Optimal reset control and disturbance compensation for advanced hard disk drives

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Optimal Reset Control and Disturbance Compensation for Advanced Hard Disk Drives

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A thesis submitted to the Nanyang Technological University

in fulfilment of the requirements for the degree of

Doctor of Philosophy

2010

Statement of Originality

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

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Date

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Li Hui

*To my family,
for their encouragement and love.*

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Summary

The demand for better servo control technology for hard disk drives (HDDs) never ceases. One important element for HDDs servo control is fast read/write speed which demands the fast and swift motion of HDD servo system. The other important requirement for HDDs is better precision for future drives with ultrahigh densities.

In order to achieve fast and high accurate performance, a framework of optimal reset control is proposed for HDD. In such theory framework, LQR optimal reset control and H_2 optimal reset control are included. LQR optimal reset control can improve transient response performance; while H_2 reset control can achieve better control accuracy and disturbance attenuation performance. A unified reset control is design to combine the merits of the two control and achieve a unified performance via a soft switching mechanism.

The dual stage hard disk can achieve higher bandwidth than the single one. The LQR optimal reset control is also designed for the dual stage HDD servo systems to achieve better transient performance.

In order to attenuate different disturbances, two add-on disturbance rejection methods are proposed. One "add-on" disturbance method which can cancel most of the components in periodic disturbance has been proposed. This method has rebuilt a signal which can approximately represent the disturbances thus cancel the disturbances by adding the opposite value of this signal. The other "add-on" disturbance compensation method is a general form of disturbance observers. This method does not need to solve the plant model inverse, and uses H_∞ control method to design the Q-filter in the disturbance observer. After using these two methods, the disturbances can be compensated well in each case.

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Nomenclature

The following abbreviations are frequently used throughout this thesis:

2-DOF	Two-degree-of-freedom
A/D	Analog to Digital
BPI	Bits Per Inch
BPSI	Bits Per square Inch
D/A	Digital to Analog
DAC	Digital-to-analog converter
DISO	Dual input single output
DOB	Disturbance Observer
DSA	Dual Stage Actuator
DSP	Digital Signal Processor
HDD	Hard Disk Drive
IVC	Initial Value Compensation
KYP	Kalman-Yakubovich-Popov
LDV	Laser Doppler Vibrometer
LMI	Linear Matrix Inequality
LQG	Linear Quadratic Gaussian
LQR	Linear Quadratic Regulation
LSM	Least-Squares method
LTR	Loop Transfer Recovery
NRRO	Non Repeatable Runout
ORL	Optimal Reset Law
PES	Position Error Signal
PID	Proportional Integral Derivative
PTC	Perfect Tracking Control

PTOS	Proximate Time Optimal Servomechanism
PZT	Lead-zirconate-titanate
RPM	Revolution per minute
RPT	Robust and Perfect Tracking
RRO	Repeatable Runout
R/W	Read/Write
SNR	Signal Noise Ratio
STW	Servo Track Writer
TMR	Track Misregistration
TPI	Track Per Inch
UCS	Unified Control Scheme
VCM	Voice Coil Motor
ZPETC	Zero Phase Error Tracking Control

Chapter 1

Introduction

1.1 Background

A hard disk drive or HDD is a device that uses one or more rotating magnetic disks ("magnetic media") to store and allow fast access to data [1]. Hard disk drives provide non-volatile data storage, which means that the data remains present when power is no longer applied to the device. The most common hard drives prevailing today are 3.5-inch and 2.5-inch form factor drives, having capacities ranging from 40 gigabytes ("GB") to 2 terabytes ("TB"), nominal rotation speeds up to 10,000 revolutions per minute ("RPM"), and offer interfaces including both Enhanced Integrated Drive Electronics ("EIDE") and Serial Advanced Technology Attachment ("SATA"). HDDs play an important role in the modern era of digital technology. They are found in desktop and notebook computers, corporate networks, and home entertainment systems, as well as sophisticated medical, military/aerospace, and telecommunications systems. They are also used as personal storage for backup and portable application.

It is reported that the world's total yearly production of print, film, optical,

and magnetic content would require roughly 1.5 billion GB of storage, which is the equivalent of 250 MB per person on earth [2]. The digital universe in 2007 is at 2.25×10^{1021} bits (281 exabytes or 281 billion gigabytes). The resizing comes as a result of faster growth in cameras, digital TV shipments, and better understanding of information replication. By 2011, the digital universe will be 10 times the size it was in 2006 [3]. Such large demanding needs hard disk drive to increase capacity. As for a significant reduction in data access time, track seeking times achieved using bounded control effort have to be shorten.

To assist HDDs to maintain a significant market share for desktop and home multimedia storage applications in future, new control strategies will need to be formulated to achieve higher track following accuracy as well as faster and smoother track seeking motion. This dissertation is concerned with the development of new, effective and practical control algorithms for fast and high storage density future HDDs.

1.2 Hard Disk Drive Servo System Overview

1.2.1 Hard Disk Drive Trends

The HDD industry has a relatively short but fascinating history. The fascinating journey of this marvelous device began with a huge, monstrous equipment called RAMAC (Random Access Method of Accounting and Control), which used the very first non-volatile direct access storage device introduced by IBM in 1956 [4]. The first hard disk drive (RAMAC) supported an areal density of 2000 *bits/in*² only. Today the density has escalated to a high 400 *Gbits/in*², which is an increase by a factor of 20 million. Table 1.1 shows several historic hard disk drive storage capacities, densities and prices. We can see that the area densities increase exponentially

accompany with prices decrease exponentially. This phenomenal increase in areal density have been achieved by a coordinated efforts in improving all aspects of the hard drive. Head technology has been continuously improved to be able to read and write smaller bits. Improved technologies have enabled new generation media to reliably hold magnetic domains of smaller size. Track density has been increased by reducing component disturbances that move the head off track and by improving the servo controllers ability to regulate the head on narrower tracks. Smoother disk surface, better quality of lubricant and better air bearing technologies allow the slider to fly in closer proximity of the disk so that the bit size is reduced. Typical fly heights in 1997 were 25 nanometers (nm). Today they are about 3 nm. The read-write electronics and data encoding schemes have played their part in improving bit density by enabling detection of information reliably from ever smaller data signals contaminated by the surrounding noise. Excellence in design and production maintained the steady growth in the areal density which was accelerated time to time by availability of new technologies. Introduction of new technologies has always made an impact on the strive for improving areal density. Cumulative average growth rate of the areal density was 25% in the seventies and eighties. Drives in those days used inductive heads for both reading and writing. MR heads, introduced in early 1990s, boosted the growth rate to 60%. The giant MR heads (GMR) appeared in the drives in later half of 1990s, and the industry experienced a growth rate of more than 100%.

Another significant change that the HDD industry experienced in the past is the trend in the access time and data transfer rate. For applications that require faster data rates, speeding up the disk rotational speed has reduced the latency component of access time and increased the speed of data flow from the heads. There has been a steady progression over the years from 4,500, 5,400, 7,200, 10,000, to now 15,000 RPM hard drives. This has posed challenges to keeping the windage off-track disturbances to an acceptable level. Fluid dynamic bearings have replaced

Table 1.1: Hard Disk Drive Storage Capacity and Price : Some Historic Examples [5, 6]

Year	Manufacturer	Capacity	Form factor	Area Density	Price Per MB
1956	IBM	12MB	14	$2KB/in^2$	\$6322
1982	Lobo	19MB	$5\frac{1}{4}$	$6MB/in^2$	\$289
1988	Seagate	30MB	$5\frac{1}{4}$	$10MB/in^2$	\$9.97
1994	Quantum	1800MB	$3\frac{1}{2}$	$306MB/in^2$	\$0.536
2000	Maxtor	80GB	$3\frac{1}{2}$	$13.6GB/in^2$	\$0.536
2006	Western Digital	400GB	$3\frac{1}{2}$	$68.3GB/in^2$	\$0.000265

ball bearing spindles to reduce runout (off-track movement of the head or disk) at high RPM. The higher data rates coming into the head due to the higher rotational speed and bit density have introduced challenges in drive electronics to be able to reliably process the data.

On the disk surfaces, data are recorded in tightly-packed concentric rings called tracks. An increase of HDD storage capacity can be achieved by the ability to record more bits of information onto a single disk, i.e. to increase the areal density of the disks. Areal density is measured in bits per square inch (BPSI), which is given by

$$BPSI = BPI \times TPI \quad (1.1)$$

Thus, higher BPSI can be achieved by either increasing BPI or TPI, or even both.

Figure 1.1 shows the HDD capacity increasing trade. While new recording media and technologies are being consistently developed to achieve ever higher Bits per inch (BPI), the BPI is anticipated to reach the upper limit soon, which is the result of thermal instability of the recording media. Thus, to avoid the thermal degradation in magnetic recording, higher storage density can be achieved by pushing for higher track density, measured by track per inch (TPI). And along with

Figure 1.1: Hard disk capacity increasing trade

higher TPI comes the need for better servo control for better accuracy to reduce track misregistration (TMR) during track following.

The HDDs never cease to strive for smaller form factors, faster data rate, denser areal density and cheaper price. The rapid evolution of HDDs in performance and cost requires many cross-discipline technological innovations from the recording media, magnetic heads, disk technology, head-disk interface, spindle motor, signal processing, electronic circuit, mechanics and servo technologies.

1.2.2 Structure of Hard Disk Drives

The HDD is a typical servomechanism with the specific task of controlling the position of the read/write heads. Figure 1.2 shows the components in a typical HDD. It consists of a stack of disks, sputtered with a thin magnetic layer or recording medium, which are written with data in concentric circles called tracks. The disks

Figure 1.2: Components of a HDD with top cover removed [7].

are stacked along a spindle and rotated by an electric motor inside the spindle, called the spindle motor. Electromagnetic read/write heads are used to either write data onto the disks or read information from them. The head is attached to a slider, an aerodynamically shaped block that allows the head to maintain a consistent flying height of only several nanometers on an air bearing above the disk surface. The slider is connected to a suspension which is swaged onto the arm, or E-block, whose name comes from the E shape in the side view. The E-block is attached to a fixed pivot bearing and is driven by a voice coil motor (VCM) that swings between two permanent magnets. The VCM is a rotary actuator, which moves the head over the disk surface from the inner to the outer diameter by rotating the E-block about the pivot. The flexible printed circuit (FPC) attached to the side of the E-block conveys all electrical signals between the E-block and the stationary electronics block fixed on the baseplate.

In this thesis, the HDD servo system refers to the VCM actuator servo system, or the head positioning servomechanism. The traditional disk drive system is a non-collocated control system where the measured output occurs at the head which is not directly actuated, but the control effort is applied to the VCM. Hence, the mechanical resonance modes of the arm and suspension which are located between the VCM and the head would cause vibrations that affect the head position. The resonance modes in the total system must be taken into account during the control design.

The head position is not measured directly in HDDs. Instead, the servo controller must work on the position error signal (PES), which is the difference between the desired track center and the actual head position. A schematic drawing of the HDD servo is shown in Figure 1.3. The PES is demodulated from the servo sectors by a data channel. At the time of manufacture, the HDD is placed in a servo track writer (STW) that embeds the PES bursts and track numbers directly on the disk at regular intervals between user data. The PES can be demodulated from the read back signal of the servo sectors as discrete-time signals. Thus, the HDD servomechanism is a sampled-data system in nature. The sampling rate of the PES can be computed by

$$f_s = N_{sector} \frac{RPM}{60}, \quad (1.2)$$

where f_s is the sampling rate in Hertz, N_{sector} is the number of the servo sectors along one track, and RPM is the spindle speed in revolution per minute. The number of the sectors should be minimized so that a larger area can be used for recording data. Thus, it is a challenge to ensure acceptable servo control performance with low sampling rates.

The servo controller is programmed into a chip located on the printed circuit board. The servo controller is designed to minimize the PES by generating appropriate voltage command. The voltage demand is then fed through the power

Figure 1.3: Embedded servo scheme.

amplifier via a digital-to-analog converter (DAC). The output current of the power amplifier passes the coils of the VCM and produces torque to drive the head into proper position. The controller operates in a track-following mode when reading data on a track and in a track-seeking mode when moving to another track.

1.2.3 HDD Head Positioning Mechatronics

The generalized diagram of a hard disk drive servo control loop is illustrated in Figure 1.4. The system includes the actuator and driver, PES demodulator, timing generation and position control subsystems. As the disk is rotating in an embedded servo system, the recording head, which is capable of either writing data or reading data while following the track, will cross servo information at regular intervals. These intervals are referred to as system sampling periods. Each time a servo wedge is crossed; specially encoded servo data is read, amplified and output to the PES demodulator that converts the encoded servo data to a digital position error

Figure 1.4: Generalized diagram of a HDD servo control loop [6].

signal. The PES is then fed into the controller, which is loaded with several control algorithms. Subsequently, the output of the servo controller is converted and held to an equivalent analog value that is smoothed and converted to a current by the voice coil motor (VCM) driver in order to move the actuator in the direction that reduces the PES.

For control algorithm, it contains three stages or aspects, which called track seeking, track settling and track following.

- Track seeking moves the R/W head from the present track to a specified destination track fast and smoothly using a bounded control effort.
- Track following maintains the head as close as possible to the destination track center while information is being read from or written to the disk. Track following is always considered as disturbance rejection.
- One additional stage represents smooth transition from the non-linear mode

(seek mode) to the linear mode (track-following mode) during a single servo action. This stage always is always called track settling.

Figure 1.5 demonstrates servo modes in an HDD. For each stage, a lot of approaches have been proposed to improve the servo performance. A summary of the servo technologies for HDDs can be found in [5, 6, 8, 9]. Reviews of the servo algorithms for each servo mode are given as follows.

Figure 1.5: Servo modes in an HDD

1.2.4 Track-Seeking Control

The objective of track-seeking control is to position the read/write head to the designation track quickly and smoothly using bounded control effort. Fast seeking would reduce the time in preparation for data reading/writing; and smooth seeking such that the ideal angular velocity profile with minimum acoustic noise will be guaranteed to make HDD work quietly in consumer electronic appliances.

1.2.4.1 Proximate Time Optimal Servomechanism

The time optimal control or bang-bang control is a well known solution for such point-to-point maneuver with limited control authority. Practical realization of the bang-bang control has several drawbacks such as control chatter and sensitivity to parameter variations. In improvement, the proximate time optimal servomechanism (PTOS) [6, 10] has been a popular and successful method. In the acceleration mode, the head is driven by maximum force until the velocity error becomes so large that saturation occurs. The servo loop is opened, and once the error has become small, the inner velocity feedback loop can follow the reference velocity trajectory that is based on the time optimal curve of the inertia model. Once the distance to the target track is under a certain threshold, the actuator is decelerated and prepares for switching to track-following. For long distance track-seeking, the PTOS is an excellent servo design because the saturation problem and jerk minimization have been solved.

1.2.4.2 2-Degree of Freedom

In the case that the power amplifier is not saturated during track-seeking, usually in short distance track-seeking, the two-degree-of-freedom (2DOF) control has already been a very popular servo method [11–13]. The two-degree-of-freedom (2-DOF) control is to design a stable feedforward compensator that produces an inverse dynamic of the closed loop system to shorten the transient response of a stable feedback servo loop. The key problem in 2-DOF is how to produce a precise inverse model of the closed loop system. Hence, the zero phase tracking control (ZPETC) [14] has been used to design a single-rate feedforward digital compensator.

1.2.4.3 Robust and Perfect Tracking

Another method for short-distance track seeking, a robust and perfect tracking (RPT) [15, 16] controller was proposed. Such method can track a given signal with arbitrarily fast settling time in the face of external disturbances and initial conditions. Since the ZPETC is basically a design method for a single-rate system, the perfect tracking control (PTC) was proposed to improve the transient response using multi-rate control method [17–19].

Chapter 3 in this thesis will discuss HDDs working in seeking mode. It presents a non-linear control method which can improve transient response.

1.2.5 Track-Settling Control

Once the R/W heads have reached the target track, the HDD servo system will change its mode of operation to track following. A fast and smooth operation mode transition will guarantee minimum time delay for fast data access. However, during mode switching, the states variables of VCM actuator are usually at some non-zero positions. Given the fact that most VCM have very bad and unwanted dynamics, the non-zero states of the plant will ensure the mode switching process to be slow and jerky.

1.2.5.1 Initial Value Compensation

In 1996, a method called initial value compensation (IVC) [20] is developed to compensate the undesirable effects of the non-zero plant state variables during mode switching. Initially, the design for IVC control methods aims to cancel the undesirable dynamics of responses owing to non-zero plant state variables upon switching. In its later development, optimal control theories [21–23] have been applied to en-

hance the performance of IVC.

1.2.5.2 Sliding Mode Control

Sliding mode control is another way for such mode switching [24, 25]. Based on time-varying sliding surfaces, the proposed nonlinear control approach eliminates the switching mechanism and thus realizes the smooth mode transit. A time optimal-based discrete-time sliding mode control (DSMC) scheme with application to hard disk drives (HDDs) is developed in [26].

1.2.5.3 Composite Nonlinear Feedback Control

Recently, a new nonlinear control technique [27–35], the composite nonlinear feedback control, had been studied and applied in HDD servo tracking, for a class of linear systems with actuator nonlinearities. The simulation and implementation results show that the new technique has outperformed the conventional method by more than 30%. Furthermore, it has also been shown by an example that the CNF control is capable of beating the well known time-optimal control (or bang-bang control) [36] in asymptotic tracking.

Chapter 5 in this thesis will discuss HDDs working in following mode. It presents a method which can make a soft switch between two non-linear control methods.

1.2.6 Track-Following Control

The aim of track-following control is to maintain the read/write head stationary over the center of the data track under variations of environment around the HDD. During track-following, various disturbances and noises could cause the head to

deviate off the target track. Thus, the controller must move the actuator swiftly to reduce the effects of disturbances and noises and keep the head on the track within the allowable error tolerance. The track-following performance is evaluated by the term of track misregistration (TMR), which is the offset between the actual head position and the track centre. The TMR is normally reported as a variance parameter per sample of the disk servo, defined as the following:

$$TMR = 3\sigma = 3\sqrt{\int_0^\infty H(\omega)d\omega} \quad (1.3)$$

where ω is the radian frequency; $H(\omega)$ is the power spectrum of PES; σ is the standard deviation of PES. The TMR must be less than 10% of the track pitch for acceptable tracking performance, thus, the objective of track-following is to ensure that the value of 3σ to be as small as possible. There are generally two types of disturbances that contribute to TMR; the internal and external disturbances. The internal disturbances are largely due to the rotation of the disks on the spindle and the reaction forces on the drive baseplate and housing. These disturbances cause the synchronous position error (periodic) and non-synchronous position error (random). The synchronous position error is due to repeatable runout (RRO), which is due to the data track not being perfectly round in servo writing or perfectly concentric with the axis of rotation of the disk. The non-synchronous position error is largely due to the non-repeatable runout (NRRO) from PES generation noise, disk vibration, windage, and reaction forces from the actuator [37]. The external disturbances are from environmental shock and vibration such as factory floor environment, and sudden motion of shock.

Various feedback control strategies have been developed to reduce the effects of disturbances and noises, so that TMR may be lowered on a feedback system, the sensitivity function from the disturbances to the output indicates the capability of disturbance rejection. If the largest singular value of the sensitivity function is less

than unity over a frequency range, the effect of disturbance at the plant output are effectively desensitized over that frequency range.

1.2.6.1 Classical Control

Classical design approaches [38] have been used for many years. The common ones are proportional-integral-derivative (PID) control, lead-lag compensator and notch filters, which are used to stabilize the system and increase the servo bandwidth.

1.2.6.2 Notch Filters

Increasing servo bandwidth is crucial to improving positioning accuracy. There are two major obstacles to increase the servo bandwidth, which are the sampling frequency and mechanical resonance. Notch filtering is a popular gain stabilization method that has been widely used to compensate for mechanical resonances. Notch filters ensure a large stability margin according to small gain theorem but the drawback is the phase shift at around the crossover frequency and this limits the servo bandwidth [39].

1.2.6.3 Model-based Control

A typical model-based control design is the disturbance observer (DOB), which uses the PES and a nominal plant model to create an estimate of the disturbance. This estimate is then used to compensate for the disturbance effects. The effectiveness of the DOB in rejecting shock and vibration disturbance below 100 Hz was demonstrated in [40]. The pivot friction compensation using an accelerometer and a DOB is proposed in [41]. The repetitive control [42] has been applied to the track-following mode for RRO compensation. The internal-model-based repetitive controller with a loop shaping filter is further developed for RRO and NRRO compensation [43].

Chapter 8 in this thesis proposes a simple disturbance observer for disturbance compensation, Chapter 7 also takes such structure as reference, and design a disturbance rebuilder to compensate periodic external disturbance.

1.2.6.4 Multi-rate Control

Although the sampling period of the position signal is restricted, the sampling period of the control input is only limited by the execution speed of the microprocessor. As a result, the sampling period of the control input is much shorter than that of the position signal. A multirate feedback using an observer that outputs an estimated position signal two to four times during a sampling period can reduce phase shift and increase servo bandwidth [44]. [23,45] proposes a novel method for short seeking control based on multirate track following control and initial value adjustment. [46–48]discusse the intersample ripple problem in multirate control design. [49] considers multirate control from the viewpoint of computation saving.

1.2.6.5 Advanced Optimal Control

Advanced control methods such as Linear Quadratic Gaussian (LQG), Loop Transfer Recovery (LTR) [50, 51], H_2/H_∞ [9, 52–55] and Disturbance Decoupling / Almost Disturbance Decoupling have been used to optimize the servo loop with robust considerations. These modern approaches optimize the loop shaping design. LQG design optimizes the linear quadratic criterion while LTR method generates the estimator to make the output feedback loop to be the same with the full state feedback system. H_2/H_∞ controllers are derived based on the minimization of the 2-norm and ∞ -norm of certain system transfer function. H_∞ robust design deals with systems having uncertainty in the model definition and when taken with the structured singular value analysis method, the technique provides a design procedure with mathematically proven results. However, designs are more sensitivity than

LQG systems regarding the choice of cost weighting, more complicated and abstract, and may result in poor robustness unless care is taken in the selection of the cost weighting. Chapter 4 and Chapter 8 in this thesis will utilize H_2/H_∞ optimal methods as tools to solve problems of each.

1.2.6.6 Nonlinear Controller

Currently, prevailing HDD servos are linear. Previous studies showed that there are various limitations and tradeoffs associated with linear control systems [56, 57]. Typically, Bode gainphase relation, Bode's integral theorem [58] and time-domain constraint. In recent years, some new nonlinear control algorithms [59, 60] had been introduced and developed that aims to improve HDD servo performance by means of breaking the performance limitations in linear control systems.

1.2.6.7 Adaptive Feedback Control

In the HDD servo, it is not easy to realize robust performance under external vibration uncertainties using fixed controllers, especially where high performance is required. Adaptive feedback control [61–66] was designed to compensate selective harmonics and NRRO, as well as to reject narrow band vibration due to windage and disk fluttering.

1.2.6.8 Artificial Intelligence

Several artificial intelligence methods are also attempted for improving HDD performance. For instance, An adaptive NN demand tracking controller [67, 68] has been applied to a VCM-actuator incorporating a solution for high-frequency resonance cancellation. A new multiobjective genetic algorithm (GA) [69] was used for tuning servo controller parameters such that the resultant performance can satisfy

all the desired requirements. [70] uses two adaptive neural networks, each of which is tailored for a specific task, to compensate disturbance and frictions in HDD servo system.

1.2.7 Dual-Stage Actuator HDD

Conventional voice coil motor (VCM) only single-stage servo is hard to provide high servo performance due to its mechanical resonance modes, various disturbances and noise in HDDs. Dual-stage actuators (DSA) are introduced to overcome the limitation [66, 71, 72]. By combining the DSA system with properly designed controllers, the two actuators are complementary to each other and the defects of one actuator can be compensated by the merits of the other one. Therefore, the DSA system can provide both fast track seeking and high positioning accuracy. There are three categories of dual-stage actuator servos: a suspension-driven type [73], a slider-driven type [74] and a head element-driven type [75]. The piezoelectric (PZT), electrostatic and electromagnetic actuators have been used as the second stage actuator. The type of actuator is selected according to cost and reliability. Figure 1.6 shows a typical secondary piezoelectric actuator.

Plenty of work has been paid to the control design of the dual-stage servo systems [77–88]. In most cases, PZT microactuator output and VCM actuator output are not measured separately. Only the total output of the two actuators, i.e., the sum of PZT actuator and VCM actuator outputs, is available for the servo controller. A one-track seeking controller was designed in [89]. A decoupled master-slave structure has been proposed, and a three-step design method has also been presented. [90] proposes a compensator-based strategy for design of a track-seeking and track-following control system for a dual-stage servo actuator in hard disk drives. The proposed control system improves performance of both long-span seeking and short-span seeking and also achieves high-bandwidth track following performance.

Figure 1.6: A typical secondary piezoelectric actuator [76].

To further reduce the settling time, [91] design the VCM actuator controller to yield a closed-loop system with a small damping ratio for a fast rise time and certain allowable overshoot. Then, a composite nonlinear control law is designed for the PZT actuator to reduce the overshoot caused by the VCM actuator as the system output approaches the target track.

Chapter 6 will discuss the issue of this kind HDD. It presents a control method which can improve transient response for such dual input single output plant.

1.3 Contributions of the Thesis

As mentioned earlier in the introduction, new and effective servo control algorithms need to be developed to help HDD to maintain its leadership in market share of data storage for desktop and multimedia applications. This research will do just that. In order to avoid the limitation of linear controllers, a theoretical system of optimal reset control is developed. The theoretical system includes discrete-time

linear quadratic regulation (LQR) optimal reset control, discrete H_2 optimal reset control, and unified reset control. The plants of such issue include single stage HDD system and dual stage HDD system. The LQR optimal reset control will provide new solutions to improve transient response performance. The H_2 optimal reset control will provide new loop shaping solutions to improve the disturbance attenuation. A novel nonlinear time-varying control algorithm is developed, which will be used to design a unified control scheme that combines the previous two control algorithm and achieves fast track seeking, swift track settling as well as accurate track following performance. The thesis also has come up with two new add-on methods for improving servo performance in advance. These new add-on servo solutions are capable of enhancing any existing HDD servo system while preserving the servo stability margins of the main feedback loop. The contributions made by this thesis are summarized below:

- A discrete-time linear quadratic regulation (LQR) reset control algorithm to achieve better transient response performance. This method can overcome the limitation which the linear control have. The design method can guarantee the system stability and the solution of the controller is obtained by solving Riccati equations. The proposed reset controller design method is suitable to apply to short-span and long-span track seeking of a hard disk drive servo system. The proposed design is much more capable of improving transient response than traditional control design methods.
- A discrete time H_2 reset control algorithm to achieve better disturbance attenuation. With a base linear system designed conventionally, the discrete-time reset law design aims at improving disturbance suppression of the system. The design method can be converted to a standard H_2 optimal control problem and the solution of the controller is obtained by solving Riccati equations. The proposed reset controller design method is applied to track following stage of

a hard disk drive servo system.

- A unified reset control design method to achieve both track-seeking and track-following performance in hard disk drive servo systems. It involves previous two reset methods. The unified control makes a soft switch from LQR reset controller to H_2 reset controller, which avoids sharp vibrations. The unified reset controller inherits the merits of both discrete LQR reset controller and discrete H_2 reset controller, and achieves both swift seeking and accurate following performance.
- The LQR reset method also used in dual-stage HDD servo system. The dual-stage system consists of a voice coil motor (VCM) as a first stage actuator and a push-pull-type piezo-electric transducer Lead-zirconate-titanate (PZT) as a second-stage actuator. It is considered as dual-input single-output (DISO) problem. We designed two optimal reset controllers for VCM and PZT, and combined the two components with a decoupled master-slave structure. With a base linear system designed conventionally, the discrete-time reset law design aims at improving the system transient response. The design method can guarantee the system stability and the solution of the controller is also obtained by solving Riccati equations. The proposed method is able to achieve short settling time and low overshoot.
- An “add on” disturbance method which can cancel most of the components in it has been proposed. This method reconstructs a signal which can approximately represent the disturbances thus cancel the disturbances by adding the opposite value of this signal. When rebuilding this signal, internal model principle and magnitude/phase adaptive estimation algorithm is used to recreate the disturbance signal. Moreover, a polynomial adaptive fitting method which does not need frequency estimation is proposed to deal with a time delay problem which exists in the signal recreation stage. It turns out that

the adaptive fitting method is much more effective to deal with multiple frequency disturbances. The cancellation is effective for single frequency and multiple frequency disturbances even when the disturbance is time-varying in both frequency and amplitude/phase.

- A general form of disturbance observers, which does not need to solve the plant model inverse, and uses H_∞ control method to design the Q-filter in the disturbance observer. The conventional disturbance observer is proved to be a special case of the generalized form. Comparison to the conventional form of disturbance observers is made and the significance of the proposed method in DOB design and disturbance attenuation is verified via simulation and experimental results.

1.4 Organization of the Thesis

The organization of this thesis is as follows:

This chapter has given the background of the HDD technology and HDD servo system. Previous research of the HDD servo technologies has been reviewed and some alternative HDD structures for specific high performance are introduced. The research motivation and objectives have been discussed as well.

Chapter 2 presents system modeling and application. Both least squares method and manually zero-pole assignment are attempted. Notch filter are applied to simplify the system to a second order system. Both single stage system and dual stage system modeling are included in this chapter. Noises and disturbances modeling are also included here. A detailed description of our experimental platform will be provide in this chapter as well. This chapter is a preparation for the following chapters.

Chapters 3-6 will structure a theoretical framework of optimal reset control and use them to solve the issues in HDD drive, improve the preference for different cases.

Chapter 3 proposes a design method of discrete-time reset control systems with the reset time instants pre-specified. With a base linear system designed conventionally, the discrete-time reset law design aims at improving the system transient responses. The design method can guarantee the system stability and the solution of the controller is obtained by solving Riccati equations. The proposed reset controller design method is applied to short-span and long-span track seeking of a hard disk drive servo system.

In Chapter 4, in order to suppress the internal and external disturbance, we have studied discrete H_2 optimal reset law design of reset control systems. Firstly, we transferred closed-loop reset control system to discrete linear system with reset value as control input. Then we convert the optimal reset problem to a standard H_2 optimal control problem. Based on this, the optimal reset laws were given in terms of Riccati equations.

Since the discrete optimal reset control which is presented in Chapter 3 for track-seeking and the discrete H_2 reset control which is presented in Chapter 4 for track-following have their own merits and weakness. The unified reset control law has been studied in Chapter 5. A soft switch is designed to apply the discrete LQR reset controller for track-seeking and the discrete H_2 reset controller for track-following together in order to achieve better performance for each stage.

Chapter 6 presents an optimal reset control (ORC) method for a dual-stage actuator servo system in hard disk drives (HDDs). The dual-stage actuators are introduced to overcome the limitation. By combining the dual-stage actuator system with properly designed controllers, the two actuators are complementary to each other and the defects of one actuator can be compensated by the merits of the other

one. Therefore, the dual-stage actuator system can provide both fast track seeking and high positioning accuracy. We design an optimal reset control for such dual input single output system.

Chapters 7 and 8 present two different “add on” disturbance rejection methods. Chapter 7 shows a method which can cancel most of the components in it. In this method, we suppose to rebuild a signal which can approximately represent the disturbances thus cancel the disturbances by adding the opposite value of this signal. When rebuilding this signal, internal model principle and magnitude/phase adaptive estimation algorithm is used to recreate the disturbance signal. A polynomial adaptive fitting method which does not need frequency estimation is proposed to deal with time delay problem which exists in the signal recreation stage. It turns out that the adaptive fitting method is much more effective to deal with multiple frequency disturbances. Simulation results show the cancellation effectiveness for single frequency and multiple frequency disturbances even when the disturbance is time-varying in both frequency and amplitude/phase.

Chapter 8 proposes a general form of disturbance observers, which does not need to solve the plant model inverse, and uses H_∞ control method to design the Q -filter in the disturbance observer. The conventional disturbance observer is proved to be a special case of the generalized form. Comparison to the conventional form of disturbance observers is made and the significance of the proposed method in DOB design and disturbance attenuation is verified via simulation and experimental results.

Lastly, chapter 9 contains the conclusion of the thesis and proposes topics for future research.

Chapter 2

Identification and Simplification of Model Transfer Function

This chapter studies the modeling of the plants which are investigated in this dissertation. The plants include HDD single stage VCM, HDD dual stage components, and the disturbances which are acting on the plants.

Control design should take plant models as the objective for study. Generally, there are two ways to obtain plant models: analytical derivation and system identification in frequency/time domain. The analytical derivation needs a lot of knowledge about the material and physics of the plants to work out reasonable models. In control perspective, more concerns are the input-output characteristics of the plants. Mathematical model may be developed either by use of “laws of nature”, commonly known as modeling, or based on the experimentation, which is known as system identification. This section will present the modeling of single-stage servo system and dual-stage actuator system with system identification in frequency domain. This method takes the plant as a black box and measures the steady-state output response of the actuator to a sinusoidal input over a given frequency band. The result is considered in a Bode plot.

2.1 Setup for Measurement and Testing

Figure 2.1 and Figure 2.2 show the configuration of a HDD system during experiment. The equipment and software used for experiments are:

1. **MATLAB 7.0.4** It has been used extensively for the design, development as well as effective verification via simulations of the proposed control algorithms in this thesis. Some of its most frequently used toolboxes and packages include, CONTROL SYSTEMS TOOLBOX, LINEAR MATRIX INEQUALITY (LMI) CONTROL TOOLBOX, FILTERS DESIGN TOOLBOX and SIMULINK.
2. **dSPACE DSP System.** A dSPACE digital signal processor (DSP) system is used in the actual implementation throughout the entire course of the research. Some of the major components of the system includes:
 - *dSPACE Add-on Card, DS1103.* The most important component within the system. This card is built using a Texas Instruments TMS320C240 floating point DSP board. The DSP has been supplemented by a set of analog to digital (A/D) and digital to analog (D/A) converters.
 - *Real-Time Interface (RTI) and Real-Time Workshop (RTW).* Acting as a link between Simulink and dSPACE DSP system, RTI and RTW automatically generate real-time codes from SIMULINK models and implement these codes on the dSPACE real-time hardware.
 - *dSPACE ControlDesk.* A software platform that enables controlling and monitoring of the implementation process of the dSPACE DSP system.
3. **Polytec Scanning Vibrometer PSV-200.** [92] It is a scanning laser doppler vibrometer (LDV), an laser optical device used for high accuracy, but short range, displacement and velocity measurement.

4. **HP 35670A Dynamic Signal Analyzer** . [93] The HP Dynamic Signal Analyzer is a dynamic monitor and measuring instrument that can be used for analysis of stability as well as performance of a control system. It is also used to obtain frequency response of our plant to be controlled in order to perform system identification.
5. **LeCroy 5005A Disk Drive Analyzer**. [94] An instrument that can be used to monitor or measure the voltage of any connecting junction within the control system in any implementation. During implementations, it serves both as a tool for identifying mistakes within a control system as well as a device for step-response performance measurement.
6. **Vibration Isolation Platform**. To ensure accurate measurement of very small displacement of less than $1\text{ }\mu\text{m}$ of R/W head, there is a need to isolate the experimental implementation from all external vibrations. Thus, a Newport vibration isolation platform is used during any implementation.

In the experimental study, the frequency characteristics of the Seagate HDD (ST 31276A) have been obtained using a Laser Doppler Vibrometer (LDV) and an HP Dynamic Signal Analyzer. The LDV, which allows non-contacting velocity and displacement measurements, was used to measure the movement of HDD actuator. To keep the input of the actuator to be a sinusoidal wave within a desired frequency band, the DSA is used and plots out the frequency response on the screen. There is no need for additional sensor/transducer to be attached to the device, which may change some dynamics of the measured mechanical system. Although this method to set up a servo feedback loop is different from the PES generation in actual HDDs, LDV measurement is considered as a very good alternative of servo implementation in laboratory surroundings [80].

Figure 2.1: Experimental setup for plant identification.

Figure 2.2: Experimental setup for designed controller verification.

2.2 System Modeling and Compensation for Single Stage HDD

2.2.1 Algorithm of Frequency Domain Identification

With the measured frequency response data of actuators, there are well established methods for identification of a transfer function from frequency response data [5]. These methods finds the coefficients of the identified transfer function $G(s) = \frac{B(s)}{A(s)}$ match as closely as possible to the frequency response obtained experimentally.

Suppose the transfer function model $G(s)$ is with the division of two polynomials,

$$G(s) = \frac{B(s)}{A(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_m}{s^n + a_1 s^{n-1} + \dots + a_n}, \quad (2.1)$$

with $m \leq n$ for proper transfer function. The frequency response of a system is equal to its transfer function evaluated at the points along the positive imaginary axis of the complex plane, i.e., the response at any frequency ω_k is

$$\frac{b_0(j\omega_k)^m + b_1(j\omega_k)^{m-1} + \dots + b_m}{(j\omega_k)^n + a_1(j\omega_k)^{n-1} + \dots + a_n} = G_{fr}(\omega_k) \quad (2.2)$$

The parameters of the transfer function $G_{fr}(\omega_k)$ can be obtained by solving the least squares estimation problem that minimizes the error criterion.

$$\sum_{k=1}^N \left| \frac{B(j\omega_k)}{A(j\omega_k)} - G_{fr}(\omega_k) \right|^2 \quad (2.3)$$

This is a nonlinear least squares problem and can be solved iteratively.

$$b_0(j\omega_k)^m + \dots + b_m - G_{fr}(\omega_k)(a_1(j\omega_k)^{n-1} + \dots + a_n) = (j\omega_k)^n G_{fr}(\omega_k) \quad (2.4)$$

Let $\phi = [b_0, b_1, \dots, b_m, a_1, \dots, a_m]'$, and

$$\theta = [(j\omega_k)^m, (j\omega_k)^{m-1}, \dots, 1, -G_{fr}(j\omega_k)^{m-1}, \dots, -G_{fr}]$$

then suppose there are N different frequencies, the equation can be represented as

$$\phi_{2N \times n_p} \theta_{n_p \times 1} = Y_{2N \times 1} \quad (2.5)$$

Y have $2N$ dimension because each equation should have real part and imaginary part equal respectively. n_p is the number of parameters to be identified. This is a linear model in parameters and can be solved using linear least-squares method (LSM), i.e, to find the estimate $\hat{\theta}$ of the parameter vector by minimizing the cost function,

$$J_{LS} = (Y - \phi\hat{\theta})^T (Y - \phi\hat{\theta}) \quad (2.6)$$

Solution of this least square problem is,

$$\hat{\theta} = (\phi^T \phi)^{-1} \phi^T Y \quad (2.7)$$

A typical frequency response of VCM measured with LDV is shown in Figure 2.3. The frequency range of input sinusoidal signal is from 10 Hz to 4.5 kHz. Higher order or wider frequency range modeling can also be obtained by this linear least-square method, but the fitting result is not good. That is because for higher frequency range, the frequency response is varying sharply, so more sampling data and larger weighting should be considered. There is a simple method for system modeling which avoids to set weighting or take more sampling data. It is shown in next section.

Figure 2.3: Frequency responses of the actual and identified VCM actuator models (LSM).

Figure 2.4: Frequency responses of the actual and identified VCM actuator models.

2.2.2 Manual Zero-Pole Assignment

For higher frequency range, since the model has many resonances, it is hardly use the above algorithm to identify model parameters, thus we should make a conversion. Zero-Pole assignment by hand is a simple and feasible method. Higher frequency response is shown in Figure 2.4. It can be seen in Figure 2.4, the resonances occur around $8.3kHz$, $12kHz$, $12.6kHz$, $15.3kHz$, $17kHz$, $18.5kHz$ respectively. So the higher order poles in VCM model should be at these frequencies, while adjusting the damping ratio in these denominator terms make suitable magnitude of the resonances and placing the suitable zeros to match phase of simple data frequency response. A time delay component is also added to match the phase of simple resonances.

Then the much higher order system model can be obtained as follow,

$$\begin{aligned}
 G(s) = & 0.25875 \times \frac{s^2 - 2 \times 0.3 \times 2\pi \times 14700s + (2\pi \times 14700)^2}{s^2 + 2 \times 0.27 \times 2\pi \times 100s + (2\pi \times 100)^2} \\
 & \times \frac{s^2 + 2 \times 0.3 \times 2\pi \times 16000s + (2\pi \times 16000)^2}{s^2 + 2 \times 0.013 \times 2\pi \times 15300s + (2\pi \times 15300)^2} \\
 & \times \frac{s^2 + 2 \times 0.9 \times 2\pi \times 20000s + (2\pi \times 20000)^2}{s^2 + 2 \times 0.01 \times 2\pi \times 17000 \times s + (2\pi \times 17000)^2} \\
 & \times \frac{s^2 + 2 \times 0.05 \times 2\pi \times 18000s + (2\pi \times 18000)^2}{s^2 + 2 \times 0.009 \times 2\pi \times 18500s + (2\pi \times 18500)^2} \\
 & \times \frac{s^2 + 2 \times 0.009 \times 2\pi \times 12200s + (2\pi \times 12200)^2}{s^2 + 2 \times 0.034 \times 2\pi \times 12600s + (2\pi \times 12600)^2} \\
 & \times \frac{s^2 + 2 \times 0.3 \times 2\pi \times 11300 + (2\pi \times 11300)^2}{s^2 + 2 \times 0.024 \times 2\pi \times 12000 \times s + (2\pi \times 12000)^2}
 \end{aligned} \tag{2.8}$$

The frequency response of this model is also shown in Figure 2.4.

2.2.3 Notch Filter and Compensation

For the resonances in a higher order system, if we use a compensator with cascaded notch filter to actively damp the resonances, the transfer function of the resonance

Figure 2.5: Frequency responses of VCM model by using compensator

Figure 2.6: Simplified system frequency response.

compensator is given by

$$F_v = \prod_{i=1}^n \frac{s^2 + 4\pi\zeta_{1i}f_i s + (2\pi f_i)^2}{s^2 + 4\pi\zeta_{2i}f_i s + (2\pi f_i)^2}, \zeta_{1i} < \zeta_{2i} \quad (2.9)$$

where $f_i(\text{Hz})$ denotes the resonance frequency, ζ_{1i} and ζ_{2i} are the damping ratios chosen at notch the resonance peak. We have implemented an F_v to moderately damp the main resonance modes at 8.3kHz , 12kHz , 12.6kHz , 15.3kHz , 17kHz and 18.5kHz respectively. Figure 2.5 shows the frequency response of VCM model after using compensator compare with the fundamental system. It is shown that the most effect of the resonances is reduced and the compound system can achieve the performance similar to that of a 2nd-order system.

As we mentioned before, the system with notch filter compensator can achieve the performance similar to that of a 2nd-order system, so we use a 2nd-order model to simplify the system. A model with the 2nd-order transfer function is given by (2.10).

$$G(s) = \frac{-7.481 \times 10^9}{s^2 + 339.3s + 3.948 \times 10^5} \quad (2.10)$$

Figure 2.6 shows the frequency response of the simplified system together with one of fundamental system with notch filter compensator.

$$\begin{pmatrix} \dot{y} \\ \dot{v} \end{pmatrix} = \begin{bmatrix} 0 & 1 \\ -3.9478 \times 10^5 & 339.3 \end{bmatrix} \begin{pmatrix} y \\ v \end{pmatrix} + \begin{pmatrix} 0 \\ -7.481 \times 10^9 \end{pmatrix} \quad (2.11)$$

where u is the actuator input (in volts), y and v are the position (in tracks) and the velocity of the R/W head.

2.3 System Modeling and Compensation for Dual Stage HDD

This section introduces system identification of dual stage HDD servo system, the compensation and simplification procedures are also shown.

2.3.1 Manual Zero-Pole Assignment

In this dissertation, we study a dual-stage HDD with a push-pull PZT microactuator as shown in Figure 2.7. It consists of a VCM actuator as the primary stage and a PZT actuator as the secondary stage. The PZT is located between the suspension and the E block, which is moved by the VCM. The two actuators are, respectively, driven through a PZT amplifier and a VCM driver. The head position is also measured using a laser Doppler vibrometer (LDV) in our experiment setup. Throughout this dissertation, we will refer to the combined VCM and E-block assembly as VCM actuator and the combined PZT and suspension assembly as PZT actuator. We assume that the coupling effects between the two actuators are negligible, and then, measure the frequency responses of the VCM and PZT actuator separately, i.e, which are shown with the solid lines in Figure 2.8 and Figure 2.9. The transfer functions of the VCM plant and PZT plant are represented as

$$\begin{aligned}
 G_{VCM}(s) = & 3.70 \times 10^3 \times \frac{s - 2 \times 3000 \times \pi}{s + 2 \times 3000 \times \pi} \frac{1}{s^2 + 2 \times 0.3 \times 2\pi \times 100 + (2\pi \times 100)^2} \\
 & \times \frac{4200^2}{3000^2} \times \frac{s^2 + 2 \times 0.6 \times 2\pi \times 3000 + (2\pi \times 3000)^2}{s^2 + 2 \times 0.17 \times 2\pi \times 4200 + (2\pi \times 4200)^2} \\
 & \times \frac{10500^2}{7000^2} \times \frac{s^2 + 2 \times 0.2 \times 2\pi \times 7000 + (2\pi \times 7000)^2}{s^2 + 2 \times 0.05 \times 2\pi \times 10500 + (2\pi \times 10500)^2} \\
 & \times \frac{16500^2}{14000^2} \frac{s^2 + 2 \times 0.04 \times 2\pi \times 14000 + (2\pi \times 14000)^2}{s^2 + 2 \times 0.15 \times 2\pi \times 16500 + (2\pi \times 16500)^2}
 \end{aligned} \tag{2.12}$$

$$\begin{aligned}
G_{PZT} = & 0.28 \times \frac{6700^2}{7000^2} \times \frac{s^2 + 2 \times 2\pi \times 0.07 \times 7000s + (2\pi \times 7000)^2}{s^2 + 2 \times 2\pi \times 0.069 \times 6700s + (2\pi \times 6700)^2} \\
& \times \frac{11200^2}{18500^2} \times \frac{s^2 + 2 \times 2\pi \times 0.023 \times 18500s + (2\pi \times 18500)^2}{s^2 + 2 \times 2\pi \times 0.018 \times 11200s + (2\pi \times 11200)^2} \\
& \times \frac{5200^2}{4500^2} \times \frac{s^2 + 2 \times 2\pi \times 0.7 \times 4500s + (2\pi \times 4500)^2}{s^2 + 2 \times 2\pi \times 0.5 \times 5200s + (2\pi \times 5200)^2} \\
& \times \frac{12000^2}{11000^2} \times \frac{s^2 + 2 \times 2\pi \times 0.02 \times 11000s + (2\pi \times 11000)^2}{s^2 + 2 \times 2\pi \times 0.04 \times 12000s + (2\pi \times 12000)^2} \\
& \times \frac{5200^2}{4500^2} \times \frac{s^2 + 2 \times 2\pi \times 0.7 \times 4500s + (2\pi \times 4500)^2}{s^2 + 2 \times 2\pi \times 0.5 \times 5200s + (2\pi \times 5200)^2} \\
& \times \frac{12000^2}{11000^2} \times \frac{s^2 + 2 \times 2\pi \times 0.02 \times 11000s + (2\pi \times 11000)^2}{s^2 + 2 \times 2\pi \times 0.04 \times 12000s + (2\pi \times 12000)^2} \\
& \times \frac{2\pi \times 20000}{s^2 + 2 \times 2\pi \times 0.02 \times 20000s + (2\pi \times 20000)^2} \\
& \times \frac{-s + 2\pi \times 33000}{s + 2\pi \times 33000}
\end{aligned} \tag{2.13}$$

Figure 2.7: HDD DSA with a PZT microactuator.

2.3.2 Resonance Compensation

The resonances of the E-block and the suspension exert adverse effects on the tracking performance. Here, we use a compensator with cascaded notch filters to actively damp the resonances. The transfer function of the resonance compensator is given by

$$F_{V,P} = \prod_{i=1}^l \frac{s^2 + 4\pi\zeta_{1i}f_i s + (2\pi f_i)^2}{s^2 + 4\pi\zeta_{2i}f_i s + (2\pi f_i)^2}, \quad \zeta_{1i} < \zeta_{2i} \quad (2.14)$$

where f_i (in hertz) denotes the resonance frequency, and ζ_{1i} and ζ_{2i} are the damping ratios chosen to notch the resonance peak. We have implemented an F_V to moderately dampen the main resonance modes in the VCM actuator such that the compensated VCM model involves no extra phase lag within 1 kHz while the compensated modes cause insignificant vibrations. Similarly, an F_P is implemented to largely dampen the resonance modes in the PZT actuator.

Figure 2.10 and Figure 2.11 show that the simplified model (dash lines) can match the model with resonance compensator (dash-dotted lines) precisely in the frequency range of interest. Thus, from now on, we will take the DSA system in Equation (2.15) as the plant model for further control design.

Suppose the high frequency resonances for both model can be compensated by a series of notch filters, then the system model can be simplified. The dash-dotted lines in Figure 2.10 and Figure 2.11 show the identified frequency responses of the VCM and PZT actuator after resonance compensation. From the identified response with resonance compensation, we observe that the compensated VCM actuator and PZT actuator can be reliably approximated as a pure double integrator and a second-order model, respectively, in the frequency range of interest. Furthermore, we place two saturation blocks before the resonance compensators and set the saturation levels $\bar{u}_1$ and $\bar{u}_2$ sufficiently small such that the control signals u_V and u_P never exceed the voltage limits of the VCM driver and PZT amplifier, respectively. In this

way, we can treat the DSA with resonance compensation as a linear DISO system, which is represented in a state space form as

$$\left\{ \begin{array}{l} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ \quad + \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} \begin{pmatrix} sat(u_1) \\ sat(u_2) \end{pmatrix} \\ y = y_v + y_p = C_1 x_1 + C_2 x_2 \end{array} \right. \quad (2.15)$$

where the state $x_1 = [y_v, \dot{y}_v]^T$, $x_2 = [y_p, \dot{y}_p]^T$,

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ b_1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (2.16)$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad (2.17)$$

and the saturation function $sat(u_i)$ ($i = 1, 2$) is defined as

$$sat(u_i) = sgn(u_i) \min(\bar{u}_i, |u_i|) \quad (2.18)$$

where $\bar{u}_i$ is the saturation level of the i th control input.

The DSA model parameters in Equations (2.16), (2.17), (2.18) are identified to match the responses with resonance compensator, which are given by

$$\begin{aligned} b_1 &= 5.7 \times 10^7, a_1 = -9.8 \times 10^8, a_2 = -5 \times 10^4, \\ b_2 &= 1.9 \times 10^8, \bar{u}_1 = 5V, \bar{u}_2 = 5V. \end{aligned} \quad (2.19)$$

Figure 2.10 and Figure 2.11 shows that the identified models (dash lines) can

match the identified ones (solid lines) precisely in the frequency range of interest. Thus, from now on, we will take the DSA system in Equation (2.15) as the plant model for further control design.

2.4 Noise and Disturbance Modeling

The disturbances in self-servo track writing are identified and modeled in [95].

Figure 2.12 shows the closed-loop NRRO spectrum of a 3.5-inch HDD with a VCM actuator. The NRRO spectrum is given by

$$S_{pes} = |P(z)S(z)|^2 |D_i(z)|^2 + |S(z)|^2 |N(z)|^2 + |S(z)|^2 |D_o(z)|^2 \quad (2.20)$$

where $S(z)$ is the sensitivity function. The procedure to obtain $D_i(z)$, $D_o(z)$ and $N(z)$, as shown in [95], is listed in the following:

- i. Find the baseline curve S_b of the spectrum

$$S_b(j) = \min_{i=1+(j-1)m}^{jm} S_{pes}(i), j = 1, 2, \dots, L/m$$

where L is the length of S_{pes} and m is a sufficiently small integer;

- ii. Compute $D_i(z)$ and $N(z)$ using

$$|D_i(z)|^2 = \frac{W_L S_b}{|P(z)S(z)|^2}; \quad (2.21)$$

$$|N(z)|^2 = \frac{W_H S_b}{|S(z)|^2} \quad (2.22)$$

where $W_L(W_H)$ is a low (high)-pass weighting function used to select S_b of low (high) frequencies corresponding to the two dominant peaks. Till now, S_b can be fit well by the identified $D_i(z)$ and $N(z)$.

iii. The spikes on the spectrum, especially between 0 to $2.5kHz$, are regarded as do effect. Thus,

$$|D_o(z)|^2 = \frac{S_{pes} - [|P(z)S(z)|^2 |D_i(z)|^2 + |S(z)|^2 |N(z)|^2]}{|S(z)|^2} \quad (2.23)$$

where S_{pes} is the whole NRRO spectrum as in Figure 2.12. For simplicity, a low order model is used to describe d_o . The identified disturbances and noise models are as follows:

$$\begin{aligned} D_1(s) &= \frac{12.79}{s^2 + 113.1s + 3.198 \times 10^5}, N(s) = 0.0023 \\ D_2(s) &= \frac{0.0019(s^2 + 381s + 7.012 \times 10^5)}{s^2 + 8.168s + 6.672 \times 10^5} \\ &\times \frac{(s^2 + 2276s + 1.602 \times 10^7)(s^2 + 2640s + 5.647 \times 10^8)}{(s^2 + 245s + 1.668 \times 10^7)(s^2 + 477.5s + 5.701 \times 10^8)} \end{aligned} \quad (2.24)$$

2.5 Summary

System modeling and identification of single stage HDD are shown in this chapter. Both least squares method and manually zero-pole assignment are attempted. Notch filter are applied to simplify the system to a second order system. Dual stage HDD servo systems and disturbances are also modeled in this chapter. This chapter is a preparation for the following chapters.

Figure 2.8: Frequency responses of a HDD VCM actuator system.

Figure 2.9: Frequency responses of a HDD PZT actuator system.

Figure 2.10: Frequency responses of a HDD VCM actuator system and compensated modeling.

Figure 2.11: Frequency responses of a HDD PZT actuator system and compensated modeling.

Figure 2.12: Nonrepeatable runout (NRRO) power spectrum calculated from measured PES signal without servo control, reflecting the disturbance distribution of the STW platform.

Chapter 3

Discrete-time LQR Optimal Reset Control for the Improvement of HDD Servo Control Transient Performance

As discussed in Chapter 1, a fast data access rate is an essential requirement of HDD performance. It demands the HDD servo system have fast transient response. This chapter proposes a design method for discrete-time LQR reset control systems, aiming at improving the system transient responses.

3.1 Introduction

3.1.1 Introduction of Reset Control

Reset control was initially proposed by [96] to overcome limitations of linear control. This reset controller, termed as Clegg-integrator, consists of an integrator and a reset law which resets the output of the integrator to zero when its input vanishes. Equation (3.1) shows a typical Clegg-integrator.

$$\begin{cases} \dot{z}(t) = e(t), & e(t) \neq 0 \\ z(t^+) = 0, & e(t) = 0 \end{cases} \quad (3.1)$$

From the basic idea of reset control, one can see that reset control is helpful in reducing windup caused by integration. Moreover, a Clegg integrator has a similar magnitude-frequency response as a pure integrator, but with 51.9° less phase lag. This favourable property helps increase the phase margin of a system. In [97], Krishman and Horowitz developed a quantitative control design procedure of Clegg integrator. In [98], Horowitz and Rodenbaum generalized the concept of reset control to higher order systems. More details can be found in [99].

A lot of work has shown the advantages of reset control over linear control. For instance, in [100], an example was presented to show that reset control can achieve some control specifications which cannot be achieved by any ordinary linear control. Besides, [101] shows that reset control can achieve much better sensor noise suppression without degrading disturbance rejection or losing margins. These advantages make reset control an important technique for performance improvement. See [102], [103] and [104] for instance. Recently, reset control was introduced to hard disk drive servo systems [105, 106].

There are in general two steps in reset control design [107]: linear compensator

design and reset element design. The linear compensator is designed to meet all performance specifications other than output overshoot, then reset element is designed to reduce the overshoot. A reset controller can improve closed-loop performance only when the reset law interacts well with the base linear system. In other words, if the reset controller is not appropriately designed, it may have little contribution to the performance improvement, or even cause performance degradation. For example, reset control may destroy the stability of the closed-loop system if it does not cooperate well with the base linear system.

In reset control system design, there are three basic problems: stability analysis, base linear system design and reset law design.

For the stability analysis, there are many papers in literature addressing this issue, see [108–111], etc. Most of these existing results require the base linear systems to be stable. However, stability of a reset control system depends on both the base linear system and the reset actions. Each of these two factors can contribute to or destroy the stability of the overall system. Note that reset control systems are also known as impulsive systems and their stability issues have been addressed in [112].

For the reset control law design, most efforts are put on the design of base linear system in existing literatures [99, 101, 105]. The reset law adopted is in general the traditional one, i.e., reset the state of controller to zero when tracking error crosses zero. Base linear system is then designed to interact well with the reset law. We refer to this kind of reset control as the *traditional reset control* in this dissertation. Actually, reset control can be more general, for instance, the time and the amount of reset can both be designed so that the reset law and the base linear system can cooperate better with each other. [113] investigates frequency-domain properties of reset systems with arbitrary reset matrices and their application in disturbance rejection.

3.1.2 Overview of This Chapter

The purpose of this chapter is to propose a novel approach to reset control law design for a reset control system and apply the proposed method to improve the transient response of HDD systems. We focus on systems of which the base linear systems are already appropriately designed and the reset time instants are pre-specified. The design of the reset law aims to minimize some performance index and its solution is obtained by solving a Riccati equation. In case of equidistant reset control, we show that the resulting reset law is time invariant. This chapter shows discrete-time reset control law design method which is suitable for digital sampling control system.

This chapter is arranged as follows. In Section 3.2, we set up the problem studied in general cases. In Section 3.3, we investigate optimal reset law design problem. Section 3.4 gives the application of the proposed design method to HDD systems. Some concluding remarks are made in Section 3.5.

3.2 LQR Optimal Reset Control Problem Setting

In this section, we consider a general control problem. A typical reset control system is depicted in Figure 3.1. The dynamics of the plant is described by

$$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx, \end{cases} \quad (3.2)$$

where $x \in \mathbb{R}^{n_x}$ is the state, $u \in \mathbb{R}^{n_u}$ the control input and $y \in \mathbb{R}^{n_y}$ the output of the system. The zero-order-hold equivalent description of the plant (5.1) with the sampling time T_s is given by

$$\begin{cases} x(i+1) &= A_d x(i) + B_d u(i) \\ y(i) &= C x(i), \end{cases} \quad (3.3)$$

where $A_d = e^{AT_s}$, $B_d = \int_0^{T_s} e^{A(T_s-\tau)} B d\tau$.

Define T_{rst} as the reset period. We assume that the reset interval T_{rst} is at every n sampling intervals, i.e., $T_{rst} = nT_s$, as illustrated below.

The variables k and i are used to represent the time instant $t = kT_{rst} + iT_s$ with $i = 0, 1, 2, \dots, n-1$. We then introduce the following discrete reset controller:

$$\begin{cases} q(k, i) = Dq(k, i-1) + Ee(i-1), & (i = 1, 2, \dots, n-1) \\ q(k, 0) = \rho_k(x(k-1, n-1), q(k-1, n-1), r), \\ u(k, i) = Gx(k, i) + Hq(k, i) + Me(k, i), \end{cases} \quad (3.4)$$

where $q \in \mathbb{R}^{n_q}$ is the dynamic state in the controller, r is the reference signal which is assumed to be constant, $\rho_k(x, q, r)$ is the reset value at time instant $t = kT_{rst}$ and it is the so-called the reset law, D , E , G , H and M are all constant matrices with compatible dimensions, and $e(k, i) = r - y(k, i)$ is the tracking error.

Next we combine (3.3) and the reset controller (3.4) as follows:

$$\begin{cases} \bar{x}(k, i) = \bar{A}\bar{x}(k, i-1) + \bar{B}r, & (i = 1, 2, \dots, n-1) \\ q(k, 0) = \rho_k(\bar{x}(k-1, n-1), r), \\ \bar{y}(k, i) = \bar{C}\bar{x}(k, i), \end{cases} \quad (3.5)$$

Figure 3.1: Block diagram of Discrete-time LQR reset control system.

where $\bar{x}(k, i) = (x(k, i)^T \ q(k, i)^T)^T$, $\bar{C} = (C, \ 0_{p \times q})$,

$$\bar{A} = \begin{pmatrix} A_d + B_d G - B_d M C & B_d H \\ -E C & D \end{pmatrix}, \bar{B} = \begin{pmatrix} B_d M \\ E \end{pmatrix}. \quad (3.6)$$

Before we introduce a cost function for the reset controller design, we need to make an assumption on the steady state of the system (3.5).

Assumption 3.2.1. *For any $r \in \mathbb{R}^{n_r}$, there exists $\bar{\mathbf{x}}_r = (\mathbf{x}_r^T, \mathbf{q}_r^T)^T \in \mathbb{R}^{n+q}$, such that*

$$\bar{A}\bar{\mathbf{x}}_r + \bar{B}r = \bar{\mathbf{x}}_r,$$

$$\bar{C}\bar{\mathbf{x}}_r - r = 0.$$

Remark 3.2.1. *Assumption guarantees the system has steady state for each given r . We assume the refer input r is constant, thus $\bar{\mathbf{x}}_r$ the steady-state value of the closed-loop is also a constant. The above assumption is actually an necessary condition for perfect tracking.*

Not all the system satisfies such assumption, only the internal stabled system satisfies such assumption. But for a BIBO stabled SISO system, many system realizations can achieve internal stable, such as controllability canonical form, observability canonical form.

With the above Assumption, the control input u_r in the steady state is given by

$$u_r = Gx_r + Hq_r. \quad (3.7)$$

For each time instant $t = kT_{rst} + iT_s$, we define a cost function $J(k, i)$ as

$$J(k, i) = e^T(k, i)Q_{k,i}e(k, i) + (u(k, i) - u_r)^T R_{k,i}(u(k, i) - u_r)$$

Here $Q_{k,i}$ and $R_{k,i}$ are positive semi-definite matrices. The optimal reset law (ORL) design problem considered in this paper is thus formulated as follows.

Problem 3.2.1. *Design ρ_k , such that the resulting control system is asymptotically stable and meanwhile the cost function $J(\infty)$ is minimized, where*

$$J(\infty) := \sum_{k=0}^{\infty} \sum_{i=0}^{n-1} J(k, i) \quad (3.8)$$

3.3 Discrete-time LQR Optimal Reset Law Design

In this section, we suppose to solve the optimal reset law (ORL) design problem stated in the previous section. It will be proved that this problem can be converted to a standard LQR problem.

We make a coordinate transformation as follows.

$$\begin{cases} \xi_x(k, i) = x(k, i) - \mathbf{x}_r \\ \xi_q(k, i) = q(k, i) - \mathbf{q}_r \end{cases} \quad (3.9)$$

where the steady-state values $\mathbf{x}_r$ and $\mathbf{q}_r$ are defined in Assumption 1. From Equation (3.5), we have

$$\begin{cases} \xi(k, i+1) = \bar{A}\xi(k, i) \\ \xi_q(k, 0) = \bar{\rho}(k, 0) \\ e(k, i) = -\bar{C}\xi(k, i) \end{cases} \quad (3.10)$$

where $\xi(k) = (\xi_x(k)^T \ \xi_q(k)^T)^T$ and $\bar{\rho}(k, 0) = \rho(k, 0) - q_r$. From Equations (3.4) and (3.7), it follows that

$$u(k) - u_r = \begin{bmatrix} G - MC & H \end{bmatrix} \begin{pmatrix} \xi_x(k) \\ \xi_q(k) \end{pmatrix}. \quad (3.11)$$

Let $N = \begin{bmatrix} G - MC & H \end{bmatrix}$. For a reset interval $(iT_{rst}, (i+1)T_{rst}]$, the cost function should be

$$\begin{aligned} \sum_{i=0}^{n-1} J(k, i) &= \sum_{i=0}^{n-1} e^T(k, i) Q_{k,i} e(k, i) + (u(k, i) - u_r)^T R_{k,i} (u(k, i) - u_r) \\ &= \sum_{i=0}^{n-1} \xi_x^T(k, i) C^T Q_{k,i} C \xi_x(k, i) + \xi(k, i)^T N^T R_{k,i} N \xi(k, i) \\ &= \sum_{i=0}^{n-1} \xi^T(k, i) (\bar{C}^T Q_{k,i} \bar{C} + N^T R_{k,i} N) \xi(k, i) \\ &= \sum_{i=0}^{n-1} \xi^T(k, 0) (\bar{A}^T)^i (\bar{C}^T Q_{k,i} \bar{C} + N^T R_{k,i} N) (\bar{A})^i \xi(k, 0) \\ &= \xi^T(k, 0) \sum_{i=0}^{n-1} [(\bar{A}^T)^i (\bar{C}^T Q_{k,i} \bar{C} + N^T R_{k,i} N) (\bar{A})^i] \xi(k, 0) \\ &= \xi^T(k, 0) \Theta_k \xi(k, 0). \end{aligned} \quad (3.12)$$

with $\Theta_k = \sum_{i=0}^{n-1} [(\bar{A}^T)^i (\bar{C}^T Q_{k,i} \bar{C} + N^T R_{k,i} N) (\bar{A})^i]$.

Hence $J(\infty)$ is the sum of $\xi^T(k, 0) \Theta_k \xi(k, 0)$ at each reset time instant kT_{rst} ,

and given by

$$J(\infty) = \sum_{k=0}^{\infty} \xi^T(k, 0) \Theta_k \xi(k, 0) \quad (3.13)$$

On the other hand, with the reset interval T_{rst} as the sampling interval the system (3.10) can be written as

$$\begin{pmatrix} \xi_x(k+1, 0) \\ \xi_q(k+1, 0) \end{pmatrix} = (\bar{A})^n \begin{pmatrix} \xi_x(k, 0) \\ \bar{\rho}(k, 0) \end{pmatrix} \quad (3.14)$$

Partition $(\bar{A})^n$ as follows,

$$(\bar{A})^n = \begin{pmatrix} \Gamma_A & \Gamma_B \\ * & * \end{pmatrix}, \quad (3.15)$$

then we have

$$\xi_x(k+1, 0) = \Gamma_A \xi_x(k, 0) + \Gamma_B \bar{\rho}(k, 0) \quad (3.16)$$

Note that system (3.16) is simply a linear discrete system with state variable ξ_x and control input $\bar{\rho}$. Consider the following linear quadratic regulation (LQR) problems of system (3.16).

Problem 3.3.1. *Design a control sequence $\rho(i)$, $i = 0, 1, \dots$, for system (3.16) such that the resulting system is asymptotically stable and the following quadratic performance index is minimized,*

$$J(\infty) = \sum_{k=0}^{\infty} \begin{pmatrix} \xi_x(k, 0) \\ \bar{\rho}(k, 0) \end{pmatrix}^T \Theta_k \begin{pmatrix} \xi_x(k, 0) & \bar{\rho}(k, 0) \end{pmatrix}^T.$$

Proposition 3.3.1. *Suppose that there exist positive numbers $\varepsilon > 0$ such that*

$$\lambda_{\min}(\Gamma_B^T \Gamma_B) \geq \varepsilon > 0. \quad (3.17)$$

Then the ORL Problem 3.2.1 is equivalent to LQR Problem 3.3.1.

Proof: We only need to prove that under the above condition, system (3.16) is asymptotically stable if and only if system (3.10) is asymptotically stable. It is obvious that if system (3.10) is asymptotically stable, then system (3.16) is asymptotically stable. In the following, we assume that system (3.16) is asymptotically stable. Then we have

$$\begin{aligned}\lim_{k \rightarrow \infty} \xi_x(k, 0) &= 0, \\ \lim_{k \rightarrow \infty} \Gamma_B \bar{\rho}(k, 0) &= 0.\end{aligned}$$

According to condition (3.17), we have

$$\lim_{k \rightarrow \infty} \bar{\rho}(k, 0) = 0.$$

Note that $\xi_q(k, 0) = \bar{\rho}(k, 0)$, we have

$$\lim_{k \rightarrow \infty} \xi(k, 0) = 0,$$

and thus system (3.10) is asymptotically stable. $\square$

Remark 3.3.1. *In this case, Γ_A and Γ_B are constant and the conditions of Proposition 3.3.1 hold naturally provided that Γ_B is of full rank. Actually, we can always make Γ_B to be of full rank. Otherwise, the system (3.16) would have redundant inputs in $\bar{\rho}(k, 0)$. In this case, we can put the redundant components of $\bar{\rho}(k, 0)$ and $\xi_x(k, 0)$ into $\xi_q(k, 0)$ by rearrangement and transformations, then repartition $(\bar{A})^n$.*

According the above proposition, to solve the ORL problem we only need to solve the corresponding LQR problem 3.3.1, which is standard linear quadratic regulation problems, and thus we can obtain the optimal solutions directly by solving some Riccati equations. Partition Θ_k as

$$\Theta_k = \begin{pmatrix} \bar{Q} & \bar{T} \\ \bar{T}^T & \bar{R} \end{pmatrix} \quad (3.18)$$

Define

$$\begin{aligned} \bar{\Gamma}_A &= \Gamma_A - \Gamma_B \bar{R}^{-1} \bar{T}^T, \\ \tilde{Q} &= \bar{Q} - \bar{T} \bar{R}^{-1} \bar{T}^T \end{aligned} \quad (3.19)$$

The solution of the LQR problem can be obtained immediately as follows [114].

Proposition 3.3.2. *Assume that (Γ_A, Γ_B) is controllable and the matrices $\bar{R}$ and $\tilde{Q}$ are positive definite. Then the optimal reset law which stabilizes system (3.2) and minimizes $J(\infty)$ is given by*

$$\rho^*(k, 0) = -K(x(k, 0) - \mathbf{x}_r) + \mathbf{q}_r, \quad (3.20)$$

where K is determined by

$$K = (\Gamma_B^T S \Gamma_B + \bar{R})^{-1} (\Gamma_B^T S \Gamma_A + \bar{T}^T) \quad (3.21)$$

and S is the solution of the Riccati equation

$$S = \tilde{Q} + \bar{\Gamma}_A^T S \bar{\Gamma}_A - \bar{\Gamma}_A^T S \bar{\Gamma}_B (\Gamma_B^T S \Gamma_B + \bar{R})^{-1} \Gamma_B^T S \bar{\Gamma}_A. \quad (3.22)$$

Furthermore, the minimum of $J(\infty)$ is given by

$$J^*(\infty) = (x(0) - \mathbf{x}_r)^T S (x(0) - \mathbf{x}_r). \quad (3.23)$$

Then, the whole process of LQR optimal reset law is achieved. Different with the design of standard LQR problem design, our approach is designed based on an existing linear controller which industry always uses, and improves the performance. So the design doesn't change the controller much, which is a benefit for industrial

implementation.

3.4 Application to HDD Servo System

3.4.1 System Modeling

The frequency responses of a HDD voice coil motor (VCM) actuator have been measured using a Laser Doppler Vibrometer and a Dynamic Signal Analyzer. Based on the measured frequency responses of the actuator, its transfer function is shown in Equation (2.8). Suppose by adding notch filters, the system is compensated and simplified as a 2nd order system (2.11), which is called the nominal plant and whose frequency response can be seen in Figure 2.6. The model of the nominal plant is described by (2.11).

Our target is to design a seeking controller for the VCM actuator to have fast positioning without overshoot.

3.4.2 Reset Control Law Design

The controller structure is depicted in Figure 3.2. The reset controller is given by

$$\begin{cases} q(k, i) = Dq(k, i-1) + Ee(k, i-1), \\ q(k, 0) = \rho_k(x(k-1, n-1), q(k-1, n-1), r), \\ v(k, i) = \alpha e(k, i) + \beta x_2(k, i) + \gamma q(k, i), \end{cases} \quad (3.24)$$

with $D = 1$, $E = T_s$, $\alpha = 0.5$, $\beta = 4 \times 10^{-4}$, and $\gamma = 6000$.

The base linear controller is given by the first and the third equations in (3.24).

For the reset law design, we assume $R_{k,i}$ and $Q_{k,i}$ are time-invariant. The reset

Figure 3.2: The structure of reset controller.

law is given by

$$\rho_k^* = -k_1(x_1 - r) - k_2\hat{x}_2 + \frac{a}{c\gamma}r. \quad (3.25)$$

In the reset controller (3.25), the velocity x_2 is replaced with its estimated value $\hat{x}_2$. An state observer is needed to obtain the estimated velocity.

3.4.3 Simulation Results

The input disturbance, output disturbance and measurement noise are added in the simulation. output the sampling period T_s is 2.5×10^{-5} sec and $Q = 1$, $R = 17$. The reset interval $T_{rst} = 2 \times T_s$. The reset law is given by (3.25). By solving Riccati equation (3.21) and (3.22), we have

$$\begin{aligned} k_1 &= -3.6049 \times 10^{-5}, \\ k_2 &= -4.6124 \times 10^{-8}. \end{aligned}$$

Figure 3.3 gives the step responses for the base linear control, traditional reset control and the optimal reset control. We can see that the proposed optimal reset control almost removes the overshoot completely and at the same time reduces the

rise time significantly.

Figure 3.4 gives the responses for different reset intervals $T_{rst} = 2T_s, 5T_s, 20T_s$ respectively. The transient performance is better when reset interval is smaller. In Figure 3.5, step responses for different R and fixed value $Q = 1$ are given.

Compared with [115], the advantage of the proposed optimal reset control in this chapter is that the design process is more simple. We do not need to worry about the stability since the stability is naturally assured.

Remark 3.4.1. *In the short-span seeking, we find that the performance is not sensitive to T_{rst} ($T_{rst} = nT_s$) with $n \leq 3$. But if $n > 3$, the performance will become worse. Since the optimal cost function weights R and Q should be set for different input levels, in order to tradeoff the ratio of input and error signal. we need different values of R/Q for different reference levels r so as to achieve the best performance.*

Remark 3.4.2. *For case of $T_{rst} = 20T_s$ in Figure 3.4, when reset is injected, sharp changes in the output of the reset element occur, and then the abrupt responses cause big spikes in the control signal.*

In the long-span seeking control, we choose $Q = 1$ and $R = 12$. We add the control signal saturation $|u| < 2.5$ as the bounded input effort. We found that in this range, the control input has great efficient during rising stage (see the control input signals in Figure 3.6). Compared to the design of optimal time control, we do not need to design a transitional controller to smooth the input in order to avoid chattering, thus the design process is more simple.

Remark 3.4.3. *In simulations, the performance is also not sensitive to T_{rst} ($T_{rst} = nT_s$) with $n \leq 10$. But if $n > 10$, the performance will become worse. In the long-span seeking control, the performance is not sensitive to R/Q . The control signal will not be amplified boundlessly as a result of the saturation part, so the R/Q can be fixed without changing the performance.*

Figure 3.3: Step responses for base linear control, traditional reset control and proposed discrete optimal reset control.

3.4.4 Experimental Results

The discrete optimal reset control law is verified via implementation in the HDD servo system and the seeking performance is investigated.

Figure 3.7 shows the seeking result for $r = 0.25\mu m$ with comparison to the linear PID control. It is seen that the rising time of the linear controller and the reset controller are similar, but when reaching the target, the response with the ORL method is swiftly settled to the target with less overshoot. The settling time is $0.33ms$. Figure 3.8 shows the seeking performance for $r = 4\mu m$. The performance of this longer span seeking with the reset control is also much better than that with the PID control. No overshoot is noticed and the settling time is less than $0.5ms$. The control effort of ORL method have much more efficiency since it is minimized via the cost function including the control input term.

Figure 3.4: Step responses for optimal reset control system with different T_{rst} for $r = 1$.

3.4.5 Investigation of Track Following

Finally, we evaluate the position error signal (PES) of the proposed DSA control system by injecting the input disturbance, output disturbance and measurement noise which is shown in Equation (2.24) and setting $y_r = 0$. In Figure 3.9 and Table 3.1, It is shown that the LQR reset control is some what worse than the PID control in disturbance rejection, thus the LQR reset control is just suitable for track seeking stage, for track following stage, we should find a better way to attenuate the disturbance. For the LQR reset control, it is a feedback form which makes it possible to utilize the optimal reset control structure in track-following mode as well.

Figure 3.5: Step responses for different R/Q .Table 3.1: The PES and contribution of disturbances and noise in the error, denoted by $e^{(*)}$, in $\mu\text{ m}$

Cases	e	$e(d_1)$	$e(d_2)$	$e(n)$
PID	0.01150	0.01079	0.001999	0.002271
LQR Reset Control	0.01894	0.01466	0.002078	0.003671

3.5 Summary

In this chapter, we have studied discrete-time optimal reset law design for reset control systems. Firstly, we transferred closed-loop reset control system to discrete-time linear system with reset value as control input. Then we proved that ORL design problem is equivalent to LQR problem. Based on this, the optimal reset laws were given in terms of Riccati equations. The design process of the proposed reset law is very simple. The resulting reset law is of feedback form and thus can be easily

Figure 3.6: Displacements and control signals for $r = 10$.

implemented in real-time.

The proposed design was applied to seeking control of single-stage hard disk drive systems. Both short-span and long-span seeking control were studied. Simulation results and experiment results show that the proposed design can achieve better seeking performance.

Since the LQR reset control is not a better way for disturbance rejection, the next chapter will show an optimal reset method which is for improving the disturbance attenuation performance.

Figure 3.7: Displacements and control signals for $r = 0.25\mu m$.

Figure 3.8: Displacements and control signals for $r = 4\mu m$.

Figure 3.9: Disturbance cancelation comparison between PID and LQR reset.

Chapter 4

Discrete-time H_2 Optimal Reset Control with application to HDD Track-following

The previous chapter presents a discrete-time LQR optimal reset control method to improve the transient performance which is suitable for HDD system track seeking stage. But for track following, since discrete-time LQR optimal reset control has low bandwidth, its disturbance rejection ability has limitations.

In order to achieve better disturbance attenuation, another reset control method is presented in this chapter. This method has the similar structure with LQR optimal reset but with different optimality criterion.

4.1 Introduction

The purpose of this chapter is to propose a novel approach to reset control law design for a reset control system and apply the proposed method to improve the disturbance

suppression performance of HDD systems. The design of the reset law aims to minimize H_2 norm of the transfer function from disturbance to system output and the solution can be obtained by solving two Riccati equations. H_2 optimal control method is also widely used in other control issues of HDD systems, see [116–118].

This chapter is arranged as follows. In Section 4.2, we set up the problem to be studied in this chapter. In Section 4.3, we investigate optimal reset law design problem. Section 4.4 gives the application of the proposed design method to HDD systems. Some concluding remarks are made in Section 4.5.

4.2 Discrete-time H_2 Optimal Reset Control Problem Setting

In this section, we consider a general control problem regardless the HDD plant specifications. This model has considered input disturbance, output disturbance and measurement noise which is shown in Figure 4.1. The dynamics of the plant is described by

$$\begin{cases} x(k+1) &= Ax(k) + B_1u(k) + B_2w(k) \\ e(k) &= C_1x(k) + D_{12}w(k) \\ z(k) &= C_2x(k) + D_{21}u(k) + D_{22}w(k) \end{cases} \quad (4.1)$$

where $x(k) \in R^{n_x}$ is the state, $u(k) \in R^{n_u}$ is the control input, $w(k) \in R^{n_w}$ is the disturbance input, $y(k) \in R^{n_y}$ the measurement output and $z(k) \in R^{n_z}$ is the controlled output of the system. $e(k) \in R^{n_e}$ is the error signal, the difference between this $e(k)$ and the one in previous chapter is that $e(k)$ in this chapter considers the input disturbance, output disturbance and measurement noise in it. The plant is with the sampling time T_s .

Define T_{rst} as the reset period. We assume that the reset interval T_{rst} is also

Figure 4.1: Block diagram of Discrete-time H_2 reset control system.

at every n sampling intervals, i.e., $T_{rst} = nT_s$.

The variables k and i are used to represent the time instant $t = kT_{rst} + iT_s$ with $i = 0, 1, 2, \dots, n-1$. We then introduce the following discrete reset controller:

$$\begin{cases} q(k, i) = Eq(k, i-1) + Fe(k, i-1), \\ q(k, 0) = \rho_k(x(k-1, l)), \\ u(k, i) = Hq(k, i) + Me(k, i), \end{cases} \quad l = 1, 2, \dots, n-1 \quad (4.2)$$

where $q \in R^{n_q}$ is the dynamic state in the controller, r is the reference signal which is assumed to be constant, $\rho_k(x, q)$ is the reset value at time instant $t = kT_{rst}$ and it is the so-called the reset law. E , F , H and M are all constant matrices with compatible dimensions.

Denote T_{zw} the transfer function matrix from w to z , The discrete-time H_2 optimal reset control problem is then states as: *Design a reset control law $\rho_k(x(k-1, n-1), z(k-1, n-1))$ such that the H_2 norm $\|T_{zw}\|_2$ is minimized.*

4.3 H_2 Optimal Reset Law Design

In this section, we turn to solve the optimal reset law problem proposed in the previous section. The main idea is to transfer these problems to standard H_2 optimal control problems.

4.3.1 Problem Transformation

Combining system (4.1) and controller (4.2), we have

$$\begin{cases} \bar{x}(k, i) = \bar{A}\bar{x}(k, i-1) + \bar{B}w(k, i-1), \\ q(k, 0) = \rho_k(x(k-1, l)), & l = 1, 2, \dots, n-1 \\ y(k, i) = \bar{C}_1\bar{x}(k, i) + \bar{D}_{12}w(k, i-1) \\ z(k, i) = \bar{C}_2\bar{x}(k, i) + \bar{D}_{22}w(k, i-1) \end{cases} \quad (4.3)$$

where $\bar{x}(k, i) = (x(k, i)^T \ q(k, i)^T)^T$,

$$\bar{A} = \begin{pmatrix} A + B_1MC_1 & B_1H \\ FC_1 & E \end{pmatrix}, \bar{B} = \begin{pmatrix} B_1MD_{12} + B_2 \\ FD_{12} \end{pmatrix}.$$

$$\bar{C}_1 = (C_1, 0_{v \times r}), \bar{C}_2 = (C_2 + D_{21}MC_1, D_{21}H)$$

$$\bar{D}_{12} = D_{12}, \bar{D}_{22} = D_{21}MD_{12} + D_{22}.$$

We then convert the system to a discrete system which considers the reset period as sampling period, the system becomes

$$\begin{cases} \bar{x}(k+1, 0) = \tilde{A}\bar{x}(k, 0) + \tilde{B}_2\bar{w}(k, n-1), \\ q(k, 0) = \rho_k(x(k-1, l)), & l = 1, 2, \dots, n-1, \\ \bar{y}(k, n-1) = \tilde{C}_1\bar{x}(k, 0) + \tilde{D}_{12}\bar{w}(k, n-1) \\ \bar{z}(k, n-1) = \tilde{C}_2\bar{x}(k, 0) + \tilde{D}_{22}\bar{w}(k, n-1) \end{cases} \quad (4.4)$$

where

$$\bar{w}(k, 0) = (w(k, 0)^T, w(k, 1)^T, \dots, w(k, n-1)^T)^T,$$

$$\bar{y}(k, n-1) = (y(k, 0)^T, y(k, 1)^T, \dots, y(k, n-1)^T)^T,$$

$$\bar{z}(k, n-1) = (z(k, 0)^T, z(k, 1)^T, \dots, z(k, n-1)^T)^T,$$

$$\text{and } \tilde{A} = \bar{A}^n, \tilde{B}_2 = [\bar{A}^{n-1}\bar{B} \quad \bar{A}^{n-2}\bar{B} \quad \dots \quad \bar{B}],$$

$$\tilde{C}_1 = \begin{bmatrix} \bar{C}_1 \\ \bar{C}_1\bar{A} \\ \dots \\ \bar{C}_1(\bar{A})^{n-1} \end{bmatrix}, \tilde{C}_2 = \begin{bmatrix} \bar{C}_2 \\ \bar{C}_2\bar{A} \\ \dots \\ \bar{C}_2(\bar{A})^{n-1} \end{bmatrix}$$

$$\tilde{D}_{12} = \begin{bmatrix} \bar{D}_{12} & 0_{r \times l} & \dots & 0_{r \times l} \\ \bar{C}_1\bar{B} & \bar{D}_{12} & \dots & 0_{r \times l} \\ & \ddots & & \\ & & \ddots & \\ \bar{C}_1(\bar{A})^{n-2}\bar{B} & \bar{C}_1(\bar{A})^{n-3}\bar{B} & \dots & \bar{D}_{12} \end{bmatrix},$$

$$\tilde{D}_{22} = \begin{bmatrix} \bar{D}_{22} & 0_{r \times p} & \dots & 0_{r \times p} \\ \bar{C}_2\bar{B} & \bar{D}_{22} & \dots & 0_{r \times p} \\ & \ddots & & \\ & & \ddots & \\ \bar{C}_2(\bar{A})^{n-2}\bar{B} & \bar{C}_2(\bar{A})^{n-3}\bar{B} & \dots & \bar{D}_{22} \end{bmatrix}.$$

Partition $\tilde{A}$, $\tilde{C}_1$ and $\tilde{C}_2$ as a form of follows:

$$\tilde{A} = \begin{pmatrix} \Gamma_A & \Gamma_{B_1} \\ * & * \end{pmatrix},$$

$$\tilde{C}_1 = \begin{pmatrix} \Gamma_{C_1} & \Gamma_{D_{11}} \end{pmatrix},$$

$$\tilde{C}_2 = \begin{pmatrix} \Gamma_{C_1} & \Gamma_{D_{21}} \end{pmatrix},$$

Use ρ_k to replace $\rho_k(x(k-1, n-1), q(k-1, n-1))$, let $\Gamma_{B_2} = \tilde{B}_2, \Gamma_{D_{12}} = \tilde{D}_{12}$ and $\Gamma_{D_{22}} = \tilde{D}_{22}$, then the system becomes

$$\begin{cases} x(k+1, 0) &= \Gamma_A x(k, 0) + \Gamma_{B_1} \rho_k + \Gamma_{B_2} \bar{w}(k, n-1) \\ \bar{y}(k, n-1) &= \Gamma_{C_1} x(k, 0) + \Gamma_{D_{12}} \bar{w}(k, n-1) \\ \bar{z}(k, n-1) &= \Gamma_{C_2} x(k, 0) + \Gamma_{D_{21}} \rho_k + \Gamma_{D_{22}} \bar{w}(k, n-1) \end{cases} \quad (4.5)$$

Thus the problem convert to *design a reset control law ρ_k such that the H_2 norm $\|T_{zw}\|_2$ is minimized*, which is a standard H_2 problem.

4.3.2 Solutions to H_2 Optimal Reset Control Problems

We let Σ_P be the subsystem characterized by $(\Gamma_A, \Gamma_{B_2}, \Gamma_{C_1}, \Gamma_{D_{12}})$ and Σ_Q be the subsystem characterized by $(\Gamma_A, \Gamma_B, \Gamma_{C_2}, \Gamma_{D_{21}})$. The system block is show in Figure 4.2. Without the lose of generality, we assume that $\Gamma_{D_{22}} = 0$, thus we have the following solutions.

Theorem 4.3.1. [9](Regular Case) *The system satisfies the following assumptions*

1. Σ_P has no invariant zeros on the imaginary axis and is left invertible;
2. Σ_Q has no invariant zeros on the imaginary axis and is right invertible;

then the optimal controller is given by Σ_{cmp} :

$$\begin{cases} v(k+1) &= (\Gamma_A + \Gamma_B F + K \Gamma_{C_1} - \Gamma_B N \Gamma_{C_1}) v(k) \\ &+ (\Gamma_B N - K) y(k, 0) \\ \rho_k &= (F - N \Gamma_{C_1}) v(k) + N y(k, 0) \end{cases} \quad (4.6)$$

Figure 4.2: Block diagram of standard H_2 optimal control.

where

$$F = -(\Gamma_{B_1}^T P \Gamma_{B_1} + \Gamma_{D_{21}}^T \Gamma_{D_{21}})^{-1}(\Gamma_{B_1}^T P \Gamma_A + \Gamma_{D_{21}}^T \Gamma_{C_2})$$

$$K = -(\Gamma_A Q \Gamma_{C_1}^T + \Gamma_{B_2} \Gamma_{D_{12}}^T)(\Gamma_{D_{12}} \Gamma_{D_{12}}^T + \Gamma_{C_1} Q \Gamma_{C_1}^T)^{-1}$$

and

$$N = -(\Gamma_{B_1}^T P \Gamma_{B_1} + \Gamma_{D_{21}}^T \Gamma_{D_{21}})^{-1}[(\Gamma_{B_1}^T P \Gamma_A + \Gamma_{D_{21}}^T \Gamma_{C_2}) \\ \times Q \Gamma_{C_1} + \Gamma_{B_1}^T P \Gamma_{B_2} \Gamma_{D_{12}}^T](\Gamma_{D_{12}} \Gamma_{D_{12}}^T + \Gamma_{C_1} Q \Gamma_{C_1}^T)^{-1}$$

and where $P = P^T \geq 0$ and $Q = Q^T \geq 0$ are, respectively, the stabilizing solutions of the following Riccati equations:

$$P = \Gamma_A^T P \Gamma_A + \Gamma_{C_2}^T \Gamma_{C_2} - (\Gamma_{C_2}^T \Gamma_{D_{21}} + \Gamma_A^T P \Gamma_{B_1}) \\ \times (\Gamma_{D_{21}}^T \Gamma_{D_{21}} + \Gamma_{B_1}^T P \Gamma_{B_1})^{-1}(\Gamma_{D_{21}}^T \Gamma_{C_2} + \Gamma_{B_1}^T P \Gamma_A)$$

$$Q = \Gamma_A Q \Gamma_A^T + \Gamma_{B_2} \Gamma_{B_2}^T - (\Gamma_{B_2} \Gamma_{D_{12}}^T + \Gamma_A Q \Gamma_{C_1}^T) \\ \times (\Gamma_{D_{12}} \Gamma_{D_{12}}^T + \Gamma_{C_1} Q \Gamma_{C_1}^T)^{-1}(\Gamma_{D_{12}} \Gamma_{B_2}^T + \Gamma_{C_1} Q \Gamma_A^T)$$

If the system does not satisfy the conditions, we have the follow result to find a suboptimal solutions.

Theorem 4.3.2. [9](Singular Case) *The system does not satisfy the conditions states in Theorem 4.3.1. The following perturbation approach would yield a sub-optimal controller for the general discrete-time system, given any $\varepsilon > 0$, define*

$$\hat{B}_2 = [\Gamma_{B_2}, \varepsilon I, 0]; \hat{D}_{12} = [\Gamma_{D_{12}}, 0, \varepsilon I]; \hat{C}_2 = \begin{bmatrix} \Gamma_{C_2} \\ \varepsilon I \\ 0 \end{bmatrix}; \hat{D}_{21} = \begin{bmatrix} \Gamma_{D_{21}} \\ 0 \\ \varepsilon I \end{bmatrix}; \text{ replace } \Gamma_{B_2},$$

$\Gamma_{D_{12}}, \Gamma_{C_2}$ and $\Gamma_{D_{21}}$ in Theorem 4.3.1 by $\hat{B}_2, \hat{D}_{12}, \hat{C}_2$ and $\hat{D}_{21}$ respectively, then the suboptimal solutions are similar with the solutions in Theorem 4.3.1.

Theorem 4.3.1 and Theorem 4.3.2 give the solutions of the case that $\Gamma_{D_{22}} = 0$ is hold. It is show in [119] that for more general case that $\Gamma_{D_{22}} \neq 0$, Since (Γ_A, Γ_{B_1}) is stabilizable, There is a constant matrix F_b to make $\Gamma_A + \Gamma_{B_1}F_b$ stable. let $v(k) = u(k) - F_b x(k)$, then the system separates to two subsystems,

$$\begin{bmatrix} x(k+1) \\ z(k) \end{bmatrix} = \begin{bmatrix} \Gamma_A + \Gamma_{B_1}F_b & \Gamma_{B_1} & \Gamma_{B_2} \\ \Gamma_{C_2} + \Gamma_{D_{21}}F_b & \Gamma_{D_{21}} & \Gamma_{D_{22}} \end{bmatrix} \begin{bmatrix} x(k) \\ v(k) \\ w(k) \end{bmatrix}$$

and

$$\begin{bmatrix} x(k+1) \\ y(k) \\ v(k) \end{bmatrix} = \begin{bmatrix} \Gamma_A & \Gamma_{B_1} & \Gamma_{B_2} \\ \Gamma_{C_1} & 0 & \Gamma_{D_{12}} \\ -F_b & I & 0 \end{bmatrix} \begin{bmatrix} x(k) \\ u(k) \\ w(k) \end{bmatrix}$$

$$\text{where } G_1 = \begin{bmatrix} \Gamma_A + \Gamma_{B_1}F_b & \Gamma_{B_2} & \Gamma_B \\ \Gamma_{C_2} + \Gamma_{D_{21}}F_b & \Gamma_{D_{22}} & \Gamma_{D_{21}} \end{bmatrix}, G_{tmp} = \begin{bmatrix} \Gamma_A & \Gamma_{B_1} & \Gamma_{B_2} \\ \Gamma_{C_1} & 0 & \Gamma_{D_{12}} \\ -F_b & I & 0 \end{bmatrix}.$$

Figure 4.3 shows the structure of H_2 optimal control for $\Gamma_{D_{22}} \neq 0$ case.

Another approach to solve such optimization problem by using Linear Matrix Inequalities (LMIs) is achieved in [120] which follows such theorem.

Figure 4.3: Block diagram of H_2 optimal control for $\Gamma_{D22} \neq 0$.

Theorem 4.3.3. (H_2 output-feedback) All controllers in the form 4.5 such that the inequality $\|T_{\bar{w}\bar{z}}(\zeta)\|_2^2 < \mu$ holds are parameterized by the LMI. The symbol $T_{\bar{w}\bar{z}}(\zeta)$ denotes the transfer function from the input w to the output z .

$$\text{trace}W < \mu \quad (4.7)$$

$$\begin{bmatrix} W & \Gamma_{C_2}X + \Gamma_{D_{21}}L & \Gamma_{C_2}X + \Gamma_{D_{21}}R\Gamma_{C_1} \\ (\bullet)' & X + X' - P & I + S' - J \\ (\bullet)' & (\bullet)' & Y + Y' - H \end{bmatrix} > 0 \quad (4.8)$$

$$\begin{bmatrix} P & J & \Gamma_A X + \Gamma_{B_1}L & \Gamma_A + \Gamma_{B_1}R\Gamma_{C_1} & \Gamma_{B_2} + \Gamma_{B_1}R\Gamma_{D_{12}} \\ (\bullet)' & H & Q & Y\Gamma_A + F\Gamma_{C_1} & Y\Gamma_{B_2} + F\Gamma_{D_{12}} \\ (\bullet)' & (\bullet)' & X + X' - P & I + S' - J & 0 \\ (\bullet)' & (\bullet)' & (\bullet)' & Y + Y' - H & 0 \\ (\bullet)' & (\bullet)' & (\bullet)' & (\bullet)' & I \end{bmatrix} > 0 \quad (4.9)$$

$$\Gamma_{D_{22}} + \Gamma_{D_{21}}R\Gamma_{D_{12}} = 0 \quad (4.10)$$

where the matrices X, L, Y, F, Q, R, S, J and the symmetric matrices P, H and W are the variables.

Figure 4.4: HDD servo loop with disturbance models

4.4 Application to HDD Systems

4.4.1 System Modeling

A typical HDD servo loop with disturbance models is shown in Figure 4.4. $P(z)$ represents the actuator plant, v includes all torque disturbances, d represents disturbances that are due to nonrepeatable disk and spindle motions, n represents the PES demodulation and electrical noise. D_1 , D_2 and N are the models of v , d , and n , and $w_k (k = 1, 2, 3)$ is a white noise with variance equal to 1. For track following, the reference signal $r = 0$.

The frequency characteristics of the HDD VCM actuator have been measured using a LDV and a Dynamic Signal Analyzer. Based on the measured frequency responses, its transfer function can be obtained in Chapter 2 by Equation (2.8).

The disturbance and noise models ($D_1(s)$, $D_2(s)$, $N(s)$) are identified based on the PES NRRO power spectrum measured from our STW platform and given by Equation (2.24).

4.4.2 Control Problem Formulation

We apply a discrete controller with the form of $u(k, i) = Hq(k, i) + My(k, i) = k_i q(k, i) + k_p y(k, i)$. We now derive the state-space representation (4.5) for the system in Figure 4.4 with

$$x = [x_p^T, x_v^T, x_d^T, x_n^T]^T$$

$$w = [w_1, w_2, w_3]^T$$

(A_p, B_p, C_p, D_p) , (A_v, B_v, C_v, D_v) , (A_d, B_d, C_d, D_d) , (A_n, B_n, C_n, D_n) are respectively the state-space models of $P(z)$, $V(z)$, $D(z)$, $N(z)$. Thus for our problem in system (4.1)

$$A = \begin{bmatrix} A_p & B_p C_v & 0 & 0 \\ 0 & A_v & 0 & 0 \\ 0 & 0 & A_d & 0 \\ 0 & 0 & 0 & A_n \end{bmatrix},$$

$$B_2 = \begin{bmatrix} B_p D_v & 0 & 0 \\ B_v & 0 & 0 \\ 0 & B_d & 0 \\ 0 & 0 & B_n \end{bmatrix}, B_1 = \begin{bmatrix} B_p & 0 & 0 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -C_p & 0 & -C_d & C_n \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -C_p & 0 & -C_d & 0 \end{bmatrix}, D_{21} = \begin{bmatrix} 0 \end{bmatrix},$$

$$D_{12} = \begin{bmatrix} 0 & -D_d & D_n \end{bmatrix}, D_{22} = \begin{bmatrix} 0 & -D_d & 0 \end{bmatrix},$$

We investigate 2 cases, the matrices are shown as follows,

- $n = 1$, in the system (4.5),

$$\bar{w}(k) = w(k, 0), \bar{y}(k) = y(k, 0), \bar{z}(k) = z(k, 0),$$

$$\Gamma_A = A + B_1 M C_1 + B_1 G, \Gamma_{B_1} = B_1 H,$$

$$\Gamma_{B_2} = B_1 M D_{12} + B_2, \Gamma_{C_1} = C_1, \Gamma_{D_{12}} = D_{12},$$

$$\Gamma_{C_2} = C_2 + D_{21} M C_1, \Gamma_{D_{21}} = D_{21} H,$$

$$\Gamma_{D_{22}} = D_{21} M D_{12} + D_{22}.$$

- $n = 2$, in the system (4.5),

$$\bar{w}(k) = (w(k, 0)^T, w(k, 1)^T)^T, \bar{e}(k) = (e(k, 0)^T, e(k, 1)^T)^T,$$

$$\bar{z}(k) = (z(k, 0)^T, z(k, 1)^T)^T, \tilde{A} = \bar{A}^n, \tilde{B}_2 = [\bar{A}\bar{B} \quad \bar{B}],$$

$$\tilde{C}_1 = \begin{bmatrix} \bar{C}_1 \\ \bar{C}_1 \bar{A} \end{bmatrix}, \tilde{C}_2 = \begin{bmatrix} \bar{C}_2 \\ \bar{C}_2 \bar{A} \end{bmatrix}, \tilde{D}_{12} = \begin{bmatrix} \bar{D}_{12} & 0_{v \times s} \\ \bar{C}_1 \bar{B} & \bar{D}_{12} \end{bmatrix}, \tilde{D}_{22} = \begin{bmatrix} \bar{D}_{22} & 0_{v \times p} \\ \bar{C}_2 \bar{B} & \bar{D}_{22} \end{bmatrix}.$$

Matrix $\Gamma_A, \Gamma_{B_1}, \Gamma_{B_2}, \Gamma_{C_1}, \Gamma_{C_2}, \Gamma_{D_{11}}, \Gamma_{D_{21}}$ can be obtained by partitioning $\tilde{A}$, $\tilde{B}_2$, $\tilde{C}_1$ and $\tilde{C}_2$:

$$\tilde{A} = \begin{pmatrix} \Gamma_A & \Gamma_{B_1} \\ * & * \end{pmatrix}, \tilde{B}_2 = \begin{pmatrix} \Gamma_{B_2} \\ * \end{pmatrix},$$

$$\tilde{C}_1 = \begin{pmatrix} \Gamma_{C_1} & \Gamma_{D_{11}} \end{pmatrix}, \tilde{C}_2 = \begin{pmatrix} \Gamma_{C_2} & \Gamma_{D_{21}} \end{pmatrix},$$

$$\Gamma_{D_{12}} = \tilde{D}_{12}, \Gamma_{D_{22}} = \tilde{D}_{22}$$

Then we can get the solution from the procedures in Subsection 4.3.2.

4.4.3 Implementation Results

Figures 4.5, 4.6 and 4.7 show the comparison of output displacement, control signal and output spectrum between baseline PID and H_2 reset control using single sampling rate and double sampling rate respectively. Figure 4.6 shows that H_2 reset

Figure 4.5: Output comparison

control has larger control effort than PID control, the single sampling reset control has larger control than double reset control. Figure 4.7 shows that the disturbance attenuation is much improved in lower frequency.

Figure 4.8 shows the sensitivity functions with baseline PID control, LQR reset control in previous chapter and the H_2 reset control. Since the proposed LQR reset control and H_2 reset method are non-linear method, we use frequency scanning method to generate it. It is shown that proposed H_2 reset method has increased the disturbance rejection capability in the low frequency range. It also shows the different effect of single sampling rate reset and double sampling rate reset. We can observe that double sampling rate reset has higher bandwidth, and in mid-frequency band $< 1k$ Hz, the sensitivity function gain with the double sampling reset is lower than the single one. Since in double sampling rate reset, reset mechanism and linear

Figure 4.6: Control signal comparison.

Figure 4.7: Spectrum comparison

Figure 4.8: Sensitivity function

controller take place by turns and the two strategies can coordinate and cooperate with each other, the total effect is improved.

Since the disturbance concentrates on the lower frequency band (< 200 Hz) as seen in Figure 4.7, based on the sensitivity functions in Figure 4.8 the single rate reset has better suppression effect than the double rate reset, which is noticed in Figure 4.5 and Figure 4.7. However, Figure 4.8 also shows that the double rate reset has better attenuation capability in middle frequency band ($200\text{Hz}, 1\text{kHz}$). Thus it implies a potential to reject disturbances in wider frequency range. It would be then significant to further investigate the attenuation capability of the optimal reset control to disturbances in the overall frequency range.

Since the H_2 reset control can achieve much better disturbance attenuation ability than LQR reset in almost all frequency range which shown in Figure 4.8, it is possible to design a unified method which combines the merits of two methods together and achieve a better performance in a whole. The next chapter will show

a unified control, in which, a soft switch is designed to apply the two optimal reset controllers together in order to achieve better performance for each stage.

4.5 Summary

In this chapter, in order to suppress the internal and external disturbance, we have studied discrete optimal reset law design of reset control systems. Firstly, we transferred closed-loop reset control system to discrete linear system with reset value as control input. Then we convert the optimal reset problem to a standard H_2 optimal control problem. Based on this, the optimal reset laws were given in terms of Riccati equations. The design process of the proposed reset law is very simple. The resulting reset law is of feedback form and thus can easily be implemented online.

The previous chapter and this chapter have presented two types of optimal reset control methods. The first one improved transient response performance; the second one improved the disturbance attenuation performance. The next chapter will represent a unified control method to combine the merits of the two method which can achieve fast seeking, smooth settling and precise following.

Chapter 5

A Unified Controller Design Using Two Different Types of Optimal Reset Control Laws for HDD Servo Systems

This chapter proposes a unified reset control design method to achieve both track-seeking and track-following performance in hard disk drive servo systems. It involves a discrete LQR reset controller which has been shown in Chapter 3 and an H_2 reset controller which has been shown in Chapter 4. The LQR reset controller is more capable of improving transient response performance which includes fast raise time and little overshoot. The discrete-time H_2 reset controller performs a better disturbance rejection performance which is more suitable for track following stage. The unified control makes a soft switch from LQR reset controller to H_2 reset controller, which avoids sharp vibrations.

5.1 Introduction

The previous two chapters have shown two reset methods which achieve good performance in one stage each. This chapter aims at combining the two controllers together, letting them get good performance in each stage and achieve a unified performance. The issue of unified servo control leads to a mode switch control. A lot of literature is relevant. [20] solves such issue by using initial value compensation (IVC). [121] improves the IVC performance by solving LMIs. [31] proposes a new nonlinear time-varying unified control scheme (UCS) to compile a normal controller and H_∞ controller in HDDs so as to avoid the problem of mode switching. The unified reset control proposed in this chapter utilizes the advantages of reset control and avoids the mode switching problem as well.

This chapter is arranged as follows. In Section 5.2, we introduce the structure of unified reset control. Section 5.3 presents the design method of the proposed controller. Section 5.4 gives the application of the proposed design method to HDD systems. Some concluding remarks are made in Section 5.5.

5.2 Unified Control Scheme for HDD Servo System

The unified reset control system is similar with Figure 4.1 in the previous chapter.

The dynamics of the plant system is described by

$$\begin{cases} x(k+1) &= Ax(k) & + B_1 u(k) & + B_2 w(k) \\ e(k) &= C_1 x(k) & & + D_{12} w(k) \\ z(k) &= C_2 x(k) & + D_{21} u(k) & + D_{22} w(k) \end{cases} \quad (5.1)$$

with

$$x = [x_p^T \ x_{d_1}^T \ x_{d_2}^T \ x_n^T]^T,$$

$$w = [w_1 \ w_2 \ w_3]^T,$$

where $x(i) \in \mathbb{R}^l$ is the state, $y(i) \in \mathbb{R}^s$, $z(i) \in \mathbb{R}^p$, $u(i) \in \mathbb{R}^m$ is the control input. (A_p, B_p, C_p, D_p) , $(A_{d_1}, B_{d_1}, C_{d_1}, D_{d_1})$, $(A_{d_2}, B_{d_2}, C_{d_2}, D_{d_2})$, (A_n, B_n, C_n, D_n) are respectively the state-space models of $P(z)$, $D_1(z)$, $D_2(z)$, $N(z)$. x_p^T , $x_{d_1}^T$, $x_{d_2}^T$ and x_n^T are respectively the states of $P(z)$, $D_1(z)$, $D_2(z)$, $N(z)$. The state matrix of system (5.1) can be written as

$$A = \begin{bmatrix} A_p & B_p C_{d_1} & 0 & 0 \\ 0 & A_{d_1} & 0 & 0 \\ 0 & 0 & A_{d_2} & 0 \\ 0 & 0 & 0 & A_n \end{bmatrix},$$

$$B_1 = \begin{bmatrix} B_p \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} B_p D_{d_1} & 0 & 0 \\ B_{d_1} & 0 & 0 \\ 0 & B_{d_2} & 0 \\ 0 & 0 & B_n \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -C_p & 0 & -C_{d_2} & C_n \end{bmatrix},$$

$$C_2 = \begin{bmatrix} -C_p & 0 & -C_{d_2} & 0 \end{bmatrix}, D_{21} = \begin{bmatrix} 0 \end{bmatrix},$$

$$D_{12} = \begin{bmatrix} 0 & -D_{d_2} & D_n \end{bmatrix}, D_{22} = \begin{bmatrix} 0 & -D_{d_2} & 0 \end{bmatrix}.$$

The sampling time of the baseline controller is denoted as T_s . We assume that the reset interval T_{rst} is at every η sampling intervals, i.e., $T_{rst} = \eta T_s$, as the same with pervious chapter.

The variables k and i are used to represent the time instant $t = kT_{rst} + iT_s$

with $i = 0, 1, 2, \dots, \eta - 1$.

Here, the design of nonlinear time-varying control law is to achieve a fast and smooth track-seeking as well as high precision track-following performance. Motivated by the previous works [27, 31], we have the following control law for the proposed unified control scheme:

$$\begin{cases} q(k, i) = Eq(k, i - 1) + Fe(k, i - 1), \\ q(k, 0) = \rho_{LQR}(k, 0) + g(k)\rho_{H_2}(k, 0), \\ u(k, i) = Hq(k, i) + Me(k, i) + Gx(k, i), \end{cases} \quad (5.2)$$

where $q(k, i) \in \mathbb{R}^v$ is the dynamic state in the controller, E, F, G, H and M are all constant matrices with compatible dimensions. $\rho_{LQR}(k, 0)$ is discrete-time LQR reset value at time instant $t = kT_{rst}$ which is designed to achieve fast and smooth track-seeking, $\rho_{H_2}(k, 0)$ is discrete-time H_2 reset control value which is designed to achieve high precision track-following performance. $g(k)$ is a scalar function which satisfies the following conditions:

- 1) $g(k) \in \mathbb{R}$;
- 2) $g(k) = 0 \quad \forall k < k_s$;
- 3) $g(k) - g(k - 1) \geq 0 \quad \forall k$;
- 4) $\lim_{k \rightarrow \infty} g(k) = 1$.

k_s is the settling time, i.e., when the system output is approaching the target. The scalar function $g(k)$ in this chapter is designed as follows

$$\begin{cases} g(k) = 0, & k < \bar{k}_s, \\ g(k) = 1 - \exp[-\kappa((k - \bar{k}_s)T_{rst})^2], & \text{else,} \end{cases}$$

where $\bar{k}_s = k_s + \epsilon$, $\epsilon > 0$, and $\kappa > 0$. These two parameters determine how fast $g(k)$ approaches unity, which in turn decide how fast the set of control parameters of the

unified controller is tuned from the one that is optimized for rapid track-seeking to another which is optimized for high accuracy track-following.

5.3 Design of the Unified Control

5.3.1 Design of the Baseline Linear Control

The baseline linear controller is given by the first and the third equations in Equation (5.2). We design a normal PID control which can achieve short rising time.

$$u(k, i) = \alpha e(k, i) + \beta x_2(k, i) + \gamma q(k, i) \quad (5.3)$$

x_2 is the second state in x_p which stands for the velocity of plant motion. The PID gains α, β, γ are corresponding to the parameters in matrices G, H and M .

5.3.2 Design of ρ_{LQR}

For track seeking, we propose a discrete-time LQR reset control scheme. Our target is to achieve swift track seeking performance including fast rising time together with less overshoot. We simplify the plant model as a second order system, assuming that the higher order resonances have been compensated by notch filters. The disturbances and measurement noise, which may impact on the position error in track-following, are not considered at this stage. Thus, the dynamics of the plant is described by

$$\begin{cases} x_p(i+1) &= A_p x_p(i) + B_p u(i) \\ y(i) &= C_p x_p(i). \end{cases} \quad (5.4)$$

We then introduce the following discrete reset controller:

$$q(k, 0) = \rho_{LQR}(k). \quad (5.5)$$

Our target for designing this controller is to minimize the error for tracking a reference with the control effort constraint, thus we consider such a cost function which includes error quadratic term and control effort quadratic term. For each time instant $t = kT_{rst} + iT_s$, we define the cost function $J(k, i)$ as

$$J(k, i) = e^T(k, i)Qe(k, i) + (u(k, i) - u_r)^T R(u(k, i) - u_r)$$

where Q and R are positive semi-definite matrices, and u_r is the steady state of control effort. Thus the overall cost function should be

$$J(\infty) := \sum_{k=0}^{\infty} \sum_{i=0}^{n-1} J(k, i). \quad (5.6)$$

To minimize such cost function, we find a method to convert this problem to a standard LQR problem. Chapter 3 presents the equivalence between the cost function minimization problem and LQR problem, and the design procedures. In order to transfer the steady-state of the system to original, we make a coordinate transformation as follows:

$$\begin{cases} \xi_x(k, i) = x_p(k, i) - \mathbf{x}_r \\ \xi_q(k, i) = q(k, i) - \mathbf{q}_r \end{cases} \quad (5.7)$$

where $\mathbf{x}_r$ and $\mathbf{q}_r$ are the steady-state values. After several equivalent transformation and deduction, the system transfer function has been converted to a discrete one which has reset interval as its sampling interval,

$$\xi_x(k+1) = \Gamma_A \xi_x(k) + \Gamma_{B_1} \rho_{LQR}(k) \quad (5.8)$$

where Γ_A and Γ_{B_1} are derived as in Chapter 3. The optimal reset value for each reset moment is given by

$$\rho_{LQR}(k) = -K_s \xi_x(k) + \mathbf{q}_r, \quad (5.9)$$

where K_s is obtained by solving Riccati equations.

5.3.3 Design of ρ_{H_2}

For track-following, we propose a discrete H_2 reset control scheme. Our target in this scheme is to achieve a better performance for disturbance rejection. So the input disturbance, output disturbance and measurement noise are included in the system modeling.

Then we introduce the following discrete-time reset controller which has the same structure as an LQR reset controller:

$$q(k, 0) = \rho_{H_2}(k). \quad (5.10)$$

After several equivalent transformation including replace x_p by ξ_x in x , the system function becomes

$$\begin{cases} \bar{x}(k+1) = \tilde{\Gamma}_A \bar{x}(k) + \tilde{\Gamma}_{B_1} \rho_{H_2}(k) + \tilde{\Gamma}_{B_2} w(k) + \tilde{\Gamma}_{B_1} \mathbf{q}_r \\ e(k) = \tilde{\Gamma}_{C_1} \bar{x}(k) + \tilde{\Gamma}_{D_{11}} \rho_{H_2}(k) + \tilde{\Gamma}_{D_{12}} w(k) \\ z(k) = \tilde{\Gamma}_{C_2} \bar{x}(k) + \tilde{\Gamma}_{D_{21}} \rho_{H_2}(k) + \tilde{\Gamma}_{D_{22}} w(k) \end{cases} \quad (5.11)$$

where

$$\bar{x} = \begin{bmatrix} \xi_x^T & x_{d_1}^T & x_{d_2}^T & x_n^T \end{bmatrix}^T,$$

$$\tilde{\Gamma}_A = \begin{bmatrix} \Gamma_A - \Gamma_{B_1} K_s & \Gamma_{B_1} C_{d_1} & 0 & 0 \\ 0 & A_{d_1} & 0 & 0 \\ 0 & 0 & A_{d_2} & 0 \\ 0 & 0 & 0 & A_n \end{bmatrix},$$

$$\tilde{\Gamma}_{B_1} = \begin{bmatrix} \Gamma_{B_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tilde{\Gamma}_{B_2} = \begin{bmatrix} \Gamma_{B_1} D_{d_1} & 0 & 0 \\ B_{d_1} & 0 & 0 \\ 0 & B_{d_2} & 0 \\ 0 & 0 & B_n \end{bmatrix},$$

$$\tilde{\Gamma}_{C_1} = C_1, \tilde{\Gamma}_{C_2} = C_2, \tilde{\Gamma}_{D_{21}} = [0], \tilde{\Gamma}_{D_{12}} = D_{12}, \tilde{\Gamma}_{D_{22}} = D_{22}.$$

Denote T_{zw} as the transfer function matrix from w to z . The discrete-time H_2 optimal reset control problem is then stated as: *Design a reset control law $\rho_{H_2}(k)$ such that the H_2 norm $\|T_{zw}\|_2$ is minimized.*

To minimize the H_2 norm, we find a method to convert this problem to a standard H_2 problem. Chapter 4 shows the equivalence between these two problems, and the design procedures. We just show the conclusion in this chapter. The optimal reset value for each reset moment is given by solving Riccati equations or LMIs in Chapter 4 and in terms of feedback from $x(k)$,

$$\rho_{H_2}(k) = -K_f \xi_x(k) = -\bar{K}_f \bar{x}(k) \quad (5.12)$$

where $\bar{K}_f = \begin{bmatrix} K_f & 0 & 0 & 0 \end{bmatrix}$.

5.3.4 Stability for the Unified Control Scheme

For the stability issues, we can omit the effect of disturbance and noise, and just consider such a system,

$$\bar{x}(k+1) = \tilde{\Gamma}_A \bar{x}(k) + \tilde{\Gamma}_{B_1} \rho_{H_2}(k).$$

For a nonlinear controller or a switching controller, there is a sufficient condition for the stability. We introduce such a lemma.

Lemma 5.3.1. *Suppose systems*

$$\mathcal{X}(k+1) = \mathcal{A} \mathcal{X}(k) \tag{5.13}$$

and

$$\mathcal{X}(k+1) = (\mathcal{A} + \mathcal{B}) \mathcal{X}(k) \tag{5.14}$$

are asymptotically stable. Then the system

$$\mathcal{X}(k+1) = (\mathcal{A} + \mu \mathcal{B}) \mathcal{X}(k) \quad (0 \leq \mu \leq 1) \tag{5.15}$$

is asymptotically stable if the system (5.13) and the system (5.14) can find a common Lyapunov function.

Proof: Suppose we have

$$\mathcal{A}' P \mathcal{A} - P = -W_1 < 0$$

$$(\mathcal{A} + \mathcal{B})' P (\mathcal{A} + \mathcal{B}) - P = -W_2 < 0$$

then

$$\begin{aligned}
& (\mathcal{A} + \mu\mathcal{B})'P(\mathcal{A} + \mu\mathcal{B}) - P \\
&= \mathcal{A}'P\mathcal{A} + 2\mu\mathcal{B}'P\mathcal{A} + \mu^2\mathcal{B}'P\mathcal{B} - P \\
&= \mu\mathcal{A}'P\mathcal{A} + 2\mu\mathcal{B}'P\mathcal{A} + \mu\mathcal{B}'P\mathcal{B} + (1 - \mu)\mathcal{A}'P\mathcal{A} - P - (\mu - \mu^2)\mathcal{B}'P\mathcal{B} \\
&= -(1 - \mu)W_1 - \mu W_2 - (\mu - \mu^2)\mathcal{B}'P\mathcal{B} < 0
\end{aligned}$$

Thus the system

$$\mathcal{X}(k+1) = (\mathcal{A} + \mu\mathcal{B})\mathcal{X}(k) \quad (0 \leq \mu \leq 1) \quad (5.16)$$

is asymptotically stable. $\square$

We consider the system $\bar{x}(k+1) = \tilde{\Gamma}_A\bar{x}(k)$ as the system (5.13) in Lemma 5.3.1. The system's stability is guaranteed according to Section 3.3. With $\rho(k) = -\bar{K}_f\bar{x}(k)$ as the control effort to the system $\bar{x}(k+1) = \tilde{\Gamma}_A\bar{x}(k) + \tilde{\Gamma}_{B_1}\rho_{H_2}(k) = (\tilde{\Gamma}_A - \tilde{\Gamma}_{B_1}\bar{K}_f)\bar{x}(k)$, its stability is guaranteed since the H_2 optimal control problem is solved based on Lyapunov stability. According to Lemma 5.3.1 as $0 < g(k) \leq 1$, the system can also be stabilized by such a control effort $\rho(k) = -g(k)\bar{K}_f\bar{x}(k)$.

5.4 System Identification and Controller Design

5.4.1 System Modeling

Consider a typical HDD servo loop. In Figure 4.1, D_1 includes all torque disturbances, D_2 represents disturbances that are due to nonrepeatable disk and spindle motions, N represents the PES demodulation and electrical noise.

Based on the measured frequency response, its transfer function is obtained as Equation (2.8). with the frequency responses plotted in Figure 2.4.

The disturbance and noise models $(D_1(s), D_2(s), N(s))$ are identified based on the PES NRRO power spectrum measured from the STW platform and given by [65].

Figure 5.1: Displacement for three kinds of controllers.

5.4.2 Controller Design for HDD Servo System

Our sampling time is $T_s = 2.5 \times 10^{-5}$ sec. Six 2nd order notch filters are used to compensate the resonances.

The reset controller is given by

$$\begin{cases} q(k, i) = Eq(k, i-1) + Fe(k, i-1), \\ q(k, 0) = \rho_{LQR}(k) + g(k)\rho_{H_2}(k) \\ u(k, i) = \alpha e(k, i) + \beta x_2(k, i) + \gamma q(k, i), \end{cases} \quad (5.17)$$

with $E = 1$, $F = T_s$, $\alpha = 0.0018$, $\beta = 1 \times 10^{-6}$, and $\gamma = 5$.

By using the design method in Chapter 3, The LQR reset law and the H_2 reset law is obtained respectively by solving Riccati equations and LMIs.

In $g(k)$, $\kappa = 2 \times 10^3$, $\bar{k}_s = 80$. Though a large κ ensures a fast switch between different modes of operation, it is noted if it varies too fast, a rapid change in control signal may cause undesirable excitations of poorly damped actuator modes to occur.

Figure 5.2: Displacement for three kinds of controllers (zoomed-in).

5.4.3 Implementation Results

In our implementation,

$$\rho_{LQR}(k) = -6.056 \times 10^{-4}(x_1 - r) - 2.618 \times 10^{-8}\hat{x}_2 + 1.056 \times 10^{-5}r,$$

$$\rho_{H_2}(k) = \frac{0.006479z^4 - 0.01497z^3 + 0.008579z^2 + 0.002095z - 0.00218}{z^4 - 0.6324z^3 - 1.323z^2 + 0.5529z + 0.4033}e(k).$$

The order of $\rho_{H_2}(k)$ is always the same as the one of the plant in general, but the order can be reduced, since there are zeros and poles cancelation.

Figures 5.1-5.2 show the comparison of step responses using the unified control, H_2 reset control and LQR reset control, respectively. From these two figures, the unified controller achieves both the rapid track-seeking and high accuracy track-following while the discrete LQR reset controller and the discrete H_2 reset controller achieve their own purposes. Figures 5.3-5.4 show the comparison of control signals of the unified control, the H_2 reset control and the LQR reset control used in track-seeking and track-following, respectively. It is seen that the unified reset control has smaller control effort at track-seeking stage which is inherited from the discrete LQR reset control and larger control effort at track following stage which is inherited from discrete H_2 reset control.

Figure 5.3: Track seeking control signals using three controllers respectively.

Figure 5.4: Track following control signals using three controllers respectively.

5.5 Summary

In this chapter, we have studied the unified reset control law for HDD track-seeking and track-following. It includes the discrete LQR reset controller for track-seeking and the discrete-time H_2 reset controller for track-following. A soft switch is designed for the two optimal reset controllers together in order to achieve better performance for each stage. The simulation results have shown that the unified reset controller is able to achieve both swift track-seeking and accurate track-following.

Next chapter will present the optimal reset control design for HDD dual stage servo system.

Chapter 6

Optimal Reset Control for a Hard Disk Drive Dual-stage Actuator System

The previous three chapters present a series of optimal reset controller design approaches which is implied in single stage HDD servo system. They aim at improving the system performance for single-stage hard disk drive. Chapter 3 was about to improve the transient response performance. Chapter 4 proposed a H_2 reset method for improving track following performance. A soft switch is designed in Chapter 5 to apply the two optimal reset controllers together in order to achieve better performance for each stage. The performance of HDD servo systems still can be improved by structure and more sensors. One of the best ways to increase the positioning accuracy is to use a micro-actuator as the second stage actuator which permits finer head positioning over closely-spaced data tracks.

This chapter presents an optimal reset control (ORC) method for a dual-stage actuator servo system (DSA) in HDDs. The dual-actuator system consists of a VCM as a first stage actuator and a push-pull-type piezo-electric transducer

(PZT) as a second-stage actuator. It is considered as Dual-input Single-output (DISO) problem. We designed two optimal reset controllers for VCM and PZT, and combined the two components with a decoupled master-slave structure. With a base linear system designed conventionally, the discrete-time reset law design aims at improving the system transient responses in advance.

6.1 Introduction

This chapter aims to improve the control performance for a dual-stage actuation system in hard disk drives in track-seeking. We design an optimal reset method which is suitable for a DISO system and applied to improve the transient response performance of the dual-stage actuation system.

This chapter is arranged as follows. In Section 6.2, we present the plant modeling and propose a reset control structure. In Section 6.3, we propose the design method of the optimal reset control law. Section 6.4 gives the application results of the proposed design method to HDD track-seeking. Some concluding remarks are made in Section 6.5.

6.2 Proposed Control Structure

The structure of dual-stage tracking control system is shown in Figure 6.1 which is with a structure of Decoupled Master-Slave (DMS) scheme shown in Figure 6.2. In this figure, G_v , G_p , F_v and F_p are the VCM plant model, PZT plant, resonance notch filter for VCM and resonance notch filter for PZT. The performance of the master-slave configuration depends on the estimator of micro actuator output. The DMS scheme is more preferable than the coupled one, because it splits the dual-stage loop into two independent loops and the stability will be guaranteed by

two stable loops. Suppose two linear controllers for VCM and PZT are designed separately, then we design the reset law which determines the reset value to change dynamic state value in the linear controllers.

Figure 6.1: Structure of LQR reset control for dual-stage tracking.

Suppose two linear controllers for each actuator are designed as

$$\begin{cases} \dot{z}_1 = Dz_1 + Ee_1, \\ u_1 = G_1x_1 + H_1z_1 + M_1e_1, \\ e_1 = y_r - \hat{y}_v = y_r - y + \hat{y}_p \end{cases} \quad (6.1)$$

$$\begin{cases} \dot{z}_2 = Dz_2 + Ee_2, \\ u_2 = G_2x_2 + H_2z_2 + M_2e_2, \\ e_2 = y_r - y \end{cases} \quad (6.2)$$

Figure 6.2: Master slave structure for dual-stage tracking.

where z_i, u_i and e_i are the dynamic state in controller, control input error signal respectively. $\hat{y}_v$ and $\hat{y}_p$ are the estimation of the displacements which are introduced by VCM and PZT respectively. D, E, G_i, H_i and M_i are all constant matrices with compatible dimensions. We add reset element for each controller as

$$z_i(t_k^+) = \rho_i(x, y_r), t = t_k^+ \quad i = 1, 2 \quad (6.3)$$

where t_k s are the reset moments. To simplify the problem and reduce the difficulty in the design procedure, we assume that the periods between two contiguous reset moments are equivalent, i.e, $t_{k+1} - t_k = T_{rst}$.

Our objective of ORL design is to find such reset value series ρ_i .

Remark 6.2.1. *Since the estimation of PZT displacement is much more accurate and faster converging speed than the one of VCM displacement, we use $y - \hat{y}_p$ to represent the $\hat{y}_v$ which also coincides with the decoupled master-slave structure.*

6.3 Optimal Reset Law Design

The design of ORC is based on a traditional linear controller which is appropriately predesigned. The ORC algorithm is to reset the dynamic states in the linear controller to optimal values at pre-specified reset moment. The reset mechanism aims to minimize some performance index and the solution is obtained by solving a Riccati equation. Before we proceed to design the reset law, we would like to discuss about the resonance compensation and the state estimation.

6.3.1 State Estimator Design

Since the head position y is the only measurable signal for feedback control, we need to estimate the VCM and PZT actuator states by using the identified DSA model in Equation (2.15). The state estimator is given by

$$\hat{\dot{x}} = A_D \hat{x} + B_D \text{sat}(u) + L(y - C_D \hat{x}) \quad (6.4)$$

where $\hat{x} = [\hat{y}_v \ \hat{\dot{y}}_v \ \hat{y}_p \ \hat{\dot{y}}_p]^T$, $u = [u_1 \ u_2]^T$, $A_D = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$, $B_D = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$, $C_D = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$. The state estimator gain L can be calculated using the pole placement method by selecting the well-damped estimator poles. Such poles are with the frequencies as two to six times faster than the DSA servo bandwidth.

6.3.2 Optimal Reset Law Design

By combining system (2.15), controllers (6.1), (6.2) and reset element (6.3), we have

$$\begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}y_r, & t \neq t_k \\ z(t_k^+) = \rho(x, y_r), & t = t_k^+ \\ y = \bar{C}\bar{x} \end{cases} \quad (6.5)$$

where

$$\bar{x} = \begin{bmatrix} y_v & \dot{y}_v & y_p & \dot{y}_p & z_1^T & z_2^T \end{bmatrix}^T, z = \begin{bmatrix} z_1^T & z_2^T \end{bmatrix}^T$$

$$\bar{A} = \begin{bmatrix} \bar{A}_{11} & 0 & B_1 H_1 & 0 \\ 0 & \bar{A}_{22} & 0 & B_2 H_2 \\ -EC_1 & 0 & D & 0 \\ -EC_1 & -EC_2 & 0 & D \end{bmatrix}, \bar{A}_{11} = A_1 + B_1 G_1 - B_1 M_1 C_1, \bar{A}_{22} = A_2 +$$

$$B_2 G_2 - B_2 M_2 C_2, \bar{B} = \begin{bmatrix} B_1 M_1 & B_2 M_2 & E & E \end{bmatrix}^T, \bar{C} = [C_1 \ C_2].$$

Assumption 6.3.1. For any $y_r \in \mathbb{R}^p$, there exists $\bar{x}_r = (x_r^T \ z_r^T)^T \in \mathbb{R}^{n+q}$, which satisfies

$$\begin{cases} \bar{A}\bar{x}_r + \bar{B}y_r = 0 \\ \bar{C}\bar{x}_r - y_r = 0 \end{cases} \quad (6.6)$$

With the above Assumption, the control input $u_r = [u_{1r} \ u_{2r}]^T$ in the steady state is given by

$$u_r = Gx_r + Hz_r \quad (6.7)$$

$$\text{where } G = \begin{bmatrix} G_1 & 0 \\ 0 & G_2 \end{bmatrix} \text{ and } H = \begin{bmatrix} H_1 & 0 \\ 0 & H_2 \end{bmatrix}.$$

Let $u = [u_1 \ u_2]^T$, then for each reset time interval, we consider a cost function as

$$J_k = \int_{t_k}^{t_{k+1}} e_2^T(t) Q_k e_2(t) + (u(t) - u_r)^T R_k (u(t) - u_r) dt \quad (6.8)$$

The purpose of the reset control law is to minimize the overall cost function

$$J(\infty) = \sum_{k=0}^{\infty} J_k. \quad (6.9)$$

Let $\xi_x = x - x_r$, $\xi_z = z - z_r$. System (6.5) becomes

$$\begin{aligned}\dot{\xi} &= \bar{A}\xi, \quad t \neq t_k \\ \xi_z(t_k^+) &= \bar{\rho}\end{aligned}\tag{6.10}$$

where $\xi = [\xi_x^T \ \xi_z^T]^T$ and $\bar{\rho} = \rho - z_r$, $\rho = [\rho_1 \ \rho_2]^T$.

Replacing e_2 and u in Equation (6.2) in cost function J_k ,

$$J_k = \xi^T(t_k^+) \Theta_k \xi(t_k^+)\tag{6.11}$$

with $\Theta_k = \int_0^{T_{rst}} e^{\bar{A}^T t} \Omega_k e^{\bar{A} t} dt$, $\Omega_k = \begin{pmatrix} \Omega_k^{11} & \Omega_k^{12} \\ \Omega_k^{12^T} & \Omega_k^{22} \end{pmatrix}$, $\Omega_k^{11} = \bar{C}^T Q \bar{C} + (G - M \bar{C})^T R (G - M \bar{C})$, $\Omega_k^{12} = (G - M \bar{C})^T R H$, $\Omega_k^{22} = H^T R H$ and $M = \text{diag}\{M_1, \ M_2\}$.

On the other hand, from system (6.10), we have

$$\begin{pmatrix} \xi_x(t_{k+1}) \\ \xi_z(t_{k+1}) \end{pmatrix} = e^{\bar{A} T_{rst}} \begin{pmatrix} \xi_x(t_k) \\ \xi_z(t_k) \end{pmatrix}.\tag{6.12}$$

Let $e^{\bar{A} T_{rst}} = \begin{pmatrix} \Gamma_A & \Gamma_B \\ * & * \end{pmatrix}$. Equation (6.12) yields

$$\xi_x(t_{k+1}) = \Gamma_A \xi_x(t_k) + \Gamma_B \bar{\rho}.\tag{6.13}$$

System (6.13) is simply a linear discrete-time system with state variable $\xi_x(t_k)$ and control input $\bar{\rho}$. The system (6.5) and system (6.13) are asymptotically stable at the same time under some mild conditions.

Proposition 6.3.1. *Suppose that there exist positive numbers $\varepsilon > 0$ such that*

$$\lambda_{\min}(\Gamma_B^T \Gamma_B) \geq \varepsilon > 0. \quad (6.14)$$

Then system (6.5) is asymptotically stable if and only if system (6.13) is asymptotically stable.

Proof: It is obvious that if system (6.5) is asymptotically stable, then system (6.13) is asymptotically stable. In the following, we assume that system (6.13) is asymptotically stable. Then we have

$$\begin{aligned} \lim_{k \rightarrow \infty} \xi_x(t_k) &= 0, \\ \lim_{k \rightarrow \infty} \Gamma_B \bar{\rho} &= 0. \end{aligned}$$

According to condition (6.14), we have

$$\lim_{k \rightarrow \infty} \bar{\rho} = 0.$$

Note that $\xi_z(t_k) = \bar{\rho}$, we have

$$\lim_{k \rightarrow \infty} \xi(t_k) = 0,$$

and thus system (6.13) is asymptotically stable. $\square$

The problem becomes to find a control law to stabilize system (6.13) and minimize the cost function $J(\infty)$. This is a standard linear quadratic regulation (LQR) problem. Thus the reset control law can be obtained by solving a Riccati equation. The reset value sequence is given by

$$\rho = -K(x(t_k) - x_r) + z_r, \quad k = 0, 1, 2, \dots \quad (6.15)$$

where K is determined by

$$K = (\Gamma_B^T S \Gamma_B + \bar{R})^{-1} (\Gamma_B^T S \Gamma_A + \bar{T}^T) \quad (6.16)$$

and $S > 0$ is the solution of the Riccati equation

$$S = \tilde{Q} + \bar{\Gamma}_A^T S \bar{\Gamma}_A - \bar{\Gamma}_A^T S \Gamma_B (\Gamma_B^T S \Gamma_B + \bar{R})^{-1} \Gamma_B^T S \bar{\Gamma}_A \quad (6.17)$$

with $\bar{\Gamma}_A^T = \Gamma_A - \Gamma_B^T R^{-1} \bar{T}$, $\tilde{Q} = \bar{Q} - \bar{T} \bar{R}^{-1} \bar{T}^T$ and the matrices $\bar{R}$ and $\bar{Q}$ are obtained by partitioning Θ_k as $\Theta_k = \begin{bmatrix} \bar{Q} & \bar{T}^T \\ \bar{T} & \bar{R} \end{bmatrix}$.

6.4 Simulation and Implementation Results

6.4.1 Simulation Results

The control design is illustrated by the simulation results using Matlab and Simulink.

The baseline linear controllers are designed as

$$\begin{aligned} D &= 0, \quad E = 1, \quad M_1 = 4, \quad H_1 = 5 \times 10^3, \quad M_2 = 200, \\ G_1 &= [0 \quad 0.6 \times 10^{-3}], \quad H_2 = 3 \times 10^4, \quad G_2 = [0 \quad 0]. \end{aligned} \quad (6.18)$$

The resonance compensators are designed as

$$\begin{aligned} F_V &= \frac{s^2 + 8972s + 6.964 \times 10^8}{s^2 + 3.167s + 6.964 \times 10^8} \\ &\times \frac{s^2 + 6597s + 4.352 \times 10^9}{s^2 + 79170s + 4.352 \times 10^9} \\ &\times \frac{s^2 + 3.11 \times 10^4 s + 1.075 \times 10^{10}}{s^2 + 1.244 \times 10^5 s + 1.075 \times 10^{10}}, \end{aligned} \quad (6.19)$$

$$\begin{aligned}
F_P &= \frac{s^2 + 5809s + 1.772 \times 10^9}{s^2 + 5.052 \times 10^4 s + 1.772 \times 10^9} \\
&\times \frac{s^2 + 2533s + 4.952 \times 10^9}{s^2 + 8.445 \times 10^4 s + 4.952 \times 10^9} \\
&\times \frac{s^2 + 3.267 \times 10^4 s + 1.605 \times 10^9}{s^2 + 4.574 \times 10^4 s + 1.605 \times 10^9} \\
&\times \frac{s^2 + 6032s + 5.685 \times 10^9}{s^2 + 1.508 \times 10^4 s + 5.685 \times 10^9}.
\end{aligned} \tag{6.20}$$

A full order estimator is designed as a Luenberger observer. The state estimator gain is obtained as

$$L = \begin{bmatrix} 200 & 1 \times 10^4 & 10 & 1 \times 10^4 \end{bmatrix}.$$

Reset interval is chosen as two times of sampling time interval $T_{rst} = 5 \times 10^{-5}$ sec.

The weighting matrices Q and $R = \text{diag}\{R_1, R_2\}$ determine the tradeoff among the control effort of VCM, control effort of PZT within the control limitation. Tuning the ratio R_1/R_2 determines how many contributions the two actuators make to the overall output. Since the DSA system is a DISO system, which means that, for a given desired trajectory, alternative control inputs to the two actuators are not unique.

Table 6.1: Rising time for different R_1 and R_2

$R_2 \backslash R_1$	0.7	0.07	7×10^{-3}	7×10^{-4}	7×10^{-5}	7×10^{-6}
10^{-2}	0.180	0.135	0.128	0.125	0.126	0.180
10^{-4}	0.205	0.200	0.220	6.01	-	-
10^{-5}	0.20	0.20	0.18	0.19	0.19	0.19
10^{-6}	0.20	0.20	0.20	0.18	0.120	0.10
10^{-8}	0.20	0.20	0.198	0.17	0.10	0.10

In Chapter 3, it is discussed how the performance changes with different R/Q ratio in a single stage HDD system. In the dual-stage HDD systems, the seeking performance with different R/Q ratio are investigated. Since such performance is not related to the absolute value but related to the ratio, we can fix the $Q = 1$ and

Table 6.2: Settling time for different R_1 and R_2

$R_2 \backslash R_1$	0.7	0.07	7×10^{-3}	7×10^{-4}	7×10^{-5}	7×10^{-6}
10^{-2}	0.245	0.245	-	0.432	0.433	0.433
10^{-4}	0.285	0.250	0.452	1.01	-	-
10^{-5}	0.342	0.250	0.202	0.220	0.300	0.257
10^{-6}	0.342	0.388	0.263	0.208	0.620	0.611
10^{-8}	0.341	0.338	0.348	0.243	1.75	-

Table 6.3: Overshoot for different R_1 and R_2

$R_2 \backslash R_1$	0.7	0.07	7×10^{-3}	7×10^{-4}	7×10^{-5}	7×10^{-6}
10^{-2}	6.2%	12.8%	30%	36%	37%	38%
10^{-4}	8%	6.0%	2.5%	10%	60%	80%
10^{-5}	3.0%	3.0%	2.2%	3%	3%	3.2%
10^{-6}	4.1%	4.1%	3.3%	2.2%	50%	50%
10^{-8}	4.2%	4.1%	3.9%	2.4%	20%	70%

verify the different R_1 and R_2 . The rising time, settling time (error within 2% of seek length) and overshoot versus selected R_1 s, R_2 s and Q s with $y_r = 1$ are shown in Table 6.1, 6.2 and 6.3.

From the figures, we find larger R_2/R_1 will downgrade the performance and achieve large overshoot, but if R_2/R_1 is smaller than a certain level, decreasing R_2/R_1 will not bring a significant improvement. Thus only a well-tuned R_2/R_1 ratio can achieve better performance.

The ratios R_1/Q and R_2/Q determine to what extent the control gain will reach. Either too big or too small value of R_1/Q and R_2/Q will downgrade the performance and only moderate value can achieve best performance.

Observed from the Table 6.1, 6.2 and 6.3, it is clear that the optimal reset control performs well when $R_1 \in [y_r \times 0.0007 \ y_r \times 0.007]$ and $R_2 \in [y_r \times 10^{-6} \ y_r \times 10^{-5}]$. In this range, the settling time is about 0.23 ms and overshoot is less than

2.5%. In simulations and experiments, we choose $R = \text{diag}\{y_r \times 0.007, y_r \times 10^{-5}\}$. The displacement and control signals for the seeking target $y_r = 0.2$ are shown in Figure 6.3. Figure 6.4 and Figure 6.5 show the comparison between the optimal reset and baseline PID control. It is noticed that a large overshoot is achieved with the designed baseline PID control. That is because the baseline PID control is design to achieve short rising time regardless the large overshoot. And when using LQR reset control, the rising time should be similar with baseline PID control. When reaching the target, the optimal reset control can settle swiftly, no overshoot is noticed. Figure 6.6 shows the displacement for different seeking length which ranged from 0.1 to 5. We find for different seeking length, the proposed method can achieve good seeking performance. The settling time is from 0.1 *ms* to 0.5 *ms*.

6.4.2 Experimental Result

The proposed LQR reset control has been applied to the dual-stage HDD in Figure 2.7. The controller is implemented on real-time DSP system (dSPACE 1103) with a sampling rate of 40 kHz. Reset interval is chosen as two times of the sampling time interval T_s , i.e, $T_{rst} = 2T_s = 5 \times 10^{-5}$ sec. We use Laser Doppler LDV to measure the displacement of HDD read-write head and the displacement range is $1\mu m/v$.

Figure 6.7 shows the displacements and control signals for the seeking target $y_r=0.2 \mu m$. A little overshoot is noticed and the settling time is less than 0.25 ms. Compared with the baseline PID control performance which is shown in Fig 6.9, the raise time with optimal reset control is similar but when reaching the target, the optimal reset control can settle swiftly and smoothly. Figure 6.8 shows the control effort for each stage. Figure 6.10 shows the displacement for the seeking target $y_r=2 \mu m$. Compared baseline linear control effect which is shown in Figure 6.11, the proposed method can also get significant improvement. As seen in Figure 6.10, the overshoot is about 2% of $2 \mu m$, which is about $0.04 \mu m$.

Figure 6.3: Displacements and control signals for seeking target $y_r = 0.2$ with reset control.

Compared with the typical track-seeking control for dual-stage systems in [89], which uses feedforward and reference trajectory method, the optimal reset control proposed in this method is just a feedback form. This makes it possible to utilize the optimal reset control structure in track-following mode as well. In terms of performance, the settling time for $2 \mu m$ ($0.2 \mu m$) seeking is $0.32 ms$ ($0.25 ms$) achieved in this chapter, which is comparable with $0.3 ms$ for $1 \mu m$ and $3 \mu m$ seeking in [89].

Figure 6.4: Displacements and control signals for seeking target $y_r = 2$ with reset control.

6.5 Summary

In this chapter, we have studied an optimal reset control method for a dual-stage actuator system in hard disk drives for track-seeking. It is considered as a Dual-input Single-output reset control problem. We designed two optimal reset controllers for VCM and PZT, and combined the two components with a decoupled master-slave structure. The proposed design method can guarantee the system stability and the solution of the controller has been obtained by solving Riccati equations. Track-seeking performance has been investigated via simulation and experiment. It has been shown that the proposed method is able to achieve short settling time and low overshoot.

The previous chapters have shown a series of optimal reset control and applications. In the next two chapters, two other new control methods that can be

Figure 6.5: Displacement and control signal for seeking target $y_r = 2$ with PID control.

easily implemented by “adding” them onto existing HDD servo loops have been developed. These two new “add-on” solutions are expected to be more useful to the HDD industry.

Figure 6.6: Displacements and control signals for different seeking target.

Figure 6.7: Seeking responses to $y_r = 0.2 \mu\text{m}$ using proposed optimal reset control. The voltage resolution: 50 mV/div, time resolution: 1 ms/div. Within the PZT stroke limit, the settling time is approximately 0.25 ms. (a) Ch3-reference (b) Ch4-output y .

Figure 6.8: Control signal for $y_r = 0.2 \mu\text{m}$. The voltage resolution: 50 mv/div, time resolution: 1 ms/div. (a) Ch3–control signal for VCM (b) Ch4–control signal for PZT.

Figure 6.9: Seeking responses to $y_r = 0.2 \mu\text{m}$ using conventional PID control. The voltage resolution: 50 mv/div, time resolution: 1 ms/div. (a) Ch3–reference (b) Ch4–output y .

Figure 6.10: Seeking responses to $y_r = 2 \mu\text{m}$ using proposed optimal reset control. The voltage resolution: 1 v/div, time resolution: 1 ms/div. Within the PZT stroke limit, the settling time is approximately 0.32 ms. (a) Ch3–reference (b) Ch4–output y .

Figure 6.11: Seeking responses to $y_r = 2 \mu\text{m}$ using conventional PID control. The voltage resolution: 1 v/div, time resolution: 1 ms/div. (a) Ch3–reference (b) Ch4–output y .

Chapter 7

A Novel Algorithm for Unknown Periodic Disturbance Cancellation in HDDs

Previous chapters structured a theoretical framework of optimal reset control and used them to solve several problems in HDD drive, improve the performance for different stages. When the control system is designed, for different HDD implementations, the performance can still be improved by using “add-on” strategy. This chapter and next chapter will present two “add-on” strategies which can attenuate internal and external disturbances effectively.

This chapter focuses on dealing with periodic disturbances caused by different sources. These disturbances which do not change drastically will degrade the performance of voice coil motor actuator based servo system so that significant repeatable run-out is formed in position error signal. This chapter shows a method which can cancel most of the components in it. In this method, we suppose to rebuild a signal which can approximately represent the disturbances thus cancel the disturbances by adding the opposite value of this signal. When rebuilding this signal, the internal

model principle and magnitude/phase adaptive estimation algorithm are used to recreate the disturbance signal. A polynomial adaptive fitting method which does not need frequency estimation is proposed to deal with the time delay problem which exists in the signal recreation stage. It turns out that the adaptive fitting method is much more effective to deal with multiple frequency disturbances.

7.1 Introduction

The disturbances affecting the performance of HDD head positioning servomechanism are contributed by some internal components of the drive as well as sources external to the HDD. The external disturbances are typically in the form of shock and vibration that come from the environment. With the growing necessity of HDD used in extreme environment, the servo systems have to deal with more periodic disturbances in addition to internal disturbances. The frequency, amplitude and phase of these periodic disturbances are often time-varying or even unknown. Identification and then compensation of such unknown disturbances in HDDs are thus quite important. Therefore, it is necessary to find effective algorithms to suppress the impact from these periodic disturbances.

There are quite a lot of studies on rejecting unknown periodic disturbances. References [122], [123] propose to use an adaptive feedforward controller to reduce the effect of periodic disturbances, while in [124] by using the adaptive feedforward controller an improved performance is achieved. In [125] a discrete-time self-tuning repetitive controller is used to deal with periodic disturbances. In [126–130] an internal model principle is proposed which can estimate the disturbance frequency and cancel it in one procedure. To deal with repeatable runout, [131] tries two kinds of iterative learning control methods and gives the comparison. [132] deduces internal model principle in discrete time representation and solves the periodic disturbance

problem involved in a mobile HDD, but the method will not work if the disturbances are at multi-frequencies and the number of filters is less than the number of frequencies.

In our work, the internal model principle is used to estimate the frequency of unknown periodic disturbance and the Recursive Least Square algorithm is used to estimate its magnitude and phase adaptively so as to rebuild the disturbance signal and then cancel it by adding the opposite value of this signal to the command signal. The cancellation scheme results effectively when the disturbance is time-varying in frequency, amplitude as well as phase. To increase the performance for multi-frequency disturbance suppression, a polynomial adaptive fitting method which does not need frequency estimation is proposed to overcome the delay problem in the stage of disturbance rebuilding. It turns out that the adaptive fitting method is more effective to deal with multiple frequency disturbances.

7.2 Structure of Control System

In order to cancel the periodic disturbances, we suppose to build a new signal which is approximate to the disturbance and add its opposite value to the command signal. Figure 7.1 shows the system structure of the proposed algorithm, where r , e , u , y and w are the system command, position error signal (PES), system input, system output and disturbance respectively. The internal model principle can effectively estimate the dominate frequency of PES, which is also the fundamental frequency of disturbance; while a disturbance rebuilder uses the frequency estimation and a signal generated by the difference of output passing through the inverse of plant and input signal to rebuild a signal which approximates the periodic disturbance. Then the disturbance will be cancelled by adding the opposite value of this signal to the system input signal. For the multi-frequency case, sometimes the frequencies cannot

be estimated correctly even if there is enough number of frequency estimators. Since each signal can be represented by an infinite order polynomial function according to Taylor's series, we use the 3rd order polynomial spline to replace sine wave spline which can approximate the disturbance and avoid to use frequency estimators.

Figure 7.1: The structure of track following system for disturbance cancellation.

7.3 Adaptive Methods for Periodic Disturbance Cancellation

7.3.1 Internal Model Principle for Frequency Estimation

Frequency estimation can be achieved by using internal model. If the disturbance has simple component or dominating component such as $w = a \sin(\omega t + \phi)$, according to [128] we can use an internal model to represent it. The internal model has the

following discrete state-space representation,

$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = A_d \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + B_d e(k) \\ y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \end{cases} \quad (7.1)$$

$$A_d = \begin{bmatrix} \cos(\omega T) & \sin(\omega T) \\ -\sin(\omega T) & \cos(\omega T) \end{bmatrix} \quad B_d = \frac{K_f}{\omega} \begin{bmatrix} 1 - \cos(\omega T) \\ \sin(\omega T) \end{bmatrix} \quad (7.2)$$

where $x_1(k+1), x_2(k+1)$ are the state variables, ω and T are the frequency estimate and the sampling time of the controller, respectively. The frequency estimate can be gained as following,

$$\omega(k+1) = \omega(k) - K_e \frac{K_f e(k) \omega(k) x_1(k)}{\omega(k)^2 x_1(k)^2 + x_2(k)^2} \quad (7.3)$$

Remark 7.3.1. *This frequency estimation is deduced from continuous time form [127], the continuous time internal model of (7.1) can be represented as*

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ K_f \end{bmatrix} e(k) \\ y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases} \quad (7.4)$$

If input is $w = a \sin(\omega t + \phi)$, then $x_1(t)$ and $x_2(t)$ converge exponentially fast to

$$x_1(t) = A \sin(\omega t + \phi + \psi)$$

$$x_2(t) = A \omega \cos(\omega t + \phi + \psi)$$

The steady state of ω is

$$\omega = \frac{d}{dt} \left(\angle \left(x_1(t) + j \frac{x_2(t)}{\omega} \right) \right)$$

let $\hat{\omega}$ be the estimation of ω ,

$$\hat{\omega} = \frac{d}{dt} \left(\tan^{-1} \frac{\omega x_1}{x_2} \right) = \frac{\omega (\dot{x}_1 x_2 - x_1 \dot{x}_2)}{(\omega x_1)^2 + x_2^2}$$

substituting $\dot{x}_1$ and $\dot{x}_2$ by (7.4) gives

$$\hat{\omega} = \omega - \frac{\omega K_f e x_1}{(\omega x_1)^2 + x_2^2}$$

Let $\varepsilon = \hat{\omega} - \omega$, then ω can be updated by

$$\frac{d\omega}{dt} = -K_e \varepsilon = -K_e \frac{\omega K_f e x_1}{(\omega x_1)^2 + x_2^2}$$

Thus, the discrete time of such estimation can be represented as (7.3).

The gains K_f and K_e determine the convergence and the stability of the frequency estimator. If we replace $\omega(k)x_1(k)$, $x_2(k)$ and $K_e K_f$ to $X_1(k)$, $X_2(k)$ and K'_f , then the Equation (7.1) is deduced as,

$$\begin{cases} X_1(k) = 2 \cos(\omega T) X_1(k-1) - X_1(k-2) \\ \quad + K'_f (1 - \cos(\omega T)) (e(k-1) + e(k)) \\ X_2(k) = 2 \cos(\omega T) X_2(k-1) - X_2(k-2) \\ \quad + \frac{K'_f}{\omega} \sin(\omega T) (e(k-1) + e(k)) \end{cases} \quad (7.5)$$

By using these equation iteratively, we get the estimation of the dominate frequency.

When getting the plant inverse to guarantee the system causal, there will be

several samples of time delay. The estimated signal cannot be added to the control signal since it cannot match the current disturbance infection. Thus it is needed to predict the signal of next several sample instants, so that can exactly match the signal. Here we apply two methods to achieve the prediction, magnitude, phase estimation and polynomial spline fitting.

7.3.2 Magnitude and Phase Estimation

For disturbance which has single frequency component, magnitude and phase estimation can be achieved using two parameter estimation [133]. Suppose $w = \alpha \cos(\omega_c t) + \beta \sin(\omega_c t)$, $\Theta(k) = [\alpha(k), \beta(k)]^T$ are the coefficients of $\cos(\omega_c t)$ and $\sin(\omega_c t)$. Let $\Phi(k) = [\cos(\hat{\omega}_c kT), \sin(\hat{\omega}_c kT)]^T$, Then $\Theta(k)$ can be estimated using the Recursive Least Square parameter adaptation algorithm. In this LS method, the criterion to be minimized is

$$J(t) = \sum_{i=1}^t \lambda^{t-k} [P(z)^{-1}y(i) - u(i) - \Phi(i)^T \hat{\Theta}(i)]^2 \quad (7.6)$$

Denote $\tilde{y}(i) = P(z)^{-1}y(i) - u(i)$. λ is called the forgetting factor. The value of $\hat{\Theta}$ is obtained by seeking the value that make $\partial J(t)/\partial \hat{\Theta} = 0$:

$$\frac{\partial J(t)}{\partial \hat{\Theta}} = -2 \sum_{i=1}^t [\tilde{y}(i) - \hat{\Theta}(t)^T \Phi(i-1)] \Phi(i-1) = 0 \quad (7.7)$$

So that

$$\left[\sum_{i=1}^t \Phi(i-1) \Phi(i-1)^T \right] \hat{\Theta}(t) = \sum_{i=1}^t \tilde{y}(i) \Phi(i-1) \quad (7.8)$$

It follows that,

$$\begin{aligned} \hat{\Theta}(t) &= \left[\sum_{i=1}^t \Phi(i-1) \Phi(i-1)^T \right]^{-1} \sum_{i=1}^t \tilde{y}(i) \Phi(i-1) \\ &= F(t) \sum_{i=1}^t \tilde{y}(i) \Phi(i-1) \end{aligned} \quad (7.9)$$

in which $F(t)^{-1} = [\sum_{i=1}^t \Phi(i-1)\Phi(i-1)^T]$, which leads to,

$$\hat{\Theta}(t+1) = F(t+1) \sum_{i=1}^{t+1} \tilde{y}(i)\Phi(i-1) \quad (7.10)$$

Since

$$F(t+1)^{-1} = \left[\sum_{i=1}^{t+1} \Phi(i-1)\Phi(i-1)^T \right] = F(t)^{-1} + \Phi(t)\Phi(t)^T \quad (7.11)$$

and

$$\begin{aligned} \sum_{i=1}^{t+1} \tilde{y}(i)\Phi(i-1) &= \sum_{i=1}^t \tilde{y}(i)\Phi(i-1) + \tilde{y}(t+1)\Phi(t) \\ &\quad + \Phi(t)\Phi(t)^T \hat{\Theta}(t) - \Phi(t)\Phi(t)^T \hat{\Theta}(t) \end{aligned} \quad (7.12)$$

Considering Equation (7.9) and Equation (7.10), Equation (7.12) may be rewritten as:

$$\begin{aligned} \sum_{i=1}^{t+1} \tilde{y}(i)\Phi(i-1) &= F(t+1)^{-1} \hat{\Theta}(t+1) \\ &= F(t)^{-1} \hat{\Theta}(t) + \Phi(t)\Phi(t)^T \hat{\Theta}(t) + \Phi(t)[\tilde{y}(t+1) - \Phi(t)^T \hat{\Theta}(t)] \end{aligned} \quad (7.13)$$

so that

$$\begin{aligned} F(t+1)^{-1} \hat{\Theta}(t+1) &= F(t+1)^{-1} \hat{\Theta}(t) + \Phi(t)\epsilon^0(t+1) \\ \hat{\Theta}(t+1) &= \hat{\Theta}(t) + F(t+1)\Phi(t)\epsilon^0(t+1) \end{aligned} \quad (7.14)$$

where $\epsilon^0(t) = P(z)^{-1}y(t-1) - u(t-1) - \Phi(t-1)^T \hat{\Theta}(t-1)$ Since $(F^{-1} + \phi\phi^{-1})^{-1} = F - \frac{F\phi\phi^T F}{1 + \phi^T F \phi}$, so that

$$F(t+1) = F(t) - \frac{F(t)\Phi(t)\Phi^T(t)F(t)}{1 + \Phi^T(t)F(t)\Phi(t)} \quad (7.15)$$

The recursive formula for the inverse of the adaptation gain $F(t+1)^{-1}$ is generalized by introducing a weighting λ , as indicated below:

$$F(t+1)^{-1} = \lambda F(t)^{-1} + \Phi(t)\Phi(t)^T \quad (7.16)$$

so that

$$F(t+1) = \frac{1}{\lambda} \left[F(t) - \frac{F(t)\Phi(t)\Phi^T(t)F(t)}{\lambda + \Phi^T(t)F(t)\Phi(t)} \right] \quad (7.17)$$

Then Θ can be estimated by iterating Equation (7.14) and Equation (7.17). In this method, convergence can be controlled by adjusting the value of λ . The effect of $\lambda < 1$ is to introduce increasingly weaker weighting on the old data ($i < t$). The maximum weight is given to the most recent error. So the fitting result will reflect the most recent signal varying trend.

For the disturbance which has harmonic components, with the frequency estimation method in subsection 7.3.1 used to get the fundamental frequency, the adaptation algorithm can be applied to estimate the magnitude and phase corresponding to several main harmonic components.

For disturbance having multi-frequency components which are not harmonics, there should be several frequency estimators to get the different frequencies, and meanwhile Equations (7.14) and (7.17) is used to estimate the corresponding magnitude and phase. This will certainly increase computation complexity. Moreover, even the estimator number is sufficient, it can not be sure that estimator can converge to a different frequency. Thus with the signal frequency, magnitude and phase estimation method, the accuracy of the signal estimation will be lower, which will lead to poor compensation effectiveness.

7.3.3 Polynomial Spline Fitting Principle

As mentioned previously, for multi-frequency disturbances, the above adaptive algorithm will suffer from discounted compensation effect. Hence in this section a polynomial method is proposed since each signal can be represented by a polynomial function with little error according to Taylor's series. In mathematics, the

Taylor series is a representation of a function as an infinite sum of terms calculated from the values of its derivatives at a single point [134]. Here a 3rd order polynomial function of the current time instant which has time-varying coefficients is used to approximately represent the signal. The estimated signal is given by

$$\hat{w}(k) = \Phi(k)^T \hat{\Theta}(k) \quad (7.18)$$

where $\hat{\Theta}(k) = [a_1(k), a_2(k), a_3(k), a_4(k)]^T$ and $\Phi(k) = [1, kT, (kT)^2, (kT)^3]^T$, $\hat{\Theta}(k)$ is updated according to the Recursive Least Square parameter adaptation algorithm (7.14) and (7.17). In order to guarantee the convergence, $kT < 1$ is needed.

Figure 7.2 shows the block diagram of polynomial spline fitting method. It should be noted that the method does not need to estimate the disturbance frequency.

Figure 7.2: The structure of polynomial spline fitting disturbance cancellation.

7.3.4 Plant Model Inverse

When rebuilding the disturbance signal, it needs to find the inverse of the plant. Since there may be unstable zeros in the system model, the plant inverse cannot be solved directly. In this case, the ZPETC method [135] can be used to obtain the plant inverse.

Suppose the plant transfer function is given as follows:

$$P(z) = \frac{B_s(z)B_u(z)}{A(z)} \quad (7.19)$$

where $B_s(z)$ and $B_u(z)$ represent stable and unstable parts respectively and μ is the order of $B_u(z)$. $P(z)^{-1}$ can be represented by

$$P(z)^{-1} = \frac{A(z)B_u^*(z)}{z^{d+2u}B_s(z)} \quad (7.20)$$

where $B_u^*(z)$ is the complex conjugate of $B_u(z)$, and d is the difference between the orders of $A(z)$ and $B(z) = B_s(z)B_u(z)$. The phase of the shaped plant $P(z)P(z)^{-1}$ is z^{-d} , which is near zero phase in the low frequency area and will drop down exponentially when the frequency increases. Therefore the condition that $P(z)P(z)^{-1}$ is real cannot be guaranteed over high frequency range.

Remark 7.3.2. *Since the plant has unstable zeros, the true plant inverse is unstable.*

The plant inverse in (7.20) is not the true inverse. In this inverse,

$$B_u^*(z) = \frac{z^{2u}B_u(z^{-1})}{B_u(1)^2}$$

Thus,

$$\frac{B_u^*(z)}{z^{2u}}B_u(z) = \frac{B_u(z^{-1})B_u(z)}{B_u(1)^2}$$

The following relation can be easily verified by direct substitution of $z = e^{j\omega T}, 0 \leq$

$\omega < \pi/T$.

$$\text{Im}[B_u(e^{-j\omega T})B_u(e^{j\omega T})/B_u(1)^2] = 0, 0 \leq \omega \leq \pi/T$$

Equation implies that, there is no phase shift through $P(z)P(z)^{-1}$. The ZPETC cancels the poles and cancellable zeros of the closed-loop system and compensates for phase shifts induced by uncancellable zeros. Because of this property, such $P(z)^{-1}$ is called the zero phase error tracking controller (ZPETC) [136].

7.4 Simulation Results

7.4.1 System Modeling and Controller

With the measured frequency response data of actuators, the transfer function can be obtained as follows, whose frequency response has been shown in Figure 2.4.

We apply a discrete controller with the form of

$$\frac{-9.999 \times 10^{-7} z^2 + 1.958 \times 10^{-6} z - 9.582 \times 10^{-6}}{2.218 \times 10^{-5} z^2 - 2.118 \times 10^{-5} z - 1 \times 10^{-6}}$$

and sampling time $2.2178 \times 10^{-5} \text{sec}$. The system open loop frequency response is shown in Figure 7.3.

7.4.2 Single Frequency Case

For the cancellation of the periodic disturbance which has a simple frequency or a domain frequency, the AIM method is used to estimate the frequency.

When using the AIM method, frequency estimation will not converge in the first stage if we choose a larger estimation gain in order to achieve high convergence speed. This problem can be solved if we know the range of the disturbance frequency

Figure 7.3: System open loop frequency response.

and add an estimator frequency band constrain. Figure 7.4 and Figure 7.5 show estimation of disturbance frequency at 700 Hz and 2000 Hz. When adding 200 Hz and 500 Hz lower band constrains respectively, the frequency estimation speeds advances significantly.

For the case of single frequency simulation, a sine wave with 200Hz frequency is added to the system input signal as disturbance while a white noise with $\frac{1}{10}$ magnitude of the sine wave signal is also added in. Since the frequency estimation will affect the disturbance rebuilder's work, this disturbance rebuilder should be added to the system approximately at 50ms which is after the estimation convergence. Figure 7.6 reflects the effect of the proposed algorithms dealing with the single frequency disturbance which has phase change $\frac{\pi}{2}$ and Figure 7.7 reflects the one with magnitude change to its twice at 100 ms. Figure 7.6 and Figure 7.7 shows that

for single frequency disturbance or single dominate frequency disturbance case, this method can effectively cancel the disturbance effect and the converge time can be less than a disturbance cycle.

7.4.3 Multiple Frequency Case

For the multiple frequency case, a disturbance with different frequencies is added to the system input signal as disturbance while a white noise with $\frac{1}{10}$ magnitude of the sine wave signal is also added in. We apply the two methods which are mentioned before. For the frequency estimation and cancellation method, the disturbance rebuilder should be added to the system approximately at 50 ms which is after the estimation convergence, while for the polynomial spline fitting method, the disturbance rebuilder can be added in the very beginning.

Figure 7.8 and Figure 7.9 show results to deal with multi-frequency disturbance cases. In Figure 7.8, a harmonics disturbance with the frequency of 200 Hz, 400 Hz and 600 Hz was added in the simulation, while Figure 7.9 presents a disturbance which has components with frequencies of 200 Hz, 240 Hz, 260 Hz in it. Since the spindle speed of HDD is 12000 RPM, the base disturbance frequency is 200 Hz, thus we choose 200 Hz as base harmonics frequency in this simulation. Frequency estimation and cancellation methods and 3rd order polynomial spline fitting methods have been used in the simulations. Result presents that the 3rd order polynomial fitting method has better performance and robustness to observer noise.

For the multi-frequency case, one frequency estimator can not bring out all the frequencies in disturbance. Even the estimator number is sufficient, it can not be sure that estimator can converge to a different frequency. Thus with the signal frequency, magnitude and phase estimation method, the signal estimation will not perform as well as the spline fitting method which does not need frequency estimation.

How the different parameters will impact on the convergence performance is investigated in Figure 7.10, Figure 7.11 and Figure 7.12. Figure 7.10 shows the time delay d in Block z^{-d} in Figure 7.2 impacts on the convergence performance. Non-harmonics disturbance $w = \sin(200\pi t) + 0.5\sin(240\pi t) + 0.3\sin(260\pi t) + v$ was added. It is obvious that the larger d degrade the convergence performance. But the performance is acceptable when $d \leq 10$. Figure 7.11 shows how the different weighting λ impacts on the convergence performance. Non-harmonics disturbance $w = \sin(200\pi t) + 0.5\sin(240\pi t) + 0.3\sin(260\pi t) + v$ was also added. The smaller λ can achieve better performance. The non-harmonics disturbance $w = \sin(200\pi t) + 0.5\sin(240\pi t) + 0.3\sin(260\pi t) + B_nv$. We define Signal Noise ratio as $SNR = 20 \log_{10} B_n$ in this case. We can see the performance shown in Figure 7.12. This method can work even with SNR as low as $-20dB$.

7.5 Summary

In this chapter, unknown periodic disturbances have been rebuilt and canceled based on the internal model principle to adaptively estimate frequency and the Recursive Least Square algorithm to adaptively estimate magnitude and phase. This adaptive cancellation scheme can effectively reduce the disturbance effect on position error signal when the disturbance signal is time-varying in both frequency and amplitude/phase and in a wide frequency range. In order to increase the performance of the above cancellation scheme for multiple frequency disturbance, the polynomial fitting method with coefficients adaptively updated has been proposed. It has been shown that such a scheme is able to cancel the multi-frequency disturbance better when it has harmonic or non-harmonic components.

Next chapter will present another add-on strategy for the disturbance attenuation improvement which is called general disturbance observer.

Figure 7.4: Frequency estimation improvement by adding band constraint $f > 200\text{Hz}$.

Figure 7.5: Frequency estimation improvement by adding band constraint $f > 500\text{Hz}$.

Figure 7.6: System output by canceler for phase change disturbance.

Figure 7.7: System output by canceler for magnitude change disturbance.

Figure 7.8: System output with frequency estimation and cancellation and with 3rd order polynomial fitting cancellation for harmonic case.

Figure 7.9: System output with frequency estimation and cancellation and with 3rd order polynomial fitting cancellation for nonharmonic case.

Figure 7.10: System output with 3rd order polynomial fitting cancellation with different time delay in model.

Figure 7.11: System output with 3rd order polynomial fitting cancellation with different forgetting factors.

Figure 7.12: System output with 3rd order polynomial fitting cancellation with different SNR.

Chapter 8

A Novel Disturbance Observer Design and Application in HDDs for Disturbance Compensation

This chapter will introduce another add-on strategy for the disturbance attenuation improvement which is called general disturbance observer. Conventional disturbance observer requires plant model inverse to retrieve the input to the actual plant in order to estimate the input disturbance imposed on the plant for compensation. This chapter proposes a general form of disturbance observers, which does not need to solve the plant model inverse, and uses H_∞ control method to design the Q -filter in the disturbance observer. The conventional disturbance observer is proved to be a special case of the generalized form.

8.1 Introduction

The idea of using disturbance observer to improve the performance of servomechanism was introduced as early as in [137,138], where with the disturbances assumed to be generated by a linear time-invariant dynamic system and then estimated from the system measurements using Luenberger observer, the effect of the disturbances could be compensated by feeding back the disturbance estimates into the system. However, it is not always realistic to assume that disturbance model is linear time-invariant and the identification of disturbance model is not always easy. Subsequently, a new type of disturbance observer (DOB) have been introduced [139]. This new method does not require the full information of the disturbance model or assume it to be linear time-invariant. However, it requires the model of controlled plant to be accurately known and invertible, at least within the bandwidth of interest [40,140]. Later on, it has been proven that under certain assumptions imposed on the plant and disturbance models, the two different methods are equivalent. In this paper, we study the latter method while a general form of disturbance observer, which does not need to solve the plant model inverse, is proposed. H_∞ control method is then used to design the Q -filter in the general disturbance observer.

It has been shown that a DOB is capable of estimating disturbances and modeling error [139]. Hence, DOB can be used to increase the read/write (R/W) head-positioning accuracy in HDDs by using its estimation results to cancel away the effect of the disturbances and modeling [40,139]. Further, due to its cost effectiveness and easy “add-on” implementation with minimal change required to the existing feedback controller, the disturbance observer without using additional sensors is frequently used to enhance the tracking performance of a hard disk drive servo system, such as the attenuation of disturbances [141,142], and the compensation of pivot friction [41].

As for the design method of DOB, it is well understood that at any frequency range whereby the Q -filter is set to be unity, the disturbances can be completely canceled, but the measurement noise is unattenuated. If majority of the disturbances are of relatively lower frequency, when a DOB is added on to attenuate the effect of disturbances, the standard and conventional way of designing a Q -filter is to design it to be a low-pass filter with unity DC gain [139]. In this paper, we introduce the conventional disturbance observer first, and then propose a general form of disturbance observer. H_∞ performance in terms of disturbances and noise attenuation is used to formulate the H_∞ control in order to design the Q -filter. The proposed DOB is applied to a HDD servo system and its effectiveness in disturbance attenuation is demonstrated by illustrating simulation and experimental results. It is also compared to the conventional form in both design method and disturbance attenuation effectiveness through simulation and implementation.

8.2 A General Disturbance Observer and Design Method

8.2.1 Conventional Disturbance Observer

Figure 8.1 shows the block diagram of the conventional disturbance observer (DOB) structure [139], where C is the feedback controller, P_n is the nominal model of the plant P , $\hat{d}_1$ is an estimate of the input disturbance d_1 , and P_n^{-1} is the inverse of P_n which can be obtained using ZPETC method in [135]. d represents a delay in the plant P . Theoretically, $P(z) = z^{-d}P_n(z)$. Ignoring the nominal feedback loop with C , the transfer functions from the disturbance d_1 and noise n to the output y are given by

$$T_{ed_1} = \frac{-PP_n(1 - Qz^{-d})}{P_n + Q(P - P_nz^{-d})}, \quad (8.1)$$

$$T_{en} = \frac{PQ}{P_n + Q(P - P_nz^{-d})}. \quad (8.2)$$

To reject the disturbance d_1 , Q can be set as unity because $T_{ed_1} \approx 0$ when the delay is negligible. However, when $Q = 1$, $T_{en} \approx -1$ which means the measurement noise n is not attenuated. Thus, to eliminate the noise effect, it is known from Equation (8.2) that an ideal solution of Q is zero, but this will mean $T_{ed_1} \approx -P$ and hence the disturbance will be amplified.

Figure 8.1: Block diagram of control loop with conventional disturbance observer.

On the other hands, considering the overall system in Figure 8.1, the sensitivity function is given by

$$S(z) = \frac{1 - Qz^{-d}}{1 - Qz^{-d} + PC + PQP_n^{-1}}. \quad (8.3)$$

Theoretically, the plant model is $P = P_nz^{-d}$, thus Equation (8.3) becomes

$$S(z) = \frac{1 - Qz^{-d}}{1 + PC}, \quad (8.4)$$

which is stable as long as Q is stable since the loop is stable before the disturbance

observer is added on. Moreover, it is deduced from Equation (8.3) that when $Qz^{-d} = 1$ with zero phase around the disturbance frequency, the disturbance can be rejected because $S(z) \rightarrow 0$. Hence Q is designed such that the phase of Qz^{-d} is almost zero degree and the magnitude is close to one in the frequency range where the disturbance d_1 dominates. Note that in Figure 8.1, it is needed to solve the inverse P_n^{-1} of the nominal plant model. In what follows, a general form of the disturbance observer is proposed and the plant model inverse is not needed.

Figure 8.2: Block diagram of control loop with the general disturbance observer.

8.2.2 A General Disturbance Observer

Let the model of the plant P be expressed as

$$P_n(z) = B(z)/A(z), \quad (8.5)$$

with

$$B(z) = b_m z^m + b_{m-1} z^{m-1} + \cdots + b_0,$$

$$A(z) = z^n + a_{n-1} z^{n-1} + \cdots + a_0$$

be the nominal model of the plant P . To be more realistic, an output disturbance d_2 is considered in Figure 8.2. It shows a general form of disturbance observer where

$$M(z) = \frac{B(z)}{z^{d_m}}, \quad N(z) = \frac{A(z)}{z^{d_n}}. \quad (8.6)$$

In the conventional disturbance observer in Figure 8.1,

$$M(z) = z^{-d}, \quad N(z) = P_n^{-1}(z). \quad (8.7)$$

Denote the state-space descriptions: $P(z) : (A_p, B_p, C_p, D_p)$, $C(z) : (A_c, B_c, C_c, D_c)$, $M(z) : (A_M, B_M, C_M, D_M)$, $N(z) : (A_N, B_N, C_N, D_N)$. From Figure 8.2,

$$x(k+1) = Ax(k) + B_1w(k) + B_2u_q(k), \quad (8.8)$$

$$y_q(k) = C_yx(k) + D_{yw}w(k) + D_{yu}u_q(k), \quad (8.9)$$

$$e(k) = C_zx(k), \quad (8.10)$$

where

$$A = \begin{bmatrix} A_p - B_pD_cC_p & B_pC_c & 0 & 0 \\ -B_cC_p & A_c & 0 & 0 \\ -B_MD_cC_p & B_MC_c & A_M & 0 \\ -B_NC_p & 0 & 0 & A_N \end{bmatrix}, \quad (8.11)$$

$$B_1 = \begin{bmatrix} B_p & -B_pD_c & B_pD_c \\ 0 & -B_c & B_c \\ 0 & -B_MD_c & B_MD_c \\ 0 & -B_N & B_N \end{bmatrix}, \quad B_2 = \begin{bmatrix} -B_p \\ 0 \\ -B_M \\ 0 \end{bmatrix}, \quad (8.12)$$

$$C_y = \begin{bmatrix} -D_N C_p - D_M D_c C_p & D_M C_c & C_M & C_N \end{bmatrix}, \quad (8.13)$$

$$D_{yw} = \begin{bmatrix} 0 & -(D_N + D_M D_c) & D_N + D_M D_c \end{bmatrix},$$

$$D_{yu} = -D_M,$$

$$C_z = \begin{bmatrix} C_p & 0 & 0 & 0 \end{bmatrix}, \quad D_{zw} = [0 \ 1 \ 0], \quad D_{zu} = 0, \quad (8.14)$$

and $x^T(k) = [x_p^T(k) \ x_c^T(k) \ x_M^T(k) \ x_N^T(k)]$ is the augmented state of $P(z)$, $C(z)$, $M(z)$, and $N(z)$, and $w^T(k) = [d_1(k) \ d_2(k) \ n(k)]$.

Denote the transfer function from w to e as $T_{ew} = [T_{ed_1} \ T_{ed_2} \ T_{en}]^T$. H_∞ control method will be used to design $Q(z)$ to minimize the H_∞ norm

$$\|[W_{d_1} T_{ed_1} \ W_{d_2} T_{ed_2} \ W_n T_{en}]^T\|_\infty. \quad (8.15)$$

W_{d_1} , W_{d_2} and W_n are weightings respectively to d_1 , d_2 and n . Here the models for disturbances d_1 , d_2 and noise n are not needed, whereas in order to have a desired suppression of the disturbances d_1 , d_2 and the noise n , appropriately choosing weightings W_{d_1} , W_{d_2} and W_n are required.

Different from the conventional disturbance observer, the general disturbance observer does not need the inverse of the nominal plant model. As shown later, it is still able to suppress the disturbances in the frequency range lower than 1 kHz and without much degradation to higher frequency disturbances and noise.

The objective of the general disturbance observer design is then stated as: *Given a positive scalar γ and appropriate weightings W_{d_1} , W_{d_2} and W_n , design a stable $Q(z) : (A_Q, B_Q, C_Q, D_Q)$ such that $\|[W_{d_1} T_{ed_1} \ W_{d_2} T_{ed_2} \ W_n T_{en}]^T\|_\infty < \gamma$.* The H_∞ control design problem can be solved via the LMI approach as stated in [143].

Remark 8.2.1. *The sensitivity function of the control system in Figure 8.2 with the general disturbance observer is given by*

$$S(z) = \frac{1 + QM}{1 + PC + QM - PQN}. \quad (8.16)$$

With the conventional disturbance observer with $M(z)$ and $N(z)$ in Equation (8.7), (8.16) is equal to

$$S(z) = \frac{1 + Qz^{-d}}{1 + PC + Qz^{-d} - PQP_n^{-1}}. \quad (8.17)$$

Assume that in Equation (8.16)

$$Q(z) = Q_n(z) \times \frac{z^{d_n}}{B(z)}, \quad P = P_n, \quad (8.18)$$

then

$$S(z) = \frac{1 + Q_n z^{d_n} / z^{d_m}}{1 + PC + Q_n z^{d_n} / z^{d_m} - Q_n}, \quad (8.19)$$

which recovers the form in Equation (8.17) when $d_m \geq d_n$, meaning that the conventional disturbance observer is a special case of the general disturbance observer with M and N in Equation (8.6).

Remark 8.2.2. In Figure 8.2, if the loop is cut at u and the transfer function from u to e or y is considered as a new plant, it is derived that

$$P_{equ} = \frac{y}{u} = \frac{P}{1 - (M - PN)Q}. \quad (8.20)$$

An equivalent open loop is denoted by $T_{EQ-OL}(z)$, and

$$T_{EQ-OL}(z) = P_{equ}C = \frac{P}{-1 + (M - PN)Q}C. \quad (8.21)$$

Assume ideally that $P_n = P$ and $d_m = d_n$. Equation (8.20) becomes $P_{equ} = P$, which

implies that the proposed general disturbance observer does not influence the characteristics of open loop $P(z)C(z)$ greatly and thus the stability performance achieved by the nominal control loop with controller $C(z)$ is maintained. This is similar to the conventional disturbance observer [40].

8.3 Application Results

Figure 8.3: Frequency response of the VCM actuator.

We consider the VCM plant $P(s)$ model in Figure 8.3. The discretized model $P_n(z)$ with $T_s = 1/30000$ sec is given by Equation (8.5) with

$$B(z) = 0.02399z^5 + 0.1868z^4 - 0.03483z^3 - 0.02165z^2 + 0.1728z + 0.02025,$$

$$A(z) = z^6 - 3.052z^5 + 4.657z^4 - 4.979z^3 + 3.997z^2 - 2.399z + 0.7765.$$

The nominal controller $C(z)$ is the combination of a PID (proportional-integral-derivative) controller and two notch filters, and given by

$$C(z) = \frac{1.474 \times 10^{-5} z^6 - 4.416 \times 10^{-5} z^5 + 6.517 \times 10^{-5} z^4 - 6.621 \times 10^{-5} z^3 + 5.015 \times 10^{-5} z^2 - 2.872 \times 10^{-5} z + 9.053 \times 10^{-6}}{3.333 \times 10^{-5} z^6 - 7.59 \times 10^{-5} z^5 + 8.081 \times 10^{-5} z^4 - 4.846 \times 10^{-5} z^3 + 1.545 \times 10^{-5} z^2 - 5.234 \times 10^{-6} z} \quad (8.22)$$

Let $M(z) = \frac{B(z)}{z^6}$, and $N(z) = \frac{A(z)}{z^6}$, $W_{d_1} = 0.3$, $W_{d_2} = 1$ and $W_n = 1.5$, a stable $Q(z)$ is obtained and the frequency response is shown in Figure 8.4. Note that the plant model inverse is not required in the general DOB. This benefit is of great significance especially for nonminimum phase plant.

The sensitivity function $|S(z)|$ is plotted in Figure 8.5, which means that the designed disturbance observer is able to suppress the disturbances at lower than 1 kHz and without much degradation for higher frequency disturbances. The servo performance such as bandwidth will change with different weightings W_{d_1} , W_{d_2} and W_n .

To demonstrate the effectiveness of the disturbance estimation in the time domain, we assume that the disturbances d_1 and d_2 and the noise n are generated by $d_1 = D_1(s)w_1$, $d_2 = D_2(s)w_2$ and $n = Nn(s)w_3$, where [65]

$$D_1(s) = \frac{0.0004(s^2 - 83.39s + 9.741 \times 10^5)(s^2 + 1616s + 9.626 \times 10^6)}{(s^2 + 125.7s + 3.948 \times 10^5)(s^2 + 10.05s + 1.011 \times 10^6)}, \quad (8.23)$$

$$D_2(s) = \frac{0.0019(s^2 + 3329s + 1.695 \times 10^7)(s^2 + 3340s + 5.61 \times 10^8)}{(s^2 + 245s + 1.668 \times 10^7)(s^2 + 477.5s + 5.701 \times 10^8)}, \quad (8.24)$$

$$Nn(s) = 0.003. \quad (8.25)$$

and $w_i (i = 1, 2, 3)$ are independent white noises with unity variance.

With the designed general DOB, the estimate $\hat{d}_1$ of the d_1 is shown in Figure 8.7. It follows the original d_1 approximately. As a result, the error signal is shown in Figure 8.8. 50% reduction is achieved. As illustrated in Table 1, the proposed DOB is more effective to compensate the input disturbance d_1 than d_2 and n . W_{d_2} and W_n are selected as 1 and 1.5 is to keep the attenuation to d_2 and n achieved by the nominal feedback controller $C(z)$. With lower W_{d_2} and W_n and higher W_{d_1} , the attenuation to d_2 and n will be degraded although more suppression to d_1 will be attained by using the DOB.

Moreover, the conventional disturbance observer is designed for comparison to the general form. $M(z)$ and $N(z)$ are given by

$$M(z) = z^{-1}, \quad (8.26)$$

$$N(z) = P_n(z)^{-1} = \frac{5.2494(z^2 - 1.983z + 0.9834)(z^2 - 1.253z + 0.9654)(z^2 + 0.1836z + 0.8179)}{z(z + 0.9501)(z + 0.1259)(z + 0.116)(z^2 - 1.22z + 0.9646)}.$$

A stable $Q(z)$ for the conventional disturbance observer is designed with the H_∞ control method. Figure 8.6 shows the resultant sensitivity function, which are similar to the one from the general disturbance observer. However, the plant model inverse needs to be calculated.

Table 8.1: Contribution of disturbances and noise in the error, denoted by e^* , in μ m

Cases	$e(d_1)$	$e(d_2)$	$e(n)$
No control	0.3139	0.0015	0.003
$C(z)$	0.0078	0.0014	0.0011
$C(z)$ &DOB	0.0029	0.0014	0.0016

Experiment has been done with a Laser Doppler Vibrometer and a dSpace 1103. The measured sensitivity functions are shown in Figure 8.9, which agrees with the simulation results in Figure 8.5. To evaluate the effect of the DOB on the stability performance achieved by the nominal controller $C(z)$, T_{EQ-OL} is measured with the general DOB and the conventional DOB, and shown in Figure 8.10 and Figure 8.11. As expected, the stability performance such as gain margin and phase margin are not affected.

Figure 8.4: Frequency response of the designed $Q(z)$.

Figure 8.5: The sensitivity functions without and with the general disturbance observer.

8.4 Summary

A general form of disturbance observer has been proposed and designed based on H_∞ control method such that the disturbance can be attenuated with appropriate selection of noise and disturbances weightings. The disturbance observer does not need to solve the plant model inverse, and thus its design is simplified and has great advantages over the conventional disturbance observer especially for nonminimum phase plant. The simulation and implementation results show that the general disturbance observer designed using the method employed in the paper is able to effectively improve the attenuation of disturbance in frequency lower than 1 kHz, and will not sacrifice the stability performance of the nominal feedback control loop.

Figure 8.6: The sensitivity function comparison with the general and the conventional disturbance observers.

Figure 8.7: Disturbance d_1 .

Figure 8.8: Error signal e .

Figure 8.9: The measured sensitivity functions without and with the general disturbance observer.

Figure 8.10: The comparison of T_{EQ-OL} without and with the general disturbance observer.

Figure 8.11: The comparison of T_{EQ-OL} without and with the conventional disturbance observer.

Chapter 9

Conclusions and Future Work

9.1 Conclusions

Future HDDs should have larger storage capacities as well as shorter data access times in order to sustain a significant global market share for desktop and home multimedia storage application. In this dissertation, various novel control algorithms have been developed for HDD servo system to achieve this goal.

A discrete time LQR reset control algorithm to achieve better transient response performance. A design method of discrete time reset control systems with the reset time instants pre specified was proposed. With a base linear system designed conventionally, the discrete-time reset law design aims at improving the system transient responses. The design method can guarantee the system stability and the solution of the controller is obtained by solving Riccati equations. The proposed reset controller design method is suitable to apply to short-span and long-span track seeking of a hard disk drive servo system. The proposed design is much more capable of improving transient response than traditional control design methods.

A discrete time H_2 reset control algorithm to achieve better disturbance at-

tenuation. A design method of discrete-time reset control systems with the reset time instants pre-specified. With a base linear system designed conventionally, the discrete-time reset law design aims at improving disturbance suppression of the system. The design method can be converted to a standard H_2 optimal control problem and the solution of the controller is obtained by solving Riccati equations. The proposed reset controller design method is applied to track following stage of a hard disk drive servo system.

A unified reset control design method to achieve both track-seeking and track-following performance in hard disk drive servo systems. It involves previous two reset method. The unified control makes a soft switch from LQR reset controller to H_2 reset controller, which avoids sharp vibrations. The unified reset controller inherits the merits of both discrete LQR reset controller and discrete H_2 reset controller, and achieves both swift seeking and accurate following performance.

The LQR reset method also used in Dual stage HDD servo system. The dual stage system consists of a voice coil motor (VCM) as a first stage actuator and a push-pull-type piezo-electric transducer (PZT) as a second-stage actuator. It is considered as Dual-input Single-output (DISO) problem. We designed two optimal reset controllers for VCM and PZT, and combined the two components with a decoupled master-slave structure. With a base linear system designed conventionally, the discrete-time reset law design aims at improving the system transient responses. The design method can guarantee the system stability and the solution of the controller is also obtained by solving Riccati equations. The proposed method is able to achieve short settling time and low overshoot.

An "add on" disturbance method which can cancel most of the components in it has been proposed. This method has rebuilt a signal which can approximately represent the disturbances thus cancel the disturbances by adding the opposite value of this signal. When rebuilding this signal, internal model principle and magni-

tude/phase adaptive estimation algorithm is used to recreate the disturbance signal. Moreover, a polynomial adaptive fitting method which does not need frequency estimation is proposed to deal with time delay problem which exists in the signal recreation stage. It turns out that the adaptive fitting method is much more effective to deal with multiple frequency disturbances. The cancellation can effectively cancel single frequency and multiple frequency disturbances even when the disturbance is time-varying in both frequency and amplitude/phase.

A general form of disturbance observers, which does not need to solve the plant model inverse, and uses H_∞ control method to design the Q-filter in the disturbance observer. The conventional disturbance observer is proved to be a special case of the generalized form. Comparison to the conventional form of disturbance observers is made and the significance of the proposed method in DOB design and disturbance attenuation is verified via simulation and experimental results.

9.2 Suggestions for Future Research

Narrow Bandwidth Disturbance Rejection with Middle and High Frequency

The narrow bandwidth disturbance rejection attracts industry's interest. Chapter 7 presents a solution of disturbance rejection but this method can just deal with the low frequency disturbances. For middle frequency and high frequency disturbance, A generalized Kalman-Yakubovic-Popov (KYP) lemma establishes the equivalence between a frequency domain inequality (FDI) for a transfer function and a LMI associated with its state-space realization. Several theorems and proof were deducted and applied in HDD servo systems, see [65, 144, 145]. Such methods can be applied in off-line calculation, but they cannot deal with time-varying disturbances.

To find an adaptive methods to deal with the middle frequency and high frequency disturbances time-varying frequencies is an important and challenging issue. In such method, it should have an on line frequency identification, such as Fourier Transformation, and then the notch filter with the dominant identified frequency. The phase in middle frequency and high frequency is very sensitive, the design of notch filter is difficult to make phase stable.

Figure 9.1: The structure of adaptive notch filter

Acoustics Reduction

Nowadays, hard disk drives (HDD) are more widely used in some video/audio devices such as DVR, video camera, MP3 players, etc.; for such applications the acoustic level of HDD is a key factor of performance. The acoustics emitted by an HDD are normally excited by structural loading, namely, the acoustics of HDD mainly comes from two sources: the spindle motor during spinning and the VCM actuator during seeking. During seeking, VCM current is the dominant excitation source. The key to improve the seeking acoustics is to reduce the rate of seeking current thus to reduce seeking power at high frequencies so that mechanical modes are not excited to generate noise.

Attempts [146,147] have been made, the acoustics in HDDs should be greatly reduced without significant increase of the seeking time.

Integrated Design of a Controller and a Structure

In head-positioning systems of hard disk drive, the negative effects of a primary resonant mode are serious problems for positioning accuracy. In order to improve positioning accuracy, the primary resonant frequency should be increased because the effects of the primary resonant mode limit the servo bandwidth. An integrated design [148] of a controller and structure for the head positioning system was developed to solve this problem. [149–151] designed a method that can suppress the vibration from high-order resonant modes in the head-positioning system of a hard disk drive. The mechanical structure improvement can widen the space for servo control performance. Thus, to propose a mechanical structure with certain frequency response which can provide a better model for hdd control design is an interesting issue for the advanced research and design.

Dual-Stage Reset Control for Track following

In order to increase HDD TPI, it cannot be avoided to turn to dual-stage as solution. The dual-stage plant can get much higher bandwidth the single one, and the extra degree of freedom provides wide design space for improve the performance. [152,153] proposed PQ Method which is a technique for transforming the problem of designing controllers for dual-input/single-output (DISO) systems to two single-input/single-output (SISO) design problems, and the sbode plot is a new modification of the traditional bode plot which facilitates loop shaping design for SISO systems. That is an control allocation issue which means how you divide the responsibility for each stage.

Usually we divide the responsibility as that VCM work at low frequency and MA work at high frequency. For our reset control, LQR reset control has low bandwidth but good transient response performance which is suitable for track seeking, while H_2 reset control has higher bandwidth which is suitable for track following. Thus to design LQR reset control for VCM to improve seeking performance and de-

sign a H_2 rest controller for MA to let them work in higher frequency and improve track following performance can be considered as a new solution.

Author's Publications

The research work related to this thesis has been reported in the following publications:

1. Hui Li, Chunling Du, Youyi Wang and Yuqian Guo, "Discrete-Time Optimal Reset Control for Hard Disk Drive Servo Systems," *IEEE Transactions on Magnetics*, Vol.45, No. 11, 5104-5107, Nov 2009.
2. Chunling Du, Hui Li, Thum, C.K., Lewis, F.L., Youyi Wang, "Simple Disturbance Observer for Disturbance Compensation," *IET Journal of Control Theory and Application*, Vol. 4, Iss. 9, pp. 1748-1755, 2010.
3. Hui Li, Chunling Du, Youyi Wang, "Optimal Reset Control for a Dual-stage Actuator System in HDDs," *IEEE Transactions on Mechatronics*, Vol. 16, No. 3, pp. 480-488, Jun, 2011.
4. Hui Li, Chunling Du and Youyi Wang, A Novel Algorithm for Unknown Periodic Disturbance Cancellation in HDDs," *In Proceedings of 10th Intl. Conf. on Control, Automation, Robotics and Vision*, pp. 97-102, Hanoi, Vietnam, Dec. 17-20, 2008.
5. Hui Li, Chunling Du, Youyi Wang and Yuqian Guo, "Optimal Reset Control for a Hard Disk Drive Dual-stage Actuator System," *In Proceedings of Asia Pacific Magnetic Recording Conference*, Singapore, Jan. 14-16 2009.
6. Hui Li, Chunling Du, Youyi Wang and Yuqian Guo, "Discrete-time Optimal Reset Control for the Improvement of HDD Servo Control Transient Performance," *In Proceedings of American Control Conference*, St. Louis, USA, Jun. 10-12 2009.
7. Hui Li, Chunling Du, and Youyi Wang, "Discrete-time H2 Optimal Reset Control with Application to HDD Track-following," *In Proceedings of 2009 Chinese Control and Decision Conference*, pp 3613-17, 2009.
8. Chunling Du, Hui Li, Thum, C.K., Lewis, F.L., Youyi Wang, "A novel disturbance observer design and application in HDDs for disturbance compensation," *In proceedings of TENCON 2009*. Singapore, 2009.
9. Hui Li, Chunling Du, and Youyi Wang. "A Unified Controller Design Using Two Different Types of Optimal Reset Control Laws for HDD Servo Systems,"

In proceedings of 11th Intl. Conf. on Control, Automation, Robotics and Vision, pp. 2242-2247, Singapore, Dec. 7-10 2010.

Bibliography

- [1] Western Digital Co., *Western Digital 2009 Annual Report*. Lake Forest, USA: Western Digital, 2009.
- [2] Unescopress, Retrieved: Nov 2010. Digital Information Poses Problems for Conservationists. [Online] Available: <http://portal.unesco.org/en/ev.php-URL-ID-4805-URL-DO-DO-TOPIC-URL-SECTION-201.html>.
- [3] IDC, Retrieved: Mar 2008. An Updated Forecast of Worldwide Information Growth Through 2011. [Online] Available: <http://www.emc.com/collateral/analyst-reports/diverse-exploding-digital-universe.pdf>.
- [4] Wikipedia, Retrieved: Jul 2007. IBM 305 RAMAC. [Online] Available: http://en.wikipedia.org/wiki/IBM_305.
- [5] A. A. Mamun, G. Guo, and C. Bi, *Hard Disk Drive: Mechatronics and Control*. Florida, USA: CRC Press, 2006.
- [6] D. Abramovitch and G. Franklin, “A brief history of disk drive control,” *IEEE Control Systems Magazine*, vol. 22, no. 3, pp. 28 – 42, 2002.
- [7] Keppanet, Retrieved: June 2010. HARD DISK INSIDE. [Online] Available: <http://keppanet.netfirms.com/keppanet/harddisk/hdinside.htm>.
- [8] T. Yamaguchi, “Modelling and control of a disk file head-positioning system,” *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 215, no. 6, pp. 549–568, 2001.
- [9] B. M. Chen, T. H. Lee, K. Peng, and V. Venkataramanan, *Hard Disk Drive Servo Systems, 2nd edition*. New York, USA: Springer, 2006.
- [10] M. L. Workman, *Adaptive Proximate Time Optimal Servomechanisms*. PhD thesis, Stanford University, 1987.

- [11] L. Yi and M. Tomizuka, "Two-degree-of-freedom control with robust feedback control for hard disk servo systems," *IEEE/ASME Transactions on Mechatronics*, vol. 4, pp. 17–24, Mar 1999.
- [12] S. Hara, T. Hara, L. Yi, and M. Tomizuka, "Novel reference signal generation for two-degree-of-freedom controllers for hard disk drives," *IEEE/ASME Transactions on Mechatronics*, vol. 5, pp. 73–78, Mar. 2000.
- [13] Y. Li, *Two Degree of Freedom Control for Disk Drive Servo Systems*. PhD thesis, University of California, Berkeley, USA, 2000.
- [14] M. Tomizuka, "On the design of digital tracking controllers," *Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control*, vol. 115, no. 2B, pp. 412–418, Jun 1993.
- [15] T. Goh, Z. Li, B. Chen, T. H. Lee, and T. Huang, "Design and implementation of a hard disk drive servo system using robust and perfect tracking approach," *IEEE Transactions on Control Systems Technology*, vol. 9, pp. 221–233, Mar. 2001.
- [16] K. Liu, B. Chen, and Z. Lin, "On the problem of robust and perfect tracking for linear systems with external disturbances," *International Journal of Control*, vol. 74, no. 2, pp. 158–174, Jan 2001.
- [17] H. Fujimoto, Y. Hori, and A. Kawamura, "Perfect tracking control based on multirate feedforward control with generalized sampling periods," *IEEE Transactions on Industrial Electronics*, vol. 48, no. 3, pp. 636–644, 2001.
- [18] H. Fujimoto, "Rro compensation of hard disk drives with multirate repetitive perfect tracking control," *IEEE Transactions on Industrial Electronics*, vol. 56, pp. 3825–3831, Oct. 2009.
- [19] R. Chen, G. Guo, T. Huang, T.-S. Low, and S. Weerasooriya, "Optimal multirate control design for hard disk drive servo systems," *IEEE Transactions on Magnetics*, vol. 34, no. 4, pp. 1898–1900, 1998, 07.
- [20] T. Yamaguchi, K. Shishida, S. Tohyama, and H. Hirai, "Mode switching control design with initial value compensation and its application to head positioning control on magnetic disk drives," *IEEE Transactions on Industrial Electronics*, vol. 43, pp. 65–73, Feb. 1996.

- [21] T. Yamaguchi, H. Numasato, and H. Hirai, "A mode-switching control for motion control and its application to disk drives: Design of optimal mode-switching conditions," *IEEE/ASME Transactions on Mechatronics*, vol. 3, no. 3, pp. 202–209, 1998.
- [22] M. Hirata and M. Tomizuka, "Short track seeking of hard disk drives under multirate control-computationally efficient approach based on initial value compensation," *IEEE/ASME Transactions on Mechatronics*, vol. 10, no. 5, pp. 535–545, 2005.
- [23] L. Yang and M. Tomizuka, "Short seeking by multirate digital controllers for computation saving with initial value adjustment," *IEEE/ASME Transactions on Mechatronics*, vol. 11, pp. 9 – 16, Feb. 2006.
- [24] J. Zhou and Y. Wang, "Fast sliding-mode seek control of hard disk servos," *Microsystem Technologies*, vol. 10, no. 1, pp. 35–41, Dec. 2003.
- [25] W. C. Wu and T. S. Liu, "Sliding-mode based learning control for track-following in hard disk drives," *Mechatronics*, vol. 14, no. 8, pp. 861–876, 2004.
- [26] Q. Hu, C. Du, L. Xie, and Y. Wang, "Discrete-time sliding mode control with time-varying surface for hard disk drives," *IEEE Transactions on Control Systems Technology*, vol. 17, pp. 175 –183, Jan. 2009.
- [27] B. M. Chen, T. H. Lee, K. Peng, and V. Venkataramanan, "Composite nonlinear feedback control for linear systems with input saturation: Theory and an application," *IEEE Transactions on Automatic Control*, vol. 48, no. 3, pp. 427–439, 2003.
- [28] V. Venkataramanan, K. Peng, B. M. Chen, and T. H. Lee, "Discrete-time composite nonlinear feedback control with an application in design of a hard disk drive servo system," *IEEE Transactions on Control Systems Technology*, vol. 11, no. 1, pp. 16–23, 2003.
- [29] K. Peng, B. Chen, G. Cheng, and T. Lee, "Modeling and compensation of nonlinearities and friction in a micro hard disk drive servo system with nonlinear feedback control," *IEEE Transactions on Control Systems Technology*, vol. 13, pp. 708 – 721, Sept. 2005.
- [30] W. Lan, C. K. Thum, and B. Chen, "A hard-disk-drive servo system design using composite nonlinear-feedback control with optimal nonlinear gain tuning

- methods,” *IEEE Transactions on Industrial Electronics*, vol. 57, pp. 1735 – 1745, May 2010.
- [31] C. K. Thum, C. Du, B. Chen, E. H. Ong, and K. P. Tan, “A unified control scheme for track seeking and following of a hard disk drive servo system,” *IEEE Transactions on Control Systems Technology*, vol. 18, pp. 294 –306, Mar. 2010.
- [32] G. Cheng and K. Peng, “Robust composite nonlinear feedback control with application to a servo positioning system,” *IEEE Transactions on Industrial Electronics*, vol. 54, no. 2, pp. 1132–1140, 2007.
- [33] G. Cheng, K. Peng, B. M. Chen, and T. H. Lee, “Improving transient performance in tracking general references using composite nonlinear feedback control and its application to high-speed XY-table positioning mechanism,” *IEEE Transactions on Industrial Electronics*, vol. 54, no. 2, pp. 1039–1051, 2007.
- [34] C. Thum, *Advanced Control Techniques For Future Storage Devices*. PhD thesis, National University of Singapore, Singapore, 2008.
- [35] Y. Li, V. Venkataramanan, G. Guo, and Y. Wang, “Dynamic nonlinear control for fast seek-settling performance in hard disk drives,” *IEEE Transactions on Industrial Electronics*, vol. 54, no. 2, pp. 951–962, 2007.
- [36] H. Ho, “Fast servo bang-bang seek control,” *IEEE Transactions on Magnetics*, vol. 33, no. 6, pp. 4522 – 7, 1997, 11.
- [37] H. Ho, “Noise impact on servo tmr,” *Proceedings of the 1997 American Control Conference*, vol. 5, pp. 2906 – 2909, 1997.
- [38] G. F. Franklin, J. D. Powell, and M. L. Workman, *Digital Control of Dynamic Systems*, 3rd ed. Massachusetts: Addison-Wesley, 1998.
- [39] T. Yamaguchi and S. Nakagawa, “Recent control technologies for fast and precise servo system of hard disk drives,” in *6th International Workshop on Advanced Motion Control. Proceedings*, pp. 69 – 73, 2000.
- [40] M. T. White, M. Tomizuka, and C. Smith, “Improved track following in magnetic disk drives using a disturbance observer,” *IEEE/ASME Transactions on Mechatronics*, vol. 5, no. 1, pp. 3–11, 2000.

- [41] J. Ishikawa and M. Tomizuka, "Pivot friction compensation using an accelerometer and a disturbance observer for hard disk drives," *IEEE/ASME Transactions on Mechatronics*, vol. 3, no. 3, pp. 194–201, 1998.
- [42] M. Tomizuka, T. Tsao, and K. Chew, "Analysis and synthesis of discrete-time repetitive controllers," *ASME Journal of Dynamic Systems, Measurement and Control*, vol. 111, pp. 353–358, Sep. 1989.
- [43] L. Guo, "Reducing the manufacturing costs associated with hard disk drives—a new disturbance rejection control scheme," *IEEE/ASME Transactions on Mechatronics*, vol. 2, no. 2, pp. 77–85, 1997.
- [44] D. Phan, "The design and modeling of multirate digital control systems for disk drive applications," in *Asia-Pacific Workshop on Advances in Motion Control Proceedings*, pp. 189 – 205, 1993.
- [45] W.-J. Cao, Q. Bi, X. Liu, C. Lim, and Y. C. Soh, "Frequency-domain transfer function of digital multirate controller with current estimator," *IEEE Transactions on Control Systems Technology*, vol. 13, pp. 131 – 137, Jan. 2005.
- [46] Q.-W. Jia, K.-K. Ooi, and D.-W. Wu, "Multirate control of an hdd servo system with intersample ripple free," *IEEE Transactions on Magnetics*, vol. 38, pp. 2192 – 2194, Sept. 2002.
- [47] K. Ohno and R. Horowitz, "A variable structure multirate state estimator for seeking control of hdds," *IEEE Transactions on Control Systems Technology*, vol. 13, pp. 233 – 245, Mar. 2005.
- [48] Q.-W. Jia, "Intersample ripple-free multirate control with application to a hard disk drive servo," *IEEE/ASME Transactions on Mechatronics*, vol. 10, pp. 341 – 345, Jun. 2005.
- [49] J. Ding, F. Marcassa, S.-C. Wu, and M. Tomizuka, "Multirate control for computation saving," *IEEE Transactions on Control Systems Technology*, vol. 14, pp. 165 – 169, Jan. 2006.
- [50] S. Weerasooriya and D. T. Phan, "Discrete-time LQG/LTR design and modeling of a disk drive actuator tracking servo system," *IEEE Transactions on Industrial Electronics*, vol. 42, no. 3, pp. 240–247, 1995.
- [51] B. M. Chen and Y.-L. Chen, "Loop transfer recovery design via new observer-based and css architecture-based controllers," *International Journal of Robust and Nonlinear Control*, vol. 5, no. 7, pp. 649 – 669, 1995.

- [52] B. M. Chen, *Robust and H_∞ Control*. London: Springer, 2000.
- [53] Y. Teo and T. Tay, "Application of the l_1 -optimal regulation strategy to a hard disk servo system," *IEEE Transactions on Control Systems Technology*, vol. 4, pp. 467 – 472, Jul. 1996.
- [54] Z. Li, G. Guo, B. Chen, and T. Lee, "Optimal track-following design for the highest tracks per inch in hard disk drives," *Journal of Information Storage and Processing Systems*, vol. 3, no. 1-2, pp. 27 – 41, Jan. 2001.
- [55] J. Teoh, *High Precision Motion Control for Hard Disk Drive Systems*. PhD thesis, Nanyang Technological University, Singapore, 2007.
- [56] R. Middleton, "Trade-offs in linear control system design," *Automatica*, vol. 27, no. 2, pp. 281 – 292, 1991.
- [57] G. Goodwin and M. Seron, "Fundamental design tradeoffs in filtering, prediction, and smoothing," *IEEE Transactions on Automatic Control*, vol. 42, no. 9, pp. 1240 – 51, Sept. 1997.
- [58] J. Chen, "Sensitivity integrals and transformation techniques: A new perspective," *IEEE Transactions on Automatic Control*, vol. 42, no. 7, pp. 1037–1044, 1997.
- [59] Y. Li, *Nonlinear Control Designs For Hard Disk Drive Servos*. PhD thesis, Nanyang Technological University, Singapore, 2005.
- [60] Y. Li, G. Guo, and Y. Wang, "Nonlinear loop shaping for suppressing sensitivity hump in hard disk drive servo systems," in *Proceedings of the 5th International Conference on Control and Automation, ICCA'05*, (Budapest, Hungary), pp. 800 – 806, 2005.
- [61] R. Chen, *Advanced Control Designs For Hard Disk Drive Servo Systems*. PhD thesis, National University of Singapore, Singapore, 2001.
- [62] S. Weerasooriya, J. Zhang, and T. Low, "Efficient implementation of adaptive feedforward runout cancellation in a disk drive," *IEEE Transactions on Magnetics*, vol. 32, no. 5, pp. 3920 – 3922, Sept. 1996.
- [63] K. Ohno and T. Hara, "Adaptive resonant mode compensation for hard disk drives," *IEEE Transactions on Industrial Electronics*, vol. 53, pp. 624 – 630, April 2006.

- [64] H. Taghirad and E. Jamei, “Robust performance verification of adaptive robust controller for hard disk drives,” *IEEE Transactions on Industrial Electronics*, vol. 55, pp. 448–456, Jan. 2008.
- [65] C. Du, L. Xie, J. Zhang, and G. Guo, “Disturbance rejection for a data storage system via sensitivity loop shaping and adaptive nonlinear compensation,” *IEEE/ASME Transactions on Mechatronics*, vol. 13, pp. 493–501, Oct. 2008.
- [66] S. Koganezawa, Y. Uematsu, T. Yamada, H. Nakano, J. Inoue, and T. Suzuki, “Dual-stage actuator system for magnetic disk drives using a shear mode piezoelectric microactuator,” *IEEE Transactions on Magnetics*, vol. 35, pp. 988–992, Mar. 1999.
- [67] G. Herrmann, S. Ge, and G. Guo, “Practical implementation of a neural network controller in a hard disk drive,” *IEEE Transactions on Control Systems Technology*, vol. 13, pp. 146–154, Jan. 2005.
- [68] M. Cimino and P. Pagilla, “A design technique for multirate linear systems,” *IEEE Transactions on Control Systems Technology*, vol. 17, pp. 1342–1349, Nov. 2009.
- [69] K.-S. Low and T.-S. Wong, “A multiobjective genetic algorithm for optimizing the performance of hard disk drive motion control system,” *IEEE Transactions on Industrial Electronics*, vol. 54, pp. 1716–1725, Jun. 2007.
- [70] C. Y. Lai, F. Lewis, V. Venkataramanan, X. Ren, S. S. Ge, and T. Liew, “Disturbance and friction compensations in hard disk drives using neural networks,” *IEEE Transactions on Industrial Electronics*, vol. 57, pp. 784–792, Feb. 2010.
- [71] T. Semba, T. Hirano, J. Hong, and L. Fan, “Dual-stage servo controller for HDD using MEMS microactuator,” *IEEE Transactions on Magnetics*, vol. 35, pp. 2271–2273, Sept. 1999.
- [72] Y. Soeno, S. Ichikawa, T. Tsuna, Y. Sato, and I. Sato, “Piezoelectric piggyback microactuator for hard disk drive,” *IEEE Transactions on Magnetics*, vol. 35, pp. 983–987, Mar. 1999.
- [73] Y. Li, R. Horowitz, and R. Evans, “Vibration control of a pzt actuated suspension dual-stage servo system using a pzt sensor,” *IEEE Transactions on Magnetics*, vol. 39, pp. 932–937, Mar. 2003.

- [74] T. Hirano, M. White, H. Yang, K. Scott, S. Pattanaik, S. Arya, and F.-Y. Huang, "A moving-slider mems actuator for high-bandwidth hdd tracking," *IEEE Transactions on Magnetics*, vol. 40, pp. 3171 – 3173, jul. 2004.
- [75] T. Imamura, M. Katayama, Y. Ikegawa, T. Ohwe, R. Koishi, and T. Koshikawa, "Mems-based integrated head/actuator/slider for hard disk drives," *IEEE/ASME Transactions on Mechatronics*, vol. 3, pp. 166 –174, Sept 1998.
- [76] W. Digital, Retrieved: Nov 2010. Western Digital Caviar Black 2TB Hard Drive Review. [Online] Available: <http://www.hardwarecanucks.com/forum/hardware-canucks-reviews/24310-western-digital-caviar-black-2tb-hard-drive-review-3.html>.
- [77] D. Horsley, R. Horowitz, and A. Pisano, "Microfabricated electrostatic actuators for hard disk drives," *IEEE/ASME Transactions on Mechatronics*, vol. 3, pp. 175 –183, Sept. 1998.
- [78] B. M. Chen, T. H. Lee, C. C. Hang, Y. Guo, and S. Weerasooriya, " H_∞ almost disturbance decoupling robust controller design for a piezoelectric bimorph actuator with hysteresis," *IEEE Transactions on Control Systems Technology*, vol. 7, pp. 160–174, 1999.
- [79] Y. Li and R. Horowitz, "Mechatronics of electrostatic microactuators for computer disk drive dual-stage servo systems," *IEEE/ASME Transactions on Mechatronics*, vol. 6, pp. 111 –121, Jun. 2001.
- [80] D. Wu, *Study on High Performance Dual-stage Servomechanism in Hard Disk Drives*. PhD thesis, National University of Singapore, Singapore, 2002.
- [81] G. Herrmann, M. Turner, I. Postlethwaite, and G. Guo, "Practical implementation of a novel anti-windup scheme in a hdd-dual-stage servo-system," *IEEE/ASME Transactions on Mechatronics*, vol. 9, pp. 580 –592, Sept. 2004.
- [82] D. Wu, G. Guo, and T. C. Chong, "Comparative analysis on resonance compensation in HDD dual-stage actuation systems," *IEEE Transactions on Industrial Electronics*, vol. 50, no. 6, pp. 2189–2191, 2003.
- [83] S.-H. Lee and Y.-H. Kim, "Minimum destructive interference design of dual-stage control systems for hard disk drives," *IEEE Transactions on Control Systems Technology*, vol. 12, pp. 517 – 531, Jul. 2004.

- [84] S. H. Lee, C. C. Chung, and C. W. Lee, "Active high-frequency vibration rejection in hard disk drives," *IEEE/ASME Transactions on Mechatronics*, vol. 11, no. 3, pp. 339–345, 2006.
- [85] C. Du and G. Guo, "Lowering the hump of sensitivity functions for discrete-time dual-stage systems," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 5, pp. 791–797, 2005.
- [86] G. Herrmann, B. Hredzak, M. Turner, I. Postlethwaite, and G. Guo, "Discrete robust anti-windup to improve a novel dual-stage large-span track-seek/following method," *IEEE Transactions on Control Systems Technology*, vol. 16, pp. 1342–1351, Nov. 2008.
- [87] H. Numasato and M. Tomizuka, "Settling control and performance of a dual-actuator system for hard disk drives," *IEEE/ASME Transactions on Mechatronics*, vol. 8, pp. 431–438, Dec. 2003.
- [88] K. Chan, W. Liao, and I. Shen, "Precision positioning of hard disk drives using piezoelectric actuators with passive damping," *IEEE/ASME Transactions on Mechatronics*, vol. 13, pp. 147–151, Feb. 2008.
- [89] M. Kobayashi and R. Horowitz, "Track seek control for hard disk dual-stage servo systems," *IEEE Transactions on Magnetics*, vol. 37, pp. 949–954, Mar. 2001.
- [90] B. Hredzak, G. Hermann, and G. Guo, "A proximate-time-optimal-control design and its application to a hard disk drive dual-stage actuator system," *IEEE Transactions on Magnetics*, vol. 42, no. 6, pp. 1708–1715, Jun. 2006.
- [91] J. Zheng, M. Fu, Y. Wang, and C. Du, "Nonlinear tracking control for a hard disk drive dual-stage actuator system," *IEEE/ASME Transactions on Mechatronics*, vol. 13, pp. 510–518, Oct. 2008.
- [92] Polytec, *POLYTEC Scanning vibrometer PSV-200 / Controller OFV-3001-S: Operator's manual*. Waldbronn, Germany: Polytec GmbH, 1999.
- [93] Hewlett Packard Co., *HP 35670A : Operator's guide*. Washington, USA: Hewlett Packard Co., 1994.
- [94] Lecroy Corporation, *LeCroy Digital Oscilloscopes 9350/54 Series : Operator's manual*. New York, USA: Lecroy Corporation, 1996.

- [95] C. Du, J. Zhang, and G. Guo, "Disturbance modeling and control design for self-servo track writing," *IEEE/ASME Transactions on Mechatronics*, vol. 10, no. 1, pp. 122–127, 2005.
- [96] J. C. Clegg, "A nonlinear integrator for servomechanism," *Transactions of the American Institute of Electrical Engineers*, vol. 77, pp. 41–42, Nov. 1958.
- [97] K. R. Krishnan and I. M. Horowitz, "Synthesis of a nonlinear feedback system with significant plant-ignorance for prescribed system tolerance," *International Journal of Control*, vol. 19, pp. 689–706, 1974.
- [98] I. Horowitz and P. Rosenbaum, "Nonlinear design for cost of feedback reduction in systems with large parameter uncertainty," *International Journal of Control*, vol. 21, pp. 977–1001, 1975.
- [99] Y. Chait and C. V. Hollot, "On horowitz's contributions to reset control," *International Journal of Robust and Nonlinear Control*, vol. 12, pp. 335–355, 2002.
- [100] O. Beker, C. V. Hollot, and Y. Chait, "Plant with integrator: an example of reset control overcoming limitations of linear feedback," *IEEE Transaction on Automatic Control*, vol. 46(11), pp. 1797–1799, 2001.
- [101] Y. Zheng, Y. Chait, C. V. Hollot, M. Steinbuch, and M. Norg, "Experimental demonstration of reset control design," *IFAC J. Control Engineering Practice*, vol. 8(2), pp. 113–120, 2000.
- [102] J. E. Bobrow, F. Jabbari, and K. Thai, "An active truss element and control law for vibration suppression," *Smart Materials and Structures*, vol. 4, pp. 264–269, 1995.
- [103] R. T. Bupp, D. S. Bernstein, and V. C. and W. M. Haddad, "Resetting virtual absorbers for vibration control," in *Proceedings of the American Control Conference. Albuquerque, NM*, pp. 2647–2651, Jul. 1997.
- [104] W. M. Haddad, V. Chellaboina, and N. A. Kablar, "Active control of combustion instabilities via hybrid resetting controllers," in *Proceedings of the American Control Conference, Chicago, IL*, 2000.
- [105] D. Wu, G. Guo, and Y. Wang, "Reset integral-derivative control for hdd servo systems," *IEEE Transactions on Control Systems Technology*, vol. 15, pp. 161–167, Jan. 2007.

- [106] Y. Li, G. Guo, and Y. Wang, “Nonlinear mid-frequency disturbance compensation in hdds,” in *in the 16th IFAC Triennial World Congress, Praha, Crech*, pp. 4–8, Jul. 2005.
- [107] Q. Chen, Y. Chait, and C. V. Hollot, “Analysis of reset control systems consisting of a fore and second order loop,” *Journal of Dynamic Systems, Measurement, and Control*, Jun. 2001.
- [108] O. Beker, C. V. Hollot, Q. Chen, and Y. Chait, “Stability of a reset control system under constant inputs,” in *Proceedings of the American Control Conference, San Diego, California*, pp. 3044–3045, Jun. 1999.
- [109] O. Beker, C. V. Hollot, and Y. Chait, “Stability of a mimo reset control system under constant inputs,” in *Proceedings of the 38th Conference on Decision & Control, Phoenix, Arizona USA*, pp. 2780–2781, Dec. 1999.
- [110] Q. Chen, C. V. Hollot, and Y. Chait, “Stability and asymptotic performance analysis of a class of reset control systems,” in *Proceedings of the 39th IEEE Conference on Decision and Control, Sydney, Australia*, pp. 251–256, Dec. 2000.
- [111] O. Beker, C. V. Hollot, Y. Chait, and H. Han, “Fundamental properties of reset control systems,” *Automatica*, vol. 40, pp. 905–915, Jun. 2004.
- [112] T. Yang, *Impulsive Control Theory*. New York: Springer-Verlag Berlin Heidelberg, 2001.
- [113] Y. Guo, Y. Wang, and L. Xie, “Frequency-domain properties of reset systems with application in hard-disk-drive systems,” *IEEE Transactions on Control Systems Technology*, vol. 17, pp. 1446–1453, Nov. 2009.
- [114] V. Strejc, *State Space Theory of Discrete Linear Control*. Chichester: John Wiley & Sons, 1981.
- [115] Y. Guo, Y. Wang, L. Xie, and J. Zheng, “Stability analysis and design of reset systems: Theory and an application,” *Automatica*, vol. 45, no. 2, pp. 492 – 497, 2009.
- [116] Z. Li, G. Guo, B. M. Chen, and T. H. Lee, “Optimal track-following design for the highest tracks per inch in hard disk drives,” *Journal of Information Storage and Processing Systems*, vol. 3, no. 1, pp. 27–41, 2001.

- [117] C. Du, L. Xie, J. N. Teoh, and G. Guo, "An improved mixed H_2/H_∞ control design for hard disk drives," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 5, pp. 832–839, 2005.
- [118] C. Du, L. Xie, J. N. Teoh, and G. Guo, "H2 control for head positioning in axial and radial dimensions for self-servo track writing," *IEEE Transactions on Control Systems Technology*, vol. 16, pp. 177 –181, Jan. 2008.
- [119] K. Zhou, J. C. Doyle, and K. Glover, *Robust and Optimal Control*. Englewood Cliffs, NJ: Prentice-Hall, 1996.
- [120] M. de Oliveira, J. Geromel, and J. Bernussou, "An lmi optimization approach to multiobjective controller design for discrete-time systems," in *Proceedings of the 38th IEEE Conference on Decision and Control*, vol. 4, pp. 3611 – 16, 1999.
- [121] T. Doh and J. Ryoo, "A linear matrix inequality approach to initial value compensation for mode switching control in hard disk drive servo systems," *IEEE Transactions on Magnetics*, vol. 45, pp. 3362–3364, Sep. 2009.
- [122] M. Bodson, "Performance of an adaptive algorithm for sinusoidal disturbance rejection in high noise," *Automatica*, vol. 37, no. 7, pp. 1133 – 1140, 2001.
- [123] M. Bodson and S. Douglas, "Adaptive algorithms for the rejection of sinusoidal disturbances with unknown frequency," in *Proceedings of the 13th World Congress International Federation of Automatic Control. Vol.K. Adaptive Control*, (Oxford, UK), pp. 229 – 34, 1997.
- [124] T.-C. Tsao, Y.-X. Qian, and M. Nemani, "Repetitive control for asymptotic tracking of periodic signals with an unknown period," *Transactions of the ASME, Journal of Dynamic Systems, Measurement and Control*, vol. 122, no. 2, pp. 364 – 369, 2000.
- [125] J.-X. Xu, T. H. Lee, and H.-W. Zhang, "Comparative studies on repeatable runout compensation using iterative learning control," in *Proceedings of the 2001 American Control Conference.*, vol. 4, pp. 2834 – 2839, 2001.
- [126] L. Brown and Y. Ma, "Identification and cancellation of disturbances having two close sinusoidal components," in *Proceedings of 2006 American Control Conference*, (Minneapolis, MN, USA), pp. 5–8, 2006.

- [127] L. Brown and Q. Zhang, "Periodic disturbance cancellation with uncertain frequency," *Automatica*, vol. 40, no. 4, pp. 631 – 637, Apr. 2004.
- [128] L. Brown and Q. Zhang, "Identification of periodic signals with uncertain frequency," *IEEE Transactions on Signal Processing*, [see also *IEEE Transactions on Acoustics, Speech, and Signal Processing*], vol. 51, pp. 1538–1545, June 2003.
- [129] Q. Zhang and L. Brown, "Noise analysis of an approach for frequency identification," in *2004 43rd IEEE Conference on Decision and Control (CDC)*, vol. 1, pp. 839 – 844, 2004.
- [130] Q. Zhang and L. Brown, "Designing of adaptive bandpass filter with adjustable notch for frequency demodulation," *American Control Conference, 2003. Proceedings of the 2003*, vol. 4, pp. 2931–2936, June 2003.
- [131] C. Duan, G. Gu, C. Du, and T. C. Chong, "Robust compensation of periodic disturbances by multirate control," *IEEE Transactions on Magnetics*, vol. 44, pp. 413 –418, Mar. 2008.
- [132] M. Nagashima, K. Usui, and M. Kobayashi, "Rejection of unknown periodic disturbances in magnetic hard disk drives," *Magnetics, IEEE Transactions on*, vol. 43, pp. 3774–3778, Sept. 2007.
- [133] I. Landau, *System Identification and Control Design*. Englewood Cliffs: NJ: Prentice Hall, 1990.
- [134] Wikipedia, Retrieved: June 2008. Taylor series. [Online] Available: <http://en.wikipedia.org/wiki/Taylorseries>.
- [135] M. Tomizuka, "Zero phase error tracking algorithm for digital control," *Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control*, vol. 109, no. 1, pp. 65 – 68, Mar 1987.
- [136] M. Tomizuka, "On the design of digital tracking controllers," *Journal of Dynamic Systems, Measurement, and Control*, vol. 115, no. 3, pp. 412–418, JUNE 1993.
- [137] C. Johnson, "Accommodation of external disturbances in linear regulator and servomechanism problems," *IEEE Transactions on Automatic Control*, vol. 163, no. 6, pp. 635–644, 1971.

- [138] E. Davison, "The robust decentralized control of a general servomechanism problem," *IEEE Transactions on Automatic Control*, vol. 21, no. 1, pp. 14–24, 1976.
- [139] K. Ohnishi, "A new servo method in mechatronics," *Trans. Jpn. Soc. Elect. Eng.*, vol. 107-D, no. 4, pp. 83–86, 1987.
- [140] E. Schrijver and J. van Dijk, "Disturbance observers for rigid mechanical systems: equivalence, stability, and design," *Transactions of the ASME. Journal of Dynamic Systems, Measurement and Control*, vol. 124, no. 4, pp. 539 – 48, Dec. 2002.
- [141] T. Semba, "A disturbance observer to suppress vibration effects of a hdd in a disk array system," in *Proceedings of the American Control Conference*, vol. 2, pp. 1362 – 1367, 2003.
- [142] S.-W. Park, H. S. Yang, T.-W. Park, and Y.-P. Park, "Disturbance observer design for enhanced track following using accelerometer in hdd," in *Asia-Pacific Magnetic Recording Conference 2004. Digest of APMRC 2004*, pp. 52–53, 2004.
- [143] C. Du and G. Guo, "Lowering the hump of sensitivity functions for discrete-time dual-stage systems," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 5, pp. 791–797, 2005.
- [144] C. Du, L. Xie, G. Guo, and J. Nee Teoh, "A generalized kyp lemma based approach for disturbance rejection in data storage systems," *Automatica*, vol. 43, no. 12, pp. 2112 – 2118, 2007.
- [145] C. Du, J. N. Teoh, and L. Xie, "Positioning error minimization with specific narrowband disturbance rejections using a low-order H-2 and KYP lemma based control," *Mechatronics*, vol. 20, pp. 293–302, Mar. 2010.
- [146] J. Wang and Y. Wang, "Add-on feedforward disturbance compensation for vibration suppression in hdd," in *2009 IEEE International Conference on Control and Automation*, pp. 355 – 360, 2009.
- [147] J. Wang, Q. Jia, and Y. Wang, "Hard disk drive seeking acoustics reduction by a new two-degree-of-freedom seeking controller," *IEEE Transactions on Magnetics*, vol. 46, no. 6, pp. 1587 – 1590, Jun, 2010.

- [148] T. Atsumi, T. Shimizu, T. Arisaka, and H. Masuda, “Integrated design of a controller and a structure for head-positioning in hard disk drives,” *Journal of Vibration and Control*, vol. 12, no. 7, pp. 713 – 36, Jul, 2006.
- [149] T. Atsumi, “Feedforward control using sampled-data polynomial for track seeking in hard disk drives,” *IEEE Transactions on Industrial Electronics*, vol. 56, pp. 1338 –1346, May 2009.
- [150] T. Atsumi, A. Okuyama, and S. Nakagawa, “Vibration control above the nyquist frequency in hard disk drives,” *IEEE Transactions on Industrial Electronics*, vol. 55, pp. 3751 –3757, Oct. 2008.
- [151] T. Atsumi, T. Arisaka, T. Shimizu, and T. Yamaguchi, “Vibration servo control design for mechanical resonant modes of a hard-disk-drive actuator,” in *JSME International Journal, Series C*, pp. 819 – 827, Sept. 2003.
- [152] S. J. Schroeck, W. C. Messner, and R. J. McNab, “On compensator design for linear time-invariant dual-input single-output systems,” *IEEE Transactions on MEchatronics*, vol. 6, no. 1, pp. 412–418, March 2001.
- [153] S. J. Schroeck, W. C. Messner, and R. J. McNab, “Designing controllers for two stage disk drive actuator systems using the pq method and the sbode plot,” *IEEE Transactions on Magnetics*, vol. 37, no. 2, pp. 50–57, March 2001.