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Optical Detection of Optical Code Division Multiple Access

Kok Shaw Wei

School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University

in fulfillment of the requirement for the degree of

Doctor of Philosophy

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*To my parents, my siblings and my friends
for their encouragement and love.*

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Summary

This thesis studies and analyzes the optical detection for optical code division multiple access (OCDMA) communication system. Unlike the conventional detection scheme which uses optical pulses, our study addresses the optical detection that is based on the number of photon counts.

For the first time, we investigate the optical detection under the condition where the probabilities of data bit transmission is non-equiprobable. The difficulty of this problem is due to the complex expression of the likelihood probabilities of the number of photons received which consists of a mixed Poisson-binomial term. Hence, two auxiliary functions are introduced that will help us in simplifying the complex expression of the mixed Poisson-binomial term into a more manageable expression. By establishing important properties that are derived from the two auxiliary functions, we are able to obtain the monotonicity property for the likelihood probabilities of number of photons received. With the obtained monotonicity property, the problem of optical detection in the presence of unbalanced transmission probabilities of data bit can be addressed. Given a range of transmission probabilities of data bit, a BER bound can be attained using the monotonicity property. The theory is established for two typical detectors, namely chip-level and correlation detector which are respectively effective in combating against multi-user interferences (MUI) and robust against noise.

The results achieved for one dimension (1D) optical detection are extended to the detection using 2D code which utilizes wavelength in addition to time to form the code.

2D coding gives an advantage over the conventional 1D code, which is constructed using only time chips, to enhance the communication capacity more efficiently. For 2D coding, a dual detection scheme is proposed in this thesis by considering the merits of the chip-level and the correlation detector. The implementation issue of 2D optical detector is addressed by exploring the use of optical interleaver which is capable of splitting dense multiple wavelengths. A systematic design of the Fourier Filter Flattop (F^3T) optical interleaver is developed and analytical solution to the design parameters of F^3T were obtained.

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Symbols and Acronyms

Symbols

E	Error in bit detection
n	Number of photons detected
L	Length of signature code
ω	Weights of the signature code
$r(t)$	Received signal at the detector
ζ	Interference pulses in each weighted chip
Q	Average number of photons produced by a pulse
N	Number of users
$\Lambda(n)$	Likelihood ratio
κ_i	Coupling ratio
E_i	Electric field
ΔL	Physical length difference of the differential delays
α	The level of unflatness of passband
λ	Wavelength
τ	Time chip
A	Total number of chips in two dimension signature code
M	Total number of interference pulses

Acronyms

OCDMA	Optical code division multiple access
MUI	Multi-user interference
BER	Bit error rate
OOC	Optical orthogonal codes
OOK	On-off-keying
PPM	Pulse position modulation
MAP	Maximum <i>a posterior</i> probability
ML	Maximum-likelihood
2D	Two dimension

Chapter 1

Introduction

1.1 Motivation

Code division multiple access (CDMA) is a multiplexing technique which is investigated and successfully used in satellite [1] and digital cellular communication [2]. It has the advantage over other conventional multiplexing techniques in enhancing the transmission capacity by increasing the number of users simultaneously accessing the network. This is achieved by assigning unique signature code to individual user for encoding the bit symbol before transmitting it over the network. Over the recent years, CDMA has attracted increasing attention in the optical community [3]-[14]. This is because optical fiber is equipped with immense bandwidth and is capable of providing the required bandwidth for transmitting the signal using CDMA technique. This gives rise to optical CDMA (OCDMA). For an OCDMA system, each user will be assigned a unique signature code sequence, which consists of optical pulses, to encode the bit before it is being transmitted over the network [5].

In order to implement CDMA in optical domain, some enabling technologies are needed to be first established due to the different natures of the optical signals and

electric signals. The first enabling technology concerns the signature code used for the OCDMA communication. Due to the nature and properties of light, optical signals used to represent the codes always take non-negative values in optical domain. The bi-polar codes which are used in wireless CDMA to construct the signature code are not suitable to be used in the optical domain. Thus, coding schemes are needed to be redefined for the OCDMA system. When and after the coding schemes are developed, the second enabling technology coming into a play is to address the implementation of the coding scheme in optical domain. The methods used for implementing the signature code for wireless CDMA is no more appropriate for the OCDMA counterpart. It is highly desired to implement the OCDMA codes without using any conversion of optical signals to electrical signals, in order to take the advantage of the large bandwidth and high transmission speed of optical medium. However, regardless of the coding schemes and their implementation methods, the most critical enabling technology for the OCDMA system is the optical detection. Although the methodologies for the wireless CDMA can be borrowed, the optical detection for OCDMA communication is very different as it involves the optical signals such as the photon counts and optical pulses which take non-negative values and have different statistics. In addition, it is a common problem, to both wireless CDMA and optical CDMA communication, to estimate the bit error rate (BER) bounds in the presence of non-equal probabilities of data bit transmission. This problem is important since in CDMA system, the transmission of the data bit is usually non-equiprobable among different users. In fact, the problem is even more difficult for OCDMA communication when the photon counts are used in the optical detection. It is because the received photon counts usually have more complicated statistics than the Gaussian ones. So far, no work has been done regarding to this problem. The problem of optical detection using photon counts motivates the research of this thesis.

1.2 State-of-arts of OCDMA

Since OCDMA communication is proposed two decades ago [5], it has attracted intensive interest in optical communication community. Many research work have been done around focuses on the three enabling technologies mentioned earlier on. First, various coding schemes are developed. One of the earliest coding method proposed for OCDMA is optical orthogonal code (OOC) [5]-[7], [22]-[39]. Due to its simplicity, it is one of the most widely used OCDMA coding schemes till now. However, its capacity and efficiency are limited by the length of signature code and the weights. To overcome this problem, other codes like prime codes [40]-[50] are proposed. Recently, taking the advantage that wavelength is available in optical domain in addition to traditional parameters such as frequency, amplitude and phase, 2D signature codes are suggested [81]-[85]. For OCDMA 2D codes have the edge over the conventional 1D signature code in that it is constructed using both time chips and wavelengths bands. Thus, it enhances the transmission capacity more efficiently. Working with the signature codes, various modulation techniques are developed as well in the OCDMA system to transmit the signals over fiber links. The most commonly used modulation techniques include on-off-keying (OOK) [51]-[54], pulse position modulation (PPM) [54], [55]-[58] and overlapping PPM (OPPM) [59]-[61].

Secondly, various implementation schemes of the OCDMA coding and decoding are developed. The implementing schemes can be basically classified into three groups depending on the technology used. In the first group is the use of time delay lines technology [3], [74]. This method is commonly used to construct OCDMA signature code that spread in time. It consists of an encoder which has a $1 \times \omega$ optical power splitter, ω delay lines, and a $\omega \times 1$ optical power combiner where ω is the weights of the signature codes. An optical pulse will be split into ω pulses which are transmitted over each delay line. Then, the optical pulses will be combined at the combiner to form the signature codes. Time delay line can also be used to form 2D codes if different

wavelengths are used. The next group for implementing OCDMA signature codes is the use of fiber Bragg grating (FBG) [69]-[72]. The basic principle of FBG is to extract a single wavelength from multiple wavelengths. By cascading together several FBGs, each extracts of which a different wavelength, OCDMA signature codes using spectral encoding can be constructed. This method can be further extended to encode 2D signature codes as well. In addition, OCDMA signature code constructed using multiple wavelengths encoding can also be achieved using planar lightwave circuit (PLC) [73]. All these three methods do not consist of any conversion of optical signals to electrical signals. Hence, they are sought out to implement OCDMA signature codes.

However, as the heart of OCDMA, optical detection methods received less attention as compared to the progresses in coding and decoding. So far the most commonly used OCDMA receiver is the correlation receiver [5]-[6]. The idea of the correlation detection method is borrowed from the wireless CDMA. The principle is to correlate the received optical signal with the desired user's OCDMA signature code. Then, the optical signals can be correctly retrieved. Although it is simple to operate, its performance diminished with multi-user interferences (MUIs) caused by multiple simultaneous accesses to the network. Thus, optical hardlimiters (OHL) [62]-[64] are proposed to work with the correlation receiver to improve the BER performance. A greater improvement in performance can be further achieved by using two OHLs [65]-[68] to reduce the noise and the MUI. To further enhance the performance of the optical detection in the presence of MUI, optimum detector [75] and multiuser detection [76]-[78] are proposed. They are effective in achieving a better performance but the complexity of these detectors increases with the number of users simultaneously accessing the network which makes them impractical. Recently, a new detection scheme called chip-level receiver [79]-[80] is developed. It is shown that chip-level receiver has a good performance in combating against MUI. Worthy to mention that the developed detection methods are designed on the assumption that the data bit

transmission is equiprobable. In addition, most of the above detection methods are based on the number of optical pulses instead of photons counting. But in OCDMA, transmission of the data bit is usually non-equiprobable among different users. Till now this is still an open problem in optical communication.

1.3 Objective

The main objective of this thesis is to analyze the optical detection for OCDMA communication systems based on the number of photon counts under the circumstance where the data bit transmission is non-equiprobable. To achieve the objective of this thesis, the critical point is to establish the monotonicity of the likelihood probabilities of the number of photons received. However, the complex expression of the likelihood probabilities of the number photons received consists of a Poisson-binomial term. So far no effective approaches are available to handle Poisson-binomial terms. This constitutes the difficulty to address the monotonicity. Thus, the first step of our research is to devise an approach to simplify the complex expression of the mixed Poisson-binomial term into a more manageable expression. With the simplified expression, we are then able to evaluate the optimal performance of the optical detectors. Next, a relationship such as the monotonicity property for the likelihood probabilities of the number of photons received is needed to address the problem of optical detection in the presence of unbalanced transmission probabilities of data bits. The theory and investigation for optical detection established here is for two typical detectors, namely chip-level [79]-[80] and correlation detectors [5]-[6]. With the results obtained, this thesis will also look into several implementation issues of OCDMA communication systems. Following this routine, the research in this thesis is carried out with the following sub-objectives.

- To devise a manageable approximate method to address the mix Poisson-binomial

probability and uses it to analyze the optimal optical detection of chip-level receivers. The difficulty of this problem lies with the complex expression of the likelihood probabilities of the number of photons received which consists of a Poisson-binomial term. Thus, an approach is proposed to simplify the complex expression. In addition, it is highly complex to evaluate the BER performance of a chip-level receiver. So an approximate BER performance by establishing optimality in each weighted chip for the condition where the data bit transmission is equiprobable is studied to simplify the actual BER performance analysis. By extending the results obtained for chip-level receivers in the presence of equiprobable data bit transmission to chip-level receivers in the presence of non-equiprobable data bit transmission, a BER bound can be obtained.

- To analyze the optical detection of a correlation receiver in the presence of non-equiprobable data bit transmission. By using the results obtained to handle the mix Poisson-binomial probability, the monotonicity property is established for correlation receiver. And then the BER bound for this case can be achieved.
- To investigate a new OCDMA receiver by combining the merits of chip-level and correlation receivers. Although a chip-level receiver is more effective in combating against MUI than a correlation receiver, it is less robust to dark current noise as compared to the latter. In contrast, correlation receiver is more robust to noise but more susceptible to MUI. Hence, we wish to develop a dual detector by combining the advantages of both chip-level and correlation receivers. In order to take the advantage of the 2D OCDMA signature code constructed using multiple wavelengths and time chips, the dual detector concept can be extended to implement a 2D dual detector. The principle of this 2D dual detector is that the weighted time chips over one wavelength is first correlated. Then, the chip-level detection for each summed wavelength is performed.
- To address some implementation issues of OCDMA detection. In this thesis,

the design problem of Fourier Filter Flattop (F^3T) interleaver [95] - [101] is studied. As 2D coding consists of multiple wavelengths, the optical receiver will have to retrieve individual wavelength. Hence, an F^3T interleaver is being looked into for implementing the 2D detection method since it can split up dense wavelengths. So we want to study and analyze the problems in designing F^3T interleavers. So that a systematic design of the optical interleaver can be developed and an analytical solution to the design parameters of F^3T can be obtained.

1.4 Major Contributions

The following original contributions have been made to the OCDMA detection analysis.

- For the first time, the problem of optimal BER for both chip-level and correlation detector in the presence of unbalanced transmission probabilities of data bit is addressed. A systematic approach is developed to analyze the BER performance of chip-level and correlation receivers.
- A computational approach is proposed to simplify the mixed Poisson-binomial type likelihood probabilities of the number of photons received. By introducing two auxiliary functions, the complex expression which consists of the mixed Poisson-binomial term is reduced to a more manageable expression.
- Proposal of an approach to estimate the BER of chip-level receiver. As it is very computational expensive and difficult to evaluate the BER performance of chip-level detectors, an approximate optimal BER approach is proposed to simplify its performance analysis by establishing optimality in each weighted chip.

- Given a range of *a priori* probabilities data bit transmission, the BER bounds for both chip-level and correlation receivers are found without the need to perform exhaustive searching with the *a priori* probabilities range.
- A novel dual detection scheme is proposed by combining the merits of chip-level and correlation receivers.
- A systematic design of the F^3T optical interleaver is developed and an analytical solution to the design parameters of F^3T is obtained.

1.5 Organization

The rest of this thesis is organized as follows.

Chapter 2 gives an overview of the current technology used for OCDMA decoder. The requirements need to overcome the detection problem are found and justification on the model and methods use in this thesis are given.

Chapter 3 is devoted entirely to chip-level detection where transmitting of a bit “0” and bit “1” is equiprobable. The objective is to investigate the optimality of chip-level receivers.

The next chapter extends the research work of chip-level receivers into the case where transmitting a bit “0” and bit “1” is not equiprobable. The optimality issue for this condition is investigated. The BER bound calculation is also given in this chapter when only a range of transmission probabilities is given.

The research work in optimality analysis of the threshold of a chip-level receiver is extended to the case for a correlation receiver. It is being investigated in Chapter 5.

Several implementation issues of OCDMA system are discussed in Chapter 6. A dual detection scheme is proposed by taking into consideration the merits of both chip-

level and correlation receivers. Then, a fused fiber based interleaver device called F^3T interleaver is explored to address the implementation issue of 2D detector due to its capability in separating dense wavelengths. A quantitative design approach for this interleaver is proposed.

Finally, we will conclude the thesis in the last chapter and give suggestions for future research work.

Chapter 2

OCDMA Decoder System Overview

2.1 Introduction

In this chapter, we brief the principle of OCDMA and review the current OCDMA detection technologies from four perspectives including the signature code, the modulation, the detection methodology, and the implementation. The review not only provides the necessary technical background of OCDMA detection, it also addresses the inadequacy of the models and the methods used in the current OCDMA detection technologies. In the review, we analyze the effects of several key factors including network traffic, traffic bursty, code pattern, code length, and optical buffer on the performance of OCDMA detection. Our strategy to mitigate the inadequacy and consider the factors is also briefed.

2.2 Overview of Current OCDMA User Detection Technologies

An OCDMA communication system consists of the basic building blocks as shown in Figure 2.1. In the system, a unique signature code is assigned to each user of the system. It is represented by a logical sequence of “0” and “1”. When a user wants to send data which is expressed in terms of a stream of bits, the bits are encoded by the signature code of the user and then are modulated by the modulator into optical signals either in terms of pulses or continuous forms. The optical signals are multiplexed and transmitted over the fiber links to the destinations through optical amplification and optical switching. At the destination, the transmitted signal is demultiplexed into receivers, each of which usually consists of a demodulator, a decoder and a detector. In most of cases, the decoder and the detector are usually combined into one which is called the OCDMA detector. The demodulator extracts the data from the carriers; the detector decodes the data with a desired user’s signature code and then detects the user’s bits based on the decision criteria. Since users of the OCDMA system can access the OCDMA system simultaneously, the broad bandwidth of fiber links are maximally utilized. But, the signals received at the destination may come from all users. Therefore, the success of OCDMA relies on the detection of a desired user in the presence of MUIs. In this thesis, we will focus on the detection of OCDMA.

From the principle of the OCDMA, the OCDMA user detection involves several major elements including the signature code, the modulation scheme, the detection methods, and the optical implementation. In the following, we will review the current OCDMA detectors respectively according to these elements.

Figure 2.1: OCDMA communication system

2.2.1 Signature Code

The basic idea of OCDMA is to allow a number of users to access the same bandwidth simultaneously. This is achieved by assigning each of the user a unique signature code. The code is formed by dividing a bit duration or a wavelength range into many pieces, each of which is called a chip. Assigning chips with “0” or “1”, the signature code is expressed as a sequence of “0” and “1”. Thus, in order to accommodate more users while avoiding interferences among users, signature codes are of course expected to be orthogonal with each other. However, the practical code length is limited. The number of users is very limited if pure orthogonality is pursuing. Thus, signature codes in practice are usually not made purely orthogonal with each other. The non-orthogonality of the signature code in turn brings the multi-user interferences to the OCDMA detection. To assess the extend of the non-orthogonality, cross-correlation of the signature codes comes into play. It in essence reflects the total number of MUIs. The lower cross-correlation level means a lesser number of MUIs. To attain good cross-correlation properties, several commonly used signature codes are proposed which include various prime codes [3], OOCs [22], and 2D codes [81]-[85].

For prime codes, they are basically derived from the use of Galois fields $GF(P)$ given as $GF(P) = \{0, 1, \dots, j, \dots, P-1\}$ where P is a prime number and it gives the signature

code length $L = P^2$. A prime sequence $S_x = (s_{x0}, s_{x1}, \dots, s_{xj}, \dots, s_{x(P-1)})$ is constructed as shown in Table 2.1. Then, the signature code $C_x = (c_{x0}, c_{x1}, c_{x2}, \dots, c_{xi}, \dots, c_{x(N-1)})$ can be generated by using the prime sequence. For positions $i = s_{xj} + jP$ for $j = 0, 1, \dots, P - 1$, “1” is inserted whereas “0” is assigned to all other positions, in brief, C_x can be expressed as:

$$c_{xi} = \begin{cases} 1, & \text{for } i = s_{xj} + jP \\ 0, & \text{otherwise.} \end{cases} \quad (2.1)$$

		0	1	..	j	..	$P - 1$
S_x	0	0	0	..	0	..	0
	1	0	$1 \bmod P$	..	$j \bmod P$	..	$P - 1 \bmod P$
	$\vdots$	$\vdots$	$\vdots$	..	$\vdots$	..	$\vdots$
	$\vdots$	$\vdots$	$\vdots$	..	$\vdots$	..	$\vdots$
	x	0	$x \bmod P$	..	$xj \bmod P$	..	$x(P - 1) \bmod P$
	$\vdots$	$\vdots$	$\vdots$	..	$\vdots$	..	$\vdots$
	$P - 1$	0	$P - 1 \bmod P$	..	$(P - 1)j \bmod P$	..	$(P - 1)^2 \bmod P$

Table 2.1: Prime sequence

The cross-correlation function at the i -th chip position for any two distinct code sequences is found to be at most two and given as [3]

$$\sum_{i=0}^{P^2-1} c_{x1,i} c_{x2,i} \leq 2. \quad (2.2)$$

The second commonly used signature code is the OOC. So far, many methods of constructing OOC [22] have been developed. It is expressed as a sequence of L binary numbers, among which ω chips are assigned as weighted chips represented by “1”s. For any two distinct OOC code sequences a and b , their cross-correlation satisfy

[22]

$$\sum_{i=0}^{L-1} a_i b_{i+\tau} \leq \lambda_c \quad (2.3)$$

where λ_c is the cross-correlation constraint and τ is a time shift. Different OOC codes have different specifications on λ_c . For example in [79], OOC is employed as the signature code sequence for the chip-level detection. The OOC is chosen to have a cross-correlation λ_c that is bounded by one. Under this condition, the maximum number of code size will satisfy the following relationship: [22]

$$N \leq \frac{L-1}{\omega(\omega-1)}. \quad (2.4)$$

Due to the difficulty in the implementation of a time delay line in practice, the length of signature code in time dimension is limited. Taking advantage of optical communication, wavelength domain has been explored to form 2D codes in order to increase the overall code length, together with time domain. The signature codes in this case have the structure as shown in Figure 2.2.

λ_1		1		
λ_2	1			
λ_3			1	
λ_4				1
	τ_1	τ_2	τ_3	τ_4

Figure 2.2: Schematic of 2D signature codes

For the 2D codes, their cross-correlation can be expressed as [82]

$$\sum_{m=0}^{R-1} \left(\sum_{n=0}^{L-1} a_{m,n} b_{m,n+\tau} \right) \leq \lambda_c \quad (2.5)$$

where a and b are two periodic 2D codes, L is the temporal length and R is the number of rows in the codes.

Comparing these commonly used codes, OOC codes give better cross-correlation performance thus contributes fewer MUI, prime codes is simple to construct, and 2D codes can fully utilize the bandwidth of the WDM communication. However, regardless of the code format, the effect of the signature code to the OCDMA detection has two aspects. The first is that the signature code determine the number of users N that can be supported by the OCDMA system. The second is that the number of MUIs which is defined by the cross-correlation properties of the code. In all, different signature codes specify the number of MUIs and the maximum number of users to the OCDMA detection. Thus, the OCDMA detection problem can be solved by two strategies. On one hand, passively, a OCDMA detector can be designed to meet the numbers of MUIs and users, no matter which signature codes are used. On the other hand, actively, the design of an OCDMA detector should provide the requirements on the number of the MUI so that the code designers can design the satisfactory signature codes for the detector. In this thesis, the designs of the OCDMA detection assume only the number of MUIs as a general number M . Then, we derived the conditions on the number of MUIs, the number of maximal number of users, and other code parameters like L and ω . It is shown that OOC code is one of codes satisfying the derived conditions. Thus, in all simulation studies in the thesis, OOC codes are used for the analysis of the problem of optical detection for OCDMA.

2.2.2 Modulation

The OCDMA detection can be designed for different modulation. In principle, all the traditional modulation schemes including the amplitude modulation, the phase modulation, and the frequency modulation can be used in OCDMA. In addition to the traditional modulation, wavelength modulation provides a unique feature of OCDMA. But, since optical phase is difficult to be maintained in fiber links, the phase modulation is not recommended for the first principle for OCDMA unless coherent

OCDMA is considered. The detection for the rest modulation schemes, eventually, reduces to the detection based on the intensity of optical signals. This is basically the same as the detection for the amplitude modulation. Thus, to see the effects of the modulation, we consider two modulation techniques that have been currently proposed in OCDMA communication. They are the OOK [51]-[54] and PPM modulation technique [54], [55]-[58].

In OOK modulation for OCDMA, a sequence of optical pulses, each of which occupies a chip duration and appears only at the weighted chips in the signature code, is used to represent the logic “1” in a bit duration. The sequence of pulses is sent sequentially when a bit “1” is to be sent. Nothing will be transmitted if a bit “0” is to be sent. The OOK modulation can be illustrated by Figure 2.3. The detection of the OOK modulated signals is to detect the optical pulses at weight chip positions.

Figure 2.3: OOK modulation

In PPM modulation with M -ary symbols, each symbol is represented by M consecutive bits. The j -th symbol for $j \in \{0, 1, \dots, M - 1\}$ is represented by the sequence of optical pulses like that used in OOK appearing in the j -th bit while no pulses appear in the other bits. The schematic is illustrated in Figure 2.4. For PPM modulation, the detection of a symbol is to detect the presence of signature code at the j -th bit duration and absence of signature code at all other bit durations in a M consecutive bit stream. In this sense, the OCDMA detection developed for OOK can be extended for PPM.

In fact, the development of detection based on the simple OOK modulation not only provides a practical OCDMA detector, it also helps us to focus on those more

critical problems limiting the performance of optical detection in the presence of MUIs and other uncertainties including shot noise. For example in [79] where the chip-level detection is proposed to deliver better bit error rate performance than that of the traditional correlation detector, the detector is first developed for the OOK modulation to highlight the methodology of chip-level detection. The obtained results are easily extended to the PPM modulation. In [5], etc, the OCDMA detection using hard limiters are also illustrated for OOK modulation. Thus, we follow the same strategy in the thesis to focus us on the difficulty in the design of an OCDMA detector using photon counts rather than intensity.

Figure 2.4: PPM modulation

2.2.3 Methodology

As the heart of a detector, the detection methodology directly affect the performance of the detector. It specifies the way to manipulate the received optical signals and make a decision on the received bit of a desired user. So far, the OCDMA detection methodologies can be classified into two broad classes include the correlation detection and the chip-level detection.

The correlation detector is the earliest detection method proposed for OCDMA system based on which several variations are proposed later [5]-[6]. Its schematic is shown in Figure 2.5.

At a receiving end, the received signal $r(t)$ contains signals from both the desired user

Figure 2.5: Correlation receiver

and undesired users given as follow

$$r(t) = \sum_{n=1}^N S_n(t - \tau_n) \quad (2.6)$$

where N is the number of users, $S_n(t - \tau_n)$ is the signal transmitted by the n -th user and τ_n is the time delay of the n -th signal. The signal $S_n(t)$ transmitted by the n -th user is given by

$$S_n(t) = S_n b_n(t) y_n(t) \quad (2.7)$$

where S_n is the optical intensity, $b_n(t)$ is the data bit “0” or bit “1” transmitted and $y_n(t)$ is the signature code pulse sequence of the n -th user. $y_n(t)$ is given by

$$y_n(t) = \sum_{j=1}^L A_j^n P(t - jT_c) \quad (2.8)$$

where $P(t)$ denotes an optical pulse center at zero with a chip duration T_c , L is the length of the signature code, and A_j^n is the binary value at the j -th chip according to the signature code of the n -th user.

In the correlation detector, the received signal is first correlated with the desired user signature code. The resulting signal is then photodetected and integrated over one bit duration T where it is further sampled at $t = T$. At the sampling time, the output signal will be compared against a threshold value θ . If the signal is greater than θ , a bit “1” being sent is decided and bit “0” being sent otherwise.

It is shown that the correlation detector is simple and robust to system noise. But

its BER performance deteriorates rapidly when the number of users simultaneously accessing the network increases. So in order to improve the BER performance in the presence of MUIs, a kind of correlation detector with optical hard-limiters is developed [62]-[64] as shown in Figure 2.6.

Figure 2.6: Correlation receiver with one optical hard-limiter

The optical hard-limiter is defined as

$$g(x) = \begin{cases} T_h & \text{when } x \geq T_h, \\ 0 & \text{when } 0 < x < T_h \end{cases} \quad (2.9)$$

where T_h is the threshold value of the optical hard-limiters.

In this kind of correlation detector, the received signal $r(t)$ passes through an optical hard-limiter before it is correlated with the signature code. If the optical intensity of $r(t)$ is greater or equal to T_h , the optical hard-limiter clamps the intensity at T_h and if the optical intensity is smaller than T_h , the output of the optical hard-limiter is set zero. The basic idea of using hard limiter is to preventively remove some severe MUIs before the correlation so that the effect of MUI on the detection is reduced and the BER performance is improved.

For the correlation detectors in Figures 2.5 and 2.6, the analysis assumes an ideal photodetector and the optical pulses are used as the decision variables. The convenience of such an assumption lies in that the resultant bit errors occur only when bit "0"s are sent. As a result, the threshold used in the correlation detection can be conveniently set at the number of the weight chips in order to obtain the minimal

BER. However, the assumption is not valid in practice when the optical signal is not strong enough. In this case, shot noise has to be taken into account due to the physical characteristics of photodetectors.

In order to improve the detection performance of correlation detector in the presence of shot noises, the use of double optical hard-limiters is proposed [65]-[68]. The schematic of the correlation detector using double hard-limiters is illustrated by Figure 2.7.

Figure 2.7: Correlation receiver with two optical hard-limiters

As above, one optical hard-limiter is placed before the correlation to remove some severe MUI patterns. But the second optical hard-limiter is placed immediately after the correlation before the photodetector. It is used to further remove some interference patterns and the shot noise. Compared to the previous correlation detectors, this detector takes into consideration of the characteristic of the photodetector which can be characterized by a Poisson shot noise model. However, the number of photon counts is used as the decision variables in this case to compare with a threshold θ in decision. As a result, the decision error can also happen when “1” is being sent. In this case, selecting the threshold θ to yield out the minimal BER is a challenging job.

In contrast to the correlation detector methodology where the decision on a bit is made based on the correlation result without knowing the details of the bit, the second class of OCDMA detection methodology makes a decision on a bit by examining the details of a bit. It is based on detection of the weighted chips. The first chip-level detector was proposed in [79]. The detection scheme is shown in Figure 2.8. The received signal $r(t)$ is first sampled at all weighted chips. The result at each chip is photodetected and

integrated over a bit duration T . Then, the results at all weighted chips are sampled at a bit time $t = T$. Chip “0” and “1” are judged for each weight chip respectively by using a threshold θ_i . Finally, when all weight chips are “1”, a bit “1” being sent is declared. Otherwise, a bit “0” is declared. The same idea was extended to form a AND detector for 2D OCDMA in [87]. The results have shown that the chip-level detectors can significantly improve the BER performance of OCDMA detection in the presence of MUIs. However, its performance in the presence of shot noise has not be confirmed. This is due to that it is more difficult to determine the thresholds used for the detection at each weight chip.

Figure 2.8: Chip-level receiver

Summarizing the above analysis of the two methodologies for OCDMA detection, we can address the important concerns for OCDMA detection. The first concern is about the BER performance of the OCDMA. For both methodologies, it is expected that an OCDMA detector can yield the minimal BER. To this end, the maximum *a posteriori* probability (MAP) principle finds its niche in the development of detector delivering an average minimal BER. Second, as OCDMA is different from its electric counterpart, it is of importance to consider the characteristics of photodetectors. Thus, the detection design and performance analysis should be performed in the

presence of shot noise regardless of the detection methodology. In this thesis, we will consider the optimality of both the correlation detectors and the chip-level detectors that are all based on photon counts.

2.2.4 Implementation

Lastly, we review implementation schemes of the OCDMA detection. The most common method to implement the OCDMA decoder is the use of optical delay lines. Fiber Bragg grating (FBG) [72], Mach-Zehnder interferometer (MZI) [73], and normal fiber optics delay lines [3] have been explored to construct the delay. Figures 2.9 and 2.10 shows the use of fiber optics delay line to implement the correlation and chip-level receiver respectively. Although the time delay line technologies enables us to implement optical detector in optical domain, they are also imposing limitations on the optical detection. One of the limitation is that the current time delay line technologies do not support a long signature code due to the fabrication difficulty. But, we do need long signature codes to increase the number of users of the OCDMA system. To overcome this tricky problem, 2D coding is being explored as signature codes. The 2D codes involves the wavelength division in addition to the time division. A chip is indexed by time and wavelength. The code dimension can be increased without imposing strict requirements on the time delay lines. To enable 2D codes in optical detection, all-optical wavelength multiplexer/demultiplexers are needed, in addition to new coding and detection schemes. The multiplexer/demultiplexers are expected to yield out flat passband banks with enough bank isolation. Of course, the insertion loss should be as small as possible. In this thesis, this issue is addressed by proposing a 2D detector and an all-fiber optical interleaver.

Figure 2.9: Correlation receiver using fiber optics delay lines

Figure 2.10: Chip-level receiver using fiber optics delay lines

2.3 Key Factors Limiting OCDMA Detector Performance

In this section, we address some key factors limiting the performance of the OCDMA detection. The analysis helps us to see how to consider these factors in the detector design.

The first factor is the traffic loading of the network. It is reflected by the maximum number of users N who can simultaneously access the network. The number of users that can be supported is directly related to the performance of OCDMA detection method. When the number of users increases, MUI will increase and in turn increase the average bit error rate. For a given number of users, the minimal average bit error rate is usually achieved by applying the MAP principle to determine an appropriate threshold in detection. It is shown in that last section that the chip-level detectors provide better BER performance than those of the correlation detectors in

the presence of severe MUI condition.

However, when the number of users simultaneously accessing the network deviates from the value used in design at some time. Thus, the threshold used in detection is no more optimal for this situation, and in turn the BER performance will be deteriorated due to the wrong threshold. This is case for a practical network suffering from the traffic bursty. In this case, the number of users that simultaneously accessing the network at a given time will change rapidly. This change in the number of users will lead to a re-calculation of the threshold obtained. Clearly, it is time consuming and impractical to determine the threshold used OCDMA detector by numerical or experimental tests. Therefore, it is of great interest to formulate the relationship among the threshold, the number of users N , and the other detection variables so that the threshold can be obtained at least in a range to accommodate the change of the number of active users.

The next factor is the parameters of the signature codes used in OCDMA. As mentioned in the previous section, the code pattern affects the MUI specifications and the code length affects the optical implementation. Together with other parameters like the number of weight chips, these parameters directly influence the number of users N that the network can support. For example, consider using OOC as signature code, the number of users N , code length L and code weights ω satisfy the relationship

$$N \leq \frac{L - 1}{\omega(\omega - 1)} \quad (2.10)$$

if the cross-correlation is bounded by one.

From equation (2.10), by increasing either the signature code length L or decreasing the weights ω , we can increase the number of users N . Further analysis in the later part of the thesis also shows that N , ω , L , and other detection variables are the key parameters to the optimal threshold used in OCDMA detection. Any change

of these parameters will lead to a re-calculation of the optimal threshold that will minimize the BER performance. Thus, the OCDMA detection method must address this problem so as to cut down the computational time for the search of the optimal threshold. It is again required to formulate the relationship of the threshold and the parameters of the system and the detection.

The third factor affecting OCDMA performance is the representation of the optical signals. This is particularly important for chip-level detection. When the power of the light signal at a chip is strong enough, the signal received at the chip can be treated as a pulse. This requirement limits the number of chips deployed in a bit duration as the power budget of the bit is limited. Thus, weak chip signals should be considered in order to form more signatures codes for users. In this case, due to the characteristics of the photodetectors, the signal received at a chip cannot be modelled by a pulse. Its representation in terms of photon counts is more accurate. However in this case, the statistics of MUIs will change significantly from following a Binomial probability to following a mixed Poisson-binomial probability. The resultant discrepancy in MUIs may change the BER of the detection with several orders as shown in later studies in the thesis.

Another effect of the signature code is the asymmetry of the transmission probabilities of “0” and “1”. In existing analysis of OCDMA detectors, the BER performance is evaluated by assuming that the bits “0” and bits “1” are being equiprobably transmitted. However, in practice, the transmission probabilities of “0” and “1” are uncertain due to various reasons. This can be due to the unequal probable appearances of bit “0” and bit “1” in the source codewords and it can also be due to the unequal probable transmissions of source symbols. Instead, a practical assumption about the probabilities is that they change within a known range. Therefore it is of great interest to study the BER performance given a range of the transmission probabilities. This problem has not been well addressed in literatures.

2.4 Conclusion

In this chapter, we have reviewed the OCDMA detection technologies respectively from perspectives of the signature code, the modulation scheme, the detection methodology, and the optical implementation. We have also reviewed the key factors influencing the OCDMA detection. The factors include the network traffic, bursty, optical buffer, and the signature code parameters. Through the review, we address the inadequacy of the models and the methods used in current OCDMA technology. Meanwhile, we also address the requirements for an OCDMA detector to be able to mitigate the inadequacies. The requirements alternatively justify the model and methods used in this thesis. We conclude this chapter with the following points.

First, the quantitative assessment of OCDMA detection performance can only be feasible for the case which assumes ideal photodetector. However, the practical photodetector contribute shot noise characterized by a Poisson model. In this case, photon count is a good candidate used as the decision variable. Such a choice follows more closely to the actual optical detection situation. Although all the previous research has touched on the shot noise, the performance of the OCDMA detectors is only analyzed experimentally. The accurate assessment of the OCDMA detection performance is still an open problem. In this thesis, we will consider the BER performance of the chip-level detector and the correlation detectors using photon counts.

Second, as the detectors based on the MAP criterion can yield the minimal bit error rate in an average sense. The optimality of OCDMA detectors are analyzed by using the MAP criterion. To address the optimality in the average sense as specified by MAP criterion, this thesis will adopt the likelihood ratio to determine the threshold used in the OCDMA detection.

Third, as declared in Section 2.2.1, the signature code pattern only affect the number of MUIs. Thus, either the OCDMA detector to be designed should be able to handle

the specified MUIs, or the design of the detector should be able to specify the MUIs as guideline for code designer to develop new codes. The thesis follows the later requirement. A condition about the number of MUIs is given and the condition is clearly fulfilled by using OOC code with a cross-correlation bounded by one. Hence, OOC code is used as the signature code in the simulation studies to illustrate the performance of the derived detector.

Fourth, as the main problem in the detection using photon counts in the presence of MUIs lies in determining the optimal threshold by MAP criterion. In order to highlight the problem, we follow the same strategies as adopted in previous works to focus on the the development of OCDMA detectors for OOK modulation. As discussed in Section 2.2.2, the obtained results can be extended to other modulation schemes like PPM modulation later.

Last but not the least, it is noticed that the MAP criterion is usually used in OCDMA detectors to determine the optimal threshold to attain the minimal average BER of the OCDMA. However, due to signature code design and the transmission conditions of the network, the transmission probabilities of “0” and “1” are not constant. They may vary in a range from a practical point of view. In this case, it is of concern to assess the average BER of the system. So far, the current model and methods cannot answer this problem. In this thesis, we will give the range of the BER by exploring the monotonicity of the likelihood ratio of photon count.

Chapter 3

Approximate Optimal Chip-level Detection with Equiprobable Data Bit Transmission

3.1 Introduction

Optical detection plays an important role in optical communication systems in ensuring the success of OCDMA systems. In this chapter, we will investigate the problem of optical detection of chip-level receivers under the condition that the probabilities of data bit transmission are equiprobable. First, we will look at the basic principle of how to minimize the BER performance of optical detection. Then, we will examine the chip-level optical detection with the following assumptions [79]. These assumptions will be used throughout the thesis unless otherwise stated.

- All users use perfect equal power to transmit their data.
- The weights of the signature codes are evenly distributed among the code lengths.

- The system is chip-synchronous.
- Dark current and thermal noises are negligible.

The difficulty in addressing the problem of chip-level optical detection lies in the complex expression of the likelihood probabilities of photon counts received. By introducing two auxiliary functions and establishing their important properties, we attempt to address this difficulty for both OOK and PPM modulation techniques. In addition to that, it is very complex to calculate the BER for chip-level detection. Hence, we propose an approximate approach to evaluate the BER by establishing optimality in each weighted chip position.

3.2 Basic Principle of Determining Threshold and BER

For a communication system, a receiver will produce an observation data X after the received signal is correlated with the desired user's signature code. By observing this data received, the receiver will be able to make a decision on which bit symbol is transmitted. This decision rule is based on the MAP criterion. By considering binary transmission, the receiver will decide if a bit "0" or bit "1" is sent by choosing between the two corresponding *a posteriori* probabilities that has the larger value. Thus, MAP decision criterion is defined as

$$\max\{P(0|X), P(1|X)\}. \quad (3.1)$$

By using Bayes' rule, the two *a posteriori* probabilities can be expressed as

$$P(0|X) = \frac{P(X|0)P(0)}{P(X)}, \quad (3.2)$$

$$P(1|X) = \frac{P(X|1)P(1)}{P(X)} \quad (3.3)$$

where $P(X|j)$, for $j = 0, 1$, is the likelihood probabilities of observing X data given the symbol j is transmitted and $P(j)$ is the *a priori* probability of symbol j transmitted.

By substituting (3.2) and (3.3) into (3.1), MAP criterion is equivalent to

$$\max\{P(X|0)P(0), P(X|1)P(1)\} \quad (3.4)$$

as $P(X)$ is equal in calculating the *a posteriori* probabilities.

If the transmission probabilities of bit “0” and bit “1” are equal, *i.e.*

$$P(0) = P(1), \quad (3.5)$$

(3.4) can be further simplified to

$$\max\{P(X|0), P(X|1)\} \quad (3.6)$$

where MAP criterion makes the same decision as the maximum-likelihood (ML) criterion. The decision made by ML criterion depends solely on the likelihood probabilities $P(X|j)$.

In order to achieve an optimal detection, the key issue is to make a decision that will minimize the BER. A threshold has to be first located before a decision can be made. This optimal threshold shall separate the decision space into two parts which are dominated respectively by the probabilities of the receiving signals on a hypothesis of bit “0” being sent and the one under a hypothesis of bit “1” being sent. This is the optimal criterion that will give the minimum probability of error for the signal

detection. Thus, the two situations can be hypothesized as

First Hypothesis: X is observed when bit “0” is sent

Alternate Hypothesis: X is observed when bit “1” is sent

Figure 3.1 illustrates the principle of MAP criterion, which shows that the minimal decision error can be obtained by thresholding the decision variable X by a threshold θ determined at the intersection of two hypotheses.

Figure 3.1: The decision regions

Using such a detection, the BER can be accessed by

$$P(E) = P(E|0)P(0) + P(E|1)P(1) \quad (3.7)$$

where $P(E|0)$ is the probability of error under the first hypothesis and $P(E|1)$ is the probability of error under the alternate hypothesis.

In general, there are four decisions that can be made based on the decision regions

divided by the threshold θ . They are

1. Declare bit “0” is sent when bit “0” is actually sent
2. Declare bit “0” is sent when bit “1” is actually sent
3. Declare bit “1” is sent when bit “0” is actually sent
4. Declare bit “1” is sent when bit “1” is actually sent

From the above four decisions, we can see that decisions 1 and 4 are correct while decisions 2 and 3 are in error. Thus, decision 2 corresponds to $P(E|1)$ and decision 3 corresponds to $P(E|0)$.

Using the likelihood probabilities of the receiver, we are able to get

$$P(E|0) = \int_{\theta}^{\infty} P(X|0)dx, \quad (3.8)$$

$$P(E|1) = \int_{-\infty}^{\theta} P(X|1)dx \quad (3.9)$$

and (3.7) can be found.

3.3 OOK Chip-level Detection

As shown in the previous section, the threshold used in the detection is crucial to minimize the BER. The optimal threshold will minimize the BER in sense of MAP criterion. Hence, we will analyze the problem in obtaining the threshold and examine its optimality for the chip-level receiver in this section.

Each user is assigned a signature code sequence which has L chips in which ω weights are allocated. Using the signature code, the bit symbols are encoded before the user transmits them over the optical fiber network. For OOK-CDMA chip-level detection, the encoded symbol will be transmitted to represent a bit “1”, and there is no

transmission of signature code if a bit “0” is to be transmitted. At the receiving end, the desired user will decode the received symbol by correlating the obtained signals with his own signature code sequence before being photodetected by a photodetector. Then, based on the output of the photodetector, sampled for each weighted chip time, a decision on what symbol is transmitted will be made.

In an OOK-CDMA chip-level receiver, the received signal $r(t)$ will be sampled optically at the correct weighted chips using the desired user’s signature code $y_n(t)$. Each sampled signal will then be photodetected. It is then integrated over each weighted chip before sampling at the weighted time chips. Based on the number of photons collected in each weighted chip position, a decision rule can be obtained.

A threshold θ has to be set in order to decide if a bit “0” or bit “1” is sent. The threshold operation is performed in each weighted chip position. If the number of photons X_i collected is equal to or greater than θ for $i \in \psi$ ($\psi = 1, 2, \dots, \omega$), bit “1” is declared to be transmitted. Otherwise, a bit “0” is declared to be sent. Thus, the decision rule can be expressed as [79]

$$\text{Declare} \begin{cases} 1, & \text{if } X_i \geq \theta, \forall i \in \psi \\ 0, & \text{otherwise.} \end{cases} \quad (3.10)$$

The BER probability for OOK-CDMA chip-level detection using OOC as signature code is given as [79]

$$P_E = P(X_i \geq \theta, \forall i \in \psi | 0)P(0) + P(X_i < \theta, \text{ for some } i \in \psi | 1)P(1). \quad (3.11)$$

It is computational expensive to tabulate the actual BER performance of the chip-level receiver. Thus, we attempt to address the problem by approximating it. We intend to carry out the work by using the optimal chip error rate (CER) to estimate the optimal BER of a chip-level detector. In general, it is difficult to obtain an exact

relationship between CER and BER. The relationship depends on the way how the spreading signature code is constructed. However, comparing such a relationship to the one between BER and symbol error rate (SER), it can be derived for some cases of practical interest as in [102]. The first two cases consider respectively assuming the number of users is two power of the number of weighted chips and assuming any adjacent signature codes differ in only one chip position. For the first case, it can be derived as in [102] that $CER = \left(\frac{N/2}{N-1}\right) \cdot BER$ where N denotes the number of the signature codes. For the second case, the BER bounds can be derived as in [102] as $CER \leq BER \leq (\log_2 N) \cdot BER$.

Another practical case is like the one described in [87] where the receiving photon counts obtained in each chip are assumed independent of any other chip. It is valid under the two scenarios. One is that the codes are randomly generated with a uniform distribution. The other is that well designed deterministic codes are used and the receiver does not take into account the algorithmic constructions of other users' codes in the system. In this case, the BER can be approximated using the CER by

$$P_E = \frac{1}{2}[P(X_i \geq \theta|0)^\omega + (1 - (P(X_i \geq \theta|1))^\omega)] \quad (3.12)$$

where the data bit transmission is equiprobable.

Regardless which cases, the key issue is to obtain the optimal CER at weighted chips. Thus, we will first analyze the optimality in each weighted chip.

The BER performance of the chip-level receiver considered is affected by two factors. They are the receiver shot noise and MUI where they have a Poisson [94] and Binomial [79] distribution respectively. The photodetected we consider assumed a Poisson shot noise model. Thus, it has a Poisson distribution. Next, if we consider OOC as signature codes, we can see that the MUI has a multinomial distribution in a bit time. However, if we examine MUI in each chip, it can be simplified to a binomial

term. If we further assume and do not take into the algorithm of the codes used, the MUI will take a binomial term without any simplification. As seen from the BER expression, the threshold θ plays a vital role in the BER analysis. This will in turn enable us to correctly assess the transmission capacity. Hence, it is essential to correctly determine the threshold and more importantly examine its optimality which will minimize the BER performance.

From the decision rule, it can be seen that optimal detection of chip-level receiver can be obtained by achieving the optimal threshold in a weighted chip position. Therefore, the problem of obtaining an optimal detection for chip-level receiver can be described as obtaining the the optimal threshold in a weighted chip position. Clearly, by achieving an optimal threshold, it will give an optimal detection that will minimize the probability of the BER. Thus, we will first analyze the the hypothesis probabilities of the weighted chip position of the chip-level receiver.

Denote X_i for $i = 1, 2, \dots, \omega$ as the photon counts collected from the i th weighted chip position. By setting a threshold θ , the decision of declaring the presence of an optical pulse in i th chip position is decided by

$$\text{Declare} \begin{cases} \text{Presence of optical pulse}(s), & \text{if } X_i \geq \theta \\ \text{Absence of optical pulse}(s), & \text{if } X_i < \theta. \end{cases} \quad (3.13)$$

If all the weighted chip positions are declared presence of optical pulse(s), then a bit “1” is affirmed. On the other hand, if at least one weighted chip position is declared absence of optical pulse(s), chip-level receiver will decide a bit “0” is transmitted.

With the assumption of equal probability of transmitting a bit “0” and bit “1”, *i.e.* $P(0) = P(1) = \frac{1}{2}$, the decision rule will be based on the ML criterion which is given by (3.6). Now the observation data X is the photon counts collected from the i th weighted chip position of the chip-level receiver. Hence, the threshold can be found

using the decision rule given as

$$P(X_i|1) \underset{X_i < \theta}{\overset{X_i \geq \theta}{\geq}} P(X_i|0). \quad (3.14)$$

The threshold will take an integer value of the photon count n satisfying

$$\frac{P(X_i = n|1)}{P(X_i = n|0)} = 1. \quad (3.15)$$

Note that the probabilities $P(X_i = n|1)$ and $P(X_i = n|0)$ are of discrete distributions since the photon count only takes integer values. The distributions may not intersect exactly at any photon count number. Thus, an integer threshold exactly satisfying equation (3.15) may not exist. But, from the MAP principle, the threshold should minimize the decision error. Thus, we choose the decision law as $X \geq \theta$ to decide “1” else “0”. Then, it is sufficient to choose the optimal threshold θ as the minimal photon number integer n satisfying

$$P(X_i = n|1) > P(X_i = n|0). \quad (3.16)$$

Figure 3.2 gives a graphical illustration of finding the threshold satisfying the MAP criterion using discrete distributions. In the figure, the X-axis denotes the number of photon count and the Y-axis is the probability of observing the n photon counts where “dots” denote the distribution corresponding to a bit “0” being sent and “x”s denotes the distribution corresponding to “1” being sent.

Now, we will derive the likelihood probabilities for the weighted chip position. For each user, all the weights are evenly distributed among the L chips position. Hence, the probability of a pulse appearing at a chip position is $\frac{\omega}{2L}$. The interference pulses I_i in the i th chip of weight positions is a binomial random variable with parameter $\frac{\omega}{2L}$.

Figure 3.2: Threshold determination

The probability of having ζ interference pulses in i th weighted chip is

$$P(I_i = \zeta) = \binom{M}{\zeta} \left(\frac{\omega}{2L}\right)^\zeta \left(1 - \frac{\omega}{2L}\right)^{M-\zeta} \quad (3.17)$$

where $\zeta = 0, 1, \dots, M$ and M is the total number of interfering pulses in a bit duration. The derivation of equation (3.17) can be found in Section 3.12.

Since the photodetector has a Poisson characteristic, the probability of receiving n photons at the i th weighted chip position, under the condition that ζ pulses are photodetected and a symbol j is sent, is given by [79]

$$P(X_i = n|I_i = \zeta, j) = \frac{(e^{-Q\alpha_j})(Q\alpha_j)^n}{n!}, \quad \text{for } j = 0, 1 \quad (3.18)$$

where Q is the average number of photons produced by a pulse and $\alpha_j = \zeta + j$. Thus, the probability of n photons being generated in the i th weighted chip position is

$$\begin{aligned} P(X_i = n|j) &= \sum_{\zeta=0}^M P(X_i = n|I_i = \zeta, j)P(I_i = \zeta|j) \\ &= \sum_{\zeta=0}^M P(X_i = n|I_i = \zeta, j)P(I_i = \zeta). \end{aligned} \quad (3.19)$$

Under the hypothesis of a bit “0” being sent, no signature code will be transmitted and hence $\alpha_0 = \zeta$. Thus, each weighted chip position will not contain any optical pulse by the desired user. Any optical pulses detected in that chip is the interference pulses by the undesired users. Hence, substituting (3.17) and (3.18) into (3.19) gives the probability of n photons being detected when bit “0” is sent:

$$P(X_i = n|0) = \sum_{\zeta=0}^M \frac{(e^{-Q\zeta})(Q\zeta)^n}{n!} \binom{M}{\zeta} p^\zeta q^{M-\zeta} \quad (3.20)$$

where $p = \frac{\omega}{2L}$ and $q = 1 - p$.

For the hypothesis of a bit “1” being sent, the desired user’s signature code will be transmitted to represent a bit “1” and $\alpha_1 = \zeta + 1$. Each pulse chip will contain at least one pulse which belongs to the desired user. Thus, the photodetector will detect at least one pulse in addition to any other pulses produced by the undesired users. Similarly, we can obtain the probability of n photons being detected when a bit “1” is sent by substituting (3.17) and (3.18) into (3.19):

$$P(X_i = n|1) = \sum_{\zeta=0}^M \frac{(e^{-Q(\zeta+1)})(Q(\zeta+1))^n}{n!} \binom{M}{\zeta} p^\zeta q^{M-\zeta}. \quad (3.21)$$

Using the probabilities in (3.20) and (3.21), the design of the optimal chip-level receiver can thus be described as one of obtaining the optimal threshold θ which is the unique solution to (3.14). To this end, the probabilities $P(X_i = n|0)$ and $P(X_i = n|1)$ are to be compared at all possible values of n for $n = 0, 1, \dots, \infty$, in order to search for the optimal threshold. But using this method in achieving the optimal threshold involves a high computational cost and is impossible to be carried out.

With the purpose of obtaining and computing the optimal threshold, investigation of the likelihood probabilities of the weighted chip position is needed. The difficulty

in analyzing the threshold lies in the mixed Poisson-binomial terms in the likelihood probabilities as shown in (3.20) and (3.21). This makes the analyzing work an arduous job. If there is no efficient method to handle these probabilities, they have to be calculated numerically and compared indefinitely in order to attain the optimal threshold for the optical detection. So in order to acquire the optimal threshold, we first need to simplify the likelihood probabilities of the chip-level receiver for the weighted chip position. This motivates us to propose a much simpler and manageable expressions to represent the mixed Poisson-binomial terms in the likelihood probabilities.

By introducing two auxiliary functions, we attempt to make the Poisson-binomial terms in the likelihood probabilities simpler and easier to handle. It can be shown that the probabilities of the receiving photon counts can be obtained by differentiating the auxiliary functions, as described in the next section.

3.4 Two Auxiliary Functions

In this section, two auxiliary functions are introduced to handle the probabilities $P(X_i = n|0)$ and $P(X_i = n|1)$ for $n = 0, 1, \dots, \infty$. We will first show that with the help of the two auxiliary functions, the probabilities in (3.20) and (3.21) can be obtained respectively by differentiating the auxiliary functions. Then, several important properties of these auxiliary functions are derived, which play a key role in computing the optimal threshold.

First, we bring in an auxiliary function given by

$$r_0(x) = (pe^{-Qx} + q)^M. \quad (3.22)$$

The relationship between the function $r_0(x)$ and the probability $P(X_i = n|0)$ in (3.20) is established as follows.

Expanding the binomial at the right hand side of (3.22), the function $r_0(x)$ can be expressed as a polynomial

$$r_0(x) = \sum_{\zeta=0}^M (e^{-Q\zeta x}) \binom{M}{\zeta} p^\zeta q^{M-\zeta}. \quad (3.23)$$

Clearly, $P(X_i = 0|0) = r_0(1)$. To see the relationship between $P(X_i = n|0)$ and $r_0(x)$ in general, we differentiate each item in $r_0(x)$ with respect to x . As a result, we have the first order derivative given as

$$r_0^{(1)}(x) = \sum_{\zeta=0}^M (-Q\zeta)(e^{-Q\zeta x}) \binom{M}{\zeta} p^\zeta q^{M-\zeta}. \quad (3.24)$$

where the superscript (1) denotes the 1st order derivative of the function concerned.

It can be seen from (3.20) that $P(X_i = 1|0)$ can be calculated by evaluating the function $-r_0^{(1)}(x)$ at $x = 1$, that is,

$$P(X = 1|0) = -r_0^{(1)}(x) \Big|_{x=1}. \quad (3.25)$$

Differentiating (3.24) one more time gives the second order derivative

$$r_0^{(2)}(x) = \sum_{\zeta=0}^M (Q\zeta)^2 (e^{-Q\zeta x}) \binom{M}{\zeta} p^\zeta q^{M-\zeta}. \quad (3.26)$$

Compared with $P(X_i = n|0)$ in (3.20) when $n = 2$, it follows that

$$P(X_i = 2|0) = \frac{1}{2} r_0^{(2)}(x) \Big|_{x=1}. \quad (3.27)$$

We continue the same procedures to differentiate $r_0(x)$, the probability of n photons being detected under the hypothesis of a bit “0” being sent can be exactly represented

by the n th order derivative of the auxiliary function $r_0(x)$ as

$$P(X_i = n|0) = \frac{1}{n!} (-1)^n r_0^{(n)}(x) \Big|_{x=1}. \quad (3.28)$$

Now, we examine the possibility of simplifying the probability $P(X_i = n|1)$ in (3.21).

For this purpose, we introduce the second auxiliary function given by

$$r_1(x) = e^{-Qx} (pe^{-Qx} + q)^M, \quad (3.29)$$

which has an alternative polynomial expression as

$$r_1(x) = \sum_{\zeta=0}^M (e^{-Q(\zeta+1)x}) \binom{M}{\zeta} p^\zeta q^{M-\zeta}. \quad (3.30)$$

Applying the same procedures used to express $P(X_i = n|0)$ as $r_0(x)$, the probability of n photons being detected under the hypothesis of “1” being sent can be similarly represented by the n th order derivative of the second auxiliary function $r_1(x)$ as

$$P(X_i = n|1) = \frac{1}{n!} (-1)^n r_1^{(n)}(x) \Big|_{x=1}. \quad (3.31)$$

Summarizing the above discussion we have the following conclusion.

Lemma 3.1. *Let $r_0(x) = (pe^{-Qx} + q)^M$ and $r_1(x) = e^{-Qx}(pe^{-Qx} + q)^M$. Then, the probabilities $P(X_i = n|0)$ and $P(X_i = n|1)$ in (3.20) and (3.21) can be calculated by*

$$P(X_i = n|j) = \frac{1}{n!} (-1)^n r_j^{(n)}(x) \Big|_{x=1}, \quad \text{for } j = 0, 1. \quad (3.32)$$

Proof: The result follows from the derivation of (3.28) and (3.31).

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With the two introduced auxiliary functions, we can now establish some of their properties that will be shown in the following section.

3.5 Properties of Auxiliary Functions

In this section, we derive several important properties of the auxiliary functions. They are useful in obtaining the monotonicity property which is important for deriving the main result on the optimality of the threshold obtained in the next section. All these properties are respectively presented in the form of lemmas.

Lemma 3.2. *Let*

$$g_0(x) = r_0^{(1)}(x) \quad (3.33)$$

$$g_{m+1}(x) = -P_{m+2}^M(pe^{-Qx} + q)^{M-m-2}(Q^{m+2}p^{m+2}e^{-(m+2)Qx}) \quad (3.34)$$

for $m = 0, 1, \dots, M-2$, where P_{m+2}^M denotes the permutation, i.e. $P_{m+2}^M = (M)(M-1)\dots(M-m-1)$. Then, we have

$$1) \quad g_m^{(1)}(x) = -g_{m+1}(x) - (m+1)Qg_m(x), \quad \text{for } m = 0, 1, \dots, M-2 \quad (3.35)$$

$$2) \quad g_{M-1}^{(1)}(x) = -(M)Qg_{M-1}(x). \quad (3.36)$$

Moreover, let $g_M(x) = g_{M-1}^{(1)}(x)$, then

$$g_M^{(1)}(x) = -(M)Qg_M(x). \quad (3.37)$$

Proof: Using the definition of $r_0(x)$ in (3.22), $g_0(x)$ is expressed as

$$\begin{aligned} g_0(x) &= r_0^1(x) \\ &= (M)(pe^{-Qx} + q)^{M-1}(-Qpe^{-Qx}). \end{aligned} \quad (3.38)$$

Hence, the first order derivative of $g_0(x)$ is obtained as

$$\begin{aligned} g_0^{(1)}(x) = & -(M)(M-1)(pe^{-Qx} + q)^{M-2}(-Q^2p^2e^{-2Qx}) \\ & + (M)(pe^{-Qx} + q)^{M-1}(-Qpe^{-Qx})(-Q). \end{aligned} \quad (3.39)$$

Substituting $g_1(x)$ into the above equation results in

$$g_0^{(1)}(x) = -g_1(x) - Qg_0(x). \quad (3.40)$$

which proves (3.35) for $m = 0$.

Next, we differentiate $g_1(x)$ to get its first order derivative and we get

$$\begin{aligned} g_1^{(1)}(x) = & -(M)(M-1)(M-2)(pe^{-Qx} + q)^{M-3}(-Q^3p^3e^{-3Qx}) \\ & + (M)(M-1)(pe^{-Qx} + q)^{M-2}(-Q^2p^2e^{-2Qx})(-2Q). \end{aligned} \quad (3.41)$$

By the definitions of $g_1(x)$ and $g_2(x)$, we have

$$g_1^{(1)}(x) = -g_2(x) - 2Qg_1(x). \quad (3.42)$$

Repeating the above procedures, (3.35) can be obtained.

Equations (3.36) and (3.37) can be shown by substituting the definitions of $g_{M-1}(x)$ and $g_M(x)$.

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It is clear from (3.37) that higher order derivatives of $g_M(x)$ are given by

$$g_M^{(k)}(x) = (-1)^k [(M)Q]^k g_M(x), \quad (3.43)$$

where k is an integer greater than 1.

Lemma 3.3. Denote $r_1(x)$ by a new function $f_0(x)$, i.e., $f_0(x) = r_1(x)$, and define a function series as

$$f_{m+1}(x) = P_{m+1}^M(pe^{-Qx} + q)^{M-m-1}(Q^{m+1}p^{m+1}e^{-(m+2)Qx}), \quad (3.44)$$

for $m = 0, 1, \dots, M-1$. Then,

$$1) \quad f_m^{(1)}(x) = -f_{m+1}(x) - (m+1)Qf_m(x) \quad \text{for } m = 0, 1, \dots, M-1 \quad (3.45)$$

$$2) \quad f_M^{(1)}(x) = -(M+1)Qf_M(x). \quad (3.46)$$

Proof: Employing the definition of $r_1(x)$ in (3.29), we are able to express $f_0(x)$ as

$$\begin{aligned} f_0(x) &= r_1(x) \\ &= e^{-Qx}(pe^{-Qx} + q)^M. \end{aligned} \quad (3.47)$$

Taking the first order derivative of $f_0(x)$, we will get

$$\begin{aligned} f_0^{(1)}(x) &= -(M)(pe^{-Qx} + q)^{M-1}(Qpe^{-2Qx}) \\ &\quad + e^{-Qx}(pe^{-Qx} + q)^M(-Q). \end{aligned} \quad (3.48)$$

Denoting $f_1(x)$ as the first term of the above equation, (3.48) can be expressed as

$$f_0^{(1)}(x) = -f_1(x) - Qf_0(x). \quad (3.49)$$

which proves (3.45) for $m = 0$.

Next, we differentiate $f_1(x)$ to get its first order derivative

$$\begin{aligned} f_1^{(1)}(x) &= -(M)(M-1)(pe^{-Qx} + q)^{M-2}(-Q^2p^2e^{-3Qx}) \\ &\quad + (M)(pe^{-Qx} + q)^{M-1}(-Qpe^{-2Qx})(-2Q). \end{aligned} \quad (3.50)$$

By the definitions of $f_1(x)$ and $f_2(x)$, we have

$$f_1^{(1)}(x) = -f_2(x) - 2Qf_1(x). \quad (3.51)$$

Repeating the above procedures, (3.45) and (3.46) can be obtained.

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Using (3.45), the higher order derivatives of $f_M(x)$ can thus be expressed as

$$f_M^{(k)}(x) = (-1)^k ((M+1)Q)^k f_M(x) \quad (3.52)$$

where k is an integer greater than 1.

Upon closer inspection, it is found that $f_1(x) = -e^{-Qx}g_0(x)$, $f_2(x) = -e^{-Qx}g_1(x)$ and so on. With the new notations of $f_m(x)$ and $g_m(x)$, we can get the relationship

$$f_m(x) = -e^{-Qx}g_{m-1}(x) \quad (3.53)$$

for $m = 1, 2, \dots, M$. Further relationship of $f_m(x)$ and $g_m(x)$ is explored in the following Lemma.

Lemma 3.4. *Suppose the average number of photons produced by one optical pulses is controlled such that*

$$Q = \begin{cases} a \text{ value} \leq -\ln \left(1 - \frac{1}{(M+1)p} \right) & \text{if } (M+1)p > 1, \\ \text{any value} & \text{if } (M+1)p \leq 1. \end{cases} \quad (3.54)$$

Then, the functions $f_m(1)$ and $g_m(1)$ defined in Lemmas 3.2 and 3.3 satisfies

$$1) \quad -f_m^{(1)}(1) + g_m(1) > 0 \quad \text{for } m = 0, 1, \dots, M-1, \quad (3.55)$$

$$2) \quad -f_M^{(1)}(1) + g_M(1) > 0. \quad (3.56)$$

3.5 Properties of Auxiliary Functions

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Proof: For $m = 0, 1, \dots, M - 1$, it is known by letting $x = 1$ in Lemmas 3.2 and 3.3 that $-f_m^{(1)}(1) + g_m(1)$ can be expressed as

$$-f_m^{(1)}(1) + g_m(1) = f_{m+1}(1) + (m+1)Qf_m(1) + g_m(1). \quad (3.57)$$

Substituting the definitions of $f_m(x)$ and $g_m(x)$ into the right hand side of the above equation and factorizing the common factors, we have

$$\begin{aligned} & f_{m+1}(1) + (m+1)Qf_m(1) + g_m(1) \\ &= \binom{M}{m} p^m Q^{m+1} e^{-(m+1)Q} (pe^{-Q} + q)^{M-m-1} * \\ & \quad [(1 - e^{-Q})(mp + p - (M+1)p) + (m+1)(pe^{-Q} + q)]. \end{aligned} \quad (3.58)$$

Using condition (3.54), the first conclusion (3.55) of the lemma holds.

For $m = M$, it follows by using Lemmas 3.2 and 3.3 that

$$-f_M^{(1)}(1) + g_M(1) = (M+1)Qf_M(1) + g_M(1). \quad (3.59)$$

By definitions, $f_M(1)$ and $g_M(1)$ are always positive, so (3.56) is satisfied.

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Equation (3.54) gives the sufficient condition of (3.55) and (3.56). It relates the average of photons produced by one optical pulse to the number of multi-user interferences and the probability of an interference pulse appearing at a weight chip. The number of multi-user interference is determined by the cross-correlation of the signature code and the number of users. The probability of an optical pulse appearing at a chip is determined by the code parameters including the weight number and the code length. Thus, (3.54) gives the specifications on system parameters which support the conclusions derived in the lemma and the subsequent analysis. In practice, when the late two are given, we can choose appropriate photo-detector and tune the light power of

an optical pulse to control Q such that (3.54) can always be satisfied. Actually, when OOC code with cross-correlation being one is used, $(M + 1)p \leq 1$ is always satisfied regardless of Q , since in this case $M = N - 1$, $p = \frac{w}{2L}$, and (2.10) holds.

3.6 Optimality of OOK Chip-level Detection

With the preparation in the previous section, we are now in position to find the optimality of the threshold located for the chip-level receiver.

First, it is noted from (3.22) and (3.29) that the two auxiliary functions are related by

$$r_1(x) = e^{-Qx} r_0(x). \quad (3.60)$$

By using Lemma 3.1, the probabilities of receiving zero photon under different hypothesis are respective given by

$$P(X_i = 0|0) = r_0(1), \quad (3.61)$$

$$P(X_i = 0|1) = r_1(1). \quad (3.62)$$

It follows immediately by applying (3.60) that

$$P(X_i = 0|1) = e^{-Q} P(X_i = 0|0). \quad (3.63)$$

Since Q is the average number of photons produced by a pulse, it is always non-negative. Therefore, $e^{-Q} < 1$ for $Q > 0$ and in turn, we have

$$P(X_i = 0|0) > P(X_i = 0|1). \quad (3.64)$$

On the other hand, Lemma 3.4 has shown that $-f_0^{(1)}(x) + g_0(x) > 0$. Using the

definitions of $f_0(x) = r_1(x)$ and $g_0(x) = r_0^{(1)}(x)$, it follows from Lemma 3.1 that

$$P(X_i = 1|0) < P(X_i = 1|1). \quad (3.65)$$

With the results of (3.64) and (3.65), we can conclude that the threshold θ for chip-level receiver is one. More rigorously, in order to show that it is the only threshold, we need to further prove that $P(X_i = n|0) < P(X_i = n|1)$ for all $n > 1$.

Using the notations of f_m and g_m , we can represent (3.28) and (3.31) respectively as

$$P(X_i = n|0) = \frac{1}{n!}(-1)^n g_0^{n-1}(x) \Big|_{x=1}, \quad (3.66)$$

$$P(X_i = n|1) = \frac{1}{n!}(-1)^n f_0^n(x) \Big|_{x=1}. \quad (3.67)$$

Therefore, the problem of obtaining the optimal threshold can be achieved by showing

$$(-1)^n f_0^{(n)}(1) + (-1)^{n-1} g_0^{(n-1)}(1) > 0 \quad (3.68)$$

for $n > 1$ by using (3.66) and (3.67).

To this end, we need the following lemma for obtaining the monotonous relationship of $P(X_i = n|0) - P(X_i = n|1) < 0$ for all $n > 1$.

Lemma 3.5. *For all positive integers n , we have*

$$(-1)^{n+1} f_m^{(n+1)}(1) + (-1)^n g_m^{(n)}(1) > 0 \quad (3.69)$$

where $m = 0, 1, \dots, M$.

Proof: An induction strategy is adopted to prove the lemma. It is clear from Lemma 3.4 that this lemma holds for $n = 1$. Next, we prove the case when $n = 2$. This requires the proof that $f_m^{(2)}(1) - g_m^{(1)}(1) > 0$ for $m = 0, 1, \dots, M$. Using Lemmas 3.2

and 3.3, it is known that

$$f_m^{(2)}(1) - g_m^{(1)}(1) = (-f_{m+1}^{(1)}(1) + g_{m+1}(1)) + (m+1)Q(-f_m^{(1)}(1) + g_m(1)) \quad (3.70)$$

Applying the conclusions in Lemma 3.4 confirms that $f_m^{(2)}(1) - g_m^{(1)}(1) > 0$ holds for $0 \leq m \leq M-2$.

When $m = M-1$, we have

$$f_{M-1}^{(2)}(1) - g_{M-1}^{(1)}(1) = -f_M^{(1)}(1) + (M)Q(-f_{M-2}^{(1)}(1) + g_{M-2}(1)). \quad (3.71)$$

By definition, we know that $(-1)^1 f_M^{(1)}(1) > 0$. Again using Lemma 3.4 gives $-f_{M-1}^{(1)}(1) + g_{M-1}(1) > 0$. Thus, $f_{M-1}^{(2)}(1) - g_{M-1}^{(1)}(1) > 0$ is proven.

For $m = M$, we can obtain by using Lemmas 3.2 and 3.3 that

$$f_M^{(2)}(1) - g_M^{(1)}(1) = ((M+1)Q)^2 f_M(1) + (M)Q g_M(1). \quad (3.72)$$

The positivity of the above expression is obvious by the definitions of $f_M(1)$ and $g_M(1)$. Thus, we have proven that the lemma holds for $n = 2$.

To carry out the induction, we assume that the conclusion of the lemma is valid for all $n \leq k$, that is,

$$(-1)^k f_m^{(k)}(1) + (-1)^{k-1} g_{m+1}^{(k-1)}(1) > 0 \quad (3.73)$$

for $n = 1, 2, \dots, k$ and $m = 0, 1, \dots, M$.

Next, we prove the lemma is valid for $n = k+1$. In this case, we have

$$\begin{aligned} & (-1)^{k+1} f_m^{(k+1)}(1) + (-1)^k g_m^{(k)}(1) \\ &= [(-1)^k f_{m+1}^{(k)}(1) + (-1)^{k-1} g_{m+1}^{(k-1)}(1)] \\ &+ (m+1)Q[(-1)^k f_m^{(k)}(1) + (-1)^{k-1} g_m^{(k-1)}(1)] \end{aligned} \quad (3.74)$$

for $0 \leq m \leq M - 2$. When $m = M - 1$ and $m = M$, the item $(-1)^{k+1} f_m^{(k+1)}(1) + (-1)^k g_m^{(k)}(1)$ can be expressed respectively by

$$\begin{aligned} & (-1)^{k+1} f_{M-1}^{(k+1)}(1) + (-1)^k g_{M-1}^{(k)}(1) \\ &= (-1)^k f_M^{(k)}(1) + (M)Q[(-1)^k f_{M-1}^{(k)}(1) + (-1)^{k-1} g_{M-1}^{(k-1)}(1)], \end{aligned} \quad (3.75)$$

$$(-1)^{k+1} f_M^{(k+1)}(1) + (-1)^k g_M^{(k)}(1) = ((M+1)Q)^{k+1} f_M(1) + [(M)Q]^k g_M(1). \quad (3.76)$$

Using the induction assumption (3.73), the first and second components in (3.74) are positive. This in turn shows that $(-1)^{k+1} f_m^{(k+1)}(1) + (-1)^k g_m^{(k)}(1) > 0$ for $0 \leq m \leq M - 2$. Again under assumption (3.73), the second component in (3.75) is positive. Using the definition of $f_M(1)$, it is easily verified that $(-1)^k f_M^{(k)}(1) > 0$. Thus, $(-1)^{k+1} f_{M-1}^{(k+1)}(1) + (-1)^k g_{M-1}^{(k)}(1) > 0$. Lastly, (3.76) is obviously positive because of positivity of $f_M(1)$ and $g_M(1)$ defined in Lemmas 3.2 and 3.3.

Therefore, the lemma is valid for $n = k + 1$. By induction, the lemma is concluded.

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Using this lemma, the validity of (3.69) has been proven. With $x = 1$ and $m = 0$, we can conclude that

$$P(X_i = n|0) < P(X_i = n|1) \quad (3.77)$$

for $n > 1$. Therefore, the threshold given by $\theta = 1$ is unique and optimal in each weighted chip position using the ML criterion.

Remark: It is noted that the condition (3.54) is always achievable when the total number interference pulses and the signature code format (including the length and the weights number) are given. The controlling of the power of an optical pulse used to represent appearance of a chip “1” can be done using a circuit. This is to ensure that the threshold obtained is optimal and unique.

Next, the property of the optimal threshold being uniquely one is used to establish

the optimum bit error of the chip-level detection . It is seen that the optimality of BER can be obtained by proving

$$P(X_i \geq 1|0)^\omega + 1 - P(X_i \geq 1|1)^\omega \leq P(X_i \geq \theta|0)^\omega + 1 - P(X_i \geq \theta|1)^\omega \quad (3.78)$$

for any $\theta \neq 1$.

This inequality holds obviously for $\theta = 0$. Therefore, it suffices to prove its validation for $\theta \geq 2$.

Since threshold $\theta = 1$ shall minimizes the chip error rate, it follows that

$$P(X_i \geq 1|0) + 1 - P(X_i \geq 1|1) \leq P(X_i \geq \theta|0) + 1 - P(X_i \geq \theta|1). \quad (3.79)$$

On the other hand, it is obvious that for $\theta \geq 2$

$$P(X_i \geq \theta|0) \leq P(X_i \geq 1|0), \quad (3.80)$$

$$P(X_i \geq \theta|1) \leq P(X_i \geq 1|1). \quad (3.81)$$

In addition to that, the uniqueness of the optimal threshold at one implies that the two hypothesis probabilities $P(X_i|0)$ and $P(X_i|1)$ intersects only at $X_i = 1$. Since $P(X_i = 0|0) > P(X_i = 0|1)$, we can conclude that $P(X_i \geq \theta|1) > P(X_i \geq \theta|0)$ for $\theta \geq 1$, as illustrated in Figure 3.3. The figure is exaggerated in order to show the relationship of the two hypothesis probabilities. Thus, inequality (3.79) can be re-written as

$$0 \leq P(X_i \geq 1|0) - P(X_i \geq \theta|0) \leq P(X_i \geq 1|1) - P(X_i \geq \theta|1), \quad \text{for } \theta \geq 2. \quad (3.82)$$

To conclude (3.78), we introduce the following lemma.

Lemma 3.6. For the function $h(x) = x^n$ where n is an integer, it holds that

$$h(x_1) - h(x_2) \leq h(y_1) - h(y_2) \quad (3.83)$$

for any x_1, x_2, y_1 , and y_2 which satisfy

$$1) \quad x_1, x_2, y_1, y_2 \in [0, 1] \quad (3.84)$$

$$2) \quad 0 \leq x_1 - x_2 \leq y_1 - y_2 \quad (3.85)$$

$$3) \quad x_1 \leq y_1 \quad (3.86)$$

The proof of this lemma is trivial, it is omitted, while a graphic illustration using of conditions in Lemma 3.6 is given in Figure 3.4.

Figure 3.3: Chip error using different thresholds

Substituting $x_1 = P(X_i \geq 1|0)$, $x_2 = P(X_i \geq \theta|0)$, $y_1 = P(X_i \geq 1|1)$, and $y_2 = P(X_i \geq \theta|1)$ into Lemma 3.6, we have

$$0 \leq P(X_i \geq 1|0)^\omega - P(X_i \geq \theta|0)^\omega \leq P(X_i \geq 1|1)^\omega - P(X_i \geq \theta|1)^\omega \quad (3.87)$$

which concludes (3.78). Thus, we can determine the optimal BER of chip-level detection by thresholding the receiving photons of each weighted chip with a threshold

Figure 3.4: Bit errors using different threshold

of one.

Theorem 3.1. *Consider an OCDMA chip-level detector working under the conditions specified in Section 3.1 where the decision is based on photon counts. If the average photons of a optical pulse is controlled such that (3.54) is satisfied, then the optimal threshold for each chip is uniquely given by $\theta = 1$. An optimal CER will be obtained. If the photon counts received at each chip is independent of those at other chips, the minimum BER of the chip-level detection can be achieved using optimal CER and is given by*

$$P_E = \frac{1}{2}[P(X_i \geq 1|0)^\omega + (1 - (P(X_i \geq 1|1))^\omega)]. \quad (3.88)$$

From Theorem 3.1, we can see that the analytical model we introduced is effective in obtaining the BER performance of chip-level receiver. This is because the model is able to reduce the computation time on obtaining the optimal and unique threshold. The optimal threshold obtained is of value one and is regardless of the parameters Q , N , ω and L . The computational time can be further shortened as the threshold obtained in our analysis is proved to be unique.

3.7 OOK Chip-level Detection With a Limiter

In this section, we proposed a chip-level detector with a limiter. In the proposed chip-level detector, the light signal received at a chip is detected by a photo-detector. The received photon counts at each weighted chip are capped through a limiter before they are used for detection. Denote the photon counts received by the photodetector at the i -th weight chip as X_i and the capped photon counts as Y_i . They are related by

$$Y_i = \begin{cases} X_i, & \text{if } X_i < \theta_i, \\ \theta_i, & \text{otherwise.} \end{cases} \quad (3.89)$$

where θ_i is the cap level of the limiter set for the i -th chip output. Its value is chosen as a design parameter in the next section to simplify the optimal chip-level detector. Then, the signals received by any user can be represented by $\vec{Y} = [Y_1, Y_2, \dots, Y_\omega]$. The proposed chip-level detector is to use the receiving signal $\vec{Y}$ of a desired user to judge the bits sent by the user.

The optimal chip-level detector is achieved by performing MAP probability criterion as follows

$$P(\vec{Y}|1) \underset{\substack{\text{"0" sent}}}{\overset{\text{"1" sent}}{\geq}} P(\vec{Y}|0) \quad (3.90)$$

where $P(\vec{Y}|1)$ and $P(\vec{Y}|0)$ are the two likelihood probabilities. It is known that the detection satisfying the MAP criterion gives the minimal average BER.

Under the assumptions in Section 3.1, the likelihood probabilities in (3.90) can be expressed as

$$P(\vec{Y}|\alpha) = \prod_{i=1}^{\omega} P(Y_i|\alpha), \quad \alpha = 0, 1 \quad (3.91)$$

where $P(Y_i|\alpha)$ denotes the probability of receiving Y_i photons at the i -th chip when a bit α is sent. The MAP criterion (3.90) can also be expressed alternatively by using

the likelihood ratios as follows:

$$\prod_{i=1}^{\omega} \Lambda_i(n_i) \underset{\substack{\text{a bit "0" sent} \\ \text{a bit "1" sent}}}{\geq} 1 \quad (3.92)$$

where the likelihood ratio for each chip is defined as $\Lambda_i(n) = \frac{P(Y_i=n|1)}{P(Y_i=n|0)}$ and n denotes the photon counts at the i -th weighted chip.

Clearly, even the photon counts received at different weighted chips are independent as assumed in [33], the optimal chip-level detection resulting in the minimal BER cannot be easily determined, because there could be many combinations of $[n_1, n_2, \dots, n_{\omega}]$ satisfying (3.92). Next, we show that by taking advantage of using a limiter at each weighted chip, the optimal chip-level detection can be achieved by a simple AND chip-level detector. To this end, we derive first the likelihood probabilities in (3.90) and then obtain the explicit expressions of the likelihood ratios in (3.92).

Since Y_i is obtained by applying a limiter on X_i as shown in (3.89), the likelihood probabilities $P(Y_i|\alpha)$ can be obtained as

$$P(Y_i = n|\alpha) = \begin{cases} P(X_i = n|\alpha), & \text{if } n < \theta_i, \\ \sum_{n=\theta_i}^{\infty} P(X_i = n|\alpha), & \text{if } n = \theta_i. \end{cases} \quad (3.93)$$

Substituting $P(Y_i|\alpha)$ into the definition of the likelihood ratio $\Lambda_i(n)$ in (3.92), $\Lambda_i(n)$ at the j -th weighted chip can be obtained by

$$\Lambda_i(n) = \begin{cases} \frac{P(X_i=n|1)}{P(X_i=n|0)}, & \text{if } n < \theta_i, \\ \Lambda_i(\theta_i) \triangleq \frac{\sum_{n=\theta_i}^{\infty} P(X_i=n|1)}{\sum_{n=\theta_i}^{\infty} P(X_i=n|0)} = \frac{1 - \sum_{n=0}^{\theta_i-1} P(X_i=n|1)}{1 - \sum_{n=0}^{\theta_i-1} P(X_i=n|0)}, & \text{if } n = \theta_i. \end{cases} \quad (3.94)$$

As established in Appendix A, $\Lambda_i(n)$ is an increasing function of n when $n < \theta_i$. As a result, $\Lambda_i(n) \leq \max\{\Lambda_i(\theta_i - 1), \Lambda_i(\theta_i)\}$. A simple calculation shows that $\Lambda_i(0) = e^{-Q} < 1$ and $\Lambda_i(1) > 1$ for all i . Note that $\Lambda_i(\theta_i - 1)$ and $\Lambda_i(\theta_i)$ are functions of θ_i .

We can choose the cap level of the limiter θ_i and the average photon counts per pulse of the shot noise power such that

$$\max\{\Lambda_i(\theta_i - 1), \Lambda_i(\theta_i)\} \leq (\Lambda_i(0))^{-1/(\omega-1)} = e^{Q/(\omega-1)}. \quad (3.95)$$

Then, as long as there is a weighted chip such that $Y_i = 0$, the product $\prod_{i=1}^{\omega} \Lambda_i(n_i) < 1$, in turn, the likelihood ratio $\Lambda(n)$ in (3.92) is less than one and a bit “0” is judged. Otherwise, the likelihood ratio $\Lambda(n)$ is greater than one and a bit “1” is judged for the user. Therefore, the optimal chip-level detector using photon counts is a simple “AND” chip-level detector subject to the condition (3.95).

By using the monotonicity of $\Lambda_i(n)$, we can also draw some insightful remarks from the condition (3.95). Firstly, since $\Lambda_i(n)$ is an increasing function of n , it is easy to verify that $\Lambda_i(\theta_i)$ is an increasing function of θ_i . Thus, when a big cap level θ_i is used, the average photon counts per pulse Q of the shot noise must be big enough. Considering the extreme case, θ_i approaches to infinity, which means that no limiter is used. In this case, the average photon counts per pulse should be big enough in order to satisfy the condition (3.95). As a result, the signal received at a weighted chip can be treated as optical pulses. Then, the optimal chip-level detector is reduced to a simple “AND” chip-level detector without using limiters at weighted chips. This is just the conclusion of [33]. Secondly, if the average photon counts per pulse Q of the shot noise is chosen such that there is a cap level θ_i satisfying (3.95), then $\theta_i = 1$ is also a cap level satisfying the condition (3.95) with the chosen Q because of the monotonicity of $\Lambda_i(n)$. This means that the optimal chip-level detector can be achieved by performing hard limiters with a cap at $\theta_i = 1$ on weighted chips and therefore the detector is an “AND” chip-level detector.

3.8 Numerical Results

In order to verify the theoretical analysis, we consider an OCDMA system using $M = 25$, and OOC codes with $\omega = 4$ and $L = 2000$. We plot the two hypothesis probabilities against the number of photons for $Q = 25$ and $Q = 60$ in Figure 3.5 and 3.6 respectively. As can be seen in the figures, the threshold is found to be value one using the ML criterion in each weighted chip position.

Figure 3.5: Optimal threshold for chip-level receiver with $M = 25$, $\omega = 4$, $L = 2000$ and $Q = 25$

Figure 3.6: Optimal threshold for chip-level receiver with $M = 25$, $\omega = 4$, $L = 2000$ and $Q = 60$

Next, we will demonstrate the first practical case of using optimal CER to get the

BER bounds assuming the number of users is two power of the number of weighted chips and assuming any adjacent signature codes differ in only one chip position. In this case, we use $M = 32$, $\omega = 5$, $L = 2000$.

Figure 3.7: BER bounds using optimal CER

The actual BER performance and the approximated BER are compared in Figure 3.8 for the second practical case where we assume the receiving photon counts at any chip is independent of any other chip. It can be seen that the BER performance can be closely approximated by our analysis.

Figure 3.8: Actual BER and approximate BER with $M = 25$, $\omega = 4$, $L = 2000$ and $Q = 25$

The relationship of the threshold and the BER is plotted in Figures 3.9 and 3.10 using approximate BER and actual BER. By setting the optimal threshold obtained

using the ML criterion, the error probability of each chip will be minimized. With the minimized CER, this in turn will minimize the BER as shown in Theorem 3.1. Thus, by setting the optimal threshold $\theta = 1$, it will minimize the BER performance for different number of users contributing interference pulses as seen in the figures.

Figure 3.9: Relationship of threshold and approximate BER with $\omega = 4$, $L = 2000$ and $Q = 25$

Figure 3.10: Relationship of threshold and actual BER with $\omega = 4$, $L = 2000$ and $Q = 25$

Next, we analyze the effect of varying number of photons produced by each chip by comparing the actual and approximate BER with the optimal threshold $\theta = 1$. In Figure 3.11, it can be seen that the actual BER performance is closely approximated by our analysis.

Figure 3.11: Relationship of actual and approximate BER with different number of photons per chip pulse Q

We also compare the actual and approximate BER with varying weights, ω . From Figure 3.12, the approximate BER is close to the actual BER.

Figure 3.12: Relationship of actual and approximate BER with different weights ω

By taking into consideration of the signature code length L , Figure 3.13 shows the comparison of the actual and approximate BER. Again, it can be seen that the two curves are closely related.

Now, we want to examine the BER performance of the proposed chip-level detector with a limiter. We use $L = 2000$, and $\omega = 4$. However, in this case, the codes are assumed to be randomly generated by not taking into the account of the algorithmic

Figure 3.13: Relationship of actual and approximate BER different signature code length L

constructions of other users' codes as in [33]. We will compare the BER performance of the optimal chip-level detector, the AND chip-level detector, and the optimal correlation detector with the average photon counts per pulse Q taking different values. Given a value of Q , the BER for each detector is calculated against the number of interference pulses M .

Here the AND chip-level detector is constructed like the one using optical pulses in [79]. A chip "1" or "0" is decided individually at each weighted chip by

$$Y_i \underset{\substack{\text{a chip "0"}}}{\underset{\substack{\text{a chip "1"}}}{\geq}} 1 \quad (3.96)$$

Then, a bit "1" or "0" is judged by performing the "AND" operation on the decision results obtained at all weighted chips. As shown, $\Lambda_i(n)$ is an increasing function, $\Lambda_i(0) < 1$ and $\Lambda_i(1) > 1$. Therefore, the individual decision at each weighted chip is optimal in the sense of minimizing the average chip error but not the average bit error. In contrast to the chip-level detection, the optimal correlation detector uses $Y = \sum_{j=1}^{\omega} Y_j$ as the decision variable. The optimal decision is made by using the MAP criterion about Y to seek the optimal threshold to minimize the average bit error. When Y is greater than the threshold, a bit "1" is judged, otherwise, a bit "0" is

judged.

First, we consider choosing $Q = 15$ and $\theta_i = 30$ for all weighted chips, and the number of interference pulses ranges from $M = 10$ to $M = 100$. A straightforward calculation show that $\Lambda_i(29) = 36.6522$ and $\Lambda_i(30) = 40.7720$. Substituting these values into (3.95), the condition (3.95) is satisfied. This means the optimal chip-level detector is just an “AND” detector. To show this, we calculate the BERs respectively by using the optimal chip-level detector and the AND chip-level detector. In the calculation of BER of the optimal chip-level detector, we use (3.92) to judge a bit “1” or “0” directly. The BERs against the number of interference pulses are plotted in Figure 3.14 where the corresponding result of the optimal correlation detector is also plotted for comparison. The curves clearly show that the optimal chip-level detector and the AND chip-level detector yields the identical BER performance and they perform better than the optimal correlation detector. This is because the average photon counts per pulse Q is bigger enough and the shot noise effect on the signal received at weighted chips can be neglected.

Next, we reduce the average photon counts to $Q = 10$. In this case, the condition (3.95) is violated. We calculate the BERs for three detectors and plot the results as curves in Figure 3.15. Apparently, the BER of the AND chip-level detector is getting worse than the optimal chip-level detector when the number of interference pulses is fewer. To see the reason, we calculate the likelihood ratios $\Lambda_i(n)$. It is found that the AND chip-level detector does make wrong decisions with respect to the MAP criterion (3.92). For example, when the receiving signal is $\{Y_1 = 0, Y_2 = 1, Y_3 = 1, Y_4 = 1\}$, the AND chip-level detector decides a bit “0”. However, the corresponding likelihood ratios are given by $\Lambda_1(0) = 0.1353$, $\Lambda_2(1) = 20.1180$, $\Lambda_3(1) = 20.1180$, and $\Lambda_4(1) = 20.1180$, which make $\Lambda(n)$ in (3.92) greater than one. Thus, the optimal chip-level detector makes a decision of a bit “1”. Since the MAP criterion (3.92) minimizes the average bit error, the optimal chip-level detector yields the better BER performance

than the AND chip-level detector.

Lastly, we consider a worse case of the shot noise by taking $Q = 2$. The BER of the three detectors are plotted against the number of interference as shown in Figure 3.16. Comparing the curves, we can see that the AND chip-level detector loses its BER advantage in the presence of the severe shot noise while the optimal chip-level detector remains the best among the three detectors. This observation can be clearly understood by considering the amount of the information used in the detection. The AND chip-level detector only consider a local information at individual chips. Even though it achieves the local optimal chip error by fully exploiting the local information, it cannot attain the global optimality at the bit level where the correlation among chips should be taken into account. On the other hand, the optimal correlation detector goes to the other extreme case to highlight much more the global optimality without distinguishing the individual information at chips. In contrast, the optimal chip-level detector considers the global optimality by using the local chip information. This is the reason why the optimal chip-level detector always performs well for all the cases of the shot noise.

Figure 3.14: BER performance of the chip-level detectors and the correlation MAP detector (when $Q=15$)

Figure 3.15: BER performance of the chip-level detectors and the correlation MAP detector (when $Q=10$)

3.9 PPM Chip-level Detection

For enhancing the BER performance of M -ary transmission (we use D to denote the total number of symbols to avoid confusion, as M has been assigned to denote the total number of interference.), PPM-CDMA chip-level detection is suggested [55]. This detection method is similar to OOK-CDMA chip-level detection method. But in this case, the number of photons X_{ih} will be sampled at each weighted chip time i ($i = 1, 2, \dots, \omega$) for all time slot h ($h = 0, 1, 2, \dots, D-1$) after the correlation process of the received optical signal $r(t)$ with the desired user's signature code $y_g(t)$. With the number of photons collected in each time chip i and time slot h , the decision on which symbol is transmitted can be made.

The decision rule for PPM-CDMA chip-level receiver is made up of two parts in order to declare symbol d is transmitted for d in the range $1 \leq d \leq D$. The first condition is that the number of photons collected for every i chip time in the d time slot must be greater than or equal to the threshold θ . The next condition is that there must be

Figure 3.16: BER performance of the chip-level detectors and the correlation MAP detector (when $Q=2$)

at least one i chip time for all $h \neq d$ time slot having the photons number less than θ . The decision rule can be written as [79]

$$\text{Declare} \begin{cases} d, & \text{if } X_{id} \geq \theta \forall i, X_{ih} < \theta \text{ for some } i, h \neq d \\ \text{Wrong decision,} & \text{otherwise.} \end{cases} \quad (3.97)$$

The probability of a word error is upper bounded as follows [79]

$$P_E \leq P(X_{i0} < \theta, \text{ for some } i \in \iota|0) + (M-1)P(X_{i1} \geq \theta, \forall i \in \iota|0) \quad (3.98)$$

for $\iota = 1, 2, \dots, \omega$.

3.10 Optimality of PPM Chip-level Detection

The threshold of each weighted chip position of OCDMA system using PPM chip-level detection can be found by applying the MAP criterion. The optimal detection for

chip-level receiver using PPM can be assessed by investigating the optimal threshold in each weighted chip position.

Now, the threshold can be again obtained by using the MAP criterion

$$P(X_{id}|d)P(d) \underset{X_i < \theta}{\overset{X_i \geq \theta}{\gtrless}} P(X_{id} \neq d)P(\neq d). \quad (3.99)$$

With the assumption of equal probability of transmitting d symbol, $P(d) = \frac{1}{D}$ and $P(\neq d) = \frac{D-1}{D}$.

Now we will derive the likelihood probabilities $P(X_{id}|d)$ and $P(X_{id} \neq d)$ in (3.99). Note that the probability of an optical pulse appearing on a weighted chip position is $\frac{\omega}{DL}$. Thus, the probability of having ζ interference pulses in (id) -th weighted chip is

$$P(I_{id} = \zeta) = \binom{M}{\zeta} \left(\frac{\omega}{DL}\right)^\zeta \left(1 - \frac{\omega}{DL}\right)^{M-\zeta} \quad (3.100)$$

where $\zeta = 0, 1, \dots, M$.

So the likelihood probability can be obtained as

$$P(X_{id} = n|d) = \sum_{\zeta=0}^M \frac{(e^{-Q(\zeta+1)})(Q(\zeta+1))^n}{n!} \binom{M}{\zeta} p^\zeta q^{M-\zeta}, \quad (3.101)$$

$$P(X_{id} = n| \neq d) = \sum_{\zeta=0}^M \frac{(e^{-Q\zeta})(Q\zeta)^n}{n!} \binom{M}{\zeta} p^\zeta q^{M-\zeta} \quad (3.102)$$

where $p = \frac{\omega}{DL}$ and $q = 1 - p$.

Using the auxiliary functions (3.22) and (3.29), it can be seen that (3.101) and (3.102) can be respectively expressed as

$$P(X_{id} = n|d) = \frac{1}{n!} (-1)^n r_1^{(n)}(x)|_{x=1}, \quad (3.103)$$

$$P(X_{id} = n | \neq d) = \frac{1}{n!} (-1)^n r_0^{(n)}(x) \Big|_{x=1}. \quad (3.104)$$

Although the value of p is different for OOK and PPM modulation technique, it will not affect the expressions as they are independent of p .

Hence, summarizing the above discussion we have the following conclusion.

Lemma 3.7. *Let $r_0(x) = (pe^{-Qx} + q)^M$ and $r_1(x) = e^{-Qx}(pe^{-Qx} + q)^M$. Then, the probabilities $P(X_{id} = n | d)$ and $P(X_{id} = n | \neq d)$ in (3.101) and (3.102) are given*

$$P(X_{id} = n | h) = \frac{1}{n!} (-1)^n r_h^{(n)}(x) \Big|_{x=1}, \quad \text{for } h = d, \neq d. \quad (3.105)$$

Proof: The result follows from the derivation of (3.103) and (3.104).

▲▲▲

By bringing in the properties of the auxiliary functions, we will get

$$f_0 = r_1(x), \quad (3.106)$$

$$g_0 = r_0^{(1)}(x). \quad (3.107)$$

Using the MAP criterion, the threshold can be found to be

$$\frac{P(X_{id} = n | d)}{P(X_{id} = n | \neq d)} \underset{x_i < \theta}{\overset{x_i \geq \theta}{\gtrless}} \frac{P(\neq d)}{P(d)}. \quad (3.108)$$

The probabilities of receiving zero photon under different hypothesis using Lemma 3.7 are given by

$$P(X_{id} = 0 | \neq d) = r_0(1), \quad (3.109)$$

$$P(X_{id} = 0 | d) = r_1(1). \quad (3.110)$$

As the two auxiliary functions are related by (3.60), it follows that

$$\frac{r_1(1)}{r_0(1)} = e^{-Q}. \quad (3.111)$$

Substituting (3.109) and (3.110) into (3.111), we will get

$$\frac{P(X_{id} = 0|d)}{P(X_{id} = 0| \neq d)} = e^{-Q}. \quad (3.112)$$

As the probabilities of transmitting d symbols is assumed equal, it is found that

$$\frac{P(\neq d)}{P(d)} = D - 1. \quad (3.113)$$

Comparing (3.112) and (3.113) for $D > 1$,

$$\frac{P(X_{id} = 0|d)}{P(X_{id} = 0| \neq d)} < \frac{P(\neq d)}{P(d)}. \quad (3.114)$$

Next, the probabilities of receiving one photon under different hypothesis using Lemma 3.7 are given by

$$P(X_{id} = 1| \neq d) = -r_0^{(1)}(1), \quad (3.115)$$

$$P(X_{id} = 1|d) = -r_1^{(1)}(1). \quad (3.116)$$

Using the notation of f_m and g_m , we can get

$$\frac{P(X_{id} = 1|d)}{P(X_{id} = 1| \neq d)} = \frac{f_0^{(1)}}{g_0}. \quad (3.117)$$

Utilizing $0 < \omega DL - 2DL + L - \omega^2 + \omega + D - 1$ for $D > 1$, $\omega > 1$ and $L \gg \omega^2$, we

can derive

$$\frac{L-1}{\omega(\omega-1)} < \frac{\frac{1}{p}-1}{D-1}. \quad (3.118)$$

If we employed Johnson bound with OOC cross- and auto-correlation bounded by one, we are able to form the following inequality

$$M+1 \leq \frac{L-1}{\omega(\omega-1)} < \frac{\frac{1}{p}-1}{D-1} < \frac{(M+1)e^{-Q} + \frac{1}{p} - 1}{D-1} + 1. \quad (3.119)$$

Hence, it can be seen from (3.119) that

$$(M+1) < \frac{(M+1)e^{-Q} + \frac{1}{p} - 1}{D-1} + 1. \quad (3.120)$$

After manipulation of terms in (3.120), we are able to obtain

$$D-1 < \frac{(M+1)e^{-Q} + \frac{1}{p} - 1}{M}. \quad (3.121)$$

The right hand side of equation (3.121) is found to be

$$\frac{(M+1)e^{-Q} + \frac{1}{p} - 1}{M} = \frac{f_0^{(1)}}{g_0}. \quad (3.122)$$

Thus, using (3.113), (3.117), (3.121) and (3.122), we are able to attain

$$\frac{P(\neq d)}{P(d)} < \frac{P(X_{id} = 1|d)}{P(X_{id} = 1| \neq d)}. \quad (3.123)$$

Therefore, we can conclude that the threshold for PPM chip-level detection is given by $\theta = 1$ using (3.114) and (3.123) satisfying the MAP criterion (3.108). Using the analysis in the previous section, this threshold can be shown to minimize the BER performance.

3.11 Conclusion

In this chapter, OCDMA chip-level detection proposed by *Shalaby*, which is based on the photon counts, is studied. First, two auxiliary functions are introduced to simplify the complex expressions of the probability of photon counts obtained respectively under the hypothesis of bit “1” being sent and the hypothesis of bit “0” being sent. It is shown that the probabilities can be obtained exactly by differentiating the auxiliary functions. Together with the properties of the auxiliary function established, the threshold in each weighted chip position is proven to be one. This value is shown to be unique. An approximate BER performance using optimal CER is achieved by establishing optimality in each weighted chip position is proposed to simplify the performance analysis. In addition to that, this threshold is shown to minimize the chip-level detection BER. Thus, it can be concluded to be unique and optimal. The obtained threshold is the optimal threshold used to achieve an optimal OCDMA chip-level detection. The same conclusion can also be reached for the proposed chip-level receiver using a limiter. So it can be seen that regardless of the methods used for implementing chip-level receiver, the traffic loading and the signature code length and pattern, the algorithm used in our analysis is efficient for chip-level detection. Other methods are unable to prove that the threshold obtained is unique and optimal. Even though the theoretical results in this thesis are obtained for chip-level receiver, they apparently can be extended to a more general case to tackle the problem of obtaining the threshold for other type of receivers. Thus, the proposed auxiliary function scheme provides an efficient method to compute the likelihood probabilities of the receiver. This in turn enables us to assess and investigate the optimal BER performance of optical detection. The analytical model introduced also reflects more closely to the performance of the actual OCDMA system. However, the limitation of the model is the need to assume independency of the photon counts obtained in each chip.

3.12 Appendix

The derivation of equation (3.17) is given as follow.

Let κ_j be the number of interference pulses in j weighted chip where $j \in \{1, 2, \dots, \omega\}$.

Let the vector $(\kappa_1, \kappa_2, \dots, \kappa_i)^T$ be denoted by κ^i . Using OOC, κ^i is a multinomial random vector with parameters $\frac{\omega}{2L}$ and $N - 1$ [79].

$$P(\kappa^i = \zeta^i) = \frac{(N-1)!}{\zeta_1! \zeta_2! \dots \zeta_i! (N-1 - \sum_{j=1}^i \zeta_j)!} \left(\frac{\omega}{2L}\right)^{\sum_{j=1}^i \zeta_j} \left(1 - i \frac{\omega}{2L}\right)^{N-1 - \sum_{j=1}^i \zeta_j}. \quad (3.124)$$

Let $X = (\kappa_1 + \kappa_2 + \dots + \kappa_i)$ where X is the total number of interference pulses in all i chips. Thus,

$$\begin{aligned} P(X = n) &= P(\kappa_1 + \kappa_2 + \dots + \kappa_i = n) \\ &= \sum_{\zeta} \frac{(N-1)!}{\zeta_1! \zeta_2! \dots \zeta_i! (N-1 - \sum_{j=1}^i \zeta_j)!} \left(\frac{\omega}{2L}\right)^{\sum_{j=1}^i \zeta_j} \left(1 - i \frac{\omega}{2L}\right)^{N-1 - \sum_{j=1}^i \zeta_j}. \end{aligned} \quad (3.125)$$

Next, we introduce an identify to help us simplify equation (3.125).

$$\begin{aligned} 1 &= 1^n = \left(\frac{1}{i} + \frac{1}{i} + \dots + \frac{1}{i}\right)^n \\ &= \sum_{\zeta} \frac{n!}{\zeta_1! \zeta_2! \dots \zeta_i!} \left(\frac{1}{i}\right)^{\zeta_1} \left(\frac{1}{i}\right)^{\zeta_2} \dots \left(\frac{1}{i}\right)^{\zeta_i} \\ &= \sum_{\zeta} \frac{n!}{\zeta_1! \zeta_2! \dots \zeta_i!} \left(\frac{1}{i}\right)^{\sum_{j=1}^i \zeta_j}. \end{aligned} \quad (3.126)$$

From (3.126), we are able to get

$$\sum_{\zeta} \frac{1}{\zeta_1! \zeta_2! \dots \zeta_i!} = \frac{1}{n! \left(\frac{1}{i}\right)^{\sum_{j=1}^i \zeta_j}}. \quad (3.127)$$

Therefore, by substituting (3.127) into (3.125), we will get

$$P(X = n) = \frac{(N-1)!}{n!(N-1-n)!} \left(i\frac{\omega}{2L}\right)^n \left(1 - i\frac{\omega}{2L}\right)^{N-1-n} \quad (3.128)$$

where X now follows a Binomial distribution with parameters $\frac{i\omega}{2L}$ and $N-1$. Now if we look at only one chip, $i = 1$ and if we do not consider OOC, the number of interference pulses $N-1$ can be replaced by variable M . Next, by replacing X by I_i and n by ζ , equation (3.16) can be obtained.

If using OOC with cross- and auto-correlation bounded by one, the two auxiliary functions with their properties are summarized and given below.

$$g_0 = (N-1)(pe^{-Qx} + q)^{N-2}(-Qpe^{-Qx})$$

For $m < N-2$	
$g_0^{(1)} = -g_1 - Qg_0$ $g_1^{(1)} = -g_2 - 2Qg_1$ $g_2^{(1)} = -g_3 - 3Qg_2$ $g_3^{(1)} = -g_4 - 4Qg_3$ $\vdots$ $\vdots$ $g_m^{(1)} = -g_{m+1} - (m+1)Qg_m$ $\vdots$ $\vdots$ $g_{N-3}^{(1)} = -g_{N-2} - (N-2)Qg_{N-3}$	$g_1 = {}^p(N-1 \choose 2)(pe^{-Qx} + q)^{N-3}(Q^2p^2e^{-2Qx})$ $g_2 = {}^p(N-1 \choose 3)(pe^{-Qx} + q)^{N-4}(Q^3p^3e^{-3Qx})$ $g_3 = {}^p(N-1 \choose 4)(pe^{-Qx} + q)^{N-5}(Q^4p^4e^{-4Qx})$ $g_4 = {}^p(N-1 \choose 5)(pe^{-Qx} + q)^{N-6}(Q^5p^5e^{-5Qx})$ $\vdots$ $\vdots$ $g_{m+1} = {}^p(N-1 \choose m+2)(pe^{-Qx} + q)^{N-(m+3)}(Q^{m+2}p^{m+2}e^{-(m+2)Qx})$ $\vdots$ $\vdots$ $g_{N-2} = {}^p(N-1 \choose N-1)Q^{N-1}p^{N-1}e^{-(N-1)Qx}$
For $m \geq N-2$	
$g_{N-2}^{(1)} = -(N-1)Qg_{N-2}$ $g_{N-1}^{(1)} = -(N-1)Qg_{N-1}$ $g_{N-1}^{(k)} = (-1)^k((N-1)Q)^k g_{N-1}$	$g_{N-1} = g_{N-2}^{(1)}$ $g_{N-1} = (N-1)Q({}^p(N-1 \choose N-1)Q^{N-1}p^{N-1}e^{-(N-1)Qx})$

where ${}^p(N-1 \choose m+2) = (N-1)(N-2)\dots(N-(m+2))$ for $m < N-2$.

Table 3.1: Properties of auxiliary function for likelihood probability of transmitting a bit “0”

$f_0 = e^{-Qx}(pe^{-Qx} + q)^{N-1}$

For $m \leq N - 2$	
$f_0^{(1)} = -f_1 - Qf_0$ $f_1^{(1)} = -f_2 - 2Qf_1$ $f_2^{(1)} = -f_3 - 3Qf_2$ $f_3^{(1)} = -f_4 - 4Qf_3$ $\vdots$ $\vdots$ $f_m^{(1)} = -f_{m+1} - (m + 1)Qf_m$ $\vdots$ $\vdots$ $f_{N-2}^{(1)} = -f_{N-1} - (N - 1)Qf_{N-2}$	$f_1 = {}^p\binom{N-1}{1}(pe^{-Qx} + q)^{N-2}(Qpe^{-2Qx})$ $f_2 = {}^p\binom{N-1}{2}(pe^{-Qx} + q)^{N-3}(Q^2p^2e^{-3Qx})$ $f_3 = {}^p\binom{N-1}{3}(pe^{-Qx} + q)^{N-4}(Q^3p^3e^{-4Qx})$ $f_4 = {}^p\binom{N-1}{4}(pe^{-Qx} + q)^{N-5}(Q^4p^4e^{-5Qx})$ $\vdots$ $\vdots$ $f_{m+1} = {}^p\binom{N-1}{m+1}(pe^{-Qx} + q)^{N-(m+2)}(Q^{m+1}p^{m+1}e^{-(m+2)Qx})$ $\vdots$ $\vdots$ $f_{N-1} = {}^p\binom{N-1}{N-1}(Q^{N-1}p^{N-1}e^{-NQx})$
For $m = N - 1$	
$f_{N-1}^{(1)} = (-NQ)f_{N-1}$ $f_{N-1}^{(k)} = (-1)^k(NQ)^kf_{N-1}$	

where ${}^p\binom{N-1}{m+1} = (N - 1)(N - 2) \dots (N - (m + 1))$ for $m \leq N - 2$.

Table 3.2: Properties of auxiliary function for likelihood probability of transmitting a bit “1”

$f_1 = -e^{-Qx}g_0$ $f_2 = -e^{-Qx}g_1$ $f_3 = -e^{-Qx}g_2$ $f_4 = -e^{-Qx}g_3$ $\vdots$ $\vdots$ $f_{m+1} = -e^{-Qx}g_m$ $\vdots$ $\vdots$ $f_{N-1} = -e^{-Qx}g_{N-2}$
--

Table 3.3: The relationship between two auxiliary functions

$(-1)^n f_m^{(n)} + (-1)^{n-1} g_m^{(n-1)} > 0$

		n				
		1	2	..	n	..
m	0	$-f_0^{(1)} + g_0 > 0$	$f_0^{(2)} - g_0^{(1)} > 0$	..	$(-1)^n f_0^{(n)} + (-1)^{n-1} g_0^{(n-1)} > 0$	..
	1	$-f_1^{(1)} + g_1 > 0$	$f_1^{(2)} - g_1^{(1)} > 0$	..	$(-1)^n f_1^{(n)} + (-1)^{n-1} g_1^{(n-1)} > 0$	..
	:	:	:	..	:	..
	:	:	:	..	:	..
	m	$-f_m^{(1)} + g_m > 0$	$f_m^{(2)} - g_m^{(1)} > 0$	..	$(-1)^n f_m^{(n)} + (-1)^{n-1} g_m^{(n-1)} > 0$	..
	:	:	:	..	:	..
	:	:	:	..	:	..
	$N - 1$	$-f_{N-1}^{(1)} + g_{N-1} > 0$	$f_{N-1}^{(2)} - g_{N-1}^{(1)} > 0$	..	$(-1)^n f_{N-1}^{(n)} + (-1)^{n-1} g_{N-1}^{(n-1)} > 0$	..

Table 3.4: Relationship between the auxiliary function of likelihood probabilities for photon number n greater than zero

Chapter 4

Approximate Optimal Chip-level Detection in the Presence of Uncertain Transmission Probabilities

4.1 Introduction

In the previous chapter, we have investigated the optical detection for chip-level receiver and its optimality is presented. Its performance is evaluated based on the equal probability of transmitting a bit “0” and bit “1”. However, this is not the case in practical situation where some source symbols will occur more frequently than others. Moreover, non-equiprobable data transmission will also arise by employing variable length codewords for coding the source symbols. Hence, these will give rise to unequal probabilities of transmitting of a bit “0” and bit “1”.

This chapter examined the problem of optical detection of chip-level receiver where

the probabilities data bit transmission is non-equiprobable. Some sort of relationship such as the monotonicity property with respect to the likelihood probabilities of the photon counts obtained is needed to be established to address the problem. The difficulty in obtaining this monotonicity property is again due to the mixed Poisson-binomial terms in the likelihood probabilities of the respective receiver. But this difficulty can be overcome by the two auxiliary functions proposed in chapter 2. By setting up new properties for the auxiliary functions, the monotonicity property is investigated and established.

In practice, it is difficult to obtain information on the exact probability of transmitting a bit “0” and bit “1”. Thus, a range of *a priori* probabilities is used instead. In this case, only a BER bound is obtained in its place of an exact BER value. In the presence of uncertain *a priori* probabilities, the monotonicity property of the likelihood ratio is essential to locate this BER bound. In addition to that, the monotonicity property can be used to determine the robust margin of *a priori* probabilities corresponding to a given threshold or a BER level. The results acquired can be further extended to the performance analysis for the correlation receiver.

4.2 Properties of Auxiliary Functions

It is shown in the previous chapter that the two hypothesis probabilities can be represented by using $r_0(x)$ and $r_1(x)$ as

$$P(X_i = n|j) = \frac{1}{n!}(-1)^n r_j^{(n)}(x) \Big|_{x=1}, \quad \text{for } j = 0, 1 \quad (4.1)$$

where the superscript (n) denotes the n -th order derivatives of $r_j(x)$ with respect to x .

In this section, we derive several important properties of the auxiliary functions using

OOc as signature code. They are useful for deriving the monotonicity property of the likelihood ratio in the next section.

In order to handle various order derivatives of $r_0(x)$ and $r_1(x)$ in (4.1), we define two families of functions, which are respectively denoted by g and f , as follows:

$$g_0(x) = r_0^{(1)}(x), \quad (4.2)$$

$$g_{m+1}(x) = -P_{m+2}^{N-1}(pe^{-Qx} + q)^{N-m-3}(Q^{m+2}p^{m+2}e^{-(m+2)Qx}) \quad (4.3)$$

for $m = 0, 1, \dots, N-3$.

$$g_{N-1}(x) = g_{N-2}^{(1)}(x), \quad (4.4)$$

$$f_0(x) = r_1(x), \quad (4.5)$$

$$f_{m+1}(x) = P_{m+1}^{N-1}(pe^{-Qx} + q)^{N-m-2}(Q^{m+1}p^{m+1}e^{-(m+2)Qx}) \quad (4.6)$$

for $m = 0, 1, \dots, N-2$ where P_{m+2}^{N-1} denotes the permutation, i.e., $P_{m+2}^{N-1} = (N-1)(N-2)\dots(N-m-2)$.

As shown in previous chapter, the two families of functions have the following properties.

Lemma 4.1. *Using the definitions in (4.2)-(4.6), the following equations hold.*

$$1) \quad g_m^{(1)}(x) = -g_{m+1}(x) - (m+1)Qg_m(x), \quad \text{for } m = 0, 1, \dots, N-3 \quad (4.7)$$

$$2) \quad g_{N-2}^{(1)}(x) = -(N-1)Qg_{N-2}(x) \quad (4.8)$$

$$3) \quad g_{N-1}^{(1)}(x) = -(N-1)Qg_{N-1}(x) \quad (4.9)$$

$$4) \quad g_{N-1}^{(k)}(x) = (-1)^k [(N-1)Q]^k g_{N-1}(x), \quad \text{for } k > 1 \quad (4.10)$$

$$5) \quad f_m^{(1)}(x) = -f_{m+1}(x) - (m+1)Qf_m(x), \quad \text{for } m = 0, 1, \dots, N-2 \quad (4.11)$$

$$6) \quad f_{N-1}^{(1)}(x) = -NQf_{N-1}(x) \quad (4.12)$$

$$7) \quad f_{N-1}^{(k)}(x) = (-1)^k (NQ)^k f_{N-1}(x), \quad \text{for } k > 1 \quad (4.13)$$

$$8) \quad f_m(x) = -e^{-Qx} g_{m-1}(x), \quad \text{for } m = 1, 2, \dots, N-1. \quad (4.14)$$

As the subsequent analysis involves inequalities of the two families of functions, the sign properties of these functions are derived first. These properties are summarized in Lemma 4.2 to Lemma 4.4 as follows.

Lemma 4.2. *By differentiating $g_m(x)$ once, we have*

$$1) \quad g_m^{(1)}(x) > 0, \quad \text{for } m = 0, 1, \dots, N-2 \quad (4.15)$$

$$2) \quad g_{N-1}^{(1)}(x) < 0. \quad (4.16)$$

Proof: From Lemma 4.1, we can see that $g_m^{(1)}(x) = -g_{m+1}(x) - (m+1)Qg_m(x)$ for $m = 0, 1, \dots, N-3$ and $g_{N-2}^{(1)}(x) = -(N-1)Qg_{N-2}(x)$. From the definition in (4.3), $g_m(x) < 0$, for $m = 0, 1, \dots, N-2$, we can conclude $g_m^{(1)}(x) > 0$ for $m = 0, 1, \dots, N-2$. Next, we can obtain $g_{N-1}^{(1)}(x)$ by substituting $k = 1$ into (4.10). As $g_{N-1}(x) = g_{N-2}^{(1)}(x) > 0$, we will get $g_{N-1}^{(1)}(x) < 0$. Thus, Lemma 4.2 is concluded.

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Lemma 4.3. For higher order derivatives of $g_m(x)$, we have, for $m = 0, 1, \dots, N-2$,

$$(-1)^p g_m^{(p)}(x) < 0 \text{ for } p \geq 1. \quad (4.17)$$

Proof: We will prove Lemma 4.3 by induction. Using Lemma 4.2, Lemma 4.3 holds when $p = 1$. For $p = 2$, $g_m^{(2)}(x) = -g_{m+1}^{(1)}(x) - (m+1)Qg_m^{(1)}(x)$ for $m = 0, 1, \dots, N-3$, which is obtained by differentiating (4.7) once. As seen from Lemma 4.2, $g_{m+1}^{(1)}(x) > 0$ and $g_m^{(1)}(x) > 0$ and thus it gives $g_m^{(2)}(x) < 0$ for $m = 0, 1, \dots, N-3$. For $m = N-2$, it follows from (4.8) that $g_{N-2}^{(2)}(x) = -(N-1)Qg_{N-2}^{(1)}(x)$ and $g_{N-2}^{(1)}(x) = -(N-1)Qg_{N-2}(x)$. By the definition of $g_{N-2}(x)$, $g_{N-2}(x) < 0$. Thus, $g_{N-2}^{(2)}(x) < 0$. This has proven that Lemma 4.3 holds for $p = 2$.

To carry out the induction, we assume that the conclusion of Lemma 4.3 is valid for all $p \leq k$, that is,

$$(-1)^p g_m^{(p)}(x) < 0 \quad (4.18)$$

for $p = 1, 2, \dots, k$ and $m = 0, 1, \dots, N-2$.

Then, we are going to prove that Lemma 4.3 is valid for $p = k+1$. In this case, we will show

$$(-1)^{k+1} g_m^{(k+1)}(x) < 0 \quad (4.19)$$

for $m = 0, 1, \dots, N-2$.

By expanding the left hand side of (4.19), we have

$$1) \quad (-1)^{k+1} g_m^{(k+1)}(x) = (-1)^{k+1} [-g_{m+1}^{(k)}(x) - (m+1)Qg_m^{(k)}(x)] \quad (4.20)$$

$$2) \quad (-1)^{k+1} g_{N-2}^{(k+1)}(x) = (-1)^{k+1} [-(N-1)Qg_{N-2}^{(k)}(x)] \quad (4.21)$$

for $m = 0, 1, \dots, N-3$.

When k is even, $g_m^{(k)}(x) < 0$ under induction assumption (4.18) for $m = 0, 1, \dots, N-2$.

Putting $g_m^{(k)}(x) < 0$ into (4.20) and (4.21) gives $g_m^{(k+1)}(x) > 0$ for $m = 0, 1, \dots, N-2$. On the other hand, when k is odd, induction assumption (4.18) shows that $g_m^{(k)}(x) > 0$ for $m = 0, 1, \dots, N-2$. This gives $g_m^{(k+1)}(x) < 0$ for $m = 0, 1, \dots, N-2$. Hence, we can conclude that the lemma is valid for $p = k+1$, in turn, Lemma 4.3 is proven by induction.

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Lemma 4.4. *By differentiating $f_m(x)$ once, we have*

$$f_m^{(1)}(x) < 0, \text{ for } m = 0, 1, \dots, N-1. \quad (4.22)$$

Proof: From Lemma 4.1, we can see that $f_m^{(1)}(x) = -f_{m+1}(x) - (m+1)Qf_m(x)$ for $m = 0, 1, \dots, N-2$. Since $f_{m+1}(x) > 0$ and $f_m(x) > 0$, we can conclude that $f_m^{(1)}(x) < 0$ for $m = 0, 1, \dots, N-2$. For $m = N-1$, $f_{N-1}^{(1)}(x) = -NQf_{N-1}(x)$ and we can see that $f_{N-1}(x) > 0$ by the definition of $f_{N-1}(x)$. Hence, $f_{N-1}^{(1)}(x) < 0$. Thus, Lemma 4.4 is concluded.

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Lemma 4.5. *Regarding the higher order derivatives of $f_m(x)$ for $m = 0, 1, \dots, N-1$, we have,*

$$(-1)^p f_m^{(p)}(x) > 0 \text{ for } p \geq 1. \quad (4.23)$$

Proof: We will prove Lemma 4.5 by induction. From Lemma 4.4, Lemma 4.5 holds when $p = 1$. Next, we consider $p = 2$. Differentiating (4.11) once gives $f_m^{(2)}(x) = -f_{m+1}^{(1)}(x) - (m+1)Qf_m^{(1)}(x)$ for $m = 0, 1, \dots, N-2$. Using Lemma 4.4, $f_{m+1}^{(1)}(x) < 0$ and $f_m^{(1)}(x) < 0$. Hence, $f_m^{(2)}(x) > 0$ for $m = 0, 1, \dots, N-2$. For $m = N-1$, the inequality $f_{N-1}^{(2)}(x) > 0$ can be proven by substituting $k = 2$ into (4.13) and using $f_{N-1}(x) > 0$. Thus, Lemma 4.5 holds for $p = 2$.

To proceed by induction, we assume the conclusion of Lemma 4.5 is valid for all $p \leq k$,

that is,

$$(-1)^p f_m^p(x) > 0 \quad (4.24)$$

for $p = 1, 2, \dots, k$ and $m = 0, 1, \dots, N - 1$.

Then, we prove that Lemma 4.5 is valid for $p = k + 1$, that is

$$(-1)^{k+1} f_m^{k+1}(x) > 0 \quad (4.25)$$

for $m = 0, 1, \dots, N - 1$.

By using $f_m^{k+1}(x) = (f_m^k(x))'$ and Lemma 4.1, we obtain

$$1) \quad (-1)^{k+1} f_m^{(k+1)}(x) = (-1)^{k+1} [-f_{m+1}^{(k)}(x) - (m+1)Qf_m^{(k)}(x)] \quad (4.26)$$

$$2) \quad (-1)^{k+1} f_{N-1}^{(k+1)}(x) = (-1)^{k+1} [-NQf_{N-1}^{(k)}(x)] \quad (4.27)$$

for $m = 0, 1, \dots, N - 2$.

When k is even, $f_m^{(k)}(x) > 0$ for $m = 0, 1, \dots, N - 1$ under induction assumption (4.24). Thus, $f_m^{(k+1)}(x) < 0$ for $m = 0, 1, \dots, N - 1$. When k is odd, $f_m^{(k)}(x) < 0$ for $m = 0, 1, \dots, N - 1$ under induction assumption (4.24), which results in $f_m^{(k+1)}(x) > 0$ for $m = 0, 1, \dots, N - 1$. Therefore, the lemma is valid for $p = k + 1$. By induction, Lemma 4.5 is concluded.

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Using the definitions in (4.2)-(4.6), the likelihood ratio can be expressed as $\Lambda(n) = \frac{f_0^{(n+1)}(x)}{g_0^{(n)}(x)} \Big|_{x=1}$. To obtain the monotonicity of $\Lambda(n)$, we turn to establish the monotonicity of $\frac{f_{m+1}^{(n)}(x)}{g_{m+1}^{(n-1)}(x)}$ with respect to m . For this purpose, we first derive the monotonicity of $\frac{f_m^{(1)}(x)}{g_m(x)}$ with respect to m .

Lemma 4.6. *The functions $f_m^{(1)}(x)$ and $g_m(x)$ defined in Lemma 4.1 satisfy*

$$\frac{f_m^{(1)}(x)}{g_m(x)} < \frac{f_{m+1}^{(1)}(x)}{g_{m+1}(x)} \quad (4.28)$$

for $m = 0, 1, \dots, N - 3$.

Proof: As $p \leq 1 \leq pe^{-Qx} + 1$, we are able to obtain

$$\frac{Ne^{-Qx} + \frac{(m+1)}{p} - (m+1)}{N - (m+1)} < \frac{Ne^{-Qx} + \frac{(m+2)}{p} - (m+2)}{N - (m+2)} \quad (4.29)$$

for $m = 0, 1, \dots, N - 3$. The left hand side of (4.29) is found to be equal to $\frac{f_m^{(1)}(x)}{g_m(x)}$ and the right hand side is equal to $\frac{f_{m+1}^{(1)}(x)}{g_{m+1}(x)}$. Therefore, we get $\frac{f_m^{(1)}(x)}{g_m(x)} < \frac{f_{m+1}^{(1)}(x)}{g_{m+1}(x)}$ which completes the proof for Lemma 4.6.

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Now the monotonicity of $\frac{f_{m+1}^{(n)}(x)}{g_{m+1}^{(n-1)}(x)}$ with respect to m is established as in the following lemma.

Lemma 4.7. *For all positive integer n , we have*

$$\frac{f_m^{(n)}(x)}{g_m^{(n-1)}(x)} < \frac{f_{m+1}^{(n)}(x)}{g_{m+1}^{(n-1)}(x)} \quad (4.30)$$

where $m = 0, 1, \dots, N - 3$.

Proof: An induction strategy is adopted to prove the lemma. It is clear from Lemma 4.6 that this lemma holds for $n = 1$. Next, we prove the case for $n = 2$. This requires us to prove

$$\frac{f_m^{(2)}(x)}{g_m^{(1)}(x)} < \frac{f_{m+1}^{(2)}(x)}{g_{m+1}^{(1)}(x)} \quad (4.31)$$

for $m = 0, 1, \dots, N - 3$. As seen from Lemma 4.2, $g_m^{(1)}(x) > 0$ and $g_{m+1}^{(1)}(x) > 0$ for

$m = 0, 1, \dots, N - 3$. Thus, (4.31) can be expressed as

$$f_m^{(2)}(x)g_{m+1}^{(1)}(x) - f_{m+1}^{(2)}(x)g_m^{(1)}(x) < 0. \quad (4.32)$$

So proving (4.31) is equivalent to proving (4.32). By expanding the left hand side of (4.32) by using Lemma 4.1, we obtain

$$\begin{aligned} & f_m^{(2)}(x)g_{m+1}^{(1)}(x) - f_{m+1}^{(2)}(x)g_m^{(1)}(x) \\ &= (m+1)(m+2)Q^2[f_m^{(1)}(x)g_{m+1}(x) - f_{m+1}^{(1)}(x)g_m(x)] \\ & \quad + [f_{m+1}^{(1)}(x)g_{m+2}(x) - f_{m+2}^{(1)}(x)g_{m+1}(x)] \\ & \quad + (m+1)Q[f_m^{(1)}(x)g_{m+2}(x) - f_{m+2}^{(1)}(x)g_m(x)], \end{aligned} \quad (4.33)$$

for $m = 0, 1, \dots, N - 4$.

$$\begin{aligned} & f_{N-3}^{(2)}(x)g_{N-2}^{(1)}(x) - f_{N-2}^{(2)}(x)g_{N-3}^{(1)}(x) \\ &= (N-2)(N-1)Q^2[f_{N-3}^{(1)}(x)g_{N-2}(x) - f_{N-2}^{(1)}(x)g_{N-3}(x)] \\ & \quad - [f_{N-1}^{(1)}(x)g_{N-2}(x)] - (N-2)Q[f_{N-1}^{(1)}(x)g_{N-3}(x)]. \end{aligned} \quad (4.34)$$

It is noted from the definition of $g_m(x)$ in equation (4.3) that $g_m(x) < 0$ for $m = 0, 1, \dots, N - 2$. Hence, Lemma 4.6 can be written as $f_m^{(1)}(x)g_{m+1}(x) - f_{m+1}^{(1)}(x)g_m(x) < 0$ for $m = 0, 1, \dots, N - 3$. Thus, the first term of (4.33) and (4.34) are negative. Again, using Lemma 4.6, we can get $\frac{f_m^{(1)}(x)}{g_m(x)} < \frac{f_{m+1}^{(1)}(x)}{g_{m+1}(x)}$ and $\frac{f_{m+1}^{(1)}(x)}{g_{m+1}(x)} < \frac{f_{m+2}^{(1)}(x)}{g_{m+2}(x)}$ for $m = 0, 1, \dots, N - 4$. Therefore, $\frac{f_m^{(1)}(x)}{g_m(x)} < \frac{f_{m+2}^{(1)}(x)}{g_{m+2}(x)}$. We can express $\frac{f_m^{(1)}(x)}{g_m(x)} < \frac{f_{m+2}^{(1)}(x)}{g_{m+2}(x)}$ as $f_m^{(1)}(x)g_{m+2}(x) - f_{m+2}^{(1)}(x)g_m(x) < 0$ since $g_m(x) > 0$ for $m = 0, 1, \dots, N - 2$ from the definition of $g_m(x)$. Thus, the third term of (4.33) is negative. By replacing all the m by $m+1$ in (4.32), the second term of (4.33) is $f_{m+1}^{(1)}(x)g_{m+2}(x) - f_{m+2}^{(1)}(x)g_{m+1}(x) < 0$ for $m = 0, 1, \dots, N - 4$. From the definitions, $g_{N-2}(x) < 0$ and $g_{N-3}(x) < 0$ and from Lemma 4.4, $f_{N-1}^{(1)}(x) < 0$. Thus, the second term $-f_{N-1}^{(1)}(x)g_{N-2}(x)$ is negative. Since the third term in (4.34) satisfies $-f_{N-1}^{(1)}(x)g_{N-3}(x) < 0$ due to $f_{N-1}^{(1)}(x) < 0$ resulting

from Lemma 4.4, Lemma 4.7 holds for $n = 2$.

We assume that the conclusion of the lemma is valid for all $n \leq k$ in order to carry out the induction, that is,

$$\frac{f_m^{(n)}(x)}{g_m^{(n-1)}(x)} < \frac{f_{m+1}^{(n)}(x)}{g_{m+1}^{(n-1)}(x)}, \quad \text{for } n = 1, 2, \dots, k; \text{ and } m = 0, 1, \dots, N-3. \quad (4.35)$$

If n is even, $g_m^{(n-1)}(x) > 0$ and $g_{m+1}^{(n-1)}(x) > 0$ from Lemma 4.3. If n is odd, $g_m^{(n-1)}(x) < 0$ and $g_{m+1}^{(n-1)}(x) < 0$ using Lemma 4.3. In other words, regardless of n is even or odd, (4.35) can be expressed as

$$f_m^{(n)}(x)g_{m+1}^{(n-1)}(x) - f_{m+1}^{(n)}(x)g_m^{(n-1)}(x) < 0 \quad (4.36)$$

for $n = 1, 2, \dots, k$ and $m = 0, 1, \dots, N-3$.

Next, we will prove that the lemma is valid for $n = k+1$. In this case, we need to show $\frac{f_m^{(k+1)}(x)}{g_m^{(k)}(x)} < \frac{f_{m+1}^{(k+1)}(x)}{g_{m+1}^{(k)}(x)}$. By using Lemma 4.3, $\frac{f_m^{(k+1)}(x)}{g_m^{(k)}(x)} < \frac{f_{m+1}^{(k+1)}(x)}{g_{m+1}^{(k)}(x)}$ can be expressed as $f_m^{(k+1)}(x)g_{m+1}^{(k)}(x) - f_{m+1}^{(k+1)}(x)g_m^{(k)}(x) < 0$. Thus, in order to complete the induction proof, we need to show $f_m^{(k+1)}(x)g_{m+1}^{(k)}(x) - f_{m+1}^{(k+1)}(x)g_m^{(k)}(x) < 0$. By expanding the left hand side of the inequality, we have

$$\begin{aligned} & f_m^{(k+1)}(x)g_{m+1}^{(k)}(x) - f_{m+1}^{(k+1)}(x)g_m^{(k)}(x) \\ &= (m+1)(m+2)Q^2[f_m^{(k)}(x)g_{m+1}^{(k-1)}(x) - f_{m+1}^{(k)}(x)g_m^{(k-1)}(x)] \\ & \quad + [f_{m+1}^{(k)}(x)g_{m+2}^{(k-1)}(x) - f_{m+2}^{(k)}(x)g_{m+1}^{(k-1)}(x)] \\ & \quad + (m+1)Q[f_m^{(k)}(x)g_{m+2}^{(k-1)}(x) - f_{m+2}^{(k)}(x)g_m^{(k-1)}(x)] \end{aligned} \quad (4.37)$$

for $m = 0, 1, \dots, N - 4$.

$$\begin{aligned}
 & f_{N-3}^{(k+1)}(x)g_{N-2}^{(k)}(x) - f_{N-2}^{(k+1)}(x)g_{N-3}^{(k)}(x) \\
 &= (N-2)(N-1)Q^2[f_{N-3}^{(k)}(x)g_{N-2}^{(k-1)}(x) - f_{N-2}^{(k)}(x)g_{N-3}^{(k-1)}(x)] \\
 & \quad - [f_{N-1}^{(k)}(x)g_{N-2}^{(k-1)}(x)] - (N-2)Q[f_{N-1}^{(k)}(x)g_{N-3}^{(k-1)}(x)].
 \end{aligned} \tag{4.38}$$

The first terms in (4.37) and (4.38) are negative because of the induction assumption (4.36). From (4.35), we can get $\frac{f_m^{(k)}(x)}{g_m^{(k-1)}(x)} < \frac{f_{m+1}^{(k)}(x)}{g_{m+1}^{(k-1)}(x)}$ and $\frac{f_{m+1}^{(k)}(x)}{g_{m+1}^{(k-1)}(x)} < \frac{f_{m+2}^{(k)}(x)}{g_{m+2}^{(k-1)}(x)}$ for $m = 0, 1, \dots, N - 4$. As a result, $\frac{f_m^{(k)}(x)}{g_m^{(k-1)}(x)} < \frac{f_{m+2}^{(k)}(x)}{g_{m+2}^{(k-1)}(x)}$. Using Lemma 4.3, $\frac{f_m^{(k)}(x)}{g_m^{(k-1)}(x)} < \frac{f_{m+2}^{(k)}(x)}{g_{m+2}^{(k-1)}(x)}$ can be expressed as $f_m^{(k)}(x)g_{m+2}^{(k-1)}(x) - f_{m+2}^{(k)}(x)g_m^{(k-1)}(x) < 0$. Thus, the third term of (4.37) is negative. By replacing all the m by $m + 1$ in (4.36), it can be seen that the second term of (4.37) is $f_{m+1}^{(k)}(x)g_{m+2}^{(k-1)}(x) - f_{m+2}^{(k)}(x)g_{m+1}^{(k-1)}(x) < 0$ $m = 0, 1, \dots, N - 4$. Similarly using Lemmas 4.3 and 4.5, the second term $-f_{N-1}^{(k)}(x)g_{N-2}^{(k-1)}(x)$ and the third term $-f_{N-1}^{(k)}(x)g_{N-3}^{(k-1)}(x)$ in (4.38) can be shown negative. Thus, the lemma is valid for $n = k + 1$. With this, Lemma 4.7 is concluded by induction.

▲▲▲

4.3 Monotonicity and Optimality of Chip-level Detection

With the properties obtained in the previous section, we are now in the position to present the main results on the monotonicity of the likelihood ratio. These results are respectively summarized in the following theorems.

Theorem 4.1. (Monotonicity of likelihood ratio) *The likelihood ratio $\Lambda(n) \triangleq \frac{f_0^{(n+1)}(x)}{g_0^{(n)}(x)} \Big|_{x=1}$ is a monotonous function of the number of receiving photons, i.e.,*

$$\Lambda(n-1) < \Lambda(n), \tag{4.39}$$

for all integer $n \geq 0$.

Proof: By putting $m = 0$ into Lemma 4.7, we can get $\frac{f_0^{(n)}(x)}{g_0^{(n-1)}(x)} < \frac{f_1^{(n)}(x)}{g_1^{(n-1)}(x)}$. If n is even, from Lemma 4.3, $g_0^{(n-1)}(x) > 0$ and $g_1^{(n-1)}(x) > 0$. If n is odd, from Lemma 4.3, $g_0^{(n-1)}(x) < 0$ and $g_1^{(n-1)}(x) < 0$. So we are able to re-express $\frac{f_0^{(n)}(x)}{g_0^{(n-1)}(x)} < \frac{f_1^{(n)}(x)}{g_1^{(n-1)}(x)}$ as $f_0^{(n)}(x)g_1^{(n-1)}(x) < f_1^{(n)}(x)g_0^{(n-1)}(x)$. Next, multiplying both sides by -1 and adding $-Qf_0^{(n)}(x)g_0^{(n-1)}(x)$ to both sides, we have

$$f_0^{(n)}(x)[-g_1^{(n-1)}(x) - Qg_0^{(n-1)}(x)] > g_0^{(n-1)}(x)[-f_1^{(n)}(x) - Qf_0^{(n)}(x)]. \quad (4.40)$$

Using the definitions in Lemma 4.3, the above expression (4.40) can be written as

$$f_0^{(n)}(x)g_0^{(n)}(x) > g_0^{(n-1)}(x)f_0^{(n+1)}(x). \quad (4.41)$$

If n is even, from Lemma 4.3, $g_0^{(n)}(x) < 0$ and $g_0^{(n-1)}(x) > 0$. If n is odd, from Lemma 4.3, $g_0^{(n)}(x) > 0$ and $g_0^{(n-1)}(x) < 0$. For both cases, we will get $\frac{f_0^{(n)}(x)}{g_0^{(n-1)}(x)} < \frac{f_0^{(n+1)}(x)}{g_0^{(n)}(x)}$. Hence, Theorem 4.1 is proven.

▲▲▲

When $P(0)$ and $P(1)$ is not equal, the optical detection uses the MAP criterion (3.4) to maximize the probability of a correct decision and minimize the probability of error. The MAP criterion of deciding which symbol is sent by observing the number of photon counts collected from i th weighted chip is expressed as

$$P(X_i = n|0)P(0) \underset{\substack{\text{"0" being sent} \\ \text{"1" being sent}}}{\lesseqgtr} P(X_i = n|1)P(1). \quad (4.42)$$

By rearranging the terms in (4.42), MAP criterion can be written as

$$\frac{P(0)}{P(1)} \underset{\substack{\text{"0" being sent} \\ \text{"1" being sent}}}{\lesseqgtr} \frac{P(X_i = n|1)}{P(X_i = n|0)}. \quad (4.43)$$

Using the definitions in Lemmas 4.1, 4.2 and 4.3, the likelihood ratio is given as

$$\Lambda(n) \triangleq \frac{P(X_i = n|1)}{P(X_i = n|0)} = \frac{f_0^{(n+1)}(x)}{g_0^{(n)}(x)} \Big|_{x=1}. \quad (4.44)$$

Hence, the detection of “0” and “1” is executed by comparing $\Lambda(n)$ and the ratio $\frac{P(0)}{P(1)}$. Since we have established the increasing monotonicity of the likelihood ratio, the decision can be simplified by comparing the received photons number represented by X_i with a threshold n_i which is the smallest integer satisfying

$$\Lambda(n_i) > \frac{P(0)}{P(1)} \quad (4.45)$$

where $P(0)$ and $P(1)$ are the *a priori* probabilities of transmitting “0” or “1” symbols respectively. When $X_i \geq n_i$, a chip “1” is judged for the i -th chip, otherwise “0” is judged for this chip. This threshold is proven to be unique using the monotonicity property.

4.4 BER Bound and Robust Margin of Chip-level Detection

As the first application of the monotonicity of likelihood ratio, we evaluate the BER bounds in the presence of uncertain *a priori* probabilities. Suppose the *a priori* probability $P(0)$ varies within an interval given by $[a, b]$. This implies that the *a priori* probability $P(1)$ varies within an interval given by $[1 - b, 1 - a]$. Using the MAP criterion (4.44), the thresholds corresponding to *a priori* probability $P(0) = a$ and $P(0) = b$ can be uniquely determined by substituting (4.1) for $j = 0, 1$ into (4.44). We denote the two thresholds as n_a and n_b . Expressing the probabilities in

(4.44) with two auxiliary functions, the MAP decision can be expressed as:

$$\Lambda(n_a) \begin{matrix} X \geq n_a \\ \gtrless \\ X < n_a \end{matrix} \frac{P(0)}{P(1)} \quad (4.46)$$

$$\Lambda(n_b) \begin{matrix} X \geq n_b \\ \gtrless \\ X < n_b \end{matrix} \frac{P(0)}{P(1)} \quad (4.47)$$

Hence, when the *a priori* probability $P(0)$ takes any value in $[a, b]$, for example $P(0) = c$, the corresponding optimal threshold n_c must satisfy $n_a \leq n_c \leq n_b$ since the likelihood ratio corresponding to c falls into an interval given by $[\Lambda(n_c - 1), \Lambda(n_c)]$ satisfying

$$\left\{ \begin{array}{ll} \Lambda(n_a - 1) \leq \frac{a}{1-a} \leq \Lambda(n_a) \leq \dots \leq \Lambda(n_c - 1) \leq \frac{c}{1-c} \leq \Lambda(n_c) \leq \dots \\ \qquad \qquad \qquad \leq \Lambda(n_b - 1) \leq \frac{b}{1-b} \leq \Lambda(n_b) & \text{for } n_a < n_c < n_b \\ \Lambda(n_a - 1) \leq \frac{a}{1-a} \leq \frac{c}{1-c} \leq \Lambda(n_a) & \text{for } n_c = n_a \\ \Lambda(n_b - 1) \leq \frac{c}{1-c} \leq \frac{b}{1-b} \leq \Lambda(n_b) & \text{for } n_c = n_b \end{array} \right. \quad (4.48)$$

Alternatively, we have

$$\left\{ \begin{array}{ll} \frac{\Lambda(n_a - 1)}{1 + \Lambda(n_a - 1)} \leq a \leq \frac{\Lambda(n_a)}{1 + \Lambda(n_a)} \leq \dots \leq \frac{\Lambda(n_c - 1)}{1 + \Lambda(n_c - 1)} \leq c \leq \frac{\Lambda(n_c)}{1 + \Lambda(n_c)} \leq \dots \\ \qquad \qquad \qquad \leq \frac{\Lambda(n_b - 1)}{1 + \Lambda(n_b - 1)} \leq b \leq \frac{\Lambda(n_b)}{1 + \Lambda(n_b)} & \text{for } n_a < n_c < n_b \\ \frac{\Lambda(n_a - 1)}{1 + \Lambda(n_a - 1)} \leq a \leq c \leq \frac{\Lambda(n_a)}{1 + \Lambda(n_a)} & \text{for } n_c = n_a \\ \frac{\Lambda(n_b - 1)}{1 + \Lambda(n_b - 1)} \leq c \leq b \leq \frac{\Lambda(n_b)}{1 + \Lambda(n_b)} & \text{for } n_c = n_b \end{array} \right. \quad (4.49)$$

This equation suggests that the full range of *a priori* probability can be divided into intervals, each of which is represented by $\left[\frac{\Lambda(n)}{1 + \Lambda(n)}, \frac{\Lambda(n+1)}{1 + \Lambda(n+1)} \right]$ for $n = 0, 1, 2, \dots$. As the threshold takes only integer value, there are finite number of thresholds in the range $[n_a, n_b]$ when $P(0)$ varies in $[a, b]$. Denote these thresholds as n_u for $u = 1, 2, \dots, K$ with $n_1 = n_a$ and $n_K = n_b$. The corresponding *a priori* probabilities are denoted as

$P_u(0)$.

Using the above denotation, the BER resulting from the optimum detection when the *a priori* probability $P(0)$ takes a value in the range $(P_u(0), P_{u+1}(0)]$ is obtained by

$$P(E) = P(0)P(X_i \geq n_{u+1}, \forall i|0) + (1 - P(0))P(X_i < n_{u+1}, \text{for some } i|1) \quad (4.50)$$

Since $P(X_i \geq n_{u+1}, \forall i|0)$ and $P(X_i < n_{u+1}, \text{for some } i|1)$ are fixed when n_{u+1} is given, *BER* is either an increasing function or a decreasing function of $P(0)$ depending upon whether $P(X_i \geq n_{u+1}, \forall i|0) \geq P(X_i < n_{u+1}, \text{for some } i|1)$ or $P(X_i \geq n_{u+1}, \forall i|0) \leq P(X_i < n_{u+1}, \text{for some } i|1)$. This can be seen by evaluating the first order derivative of $P(E)$ with respect to $P(0)$. Therefore, *BER* is bounded by $P_u(0)P(X_i \geq n_u, \forall i|0) + (1 - P_u(0))P(X_i < n_u, \text{for some } i|1)$ and $P_{u+1}(0)P(X_i \geq n_{u+1}, \forall i|0) + (1 - P_{u+1}(0))P(X_i < n_{u+1}, \text{for some } i|1)$. This conclusion holds for all $u = 0, 1, \dots, K-1$. Thus the BER bounds can be obtained by the following theorem.

Theorem 4.2. (Bounds of BER) *Given a priori probability $P(0) = c \in [a, b]$, the threshold obtained from the MAP (4.44) is unique and satisfies $n_a \leq n_c \leq n_b$. Define*

$$B_0 \triangleq aP(X_i \geq n_1, \forall i|0) + (1 - a)P(X_i < n_1, \text{for some } i|1) \quad (4.51)$$

$$B_u \triangleq P_u(0)P(X_i \geq n_u, \forall i|0) + (1 - P_u(0))P(X_i < n_u, \text{for some } i|1) \quad (4.52)$$

$$B'_v \triangleq P_v(0)P(X_i \geq n_v, \forall i|0) + (1 - P_v(0))P(X_i < n_v, \text{for some } i|1) \quad (4.53)$$

$$B'_K \triangleq bP(X_i \geq n_K, \forall i|0) + (1 - b)P(X_i < n_K, \text{for some } i|1) \quad (4.54)$$

where $P_u(0)$ and n_u are defined above. Then the BER bounds are given by

$$BER_{upper} = \max \left\{ B_0, B'_K, \max_{1 \leq u \leq K-2} \{B_u\}, \max_{1 \leq v \leq K-1} \{B'_v\} \right\} \quad (4.55)$$

$$BER_{lower} = \min \left\{ B_0, B'_K, \min_{1 \leq u \leq K-2} \{B_u\}, \min_{1 \leq v \leq K-1} \{B'_v\} \right\}. \quad (4.56)$$

4.5 Numerical Results

To demonstrate the above results, we consider an optical CDMA system using OOK modulation with the OOC coding. In this system, there are $N = 25$ users simultaneously accessing the system and each user is assigned a signature which is coded by using $L = 2000$ chips and $\omega = 9$ weights. The average transmitted photons per chip pulse $Q = 15$. Supposed the symbol “0”s from a user are transmitted in a probability $P(0)$ in a range of $[0.3, 0.6]$. Here we choose $[0.3, 0.6]$ is just an illustrative example. It can be chosen as any other range. The range $[0.3, 0.6]$ is chosen as an example to show that we would not be able to determine how the BER performance would change in the presence of uncertain transmission probabilities, if we did not have the monotonicity property. The range can be any sub-intervals of $[0, 1.0]$. The range is usually determined from the statistics obtained from real data traffic measurement depending upon the applications. Without loss of generality, we choose $[0.3, 0.6]$ as an example to illustrate the BER bound estimation problem in the presence of uncertain transmission probabilities. For this system, the likelihood ratio is formed by (4.44) and it is plotted against the number of receiving photons in Figure 4.1. When $P(0)$ varies in the range $[0.3, 0.6]$, one threshold (*i.e.*, $K = 1$) is found which is $n_1 = 1$. It is marked by squares in the figure. Using (4.44), the *a priori* probabilities corresponding to the threshold is given by $P_2(0) = 0.9487$. It is clearly shown that $\Lambda(n)$ is a monotonous function using Theorem 4.1. Thus, the threshold resulting from $P(0)$ varying in $[0.3, 0.6]$ falls into a range $(0, 1]$.

From (4.49), it is clear that the monotonicity of the likelihood ratio also suggests the robust margin of the *a priori* probabilities given a BER level or a threshold. The robust margin defines the maximum range where *a priori* probabilities can change while the threshold remains unchanged. This conclusion is summarized into a theorem as follows.

Theorem 4.3. (Robust margin of *a priori* probabilities) *Given a threshold n ,*

Figure 4.1: Threshold bound for chip-level receiver

the robust margin of *a priori* probabilities is determined as

$$\left[\frac{\Lambda(n)}{1 + \Lambda(n)}, \frac{\Lambda(n+1)}{1 + \Lambda(n+1)} \right]. \quad (4.57)$$

Proof: It can be obtained straightforwardly from (4.48).

We further the analysis by varying the length of the signature codes L , weights ω , number of users N and the average transmitted photons per chip pulse Q .

In Figure 4.2, we can see that with a longer signature code length L , the maximum range where *a priori* probabilities can change while the threshold remains unchanged will increase.

Next, we use different number of weights ω to see the changes in the threshold bound. As seen in Figure 4.3, a lower number of ω will see a decrease in the maximum range of the *a priori* probabilities that can change while the threshold remains unchanged.

In Figure 4.4, we can see that as the number of users N increase, the maximum range

Figure 4.2: Threshold bound for chip-level receiver for different signature code length L

of the *a priori* probabilities that can change while the threshold remains unchanged will also increase.

Lastly, we plot the likelihood ratio against the number of photons by varying the average transmitted photons per chip pulse Q . It can be seen in Figure 4.6 that by lowering the value Q , the maximum range of the *a priori* probabilities that can change while the threshold remains unchanged will decrease. However, this result differ from the above in the sense that the lower threshold bound decreases with decreasing Q while the upper threshold bound remains constant for all values of Q . Figure 4.6 shows the threshold bound of chip-level receiver for $Q = 2$. It can be seen that when $P(0)$ varies between $[0.1192, 0.9494]$, there are two thresholds. The threshold of value one corresponds to $P(0)$ varies between $[0.1192, 0.9487]$ and the threshold of value two corresponds to $P(0)$ varies between $[0.9487, 0.9494]$. Each threshold will give a BER value. Hence, this will in turn give a BER bound.

Figure 4.3: Threshold bound for chip-level receiver for different weights ω

4.6 Conclusion

In this chapter, we have studied the chip-level OCDMA detection in the presence of uncertain *a priori* probabilities. The monotonicity of the likelihood ratio is first established. This is accomplished by representing the *a priori* probabilities with two auxiliary analytic functions introduced in chapter 3 and applying an induction approach. With the monotonicity property, the threshold satisfying the MAP criterion is shown to fall into a range bounded by the two boundary thresholds. These boundary thresholds are calculated by using the maximum and the minimum values that *a priori* probabilities could take. As the BER is determined when the threshold is given, the BER bound can be obtained by comparing BERs corresponding to a finite number of thresholds in the range specified by the boundary thresholds. Moreover, the monotonicity of the likelihood ratio also suggests that the full range of *a priori* probabilities can be divided into intervals during which the threshold is fixed even when the *a priori* probability changes. Thus, each interval defines the maximum range where *a priori* probabilities can change while the threshold remains unchanged. It, for

Figure 4.4: Threshold bound for chip-level receiver for different number of users N

the first time, discloses the robustness of a threshold with respect to the transmission.

Figure 4.5: Threshold bound for chip-level receiver for different number of photons per chip pulse Q

Figure 4.6: Threshold bound for chip-level receiver for $Q = 2$

Chapter 5

Optimal Correlation Detection in the Presence of Uncertain Transmission Probabilities

5.1 Introduction

In the previous chapter, we have located the threshold of the chip-level receiver and its optimality is presented for non-equiprobable data transmission. This chapter will then examine the BER performance of correlation receiver where the probabilities of transmitting of a bit “0” and bit “1” are not equal. In order to accurately obtain the BER in this situation, an optimal threshold has to be found. Hence, some sort of relationship such as the monotonicity property with respect to the likelihood probabilities is needed to be established for this case to conclude the optimality of the threshold obtained. The difficulty in obtaining this monotonicity property is again due to the mixed Poisson-binomial terms in the likelihood probabilities of the respective receiver. This difficulty can be overcome by the two auxiliary functions

proposed in chapter 2. By setting up new properties for the auxiliary functions, the monotonicity property is investigated and established.

In practice, it is difficult to obtain information on the exact probability of transmitting a bit “0” and bit “1”. Thus, a range of *a priori* probabilities is used instead. In this case, only a BER bound is obtained in its place of an exact BER value. In the presence of uncertain *a priori* probabilities, the monotonicity property of the likelihood ratio is essential to locate this BER bound. In addition to that, the monotonicity property can be used to determine the robust margin of *a priori* probabilities corresponding to a given threshold or a BER level. The results acquired can be further extended to the performance analysis for the correlation receiver.

5.2 OOK Correlation Detection

The principle of OCDMA correlation detection will be illustrated by looking at an OCDMA system utilizing OOC as a signature code and OOK as a modulation technique. In this system, information transmitted over the communication system is modulated by two symbols in bits. The symbols are denoted logically by “0” and “1” which represent respectively the absence and the presence of optical pulse(s) in a bit. Each user of the OCDMA system is assigned a unique signature to encode the optical pulses before transmitting them over the optical fiber network. The code sequence is formed by dividing one symbol bit duration into L chips which consist of ω weights. At the receiving end, the received optical signals $r(t)$ is correlated with the desired user’s signature code $y_n(t)$ where $n = 1, 2, \dots, N$. Then, the resultant signal is photodetected by a photodetector and it is summed over each bit duration. By sampling at the end of each bit duration, a decision on what bit symbol is transmitted will be made based on number of photon counts collected over all the weighted chip positions of the signature code of the desired user.

Denote Z as the photon counts collected from the correlation, which is actually the total number of photons collected from all weight positions in a bit of the desired user. The hypothesis probabilities of receiving n photons are denoted by $P(Z = n|0)$ and $P(Z = n|1)$ for the hypotheses of symbol “0” being sent and “1” being sent respectively.

In optical detection using MAP criterion, the likelihood ratio is defined as

$$\Lambda_c(n) \triangleq \frac{P(Z = n|1)}{P(Z = n|0)} \quad (5.1)$$

The detection of “0” and “1” is then executed by obtaining a threshold n -th satisfying

$$\Lambda_c(n) \begin{matrix} \geq \\ \leq \end{matrix} \frac{P(0)}{P(1)} \quad (5.2)$$

where $P(0)$ and $P(1)$ are the *a priori* probabilities of transmitting “0” or “1” symbols respectively.

Now, we derive a specific expression of the likelihood ratio for the optical correlation detection. Assume that there are N users. For each user, the weights are equiprobable distributed in the L chip positions. Hence, the probability of an optical pulse appearing in one of the weight positions is $\frac{\omega}{2L}$. As optical CDMA technique allows multiple users to transmit data simultaneously over the fiber optics network, $r(t)$ will consist of the desired user’s signal and the other undesired users’ signals. Hence, the interference by other users can be seen as overlapping of chip-pulses at the desired user’s weighted chip positions. Since OOC is used in the signature code, each undesired user will at most contribute an interference chip-pulse to the desired user as the cross- and auto-correlation are bounded by one. The interference pulses number I is a binomial random variable with parameter $\frac{\omega^2}{2L}$. Thus, the probability of having

$I = \zeta$ interference is given as

$$P(I = \zeta) = \binom{N-1}{\zeta} \left(\frac{\omega^2}{2L}\right)^\zeta \left(1 - \frac{\omega^2}{2L}\right)^{N-1-\zeta}. \quad (5.3)$$

Since the photodetector has a Poisson characteristic, the probability of receiving n photons under the condition that ζ pulses are photodetected and a symbol j is sent, is given by

$$P(Z = n|I = \zeta, j) = \frac{(e^{-Q\alpha_j})(Q\alpha_j)^n}{n!}, \quad \text{for } j = 0, 1 \quad (5.4)$$

where Q is the average number of photons produced by a pulse and $\alpha_j = \zeta + \omega j$. Thus, the probability of n photons being generated is

$$\begin{aligned} P(Z = n|j) &= \sum_{\zeta=0}^{N-1} P(Z = n|I = \zeta, j)P(I = \zeta|j) \\ &= \sum_{\zeta=0}^{N-1} P(Z = n|I = \zeta, j)P(I = \zeta). \end{aligned} \quad (5.5)$$

Under the hypothesis of a bit “0” being sent, no signature code will be transmitted and hence $\alpha_0 = \zeta$. Thus, there will not be any pulse by the desired user. Any pulses detected in that chip is the interference pulses by the undesired users. Hence, substituting (5.3) and (5.4) into (5.5) give the probability of n photons being detected when a bit “0” is sent:

$$P(Z = n|0) = \sum_{\zeta=0}^{N-1} \frac{(e^{-Q\zeta})(Q\zeta)^n}{n!} \binom{N-1}{\zeta} p^\zeta q^{N-1-\zeta} \quad (5.6)$$

where $p = \frac{\omega^2}{2L}$ and $q = 1 - p$.

For the hypothesis of a bit “1” being sent, the desired user’s signature code will be transmitted to represent a bit “1” and $\alpha_1 = \zeta + \omega$. Each pulse chip will contain at least one pulse which belongs to the desired user. Thus, the photodetector will detect

at least one pulse in addition to any other pulses produced by the undesired users. Similarly, we can obtain the probability of n photons being detected in this case by substituting (5.3) and (5.4) into (5.5):

$$P(Z = n|1) = \sum_{\zeta=0}^{N-1} \frac{(e^{-Q \cdot (\zeta + \omega)})(Q \cdot (\zeta + \omega))^n}{n!} \binom{N-1}{\zeta} p^\zeta q^{N-1-\zeta}. \quad (5.7)$$

Substituting the probabilities in (5.6) and (5.7) into (5.1), we will get

$$\Lambda_c(n) = \frac{P(Z = n|1)}{P(Z = n|0)} = \frac{e^{-Q\omega} \left[\sum_{\zeta=0}^{N-1} e^{-Q\zeta} (\omega + \zeta)^n f(\zeta) \right]}{\sum_{\zeta=0}^{N-1} e^{-Q\zeta} (\zeta)^n f(\zeta)} \quad (5.8)$$

where $f(\zeta) \triangleq \binom{N-1}{\zeta} p^\zeta (1-p)^{(N-1-\zeta)}$.

Using Figure 3.1, it can be seen that a threshold θ has to be found first before deciding which bit is sent. So if the total number of photons collected is greater than or equal to θ , a bit “1” is declared to be sent. Otherwise, bit “0” is declared to be sent. Let us denote Z_i as the photon counts collected at the i th weighted chip positions from the correlation detection. Thus, the decision rule can be shown as

$$\text{Declare} \begin{cases} 1, & \text{if } Z = \sum_{i=1}^{\omega} Z_i \geq \theta \\ 0, & \text{otherwise.} \end{cases} \quad (5.9)$$

From the decision rule (5.9), we can see that it governs the performance of OOK-OCDMA correlation receiver. The obtained threshold is optimal in the sense that it minimizes the BER of the transmission. The BER of the MAP detection is given by

$$P(E) = P(Z \geq \theta|0)P(0) + P(Z < \theta|1)P(1) \quad (5.10)$$

where $P(0)$ and $P(1)$ are the probability of transmitting a bit “0” or bit “1” respectively. $P(Z \geq \theta|0)$ and $P(Z < \theta|1)$ are respectively the probabilities of error given the condition that bit “0” and bit “1” are sent. They can be described as [55]

$$P(Z \geq \theta|0) = \sum_{\zeta=0}^{N-1} P(Z \geq \theta|0, I = \zeta) P(I = \zeta), \quad (5.11)$$

$$P(Z < \theta|1) = \sum_{\zeta=0}^{N-1} P(Z < \theta|1, I = \zeta) P(I = \zeta). \quad (5.12)$$

Substituting (5.4) into (5.11) and (5.12), we will get

$$P(Z \geq \theta|0) = \sum_{\zeta=0}^{N-1} \sum_{n=\theta}^{\infty} \frac{(e^{-Q\zeta})(Q\zeta)^n}{n!} P(I = \zeta), \quad (5.13)$$

$$P(Z < \theta|1) = \sum_{\zeta=0}^{N-1} \sum_{n=0}^{\theta-1} \frac{(e^{-Q \cdot (\zeta+\omega)})(Q \cdot (\zeta + \omega))^n}{n!} P(I = \zeta). \quad (5.14)$$

Thus, using (5.3), (5.13) and (5.14), $P(E)$ can be calculated.

5.3 Monotonicity and Optimality of Correlation Detection

Before we establish the monotonicity of $\Lambda(n)$, we first present a useful property in Lemma 5.1 by using the result in Section 4.3 on the monotonicity of the likelihood ratio $\Lambda_1(n)$ defined by

$$\Lambda_1(n) = \frac{e^{-Q} \left[\sum_{\zeta_i=0}^{N-1} e^{-Q\zeta_i} (1 + \zeta_i)^n f_1(\zeta_i) \right]}{\sum_{\zeta_i=0}^{N-1} e^{-Q\zeta_i} \zeta_i^n f_1(\zeta_i)}, \text{ for } i = 1, 2, \dots, \omega \quad (5.15)$$

where $f_1(\zeta_i) = \binom{N-1}{\zeta_i} p_1^{\zeta_i} (1-p_1)^{(N-1-\zeta_i)}$ and $p_1 = \frac{\omega}{2L}$.

Lemma 5.1. *The function $f(\zeta)$ defined in (5.8) satisfies*

$$\begin{aligned} & \sum_{\zeta=0}^{N-1} e^{-Q\zeta} \zeta^n f(\zeta) \sum_{\zeta=0}^{N-1} e^{-Q\zeta} (1+\zeta)^n \ln(1+\zeta) f(\zeta) \\ & - \sum_{\zeta=0}^{N-1} e^{-Q\zeta} (1+\zeta)^n f(\zeta) \sum_{\zeta=1}^{N-1} e^{-Q\zeta} \zeta^n \ln(\zeta) f(\zeta) > 0. \end{aligned} \quad (5.16)$$

Proof: It is shown in Section 4.3 that the likelihood ratio $\Lambda_1(n)$ in (5.15) is monotonically increasing function of n and the proof does not need the specific form of p_1 . Thus, the likelihood ratio $\Lambda_1(n)$ has a positive first order derivative with respect to n , that is,

$$\begin{aligned} \frac{d\Lambda_1(n)}{dn} = & \frac{e^{-Q[\sum_{\zeta_i=0}^{N-1} e^{-Q\zeta_i} \zeta_i^n f_1(\zeta_i)]} \sum_{\zeta_i=0}^{N-1} e^{-Q\zeta_i} (1+\zeta_i)^n \ln(1+\zeta_i) f_1(\zeta_i)}{[\sum_{\zeta_i=0}^{N-1} e^{-Q\zeta_i} \zeta_i^n f_1(\zeta_i)]^2} \\ & - \frac{\sum_{\zeta_i=0}^{N-1} e^{-Q\zeta_i} (1+\zeta_i)^n f_1(\zeta_i) \sum_{\zeta_i=1}^{N-1} e^{-Q\zeta_i} \zeta_i^n \ln(\zeta_i) f_1(\zeta_i)}{[\sum_{\zeta_i=0}^{N-1} e^{-Q\zeta_i} \zeta_i^n f_1(\zeta_i)]^2} > 0. \end{aligned} \quad (5.17)$$

where we have used the fact

$$\frac{d(x^n)}{dn} = \begin{cases} x^n \ln(x) & \text{when } x \neq 0 \text{ and } n \neq 0, \\ 0 & \text{when } x = 0 \text{ or } n = 0 \end{cases} \quad (5.18)$$

where if either x or n is equal to zero, x^n will become a constant term.

Since the above inequality relation does not depend on the specific form of p_1 , the conclusion of the lemma is reached by replacing ζ_i and $f_1(\zeta_i)$ respectively with ζ and $f(\zeta)$ defined in (5.8).

▲▲▲

With this relationship, we are now in the position to present the main result on the monotonicity of the likelihood ratio of correlation detection, as summarized in the following theorem.

Theorem 5.1. (Monotonicity of likelihood ratio) *The likelihood ratio $\Lambda_c(n)$ defined in (5.1) is a monotonically increasing function of the number of photons received, i.e.,*

$$\Lambda_c(n-1) < \Lambda_c(n), \quad (5.19)$$

for all integer $n \geq 0$.

Proof: For the convenience of derivation, we will introduce two denotations as $a = \sum_{\zeta=1}^{N-1} e^{-Q\zeta} \zeta^n \ln(\zeta) f(\zeta)$ and $b = \sum_{\zeta=0}^{N-1} e^{-Q\zeta} \zeta^n f(\zeta)$. Then, regrouping the left hand side of the inequality in Lemma 5.1, we have

$$\begin{aligned} & e^{-Q \cdot 0} (1+0)^n f(0) \left[\frac{\ln(1+0)}{a} - \frac{1}{b} \right] + e^{-Q \cdot 1} (1+1)^n f(1) \left[\frac{\ln(1+1)}{a} - \frac{1}{b} \right] + \dots \\ & + e^{-Q \cdot (N-1)} (1+N-1)^n f(N-1) \left[\frac{\ln(1+N-1)}{a} - \frac{1}{b} \right] > 0. \end{aligned} \quad (5.20)$$

As $\frac{\ln(\omega+\zeta)}{a} > \frac{\ln(1+\zeta)}{a}$ for all $\zeta \geq 0$, we can get

$$\begin{aligned} & e^{-Q \cdot 0} (\omega+0)^n f(0) \left[\frac{\ln(\omega+0)}{a} - \frac{1}{b} \right] + e^{-Q \cdot 1} (\omega+1)^n f(1) \left[\frac{\ln(\omega+1)}{a} - \frac{1}{b} \right] + \dots \\ & + e^{-Q \cdot (N-1)} (\omega+N-1)^n f(N-1) \left[\frac{\ln(\omega+N-1)}{a} - \frac{1}{b} \right] \\ & > e^{-Q \cdot 0} (1+0)^n f(0) \left[\frac{\ln(1+0)}{a} - \frac{1}{b} \right] + \dots \\ & + e^{-Q \cdot 1} (1+1)^n f(1) \left[\frac{\ln(1+1)}{a} - \frac{1}{b} \right] + \dots \\ & + e^{-Q \cdot (N-1)} (1+N-1)^n f(N-1) \left[\frac{\ln(1+N-1)}{a} - \frac{1}{b} \right] \end{aligned} \quad (5.21)$$

which indicates that

$$\begin{aligned} & e^{-Q \cdot 0} (\omega+0)^n f(0) \left[\frac{\ln(\omega+0)}{a} - \frac{1}{b} \right] + e^{-Q \cdot 1} (\omega+1)^n f(1) \left[\frac{\ln(\omega+1)}{a} - \frac{1}{b} \right] + \dots \\ & + e^{-Q \cdot (N-1)} (\omega+N-1)^n f(N-1) \left[\frac{\ln(\omega+N-1)}{a} - \frac{1}{b} \right] > 0. \end{aligned} \quad (5.22)$$

The derivation of equation (5.21) can be found in Appendix 5.7.

Similarly, by expanding the brackets in (5.22), regrouping the terms, and substituting

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a and b , we can obtain

$$\begin{aligned} & \sum_{\zeta=0}^{N-1} e^{-Q\zeta} \zeta^n f(\zeta) \sum_{\zeta=0}^{N-1} e^{-Q\zeta} (\omega + \zeta)^n \ln(\omega + \zeta) f(\zeta) \\ & - \sum_{\zeta=0}^{N-1} e^{-Q\zeta} (\omega + \zeta)^n f(\zeta) \sum_{\zeta=1}^{N-1} e^{-Q\zeta} \zeta^n \ln(\zeta) f(\zeta) > 0. \end{aligned} \quad (5.23)$$

The left hand side of (5.23) is the numerator of the the first order derivative of the likelihood ratio of correlation receiver which is found to be

$$\begin{aligned} \frac{d\Lambda_c(n)}{dn} = & \frac{e^{-Q\omega} \left[\sum_{\zeta=0}^{N-1} e^{-Q\zeta} \zeta^n f(\zeta) \sum_{\zeta=0}^{N-1} e^{-Q\zeta} (\omega + \zeta)^n \ln(\omega + \zeta) f(\zeta) \right]}{\left[\sum_{\zeta=0}^{N-1} e^{-Q\zeta} \zeta^n f(\zeta) \right]^2} \\ & - \frac{\sum_{\zeta=0}^{N-1} e^{-Q\zeta} (\omega + \zeta)^n f(\zeta) \sum_{\zeta=1}^{N-1} e^{-Q\zeta} \zeta^n \ln(\zeta) f(\zeta)}{\left[\sum_{\zeta=0}^{N-1} e^{-Q\zeta} \zeta^n f(\zeta) \right]^2}. \end{aligned} \quad (5.24)$$

Using the inequality in (5.23), we can conclude that $\frac{d\Lambda_c(n)}{dn} > 0$. With this, the theorem is proven.

▲▲▲

5.4 BER Bound, Robust Margin and Adaptive Threshold of Correlation Detection

The monotonicity of the likelihood ratio clearly shows that the optimal threshold obtained by using the MAP criterion in (5.2) is a unique optimal threshold. In addition, one more important application of the monotonicity property is to calculate the BER bounds associated with the MAP correlation detection in the presence of uncertain *a priori* probabilities. In practice, *a priori* probabilities of bit “0” and bit “1” are not equal. This is not only due to the unequal probable appearances of “0” and “1” in the source codewords, but also due to the unequal probable transmissions

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of source symbols. In these cases, only a range of the *a priori* probabilities of “0” and “1” can be estimated. Therefore, it is of great interest to obtain an accurate estimate about the bounds of BER, in order to access the capacity of the system which uses the detectors. Without the monotonicity property, exhaustive searching, which is impossible in practice, has to be performed over the range of *a priori* probabilities.

Using the monotonicity of likelihood ratio, we are able to evaluate the BER bounds in the presence of uncertain *a priori* probabilities. Suppose the *a priori* probability $P(0)$ vary within an interval given by $[a, b]$. This implies that the *a priori* probability $P(1)$ varies within an interval given by $[1 - b, 1 - a]$. Using the MAP criterion (5.2), the thresholds corresponding to *a priori* probability $P(0) = a$ and $P(0) = b$ can be uniquely determined by substituting (5.8) for $j = 0, 1$ into (5.2). We denote the two thresholds as n_a and n_b . They satisfy

$$\Lambda_c(n_a) \begin{cases} Z > n^{th} \\ \geq \\ Z < n^{th} \end{cases} \frac{P(0)}{P(1)}, \quad (5.25)$$

$$\Lambda_c(n_b) \begin{cases} Z > n^{th} \\ \geq \\ Z < n^{th} \end{cases} \frac{P(0)}{P(1)}. \quad (5.26)$$

When the *a priori* probability $P(0)$ takes any value in $[a, b]$, for example $P(0) = c$, the corresponding optimal threshold n_c must satisfy $n_a \leq n_c \leq n_b$ since the likelihood ratio corresponding to c falls into an interval given by $[\Lambda(n_c - 1), \Lambda(n_c)]$ satisfying

$$\left\{ \begin{array}{ll} \Lambda_c(n_a - 1) \leq \frac{a}{1-a} \leq \Lambda_c(n_a) \leq \dots \leq \Lambda_c(n_c - 1) \leq \frac{c}{1-c} \leq \Lambda_c(n_c) \leq \dots \\ \qquad \qquad \qquad \leq \Lambda_c(n_b - 1) \leq \frac{b}{1-b} \leq \Lambda_c(n_b) & \text{for } n_a < n_c < n_b \\ \Lambda_c(n_a - 1) \leq \frac{a}{1-a} \leq \frac{c}{1-c} \leq \Lambda_c(n_a) & \text{for } n_c = n_a \\ \Lambda_c(n_b - 1) \leq \frac{c}{1-c} \leq \frac{b}{1-b} \leq \Lambda_c(n_b). & \text{for } n_c = n_b \end{array} \right. \quad (5.27)$$

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Alternatively, we have

$$\left\{ \begin{array}{ll} \frac{\Lambda_c(n_a-1)}{1+\Lambda_c(n_a-1)} \leq a \leq \frac{\Lambda_c(n_a)}{1+\Lambda_c(n_a)} \leq \dots \leq \frac{\Lambda_c(n_c-1)}{1+\Lambda_c(n_c-1)} \leq c \leq \frac{\Lambda_c(n_c)}{1+\Lambda_c(n_c)} \leq \dots \\ \leq \frac{\Lambda_c(n_b-1)}{1+\Lambda_c(n_b-1)} \leq b \leq \frac{\Lambda_c(n_b)}{1+\Lambda_c(n_b)} & \text{for } n_a < n_c < n_b \\ \frac{\Lambda_c(n_a-1)}{1+\Lambda_c(n_a-1)} \leq a \leq c \leq \frac{\Lambda_c(n_a)}{1+\Lambda_c(n_a)} & \text{for } n_c = n_a \\ \frac{\Lambda_c(n_b-1)}{1+\Lambda_c(n_b-1)} \leq c \leq b \leq \frac{\Lambda_c(n_b)}{1+\Lambda_c(n_b)} & \text{for } n_c = n_b \end{array} \right. \quad (5.28)$$

This equation suggests that the full range of *a priori* probability can be divided into intervals, each of which is represented by $\left[\frac{\Lambda_c(n)}{1+\Lambda_c(n)}, \frac{\Lambda_c(n+1)}{1+\Lambda_c(n+1)} \right]$ for $n = 0, 1, 2, \dots$

As the threshold takes only integer value, there are finite number of thresholds in the range $[n_a, n_b]$ when $P(0)$ varies in $[a, b]$. Denote these thresholds as n_u for $u = 1, 2, \dots, K$ with $n_1 = n_a$ and $n_K = n_b$. The corresponding *a priori* probabilities are denoted as $P_u(0)$.

Using the above denotation, the BER resulting from the optimum detection when the *a priori* probability $P(0)$ takes a value in the range $(P_u(0), P_{u+1}(0)]$ is obtained by

$$P(E) = P(0)P(n \geq n_{u+1}|0) + (1 - P(0))P(n < n_{u+1}|1). \quad (5.29)$$

Since $P(Z \geq n_{u+1}|0)$ and $P(Z < n_{u+1}|1)$ are fixed when n_{u+1} is given, *BER* is either an increasing function or a decreasing function of $P(0)$ depending upon whether $P(Z \geq n_{u+1}|0) \geq P(Z < n_{u+1}|1)$ or $P(Z \geq n_{u+1}|0) \leq P(Z < n_{u+1}|1)$. This can be seen by evaluating the first order derivative of $P(E)$ with respect to $P(0)$. Therefore, *BER* is bounded by $P_u(0)P(Z \geq n_{u+1}|0) + (1 - P_u(0))P(Z < n_{u+1}|1)$ and $P_{u+1}(0)P(Z \geq n_{u+1}|0) + (1 - P_{u+1}(0))P(Z < n_{u+1}|1)$. This conclusion holds for all $j = 0, 1, \dots, K-1$. Thus the BER bounds can be obtained by the following theorem.

Theorem 5.2. (Bounds of BER) *Given a priori probability $P(0) = c \in [a, b]$, the*

threshold obtained from the MAP (5.2) is unique and it satisfies $n_a \leq n_c \leq n_b$. Define

$$B_0 \triangleq aP(Z \geq n_1|0) + (1 - a)P(n < n_1|1) \quad (5.30)$$

$$B_u \triangleq P_u(0)P(Z \geq n_u|0) + (1 - P_u(0))P(n < n_u|1) \quad (5.31)$$

$$B'_v \triangleq P_v(0)P(Z \geq n_v|0) + (1 - P_v(0))P(n < n_v|1) \quad (5.32)$$

$$B'_K \triangleq bP(Z \geq n_K|0) + (1 - b)P(n < n_K|1) \quad (5.33)$$

where $P_u(0)$ and n_u are defined above. Then the BER bounds are given by

$$BER_{upper} = \max \left\{ B_0, B'_K, \max_{1 \leq u \leq K-2} \{B_u\}, \max_{1 \leq v \leq K-1} \{B'_v\} \right\} \quad (5.34)$$

$$BER_{lower} = \min \left\{ B_0, B'_K, \min_{1 \leq u \leq K-2} \{B_u\}, \min_{1 \leq v \leq K-1} \{B'_v\} \right\}. \quad (5.35)$$

5.5 Numerical Results

To demonstrate the above results, we consider an optical CDMA system using OOK modulation with the OOC coding. In this system, each user is assigned a signature which is coded by using $L = 2000$ chips and $\omega = 9$ weights. The average transmitted photons per chip pulse $Q = 15$. Supposed that there are $N = 25$ users simultaneously accessing the system and the symbol “0”s from each user are transmitted in a probability $P(0)$ in a range of $[0.3, 0.6]$. For this system, the likelihood ratio is formed by (5.1) and it is plotted against the number of receiving photons in Figure 5.1. When $P(0)$ varies in the range $[0.3, 0.6]$, four thresholds (*i.e.*, $K = 4$) are found which are respectively $n_1 = 89$, $n_2 = 90$, $n_3 = 91$, and $n_4 = 92$. They are marked by squares in the figure. Using (5.2), the *a priori* probabilities corresponding to the four thresholds are given by $P_1(0) = 0.3066$, $P_2(0) = 0.4428$, $P_3(0) = 0.5860$, and $P_4(0) = 0.7141$, respectively. It is clearly shown that $\Lambda(n)$ is monotonous function. Thus the thresholds resulting from $P(0)$ varying in $[0.3, 0.6]$ falls into a range $[89, 92]$.

Figure 5.1: Threshold bound for correlation receiver

Next we examine the BER performance in the presence of $P(0)$ varying in $[0.3, 0.6]$. Since thresholds take only integer numbers, the threshold for $P(0)$ in a range of $(P_j(0), P_{j+1}(0)]$ is identical to the one for $P_{j+1}(0)$ for $j = 1, 2, 3$. Similarly, the threshold for $P(0)$ in a range of $(a, P_1(0)]$ is n_1 and the threshold for $P(0)$ in a range of $(P_{M-1}, b]$ is n_M . However, although the threshold is fixed for $P(0)$ varying in the ranges, the resultant BER will vary continuously with $P(0)$. Figure 5.6 depicts the relationship between the BER and the *a priori* probability $P(0)$. The curves show that the BER is continuous in ranges of $[a, P_1(0)]$, $(P_1(0), P_2(0)]$, $(P_2(0), P_3(0)]$, and $(P_3(0), b]$, respectively. However, the BER has sudden jumps at $P_1(0)$, $P_2(0)$, and $P_3(0)$. This is because the BER for $P(0)$ greater than $P_j(0)$ is calculated by using threshold for $P_{j+1}(0)$ and this threshold is not the optimal one for $P(0) = P_j(0)$. The same reason holds for $j = 1, 2, 3$. To see this clearly, one jump at $P_2(0)$ is enlarged and displayed in Figure 5.6. Using Theorem 5.2, the BER can be easily calculated.

From (5.28), it is clear that the monotonicity of the likelihood ratio also gives the robust margin of the *a priori* probabilities given a BER level or a threshold. The robust margin defines the maximum range where *a priori* probabilities can change

while the threshold remains unchanged. This conclusion is summarized in the following theorem.

Theorem 5.3. (Robust margin of a priori probabilities) *Given a BER level, equivalently given a threshold n -th, the robust margin of a priori probabilities is determined as*

$$\left[\frac{\Lambda_c(n^{th} - 1)}{1 + \Lambda_c(n^{th} - 1)}, \frac{\Lambda_c(n^{th})}{1 + \Lambda_c(n^{th})} \right]. \quad (5.36)$$

Proof: It can be obtained straightforwardly from (5.27).

Theorem 5.4. (Adaptive Threshold) *For a given OCDMA system with the users N , weights ω , signature code length L and average transmitted photons per chip pulse Q , we are able to obtain the threshold adaptively when the a priori probability $P(0)$ changes from a to b .*

Proof: It can be obtained straightforwardly from (5.27).

Before we close the discussion of this section, we will repeat the above simulation by changing the parameters L , ω , Q and N to see how these parameters affect the performance.

When the signature code length L decreases in length, the corresponding threshold values for $P(0)$ that varies in the range of $[0.3, 0.6]$ become larger. As seen in Figure 5.2, when $L = 1500$, the starting threshold become $n_1 = 92$.

In Figure 5.3, we plot the likelihood ratio against the number of photons with varying weights ω . It can be seen that if lower the value of ω , the value of the threshold that fall within the bound will shift to the right. When $\omega = 5$, the corresponding thresholds are $n_1 = 90$, $n_2 = 91$, $n_3 = 92$ and $n - 4 = 93$.

By reducing the number of users N , it will cause the value of the likelihood ratio to shift upwards with the same number of photons as seen in Figure 5.4. Thus, the

Figure 5.2: Threshold bound for correlation receiver with different signature code length L

corresponding thresholds for $P(0)$ varies in the range $[0.3, 0.6]$ will shift to the left. For $N = 20$, the thresholds will be $n_1 = 87$, $n_2 = 88$, $n_3 = 89$ and $n - 4 = 90$.

Finally, we vary the value of number of photons per chip pulse Q to see the effect on the threshold bound. As likelihood ratio $\Lambda(n)$ is a monotonously increasing function, the values of the threshold for lower Q that correspond to $P(0)$ varies in the range $[0.3, 0.6]$ will be smaller than 89.

It can be seen from all these figures that the simulation results tally with the theoretical analysis in this chapter even though the parameters L , M , Q , and N affect the performance one way and the other.

5.6 Conclusion

We have investigated the correlation OCDMA detection in the presence of uncertain *a priori* probabilities in this chapter. By extending the research work from chip-level

Figure 5.3: Threshold bound for correlation receiver with different weights ω

detection in the previous chapter to correlation receiver examined in this chapter, we have proven that the likelihood ratio used in the optical correlation detection is a monotonically increasing function of the received photon count. With this result, it is shown that the optical correlation detection has a unique optimal threshold. A more useful application of the monotonicity of the likelihood ratio is that it enables us to obtain the BER bounds and in turn the capability of OCDMA systems in the presence of uncertain *a priori* probabilities.

5.7 Appendix

The derivation of equation (5.21) is given as follow.

Since $\frac{\ln(\omega+\zeta)}{a} > \frac{\ln(1+\zeta)}{a}$ for all $\zeta > 0$, the following inequality

$$e^{-Q\zeta} f(\zeta) (\omega + \zeta)^n \left[\frac{\ln(\omega + \zeta)}{a} - \frac{1}{b} \right] > e^{-Q\zeta} f(\zeta) (1 + \zeta)^n \left[\frac{\ln(1 + \zeta)}{a} - \frac{1}{b} \right] \quad (5.37)$$

Figure 5.4: Threshold bound for correlation receiver with different number of users N

also holds as b , $e^{-Q\zeta}$, $f(\zeta)$, $(\omega + \zeta)^n$ and $(1 + \zeta)^n$ are all positive.

Hence, for $\zeta = 0$ to $N - 1$, we are able to obtain the following relationship

$$\begin{aligned}
 & e^{-Q \cdot 0} f(0)(\omega + 0)^n \left[\frac{\ln(\omega + 0)}{a} - \frac{1}{b} \right] + e^{-Q \cdot 1} f(1)(\omega + 1)^n \left[\frac{\ln(\omega + 1)}{a} - \frac{1}{b} \right] + \dots \\
 & + e^{-Q \cdot (N-1)} f(N-1)(\omega + N-1)^n \left[\frac{\ln(\omega + N-1)}{a} - \frac{1}{b} \right] \\
 & > e^{-Q \cdot 0} f(0)(1 + 0)^n \left[\frac{\ln(1 + 0)}{a} - \frac{1}{b} \right] + e^{-Q \cdot 1} f(1)(1 + 1)^n \left[\frac{\ln(1 + 1)}{a} - \frac{1}{b} \right] + \dots \\
 & + e^{-Q \cdot (N-1)} f(N-1)(1 + N-1)^n \left[\frac{\ln(1 + N-1)}{a} - \frac{1}{b} \right] > 0.
 \end{aligned} \tag{5.38}$$

Figure 5.5: Threshold bound for correlation receiver with different number of photons per chip pulse Q

5.6.1: BER for $P(0) \in (0.3, 0.6)$

5.6.2: Enlarged portion of a) at $P_2(0)$

Figure 5.6: The dependence of BER in the presence of uncertain *a priori* probability $P(0)$

Chapter 6

Several Implementation Issues of OCDMA Detectors

6.1 Introduction

In the previous chapters, the analysis on optical detection is concentrated on the 1D OCDMA communication systems. The attained results are extended to the 2D OCDMA system in this chapter. In an optical communication system, wavelength is available for constructing the signature code in addition to time. Thus, 2D OCDMA system utilize signature code that spread both in wavelength and time can enhance the transmission capacity without having to use long time delay line. In addition, it also remove the pressure of creating ultra-short time chip.

A 2D SUM and AND detector which the detection is based on optical pulses are investigated [87]. It is shown in the BER performance that AND detector is better than SUM detector in combating against MUI. The same conclusion can be achieved by considering using photon counts for optical detection. However, SUM detector is more robust than AND detector in the presence of very small channel noise. Thus, this

motivates us to investigate a dual detector using photon counts for optical detection based on the merits of SUM and AND detector. We then explore the use of optical interleaver to address the implementation issue of 2D optical detector as it is capable of splitting dense multiple wavelengths. By developing a systematic design of the F^3T optical interleaver, analytical solution to the design parameters of F^3T can be obtained.

6.2 Two Dimension Signature Code

In a 2D OCDMA system, the signature code is constructed on both multiple wavelengths and time chips and will be in a $L \times \tau$ matrix that consists of L wavelengths and τ time chips over a bit time. In the f -th wavelength and g th time chip k_{fg} , it will consist of either an optical pulse (chip “1”) or no pulse at all (chip “0”). The weights ω of the code, which is the total number of “1”s, are assumed uniformly distributed. Since the total number of chips in a bit time is A ($L \times \tau$), the probability of a pulse appearing in k_{fg} chip is $\frac{\omega}{2A}$ if the probabilities of a bit “0” and bit “1” is equiprobable. In here, I_a , where $a = 1, 2, \dots, \omega$, is denoted as the the total number of pulses collected in each weighted chip.

λ_1				1
λ_2	1			
λ_3			1	
λ_4		1		
	τ_1	τ_2	τ_3	τ_4

Figure 6.1: Two dimension signature code

6.3 2D Detector

We will consider OCDMA system utilizing OOK modulation technique. In this system, the desired user's signature code will be transmitted to represent a bit "1" sent and no transmission of signature code if a bit "0" is to be sent. At the receiving end, the received symbol will be correlated with the desired user's signature code. Then, based on the number of pulses obtained, a decision on what symbol is sent will be made.

6.3.1 SUM Detector

In a SUM detector, the received signal will first pass through a 2D correlator. After the correlation, pulses in weighted chips of the desired user will be summed and used to make a decision on "0" or "1" being sent. The SUM decision rule is given by

$$\text{bit sent} \begin{cases} \text{"1"}; I = \sum_{a=1}^{\omega} I_a \geq \omega, \\ \text{"0"}; \text{otherwise.} \end{cases} \quad (6.1)$$

The BER of the 2D SUM detector is given by

$$P_{sb}(E) = \frac{1}{2}P_{sb}(E|0) + \frac{1}{2}P_{sb}(E|1). \quad (6.2)$$

When bit "1" is sent, the pulses detected will be at least ω . Therefore, $P_{sb}(E|1)$ in (6.2) will be 0. Hence, the BER for the SUM detector simplified to

$$P_{sb}(E) = \frac{1}{2}P_{sb}(E|0). \quad (6.3)$$

As OOC is only applicable to 1D code, we cannot generalize the cross- and auto-

correlation properties of OOC to 2D code. Thus, we will not consider the algorithm of the code in this case. Hence, the cross- and auto-correlation properties are determined by the number of weights used. Assumed the pulses obtained in a chip are independent of those received at other chips, $P_{Sb}(E|0)$ can be obtained as [87]

$$P_{Sb}(E|0) = \sum_{\zeta=\omega}^{\omega(N-1)} \binom{\omega(N-1)}{\zeta} \left(\frac{\omega}{2A}\right)^{\zeta} \left(1 - \frac{\omega}{2A}\right)^{\omega(N-1)-\zeta} \quad (6.4)$$

where ζ is total number of pulses obtained.

For a 2D OCDMA system with weights ω and code dimension A , the number of user N cannot exceed $\frac{2A}{\omega} - 1$ [87]. Letting N take the maximum value of $\frac{2A}{\omega} - 1$, the BER will take the maximum value as well. If the number of users becomes very large, (6.4) will approach to

$$P_{Sb}(E|0) = 1 - \sum_{\zeta=0}^{\omega-1} \frac{\omega^{\zeta}}{e^{\omega} \zeta!}. \quad (6.5)$$

Therefore, the BER of the SUM detector is given as

$$P_{Sb}(E) = \frac{1}{2} \left(1 - \sum_{\zeta=0}^{\omega-1} \frac{\omega^{\zeta}}{e^{\omega} \zeta!}\right) \quad (6.6)$$

when there is a large number of users.

6.3.2 AND Detector

In a AND detector, all weighted chips of the desired user are detected individually. It is found that if there is at least one pulse detected in all these chips, then bit "1"

is declared. Otherwise, bit "0" is declared. Hence, the AND decision rule is

$$\text{bit sent} \begin{cases} \text{"1"}; I_a \geq 1 \forall a = 1, \dots, \omega, \\ \text{"0"}; \text{otherwise.} \end{cases} \quad (6.7)$$

The BER of the AND detector is given as

$$P_{Ab}(E) = \frac{1}{2}P_{Ab}(E|0) + \frac{1}{2}P_{Ab}(E|1). \quad (6.8)$$

Since at least one pulse will be detected in each weighted chip when a bit "1" is sent, $P_{Ab}(E|1)$ in (6.8) equals to 0. Hence, the BER becomes

$$P_{Ab}(E) = \frac{1}{2}P_{Ab}(E|0) \quad (6.9)$$

where $P_{Ab}(E|0)$ can be similarly calculated as

$$P_{Ab}(E|0) = \prod_{a=1}^{\omega} \sum_{\zeta_a=1}^{N-1} \binom{N-1}{\zeta_a} \left(\frac{\omega}{2A}\right)^{\zeta_a} \left(1 - \frac{\omega}{2A}\right)^{N-1-\zeta_a} \quad (6.10)$$

with ζ_a representing the pulses obtained in each weighted chip.

The BER of the AND detector will also approach to its maximum value if we let N takes maximum value of $\frac{2A}{\omega} - 1$. If there an increasing large number of users simultaneously accessing the network, (6.10) will become

$$P_{Ab}(E|0) = \prod_{a=1}^{\omega} \left(1 - \frac{1}{e}\right). \quad (6.11)$$

Using (6.11) into (6.9) gives the BER of AND detector as

$$P_{Ab}(E) = \frac{1}{2} \prod_{a=1}^{\omega} \left(1 - \frac{1}{e}\right) \quad (6.12)$$

when N is very large.

6.4 Dual Detection

The concept of SUM and AND detector has been given in the previous chapter. This gives us a hint on how we can propose a dual detector. We consider the system utilizing the OOK modulation technique. In such a system, the signature code will be transmitted to represent a bit symbol “1”, and there is no transmission of the signature code if the bit symbol “0” is to be transmitted. At the receiving end, each user will decode the received symbol by correlating the obtained signals with the user’s own signature code sequence. The correlation results can be photo-detected in each chip or as a whole in a bit. Then, based on the number of optical pulses obtained, a decision on what symbol is transmitted will be made. When the dark current noise become significant, the detection must be able to take into account the effect due to noise.

In order to derive a dual detector for OCDMA communication, we consider first a conventional correlation SUM detector and a chip-level AND detector respectively. For a correlation SUM detector, the received signal will be correlated with the signature code of the desired user. Then, the correlation result is photo-detected as a whole to obtain the total number of pulses in a bit duration. Denote X as the total number of receiving optical pulses. A threshold θ is set and the transmission of bit “1” or “0” is decided by

$$\text{Declare} \begin{cases} 1, & \text{if } X \geq \theta; \\ 0, & \text{otherwise.} \end{cases} \quad (6.13)$$

For a chip-level AND detector, all weighted chips are photo-detected individually. Denoted X_j , for $j = 1, 2, \dots, \omega$, as the optical pulses collected from the j th weighted

chip. The presence or absence of a chip “1” or a chip “0” at the j th chip is decided by thresholding X_j with a threshold θ_j , then, the transmission of a bit “1” or a bit “0” is decided by

$$\text{Declare} \begin{cases} 1, & \text{if } X_j \geq \theta_j, \forall j = 1, 2, \dots, \omega; \\ 0, & \text{otherwise.} \end{cases} \quad (6.14)$$

It is clear that the total number of optical pulses used in the SUM detector can also be obtained by $X = \sum_{j=1}^{\omega} X_j$. In fact, the signals obtained in the chip-level AND detector can be used to form other decision variables for a detection which can deliver an independent decision in the presence of noises. Such a decision is just a mathematical manipulation and it does not incur further hardware expenses. However, it does help to improve the performance of OCDMA detection. In the past, using either only correlation or chip-level receiver will give one resulted signal. Then, a decision will be made based on this output. However, this proposed dual detector consists of both correlation and chip-level detector. Now, a decision will be made based on the each independent output. It is like seeking a second opinion before a decision is made.

When the correlation receiver received a total number of photons equal or greater to the threshold, it will decide a bit “1” is sent. Otherwise, a bit “0” is decided to be sent. For chip-level receiver, a bit “1” is decided to be sent if all weighted chips received at least θ_j photons or else a bit “0” is declared. Since the dual detector is made up of correlation and chip-level receiver, it will receive four combinations of outcome as shown in Table 6.1.

The dual detector can functions in two ways. It can make two different decision based on different decision rule using the same design. Hence, we will investigate the two different dual detector.

First, we will examine the dual detector using decision rule 1. If both correlation and

		AND receiver decision	
		0	1
SUM receiver decision	0	00	01
	1	10	11

Table 6.1: Four combinations outcome of dual detector

chip-level receiver decide a bit “0” is sent, dual detector will decide a bit “0” sent. If both detectors decide a bit “1” is sent and in the case where correlation and chip-level receiver make a different decision, dual detector will decide bit “1” is sent. We will call this as receiver scheme as dual detector 1.

To show this, dual detector 1 is designed by having the decision rule as

$$\text{Declare} \begin{cases} 1, & \text{if } X \geq \theta \text{ or } X_j \geq \theta_j \quad \forall j = 1, 2, \dots, \omega; \\ 0, & \text{if } X < \theta \text{ and } X_j < \theta_j \text{ for some } j = 1, 2, \dots, \omega. \end{cases} \quad (6.15)$$

Denote the error probabilities of sending “0” and sending “1” respectively as $P_{sum}(E|0)$ and $P_{sum}(E|1)$ for the SUM detector, and $P_{and}(E|0)$ and $P_{and}(E|1)$ for the AND detector. According to the dual detection law in (6.15), the BER of the dual detector is obtained as

$$P_{dual}(E) = \frac{1}{2}[P_{dual}(E|0) + P_{dual}(E|1)] \quad (6.16)$$

where

$$P_{dual}(E|0) = 1 - P_{sum}(0|0)P_{and}(0|0), \quad (6.17)$$

$$P_{dual}(E|1) = P_{sum}(0|1)P_{and}(0|1). \quad (6.18)$$

In order to get a smaller BER for dual detector 1 as compared to the BER of corre-

lation or chip-level receiver, we will get

$$P_{sum}(0|1)P_{and}(0|1) + 1 - P_{sum}(0|0)P_{and}(0|0) < P_{sum}(1|0) + P_{sum}(0|1), \quad (6.19)$$

$$P_{sum}(0|1)P_{and}(0|1) + 1 - P_{sum}(0|0)P_{and}(0|0) < P_{and}(1|0) + P_{and}(0|1). \quad (6.20)$$

Hence, we will get respectively the conditions from (6.19) and (6.20)

$$\frac{P_{sum}(0|0)}{P_{sum}(0|1)} < \frac{P_{and}(1|1)}{P_{and}(1|0)}, \quad (6.21)$$

$$\frac{P_{and}(0|0)}{P_{and}(0|1)} < \frac{P_{sum}(1|1)}{P_{sum}(1|0)}. \quad (6.22)$$

Under these conditions, the dual detector will have a better BER performance than chip-level and correlation receiver.

Next, we will look at the dual detector using another rule, called decision rule 2. Now, dual detector will decide a bit “1” sent if only both correlation and chip-level receiver decide a bit “1” is sent. If either correlation or chip-level receiver makes a decision on a bit “0” is sent, dual detector will decide bit “0” is sent. We will call this as receiver scheme as dual detector 2.

To show this, dual detector 2 is designed by having the decision rule as

$$\text{Declare} \begin{cases} 1, & \text{if } X \geq \theta \text{ and } X_j \geq \theta_j \quad \forall j = 1, 2, \dots, \omega; \\ 0, & \text{if } X < \theta \text{ or } X_j < \theta_j \quad \text{for some } j = 1, 2, \dots, \omega. \end{cases} \quad (6.23)$$

Denote the error probabilities of sending “0” and sending “1” respectively as $P_{sum}(E|0)$ and $P_{sum}(E|1)$ for the SUM detector, and $P_{and}(E|0)$ and $P_{and}(E|1)$ for the AND detector. According to the dual detection law in (6.23), the BER of the dual detector

is obtained as

$$P_{dual}(E) = \frac{1}{2}[P_{dual}(E|0) + P_{dual}(E|1)] \quad (6.24)$$

where

$$P_{dual}(E|0) = P_{sum}(1|0)P_{and}(1|0), \quad (6.25)$$

$$P_{dual}(E|1) = 1 - P_{sum}(1|1)P_{and}(1|1). \quad (6.26)$$

Similarly, the BER of dual detector 2 is compared against the BER of correlation or chip-level receiver. The BER performance is better given the condition

$$P_{sum}(1|0)P_{and}(1|0) + 1 - P_{sum}(1|1)P_{and}(1|1) < P_{sum}(1|0) + P_{sum}(0|1), \quad (6.27)$$

$$P_{sum}(1|0)P_{and}(1|0) + 1 - P_{sum}(1|1)P_{and}(1|1) < P_{and}(1|0) + P_{and}(0|1). \quad (6.28)$$

Hence, the respectively conditions from (6.27) and (6.28) are

$$\frac{P_{sum}(1|1)}{P_{sum}(1|0)} < \frac{P_{and}(0|0)}{P_{and}(0|1)}, \quad (6.29)$$

$$\frac{P_{and}(1|1)}{P_{and}(1|0)} < \frac{P_{sum}(0|0)}{P_{sum}(0|1)}. \quad (6.30)$$

The dual detector will yield a better BER performance than chip-level and correlation receiver with the above conditions.

For the sake of illustrating the operation principle, the schematic of the proposed dual detector is shown in Figure 6.2. As all users of the OCDMA system are using the same detector, only the dual detector for the first user is given. The dual detector consists of a photodetector, a SUM detector, and a chip-level AND detector. During a bit period, the photodetector detects in a chip-rate ($\frac{1}{T_c}$ Hz) the light intensity on the communication link. The signal (X_j) received during T_c seconds is respectively fed into the chip detector and the SUM detector. In the chip detector, the appearance of

a laser pulse at the j th chip is decided by (6.14), and the decision outputs on chip “0” or chip “1” are stored in a decision memory. When a bit duration is over, the AND operation subject to the signature of the desired user is performed over the stored chips. A decision represented by $b_{i,1}^c$ is obtained on “0” or “1” of the received bit r_i sent by the desired user. In contrast, the received signals in all chip durations are stored first in a memory in the SUM detector without applying any decision. The SUM decision (6.13) is only performed in a bit rate, by using the signature of the desired user. The decision of the dual detector 1 and 2 is thus obtained in a bit rate by performing (6.15) and (6.23) respectively.

It is clear from the schematic that the dual detector uses the same physical components as used in a chip-level detector [79]. The SUM detection involves only several mathematical manipulations on the signals received by the chip-level detector. No extra physical components are needed. However, the incorporation of a SUM detector in a chip-level detection has maximally extracted useful information received by the chip-level detection. Therefore, the dual detector can deliver better BER performance than the conventional chip-level detectors and the SUM detectors. In turn, the dual detector can provide a higher spectral efficiency than the conventional chip-level OCDMA detectors which are running in the same chip-rate. Although the dual detector in Figure 6.2 is based on 1D detector, the result can be further extended to 2D detector.

By taking advantage of the 2D OCDMA signature code constructed using multiple wavelengths and time chips, the dual detector concept can be extended to implement a 2D dual detector. The principle of the first 2D dual detector is basically similar to the 1D counterpart where now each chip is pin pointed by two parameters which is the specific time chip and wavelength.

With the SUM and AND detector, this motivates us to examine the second type of 2D dual detector which can be termed as SUM-AND detector. The principle of this

Figure 6.2: Schematic of dual detector for multiuser detection of OCDMA

detector is that the weighted time chips over one wavelength is first correlated. Then, the chip-level detection for each summed wavelength is performed. So it is to perform the SUM operation and then followed by AND operation as seen in Figure 6.3.

Figure 6.3: Operation of SUM-AND detector

6.5 Numerical Results 1

In order to demonstrate the performance of the proposed dual detectors, we compared them with SUM and AND detectors. First, we consider the OCDMA system used in [87] for the analysis optical detection using optical pulses. Each user is assigned a 2D

signature code which consists of using $\lambda = 23$ wavelengths, $\tau = 435$ time chips and $\omega = 23$ weights. The codes used in the system are assumed to be generated randomly and the presence of an optical pulse at one chip will not affect the probability of presence of an optical pulse in another chip.

Figure 6.4: BER performance comparison of SUM, AND and dual detectors using optical pulses as optical detection

From the figure, we can see that by using dual detection method 1, it has no advantage over the AND detector. Its BER performance is comparable to SUM detector. On the other hand, using dual detector method 2 gives a better BER performance than only using SUM and AND detector alone. Hence, dual detector 2 is effective in mitigating the error probability caused by MUI.

Next, we shall compare the BER performance of the dual detectors with SUM and AND detectors using photon counts as optical detection method using 1D signature codes. In here, we consider the system used in [79] for the analysis. Each user is assigned an OOC signature code with a code length $L = 2000$ and a number of user $N = 25$ and the code weight $\omega = 9$ with a cross- and auto-correlation bounded by one. The BER performance comparison is plotted in Figure 6.5.

Figure 6.5: BER performance comparison of SUM, AND and dual detectors using photon counts as optical detection

It can be seen from the figure that for low optical energy used in each chip, SUM detector gives a better BER performance than AND detector. By using dual detector 1, it gives a better performance than SUM or AND detector until the average photons/nat $\mu = 160$. Dual detector 2 has no advantages over the SUM and AND detectors when $\mu < 160$. When a higher optical energy is used in each chip, it gives a better performance than SUM detector. However, AND detector gives the best performance in this case.

Thus, it can be seen that dual detectors are effective in improving the performance of optical detection than using either only SUM or AND detector as seen from the two figures shown above.

6.6 Multiple Wavelengths Filter

Now we examined the use of the interleaver to implement the 2D OCDMA detector. Using the interleaver, the wavelengths are first separated. Then, from the desired

user's signature codes, each wavelength is correlated. The resulting signals will be used for making decision if a bit "0" or bit "1" is transmitted.

Figure 6.6: Multiple wavelengths filter

After each wavelength has been extracted separately, it will be correlated independently according to the time chip of the desired user signature code. The resulting signal then will be added up to compare against the threshold for the correlation receiver. On the other hand, the signal in each wavelength and time chip can be used individually and obtained chip-level receiver.

6.7 Design Problems of F^3T interleavers

An F^3T interleaver is made up of three cascaded directional couplers as shown in Figure 6.7. Denote the electric fields at the i -th port E_i ($i = 1, 2, 3, 4$) respectively. They are related by

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} c_3 & js_3 \\ js_3 & c_3 \end{bmatrix} \begin{bmatrix} e^{j2\theta} & 0 \\ 0 & e^{-j2\theta} \end{bmatrix} \begin{bmatrix} c_2 & js_2 \\ js_2 & c_2 \end{bmatrix} \begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix} \begin{bmatrix} c_1 & js_1 \\ js_1 & c_1 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \quad (6.31)$$

where

$$\theta = \frac{\Delta L n \pi}{\lambda}, \quad (6.32)$$

$$c_i \triangleq \sqrt{\kappa_i}, \text{ and } s_i \triangleq \sqrt{1 - \kappa_i}, \quad (6.33)$$

with ΔL being the physical length difference of the differential delays that is associated with the phase of power spectrum θ . n is the refractive index, λ the wavelength of the incident light signal. κ_i ($i = 1, 2, 3$) is respectively the power coupling ratio of the i -th directional coupler that can be expressed in another form given by c_i which is the bar-state coupling ratio and s_i is the cross-state coupling ratio.

Figure 6.7: Fourier filter flat top interleaver

Thus, the spectrum of the power transfer functions from E_1 to E_3 and from E_1 to E_4 are respectively obtained as follows:

$$\begin{aligned}
 P_{13}(\theta) = & -2c_1s_3c_2^2s_1c_3\cos(6\theta) + 2c_1s_3^2c_2s_1s_2\cos(2\theta) \\
 & -2c_1^2s_3c_2c_3s_2\cos(4\theta) + c_1^2c_3^2c_2^2 \\
 & +2c_1c_3s_2^2s_1s_3\cos(2\theta) - 2c_1c_3^2s_2s_1c_2\cos(2\theta) \\
 & +s_1^2s_3^2c_2^2 + c_1^2s_3^2s_2^2 \\
 & +2s_1^2s_3s_2c_3c_2\cos(4\theta) + s_1^2c_3^2s_2^2,
 \end{aligned} \tag{6.34}$$

$$\begin{aligned}
 P_{14}(\theta) = & c_1^2s_3^2c_2^2 + 2c_1s_3c_2^2s_1c_3\cos(6\theta) \\
 & -2c_1s_3^2c_2s_1s_2\cos(2\theta) + 2c_1^2s_3c_2c_3s_2\cos(4\theta) \\
 & -2c_1c_3s_2^2s_1s_3\cos(2\theta) + 2c_1c_3^2s_2s_1c_2\cos(2\theta) \\
 & +s_1^2s_3^2s_2^2 - 2s_1^2s_3s_2c_3c_2\cos(4\theta) \\
 & +s_1^2c_3^2c_2^2 + c_1^2c_3^2s_2^2.
 \end{aligned} \tag{6.35}$$

Note that $P_{13}(\theta)$ and $P_{14}(\theta)$ are all periodical with respect to θ or the incident wavelength λ , and they satisfy

$$P_{13}(\theta) + P_{14}(\theta) = 1 \tag{6.36}$$

which is usually referred to as the energy conservative condition.

The interleaver design can be described as to obtain appropriate coupling ratios κ_i ($i = 1, 2, 3$) such that the spectrums of P_{13} and P_{14} have a phase difference of $\frac{\pi}{2}$ in θ , but with a symmetric identical spectral. It is also highly desired that the passbands and stopbands of P_{13} and P_{14} are as much isolated as possible to achieve a good channel discrimination, and more concerned, they have as much flat tops as possible in order to reduce power variations resulting from channel wavelength drifts.

6.7.1 Direct Design of Interleavers with Flat Top

Clearly, the first requirement on P_{13} and P_{14} shifting by $\frac{\pi}{2}$ is satisfied if

$$P_{13}(\theta) = P_{14}\left(\theta + \frac{\pi}{2}\right). \quad (6.37)$$

Substituting (6.34) and (6.35) into (6.37) gives $c_1 = s_1$. By the definitions of c_1 and s_1 in (6.33), we know that

$$c_1 = s_1 = \frac{\sqrt{2}}{2} = \sqrt{\kappa_1}, \quad (6.38)$$

which means that the first coupler in the F^3T interleaver has a $3dB$ coupling ratio. This property will be used in the subsequential analysis. An immediate conclusion from this property and the conservation condition in (6.36) is that P_{13} and P_{14} have identical spectrals with symmetric passbands and stopbands.

To proceed, we examine the derivatives of P_{13} and P_{14} with respect to θ . As P_{13} and P_{14} have identical symmetric spectrals, it suffices to examine the derivatives of $P_{13}(\theta)$. By straightforward calculations, it follows that

$$\begin{aligned} \frac{dP_{13}}{d\theta} = & 12c_1s_3c_2^2s_1c_3\sin(6\theta) - 4c_1s_3^2c_2s_1s_2\sin(2\theta) \\ & + 8c_1^2s_3c_2c_3s_2\sin(4\theta) - 4c_1c_3s_2^2s_1s_3\sin(2\theta) \\ & + 4c_1c_3^2s_2s_1c_2\sin(2\theta) - 8s_1^2s_3s_2c_3c_2\sin(4\theta) \end{aligned} \quad (6.39)$$

$$\begin{aligned}\frac{d^2 P_{13}}{d\theta^2} &= 72c_1 s_3 c_2^2 s_1 c_3 \cos(6\theta) - 8c_1 s_3^2 c_2 s_1 s_2 \cos(2\theta) \\ &\quad + 32c_1^2 s_3 c_2 c_3 s_2 \cos(4\theta) - 8c_1 c_3 s_2^2 s_1 s_3 \cos(2\theta) \\ &\quad + 8c_1 c_3^2 s_2 s_1 c_2 \cos(2\theta) - 32s_1^2 s_3 s_2 c_3 c_2 \cos(4\theta)\end{aligned}\quad (6.40)$$

$$\begin{aligned}\frac{d^3 P_{13}}{d\theta^3} &= -432c_1 s_3 c_2^2 s_1 c_3 \sin(6\theta) + 16c_1 s_3^2 c_2 s_1 s_2 \sin(2\theta) \\ &\quad - 128c_1^2 s_3 c_2 c_3 s_2 \sin(4\theta) + 16c_1 c_3 s_2^2 s_1 s_3 \sin(2\theta) \\ &\quad - 16c_1 c_3^2 s_2 s_1 c_2 \sin(2\theta) + 128s_1^2 s_3 s_2 c_3 c_2 \sin(4\theta)\end{aligned}\quad (6.41)$$

$$\begin{aligned}\frac{d^4 P_{13}}{d\theta^4} &= -2592c_1 s_3 c_2^2 s_1 c_3 \cos(6\theta) + 32c_1 s_3^2 c_2 s_1 s_2 \cos(2\theta) \\ &\quad - 512c_1^2 s_3 c_2 c_3 s_2 \cos(4\theta) + 32c_1 c_3 s_2^2 s_1 s_3 \cos(2\theta) \\ &\quad - 32c_1 c_3^2 s_2 s_1 c_2 \cos(2\theta) + 512s_1^2 s_3 s_2 c_3 c_2 \cos(4\theta).\end{aligned}\quad (6.42)$$

As the flatness of the passbands and/or stopbands is concerned, without loss of generality, we consider the derivatives at $\theta = 0$. They are given by

$$\left. \frac{dP_{13}}{d\theta} \right|_{\theta=0} = 0 \quad (6.43)$$

$$\begin{aligned}\left. \frac{d^2 P_{13}}{d\theta^2} \right|_{\theta=0} &= 72c_1 s_3 c_2^2 s_1 c_3 - 8c_1 s_3^2 c_2 s_1 s_2 + 32c_1^2 s_3 c_2 c_3 s_2 \\ &\quad - 8c_1 c_3 s_2^2 s_1 s_3 + 8c_1 c_3^2 s_2 s_1 c_2 - 32s_1^2 s_3 s_2 c_3 c_2\end{aligned}\quad (6.44)$$

$$\left. \frac{d^3 P_{13}}{d\theta^3} \right|_{\theta=0} = 0 \quad (6.45)$$

$$\begin{aligned}\left. \frac{d^4 P_{13}}{d\theta^4} \right|_{\theta=0} &= -2592c_1 s_3 c_2^2 s_1 c_3 + 32c_1 s_3^2 c_2 s_1 s_2 - 512c_1^2 s_3 c_2 c_3 s_2 \\ &\quad + 32c_1 c_3 s_2^2 s_1 s_3 - 32c_1 c_3^2 s_2 s_1 c_2 + 512s_1^2 s_3 s_2 c_3 c_2.\end{aligned}\quad (6.46)$$

It can be concluded from the form of $\frac{d^n P_{13}}{d\theta^n}$ that all odd order of derivatives are zeros. But it is impossible for $\frac{d^2 P_{13}}{d\theta^2}$ and $\frac{d^4 P_{13}}{d\theta^4}$ to vanish at the same time.

6.7.2 Interleavers of Maximally Flat Top

From the above discussion, the passband/stopband of P_{13} is at most the third order flat in a maximally flat sense. Thus, to have a maximally flat top, it must be satisfied that $\frac{d^2 P_{13}}{d\theta^2} \Big|_{\theta=0} = 0$. Using (6.38), we have

$$\frac{d^2 P_{13}}{d\theta^2} \Big|_{\theta=0} = 4 (8c_2^2 s_3 c_3 + s_3 c_3 (c_2^2 - s_2^2) + s_2 c_2 (c_3^2 - s_3^2)) = 0. \quad (6.47)$$

On the other hand, the requirement on the maximum isolation between passbands and stopbands results in

$$P_{13}(0) - P_{13}\left(\frac{\pi}{2}\right) = 1. \quad (6.48)$$

Using (6.38) again, it follows from (6.34) that

$$P_{13}(0) = \frac{1}{2} + c_3 s_3 (s_2^2 - c_2^2) + c_2 s_2 (s_3^2 - c_3^2), \quad (6.49)$$

$$P_{13}\left(\frac{\pi}{2}\right) = \frac{1}{2} - s_3 c_3 (s_2^2 - c_2^2) - s_2 c_2 (s_3^2 - c_3^2). \quad (6.50)$$

Thus, (6.48) implies that

$$c_3 s_3 (s_2^2 - c_2^2) + c_2 s_2 (s_3^2 - c_3^2) = \frac{1}{2}. \quad (6.51)$$

Substituting (6.51) into (6.47) gives

$$8c_2^2 s_3 c_3 = \frac{1}{2}. \quad (6.52)$$

Applying the definitions of c_i and s_i ($i = 2, 3$), we have

$$\begin{cases} \sqrt{\kappa_3(1-\kappa_3)}(1-2\kappa_2) + \sqrt{\kappa_2(1-\kappa_2)}(1-2\kappa_3) = \frac{1}{2} \\ 8\kappa_2\sqrt{\kappa_3(1-\kappa_3)} = \frac{1}{2}. \end{cases} \quad (6.53)$$

This actually gives two equations for the two unknowns to be determined. Simple study shows that the above equations have a unique solution at $\kappa_2 = 0.25$ and $\kappa_3 = 0.067$.

So far, we have obtained an interleaver with maximal flat passbands and stopbands. In summary, the most flattop F^3T interleaver is made of three cascaded directional fiber couplers respectively with coupling ratios 0.5, 0.25, and 0.067.

6.7.3 Interleavers with a Given Flatness of Passband

In some applications, certain degree of unflatness is acceptable in passbands/stopbands in order to obtain wider passband/stopband width than that of the interleaver with maximally flat passbands/stopbands. In this case, one hopes to design the interleaver by a given level of unflatness. To this end, a measure has to be taken. As $P_{13}(\theta)$ is at most third order of flat and all odd order derivatives of $P_{13}(\theta)$ are zeros, it is rational to choose the second order derivative $\left. \frac{d^2 P_{13}}{d\theta^2} \right|_{\theta=0}$ as a measure of flatness. Later discussion will show that this measure is exchangeable with the passband ripple which is conventionally used to describe the passband unflatness.

Using $\left. \frac{d^2 P_{13}}{d\theta^2} \right|_{\theta=0}$ as a measure, the unflatness can be specified by

$$\left. \frac{d^2 P_{13}}{d\theta^2} \right|_{\theta=0} = \alpha, \quad (6.54)$$

where α is a specification parameter given by users. It will be shown in later discussion that α can only be taking as a non-negative value.

Given an unflatness level, the interleaver design problem is reduced to find those

coupling ratios to maximize the channel isolation, that is,

$$\begin{cases} \max_{\kappa_2, \kappa_3} \{P_{13}(0) - P_{13}(\pi/2)\} \\ \text{subject to } \left. \frac{d^2 P_{13}}{d\theta^2} \right|_{\theta=0} = \alpha. \end{cases} \quad (6.55)$$

For convenience of derivation, the coupling ratios $\kappa_i (i = 2, 3)$ are expressed as $\kappa_i = \sin^2(\phi_i)$ where $\phi_i \in [0, \pi/2]$. Then all c_i and s_i can be expressed respectively by $c_i = \sin(\phi_i)$ and $s_i = \cos(\phi_i)$ because of (6.33). Substituting this into (6.49), (6.50), and (6.54), and using trigonometric, the optimization in (6.55) is rewritten as

$$\begin{cases} \max_{\kappa_2, \kappa_3} \{\sin(2\phi_2 + 2\phi_3)\} \\ \text{subject to } 16\sin(2\phi_3) - 16\cos(\phi_2)\sin(\phi_3) - 4\sin(2\phi_2 + 2\phi_3) = \alpha. \end{cases} \quad (6.56)$$

Applying Lagrange approach, the optimal solutions to (6.56) are achieved by

$$\phi_2^* = 2\phi_3^*, \quad (6.57)$$

$$48\sin^3(\phi_2^*) - 12\sin(\phi_2^*) - \alpha = 0. \quad (6.58)$$

Substituting $c_2 = \sin(\phi_2)$ into (6.58) gives

$$48(c_2^*)^3 - 12(c_2^*) - \alpha = 0. \quad (6.59)$$

As $\alpha \geq 0$, it is easy to verify that the equation (6.59) has only one non-negative real root about c_2^* . After obtaining c_2^* , the desired κ_2 and κ_3 are obtained as below

$$\kappa_2^* = \sin^2(\phi_2^*) = (c_2^*)^2 \quad (6.60)$$

$$\begin{aligned} \kappa_3^* &= \sin^2(\phi_3^*) = \frac{1}{2}(1 - \cos(2\phi_3^*)) = \frac{1}{2}(1 - \cos(\phi_2^*)) = \frac{1}{2}(1 - \sqrt{1 - \sin^2(\phi_2^*)}) \\ &= \frac{1}{2} - \frac{1}{2}\sqrt{1 - \kappa_2^*}. \end{aligned} \quad (6.61)$$

Now we prove that $\alpha \geq 0$. To this end, we examine $\frac{dP_{13}}{d\theta}$ and $P_{13}(\theta)$ when the optimality given in (6.57) and (6.59) is satisfied.

Substituting the trigonometric expressions of c_i and s_i , ($i = 2, 3$) into (6.39) gives

$$\begin{aligned} \frac{dP_{13}}{d\theta} = & 3\sin^2(\phi_2)\sin(2\phi_3)\sin(6\theta) - \cos^2(\phi_3)\sin(2\phi_2)\sin(2\theta) \\ & + \sin^2(\phi_3)\sin(2\phi_2)\sin(2\theta) - \cos^2(\phi_2)\sin(2\phi_3)\sin(2\theta). \end{aligned} \quad (6.62)$$

Applying the trigonometry formula $\sin(6\theta) = 3\sin(2\theta) - 4\sin^3(2\theta)$, we have

$$\begin{aligned} \frac{dP_{13}}{d\theta} = & \sin(2\theta) \left(-12\sin^2(\phi_2)\sin(2\phi_3)\sin^2(2\theta) + 9\sin^2(\phi_2)\sin(2\phi_3) \right. \\ & \left. - \cos^2(\phi_3)\sin(2\phi_2) + \sin^2(\phi_3)\sin(2\phi_2) - \cos^2(\phi_2)\sin(2\phi_3) \right). \end{aligned} \quad (6.63)$$

When the optimality (6.57) holds, $\frac{dP_{13}}{d\theta}$ is given by

$$\frac{dP_{13}}{d\theta} = \sin(2\theta) \left(-12\sin^3(\phi_2)\sin^2(2\theta) + 12\sin^3(\phi_2) - 3\sin(\phi_2) \right). \quad (6.64)$$

Forcing $\frac{dP_{13}}{d\theta}$ equal to zero, it is known that $P_{13}(\theta)$ has only three stationary points in $[-\pi/2, \pi/2)$. They are given by

$$\theta_1 = 0, \quad (6.65)$$

$$\theta_2 = \frac{1}{2}\cos^{-1}\left(\frac{1}{2\sin(\phi_2^*)}\right), \quad (6.66)$$

$$\theta_3 = -\frac{1}{2}\cos^{-1}\left(\frac{1}{2\sin(\phi_2^*)}\right). \quad (6.67)$$

Similar to (6.64), $P_{13}(\theta)$ under the optimality condition (6.57) can be written as

$$P_{13}(\theta) = \frac{1}{2} + \frac{1}{2}\sin(\phi_2^*)\cos(2\theta) [3 - 4\cos^2(2\theta)\sin^2(\phi_2^*)]. \quad (6.68)$$

It is obvious that $P_{13}(\theta_2) = 1$ and $P_{13}(\theta_3) = 1$, which means $P_{13}(\theta)$ reaches its maximum at $\theta = \theta_2$ and $\theta = \theta_3$. It is concluded in turn that $P_{13}(\theta)$ has a minimum

at $\theta = 0$, otherwise $P_{13}(\theta)$ would be flat over $[\theta_2, \theta_3]$ because $P_{13}(\theta)$ has only three stationary points in $[-\pi/2, \pi/2]$. Therefore, $\frac{d^2 P_{13}}{d\theta^2} \geq 0$, which completes the proof that $\alpha \geq 0$.

The fact that $\alpha \geq 0$ can also be understood as below. Note that the first order derivative of $P_{13}(\theta)$ is always zero at $\theta = 0$ which means $P_{13}(\theta)$ has an either convex or concave peak at $\theta = 0$. The convexity is determined by the second order derivative $\left. \frac{d^2 P_{13}}{d\theta^2} \right|_{\theta=0}$. Clearly, it can be expected that $P_{13}(\theta)$ is a bit concave at $\theta = 0$ in comparison with the most flat case, in order to widen the passband. Thus, α should be taking a non-negative value.

Next we explore the relationship between the passband ripple and the given unflatness level α . It is clear from the above discussion that the ripple can be specified by $1 - P(0)$ where $P(0)$ is given in (6.49). Again using trigonometric expressions of c_i and s_i , ($i = 2, 3$) gives

$$\begin{aligned} 1 - P(0) &= \frac{1}{2} - \frac{1}{2} \sin(2\phi_2^* + 2\phi_3^*) = \frac{1}{2} - \frac{1}{2} \sin(3\phi_2^*) \\ &= \frac{1}{2} (1 - 3\sin(\phi_2^*) + 4\sin^3(\phi_2^*)) \end{aligned} \quad (6.69)$$

where $\sin(\phi_2^*)$ is the non-negative root of the equation (6.58) with α given. Thus, (6.69) together with (6.58) give the relationship between the unflatness measure α and the ripple defined in a conventional sense.

To close the discussion of this section, it is noteworthy that the results obtained in Section 6.7.2 for the most flat case are special cases of the results presented in this subsection. To see this, we just need to let $\alpha = 0$ in (6.59) to obtain the resultant coupling ratios as in Section 6.7.2.

6.7.4 Numerical Results 2

To verify the theoretical analysis, simulation studies are carried out in this section for the design of an 2×2 interleaver capable of separating two sets of interleaving 50GHz signals.

In the design the differential delay ΔL is tuned for 50GHz. Using results in Section 6.7.2, the F^3T interleaver with the most flat passbands and stopbands is obtained by choosing $\kappa_1 = 0.5$, $\kappa_2 = 0.25$ and $\kappa_3 = 0.067$. The power transmission spectra of the obtained F^3T interleaver is shown in Figure 6.8, where the spectral responses of the unbalanced MZI interleavers and the integral-optimization based F^3T interleavers in [101] are also plotted for comparison. Figure 6.9 gives an enlarged version of Figure 6.8 to see clearly the top portion of the passband of those interleavers. It is clear from the figure that the obtained F^3T interleaver is attractive in flat passbands/stopbands and big isolation. Further examination shows that the 0.5dB passband width of the obtained F^3T interleaver is about 30GHz, compared to 28GHz for an integral-optimization based F^3T interleavers and 20GHz for a single unbalanced MZI interleaver. Also the most flat F^3T interleaver provides 12GHz of 25dB stopband width, larger than 4GHz for MZI interleaver. Although the 25dB passband of the most flat F^3T interleaver is smaller than 14GHz for integral-optimization based F^3T interleavers, the most flat F^3T interleaver demonstrates a bigger isolation and a wider flat stopband with zero gain.

Next, we consider ripples in interleaver design. Under a given level of ripples specified by α , the desired F^3T interleaver can be obtained by solving (6.59), (6.60) and (6.61) as discussed in Section 6.7.3. Figure 6.10 and Figure 6.11 show spectral responses of the F^3T interleaver designed with α taking values of 0.495, 1.037, and 2.580. From (6.69), it is known that these values of α correspond to 0.005dB, 0.02dB, and 0.1dB ripples respectively. As observed from the curves, the bigger the ripples in the passband, the wider the passband width, however, the poorer the isolation

between the passband and the stopband. Table 6.2 lists some performance parameters of the F^3T interleaver with the most flat passbands, the F^3T interleavers with certain passband ripples, the MZI interleavers, and the F^3T interleavers obtained by numerically optimizing an integral criteria. Compared with the most flat case, the F^3T with 0.01dB ripples possesses 31.1GHz of 0.5dB passband and 15.4GHz of 25dB stopband, much wider than those of the most flat F^3T interleaver. It confirms the theoretical analysis that acceptance of certain ripples helps to widen the passband and stopband width. Moreover, the F^3T interleaver designed with 0.013dB ripples is able to provide 31.3GHz of 25dB passband width and 15.8GHz of stopband width, all greater than 28GHz and 14GHz for the integral-optimization based interleavers, which concludes that the interleaver designed by the proposed approach demonstrates superior performance to the other existing interleavers.

Then with the interleaver, the signal that passed through the optical delay lines can be further decoded into their respective wavelengths held by the desired user's signature code. This will not caused any long delays as compared to decoding by optical delay lines as decoding of all wavelengths can be done at the same time instant.

parameters		passband ripples (dB)	passband 0.1 dB (GHz)	width 0.5 dB (GHz)	stopband 25 dB (GHz)	width 30 dB (GHz)
the most flat F^3T	interleaver	0	19.4	29.9	11.6	8.66
F^3T interleaver	$\alpha = 0.215$	0.001	20.1	30.3	13.0	10.4
	$\alpha = 0.495$	0.005	21.0	30.8	14.4	12.2
	$\alpha = 0.714$	0.01	21.6	31.1	15.4	-
	$\alpha = 0.822$	0.013	21.9	31.3	15.8	-
	$\alpha = 1.037$	0.02	22.4	31.5	-	-
MZI interleaver		-	9.6	21.4	3.6	2
optimization based F^3T		0.005	-	28	14	-

-: stands for not existing

Table 6.2: Performance comparison of several interleavers

Figure 6.8: Spectral response of the most flat F^3T interleaver

6.8 Conclusion

Based on the chip-level detection, a dual detection scheme is developed in attempts to improve the capacity of OCDMA system in the presence of dark current noise and MUI. It is accomplished by combining two conventional detectors which can provide independent decisions but have complementary BER features in the presence of dark current noise and MUI. A dual detector is constructed by using the a correlation SUM detector and a chip-level AND detector. As all signals needed in the dual detector are available in the chip-level detection, no further hardware expenses are required. Thus, the dual detection scheme can be considered as a cost-effective approach to enhance the BER performance of OCDMA receivers.

Next, we study the problem of designing interleavers with flat-top passband and stopband responses for implementing the 2D OCDMA receiver. The exact solution

Figure 6.9: Enlarged version of Figure 6.8 with top portion highlighted

to F^3T interleavers is obtained by analyzing the derivatives of interleaver spectral. To further widen the bandwidth of passband and stopband, a given level of ripples is considered. Using the second order of derivative of the interleaver spectral, an analytical approach is proposed to design F^3T interleavers with specified ripple and maximal isolation. Comparison studies show that the F^3T interleaver designed by the proposed approach has favorable performance superior to the existing interleavers that are designed by numerical methods. Even though the theoretical results in this paper are obtained for F^3T interleavers consisting of three couplers, they apparently can be extended to more general case to tackle the problem of designing optical filters with N —stages cascaded fiber couplers. Thus, it is expected that the proposed scheme enables a quantitative design of optical filters with prespecified performance.

Figure 6.10: Spectral response of F^3T interleaver with ripples

6.9 Appendix

Derivation for condition (6.21):

$$\begin{aligned}
 & P_{sum}(0|1)P_{and}(0|1) + 1 - P_{sum}(0|0)P_{and}(0|0) < P_{sum}(1|0) + P_{sum}(0|1) \\
 \Rightarrow & P_{sum}(0|1)P_{and}(0|1) + 1 - [(1 - P_{sum}(1|0))(1 - P_{and}(1|0))] < P_{sum}(1|0) + P_{sum}(0|1) \\
 \Rightarrow & P_{sum}(0|1)P_{and}(0|1) + P_{and}(1|0) - P_{sum}(1|0)P_{and}(1|0) < P_{sum}(0|1) \\
 \Rightarrow & P_{sum}(0|1)P_{and}(0|1) + P_{and}(1|0)[1 - P_{sum}(1|0)] < P_{sum}(0|1) \\
 \Rightarrow & P_{sum}(0|1)[1 - P_{and}(0|1)] > P_{and}(1|0)P_{sum}(0|0) \\
 \Rightarrow & P_{sum}(0|1)P_{and}(1|1) > P_{and}(1|0)P_{sum}(0|0) \\
 \Rightarrow & \frac{P_{sum}(0|0)}{P_{sum}(0|1)} < \frac{P_{and}(1|1)}{P_{and}(1|0)}
 \end{aligned}$$

▲▲▲

Figure 6.11: Enlarged version of Figure 6.10 with top portion highlighted

Derivation for condition (6.22):

$$\begin{aligned}
 & P_{sum}(0|1)P_{and}(0|1) + 1 - P_{sum}(0|0)P_{and}(0|0) < P_{and}(1|0) + P_{and}(0|1) \\
 \Rightarrow & P_{sum}(0|1)P_{and}(0|1) + 1 - [(1 - P_{sum}(1|0))(1 - P_{and}(1|0))] < P_{and}(1|0) + P_{and}(0|1) \\
 \Rightarrow & P_{sum}(0|1)P_{and}(0|1) + P_{sum}(1|0) - P_{sum}(1|0)P_{and}(1|0) < P_{and}(0|1) \\
 \Rightarrow & P_{sum}(0|1)P_{and}(0|1) + P_{sum}(1|0)[1 - P_{and}(1|0)] < P_{and}(0|1) \\
 \Rightarrow & P_{and}(0|1)[1 - P_{sum}(0|1)] > P_{sum}(1|0)P_{and}(0|0) \\
 \Rightarrow & P_{and}(0|1)P_{sum}(1|1) > P_{sum}(1|0)P_{and}(0|0) \\
 \Rightarrow & \frac{P_{and}(0|0)}{P_{and}(0|1)} < \frac{P_{sum}(1|1)}{P_{sum}(1|0)}
 \end{aligned}$$

▲▲▲

Derivations for condition (6.29):

$$\begin{aligned}
& P_{sum}(1|0)P_{and}(1|0) + 1 - P_{sum}(1|1)P_{and}(1|1) < P_{sum}(1|0) + P_{sum}(0|1) \\
\Rightarrow & P_{sum}(1|0)P_{and}(1|0) + 1 - [(1 - P_{sum}(0|1))(1 - P_{and}(0|1))] < P_{sum}(1|0) + P_{sum}(0|1) \\
\Rightarrow & P_{sum}(1|0)P_{and}(1|0) + P_{and}(0|1) - P_{sum}(0|1)P_{and}(0|1) < P_{sum}(1|0) \\
\Rightarrow & P_{sum}(1|0)P_{and}(1|0) + P_{and}(0|1)[1 - P_{sum}(0|1)] < P_{sum}(1|0) \\
\Rightarrow & P_{sum}(1|0)[1 - P_{and}(1|0)] > P_{and}(0|1)P_{sum}(1|1) \\
\Rightarrow & P_{sum}(1|0)P_{and}(0|0) > P_{and}(0|1)P_{sum}(1|1) \\
\Rightarrow & \frac{P_{sum}(1|1)}{P_{sum}(1|0)} < \frac{P_{and}(0|0)}{P_{and}(0|1)}
\end{aligned}$$

▲▲▲

Derivations for condition (6.30):

$$\begin{aligned}
& P_{sum}(1|0)P_{and}(1|0) + 1 - P_{sum}(1|1)P_{and}(1|1) < P_{and}(1|0) + P_{and}(0|1) \\
\Rightarrow & P_{sum}(1|0)P_{and}(1|0) + 1 - [(1 - P_{sum}(0|1))(1 - P_{and}(0|1))] < P_{and}(1|0) + P_{and}(0|1) \\
\Rightarrow & P_{sum}(1|0)P_{and}(1|0) + P_{sum}(0|1) - P_{sum}(0|1)P_{and}(0|1) < P_{and}(1|0) \\
\Rightarrow & P_{sum}(1|0)P_{and}(1|0) + P_{sum}(0|1)[1 - P_{and}(0|1)] < P_{and}(1|0) \\
\Rightarrow & P_{and}(1|0)[1 - P_{sum}(1|0)] > P_{sum}(0|1)P_{and}(1|1) \\
\Rightarrow & P_{and}(1|0)P_{sum}(0|0) > P_{sum}(0|1)P_{and}(1|1) \\
\Rightarrow & \frac{P_{and}(1|1)}{P_{and}(1|0)} < \frac{P_{sum}(0|0)}{P_{sum}(0|1)}
\end{aligned}$$

▲▲▲

Chapter 7

Conclusion and Recommendation for Further Research

7.1 Conclusion

This thesis studies and analyzes the problems of optical detection of OCDMA based on the number of photon counts received for chip-level and correlation receivers. For the first time, the investigation on the optical detection under the condition where the data bit transmission is non-equiprobable is carried out.

First, we have devised a manageable approximate method to address the mix Poisson-binomial probability and use it to analyze the optimal optical detection of chip-level receivers. We have shown that by introducing two auxiliary functions, they can help in simplifying the complex expression of the likelihood probabilities of the number of photons received. Then, important properties derived from the two auxiliary functions are established. These properties are important in obtaining the monotonicity property for the likelihood probabilities of number of photons received. We have presented that the problem of optical detection in the presence of unbalanced trans-

mission probabilities of data bit can be solved by using the monotonicity property. However, the evaluation of the BER performance of chip-level receiver is complex. Thus, we have shown that the approximate approach we proposed can simplify the analysis of the BER performance. Thus, the approximately optimal BER can be obtained by establishing the optimality on each weight chip position. Given a range of transmission probabilities of data bit, a BER bound can be attained using the monotonicity property. Hence, optimal chip-level optical detection is achieved.

Next, the optical detection of correlation receiver in the presence of non-equiprobable data bit transmission is analyzed. The results obtained to handle the mix Poisson-binomial probability is extended to establish the monotonicity property for correlation receiver. We have further shown that the optimal correlation detection under the condition where the data bit transmission is non-equiprobable can be achieved using the monotonicity property attained from chip-level detection. But in this case, the result obtained is a general result and does not depend on the BER approximation method derived for chip-level detection.

A new OCDMA receiver by combining the merits of chip-level and correlation receiver is investigated. It is found that chip-level receiver is more effective than correlation receiver in combating MUI but less robust against dark current noise as compared to the latter. In contrast, correlation receiver is more robust to noise but more susceptible to MUI. Hence, by combining the two detectors, a dual detector is obtained. The result can be further extended to give a 2D dual detector.

Lastly, some implementation issues of OCDMA detection are addressed. The design problem of F^3T interleaver is analyzed as it is being looked into for implementing the 2D detection method since it can split up dense wavelengths. As 2D signature code consists of multiple wavelengths, the wavelength can be individually obtained before correlation for each wavelength can take place. With the parameters given, the F^3T interleaver can be correctly designed and implemented.

7.2 Recommendation for Further Research

There are still many problems that are needed to be studied and explored in the area of OCDMA. We would like to investigate for the following topics in future.

- The analysis of optimal BER without the need of assuming independency between each chip has to be further examined.
- The monotonicity property for OCDMA receiver utilizing PPM and OPPM techniques have to be further analyzed.
- The BER bound of the receiver is found when the data bit transmission is not equiprobable. It is obtained under the condition only the desired user has unbalanced transmission probability. We would analysis further by considering unbalanced transmission for all users.
- The proposed dual detector will give a better BER performance than correlation or chip-level receiver under some conditions. It will be interesting to obtain the parameters that will affect these conditions and give a better BER performance for the dual detector.
- Most of the efforts have concentrated on the condition that it is chip-synchronous. We would like to extend the results of the monotonicity property of the OCDMA receiver to the case where it is chip-asynchronous.
- A more through and in-depth work has to be done on analyzing the 2D chip-level and correlation receiver with the consideration of noises. In addition, more in-depth investigation work on the proposed 2D dual detector is also needed.
- The linear phase response of the interleaver has to be considered to analyze the effects on the interleaving delay on the buffering in optical domain.

Author's Publications

- 1) S. W. Kok, Y. Zhang, C. Wen, Y. C. Soh, "Design of all-fiber optical interleavers with a given specification on passband ripples", *Optics Communications*, 2003, Vol. 226, No. 1-6, pp. 241-248.
- 2) S. W. Kok, Y. Zhang, C. Wen, Y. C. Soh, "Dual detection for optical code division multiple access communication", *Optical Engineering*, 2004, Vol. 43, No. 12, pp. 2835-2836.
- 3) S. W. Kok, Y. Zhang, C. Wen, Y. C. Soh, "Performance comparison between 2D correlation and chip-level receivers for OCDMA", *47th IEEE International Midwest Symposium on Circuits and Systems, Hiroshima, Japan, July 2004*, pp. II-533 - II-536.
- 4) S. W. Kok, Y. Zhang, C. Wen, Y. C. Soh, "Optimality of chip-level detection for OCDMA", *Optics Communications*, 2005, Vol. 255, No. 4-6, pp. 196-208.
- 5) S. W. Kok, Y. Zhang, C. Wen, Y. C. Soh, "Optimality of chip-level detection for OCDMA using OOC", Accepted *Opto Ireland 2005*.
- 6) S. W. Kok, Y. Zhang, C. Wen, Y. C. Soh, "Performance of chip-level optical CDMA detection in the presence of uncertain a priori probabilities", revised for *IEEE Transactions on Information Theory*, 2006
- 7) S. W. Kok, Y. Zhang, C. Wen, Y. C. Soh, "Monotonicity of likelihood ratio of MAP correlation detection for OCDMA communication", revised for *IEEE Transactions on Communication*, 2006

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