Optical bistability in nonlinear periodic structures : from bragg gratings to photonic crystal

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OPTICAL BISTABILITY IN PERIODIC STRUCTURES: FROM BRAGG GRATINGS TO PHOTONIC CRYSTAL

Submitted by:
Yosia

Supervised by:
Prof. Shum Ping (NTU)
Prof. Lu Chao (A*STAR[†])

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SINGAPORE

[†]Prof. Lu Chao is currently with Hong Kong Polytechnic University

Summary

In the era of advanced communications technology, the demand for bandwidth is ever increasing due to the larger number of users and the expectation for a better Quality of Services (QoS). In present communications network, optical signal still have to experience electronic conversion for processing as it propagates from the transmitter to the receiver. The conversion to electrical signal introduces speed bottleneck, and therefore results in low QoS from user's point of view. To overcome bottleneck of bandwidth requirements, the signal is expected to remain in optical domain as it propagates from transmitter to receiver without experiencing conversion to electrical domain. Consequently, there is a need to do signal processing purely in optical domain to enable all-optical technology.

The nonlinear periodic structures (photonics band-gap materials) are capable of producing plethora of optical phenomena to perform all-optical signals processing functionalities such as all-optical memory, switching, limiter, transistor, optical pulse compression, flip-flop, buffering, and even optical sequential logic device to synchronize optical signal and its clock. In parallel to the progress of the research in the fields of nonlinear periodic structure, new optical phenomena in nonlinear periodic structures to perform some optical signal processing capabilities are reported in this thesis, especially optical bistability. Some of all-optical signal processing functionalities can be derived from optical bistability such as: all-optical flip-flop, memory, transistor, and switching operations.

Time domain simulations to investigate the dynamic nonlinear behavior of multiple wave interactions in the Bragg gratings based on optical bistability principle are reported. Depending on the Full Width Half Maximum (FWHM) and peak intensity of the Gaussian pump, three unique permanent switching behaviors can be classified and function as step-up all optical switching, all-optical inverter, and all-optical limiter respectively.

Investigating materials with higher order nonlinearities, set of cubic-quintic Nonlinear Coupled Mode Equations (CQNLCMEs) is derived analytically to govern the wave propagation in the cubic-quintic grating (CQG). Pseudoatylene Tuloene Sulfonate (PTS) crystal with Bragg grating serves as the base material to represent the cubic-quintic Grating. Unlike conventional silica Fiber-Bragg Grating that only has one bistability in its transmission characteristics, *Double Optical Bistability* was found in a certain detuning range for PTS crystal with π phase-shifted Bragg grating. Similarly, the set of cubic-quintic-septic Nonlinear Coupled Mode Equations (CQSNLCMEs) is derived analytically to govern the wave propagation in chalcogenide-Bragg

grating by taking into account 3rd, 5th, and 7th order nonlinearities simultaneously that present in chalcogenide-glass.

One application of optical bistability is all-optical transistor which is demonstrated experimentally in Nonlinear DFB InGaAsP-InP waveguide with Bragg reflectors.

Next, optical bistability in Al_xGaAs Photonic Crystal are presented. Two Aluminum contents were used in the experiments: 18% and 26%. The results of optical bistability in the two aluminum content were compared in terms of their performances. Method to distinguish various origins of nonlinearity in optical bistability is also presented.

Lastly, Critical Slowing Down (CSD) phenomena from nonlinear InGaAsP-InP Photonics Crystal resonator are presented experimentally. The hallmark of CSD is the occurrence of long unstable state in the transmission artifacts before switching.

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[†] Prof. Lu Chao is currently with Hong Kong Polytechnic University

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CHAPTER 1: INTRODUCTION

Chapter 1

Introduction

1.1 Motivational Background

In the era of advanced communications technology, the users' demand for bandwidth is continuously increasing due to larger number of users and expectation for a better Quality of Services (QoS). In the present communications technology, optical signal still have to experience the electronic conversion for processing as it propagates from the transmitter to the receiver. The conversion to electrical signal from optical signal in every regenerator produces bandwidth bottleneck, and therefore results in low QoS from the user point of view.

To overcome this bandwidth bottleneck requirement, the signal is expected to remain in optical domain as it propagates from transmitter to receiver without experiencing electrical conversion. Consequently, there is a need to do signal processing purely in optical domain to enable the future all-optical technology.

The nonlinear periodic structures (photronics band-gap and resonators) are capable of producing plethora of optical phenomena to perform all-optical signals processing functionalities such as all-optical memory, switching, limiter, optical pulse compression, flip-flop, optical diode and even optical sequential logic device to synchronize optical signal and its clock [1.1-1.10], which has been shown theoretically and experimentally. In parallel to the progress of the research in the fields of nonlinear periodic structures, some new optical phenomena and their usefulness to perform some of optical signal processing functionalities are reported in this thesis.

One of the most useful nonlinear phenomena in periodic structures is *optical bistability* as many signal processing applications are derived from bistability concept, such as memory, switching, limiter, and flip-flop operation.

1.2 Objective

The aim of PhD project is to investigate new optical phenomena in nonlinear periodic structures which lay physical foundations in realizing some of the all-optical signal processing capabilities to support the future all-optical technology.

CHAPTER 1: INTRODUCTION

The origin of optical nonlinearity and some of potential applications of optical phenomena in 1-D Bragg grating and 2-D photonic crystal are investigated.

1.3 Project scope

Optical nonlinearity in 1-D Bragg grating and 2-D photonic crystal were investigated, mainly on optical bistability. Some of the potential applications of optical bistability in the context of all-optical signal processing are discussed. There are three materials used for 2-D photonic crystal experiments: Si, InGaAsP, and AlGaAs. The origin of optical nonlinearity and the method to distinguish them are discussed.

The main contributions in this thesis are firstly to propose 3 potential switching applications by optical bistability such all-optical switching, inverter, and limiter depending on the pump intensity, secondly is to show the significance of unstable state principle that determining the switching, thirdly is to show the first experimental results on all-optical transistor based on optical bistability, fourthly is to show various origins of optical nonlinearities in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ photonic crystal and the methods to distinguish them, and finally to show critical slowing down phenomena in InGaAsP photonic crystal resonator.

1.4 Thesis Organization

This thesis is intended for readers with some background knowledge on the field of nonlinear periodic structures. No discussion on the theoretical background of nonlinear periodic structure is provided. However, the necessary references and thorough literature review is given in Chapter 2 to equip the readers with necessary background knowledge to understand motivation and basic concepts used in the thesis. For theoretical background in linear Bragg grating, the readers are strongly suggested to refer [1.7]. While in nonlinear Bragg grating, the readers are strongly suggested to refer [1.8] and [1.9].

Each chapter is written to have stand-alone format having its own introduction, body (technical content), conclusion, and references.

In Chapter 2, the thorough literature review of the past researches that lead into new findings in this thesis are written in the "past findings and questions" format. The past findings are discussed and some points that have not been addressed are identified. The questions are raised to help the readers understand the issues that lead into the new findings. The thorough literature review for the nonlinear phenomena in fiber Bragg grating (FBG) as a representative of cubic nonlinear periodic medium is given in section 2.1. The past findings on the cubic quintic nonlinear periodic medium such as Pseudoatylene Toluene Sulfonate (PTS) Crystal with Bragg grating and the

CHAPTER 1: INTRODUCTION

literature review on the periodic medium with cubic quintic septic nonlinear response such as chalcogenide-Bragg grating is presented in section 2.2. Section 2.3 covers short literature review on the all-optical transistor application based on bistability principle. Lastly, section 2.4 covers short literature review on the optical bistability in silicon photonics crystal resonator and critical slowing down phenomena.

Chapter 3 discuss on the optical phenomena in cubic nonlinear periodic media such as Fiber Bragg Grating (FBG). Section 3.2 gives a brief theoretical background on the Nonlinear Coupled Mode Equations (NLCMEs) to govern the nonlinear wave propagation inside the FBG and short discussion on the time-domain numerical tool to solve the set of NLCMEs. Section 3.3 discuss on the multiple wave interactions in FBG under strong probe and strong pump condition to show light-controlling-light phenomena that serves as physical foundation to realize some of the all-optical signal processing functionalities such as all-optical switching, inverter, and limiter. Nonlinear Coupled Mode Equations for strong probe and strong pump is derived analytically and solved numerically to show permanent switching behaviors by the unstable state principle. Section 3.4 discuss on the shape of hysteresis from asymmetric fano-resonance FBG that distinguish itself from symmetric Lorentzian's counterpart. Lastly, section 3.5 covers the physical explanation behind all-optical AND gate in birefringent FBG.

Chapter 4 covers on the continuous wave (CW) nonlinear transmission characteristics for two categories of nonlinear periodic structures with single resonance, namely cubic-quintic grating and cubic-quintic-septic grating. Section 4.2 and section 4.3 covers on the CW nonlinear transmission characteristics in PTS crystal with Bragg grating to represent cubic-quintic grating. Whereas Section 4.4 and Section 4.5 covers the discussions on CW nonlinear transmission characteristics in nonlinear Chalcogenide-Bragg grating to represent cubic-quintic-septic grating. Double Optical Bistability in the CW nonlinear transmission characteristics for both media is reported.

Chapter 5 discuss on all-optical transistor operation based on bistability principle. The Nonlinear Coupled Mode Equations to govern multiple wave propagation inside InGaAsP-InP DFB waveguide is given in Section 5.2. Experimental result on all-optical transistor is presented in Section 5.3. Section 5.4 gives the physical explanation behind all-optical transistor operation.

Chapter 6 covers optical bistability discussion in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ photonic crystal resonator. Section 6.2 covers the discussion on optical bistability experiments and simulations in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ Photonics Crystal resonator. Section 6.3 covers method to distinguish between Kerr and thermal nonlinearity in optical bistability of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ Photonics Crystal resonator.

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Chapter 7 presents Critical Slowing Down (CSD) phenomena from nonlinear cavity in photonics crystal resonator. Section 7.2 presents some numerical simulations on CSD. Section 7.3 gives the mathematical and physical explanation of CSD. Section 7.4 presents experimental results of CSD from nonlinear InGaAsP-InP photonics crystal resonator. Some of possible CSD applications are presented in Section 7.5.

Finally, thesis summary is presented in Chapter 8.

CHAPTER 2: LITERATURE REVIEW

CHAPTER 2

LITERATURE REVIEW

2.1 NONLINEAR PHENOMENA IN GRATING WITH CUBIC NONLINEARITY

Fiber Bragg Grating (FBG) is 1-D periodic refractive index variation inside fiber core that exhibit band gap around Bragg wavelength [2.1]. Various type of FBG structure from uniform, phase-shifted, apodized, chirped, and superstructure grating with their respective spectrum are discussed in [2.1]. Uniform grating created band gap around Bragg wavelength with huge side-lobes. Various apodization techniques are introduced to minimize the side-lobes at the edge of the band-gap. Chirped grating is used for manipulating the dispersion property of the grating. Lastly, superstructure grating is to design multiple filters for WDM system. By transfer matrix method (TMM) or Layer Peeling Method (LP), the grating spectrum can be found for any given grating structure. Inversely, given the required grating spectrum, the grating structure can be found by inverse scattering method. However, all of these works were done in linear regime where the probe power is weak.

In nonlinear regime when the light is strong, due to self-focusing cubic nonlinearity in conventional silica fiber, the grating spectrum in linear regime is affected. Plethora nonlinear optical phenomena are observed when the light is propagating with strong intensity inside the FBG. Some of nonlinear phenomena in the periodic structure lay basic physical foundations to realize some of the all-optical signal processing functionalities such as all-optical switching, all-optical limiter, pulse compressor, flip-flop and AND gate behavior [2.2-2.6].

Having understood the spectrum of Bragg grating in nonlinear regime, all-optical switching functionalities by bistability and optical pulse compression are shown and the performance is compared in uniform and phase-shifted Bragg grating in nonlinear regime [2.2]. It is shown that bistability comes from the sidelobes of the uniform grating in nonlinear regime. For phase-shifted grating, bistability comes from resonance created in the band gap. In [2.3] and [2.4], all-optical limiter functionality is shown in nonlinear periodic structure. Similar with [2.2], bistable switching was shown in nonlinear Bragg grating [2.5].

In [2.6], all-optical AND gate by simulations is done on birefringence nonlinear Bragg grating. It is shown that when only one pulse in X or Y-polarization propagates inside the grating, it will decay.

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However, when both pulses propagate together, they manage to appear and the output of the grating although their wavelength are well inside the band gap.

In 1991, C.M. De Sterke *et al.* proposed a numerical tool to solve the Nonlinear Coupled Mode Equations (NLCMEs) to govern the nonlinear wave propagation inside the FBG by Implicit 4th Order Runge-Kutta Method [2.7]. This numerical method is able to capture the dynamic behavior of wave interactions in nonlinear FBG both time and space domain. This method is therefore superior compared to TMM or LP method introduced in [2.1] which only captures steady state value. This method is widely used to solve various nonlinear propagation problems in Bragg grating throughout this thesis.

By this numerical method, he introduced Optical Push Broom (OPB) effect where the energy stored inside the grating can be flushed by the strong Gaussian pump that operates in the nonlinear regime with wavelength detuned relatively far from the Bragg wavelength so that it propagates as if in bare fiber [2.8].

He pointed out that providing the pump pulse width is shorter than the grating length and the probe is operating at the resonance transmission in the negative detuning region, the probe velocity inside the grating increase momentarily at every location passed by the pump. Therefore the probe energy will finally pile up in front of the pump.

The hallmark of OPB is a high and narrow peak followed by a much longer dip in the probe transmission to maintain the energy conservation law. In 2000, Broderick *et al.* published the experimental results on the OPB to verify Sterke's theory [2.9].

He also pointed out that the pump wavelength does not affect the strength of cross phase modulation (XPM) with the probe as it did not enter into the equations [2.8]. However, more recent experiments in [2.10] showed results contrary to this claim.

The seemingly contradicting truth between theory and experiments were investigated by deriving Nonlinear Coupled Mode Equations by taking into account strong probe and strong pump at the same time. The result is given in Chapter 3, equation (3.5) to (3.7). It is shown that the pump wavelength affect the strength of XPM with the probe. This model is also consistent with the experimental results shown in [2.10].

Apart from that, in 1994, C.M.De Sterke *et al.* showed the solitary wave with wavelength inside the stop-band trapped inside the grating namely Gap Soliton [2.11]. In this paper, the optical

CHAPTER 2: LITERATURE REVIEW

bistability phenomenon in FBG is also discussed. The occurrence of the Gap Soliton is pointed out to be at the point near but not exactly in the switching-down threshold of hysteresis where input is equal to output. Knowing that the bistability phenomenon occurs in FBG if the detuning is chosen judiciously, Broderick *et al.* subsequently studied permanent switching in FBG by cross phase modulation (XPM) between CW probe and Gaussian pump numerically [2.12].

Extensive experiments on the permanent switching behaviors with its application as all-optical flip-flop were subsequently done independently by the group in Japan in InGaAsP-InP DFB waveguide and Semiconductor all-optical amplifier with Bragg gratings [2.13-2.16]. Various pump powers and wavelengths were used to tune the switching in InGaAsP-InP DFB waveguide to optimize and investigate relationship between pump's wavelength and power required to switch the probe from low to high state.

Although there is some consistency, no mathematical model is derived to govern the relationship between pump power and wavelength as the data were gathered experimentally. The physical explanation behind this permanent switching behavior is also missing in all of these papers. In this thesis, mathematical model based on Nonlinear Coupled Mode Equations to govern the relationship between pump power and wavelength with their effects to the probe is given in Chapter 3, equation (3.5) to (3.7). It is shown that the pump wavelength affect the strength of XPM with the probe. This model is also consistent with the experimental results shown in [2.10].

In Chapter 3 of this thesis, it is pointed out that the unstable state is responsible to determine the permanent switching behavior. Numerical simulation and experimental proof of unstable state principle that responsible for permanent switching behavior is discussed in Chapter 3 and Chapter 5 respectively.

More recent paper in 2002, Hojoon Lee *et al.* did some numerical studies to investigate the nonlinear switching characteristics and compared optical bistability phenomena in uniform and phase-shifted FBG by using Implicit 4th Order Runge-Kutta Method [2.2]. Although he compared the bistability from uniform grating and phase-shifted grating, what missing in his paper is the discussion on the unstable state inside the bistability hysteresis loop (the negative slope of the S-Curve) which exists analytically but not numerically due to the unstable nature of the branch.

Based on the past findings on the permanent switching, two main questions are still subject to further research, namely:

CHAPTER 2: LITERATURE REVIEW

1. The physical reason of permanent switching behavior that leads into all-optical flip-flop shown theoretically in [2.12] and experimentally in [2.13-2.16]. Is there any relationship between the permanent switching behavior with the unstable state that is missing in the numerical simulations in [2.2] and proven mathematically in [2.11]?

The answer is discussed in Chapter 3 by simulation results, and proven by experimental results in Chapter 5.

2. In [2.2], it is pointed out that the symmetric Lorentzian resonance can be created by inserting π phase-shift in the middle of the grating. Consequently the switching threshold is reduced at the expense of lower extinction ratio due to the field enhancement effect in the defect. Although the shape of bistability from symmetric Lorentzian resonance is well understood, there is no literature on the shape of bistability from asymmetric fano-resonance in FBG.

The hallmark of bistability from asymmetric fano-resonance in FBG that distinguishes itself from the Lorentzian's counterpart by introducing another scattering path is investigated in Chapter 3.

In 1998, J.E. Sipe and S. Pereira investigated the nonlinear pulse propagation inside the birefringent FBG [2.6]. Either X-polarized or Y-polarized pulse propagates in the grating at a certain detuning value inside the band gap will decay. However, if both nonlinear pulses launched together, they will help each other propagate inside the grating although their wavelengths are located in the stop band region. At the output, both pulses appeared compressed simultaneously.

The experiment to prove the all-optical AND-gate phenomena subsequently was done by Taverner *et al.* [2.17]. However the physical explanation on the all-Optical AND-gate phenomenon is missing in [2.17], namely the physical reason behind all-optical AND-gate phenomena in birefringent FBG shown in [2.6, 2.17].

The answers to the missing information above are discussed in Chapter 3 by investigating the picture of pulse propagation inside the grating with birefringence. In conclusion, OPB effect introduced earlier by C.M. Sterke is responsible for all-optical AND-gate phenomenon in nonlinear birefringence grating.

CHAPTER 2: LITERATURE REVIEW

2.2 NONLINEAR PHENOMENA IN GRATING WITH HIGHER ORDER NONLINEARITIES

On the basis of the experimental observations, several materials have been found not only to exhibit strong cubic Kerr nonlinearity (3rd order) like silica or III-V materials (AlGaAs or InGaAsP) but also dominant quintic nonlinearity (5th order). For short, they are called as cubic quintic (CQ) nonlinear material [2.18].

CQ nonlinear material was proposed as an empirical description of the nonlinear dielectric response of Polydiacetylene Toluene Sulfonate (PTS) crystals as well as of special glasses and organic materials [2.19].

In 2001, Atai *et al.* performed a theoretical postulation that two solitons families exist in the periodic medium with cubic quintic nonlinearity. The first family of well-known sech soliton appears when the cubic nonlinearity is dominant over the quintic counterparts. This is a general type of soliton which also appear in Silica material. On the other hand, the second families of solitons appear when quintic nonlinearity is dominant over the cubic counterparts [2.20].

The stable and unstable regions are defined for both soliton families according to their frequencies and nonlinearities. Subsequently in 2004, he investigated the interaction of the Bragg gratings solitons in the CQ medium for finite and infinite grating length [2.21]. His findings show some interesting result that the interaction of two soliton families depends on their initial phase and initial separation distance between them. The map to draw boundary between first-type and second-type soliton are presented in [2.21] including the stable and unstable regime for the two soliton families.

While there are quite a few literatures on the solitons in CQ nonlinear medium [2.18-2.21], the focus in this thesis is to study on the continuous wave (CW) nonlinear transmission characteristics (including the bistability phenomena) in the CQ medium with Bragg grating or in short we call it as cubic quintic grating (CQG).

PTS crystal parameter is chosen to represent the CQ nonlinear material. A single resonance is obtained by inserting π phase-shifted in the middle of the grating. The study on the CW nonlinear transmission characteristics and bistability phenomena in Bragg grating PTS-crystal with single resonance is presented in Chapter 4, Section 4.2 and 4.3.

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Double optical bistability from a single resonance is shown to be new physical phenomenon appear in transmission characteristics of CQG. The first bistability is derived based on cubic-self focusing nonlinearity while the second bistability is derived based on quintic-self defocusing nonlinearity.

Next, chalcogenide glass was proposed as a strong candidate to answer the need for all-optical signal processing due to its strong nonlinear coefficient as compared to the silica fiber [2.22, 2.23]. In [2.22], chalcogenide glass was reported to exhibit not only 3rd and 5th order nonlinearities but also 7th order nonlinearity. Not only chalcogenide-glass possesses until 7th order nonlinearity, the 3rd order nonlinear effect is three orders stronger than that in silica, thus require very low switching power than in silica. In other words, chalcogenide glass can be classified as a cubic-quintic-septic nonlinear material.

Several successful grating writing experiments in waveguide and fiber based on chalcogenide-glass material owing to its high photosensitivity have also been reported [2.24-2.26]. Following the success of grating writing experiment in chalcogenide-glass based materials, theoretical model to govern the wave propagation in the cubic-quintic-septic nonlinear periodic media such as chalcogenide-Bragg gratings by counting until 7th order nonlinearities is discussed in this thesis.

The Cubic-Quintic-Septic Nonlinear Coupled Mode Equations (CQSNLCMEs) to govern the wave propagation inside the chalcogenide-Bragg gratings is presented in Chapter 4. The CW nonlinear transmission characteristic in π phase-shifted chalcogenide-Bragg grating was investigated by solving CQSNLCMEs numerically by implicit 4th order Runge Kutta Method as discussed in Chapter 4, Section 4.4 and 4.5.

Again, similar with PTS crystal, double optical bistability from a single-resonance appear in its nonlinear transmission characteristics.

2.3 ALL-OPTICAL TRANSISTOR BASED ON BISTABILITY PRINCIPLE

In 2004, Mehmet F. Yanik has investigated all-optical transistor phenomena based on bistability principle in photonics crystal resonator [2.27]. He introduced from conceptual point of how all-optical transistor can be achieved by manipulating signal in one path could affect signal at the other path that operates in bistable regime.

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However, there is no clear physical explanation and parameter boundaries on which the phenomena could happen and no discussion on how much the small signal gain he could achieve in his photonic crystal structure. Moreover, his result is only simulation as the proof-of-concept that all-optical transistor application could be derived from bistable phenomenon.

In Chapter 5, we discussed the physical reason behind all-optical transistor based on bistability and demonstrate experimentally in InGaAsP-InP DFB waveguide with Bragg reflectors. By experimental result, the Gain performance as one of the key parameter in transistor application can also be measured. Beside Gain, SNR and switching time to determine the bandwidth of operation were also measured.

2.4 ALL-OPTICAL MEMORY BASED ON BISTABILITY IN $\text{Al}_x\text{Ga}_{1-x}\text{As}$ NONLINEAR PHOTONIC CRYSTAL RESONATORS

In 2005, Notomi *et al.* have achieved optical bistability experimentally from silicon photonics crystal resonator [2.28]. However it was shown by Tanabe *et al.* [2.29] that the memory time is limited to only few nanoseconds in silicon photonics crystal resonators due to competing nonlinearity between thermal and plasma carrier effect. With the motivation to improve memory time in Photonic Crystal Resonator, AlGaAs is proposed as substrate for photonic crystal resonator.

The material properties of AlGaAs and its capability to improve the memory time are investigated. Extensive investigation on switching in AlGaAs Photonic Crystal has been done by A.D. Bristow *et al.* [2.34, 2.35].

In [2.34], he investigated on the power-dependent wavelength shift of photonic coupling resonances of a two-dimensional photonic crystal waveguide by reflection geometry pump-probe measurements. The quadratic response is indicative of two-photon induced carrier creation, which alters the refractive index of the semiconductor core of the photonic lattice.

In [2.35], he used femtosecond pump-probe spectroscopy to study the ultrafast nonlinear response of AlGaAs two-dimensional photonic crystal waveguides in the near-infrared spectral region. The modulation of the reflectivity spectra due to the refractive index change produced by photogenerated carriers was measured. These results confirm the excellent potential of photonic crystal waveguides in ultrafast nonlinear switching applications.

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In Chapter 6, the optical bistability performance for all-optical memory application from $\text{Al}_x\text{Ga}_{1-x}\text{As}$ Photonic Crystal Resonator is investigated. It is shown that the memory time in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ can last indefinitely much more than silicon counterpart.

Two major nonlinearities presence in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ is Kerr and Thermal effect. Thus red-shift of the cavity is introduced in this material. The method to distinguish various origins of nonlinearity from optical bistability is also presented. It is shown that Q factor of cavity, switching power, and aluminum contents (x) are three important parameters to distinguish two major nonlinearities presence in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ cavity.

Two aluminum contents were used in the experiments, $x = 18\%$ and $x = 26\%$. More Kerr effect is found when aluminum content is higher.

2.5 CRITICAL SLOWING DOWN FROM NONLINEAR CAVITY

In [2.30-2.33] were discussed about critical slowing down (CSD) phenomena in anomalous bistable regime theoretically and experimentally in various materials and structures. CSD is a general phenomenon that happens in anomalous bistable regime of any structure that exhibit cavity with nonlinearity. In all previous chapters, the concern is about the switching at steady state and the physical explanation behind the switching. However, CSD is a phenomenon that appears during transient in bistability as explained in [2.30].

In this thesis, we discuss the hallmark of CSD phenomena and physical reason behind CSD particularly in InGaAsP photonic crystal resonator. Experimental demonstration of CSD in InGaAsP is also shown in Chapter 7. The nonlinear phenomenon in InGaAsP is based on plasma carrier effect, thus it introduces blue-shift in the cavity.

It is shown that just a slight variation of operating wavelength can cause large variation of switching time. CSD could be useful phenomenon mainly to tell us the lowest switching power for a certain detuning, to distinguish between various nonlinear effects in material, and to control switching time for optical network.

CHAPTER 3

NONLINEAR PHENOMENA IN GRATING WITH CUBIC NONLINEARITY

3.1 INTRODUCTION

This chapter covers the numerical study on the nonlinear switching dynamics based on optical bistability in the grating with cubic Kerr nonlinearity such as conventional silica-Fiber Bragg Grating (FBG). The theoretical background of nonlinear FBG is covered in section 3.2. Section 3.3 shows some numerical simulations on the multiple wave interactions inside the nonlinear Fiber Bragg Grating (FBG) under strong probe and strong pump condition. Previously only the interaction between strong pump and weak probe inside nonlinear FBG was ever reported [3.13]. Three potential applications such as: all-optical switching, inverter, and limiter are shown under strong probe and strong pump multiple wave propagation inside FBG. Section 3.4 discuss on the shape of optical bistability from asymmetric fano-resonance FBG that distinguish itself from symmetric Lorentzian's counterpart. Section 3.5 discuss on the physical reason behind all-optical AND gate phenomenon in nonlinear birefringent FBG. Lastly, conclusion, acknowledgement, and reference are provided in Section 3.6, 3.7, and 3.8 respectively.

3.2 BACKGROUND THEORY

The transmission spectrum of the typical uniform FBG is shown in Fig 3.1. The band-gap is located around Bragg wavelength at 1550 nm. The numerical simulation is done by Implicit 4th order Runge Kutta Method (RKM $\rightarrow$ + sign) [3.1] that matches with the Transfer Matrix Method (TMM $\rightarrow$ solid line) in [3.2]. This shows the accuracy of Implicit 4th order Runge Kutta Method that will be used as the numerical tool in solving various kinds of Nonlinear Coupled Mode Equations (NLCMEs) throughout this chapter. Comparisons of the two numerical tools (TMM and RKM) are given below.

Some limitations of the TMM are as follows [3.2]:

1. It can only handle the case when input is continuous wave (CW) as it only gives the steady state solution. No transient time-domain information is provided.
2. It can only handle the condition when only one input wave exists inside the grating. It cannot handle multiple wave propagations inside the grating.

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The drawbacks of Implicit 4th Order Runge Kutta Method over TMM are:

1. Longer calculation time due to larger number of iteration parameters to achieve the same accuracy as TMM.
2. Larger memory space to store the variables in computer.
3. More complex algorithm to implement.

Fig 3.1 Transmissivity of Fiber Bragg Grating in the linear regime (+ for Implicit 4th Order Runge Kutta Method and Solid Line is for TMM)

When strong wave propagates inside the grating, the refractive index is increased due to self-focusing Kerr nonlinearity as given by [3.9]:

$$n(z) = \bar{n} + n_1(z) \times \cos \left[\frac{2\pi}{\Lambda} z + \theta(z) \right] + n_2 |E(z)|^2 \quad (3.1)$$

Λ = pitch in the grating, z = propagation direction of wave, $\theta(z)$ = initial phase of grating, n_2 = nonlinear coefficient of the medium

As the wave is close to the Bragg wavelength under short period grating condition, the field inside the grating consists of two coupled counter-propagating waves. Hence, the electric field can be written as

$$E(z, t) = [A_+(z, t) \cdot \exp(iK_B z) + A_-(z, t) \cdot \exp(-iK_B z)] \cdot \exp(-i\omega_0 t) \quad (3.2)$$

$A_{\pm}$ = forward/backward propagating wave, K_B = coupling coefficient of the grating, and ω_0 = wave frequency

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Substituting (3.1) and (3.2) into the scalar Maxwell's wave equation, the following set of Nonlinear Coupled Mode Equations (NLCMEs) to govern the wave propagation inside the nonlinear FBG is obtained [9]:

$$+ i \frac{\partial A_+}{\partial z} + i \frac{1}{V_g} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- + \Gamma |A_+|^2 A_+ + 2\Gamma |A_-|^2 A_+ = 0 \quad (3.3)$$

$$- i \frac{\partial A_-}{\partial z} + i \frac{1}{V_g} \frac{\partial A_-}{\partial t} + \delta A_- + \kappa^* A_+ + \Gamma |A_-|^2 A_- + 2\Gamma |A_+|^2 A_- = 0 \quad (3.4)$$

δ is detuning, κ is coupling coefficient, Γ is self-phase modulation (SPM) coefficient.

Mathematical definitions of them can be found in Appendix A.

In linear regime, FBG exhibits bandgap, with center Bragg wavelength $\lambda_B = 2n\Lambda$. The width of the bandgap depends on the value of coupling coefficient of the grating κ . In nonlinear regime, FBG exhibits hysteresis (optical bistability) phenomenon around the switching thresholds in its CW nonlinear transmission characteristics. Optical bistability in nonlinear periodic media is a simple yet powerful phenomenon which plays a crucial role in realizing the future all-optical technology. Since first introduced by Winful *et al.* in 1979 [3.3], optical bistability has been studied theoretically and experimentally in various nonlinear periodic media such as fiber Bragg grating, photonics crystal, and GaAsInP/InP with Bragg reflectors [3.4-3.9]. The physical origin of optical bistability is from the fact that the narrow resonance spectrum will be twisted either to higher wavelength in self-focusing nonlinear material or to lower wavelength in self-defocusing nonlinear

material as long as the minimum detuning requirement is satisfied, i.e. $|\delta| \geq \frac{\sqrt{3}}{2} \Delta\delta_{FWHM}$. Without

resonance in the linear transmission curve, there will be no bistability in nonlinear regime. In the case of uniform FBG, narrow resonance refers to the side-lobe nearest to the band gap in the negative detuning region (around 1549.85nm) as shown Fig 3.2.

Fig 3.2 Transmissivity of Fiber Bragg Grating for various input intensities

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The grating parameters used in this simulation are $\bar{n} = 1.45$, $\lambda_B = 1550$ nm, $n_2 = 2.6 \times 10^{-20}$ m²/W, $L = 1$ cm, $n_1 = 2 \times 10^{-4}$. By Implicit 4th Order Runge Kutta Method [3.1], the transmissivity curve in the wavelength domain for various input intensities are obtained as shown in Fig 3.2.

To solve NLCMEs by implicit 4th Order Runge Kutta Method, by applying coordinate transformations proposed in [3.1]: $z = \varepsilon - \tau$ and $t = \varepsilon + \tau$

Therefore the nonlinear coupled mode equations can be reduced to coupled ordinary differential equations (ODEs):

$$+ i \frac{\partial A_+}{\partial \varepsilon} + \delta A_+ + \kappa A_- + \Gamma |A_+|^2 A_+ + 2\Gamma |A_-|^2 A_+ = 0$$

$$- i \frac{\partial A_-}{\partial \tau} + \delta A_- + \kappa^* A_+ + \Gamma |A_-|^2 A_- + 2\Gamma |A_+|^2 A_- = 0$$

It is trivial issue to solve ODEs by RKM, however iterations are needed since the equations are coupled. Since coordinate transformation has been applied, we need to solve equations (6) – (8) from ref [3.1] with coefficients given at Table 1 or ref [1], Iterations must be done until the value of $A_{\pm}(z, t)$ converges.

In order to get transmission spectrum, we need to calculate $|A_+(L, t)|^2$ at steady state value as $t \rightarrow \infty$.

$$\rightarrow \infty, T = \frac{|A_+(L, t)|^2}{|A_+(0, t)|^2}$$

Bistability curve is obtained by increasing input intensity in step-functions $|A_+(0, t)|^2$ up to the up-switching threshold and calculate corresponding output intensity $|A_+(L, t)|^2$. The upper branch of bistable curve is obtained by reducing input intensity in step-function and calculates corresponding output intensities until down-switching threshold is obtained. The simulation results obtained match exactly with results given in [3.1] and [3.9] using the same algorithm.

When the light intensity is at 30 GW/cm², the whole transmission spectrum is shifted to the right and the hysteresis is formed at wavelength around 1549.92 nm from the side-lobe nearest to the band gap in low wavelength regime. If the side-lobe is completely removed by apodizing the grating, the hysteresis will consequently disappear.

Fig 3.2 does not show the whole picture of the optical bistability due to the nature of the time-domain numerical tool. From analytical point of view, there should be unstable state inside the hysteresis loop (negative slope of the S-curve). The significance of the unstable state will be discussed in the Section 3.3.

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By inserting π -phase shift in the middle of the grating (phase-shifted grating), the switching threshold is reduced at the expense of reduction of extinction ratio [3.9]. Under positive Kerr nonlinearities, the bistability phenomena can only be observed in the negative detuning region as for the case of phase-shifted grating because it comes from a single symmetric Lorentzian resonance at the Bragg wavelength [3.9] as opposed to the positive detuning region for the

uniform case. The detuning condition to observe optical bistability is given as $|\delta| \geq \frac{\sqrt{3}}{2} \Delta\delta_{FWHM}$

where $\Delta\delta = \frac{\kappa}{\sinh(5\kappa L)^2}$ for π phase-shifted FBG.

3.3 MULTIPLE WAVE INTERACTIONS IN NONLINEAR FIBER BRAGG GRATING

This section covers the study on the nonlinear switching dynamics from multiple wave interactions inside nonlinear FBG. The schematic of FBG with strong pump and strong probe is given in Fig. 3.3.

Fig 3.3 Schematic of multiple wave interactions in Fiber Bragg Grating

Nonlinear Coupled Mode Equations (NLCMEs) governing the strong probe and strong pump propagation inside the FBG is given as:

$$i \frac{\partial A_+}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- + \Gamma_B \left\{ |A_+|^2 + 2|A_-|^2 + |A_P|^2 \right\} A_+ + R \Gamma_P |A_P|^2 A_+ = 0 \quad (3.5)$$

$$-i \frac{\partial A_-}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_-}{\partial t} + \delta A_- + \kappa^* A_+ + \Gamma_B \left\{ |A_-|^2 + 2|A_+|^2 + |A_P|^2 \right\} A_- + R \Gamma_P |A_P|^2 A_- = 0 \quad (3.6)$$

$$i \frac{\partial A_P}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_P}{\partial t} + \frac{1}{R} \Gamma_B \left\{ |A_+|^2 + |A_-|^2 \right\} A_P + \Gamma_P \left\{ |A_+|^2 + |A_-|^2 + |A_P|^2 \right\} A_P = 0 \quad (3.7)$$

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$$\bar{n} = \text{average refractive index of material, } c = \text{speed of light, } \Gamma_B = \text{SPM coefficient at bragg wavelength, } \Gamma_p = \text{SPM coefficient at pump wavelength, } R = \text{ratio of bragg wavelength with pump wavelength } \frac{\lambda_B}{\lambda_p} \quad (3.8)$$

The derivation of NLCMEs under strong probe and strong pump illumination and mathematical definitions of each symbols are given in Appendix A. From (3.5) and (3.6), it is shown the pump wavelength λ_p affects the strength of XPM with the probe. This result agrees with experimental results in [3.10] qualitatively, as pump power needed to induce the switch is increased as its wavelength increases.

Fig.3.4 shows optical bistability in the CW nonlinear transmission characteristics of uniform FBG by solving (3.5) and (3.6) numerically with Implicit 4th order Runge-Kutta Method [3.1]. The middle part of the S-curve is not able to be shown as it is unstable. The uniform grating parameters in our simulation are given as follows: grating length (L) = 1 cm, $n = 1.45$, $\kappa = 500/\text{m}$, $n_2 = 2.6 \times 10^{-20} \text{ m}^2/\text{W}$, Bragg wavelength (λ_B) = 1550 nm, input probe CW is at $\delta = 474/\text{m}$ ($\lambda = 1549.875 \text{ nm}$). The numerical simulations on the multiple wave interactions under strong probe and strong pump conditions are shown in Fig 3.5. Three unique nonlinear permanent switching behaviors for CW probe transmission at $18 \text{ GW}/\text{cm}^2$ by Cross Phase Modulation (XPM) with Gaussian pump with a constant full width half maximum (FWHM) at 75 ps but varying peak intensities are observed in that figure. The pump peak intensities are categorized into 3 regions: low, medium, and high to distinguish the three permanent switching behaviors.

Fig 3.4 Optical bistability in uniform Fiber Bragg Grating

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Fig 3.5 Three nonlinear switching behaviors of continuous wave probe transmission by cross phase modulation with Gaussian pump in Fiber Bragg Grating

The compression peak followed by a much longer dip in the probe transmission after the pump injection known as optical push broom (OPB) effect can be observed in all three nonlinear switching. OPB basically tell us that the presence of the pump will shift the group velocity of the probe temporarily from subluminal point to luminal point as shown in Fig. 3.6. As the pump propagates at luminal velocity, the probe energy will finally piled up just after the pump followed by a much longer dip to satisfy energy conservation law. The peak compression followed by much longer dip is the signature of OPB effect as shown in Fig 3.7. More thorough theoretical discussions and experimental observations on the OPB can be found in [3.11] and [3.12] respectively.

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Fig 3.6 Group delay in uniform Fiber Bragg Grating

Fig 3.7 (a) Optical Push Broom Effect inside Fiber Bragg Grating and (b) Optical Push Broom Effect at the Output of Fiber Bragg Grating

Similar permanent switching behavior shown Fig 3.5 has been discussed theoretically in [3.13] by XPM with Gaussian pump at FWHM of 10 ns and experimentally in [3.14-3.17]. However, there was no explanation given on why the permanent switching occurs in those literatures. Unlike [3.13], in our study the pump FWHM is set at 75ps, which is comparably in the same order as the grating length so that OPB effect can be observed clearly as it is one of the OPB requirements as mentioned in [3.11]. To explain how OPB leads into various nonlinear permanent switching behaviors, three lines of the probe transmission just after the pump injection are plotted on the same axis in Fig. 3.8. The plot is zoomed into the part just after probe compression peak, where the dip is located. The unstable state from analytical bistability solution at 4.6 GW/cm² (25.5% of probe input intensity) is drawn in that figure.

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Fig 3.8 Transient artifacts of continuous wave probe transmission after pump injection

The role of the short pump in the system that exhibits optical bistability is to cause temporal perturbation into the nonlinear system. At the time when the effect of this temporary perturbation disappears (at time around 1250 ps in Fig. 3.8), the probe transmission in the system has to decide whether to go to low or high state. At this time, if the probe dip is above the unstable state, the probe transmission goes up and settles at the high state as there are extra photons than the unstable state, else it goes to low state.

Based on [18], the mathematical prove that the middle branch indeed unstable is given below. Suppose a photon with energy hf is injected into the nonlinear system where the output is initially at the middle branch of the S-curve. Additional photon will change the output of the system to $O + hf$, where O is the original output. The change in the output will change the transmissivity of the system to $T + \frac{\partial T}{\partial O} hf$. The change in the transmissivity will subsequently

induce another change at the output by $\Delta O = \left(\frac{\partial T}{\partial O} hf \right) I$, so that the total output is given as:

$$\left(T + \frac{\partial T}{\partial O} hf \right) I = O + \frac{\partial T}{\partial O} I hf = O + \Delta O \quad (3.9)$$

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For the unstable branch $\frac{\partial I}{\partial O} < 0$, therefore $\frac{\partial I}{\partial O} T < 0$ as $T > 0$ (3.10)

Writing the input output relationship $O = IT$ (3.11)

Taking total derivative with respect to O in (3.11), (3.12) can be obtained:

$$1 = \frac{\partial I}{\partial O} T + \frac{\partial T}{\partial O} I \quad (3.12)$$

But from (3.10) $\frac{\partial I}{\partial O} T < 0$

Therefore $\frac{\partial T}{\partial O} I > 1$ (3.13)

From the inequality condition in (3.13), the ΔO in (3.9) is therefore greater than hf by $\frac{\partial T}{\partial O} I$. In

other words, the addition of a photon with energy hf when the output is in the middle branch will lead into more photons added into the output so it will finally settle at the upper hysteresis branch (higher state). Similarly, the reduction of a single photon in middle branch will lead into reduction of more photons of the output so that it will finally settle at the lower hysteresis branch (lower state).

This is what exactly happens in Fig 3.8. The Gaussian pumps acts as a temporal perturbation which add or reduce photons from the unstable state in the hysteresis once its effect disappear. If the probe intensity is lower than the unstable state after the effect of the pump perturbation finish, there are lesser photons in the probe than the unstable branch. Hence more photons will be taken away from the probe so that it settles at the lower hysteresis branch. Conversely, if the probe intensity is higher than the unstable state after the effect of the perturbation finish, there are more photons than the unstable branch. Hence more photons will be added into the probe to finally let it settle at the upper hysteresis branch. This is called "unstable state principle".

Based on unstable state principle, the switching behaviors of the probe can be manipulated by changing the pump peak intensity to modify the strength of XPM as shown in Fig 3.5. Below the low pump region, there is no switching in the output probe transmission. In low pump region ($45 \text{ GW/cm}^2 < \text{pump peak intensity} \leq 55 \text{ GW/cm}^2$), the probe compression peak is weak due to weak XPM with relatively weak pump. Consequently, the dip following the compression is shallow to balance the weak compression. At the time when the effect of the temporal disturbance disappears, the shallow dips are always above the unstable state regardless the initial state of the probe. Therefore, the final probe transmissions always settle at high states. One potential application of the FBG in low pump region is as *step-up all-optical switching*.

In medium pump region ($55 \text{ GW/cm}^2 < \text{pump peak intensity} \leq 97.5 \text{ GW/cm}^2$), when probe is at low state, the compression peak is low enough to cause the dip above the unstable state. Hence

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the probe finally settles at high state after the pump. However, when input is at high state, the compression peak is correspondingly strong due to stronger XPM with the same pump intensity. Consequently, the dip is deep enough to balance strong XPM and finally force the probe to settle at low state. In medium pump region, the FBG can function as an *all-optical inverter* due to the inverting nature between the probe states before and after the pump.

By the same reasoning, in high pump region (pump peak intensity > 97.5 GW/cm²), the probe compression peak is very high due to strong XPM with the strong pump. Correspondingly, the dips always fall below the unstable state and force the probe to settle at low state regardless its initial state before the pump. The FBG performs as an *all-optical limiter* in this region.

Needless to say, the boundaries of the pump peak intensity in the low, medium, and high pump region will change accordingly as the FWHM of Gaussian pump is changed. What really matter is whether the dip after the effect of pump disappears is above or below the unstable state to determine the final state of the probe transmission. The three permanent switching behaviors in the FBG with their potential applications are summarized in Table 3.1.

Table 3.1 Summary of three switching behaviors in the bistable regime by cross phase modulation with Gaussian pump

Probe Input	Pump Intensity	Probe Output	Potential Application
Low	Low	High	step-up all-optical switching
High		High	
Low	Medium	High	all-optical inverter
High		Low	
Low	High	Low	all-optical limiter
High		Low	

By similar XPM method, the unstable state is drawn and connected for all probe intensities inside the hysteresis as shown in Fig 3.9. The cross mark indicates the unstable state in Fig.3.8 when the probe is at 18 GW/cm².

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Fig 3.9 Optical bistability in uniform Fiber Bragg Grating with unstable state

3.4 OPTICAL BISTABILITY FROM FANO-RESONANCE FIBER BRAGG GRATING

In section 3.2 and 3.3, the optical bistability from Lorentzian resonance in phase-shifted FBG as well as from uniform FBG have been discussed. The signatures of optical bistability from the symmetric Lorentzian resonances are the occurrence of 100% transmission point near the down-switching threshold and the negative transmission slope in the upper transmission hysteresis branch, i.e. $\left(\frac{\partial T}{\partial I_{in}} < 0\right)$ [3.8].

This section discuss on the optical bistability from asymmetric fano-resonance in FBG. Asymmetric fano-resonance from FBG is created by introducing two scattering pathways. The first path consists of π phase-shifted FBG to create discrete state Lorentzian-resonance and the second path is continuum states with phase-shifter φ to introduce phase difference at the output. Due to unequal phase distribution of the Lorentzian resonance around the Bragg wavelength, the coherence interference with stable CW background light from second path will create asymmetric fano-resonance as shown in Fig 3.10 for φ at 0.2π and 0.4π . The phase-shifted grating

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parameters used in our simulation are given as follows: $L = 1$ cm, n_{AC} is at 1.25×10^{-4} , n_2 is at $2.6 \times 10^{-20} \text{ M}^2/\text{W}$ and $\bar{n}$ is set at 1.45.

Fig 3.10 Fano-resonance Fiber Bragg Grating in linear regime for various phase shifts ϕ

In nonlinear regime, one distinguishing feature of the shape of fano-bistability as compared to Lorentzian's counterpart is the positive transmission slope in the upper hysteresis branch, i.e.

$\left(\frac{\partial T}{\partial I_{in}} > 0\right)$ as shown in Fig 3.11. The operating wavelength is set to be slightly higher than

the resonance wavelength at 1550.025 nm (marked by X in Fig 3.10). A positive transmission slope in the upper transmission hysteresis branch is observed up to $22 \text{ GW}/\text{cm}^2$ before it becomes negative at even higher intensity, while the slope is always negative in the upper transmission branch of Lorentzian-bistability case [3.8]. The unstable state is obtained by the XPM with strong pump as discussed in section 3.3.

Fig 3.11 Fano-bistability with positive slope in the upper hysteresis branch

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3.5 ALL-OPTICAL AND-GATE IN BIREFRINGENT FIBER BRAGG GRATING

In 1998, S.Pereira and J. E.Sipe investigated on the nonlinear pulse propagations inside the birefringent FBG [3.19]. All-optical AND-gate inside the birefringent FBG was found theoretically by Pereira *et al.* [3.19] and shown experimentally by Taverner *et al.* [3.20]. However, the physical explanation behind the all-optical AND gate is missing in [3.19, 3.20]. RKM is used to solve nonlinear coupled mode equations for birefringent FBG in ref [3.19].

Figure 3.12 All-optical AND-gate in birefringent Fiber Bragg Grating

The physical explanation on the all-optical AND gate can be inferred by looking the pulses propagation inside the birefringent grating as the time evolves shown in Fig 3.12. Fig 3.12 (a) show when both X-polarized and Y-polarized pulse are weak. If either strong X-polarized or Y-polarized pulse will decay when propagates alone through the grating because its wavelength is within the band gap and the nonlinearity is not strong enough to shift the pulses out of the band-gap as shown in Fig 3.12 (b) and (c). However, when both pulses are presence, they will help

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each other to propagate through the grating (d). At one point of time, one pulse with stronger intensity pushes the weaker pulse forward due to the push broom principle discussed in section 3.4, while at other point of time the reverse process happens as shown in Fig 3.13. Because of this effect, both pulses prevent each other from decaying inside the grating and finally appear compressed due to solitons effect at the grating's output as shown in Fig 3.12 part (d).

Fig 3.13 All-optical AND-gate due to Optical Push Broom effect

3.6 CONCLUSIONS

This chapter covers the study on the optical phenomena in the grating with cubic Kerr nonlinearity such as conventional silica Fiber Bragg Grating. By unstable state principle that acts as bistable threshold, three nonlinear permanent switching behaviors are observed in the probe transmissions by cross phase modulation (XPM) with varying intensity Gaussian pump. Hence, in the context of all-optical signal processing technology, nonlinear FBG can offer three potential switching applications as step-up all-optical switching, all-optical inverter, and all-optical limiter. Instead of from symmetric Lorentzian resonance, the shape of optical bistability from asymmetric fano-resonance in FBG was investigated. The upper transmission hysteresis branch of fano-bistability has positive slope up to a certain intensity which distinguish its shape with the Lorentzian's counterpart. Lastly, Optical Push Broom principle is found as the physical reason behind all-optical AND gate phenomenon in birefringent FBG.

CHAPTER 3: OPTICAL PHENOMENA IN GRATING WITH CUBIC NONLINEARITY

3.7 PUBLICATION RELATED TO THIS CHAPTER

Referee's Journal

- [1] Yosia, Shum Ping, and Lu Chao, "Bistability threshold inside hysteresis loop of nonlinear fiber Bragg gratings", Optics Express **13**, No. 13, 5127-5135 (2005).
- [2] Yosia and P. Shum, "Optical Bistability in Periodic Media with Third-, Fifth-, and Seventh-Order Nonlinearities", IEEE Journal Lightwave of Technology **25**, No.3, 875-882 (2007).

International Conference

Yosia, L.Y.Mario, and P.Shum, "Optical Bistability from Asymmetric Fano-resonance of Nonlinear Fiber Bragg Grating", CPT-X 2007 Tokyo (poster).

CHAPTER 4

NONLINEAR TRANSMISSION CHARACTERISTICS IN π PHASE-SHIFTED GRATING WITH HIGHER ORDER NONLINEARITIES

4.1 INTRODUCTION

Unlike previous chapter that discuss nonlinear transmission characteristics in 3rd order nonlinear Bragg grating, the numerical study on the continuous wave (CW) nonlinear transmission characteristics in π phase-shifted Cubic Quintic Grating (CQG) counting until 5th order nonlinearity and Cubic Quintic Septic Grating (CQSG) counting until 7th order nonlinearity are presented in this chapter. The π phase shift is inserted in the middle of the grating serving as a defect to open a narrow transmission resonance at the Bragg wavelength that subsequently produces optical bistability in nonlinear regime.

The material parameters of Polydiacetylene Tuloene Sulfonate (PTS) Crystal are used to represent Cubic Quintic (CQ) nonlinear material [4.1, 4.2] and chalcogenide-glass to represent Cubic Quintic Septic (CQS) nonlinear material [4.3]. In [4.1, 4.2] discuss about nonlinear soliton for Cubic-Quintic nonlinear material while [4.3] discuss about the 7th order nonlinearity in chalcogenide glass.

In this chapter, the CW nonlinear transmission characteristics in both CQ and CQS nonlinear material are presented. The CW nonlinear transmission characteristics show a new feature whereby Double Optical Bistability from a single resonance is formed in a certain detuning range near the Bragg wavelength for both CQG and CQSG. However, unlike the case of CQG where the second optical bistability occurs at the intensity near to the first one, the second optical bistability occurs at intensity far above the first one for CQSG.

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4.2 BACKGROUND THEORY ON CUBIC QUINTIC GRATING

Materials with self-focusing (SF) cubic and self-defocusing (SDF) quintic nonlinear response, or in short called as cubic quintic (CQ) nonlinear materials, are found to be attractive media for the formations of plethora solitons families. In 2004, Gisin *et al.* found the bistable guided solitons in the CQ medium [4.4]. Atai *et al.* investigated the solitons formations as the Bragg grating is inscribed in the CQ nonlinear material, or in short we call it as cubic quintic grating (CQG). Two solitons families are formed in the CQG [4.5]. The first type is the well-known sech solitons, while the second type is the “two-tier” solitons with a sharp but nonsingular peak. Subsequently, in 2004, Atai investigated the interaction of Bragg Gratings (BG)-solitons for a finite and infinite grating length [4.6]. The parameters that determine the interaction behavior of the first type of solitons whether they repel or attract each other are the initial phase difference φ and initial separation of the adjacent solitons, whereas the interaction always results in deformation and subsequently destruction of the second type of solitons.

On the basis of experimental observations, the CQ nonlinearity was found as an empirical description of the nonlinear dielectric response of Polydiacetylene Toluene Sulfonate (PTS) crystals as well as of special glasses and organic materials [4.1]. In our study, PTS crystal is chosen as the base material to represent CQ nonlinear medium.

While the solitons formations and interactions in the CQG have been investigated thoroughly in [4.5,4.6], this chapter focus more on the CW nonlinear transmission characteristics and bistability from a single resonance by inserting π phase-shift in the middle of the Bragg grating inscribed in PTS crystal, with cubic and quintic nonlinearities.

The refractive index variation to count the cubic and quintic nonlinearities along the CQG is given by (4.1):

$$n(z) = \bar{n} + n_1(z) \times \cos \left\{ \frac{2\pi}{\Lambda} z + \theta(z) \right\} + n_2 \times |E(z)|^2 - n_3 \times |E(z)|^4, \quad n_2 \ \& \ n_3 > 0 \quad (4.1)$$

By similar assumptions in [4.7], the electric field propagating inside the grating is written as:

$$E(z,t) = [A_+(z,t). \exp(iK_B z) + A_-(z,t). \exp(-iK_B z)]. \exp(-i\omega_0 t) \quad (4.2)$$

Substituting (4.1) and (4.2) into the scalar Maxwell's wave equations, the following set of the Cubic Quintic Nonlinear Coupled Mode Equations (CQNLCMEs) is obtained:

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$$\begin{aligned}
 & + i \frac{\partial A_+}{\partial z} + i \frac{1}{V_g} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- + \Gamma_1 |A_+|^2 A_+ + 2 \Gamma_1 |A_-|^2 A_+ \\
 & - \Gamma_2 |A_+|^4 A_+ - 6 \Gamma_2 |A_+|^2 |A_-|^2 A_+ - 3 \Gamma_2 |A_-|^4 A_+ = 0
 \end{aligned} \tag{4.3}$$

$$\begin{aligned}
 & - i \frac{\partial A_-}{\partial z} + i \frac{1}{V_g} \frac{\partial A_-}{\partial t} + \delta A_- + \kappa^* A_+ + \Gamma_1 |A_-|^2 A_- + 2 \Gamma_1 |A_+|^2 A_- \\
 & - \Gamma_2 |A_-|^4 A_- - 6 \Gamma_2 |A_-|^2 |A_+|^2 A_- - 3 \Gamma_2 |A_+|^4 A_- = 0
 \end{aligned} \tag{4.4}$$

$$\text{Where } \Gamma_1 = \frac{2 \cdot \pi}{\lambda_B} n_2 \tag{4.5}$$

$$\text{And } \Gamma_2 = \frac{2 \cdot \pi}{\lambda_B} n_3 \tag{4.6}$$

Please refer to Chapter 3 for definitions of symbols in NLCMEs. The mathematical definitions and derivations of each symbol can be found in Appendix B, when $n_6 = 0$.

The CW nonlinear transmission characteristics in CQG can be found by solving the set of CQNLCMEs numerically by Implicit 4th order Runge-Kutta Method [4.8].

The parameter values of PTS crystal to represent the CQ nonlinear material are given as follows: Average refractive index ($\bar{n}$) = 1.74 obtained by fitting a Sellmeier equation for λ at 1600 nm [4.8], cubic coefficient (n_2) = $2.2 \times 10^{-16} \text{ m}^2/\text{W}$, quintic coefficient (n_3) = $2 \times 10^{-28} \text{ m}^4/\text{W}^2$ (the improvement in the n_3 from the minimum value at $8 \times 10^{-30} \text{ m}^4/\text{W}^2$ can be realized by cascaded third order processes that is designing the appropriate wave-vector mismatch and the grating length for λ at 1600 nm [4.1]).

The grating parameters are given as follows: length (L) = 1 cm, $n_1(z) = 1.5 \times 10^{-4}$ and π phase shift is inserted in the middle of the grating. TPA is reported to be negligible for 1 cm PTS Crystal at wavelength 1600 nm [4.1], hence we neglect its effect in deriving the CQNLCMEs. The CW input λ is excited at 1600.03 nm.

4.3 CONTINUOUS WAVE NONLINEAR TRANSMISSION CHARACTERISTICS IN CUBIC QUINTIC GRATING

The CW nonlinear transmission characteristic reveals Double Optical Bistability phenomenon from a single resonance as shown in Fig 4.1.

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Fig 4.1 Double optical bistability in π phase-shifted Cubic Quintic Grating

This phenomenon can be explained by the nature of the SF- cubic and SDF-quintic nonlinearity in the medium itself. The phase-shifted grating exhibits a narrow peak transmission resonance at the Bragg wavelength in the linear regime. This peak will be twisted to the direction of negative detuning region and a bistability is consequently formed when the SF-cubic nonlinear term is dominant over the SDF-quintic term. However, when the input intensity is stronger such that the quintic term is dominant over the cubic counterpart, the transmissivity curve is distorted further to the direction of negative detuning due to the SF cubic nonlinearity but at the same time it starts to have another distortion in the direction of positive detuning due to the SDF-quintic term. The second bistability is formed in the negative detuning region where the first bistability was previously exist as shown in Fig.4.2. As a result, two optical bistability from a single resonance are formed in a certain range of negative detuning range close to the Bragg wavelength. In short, the 1st bistability is produced due to the SF-cubic term and the 2nd bistability is produced due to the SDF-quintic term.

By the cross phase modulation (XPM) Method with strong Gaussian pump described in Chapter 3 section 3.3; the unstable state is drawn in Fig 4.1. The derivation of the set of CQNLCMEs for strong probe and strong pump is given in Appendix B.

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Fig 4.2 Transmissivity curve of the π phase-shifted Cubic Quintic Grating in nonlinear regime

4.4 THEORETICAL MODEL OF CUBIC QUINTIC SEPTIC GRATING

Recently, chalcogenide glass was proposed as a strong material candidate to answer the need for all-optical signal processing due to its high nonlinear coefficient as compared to the conventional silica glass [4.9, 4.10]. Chalcogenide glass was reported not only to exhibit 3rd and 5th order nonlinearities, but also 7th order nonlinearity [4.9]. In other words, chalcogenide glass can be classified as a cubic-quintic-septic nonlinear material.

Owing to its high photosensitivity, several successful grating writing experiments in waveguide and fiber based on chalcogenide-glass have been reported [4.11 - 4.13]. Following the success of these experiments, we developed a theoretical model to describe the wave propagations in the cubic-quintic-septic nonlinear periodic media.

The refractive index variation along the grating by taking into account until 7th order nonlinearities is written as:

$$n(z) = \bar{n} + n_1(z) \times \cos \left\{ \frac{2\pi}{\Lambda} z + \theta(z) \right\} + n_2 \times |E(z)|^2 + n_4 \times |E(z)|^4 + n_6 \times |E(z)|^6 \quad (4.8)$$

Where: $n_2, n_6 > 0$ and $n_4 < 0$.

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To describe nonlinear wave propagations in chalcogenide-fiber Bragg gratings, the set of Cubic Quintic Septic Nonlinear Coupled Mode Equations (CQSNLCMEs) is derived from scalar Maxwell's wave equation in Appendix B.

The set of CQSNLCMEs is written in (4.9) and (4.10):

$$\begin{aligned}
 & i \frac{\partial A_+}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- \\
 & + \{ \Gamma_1 |A_+|^2 + 2 \Gamma_1 |A_-|^2 \} A_+ \\
 & + \{ \Gamma_2 |A_+|^4 + 6 \Gamma_2 |A_+|^2 |A_-|^2 + 3 \Gamma_2 |A_-|^4 \} A_+ \\
 & + \{ \Gamma_3 |A_+|^6 + 12 \Gamma_3 |A_+|^4 |A_-|^2 + 18 |A_+|^2 |A_-|^4 + 4 \Gamma_3 |A_-|^6 \} A_+ = 0
 \end{aligned} \tag{4.9}$$

$$\begin{aligned}
 & - i \frac{\partial A_-}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_-}{\partial t} + \delta A_- + \kappa^* A_+ \\
 & + \{ \Gamma_1 |A_-|^2 + 2 \Gamma_1 |A_+|^2 \} A_- \\
 & + \{ \Gamma_2 |A_-|^4 + 6 \Gamma_2 |A_+|^2 |A_-|^2 + 3 \Gamma_2 |A_+|^4 \} A_- \\
 & + \{ \Gamma_3 |A_-|^6 + 12 \Gamma_3 |A_-|^4 |A_+|^2 + 18 |A_-|^2 |A_+|^4 + 4 \Gamma_3 |A_+|^6 \} A_- = 0
 \end{aligned} \tag{4.10}$$

Please refer to Chapter 3 for general definitions of each symbol. The mathematical derivations of each symbol can be found in Appendix B.

Unless otherwise stated, all the notations and assumptions in this section follow closely [6]. The ratio of self-focusing cubic nonlinearity is shown to be at 1:2, and self-defocusing quintic nonlinearity is at 1:6:3, while self-focusing septic nonlinearity is at 1:12:18:4. The CW nonlinear transmission characteristic in π phase-shifted chalcogenide-Bragg grating is investigated numerically from CQSNLCMEs by Implicit 4th order Runge-Kutta Method [4.8].

The chalcogenide fiber grating parameters used in our simulations are given as follows: $L = 1$ cm, $n_{AC} = 1.5 \times 10^{-4}$, $\bar{n} = 2.45$ [4.15], Bragg wavelength is set at 1550 nm, nonlinear coefficients are: third order $n_2 = 2.7 \times 10^{-13}$ cm²/W, fifth order $n_4 = -7.8 \times 10^{-23}$ cm⁴/W², and seventh order $n_6 = 7.2 \times 10^{-33}$ cm⁶/W³ [4.16], π -phase shift is inserted in the middle of grating as a defect to reduce switching threshold by field enhancement. The defect opens a narrow transmission resonance at Bragg wavelength with Q-factor given as $\frac{\lambda_B}{\Delta\lambda}$. The CW input wavelength λ is set at 1550.015 nm.

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4.5 DOUBLE OPTICAL BISTABILITY IN CHALCOGENIDE -BRAGG GRATING

Unlike conventional silica fiber Bragg gratings (FBG) with only one optical bistability appears in its CW nonlinear switching characteristics due to third order nonlinearity $\chi^{(3)}$ [4.7], Double Optical Bistability is observed in phase-shifted c-FBGs as shown in Fig. 4.3 and 4.4 for the first and second optical bistability respectively. The first optical bistability arise due to self-focusing third order nonlinearity $\chi^{(3)}$ and happens in MW/cm² intensity range as the nonlinearity of chalcogenide glass is roughly third order higher as compared to silica counterpart, whereas the second optical bistability arise due to self-defocusing fifth-order nonlinearity $\chi^{(5)}$. However, unlike the case of CQG, the second optical bistability occurs at much higher intensity from the first hysteresis because the effect of self-defocusing fifth-order nonlinearity $\chi^{(5)}$ is tempered by self-focusing seventh-order nonlinearity $\chi^{(7)}$.

First Optical Bistability in phase-shifted chalcogenide-fibre Bragg Gratings

Fig 4.3 First optical bistability in π phase-shifted c-Fiber Bragg Grating

Fig 4.4 Second optical bistability in π phase-shifted c-Fiber Bragg Grating

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4.6 CONCLUSIONS

The CW nonlinear transmission characteristic in the phase-shifted Cubic Quintic Grating shows Double Optical Bistability from a single resonance in a certain negative detuning range close to the Bragg wavelength. Double Optical Bistability also appears from a single resonance in the CW nonlinear transmission characteristics of π phase-shifted Cubic Quintic Septic Grating. However, the second optical bistability occurs at much higher intensity from the first one.

4.7 PUBLICATIONS RELATED TO THIS CHAPTER

Referee's Journal

- [1] Yosia, P. Shum, and C. Lu, "Nonlinear switching and pulse propagation in Cubic Quintic Grating", IEEE Photonics Technology Letter **17**, No.12, 2670-2672 (2005).
- [2] Yosia and P. Shum, "Optical Bistability in Periodic Media with Third-, Fifth-, and Seventh-Order Nonlinearities", IEEE Journal Lightwave of Technology **25**, No.3, 875-882 (2007).
- [3] Yosia and P. Shum, "Double Optical Bistability and Its Applications in Chalcogenide Bragg Grating ", Physica B Elsevier Science **394**, 293–296 (2007).

International Conference

- [1] Yosia, Shum Ping, and Gong Yandong, "Double Optical Bistability phenomena in Cubic Quintic Grating", BGPP/ACOFT OSA meeting Sydney, 2005 (poster).
- [2] Yosia, S. Ping, C. Lu, "Double Optical Bistability in phase-shifted chalcogenide-fiber Bragg gratings", ETOPI7 2006 Sydney (oral).

Chapter 5

All-Optical Transistor Operation based on Bistability Principle in Nonlinear DFB InGaAsP-InP Waveguide: A Transient Perspective

5.1 Introduction

While the theoretical discussions on the relationship between unstable state and the switching in the bistable system have been covered in chapter 3, this chapter shows the experimental demonstration of this concept. All-optical transistor application following this unstable state principle is shown experimentally in DFB InGaAsP-InP waveguide with Bragg reflectors in Mizumoto Laboratory, Tokyo Institute of Technology. Some of the Optical Bistability experiments in DFB InGaAsP-InP waveguide with Bragg reflectors and nonlinear properties of this material were reported before in [5.1-5.4].

The theoretical model of nonlinear wave propagation in periodic structures under strong probe and strong pump governed by Nonlinear Coupled Mode Equations (NLCMEs) is given in section 5.2. By increasing the pump power above a certain limit, the probe transmission in the switching mode is turned into transistor mode as presented in section 5.3. The experimental results are justified by solving NLCMEs numerically to investigate the probe transmissions transient artifacts in section 5.4. The unstable state is shown to play a crucial role in distinguishing the probe transmission artifacts between the switching and transistor mode. Finally, the existence of unstable state is verified in the experiments by taking the snapshots of probe transmissions as reported in section 5.5.

5.2 Background Theory

Nonlinear Coupled Mode Equations (NLCMEs) to govern the nonlinear wave propagation inside the periodic structures under paraxial approximations and weak grating assumptions are when the strong probe and strong pump propagates concurrently inside the grating are given by:

$$i \frac{\partial A_+}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- + \Gamma_B \{ |A_+|^2 + 2|A_-|^2 + |A_p|^2 \} A_+ + R\Gamma_p |A_p|^2 A_+ = 0 \dots (5.1)$$

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$$-i \frac{\partial A_-}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_-}{\partial t} + \delta A_- + \kappa^* A_+ + \Gamma_B \{ |A_-|^2 + 2|A_+|^2 + |A_p|^2 \} A_- + R \Gamma_p |A_p|^2 A_- = 0 \dots (5.2)$$

$$i \frac{\partial A_p}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_p}{\partial t} + \frac{1}{R} \Gamma_B \{ |A_+|^2 + |A_-|^2 \} A_p + \Gamma_p \{ |A_+|^2 + |A_-|^2 + |A_p|^2 \} A_p = 0 \dots (5.3)$$

Where: $R = \frac{\lambda_B}{\lambda_p}$

The above set of NLCMEs is valid when the probe wavelength is located around the band-gap while the pump wavelength is located far from the band-gap to be unperturbed by the grating. The set of NLCMEs has been discussed before in Chapter 3. The derivation and notation definition can be found in Appendix A.

5.3 Experimental Results of All-Optical Transistor Operation

The linear transmission spectrum of the DFB waveguide and probe spectrum are shown in Fig 5.1. The probe wavelength λ is set to be around the edge of the band-gap at 1566.2 nm while its power is set in the bistable regime. The Bragg wavelength λ_B of DFB waveguide is designed at 1565.9 nm.

Fig 5.1 Linear transmission spectrum of Distributed Feedback Bragg waveguide (solid) and probe spectrum (dashed)

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While the probe wavelength is located inside the band-gap, the pump wavelength is set at 1550 nm hence unperturbed by the grating. The pump power is varied to investigate various probe transmission operating modes. The probe is set at 135 ns long, while the shorter pump is set at 50 ns long and overlapped around the middle of the probe as shown in Fig 5.2.

Fig 5.2 Input probe (solid-wider pulse) at 1566.2 nm and pump (dotted-narrower pulse) at 1550 nm

Both probe and pump output is monitored by optical oscilloscope in the time averaging mode to suppress the noise. The schematic of experimental set-up is shown in Fig 5.3.

Fig 5.3 Schematic of experimental set-up to observe all-optical transistor operation

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Three pump power regimes can be distinguished clearly where the time average of probe transmissions artifacts behave differently as shown in Fig 5.4. In the low pump regime, the probe transmission is in the switching mode (a). In the medium pump regime, the time average of probe transmission after the pump is turned off (>100 ns) always varies between the low and high state (b). Lastly, in the high pump regime, the probe transmission is in the transistor mode (c).

The term of all-optical transistor operation shown in Fig 5.4 (c) is derived based on the fact that the relatively lower pump power when it is turned on can produce high probe transmission as it operates in the high state of bistable regime. However when the pump is off, the probe transmission is low as it operates in the low state of bistable regime.

The best gain (G) of the transistor operation calculated as the ratio between the high probe transmission and the pump output is estimated to be around 6 dB. However, when the pump is off, some of the probe transmission remains as it operates in the low state of bistable regime. This is regarded as the noise in the transistor operation. The best SNR, which roughly equal to the extinction ratio of bistable switching, is estimated at 12.7 dB. The rise time was measured at 3 ns; hence limit the bandwidth of our device for transistor operation at around 333 MHz.

At this point, we would like to highlight that the trace shown in Fig 5.4 is not purely for probe transmission. However, it is the sum of the probe transmission and the pump output as the optical oscilloscope cannot differentiate the two wavelengths. The averaging of the probe signal is taken so it is possible to capture unstable mode shown in Fig 5.4 (b).

Comparing the trace in Fig 5.4(a), 5.4(b), and 5.4 (c) when the pump is turned on, one can note that the trace in Fig 5.4a is the lowest (≈ 720 μ W), in Fig 5.4 (b) is the highest (≈ 750 μ W), while in Fig 5.4 (c) is in between the two (≈ 730 μ W). As the pump power is increased from low, medium, to high regime in Fig 5.4 (a), 5.4(b), and 5.4 (c) respectively, the probe transmission is reduced in the presence of pump. The stronger pump will result in more reductions of probe transmission as the bistability curve shift to the lower intensity regime hence producing lower transmission for the same input probe intensity as explained in [5.5]. We speculate that the reduction of probe transmission is lower than the increment of pump from Fig 5.4 (a) to Fig 5.4(b) so that the trace is higher in Fig 5.4 (b). However, the reduction of probe transmission is higher than the increment of pump from Fig 5.4 (b) to Fig 5.4 (c) so that the trace is lower in Fig 5.4 (c).

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Fig 5.4 Experimental results on time average probe transmissions in the switching (a), unstable (b), and transistor (c) mode when pump is in the low (a), medium (b), and high (c) regime respectively

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5.4 A Transient Perspective of All-Optical Transistor from Numerical Simulations

To understand the three probe transmission operating modes shown in Fig 5.4, numerical simulations performed by Implicit 4th Order Runge-Kutta Method [5.6] to solve the set of NLCMEs (5.1)-(5.3) under strong probe and strong pump assumptions is shown in Fig 5.5. When the pump power is in the low regime, the probe transmission exhibits switching operation. However, as the pump power is increased, it turns the probe transmission from the switching to transistor mode.

From the transient perspective, the pump can be treated as the temporary perturbation to the bistable system as it is shorter than the probe. When the pump is turned off (indicated by the circle in Fig 5.5), the probe has to decide whether to stay at the low or high state in the bistable regime. If the probe is above the unstable state after the pump is turned off, it will settle at the high state (switching mode) otherwise it will settle at the low state (transistor mode). By cross phase modulation (XPM) with low pump power, the probe transmission dip is consequently shallow to be above the unstable state when the pump is turned off as shown by dashed line in Fig 5.5. As discussed in [5.7] and re-elaborated in Section 3.3 of this thesis, the addition of photon into the unstable branch will result in the additions of more photons. Hence, the final probe transmission settles at the high state. On the contrary, the probe transmission dip is correspondingly deep by the XPM with the high pump power when the pump is turned off as shown by solid line in Fig 5.5. Hence the probe transmission is pushed below the unstable state. Similarly, the reduction of photon into the unstable branch will result in the reductions of more photons to settle the final probe transmission at the low state.

Fig 5.5 Transient perspective of all-optical transistor from numerical simulations

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Fig 5.5 show the critical cases where the pump power is set at the maximum value of low regime and minimum value of high regime for the dashed and solid line respectively so that the unstable state line can be drawn clearly to distinguish the two probe transmission transient artifacts. The big spike in the beginning of probe transmission in the simulations shown in Fig 5.5 is because of the fast rise time of the pump. In the experiment shown in Fig 5.4, the spike is smaller because TPA of InGaAsP is not counted in equation (5.1) to (5.3) as NLCMEs only counts Kerr effect.

While there are only two pump power regimes (low and high) to differentiate the probe transmissions transient artifacts by numerical simulations, three pump power regimes (low, medium, and high) are observed in the experiments as pointed out in the section 5.3. The different pump power classification between the simulations and experiments is due to the time averaging process and influence of the noise as explained below.

In the experiment, the low pump regime is defined when the probe transmission is well above the unstable state once the pump is turned off. The noise therefore seldom brings the final probe transmission to settle at the low state. By time averaging process, the probe transmission is in the switching mode as shown in Fig 5.4(a). At this point, we would like to highlight the discrepancy between the simulation artifact and experimental result in the switching mode. In Fig 5.5, the probe transmission state when the pump is on ($t \approx 50\text{-}100\text{ ns}$) is lower than after the pump is turned off ($t > 100\text{ ns}$). This is because the bistability curve is shifted to the lower intensity regime in the presence of the pump as explained in [5.5]. As the bistability curve is shifted to the left, the transmissivity and consequently the output drops for the same input power. However, the experimental result shown in Fig 5.4(a) is in contradiction with the simulation artifact, i.e. the trace in the middle state (when the pump is on) is higher than that in the final state (when the pump is turned off). The reason is because the trace in the experiment consists of the sum between the probe transmission and the pump output as the optical oscilloscope cannot differentiate the two different wavelengths.

In medium pump regime, the time averaging of probe transmissions always vary in between but settle in neither low nor high state as shown in Fig 5.4(b). This is because the noise mainly generated by EDFA often causes the probe transmissions to settle at the "wrong" state. Hence, the time averaging of probe transmissions just gives roughly the value between the low and high state. We call this as the noise-prone regime.

Lastly, in high pump regime, the probe transmission is well below the unstable state once the pump is turned off. The noise therefore seldom brings the final probe transmission to the high

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state. By time averaging process, the probe transmission show consistent all-optical transistor operation as shown in Fig 5.4 (c).

5.5 Verification of the Unstable State in Experiment

As the unstable state is shown to play a crucial role in distinguishing probe transmissions transient artifacts by numerical simulations in section 5.4, its existence is verified in the experiment by setting the probe transmission in the unstable mode as described in section 5.3, Fig 5.4(b). However, the snapshots by interleaved sampling (instantaneous measurements) are taken instead of time averaging so that at one point of time the probe transmission is in the switching operation but at other point of time it is in the transistor operation as shown in Fig 5.6.

The Snapshots of Switching (solid) and Transistor (dotted) Operations

Fig 5.6 Snapshots of probe transmissions in the switching (solid) and transistor (dotted) operations

As the probe transmission operates in the unstable mode, its transient artifact is around the lowest point during switching operation while it is around its highest point during transistor operation after turning off the pump (at $t \approx 100$ ns). Therefore, the unstable state line can be drawn clearly to distinguish the two transient artifacts by zooming in Fig 5.6 at the time when the pump is just turned off (indicated by the ellipse) as shown in Fig 5.7.

CHAPTER 5: ALL-OPTICAL TRANSISTOR IN InGaAsP-InP DFB WAVEGUIDE

Fig 5.7 Unstable state line (dashed) is drawn as boundary between switching (solid) and transistor (dotted) operations

5.6 Conclusions

The experimental demonstration of all-optical transistor operation based on bistability principle in DFB InGaAsP-InP waveguide is reported. Three pump power regimes in the experiments were observed as the probe transmissions operates in the switching, unstable, and transistor mode for the low, medium, and high pump power regime respectively. The unstable state is shown as a boundary to distinguish the probe transmission transient artifacts between the switching and transistor operations by numerical simulations and by taking probe transmission snapshots when it is operating in the unstable mode.

5.7 Publications Related to This Chapter

Referee's Journal

- [1] Yosia, Y.Akano, K. Tamura, T. Mizumoto, and P.Shum, "All-Optical Transistor Operation Based on Bistability Principle in Nonlinear DFB GaInAsP-InP Waveguide: A Transient Perspective", Journal Optical Society of America B **24**, No.7, 1584-1588 (2007).
- [2] Yosia and P. Shum, "Double Optical Bistability and Its Applications in Chalcogenide Bragg Grating ", Physica B Elsevier Science **394**, 293–296 (2007).

International Conference

Yosia, Y.Akano, K. Tamura, T. Mizumoto, and P.Shum, "All-Optical Transistor Operation Based on Bistability Principle in Nonlinear DFB GaInAsP-InP Waveguide: A Transient Perspective", OECC/IOOC 2007, Yokohama, Japan (Oral).

Chapter 6

All-Optical Bistability in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ Photonic Crystal Resonators

6.1 Introduction and Motivation

In 2005, Notomi *et al.* achieved optical bistability from silicon photonics crystal (PhC) resonator [6.1]. One promising application of optical bistability in PhC resonator is all-optical on chip memory for future all-optical network due to its ability to confine the light in very small cavity volume [6.2]. The concept behind optical memory by bistability in photonics crystal material is when a stream of bits coming from input signal, the occurrence of 1-bit in the input signal can induce the switching from low to high state when the bias operates in the bistable regime. Thus, the switching to "ON" state in the output indicates that 1-bit from the input signal has been recorded. In other words, the optical bistability phenomena can function as *1-bit all-optical memory*. However, the memory time ("ON" time) can only last for few-nanoseconds in silicon photonics crystal resonator due to the competing nature of two optical nonlinearities, namely: plasma carrier effect and thermo-optic effect [6.1].

The need to improve the memory time has motivated Photonics Nanostructure Research Group, NTT, Basic Research Laboratory, Japan to explore various new materials beside silicon such as AlGaAs and InGaAsP-InP Photonics Crystal resonator for all-optical memory application. In this chapter, all-optical memory performance based on optical bistability in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ photonics crystal resonators and its origin of nonlinearity are discussed. The optical bistability measurement is discussed in section 6.2. Section 6.3 discuss about the origin of various nonlinearities inducing bistability in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ photonics crystal resonators and the method to distinguish them.

6.2 Optical Bistability in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ Nonlinear Photonic Crystal Resonator

One of the experimental results of optical bistability in $\text{Al}_{0.18}\text{Ga}_{0.82}\text{As}$ PhC resonator is shown in Fig 6.2. The cavity property of this resonator is as follows: Quality factor $Q = 83,052$ and resonant wavelength $\lambda_R = 1588.65 \text{ nm}$.

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Several characteristics of optical bistability in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ PhC Resonator are as follows. The experimental set-up is the same as shown in Fig 5.2, but the device under test is $\text{Al}_x\text{Ga}_{1-x}\text{As}$ PhC resonator. The schematic diagram of 2-D semiconductor photonic crystal resonator is shown at Fig 6.1

Fig 6.1 Schematic of 2-D Photonic Crystal

1. The set pulse required to induce the switching is relatively long ($\geq$ ns). Below this width, the measurement becomes unstable. This is because the dominant nonlinear effect to cause the switching is *thermo-optics* effect in this material which requires a long time for the power to be accumulated in the cavity before the switching.

Fig 6.2 Red-shifted optical bistability in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ photonic crystal resonator based on Thermo-Optics Effect

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2. As the dominant nonlinear effect to cause the switching is thermo-optic effect, the output will still be at "ON" after the switching happen even when the input is turned "OFF" for time less than thermal lifetime ($\tau_{\text{THERMAL}} = 100 \text{ ns}$) as shown in Fig 6.3 for input and output respectively. The measurements were taken for various widths of set pulses. This is because the thermal effect will disappear only after one thermal relaxation time (100 ns). Fig 6.4 shows the calculation of nonlinear effect contribution from Kerr, plasma carrier, and thermo-optics effect respectively in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ PhC resonator based on coupled mode equations in [6.3]. It is obvious that the thermal nonlinearity keeps the memory at "ON" state even when the input is turned "OFF". However, it is not clear which nonlinearity induces the switching because it is mixed between thermal and Kerr effect for 1 ns long set pulse.
3. The thermo-optic and Kerr nonlinearities induce red-shifted bistability. In other words, the bistability is observed for higher wavelength than resonance wavelength.
4. The memory "ON" time can last indefinitely

Fig 6.3 Thermal effect keeps the memory time "ON" after switching even when the input pulse is reset

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Fig 6.4 Contribution of Kerr, thermal, and plasma carrier nonlinear effects (courtesy – Dr. Takasumi Tanabe NTT)

6.3 Distinguishing between Kerr and Thermal Effect

In the previous section, the switching in the bistable regime is due to mixed nonlinearities: Kerr and thermal effect as both nonlinearities induce red-shift in the resonance. However, this section discusses on how to distinguish the two, and thus how to achieve fast-Kerr dominant bistability in the future experiment. For this purpose, the input pulse is designed as shown below.

Fig 6.5 Input pulse to distinguish Kerr and Thermal effect in the bistable regime

Note:

[1] The set pulse is short (≤ 50 ps) but much stronger than bias to reduce thermal contribution.

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[2] The distance between the set pulse and reset pulse is set at several cavity lifetime to avoid accumulation of thermal effect

[3] The width of reset pulse is several cavity lifetime to eliminate Kerr nonlinearity only

The set pulse is set to be 50 ps or less as the purpose is to observe pure Kerr dominant nonlinearity. Secondly, the distance between set pulse and reset pulse is several cavity lifetime so that the thermal effect does not have time to accumulate after the output is switched. Thirdly, the reset pulse is set at several cavity lifetimes to eliminate Kerr effect only. Thus, the output after reset is only due to thermal effect. If the switching by the set pulse were motivated by Kerr effect, the output will be at "OFF" state after reset pulse. However, if the switching were motivated by thermal effect, the output will be at "ON" state. There are 2 possible methods to convert thermal dominant bistability to Kerr dominant counterpart as shown in Fig 6.6. By increasing Q factor, Kerr effect increases linearly but thermal effect reduces gradually for the same detuning. At a certain Q-factor of the cavity, the thermal dominant nonlinearity will finally be converted into Kerr dominant counterpart. Secondly by reducing the operating bistable power, the Kerr effect reduces linearly but the thermal effect reduces quadratically for the same detuning wavelength. Thus, the thermal dominant nonlinearity can be converted to the Kerr dominant counterpart for the same Q factor. The second method is more feasible for the experiment.

Fig 6.6 Method to convert Thermal dominant bistability to Kerr dominant counterpart

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Definitions of each symbol in Fig 6.6

P is the input power to cavity, n_2 is nonlinear, β_2 is two photon absorption coefficient, Q is quality factor of cavity, n is scaling factor

The simulation results for the output pulse at aluminum content $x = 18\%$ is shown in Fig 6.7 and Fig 6.8 for input power $100 \mu\text{W}$ and $25 \mu\text{W}$ respectively taking into account Kerr and thermal effect in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ PhC Resonator. Where the input is not reset, the optical bistability is shown by black line. However, when the input is reset according to the method shown in Fig 6.5, the output is shown by blue line. At Fig 6.7, because the output is at “ON” state after the reset pulse at $Q = 200,000$, thermal nonlinearity is more dominant. However, because the output is at “OFF” state after reset pulse at $Q = 210,000$, Kerr nonlinearity is more dominant. In other word, the transition from thermal to Kerr nonlinearity is at Q between 200,000 and 210,000. Similar explanation is for Fig 6.8. It is shown that by reducing the input power, Q transition has reduced from 200,000 to around 40,000.

Fig 6.7 Thermal dominant (left) converted into Kerr dominant (right) bistability for input power $100 \mu\text{W}$

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Fig 6.8 Thermal dominant (left) converted into Kerr dominant (right) bistability for lower input power $25 \mu\text{W}$

It is interesting to see how much the reduction factor in the Q-transition. Note that for the same detuning, Kerr effect is reduced by Q times the operating power P. However, thermal effect is affected by the square of the operating power, with comparably negligible effect on the Q. Therefore, *the reduction of Q-transition is roughly linear with the reduction in power*. The simulations were also done for 26% aluminum content, as 26% aluminum content has lower Two Photon Absorption (TPA) coefficient thus expected easier to observe Kerr effect. The table of $\text{Al}_x\text{Ga}_{1-x}\text{As}$ material parameters is tabulated in Table 6.1. It is shown indeed that the Q-transition reduces for the same operating power as compared to 18% aluminum content counterpart. The summary of Q-transition from thermal to Kerr dominant bistability for $x = 18\%$ and 26% for both high and low power are tabulated in Table 6.2.

Table 6.1 Material Parameters for $\text{Al}_x\text{Ga}_{1-x}\text{As}$ (Courtesy – Dr. Akihiko Shinya, NTT)

	$\text{Al}_x\text{Ga}_{1-x}\text{As}$	Reference
Eff. Carrier lifetime [ps]	12.25	[6.4-6.6]
Thermal lifetime: [ns]	100	
Thermal generation speed [ps]	12.25	Bulk = 100 [6.4-6.6]
Photon lifetime: [ps]	16.457 for $Q=20,000$	
TPA coeff. b [cm/GW]	0.4 for $x = 0.18$, 0.34 for $x = 0.26$	[6.8],[6.9] for conversion rule

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Mode volume [μm^3]	0.14	
Refractive index	3.3	[6.9] for conversion rule
Band filling & plasma effect Dn/c(N,P) (c: carrier concentration cm-3])	-2.7×10^{-21}	[6.8,6.11]
n2: cm^2/W	3.8×10^{-13} for $x = 0.18$ 3×10^{-13} for $x = 0.26$	[6.8] [6.9] for conversion rule
TO coeff. [1/K]	1.96×10^{-4} for $x = 0.26$ 1.99×10^{-4} for $x = 0.18$	[6.7]
Density [g/cm3]	5.072	5.36-1.6x [6.10]
Heat capacity [pJ/g/K]	3.57×10^{11}	$0.08 + 0.03x$ [cal/g/K] [6.10]

Table 6.2 Q-transition for $\text{Al}_x\text{Ga}_{1-x}\text{As}$

	High P (100 μ Watt)	Low P (25 μ Watt)
X = 0.18	200,000	40,000
X = 0.26	160,000	35,000

6.4 Conclusions

Optical Bistability in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ PhC Resonator is presented. Two aluminum contents were used in the experiment 18% and 26%. Various sources of nonlinearities such as: Kerr, plasma carrier, and thermal effect contribution are counted separately. It is shown by simulation that when Q factor of resonator is low, the thermal effect is more dominant than Kerr nonlinearities and vice versa. The Q-transition from thermal optic to Kerr effect happens is lower when we operate at low power bistability.

6.5 Publications Related to This Chapter

A.Shinya, Yosia, T.Tawara, T.Tanabe, K.Tateno, E. Kuramochi, M. Notomi," Optical nonlinear response and bistability in high-Q AlGaAs photonic crystal nanocavity", JSAP conference (2008).

Chapter 7

Critical Slowing Down Phenomena and Its Applications in Nonlinear InGaAsP Photonic Crystal Resonator

7.1 Introduction

While the previous chapters focus on the optical bistability phenomena, this chapter focuses more on the transient artifacts in a certain anomalous bistable regime namely Critical Slowing Down (CSD) phenomena to investigate the switching time required to settle at the high state. CSD is general phenomena in nonlinear cavity where the switching time needed to reach steady state become infinitely slow in the anomalous bistable regime [7.1-7.4].

CSD phenomena from nonlinear cavity could be observed for various detuning in anomalous bistable regime by changing the input rise time τ . Slower τ will result in smaller detuning for which CSD happens. However when τ small enough such that the anomalous bistable regime disappears, CSD does not happen. The numerical simulation on the CSD contour map for various τ is given in section 7.2. The amount of slowing down effect is longer for smaller detuning. The CSD contour map in λ , time, and τ regime is also shown in this section.

The physical explanation and mathematical insight on the CSD phenomena is presented in section 7.3. Basically CSD happens due to the balance between the coupling of light into the cavity and the leakage from it that depends on the Q-factor of the cavity itself. The hallmark of CSD is the occurrence of the output transmission in the long unstable state before the switching.

The main contributions in this chapter are the observation of CSD and anomalous switching from nonlinear cavity in InGaAsP-InP Photonics Crystal (PhC) are presented in section 7.4.

To investigate some of CSD applications, some numerical simulations on multiple pulses propagation in nonlinear cavity are discussed in section 7.5. Three pump power regimes are identified for which the probe transmission artifacts behave differently for each regime. In low pump power regime, the probe switching time is faster for stronger pump power. Conversely, in

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medium pump power regime, the probe switching time is slower for stronger pump power. In these two regimes, the device can function as amplitude-to-time converter. Lastly, in high pump power regime, the pump power can control the probe final transmission state (ON/OFF). Thus the device function as optical threshold circuitry. As the proof of this concept, controlling the switching time by set pulse for various detuning has also been observed in the experiment, thus makes the device as wavelength-to-time converter.

Finally, conclusion is given in section 7.6.

7.2 Critical Slowing Down Numerical Simulations

The nonlinear wave propagation inside periodic structure with weak refractive index modulation can be described by the set of Nonlinear Coupled Mode Equations (NLCMEs) [7.5] as described previously in Chapter 3. While the previous chapters focus on the physical reason behind nonlinear switching in the bistable system and some of its applications from nonlinear cavity, this chapter focuses more on the *transient artifacts during the switching* in a certain anomalous bistable regime.

The normalized input CW is modeled by $I(t) = 1 - \exp(-t/\tau)$. By varying τ , the switching times for various detuning during transient are investigated. As the switching power per modal volume (P/V) is equivalent for the same cavity spectrum [7.6], the discussion about the switching power, nonlinear refractive index of the material, and the device modal volume itself are neglected. The numerical simulation artifact below is generally valid for any cavity in general with dispersive nonlinearity, regardless whether the cavity is formed in the grating (weak refractive index modulation) or photonics crystal (high refractive index modulation). In our simulations, the cavity properties are given as follows: $Q = 232,000$, $\lambda_R = 1551.46$ nm, hence $\tau_{\text{CAVITY}} \approx 191$ ps. In nonlinear regime with positive Kerr nonlinearity, the peak resonance shift towards longer wavelength and the cavity becomes asymmetric as shown in Fig 7.1 by numerical tools developed in [7.7]. The gradual increase in nonlinear transmission spectrum is observed until around 1551.47 nm followed by sudden drop in transmission. The colorbar represent transmissivity. The model of nonlinear switching of resonator by Kerr effect is given by Nonlinear Coupled Mode Equations solved by Runge-Kutta Method as described in Chapter 3 before.

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Fig 7.1 Cavity in the nonlinear regime

From Fig 7.1, it is not clear how the transient switching is being slowed down. Fig 7.2(a), (b), and (c) shows the contour map of output probe transmission for $\tau = 25$ ps, 145.5 ps, and 250 ps near the edge of transmission resonance in nonlinear regime just before the drop.

Fig 7.2 (a) Input $\tau = 25$ ps, closer look reveals CSD happens at near 1551.4702 nm

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Fig 7.2 (b) Input $\tau = 145.5$ ps, CSD happens at near 1551.47 nm

Fig 7.2 (c) Input $\tau = 250$ ps, slowing down happens at near 1551.46995 nm

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Two conclusions can be drawn from Fig 7.2. Firstly, the detuning for which CSD happens is smaller for the case of higher τ (slower input rise time). Secondly, the slowing down factor (amount of slowing down) is higher for smaller detuning. However, the CSD contour shown in Fig 7.2 does not reveal the existence of anomalous bistable regime. To investigate CSD, it is more appropriate to look into the 2-D plots as shown in Fig 7.3. The plot for $\tau = 25$ ps varying λ , $\lambda = 1551.47$ nm varying τ , and $\tau = 250$ ps varying λ are given in Fig 7.3 (a), (b), and (c) respectively.

Fig 7.3 (a) Input $\tau = 25$ ps, CSD happens for $\lambda = 1551.4708$ nm

Fig 7.3 (b) CSD at $\lambda = 1551.47$ nm happens when input $\tau = 145$ ps

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Fig 7.3 (c) Input $\tau = 250$ ps, slowing down happens at $\lambda = 1551.46995$ nm

From Fig 7.3(a) and (b), before the probe transmission settle at the high state, they always pass through flat unstable state (between 1~2 ns for Fig 7.3(a) and between 1~6 ns for Fig 7.3(b)). This indicates the presence of anomalous bistable regime and thus anomalous switching. However, this is not the case in Fig 7.3(c). The output probe transmissions never pass through a flat curve (indicating the unstable state does not exist) before settle at high or low state. Hence the term CSD is not appropriate since anomalous switching does not happen. The slowing down effect is more appropriate to describe Fig 7.3 (c). Fig 7.4(a) illustrates the CSD transmission contour map for various input τ in λ and t -domain, while Fig 7.4(b) illustrates the location of anomalous bistable regime for which CSD happens.

Fig 7.4 (a) CSD contour map for various input τ

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Fig 7.4 (b) Effect of input τ in the anomalous bistable regime

7.3 Mathematical Derivation and Physical Explanation of Critical Slowing Down

The mathematical prove of CSD is given as follows.

[1] The output rate of change is an output dependent function:

$$\frac{dO}{dt} = Q(O) \dots (7.1)$$

Where O is output, and $Q(O)$ is the output dependent function

During critical slowing down $O = O^*$, hence: $\frac{dO}{dt} = Q(O^*) = 0 \dots (7.2)$

[2] By first order Taylor expansion (7.2) is written as:

$$\frac{dO}{dt} = Q(O^*) + \left. \frac{dQ}{dO} \right|_{O=O^*} (O - O^*) \dots (7.3)$$

Setting:

$$\lambda = \left. \frac{dQ}{dO} \right|_{O=O^*} \dots (7.4)$$

$$\varepsilon = (O - O^*) \dots (7.5)$$

[3] Therefore, at the point when critical slowing down happen:

$$\frac{d\varepsilon}{dt} = \lambda \varepsilon \dots (7.6)$$

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[4] The solution of 1st order ODE above is given as: $\varepsilon(t) = \varepsilon(0) \exp(\lambda t) \dots (7.7)$

There are 3 possible cases:

- $\lambda > 0 \rightarrow$ Anomalous switching to high state
- $\lambda < 0 \rightarrow$ No anomalous switching
- $\lambda = 0 \rightarrow$ Critical slowing down (at unstable state)

Looking from physical point of view, as the cavity is charged by input light $\frac{dI}{dt} > 0 \rightarrow \frac{\partial O}{\partial t} > 0$

However at the unstable state $\frac{dO}{dI} < 0$ as discussed in [7.8], that means when $\frac{dI}{dt} > 0$

$\rightarrow \frac{\partial O}{\partial t} < 0$. This represents leakage from the cavity. By balancing the two effects, the total rate of change of the output can be close to zero for which CSD happens, and the output transmission is trapped at the unstable state.

7.4 Critical Slowing Down Experimental Results from InGaAsP-InP Photonic Crystal Resonator

CSD is observed in the experiment from nonlinear cavity with Q factor around 42,300 in InGaAsP-InP photonics crystal. The input pulse is a 500 ns long probe with set pulse in the middle as shown in Fig 7.5 (1 Div = 100 ns). The experimental set-up is the same as in Chapter 5 and 6. Only the sample is changed.

Fig 7.5 Input pulse to InGaAsP photonic crystal resonator (1 Div = 100 ns)

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As the nonlinearity has negative value in this material, blue-shift bistability is observed in the experiment. As the wavelength is reduced, the output response time becomes slower as shown in Fig 7.6. At CSD, the long unstable state is observed before the output switch “ON”. After CSD happens, the output transmission is at “OFF” state for even lower wavelength.

Fig 7.6 Critical slowing down phenomena from InGaAsP photonic crystal resonator experiment

Fig 7.7 shows the anomalous switching in the certain detuning regime from resonance with Q-factor at 135,000 InGaAsP PhC resonators, where the output goes to the high state first before switch to the low state. The switching from low to high state becomes slower for increasing detuning (decreasing λ), on the contrary the switching from high to low state becomes faster.

Fig 7.7 Controlling switching time by set pulse for various λ

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7.5 Critical Slowing Down Applications

To investigate CSD application, the simulations on multiple wave propagation inside nonlinear cavity is performed by numerical tools in [7.7]. While the probe rise time τ is adjusted such that the output response time exhibits CSD, the pump power is varied. There are 3 pump power regimes: low, medium, and high for which the probe output transmission artifacts behave differently as shown in Fig 7.8(a), (b), and (c) respectively. The pump is sent when the output probe transmission is trapped in the unstable state.

Fig 7.8 (a) Low pump power regime

Fig 7.8 (b) Medium pump power regime

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Fig 7.8 (c) High pump power regime

In low pump power regime, the switching time is reduced as the pump power is increased. The reason is because the pump power adds some photons into the probe in the unstable state by cross phase modulation (XPM). Consequently, the addition of photons into the unstable state will generate more photons [7.8]. The higher pump power will add higher number of photons into the unstable state, hence the output transmission settle faster to the high state. Conversely, in medium pump power regime, Optical Push Broom (OPB) phenomena happen [7.9]. OPB will concentrate the output transmission peak at one point of time followed by the dip. The higher pump power will produce higher peak transmission and thus created shallower dip [7.10]. Since it takes more time for the output transmissions to settle at the high states for shallower dips, therefore higher pump power will result in longer switching time. In the low and medium pump power regime, the device can function as pump amplitude-to- delay converter. Finally, in high pump power regime, the dip is shallow enough so that at a certain pump power threshold the output probe transmission cannot recover above the unstable state. Therefore the final output transmission is at the OFF state. The device thus behaves as optical threshold circuitry in this regime.

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As a proof of this concept, the wavelength is varied instead of set pump power in the nonlinear experiment from InGaAsP-InP Photonics Crystal Cavity. It is shown that the switching time can be controlled by varying the wavelength (detuning) as shown in Fig 7.9. As the detuning is increased (δ), the unstable state is higher. For the same set pulse power, it is more difficult to add photons to reach beyond the unstable state. Thus, the switching time is longer as shown by the red line as compared to the black one. Thus, the device behaves as wavelength-to-time converter.

Fig 7.9 Switching time Vs detuning δ for the same set pulse (courtesy by: Dr. Akihiko Shinya – NTT)

7.6 Conclusions

CSD phenomena have been shown theoretically and experimentally in InGaAsP-InP photonics crystal resonator. CSD happens at the specific detuning, for which light coupled into the cavity balances the leakage from it. For longer detuning, faster input rise time is necessary to observe CSD. The slowing down factor is smaller for longer detuning. By varying the set pulse but fixing the wavelength or vice versa, it was shown that the switching time can be controlled in the simulations and experiments. Amplitude-to-delay or wavelength-to-delay converter suggests a prominent application of CSD phenomena.

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7.7 Publication Related to This Chapter

[1] Yosia, Akihiko Shinya, Takasumi Tanabe, Masaya Notomi, and Lu Chao, "Critical Slowing Down in InGaAsP-InP Nonlinear Photonics Crystal Resonator", CLEO/QELS, San Jose, California (2008) (oral).

[2] Akihiko Shinya, Shinji Matsuo, Yosia, Takasumi Tanabe, Eiichi Kuramochi, Tomonari Sato, Takaaki Kakitsuka, Masaya Notomi; "All-Optical On-Chip Memory Based on Ultra High Q InGaAsP Photonic Crystal Nanocavity", CLEO/QELS, San Jose, California (2008).

CHAPTER 8: CONCLUSIONS

CHAPTER 8

CONCLUSIONS AND FUTURE RESEARCH

8.1 Conclusions

The numerical studies on the continuous wave (CW) nonlinear transmission characteristics in cubic, cubic-quintic, and cubic-quintic-septic periodic structures are presented. Beside of simulations, experimental works were done to prove the validity of numerical studies on all-optical transistor operations based on bistable principle in InGaAsP DFB waveguide. The experimental results agree with simulations.

Throughout the whole PhD projects, two structures of periodic structures are used in the experiments: Grating and Photonic Crystal. As for Photonic crystal, two materials were used in the experiments to investigate various nonlinear phenomena: InGaAsP, and AlGaAs. Bistable experiments were done in InGaAsP to obtain Critical Slowing Down phenomena. While in AlGaAs, the objective is to distinguish various sources of nonlinearity in bistable curve : kerr, plasma carrier, and thermo-optic effect. Subsequently, a proposed method is introduced to get pseudo-kerr nonlinearity for bistability.

Numerical simulations were performed to support the validity and give physical explanations of experimental results in both structures.

Several simulation findings in nonlinear periodic structures are given as follow:

1. By unstable state principle that acts as bistable threshold, three nonlinear permanent switching behaviors are observed in the probe transmissions by cross phase modulation (XPM) with varying intensity Gaussian pump. Thus, in the context of all-optical signal processing technology, FBG offers three potential applications as step-up all-optical switching, all-optical inverter, and all-optical limiter.
2. The shape of optical bistability from asymmetric fano-resonance in nonlinear FBG was investigated. The upper hysteresis branch of fano-bistability has positive slope $\left(\frac{\partial T}{\partial I_{in}} > 0\right)$ up to a certain intensity which distinguish its shape with the Lorentzian's counterpart.
3. Optical Push Broom effect is found as the physical reason behind all-optical AND gate phenomena in nonlinear birefringent FBG.

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4. The CW nonlinear transmission characteristics in the phase-shifted cubic-quintic grating such as PTS-Crystal with Bragg gratings shows Double Optical Bistability phenomenon from a single resonance in a certain negative detuning window close to the Bragg wavelength. Similar phenomena is also appears in the nonlinear transmission characteristics of π phase-shifted cubic-quintic-septic grating such as chalcogenide-Bragg gratings.

Some experimental findings in periodic structures are given as follow:

5. Based on unstable state principle, all-optical transistor operation was demonstrated in InGaAsP-InP DFB waveguide with Bragg reflectors.
6. All-optical memory performances based on bistability are investigated in $\text{Al}_x\text{Ga}_{1-x}\text{As}$ photonics crystal resonator experimentally. A method to distinguish two major nonlinear effects of Kerr and thermo-optic can be distinguished by Q-factor of cavity and forming the appropriate input pulse shape into the resonator. This has been proven conceptually by simulations.
7. Critical Slowing Down (CSD) phenomena has been demonstrated experimentally in InGaAsP-InP PhC resonator. Physical explanation and mathematical derivation of CSD phenomena have been presented. The hallmark of CSD is the occurrence of long unstable state in the output transmission before the switching. CSD happens when the charging of light into the cavity balance the leakage from it.

8.2 Future Research

Optical switching by bistability in improving the memory time by optimizing the percentage of material contents (x and y) of $\text{In}_x\text{Ga}_y\text{AsP}$ for all-optical memory application is quite promising for future research direction. So far, the best memory time in $\text{In}_x\text{Ga}_y\text{AsP}$ PhC is limited to 150 ns due to competing effect between plasma carrier and thermo-optics effect in the material [8.1].

Achieving pseudo-Kerr nonlinearity in AlGaAs Photonic Crystal resonator (PhC) by increasing-Q factor method as explained in Chapter 6 experimentally is also one of the promising research direction. Although has been proven conceptually, this has not been done due to limitations of high-power set pulse in our lab during PhD. Mode locked high-power femtosecond laser can be used to generate strong and short set pulse to achive pseudo-Kerr nonlinearity in AlGaAs PhC resonator.

Optical multistability for coupled cavity resonator (CROW) is one of new research direction. Optical Bistability results from single cavity. However, coupled cavities can produce multiple-peak

CHAPTER 8: CONCLUSIONS

resonators in linear regime [8.2,8.3]. By designing appropriate shape of multiple-peak resonators, we can theoretically achieve multiple peak resonators as written in some literatures. In nonlinear regime, these multiple peak resonators can form optical multistability. Further research can open the view on the stability property of every branch in optical multistability, as well as relationship between multiple peak resonators in linear regime and shape of optical multistability.

In the area of biosensing application, linear cavity in PhC has been utilized to detect the amount of impurities by how much it detunes from original position [8.4,8.5]. While some literatures have shown biosensing application from linear cavities, few or none literature shows utilization of nonlinear cavities for biosensing applications. The advantages and disadvantages of using nonlinear cavity for biosensing application could be compared qualitatively and quantitatively with linear counterpart.

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- [1] Yosia, Shum Ping, and Lu Chao, "Bistability threshold inside hysteresis loop of nonlinear fiber Bragg gratings", *Optics Express* **13**, No. 13, 5127-5135 (2005).
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APPENDIX A: THE DERIVATION OF NLCMEs for STRONG PROBE AND STRONG PUMP

Appendix A: The Derivation of Nonlinear Coupled Mode Equations for Strong Probe and Strong Pump in Bragg Gratings

Several assumptions to derive Nonlinear Coupled Mode Equations (NLCMEs) for Multiple Waves Propagation in Bragg Grating are:

- [1] The probe and the pump are both strong (high power).
- [2] The probe is located near Bragg wavelength, hence it consist of forward and backward propagating waves.
- [3] The pump is located far from Bragg wavelength and hence unperturbed by the grating.
- [4] The probe and pump are in the same polarization state (SOP).

Writing the electric field inside the grating under 4 assumptions above:

$$E = A_+(z, t) \exp(i[K_B z - \omega_B t - \Omega_0 t]) + A_-(z, t) \exp(i[-K_B z - \omega_B t - \Omega_0 t]) + A_p(z, t) \exp(i[K_p z - \omega_p t]) \dots (1), \text{ where : } K_B = \frac{2\pi n_0}{\lambda_B} \& K_p = \frac{2\pi n_0}{\lambda_p}$$

Ω_0 Indicates that the probe is slightly detuned from the Bragg frequency.

A_+ = forward wave

A_- = backward wave

ω_B = Bragg frequency

ω_p = Pump frequency

Note: the actual propagation constant can vary from imaginary to real values depending on the probe wavelength. If the probe is inside the band gap, the propagation constant k will be imaginary and therefore the field decays inside the grating. On the contrary, if the wavelength is far from the Bragg to produce some transmissions.

APPENDIX A: THE DERIVATION OF NLCMEs for STRONG PROBE AND STRONG PUMP

To get Nonlinear Coupled Mode Equations (NLCMEs), we start from scalar Maxwell's wave equation:

$$\frac{\partial^2 E}{\partial z^2} - \frac{n^2}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \dots (2)$$

Writing scalar Maxwell's wave equation for A_+ only under assumption of slowly varying approximation:

$$\frac{\partial^2 A_+}{\partial z^2} \approx i2K_B \frac{\partial A_+}{\partial z} - K_B^2 A_+ \dots (3)$$

$$\frac{\partial^2 A_+}{\partial t^2} \approx -i2(\omega_B + \Omega_0) \frac{\partial A_+}{\partial t} - (\omega_B + \Omega_0)^2 A_+ \dots (4)$$

Writing: $n = n_0 + n_1 \frac{[\exp(iK_0) + \exp(-iK_0)]}{2} + n_2 |E|^2 \dots (5)$, where : $K_0 = 2K_B$

n_0 = average refractive index of core material

n_1 = strength of refractive index modulation

n_2 = Kerr nonlinear refractive index

Under weak perturbation and nonlinearity: $n_1 \ll n_0$ & $n_2 \ll n_0$

The second term in scalar Maxwell's wave equation can be written as:

$$-\frac{n^2}{c^2} \frac{\partial^2 A_+}{\partial t^2} \approx \frac{K_B}{c} \left\{ n_0 + n_1 [\exp(iK_0) + \exp(-iK_0)] + 2n_2 |E|^2 \right\} \left\{ i2 \left(1 + \frac{\Omega_0}{\omega_B} \right) \frac{\partial A_+}{\partial t} + \left(\omega_B + 2\Omega_0 + \frac{\Omega_0^2}{\omega_B} \right) A_+ \right\} \dots (6)$$

As the probe is slightly detuned from the Bragg condition: $\Omega_0 \ll \omega_B$

$$-\frac{n^2}{c^2} \frac{\partial^2 A_+}{\partial t^2} \approx \frac{K_B}{c} \left\{ n_0 + n_1 [\exp(iK_0) + \exp(-iK_0)] + 2n_2 |E|^2 \right\} \left\{ i2 \frac{\partial A_+}{\partial t} + (\omega_B + 2\Omega_0) A_+ \right\} \dots (7)$$

Adding (3) and (7), we arrive at (8):

APPENDIX A: THE DERIVATION OF NLCMEs for STRONG PROBE AND STRONG PUMP

$$i2K_B \frac{\partial A_+}{\partial z} - K_B^2 A_+ + \frac{K_B}{c} \left\{ n_0 + n_1 [\exp(iK_0) + \exp(-iK_0)] + 2n_2 |E|^2 \right\} \left\{ i2 \frac{\partial A_+}{\partial t} + (\omega_B + 2\Omega_0) A_+ \right\} + \text{nonlinear_term_} A_- + \text{nonlinear_term_} A_p = 0 \dots (8)$$

Simplifying (8):

$$i2K_B \frac{\partial A_+}{\partial z} - K_B^2 A_+ + K_B^2 A_+ + 2 \frac{K_B n_0 \Omega_0}{c} A_+ + \frac{K_B n_1 \omega_B}{c} A_- + 2n_2 |E|^2 \frac{K_B \omega_B}{c} A_+ + i2 \frac{n_0 K_B}{c} \frac{\partial A_+}{\partial t} + \text{nonlinear_term_} A_- + \text{nonlinear_term_} A_p = 0 \dots (9)$$

Dividing (9) by $2K_B$:

$$i \frac{\partial A_+}{\partial z} + i \frac{1}{V_g} \frac{\partial A_+}{\partial t} + \frac{\Omega_0}{V_g} A_+ + \frac{n_1 \omega_B}{2c} A_- + n_2 |E|^2 \frac{\omega_B}{c} A_+ + \frac{\text{nonlinear_term_} A_-}{2K_B} + \frac{\text{nonlinear_term_} A_p}{2K_B} = 0 \dots (10), \text{ where } V_g = \frac{c}{n_0}$$

Expressing:

$$\frac{\text{nonlinear_term_} A_-}{2K_B} = \Gamma(A_-) = \Gamma_B |A_-|^2 A_- \dots (11)$$

$$\frac{\text{nonlinear_term_} A_p}{2K_B} = \Gamma(A_p) = \frac{\lambda_B}{\lambda_p} \Gamma_p |A_p|^2 A_p \dots (12)$$

$$\text{Introducing several symbols: } \frac{\Omega_0}{V_g} = \delta, \kappa = \frac{\pi n_1}{\lambda_B}, \Gamma_B = \frac{2\pi n_2}{\lambda_B}$$

Hence (10) can be rewrite as:

$$i \frac{\partial A_+}{\partial z} + i \frac{1}{V_g} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- + \Gamma_B |E|^2 A_+ + \Gamma(A_-) + \Gamma(A_p) = 0 \dots (13)$$

$$\text{But: } |E|^2 = E \times E^* = |A_+|^2 + |A_-|^2 + |A_p|^2 + A_+ A_-^* + A_+^* A_- + A_+ A_p^* + A_+^* A_p + A_- A_p^* + A_-^* A_p \dots (12)$$

The final form of (13) after substituting (12) is:

$$i \frac{\partial A_+}{\partial z} + i \frac{1}{V_g} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- + \Gamma_B \left(|A_+|^2 + 2|A_-|^2 + |A_p|^2 \right) A_+ + R \Gamma_p |A_p|^2 A_p = 0 \dots (14), \text{ where } R = \frac{\lambda_B}{\lambda_p}$$

(14) is the first set of Nonlinear Coupled Mode Equations for strong probe and strong pump interaction in nonlinear grating.

Similarly for A_- , we can derive the second set of NLCMEs:

APPENDIX A: THE DERIVATION OF NLCMEs for STRONG PROBE AND STRONG PUMP

$$-i \frac{\partial A}{\partial z} + i \frac{1}{V_g} \frac{\partial A}{\partial t} + \delta A_- + \kappa^* A_+ + \Gamma_B \left(|A_-|^2 + 2|A_+|^2 + |A_p|^2 \right) A_+ + R \Gamma_p |A_p|^2 A_- = 0 \dots (15)$$

The third set is of NLCMEs to count the pump evolution inside the grating can be derived below:

$$\frac{\partial^2 A_p}{\partial z^2} \approx i 2 K_p \frac{\partial A_p}{\partial z} - K_p^2 A_p \dots (16)$$

$$\frac{\partial^2 A_p}{\partial t^2} \approx -i 2 \omega_p \frac{\partial A_p}{\partial t} - \omega_p^2 A_p \dots (17)$$

$$-\frac{n^2}{c^2} \frac{\partial^2 A_+}{\partial t^2} \approx \frac{K_p}{c} \left\{ n_0 + n_1 [\exp(iK_0) + \exp(-iK_0)] + 2n_2 |E|^2 \right\} \left\{ i 2 \frac{\partial A_p}{\partial t} + \omega_p A_p \right\} \dots (18)$$

Adding (16) and (18) to produce (2):

$$i 2 K_p \frac{\partial A_p}{\partial z} - K_p^2 A_p + \frac{K_p}{c} \left\{ n_0 + n_1 [\exp(iK_0) + \exp(-iK_0)] + 2n_2 |E|^2 \right\} \left\{ i 2 \frac{\partial A_p}{\partial t} + \omega_p A_p \right\} = 0 \dots (19)$$

$$i 2 K_p \frac{\partial A_p}{\partial z} - K_p^2 A_p + i 2 \frac{K_p n_0}{c} \frac{\partial A_p}{\partial t} + K_p^2 A_p + 2 \frac{n_2 K_p \omega_p}{c} |E|^2 A_p + \text{coupling_term} + \text{nonlinear_term_} A_+ + \text{nonlinear_term_} A_- = 0 \dots (20)$$

Under short grating length and loose coupling condition (large phase mismatch), the pump undergoes negligible evolutions due to interaction with strong probe.

Simplify (20) by dividing it with $2K_p$:

$$i \frac{\partial A_p}{\partial z} + i \frac{1}{V_g} \frac{\partial A_p}{\partial t} + \Gamma_p |E|^2 A_p + \frac{\text{nonlinear_term_} A_+}{2K_p} + \frac{\text{nonlinear_term_} A_-}{2K_p} = 0 \dots (21)$$

$$\frac{\text{nonlinear_term_} A_+}{2K_p} = \Gamma(A_+) = \frac{\lambda_p}{\lambda_B} \Gamma_B |A_+|^2 A_p \dots (22)$$

$$\frac{\text{nonlinear_term_} A_-}{2K_p} = \Gamma(A_-) = \frac{\lambda_p}{\lambda_B} \Gamma_B |A_-|^2 A_p \dots (23)$$

Re-writing (21) by substituting (22) and (23), the third NLCME to express the pump evolution inside the grating can be written as:

$$\pm i \frac{\partial A_p}{\partial z} + i \frac{1}{V_g} \frac{\partial A_p}{\partial t} + \Gamma_p \left(|A_+|^2 + |A_-|^2 + |A_p|^2 \right) A_p + \frac{\Gamma_B}{R} \left(|A_+|^2 + |A_-|^2 \right) A_p = 0 \dots (24)$$

APPENDIX A: THE DERIVATION OF NLCMEs for STRONG PROBE AND STRONG PUMP

Let set the three NLCMEs under strong probe and pump condition is written from (A) until (C):

$$i \frac{\partial A_+}{\partial z} + i \frac{1}{V_g} \frac{\partial A_+}{\partial t} + \delta A_+ + \kappa A_- + \Gamma_B \left(|A_+|^2 + 2|A_-|^2 + |A_p|^2 \right) A_+ + R \Gamma_p |A_p|^2 A_+ = 0 \dots (A), \text{ where : } R = \frac{\lambda_B}{\lambda_p}$$

$$-i \frac{\partial A_-}{\partial z} + i \frac{1}{V_g} \frac{\partial A_-}{\partial t} + \delta A_- + \kappa^* A_+ + \Gamma_B \left(|A_-|^2 + 2|A_+|^2 + |A_p|^2 \right) A_- + R \Gamma_p |A_p|^2 A_- = 0 \dots (B)$$

$$\pm i \frac{\partial A_p}{\partial z} + i \frac{1}{V_g} \frac{\partial A_p}{\partial t} + \frac{\Gamma_B}{R} \left(|A_+|^2 + |A_-|^2 \right) A_p + \Gamma_p \left(|A_+|^2 + |A_-|^2 + |A_p|^2 \right) A_p = 0 \dots (C)$$

$\pm$ in (C) is to handle the case when pump is propagating in the direction of forward or backward wave.

APPENDIX B: THE DERIVATION OF CQSNLCMEs

Appendix B: The Derivation of Cubic-Quintic-Septic Nonlinear Coupled Mode Equations

The Cubic-Quintic-Septic Nonlinear Coupled Mode Equations (CQNLCEs) to govern the nonlinear pulse propagation inside the Chalcogenide Fiber Bragg Gratings (C-FBG) can be derived by substituting the material refractive index variation along the grating given in (B.1) into scalar Maxwell's wave equations (B.2).

The refractive index variation along the C-FBG length is given by:

$$n(z) = \bar{n} + n_1(z) \times \cos\left[\frac{2\pi}{\Lambda} z + \theta(z)\right] + n_2 \times |E(z)|^2 + n_4 \times |E(z)|^4 + n_6 \times |E(z)|^6 \quad (\text{B.1})$$

Where: $n_2, n_6 > 0$ and $n_4 < 0$ to describe the nonlinear response of chalcogenide glass.

When $n_6 = 0$, it describes the nonlinear response of Pseudoatylene Toluene Sulfonate (PTS – crystal), and the equation becomes cubic quintic nonlinear coupled mode equations.

Scalar Maxwell's wave equation is given by:

$$\frac{\partial^2 E(z,t)}{\partial z^2} - \frac{n^2(z)}{c^2} \frac{\partial^2 E(z,t)}{\partial t^2} = 0 \quad (\text{B.2})$$

With the assumption that the input wavelength is close detuned to the Bragg wavelength, the input wave consists of two coupled counter-propagating waves for short period grating condition. Therefore electric field inside the grating is written as:

$$E(z,t) = [A_+(z,t) \cdot \exp(ik_B z) + A_-(z,t) \cdot \exp(-ik_B z)] \cdot \exp(-i\omega_0 t) \quad (\text{B.3})$$

Where $k_B = \frac{\pi}{\Lambda}$

By the paraxial approximation ($\frac{\partial^2 A_{\pm}}{\partial z^2} \ll 2K_B \frac{\partial A_{\pm}}{\partial z}$), the second order derivatives in (B.2) can be written as:

$$\frac{\partial^2 E(z,t)}{\partial z^2} \approx \left(-k_B^2 A_+ + 2ik_B \frac{\partial A_+}{\partial z} \right) \ell^{i(k_B z - \omega_0 t)} + \left(-k_B^2 A_- - 2ik_B \frac{\partial A_-}{\partial z} \right) \ell^{-i(k_B z - \omega_0 t)} \quad (\text{B.4})$$

$$\frac{\partial^2 E(z,t)}{\partial t^2} \approx -\omega_0 \left(\omega_0 A_+ + 2i \frac{\partial A_+}{\partial z} \right) \ell^{i(k_B z - \omega_0 t)} - \omega_0 \left(\omega_0 A_- + 2i \frac{\partial A_-}{\partial z} \right) \ell^{-i(k_B z - \omega_0 t)} \quad (\text{B.5})$$

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By the assumptions of weak index modulation and, small cubic, quintic and septic nonlinear refractive index as compared to the average refractive index of the C-FBG material (neglecting the square of the small terms and the cross products between them), the refractive index square can be approximated as:

$$n^2(z) \approx \bar{n}^2 + 2\bar{n}n_1(z)\cos(k_0 z) + 2\bar{n}n_2|E|^2 + 2\bar{n}n_4|E|^4 + 2\bar{n}n_6|E|^6 \quad (\text{B.6})$$

$$\text{Where } k_0 = \frac{2\pi}{\Lambda}$$

The higher order intensity terms in (B.6) can be expressed as:

$$|E|^2 = E \cdot E^* = |A_+|^2 + |A_-|^2 + A_+ A_-^* \ell^{i2k_B z} + A_+^* A_- \ell^{-i2k_B z} \quad (\text{B.7})$$

$$\begin{aligned} |E|^4 = |E|^2 |E|^2 = & |A_+|^4 + |A_-|^4 + 4|A_+|^2 |A_-|^2 + 2|A_+|^2 A_+ A_-^* \ell^{i2k_B z} + 2|A_+|^2 A_+^* A_- \ell^{-i2k_B z} + \dots \\ & 2|A_-|^2 A_+ A_-^* \ell^{i2k_B z} + 2|A_-|^2 A_+^* A_- \ell^{-i2k_B z} + A_+^2 A_-^{*2} \ell^{i4k_B z} + A_+^{*2} A_-^2 \ell^{-i4k_B z} \end{aligned} \quad (\text{B.8})$$

$$\begin{aligned} |E|^6 = |E|^2 |E|^2 |E|^2 = & |A_+|^6 + |A_-|^6 + 9|A_+|^4 |A_-|^2 + 9|A_+|^2 |A_-|^4 + \dots \\ & 3\ell^{i2k_B z} A_+ A_-^* (|A_+|^4 + |A_-|^4 + 3|A_+|^2 |A_-|^2) + 3\ell^{-i2k_B z} A_+^* A_- (|A_+|^4 + |A_-|^4 + 3|A_+|^2 |A_-|^2) + \dots \\ & 3\ell^{i4k_B z} A_+^2 A_-^{*2} (|A_+|^2 + |A_-|^2) + 3\ell^{-i4k_B z} A_+^{*2} A_-^2 (|A_+|^2 + |A_-|^2) + A_+^3 A_-^{*3} \ell^{i6k_B z} + A_-^3 A_+^{*3} \ell^{-i6k_B z} \end{aligned} \quad (\text{B.9})$$

Therefore, the 2nd term in the Maxwell's wave equation (B.2) can be expanded by combining (B.5) and (B.6):

$$\begin{aligned} \frac{n^2}{c^2} \frac{\partial^2 E(z,t)}{\partial t^2} \approx \frac{k}{\omega_0 n c} \left[\bar{n}^2 + \bar{n}n_1(z) \left(\ell^{ik_0 z} + \ell^{-ik_0 z} \right) + 2\bar{n}n_2|E|^2 + 2\bar{n}n_4|E|^4 + 2\bar{n}n_6|E|^6 \right] \\ \left\{ -\omega_0 \left(\omega_0 A_+ + 2i \frac{\partial A_+}{\partial z} \right) \ell^{i(k_B z - \omega_0 t)} - \omega_0 \left(\omega_0 A_- + 2i \frac{\partial A_-}{\partial z} \right) \ell^{-i(k_B z + \omega_0 t)} \right\} \end{aligned} \quad (\text{B.10})$$

Canceling the common terms in (B.10) and taking out the minus sign, we arrive at:

$$\begin{aligned} \approx -\frac{k}{c} \left[\bar{n} + n_1(z) \left(\ell^{ik_0 z} + \ell^{-ik_0 z} \right) + 2n_2|E|^2 + 2n_4|E|^4 + 2n_6|E|^6 \right] \\ \left\{ \left(\omega_0 A_+ + 2i \frac{\partial A_+}{\partial z} \right) \ell^{i(k_B z - \omega_0 t)} + \left(\omega_0 A_- + 2i \frac{\partial A_-}{\partial z} \right) \ell^{-i(k_B z + \omega_0 t)} \right\} \end{aligned} \quad (\text{B.11})$$

Expanding the (B.11) by substituting the higher order intensity terms (B.7) until (B.9) we arrive at:

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$$\begin{aligned}
 &= -\frac{k}{c} \cdot \left\{ \bar{n} \omega_0 A_+ + i 2 \bar{n} \frac{\partial A_+}{\partial t} + \left(2 n_2 \omega_0 |A_+|^2 + 4 n_2 \omega_0 |A_-|^2 \right) A_+ + \dots \right. \\
 &\quad \left(2 n_4 \omega_0 |A_+|^4 + 6 n_4 \omega_0 |A_-|^4 + 12 n_4 \omega_0 |A_+|^2 |A_-|^2 \right) A_+ + \dots \\
 &\quad \left(2 n_6 \omega_0 |A_+|^6 + 8 n_6 \omega_0 |A_-|^6 + 24 n_6 \omega_0 |A_+|^4 |A_-|^2 + 36 |A_+|^2 |A_-|^4 \right) A_+ + \dots \\
 &\quad \left. A_- n_1 \omega_0 \right\} \ell^{i(K_B z - \omega_0 t)} \quad (B.12)
 \end{aligned}$$

$$\left. \begin{aligned}
 &\bar{n} \omega_0 A_- + i 2 \bar{n} \frac{\partial A_-}{\partial t} + \left(2 n_2 \omega_0 |A_-|^2 + 4 n_2 \omega_0 |A_+|^2 \right) A_- + \dots \\
 &\left(2 n_4 \omega_0 |A_-|^4 + 6 n_4 \omega_0 |A_+|^4 + 12 n_4 \omega_0 |A_-|^2 |A_+|^2 \right) A_- + \dots \\
 &\left(2 n_6 \omega_0 |A_-|^6 + 8 n_6 \omega_0 |A_+|^6 + 24 n_6 \omega_0 |A_-|^4 |A_+|^2 + 36 |A_-|^2 |A_+|^4 \right) A_- + \dots \\
 &A_+ n_1 \omega_0
 \end{aligned} \right\} \ell^{-i(K_B z - \omega_0 t)} \quad (B.13)$$

Cumulating the $\ell^{i(K_B z - \omega_0 t)}$ phase term in (B.12), after combining (B.4) and (B.12) in (B.2), we arrive at:

$$\left(-k_B^2 A_+ + 2 i k_B \frac{\partial A_+}{\partial z} \right) + \frac{k}{c} \left\{ \begin{aligned}
 &\bar{n} \omega_0 A_+ + i 2 \bar{n} \frac{\partial A_+}{\partial t} + (2 n_2 \omega_0 |A_+|^2 + 4 n_2 \omega_0 |A_-|^2) A_+ \\
 &+ (2 n_4 \omega_0 |A_+|^4 + 6 n_4 \omega_0 |A_-|^4 + 12 n_4 \omega_0 |A_+|^2 |A_-|^2) A_+ \\
 &+ (2 n_6 \omega_0 |A_+|^6 + 8 n_6 \omega_0 |A_-|^6 + 24 n_6 \omega_0 |A_+|^4 |A_-|^2 + 36 n_6 \omega_0 |A_+|^2 |A_-|^4) A_+ \\
 &+ A_- n_1 \omega_0
 \end{aligned} \right\} = 0 \quad (B.14)$$

$$\text{As: } k = 2\pi \bar{n} / \lambda$$

Therefore (B.12) becomes:

$$\begin{aligned}
 &i 2 k_B \frac{\partial A_+}{\partial z} + i 2 \bar{n} \frac{k}{c} \frac{\partial A_+}{\partial t} + (k^2 - k_B^2) A_+ + \left\{ \frac{2 n_2 k^2}{n} |A_+|^2 + \frac{4 n_2 k^2}{n} |A_-|^2 \right\} A_+ + \dots \\
 &\left\{ 2 n_4 \frac{k^2}{n} |A_+|^4 + 6 n_4 \frac{k^2}{n} |A_-|^4 + 12 n_4 \frac{k^2}{n} |A_+|^2 |A_-|^2 \right\} A_+ + \dots \\
 &\left(2 n_6 \frac{k^2}{n} |A_+|^6 + 8 n_6 \frac{k^2}{n} |A_-|^6 + 24 n_6 \frac{k^2}{n} |A_+|^4 |A_-|^2 + 36 n_6 \frac{k^2}{n} |A_+|^2 |A_-|^4 \right) A_+ + A_- n_1 \frac{k^2}{n} \} = 0 \quad (B.15)
 \end{aligned}$$

APPENDIX B: THE DERIVATION OF CQSNLCMEs

Simplifying the equation by taking the factor $2k$ out from (B.15) and assuming that $k \approx K_B$

because the input wavelength is close to the Bragg wavelength, we finally arrive at:

$$\begin{aligned} i \frac{\partial A_+}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_+}{\partial t} + (k - k_B) A_+ + \left\{ \frac{n_2 k}{n} |A_+|^2 + \frac{2 n_2 k}{n} |A_-|^2 \right\} A_+ + A_- n_1 \frac{k}{2n} \\ + \left\{ n_4 \frac{k}{n} |A_+|^4 + 3 n_4 \frac{k}{n} |A_-|^4 + 6 n_4 \frac{k}{n} |A_+|^2 |A_-|^2 \right\} A_+ + \dots \\ \left(n_6 \frac{k}{n} |A_+|^6 + 4 n_6 \frac{k}{n} |A_-|^6 + 12 n_6 \frac{k}{n} |A_+|^4 |A_-|^2 + 18 n_6 \frac{k}{n} |A_+|^2 |A_-|^4 \right) A_+ = 0 \end{aligned} \quad (B.16)$$

Introducing symbols:

$$\text{Detuning: } \delta = 2\pi n \left(\frac{1}{\lambda} - \frac{1}{\lambda_B} \right) \quad (B.17)$$

$$\text{Cubic nonlinearity: } \Gamma_1 = \frac{2\pi n_2}{\lambda} \quad (B.18)$$

$$\text{Quintic nonlinearity: } \Gamma_2 = \frac{2\pi n_4}{\lambda} \quad (B.19)$$

$$\text{Septic nonlinearity: } \Gamma_3 = \frac{2\pi n_6}{\lambda} \quad (B.20)$$

$$\text{Coupling coefficient: } \kappa = \frac{\pi n_1}{\lambda} \quad (B.21)$$

Substituting the symbols in (B.17) until and (B.21) into (B.16), the 1st set of Cubic Quintic Septic Nonlinear Coupled Mode Equations (CQSNLCMEs) is written as:

$$\begin{aligned} i \frac{\partial A_+}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_+}{\partial t} + A_- \kappa + \delta A_+ + \left\{ \Gamma_1 |A_+|^2 + 2 \Gamma_1 |A_-|^2 \right\} A_+ \\ + \left\{ \Gamma_2 |A_+|^4 + 3 \Gamma_2 |A_-|^4 + 6 \Gamma_2 |A_+|^2 |A_-|^2 \right\} A_+ \\ + \left\{ \Gamma_3 |A_+|^6 + 4 \Gamma_3 |A_-|^6 + 12 \Gamma_3 |A_+|^4 |A_-|^2 + 18 |A_+|^2 |A_-|^4 \right\} A_+ \\ = 0 \end{aligned} \quad (B.22)$$

Similarly, accumulating the $e^{-i(K_B z - \omega_0 t)}$ term alone after substituting (B.4) and (B.13) in (B.2), the 2nd set of CQSNLCMEs is:

$$\begin{aligned} -i \frac{\partial A_-}{\partial z} + i \frac{\bar{n}}{c} \frac{\partial A_-}{\partial t} + \delta A_- + \left\{ \Gamma_1 |A_-|^2 + 2 \Gamma_1 |A_+|^2 \right\} A_- + A_+ \kappa^* \\ + \left\{ \Gamma_2 |A_-|^4 + 3 \Gamma_2 |A_+|^4 + 6 \Gamma_2 |A_+|^2 |A_-|^2 \right\} A_- \\ + \left\{ \Gamma_3 |A_-|^6 + 4 \Gamma_3 |A_+|^6 + 12 \Gamma_3 |A_-|^4 |A_+|^2 + 18 |A_-|^2 |A_+|^4 \right\} A_- = 0 \end{aligned} \quad (B.23)$$

(B.22) and (B.23) are forming the set of CQSNLCMEs to govern the wave propagation in Chalogenide-Bragg Grating.