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A Study of GPS Based Attitude Determination Technique

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School of Electrical & Electronic Engineering

A thesis submitted to Nanyang Technological University
in fulfillment of the requirement for the degree of
Master of Engineering

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Statement of Originality

I hereby certify that the content of this thesis is the result of work done by me and has not been submitted for a higher degree to any other University or Institution.

9-Jun-2005

Date

Dai Lin

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Summary

This thesis mainly deals with GPS-based attitude determination technique. The attitude determination process is carried out mainly in two stages. The first stage determines the correct double difference carrier phase ambiguities for the baseline vectors. The LAMBDA method is applied to solve the integer ambiguities. LAMBDA stands for Least Squares AMBiguity Decorrelation Adjustment. The LAMBDA method employs the least squares estimator to estimate integer ambiguities, using float ambiguities and corresponding variance-covariance matrix as input. It has been found that the ambiguities resolution can be carried out very fast and efficiently by means of the LAMBDA method.

After the correct integer ambiguities have been obtained, the receiver position has to be calculated. Two methods for state estimation are applied for the receiver position in this study. They are the Receding-horizon State Estimation (RHSE) and the Kalman Filter. RHSE uses a finite number of past measurement samples to estimate the state vector. It is a recursive algorithm which can be employed for real-time applications. Kalman Filter is also a recursive method and has been used extensively for dynamic state estimation in real-time. These two methods were compared. Experiments were carried out using different types of off-the-shelf GPS receivers. The attitude parameters were estimated from the two baselines formed by the receivers. Based on our experimental study, the states calculated by RHSE always converge, while those of the Kalman Filter may diverge if the filter parameters were not selected appropriately. Furthermore, the accuracy of Receding-horizon State Estimation is comparable to those obtained by Kalman Filter.

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Chapter 1

Introduction

1.1 Motivation

Global Positioning System, or GPS, was conceived as a ranging system from known positions of satellites in space to unknown positions on land, sea, in air and space. Its original objectives were the instantaneous determination of position and velocity, as well as the precise coordination of time. The GPS satellites orbit in almost circular orbits with an altitude of about 20200 km above the earth and a period of approximately 12 sidereal hours [3]. There are 24 satellites deployed in six evenly spaced planes (A to F, with four satellites per plane) inclined at a 55 degree angle with respect to the equator. The satellite orbits are designed in such a way that at least four satellites are visible at anytime over any location on the earth's surface. Furthermore, there are four spare satellites, which were designated to replace malfunctioned satellites. There are five classes or types of GPS satellites: Block I, Block II, Block IIA, Block IIR, and Block IIF satellite. GPS signals are available at two frequency: $L1=1575.42$ MHz and $L2=1227.60$ MHz. The dual frequencies are essential for eliminating source of the major error: the ionospheric refraction.

GPS receiver technology has experienced rapid developments in recent years in terms of hardware and signal processing techniques. From the very low cost GPS sensors or boards (e.g. Garmin25 GPS receiver [48]) which only possess basic navigation functions to expensive dual frequency geodetic receivers (e.g. Novatel GPS receiver [49]), a wide selection is available to meet the needs of different applications. GPS receivers can be classified into four groups: (1) C/A-code pseudorange, (2) C/A-code carrier phase, (3) P-code carrier phase, and (4) Y-code carrier phase measuring instruments [3]. The GPS system gives users a variety of observations, including pseudoranges derived from the C/A-code and P-code, and carrier phase observations of the L1 and L2 carriers.

In recent years, numerous GPS applications have blossomed and amongst the many innovative applications of GPS is attitude determination. Although GPS was primarily designed for precise position measurement, with three or more GPS sensors properly configured in a plane or space, a multi-GPS sensors system can be used to determine the attitude parameters of the platform. It has the potential of replacing existing sophisticated and expensive attitude sensors such as Inertia Navigation Systems (INS) for air and marine applications [13] [43].

GPS based attitude determination was proposed in the late 1970s [4] [5] [6]. GPS-based attitude determination has potential application for micro-satellite. Using GPS receivers on a Low Earth Orbit (LEO) micro-satellite has the advantage of using a single lightweight, low-cost sensor for a multitude of functions.

The first attitude determination results using real GPS receivers phase measurements were reported by Evan [14] who proposed a method to measure platform attitude angles with a single antenna that periodically rotates in a plane. The first prototype GPS multi-antenna receiver was manufactured in 1988 and tested in a dynamic marine environment [15] [16]. It was an 18 channel receiver consisting of

three antennas with a reported accuracy for roll, pitch and heading determination of a few degrees for antenna separations ranging from 40 to 60cm. In addition, Cannon et al [10] had successfully developed a heading determination system with two OEM GPS sensors, while Lu et al [17] had tested a prototype attitude determination system consisting of three OEM GPS sensors over an antenna separation from 4 to 5 meters.

For accurate attitude determination using GPS from relatively short baselines on a moving platform, C/A code carrier phase have to be used. At present, two types of specially designed receivers are commercially available. One is the Ashtech 3DF receiver [2] and the other is the Trimble TANS Vector attitude receiver [44]. Both receivers are single frequency C/A code receivers which tracks C/A code and provides C/A code pseudorange and L1 carrier phase measurement. Both receivers have 24 dedicated tracking channels which are divided into four banks, with each bank being assigned to an antenna. A unique feature of these receivers is that a single oscillator is used to control all the tracking channels of the multiple antennas. Theoretically, the biases associated with the oscillator, i.e. the receiver clock, will cancel out when single difference observations between two antennas are formed. Using single difference carrier phase measurements has the advantage that it has a lower noise than its double differences counterpart. Unfortunately, as seen from experiments [18], significant residual receiver clock biases still exist between different antenna banks. Although single difference can be used, double difference is usually employed in attitude determination applications. According to [12], the limiting factor for high accuracy attitude determination is carrier phase multipath rather than receiver noise .

At least two types of GPS based attitude sensors are commercially available: the LABEN-Tensor and the SSTL SGR-20 Space GPS Receiver. LABEN-Tensor is a

kind of space-qualified, GPS attitude and orbit determination receiver. It provides the unique feature to offer 13 output states of navigation and attitude information, for guidance and control of space vehicles. GPS Tensor is an ideal replacement for conventional attitude determination equipment, such as Earth sensors, Sun sensors, and gyros. It delivers position, velocity, time and filtered orbit determination using a continuous-tracking, 9-channels L1 C/A code receiver. When configured with four antennas, GPS Tensor provides platform roll, pitch, and yaw angles and rates. The GPS Tensor has been selected as navigation receiver on board several satellites already launched or under development.

STL SGR-20 Space GPS Receiver is a spacecraft orbit determination sub-system designed for small and large LEO satellite applications. The SGR-20 provides GPS standard time, position and velocity in a compact qualified unit, proven in orbit on numerous missions. The SGR-20 has 24 channels enabling a very fast time to first fix, and supports multiple antennas to improve visibility under changing vehicle orientations and attitude determination.

There are several receiver structures to measure differential phase. The simplest implementation of an attitude receiver, is the combination of several survey-quality receivers together, each running independently, which is called integrated receiver. Lu's experiment [7] has shown the accuracy of this configuration is close to a dedicated attitude receiver. But this configuration produces a new error due to the un-synchronized measurements. Furthermore, it needs one more satellite to perform double difference. This kind of receiver also can be implemented by standard (non-survey-class) GPS receiver for the purpose of reducing cost.

A more integrated architecture, called parallel structure, is used in splitting local oscillator as it is distributed to the dedicated down-converter for each antenna. The structure was implemented in Trimble Vector Receiver [8]. A conceptual lay-

Figure 1.1: Vector Receiver Block Diagram[47]

out of the common local oscillator architectures of Vector Receiver is shown in Figure (1.1). The receiver features a common local oscillator for the RF sections and a correlator specifically designed for differential carrier phase measurement. Three down-converted RF signals are set through a band pass filter, converted from analog to digital, and distributed to 20 GPS channels where the carrier phase and code phase measurements are conducted. These GPS channels are organized into two banks of 5 differential phase measurement units, which consist of 2 correlation channels. The master and slave antennae share same the CNCO which is adjusted to tracking the master signal. The decoder can detect the phase differential between slave signal and CNCO. The advantage of this configuration is that the common mode oscillator error is excluded out, and the measurement is much less sensitive to the variety of environment. But it needs a special designed correlator. The number of correlator channels determines the number of baselines. This limits the flexibility in the baseline configuration.

An alternative implementation scheme called multiplexing antenna receiver is pro-

posed by Cohen [9]. This kind of receiver is based on a standard survey-quality positioning GPS receiver. A RF switch is used to time-multiplex the master and slave antenna signals shown as Figure (1.2). When the master antenna pass through the signal path, the PLL runs closed loop, and tracks the master carrier. The output of the correctors at this time is the carrier phase tracking error. When the signal of any slave antenna pass through the hardware channel, the PLL runs open loop. The output of the correlator at this time corresponds to differential carrier phase between a slave antenna and the master antenna. Cohen [9] claimed that the performance-to-cost ratio is improved substantially by using this multiplexing architecture. The overall size, weight and power are effectively reduced. However, the disadvantage of this architecture can not be ignored. First, the carrier tracking do not work continuously. Only when master antenna is connected to the receiver, the loops are closed. For example, if there are 4 antennae, the time when the tracking loops are closed is less than what a typical receiver tracking loops achieves. This reduces robustness of the carrier tracking. Second, the carrier phases coming from different antennae are measured in different time, which produces a synchronization error. In order to reduce the error, in [9], an estimator of differential phase is proposed. It increases the burden of computation and needs a high speed RF switch.

Figure 1.2: Antenna multiplexing receiver diagram[47]

There is no doubt that GPS sensor technology will continue to improve and the cost

1.1 Motivation

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of GPS sensors will continue to decrease. Thus, it would be desirable if accurate platform attitude determination could also be obtained by using three or four non-dedicated, off-the-shelf low-cost GPS sensors with high quality carrier phase output. It has the advantages of being relatively inexpensive and accurate. Sub-centimetre level relative positioning between multiple antennas is the key to precise attitude determination using GPS. To achieve this level of accuracy, carrier phase observations with correctly resolved integer cycle ambiguities have to be used.

The carrier phase measurement equation is

$$\phi_r^s(t) = \varrho_r^s(t) + \lambda N_r^s + c(\delta^s(t) - \delta_r(t)) - I_r^s + T_r^s + \varepsilon_r^s \quad (1.1)$$

where $\phi_r^s(t)$ is the carrier phase measurement (in meter); $\varrho_r^s(t)$ represents the distance between the satellite 's' and the receiver 'r'; N_r^s is the integer ambiguity, the unknown integer number of carrier cycles between satellite and receiver; $\delta^s(t) - \delta_r(t)$ is the satellite clock delay minus receiver clock delay; c is the speed of light; λ is the wavelength; I_r^s is the delay of the carrier due to the ionosphere and T_r^s is the delay of the carrier due to the troposphere.

In this equation, the carrier phase integer ambiguity N_r^s need to be solved. With double differential carrier phase measurements, an effective method, the LAMBDA method [30], is applied to solve the integer ambiguity N . The LAMBDA method has been developed for surveying and there are limitations, such as the number of epochs used, when applying to attitude determination of micro-satellites. After that, sub-centimetre relative positioning can be dealt with using Kalman Filter or Receding-horizon State Estimation.

The main steps of processing GPS data to estimate attitude parameters are shown in Figure (1.3). First, GPS signal raw data is collected using GPS receivers. Sec-

ond, double difference equations of carrier phase measurement are formed, in order to eliminate noises and errors. Third, the integer ambiguities are solved with the LAMBDA method. Fourth, two baseline vector can be calculated after obtaining three receiver positions. Last, with two baselines, the attitude of the baselines are computed.

Figure 1.3: System Steps

1.2 Objective

The main objective of this research is to compare the performance of the two state estimation methods: the Kalman Filter, and the Receding-horizon State Estimation (RHSE) when applied to attitude determination using off-the-shelf GPS receivers. The merits and demerits of the two methods are compared. This can help to select appropriate method in different systems and situations, so that a better solution can be obtained.

1.3 Major Contribution of this Research

In the course of this work, a real-time GPS based attitude determination system using off-the-shelf GPS receivers has been developed to assess the system performance that is achievable. The system can directly estimate the attitude parameters using carrier phase and pseudorange observations from the GPS receivers. Efficient and reliable real-time carrier phase ambiguity resolution techniques were reviewed and the LAMBDA method for integer ambiguity resolution is adopted in this work.

In this project, the main task is to compare the Kalman Filter and the Receding-horizon State Estimation. Although Kalman Filter is applied frequently, divergence may occur when the parameters of the Kalman Filter are not selected properly. The Kalman Filter's power is in its ability to take past measurements and aid the current epoch. It does this by propagating past measurements to the present. If the model is inaccurate, the solution will be suboptimal even with good measurements. Thus it is imperative to do a lot of simulation experiments to generate the dynamics model and the input process noise for the Kalman Filter, so that the dynamics model and process noises match reality [24]. One of the main advantages of a Kalman Filter is that the Kalman Filter has the history included without the need for storing data for the history (as it is necessary for the Batch Least Square).

Besides Kalman Filter, Receding-horizon State Estimation (RHSE) is also applied to this attitude determination application. RHSE is another recursive method for state estimation, and it performs well with this application. It is shown that RHSE can also reach the same accuracy level of the Kalman Filter.

1.4 Thesis Outline

Chapter 1 introduces the motivation and objective about attitude determination in this research. The main contribution is described as well.

In Chapter 2, the coordinate transformations and parameterizations are discussed. The Earth-fixed, local level and body frames are defined. The rotation matrix between the frames is derived. The technique of attitude determination using GPS is also described. The direct method, which uses three antennas to determine the attitude parameters, is reviewed. In addition, the single and double difference GPS observation models are examined along with the interferometric models. The linearization equation of these models are discussed, as well as methods for integer ambiguity resolution. The Kalman Filter and Receding-horizon State Estimation techniques are outlined.

In Chapter 3, the experimental setups, results and analysis are given. Both the static and the dynamic cases are studied. The static test was conducted with Trimble and Leica antennas and receivers. The dynamic tests were conducted with the Ashtech receivers using a GPS simulator. The attitude parameters calculated by the Kalman Filter and Receding-horizon State Estimation method are compared.

In the last chapter, conclusion and future work are discussed.

Appendix A contains a brief description of the Welnav GPS simulator GS-1010.

Chapter 2

GPS-based Attitude Determination

2.1 Attitude Determination Using GPS

GPS based systems can derive attitude solutions by using an array of 3 or more antennas in a precisely known layout. To generate accurate solution, a process known as integer ambiguity resolution must be performed before using the highly accurate carrier phase observations. The attitude obtained could form the basis for controlling the attitude of micro-satellite. There are several methods to determine the attitude using GPS data, e.g. the direct method [18] and least square method [3]. Another technique, SAM attitude method [42], which embeds the attitude (the angles of pitch, roll and yaw) into the equations directly has also been proposed.

In order to determine the attitude, several coordinate systems as shown in Figure (2.1) need to be defined and their relationship determined.

World Geodetic System 1984 (WGS84) [18] [50] is an earth-fixed cartesian coor-

Figure 2.1: the Geocentric System and the Local Level System [18]

dinate system which all GPS positioning information is referred. Suppose that a baseline vector from the master antenna to the slave antenna is determined by GPS in the WGS84 system. In order to use this baseline vector for attitude determination, it needs to be transformed into the local level coordinate system.

The local level system is used as the reference to measure the attitude of a platform. It is a topocentric system defined on the best-fitting ellipsoid (e.g. WGS84) and rotates with the earth. For attitude determination, the origin of local level coordinate system is at the master antenna, with the X^L -axis is towards ellipsoidal east, Y^L -axis is towards ellipsoidal north, and Z^L -axis points upwards along the ellipsoidal normal and forming a right hand system with X^L -axis and Y^L -axis.

With the master antenna at the origin of the local level system, the cartesian coordinate of the slave antenna can be converted to the local level system using

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the relation

$$\begin{bmatrix} y \\ x \\ z \end{bmatrix} = T \begin{bmatrix} X_s - X_m \\ Y_s - Y_m \\ Z_s - Z_m \end{bmatrix} \quad (2.1)$$

where the rotation matrix T is given by

$$T = \begin{bmatrix} -\sin \varphi \cos \lambda & -\sin \varphi \sin \lambda & \cos \varphi \\ -\sin \lambda & \cos \lambda & 0 \\ \cos \varphi \cos \lambda & \cos \varphi \sin \lambda & \sin \varphi \end{bmatrix} \quad (2.2)$$

(X_m, Y_m, Z_m) and (X_s, Y_s, Z_s) denote the WGS84 coordinates of the master and slave antenna respectively. The latitude φ and longitude λ of the local level coordinate system can be found, given the Cartesian coordinates of the master GPS receiver, by the relation

$$\begin{bmatrix} X_m \\ Y_m \\ Z_m \end{bmatrix} = \begin{bmatrix} (N + h) \cos \varphi \cos \lambda \\ (N + h) \cos \varphi \sin \lambda \\ (\frac{Nb^2}{a^2} + h) \sin \varphi \end{bmatrix} \quad (2.3)$$

where,

$a = 6378137.0m$, (the semi-major axis of the ellipsoid);

$b = 6356752.3142m$, (the semi-minor axis of the ellipsoid);

$N = \frac{a}{(1 - e^2 \sin^2 \varphi)^{\frac{1}{2}}}$, (the prime vertical radius of curvature at point P);

$e = \sqrt{\frac{a^2 - b^2}{a^2}}$, (the eccentricity of the ellipsoid); and

h is geodetic height of the master GPS receiver (Figure 2.1).

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Based on equation (2.3), the ellipsoidal coordinates φ , λ , h can be computed as follows:

Firstly, given X_m and Y_m , the longitude λ is given by,

$$\tan \lambda = \frac{Y_m}{X_m} \quad (2.4)$$

From equation (2.3), we see that

$$\sqrt{X_m^2 + Y_m^2} = (N + h) \cos \varphi \quad (2.5)$$

Equation (2.5) can be rearranged as

$$h = \frac{\sqrt{X_m^2 + Y_m^2}}{\cos \varphi} - N \quad (2.6)$$

so that the ellipsoidal height appears explicitly.

Introducing the term eccentricity e ,

$$e^2 = \frac{a^2 - b^2}{a^2} \quad (2.7)$$

the Z coordinate in equation (2.3) can be written as,

$$Z_m = (N + h - e^2 N) \sin \varphi = (N + h) \left(1 - e^2 \frac{N}{N + h}\right) \sin \varphi \quad (2.8)$$

From equation (2.5), and equation (2.8), we have

$$\tan \varphi = \frac{Z_m}{\sqrt{X_m^2 + Y_m^2}} \left(1 - e^2 \frac{N}{N + h}\right)^{-1} \quad (2.9)$$

Equations (2.6) and (2.9) can be solved iteratively for h and φ .

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So, the quantities λ , φ and h can be computed using the following iterative method

1. Compute $\lambda = \arctan \frac{Y_m}{X_m}$.
2. Compute an approximate value $\varphi_{(0)}$ from $\tan \varphi_{(0)} = \frac{Z_m}{\sqrt{X_m^2 + Y_m^2}} (1 - e^2)^{-1}$.
3. Compute an approximate value $N_{(0)}$ from $N_{(0)} = \frac{a^2}{\sqrt{a^2 \cos^2 \varphi_{(0)} + b^2 \sin^2 \varphi_{(0)}}}$.
4. Compute the ellipsoidal height by $h = \frac{\sqrt{X_m^2 + Y_m^2}}{\cos \varphi_{(0)}} - N_{(0)}$.
5. Compute an improved value for the latitude by $\tan \varphi = \frac{Z_m}{\sqrt{X_m^2 + Y_m^2}} (1 - e^2 \frac{N_{(0)}}{N_{(0)} + h})^{-1}$.
6. Check for another iteration step: if $\varphi = \varphi_{(0)}$ then the iteration is completed otherwise set $\varphi_{(0)} = \varphi$ and continue with step 3.

Figure 2.2: Antenna Body Frame Coordinate System [18]

To define the platform attitude precisely, we now need to define the body frame coordinate system. With three GPS receivers set up on a platform, two baseline vectors, one from the master receiver to slave receiver 1 and the other from master receiver to slave receiver 2 can be formed. As shown in Figure (2.2), the body frame coordinate system has the origin at the master antenna with one baseline as the y-axis.

When three antennas are available, the direct method can be used to compute

the attitude of the body frame directly. Otherwise, the least square method can be used if more than three antennas are used in the body frame system. In this thesis, we assume only three antennas and hence the direct method is adopted to solve for the attitude parameters.

2.2 Direct Method

From Figure (2.2), L_{12} is the distance between the master antenna and the slave antenna 1, and can be computed as, $L_{12} = \sqrt{(X_{s1} - X_m)^2 + (Y_{s1} - Y_m)^2 + (Z_{s1} - Z_m)^2}$. Similarly, the distance between the master antenna and the slave antenna 2, can be computed as $L_{13} = \sqrt{(X_{s2} - X_m)^2 + (Y_{s2} - Y_m)^2 + (Z_{s2} - Z_m)^2}$, and the distance between the slave antenna 1 and the slave antenna 2, can be computed as $L_{23} = \sqrt{(X_{s1} - X_{s2})^2 + (Y_{s1} - Y_{s2})^2 + (Z_{s1} - Z_{s2})^2}$.

In the body frame coordinates: the master antenna is at $b_m = [0, 0, 0]^T$, the slave antenna 1 is at $b_1 = [0, L_{12}, 0]^T$, and the slave antenna 2 is at $b_2 = [L_{13} \sin(\alpha), L_{13} \cos(\alpha), 0]^T$, where the α is the angle between two baselines which can be computed as

$$\alpha = \arccos \frac{L_{12}^2 + L_{13}^2 - L_{23}^2}{2L_{12}L_{13}} \quad (2.10)$$

Suppose the corresponding GPS-derived local level coordinates for the two slave antennas are $u_1 = [x_1, y_1, z_1]^T$, and $u_2 = [x_2, y_2, z_2]^T$. The local level frame coordinates (u_i) for each slave GPS antenna can be rotated into the corresponding body frame coordinates (b_i) by using the rotation,

$$b_i = R_{312}(y, p, r)u_i \quad (2.11)$$

through the attitude matrix $R_{312}(y, p, r) = R_2(r)R_1(p)R_3(y)$ where y denotes yaw,

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p denotes pitch and r denotes roll;

$$R_1(p) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(p) & \sin(p) \\ 0 & -\sin(p) & \cos(p) \end{bmatrix} \quad (2.12)$$

is the rotation matrix about the x-axis, and

$$R_2(r) = \begin{bmatrix} \cos(r) & 0 & -\sin(r) \\ 0 & 1 & 0 \\ \sin(r) & 0 & \cos(r) \end{bmatrix} \quad (2.13)$$

the rotation matrix about the y-axis, and

$$R_3(y) = \begin{bmatrix} \cos(y) & \sin(y) & 0 \\ -\sin(y) & \cos(y) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2.14)$$

the rotation matrix about the z-axis. and

$$R_{312}(y, p, r) = \begin{bmatrix} \cos r \cos y - \sin r \sin p \sin y & \cos r \sin y + \sin r \sin p \cos y & -\sin r \cos p \\ -\cos p \sin y & \cos p \cos y & \sin p \\ \sin r \cos y + \cos r \sin p \sin y & \sin r \sin y - \cos r \sin p \cos y & \cos r \cos p \end{bmatrix}.$$

Substituting slave antenna-1 coordinates b_1 and u_1 into equation (2.11) and using the orthogonality of attitude matrix $R_{312}(y, p, r)$, yaw and pitch can be obtained as,

$$\tan y = \frac{x_1}{y_1} \quad (2.15)$$

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$$\tan p = -\frac{z_1}{\sqrt{x_1^2 + y_1^2}} \quad (2.16)$$

Thus the baseline between master antenna, slave antenna 1 actually determines the yaw and pitch of the antenna platform.

After the yaw and pitch are obtained, the local-level coordinates $u_2 = [x_2, y_2, z_2]^T$ of slave antenna 2 is first rotated about the local level z-axis by an amount yaw, and then rotated again about the rotated local level x-axis by an amount pitch. The resultant coordinates of the slave antenna 3 after these two rotations are denoted by $[x_2'', y_2'', z_2'']^T$.

$$\begin{bmatrix} x_2'' \\ y_2'' \\ z_2'' \end{bmatrix} = R_1 R_3 \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \quad (2.17)$$

A third rotation, $R_2(r)$, rotates $[x_2'', y_2'', z_2'']$ to its body frame coordinates $b_2 = [L_{13} \sin(\alpha), L_{13} \cos(\alpha), 0]^T$, namely

$$\begin{bmatrix} L_{13} \sin(\alpha) \\ L_{13} \cos(\alpha) \\ 0 \end{bmatrix} = R_2 \begin{bmatrix} x_2'' \\ y_2'' \\ z_2'' \end{bmatrix} \quad (2.18)$$

According to the third row of equation (2.18), roll can be obtained as

$$-\tan r = \frac{z_2''}{x_2''} \quad (2.19)$$

Equations (2.15), (2.16) and (2.19) are the formulas to directly compute yaw, pitch and roll.

When it comes to assessing the accuracy of yaw, pitch, roll, the standard deviation

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of yaw, pitch and roll are given by,

$$\sigma_y = \sqrt{\cos^2(y)\sigma_{\lambda 1}^2 + \sin^2(y)\sigma_{\varphi 1}^2 / L_{12} \cos(p)} \quad (2.20)$$

$$\sigma_p = \sqrt{\cos^2(p)\sigma_{h 1}^2 + \sin^2(p)\cos^2(y)\sigma_{\lambda 1}^2 + \sin^2(p)\sin^2(y)\sigma_{\varphi 1}^2 / L_{12}} \quad (2.21)$$

$$\sigma_r = \sqrt{\cos^2(r)\sigma_{z'2}^2 + \sin^2(y)\sigma_{x'2}^2 / L_{13} \cos(\alpha)}, \quad (2.22)$$

while

$$\sigma_{x'2}^2 = \cos^2(y)\sigma_{\lambda 2}^2 + \sin^2(y)\sigma_{\varphi 2}^2$$

$$\sigma_{z'2}^2 = \sin^2(p)\sin^2(y)\sigma_{\lambda 2}^2 + \sin^2(p)\cos^2(y)\sigma_{\varphi 2}^2 + \cos^2(p)\sigma_{h 2}^2$$

$\sigma_{\lambda 2}$, $\sigma_{\varphi 2}$, $\sigma_{h 2}$ equal the standard deviations of longitude, latitude and height of slave antenna 2 in the local level. Similarly, $\sigma_{\lambda 1}$, $\sigma_{\varphi 1}$, $\sigma_{h 1}$ equal those of slave antenna 1 in the local level system.

Least Square Approach

When there are more than two baselines, then the least square approach can be used to determine the attitude. The least square approach can be used. Assuming that there are n independent baselines, equation (2.11) can be rewritten in the new form:

$$A_b = R_{312}(y, p, r)A_u \quad (2.23)$$

A_b is a $3 \times n$ matrix with the baseline vectors in the body frame as column vectors. A_u is the matrix containing the baseline vectors in the local level frame. Then R_{312}

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is given by,

$$R_{312} = A_b A_u^T (A_u A_u^T)^{-1} \quad (2.24)$$

Denoting the elements of the matrix R_{312} by R_{ij} , the attitude parameters are obtained by

$$\begin{aligned} \tan r &= -\frac{R_{13}}{R_{33}} \\ \tan p &= \frac{R_{23}}{\sqrt{R_{21}^2 + R_{22}^2}} \\ \tan y &= -\frac{R_{21}}{R_{22}} \end{aligned} \quad (2.25)$$

which may be verified by inspecting the matrix

$$R_{312} = \begin{bmatrix} \cos r \cos y - \sin r \sin p \sin y & \cos r \sin y + \sin r \sin p \cos y & -\sin r \cos p \\ -\cos p \sin y & \cos p \cos y & \sin p \\ \sin r \cos y + \cos r \sin p \sin y & \sin r \sin y - \cos r \sin p \cos y & \cos r \cos p \end{bmatrix}.$$

Because the least square technique computes the best estimate using n independent equations, it is less sensitive to noisy measurements. But the computation process is not as easy as the direct method when there are only three antennas in the set up.

Besides the least squares method, there are also TRIAD [25], QUEST [46][27] methods can also be used for attitude determination.

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From the previous sections, we know that after obtaining the positions of the three GPS receivers, the attitude of the platform can be calculated. In order to get

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accurate positions, the carrier phase observation has to be used. From equation (1.1), the carrier phase observation contains the integer ambiguity which needs to be resolved. To solve the integer ambiguity accurately, the double difference observation model should be formed first.

2.3.1 Single-Difference

The observation model for pseudorange measurement, $\rho_r^s(t)$, between satellite 's' and receiver 'r' is

$$\rho_r^s(t) = \varrho_r^s(t) + c(\delta^s(t) - \delta_r(t)) + I_r^s + T_r^s + \varepsilon_r^s \quad (2.26)$$

where

$\varrho_r^s(t)$ represents the actual distance between the satellite 's' and the receiver 'r',

$\delta^s(t) - \delta_r(t)$ is the satellite clock delay minus receiver clock delay,

c is the speed of light,

I_r^s is the delay of the carrier due to the ionosphere, and

T_r^s is the delay of the carrier due to the troposphere,

ε_r^s is the measurement noise.

Compared with pseudorange measurement, the main advantages of carrier phase are the low measurement noise level and the low multipath effects. So implementing the carrier phase observations is for the accurate platform attitude determination. The carrier phase measurement equation is

$$\phi_r^s(t) = \varrho_r^s(t) + \lambda N_r^s + c(\delta^s(t) - \delta_r(t)) - I_r^s + T_r^s + \varepsilon_r^s \quad (2.27)$$

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$\phi_r^s(t)$ is the carrier phase measurement (in meters); N is the unknown integer number of carrier cycles between satellite and receiver, known as the integer ambiguity; λ is the wavelength.

Raw pseudorange and carrier phase measurements given by equation (2.26) and equation (2.27) are affected by a number of errors. Differencing the measurements collected simultaneously by two receivers with respect to the same satellite will eliminate the satellite clock error and greatly reduce the atmospheric errors. This process is called single difference between receivers. If two single difference measurements related to two different satellites are further differenced, the receiver clock error is further eliminated and the resultant observation is called the double difference.

Assuming two receivers A and B, and two satellites j and k, two measurement equations can be formed:

$$\phi_A^j(t) = \varrho_A^j(t) + \lambda N_A^j - c(\delta_A(t) - \delta^j(t)) - (I_A^j - T_A^j - \varepsilon_A^j) \quad (2.28)$$

$$\phi_B^j(t) = \varrho_B^j(t) + \lambda N_B^j - c(\delta_B(t) + \delta^j(t)) - (I_B^j - T_B^j - \varepsilon_B^j) \quad (2.29)$$

The time independent phase ambiguities N_A^j , N_B^j are integers.

From equations (2.28) and (2.29), we obtain

$$\phi_{AB}^j(t) = \varrho_{AB}^j(t) + \lambda N_{AB}^j - c\delta_{AB}(t) \quad (2.30)$$

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where

$$\phi_{AB}^j(t) = \phi_B^j(t) - \phi_A^j(t)$$

$$\delta_{AB}(t) = \delta_B(t) - \delta_A(t)$$

$$N_{AB}^j = N_B^j - N_A^j$$

$$\varrho_{AB}^j(t) = \varrho_B^j(t) - \varrho_A^j(t)$$

Equation (2.30) is referred to as the single-difference equation. The single-difference equation has an additional unknown parameter due to the receiver clock offset which prevents an effective separation of the integer ambiguities from the clock offset. Using double difference will eliminate the clock offset term and the isolation of the integer ambiguities becomes possible.

2.3.2 Double-difference

To obtain the double-difference equation, two single-difference equations are subtracted,

$$\phi_{AB}^k(t) - \phi_{AB}^j(t) = [\varrho_{AB}^k(t) - \varrho_{AB}^j(t)] + \lambda N_{AB}^k - \lambda N_{AB}^j \quad (2.31)$$

which can be written compactly as

$$\phi_{AB}^{jk}(t) = \varrho_{AB}^{jk}(t) + \lambda N_{AB}^{jk} \quad (2.32)$$

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where

$$\begin{aligned}\phi_{AB}^{jk}(t) &= \phi_B^k(t) - \phi_B^j(t) - \phi_A^k(t) + \phi_A^j(t) \\ \varrho_{AB}^{jk}(t) &= \varrho_B^k(t) - \varrho_B^j(t) - \varrho_A^k(t) + \varrho_A^j(t) \\ N_{AB}^{jk} &= N_B^k - N_B^j - N_A^k + N_A^j\end{aligned}$$

2.3.3 Linearization

The distance between satellite j to receiver A can be expressed in terms of the coordinates of the satellite (X^j, Y^j, Z^j) , and the GPS receiver (X_A, Y_A, Z_A) as

$$\varrho_A^j = |\mathbf{r}_j - \mathbf{r}_A| = \sqrt{(X_j - X_A)^2 + (Y_j - Y_A)^2 + (Z_j - Z_A)^2} \quad (2.33)$$

where

$\mathbf{r}_j = (X_j, Y_j, Z_j)$ is the coordinates of satellite j , and

$\mathbf{r}_A = (X_A, Y_A, Z_A)$ is the coordinates of receiver A

Suppose the approximate coordinates of receiver A is $\mathbf{r}_{A0} = (X_{A0}, Y_{A0}, Z_{A0})$, then we can express the approximate distance from satellite j to receiver A as

$$\varrho_{A0}^j = |\mathbf{r}_j - \mathbf{r}_{A0}| = \sqrt{(X_j - X_{A0})^2 + (Y_j - Y_{A0})^2 + (Z_j - Z_{A0})^2}$$

Then by Taylor series expansion of equation (2.33) up to first order, we have

$$\varrho_A^j = \varrho_{A0}^j + \left(\frac{\partial \varrho_A^j}{\partial \mathbf{r}_A} \right)^T \bigg|_{\mathbf{r}_{A0}} \cdot \delta \mathbf{r}_A$$

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where

$$\left(\frac{\partial \varrho_A^j}{\partial \mathbf{r}_A}\right)^T|_{\mathbf{r}_{A0}} = \left[\frac{\partial \varrho_A^j}{\partial X_A} \quad \frac{\partial \varrho_A^j}{\partial Y_A} \quad \frac{\partial \varrho_A^j}{\partial Z_A} \right]|_{\mathbf{r}_{A0}}$$

and

$$\frac{\partial \varrho_A^j}{\partial X_A}|_{\mathbf{r}_{A0}} = \frac{1}{2} \frac{1}{\varrho_A^j}|_{A0} \cdot 2(X_j - X_A) = \frac{X_j - X_{A0}}{\varrho_{A0}^j}$$

Similarly,

$$\frac{\partial \varrho_A^j}{\partial Y_A}|_{\mathbf{r}_{A0}} = \frac{Y_j - Y_{A0}}{\varrho_{A0}^j} \quad \text{and} \quad \frac{\partial \varrho_A^j}{\partial Z_A}|_{\mathbf{r}_{A0}} = \frac{Z_j - Z_{A0}}{\varrho_{A0}^j}$$

Thus,

$$\begin{aligned} \varrho_{AB}^{jk} &= (\varrho_B^k - \varrho_B^j) - (\varrho_A^k - \varrho_A^j) \\ &= (\varrho_{B0}^k - \varrho_{B0}^j) - (\varrho_{A0}^k - \varrho_{A0}^j) \\ &\quad + \left(\left(\frac{\partial \varrho_B^k}{\partial \mathbf{r}_B} \right)^T|_{\mathbf{r}_{B0}} \cdot \delta \mathbf{r}_B - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B} \right)^T|_{\mathbf{r}_{B0}} \cdot \delta \mathbf{r}_B \right) - \left(\left(\frac{\partial \varrho_A^k}{\partial \mathbf{r}_A} \right)^T|_{\mathbf{r}_{A0}} \cdot \delta \mathbf{r}_A - \left(\frac{\partial \varrho_A^j}{\partial \mathbf{r}_A} \right)^T|_{\mathbf{r}_{A0}} \cdot \delta \mathbf{r}_A \right) \end{aligned}$$

So equation (2.32) becomes

$$\begin{aligned} &\varphi_{AB}^{jk} - (\varrho_{B0}^k - \varrho_{B0}^j) + (\varrho_{A0}^k - \varrho_{A0}^j) \\ &= \left(\left(\frac{\partial \varrho_B^k}{\partial \mathbf{r}_B} \right)^T|_{\mathbf{r}_{B0}} \cdot \delta \mathbf{r}_B - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B} \right)^T|_{\mathbf{r}_{B0}} \cdot \delta \mathbf{r}_B \right) - \\ &\quad \left(\left(\frac{\partial \varrho_A^k}{\partial \mathbf{r}_A} \right)^T|_{\mathbf{r}_{A0}} \cdot \delta \mathbf{r}_A - \left(\frac{\partial \varrho_A^j}{\partial \mathbf{r}_A} \right)^T|_{\mathbf{r}_{A0}} \cdot \delta \mathbf{r}_A \right) + \lambda N_{AB}^{jk} \end{aligned} \quad (2.34)$$

For relative positioning, we may assume $\delta \mathbf{r}_A = 0$. Then equation (2.34) becomes

$$\varrho_{AB}^{jk} = \left[\left(\frac{\partial \varrho_B^k}{\partial \mathbf{r}_B} \right)^T|_{\mathbf{r}_{B0}} - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B} \right)^T|_{\mathbf{r}_{B0}} \right] \cdot \delta \mathbf{r}_B + \lambda N_{AB}^{jk} \quad (2.35)$$

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where

$$\ell_{AB}^{jk} = \phi_{AB}^{jk} - (\varrho_{B0}^k - \varrho_{B0}^j) + (\varrho_{A0}^k - \varrho_{A0}^j)$$

contains data provided by the GPS receivers.

In a LEO, usually 6 to 8 satellites are visible if the GPS antenna array is place in an optimal way. For simplicity, assume that at any given epoch, we can observe 4 satellites j, k, l, m , then from equation (2.35), we can get the following set of double difference equations,

$$\begin{bmatrix} \ell_{AB}^{jk} \\ \ell_{AB}^{jl} \\ \ell_{AB}^{jm} \end{bmatrix} = \begin{bmatrix} \left(\frac{\partial \varrho_B^k}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} \\ \left(\frac{\partial \varrho_B^l}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} \\ \left(\frac{\partial \varrho_B^m}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} \end{bmatrix} \cdot \delta \mathbf{r}_B + \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \begin{bmatrix} N_{AB}^{jk} \\ N_{AB}^{jl} \\ N_{AB}^{jm} \end{bmatrix}$$

Or in vector notation,

$$\underline{\ell} = c\underline{x} + \lambda \underline{I} \underline{N} \quad (2.36)$$

where

$$\begin{aligned} \underline{\ell} &= \begin{bmatrix} \ell_{AB}^{jk} & \ell_{AB}^{jl} & \ell_{AB}^{jm} \end{bmatrix}^T \\ \underline{x} &= \delta \mathbf{r}_B = \begin{bmatrix} \delta X_B & \delta Y_B & \delta Z_B \end{bmatrix}^T \\ \underline{N} &= \begin{bmatrix} N_{AB}^{jk} & N_{AB}^{jl} & N_{AB}^{jm} \end{bmatrix}^T \\ c &= \begin{bmatrix} \left(\frac{\partial \varrho_B^k}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} \\ \left(\frac{\partial \varrho_B^l}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} \\ \left(\frac{\partial \varrho_B^m}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} - \left(\frac{\partial \varrho_B^j}{\partial \mathbf{r}_B}\right)^T |_{\mathbf{r}_{B0}} \end{bmatrix} \end{aligned}$$

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In equation (2.36), for static case where both receiver A and B are fixed, if there are p satellites in view, with one baseline configuration then we can form $(p-1)$ double difference equations with $3+p-1$ unknowns. In addition, if we observe for q epoch then we will have $q(p-1)$ equations and $(p-1)+3$ unknowns. In order that equation (2.36) has a unique solution, the number of equation must at least equal to the number of unknowns, i.e. $q(p-1) \geq (p-1)+3$. Thus, given that p satellites are in view, we will need $q \geq \frac{p+2}{p-1}$ and $p \geq 2$ epoch to resolve the integer ambiguity.

On the other hand, for kinematic case, receiver A of the baseline vector remains fixed, but the receiver B moves, and its position is to be determined as well. In this case, with p satellites and q epoch, $q(p-1)$ equations can be formed, and the total number of unknown is $(p-1)+3q$. Thus we need $q \geq \frac{p-1}{p-4}$, and $p \geq 5$, in order that a unique solution can be determined.

Furthermore, PDOP (Position Dilution of Precision) can be calculated as,

$$PDOP = \sqrt{V_x + V_y + V_z} \quad (2.37)$$

where V_x , V_y and V_z denote the diagonal elements of the cofactor matrix

$$Q_X = (c^T c)^{-1} \quad (2.38)$$

where c is determined from equation (2.36).

This section establishes the background knowledge for forming the linearized equation for state estimation. There are two sets of unknowns: the integer ambiguity $(N_{AB}^{jk}, N_{AB}^{jl}, N_{AB}^{jm})$ and the receiver coordinates $\delta \mathbf{r}_B = (\delta X_B, \delta Y_B, \delta Z_B)$. How to solve the equation (2.36) will be described next.

2.4 GPS Carrier Phase Ambiguity Resolution

To achieve a highly accurate result, the carrier phase observables have to be used. However, the integer ambiguity is the most important parameter in the carrier phase observables. There are three main steps (see Figure (2.3)) for the ambiguity solution: a priori approximate solution, ambiguity search strategy, and determine correct ambiguities.

Figure 2.3: General Concept of Ambiguity Search

2.4.1 General Concept

The GPS double difference (DD) carrier phase measurements (see Equation 2.32) are ambiguous by an unknown integer number of cycles. The baseline solution can be strengthened by using the priori knowledge of the integer nature of the ambiguities. This is of particular relevance to applications where use is made of

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short observation time spans. Up to now, there is no deterministic method existing for the computation of the most likely integer ambiguity estimates.

Many techniques have been formed for on-the-fly carrier phase ambiguity resolution. Among these methods are the ambiguity function technique [41] [28], the narrow-lane and extrawide-lane technique, the Hatch's least-squares ambiguity search method [20] [21] and the fast ambiguity search filter technique [17]. Although the ambiguity function technique is not sensitive to cycle slips, it needs extensive computation time (1 to 2 minutes), even for a 1-metre search cube. In this case, the ambiguity function method is not fit for use in GPS multi-antenna systems for micro-satellite attitude determination. The extrawide-lane technique is primarily developed for working only with dual frequency P-code receivers which are not used for platform attitude determination cases. The most tempting techniques used in GPS multi-antenna systems are the Hatch's least-squares ambiguity search method and the fast ambiguity search filter method. These two are based on the (sequential) least-squares adjustment. Within a properly defined ambiguity search space and under normal error distributions, the correct ambiguity set can be contained in the search space so as to give the smallest sum of squares of carrier phase residuals among all the potential ambiguity sets [23].

The conceptual steps for on-the-fly ambiguity resolution by least-squares method are given in Figure (2.3). The process starts at a certain epoch with the GPS carrier phase and pseudorange observations plus the available a priori information. Generally, a least-squares adjustment is performed based upon all available data including carrier phases. The ambiguity search space, i.e., the change intervals of integer ambiguity, is obtained from the related statistics. The potential ambiguity sets within the ambiguity search space are tested to determine the correct one. The testing criteria normally depends on statistics, for example: the sum of

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squares of carrier phase residuals and floating ambiguity confidence intervals. In order to speed up the computation and improve efficiency, some special numerical algorithms can be applied or even the determination of search space and the testing of potential ambiguities can be interwoven so that an early exit or the inability of ambiguity resolution can be formed before all the potential ambiguity sets are tried [11] [22]. The major difference between Hatch's ambiguity search method and the fast ambiguity search filter technique exists in the ways the ambiguity search space are constructed. In Hatch's method, only three ambiguity change intervals related to the four primary satellites are determined using the available information up to that epoch. But, in the fast ambiguity search filter technique, the change interval for each ambiguity is determined sequentially as if the ambiguity parameters were eliminated or set to integers one by one [17].

Ambiguity resolution on-the-fly is not an easy job. It is impacted by a lot of factors typically including errors and biases, satellite geometry and the search algorithms, which are used. In least-squares adjustment and statistical testing, the errors are normally assumed to be Gaussian and the derived statistics are tested under this assumption. In GPS positioning and attitude determination, however, this is not always true due to the corruption of multi-path effects which tend to be cyclic in nature. Therefore, there exists a possibility that incorrect ambiguities may be chosen due the abnormal behavior of the statistics in the presence of excessive non-Gaussian errors. The satellite geometry, including the number of satellites, also affects the reliability of on-the-fly ambiguity resolution. The bottom line is that enough geometric information has to be obtained or accumulated in the observations so that all the errors can be suppressed or smoothed out and the correct ambiguities become separable with the other alternatives. The general trend is that more satellites in view and more geometry information accumulated result in better reliability for ambiguity resolution. The algorithm development for on-the-

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fly ambiguity resolution has been the focus on high accuracy (cm-level) kinematic positioning for many years. Generally speaking, an optimal on-the-fly technique for kinematic or real-time applications must have the following properties:

- (1) Fast in computation and short in observation time span required for ambiguity resolution
- (2) High reliability to ensure the correct ambiguity resolution and robust in the presence of non-Gaussian errors, and
- (3) Quality control against incorrect ambiguities and cycle slips [17].

Obviously, there are contradictions in some of the above requirements and trade-off is necessary. For instance, higher reliability of ambiguity resolution may require a longer time span of observations, and real-time applications need the shorter observation span for ambiguity resolution. Up to now, there is no ambiguity resolution method meeting all the above requirements and improvements for the existing methods are needed [18].

2.4.2 LAMBDA Method

LAMBDA method was proposed by Teunissen [30]. LAMBDA stands for the Least-squares AMBiguity Decorrelation Adjustment. This method is a simple mathematical model that is suitable for short baselines. there are two types of unknowns occur in the model: baseline coordinates (collected in the vector b) and double difference ambiguities (collected in α). Other unknowns, such as atmospheric delay parameters, can be included in b as well. Carrier phase observations are assumed to be available. Code observations can also be included, however, they are not related to the ambiguity terms [31]. The linearized double difference observation equations (see Equation 2.36) can be formed in the following linear

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system of equations :

$$Y = Bb + A\alpha + e \quad (2.39)$$

where:

Y is the vector of observed minus computed double difference measurements; $Y \in R^m$;

b is the vector that contains the increments of the p baseline coordinates;

α is the vector of n double difference ambiguities;

B is the $m \times p$ design matrix for the baseline coordinates;

A is the $m \times n$ design matrix for the ambiguities; and

e is the vector of unmodelled effects and measurement noise.

From equation (2.36), $Y=\underline{\ell}$, $B=c$, $b=\underline{x}$, $A=\lambda I$, $\alpha = \underline{N}$

The $m \times (p + n)$ design matrix $[B \ A]$ is assumed to be full rank equal $p+n$. ie. enough observations have been made to determine the baseline coordinates and the double difference ambiguities. Q_Y is the covariance matrix of the observables Y and this matrix is symmetric and positive definite. The double difference ambiguities are placed at the end for reasons of computational efficiency [33]. The formula is rearranged from equation 2.36 :

$$\ell = \begin{bmatrix} c & \lambda I \end{bmatrix} \begin{bmatrix} \underline{x} \\ \underline{N} \end{bmatrix} \quad (2.40)$$

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LAMBDA method have two steps. The first step is the transformation step that an ambiguity transformation Z^* is constructed to allow one to reparameterize the original double-difference ambiguities[34]. The original integer ambiguity vector α , its real-valued least-squares estimate $\hat{\alpha}$ and corresponding variance-covariance matrix $Q_{\hat{\alpha}}$ are transformed:

$$z = Z^* \alpha, \hat{z} = Z^* \hat{\alpha}, Q_{\hat{z}} = Z^* Q_{\hat{\alpha}} Z \quad (2.41)$$

The quantity z denotes the transformed ambiguities. The ambiguity transformation Z^* , which must be integer and volume preserving, doesn't require the use of dual-frequency carrier phase data per se, which also allows one to take care of the slowly changing receiver-satellite geometry, which is so emphatically that can present in the ambiguity variance-covariance matrix $Q_{\hat{\alpha}}$ [39].

The second is a search step (Figure 2.4). As soon as the transformed ambiguities have been obtained, the actual search for the integer least-squares ambiguities is performed in the second step. The search is based on a sequential conditional least-squares adjustment of $\hat{z}$ and it gets its efficiency from the properties of the transformed ambiguities. The transformed ambiguity search space for the conditional least-squares ambiguity estimates can be described by the inequality [39].

$$\sum_{i=1}^n (\hat{z}_{i|I} - z_i)^2 / \sigma_{\hat{z}_{(i|1,i|I)}} \leq \chi^2 \quad (2.42)$$

where, $\hat{z}_{i|I}$ is the least-squares estimate of the i th ambiguity conditioned on the first $(i-1)$ number of ambiguities and $\sigma_{\hat{z}_{(i|1,i|I)}}$ is its variance. The χ^2 is an appro-

privately chosen positive constant, which can make sure that the ambiguity search space indeed includes the sought for integer least-squares ambiguities. The sum of squares structure in the inequality allows one to formulate bounds for the individual ambiguities, so enabling that a search for the transformed integer least-squares ambiguity $\tilde{z}$ can be performed [32]. Once the integer estimate $\tilde{z}$ has been determined, the corresponding baseline solution $\tilde{b}$ can be recovered as

$$\tilde{b} = \hat{b} - Q_{\hat{b}\tilde{z}} Q_{\tilde{z}}^{-1} (\hat{z} - \tilde{z}) \quad (2.43)$$

where, $\hat{b}$ is the non-fixed least-squares baseline solution.

An ellipsoidal region is the ambiguity search ellipsoid. The shape and orientation of the ellipsoid are governed by the covariance matrix of the ambiguities [36]. The decorrelation realizes an ellipsoid that is very much sphere-like. It can be searched through very efficiently. The size of the ellipsoid can be controlled prior to the search using the volume function. The volume offers an indication of the number of candidates contained in the ellipsoid. A request for only a few candidates can be formed. It enables a straightforward implementation of the search. Only a limited number of candidates will be output of which one is the integer least-squares estimate for the vector of ambiguities. The LAMBDA method provides, which are based upon the float ambiguities and their covariance matrix, the integer least-squares estimate for the ambiguities. By this decorrelation, the integer estimation can be carried out efficiently and very fast [38].

The search for the transformed integer least-squares ambiguities already commences with tight bounds, so as to assure the solution can be found in a highly efficient manner. From a practical point of view, it is very apparently here that

Figure 2.4: Search Regions Before and After LAMBDA

the need for decorrelating Z-transformation. Positioning results must be available in real-time kinematic and navigation applications. Therefore the hard part of the parameter estimation is that the ambiguity resolution has to be done on-the-fly. The integer least-squares estimator is best in the sense of maximizing the probability of correct integer estimation. It shows that the success rate of any other integer estimator of the carrier phase ambiguities will be smaller than or at the most equal to the ambiguity success rate of the integer least-squares estimator.

2.4.3 Validation of the Integer Ambiguity Solution

In order to validate the correct integer ambiguity set, one approach is to make use of the triangular closed loop formed by the three antennas (Figure 2.2). The misclosure test can be implemented to reinforce the integrity of ambiguity resolution results [13]. The principle of misclosure check is that any number of baselines which form a closed loop must have zero ambiguity summation on frequency of L1 or L2 for any common satellite pair in the double difference. From the closed loop, one constraint equation can be formed, i.e. the ambiguities for any common pair of satellites should satisfy the conditions below [29]

$$\nabla \Delta N_{master-slave1} + \nabla \Delta N_{slave1-slave2} + \nabla \Delta N_{slave2-master} = 0 \quad (2.44)$$

2.4 GPS Carrier Phase Ambiguity Resolution

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Above constraint equation can be set up as a quality control. In practice, the following equation with a threshold (here 1 cycle) is set up as standard of quality control

$$\nabla \Delta N_{master-slave1} + \nabla \Delta N_{slave1-slave2} + \nabla \Delta N_{slave2-master} < 1(cycle) \quad (2.45)$$

If above equation is fulfilled, then ambiguities are correct; if the above condition is not satisfied, at least one of the baselines must have included incorrect ambiguity solutions. Further validations then have to be conducted under this situation.

2.4.4 Summary of the LAMBDA Method

In this project, the following steps were adopted to resolve the integer ambiguity

1. First, solve for an approximate integer value of $\underline{N}$, by assuming $\delta \mathbf{r}_B = \mathbf{0}$ in equation (2.36).
2. Second, we can obtain the accurate integer ambiguity, $[N]$, which represents integer value of N , from the LAMBDA method, and then check the quality of the ambiguity solution with equation (2.45).
3. Lastly, substitute the accurate integer ambiguity $[N]$ into equation (2.36), and re-arrange equation (2.36) to become

$$\underline{\ell} - \lambda I[N] = c\underline{x} \quad (2.46)$$

The quantity on the LHS of equation (2.46) are all known. The unknown is $\underline{x}$, which can be solved using the Kalman Filter or RHE method, detailed in the next section.

2.5 Kalman Filter and Receding-horizon State Estimation Methods

When it comes to real-time system, it requires processing of data in a very short time period as and when data is acquired, and it is allowed to change over time. Whenever data is not available, one needs to predict the state based on the past measurements. And when considering a dynamic or kinematic system, for example, a moving micro-satellite, because of the dynamics of the state with the constant velocity, using Kalman Filter or Receding-horizon State Estimation can make good use of the previous data. They could be believed as extensions of least squares method to deal with a kinematic problem instead of a static one [19].

The theory of Kalman Filter is well-known. Given a dynamic system in the state space form:

$$\begin{aligned} x_{k+1} &= A_k x_k + w_k \\ y_k &= c_k x_k + e_k \end{aligned} \quad (2.47)$$

The Kalman Filter equations for estimating the state vector x_k are

$$x_{k+1}^- = A_k x_k^+ \quad (2.48)$$

$$Q_{x,k+1}^- = A_k Q_{x,k}^+ A_k^T \quad (2.49)$$

$$K_{k+1} = Q_{x,k+1}^- c_{k+1}^T (c_{k+1} Q_{x,k+1}^- c_{k+1}^T + Q_{e,k+1})^{-1} \quad (2.50)$$

$$x_{k+1}^+ = x_{k+1}^- + K_{k+1} (y_{k+1} - c_{k+1} x_{k+1}^-) \quad (2.51)$$

$$Q_{x,k+1}^+ = (I - K_{k+1} c_{k+1}) Q_{x,k+1}^- \quad (2.52)$$

where Q_x and Q_e are the covariance matrices of the state vector and measurement noise respectively; however, they are often difficult to model and have to frequently be determined by empirical means.

If the platform is assumed to be an integrator driven by unknown input w , i.e.

$$\dot{x}(s) = \frac{1}{s}w(s)$$

Then, in discrete-time, we can write

$$x_{k+1} = x_k + w_k$$

where we have used a normalised sampling period $h = 1$. Thus, in the Kalman filter framework, we have $A_k = I$.

In the case of attitude determination, we recall that our observation equation (2.46), after the integer ambiguity has been resolved, was $\underline{\ell} - \lambda I[N] = c\underline{x}$, where the state vector $\underline{x}$ is the adjustment from the approximate value $\underline{x}_0$, which can be obtained from pseudorange measurement.

Once the state $\underline{x} = (\delta X, \delta Y, \delta Z)$ is obtained, the coordinates of the GPS receiver can be determined by $\underline{X} = \underline{x}_0 + \underline{x}$. With $\underline{X}$ of the 3 GPS receivers determined, baseline vectors can be formed and the platform attitude can be computed.

2.6 Receding-horizon State Estimation

The receding-horizon state estimation can be considered to overcome the problem of divergence in the state estimates. To estimate the state vector based on a finite number of past measurement samples is the idea of receding horizon state

2.6 Receding-horizon State Estimation

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estimation [26]. So the oldest measurement sample is discarded as a new sample becomes available. Thus, the state estimator memory is fixed. The state estimation horizon determines the number of past measurement samples used to reconstruct the state vector, so it can be chosen as an alternative tuning parameter for the batch least squares state observer.

Receding-horizon State Estimation is a method to address the problem of estimating the state vector x with a finite number of samples of the system output y .

The system is represented by:

$$\begin{aligned} x_{k+1} &= Ax_k \\ y_k &= c_k x_k \end{aligned} \tag{2.53}$$

where, x_k is state vector (n -vector); y_k is output vector (m -vector); A is a $n \times n$ nonsingular matrix, and c is a $m \times n$ matrix.

2.6.1 Batch Least Squares

By setting up $N+1$ observation equations, we have

$$Y_{k,N} = M_N x_{k-N} \tag{2.54}$$

where

$$Y_{k,N} = \begin{bmatrix} y_k & y_{k-1} & \cdots & y_{k-N} \end{bmatrix}^T \tag{2.55}$$

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and

$$M_N = \begin{bmatrix} (c_k A^N)^T & (c_{k-1} A^{N-1})^T & \cdots & c_{k-N}^T \end{bmatrix}^T \quad (2.56)$$

Here, N is the tuning parameter known as the state estimation horizon. It determines the number of past measurements used to reconstruct the state estimates.

Because there are more equations than unknowns, a least squares solution could be sought, giving

$$\hat{x}_k = A^N \hat{x}_{k-N} = A^N P_N M_N^T Y_{k,N} \quad (2.57)$$

where

$$\begin{aligned} P_N &= (M_N^T M_N)^{-1} = (P_{N-1}^{-1} + (c_k A^N)^T c_k A^N)^{-1} \\ &= P_{N-1} - P_{N-1} (c_k A^N)^T (I + c_k A^N P_{N-1} (c_k A^N)^T)^{-1} c_k A^N P_{N-1} \end{aligned} \quad (2.58)$$

With N fixed, the state estimate at $k+1$ could be computed as

$$\hat{x}_{k+1} = A^N P_N M_N^T Y_{k+1,N} \quad (2.59)$$

where

$$Y_{k+1,N} = \begin{bmatrix} y_{k+1} & y_k & \cdots & y_{k+1-N} \end{bmatrix}^T \quad (2.60)$$

Discarding the oldest observation, and adding the newest observation at the same time. It is a receding horizon, or moving window approach and the resulting state observer is termed the moving window or receding horizon state observer.

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2.6.2 Recursive Solution

It is very clearly seen that the equation (2.57) is an inefficient way of actually computing an estimate. It is also not suitable if the estimation horizon N is to be used as an online tuning parameter for the performance of the proposed state observer. In many engineering applications, the observations are obtained sequentially. It is desirable to calculate, when new observations are obtained, the current state estimate based on the previous state estimate.

A technique, which is generally applied to derive a recursive solution from equation (2.59), is the so-called growing memory strategy. The following shows that this gives the conventional dynamic state estimation equation.

The equation (2.59) can be rewritten as

$$\begin{aligned}\hat{x}_{k+1} &= A^N P_N Y_{k+1,N} \\ &= A^N P_N M_{N-1}^T (Y_{k,N-1} + (c_k A^N)^T y_{k+1}) \\ &= A^N P_N M_{N-1}^T Y_{k,N-1} + A^N P_N (c_k A^N)^T y_{k+1}\end{aligned}\tag{2.61}$$

Let

$$\begin{aligned}L_N &= A^N P_N (c_k A^N)^T \\ &= A^N (P_{N-1} - P_{N-1} (c_k A^N)^T (I + c_k A^N P_{N-1} (c_k A^N)^T)^{-1} c_k A^N P_{N-1}) (c_k A^N)^T \\ &= A^N P_{N-1} (c_k A^N)^T (I + c_k A^N P_{N-1} (c_k A^N)^T)^{-1}\end{aligned}\tag{2.62}$$

2.6 Receding-horizon State Estimation

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Then,

$$\begin{aligned}
 & A^N P_N M_{N-1}^T Y_{k,N-1} \\
 &= A^N (P_{N-1} - P_{N-1} (c_k A^N)^T (I + c_k A^N P_{N-1} (c_k A^N)^T)^{-1} c_k A^N P_{N-1}) M_{N-1}^T Y_{k,N-1} \\
 &= (A - A^N P_{N-1} (c_k A^N)^T (I + c_k A^N P_{N-1} (c_k A^N)^T)^{-1} c_k A) A^{N-1} P_{N-1} M_{N-1}^T Y_{k,N-1} \\
 &= (A - L_N c_k A) \hat{x}_k
 \end{aligned} \tag{2.63}$$

where $\hat{x}_k$ substitutes $A^{N-1} P_{N-1} M_{N-1}^T Y_{k,N-1}$ from equation (2.59). So the equation (2.56) can be rewritten as

$$\hat{x}_{k+1} = (A - L_N c_k A) \hat{x}_k + L_N y_{k+1} \tag{2.64}$$

This formula corresponds to the conventional dynamic state estimator. For the attitude determination application in this work it is assumed that $A = I$ and equation (2.58) reduces to

$$P_N = P_{N-1} - P_{N-1} c_k^T (I + c_k P_{N-1} c_k^T)^{-1} c_k P_{N-1}$$

and equation (2.62) reduces to

$$L_N = P_N c_k^T = P_{N-1} c_k^T (I + c_k P_{N-1} c_k^T)^{-1}$$

The Kalman Filter and the RHSE are related. The Kalman Filter's gain is chosen by user selecting Q_x and Q_e , where the gain of the RHSE is chosen by user selecting N .

Chapter 3

Experiment Description and Results

In order to assess the performance of GPS multi-antenna systems, gauge the achievable system accuracy of the ambiguity resolution routines, and test the design of the Kalman Filter and Receding-horizon State Estimation, a series static and dynamic tests were conducted. During the static test, two Trimble GPS receivers (Trimble 4700 and Trimble 4800) and one Leica (SR530) were used. In the dynamic test, two Ashtech G12 Sensors were used.

In this chapter, descriptions for the static and dynamic tests are provided. The test results as well as the accuracy comparisons are also shown.

3.1 Design and Flowchart

3.1.1 GPS-based Attitude Determination

We use the GPS receivers and antennas to collect the GPS carrier phase and pseudorange. Here, we set up a coordinate system in the plane of three antennas. The experiment setup for the static test is shown in Figure 3.2. In this test, the origin is defined at the center of the master antenna. At the same time the local level frame's origin is also at the center of the master antenna. The attitude of the three antennas platform based on the techniques described in Chapter 2.

3.1.2 Flowchart

The flowchart for the software designed for attitude determination is shown in Figure (3.1). The major functions are described below.

As Figure (3.1) shows, first step is to use three antennas to get GPS raw data. Therefore, collecting enough epoch is necessary to form double difference equations and then linearize them, which will be used in the later calculation. We select the satellite, which has the largest elevation, as the reference satellite. Next is to apply the LAMBDA method to calculate the integer ambiguities, and Kalman Filter or Receding-horizon Recursive State Estimation is to calculate the state vector

$(\delta X, \delta Y, \delta Z)$ and then obtain the three positions by using
$$\begin{bmatrix} X = X_0 + \delta X \\ Y = Y_0 + \delta Y \\ Z = Z_0 + \delta Z \end{bmatrix}.$$
 The

last step is to obtain the platform attitude from the receiver positions in local level system and those of the body frame.

Figure 3.1: Attitude Determination System Flowchart

3.2 Static Test

The purpose of this test was to verify the LAMBDA method, Kalman Filter, and the direct method.

3.2.1 Test Description

In order to show the performance of attitude determination using the methods described in chapter two, one static test was done on January 29, 2003. This test was conducted on the roof top of the S2 building of the Nanyang Technological University, Singapore. The test environment is relatively free of tall buildings, trees and traffic.

Figure 3.2: Static Test

Figure (3.3) is the baseline formation.

Figure 3.3: Antennas formation

Table 3.1: GPS Receiver and Antenna

	Antenna Type	Receiver Type
Master Antenna	Trimble 4800 Internal	Trimble 4800
Slave Antenna 1	MICRO-CTR L1/L2 GND	Trimble 4700
Slave Antenna 2	MICRO-CTR L1/L2 GND	Leica AT502

For this test, data was collected with three pairs of receivers and antennas as listed in Table 3.1. Data was collected at a rate of 1 Hz in static mode with the receivers and antennas stationary for two hours, from 13:40 to 15:47 local time.

The Trimble 4700, 4800 receiver and Leica AT502 are dual frequency receivers. Dual frequency GPS receivers offer two major advantages over single frequency equipment: Ionospheric errors that are inherent in all GPS observations can be modelled and significantly reduced by combining satellite observations made on two different frequencies, and observations on two frequencies allow for faster ambiguity resolution times.

In the static test, the raw data of the three GPS receivers were processed by

GeoGenius reference in static mode. Attitude values were computed with the baseline results. The baselines were calculated with the pseudorange, carrier phase and the accuracy parameters $(\sigma_{\lambda 1}, \sigma_{\varphi 1}, \sigma_{h1})$ and $(\sigma_{\lambda 2}, \sigma_{\varphi 2}, \sigma_{h2})$ (Equation 6, 7 and 8) which were obtained by GeoGenius. The roll is 62.027 degree, the pitch is 20.263 degree, and the yaw is -21.114 degree. Their accuracies are: $\sigma_{roll} = 0.1049$ degree, $\sigma_{pitch} = 0.0526$ degree, $\sigma_{yaw} = 0.0848$ degree.

3.2.2 Data Processing

The results of the the PDOP value and the number of satellites tracked are shown in Figure (3.4). During the test, the number of satellites tracked was mostly around six to eight. In general, the number of satellites tracked averages a little over seven, and the average PDOP was around 2.1. The PDOP was calculated using equation (2.37).

Figure 3.4: PDOPs and SV Numbers During the Test

The first several epoches data are used to calculate the initial data for the estimation. We use LAMBDA method, Kalman Filter and direct method described

in chapter two to calculate the attitude parameters yaw, pitch and roll. The estimated platform yaw, pitch and roll at every epoch were plotted in Figures (3.5), (3.6) and (3.7). The standard deviations are 0.303 degree, 0.531 degree and 0.573 degree for yaw, pitch and roll, respectively. Figures (3.8), (3.9) and (3.10) show the accuracy of the estimated platform yaw, pitch and roll which were computed using equations (2.20) (2.21) and (2.22). It can be seen that the accuracy of GPS derived attitude is within one degree. The accuracy of the platform yaw and roll (Figure 3.8 to 3.10) is almost within 0.5 degree. The accuracy of the platform pitch is almost within 0.75 degree. We haven't calibrated baseline. The accuracies plotting in Fig.(3.8) to (3.10) are the relative accuracies calculated by equation (2.20), (2.21) and (2.22). They are not absolute accuracy. In this static test, we collected GPS data over a very long period and with the software named GeoGenius to get the standard deviations of longitude, latitude and height of slave antennas.

Figure 3.5: The Platform Yaw for 29th, January 2003

3.3 Dynamic Simulation

Tests were conducted using the Welnav GPS simulator GS-1010 (for details about the GPS simulator, please see Appendix) to simulate the dynamic vehicle performance. Dynamic tests of the attitude system were conducted using Ashtech G12

Figure 3.6: The Platform Pitch for 29th, January 2003

Figure 3.7: The Platform Roll for 29th, January 2003

Figure 3.8: The Relative Accuracy of Platform Yaw as Computed by Equation (2.20)

Figure 3.9: The Relative Accuracy of Platform Pitch as Computed by Equation (2.21)

Figure 3.10: The Relative Accuracy of Platform Roll as Computed by Equation (2.22)

Table 3.2: Track Design

Cruise Duration	200 second	
Terminal Speed	500m/s	acceleration $1m/s^2$
Change in Heading	90 degree	acceleration $1m/s^2$
Cruise Duration	100 second	

sensors. We use the GPS simulator to setup two baselines, (master receiver—slave 1 receiver) and (master receiver—slave 2 receiver).

Figure 3.11: Simulation Setup

3.3.1 Test Description

Trajectory of the vehicle is shown in Figure (3.12). The vehicle was stationary for 200 seconds at the beginning of the experiment. Then, it moved North with an acceleration of $1m/s^2$ until it reached a terminal speed of 500m/s. Then it took a 90 degree turn (see table 3.3). After its 90 degree's turning, it kept moving for 100 seconds with the constant speed of 500 m/s.

The tracks of the three moving points are the same as the Table (3.2) shows, Table 3.3 shows the master receiver position at the start of the experiment. The

Figure 3.12: The Designed Trace

Table 3.3: Master Receiver Position

Latitude	N 1°20'34.1022
Longitude	E 103°40'4.7147
Height	83.52 m

master receiver coordinates in the ECEF system are (-1507962.645, 6195607.656, 148468.451) in ECEF system. Slave 1 receiver start position = (-1507963.845, 6195607.656, 148468.451) in ECEF system and slave 2 receiver position = (-1507962.645, 6195608.856, 148468.451) in ECEF system. The baseline distance between the master receiver and slave receiver is 1.2 meter.

The roll, pitch and yaw value of the platform attitude could be directly obtained from the three receiver positions given by the GPS simulator. The test data were recorded at an epoch rate of 0.1 second.

Figure 3.13: The Roll of Original Platform Attitude

Figure 3.14: The Pitch of Original Platform Attitude

Figure 3.15: The Yaw of Original Platform Attitude

3.3.2 Data Processing

Two methods were used to process the GPS data. One is the Kalman Filter, the other is the RHSE.

Figure 3.16: The Number of satellites for 24th, March 2004

The number of satellites tracked during the test are presented in Figure 3.16, which shows that the number of satellites tracked was between 4 to 11. After the first 200

Table 3.4: Results of Kalman Filter

	Mean (P)	Mean (R)	Mean (Y)	SD (P)	SD (R)	SD (Y)	RSS
$Q_e=0.010I$	-0.041	0.059	0.042	2.212	1.987	2.006	3.587
$Q_e=0.009I$	-0.040	-0.051	0.029	2.108	2.003	1.998	3.528
$Q_e=0.008I$	0.043	0.041	-0.039	1.965	1.875	1.901	3.315
$Q_e=0.007I$	0.017	-0.040	-0.044	1.801	1.752	1.886	3.142
$Q_e=0.006I$	0.038	0.043	0.026	1.756	1.687	1.769	3.010
$Q_e=0.005I$	-0.026	0.029	-0.039	1.501	1.594	1.650	2.742
$Q_e=0.004I$	0.039	0.020	-0.036	1.453	1.421	1.497	2.524
$Q_e=0.003I$	0.020	-0.019	-0.022	1.399	1.432	1.445	2.469
$Q_e=0.002I$	0.0073	0.019	0.013	1.402	1.398	1.399	2.424
$Q_e=0.001I$	-0.015	-0.026	0.031	1.401	1.411	1.402	2.433

epoch, the number of satellites tracked was around 10. This is because the GPS simulator needed time to warm up to generate the correct GPS satellite signals.

Kalman Filter

The covariance matrix of the system noise, Q_e is difficult to model and have to be obtained by empirical means. The Table 3.4 shows the attitude results, when Q_e is selected from 0.001I to 0.010I where I is the identity matrix

The Mean (P), Mean (R), Mean (Y) are the mean of errors for the Pitch, Roll and Yaw, respectively. The SD (P), SD (R), SD (Y) are the standard deviation of errors for the Pitch, Roll and Yaw, respectively. RSS is the Root-Sum-Square of the SD (P), SD (R) and SD (Y).

Definition of SD is

$$SD = (\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2)^{\frac{1}{2}} \tag{3.1}$$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i \tag{3.2}$$

$\bar{x}$ is the mean values of the elements along different dimensions of an array.

$n=14855$, is the total number of epoch. When Q_e is between $0.001I$ and $0.01I$, the results are acceptable as can be seen from Table 3.4. When Q_e is larger than $0.01I$, the results are divergent (Figure 3.29), which is not acceptable. The mean of the attitude errors for pitch, roll and yaw ($\text{mean}(P)$, $\text{mean}(R)$, and $\text{mean}(Y)$) change little in the range between $0.001I$ and $0.01I$, but there still has a decrease of the errors. And the standard deviation of errors is decreasing too, when Q_e changed from $0.010I$ to $0.001I$.

The mean of the attitude parameters errors are the smallest, when Q_e was $0.002I$, and the Root-Sum-Square (RRS) in the Table 3.4 is also the smallest.

Figure 3.17 – 3.22 show the pitch, roll and yaw of the platform when $Q_e = 0.002I$

Figure 3.17: The Platform Pitch

Figure 3.18: The Residuals error of Platform Pitch between Fig.(3.14) and (3.17)

Figure 3.19: The Platform Roll

Figure 3.20: The Residuals error of Platform Roll between Fig.(3.13) and (3.19)

Figure 3.21: The Platform Yaw

Figure 3.22: The Residuals error of Platform Yaw between Fig.(3.15) and (3.21)

Receding-horizon State Estimation

The same set data was also processed using RHSE method. And the Figure 3.23 – 3.28 show the best results when N is 30.

Figure 3.23: The Platform Pitch, when N was 30

Figure 3.24: The Residuals error of Platform Pitch between Fig.(3.14) and (3.23), when N was 30

From Table 3.5, the accuracy is around ± 1.7 degree when the platform of the three points stationary. However, it increased to ± 2.3 degree when the platform is moving.

N should be chosen no less than the dimension of the state vector. Here N should no less than 3. The closer the N is to 30, the smaller the mean and standard deviation of the errors are. The Table 3.5 shows that when N was 30, the mean of the attitude parameters (pitch, yaw, and roll) errors is the smallest. and the SD of the attitude parameters is around 0.96, which is the smallest in the table. Furthermore, the RRS of N=30 is also the smallest. So the results are the best with

Figure 3.25: The Platform Roll, when N was 30

Figure 3.26: The Residuals error of Platform Roll between Fig.(3.13) and (3.25), when N was 30

Figure 3.27: The Platform Yaw, when N was 30

Figure 3.28: The Residuals error of Platform Yaw between Fig.(3.15) and (3.27), when N was 30

Table 3.5: Results of Receding-horizon State Estimation

	Mean (P)	Mean (R)	Mean (Y)	SD (P)	SD (R)	SD (Y)	RSS
N=3	10.98	13.01	12.31	17.971	18.746	18.178	31.699
N=5	5.89	6.45	5.87	10.991	9.787	9.197	17.354
N=8	2.28	2.87	2.54	6.152	6.173	6.024	10.594
N=10	1.83	2.07	1.56	4.754	4.589	4.873	8.210
N=15	0.98	0.87	0.26	2.554	2.8687	2.101	4.378
N=20	0.28	0.073	-0.186	1.8513	1.8687	1.8709	3.228
N=30	-0.024	-0.019	-0.009	0.9635	0.9634	0.9660	1.670
N=40	-0.024	0.019	-0.026	1.2989	1.3299	1.3203	2.280
N=50	0.041	-0.046	-0.057	1.4001	1.3895	1.4156	2.428

3.3 Dynamic Simulation

N=30. Furthmore, our experience showed that RHSE method always converge, even when N is larger than 50.

The two methods, Kalman Filter and RHSE, have been applied. Both of them can reach the same accuracy level. Compared with RHSE, although Kalman Filter is good in dynamic system, the results will diverge when the parameter Q_e is not selected appropriately.

The RHSE always converge, and it can reach the accuracy level of the Kalman Filter. So it is a promising method for attitude determination application.

Figures 3.29, 3.30 shows a pair of baseline vectors results of Kalman Filter and RHSE. The result of Kalman Filter (Figure 3.29) is divergent, when Q_e is 0.5I, when the RHSE result (Figure 3.30) is convergent with N=10.

Figure 3.29: The Divergent Result of Kalman Filter

Figure 3.30: The Convergent Result of Receding-horizon State Estimation

Chapter 4

Conclusions and Recommendations

4.1 Conclusions

In this project, a GPS-based attitude determination method has been developed. Its aim is to establish the feasibility of real-time attitude determination using GPS signal. The double difference carrier phase observations model was used, so that the attitude could be solved accurately. LAMBDA method was implemented to solve the integer ambiguities. Then, Kalman Filter and Receding-horizon State Estimation were applied to obtain the baseline vectors. This, together with direct method, makes it easier and faster to solve the attitude. The system was designed and built by the author, and the software of the Matlab Satellite Navigation Toolbox Vision 2.0 was also used.

One static experiment has been conducted with three receivers on the roof top of S2 building, in Nanyang Technological University, Singapore. Real data of GPS signal has been collected with three GPS receivers and antennas. The dynamic

test was also simulated by the GPS simulator with two GPS receivers. Both the static and dynamic tests were performed to gauge the algorithms and models.

4.1.1 Static Results

It has been demonstrated that the use of three-antenna on a static platform with a baseline of about 1m can yield an accuracy of one degree. The experimental results showed that the yaw, pitch and roll angles were 21.5°, 20° and 63° respectively. The attitude determination accuracy was about 0.5° in yaw, 0.75° in pitch and 0.5° in roll.

4.1.2 Dynamic Results

The two methods, Kalman Filter and Receding-horizon State Estimation, were applied to determine the attitude. The accuracy achieved by both methods were similar. RHSE always produce convergent results, but Kalman Filter may diverge if the parameter Q_e is not selected approximately.

However, because of the requirement of equipment performance, cost and accuracy, the different equipment may produce different accuracy level. When high accurate results are required, the high accuracy receiver can be used. For example, Novtel GPS receiver can provide double frequency (L1, L2) pseudorange and carrier phase signal. On the other hand, Ashtech and Garmin GPS receivers provide only single frequency L1 pseudorange and carrier phase signals, but they cost less than the Novatel receivers, although the advantages of having dual frequency carrier phase measurements are that the relative ionospheric corrections can be computed and the wide-lane and extra-wide-lane ambiguity resolution techniques can be applied [1] [45].

4.2 Future Works

The eventual aim of the project is to provide an accurate real-time attitude for micro-satellites. The attitude information can be used for controlling the micro-satellite. To realize this goal, many issues need to be solved to improve the performance of GPS attitude determination since it is a function of receiver firmware, satellite geometry, antenna carrier phase stability, and multi-path rejection ability. The future work will be focused on the improvement of the reliability and accuracy of attitude determination. Real time ambiguity resolution is one of the most important aspects of the GPS attitude determination, especially for high dynamic LEO micro-satellite.

4.2.1 GPS Receiver Configuration

Attitude determination accuracy is an important issue. It is dependent on factors like, GPS receiver firmware, GPS satellite geometry, baseline length, multipath rejection ability, microsatellite dynamics and vibration. Due to the high speed of microsatellite, it is important to analyze the visibility of the onboard GPS receiver. During the design stage of setting up a GPS multi-antenna system, it is necessary to properly analyze, simulate and select the antenna configuration to minimize the impact of errors on attitude determination. For the micro-satellite, especially with short baselines, the baseline arrangement should be analyzed to find an optimal configuration.

4.2.2 Use a Simple Transform Matrix between the Geocentric and Local level System

Besides the method presently used to obtain the transform matrix, an alternate way to transform the geocentric coordinates to the local level coordinate is as follows [51]. Since

$$\begin{aligned} T &= \begin{bmatrix} \hat{X}_\ell^T & \hat{Y}_\ell^T & \hat{Z}_\ell^T \end{bmatrix}_{3 \times 3}^T \\ \hat{Z}_\ell &= \hat{P} \\ \hat{X}_\ell &= \frac{\hat{N} \times \hat{P}}{|\hat{N} \times \hat{P}|} \\ \hat{Y}_\ell &= \hat{Z}_\ell \times \hat{X}_\ell \end{aligned}$$

where $\hat{P} = (X, Y, Z)_m$ is the master receiver position in the geocentric system, $\hat{N} = (0, 0, 1)$ is known.

This method can be tried in later research. Whether it may reduce the calculation time need to be investigated.

4.2.3 Incorporate the Angle in LAMBDA Method

A variation of the LAMBDA ambiguity resolution method is proposed for future work. One idea is to incorporate the pitch, yaw, and roll angles directly in the LAMBDA method. If it is feasible, then it will save time and eliminate the coordinate system computation. It may help the system to get a more accurate attitude result.

One technique, SAM [42] attitude method, should also be investigated. It embeds the attitude (the angles of pitch, roll and yaw) into the equations directly.

4.2.4 Further Receding-horizon State Estimation Application and Development

In this thesis, both the theory and application of RHSE have been described. The results showed that, RHSE is a promising method for attitude determination application. More work should be carried out to investigate the strength and weaknesses of RHSE. The Kalman Filter and RHSE should be further investigated to see if it could detect cycle slips.

4.2.5 Analyze Different Noises

Because the GPS satellite simulator can inject several kinds of noises, such as the ionospheric delays, the tropospheric delays, satellite ephemeris errors, ranging errors caused by multi-path, and some code or carrier phase measurement noise. It requires a lot more simulations to understand and use them properly. Simulating a true condition is possible with the simulator. With these noises, experiments will be done to explore how to avoid or eliminate them in the true condition, so as to get a precise attitude result.

4.2.6 Quaternion Method

Quaternion method uses the Euler symmetric parameters which are based on Euler's theorem that given two coordinate systems, there is one invariant axis, along which measurements are the same in both coordinate systems and that it is possible to move from one coordinate system to the other through one rotation, β about the invariant axis. The invariant axis is referred to as the Euler axis. Given a unit vector, e along the Euler axis, the quaternion is defined as

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} e_x \sin \frac{\beta}{2} \\ e_y \sin \frac{\beta}{2} \\ e_z \sin \frac{\beta}{2} \\ \cos \frac{\beta}{2} \end{bmatrix} \quad (4.1)$$

The quaternion components therefore satisfy the following constraint:

$$q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1 \quad (4.2)$$

The attitude transformation matrix can then be written in quaternion form:

$$A = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix} \quad (4.3)$$

The work described in this thesis should, in principle, be applicable to a quaternion formulation and this should be investigated.

Appendix A

GS-1010

The GPS simulator is shown in Figure (A.1)

Figure A.1: GPS Simulator

A graphical interface provides a real-time display of the receiver's performance under test. The following RS-232 formats are currently supported: NMEA, Leica MX 9400, Rockwell Zodiac and Navcore-V. In 1 or 2 chassis, upto 48 channels can

Table A.1: GPS Simulator 1

Velocity	$\pm 6,000\text{ m/s}$ (12,000 m/s option)
Acceleration	$\pm 1,000\text{m/s/s}$
Jerk	$\pm 1,000\text{m/s/s/s}$
Pseudorange	$\leq \pm 0.01\text{m}$
Pseudorange Rate	$\leq \pm 0.001\text{m/s}$
Delta Pseudorange	$\pm 0.012\text{m}$
Interchannel Bias	± 0.0126
Level Accuracy	$+0.2\text{dB RSS}$
Output Impedance	50ohms
Power Level, L1 C/A	$-147.5\text{dBm to } -100\text{dBm}$
Spurious	$< -30\text{dBc}$
VSWR	1.5 : 1 max
Accuracy	$5 \times 10^{-9}\text{aging/day}$
1PPS	$< 100\text{ns}$

Table A.2: GPS Simulator 2

RF Simulator Power	110 to 230 V	Auto-detect
Computer Power	110 to 240 V	Switch
Monitor Power	110 to 240 V	Switch

be distributed into 4 phase coherent RF outputs. This provides a test platform for differential GPS testing, mulitple vehicle tracking, and attitude determination. Differential corrections can be implemented through dual RF outputs, or generated by a serial port in RTCM-104 or SCAT-I format. For automotive applications, ana-log rate table, map-matching software, and wheel speed sensors can be integrated into the test suite. A 64 Hz closed-loop operating mode with less than 10 msec transport delay is also available. Vehicle dynamics can be generated interactively using point-and-shoot software or joystick.

Table A.1 and A.2 are the simulator characteristics.

Signals of 12-24 simulated L1=1574.42 MHz C/A GPS satellites can be simulated. The data cable is RTCM104/SCAT-I via RS232 and various receiver support is via RS232. The simulator has available options including: additional channels 12 to 24/chasis (space velocity and jammer input), multiple RF outputs (10 MHz,

ARINC-429), and automotive package, which includes: real-time closed-loop and computer upgrades.

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