Advection and dispersion of pollutants and small rigid objects under regular and random waves

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Advection and Dispersion of Pollutants and Small Rigid Objects under Regular and Random Waves

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School of Civil and Environmental Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
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LIST OF SYMBOLS

a	=	wave amplitude (m);
A_b	=	area of the bottom surface (m ²);
A_s	=	area of the side surface (m ²);
A_w	=	wetted area (m ²);
A_{ws}	=	area of the wetted side surface (m ²);
c	=	wave celerity (m s ⁻¹);
C	=	an arbitrary constant;
C_{f0}	=	skin friction coefficient associated with the W_{10} wind speed;
C_d	=	drag force coefficient;
C_{dx}	=	drag force coefficient in x direction;
C_{dz}	=	drag force coefficient in z direction;
C_m	=	added mass coefficient;
cn	=	elliptical cosine function;
d	=	water depth (m);
D	=	dispersion coefficient (m ² s ⁻¹);
$D_{xx}, D_{xy}, D_{yx}, D_{yy}$	=	diffusion tensors (m ² s ⁻¹);
E	=	2 nd kind elliptical integral;
e_i	=	random phase of each component wave;
F_d	=	horizontal component of drag force (N);
F_g	=	horizontal force of small object caused by the gravity (N);
$f(k)$	=	wave number spectrum;
$f(\omega)$	=	frequency spectrum;
g	=	gravitational acceleration (m s ⁻²);
H	=	wave height (m);
$\overline{H}$	=	mean wave height (m);
H_s	=	significant wave height (m);

J	=	Jacobian of coordinate transformation;
K	=	dispersive coefficient caused by horizontal drift;
K	=	1 st kind elliptical integral;
k	=	wave number (m^{-1});
KC	=	Keulegan-Carpenter number;
L	=	wave length (m);
L_g	=	longitudinal length of the polyethylene sheet/small floating object (m);
L_t	=	transverse length of small floating object (m);
m	=	mass of small object (kg);
Δr	=	displacement of particles in the time interval Δt (m);
R	=	real part of what follows;
Re	=	Reynolds number;
R_t	=	tangential resistance force (N);
R_n	=	normal resistance force (N);
s	=	density ratio ρ_s/ρ ;
$s(\omega)$	=	frequency spectrum;
$\hat{s}(\hat{\omega})$	=	dimensionless frequency spectrum;
t	=	time (s);
t^*	=	non-dimensional time t/T ;
Δt	=	time interval of observation (s);
T	=	wave period (s);
$\vec{V}$	=	velocity tensor (m s^{-1});
u_s	=	Stokes drift velocity (m s^{-1});
u_s^*	=	dimensionless Stokes drift velocity;
u_d	=	drift velocity including the viscous effect (m s^{-1});
u_m	=	depth-averaged drift (m s^{-1});
u_m^*	=	dimensionless depth-averaged drift;
u_s'	=	drift-velocity fluctuations (m s^{-1});
u, w	=	velocity in horizontal and vertical direction, respectively (m s^{-1});

u, w	=	velocity in horizontal and vertical direction respectively on (ξ, η) coordinate system (m s^{-1});
$U_{19.5}$	=	wind velocity above the sea surface at 19.5 m (m s^{-1});
V	=	surface water particle velocity in horizontal direction;
W_{10}	=	wind speed at the standard anemometer height of 10 m (m s^{-1});
x, z	=	horizontal and vertical Cartesian coordinates (m);
x', z'	=	dimensionless horizontal and vertical Cartesian coordinates;

Greek Letters

β	=	damping coefficient $\sqrt{\omega / 2\nu}$;
ν	=	kinematic viscosity of fluid ($\text{m}^2 \text{s}^{-1}$);
ϕ	=	velocity potential;
$\phi^{(1)}, \phi^{(2)}$	=	first- and second-order velocity potential;
$\Phi(\theta)$	=	frequency-independent directional spreading factor;
Ψ	=	stream function;
ψ_1, ψ_2	=	first- and second-order stream function;
ω	=	angular frequency (s^{-1});
$\hat{\omega}$	=	dimensionless angular frequency;
Ω	=	vorticity;
ε	=	wave slope;
ℓ	=	submergence of the small object (m);
θ_1, θ_2	=	wave directional angle;
μ	=	dynamic viscosity (N s m^{-2});
τ	=	integral autocorrelation time of the fluctuations (s);
τ_s	=	shear stress (N m^{-2});
ξ, η	=	horizontal and vertical coordinates;
ρ_s, ρ_a and ρ	=	mass density of small object, air and water, respectively (kg m^{-3});
Γ	=	a complex constant;
Δ	=	thickness of the small object (m);
δ	=	dimensionless thickness of the wave boundary layer;

Subscription

0	=	initial value of relative variable;
1	=	physical value of the 1 st component wave;
2	=	physical value of the 2 nd component wave;
i	=	physical value of the i component wave;
j	=	physical value of the j component wave;

Superscription

(1)	=	1 st order variable;
(2)	=	2 nd order variable;
+	=	sum of two variables;
-	=	difference of two variables;

Vector operators

∇	=	$\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$;
∇^2	=	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

SUMMARY

The objective of this project is to investigate, analytically and experimentally, the advection and dispersion processes of pollutants in the ocean due to surface waves. The wave-induced drift of small floating objects is studied as well.

In the first part of this study, the drift velocity under random waves is investigated analytically. Some observations and measurements of the drift velocity of an oil patch (modeled as a floating polyethylene sheet) are presented under both deep and shallow water wave conditions. Two unique aspects of the experimental study are noted. First, we used a motion monitoring system with two infra-red cameras to record accurately the position of the polyethylene sheet at 50Hz under deep water waves. The recording provided the entire time history of the movement of the floating sheet after the wave train was initiated. The time history shows that the polyethylene sheet established a quasi-steady drift velocity shortly after the waves arrived at the floating sheet. Second, experimental measurements of the drift velocity were taken under shallow water conditions. These shallow water measurements were not available in the past. It can be observed from the experimental data that the drift velocity decreased when the water depth increased.

In the second part, whether surface floating substances would disperse in a random wave field due to Stokes drift is investigated. Based on the random-walk theory, Herterich and Hasselmann (1982) suggested that the fluctuations of Stokes-drift velocity in random waves should lead to horizontal diffusion. However, the present investigation shows that the drift velocity is actually a time-independent value that does not possess any fluctuation given a specific spectrum. Since the fluctuation is a necessary mechanism for the dispersive effect, the conclusion is that the Stokes drift in random waves would not lead to a dispersive effect on surface floating substances. Experimental results are presented to support the theoretical analysis. In addition, following a similar approach as Law (2000), the Taylor dispersion of pollutants under random wave condition is studied. The analytical results show that the Taylor dispersive effect is much more significant in random waves than in regular waves.

In the last part, we investigate the drift velocity of a small rigid floating object driven by regular waves in deep water. Experimental observations were first presented on the drift behavior of square, circular and elliptical objects by means of an infrared motion monitoring system. These objects possessed different drag and added mass coefficients, and the measurements on the drift velocity of these objects clarified some outstanding issues. The theoretical drift of a small thin flat plate was then derived. The comparison between the experimental drift and simulation results of small circle plates is given to verify the theoretical predictions.

Chapter

1

INTRODUCTION

In this chapter, the scope of the present study is introduced. It consists of both the wave-induced advection and dispersion of pollutants in the sea, and also the wave-induced drift of small floating objects in deep water waves. The layout of this report is also presented.

1.1 Advection and dispersion of pollutants in the sea

The rapid development of offshore oil production and maritime transportation heightens the worldwide risk of oil spills. Oil spill contamination is a serious hazard that threatens the marine life ecology in both deep seas and coastal areas. Once spillage occurs, the floating oil patches are transported by the combined actions of winds, currents and waves. The physical properties of crude oil, summarized in Table 1.1 (Sobey, 1992), are different from the sea water. In general, oil is significantly lighter, immiscible, and can spread as a thin layer on the water surface.

The long term fate of a surface oil patch depends on a range of weathering processes

(evaporation, emulsification, biodegradation etc.). Evaporation typically accounts for about half the spill mass. Except for evaporation, the other physical transport processes will not alter the volume of the patch. In the short term, an oil patch is subjected to both free and forced advection processes. Free advection includes the buoyant spreading (e.g. Hoult, 1972) of the oil patch under the influence of gravity, viscous and surface tension forces, in order of importance. Forced advection is the transport by local currents, winds and waves. Unless the environmental conditions are sufficiently calm, forced advection will be predominant. The centre of mass of an oil patch moves in response to the range of the forced advection processes. The spatial extent of the patch will enlarge in response to buoyant spreading and to shear flow dispersion associated with spatial gradients in the surface current field.

Intense turbulent mixing associated with breaking waves or perhaps ship propellers may force the local break-up of oil patches into droplets. These droplets may be mixed deeper into the water column by turbulence but tend to migrate back to the surface under buoyant forces. The problem remains essentially one of surface transport.

Table 1.1 Physical properties of crude oil and salt water (Sobey, 1992)

Parameters	Crude oil (typically)	Salt water (typically)
Mass density ρ	890 kg/m ³	1025 kg/m ³
Dynamic viscosity μ	0.08 kg/m s	0.001 kg/m s
Surface tension σ	0.027 kg/s ²	0.074 kg/s ²

1.1.1 Drift by Wind

Wind is an important factor for the advection of surface oil slicks. Wind blowing over the water surface initiates a stress at the water surface, see Fig. 1.1(a). The wind speed

in the lower atmospheric boundary increases with height above the water surface. The characteristic wind speed is identified as the speed at the standard anemometer height of 10 m; the wind speed is denoted as W_{10} . Assuming a quadratic form for the friction, the shear stress at the water surface is

$$\tau_s = f_a \frac{1}{2} \rho_a W_{10}^2 = C_{10} \rho_a W_{10}^2 \quad (1.1)$$

where f_a is a friction factor, ρ_a the mass density of air and C_{10} the skin friction coefficient associated with the W_{10} wind speed. C_{10} is a coefficient of the order 0.002 (Sobey, 1992).

The shear stress τ_s drives a surface current u_s and transfers momentum to the water column over another boundary layer at the water surface as sketched in Fig. 1.1(b).

Figure 1.1 Drift by wind

Assuming again a quadratic friction representation in the water boundary layer, the shear at the water surface is

$$\tau_s = f_w \frac{1}{2} \rho u_s^2 \quad (1.2)$$

where f_w is another friction factor and ρ is the mass density of water. Equating the two representations for the surface wind shear stress gives

$$u_s = \left(\frac{f_a}{f_w} \right)^{1/2} \left(\frac{\rho_a}{\rho} \right)^{1/2} W_{10} \quad (1.3)$$

With $f_a \approx f_w$, $\rho_a \approx 1.2 \text{ kg/m}^3$ and $\rho \approx 1025 \text{ kg/m}^3$, the surface wind drift is thus approximately

$$u_s = 0.03 W_{10} \quad (1.4)$$

which is often referred to as “the 3% rule” (Sobey, 1992). The wind shear at the water surface will depend on the turbulence and mean flow structure of both the atmospheric and water boundary layers and the roughness (wave activity) at the water surface. Eq. (1.4) is a reasonable order of magnitude estimate of the wind drift under these widely varying conditions and is commonly adopted for modeling purposes (Spaulding, 1988).

1.1.2 Drift by Currents

In a coastal environment, the currents may be driven by tides, wind and waves. Both tide and wind-driven circulation can be predicted by numerical hydrodynamic models based on the long wave equations. The circulation patterns are sensitive to the details of the tidal forcing (especially the diurnal inequality and the spring-neap cycle) and the nature of the wind forcing (direction, magnitude and duration).

The structure of flow currents is complex and three-dimensional, given the interaction between the wind, waves, currents and the generation of turbulent kinetic energy by shear and breaking wave activity. To simplify the problem, the currents induced by winds and waves are normally lumped together and represented by an

empirically-based drift factor as well as deflection angle that are dependent on the local wind speed and direction (Stolzenbach et al. 1977; Huang and Monastero 1982). The drift speed typically varies from 2.5% to 4.4% of the wind speed, with a mean value of 3-3.5%. The deflection angle varies between 0 and 25° to the right/left of the wind direction (northern/southern hemisphere) with a mean value of about 15°.

1.1.3 Drift by waves

In many previous studies, the wave-driven transport of surface oil was often ignored. Yet it can be a dominant mechanism transporting oil to open coast shorelines as suggested by Sobey and Baker (1997) and Cheng et al. (2003). This is because, in the absence of wind, tidal currents are naturally confined to be parallel to the adjacent shorelines and cannot carry an oil patch onto the beach. Waves, however, are directed towards fringing beaches or wetlands by the refraction mechanism and can in fact be the main agent that moves the oil ashore. Fig. 1.2 shows a schematic sketch of how an oil patch is transported from deep to shallow water and then onto the shore under currents and waves.

Figure 1.2 Wave-driven transport of surface oil spill (from Law and Huang, 2007)

Order of magnitude computations suggest that the net wave drift in the direction of wave propagation would be in the order of 0.5 km/hr for a low swell. This is at least comparable in magnitude to tidally-averaged excursions and wind drifts driven by sea breezes.

It is often assumed that the wave-induced transport can be included in the wind drift by adjusting the 0.03 factor in Eq. (1.4). This may be appropriate when the waves are driven by the local wind. Swell waves approaching an open coast are completely uncorrelated with the local wind and also drive an onshore transport. These swells are not accounted for in most of the trajectory models that focus only on wind and current-induced drift.

Although the drift due to surface waves can have the same order of magnitude as wind and currents, a satisfactory analysis and predictive capability for wave-driven surface oil patches does not exist. Hence, this study focuses on the wave factor.

In the prediction of the wave-induced drift velocity, a usual practice has been to adapt the Stokes' prediction of $u_s/c = (ka)^2$, where c the wave celerity, a the wave amplitude and k the wave number, which is based on the Stokes wave theory under the assumption of inviscid fluid. It has been known that when the free surface is covered with a viscous layer such as oil slicks or other contaminants, the Stokes drift would underestimate the drift velocity. After considering the viscous boundary layer, Longuet-Higgins (1952) proposed that the drift velocity for a viscous but clean surface should be $u_s/c = \frac{5}{4}(ka)^2$. With a contaminated surface, Phillips (1977) suggested that the drift velocity for a viscous surface layer would increase to $u_s/c = \frac{7}{4}(ka)^2$, or $\frac{7}{4}$ times greater than that given by the inviscid theory of Stokes. Based on Phillips' work, Lee and Kang (1996) obtained additional empirical

formulations on the drift of floating thin sheets in waves. Law (1999) conducted an analysis to the 3rd order of the wave steepness and found an additional 3rd order pressure component that drives the patch forward. He estimated the drift velocity of a long covering patch due to progressive waves to be $u_d / c = \frac{5}{2}(ka)^2$.

All the former work focused on a continuous free or contaminated surface. In reality, the oil patches on wave surface are finite in size (see Fig. 1.3). Thus, in the present study, further investigations were conducted on the effect due to the finite extent.

Figure 1.3 An oil patch on the water surface

1.1.4 Wave-driven transport in the shallow water

Comparing to oil patches drifting in deep water, environmentalists perhaps have more concerns on the oil patches moving ashore (see Fig. 1.2). The oil being transported to the shore can account for a significant proportion of the total spill volume. For example, in the 1978 Amoco Cadiz spill along the Brittany coast of France, Gundlach et al. (1983) reported that an estimate of 28% of the spilled oil came ashore.

There are a variety of environmental forcing mechanisms that caused an oil patch moving towards the beach, i.e. winds, currents and waves etc. Winds can certainly drive the oil spill transport onshore as reported by many investigators, e.g. Gill (1982), Sobey (1992) etc. Tidal currents naturally flow parallel to the shoreline and would

have little contribution driving the oil spill ashore (see Fig. 1.2). Wave refraction and the mass transport in progressive waves are thus the main agents driving the oil spill ashore. Especially in the absence of onshore winds, swell waves will provide a continuous mechanism for oil spill beaching. Note that in shallow waters, the wave profile will deform and become non-linear, while wave breaking may also occur, which will cause the problem further complicated.

1.1.5 Dispersion of an oil patch

Besides drift, an oil patch also disperses in the ocean. There are a number of processes contributing to the total dispersion, i.e. tidal flows, ocean currents and turbulence, which affect the displacement of particles on a large scale. On a small scale of up to a few kilometers, the action of water waves has a significant role. However, the action of wave motion is normally not considered in numerical simulations and models of pollution dispersion so far.

On the possibility of the dispersion of a surface oil patch due to the wave motion, Herterich and Hasselmann (1982) suggested a hypothesis based on the random fluctuations of Stokes drift, which is widely agreed in the literature (Buick et al., 2001). However, whether this hypothesis by Herterich and Hasselmann is valid or not is examined in detail in the study. Meanwhile, the dispersion of soluble pollutants due to the Stokes drift in random waves is also investigated.

1.2 Drift of small floating objects in deep water waves

With increasing ocean oil drilling activities, more platforms and offshore structures have been installed in the past two decades. Several of these marine structures were located in the Arctic or cold regions, e.g. near Sakhalin Island. In such regions, the drifting ice floe can be extremely hazardous to offshore structures. In order to determine the ice force resulting from the interaction, the knowledge of the velocity

and hydrodynamic characteristics of the iceberg or ice floe is necessary. Generally, the iceberg drift was regarded to be governed primarily by the wind and ocean current (e.g. El-Tahan 1983). Cheema and Ahuja (1977) and Dempster (1974) presented a relatively simple approach of predicting the drift of the ice floes involving a kinematic model, in which the velocity of the ice floes is expressed directly in terms of the velocities of the winds and currents. Based on these works, dynamic models were developed by Smith and Banke (1981), Mountain (1980), Sodhi and El-Tahan (1980) and Gaskill and Rochester (1984). These dynamic models have a better physical foundation, in that the drift of ice floes is computed by considering various environmental forces acting on ice floes:

$$m \frac{du_d}{dt} = F_{wind} + F_{current} + F_{Coriolis} \quad (1.5)$$

where m is the mass of the ice floe combined with its added mass, u_d the drift velocity of the ice floe, F_{wind} the wind drag, $F_{current}$ the current drag and $F_{Coriolis}$ the combined force associated with the Coriolis force and the effect of the sea surface slope.

For ice floes in tens-of-meters size range, Wadhams (1983) pointed out that the wave-induced drift can be dominating even under strong winds. Based solely on theoretical arguments, Arikainen (1972) drew a similar conclusion that the wave-induced drift can actually be as large as the wind-caused drift and thus should not be neglected. Harms (1986) performed laboratory measurements on the drift of wavelength-size ice floe models under regular wave conditions. He obtained relatively simple empirical formulas that can predict the wave-induced drift of these ice floes. Since then, few studies have paid attention to the drift of small floating objects.

1.3 Objective

In summary, there are two primary objectives in this study. The first objective focuses on the drift and dispersion of pollutants due to surface waves. We aim to improve the knowledge of drift and dispersion of surface pollutants such as floating oil, and also the dispersion of soluble pollutants in random waves. The second objective is to investigate the wave-induced drift of small floating objects. These two objectives are closely linked as they both necessitate the analysis of the second order net drift velocity under wave action.

1.4 Layout of the thesis

The layout of the thesis is as follows:

In this chapter, the background of this study is introduced. The objectives are also presented.

Chapter 2 reviews the relevant theories and experimental results on the advection and dispersion of pollutants in the sea. Previous investigations on the drift of small object are also highlighted.

Chapter 3 describes in detail the drift velocity of regular Stokes waves under the influence of a surface boundary layer. The drift velocity in random waves is also derived in order to verify whether the drift velocity leads to dispersion on the surface.

Chapter 4 investigates the drift of two-dimensional floating surface films by both deep and shallow water waves in the laboratory. The focus is on the observations and measurements on the drift behavior and to clarify some outstanding issues. Thin polyethylene sheets with reflective markers were used to simulate the inextensible surface films.

In Chapter 5, whether the Stokes drift due to random waves leads to surface dispersion is discussed. After clarifying the concepts with the mathematical deduction

and experimental observations, it is shown that the Stokes drift under random waves would not induce surface dispersion, contrary to the Herterich and Hasselmann's (1982) theory. The longitudinal dispersion (i.e. the so-called Taylor dispersion) in random waves is also investigated.

Chapter 6 studies the drift velocity of small rigid floating objects driven by regular waves in deep water.

Finally, the conclusions and recommendations are outlined in Chapters 7 and 8 respectively.

Chapter

2

LITERATURE REVIEW

2.1 Surface drift of inviscid fluid

As pointed out by Stokes in his classic memoir (1847), individual particles in a progressive, irrotational wave do not undergo exactly closed paths. Besides their orbital motion, they also possess a second-order mean velocity (called the mass-transport velocity) in the direction of wave propagation. Figure 2.1 shows the coordinate system for the analysis. If the surface displacement is expressed as:

$$\eta = a \cos(kx - \omega t) \quad (2.1)$$

where x and η are horizontal and vertical co-ordinates, t is the time, $k = 2\pi / L$, where L is the wave length, and $\omega = 2\pi / T$, then the Stokes' expansion for the mass-transport velocity $u_s(z)$ is equivalent to:

$$u_s(z) = \frac{a^2 k \omega \cosh 2k(d+z)}{2 \sinh^2(kd)} + C \quad (2.2)$$

where d is the water depth, C is an arbitrary constant, and z is the vertical co-ordinate measured positively upwards from the mean surface (see Fig. 2.1). If it is assumed that the total depth-wise horizontal transport is zero such as in a wave flume, we must have

$$C = -\frac{a^2 \omega \sinh 2kd}{4d \sinh^2 kd} = -\frac{a^2 \omega}{2d} \coth kd \quad (2.3)$$

In deep ocean ($kd \gg 1$), Equation (2.2) simply becomes

$$u_s(z) = a^2 k \omega e^{-2kz} \quad (2.4)$$

Figure 2.1 Coordinate systems.

Figure 2.2 A typical profile of the mass transport velocity in a progressive, irrotational wave.

The typical velocity profile of the mass transport in a progressive irrotational wave is shown in Figure 2.2. It can be seen that the velocity increases steadily with height above the bottom, and that on the bottom itself the velocity gradient is zero. Both features can be shown to be necessary consequences of the irrotational character of the motion, and do not depend on the smallness of the wave amplitude as assumed by Stokes (an elegant geometrical proof for waves in deep water was given by Rayleigh (1876); and proofs for finite depths were given by Ursell (1953) and Longuet-Higgins (1953)).

The irrotational wave is not the only type of wave theoretically possible in an ideal flow. In the exact solution of Gerstner (1809) and Rankine (1863), the particles undergo exactly circular orbits, and the mass-transport velocity vanishes identically. Indeed, Dubreil-Jacotin (1934) showed that a wave motion may be superposed upon a steady stream having an arbitrary velocity distribution, so that the mass-transport velocity could take any desired value. The hypothesis of irrotational motion was assumed by Stokes on the ground that, under conservative forces, no vorticity can be generated in the interior of a uniform fluid, even with viscosity. Therefore, if the motion is started from rest, it must be initially irrotational. The mass-transport is then uniquely determined.

2.2 Surface drift of viscous fluid

2.2.1 Free surface drift of viscous fluid

From the literature, it had been observed that the mass transport velocities measured in some previous laboratory experiments were different quantitatively from the Stokes' irrotational theory. For example, Bagnold (1947) found a strong forward velocity near the bottom and a weaker backward velocity at higher levels-the exact opposite of the Stokes velocity distribution (see Fig. 2.3). Other observers (Caligny 1878; King 1948) found a forward drift both near the bottom and the free surface, with a backward drift in between (see Fig. 2.4).

Figure 2.3 The mass-transport velocity profile of Bagnold (1947)

Figure 2.4 The mass-transport velocity profile of Caligny (1878)

The discrepancy is obviously caused by the existence of finite viscosity in real fluids. This viscous effect dominates inside the boundary layers and, by diffusion, might influence the whole fluid region, especially in shallow water.

Although several attempts had been made to include the effect of viscosity in the wave drift velocity problem, real progress in the field had to await the work of Longuet-Higgins (1953). He was the first researcher to introduce the concept of boundary layer theory into the calculations of drift velocity. He considered two-dimensional motion for waves of small amplitude in a finite depth, and included the oscillatory boundary layer with thickness $O(\delta)$ adjacent to both the bottom and free surface. The viscosity alters the flow in the boundary layers and affects the entire fluid region through the diffusion of vorticity.

Near the bottom, Longuet-Higgins (1953, 1960) first estimated that the mass transport velocity in a viscous fluid is $5/2$ times the inviscid value:

$$u_d(z = -d) = \frac{5a^2 k \omega}{4 \sinh^2(kd)} \quad (2.5)$$

where u_d is the drift velocity in a viscous fluid.

Near the free surface, the mass transport gradient profile is twice the value predicted by Stokes:

$$\left. \frac{d[u_d(z)]}{dz} \right|_{z=0} = 4a^2 \omega k^2 \coth kd \quad (2.6)$$

The mass transport velocity profile for a closed wave flume is:

$$u_d(z) = \frac{\omega a^2 \coth kd}{d^2} (z + d) \left[\frac{3}{4} \left(\frac{z}{d} - 1 \right) + \frac{1}{2} k^2 d^2 \left(\frac{3z}{d} + 1 \right) \right] + \frac{\omega a^2 k}{2 \sinh^2 kd} \left\{ \frac{9}{4} \left(\frac{z}{d} \right)^2 - \frac{3}{4} + \cosh[2k(z + d)] \right\} \quad (2.7)$$

From Longuet-Higgins' result (Eq. 2.7), it can be observed that: for large depths (relative to the wavelength), the surface drift velocity increases steadily with the depth-to-wavelength ratio; while at small depths, this solution predicts negative drift velocity. This prediction is different from the experimental measurements. Swan and Sleath (1990) derived a second approximation to the Longuet-Higgins' solution, which is fourth-order in wave steepness. Experimental measurements were also presented to verify the theoretical analysis shown in Fig. 2.5. The drift measurements at large value of kd are approaching a constant value that is significantly lower than Longuet-Higgins' prediction. On the other hand, the experimental data showed good consistency with the fourth-order solution.

Fig. 2.5 Comparison of analytical and experimental results by Swan and Sleath (1990)

Note that in this figure, h is the water depth.

For progressive waves of sufficiently large amplitudes such that the convection effects of mean vorticity dominate those of viscous diffusion in the interior of the fluid, the conduction solution (Longuet-Higgins, 1953) is not expected to be universally valid. Based on Stuart's (1966) theory, Dore (1977) attempted to obtain a convection solution valid for such waves in deep water. Laminar flow is assumed to exist throughout the fluid.

In order to yield the convection solution for progressive gravity waves, Stuart (1966) commented on the possible application of the double boundary-layer structure. Beneath and adjacent to the oscillatory free-surface layer of thickness $O(\delta)$, there exists a second (or outer) boundary layer, of thickness generally much greater than δ , within which the secondary vorticity field decays to zero. However, the most salient simplifying feature of this theory concerns the region of wave generation. In other words, the secondary boundary-layer thickness depends on the horizontal coordinate (see Fig. 2.6).

Figure 2.6 Schematic diagram of double boundary layer

which x' and z' are dimensionless coordinates measuring the distance along and perpendicular to the mean surface level respectively, δ is the dimensionless thickness of the wave boundary layers, and ϵ is the Strouhal number, i.e. the ratio of the advective terms in the momentum equations to the time-derivative terms.

The wave-induced mass transport has also been studied by Dore (1970, 1978), Unluata and Mei (1970) and Craik (1982) etc. It is conjectured that the negligence of viscosity in the Stokes theory could be a major culprit that explains the difference between the Stokes drift and real measurements, especially when the surface is covered with a thin film. When a liquid surface is contaminated by a thin layer of oil, the characteristics of the viscous boundary layer near the surface could be significantly altered. Now, the surface may be able to sustain tangential stresses due to the change in the surface tension associated with the presence of surface contaminants. The surface acts essentially as an elastic membrane that resists extension and contraction.

2.2.2 Surface drift of an inextensible film

If the free surface is dirty or covered by an inextensible thin film, the character of the viscous boundary layer near the surface will be significantly altered. The surface may be able to sustain tangential stresses due to the existence of surface tension, and the drift velocity will be different from the one with a clean free surface. The fact that the degree of contamination of the free surface has a definite influence on the drift velocity is clearly demonstrated in Fig. 2.7.

Figure 2.7 Effect of surface contamination on wave-induce mean drift velocity

Note that in this figure, h is the water depth. (Kang and Lee, 1995)

Phillips (1977) utilized a typical conformal mapping accurate to the first order of wave steepness to analyze the drift velocity with a contaminated surface. A curvilinear moving coordinate system following the wave surface was employed. The governing equation in this curvilinear coordinate was derived under the boundary layer approximations. The fluid motion inside the boundary layer was represented with a stream function which consists of the irrotational and rotational part of the motion. The stream function was subsequently expanded with respect to the perturbed vorticity equation. The leading-order equation of the perturbed vorticity equation was solved by using the no-slip condition at the free surface, assuming the surface was inextensible. Finally, he estimated that the mean Eulerian velocity difference across the boundary layer would be $(3/4)u_s$. Hence, coupled with the Lagrangian transport component, the drift velocity was predicted to be $(7/4)u_s$. In other words, the film will drift at 7/4 times the Stokes drift velocity for deep-water waves.

$$\frac{u_d}{c} = \frac{7}{4}(ka)^2 \quad (2.8)$$

An approach was suggested by Kamata (1982) to predict the wave-induced drift velocity by balancing the mean shear force induced by the wave motion on a viscous surface with the frictional drag of a flat plate subject to a uniform flow equal to the drift velocity. To validate his heuristic approach, Kamata used vinyl sheets to substitute a contaminated surface film on the water surface in his experiments. Earlier, vinyl sheets were used by Teeson et al. (1970) and Alofs and Reisbig (1972) to obtain the wave-induced drift velocity of oil slicks.

In the investigations of Kang and Lee (1993, 1994, 1995, 1996), the boundary layer formed beneath the water surface was analyzed to calculate the mass transport velocity due to unidirectional oscillatory waves small amplitude. They demonstrated

the analytical procedures of the analysis by Phillips, pointing out particularly the importance of the special treatment of the boundary conditions and time-averaging in the Lagrangian frame. The Phillips' estimation on drift in Eq. (2.8) was verified by their work. The force acting on a surface layer was obtained using a relationship between the energy and momentum of the wave motion, after finding the energy loss which is associated with friction in the surface boundary layer.

Kang and Lee (1995) also performed experiments in a wave tank (see Figures 2.9 and 2.10) and derived semi-empirical equations that took into account the size of the oil patch. Rectangular polyethylene sheets were used to simulate the oil patches. They separated the wave-induced flow conditions into laminar and turbulent regimes, and proposed curve fitting equations to predict the wave-induced surface drift of the polyethylene sheets. Their results (Kang and Lee, 1995) for the laminar regime were:

$$\frac{u_d}{c} = 0.7 \left(\frac{L_g}{L} \right)^{1/3} (ka)^{4/3} \quad \text{for } \frac{L_g}{L} < 1.5 \quad \text{and} \quad \frac{u_d L_g}{\nu} < 10^6 \quad (2.9)$$

and

$$\frac{u_d}{c} = 0.8(ka)^{4/3} \quad \text{for } \frac{L_g}{L} \geq 1.5 \quad \text{and} \quad \frac{u_d L_g}{\nu} < 10^6 \quad (2.10)$$

where L_g / L represents the longitudinal length of the polyethylene sheet normalized against the wavelength, and ν is the kinematic viscosity.

Turbulent condition is generally unachievable in practice due to laboratory constraints. The form of Eq. (2.9) was derived by force balance - equating the driving force and the resistive drag acting on the sheet. In deriving the resistive force, a flat plate traveling along the horizontal water surface was assumed. This shows that, in reality,

the derivation is based on the assumption that the thickness of the viscous shear layer would be similar to a Blasius layer (shown in Fig. 2.8). However, most of the experimental studies that attempted to verify the drift estimate, including Kang and Lee (1995), were conducted within one to two minutes after the wave train was initiated. The short duration was to prevent the interference by the reflected waves off the end of the wave flume. With the short time duration, the thickness of the viscous boundary layer would probably not be able to grow to the Blasius thickness assumed. In any case, the application of Eq. (2.9) has to be restricted to some finite L_g to prevent the predicted u_d from growing to infinity. Implicit in Eq. (2.10) is the independence of wave drift on the sheet length at $L_g > 1.5L$. Incidentally, Kang and Lee (1995) asserted that the drift velocity would tend to zero as $L_g \rightarrow +\infty$ due to wave damping effects. At the other extreme, as $L_g \rightarrow 0^+$, Eq. (2.9) predicts zero mass transport which contradicts to Stokes theory. The influence of sheet width on the drift velocity was not extensively investigated but it was suggested that the effect was insignificant.

Figure 2.8 Schematic diagram of boundary layer by Kang and Lee (1995)

Figure 2.9 Drift velocity of vinyl sheets, $L_g/L < 1.5$ (Kang and Lee, 1995)

Figure 2.10 Drift velocity of vinyl sheets, $L_g/L \geq 1.5$

Note that in this figure, h is the water depth. (Kang and Lee, 1995)

Law (1999) performed both theoretical analysis and experiments on the problem by using rectangular pieces of polyethylene sheets. In his study, the drift velocity of a thin surface inextensible film due to progressive waves up to the 3rd order of wave steepness was investigated in detail. The results show similar 1st order oscillatory boundary layer and 2nd streaming to most of the past studies. However, a 3rd order pressure component was found to be in phase with the wave slope. The net pressure force created was comparable to the viscous drag due to 2nd order streaming. Combining the 3rd pressure force, 2nd viscous drag and stress due to wave attenuation, an estimate of the drift velocity of a long covering thin film due to progressive waves was provided:

$$\frac{u_d}{c} = \frac{5}{2}(ka)^2 \quad (2.11)$$

This estimate is faster than the classical 7/4 predicted by Phillips (1977) in Eq. 2.8. Experimental findings from a total of 45 tests performed in the study by Law (1999) were used to verify the estimation of the drift velocity. Focusing on these experimental data (see Fig. 2.11) with the longer piece (i.e. sheet length is more than twice the wavelength), it can be observed that in overall, the 5/2 factor based on the study is in closer consistency to the experimental measurements than that of Phillips (1977).

Figure 2.11 Comparison between the experimental results and the analytical estimate (Law, 1999)

Law (1999) obtained his result by assuming that the vorticity is confined to the wave boundary layer. In other words, the viscous shear layer was assumed to be of the same thickness as the wave boundary layer. The assumption was drawn based on the experimental observations that the wave-induced drift can be much more than Phillips' estimate in many cases. Although the assumption of the boundary layer thickness might be acceptable during the short period of his experiments, the vorticity that is generated by the patch movement will eventually diffuse outside the wave boundary layer which then renders the assumption to be unreasonable.

In all previous experiments, rectangular sheets were used as a two-dimensional representation of an 'inextensible' oil patch. Field observations however have shown that the shape of the oil patches in the ocean can be irregular and closer to an elliptical form. The National Oceanic and Atmospheric Administration (NOAA) classifies the

shape of floating oils into ‘pancakes’, ‘streamers’, ‘tarballs’ and ‘windrows’ (HAZMAT, 1996) which can be approximated with ellipses. Wong and Law (2003) used elliptical pieces of polyethylene sheets to simulate the surface contaminant. The results showed that the drift velocity increases with the longitudinal length of the polyethylene sheet until approximately 0.8 times the wavelength. Beyond that, any further increment would not result in a substantial increase in the drift velocity. The effect due to the normalized transverse width was found to be significant within a confined range. At large wave steepness, the drift velocity appears to be limited by the Stokes drift. A set of best-fit empirical equations based on the sigmoidal function was introduced for oil spill trajectory predictions.

The above theoretical analysis typically follows a conformal approach that assumes the wave height to be constant during the propagation. In reality, the presence of viscous dissipation will attenuate the wave height underneath the patch. With momentum conservation, an Eulerian mean current must then be generated by the attenuation. Weber (2001) and Christensen (2005) investigated the drift of an inextensible film with the damped waves. In particular, Weber (2001) suggested that in a typical laboratory situation when waves are generated at one end of a wave tank while the surface film is free to drift, the waves will attenuate in space rather than time.

Based on the theory of Weber (2001), Christensen and Weber (2006) further derived the surface drift velocity of an inextensible film as shown in Eq. (2.12):

$$U(z=0) = Au_s \left\{ \sqrt{t/T} - 1/4 + \frac{(2\omega)^{1/2}}{\pi} \int_0^\infty \left[\frac{\xi + \omega}{\xi^{1/2}(\xi^2 + \omega^2)} - \frac{3}{4\xi^{1/2}(\xi + 2\omega)} \right] e^{-\xi t} d\xi \right\} \quad (2.12)$$

where A is the coefficient related to amplitude damping over the film, ω the angular frequency, t the time from the start of the wave train.

It can be seen that Eq. (2.12) implies an accelerating drift with time. This is caused by the assumption that as the vorticity generated by the drift motion is continuously diffused into the interior domain, an ever-increasing drift velocity is needed to counter the imposed viscous shear by the progressive waves. In other words, the boundary layer thickness increases with the time that the vorticity diffuses. The schematic diagram of boundary layer is shown in Fig. 2.12.

Figure 2.12 Schematic diagram of boundary layer by Christensen and Weber (2006)

The acceleration implied in Eq. (2.12) can be checked against a wave drift experiment. For example, the drift velocity would be increased by a factor of about 1.7 between 20 and 60 seconds after the wave train arrives at the patch. However, there is still no remark on the observation of this acceleration in past experimental studies on wave drift. Christensen and Weber (2006) postulated the reason that past experiments typically relied on video-recording the sheet movement and subsequently determining the average drift velocity by digitizing the images within a wide time interval. In doing so, a steady wave drift was already inherently assumed in the data analysis.

In an effort to confirm that the magnitude of the drift velocity predicted by Eq. (2.12) is reasonable, Christensen and Weber (2006) compared the measurements from Law (1999) with the theoretical results of Eq. (2.12), taking the wave damping to be small such that $A = 1$. They showed that all but two data points of Law (1999) are within the theoretical bounds defined by the Eq. (2.12) evaluated at $t = 5$ and 50 seconds. Whether the drift accelerates or not has important implications on the dynamics of the surface layer under the wave motion. Clearly, additional investigation is needed to clarify the issue, which shall be pursued in the present study (discuss in Chapter 4).

2.3 Wave-induced Drift in Shallow Water

As the waves propagate into the shallow water zone, the wave profile begins to deviate from symmetry with respect to the mean water level. The wave crest will become significantly steeper, taller and narrower than the wave trough. As discussed in Chapter 1 and shown in Figure 1.2, the drift velocity of the patch induced by the shallow water waves is of interest to the overall beaching analysis of the oil patch.

In the previous theoretical study on the shallow water waves, the horizontal velocity of water particle was derived up to the second order by Laitone (1961):

$$u = \sqrt{gd} \left[\frac{\eta}{d} - \frac{1}{4} \left(\frac{\eta}{d} \right)^2 + \left(\frac{d}{3} - \frac{(d+z)^2}{2d} \right) \eta_{,xx} \right] \quad (2.13)$$

For Cnoidal wave, the water surface elevation η can be expressed as

$$\eta = H \left(\frac{1}{m^2} - 1 - \frac{E}{m^2 K} \right) + Hcn^2 \left[\frac{2K}{L} (x - ct) \right] \quad (2.14)$$

where cn is the elliptical cosine function, K and E are the 1st and 2nd kind elliptical integral respectively.

Although the velocities of water particle in shallow water are known, there is no theory to predict the related wave-induced drift at present. Law and Huang (2007) presented some experimental findings which will be summarized in Chapter 4.

2.4 Dispersion due to Surface Drift

2.4.1 Background

A surface oil patch will undergo advection and dispersion simultaneously due to the ambient water motion. Regarding surface dispersion, a number of mechanisms have been identified to contribute to the total dispersion of an oil patch, namely turbulence (Maksimenko, 1991; Craig and Banner, 1994) and shear dispersion due to ocean currents and tidal flows (Sanderson and Pall, 1990). These factors affect the enlargement of the size of a pollutant patch in a large scale. When a small scale of influence (up to a few kilometers) is considered, the action of water waves has been suggested to be a possible mechanism with the same order of significance that is comparable to the others, i.e. Herterich and Hasselmann (1982) etc. A brief review of Herterich and Hasselmann's (1982) theory is given in the following section.

2.4.2 Theory of Herterich and Hasselmann (1982)

Most of the recent literatures on the dispersion of floating substances at the sea surface under the random wave condition, such as Mesquita et al. (1992) and Buick et al. (2001), were based on the theoretical development by Herterich and Hasselmann (1982). According to their theory, the dispersion mechanism can be readily understood physically through the random fluctuations of the wave-induced Stokes-drift current as detailed below.

In a random wave field in deep ocean, the ensemble-mean Stokes-drift velocity can be

obtained by

$$\langle u_s(z) \rangle = 2 \int_0^\infty f(k) \omega k e^{2kz} dk \quad (2.15)$$

where $\langle u_s(z) \rangle$ is the drift velocity, with angle brackets denoting ensemble averaging, $f(k)$ is the wave number spectrum.

Herterich and Hasselmann (1982) suggested that this ensemble mean represents the time average drift-velocity, and that the individual particles at different positions of the wave train will have different instantaneous drift velocity u_s , such that they experience drift-velocity fluctuations u_s' ($u_s' = u_s - \langle u_s \rangle$) due to the fluctuations of the local Rayleigh-distributed wave amplitudes in a random sea. The random fluctuations in the drift velocity would then lead to dispersion following the random-walk theory. The dispersion coefficient, D is given by

$$D = \langle u_s'^2 \rangle \tau \quad (2.16)$$

where τ represents the integral autocorrelation time of the fluctuations.

Finally, in a multi-directional sea, the diffusion tensor can be obtained by

$$\begin{pmatrix} D_{xx} & D_{xy} \\ D_{yx} & D_{yy} \end{pmatrix} = \frac{\pi}{4g^2} \int_0^\infty \omega^6 f^2(\omega) d\omega \int_{-\pi}^\pi \int_{-\pi}^\pi \Phi(\theta_1) \Phi(\theta_2) [1 + \cos(\theta_1 - \theta_2)]^2 \\ \times \begin{pmatrix} (\cos \theta_1 + \cos \theta_2)^2, & (\cos \theta_1 + \cos \theta_2)(\sin \theta_1 + \sin \theta_2) \\ (\cos \theta_1 + \cos \theta_2)(\sin \theta_1 + \sin \theta_2), & (\sin \theta_1 + \sin \theta_2)^2 \end{pmatrix} d\theta_1 d\theta_2 \quad (2.17)$$

where $f(\omega)$ is a frequency spectrum, $\Phi(\theta)$ is a frequency-independent directional spreading factor, and D_{ij} are diffusion tensors.

Buick et al. (2001) presented some experimental results to verify the theoretical

predictions of Herterich and Hasselmann (1982) for the case of a pair of particles under 3-D random wave condition. The experimental results appeared to agree with the theoretical predictions at lower values of the particle separating distance. However, it was also obvious that the measurements began to deviate substantially with the prediction when the distance increased.

The predictions by Herterich and Hasselmann (1982) will be further examined in Chapter 3 and the concepts clarified in Chapter 5. It will be shown that the Stokes drift velocity obtained in the frequency domain is not only a mean value but an instantaneous one as well. Thus, the random fluctuations of drift velocity should not exist and there should not be any dispersion due to surface Stokes drift.

2.5 Taylor Dispersion

Besides research on the dispersion of oil patches on the ocean surface, the dispersion of soluble pollutants due to Stokes drift is also studied in this project. When soluble pollutants are discharged into a water body, they will be passively advected and dispersed by the water motion. The dispersion mechanisms are mainly through turbulent diffusion and longitudinal dispersion with tidal and wind-induced shear currents. This project focuses on the longitudinal dispersive effect due to mass transport induced by progressive surface waves, following the approach presented by Law (2000).

Since the classic work of Taylor (1953), there has been extensive research on the longitudinal dispersion generated by turbulent and laminar shear flows. Both chemical and hydraulic engineers are interested in the resulting variation in velocity profile due to the shear stresses. In hydraulic engineering, the studies concentrated on the effect of steady shear currents in rivers (Fischer et al. 1979). This is obviously driven by the important role longitudinal mixing plays in transport of pollutants in rivers.

In addition to the steady flow analysis, Taylor dispersion mechanism was also examined under purely oscillatory shear flows. Watson (1983) stated that the longitudinal turbulent diffusion of a contaminant cloud would increase in an oscillatory flow when a linear concentration gradient exists in that direction. Yasuda (1984) not only demonstrated that longitudinal dispersion occurs for both the steady and oscillatory shear currents, but also analyzed the dispersion process from initial to stationary stages. Smith (1982) and Van Den Broeck (1990) also studied the dispersion due to oscillatory shear flows.

Most of the investigations on Taylor dispersion involve cross-sectional velocity variations due to turbulent or laminar shear stresses. This is the reason why this dispersion mode is regarded as “shear dispersion” (Fischer et al., 1979). In reality, there are two necessary factors for Taylor dispersion to occur. One is the background diffusion, the other is that of a cross-sectional variation of the flow profile in steady currents, and a cross-sectional variation of magnitude of the oscillation in oscillatory currents. In general, the stronger the cross-sectional variation, the larger the dispersion coefficient will be. Undoubtedly, this cross-sectional velocity variation can be created by turbulent and laminar shear stresses. Such a variation can also exist in irrotational water motion (Dean and Dalrymple, 1991). A clear example is surface gravity waves that are commonly considered to be irrotational except near boundaries.

The existence of Lagrangian drift contributes significantly to the longitudinal dispersion of a pollutant cloud in a water body with very low diffusivity such as in a wave flume. The absolute magnitude of the longitudinal dispersive coefficient K maybe small, but if K/D is large, then the Taylor mechanism would still provide a leading role in the mixing of a pollutant cloud along the longitudinal direction. Analytical investigation of Taylor dispersion under regular waves conditions were presented by Law (2000). In the study, different drift profiles were concerned: a Stokes drift profile, a drift profile with viscous effect derived by Longuet-Higgins (1953), Unluata and Mei (1970) and Craik (1982), and also a profile under the cover

of a contaminated inextensible surface obtained by Craik (1982). The approach developed by Law (2000) to analyze the longitudinal dispersion is extended to the case of random waves in chapter 5.

2.6 Drift of the small floating objects

As mentioned in Chapter 1, an iceberg or ice floe is driven by winds, surface currents and water waves. The factors of winds and currents were examined by most of the previous researchers. In this study, we will focus on the drift of an isolated floating ice induced by surface waves. Floating ice floes have different sizes ranging from several meters to hundreds of meters. Generally, there are two existing methods to examine the hydrodynamic forces on a marine object under wave action. One is the potential flow theory for large objects, the other is the Morison's equation for small objects. The first method solves the velocity and pressure fields surrounding the large object using the potential flow theory (e.g. Losada et al. 1993) with the object as flow boundaries. These large objects reflect, refract and scatter waves (e.g. Squire 2007). As to the small objects, their disturbance to the wave field is usually neglected. The Morison's equation can then be used to predict the wave forces on these objects (e.g. Sorensen 1978). Both methods had been applied extensively to compute the wave-induced loading. However, they have not been well explored in terms of the time average behavior of the wave-induced drifting of these objects. The most relative studies of drift of small floating objects are reviewed in the following subsections.

2.6.1 Theory of Shen and Zhong (2001)

The wave-induced drifting motion of small rigid floating objects has received little attention so far. Rumer et al (1979) was probably the first to extend the Morison's equation to investigate the wave-induced drift of small floating objects. In their approach, the wave surface is considered to be an oscillating slope, with the gravity component normal to the slope balanced by the buoyancy; and only the component

tangent to the surface slope causes the movement of the object. The mechanism is illustrated in Figure 2.13.

Figure 2.13 Schematics of wave-induced gravity component in horizontal direction

Based on the work of Rumer et al. (1979), Shen and Zhong (2001) derived the drift of small rigid floating objects on a one-dimension propagating monochromatic wave as follows.

The equation of force balanced in wave propagating direction is

$$m(1+C_m)\frac{d^2x}{dt^2}=F_g+F_d \quad (2.18)$$

where m is the mass of the small object, F_g the horizontal force of the small object caused by the gravity, F_d the horizontal component of the drag force and C_m the added mass coefficient.

The horizontal gravity force F_g and drag force F_d are defined as:

$$F_g=-\frac{1}{2}m(g+\frac{\partial^2\eta}{\partial t^2})\sin[2(\tan^{-1}\frac{\partial\eta}{\partial x})] \quad (2.19)$$

and

$$F_d = \rho C_d A |V - u| (V - u) \quad (2.20)$$

where ρ is the water density, C_d the drag force coefficient, A the wetted surface area, V the surface water particle velocity in the horizontal direction, and $u = dx/dt$ the horizontal velocity of the small floating object.

The surface profile and velocity are obtained by assuming a small amplitude wave:

$$\eta = a \sin(kx - \omega t) \quad (2.21)$$

$$\left. \frac{\partial^2 \eta}{\partial t^2} \right|_{\max} \approx \omega^2 \cdot 2a = 2ka g \ll g \quad (2.22)$$

$$V = \frac{\omega \eta}{\tanh kd} + O(ka)^2 \quad (2.23)$$

Substituting Eq. (2.21) – (2.23) into (2.18), up to the first order of ka , the governing equation (2.18) becomes:

$$(1 + C_m) \frac{du}{dt} = -g \frac{\partial \eta}{\partial x} + \frac{\rho C_d}{\rho_s \Delta} \left| \frac{\omega \eta}{\tanh kd} - u \right| \left(\frac{\omega \eta}{\tanh kd} - u \right) \quad (2.24)$$

where ρ_s is the density of the small object and Δ the thickness of the small object.

Based on the governing equation (2.24), Shen and Zhong (2001) obtained the approximate analytical solutions for two extreme cases where either the added mass coefficient C_m or the drag coefficient C_d vanishes on small wave amplitude and deepwater conditions.

Assuming $C_m = 0$, Eq. (2.24) becomes

$$\frac{du}{dt} = -gka \cos(kx - \omega t) + \frac{\rho C_d}{\rho_s \Delta} [\omega a \sin(kx - \omega t) - u] \cdot [\omega a \sin(kx - \omega t) - u] \quad (2.25)$$

The analytical solution for the velocity of the object is:

$$u = \omega a \sin(kx - \omega t) \quad (2.26)$$

It is known that Eq. (2.26) is the first order velocity of a sinusoidal wave. In other words, the small object possesses the Stokes drift as the added mass coefficient vanishes.

For the case of $C_d = 0$, Eq. (2.24) becomes

$$(1 + C_m) \frac{du}{dt} = -gka \cos(kx - \omega t) \quad (2.27)$$

The solution of the above equation will depend on the initial condition. This means that the drift velocity under this condition can be positive or negative depending on the initial position and initial velocity of the object. Shen and Zhong (2001) solved the drift velocity of different objects within the range of realistic added mass and drag coefficients numerically and concluded that the wave-induced drift of floating objects will decrease if the added mass coefficient and/or the drag coefficient increase.

Their prediction is however contrary to the experimental observations obtained in this study. The comparison will be discussed in details in Chapter 6.

2.6.2 Investigation of Grotmaack and Meylan (2006)

Grotmaack and Meylan (2006) pointed out the error in the slope sliding model used by Rumer et al. (1979) and Shen and Zhong (2001). The vertical inertia was used

incorrectly instead of the centripetal force in the derivation of Rumer et al. (1979) and Shen and Zhong (2001). The mechanism is illustrated in Figure 2.14, in which the three forces are shown: the inertia force in the horizontal direction, the force due to gravity mg and the centripetal force ma_n .

Figure 2.14 Three forces: inertia, gravity and centripetal (Grotmaack and Meylan 2006)

After correcting the model used by Shen and Zhong (2001), the governing equation is modified to:

$$m(1 + C_m) \frac{du}{dt} = -mgka \cos(kx - \omega t) + \rho AC_d |kca \sin(kx - \omega t) - u| \times [kca \sin(kx - \omega t) - u] \quad (2.28)$$

The analytical solution for the above governing equation cannot be obtained easily. Therefore, numerical methods were used to determine the drift of small objects. Grotmaack and Meylan (2006) results showed that the drift of the small object is independent of the initial conditions of object but slightly dependent on the magnitude of the drag coefficient. The current understanding on the drift of the small object will be further advanced in chapter 6.

Chapter

3

SURFACE DRIFT IN REGULAR AND RANDOM WAVES

In regular waves, the 2nd order velocities were derived originally by Stokes (1847), and the mass transport (or drift) is taken as a mean velocity and also a component in the instantaneous orbital velocity. Under random wave condition, Herterich and Hasselmann (1982) suggested that there exists a drift fluctuation (i.e. the drift is time dependent) in the 2nd order with a mixed Lagrangian-Eulerian approach that would then lead to a wave-induced dispersion at the water surface. This suggestion by Herterich and Hasselmann has been commonly adopted in many theoretical works on wave-induced surface dispersion in the last two decades. The objective of this chapter is to clarify whether the drift fluctuation exists in irregular waves up to the 2nd order velocities by analytical derivations.

3.1 Drift velocity due to regular waves with a clean surface

Assuming the fluid to be incompressible, the continuity equation for the wave motion can be expressed as

$$\nabla \cdot \vec{V} = 0 \quad (3.1)$$

If the flow is irrotational, i.e. $\nabla \times \vec{V} = 0$, then there exists a velocity potential ϕ such that the velocity can be expressed as $\vec{V} = -\nabla \phi$. Substituting this relationship into Eq. (3.1):

$$\nabla^2 \phi = 0 \quad (3.2)$$

This is the well-known Laplace equation, which is the governing equation of the wave motion. The kinematic and dynamic boundary conditions on the water surface can be expressed as Eqs. (3.3) and (3.4) respectively,

$$\frac{\partial \eta}{\partial t} + \nabla \phi \cdot \nabla \eta = \frac{\partial \phi}{\partial z} \quad (3.3)$$

$$\frac{\partial \phi}{\partial t} + \frac{1}{2} |\nabla \phi|^2 + g\eta = 0 \quad (3.4)$$

Perturbation technique can be used to solve the equations, and the 1st and 2nd order velocity potential can be obtained respectively (Dean and Dalrymple, 1991):

$$\phi^{(1)} = \frac{a\omega}{k} \frac{\cosh k(z+d)}{\sinh kd} \sin \theta \quad (3.5)$$

$$\phi^{(2)} = \frac{a\omega}{k} \frac{3}{8} ak \frac{\cosh 2k(z+d)}{\sinh^4 kd} \sin 2\theta \quad (3.6)$$

where $\theta = kx - \omega t$.

Hence, the Eulerian velocities up to the 2nd order are:

$$\begin{aligned} u(x, z, t) &= \frac{\partial \phi}{\partial x} = \frac{\partial(\phi^{(1)} + \phi^{(2)})}{\partial x} \\ &= a\omega \left(\frac{\cosh k(z+d)}{\sinh kd} \cos \theta + \frac{3}{4} ak \frac{\cosh 2k(z+d)}{\sinh^4 kd} \cos 2\theta \right) \end{aligned} \quad (3.7)$$

$$\begin{aligned}
 w(x, z, t) &= \frac{\partial \phi}{\partial z} = \frac{\partial(\phi^{(1)} + \phi^{(2)})}{\partial z} \\
 &= a\omega \left(\frac{\sinh k(z+d)}{\sinh kd} \sin \theta + \frac{3}{4} ak \frac{\sinh 2k(z+d)}{\sinh^4 kd} \sin 2\theta \right)
 \end{aligned} \quad (3.8)$$

$$x - x_0 = \int_0^t u dt \cong -a \frac{\cosh k(z+d)}{\sinh kd} \sin \theta \quad (3.9)$$

$$z - z_0 = \int_0^t w dt \cong a \frac{\cosh k(z+d)}{\sinh kd} \cos \theta \quad (3.10)$$

where x_0, z_0 are the horizontal and vertical coordinates from the initial displacement respectively.

Given the Eulerian velocities and displacements, the Lagrangian velocity can be derived using Taylor's expansion as:

$$\begin{aligned}
 u(x, z, t) &= u(x_0, z_0, t) + \left(\frac{\partial u}{\partial x} \right)_{(x_0, z_0)} (x - x_0) + \left(\frac{\partial u}{\partial z} \right)_{(x_0, z_0)} (z - z_0) + o(a^3 k^3) \\
 &= c \left[\underbrace{ak \frac{\cosh k(z_0+d)}{\sinh kd} \cos \theta_0}_{\text{1st harmonic}} + \underbrace{\frac{(ak)^2}{2 \sinh^2 kd} \left(\frac{3 \cosh 2k(z_0+d)}{2 \sinh^2 kd} - 1 \right) \cos 2\theta_0}_{\text{2nd harmonic}} \right. \\
 &\quad \left. + (ka)^2 \frac{\cosh 2k(z_0+d)}{2 \sinh^2 kd} + o(ka)^3 \right] \\
 &\quad \text{mass transport (drift)}
 \end{aligned} \quad (3.11)$$

where $\theta_0 = kx_0 - \omega t$.

Hence, the movement of the individual particle, besides its orbital motion, also possesses a 2nd order and time-independent velocity (called the mass-transport velocity) in the direction of wave propagation:

$$u_s = c(ka)^2 \frac{\cosh 2k(z_0+d)}{2 \sinh^2 kd} \quad (3.12a)$$

At the mean sea level where $z=0$, the surface Stokes drift velocity is then

$$u_s = c(ka)^2 \quad (3.12b)$$

3.2 Drift velocity under random waves with a clean surface

The ocean surface waves in reality are a combination of many wave components of various frequencies and irregular. The superposition method, which treats irregular waves as a superposition of linear waves with different frequencies, amplitudes and phases, is often used for analysis. We shall first derive the nonlinear-wave solution from a combination of two sinusoidal waves. Then, the solution will be generalized to a spectrum of irregular waves.

3.2.1 2nd order velocity potential of two wave components

The surface boundary condition that a progressive wave shall satisfy up to 2nd order has been discussed extensively (e.g. Dean and Darymple, 1991).

$$\begin{aligned} \frac{\partial^2 \phi^{(2)}}{\partial t^2} + g \frac{\partial \phi^{(2)}}{\partial z} = & -2 \left(\frac{\partial \phi^{(1)}}{\partial x} \frac{\partial^2 \phi^{(1)}}{\partial x \partial t} + \frac{\partial \phi^{(1)}}{\partial z} \frac{\partial^2 \phi^{(1)}}{\partial z \partial t} \right) \\ & + \frac{1}{g} \frac{\partial \phi^{(1)}}{\partial t} \left(g \frac{\partial^2 \phi^{(1)}}{\partial z^2} + \frac{\partial^3 \phi^{(1)}}{\partial z \partial t^2} \right) \end{aligned} \quad (3.13)$$

With two sinusoidal waves of different frequencies propagating along the x axis, the combined velocity potential by superposition following Eq. (3.5) is:

$$\phi^{(1)}(x, z, t) = \sum_{i=1}^2 \frac{ga_i}{\omega_i} \frac{\cosh k_i(z+d)}{\cosh k_i d} \sin(k_i x - \omega_i t) \quad (3.14)$$

Then, the differential items can be derived as follows:

$$\frac{\partial \phi^{(1)}}{\partial x} = \sum_{i=1}^2 \frac{g a_i k_i}{\omega_i} \frac{\cosh k_i(z+d)}{\cosh k_i d} \cos(k_i x - \omega_i t) \quad (3.15)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial x \partial t} = \sum_{i=1}^2 g a_i k_i \frac{\cosh k_i(z+d)}{\cosh k_i d} \sin(k_i x - \omega_i t) \quad (3.16)$$

$$\frac{\partial \phi^{(1)}}{\partial z} = \sum_{i=1}^2 \frac{g a_i k_i}{\omega_i} \frac{\sinh k_i(z+d)}{\cosh k_i d} \sin(k_i x - \omega_i t) \quad (3.17)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial z \partial t} = \sum_{i=1}^2 -g a_i k_i \frac{\sinh k_i(z+d)}{\cosh k_i d} \cos(k_i x - \omega_i t) \quad (3.18)$$

$$\frac{\partial^2 \phi^{(1)}}{\partial z^2} = \sum_{i=1}^2 \frac{g a_i k_i^2}{\omega_i} \frac{\cosh k_i(z+d)}{\cosh k_i d} \sin(k_i x - \omega_i t) \quad (3.19)$$

$$\frac{\partial^3 \phi^{(1)}}{\partial z \partial t^2} = \sum_{i=1}^2 -g a_i k_i \omega_i \frac{\sinh k_i(z+d)}{\cosh k_i d} \sin(k_i x - \omega_i t) \quad (3.20)$$

$$\frac{\partial \phi^{(1)}}{\partial t} = \sum_{i=1}^2 -g a_i \frac{\cosh k_i(z+d)}{\cosh k_i d} \cos(k_i x - \omega_i t) \quad (3.21)$$

At the mean sea level, $z=0$ and substituting all the above items into Equation (3.13):

$$\begin{aligned} & \frac{\partial^2 \phi^{(2)}}{\partial t^2} + g \frac{\partial \phi^{(2)}}{\partial z} \\ &= \frac{1}{2} \sin 2\theta_1 \left[3 \frac{g^2 a_1^2 k_1^2}{\omega_1} (\tanh^2 k_1 d - 1) \right] + \frac{1}{2} \sin 2\theta_2 \left[3 \frac{g^2 a_2^2 k_2^2}{\omega_2} (\tanh^2 k_2 d - 1) \right] \\ &+ \frac{1}{2} \sin(\theta_1 + \theta_2) \left[-2g^2 a_1 a_2 k_1 k_2 \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) + 2g^2 a_1 a_2 k_1 k_2 \tanh k_1 d \tanh k_2 d \left(\frac{1}{\omega_1} + \frac{1}{\omega_2} \right) \right. \\ &\left. - g^2 a_1 a_2 \left(\frac{k_1^2}{\omega_1} + \frac{k_2^2}{\omega_2} \right) + g a_1 a_2 (k_1 \omega_1 \tanh k_1 d + k_2 \omega_2 \tanh k_2 d) \right] \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{2} \sin(\theta_1 - \theta_2) \left[-2g^2 a_1 a_2 k_1 k_2 \left(\frac{1}{\omega_1} - \frac{1}{\omega_2} \right) + 2g^2 a_1 a_2 k_1 k_2 \tanh k_1 d \tanh k_2 d \left(\frac{1}{\omega_1} - \frac{1}{\omega_2} \right) \right. \\
 & \left. - g^2 a_1 a_2 \left(\frac{k_1^2}{\omega_1} - \frac{k_2^2}{\omega_2} \right) + g a_1 a_2 (k_1 \omega_1 \tanh k_1 d - k_2 \omega_2 \tanh k_2 d) \right] \quad (3.22)
 \end{aligned}$$

where $\theta_1 = k_1 x - \omega_1 t$, $\theta_2 = k_2 x - \omega_2 t$.

In order to simplify Eq. (3.22), define:

$$g_{ij}^{\pm} = g \left[\frac{g k_i k_j}{\omega_i} (\tanh(k_i d) \tanh(k_j d) \mp 1) \pm \frac{1}{2} \omega_j k_j \tanh(k_j d) \mp \frac{k_j^2}{\omega_j} \right] \quad (3.23)$$

and

$$\begin{aligned}
 G^{\pm}(\omega_1, \omega_2) &= g_{12}^{\pm} \pm g_{21}^{\pm} \\
 &= -g^2 \left[k_1 k_2 \left(\frac{1}{\omega_1} \pm \frac{1}{\omega_2} \right) (1 \mp \tanh k_1 d \tanh k_2 d) \right. \\
 &\quad \left. + \frac{1}{2} \left(\frac{k_1^2}{\omega_1 \cosh^2 k_1 d} \pm \frac{k_2^2}{\omega_2 \cosh^2 k_2 d} \right) \right] \quad (3.24)
 \end{aligned}$$

then Eq. (3.22) can be simplified as:

$$\begin{aligned}
 \frac{\partial^2 \phi_2}{\partial t^2} + g \frac{\partial \phi_2}{\partial z} &= a_1 a_2 G^{\pm}(\omega_1, \omega_2) \sin(k_{\pm} x - \omega_{\pm} t) + \frac{1}{2} a_1^2 G^{+}(\omega_1, \omega_1) \sin 2(k_1 x - \omega_1 t) \\
 &\quad + \frac{1}{2} a_2^2 G^{+}(\omega_2, \omega_2) \sin 2(k_2 x - \omega_2 t) \quad (3.25)
 \end{aligned}$$

where $k_{\pm} = k_1 \pm k_2$, $\omega_{\pm} = \omega_1 \pm \omega_2$.

Assuming a solution of 2nd velocity potential with the form of:

$$\begin{aligned}
 \phi^{(2)}(x, z, t) &= a_1 a_2 E_{12}^{\pm} \frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm} d} \sin(k_{\pm} x - \omega_{\pm} t) \\
 &\quad + \frac{1}{2} a_1^2 E_{11}^{+} \frac{\cosh 2k_1(z+d)}{\cosh 2k_1 d} \sin 2(k_1 x - \omega_1 t)
 \end{aligned}$$

$$+\frac{1}{2}a_2^2 E_{22}^+ + \frac{\cosh 2k_2(z+d)}{\cosh 2k_2 d} \sin 2(k_2 x - \omega_2 t) \quad (3.26)$$

where $E_{11}^+, E_{12}^\pm, E_{22}^+$ are unknown variables, it can be shown that this solution would satisfy the governing equation as well as the boundary conditions:

$$\nabla^2 \phi^{(2)} = 0 \quad -d < z < 0 \quad (3.27)$$

$$\begin{aligned} \frac{\partial^2 \phi^{(2)}}{\partial t^2} + g \frac{\partial \phi^{(2)}}{\partial z} &= a_1 a_2 G^\pm(\omega_1, \omega_2) \sin(k_\pm x - \omega_\pm t) + \frac{1}{2} a_1^2 G^+(\omega_1, \omega_1) \sin 2(k_1 x - \omega_1 t) \\ &+ \frac{1}{2} a_2^2 G^+(\omega_2, \omega_2) \sin 2(k_2 x - \omega_2 t) \quad z = 0 \quad (3.28) \end{aligned}$$

$$\frac{\partial \phi^{(2)}}{\partial z} = 0 \quad z = -d \quad (3.29)$$

where (3.28) is the dynamic boundary condition at the water surface and (3.29) is the no-flow boundary condition at the sea bed.

Substituting the special solution into the equations, $E_{11}^+, E_{12}^\pm, E_{22}^+$ can be obtained as:

$$E_{12}^\pm = \frac{G^\pm(\omega_1, \omega_2)}{D^\pm(\omega_1, \omega_2)} \quad (3.30)$$

$$E_{11}^+ = \frac{G^+(\omega_1, \omega_1)}{D^+(\omega_1, \omega_1)} \quad (3.31)$$

$$E_{22}^+ = \frac{G^+(\omega_2, \omega_2)}{D^+(\omega_2, \omega_2)} \quad (3.32)$$

where $D^\pm(\omega_1, \omega_2) = gk_\pm \tanh(k_\pm d) - \omega_\pm^2$.

Finally, the solution of 2nd velocity potential is:

$$\begin{aligned}
 \phi^{(2)}(x, z, t) = & a_1 a_2 \frac{G^{\pm}(\omega_1, \omega_2)}{D^{\pm}(\omega_1, \omega_2)} \frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm}d} \sin(k_{\pm}x - \omega_{\pm}t) \\
 & + \frac{1}{2} a_1^2 \frac{G^+(\omega_1, \omega_1)}{D^+(\omega_1, \omega_1)} \frac{\cosh 2k_1(z+d)}{\cosh 2k_1d} \sin 2(k_1x - \omega_1t) \\
 & + \frac{1}{2} a_2^2 \frac{G^+(\omega_2, \omega_2)}{D^+(\omega_2, \omega_2)} \frac{\cosh 2k_2(z+d)}{\cosh 2k_2d} \sin 2(k_2x - \omega_2t)
 \end{aligned} \quad (3.33)$$

3.2.2 2nd order velocity potential in irregular waves

Using the same superposition method, if a spectrum of wave components with different initial phase are considered, then the 2nd order potential under irregular wave condition can be obtained as follow:

$$\begin{aligned}
 \phi^{(2)}(x, z, t) = & \sum_i \sum_{j>i} a_i a_j \frac{G^{\pm}(\omega_i, \omega_j)}{D^{\pm}(\omega_i, \omega_j)} \frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm}d} \sin(k_{\pm}x - \omega_{\pm}t + e_{\pm}) \\
 & + \frac{1}{2} \sum_i a_i^2 \frac{G^+(\omega_i, \omega_i)}{D^+(\omega_i, \omega_i)} \frac{\cosh 2k_i(z+d)}{\cosh 2k_id} \sin 2(k_ix - \omega_it + e_i)
 \end{aligned} \quad (3.34)$$

where $e_{\pm} = e_i \pm e_j$ and e_i, e_j are initial phases of different wave components.

For a random field of ocean surface waves, the velocity potential and Eulerian velocity up to 2nd order are:

$$\eta^{(1)}(x, t) = \sum_i a_i \cos(k_ix - \omega_it + e_i) \quad (3.35)$$

$$\phi^{(1)}(x, z, t) = \sum_i \frac{a_i g}{\omega_i} \frac{\cosh k_i(z+d)}{\cosh k_id} \sin(k_ix - \omega_it + e_i) \quad (3.36)$$

$$\begin{aligned}
 \phi^{(2)}(x, z, t) = & \sum_i \sum_{j>i} a_i a_j \frac{G^{\pm}(\omega_i, \omega_j)}{D^{\pm}(\omega_i, \omega_j)} \frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm}d} \sin(k_{\pm}x - \omega_{\pm}t + e_{\pm}) \\
 & + \frac{1}{2} \sum_i a_i^2 \frac{G^+(\omega_i, \omega_i)}{D^+(\omega_i, \omega_i)} \frac{\cosh 2k_i(z+d)}{\cosh 2k_id} \sin 2(k_ix - \omega_it + e_i)
 \end{aligned} \quad (3.37)$$

$$u^{(1)}(x, z, t) = \frac{\partial \phi^{(1)}}{\partial x} = \sum_i a_i \omega_i \frac{\cosh k_i(z+d)}{\sinh k_i d} \cos(k_i x - \omega_i t + e_i) \quad (3.38)$$

$$\begin{aligned} u^{(2)}(x, z, t) = \frac{\partial \phi^{(2)}}{\partial x} = & \sum_i \sum_{j>i} a_i a_j k_{\pm} \frac{G^{\pm}(\omega_i, \omega_j)}{D^{\pm}(\omega_i, \omega_j)} \frac{\cosh k_{\pm}(z+d)}{\cosh k_{\pm} d} \cos(k_{\pm} x - \omega_{\pm} t + e_{\pm}) \\ & + \frac{1}{2} \sum_i a_i^2 \frac{G^+(\omega_i, \omega_i)}{D^+(\omega_i, \omega_i)} \frac{\cosh 2k_i(z+d)}{\cosh 2k_i d} \times 2k_i \cos 2(k_i x - \omega_i t + e_i) \end{aligned} \quad (3.39)$$

$$x - x_0 = \int_0^t u dt \cong \sum_i -a_i \frac{\cosh k_i(z+d)}{\sinh k_i d} \sin(k_i x_0 - \omega_i t + e_i) \quad (3.40)$$

$$z - z_0 = \int_0^t w dt \cong \sum_i a \frac{\cosh k_i(z+d)}{\sinh k_i d} \cos(k_i x_0 - \omega_i t + e_i) \quad (3.41)$$

where

$$\omega_{\pm} = \omega_i \pm \omega_j, k_{\pm} = k_i \pm k_j, e_{\pm} = e_i \pm e_j, D^{\pm}(\omega_i, \omega_j) = gk_{\pm} \tanh(k_{\pm} d) - \omega_{\pm}^2,$$

$$G^{\pm}(\omega_i, \omega_j) = -g^2 \left[\frac{k_i k_j}{\omega_i \omega_j} \omega_{\pm} (1 \mp \tanh(k_i d) \tanh(k_j d)) \right.$$

$$\left. + \left(\frac{k_i^2}{2\omega_i \cosh^2 k_i d} \pm \frac{k_j^2}{2\omega_j \cosh^2 k_j d} \right) \right].$$

Again the horizontal Lagrangian velocity can be derived from the Taylor's expansion:

$$\begin{aligned} u(x, z, t) = & u(x_0, z_0, t) + \left(\frac{\partial u}{\partial x} \right)_{(x_0, z_0)} (x - x_0) + \left(\frac{\partial u}{\partial z} \right)_{(x_0, z_0)} (z - z_0) + o(\varepsilon^3) \\ = & \sum_i c_i (a_i k_i) \frac{\cosh k_i(z_0+d)}{\sinh k_i d} \cos(k_i x_0 - \omega_i t + e_i) \\ & + \sum_i \sum_{j>i} a_i a_j k_{\pm} \frac{G^{\pm}(\omega_i, \omega_j)}{D^{\pm}(\omega_i, \omega_j)} \frac{\cosh k_{\pm}(z_0+d)}{\cosh k_{\pm} d} \cos(k_{\pm} x_0 - \omega_{\pm} t + e_{\pm}) \\ & + \sum_i a_i^2 k_i \frac{G^+(\omega_i, \omega_i)}{D^+(\omega_i, \omega_i)} \frac{\cosh 2k_i(z_0+d)}{\cosh 2k_i d} \cos 2(k_i x_0 - \omega_i t + e_i) \end{aligned}$$

$$\begin{aligned}
 & + \sum_i \sum_{j>i} \frac{a_i a_j}{2} \frac{(k_i \omega_i + k_j \omega_j)}{\sinh k_i d \cdot \sinh k_j d} [\cos(k_- x_0 - \omega_- t + e_-) \cosh k_+ (z_0 + d) \\
 & - \cos(k_+ x_0 - \omega_+ t + e_+) \cosh k_- (z_0 + d)] \\
 & + \sum_i c_i \frac{(a_i k_i)^2}{2 \sinh^2 k_i d} \cos 2(k_i x_0 - \omega_i t + e_i) \\
 & + \sum_i c_i (k_i a_i)^2 \frac{\cosh 2k_i (z_0 + d)}{2 \sinh^2 k_i d} + o(\varepsilon^3)
 \end{aligned} \tag{3.42}$$

In this expression of Lagrangian velocity, the first item denotes the 1st harmonic, the third and fifth items the 2nd harmonic, and the second and fourth items the interaction of different wave components. Only the last item is a time-independent value, which is the mass transport velocity in the random wave field. Hence, it can be concluded that the mass transport under random waves is given by the sum of the contributions from all individual wave components, i.e.,

$$\begin{aligned}
 u_s &= \sum_{i=1}^{\infty} c_i (k_i a_i)^2 \frac{\cosh 2k_i (z_0 + d)}{2 \sinh^2 k_i d} \\
 &= 2 \int_0^{\infty} s(\omega) \omega k \frac{\cosh 2k (z_0 + d)}{2 \sinh^2 k d} d\omega
 \end{aligned} \tag{3.43}$$

where $s(\omega)$ is the wave energy spectrum.

For deep-water random waves, the surface drift velocity at $z=0$ is then

$$u_s = 2 \int_0^{\infty} s(\omega) \omega k d\omega \tag{3.44}$$

The wave spectrum and its parameter can be determined from a wave record. Hence, the corresponding drift velocity from Eq. (3.44) would become a time-independent quantity. In other words, the Stokes drift should not possess any fluctuation component. This conclusion has an important implication towards whether the random waves would exhibit a surface dispersive effect or not.

Chapter

4

EXPERIMENTAL STUDY ON THE DRIFT OF AN OIL PATCH

In this chapter, some observations and measurements of the drift velocity of an oil patch modeled using a polyethylene sheet are presented under both deep and shallow regular wave conditions. Particularly, two unique aspects of the present study are noted. First, we used a motion monitoring system with two infra-red cameras to record accurately the position of the polyethylene sheets at 50Hz under deep water waves. The recording provided the entire time history of the movement of the floating sheet after the wave train was initiated. As will be illustrated in the following in this chapter, the time history showed a quasi-steady drift velocity without acceleration being developed shortly after the waves arrived at the floating sheet. Second, experimental measurements of the drift velocity were taken under shallow water conditions. These shallow water measurements were not available in the past. Some observations between the wave drift and shallow water characteristics are summarized.

4.1 Experimental setup

Experiments were conducted in a wave flume measuring 45 m long, 1.55 m wide and 1.5 m height (shown in Fig. 4.1). The large flume size allowed deep-water waves to be generated. One end of the flume housed a two-dimensional piston type random wave-maker. The wave piston was controlled directly using the DHI Active Wave Absorption Control System (AWACS). Meanwhile, DHI wave synthesizer software was used to specify the types of wave required as well as their respective parameters. At the other end of the wave flume, an artificial absorbent beach was created to provide an efficient dissipation of the wave energy (in Fig. 4.1).

Four sensitive capacitance wave probes were mounted on a steel frame positioned at a range of 1.2 m to 2.2 m from the wave paddle (in Fig. 4.1). The wave probes were capable of measuring wave heights to the nearest 0.5 mm. The probes had a small diameter (approximately 0.6 cm) such that their placement would not result in significant modifications of the wave profile.

Figure 4.1 Schematic layout of the wave flume

For the deep water tests, a motion monitoring system (Qualisys Track Manager) was installed to capture the movement history of the floating patches. The system consisted of 2 ProReflex infrared cameras, a portable measurement computer, and several markers coated with reflective material. The two cameras were angled at each other to cover an intersecting span of about 4 m viewable by both cameras (shown in

Fig. 4.2). Typically, a group of five markers was placed on the sheet at the up-wave end. The images of the markers group were continuously acquired by the cameras and the signal was processed by the system software to give the instantaneous position of the markers group in a pre-calibrated coordinate system. With this setup, the vertical displacement of the markers group directly gave the wave height before attenuation. At the same time, the drift behavior of the polyethylene sheet was detailed by tracking the horizontal motion of these markers. For the shallow water tests, however, the common video-based approach as described in Law (1999) and Wong and Law (2003) was used instead because the infra-cameras were not available during the test period.

Polyethylene sheets with a density of 0.9 g cm^{-3} and thickness 0.08 mm were used to simulate the inextensible condition experienced by the water surface covered with an oil patch. In order to satisfy the 2-D condition, all sheets occupied essentially the full width of the wave flume, leaving only a 1-cm gap between the sheets and the side wall to ensure free movement without excessive energy transmission through the gap. Different longitudinal lengths of the polyethylene sheet L_g , from 0.465 m to 6.0 m , were tested in the experiments. The definition of L_g is also shown in Fig. 4.2.

Figure 4.2 Schematic layout of the experimental setup

4.2 Experimental procedures

The experimental procedures were divided into two parts: the calibration of equipments and the actual running of the tests. Before the commencement of an experiment series, the water depths in the experiments (shown in Table 4.1 and 4.2) were first established in the wave flume. The wave generation characteristics (i.e. wave height and wave period) by the wave maker were then verified with the specific water depth. For the shallow water tests, we adopted similar procedures for tracking the sheet movement as in Law (1999) and Wong and Law (2003). For the deep water tests, which involved the deployment of a motion monitoring system, procedures will be described in details in the following.

4.2.1 Calibration of equipments

Prior to each test, the wave-maker and the motion monitoring system were calibrated. Typically, the wave-maker was calibrated firstly, followed by the set-up and calibration of the motion monitoring system. For the deep water tests, the two infrared cameras were placed next to the wave flume at a longitudinal location about 6 m from the wave generator. The infrared camera system was calibrated to create a distorted coordinate system that covered the measurement span. After the calibration was completed, a polyethylene sheet with a specific length L_g and with the 5 reflective markers on top was placed onto the water surface within the test span (shown in Fig. 4.3). Then this polyethylene sheet with markers can be acquired by the QTM software in the distorted coordinate system (shown in Fig. 4.4). The x -coordinate is the same as wave propagating direction in this coordinate system.

Fig. 4.3 Polyethylene sheet test sample with passive markers on top

Figure 4.4 Polyethylene sheet test sample in the calibrated-coordinate system

4.2.2 Tests running

After the polyethylene sheet was acquired by the motion monitoring system (see Fig. 4.4), checking would be performed to verify that the water body was sufficiently still, by inspecting the motion of the markers for a minute or two. Any discernible movement would suggest the presence of residual current. After it was confirmed that there was no residual current, the wave generator was activated to start the test. During the test, the infra-red cameras captured the images of the reflective markers on

the polyethylene sheets at 50Hz. These images were processed to produce the displacement data for the entire process. The data were subsequently analyzed and plotted to reveal the drift behavior. A typical sheet motion sequence in the calibrated-coordinate is shown in Fig. 4.5.

Figure 4.5 Coordinates of sheet at different time $t = 0, 10, 20$ and 30 s

4.3 Results and discussion

4.3.1 Deep water waves

Table 4.1 lists the experimental parameters for the drift measurements under the deep water regular wave condition. The water depth was constant at $d = 0.8$ m and the wave period was fixed at $T = 1.0$ s, yielding a wave length of $L = 1.52$ m by the dispersion relationship. The wave height H ranged from 0.02 to 0.06 m with a 0.01 m

interval.

Table 4.1 Experimental parameters for deep water condition

Expt. Series	H (m)	T (s)	d (m)	L (m)	H/L	d/L
DA	0.02	1.0	0.80	1.52	0.013	0.53
DB	0.03	1.0	0.80	1.52	0.020	0.53
DC	0.04	1.0	0.80	1.52	0.026	0.53
DD	0.05	1.0	0.80	1.52	0.033	0.53
DE	0.06	1.0	0.80	1.52	0.040	0.53
DF	0.07	1.0	0.80	1.52	0.046	0.53
DG	0.08	1.0	0.80	1.52	0.053	0.53

4.3.1.1 Displacement history

A typical longitudinal trajectory of the polyethylene sheet is shown in Fig. 4.6. The data are extracted from the results in the Experimental Series DB with $L_g = 3$ m. Time $t = 0$ in Figure 4.6 corresponded to the time when the wave front arrived at the floating sheet. It can be observed that the magnitude of the vertical oscillation reached the wave height quickly within a few wave cycles. The horizontal displacement increased with time parabolically in the first approximately 30 seconds, implying that the sheet was accelerating and that the wave imposed driving force exceeded the viscous resistance by the sheet movement during this period. Subsequently, the quasi-steady longitudinal trajectory increased almost linearly with time, which confirmed that the sheet was now moving with a mean drift velocity. It should be emphasized that the behavior was typical for most of the tests conducted. In fact, to ensure that the observations were valid, all the tests were performed 2-3 times to verify that the results were repeatable.

Figure 4.6 A typical longitudinal trajectory of the polyethylene sheet

The history of the period-average velocity based on Figure 4.6 is plotted in Figure 4.7. The period-average velocity was calculated by noting the difference in the maximum longitudinal position of the sheet in a time interval of one wave period, or 1 second in this case. Similar to Figure 4.6, Figure 4.7 shows that the sheet accelerated from the start but reached a steady drift motion in about 30 seconds. The magnitude of the mean drift stayed roughly the same until the end of the measurement period which was 90 seconds.

Figure 4.7 The history of period-average velocity

The evidence presented here shows that the sheet did not accelerate, contrary to the prediction by Christiansen and Weber (2005). Rather, the measurements imply that the wave-imposed driving force on the sheet was quickly balanced by the viscous resistance induced by the sheet movement, such that the sheet experienced a zero net force towards a steady drift. The quantitative results were consistent with the qualitative observations by Wong and Law (2003) during their tests on elliptical patches. If the wave-imposed force on the sheet is indeed constant as asserted by almost all previous researchers (Kang and Lee, 1996; Law, 1999; Christiansen and Weber, 2005), then the puzzling question would be why the vorticity generated by the sheet movement did not diffuse continuously into the interior domain of the wave train which would surely lead to a time-varying drift as suggested by Christiansen and Weber (2005). Addressing the question requires further understanding on the issue and is the scope of another study currently underway.

Another note to make is on the effects by the reflected waves. Prior to the study, we had examined the effectiveness of the artificial beach installed in terms of dissipating the incoming waves. It was found that the beach was quite effective and the reflected wave height was less than 10% within the range of the wave conditions tested. Given the wave period of 1.0 second, the group velocity of the wave train would be equal to 0.78 m/s. The distance between the test span and the end of the flume was approximately 30 m. Hence, it would take about 80 seconds for the wave train to cover the distance to-and-fro. Since our experimental duration was less than 100 seconds for all tests, the reflected waves were not expected to be a significant factor to be considered. Even when the reflected waves reached the sheet, the induced wave drift would be very small given their small amplitude.

The mean drift velocity can be determined by best-fitting the displacement curve in the quasi-steady stage. For example, Figure 4.8 illustrates the results of the best fit for the experimental data shown in Figure 4.6. With the existence of the steady drift, it would now be meaningful to compare the magnitude of the drift with the various wave parameters. Some of the observations are summarized in the following sections.

Figure 4.8 Best-fitting equation of the displacement curve in quasi-steady stage

4.3.1.2 Effect of sheet length on drift velocity

We first investigate the dependency of the drift velocity on the sheet length under varying wave steepness (Fig. 4.9). From the figure, it is obvious that the size ratio L_g/L had a direct influence on the normalized drift velocity u_d/c . For all cases, the drift increased with the size ratio until it reached a certain limiting value of the order of one wave length. As shown in Fig. 4.9, the limiting value increased with the wave steepness, from 0.8 for $ka = 0.062$ to 1.4 for $ka = 0.124$. The plateau was reached earlier than Kang & Lee (1995) who suggested that the dependency would cease after $L_g/L \geq 1.5$.

Figure 4.9 Effect of longitudinal sheet length on drift velocity

Due to the different behavior, we shall separate the observations and results into two batches: sheets with size $L_g/L < 1.0$ were defined as short and $L_g/L \geq 1.0$ long. The demarcation makes it easier to visualize the results in the two different ranges.

4.3.1.3 Effect of wave steepness on drift velocity

The celerity-normalized drift velocity u_d / c is plotted against the wave steepness ka for the short and long sheet size ratios L_g / L in Figure 4.10(a) and 4.10(b) respectively. The figures include the measurements in the present study as well as Kang and Lee (1996). It is clear that, for Kang & Lee's study and this investigation, the drift velocity generally increases with the wave steepness ka , but the dependency is not exactly to the second power as stipulated in the form of Stokes drift. At a small wave steepness, the drift velocity increases at a much faster rate, particularly for the long sheets. However, the rate of increase diminishes significantly at a higher steepness of $ka > 0.12$. It should be noted that the larger ka implies a modification of the wave shape which is normally not taken into account in the analytical derivations. Also, it can be observed that Kang and Lee (1996) results are generally higher than the present study particularly for the long sheets in Figure 4.10(b). The reason is unclear as both studies were conducted under deep water condition, although the accuracy of the infra-red measurements by the monitoring system in the present study is expected to be higher.

More information can be obtained by plotting the drift velocity normalized by the Stokes drift against the wave steepness. The graph is shown in Fig. 4.11 categorized in terms of L_g / L . Towards the left for smaller ka , the large values of u_d / u_s are due to the relatively large drift coupled by the smallness of the wave steepness, particularly for $L_g / L \geq 1.0$. Despite some scatters due to experimental uncertainties, it appears that the ratio of the drift velocity with the Stokes drift generally decreases exponentially, and becomes lower than the Phillips's estimate for long sheets when $ka \geq 0.14$.

(a)

(b)

Figure 4.10 Effect of wave steepness on drift velocity: (a) short sheets (b) long sheets

Figure 4.11 Effect of wave steepness on Stokes-normalized drift velocity

4.3.2 Shallow water waves

For shallow water waves, Laitone (1962) pointed out that cnoidal waves would be generated if: 1) Ursell number $Ur = \frac{L^2 H}{d^3}$ is greater than 10; 2) d/L ratio is less than 0.125. In this study, we selected a range of experimental conditions given in Table 4.2 to simulate the Cnoidal wave condition in the laboratory. As shown in the table, the water depth d was set at three different values of 0.35, 0.30 and 0.25 m. The wave period T was chosen as 1.7, 1.8 or 1.9 s, and the height was varied from 0.06 to 0.12 m at 0.01 m interval.

We limited the scope of the investigation for long sheets only due to time constraint. Based on the deep water test results, it is expected that the drift velocity would not be dependent on L_g/L for long sheets, and would only depend on the wave steepness. To achieve that, we chose the sheet length to exceed at least 3 times the wavelength in

all shallow water tests.

Table 4.2 Experimental parameters for shallow water condition

Expt.	H (m)	T (s)	Ursell No.	d (m)	L (m)	c (m/s)	d/L
SA1	0.06	1.7	5	0.35	NA	NA	NA
SA2	0.07	1.7	10	0.35	2.475	1.456	0.141
SA3	0.08	1.7	12	0.35	2.536	1.492	0.138
SA4	0.09	1.7	15	0.35	2.673	1.572	0.131
SA5	0.10	1.7	17	0.35	2.700	1.588	0.130
SA6	0.08	1.8	14	0.35	2.739	1.522	0.128
SA7	0.09	1.8	17	0.35	2.846	1.581	0.123
SA8	0.10	1.8	16	0.35	2.619	1.455	0.134
SA9	0.11	1.8	27	0.35	3.244	1.802	0.108
SA10	0.12	1.8	25	0.35	2.989	1.660	0.117
SA11	0.07	1.9	15	0.35	3.031	1.595	0.115
SA12	0.08	1.9	23	0.35	3.511	1.848	0.100
SA13	0.09	1.9	18	0.35	2.928	1.541	0.120
SA14	0.10	1.9	26	0.35	3.339	1.757	0.105
SA15	0.11	1.9	25	0.35	3.122	1.643	0.112
SA16	0.12	1.9	26	0.35	3.048	1.604	0.115
SB1	0.09	1.7	22	0.30	2.725	1.603	0.110
SB2	0.10	1.7	23	0.30	2.492	1.466	0.120
SB3	0.11	1.7	25	0.30	2.477	1.457	0.121
SB4	0.12	1.7	28	0.30	2.510	1.476	0.120
SB5	0.13	1.7	31	0.30	2.537	1.493	0.118
SB6	0.08	1.8	23	0.30	2.786	1.548	0.108
SB7	0.09	1.8	25	0.30	2.739	1.521	0.110
SB8	0.10	1.8	23	0.30	2.492	1.384	0.120
SB9	0.11	1.8	34	0.30	2.889	1.605	0.104
SB10	0.12	1.8	36	0.30	2.846	1.581	0.105
SB11	0.07	1.9	18	0.30	2.635	1.387	0.114
SB12	0.08	1.9	22	0.30	2.725	1.434	0.110
SB13	0.09	1.9	30	0.30	3.000	1.579	0.100
SB14	0.10	1.9	32	0.30	2.939	1.547	0.102
SB15	0.11	1.9	35	0.30	2.931	1.543	0.102
SC1	0.08	1.7	30	0.25	2.421	1.424	0.103
SC2	0.09	1.7	33	0.25	2.394	1.408	0.104
SC3	0.10	1.7	39	0.25	2.469	1.452	0.101
SC4	0.11	1.7	41	0.25	2.413	1.420	0.104
SC5	0.12	1.7	45	0.25	2.421	1.424	0.103
SC6	0.07	1.8	24	0.25	2.315	1.286	0.108
SC7	0.08	1.8	30	0.25	2.421	1.345	0.103
SC8	0.09	1.8	32	0.25	2.357	1.309	0.106
SC9	0.10	1.8	40	0.25	2.500	1.389	0.100
SC10	0.11	1.8	44	0.25	2.500	1.389	0.100
SC11	0.06	1.9	22	0.25	2.394	1.260	0.104
SC12	0.07	1.9	20	0.25	2.113	1.112	0.118
SC13	0.08	1.9	30	0.25	2.421	1.274	0.103
SC14	0.09	1.9	35	0.25	2.465	1.297	0.101
SC15	0.10	1.9	36	0.25	2.372	1.248	0.105

4.3.2.1 Effect of water depth on drift velocity

In Figs. 4.12(a), 4.12(b) and 4.12(c), the drift velocity u_d is plotted against the wave height H at the three different water depths d with the wave period $T = 1.7, 1.8$ and 1.9 s respectively. A few trends can be noted from the graphs. For all three wave periods, the drift velocity of polyethylene sheet increased almost linearly with the wave height. This is contrary to the deep water cases whereby in general the drift velocity is increased to the second order of the wave steepness (i.e. Figure 4.10). In addition, it was surprising to find that the drift velocity actually became faster at a lower water depth. If the dependency with the wave steepness was a proportional relationship, then the drift velocity should have decreased instead since the wave celerity should decrease with the lower water depth (Laitone, 1961). This observation shows that the distorted wave shape in shallow water plays a dominant role in controlling the drift motion. In addition, it implies that the drift of an oil patch in the near-shore area towards the beach could be much faster than the drift of the patch in deeper water.

(a)

(b)

(c)

Figure 4.12 Effect of water depth on drift velocity in shallow water:

(a) $T = 1.7$ s (b) $T = 1.8$ s (c) $T = 1.9$ s

4.3.2.2 Effect of Ursell number on drift velocity

In shallow water waves, the characteristics are often correlated to H/d as well as Ursell number (see “Coastal Engineering Manual - Part II”, US Army Corps of Engineers, 2002). Fig. 4.13 plots the celerity-normalized drift velocity u_d/c versus

Ursell number $\frac{L^2 H}{d^3}$ under a range of H/d . Within this range, the relationship shows an approximate linear trend of drift velocity increasing with the Ursell number. For bulk estimate purpose, a best-fitting equation was then fitted in Figure 4.13:

$$\frac{u_d}{c} = 0.0007 \text{ Ur} + 0.0123 \quad \text{for } \text{Ur} > 10 \quad (4.1)$$

It should be noted, however, that Equation (4.1) only estimates the progressive wave drift in shallow water. The potential nearshore circulation driven by the wave motion has not been included.

Figure 4.13 Effect of Ursell number on drift velocity in shallow water

4.4 Discussion

1. For the measurements under deep water waves, a motion monitoring system with two infra-red cameras is used to record accurately the position of the polyethylene sheet at 50Hz. This implies that the accurate coordinates of a polyethylene sheet can be reported every 0.02 seconds. In the previous investigation, the movement of a polyethylene sheet was recorded by a video camera. The position of the polyethylene sheet was then estimated through the video clips. That is the reason why the accuracy of infrared measurements in the present study is expected to be higher.

2. The kink at $L_g/L \sim 1$ could be due to the discontinuity in the kinematic boundary condition at the edge of the sheet. The 'edge effect' greatly diminishes when the longitudinal length of the sheet exceeds one wave length.

3. For the drift velocity in shallow water waves, it is emphasized that this is only for bulk estimate (e.g. oil beaching analysis) based on the data trend. If additional experimental or field data are available in the future, there is no doubt that a more refined analysis can then be performed.

Chapter

5

DISPERSION DUE TO WAVE DRIFT

In this chapter, we investigate whether surface floating particles would disperse in a random wave field due to Stokes drift. From the theoretical analysis, it will be shown that since the surface Stokes drift velocity is in fact a time-independent value that does not possess any fluctuation, it should not cause any dispersive effect contrary to what has been suggested in the literature. Meanwhile, the study also addresses the Taylor dispersion of soluble contaminants due to random waves.

5.1 Summary of drift velocity

In Chapter 3, the Lagrangian velocities under regular waves are shown in Eq. (3.11).

There is a time-independent Stokes drift component:

$$u_s = \left\{ \varepsilon^2 \frac{\cosh 2kz}{2 \sinh^2 kd} \right\} c \quad (5.1)$$

It is also shown in Chapter 3 that the drift velocity under random waves can be expressed as:

$$u_s = 2 \int_0^\infty s(\omega) \omega k \frac{\cosh 2kz}{2 \sinh^2 kd} d\omega \quad (5.2)$$

5.2 Stokes drift and surface dispersion

As mentioned in the literature review in Chapter 2, the key reference to the analysis of the effect of drift on surface dispersion is that of Herterich and Hasselmann's (1982), based on the central conjecture that the Stokes drift velocity fluctuates in a random manner under the multi-directional sea. The analysis thus hinges on whether the conjecture is accurate. Before we discuss the issue, it is essential to review some fundamental concepts on the analysis of random waves.

It is well known that there are two possible approaches to analyze the time series of a random wave field, namely time-domain and frequency-domain analysis (Coastal Engineering Manual, 2002). In the time-domain analysis, or commonly known as the zero-crossing method, the time series of surface elevation record contains a number of individual waves which have different wave heights and periods based on the zero-crossing intervals. Longuet-Higgins (1952) have shown that the probability density of the individual zero-crossing wave heights follow the Rayleigh distribution. On the other hand, in the frequency-domain analysis or commonly known as the spectral analysis, the surface elevation is expanded in the form of a Fourier series:

$$\eta(t) = \sum_{i=1}^{\infty} a_i \cos(k_i x - \omega_i t + e_i) = \sum_{i=1}^{\infty} \sqrt{2s(\hat{\omega}_i) \Delta \omega_i} \cos(\tilde{k}_i x - \tilde{\omega}_i t + e_i) \quad (5.3)$$

where a_i, k_i, ω_i, e_i are amplitude, wave number, angular frequency and random phase of each component wave, respectively; $s(\omega)$ is frequency spectrum.

The functional form implies that the time series is composed of an infinite number of linear waves (component waves) with different frequencies. The two different approaches are typically employed independently, although it has been shown that the significant wave height can be obtained with similar values from both analyses in deep waters (i.e. Yu, 2000).

In Herterich and Hasselmann's theory (1982), the ensemble-mean Stokes drift velocity is first obtained in the frequency-domain, and then the 'instantaneous' fluctuating drift velocity in the time-domain. This cross-over poses an ambiguity in the derived relationship for the dispersion effect.

In Chapter 3, it is demonstrated that the Stokes drift velocity obtained in the frequency domain is not only a mean value, but an instantaneous one as well. Hence, the random fluctuations of drift velocity should not exist. Consequently, the surface dispersive effect should not exist contrary to the suggestion by Herterich and Hasselmann (1982) and others.

5.3 Experiments on surface dispersion

In Section 5.2, it has been concluded that there should not be a dispersion effect to surface drift. To further verify this issue, experiments were performed as described in the following.

5.3.1 Experimental setup

The experiments were conducted in the same wave flume described in Chapter 4. Here, random waves were run during the experiments on surface dispersion. The Pierson-Moscowitz (P-M) wave spectrum was chosen.

Small vinyl marks with a radius of about 1 cm were used as floating particles in the experiments. The vinyl marks had a density of 0.9 g cm^{-3} and they were used to simulate the surface contaminant particles. They floated on the water surface without any relative movement to the surface water particles.

A ruler is fixed on the wall of one wave flume side to measure the position of markers. On the opposite, a digital video camera, mounted on a tripod, was positioned beside the measurement area to capture the images of the particles' movement. The field of view of the camera covered a span of about 2 m in order to identify the position of the small vinyl marks and their coordinates in the images. The schematic layout of the experimental set-up is shown in Fig. 5.1.

Figure 5.1 Schematic layout of experimental set-up

5.3.2 Experimental procedures

Two pieces of vinyl marks were placed along the wave flume on the water surface. According to Eq. (3.9), the 1st order horizontal displacement is the same order as wave height (0.05 m in this study). The initial distance for the marks was thus set to be larger at 0.2 m to avoid collision. The experiment was initiated only after the water was sufficiently still. This can be judged by visually inspecting the motion of the marks for one minute or two. Any discernible movement would indicate the presence of residual current. After it was ensured that there were no residual currents, the digital video camera was switched on and the wave generator was activated.

The experimental duration is determined base on Wong and Law's (2003) experiments conclusion. Upon the activation of the wave-maker, there were three stages of the variation in the drift velocity's magnitude: the transient accelerating stage, the steady state stage and the stage with return current. The first stage was caused by a large initial acceleration of the water body which brings the drift to a steady velocity in a very short period. The effect of this acceleration to the two particle marks was the same when the wave evolution was not considered. In the second stage, the drift would proceed at a velocity equal to the open sea value. With the reflection of waves and the return flow of accumulated mass at the far end of the wave flume, the drift velocity would begin to decrease. This is the third stage. In order to avoid the effect of the third stage, the experimental time was chosen to be 100 s.

5.3.3 Experimental results

In the experiments, the Pierson-Moscowitz spectrum was used to generate the random waves, and the spectral parameters were $H_s=0.03, 0.04$ and 0.05m and $d=0.8\text{ m}$. The wave-maker spectra, plotted in the following (Figs. 5.2), show that the spectrum generated in the tank agrees well with the theoretical Pierson-Moscowitz (P-M) spectrum.

Figure 5.2 Wavemaker spectrum (P-M): (a) $H_s=0.03\text{m}$ (b) $H_s=0.04\text{m}$ (c) $H_s=0.05\text{m}$

In order to verify the repetition of experiments, three tests have been done for each wave height $H_s=0.03, 0.04$ and 0.05m . The relative distance of markers can be determined by the coordinate-difference of them in video pictures.

The relative distances of the marker particles at different times are shown in Appendix A. It can be observed that the average distances were almost unchanged after 100 seconds. The distances only oscillated within a small range (-0.05 to 0.05m in these experiments, see Appendix A). These experimental observations were distinctly different from those of Buick et al. (2001) who reported that the relative distance of two marker particles increased with time.

Following Herterich and Hasselmann's (1982) theory and for the given experimental conditions, the relative distance would have increased by 8 cm, 10 cm and 12 cm corresponding to the significant wave height 0.03 m, 0.04 m and 0.05m respectively (see Appendix B). This predicted increase was however not observed in our experiments. The comparison between the experimental observations and Herterich and Hasselmann's (1982) prediction is presented in Figs. 5.3. The present results thus reinforce the conclusion that the stochastic Stokes drift does not lead to a surface dispersive effect.

(a)

(b)

(c)

Figure 5.3 Comparison between experimental observations (the 2nd test) and Herterich & Hasselmann's (1982) theory: (a) $H_s=0.03\text{m}$ (b) $H_s=0.04\text{m}$ (c) $H_s=0.05\text{m}$

5.4 Taylor dispersion due to wave action

The mechanism of Taylor dispersion, introduced in Chapter 2, is due to the presence of a velocity gradient in the vertical direction. It is known from the classic wave theory that there exists a gradient of 1st order orbital velocities (Dean and Dalrymple, 1991). However, it should be noted that the trajectory of the particles movement is close at this order. Hence, the 1st order orbital motion of the pollutant particles can be temporarily ignored, and the wavy surface can also be ignored. This is analogous to the approaches towards analyzing the persistence release of pollutants into a water body by considering only the time-mean velocities (Csanady 1983).

In a progressive wave environment, as the pollutant particles move passively in phase and magnitude with the water particles, they also undergo the second order period-averaged drift displacement. Since the motion of a pollutant particle is followed, the drift displacement should obey the Lagrangian pattern. The Taylor dispersive effect due to the drift pattern is discussed in the following as described by Law (2000).

The depth-wise variation of the drift velocity gives rise to a dispersive effect in the longitudinal direction. Assuming a velocity scale of $\overline{u_d}$, the longitudinal dispersion coefficient due to the effect can be calculated as (Van den Broeck, 1990):

$$K = \overline{u_d}^2 d^2 \int_0^{\infty} \frac{\left\{ \int_0^z [u'(z'') - u'_m] p(z'') dz'' \right\}^2}{Dp_s(z')} dz' \quad (5.4)$$

where D is the molecular (laminar) or ambient eddy diffusivity (turbulent) that for this analysis is assumed to be isotropic in all directions and constant throughout the water

depth, u_m the non-dimensional depth averaged velocity, u' the non-dimensional drift, and $z'=z/d$. $p_s(z')$ is the steady state probability density function of a pollutant particle in the vertical direction that can be expressed as:

$$p_s(z') = \lim_{t \rightarrow \infty} p(z', t | z_0', t_0) \quad (5.5)$$

where p is the probability density function of the particle transitioning from z_0' at t_0' to z' at t . The pollutant particle will not be able to cross both the water surface and the impermeable sea bed. Thus both boundaries are totally reflective. Assuming small amplitude wave motion and considering only the time mean position, the reflective conditions can be expressed as:

$$\frac{\partial p(z', t | z_0', t_0)}{\partial z'} = 0 \text{ at } z'=0,1 \quad (5.6)$$

Since the 1st order orbital motion of the particle is ignored, hence, the particle only translates vertically based on diffusive movement. With this assumption, the vertical movement of the pollutant particle will simply be a Markov process. According to Van Den Broeck's (1982) theory, the depth-wise probability density function through the Fokker-Planck representation can be expressed as:

$$\frac{\partial p(z', t | z_0', t_0)}{\partial t} = \frac{D}{d^2} \frac{\partial^2 p(z', t | z_0', t_0)}{\partial z'^2}; \quad (5.7)$$

$$p(z', t | z_0', t_0) = \delta(z' - z_0')$$

The solution of (5.7) that satisfies the top and bottom reflective boundary condition is:

$$p(z', t | z_0', t_0) = 1 + 2 \sum_{n=1}^{\infty} e^{-\frac{\pi^2 n^2 D(t-t_0)}{d^2}} \quad (5.8)$$

From (5.8), when t is large, we have

$$p_s(z') = 1 \quad (5.9)$$

Substituting (5.9) into (5.4) yields

$$\frac{K}{D} = \frac{\overline{u_d}^2 d^2}{D^2} \int_0^1 \left\{ \int_0^1 [u'(z'') - u'_m] dz'' \right\}^2 dz' \quad (5.10)$$

where K is the Taylor dispersion coefficient, u_m the depth-averaged horizontal drift,

$\overline{u_d}$ the drift velocity scale ($\overline{u_d} = u_m$), u' non-dimensional drift, u'_m the depth-averaged non-dimensional drift.

5.4.1 Taylor dispersion in regular waves

The longitudinal dispersion based on the profile of a second order Stokes drift in regular waves was investigated in details in Law (2000). In Chapter 3, it is shown that there is a time-independent drift velocity in a regular wave motion trajectory. The depth-averaged drift velocity, without a pressure gradient is known to be (Fredsoe and Diegaard, 1992):

$$u_m = \frac{\pi H^2}{4Td} \frac{1}{\tanh(kd)} \quad (5.11)$$

Using u_m as the non-dimensional scale for velocity, the drift profile can be expressed as:

$$u' = \frac{u_s}{u_m} = kd \left(\frac{\cosh(kd \cdot z') + \sinh(kd \cdot z')}{\sinh(kd) \cosh(kd)} \right) \quad (5.12)$$

Substituting (5.12) into (5.10) and integrating yield:

$$\frac{K}{D} = \Omega^4 f_s(kd) \quad (5.13)$$

where

$$\Omega = \frac{H}{\sqrt{DT}}$$

represents the ratio of the wave height and the diffusive movement within one wave period. The function can be integrated as (Law, 2000):

$$f_s = \frac{\pi^2}{16 \tanh^2(kd)} \left(\frac{\sinh(4kd) - 4kd}{4kd(\cosh(4kd) - 1)} + \frac{1}{2(kd)^2} - \frac{\cosh(2kd)}{kd \sinh(2kd)} + \frac{1}{3} \right) \quad (5.14)$$

5.4.2 Taylor dispersion in random waves

Following a similar approach as Law (2000), we proceed to investigate the Taylor dispersion in random waves in this study. For random waves, in order to obtain the Taylor dispersive coefficient in Eq. (5.10), the expressions of $\overline{u_d}$, u' and u'_m need to be resolved first.

The drift velocity profile under random waves has previously been obtained in Eq. (3.43). The depth-averaged drift velocity u_m can be expressed as:

$$\begin{aligned} u_m &= \frac{1}{d} \int_0^d u_s dz \\ &= \frac{1}{d} \int_0^d 2 \int_0^\infty s(\omega) \omega k \frac{\cosh(2kz)}{2 \sinh^2(kd)} d\omega dz \\ &= \frac{1}{d} \int_0^\infty s(\omega) \omega \frac{1}{\tanh(kd)} d\omega \end{aligned} \quad (5.15)$$

It is necessary to express the wave spectrum in non-dimensional manner for the numerical calculation as follow:

$$\hat{s}(\hat{\omega}) = \frac{s(\omega) \omega_m}{m_0} \quad \text{where } m_0 = H_s^2 / 16 \quad (5.16)$$

$$\hat{\omega} = \omega / \omega_m \quad (5.17)$$

where ω_m is the peak frequency of wave spectrum and H_s the significant wave height.

Substituting Eq.(5.16) and (5.17) into Eq. (5.15):

$$\begin{aligned}
 u_m &= \frac{1}{d} \int_0^\infty \frac{\hat{s}(\hat{\omega}) m_0}{\omega_m} (\hat{\omega} \omega_m) \frac{1}{\tanh(kd)} d(\hat{\omega} \omega_m) \\
 &= \frac{m_0 \omega_m}{d} \int_0^\infty \frac{\hat{s}(\hat{\omega}) \hat{\omega}}{\tanh(kd)} d\hat{\omega}
 \end{aligned} \tag{5.18}$$

Substituting $m_0 = H_s^2/16$ into Eq. (5.18) and note that $\overline{u_d} = u_m$:

$$\begin{aligned}
 \overline{u_d}^2 &= \left(\frac{(H_s^2/16)(2\pi/T_m)}{d} \right)^2 \left(\int_0^\infty \frac{\hat{s}(\hat{\omega}) \hat{\omega}}{\tanh(kd)} d\hat{\omega} \right)^2 \\
 &= \left(\frac{H_s}{\sqrt{dT_m}} \right)^4 \frac{\pi^2}{64} \left(\int_0^\infty \frac{\hat{s}(\hat{\omega}) \hat{\omega}}{\tanh(kd)} d\hat{\omega} \right)^2
 \end{aligned} \tag{5.19}$$

where T_m is the wave period relative to peak frequency.

$$\therefore \frac{\overline{u_d}^2 d^2}{D^2} = \Omega_r^4 f_1(\varepsilon_m) \tag{5.20}$$

where

$$\Omega_r = \frac{H_s}{\sqrt{DT_m}} \tag{5.21}$$

represents the ratio of the significant wave height and the diffusive movement corresponding to one dominant wave period in the random sea.

$$f_1(\varepsilon_m) = \frac{\pi^2}{64} \left(\int_0^\infty \frac{\hat{s}(\hat{\omega}) \cdot \hat{\omega}}{\tanh(kd)} d\hat{\omega} \right)^2 \tag{5.22}$$

Eq. (5.22) is a function of dimensionless water depth.

The non-dimensional drift u' can be obtained based on Eq. (3.43) & Eq. (5.15):

$$u' = \frac{u_s}{u_m} = \frac{2 \int_0^\infty s(\omega) \omega k \frac{\cosh(2kz)}{2 \sinh^2(kd)} d\omega}{\frac{1}{d} \int_0^\infty s(\omega) \omega \frac{1}{\tanh(kd)} d\omega} \tag{5.23}$$

$$= \frac{\int_0^\infty s(\omega) \omega (kd) \frac{\cosh(2kd \cdot z')}{\sinh^2(kd)} d\omega}{\int_0^\infty s(\omega) \omega \frac{1}{\tanh(kd)} d\omega}$$

where $z' = \frac{z}{d}$.

Substituting Eq. (5.16) and (5.17) into Eq. (5.23) to dimensionless the wave spectrum:

$$\begin{aligned} u' &= \frac{\int_0^\infty \frac{\hat{s}(\hat{\omega}) m_0}{\omega_m} (\omega_m \hat{\omega}) \cdot (kd) \frac{\cosh(2kd \cdot z')}{\sinh^2(kd)} d(\omega_m \hat{\omega})}{\int_0^\infty \frac{\hat{s}(\hat{\omega}) m_0}{\omega_m} (\omega_m \hat{\omega}) \frac{1}{\tanh(kd)} d(\omega_m \hat{\omega})} \\ &= \frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot (kd) \frac{\cosh(2kd \cdot z')}{\sinh^2(kd)} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} \end{aligned} \quad (5.24)$$

In addition, the depth-averaged non-dimensional drift u'_m is equal to 1 by definition:

$$u'_m = \frac{1}{d} \int_0^d \frac{u_s}{u_m} dz = \frac{\frac{1}{d} \int_0^d u_s dz}{u_m} = \frac{u_m}{u_m} = 1 \quad (5.25)$$

Substituting the expressions of Eq. (5.19), Eq. (5.24) and Eq (5.25) into the governing equation Eq. (5.10):

$$\frac{K}{D} = \Omega_r^4 f(\varepsilon_m) = \Omega_r^4 f_1(\varepsilon_m) \cdot f_2(\varepsilon_m) \quad (5.26)$$

where

$$\Omega_r = \frac{H_s}{\sqrt{DT_m}}$$

$$\varepsilon_m = \frac{\omega_m^2 d}{g}$$

$$f_1(\varepsilon_m) = \frac{\pi^2}{64} \left(\int_0^\infty \frac{\hat{s}(\hat{\omega}) \hat{\omega}}{\tanh(kd)} d\hat{\omega} \right)^2$$

$$f_2(\varepsilon_m) = \int_0^1 \left\{ \int_0^1 [u'(z'') - 1] dz'' \right\}^2 dz'$$

The $f_2(\varepsilon_m)$ can be solved according to Eqs. (5.24) and (5.25):

$$\begin{aligned} f_2(\varepsilon_m) &= \int_0^1 \left(\int_0^1 [u'(z'') - u'_m] dz'' \right)^2 dz' \\ &= \int_0^1 \left(\frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \int_0^1 (kd) \frac{\cosh(2kd \cdot z'')}{\sinh^2(kd)} dz'' \cdot d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} - \int_0^1 u'_m dz'' \right)^2 dz' \\ &= \int_0^1 \left(\frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\sinh(2kd \cdot z')}{2 \sinh^2(kd)} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} - z' \right)^2 dz' \\ &= \int_0^1 \left(\frac{\left(\frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\sinh(2kd \cdot z')}{2 \sinh^2(kd)} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} \right)^2}{-z' \frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\sinh(2kd \cdot z')}{\sinh^2(kd)} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} + z'^2} \right) dz' \\ &= \int_0^1 \left(\frac{\left(\frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\sinh(2kd \cdot z')}{2 \sinh^2(kd)} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} \right)^2}{-z' \frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\sinh(2kd \cdot z')}{\sinh^2(kd)} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} + z'^2} \right) dz' \end{aligned} \quad (5.27)$$

In the above equation, the 2nd and 3rd terms of the RHS can be solved analytically as follows:

The 2nd term:

$$\int_0^{\infty} z' \cdot \sinh(2kd \cdot z') dz' = \frac{\cosh(2kd)}{2kd} - \frac{\sinh(2kd)}{(2kd)^2} \quad (5.28)$$

The 3rd term:

$$\int_0^{\infty} z'^2 dz' = \frac{1}{3} \quad (5.29)$$

The expression of $f_2(\varepsilon_m)$ in Eq. (5.27) then becomes:

$$\begin{aligned} f_2(\varepsilon_m) = & \int_0^{\infty} \left(\frac{\int_0^{\infty} \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\sinh(2kd \cdot z')}{2 \sinh^2(kd)} d\hat{\omega}}{\int_0^{\infty} \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} \right)^2 dz' \\ & - \frac{\int_0^{\infty} \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\cosh(2kd)}{2kd} - \frac{\sinh(2kd)}{(2kd)^2} \cdot d\hat{\omega}}{\int_0^{\infty} \hat{s}(\hat{\omega}) \cdot \hat{\omega} \frac{1}{\tanh(kd)} d\hat{\omega}} + \frac{1}{3} \end{aligned} \quad (5.30)$$

It is known that the dispersive relationship can be stated as:

$$\omega^2 = gk \tanh(kd) \quad (5.31)$$

The first order approximation of the above dispersive relationship was derived by Eckart (1952) in the following equation with an error within 5%:

$$kd \cong \frac{\omega^2 d / g}{[\tanh(\omega^2 d / g)]^{1/2}} \quad (5.32)$$

Substituting the dimensionless frequency, $\hat{\omega} = \omega / \omega_m$, Eq. (5.32) becomes:

$$kd \cong \frac{\omega_m^2 d / g \cdot \hat{\omega}^2}{[\tanh(\omega_m^2 d / g \cdot \hat{\omega}^2)]^{1/2}} = \frac{\varepsilon_m \cdot \hat{\omega}^2}{[\tanh(\varepsilon_m \cdot \hat{\omega}^2)]^{1/2}} \quad (5.33)$$

where $\varepsilon_m = \omega_m^2 d / g$ relate to dimensionless water depth.

Substituting Eq. (5.33) into $f_2(\varepsilon_m)$:

$$f_2(\varepsilon_m) = \int_0^1 \left(\frac{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{\sinh \left\{ 2 \cdot \frac{\varepsilon_m \cdot \hat{\omega}^2}{[\tanh(\varepsilon_m \cdot \hat{\omega}^2)]^{1/2}} \cdot z' \right\}}{2 \sinh^2 \left\{ \frac{\varepsilon_m \cdot \hat{\omega}^2}{[\tanh(\varepsilon_m \cdot \hat{\omega}^2)]^{1/2}} \right\}} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \cdot \hat{\omega} \cdot \frac{1}{\tanh \left\{ \frac{\varepsilon_m \cdot \hat{\omega}^2}{[\tanh(\varepsilon_m \cdot \hat{\omega}^2)]^{1/2}} \right\}} d\hat{\omega}} dz' \right)^2$$

$$+ \frac{\int_0^\infty \hat{s}(\hat{\omega}) \hat{\omega} \cdot \frac{\cosh \left\{ \frac{2\varepsilon_m \hat{\omega}^2}{[\tanh(2\varepsilon_m \hat{\omega}^2)]^{1/2}} \right\} \sinh \left\{ \frac{2\varepsilon_m \hat{\omega}^2}{[\tanh(\varepsilon_m \hat{\omega}^2)]^{1/2}} \right\}}{2\varepsilon_m \hat{\omega}^2 \cdot \frac{[\tanh(\varepsilon_m \hat{\omega}^2)]^{1/2}}{\sinh^2 \left\{ \frac{\varepsilon_m \hat{\omega}^2}{[\tanh(\varepsilon_m \hat{\omega}^2)]^{1/2}} \right\}} \cdot \left\{ \frac{\varepsilon_m \hat{\omega}^2}{[\tanh(\varepsilon_m \hat{\omega}^2)]^{1/2}} \right\}^2} d\hat{\omega}}{\int_0^\infty \hat{s}(\hat{\omega}) \hat{\omega} \cdot \frac{1}{\tanh \left\{ \frac{\varepsilon_m \hat{\omega}^2}{[\tanh(\varepsilon_m \hat{\omega}^2)]^{1/2}} \right\}} d\hat{\omega}} + \frac{1}{3} \quad (5.34)$$

Numerical method can be used to solved $f(\varepsilon_m)$ and $\frac{K}{D}$ with MATLAB. As an illustration, the following describes the results with two spectrums that were reported in the literature. The first is the popular Pierson-Moscowitz spectrum for fully developed sea, and the second is the Wen spectrum for intermediate water waves.

(a) Pierson-Moscowitz spectrum

The random wave condition can be expressed by using the power spectrum, since this provides a convenient frequency domain description of the sea state. A number of parameterized formulas for such power spectrum have been derived from observations and theoretical work. When considering these spectra, it is important to distinguish whether they are valid for a developing or fully developed sea. A fully developed sea exists for a condition where energy input from wind into surface waves is balanced by the dissipation of wave energy in breaking. Wave spectra under fully developed condition have been proposed by various researchers, such as Bretschneider (1959), Mitsuyasu (1980) and Scott (1965).

In 1964, Pierson and Moscowitz (1964) proposed a new formula for an energy spectrum distribution of a wind generated sea state based on the similarity theory of Kitaigorodskii and the site measurements of Moscowitz (1964). This spectrum, commonly known as P-M model, has since been extensively used by ocean engineers as one of the most representative all over the world.

The P-M spectral model describes a fully-developed sea determined by one parameter, namely the wind speed (see Eq. 5.35). The fetch and duration are considered infinite. For the applicability of such a model, the wind has to blow over a large area at a nearly constant speed for many hours prior to the time when the wave record is obtained, and the wind should not change its direction more than a certain specified percentage. In spite of these assumptions, the P-M model has been found to be useful in common engineering applications.

The P-M spectrum model is written as:

$$s(\omega) = \frac{\alpha g^2}{\omega^5} \exp\left[-\beta \left(\frac{g}{W_{19.5}\omega}\right)^4\right] \quad (5.35)$$

where $\alpha = 8.1 \times 10^{-3}$, $\beta = 0.74$, $\alpha g^2 = 0.78$ (Yu, 2000) and $W_{19.5}$ the wind velocity above the sea surface at 19.5 m.

Note that given the wind velocity $W_{19.5}$, the P-M spectrum can be calculated by using Eq. (5.35). The dimensionless P-M spectrum can be further expressed as:

$$\hat{s}(\hat{\omega}) = 0.71 \times 7.14 \hat{\omega}^{-5} e^{-1.25 \hat{\omega}^{-4}} \quad (5.36)$$

Substituting the dimensionless P-M spectrum Eq. (5.36) into the governing equation Eq. (5.26), the dispersion coefficient can be solved by using MATLAB numerically.

Figure 5.4 shows the relationship between $f(\varepsilon_m)$ and ε_m . $f(\varepsilon_m)$ increases monotonically with ε_m implying that the dispersive effect is more prominent in transitional water.

Figure 5.4 Variation of $f(\varepsilon_m)$ versus ε_m (P-M spectrum)

The relationship between $\frac{K}{D}$ and Ω_r in Eq. (5.26) is illustrated for the P-M spectrum in Figure 5.5. From the figure that, the dispersive effect is negligible when Ω_r is less than 2. In general, the magnitude of the longitudinal dispersion coefficient is an order of magnitude larger than in regular wave condition. Hence, it can be inferred that the

Taylor dispersive effect would be much more significant in random waves.

Figure 5.5 Variation of K / D versus Ω_r for random waves (P-M spectrum)

(b) Wen spectrum

Obviously, the fully developed condition can only be approached asymptotically and there are doubts as to the actual existence of such a state. Thus, many wave spectra under developing sea state were also studied, including the famous JONSWAP spectrum (Hasselmann et al., 1973), Wallops spectrum (Huang et al., 1981) etc. The JONSWAP (Joint North Sea Wave Project) spectrum is perhaps the most popular in such spectra family. However, the JONSWAP spectrum is restricted to the deep water condition.

Wen et al. (1994, 1996) presented a wave spectrum of a developing sea state under intermediate water depth by introducing a depth factor $H^* = \frac{0.626H_s}{d} = \frac{\overline{H}}{d}$ to the spectrum as follows:

$$\hat{s}(\hat{\omega}) = \frac{Q}{R_m^2 - 1} P_m \hat{\omega}^{-p_m} \exp \left[-\frac{P_m}{q_m} (\hat{\omega}^{-q_m} - 1) \right] + \frac{3.62(1 - 0.615H^*)}{Q^{0.35}} \left(1 - \frac{Q}{R_m^2 - 1} \right) \cdot \exp \left[-\frac{41.1(1 - 0.615H^*)^2}{Q^{0.699}} (\hat{\omega} - 1)^2 \right] \quad (5.37)$$

where

$$\left. \begin{aligned} P_m &= 1.444 + 0.888H^* \\ R_m &= 1.44 + 0.113e^{6.38H^*} \\ p_m &= 4.25 - 2.5H^* \\ q_m &= 5.5 - 1.78H^* + 1.1(H^*)^2 \\ Q &= 4.14[1 + 1.5(H^*)^{1.85}]P^{-4.8(H^*)^{1.5}}e^{-0.809P^{1.766}} \\ P &= 2 \end{aligned} \right\} \quad (5.38)$$

In deep water condition where $H^* \cong 0$, there is no significant difference between the dimensionless Wen spectrum (in solid) and P-M spectrum (in dash) as shown in Fig. 5.6.

Figure 5.6 Comparison between P-M spectrum and Wen spectrum in deep water
($H^*=0$)

The profiles of the dimensionless Wen spectrum with $H^* = 0, 0.05$ and 0.1 are plotted in Fig. 5.7. As H^* increases, the peak value of the spectrum becomes higher, and the distribution narrower.

Figure 5.7 Comparison of Wen spectrum profile with $H^*=0, 0.05$ and 0.1

Wen spectrum (Eq. 5.37) can be substituted into the governing equation (Eq. 5.26) to solve the dispersion coefficient for this case. Figure 5.8 illustrates the relationship between $f(\varepsilon_m)$ and ε_m for the Wen spectrum with different H^* values (0, 0.05 and 0.1). The results show that $f(\varepsilon_m)$ generally increases with higher H^* value. This implies that, with a constant water depth, a larger wave height will cause stronger dispersive effect in transitional water, which is physically reasonable. At the same time, given a constant wave height, the dispersive effect would increase with a smaller water depth.

Figure 5.8 Variation of $f(\varepsilon_m)$ versus ε_m (Wen spectrum)

The relationship between $\frac{K}{D}$ and Ω_r is plotted in Figure 5.9 using the Wen spectrum with $H^*=0.1$. It can be noted that, the dispersive effect is again negligible when Ω_r is less than 2, which is similar to the results from the P-M spectrum. Comparing with Fig. 5.5, the longitudinal dispersion coefficient in the intermediate water depth condition is generally higher than in the deep water condition.

Figure 5.9 Plot of K/D versus Ω_r for random waves (Wen spectrum, $H^*=0.1$)

Chapter

6

WAVE-INDUCED DRIFT OF SMALL FLOATING OBJECTS IN DEEP WATER WAVES

In this chapter, we investigate the drift velocity of a small rigid floating object driven by regular waves in deep water. Experimental observations are first presented on the drift behavior of square, circular and elliptical objects by means of an infrared motion monitoring system. These objects possess different drag and added mass coefficients, and the measurements on the drift velocity of these objects clarify some outstanding issues. The theoretical drift of a small thin flat plate is then derived. The comparison between the experimental drift and simulation results of small circular plates is given to verify the theoretical prediction.

6.1 Introduction

As discussed earlier in Chapters 1 and 2, the drift of floating objects needs to be considered in some engineering applications. For example, in the cold region, the drifting ice floes can be extremely hazardous to offshore structures. Generally, the wind and ocean current are considered as the primary factors causing iceberg drift (e.g. El-Tahan 1983). At the same time, researchers such as Wadhams (1983) and Arikainen (1972) have demonstrated that the wave-induced drift can actually be as large as the wind-caused drift and should not be neglected. Harms (1986) performed laboratory measurements on the drift of two-dimensional wavelength-size ice floe models under regular wave conditions. He obtained relatively simple empirical formulas that can predict the wave-induced drift of these two-dimensional ice floes. However, few works have been made on the drift of three-dimensional floating objects that better resemble the practical situations.

This study investigates the drift of three-dimensional small rigid floating objects. In the theoretical part, the drift velocity of small floating objects of an arbitrary shape and small submergence was derived. Experimental work was then conducted to determine the time history of the wave-induced drifting of these small rigid floating objects which were not available in the past. The experimental data were also compared with the theoretical results. As illustrated in this chapter, the time history indeed showed a quasi-steady drift velocity without acceleration shortly after the waves arrived at the floating objects.

6.2 Experimental investigation

6.2.1 Experimental setup and preparation

The same wave flume and motion monitoring system (Qualisys Track Manager), which was introduced in Chapter 4, were used in the current experiments. However, the

objects investigated as well as the wave conditions in this experimental series were different from that in Chapter 4. The layout of the experimental setup is shown in Figure 6.1 with two cameras set at an optimum angle to cover an intersecting span of about 3 m.

Figure 6.1 Schematic layout of the experimental setup

In these experiments, Stokes waves with 1.2 and 1.5 m wavelength were generated. Polyethylene plates with a density of 0.96 g cm^{-3} and uniform thickness (Δ) of 3.0 and 4.5 cm were used to simulate the small objects. Square, circular and elliptical shape models that were used in the experiments are shown in Fig. 6.2. All these shapes were cut out precisely from AutoCAD generated templates. The dimensions of the plates used in the experiments are listed in Table 6.1, where L_g is the longitudinal axis length of the square, circular and elliptical polyethylene plates parallel to the direction of wave propagation; L_t the transverse axis length perpendicular to the direction of wave propagation. Since it is generally accepted that the effect of diffraction becomes important as the ratio $L_g/L > 0.2$ (Isaacson, 1979), where L is wavelength, therefore the longitudinal length of the plate L_g was chosen as 0.2 m to ensure that the diffraction would be insignificant.

Table 6.1 The dimensions of the objects used in the experiments

Shape series	Shape codes	L_g (mm)	L_t (mm)	Δ (mm)
Square	square1	200	200	45
	square1'	200	200	30
Circle	circle1	200	200	45
	circle1'	200	200	30
Ellipse I	ellipse1	200	150	45
	ellipse1'	200	150	30
Ellipse II	ellipse2	200	100	45
	ellipse2'	200	100	30

Figure 6.2 Shapes of plates used in the experiments: (a) Square, (b) Circle, (c) Ellipse I and (d) Ellipse II

6.2.2 Experimental procedures

The experimental procedures were divided into two parts: the calibration of equipments and the actual running of the tests. Prior to the commencement of an experiment series, the experimental water depth of 0.8 m was set in the wave flume. The wave generation characteristics by the wave maker were then verified with the specific water depth. Two infrared cameras were placed next to the wave flume at a longitudinal location about 7 m from the wave generator. The infrared camera system was calibrated with floating objects of known sizes to create a distorted coordinate system that covered the measurement span. After the calibration was completed, a small object with the 5 reflective markers on top was placed onto the water surface within the test span. Checking was then performed to verify that the water body was sufficiently still by inspecting the motion of the markers for a minute or two. Any discernible movement would suggest the presence of residual current. After it was confirmed that there was no residual current, the wave generator was activated to start the test. During the test, the infra-red cameras captured the images of the reflective markers on the small objects at 50Hz. These images were processed to produce the displacement data for the entire process. The data were subsequently analyzed and plotted to reveal the drift behavior. To avoid a possible disturbance in the wave tank, there was a lapse of at least 15 minutes between runs.

6.2.3 Horizontal movement of a polyethylene plate

The effect of wave reflection was investigated for all runs. With wave period of 1.0 and 0.9 second in the experiments, the group velocity of the wave train was equal to 0.78 and 0.70 m/s, respectively. The distance from the test span to the end of wave flume was approximate 35 m. Therefore, it would take about 90 seconds for the wave train to cover the distance to-and-fro. Since the experimental duration in our experiments was 80 seconds for all runs, the wave reflection should not be present.

In the beginning of an experiment, the polyethylene plate was placed on the water surface in still water with its longitudinal axis parallel to wave propagation direction within a small tolerance as an absolute parallel was difficult. After the waves were generated, the polyethylene plate began to drift along the wave flume. Figure 6.3 illustrates how the motion of a polyethylene plate moved from a still position in the oscillating waves. The ellipse II with $L_g = 200$ mm, $L_t = 100$ mm and $\Delta = 30$ mm (see Table 6.1), in the figure was advected by waves with $ka = 0.08$ propagating from left to right. Observations showed that the longitudinal axis of the ellipse II kept parallel to the wave propagating direction during the entire run. Similar observations were spotted for plates with square and circular shapes.

Figure 6.3 Typical movement of ellipse2' (Table 6.1) tracked by a fixed video camera.
($t' = t/T$ is the non-dimensional time)

After each test, the trajectory of the horizontal displacement of the floating object was analysed by the motion monitoring system. An example is shown in Fig. 6.4. Without the effect of wave reflection, two stages are identified in the horizontal trajectory: during 0 ~ 25 seconds, the small object moved with a significant acceleration; thereafter, from 25 to 80 seconds, the motion achieved a quasi-steady state.

Figure 6.4 Time history of horizontal movement recorded by motion monitoring system

The constant drift velocity in the quasi steady state can be computed with two approaches. One is to obtain the mean velocity in one wave period within the quasi-steady state stage. The period-mean drift velocity can be calculated as the horizontal distance between two neighbor wave crests divided by one wave period. An example is shown in Fig. 6.5, corresponding to the trajectory in Fig. 6.4. Choosing a quasi-steady time interval, from 25 to 45 seconds, the average period-mean velocity calculated from Fig. 6.5 is 0.019 m/s. The other way is to linearize the trajectory in the quasi-steady stage, and then determine the slope of the best-fit linear trend line as the drift velocity. An example is shown in Fig. 6.6. After linearizing the trajectory from the 25 second to the 45 second, a drift velocity of 0.020 m/s is obtained. The drift velocities determined by both approaches are almost identical. Hence the drift velocities in this study are determined with the first approach only.

Figure 6.5 Drift velocity determined by wave crest

Figure 6.6 Drift velocity determined by the linear trend line

6.2.4 Experimental results comparing with Shen and Zhong (2001)

In the theoretical study of Shen and Zhong (2001), approximate analytical solutions were obtained for two extreme conditions: the added mass coefficient C_m is zero and the drag coefficient C_d is zero. With $C_m = 0$, the computed drift velocity was independent of the drag coefficient and equal to the Stokes drift. With $C_d = 0$, however, the analytical solution of the drift velocity was dependent on the initial conditions. For realistic cases in which both added mass coefficient and drag coefficient were non-zero, the drift was computed numerically. When the small object started from still (both velocity and acceleration are zero), Shen and Zhong's (2001) numerical results showed that the celerity-normalized drift velocity was always smaller than Stokes drift. This implies that the drift will be reduced with a finite added mass coefficient and drag coefficient. This present experimental data however showed a very different pattern to the theoretical results suggested by Shen and Zhong (2001).

The celerity-normalized drift velocity, u_d / c , was plotted against the wave slope ka for the various circular and elliptical shapes in Figures 6.7(a) and 6.7(b) respectively. Firstly, the present measurements were compared with Stokes drift. It was found that, for these realistic small objects with nonzero C_m and C_d , the experimental drift velocity was significantly faster than Stokes drift, which is exactly opposite to the predictions based on existing approaches such as Shen and Zhong (2001). This phenomenon can be seen more clearly in Fig. 6.8 which the Stokes-normalized drift velocity u_d / u_s is plotted against the wave slope. Another important observation in Fig. 6.7 is that, the celerity-normalized drift velocity shows a similar increasing trend to the Stokes drift.

Figure 6.7 Effect of wave slope on celerity-normalized drift velocity for different shapes of polyethylene plates: (a) Circle and EllipseI&II (b) Circle and Square

Figure 6.8 Effect of wave slope on Stokes-normalized drift velocity for different shapes of polyethylene plates: (a) Circle and Ellipse1&II (b) Circle and Square

In other words, the drift of small rigid floating object increases approximately with the square of the wave slope, similar to the Stokes drift.

6.3 Analysis

6.3.1 Description of the problem

For analysis we examine the motion of a small floating object riding on a train of regular waves with the water surface described by η as shown in Figure (6.9). Rather than adopting Shen and Zhong (2001) which represented the majority opinion in the literature in the development of the subject so far, we select a more recent approach that was proposed conceptually by Marchenko (1999) and Grotmaack and Meylan (2006) though they did not provide any verification in terms of experimental or field evidence. The motion of the floating object is observed in a coordinate system (x', z) which is moving at the wave phase velocity c . In this coordinate system, the water surface is stationary. The relation between the earth-fix coordinate x and x' is $x' = x - ct$. The kinematic surface boundary condition specifies that water particles on the surface can only slide along the water surface, i.e. the velocity normal to the water surface must be zero. The unit vectors normal and tangential to the water surface can be expressed as

$$\vec{n} = \frac{(-\frac{\partial \eta}{\partial x'}, 1)}{\sqrt{1 + (\frac{\partial \eta}{\partial x'})^2}}, \quad \vec{\tau} = \frac{(1, \frac{\partial \eta}{\partial x'})}{\sqrt{1 + (\frac{\partial \eta}{\partial x'})^2}} \quad (6.1)$$

Figure 6.9 A small floating object viewed in a moving coordinate system

Let the velocity of the object be $\vec{u}$ and the velocity of the water particle on the surface by $\vec{V}$. The tangential and normal velocities of the object are

$$u_n = \vec{n} \cdot \vec{u} = 0, \quad u_t = \vec{t} \cdot \vec{u} \quad (6.2)$$

Similarly, the tangential and normal velocities of the water particle are

$$V_n = \vec{n} \cdot \vec{V} = 0, \quad V_t = \vec{t} \cdot \vec{V} \quad (6.3)$$

In the following, the equations for the motion of the floating object will be derived.

The angle α in Figure (6.9) can be obtained

$$\tan(\alpha) = \frac{\partial \eta}{\partial x} \quad (6.4)$$

or

$$\alpha = \arctan\left(\frac{\partial \eta}{\partial x}\right) \quad (6.5)$$

6.3.2 Governing equations

We first assume that the pitching can be neglected. The heave (motion normal to the moving water surface) will always be important since if the object is sliding along the water surface, there will be a variable centrifugal force that cause heaving motion. If the pitching motion is ignored, a constant buoyancy force cannot balance the variable centrifugal force, i.e. there must be a heaving motion caused by the variable centrifugal force.

We shall now derive the equations for the case where heaving is kept but the pitching is ignored. The definition sketch is shown in Figure (6.10), where $\ell(t)$ is the variable submergence. The bottom of the floating object is assumed symmetrical about the surface passing through the axis $O\zeta$ with O being at the moving surface. As pitching is ignored, the bottom of the floating object is always parallel to the AB, which is tangent to the moving surface at point O .

Figure 6.10 Floating object with heaving

6.3.2.1 Equations of motion

Within the wave-moving coordinates, the equation governing the motion of this object is (here the heaving is included for the moment)

$$m(1+C_m)\left(\frac{d\vec{u}_\tau}{dt} + \frac{d\vec{u}_n}{dt}\right) = \vec{F}_\tau + \vec{F}_n \quad (6.6)$$

where $\vec{F}_\tau$ is the force tangential to the water surface and $\vec{F}_n$ the force normal to the water surface. Note that

$$\frac{d\vec{u}_\tau}{dt} = \frac{du_\tau}{dt} \vec{\tau} + u_\tau \frac{d\vec{\tau}}{dt} = \frac{du_\tau}{dt} \vec{\tau} + \omega\beta_\tau u_\tau \vec{n} \quad (6.7)$$

$$\frac{d\vec{u}_n}{dt} = \frac{du_n}{dt} \vec{n} + u_n \frac{d\vec{n}}{dt} = \frac{du_n}{dt} \vec{n} + \omega\beta_n u_n \vec{\tau} \quad (6.8)$$

where both β_τ and β_n are calculated by using η and change with time. Thus from equations (6.6), (6.7) and (6.8), we have

$$m(1+C_m)\left(\frac{du_\tau}{dt} + \omega\beta_n u_n\right) = F_\tau \quad (6.9)$$

and

$$m(1+C_m)\left(\frac{du_n}{dt} + \omega\beta_\tau u_\tau\right) = F_n \quad (6.10)$$

In view of the following identity

$$\frac{d\vec{e}}{dt} = \vec{\Omega} \times \vec{e} \quad (6.11)$$

By substituting into equation (6.5), we obtain:

$$\omega\beta_n = -\frac{\frac{d^2\eta}{dx^2}}{1 + \left(\frac{d\eta}{dx}\right)^2} \quad (6.12)$$

$$\omega\beta_\tau = \frac{\frac{d^2\eta}{dx^2}}{1 + \left(\frac{d\eta}{dx}\right)^2} \quad (6.13)$$

The total force acting on the object includes the pressure force $\vec{P}$, the gravitational force $-m\vec{g}$ and the resistance force $\vec{R}$ due to viscosity. It then follows that the two components of the total force on the object can be written as

$$F_\tau = (\vec{P} - m\vec{g} + \vec{R}) \cdot \vec{\tau} = P_\tau - mg \frac{\frac{\partial\eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial\eta}{\partial x'}\right)^2}} + R_\tau \quad (6.14)$$

and

$$F_n = (\vec{P} - m\vec{g} + \vec{R}) \cdot \vec{n} = P_n - \frac{mg}{\sqrt{1 + \left(\frac{\partial\eta}{\partial x'}\right)^2}} + R_n \quad (6.15)$$

6.3.2.2 Pressure force

The total pressure on the moving water surface is the constant atmospheric pressure p_a . The total pressure acting on the side surface of the floating object of small thickness will be nearly the same as the atmospheric pressure p_a . If the pitching is ignored, the tangential component of the pressure force will be zero to the order considered in the present problem ($O(ka)$ where ka is the wave slope), i.e. $P_\tau = 0$.

The normal pressure force P_n is simplified as

$$P_n = \rho g \frac{-\ell}{\sqrt{1 + \left(\frac{\partial\eta}{\partial x}\right)^2}} A_b \quad (6.16)$$

where A_b is the area of the bottom surface, see Figure (6.10).

Eq. (6.16) can also be converted to:

$$P_n = \rho g \Delta A_b \frac{-\ell / \Delta}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x}\right)^2}} = mg \frac{-\ell / \Delta}{s \sqrt{1 + \left(\frac{\partial \eta}{\partial x}\right)^2}} \quad (6.17)$$

where $s = \frac{\rho_s}{\rho}$, ρ_s is the density of the small object.

6.3.2.3 Resistance force

The two components of the resistance force are modelled by the following equations

$$R_\tau = \rho C_{dx} \cdot A_w |V_\tau - u_\tau| (V_\tau - u_\tau) \quad (6.18)$$

where A_w is the wetted area and C_{dx} the drag coefficient in x direction.

The normal resistance force is expressed as

$$R_n = \rho C_{dz} A_{wx} |V_n - u_n| (V_n - u_n) \quad (6.19)$$

where C_{dz} the drag coefficient in z direction, $A_{wx} = A_s |\ell| / \Delta$ is the area of the wetted side surface. Here, A_s is the area of the side surface.

R_n and R_τ can also be expressed as

$$R_\tau = \rho C_{dx} A_w |V_\tau - u_\tau| (V_\tau - u_\tau) = m C_{dx} \frac{A_w}{s \Delta A_b} |V_\tau - u_\tau| (V_\tau - u_\tau) \quad (6.20)$$

where A_b is the area of the bottom surface. Note that $A_w = A_{wx} + A_b$, we have

$$\begin{aligned}
 R_\tau &= mC_{dx} \frac{1 + A_{vx} / A_b}{s\Delta} |V_\tau - u_\tau| (V_\tau - u_\tau) \\
 &= mC_{dx} \frac{1 + (A_s / A_b) (|\ell| / \Delta)}{s\Delta} |V_\tau - u_\tau| (V_\tau - u_\tau)
 \end{aligned} \tag{6.21}$$

and

$$\begin{aligned}
 R_n &= \rho_s C_{dz} A_s \frac{|\ell|}{s\Delta} |V_n - u_n| (V_n - u_n) \\
 &= mC_{dz} \frac{A_s}{\Delta A_b} \frac{|\ell|}{s\Delta} |V_n - u_n| (V_n - u_n)
 \end{aligned} \tag{6.22}$$

6.3.2.4 Equations in dimensionless form

In summary, the equations of motion can be written as

$$\begin{aligned}
 m(1 + C_m) \left(\frac{du_\tau}{dt} + \omega \beta_n u_n \right) &= -mg \frac{\frac{\partial \eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'} \right)^2}} \\
 + mC_{dx} \frac{1 + (A_s / A_b) (|\ell| / \Delta)}{s\Delta} |V_\tau - u_\tau| (V_\tau - u_\tau)
 \end{aligned} \tag{6.23}$$

and

$$\begin{aligned}
 m(1 + C_m) \left(\frac{du_n}{dt} + \omega \beta_\tau u_\tau \right) &= mg \frac{-\ell / \Delta}{s \sqrt{1 + \left(\frac{\partial \eta}{\partial x} \right)^2}} \\
 - \frac{mg}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'} \right)^2}} + mC_{dz} \frac{A_s}{s\Delta A_b} \frac{|\ell|}{\Delta} |V_n - u_n| (V_n - u_n)
 \end{aligned} \tag{6.24}$$

It is convenient now to make the equations dimensionless by choosing characteristic scales for all the variables in the problem. We scale the length by the wave length L , the time by the wave period T and the speed by the wave celerity $c = L/T$.

When heaving is considered and pitching ignored, the pressure forces will be normal to the moving surface. In this case, the equations governing u_x and u_n are

$$(1 + C_m) \left(\frac{du_x}{dt} + 2\pi\beta_n u_n \right) = - \frac{2\pi}{\tanh(kd)} \frac{\frac{\partial \eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'}\right)^2}} + \frac{C_{dx}}{s(\Delta/L)} \left(1 + \frac{A_s}{A_b} \frac{|\ell|}{\Delta/L} \right) |V_x - u_x| (V_x - u_x) \quad (6.25)$$

$$\frac{dx'}{dt} = \frac{u_x}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'}\right)^2}} \quad (6.26)$$

And

$$(1 + C_m) \left(\frac{du_n}{dt} + 2\pi\beta_s u_s \right) = - \frac{2\pi}{\tanh(kd)} \frac{1}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'}\right)^2}} \left(1 - \frac{|\ell|}{s(\Delta/L)} \right) + C_{dz} \frac{A_s}{A_b} \frac{|\ell|}{s(\Delta/L)^2} |V_n - u_n| (V_n - u_n) \quad (6.27)$$

$$\frac{d\ell}{dt} = u_n, \quad s = \frac{\rho_s}{\rho} \quad (6.28)$$

where all the variables are dimensionless.

6.3.2.5 Simulation of one particle drift on the water surface

A particle on the surface with the same density as water is examined firstly by using the above model. Note that for this case there is no added mass force, drag force and heaving.

We examine a simple case where the surface profile of a train of deep water waves can be described by

$$\eta = a \sin(kx'), \quad \frac{\partial \eta}{\partial x'} = ka \cos(kx') \quad k = 2\pi / L \quad (6.29)$$

The wave dissipation can be considered later by allowing the wave amplitude a to vary slowly with x' since x' is the horizontal coordinate of the floating object and is a function of time t . The absolute location of the object x can then be recovered by

$$x = x' + t \quad (6.30)$$

from which the drift velocity can be calculated. The initial conditions are specified:

$$x' = 0, \quad u_x = -1 \quad \text{at } t = 0 \quad (6.31)$$

i.e. the wave orbital velocity at $t = x' = 0$ is zero for the given wave form, Equation (6.29). Within the move-coordinate, $u_x = -1$.

For deep water waves, it is renown that the surface Stokes drift is

$$\bar{u}_s = (ka)^2 \quad (6.32)$$

where $\bar{u}_s$ is the dimensionless Stokes drift velocity. Equations (6.25) and (6.27) can be solved with MATLAB function ODE45. The options for the ODE solver are: relative error= 10^{-10} ; initial step size= 10^{-7} . This controls the accuracy of the integrator.

Figure 6.11 shows a time history of the horizontal displacement x of the floating object for $ka = 0.1$. By fitting the displacement with a linear function, the dimensionless surface drift is obtained

$$\bar{u}_d = 0.01004 \quad (6.33)$$

which is the same as the Stokes drift. This example shows that the wave force is correctly modeled in equations (6.25) and (6.27).

Figure 6.11 Time history of x for $ka = 0.1$

6.3.2.6 Equations with drag force, added mass and damping considered

The dimensional equations for a small floating object were given in Marchenko (1999), who considered the drag force in his model. Grotmaack and Meylan (2006) advanced the approach further and considered both the drag force and the added mass. Note that both studies focused on the conceptual framework and no verification was performed. Here, we take the approach further by considering heaving of the small object as well as the presence of wave damping. Again, we normalize the length by the wave length L , time by wave period T and the velocity by the wave celerity $c = L/T$, the dimensionless momentum equation governing the small floating body can be written as

$$(1 + C_m) \frac{du_\tau}{dt} = -\frac{2\pi}{\tanh(kd)} \frac{\frac{\partial \eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'}\right)^2}} + \frac{C_{dv}}{s(\Delta/L)} \int_S |V_\tau - u_\tau| (V_\tau - u_\tau) dS / S_0 \quad (6.34)$$

All the variables in equation (6.34) are dimensionless. The drag force will depend on the shape and draft of the floating object. To perform the integration, a local coordinate system is needed. One thing to note is that equation (6.34) is derived by assuming that the object is sliding along the water surface.

For simplicity, we first assume that the drag force is uniform over the bottom of the floating object. The integration of the drag force in equation (6.34) can be given explicitly as

$$(1 + C_m) \frac{du_r}{dt} = -\frac{2\pi}{\tanh(kd)} \frac{\frac{\partial \eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'}\right)^2}} + \frac{C_{dx}}{s(\Delta/L)} |V_r - u_r| (V_r - u_r) \quad (6.35)$$

For convenience, we rewrite the above equation in the following form

$$\frac{du_r}{dt} = \alpha_1 \frac{\frac{\partial \eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'}\right)^2}} + \alpha_2 |V_r - u_r| (V_r - u_r) \quad (6.36)$$

$$\frac{dx'}{dt} = \frac{u_r}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'}\right)^2}} \quad (6.37)$$

$$\alpha_1 = -\frac{2\pi}{\tanh(kd)(1 + C_m)}, \quad \alpha_2 = \frac{C_{dx}}{s(\Delta/L)(1 + C_m)} \quad (6.38)$$

To simplify the problem, we assume that the object is introduced in such a way that the initial velocity of the object is the same as that of the wave orbital velocity.

Now we examine the relative importance of the gravity force and the drag force.

Suppose that the drag force is larger than the gravity force, we have

$$\frac{\alpha_2 |V_r - u_r| (V_r - u_r)}{\alpha_1 \frac{\partial \eta / \partial x'}{\sqrt{1 + (\partial \eta / \partial x')^2}}} = O\left(\frac{\Delta U^2 \alpha_2}{\alpha_1 ka}\right) > 1 \quad (6.39)$$

where ΔU is the scale of the velocity difference. It then follows that

$$\Delta U > \sqrt{2\pi \frac{ka}{C_{dx}} \frac{\Delta}{L}} \quad (6.40)$$

where (6.38) has been used.

If the object is going to follow the wave orbital motion and the drift motion, we expect that

$$\Delta U \ll u_s = O(ka)^2 = 0.01 \quad (6.41)$$

Thus it is expected that the drag force has only a second order effect on the floating object, except during the initial stage when the initial velocity specified is greatly different from the wave orbital velocity. Another point is that if the initial condition is not correctly specified, it may take quite a long time for the object to adjust to the velocity of the flow. However, as $\Delta U \rightarrow 0$ with time, the viscous effects will become weaker and weaker.

It is known that wave damping will impose a positive stress on the water surface and induce a mean current. For the experiments in a wave flume, both the bottom and the sidewalls contribute to wave damping. The damping coefficient can be calculated from explicit formula given in standard textbooks. We shall now estimate the importance of the wave damping in the present problem.

In the wave-moving coordinates, for monochromatic waves, the surface revelation can be written with damping as

$$\eta = \frac{H/L}{2} e^{-2\beta\pi x} \cos(2\pi x) \quad (6.42)$$

where $\beta \ll 1$ is the damping coefficient. It follows that

$$\frac{\partial \eta}{\partial x} = -\frac{H}{L} \pi e^{-2\beta\pi x} (\beta \cos(2\pi x) + \sin(2\pi x)) \quad (6.43)$$

Wave damping may change the gravitational force and the cumulative impact of the wave damping will be a change in the surface drift.

The damping due to the laminar bottom boundary layer can be represented by

$$\frac{H}{H_0} = e^{-2\beta\pi x}, \quad \beta = \frac{\sqrt{2\nu/\omega}/L}{2kd + \sinh(2kd)} \quad (6.44)$$

where x is normalized by wave length (see Dean and Dalrymple (1991), page 267).

For the laboratory condition, the above approximation is sufficient.

6.4 Comparing with the experimental drift of a circular plate

The dimensionless water depth kd , density ratio $s = \frac{\rho_s}{\rho}$, dimensionless thickness of the circular plate $\frac{\Delta}{L}$, damping coefficient β , added mass coefficient C_m and drag coefficients C_{dx} and C_{dz} need to be determined when calculating the theoretical drift of small floating objects.

Based on the experimental setup, the dimensionless water depth kd , the density ratio s , and the dimensionless thickness of the circular plate $\frac{\Delta}{L}$ are presented as follow:

$$kd = \frac{2\pi}{L} \cdot d \cong 2, \quad s = 0.96$$

$$\left(\frac{\Delta}{L}\right)_{\Delta=3cm} = 0.02, \quad \left(\frac{\Delta}{L}\right)_{\Delta=4.5cm} = 0.03$$

According to Eq. (6.44), it is not difficult to obtain the damping coefficient of the bottom boundary layer. In the current experiments, each submerged side wall of the wave flume had a similar size to the width of the bottom. The damping coefficient β in the simulation was studied in a reasonable range from three times of the bottom damping coefficient $\beta = 3 \cdot \frac{\sqrt{2\nu/\omega} / L}{2kd + \sinh(2kd)} \cong 0.00004$ to $\beta = 0.0002$ (shown in Table 6.2). The results showed that the damping coefficient in such a range has an insignificant effect on the simulation.

Table 6.2 Effect of damping coefficient on the simulation results

C_{dx}	C_{dz}	C_m	Δ/L	β	ka	u_s/c	u_d/c	u_d/u_s
0	0.5	0.15	0.02	0.00004	0.04	0.0016	0.0072	4.50
					0.06	0.0036	0.0120	3.33
					0.08	0.0064	0.0180	2.81
					0.10	0.0100	0.0250	2.50
					0.12	0.0144	0.0330	2.29
0	0.5	0.15	0.02	0.00002	0.04	0.0016	0.00729	4.55
					0.06	0.0036	0.0124	3.44
					0.08	0.0064	0.0186	2.90
					0.10	0.0100	0.0257	2.57
					0.12	0.0144	0.0337	2.34
0	0.5	0.15	0.03	0.00004	0.04	0.0016	0.0071	4.45
					0.06	0.0036	0.0120	3.33
					0.08	0.0064	0.0180	2.81
					0.10	0.0100	0.0250	2.50
					0.12	0.0144	0.0320	2.23
0	0.5	0.15	0.03	0.00002	0.04	0.0016	0.0071	4.45
					0.06	0.0036	0.0121	3.36
					0.08	0.0064	0.0180	2.81
					0.10	0.0100	0.0250	2.50
					0.12	0.0144	0.0324	2.25
0.015	0.5	0.15	0.02	0.00004	0.04	0.0016	0.00591	3.69
					0.06	0.0036	0.0095	2.64
					0.08	0.0064	0.0135	2.10
					0.10	0.0100	0.0180	1.80
					0.12	0.0144	0.0230	1.60
0.015	0.5	0.15	0.02	0.00002	0.04	0.0016	0.00597	3.73
					0.06	0.0036	0.0096	2.67
					0.08	0.0064	0.0137	2.14
					0.10	0.0100	0.0183	1.83
					0.12	0.0144	0.0235	1.63
0.015	0.5	0.15	0.03	0.00004	0.04	0.0016	0.00614	3.84
					0.06	0.0036	0.00994	2.76
					0.08	0.0064	0.0142	2.22
					0.10	0.0100	0.0190	1.90
					0.12	0.0144	0.0245	1.71
0.015	0.5	0.15	0.03	0.00002	0.04	0.0016	0.0062	3.87
					0.06	0.0036	0.0101	2.80
					0.08	0.0064	0.0145	2.27
					0.10	0.0100	0.0195	1.95
					0.12	0.0144	0.0250	1.74

The added-mass and drag coefficients of objects in a wave environment have been studied extensively. These previous studies involved either submerged or large-sized floating objects. So far, there is no effective method to evaluate the added-mass and drag coefficients of a small object floating on the water surface. Thus, the key point in

the drift calculation is to make the correct selection of the added-mass and drag coefficients.

The added mass coefficient C_m is always presented under wave conditions. Studies show that its value depends on both the Re and KC numbers. Its average values under various wave conditions are between 0.2 to 1.0, as described by Li and Teng (2002). However, the small object is in floating condition in the present investigation, which means the added mass coefficient C_m should be smaller than submerged condition. Thus, C_m should be less than 0.2 in the simulation.

Because the heaving is considered in the analysis, the drag coefficient is divided into x - and z -axis components, C_{dx} and C_{dz} respectively. In general, C_{dz} is regarded as a function of Re number (Li and Teng, 2002), but it does not change sharply, for a smooth circular cylinder, when the Re number varies within 10^2 - 10^5 . As for C_{dx} , it is regarded as a constant according to various authors (Teng and Li, 2001, Takagi et al., 2004, etc). For small floating objects, the drag coefficients are more complex. Because the small circular plate is floating on the surface, the drag coefficients should be smaller than those under submerged fixed cases. Moreover, C_{dz} should be larger than C_{dx} due to the heaving motion.

In summary, the parameters used in predicting the circular plate drift are shown in Table 6.3. During a particular simulation, C_m , C_{dx} and C_{dz} were kept the same as constants.

Table 6.3 Comparison of experimental and simulation results

C_{dx}	C_{dz}	C_m	Δ/L	β	ka	u_s/c	u_d/c	u_d/u_s	u_d/u_s (Exp.)	Diff.(%)
0	0.5	0.15	0.02	0.00004	0.04	0.0016	0.0072	4.50	3.01	50
					0.06	0.0036	0.0120	3.33	2.66	25
					0.08	0.0064	0.0180	2.81	2.14	31
					0.10	0.0100	0.0250	2.50	1.94	29
					0.12	0.0144	0.0330	2.29	1.90	21
0	0.5	0.15	0.03	0.00004	0.04	0.0016	0.0071	4.45	3.59	24
					0.06	0.0036	0.0120	3.33	2.84	17
					0.08	0.0064	0.0180	2.81	2.26	24
					0.10	0.0100	0.0250	2.50	2.11	18
					0.12	0.0144	0.0320	2.23	2.00	12
0.015	0.5	0.15	0.02	0.00004	0.04	0.0016	0.00591	3.69	3.01	22
					0.06	0.0036	0.0095	2.64	2.66	-1
					0.08	0.0064	0.0135	2.10	2.14	-2
					0.10	0.0100	0.0180	1.80	1.94	-7
					0.12	0.0144	0.0230	1.60	1.90	-15
0.015	0.5	0.15	0.03	0.00004	0.04	0.0016	0.00614	3.84	3.59	7
					0.06	0.0036	0.00994	2.76	2.84	-3
					0.08	0.0064	0.0142	2.22	2.26	-2
					0.10	0.0100	0.0190	1.90	2.11	-9
					0.12	0.0144	0.0245	1.71	2.00	-13

As shown in Table 6.3, the simulation results are sensitive to the change of C_{dx} . In general, the simulation results agree well with the experimental data when the C_{dx} is between 0 and 0.015. More detailed comparisons are given in Figs. 6.12 – 6.15. The experimental drift under higher wave slope looks larger than the simulation results. The main reason may be that the small circular plate drift is affected by the pitch, which may appear under large wave slope conditions.

Figure 6.12 u_d/c versus ka of the circular object ($\Delta/L=0.02$)

Figure 6.13 u_d/c versus ka of circular object ($\Delta/L=0.03$)

Figure 6.14 u_d / u_s versus ka of circular object ($\Delta/L=0.02$)

Figure 6.15 u_d / u_s versus ka of circular object ($\Delta/L=0.03$)

6.5 Discussion

Assuming small amplitude waves, the drift velocity of a small object modelled by a thin flat plate of an arbitrary shape was investigated experimentally under the deep water condition. The drift velocity increased from zero to a quasi-steady magnitude within a short time. The quasi-steady drift velocity was found to increase with the wave slope at a quadratic rate, and was higher than the Stokes drift in all cases. The comparison between the experimental results and theoretical prediction implies that, the theoretical models conceptualized in this study are reasonable to predict the surface drift of small objects. Furthermore, this method can be extended easily to waves in limited depth condition.

6.5.1 Discussion on numerical model

The reasons why previous models in the literature failed to predict the correct surface drift may be due to

1. Water particle velocity of the mean water level instead of the real surface was used in the initial condition and the drag formulation;
2. Wave damping was not considered in previous works;
3. The previous numerical runs did not have a sufficient time for the simulation.

6.5.2 Discussion on Figures 6.14 and 6.15

The oscillating velocity of the water particle can be expressed:

$$u = c(ka) \cos(kx - \omega t) \quad (6.45)$$

The governing equation in the tangential direction is Eq. (6.23):

$$m(1 + C_m) \left(\frac{du_x}{dt} + \omega \beta_n u_n \right) = -mg \frac{\frac{\partial \eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'} \right)^2}} + mC_{dx} \frac{1 + (A_s / A_b)(|\ell| / \Delta)}{s\Delta} |V_x - u_x| (V_x - u_x) \quad (6.46)$$

In the tangential direction,

$$-mg \frac{\frac{\partial \eta}{\partial x'}}{\sqrt{1 + \left(\frac{\partial \eta}{\partial x'} \right)^2}} \cong m \frac{dV_\tau}{dt} \quad (6.47)$$

Assuming $C_m = 0$ and substituting Eq. (6.47) into (6.46) yields:

$$\frac{d}{dt} (u_x - V_\tau) = \alpha \cdot |V_x - u_x| (V_x - u_x) - \omega \beta_n u_n \quad (6.48)$$

$$\text{where } \alpha = C_{dx} \frac{1 + (A_s / A_b)(|\ell| / \Delta)}{s\Delta}.$$

The relationship between the relative velocity and the water particle is plotted under both lower ($ka = 0.04$) and higher ($ka = 0.10$) wave steepness in Figure 6.16 with the parameters ($kd = 2$, $C_{dx} = 0.015$, $C_{dz} = 0.5$, $C_m = 0.15$, $\Delta/L = 0.02$, $\rho_s/\rho = 0.96$, $\beta = 0.00004$, $U_m = 0$). The figures show that the object velocity dividing by the water particle velocity is oscillating around 0.2 and the amplitude is approximate 0.7. It implies that the relative velocity consisted of a drift component and an oscillating velocity as:

$$\Delta U(t') = u_\tau(t') - V_\tau(t') = u_{diff} + A \sin(\omega t') \quad (6.49)$$

where t' is a relative time coordinate, $t' = t - t_0$. t_0 is a function of the geometric parameters and $dt' = dt$.

(a) $ka = 0.04$

(b) $ka = 0.10$

Figure 6.16 Plots of relative velocity against time

Integrating Eq. (6.48) in one wave period:

$$\begin{aligned}
 u_{diff} &\equiv \int_0^T \alpha |\Delta U| \Delta U dt' - \int_0^T \omega \beta_n u_n dt' \\
 &= \begin{cases} \alpha \cdot \int_0^T [u_{diff} + A \sin(\omega t')]^2 dt' & (\Delta U > 0) \\ \alpha \cdot \int_0^T -[u_{diff} + A \sin(\omega t')]^2 dt' & (\Delta U < 0) \end{cases} \\
 &\equiv \begin{cases} \alpha \cdot \frac{1}{2} A^2 \cdot T & (\Delta U > 0) \\ -\alpha \cdot \frac{1}{2} A^2 \cdot T & (\Delta U < 0) \end{cases} \quad (6.50)
 \end{aligned}$$

Scaling Eq. (6.50) by Stokes drift:

$$\frac{u_{diff}}{u_s} \cong \frac{\alpha}{2} \cdot A^2 (ka)^{-2} e^{-1} \cdot T \quad (6.51)$$

Simplifying Eq. (6.51) yields

$$\frac{u_{diff}}{u_s} \cong \frac{\alpha}{2} \cdot A^2 (ka)^{-2} \frac{T^2}{L} \quad (6.52)$$

For constant wave length L , the relationship between object drift and wave steepness can be plotted in Figure 6.17.

Figure 6.17 Plot of u_{diff}/u_s against ka

Thus for long waves whereby k is small while L and T large, the combined effect from Eq. (6.52) is that the non-dimensional difference in drift between the object and the water particle would increase. The trend shown in Figure 6.17 is observed and presented in Figs 6.14 and 6.15.

Chapter

7

CONCLUSIONS

7.1 Wave-induced drift of a surface oil patch

Some observations and measurements are reported in this study on the wave-induced drift of two-dimensional floating oil patches that were simulated using polyethylene sheets by both deep and shallow water waves. It was found that the drift velocity increased with the longitudinal length of the polyethylene sheet until the sheet length was approximately the same magnitude as the wavelength. Lengthening the sheet further did not lead to further significant changes in the drift velocity. For shallow water waves, the drift velocity decreased when the water depth increased. For bulk estimate purpose, it was concluded that the drift velocity increases linearly with the Ursell number in shallow water.

7.2 Dispersion due to wave drift

The dispersion due to wave drift under random wave conditions was investigated in two parts: (i) the first part was on the surface dispersive effect on a surface oil patch; and (ii) the second was the Taylor longitudinal dispersion of soluble contaminants.

7.2.1 Surface dispersion

On the analysis of surface dispersion, the Stokes drift velocity obtained in the frequency domain was found to be not only a mean value, but an instantaneous one as well, and hence the random fluctuations of drift velocity should not exist. Thus, it was concluded that there should not be any dispersion due to surface Stokes drift in random wave which is contrary to existing opinions in the literature. The experimental observations and measurements also reinforced the conclusion that the stochastic Stokes drift does not lead to a surface dispersive effect.

7.2.2 Taylor dispersion

For soluble contaminants, the Taylor dispersion due to wave drift by random waves was studied. Based on the analytical derivation, numerical results of dispersive coefficients for both Pierson-Moskowitz spectrum and Wen spectrum were obtained. The results show that the longitudinal dispersion is much more significant in random waves than in regular waves. The results based on the Wen spectrum also show that the Taylor dispersive effect in the intermediate water depth condition is generally stronger than in the deep water condition.

7.3 Wave-induced drift of a small floating object

Both experimental and analytical investigations on the drift of a small floating object were presented under the deep water wave condition. In the experimental part,

systematic laboratory measurements were performed to determine the drift of small floating objects in regular waves. Two key observations were made: first, in all experiments, the drift velocity of small objects increased quickly at the beginning and reached a quasi-steady mean value. Second, the drift velocity was found to increase with the wave slope at a quadratic rate but higher than the Stokes drift.

In the analytical analysis, a slope sliding model was established to describe the motion of small floating objects in waves. The model advances the current conceptual understanding in the literature and includes all relevant mechanisms of drag, damping and heave. A numerical solution for a circular shape object was then presented to compare with the experimental results. The comparison shows that, the theoretical model developed in this study is able to predict the surface drift of small objects. Furthermore, this model can be easily extended to waves in finite depth water condition.

Chapter

8

RECOMMENDATIONS

8.1 Wave-induced drift of a surface oil patch

The observations and measurements on the wave-induced drift of a surface floating patch showed that a quasi-steady drift velocity without acceleration was developed shortly after the waves arrived at the floating sheet. Analytical analysis on the boundary layer structure under the floating sheet is recommended in the future study to understand the mechanism that leads to the steady drift.

8.2 Dispersion due to wave drift

8.2.1 Surface dispersion

The present analysis is based on the assumption of inviscid and irrotational flow. The

waves in the experiments were small amplitude waves. However, in reality, it is more likely that turbulence would exist in the water domain. Further study should consider how the turbulence will affect the surface dispersion under waves.

8.2.2 Taylor dispersion

On the analysis of Taylor dispersion under random waves, it should be noted that, the depth-wise variation of the eddy diffusivity was not addressed as the diffusivity value was assumed to be a constant in space. In reality, the turbulent eddy diffusivity would likely vary with depth and this should be addressed in the future studies. In addition, control experiments should also be performed in the future to verify the analytical results.

8.3 Wave-induced drift of a small floating object

The experimental studies on the surface drift of a small object were carried out under small amplitude wave conditions. The comparison between the experimental data and the theoretical models conceptualized in this study implies that the present model gives reasonable predictions. However, the determination of the added-mass (C_m) and drag (C_d) coefficients are the most difficult issues. So far, only empirical data of the added-mass (C_m) and drag (C_d) coefficients for small fixed objects are available in the literature. Thus, experimental studies on these hydrodynamic coefficients are recommended for the further research on the drift motion of small floating objects.

REFERENCES

Alofs, D.J. and Reisbig, R.L. (1972) "An experimental evaluation of oil slick movement caused by waves", J. Phys. Oceanogr., Vol. 2, pp. 439-443.

Arikainen, A.I. (1972) "Wave drift of an isolated floe", AIDJEX Bulletin No. 16 (Translation of TRUDY Proceedings, AANII, Vol. 303, Leningrad), Division of Marine Resources, Univ. of Washington, Seattle, Wash., pp. 125-131.

Bagnold, R.A. (1947) "Sand movement by waves: some small-scale experiments with sand of very low density", J. Inst. Civil Engrs., Vol. 27, pp. 447-469.

Bretschneider, C.L. (1959) "Wave variability and wave spectra for wind-generated gravity waves", Beach Erosion Board, U.S. Army Corps. of Engineers, Tech. Mem., Vol. 118.

Buick, J.M., Morrison, I.G., Durrani, T.S. and Greated, C.A. (2001) "Particle diffusion on a three-dimensional random sea", Experiments in Fluids, Vol. 30, pp. 88-92.

Caligny, A.F.H. (1878) "Experiences sur les mouvements des molecules liquides des ondes courantes, considerees dans leur mode d'action sur la marche des navires", C.R. Acad. Sci., Paris, Vol. 87, pp. 1019-1023.

Cheema, P.S. and Ahuja, H.N. (1977) "Computer simulation of iceberg drift", Proceedings of the Offshore Technology Conference, Paper No. OTC 2951, Houston, Tex., May, 1977.

Cheng, N.S., Law, A.W.K. and Findikakis, A.N. (2003) "Oil transport in surf zone", Journal of Hydraulic Engineering, ASCE, Vol. 126, pp. 803-809.

Christensen, K.H. (2005) "Transient and steady drift currents in waves damped by surfactants", Phys. Fluids, Vol. 17, No. 4.

Christensen, K.H. and Weber, J.E. (2005) "Drift of an inextensible sheet caused by surface waves", Journal of Environmental Fluid Mechanics Vol. 5, pp. 495-505.

Cieslikiewicz, W. and Gudmestad, O.T. (1994) "Mass transport within the free surface zone of water waves", Wave Motion, Vol. 19, pp. 145-158.

Craig, P.D. and Banner, M.L. (1994) "Modeling wave-enhanced turbulence in the ocean surface layer", J. Phys. Oceanogr., Vol. 24, pp. 2546-2559.

Craik, A.D.D. (1982) "The drift velocity of water waves", Journal of Fluid Mech., Vol. 116, pp. 187-205.

Csanady, G.T. (1983) "Dispersion by randomly varying currents", Journal of Fluid Mech., Vol. 133, pp. 233-244.

Cummins, I. and Swan, C. (1995) "Vorticity effects in combined waves and currents", Coastal Engineering, Vol. 19, pp. 113-127.

Dean, R.G. and Dalrymple, R.A. (1991) Water wave mechanics for engineers and scientists, World Scientific Publishing, 353 pages.

Dempster, R.T. (1974) "The measurement and modeling of iceberg drift", Proceedings of IEEE Conference: Oceans'74, Vol. 1, Halifax, Nova Scotia, pp. 125-129.

Donelan, M.A., Anctil, F. and Doering, J.C. (1992) "A simple method for calculating the velocity field beneath irregular waves", Coastal Engineering, Vol. 16, pp. 399-424.

Dore, B.D. (1970) "Mass transport in layered fluid systems", Journal of Fluid Mech., Vol. 40, pp. 113-126.

Dore, B.D. (1973) "On mass transport induced by interfacial oscillations at a single frequency", Proc. Cambridge Phil. Soc., Vol. 74, pp. 333-347.

Dore, B.D. (1970) "Some effects of the air-water interface on gravity waves", Geophys. Astrophys. Fluid Dynamics, Vol. 10, pp. 215-230.

Dubreil-Jacotin, M.L. (1934) "Sur la détermination rigoureuse des ondes permanentes périodiques d'ampleur finie", J. Math. Pures Appl., Vol. 13, pp. 217-291.

Eckart, C. (1952) "The propagation of gravity waves from deep to shallow water", Nat'l Bur of Stds., Circular 521, Washington D.C., pp. 165-173.

El-Tahan, M.S. and El-Tahan, H.W. (1983) "Forecast of iceberg ensemble drift", Paper No. OTC-4460, 15th offshore Tech. Conf., Houston, Tex.

Fischer, H.B., List, E.J., Koh, R.C.Y., Imberger, J. and Brooks, N.H. (1979) Mixing of Inland and Coastal Waters, Academic Press, 483 pages.

Fredsoe, J. and Deigaard, R. (1992) Mechanics of coastal sediment transport, World Scientific Publishing, 369 pages.

Gaskill, H.S. and Rochester, J. (1984) "A new technique for iceberg drift prediction", Cold Regions Science and Technology, Vol. 8, pp. 223-234.

Gerstner F.J. von (1809) "Theorie der wellen" Ann. Phys., Vol. 32, pp. 412-440.

Giarrusso, C.C., Pugliese Carratelli, E. and Spulsi, G. (2001) "On the effects of wave drift on the dispersion of floating pollutants", Ocean Engineering, Vol. 28, pp. 1339-1348.

Gill, A.E. (1982) Atmosphere-Ocean Dynamics, Academic Press, New York.

Grotmaack, R. and Meylan, M.H. (2006) "Wave forcing of small floating bodies", Journal of Waterway, Port, Coastal, and Ocean Engineering, ASCE, Vol. 132 (3), pp. 192-198.

Gullenberg, G. (1982) "Physical Processes", Chapter 1 in Pollutant Transfer and Transport in the Sea, Vol. 1, edited by Gullenberg, G. CRC Press, pp. 1-89.

Gundlach, E.R., Boehm, P.D., Marchand, M., Atlas, R.M., Ward, D.M. and Wolfe, D.A. (1983) "The fate of Amoco Cadiz oil", Science, Vol. 221, pp.122-129.

Gwinn, A.W. and Jacobs, S.J. (1997) "Mass transport in viscous flow under a progressive water wave", Journal of Fluid Mech., Vol. 340, pp. 61-82.

Harms, V.W. (1987) "Steady wave-drift of modeled ice floes", Journal of Waterway, Port, Coastal, and Ocean Engineering, ASCE, Vol. 113 (6), pp. 606-622.

Hasselmann, K. (1973) "Measurements of wind wave growth and swell decay during the Joint North Sea Wave Project (JONSWAP)", Deutsches Hydrograph. Inst., Vol. 12, pps. 95.

Herterich, K. and Hasselmann, K. (1982) "The horizontal diffusion of tracers by surface waves", J. Phys. Oceanogr., Vol. 12, pp. 704-711.

Hoult, D.P. (1972) "Oil spreading on the sea", Annual Review of Fluid Mechanics, Vol. 4, pp. 341-368.

Hsu, T.W. and Ou, S.H. (1994) "On the mass transport of water waves in a turbulent boundary layer", Ocean Engineering, Vol. 21, pp. 195-206.

Huang, J.C. and Monastero, F.C. (1982) "Review of the state-of-the-art of oil spill simulation models", Final Rep. Submitted to the Am. Petr. Inst., Raytheon Ocean Systems Co., East Providence, R.I.

Huang, N.E., Long, S.R., Tung, C.C., Yuen, Y. and Bliven, L.F. (1981) "A unified two-parameter wave spectral model for a general sea state", Journal of Fluid Mech., Vol. 112, pp. 203-224.

Isaacson, M. (1979) "Wave-induced forces in the diffraction regime", In: Mechanics of Wave-Induced Forces on Cylinders, (Ed. T.L. Shaw), Pitman Advanced Publishing Program, pp. 68-89.

Jansons, K.M. and Lythe, G.D. (1998) "Stochastic stokes drift", Physical Review Letters, Vol. 81, pp. 3136-3139.

Kamata, M. (1982) "Oil slick behavior in waves", Ph.D. Thesis, Rice University, Houston, Texas, U.S.A.

Kang, K.H. and Lee, C.M. (1993) "Investigation of drift velocity of spilt oil in waves", In: Proceedings of the Second Japan-Korea Joint Workshop on Ship and Marine Hydrodynamics, Osaka, Japan, pp. 275-284.

Kang, K.H. and Lee, C.M. (1994) "Wave-induced mass transport in the boundary layer at an oscillating surface", In: Proceedings of KSME Spring Meeting, pp. 503-508.

Kang, K.H. and Lee, C.M. (1994) "On wave-induced drift of spilt oil", In: Proceedings of the Third JSME-KSME Fluids Engineering Conference, Sendai, Japan, pp. 420-425.

Kang, K.H. and Lee, C.M. (1995) "Steady streaming of viscous surface layer in waves", J. Marine Sci. Technology, Vol. 1, pp. 3-12.

Kang, K.H. and Lee, C.M. (1996) "Prediction of drift in a free surface", Ocean Engineering, Vol. 23, pp. 243-255.

King, C.A.M. (1948) Ph.D. Dissertation, Cambridge.

Laitone, E.V. (1961) "The second approximation to cnoidal and solitary waves", Journal of Fluid Mech., Vol. 9, pp. 430-444.

Laitone, E.V. (1962) "Limiting conditions for cnoidal and Stokes waves", Journal of Geophysical Research, Vol. 67, pp. 1555-1564.

Law, A.W.K. (1999) "Wave-induced surface drift of an inextensible thin film", Ocean Engineering, Vol. 26, pp. 1145-1168.

Law, A.W.K. (2000) "Taylor dispersion of contaminants due to surface waves", J. Hydraulic Research, Vol. 38, pp. 41-48.

Law, A.W.K. and Huang G. (2007) "Observations and measurements of wave-induced drift of surface inextensible film in deep and shallow waters", Ocean Engineering, Vol. 34, pp. 94-102.

Lee, C.M. and Kang, K.H. (2002) "Floating thin sheets in waves: drift force, drift velocity and wave damping", 21st International Conference on OMAE, OMAE2002-28473.

Li, Y.C. and Teng, B. (2002) Wave action on the maritime structures, 2nd Ed., The Ocean Press, Beijing, pp. 250-265 (in chinese).

Longuet-Higgins, M.S. (1952) "On the statistical distribution of the height of sea waves", Journal of Geophysical Research, Vol. 11, pp. 245-266.

Longuet-Higgins, M.S. (1953) "Mass transport in water waves", Phil. Trans. Roy. Soc. (London), Vol. A245, pp. 535-583.

Losada, I.J., Dalrymple, R.A. and Losada, M.A. (1993) "Water waves on crown breakwaters", Journal of Waterway, Port, Coastal, and Ocean Engineering, ASCE, Vol. 119 (4), pp. 367-380.

Marchenko, A. V. (1999) "The floating behavior of a small body acted upon by a surface wave", J. Appl. Maths, Vol. 63, No. 3, pp471-478.

Maksimenko, N.A. (1991) "Comparative analysis of Lagrangian statistical characteristics for synoptic-scale currents in hydro-physical study areas", Oceanology, Vol. 30, pp. 5-9.

Mesquita, O.N., Kane, S. and Gollub, J.P. (1992) "Transport by capillary waves: Fluctuating Stokes drift", Physical Review A, Vol. 45, pp. 3700-3705.

Mitsuyasu, H. (1980) "Observation of the power spectrum of ocean waves using a cloverleaf buoy", J. Phys. Oceanogr., Vol. 10(2), pp. 286-296.

Moscowitz, L. (1964) "Estimates of the power spectrums for fully developed seas for wind speeds of 20 to 40 knots", Journal of Geophysical Research, Vol. 69(24), pp. 5161-5179.

Mountain, D.G. (1980) "On predicting iceberg drift", Cold Regions Science and Technology, Vol. 1, pp. 273-282.

Neumann, G. (1953) "On ocean wave spectra and a new method of forecasting wind-generated sea", Beach Erosion Board, U.S. Army Corps. of Engineers, Tech. Mem., Vol. 43, pps. 42.

Phillips, O.M. (1977) The Dynamics of the Upper Ocean, 2nd ed. Cambridge University Press, UK.

Pierson, W.J. and Moscowitz, L. (1964) "A proposed spectral form for fully developed wind seas based on the similarity theory of S.A. Kitaigorodskii", Journal of Geophysical Research, Vol. 69(24), pp. 5181-5190.

Rankine, W. J. M. (1863) "On the exact form of waves near the surface of deep water", Phil. Trans. Roy. Soc. London, Vol. 153, pp. 127-138.

Rayleigh, Lord (1876) "On waves", Phil. Mag. (5), Vol. 1, pp. 257-279.

Rumer, R.R., Crissman, R. and Wake, A. (1979) "Ice transport in great lakes". Water Resour. and Envir. Engrg. Res. Rep. No. 79-3, State University of New York, Buffalo.

Sanderson, B.G. and Pal, B.K. (1990) "Patch diffusion computed from Lagrangian data, with application to the Atlantic equatorial undercurrent". Atmos Ocean, Vol. 28, pp. 444-465.

Scott, J.R. (1965) "A sea spectrum for model tests and long-term ship prediction". J. Ship Res., Vol. 9(13), pp. 145-152.

Shen, H.H. and Zhong, Y. (2001) "Theoretical study of drift of small rigid floating objects in wave fields". Journal of Waterway, Port, Coastal, and Ocean Engineering, ASCE, Vol. 127 (6), pp. 343-351.

Smith, R. (1982) "Contaminant dispersion in oscillatory flows". Journal of Fluid Mech., Vol. 114, pp. 379-398.

Smith, S.D. and Banke, E.G. (1981) "A numerical model of iceberg drift". Proceedings of the 6th International Conference on Port and Ocean Engineering under Arctic Conditions, Vol. 2, pp. 1001-1011, Quebec City, Quebec.

Smith, S.D. and Banke, E.G. (1982) "The influence of winds, currents and towing forces on the drift of icebergs". Cold Regions Science and Technology, Vol. 6, pp. 241-255.

Sobey, R.J. (1992) Physical processes in the transport of surface oil slicks, University of California, Berkeley report: 14 pages.

Sobey, R. J. and Barker, C. H. (1997) "Wave-driven transport of surface oil". Journal of Coastal Research, Vol. 13, pp. 490-496.

Sodhi, D.S. and El-Tahan, M. (1980) "Prediction of an iceberg drift trajectory during a storm". Annals of Glaciology, Vol. 1, pp. 77-82.

Sorensen, R.M. (1978) Basic coastal engineering, Wiley-Intersciences, New York.

Spaulding, M.L. (1988) "A state-of-the-art review of oil spill trajectory and fate modeling". Oil and Chemical Pollution, Vol. 4, pp. 39-55.

Squire, V.A. et. al (1995), "Of ocean waves and sea ice", Annu. Rev. Fluid Mech., Vol. 27, pp. 115-168.

Squire, V.A. (2007) "Of ocean waves and sea-ice revisited", Cold Regions Science and Technology, Vol. 49, pp. 110-133.

Stokes, G.G. (1847) "On the theory of oscillatory waves". Trans. Cambridge Phil. Soc., Vol. 8, pp. 441-455.

Stolzenbach, K.D., Madsen, O.S., Adams, E.E., Pollack, A.M. and Cooper, C.K. (1977) "A review and evaluation of basic techniques for predicting the behavior of surface oil slicks", Rep. No. 222, Dept. of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

Stuart, J.T. (1966) "Double boundary layers in oscillatory viscous flow". Journal of Fluid Mech. Vol. 24, pp. 673-687.

Sumer, B.M. (1997) Hydrodynamics around cylindrical structures, World Scientific, Singapore.

Takagi, T., Shimizu, T., Suzuki, K., Hiraishi, T. and Yamamoto, K. (2004) "Validity and layout of "NaLA": a net configuration and loading analysis system", Fisheries Research Vol. 66, pp. 235-243.

Taylor, G.I. (1953) "Dispersion of soluble matter in solvent flowing slowly through a tube", Proc. Roy. Soc. (London), Ser. A.219, pp. 186-203.

Teng, B. and Li, Y. C. (1991) "The force coefficients on inclined cylinder in random waves and current", Journal of Hydrodynamics B Vol. 3(3), pp. 34-36.

Teeson, D., White, F.M. and Schenck, H. (1970) "Studies of the simulation of drifting oil by polyethylene sheets", Ocean Engineering, Vol. 2, pp. 1-11.

Unluata, U. and Mei C.C. (1970) "Mass transport in water waves", Journal of Geophysical Research, Vol. 75, pp. 7611-7618.

Ursell, F. (1953) "Short surface waves due to an oscillating immersed body", Proc. Roy. Soc. (London), Ser. A.220, pp. 90-103.

Van Den Broeck, C. (1982) "A stochastic description of longitudinal dispersion in uniaxial flows", Physica A, Vol. 112, pp. 343-352.

Van Den Broeck, C. (1990) "Taylor dispersion revisited", Physica A, Vol. 168, pp. 677-696.

Wadhams, P. (1983) "A mechanism for the formation of ice edge bands", Journal of Geophysical Research, Vol. 88 (C5), pp. 2813-2818.

Watson, E.J. (1983) "Diffusion in oscillatory pipe flow", Journal of Fluid Mech., Vol. 133, pp. 233-244.

Weber, J.E. (2001) "Virtual wave stress and mean drift in spatially damped surface waves", Journal of Geophysical Research, Vol. 106(C6), pp. 11653-11657.

Wen, S.C., Zhang, D.C., Guo, P. and Chen, B. (1994) "Form of deep-water wind-wave frequency spectrum (I): Derivation of spectrum", Progress in Natural Science, Vol. 4, pp. 407-427.

Wen, S.C., Guan, C.L., Sun, S.C., Wu, K.J. and Zhang, D.C. (1996) "Effect of water depth on wind-wave frequency spectrum (I): Spectral form", Chinese Journal of Oceanology and Limnology, Vol. 14(2), pp. 97-105.

Wong, P.C.Y. and Law, A.W.K. (2003) "Wave-induced drift of an elliptical surface film", Ocean Engineering, Vol. 30, pp. 413-436.

Wood, I.R., Bell, R.G. and Wilkinson, D.L. (1993) Ocean disposal of wastewater, World Scientific, 425 pages.

Yasuda, H. (1984) "Longitudinal dispersion of matter due to the shear effect of steady and oscillatory currents", Journal of Fluid Mech., Vol. 148, pp. 383-403.

Yeh, H.H. (1991) "Vorticity-generation mechanisms in bores", Proc. Roy. Soc. (London), Ser. A.432, pp. 215-231.

Yu, Y.X. (2000) Random Wave and Its Applications for Engineering, Dalian University of Technology Press, China.

APPENDICES

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APPENDIX A

EXPERIMENTAL RESULTS

ON SURFACE DISPERSION

Table A.1 The 1st test for $H_s=0.03\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	-0.01
2	0	54	0.01
4	0	56	0
6	0.02	58	-0.01
8	0.03	60	0.03
10	0.01	62	0.04
12	0.01	64	-0.01
14	0.02	66	0.04
16	0.03	68	0
18	0.02	70	0.02
20	0	72	-0.02
22	0.03	74	0.01
24	0	76	0.02
26	0.01	78	0.01
28	0.02	80	0
30	-0.01	82	-0.02
32	-0.02	84	-0.03
34	-0.02	86	0
36	0	88	0.02
38	-0.01	90	0.01
40	-0.02	92	0.01
42	0	94	0
44	0	96	0
46	0.01	98	0.01
48	0.02	100	0.02
50	0

Fig. A.1 Change of relative distance ($H_s=0.03\text{m}$)

Table A.2 The 2nd test for $H_s=0.03\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	-0.01
2	0	54	0
4	0.01	56	0.01
6	-0.01	58	0.01
8	-0.01	60	-0.02
10	0	62	0
12	-0.02	64	-0.01
14	0	66	0
16	-0.02	68	0.01
18	0	70	-0.02
20	-0.02	72	0.02
22	-0.01	74	-0.02
24	0	76	-0.02
26	0	78	0
28	0.01	80	-0.04
30	-0.03	82	-0.03
32	0	84	-0.02
34	0.01	86	0.02
36	-0.02	88	-0.01
38	0.02	90	0
40	-0.02	92	-0.01
42	0	94	-0.01
44	-0.02	96	0
46	0.01	98	0.02
48	0.02	100	0
50	-0.02

Fig. A 2 Change of relative distance ($H_s=0.03\text{m}$)

Table A.3 The 3rd test for $H_s=0.03\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	0.01
2	0	54	-0.05
4	0	56	-0.03
6	0.01	58	-0.03
8	0	60	-0.04
10	-0.01	62	-0.01
12	-0.01	64	-0.05
14	-0.01	66	-0.03
16	0	68	0
18	-0.01	70	0
20	0	72	0
22	-0.01	74	0.01
24	-0.02	76	0
26	-0.02	78	0.02
28	-0.03	80	0
30	-0.01	82	-0.01
32	-0.03	84	0
34	-0.02	86	0.02
36	-0.01	88	0.01
38	-0.02	90	0
40	0.01	92	-0.01
42	0	94	0
44	-0.02	96	0
46	-0.01	98	0
48	-0.02	100	0.02
50	-0.02

Fig. A 3 Change of relative distance ($H_s=0.03\text{m}$)

Table A.4 The 1st test for $H_s=0.04\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	0.01
2	0	54	-0.02
4	0	56	0
6	0.01	58	0.03
8	-0.01	60	-0.02
10	-0.02	62	-0.02
12	0	64	0
14	-0.02	66	0
16	0	68	-0.01
18	-0.02	70	-0.01
20	0	72	0
22	0.01	74	0
24	0.02	76	-0.03
26	-0.02	78	0
28	-0.02	80	0.02
30	0	82	0
32	0	84	0
34	0	86	-0.01
36	0.01	88	0
38	-0.01	90	0.02
40	-0.02	92	0
42	-0.01	94	-0.01
44	-0.03	96	0
46	-0.02	98	-0.01
48	0	100	-0.02
50	0

Fig. A.4 Change of relative distance ($H_s=0.04\text{m}$)

Table A.5 The 2nd test for $H_s=0.04\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	0.01
2	0	54	0
4	0	56	-0.01
6	0.01	58	0
8	0	60	0
10	-0.01	62	-0.03
12	-0.01	64	-0.02
14	0.01	66	0
16	-0.02	68	0
18	0.01	70	-0.02
20	0	72	-0.01
22	0.01	74	-0.02
24	-0.01	76	-0.04
26	-0.03	78	-0.01
28	0.02	80	0
30	0	82	0
32	0.01	84	0
34	0.01	86	-0.02
36	0.01	88	0
38	0.01	90	0
40	-0.01	92	-0.01
42	0.01	94	0
44	0	96	0
46	-0.02	98	-0.02
48	0	100	0
50	-0.02

Fig. A 5 Change of relative distance ($H_s=0.04\text{m}$)

Table A.6 The 3rd test for $H_s=0.04\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	0
2	0	54	0.02
4	-0.02	56	0.02
6	-0.02	58	-0.03
8	-0.01	60	0
10	-0.02	62	0
12	-0.04	64	-0.01
14	-0.02	66	0
16	-0.03	68	0.01
18	-0.04	70	-0.01
20	-0.01	72	-0.01
22	-0.02	74	-0.03
24	-0.01	76	0
26	-0.03	78	0
28	-0.01	80	-0.01
30	0	82	-0.02
32	0	84	0
34	-0.01	86	0
36	-0.01	88	-0.03
38	0	90	-0.02
40	0	92	-0.03
42	0.02	94	-0.03
44	0.02	96	-0.03
46	-0.01	98	-0.03
48	0	100	-0.02
50	-0.01

Fig. A.6 Change of relative distance ($H_s=0.04\text{m}$)

Table A.7 The 1st test for $H_s=0.05\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	-0.01
2	0	54	0
4	0	56	0
6	-0.01	58	-0.01
8	0.01	60	0.02
10	0	62	0
12	0.01	64	0.02
14	0	66	0.01
16	-0.01	68	-0.02
18	0	70	-0.03
20	0.01	72	-0.02
22	-0.03	74	-0.02
24	-0.04	76	-0.02
26	-0.01	78	-0.03
28	0.01	80	-0.02
30	-0.01	82	-0.03
32	0	84	0
34	0	86	-0.02
36	0	88	-0.01
38	0	90	-0.01
40	0	92	-0.01
42	0	94	-0.02
44	-0.02	96	-0.03
46	0.01	98	-0.02
48	-0.01	100	-0.03
50	-0.02

Fig. A 7 Change of relative distance ($H_s=0.05\text{m}$)

Table A.8 The 2nd test for $H_s=0.05\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	-0.01
2	0	54	-0.03
4	-0.02	56	-0.01
6	-0.03	58	-0.03
8	-0.02	60	-0.01
10	-0.01	62	0
12	0	64	-0.01
14	-0.01	66	0
16	0	68	-0.03
18	-0.01	70	0
20	0	72	0
22	0	74	-0.01
24	0	76	0
26	-0.02	78	-0.04
28	-0.01	80	-0.01
30	0	82	0
32	0	84	0.01
34	0.01	86	0.01
36	-0.02	88	-0.01
38	0	90	0
40	-0.01	92	0
42	-0.02	94	0
44	0	96	0
46	0	98	0
48	0.01	100	-0.02
50	-0.01

Fig. A.8 Change of relative distance ($H_s=0.05\text{m}$)

Table A.9 The 3rd test for $H_s=0.05\text{m}$

Time (s)	Change of Relative Distance (m)	Time (s)	Change of Relative Distance (m)
0	0	52	0.02
2	0	54	0.02
4	0	56	0
6	-0.01	58	-0.01
8	0	60	0
10	0.01	62	0
12	-0.01	64	-0.01
14	-0.01	66	0
16	-0.03	68	0.01
18	0	70	0
20	0	72	0.02
22	0	74	0.01
24	-0.02	76	0.01
26	-0.05	78	0.01
28	0	80	0.01
30	0.01	82	-0.01
32	-0.01	84	0.01
34	0.01	86	0.01
36	-0.01	88	0.01
38	0.01	90	-0.01
40	0	92	0.01
42	0.01	94	0.01
44	0.02	96	-0.01
46	0.01	98	0.03
48	0.01	100	0.05
50	0.02

Fig. A.9 Change of relative distance ($H_s=0.05\text{m}$)

APPENDIX B

MATHEMATICAL DERIVATION ON SURFACE DISPERSIVE COEFFICIENTS UNDER RANDOM WAVES WITH HERTERICH AND HASSELMANN'S (1982) THEORY

Based on the classical theory of Einstein (1905) and Taylor (1921) on diffusion by random continuous motion, Herterich and Hasselmann (1982) pointed out a particle with velocity $u_i(t)$ ($i=1,2,3$) undergoing statistically stationary fluctuations generally follows a random-walk path $x_i(t)$ which is characterized by non-stationary statistics. For large time t , both the mean and variance of the displacement increase linearly with time:

$$\langle x_i \rangle = \langle u_i \rangle t \quad (\text{A.1})$$

$$\langle (x_i - \langle x_i \rangle)(x_j - \langle x_j \rangle) \rangle = 2D_{ij}t \quad (\text{A.2})$$

where $x_i - \langle x_i \rangle$ is the fluctuation of a random-walk path, the angle brackets denote the average of a large ensemble.

In the x direction:

$$2D_{xx}t = (\Delta r_x)(\Delta r_x) \quad (\text{A.3})$$

where $\Delta r_x = x_i - \langle x_i \rangle$ is fluctuation of random walk.

The dispersive coefficient in 3-D random wave condition was formulated by Herterich and Hasselmann (1982) as follows:

$$D_{xx} = \frac{\pi}{4g^2} \int_0^\infty d\omega \omega^6 s^2(\omega) \int_{-\pi}^\pi d\theta_1 G(\theta_1) \int_{-\pi}^\pi d\theta_2 G(\theta_2) [1 + \cos(\theta_1 - \theta_2)]^2 (\cos \theta_1 + \cos \theta_2)^2 \quad (\text{A.4})$$

When the waves only travel in one direction such as in a wave flume, the spreading functions $G(\theta_1)$ and $G(\theta_2)$ will become δ -function:

$$G(\theta_1) = \begin{cases} \infty & \theta_1 = 0 \\ 0 & \theta_1 \neq 0 \end{cases} \quad \text{and} \quad \int_{-\pi}^{\pi} G(\theta_1) d\theta_1 = 1 \quad (\text{A.5})$$

$$G(\theta_2) = \begin{cases} \infty & \theta_2 = 0 \\ 0 & \theta_2 \neq 0 \end{cases} \quad \text{and} \quad \int_{-\pi}^{\pi} G(\theta_2) d\theta_2 = 1 \quad (\text{A.6})$$

In order to solve (A.4), part of this equation is expanded:

$$\begin{aligned} & \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) [1 + \cos(\theta_1 - \theta_2)]^2 (\cos \theta_1 + \cos \theta_2)^2 \\ &= \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) (1 + 2 \cos \theta_1 \cos \theta_2 + 2 \sin \theta_1 \sin \theta_2 + \cos^2 \theta_1 \cos^2 \theta_2 + \sin^2 \theta_1 \sin^2 \theta_2 \\ &+ 2 \sin \theta_1 \cos \theta_1 \sin \theta_2 \cos \theta_2) (\cos^2 \theta_1 + 2 \cos \theta_1 \cos \theta_2 + \cos^2 \theta_2) \\ &= \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) [(\cos^2 \theta_1 + 2 \cos \theta_1 \cos \theta_2 + \cos^2 \theta_2) \\ &+ (2 \cos^3 \theta_1 \cos \theta_2 + 4 \cos^2 \theta_1 \cos^2 \theta_2 + 2 \cos \theta_1 \cos^3 \theta_2) \\ &+ (2 \sin \theta_1 \cos^2 \theta_1 \sin \theta_2 + 4 \sin \theta_1 \cos \theta_1 \sin \theta_2 \cos \theta_2 + 2 \sin \theta_1 \sin \theta_2 \cos^2 \theta_2) \\ &+ (\cos^4 \theta_1 \cos^2 \theta_2 + 2 \cos^3 \theta_1 \cos^3 \theta_2 + \cos^2 \theta_1 \cos^4 \theta_2) \\ &+ (\sin^2 \theta_1 \cos^2 \theta_1 \sin^2 \theta_2 + 2 \sin^2 \theta_1 \cos \theta_1 \sin^2 \theta_2 \cos \theta_2 + \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \theta_2) \\ &+ (2 \sin \theta_1 \cos^3 \theta_1 \sin \theta_2 \cos \theta_2 + 4 \sin \theta_1 \cos^2 \theta_1 \sin \theta_2 \cos^2 \theta_2 + 2 \sin \theta_1 \cos \theta_1 \sin \theta_2 \cos^3 \theta_2) \end{aligned}$$

Calculating the above integral item by item:

$$I_1 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^2 \theta_1 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos^2 \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) = 1$$

$$I_2 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \cos \theta_1 \cos \theta_2 = 2 \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos \theta_2 = 2$$

$$I_3 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^2 \theta_2 = 1$$

$$I_4 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \cos^3 \theta_1 \cos \theta_2 = 2 \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos^3 \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos \theta_2 = 2$$

$$I_5 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 4 \cos^2 \theta_1 \cos^2 \theta_2 = 4 \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos^2 \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^2 \theta_2 = 4$$

$$I_6 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \cos \theta_1 \cos^3 \theta_2 = 2 \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^3 \theta_2 = 2$$

$$I_7 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \sin \theta_1 \cos^2 \theta_1 \sin \theta_2 = 0$$

$$I_8 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 4 \sin \theta_1 \cos \theta_1 \sin \theta_2 \cos \theta_2 = 0$$

$$I_9 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \sin \theta_1 \sin \theta_2 \cos^2 \theta_2 = 0$$

$$I_{10} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^4 \theta_1 \cos^2 \theta_2 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos^4 \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^2 \theta_2 = 1$$

$$I_{11} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \cos^3 \theta_1 \cos^3 \theta_2 = 2 \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos^3 \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^3 \theta_2 = 2$$

$$I_{12} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^2 \theta_1 \cos^4 \theta_2 = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \cos^2 \theta_1 \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \cos^4 \theta_2 = 1$$

$$I_{13} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \sin^2 \theta_1 \cos^2 \theta_1 \sin^2 \theta_2 = 0$$

$$I_{14} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \sin^2 \theta_1 \cos \theta_1 \sin^2 \theta_2 \cos \theta_2 = 0$$

$$I_{15} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) \sin^2 \theta_1 \sin^2 \theta_2 \cos^2 \theta_2 = 0$$

$$I_{16} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \sin \theta_1 \cos^3 \theta_1 \sin \theta_2 \cos \theta_2 = 0$$

$$I_{17} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 4 \sin \theta_1 \cos^2 \theta_1 \sin \theta_2 \cos^2 \theta_2 = 0$$

$$I_{18} = \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) 2 \sin \theta_1 \cos \theta_1 \sin \theta_2 \cos^3 \theta_2 = 0$$

So, the total value is:

$$\int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) [1 + \cos(\theta_1 - \theta_2)]^2 (\cos \theta_1 + \cos \theta_2)^2 = \sum_{k=1}^{18} I_k = 16 \quad (\text{A.7})$$

The dispersion coefficient on 2-D condition can be obtained as:

$$\begin{aligned} D_{xx} &= \frac{\pi}{4g^2} \int_0^{\infty} d\omega \omega^6 s^2(\omega) \int_{-\pi}^{\pi} d\theta_1 G(\theta_1) \int_{-\pi}^{\pi} d\theta_2 G(\theta_2) [1 + \cos(\theta_1 - \theta_2)]^2 (\cos \theta_1 + \cos \theta_2)^2 \\ &= \frac{\pi}{4g^2} \int_0^{\infty} d\omega \omega^6 s^2(\omega) \cdot 16 \\ &= \frac{4\pi}{g^2} \int_0^{\infty} d\omega \omega^6 s^2(\omega) \end{aligned} \quad (\text{A.8})$$

Substituting the P-M spectrum $s(\omega) = \frac{0.78}{\omega^5} \exp(-\frac{3.11}{\omega^4 H_s^2})$ into the above equation:

$$D_{xx} = \frac{4\pi}{g^2} \int_0^{\infty} 0.78^2 \omega^{-4} \exp(-\frac{6.22}{\omega^4 H_s^2}) d\omega \quad (\text{A.9})$$

Given the significant wave height H_s , the dispersive coefficient can be solved numerically by Eq. (A.9). The change of relative distance in time $t=100\text{s}$ calculated by (A.3) is:

$$(\Delta r_x)_{H_v=0.03m}=8cm$$

$$(\Delta r_x)_{H_v=0.04m}=10cm$$

$$(\Delta r_x)_{H_v=0.05m}=12cm$$