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NEW TECHNIQUES AND STRUCTURES FOR ARRAY SIGNAL PROCESSING WITH ENHANCED ROBUSTNESS

ZHANG YING

School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
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Acknowledgements

When I am compiling this Ph.D. thesis, I recall many scenario during this hard period, the day when I left my parents, the day when I met my current supervisor, the day when I started to be a Ph.D. student in NTU. In the last three years, I am pursuing not only the highest academic degree in my life, but also the dream of becoming a successful scientist. The academic degree is up to come true, while the dream is still on her way.

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Summary

The content of this thesis comprises original research work on the investigations of new structures and algorithms to enhance robustness of array signal processing.

To suppress the high sidelobe level of the Minimum Variance and Distortionless Response (MVDR) beamformer, two beamformers imposed with sparse constraint on the beampattern are proposed. In the first algorithm, the sparse constraint is imposed on the beampatterns of the beamformer with respect to interference directions, which results in a beamforming algorithm with lower sidelobe level and better robustness against look direction mismatch. When array imperfection is taken into account, a modified algorithm is proposed in order to improve the performance of the beamformer when the interference signals are not located perfectly at the presumed directions. The modified algorithm maintains distortionless response in a wide angular range. It has been demonstrated by computer simulations that the proposed algorithms are valid for sidelobe suppression. Their robustness against steering vector error is also verified.

Many conventional algorithms suffer from performance degradation in the presence of steering vector error. To enhance the robustness against the steering vector error, a new ℓ_2 -norm beamformer is proposed. Different from the MVDR beamformer, the proposed beamformer minimizes not only the beamformer's output power, but also the cross-correlation of the real and imaginary parts of the beamformer's output. When

the proposed beamformer is combined with sphere constraint on steering vector error, its performance can be further improved. Beside the ℓ_2 -norm beamformer, a robust ℓ_1 -norm beamformer which is capable of working satisfactorily in the presence of steering vector error and intermittent impulsive noise is also derived. Computer simulations are conducted to verify validity and advantage of the proposed algorithms.

For Direction-of-Arrival (DOA) estimation, a Multiple Signal Classification (MUSIC) - like DOA estimation algorithm without estimating the number of sources is proposed. Compared with the MUSIC algorithm which is sensitive to the estimated number of sources, the proposed algorithm is robust against incorrect estimation of the number of sources. We theoretically prove the effectiveness of the new algorithm. The bounds of the parameter which plays an important role in the algorithm are derived. It is demonstrated that the proposed algorithm has lower computational load than that of the MUSIC algorithm when the number of sensors is large. For correlated sources, the Spatial Smoothing (SS) technique which is valid for the MUSIC algorithm can also be applied to the proposed algorithm.

To enhance the robustness of the Frost beamformer, we propose two sub-optimal block cascaded structures for broadband beamforming. The new beamformers possess information locally instead of globally, thereby brings better robustness against the steering vector error. It is demonstrated that the computational complexity and the calculating time are significantly reduced using the new structures. Furthermore, parallel update of the basic blocks is applicable, which is favorable for real-time implementation. Computer simulations are conducted to demonstrate the performance of the new structures.

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List of Notations

- j Imaginary unit, $j = \sqrt{-1}$.
- $\mathbf{I}$ Identity matrix
- $\exists x > a$ There exists a x satisfying $x > a$.
- $\nabla_{\mathbf{x}}$ Gradient with respect to $\mathbf{x}$.
- $E[x]$ Expectation of x .
- $\mathbf{x}_i$ The i^{th} element of $\mathbf{x}$.
- x^* Conjugate of x .
- $\mathbf{x}^*$ Element-by-element conjugate of $\mathbf{x}$.
- $\mathbf{x} \otimes \mathbf{y}$ Kronecker product of $\mathbf{x}$ and $\mathbf{y}$.
- $\mathbf{x} \cdot \mathbf{y}$ Inner product of $\mathbf{x}$ and $\mathbf{y}$.
- $x(t) * y(t)$ Linear convolution of $x(t)$ and $y(t)$.
- $|x|$ Absolute value of x .
- $\|\mathbf{x}\|_2$ Euclidean norm of $\mathbf{x}$.
- $\|\mathbf{x}\|_p$ ℓ_p -norm of $\mathbf{x}$.
- $\text{Real}\{x\}$ Real part of x .
- $\text{Imag}\{x\}$ Imaginary part of x .
- $\mathbf{X}_{i,j}$ The element of the i^{th} row and the j^{th} column of $\mathbf{X}$.
- $\mathbf{X}^{-1}$ Inverse of $\mathbf{X}$.
- $\mathbf{X}^H$ Conjugate transpose of $\mathbf{X}$.

$\mathbf{X}^T$ Symmetrical transpose of $\mathbf{X}$.

List of Abbreviations

AIC	Akaike Information Criterion
BSS	Blind Source Separation
CPLD	Complex Programmable Logic Device
DFT	Discrete Fourier Transform
DOA	Direction of Arrival
DoF	Degree of Freedom
EM	Expectation Maximization
ESPRIT	Estimation of Signal Parameters using Rotational Invariance Techniques
FIR	Finite Impulse Response
FLOS	Fractional Lower Order Statistics
FPGA	Field Programmable Gate Array
GMM	Gaussian Mixture Model
GSC	Generalized Sidelobe Canceler
HF	High Frequency

HOS	High Order Statistics
ICA	Independent Component Analysis
IIR	Infinite Impulse Response
INR	Interference to Noise Ratio
KKT	Karush-Kuhn-Tucker
LMP	Least Mean ℓ_p -norm
LMS	Least Mean Square
LS	Least Squares
MCA	Minor Component Analysis
MDL	Minimum Description Length
MI	Mutual Information
MISO	Multiple Input Single Output
MLE	Maximum Likelihood Estimate
MUSIC	MUltiple SIgnal Classification
MVDR	Minimum Variance and Distortionless Response
NINC	Nonlinear Inverse Noise Cancelation
pdf	Probability Density Function
PSD	Power Spectral Density
RMSE	Root Mean Square Error

SINR	Signal to Interference plus Noise Ratio
SIR	Signal to Interference Ratio
SMI	Sample Matrix Inverse
SNR	Signal to Noise Ratio
SOI	Signal of Interest
TDE	Time Delay Estimation
TDL	Tapped Delay Line
ULA	Uniform Linear Array
UWB	UltraWide Band

Chapter 1

Introduction

1.1 Background

A sensor array consists of several sensors located at different positions. Propagating signals received by the sensor array contain much information about the sources which produce them. Array signal processing aims to exploit the information from signals/data collected by a sensor array or several sensor arrays. Array signal processing is also called "spatial processing" which can be conducted in space-time domain or space-frequency domain. Conventional applications of array signal processing include radar and sonar, communications, imaging, geophysical, astrophysical exploration and biomedical systems, etc [1].

Two main categories of array signal processing are beamforming and Direction-of-Arrival (DOA) estimation [2].

Beamforming is one of the most important research topics in array signal processing. It is the name to a wide variety of array processing algorithms which focus on the array's signal-capturing abilities in a particular direction. In real world, propagating

signals come from many different directions, and are often contaminated by noise. The main objective of beamforming is to enhance desired signals impinging on an array from certain directions, the so called spatial-temporal filtering [3]. A beamforming algorithm points the array's spatial filter toward desired directions algorithmically rather than physically. Essentially, the underlying idea of beamforming is very simple: if a propagating signal is present in an array's aperture, the sensor outputs, delayed by appropriate amounts, weighted and added together, reinforce the signal with respect to noise and waves propagating in other directions.

DOA estimation, from its name, is to find the directions of signals impinging on the array. It is realized by exploiting the one-to-one relationship between the direction of a signal and the associated received steering vector. It is equivalent to spectral estimation when we see that there is a Fourier relationship between the beampattern and the excitation at the array. In practice, the estimation is made difficult by the fact that there are usually an unknown number of signals impinging on the array simultaneously, each from unknown directions and with unknown amplitudes. Also, the received signals are always corrupted by noise.

1.2 Overview of Array Signal Processing

1.2.1 Beamforming Techniques

Beamforming techniques can be divided into two classes: the data-independent beamformers and data-dependent/adaptive beamformers [4]. The standard data-independent beamformers include the delay-and-sum approach [5] as well as methods based on various weights vectors for sidelobe control [6–8]. The data-dependent/adaptive beamformers select the weight vector as a function of the data to optimize the perfor-

mance subject to various constraints designed according to specific applications [9,10]. The adaptive beamformers can have better resolution and much better interference rejection capability than the data-independent beamformers. However, the former are much more sensitive to errors, such as the array steering vector errors caused by imprecise sensor calibrations, than the latter. Over the last decades, much effort has been devoted to devise robust adaptive beamformers [4].

A. Robust Beamforming Techniques Against the Steering Vector Error

The Minimum Variance Distortionless Response (MVDR) beamformer is designed to minimize the output power of interferences and noises, and simultaneously generates undistorted response at the look direction [9]. When the actual steering vector at look direction differs from the nominal steering vector, the desired signal may be suppressed as interference and the performance of the MVDR beamformer is known to degrade dramatically [11].

To address uncertainty in the array steering vector or DOA of the desired signal, a set of unity-gain constraints for a small spread of angles around the nominal look direction can be imposed, which are known as point mainbeam constraints or neighboring location constraints [12]. Besides, the derivative of the weighted array output can also be constrained to zero at the desired look angle, which is called derivative mainbeam constraints [13–15]. Each constraint removes one of the remaining Degrees of Freedom (DoF) available to reject undesired signals, which is particularly significant for an array with a small number of elements. To overcome this limitation, a low-rank approximation to the constraints has been proposed [16]. The drawback of the low-rank approximation is the implicit relationship between satisfactory performance and the selected rank. Some regularization methods [17] are also proposed to enhance the robustness of the MVDR beamformer. These algorithms are investigated in detail

in [18–20]. Beamformers using eigenvalue thresholding methods to achieve robustness have also been used [21].

The aforementioned algorithms do not explicitly use information about the variation in the array response. Jian Li and Peter Stoica propose a series of robust beamforming algorithms by modeling the variation of array response explicitly in an uncertainty ellipsoid [4]. By doing so, optimization parameters used in the robust algorithms which have to be picked empirically in previous techniques can now be selected precisely based on mathematical formulation.

Since the aim of adaptive beamformer is to maximize the output Signal to Interference plus Noise Ratio (SINR), the optimal weight vector can be derived by means of maximizing the output SINR [22, 23]. For a single point source, this approach is equivalent to the MVDR beamformer. For multiple sources, the solution of the algorithm can be derived by solving a generalized eigenvalue problem [18, 24]. With sample covariance matrix, these solutions are usually referred to as the Sample Matrix Inverse (SMI) beamformers [25]. To enhance its robustness against limited number of snapshots, diagonal loading methods can be applied to the SMI beamformers [26–29].

The main shortcoming of traditional diagonal loading-based techniques is that there is no rigorous way of choosing the loading parameter. To settle this problem, a more powerful and theoretically rigorous worst-case performance optimization-based approach for robust adaptive beamforming has been recently developed [24, 30, 31]. The main modification in such an algorithm with respect to the original formulation of the SMI beamformers is that instead of requiring fixed distortionless response towards the single presumed steering vector, such distortionless response is now maintained by means of inequality constraints for a continuum of all possible steering vectors that

belong to the spherical uncertainty set.

B. Robust Adaptive Beamforming Techniques Against Impulsive Noise and Chaotic Noise

The issue of noise suppression is another important problem in the field of signal processing and is relevant to many applications such as radar, speech processing and wireless communications. Methods proposed in the past tend to be based on the Gaussian noise assumption.

When the noise exhibits chaotic behavior (e.g. noise from an electronic oscillator, engine noise from mechanical system, sea clutter in radar and fluid turbulence in sonar) [32], the stochastic-based noise suppression technique will become less effective. In order to obtain effective noise suppression, a class of nonlinear prediction filters which exploit the chaotic properties of the noise process has been studied extensively [33–35]. The Nonlinear Inverse Noise Cancellation (NINC) technique [34] is effective for chaotic noise. With the a priori knowledge, the desired signal is blocked using a bandstop filter. Since the in-band noise is also filtered, a nonlinear filter is adopted to recover the in-band noise from the out-band noise. Linear filter can only suppress the out-band noise as much as possible while has no effect on the in-band noise cancellation. Nonlinear inverse is not applicable for Gaussian noise case, because the frequency components of Gaussian signal are mutually independent.

There have been some literature demonstrating that impulsive noise appears on the signal at wireless receiver in a form of impulsive noise bursts [36–38]. These bursts may appear at some time and disappear at a later time. All the beamformers based on the second-order statistics can not perform well. To enhance the performance of beamformers in the presence of impulsive noise, some robust beamforming algorithms

have been introduced. These algorithms can be differentiated according to their means of modeling the impulsive noise. In [39–42], impulsive noise is modeled by a α -stable random process. Similar to the Least Mean Square (LMS) algorithm which minimizes the variance of the noise, "dispersion" is defined and used to characterize the impulsive noise. Dispersion is defined as an ℓ_p -norm where $p < \alpha$ holds. In such a case, ℓ_p -norm is usually adopted as the criterion to suppress the noise. ℓ_p -norm is sometimes considered as Fractional Lower Order Statistics (FLOS). Since only when $p < \alpha$ holds, the methods using ℓ_p -norm can perform well, a convenient choice of p is 0. However, ℓ_0 -norm means exhaustive search in the solution space, so some alternatives have to be used, such as geometric power [43]. Beside the α -stable random process, impulsive noise can also be modeled as mixture of Gaussian variables [44], where the Expectation Maximization (EM) algorithm can be applied.

1.2.2 DOA Estimation Algorithms

The simplest way of estimating DOA is to use the correlation technique. By the Cauchy-Schwarz inequality [45], the correlation of two vectors has a maximum value when the two vectors have the same direction. In essence, the correlation technique is the estimation of the spectrum of the incoming data, and the peaks of the plot indicate the estimated DOA. In the case of linear, uniformly spaced array, the steering vector is equivalent to Fourier coefficients, i.e., the correlation computation is equivalent to a Discrete Fourier Transform (DFT) [46] of the data vector.

Another way of estimating the DOA of an incoming signal is to maximize the likelihood with respect to that particular angle which the signal comes from. The solution derived by maximizing the likelihood is named as Maximum Likelihood Estimate (MLE) [47]. An interesting aspect of this estimator is that if there is only one

source and the spatial noise is white, the correlation matrix is diagonal and the MLE approach degenerates to the correlation technique. The MLE approach is optimal in the maximum likelihood sense. However, it is an impractical algorithm. The algorithm assumes knowledge of the interference covariance matrix, something that is not available in practice. Also, the algorithm is highly computationally intensive.

The MUltiple SIgnal Classification (MUSIC) algorithm [48] is probably the most popular technique for DOA estimation. It is dependent on the covariance matrix of the data. It has been demonstrated that the eigenvector matrix of the data covariance matrix can be decomposed into two orthogonal parts: the so-called signal subspace and the noise subspace. Making use of this property, the DOA of sources are determined by finding the steering vectors which are orthogonal to the noise subspace as much as possible. In practice, the data covariance matrix is unknown and has to be estimated from the received data. Due to limited number of snapshots, the noise eigenvalues are more of a continuum. There is no clear distinction between the signal and noise eigenvalues, which makes determination of the number of sources difficult. Therefore, some order selection algorithms, e.g., Akaike Information Criterion (AIC) [49] and Minimum Description Length (MDL) [50], should be used to determine the effective order of the model before the MUSIC algorithm is implemented.

There is a significant problem with the MUSIC algorithm. The accuracy is limited by the discretization at which the MUSIC spectrum is evaluated. More importantly, it requires either human interaction to decide on the largest M peaks or a comprehensive search algorithm to determine these peaks. This is an extremely computationally intensive process. When a Uniform Linear Array (ULA) is used, the root-MUSIC algorithm [51, 52] which solves equation of a polynomial is a better choice.

The Estimation of Signal Parameters using Rotational Invariance Techniques (ESPRIT) [53] is another parameter estimation technique. It is based on the fact that in the steering vector, the signal at one element is a constant phase shift from the earlier element. Therefore, the covariance matrices of the two overlapped subarrays span the same subspace. The phase shift can be determined from the two sets of eigenvalues with respect to the two subarrays. Note that ESPRIT requires a significantly greater computational load than the MUSIC algorithm. This is because we need two eigen-decompositions. To save computational load, an algorithm using the matrix pencil has been proposed [54]. Unlike ESPRIT working with the signal subspace as defined by the correlation matrix, the matrix pencil algorithm works with observation data directly.

1.3 Motivation and Objectives

In practical scenarios, the performance degradation of traditional array signal processing algorithms is pronounced because most of these algorithms are based on the assumption of an accurate knowledge of the array. However, there is very likely a mismatch between the presumed and the actual array model which can be caused by look direction/pointing errors, an imperfect array calibration, unknown wavefront distortions and signal fading, near-far wavefront mismodeling, local scattering, as well as other facts. Traditional algorithms are known to be very sensitive to slight mismatches of such type. For example, in the presence of steering vector error, an adaptive beamformer tends to interpret the Signal of Interest (SOI) as an interfering source and, consequently, suppresses the desired signal instead of maintaining the distortionless response to it. To enhance the output SINR, the beamformer should formulate its objective function properly according to the noise environment assumption. For traditional beamformers, Gaussian noise assumption is usually made. However, the real

facts may deviate from this assumption that the environment noise sometimes shows the property of super Gaussian or impulsive. When impulsive noise appears, traditional beamformers which are based on output power minimization or second-order statistics optimization usually fail to give satisfactory results. Similarly, for DOA estimation, if the order of the presumed model is different from the real one, the MUSIC algorithm tends to fit its result to the incorrect model order estimation. As a result, erroneous DOA estimation results may be obtained. Our research is dedicated to solve these problems. Up to now, several methods have been proposed, while there is still a long way to go.

Firstly, we notice that one of disadvantages of the MVDR beamformer is its high sidelobe level which could result in significant performance degradation in case of unexpected interferences or increase of the noise power [55]. In practice, look direction mismatch problem caused by imprecise knowledge of the steering vector may also occur. In such a case, the SOI will be mistaken as interference and the performance of the MVDR beamformer is known to degrade dramatically [11]. It is known that "sparsity" measures the number of zero elements in a vector, i.e., the more the number of zero elements, the higher the sparsity of the vector. If we consider array response gain as a vector, we hope that the elements of such a vector approach zero when the locations of these elements correspond to interference directions. This motivates us to apply the sparsity constraint to the array response gain associated with directions of interferences. When the sparsity measurement is applied, the gains of array with respect to interference directions are forced to zero, which in turn suppresses the sidelobe level of the beampattern.

The conventional beamformer only imposes a constraint on output power of the beamformer. To enhance the performance of the beamformer, we propose a robust

beamformer which additionally minimizes the cross-correlation of the real and imaginary parts of the beamformer's output. It is found that by doing so, the new beamformer shows better robustness against steering vector error. For impulsive noise cancellation, the ℓ_p -norm beamformers are expected to work satisfactorily in the presence of impulsive noise. However, these beamforming methods against impulsive noise degrade when the array steering vector is not completely known, which motivates us to propose a new beamforming algorithm against both impulsive noise and steering vector error. By modeling the variation of the actual steering vector from the nominal steering vector explicitly, a new algorithm which works well in the presence of impulsive noise and steering vector error is possible to devise.

The AIC [49] and MDL [50] criteria using asymptotic arguments are the most frequently used methods to estimate the number of sources. However, experimental evidence shows that they do indeed tend to estimate a wrong number of components for a small sample size and a low Signal-to-Noise Ratio (SNR) [56]. To facilitate estimation of DOA, new algorithms which do not require source number estimation are necessary. It is noticed that beamforming techniques are able to estimate DOA of sources without knowing the number of sources. Therefore, it motivates us to combine the beamforming technique and the DOA estimation algorithm to design an algorithm which inherits the merit of both methods.

For broadband beamforming, in perspective of hardware implementation, the development of Field - Programmable Gate Array (FPGA) and Complex Programmable Logic Device (CPLD) makes efficient and high-speed computation feasible, but more hardware resources are required to realize a high-order Finite Impulse Response (FIR) filter [57]. For the design of Infinite Impulse Response (IIR) filter, since FPGA processes digital signal which is obtained by quantization, the error induced by quanti-

zation will increase error in the frequency response, and even cause shifts of the poles of the IIR filter out of the unit circle. Therefore, either theoretically or practically, an efficient and easy implementation of broadband beamforming is necessary. The desired beamformer should facilitate hardware implementation and enjoy moderate computational complexity especially when the number of taps is large. Besides, a new broad-band beamforming technique is expected to be robust against array imperfection. It is learned from the gossiping algorithms that local computation shows improved robustness against the system imperfection. Therefore, in this thesis, we propose two block cascaded structures to realize broadband beamforming.

1.4 Contributions of the Thesis

The main contributions of the thesis are listed below:

A. ℓ_p -norm Constraint to Enhance the Robustness of the Beamformer Against the Steering Vector Error and High Sidelobe Level

Two disadvantages of the Minimum Variance Distortionless Response (MVDR) beamformer are its high sidelobe level and high sensitivity to look direction mismatch problem caused by imprecise knowledge of the steering vector. In this thesis, we propose two beamforming algorithms which impose sparse measurement constraint on the beampattern in order to suppress the sidelobe level and simultaneously enhance the robustness of beamformers. In the first algorithm, the sparse measurement constraint is added to the conventional MVDR beamformer, which results in a new beamforming algorithm with lower sidelobe level and better robustness against look direction mismatch. Taking array steering vector error into account, we further extend the first algorithm from sidelobe suppression only to both sidelobe suppression and mainlobe

control. Instead of maintaining distortionless response on one look direction, the second algorithm attempts to maintain distortionless response on a wide angular range so that sources impinging on the array from nearby directions of look direction can be retained. Computer simulations show that the proposed algorithms can not only reduce the sidelobe level of the MVDR beamformer, but also show robustness against steering vector error.

B. ℓ_1 -norm and ℓ_2 -norm Beamformers with Sphere Constraint on the Steering Vector Error to Enhance the Robustness Against Steering Vector Error and Impulsive Noise

To enhance robustness of the beamformer against array steering vector error, we propose a new ℓ_2 -norm beamformer which not only minimizes the output power of the beamformer, but also minimizes the cross-correlation of the real and imaginary part of the beamformer's output. Computer simulations show that the new beamformer is less sensitive to steering vector error compared with the MVDR beamformer. Furthermore, we extend this idea to propose a new beamforming algorithm against both impulsive noise and steering vector error. This new beamformer iteratively minimizes the ℓ_1 -norm of the beamformer's output to suppress the impulsive noise, subject to a prespecified set of quadratic constraints which reduce the influence caused by steering vector error. Simulations show that the proposed algorithm performs better than the MVDR beamformer and the MVDR beamformer with norm constraint [58].

C. Design of a DOA Estimator Without Knowing the Number of Sources in the Framework of Beamforming

To realize DOA estimation, we propose a new optimization problem which does not require estimation of the number of sources. The solution to the new optimization problem is the eigenvector associated with the minimum eigenvalue of the matrix

which depends on the look direction and the array correlation matrix. It is shown that the new algorithm is similar to the MUSIC algorithm in two aspects: quiescent array response and structure of the optimal weight vector associated with the look direction. Therefore, we name the proposed algorithm as the MUSIC-like algorithm. Theoretical investigation is conducted to demonstrate the effectiveness of the proposed algorithm. Firstly, when the steering vector lies in the signal subspace, the optimal weight vector is shown to be located in the noise subspace so that the defined direction finding function shows a peak to indicate the presence of a target. Secondly, when there is no target at the look direction and the corresponding steering vector is not perpendicular to any noise eigenvector, it is proved that the proposed algorithm prevents the presence of spurious peaks. Finally, we investigate the case when the steering vector is not in the signal subspace and is perpendicular to some of the noise eigenvectors. In this case, the MUSIC algorithm can give spurious peaks if the number of sources is overestimated. However, for the proposed algorithm, it is shown that when certain condition is satisfied, the problem of spurious peaks will not occur.

D. Block Cascaded Structures for Broadband Beamforming

We propose two sub-optimal structures which process information locally instead of globally, ameliorate calibration difficulty, and bring better robustness against the steering vector error. The sub-optimal processing also facilitates hardware implementation of broadband beamforming. The basic blocks are cascaded to form a Frost-like beamformer. The basic blocks are themselves sub-Frost beamformers whose number of input channels and tapped delays is small. The independence between the basic blocks makes parallel computation of the weights feasible, which is very favorable for real-time implementation. The design of the beamformer can be very flexible to meet the requirement of various working scenario.

1.5 Organization of the Thesis

In this thesis, we start with introducing fundamental knowledge of array signal processing. Then, we focus on narrowband beamforming. Based on the formulation of the MVDR beamformer, we propose two novel beamforming algorithms in Chapter 3 to enhance robustness of the MVDR beamformer against high sidelobe level and steering vector error. In Chapter 4, our effort is dedicated to improve beamformers' performance in the presence of both of steering vector error and impulsive noise. Notice that the beamforming techniques can be applied to DOA estimation without estimation of the number of sources. We subsequently propose a new DOA estimation algorithm which realizes DOA estimation in the framework of beamforming in Chapter 5. Since this new algorithm is essentially a beamformer, it is robust against erroneous estimation of the number of sources. In Chapter 6, we aim to find robust broadband beamformer by introducing two sub-optimal broadband beamforming structures. Discussions on future work are given in Chapter 7.

The outline of the thesis is listed below:

In Chapter 2, we briefly review fundamentals on array signal processing. The definition of sensor array is given. Some applications of array signal processing are introduced. For easy understanding of the subsequent studies, basic knowledge on array is given. Besides, some conventionally used beamforming techniques and DOA estimation algorithms are introduced.

In Chapter 3, we introduce two new robust beamforming techniques imposed with sparse constraint on the beampattern. Mathematical formulation of the problem is given, and the update formula are derived using the Lagrange multiplier technique. The choice of some important parameters used in the proposed algorithm is addressed.

Computer simulations are conducted to verify the validity of the new algorithm. Comparison with other algorithms are made to show superiority of the new algorithm. Finally, conclusions are drawn.

Chapter 4 introduces several new algorithms to enhance robustness of the MVDR beamformer. Firstly, some existing robust beamforming techniques are reviewed. Then, formulation of the new algorithms are presented. Details on derivation of update formula of new algorithms are shown. Determination of some important parameters used in the algorithms is also addressed. Computer simulations are conducted to verify the validity of the new algorithms. Comparison with other algorithms shows advantages of the proposed algorithms. At the end of the chapter, conclusions are drawn.

Chapter 5 presents a new DOA estimation algorithm. Brief review of existing DOA algorithms is given, and their relationship between each other is also addressed. The new algorithm is formulated and analyzed. The quiescent response is given and compared to that of the MUSIC algorithm. Furthermore, rigorous theoretical deduction of the bounds of a critical parameter is shown. Computational complexity comparison is made to show advantage of the new algorithm. Computer simulations prove the validity and advantages of the new algorithm.

In Chapter 6, we firstly discuss the drawbacks and limitations of existing broadband beamformers. The Frost beamformer is then reviewed, and new structures for broadband beamforming are presented with the functional block diagram and methods to update the coefficients of the beamformer. Two ways of update are applicable, whose advantages are addressed respectively. To prove that the proposed structures have lower computational load than that of the Frost beamformer, we mathematically

analyze the computational complexity. Tables and figures are given to illustrate the derived results, and comparison with the Frost beamformer is also given. Theoretical output Signal-to-Interference plus Noise Ratio (SINR) of the proposed algorithms are addressed in detail. Computer simulations on narrowband sources, broadband sources, and in the presence of steering vector error, are conducted to demonstrate the performance of the new structures. Conclusions are drawn at the end of the chapter.

Finally, in Chapter 7, we make conclusions of all the aforementioned studies. Suggestions are also given for future research.

Chapter 2

Fundamentals on Array Signal Processing

2.1 Introduction

This chapter prepares some fundamental knowledge of array signal processing for the studies in the subsequent chapters. It is organized as follows: Section 2.2 introduces array fundamentals, including definition of sensor array and array signal processing, coordinate systems, wave propagation model, array geometry and beam pattern. Section 2.3 presents some existing beamforming techniques, including conventional beamformers and robust beamformers. Section 2.4 introduces some conventional DOA estimation algorithms. Finally, conclusions are given in Section 2.5.

2.2 Array Fundamentals

2.2.1 Coordinate System and Wave Propagation Model

A. Coordinate System

Cartesian coordinates and spherical coordinates [59] are the two most usually used coordinate systems, as shown in Figure 2.1.

Figure 2.1: Cartesian coordinates and spherical coordinates.

In most situations, a three-dimensional Cartesian grid represents space, with time being the fourth dimension. A space-time signal is written as $s(x, y, z, t)$, for example, with x , y and z being the three spatial variables in a right-handed orthogonal coordinate system. We shall use the position vector $\mathbf{x} = (x, y, z)$ to denote the set of spatial variables (x, y, z) . Hence, we can write $s(x, y, z, t)$ as $s(\mathbf{x}, t)$.

For spherical coordinate system, a point is represented by its distance r from the origin, its azimuth θ within an equatorial plane containing the origin, and its polar angle φ down from the vertical axis.

The spherical coordinates of a point are related to the Cartesian coordinates by simple trigonometric formulas:

$$x = r \sin \varphi \cos \theta, \quad (2.1a)$$

$$y = r \sin \varphi \sin \theta, \quad (2.1b)$$

$$z = r \cos \varphi, \quad (2.1c)$$

and

$$r = \sqrt{x^2 + y^2 + z^2}, \quad (2.2a)$$

$$\theta = \cos^{-1} \left(\frac{x}{\sqrt{x^2 + y^2}} \right) = \sin^{-1} \left(\frac{y}{\sqrt{x^2 + y^2}} \right), \quad (2.2b)$$

$$\varphi = \cos^{-1} \left(\frac{z}{\sqrt{x^2 + y^2 + z^2}} \right). \quad (2.2c)$$

Here, the (x, y) plane forms the equatorial plane, and azimuths are measured as angles counterclockwise from the x axis.

B. Wave Propagation Model

Wave types include electromagnetic, acoustic etc. They travel at different speeds in different media [60]. For example, electromagnetic wave travels at the speed of $3 \times 10^8 \text{ m/s}$ (meter/second) in free space. Acoustic wave travels at about 330.7 m/s in the air, while at about 1498 m/s in sea water. Information about distant events is carried to our sensors by propagating waves.

The wave equation is given by

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 s}{\partial t^2}. \quad (2.3)$$

One solution of the wave equation leads to the plane wave:

$$s(\mathbf{x}, t) = s(t - \boldsymbol{\alpha} \cdot \mathbf{x}), \quad (2.4)$$

where $\boldsymbol{\alpha} = (\alpha_x, \alpha_y, \alpha_z)$ is called the slowness vector with

$$\|\boldsymbol{\alpha}\|_2 = 1/c, \quad (2.5)$$

where c is the speed of propagating plane wave.

The term 'plane wave' arises because at any instant of time t_0 , the value of $s(x, y, z, t_0)$ is the same at all points lying in a plane given by

$$\alpha_x x + \alpha_y y + \alpha_z z = C, \quad (2.6)$$

where C is a constant.

A typical plane wave is the exponential plane wave:

$$s(\mathbf{x}, t) = A \exp \{j(\omega t - \mathbf{k} \cdot \mathbf{x})\}, \quad (2.7)$$

where $\mathbf{k} = (k_x, k_y, k_z)$ is called the wavenumber vector with

$$|\mathbf{k}| = 2\pi/\lambda, \quad (2.8)$$

where λ is the wavelength.

$\mathbf{k}$ and $\boldsymbol{\alpha}$ is related by

$$\mathbf{k} = \omega\boldsymbol{\alpha}. \quad (2.9)$$

In this thesis, we assume that far field conditions hold, i.e., signal sources are far from the sensor array. Therefore, the propagating wave incident on the array has the plane wavefront rather than the spherical wavefront.

2.2.2 Array Geometry and Array Pattern

Array geometry is the geometrical form in which array elements are placed according to the application of scenario or the pattern to be synthesized. Linear array, circular array and rectangular array are common geometries of interest. Linear array, due to its simplicity and usefulness, has been widely studied and used in real applications. Figure 2.2 depicts the geometry of a linear array. In this thesis, we mainly focus on the use of the linear array. In this type of geometry, the array elements are deployed along a straight line with the uniform or nonuniform inter-element spacing.

Figure 2.2: Geometry of a linear array.

The array pattern [61–63] is a quantity through which the amplitude and phase of the beamformed signal when the wavefield consists of a single plane wave can be determined. For an L -element array, it can be expressed as

$$W(\mathbf{k}) = \sum_{l=0}^{L-1} w_l \exp \{j \mathbf{k} \cdot \mathbf{x}_l\}, \quad (2.10)$$

where w_l is the beamforming weight.

$W(\mathbf{k})$ is called the array pattern because, through the quantity $W(\omega^0 \boldsymbol{\alpha} - \mathbf{k}^0)$, where ω^0 denotes the angular frequency of the source and $\mathbf{k}^0$ is the wavenumber vector, it determines the amplitude and phase of the beamformed signal when the wavefield consists of a single plane wave. In Figure 2.3, we plot beampatterns of several uniformly weighted linear arrays with sensors located at:

$$\mathbf{y} = [0 \ 1 \ 2 \ 3 \ 4]^T \times \lambda/2,$$

$$\mathbf{y} = [0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7]^T \times \lambda/2,$$

$$\mathbf{y} = [0 \ 2 \ 4 \ 6 \ 7]^T \times \lambda/2.$$

Figure 2.3: Beam patterns of different linear arrays.

From Figure 2.3, we see that the ULA with eight sensors has narrower mainlobe than that of the ULA with five sensors, which means increasing number of sensors (large aperture length) in a ULA can improve resolution capability of the array. Deploying a non-uniform array may increase resolution, but usually shows higher side lobes.

2.3 Beamforming Algorithms

Several beamforming techniques are reviewed in this section. It should be mentioned that there are many beamformers in real applications, while not all of them are discussed here. The beamformers introduced in this section are relevant to our subsequent studies.

2.3.1 Conventional Adaptive Beamforming Techniques

First of all, we review some conventional adaptive beamforming techniques, including MVDR beamforming [4] and Frost beamforming [64, 65].

Many adaptive array signal processing techniques can be derived by solving a constrained mean-squared optimization problem. Let $\mathbf{x}$ denote the vector of observations, lying in either frequency or time domains, obtained from the entire array, $\mathbf{w}$ the weight vector to the observation vector, the correlation matrix of the observation $\mathbf{R} = E[\mathbf{x}\mathbf{x}^H]$. The general optimization problem for adaptive array processing is

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (2.11a)$$

$$\text{subject to } \mathbf{C} \mathbf{w} = \mathbf{c}, \quad (2.11b)$$

where $\mathbf{C}$ is the constraint matrix with linearly independent rows, and $\mathbf{c}$ is a column vector of constraining values.

To solve the above constrained optimization problem, the classical Lagrange multiplier method can be used:

$$L(\mathbf{w}, \boldsymbol{\lambda}) = \mathbf{w}^H \mathbf{R} \mathbf{w} - \boldsymbol{\lambda}^H (\mathbf{C} \mathbf{w} - \mathbf{c}) + \boldsymbol{\lambda}^T (\mathbf{C}^* \mathbf{w}^* - \mathbf{c}^*), \quad (2.12)$$

where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers.

The solution to the constrained optimization problem (2.11) is

$$\mathbf{w}_{opt} = \mathbf{R}^{-1} \mathbf{C}^H (\mathbf{C} \mathbf{R}^{-1} \mathbf{C}^H)^{-1} \mathbf{c}, \quad (2.13)$$

and the array output power is

$$P = \mathbf{w}_{opt}^H \mathbf{R} \mathbf{w}_{opt} = \mathbf{c}^H (\mathbf{C} \mathbf{R}^{-1} \mathbf{C}^H)^{-1} \mathbf{c}. \quad (2.14)$$

These results lie at the heart of many important adaptive beamforming techniques.

Figure 2.4: Decomposition of adaptive beamformer into two orthogonal parts.

The projection matrix associated with the constraint equation is

$$\mathbf{P}_C = \mathbf{C}^H (\mathbf{C} \mathbf{C}^H)^{-1} \mathbf{C}. \quad (2.15)$$

This matrix serves to decompose the optimum weight vector into two orthogonal components (shown in Figure 2.4). The projection of the optimum weight vector onto the constraint is

$$\mathbf{w}_c = \mathbf{P}_C \mathbf{w}_{opt} = \mathbf{C}^H (\mathbf{C} \mathbf{C}^H)^{-1} \mathbf{c}. \quad (2.16)$$

Note that this vector does not depend on the spatial correlation matrix and therefore describes the nonadaptive component of the solution. The remaining component does depend on the observations and represents the purely adaptive component of array processing algorithms based on constrained optimization:

$$\mathbf{w}_a = (\mathbf{I} - \mathbf{P}_C) \mathbf{w}_{opt}. \quad (2.17)$$

A. MVDR beamformer

Figure 2.5 shows the structure of an L -element narrowband adaptive antenna array processor.

Figure 2.5: Structure of a narrowband adaptive array beamformer.

The output of a beamformer can be represented by

$$y(t) = \mathbf{w}^H \mathbf{x}(t), \quad (2.18)$$

where $\mathbf{w} = [w_0, \dots, w_{L-1}]^T$ represents the weight vector.

The MVDR beamformer (also called Capon's method) imposes a constraint on $\mathbf{w}$ to keep the desired signal distortionless:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (2.19a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}_0(\theta) = 1, \quad (2.19b)$$

where $\mathbf{a}_0(\theta)$ is the nominal steering vector and θ denotes the desired look direction.

Using the Lagrange multiplier technique, the optimal weight vector of the MVDR beamformer is given by

$$\mathbf{w}_{opt} = \alpha \mathbf{R}^{-1} \mathbf{a}_0(\theta), \quad (2.20)$$

where $\alpha = \frac{1}{\mathbf{a}_0^H(\theta) \mathbf{R}^{-1} \mathbf{a}_0(\theta)}$ is the array gain. The output power with respect to $\mathbf{w}_{opt}$ is

$$P = \frac{1}{\mathbf{a}_0^H(\theta) \mathbf{R}^{-1} \mathbf{a}_0(\theta)}. \quad (2.21)$$

A measurement of the effectiveness of a beamformer is given by the output SINR:

$$SINR = \frac{\sigma_0^2 |\mathbf{w}^H \mathbf{a}_0(\theta)|^2}{\mathbf{w}^H \mathbf{R}_{nn} \mathbf{w}}, \quad (2.22)$$

where σ_0^2 is the power of the SOI, $\mathbf{R}_{nn}$ is the correlation matrix of the interference plus noise.

To illustrate the performance of the MVDR beamformer, a ULA with ten half-wavelength spaced sensors is used. The SOI and interference impinge on the array from 20° , -30° , -10° , 0° and 50° , respectively. The SINR and SNR are -10 dB and 3 dB, respectively. Figure 2.6 shows the derived beampattern of the MVDR beamformer. Figure 2.7 plots the output SINR versus SNR. N denotes the number of snapshots.

Figure 2.6: Beampattern of the MVDR beamformer.

It is observed that the MVDR beamformer has 0 dB gain at 20° , while casts deep nulls at directions of interference signals. From Figure 2.7, we see that the theoretical performance of the MVDR beamformer is optimal. In practical applications, only finite number of snapshots are available. It is observed that the performance of the MVDR beamformer becomes constant when the value of SNR is high. This is because when the noise power is small enough, the expectation of the array gain is proportional to the number of snapshots and is not influenced by the value of SNR [66]. Furthermore, the smaller the number of snapshots, the poorer the performance of the MVDR beamformer.

In real applications, the assumed look directional steering vector $\mathbf{a}_0(\theta)$ may differ from the actual one for a host of reasons including imprecise knowledge of the signal's

DOA. The SINR of the MVDR beamformer deteriorates catastrophically for modest differences between the assumed and actual steering vectors.

Figure 2.7: Output SINR versus SNR of the MVDR beamformer.

B. Frost beamformer

Frost beamforming is a classical broadband beamforming technique. Figure 2.8 shows the structure of a Frost beamformer with L sensors and J tapped delays [67].

Figure 2.8: Structure of the Frost beamformer.

The constrained optimization problem is posed as

$$\min_{\mathbf{w}} \quad \mathbf{w}^T \mathbf{R} \mathbf{w}, \quad (2.23a)$$

$$\text{subject to} \quad \mathbf{C}^T \mathbf{w} = \mathbf{f}, \quad (2.23b)$$

where the $LJ \times J$ matrix $\mathbf{C}$ is known as the constraint matrix, whose j^{th} column consists of zeros except for the j^{th} group of L elements equal to 1, and $\mathbf{f}$ specifies the frequency response of the beamformer at the look direction. Usually, only one of the elements in $\mathbf{f}$ equals to 1 and the others equal to 0, or, it can be optimized [68].

With the Lagrange multiplier technique and setting its gradient with respect to $\mathbf{w}$ to zero, one obtains the optimum weight vector as

$$\mathbf{w}_{opt} = \mathbf{R}^{-1} \mathbf{C} [\mathbf{C}^T \mathbf{R} \mathbf{C}]^{-1} \mathbf{f}. \quad (2.24)$$

To avoid calculating a matrix inverse, the gradient descent technique may be used

to find the update formula of $\mathbf{w}$:

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu \nabla_{\mathbf{w}} [\mathbf{w}^H(k) \mathbf{R} \mathbf{w}(k) + \lambda (\mathbf{C}^T \mathbf{w}(k) - \mathbf{f})], \quad (2.25)$$

where μ denotes the stepsize and λ is the Lagrange multiplier. (2.25) results in

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \mu [\mathbf{I} - \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T] \mathbf{R} \mathbf{w}(k) + \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} [\mathbf{f} - \mathbf{C}^T \mathbf{w}(k)]. \quad (2.26)$$

Defining

$$\mathbf{F} = \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} \mathbf{f}, \quad (2.27)$$

and

$$\mathbf{P} = \mathbf{I} - \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T, \quad (2.28)$$

(2.26) can be expressed as

$$\mathbf{w}(k+1) = \mathbf{P} [\mathbf{w}(k) - \mu \mathbf{R} \mathbf{w}(k)] + \mathbf{F}, \quad (2.29)$$

which is called the deterministic constrained gradient descent algorithm.

Replacing $\mathbf{R}$ by $\mathbf{x}(k)\mathbf{x}^T(k)$, the stochastic constrained LMS algorithm is derived as

$$\mathbf{w}(k+1) = \mathbf{P} [\mathbf{w}(k) - \mu y(k) \mathbf{x}(k)] + \mathbf{F}. \quad (2.30)$$

The output SINR for evaluating performance of the beamformer is given by

$$SINR = 10 \log_{10} \left(\frac{\mathbf{w}^T \mathbf{R}_{ss} \mathbf{w}}{\sum_{j=1}^J \mathbf{w}^T \mathbf{R}_{II} \mathbf{w} + \mathbf{w}^T \mathbf{R}_{nn} \mathbf{w}} \right) dB, \quad (2.31)$$

where $\mathbf{R}_{ss}$, $\mathbf{R}_{II}$ and $\mathbf{R}_{nn}$ denote covariance matrices of the SOI, interferences and spatial noise, respectively.

Figure 2.9: Beampattern of the Frost beamformer.

To illustrate the performance of the Frost beamformer, we assume a ULA with eight sensors. The inter-element spacing is half-wavelength with respect to the highest frequency. The broadband sources are supposed to have normalized central frequency at 0.3 with bandwidth equal to 0.2. The SOI impinges on the array from broadside direction, and the two interferences are located at -40° and -60° . The SNR and SINR are assumed to be 20 dB and -20 dB, respectively. The number of snapshots is 2000, and the number of taps used in the Frost beamformer is 7. Figure 2.9 shows the derived beampattern. Figure 2.10 depicts the output SINR versus iteration number using 200 independent trials, where the stepsize is chosen as 0.009.

Figure 2.10: Output SINR versus iteration number of the Frost beamformer.

It is observed from Figure 2.9 that the Frost beamformer is capable of suppressing interferences and simultaneously maintain distortionless response of the SOI. From Figure 2.10, we see the output SINR of the Frost beamformer increases with the number of iterations.

The weight vector $\mathbf{w}(k)$ obtained by using the stochastic algorithm (2.30) is a random vector. Convergence of the mean weight vector to the optimum is demonstrated by showing that the length of the difference vector between the mean weight vector and the optimum (2.24) asymptotically approaches zero [99].

It has been demonstrated that the convergence of the mean weight vector to the

optimum weight vector along any eigenvector of $\mathbf{PRP}$ is geometric with geometric ratio $(1 - \mu\sigma_i)$, where σ_i is the eigenvalue of $\mathbf{PRP}$. If μ is chosen so that

$$0 < \mu < 1/\sigma_{max},$$

we have

$$\lim_{k \rightarrow \infty} \|E[\mathbf{w}(k)] - \mathbf{w}_{opt}\| = 0. \quad (2.32)$$

Therefore, the mean weight vector asymptotically converges to the optimal solution.

Furthermore, by analyzing with respect to stationary and nonstationary environment, the best steady-state performance of the Frost beamformer can be obtained by making μ as small as possible. However, small values of μ will slow down the convergence of the algorithm (2.30).

2.3.2 Robust Beamforming Techniques

In real applications, the aforementioned beamforming techniques are sensitive to errors, such as the array steering vector errors caused by imprecise sensor calibrations. Therefore, much effort has been devoted to devise robust adaptive beamformers. In this section, we briefly introduce some robust beamforming techniques which are relevant to our subsequent studies.

A. Capon Beamforming with Norm Constraint

In the presence of array steering vector errors, the MVDR beamformer may attempt to suppress the SOI as if it were an interference. Since the true steering vector $\mathbf{a}(\theta)$ is usually close to the nominal steering vector $\mathbf{a}_0(\theta)$, the Euclidean norm of the resulting weight vector can become very large in order to satisfy the distortionless constraint

$\mathbf{w}^H \mathbf{a}_0(\theta) = 1$ and at the same time suppress the SOI, that is, $\mathbf{w}^H \mathbf{a}(\theta) \approx 0$.

The goal of the MVDR beamformer with norm constraint is to impose an additional constraint on the Euclidean norm of $\mathbf{w}$ for the purpose of improving the robustness of the MVDR beamformer against SOI steering vector errors and control the white noise gain. The robust beamforming problem with norm constraint is formulated as follows [4]:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (2.33a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}_0(\theta) = 1, \quad \|\mathbf{w}\|_2^2 \leq \varepsilon. \quad (2.33b)$$

Note that the quadratic inequality constraint can be interpreted as constraining the white noise gain at the output.

Using the Lagrange multiplier technique, the solution to (2.33) is found as

$$\mathbf{w}_{opt} = \frac{(\mathbf{R} + \lambda \mathbf{I})^{-1} \mathbf{a}_0(\theta)}{\mathbf{a}_0(\theta)^H (\mathbf{R} + \lambda \mathbf{I})^{-1} \mathbf{a}_0(\theta)}, \quad (2.34)$$

where λ is determined by ε [4].

The problem of the MVDR beamforming with a norm constraint is that the choice of ε is not easy to make. In particular, this choice is not directly linked to the uncertainty of the SOI steering vector.

B. Robust Minimum Variance Beamforming

A generalization of (2.19) which aims to minimize the weighted power output of the array in the presence of uncertainties in $\mathbf{a}(\theta)$ is:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (2.35a)$$

$$\text{subject to } \text{Real}\{\mathbf{w}^H \mathbf{a}_0(\theta)\} \geq 1, \forall \mathbf{a}_0(\theta) \in \xi, \quad (2.35b)$$

where $\xi = \{\mathbf{A}\mathbf{u} + \mathbf{a}_0(\theta) \mid \|\mathbf{u}\|_2 \leq 1\}$ is an ellipsoid that covers the possible range of values of $\mathbf{a}_0(\theta)$ due to imprecise knowledge of the array manifold $\mathbf{a}_0(\theta)$, uncertainty in the angle of arrival θ , or other factors.

Using the Lagrange multiplier technique, the solution to (2.35) is given by

$$\mathbf{x} = -\lambda (\mathbf{R} + \lambda \mathbf{Q})^{-1} \mathbf{c}_0(\theta), \quad (2.36)$$

where $\mathbf{Q} = \mathbf{A}\mathbf{A}^T - \mathbf{c}_0(\theta)\mathbf{c}_0(\theta)^T$, $\mathbf{c}_0(\theta) = [\text{Real}(\mathbf{a}_0(\theta)), \text{Imag}(\mathbf{a}_0(\theta))]$, and λ is the root of equation

$$f(\lambda) = \lambda^2 \mathbf{c}_0(\theta)^T (\mathbf{R} + \lambda \mathbf{Q})^{-1} \mathbf{Q} (\mathbf{R} + \lambda \mathbf{Q})^{-1} \mathbf{c}_0(\theta) - 2\lambda \mathbf{c}_0(\theta)^T (\mathbf{R} + \lambda \mathbf{Q})^{-1} \mathbf{c}_0(\theta) - 1. \quad (2.37)$$

The advantage of the robust minimum variance beamformer lies in that the weight selection uses the a priori uncertainties in the array manifold in a precise way.

C. Robust Beamformers in the Presence of Impulsive Noise

The aforementioned algorithms aim to enhance robustness of the beamformer against steering vector error and are all based on Gaussian noise assumption, while in real applications, impulsive noise (caused by man-made or natural electromagnetic distur-

bances [36-38, 68]) usually appears, such as thunderstorm, car ignition, microwave over, office equipments, etc. To improve the performance of beamformers in the presence of impulsive noise, some robust algorithms have been proposed.

It is known that impulsive noise can be characterized as a α -stable process, where minimization of FLOS is usually chosen as criterion when designing beamformers. The optimization problem of a class of robust beamformers based on FLOS is given by

$$\min_{\mathbf{w}} E[|\mathbf{w}^H \mathbf{x}|^p], \quad (2.38a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}_0(\theta) = 1, \quad (2.38b)$$

where $p < \alpha$.

The solution of (2.38) can be iteratively found by

$$\mathbf{w}(k+1) = \mathbf{P} [\mathbf{w}(k) - \mu |\mathbf{y}^T(k)\mathbf{x}(k)|^{-1} \mathbf{y}^T(k)\mathbf{x}(k)] + \mathbf{f}, \quad (2.39)$$

where

$$\mathbf{P} = \mathbf{I} - \mathbf{a}_0(\theta)\mathbf{a}_0^T(\theta)/(\mathbf{a}_0^T(\theta)\mathbf{a}_0(\theta)),$$

$$\mathbf{f} = \mathbf{a}_0(\theta)/(\mathbf{a}_0^T(\theta)\mathbf{a}_0(\theta)),$$

and μ is the stepsize.

2.4 DOA Estimation Algorithms

2.4.1 CRLB for DOA Estimation

The Cramer Rao Lower Bound (CRLB) [69, 70] is an important parameter used to evaluate performance of any unbiased estimators. It tells us that estimating parameters from noisy data will necessarily result in noisy estimates. Furthermore, the CRLB is the best we can possibly do in minimizing the residual noise in unbiased estimates.

The CRLB theorem: Given a length- N vector of received signals $\mathbf{x}$ dependent on a set of P parameters $\boldsymbol{\theta} = [\theta_1, \dots, \theta_P]^T$, corrupted by additive noise,

$$\mathbf{x} = \mathbf{v}(\boldsymbol{\theta}) + \mathbf{n}, \quad (2.40)$$

where $\mathbf{v}(\boldsymbol{\theta})$ is a known function of the parameters, the variance of an unbiased estimate of the p^{th} parameter θ_p , is greater than the CRLB

$$\text{var}(\theta_p) \geq \mathbf{J}_{pp}^{-1}, \quad (2.41)$$

where $\mathbf{J}_{pp}$ is the p^{th} diagonal entry of the inverse of the Fisher information matrix $\mathbf{J}$ whose $(i, j)^{th}$ is given by

$$\mathbf{J}_{ij} = E \left\{ \frac{\partial^2}{\partial \theta_i \partial \theta_j} [\ln f_{\mathbf{x}}(\mathbf{x}|\boldsymbol{\theta})] \right\}, \quad (2.42)$$

where $f_{\mathbf{x}}(\mathbf{x}|\boldsymbol{\theta})$ is the pdf of the received vector given the parameters $\boldsymbol{\theta}$.

Due to the fact that the minimum variance is dependent on the inverse of the Fisher information matrix, we cannot ignore parameters that we are not interested in. The vector $\boldsymbol{\theta}$ must include all the parameters in the model for $\mathbf{v}$.

The CRLB for DOA estimation of multiple uncorrelated signals is given by [71]:

$$CRLB(\theta) = \frac{6\sigma^2}{|\alpha|^2 L(L^2 - 1)(kd)^2 \sin^2 \theta}, \quad (2.43)$$

where σ^2 is the power of the additive white noise, α denotes the amplitude of the source, $k = 2\pi/\lambda$ is the wavenumber, and d denotes the inter-element spacing.

We now have the CRLB of DOA estimators. Note that there are clear physical interpretations to be made. The CRLB sets the best possible estimation. As expected, as the SNR ($\frac{|\alpha|^2}{\sigma^2}$) increases, the CRLB is reduced. Further, the denominator is approximately proportional to $[(L-1)kd]^2$, which is proportional to the electrical length of the array, the length in terms of wavelength, i.e. as the array size increases, we can form a better estimate. The L term suggests that for a given overall electrical length the more samples (elements) we have the better the estimate we can obtain. Finally, the $\sin(\theta)$ term represents the fact that for a linear array, as we scan off broadside the beamwidth increases in θ terms, i.e., this represents the beam broadening factor making DOA estimates that much worse.

2.4.2 MUSIC Algorithm

The MUSIC algorithm, which is dependent on the correlation matrix of the data, is one of the most well-known DOA estimation algorithms. It plots the pseudo-spectrum as

$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{a}^H(\theta) \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta)}, \quad (2.44)$$

where $\mathbf{E}_n$ consists of all noise eigenvectors. Note that since the eigenvectors making up $\mathbf{E}_n$ are orthogonal to the signal steering vectors, the denominator becomes zero

when θ is a signal direction. Therefore, the estimated signal directions are the M largest peaks in the pseudo-spectrum.

In a communication situation, assuming flat fading, there may be multipath components from many directions. These components would be correlated with each other. Correlated components reduce the rank of the signal correlation matrix $\mathbf{R}_s$, resulting in more than $(N - M)$ noise eigenvalues. In order to solve this problem, the Spatial Smoothing (SS) technique is proposed in the literature.

For the SS technique, the L elements are subdivided into Q overlapping subarrays, each with P elements. For example, subarray 0 would include elements 0 through $P - 1$, subarray 1 include elements 1 through P , etc. Therefore, $Q = L - P + 1$. Using the data from each subarray, Q correlation matrices are estimated, each of dimension $P \times P$. The MUSIC algorithm then continues using a smoothed correlation matrix correlation matrix

$$\mathbf{R}_Q = \frac{1}{Q} \sum_{q=0}^{Q-1} \mathbf{R}_q, \quad (2.45)$$

This formulation can detect the DOA of up to $Q - 1$ correlated signals. This is because the signal correlation matrix component of $\mathbf{R}_Q$ becomes full rank again [72].

2.4.3 An Alternative Formulation for the MUSIC Estimator

Consider the following constrained optimization problem for an array beamformer:

$$\min_{\mathbf{w}(\theta)} \|\mathbf{w}(\theta) - \mathbf{a}(\theta)\|_2^2, \quad (2.46a)$$

$$\text{subject to } \mathbf{E}_s^H \mathbf{w}(\theta) = 1. \quad (2.46b)$$

where $\mathbf{E}_s$ is the signal subspace matrix of the correlation matrix $\mathbf{R}$.

Using the method of Lagrange multipliers, and the unitary property of orthonormal eigenvectors, one obtains

$$\mathbf{w}(\theta) = \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta), \quad (2.47)$$

where $\mathbf{E}_n$ is the noise subspace matrix of the correlation matrix $\mathbf{R}$.

If this array weight vector is now used to scan the array at a particular direction θ , one obtains the following array gain expression for that direction:

$$G(\theta) = \mathbf{a}^H(\theta) \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta). \quad (2.48)$$

From (2.48), one can see that if we were to take the inversion of the expression, one actually obtains the spatial spectrum of the MUSIC estimator, i.e., $P_{MUSIC}(\theta) = 1/G(\theta)$.

2.4.4 Other Conventional DOA Estimation Algorithms

A. DOA estimation using correlation

The correlation method plots $P_{corr}(\theta)$ versus θ given by

$$P_{corr}(\theta) = \mathbf{a}^H(\theta) \mathbf{x}, \quad (2.49)$$

where $\mathbf{x}$ denotes the array snapshot. $P_{corr}(\theta)$ is a non-adaptive estimate of spectrum of the incoming data. The peaks of this plot are the estimated DOA.

B. Maximum likelihood estimator

For MLE, we have two unknown parameters, the magnitude and DOA. The MLE is given by

$$\hat{\boldsymbol{\theta}}, \hat{\boldsymbol{\alpha}} = \arg \max_{\boldsymbol{\theta}, \boldsymbol{\alpha}} [f_{\mathbf{x}}(\mathbf{x}|\boldsymbol{\theta}, \boldsymbol{\alpha})], \quad (2.50)$$

where $f_{\mathbf{x}}(\mathbf{x}|\boldsymbol{\theta}, \boldsymbol{\alpha})$ is the probability density function (pdf) of the data vector $\mathbf{x}$ given parameters $\boldsymbol{\theta}$ and $\boldsymbol{\alpha}$.

Solving (2.50) yields the estimation of $\boldsymbol{\theta}$ which is given by

$$\hat{\boldsymbol{\theta}} = \arg \max_{\boldsymbol{\theta}} \left[\frac{\mathbf{a}^H(\boldsymbol{\theta}) \mathbf{R}_n^{-1} \mathbf{x}}{\mathbf{a}^H(\boldsymbol{\theta}) \mathbf{R}_n^{-1} \mathbf{a}(\boldsymbol{\theta})} \right], \quad (2.51)$$

where $\mathbf{R}_n$ denotes the interference plus noise correlation matrix.

An interesting aspect of this estimator is that if there is only one user and $\mathbf{R}_n = \sigma^2 \mathbf{I}$, where σ^2 denotes the power of the noise, the correlation matrix is diagonal and the MLE reduces to the correlation technique. This is expected because the correlation technique is equivalent to the matched filter, which is optimal in the single user case (more generally, in any case where the received data is a single data vector plus white Gaussian noise).

The ML approach is optimal in the maximum likelihood sense. However, it is an impractical algorithm. The algorithm assumes knowledge of $\mathbf{R}_n$, the interference covariance matrix, something that is not available in practice.

C. ESPRIT: Estimation of Signal Parameters using Rotational In- variance Techniques

ESPRIT is another parameter estimation technique, based on the fact that in the steering vector, the signal at one element is a constant phase shift from the earlier element.

Suppose $\mathbf{A}_0$ and $\mathbf{A}_1$ consist of the 1^{st} to the $(L-1)^{th}$ and 2^{nd} to the L^{th} rows of the array manifold $\mathbf{A}$, respectively. It is known that $\mathbf{A}_1 = \mathbf{A}_0\Phi$. The ESPRIT algorithm begins by recognizing that the steering vectors in matrix $\mathbf{A}$ span the same subspace. Since both these matrices span the same subspace, there exists an invertible matrix $\mathbf{C}$ such that $\mathbf{Q}_s = \mathbf{AC}$.

Defining matrices $\mathbf{Q}_0$ and $\mathbf{Q}_1$ derived from $\mathbf{Q}$ just as $\mathbf{A}_0$ and $\mathbf{A}_1$ were derived from $\mathbf{A}$. It can be shown that

$$\mathbf{Q}_1 = \mathbf{Q}_0\mathbf{\Psi}, \quad (2.52a)$$

$$\mathbf{\Psi} = \mathbf{C}^{-1}\Phi\mathbf{C}. \quad (2.52b)$$

Therefore, for ESPRIT algorithm, we construct $\mathbf{Q}_0$ and $\mathbf{Q}_1$ from $\mathbf{Q}$. Then, using (2.52a), we can compute $\mathbf{\Psi}$. It is observed from (2.52b) that the matrix Φ is a diagonal matrix of the eigenvalue of $\mathbf{\Psi}$. Finding eigenvalues of $\mathbf{\Psi}$ gives the estimates of DOA.

2.5 Conclusions

In this chapter, we briefly introduce fundamentals on array signal processing, and review some existing beamforming algorithms, including conventional adaptive beamforming algorithms, robust beamforming algorithms against steering vector error and

impulsive noise. For DOA estimation, we introduce the CRLB which is important to evaluate performance of DOA estimators. Besides, the MUSIC algorithm and some other conventional DOA estimation algorithms are also presented.

After reviewing the existing algorithms, we aim to propose new robust array processing algorithms from the following perspectives: 1) It is observed from (2.19) that the MVDR beamformer neither imposes explicit constraint on the interference signals, nor introduces constraint on sidelobe. In practice, it is possible that the receiver has a prior knowledge on the directions of interference signals, which can be used to enhance the performance of the beamformer. Even if this knowledge is unavailable, sidelobe suppression is necessary to avoid a high value of the gain of the array at the unwanted directions. 2) For the robust beamformers against the steering vector error, ellipsoid constraint has been introduced. For the existing algorithms, robust algorithms against the steering vector error are derived with Gaussian noise assumption. However, due to man-made or natural electromagnetic discharges, impulsive noise environment is also necessary to be taken into account. Therefore, formulation of these algorithms in impulsive noise environment is meaningful. 3) From the fact that the beamforming techniques can be implemented on DOA estimation without knowing the number of sources, we attempt to design a new DOA estimator in the framework of beamforming. 4) For efficient implementation of the Frost beamformer, we dedicate ourselves to construct new structures to realize broadband beamforming. All these ideas will be presented in details in the subsequent chapters.

Chapter 3

ℓ_p -norm Constraint to Enhance the Robustness of the Beamformer Against the Steering Vector Error and High Sidelobe Level

3.1 Introduction

Adaptive beamforming is an important research topic in array signal processing. The objective of adaptive beamforming is to enhance the source from the desired direction, while suppressing the background noise and all the interferences from other directions. This objective can be realized by MVDR beamformer [4] which casts deep nulls in the directions of strong interferences and at the meantime keeps the desired signal distortionless.

One of the disadvantages of the MVDR beamformer is its high sidelobe level which could result in significant performance degradation in case of unexpected interference

or increase of the noise power [55]. In practice, the mismatch problem caused by imprecise knowledge of the steering vector might occur. In such cases, the SOI will be mistaken as interference and the performance of the MVDR beamformer is known to degrade dramatically [11].

In this chapter, we propose two new beamforming algorithms which are robust against look direction mismatch and steering vector error. The achieved beampatterns have lower sidelobe level compared with the MVDR beamformer. Sidelobe suppression is realized via adding a sparse measurement on beampattern. Due to imperfect knowledge of the array, the presumed directions of signals maybe vary from the real ones, thereby causes insufficient cancelation of interference. To solve this problem, we extend the first algorithm from sidelobe suppression only to both sidelobe suppression and mainlobe control. Instead of maintaining distortionless response on one look direction, the second algorithm attempts to maintain distortionless response on a wide angular range so that sources impinging on the array from nearby directions of look direction can be retained. The proposed optimization problems can be solved iteratively using the fixed point algorithms. The validity and the advantages of the new algorithms are verified via computer simulations.

The rest of this chapter is organized as follows: Chapter 3.2 presents the proposed beamforming technique with sidelobe suppression using an ℓ_p -norm constraint on the beampattern. Chapter 3.3 extends this algorithm with robustness consideration. In Chapter 3.4, parameter selection of these new algorithms is discussed. Computer simulations are conducted in Chapter 3.5. Finally, conclusions are drawn in Chapter 3.6.

3.2 Algorithm I: Sidelobe Suppression with Sparse Constraint on Beampattern

The mathematical formulation of the MVDR beamformer is given by

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (3.1a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}(\theta_0) = 1. \quad (3.1b)$$

From the perspective of the beampattern, it is observed from (3.1b) that there is only explicit constraint on the look direction, i.e., θ_0 , while no constraint is imposed on the directions of interference. To repair this drawback, we propose the following optimization problem with an additional constraint on the sparsity of the beampattern with respect to potential interference directions:

$$\min_{\mathbf{w}} \frac{1}{2} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (3.2a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}(\theta_0) = 1, \quad (3.2b)$$

$$\| \mathbf{w}^H \mathbf{A} \|_p^p < \epsilon, \quad (3.2c)$$

where $\mathbf{A}$ is an $L \times N$ matrix which consists of array steering vectors in the angular range which contains all the possible interference directions, L is the number of sensors, and N denotes the number of samples over the angular range with respect to the interference. Different from the MVDR beamformer whose optimization problem is formulated by (3.2a) and (3.2b), we introduce a sparse measurement constraint (3.2c) on array response gain for the interference directions. In the following sections, we will show that robustness against look direction mismatch can be obtained using this new constraint.

In (3.2c), $\|\mathbf{x}\|_p = (\sum_i |x_i|^p)^{\frac{1}{p}}$ is the ℓ_p -norm of the vector $\mathbf{x}$. When $p < 2$, the ℓ_p -norm can be defined as the "dispersion" for super-Gaussian distribution. When $p \leq 1$, the ℓ_p -norm can be interpreted as the diversity measurement [73]. The smaller the $\|\mathbf{x}\|_p^p$, the sparser the $\mathbf{x}$ is, which means the number of trivial entries in $\mathbf{x}$ is larger. As for beamforming, the smaller the term $\|\mathbf{w}^H \mathbf{A}\|_p^p$, the lower the side-lobe level, since most of the entries in $\mathbf{w}^H \mathbf{A}$ are forced to some trivial values. To suppress the sidelobe level, $p < 2$ is used in this thesis. From computer simulations, we see that the proposed algorithm is not sensitive to the value of p , so p can be chosen empirically.

The optimization problem (3.2) can be simplified to

$$\min_{\mathbf{w}} \frac{1}{2} \mathbf{w}^H \mathbf{R} \mathbf{w} + \lambda \|\mathbf{w}^H \mathbf{A}\|_p^p, \quad (3.3a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}(\theta_0) = 1. \quad (3.3b)$$

In (3.3a), λ is a positive regularization parameter.

To derive the solution of (3.3), the Lagrange multiplier technique is used:

$$J(\mathbf{w}) = \frac{1}{2} \mathbf{w}^H \mathbf{R} \mathbf{w} + \lambda \|\mathbf{w}^H \mathbf{A}\|_p^p + \gamma (\mathbf{w}^H \mathbf{a}(\theta_0) - 1), \quad (3.4)$$

where γ is the Lagrange multiplier.

If we define $\overline{\mathbf{A}} = [\mathbf{A} \mid \alpha \cdot \mathbf{a}(\theta_0)]$, $\mathbf{d}^T = [\mathbf{0}^T \mid \alpha \cdot 1]$, where $\alpha = \gamma/\lambda$, then (3.4) is equivalent to

$$J(\mathbf{w}) = \frac{1}{2} \mathbf{w}^H \mathbf{R} \mathbf{w} + \lambda \|\overline{\mathbf{A}}^H \mathbf{w} - \mathbf{d}^*\|_p^p, \quad (3.5)$$

where $*$ denotes the complex conjugate.

Calculating the gradient of $J(\mathbf{w})$ with respect to $\mathbf{w}$, we have

$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \mathbf{R}\mathbf{w} + \tilde{\lambda} \bar{\mathbf{A}} \mathbf{\Pi}(\mathbf{w}) (\bar{\mathbf{A}}^H \mathbf{w} - \mathbf{d}^*), \quad (3.6)$$

where $\tilde{\lambda} = \lambda p$, $\mathbf{\Pi}(\mathbf{w}) = \text{diag} \left\{ \left| \left(\bar{\mathbf{A}}^H \mathbf{w} - \mathbf{d}^* \right)_1 \right|^{p-2}, \dots, \left| \left(\bar{\mathbf{A}}^H \mathbf{w} - \mathbf{d}^* \right)_N \right|^{p-2} \right\}$. It is observed from (3.6) that $\mathbf{w}$ is contained in a sign function which prevents an analytical solution.

Setting (3.6) to zero, and using the gradient factorization approach [73], the update formula of $\mathbf{w}$ is derived as

$$\mathbf{w}(i+1) = \tilde{\lambda} \left(\mathbf{R} + \tilde{\lambda} \bar{\mathbf{A}} \mathbf{\Pi}(\mathbf{w}(i)) \bar{\mathbf{A}}^H \right)^{-1} \bar{\mathbf{A}} \mathbf{\Pi}(\mathbf{w}(i)) \mathbf{d}^*, \quad (3.7)$$

where i denotes the iteration index.

3.3 Algorithm II: Sidelobe Suppression with Robustness Consideration

Notice that the above formulation is presented under the assumption that the actual steering vector is known. When the presumed steering vector is used instead of the actual steering vector, the above formulation may break down when the interference constraint formulated in $\mathbf{A}$ contains the actual steering vector of the desired signal. This happens when the presumed look direction is erroneous.

To overcome this, we generalize the above formulation by expanding the desired signal constraint. Consider the case where there is uncertainty in the look direction:

the DOA of the desired signal is assumed to be within a spatial sector instead of a particular direction. Let Θ_s denote the spatial sector where the DOA of the desired signal is assumed to be inside. The extension of the optimization in (3.2) is as follows:

$$\min_{\mathbf{w}} \frac{1}{2} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (3.8a)$$

$$\text{subject to } \mathbf{A}_s^H \mathbf{w} = \mathbf{1}, \quad (3.8b)$$

$$\| \mathbf{w}^H \mathbf{A} \|_p^p < \epsilon, \quad (3.8c)$$

where $\mathbf{A}_s = [\bar{\mathbf{a}}(\theta_{s,1}), \dots, \bar{\mathbf{a}}(\theta_{s,N_s})]$ are the possible steering vectors of the SOI within the spatial sector Θ_s , and $\mathbf{1}$ is a vector consisting of all 1s.

The solution to the above optimization can be derived, similar to the derivation presented in Chapter 3.2. Firstly, we combine the constraints (3.8c) to (3.8b):

$$\min_{\mathbf{w}} \frac{1}{2} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (3.9a)$$

$$\text{subject to } \| \mathbf{\Gamma}^H \mathbf{w} - \mathbf{d} \|_p^p < \epsilon, \quad (3.9b)$$

where $\mathbf{\Gamma} = [\mathbf{A}_i \mid 1/\gamma \cdot \mathbf{A}_s]$, $\mathbf{d}^T = [\mathbf{0}^T \mid 1/\gamma \cdot \mathbf{1}]$. Next, we formulate the Lagrange function and equate the gradient to zero. Finally, the weight vector update equation is obtained as follows:

$$\mathbf{w}(i+1) = \tilde{\lambda} \left(\mathbf{R} + \tilde{\lambda} \mathbf{\Gamma} \mathbf{\Pi}(\mathbf{w}(i)) \mathbf{\Gamma}^H \right)^{-1} \mathbf{\Gamma} \mathbf{\Pi}(\mathbf{w}(i)) \mathbf{d}^*. \quad (3.10)$$

It is worth mentioning that the constraint matrix $\mathbf{\Gamma}$ can be implemented by considering only a few directions representing the directions of the SOI within the spatial sector Θ_s . In fact, the number of directions taken within Θ_s can be as small as two,

i.e. $N_s = 2$. The first direction $\theta_{s,1}$ is taken at the left boundary while the other $\theta_{s,2}$ is taken at the right boundary.

3.4 Implementation of the Proposed Algorithms

To implement the proposed algorithms, we should properly choose the values of parameters appearing in the iterative algorithms.

From (3.7) and (3.10), it is observed that there are three parameters: p , λ and γ , impacting the performance of the proposed beamformers. The parameter p determines the sparsity of the beampattern. The smaller the value of p , the sparser the derived beampattern (or equivalently, the lower the sidelobe level). The parameter λ determines the effectiveness of the sparse constraint. Using a large λ emphasizes the impact of the sparse constraint and will result a trivial solution $\mathbf{w}$ ($\mathbf{w} = \mathbf{0}$ gives the smallest value of $\|\mathbf{w}^H \mathbf{A}\|_p^p$), since the constraint on distortionless response in (3.2b) and (3.8b) will be neglected in this situation. Similarly, using a large γ emphasizes the impact of the distortionless response constraint and will attenuate the effect of the sparse constraint. Therefore, these parameters should be chosen carefully in order to compromise between suppression of interferences and maintenance of the desired source.

For the first algorithm, in order to appropriately set λ , we compute the gradient of (3.5) with respect to λ and equate its to zero:

$$\|\bar{\mathbf{A}}^H \mathbf{w} - \mathbf{d}^*\|_p^p = 0. \quad (3.11)$$

Left multiplying (3.6) with $\mathbf{w}^H$ yields

$$\mathbf{w}^H \mathbf{R} \mathbf{w} + \tilde{\lambda} \mathbf{w}^H \bar{\mathbf{A}} \Pi(\mathbf{w}) (\bar{\mathbf{A}}^H \mathbf{w} - \mathbf{d}^*) = 0. \quad (3.12)$$

With the definition of $\bar{\mathbf{A}}$ and $\mathbf{d}$, we may equivalently express (11) as

$$\| \mathbf{A}^H \mathbf{w} \|_p^p + \alpha^p | \mathbf{a}^H(\theta_0) \mathbf{w} - 1 |^p = 0. \quad (3.13)$$

Also, (3.12) is equivalent to

$$\begin{aligned} & \mathbf{w}^H \mathbf{R} \mathbf{w} + \| \mathbf{A}^H \mathbf{w} \|_p^p + \alpha^p \mathbf{w}^H \mathbf{a}(\theta_0) \\ & | \mathbf{a}^H(\theta_0) \mathbf{w} - 1 |^{p-2} (\mathbf{a}^H(\theta_0) \mathbf{w} - 1) = 0, \end{aligned} \quad (3.14)$$

which can be written as

$$\begin{aligned} & \mathbf{w}^H \mathbf{R} \mathbf{w} + \| \mathbf{A}^H \mathbf{w} \|_p^p + \alpha^p | \mathbf{a}^H(\theta_0) \mathbf{w} - 1 |^p + \\ & \alpha^p | \mathbf{a}^H(\theta_0) \mathbf{w} - 1 |^{p-2} (\mathbf{a}^H(\theta_0) \mathbf{w} - 1) = 0. \end{aligned} \quad (3.15)$$

With (3.13), (3.15) can be simplified as

$$\mathbf{w}^H \mathbf{R} \mathbf{w} + \alpha^p | \mathbf{a}^H(\theta_0) \mathbf{w} - 1 |^{p-2} (\mathbf{a}^H(\theta_0) \mathbf{w} - 1) = 0. \quad (3.16)$$

Solving (3.16) gives solution of λ as

$$\lambda = \gamma \left(\frac{\mathbf{w}^H \mathbf{R} \mathbf{w}}{| \mathbf{a}^H(\theta_0) \mathbf{w} - 1 |^{p-2} (\mathbf{a}^H(\theta_0) \mathbf{w} - 1)} \right)^{1-p}. \quad (3.17)$$

When implementing the algorithm, we may alternatively update $\mathbf{w}(i)$ and $\lambda(i)$ or $\gamma(i)$ using (3.7) and (3.17), respectively. For the second algorithm, the relationship

between λ or γ is similar to (3.17) by replacing $\bar{\mathbf{A}}$ with $\mathbf{\Gamma}$ and setting the denominator as $\|\mathbf{A}_s^H \mathbf{w} - \mathbf{d}\|^{p-1}$.

Because we have deduced the relationship between λ and γ , we may properly set one of them and compute the other one using (3.17). Based on the existing literature, when p is fixed, some principles of choosing a proper λ , such as the L-curve [73], have been introduced. Therefore, when implementing the algorithm, we may firstly choose a proper value of p . Then, we use exiting principles to determine λ . When λ is derived, we use (3.17) to compute γ . It should be emphasized that how to precisely choose λ is an issue under study. Current algorithms do not guarantee optimal results. Fortunately, from our computer simulations, we see that the proposed algorithms are not sensitive to the choice of γ and λ . Therefore, they can be chosen empirically. When doing so, we should keep in mind that if a larger λ or a smaller p is used, the value of γ should increase so that a trivial $\mathbf{w}$ can be avoided. (From (3.5), we see that large value of λ emphasize the second term of $J(\mathbf{w})$ and also causes $\mathbf{d}$ to be a zero vector, thereby causes zero or trivial solution to the problem. In order to avoid its happening, we may increase the value of γ so that $\mathbf{d}$ is not a zero vector.)

We summarize the proposed algorithms in Table I. The stopping criterion can be chosen as $i > N_{iter}$, where N_{iter} is a preset number of iterations. The other usually adopted stopping rule is to evaluate the value of $\|\mathbf{w}(i+1) - \mathbf{w}(i)\|_2^2 / \|\mathbf{w}(i)\|_2^2$. If the value is smaller than a preset threshold, terminate the algorithm.

Table 3.1: Summary of the Proposed Algorithms

Step 1: Parameter setting:
1) Setting up p , λ and γ (λ and γ can be chosen empirically);
2) Generating $\overline{\mathbf{A}}$ according to the presumed look direction and sidelobe sector;
Step 2: Initialization: $i = 0$, $\mathbf{w}(0)$;
Step 3: Iterations:
1) Computing $\Pi(\mathbf{w}(i))$;
2) Updating $\mathbf{w}(i)$ using (3.7) or (3.10);
3) If λ is not set empirically, using L-curve principle to determine $\lambda(i)$;
4) (3.17) is used to compute $\gamma(i)$;
5) $i = i + 1$;
Step 4: Terminating the algorithm if stopping criterion is satisfied, otherwise, going back to Step 3.

3.5 Simulation Results

3.5.1 Evaluation of Algorithm I

1. Implementation of Algorithm I to Array Synthesis

It is observed that the proposed algorithms can also be applied to array synthesis algorithms [74–76] with $\mathbf{R} = \mathbf{I}$ in (3.2). The sparse constraint (3.2c) plays a role to suppress the assigned sidelobe sector. To verify this, we conduct the following simulations. In all the simulations, a ULA with 32 half-wavelength spaced sensors is assumed. α is fixed to 50.

Firstly, we assume sidelobe suppression sector as $[-90^\circ, -1^\circ] \cup [1^\circ, 90^\circ]$. The array beampattern obtained using the proposed algorithm is shown in Figure 3.1.

Figure 3.1: Array beampatterns for a 32-sensor ULA with sidelobe suppression sector $[-90^\circ, -1^\circ] \cup [1^\circ, 90^\circ]$.

One can see from Figure 3.1 that the proposed algorithm reduces the sidelobe level several decibels from the original beampattern. Also it is noticed that the smaller the value of p , the greater the sidelobe suppression. This coincides with our previous analysis that p controls the sparsity of the beampattern.

In certain applications, one may like to maintain the mainlobe of the array response at a certain width to ensure that the signal coming from the desired look direction will be received by the array with sufficient array gain. In Figure 3.2, the proposed algorithm is used in the design of array pattern under this situation. The sidelobe suppression sector is chosen to be $[-90^\circ, -5^\circ] \cup [5^\circ, 90^\circ]$.

Figure 3.2: Array beampatterns for a 32-sensor ULA with sidelobe suppression sector $[-90^\circ, -5^\circ] \cup [5^\circ, 90^\circ]$.

It is seen that compared with the original beampattern, the beamwidth is increased slightly, while the sidelobe level is improved by the order of tens of decibels. The sidelobes beside the mainlobe is significantly improved by more than 5 dB.

2. Implementation of Algorithm I to Beamforming

In this section, a ULA with eight half-wavelength inter-element spaced sensors is used. The DOA of the desired signal is supposed to be 0° , and the DOAs of three interferences are set to -30° , 30° and 70° . The SNR is set to 10 dB, and the SINR is -20 dB. 100 snapshots are used to compute the covariance matrix $\mathbf{R}$. The regularization parameter λ is set to 0.2 in all the simulations. The matrix $\mathbf{A}$ consists of array steering vectors in the angular range from $[-90^\circ, 0^\circ)$ and $(0^\circ, 90^\circ]$ with 1° sampling interval. In this case, all the potential interferences are contained in $\mathbf{A}$ and the sidelobe level

of the whole angular range is under control.

Figure 3.3: Beam patterns for SNR=10 dB in the no mismatch case.

In the first simulation, we assume no mismatch problem happens. The proposed method is compared with the MVDR beamformer. With different p and α , the derived beam patterns are shown in Figure 3.3.

It is observed that all of the beamformers cast deep nulls in directions of interference and generate distortionless response at the look direction. However, the sidelobe level of MVDR beamformer is much higher than that of the proposed algorithm. For the proposed beamformer, the sidelobe level decreases as p decreases. This is because the smaller the p , the smaller the values of the beam pattern are. This simulation result coincides with our previous analysis.

Figure 3.4: Plots of output SINR versus SNR in no mismatch case.

Figure 3.4 shows the plots of output SINR versus SNR with 1000 independent trials. The output SINR is calculated via

$$SINR_{output} = \frac{\sigma_s^2 \mathbf{w}^H \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) \mathbf{w}}{\mathbf{w}^H \left(\sum_{j=1}^J \sigma_j^2 \mathbf{a}(\theta_j) \mathbf{a}^H(\theta_j) + \sigma_n^2 \mathbf{I} \right) \mathbf{w}}, \quad (3.18)$$

where J denotes the number of interferences.

It is noted that the proposed beamformer can achieve an output SINR approximately 6 dB larger than that of MVDR beamformer for all of the SNRs. For different p and α , the proposed algorithm shows similar performance results.

In the second simulation, we assume there is a 3° mismatch in the look direction. The desired signal impinges the array from broadside, while the look direction of the beamformer is assumed to be 3° . Figure 3.5 shows the obtained beampatterns of different algorithms.

Figure 3.5: Beampatterns for SNR=10 dB in 3° mismatch case.

It is observed from Figure 3.5 that the MVDR beamformer introduced a deep null at 0° , which means the desired source is suppressed due to imprecise knowledge of the look direction. In contrast, the proposed beamformer only slightly shifts its maximum point of the beampattern about 3° from broadside direction, and is still capable of generating distortionless response at the direction of SOI. At interference directions, all the algorithms are able to cast deep nulls.

Figure 3.6: Plots of output SINR versus SNR in 3° mismatch case.

Figure 3.6 plots the output SINR versus SNR with 1000 independent trials. From the figure, we see that the performance of MVDR beamformer and the proposed one differ a lot. It is observed that the proposed algorithm gives an output SINR of about 10 dB, while the MVDR beamformer fails to work in all the cases. Similar to Figure 3.2, for different p and α , the proposed algorithm has very similar performance.

3.5.2 Evaluation of Algorithm II

In this section, we evaluate the performance of the proposed algorithm when directions of interferences and SOI are not precisely known. A ULA with ten half-wavelength inter-element spaced sensors is used. The actual source DOA is supposed to be 0° , and the DOAs of four interferences are set to $-30^\circ, -10^\circ, 20^\circ, 45^\circ$. The SINR is assumed

to be -20 dB, and the SNR is assumed to be 10 dB. The regularization parameter α is set to 50 in all the simulations. The matrix $\mathbf{A}$ consists of array steering vectors in the DOA range $[-90^\circ, -4^\circ)$ and $(4^\circ, 90^\circ]$ with 10° sampling interval, while the matrix $\mathbf{A}_s$ consists of only two array steering vectors that is defined in -3° and 3° .

Note that unlike the simulation settings in the previous section, we greatly relax our interferences' constraints by using 10° sampling interval. Also, the interference from 45° does not fall in the spatial sampling grid.

In the first simulation, we assume a 3° look mismatch, i.e., the SOI impinges the array from broadside, while we assume it comes from 3° . Figure 3.7 shows the derived beampatterns using different algorithms.

Figure 3.7: Beampatterns for SNR=10 dB in 3° mismatch case.

It is observed from Figure 3.7 that the MVDR casts a deep null at 0° , while the proposed algorithms still perform satisfactorily in this case. For interference directions, all the algorithms cast deep nulls.

Figure 3.8 plots the output SINR versus SNR with 1000 independent trials.

Figure 3.8: Plots of output SINR versus SNR in 3° mismatch case.

We see that the proposed algorithm outperforms the MVDR beamformer. The MVDR beamformer almost fails to work in all the cases. As SNR increases, the performance of the MVDR beamformer is more significantly influenced by the look direction mismatch so that its performance degrades. The proposed algorithms with different values of p again achieve similar performance to each other.

Figure 3.9: Beam patterns for SNR=10 dB in 3° mismatch and sensor position error.

In the second simulation, besides the look direction mismatch, we further assume sensor position error which is a Gaussian variable with zero mean and standard deviation 0.1 times the sensor spacing appears in the steering vector. Figure 3.9 depicts the obtained beam patterns using different algorithms. Figure 3.10 shows the output SINR versus SNR with 1000 independent trials.

Figure 3.10: Plots of output SINR versus SNR in 3° mismatch and sensor position error.

From Figure 3.9, we see that the proposed algorithm is capable of maintaining distortionless response at the direction of the SOI, while the MVDR beamformer suppresses the SOI about 25 dB in order to yield distortionless response at the look direction. Figure 3.10 clearly demonstrates that the proposed algorithm is robust against look direction error and sensor position error, while the MVDR beamformer is very sensitive to these errors.

3.6 Conclusions

In this chapter, we devise two beamformers with explicit constraint on interference directions using the ℓ_p -norm. With imprecise knowledge of DOA of interferences,

distortionless response can be imposed on a relatively wide angular range instead of only one look direction. Computer simulations show that the proposed algorithm can give better sidelobe suppression effect than that of the MVDR beamforming technique. It has also been demonstrated that the proposed algorithms are robust against look direction error and sensor position error. With $\mathbf{R}$ replaced by $\mathbf{I}$, the proposed algorithm can be applied to array synthesis, which satisfactorily suppresses the sidelobe level at the assigned sidelobe sector.

Since the ℓ_p -norm can also be used as objective function to minimize the impulsive noise, we will introduce new robust beamformers against impulsive noise using the ℓ_p -norm in the next chapter. Different from this chapter that $p < 2$ is assumed, we will assume $p = 1$ in Chapter 4 so that a non-concave optimization problem can be derived.

Chapter 4

ℓ_1 -norm and ℓ_2 -norm Beamformers with Sphere Constraint on the Steering Vector Error to Enhance the Robustness Against the Steering Vector Error and Impulsive Noise

4.1 Introduction

In this chapter, we aim to improve the performance of the beamformer in the presence of steering vector error and impulsive noise.

It is known that in real applications, imprecise knowledge of the array steering

vector causes the nominal and actual steering vector to be distinct. In such a situation, performance of the MVDR beamformer is shown to deteriorate significantly [11]. To amend this drawback, some robust beamforming techniques have been introduced [4]. These robust beamformers add new constraints to the original cost function of the MVDR beamformer to enhance beamformer's robustness against steering vector error. Increased constraints sacrifice the DoF of the beamformer to suppress interferences, which is especially important when the number of sensors is small.

There have been some literature demonstrating that impulsive noise appears on the signal at wireless receiver in a form of impulsive noise bursts [36–38]. These bursts may appear at some time and disappear at a later time. All the beamformers mentioned in previous section can not perform well, because they are all based on the second-order statistics. To enhance the performance of beamformers in the presence of impulsive noise, some new beamforming algorithms have been introduced. These algorithms can be differentiated according to their modeling of impulsive noise. In [39–42], impulsive noise is modeled by a α -stable random process. Similar to the LMS algorithm which minimizes the variance of the noise, 'dispersion' is defined and used to characterize the noise. Dispersion is defined as an ℓ_p -norm where $p < \alpha$ holds. In such a case, ℓ_p -norm is usually adopted as the criterion to suppress the noise. ℓ_p -norm is sometimes considered as FLOS. Since only when $p < \alpha$ holds, the methods using ℓ_p -norm can perform well, a convenient choice of p is 0. However, the ℓ_0 -norm means an exhaustive search in the solution space, so some alternatives have been proposed, such as geometric power [43]. Besides the α -stable random process, impulsive noise can also be modeled as mixture of Gaussian variables [77], where the EM algorithm can be applied. The aforementioned beamforming methods are expected to degrade in performance when the array steering vector is not completely known.

In this chapter, we design robust ℓ_1 -norm and ℓ_2 -norm beamformers which perform well against steering vector error and impulsive noise. In Section 4.2, we propose a robustness enhanced ℓ_2 -norm beamformer which aims to minimizing the correlation of the real and the imaginary part of the desired signal in the objective function as well as the beamformer's output. Computer simulations show that the proposed algorithm outperforms the MVDR beamformer and the norm constrained MVDR beamformer [58] in the presence of steering vector error. Section 4.3 presents the proposed ℓ_1 -norm beamformer with sphere constraint on the steering vector error to suppress the impulsive noise [37, 38, 44] and the steering vector error. In Section 4.4, computer simulations are conducted to verify validity and advantage of the proposed algorithms. Conclusions are drawn in Section 4.5.

4.2 Robust ℓ_2 -norm Beamformer

4.2.1 Problem Formulation

For M narrowband sources, the input of an array with L sensors can be formulated as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (4.1)$$

where $\mathbf{A} = [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_M)]$ is the array manifold, $\mathbf{s}(t) = [s_1(t), \dots, s_M(t)]^T$ is a vector containing the desired signal $s_1(t)$ and the interferences, and $\mathbf{n}(t)$ denotes the spatial noise. The real steering vector $\mathbf{a}(\theta)$ is associated with the nominal steering vector $\mathbf{a}_0(\theta)$ by

$$\mathbf{a}(\theta) = \mathbf{a}_0(\theta) + \mathbf{e}, \quad (4.2)$$

where $\mathbf{e}$ denotes the error.

The objective of a beamformer is to recover the desired signal $s_1(t)$ by

$$\hat{s}_1(t) = \mathbf{w}^H \mathbf{x}(t), \quad (4.3)$$

where $\mathbf{w}$ is the weight vector, and $\hat{s}_1(t)$ is the beamformer output, which is an estimate of $s_1(t)$.

4.2.2 Optimization Function and its Solution

The proposed beamformer is designed using the following optimization problem in real-valued vectors as:

$$\min_{\mathbf{z}} E(|\mathbf{z}^T \mathbf{y}|^2), \quad (4.4a)$$

$$\text{subject to } \mathbf{z}^T \mathbf{c}_0(\theta) = 1, \quad (4.4b)$$

where

$$\begin{aligned} \mathbf{z} &= [\text{Real}(\mathbf{w})^T \text{Imag}(\mathbf{w})^T]^T, \\ \mathbf{y} &= \begin{bmatrix} \text{Real}(\mathbf{x}) & \text{Imag}(\mathbf{x}) \\ \text{Imag}(\mathbf{x}) & -\text{Real}(\mathbf{x}) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ \mathbf{c}_0(\theta) &= [\text{Real}(\mathbf{a}_0(\theta))^T \text{Imag}(\mathbf{a}_0(\theta))^T]^T. \end{aligned}$$

Using the Lagrange multiplier technique, the solution of (4.4) is given by

$$\mathbf{z}_{opt} = \frac{\mathbf{R}_{yy}^{-1} \mathbf{c}_0(\theta)}{\mathbf{c}_0^T \mathbf{R}_{yy}^{-1} \mathbf{c}_0(\theta)}. \quad (4.5)$$

After deriving $\mathbf{z}_{opt} = [\text{Real}(\mathbf{w}_{opt})^T \text{Imag}(\mathbf{w}_{opt})^T]^T$, the desired signal is recovered using

$$\hat{s}(t) = (\text{Real}(\mathbf{w}_{opt}) + j \text{Imag}(\mathbf{w}_{opt}))^H \mathbf{x}(t). \quad (4.6)$$

4.2.3 Analysis of the Proposed Algorithm

To analyze the performance of the proposed algorithm, we firstly convert the MVDR beamformer in real-valued form.

The optimization problem of the MVDR beamformer is given by

$$\min_{\mathbf{w}} \quad \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w}, \quad (4.7a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}_0(\theta) = 1. \quad (4.7b)$$

It has been demonstrated that (4.4b) and (4.7b) are equivalent for implementing the MVDR beamformer [4].

Denote the real and imaginary part of $\mathbf{x}$ as $\mathbf{u}$ and $\mathbf{v}$, respectively: $\mathbf{x} = \mathbf{u} + j\mathbf{v}$. $\mathbf{R}_{xx}$ is calculated via

$$\begin{aligned} \mathbf{R}_{xx} &= E[\mathbf{x}\mathbf{x}^H] \\ &= E[(\mathbf{u} + j\mathbf{v})(\mathbf{u}^T - j\mathbf{v}^T)] \\ &= E[\mathbf{u}\mathbf{u}^T + \mathbf{v}\mathbf{v}^T + j\mathbf{v}\mathbf{u}^T - j\mathbf{u}\mathbf{v}^T] \\ &= \mathbf{R}_{uu} + \mathbf{R}_{vv} + j(\mathbf{R}_{vu} - \mathbf{R}_{uv}). \end{aligned} \quad (4.8)$$

Denote the real and imaginary part of $\mathbf{w}$ as $\mathbf{a}$ and $\mathbf{b}$, respectively: $\mathbf{w} = \mathbf{a} + j\mathbf{b}$. The output power of the MVDR beamformer is expressed as

$$\begin{aligned} \text{Power}(\mathbf{a}, \mathbf{b}) &= \mathbf{w}^H \mathbf{R}_{xx} \mathbf{w} \\ &= (\mathbf{a}^T - j\mathbf{b}^T) \mathbf{R}_{xx} (\mathbf{a} + j\mathbf{b}) \\ &= \mathbf{a}^T (\mathbf{R}_{uu} + \mathbf{R}_{vv}) \mathbf{a} + \mathbf{b}^T (\mathbf{R}_{uu} + \mathbf{R}_{vv}) \mathbf{b} + 2\mathbf{a}^T (\mathbf{R}_{uv} - \mathbf{R}_{vu}) \mathbf{b}. \end{aligned} \quad (4.9)$$

Therefore, the optimization problem of the MVDR beamformer using real-valued

constraint is given by

$$\min_{\mathbf{a}, \mathbf{b}} \text{Power}(\mathbf{a}, \mathbf{b}), \quad (4.10a)$$

$$\text{subject to } \mathbf{c}_0^T(\theta) \begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix} = 1. \quad (4.10b)$$

Using the Lagrange multiplier technique, we have

$$\begin{pmatrix} \mathbf{a} \\ \mathbf{b} \end{pmatrix}_{opt} = \frac{\begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix}^{-1} \mathbf{c}_0(\theta)}{\mathbf{c}_0^T(\theta) \begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix}^{-1} \mathbf{c}_0(\theta)}, \quad (4.11)$$

where

$$\mathbf{A} = \begin{pmatrix} \mathbf{R}_{uu} + \mathbf{R}_{vv} & \mathbf{R}_{uv} - \mathbf{R}_{vu} \end{pmatrix}, \quad (4.12)$$

$$\mathbf{B} = \begin{pmatrix} \mathbf{R}_{vu} - \mathbf{R}_{uv} & \mathbf{R}_{uu} + \mathbf{R}_{vv} \end{pmatrix}. \quad (4.13)$$

For the proposed algorithm (4.4), $\mathbf{y}$ is defined as

$$\mathbf{y} = \begin{pmatrix} \mathbf{u} + \mathbf{v} \\ \mathbf{v} - \mathbf{u} \end{pmatrix}. \quad (4.14)$$

Supposing the weight vector composes of two parts as $\mathbf{z} = \begin{pmatrix} \mathbf{c} \\ \mathbf{d} \end{pmatrix}$, the output of the

proposed beamformer is given by

$$\begin{aligned}\text{Output}(\mathbf{c}, \mathbf{d}) &= \mathbf{z}^T \mathbf{R}_{yy} \mathbf{z} \\ &= \text{Power}(\mathbf{c}, \mathbf{d}) + \mathbf{c}^T (\mathbf{R}_{uv} + \mathbf{R}_{vu}) \mathbf{c} \\ &\quad - \mathbf{d}^T (\mathbf{R}_{uv} + \mathbf{R}_{vu}) \mathbf{d} + 2\mathbf{c}^T (\mathbf{R}_{vv} - \mathbf{R}_{uu}) \mathbf{d}.\end{aligned}\quad (4.15)$$

Also using the Lagrange multiplier technique, we derive the optimal solution of $\mathbf{z}$ which is given by

$$\begin{pmatrix} \mathbf{c} \\ \mathbf{d} \end{pmatrix}_{opt} = \frac{\begin{pmatrix} \mathbf{A} + \mathbf{C} \\ \mathbf{B} + \mathbf{D} \end{pmatrix}^{-1} \mathbf{c}(\theta)}{\mathbf{c}^T(\theta) \begin{pmatrix} \mathbf{A} + \mathbf{C} \\ \mathbf{B} + \mathbf{D} \end{pmatrix}^{-1} \mathbf{c}(\theta)}, \quad (4.16)$$

where

$$\mathbf{C} = \begin{pmatrix} \mathbf{R}_{uv} + \mathbf{R}_{vu} & \mathbf{R}_{vv} - \mathbf{R}_{uu} \end{pmatrix}, \quad (4.17)$$

$$\mathbf{D} = \begin{pmatrix} \mathbf{R}_{vv} - \mathbf{R}_{uu} & -\mathbf{R}_{uv} - \mathbf{R}_{vu} \end{pmatrix}. \quad (4.18)$$

It is observed from (4.11) and (4.16) that the proposed algorithm differs from the MVDR beamformer in that an additional matrix $\begin{pmatrix} \mathbf{C} \\ \mathbf{D} \end{pmatrix}$ appears in the solution.

It can be deduced that

$$\begin{pmatrix} \mathbf{A} \\ \mathbf{B} \end{pmatrix} = E \left[\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} \begin{pmatrix} \mathbf{u}^T & \mathbf{v}^T \end{pmatrix} + \begin{pmatrix} -\mathbf{v} \\ \mathbf{u} \end{pmatrix} \begin{pmatrix} -\mathbf{v}^T & \mathbf{u}^T \end{pmatrix} \right] \quad (4.19)$$

and

$$\begin{pmatrix} \mathbf{C} \\ \mathbf{D} \end{pmatrix} = E \left[\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} \begin{pmatrix} \mathbf{v}^T & -\mathbf{u}^T \end{pmatrix} + \begin{pmatrix} \mathbf{v} \\ -\mathbf{u} \end{pmatrix} \begin{pmatrix} \mathbf{u}^T & \mathbf{v}^T \end{pmatrix} \right]. \quad (4.20)$$

Denoting the optimal weight vector of the beamformer as $\mathbf{w}_{opt} = \mathbf{a}_{opt} + j\mathbf{b}_{opt}$, it is observed that

$$\begin{pmatrix} \mathbf{a}_{opt} \\ \mathbf{b}_{opt} \end{pmatrix}^T \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix} = \text{Real}(\hat{s}_1(t)), \quad (4.21a)$$

$$\begin{pmatrix} \mathbf{a}_{opt} \\ \mathbf{b}_{opt} \end{pmatrix}^T \begin{pmatrix} \mathbf{v} \\ -\mathbf{u} \end{pmatrix} = \text{Imag}(\hat{s}_1(t)), \quad (4.21b)$$

which means that $\mathbf{z}^T \begin{pmatrix} \mathbf{C} \\ \mathbf{D} \end{pmatrix} \mathbf{z}$ is a measurement of correlation of the real part and imaginary part of the desired source in essence. Therefore, the proposed optimization problem (4.2) differs from the MVDR beamformer in that the proposed algorithm not only minimizes the output power, but also minimizes correlation of the real part and imaginary part of the beamformer's output. Since the real part and imaginary part of the beamformer's output should be orthogonal to each other, the optimal solution of the proposed beamformer belongs to the subset of solutions of the MVDR beamformer. Therefore, the proposed algorithm performs as well as the MVDR beamformer if not better than.

4.2.4 Robust Beamforming Algorithm with Explicit Constraints on the Steering Vector Error

The proposed algorithm can be extended to incorporate constraints on the steering vector error to further enhance its robustness.

Supposing that the steering vector uncertainty is contained in a sphere [26], the proposed robust beamformer can be formulated as

$$\min_{\mathbf{z}} E(|\mathbf{z}^T \mathbf{y}|^2), \quad (4.22a)$$

$$\text{subject to } \mathbf{z}^T \mathbf{c}(\theta) \geq 1, \quad (4.22b)$$

$$\mathbf{c}(\theta) = \mathbf{c}_0(\theta) + \mathbf{e},$$

$$\|\mathbf{e}\|_2 \leq \varepsilon, \quad \varepsilon > 0.$$

The constraints given in (4.22b) can be equivalently expressed as

$$\mathbf{z}^T \mathbf{c}_0(\theta) - 1 \geq \mathbf{z}^T \mathbf{e}, \quad \text{for all } \|\mathbf{e}\|_2 \leq \varepsilon. \quad (4.23)$$

By the Cauchy-Schwartz inequality, we see that (4.23) is equivalent to the constraint

$$\mathbf{z}^T \mathbf{c}_0(\theta) - 1 \geq \varepsilon \|\mathbf{z}\|_2. \quad (4.24)$$

Hence, we express the robust ℓ_2 -norm beamforming problem as

$$\min_{\mathbf{z}} E(|\mathbf{z}^T \mathbf{y}|^2), \quad (4.25a)$$

$$\text{subject to } \mathbf{z}^T \mathbf{c}_0(\theta) - 1 \geq \varepsilon \|\mathbf{z}\|_2. \quad (4.25b)$$

Since (4.25a) is convex, and (4.25b) precludes the trivial minimizer of $\|\mathbf{z}^T \mathbf{y}\|$, this constraint will be tight for any optimal solution ((4.25b) is satisfied with equality at the optimal $\mathbf{z}$). We may equivalently express (4.25) as

$$\min_{\mathbf{z}} E(\|\mathbf{z}^T \mathbf{y}\|^2), \quad (4.26a)$$

$$\text{subject to } \mathbf{z}^T \mathbf{c}_0(\theta) - 1 = \varepsilon \|\mathbf{z}\|_2. \quad (4.26b)$$

Using the Lagrange multiplier technique, its solution is derived as

$$\mathbf{z} = -\lambda (\mathbf{R}_{yy} + \lambda \mathbf{Q})^{-1} \mathbf{c}_0(\theta), \quad (4.27)$$

where λ is the root of the equation:

$$f(\lambda) = \lambda^2 \sum_{i=1}^{2L} \frac{c_i^2 \gamma_i}{(1 + \lambda \gamma_i)^2} - 2\lambda \sum_{i=1}^{2L} \frac{c_i^2}{(1 + \lambda \gamma_i)} - 1. \quad (4.28)$$

In (4.27), $\mathbf{Q} = \varepsilon^2 \mathbf{I} - \mathbf{c}_0(\theta) \mathbf{c}_0^T(\theta)$. In (4.28), c_i is the i^{th} element of $\mathbf{c} = \mathbf{V}^T \mathbf{R}_{yy}^{-1/2} \mathbf{c}_0(\theta)$, γ_i is the i^{th} element of the diagonal elements of $\mathbf{\Gamma}$. $\mathbf{V}$ and $\mathbf{\Gamma}$ denote the matrices consisting of eigenvectors and eigenvalues of $\mathbf{R}_{yy}^{-1/2} \mathbf{Q} (\mathbf{R}_{yy}^{-1/2})^T$, respectively. Note that $\mathbf{R}_{yy} = E[\mathbf{y} \mathbf{y}^T]$.

4.3 ℓ_1 -norm Beamformer with Sphere Constraint

4.3.1 The Proposed Optimization Problem

In the presence of impulsive noise and steering vector error, the proposed beamformer is designed using the following optimization problem in real-valued vectors:

$$\min_{\mathbf{z}} E(|\mathbf{z}^T \mathbf{y}|), \quad (4.29a)$$

$$\text{subject to } \mathbf{z}^T \mathbf{c}(\theta) \geq 1, \quad (4.29b)$$

$$\mathbf{c}(\theta) = \mathbf{c}_0(\theta) + \mathbf{e},$$

$$\|\mathbf{e}\|_2 \leq \varepsilon, \quad \varepsilon > 0.$$

Using the result we derived in last section, the equivalent expression of the robust ℓ_1 -norm beamforming problem is given as

$$\min_{\mathbf{z}} E(|\mathbf{z}^T \mathbf{y}|), \quad (4.30a)$$

$$\text{subject to } \mathbf{z}^T \mathbf{c}_0(\theta) - 1 = \varepsilon \|\mathbf{z}\|_2. \quad (4.30b)$$

4.3.2 Lagrange Multiplier Techniques

In this section, we focus on finding solution of the proposed robust ℓ_1 -norm beamformer. If we impose $\mathbf{z}^T \mathbf{c}_0(\theta) \geq 1$ on (4.30b), the proposed optimization problem becomes

$$\min_{\mathbf{z}} E(|\mathbf{z}^T \mathbf{y}|), \quad (4.31a)$$

$$\text{subject to } (\mathbf{z}^T \mathbf{c}_0(\theta) - 1)^2 = \varepsilon^2 \|\mathbf{z}\|_2^2. \quad (4.31b)$$

Then, we define the instantaneous scalar function as

$$\begin{aligned} L(\mathbf{z}, \lambda) &= |\mathbf{z}^T \mathbf{y}| + \lambda (\varepsilon^2 \|\mathbf{z}\|_2^2 - (\mathbf{z}^T \mathbf{c}_0(\theta) - 1)^2) \\ &= |\mathbf{z}^T \mathbf{y}| + \lambda (\mathbf{z}^T \mathbf{Q} \mathbf{z} + 2\mathbf{c}_0^T(\theta) \mathbf{z} - 1), \end{aligned} \quad (4.32)$$

where $\mathbf{Q} = \varepsilon^2 \mathbf{I} - \mathbf{c}_0(\theta) \mathbf{c}_0^T(\theta)$.

Setting the gradient of (4.32) with respect to $\mathbf{z}$ and λ to 0 yields

$$\nabla_{\mathbf{z}}|_{\mathbf{z}=\mathbf{z}_{opt}} = \text{sign}(\mathbf{z}_{opt}^T \mathbf{y}) \mathbf{y} + 2\lambda (\mathbf{Q} \mathbf{z}_{opt} + \mathbf{c}_0(\theta)) = 0 \quad (4.33)$$

and

$$\mathbf{z}_{opt}^T \mathbf{Q} \mathbf{z}_{opt} + 2\mathbf{c}_0^T(\theta) \mathbf{z}_{opt} - 1 = 0. \quad (4.34)$$

However, the sign function with respect to $\mathbf{z}$ in (4.33) prevents us from deriving the analytical solution of $\mathbf{z}_{opt}$. Therefore, we adopt an iterative algorithm to look for the stationary point of (4.31):

$$\mathbf{z}(k+1) = \mathbf{z}(k) - \mu(k) \nabla_{\mathbf{z}}|_{\mathbf{z}=\mathbf{z}(k)}, \quad (4.35)$$

where $\mu(k)$ denotes the stepsize.

4.3.3 Determination of the Lagrange Multiplier λ and the Stepsize $\mu(k)$

It is noted that $\mathbf{z}(k+1)$ should satisfy (4.34). Substituting (4.35) into (4.34) yields

$$\mu^2(k) \nabla_{\mathbf{z}}^T|_{\mathbf{z}=\mathbf{z}(k)} \mathbf{Q} \nabla_{\mathbf{z}}|_{\mathbf{z}=\mathbf{z}(k)} - 2\mu(k) \nabla_{\mathbf{z}}^T|_{\mathbf{z}=\mathbf{z}(k)} \mathbf{c}_0(\theta) - 2\mu(k) \mathbf{z}^T(k) \mathbf{Q} \nabla_{\mathbf{z}}|_{\mathbf{z}=\mathbf{z}(k)} = 0, \quad (4.36)$$

which is a quadratic function of $\mu(k)$. Substituting

$$\nabla_{\mathbf{z}}|_{\mathbf{z}=\mathbf{z}} = \text{sign}(\mathbf{z}^T \mathbf{y}) \mathbf{y} + 2\lambda (\mathbf{Q}\mathbf{z} + \mathbf{c}_0(\theta))$$

into (4.36), we get an equation for λ :

$$A_1 \lambda^2 + B_1 \lambda + C_1 = 0, \quad (4.37)$$

where

$$A_1 = \mu^2(k) \mathbf{f}^T(k) \mathbf{Q} \mathbf{f}(k), \quad (4.38a)$$

$$B_1 = [\text{sign}(\mathbf{z}(k)^T \mathbf{y}) \mathbf{f}^T(k) \mathbf{Q} \mathbf{y} \mu^2(k) - \mu(k) \|\mathbf{f}(k)\|_2^2], \quad (4.38b)$$

$$C_1 = \mu^2(k) \mathbf{y}^T \mathbf{Q} \mathbf{y} - \mu(k) \text{sign}(\mathbf{z}(k)^T \mathbf{y}) \mathbf{f}^T \mathbf{y}, \quad (4.38c)$$

$$\mathbf{f}(k) = 2(\mathbf{Q}\mathbf{z}(k) + \mathbf{c}_0(\theta)). \quad (4.38d)$$

The condition of existence of solutions to (4.37) is given by:

$$\Delta_\lambda = B_1^2 - 4A_1C_1 \geq 0. \quad (4.39)$$

We further define

$$A_2 = 4\mathbf{f}^T(k) \mathbf{Q} (\mathbf{y} \mathbf{f}^T(k) - \mathbf{f}(k) \mathbf{y}^T) \mathbf{Q} \mathbf{y}, \quad (4.40a)$$

$$B_2 = -4\text{sign}(\mathbf{z}(k)^T \mathbf{y}) \mathbf{f}^T(k) \mathbf{Q} (\mathbf{y} \mathbf{f}^T(k) - \mathbf{f}(k) \mathbf{y}^T) \mathbf{f}(k) \quad (4.40b)$$

$$C_2 = \|\mathbf{f}(k)\|_2^4. \quad (4.40c)$$

Substituting (4.38a)-(4.38c) into (4.39) yields

$$A_2 \mu^2(k) + B_2 \mu(k) + C_2 \geq 0. \quad (4.41)$$

The solution of (4.41) is

$$\begin{cases} \mu(k) > 0, & A_2 > 0 \\ 0 < \mu(k) \leq \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\}, & A_2 < 0. \end{cases} \quad (4.42)$$

where $\mu_{1,2}^{(2)}$ are the roots of the equation $A_2\mu^2(k) + B_2\mu(k) + C_2 = 0$. Since $A_2 < 0$ and $C_2 > 0$, a solution for $\mu(k)$ satisfying (4.42) always exists.

When (4.41) is satisfied, we solve (4.37) to get the values of λ and choose the one which minimizes (4.31a) and simultaneously satisfies

$$\mathbf{z}(k+1)^T \mathbf{c}_0(\theta) - 1 \geq 0. \quad (4.43)$$

Now, we will discuss how to choose $\mu(k)$ such that (4.43) holds.

Let us define the nonnegative function of λ in (4.43) as

$$\begin{aligned} g(\lambda) &= \mathbf{z}(k+1)^T \mathbf{c}_0(\theta) - 1 \\ &= \mathbf{z}^T(k) \mathbf{c}_0(\theta) - \mu(k) \text{sign}(\mathbf{z}(k)^T \mathbf{y}) \mathbf{y}^T \mathbf{c}_0(\theta) - \\ &\quad \lambda 2\mu(k) \left(\mathbf{z}^T(k) \mathbf{Q} \mathbf{c}_0(\theta) + \|\mathbf{c}_0(\theta)\|_2^2 \right) - 1. \end{aligned} \quad (4.44)$$

Assuming λ_1 and λ_2 are the roots of (4.37), they should satisfy

$$\lambda_1 + \lambda_2 = -B_1/A_1, \quad (4.45a)$$

$$\lambda_1 \lambda_2 = C_1/A_1. \quad (4.45b)$$

The multiplication of $g(\lambda_1)g(\lambda_2)$ is then given by

$$\begin{aligned} g(\lambda_1)g(\lambda_2) &= 4\lambda_1\lambda_2\mu^2(k) \left(\mathbf{z}^T(k)\mathbf{Q}\mathbf{c}_0(\theta) + \|\mathbf{c}_0(\theta)\|_2^2 \right)^2 - \\ &\quad 2(\lambda_1 + \lambda_2)\mu(k) \left(\mathbf{z}^T(k)\mathbf{Q}\mathbf{c}_0(\theta) + \|\mathbf{c}_0(\theta)\|_2^2 \right) \\ &\quad \left(\mathbf{z}^T(k)\mathbf{c}_0(\theta) - \mu(k)\text{sign}(\mathbf{z}(k)^T\mathbf{y})\mathbf{y}^T\mathbf{c}_0(\theta) - 1 \right) + \\ &\quad \left(\mathbf{z}^T(k)\mathbf{c}_0(\theta) - \mu(k)\text{sign}(\mathbf{z}(k)^T\mathbf{y})\mathbf{y}^T\mathbf{c}_0(\theta) - 1 \right)^2. \end{aligned} \quad (4.46)$$

Substituting (4.45) into (4.46) yields

$$g(\lambda_1)g(\lambda_2) = A_3\mu^2(k) + B_3\mu(k) + C_3, \quad (4.47)$$

where

$$A_3 = \frac{\mathbf{y}^T\mathbf{Q}\mathbf{y}D^2 - 2\mathbf{f}^T(k)\mathbf{Q}\mathbf{y}\mathbf{y}^T\mathbf{c}_0(\theta)D}{\mathbf{f}^T(k)\mathbf{Q}\mathbf{f}(k)} + \|\mathbf{y}^T\mathbf{c}_0(\theta)\|^2, \quad (4.48a)$$

$$\begin{aligned} B_3 &= \frac{-\text{sign}(\mathbf{z}(k)^T\mathbf{y})\mathbf{f}^T(k)\mathbf{y}D^2}{\mathbf{f}^T(k)\mathbf{Q}\mathbf{f}(k)} + \frac{\|\mathbf{f}(k)\|_2^2 \text{sign}(\mathbf{z}(k)^T\mathbf{y})\mathbf{y}^T\mathbf{c}_0(\theta)D}{\mathbf{f}^T(k)\mathbf{Q}\mathbf{f}(k)} - \\ &\quad 2 \left(\mathbf{z}^T(k)\mathbf{c}_0(\theta) - 1 \right) \text{sign}(\mathbf{z}(k)^T\mathbf{y})\mathbf{y}^T\mathbf{c}_0(\theta) + \frac{2\text{sign}(\mathbf{z}(k)^T\mathbf{y})\mathbf{f}^T(k)\mathbf{Q}\mathbf{y}}{\mathbf{f}^T(k)\mathbf{Q}\mathbf{f}(k)} \left(\mathbf{z}^T(k)\mathbf{c}_0(\theta) - 1 \right) D, \end{aligned} \quad (4.48b)$$

$$C_3 = \frac{- \left(\mathbf{z}^T(k)\mathbf{c}_0(\theta) - 1 \right) \|\mathbf{f}(k)\|_2^2 D}{\mathbf{f}^T(k)\mathbf{Q}\mathbf{f}(k)} + \left(\mathbf{z}^T(k)\mathbf{c}_0(\theta) - 1 \right)^2, \quad (4.48c)$$

$$D = 2 \left(\mathbf{z}^T(k)\mathbf{Q}\mathbf{c}_0(\theta) + \|\mathbf{c}_0(\theta)\|_2^2 \right). \quad (4.48d)$$

Four possible solutions of (4.47) are discussed as follows:

Case 1: $A_3 < 0, C_3 > 0$.

Subcase 1: One of $\lambda_{1,2}$ satisfies (4.43):

In this case, $\exists \mu(k) \geq \mu_{max}$ so that $g(\lambda_1)g(\lambda_2) \leq 0$ holds, where $\mu_{max} = \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}$ and $\mu_{1,2}^{(3)}$ are the roots of the equation $A_3\mu^2(k) + B_3\mu(k) + C_3 = 0$.

With (4.42), $\mu(k)$ should be chosen based on the inequality:

$$\max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\} \leq \mu(k), \text{ when } A_2 > 0, \quad (4.49a)$$

$$\max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\} \leq \mu(k) \leq \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\}, \text{ when } A_2 < 0. \quad (4.49b)$$

If $\max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}$ is larger than the value of the right side of (4.49b), we need to consider another case.

Subcase 2: $\lambda_{1,2}$ satisfies (4.43):

In this case, $\exists 0 < \mu(k) \leq \mu_{max}$ so that $g(\lambda_1)g(\lambda_2) \geq 0$ holds. To guarantee (4.43), $g(\lambda_1) + g(\lambda_2) \geq 0$ should also hold.

This gives the inequality:

$$\mu(k) \leq J, \text{ when the denominator of } J \text{ is negative}, \quad (4.50a)$$

$$\mu(k) \geq J, \text{ when the denominator of } J \text{ is positive}. \quad (4.50b)$$

In (4.50), J is given by

$$J = \frac{\| \mathbf{f}(k) \|_2^2 D / (\mathbf{f}^T(k) \mathbf{Q} \mathbf{f}(k)) - 2 (\mathbf{z}^T(k) \mathbf{c}_0(\theta) - 1)}{2 \text{sign}(\mathbf{z}(k)^T \mathbf{y}) (\mathbf{f}^T(k) \mathbf{Q} \mathbf{y} D / (\mathbf{f}^T(k) \mathbf{Q} \mathbf{f}(k)) - \mathbf{y}^T \mathbf{c}_0(\theta))}. \quad (4.51)$$

The appropriate value of $\mu(k)$ should satisfy (4.42) and (4.50).

Case 2: $A_3 < 0, C_3 \leq 0$.

In this case, $g(\lambda_1)g(\lambda_2) \leq 0$ holds for every $\mu(k) \geq 0$. Therefore, the inequality that $\mu(k)$ should satisfy is (4.42).

Case 3: $A_3 > 0, C_3 < 0$.

In this case, $\exists 0 < \mu(k) \leq \mu_{max}$ so that $g(\lambda_1)g(\lambda_2) \leq 0$ holds.

With (4.42), $\mu(k)$ should be chosen based on the inequality:

$$0 < \mu(k) \leq \min \left\{ \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}, \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\} \right\}, \text{ when } A_2 < 0, \quad (4.52a)$$

$$0 < \mu(k) \leq \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}, \text{ when } A_2 > 0. \quad (4.52b)$$

Since $\max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\} > 0$ holds, (4.52) is applicable. Hence, we do not need to discuss the case when the two roots of (4.37) satisfy (4.43).

Case 4: $A_3 > 0, C_3 \geq 0$.

Subcase 1: $B_3 > 0$:

In this case, $\forall \mu(k) > 0$, $g(\lambda_1)g(\lambda_2) \geq 0$ holds. Therefore, we require $g(\lambda_1) + g(\lambda_2) \geq 0$ hold in order to satisfy (4.43). This gives the same inequality shown in (4.50). The appropriate value of $\mu(k)$ should satisfy (4.42) and (4.50).

Subcase 2: $B_3 < 0$:

In this case, if $A_2 > 0$, we may simply choose $\mu(k)$ to satisfy:

$$\min \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\} \leq \mu(k) \leq \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}. \quad (4.53)$$

Otherwise, (i) if $\max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\} \geq \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}$, we may choose $\mu(k)$ according to (4.53); (ii) if $\min \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\} \leq \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\} \leq \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}$, we may choose $\mu(k)$ to satisfy:

$$\min \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\} \leq \mu(k) \leq \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\}; \quad (4.54)$$

(iii) if $\max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\} \leq \min \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}$, we choose $\mu(k)$ to satisfy:

$$\mu(k) \leq \min \left\{ J, \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\} \right\}, \text{ when the denominator of } J \text{ is negative,} \quad (4.55a)$$

$$J \leq \mu(k) \leq \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\}, \text{ when the denominator of } J \text{ is positive.} \quad (4.55b)$$

Proposition: If $\mathbf{z}^T(0)\mathbf{c}_0(\theta) - 1 \geq 0$ holds, $\mu(k)$ exists for all the cases.

Proof:

In Case 2 and Case 3, $\mu(k)$ can be chosen according to (4.42) and (4.50), respectively. We address the existence of $\mu(k)$ in Case 1 and Case 4 here.

Because $C_3 > 0$ and $\mathbf{z}^T(0)\mathbf{c}_0(\theta) - 1 \geq 0$, when $k = 0$, we have

$$\mathbf{z}^T(0)\mathbf{c}_0(\theta) - 1 > \frac{\|f(k)\|_2^2 D}{\mathbf{f}^T(k)\mathbf{Q}\mathbf{f}(k)}. \quad (4.56)$$

Therefore,

$$2(\mathbf{z}^T(0)\mathbf{c}_0(\theta) - 1) > \frac{\|f(k)\|_2^2 D}{\mathbf{f}^T(k)\mathbf{Q}\mathbf{f}(k)}. \quad (4.57)$$

surely holds, which means the numerator of J is negative. Equivalently, for (4.50a), the right side is a positive value. For (4.50b), the right side is a negative value.

Therefore, when J is positive, the bounds of $\mu(k)$, $k = 1$ in Case 1 and Case 4 are given by

$$0 < \mu(k) \leq \min \left\{ J, \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\} \right\}, \text{ when } A_2 > 0. \quad (4.58a)$$

$$0 < \mu(k) \leq \min \left\{ J, \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\} \right\}, \text{ when } A_2 < 0. \quad (4.58b)$$

When J is negative, the bounds of $\mu(k)$, $k = 1$ in Case 1 and Case 4 is given by

$$0 < \mu(k) \leq \max \left\{ \mu_1^{(3)}, \mu_2^{(3)} \right\}, \text{ when } A_2 > 0. \quad (4.59a)$$

$$0 < \mu(k) \leq \max \left\{ \mu_1^{(2)}, \mu_2^{(2)} \right\}, \text{ when } A_2 < 0. \quad (4.59b)$$

It is known that $\mu(k)$, $k = 1$ chosen based on (4.58) or (4.58) satisfies (4.43). Therefore, $\mu(k)$, $k = 2, \dots$ can be shown to satisfy (4.43) in the same way as above.

□

4.3.4 Summary of the Algorithm

We summarize the procedure of the proposed algorithm here.

Step 1: Choose a proper $\mathbf{z}(0)$ to initialize the algorithm. In our simulations, we choose

$$\mathbf{z}(0) = \frac{1}{(\|\mathbf{c}_0(\theta)\|_2 - \varepsilon) \|\mathbf{c}_0(\theta)\|_2} \mathbf{c}_0(\theta)$$

to guarantee a solution that satisfies (4.34) and (4.43).

Step 2: For each iteration, compute A_3 and C_3 .

Step 3: Depending on the value of A_3 and C_3 , one of the cases elaborated above will be satisfied. Determine the value of $\mu(k)$ based on the inequality given in the corresponding case.

Step 4: Solving (4.37) to determine λ .

Step 5: Update $\mathbf{z}(k)$ using (4.35).

Step 6: Terminate the iterative algorithm until $k = N - 1$ or $\|\mathbf{z}(k + 1) - \mathbf{z}(k)\|_2 / \|\mathbf{z}(k)\|_2 \leq \zeta$, where ζ is a preset small value.

4.4 Computer Simulations

In order to verify the validity of the proposed algorithm, we design the following simulation scenarios. A ULA with 8 half-wavelength spaced sensors is considered. Five sources from $20^\circ, -30^\circ, 0^\circ, -10^\circ, 50^\circ$ are assumed. The source from 20° is the SOI, while the rest are considered to be interference signals. The SINR is -20 dB.

Figure 4.1: Array gain of algorithms in ideal case.

Additive Gaussian noise is added. When plotting all the beampatterns, the SNR is set to 3 dB. For comparison, the MVDR beamformer and the norm constrained MVDR beamformer [58] are taken into account. 500 independent trials are conducted to compute output SINR.

4.4.1 The Proposed ℓ_2 -norm Beamformers

In this simulation, we investigate the performance of the proposed ℓ_2 -norm beamformers with and without robust constraint. The number of snapshots is 1000.

Figure 4.2: Output SINR versus SNR in ideal case.

1. Ideal Case

First of all, precise knowledge of array is assumed. Beampatterns of the proposed ℓ_2 -norm beamformer, the MVDR beamformer and the MVDR beamformer with norm constraint are shown in Figure 4.1, where $\text{SNR} = 3$ dB. Figure 4.2 plots the output SINR versus SNR of all the algorithms.

It is observed from Figure 4.1 that all the ℓ_2 -norm algorithms have similar beam-patterns in ideal case. They all cast deep nulls at interference directions, and give 0 dB gain at the look direction. Figure 4.2 further proves these conclusions. All the ℓ_2 beamformers have nearly identical output SINR. When the noise power is small enough, the array gain is proportional to the number of snapshots and is not influenced by the value of SNR [66].

Figure 4.3: Array response gain of algorithms in the presence of steering vector error.

2. In the Presence of Steering Vector Error

In this simulation, we evaluate the performance of the proposed algorithms in the presence of steering vector error. Steering vector error consists of two ingredients: sensor position error and look mismatch. Sensor position error is assumed to be a Gaussian variable with zero mean and standard deviation 10 percent of inter-element spacing. For look mismatch, we assume that the look direction deviates from the direction of SOI by Δ degrees, where Δ is a Gaussian variable with zero mean and standard deviation $\sqrt{2}$ degrees.

Figure 4.3 shows beampatterns of all algorithms with $\text{SNR} = 3$ dB. In Figure 4.4, we plot the output SINR versus SNR of all algorithms.

Figure 4.4: Output SINR versus SNR in the presence of steering vector error.

It is noted that all the algorithms cast deep nulls at the directions of interferences. At the direction of the desired signal, the gains are less than 0 dB for all the algorithms, which is caused by the influence of the steering vector error. Among the algorithms, the proposed algorithm with constraint on steering vector error performs the best, and the proposed algorithm without such constraint has higher gain at the signal direction than that of the MVDR beamformer and the norm constrained MVDR beamformer. The better performance of the proposed algorithms is further demonstrated by Figure 4.4. The output SINR of the proposed algorithms increase with the SNR and is higher than that of the MVDR beamformer and the MVDR beamformer with norm constraint. It is noticed that the performance of the MVDR beamformer degrades significantly when SNR is high [78]. However, the proposed algorithms show much better robustness in this case.

Figure 4.5: Array response gain of algorithms in the presence of sensor position error.

4.4.2 The Proposed ℓ_1 -norm Beamformer

In this simulation, we investigate the performance of the proposed robust ℓ_1 -norm beamformers. The number of snapshots used for all the algorithms is 2000.

1. In the Presence of Steering Vector Error Only

Simulation 1a: First of all, the performance of the proposed algorithm in the presence of sensor position error is evaluated. Sensor position error is assumed to be a Gaussian variable with standard deviation 10 percent of sensor spacing.

Figure 4.5 shows the performance of various beamformers. It is noted that all the algorithms cast deep nulls at the directions of the interference. Although the MVDR beamformer and the norm constrained MVDR beamformer cast deeper nulls than

Figure 4.6: Output SINR of various algorithms in the presence of sensor position error.

the proposed algorithm, the nulls of the proposed algorithm are sufficient to suppress the interference. The curve in Figure 4.6 shows the output SINR of the proposed algorithm against the iteration number. We see that the final output SINR of the proposed algorithm is higher than that of the other methods.

Simulation 1b: Secondly, we assume look direction mismatch appears. We choose the look direction of the array as $20^\circ + \Delta$, where Δ is a Gaussian random variable with zero mean and standard deviation $\sqrt{2}^\circ$.

Figure 4.7 shows the beampatterns of three algorithms with $\Delta = 3^\circ$. It is observed that although the three algorithms have similar performance in suppressing interference, the MVDR beamformer attenuates the desired signal about 10 dB. The MVDR

Figure 4.7: Array response gain of algorithms in the presence of sensor position error and look direction mismatch.

beamformer with norm constraint attenuates the SOI by about 5 dB. From Figure 4.8, we can see that the proposed algorithm outperforms the other beamformers by giving the highest output SINR.

2. In the Presence of Both the Intermittent Impulsive Noise and Steering Vector Error

In this simulation, we take both sensor failure and steering vector error into account. In the simulation, we generate impulsive noise using ϵ mixture of two Gaussian random variables [79,80]. The pdf of the noise is given by $0.1 \frac{e^{-x^2/2}}{\sqrt{2\pi}} + 0.9 \frac{e^{-x^2/6000}}{\sqrt{6000\pi}}$. We assume that the 1st, 5th and 6th sensors are with impulsive noise. Because impulsive noise is taken into account, we define an output SINR with ℓ_1 -norm to evaluate the performance of the proposed algorithm:

Figure 4.8: Output SINR of various algorithms in the presence of sensor position error and look direction mismatch.

$$SINR_{t_1} = \frac{| \mathbf{w}^H \mathbf{a}(\theta_0) s(t) |}{\sum_{i=1}^J | \mathbf{w}^H \mathbf{a}(\theta_i) s_i(t) + \mathbf{w}^H \mathbf{n}(t) |}, \quad (4.60)$$

where $s(t)$ denotes the SOI, $s_i(t)$ denotes the i^{th} interference, and $\mathbf{n}(t)$ is the spatial noise.

Simulation 2a: Firstly, beside the impulsive noise, only position error is assumed. Figure 4.9 shows the array gain using different methods. It is observed that at look direction, the MVDR beamformer and the robust MVDR beamformer suppressed the signal about 10 dB, while the proposed algorithm maintains array gain larger than 0 dB. At the directions of interference, all the methods cast deep nulls. The output SINR of the proposed algorithm is higher than that of the other methods, as shown

Figure 4.9: Performance of the algorithms in the presence of sensor position error and impulsive noise.

in Figure 4.10 and Figure 4.11.

Simulation 2b: Secondly, look direction mismatch is also taken into account. Figure 4.12 plots the array gain using different methods with $\Delta = 3^\circ$. It is observed that the MVDR beamformer casts a null about -15 dB at 20° which means the desired signal is suppressed significantly. Robust MVDR beamformer casts a null about -10 dB at 20° . By contrast, the proposed algorithm does not suppress the desired signal. The gain at 20° is larger than 0 dB. For the interference directions, all the methods cast deep nulls. The performance of algorithms can be illustrated by Figure 4.13 and Figure 4.14. The output SINR of the proposed algorithm is the highest among all three methods.

Figure 4.10: Output SINR of various algorithms in the presence of sensor position error and impulsive noise.

In order to investigate the performance of the algorithms in all the simulations clearly, we show some statistics of the 500 independent trials in Table 4.1. The output SINR values with respect to 10% or 90% denote the values which 10% or 90% of 500 output SINR are not larger than. It is observed that the proposed algorithm performs the best among the three. Comparing the results in Simulations A and B, it is found that impulsive noise is more difficult to suppress than the Gaussian noise for all three approaches.

Figure 4.11: Output SINR with ℓ_1 -norm definition of various algorithms in the presence of sensor position error and impulsive noise.

4.5 Conclusions

In this chapter, we introduce ℓ_1 -norm and ℓ_2 -norm beamformers robust against the steering vector error and impulsive noise. The proposed ℓ_2 -norm beamformer enhances its robustness by incorporating a term which aims to minimize the correlation of the real and imaginary parts of the desired signal in the objective function. Extension of the proposed ℓ_2 -norm beamformer by incorporating sphere constraint on the steering vector error is also addressed. Computer simulations show that the proposed algorithms perform similar to the MVDR beamformer in ideal case. In the presence of steering vector error, the proposed beamformers outperform the MVDR beamformer and the MVDR beamformer with norm constraint. For the robust ℓ_1 -norm beamformer, it has the following advantages over the MVDR beamformer and the MVDR

Figure 4.12: Performance of the algorithms in the presence of steering vector error and impulsive noise.

beamformer with norm constraint in the presence of steering vector error and impulsive noise: 1) The proposed beamformer is capable of maintaining the gain of the desired signal. 2) The output SINR of the proposed beamformer is higher than that of the MVDR beamformer and the MVDR beamformer with norm constraint.

In the next chapter, we will also start from the MVDR beamformer and aim to devise a DOA estimator in the framework of beamforming. It is known that beamforming technique is able to estimate sources DOA without knowing the number of sources. However, its resolution is usually poorer than that of the MUSIC algorithm. By modifying the optimization problem of the beamformer, we devise a new DOA estimator which has satisfactory resolution capability and also does not require to know the number of sources.

Figure 4.13: Output SINR of the proposed algorithm against iteration number in the presence of steering vector error and impulsive noise.

Figure 4.14: Output SINR with ℓ_1 -norm definition of the proposed algorithm against iteration number in the presence of steering vector error and impulsive noise.

Table 4.1: Statistics of the output SINR (dB) for 500 independent trials.

Case	Statistical Parameter	Proposed Method	MVDR	Robust MVDR
Simulation A1	10%	2.66	-8.04	-3.27
	90%	7.95	0.57	4.84
	mean	5.05	-3.71	0.79
	standard deviation	2.42	3.62	3.42
Simulation A2	10%	-0.27	-10.74	-5.54
	90%	6.29	-2.62	2.50
	mean	4.89	-6.01	-0.85
	standard deviation	3.29	3.89	2.50
Simulation B1	10%	-2.91	-9.27	-5.19
	90%	4.93	3.07	3.83
	mean	2.23	-1.89	-0.09
	standard deviation	3.14	4.65	3.77
Simulation B2	10%	-7.25	-12.18	-9.30
	90%	7.43	1.77	3.81
	mean	2.05	-5.95	-2.31
	standard deviation	5.26	5.37	5.07

Chapter 5

Design of a DOA Estimator Without Knowing the Number of Sources in the Framework of Beamforming

5.1 Introduction

The problem of DOA estimation has been intensively studied for a long period. Many DOA estimation algorithms have been proposed. Among them, many are based on eigenstructure analysis [53,81]. These methods rely on the property of the array correlation matrix that the vector space spanned by its eigenvectors may be partitioned into two disjoint parts: the so-called signal subspace and the noise subspace. In principle, the eigenstructure-based methods search for directions such that the steering vectors associated with these directions are orthogonal to the noise subspace and consequently belong to the signal subspace. The most representative one is the MUSIC algorithm proposed by Schmidt R.O. in 1986 [48]. The advantage of the MUSIC algorithm is

its easy implementation and high resolution. However, all the eigenstructure-based algorithms require exact determination of effective rank of the correlation matrix, which is hard to establish without a prior knowledge of the signal environment. If the number of sources is incorrectly estimated, the performance of these algorithms will deteriorate significantly.

Figure 5.1: Number of sources estimation using the AIC and MDL criteria with SNR = -10 dB.

The Akaike Information Criterion (AIC) [49] and the Minimum Description Length (MDL) [50] which are designed criteria using asymptotic arguments are the most frequently used algorithms to estimate the number of sources. However, experimental evidence shows that they do indeed tend to estimate a wrong number of components for a small sample size and a low SNR [56]. To illustrate this, we implement the AIC

and MDL criteria in the following environment. A ULA with eight half-wavelength spaced sensors is used. The number of snapshots is assumed to be 100. The SNR is -10 dB. Three sources from 50° , 65° and 110° are supposed to impinge the array. Figure 5.1 shows the estimation results of the AIC and MDL criteria. From it, we see that the two criteria are not able to give correct estimation results. (The estimated number of sources should correspond to the x-axis value when the minimal value of the criterion is reached.) Therefore, if these incorrect estimation results are used to implement the MUSIC algorithm, the DOA estimation results are expected to be erroneous.

To avoid source number estimation, beamforming techniques, such as the MVDR beamformer, alternatively called Capon's beamforming [9], can be applied to DOA estimation problem. When implemented, the beamformer scans all the possible directions. The output power with respect to every direction is computed and compared. The peaks of the output power indicate presence of potential sources. However, Capon's beamforming technique could not provide resolution as high as that of the MUSIC algorithm [82]. Related to the MUSIC algorithm and the MVDR beamformer, Choi et.al. [83] present a DOA estimation algorithm which does not require estimation of the number of sources. However, that method assumes equality of the noise eigenvalues which conflicts the fact in real applications. Moreover, how to determine the value of order n is not addressed. Besides beamforming techniques, some other DOA estimation algorithms without source number estimation have been proposed recently [84], [85]. For [85], it works only in the case when the number of sources is no larger than half of the number of sensors. Furthermore, how to determine the threshold and the pre-estimation of the number of sources is not addressed. For [84], it is an extension of the Reversible Jump Markov Chain Monte Carlo (RJMCMC) method which is very inefficient and computationally intensive [86].

It is shown in [68] that the MUSIC algorithm can be expressed as a beamforming problem, where subspace decomposition is required to determine the constraint, whereas in this chapter, we propose a new optimization problem which is independent on subspace decomposition so that the number of sources is not required. The solution to the new optimization problem is the eigenvector associated with the minimum eigenvalue of the matrix which depends on the look direction and the array correlation matrix. It is shown that the new algorithm is similar to the MUSIC algorithm in two aspects: quiescent array response and structure of the optimal weight vector associated with the look direction. Therefore, we name the proposed algorithm as the MUSIC-like algorithm. A theoretical investigation is elaborately conducted to demonstrate the effectiveness of the proposed algorithm. Firstly, when the steering vector lies in the signal subspace, the optimal weight vector is shown to be located in the noise subspace so that the defined direction finding function shows a peak to indicate the presence of a target. Secondly, when there is no target at the look direction and the corresponding steering vector is not perpendicular to any noise eigenvector, it is proved that the proposed algorithm prevents the presence of spurious peaks. Finally, we investigate the case when the steering vector is not in the signal subspace and is perpendicular to some of the noise eigenvectors. In this case, the MUSIC algorithm can give spurious peaks if the number of sources is overestimated. However, for the proposed algorithm, it is shown that when certain condition is satisfied, the problem of spurious peak will not occur.

The rest of the chapter is organized as follows. Section 5.2 presents the proposed method, and gives derivation of its solution. In Section 5.3, the proposed algorithm is elaborately investigated, and the bound of an important parameter is derived. Section 5.4 discusses computational complexity of the proposed algorithm. To verify the va-

validity of the proposed algorithm, computer simulations are conducted, and the results are given in Section 5.5. Finally, we make conclusions in Section 5.6.

5.2 MUSIC-like DOA Estimation Algorithm Without Estimating the Number of Sources

In this section, we propose a new algorithm to realize DOA estimation in the framework of beamforming so that the requirement of estimating the number of sources is eliminated.

The proposed optimization problem is given as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w}, \quad (5.1a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{s}(\theta) \mathbf{s}^H(\theta) \mathbf{w} + \beta \|\mathbf{w}\|_2^2 = c, \quad (5.1b)$$

where $c, \beta > 0$ are constants. It is observed that (5.1) is equivalent to a MVDR beamformer with norm constraint except that the square of the gain of array at the SOI is used. The reason of using the square of the gain of array at the SOI is based on the observation that the output power is of more concern than the output signal copy itself for a DOA estimation problem. Therefore, the constraint on distortionless response of the MVDR beamformer is unnecessary for DOA estimation. For the norm constraint on the weight vector, it provides two advantages on the proposed estimator. Firstly, it enhances robustness of the beamformer when two sources are closely spaced. Secondly, it guarantees invertibility of the matrix used in the subsequent study.

Using the Lagrange multiplier technique, we obtain the following Lagrangian:

$$f(\mathbf{w}) = \mathbf{w}^H \mathbf{R} \mathbf{w} - \lambda (\mathbf{w}^H \mathbf{s}(\theta) \mathbf{s}^H(\theta) \mathbf{w} + \beta \|\mathbf{w}\|_2^2 - c), \quad (5.2)$$

where λ is the Lagrange multiplier.

Setting the gradient of (5.2) with respect to $\mathbf{w}$ to zero yields

$$\mathbf{R} \mathbf{w} = \lambda (\mathbf{s}(\theta) \mathbf{s}^H(\theta) + \beta \mathbf{I}) \mathbf{w}, \quad (5.3)$$

which means the optimal weight vector $\mathbf{w}$ is given by the generalized eigenvector associated with the minimum generalized eigenvalue of the matrix pencil $\{\mathbf{R}, (\mathbf{s}(\theta) \mathbf{s}^H(\theta) + \beta \mathbf{I})\}$. Because $(\mathbf{s}(\theta) \mathbf{s}^H(\theta) + \beta \mathbf{I})$ is invertible, it is equivalent to find the minimum eigenvector of $(\mathbf{s}(\theta) \mathbf{s}^H(\theta) + \beta \mathbf{I})^{-1} \mathbf{R}$. Therefore, the value of c is not critical to the proposed algorithm. The choice of β is to be addressed in Section 5.3.

With the optimal weight vector $\mathbf{w}$, the direction finding function (the spatial spectrum) is defined as

$$P(\theta) = \frac{1}{|\mathbf{w}^H \mathbf{s}(\theta)|^2}. \quad (5.4)$$

The DOA is then estimated by finding the peaks of $P(\theta)$.

Now, it is straightforward to derive the quiescent array response of the new algorithm. Setting $\mathbf{R} = \sigma^2 \mathbf{I}$, where σ^2 denotes the variance of the spatial noise, and substituting it into (7.8), one obtains

$$\mathbf{w}_q = \frac{\mathbf{s}(\theta)}{\sqrt{L}}, \quad (5.5)$$

where the subscript q stands for quiescent response. Then, the array gain is given by $|\mathbf{w}_q^H \mathbf{s}(\theta)| = \|\mathbf{s}(\theta)\|_2^2 / \sqrt{L} = \sqrt{L}$. From (5.4), the corresponding direction finding function is then

$$P(\theta) = \frac{1}{L}, \quad (5.6)$$

which is identical to that of the MUSIC algorithm in the quiescent condition.

5.3 Analysis of the Proposed Algorithm

In this section, we present theoretical analysis on the new algorithm in three different cases, and derive the theoretical bounds of the parameter β .

Using the Sherman-Morrison formula

$$(\mathbf{A} + \mathbf{xy}^H)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{xy}^H\mathbf{A}^{-1}}{1 + \mathbf{y}^H\mathbf{A}^{-1}\mathbf{x}},$$

we have

$$(\beta\mathbf{I} + \mathbf{s}(\theta)\mathbf{s}^H(\theta))^{-1} = \beta^{-1}\mathbf{I} - \alpha\mathbf{s}(\theta)\mathbf{s}^H(\theta), \quad (5.7)$$

where

$$\alpha = \frac{\beta^{-2}}{1 + \beta^{-1} \|\mathbf{s}(\theta)\|_2^2} = \frac{\beta^{-2}}{1 + \beta^{-1}L}$$

is a constant.

The eigen-decomposition of the array correlation matrix can be represented as $\mathbf{R} = \sum_{i=1}^L \xi_i \mathbf{e}_i \mathbf{e}_i^H$, where ξ_i and $\mathbf{e}_i$ denote the eigenvalue and eigenvector of $\mathbf{R}$, respectively. In the presence of spatial noise, $\mathbf{R}$ is a full-rank matrix, its eigenvectors form an orthogonal basis of the L -dimensional space. Therefore, we may represent $\mathbf{s}(\theta)$ using

the eigenvectors of $\mathbf{R}$, given by

$$\mathbf{s}(\theta) = \sum_{i=1}^L \mathbf{e}_i^H \mathbf{s}(\theta) \mathbf{e}_i. \quad (5.8)$$

Accordingly, we have

$$\mathbf{s}(\theta) \mathbf{s}^H(\theta) = \sum_{i=1}^L \sum_{j=1}^L a_i a_j^* \mathbf{e}_i \mathbf{e}_j^H = \sum_{i=1}^L |a_i|^2 \mathbf{e}_i \mathbf{e}_i^H + \sum_{i=1}^L \sum_{j=1, j \neq i}^L a_i a_j^* \mathbf{e}_i \mathbf{e}_j^H, \quad (5.9)$$

where $a_i = \mathbf{e}_i^H \mathbf{s}(\theta)$.

Substituting (5.9) into (5.7) and with the eigen-decomposition of $\mathbf{R}$, one obtains

$$(\beta \mathbf{I} + \mathbf{s}(\theta) \mathbf{s}^H(\theta))^{-1} \mathbf{R} = \sum_{i=1}^L (\beta^{-1} - \alpha |a_i|^2) \xi_i \mathbf{e}_i \mathbf{e}_i^H - \alpha \sum_{i=1}^L \sum_{j=1, j \neq i}^L a_i a_j^* \xi_j \mathbf{e}_i \mathbf{e}_j^H. \quad (5.10)$$

To perform eigen-decomposition of (5.10), we are to solve the following equation:

$$\left(\sum_{i=1}^L (\beta^{-1} - \alpha |a_i|^2) \xi_i \mathbf{e}_i \mathbf{e}_i^H - \alpha \sum_{i=1}^L \sum_{j=1, j \neq i}^L a_i a_j^* \xi_j \mathbf{e}_i \mathbf{e}_j^H \right) \mathbf{x} = \gamma \mathbf{x}, \quad (5.11)$$

where γ and $\mathbf{x}$ denote the eigenvalue and the eigenvector, respectively. It is now easy to show that the eigenvector $\{\mathbf{e}_i \mid \forall i\}$ is not the solution of (5.11). Instead, the optimal weight vector is given by the weighted sum of the eigenvectors, denoted as $\mathbf{w} = \mathbf{x}_{\min} = \sum_{i=1}^L b_i \mathbf{e}_i$. To find the new set of eigenvectors, substituting $\mathbf{x} = \sum_{i=1}^L b_i \mathbf{e}_i$ into (5.11) yields a new set of eigen-equations:

$$\tilde{\mathbf{B}} \mathbf{b} = \gamma \mathbf{b}. \quad (5.12)$$

The vector $\mathbf{b} = [b_1, \dots, b_L]^T$ is the eigenvector with respect to the minimum eigenvalue

of the matrix $\tilde{\mathbf{B}}$ whose elements are given by

$$\tilde{b}_{ij} = \begin{cases} (\beta^{-1} - \alpha |a_i|^2) \xi_i, & i = j \\ -\alpha a_i a_j^* \xi_j, & i \neq j \end{cases}, i, j = 1, \dots, L. \quad (5.13)$$

In the following discussion, we investigate the eigenvector of $\tilde{\mathbf{B}}$ with respect to its minimum eigenvalue in three cases. Without loss of generality, assuming that the eigenvalues of $\mathbf{R}$ are in descending order, i.e., $\xi_1 \geq \dots \geq \xi_M > \xi_{M+1} \geq \dots \geq \xi_L$, we define $\mathbb{S} = \{1, \dots, M\}$ and $\mathbb{N} = \{M+1, \dots, L\}$ as signal subscript and noise subscript, respectively. Accordingly, $\{\xi_i \mid \forall i \in \mathbb{S}\}$ are the signal eigenvalues, and $\{\xi_i \mid \forall i \in \mathbb{N}\}$ are the noise eigenvalues.

5.3.1 The Steering Vector Resides Completely in the Signal Subspace.

In this case, $a_i = 0, \forall i \in \mathbb{N}$, and from (5.13), we have

$$\tilde{b}_{ij} = \begin{cases} \beta^{-1} \xi_i, & i = j \\ 0, & i \neq j \end{cases}, \forall i, j \in \mathbb{N}. \quad (5.14)$$

The matrix $\tilde{\mathbf{B}}$ has the following structure

$$\tilde{\mathbf{B}} = \begin{bmatrix} \tilde{b}_{11} & \dots & \tilde{b}_{1M} & \dots & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \tilde{b}_{M1} & \dots & \tilde{b}_{MM} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & \dots & \tilde{b}_{LL} \end{bmatrix}, \quad (5.15)$$

which is a diagonal block matrix.

It is straightforward to see that $\{\beta^{-1}\xi_i \mid \forall i \in \mathbb{N}\}$ are eigenvalues of (5.15). We will discuss two possibilities on these eigenvalues.

Firstly, if $\exists i \in \mathbb{N}$ so that $\beta^{-1}\xi_i$ is the minimum eigenvalue of (5.15), then its corresponding element in $\mathbf{b}$, i.e., b_i , can not be zero, and since (5.15) without the i^{th} column and i^{th} row is with rank $L-1$, all the other elements of $\mathbf{b}$ should be equal to 0, which means the solution of (5.1) is in the same direction of $\mathbf{e}_i$. Because $\mathbf{s}(\theta)$ belongs to the signal subspace, which thereby is perpendicular to $\mathbf{e}_i, \forall i \in \mathbb{N}$, the direction finding function given by (5.4) is expected to show a peak at the look direction θ . A particular case is that the number of snapshots is large and the noise is spatially white so that $\xi_{M+1} = \dots = \xi_L = \xi_N$ holds. In this case, when $\beta^{-1}\xi_N$ is the minimum eigenvalue, $\{b_i \mid \forall i \in \mathbb{N}\}$ are nonzero. It means that the optimal weight vector is given by the weighted sum of all the noise eigenvectors, which is similar to that of the MUSIC algorithm. As a result, the spatial spectrum shows a peak at the look direction.

Secondly, if $\beta^{-1}\xi_i, \forall i \in \mathbb{N}$ is not the minimum eigenvalue, then $b_i = 0, \forall i \in \mathbb{N}$ holds, which means that the desired weight vector is contained in the signal subspace. The nonzero elements in $\mathbf{b}$ constitute the eigenvector with respect to the minimum eigenvalue of the following matrix:

$$\begin{bmatrix} (\beta^{-1} - \alpha |a_1|^2)\xi_1 & \dots & -\alpha a_1 a_M^* \xi_M \\ \vdots & \ddots & \vdots \\ -\alpha a_M a_1^* \xi_1 & \dots & (\beta^{-1} - \alpha |a_M|^2)\xi_M \end{bmatrix}. \quad (5.16)$$

As a result, the dependence between the weight vector and $\mathbf{s}(\theta)$ impedes a peak in the direction finding function at θ . To prevent its happening, we impose a constraint on the value of β .

Since $(\beta^{-1}\mathbf{I} - \alpha\mathbf{s}(\theta)\mathbf{s}^H(\theta))$ as well as $\mathbf{R}$ are Hermitian matrices, the following inequality holds [87]:

$$\min_i \{\beta^{-1} - \alpha |a_i|^2 | i \in \mathbb{S}\} \min_i \{\xi_i | i \in \mathbb{S}\} \leq \tilde{\gamma}_{min}, \quad (5.17)$$

where $\tilde{\gamma}_{min}$ stands for the minimum eigenvalue of (5.16). To prevent the optimal weight vector from being located in the signal subspace, the following constraint is imposed on β :

$$\{\beta^{-1}\xi_i | \exists i \in \mathbb{N}\} < \min_i \{(\beta^{-1} - \alpha |a_i|^2)\xi_i | i \in \mathbb{S}\}, \quad (5.18)$$

which guarantees that the minimum eigenvalue of (5.15) is contained in the set of $\{\beta^{-1}\xi_i | i \in \mathbb{N}\}$. Solving (5.18) gives the lower bound of β as

$$\beta > \frac{L}{\xi_M/\xi_i - 1}, \quad i \in \mathbb{N}. \quad (5.19)$$

Since the value of M and hence ξ_M is unknown, an alternative is required.

The lowest bound of β is given by

$$\beta > \frac{L}{\xi_M/\xi_L - 1}. \quad (5.20)$$

When (5.20) is satisfied, the eigenvector with respect to the minimum eigenvalue of (5.15) has the same direction of $\mathbf{e}_L$ which belongs to the noise subspace so that

$\mathbf{w}^H \mathbf{s}(\theta) = 0$ holds.

Because $\xi_{L-1} \leq \xi_M$, we propose to set β using

$$\beta = \frac{kL}{\xi_{L-1}/\xi_L - 1}, \quad k \leq 1. \quad (5.21)$$

How to properly set k will be addressed in the subsequent discussion.

5.3.2 The Steering Vector is not Residing Completely in the Signal Subspace and is not Perpendicular to any of the Noise Eigenvector.

In this case, $\tilde{\mathbf{B}}$ is no longer block diagonal. The solution of (5.1) lies in the same space of $\mathbf{s}(\theta)$. Define $\mathbf{a} = [a_1, \dots, a_L]^T$. To prevent a spurious peak, $\mathbf{s}(\theta)$ and $\mathbf{w}$ should not be perpendicular, or equivalently, $\mathbf{s}(\theta)^H \mathbf{w} = \mathbf{a}^H \mathbf{b}$, should be nonzero. Now, we will show that the proposed method satisfies this condition.

Supposing γ_{min} is the minimum eigenvalue of $\tilde{\mathbf{B}}$. To derive its corresponding eigenvector, we substitute it into the characteristic equation. It follows that

$$|\tilde{\mathbf{B}} - \gamma_{min} \mathbf{I}| = 0,$$

and the matrix $(\tilde{\mathbf{B}} - \gamma_{min} \mathbf{I})$ is of rank $L - p$, where p is the multiplicity of γ_{min} . Without loss of generality, we assume that $p = 1$. After some basic matrix manipulations,

$\tilde{\mathbf{B}} - \gamma_{\min} \mathbf{I}$ can be transformed to

$$\begin{bmatrix} \tilde{b}_{11} - \gamma_{\min} & \dots & \tilde{b}_{1M} & \dots & \tilde{b}_{1L} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{b}_{L1} & \dots & \tilde{b}_{LM} & \dots & \tilde{b}_{LL} - \gamma_{\min} \end{bmatrix}, \quad (5.22)$$

where the i^{th} row, $i \in (\mathbb{N} \cup \mathbb{S})$ becomes all zeros. Defining (5.22) without the i^{th} row and i^{th} column as $\mathbf{B}_{(-i)}$, it is easy to show that $\mathbf{b} = [b_1, \dots, b_{i-1}, 1, b_{i+1}, \dots, b_L]^T = [\mathbf{b}_{(1:i-1)}^T, 1, \mathbf{b}_{(i+1:L)}^T]^T$ can be derived from the equation

$$\mathbf{B}_{(-i)} \begin{bmatrix} \mathbf{b}_{(1:i-1)} \\ \mathbf{b}_{(i+1:L)} \end{bmatrix} = \alpha a_i^* \xi_i \begin{bmatrix} \mathbf{a}_{(1:i-1)} \\ \mathbf{a}_{(i+1:L)} \end{bmatrix}, \quad (5.23)$$

where $\mathbf{a}_{(1:i-1)} = [a_1, \dots, a_{i-1}]^T$, and $\mathbf{a}_{(i+1:L)} = [a_{i+1}, \dots, a_L]^T$.

Since $\mathbf{B}_{(-i)}$ is invertible (proof is shown in Appendix A), we have

$$\begin{bmatrix} \mathbf{b}_{(1:i-1)} \\ \mathbf{b}_{(i+1:L)} \end{bmatrix} = \alpha a_i^* \xi_i \mathbf{B}_{(-i)}^{-1} \begin{bmatrix} \mathbf{a}_{(1:i-1)} \\ \mathbf{a}_{(i+1:L)} \end{bmatrix}. \quad (5.24)$$

It follows from (5.24) that the following equation holds:

$$\begin{aligned} \mathbf{a}^H \mathbf{b} &= [\mathbf{a}_{(1:i-1)}^H, \mathbf{a}_{(i+1:L)}^H, a_i^*] \begin{bmatrix} \mathbf{b}_{(1:i-1)} \\ \mathbf{b}_{(i+1:L)} \\ 1 \end{bmatrix} \\ &= [\mathbf{a}_{(1:i-1)}^H, \mathbf{a}_{(i+1:L)}^H] \alpha a_i^* \xi_i \mathbf{B}_{(-i)}^{-1} \begin{bmatrix} \mathbf{a}_{(1:i-1)} \\ \mathbf{a}_{(i+1:L)} \end{bmatrix} + a_i^*. \end{aligned} \quad (5.25)$$

If (5.25) is equal to zero, then

$$[\mathbf{a}_{(1:i-1)}^H, \mathbf{a}_{(i+1:L)}^H] \mathbf{B}_{(-i)}^{-1} \begin{bmatrix} \mathbf{a}_{(1:i-1)} \\ \mathbf{a}_{(i+1:L)} \end{bmatrix} = \frac{-1}{\alpha \xi_i} \quad (5.26)$$

must hold. However, because $\mathbf{B}_{(-i)}$ is positive (proof is shown in Appendix B), (5.26) can not be satisfied, which means that $\mathbf{a}^H \mathbf{b} = \mathbf{s}^H(\theta) \mathbf{w} \neq 0$ must hold. Therefore, spurious peaks will not occur.

When $p > 1$, the proof is similar. In this case, there will be $p > 1$ zero rows in (5.22). Without loss of generality, setting $b_i = 1$ and $b_j = 0$ which corresponds to the other $p - 1$ zero rows of (5.22), similar results as (5.23)-(5.26) can be derived.

5.3.3 The Steering Vector is not in the Signal Subspace and is Perpendicular to Some of the Noise Eigenvectors.

For this case, if the number of sources is overestimated, the MUSIC algorithm will show a spurious peak in its spatial spectrum if the omitted noise eigenvector is precisely perpendicular to the steering vector.

For the proposed algorithm, the discussion is similar to that in the previous subsection. Let $\mathbb{N}_{\parallel}$ be the integer set such that $\{\mathbf{e}_i \mid i \in \mathbb{N}_{\parallel}\}$ are not perpendicular to the steering vector, and $\mathbb{N}_{\perp}$ be the set such that $\{\mathbf{e}_i \mid i \in \mathbb{N}_{\perp}\}$ represent the noise eigenvectors perpendicular to the steering vector.

Similar to (5.15), the matrix $\tilde{\mathbf{B}}$ remains block diagonal with block size M plus the dimension of $\mathbb{N}_{\parallel}$ for all $i \in (\mathbb{S} \cup \mathbb{N}_{\parallel})$, and 1 for all $i \in \mathbb{N}_{\perp}$. Therefore, $\{\beta^{-1}\xi_i \mid i \in \mathbb{N}_{\perp}\}$ are the eigenvalues of the matrix $\tilde{\mathbf{B}}$. If none of them is the minimum eigenvalue, $b_i = 0$ holds for all $i \in \mathbb{N}_{\perp}$. It means that the optimal weight vector lies in the space spanned by $\{\mathbf{e}_i \mid i \in (\mathbb{S} \cup \mathbb{N}_{\parallel})\}$ which is in the same space of $\mathbf{s}(\theta)$. Therefore, for the proposed method, the spurious peak will not occur. The proof is similar to that in the previous subsection, and is omitted here.

However, if $\exists i \in \mathbb{N}_{\perp}$ so that $\beta^{-1}\xi_i$ is the minimum eigenvalue of $\tilde{\mathbf{B}}$, a spurious peak will appear due to $\mathbf{s}(\theta) \perp \mathbf{e}_{\mathbb{N}_{\perp}}$. To prevent $\{\beta^{-1}\xi_i \mid i \in \mathbb{N}_{\perp}\}$ being the minimum eigenvalue, we impose the following constraint on β :

$$\{(\beta^{-1} - \alpha\kappa L)\xi_i \mid \exists i \in \mathbb{N}_{\parallel}\} < \min_i \{\beta^{-1}\xi_i \mid i \in \mathbb{N}_{\perp}\}, \quad (5.27)$$

where $0 < \kappa \leq 1$. For a certain $i \in \mathbb{N}_{\parallel}$, when $\xi_i \leq \xi_{\mathbb{N}_{\perp}^{\min}}$ holds, (32) is satisfied for arbitrary β , where $\xi_{\mathbb{N}_{\perp}^{\min}}$ represents the minimum value of $\{\xi_i \mid \forall i \in \mathbb{N}_{\perp}\}$. Otherwise, β is upper bounded by

$$\beta < \frac{L}{\xi_{\mathbb{N}_{\parallel}}/\xi_{\mathbb{N}_{\perp}^{\min}} - 1} \leq \frac{L}{\xi_{L-1}/\xi_L - 1}. \quad (5.28)$$

Now, we will address how to set k in (5.21). Assuming $\xi_M = c_1\xi_L$ and $\xi_{L-1} = c_2\xi_L$, where $c_1 \geq c_2 \geq 1$. For a high SNR environment, $c_1 \gg c_2 \approx 1$, the upper bound

of β approaches infinity. It is observed that when $\beta \rightarrow \infty$, the proposed method degenerates to the MUSIC algorithm only using the eigenvector with respect to the minimum eigenvalue of $\mathbf{R}$ to characterize the noise subspace. In this case, only the lower bound of β need to be taken into account. It is noted that the lowest bound given by (5.20) is $\frac{L}{c_1-1}$ which is much smaller than $\frac{L}{c_2-1}$. Therefore, setting k to a not too small value guarantees that (5.20) holds. For moderate or low SNR environment, c_1 is moderately or slightly larger than c_2 , which means choosing k close to 1 can guarantee that (5.20) and (5.28) hold.

5.4 Complexity Analysis and Summary of the Proposed Algorithm

The proposed algorithm requires a generalized eigen-decomposition for every direction. Therefore, a fast generalized eigen-decomposition is crucial to the proposed algorithm. Y.N.Rao et.al. [88] propose a fast algorithm whose computational complexity is about $O(L^2)$ to realize a generalized eigen-decomposition. Besides, a fast Minor Component Analysis (MCA) algorithm recently proposed by Attallah [89] can also be applied to the proposed algorithm. It has been demonstrated that the computational complexity of deriving p minor eigenvectors is $O(n) + O(np^2)$ by using their method, where n is the size of the observation vector. The computational complexity of computing the covariance matrix of the observation can be reduced from $O(n^2)$ to $O(n)$, if the temporal data covariance matrix has the property of shift-invariance [90]. For the proposed algorithm, it is good to see that $\mathbf{R}$ as well as the matrix $(\mathbf{s}(\theta)\mathbf{s}^H(\theta) + \beta\mathbf{I})$ have this property.

In order to reduce the computational complexity of the proposed algorithm, we

may combine the algorithms given in [88] and [89] together. Specifically, [88] is used for some values of θ_i , where θ_i 's are chosen to be far apart. Then, one applies [89] to compute the smallest eigenvector with respect to θ_{ij} , where θ_{ij} are in the neighboring of θ_i . To accelerate the convergence of the adaptive MCA algorithm, the derived smallest eigenvector with respect to θ_i can be used to initialize the algorithm when computing the eigenvector with respect to θ_{ij} . Supposing that [88] is implemented N_{rough} times and [89] is implemented N_{fine} times, the total computational load of the proposed algorithm is given by $O(N_{rough}L^2 + 2N_{fine}N_{iter}L + (N_{rough} + N_{fine})L)$, where N_{iter} denotes the number of iterations for the MCA algorithm to converge.

For the MUSIC algorithm, using the fast subspace decomposition technique [91], the computational complexity of eigen-decomposition is $O(ML^2)$. It requires $L(L - M)$ multiplies to compute the spatial spectrum. Therefore, the total computational complexity of the MUSIC algorithm is $O(ML^2 + L(L - M)N + (L - M)N)$.

For the MVDR beamformer, the computational complexity is $O(L^3 + L^2N + LN)$.

Figure 5.2: Computational complexity of various algorithms.

For 10° rough sampling and 1° fine sampling, i.e., $N_{rough} = 18$, $N_{fine} = 162$ and $N = 180$, the computational complexity of various algorithms are plotted in Figure 5.2. Although we set $N_{iter} = 20$ for plotting Figure 5.2, in our simulations, we find that N_{iter} is always less than 20 (shown in Figure 5.3). The number of sources is assumed to be $(L - 5)$ for the MUSIC algorithm. It can be seen that when the number of sensors is large, the proposed algorithm has the smallest computational complexity among the three algorithms. For a small number of sensors, the MUSIC algorithm has the computational advantage over the proposed algorithm. However, in this case, the computational complexity is not vary large for either approach.

Figure 5.3: Convergence of the combination of fast generalized eigendecomposition algorithms.

The implementation of the new algorithm is summarized as follows:

Step 1. Choose an appropriate β according to (26);

Step 2. Compute $\tilde{\mathbf{B}} = (\beta^{-1}\mathbf{I} - \alpha\mathbf{s}(\theta)\mathbf{s}^H(\theta))\mathbf{R}$;

Step 3. Derive the eigenvector with respect to the minimum eigenvalue of $\tilde{\mathbf{B}}$;

Step 4. Compute the spatial spectrum based on (9).

5.5 Computer Simulations

In this section, we devise several simulation scenarios to verify the validity of the proposed algorithm. We also compare the proposed algorithm with the MUSIC algorithm and the MVDR beamformer.

In all the simulations, the Root-Mean-Square-Error (RMSE) is calculated as

$$RMSE(\theta) = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{\theta}^i - \theta)^2}, \quad (5.29)$$

where $N = 500$ for our simulation, θ is the true DOA and $\hat{\theta}^i$ represents the estimated value of the DOA of the i^{th} trial.

5.5.1 Comparison with the MUSIC Algorithm and the MVDR Beamformer in High SNR Environment

In this simulation, we assume a ULA with eight half-wavelength spaced sensors. Three uncorrelated narrowband sources with identical power equal to 0 dB are assumed to impinge the array from 50° , 65° and 110° . It is supposed that the number of sources estimated to implement the MUSIC algorithm is precisely equal to 3. Therefore, the MUSIC algorithm works in the ideal situation.

Table 5.1: RMSE (degree) of the estimation for DOA=50°, 65° and 110° with SNR= 0 dB.

Number of snapshots	DOA (o)	the MUSIC algorithm (o)	the Capon's method (o)	the proposed method (o)
40	50	0.807	0.915	0.890
	65	0.584	0.740	0.629
	110	0.377	0.526	0.387
60	50	0.577	0.828	0.700
	65	0.520	0.732	0.604
	110	0.335	0.470	0.354
80	50	0.557	0.724	0.638
	65	0.424	0.669	0.547
	110	0.287	0.379	0.287
100	50	0.512	0.646	0.602
	65	0.400	0.586	0.527
	110	0.282	0.304	0.283

The RMSE of various algorithms versus the number of snapshots are presented in Table 5.1, where the SNR is set to 0 dB. It is observed from Table 5.1 that as the number of snapshots increases, the performance of all the algorithms are improved. The MUSIC algorithm slightly outperforms the proposed algorithm. Both of them perform better than the MVDR beamformer.

Table 5.2: RMSE (degree) of the estimation for DOA=50°, 65° and 110° with No.of Snapshots=100.

SNR (dB)	DOA (o)	the MUSIC algorithm (o)	the Capon's method (o)	the proposed method (o)
0	50	0.512	0.646	0.602
	65	0.400	0.586	0.527
	110	0.282	0.304	0.283
10	50	0.516	0.663	0.564
	65	0.387	0.540	0.433
	110	0.162	0.226	0.173
20	50	0.123	0.206	0.141
	65	0.001	0.128	0.050
	110	0.001	0.061	0.001

Table 5.2 shows the RMSE with varied SNR when the number of snapshots is fixed to 100. It is observed that the performance of all the algorithms are ameliorated as SNR increases. The performance of the proposed algorithm get close to that of the MUSIC algorithm when the SNR equals to 20 dB.

Figure 5.4: RMSE against separation angle with respect to the source at 50° .

To more clearly see the performance of the proposed algorithm and the MUSIC algorithm in high SNR environment, we plot the estimation RMSE against the separation of angle in Figure 5.4 and Figure 5.5, where SNR equals to 10 dB and the number of sensors is four. The two sources are assumed to be located at 50° and $50^\circ + \Delta\theta$, where $\Delta\theta$ varies from 5° to 90° . The CRLB [71] of the estimation is also computed to illustrate the performance of algorithms. It is observed that the two algorithms achieve similar performance.

Figure 5.5: RMSE against separation angle with respect to the source at $50^\circ + \Delta\theta$.

5.5.2 Comparison with the MUSIC Algorithm in Low SNR Environment

In this simulation, we evaluate the performance of the algorithms when $\text{SNR} = -10$ dB so that the AIC and MDL fail to work (shown in Figure 5.1).

Table 5.3: RMSE (degree) of the estimation of the MUSIC algorithm when the number of sources is incorrectly estimated. SNR= -10 dB .

Number of snapshots	DOA (o)	$M_{est} = 4$ (o)	$M_{est} = 5$ (o)	the proposed algorithm (o)
80*	50	1.956	3.667	1.259
	65	1.517	1.708	1.062
	110	1.083	1.543	0.842
90*	50	1.765	3.315	1.217
	65	1.412	1.339	0.997
	110	0.807	1.205	0.772
100*	50	1.161	1.675	1.019
	65	0.907	1.122	0.908
	110	0.719	0.802	0.661

Table 5.3 shows the estimation RMSE of the MUSIC algorithm when the number of sources is incorrectly estimated. Compared with the proposed algorithm, the MUSIC algorithm has larger estimation RMSE than that of the proposed algorithm.

Figure 5.6: Direction finding comparison with $L = 8$, No. of snapshots=100 and SNR=-10 dB.

Figure 5.6 presents the derived spatial spectra of the three algorithms in decibels, i.e., $10 \log_{10} P(\theta)$, where $P(\theta)$ is given by (7.1), (7.2) or (7.9). SNR is assumed to be -10 dB and the number of the snapshots is 100. It is observed that spurious peaks appear in the MUSIC spectra when the number of sources are incorrectly estimated, while the proposed algorithm is capable of performing satisfactorily.

5.5.3 Comparison with the MUSIC Algorithm When the Number of Sources is Incorrectly Estimated.

To further illustrate the superiority of the proposed algorithm when the number of the sources is incorrectly estimated, we do the following simulations in this subsection. Five uncorrelated narrowband sources with DOAs 30° , 50° , 65° , 110° and 120° are

Figure 5.7: Direction finding comparison with $L = 8$ and No.of snapshots=40. M_{est} in the figure represents the estimated number of the targets for implementation of the MUSIC algorithm.

assumed impinging a ULA with eight half-wavelength spaced sensors. The SNR is set to 8.5 dB for the target from 30°, 10 dB for the sources from 50°, 65°, 110°, and 20 dB for the one from 120°. 40 snapshots are used.

Figure 5.7 shows the spatial spectra of various algorithms. Underestimation as well as overestimation of number of sources is considered. It is observed from Figure 5.7 that there are M_{est} peaks shown in the direction finding function of the MUSIC algorithm. When M_{est} is underestimated, the MUSIC algorithm can only locate the target with the highest SNR and gives a rough estimation of the location of other targets. Its performance is satisfactory only when the number of targets is correctly estimated. When the number of sources is overestimated, a spurious peak occurs, as

Figure 5.8: Distribution of the number of peaks in the spatial spectrum of 500 independent trials. From left to right, each group of the bars represents the MUSIC algorithm with estimated number of sources equal to 3, 4, 5, 6 and the proposed algorithm, respectively. From left to right, the bar in each group represents the number of peaks equal to 2 to 6, respectively.

shown in Figure 5.7.

The groups of bars in Figure 5.8 present the percentage of occurrence of peaks in the spatial spectra of the proposed algorithm and the MUSIC algorithm with the estimated number of sources equal from 3 to 6. From Figure 5.8, it is noted that in most of cases, the MUSIC algorithm is incapable of locating the omitted targets if the number of the sources are underestimated. When $M_{est} = 3$, the number of peaks in the corresponding spatial spectrum is 4 at most with the percentage occurrence less than 10%. When $M_{est} = 4$, there is only about a 10% chance to locate the five targets.

If overestimation happens, more than 65% of trials show spurious peaks. However, for the proposed algorithm, over 80% of 500 independent trials give good results.

5.5.4 Capability to Cope with Correlated Sources.

Figure 5.9: Direction finding with correlated sources.

The SS technique [92] which is valid for the MUSIC algorithm can also be applied to the proposed algorithm when correlated sources appear. To verify this, we assume that the two sources at 50° and 65° in Subsection A of Section 6.5 are correlated (correlation coefficient equals to 0.99). Figure 5.9 shows the spatial spectra of the proposed algorithm and the MUSIC algorithm using spatial smoothing technique. It is noted that the SS technique can also be applied to the proposed algorithm. From Figure 5.9, we see that without knowing the number of sources, the proposed algorithm is able to give correct estimation of sources' DOA.

5.6 Conclusions

In this chapter, we propose a new DOA estimation algorithm without estimating the number of sources. The proposed algorithm is similar to the MUSIC algorithm in two aspects. Firstly, they share an identical quiescent array response. Secondly, the optimal weight vector of the two algorithms are all given by the weighted sum of noise eigenvectors. The appealing advantage of the proposed algorithm lies in the fact that the number of sources is not required to estimate the DOAs. Using the recently proposed MCA algorithm, the proposed algorithm has lower computational complexity than that of the MUSIC algorithm and the MVDR beamformer. Spurious peaks are avoided in the new algorithm, and rigorous theoretical analysis has been given to support this. Simulations have demonstrated the superiority of the new algorithm compared to the MUSIC algorithm when the number of sources is incorrectly estimated.

Up to now, we dedicate ourselves to propose new optimization problems for narrowband source case. In the next chapter, we will do some work in broadband case. Two new structures which facilitate the implementation of broadband beamforming will be given.

Chapter 6

Block Cascaded Structures for Broadband Beamforming

6.1 Introduction

Beamforming, a signal processing technique used in sensor arrays for directional signal transmission or reception, has been widely used in communication and navigation systems. These applications include radio communications, sonar, radar, seismology, acoustics, and microphone arrays [93,94]. The objective of beamforming is to enhance the source from the desired direction, while suppressing all the other sources as well as the background noise, i.e., enhance the SINR. In recent years, rapid developments of broadband systems appear because of demanding for high data rates and bandwidths. It has been shown that the narrowband beamformer with a desired spatial response for a particular frequency causes varied mainlobe shape and sidelobe/null locations at other different frequencies. Therefore, various broadband beamforming techniques have been proposed to achieve satisfactory performance. A commonly used broadband beamformer is the Frost beamformer [64], which uses an FIR filter in each channel to achieve good interference suppression performance over a wide bandwidth. Usually,

the constrained LMS algorithm is implemented to compute the optimum weights. Its computational complexity of one-step update is $O(L^2J)$, where L and J denote the number of sensors and filter taps, respectively.

As pointed out in [95], the number of taps required to achieve satisfactory interference suppression increases with the increase of the operating bandwidth. Too many adaptive weights will slow down the convergence rate and increase the computational complexity. To achieve better SINR without increasing the number of taps greatly, the infinite-impulse-response (IIR) filter-based broadband beamformer is proposed [96]. It has been demonstrated that this IIR-based beamformer is capable of using fewer adaptive weights to achieve higher output SINR compared to the Frost. However, the improved performance is obtained at the cost of a more complicated iteration procedure. The number of multiplications is $O(3L^3J^2)$ to perform a one-step update of the beamformer weights, and $O(LJ^2 + 5J^2)$ to perform beamforming. Therefore, the computational complexity is still very large.

For the perspective of hardware implementation, the development of FPGA and CPLD make efficient and high-speed computation feasible, but more hardware resources are required to realize a high-order FIR filter [57]. For the design of an IIR filter, finite-precision effects are much more of a concern with IIR filters than with FIR filters, since the effects are more difficult to analyze and minimize, coefficient quantization errors can cause the filters to become unstable. Therefore, either theoretically or practically, an efficient and easy implementation of broadband beamforming is essential. The desired beamformer should facilitate hardware implementation and enjoy moderate computational complexity especially when the number of taps is large.

In this chapter, we propose two new structures to realize broadband beamforming.

The basic blocks are cascaded to form a Frost-like beamformer. The basic blocks are themselves sub-Frost beamformers whose number of input channels and tapped delays is small. The independence between the basic blocks makes parallel computation of the weights feasible, which is very favorable for real-time implementation. The design of the beamformer can be very flexible to meet the requirement of various working scenarios. Meanwhile, the sub-optimal structure, which processes information locally instead of globally, decreases calibration difficulties, and brings better robustness against steering vector error. The above mentioned advantages are achieved with the reduction of the output SINR due to its sub-optimal processing in essence. However, the output SINR decreases insignificantly. Furthermore, the sub-optimal processing shows better robustness against steering vector error compared with the Frost beamformer.

In the remaining parts of the chapter, Section 6.2 briefly reviews the broadband beamforming techniques. In Section 6.3, the proposed structures are presented in details. Section 6.4 elaborately discusses the computational load of the proposed structures. In Section 6.5, theoretical output SINR of the proposed structures is derived. Section 6.6 gives simulation results. Finally, conclusions are drawn in Section 6.7.

6.2 Review of Broadband Beamforming Techniques

Figure 6.1: The structure of the Frost beamformer.

Figure 6.1 depicts a Frost beamformer with L channels and J taps. The intertap delay spacing is equal to T . It is assumed that the input signals have been presteered, so the pure time delays in front of each element are used to align the look direction signal temporally. After alignment, identical signal components appear simultaneously on the first taps and parade in parallel down the tapped delay lines following each sensor. In contrast, interferences arriving from the non-look direction will not induce equal voltage components on each column of the taps. The tap voltages (signal and interferences) are then weighted and summed to produce the array output. As far as the desired look direction signal is concerned, the processor is equivalent to a single tapped delay line, where each weight is equal to the sum of weights in each vertical column. The weights in the equivalent tapped delay line therefore specify the frequency response characteristic for the look direction. It should be noted that all the tap weights in the broadband processor are real compared to the complex weights of the narrowband beamformer.

For an L sensors processor with J taps each, J constraints are used to specify the look direction response and the remaining $LJ - J$ DoF in choosing the weights may be used to minimize the output power of the array. With the look direction frequency response maintained by the J constraints, minimization of the processor output is equivalent to minimizing the non-look direction interferences.

Denoting the LJ -dimensional vector $\mathbf{x}(k) = [x_{11}(k), x_{21}(k), \dots, x_{LJ}(k)]^T$ as the collection of tap voltages, and the collection of tap weights as $\mathbf{w} = [w_{11}, w_{21}, \dots, w_{LJ}]^T$. The output of the array is given by

$$y(k) = \mathbf{w}^T \mathbf{x}(k), \quad (6.1)$$

and the mean output power of the array is

$$E[y^2(k)] = \mathbf{w}^T \mathbf{R} \mathbf{w}. \quad (6.2)$$

The constrained optimization problem is posed as

$$\min_{\mathbf{w}} \mathbf{w}^T \mathbf{R} \mathbf{w}, \quad (6.3a)$$

$$\text{subject to } \mathbf{C}^T \mathbf{w} = \mathbf{f}, \quad (6.3b)$$

where the $LJ \times J$ matrix $\mathbf{C}$ is known as the constraint matrix [64], and the vector $\mathbf{f}$ is the specified impulse response at the look direction.

6.3 The Proposed New Structure Beamformer

We introduce two new structures for broadband beamforming in this section: the Tree structure and the modified Tree structure.

6.3.1 New Structures

Figure 6.2: Tree structure beamformer.

Figure 6.2 shows the proposed Tree structure beamformer. L sensors are divided into Q subarrays. If n_i denotes the number of sensors of the i^{th} subarray, then $\sum_{i=1}^Q n_i = L$ holds. The subarrays conduct beamforming independently in the 1st stage, and then are combined in the subsequent stages. The total number of stages is P which is dependent on L and n_i . Each rectangle in Figure 6.2 represents a basic block with n_i channels and J_p tapped delays. Practically, we may set $n_i \ll L$, $J_p \ll J$. Therefore, hardware implementation of the basic blocks is very easy to realize. The choice of n_i and J_p is very flexible, according to different requirements. The Tree structure can

be understood as processing signals stage-by-stage, i.e., interferences are canceled in each stage, then forwarded to the next one.

Figure 6.3: Modified Tree structure beamformer.

Figure 6.3 illustrates the modified Tree beamformer. It is different from the Tree beamformer in that its subarrays can be overlapped. The advantage of doing so is to increase the DoF so as to achieve better performance.

6.3.2 Filter Weights Updating Procedure

Since every basic block is a beamformer, the update formula of the Frost beamformer are accordingly applicable for these blocks. Because the blocks in the fixed stage are independent of each other, parallel updating is feasible. For the update between stages, we propose the two following strategies.

1) Stage-by-stage update: The $(p + 1)^{th}$ stage is processed only if the weights update in the p^{th} stage has been completed. The final output of the p^{th} stage is used as the input of the $(p + 1)^{th}$ stage, i.e., $x^{(p+1)}(k) = \mathbf{w}^{(1:p)T} \mathbf{x}^{(p)}(k)$, where $\mathbf{w}^{(1:p)}$ denotes

the final weights in the first p^{th} stage.

The stage-by-stage update permits parallel updating of the basic blocks in each stage. It accelerates the convergence of the following-up stages by optimizing the output of the current stage. If addition is conducted at the output of the current stage and satisfactory interference suppression has been obtained, the update of the subsequent stages can be canceled. Therefore, it provides a way to flexibly operate the beamformer.

2) Iteration-by-iteration update: The current output of the p^{th} stage is used as the input of the $(p + 1)^{th}$ stage, i.e., $x^{(p+1)}(k) = \mathbf{w}^{(1:p)T}(k)\mathbf{x}^{(p)}(k)$.

For an iteration-by-iteration update, if the computational complexity of all the stages are identical, parallel update of all the blocks is feasible, which means the time expenditure of the overall beamformer is the same as that of a basic block. Therefore, the time expenditure is significantly reduced. If the computational complexity is different between stages, overlaps between computation time can still be achieved so that the overall computation time can also be reduced.

6.4 Computational Complexity

Since all the proposed structures are using the sub-Frost beamformer as the basic building block, we shall firstly work out its computational load. From the expression of the update formula using the constrained LMS algorithm [64], the computational load required to derive $\mathbf{w}(k + 1)$ from $\mathbf{w}(k)$ for the Frost beamformer with L channels and J taps is $1 + 2LJ + L^2J$.

For analysis convenience, we assume that all basic blocks of the proposed structures are with two input channels, i.e., $n_1 = \dots = n_Q = 2$ and $J_1 = \dots = J_P = n$ (see Figure 6.6 and Figure 6.7 for a ULA with four sensors). For fair comparison, the simulations are conducted under the condition that the number of overall taps of the Frost beamformer is identical to that of the proposed structures.

Since it has been assumed that n taps are used in the basic blocks of the Tree structure, the number of overall taps is then given by $(n - 1)P + 1$. Therefore, we compare it with a Frost beamformer having $(n - 1)P + 1$ taps.

Table 6.1: Computational load comparison of one-step iteration of the Tree structure and the equivalent Frost beamformer with L sensors.

Structure	Number of taps in a basic block	Number of Basic blocks	Number of Multiplica- tions
Tree structure	n	$L - 1$	$(L - 1)(1 + 8n)$
Frost	$N(n - 1) + 1$	1	$1 + 2L[N(n - 1) + 1] + L^2[N(n - 1) + 1]$

The computational load of the Tree structure are tabulated in Table 6.1. For computational convenience, we assume that $L = 2^N$, where N is a positive integer. It is observed from Table 6.1 that the computational complexity of the Tree beamformer is $O(8Ln)$, while it is $O(L^2Nn + 2LNn)$ for the Frost beamformer.

Figure 6.4: Computational load with given number of sensors (dash line: Tree structure beamformer, solid line: Frost beamformer. circle: $L = 2^2$, square: $L = 2^3$, triangle: $L = 2^4$).

Figure 6.4 shows the number of multiplications required for a one-step update of the Tree structure and the Frost structure. It is noted from the table as well as the plots that the computational load of the Frost beamformer relative to that of the Tree structure increases significantly with N .

For the modified Tree structure, the computational load are tabulated in Table 6.2. Figure 6.5 shows the number of multiplications required for one-step update of the modified Tree structure.

Table 6.2: Computational load comparison of one-step iteration of the modified Tree structure and the equivalent Frost beamformer with L sensors

Structure	Number of taps in a basic block	Number of Basic blocks	Number of Multiplica- tions
Modified Tree structure	n	$L(L-1)/2$	$L(L-1)(1+8n)/2$
Frost	$(L-1)(n-1)+1$	1	$1+2L[(L-1)(n-1)+1]+L^2[(L-1)(n-1)+1]$

Figure 6.5: Computational load with given number of sensors (dash line: Modified Tree structure beamformer, solid line: Frost beamformer. circle: $L = 4$, square: $L = 5$, triangle: $L = 6$).

The computational complexity of the modified Tree structure is $O(4L^2n)$, while is $O(L^3n + L^2n)$ for its counterpart. It is natural to see that the computational complexity of the modified Tree structure exceeds that of the Tree structure, since the stages of the former is more than that of the latter. The exceeding computational load of the Frost beamformer over that of the modified Tree structure increases as L increases.

6.5 Theoretical Output SINR and Robustness to Steering Vector Error

In this section, we derive the theoretical output SINR of the proposed structures.

6.5.1 Theoretical Output SINR

The theoretical output SINR of the beamformer is calculated by

$$SINR = \frac{\mathbf{w}_{opt}^T \mathbf{R}_{ss} \mathbf{w}_{opt}}{\mathbf{w}_{opt}^T (\mathbf{R}_{ii} + \mathbf{R}_{nn}) \mathbf{w}_{opt}}, \quad (6.4)$$

where $\mathbf{w}_{opt}$ denotes the optimal weights vector, $\mathbf{R}_{ss}$, $\mathbf{R}_{ii}$ and $\mathbf{R}_{nn}$ represent correlation matrices of the desired signal, the interferences and the spatially white noise, respectively.

For a broadband source with flat spectrum, the correlation matrix is calculated as the integral within the interested frequency band [82], i.e.,

$$\mathbf{R} = \int_{f_l}^{f_u} \mathbf{R}(f) df. \quad (6.5)$$

According to [82], $\mathbf{R}(f)$ is expressed as

$$\mathbf{R}(f) = S(f)\mathbf{a}(f, \theta)\mathbf{a}^H(f, \theta), \quad (6.6)$$

where $S(f) = \sigma^2/2B$ is the spectral density, $B = f_u - f_l$ denotes the bandwidth, and $\mathbf{a}(f, \theta)$ is the space-time steering vector which is computed by the Kronecker product of the temporal and spatial steering vectors.

More specifically, the $(m, n)^{th}$ element of $\mathbf{R}$ is given by

$$[\mathbf{R}]_{m,n} = \frac{\sigma^2}{B} [f_u \text{sinc}(2f_u \varsigma_{m,n}) - f_l \text{sinc}(2f_l \varsigma_{m,n})] \quad (6.7)$$

where $\text{sinc}(x) = \sin(\pi x)/\pi x$, $\varsigma_{m,n}$ is the time delay between the m^{th} and the n^{th} element of $\mathbf{a}(f, \theta)$. Substituting the σ^2 , $\varsigma_{m,n}$ with respect to the desired source, interferences into (6.7), $\mathbf{R}_{ss}$ and $\mathbf{R}_{ii}$ can be derived. Usually, $\mathbf{R}_{nn} = \sigma_n^2 \mathbf{I}$ is assumed.

Besides the correlation matrices, the overall optimal weight vector is needed to calculate the SINR. Its derivation is given below.

Suppose that the frequency response of the first $p - 1$ stages with respect to the i^{th} block, denoted as $\mathbf{H}_{opt,i}^{(1:p-1)}$, is known, the input of the i^{th} block is calculated as

$$\mathbf{x}_i^{(p)}(f) = \mathbf{H}_{opt,i}^{(1:p-1)} \mathbf{x}(f), \quad (6.8)$$

where $\mathbf{H}_{opt,i}^{(1:p-1)} = \mathbf{w}_{opt,i}^{(1:p-1)H} \mathbf{a}^{(1:p-1)}(f, \theta)$, $\mathbf{w}_{opt,i}^{(1:p-1)}$ and $\mathbf{a}^{(1:p-1)}(f, \theta)$ are the optimal weight vector and the space-time steering vector of the first $p - 1$ stages with respect to the i^{th} block, respectively.

To compute the optimal weight vector of the i^{th} block, denoted as $\mathbf{w}_{opt,i}^{(p)}$, the input correlation matrix of the p^{th} stage with respect to the i^{th} block is required [64]. It can be computed from (6.8) as

$$\mathbf{R}_i^{(p)}(f) = E [\mathbf{x}_{in}^{(p)}(f) \mathbf{x}_{in}^{(p)T}(f)] = \left(\mathbf{1} \otimes \mathbf{H}_{opt,i}^{(1:p-1)} \right) E [\mathbf{x}_n(f) \mathbf{x}_n^T(f)] \left(\mathbf{1} \otimes \mathbf{H}_{opt,i}^{(1:p-1)} \right)^T, \quad (6.9)$$

where $\mathbf{1}$ is an $n \times 1$ vector with all ones, $\otimes$ represents the Kronecker product, $\mathbf{x}_{in}^{(p)}(f)$ and $\mathbf{x}_n(f)$ denote the temporally stacked $\mathbf{x}_i^{(p)}(f)$ and $\mathbf{x}(f)$, respectively.

To illustrate how to work out $\mathbf{H}_{opt,i}^{(1:p-1)}$, we will concentrate on a ULA with four sensors for simplicity.

Figure 6.6: Tree structure for a ULA with four sensors.

Figure 6.7: Modified Tree structure for a ULA with four sensors.

For a four-sensor ULA, the proposed structures are shown in Figure 6.6 and Figure 6.7. $H_{iU}(f)$ denotes the frequency response of the upper channel of the i^{th} basic block, and $H_{iL}(f)$ denotes the frequency response of the lower channel.

Tree structure

By calculating the correlation matrices with respect to the first two and the last two sensors, $\mathbf{H}_{opt,1}^{(1)}(f) = \begin{bmatrix} H_{1U}(f) \\ H_{1L}(f) \end{bmatrix}$ and $\mathbf{H}_{opt,2}^{(1)}(f)$ can be determined.

From the Tree structure, we can show that

$$\mathbf{H}_{opt,3}^{(1:1)} = \begin{bmatrix} H_{1U}(f) & H_{1L}(f) & 0 & 0 \\ 0 & 0 & H_{2U}(f) & H_{2L}(f) \end{bmatrix} \quad (6.10)$$

Substituting (6.10) into (6.9) and using (6.7), $\mathbf{H}_3(f)$ can be computed.

Based on the structure, the overall optimum weights of the Tree beamformer, denoted as $\mathbf{H}_{opt,overall}^{(1:2)}(f)$, is given by

$$\mathbf{H}_{opt,overall}^{(1:2)}(f) = \begin{bmatrix} H_{1U}(f)H_{3U}(f) \\ H_{1L}(f)H_{3U}(f) \\ H_{2U}(f)H_{3L}(f) \\ H_{2L}(f)H_{3L}(f) \end{bmatrix}. \quad (6.11)$$

Multiplication of the frequency response $H_i(f)$ means convolution of the corresponding weights. Therefore, based on (6.11), the overall optimal weights vector can be derived. Vectorizing it and substituting into (6.4), the theoretical output SINR of the Tree structure can be calculated.

Figure 6.8: Comparison of the theoretical output SINR of the Tree structure and the Frost beamformer. (Upper figure: $\sigma_n^2 = -20$ dB, Lower figure: $\sigma_n^2 = -10$ dB.)

Figure 6.8 shows the theoretical output SINR of the Tree structure and the Frost beamformer versus the incident angle of the interference with $\sigma_s^2 = \sigma_i^2 = 0$ dB. The bandwidth of the real signal is assumed to be $B = 0.2$ and centered at $f_c = 0.3$. The number of taps in the basic block and the Frost beamformer is $n = 3$ and $J = 5$. It is observed that, theoretically, the Frost beamformer outperforms the Tree structure by about 2 to 4 dB with $\sigma_n^2 = -20$ dB. As the interference becomes far away from the broadside direction, the performance of the Frost and the proposed gets closer. As the power of the background noise increases, the performance of the two beamformers get closer.

Figure 6.9: Comparison of the theoretical output SINR of the Tree structure and the Frost beamformer with 3° look direction mismatch. (Upper figure: $\sigma_n^2 = -20$ dB, Lower figure: $\sigma_n^2 = -10$ dB.)

Figure 6.9 shows the theoretical output SINR of the Tree structure and the Frost

beamformer versus the incident angle of the interference when 3° look direction mismatch occurs. All the simulations parameters are the same with the previous simulation. It is observed that the Frost beamformer degrades significantly in the presence of look direction mismatch. The Tree structure shows better robustness against look direction mismatch compared to the Frost beamformer.

The modified Tree structure

For the modified Tree structure, $\mathbf{H}_1(f)$, $\mathbf{H}_2(f)$ and $\mathbf{H}_3(f)$ can be determined by computing the correlation matrices with respect to the first two, mid two and the last two sensors of the array.

Define $\mathbf{H}_{opt,4}^{(1:1)}$ and $\mathbf{H}_{opt,5}^{(1:1)}$ as

$$\begin{aligned}\mathbf{H}_{opt,4}^{(1:1)} &= \begin{bmatrix} H_{1U}(f) & H_{1L}(f) & 0 & 0 \\ 0 & H_{2U}(f) & H_{2L}(f) & 0 \end{bmatrix}, \\ \mathbf{H}_{opt,5}^{(1:1)} &= \begin{bmatrix} 0 & H_{2U}(f) & H_{2L}(f) & 0 \\ 0 & 0 & H_{3U}(f) & H_{3L}(f) \end{bmatrix}.\end{aligned}\quad (6.12)$$

With (6.9), $\mathbf{R}_4^{(2)}$ and $\mathbf{R}_5^{(2)}$ are computed. Accordingly, we can derive $\mathbf{H}_4(f)$ and $\mathbf{H}_5(f)$.

Similarly, by defining

$$\mathbf{H}_{opt,6}^{(1:2)} = \begin{bmatrix} \mathbf{h}_{opt,6}^{(1:2)T}(1) \\ \mathbf{h}_{opt,6}^{(1:2)T}(2) \end{bmatrix}, \quad (6.13)$$

where

$$\mathbf{h}_{opt,6}^{(1:2)}(1) = \begin{bmatrix} H_{1U}(f)H_{4U}(f) \\ H_{1L}(f)H_{4U}(f) + H_{2U}(f)H_{4L}(f) \\ H_{2L}(f)H_{4L}(f) \\ 0 \end{bmatrix},$$

$$\mathbf{h}_{opt,6}^{(1:2)}(2) = \begin{bmatrix} 0 \\ H_{2U}(f)H_{5U}(f) \\ H_{2L}(f)H_{5U}(f) + H_{3U}(f)H_{5L}(f) \\ H_{3L}(f)H_{5L}(f) \end{bmatrix}$$

$\mathbf{H}_6(f)$ is ready to be obtained from (6.9).

The overall frequency response can be calculated via

$$\mathbf{H}_{opt,overall}^{(1:3)}(f) = \begin{bmatrix} H_{1U}(f)H_{4U}(f)H_{6U}(f) \\ H_{1L}(f)H_{4U}(f)H_{6U}(f) + \\ H_{2U}(f)[H_{4L}(f)H_{6U}(f) + H_{5U}(f)H_{6L}(f)] \\ H_{3U}(f)H_{5L}(f)H_{6L}(f) + \\ H_{2L}(f)[H_{4L}(f)H_{6U}(f) + H_{5U}(f)H_{6L}(f)] \\ H_{3L}(f)H_{5L}(f)H_{6L}(f) \end{bmatrix}. \quad (6.14)$$

Accordingly, the optimum weights as well as the output SINR can be obtained.

Figure 6.10: Comparison of the theoretical output SINR of the modified Tree structure and the Frost beamformer (Upper figure: $\sigma_n^2 = -20$ dB, Lower figure: $\sigma_n^2 = -10$ dB.)

Figure 6.10 shows the theoretical output SINR of the modified Tree structure and the Frost beamformer. The number of taps for the Frost beamformer is $J = 7$. Other parameters are the same as those in calculation with respect to the Tree structure. It is observed from the figure that the theoretical output SINR of the Frost beamformer is higher than that of the Tree structure. Comparing Figure 6.10 with Figure 6.8, it is observed that the modified Tree structure has higher output SINR than that of the Tree structure.

Figure 6.11: Comparison of the theoretical output SINR of the modified Tree structure and the Frost beamformer with 3° look direction mismatch. (Upper figure: $\sigma_n^2 = -20$ dB, Lower figure: $\sigma_n^2 = -10$ dB.)

Figure 6.11 shows the theoretical output SINR of the modified Tree structure and the Frost beamformer versus the incident angle of the interference when 3° look direction mismatch occurs. All the simulations parameters are the same with the previous simulation. It is observed that the Frost beamformer degrades significantly in the presence of look direction mismatch. As for comparison, the modified Tree structure shows better robustness against look direction mismatch compared to the Frost beamformer.

It is known that by increasing the number of taps can the output SINR increases. Also, noted from Figure 6.4 and Figure 6.5, the number of taps in the basic blocks of

the proposed structures can be increased somewhat. Therefore, improved theoretical output SINR is expected.

6.6 Experimental results

To verify the validity of the proposed structures, we devise simulation scenarios with narrowband sources, broadband sources, and include the presence of the steering vector errors. A half-wavelength spaced ULA with four sensors is assumed in all cases (see Figure 6.6 and Figure 6.7). The algorithms are written in Matlab code, and implemented on a Personal Computer (PC) with a 3GHz CPU. Parallel updates of the basic blocks are implemented. We compare the performance of various methods in the aspect of time expenditure, output SINR, beampattern and power response.

The output SINR of the k^{th} iteration is calculated via

$$SINR(k) = 10 \log_{10} \frac{\mathbf{w}(k)^T \mathbf{R}_{ss} \mathbf{w}(k)}{\mathbf{w}(k)^T (\mathbf{R}_{ii} + \mathbf{R}_{nn}) \mathbf{w}(k)}. \quad (6.15)$$

The beampattern is calculated as

$$BP(\theta) = 20 \log_{10} \int_B |\mathbf{w}_{final}^T \mathbf{a}(f, \theta)| df, \quad (6.16)$$

and the power response with respect to the signal from θ_0 is given by

$$P_{\theta_0}(f) = 10 \log_{10} |\mathbf{w}_{final}^T \mathbf{a}(f, \theta_0)|^2, \quad (6.17)$$

where $\mathbf{w}_{final}$ denotes the weights vector when the iterations are ended.

6.6.1 Narrowband Sources

The Tree structure is considered in this experiment because it has fewer stages than that of the modified Tree structure which favors narrowband beamforming.

A 500Hz sinusoid with unit variance is assumed to impinge the array from the broadside direction. An interference sinusoid with the same frequency is assumed, whose direction-of-arrival is set to 40° away from the broadside direction. The Signal to Interference Ratio (SIR) is -20dB. Spatially white noise is added into the received signal so that the SNR is 20dB. The Frost beamformer is implemented with $J = 2$ taps, which is equivalent to a MVDR beamformer [9]. The taps in the basic blocks of the Tree structure is set to $n = 2$, i.e. every basic block is a narrowband beamformer. All of the algorithms are conducted using the normalized LMS algorithm [97].

Table 6.3: Tree structure: comparison of the output SINR and the elapsed calculating time with single narrowband interference. The asterisks in the table highlight the converging (not converged) output SINR.

Number of snapshots	-	Frost	Tree	Modified Tree
50	Output SINR (dB)	-10.87*	-9.69*	-12.52*
	Computing time (s)	0.0043	0.0022	0.0010
80	Output SINR (dB)	5.76*	5.49*	2.15*
	Computing time (s)	0.0055	0.0028	0.0015
100	Output SINR (dB)	6.01	5.66	5.78
	Computing time (s)	0.0065	0.0034	0.0017

Table 6.3 shows the output SINR and the elapsed calculating time after the 50th, 80th, and 100th snapshot, respectively. Tree: stage-by-stage indicates that stage-by-stage update is used, while Tree: Iter-by-Iter indicates that iteration-by-iteration

update is used. The output SINR and the elapsed calculating time are averaged over 100 independent trials.

It is observed from Table 6.3 that the converged output SINR of the Frost beamformer slightly exceeds that of the proposed structure, while the calculating time of the proposed structure is much less.

Figure 6.12: Comparison of the beampatterns with narrowband sources.

Figure 6.12 shows the final beampatterns. It is shown that all the methods are capable of nulling the interference from 40° . At the meantime, the signal from broadside direction is maintained.

6.6.2 Broadband Sources

A. Single interference

In this experiment, we assume that a broadband interference comes from 40° impinging the array. The normalized bandwidth of the desired signal as well as the interference is $B = 0.2$ and centered at $f_c = 0.3$. The SIR and SNR is set to -20dB, 20dB, respectively. We use 5 taps in each channel of the Frost beamformer, and 3 in every basic block of the Tree structure. 500 independent trials were conducted.

Figure 6.13: Tree structure: the averaged beampatterns with single broadband interference.

Figure 6.13 shows the averaged beampattern over all the trials. Figure 6.14 is the averaged power response with respect to the sources. The output SINR versus iteration is plotted in Figure 6.15. Table 6.4 presents the elapsed calculating time

after the 400th, 700th and 1000th snapshot.

Figure 6.14: Tree structure: the averaged power response with single broadband interference.

Figure 6.15: Tree structure: the averaged output SINR with single broadband interference.

It should be mentioned that the output SINR of the stage-by-stage update is calculated at the second stage of the Tree structure, i.e., after the convergence of the first stage. This is why the initial output SINR of the stage-by-stage update is higher than the others. From Figure 6.13, we see that the Frost beamformer locates the deepest null at 40° compared with the proposed structure. In Figure 6.14, all the algorithms satisfactorily suppress the interference in the interested frequency band. Figure 6.15 shows that the Frost beamformer has the highest output SINR. This coincides with our previous theoretical analysis. To update stage-by-stage or iteration-by-iteration, the output SINR after convergence is about 1dB or 2dB less than that of the Frost, respectively. The corresponding saved calculating time is approximately 10% and 50%.

Table 6.4: Tree structure: the output SINR and the elapsed calculating time with single broadband interference. The asterisks in the table highlight the converging (not converged) output SINR.

Number of snapshots	-	Frost	Tree	Modified Tree
400	Output SINR (dB)	1.03*	-1.39*	-1.63*
	Computing time (s)	0.0170	0.0132	0.0068
700	Output SINR (dB)	1.828*	-0.41*	-0.86*
	Computing time (s)	0.0283	0.0241	0.0126
1000	Output SINR (dB)	2.14*	0.97*	-0.12*
	Computing time (s)	0.0337	0.0310	0.0170

Figure 6.16: Modified Tree structure: the averaged output SINR with single broadband interference.

For the modified Tree structure, we keep $n = 3$ and set $J = 7$ for the Frost beamformer, so that the number of the tapped delays is identical for both of the beamformers. Figure 6.16 shows the output SINR. The time expenditure using stage-by-stage update is half more than that of the Tree structure, while remains the same when iteration-by-iteration update is implemented. Compared to the Tree structure, the performance of the modified Tree is improved so that it achieves very close output SINR values to the Frost beamformer.

B. Double interferences

Two broadband interferences from 30° and 60° are assumed in this simulation. The other simulation parameters are identical to those in the case of single broadband interference.

Table 6.5: Tree structure: the output SINR and the elapsed calculating time with double broadband interferences. The asterisks in the table highlight the converging (not converged) output SINR.

Number of snapshots	-	Frost	Tree	Modified Tree
1000	Output SINR (dB)	-3.08*	-4.95*	-6.35*
	Computing time (s)	0.0306	0.0280	0.0142
2000	Output SINR (dB)	-1.99*	-3.71*	-4.98*
	Computing time (s)	0.0597	0.0577	0.0312
3000	Output SINR (dB)	-0.49*	-2.52*	-2.93*
	Computing time (s)	0.0861	0.0819	0.0431

We still compute the output SINR and the elapsed calculating time, which are presented in Table 6.5. The averaged beampattern is shown in Figure 6.17. The

output SINR is plotted in Figure 6.18.

Figure 6.17: Tree structure: the averaged beampatterns with double broadband interferences.

The simulation results are similar to that of the single broadband source. The Frost gains about 3dB higher SINR at the cost of double calculation time compared with the iteration-by-iteration update. Therefore, it is more reasonable to use the proposed structure when processing time is crucial.

Figure 6.18: Tree structure: the averaged output SINR with double broadband interferences.

For the modified Tree structure, we broaden the interested frequency band to $[0.1, 0.4]\text{Hz}$. The number of taps is set to $J = 11$ and $n = 6$ for the Frost beamformer and the modified Tree structure, respectively. Figure 6.19 presents the output SINR. The time expenditure as well as the output SINR after the 500^{th} , 1000^{th} and the 2000^{th} snapshot are tabulated in Table 6.6.

Figure 6.19: Modified Tree structure: the averaged output SINR with double broad-band interferences.

Table 6.6: Modified Tree structure: the output SINR and the elapsed calculating time with double broadband interferences. The asterisks in the table highlight the converging (not converged) output SINR.

Number of snapshots	-	Frost	Tree	Modified Tree
500	Output SINR (dB)	-9.89*	-7.30*	-12.50*
	Computing time (s)	0.0192	0.0281	0.0074
1000	Output SINR (dB)	-7.73*	-6.80*	-11.28*
	Computing time (s)	0.0342	0.0453	0.0177
2000	Output SINR (dB)	-7.54*	-5.68*	-9.42*
	Computing time (s)	0.0699	0.0873	0.0344

From Table 6.6, we see that the stage-by-stage update achieves higher output SINR than that of the rest, while iteration-by-iteration update reduces half of the computation time than that of the rest in sacrifice of about 2dB output SINR. Therefore, it is flexible to tradeoff between the output SINR and the calculating time by combining the stage-by-stage update and iteration-by-iteration update.

6.6.3 Performance in the Presence of Steering Vector Error

To examine the performance of various beamformers in the imperfect system, we conduct the following experiments. A single broadband interference is considered. The parameters remains the same as those in the previous simulations.

Firstly, the performance of various beamformers with pointing error [11] is examined. The pointing error is generated from a Gaussian random process with 3° mean and standard variance $\sqrt{2}^\circ$. 100 independent trials were conducted and averaged. Table 6.7 shows the convergence performance of various algorithms. Secondly, the gain and phase error is taken into account. Table 6.8 shows the convergence performance of the algorithms. The error is assumed to be angle-independent, and is uniformly distributed random variables within $[0.7, 1.3]$ and $[0, 2\pi]$, respectively. 100 independent trials were conducted and averaged to give the results. Finally, Table 6.9 presents the results derived under array location error which is assumed to be uniformly distributed $\pm 0.2d$ around the ideal sensor location, where d stands for the sensor spacing.

It is observed from the tables that different kinds of errors brings different influences on the performance of the algorithms. The Frost beamformer continues to work under both pointing error and sensor location error. However, its performance deteriorates from that of the ideal case (see Figure 6.16) more conspicuously than the Tree

Table 6.7: Evaluation of convergence performance with pointing error. The asterisks in the table highlight the converging (not converged) output SINR.

	stepsize μ	N_c	Output SINR (dB)
Frost	0.01	2500	2.01
	0.02	1200	1.26
	0.05	600	-4.07
Tree: stage-by-stage	0.01	3000	2.60*
	0.02	1800	3.10
	0.05	1200	3.31
Tree: Iter-by-Iter	0.01	3000	-0.61*
	0.02	1800	0.12*
	0.05	1200	0.98

Table 6.8: Evaluation of convergence performance with phase and gain error. The asterisks in the table highlight the converging (not converged) output SINR.

	stepsize μ	N_c	Output SINR (dB)
Frost	0.001	--	-13.82
	0.005	--	-14.44
	0.01	--	-14.09
Tree: stage-by-stage	0.001	3000	-6.31*
	0.005	3000	-0.56*
	0.01	1500	-4.01
Tree: Iter-by-Iter	0.001	3000	-9.67*
	0.005	3000	-7.71*
	0.01	3000	-5.98*

structure. When gain and phase error appears, it is noted that the Frost beamformer failed for all the examined stepsizes, even for a smaller value 0.0001 (not shown in the table). By contrast, the Tree structure can still work in this case, although with some output SINR degradation.

Comparing the stage-by-stage update and the iteration-by-iteration update, we

Table 6.9: Evaluation of convergence performance with location error. The asterisks in the table highlight the converging (not converged) output SINR.

	stepsize μ	N_c	Output SINR (dB)
Frost	0.001	3000	-1.65*
	0.005	3000	2.42*
	0.01	3000	2.60*
Tree: stage-by-stage	0.001	3000	-3.19*
	0.005	3000	1.00*
	0.01	3000	3.50*
Tree: Iter-by-Iter	0.001	3000	-3.26*
	0.005	3000	-1.59*
	0.01	3000	0.94*

found that the former is more robust against the steering vector error. This is easy to understand. The stage-by-stage update is a truly subarray processor. When a given block is working, it does not use information from other blocks until it terminates. Therefore, the error of the other subarrays will not appear in the update procedure. The block gives its best performance as much as possible subject to its local conditions. However, the iteration-by-iteration update is essentially a hybrid scheme. The subsequent stages are influenced by their predecessor. It can introduce errors to other blocks, which in turn deteriorates the performance.

6.7 Conclusions and Discussions

In this chapter, we analyze the two new structures for broadband beamforming. The advantages of the new structures lie in their reduced computational complexity, easier hardware implementation and enhanced robustness against steering vector error. Moreover, parallel update of the basic blocks is feasible which significantly decreases the time expenditure. Experimental results have demonstrated the validity and supe-

riority of the proposed structures. The time expenditure of the proposed structures can be reduced as much as half of that of the Frost beamformer in sacrifice of output SINR in few dBs. The loss of output SINR can be compensated by increasing the number of taps of the basic blocks. As illustrated, the exceeding computational load of the Frost to the Tree structure increases exponentially with N . Therefore, it is expected that the proposed structure will save much more calculating time than that of the Frost when it is applied to a large scale array. Furthermore, the subarray eases difficulty in array calibration, which in turn alleviates the influence caused by the steering vector error.

Up to now, we dedicate ourselves to enhance robustness of beamformers against steering vector error. It is noted that the beamformers can also be used to DOA estimation problems. One of the attractive merit of applying the beamforming techniques to DOA estimation is no need of estimating the number of sources. However, the conventional beamformers do not have satisfactory resolution capability when resolving closely spaced signals. In the next chapter, we aim to design a DOA estimation algorithm which does not require the number of sources, and at the meantime, has satisfactory resolution capability.

Chapter 7

Conclusions and Future Work

In this chapter, we make conclusions on studies presented in this thesis and provide some suggestions on the future research.

7.1 Conclusions

In Chapter 3, we have proposed two robust beamformers with sparse constraint on beampattern to suppress the sidelobe level of the beamformer. It has been shown that the proposed algorithms give better sidelobe suppression effect than that of the MVDR beamforming technique. Furthermore, mismatch problem can also be alleviated using the new algorithm. It has been demonstrated by simulations that these new algorithms perform better than the MVDR beamformer.

In Chapter 4, we introduce ℓ_1 -norm and ℓ_2 -norm beamforming algorithms which are robust against array steering vector error. The proposed ℓ_2 -norm algorithm enhances its robustness by incorporating a term which aims to minimize the correlation of the real and imaginary parts of the desired signal in the objective function. For the ℓ_2 -norm beamformer, computer simulations show that the proposed algorithms perform

similarly to the MVDR beamformer in ideal case. In the presence of steering vector error, the proposed ℓ_2 -norm beamformers outperform the MVDR beamformer and the MVDR beamformer with norm constraint. For the robust ℓ_1 -norm beamformer, it has the following advantages over the MVDR beamformer and the MVDR beamformer with norm constraint: 1) In the presence of steering vector error and impulsive noise, the proposed beamformer is capable of maintaining the response gain at the desired signal direction. 2) The output SINR of the proposed beamformer is higher than that of the MVDR beamformer and the robust MVDR beamformer.

In Chapter 5, we propose a new DOA estimation algorithm without estimating the number of sources. The algorithm realized DOA estimation in the framework of beamforming. The proposed algorithm is similar to the MUSIC algorithm in two aspects. Firstly, they share an identical quiescent array response. Secondly, the optimal weight vector of the two algorithms are all given by the weighted sum of noise eigenvectors. The appealing advantage of the proposed algorithm lies in that the number of sources is not required to estimate the DOAs. Using the recently proposed MCA algorithm, the proposed algorithm has less computational complexity than that of the MUSIC algorithm and the MVDR beamformer. Spurious peaks are avoided in the new algorithm, and rigorous theoretical analysis has been given to support this. Simulations have demonstrated the superiority of the new algorithm compared to the MUSIC algorithm when the number of sources is incorrectly estimated.

In Chapter 6, we analyze two block cascaded structures for broadband beamforming. The advantages of the new structures lie in their reduced computational complexity, easier hardware implementation and enhanced robustness against steering vector error. Moreover, parallel update of the basic blocks is feasible which significantly decreases the time expenditure. Experimental results have demonstrated the validity

and superiority of the proposed structures. The time expenditure of the proposed structures can be reduced as much as half of that of the Frost beamformer in sacrifice of output SINR in few dBs. The loss of output SINR can be compensated by increasing the number of taps of the basic blocks. As illustrated, the exceeding computational load of the Frost algorithm to the Tree structure increases exponentially with N . Therefore, it is expected that the proposed structure will save much more calculation time than that of the Frost algorithm when it applies to large scale array. Furthermore, the subarray eases difficulty in array calibration, which in turn alleviates the influence caused by the steering vector error.

7.2 Future Work

7.2.1 Application of Volterra Filter Structure to DOA Estimation

From computer simulations in Chapter 5, we see that the resolution capability of the MUSIC-like algorithm is slightly poorer than that of the MUSIC algorithm. It is known that high-order statistics is insensitive to Gaussian noise and can improve resolution capability of array signal processing algorithms. In order to improve resolution capability of the proposed algorithm, we may consider the application of Volterra filter structure to our algorithm. Different from conventional high-order statistics algorithms which directly utilize high-order statistics instead of covariance matrix to estimate DOA of sources, we construct snapshots in a high-order form.

Potential application of the Volterra filter structure to DOA estimation is introduced as follows.

Observation Model in Volterra Filter Structure

The Volterra filter belongs to a class of nonlinear filters which deal with nonlinear models [98]. In particular, this model is useful for nonlinear adaptive filtering because the classical formulation of linear adaptive filters can be easily extended to fit this model. The Volterra series expansion for a nonlinear system with output $y[n]$ and input $x[n]$ is given by

$$\begin{aligned}
 y[n] = & h_0 + \sum_{m_1=0}^{\infty} h_1[m_1]x[n-m_1] + \\
 & \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} h_2[m_1, m_2]x[n-m_1]x[n-m_2] + \dots \\
 & + \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \dots \sum_{m_p=0}^{\infty} h_p[m_1, \dots, m_p]x[n-m_1] \dots x[n-m_p] + \dots
 \end{aligned} \tag{7.1}$$

In (7.1), $h_p[m_1, \dots, m_p]$ is known as the p^{th} order Volterra kernel of the system.

In practice, the truncated form of (7.1) is used:

$$\begin{aligned}
 y[n] = & h_0 + \sum_{m_1=0}^N h_1[m_1]x[n-m_1] + \\
 & \sum_{m_1=0}^N \sum_{m_2=0}^N h_2[m_1, m_2]x[n-m_1]x[n-m_2] + \dots \\
 & + \sum_{m_1=0}^N \sum_{m_2=0}^N \dots \sum_{m_p=0}^N h_p[m_1, \dots, m_p]x[n-m_1] \dots x[n-m_p] + \dots
 \end{aligned} \tag{7.2}$$

The number of coefficients in this polynomial expansion is proportional to N^p , where N denotes the order of the polynomial expansion.

In our study, we set $N = 2$, i.e., a second-order Volterra filter structure is used. The following assumptions are made: *A1*) Sources are mutually uncorrelated Gaussian sources. *A2*) Sources are independent from the spatial noise.

Take an array with two sensors as an example. Supposing there are two sources, the data received by the array can be expressed as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \quad (7.3)$$

where $\mathbf{x}(t) = [x_1(t), x_2(t)]^T$, $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2)]$ and $\mathbf{s}(t) = [s_1(t), s_2(t)]^T$, $\mathbf{n}(t)$ denotes the spatial noise vector.

We transform the observed data into the following second-order Volterra structure:

$$\mathbf{y}(t) = \mathbf{B}\mathbf{z}(t) + \mathbf{v}(t), \quad (7.4)$$

where

$$\mathbf{y}(t) = [x_1(t), x_2(t), x_1^2(t), |x_1(t)|^2, x_1(t)x_2(t), x_1(t)x_2^*(t), x_2^2(t), |x_2(t)|^2]^T, \quad (7.5)$$

$$\mathbf{B} = \begin{bmatrix} \tilde{\mathbf{B}}(\theta_1) & \tilde{\mathbf{B}}(\theta_2) & \mathbf{b}(\theta_1, \theta_2) \end{bmatrix}, \quad (7.6)$$

with

$$\tilde{\mathbf{B}}(\theta_i) = \begin{bmatrix} a_1(\theta_i) & 0 \\ a_2(\theta_i) & 0 \\ 0 & a_1^2(\theta_i) \\ 0 & 1 \\ 0 & a_1(\theta_i)a_2(\theta_i) \\ 0 & a_1(\theta_i)a_2^*(\theta_i) \\ 0 & a_2^2(\theta_i) \\ 0 & 1 \end{bmatrix}, \quad i = 1, 2. \quad (7.7)$$

$$\mathbf{b}(\theta_1, \theta_2) = [0, 0, 2a_1(\theta_1)a_1(\theta_2), 2\text{Re}\{a_1(\theta_1)a_1^*(\theta_2)\}, a_1(\theta_1)a_2(\theta_2) + a_2(\theta_1)a_1(\theta_2), \\ a_1(\theta_1)a_2^*(\theta_2) + a_2^*(\theta_1)a_1(\theta_2), 2a_2(\theta_1)a_2(\theta_2), 2\text{Re}\{a_2(\theta_1)a_2^*(\theta_2)\}]^T \quad (7.8)$$

$$\mathbf{z}(t) = [s_1(t), s_1^2(t), s_2(t), s_2^2(t), s_1(t)s_2(t)]^T, \quad (7.9)$$

and

$$\mathbf{v}(t) = [n_1(t), n_2(t), n_1^2(t), |n_1(t)|^2, n_1(t)n_2(t), n_1(t)n_2^*(t), n_2^2(t), |n_2(t)|^2]^T. \quad (7.10)$$

Since for DOA estimation, we are only interested in signal amplitude or its power, we consider $\tilde{\mathbf{B}}(\theta_i)$ associated with $s_i(t)$ and $s_i^2(t)$ as "steering vector".

With this steering vector, the proposed MUSIC-like algorithm with Volterra filter structure can be implemented as the steps given in Section 5.4. We give the summary of the new algorithm here.

Step 1. Construct $\mathbf{y}(t)$ from $\mathbf{x}(t)$ using (7.5);

Step 2. Estimate the covariance matrix of $\mathbf{y}(t)$ via

$$\mathbf{R}_{yy} = \frac{1}{N} \sum_{i=1}^N \mathbf{y}(t) \mathbf{y}^H(t);$$

Step 3. For every θ , compute the eigenvector $\mathbf{e}_n$ associated with the minimum eigenvalue of $\left(\tilde{\mathbf{B}}(\theta) \tilde{\mathbf{B}}^H(\theta) + \beta \mathbf{I} \right)^{-1} \mathbf{R}_{yy}$

Step 4. Compute the value of $-10 \log_{10} \| \tilde{\mathbf{B}}(\theta)' * \mathbf{e}_n \|_2^2$.

Some Implementation Issues

Several issues on implementation of the algorithm are addressed here.

The dimension of $\mathbf{y}(t)$ is $(L+1)^2 \times 1$ which dramatically increases the computational complexity of the algorithm. It is noted from $\tilde{\mathbf{B}}(\theta_i)$ that some of its rows are constant which are independent of θ_i . Therefore, they can be removed, and only multiplication terms $x_i(t)x_j(t), \forall i, j$ and cross conjugate multiplication terms $x_i(t)x_j^*(t), \forall i, j, i \neq j$ remain. By doing so, the computational complexity of the algorithm is reduced because of the reduced dimension of $\mathbf{y}(t)$ to $(L+1)^2 - L$. However, the computational complexity is still large. From simulations, we find that $x_i(t)x_j^*(t), \forall i, j, i \neq j$ can also be removed from $y(t)$ without affecting the performance of the proposed algorithm. Accordingly, the computational load can be further reduced to $L(L+1)/2$. Besides, it is found that the condition number of $\mathbf{R}_{yy}$ is improved so that the algorithm becomes more stable.

Furthermore, based on A1, we notice that the rank of $\mathbf{R}_{zz}$ is $2L$ due to the spatial noise. Hence, $\mathbf{e}_n$ in *Step 3* can be chosen as the eigenvector associated with the $2L^{th}$ maximum eigenvalue of $\left(\tilde{\mathbf{B}}(\theta) \tilde{\mathbf{B}}^H(\theta) + \beta \mathbf{I} \right)^{-1} \mathbf{R}_{yy}$. Doing this can also make the

algorithm more stable.

Computer Simulations

In this section, we devise several simulation scenarios to verify the validity of the proposed algorithm. We compare the proposed algorithm with the MUSIC algorithm and Capon's method.

A. Comparison of the performance of various algorithms with closely spaced targets

In this simulation, we assume a ULA with three half-wavelength spaced sensors. Two uncorrelated narrowband sources with identical power equal to 0dB are assumed to impinge the array from 50° and 65° . It is supposed that the number of sources estimated to implement the MUSIC algorithm is precisely equal to 2. Therefore, the MUSIC algorithm works in the optimal situation. 1000 independent trials are conducted.

Figure 7.1: Direction finding comparison with $L = 3$, the number of snapshots=50 and SNR=20dB.

Figure 7.1 shows the spatial spectra using different algorithms. The RMSE of various algorithms versus the number of the snapshots are presented in Figure 7.2, where the SNR is set to 20dB and the number of snapshots is 50.

Figure 7.2: RMSE of the estimations against the number of snapshots with SNR=20 dB.

B. Comparison of various algorithms with very closely spaced sources.

In this simulation, we evaluate the performance of the proposed algorithm and the MUSIC algorithm with very closely spaced sources. The array is identical to that in the previous simulation. Two narrowband sources are assumed to impinge the array from 30° and 33° . The SNR is set to 20dB. The number of the estimated sources are assumed to be correct for the MUSIC algorithm. 1000 independent trials are conducted.

Figure 7.3 depicts the result using the AIC and MDL criteria to estimate the number of sources. It is found that in this case, both criteria fail to work. Both

criteria give the estimated number of sources as 1. (The estimated number of sources should correspond to the x-axis value when the minimal value of the criterion is reached.) Figure 7.4 shows the power spectra of the MUSIC algorithm, the MVDR beamformer and the proposed algorithm for 50 independent trials.

Figure 7.3: Estimated number of sources using the AIC and MDL criteria.

Figure 7.4: Direction finding comparison with $L = 3$, the number of snapshots=50 and SNR=20 dB.

Figure 7.5: RMSE of the estimations against the number of snapshots with SNR=20 dB.

7.2.2 Investigating the Performance of the MUSIC-like DOA Estimation Algorithm to Robust Beamforming Against Steering Vector Error

It is noted that the MVDR beamformer with norm constraint can improve robustness of the beamformer. From this observation, we propose to use the following optimization function to design a robust beamformer:

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} + \alpha \| \mathbf{w}^H \mathbf{A}(\theta) \|^2_2, \quad (7.11a)$$

$$\text{subject to } \mathbf{w}^H \mathbf{a}(\theta_0) \mathbf{a}^H(\theta_0) \mathbf{w} + \beta \| \mathbf{w} \|^2_2 = c, \quad (7.11b)$$

where α is a regularization parameter which compromises between interferences cancellation and sidelobe suppression, and $\mathbf{A}(\theta)$ consists of all nominal steering vectors at interference directions. It is observed from (7.11) that this new algorithm aims to minimizing the power of interferences and additive noise as well as suppressing the sidelobe level of the beampattern with constraint on maintaining the output gain at the look direction and avoiding self-cancellation of the SOI (achieved by the term $\|\mathbf{w}\|_2^2$). Different from the MVDR beamforming, the array gain at the look direction is not imposed to 0 dB. Instead, the array gain is adaptive to the level of white noise. The lower the level of white noise, the higher the array gain.

To guarantee satisfactory performance, we should choose a proper value of β : i) When the steering vector points at interference directions (i.e., in the signal subspace of $\mathbf{R} + \alpha\mathbf{A}(\theta)\mathbf{A}^H(\theta)$), β should be bounded by

$$\beta > \frac{L}{\xi_M/\xi_i - 1}, \quad i \in \mathbb{N}, \quad (7.12)$$

where $\mathbb{N}$ represents noise subspace indices. In this case, the optimal weight vector $\mathbf{w}_{opt}$ is orthogonal to the steering vector, which means that array gain at interference directions is very small. ii) When array is imprecise, the steering vector points at a direction near the SOI. In this case, due to the nonnegative property of $\mathbf{B}_{-i}$ ($\mathbf{B}_{-i}$ denotes the matrix $\tilde{\mathbf{B}} - \gamma_{min}\mathbf{I}$ with the i^{th} row and column omitted), the array gain with respect to this direction can be maintained. It means even if the real direction of the SOI is somewhat different from the presumed direction, the proposed beamformer will not cast a null at the direction of the SOI. iii) When the steering vector points at the direction of SOI, β should be bounded by

$$\beta < \frac{L}{\xi_{N_{\parallel}}/\xi_{N_{\perp}^{min}} - 1} \leq \frac{L}{\xi_{L-1}/\xi_L - 1}. \quad (7.13)$$

In this case, the array gain with respect to the SOI can be maintained. Combining (7.12) and (7.13), we propose to choose β using

$$\beta = \frac{kL}{\xi_{L-1}/\xi_L - 1}, k \leq 1. \quad (7.14)$$

Computer simulations

To verify the validity and advantages of the proposed algorithm, we devise the following simulation scenarios. A ULA with ten half-wavelength spaced sensors is used. Five sources from $0^\circ, -30^\circ, -10^\circ, 20^\circ, 50^\circ$ are assumed to impinge on the array. The source from broadside is the SOI, while the others are interferences. The SINR is -20 dB. The number of snapshots is 100. The sidelobe suppression vector is designed as $[-90^\circ, -10^\circ] \cup [10^\circ, 90^\circ]$ with 5° interval.

A. Ideal case

First of all, the MVDR beamformer, the proposed algorithm and the MUSIC algorithm are evaluated for the ideal case.

Figure 7.6 shows the obtained beampatterns of different algorithms. Figure 7.7 shows the output SINR versus SNR with 1000 independent trials.

Figure 7.6: Beampatterns of different algorithms in ideal case.

Figure 7.7: Output SINR versus SNR in ideal case.

B. In the presence of look direction mismatch

In this simulation, we evaluate the performance of various algorithms in the presence of look direction mismatch. Simulation parameters are identical to those in the previous simulation.

Figure 7.8: Beampatterns of different algorithms in the presence of look direction mismatch.

Figure 7.8 depicts beampatterns of different algorithms with 3° look direction mismatch. To more clearly see the performance, we plot output SINR versus SNR with 1000 independent trials in Figure 7.9.

Figure 7.9: Output SINR versus SNR in the presence of look direction mismatch.

C. In the presence of look direction mismatch and sensor position error

In this simulation, we evaluate the performance of various algorithms in the presence of look direction mismatch and sensor position error. Simulation parameters are identical to those in the previous simulation. The sensor position error is assumed to be a Gaussian variable with zero mean and standard deviation 0.1 times the inter-element spacing.

Figure 7.10: Beampatterns of different algorithms in the presence of look direction mismatch and sensor position error.

Figure 7.11: Output SINR versus SNR in the presence of look direction mismatch and sensor position error.

Figure 7.10 depicts the beampatterns of different algorithms with 3° look direction mismatch. To more clearly see the performance, we plot the output SINR versus SNR with 1000 independent trials in Figure 7.11.

7.2.3 Other Researches

Based on the studies in this thesis, several suggestions for the future research work are proposed as follows:

For the proposed new structures for broadband beamforming, since the proposed Tree structure and the modified Tree structure consist of sub-Frost beamformers, they may achieve some local advantages over the conventional Frost beamformer. Firstly, array frequency response can be optimized based on the local information of

the array. Different basic blocks may be designed to have different array responses so that the overall structure achieves desired performance. Secondly, in a near-field condition, it is possible that interferences are not present simultaneously on all sensors. Some sensors near the interference are under its influence, while the rest are not. In this case, Frost beamforming and the proposed beamforming techniques may achieve different performance. Whether global processing or local processing is more favorable requires further investigation. Furthermore, as Generalized Sidelobe Canceler (GSC) [99, 100] structure can be applied to the Frost beamformer, it can also be applied to the proposed structures by simply replacing the FIR channels of the GSC structure with the proposed structures. Whether the proposed algorithm can achieve good performance requires further investigation.

For the MUSIC-like DOA estimation algorithm, it is known that the computational load is higher than that of the MUSIC algorithm when the number of sensors is not large. To reduce the computational load, the Conjugate Gradient (CG) [101, 102] algorithm may be applied. Since the CG algorithm can be used to find the generalized eigenvector, it also can be used in our problem. Besides narrowband sources processing, it is possible to extend our algorithm to broadband case. [103] has introduced a MUSIC approach for estimation of directions of arrival of multiple narrowband and broadband sources. The algorithm performs DOA estimation in the framework of beamforming. By imposing array output with respect to derivatives of steering vectors to zero, the algorithm is able to realize broadband DOA estimation. For the proposed MUSIC-like DOA estimation algorithm, this idea can be borrowed.

The issue of noise suppression is an important problem in the field of signal processing and is relevant to many applications such as radar, speech processing and wireless communications. Methods proposed in the past tend to be based on Gaussian noise as-

sumption. However, when the noise process exhibits chaotic behavior (e.g. noise from electronic oscillator, engine noise from mechanic system, sea clutter in radar and fluid turbulence in sonar), the stochastic-based noise suppression technique will become less effective. To have better effective noise suppression, a class of nonlinear prediction filters which exploit the chaotic properties has been studied extensively [33–35]. The recent work by Broomhead et al. [34] shows that it is possible to separate a narrow-band signal from a chaotic noise in a single-channel receiving system. The method uses nonlinear inverse filter to reconstruct the chaotic noise. Then, the clean narrow-band signal can be obtained by removing the reconstructed noise from the received signal. For multi-channel receiving system, beamforming techniques can be utilized to enhance the desired signal and suppress the noise. Compared to single-channel signal enhancement techniques, the beamformers promise higher SNR improvement because the spatial information of the desired signal is exploited. However, when the noise is chaotic and directional, further SNR improvement will be possible. To achieve that, we need to devise a method that exploits both the spatial information and the coherence of the chaotic process that generates the noise. To exploit spatial information of the interference, nonlinear Support Vector Regression (SVR) [104–106] may be used to restore the in-band chaotic noise from its multiple out-band counterparts. Our initial simulations have shown that the SVR beamformer is capable of suppressing the chaotic and impulsive noise better than conventional beamformers. Further investigation may be done along this direction.

Appendix A: Proof of the Invertibility of the Matrix $\mathbf{B}_{(-i)}$

From (5.13), we may rewrite $\tilde{\mathbf{B}}$ as

$$\tilde{\mathbf{B}} = \mathbf{A}\mathbf{\Lambda}$$

where

$$a_{ij} = \begin{cases} (\beta^{-1} - \alpha |a_i|^2), & i = j \\ -\alpha a_i a_j^*, & i \neq j \end{cases},$$

and $\mathbf{\Lambda} = \text{diag}\{\xi_1, \dots, \xi_L\}$. The eigenvalues of $\tilde{\mathbf{B}}$ satisfy

$$|\tilde{\mathbf{B}} - \gamma \mathbf{I}| = |\mathbf{A}\mathbf{\Lambda} - \gamma \mathbf{I}| = 0,$$

which is equivalent to

$$|\tilde{\mathbf{B}} - \gamma \mathbf{I}| |\mathbf{\Lambda}^{-1}| = |\mathbf{A} - \gamma \mathbf{\Lambda}^{-1}| = 0.$$

Similar to (5.22), we assume that the i^{th} row of $\mathbf{A} - \gamma \mathbf{\Lambda}^{-1}$ is zero. Since $\mathbf{A} - \gamma \mathbf{\Lambda}^{-1}$ is hermitian, the i^{th} column of $\mathbf{A} - \gamma \mathbf{\Lambda}^{-1}$ can also be transformed to zero with basic matrix manipulation in column direction. It means the row rank and column rank of

$\mathbf{A} - \gamma\mathbf{\Lambda}^{-1}$ is the same. Taking the consideration that

$$\text{rank}(\tilde{\mathbf{B}} - \gamma_{\min}\mathbf{I}) = L - 1,$$

and $\mathbf{\Lambda}$ is a diagonal matrix, the row rank and column rank of $\mathbf{A} - \gamma\mathbf{\Lambda}^{-1}$ should be $L - 1$. Therefore, the column rank of $(\tilde{\mathbf{B}} - \gamma_{\min}\mathbf{I})$ is $L - 1$. Since the size of $\mathbf{B}_{(-i)}$ is $(L - 1) \times (L - 1)$, it must be invertible.

Appendix B: Proof of the Positivity of the Matrix $\mathbf{B}_{(-i)}$

From (5.15), it is straightforward that

$$\mathbf{B}_{(-i)} = \tilde{\mathbf{B}}_{(-i)} - \gamma_{\min} \mathbf{I}_{L-1},$$

where $\tilde{\mathbf{B}}_{(-i)}$ is derived by canceling the i^{th} row and i^{th} column of $\tilde{\mathbf{B}}$.

Supposing that the minimum eigenvalue of $\tilde{\mathbf{B}}_{(-i)}$, denoted as $\gamma_{\min(-i)}$, is smaller than $\gamma_{\min}$ which is the minimum eigenvalue of $\tilde{\mathbf{B}}$, we conclude that $\tilde{\mathbf{B}} - \gamma_{\min(-i)} \mathbf{I}$ is a positive matrix. Then, $|\tilde{\mathbf{B}}_{(-i)} - \gamma_{\min(-i)} \mathbf{I}_{L-1}| > 0$ holds, because it is an $(L-1)$ -order leading principle minor of the positive matrix $\mathbf{U}_{(i,L)} \left(\tilde{\mathbf{B}}_{(-i)} - \gamma_{\min(-i)} \mathbf{I}_{L-1} \right) \mathbf{U}_{(i,L)}^H$, where $\mathbf{U}_{(i,L)}$ denotes the identity matrix with the i^{th} column and the L^{th} column exchanged. However, since $\gamma_{\min(-i)}$ is the minimum eigenvalue of $\tilde{\mathbf{B}}_{(-i)}$, $|\tilde{\mathbf{B}}_{(-i)} - \gamma_{\min(-i)} \mathbf{I}_{L-1}| = 0$ should hold, which contradicts the previous conclusion. Therefore, $\gamma_{\min(-i)}$ is no smaller than $\gamma_{\min}$, and $\mathbf{B}_{(-i)}$ is nonnegative. Notice that $\mathbf{B}_{(-i)}$ is invertible, it must be positive.

Author's Publications

- [1] Ying Zhang, Boon Poh Ng and Qun Wan, "Sidelobe Suppression for Adaptive Beamforming with Sparse Constraint on Beam Pattern," *Electronics Letters*, Vol. 44, No. 10, 615–616, May 8, 2008.
- [2] Ying Zhang, and Boon Poh Ng, "MUSIC-like DOA Estimation Without Estimating the Number of Sources," *IEEE Transactions on Signal Processing*, Vol. 58, No. 3, 1668–1676, March, 2010.
- [3] Ying Zhang, Joni Polili Lie, Chong-Meng See, Boon Poh Ng, "Robust Minimum ℓ_1 -Norm Adaptive Beamformer Against Intermittent Sensor Failure and Steering Vector Error," *IEEE Transactions on Antenna and Propagations*, Vol. 58, No. 5, 1796–1801, May, 2010.
- [4] Ying Zhang, Joni Polili Lie, Boon Poh Ng, and Qun Wan, "Robust Beamforming Technique With Sidelobe Suppression Using Sparse Measurement on Beampattern," *The Applied Computational Electromagnetics Society*, accepted for publication.
- [5] Ying Zhang, Boon Poh Ng, "Block Cascaded Structures for Broadband Beamforming," *Journal of Signal Processing Systems*, submitted for publication.
- [6] Vinod Reddy, Ying Zhang, Boon Poh Ng, "DOA Estimation of Wideband and Narrowband Sources Without Estimating the Number of Sources," *IEEE Transactions on Signal Processing*, submitted for publication.

Bibliography

- [1] W.L.Stutzman and G.A.Thiele, *Antenna Theory and Design*. New York: Wiley, 1981.
- [2] D.Jonson and D.Dudgeon, *Array Signal Processing: Concepts and Techniques*. Signal Processing Series, Prentice Hall, Englewood Cliffs, 1996.
- [3] B.D.V.Veen and K.M.Buckley, "Beamforming: a versatile approach to spatial filtering," *IEEE ASSP Magazine*, vol. 5, no. 2, pp. 4–24, April 1988.
- [4] J. Li and P. Stoica, *Robust Adaptive Beamforming*. Wiley, 2006.
- [5] D. Dudgeon, "Fundamentals of digital array processing," *Proceedings of the IEEE*, vol. 65, no. 6, pp. 898–904, June 1977.
- [6] W. Sutzman, "Sidelobe control of antenna patterns," *IEEE Transactions on Antennas and Propagation*, vol. 20, no. 1, pp. 102–104, January 1972.
- [7] W. Kummer, "Basic array theory," *Proceedings of the IEEE*, vol. 80, no. 1, pp. 127–140, January 1992.
- [8] S. Sanzgiri and J. Butler, "A variational technique for antenna sidelobe control," *Antennas and Propagation Society International Symposium*, vol. 9, pp. 231–234, September 1971.
- [9] J.Capon, "High resolution frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, vol. 57, no. 8, pp. 1408–1418, August 1969.
- [10] L. Griffiths, "Adaptive monopulse beamforming," *Proceedings of the IEEE*, vol. 64, no. 8, pp. 1260–1261, August 1976.
- [11] H. Cox, "Resolving power and sensitivity to mismatch of optimum array processors," *J. Acoust. Soc. Am.*, vol. 54, no. 3, pp. 771–785, 1973.
- [12] J. Krolik, "The performance of matched-field beamformers with mediterranean vertical array data," *IEEE Transactions on Signal Processing*, vol. 44, no. 10, pp. 2605–2611, 1996.

- [13] K. Buckley and L. Griffiths, "An adaptive generalized sidelobe canceller with derivative constraints," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 311–319, March 1986.
- [14] A. Steele, "Comparison of directional and derivative constraints for beamformers subject to multiple linear constraints," *IEE Proceedings F Communications, Radar and Signal Processing*, vol. 130, no. 1, pp. 41–45, February 1983.
- [15] M.H.Er, B.P.Ng, and A.Cantoni, "A new set of constraints for derivative-constrained broad-band beamformers," *Journal of Electrical and Electronics Engineering*, vol. 11, no. 2, pp. 87–101, June 1991.
- [16] J. Krolik, "Matched-field minimum variance beamforming," *Journal of the Acoustic Society of America*, vol. 92, no. 3, pp. 1406–1419, 1992.
- [17] A. Tikhonov and Y. Arsenin, *Solution of Ill-posed Problems*. V.H.Winston and Sons, 1977.
- [18] A. Gershman, "Robust adaptive beamforming in sensor arrays," *AEU-International Journal of Electronics and Communications*, vol. 53, no. 6, pp. 305–314, 1999.
- [19] D. Johnson, "The application of spectral estimation methods to bearing estimation problems," *Proceedings of the IEEE*, vol. 70, no. 9, pp. 1018–1028, September 1982.
- [20] S. Haykin, *Adaptive Filter Theory*. Prentice Hall, Englewood Cliffs, 1996, ch. Information and System Sciences Series.
- [21] K. Harmanci, "Relationships between adaptive minimum variance beamforming and optimal source localization," *IEEE Transactions on Signal Processing*, vol. 48, no. 1, pp. 1–13, 2000.
- [22] R. Monzingo and T. Miller, *Introduction to Adaptive Arrays*. Willey, NY, 1980.
- [23] H. V. Trees, *Optimum Array Processing*. Willey, NY, 2002.
- [24] S. Shahbazpanahi, A. Gershman, Z. Luo, and K. Song, "Robust adaptive beamforming for general-rank signal models," *IEEE Transactions on Signal Processing*, vol. 51, pp. 2257–2269, September 2003.
- [25] I. Reed, J. Mallett, and L. Brennan, "Rapid convergence rate in adaptive arrays," *IEEE Transactions on Aerospace and Electronics System*, vol. 10, pp. 853–863, November 1974.
- [26] H. Cox, R. Zeskind, and M. Owev, "Robust adaptive beamforming," *IEEE Transactions on Acoustic, Speech, Signal Processing*, vol. 35, pp. 1365–1376, October 1987.

- [27] B. Carlson, "Covariance matrix estimation errors and diagonal loading in adaptive arrays," *IEEE Transactions on Aerospace and Electronics Systems*, vol. 24, pp. 397–401, July 1988.
- [28] W. Gabriel, "Spectral analysis and adaptive array superresolution techniques," *Proceedings of IEEE*, vol. 68, pp. 654–666, June 1980.
- [29] Y. Abramovich, "Controlled method for adaptive optimization of filters using the criterion of maximum snr," *Radio Engineering and Electronic Physics*, vol. 26, pp. 87–95, March 1981.
- [30] S. Vorobyov, A. Gershman, and Z. Luo, "Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *IEEE Transactions on Signal Processing*, vol. 51, pp. 313–324, February 2003.
- [31] S. Vorobyov, A. Gershman, Z. Luo, and N. Ma, "Adaptive beamforming with joint robustness against mismatched signal steering vector and interference non-stationarity," *IEEE Signal Processing Letters*, vol. 11, pp. 108–111, February 2004.
- [32] C. Werndl, "What are the new implications of chaos for unpredictability," *The British Journal for the Philosophy of Science*, no. 60, pp. 195–220, 2009.
- [33] W. W. Taylor, *Nonlinear Modeling and Forecasting*. Addison Wesley, 1992, ch. Application of Nonlinear Prediction to Signal Separation.
- [34] D. Broomhead, J. Huke, and M. Potts, "Cancelling deterministic noise by constructing nonlinear inverses to linear filters," *Physica D*, vol. 89, pp. 439–458, 1996.
- [35] M. R. Cowper and B. Mulgrew, "The application of a nonlinear inverse noise cancellation technique to maritime surveillance radar," *Proceedings of the 10th IEEE Workshop on Statistical Signal and Array Processing*, pp. 267–271, 2000.
- [36] E. Skomal and A. Smith, *Measuring the Radio Frequency Environment*. Van Nostrand Reinhold Company, 1985.
- [37] K. Blackard, T. Rappaport, and C. Bostian, "Measurements and models of radio frequency impulsive noise for indoor wireless communications," *IEEE Journal on Selected Areas of Communications*, vol. 11, no. 7, pp. 991–1001, September 1993.
- [38] R. Shepherd, "Measurements of amplitude probability distributions and power of automobile ignition noise at hf," *IEEE Transactions on Vehicular and Technology*, vol. 23, no. 3, pp. 72–83, August 1974.

- [39] M. Shao and C. L. Nikas, "Signal processing with fractional lower order moments: Stable processes and their applications," *Proceedings of the IEEE*, vol. 81, no. 7, pp. 986–1010, July 1993.
- [40] G. Aydin, O. Arikan, and C. A. Enis, "Robust adaptive filtering algorithms for α -stable random processes," *IEEE Transactions on Circuits and Systems-II: Analog and Digital Signal Processing*, vol. 46, no. 2, pp. 198–202, February 1999.
- [41] Y.-C. Hsu, "Fractional lower order moment based adaptive algorithms," *13th Asilomar Conference on Signals, Systems and Computers*, pp. 334–338, Oct.-Nov 1996.
- [42] J. Kivinen, M. Warmuth, and B. Hassibi, "The p -norm generalization of the lms algorithm for adaptive filtering," *IEEE Transactions on Signal Processing*, vol. 54, no. 5, pp. 1782–1793, May 2006.
- [43] J. He, Z. Liu, and K. T. Wong, "Linearly constrained minimum-"geometric power" adaptive beamforming using logarithmic moments of data containing heavy-tailed noise of unknown statistics," *IEEE Antenna and Wireless Propagation Letters*, vol. 6, pp. 600–603, 2007.
- [44] B. Hu and N. Beaulieu, "On characterizing multiple access interference in th-uwb systems with impulsive noise models," *RWS 2008*, pp. 879–882, 2008.
- [45] Bertsekas, P. Demitri, Tsitsiklis, and N. John, *Parallel and Distributed Computation: Numerical Methods*. Massachusetts: Athena Scientific, 1997.
- [46] A. Oppenheim, R. Schaffer, and J. Buck, *Discrete-time signal processing*. N.J. Prentice Hall, 1999.
- [47] S. Pillai, *Array Signal Processing*. NY: Springer-Verlag New York Inc., 1989.
- [48] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, March 1986.
- [49] H. Akaike, "A new look at the statistical model identification," *IEEE Transactions on Automatic Control*, vol. 19, no. 6, pp. 716–723, 1974.
- [50] P. Grünwald, *the Minimum Description Length principle*. MIT Press, 2007.
- [51] B. Rao and K. Hari, "Performance analysis of root-music," *IEEE Transactions on Acoustic, Speech, Signal Processing*, vol. 37, no. 12, p. 1939C1949, December 1989.
- [52] M. Pesavento, A. Gershman, and M. Haardt, "Unitary root-music with a real valued eigendecomposition: a theoretical and experimental performance study," *IEEE Transactions on Signal Processing*, vol. 48, no. 5, p. 1306C1314, May 2000.

- [53] A. Paulraj, R. Roy, and T. Kailath, "Estimation of signal parameters via rotational invariance techniques - esprit," *Proceeding of 19th Asilomar Conference on Signals, Systems and Computers*, pp. 83–89, November 1985.
- [54] N. Yilmazer, S. Burintramart, and T. Sarkar, "Performance analysis of direct data domain approach and esprit method for direction-of-arrival estimation," *IEEE Antennas and Propagation International Symposium*, pp. 5295 – 5298, 9-15 June 2007.
- [55] M. Wax and Y. Anu, "Performance analysis of the minimum variance beamformer," *IEEE Transactions on Signal Processing*, vol. 44, no. 4, pp. 928–937, 1996.
- [56] P. Djuric, "A model selection rule for sinusoids in white gaussian noise," *IEEE Transactions on Signal Processing*, vol. 44, pp. 1744–1751, July 1996.
- [57] Yang and T. Yoshino, "Firgen: a computer-aided design system for high performance fir filter integrated circuits," *IEEE Transactions on Signal Processing*, vol. 39, pp. 1655–1668, July 1991.
- [58] J. Li, P. Stoica, and Z. Wang, "Doubly constrained robust capon beamformer," *IEEE Transactions on Signal Processing*, vol. 52, no. 9, pp. 2407–2423, September 2004.
- [59] S. Morita, T. Nagase, and K. Nomizu, *Geometry of Differential Forms*. American Mathematical Society Bookstore, 2001.
- [60] A. Love, *A Treatise on The Mathematical Theory of Elasticity*. New York: Dover.
- [61] C. Balanis, *Antenna Theory, Analysis and Design*. John Wiley and Sons, Inc., 2nd ed, 1982.
- [62] D. Cheng, *Field and Wave Electromagnetics*. Addison-Wesley Publishing Company Inc., Edition 2, 1998.
- [63] E. Jordan and K. Balmain, *Electromagnetic Waves and Radiating Systems*. Prentice-Hall, Edition 2, 1968.
- [64] O. L. Frost, "An algorithm for linearly constrained adaptive array processing," *Proceedings of the IEEE*, vol. 60, no. 8, pp. 926–935, August 1972.
- [65] M. Strupl and P. Sovka, "Analysis and simulation of frost's beamformer," *Radio Engineering*, vol. 12, no. 2, pp. 1–9, June 2003.
- [66] K. J. Raghunath and V. U. Reddy, "Finite data performance analysis of mvdr beamformer with and without spatial smoothing," *IEEE Transactions on Signal Processing*, vol. 40, no. 11, pp. 2126–2136, November 1992.

- [67] J.T.Mayhan, A.J.Simmons, and W.C.Cummings, "Wide-band adaptive antenna nulling using tapped delay lines," *IEEE Transactions on Antennas and Propagation*, vol. 29, no. 6, pp. 923–936, November 1981.
- [68] B. P. Ng and H. Duan, "Designing amplitude/phase response of the frost beam-former," *IEEE Transactions on Signal Processing*, vol. 55, no. 5, pp. 1944–1949, May 2007.
- [69] Cramer and Herald, *Mathematical Methods of Statistics*. Princeton, NJ: Princeton Univ. Press., 1946.
- [70] S.M.Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Prentice Hall, 1993.
- [71] P. Stoica and A. Nehorai, "Music, maximum likelihood, and cramer-rao bound," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 37, no. 5, pp. 720–741, May 1989.
- [72] J. Liberti and T. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications*. Upper Saddle River, New Jersey: Prentice-Hall, Inc., 1997.
- [73] B.D.Rao, K.Engan, S.F.Cotter, J.Palmer, and K.K.Delgado, "Subset selection in noise based on diversity measure minimization," *IEEE Transactions on Signal Processing*, vol. 51, no. 3, pp. 760–770, March 2003.
- [74] L.J.Griffiths and K.M.Buckley, "Quiescent pattern control in lineary constrained adpatively arrays," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. 35, no. 7, pp. 917–926, July 1987.
- [75] B.P.Ng, "Designing array patterns with optimum inter-element spacings and optimum weights using a computer-aided approach," *International Journal of Electronics*, vol. 73, no. 3, pp. 653–664, June 1992.
- [76] —, "Array synthesis using a simple computer-aided approach," *Electronics Letters*, vol. 26, no. 5, pp. 337–339, March 1990.
- [77] V. Bhatia and B. Mulgrew, "Non-parametric likelihood based channel estimator for gaussian mixture noise," *Signal Processing*, vol. 87, no. 11, pp. 2569–2586, November 2007.
- [78] H. Cox, "Resolving power and sensitivity to mismatch of optimum array processors," *J. Acoust. Soc. Amer.*, vol. 54, pp. 771–785, September 1973.
- [79] C. J. Masreliez, "Approximate non-gaussian filtering with linear state and observation relations," *IEEE Transactions on Automatic Control*, vol. 20, pp. 107–110, September 1975.

- [80] H. W. Sorenson and D. L. Alspach, "Recursive bayesian estimation using gaussian sums," *Automatica*, vol. 7, pp. 465–479, September 1971.
- [81] R. Kumaresan and D. W. Tufts, "Estimating the angles of arrival of multiple plane waves," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 19, pp. 134–139, January 1983.
- [82] L. C. Godara, *Smart Antennas*. CRC Press LLC, 2004.
- [83] J. Choi, I. Song, and H. M. Kim, "On estimating direction of arrival when number of signal sources is unknown," *Signal Processing*, vol. 34, pp. 193–205, November 1993.
- [84] M. N. Jin, Y. J. Zhao, and D. H. Li, "Joint source number detection and doa estimation via reversible jump mcmc," *Proc. Microw. Millimeter Wave Technol. Pacific-Asia Workshop Computational Intell. Ind. Appl. (PACIIA)*, vol. 1, pp. 859–863, December 2008.
- [85] C. Qi, Y. Zhang, Y. Han, and X. Chen, "An algorithm on high resolution doa estimation with unknown number of signal sources," *ICMMT 4th International Conference on Microwave and Millimeter Wave Technology*, pp. 227–230, August 18–21 2004.
- [86] P. Green, "Reversible jump markov chain monte carlo computation and bayesian model detection," *Biometrika*, vol. 82, no. 4, pp. 711–732, August 18–21 1995.
- [87] S. Huyun, "Estimation of the eigenvalues of ab for $a > 0$, $b > 0$," *Linear Algebra and Its Applications*, vol. 73, pp. 147–150, September 1986.
- [88] Y. Rao and J. Principe, "An rls type algorithm for generalized eigendecomposition," *Proceedings of the 2001 IEEE Signal Processing Society Workshop on Neural Networks for Signal Processing XI*, pp. 263–272, September 2001.
- [89] S. Attallah and K. Meraim, "A fast algorithm for the generalized symmetric eigenvalue problem," *IEEE Signal Processing Letters*, vol. 15, pp. 797–800, 2008.
- [90] C. Davila, "Efficient, high performance, subspace tracking for time-domain data," *IEEE Transactions on Signal Processing*, vol. 48, no. 12, pp. 3307–3315, December 2000.
- [91] G. H. Xu and T. Kailath, "Fast subspace decomposition," *IEEE Transactions on Signal Processing*, vol. 42, no. 3, pp. 539–551, March 1994.
- [92] V. Reddy, A. Paulraj, and T. Kailath, "Performance analysis of the optimum beamformer in the presence of correlated sources and its behavior under spatial smoothing," *IEEE Transactions on Acoustics, Speech and Signal Processing*, vol. ASSP-35, no. 7, pp. 574–584, July 1987.

- [93] L. C. Godara, "Application of antenna arrays to mobile communications, part ii: Beam-forming and direction-of-arrival considerations," *Proceedings of the IEEE*, vol. 85, no. 8, pp. 1195–1245, August 1997.
- [94] T. Do-Hong and P. Russer, "Signal processing for wideband smart antenna array applications," *IEEE Microwave Magazine*, vol. 5, no. 1, pp. 57–67, March 2007.
- [95] F. W. Vook and J. R. T. Compton, "Bandwidth performance of linear adaptive arrays with tapped delay-line processing," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, no. 3, pp. 901–908, July 1992.
- [96] H.P.Duan, B.P.Ng, C.M.S.See, and J.Fang, "Broadband beamforming using tdl-form iir filters," *IEEE Transactions on Signal Processing*, vol. 55, no. 3, pp. 990–1002, March 2007.
- [97] D.T.M.Slock, "On the convergence behavior of the lms and the normalized lms algorithms," *IEEE Transactions on Signal Processing*, vol. 42, no. 9, pp. 2811–2825, September 1993.
- [98] V. Mathews, "Adaptive polynomial filters," *IEEE Signal Processing Magazine*, pp. 10–26, July 1991.
- [99] L. Griffiths and C. Jim, "An alternative approach to linearly constrained adaptive beamforming," *IEEE Transactions on Antennas Propagation*, vol. AP-30, no. 1, pp. 27–34, January 1982.
- [100] N. Jablon, "Adaptive beamforming with the generalized sidelobe canceller in the presence of array imperfections," *IEEE Transactions on Antennas Propagation*, vol. 34, no. 8, pp. 996–1012, August 1986.
- [101] H. Semira, H. Belkacemi, and SylvieMarcos, "High-resolution source localization algorithm based on the conjugate gradient," *EURASIP Journal on Advances in Signal Processing*, vol. 2007, pp. 1–9.
- [102] L. L. Scharf, E. K. P. Chong, M. D. Zoltowski, J. S. Goldstein, and I. S. Reed, "Subspace expansion and the equivalence of conjugate direction and multistage wiener filters," *IEEE Transactions on Signal Processing*, vol. 56, no. 10, pp. 5013–5019, October 2008.
- [103] B.P.Ng, "A music approach for estimation of directions of arrival of multiple narrowband and broadband sources," *Signal Processing*, vol. 40, pp. 319–323, 1994.
- [104] H. Drucker, C. J. Burges, L. Kaufman, A. Smola, and V. Vapnik, "Support vector regression machines," *Advances in Neural Information Processing Systems*, no. 9, pp. 155–161, 1997.

- [105] M. M. Ram, N. Xu, and C. G. Christodoulou, "Beamforming using support vector machines," *IEEE Antennas and Wireless Propagation Letters*, vol. 4, pp. 439–442, 2005.
- [106] M. Martinez-Ramon, J. L. Rojo-alvarez, G. Camps-Valls, and C. G. Christodoulou, "Kernel antenna array processing," *IEEE Transactions on Antennas and Propagation*, vol. 55, no. 3, pp. 642–650, March 2007.