two solid boundaries.

Applying it to the Head-Disk Interface (HDI), the non-dimensional Reynolds equation can be modified as the following

$$\frac{\partial}{\partial X} \left(PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = \sigma \frac{\partial(PH)}{\partial T} \quad (2-1)$$

where, $X = x / L$, $Y = y / L$, $H = h / h_m$, $P = p / p_a$, and $T = \omega t$, P , H , X and Y are the non-dimensionalized pressure, flying height, coordinate in the slider length direction and coordinate in the slider width direction, respectively, p_a is the ambient atmospheric pressure, h_m is the reference clearance at the trailing edge center, and L is the length of the slider. $\Lambda_x = (6\mu U_x L) / (p_a h_m^2)$ and $\Lambda_y = (6\mu U_y L) / (p_a h_m^2)$ are called bearing numbers in the x and y directions, respectively; $\sigma = (12\mu \omega L^2) / (p_a h_m^2)$ is the squeeze number.

Equation (2-1) is derived by assuming the gas to be a continuous medium with non-slip at the solid boundary. This equation yields accurate results when K_n is less than 0.001, which corresponds to the continuum flow regime. The Knudsen number which is the ratio of the mean free path of gas molecules γ to the available range of movement h is used to measure the degree of gas rarefaction at thin film thicknesses ($K_n = \gamma/h$). The compressible hydrodynamic lubrication of gas bearings is characterized by extremely small film thicknesses which can restrict the free movement of individual gas

molecules. The mean free path of molecules which can be reduced with higher operating pressures is $0.064\mu\text{m}$ in air at atmospheric pressure. When the Knudsen number is in the range between $0.01 < K_n < 0.1$ in the atmospheric air, a thin layer of gas close to the solid surface loses its fluid characteristic and is capable of ‘slipping’ against the surface. This means that the effective viscosity of the gas declines with its proximity to the surface. When the film thickness becomes comparable to this value, the gas loses its continuity and be subjected to rarefaction [39].

For current HDIs, the flying height is down to 8 nm or even less and the gas may no longer be a continuum which can be described by the macroscopic equations of fluid dynamics with a group of macroscopic variables such as density, velocity and pressure. The best formulation available for the air bearing problem in the HDI is the Boltzmann equation, which is obtained from a more fundamental and microscopic point of view. The macroscopic equations for fluids, such as the continuous equation, the momentum and energy equations are simply a special case of the Boltzmann equation. However, the solution to the Boltzmann equation is, in general, very difficult to obtain even for the physically simple situations. In order to solve the air bearing problem still using a Reynolds-type lubrication equation and incorporating the rarefaction effect, several authors introduced the generalized Reynolds equation with different slip correction models.

If the mean free path of the molecules becomes comparable to the film thickness, slipping between the gas and the walls takes place, which produces an effect similar to reduce the viscosity. Another effect is that the heat condition is characterized by a discontinuity in temperature between the solid boundary and the gas. The

magnitude of the slip velocity and of the temperature discontinuity can be evaluated from the kinetic theory of gases.

$$u_{slip} = \sigma \frac{(2-f)}{f} \lambda \frac{\partial u}{\partial z} + \dots \quad (2-2)$$

$$\lambda = \frac{1}{\sqrt{2}ns} \quad (2-3)$$

where σ is a numerical constant, f is the surface accommodation coefficient, u is the flow velocity of lubricant in x direction, λ is the molecular mean free path, n is the number of molecules per unit volume, and s is the mutual collision cross section. The velocity is estimated by expanding it in a Taylor series about the wall position and retaining the zeroth and first order terms. Higher order terms are not considered. Using this slip modification combining with the general Reynolds equation, Burgdorfer first-order model can be obtained as following

$$\frac{\partial}{\partial X}(Q_p PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH) + \frac{\partial}{\partial Y}(Q_p PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH) = \sigma \frac{\partial PH}{\partial T} \quad (2-4)$$

where $Q_p = 1 + (6K_n)/(PH)$. Q_p is the Poiseuille flow rate coefficient, which reflects the type of slip correction used.

It should be noted as the Knudsen number approaches unity, the second-order slip effects should not be neglected. The third or the higher order slip effects do not occur since the third or higher derivatives of the velocity are identically equal to zero. The second order slip effects are approximated including the surface accommodation coefficient. The main effect of the second order slip term is to increase the slip effects

by permitting more side leakage flow resulting in a decrease in the load carrying capacity for a fixed clearance height. Using this slip modification combining with the general Reynolds equation, Hsia modified the first-order model to a second-order Model by setting $Q_p = 1 + (6K_n)/(PH) + 6[(K_n)/(PH)]^2$ [40].

To increase the accuracy of the slip-flow model, the second-order modified Reynolds equation was derived using slip-flow boundary conditions for both shear and pressure flows, while the first-order modified Reynolds equation was derived using only those for the shear flow. The second-order modified Reynolds equation, however, involves two principal problems. First, since the slip-flow mode for the pressure flow was deduced through analogy to that for a shear flow, the physical background has remained uncertain. Second, accuracy was not necessarily enhanced contrary to the aim of the higher-order slip-flow model. Mitsuya in 1993 introduced the 1.5-order slip model in order to predict the load capacity more accurately from the physical considerations that taking account of the accommodation coefficient into account [41]. This model features two key differences compared with the second-order model. One is that accommodation coefficient is involved and the other is that the second-order coefficient is 4/9 times smaller. A molecule colliding with the surface has a momentum given at the last collision, which occurred away from the surface at a distance of λ . He suggested that λ should be replaced by $2\lambda/3$. The correction coefficient for the 1.5-order slip model can be expressed as following

$$Q_p = 1 + \frac{6aK_n}{PH} + \frac{8}{3} \left(\frac{K_n}{PH} \right)^2 \quad (2-5)$$

where $a = (2 - \alpha)/(\alpha)$. Generally, surface accommodation α coefficient equal to 0.89.

These comparisons confirm that the present 1.5-order modified Reynolds equation provides intermediate characteristics between those derived from the first and the second order slip flow models and produces an approximation closer to the exact solution resulting from the Boltzmann-Reynolds equation.

As both equations in above introduction were originally derived by considering the slip flow boundary condition for $K_n \ll 1$, it is not clear whether they apply to a high Knudsen number. Even if the equation leads to numerical results coinciding with experimental results, it may not be justified physically. To accurately examine the applicability of the modified Reynolds equation to Ultra-low clearances, it has become essential to establish a generalized gas film lubrication equation for arbitrary Knudsen number. Gans, who first treated the linearized Boltzmann equation as a basic equation, derived the approximation lubrication equation analytical using a successive approximation method [42]. He derived the linearized version of the Boltzmann equation using the Bhatnagar-Gross-Krook (BGK) model and he solved by taking two moments of this equation and using successive approximations. As a result, a lubrication equation was derived, which is similar to Burgdorfer's first order modified Reynolds equation. Therefore, Gans claimed that this equation should be valid for all Knudsen number.

Fukui and kaneko started from the linearized Boltzmann equation based on the BGK model. The basic equation is decomposed so as to describe the fundamental flows that depend on the gradients of temperature, pressure and velocity. By considering mass conservation, these flows are incorporated later into a generalized Reynolds equation [43]. The generalized Reynolds equation based on the Boltzmann equation can be expressed as following

$$\begin{aligned} & \frac{\partial}{\partial X} \left\{ \left[\bar{Q}_p(D_0 PH) \frac{\partial P}{\partial X} - \bar{Q}_r(D_0 PH) P \frac{\partial \tau_w}{\partial X} \right] PH^3 - \Lambda_x PH \right\} \\ & + \frac{\partial}{\partial Y} \left\{ \left[\bar{Q}_p(D_0 PH) \frac{\partial P}{\partial Y} - \bar{Q}_r(D_0 PH) P \frac{\partial \tau_w}{\partial Y} \right] PH^3 - \Lambda_y PH \right\} = 0 \end{aligned} \quad (2-6)$$

where Λ and τ_w are the bearing number and the non-dimensional boundary temperature (T_w/T_0-1). To overcome the difficulties in solving the BGK model, Fukui and Kaneko introduced the use of a Poiseuille flow database to allow a quicker computation of a generalized lubrication equation for high K_n number gas bearing.

In all above models, the concept of the molecular mean free path is very important. It is the key coefficient to describe the microscopic behavior of gas molecules in physics and to define another important parameter, Knudsen number. The molecular mean free path is defined as the average distance that a molecule travels between two collisions of molecules. However, to increase the magnetic storage areal density to the range of 1Tbit/in², a flying height around 3 nm is believed to be necessary [44].

2.3.3 Suspension Modeling

2.3.3.1 Structure of a Head-gimbal Assembly

Fig. 2.6 Schematic of a Head-gimbal Assembly

A head-gimbal assembly of a HDD as shown in Fig. 2.6 includes a loadbeam connected to a pivot arm, a slider on which a magnetic head is mounted, an elastic gimbal having one end coupled to the loadbeam and the other free end portion at which the slider is supported. The gimbal allows a certain degree of head slider/disk surface compliance. Dimple is a convex dome feature formed in gimbal or load beam that transmits the load to the slider. Suspension is the assembly made up of the load beam and the gimbal.

How to model the suspension properly is crucial for simulating the L/UL processes. Although the air bearing models are unanimous, the suspension models are diverse.

2.3.3.2 Comprehensive Finite Element Models

Jeong and Bogy use a comprehensive finite element model to simulate the motion of a suspension [45]. Although being included in the model of the suspension, the effects from different parameters cannot be easily identified. It is inconvenient to be adopted in the design process and time-consuming to be calculated.

In order to simplify the whole process of the analysis and clarify the crucial parameter effects, lumped parameter models and simplified finite element models are used to analyze the dynamics during the load/unload process.

2.3.3.3 Simplified Finite Element Models

Stefan Weissner et al. presented a load beam/gimbal model that is based on C1 continuous finite beam elements as shown in Fig.2.7.[46] Not only the unknown variable but also its first derivative is continuous across element boundaries in the model in which the unknown variables are the deflections and the first derivatives are the slopes. There is an advantage the motion on the ramp can be easily modeled by changing the boundary condition at the lift tab end. The load beam/gimbal mass is automatically included. The dimple and the limiter can be easily modeled using rod

elements.

Fig. 2.7 FEM beam model

2.3.3.4 Lumped Parameter Models

Hu et al.'s modeled the suspensions in 1998 as three decoupled springs and dampers, and the L/UL tab-ramp interaction was modeled by the “de-gramming” rate. The effects of the suspensions are not sufficiently included in their simulations. In 1999, Zeng et al modeled the suspensions as three springs and dampers with varied parameters, and the three degrees-of-freedom (DOFs) were de-coupled by defining offsets x_d and y_d in the X and Y directions. The L/UL tab-ramp interaction was modeled by moving the upper ends of the springs and dampers. However, the effects of the slider's pitch change due to the movement of the L/UL tab are not included in this model. In 2000, Zeng et al proposed another simplified suspension model for the L/UL simulation. The model has four DOFs. The force applied by the ramp can be directly obtained from this model shown in Fig. 2.8. The effects of the pitch static attitude (PSA), the roll static attitude (RSA), and the initial disturbances on the loading process are considered [47].

Fig. 2.8 Zeng' four DOF model

In 2001, Wei Hua et al proposed a 9D model illustrated in Fig. 2.9 to uncover the L/UL details and further understand the L/UL process. The suspension is divided into two parts as their displacement can be 1000 times larger than the steady flying height of the slider, according to the suction force produced in the unload process and the stiffness of local suspension. Because each component has three degrees of freedom (vertical displacement, pitch and roll), there are totally nine degrees of freedom for the L/UL simulation in the system. In the 9D model, there are 6D addressing the suspension. Therefore, the effect of the suspension on the unloading performance can be evaluated.

Fig. 2.9 Wei's nine DOF model

In 2003, M. Honchi presented a suspension model which is described by five simple DOF model (slider vertical, pitch, roll, load beam vertical, and arm vertical direction) as shown in Fig. 2.10. The slider and the load beam are connected by flexure spring, dimple spring and limiter spring. At the steady state of the slider, the dimple spring is contacting the slider with load force plus preload. When the contact force of dimple spring is decreased to zero, the dimple spring separates from the slider. When the spacing between limiter and slider becomes zero, the limiter spring starts to work. The air bearing force and contact force are inputted as vertical force and moment to the equation [48].

Fig. 2.10 Honchi's five DOF model

2.4 LITERATURE REVIEW ON UNLOADING PROCESSES

The ramp is mounted at the edge of the disk. The unloading zone is used for load/unload and no data is stored on. A typical unloading process is illustrated as shown in Fig. 2.11.

Fig. 2.11 A typical unloading process

There have been a number of investigations of the effects of the various unloading parameters on the ramp L/UL process. Zeng and Bogoy designed sliders specifically for

L/UL applications to improve the unloading performance [49]. Bogy and Zeng [50] and Zeng and Bogy [51] investigated the effects of air-bearing surface design, suspension limiters, static slider attitudes, slider burnish, disk RPM, L/UL velocity, ramp profile, and dimple preload on L/UL performance. Jeong and Bogy studied L/UL systems experimentally to determine the conditions required to prevent slider-disk contact [52]. Suk and Gillis [53] experimentally found that an optimally burnished slider improves the reliability of the L/UL process. The liftoff force increase due to a high unloading velocity degrades the stability of the unloading process and increases the probability of slider-disk contact. Contact due to the rebound of the slider from the dimple also occurs during the unloading process. Such contacts can damage the disk surface and greatly affect unloading performance [54]. Y. Lee, et al., pointed out that an effective disk bump design for the unloading zone of the disk surface can improve the unloading performance [55]. Zeng and Bogy investigated the dynamics of negative pressure sliders during unloading processes by simulations and experiments. They concluded that the air bearing suction forces result in dimple separation during the unloading process and the quick release of the air bearing results in a high speed impact between the slider and the load beam. It is possible for the slider to hit the disk. Selecting a suitable RPM can decrease the lift-off force [56]. T. Hideaki, etc., experimentally studied the slider behavior during the unloading processes with a laser Doppler vibrometer. Three stages were observed. He also concluded that as the negative pressure force of the slider decreases, slider unloading occurs early [57]. J. R. Yaeger studied the unloading distance for different subambient style air-bearing designs with multiple unloading parameters. He figured out that footprint lengths and post-lift slider motion increase with increasing unload velocity for nominal PSA/RSA values, but slider motion decreases when the limiter engages. Footprint can be minimized by keeping

subambient pressure rearward, but limiter-free unloading is an unlikely event for all values of PSA/RSA due to manufacturing tolerances. A leading-edge limiter is required to strip the slider from the disk in these cases [58]. G. Sheng experimentally studied the dimple separation and head–disk impact of a negative pressure slider in unloading processes. The dimple impacts, which reflect the close of separated dimple in unloading processes, were observed in most of the tested cases by using AE measurements. The head–disk impacts were occasionally observed for HGA with gaped limiter under larger unloading speed by using electrical resistance measurements [59]. The investigation conducted by Wang, et al. [60] revealed that a relatively high unloading velocity could lead to a late air-bearing separation, even at the disk outer edge. They observed that the unloading distance increased with an increase of the lateral velocity and that the unloading time depended strongly on the suspension stiffness.

2.5 DRAWBACK OF EXISTING METHODOLOGIES AND SOLUTION

Types of suspension models and air bearing models have been used in HDD industry. The suspension models, from simplified lumped parameter models to comprehensive finite element models, address the structural mechanics of the suspension at different levels. The comprehensive finite element models, more precise than the simplified lumped parameter models, were adopted to address the overall performance of the structure. However these kinds of models will consume more computational time during the simulation. The lumped parameter models, which are less precise but also consume less computational time, can be used to target key points on the structure. The air bearing models are unanimously modified from Reynolds equation by addressing the slip effect in nano-scale thin film. Different strategies of the

corrections, from derivations to experiments, were adopted.

All models address the HDIs correctly in some sense. Conventionally, the existing methods simulate the L/UL behaviors of a slider by iteratively obtaining the instantaneous attitude of a slider through repeatedly solving a modified Reynolds equation which is coupled with the dynamics of the suspension. This approach requires a huge amount of computational time and power.

In the study, we prefer to develop an efficient methodology and construct our own models by modifying some existing suspension and air bearing models.

CHAPTER THREE

HEAD DISK INTERFACE AND SUSPENSION MODELING FOR UNLOADING PROCESSES

In this chapter, the structures of a typical suspension and a ramp load/unload mechanism were briefly introduced. A dual scale model for analyzing unloading processes was proposed and implemented by considering the incomparability of milli-scale deformation of the suspension and the nanometer scale variation of the air bearing gap in a head disk interface (HDI). The millimeter scale model for the suspension was presented in this chapter. The suspension was modeled as a 3-DOF lumped parameter dynamic system. Three stages in an unload process and transitional conditions were discussed based on the 3 DOF model. The nanometer scale model for the slider air bearing will be presented in chapter 5 and 6.

3.1 Mechanical Structure of HDA, HSA, HGA

A typical hard disk drive comprises a head disk assembly (HDA) and a printed circuit board assembly (PCB). The PCB controls the HDA functions and provides an interface between the disk drive and its host computer. A HDA includes at least one magnetic disk for data storage, a spindle motor for rotating the disk, and a head stack assembly (HSA) on which a slider is mounted for read/write element.

3.1.1 Head Stack Assembly (HSA)

The head stack assembly is controlled by a servo system to position the read/write

elements precisely on particular tracks on the disk. A typical HSA as shown in Fig. 3.1 consists of an actuator assembly controlled by servo control command, a head gimbal assembly (HGA) extending from the actuator assembly and forcing the slider toward the disk, and a flexible cable assembly. The actuator arms are made of solid steel or aluminum of significant thickness unsuitable for holding the read-write heads over the disk surface. An extended arm, known as the suspension, carries the head slider. The suspension is made of thin sheet of stainless steel and is attached to the actuator arm. Compared with the suspension, the actuator arm is stiff.

Fig. 3.1 A typical structure of head stack assembly (HSA)

3.1.2 Head Gimbal Assembly (HGA)

In an operating disk drive, the slider is loaded and floats over a rapidly spinning uneven disk. The suspension provides a force on the slider in the direction into the disk to keep the read-write sensor close to the surface of the disk. The downward suspension force is counteracted by the upward aerodynamic forces of the air bearing surface. These

forces must precisely balance each other in the proper magnitude and location; otherwise twisting force acting on the slider will cause unwanted slant flying attitude and unbalanced force may lead to instability of the slider. Besides providing this downward force, the suspension must allow the compliance of the slider to fit for the uneven disk surface. Namely, the suspension should accommodate the rotation of the slider in the pitch and roll directions so that it can stay close to the surface despite the presence of asperities on the disk surface. Any additional torque on the slider applied by the suspension may obstruct the adjustment of the slider. From this standpoint, a good suspension design should provide enough compliance that is achieved through low rotational stiffness design. Then the magnitude of the torque caused by slight deviation of static attitude from the nominal is minimized. Although a low rotational stiffness design is required to prevent the slider crash, the servomechanism asks for a high stiffness design in the horizontal direction. Therefore, it can swing back and forth rapidly during track seeking, settling, and following without producing excessive vibration. The servo control system cannot swiftly and accurately position the head on the right track if the stiffness between the sensing point and the actuating point is too low. To address this difficulty, suspension arms are usually designed with two separate components, a gimbal and a load beam. The gimbal is made of thin material to give low pitch and roll stiffness and the load beam is made of relatively thick material to provide high stiffness in the other directions that are required for the servo. The slider is adhesively bonded to the gimbal which is laser welded to the load beam. A typical structure of head gimbal assembly is shown in Fig. 3.2.

Fig. 3.2 A typical structure of head gimbal assembly (HGA)

Commercially, HGAs can be provided in various ways, including a two-piece design, a three-piece design, and a four-piece design. The four piece design, the concern of the study, comprises a load beam, a gimbal, an actuator arm, and a hinge piece. The load beam, which is a relatively thick trapezoid piece, ends with a pre-bent hinge. To increase the stiffness of the load beam, its edges are often bent to form a half-I shape beam. The hinge provides the suspension a spring force to create the gram load to position the head slider at a desired fly height and allow the load beam waggle vertically. The vertical movement of the suspension is necessary to accommodate unevenness of the disk surface and flutter of disk. Whenever the slider encounters a slowly increasing bump or contour on the disk surface, the aerodynamic force pushes the slider to follow the uneven of the disk. The narrow scope of vertical movement of the suspension accommodates these motions.

A typical gimbal welded to the load beam consists of a resilient tongue and two resilient arms as shown in Fig. 3.9 (a). The slider is attached to the tongue, which is in turn connected to the load beam by the arms. The gram load provided by the bend of the hinge is transferred to the gimbal via a downward pointing protrusion called a dimple that is located between the rigid region of the load beam and the gimbal. The gimbal arms allow the resilient tongue with the flying slider to rotate in pitch and roll directions to accommodate surface variations in the spinning magnetic disk. The gimbal arms also

force a return of the slider to a nominal position when air bearing force is absent. The roll axis is a central longitudinal axis of the head suspension. The pitch axis is perpendicular to the roll axis. That means the pitch axis is transverse to the longitudinal axis of the load beam and crosses the roll axis at or around the head slider.

Since the slider is attached to the load beam by a relatively soft gimbal, the pitch static attitude (PSA) and the roll static attitude (RSA) are formed naturally. PSA is the pitch angle whom the slider is naturally positioned by load beam-gimbal assembly without being constrained by the air bearing force. During flying, an additional pitch moment on the slider caused by the variation of the pitch angle has to be balanced by the air bearing pressure. Thus, the PSA affects the steady state flying attitude of the slider significantly. During the loading process when the slider approaches the disk, the PSA strongly affects the history of the slider attitude. A positive PSA help the slider builds up positive pitch angle as the air bearing starts, while a negative PSA causes a negative pitch angle. A negative pitch angle during loading results in a negative net air bearing force that cannot decelerate the downward motion of the slider which may lead head and disk damage. In current hard disk drive industry, the pitch static attitude (PSA) and the roll static attitude (RSA) affect the flying stability of the slider critically. The ability of the magnetic head to read from and write to the magnetic disk will be affected if the flying is not stable. Large variations in the head flying ability could lead to an undesirable disk crash. This leads to head damage, further negatively affecting disk drive reliability. To enable the slider proximately flying above the disk places in a stable manner, stringent requirements on the mechanical geometric design of the suspension, such as providing a proper range of its vertical stiffness, gimbal pitch and roll stiffness, gimbal pitch/roll static attitude

(PSA/RSA), operational shock performance (G/gram) and the like are raised.

A lift-tab used for L/UL processes extends from the end of the load beam as shown in Fig. 3.9 (a), which is set next to the end of load beam upside or downside or in the plane of the end of the load beam. The contact surface of the lift-tab is spherical to reduce the frictional force between the lift-tab and the ramp. Besides, a lower stiffness of the gimbal will lead a longer L/UL time, which requires more space for L/UL zone. Limiters used to help lift the subambient slider are typically used in L/UL mechanism to limit the separation between gimbal and dimple that may occur during the unloading process. The limiters are small hooks attached to the resilient tongue that engages with the main suspension in case of a too large separation between gimbal and dimple. Limiters can be located at proximal or distal of the gimbal, or at any location between. While the limiters provide valuable protection against damage, they can also restrict or prevent an enough range of movement for pitch and roll adjustment of the head suspension, because of the limit on movement allowed by the limiters [61-66].

3.2 Ramp Load/unload Mechanism

Ramp mechanism with a ramp at the out edge of the disk is one convenient and popular design using a rotary actuator to move the suspension along a ramp to L/UL a slider. This design also makes shock-induced slider-disk impact unlikely by keeping the slider away from the disk during off period. Ramp L/UL systems can be designed with several different structures as end-lift and mid-lift designs. End-lift designs have a lift-tab extension on the end of the load beam while mid-lift designs have a feature on the load beam between the suspension hinge and the head. End-lift designs facilitate the installation

of the actuator during drive assembly by allowing the use of backside merge ramps on the ramp structure. However, the extra mass, even a small mass, added at the end of the load beam creates a measurable increase in actuator inertia and lowers the resonant frequencies of suspension modes. End-lifts can be designed as those with centered and offset lift features. A centerline lift-tab design is simplicity, low cost and generally with a lower mass, and deflect least under load. In order to lift the head at the proper position, the ramp used in the centerline end- lift design must overhang its associated disk surface. However, debris produced during the ramp-lift-tab contact may spread on the surface of the disk and damage the surface. A typical centerline end-lift ramp load/unload mechanism is shown in Fig. 3.3. In our work, we mainly concentrate on this type of design. The instant positions of the actuator in ramp load/unload mechanism are shown in Fig. 3.4 [67-69].

Fig. 3.3 A typical Centerline End- lift Ramp Load/Unload Mechanism

(a) the slider flies over the disk (b) load/unload processes (c) lift-tab rests on the ramp

Fig. 3.4 Instant Position of the Actuator in Ramp Load/Unload Mechanism

During the unloading process, the deformation of the HGA caused by the raised lift tab affects the deflection of the hinge apparently. This leads the changes of the vertical/angular equilibrium position of the load beam and the slider to be considerable as shown in Fig. 3.5. The gimbal, which is a soft component, also plays a key role in the deformation of the HGA during the L/UL processes. The deformation of the actuator arm can be ignored due to its high stiffness compared to other soft parts of the HSA. Although the load beam is strong compared to the hinge and the gimbal, its local deformation will affect the L/UL processes significantly and cannot be neglected. During a typical unloading process, suspension exhibits different states depending on dimple or limiter engagement.

3.3 Dual-Scale Model for Unloading Process Studies

In current hard disk drives, the L/UL zone used for the load/unload processes is not used for data recording due to potential data loss caused by contacts and wears between the slider and the disk. How to use the L/UL zone for data storage by optimizing the load/unload behavior of a slider and avoiding head-disk contacts becomes an efficient way to increase the total recording capacity. Since this project mainly concerns developing an efficient method to study how to unload a slider swiftly and smoothly, efficient models are needed to simulate the unloading processes. As mentioned before, the behavior of the slider is obtained by coupling fluid dynamics of the air bearings and structural dynamics of the suspensions. Although modified Reynolds equations are commonly accepted to describe the air bearing, the suspension is modeled with diverse methods in current HDD industrial.

The behavior of the slider cannot be fully determined before it is attached to the suspension. A method efficiently appraising the slider design, the suspension design and the head-gimbal assembly is desirable to shorten the design circle. If we can decouple the air

bearing analysis and the suspension analysis, the computational time required for solving the Reynolds equation can be reduced significantly.

Normally, a dynamic load which changes with time can have a significantly larger effect than a static load of the same magnitude due to the structure's inability to respond quickly to the loading. If it changes slowly, the structure's response may be determined with static analysis. Normally, a modern negative pressure slider can generate very stiff air bearings with resonance frequencies in the range from 80 kHz to 180 kHz for 30% sliders [70, 71] while a normal unloading process may take several milli-second. Quasi-static analyses could be used to simplify the study of the dynamics of a suspension and slider.

Since the unloading process is affected by many factors, further simplification is necessary. As the air bearing flying height (a few nanometer levels) and the deformation of suspensions (micro- or milli-meter level) run at two different scales, a dual-scale model is proposed. General structural relationship between the forces/moments and the translation/angular displacements can be expressed by the following equation.

$$\Delta F = \sum_{i=1}^n k_i \cdot \Delta z_i \quad (3-1)$$

where k_i denotes the stiffnesses and Δz denotes the displacements.

Fig. 3.5 Variation of the vertical/angular equilibrium positions for suspension

For simplification, a three-independent-variable system, i.e. the position and the attitude of the slider, and the position of the lift tab, is considered and expressed as the following equation.

$$\Delta F = k_r \cdot \Delta z_r + k_s \cdot \Delta z_s + k_\theta \cdot \Delta \theta \quad (3-2)$$

where k_r , k_s and k_θ denote the translational/rotational stiffness coefficients of a suspension. Δz_r denotes the displacement of the lift tab at the ramp-lift tab touch point (for short, the displacement of the lift tab in following chapters), which changes in the micro- or milli-meter scale. Δz_s denotes the displacement of the center of the slider (for short, the displacement of the slider in following chapters). $\Delta \theta$ represents the attitude change of the slider in pitch direction. Δz_s changes in nano-meter scale. $\Delta \theta$ changes in micro-radian scale. Because k_r , k_s and k_θ (stiffnesses of the suspension) are in the comparable order of magnitude while Δz_r is much larger than the order of Δz_s and $\Delta \theta$, Δz_s and $\Delta \theta$ have little effects on the total force and can be neglected. Thus eq. (3-2) can be reduced to the following

$$\Delta F \approx k_r \cdot \Delta z_r \quad (3-3)$$

The nano-scale air bearings are designed with strong stiffnesses to enhance the slider's resistance to external disturbances in normal operational condition, whereas the suspensions are designed with enough compliance to follow the long-wave vibration. Since the incomparability of the stiffnesses between these two components, the nano-scale minimum flying height affect the milli- or micro- scale deformation of the suspension slightly. The suspension displacement can be 1000 times larger than the steady flying height of slider. From the perspective of the suspension, the change of the slider attitude before air bearing separation can be neglected. Therefore, when we study the suspension,

we can assume the slider is just held by displacement constraints. Combining with the quasi-static analysis, a dual scale model, which studies the suspension (called global model) and the air bearing (called nano-scale model) separately, is enabled.

During L/UL, the lift force and moment on the slider come from the deformation of the suspension. Slider adjusts its attitude according to the lift force and moment. The gap of the air bearing and the angular change of the slider are very small compared with the deformation of the suspension. From the viewpoint of the micro- or milli-meter scale model, the effect from the air bearing is represented by displacement constraints while the gap and the angular change are neglected. Thus, it is assumed that the slider makes completely contact with the disk surface before the instant of air bearing separation as viewed from the micro- or milli-meter scale. The instantaneous forces and the moments acting on the slider at this stage can be determined from the deformations of the suspension. Subsequently, the forces and the moments are transferred to the nano-scale part and used to determine the flying attitudes of the slider. From the viewpoint of the nano-scale model, the relationship between the forces/moments and the flying heights/pitch angles is obtained by solving the modified Reynolds equation. Having known the flying attitudes of slider, the maximum suction force can be calculated. Thus the air bearing separation time can be determined by transferring the maximum suction force back to the global model. With this approximation, the global model (used for studying the suspension) and the nano-scale model (used for studying the air bearing) can be studied separately in most calculation, leading to appreciable simplification of the dynamic problem. After air bearing separation, the slider is pulled off from the disk and begins to vibrate about its equilibrium position. The whole idea and simulation steps can be described by as Fig. 3.6 [72].

(a) dual scale model before air separation

(b) model after air separation

Fig. 3.6 Milli-scale model of suspension and Nano-scale model of air bearing

3.4 ANALYSIS OF UNLOADING PROCESSES

An unloading time is defined as the time elapsed between the instant when the lift-tab touches the ramp and the instant when the slider separates from the disk, where the first instant is defined as time zero. The unloading distance shown in Fig. 3.7 is defined as the lateral span that the slider runs during the unloading time. It is desirable to enable the head-disk separation quickly after the lift-tab engages with the ramp while the slider is still flying on the disk. The worst situation is the slider cannot be pulled off from the disk after the lift-tab reaches the plateau region of the ramp. At this region, the lift-tab actually leaves off the slope of the ramp and reaches its acme. Since the lift-tab cannot be raised anymore, no extra force leading air bearing separation will be introduced onto the slider. For such a case, the slider travels to the outer edge of the disk to break down the air bearing function.

The situation will be more complicated and out of the scope of the current researches [73-75].

Fig. 3.7 Definition of unloading distance

Normally, the HGA experiences four stages during unloading process. In the normal operational state as shown in Fig. 3.8, the slider connected to the suspension through a gimbal turns around the dimple and flies over the disk. The whole suspension actually swings vertically at the end of the actuator arm, while the hinge provides enough flexibility for the movement. At this state, the suspension assembly with a subambient pressure slider is supported only by the air bearing force and actuator arm. The lift-tab is left free at the end of the load beam. The stiffness of the suspension at this state is relatively low compared with that in the load/unload process because in the load/unload process the suspension beam is supported on both ends.

Fig. 3.8 Normal operational condition

Fig. 3.9 Stages for unloading process

During unloading, as the slider is driven outward to the ramp with a lateral unloading velocity, the lift-tab of the suspension first touches the slope of the ramp. Because the preload force or the slider suction force exists, the slider with subambient pressure design stays in the proximity of the disk. It will not move immediately and apparently as the lift-tab is raised by the slope of the ramp. The positive air bearing force still exists and the slider stays in the proximity of the disk. However, since the forces on the suspension assembly have been changed, the deformation of the assembly is redistributed. This stage is named as dimple engaged shown in Fig. 3.9 (a). When the load beam continues to be raised, the preload force on the slider is gradually taken over by the lift-tab until no air bearing force on the slider. As the lift-tab moves up further on the ramp, the suspension preload on the slider is reduced from a press force to a lift force because the preload force is totally shared by the lift-tab. However, the slider is still engaged with the dimple since the preload force in the load beam-flexure assembly also should be overcome. As this force is released gradually by the further deformation of the suspension, the contact between gimbal and dimple fades out and disappears at last. After that only the gimbal takes effect to balance the air bearing suction force, resulting in a further increase of the lift tab force, but with a

much smaller slope. This stage called dimple separated is shown in Fig. 3.9 (b). As the clearance between the slider and the load beam increases, the spring force from the gimbal increases. If the gimbal force is large enough to pull the slider off from the surface of the disk at this stage, the limiter engagement will not happen, and the unloading process is terminated. If the gimbal itself cannot produce enough force to lift the slider away from the surface of the disk, the limiter engagement with the stiff load beam called limiter engaged shown in Fig. 3.9 (c), may occur. A displacement limiter is designed to avoid excessive extension of the gimbal. During this stage, the air bearing force remains negative to suck the slider. The suspension spring force continues to grow in magnitude with a steep slope until the lift force caused by the deformation of the suspension overcomes the air bearing suction force. Then air bearing breaks up, as indicated by an abrupt return of the air bearing force to zero demonstrated by Fig. 3.9 (d). Then the deformed flexure forces the reengagement of the dimple and the slider, leading the bounces between the slider and the load beam. The bouncing slider can either collide with the disk or vibrate freely. The suspension assembly shows a different effective vertical stiffness and rotational stiffness in the whole process. In the last stage, the slider strongly vibrates due to the combination effects of the slider inertia and suspension forces.

3.5 THREE DOF GLOBAL MATRIX MODEL FOR SUSPENSIONS

Head gimbal assembly (HGA) is fixed on a pivot with a nut. In the normal operational condition, the lift tab is left free and the air bearing force which can be equivalently concentrated as a force F_s and two moments, M_β and M_θ , distributes on the slider's surface. In L/UL processes, an additional force F_r from the ramp acts on the lift tab. The external forces and moments on the suspension during the L/UL processes can be illustrated by Fig.

3.10.

Fig. 3.10 External forces and moments on a suspension

Its 3-DOF lumped parameter model addressing the head disk interface (HDI) can be illustrated by Fig. 3.11.

Fig. 3.11 3-DOF global lumped parameter model for a suspension

It is difficult to use analytical structural analysis to study a suspension due to its non-uniformity of the cross sections. Most analytical methods were developed to analyze particular and simple types of structures and intended for manual calculations. They often involve certain assumptions to reduce the computational effort required for the analysis. In contrast to analytical methods, matrix methods were specifically developed for computer implementation.

The finite element method and the lumped parameters matrix model are adopted in the study. Although the finite element model is more comprehensive and precise than the lumped parameter model, the dynamic simulation of the model is time-consuming. Lumped parameter model is a simplified mathematical model of a physical system. In the analysis

of the load/unload process, we prefer to use both of them. Efficient and fairly accurate simulations can be performed using the lumped model together with its realistic and accurate parameters fed by the finite element model and checked with experiments.

Since one end of the suspension is fixed to the pivot as shown in Fig. 3.10, $\Delta z_{ax} = 0$, $\Delta z_{ay} = 0$, $\Delta \alpha = 0$, and $\Delta \gamma = 0$. As the forces and moments on the pivot are not the focus in the study, the suspension can be structurally modeled as

$$\begin{Bmatrix} \Delta F_r \\ \Delta F_s \\ \Delta M_\theta \\ \Delta M_\beta \end{Bmatrix} = \begin{bmatrix} k_{rr} & k_{rs} & k_{r\theta} & k_{r\beta} \\ k_{sr} & k_{ss} & k_{s\theta} & k_{s\beta} \\ k_{\theta r} & k_{\theta s} & k_{\theta\theta} & k_{\theta\beta} \\ k_{\beta r} & k_{\beta s} & k_{\beta\theta} & k_{\beta\beta} \end{bmatrix} \begin{Bmatrix} \Delta z_r \\ \Delta z_s \\ \Delta \theta \\ \Delta \beta \end{Bmatrix} \quad (3-4)$$

There are two equivalent forces, F_r and F_s , and two equivalent moments, M_β and M_θ , applied on the HGA. ΔF_r is the change of the force applied on the lift tab in the vertical direction, and ΔF_s is the change of the force applied on the slider in the vertical direction. Moments ΔM_θ and ΔM_β is the change of the moments applied on the slider in the pitch and roll directions, respectively. Δz_r , Δz_s , $\Delta \theta$, and $\Delta \beta$ are variations of their corresponding translational displacement, pitch angle, and roll angle. All the variations are measured from the equilibrium position in normal operational condition.

In this study, the effect from the roll angle in eq. (3-4) is neglected for simplification (The reason is discussed in chapter 5). Thus, eq. (3-4) reduced to:

$$\begin{Bmatrix} \Delta F_r \\ \Delta F_s \\ \Delta M_\theta \end{Bmatrix} = \begin{bmatrix} k_{rr} & k_{rs} & k_{r\theta} \\ k_{sr} & k_{ss} & k_{s\theta} \\ k_{\theta r} & k_{\theta s} & k_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \Delta z_r \\ \Delta z_s \\ \Delta \theta \end{Bmatrix} \quad (3-5)$$

where, ΔF_θ and ΔM_θ denote the variations of the air bearing force and moment from the nanometer scale gap. ΔF_s and ΔM_θ denote the variation of their equivalent displacement

constraint force and moment in the dual scale model. Since the variations of Δz_s and $\Delta\theta$ are in the nanometer scale and the micro radian scale, respectively, they affect the performance of the suspension little and can be regarded as zero in the global model.

3.6 UNLOADING PROCESS ANALYSIS WITH 3-DOF MODEL

The suspension may experience three stages in unloading processes, namely dimple engaged, dimple separated, and limiter engaged. The conditions for stage transition were discussed with the help of the suspension model shown in Fig. 3.12.

(1) Nominal working condition

Since the lift tab was kept free in nominal working condition, there was no force acting on it. The air bearing force can be calculated with the deformation of the suspension expressed by the following equation.

$$F_s = k_d (L_d - L_a) - k_g (L_a - L_g) \quad (3-6)$$

where L_d and L_g are the free lengths of the dimple and the gimbal, respectively, L_a is the gap between the load beam and the slider, k_d and k_g are the stiffnesses of the dimple and the gimbal, respectively. Since the dimple is pressed by the gimbal after the components are assembled, L_a is smaller than L_d if no external force exists.

Fig. 3.12 schematic of the local 3-DOF lumped parameter model for a suspension

(2) Dimple engaged

When an unloading process commenced, a part of the preload force began to be gradually shared by the lift tab. Although L_a was increased at this point, (3-6) remains. As the load beam continued to be raised up, the lift-tab took over the HSA preload force until the suspension preload force equaled to the lift-off force and F_s reached zero. The strength force on the gimbal equaled to the compress force on the dimple as expressed in (3-7).

$$k_d (L_d - L_a) - k_g (L_a - L_g) = 0 \quad (3-7)$$

At this point, $L_d > L_a$ while $L_g < L_a$, namely the dimple still engaged with the slider.

(3) Dimple separated

When L_a were stretched to the value of L_d , namely $L_a = L_d$, the dimple separated from the slider. The instant air bearing force was worked out with (3-8).

$$F_s = -k_g (L_d - L_g) \quad (3-8)$$

After that, the gimbal alone took effect to balance the air bearing force which functioned as a suction force at the stage expressed with (3-9).

$$F_s = -k_g (L_a - L_g) \quad (3-9)$$

(4) Limiter engaged

When $L_b + L_a \geq L_L$, the limiter engaged with the local load beam. The limiter and the gimbal together took effect to balance the air bearing force expressed with (3-10).

$$F_s = -k_g (L_a - L_g) - k_L (L_b + L_a - L_L) \quad (3-10)$$

L_L are the free lengths of limiter, L_b is the thickness of the slider, k_L the stiffnesses of the limiter.

(5) Air bearing separation

When the suspension lift force on the slider was larger than maximum air bearing suction force F_{smax} , no extra air bearing force could be provided. The air bearing broke up, indicated by an abrupt return of the air bearing force to zero.

Although the free length of each component cannot be obtained easily, the equivalent length from the perspective of the lift tab can be measured using FEM model and experiments. The equivalent results are used in the following simulation.

CHAPTER FOUR

PARAMETER ESTIMATION USING FINITE ELEMENT MODEL AND EXPERIMENTS

In chapter 3, the structural dynamics of a suspension is modelled as a 3-DOF lumped parameters system with a stiffness matrix. A comprehensive finite element model, including all major components of a suspension, was constructed in this chapter to perform static analyses of the suspension structure and obtain the stiffness matrices as well as the equivalent lengths of the suspension at different unloading stages. Static analyses were performed to obtain the stiffness matrices and the equivalent lengths of the suspension in different unloading stages. Quasi-static analyses were performed to study the behaviors of the suspension in unloading processes. Stages in the unloading processes were discussed. Three experiments were designed and conducted to verify the equivalent lengths and a part of parameters in the stiffness matrix.

4.1 Comprehensive Finite Element Model for a Suspension

Commercial FEM software, ANSYS, is used to model a suspension shown as Fig. 4.1. ANSYS program does not assume a system of units for analyses. Considering the actual size of the HGA and for the consistency of the units to be used, gram, millimeter and microsecond, instead of meter, kilogram and second, which are adopted in SI, are adopted in the suspension FEM model. Thus the corresponding units for the force and the pressure in the model are Newton and MPa, respectively.

Fig. 4.1 Suspension FE Model

First, the components of a HGA, i.e. the hinge, the load beam, the gimbal and the dimple, are modeled geometrically. Then, the components are meshed with proper elements and coupled together with hard points. Considering the geometric structure of the HGA, the higher order element SOLID95 defined by 20 nodes is used to model the actuator arm, the dimple, the limiter, and the slider. It has plasticity, large deflection, and large strain capabilities and can tolerate irregular shapes without as much loss of accuracy. The SHELL63 element, which accommodates both bending and membrane capabilities and allows both in-plane and normal loads, is used to model the hinge, the gimbal and the load beam. Stress stiffening and large deflection capabilities are included.

The actuator arms are made of bulk steel with significant thickness and the extended suspension made of thin sheet of stainless steel. The slider is fabricated with titanium carbide. The Young's modulus of the Steel is $1.90 \times 10^5 \text{ N/mm}^2$, its density is 0.00785 g/mm^3 , and its Poisson ratio is 0.33. Since ANSYS takes no unit, 1.90e5, 0.00785 and 0.33 are input as the Young's modulus, density and Poisson ratio for steel, respectively. 4.39e5, 0.004938 and 0.18 are input as the Young's modulus, density and Poisson ratio for titanium carbide, respectively.

Since the dimple may contact with the gimbal tongue while the limiter may contact with the top surface of the load beam during the load/unload processes, the contact pairs paired via a shared real constant set are included to model the contact interface as shown in Fig. 4.2. Since the limiter and the load beam are all made from steel, flexible-to-flexible contact is chosen. The dimple which has a convex surface and the limiter which is modeled with a high-order solid element are modeled with 3D contact element TARGET170 as the target surfaces. The top surface of the load beam and the back of the gimbal tongue are modeled with 3-D Surface-to-Surface 8-node Contact element CONTA174 as the contact surfaces. To ensure the contact pairs work properly, it is paramount that the normals of the contact elements be oriented correctly for proper contact detection. If the element normals do not point toward the contact surface, select this element and reverse the direction of the surface normals by setting Surf to Surf property.

4.2 ESTIMATION OF THE SUSPENSION STIFFNESS MATRICES WITH FINITE ELEMENT MODEL

4.2.1 Static Analysis with FEM

The static analyses based on the FEM model can be used to obtain the desired stiffness matrices and the flexibility matrices. To obtain the stiffness matrices directly, nodal imposed (nonzero) displacements, instead of the steady loading forces, are applied on the four corners of the slider and the lift tab-ramp contact point on the ramp in the Z direction. Their corresponding nodal reaction forces are checked. The nodal imposed

displacements and the DOF constraint are set up carefully to mimic different states in unloading processes as shown in Fig. 4.3.

(a) limiter-load beam contact pair

(b) dimple-gimbal contact pair

Fig. 4.2 Contact Pairs of the FEM

(a) Boundary Conditions

(b) Dimple engaged

(c) Dimple separated

(d) Limiter engaged

Fig. 4.3 FEM with unloading Stages

4.2.2 Estimation of the Suspension Stiffness Matrices

From the previous chapters, the suspension can be simplified as an equivalent structure that can be expressed as:

$$\begin{Bmatrix} \Delta F_r \\ \Delta F_s \\ \Delta M_\theta \end{Bmatrix} = \begin{bmatrix} k_{rr} & k_{rs} & k_{r\theta} \\ k_{sr} & k_{ss} & k_{s\theta} \\ k_{\theta r} & k_{\theta s} & k_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \Delta z_r \\ \Delta z_s \\ \Delta \theta \end{Bmatrix} \quad (4-1)$$

To estimate the stiffness matrix using the suspension FEM model, an efficient procedure is to establish the relationship between the deformation of the suspension depicted by displacements of several key nodes and the corresponding forces acting on them. Moments are calculated with respect to the center of the slider. To mimic the various stages of the suspension unloading process precisely, displacement loadings (constraints) are applied on the slider to adjust its attitude and on the ramp to simulate different unloading position. Their corresponding reaction forces, which will be converted to equivalent forces and moment, can be obtained from the finite element analysis. It should be noticed that the stiffness matrix changes with the various stages of the suspension movement. In our case, four slider corner nodes, node 16652 and 16653 at the trailing edge and node 18909 and 18914 at the leading edge, and node 11032 at the central lift tab are applied the displacement loadings as shown in Fig. 4.4.

Fig. 4.4 Nodes for the displacement loadings/constraints

Since the roll angle has not been considered in the study, the displacements at node 18909 and 16652 is identical to those at node 18914 and 16653, respectively. For simplification, the geometric center of the slider is assumed to be the center of gravity. So the attitude of the slider can be expressed as the follows.

$$z_s = \frac{z_l + z_t}{2} \quad (4-2)$$

$$\theta = \arctan \frac{z_t - z_l}{L} \quad (4-3)$$

where z_l is the absolute translational displacement of the slider leading edge with respect to its free state. z_t is the absolute translational displacement of the slider trailing edge with respect to its free state. L is the length of the slider. z_s denotes the absolute translational displacement of the center of the mass with respect to its free state. θ denotes the absolute angular displacement of the slider in pitch direction with respect to its free state. z_r denotes the absolute translational displacement of a central point of the lift tab with respect to its free state.

Two approaches can be used to determine the stiffness matrices efficiently. One is solving equation with all independent variables Δz_r , Δz_s , and $\Delta \theta$ being changed simultaneously. The other is changing only one independent variable each time. The latter is used here.

For instance, in order to obtain k_{rr} , k_{sr} , $k_{\theta r}$, z_r is varied while z_s and θ are kept constant. Corresponding forces and moment for each configuration are calculated. The variation of those values can be used to determine the k_{rr} , k_{sr} , and $k_{\theta r}$ based on the following equation

$$\begin{Bmatrix} \Delta F_r \\ \Delta F_s \\ \Delta M_\theta \end{Bmatrix} = \begin{bmatrix} k_{rr} & k_{rs} & k_{r\theta} \\ k_{sr} & k_{ss} & k_{s\theta} \\ k_{\theta r} & k_{\theta s} & k_{\theta\theta} \end{bmatrix} \begin{Bmatrix} \Delta z_r \\ 0 \\ 0 \end{Bmatrix} \quad (4-4)$$

Similarly, all components of the stiffness matrices can be determined likewise.

As mentioned earlier, there are three stages in the slider unload processes. For each stage, the structure of the suspension system is changed to certain extent (dimple, limiter). Therefore, the stiffness matrix has to be determined separately for each stage following the above-mentioned procedure.

4.2.2.1 Normal Operational Condition

In the normal operational condition, since it does not touch the ramp, there is no external force, namely no displacement constraint on the lift tab. It oscillates freely at the end of the suspension. The four corners of the slider are applied displacement loading to ensure the normal operational state. The displacements are given with respect to the free state of the suspension. The displacement constraints and their corresponding forces are shown in Table 4.1.

Table 4.1. Configuration for normal operational condition

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.595
	Force(N)	3.27100×10^{-3}	3.17890×10^{-3}	3.41710×10^{-3}	3.35930×10^{-3}	0

Fig. 4.5 Configuration and stress contour for the normal operational condition

Absolute translational/angular displacements with respect to the Free State of the

suspension are: $z_r = 1.595(\text{mm})$, $z_s = 1.129 (\text{mm})$, $\Delta\theta = 0.175 (\text{radian})$.

Their corresponding forces/moment are:

$$F_r = 0 (\text{N}), \quad \Delta F_s = 1.323 \times 10^{-2} (\text{N}), \quad \Delta M_\theta = 2.013 \times 10^{-7} (\text{N} \cdot \text{m})$$

The preload force is 1.35 gram force and the preload moment is $2.013 \times 10^{-7} \text{ N} \cdot \text{m}$.

4.2.2.2 Dimple Engaged

Downward displacement loadings instead of upward displacement loadings are applied on the lift tab to ensure the dimple engages with the gimbal tongue steadily. The displacement constraints and their corresponding forces are shown in Table 4.2.

Table 4.2. Configuration with dimple engaged for k_{rr} , k_{sr} , $k_{\theta r}$ estimation

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.5
	Force(N)	6.32930×10^{-2}	6.16840×10^{-2}	6.17820×10^{-2}	6.10430×10^{-2}	-1.75140×10^{-2}
Setup 2	Disp(mm)	1.02	1.02	1.238	1.238	1.58
	Force(N)	1.27420×10^{-2}	1.24110×10^{-2}	1.26270×10^{-2}	1.24620×10^{-2}	-2.76290×10^{-2}

Fig. 4.6 Configuration with dimple engaged for k_{rr} , k_{sr} , $k_{\theta r}$ estimation

$$\Delta z_r = z_r^{(2)} - z_r^{(1)} \quad (4-5)$$

$$\Delta z_s = z_s^{(2)} - z_s^{(1)} \quad (4-6)$$

$$\Delta\theta = \theta^{(2)} - \theta^{(1)} \quad (4-7)$$

$$\Delta F_r = F_r^{(2)} - F_r^{(1)} \quad (4-8)$$

$$\Delta F_s = F_s^{(2)} - F_s^{(1)} \quad (4-9)$$

$$\Delta M_{\theta} = M_{\theta}^{(2)} - M_{\theta}^{(1)} \quad (4-10)$$

where, the superscripts denote the serial number of the setups. Δz_r is the relative translational displacement of the lift tab as shown in Fig. 3.12, Δz_s is the relative translational displacement of the center of the slider, $\Delta \theta$ is the relative angular displacement of the slider, ΔF_r is the relative variation of the force acting on the lift tab, ΔF_s is the relative variation of the total constraint force acting on the slider, ΔM_{θ} is the relative variation of the total constraint moment acting on the slider. Since $\Delta z_s = 0$ and $\Delta \theta = 0$,

$$k_{rr} = \frac{\Delta F_r}{\Delta z_r} \quad (4-11)$$

$$k_{rs} = \frac{\Delta F_s}{\Delta z_r} \quad (4-12)$$

$$k_{r\theta} = \frac{\Delta M_{\theta}}{\Delta z_r} \quad (4-13)$$

Relative translational/angular displacements of the suspension of the setups are:

$$\Delta z_r = -0.1(\text{mm}), \quad \Delta z_s = 0 \text{ (radian)}, \quad \Delta \theta = 0 \text{ (radian)}.$$

Their relative corresponding forces/moment are:

$$\Delta F_r = -0.1847 \text{ (N)}, \quad \Delta F_s = 0.24723 \text{ (N)}, \quad \Delta M_{\theta} = -1.6078\text{E-}06 \text{ (N}\cdot\text{m)}.$$

The stiffnesses are:

$$k_{rr} = 1.847 \times 10^3 \text{ (N/m)}, \quad k_{sr} = -2.47238 \times 10^3 \text{ (N/mm)}, \quad k_{\theta r} = 1.6078 \times 10^{-2} \text{ (N)}$$

4.2.2.3 Dimple Separated

Table 4.3. Configuration with dimple separated for k_{rr} , k_{sr} , $k_{\theta r}$ estimation

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp (mm)	1.02	1.02	1.238	1.238	1.61
	Force (N)	-4.09760×10^{-4}	-4.10000×10^{-4}	-1.41080×10^{-4}	-1.58280×10^{-4}	1.07610×10^{-2}
Setup2	Disp (mm)	1.02	1.02	1.238	1.238	1.66
	Force (N)	-6.16650×10^{-4}	-6.17830×10^{-4}	-2.28380×10^{-4}	-2.49190×10^{-4}	1.14950×10^{-2}

(a) setup 1

(b) setup 2

Fig. 4.7 Configuration with dimple separated for k_{rr} , k_{sr} , $k_{\theta r}$ estimation

Relative translational/angular displacements of the suspension are:

$$\Delta z_r = 0.05 \text{ (mm)}, \quad \Delta z_s = 0 \text{ (radian)}, \quad \Delta \theta = 0 \text{ (radian)}.$$

Their relative corresponding forces/moment are:

$$\Delta F_r = 7.341 \times 10^{-4} \text{ (N)}, \quad \Delta F_s = -5.929 \times 10^{-4} \text{ (N)}, \quad \Delta M_\theta = 1.458 \times 10^{-7} \text{ (N}\cdot\text{m)}.$$

The coupling stiffnesses are:

$$k_{rr} = 0.01468 \times 10^3 \text{ (N/m)}, \quad k_{sr} = -0.01186 \times 10^3 \text{ (N/m)}, \quad k_{\theta r} = 2.916 \times 10^{-3} \text{ (N)}$$

4.2.2.4 Limiter Engaged

Table 4.4. Configuration with limiter engaged for k_{rr} , k_{sr} , $k_{\theta r}$ estimation

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.9
	Force(N)	-3.49380×10^{-2}	-3.46050×10^{-2}	3.81390×10^{-3}	3.78840×10^{-3}	5.24950×10^{-2}
Setup 2	Disp(mm)	1.02	1.02	1.238	1.238	2.2
	Force(N)	-1.18650×10^{-1}	-1.16200×10^{-1}	1.45000×10^{-2}	1.40740×10^{-2}	1.50850×10^{-1}

 Fig. 4.8 Configuration with limiter engaged for k_{tr} , k_{sr} , $k_{\theta r}$ estimation

Relative translational/angular displacements of the suspension are:

$$\Delta z_r = 0.3 \text{ (mm)}, \quad \Delta z_s = 0 \text{ (radian)}, \quad \Delta \theta = 0 \text{ (radian)},$$

Their relative corresponding forces/moment are:

$$\Delta F_r = 9.836 \times 10^{-2} \text{ (N)}, \quad \Delta F_s = -1.443 \times 10^{-1} \text{ (N)}, \quad \Delta M_\theta = 1.148 \times 10^{-4} \text{ (N}\cdot\text{m)}$$

The coupling stiffnesses are:

$$k_{rr} = 0.3279 \times 10^3 \text{ (N/m)}, \quad k_{sr} = -0.4811 \times 10^3 \text{ (N/m)}, \quad k_{\theta r} = 3.828 \times 10^{-1} \text{ (N)}$$

4.2.2.5 Stiffness Matrices for Three Stages

Similarly, all the elements of the stiffness matrices can be obtained by setting non-zero Δz_s and $\Delta \theta$ separately. The stiffness matrices for three stages are expressed as eq. (4-14) to eq. (4-15). The detailed data are shown in appendix C.

The stiffness matrix $K^{(d)}$ for dimple engaged

$$K^{(d)} = \begin{bmatrix} 1.8470 \times 10^3 & -2.4760 \times 10^3 & 1.6078 \times 10^{-2} \\ -2.4724 \times 10^3 & 3.3276 \times 10^3 & -2.4228 \times 10^{-2} \\ 1.6728 \times 10^{-2} & -2.3345 \times 10^{-2} & 1.7317 \times 10^{-5} \end{bmatrix} (N, m, \text{radian}) \quad (4-14)$$

The stiffness matrix $K^{(g)}$ for dimple separated

$$K^{(g)} = \begin{bmatrix} 0.01468 \times 10^3 & -0.01190 \times 10^3 & 3.0041 \times 10^{-3} \\ -0.01186 \times 10^3 & 0.01694 \times 10^3 & -6.6635 \times 10^{-3} \\ 2.9162 \times 10^{-3} & -6.4951 \times 10^{-3} & 1.7169 \times 10^{-5} \end{bmatrix} (N, m, \text{radian}) \quad (4-15)$$

The stiffness matrix $K^{(L)}$ for limiter engaged

$$K^{(L)} = \begin{bmatrix} 0.3279 \times 10^3 & -0.4869 \times 10^3 & 3.6862 \times 10^{-1} \\ -0.4811 \times 10^3 & 0.7399 \times 10^3 & -5.6544 \times 10^{-1} \\ 3.8280 \times 10^{-1} & -5.8439 \times 10^{-1} & 4.7012 \times 10^{-1} \end{bmatrix} (N, m, radian) \quad (4-16)$$

4.3 UNLOADING PROCESS ANALYSIS WITH FINITE ELEMENT MODEL

4.3.1 Overview of the Unloading Process

As a complement to chapter 3, more details, such as the equivalent lengths, the histories of the forces and the moments on the suspension, and transitional conditions, were revealed through the unloading analyses with the FEM. As the gimbal and the load beam are spot-welded together, the gimbal tongue is pushed by the protrusive dimple and γ_1 and γ_2 are formed as shown in Fig. 4.9, where γ_1 is the angle formed between the load beam and the gimbal arm and γ_2 is the angle formed between the gimbal arm and the gimbal tongue.

Fig. 4.9 Structure of Suspension

During the normal operational state, γ_1 and γ_2 have very tiny changes since the attitude of the slider varies in micro radian scale. During the unloading process, as the lift tab is raised up while the slider is still sucked by the negative force, γ_1 and γ_2 increase apparently and monotonously. The force on the lift-tab also increases monotonically but with different slopes at different stages as shown in Fig. 4.10. The different slopes

indicate different unload stages. The equivalent free lengths of each component from the perspective of the lift tab can be estimated by considering the position of the turning points of the segmented lines.

Fig. 4.10 histories of the force on the lift tab in the unloading process

To balance the gradually increased upward lift force caused by the enlarged γ_1 and the gradually increased clockwise gimbal moment caused by the enlarged γ_2 , an increasing downward constraint (suction) force shown in Fig. 4.11 and an increasing anti-clockwise air bearing moment shown in Fig. 4.12 are required on the bottom surface of the slider.

(a) Net constraint force on the slider

(b) Constraint force at the leading edge

(c) Constraint force at the trailing edge

Fig. 4.11 histories of constraint force on the slider during the unloading process

(a) Net constraint moment on the slider

(b) moment caused by the leading forces

(c) moment caused by the trailing forces

Fig 4.12 Histories of constraint moment during the unloading process

Because of the mechanical design of the suspension and the design of the slider surface profile, the forces and the moment at the leading edge and trailing edge as shown in Fig. 4.11 (b), Fig. 4.11 (c), Fig. 4.12 (b), and Fig. 4.12 (c) exhibits totally different behaviors.

4.3.2 Dimple Engaged

During the unloading process, as the actuator arm is driven outward by the voice coil motor to the ramp with a lateral unloading velocity, the lift-tab first touches the slope of the ramp. The whole HGA is bending by the slope upward. Although the lift tab gradually

shares a part of hinge preload force, the subambient pressure slider is still pressed by residual hinge preload force and stays in the proximity of the disk. In this stage, although the positive air bearing forces at both the leading edge and the trailing edge are diminishing, they still dominate the air bearing and pushes the slider toward the disk. The dimple continues to contact with the gimbal tongue since its tip still bears non-zero force as shown in Fig 4.13 (a).

(a) constraint force

(b) constraint moment

Fig. 4.13 Histories of constraint force and moment in dimple engaged stage

It is also shown in Fig. 4.14 that the tip of the dimple almost keeps motionless before the air bearing separation, which means the dimple does not move immediately and

apparently with the rise of the lift tab. The air bearing moment keeps increasing as shown in Fig. 4.13 (b) in anticlockwise direction to balance the extra moment caused by the deformation of the gimbal. However, since the constraints on the suspension assembly have been changed, the deformation and the reaction force are redistributed.

Fig. 4.14 Histories of contact force on the tip of the dimple in dimple engaged stage

4.3.3 Dimple separating

As the load beam continues to be raised, the lift tab gradually takes over the hinge preload force. When the lift tab shares all the hinge preload force alone, the air bearing force is attenuated from positive to zero, and the slider is ready to be pulled away from the disk. However, the dimple still engages with the tongue of the gimbal at this point because of the preload from the gimbal-load beam assembly. As the lift force increases continuously, the forces at the leading area and at trailing area reverse to negative values subsequently, and the negative force gradually dominates the air bearing in all the area. The slider commences to be attracted by the suction force. When the gap between the load beam and the tongue of the gimbal is larger than the free length of the dimple

eventually, the stretching forces, namely the positive lift force and the negative suction force, on the HGA lead dimple separation and the force on the dimple disappears. The transitional process is illustrated in Fig. 4.15 (a). The air bearing moment as shown in Fig. 4.15 (b) keeps increasing in anticlockwise direction to balance the extra moment caused by the deformation of the gimbal.

Fig. 4.15 Histories of constraint force and moment in dimple separating stage

4.3.4 Dimple separated

After the dimple separation, only the gimbal takes effect to balance the air bearing

suction force as shown in Fig. 4.16 (a). At this stage, the air bearing suction force increases monotonically to balance the increscent lift force. Since only the relatively soft gimbal takes effect during this period, the forces change with gentle inclines as shown in Fig. 4.16 (a). The air bearing forces on the area redistribute to balance the extra moment caused by the further deformation of the gimbal.

Fig. 4.16 Histories of constraint force and moment in dimple separated stage

4.3.5 Limiter Engaging

A displacement limiter with a high stiffness is designed to avoid excessive extension of the gimbal. When the gap between the slider and the load beam is larger than the free

length of the limiter, the limiter engages with the top side of the load beam. It helps the gimbal lift the slider as shown in Fig. 4.17.

(a) constraint force

(b) constraint moment

Fig. 4.17 Histories of constraint force and moment in limiter engaging stage

As the histories of the constraint force and moment shown in Fig. 4.17 (a) and Fig. 4.17 (b), a larger suction force at the leading edge is required to balance the force because the limiter is mounted at the leading edge of the slider. The trailing edge will bear positive pressures again because of a large pitch angle.

4.3.6 Limiter Engaged

After the limiter engages with the load beam, the force and moment changes steadily as shown in Fig. 4.18. During this stage, the air bearing force remains negative to suck the slider and the suspension spring force continues to grow with a steep slope until the lift force caused by the deformation of the suspension overcomes the air bearing suction force. The air bearing breaks up, indicated by an abrupt return of the air bearing force to zero. Then the deformed flexure forces the reengagement of the dimple and the slider, leading the bounces between the slider and the load beam. The bouncing slider can either collide with the disk or vibrate freely. The suspension assembly shows a different effective vertical stiffness and rotational stiffness in the whole process.

Fig. 4.18 Histories of constraint forces and moment in limiter engaged stage

As the clearance between the gimbal and the dimple increases, the spring force from

the gimbal increases. If the gimbal force itself is large enough to pull the slider off from the surface of the disk at this stage, the limiter engagement will not happen, and the unloading process is terminated. If the gimbal itself cannot produce enough force to lift the slider away from the surface of the disk, the limiter engagement with the stiff load beam may occur.

4.3.7 Air Bearing Separation

When the suction force reaches its maximum point, the air bearing can no longer provide force to counteract the increasing lift force, leading the air bearing breaks up abruptly. Then the slider is dragged toward the load beam by the spring force, and dimple-gimbal engagement recurs. The slider vibrates strongly due to the combination effects of the slider inertia and suspension forces and may hit the disk. In this project, the unloading time and the behavior before air bearing separation are focused. So this stage is out of the discussion of the dissertation.

4.4 ESTIMATION OF SUSPENSION PARAMETERS WITH EXPERIMENTS

The equivalent lengths and a part of stiffness coefficients obtained through the finite element analyses were verified with those from experiments.

4.4.1 Design of the Experiments

To verify the stiffness matrices obtained from the FEM model, a set of experiments

are conducted. The ideal way to estimate the stiffness matrix experimentally is measuring the translational displacements, angular displacements, and their corresponding forces and moments directly. Since the moments are normally difficult to be measured experimentally, the elements in the stiffness matrices relating to angular displacements and moments are skipped by setting $\Delta\theta=0$ in the project, thus reducing eq. (4-4) to eq. (4-17).

$$\begin{Bmatrix} \Delta F_r \\ \Delta F_s \end{Bmatrix} = \begin{bmatrix} k_{rr} & k_{rs} \\ k_{sr} & k_{ss} \end{bmatrix} \begin{Bmatrix} \Delta z_r \\ \Delta z_s \end{Bmatrix} \quad (4-17)$$

The stiffness matrix in eq. (4-17) can be obtained by changing the configuration of Δz_r and Δz_s and measuring the resulting force ΔF_r and ΔF_s simultaneously using two cantilever beam sensor. However, it is relatively costly. It should be pointed out that the relative values instead of absolute values are used in the experiments. In the study, one cantilever beam sensor with three experiments is employed to figure all the elements out by counting on the help of the flexibility matrices expressed as

$$\begin{Bmatrix} \Delta z_r \\ \Delta z_s \end{Bmatrix} = \begin{bmatrix} \lambda_{rr} & \lambda_{rs} \\ \lambda_{sr} & \lambda_{ss} \end{bmatrix} \begin{Bmatrix} \Delta F_r \\ \Delta F_s \end{Bmatrix} \quad (4-18)$$

With one cantilever beam sensor alone, ΔF_r or ΔF_s can be measured depending on the configuration of the experiments. With the eq. (4-17), k_{rr} / k_{rs} or k_{sr} / k_{ss} pairs can be obtained. With the eq. (4-18), λ_{rr} and λ_{ss} can be obtained. Concretely, three experiments are designed for the purpose. In Experiment One, λ_{rr} is figured out by applying a force on the lift tab alone and measuring the reaction force. Since $\Delta F_s = 0$,

$$\lambda_{rr} = \frac{\Delta z_r}{\Delta F_r} \quad (4-19)$$

In Experiment Two, λ_{ss} is figured out by applying a force on the slider alone and

measuring the reaction force. Since $\Delta F_r = 0$,

$$\lambda_{ss} = \frac{\Delta z_s}{\Delta F_s} \quad (4-20)$$

In Experiment Three as shown in Fig. 4.24, the external force is applied on the slider alone with the help of a self-made fixture. By changing the configuration of Δz_r and Δz_s thrice and measuring the reaction force, k_{sr} and k_{ss} are figured out with .

$$\Delta F_s = k_{sr} \Delta z_r + k_{ss} \Delta z_s \quad (4-21)$$

Since

$$\begin{bmatrix} k_{rr} & k_{rs} \\ k_{sr} & k_{ss} \end{bmatrix} = \begin{bmatrix} \lambda_{rr} & \lambda_{rs} \\ \lambda_{sr} & \lambda_{ss} \end{bmatrix}^{-1} \quad (4-22)$$

We can obtain

$$\lambda_{rr} = \frac{k_{ss}}{k_{rr}k_{ss} - k_{sr}k_{rs}} \quad (4-23)$$

$$\lambda_{ss} = \frac{k_{rr}}{k_{rr}k_{ss} - k_{sr}k_{rs}} \quad (4-24)$$

Thus

$$k_{rr} = \frac{k_{ss} \lambda_{ss}}{\lambda_{rr}} \quad (4-25)$$

$$k_{rs} = \left(k_{rr}k_{ss} - \frac{k_{ss}}{\lambda_{rr}} \right) / k_{sr} \quad (4-26)$$

Until now, all elements in the stiffness matrix and flexibility matrix are obtained. Because of the limitation of the experiments, only the matrix for the dimple engaged stage is checked currently. In the preliminary step, the experimental apparatuses were calibrated and showed no hysteresis effect, no drift, and were almost linear.

4.4.2 Obtaining flexibility coefficient λ_{rr} (Experiment One)

(a) Schematic diagram for the Experiment One

(b) Model for the Experiment One

(c) Experimental setup one

Fig. 4.19 Setup for Experiment One to measure λ_{rr}

The setup for Experiment One is schematized as Fig. 4.19 (a) and constructed as shown in Fig. 4.19 (c). A micrometer stage is used to push the lift tab to change z_r and a cantilever beam sensor is used to measure the external force F_r acting on the lift tab. Since there is no force on the slider to drag the dimple-gimbal contact apart in the experiment, only λ_{rr} for dimple engaged can be obtained. The experimental results are plotted in Fig. 4.20. As the result shown, it is almost linear. Then $\lambda_{rr} = 178 \text{ mm/N}$.

 Fig. 4.20 Force V.S displacement for λ_{tr}

4.4.3 Obtaining flexibility coefficient λ_{ss} (Experiment Two)

(a) Schematic diagram for the Experiment Two

(b) Model for the Experiment Two

(c) Experimental setup two

 Fig. 4.21 Setup for Experiment Two to measure λ_{ss}

The setup for Experiment Two is schematized as Fig. 4.21 (a) and constructed as shown in Fig. 4.21 (c). A micrometer stage is used to push the lift tab to change z_s and a

cantilever beam sensor is used to measure the external force F_s acting on the slider. There is no force on the slider. The experimental results are plotted in Fig. 4.22. As the result shown, it is almost linear. Then $\lambda_{ss} = 93.53 \text{ mm/N}$.

Fig. 4.22 Force V.S displacement for λ_{tr}

4.4.4 Obtain stiffness coefficients k_{sr} and k_{ss} (Experiment Three)

If the setups used in previous two experiments are still used, and two external forces are applied on the slider and the lift tab simultaneously, which is modeled as Fig. 4.23, there is no way to disentangle the forces with only one cantilever beam sensor.

Fig. 4.23 Simplified lumped parameter model for setup without auxiliary fixture

It is desired and possible to design and fabricate a fixture attaching to the cantilever beam sensor to disentangle the two forces by converting one external force to an

internal one. The hook like fixture shown in Fig. 4.24 (a) is used to convert the external force on the lift tab to an internal force. With the help of the fixture, the force on the slider can be measured alone. The setup for the Experiment Three is shown as Fig. 4.24 (b) and Fig. 4.24 (c).

Fig. 4.24 Setup for Experiment Three to obtain stiffness coefficients k_{sr} and k_{ss}

Extra errors are introduced into the results by adding the fixture into the setup. How to minimize the effects of the extra fixture is critical for the experiment. The setup for the experiment can be modeled as Fig. 4.25. Where, d_m is adjustable with a strong micrometer head, Δz_f is the deformation of the hook-shaped fixture, k_f is the stiffness of the fixture.

Fig. 4.25 Simplified lumped parameter model for the hook fixture setup

When the slider is kept stationary and the position of the lift-tab is adjusted, $k_f \Delta z_f = k_r \Delta z_r$. When $k_f \gg k_r$, $\Delta z_f \ll \Delta z_r$. The deformation of the fixture affects the results little. When d_m is kept stationary while the position of the slider is changed, if $k_f \gg k_s$, Δz_f also can be ignored. Thus, to ensure the setup work accurately, a relatively strong fixture is preferred and the error on the results caused by the deformation of the fixture is small and negligible. However, remember a too heavy fixture may damage the cantilever beam sensor or lead the measurement beyond the range of the sensor.

In Experiment Three, two steps are applied. Firstly, the slider is kept stationary and the lift tab is raised. k_{sr} is estimated with $\Delta F_s / \Delta Z_r^{(f)}$. The results are plotted as shown in Fig. 4.26. From the experiment we know, $k_{sr}^{(d)} = -1.0063$, $k_{sr}^{(g)} = -0.02759$, and $k_{sr}^{(L)} = -0.25707$. Equivalent free length of the dimple and the limiter from the perspective of the lift tab which is used for simulations can be obtained as $L^{(d)} = 0.02\text{mm}$ and $L^{(g)} = 0.14\text{mm}$, respectively.

 Fig. 4.26 k_{sr} , $L^{(d)}$, $L^{(d)}$ estimation

Secondly, the lift tab is kept stationary and the slider position is changed. k_{ss} is estimated with $\Delta F_s / \Delta Z_s$. From the experiment we know, $k_{ss}^{(d)} = 1.62691$, $k_{ss}^{(g)} = 0.02147$, and $k_{ss}^{(L)} = 0.35798$

 Fig. 4.27 k_{ss} estimation

In Experiment Three, three stages were observed. The stiffness of the suspension changed apparently according to different combinations of the components.

4.4.5 Derivation of stiffness coefficients k_{rr} and k_{rs}

$$k_{rr} = \frac{k_{ss} \lambda_{ss}}{\lambda_{rr}} = \frac{1.62691 \times 92.35}{194} = 0.775$$

$$k_{rs} = \left(k_{rr} k_{ss} - \frac{k_{ss}}{\lambda_{rr}} \right) / k_{sr} = \left[0.775 \times 1.62691 - \frac{1.62691}{194} \right] / (-1.0063) = -1.24463$$

Thus the stiffness matrix for dimple engaged was obtained as

$$K^{(d)} = \begin{bmatrix} 0.775 & -1.245 \\ -1.001 & 1.229 \end{bmatrix}$$

4.5 COMPARISON OF THE RESULTS FROM THE FE MODEL WITH THOSE FROM EXPERIMENTS

There are several major factors affecting the accuracy of the measurement: (1) error caused by the tolerance of the mechanical design; (2) error caused by the fabrication of the parts; (3) error caused by the setup of the assembly.

Furthermore, there are several major factors affecting the accuracy of the FEM solution: (1) error caused by the simplification of the physical structure; (1) error cause by the simplification of the element type; (2) error caused by the round off; (3) error caused by numerical solution schemes.

All the errors affect the results. However, in the study, since the fabrication and assembly of the fixture and parts were not precise enough, more errors may be introduced into the results while the finite element model was constructed with commercial software, we can trust it more. So we think more errors come from the experiments. Considering the errors in the experimental installations and devices and the numerical errors, the results from the experiments are comparable to the results from the finite element model. Therefore, data from the FEM are usable for further simulation.

Table 4.5 Comparison of the data from the FEM and the experiments

	From Finite Element Model	From Experiments
$K^{(d)}$ (N/mm)	$\begin{bmatrix} 1.8470 & -2.4760 \\ -2.4724 & 3.3276 \end{bmatrix}$	$\begin{bmatrix} 0.775 & -1.245 \\ -1.001 & 1.229 \end{bmatrix}$
$L^{(d)}$ (mm)	0.02	0.005
$L^{(g)}$ (mm)	0.14	0.175

In table 4.5, the equivalent free length of the dimple (0.02mm) calculated from the FEM is much larger (0.005mm) than that measured from the experiments. The discrepancy is mainly caused by the resolution of the experiments. However, since the stage in dimple engaged is very short and affects the whole unloading process little, we ignore the discrepancy.

CHAPTER FIVE

AIR BEARING MODELING AND NUMERICAL SOLUTIONS

In this chapter, head disk interfaces (slider air bearing) in modern HDD were modeled through the modified Reynolds equations. Finite difference method and finite volume method were employed to solve the modified Reynolds equation numerically in the chapter. Non-uniform grids were arranged for the meshing scheme to accommodate the nonlinear air bearing pressure distribution of subambient sliders with steep walls.

5.1 INTRODUCTION TO THE AIR BEARING MODEL AND NUMERICAL SOLUTION

In hard disk drives (HDDs), the read-write heads attached to the slider are lifted from the disk surface by the air-bearing force. The air bearing force is generated by a thin air layer squeezed into the narrow space between the slider and disk surface due to the rotating disk. The governing equation for obtaining the pressure distribution of the head-disk interface (HDI) is a nonlinear partial differential equation, called Reynolds-type lubrication equation, which was first developed from the Navier-Stokes equations by Reynolds in 1886. The Reynolds Equation forms the foundation of the fluid film lubrication theory. Since the analytical solutions exist only for very simple bearing

conditions, the generalized lubrication equation for complex bearing geometries has to be solved through numerical methods.

There are several numerical methods available for the solutions of the generalized lubrication equation. According to the way of discretization, these methods can mainly be classified into the finite difference method (FDM), the finite element method (FEM), and the finite volume method (FVM). Generally, FEM is the method of choice in all types of analysis in structural mechanics (i.e. solving for deformation and stresses in solid bodies or dynamics of structures) while computational fluid dynamics (CFD) tends to use FDM and FVM.

Research on the numerical solution of air bearing slider problems has been ongoing over past decades. Garcia-Suarez et al. proposed a finite element method with an upwind scheme for air bearing simulations [76]. Miu and Bogy simulated taped-flat sliders using the factored implicit scheme of White and Nigam [77]. Ruiz and Bogy implemented the second order slip correction and the Fukui and Kaneko model [78]. A factored implicit scheme for irregular rail geometry has been developed by Cha and Bogy [79]. The method is based on a control volume formulation of the linearized Reynolds equation. It also implemented the power-law scheme in mass flow calculations to enhance the stability of the algorithm. The steady state solution is achieved using an alternating direction implicit method with time stepping [80]. A control volume formulation is adapted to analyze shaped-rail air bearing at ultra low spacing. Patanker has formulated rules, which yield robust finite volume calculation schemes [81]. X.H. Li adopted the mesh-free method to develop an efficient and powerful methodology that is capable of handling simulation and design of the air bearing sliders with complex geometry shapes [82].

In FDM, derivatives are approximated by nodal values of functions defined on a grid. Algebraic equations involve variable value at the grid-point and neighboring points result. This method is only suitable for structured grids and limited to simple geometries. However, it is simple, effective, and easily extended to higher-order schemes. In FVM, domain is divided into control volumes (CV) and conservation form is applied to each CV. Variables are defined at the center of the CV, interpolation and quadrature are used to define surface and volume integrals in terms of CV-centre values. It is suitable for any grid. However, higher order methods in 3D are difficult due to need for interpolation, differentiation and integration. Mesh-free methods eliminate some or all the traditional mesh-based views of the computational domain and rely on a particle (either Lagrangian or Eulerian) view of the field problem. A goal of mesh-free methods is to facilitate the simulation of increasingly demanding problems that require the ability to treat large deformations, advanced materials, complex geometry, nonlinear material behavior, discontinuities and singularities.

Two methods commonly used in HDD industry, FDM and FVM, are discussed in detail in this thesis.

5.2 MODIFIED REYNOLDS EQUATIONS FOR HEAD DISK INTERFACE

The general Reynolds equation can be derived by combining the Navier-Stokes equations with the continuity equation as:

$$\begin{aligned} & \frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right) \\ &= \frac{\partial}{\partial x} \left[\frac{\rho h (V_{xa} + V_{xb})}{2} \right] + \frac{\partial}{\partial y} \left[\frac{\rho h (V_{ya} + V_{yb})}{2} \right] + \rho \left[(V_{za} - V_{zb}) - V_{xa} \frac{\partial h}{\partial x} - V_{ya} \frac{\partial h}{\partial y} \right] + h \frac{\partial \rho}{\partial t} \end{aligned} \quad (5-1)$$

For a gas-lubricated bearing with perfect gas,

$$p = \rho RT$$

where R is the gas constant and T is the absolute temperature. Therefore, ρ is replaced by p in the Reynolds equation. Applying it to the Head-Disk Interface (HDI), the general Reynolds equation can be modified as the following

$$\frac{\partial}{\partial x} \left(ph^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(ph^3 \frac{\partial p}{\partial y} \right) = 6V_x \eta \frac{\partial}{\partial x} [ph] + 6V_y \eta \frac{\partial}{\partial y} [ph] + 12\eta \frac{\partial}{\partial t} [ph] \quad (5-2)$$

Letting $X = x / L$, $Y = y / L$, $H = h / h_m$, $P = p / p_a$, and $T = \omega t$, where P , H , X and Y are the non-dimensionalized pressure, flying height, coordinate in the slider length direction and coordinate in the slider width direction, respectively. p_a is the ambient atmospheric pressure, h_m is the reference clearance at the trailing edge center, and L is the length of the slider. Substitute them into the equation, the non-dimensional Reynolds equation can be written as

$$\frac{\partial}{\partial X} \left(PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = \sigma \frac{\partial (PH)}{\partial T} \quad (5-3)$$

where, $\Lambda_x = (6\mu U_x L) / (p_a h_m^2)$ and $\Lambda_y = (6\mu U_y L) / (p_a h_m^2)$ are called bearing numbers in the x and y directions, respectively; $\sigma = (12\mu \omega L^2) / (p_a h_m^2)$ is the squeeze number.

Equation (5-3) is derived by assuming the gas to be a continuous medium with non-slip at the solid boundary. This equation yields accurate results when K_n is less than 0.001. When the Knudsen number is in the range between $0.01 < K_n < 0.1$ in the atmospheric air, a thin layer of gas close to the solid surface loses its fluid characteristic and is capable of ‘slipping’ against the surface.

For current HDIs, the flying height is down to 8 nm or even less and the gas may no longer be a continuum. In order to solve the air bearing problem still using a Reynolds-type lubrication equation and incorporating the rarefaction effect, several

authors introduced the generalized Reynolds equation with different slip correction models.

Burgdorfer first-order model can be obtained as following

$$\frac{\partial}{\partial X}(Q_p PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH) + \frac{\partial}{\partial Y}(Q_p PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH) = \sigma \frac{\partial PH}{\partial T} \quad (5-4)$$

where $Q_p = 1 + (6K_n)/(PH)$. Q_p is the Poiseuille flow rate coefficient, which reflects the type of slip correction used.

Hsia modified the first-order model to a second-order Model by setting

$$Q_p = 1 + (6K_n)/(PH) + 6[(K_n)/(PH)]^2 \quad (5-5)$$

Mitsuya in 1993 introduced the 1.5-order slip model in order to predict the load capacity more accurately from the physical considerations that taking account of the accommodation coefficient into account. The correction coefficient for the 1.5-order slip model can be expressed as following

$$Q_p = 1 + \frac{6aK_n}{PH} + \frac{8}{3} \left(\frac{K_n}{PH} \right)^2 \quad (5-6)$$

where $a = (2 - \alpha)/(\alpha)$. Generally, surface accommodation α coefficient equals to 0.89.

Fukui and kaneko started from the linearized Boltzmann equation based on the BGK model. To overcome the difficulties in solving the BGK model, Fukui and Kaneko introduced the use of a Poiseuille flow database to allow a quicker computation of a generalized lubrication equation for high K_n number gas bearing.

More details about the derivation of the equations can be referred to Appendix F.

5.3 FINITE DIFFERENCE METHOD (FDM)

5.3.1 Discretization of the Modified Reynolds Equation

White and Nigam presented the factored implicit scheme which actually is a kind of FDM for discretizing the Reynolds equation at very low spacing in 1980 [83]. The method is second-order time-accurate, non-iterative and allows the calculation of finite width films while requiring only tridiagonal matrix inverse. Miu and Bogoy implemented this method for the numerical simulation of taper-flat sliders in 1986. Ruiz and Bogoy made several improvements to the Miu's program. They implemented the second order slip and the Fukui and Kaneko modification to the classical Reynolds's equation. They also implemented the slider-disk contact model of Kane and Levinson, as well as the surface roughness effects. Jeong and Bogoy simulated slider-disk interactions during dynamic load-unload using this program. This method showed some drawbacks in late studies at a much higher air bearing number. The distance for the load/unload process is much higher than the air bearing gap. This method is still suitable to simulate the air bearing dynamics which will combine the simultaneous solution of the slider dynamics during the load/unload processes.

The fundamental of this method is that some modifications are made to the basic algorithms in order to obtain efficient solver for practical multidimensional applications. It has the following attractive features [84]: The method is implicit; it has fewer problems with numerical instability than explicit methods; The method can be used to study one-, two-, and three-dimensional flows; The method is efficient because it transforms the nonlinear equations describing a multi-dimensional flow problem into linear equations corresponding to one-dimensional problems; The method permits the use of finite-difference equations that can be expressed in delta form.

A first order model expressed by the following equation is used to model the HDI.

$$\begin{aligned}
 & \nabla_{\text{dim}} \cdot (h_{\text{dim}}^3 P_{\text{dim}} \nabla_{\text{dim}} P_{\text{dim}}) + 6\lambda_a P_a \nabla_{\text{dim}} \cdot (h_{\text{dim}}^2 \nabla_{\text{dim}} P_{\text{dim}}) \\
 & = 6\mu \vec{V} \cdot \nabla_{\text{dim}} P_{\text{dim}} h_{\text{dim}} + 12\mu \frac{\partial P_{\text{dim}} h_{\text{dim}}}{\partial t_{\text{dim}}}
 \end{aligned} \tag{5-7}$$

where $\nabla_{\text{dim}} = (\partial / \partial x_{\text{dim}})i + (\partial / \partial y_{\text{dim}})j$, $\vec{V} = V_x i + V_y j$, and the subscript *dim* means dimensional variables.

The discretized equation of the (5-7) could be expressed with

$$[1 - L_1][1 - L_2]\Delta Z^n = \phi \tag{5-8}$$

where

$$\begin{aligned}
 & (1 - L_1)(\cdot)_{i,j} \\
 & = -a \left\{ \left(2hZ_x - Zh_x - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \right) \cdot C_3(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot A_3(i) \right\}^{(n)} \cdot (\cdot)_{i-1,j} \\
 & + \left\{ \begin{aligned} & 1 - a \left(hZ_{xx} - Z_x h_x - 2Zh_{xx} - 6 \frac{\lambda_a}{h_{\min}} h_{xx} \right) \\ & - a \left(2hZ_x - Zh_x - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \right) \cdot C_2(i) - a \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot A_2(i) \end{aligned} \right\}^{(n)} \cdot (\cdot)_{i,j} \\
 & - a \left\{ \left(2hZ_x - Zh_x - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \right) \cdot C_1(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot A_1(i) \right\}^{(n)} \cdot (\cdot)_{i+1,j}
 \end{aligned} \tag{5-9}$$

$$\begin{aligned}
 & (1 - L_2)(\cdot)_{i,j} \\
 & = -a \left\{ \left(2hZ_y - Zh_y - 6 \frac{\mu L_y}{h_{\min}^2 P_a} V_y \right) \cdot D_3(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot B_3(i) \right\}^{(n)} \cdot (\cdot)_{i-1,j} \\
 & + \left\{ \begin{aligned} & 1 - a \left(hZ_{yy} - Z_y h_y - 2Zh_{yy} - 6 \frac{\lambda_a}{h_{\min}} h_{yy} \right) \\ & - a \left(2hZ_y - Zh_y - 6 \frac{\mu L_y}{h_{\min}^2 P_a} V_y \right) \cdot D_2(i) - a \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot B_2(i) \end{aligned} \right\}^{(n)} \cdot (\cdot)_{i,j} \\
 & - a \left\{ \left(2hZ_y - Zh_y - 6 \frac{\mu L_y}{h_{\min}^2 P_a} V_y \right) \cdot D_1(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot B_1(i) \right\}^{(n)} \cdot (\cdot)_{i+1,j}
 \end{aligned} \tag{5-10}$$

$$\begin{aligned}
 & \phi^{(n)}(x, y, t^n, t^{n+1}) \\
 &= a \left\{ \begin{aligned} & \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) \left[\begin{aligned} & (Z_{xx} + Z_{yy})(h^{(n)} + h^{(n+1)}) \\ & -Z(h_{xx}^{(n)} + h_{yy}^{(n)} + h_{xx}^{(n+1)} + h_{yy}^{(n+1)}) \end{aligned} \right] \\ & + Z_x \left[\begin{aligned} & (h^{(n)} + h^{(n+1)})Z_x - Z(h_x^{(n)} + h_x^{(n+1)}) - 12 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \end{aligned} \right] \\ & + Z_y \left[\begin{aligned} & (h^{(n+1)} + h^{(n)})Z_y - Z(h_y^{(n)} + h_y^{(n+1)}) - 12 \frac{\mu L_y}{h_{\min}^2 P_a} V_y \end{aligned} \right] \end{aligned} \right\}^{(n)} \quad (5-11)
 \end{aligned}$$

The factor solution is achieved by the following sequence:

$$[1 - L_1] \Delta Z^* = \phi \quad (5-12)$$

$$[1 - L_2] \Delta Z^n = \Delta Z^* \quad (5-13)$$

Eqs. (5-12) and (5-13) are seen to be equivalent to eq. (5-11) by operating on eq. (5-13) with $[1 - L_1]$ and the adding eq. (5-12). The factored form of the two-dimensional problem as shown in eq. (5-11) has thus been reduced to the solution of two one-dimensional problems. Eq. (5-12) represents a number of one-dimensional problems in the x row direction while eq. (5-13) uses the solution of eq. (5-12) to define a number of one-dimensional problems in the y column direction.

More details about the discretization of the Modified Reynolds equation and the meanings of the parameters can be found in Appendix F.

5.3.2 Grid, Wall and Corner Consideration

As negative pressure sliders are designed and used in current hard disk drives, some modification should be used to ameliorate the stability of the factored implicit scheme. The negative pressure sliders will introduce recess to form suction force. The wall profile and the corner formed by the recess as shown in Fig. 5.1 should be treated specially. In our code, we use several methods to deal with this problem, including

adjusting the grids for the wall profiles, merging several grids and re-dividing them unequally to get a smooth change, adding more grid points to represent the wall profiles, and treating the corner of the recess specially. The air bearing was meshed with geometric series grids along x direction as shown in Fig. 5.2.

Fig.5.1 A simplified subambient pressure slider

Fig.5.2 Geometric series grid for the air bearing

5.3.2.1 Grid adjustment for the wall profiles

Fig. 5.3 Adjustment of the steep wall on a grid point

After the whole area of the slider is divided into grids, wall profiles may be trapped between the gaps formed by two neighboring grid points as shown in Fig. 5.3.

In order to represent the position of the wall profile correctly, some grid points will be changed to reflect the right position of these profiles. We can move the coordinate of the grid points just before the wall profile to the coordinate of the profile. Also we can move the coordinate of the grid point just next the wall profile to the coordinate of the profile. The latter method was used in our modification.

5.3.2.2 Merge and re-divide grid points in the vicinity of the wall profiles

After we reassign the coordinates for the wall profiles, abrupt changes of the coordinates which may cause the algorithm unstable may happen. To avoid such a problem, we may merge several consecutive grids near the wall profiles and re-divide them to form a gradual change in these areas.

Fig. 5.4 merged assignment of grid points

The serial number of the grid points will be changed after the grids are merged. As shown in Fig. 5.4, we assume the number of the merged grid near a wall profile is n ,

where n is an odd number. The points before the point $\left(i - \frac{(n+1)}{2}\right)$ will keep their original serial numbers. The serial number of the wall profile will become $i - \frac{(n+1)}{2} + 1$.

The previous point $\left(i + \frac{(n-1)}{2}\right)$ becomes current point $\left(i - \frac{(n+1)}{2} + 2\right)$.

Then re-division of the merged grid is required to form a gradual change of the grid in this area. We use unequal grids to redivide the merged grids. Denser points will be assigned in the proximity of the wall profiles. The strategy of this division is similar to

the division of the whole area of the slider. We assume “*extn*” point will be added to each grid beside the wall profiles. The new serial numbers of the points are shown in Fig. 5.5.

Fig. 5.5 serial numbers for extensional points

The points before the point $\left(i - \frac{(n+1)}{2}\right)$ will still keep their original serial numbers.

The serial number of the wall profile will become $i - \frac{(n+1)}{2} + 1 + n_{ext}$. The previous point

$\left(i + \frac{(n-1)}{2}\right)$ becomes current point $\left(i - \frac{(n+1)}{2} + 2 + 2 \times n_{ext}\right)$.

Normally, there are several wall profiles along one direction. The grid numbering of front profiles will affect the grid numbering of the subsequent walls. First, let us consider two wall profiles in a row as shown in Fig. 5.6.

Fig. 5.6 The series number assignment for the first two steep wall profiles

For a general case, we consider arbitrary two points in a row. The coordinate assignment is shown in Fig. 5.7

Fig. 5.7 the series number assignment for two arbitrary points in a row

5.3.2.3 Unequalization of the grid points in the vicinity of the wall profiles

For the steep sides, we should treat them specially. Finer unequal grids are applied in their vicinity.

$$L = \Delta_{\min} * (1 + s + s^2 + \dots + s^n) \quad (5-14)$$

$$\Delta_{\min} = \frac{L}{(1 + s + s^2 + \dots + s^n)} \quad (5-15)$$

On the left, it decreases gradually. On the right, it increases gradually. 2 points are expanded to $2n+1$ points. The resulted unequal grid is shown in Fig. 5.8.

Fig. 5.8 Unequal grids for the vicinity of the wall profiles

5.3.2.4 Add more points to smoothen the wall profile

Because a wall profile is a vertical plane which can be represented by a grid point along one direction, the change of the height at the wall position will be very large, which may cause the algorithm unstable. To avoid such a problem, several grids near the wall profile can be used to describe the wall profile together. Because the distance of the grids near the wall profiles is small, the modification is reasonable and approachable. The smoothening treatment for steep wall profiles is illustrated in Fig. 5.9.

Fig. 5.9 Smoothing treatment for steep wall profiles

5.3.2.5 Treatment for corner points

When two surfaces along two perpendicular wall profiles meet together, a special area, actually at the corner of the recess or embossment, will be formed. To improve the stability of the code, this area should be treated specially. First of all, we should identify where there is a corner. Consequently, we need to recognize which kind the corner is. Here, we use $\oplus$ denotes embossment, $\ominus$ denotes recess, $\bullet$ denotes a reference point. Considering an area with four points, there are several different cases. Fig 5.10 enumerates the different circumstances according to the different height of the four points. (a) denote the points at the boundary but not at the corner; (b) denote the central points not at the boundary; (c) may not exist at the real slider design; (d) denote the different cases of the corner.

To identify the boundary, the corner or the central of the area, for those the corner at the right bottom as shown in Fig 5.11. From the x direction and y direction, we can see the height will change gradually. So the algorithm becomes more robust. Furthermore, the treatment is shown as Fig 5.12 for those the step is at the left bottom of the four points.

Fig. 5.10 corner discriminations

Fig. 5.11 Assignment of the height in the corner area

(a) concave corner (b) convex corner

Fig. 5.12 Diagram of the corner area

5.3.3 Flowchart of the Finite Different Scheme

Fig. 5.13 Flowchart for the finite different scheme

5.3.4 Numerical Solutions for Simple Cases

The numerical solver can be used to solve the dynamic gas lubrication problems with sophisticated surface at very low clearances. Considering a cosine bump on the moving surface pass through a plane as shown in Fig. 5.14, the variation of the pressure distribution acting on the surface during the whole process can be shown in Fig. 5.15.

Fig. 5.14 A cosine bump fly over the slider-disk interface

(a) Peak of bump at bearing inlet (b) Peak of bump at center of bearing
(c) Peak of bump at center of bearing (d) Peak of bump go out of the bearing

Fig 5.15 Pressure distribution over plane wedge bearing due to cosine bump disturbance

Considering a simplified slider with a cavity in the center shown in Fig. 5.16, when the pitch angle and minimal flying height are fixed, the air pressure distribution on the slider can be obtained using the numerical solver. Different parameters result in different air pressure distribution, which is shown in Fig. 5.17.

Fig. 5.16 height distribution from the surface of the slider to the surface of the disk

(a) Minimal flying height: 10nm;
pitch angle (radian): 1×10^{-4}

(b) Minimal flying height: 50nm;
pitch angle (radian): 1×10^{-4}

Fig. 5.17 Pressure distributions for different minimal flying heights and pitch angles

5.4 FINITE VOLUME METHOD (FVM)

The surface of the subambient (negative) pressure air bearing slider has geometrical (clearance) discontinuities. This causes numerical difficulty for finite difference methods based on the differential form of the modified Reynolds equation. To solve the problem, an artificial smooth profile has to be used to smooth the steep wall profile which reduces the accuracy of the numerical solution. By utilizing FVM, this difficulty can be avoided effectively [75].

The Tri-diagonal matrix algorithm (TDMA) can be applied iteratively to solve a system of equations for two-dimensional problems by alternating the sweep direction. The meaning of each parameter and the derivation of the equations can be referred to Appendix H. For the first run, the equation can be rearranged in the form to solve x direction. Equation (i.15) can be remodeled into:

$$-a_{w(i,j-1)}P_{i,j-1} + a_{p(i,j)}P_{i,j} - a_{e(i,j+1)}P_{i,j+1} = a_{s(i-1,j)}P_{i-1,j} + a_{n(i+1,j)}P_{i+1,j} + b_{i,j} \quad (5-15)$$

Its equations format can be expressed as

$$\left\{ \begin{array}{l} -a_{w(1,j)}P_{1,j} + a_{p(2,j)}P_{2,j} - a_{e(3,j)}P_{3,j} = a_{s(2,j-1)}P_{2,j-1} + a_{n(2,j+1)}P_{2,j} + b_{2,j} \\ -a_{w(2,j)}P_{2,j} + a_{p(3,j)}P_{3,j} - a_{e(5,j)}P_{4,j} = a_{s(3,j-1)}P_{3,j-1} + a_{n(3,j+1)}P_{3,j} + b_{3,j} \\ \dots \\ -a_{w(i-1,j)}P_{i-1,j} + a_{p(i,j)}P_{i,j} - a_{e(i+1,j)}P_{i+1,j} = a_{s(i,j-1)}P_{i,j-1} + a_{n(i,j+1)}P_{i,j} + b_{i,j} \\ \dots \\ -a_{w(k-2,j)}P_{k-2,j} + a_{p(k-1,j)}P_{k-1,j} - a_{e(k,j)}P_{k,j} = a_{s(k-1,j-1)}P_{k-1,j-1} + a_{n(k-1,j+1)}P_{k-1,j} + b_{k-1,j} \\ -a_{w(k-1,j)}P_{k-1,j} + a_{p(k,j)}P_{k,j} - a_{e(k+1,j)}P_{k+1,j} = a_{s(k,j-1)}P_{k,j-1} + a_{n(k,j+1)}P_{k,j+1} + b_{k,j} \end{array} \right. \quad (5-17)$$

where k is the number of the grid point along x direction. In the above set of equations,

$P_{1,j}$ and $P_{k+1,j}$ are known as boundary condition. Actually, they are the ambient

pressure. Its matrix format can be expressed as

$$A\vec{x} = \vec{y} \quad (5-18)$$

where

$$A = \begin{bmatrix} a_{p(2,j)} & -a_{e(3,j)} & 0 & \cdot & 0 & 0 & 0 & \cdot & 0 & 0 \\ -a_{w(2,j)} & a_{p(3,j)} & -a_{e(5,j)} & \cdot & 0 & 0 & 0 & \cdot & 0 & 0 \\ 0 & -a_{w(3,j)} & a_{p(5,j)} & \cdot & 0 & 0 & 0 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & -a_{w(i-1,j)} & a_{p(i,j)} & -a_{e(i+1,j)} & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 0 & 0 & 0 & \cdot & a_{p(k-1,j)} & -a_{e(k,j)} \\ 0 & 0 & 0 & \cdot & 0 & 0 & 0 & \cdot & -a_{w(k-1,j)} & a_{p(k,j)} \end{bmatrix}_{(k-1) \times (k-1)}$$

$$\vec{x} = \begin{Bmatrix} P_{2,j}^{(new)} \\ P_{3,j}^{(new)} \\ P_{4,j}^{(new)} \\ \dots \\ P_{i,j}^{(new)} \\ \dots \\ P_{k-1,j}^{(new)} \\ P_{k,j}^{(new)} \end{Bmatrix}, \vec{y} = \begin{Bmatrix} a_{w(1,j)}P_{1,j}^{(boundary)} + a_{s(2,j-1)}P_{2,j-1}^{(old)} + a_{n(2,j+1)}P_{2,j+1}^{(old)} + b_{2,j} \\ a_{s(3,j-1)}P_{3,j-1}^{(old)} + a_{n(3,j+1)}P_{3,j+1}^{(old)} + b_{3,j} \\ a_{s(5,j-1)}P_{4,j-1}^{(old)} + a_{n(5,j+1)}P_{4,j+1}^{(old)} + b_{4,j} \\ \dots \\ a_{s(i,j-1)}P_{i,j-1}^{(old)} + a_{n(i,j+1)}P_{i,j+1}^{(old)} + b_{i,j} \\ \dots \\ a_{s(k-1,j-1)}P_{k-1,j-1}^{(old)} + a_{n(k-1,j+1)}P_{k-1,j+1}^{(old)} + b_{k-1,j} \\ a_{e(k+1,j)}P_{k+1,j}^{(boundary)} + a_{s(k,j-1)}P_{k,j-1}^{(old)} + a_{n(k,j+1)}P_{k,j+1}^{(old)} + b_{k,j} \end{Bmatrix}$$

Therefore, temporary pressures at the central point are obtained with eq. (5-19) line by line.

$$\vec{x} = A^{-1}\vec{y} \quad (5-19)$$

For the second run, the equation can be rearranged in the form as eq. (5-20) to obtain the pressure along y direction:

$$-a_{s(i,j-1)}P_{i,j-1} + a_{p(i,j)}P_{i,j} - a_{n(i,j+1)}P_{i,j+1} = a_{w(i-1,j)}P_{i-1,j} + a_{e(i+1,j)}P_{i+1,j} + b_{i,j} \quad (5-20)$$

For boundary condition

$$\left\{ \begin{array}{l} -a_{s(i,1)}P_{1,j} + a_{p(i,2)}P_{i,2} - a_{n(i,3)}P_{i,3} = a_{w(i-1,2)}P_{i-1,2} + a_{e(i+1,2)}P_{i+1,2} + b_{i,2} \\ -a_{s(i,2)}P_{2,j} + a_{p(i,3)}P_{i,3} - a_{n(i,4)}P_{i,4} = a_{w(i-1,3)}P_{i-1,3} + a_{e(i+1,3)}P_{i+1,3} + b_{i,3} \\ \dots \\ -a_{s(i,j-1)}P_{i,j-1} + a_{p(i,j)}P_{i,j} - a_{n(i,j+1)}P_{i,j+1} = a_{w(i-1,j)}P_{i-1,j} + a_{e(i+1,j)}P_{i+1,j} + b_{i,j} \\ \dots \\ -a_{s(i,m-2)}P_{i,m-2} + a_{p(i,m-1)}P_{i,m-1} - a_{n(i,m)}P_{i,m} = a_{w(i-1,m-1)}P_{i-1,m-1} + a_{e(i+1,m-1)}P_{i+1,m-1} + b_{i,m-1} \\ -a_{s(i,m-1)}P_{i,m-1} + a_{p(i,m)}P_{i,m} - a_{n(i,m+1)}P_{i,m+1} = a_{w(i-1,m)}P_{i-1,m} + a_{e(i+1,m)}P_{i+1,m} + b_{i,m} \end{array} \right. \quad (5-21)$$

Its matrix format can be expressed as

$$B\vec{u} = \vec{v} \quad (5-22)$$

where

$$B = \begin{bmatrix} a_{p(i,2)} & -a_{n(i,3)} & 0 & \cdot & 0 & 0 & 0 & \cdot & 0 & 0 \\ -a_{s(i,2)} & a_{p(i,3)} & -a_{n(i,4)} & \cdot & 0 & 0 & 0 & \cdot & 0 & 0 \\ 0 & -a_{s(i,3)} & a_{p(i,4)} & \cdot & 0 & 0 & 0 & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & -a_{s(i,j-1)} & a_{p(i,j)} & -a_{n(i,j+1)} & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & 0 & 0 & 0 & \cdot & a_{p(i,m-1)} & -a_{n(i,m)} \\ 0 & 0 & 0 & \cdot & 0 & 0 & 0 & \cdot & -a_{s(i,m-1)} & a_{p(i,m)} \end{bmatrix}_{(m-1) \times (m-1)}$$

$$\vec{u} = \begin{Bmatrix} P_{i,2}^{(new)} \\ P_{i,3}^{(new)} \\ P_{i,4}^{(new)} \\ \dots \\ P_{i,j}^{(new)} \\ \dots \\ P_{i,m-1}^{(new)} \\ P_{i,m}^{(new)} \end{Bmatrix}, \vec{v} = \begin{Bmatrix} a_{s(i,1)}P_{1,j}^{(boundary)} + a_{w(i-1,2)}P_{i-1,2}^{(old)} + a_{e(i+1,2)}P_{i+1,2}^{(old)} + b_{i,2} \\ a_{w(i-1,3)}P_{i-1,3}^{(old)} + a_{e(i+1,3)}P_{i+1,3}^{(old)} + b_{i,3} \\ a_{w(i-1,4)}P_{i-1,4}^{(old)} + a_{e(i+1,4)}P_{i+1,4}^{(old)} + b_{i,4} \\ \dots \\ a_{w(i-1,j)}P_{i-1,j}^{(old)} + a_{e(i+1,j)}P_{i+1,j}^{(old)} + b_{i,j} \\ \dots \\ a_{w(i-1,m-1)}P_{i-1,m-1}^{(old)} + a_{e(i+1,m-1)}P_{i+1,m-1}^{(old)} + b_{i,m-1} \\ a_{n(k+1,j)}P_{k+1,j}^{(boundary)} + a_{w(i-1,m)}P_{i-1,m}^{(old)} + a_{e(i+1,m)}P_{i+1,m}^{(old)} + b_{i,m} \end{Bmatrix}$$

Therefore, pressures at the central point are obtained with eq. (5-23) line by line.

$$\vec{u} = B^{-1}\vec{v} \quad (5-23)$$

As this research is supported by Seagate Company, Berkeley CML code, which employs the FVM scheme, is used for further studies of slider air bearings in the dissertation under their permission.

CHAPTER SIX

PERFORMANCE FUNCTIONS FOR CHARACTERIZING NONLINEARITIES OF AIR BEARING FORCES AND MOMENTS

In the chapter, a novel surface fitting scheme was introduced to characterize and approximate the nonlinear variations of the air bearing forces and moments with flying attitude in unloading processes. The air bearing forces and the moments acting on a subambient slider were studied by separately calculating the contribution of positive and negative pressures on the slider. First, a family of exponential-type functions was employed to fit the air bearing forces and moments with the minimal flying height chosen as an independent variable. Second, the pitch angle was introduced as another independent variable by fitting the coefficients in the exponential-type functions. Consequently, performance functions for characterizing and calculating the nonlinear air bearing forces and moments in unloading processes were obtained. Three subambient pressure sliders with simple as well as complex surface design were studied using the performance functions approach.

6.1 PHILOSOPHY OF DEVELOPING AN EFFICIENT SCHEME FOR CALCULATING NONLINEAR AIR BEARING FORCES/MOMENTS

It is inevitable that sliders are required to fly below 5 nm with the rapid increase in areal density. At such a low flying height, it is a big challenge to load/unload a slider safely. The suspension design and the air bearing slider design affect the load/unload behaviors predominantly. Both simulation and experiment are widely used to study the

behavior of a slider in current HDD industry. In previous simulation works reported in literature, the dynamic behavior of a slider is obtained by iteratively solving the coupled equations of the suspension dynamics and air bearing fluid dynamics. The whole process of the simulation is time-consuming because of intensive computation effort required, especially the numerical solution for air bearing governing equations. Even a small modification of the head disk interface design may require the whole process to be re-run, leading to a longer design cycle. It is critical and desirable for practical designs to significant reduce the computational time required for simulation of dynamic behavior of a slider, especially which for solving the Reynolds equation.

Although the slider air bearing working at near contact is treated as linear model in some analyses, it is also found the nonlinear effects cannot be neglected during the load/unload processes for negative pressure air-bearing sliders, especially when the sliders work at the low flying height. A conventional approach by simplifying the air bearing as a spring-damper system is unsuitable for load/unload analysis.

The dual-scale model is proposed to study the linear suspension and nonlinear air bearing separately. As mentioned in chapter 3, for simulating the load/unload process, the structural dynamics model of the suspension has to be coupled with the air bearing forces/moments that are obtained through solving the modified Reynolds equation iteratively. For each change of the slider's flying attitude, the Reynolds equation has to be re-solved computationally in order to obtain the air bearing forces/moments. This is time-consuming and needs a lot of computing capacity. However, especially for load/unload simulation, the main variables in flying attitude are only the flying height and pitch angle in the air bearing calculation. Hence, developing an efficient approach to calculate or approximate the air bearing forces/moments against the variation of flying height and pitch angle is very much desired and of practical significant.

In this dissertation, the concept of performance functions is introduced to represent the air bearing forces and moments as functions of the attitude variables, mainly the minimum flying height and the pitch angle. In principle, it is possible to extend the proposed performance functions approach to a three-parameter formulation including the roll angle. The performance functions can be obtained by fitting discrete numerical results to a continuous function. Then, the dynamics of the slider can be studied by coupling the best-fitted functions with the suspension structural dynamics model. Although this scheme is quasi-static, it is feasible and reasonable to use the scheme in studying the relatively low frequency process of the load/unload operation.

The air bearing moments are also calculated as two parts to facilitate the curve fitting, an equivalent counterclockwise positive moment and an equivalent clockwise negative moment. The geometric center is treated as the center of the gravity in the project.

Using the FDM and FVM methods discussed in preceding chapters, the air bearing pressure distributions on the slider surface at different flying attitudes can be obtained, from which the instantaneous forces and moments acting on the slider can be calculated by integrating the pressure on the discrete area. The structural force and moment from the suspension can be pinpointed by considering its deformation.

Since the purpose of this research is to develop a simple and efficient approach to obtain the dynamic behaviors of the sliders during the load/unload processes, the performance surface functions should be simple enough for analytical or numerical solution. Furthermore, the performance surface functions should be relatively precise, meaningful and can predict the trend of the variation of the air bearing forces and moments well. Therefore, some simple functions are preferable.

There are two different approaches for curve fitting. One is least-squares regression.

The other is interpolation. The strategy used in the least-squares regression is to derive a single curve that represents the general trend of the data. Because any individual data point may not be exactly precise, we make no effort to intersect every point. On the other hand, the strategy used in the interpolation is to fit a curve or a series of curves that pass directly through each of the specified discrete points. For curve fitting in the work, the least-squares regression approach is chosen.

As two variables, the minimal flying height and the pitch angle, are involved, the whole procedure of the surface fitting is divided into two steps. First, the curves against the variation of minimal flying height h_{min} with fixed pitch angles will be fitted and the functions with varied h_{min} will be obtained. Second, the effect of the pitch angle θ is introduced into the function to form a two-variable function.

6.2 GEOMETRY AND WORKING CONDITION

Currently, subambient pressure sliders, whose air bearing surfaces are typically etched with embossed pads and recessed cavities, are widely used in products to provide very low and stable flying heights. Under the flying circumstance, negative (subambient) pressure tends to be generated at the cavity area and the positive (above-ambient) pressure forms at the pad areas. Both the finite difference method and the finite volume method discussed in preceding chapters can help us to figure out the air bearing pressure distributions over the slider surface in flying. The instantaneous forces and moments acting on the slider can be calculated out by integrating the pressure over the slider area. But the subambient pressure design negatively affects the performance of the slider during the load/unload processes. The increasing effort to use sub-ambient pressure air bearing sliders for dynamic load/unload applications in magnetic hard disk drives requires desirable air bearing characteristics during the dynamic unload event

[79].

A commercial Pico-slider shown in Fig. 6.1 is used for the simulation. The length, width, and height of the slider are 1.233mm, 1.01 mm, and 0.2mm, respectively. Since the load/unload mechanism always situates at the out edge of the disk, the point of interest is set on a 1 inch disk at radius of 11.682 mm with skew angle of 9.861 degree shown in Fig. 6.2. Skew Angle is the angle between the centerline of the slider and the centerline of the arm. The disk rotational speed is 3600 rpm (revolutions per minute). The slider is considered flying with a zero roll angle. Roll angle is the angular displacement in μrad about the x-axis. For the positive roll, the inner edge spacing is larger than the outer edge spacing. The ambient pressure, the mean free path (MFP), and the dynamic viscosity are 101350 Pa, 63.5 nm, and 1.806×10^{-5} Pa·s, respectively. The air pressure distribution is obtained by solving the steady state Reynolds equation.

Fig. 6.1 Surface profiles of the S1 slider

The air bearing forces and moments for each attitude are obtained by fixing the pitch angle and the minimal flying height. Pitch angle is the angular displacement in μrad about the y-axis. For positive pitch angle, the leading edge spacing is larger than the trailing edge spacing. Flying height is the height in nm of the nominal trailing edge center.

From the design, the load point offsets in length direction and in width direction from the center of the slider is 0. The preload force is 1.35gram force.

Fig. 6.2 Skew angle and position for the load/unload process

Therefore, numerical investigations of the load/unload process are highly demanded in order to optimize the design parameters. These parameters include the design of the slider air bearing (shape of the air bearing surface) and the suspension characteristics (suspension design), disk speed, and vertical slider velocity.

6.3 FORCES AND MOMENTS WITH VARYING h_{min} AND θ

6.3.1 Air Pressure Distribution, Air Bearing Forces and Moments

Fig. 6.3 Air pressure distribution when $h_{min} = 100$ nm and $\theta = 100$ μ rad

A typical air pressure distribution obtained by solving the steady state Reynolds equation with finite volume method is shown in Fig. 6.3.

The total air bearing force and moment on the slider is obtained by integrating the pressure over the entire slider area on both sides. Thus, the total force acting on the slider can be expressed as the following.

$$F_a = \iint [p(x, y) - 1] dx dy \quad (6-1)$$

The unit of the force is N, the unit of the moment is N·m. the unit of the minimal flying height is nm, and the unit of the pitch angle is micro-radian (μrad).

The moment for the slider in pitch direction can be expressed as:

$$M_\theta = \iint (x - \frac{L_x}{2}) [p(x, y) - 1] dx dy \quad (6-2)$$

The equivalent arm for the force can be expressed as:

$$\bar{X} = \frac{\left(\iint (x - \frac{L_x}{2}) [p(x, y) - 1] dx dy \right)}{F_a} \quad (6-3)$$

Fig. 6.4 Variables assignment for the calculation

The discretized expressions of the above forces and moments can be expressed as follows:

$$F_{i,j} = \frac{(p_{i,j} + p_{i+1,j} + p_{i,j+1} + p_{i+1,j+1})}{4} \Delta x_{i,j} \Delta y_{i,j} \quad (6-4)$$

$$M_{i,j} = F_{i,j} \cdot \left(x_{i,j} + \frac{\Delta x_{i,j}}{2} \right) \quad (6-5)$$

$$F_a = \sum F_{i,j} \quad (6-6)$$

$$M_\theta = \sum M_{i,j} \quad (6-7)$$

where, $F_{i,j}$ is an local air bearing force in one grid; $M_{i,j}$ is the air bearing moment caused by the local force; F_a denotes the total air bearing force on the slider; M_θ denotes the total air bearing moment in pitch direction on the slider.

$$F_a = \sum p(i+1/2, j+1/2) \cdot Area(p(i+1/2, j+1/2)) \quad (6-8)$$

$$\bar{X} = \frac{(\sum p(i+1/2, j+1/2) \cdot Area(p(i+1/2, j+1/2)) \cdot (x(i, j) + \Delta x(i, j)/2))}{\sum p(i+1/2, j+1/2) \cdot Area(p(i+1/2, j+1/2))} \quad (6-9)$$

where,

$$p(i+1/2, j+1/2) = \frac{1}{4} (p(i, j) + p(i+1, j) + p(i, j+1) + p(i+1, j+1)) - 1 \quad (6-10)$$

$$Area(p(i+1/2, j+1/2)) = \Delta x(i, j) \cdot \Delta y(i, j) \quad (6-11)$$

Similarly

$$\bar{Y} = \frac{(\sum p(i+1/2, j+1/2) \cdot Area(p(i+1/2, j+1/2)) \cdot (y(i, j) + \Delta y(i, j)/2))}{\sum p(i+1/2, j+1/2) \cdot Area(p(i+1/2, j+1/2))} \quad (6-12)$$

6.3.2 Positive Forces Analysis

The positive air bearing distribution can be demonstrated by taking the positive part out of the entire air bearing distribution and setting the value of the negative air bearing grids to zero as illustrated in Fig. 6.5.

The total positive air bearing force can be figured out numerically as:

$$F_p = \sum_{i=1}^{160} \sum_{j=1}^{160} \frac{(p_p^{i,j} + p_p^{i+1,j} + p_p^{i,j+1} + p_p^{i+1,j+1})}{4} \cdot \Delta x_{i,j} \Delta y_{i,j} \quad (6-13)$$

where F_p is the total air bearing positive force; P_p is the air bearing pressure at the grid points. Δx and Δy are the length and width of each grid, respectively.

Fig. 6.5 Positive air pressure distribution $p_p^{i,j}$ when $h_{min} = 100 \text{ nm}$ & $\theta = 100 \mu\text{rad}$

The positive forces for different minimal flying heights and pitch angles are tabulated in Table 6.1 and illustrated as Fig. 6.6.

Fig. 6.6 positive forces for varying minimal flying height and pitch angle

6.3.3 Negative Forces Analysis

The negative air bearing distribution can be demonstrated by taking the negative part out of the entire air bearing distribution and setting the value of the positive air bearing grids to zero as illustrated in Fig. 6.7.

Fig. 6.7 Negative air pressure distribution $p_n^{i,j}$ when $h_{min} = 100 \text{ nm}$ & $\theta = 100 \text{ } \mu\text{rad}$

The negative air bearing force can be figured out numerically as:

$$F_n = \sum_{i=1}^{160} \sum_{j=1}^{160} \frac{(p_n^{i,j} + p_n^{i+1,j} + p_n^{i,j+1} + p_n^{i+1,j+1})}{4} \cdot \Delta x_{i,j} \Delta y_{i,j} \quad (6-14)$$

where F_n the total air bearing negative force; P_n is the negative air bearing pressure at the grid points. Δx and Δy are the length and width of each grid, respectively.

The negative forces for different minimal flying heights and pitch angles are tabulated in Table 6.2 and illustrated as Fig. 6.8.

6.3.4 Analysis of the Positive Moments in Pitch direction

Both the positive force at the left-hand side of the center of the gravity and the negative force at the right-hand side of the center of the gravity will cause negative moments to rotate the slider counterclockwise. Typical negative moments caused by the forces are shown in Fig. 6.9

Fig. 6.8 Negative forces for varying minimal flying height and pitch angle

(a) Moment caused by left negative pressure (b) Moment caused by right positive pressure

 Fig. 6.9 positive moments when $h_{min} = 100 \text{ nm}$ & $\theta = 100 \mu\text{rad}$

The total positive moment can be obtained by adding them together as:

$$M_p = M_{pp} + M_{pn} = \sum_{i=81}^{160} \sum_{j=80}^{160} F_p^{i,j} \cdot x_{i+\frac{1}{2}, j+\frac{1}{2}} + \sum_{i=1}^{80} \sum_{j=1}^{80} F_n^{i,j} \cdot x_{i+\frac{1}{2}, j+\frac{1}{2}} \quad (6-14)$$

where M_p is the total positive moment; M_{pp} is the positive moment caused right-hand side positive force; M_{pn} is the positive moment caused left-hand side negative force.

The total positive moments for different minimal flying heights and pitch angles are tabulated in Table 6.3 and illustrated as Fig. 6.10.

Fig. 6.10 Positive moment for varying minimal flying height and pitch angle

6.3.5 Analysis of the Negative Moment in Pitch Direction

Both the positive force at the left-hand side of the center of the gravity and the negative force at the right-hand side of the center of the gravity will cause negative moments to rotate the slider counterclockwise. Typical negative moments caused by the forces are shown in Fig. 6.11

(a) Moment caused by left positive pressure (b) Moment caused by right positive pressure

Fig. 6.11 negative moments when $h_{min} = 100$ nm & $\theta = 100$ μ rad

The total negative moment can be obtained by adding them together as:

$$M_n = M_{np} + M_{nn} = \sum_{i=81}^{160} \sum_{j=80}^{160} F_n^{i,j} \cdot x_{i+\frac{1}{2}, j+\frac{1}{2}} + \sum_{i=1}^{80} \sum_{j=1}^{80} F_p^{i,j} \cdot x_{i+\frac{1}{2}, j+\frac{1}{2}} \quad (6-15)$$

where M_n is the total negative moment; M_{np} is the negative moment caused left-hand side positive force; M_{nn} is the negative moment caused right-hand side negative force.

The total negative moments for different minimal flying heights and pitch angles

are tabulated in Table 6.4 and illustrated as Fig. 6.12.

Fig. 6.12 negative moments for varying minimal flying height and pitch angle

6.4 SURFACE FITTING FOR FORCES AND MOMENTS

6.4.1 General Procedure for Surface Fitting

The fitting was performed for the minimum flying height h and the pitch angle θ . The positive force with a fixed pitch angle decreases as the minimum flying height increases, and the corresponding negative force increases as the minimum flying height increases. A family of exponential-type functions sharing the following formats for different pitch angles can be used to fit these data

$$F_p(h) = a \cdot e^{-b \cdot h^m}$$

(6-16)

$$F_n(h) = c \cdot e^{-d \cdot h^n}$$

(6-17)

where, F_p and F_n denote the positive and the negative air bearing forces, respectively, a , b , c and d are the variable coefficients changing with θ , and m and n are constants for a slider adjusting the curvature of the fitting.

The effect from the pitch angle is, then, introduced by fitting the coefficients a , b , c and d with simple functions, such as exponential or polynomial functions

$$F_p(h, \theta) = a(\theta) \cdot e^{-b(\theta) \cdot h^m}$$

(6-18)

$$F_n(h, \theta) = c(\theta) \cdot e^{-d(\theta) \cdot h^n}$$

(6-19)

Since the moments show the same trend as the forces do, method can be used to fit the moments. Thus,

$$M_p(h, \theta) = f(\theta) \cdot e^{-u(\theta) \cdot h^i}$$

(6-20)

$$M_n(h, \theta) = w(\theta) \cdot e^{-v(\theta) \cdot h^j}$$

(6-21)

where, M_p and M_n denote the positive and the negative air bearing moments, respectively.

6.4.2 Surface Fitting for Air Bearing Forces

A family of exponential-type functions is used to fit the forces and moments for individual pitch angles as shown in Fig. 6.13. These functions are:

$$F_p(h_{\min}) = a \cdot e^{-b \cdot h_{\min}^{0.5}} \quad (6-22)$$

$$F_n(h_{\min}) = c \cdot e^{-d \cdot h_{\min}^{0.9}} \quad (6-23)$$

The effect from the pitch angle is, then, introduced by fitting the coefficients a , b , c and d shown in Table 6.5 with simple functions, such as exponential or polynomial functions. The result is shown in Fig. 6.14.

Fig. 6.13 Curve fitting in the minimum flying height direction

Table 6.5 Forces curve fitting coefficients a , b , c , and d (with 95% confidence bounds)

pitch angle (μradian)	a	b	c	d
25	0.1892	0.3094	-0.02214	0.007804
50	0.09826	0.2534	-0.02085	0.008344
77	0.06734	0.2317	-0.01908	0.008682
100	0.05434	0.2229	-0.01742	0.008772
125	0.04417	0.2115	-0.01587	0.009015
150	0.03713	0.2024	-0.01445	0.009314
175	0.03221	0.1942	-0.01309	0.009603

$$a(\theta) = 0.257 \cdot e^{-0.01542\theta} \quad (6-24)$$

$$b(\theta) = (5.561 \times 10^{-6})\theta^2 - 0.0012\theta + 0.341 \quad (6-25)$$

$$c(\theta) = (6.182 \times 10^{-5})\theta - 0.02376 \quad (6-26)$$

$$d(\theta) = (1.101 \times 10^{-5})\theta + 0.007687 \quad (6-27)$$

Fig. 6.14 Coefficients fitting in the pitch angle direction

The units for flying height, pitch angle and force are nm, μrad and N, respectively

$$F_p(h_{\min}, \theta) = 0.2569 \cdot e^{-0.01542\theta - [(5.561\text{e-}006)\theta^2 - 0.001784\theta + 0.3414] \cdot h_{\min}^{0.5}} \quad (6-28)$$

$$F_n(h_{\min}, \theta) = [(6.182\text{e-}005)\theta - 0.02376] \cdot e^{-((1.101\text{e-}005)\theta + 0.007687) \cdot h_{\min}^{0.9}} \quad (6-29)$$

$$F(h_{\min}, \theta) = F_p(h_{\min}, \theta) + F_n(h_{\min}, \theta) \quad (6-30)$$

(a) Positive air bearing force

(b) Negative air bearing force

(c) Total air bearing force

Fig. 6.15 Fitting surface for air bearing force

6.4.3 Surface Fitting for Air Bearing Moments

Fig. 6.16 Curve fitting in the minimum flying height direction

A family of exponential-type functions is used to fit for different pitch angles as shown in Fig. 6.16. These functions are expressed as

$$M_p(h_{\min}) = 10^{-4} \cdot f \cdot e^{-u \cdot h_{\min}^{0.2}} \quad (6-31)$$

$$M_n(h_{\min}) = 10^{-3} \cdot w \cdot e^{-v \cdot h_{\min}} \quad (6-32)$$

The effect from the pitch angle is, then, introduced by fitting the coefficients f , u , w and v shown in Table 6.6 with simple functions, such as exponential or polynomial functions. The result is shown in Fig. 6.17.

Table 6.6 Moments curve fitting coefficients f , u , w , and v (with 95% confidence bounds)

pitch angle (μradian)	f	u	w	v
25	5.166	2.485	-0.02121	0.01764
50	2.53	2.145	-0.01283	0.0125
77	1.538	1.915	-0.009267	0.01046
100	1.197	1.807	-0.007489	0.00953
125	0.9631	1.713	-0.006155	0.008803
150	0.7951	1.64	-0.005187	0.008498
175	0.6751	1.576	-0.004427	0.008311

$$f(\theta) = 7.926e^{-0.01936\theta} \quad (6-33)$$

$$u(\theta) = 0.000041 \times \theta^2 - 0.01385 \times \theta + 2.772 \quad (6-34)$$

$$w(\theta) = (-0.02572) \cdot e^{-0.0115\theta} \quad (6-35)$$

$$v(\theta) = 6.361 \times 10^{-7} \times \theta^2 - 0.0001813 \times \theta + 0.02102 \quad (6-36)$$

Fig. 6.17 Coefficients fitting in the pitch angle direction

$$M_p(h_{\min}) = 0.0007926e^{-0.01936\theta - (0.000041 \times \theta^2 - 0.01385 \times \theta + 2.772)} h_{\min}^{0.2} \quad (6-37)$$

$$M_n(h_{\min}, \theta) = -0.00002572e^{-0.0115\theta - (6.361 \times 10^{-7} \times \theta^2 - 0.0001813 \times \theta + 0.02102)} h_{\min} \quad (6-38)$$

$$M(h_{\min}, \theta) = M_p(h_{\min}, \theta) + M_n(h_{\min}, \theta) \quad (6-39)$$

(a) Positive air bearing moment

(b) Negative air bearing moment

(c) Total air bearing moment

Fig 6.18 Fitting surface for air bearing moment in pitch direction

6.4.4 Error Analyses

The Root Mean Square deviation of the air bearing forces is found to be 0.004757 (N). The Root Mean Square deviation of the air bearing moments is 9.63E-07 (N·m). The error can be improved by optimizing factors m , n , i , j . The calculation of the errors is shown in Appendix E.

6.5 GENERALITY OF THE SURFACE FITTING SCHEME FOR SUBAMBIENT PRESSURE SLIDERS

The abovementioned methodology could be generally employed to study other subambient pressure sliders with simple or complex surface profiles. Two more subambient sliders were exemplified.

6.5.1 Forces and Arm of the Forces obtained from the FDM Method

Fig 6.19 Profile for an simplified subambient slider

(a) Air bearing forces for a simplified slider

(b) Arms of air bearing forces for a simplified slider

Fig. 6.20 Air bearing forces and arms of forces for a simplified slide

A simplified subambient Pico-slider was studied in chapter 5 using the finite difference method. The total air bearing forces and the arms of the forces over a spectrum of minimal flying heights and pitch angles as illustrated in Fig. 6.20 show similar trends as the S1 slider does. A similar analysis can be performed on the slider. Since the moments instead of the arms of the forces are directly used to determine the attitudes of a slider, the total air bearing moments are preferably adopted in the simulations.

6.5.2 Forces and Arm of the Forces for Travel Star Slider

A commercial travel star slider once widely used shown in Fig. 6.21 is used to recheck the idea. All the forces and moment show the same trend and can be fitted with the same procedure.

Fig. 6.21 Profile for a commercial travel star slider

CHAPTER SEVEN

UNLOADING PROCESS SIMULATIONS, TREND ANALYSES AND PARAMETRIC STUDIES

In chapter 3 and 4, the structural dynamics of a suspension was modeled as a 3-DOF lumped parameters model, where the stiffness matrices and equivalent length were obtained through finite element analysis of the suspension. In chapter 5 and 6, the air bearing forces/moments, required for simulating the load/unload process of a slider, were calculated through a newly proposed simple and efficient approach based on curve fitting of performance functions. In this chapter, the behaviors of a slider in an unloading process were studied efficiently by coupling the 3-DOF lumped parameter model with the performance functions for calculating the air bearing forces and moments. A typical unloading process was simulated. Parametric studies and trend analyses were performed to demonstrate the efficiency of the method and its potential for practical applications.

7.1 COUPLING OF DUAL-SCALE MODEL FOR APPLICATION TO UNLOADING PROCESS ANALYSIS

Unloading process, with a very large suspension deformation compared to a small slider-disk gap, is not a steady state process. Due to the nonlinearity of the air bearing, conventionally, it is difficult to predict exactly how the air bearing affects the slider behaviors. With the dual scale model, the behavior of the slider during an unloading process can be predicted efficiently by coupling the separately studied air bearing fluid

force/moment with the suspension spring force/moment. The HDI in an unloading process can be shown as Fig.7.1.

Fig. 7.1 Head-disk interface

As mentioned in chapter 5, the air bearing forces can be generally expressed by the following equations

$$F_p(h_{min}, \theta) = a(\theta) \cdot e^{-b(\theta) \cdot h_{min}^m} \quad (7-1)$$

$$F_n(h_{min}, \theta) = c(\theta) \cdot e^{-d(\theta) \cdot h_{min}^n} \quad (7-2)$$

$$F_a(h_{min}, \theta) = F_p(h_{min}, \theta) + F_n(h_{min}, \theta) \quad (7-3)$$

and the moments can be generally expressed by

$$M_p(h_{min}, \theta) = f(\theta) \cdot e^{-u(\theta) \cdot h_{min}^i} \quad (7-4)$$

$$M_n(h_{min}, \theta) = w(\theta) \cdot e^{-v(\theta) \cdot h_{min}^j} \quad (7-5)$$

$$M_a(h_{min}, \theta) = M_p(h_{min}, \theta) + M_n(h_{min}, \theta) \quad (7-6)$$

The suspension is studied by substituting the air bearing effects with structural constraints expressed as

$$\begin{Bmatrix} F_r - F_{r0} \\ F_s - F_{s0} \\ M_\theta - M_{\theta0} \end{Bmatrix} = \begin{bmatrix} k_{rr} & k_{rs} & k_{r\theta} \\ k_{sr} & k_{ss} & k_{s\theta} \\ k_{\theta r} & k_{\theta s} & k_{\theta\theta} \end{bmatrix} \begin{Bmatrix} z_r - z_{r0} \\ z_s - z_{s0} \\ \theta - \theta_0 \end{Bmatrix} \quad (7-7)$$

Assuming the instant that the lift tab touches the ramp is the commencing point of the unloading process and t denotes the unloading time, from the dual-scale model we

know: $z_s|_{t=0} = z_{s0}$, $\theta|_{t=0} = \theta_0$, $z_{r0} = 0$, $F_{r0} = 0$,

$$F_{s0} = F_{a0} = a(\theta_0) \cdot e^{-b(\theta_0) \cdot h_0^m} + c(\theta_0) \cdot e^{-d(\theta_0) \cdot h_0^n} \quad (7-8)$$

and

$$M_{\theta 0} = M_{a0} = f(\theta_0) \cdot e^{-u(\theta_0) \cdot h_0^i} + w(\theta_0) \cdot e^{-v(\theta_0) \cdot h_0^j} \quad (7-9)$$

where, h_0 and θ_0 are the minimal flying height and pitch angle in normal operational condition, respectively. F_{s0} and $M_{\theta 0}$ are the equivalent constraint force and moment for normal operational condition, respectively. F_{a0} and M_{a0} are the air bearing force and moment for normal operational condition, respectively.

Given the lateral velocity of the suspension lift-tab v_y , the vertical velocity of the lift-tab v_r is given by $v_r = v_y \cdot \tan \beta$, where β is the ramp angle. Thus, $z_r = v_r t = v_y \cdot t \cdot \tan \beta$, where t is the unloading time. So the position of the lift tab is given

$$\Delta z_r = z_r = v_r(t) \cdot t \quad (7-10)$$

As we only care about the forces and moments on the slider,

$$\begin{cases} F_a - F_{a0} = F_s - F_{s0} = k_{sr} z_r \\ M_a - M_{a0} = M_\theta - M_{\theta 0} = k_{\theta r} z_r \end{cases} \quad (7-11)$$

thus

$$\begin{cases} a(\theta) \cdot e^{-b(\theta) h^m} + c(\theta) \cdot e^{-d(\theta) h^n} = k_{sr} z_r + F_{s0} \\ f(\theta) \cdot e^{-u(\theta) h^i} + w(\theta) \cdot e^{-v(\theta) h^j} = k_{\theta r} z_r + M_{\theta 0} \end{cases} \quad (7-12)$$

where F_r is the force applied by the ramp (up is positive), and z_r (up is positive) is the displacement at the tab measured from the equilibrium position. F_s is the force applied on the slider (up is positive), and z_s (up is positive) is the displacement of the center of the mass of the slider measured from the equilibrium position. M_θ is the moment applied on the slider from the air bearing (counter-clockwise is positive), and θ (clock-wise is positive) is the variation of the pitch angle of the slider measured from the equilibrium position.

Apparently, this is a nonlinear system with two equations and two unknown variables. In numerical method, solving a nonlinear system of equations $\vec{F}_v(h_{\min}, \theta)$ involves finding a solution such that every equation in the nonlinear system is 0. The objective is to find a $h_{\min}$ and a θ to let $\vec{F}_v(h_{\min}, \theta) = \begin{Bmatrix} F(h_{\min}, \theta) \\ M(h_{\min}, \theta) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$, where $F(h_{\min}, \theta)$ is the net force acting on the slider from both the suspension side and the air bearing side. $M(h_{\min}, \theta)$ is the net moment acting on the slider from both the suspension side and the air bearing side. The MATLAB routine 'fsolve' is used to solve sets of nonlinear algebraic equations. Actually, the unconstrained optimization is involved to solve the nonlinear n equations with n unknowns. Thus the coupled unloading simulation model for the ST1 head disk interface can be expressed as eq. (7-13)

$$\begin{cases} k_{sr}z_r + F_{s0} = 0.2569e^{-0.01542\theta - (0.000005561\theta^2 - 0.001784\theta + 0.3414)h_{\min}^{0.5}} \\ \quad + (0.00006182\theta - 0.02376)e^{-(0.00001101\theta + 0.007687)h_{\min}^{0.9}} \\ k_{\theta r}z_r + F_{\theta 0} = 0.00007920e^{-0.01936\theta - (0.00004101\theta^2 - 0.01385\theta + 2.772)h_{\min}^{0.2}} \\ \quad - 0.00002572e^{-0.01150\theta - (0.0000006361\theta^2 - 0.0001813\theta + 0.02102)h_{\min}} \end{cases} \quad (7-13)$$

where k_{sr} and $k_{\theta r}$ are coupling stiffness coefficients changeable with different stages.

7.2 UNLOADING SIMULATION WITH NORMAL CONDITIONS

As the results from the FEM model as expressed by eq. (4-14) to (4-16), the coupling stiffnesses for the suspension in different stages are

$$k_{sr}^{(d)} = -2.4724 \times 10^3 \text{ (N/m)}, k_{\theta r}^{(d)} = 1.6078 \times 10^{-2} \text{ (N)};$$

$$k_{sr}^{(g)} = -0.0119 \times 10^3 \text{ (N/m)}, k_{\theta r}^{(g)} = 2.9162 \times 10^{-3} \text{ (N)};$$

$$k_{sr}^{(L)} = -0.4811 \times 10^3 \text{ (N/m)}, k_{\theta r}^{(L)} = 3.8280 \times 10^{-1} \text{ (N)};$$

Given a lateral velocity of the suspension lift-tab v_y , the vertical velocity of the lift-tab v_z is given by $v_z = v_y \tan \beta$, where β is the ramp angle. Thus, $z_r = v_z t = v_y t \tan \beta$, where t is the process time. For any position of the lift-tab z_r , the attitude of the slider can be determined by solving (7-13) for h and θ . The MATLAB routine 'fsolve' is used to solve sets of nonlinear algebraic equations. When the lateral velocity is given as 0.1 m/s and the ramp slope is 12° , the air bearing separates at the position with the minimum flying height of 79 nm and the pitch angle of $115 \mu\text{rad}$. The unloading time is 8.8 ms. The history of h and F_a of the simulation are shown in Fig. 7.2.

Fig. 7.2 Histories of h and F_a

As a comparison, the simulation results for a similar design but with different parameters are presented in Fig. 7.3. The curves were obtained from UC Berkeley, CML code. Three unloading stages were also observed [79].

Fig. 7.3 Histories of the air bearing force from CML code

The limiter takes effect in helping lift the slider with this design. The histories of

the forces and moments tracked a 3D curve on the performance surface as shown in Fig. 7.4 and Fig. 7.5.

Fig. 7.4 Histories of air bearing forces on the performance surface

Fig. 7.5 Histories of air bearing moments on the performance surface

The net air bearing force changed with a strong nonlinearity in the unloading process. Having known the histories of the flying attitude of a slider, the maximum suction force and the air bearing separation time can be obtained.

Traditionally, a similar unloading simulation by previous conventional approaches might take hours or even days. As tested on a PC computer with a 1.86-GHz Pentium processor and a 1-GB RAM, the calculation for the unloading duration (up to the air bearing separation) took only several seconds with the new scheme proposed in this study or no more than one minute depending on the number of the numerical steps and the parameters of the suspension.

7.3 Parametric Studies

7.3.1 Effect of $k_{sr}^{(g)}$ on unloading processes

Since the unloading duration in dimple engaged stage is very short and plays a trivial part to the whole unloading time, we concentrated the studies on the dimple separation stage. By increasing $k_{sr}^{(g)}$ up to -23N/mm and keeping other parameters unchanged, limiter never touched the load beam in the unloading process. Only two stages were observed as shown in Fig. 7.6 (a). As k_{sr} were increased to a higher value, the slider separated from the disk at a lower minimum flying height with a shorter unloading time. The value of $k_{sr}^{(g)}$ affected the unloading processes significantly. The change of the pitch angle may go inconsistently with the increase of the minimum flying height but always downward at the end as shown in Fig 7.6. (b).

Fig. 7.6 Histories of h and θ for varying k_{sr}

θ vs. h for varying k_{sr} (N) were shown in Fig. 7.7.

Fig. 7.7 θ vs. h for varying k_{sr}

7.3.2 Effect of $k_{\theta r}^{(g)}$ on unloading processes

Assume a dual stage design with $k_{sr}^{(g)} = -30\text{N/mm}$ were realized. By changing the stiffness coefficient $k_{\theta r}^{(g)}$ to a larger value and keeping other parameters unchanged, the air bearing broke up with a shorter unloading as shown in Fig. 7.8.

Fig. 7.8 Histories of h and θ for varying $k_{\theta r}$

The value of $k_{\theta r}^{(g)}$ also affected the unloading processes significantly and the change of the pitch angle also presented a downward trend at the end of the unloading process.

7.4 Trend Analyses

7.4.1 Air Bearing Separation Point

When the air bearing force F_a reaches the maximum suction force, the air bearing

breaks up and the slider is pulled off from the disk. Since the pitch angle θ changes in the unloading process, the maximum suction force can be different from that at the minimum point, h_s , in the air bearing force curve for a constant θ , such as that shown in Fig. 7.9.

Fig. 7.9 History of F_a for a fixed pitch angle

The condition for a maximum suction force to occur is given by

$$\frac{\delta F_a(h, \theta(h))}{\delta h} = \frac{\partial F_a}{\partial h} + \frac{\partial F_a}{\partial \theta} \cdot \frac{\partial \theta}{\partial h} = 0 \quad (7-14)$$

Assume that $\partial F_a / \partial h < 0$, as commonly expected. If $\theta'(h) > 0$, $\partial F_a / \partial \theta > 0$, and the air bearing breaks down at the uphill side of the curve. If $\theta'(h) < 0$, $\partial F_a / \partial \theta < 0$, and the air bearing breaks down at the downhill side, causing an earlier separation. Where $g'(h)$

7.4.2 Sufficient Condition for No Head-Disk Contact

Look at the history of h shown in Fig. 7.1, the slider went down as the lift tab was raised at the end of dimple engaged stage. To avoid the head-disk contact, it is preferred that the minimum flying height goes up or does not go down too much as the lift tab was raised. To ensure the slider to go up during the whole unloading processes, (7-15) should be satisfied.

$$\frac{\delta F_a(h, \theta(h))}{\delta h} = \frac{\partial F_a}{\partial h} + \frac{\partial F_a}{\partial \theta} \cdot \frac{\partial \theta}{\partial h} \leq 0 \quad (7-15)$$

Based on the data in Fig. 6.13 and 6.16, $\partial F_a/\partial h \leq 0$ and $\partial F_a/\partial \theta < 0$ in normal unloading processes before the air bearing breaks up. Thus,

$$\frac{\partial \theta}{\partial h} \geq - \left| \frac{\partial F_a}{\partial h} \right| / \left| \frac{\partial F_a}{\partial \theta} \right| \quad (7-16)$$

This means that if θ decreases too steeply when the minimum flying height increases, the slider may go down instead of going up, potentially threatening the disk.

7.5 Error Analyses

There are several major factors affecting the accuracy of the simulation: Errors caused by simplification of the physical structure; Errors caused by numerical solution schemes for the air bearing model and the suspension model; Errors caused by the curve fitting and the surface fitting; Errors caused by the round-off of the coefficients in the fitting functions. Errors caused by the quasi-static simplification.

7.5.1 Errors caused by simplification of the physical structure

The air bearing was modeled with 1.5 order modified Reynolds equation which was widely accepted by HDD industrials. The comprehensive finite element suspension model was constructed with a commercial software package. The parameters for the simplified lumped parameter suspension model were obtained from the FEM model and verified with experiments. So we think we can trust the models.

7.5.2 Error caused by numerical solution schemes for the air bearing model and the suspension model

The modified Reynolds equation was solved with Berkeley CML code, which employs finite volume method. The finite element suspension model was solved with the solver embedded in the commercial FEM software package. So the results could be trusted.

7.5.3 Errors caused by the curve fitting and the surface fitting

To facilitate the analysis of the air bearing design, we prefer to use some simple functions to characterize the air bearing forces and moments. We used m , n , i , and j to adjust the curvature of the curves and surfaces to fit the data better. If more precision is required, segmented curves and surfaces could also be considered. Anyway, we think we can have some ways to fit the data precisely without fundamentally changing the philosophy of the method.

7.5.4 Errors caused by the round-off of the coefficients in the fitting functions

The rounding error may exert additional errors into the fitting functions. We can improve the functions by increasing the precision of the floating point number.

7.5.5 Errors caused by the quasi-static simplification

Normally, dynamic force will have a larger effect by considering the mass and the damping coefficient of a system. However, if the velocity and the acceleration are low, the system can be studied quasi-statically by ignoring the dynamic effects. Although the quasi-static simplification exerts additional errors to the simulation, the procedure simplifies the simulation significantly. In the simulation, we assume the unloading velocity is a constant and relatively low compared to the high frequency vibration. In

the paper, we discussed the problem, and it becomes the fundamental to the dual-scale model.

CHAPTER EIGHT

CONCLUSIONS AND FUTURE WORK

8.1 CONCLUSIONS

In this work, a novel and efficient methodology was developed to analyze the behavior of a subambient pressure slider during the unloading process.

After analyzing the structure of the suspension and the characteristics of the air bearing, a dual-scale model was proposed to model the head disk interface. The slider air bearing and the structure of the suspension are modeled and analyzed separately. The milli-meter scale suspension, whose parameters were estimated using FEM and experiments, was based on a 3-DOF lumped parameter dynamic model. The slider air bearing, modeled by modified Reynolds equations, were solved by the finite difference method and the finite volume method. The air bearing forces/moments of a slider obtained from the nanometer scale air bearing model during the L/UL was generally represented by analytical functions obtained by fitting two-dimensional discrete data. Coupling the dual-scale models, the instantaneous flying attitude of the slider and the unloading time were obtained efficiently and fairly accurately with short computation time and power required. This proposed new methodology provides a simple and efficient tool for parametric analyses in practical design stage.

With the simulation, we can see that: (1) the methodology is efficient for simulation; (2) The existing HDI design experiences three stages in a normal unloading process. (3) The air bearing suction force leads air bearing separation; (4) the slider may go down when the lift-tab goes up, which potentially threaten the disk surface; (5) The stiffness coefficients of the suspension affect unloading process apparently. By changing the stiffness of the suspension, a three stage design can be degraded to a two stage process. (6) The parametric studies and optimizations based on trend analyses are instinctually available.

8.2 FUTURE WORKS

To verify the results from the efficient scheme, a full model by coupling the solution of the dynamic air bearing model with the solution of the dynamic 3DOF suspension model is expected to be developed. As complementary works to the simulations, experiments play key roles in improving the hard disk drive design. In this work, a drive level testing platform was earlier designed and fabricated. A revised platform is proposed here for possible future work.

8.2.1 Drive Level Testing Platform

As complementary works to the simulations, experiments play key roles in improving the hard disk drive design. In this work, a drive level testing platform was earlier designed and fabricated. A revised platform is proposed here for possible future work.

As the actuator arm, the spindle and the ramp in a hard disk drive are all mounted on a single base plate, it is difficult to decouple the factors affecting the tribological phenomena during the load/unload processes. An effective way to monitor the behavior of the load/unload processes is by cutting the base plate into several pieces and studying them individually. In practice, the ramp is cut off from the base plate. Then their original spatial positions need to be recovered with additional mechanical supporters. The original servo control system is still kept as a whole to realize the load/unload processes. The histories of the load force on the ramp will be recorded with a cantilever beam sensor as shown in fig. 8.1 (a) while the pitch angle and the roll angle are measured with an existing optical system as shown in fig. 8.1 (b). More details can be revealed with the real time data obtained from the load/unload processes. During the cut off, major components, such as the spindle motor and the PCB, must be kept integral. The flexible cable cannot be distorted intensively. Enough clearance must be hold for the route of the laser beam.

(a) ramp load force measurement

(b) pitch/roll angle measurement

Fig. 8.1 parameter measurement for the load/unload processes

(a) Assembly drawing for the drive level testing platform

(b) Schematic for adjusting HDD supporter (c) Schematic for adjusting ramp supporter

Fig 8.2 Drive level testing platform

A drive level testing platform shown in Fig. 8.2 has been fabricated to recover the original position relationship between the ramp and the cut off base plate by adjusting the hard disk drive supporter and ramp supporter with slots and micrometer stages.

When the mechanical components are detached for individual studies, the servo loop for the HDD control has also been damaged with this design, disabling the original control system. Thus a revised mechanical platform with an isolated control and measurement system is raised. In the revised platform, the HDDs are divided into three pieces, the actuator arm part, the disk part and the ramp part. Each component is supported with an adjustable supporter shown as fig. 8.3.

Fig. 8.3 Revised drive level testing platform

The isolated measurement and control system is shown in fig 8.4.

Fig 8.4 Measurement & control system for revised platform

8.2.2 Altitude Chamber

As the flying height goes lower, an effective way using an existing slider to simulate a future slider lowers the air density by putting the drive level testing platform into an altitude chamber. The partial contact conditions at the head-disk interface due to the vacuum will give an opportunity to study similar conditions encountered in future generation hard disk drives with a drastic reduction of the head disk spacing. The trends of the lubricant transfer and debris deposition on the air bearing surface of the head slider might be closely observed with an optical system during and after repeated

load/unload cycles. The evolution of the material transfer process will be examined together with its impact on the tribological performance of the head-disk interface. Since we should simulate a miniature hard disk drive working at the height of 10,000 meters (30,000 feet), the pressure at an altitude of 10 km is about 22600 Pa, in the region of Low vacuum. A prototype of the altitude chamber is illustrated as fig. 8.5.

Fig. 8.5 structure of the altitude chamber

It is designed with a stainless frame. Transparent acryl cover gives the observer a chance to check the status inside the chamber. Three round holes with embedded special transparent glass let the laser beam from a laser Doppler vibrometer going through with a low refractive index.

8.2.3 Simulation for Loading Process

Because of the complexity of the head disk interface, a loading process illustrated as fig. 8.6 is not a simply reverse process of an unloading process and it must be studied separately. A loading process may experience different stages depending on the setup and the design of the HDI. The efficient scheme for analyzing the nonlinear

air bearing force and moment is still usable for loading analyses, but the model for the suspension may be revised.

Fig 8.6. An assumed loading process

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Appendix A Air bearing force/moment for varying h_{min} and θ

h_{min} denotes minimal flying height; θ denotes pitch angle

Table A_1 air bearing forces/moments for varying h_{min} when $\theta = 25\mu\text{rad}$

h_{min} (nm)	positive force (N)	negative force (N)	positive moment (N·m)	negative moment (N·m)
5	0.096469	-0.020451	1.70×10^{-5}	-2.04×10^{-5}
6	0.089335	-0.02054	1.49×10^{-5}	-1.97×10^{-5}
7	0.083604	-0.020497	1.33×10^{-5}	-1.91×10^{-5}
8	0.078385	-0.020543	1.19×10^{-5}	-1.85×10^{-5}
9	0.07375	-0.020614	1.07×10^{-5}	-1.80×10^{-5}
10	0.069907	-0.020584	9.82×10^{-6}	-1.75×10^{-5}
20	0.045811	-0.020149	5.18×10^{-6}	-1.37×10^{-5}
30	0.034022	-0.019361	3.58×10^{-6}	-1.13×10^{-5}
40	0.02664	-0.018466	2.76×10^{-6}	-9.56×10^{-6}
50	0.021552	-0.017638	2.27×10^{-6}	-8.29×10^{-6}
60	0.017822	-0.016746	1.95×10^{-6}	-7.26×10^{-6}
70	0.014989	-0.015913	1.72×10^{-6}	-6.43×10^{-6}
80	0.012767	-0.015177	1.56×10^{-6}	-5.76×10^{-6}
90	0.010989	-0.014488	1.44×10^{-6}	-5.19×10^{-6}
100	0.0095395	-0.0138	1.34×10^{-6}	-4.69×10^{-6}
200	0.0030902	-0.0083206	9.36×10^{-7}	-1.94×10^{-6}
300	0.0013807	-0.0051578	7.81×10^{-7}	-9.06×10^{-6}
400	0.00082752	-0.0033532	6.72×10^{-7}	-4.57×10^{-7}
500	0.00064965	-0.0023267	5.96×10^{-7}	-2.54×10^{-7}
600	0.00059836	-0.0016884	5.31×10^{-7}	-1.54×10^{-7}
700	0.00058969	-0.0012735	4.75×10^{-7}	-1.03×10^{-7}
800	0.00059304	-0.00098212	4.23×10^{-7}	-7.46×10^{-8}
900	0.00059595	-0.0007689	3.77×10^{-7}	-5.78×10^{-8}
1000	0.00058952	-0.00061477	3.36×10^{-7}	-4.72×10^{-8}

Table A_2 air bearing forces/moments for varying h_{min} when $\theta = 50\mu\text{rad}$

h_{min} (nm)	positive force (N)	negative force (N)	positive moment (N·m)	negative moment (N·m)
5	0.056223	-0.019749	1.27×10^{-5}	-1.23×10^{-5}
6	0.053038	-0.019765	1.27×10^{-5}	-1.22×10^{-5}
7	0.050271	-0.019651	1.05×10^{-5}	-1.19×10^{-5}
8	0.047855	-0.019605	9.68×10^{-6}	-1.17×10^{-5}
9	0.045709	-0.019568	8.95×10^{-6}	-1.15×10^{-5}
10	0.043779	-0.019353	8.33×10^{-6}	-1.13×10^{-5}
20	0.031297	-0.018595	4.88×10^{-6}	-9.68×10^{-6}
30	0.024379	-0.017664	3.46×10^{-6}	-8.40×10^{-6}
40	0.019775	-0.016789	2.69×10^{-6}	-7.39×10^{-6}
50	0.016436	-0.016016	2.22×10^{-6}	-6.57×10^{-6}
60	0.013903	-0.015182	1.91×10^{-6}	-5.87×10^{-6}
70	0.011909	-0.014479	1.69×10^{-6}	-5.29×10^{-6}
80	0.010308	-0.013773	1.52×10^{-6}	-4.79×10^{-6}
90	0.0089948	-0.013108	1.40×10^{-6}	-4.35×10^{-6}
100	0.0079032	-0.012468	1.30×10^{-6}	-3.96×10^{-6}
200	0.0028014	-0.007389	8.92×10^{-7}	-1.69×10^{-6}
300	0.0013581	-0.0045599	7.43×10^{-7}	-7.94×10^{-7}

400	0.00088541	-0.0029633	6.43×10^{-7}	-4.07×10^{-7}
500	0.00072653	-0.0020619	5.72×10^{-7}	-2.31×10^{-7}
600	0.00067437	-0.0015027	5.12×10^{-7}	-1.44×10^{-7}
700	0.00066046	-0.0011355	4.58×10^{-7}	-9.89×10^{-8}
800	0.00065477	-0.00087796	4.09×10^{-7}	-7.32×10^{-8}
900	0.00064929	-0.00068922	3.65×10^{-7}	-5.78×10^{-8}
1000	0.00063554	-0.0005529	3.26×10^{-7}	-4.79×10^{-8}

Table A_3 air bearing forces/moments for varying h_{min} when $\theta = 77\mu\text{rad}$

h_{min} (nm)	positive force (N)	negative force (N)	positive moment (N·m)	negative moment (N·m)
5	0.040417	-0.01816	1.08×10^{-5}	-8.95×10^{-6}
6	0.038336	-0.017992	9.94×10^{-6}	-8.81×10^{-6}
7	0.036526	-0.018033	9.20×10^{-6}	-8.73×10^{-6}
8	0.034912	-0.017772	8.56×10^{-6}	-8.57×10^{-6}
9	0.033476	-0.017702	8.00×10^{-6}	-8.47×10^{-6}
10	0.032178	-0.017595	7.50×10^{-6}	-8.35×10^{-6}
20	0.023616	-0.016804	4.63×10^{-6}	-7.40×10^{-6}
30	0.018783	-0.015961	3.35×10^{-6}	-6.59×10^{-6}
40	0.015514	-0.015165	2.63×10^{-6}	-5.91×10^{-6}
50	0.013105	-0.014407	2.1×10^{-6}	-5.33×10^{-6}
60	0.011242	-0.013688	1.87×10^{-6}	-4.83×10^{-6}
70	0.0097536	-0.013011	1.65×10^{-6}	-4.39×10^{-6}
80	0.008539	-0.012357	1.49×10^{-6}	-4.00×10^{-6}
90	0.0075314	-0.011728	1.36×10^{-6}	-3.66×10^{-6}
100	0.0066849	-0.011125	1.27×10^{-6}	-3.34×10^{-6}
200	0.0025871	-0.0064803	8.54×10^{-7}	-1.45×10^{-6}
300	0.0013781	-0.0039479	7.03×10^{-7}	-6.89×10^{-7}
400	0.00096211	-0.002593	6.17×10^{-7}	-3.63×10^{-7}
500	0.00081443	-0.0018103	5.52×10^{-7}	-2.11×10^{-7}
600	0.00075927	-0.0013271	4.95×10^{-7}	-1.36×10^{-7}
700	0.00073793	-0.0010034	4.43×10^{-7}	-9.59×10^{-8}
800	0.00072189	-0.00077811	3.96×10^{-7}	-7.26×10^{-8}
900	0.00070705	-0.00061327	3.54×10^{-7}	-5.84×10^{-8}
1000	0.00068542	-0.00049427	3.17×10^{-7}	-4.93×10^{-8}

Table A_4 air bearing forces/moments for varying h_{min} when $\theta = 100\mu\text{rad}$

h_{min} (nm)	positive force (N)	negative force (N)	positive moment (N·m)	negative moment (N·m)
5	0.032886	-0.016581	9.70×10^{-6}	-7.22×10^{-6}
6	0.032886	-0.01649	8.99×10^{-6}	-7.14×10^{-6}
7	0.029898	-0.01642	8.38×10^{-6}	-7.06×10^{-6}
8	0.028652	-0.016328	7.85×10^{-6}	-6.98×10^{-6}
9	0.027529	-0.016256	7.38×10^{-6}	-6.90×10^{-6}
10	0.026511	-0.016256	6.97×10^{-6}	-6.83×10^{-6}
20	0.019701	-0.01541	4.44×10^{-6}	-6.14×10^{-6}
30	0.015816	-0.014597	3.26×10^{-6}	-5.53×10^{-6}
40	0.013176	-0.013861	2.57×10^{-6}	-5.01×10^{-6}
50	0.011222	-0.013166	2.14×10^{-6}	-4.56×10^{-6}
60	0.0097031	-0.012492	1.84×10^{-6}	-4.15×10^{-6}
70	0.0084845	-0.011843	1.62×10^{-6}	-3.79×10^{-6}

80	0.0074844	-0.01122	1.46×10^{-6}	-3.47×10^{-6}
90	0.0066499	-0.010625	1.34×10^{-6}	-3.18×10^{-6}
100	0.0059449	-0.010058	1.24×10^{-6}	-2.91×10^{-6}
200	0.0024646	-0.0057828	8.25×10^{-7}	-1.28×10^{-6}
300	0.0014058	-0.0035093	6.78×10^{-7}	-6.14×10^{-7}
400	0.0010359	-0.0023118	5.99×10^{-7}	-3.31×10^{-7}
500	0.00089281	-0.0016215	5.37×10^{-7}	-1.98×10^{-7}
600	0.00083122	-0.001196	4.82×10^{-7}	-1.31×10^{-7}
700	0.00080032	-0.00090779	4.32×10^{-7}	-9.42×10^{-8}
800	0.00077939	-0.00070314	3.87×10^{-7}	-7.28×10^{-8}
900	0.00075664	-0.00055642	3.46×10^{-7}	-5.96×10^{-8}
1000	0.00072782	-0.00045021	3.10×10^{-7}	-5.08×10^{-8}

Table A_5 air bearing forces/moments for varying h_{min} when $\theta = 125\mu\text{rad}$

h_{min} (nm)	positive force (N)	negative force (N)	positive moment (N·m)	negative moment (N·m)
5	0.027527	-0.015083	8.78×10^{-6}	-5.94×10^{-6}
6	0.026271	-0.015013	8.20×10^{-6}	-5.88×10^{-6}
7	0.025156	-0.014942	7.69×10^{-6}	-5.83×10^{-6}
8	0.024155	-0.014871	7.25×10^{-6}	-5.77×10^{-6}
9	0.0236	-0.014835	7.00×10^{-6}	-5.74×10^{-6}
10	0.023248	-0.0148	6.85×10^{-6}	-5.71×10^{-6}
20	0.016822	-0.013988	4.26×10^{-6}	-5.14×10^{-6}
30	0.013593	-0.013256	3.16×10^{-6}	-4.67×10^{-6}
40	0.011396	-0.012565	2.52×10^{-6}	-4.26×10^{-6}
50	0.0097699	-0.011898	2.10×10^{-6}	-3.89×10^{-6}
60	0.0085043	-0.011257	1.81×10^{-6}	-3.56×10^{-6}
70	0.0074866	-0.010644	1.60×10^{-6}	-3.26×10^{-6}
80	0.0066488	-0.01006	1.44×10^{-6}	-2.99×10^{-6}
90	0.0059559	-0.009483	1.31×10^{-6}	-2.74×10^{-6}
100	0.0053636	-0.0089437	1.22×10^{-6}	-2.51×10^{-6}
200	0.002379	-0.0050916	7.98×10^{-7}	-1.11×10^{-6}
300	0.0014561	-0.0030855	6.57×10^{-7}	-5.45×10^{-7}
400	0.0011225	-0.002041	5.83×10^{-7}	-3.02×10^{-7}
500	0.00098092	-0.0014407	5.24×10^{-7}	-1.86×10^{-7}
600	0.00091366	-0.0010649	4.70×10^{-7}	-1.27×10^{-7}
700	0.00087257	-0.00081115	4.22×10^{-7}	-9.34×10^{-8}
800	0.00084142	-0.00063085	3.78×10^{-7}	-7.37×10^{-8}
900	0.00080959	-0.00050138	3.38×10^{-7}	-6.12×10^{-8}
1000	0.00077295	-0.00040745	3.04×10^{-7}	-5.27×10^{-8}

Table A_6 air bearing forces/moments for varying h_{min} when $\theta = 150\mu\text{rad}$

h_{min} (nm)	positive force (N)	negative force (N)	positive moment (N·m)	negative moment (N·m)
5	0.023772	-0.013776	8.04×10^{-6}	-5.02×10^{-6}
6	0.022743	-0.013705	7.55×10^{-6}	-4.97×10^{-6}
7	0.021822	-0.013635	7.12×10^{-6}	-4.93×10^{-6}
8	0.020998	-0.01354	6.74×10^{-6}	-4.88×10^{-6}
9	0.020241	-0.01347	6.40×10^{-6}	-4.83×10^{-6}
10	0.019548	-0.013399	6.09×10^{-6}	-4.79×10^{-6}
20	0.014785	-0.012698	4.09×10^{-6}	-4.37×10^{-6}

30	0.012013	-0.011991	3.07×10^{-6}	-3.99×10^{-6}
40	0.010122	-0.011331	2.46×10^{-6}	-3.65×10^{-6}
50	0.0087231	-0.010698	2.06×10^{-6}	-3.34×10^{-6}
60	0.0076354	-0.010095	1.78×10^{-6}	-3.06×10^{-6}
70	0.0067604	-0.0095217	1.57×10^{-6}	-2.81×10^{-6}
80	0.0060498	-0.0089558	1.41×10^{-6}	-2.58×10^{-6}
90	0.0054481	-0.0084271	1.29×10^{-6}	-2.36×10^{-6}
100	0.0049371	-0.0079312	1.19×10^{-6}	-2.17×10^{-6}
200	0.0023326	-0.0044707	7.75×10^{-7}	-9.71×10^{-7}
300	0.0015243	-0.0027122	6.42×10^{-7}	-4.88×10^{-7}
400	0.0012141	-0.0018016	5.70×10^{-7}	-2.78×10^{-7}
500	0.0010726	-0.0012786	5.12×10^{-7}	-1.77×10^{-7}
600	0.00099596	-0.00094872	4.60×10^{-7}	-1.24×10^{-7}
700	0.00094394	-0.00072541	4.13×10^{-7}	-9.34×10^{-8}
800	0.00090245	-0.0005665	3.70×10^{-7}	-7.50×10^{-8}
900	0.00086127	-0.00045227	3.32×10^{-7}	-6.31×10^{-8}
1000	0.00081697	-0.00036916	2.98×10^{-7}	-5.48×10^{-8}

Table A_7 air bearing forces/moments for varying h_{min} when $\theta = 175\mu\text{rad}$

h_{min} (nm)	positive force (N)	negative force (N)	positive moment (N·m)	negative moment (N·m)
5	0.021021	-0.01248	7.44×10^{-6}	-4.29×10^{-6}
6	0.020155	-0.01241	7.02×10^{-6}	-4.25×10^{-6}
7	0.019376	-0.012341	6.65×10^{-6}	-4.21×10^{-6}
8	0.018668	-0.012273	6.31×10^{-6}	-4.17×10^{-6}
9	0.018021	-0.012204	6.01×10^{-6}	-4.14×10^{-6}
10	0.017426	-0.012136	5.74×10^{-6}	-4.10×10^{-6}
20	0.013296	-0.011437	3.94×10^{-6}	-3.75×10^{-6}
30	0.010852	-0.010786	2.99×10^{-6}	-3.43×10^{-6}
40	0.0091851	-0.010162	2.41×10^{-6}	-3.14×10^{-6}
50	0.0079531	-0.0095691	2.02×10^{-6}	-2.88×10^{-6}
60	0.0069961	-0.0090069	1.75×10^{-6}	-2.64×10^{-6}
70	0.0062389	-0.0084465	1.54×10^{-6}	-2.42×10^{-6}
80	0.0055951	-0.007955	1.39×10^{-6}	-2.23×10^{-6}
90	0.0050782	-0.0074505	1.27×10^{-6}	-2.04×10^{-6}
100	0.0046284	-0.0070033	1.17×10^{-6}	-1.88×10^{-6}
200	0.00232	-0.0039151	7.57×10^{-7}	-8.50×10^{-7}
300	0.0016044	-0.0023831	6.31×10^{-7}	-4.40×10^{-7}
400	0.0013088	-0.0015921	5.61×10^{-7}	-2.59×10^{-7}
500	0.0011614	-0.0011381	5.04×10^{-7}	-1.70×10^{-7}
600	0.0010771	-0.00084557	4.52×10^{-7}	-1.22×10^{-7}
700	0.0010135	-0.00064916	4.05×10^{-7}	-9.40×10^{-8}
800	0.00096162	-0.00050925	3.63×10^{-7}	-7.66×10^{-8}
900	0.00091136	-0.00040843	3.26×10^{-7}	-6.52×10^{-8}
1000	0.00085956	-0.00033492	2.93×10^{-7}	-5.71×10^{-8}

Appendix B. Configurations and reaction forces of the suspension for stiffness estimation

The four corners of the slider, node 16652 and 16653 at the leading edge and node 18909 and 18914 at the trailing edge, and a central point of the lift tab, node 11032, are constrained with displacement constraints with respect to their free states. Their reaction forces are given out with ANSYS finite element model.

Table B_1 configurations/reaction forces for normal operating condition

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.595
	Force(N)	3.27100×10^{-3}	3.17890×10^{-3}	3.41710×10^{-3}	3.35930×10^{-3}	0
Displacement (mm)		Force/moment (N/N·m)				
z_r	1.595	F_r	0			
z_s	1.129	F_s	1.32263×10^{-2}			
θ (radian)	0.174996048	M_θ (N·m)	2.01287×10^{-7}			

Table B_2 configurations/reaction forces for estimating k_{rr} , k_{sr} , $k_{\theta r}$ with dimple engaged

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.58
	Force(N)	1.27420×10^{-2}	1.24110×10^{-2}	1.26270×10^{-2}	1.24620×10^{-2}	-2.76290×10^{-1}
Setup2	Disp(mm)	1.02	1.02	1.238	1.238	1.5
	Force(N)	6.32930×10^{-2}	6.16840×10^{-2}	6.17820×10^{-2}	6.10430×10^{-2}	-1.75140×10^{-1}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r	-0.1	ΔF_r	-0.1847	k_{rr}	1.847×10^3 (N/m)	
Δz_s	0	ΔF_s	0.24723	k_{sr}	-2.47238×10^3 (N/m)	
$\Delta \theta$ (radian)	0	ΔM_θ (N·m)	-1.6078E-06	$k_{\theta r}$	1.60783×10^{-2} (N)	

Table B_3 configurations/reaction forces for estimating k_{rs} , k_{ss} , $k_{\theta s}$ with dimple engaged

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.5
	Force(N)	6.3293×10^{-2}	6.1684×10^{-2}	6.1782×10^{-2}	6.1043×10^{-2}	-1.7514×10^{-1}
Setup2	Disp(mm)	1.04	1.04	1.258	1.258	1.5
	Force(N)	8.0342×10^{-2}	7.8304×10^{-2}	7.8320×10^{-2}	7.7388×10^{-2}	-2.2466×10^{-1}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r	0	ΔF_r	-4.95200×10^{-2}	k_{rs}	-2.476×10^3 (N/m)	

Δz_s	0.02	ΔF_s	6.65520×10^{-2}	k_{ss}	$3.3276 \times 10^3 (\text{N/m})$
$\Delta \theta$ (radian)	0	$\Delta M_\theta (\text{N} \cdot \text{m})$	-4.84569×10^{-7}	$k_{\theta s}$	$-2.42284 \times 10^{-2} (\text{N})$

Table B_4 configurations/reaction forces for estimating $k_{r\theta}$, $k_{s\theta}$, $k_{\theta\theta}$ with dimple engaged

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.5
	Force(N)	6.3293×10^{-2}	6.1684×10^{-2}	6.17820×10^{-2}	6.10430×10^{-2}	-1.7514×10^{-1}
Setup2	Disp(mm)	1.03	1.03	1.228	1.228	1.5
	Force(N)	6.3551×10^{-2}	6.1942×10^{-2}	6.18170×10^{-2}	6.10810×10^{-2}	-1.7553×10^{-1}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r	0	ΔF_r	-0.00026	$k_{r\theta}$	$1.6728 \times 10^{-2} (\text{N})$	
Δz_s	0	ΔF_s	0.000368	$k_{s\theta}$	$-2.3345 \times 10^{-2} (\text{N})$	
$\Delta \theta$ (radian)	-0.01577147	$\Delta M_\theta (\text{N} \cdot \text{m})$	-2.73109×10^{-7}	$k_{\theta\theta}$	$1.7317 \times 10^{-5} (\text{N} \cdot \text{m})$	

Table B_5 configurations/reaction forces for estimating k_{rr} , k_{sr} , $k_{\theta r}$ with dimple separated

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.61
	Force(N)	-4.0976×10^{-4}	-4.1000×10^{-4}	-1.4108×10^{-4}	-1.5828×10^{-4}	1.0761×10^{-2}
Setup2	Disp(mm)	1.02	1.02	1.238	1.238	1.66
	Force(N)	-6.1665×10^{-4}	-6.1783×10^{-4}	-2.2838×10^{-4}	-2.4919×10^{-4}	1.1495×10^{-2}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r	0.05	ΔF_r	0.000734	k_{rr}	$0.01468 \times 10^3 (\text{N/m})$	
Δz_s	0	ΔF_s	-0.00059293	k_{sr}	$-0.01186 \times 10^3 (\text{N/m})$	
$\Delta \theta$ (radian)	0	$\Delta M_\theta (\text{N} \cdot \text{m})$	1.45808×10^{-7}	$k_{\theta r}$	$2.91617 \times 10^{-3} (\text{N})$	

Table B_6 configurations/reaction forces for estimating k_{rs} , k_{ss} , $k_{\theta s}$ with dimple separated

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.66
	Force(N)	-6.1665×10^{-4}	-6.1783×10^{-4}	-2.2838×10^{-4}	-2.4919×10^{-4}	1.1495×10^{-2}
Setup2	Disp(mm)	1.04	1.04	1.258	1.258	1.66
	Force(N)	-4.7990×10^{-4}	-4.7986×10^{-4}	-1.9764×10^{-4}	-2.1592×10^{-4}	1.1257×10^{-2}
Displacement (mm)		Force/moment (N/N·m)				

Δz_r	0	ΔF_r	-0.000238	k_{rs}	-0.0119×10^3 (N/m)
Δz_s	0.02	ΔF_s	0.00033873	k_{ss}	0.0169365×10^3 (N/m)
$\Delta \theta$ (radian)	0	ΔM_θ (N·m)	-1.29903×10^{-7}	$k_{\theta s}$	-6.49514×10^{-3} (N)

Table B_7 configurations/reaction forces for estimating $k_{r\theta}$, $k_{s\theta}$, $k_{\theta\theta}$ with dimple separated

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.66
	Force(N)	-6.1665×10^{-4}	-6.1783×10^{-4}	-2.2838×10^{-4}	-2.49190×10^{-4}	1.1495×10^{-2}
Setup2	Disp(mm)	1.04	1.04	1.218	1.218	1.66
	Force(N)	-3.4738×10^{-4}	-3.4141×10^{-4}	-3.9986×10^{-4}	-4.12680×10^{-4}	1.1400×10^{-2}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r (mm)	0	ΔF_r (N)	-9.5×10^{-5}	$k_{r\theta}$	3.0041×10^{-3} (N)	
Δz_s (mm)	0	ΔF_s (N)	0.00021072	$k_{s\theta}$	-6.66345×10^{-3} (N)	
$\Delta \theta$ (radian)	-0.03162323	ΔM_θ (N·m)	-5.42927×10^{-7}	$k_{\theta\theta}$	1.71686×10^{-3} (N·m)	

Table B_8 configurations/reaction forces for estimating k_{rr} , k_{sr} , $k_{\theta r}$ with Limiter engaged

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.9
	Force(N)	-3.4938×10^{-2}	-3.4605×10^{-2}	3.8139×10^{-3}	3.7884×10^{-3}	5.2495×10^{-2}
Setup2	Disp(mm)	1.02	1.02	1.238	1.238	2.2
	Force(N)	-1.1865×10^{-1}	-1.1620×10^{-1}	1.4500×10^{-2}	1.4074×10^{-2}	1.5085×10^{-1}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r	0.3	ΔF_r	9.83550×10^{-2}	k_{rr}	0.32785×10^3 (N/m)	
Δz_s	0	ΔF_s	-1.44335×10^{-1}	k_{sr}	-0.4811×10^3 (N/m)	
$\Delta \theta$ (radian)	0	ΔM_θ (N·m)	0.000114841	$k_{\theta r}$	3.82803×10^{-1} (N)	

Table B_9 configurations/reaction forces for estimating k_{rs} , k_{ss} , $k_{\theta s}$ with Limiter engaged

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.9
	Force(N)	-3.4938×10^{-2}	-3.4605×10^{-2}	3.8139×10^{-3}	3.7884×10^{-3}	5.2495×10^{-2}

Setup2	Disp(mm)	1.04	1.04	1.258	1.258	1.9
	Force(N)	-2.6454×10^{-2}	-2.6210×10^{-2}	2.7738×10^{-3}	2.7490×10^{-3}	4.2757×10^{-2}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r	0	ΔF_r	-9.73800×10^{-3}	k_{rs}	-0.4869×10^3 (N/m)	
Δz_s	0.02	ΔF_s	1.47995×10^{-2}	k_{ss}	0.73997×10^3 (N/m)	
$\Delta \theta$ (radian)	0	ΔM_θ (N·m)	-1.16879×10^{-5}	$k_{\theta s}$	5.8439×10^{-1} (N)	

Table B_10 configurations/reaction forces for estimating $k_{r\theta}$, $k_{s\theta}$, $k_{\theta\theta}$ with Limiter engaged

		node 16652	node 16653	node 18909	node 18914	node 11032
Setup 1	Disp(mm)	1.02	1.02	1.238	1.238	1.66
	Force(N)	-6.1665×10^{-4}	-6.1783×10^{-4}	-2.2838×10^{-4}	-2.4919×10^{-4}	1.1495×10^{-2}
Setup2	Disp(mm)	1.04	1.04	1.218	1.218	1.9
	Force(N)	-2.4407×10^{-2}	-2.4138×10^{-2}	2.2587×10^{-3}	2.2268×10^{-3}	4.0838×10^{-2}
Displacement (mm)		Force/moment (N/N·m)				
Δz_r (mm)	0	ΔF_r (N)	-1.16570×10^{-2}	$k_{r\theta}$	3.6862×10^{-1} (N)	
Δz_s (mm)	0	ΔF_s (N)	1.78812×10^{-2}	$k_{s\theta}$	-5.6544×10^{-1} (N)	
$\Delta \theta$ (radian)	-0.03162323	ΔM_θ (N·m)	-1.48668×10^{-5}	$k_{\theta\theta}$	4.70122×10^{-1} (N·m)	

Appendix C Data for Unloading Process Analyses using FEM

Assume that the slider is sucked by the air bearing force on the proximity of the surface of the disk. The displacement constraints at node 11652, 11653, 18909, and 18914 are given as 1.02mm, 1.02mm, 1.238mm, and 1.238mm, respectively. Notice that when the displacement of the lift tab is given 1.5950mm, there is no external force on it. It is considered as the starting point of the unloading process.

Table C_1 Forces on the slider nodes with respect to varying lift tab positions

Displacement of lift tab 11032 (mm)	Force on the Slider node 16652 (N)	Force on the Slider node 16653 (N)	Force on the Slider node 18909 (N)	Force on the Slider node 18914 (N)	Force on the lift tab node 11032 (N)
1.595	3.27100×10^{-3}	3.17890×10^{-3}	3.41710×10^{-3}	3.35930×10^{-3}	0.00000
1.596	2.63640×10^{-3}	2.56030×10^{-3}	2.79990×10^{-3}	2.74930×10^{-3}	1.85130×10^{-3}
1.597	2.00480×10^{-3}	1.94470×10^{-3}	2.18580×10^{-3}	2.14240×10^{-3}	3.69340×10^{-3}
1.598	1.37330×10^{-3}	1.32920×10^{-3}	1.57170×10^{-3}	1.53540×10^{-3}	5.53550×10^{-3}
1.599	7.41810×10^{-4}	7.13610×10^{-4}	9.57660×10^{-4}	9.28520×10^{-4}	7.37750×10^{-3}
1.6	1.10320×10^{-4}	9.80690×10^{-5}	3.43590×10^{-4}	3.21610×10^{-4}	9.21950×10^{-3}
1.6005	-2.05430×10^{-4}	-2.09690×10^{-4}	3.65660×10^{-5}	1.81590×10^{-5}	1.01410×10^{-2}
1.6007	-3.31720×10^{-4}	-3.32800×10^{-4}	-8.62440×10^{-5}	-1.03220×10^{-4}	1.05090×10^{-2}
1.601	-3.72520×10^{-4}	-3.72590×10^{-4}	-1.25370×10^{-4}	-1.41910×10^{-4}	1.06280×10^{-2}
1.61	-4.09760×10^{-4}	-4.10000×10^{-4}	-1.41080×10^{-4}	-1.58280×10^{-4}	1.07610×10^{-2}
1.62	-4.51130×10^{-4}	-4.51570×10^{-4}	-1.58540×10^{-4}	-1.76460×10^{-4}	1.09070×10^{-2}
1.63	-4.92510×10^{-4}	-4.93130×10^{-4}	-1.76000×10^{-4}	-1.94640×10^{-4}	1.10540×10^{-2}
1.64	-5.33890×10^{-4}	-5.34700×10^{-4}	-1.93460×10^{-4}	-2.12830×10^{-4}	1.12010×10^{-2}
1.66	-6.16650×10^{-4}	-6.17830×10^{-4}	-2.28380×10^{-4}	-2.49190×10^{-4}	1.14950×10^{-2}
1.68	-6.99400×10^{-4}	-7.00970×10^{-4}	-2.63300×10^{-4}	-2.85560×10^{-4}	1.17880×10^{-2}
1.7	-7.82160×10^{-4}	-7.84100×10^{-4}	-2.98220×10^{-4}	-3.21920×10^{-4}	1.20820×10^{-2}
1.75	-9.89050×10^{-4}	-9.91930×10^{-4}	-3.85510×10^{-4}	-4.12840×10^{-4}	1.28160×10^{-2}
1.76	-1.03040×10^{-3}	-1.03350×10^{-3}	-4.02970×10^{-4}	-4.31020×10^{-4}	1.29620×10^{-2}
1.77	-1.07180×10^{-3}	-1.07510×10^{-3}	-4.20430×10^{-4}	-4.49200×10^{-4}	1.31090×10^{-2}
1.775	-1.85400×10^{-3}	-1.77480×10^{-3}	-3.15180×10^{-4}	-3.67540×10^{-4}	1.39830×10^{-2}
1.78	-3.19530×10^{-3}	-3.10470×10^{-3}	-1.35520×10^{-4}	-1.87280×10^{-4}	1.55150×10^{-2}
1.785	-4.53610×10^{-3}	-4.44460×10^{-3}	4.53480×10^{-5}	-2.53690×10^{-6}	1.70500×10^{-2}
1.79	-5.69800×10^{-3}	-5.44620×10^{-3}	2.00390×10^{-4}	1.05910×10^{-4}	1.83210×10^{-2}
1.8	-8.37550×10^{-3}	-8.11810×10^{-3}	5.63880×10^{-4}	4.75900×10^{-4}	2.13840×10^{-2}
1.85	-2.16568×10^{-2}	-2.13616×10^{-2}	2.18889×10^{-3}	2.13215×10^{-3}	3.69395×10^{-2}
1.9	-3.49380×10^{-2}	-3.46050×10^{-2}	3.81390×10^{-3}	3.78840×10^{-3}	5.24950×10^{-2}
1.95	-4.86650×10^{-2}	-4.80870×10^{-2}	5.52765×10^{-3}	5.46960×10^{-3}	6.85805×10^{-2}
2	-6.23920×10^{-2}	-6.15690×10^{-2}	7.24140×10^{-3}	7.15080×10^{-3}	8.46660×10^{-2}

Table C_2 Forces on the slider and the lift tab with respect to varying lift tab positions

Displacement of lift tab (mm)	Forces at leading edge (N)	Forces at trailing edge (N)	Net force on slider (N)	Force on the lift tab (N)
1.5950	6.44990×10^{-3}	6.77640×10^{-3}	1.32263×10^{-2}	0.00000
1.5960	5.19670×10^{-3}	5.54920×10^{-3}	1.07459×10^{-2}	1.85130×10^{-3}
1.5970	3.94950×10^{-3}	4.32820×10^{-3}	8.27770×10^{-3}	3.69340×10^{-3}

1.5980	2.70250×10^{-3}	3.10710×10^{-3}	5.80960×10^{-3}	5.53550×10^{-3}
1.5990	1.45542×10^{-3}	1.88618×10^{-3}	3.34160×10^{-3}	7.37750×10^{-3}
1.6000	2.08389×10^{-4}	6.65200×10^{-4}	8.73589×10^{-4}	9.21950×10^{-3}
1.6005	-4.15120×10^{-4}	5.47250×10^{-5}	-3.60395×10^{-4}	1.01410×10^{-2}
1.6007	-6.64520×10^{-4}	-1.89464×10^{-4}	-8.53984×10^{-4}	1.05090×10^{-2}
1.6010	-7.45110×10^{-4}	-2.67280×10^{-4}	-1.01239×10^{-3}	1.06280×10^{-2}
1.6100	-8.19760×10^{-4}	-2.99360×10^{-4}	-1.11912×10^{-3}	1.07610×10^{-2}
1.6200	-9.02700×10^{-4}	-3.35000×10^{-4}	-1.23770×10^{-3}	1.09070×10^{-2}
1.6300	-9.85640×10^{-4}	-3.70640×10^{-4}	-1.35628×10^{-3}	1.10540×10^{-2}
1.6400	-1.06859×10^{-3}	-4.06290×10^{-4}	-1.47488×10^{-3}	1.12010×10^{-2}
1.6600	-1.23448×10^{-3}	-4.77570×10^{-4}	-1.71205×10^{-3}	1.14950×10^{-2}
1.6800	-1.40037×10^{-3}	-5.48860×10^{-4}	-1.94923×10^{-3}	1.17880×10^{-2}
1.7000	-1.56626×10^{-3}	-6.20140×10^{-4}	-2.18640×10^{-3}	1.20820×10^{-2}
1.7500	-1.98098×10^{-3}	-7.98350×10^{-4}	-2.77933×10^{-3}	1.28160×10^{-2}
1.7600	-2.06390×10^{-3}	-8.33990×10^{-4}	-2.89789×10^{-3}	1.29620×10^{-2}
1.7700	-2.14690×10^{-3}	-8.69630×10^{-4}	-3.01653×10^{-3}	1.31090×10^{-2}
1.7750	-3.62880×10^{-3}	-6.82720×10^{-4}	-4.31152×10^{-3}	1.39830×10^{-2}
1.7800	-6.30000×10^{-3}	-3.22800×10^{-4}	-6.62280×10^{-3}	1.55150×10^{-2}
1.7850	-8.98070×10^{-3}	4.28111×10^{-5}	-8.93789×10^{-3}	1.70500×10^{-2}
1.7900	-1.11442×10^{-2}	3.06300×10^{-4}	-1.08379×10^{-2}	1.83210×10^{-2}
1.8000	-1.64936×10^{-2}	1.03978×10^{-3}	-1.54538×10^{-2}	2.13840×10^{-2}
1.8500	-4.30183×10^{-2}	4.32104×10^{-3}	-3.86973×10^{-2}	3.69395×10^{-2}
1.9000	-6.95430×10^{-2}	7.60230×10^{-3}	-6.19407×10^{-2}	5.24950×10^{-2}
1.9500	-9.67520×10^{-2}	1.09973×10^{-2}	-8.57548×10^{-2}	6.85805×10^{-2}
2.0000	-1.23961×10^{-1}	1.43922×10^{-2}	-1.09569×10^{-1}	8.46660×10^{-2}

Table C_3 Forces and displacement of the tip of the dimple before dimple separation

Displacement of lift tab (mm)	Force on the tip of the dimple (N)	Displacement of lift tab (mm)
1.595	1.25620×10^{-2}	1.18750
1.596	1.03740×10^{-2}	1.18750
1.597	8.19910×10^{-3}	1.18750
1.598	6.02240×10^{-3}	1.18750
1.599	3.84410×10^{-3}	1.18750
1.6	1.66430×10^{-3}	1.18750
1.6005	5.73850×10^{-4}	1.18750
1.6007	1.37550×10^{-4}	1.18770
1.601	0	1.19430
1.61	0	1.20170
1.62	0	1.20910
1.63	0	1.23120

Table C_4 Moments on the slider with respect to varying lift tab positions

Displacement of lift tab (mm)	Moments caused by leading force (N.m)	Moments caused by trailing force (N.m)	Net moment on slider (N)
1.5950	-3.97636×10^{-3}	4.17765×10^{-3}	2.01287×10^{-4}
1.5960	-3.20377×10^{-3}	3.42108×10^{-3}	2.17316×10^{-4}
1.5970	-2.43487×10^{-3}	2.66834×10^{-3}	2.33469×10^{-4}

APPENDIX C DATA FOR UNLOADING PROCESS ANALYSIS

1.5980	-1.66609×10^{-3}	1.91553×10^{-3}	2.49436×10^{-4}
1.5990	-8.97266×10^{-4}	1.16283×10^{-3}	2.65564×10^{-4}
1.6000	-1.28472×10^{-4}	4.10096×10^{-4}	2.81624×10^{-4}
1.6005	2.55921×10^{-4}	3.37380×10^{-5}	2.89659×10^{-4}
1.6007	4.09677×10^{-4}	-1.16805×10^{-4}	2.92872×10^{-4}
1.6010	4.59360×10^{-4}	-1.64778×10^{-4}	2.94582×10^{-4}
1.6100	5.05382×10^{-4}	-1.84555×10^{-4}	3.20827×10^{-4}
1.6200	5.56515×10^{-4}	-2.06528×10^{-4}	3.49987×10^{-4}
1.6300	6.07647×10^{-4}	-2.28500×10^{-4}	3.79148×10^{-4}
1.6400	6.58786×10^{-4}	-2.50478×10^{-4}	4.08308×10^{-4}
1.6600	7.61057×10^{-4}	-2.94422×10^{-4}	4.66635×10^{-4}
1.6800	8.63328×10^{-4}	-3.38372×10^{-4}	5.24956×10^{-4}
1.7000	9.65599×10^{-4}	-3.82316×10^{-4}	5.83283×10^{-4}
1.7500	1.22127×10^{-3}	-4.92183×10^{-4}	7.29091×10^{-4}
1.7600	1.27239×10^{-3}	-5.14155×10^{-4}	7.58240×10^{-4}
1.7700	1.32356×10^{-3}	-5.36127×10^{-4}	7.87437×10^{-4}
1.7750	2.23716×10^{-3}	-4.20897×10^{-4}	1.81626×10^{-3}
1.7800	3.88395×10^{-3}	-1.99006×10^{-4}	3.68494×10^{-3}
1.7850	5.53660×10^{-3}	2.63930×10^{-5}	5.56299×10^{-3}
1.7900	6.87040×10^{-3}	1.88834×10^{-4}	7.05923×10^{-3}
1.8000	1.01683×10^{-2}	6.41024×10^{-4}	1.08093×10^{-2}
1.8500	2.65208×10^{-2}	2.66392×10^{-3}	2.91847×10^{-2}
1.9000	4.28733×10^{-2}	4.68682×10^{-3}	4.75601×10^{-2}
1.9500	5.96476×10^{-2}	6.77980×10^{-3}	6.64274×10^{-2}
2.0000	7.64220×10^{-2}	8.87279×10^{-3}	8.52947×10^{-2}

Appendix D. Stress Contour for Varing Configurations

Fig D_1 Stress contour for normal operational condition ($z_l = 1.02\text{mm}$ $z_t = 1.238\text{mm}$)

Fig D_2 Stress contour for $z_l = 1.02\text{ mm}$, $z_t = 1.238\text{ mm}$, and $z_r = 1.5\text{mm}$ (dimple engaged)

Fig D_3 Stress contour for $z_l = 1.02\text{ mm}$, $z_t = 1.238\text{ mm}$, and $z_r = 1.58\text{ mm}$ (dimple engaged)

Fig D_4 Stress contour for $z_l = 1.04\text{ mm}$ $z_t = 1.258\text{ mm}$, and $z_r = 1.5\text{ mm}$ (dimple engaged)

Fig D_5 Stress contour for $z_l = 1.03\text{ mm}$ $z_t = 1.228\text{ mm}$, and $z_r = 1.5\text{ mm}$ (dimple engaged)

Fig D_6 Stress contour for $z_l = 1.02\text{ mm}$ $z_t = 1.238\text{ mm}$, and $z_r = 1.61\text{ mm}$ (dimple separated)

Fig D_7 Stress contour for $z_l = 1.02$ mm $z_t = 1.238$ mm, and $z_r = 1.66$ mm (dimple separated)

Fig D_8 Stress contour for $z_l = 1.04$ mm $z_t = 1.258$ mm, and $z_r = 1.66$ mm (dimple separated)

Fig D_9 Stress contour for $z_l = 1.04$ mm $z_t = 1.218$ mm, and $z_r = 1.66$ mm (dimple separated)

Fig D_10 Stress contour for $z_l = 1.02$ mm $z_t = 1.238$ mm, and $z_r = 1.8$ mm (limiter engaged)

Fig D_11 Stress contour for $z_l = 1.02$ mm $z_t = 1.238$ mm, and $z_r = 1.9$ mm (limiter engaged)

Fig D_12 Stress contour for $z_l = 1.05$ mm $z_t = 1.268$ mm, and $z_r = 1.9$ mm (limiter engaged)

Fig D_13 Stress contour for $z_l = 1.05$ mm $z_t = 1.208$ mm, and $z_r = 1.9$ mm (limiter engaged)

Appendix E. Calculation of ErrorTable E_1 air bearing forces/moments for varying h_{min} when $\theta = 25\mu\text{rad}$

h_{min} (n m)	Calculated force (N)	fitting force (N)	error (N)	calculated moment (N·m)	fitting moment (N·m)	error (N·m)
8	0.050692	0.057842	0.00715	-6.6×10^{-6}	-4.91×10^{-6}	1.69×10^{-6}
9	0.047043	0.053136	0.006093	-7.3×10^{-6}	-5.64×10^{-6}	1.66×10^{-6}
10	0.043779	0.049323	0.005544	-7.7×10^{-6}	-6.21×10^{-6}	1.49×10^{-6}
20	0.023067	0.025662	0.002595	-8.5×10^{-6}	-8.11×10^{-6}	3.9×10^{-7}
30	0.012465	0.014661	0.002196	-7.7×10^{-6}	-7.75×10^{-6}	5×10^{-8}
40	0.006082	0.008174	0.002092	-6.8×10^{-6}	-6.91×10^{-6}	1.1×10^{-7}
50	0.00193	0.003914	0.001984	-6×10^{-6}	-5.99×10^{-6}	1×10^{-8}
60	-0.00089	0.001076	0.001966	-5.3×10^{-6}	-5.12×10^{-6}	1.8×10^{-7}
70	-0.00286	-0.00092	0.00194	-4.7×10^{-6}	-4.33×10^{-6}	3.7×10^{-7}
80	-0.00425	-0.00241	0.00184	-4.2×10^{-6}	-3.64×10^{-6}	5.6×10^{-7}
90	-0.00523	-0.0035	0.00173	-3.8×10^{-6}	-3.04×10^{-6}	7.6×10^{-7}
100	-0.00592	-0.00426	0.00166	-3.4×10^{-6}	-2.53×10^{-6}	8.7×10^{-7}

Table E_2 air bearing forces/moments for varying h_{min} when $\theta = 50\mu\text{rad}$

h_{min} (n m)	Calculated force (N)	fitting force (N)	error (N)	calculated moment (N·m)	fitting moment (N·m)	error (N·m)
8	0.02825	0.03201	0.00376	-2×10^{-6}	-1.91×10^{-6}	9×10^{-8}
9	0.026141	0.029581	0.00344	-2.6×10^{-6}	-2.58×10^{-6}	2×10^{-8}
10	0.024426	0.027403	0.002977	-3×10^{-6}	-3.12×10^{-6}	1.2×10^{-7}
20	0.012702	0.013484	0.000782	-4.8×10^{-6}	-5.34×10^{-6}	5.4×10^{-7}
30	0.006715	0.006289	0.000426	-4.9×10^{-6}	-5.57×10^{-6}	6.7×10^{-7}
40	0.002986	0.001942	0.001044	-4.7×10^{-6}	-5.26×10^{-6}	5.6×10^{-7}
50	0.00042	-0.00088	0.0013	-4.4×10^{-6}	-4.79×10^{-6}	3.9×10^{-7}
60	-0.00128	-0.00279	0.00151	-4×10^{-6}	-4.28×10^{-6}	2.8×10^{-7}
70	-0.00257	-0.0041	0.00153	-3.6×10^{-6}	-3.77×10^{-6}	1.7×10^{-7}
80	-0.00347	-0.00502	0.00155	-3.3×10^{-6}	-3.30×10^{-6}	0
90	-0.00411	-0.00565	0.00154	-3×10^{-6}	-2.86×10^{-6}	1.4×10^{-7}
100	-0.00456	-0.00607	0.00151	-2.7×10^{-6}	-2.48×10^{-6}	2.2×10^{-7}

Table E_3 air bearing forces/moments for varying h_{min} when $\theta = 77\mu\text{rad}$

h_{min} (n m)	Calculated force (N)	fitting force (N)	error (N)	calculated moment (N·m)	fitting moment (N·m)	error (N·m)
8	0.01714	0.017362	0.000222	-1×10^{-8}	-3.77×10^{-7}	3.67×10^{-7}
9	0.015774	0.015806	3.2×10^{-5}	-4.7×10^{-7}	-9.08×10^{-7}	4.38×10^{-7}
10	0.014583	0.014411	0.000172	-8.5×10^{-7}	-1.35×10^{-6}	5×10^{-7}

20	0.006812	0.005501	0.001311	-2.8×10^{-6}	-3.38×10^{-6}	5.8×10^{-7}
30	0.002822	0.000924	0.001898	-3.2×10^{-6}	-3.84×10^{-6}	6.4×10^{-7}
40	0.000349	-0.0018	0.002149	-3.3×10^{-6}	-3.83×10^{-6}	5.3×10^{-7}
50	-0.0013	-0.00354	0.00224	-3.2×10^{-6}	-3.64×10^{-6}	4.4×10^{-7}
60	-0.00245	-0.00468	0.00223	-3×10^{-6}	-3.38×10^{-6}	3.8×10^{-7}
70	-0.00326	-0.00543	0.00217	-2.7×10^{-6}	-3.09×10^{-6}	3.9×10^{-7}
80	-0.00382	-0.00592	0.0021	-2.5×10^{-6}	-2.80×10^{-6}	3×10^{-7}
90	-0.0042	-0.00623	0.00203	-2.3×10^{-6}	-2.52×10^{-6}	2.2×10^{-7}
100	-0.00444	-0.0064	0.00196	-2.1×10^{-6}	-2.25×10^{-6}	1.5×10^{-7}

Table E_4 air bearing forces/moments for varying h_{min} when $\theta = 100\mu\text{rad}$

h_{min} (n m)	Calculated force (N)	fitting force (N)	error (N)	calculated moment (N·m)	fitting moment (N·m)	error (N·m)
8	0.012324	0.008533	0.003791	8.7×10^{-7}	-2.38×10^{-8}	8.94×10^{-7}
9	0.011273	0.007474	0.003799	4.8×10^{-7}	-4.28×10^{-7}	9.08×10^{-7}
10	0.010255	0.006525	0.00373	1.4×10^{-7}	-7.66×10^{-7}	9.06×10^{-7}
20	0.004291	0.00052	0.003771	-1.7×10^{-6}	-2.41×10^{-6}	7.1×10^{-7}
30	0.001219	-0.00249	0.003709	-2.3×10^{-6}	-2.86×10^{-6}	5.6×10^{-7}
40	-0.00069	-0.00423	0.00354	-2.4×10^{-6}	-2.95×10^{-6}	5.5×10^{-7}
50	-0.00194	-0.00529	0.00335	-2.4×10^{-6}	-2.87×10^{-6}	4.7×10^{-7}
60	-0.00279	-0.00594	0.00315	-2.3×10^{-6}	-2.72×10^{-6}	4.2×10^{-7}
70	-0.00336	-0.00633	0.00297	-2.2×10^{-6}	-2.54×10^{-6}	3.4×10^{-7}
80	-0.00374	-0.00654	0.0028	-2×10^{-6}	-2.35×10^{-6}	3.5×10^{-7}
90	-0.00398	-0.00664	0.00266	-1.8×10^{-6}	-2.16×10^{-6}	3.6×10^{-7}
100	-0.00411	-0.00664	0.00253	-1.7×10^{-6}	-1.97×10^{-6}	2.7×10^{-7}

Table E_5 air bearing forces/moments for varying h_{min} when $\theta = 125\mu\text{rad}$

h_{min} (n m)	Calculated force (N)	fitting force (N)	error (N)	calculated moment (N·m)	fitting moment (N·m)	error (N·m)
8	0.009284	0.001917	0.007367	1.48×10^{-6}	-1.80×10^{-7}	1.66×10^{-6}
9	0.008765	0.001223	0.007542	1.26×10^{-6}	-4.59×10^{-7}	1.72×10^{-6}
10	0.008448	0.000605	0.007843	1.14×10^{-6}	-6.94×10^{-7}	1.83×10^{-6}
20	0.002834	-0.00323	0.006064	-8.8×10^{-7}	-1.86×10^{-6}	9.8×10^{-7}
30	0.000337	-0.00506	0.005397	-1.5×10^{-6}	-2.20×10^{-6}	7×10^{-7}
40	-0.00117	-0.00603	0.00486	-1.7×10^{-6}	-2.28×10^{-6}	5.8×10^{-7}
50	-0.00213	-0.00657	0.00444	-1.8×10^{-6}	-2.24×10^{-6}	4.4×10^{-7}
60	-0.00275	-0.00683	0.00408	-1.8×10^{-6}	-2.15×10^{-6}	3.5×10^{-7}
70	-0.00316	-0.00694	0.00378	-1.7×10^{-6}	-2.03×10^{-6}	3.3×10^{-7}
80	-0.00341	-0.00693	0.00352	-1.6×10^{-6}	-1.90×10^{-6}	3×10^{-7}
90	-0.00353	-0.00686	0.00333	-1.4×10^{-6}	-1.76×10^{-6}	3.6×10^{-7}
100	-0.00358	-0.00673	0.00315	-1.3×10^{-6}	-1.63×10^{-6}	3.3×10^{-7}

Table E_6 air bearing forces/moments for varying h_{min} when $\theta = 150\mu\text{rad}$

h_{min} (n m)	Calculated force (N)	fitting force (N)	error (N)	calculated moment (N·m)	fitting moment (N·m)	error (N·m)
8	0.007458	-0.00232	0.009778	1.86×10^{-6}	-5.32×10^{-7}	2.39×10^{-6}
9	0.006771	-0.00277	0.009541	1.57×10^{-6}	-7.11×10^{-7}	2.28×10^{-6}
10	0.006149	-0.00317	0.009319	1.3×10^{-6}	-8.61×10^{-7}	2.16×10^{-6}
20	0.002087	-0.00554	0.007627	-2.8×10^{-7}	-1.60×10^{-6}	1.32×10^{-6}
30	2.2×10^{-5}	-0.00657	0.006592	-9.2×10^{-7}	-1.80×10^{-6}	8.8×10^{-7}
40	-0.00121	-0.00703	0.00582	-1.2×10^{-6}	-1.83×10^{-6}	6.3×10^{-7}
50	-0.00197	-0.00721	0.00524	-1.3×10^{-6}	-1.78×10^{-6}	4.8×10^{-7}
60	-0.00246	-0.00722	0.00476	-1.3×10^{-6}	-1.70×10^{-6}	4×10^{-7}
70	-0.00276	-0.00713	0.00437	-1.2×10^{-6}	-1.60×10^{-6}	4×10^{-7}
80	-0.00291	-0.00698	0.00407	-1.2×10^{-6}	-1.49×10^{-6}	2.9×10^{-7}
90	-0.00298	-0.0068	0.00382	-1.1×10^{-6}	-1.39×10^{-6}	2.9×10^{-7}
100	-0.00299	-0.00659	0.0036	-9.8×10^{-7}	-1.28×10^{-6}	3×10^{-7}

Table E_7 air bearing forces/moments for varying h_{min} when $\theta = 175\mu\text{rad}$

h_{min} (n m)	Calculated force (N)	fitting force (N)	error (N)	calculated moment (N·m)	fitting moment (N·m)	error (N·m)
8	0.006395	-0.00478	0.011175	2.14×10^{-6}	-8.39×10^{-7}	2.98×10^{-6}
9	0.005817	-0.00507	0.010887	1.87×10^{-6}	-9.45×10^{-7}	2.82×10^{-6}
10	0.00529	-0.00532	0.01061	1.64×10^{-6}	-1.03×10^{-6}	2.67×10^{-6}
20	0.001859	-0.00673	0.008589	1.9×10^{-7}	-1.43×10^{-6}	1.62×10^{-6}
30	6.6×10^{-5}	-0.00722	0.007286	-4.4×10^{-7}	-1.51×10^{-6}	1.07×10^{-6}
40	-0.00098	-0.00735	0.00637	-7.3×10^{-7}	-1.48×10^{-6}	7.5×10^{-7}
50	-0.00162	-0.0073	0.00568	-8.6×10^{-7}	-1.41×10^{-6}	5.5×10^{-7}
60	-0.00201	-0.00715	0.00514	-8.9×10^{-7}	-1.32×10^{-6}	4.3×10^{-7}
70	-0.00221	-0.00695	0.00474	-8.8×10^{-7}	-1.23×10^{-6}	3.5×10^{-7}
80	-0.00236	-0.00671	0.00435	-8.4×10^{-7}	-1.13×10^{-6}	2.9×10^{-7}
90	-0.00237	-0.00647	0.0041	-7.7×10^{-7}	-1.04×10^{-6}	2.7×10^{-7}
100	-0.00237	-0.00621	0.00384	-7.1×10^{-7}	-9.51×10^{-7}	2.41×10^{-7}

APPENDIX F**MODIFIED REYNOLDS EQUATIONS FOR HEAD DISK INTERFACE**

The Navier-Stokes equations are the equations of the motion for a viscous fluid which consist of two equations with three unknowns are the equations of motion for a viscous fluid. They are derived by considering the dynamic equilibrium of a fluid element. Surface, inertia and body forces are included. A third equation is needed to develop a complete governing differential fluid flow equation, i.e. Reynolds equation, which can be derived either from the Navier-Stokes equations of fluid motion and the continuity equation or from the laws of viscous flow and from the principles of mass conservation and the laws of viscous flow. From the point of view of physical meaning, the Reynolds equation describes a combination of two types of fluid flow: Poiseuille flow and Couette flow. The Poiseuille flow terms describe the flow due to pressure gradients within the lubricated area, while the Couette flow term describe the shear flow due to the relative motion of the two solid boundaries. Originally, it is built for incompressible fluid. Then the effects of compressibility were included. This equation establishes a relation between the geometry of the surfaces, relative sliding velocity, the property of the fluid and the magnitude of the normal load the bearing can support.

For simplification, a number of justifiable assumptions are made for the case of slow viscous motion in the lubrication analysis in which pressure and viscous terms predominate. It is assumed that: The surfaces are smooth; The fluid is Newtonian and the flow is laminar; Inertia forces resulting from acceleration of the liquids and body forces are small compared with the surface forces and may be neglected; Surface

tension effects are negligible; The fluid film thickness is much smaller than other bearing dimensions so that curvature of the fluid film can be ignored and the pressure, density, and viscosity are constant across the fluid film at any location; Nonslip boundary conditions are obeyed at the walls; Compared with the two velocity gradients, all other velocity gradients are negligible and thickness z dimension is much smaller than x and y . from the equilibrium of an element.

Based on the assumptions, the simplified Navier-Stokes equations can be written as:

$$\begin{aligned}\frac{\partial p}{\partial x} &= \frac{\partial}{\partial z} \left(\eta \frac{\partial V_x}{\partial z} \right) \\ \frac{\partial p}{\partial y} &= \frac{\partial}{\partial z} \left(\eta \frac{\partial V_y}{\partial z} \right) \\ \frac{\partial p}{\partial z} &= 0\end{aligned}\tag{f-1}$$

Where, x, y, z are the Cartesian coordinates; V_x, V_y, V_z are the gas velocities in the x, y and z directions, respectively. p is the pressure; η is the absolute (dynamic) viscosity ($\text{g cm}^{-1}\text{s}^{-1}$)

The continuity equation is based on the principle of mass conservation in a volume of element which requires that the net outflow of mass per unit time from a volume element of fluid equals to the decrease of mass within the volume. This can be expressed by the following equation:

$$\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} + \frac{\partial(\rho V_z)}{\partial z} + \frac{\partial \rho}{\partial t} = 0\tag{f-2}$$

The general Reynolds equation can be derived by combining the Navier-Stokes equations (f-1) with the continuity equation (f-2) as:

$$\begin{aligned} & \frac{\partial}{\partial x} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(\frac{\rho h^3}{12\eta} \frac{\partial p}{\partial y} \right) \\ &= \frac{\partial}{\partial x} \left[\frac{\rho h (V_{xa} + V_{xb})}{2} \right] + \frac{\partial}{\partial y} \left[\frac{\rho h (V_{ya} + V_{yb})}{2} \right] + \rho \left[(V_{za} - V_{zb}) - V_{xa} \frac{\partial h}{\partial x} - V_{ya} \frac{\partial h}{\partial y} \right] + h \frac{\partial \rho}{\partial t} \end{aligned} \quad (\text{f-3})$$

We can apply the principle of mass conservation to a control volume fixed in space and extended across the fluid film. For a gas-lubricated bearing with perfect gas,

$$p = \rho RT$$

where R is the gas constant and T is the absolute temperature. Therefore, ρ is replaced by p in the Reynolds equation.

Applying it to the Head-Disk Interface (HDI), the general Reynolds equation can be modified as the following

$$\frac{\partial}{\partial x} \left(p h^3 \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left(p h^3 \frac{\partial p}{\partial y} \right) = 6V_x \eta \frac{\partial}{\partial x} [ph] + 6V_y \eta \frac{\partial}{\partial y} [ph] + 12\eta \frac{\partial}{\partial t} [ph] \quad (\text{f-4})$$

Non-dimensionalization is the substitution of all real variables in an equation, e.g. pressure, film thickness, etc., by dimensionless fractions of two or more real parameters. This process extends the generality of a numerical solution. A basic disadvantage of a numerical solution is that data is only provided for specific values of controlling variables. A computer program would have to be executed for literally thousands of cases to provide a comprehensive coverage of all the controlling parameters. The benefit of non-dimensionalization is that the number of controlling parameters is reduced and a relatively limited data set provides the required information on any bearing.

Letting $X = x / L$, $Y = y / L$, $H = h / h_m$, $P = p / p_a$, and $T = \omega t$, where P , H , X and Y are the non-dimensionalized pressure, flying height, coordinate in the slider length direction and coordinate in the slider width direction, respectively. p_a is the ambient atmospheric pressure, h_m is the reference clearance at the trailing edge

center, and L is the length of the slider. Substitute them into the equation, the non-dimensional Reynolds equation can be written as

$$\frac{\partial}{\partial X} \left(PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial Y} \left(PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right) = \sigma \frac{\partial(PH)}{\partial T} \quad (f-5)$$

where, $\Lambda_x = (6\mu U_x L) / (p_a h_m^2)$ and $\Lambda_y = (6\mu U_y L) / (p_a h_m^2)$ are called bearing numbers in the x and y directions, respectively; $\sigma = (12\mu\omega L^2) / (p_a h_m^2)$ is the squeeze number.

Equation (f-5) is derived by assuming the gas to be a continuous medium with non-slip at the solid boundary. This equation yields accurate results when K_n is less than 0.001, which corresponds to the continuum flow regime. The Knudsen number which is the ratio of the mean free path of gas molecules γ to the available range of movement h is used to measure the degree of gas rarefaction at thin film thicknesses ($K_n = \gamma/h$). The compressible hydrodynamic lubrication of gas bearings is characterized by extremely small film thicknesses which can restrict the free movement of individual gas molecules. The mean free path of molecules which can be reduced with higher operating pressures is $0.064\mu\text{m}$ in air at atmospheric pressure. When the Knudsen number is in the range between $0.01 < K_n < 0.1$ in the atmospheric air, a thin layer of gas close to the solid surface loses its fluid characteristic and is capable of ‘slipping’ against the surface. This means that the effective viscosity of the gas declines with its proximity to the surface. When the film thickness becomes comparable to this value, the gas loses its continuity and be subjected to rarefaction.

For current HDIs, the flying height is down to 8 nm or even less and the gas may no longer be a continuum which can be described by the macroscopic equations of fluid dynamics with a group of macroscopic variables such as density, velocity and pressure. The best formulation available for the air bearing problem in the HDI is the Boltzmann

equation, which is obtained from a more fundamental and microscopic point of view. The macroscopic equations for fluids, such as the continuous equation, the momentum and energy equations are simply a special case of the Boltzmann equation. However, the solution to the Boltzmann equation is, in general, very difficult to obtain even for the physically simple situations. In order to solve the air bearing problem still using a Reynolds-type lubrication equation and incorporating the rarefaction effect, several authors introduced the generalized Reynolds equation with different slip correction models.

If the mean free path of the molecules becomes comparable to the film thickness, slipping between the gas and the walls takes place, which produces an effect similar to reduce the viscosity. Another effect is that the heat condition is characterized by a discontinuity in temperature between the solid boundary and the gas. The magnitude of the slip velocity and of the temperature discontinuity can be evaluated from the kinetic theory of gases.

$$u_{slip} = \sigma \frac{(2-f)}{f} \lambda \frac{\partial u}{\partial z} + \dots \quad (f-6)$$

$$\lambda = \frac{1}{\sqrt{2}ns} \quad (f-7)$$

where σ is a numerical constant, f is the surface accommodation coefficient, u is the flow velocity of lubricant in x direction, λ is the molecular mean free path, n is the number of molecules per unit volume, and s is the mutual collision cross section. The velocity is estimated by expanding it in a Taylor series about the wall position and retaining the zeroth and first order terms. Higher order terms are not considered. Using this slip modification combining with the general Reynolds equation, Burgdorfer first-order model can be obtained as following

$$\frac{\partial}{\partial X}(Q_p PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH) + \frac{\partial}{\partial Y}(Q_p PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH) = \sigma \frac{\partial PH}{\partial T} \quad (f-8)$$

where $Q_p = 1 + (6K_n)/(PH)$. Q_p is the Poiseuille flow rate coefficient, which reflects the type of slip correction used.

It should be noted as the Knudsen number (especially the local Knudsen number) approaches unity, the second-order slip effects should not be neglected. The third or the higher order slip effects do not occur since the third or higher derivatives of the velocity are identically equal to zero. The second order slip effects are approximated including the surface accommodation coefficient. The main effect of the second order slip term is to increase the slip effects by permitting more side leakage flow resulting in a decrease in the load carrying capacity for a fixed clearance height. Using this slip modification combining with the general Reynolds equation, Hsia modified the first-order model to a second-order Model by setting $Q_p = 1 + (6K_n)/(PH) + 6[(K_n)/(PH)]^2$.

To increase the accuracy of the slip-flow model, the second-order modified Reynolds equation was derived using slip-flow boundary conditions for both shear and pressure flows, while the first-order modified Reynolds equation was derived using only those for the shear flow. The second-order modified Reynolds equation, however, involves two principal problems. First, since the slip-flow mode for the pressure flow was deduced through analogy to that for a shear flow, the physical background has remained uncertain. Second, accuracy was not necessarily enhanced contrary to the aim of the higher-order slip-flow model. Mitsuya in 1993 introduced the 1.5-order slip model in order to predict the load capacity more accurately from the physical considerations that taking account of the accommodation coefficient into account. This model features two key differences compared with the second-order model. One is that accommodation coefficient is involved and the other is that the second-order coefficient

is 4/9 times smaller. A molecule colliding with the surface has a momentum given at the last collision, which occurred away from the surface at a distance of λ . He suggested that λ should be replaced by $2\lambda/3$. The correction coefficient for the 1.5-order slip model can be expressed as following

$$Q_p = 1 + \frac{6aK_n}{PH} + \frac{8}{3} \left(\frac{K_n}{PH} \right)^2 \quad (\text{f-9})$$

where $a = (2 - \alpha)/(\alpha)$. Generally, surface accommodation α coefficient equal to 0.89.

These comparisons confirm that the present 1.5-order modified Reynolds equation provides intermediate characteristics between those derived from the first and the second order slip flow models and produces an approximation closer to the exact solution resulting from the Boltzmann-Reynolds equation.

As both equations in above introduction were originally derived by considering the slip flow boundary condition for $K_n \ll 1$, it is not clear whether they apply to a high Knudsen number. Even if the equation leads to numerical results coinciding with experimental results, it may not be justified physically. To accurately examine the applicability of the modified Reynolds equation to Ultra-low clearances, it has become essential to establish a generalized gas film lubrication equation for arbitrary Knudsen number. Gans, who first treated the linearized Boltzmann equation as a basic equation, derived the approximation lubrication equation analytical using a successive approximation method. He derived the linearized version of the Boltzmann equation using the Bhatnagar-Gross-Krook (BGK) model and he solved by taking two moments of this equation and using successive approximations. As a result, a lubrication equation was derived, which is similar to Burgdorfer's first order modified Reynolds equation. Therefore, Gans claimed that this equation should be valid for all Knudsen number.

Fukui and kaneko started from the linearized Boltzmann equation based on the BGK model. The basic equation is decomposed so as to describe the fundamental flows that depend on the gradients of temperature, pressure and velocity. By considering mass conservation, these flows are incorporated later into a generalized Reynolds equation. The generalized Reynolds equation based on the Boltzmann equation can be expressed as following

$$\begin{aligned} \frac{\partial}{\partial X} \left\{ \left[\bar{Q}_p (D_0 PH) \frac{\partial P}{\partial X} - \bar{Q}_T (D_0 PH) P \frac{\partial \tau_w}{\partial X} \right] PH^3 - \Lambda_x PH \right\} \\ + \frac{\partial}{\partial Y} \left\{ \left[\bar{Q}_p (D_0 PH) \frac{\partial P}{\partial Y} - \bar{Q}_T (D_0 PH) P \frac{\partial \tau_w}{\partial Y} \right] PH^3 - \Lambda_y PH \right\} = 0 \end{aligned} \quad (f-10)$$

where Λ and τ_w are the bearing number and the non-dimensional boundary temperature (T_w/T_0-1). To overcome the difficulties in solving the BGK model, Fukui and Kaneko introduced the use of a Poiseuille flow database to allow a quicker computation of a generalized lubrication equation for high K_n number gas bearing.

Appendix G. Factored Implicit Scheme

For the implementation using the factored implicit method, a first order model expressed by the following equation is used.

$$\begin{aligned} & \nabla_{\text{dim}} \cdot (h_{\text{dim}}^3 P_{\text{dim}} \nabla_{\text{dim}} P_{\text{dim}}) + 6\lambda_a P_a \nabla_{\text{dim}} \cdot (h_{\text{dim}}^2 \nabla_{\text{dim}} P_{\text{dim}}) \\ &= 6\mu \bar{\mathbf{V}} \cdot \nabla_{\text{dim}} P_{\text{dim}} h_{\text{dim}} + 12\mu \frac{\partial P_{\text{dim}} h_{\text{dim}}}{\partial t_{\text{dim}}} \end{aligned} \quad (g.1)$$

where $\nabla_{\text{dim}} = (\partial / \partial x_{\text{dim}})i + (\partial / \partial y_{\text{dim}})j$, $\bar{\mathbf{V}} = V_x i + V_y j$, and the subscript *dim* means dimensional variables. Compared with the four slip correction models, eq. (g.1) is a first order slip model. The slip term in this equation is $6\lambda_a P_a \nabla_{\text{dim}} \cdot (h_{\text{dim}}^2 \nabla_{\text{dim}} P_{\text{dim}})$. When we just consider the *x* direction for clarification and simplification, the term can be expressed as $6\lambda_a P_a \left\{ \partial \left[h_{\text{dim}}^2 (\partial P_{\text{dim}} / \partial x) \right] / \partial x \right\}$. For the first order model,

$$\frac{\partial}{\partial X} \left(\left(1 + 6a \frac{K_n}{PH} \right) PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) = \sigma \frac{\partial PH}{\partial T} \quad (g.2)$$

$$\frac{\partial}{\partial X} \left(PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right) + \frac{\partial}{\partial X} \left(6a K_n H^2 \frac{\partial P}{\partial X} \right) = \sigma \frac{\partial PH}{\partial T} \quad (g.3)$$

Its slip term is $6a K_n \frac{\partial}{\partial X} \left(H^2 \frac{\partial P}{\partial X} \right)$. Both (g.2) and (g.3) are the first order model.

F.1. Non-dimensionalization of the Modified Reynolds Equation

Letting

$$P = P_{\text{dim}} / P_a, \quad h = h_{\text{dim}} / h_{\text{min}}, \quad x = x_{\text{dim}} / L_x, \quad y = y_{\text{dim}} / L_x,$$

$$\Delta t = \Delta t_{\text{dim}} / t_{\text{scale}}, \quad t = t_{\text{dim}} / t_{\text{scale}}, \quad \nabla_{\text{dim}} = i \frac{\partial}{L_x \partial x} + j \frac{\partial}{L_x \partial y} = \frac{1}{L_x} \left(i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} \right) = \frac{1}{L_x} \nabla$$

Substituting them into the dimensional compressible lubrication equation (g.1), the non-dimensional compressible lubrication equation can be obtained as

$$\begin{aligned}
 & \frac{\partial}{\partial x} \left((h^3 P) \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left((h^3 P) \frac{\partial P}{\partial y} \right) + 6 \frac{\lambda_a}{h_{\min}} \left[\frac{\partial}{\partial x} \left(h^2 \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left(h^2 \frac{\partial P}{\partial y} \right) \right] \\
 &= 6 \frac{\mu L_x}{h_{\min}^2 P_a} \left(V_x \frac{\partial Ph}{\partial x} + V_y \frac{\partial Ph}{\partial y} \right) + 12 \frac{\mu L_x^2}{h_{\min}^2 P_a t_{scale}} \frac{\partial (Ph)}{\partial t}
 \end{aligned} \tag{g.4}$$

F.2. Discretization of the Modified Reynolds Equation

Letting $Z(x, y, t) = P \cdot h$ and substituting it into (g.4) yields:

$$\begin{aligned}
 & \frac{\partial}{\partial x} \left[(hZZ_x - Z^2 h_x) + 6 \frac{\lambda_a}{h_{\min}} (hZ_x - Zh_x) - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x Z \right] \\
 & + \frac{\partial}{\partial y} \left[(hZZ_y - Z^2 h_y) + 6 \frac{\lambda_a}{h_{\min}} (hZ_y - Zh_y) - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_y Z \right] = 12 \frac{\mu L_x^2}{h_{\min}^2 P_a t_{scale}} \frac{\partial Z}{\partial t}
 \end{aligned} \tag{g.5}$$

Letting

$$\begin{aligned}
 F &= (hZZ_x - Z^2 h_x) + 6 \frac{\lambda_a}{h_{\min}} (hZ_x - Zh_x) - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x Z \\
 G &= (hZZ_y - Z^2 h_y) + 6 \frac{\lambda_a}{h_{\min}} (hZ_y - Zh_y) - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_y Z
 \end{aligned}$$

Eq. (g.5) can be expanded as

$$\frac{12 \mu L_x^2}{h_{\min}^2 P_a t_{scale}} Z_t = F_x + G_y \tag{g.6}$$

Expanding Z and Z_t about the (n) time level, this results

$$Z_t = \frac{\partial Z}{\partial t} = \frac{h_{\min}^2 P_a t_{scale}}{12 \mu L_x^2} (F_x + G_y) \tag{g.7}$$

$$\frac{12 \mu L_x^2}{h_{\min}^2 P_a t_{scale}} Z_t^{(n)} = F_x^{(n)} + G_y^{(n)}, \quad \frac{12 \mu L_x^2}{h_{\min}^2 P_a t_{scale}} Z_t^{(n+1)} = F_x^{(n+1)} + G_y^{(n+1)}$$

According to the Taylor series:

$$Z^{n+1} = Z^n + Z_t^n \Delta t + O(\Delta t^2), \quad Z^n = Z^{n+1} - Z_t^{n+1} \Delta t + O[(-\Delta t)^2]$$

We can obtain

$$Z^{(n+1)} = Z^{(n)} + \frac{\Delta t}{2} (Z_t^{(n)} + Z_t^{(n+1)}) + O(\Delta t^3) \tag{g.8}$$

where $\Delta t = (\Delta t_{\text{dim}}) / (t_{\text{scale}})$ is a non-dimensional time element.

The final numerical form will be achieved by substituting eq. (g.6) and eq. (g.7) into equation (g.8), and casting the result in a finite difference form without increasing the order of the truncation error which appears in eq. (g.8). Making the substitution, equation (g.8) appears as

$$Z^{(n+1)} = Z^{(n)} + \frac{h_{\min}^2 P_a t_{\text{scale}} \Delta t}{24 \mu L_x^2} (F_x^{(n)} + G_y^{(n)} + F_x^{(n+1)} + G_y^{(n+1)}) + O(\Delta t^3) \quad (g.9)$$

When we assume $\Delta t = 1$, this means $t_{\text{scale}} = \Delta t_{\text{dim}}$

Since F and G are nonlinear functions of Z , eq. (g.9) should be expanded about the current time level. $F^{(n)}$ and $G^{(n)}$ are used to express $F^{(n+1)}$ and $G^{(n+1)}$.

Assume $t^{(n)} \leq t \leq t^{(n+1)}$, h can be expressed as

$$h - h^{(n)} = h_t \Delta t + \frac{h_{tt}}{2!} \Delta t^2 + \dots$$

Similar rule can be applied to $(h_x - h_x^{(n)}), (Z - Z^{(n)}), (Z_x - Z_x^{(n)})$

Ignoring its higher order terms, the follows can be obtained:

$$\begin{aligned} F(x, y, t) = & F(x, y, t^{(n)}) + \frac{\partial F^{(n)}}{\partial h} \cdot (h - h^{(n)}) + \frac{\partial F^{(n)}}{\partial h_x} \cdot (h_x - h_x^{(n)}) \\ & + \frac{\partial F^{(n)}}{\partial Z} \cdot (Z - Z^{(n)}) + \frac{\partial F^{(n)}}{\partial Z_x} \cdot (Z_x - Z_x^{(n)}) + O(\Delta t^2) \end{aligned}$$

(g.10)

$$\begin{aligned} G(x, y, t) = & G(x, y, t^{(n)}) + \frac{\partial G^{(n)}}{\partial h} \cdot (h - h^{(n)}) + \frac{\partial G^{(n)}}{\partial h_y} \cdot (h_y - h_y^{(n)}) \\ & + \frac{\partial G^{(n)}}{\partial Z} \cdot (Z - Z^{(n)}) + \frac{\partial G^{(n)}}{\partial Z_y} \cdot (Z_y - Z_y^{(n)}) + O(\Delta t^2) \end{aligned} \quad (g.11)$$

Intermediate variables A, B, C, D, I, J, K, L are introduced to clarify the derivation.

$$\frac{\partial F^{(n)}}{\partial h} = A, \frac{\partial F^{(n)}}{\partial h_x} = B, \frac{\partial F^{(n)}}{\partial Z} = C, \frac{\partial F^{(n)}}{\partial Z_x} = D, \frac{\partial G^{(n)}}{\partial h} = I, \frac{\partial G^{(n)}}{\partial h_y} = J, \frac{\partial G^{(n)}}{\partial Z} = K, \frac{\partial G^{(n)}}{\partial Z_y} = L$$

Eq. (g.10) and eq. (g.11) can be re-written as

$$F(x, y, t^{(n+1)}) = F(x, y, t^{(n)}) + A \cdot (h^{(n+1)} - h^{(n)}) + B \cdot (h_x^{(n+1)} - h_x^{(n)}) + C \cdot (Z^{(n+1)} - Z^{(n)}) + D \cdot (Z_x^{(n+1)} - Z_x^{(n)}) + O(\Delta t^2) \quad (g.12)$$

$$G(x, y, t^{(n+1)}) = G(x, y, t^{(n)}) + I \cdot (h^{(n+1)} - h^{(n)}) + J \cdot (h_y^{(n+1)} - h_y^{(n)}) + K \cdot (Z^{(n+1)} - Z^{(n)}) + L \cdot (Z_y^{(n+1)} - Z_y^{(n)}) + O(\Delta t^2) \quad (g.13)$$

where, $A = \{ZZ_x + 6 \frac{\lambda_a}{h_{\min}} Z_x\}^{(n)}$, etc

Their first order partial derivatives are:

$$F_x = hZ_x^2 + hZZ_{xx} - Z^2 h_{xx} - h_x ZZ_x + 6 \frac{\lambda_a}{h_{\min}} (hZ_{xx} - Zh_{xx}) - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x Z_x \quad (g.14)$$

$$G_y = hZ_y^2 + hZZ_{yy} - ZZ_y h_y - Z^2 h_{yy} + 6 \frac{\lambda_a}{h_{\min}} (hZ_{yy} - Zh_{yy}) - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_y Z_y \quad (g.15)$$

Letting $\delta_x = \frac{\partial}{\partial x}$, $\delta_y = \frac{\partial}{\partial y}$

$$F^{(n+1)} = F^{(n)} + A^{(n)} \cdot (h^{(n+1)} - h^{(n)}) + B^{(n)} \cdot (h_x^{(n+1)} - h_x^{(n)}) + C^{(n)} \cdot (Z^{(n+1)} - Z^{(n)}) + D^{(n)} \cdot (Z_x^{(n+1)} - Z_x^{(n)}) + O(\Delta t^2) \quad (g.16)$$

$$G^{(n+1)} = G^{(n)} + I^{(n)} \cdot (h^{(n+1)} - h^{(n)}) + J^{(n)} \cdot (h_y^{(n+1)} - h_y^{(n)}) + K^{(n)} \cdot (Z^{(n+1)} - Z^{(n)}) + L^{(n)} \cdot (Z_y^{(n+1)} - Z_y^{(n)}) + O(\Delta t^2) \quad (g.17)$$

The derivation of $F_x^{(n+1)}$ and $G_y^{(n+1)}$ can be expressed as

$$F_x^{(n+1)} = \delta_x F^{(n)} + \delta_x [A \cdot (h^{(n+1)} - h^{(n)})] + \delta_x [B \cdot (h_x^{(n+1)} - h_x^{(n)})] + \delta_x [C \cdot (Z^{(n+1)} - Z^{(n)})] + \delta_x [D \cdot (Z_x^{(n+1)} - Z_x^{(n)})] + O(\Delta t^2) \quad (g.18)$$

Reform it:

$$F_x^{(n+1)} = \delta_x F^{(n)} + \delta_x [A \cdot h^{(n+1)}] - \delta_x [A \cdot h^{(n)}] + \delta_x [B \cdot \delta_x h^{(n+1)}] - \delta_x [B \cdot \delta_x h^{(n)}] + \delta_x [C \cdot Z^{(n+1)}] - \delta_x [C \cdot Z^{(n)}] + \delta_x [D \cdot \delta_x Z^{(n+1)}] - \delta_x [D \cdot \delta_x Z^{(n)}] + O(\Delta t^2) \quad (g.19)$$

$$G_y^{(n+1)} = \delta_y G^{(n)} + \delta_y [I \cdot h^{(n+1)}] - \delta_y [I \cdot h^{(n)}] + \delta_y [J \cdot h_y^{(n+1)}] - \delta_y [J \cdot h_y^{(n)}] \\ + \delta_y [K \cdot Z^{(n+1)}] - \delta_y [K \cdot Z^{(n)}] + \delta_y [L \cdot \delta_y Z^{(n+1)}] - \delta_y [L \cdot \delta_y Z^{(n)}] + O(\Delta t^2)$$

(g.20)

Letting $\Delta t = 1$, each calculation step is t_{scale} ,

$$a = \frac{h_{min}^2 P_a t_{scale} \Delta t}{24 \mu L_x^2} = \frac{h_{min}^2 P_a t_{scale}}{24 \mu L_x^2}$$

Eq. (g. 9) is rewritten as

$$Z^{(n+1)} = Z^{(n)} + a(F_x^{(n)} + G_y^{(n)} + F_x^{(n+1)} + G_y^{(n+1)}) + O(\Delta t^3)$$

(g.21)

Actually, $\delta_x A(\cdot)$ can be expressed as $\delta_x A(\cdot) = \delta_x [A \cdot (\cdot)] = A_x \cdot (\cdot) + A \cdot \delta_x (\cdot)$, etc

So eq. (g.21) can be expressed as

$$Z^{n+1} = Z^n + a \left\{ \begin{aligned} &2F_x^n + 2G_y^n + \delta_x [A^{(n)} \cdot (h^{n+1} - h^n)] + \delta_x [B^{(n)} \cdot \delta_x (h^{n+1} - h^n)] \\ &+ \delta_x [C^{(n)} \cdot (Z^{n+1} - Z^n)] + \delta_x [D^{(n)} \cdot \delta_x (Z^{n+1} - Z^n)] \\ &+ \delta_y [I^{(n)} \cdot (h^{n+1} - h^n)] + \delta_y [J^{(n)} \cdot \delta_y (h^{n+1} - h^n)] \\ &+ \delta_y [K^{(n)} \cdot (Z^{n+1} - Z^n)] + \delta_y [L^{(n)} \cdot \delta_y (Z^{n+1} - Z^n)] \end{aligned} \right\} + O(\Delta t^3)$$

Simplifying and rearranging it yields

$$\begin{aligned} &[1 - a(\delta_x C + \delta_x D \delta_x + \delta_y K + \delta_y L \delta_y)] Z^{n+1} \\ &= 2a[F_x^n + G_y^n] + [1 - a(\delta_x C + \delta_x D \delta_x + \delta_y K + \delta_y L \delta_y)] Z^n \\ &+ a \left[\begin{aligned} &\delta_x A(h^{n+1} - h^n) + \delta_x B(h_x^{n+1} - h_x^n) \\ &+ \delta_y I(h^{n+1} - h^n) + \delta_y J(h_y^{n+1} - h_y^n) \end{aligned} \right] + O(\Delta t^3) \end{aligned}$$

(g.22)

Where δ_x and δ_y are finite difference derivative approximations in the x and y directions, respectively. In this analysis, h^{n+1} is treated as known either from the profile of a fixed rigid bearing or from a solution of the slider bearing dynamics solved simultaneously with the lubrication equation. For convenience, a new function is

introduced

$$\begin{aligned} & \phi(x, y, t^{(n)}, t^{(n+1)}) \\ &= 2a(F_x^{(n)} + G_y^{(n)}) + a \left\{ \begin{aligned} & \delta_x A(h^{(n+1)} - h^{(n)}) + \delta_x B(h_x^{(n+1)} - h_x^{(n)}) \\ & + \delta_y I(h^{(n+1)} - h^{(n)}) + \delta_y J(h_y^{(n+1)} - h_y^{(n)}) \end{aligned} \right\} \end{aligned} \quad (g.23)$$

and the linear operators

$$L_1(x) = a \{ \delta_x C + \delta_x D \delta_x \} \quad (g.24)$$

$$L_2(x) = a \{ \delta_y K + \delta_y L \delta_y \}$$

For clarification,

$$L_1^{(n)}(\cdot)_{i,j} = a \{ \delta_x C + \delta_x D \delta_x \}^{(n)}(\cdot)_{i,j} \quad (g.25)$$

$$L_2^{(n)}(\cdot)_{i,j} = a \{ \delta_y K + \delta_y L \delta_y \}^{(n)}(\cdot)_{i,j}$$

Eq. (g.24) also can be expressed as

$$L_1^{(n)}(\cdot)_{i,j} = a \left\{ \delta_x \left[C \cdot (\cdot)_{i,j} \right] + \delta_x \left[D \cdot \delta_x (\cdot)_{i,j} \right] \right\}^{(n)} \quad (g.26)$$

$$L_2^{(n)}(\cdot)_{i,j} = a \left\{ \delta_y \left[K \cdot (\cdot)_{i,j} \right] + \delta_y \left[L \cdot \delta_y (\cdot)_{i,j} \right] \right\}^{(n)}$$

With the linear operators, eq. (g.25) then appears as

$$[1 - L_1 - L_2]^{(n)} Z^{n+1} = [1 - L_1 - L_2]^{(n)} Z^n + \phi^{(n)} + O(\Delta t^3) \quad (g.27)$$

Factoring the derivative operation in eq. (g-27), we obtain

$$[1 - L_1][1 - L_2] \Delta Z^n = L_1 L_2 (Z^{n+1} - Z^n) + \phi + O(\Delta t^3) \quad (g.28)$$

where $\Delta Z^n = Z^{n+1} - Z^n$

The first term on the right-hand side of eq. (g.28) is multiplied and divided by Δt , and

then rearranged to appear as

$$\begin{aligned} & L_1 L_2 (Z^{n+1} - Z^n) \\ &= \left(\frac{h_{\min}^2 P_a t_{scale} \Delta t}{24 \mu L_x^2} \right)^2 \Delta t [\delta_x C + \delta_x D \delta_x] [\delta_y K + \delta_y L \delta_y] [Z_t + O(\Delta t)] \end{aligned} \quad (g.29)$$

The order of magnitude of eq. (g.29) is the same as the truncated term in eq. (g.28) and can be omitted without increasing the order of the truncated error. This completes our proof of the second –order time accuracy of the numerical scheme which appears now as

$$[1 - L_1][1 - L_2]\Delta Z^n = \phi \quad (g.30)$$

It is interesting to note that the asymptotic steady state solution of eq. (g.30) is independent of the size of the time increment Δt .

The factor solution is achieved by the following sequence:

$$[1 - L_1]\Delta Z^* = \phi \quad (g.31)$$

$$[1 - L_2]\Delta Z^n = \Delta Z^* \quad (g.32)$$

Eqs. (g.31) and (g.32) are seen to be equivalent to eq. (g.30) by operating on eq. (g.32) with $[1 - L_1]$ and the adding eq. (g.31). The factored form of the two-dimensional problem as shown in eq. (g.30) has thus been reduced to the solution of two one-dimensional problems. Eq. (g.31) represents a number of one-dimensional problems in the x row direction while eq. (g.32) uses the solution of eq. (g.31) to define a number of one-dimensional problems in the y column direction.

If three point finite difference derivatives are used, each dimensional solution will require a tridiagonal inversion.

Letting $\Delta x(i) = x(i+1) - x(i)$, $\Delta x(i-1) = x(i) - x(i-1)$, the finite difference derivatives are expressed for unequal grid increment as

$$Z_x(i, j) = \frac{[\Delta x(i-1)]^2 Z(i+1, j) - [\Delta x(i)]^2 Z(i-1, j) - [\Delta x(i-1)]^2 Z(i, j) + [\Delta x(i)]^2 Z(i, j)}{[\Delta x(i-1)]^2 [\Delta x(i)] + [\Delta x(i-1)][\Delta x(i)]^2} \quad (g.33)$$

$$Z_{xx}(i, j) = \frac{2[\Delta x(i-1)]Z(i+1, j) - 2\{[\Delta x(i)] + [\Delta x(i-1)]\}Z(i, j) + 2[\Delta x(i)]Z(i-1, j)}{\{[\Delta x(i)][\Delta x(i-1)]^2 + [\Delta x(i-1)][\Delta x(i)]^2\}} \quad (g.34)$$

Letting $\Delta y(i) = y(i+1) - y(i)$, $\Delta y(i-1) = y(i) - y(i-1)$

$$Z_y = \frac{\partial Z(i, j)}{\partial y} = \sum_{k=1}^3 D_k(j) Z(i, j+2-k) \quad (g.35)$$

where

$$D_1(j) = \frac{\Delta y(j-1)}{\Delta y(j)[\Delta y(j) + \Delta y(j-1)]}, \quad D_2(j) = \frac{\Delta y(j) - \Delta y(j-1)}{\Delta y(j)\Delta y(j-1)}, \quad D_3(j) = \frac{-\Delta y(j)}{\Delta y(j-1)[\Delta y(j) + \Delta y(j-1)]}$$

$$Z_{yy} = \frac{\partial^2 Z(i, j)}{\partial y^2} = \sum_{k=1}^3 B_k(j) Z(i, j+2-k) \quad (g.36)$$

where

$$B_1(j) = \frac{2}{\Delta y(j)[\Delta y(j-1) + \Delta y(j)]}, \quad B_2(j) = \frac{-2}{\Delta y(j-1)\Delta y(j)}, \quad B_3(j) = \frac{2}{\Delta y(j-1)[\Delta y(j) + \Delta y(j-1)]}$$

$$L_1(\cdot)_{i,j} = a \left\{ \delta_x C(\cdot)_{i,j} + C \cdot \delta_x(\cdot)_{i,j} + \delta_x D \cdot \delta_x(\cdot)_{i,j} + D \cdot \delta_{xx}(\cdot)_{i,j} \right\} \quad (g.37)$$

For $\delta_x(\cdot)_{i,j}$ and $\delta_{xx}(\cdot)_{i,j}$

$$\delta_x(\cdot)_{i,j} = \left[C_1(i) \cdot (\cdot)_{i+1,j} + C_2(i) \cdot (\cdot)_{i,j} + C_3(i) \cdot (\cdot)_{i-1,j} \right]$$

$$\delta_{xx}(\cdot)_{i,j} = \left[A_1(i) \cdot (\cdot)_{i+1,j} + A_2(i) \cdot (\cdot)_{i,j} + A_3(i) \cdot (\cdot)_{i-1,j} \right]$$

$$(1 - L_1)(\cdot)_{i,j}$$

$$= -a \left\{ \left(2hZ_x - Zh_x - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \right) \cdot C_3(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot A_3(i) \right\}^{(n)} \cdot (\cdot)_{i-1,j}$$

$$+ \left\{ \begin{aligned} &1 - a \left(hZ_{xx} - Z_x h_x - 2Zh_{xx} - 6 \frac{\lambda_a}{h_{\min}} h_{xx} \right) \\ &- a \left(2hZ_x - Zh_x - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \right) \cdot C_2(i) - a \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot A_2(i) \end{aligned} \right\}^{(n)} \cdot (\cdot)_{i,j} \quad (g.38)$$

$$- a \left\{ \left(2hZ_x - Zh_x - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \right) \cdot C_1(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot A_1(i) \right\}^{(n)} \cdot (\cdot)_{i+1,j}$$

$$\begin{aligned}
 & (1-L_2)(\cdot)_{i,j} \\
 &= -a \left\{ \left(2hZ_y - Zh_y - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_y \right) \cdot D_3(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot B_3(i) \right\}^{(n)} \cdot (\cdot)_{i-1,j} \\
 &+ \left\{ \begin{aligned} & 1 - a \left(hZ_{yy} - Z_y h_y - 2Zh_{yy} - 6 \frac{\lambda_a}{h_{\min}} h_{yy} \right) \\ & - a \left(2hZ_y - Zh_y - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_y \right) \cdot D_2(i) - a \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot B_2(i) \end{aligned} \right\}^{(n)} \cdot (\cdot)_{i,j} \\
 &- a \left\{ \left(2hZ_y - Zh_y - 6 \frac{\mu L_x}{h_{\min}^2 P_a} V_y \right) \cdot D_1(i) + \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) h \cdot B_1(i) \right\}^{(n)} \cdot (\cdot)_{i+1,j}
 \end{aligned} \tag{g.39}$$

$$\begin{aligned}
 & \phi^{(n)}(x, y, t^n, t^{n+1}) \\
 &= a \left\{ \begin{aligned} & \left(Z + 6 \frac{\lambda_a}{h_{\min}} \right) \left[(Z_{xx} + Z_{yy})(h^{(n)} + h^{(n+1)}) \right. \\ & \quad \left. - Z(h_{xx}^{(n)} + h_{yy}^{(n)} + h_{xx}^{(n+1)} + h_{yy}^{(n+1)}) \right] \\ & + Z_x \left[(h^{(n)} + h^{(n+1)})Z_x - Z(h_x^{(n)} + h_x^{(n+1)}) - 12 \frac{\mu L_x}{h_{\min}^2 P_a} V_x \right] \\ & + Z_y \left[(h^{(n+1)} + h^{(n)})Z_y - Z(h_y^{(n)} + h_y^{(n+1)}) - 12 \frac{\mu L_x}{h_{\min}^2 P_a} V_y \right] \end{aligned} \right\}^{(n)}
 \end{aligned} \tag{g.40}$$

calculated as following:

$$\begin{aligned}
 x_{totrailing}(k) &= \Delta x_{\min} \times x_{scale}^0 \\
 &\dots \\
 x_{totrailing}(i) &= \Delta x_{\min} \times x_{scale}^0 + \Delta x_{\min} \times x_{scale}^1 + \dots + \Delta x_{\min} \times x_{scale}^{k-i} \\
 &\dots \\
 x_{totrailing}(1) &= L_x
 \end{aligned} \tag{h.2}$$

The x coordinate of each grid point is:

$$\begin{aligned}
 x(k+1) &= x_{length} \\
 &\dots \\
 x(i) &= x_{length} - x_{totrailing}(i) \\
 &= x_{length} - \Delta x_{\min} \times x_{scale}^0 - \Delta x_{\min} \times x_{scale}^1 - \dots - \Delta x_{\min} \times x_{scale}^{k-i} \\
 &\dots \\
 x(1) &= 0
 \end{aligned} \tag{h.3}$$

where

$$\Delta x(i) = x(i+1) - x(i)$$

$$\Delta x(i-1) = x(i) - x(i-1)$$

Similarly, the coordinate of each grid point along y direction is:

$$\begin{aligned}
 \Delta y(m) &= y(m+1) - x(m) = \Delta y_{\min} \times y_{scale}^0 \\
 &\dots \\
 \Delta y(j) &= \Delta y_{\min} \times y_{scale}^{k-i} \\
 &\dots \\
 \Delta y(1) &= y(2) - y(1) = y_{length} - \Delta y_{\min} \times y_{scale}^0 - \Delta y_{\min} \times y_{scale}^1 - \dots - \Delta y_{\min} \times y_{scale}^{k-2} \quad (h.4)
 \end{aligned}$$

where

$$\Delta y(i) = y(i+1) - y(i)$$

$$\Delta y(i-1) = y(i) - y(i-1)$$

Appendix I. Finite Volume method

The surface of the sub-ambient (negative) pressure air bearing slider has geometrical (clearance) discontinuities. This causes numerical difficulty for finite difference methods based on the differential form of the modified Reynolds equation. To solve the problem, an artificial smooth profile has to be used to smooth the steep wall profile which reduces the accuracy of the numerical solution. By utilizing FVM, this difficulty can be avoided effectively.

I.1 Introduction of the Control Volume Scheme

The general transport equations for Φ can be expressed as

$$\frac{\partial(\rho\phi)}{\partial t} + \text{div}(\rho\phi\vec{u}) = \text{div}(\Gamma \text{grad}\phi) + S_\phi \quad (i.1)$$

where, $\text{div}(\rho\phi\vec{u})$ is the convection term and $\text{div}(\Gamma \text{grad}\Phi)$ is the diffusive term. In two dimensions, the equation without the source term can be written as

$$\frac{\partial}{\partial X} \left(\Gamma \frac{\partial \phi}{\partial X} - \rho\phi u \right) + \frac{\partial}{\partial Y} \left(\Gamma \frac{\partial \phi}{\partial Y} - \rho\phi v \right) = \frac{\partial(\rho\phi)}{\partial t} \quad (i.2)$$

The key step of the finite volume method is the integration of equation over a three-dimensional control volume (CV), yielding

$$\int_{CV} \frac{\partial(\rho\phi)}{\partial t} dV + \int_{CV} \text{div}(\rho\phi\vec{u}) dV = \int_{CV} \text{div}(\Gamma \text{grad}\phi) dV + \int_{CV} S_\phi dV \quad (i.3)$$

Applying Gauss' divergence Theorem,

$$\frac{\partial}{\partial t} \int_{CV} \rho\phi dV + \int_A \vec{n} \cdot (\rho\phi\vec{u}) dA = \int_A \vec{n} \cdot (\Gamma \text{grad}\phi) dA + \int_{CV} S_\phi dV \quad (i.4)$$

For steady state problem,

$$\int_A \vec{n} \cdot (\rho\phi\vec{u}) dA = \int_A \vec{n} \cdot (\Gamma \text{grad}\phi) dA + \int_{CV} S_\phi dV \quad (i.5)$$

For transient problem, it is also necessary to integrate with respect to time t over a small

interval Δt from t to $t+\Delta t$.

$$(5-62) \quad \int_{\Delta t} \frac{\partial}{\partial t} \left(\int_{CV} (\rho \phi) dV \right) dt + \int_{\Delta t} \int_A \vec{n} \cdot (\rho \phi \vec{u}) dA dt = \int_{\Delta t} \int_A \vec{n} \cdot (\Gamma_{\phi} \text{grad} \phi) dA dt + \int_{\Delta t} \int_{CV} S_{\phi} dV dt$$

For a two dimensional problem, the Green' theorem can be used

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy = \oint_l P dx + Q dy \quad (i.6)$$

I.2 Integration of the Modified Reynolds Equation with Boundary Slip Correction for Steady State

Letting

$$\phi = P, \rho = H, u = \Lambda_x, v = \Lambda_y, \Gamma = Q_p PH^3$$

For the head disk interface, the steady state problem is expressed as

$$\frac{\partial}{\partial X} (Q_p PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH) + \frac{\partial}{\partial Y} (Q_p PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH) = 0 \quad (i.7)$$

Its integral form can be written as:

$$\begin{aligned} & \iint_D \left(\frac{\partial J_x}{\partial X} + \frac{\partial J_y}{\partial Y} \right) dX dY \\ &= \oint_l -J_y dX + J_x dY \\ &= \int_e J_x dY + \int_w J_x dY - \int_n J_y dX - \int_s J_y dX \\ &= J_{x=e} \Delta Y - J_{x=w} \Delta Y + J_{y=n} \Delta X - J_{y=s} \Delta X = 0 \end{aligned} \quad (i.8)$$

where $J_x = Q_p PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH$, $J_y = Q_p PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH$.

The definition and the direction of the volume can be defined as shown in Fig. i.1 and Fig. i.2.

Fig. i.1 Definition of a control volume

Fig. i.2 Definition of the direction of the volume

Let $J_e = J_{x=e} \Delta Y$, $J_w = J_{x=w} \Delta Y$, $J_n = J_{y=n} \Delta X$, $J_s = J_{y=s} \Delta X$

$$J_e - J_w + J_n - J_s = 0 \quad (i.9)$$

$$J_e \equiv J_{x=e} \Delta Y = \left(Q_p PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right)_{x=e} \Delta Y = \left(Q_p PH^3 \frac{\partial P}{\partial X} \right)_e \Delta Y - (\Lambda_x PH)_e \Delta Y \quad (i.10)$$

Similarly, we can obtain the expressions for J_w, J_n, J_s

$$J_w = J_{x=w} \Delta Y = \left(Q_p PH^3 \frac{\partial P}{\partial X} - \Lambda_x PH \right)_{x=w} \Delta Y \quad (i.11)$$

$$J_n = J_{y=n} \Delta X = \left(Q_p PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right)_{y=n} \Delta X \quad (i.12)$$

$$J_s = J_{y=s} \Delta X = \left(Q_p PH^3 \frac{\partial P}{\partial Y} - \Lambda_y PH \right)_{y=s} \Delta X \quad (i.13)$$

Peclet number measures the relative strength between convection and diffusion and is defined as:

$$Pe \equiv F / D$$

Where the convection coefficients are

$$F_e = (A_x H)_e \Delta Y, \quad F_w = (A_x H)_w \Delta Y, \quad F_n = (A_y H)_n \Delta X, \quad F_s = (A_y H)_s \Delta X$$

and the diffusion coefficients are

$$D_e = \Gamma_e \Delta Y / (\delta X)_e, \quad D_w = \Gamma_w \Delta Y / (\delta X)_w, \quad D_n = \Gamma_n \Delta X / (\delta Y)_n, \quad D_s = \Gamma_s \Delta X / (\delta Y)_s$$

It is very important that the relationship between the directionality of influencing and the flow direction and magnitude of the Peclet number is borne out in the discretization scheme. P and H are updated with the newest values.

I.3 The Difference Scheme

One problem raised is the negative coefficients violate one of the basic rules of discretization stated by Patankar and lead to divergence of the solution [74]. In high speed gas lubricated bearings, a simplistic treatment of the convective term using central difference often leads to numerical instability. Lu compared several different differencing schemes and concluded that the hybrid scheme is superior to other schemes, such as modified central difference, upwind, hybrid, power-law and QUICK scheme, in terms of stability, accuracy and computational efficiency.

For the unequal grids, the intervals between the grid points can be expressed as

$$\delta X_{wP} = \frac{1}{2} \delta X_{WP}, \quad \delta X_{Pe} = \frac{1}{2} \delta X_{PE}, \quad \delta Y_{sP} = \frac{1}{2} \delta Y_{SP}, \quad \delta Y_{Pn} = \frac{1}{2} \delta Y_{PN},$$

$$\Delta X = \delta X_{wP} + \delta X_{Pe} = \frac{1}{2} (\delta X_{WP} + \delta X_{PE}), \quad \Delta Y = \delta Y_{sP} + \delta Y_{Pn} = \frac{1}{2} (\delta Y_{SP} + \delta Y_{PN})$$

For the central differencing scheme:

$$\left(\frac{\partial P}{\partial X} \right)_w = \frac{P_P - P_W}{\delta X_{WP}} = \frac{P_P - P_W}{\delta X_w}, \quad \left(\frac{\partial P}{\partial X} \right)_e = \frac{P_E - P_P}{\delta X_{PE}} = \frac{P_E - P_P}{\delta X_e},$$

$$\left(\frac{\partial P}{\partial Y} \right)_s = \frac{P_P - P_S}{\delta Y_{SP}} = \frac{P_P - P_S}{\delta Y_s}, \quad \left(\frac{\partial P}{\partial Y} \right)_n = \frac{P_N - P_P}{\delta Y_{PN}} = \frac{P_N - P_P}{\delta Y_n}$$

$$P_w = \frac{P_p + P_w}{2}, \quad P_e = \frac{P_E + P_p}{2}, \quad P_s = \frac{P_p + P_s}{2}, \quad P_n = \frac{P_N + P_p}{2}$$

For the upwind scheme, when the flow is in the positive direction:

$$P_w = P_w, \quad P_e = P_p, \quad P_s = P_s, \quad P_n = P_p$$

For the upwind scheme, when the flow is in the negative direction:

$$P_w = P_p, \quad P_e = P_E, \quad P_s = P_p, \quad P_n = P_N$$

The hybrid differencing scheme is based on a combination of the central and upwind differencing schemes because one of the major inadequacies of the central differencing scheme is its inability to identify flow direction. The upwind differencing scheme takes into account the flow direction when determining the value at a cell face. The central differencing scheme, which is accurate to second-order, is employed for small Peclet number ($Pe < 2$). It is equivalent to using central differencing for the convection and diffusion terms. The upwind scheme, which is accurate to first order but accounts for transportiveness, is employed for large Peclet numbers ($Pe \geq 2$). It is equivalent to using upwind scheme for convection and setting the diffusion to zero.

I.4 Discretization of the integrated Reynolds Equation

By using the central difference scheme, the modified Reynolds equation can be discretized as eq. (i.15)

$$a_p P_p = a_E P_E + a_w P_w + a_N P_N + a_S P_s + b \quad (i.15)$$

Where, $a_E = D_e - \frac{F_e}{2}$, $a_N = D_n - \frac{F_n}{2}$, $a_w = D_w + \frac{F_w}{2}$, $a_S = D_s + \frac{F_s}{2}$, $b=0$,

$$a_p = a_E + a_w + a_N + a_S + (F_e - F_w + F_n - F_s)$$

A number of schemes have been studied to overcome the stability problem of the central difference scheme and the potential problem caused by non-zero sum of $(F_e - F_w + F_n - F_s)$. The resulting coefficients for the hybrid differencing schemes are

summarized in table *i.1*.

Table. *i.1* hybrid differencing formula

a_E	$D_e \max \left[0, \left(1 - 0.5 P_{ee} \right) \right] + \max (-F_e, 0) \quad \text{or} \quad \max \left(-F_e, \left(D_e - \frac{F_e}{2} \right), 0 \right)$
a_W	$D_w \max \left[0, \left(1 - 0.5 P_{ew} \right) \right] + \max (F_w, 0) \quad \text{or} \quad \max \left(F_w, \left(D_w + \frac{F_w}{2} \right), 0 \right)$
a_N	$D_n \max \left[0, \left(1 - 0.5 P_{en} \right) \right] + \max (-F_n, 0) \quad \text{or} \quad \max \left(-F_n, \left(D_n - \frac{F_n}{2} \right), 0 \right)$
a_S	$D_s \max \left[0, \left(1 - 0.5 P_{es} \right) \right] + \max (F_s, 0) \quad \text{or} \quad \max \left(F_s, \left(D_s + \frac{F_s}{2} \right), 0 \right)$
a_P	$a_E + a_W + a_N + a_S + \max (0, F_e - F_w + F_n - F_s)$
b	$\max (0, -F_e + F_w - F_n + F_s) \cdot P_P$

Appendix J. Air Bearing Forces and Moments

Table j.1 Positive forces for different minimal flying heights and pitch angles

N	Pitch Angle (μ rad)							
		25	50	75	100	125	150	175
Minimal Flying Height (mm)	5	0.096469	0.056223	0.040417	0.032886	0.027527	0.023772	0.021021
	6	0.089335	0.053038	0.038336	0.032886	0.026271	0.022743	0.020155
	7	0.083604	0.050271	0.036526	0.029898	0.025156	0.021822	0.019376
	8	0.078385	0.047855	0.034912	0.028652	0.024155	0.020998	0.018668
	9	0.073750	0.045709	0.033476	0.027529	0.023600	0.020241	0.018021
	10	0.069907	0.043779	0.032178	0.026511	0.023248	0.019548	0.017426
	20	0.045811	0.031297	0.023616	0.019701	0.016822	0.014785	0.013296
	30	0.034022	0.024379	0.018783	0.015816	0.013593	0.012013	0.010852
	40	0.026640	0.019775	0.015514	0.013176	0.011396	0.010122	0.009185
	50	0.021552	0.016436	0.013105	0.011222	0.009770	0.008723	0.007953
	60	0.017822	0.013903	0.011242	0.009703	0.008504	0.007635	0.006996
	70	0.014989	0.011909	0.009754	0.008485	0.007487	0.006760	0.006239
	80	0.012767	0.010308	0.008539	0.007484	0.006649	0.006050	0.005595
	90	0.010989	0.008995	0.007531	0.006650	0.005956	0.005448	0.005078
	100	0.009540	0.007903	0.006685	0.005945	0.005364	0.004937	0.004628
	200	0.003090	0.002801	0.002587	0.002465	0.002379	0.002333	0.002320
	300	0.001381	0.001358	0.001378	0.001406	0.001456	0.001524	0.001604
	400	0.000828	0.000885	0.000962	0.001036	0.001123	0.001214	0.001309
	500	0.000650	0.000727	0.000814	0.000893	0.000981	0.001073	0.001161
	600	0.000598	0.000674	0.000759	0.000831	0.000914	0.000996	0.001077
	700	0.000590	0.000660	0.000738	0.000800	0.000873	0.000944	0.001014
	800	0.000593	0.000655	0.000722	0.000779	0.000841	0.000902	0.000962
	900	0.000596	0.000649	0.000707	0.000757	0.000810	0.000861	0.000911
	1000	0.000590	0.000636	0.000685	0.000728	0.000773	0.000817	0.000860

Table j.2 Negative forces for different minimal flying heights and pitch angles

N	Pitch Angle (μ rad)							
		25	50	75	100	125	150	175
Minimal Flying Height (mm)	5	-0.0207	-0.019749	-0.018160	-0.016581	-0.015083	-0.013776	-0.012480
	6	-0.02069	-0.019765	-0.017992	-0.016490	-0.015013	-0.013705	-0.012410
	7	-0.02068	-0.019651	-0.018033	-0.016420	-0.014942	-0.013635	-0.012341
	8	-0.02064	-0.019605	-0.017772	-0.016328	-0.014871	-0.013540	-0.012273
	9	-0.02061	-0.019568	-0.017702	-0.016256	-0.014835	-0.013470	-0.012204
	10	-0.02058	-0.019353	-0.017595	-0.016256	-0.014800	-0.013399	-0.012136
	20	-0.02015	-0.018595	-0.016804	-0.015410	-0.013988	-0.012698	-0.011437
	30	-0.01936	-0.017664	-0.015961	-0.014597	-0.013256	-0.011991	-0.010786
	40	-0.01847	-0.016789	-0.015165	-0.013861	-0.012565	-0.011331	-0.010162
	50	-0.01764	-0.016016	-0.014407	-0.013166	-0.011898	-0.010698	-0.009569
	60	-0.01675	-0.015182	-0.013688	-0.012492	-0.011257	-0.010095	-0.009007
	70	-0.01591	-0.014479	-0.013011	-0.011843	-0.010644	-0.009522	-0.008447
	80	-0.01518	-0.013773	-0.012357	-0.011220	-0.010060	-0.008956	-0.007955
	90	-0.01449	-0.013108	-0.011728	-0.010625	-0.009483	-0.008427	-0.007451
	100	-0.0138	-0.012468	-0.011125	-0.010058	-0.008944	-0.007931	-0.007003
	200	-0.00832	-0.007389	-0.006480	-0.005783	-0.005092	-0.004471	-0.003915
	300	-0.00516	-0.004560	-0.003948	-0.003509	-0.003086	-0.002712	-0.002383

400	-0.00335	-0.002963	-0.002593	-0.002312	-0.002041	-0.001802	-0.001592
500	-0.00233	-0.002062	-0.001810	-0.001622	-0.001441	-0.001279	-0.001138
600	-0.00169	-0.001503	-0.001327	-0.001196	-0.001065	-0.000949	-0.000846
700	-0.00127	-0.001136	-0.001003	-0.000908	-0.000811	-0.000725	-0.000649
800	-0.00098	-0.000878	-0.000778	-0.000703	-0.000631	-0.000567	-0.000509
900	-0.00077	-0.000689	-0.000613	-0.000556	-0.000501	-0.000452	-0.000408
1000	-0.00062	-0.000553	-0.000494	-0.000450	-0.000407	-0.000369	-0.000335

Table j.3 positive moments for varying minimal flying height and pitch angle

N m	Pitch Angle (μrad)							
		25	50	75	100	125	150	175
Minimal Flying Height (mm)	5	1.69590×10 ⁻⁵	1.27250×10 ⁻⁵	1.08190×10 ⁻⁵	9.70120×10 ⁻⁶	8.77890×10 ⁻⁶	8.04220×10 ⁻⁶	7.44080×10 ⁻⁶
	6	1.48730×10 ⁻⁵	1.27250×10 ⁻⁵	9.94250×10 ⁻⁶	8.99450×10 ⁻⁶	8.19860×10 ⁻⁶	7.55390×10 ⁻⁶	7.02180×10 ⁻⁶
	7	1.32920×10 ⁻⁵	1.05190×10 ⁻⁵	9.19680×10 ⁻⁶	8.38470×10 ⁻⁶	7.69210×10 ⁻⁶	7.12320×10 ⁻⁶	6.64900×10 ⁻⁶
	8	1.19140×10 ⁻⁵	9.67510×10 ⁻⁶	8.55500×10 ⁻⁶	7.85340×10 ⁻⁶	7.24520×10 ⁻⁶	6.74140×10 ⁻⁶	6.31470×10 ⁻⁶
	9	1.07490×10 ⁻⁵	8.95220×10 ⁻⁶	7.99590×10 ⁻⁶	7.38470×10 ⁻⁶	7.00000×10 ⁻⁶	6.39750×10 ⁻⁶	6.01280×10 ⁻⁶
	10	9.82460×10 ⁻⁶	8.32770×10 ⁻⁶	7.50450×10 ⁻⁶	6.96830×10 ⁻⁶	6.84760×10 ⁻⁶	6.08700×10 ⁻⁶	5.73860×10 ⁻⁶
	20	5.17770×10 ⁻⁶	4.88440×10 ⁻⁶	4.63040×10 ⁻⁶	4.44310×10 ⁻⁶	4.25930×10 ⁻⁶	4.09000×10 ⁻⁶	3.93620×10 ⁻⁶
	30	3.57560×10 ⁻⁶	3.45920×10 ⁻⁶	3.34630×10 ⁻⁶	3.25660×10 ⁻⁶	3.16190×10 ⁻⁶	3.07190×10 ⁻⁶	2.98730×10 ⁻⁶
	40	2.75770×10 ⁻⁶	2.69460×10 ⁻⁶	2.62920×10 ⁻⁶	2.57490×10 ⁻⁶	2.51660×10 ⁻⁶	2.46040×10 ⁻⁶	2.40690×10 ⁻⁶
	50	2.26640×10 ⁻⁶	2.22330×10 ⁻⁶	2.17730×10 ⁻⁶	2.13850×10 ⁻⁶	2.09730×10 ⁻⁶	2.05750×10 ⁻⁶	2.01960×10 ⁻⁶
	60	1.94710×10 ⁻⁶	1.91030×10 ⁻⁶	1.87070×10 ⁻⁶	1.83920×10 ⁻⁶	1.80660×10 ⁻⁶	1.77530×10 ⁻⁶	1.74590×10 ⁻⁶
	70	1.72360×10 ⁻⁶	1.68730×10 ⁻⁶	1.65160×10 ⁻⁶	1.62380×10 ⁻⁶	1.59550×10 ⁻⁶	1.56870×10 ⁻⁶	1.54270×10 ⁻⁶
	80	1.56020×10 ⁻⁶	1.52330×10 ⁻⁶	1.48890×10 ⁻⁶	1.46270×10 ⁻⁶	1.43650×10 ⁻⁶	1.41090×10 ⁻⁶	1.39080×10 ⁻⁶
	90	1.43600×10 ⁻⁶	1.39840×10 ⁻⁶	1.36430×10 ⁻⁶	1.33870×10 ⁻⁶	1.31220×10 ⁻⁶	1.28950×10 ⁻⁶	1.26940×10 ⁻⁶
	100	1.33970×10 ⁻⁶	1.30110×10 ⁻⁶	1.26650×10 ⁻⁶	1.24090×10 ⁻⁶	1.21500×10 ⁻⁶	1.19290×10 ⁻⁶	1.17360×10 ⁻⁶
	200	9.35660×10 ⁻⁷	8.92460×10 ⁻⁷	8.53660×10 ⁻⁷	8.25400×10 ⁻⁷	7.98420×10 ⁻⁷	7.75300×10 ⁻⁷	7.57450×10 ⁻⁷
	300	7.81010×10 ⁻⁷	7.42950×10 ⁻⁷	7.03030×10 ⁻⁷	6.78330×10 ⁻⁷	6.57280×10 ⁻⁷	6.41990×10 ⁻⁷	6.31240×10 ⁻⁷
	400	6.72170×10 ⁻⁷	6.42820×10 ⁻⁷	6.16780×10 ⁻⁷	5.98780×10 ⁻⁷	5.82520×10 ⁻⁷	5.70230×10 ⁻⁷	5.60790×10 ⁻⁷
	500	5.95860×10 ⁻⁷	5.72390×10 ⁻⁷	5.51500×10 ⁻⁷	5.36580×10 ⁻⁷	5.23520×10 ⁻⁷	5.12290×10 ⁻⁷	5.03830×10 ⁻⁷
	600	5.30790×10 ⁻⁷	5.11820×10 ⁻⁷	4.94690×10 ⁻⁷	4.82480×10 ⁻⁷	4.70410×10 ⁻⁷	4.60260×10 ⁻⁷	4.51820×10 ⁻⁷
	700	4.74570×10 ⁻⁷	4.58150×10 ⁻⁷	4.42870×10 ⁻⁷	4.32300×10 ⁻⁷	4.21770×10 ⁻⁷	4.12820×10 ⁻⁷	4.05260×10 ⁻⁷
	800	4.23190×10 ⁻⁷	4.09230×10 ⁻⁷	3.96150×10 ⁻⁷	3.86570×10 ⁻⁷	3.77610×10 ⁻⁷	3.69930×10 ⁻⁷	3.63370×10 ⁻⁷
	900	3.76670×10 ⁻⁷	3.64920×10 ⁻⁷	3.53940×10 ⁻⁷	3.45900×10 ⁻⁷	3.38320×10 ⁻⁷	3.31760×10 ⁻⁷	3.26110×10 ⁻⁷
	1000	3.35900×10 ⁻⁷	3.26000×10 ⁻⁷	3.16770×10 ⁻⁷	3.09980×10 ⁻⁷	3.03540×10 ⁻⁷	2.97940×10 ⁻⁷	2.93080×10 ⁻⁷

Table j.4 negative moments for varying minimal flying height and pitch angle

	Pitch Angle (μrad)							
		25	50	77	100	125	150	175
Minimal Flying Height (mm)	5	-2.04020×10 ⁻⁵	-1.23480×10 ⁻⁵	-8.95470×10 ⁻⁶	-7.21980×10 ⁻⁶	-5.94280×10 ⁻⁶	-5.02000×10 ⁻⁶	-4.28540×10 ⁻⁶
	6	-1.97430×10 ⁻⁵	-1.21500×10 ⁻⁵	-8.81400×10 ⁻⁶	-7.13630×10 ⁻⁶	-5.88480×10 ⁻⁶	-4.97390×10 ⁻⁶	-4.24740×10 ⁻⁶
	7	-1.91170×10 ⁻⁵	-1.19230×10 ⁻⁵	-8.73320×10 ⁻⁶	-7.05940×10 ⁻⁶	-5.82760×10 ⁻⁶	-4.92850×10 ⁻⁶	-4.20960×10 ⁻⁶
	8	-1.85440×10 ⁻⁵	-1.17250×10 ⁻⁵	-8.57210×10 ⁻⁶	-6.97800×10 ⁻⁶	-5.77080×10 ⁻⁶	-4.87830×10 ⁻⁶	-4.17240×10 ⁻⁶
	9	-1.80080×10 ⁻⁵	-1.15360×10 ⁻⁵	-8.46570×10 ⁻⁶	-6.90340×10 ⁻⁶	-5.74080×10 ⁻⁶	-4.83370×10 ⁻⁶	-4.13550×10 ⁻⁶
	10	-1.75120×10 ⁻⁵	-1.13050×10 ⁻⁵	-8.35120×10 ⁻⁶	-6.82980×10 ⁻⁶	-5.71490×10 ⁻⁶	-4.78960×10 ⁻⁶	-4.09900×10 ⁻⁶
	16	-1.36580×10 ⁻⁵	-9.68170×10 ⁻⁶	-7.39690×10 ⁻⁶	-6.14040×10 ⁻⁶	-5.13620×10 ⁻⁶	-4.37160×10 ⁻⁶	-3.74750×10 ⁻⁶
	20	-1.12680×10 ⁻⁵	-8.39970×10 ⁻⁶	-6.59010×10 ⁻⁶	-5.53120×10 ⁻⁶	-4.67030×10 ⁻⁶	-3.99040×10 ⁻⁶	-3.43130×10 ⁻⁶
	30	-9.56270×10 ⁻⁶	-7.38560×10 ⁻⁶	-5.91240×10 ⁻⁶	-5.01210×10 ⁻⁶	-4.25870×10 ⁻⁶	-3.64980×10 ⁻⁶	-3.14390×10 ⁻⁶
	40	-8.29260×10 ⁻⁶	-6.57170×10 ⁻⁶	-5.33250×10 ⁻⁶	-4.55810×10 ⁻⁶	-3.88940×10 ⁻⁶	-3.34160×10 ⁻⁶	-2.88250×10 ⁻⁶
	50	-7.26350×10 ⁻⁶	-5.86890×10 ⁻⁶	-4.83060×10 ⁻⁶	-4.15430×10 ⁻⁶	-3.55700×10 ⁻⁶	-3.06230×10 ⁻⁶	-2.64460×10 ⁻⁶

APPENDIX J AIR BEARING FORCES AND MOMENTS

60	-6.43330×10^{-6}	-5.29300×10^{-6}	-4.39260×10^{-6}	-3.79320×10^{-6}	-3.25690×10^{-6}	-2.80860×10^{-6}	-2.42430×10^{-6}
70	-5.75510×10^{-6}	-4.78880×10^{-6}	-4.00380×10^{-6}	-3.46890×10^{-6}	-2.98530×10^{-6}	-2.57570×10^{-6}	-2.22570×10^{-6}
80	-5.18550×10^{-6}	-4.35050×10^{-6}	-3.65640×10^{-6}	-3.17670×10^{-6}	-2.73660×10^{-6}	-2.36240×10^{-6}	-2.04150×10^{-6}
90	-4.68900×10^{-6}	-3.96290×10^{-6}	-3.34480×10^{-6}	-2.91260×10^{-6}	-2.50990×10^{-6}	-2.16870×10^{-6}	-1.87630×10^{-6}
100	-1.94450×10^{-6}	-1.68640×10^{-6}	-1.44990×10^{-6}	-1.27730×10^{-6}	-1.11320×10^{-6}	-9.71330×10^{-7}	-8.49600×10^{-7}
200	-9.05560×10^{-7}	-7.93600×10^{-7}	-6.89300×10^{-7}	-6.14190×10^{-7}	-5.45060×10^{-7}	-4.87550×10^{-7}	-4.39600×10^{-7}
300	-4.56830×10^{-7}	-4.07010×10^{-7}	-3.62790×10^{-7}	-3.30760×10^{-7}	-3.01800×10^{-7}	-2.77810×10^{-7}	-2.58600×10^{-7}
400	-2.53930×10^{-7}	-2.31430×10^{-7}	-2.11330×10^{-7}	-1.97640×10^{-7}	-1.85990×10^{-7}	-1.77010×10^{-7}	-1.69940×10^{-7}
500	-1.54270×10^{-7}	-1.44270×10^{-7}	-1.36140×10^{-7}	-1.30850×10^{-7}	-1.26710×10^{-7}	-1.23860×10^{-7}	-1.22080×10^{-7}
600	-1.02990×10^{-7}	-9.89230×10^{-8}	-9.59380×10^{-8}	-9.42340×10^{-8}	-9.34430×10^{-8}	-9.34140×10^{-8}	-9.39660×10^{-8}
700	-7.45900×10^{-8}	-7.31960×10^{-8}	-7.26140×10^{-8}	-7.28260×10^{-8}	-7.36580×10^{-8}	-7.49650×10^{-8}	-7.66250×10^{-8}
800	-5.77590×10^{-8}	-5.77550×10^{-8}	-5.84350×10^{-8}	-5.95590×10^{-8}	-6.11710×10^{-8}	-6.30740×10^{-8}	-6.51900×10^{-8}
900	-4.71820×10^{-8}	-4.79170×10^{-8}	-4.92690×10^{-8}	-5.08050×10^{-8}	-5.27230×10^{-8}	-5.48320×10^{-8}	-5.70660×10^{-8}

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