Reflectometric measurements of polarization properties in optical fiber links using Polarimetric Optical Time Domain Reflectometry (POTDR)

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**Reflectometric Measurements of
Polarization Properties in Optical Fiber
Links Using Polarimetric Optical Time
Domain Reflectometry (POTDR)**

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ABSTRACT

Polarimetric optical time-domain reflectometry (POTDR) was proposed in 1981 for measuring the spatial distributions of local polarization properties of single-mode fibers (SMF). This PhD research project aims to investigate some fundamental issues in POTDR technology and the main achievements are summarized as follows.

Firstly, to improve the measurement accuracy, the optimizations of input states of polarization (SOP) and the polarimeter used in POTDR are investigated theoretically. Results show that 1) for the measurement of the 3×3 Mueller matrix of a pure birefringent SMF system, using two orthogonal input SOPs will statistically deliver the best measurement accuracy. 2) for the measurement of the 4×4 Mueller matrix of a SMF system having both birefringence and polarization dependent loss (PDL), the three optimized input SOPs are equally-spaced on the Poincaré sphere and centered on the reversed PDL vector of the system under test. 3) For the type of polarimeter commonly used in POTDR systems, when thermal noise is dominant in the photodetector and angular orientation errors can be neglected, the optimum angles of the waveplate and the polarizer can be found to minimize the SOP measurement noise. These conclusions are especially useful for optimizing a POTDR system since POTDR signals are quite noisy.

Secondly, novel approaches for spatially- and spectrally-resolved polarization mode dispersion measurement techniques are proposed in SMF systems without and with PDL. The experimental results are also presented to confirm the validity of the proposed techniques.

Thirdly, a novel approach for single-end PDL measurements in SMF links is demonstrated theoretically and experimentally. Further, we demonstrate by simulation

that this technique has the potential to be extended to measure the PDL distribution in a SMF link.

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ACRONYMS

A

AOM Acoustic-Optical Modulator

C

CN Condition Number

CW Continuous Wave

D

DAS Differential Attenuation Slope

DCF Dispersion Compensation Fiber

DFB Distributed Feedback (Laser)

DGD Differential Group Delay

DOP Degree of Polarization

DSF Dispersion Shifted Fiber

E

EDFA Erbium-Doped Fiber Amplifier

F

FUT Fiber Under Test

M

MMM Mueller Matrix Method

M³E Mueller Matrix Measurement Error

N

NEP Noise Equivalent Power

O

OTDR Optical Time Domain Reflectometer

P

PDL Polarization Dependent Loss

PDL/G Polarization Dependent Loss/Gain

PMD Polarization Mode Dispersion

PMF Polarization Maintaining Fiber

POTDR Polarimetric Optical Time Domain Reflectometry

PSP Principal State of Polarization

R

RS-POTDR Random-Scrambling POTDR

S

SMF Single-Mode Fiber

SNR Signal-to-Noise Ratio

SOA Semiconductor Optical Amplifier

SOP State of Polarization

V

VMMM Virtual Mueller Matrix Method

VGMMM Virtual Generalized Mueller Matrix Method

LIST OF SYMBOLS

$\mathbf{M}$	4×4 Mueller matrix
$\Delta \mathbf{M}$	4×4 Mueller matrix measurement error
$\mathbf{m}_B$ or $\mathbf{m}$	3×3 Mueller matrix
$\Delta \mathbf{m}_B$ or $\Delta \mathbf{m}$	3×3 Mueller matrix measurement error
$\vec{S} = (s_0, s_1, s_2, s_3)^T$	Four-dimensional (4D) Stokes vector
$\Delta \vec{S}$	Measurement error of 4D Stokes vector
$\vec{S}_{\text{in}}, \vec{T}_{\text{in}}, \vec{U}_{\text{in}}$	Input 4D Stokes vector
$\vec{S}_{\text{out}}, \vec{T}_{\text{out}}, \vec{U}_{\text{out}}$	Output 4D Stokes vector
x	A physical parameter, such as fiber length
Δx	Increment of x
$\mathbf{M}_x = \mathbf{M}(x + \Delta x) \mathbf{M}^{-1}(x)$	x -domain Mueller matrix
z	Fiber length
ω	Optical angular frequency
$\rho, \text{DOP} = \sqrt{s_1^2 + s_2^2 + s_3^2} / s_0$	Degree of polarization
$ \mathbf{A} $	Determinant of matrix $\mathbf{A}$
$\mathbf{I}$	Identity matrix
$\vec{s} = (s_1, s_2, s_3)^T$	Three-dimensional (3D) Stokes vector

$\Delta \vec{s}$	Measurement error of 3D Stokes vector
$\vec{s}_{\text{in}}, \vec{t}_{\text{in}}$	Input 3D Stokes vector
$\vec{s}_{\text{out}}, \vec{t}_{\text{out}}$	Output 3D Stokes vector
$\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)^T$	PMD vector or the real part of the complex PMD vector
$\vec{\Lambda} = (\Lambda_1, \Lambda_2, \Lambda_3)^T$	Imaginary part of the complex PMD vector
$\vec{W} = \vec{\Omega} + i\vec{\Lambda}$	Complex PMD vector
$\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)^T$	Local PDL vector
$\vec{\beta} = (\beta_1, \beta_2, \beta_3)^T$	Local birefringence vector
$\ \mathbf{A}\ = \sqrt{\text{Tr}(\mathbf{A}^H \mathbf{A})}$	Frobenius matrix norm of matrix $\mathbf{A}$
$\langle \cdot \rangle$	Averaging operator
$\text{Cond}(\mathbf{A}) = \ \mathbf{A}\ \cdot \ \mathbf{A}^{-1}\ $	Condition number of matrix $\mathbf{A}$
$\mathbf{R}$	Diagonal matrix $\text{diag}(1, 1, 1, -1)$
$\mathbf{M}_{\text{B}}$	Round-trip Mueller matrix
$\vec{\alpha}_{\text{B}} = (\alpha_{\text{B1}}, \alpha_{\text{B2}}, \alpha_{\text{B3}})^T$	Round-trip PDL vector
$\vec{\beta}_{\text{B}} = (\beta_{\text{B1}}, \beta_{\text{B2}}, \beta_{\text{B3}})^T$	Round-trip birefringence vector
$\vec{\Omega}_{\text{B}}$	Round-trip PMD vector
$\vec{W}_{\text{B}} = \vec{\Omega}_{\text{B}} + i\vec{\Lambda}_{\text{B}}$	Round-trip complex PMD vector
$\vec{D} = (D_1, D_2, D_3)^T$	Global PDL vector of a SMF link

D	Modulus of $\vec{D}$
D_{B}	Modulus of round-trip global PDL vector $\vec{D}_{\text{B}}$
$\theta, \alpha, \beta, \gamma$	Angles between two Stokes vectors
I	Electric current
ΔI	Measurement error of electric current
σ^2	Variance of a random variable
Θ	Measurement matrix for SOP measurement

Chapter 1 INTRODUCTION

1.1 Background and Motivation

Single-mode optical fibers (SMF) have been widely used in telecommunication and sensing systems. SMFs are produced by drawing fused silica preforms. Although fused silica is an isotropic material, SMFs are birefringent. The birefringence in a SMF may be induced by the following: 1) imperfect core and cladding geometrics (geometric deformation), 2) stress anisotropy due to the thermal expansion difference between core and cladding, 3) external stress, tension, bending and twisting, 4) external electric field and magnetic field [1]. If a SMF is bent with a diameter less than 5 cm, another polarization effect, polarization-dependent loss (PDL) can be observed [2]. Some commonly-used optical components, such as optical isolators and optical couplers, also have a small PDL. When the bandwidth of a light source is relatively large or the SMF link under investigation is time variant, depolarization can also occur in a SMF link [3]. In summary, three fundamental polarization effects, viz. birefringence, PDL and depolarization, may exist in SMFs.

Polarization effects in SMFs have two special features. The first feature is that the three polarization effects are distributed along the fiber length. This means that a SMF can be treated as a cascade of many uniform segments and each segment has a different realization of the three fundamental polarization effects. The second feature is that these polarization effects are very sensitive to the environmental factors, such as temperature, stress, etc. The first feature results in a well-known phenomenon in fiber-optic communication systems: polarization mode dispersion (PMD) and the second feature means that PMD is not deterministic but statistical [4, 5]. PMD has been considered to be a very important limiting factor in a long-haul high-speed optical fiber communication system. As same as the three fundamental polarization

effects, PMD is also distributed along the fiber length. Some fiber sections in a SMF link may have large PMD values. Finding such “bad” fiber sections requires a technology which can measure the distributed polarization effects in SMFs. Furthermore, if the fundamental polarization effects of each individual SMF segment can be measured, a SMF can be used as a distributed sensor by detecting its polarization effect distributions.

In 1981, a novel technology, polarimetric optical time domain reflectometry (POTDR), was proposed by A. J. Roger for measuring the spatial distributions of ambient physical fields that can affect the local polarization properties of SMFs [6]. In POTDR measurements, optical pulses are launched into a SMF; then, backreflected optical signals are generated by Rayleigh backscattering along the fiber or Fresnel backreflection at the fiber junctions. By detecting the state of polarization (SOP) of the backreflected signals from the fiber, the distributed polarization parameters can be measured, which include linear birefringence, twist-induced circular birefringence, PDL and PMD. In particular, PMD distribution measurements using POTDR have been extensively investigated because PMD limits the maximum bit rate in long-haul optical fiber communication systems and POTDR can be used to identify the “bad” fiber sections having larger PMD values. Furthermore, certain physical parameters, such as temperature and stress, can be inferred from the measured fiber polarization parameters. Based on such a principle, POTDR-based sensing technologies have been developed for measuring distributed variations of pressure, temperature, magnetic field, electric field, Faraday effect, vibration and so on [6-10]. However, although almost thirty years passed, POTDR technology is still not mature. To my knowledge, there are currently no commercialized POTDR-based fiber sensors in the market. For PMD distribution measurements, EXFO recently proposed a random-scrambling POTDR (RS-POTDR) which can find the SMF sections with a relatively large PMD

[11]. A prototype instrument of this RS-POTDR has been produced and tested. This seems to be the dawn of POTDR technology towards real applications.

1.2 Objective

The objective of this PhD research is to investigate some fundamental issues in POTDR technology. POTDR system optimization and novel measurement algorithms will be the main targets of our work.

1.3 Project scope

The scope of this PhD research is to investigate some fundamental issues on reflectometric measurements of polarization properties in optical fiber links using POTDR.

Optical signals in POTDR measurements are weak and noisy. Therefore, the optimization of POTDR systems is very important. In this thesis, we will theoretically optimize the polarimeter used in POTDR systems and the input SOPs for Mueller matrix measurement in order to achieve the best accuracy.

PMD in SMFs is a function of optical angular frequency. However, the existing POTDR techniques can only measure the average value of PMD. In this thesis, spectrally-resolved reflectometric PMD measurement techniques using POTDR will be presented for SMF links.

PDL is also an important parameter which affects the performance of fiber-optic communication and sensing systems. To date, reflectometric measurements of PDL have been investigated only by theoretical analysis. In this thesis, we will present a novel measurement algorithm with experimental verification.

1.4 Thesis organization

This thesis starts by introducing the background and motivation of current POTDR technologies. The objective of the research is stated, and the scope is elaborated, which is followed by the organization of the thesis in Chapter 1.

In Chapter 2, a general model for polarization effect measurements in SMFs is proposed. x -domain Mueller matrix and x -domain polarization effects will be defined. Then, general methods for the measurements of x -domain Mueller matrix and x -domain polarization effects will be proposed for two kinds of SMF systems: a pure birefringent system and a nondepolarizing system (a system with both birefringence and PDL). These definitions and methods are the fundamental parts of the fiber polarization optics. They will be helpful for POTDR system optimization, measurement algorithms and signal interpretation.

In Chapter 3, the effect of the input SOPs on the Mueller matrix measurement error (M^3E) is theoretically analyzed. Both theoretical and simulation results show that the M^3E is greatly affected by the choice of input SOPs. The optimized input SOPs can be identified. These conclusions are generally applicable to fiber polarization optics; and they are especially useful for POTDR system optimization since the measured signal in POTDR is quite noisy. The optimization of the polarimeter in a POTDR measurement is also presented in this chapter. The minimum noise of the polarimeter is given; and the optimized polarimeter is demonstrated to have very simple statistical properties.

In Chapter 4, spectrally- and spatially-resolved PMD measurement techniques are presented, together with preliminary experimental results. These techniques are designed for a pure birefringent SMF system and a nondepolarizing SMF system, respectively. Although the direction of the PMD vector still cannot be obtained, the

differential group delay (DGD) distribution can be explicitly measured using these algorithms. This is enough to meet the original objective: to identify the fiber sections with large PMD values.

In Chapter 5, a novel algorithm for reflectometric and spectrally-resolved PDL measurements is proposed. This measurement algorithm realizes such a measurement by detecting the round-trip PDL evolution in the terminal (close to far end) purely-birefringent fiber section of the SMF link under test. This technique can also be extended to measure the PDL distribution in an SMF link where SMF sections and PDL sections are interleaved.

In Chapter 6, we summarize the achievements and present the plan for further work.

Chapter 2 GENERAL MODEL OF POLARIZATION EFFECT MEASUREMENT

2.1 Introduction

The general schematic configuration for polarization effect measurement is illustrated in Fig. 2.1 (a). The light source can be a continuous-wave (CW) source or a pulsed source. The SOP generator is used to generate several different input SOPs. The SOP analyzer (also termed the polarization analyzer or the polarimeter) is used to measure the output SOPs. This general configuration is also valid for reflectometric measurements of polarization effects using POTDR. In POTDR, the SMF system under test, which is shown in the dashed box in Fig. 2.1(a), should be deployed as illustrated in Fig. 2.1(b); the whole SMF system under test should also include an optical circulator (or an optical coupler). Port 1 and port 3 of the circulator are the input and output ends of the SMF system under test, respectively. Therefore, a POTDR system can be considered as a special case of the general configuration for polarization effect measurement.

Fig. 2.1 (a) General configuration of polarization effect measurement system; (b) SMF system under test in POTDR.

In a measurement, if the input SOP is fixed and the output SOP varies with respect to a physical parameter, it can be concluded that polarization effects are present in the system. For example, PMD is just a polarization effect where the output SOP is related to the optical frequency of the light. From this point of view, the three fundamental polarization effects, birefringence, PDL and depolarization, can be considered as the polarization effects that make the output SOP vary with respect to fiber length. Then, a general model for polarization effect measurements can be proposed.

Please note, I am the main contributor for the contents in section 2.2, 2.3 and 2.4. These works have been reported in the following papers where I am the first author: [5] in Journal Publications; [15], [16], [17] and [18] in References.

2.2 Definition of x -domain Mueller matrix

It is well known that the SOP of a light can be depicted by four Stokes parameters and the polarization properties of an optical system can be completely described by a 4×4 Mueller matrix $\mathbf{M}$ [10, 12]. Usually, the four Stokes parameters are written as a vector $\vec{S} = (s_0, s_1, s_2, s_3)^T$, which is called the Stokes vector. In this thesis, the superscript “T” denotes the matrix transpose. However, $\vec{S}$ is not a real vector because two Stokes vectors can not be added unless the two light sources are completely incoherent [12]. In an optical system, the input Stokes vector $\vec{S}_{\text{in}}$ and the output Stokes vector $\vec{S}_{\text{out}}$ are related by [10, 12]

$$\vec{S}_{\text{out}} = \mathbf{M} \vec{S}_{\text{in}} \quad (2.1)$$

When $\vec{S}_{\text{in}}$ is fixed, $\vec{S}_{\text{out}}$ may vary with respect to a physical parameter x . When x has an increment Δx , it can be easily derived that

$$\vec{S}_{\text{out}}(x + \Delta x) = \mathbf{M}(x + \Delta x) \mathbf{M}^{-1}(x) \vec{S}_{\text{out}}(x) \quad (2.2)$$

In this thesis, the superscript “-1” denotes the inverse matrix. Equation (2.2) means we can define a new matrix $\mathbf{M}_x = \mathbf{M}(x + \Delta x) \mathbf{M}^{-1}(x)$. If the parameter x is the fiber length z , $\mathbf{M}_z$ is just the Mueller matrix $\mathbf{M}$ of the fiber section from z to $z + \Delta z$; and $\vec{S}_{\text{in}}$ can be considered as $\vec{S}_{\text{out}}(0)$. If x is the optical angular frequency ω , $\mathbf{M}_\omega$ contains the full information of PMD. Therefore, $\mathbf{M}_x$ can be considered as a general matrix governing the polarization properties of an optical system. In this thesis, it is termed the x -domain Mueller matrix. Then, the traditional Mueller matrix $\mathbf{M}$ is just the z -domain Mueller matrix. In POTDR measurements, $\mathbf{M}_x$ will represent a round-trip x -domain Mueller matrix.

The degree of polarization (DOP) of a light is defined by $\text{DOP} = \sqrt{s_1^2 + s_2^2 + s_3^2} / s_0$ [3, 12]. In an optical system having no depolarization effect, if the DOP of the input light is 1, then the DOP of the output light is still 1. Such a system can have both birefringence and PDL in the z -domain. Generally speaking, if the DOP of the output light can remain to be unity when a physical parameter x varies, then $\mathbf{M}_x$ represents a non-depolarizing matrix. From the definition of DOP, we can derive the following relation

$$\begin{aligned} & s_{\text{out}0}^2(x + \Delta x) - s_{\text{out}1}^2(x + \Delta x) - s_{\text{out}2}^2(x + \Delta x) - s_{\text{out}3}^2(x + \Delta x) \\ &= s_{\text{out}0}^2(x) - s_{\text{out}1}^2(x) - s_{\text{out}2}^2(x) - s_{\text{out}3}^2(x) = 0 \end{aligned} \quad (2.3)$$

Equation (2.3) implies that $\mathbf{M}_x$ has some special features. In fact, $\mathbf{M}_x$ should satisfy the Lorentz transformation, which has been theoretically demonstrated using a rigorous mathematical analysis [13]. For this reason, we may rewrite the Stokes parameters as a complex vector $\vec{S} = (is_0, s_1, s_2, s_3)^T$ to make some equations, such as Eq. (2.5), more concise. Here, $i = \sqrt{-1}$. Accordingly, the x -domain Mueller matrix is rewritten as

$$\mathbf{M}_x = \begin{pmatrix} m_{x11} & im_{x12} & im_{x13} & im_{x14} \\ -im_{x21} & m_{x22} & m_{x23} & m_{x24} \\ -im_{x31} & m_{x32} & m_{x33} & m_{x34} \\ -im_{x41} & m_{x42} & m_{x43} & m_{x44} \end{pmatrix} \quad (2.4)$$

where m_{xjk} ($j, k = 1, 2, 3, 4$) are the elements of the real x -domain Mueller matrix. In Chapter 2 and some sections of Chapter 3 of this thesis, we use the complex Stokes vector and the complex x -domain Mueller matrix. With these definitions, we have [14, 15]

$$\mathbf{M}_x^T \cdot \mathbf{M}_x = \sqrt{|\mathbf{M}_x|} \mathbf{I} \quad (2.5)$$

where $|\mathbf{M}_x|$ denotes the determinant of $\mathbf{M}_x$ and $\mathbf{I}$ is the identity matrix. Equation (2.5) is just the mathematical description of a Lorentz transformation, which is the most important feature of an optical system with both birefringence and PDL.

2.3 Polarization effects and x -domain Mueller matrix

So-called polarization effects can be defined based on the x -domain Mueller matrix.

When the increment $\Delta x \rightarrow 0$, obviously the x -domain Mueller matrix $\mathbf{M}_x \rightarrow \mathbf{I}$.

Then, the limit $\lim_{\Delta x \rightarrow 0} \frac{\mathbf{M}_x - \mathbf{I}}{\Delta x}$ is used to define the polarization effects in the x -domain.

Based on Eqs. (2.2) and (2.5), we have [16]

$$\lim_{\Delta x \rightarrow 0} \frac{\mathbf{M}_x - \mathbf{I}}{\Delta x} = \frac{d\mathbf{M}_x}{dx} = \begin{pmatrix} \eta_x & iA_1 & iA_2 & iA_3 \\ -iA_1 & \eta_x & -B_3 & B_2 \\ -iA_2 & B_3 & \eta_x & -B_1 \\ -iA_3 & -B_2 & B_1 & \eta_x \end{pmatrix} \quad (2.6)$$

where

$$\begin{cases} \eta_x = \left(\ln \sqrt[4]{|\mathbf{M}|} \right)' \\ A_{q-1} = \left(-m'_{11}m_{q1} + m'_{12}m_{q2} + m'_{13}m_{q3} + m'_{14}m_{q4} \right) / \sqrt{|\mathbf{M}|} \quad q = 2, 3, 4 \\ B_1 = \left(-m'_{41}m_{31} + m'_{42}m_{32} + m'_{43}m_{33} + m'_{44}m_{34} \right) / \sqrt{|\mathbf{M}|} \\ B_2 = \left(-m'_{21}m_{41} + m'_{22}m_{42} + m'_{23}m_{43} + m'_{24}m_{44} \right) / \sqrt{|\mathbf{M}|} \\ B_3 = \left(-m'_{31}m_{21} + m'_{32}m_{22} + m'_{33}m_{23} + m'_{34}m_{24} \right) / \sqrt{|\mathbf{M}|} \end{cases} \quad (2.7)$$

In Eq. (2.7), $m_{jk} (j, k = 1, 2, 3, 4)$ are the elements of the z -domain Mueller matrix $\mathbf{M}$.

The primes denote the derivatives with respect to the physical parameter x .

In Eq. (2.6), two vectors, $\vec{A} = (A_1, A_2, A_3)^T$ and $\vec{B} = (B_1, B_2, B_3)^T$, represent two fundamental polarization effects in the x -domain. For example, if $x = z$, $\vec{A}$ is the local polarization-dependent loss or gain (PDL/G) vector $\vec{\alpha}$, and $\vec{B}$ is the local birefringence vector $\vec{\beta}$; if $x = \omega$, $\vec{A} = \vec{\Lambda}$ and $\vec{B} = \vec{\Omega}$ are two parts of the complex PMD vector [16, 18]. Therefore, different polarization effects can be defined using Eq. (2.6) in different domains. In POTDR measurements, $\vec{A}$ and $\vec{B}$ will be the round-trip x -domain polarization effect vectors.

2.4 Measurement of polarization effects

Many methods have been proposed for the measurement of different polarization effects in SMFs. Among them, Mueller matrix-based methods have been experimentally verified to be better than SOP differentiation-based methods [17, 18]. In this thesis, we briefly introduce the Mueller matrix-based methods for polarization effect measurements in the x -domain in two kinds of optical systems: a pure birefringent system and a nondepolarizing system.

2.4.1 Pure birefringent optical system

Because there is no PDL in a pure birefringent optical system, a 3×3 Mueller matrix $\mathbf{m}_B$ is enough to describe it. This 3×3 Mueller matrix is just the sub-matrix of the

matrix in Eq. (2.4). $\mathbf{m}_B$ represents a rotation in Stokes space [12, 19]. So, it can be written as [19]

$$\mathbf{m}_B = \cos \phi \mathbf{I} + (1 - \cos \phi) \hat{r} \hat{r} - \sin \phi \hat{r} \times \quad (2.8)$$

where ϕ is the rotation angle, $\hat{r} = (r_1, r_2, r_3)^T$ is the rotation axis, $\hat{r} \hat{r} = \begin{pmatrix} r_1 r_1 & r_1 r_2 & r_1 r_3 \\ r_2 r_1 & r_2 r_2 & r_2 r_3 \\ r_3 r_1 & r_3 r_2 & r_3 r_3 \end{pmatrix}$ is

a dyadic and $\hat{r} \times$ is a cross-product operator. Accordingly, the 3-dimensional Stokes vector $\vec{s} = (s_1, s_2, s_3)^T$ should be used.

To measure the x -domain Mueller matrix of such a system, at least two different input SOPs $\vec{s}_{\text{in}}$ and $\vec{t}_{\text{in}}$ are required. For each input SOP, the output SOP corresponding to different x should be measured. Then the matrix $\mathbf{m}_B$ can be calculated using [20]

$$\mathbf{m}_B = \mathbf{f}^{-1}(x + \Delta x) \cdot \mathbf{f}(x) \quad (2.9)$$

where $\mathbf{f}(x) = \begin{pmatrix} s_{\text{out}1}(x) & s_{\text{out}2}(x) & s_{\text{out}3}(x) \\ t_{\text{out}1}(x) & t_{\text{out}2}(x) & t_{\text{out}3}(x) \\ a_{\text{out}1}(x) & a_{\text{out}2}(x) & a_{\text{out}3}(x) \end{pmatrix}$ and $\vec{a}_{\text{out}}(x) = \vec{s}_{\text{out}}(x) \times \vec{t}_{\text{out}}(x)$. Using the

obtained matrix $\mathbf{m}_B$, the polarization effect vector can be calculated based on Eq. (2.8) as [21]

$$\vec{B} = \phi \hat{r} / \Delta x \quad (2.10)$$

For PMD measurements, the method based on the above theory is called the virtual Mueller matrix method (VMMM) [17]. In Fig. 2.2, the PMD vector measurement for a SMF link using the VMMM is presented [17]. The VMMM is superior to other PMD vector measurement methods in that it does not require the knowledge of input polarization states. Further, the VMMM can use a large step Δx to attain low-noise PMD vector measurement data [17].

Fig. 2.2 PMD vector measurement result of a SMF link using the VMMM [17]. $|\Omega|$ is the amplitude of the PMD vector $\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$.

2.4.2 Nondepolarizing optical system

To measure such an optical system, at least three different input SOPs $\vec{S}_{in}$, $\vec{T}_{in}$ and $\vec{U}_{in}$ are required [15]. By measuring the output SOPs corresponding to different x and using Eq. (2.5), the x -domain Mueller matrix can be calculated as [22]

$$\mathbf{M}_x = \mathbf{F}^{-1}(x + \Delta x) \cdot \mathbf{F}(x) \quad (2.11)$$

where

$$\mathbf{F}(x) = \frac{1}{\sqrt{|\mathbf{M}_x|}} \begin{pmatrix} is_{out0}(x) & s_{out1}(x) & s_{out2}(x) & s_{out3}(x) \\ it_{out0}(x) & t_{out1}(x) & t_{out2}(x) & t_{out3}(x) \\ iu_{out0}(x) & u_{out1}(x) & u_{out2}(x) & u_{out3}(x) \\ \frac{A_{out0}(x)}{\sqrt{|\mathbf{M}_x|}} & \frac{A_{out1}(x)}{\sqrt{|\mathbf{M}_x|}} & \frac{A_{out2}(x)}{\sqrt{|\mathbf{M}_x|}} & \frac{A_{out3}(x)}{\sqrt{|\mathbf{M}_x|}} \end{pmatrix} \text{ and}$$

$$A_{\text{out}0}(x) = - \begin{vmatrix} s_{\text{out}1}(x) & s_{\text{out}2}(x) & s_{\text{out}3}(x) \\ t_{\text{out}1}(x) & t_{\text{out}2}(x) & t_{\text{out}3}(x) \\ u_{\text{out}1}(x) & u_{\text{out}2}(x) & u_{\text{out}3}(x) \end{vmatrix}, \quad A_{\text{out}1}(x) = i \begin{vmatrix} s_{\text{out}0}(x) & s_{\text{out}2}(x) & s_{\text{out}3}(x) \\ t_{\text{out}0}(x) & t_{\text{out}2}(x) & t_{\text{out}3}(x) \\ u_{\text{out}0}(x) & u_{\text{out}2}(x) & u_{\text{out}3}(x) \end{vmatrix},$$

$$A_{\text{out}2}(x) = -i \begin{vmatrix} s_{\text{out}0}(x) & s_{\text{out}1}(x) & s_{\text{out}3}(x) \\ t_{\text{out}0}(x) & t_{\text{out}1}(x) & t_{\text{out}3}(x) \\ u_{\text{out}0}(x) & u_{\text{out}1}(x) & u_{\text{out}3}(x) \end{vmatrix}, \quad A_{\text{out}3}(x) = i \begin{vmatrix} s_{\text{out}0}(x) & s_{\text{out}1}(x) & s_{\text{out}2}(x) \\ t_{\text{out}0}(x) & t_{\text{out}1}(x) & t_{\text{out}2}(x) \\ u_{\text{out}0}(x) & u_{\text{out}1}(x) & u_{\text{out}2}(x) \end{vmatrix}.$$

The determinant of the x -domain Mueller matrix can be calculated from

$$\sqrt{|\mathbf{M}_x|} = \frac{\vec{S}_{\text{out}}(x+\Delta x) \cdot \vec{T}_{\text{out}}(x+\Delta x) + \vec{S}_{\text{out}}(x+\Delta x) \cdot \vec{U}_{\text{out}}(x+\Delta x) + \vec{T}_{\text{out}}(x+\Delta x) \cdot \vec{U}_{\text{out}}(x+\Delta x)}{\vec{S}_{\text{out}}(x) \cdot \vec{T}_{\text{out}}(x) + \vec{S}_{\text{out}}(x) \cdot \vec{U}_{\text{out}}(x) + \vec{T}_{\text{out}}(x) \cdot \vec{U}_{\text{out}}(x)} \quad (2.12)$$

Because Δx has a finite value in a practical measurement, the polarization effect vectors $\vec{A}$ and $\vec{B}$ cannot be calculated using Eq. (2.6). We have to use the polar decomposition of $\mathbf{M}_x$. It has been demonstrated that $\mathbf{M}_x$ can be decomposed as [19, 22]

$$\mathbf{M}_x = T_u \begin{pmatrix} 1 & \vec{0}^T \\ \vec{0} & \mathbf{m}_B \end{pmatrix} \begin{pmatrix} 1 & i\vec{C}^T \\ -i\vec{C} & \mathbf{m}_A \end{pmatrix} \quad (2.13)$$

where T_u is the loss for unpolarized light and $\vec{0}$ is the zero vector, and

$$\mathbf{m}_A = \sqrt{1-C^2} \mathbf{I} + (1-\sqrt{1-C^2}) \hat{C} \hat{C} \quad (2.14)$$

where $\hat{C} = \vec{C} / C$. After such matrix decomposition, the two polarization effect vectors can be calculated as [23]

$$\vec{A} = T_u \vec{C} / \Delta x, \quad \vec{B} = \phi \hat{r} / \Delta x \quad (2.15)$$

For PMD measurements, the method based on the above theory is called the virtual generalized Mueller matrix method (VGMMM) [18]. In Fig. 2.3, the measurement result of two parts of the complex PMD vector $\vec{W} = \vec{\Omega} + i\vec{\Lambda}$ of a SMF link using the VGMMM is presented [18]. The VGMMM can use a relatively large frequency step to attain low-noise PMD vector data without the knowledge of input polarization

states. VGMMM combines the advantages of both matrix-based methods and SOP differentiation-based methods and overcomes their shortcomings [18].

Fig. 2.3 Complex PMD vector measurement result of a SMF link using the VGMMM [18].

$\vec{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$ and $\vec{\Delta} = (\Delta_1, \Delta_2, \Delta_3)$ are the real part and imaginary part of the complex PMD vector, respectively.

To measure high-order polarization effects, especially high-order PMD vectors, measurement algorithms based on the Magnus expansion have been proposed [24,

25], which require highly accurate measurement results of Mueller matrices. In POTDR measurements, since the detected signals are quite noisy, these algorithms are difficult to implement. Hence, we do not introduce them in this thesis.

Chapter 3 OPTIMIZATION OF INPUT SOPS AND POLARIMETER IN POTDR

3.1 Introduction

From the general configuration of polarization effect measurements shown in Fig. 2.1, to improve the measurement accuracy, the light source, the SOP generator and the SOP analyzer should have high performance. However, to propose novel devices is not in our research area. In this thesis, we just intend to investigate how to improve the measurement accuracy by using commonly available devices.

In this chapter, firstly, the effect of input SOPs on the M^3E will be investigated. Optimized input SOPs are theoretically found for optical systems without and with PDL/G. Theoretical and simulation results show that the optimized input SOPs can obviously suppress the M^3E . These results are especially useful for POTDR measurements because the POTDR signals are quite noisy. Secondly, the type of polarimeter commonly used in POTDR setups is also optimized by theoretical analysis. This polarimeter consists of a quarter-wave plate, a linear polarizer and a photodetector. The optimum angles of the waveplate and the polarizer have been theoretically found for POTDR measurements under the two conditions that thermal noise is the dominant noise in the photodetector and the angular position errors of the waveplate and the polarizer can be neglected compared to the photodetector noise. In an optimized polarimeter, the SOP measurement noise is suppressed to the minimum value.

Please note, I am the main contributor for the contents in section 3.2, 3.3 and 3.4. These works have been reported in the following papers where I am the first author: [1], [3], [4] and [5] in Journal Publications.

3.2 Pure birefringent optical system

3.2.1 Theoretical analysis

As we have mentioned in Chapter 2, a 3×3 Mueller matrix can be used to depict a pure birefringent optical system. In real measurements, various sources of noise always exist, which lead to the measurement errors $\Delta \vec{s}_{\text{out}}(x)$, $\Delta \vec{s}_{\text{out}}(x + \Delta x)$, $\Delta \vec{t}_{\text{out}}(x)$ and $\Delta \vec{t}_{\text{out}}(x + \Delta x)$, and thus the errors $\Delta \mathbf{f}(x)$ and $\Delta \mathbf{f}(x + \Delta x)$. Hence, the actual form of Eq. (2.9) should be

$$\left[\mathbf{f}(x + \Delta x) + \Delta \mathbf{f}(x + \Delta x) \right] (\mathbf{m}_B + \Delta \mathbf{m}_B) = \mathbf{f}(x) + \Delta \mathbf{f}(x) \quad (3.1)$$

where $\Delta \mathbf{m}_B$ is the measurement error of the x -domain Mueller matrix $\mathbf{m}_B$. Ignoring the high-order error $\Delta \mathbf{f}(x + \Delta x) \cdot \Delta \mathbf{m}_B$, we obtain

$$\begin{aligned} \Delta \mathbf{m}_B &= \mathbf{m}_B \mathbf{f}(x)^{-1} \left[\Delta \mathbf{f}(x) - \Delta \mathbf{f}(x + \Delta x) \mathbf{m}_B \right] \\ &= \mathbf{m}(x + \Delta x) \mathbf{f}_{\text{in}}^{-1} \left[\Delta \mathbf{f}(x) - \Delta \mathbf{f}(x + \Delta x) \mathbf{m}_B \right] \end{aligned} \quad (3.2)$$

where $\mathbf{m}$ is the z -domain Mueller matrix, $\mathbf{f}_{\text{in}} = \begin{pmatrix} s_{\text{in}1} & s_{\text{in}2} & s_{\text{in}3} \\ t_{\text{in}1} & t_{\text{in}2} & t_{\text{in}3} \\ a_{\text{in}1} & a_{\text{in}2} & a_{\text{in}3} \end{pmatrix}$ and $\vec{a}_{\text{in}} = \vec{s}_{\text{in}} \times \vec{t}_{\text{in}}$. To

evaluate $\mathbf{M}^3 \mathbf{E} \Delta \mathbf{m}_B$, we take the Frobenius matrix norms at both sides of Eq. (3.2).

Based on the definition of Frobenius matrix norm $\|\mathbf{A}\| = \sqrt{\sum_{i=1}^3 \sum_{j=1}^3 |a_{ij}|^2} = \sqrt{\text{Tr}(\mathbf{A}^T \mathbf{A})}$

(“Tr” denotes the trace of a square matrix) [25, 26], we have

$$\begin{aligned} \|\Delta \mathbf{m}_B\| &= \left\| \mathbf{m}(x + \Delta x) \mathbf{f}_{\text{in}}^{-1} \left[\Delta \mathbf{f}(x) - \Delta \mathbf{f}(x + \Delta x) \mathbf{m}_B \right] \right\| \\ &= \sqrt{\text{Tr} \left\{ \left[\Delta \mathbf{f}(x) - \Delta \mathbf{f}(x + \Delta x) \mathbf{m}_B \right]^T \left(\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T \right)^{-1} \left[\Delta \mathbf{f}(x) - \Delta \mathbf{f}(x + \Delta x) \mathbf{m}_B \right] \right\}} \\ &= \sqrt{\text{Tr} \left[\Delta \mathbf{f}^T(x) \left(\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T \right)^{-1} \Delta \mathbf{f}(x) \right] + \text{Tr} \left[\Delta \mathbf{f}^T(x + \Delta x) \left(\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T \right)^{-1} \Delta \mathbf{f}(x + \Delta x) \right] - 2 \text{Tr} \left[\Delta \mathbf{f}^T(x) \left(\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T \right)^{-1} \Delta \mathbf{f}(x + \Delta x) \mathbf{m}_B \right]} \end{aligned} \quad (3.3)$$

From the definition of $\mathbf{f}_{\text{in}}$, it can be derived that

$$(\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T)^{-1} = \begin{pmatrix} 1 & -\cos \theta & 0 \\ -\cos \theta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} / \sin^2 \theta \quad (3.4)$$

where $\cos \theta = \vec{s}_{\text{in}} \cdot \vec{t}_{\text{in}}$, θ is the angle between the two input SOPs in Stokes space [12].

Equations (3.3) and (3.4) show that the M^3E , which is expressed by the Frobenius matrix norm, only depends on the angle θ , and has no relation to the exact input SOPs.

The three components in Eq. (3.3) can be calculated as

$$\begin{aligned} \text{Tr} \left[\Delta \mathbf{f}^T(x) (\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T)^{-1} \Delta \mathbf{f}(x) \right] &= \frac{\Delta s^2(x) + \Delta t^2(x) + \Delta a^2(x) - 2 \cos \theta \Delta \vec{s}(x) \cdot \Delta \vec{t}(x)}{\sin^2 \theta} \\ &= \frac{\|\Delta \mathbf{f}(x)\|^2 - 2 \cos \theta (\Delta \vec{s} \cdot \Delta \vec{t})}{\sin^2 \theta} \end{aligned} \quad (3.5)$$

$$\begin{aligned} &\text{Tr} \left[\Delta \mathbf{f}^T(x + \Delta x) (\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T)^{-1} \Delta \mathbf{f}(x + \Delta x) \right] \\ &= \frac{\|\Delta \mathbf{f}(x + \Delta x)\|^2 - 2 \cos \theta \Delta \vec{s}(x + \Delta x) \cdot \Delta \vec{t}(x + \Delta x)}{\sin^2 \theta} \end{aligned} \quad (3.6)$$

$$\begin{aligned} &\text{Tr} \left[\Delta \mathbf{f}^T(x) (\mathbf{f}_{\text{in}} \mathbf{f}_{\text{in}}^T)^{-1} \Delta \mathbf{f}(x + \Delta x) \mathbf{m}_B \right] \\ &= \sum_{i=1}^3 \sum_{j=1}^3 \left\{ \begin{aligned} &\Delta s_i(x) [c_1 \Delta s_j(x + \Delta x) + c_2 \Delta t_j(x + \Delta x) + c_3 \Delta a_j(x + \Delta x)] \\ &+ \Delta t_i(x) [c_4 \Delta s_j(x + \Delta x) + c_5 \Delta t_j(x + \Delta x) + c_6 \Delta a_j(x + \Delta x)] \\ &+ \Delta a_i(x) [c_7 \Delta s_j(x + \Delta x) + c_8 \Delta t_j(x + \Delta x) + c_9 \Delta a_j(x + \Delta x)] \end{aligned} \right\} / \sin^2 \theta \end{aligned} \quad (3.7)$$

In Eq. (3.7), $c_i, i=1 \cdots 9$, are the constants containing θ and components of the matrix

$\mathbf{m}_B$. Obviously, $\|\Delta \mathbf{m}_B\|$ depends on the actual noise realization and the matrix $\mathbf{m}_B$.

But based on the property of the matrix norm and Eq. (3.2), we can get the upper bound of M^3E as

$$\|\Delta \mathbf{m}_B\| \leq \frac{3}{|\sin \theta|} \left\{ \|\Delta \mathbf{f}(x)\| + \sqrt{3} \|\Delta \mathbf{f}(x + \Delta x)\| \right\} \quad (3.8)$$

In deriving Eq. (3.8), the relation $\|\mathbf{m}(x + \Delta x)\| = \|\mathbf{m}_B\| = \sqrt{3}$ was used because they are 3×3 orthogonal matrices. We find that this upper bound does not depend on the matrix $\mathbf{m}_B$.

When many measurements are repeated, we assume that $\Delta s_i(x)$, $\Delta s_i(x + \Delta x)$, $\Delta t_i(x)$ and $\Delta t_i(x + \Delta x)$ with $i = 1, 2, 3$ independently and identically follow the Gaussian distribution $N(0, \sigma^2)$. This means that we assume that an ideal SOP analyzer, which has such statistical properties, is used in the measurement system. It is easy to know $\langle \Delta \mathbf{m}_B \rangle = 0$; but what we really need,

$\langle \|\Delta \mathbf{m}_B\| \rangle$, cannot be analytically calculated from Eq. (3.3). Because $\langle \|\Delta \mathbf{m}_B\| \rangle \leq \sqrt{\langle \|\Delta \mathbf{m}_B\|^2 \rangle}$, we can calculate the upper bound of $\langle \|\Delta \mathbf{m}_B\| \rangle$.

Considering $\langle \Delta s_i(x) \rangle = \langle \Delta s_i(x + \Delta x) \rangle = \langle \Delta t_i(x) \rangle = \langle \Delta t_i(x + \Delta x) \rangle = 0$, we can conclude that all components including $\langle \Delta s_i(x) \rangle, \langle \Delta s_i(x + \Delta x) \rangle, \langle \Delta t_i(x) \rangle, \langle \Delta t_i(x + \Delta x) \rangle$ are 0 in Eqs. (3.5), (3.6) and (3.7). Then we have

$$\begin{aligned} \langle \|\Delta \mathbf{m}_B\| \rangle &\leq \sqrt{\langle \|\Delta \mathbf{m}_B\|^2 \rangle} \\ &= \frac{\sqrt{\langle \Delta s(x)^2 + \Delta t(x)^2 + \Delta s(x + \Delta x)^2 + \Delta t(x + \Delta x)^2 + \Delta a(x + \Delta x)^2 \rangle}}{|\sin \theta|} \end{aligned} \quad (3.9)$$

In Eq. (3.9), $\Delta s(x)^2 + \Delta t(x)^2 + \Delta s(x + \Delta x)^2 + \Delta t(x + \Delta x)^2$ is distributed as a chi-square distribution (also termed χ^2 distribution) with the parameter $k = 12$ since it is composed of 12 independent and identical Gaussian distributions [27]. Considering

$\langle \Delta s_i(x)^2 \rangle = \langle \Delta s_i(x + \Delta x)^2 \rangle = \langle \Delta t_i(x)^2 \rangle = \langle \Delta t_i(x + \Delta x)^2 \rangle = \sigma^2$, we have

$$\langle \Delta s(x)^2 + \Delta t(x)^2 + \Delta s(x + \Delta x)^2 + \Delta t(x + \Delta x)^2 \rangle = 12\sigma^2 \quad (3.10)$$

Similarly, we have the following equation

$$\langle \Delta a(x)^2 \rangle = \langle \Delta a(x + \Delta x)^2 \rangle = 2\sigma^2 \quad (3.11)$$

Finally we obtain

$$\langle \|\Delta \mathbf{m}_B\| \rangle \leq \sqrt{\langle \|\Delta \mathbf{m}_B\|^2 \rangle} = 4\sigma / |\sin \theta| \quad (3.12)$$

Equation (3.12) means that the upper bound of $\langle \|\Delta \mathbf{m}_B\| \rangle$ depends only on the angle θ and the standard deviation of measurement noise σ .

Fig. 3.1 The curve of $1/\sin \theta$ as a function of θ in the range of 0° to 180° .

The curve of $1/\sin \theta$ as a function of θ from 0° to 180° is plotted in Fig. 3.1. This curve implies that the measurement error should have a bathtub-shaped relationship with the angle θ . But we cannot confirm this conclusion only based on Eq. (3.12) because it only presents the upper bound of the M^3E .

3.2.2 Simulation results

To confirm the above-mentioned findings, simulations need to be performed. Since the relationship under investigation is independent of the specific parameter x , we can choose $x = z$ in the simulations, viz., $\mathbf{m}_B = \mathbf{m}$ is the Mueller matrix. According to the

waveplate model of a SMF [5], the sample under simulation is a SMF link composed of 1000 randomly oriented, linearly birefringent and lossless fiber sections with a total DGD value of 10.6ps at 1550nm.

The simulation program is written using Matlab. Firstly, the Mueller matrices of the above-mentioned 1000 fiber sections are generated; then the product of these Mueller matrices is just the error-free Mueller matrix representing the SMF link under simulation. Secondly, 181 pairs of input SOPs are generated with the angle θ of 0° , 1° , $\dots$, 179° , 180° , respectively; and the output SOPs are calculated by multiplying the error-free Mueller matrix with the input SOPs. Thirdly, Gaussian noise with a standard variance of 2% is added to every component of the input SOPs and the output SOPs. Then, the Mueller matrix with errors is calculated using Eq. (2.9) and $\Delta \mathbf{m}$ can be obtained. Using such a program, the relative errors (1)

$$SR1 = \langle \|\Delta \mathbf{m}\| \rangle / \langle \|\mathbf{m}\| \rangle \quad (2) \quad SR2 = \sqrt{\langle \|\Delta \mathbf{m}\|^2 \rangle} / \langle \|\mathbf{m}\| \rangle$$

which are averaged over 10000

independent noise realizations and (3) the theoretical result $THR = \frac{4\sigma}{\|\mathbf{m}\| \cdot |\sin \theta|}$ are

shown in Fig. 3.2 as functions of the angle θ . Please note that $\|\mathbf{m}\| = \sqrt{3}$ since $\mathbf{m}$ is an orthogonal matrix.

Fig. 3.2 Comparison of the relative errors SR1, SR2 and THR as functions of the angle θ .

Obviously, all relative errors have bathtub-shaped relationship with the angle θ , and $\theta = 90^\circ$ is the optimum angle statistically corresponding to the minimum measurement error. But even if $\theta = 60^\circ$ or 120° , the relative error $SR1 = \langle \|\Delta \mathbf{m}\| \rangle / \langle \|\mathbf{m}\| \rangle$ only increases slightly from 4.2% to 4.8%. Equation (3.12) is also confirmed to be the upper bound of the expected value of measurement error. For a single noise realization, viz., a single measurement, we show two simulation results regarding two different noise realizations in Fig. 3.3. Figure 3.3(a) indicates that the optimum angle is approximately 55° , less than 90° and Fig 3.3(b) tells us the optimum angle also can be larger than 90° , approximately 128° . Actually we can find the optimum angle θ_{opt} has a Gaussian distribution as shown in Fig. 3.4; and $\theta = 90^\circ$ is the angle which has the maximum probability to achieve the minimum measurement error.

Fig. 3.3 Two examples where the optimum angles deviate from 90° in a single test.

Fig. 3.4 The distribution of the optimum angle in 10000 noise realizations.

We repeat the simulations using different input SOPs, two of which are even chosen to be aligned with the birefringent axes of the system. Simulation results show that the relationship has nothing to do with the exact input SOPs and can always be expressed by Fig. 3. 2, Fig. 3. 3 and Fig. 3. 4.

Now we can draw conclusions that for the measurement of matrix $\mathbf{m}_B$: 1) The measurement error only depends on θ and has nothing to do with the exact input SOPs; 2) $\theta = 90^\circ$ statistically has the maximum probability to result in the minimum measurement error; 3) In a single test, θ_{opt} corresponding to the optimum measurement accuracy can greatly depart from 90° , which depends on the actual noise realization in this test; 4) Whether in a single test or for the averaged value of lots of tests, the curve of the measurement error (single or averaged) versus θ looks like a bathtub with a flat bottom.

3.3 Nondepolarizing optical system

3.3.1 Optimized input SOPs by using condition number as the objective function

In such an optical system, when SOP measurement errors exist, the measurement error of the x -domain Mueller matrix should be

$$\Delta \mathbf{M}_x = \mathbf{F}_{\text{out}}^{-1}(x + \Delta x) [\Delta \mathbf{F}_{\text{out}}(x) - \Delta \mathbf{F}_{\text{out}}(x + \Delta x) \mathbf{M}_x - \Delta \mathbf{F}_{\text{out}}(x + \Delta x) \Delta \mathbf{M}_x] \quad (3.13)$$

Equation (2.11) can be rewritten as a linear equation group $\mathbf{F}_{\text{out}}(x + \Delta x) \cdot \mathbf{M}_x = \mathbf{F}_{\text{out}}(x)$.

When $\|\mathbf{F}_{\text{out}}^{-1}(x + \Delta x)\| \cdot \|\Delta \mathbf{F}_{\text{out}}(x + \Delta x)\| < 1$, $\|\Delta \mathbf{M}_x\|$ is bounded by [26]

$$\frac{\|\Delta \mathbf{M}_x\|}{\|\mathbf{M}_x\|} \leq \frac{\text{Cond}[\mathbf{F}_{\text{out}}(x + \Delta x)]}{1 - \text{Cond}[\mathbf{F}_{\text{out}}(x + \Delta x)] \frac{\|\Delta \mathbf{F}_{\text{out}}(x + \Delta x)\|}{\|\mathbf{F}_{\text{out}}(x + \Delta x)\|}} \left(\frac{\|\Delta \mathbf{F}_{\text{out}}(x)\|}{\|\mathbf{F}_{\text{out}}(x)\|} + \frac{\|\Delta \mathbf{F}_{\text{out}}(x + \Delta x)\|}{\|\mathbf{F}_{\text{out}}(x + \Delta x)\|} \right) \quad (3.14)$$

where, $\text{Cond}[\mathbf{F}_{\text{out}}(x + \Delta x)] = \|\mathbf{F}_{\text{out}}(x + \Delta x)\| \cdot \|\mathbf{F}_{\text{out}}^{-1}(x + \Delta x)\|$ is the condition number (CN) [26]. It is obvious that this upper bound seriously depends on the value of CN.

The smaller CN is, the more likely it is that $\|\Delta \mathbf{M}_x\| / \|\mathbf{M}_x\|$ will be small in a single test, regardless of the actual noise realizations of $\|\Delta \mathbf{F}_{\text{out}}(x)\| / \|\mathbf{F}_{\text{out}}(x)\|$ and $\|\Delta \mathbf{F}_{\text{out}}(x + \Delta x)\| / \|\mathbf{F}_{\text{out}}(x + \Delta x)\|$. Actually, the CN has been widely used to evaluate the measurement uncertainty of a measurement system. Please note, from Eq. (3.15) to Eq. (3.21), the tag “ $(x + \Delta x)$ ” in all output parameters is neglected to shorten equations.

When the Frobenius matrix norm is adopted, the detailed expression of the CN is calculated as [28]

$$\begin{aligned}
 \text{Cond}(\mathbf{F}_{\text{out}}) = & \sqrt{(1+\rho_{\text{out}}^s)^2 s_{\text{out}0}^2 + (1+\rho_{\text{out}}^t)^2 t_{\text{out}0}^2 + (1+\rho_{\text{out}}^u)^2 u_{\text{out}0}^2 + \frac{s_{\text{out}0}^2 t_{\text{out}0}^2 u_{\text{out}0}^2 (B_{\text{out}1}^2 + B_{\text{out}2}^2)}{|\tilde{\mathbf{M}}|}} \times \\
 & \sqrt{\left\{ s_{\text{out}0}^2 t_{\text{out}0}^2 \left[(1+\rho_{\text{out}}^s)^2 (1+\rho_{\text{out}}^t)^2 - (1+\rho_{\text{out}}^s \rho_{\text{out}}^t \cos \alpha_{\text{out}})^2 \right] + \right.} \\
 & \left\{ s_{\text{out}0}^2 u_{\text{out}0}^2 \left[(1+\rho_{\text{out}}^s)^2 (1+\rho_{\text{out}}^u)^2 - (1+\rho_{\text{out}}^s \rho_{\text{out}}^u \cos \beta_{\text{out}})^2 \right] + (B_{\text{out}1}^2 + B_{\text{out}2}^2) - \right. \\
 & \left. \left. t_{\text{out}0}^2 u_{\text{out}0}^2 \left[(1+\rho_{\text{out}}^t)^2 (1+\rho_{\text{out}}^u)^2 - (1+\rho_{\text{out}}^t \rho_{\text{out}}^u \cos \gamma_{\text{out}})^2 \right] \right\} \right. \\
 & 4 \left[s_{\text{out}0}^2 t_{\text{out}0}^2 a_{\text{out}}^2 + s_{\text{out}0}^2 u_{\text{out}0}^2 b_{\text{out}}^2 + t_{\text{out}0}^2 u_{\text{out}0}^2 c_{\text{out}}^2 \right] B_{\text{out}1}^2 + \\
 & \left. \left| \tilde{\mathbf{M}} \left[\begin{aligned} & \left((1+\rho_{\text{out}}^s)^2 (1+\rho_{\text{out}}^t)^2 (1+\rho_{\text{out}}^u)^2 - (1+\rho_{\text{out}}^s \rho_{\text{out}}^t \cos \alpha_{\text{out}})^2 - \right. \right. \\ & \left. \left. (1+\rho_{\text{out}}^t)^2 (1+\rho_{\text{out}}^s \rho_{\text{out}}^u \cos \beta_{\text{out}})^2 - (1+\rho_{\text{out}}^s)^2 (1+\rho_{\text{out}}^t \rho_{\text{out}}^u \cos \gamma_{\text{out}})^2 + \right. \right. \\ & \left. \left. 2(1+\rho_{\text{out}}^s \rho_{\text{out}}^t \cos \alpha_{\text{out}})(1+\rho_{\text{out}}^s \rho_{\text{out}}^u \cos \beta_{\text{out}})(1+\rho_{\text{out}}^t \rho_{\text{out}}^u \cos \gamma_{\text{out}}) \right] \right. \right. \\ & \left. \left. s_{\text{out}0} t_{\text{out}0} u_{\text{out}0} |B_{\text{out}1}^2 - B_{\text{out}2}^2| \right] \right\} \quad (3.15)
 \end{aligned}$$

where

$$\begin{cases} B_{\text{out}1}^2 = \rho_{\text{out}}^s{}^2 \rho_{\text{out}}^t{}^2 \rho_{\text{out}}^u{}^2 (1 - \cos^2 \alpha_{\text{out}} - \cos^2 \beta_{\text{out}} - \cos^2 \gamma_{\text{out}} + 2 \cos \alpha_{\text{out}} \cos \beta_{\text{out}} \cos \gamma_{\text{out}}) \\ B_{\text{out}2}^2 = (a_{\text{out}} + b_{\text{out}} + c_{\text{out}})(-a_{\text{out}} + b_{\text{out}} + c_{\text{out}})(a_{\text{out}} - b_{\text{out}} + c_{\text{out}})(a_{\text{out}} + b_{\text{out}} - c_{\text{out}})/4 \\ a_{\text{out}} = \sqrt{\rho_{\text{out}}^s{}^2 + \rho_{\text{out}}^t{}^2 - 2\rho_{\text{out}}^s \rho_{\text{out}}^t \cos \alpha_{\text{out}}} \\ b_{\text{out}} = \sqrt{\rho_{\text{out}}^s{}^2 + \rho_{\text{out}}^u{}^2 - 2\rho_{\text{out}}^s \rho_{\text{out}}^u \cos \beta_{\text{out}}} \\ c_{\text{out}} = \sqrt{\rho_{\text{out}}^t{}^2 + \rho_{\text{out}}^u{}^2 - 2\rho_{\text{out}}^t \rho_{\text{out}}^u \cos \gamma_{\text{out}}} \end{cases} \quad (3.16)$$

It can be easily noticed that the CN is completely determined by three output Stokes vectors at $x + \Delta x$, including their powers $(s_{\text{out}0}, t_{\text{out}0}, u_{\text{out}0})$, their DOPs $(\rho_{\text{out}}^s, \rho_{\text{out}}^t, \rho_{\text{out}}^u)$ and the three angles $(\alpha_{\text{out}}, \beta_{\text{out}}, \gamma_{\text{out}})$ between any two of them in Stokes space. From Eqs. (3.15) and (3.16), it can be observed that the value of the CN will remain unchanged when any two output Stokes vectors are interchanged. Therefore, the CN in Eq. (3.15) is a symmetric function of three output Stokes vectors. According to the Purkiss Principle [29], the CN must have a local maximum or minimum when

$$\begin{cases} s_{\text{out}0} = t_{\text{out}0} = u_{\text{out}0} \\ \rho_{\text{out}}^s = \rho_{\text{out}}^t = \rho_{\text{out}}^u \\ \alpha_{\text{out}} = \beta_{\text{out}} = \gamma_{\text{out}} \end{cases} \quad (3.17)$$

In fact, it is easy to verify that this is a local minimum. In this thesis, we do not demonstrate whether this local minimum is the global minimum or not; we are only interested in the relationship among the three input Stokes vectors when this local minimum is achieved.

On the other hand, in the z -domain, the input and output SOPs are related by the traditional Mueller matrix. Based on Eq. (2.13), three equation groups, relating the input and output parameters, can be derived as [28]

$$\begin{cases} s_{\text{out}0} = T_u s_{\text{in}0} (1 + \rho_{\text{in}}^s D \cos \theta_s) \\ t_{\text{out}0} = T_u t_{\text{in}0} (1 + \rho_{\text{in}}^t D \cos \theta_t) \\ u_{\text{out}0} = T_u u_{\text{in}0} (1 + \rho_{\text{in}}^u D \cos \theta_u) \end{cases} \quad (3.18)$$

$$\begin{cases} \rho_{\text{out}}^s = \sqrt{1 - (1 - D^2)(1 - \rho_{\text{in}}^{s^2}) / (1 + \rho_{\text{in}}^s D \cos \theta_s)^2} \\ \rho_{\text{out}}^t = \sqrt{1 - (1 - D^2)(1 - \rho_{\text{in}}^{t^2}) / (1 + \rho_{\text{in}}^t D \cos \theta_t)^2} \\ \rho_{\text{out}}^u = \sqrt{1 - (1 - D^2)(1 - \rho_{\text{in}}^{u^2}) / (1 + \rho_{\text{in}}^u D \cos \theta_u)^2} \end{cases} \quad (3.19)$$

$$\begin{cases} \rho_{\text{out}}^s \rho_{\text{out}}^t \cos \alpha_{\text{out}} = 1 - \frac{(1 - D^2)(1 - \rho_{\text{in}}^s \rho_{\text{in}}^t \cos \alpha_{\text{in}})}{(1 + \rho_{\text{in}}^s D \cos \theta_s)(1 + \rho_{\text{in}}^t D \cos \theta_t)} \\ \rho_{\text{out}}^s \rho_{\text{out}}^u \cos \beta_{\text{out}} = 1 - \frac{(1 - D^2)(1 - \rho_{\text{in}}^s \rho_{\text{in}}^u \cos \beta_{\text{in}})}{(1 + \rho_{\text{in}}^s D \cos \theta_s)(1 + \rho_{\text{in}}^u D \cos \theta_u)} \\ \rho_{\text{out}}^t \rho_{\text{out}}^u \cos \gamma_{\text{out}} = 1 - \frac{(1 - D^2)(1 - \rho_{\text{in}}^t \rho_{\text{in}}^u \cos \gamma_{\text{in}})}{(1 + \rho_{\text{in}}^t D \cos \theta_t)(1 + \rho_{\text{in}}^u D \cos \theta_u)} \end{cases} \quad (3.20)$$

where D is the value of PDL/G; PDL/G in dB is $\text{PDL/G} = 10 \log_{10} \frac{1+D}{1-D}$. From Eqs.

(3.18), (3.19) and (3.20), when the above-mentioned local minimum is achieved, it

can be seen that 1) if $s_{\text{in}0} = t_{\text{in}0} = u_{\text{in}0}$ and $\rho_{\text{in}}^s = \rho_{\text{in}}^t = \rho_{\text{in}}^u$ are satisfied, we

have $\theta_s = \theta_t = \theta_u$ and $\alpha_{in} = \beta_{in} = \gamma_{in}$; 2) when the input powers or DOPs of the three inputs are different, the relative relationship among the three input SOPs and between the three inputs and the PDL/G vector will become complicated.

Fortunately, most of the light sources used in modern polarization measurement systems are completely polarized. Moreover, the input power can remain unchanged by using some well-designed polarization state generation approaches. Therefore, in the rest of this thesis, we consider that $s_{in0} = t_{in0} = u_{in0}$ and $\rho_{in}^s = \rho_{in}^t = \rho_{in}^u = 1$ are always satisfied for both theoretical analysis and simulations. Further, since all input and output Stokes vectors can be normalized by the input power, $s_{in0} = t_{in0} = u_{in0} = 1$ can be adopted without any influence on the results.

Under the above conditions, Eqs. (3.15) and (3.16) can be simplified as [28]

$$\text{Cond}(\mathbf{F}_{out}) = \sqrt{2(D_s^2 + D_t^2 + D_u^2) + D_s^2 D_t^2 D_u^2 (B_{out1}^2 + B_{out2}^2) / (1 - D^2)^2} \times$$

$$\sqrt{\left\{ \begin{aligned} &D_s^2 D_t^2 [4 - (1 + \cos \alpha_{out})^2] + D_s^2 D_u^2 [4 - (1 + \cos \beta_{out})^2] + \\ &D_t^2 D_u^2 [4 - (1 + \cos \gamma_{out})^2] \end{aligned} \right\} (B_{out1}^2 + B_{out2}^2) -$$

$$8[D_s^2 D_t^2 (1 - \cos \alpha_{out}) + D_s^2 D_u^2 (1 - \cos \beta_{out}) + D_t^2 D_u^2 (1 - \cos \gamma_{out})] B_{out1}^2 +$$

$$2(1 - D^2)^2 \left[4 + (1 + \cos \alpha_{out})(1 + \cos \beta_{out})(1 + \cos \gamma_{out}) - (1 + \cos \alpha_{out})^2 - \right.$$

$$\left. (1 + \cos \beta_{out})^2 - (1 + \cos \gamma_{out})^2 \right] \Bigg| \over D_s D_t D_u |B_{out1}^2 - B_{out2}^2|} \quad (3.21)$$

and

$$\begin{cases} B_{out1}^2 = 1 - \cos^2 \alpha_{out} - \cos^2 \beta_{out} - \cos^2 \gamma_{out} + 2 \cos \alpha_{out} \cos \beta_{out} \cos \gamma_{out} \\ B_{out2}^2 = 4(1 - \cos \beta_{out})(1 - \cos \gamma_{out}) - (1 + \cos \alpha_{out} - \cos \beta_{out} - \cos \gamma_{out})^2 \end{cases} \quad (3.22)$$

where $D_s = 1 + D \cos \theta_s$, $D_t = 1 + D \cos \theta_t$ and $D_u = 1 + D \cos \theta_u$. $\theta_s, \theta_t, \theta_u$ are the angles between the three input Stokes vectors and the PDL/G vector in Stokes space, respectively. According to the Purkiss Principle, the CN in Eq. (3.21) has a local

maximum or minimum when $\theta_s = \theta_t = \theta_u = \theta$ and $\alpha_{in} = \beta_{in} = \gamma_{in} = \alpha$. Consequently, θ and α are actually related by

$$\cos \theta = \pm \sqrt{\frac{1 + 2 \cos \alpha}{3}} \quad (3.23)$$

By substituting Eq. (3.23) into Eq. (3.21) and doing numerical calculations with different values of PDL/G, we find that 1) this is a local minimum and it is achieved when the minus sign is chosen in Eq. (3.23); 2) this local minimum is indeed the global minimum. Under this condition, the relationships between the optimum angles α_{opt} , θ_{opt} and the PDL/G (in dB) are calculated and plotted in Fig. 3.5.

Fig. 3.5 The relationships between the optimum angles of inputs and the values of PDL/G when the CN takes the global minimum.

It is evident that α_{opt} is close to 120° when PDL/G is small. When PDL/G increases, the optimum angle α_{opt} decreases. For a given PDL/G vector, the three optimum input Stokes vectors should be equally-spaced on the Poincaré sphere (It is just a unit sphere in Stokes space) [12] and centered on the reversed PDL/G vector as shown in Fig. 3.6.

Fig. 3.6 The relative relationship of three input Stokes vectors and the PDL/G vector on the Poincaré sphere to achieve the minimum of the CN.

Figure 3.7 shows that, for different values of PDL/G, the minimum of the CN is also different. It is obvious that M^3E will dramatically increase when the value of PDL/G is up to tens of dB. Therefore, when the system under test has such a big PDL/G, its Mueller matrix cannot be accurately measured by using only three inputs in a single test.

Fig. 3.7 The relationship between the minimum of the CN and the values of PDL/G.

Since the CN in Eq. (3.21) is a function of six angles $\theta_s, \theta_t, \theta_u, \alpha_{in}, \beta_{in}, \gamma_{in}$, it is impossible to illustrate the whole function. In Fig. 3.8, we only present a curve to

partially show this function, where $D=0.5195$ (5 dB), $\theta_s = \theta_t = \theta_u = \theta$ and $\alpha_{in} = \beta_{in} = \gamma_{in} = \alpha$. Please note, in Fig. 3.8(a), α is related to θ by Eq. (3.23) with the minus sign.

Fig. 3.8 The relationships between the CN and (a) the angle α and (b) the angle θ when the value of PDL/G is 5 dB. The insets show the “zoom in” views of the same data.

In this section, we use the CN as the criterion to find the appropriate input SOPs. The results show that the minimum CN is achieved when the three input SOPs are equally-spaced on the Poincaré sphere and centered on the reversed PDL vector. The larger the PDL, the closer the three input SOPs are to the reversed PDL vector.

3.3.2 Optimized input SOPs in POTDR measurements

From Eq. (3.13), M^3E depends not only on the CN, but also on the noise realization. When many tests can be performed as in POTDR measurements, the mean of M^3E should be investigated. Similarly, we assume that all Stokes parameter measurement errors independently and identically follow the Gaussian distribution $N(0, \sigma^2)$. In the following, we consider that the measurement errors of the output SOPs are far larger than those of the input SOPs so that the input SOP errors can be neglected. In POTDR measurements, this condition is satisfied very well. This also means that we only discuss the statistical properties of the traditional z -domain Mueller matrix measurement error in this thesis. Under the conditions $\Delta s_{outj}, \Delta t_{outj}, \Delta u_{outj} (j=1, 2, 3, 4) \sim N(0, \sigma^2)$, we know that $\langle \Delta \sqrt{|\mathbf{M}|} \rangle = 0$. Hence, the variance of $\Delta \sqrt{|\mathbf{M}|}$ is needed to evaluate the measurement uncertainty. It can be derived as [28]

$$\text{Var}(\Delta \sqrt{|\mathbf{M}|}) = 2T_u^2 \sigma^2 K_{\Delta|\mathbf{M}|} \quad (3.24)$$

where

$$K_{\Delta|\mathbf{M}|} = \frac{(D_s + D_t)^2 + (D_s + D_u)^2 + (D_t + D_u)^2 - (1 - D^2)(3 - \cos \alpha_{in} - \cos \beta_{in} - \cos \gamma_{in})}{(3 - \cos \alpha_{in} - \cos \beta_{in} - \cos \gamma_{in})^2} \quad (3.25)$$

Obviously, $K_{\Delta|\mathbf{M}|}$ is also a symmetric function of the three input SOPs. Based on the Purkiss Principle and numerical calculations, its global minimum is also achieved when $\theta_s = \theta_t = \theta_u = \theta$, $\alpha_{in} = \beta_{in} = \gamma_{in} = \alpha$. When $K_{\Delta|\mathbf{M}|}$ takes the global minimum, the relationships between the optimum angles and the PDL are shown in Fig. 3.9 using solid lines. As a comparison, the optimum angles, corresponding to the CN, are also

plotted in Fig. 3.9 using dashed lines. It is clear that they are different corresponding to the same value of PDL.

Fig. 3.9 The relationships between the optimum angles and the PDL/G. Solid lines are based on the minimum of $K_{\Delta|M|}$ and dash lines are based on the minimum of the CN.

When only the output errors are considered and D has a finite value, from Eq. (3.13), we have

$$\|\Delta \mathbf{M}\| = \|\mathbf{M} \mathbf{F}_{\text{in}}^{-1} \Delta \mathbf{F}_{\text{out}} \mathbf{M}\| \quad (3.26)$$

Due to the existence of the Mueller matrix $\mathbf{M}$, it is difficult to calculate $\langle \|\Delta \mathbf{M}\| \rangle$ starting from Eq. (3.26). To overcome this mathematical difficulty, we start from the following equation

$$\mathbf{F}_{\text{in}} \mathbf{M}^{-1} = \mathbf{F}_{\text{out}} \quad (3.27)$$

Based on the relations that $\mathbf{M}^{-1} = \mathbf{M}^T / \sqrt{|\mathbf{M}|}$ and $(\mathbf{M} + \Delta \mathbf{M})^T = \mathbf{M}^T + \Delta \mathbf{M}^T$ [26], it can be derived that

$$\Delta \mathbf{M}^T = \mathbf{F}_{\text{in}}^{-1} \Delta \mathbf{F}_{\text{out}} \quad (3.28)$$

where

$$\tilde{\mathbf{F}}_{\text{out}} = \begin{pmatrix} is_{\text{out}0} & s_{\text{out}1} & s_{\text{out}2} & s_{\text{out}3} \\ it_{\text{out}0} & t_{\text{out}1} & t_{\text{out}2} & t_{\text{out}3} \\ iu_{\text{out}0} & u_{\text{out}1} & u_{\text{out}2} & u_{\text{out}3} \\ \frac{A_{\text{out}0}}{\sqrt{|\mathbf{M}|}} & \frac{A_{\text{out}1}}{\sqrt{|\mathbf{M}|}} & \frac{A_{\text{out}2}}{\sqrt{|\mathbf{M}|}} & \frac{A_{\text{out}3}}{\sqrt{|\mathbf{M}|}} \end{pmatrix} \quad (3.29)$$

$$\Delta\tilde{\mathbf{F}}_{\text{out}} = \Delta\tilde{\mathbf{F}}_{\text{out}1} - \Delta\tilde{\mathbf{F}}_{\text{out}2} \quad (3.30)$$

In Eq. (3.30),

$$\Delta\tilde{\mathbf{F}}_{\text{out}1} = \begin{pmatrix} i\Delta s_{\text{out}0} & \Delta s_{\text{out}1} & \Delta s_{\text{out}2} & \Delta s_{\text{out}3} \\ i\Delta t_{\text{out}0} & \Delta t_{\text{out}1} & \Delta t_{\text{out}2} & \Delta t_{\text{out}3} \\ i\Delta u_{\text{out}0} & \Delta u_{\text{out}1} & \Delta u_{\text{out}2} & \Delta u_{\text{out}3} \\ \frac{\Delta A_{\text{out}0}}{\sqrt{|\mathbf{M}|}} & \frac{\Delta A_{\text{out}1}}{\sqrt{|\mathbf{M}|}} & \frac{\Delta A_{\text{out}2}}{\sqrt{|\mathbf{M}|}} & \frac{\Delta A_{\text{out}3}}{\sqrt{|\mathbf{M}|}} \end{pmatrix} \text{ and } \Delta\tilde{\mathbf{F}}_{\text{out}2} = \frac{\Delta\sqrt{|\mathbf{M}|}}{|\mathbf{M}|} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_{\text{out}0} & A_{\text{out}1} & A_{\text{out}2} & A_{\text{out}3} \end{pmatrix}.$$

Based on the definition of the Frobenius matrix norm, it is easy to show that

$\|\Delta\mathbf{M}\| = \|\Delta\mathbf{M}^T\|$ [26]. Then, by substituting Eq. (3.30) into Eq. (3.28), we have

$$\begin{aligned} \|\Delta\mathbf{M}\| &= \|\Delta\mathbf{M}^T\| = \|\mathbf{F}_{\text{in}}^{-1} \Delta\tilde{\mathbf{F}}_{\text{out}1} - \mathbf{F}_{\text{in}}^{-1} \Delta\tilde{\mathbf{F}}_{\text{out}2}\| \\ &= \sqrt{\text{Tr}(\Delta\tilde{\mathbf{F}}_{\text{out}1}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}1} + \Delta\tilde{\mathbf{F}}_{\text{out}2}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}2} - \Delta\tilde{\mathbf{F}}_{\text{out}1}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}2} - \Delta\tilde{\mathbf{F}}_{\text{out}2}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}1})} \end{aligned} \quad (3.31)$$

where $\mathbf{F} = (\mathbf{F}_{\text{in}}^{-1})^H \mathbf{F}_{\text{in}}^{-1}$.

Unfortunately, we cannot directly calculate $\langle \|\Delta\mathbf{M}\| \rangle$ because of the difficulties in mathematics. As an alternative, we can calculate an upper bound as

$$\begin{aligned} \langle \|\Delta\mathbf{M}\| \rangle &\leq \sqrt{\langle \|\Delta\mathbf{M}\|^2 \rangle} \\ &= \sqrt{\langle \text{Tr}(\Delta\tilde{\mathbf{F}}_{\text{out}1}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}1}) \rangle + \langle \text{Tr}(\Delta\tilde{\mathbf{F}}_{\text{out}2}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}2}) \rangle - \langle \text{Tr}(\Delta\tilde{\mathbf{F}}_{\text{out}1}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}2}) \rangle - \langle \text{Tr}(\Delta\tilde{\mathbf{F}}_{\text{out}2}^H \mathbf{F} \Delta\tilde{\mathbf{F}}_{\text{out}1}) \rangle} \end{aligned} \quad (3.32)$$

The four terms in Eq. (3.32) are calculated as [28]

$$\begin{aligned} & \left\langle \text{Tr}(\Delta \tilde{\mathbf{F}}_{\text{out1}}^H \mathbf{F} \Delta \tilde{\mathbf{F}}_{\text{out1}}) \right\rangle = \\ & 2\sigma^2 \left\{ 2 \sum_{j=1}^3 f_{jj} + f_{44} \left\{ 4 \left[D_s^3 D_t^3 (1 - \cos \alpha_{\text{in}}) + D_s^3 D_u^3 (1 - \cos \beta_{\text{in}}) + D_t^3 D_u^3 (1 - \cos \gamma_{\text{in}}) \right] / (1 - D^2)^3 - \right\} \right. \\ & \left. \left[D_s^4 D_t^4 (1 - \cos \alpha_{\text{in}})^2 + D_s^4 D_u^4 (1 - \cos \beta_{\text{in}})^2 + D_t^4 D_u^4 (1 - \cos \gamma_{\text{in}})^2 \right] / (1 - D^2)^4 \right\} \end{aligned} \quad (3.33)$$

$$\left\langle \text{Tr}(\Delta \tilde{\mathbf{F}}_{\text{out2}}^H \mathbf{F} \Delta \tilde{\mathbf{F}}_{\text{out2}}) \right\rangle = f_{44} \frac{D_s^2 D_t^2 D_u^2 (B_{\text{out1}}^2 + B_{\text{out2}}^2)}{(1 - D^2)^3} \frac{\text{Var}(\Delta \sqrt{|\mathbf{M}|})}{\sqrt{|\mathbf{M}|}} \quad (3.34)$$

$$\left\langle \text{Tr}(\Delta \tilde{\mathbf{F}}_{\text{out1}}^H \mathbf{F} \Delta \tilde{\mathbf{F}}_{\text{out2}}) \right\rangle = \left\langle \text{Tr}(\Delta \tilde{\mathbf{F}}_{\text{out2}}^H \mathbf{F} \Delta \tilde{\mathbf{F}}_{\text{out1}}) \right\rangle = 0 \quad (3.35)$$

In Eqs. (3.33) and (3.34), f_{jj} are the diagonal elements of the matrix $\mathbf{F}$, which are

$$\begin{cases} f_{11} = \left\{ \left[4 - (1 + \cos \gamma_{\text{in}})^2 \right] (B_{\text{in1}}^2 + B_{\text{in2}}^2) - 8(1 - \cos \gamma_{\text{in}}) B_{\text{in1}}^2 \right\} / (B_{\text{in1}}^2 - B_{\text{in2}}^2)^2 \\ f_{22} = \left\{ \left[4 - (1 + \cos \beta_{\text{in}})^2 \right] (B_{\text{in1}}^2 + B_{\text{in2}}^2) - 8(1 - \cos \beta_{\text{in}}) B_{\text{in1}}^2 \right\} / (B_{\text{in1}}^2 - B_{\text{in2}}^2)^2 \\ f_{33} = \left\{ \left[4 - (1 + \cos \alpha_{\text{in}})^2 \right] (B_{\text{in1}}^2 + B_{\text{in2}}^2) - 8(1 - \cos \alpha_{\text{in}}) B_{\text{in1}}^2 \right\} / (B_{\text{in1}}^2 - B_{\text{in2}}^2)^2 \\ f_{44} = 2 \left[\frac{4 + (1 + \cos \alpha_{\text{in}})(1 + \cos \beta_{\text{in}})(1 + \cos \gamma_{\text{in}}) - (1 + \cos \alpha_{\text{in}})^2 - (1 + \cos \beta_{\text{in}})^2 - (1 + \cos \gamma_{\text{in}})^2}{(B_{\text{in1}}^2 - B_{\text{in2}}^2)^2} \right] \end{cases} \quad (3.36)$$

where

$$\begin{cases} B_{\text{in1}}^2 = 1 - \cos^2 \alpha_{\text{in}} - \cos^2 \beta_{\text{in}} - \cos^2 \gamma_{\text{in}} + 2 \cos \alpha_{\text{in}} \cos \beta_{\text{in}} \cos \gamma_{\text{in}} \\ B_{\text{in2}}^2 = 4(1 - \cos \beta_{\text{in}})(1 - \cos \gamma_{\text{in}}) - (1 + \cos \alpha_{\text{in}} - \cos \beta_{\text{in}} - \cos \gamma_{\text{in}})^2 \end{cases} \quad (3.37)$$

Finally, we have

$$\langle \|\Delta \mathbf{M}\| \rangle \leq \sqrt{\langle \|\Delta \mathbf{M}\|^2 \rangle} = K_{\|\Delta \mathbf{M}\|} \sigma \quad (3.38)$$

where

$$K_{\|\Delta\mathbf{M}\|} = \sqrt{2} \left[2 \sum_{j=1}^3 f_{jj} + f_{44} \right] \left\{ \frac{4 \left[D_s^3 D_t^3 (1 - \cos \alpha_{in}) + D_s^3 D_u^3 (1 - \cos \beta_{in}) + D_t^3 D_u^3 (1 - \cos \gamma_{in}) \right]}{(1 - D^2)^3} - \frac{D_s^4 D_t^4 (1 - \cos \alpha_{in})^2 + D_s^4 D_u^4 (1 - \cos \beta_{in})^2 + D_t^4 D_u^4 (1 - \cos \gamma_{in})^2 - D_s^2 D_t^2 D_u^2 (B_{out1}^2 + B_{out2}^2) K_{\Delta|\mathbf{M}|}}{(1 - D^2)^4} \right\} \quad (3.39)$$

Apparently, this upper bound is completely determined by $\alpha_{in}, \beta_{in}, \gamma_{in}$ and $\theta_s, \theta_t, \theta_u$. And it is easy to show that $K_{\|\Delta\mathbf{M}\|}$ is also a symmetric function of $(\alpha_{in}, \beta_{in}, \gamma_{in})$ and $(\theta_s, \theta_t, \theta_u)$. Also from the Purkiss Principle and numerical calculation, $K_{\|\Delta\mathbf{M}\|}$ takes its global minimum when $\alpha_{in} = \beta_{in} = \gamma_{in} = \alpha$, $\theta_s = \theta_t = \theta_u = \theta$. From Eq. (3.39), the relationships between the optimum angles and the value of PDL/G, when $K_{\|\Delta\mathbf{M}\|}$ takes the global minimum, are shown in Fig. 3.10 using solid lines. As a comparison, the optimum angles, corresponding to the CN and $K_{\Delta|\mathbf{M}|}$, are also plotted in Fig. 3.10 using dashed lines. It is clear that, although the value of PDL/G is the same, different criteria lead to different optimum angles.

Fig. 3.10 The relationships between the optimum angles and the PDL/G. Solid lines are based on the minimum of $K_{\parallel\Delta M}$ and dash lines are based on the minimum of the CN and $K_{\Delta|M|}$.

For different values of PDL/G, the minimum of $K_{\parallel\Delta M}$ is also different. As shown in Fig. 3.11, M^3E will dramatically increase when the value of PDL/G is up to tens of dB. Therefore, when the system under test has such a big PDL/G, the mean of the Mueller matrix cannot be accurately measured.

Fig. 3.11 The relationship between the minimum of $K_{\parallel\Delta M}$ and the values of PDL/G.

When the value of PDL/G is 5 dB, the relationship between $K_{\parallel\Delta M}$ and the angle α is shown in Fig. 3.12. The curve in Fig. 3.12 gives us partial information of $K_{\parallel\Delta M}$.

Fig. 3.12 The relationship between $K_{||\Delta M||}$ and the angle α when the value of the PDL/G is 5 dB.

The inset shows the “zoom in” view of the same data.

3.3.3 Simulation results

To verify the theoretical findings, simulations are performed. The parameters of the system under simulation are 1) Birefringence: $\phi = 5\pi/3$, $r = (0.66, 0.74, -0.1296)$; 2) PDL/G: $0 < D < 1$, $\hat{D} = (0, 0, 1)^T$ and 3) Attenuation for unpolarized light: $T_u = 1$. The theoretical result shows that the upper bound does not depend on T_u . Then, we take $T_u = 1$ in the following simulations. Here, we only show the simulation results when $\alpha_{in} = \beta_{in} = \gamma_{in} = \alpha$ and $\theta_s = \theta_t = \theta_u = \theta$. Then, the three input SOPs can be written as

$$\begin{cases} \vec{S}_{in} = (i, \sin \theta, 0, \cos \theta)^T \\ \vec{T}_{in} = (i, -\sin \theta / 2, \sqrt{3} \sin \theta / 2, \cos \theta)^T \\ \vec{U}_{in} = (i, -\sin \theta / 2, -\sqrt{3} \sin \theta / 2, \cos \theta)^T \end{cases} \quad (3.40)$$

The simulation program is written using Matlab. Firstly, the Mueller matrix under simulation is generated using the above-mentioned parameters, which is just the error-free Mueller matrix. Secondly, input SOPs $\vec{S}_{in}$, $\vec{T}_{in}$ and $\vec{U}_{in}$ are generated based on Eq.

(3.40) with the angle θ of $0^\circ, 1^\circ, \dots, 179^\circ, 180^\circ$, respectively; and the output SOPs are calculated by multiplying the error-free Mueller matrix with the input SOPs. Thirdly, Gaussian noise with a standard variance of 3% is added to every component of the output SOPs. Then the Mueller matrices with errors are calculated using Eq. (2.11) and the Mueller matrix measurement errors $\Delta\tilde{\mathbf{M}}$ can be obtained. In the simulations, $\langle\|\Delta\tilde{\mathbf{M}}\|\rangle$ is calculated using 10000 independent noise realizations. For three output Stokes vectors, this means that 120000 random values have been generated. In Fig. 3.13, the theoretical upper bound (in red) and the simulation results (in blue) of $\langle\|\Delta\tilde{\mathbf{M}}\|\rangle$ are plotted for three values of PDL/G: 5, 10 and 13 dB. Please note, in Fig. 13(a), α is related to θ by Eq. (3.23) with the minus sign. It is evident that the simulation results have the same profiles as the theoretical upper bounds, and the minimum M³Es are achieved with the angles determined by the solid lines in Fig. 3.10.

Fig. 3.13 The relationships between the theoretical upper bound (in red), simulation results (in blue) of $\langle \|\Delta \mathbf{M}\| \rangle$ and (a) the angle α and (b) the angle θ when the values of PDL/G are 5 dB, 10 dB and 13 dB, respectively.

Because birefringence and PDL are distributed along a SMF link, the PDL vector also varies along the fiber length. Then, it is impossible to find the optimum input SOPs which are suitable for all fiber sections in POTDR measurements. Therefore, the standard input SOPs $(1, 1, 0, 0)^T$, $(1, -0.5, 0.866, 0)^T$ and $(1, -0.5, -0.866, 0)^T$ can be suggested in POTDR measurements since they have obviously a larger probability of resulting in better Mueller matrix measurement accuracy [22, 28].

3.4 Noise minimization and statistical properties of a polarimeter used in POTDR

A complete POTDR system employs a polarimeter to measure the SOP of backscattered optical signal [30, 31], which allows us to acquire the most integrated polarization properties of the SMF under test [10, 32, 33]. One type of polarimeter commonly used in POTDR consists of one quarter-wave plate in conjunction with one polarizer and one photodetector [10, 31-35]. Measurement of the SOP is performed by rotating the waveplate and the polarizer to at least four pairs of angular orientations giving four optical powers measured using the photodetector. Then, the SOP, which is expressed by four Stokes parameters, can be calculated using a measurement matrix [36, 37]. This measurement matrix is the function of four pairs of angular orientations of the waveplate and the polarizer. For a single test using this polarimeter, some sets of optimum angles have been theoretically found by using the determinant and the CNs of the measurement matrix as objective functions [36, 37]. However, this is not the case when the polarimeter is used in POTDR.

In POTDR, the SOP of the backscattered optical signal rapidly changes with respect to time. Then, after passing through the polarizer in the polarimeter, the optical power will fluctuate at a frequency which may be up to tens of MHz. To detect this wideband signal, the photodetector should have an adequate bandwidth. Unfortunately, such a photodetector has also large noise because its noise is almost proportional to its bandwidth [38]. On the other hand, the power of the backscattered optical signal in POTDR is very low because it results from distributed Rayleigh backscattering and discrete Fresnel reflections in a SMF link. For these reasons, the signal-to-noise ratio (SNR) of a SOP measurement in POTDR will be very small for a single test. Hence, to acquire an acceptable SNR, averaging over many POTDR traces

is usually implemented [39]. As a consequence, the performance of the polarimeter is determined by its statistical properties. Optimization of the polarimeter is especially significant in POTDR due to the weak signal and the large noise of the photodetector. In this thesis, it will be concluded that the optimum angles for the polarimeter can be found only when thermal noise dominates in the photodetector and the angular position errors of the waveplate and the polarizer are sufficiently small compared to the photodetector noise. The optimum angles, which lead to the minimum variance of SOP measurement error (measurement noise), are as the same as those obtained in [36] and [37] by using the determinant of the measurement matrix as the objective function. Furthermore, when the polarimeter is optimized, the variance of the measurement error of each Stokes parameter and the covariance between measurement errors of any two Stokes parameters may also be derived. For a polarimeter in which both waveplate and polarizer are rotatable, the measurement noise of the DOP and the normalized Stokes vector can be calculated.

3.4.1 Noise of photodetector

Since the noise of the polarimeter is seriously affected by noise properties of the photodetector, a brief review of photodetector noise is presented in this section. A photodetector is used to convert optical power into electric current I . The current measurement error ΔI is a stationary random process. For a well-calibrated photodetector, ΔI should have a zero mean $\langle \Delta I \rangle = 0$ [38]. The measurement noise of a photodetector is depicted by the variance $\sigma^2 = \langle \Delta I^2 \rangle$ [38]. In a photodetector, thermal noise and shot noise are two fundamental noise sources. The variance of thermal noise is [38]

$$\sigma_{th}^2 = \langle \Delta I_{th}^2 \rangle = 4kTF_n \Delta f / R_L \quad (3.41)$$

where, Δf is the effective noise bandwidth, $k = 1.38 \times 10^{-23} J/K$ is the Boltzmann constant, T is the absolute temperature, R_L is the load resistance and F_n is the factor by which the thermal noise is enhanced by various resistors used in pre- and main amplifiers. The variance of shot noise is [38]

$$\sigma_s^2 = \langle \Delta I_s^2 \rangle = 2q(I_d + I_p)\Delta f \quad (3.42)$$

where, $q = 1.6 \times 10^{-19} C$ is the elementary charge, I_d is the dark current, I_p is the signal current. The variance of the total noise of a photodetector is [38]

$$\sigma^2 = \sigma_s^2 + \sigma_{th}^2 \quad (3.43)$$

It is easy to see that thermal noise is independent of the signal current I_p , so thermal noise does not depend on the power of optical signal under test. However, shot noise is obviously a function of optical power. Actually, in most cases of practical interest, thermal noise dominates the photodetector's performance, viz. $\sigma_{th}^2 \gg \sigma_s^2$ [38, 40]. For example, a photodetector from New Focus (model: 1811-FC) has a minimum noise equivalent power (NEP, it is used to depict the thermal noise [38]) of $2.5 pW / \sqrt{Hz}$ and a bandwidth of 125 MHz. Then, the minimum power of the thermal noise is about 28 nW. On the other hand, we can also estimate the power of the shot noise of this photodetector in POTDR measurements based on Eq. (3.42). The typical Rayleigh backscattering coefficient at 1550nm in a SMF is around $10^{-7} / m$ [41]. The typical optical pulse width used in POTDR is from 10ns to 100ns, corresponding to 1m to 10m spatial resolution. If the optical power of the pulse is 20 dBm (100 mW), the power of the backscattered optical signal will be 10 ~ 100 nW (in practice, the power will be much less than this value because SMF and other optical components have losses). The responsivity of this photodetector is 1A/W at 1550nm. Then, the power of the shot noise can be calculated as 0.63 nW (1m resolution) and 2 nW (10m

resolution), which are far less than the minimum power of thermal noise. Hence, in this photodetector, thermal noise dominates.

3.4.2 SOP measurement error

In a POTDR measurement, an optical pulse, with a given SOP, is fed into the SMF under test. At length z of the SMF, the backscattered optical signal is generated by Rayleigh backscattering or Fresnel backreflection. When this signal reaches the polarimeter, its SOP will be $\vec{S}(t)$, $t = 2z/v$ (v is the velocity of light in a SMF).

Fig. 3.14 Polarimeter configuration.

As shown in Fig. 3.14, the polarimeter is constructed with one quarter-wave plate, one linear polarizer and one photodetector. In Fig. 3.14(a), the polarizer is fixed, only the waveplate is rotatable. In Fig. 3.14(b), both the waveplate and the polarizer are free to rotate. Hereafter, the polarimeters in Fig. 3.14(a) and Fig. 3.14(b) are termed “polarimeter (a)” and “polarimeter (b)”, respectively. By rotating the waveplate and the polarizer to four pairs of angular orientations, four optical powers will be measured by the photodetector. Then, it has [37]

$$\vec{I} = \mathbf{\Theta} \vec{S} \quad (3.44)$$

where, $\vec{I} = (I_1, I_2, I_3, I_4)^T$. The measurement matrix is [37]

$$\Theta = \frac{1}{2} \begin{pmatrix} 1 & \cos \alpha_1 \cos \beta_1 & \sin \alpha_1 \cos \beta_1 & \sin \beta_1 \\ 1 & \cos \alpha_2 \cos \beta_2 & \sin \alpha_2 \cos \beta_2 & \sin \beta_2 \\ 1 & \cos \alpha_3 \cos \beta_3 & \sin \alpha_3 \cos \beta_3 & \sin \beta_3 \\ 1 & \cos \alpha_4 \cos \beta_4 & \sin \alpha_4 \cos \beta_4 & \sin \beta_4 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & \vec{V}_1 \\ 1 & \vec{V}_2 \\ 1 & \vec{V}_3 \\ 1 & \vec{V}_4 \end{pmatrix} \quad (3.45)$$

In Eq. (3.45), $\alpha_i = 2\theta_i$, $\beta_i = 2(\phi_i - \theta_i)$, $i = 1, 2, 3, 4$. θ_i and ϕ_i are the angular orientations of the waveplate and the polarizer, respectively. For polarimeter (a), we can choose $\phi_i = 0$ [36]. Please note that the isotropic attenuations of the waveplate and the polarizer are ignored since they have no influence on the conclusions. It is easy to see that the vector $\vec{V}_i = (\cos \alpha_i \cos \beta_i, \sin \alpha_i \cos \beta_i, \sin \beta_i)$ is unitary. Hence, it can be expressed by a dot on the surface of the Poincaré sphere [40]. For polarimeter (b), this vector can span the whole surface of the Poincaré sphere [37]. However, for polarimeter (a), this vector can only draw an “8”-shaped curve on Poincaré sphere as shown in Fig. 3.15.

Fig. 3.15 For polarimeter (a), the vector $\vec{V}$ can only draw an “8”-shaped curve on the Poincaré sphere.

In this polarimeter, the SOP measurement error is induced by the photodetector noise and the angular orientation errors of the waveplate and the polarizer [36, 37].

Considering these errors, Equation (3.44) should be

$$\vec{I} + \Delta\vec{I} = (\Theta + \Delta\Theta)(\vec{S} + \Delta\vec{S}) \quad (3.46)$$

where, $\Delta\vec{I}$ is the current measurement error, $\Delta\Theta$ is the error of matrix Θ and $\Delta\vec{S}$ is the resultant SOP measurement error. The matrix $\Delta\Theta$ is

$$\begin{aligned} \Delta\Theta &= \Delta\Theta_\phi + \Delta\Theta_\theta \\ &= \begin{pmatrix} 0 & -\cos\alpha_1 \sin\beta_1 \Delta\phi_1 & -\sin\alpha_1 \sin\beta_1 \Delta\phi_1 & \cos\beta_1 \Delta\phi_1 \\ 0 & -\cos\alpha_2 \sin\beta_2 \Delta\phi_2 & -\sin\alpha_2 \sin\beta_2 \Delta\phi_2 & \cos\beta_2 \Delta\phi_2 \\ 0 & -\cos\alpha_3 \sin\beta_3 \Delta\phi_3 & -\sin\alpha_3 \sin\beta_3 \Delta\phi_3 & \cos\beta_3 \Delta\phi_3 \\ 0 & -\cos\alpha_4 \sin\beta_4 \Delta\phi_4 & -\sin\alpha_4 \sin\beta_4 \Delta\phi_4 & \cos\beta_4 \Delta\phi_4 \end{pmatrix} + \end{aligned} \quad (3.47)$$

$$\begin{pmatrix} 0 & \sin(\beta_1 - \alpha_1) \Delta\theta_1 & \cos(\beta_1 - \alpha_1) \Delta\theta_1 & -\cos\beta_1 \Delta\theta_1 \\ 0 & \sin(\beta_2 - \alpha_2) \Delta\theta_2 & \cos(\beta_2 - \alpha_2) \Delta\theta_2 & -\cos\beta_2 \Delta\theta_2 \\ 0 & \sin(\beta_3 - \alpha_3) \Delta\theta_3 & \cos(\beta_3 - \alpha_3) \Delta\theta_3 & -\cos\beta_3 \Delta\theta_3 \\ 0 & \sin(\beta_4 - \alpha_4) \Delta\theta_4 & \cos(\beta_4 - \alpha_4) \Delta\theta_4 & -\cos\beta_4 \Delta\theta_4 \end{pmatrix}$$

By ignoring the high-order error $\Delta\Theta \cdot \Delta\vec{S}$ and using Eq. (3.44), we have

$$\Delta\vec{I} = \Theta \Delta\vec{S} + \Delta\Theta \vec{S} \quad (3.48)$$

Here, we use 2-norm to evaluate $\Delta\vec{I}$, namely $\|\Delta\vec{I}\| = \sqrt{\sum_{i=1}^4 \Delta I_i^2}$ [26]. Correspondingly,

the Frobenius matrix norm, $\|\Theta\| = \sqrt{\sum_{i=1}^4 \sum_{j=1}^4 \Theta_{ij}^2}$, is used to evaluate Θ because the

Frobenius matrix norm is the consistent matrix norm with the 2-norm of a vector [26].

From Eq. (3.48), it can be derived that

$$\|\Delta\vec{I}\|^2 = \|\Theta \Delta\vec{S}\|^2 + \|\Delta\Theta \vec{S}\|^2 + 2\Theta \Delta\vec{S} \cdot \Delta\Theta \vec{S} \quad (3.49)$$

In a real measurement, after the waveplate and the polarizer are rotated to and fixed at a set of angular orientations, many POTDR traces are measured by the photodetector and recorded for averaging. Hence, in such a measurement, $\Delta\vec{I}$ is a vector of random variables, but $\Delta\Theta$ is actually a constant matrix. By taking the average of Eq. (3.49), we have

$$\langle \|\Delta\vec{I}\|^2 \rangle = \langle \|\Theta \Delta\vec{S}\|^2 \rangle + \|\Delta\Theta \vec{S}\|^2 + 2\Theta \langle \Delta\vec{S} \rangle \cdot \Delta\Theta \vec{S} \quad (3.50)$$

3.4.3 Polarimeter optimization

The second and third terms on the right-hand side of Eq. (3.50) are related to the SOP $\vec{S}$ under test. However, as we have mentioned above, $\vec{S}$ rapidly changes with time in POTDR measurements. With these terms, it is impossible to optimize this polarimeter. Fortunately, the upper bound of the second term can be derived using the properties of norm and Eq. (3.47) as

$$\begin{aligned} \|\Delta\Theta\vec{S}\| &\leq \|\Delta\Theta\| \cdot \|\vec{S}\| \leq (\|\Delta\Theta_\phi\| + \|\Delta\Theta_\theta\|) \sqrt{1 + DOP^2} S_0 \\ &\leq \sqrt{\sum_{i=1}^4 [(1 + \cos^2 \beta_i) \Delta\theta_i^2 + \Delta\phi_i^2]} \sqrt{1 + DOP^2} S_0 \leq 2\sqrt{6} S_0 \Delta_{\max} \end{aligned} \quad (3.51)$$

where, $\Delta_{\max}$ is the maximum absolute value of all angular errors and $DOP = \sqrt{S_1^2 + S_2^2 + S_3^2} / S_0$ denotes the DOP of a light. If the waveplate and the polarizer are rotated manually, $\Delta_{\max} \leq 0.5^\circ$ can be achieved. Then, we have $\|\Delta\Theta\vec{S}\| \leq 0.043 S_0$. If they are rotated using electrically-controlled rotation stages, $\Delta_{\max} \leq 0.01^\circ$ is achievable. In this case, $\|\Delta\Theta\vec{S}\| \leq 0.00086 S_0$. If the optical power S_0 is still in the range of 10 ~ 100 nW as we have calculated above, $\|\Delta\Theta\vec{S}\| \leq 0.43 \sim 4.3 \text{ nW}$ when $\Delta_{\max} = 0.5^\circ$ and $\|\Delta\Theta\vec{S}\| \leq 8.6 \sim 86 \text{ pW}$ when $\Delta_{\max} = 0.01^\circ$. On the other hand, $\sqrt{\langle \|\Theta\Delta\vec{S}\|^2 \rangle}$ includes the contributions of both error sources. Even if the angular errors do not exist, the minimum value of $\sqrt{\langle \|\Theta\Delta\vec{S}\|^2 \rangle}$, which is thermal noise of the photodetector, is still 28 nW for the New Focus detector (model: 1811-FC) as we have calculated above. Further, when $\Delta_{\max}$ is small enough, $\langle \Delta\vec{S} \rangle$ in the third term of Eq. (3.50) will also be close to zero. Therefore, in real measurements, the angular errors can be neglected as long as they are small enough. Based on the above analysis, this is not a very difficult task. By omitting $\Delta\Theta\vec{S}$ in Eq. (3.48), we have

$$\Delta \vec{S} = \Theta^{-1} \Delta \vec{I} \quad (3.52)$$

Obviously, $\langle \Delta \vec{S} \rangle = 0$. By analogy with the photodetector noise [38, 40], $\langle \|\Delta \vec{S}\|^2 \rangle$ can

be defined as the noise of a polarimeter. Then, it can be derived that

$$\langle \|\Delta \vec{S}\|^2 \rangle = \sum_{i=1}^4 \Omega_{ii} \langle \Delta I_i^2 \rangle \quad (3.53)$$

where, Ω_{ii} are the diagonal elements of matrix Ω , which is

$$\Omega = (\Theta \Theta^T)^{-1} = \frac{1}{4} \begin{pmatrix} 2 & 1 + \cos \gamma_{12} & 1 + \cos \gamma_{13} & 1 + \cos \gamma_{14} \\ 1 + \cos \gamma_{12} & 2 & 1 + \cos \gamma_{23} & 1 + \cos \gamma_{24} \\ 1 + \cos \gamma_{13} & 1 + \cos \gamma_{23} & 2 & 1 + \cos \gamma_{34} \\ 1 + \cos \gamma_{14} & 1 + \cos \gamma_{24} & 1 + \cos \gamma_{34} & 2 \end{pmatrix}^{-1} \quad (3.54)$$

where, γ_{ij} is the angle between vectors $\vec{V}_i$ and $\vec{V}_j$. As we have mentioned above,

thermal noise dominates in the photodetector, so $\langle \Delta I_i^2 \rangle = \sigma_{th}^2$. Then, Equation (3.53)

becomes

$$\langle \|\Delta \vec{S}\|^2 \rangle = \sigma_{th}^2 \sum_{i=1}^4 \Omega_{ii} \quad (3.55)$$

Based on Eq. (3.55), to achieve the minimum value of $\langle \|\Delta \vec{S}\|^2 \rangle$ is equivalent to finding

the minimum of $\sum_{i=1}^4 \Omega_{ii}$. Please note, $\sum_{i=1}^4 \Omega_{ii}$ is just $\|\Theta^{-1}\|^2$. Because $\|\Theta\| = \sqrt{2}$,

minimization of $\sum_{i=1}^4 \Omega_{ii}$ is just equivalent to minimize the CN $\|\Theta\| \cdot \|\Theta^{-1}\|$. For

polarimeter (b), it can be demonstrated that $\left(\sum_{i=1}^4 \Omega_{ii} \right)_{\min}$ is achieved when $(|\Theta|^2)_{\max}$ is

reached (please see appendix of this chapter). According to the conclusion in [37],

any set of angles that leads to a circumscribed uniform tetrahedron within the

Poincaré sphere is a valid angle set to result in $\left(\sum_{i=1}^4 \Omega_{ii} \right)_{\min}$, then $\langle \|\Delta \vec{S}\|^2 \rangle_{\min}$. For

polarimeter (a), by direct calculation, we can find $\left\langle \|\Delta \vec{S}\|^2 \right\rangle_{\min}$ is achieved at two sets of angles $\theta : (-51.69^\circ, -15.12^\circ, 15.12^\circ, 51.69^\circ)$ and $(-74.88^\circ, -38.31^\circ, 38.31^\circ, 74.88^\circ)$, which are as same as the angles for which $(|\Theta|^2)_{\max}$ is reached [36]. Using these optimum sets of angles, the minimum variances are

$$\left\langle \|\Delta \vec{S}\|^2 \right\rangle_{\min} = \begin{cases} 20.6664\sigma_{th}^2 & \text{polarimeter (a)} \\ 10\sigma_{th}^2 & \text{polarimeter (b)} \end{cases} \quad (3.56)$$

Equation (3.56) represents the minimum noise of polarimeter (a) and (b) in POTDR measurements.

3.4.4 Statistical properties of optimized polarimeter

When the angular errors of the waveplate and the polarizer are small enough, we have

$\langle \Delta \vec{I} \rangle = 0$. From Eq. (3.52), the SOP measurement errors are also zero, which means

$\langle \Delta \vec{S} \rangle = 0$. Next, the variance of ΔS_i and the covariance between ΔS_i and ΔS_j can be

deduced as

$$\begin{aligned} \left\langle \begin{pmatrix} \Delta S_0 \\ \Delta S_1 \\ \Delta S_2 \\ \Delta S_3 \end{pmatrix} \begin{pmatrix} \Delta S_0 & \Delta S_1 & \Delta S_2 & \Delta S_3 \end{pmatrix} \right\rangle &= \begin{pmatrix} \langle \Delta S_0 \cdot \Delta S_0 \rangle & \langle \Delta S_0 \cdot \Delta S_1 \rangle & \langle \Delta S_0 \cdot \Delta S_2 \rangle & \langle \Delta S_0 \cdot \Delta S_3 \rangle \\ \langle \Delta S_1 \cdot \Delta S_0 \rangle & \langle \Delta S_1 \cdot \Delta S_1 \rangle & \langle \Delta S_1 \cdot \Delta S_2 \rangle & \langle \Delta S_1 \cdot \Delta S_3 \rangle \\ \langle \Delta S_2 \cdot \Delta S_0 \rangle & \langle \Delta S_2 \cdot \Delta S_1 \rangle & \langle \Delta S_2 \cdot \Delta S_2 \rangle & \langle \Delta S_2 \cdot \Delta S_3 \rangle \\ \langle \Delta S_3 \cdot \Delta S_0 \rangle & \langle \Delta S_3 \cdot \Delta S_1 \rangle & \langle \Delta S_3 \cdot \Delta S_2 \rangle & \langle \Delta S_3 \cdot \Delta S_3 \rangle \end{pmatrix} \\ &= \Theta^{-1} \begin{pmatrix} \langle \Delta I_1 \cdot \Delta I_1 \rangle & \langle \Delta I_1 \cdot \Delta I_2 \rangle & \langle \Delta I_1 \cdot \Delta I_3 \rangle & \langle \Delta I_1 \cdot \Delta I_4 \rangle \\ \langle \Delta I_2 \cdot \Delta I_1 \rangle & \langle \Delta I_2 \cdot \Delta I_2 \rangle & \langle \Delta I_2 \cdot \Delta I_3 \rangle & \langle \Delta I_2 \cdot \Delta I_4 \rangle \\ \langle \Delta I_3 \cdot \Delta I_1 \rangle & \langle \Delta I_3 \cdot \Delta I_2 \rangle & \langle \Delta I_3 \cdot \Delta I_3 \rangle & \langle \Delta I_3 \cdot \Delta I_4 \rangle \\ \langle \Delta I_4 \cdot \Delta I_1 \rangle & \langle \Delta I_4 \cdot \Delta I_2 \rangle & \langle \Delta I_4 \cdot \Delta I_3 \rangle & \langle \Delta I_4 \cdot \Delta I_4 \rangle \end{pmatrix} (\Theta^{-1})^T = (\Theta^T \Theta)^{-1} \sigma_{th}^2 \end{aligned} \quad (3.57)$$

For an optimized polarimeter (a), the matrix $(\Theta^T \Theta)^{-1}$ is

$$(\Theta^T \Theta)^{-1} = \begin{pmatrix} 2.3332 & -3.3332 & 0 & 0 \\ -3.3332 & 8.3336 & 0 & 0 \\ 0 & 0 & 8.3336 & -0.0008 \\ 0 & 0 & -0.0008 & 1.6664 \end{pmatrix} \quad (3.58)$$

For an optimized polarimeter (b), if the circumscribed uniform tetrahedron shown in [37] is symmetric with respect to one of three axes, $(\Theta^T \Theta)^{-1}$ will be a diagonal matrix. For example, when (θ, ϕ) takes $(0^\circ, 45^\circ)$, $(0^\circ, -9.735^\circ)$, $(60^\circ, 50.265^\circ)$ and $(120^\circ, 110.265^\circ)$, it has

$$(\Theta^T \Theta)^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{pmatrix} \quad (3.59)$$

Equations (3.57) and (3.59) mean that $\langle \Delta S_0^2 \rangle = \sigma_{th}^2$, $\langle \Delta S_1^2 \rangle = \langle \Delta S_2^2 \rangle = \langle \Delta S_3^2 \rangle = 3\sigma_{th}^2$ and $\langle \Delta S_i \cdot \Delta S_j \rangle = 0, i \neq j$. From Eq. (3.56), an optimized polarimeter (b) has a smaller SOP measurement noise than an optimized polarimeter (a). From Eqs. (3.58) and (3.59), an optimized polarimeter (b) may have very simple statistical properties.

3.4.5 Measurement noise of DOP and normalized Stokes vector for optimized polarimeter (b)

In reported POTDR technologies, when there is no PDL in a SMF link, the normalized Stokes vector $\hat{s} = \frac{(S_1, S_2, S_3)^T}{S_0 \cdot DOP}$ is often used for the calculation of fiber polarization properties [10, 31]. On the other hand, the DOP of the backscattered optical signal is also used for evaluating PMD values [34]. From Eq. (3.59), an optimized polarimeter (b) with specific angular orientations has simple statistical properties. Then, for such an optimized polarimeter (b), it can be calculated that

$$\langle \Delta DOP^2 \rangle = \frac{(3 + DOP^2) \sigma_{th}^2}{S_0^2} = k_{DOP} \frac{\sigma_{th}^2}{S_0^2} \quad (3.60)$$

$$\langle \|\Delta \hat{s}\|^2 \rangle = \frac{4DOP^2 (DOP^2 + 9/2) \sigma_{th}^2}{S_0^2} = k_s \frac{\sigma_{th}^2}{S_0^2} \quad (3.61)$$

As shown in Fig. 3.16, when the DOP of the backscattered optical signal varies from 0 to 1, the DOP measurement noise only slightly increases. However, the measurement noise of the normalized Stokes vector is far larger than that of the DOP when the backscattered optical signal is completely polarized. Therefore, from a measurement noise point of view, algorithms based on DOP measurements using wide optical pulses [34] are better than those based on normalized Stokes vector measurements using narrow optical pulses [35].

Fig. 3.16 Measurement noises of DOP and normalized Stokes vector versus DOP of backscattered optical signal in polarimeter (b).

In some POTDR setups, a four-detector polarimeter has been used [39]. A possible structure of such a polarimeter can be found in [40]. By splitting the optical beam into four beams, four optical powers can be measured in parallel through a suitable set of fixed waveplates and polarizers. The advantage of this polarimeter is that its speed is limited only by electronic data acquisition. Following the above procedure, it is also possible to optimize such a polarimeter in POTDR measurements. However, in practice, it is difficult to find four photodetectors which have exactly the same noise properties. Then, the optimum angles of the waveplates and the polarizers will depend on the exact values of noise of the four photodetectors.

Appendix

If $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are four eigenvalues of the matrix $\Theta\Theta^T$, $\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$ because $\Theta\Theta^T$ is a real symmetric matrix [26]. Based on Eq. (3.54) and the properties of a matrix [26], we have

$$\begin{cases} \sum_{i=1}^4 \lambda_i = \text{Tr}(\Theta\Theta^T) = 2 \\ \prod_{i=1}^4 \lambda_i = |\Theta\Theta^T| = |\Theta|^2 > 0 \end{cases} \quad (\text{A1})$$

From the second equation of Eq. (A1), $\lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0$. On the other hand, the four eigenvalues of the matrix $\Omega = (\Theta\Theta^T)^{-1}$ are $1/\lambda_1, 1/\lambda_2, 1/\lambda_3, 1/\lambda_4$ [26]. Then, it has

$$\sum_{i=1}^4 \Omega_{ii} = \text{Tr}(\Omega) = \sum_{i=1}^4 \frac{1}{\lambda_i} \geq 4 \sqrt[4]{\frac{1}{\prod_{i=1}^4 \lambda_i}} = 4 \sqrt[4]{\frac{1}{|\Theta|^2}} \quad (\text{A2})$$

In Eq. (A2), the equal sign is valid only when $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$. According to the result in [37], $(|\Theta|^2)_{\max} = 0.037$ can be achieved for polarimeter (b). However, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$

cannot be satisfied when $(|\Theta|^2)_{\max}$ is reached. Therefore, $\left(\sum_{i=1}^4 \Omega_{ii}\right)_{\min}$ should satisfy

$$\left(\sum_{i=1}^4 \Omega_{ii}\right)_{\min} > 4 \sqrt[4]{\frac{1}{(|\Theta|^2)_{\max}}} = 9.1203 \quad (\text{A3})$$

Equation (A3) only presents a lower bound of $\sum_{i=1}^4 \Omega_{ii}$. To find the exact value of

$\left(\sum_{i=1}^4 \Omega_{ii}\right)_{\min}$, numerical calculation has to be implemented. By fixing (α_1, β_1) at $(0^\circ, 90^\circ)$

and randomly generating $(\alpha_i, \beta_i), i=2,3,4$, $\left(\sum_{i=1}^4 \Omega_{ii}\right)_{\min} = 10$ can be found when

$(\alpha_i, \beta_i), i=2,3,4$ are $(0^\circ, -19.47^\circ)$, $(120^\circ, -19.47^\circ)$ and $(240^\circ, -19.47^\circ)$, respectively. They

are the same angle set at which $\left(|\Theta|^2\right)_{\max}$ is reached [37]. From Eq. (14), the matrix Ω only depends on the relative relations among $(\alpha_i, \beta_i), i=1,2,3,4$; it is independent of the exact values of $(\alpha_i, \beta_i), i=1,2,3,4$. Therefore, any angle set which leads to $\left(|\Theta|^2\right)_{\max}$ also results in $\left(\sum_{i=1}^4 \Omega_{ii}\right)_{\min}$.

Chapter 4 REFLECTOMETRIC MEASUREMENT OF PMD USING POTDR

4.1 Introduction

Most PMD measurement techniques require access to both fiber ends to implement the tests [17, 18, 21, 23-25]. Consequently, field tests on installed cables are time consuming and not simple [42]. More importantly, these techniques cannot be used to perform a distributed PMD measurement. In the past two decades, many reflectometric techniques have been proposed to realize the PMD distribution measurement by using POTDR [11, 34, 35, 42-53]. Due to the round-trip effect, only partial information of the PMD vector can be obtained from reflectometric measurements. Then, all reported techniques can only measure the averaged value of DGD, which is the magnitude of the PMD vector.

However, PMD in an optical fiber link is a function of optical wavelength. Spectrally-resolved measurements of the overall PMD in optical fiber links have been usually performed by accessing both fiber ends [17, 18, 21, 23-25]. But, to our knowledge, reflectometric techniques using POTDR, which can realize the spatially- and spectrally-resolved PMD measurement, have not been reported. Further, when there is PDL in the optical fiber link under test, reflectometric measurement of PMD seems to be impossible because of the lack of necessary information [54].

In this chapter, the spectrally- and spatially-resolved PMD measurement technique will be proposed firstly for optical fiber links having no PDL. Next, a similar technique will be demonstrated to be valid also for optical fiber links having PDL. Both theoretical analysis and experimental results will be presented to verify the proposed techniques.

Please note, I am the main contributor for the contents in section 4.2, 4.3 and 4.4. These works have been reported in the following papers where I am the first author: [6] and [7] in Journal Publications; [2] in Conference Publications.

4.2 Relationship between forward and round-trip polarization effect vectors

Before we start to investigate the reflectometric measurement of PMD using POTDR, we have to present the relationship between the forward polarization effects (which are what we want to measure) and the round-trip polarization effects (which are what can be directly measured using POTDR).

Fig. 4.1 A typical POTDR configuration.

A typical POTDR configuration is shown in Fig. 4.1. We denote $\mathbf{M}$ as the Mueller matrix relating the light entering the fiber under test (FUT) and that just reflecting at the fiber length z (in forward direction), $\mathbf{M}_1$ as the Mueller matrix relating light leaving the polarization controller and that at the input end of the FUT (in forward direction), and $\mathbf{M}_2$ as the Mueller matrix relating light leaving the input end of FUT and that at the input port of SOP analyzer (in backward direction). The total round-trip Mueller matrix should be $\mathbf{M}_B = \mathbf{M}_2 \mathbf{R} \mathbf{M}^T \mathbf{R} \mathbf{M} \mathbf{M}_1$ with $\mathbf{R} = \text{diag}(1, 1, 1, -1)$ [54]. If depolarization effects are not considered, from Eq. (2.6), we have

$$\frac{d\mathbf{M}}{dz}\mathbf{M}^{-1} = \begin{pmatrix} \eta_z & \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_1 & \eta_z & -\beta_3 & \beta_2 \\ \alpha_2 & \beta_3 & \eta_z & -\beta_1 \\ \alpha_3 & -\beta_2 & \beta_1 & \eta_z \end{pmatrix} \quad (4.1)$$

where $\eta_z = d(\ln \sqrt[4]{|\mathbf{M}|})/dz$, $\vec{\alpha} = (\alpha_1, \alpha_2, \alpha_3)^T$ and $\vec{\beta} = (\beta_1, \beta_2, \beta_3)^T$ are the local PDL vector and local birefringence vector, respectively. Similarly, for the round-trip Mueller matrix, we also have

$$\frac{d\mathbf{M}_B}{dz}\mathbf{M}_B^{-1} = \begin{pmatrix} \eta_{zB} & \alpha_{B1} & \alpha_{B2} & \alpha_{B3} \\ \alpha_{B1} & \eta_{zB} & -\beta_{B3} & \beta_{B2} \\ \alpha_{B2} & \beta_{B3} & \eta_{zB} & -\beta_{B1} \\ \alpha_{B3} & -\beta_{B2} & \beta_{B1} & \eta_{zB} \end{pmatrix} \quad (4.2)$$

Where $\eta_{zB} = d(\ln \sqrt[4]{|\mathbf{M}_B|})/dz$, $\vec{\alpha}_B = (\alpha_{B1}, \alpha_{B2}, \alpha_{B3})^T$ and $\vec{\beta}_B = (\beta_{B1}, \beta_{B2}, \beta_{B3})^T$ are the local round-trip PDL vector and birefringence vector, respectively. On the other hand, we have

$$\frac{d\mathbf{M}_B}{dz}\mathbf{M}_B^{-1} = \mathbf{M}_2\mathbf{R}\mathbf{M}^T \left((\mathbf{M}^T)^{-1} \frac{d\mathbf{M}^T}{dz} + \mathbf{R} \frac{d\mathbf{M}}{dz} \mathbf{M}^{-1} \mathbf{R} \right) (\mathbf{M}_2\mathbf{R}\mathbf{M}^T)^{-1} \quad (4.3)$$

The first item in the bracket is

$$(\mathbf{M}^T)^{-1} \frac{d\mathbf{M}^T}{dz} = \left(\frac{d\mathbf{M}}{dz} \mathbf{M}^{-1} \right)^T \quad (4.4)$$

Then, the sum of the two items in the bracket is

$$(\mathbf{M}^T)^{-1} \frac{d\mathbf{M}^T}{dz} + \mathbf{R} \frac{d\mathbf{M}}{dz} \mathbf{M}^{-1} \mathbf{R} = 2 \begin{pmatrix} \eta_z & \alpha_1 & \alpha_2 & 0 \\ \alpha_1 & \eta_z & 0 & -\beta_2 \\ \alpha_2 & 0 & \eta_z & \beta_1 \\ 0_3 & \beta_2 & -\beta_1 & \eta_z \end{pmatrix} \quad (4.5)$$

From Eqs. (4.1)-(4.5), we know

$$\begin{pmatrix} \eta_{zB} & \alpha_{B1} & \alpha_{B2} & \alpha_{B3} \\ \alpha_{B1} & \eta_{zB} - \beta_{B3} & \beta_{B2} & \beta_{B1} \\ \alpha_{B2} & \beta_{B3} & \eta_{zB} & -\beta_{B1} \\ \alpha_{B3} - \beta_{B2} & \beta_{B1} & \eta_{zB} & \end{pmatrix} = 2\mathbf{M}_2\mathbf{R}\mathbf{M}^T \begin{pmatrix} \eta_z & \alpha_1 & \alpha_2 & 0 \\ \alpha_1 & \eta_z & 0 & -\beta_2 \\ \alpha_2 & 0 & \eta_z & \beta_1 \\ 0 & \beta_2 & -\beta_1 & \eta_z \end{pmatrix} (\mathbf{M}_2\mathbf{R}\mathbf{M}^T)^{-1} \quad (4.6)$$

Equation (4.6) means the two matrices inside the brackets are similar matrices [26].

Because the traces and eigenvalues of similar matrices are the same [26], we obtain the following equations

$$\begin{cases} \eta_{zB} = 2\eta_z \\ \lambda^4 + (\beta_B^2 - \alpha_B^2)\lambda^2 - (\vec{\beta}_B \cdot \vec{\alpha}_B)^2 = \lambda^4 + 4(\beta_L^2 - \alpha_L^2)\lambda^2 - 16(\vec{\beta}_L \cdot \vec{\alpha}_L)^2 \end{cases} \quad (4.7)$$

where λ denotes the eigenvalues of the two matrices;

$\vec{\alpha}_L = (\alpha_1, \alpha_2, 0)^T$, $\vec{\beta}_L = (\beta_1, \beta_2, 0)^T$ and $\alpha_L = |\vec{\alpha}_L|$, $\beta_L = |\vec{\beta}_L|$, $\alpha_B = |\vec{\alpha}_B|$, $\beta_B = |\vec{\beta}_B|$.

Equating the coefficients, we have

$$\begin{cases} \beta_B^2 - \alpha_B^2 = 4(\beta_L^2 - \alpha_L^2) \\ \vec{\beta}_B \cdot \vec{\alpha}_B = 4\vec{\beta}_L \cdot \vec{\alpha}_L \end{cases} \quad (4.8)$$

The two equations in Eq.(4.8) have been found by A. Galtarossa using the Jones matrix formalism [54]; here we present an alternative deduction using the Mueller matrix formalism. Based on the general model proposed in Chapter 2, similar equations also exist between the forward and the round-trip complex PMD vectors as

$$\begin{cases} \Omega_B^2 - \Lambda_B^2 = 4(\Omega_L^2 - \Lambda_L^2) \\ \vec{\Omega}_B \cdot \vec{\Lambda}_B = 4\vec{\Omega}_L \cdot \vec{\Lambda}_L \end{cases} \quad (4.9)$$

Equations (4.8) and (4.9) show that only the linear components of the forward polarization effect vectors are related to the round-trip polarization effect vectors.

4.3 Spatially- and spectrally-resolved PMD measurement in SMF links

4.3.1 Theoretical analysis

When there is no PDL, the 3×3 Mueller matrix and 3-dimensional Stokes vector can be used to describe the polarization effects in a SMF link. Due to the round-trip effect, the round-trip PMD vector $\vec{\Omega}_B$, which can be measured directly in the reflectometric test, is only explicitly related to the first two elements of the PMD vector $\vec{\Omega}$ as [35]

$$\vec{\Omega}_B = 2\mathbf{RM}^T \vec{\Omega}_L = 2\mathbf{RM}^T (\Omega_1, \Omega_2, 0)^T \quad (4.10)$$

As a consequence, only a statistical relation $\langle \Delta\tau \rangle = \frac{2}{\pi} \langle \Delta\tau_B \rangle$ ($\Delta\tau = |\vec{\Omega}|$ and $\Delta\tau_B = |\vec{\Omega}_B|$) can be derived to give the mean value of DGD [35]. Because averaging is performed over many optical wavelengths [42], the relation between $\Delta\tau$ and optical wavelength cannot be obtained. From the theoretical analysis of backscattering polarization evolution, two equations have been derived as [35]

$$\vec{\Omega}_B = 2\mathbf{RM}^T \vec{\Omega}_L, \quad \vec{\beta}_B = 2\mathbf{RM}^T \vec{\beta}_L \quad (4.11)$$

Taking the derivatives of Eq. (4.11), we obtain

$$\begin{cases} \frac{\partial \vec{\Omega}_B}{\partial z} = 2\mathbf{RM}^T \left(\frac{\partial \Omega_1}{\partial z} + \beta_3 \Omega_2, \frac{\partial \Omega_2}{\partial z} - \beta_3 \Omega_1, \beta_2 \Omega_1 - \beta_1 \Omega_2 \right)^T \\ \frac{\partial \vec{\beta}_B}{\partial \omega} = 2\mathbf{RM}^T \left(\frac{\partial \beta_1}{\partial \omega} + \beta_2 \Omega_3, \frac{\partial \beta_2}{\partial \omega} - \beta_1 \Omega_3, \beta_1 \Omega_2 - \beta_2 \Omega_1 \right)^T \end{cases} \quad (4.12)$$

Considering the sum of two equations in Eq. (4.11), we have

$$\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} = 2\mathbf{RM}^T \left(\frac{\partial \beta_1}{\partial \omega} + \frac{\partial \Omega_1}{\partial z} + \beta_3 \Omega_2 + \beta_2 \Omega_3, \frac{\partial \beta_2}{\partial \omega} + \frac{\partial \Omega_2}{\partial z} - \beta_3 \Omega_1 - \beta_1 \Omega_3, 0 \right)^T \quad (4.13)$$

From the PMD dynamical equation $\frac{\partial \vec{\Omega}}{\partial z} = \frac{\partial \vec{\beta}}{\partial \omega} + \vec{\beta} \times \vec{\Omega}$ [55], two equations can be derived as

$$\frac{\partial \Omega_1}{\partial z} + \beta_3 \Omega_2 = \frac{\partial \beta_1}{\partial \omega} + \beta_2 \Omega_3, \quad \frac{\partial \Omega_2}{\partial z} - \beta_3 \Omega_1 = \frac{\partial \beta_2}{\partial \omega} - \beta_1 \Omega_3 \quad (4.14)$$

Substituting Eq. (4.14) into Eq. (4.13), an equation including Ω_3 is achieved as

$$\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} = 4\mathbf{R}\mathbf{M}^T \left(\frac{\partial \beta_1}{\partial \omega} + \beta_2 \Omega_3, \frac{\partial \beta_2}{\partial \omega} - \beta_1 \Omega_3, 0 \right)^T \quad (4.15)$$

Taking the norms of both sides of Eq. (4.15) and considering $\mathbf{M}$ is an orthogonal matrix, we have

$$\left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} \right) \cdot \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega} \right) = 16 \left[\left(\frac{\partial \beta_1}{\partial \omega} \right)^2 + \left(\frac{\partial \beta_2}{\partial \omega} \right)^2 + \beta_1^2 \Omega_3^2 + 2 \left(\beta_2 \frac{\partial \beta_1}{\partial \omega} - \beta_1 \frac{\partial \beta_2}{\partial \omega} \right) \Omega_3 \right] \quad (4.16)$$

For fibers and components used in optical communications, at least over the wavelength range of interest, it is an excellent approximation that the DGD between two polarization modes is independent of optical frequency in a short fiber trunk [56, 57]. This means the local birefringence must have a linear relationship with optical frequency. A short fiber section can be treated as a waveplate with fixed fast and slow axes in the wavelength range of interest [58]. Furthermore, if this fiber section is not affected by an external magnetic field or twist, the circular birefringence $\beta_3 = 0$. By

summarizing the above discussion, we can draw a conclusion that $\frac{\partial \vec{\beta}_L}{\partial \omega}$ is in parallel to $\vec{\beta}_L$. Then we easily show

$$\beta_2 \frac{\partial \beta_1}{\partial \omega} - \beta_1 \frac{\partial \beta_2}{\partial \omega} = 0, \quad \left(\frac{\partial \beta_1}{\partial \omega} \right)^2 + \left(\frac{\partial \beta_2}{\partial \omega} \right)^2 = \left| \frac{\partial \vec{\beta}_L}{\partial \omega} \right|^2 = \left(\frac{\partial \beta_L}{\partial \omega} \right)^2 \quad (4.17)$$

Due to $\beta_B = 2\beta_L$ with $\beta_B = |\vec{\beta}_B|$ and $\beta_L = |\vec{\beta}_L|$, Ω_3^2 can eventually be solved as

$$\Omega_3^2 = \frac{\left| \frac{\partial \vec{\Omega}_B}{\partial z} \right|^2 + \left| \frac{\partial \vec{\beta}_B}{\partial \omega} \right|^2 + 2 \frac{\partial \vec{\Omega}_B}{\partial z} \cdot \frac{\partial \vec{\beta}_B}{\partial \omega} - 4 \left(\frac{\partial \beta_B}{\partial \omega} \right)^2}{4\beta_B^2} \quad (4.18)$$

Equation (4.18) can be combined with $\Omega_L = \Omega_B / 2$, and the DGD is deduced to be

$$\Delta\tau = \sqrt{\Omega_L^2 + \Omega_3^2} = \frac{\sqrt{\beta_B^2 \Omega_B^2 + \left| \frac{\partial \vec{\Omega}_B}{\partial z} \right|^2 + \left| \frac{\partial \vec{\beta}_B}{\partial \omega} \right|^2 + 2 \frac{\partial \vec{\Omega}_B}{\partial z} \cdot \frac{\partial \vec{\beta}_B}{\partial \omega} - 4 \left(\frac{\partial \beta_B}{\partial \omega} \right)^2}}{2\beta_B} \quad (4.19)$$

4.3.2 Experimental verification

Fig. 4.2 Experimental setup for backreflection and forward measurement of PMD using continuous-wave signal.

To verify the proposed technique, two experiments are carried out using two different setups. The first experimental setup is shown in Fig. 4.2, which employs a CW signal and Fresnel reflection from the far end of the fiber. The tunable laser source, the polarization state generator (a rotatable polarizer followed by a rotatable quarter-wave plate) and the polarimeter are controlled and synchronized by a computer. The polarization state at port 1 of the circulator is tuned by the polarization state generator. At port 2, an FC/APC connector is used to reduce the near-end Fresnel reflection. The FUT is composed of step index SMFs, dispersion shifted fibers (DSF) and dispersion compensation fibers (DCF), which are spliced together with a total length of 35km. At the far-end of FUT, a mounting bracket is employed to guarantee the fiber far end is fixed in the measurement. A 1m-long SMF can be connected and disconnected to the FUT far end in order to change the fiber length. Because the total fiber length is 35km, the optical power contribution of Rayleigh backscattering can be neglected compared with the far-end Fresnel reflection [42].

To measure $\vec{\Omega}_B$ and $\vec{\beta}_B$, the Mueller matrix method (MMM) is employed [21]. The test is divided into three steps. Firstly the polarimeter is connected to port 3 of the circulator and the 1m-long SMF is connected to the far-end of the FUT. Now the far-end Fresnel backreflection is mainly generated by the glass-air interface of the far-end of the 1m-long SMF. The wavelength is tuned from 1550 to 1555nm with a 0.1nm step size and the polarimeter measures the evolution of polarization states at port 3 with respect to optical wavelength using the two input polarization states $(1, 0, 0)^T$ and $(0, 1, 0)^T$ defined in the MMM [21]. Secondly the 1m-long SMF is removed and then the measurement is repeated with the same parameters. Now the DGD evolution with respect to optical wavelength can be calculated based on Eq. (4.19).

Finally, the polarimeter is connected to the far-end of FUT and the measurement is repeated in the forward direction. Then, the forward measurement presents the DGD value at every wavelength to compare with the one measured using the backreflection technique. The experimental results are shown in Fig. 4.3, and good agreement is observed.

Fig. 4.3 Comparison of DGD evolution with respect to optical wavelength measured using forward and backreflection techniques using CW probe light.

In the above measurement, two MMM measurement processes are performed and a short SMF is required to be connected and removed at the far-end of FUT. This is a consequence of using CW probe light, which makes the experimental setup simpler. If a pulsed tunable laser source and a real-time polarization analyzer are employed, we can measure the polarization states of pulses reflected from two ends of the short SMF simultaneously, and then the two above-mentioned issues can be overcome. Figure 4.4 shows the second experimental setup, which employs the pulsed signal and Fresnel reflections of two ends of the short SMF. Compared with the first experimental setup, this setup is more complex.

Fig. 4.4 Experimental setup for backreflection and forward measurement of PMD using pulsed signals.

The pulse generator generates pulses with 8ns pulse width at a 10 kHz repetition rate, and then is used to directly modulate the tunable laser source to emit the optical pulses. The polarization analyzer is composed of a rotatable quarter-wave plate and a fixed polarizer as well as a 125MHz photodetector. Based on this setup, the test is divided into two steps. Firstly the polarization analyzer is connected to port 3 of the circulator. The wavelength range is again from 1550nm to 1555nm with a 0.1nm step size. The evolutions of the polarization states of optical pulses reflected from two ends of the 1m-long SMF with respect to optical wavelength are measured using the polarization analyzer at port 3. Then the spectrally-resolved DGD is obtained from the

reflectometric measurement using Eq. (4.19). Secondly the forward measurement is performed also using optical pulses for comparison. The experimental result is shown in Fig. 4.5. Although the agreement between two methods is not as good as that using a CW signal, it still confirms the validity of our proposed technique.

Fig. 4.5 Comparison of DGD evolution with respect to optical wavelength measured using backreflection and forward techniques and pulsed probe light.

4.3.3 PMD distribution measurement

The experimental setup is just shown in Fig. 4.4 and the SMF link under test is shown in Fig. 4.6. It is composed of three fiber sections: 25 km DSF, 10 km G. 652 SMF and 3 km DCF with a polarization-maintaining fiber (PMF) section inside. The PMF section is used to increase the PMD value of the third fiber section. Please note, Fresnel backreflection, not Rayleigh backscattering, is used as the measurement signals; and pulsed signals are used to avoid connecting and disconnecting fiber sections.

Fig. 4.6 The SMF link under test for spatially- and spectrally-resolved PMD measurement.

Measurement results of the DGD distribution are shown in the wavelength range from 1550 nm to 1555 nm in Fig. 4.7(a)(c) and from 1550 nm to 1560 nm in Fig. 4.7(b). Good agreement between the results of forward and reflectometric measurements can be observed [59].

(a) 25 km DSF

(b) 10 km SMF

(c) 3 km DCF with PMF

Fig. 4.7 Spatially- and spectrally-resolved PMD measurement results. DGD values of each fiber section are also compared with those measured using forward methods.

4.4 Spectrally-resolved PMD measurement in SMF links having PDL

4.4.1 Theoretical analysis

Previously, the reflectometric technique could only be used to measure an optical fiber link without PDL. In an optical fiber link with PDL, due to the round-trip effect, the round-trip complex PMD vector $\vec{W}_B = \vec{\Omega}_B + i\vec{\Lambda}_B$, which can be measured directly in the test, is only explicitly related with the first two elements of the complex PMD vector $\vec{W} = \vec{\Omega} + i\vec{\Lambda}$ as shown in Eq. (4.9). Due to the lack of equations, Eq. (4.9) cannot be uniquely solved. Even if it could be uniquely solved using some assumptions, a spectrally-resolved PMD measurement still cannot be made due to the lack of information about the third elements of the complex PMD vector, Ω_3 and Λ_3 . In this thesis, we demonstrate theoretically and experimentally that a spectrally-resolved reflectometric PMD measurement in optical fibers with PDL can be accomplished. Based on the PMD dynamical equation, two equations can be obtained to relate $\Omega_3\Lambda_3$, $\Omega_3^2 - \Lambda_3^2$ with the round-trip complex PMD vector and the round-trip local birefringence vector which can be determined by carrying out reflectometric measurements simultaneously in the optical frequency domain and the fiber length domain. Then, the spectrally-resolved DGD and differential attenuation slope (DAS) can be deterministically achieved, although the principal states of polarization (PSP), which is defined in [4], still cannot be determined.

Optical fibers used in telecommunication systems always have negligible PDL values unless a fiber is bent with a diameter less than 5 cm. The PDL in an optical fiber system is mainly caused by optical components, such as couplers, filters and so on. For a single optical component, a typical PDL value is around 0.2 dB, and so the total PDL value of an optical fiber system with many PDL components may be large. From the previously reported measurement results using the forward measurement

technique, we know that even a PDL of 0.1 dB can have an obvious impact on the PMD measurement [60]. Therefore, the PDL effect on the reflectometric measurements of PMD should be taken into consideration even if the PDL of the fiber link under test is not very large.

As we discussed in Chapter 2, for a fiber system with both birefringence and PDL, its Mueller matrix $\mathbf{M}$ meets the Lorentz transformation. Thus, if we normalize $\mathbf{M}$ to make $|\mathbf{M}| = 1$, then

$$\mathbf{B} = \frac{d\mathbf{M}}{dz} \mathbf{M}^{-1} = \begin{pmatrix} 0 & \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_1 & 0 & -\beta_3 & \beta_2 \\ \alpha_2 & \beta_3 & 0 & -\beta_1 \\ \alpha_3 & -\beta_2 & \beta_1 & 0 \end{pmatrix}, \quad \mathbf{P} = \frac{d\mathbf{M}}{d\omega} \mathbf{M}^{-1} = \begin{pmatrix} 0 & \Lambda_1 & \Lambda_2 & \Lambda_3 \\ \Lambda_1 & 0 & -\Omega_3 & \Omega_2 \\ \Lambda_2 & \Omega_3 & 0 & -\Omega_1 \\ \Lambda_3 & -\Omega_2 & \Omega_1 & 0 \end{pmatrix} \quad (4.20)$$

In reflectometric measurements, we have

$$\mathbf{B}_B = \frac{d\mathbf{M}_B}{dz} \mathbf{M}_B^{-1} = \begin{pmatrix} 0 & \alpha_{B1} & \alpha_{B2} & \alpha_{B3} \\ \alpha_{B1} & 0 & -\beta_{B3} & \beta_{B2} \\ \alpha_{B2} & \beta_{B3} & 0 & -\beta_{B1} \\ \alpha_{B3} & -\beta_{B2} & \beta_{B1} & 0 \end{pmatrix}, \quad \mathbf{P}_B = \frac{d\mathbf{M}_B}{d\omega} \mathbf{M}_B^{-1} = \begin{pmatrix} 0 & \Lambda_{B1} & \Lambda_{B2} & \Lambda_{B3} \\ \Lambda_{B1} & 0 & -\Omega_{B3} & \Omega_{B2} \\ \Lambda_{B2} & \Omega_{B3} & 0 & -\Omega_{B1} \\ \Lambda_{B3} & -\Omega_{B2} & \Omega_{B1} & 0 \end{pmatrix} \quad (4.21)$$

The differentiations of Eq. (4.21) are

$$\begin{cases} \frac{\partial \mathbf{P}_B}{\partial z} = 2\mathbf{R}\mathbf{M}^T \mathbf{R} \left(\mathbf{R}\mathbf{B}^T \mathbf{R} \mathbf{P}_L - \mathbf{P}_L \mathbf{R}\mathbf{B}^T \mathbf{R} + \frac{\partial \mathbf{P}_L}{\partial z} \right) (\mathbf{R}\mathbf{M}^T \mathbf{R})^{-1} \\ \frac{\partial \mathbf{B}_B}{\partial \omega} = 2\mathbf{R}\mathbf{M}^T \mathbf{R} \left(\mathbf{R}\mathbf{P}^T \mathbf{R} \mathbf{B}_L - \mathbf{B}_L \mathbf{R}\mathbf{P}^T \mathbf{R} + \frac{\partial \mathbf{B}_L}{\partial \omega} \right) (\mathbf{R}\mathbf{M}^T \mathbf{R})^{-1} \end{cases} \quad (4.22)$$

$$\text{where, } \mathbf{B}_L = \begin{pmatrix} 0 & \alpha_1 & \alpha_2 & 0 \\ \alpha_1 & 0 & 0 & -\beta_2 \\ \alpha_2 & 0 & 0 & \beta_1 \\ 0 & \beta_2 & -\beta_1 & 0 \end{pmatrix}, \quad \mathbf{P}_L = \begin{pmatrix} 0 & \Lambda_1 & \Lambda_2 & 0 \\ \Lambda_1 & 0 & 0 & -\Omega_2 \\ \Lambda_2 & 0 & 0 & \Omega_1 \\ 0 & \Omega_2 & -\Omega_1 & 0 \end{pmatrix}. \text{ Taking the sum of two}$$

equations in Eq. (4.22), we obtain

$$\frac{\partial \mathbf{P}_B}{\partial z} + \frac{\partial \mathbf{B}_B}{\partial \omega} = 2\mathbf{R}\mathbf{M}^T\mathbf{R} \begin{pmatrix} 0 & C_1 & C_2 & 0 \\ C_1 & 0 & 0 & D_2 \\ C_2 & 0 & 0 & -D_1 \\ 0 & -D_2 & D_1 & 0 \end{pmatrix} (\mathbf{R}\mathbf{M}^T\mathbf{R})^{-1} \quad (4.23)$$

where $C_1 = \beta_3\Lambda_2 + \alpha_3\Omega_2 + \alpha_2\Omega_3 + \beta_2\Lambda_3 + \frac{\partial\alpha_1}{\partial\omega} + \frac{\partial\Lambda_1}{\partial z}$,

$$C_2 = -\beta_3\Lambda_1 - \alpha_3\Omega_1 - \alpha_1\Omega_3 - \beta_1\Lambda_3 + \frac{\partial\alpha_2}{\partial\omega} + \frac{\partial\Lambda_2}{\partial z} ,$$

$$D_1 = \beta_3\Omega_2 - \alpha_3\Lambda_2 - \alpha_2\Lambda_3 + \beta_2\Omega_3 + \frac{\partial\beta_1}{\partial\omega} + \frac{\partial\Omega_1}{\partial z} ,$$

$$D_2 = -\beta_3\Omega_1 + \alpha_3\Lambda_1 + \alpha_1\Lambda_3 - \beta_1\Omega_3 + \frac{\partial\beta_2}{\partial\omega} + \frac{\partial\Omega_2}{\partial z} .$$

On the other hand, we can easily derive the PMD dynamical equation as [57]

$$\frac{\partial \vec{\Omega}}{\partial z} = \frac{\partial \vec{\beta}}{\partial \omega} + \vec{\beta} \times \vec{\Omega} - \vec{\alpha} \times \vec{\Lambda} , \quad \frac{\partial \vec{\Lambda}}{\partial z} = \frac{\partial \vec{\alpha}}{\partial \omega} + \vec{\beta} \times \vec{\Lambda} + \vec{\alpha} \times \vec{\Omega} \quad (4.24)$$

In optical fibers and components used in optical communications, at least over the wavelength range of interest, we have [57]

$$\frac{\partial \vec{\alpha}}{\partial \omega} = 0 \quad \text{and} \quad \frac{\partial \vec{\beta}}{\partial \omega} = \frac{\vec{\beta}}{\omega} \quad (4.25)$$

By applying Eqs. (4.24) and (4.25), the elements in Eq. (4.23) can be simplified as

$$\begin{cases} C_1 = 2(\alpha_2\Omega_3 + \beta_2\Lambda_3) \\ C_2 = -2(\alpha_1\Omega_3 + \beta_1\Lambda_3) \\ D_1 = 2\left(\frac{\beta_1}{\omega} + \beta_2\Omega_3 - \alpha_2\Lambda_3\right) \\ D_2 = 2\left(\frac{\beta_2}{\omega} + \alpha_1\Lambda_3 - \beta_1\Omega_3\right) \end{cases} \quad (4.26)$$

Because the right and left sides of Eq. (4.23) are similar matrices, they should have the same eigenvalues [26]. Then two equations can be derived as

$$\begin{cases}
 16(\vec{\alpha}_L \cdot \vec{\beta}_L)(\Omega_3^2 - \Lambda_3^2) + 16(\beta_L^2 - \alpha_L^2)\Omega_3\Lambda_3 + 16\frac{|\vec{\alpha}_L \times \vec{\beta}_L|}{\omega}\Omega_3 \\
 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega}\right) \cdot \left(\frac{\partial \vec{\Lambda}_B}{\partial z} + \frac{\partial \vec{\alpha}_B}{\partial \omega}\right) \\
 16(\beta_L^2 - \alpha_L^2)(\Omega_3^2 - \Lambda_3^2) - 64(\vec{\alpha}_L \cdot \vec{\beta}_L)\Omega_3\Lambda_3 + 16\frac{\beta_L^2}{\omega^2} + 32\frac{|\vec{\alpha}_L \times \vec{\beta}_L|}{\omega}\Lambda_3 \\
 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega}\right)^2 - \left(\frac{\partial \vec{\Lambda}_B}{\partial z} + \frac{\partial \vec{\alpha}_B}{\partial \omega}\right)^2
 \end{cases} \quad (4.27)$$

Due to $\beta_B^2 - \alpha_B^2 = 4(\beta_L^2 - \alpha_L^2)$ and $\vec{\beta}_B \cdot \vec{\alpha}_B = 4\vec{\beta}_L \cdot \vec{\alpha}_L$, Eq. (4.27) becomes

$$\begin{cases}
 4(\vec{\alpha}_B \cdot \vec{\beta}_B)(\Omega_3^2 - \Lambda_3^2) + 4(\beta_B^2 - \alpha_B^2)\Omega_3\Lambda_3 + 16\frac{|\vec{\alpha}_L \times \vec{\beta}_L|}{\omega}\Omega_3 \\
 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega}\right) \cdot \left(\frac{\partial \vec{\Lambda}_B}{\partial z} + \frac{\partial \vec{\alpha}_B}{\partial \omega}\right) \\
 4(\beta_B^2 - \alpha_B^2)(\Omega_3^2 - \Lambda_3^2) - 16(\vec{\alpha}_B \cdot \vec{\beta}_B)\Omega_3\Lambda_3 + 16\frac{\beta_L^2}{\omega^2} + 32\frac{|\vec{\alpha}_L \times \vec{\beta}_L|}{\omega}\Lambda_3 \\
 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega}\right)^2 - \left(\frac{\partial \vec{\Lambda}_B}{\partial z} + \frac{\partial \vec{\alpha}_B}{\partial \omega}\right)^2
 \end{cases} \quad (4.28)$$

In telecommunication optical fiber links, optical fibers have negligible PDL, and PDL mainly exists in some optical components. Therefore, if the fiber section from z to $z + \Delta z$ has no PDL, Eq. (4.28) can be simplified as

$$\begin{cases}
 4\beta_B^2\Omega_3\Lambda_3 = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega}\right) \cdot \frac{\partial \vec{\Lambda}_B}{\partial z} \\
 4\beta_B^2(\Omega_3^2 - \Lambda_3^2) + \frac{4\beta_B^2}{\omega^2} = \left(\frac{\partial \vec{\Omega}_B}{\partial z} + \frac{\partial \vec{\beta}_B}{\partial \omega}\right)^2 - \left(\frac{\partial \vec{\Lambda}_B}{\partial z}\right)^2
 \end{cases} \quad (4.29)$$

Thus, $(\Omega_3^2 - \Lambda_3^2)$ and $\Omega_3\Lambda_3$ can be solved from Eq. (4.29) once $\vec{\Omega}_B$, $\vec{\Lambda}_B$ and $\vec{\beta}_B$ are measured. Based on Eq. (4.9) and (4.29), we can calculate $\Omega^2 - \Lambda^2 = (\Omega_L^2 - \Lambda_L^2) + \Omega_3^2 - \Lambda_3^2$ and $\vec{\Omega} \cdot \vec{\Lambda} = \vec{\Omega}_L \cdot \vec{\Lambda}_L + \Omega_3\Lambda_3$. Furthermore we already know that DGD and DAS can be expressed as [60]

$$DGD = \sqrt{\frac{1}{2} \left[\Omega^2 - \Lambda^2 + \sqrt{(\Omega^2 - \Lambda^2)^2 + 4(\vec{\Omega} \cdot \vec{\Lambda})^2} \right]}, \quad DAS = \vec{\Omega} \cdot \vec{\Lambda} / DGD \quad (4.30)$$

Finally spectrally-resolved DGD and DAS can be clearly measured.

4.4.2 Experimental verification

Experiments have been carried out to verify the above theory. The experimental setup was shown in Fig. 4.4. Please note, Fresnel backreflections, not Rayleigh backscattering, is also used as the measurement signal. The FUT is composed of SMF, DSF and DCF, which are spliced together with a total length of 35 km. PDL is induced by bending the fibers in a diameter less than 2 cm at two positions of the fiber link, and the measured PDL, using the forward technique described in [15], is plotted in Fig. 4.8. The spectrally-resolved DGD and DAS measurement results are shown in Fig. 4.9. Forward measurement results are also shown for comparison. Good agreement between reflectometric and forward measurement results can be observed for both DGD and DAS.

Fig. 4.8 PDL in the fiber link induced by bending the fibers.

Fig. 4.9 Spectrally-resolved (a) DGD and (b) DAS measurement results by the reflectometric and forward measurements in the wavelength range from 1552 to 1557 nm.

If a SMF link is interleaved with SMF sections and PDL components, the above technique has the potential to be used to realize the spectrally- and spatially-resolved PMD measurement in this SMF link by detecting the Rayleigh backscattering signals.

Chapter 5 REFLECTOMETRIC MEASUREMENT OF PDL USING POTDR

5.1 Introduction

Standard SMFs have no PDL unless they are bent in a diameter less than 5 cm [2]. However, in a fiber communication system, SMFs are used to transmit optical signals together with optical components, such as optical couplers and filters. These optical components may have non-negligible values of PDL. Although the typical PDL value of a single optical component is only 0.1-0.5 dB, the global PDL value of a fiber link may be up to several dB when many optical components are used. Investigation of PDL is very important in an optical fiber communication system due to its adverse effect on both analog and digital optical signals [61, 62]. The combined effect of PDL and PMD gives rise to anomalous pulse spreading and deteriorates the bit error rate [63]. Several techniques have been proposed to measure the global PDL in optical fiber links [15, 64-67], but all of them require access to both ends of the fiber link under test; one end is connected to the optical source and the other is connected to the optical receiver. Such a configuration may be difficult to implement in field tests because the two fiber ends may be tens of kilometers apart from each other. To realize a single-end measurement of PDL, A. Galtarossa *et al* have theoretically proposed a technique to obtain the mean value of the forward PDL using the statistical relationship between the forward and round-trip mean PDL values [54, 68]. This relationship is valid when the forward mean PDL value is less than 10 dB [68]. However, the authors did not mention how to realize the mean value measurement in practice. Probably for this reason, this technique was only demonstrated theoretically and there has been no experimental verification reported, to our knowledge.

In this thesis, we propose a single-end global PDL measurement approach [69]. This

technique is valid when the terminal fiber section (close to far end of the fiber link) is purely birefringent. In this technique, first the round-trip PDL varying along the terminal fiber section is obtained by measuring the Rayleigh backscattering optical powers corresponding to four specific input SOPs. Then, the forward global PDL is calculated using the maximum value of the measured round-trip PDL.

If PDL sections and birefringent SMF sections are interleaved in an SMF link, then the above technique can be used to measure the PDL distribution in the fiber link. But in this case, the SOPs of the Rayleigh backscattering signals must be measured.

Please note, I am the main contributor for the contents in section 5.2 and 5.3. This work has been reported in the following paper where I am the first author: [2] in Journal Publications.

5.2 Single-end PDL measurement

5.2.1 Theoretical analysis

In the following analysis, $\mathbf{M}$ denotes the forward Mueller matrix of a fiber link which contains all PDL components exclusive of the terminal fiber section; $\mathbf{M}_E(z)$ denotes the forward Mueller matrix of the fiber section from the end position of the above-mentioned PDL fiber link to a position z in the terminal fiber section. The Mueller matrix $\mathbf{M}$ can be decomposed as [22]

$$\mathbf{M} = T \begin{pmatrix} 1 & \vec{D}^T \\ \mathbf{m}_R \vec{D} & \mathbf{m}_R \mathbf{m}_D \end{pmatrix} \quad (5.1)$$

where, T is the attenuation for unpolarized light. $\vec{D} = (D_1, D_2, D_3)^T$ is the forward PDL vector [22, 65]; PDL in dB is related to $D = |\vec{D}|$ by $\text{PDL} = 10 \log_{10} \frac{1+D}{1-D}$. The matrix $\mathbf{m}_R$ is an orthogonal matrix and $\mathbf{m}_D = \sqrt{1-D^2} \mathbf{I} + (1-\sqrt{1-D^2}) \hat{D} \hat{D}^T$ ($\mathbf{I}$ is the 3×3 identity matrix and $\hat{D} = \vec{D}/D$) [22]. Because $\mathbf{M}_E$ represents a purely birefringent Mueller

matrix, it can be written as [22]

$$\mathbf{M}_E = T_E \begin{pmatrix} 1 & \vec{0}^T \\ \vec{0} & \mathbf{m}_E \end{pmatrix} \quad (5.2)$$

where, T_E stands for the attenuation and $\mathbf{m}_E$ is an orthogonal matrix; $\vec{0}^T = (0, 0, 0)$.

Based on Eq. (5.1) and (5.2), the Mueller matrix of the fiber link from the input end to the position z in the terminal fiber section is

$$\mathbf{M}_E \mathbf{M} = T_E T \begin{pmatrix} 1 & \vec{D}^T \\ \mathbf{m}_E \mathbf{m}_R \vec{D} & \mathbf{m}_E \mathbf{m}_R \mathbf{m}_D \end{pmatrix} \quad (5.3)$$

On the other hand, the round-trip Mueller matrix is

$$\mathbf{M}_B = \mathbf{R} \mathbf{M}^T \mathbf{M}_E^T \mathbf{R} \mathbf{M}_E \mathbf{M} \quad (5.4)$$

where, $\mathbf{R} = T_R \begin{pmatrix} 1 & \vec{0}^T \\ \vec{0} & \mathbf{r} \end{pmatrix}$. $\mathbf{r} = \text{diag}(1, 1, -1)$ and T_R is the Rayleigh backscattering coefficient.

Using Eq. (5.3) and Eq. (5.4), the round-trip Mueller matrix $\mathbf{M}_B$ is derived to be

$$\mathbf{M}_B = T^2 T_R^2 T_E^2 \begin{pmatrix} 1 + (\mathbf{m}_R \vec{D})^T \mathbf{m}_E^T \mathbf{r} \mathbf{m}_E (\mathbf{m}_R \vec{D}) & \vec{D}^T (\mathbf{I} + \mathbf{m}_R^T \mathbf{m}_E^T \mathbf{r} \mathbf{m}_E \mathbf{m}_R \mathbf{m}_D) \\ \mathbf{r} (\mathbf{I} + \mathbf{m}_D^T \mathbf{m}_R^T \mathbf{m}_E^T \mathbf{r} \mathbf{m}_E \mathbf{m}_R) \vec{D} & \mathbf{r} (\vec{D} \vec{D}^T + \mathbf{m}_D^T \mathbf{m}_R^T \mathbf{m}_E^T \mathbf{r} \mathbf{m}_E \mathbf{m}_R \mathbf{m}_D) \end{pmatrix} \quad (5.5)$$

From another point of view, $\mathbf{M}_B$ can also be decomposed as

$$\mathbf{M}_B = T_B \begin{pmatrix} 1 & \vec{D}_B^T \\ \mathbf{m}_{BR} \vec{D}_B & \mathbf{m}_{BR} \mathbf{m}_{BD} \end{pmatrix} \quad (5.6)$$

where elements with the subscript “B” denote the round-trip parameters with the similar definitions as those in Eq. (5.1). In particular, $\vec{D}_B$ is the round-trip PDL vector.

Comparing Eq. (5.5) and Eq. (5.6), we have

$$T_B = T^2 T_R^2 T_E^2 \left[1 + (\mathbf{m}_R \vec{D})^T \mathbf{m}_E^T \mathbf{r} \mathbf{m}_E (\mathbf{m}_R \vec{D}) \right] \quad (5.7)$$

From Eq. (5.6), the determinant of $\mathbf{M}_B$ is found to be [22]

$$|\mathbf{M}_B| = T_B^4 (1 - D_B^2)^2 \quad (5.8)$$

On the other hand, it can be easily derived from Eq. (5.4) that

$$|\mathbf{M}_B| = T_R^8 T_E^8 |\mathbf{M}|^2 = T_R^8 T_E^8 T^8 (1 - D^2)^4 \quad (5.9)$$

From Eqs. (5.7), (5.8) and (5.9), it can be shown that

$$D_B = \sqrt{1 - \frac{(1 - D^2)^2}{\left[1 + \tilde{\tilde{D}}^T \mathbf{m}_E^T \mathbf{r} \mathbf{m}_E \tilde{\tilde{D}}\right]^2}} \quad (5.10)$$

where, $\tilde{\tilde{D}} = \mathbf{m}_R \tilde{D}$. Since the orthogonal matrix $\mathbf{m}_R$ does not change the modulus of a vector [26], $\tilde{D} = D$. If the Mueller matrix $\mathbf{m}_E$ is written as $\mathbf{m}_E = \begin{pmatrix} m_1 & m_2 & m_3 \\ m_4 & m_5 & m_6 \\ m_7 & m_8 & m_9 \end{pmatrix}$, it can be

shown that [70]

$$\mathbf{m}_E^T \mathbf{r} \mathbf{m}_E = \begin{pmatrix} 1 - 2m_7^2 & -2m_7m_8 & -2m_7m_9 \\ -2m_7m_8 & 1 - 2m_8^2 & -2m_8m_9 \\ -2m_7m_9 & -2m_8m_9 & 1 - 2m_9^2 \end{pmatrix} \quad (5.11)$$

Substituting Eq. (5.11) into Eq. (5.10), we have

$$D_B = \sqrt{1 - \frac{(1 - D^2)^2}{\left[1 + D^2 - 2(\vec{m} \cdot \vec{\tilde{D}})^2\right]^2}} \quad (5.12)$$

where, $\vec{m}(z) = (m_7, m_8, m_9)^T$ is a unit vector and $\vec{m}(0) = (0, 0, 1)^T$. Since $\vec{m}$ is a function of the terminal fiber length z , D_B also varies with respect to z . If $\vec{m}$ is parallel to $\vec{\tilde{D}}$, we have $D_B = 0$; if $\vec{m} \cdot \vec{\tilde{D}} = 0$, the maximum value of D_B will be achieved and D can be calculated from

$$D = \frac{1 - \sqrt{1 - D_{B\max}^2}}{D_{B\max}} \quad (5.13)$$

Equation (5.13) means that the forward global PDL can be obtained by measuring $D_B(z)$ and finding its maximum. Using the notation of PDL expressed in dB, Eq. (5.13) can be rewritten as $PDL = PDL_{B\max} / 2$.

Please note, the vector $\vec{D}$ defined in this thesis is the input PDL vector. The output PDL vector, which is defined in [54] and [68], should be $\vec{\Gamma} = \mathbf{m}_E \mathbf{m}_R \vec{D}$. Then, it is easy to verify that Eq. (5.12) and Eq. (5.13) present the same conclusions as those in [54] in a simpler case: purely birefringent fiber section.

There are three methods for making forward global PDL measurement in optical fiber links [64-66]. They need two [66], three [64] and four [65] specific input polarization states to accomplish the measurement, respectively. All of them can be used to measure the round-trip PDL D_B by using a POTDR setup. However, the two-input and the three-input methods are needed to measure the SOPs of the Rayleigh backscattering signals. Due to the large noise in POTDR measurements, the SOP of Rayleigh backscattering light is difficult to measure accurately. Fortunately, the four-input method only needs to measure the optical powers of the Rayleigh backscattering signals. Therefore, we use the four-input method to measure D_B .

The condition $\vec{m} \cdot \vec{D} = 0$ to achieve $D_{B\max}$ means that $\vec{m}$ should be orthogonal to $\vec{D}$ in Stokes space. On the Poincaré sphere, all unit vectors which are orthogonal to $\vec{D}$ will form a great circle. In a long SMF, $\vec{m}(z)$ has been experimentally verified to span the entire Poincaré sphere [70]. Hence $\vec{m} \cdot \vec{D} = 0$ must be reached at some positions if the terminal fiber section is long enough. In a real measurement, the terminal fiber section is probably not long enough to meet $\vec{m} \cdot \vec{D} = 0$. Further, the exact position satisfying $\vec{m} \cdot \vec{D} = 0$ may be missed due to discrete data acquisition. In such cases, the maximum angle between $\vec{m}$ and $\vec{D}$ is not 90° , but $90^\circ - \theta$. In table 5.1, the relative PDL measurement error $\Delta D / D$ really from using different θ is calculated. Because $\Delta D / D$ is also a function of D , the maximum error $|\Delta D / D|_{\max}$ is presented in table 5.1. Hence, even if the terminal fiber section is not long enough in real measurements, it is still

possible to measure the forward PDL with an acceptable accuracy as long as $\vec{m}$ can be close to $\vec{\tilde{D}}$.

Table 5.1 Relationship between angle θ and maximum PDL measurement error

θ (degree)	5	10	15	20
$ \Delta D / D _{\max}$	0.76%	2.97%	6.46%	10.93%

5.2.2 Experimental verification

An experimental setup is established as shown in Fig. 5.1. The use of a tunable laser source allows us to perform a spectrally-resolved measurement. The pulse generator generates electrical pulses with 10ns pulse width and 1 kHz repetition rate, and then modulates the tunable laser source to emit optical pulses. After amplification by an Erbium doped fiber amplifier (EDFA), optical pulses pass through an acoustic optic modulator (AOM) to enhance the extinction ratio. The rotatable polarizer and the quarter-wave plate are used to generate the four specific input polarization states $(1, 1, 0, 0)^T$, $(1, -1, 0, 0)^T$, $(1, 0, 1, 0)^T$ and $(1, 0, 0, 1)^T$ for the forward and round-trip PDL measurements [65]. The polarization controller is used to ensure that the four inputs have the same equal powers. Rayleigh backscattering signals are detected by an optical receiver with 125 MHz bandwidth.

Fig. 5.1 Experimental setup for the single-end measurement of the global PDL.

The fiber link under test is arranged in a series of a 0.1m-long polarizing fiber, a 2m-

long PMF, a 10km-long standard SMF, another 0.1m-long polarizing fiber and a 50m-long standard SMF. All these fibers are spliced together according to the above sequence. PDL is induced by two short polarizing fibers. The global PDL of this fiber link is firstly measured using the forward approach in the wavelength range from 1550nm to 1560nm with a 0.5nm step. The result is plotted in Fig. 5.2 as a solid line. The circulator is not included in the forward measurement.

In the single-end measurement, optical powers of signals reflected from the terminal 50m-long SMF are detected and recorded with a 1m step. To enhance the SNR, the signals are averaged over 10000 optical pulses. The single-end measurement is also implemented at the same wavelengths as mentioned above. The single-end PDL results are calculated and plotted in Fig. 5.2 using the dashed line. It is evident that the single-end results are still in agreement with the forward results considering OTDR measurements are quite noisy. The discrepancy between two results partially results from the PDL in the circulator, which is around 0.1-0.2 dB.

Fig. 5.2 PDL measurement results using forward and single-end techniques.

5.3 PDL distribution measurement

Obviously, when PDL sections and birefringent sections are interleaved in the fiber link under test, the above technique can be used to measure the PDL distribution. However, the four-input approach for the round-trip PDL measurement is not valid in this case. The SOPs of Rayleigh backscattering signals must be measured.

5.3.1 Theory

An optical fiber link with PDL is illustrated in Fig. 5.3. The black regions denote the sub-fiber links having PDL components. They are connected by the SMFs, which are purely birefringent.

Fig. 5.3 An optical fiber link with discrete PDLs distributed along the fiber length.

As we have mentioned in the last section, in a reflectometric measurement, the measurand is not D but D_B . From Fig. 5.3, if we measure D in the forward direction along the SMF between two PDL regions, for example SMF1, its value will remain unchanged. However, if we measure D_B along the same SMF, its value will vary. In the last section, we have demonstrated that D can be related with D_B by Eq. (5.13). For a distributed measurement, three input SOPs are employed and corresponding SOPs of Rayleigh backscattered signals along the whole fiber link can be measured using POTDR. To obtain the PDL value of the n th-PDL region, the measured SOPs, corresponding to an arbitrary position of SMFn, are treated as the “input” SOPs of this region. The measured SOPs along SMFn+1 are considered as the “outputs”. Then, the round-trip Mueller matrix of the n th-PDL region can be calculated using the three-input PDL measurement method [15, 64]. From the measured Mueller matrix, D_B^n of

the n th-PDL region can be extracted [22] and D_{Bmax}^n can be found. Finally, using Eq. (5.13), D^n can be found.

5.3.2 Simulation

As preliminary work, we perform some simulations to verify the proposed technique. The whole fiber link is composed of 1000 randomly oriented linear birefringent fiber sections with the same length of 10m. Four hundred PDL components with small PDL values are interleaved in fiber sections randomly. Firstly, by using the three-input forward measurement technique [15, 64], the error-free PDL values in frequency range from 1550 to 1560 nm are calculated as the benchmark for two PDL regions of the fiber link. Secondly, the proposed distributed measurement technique is implemented. Since POTDR signals are noisy, Gaussian noises with a standard variance of 1% are added to the four components of the “measured” SOPs of the backscattered signals. Then, the PDL values using the reflectometric measurement approach can be obtained using the algorithm described in Section 5.3.1. In Fig. 5.4, solid lines denote the benchmarks “measured” using the forward measurement approach and dashed lines show the reflectometric “measurement” results using the proposed technique. An agreement between two lines is evident.

Fig. 5.4 Simulation results (a) PDL values of the fiber link from the input end to 3 km; (b) PDL values of the fiber link from 6 km to 9 km.

Chapter 6 CONCLUSION AND FUTURE WORK

6.1 Conclusion

We have investigated some fundamental issues on making reflectometric measurements of polarization properties of SMF links using POTDR. The main achievements include the optimization of the POTDR system and novel reflectometric measurement techniques of PMD and PDL using POTDR.

1. For the measurements of the Mueller matrix and other polarization effect matrices in a birefringent optical system, such as a SMF link, we demonstrated by theoretical analysis and simulations that two input SOPs which are mutually orthogonal in Stokes space statistically have the maximum probability to achieve the optimum measurement accuracy; but in a single test, evident deviation of two optimum input SOPs from perpendicularity can be observed for achieving the minimum measurement error. For measurements of both the polarization effect matrices and the polarization effect vectors, we find that all measurement errors have a bathtub-shaped relationship with the angle between two input SOPs in the range from 0° to 180° . This means a limited offset from perpendicularity will only slightly increase the measurement error in a single test.
2. We presented the theoretical and simulation results of the relationship between three input SOPs and the M^3E in an optical system having birefringence and PDL/G. By using the CN as the criterion, it can be theoretically demonstrated that the three input SOPs should be equally-spaced on the Poincaré sphere and centered on the reversed PDL/G vector to achieve better measurement

accuracy in a single test. Further, an upper bound of the mean of the M^3E is derived when the measurement errors of the output Stokes parameters independently and identically follow the same Gaussian distribution. This upper bound also shows that the statistically best Mueller matrix measurement accuracy can be obtained when the three input SOPs have the same relationship mentioned above. Simulation results confirm the validity of the theoretical findings.

3. A polarimeter commonly used in POTDR consists of one quarter-wave plate in conjunction with one linear polarizer and one photodetector. When thermal noise dominates in the photodetector and the angular orientation errors are small enough, the optimum angles of the waveplate and the polarizer can be theoretically found in POTDR measurements. Using these optimum angles, the minimum measurement noises of Stokes parameters are deduced. When both waveplate and polarizer are free to rotate, the optimized polarimeter may have very simple statistical properties of measurement errors. For such a polarimeter, the measurement noises of DOP and the normalized Stokes vector in POTDR measurements can be derived. It is evident that the measurement noise of DOP is far smaller than that of normalized Stokes vector when the DOP of backscattered optical signal is close to unity.
4. A reflectometric technique was proposed to perform the spectrally-resolved measurement of PMD in optical fibers having no PDL. This technique is based on the PMD dynamical equation and realized by measuring the SOP evolutions of the reflected signal in both frequency and fiber length domains. Two experimental setups, employing the far-end Fresnel reflection, are constructed to verify this technique. Based on this technique, we proposed the approach for spatially- and spectrally-resolved PMD measurement in optical

fibers. It can provide DGD values in a wide optical wavelength range, not only the average value. This technique can be used to detect the “bad” fiber sections in a fiber link and potentially employed in distributed fiber optic sensing systems.

5. In optical fiber links having PDL, we demonstrated that, although the complex PMD vector cannot be fully obtained by the reflectometric measurement, the spectrally-resolved DGD and DAS can be explicitly determined by such measurements performed simultaneously in the frequency domain and the fiber length domain. In principle, this technique can be used to realize spectrally- and spatially-resolved measurements of DGD and DAS in an optical fiber link having PDL based on distributed Rayleigh backscattering.
6. A single-end technique was proposed to measure the global PDL in an optical fiber link. In this technique, the terminal fiber section of the fiber link under test is required to be purely birefringent. Then, the global PDL will be determined by the maximum of the round-trip PDL. In practice, the round-trip PDL can be obtained by measuring Rayleigh backscattering optical powers corresponding to four specific input SOPs. This technique can also be extended to measure the PDL distribution in an SMF link where SMF sections and PDL sections are interleaved.

6.2 Future Work

1. Experimental verification of the theoretical findings in section 3.4 will be carried out. The optimum input SOPs for Mueller matrix measurement using the optimized polarimeter presented in section 3.4 will also be investigated.
2. SNR enhancement will be further investigated in two kinds of POTDR systems: completely coherent and completely incoherent systems.

3. In completely incoherent POTDR systems, the SMF link under test will be a depolarizing optical system. Then, the signal interpretations will be more complicated than those in the reported works. Depolarization effects will be the research focus in such a system.
4. In completely coherent POTDR systems, depolarization will never happen. However, the other two fundamental polarization effects, birefringence and PDL, seems to be impossible to be obtained in POTDR measurements because of the coherence problems of the Rayleigh backscattering signals. So, such a system is not capable for PMD and PDL measurements. But, it may be used as a fiber-optic sensing system by establishing the connection between sensing parameters and redefined polarization parameters.

PUBLICATIONS

Journal

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