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Optical Filter Banks: Structure, Design and Implementation

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

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*To my parents, my wife and my sister
for their encouragement and love.*

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Summary

To meet the explosive bandwidth demand in fiber optic communications, Wavelength Division Multiplexing (WDM) communication systems must offer increasing channel counts and channel speeds. Since optical filter banks (FBs), which perform as a bank of optical filters, have the advantages of very high time-bandwidth product capabilities, multi-channel signal processing, and versatile frequency responses that can be tailored as specified, they can be used to overcome the bottleneck caused by Optical to Electrical (O/E) and Electrical to Optical (E/O) conversion and in turn increase the system capacity. Thus, optical filter banks are becoming an enabling technology to revolutionize WDM systems from point-to-point systems to transparent all-optical networks. Taking these advantages into account, this thesis aims to systematically study the structure, the design, and the implementation of optical filter banks, particularly for the applications in WDM optical communications.

In the design scheme, the main idea for the design of optical filter bank is to construct a multi-input multi-output (MIMO) filter bank system into a series of single-input single-output (SISO) systems. With this design strategy, the design problem is converted into SISO optical filter design which greatly simplifies the design procedure as well as the implementation of optical filter bank.

To illustrate the design scheme, we first systematically studied the optical filter bank of a simple case with two-input and two-output ports. In order to meet different design specifications, the designs of both finite impulse response (FIR) and infinite impulse response (IIR) filter banks are considered. For the implementation of two-port fiber filter banks with all-fiber structures, an innovative fabrication technology is developed.

To further explore the theory and design of optical filter banks, multi-port optical

filter banks, particularly the three-port ones, are theoretically studied. A novel 3×3 lattice structure of optical delay-line circuit is proposed to implement three-port FIR optical filter banks and a general synthesis approach is devised for deriving the design parameters of such filter banks. Efficient IIR architectures for three-port optical filter bank are also examined to further improve the performance of optical filter banks.

To illustrate an important application of the optical filter banks, spectrum equalization is studied in depth by using two-port optical filter banks in both FIR and IIR structures. In this case, one port of the optical filter bank is extracted for the design of spectrum equalization filter.

In conclusion, the thesis is concerned with the *structural design, performance* and *implementation* of optical filter bank systems, particularly for the applications in WDM optical communications.

Chapter 1

Introduction

With the development of low loss optical fibers and broadband optical amplifiers, wavelength division multiplexing (WDM) communication systems have the potential to harness a huge bandwidth [1]. Optical filters have wide bandwidths in comparison with their electronic counterparts and their frequency characteristics can be tailored to a desired response, hence they have become an enabling technology to exploit the full bandwidth potential of optical WDM communication [2–5]. In addition to the applications in optical communications, optical filters are also widely employed in many other applications including optical signal processing [6], bio-sensing technology [7], precision measurement [8], environmental monitoring [9], spectroscopy [10], astronomy [11], and many others.

Among the various optical filters, optical filter bank has the superior ability to process two or multiple bands of optical signals simultaneously. Hence, research in this area has attracted significant amount of attentions recently [12–14] because of their efficiency and simplicity, particularly in upgrading WDM communication systems where a series bands of multiple wavelength-multiplexed signals are to be combined or separated [15, 16]. When the spectral performances of the filters are of great concern, the term of optical filter banks are generally denoted by multiplexers

(for optical filter banks with many outputs) or interleavers (for optical filter banks with several output ports) in the optical communications community. In WDM communications, the most obvious application of optical filter bank is for multiplexing/demultiplexing a set of closely spaced channels [17–20]. They can also play an important role in gain equalization [21–23], dispersion compensation (DC) [24–27], multi-wavelength fiber laser [28–31] and so on. The general applications of an optical filter banks [1] in a simplified WDM system are illustrated in Fig. 1.1 in terms of their deployment in optical transmitter, optical transmission channel and optical receiver. At the optical transmitter part of the WDM communication system, optical filter banks can be used to generate multi-wavelength optical signals carrying end-user information for the transmission [30, 31]. These multiple sources with different central wavelengths may then be combined into one optical fiber with the help of an $N \times 1$ multiplexer which is also an optical filter bank. To increase the transmission capacity and decrease the deployment cost of the WDM communication systems, several pieces of optical fiber are combined together in the optical transmission part, where multi-channel devices such as multi-channel gain equalizers [22, 23] and dispersion compensators [26, 27] with multiple-input-multiple-output are used to reshape the amplitude and phase responses of each of the optical channels, respectively. The development of these high performance multi-channel devices to meet the stringent requirements of WDM communications relies heavily on the design of optical filter banks. At the receiving end, an $N \times 1$ de-multiplexer that has a reverse function of an $N \times 1$ multiplexer is used to separate the set of optical signals to the respective end uses.

Due to the advantages and wide applications of optical filter banks, this thesis aims to study, theoretically and experimentally, the structure, the design and the implementation of optical filter banks, particularly for applications in optical communications.

1.1 Motivation

Fiber optic networks are becoming increasingly popular in the recent years due to their high speed and high data transmission capacity, particularly since the early 90's when the requirement of high-capacity data services for world wide internet has been dramatically increased [1–4,15]. To meet the explosive bandwidth demand, fiber optic communication systems must offer both higher channel counts and higher transmission speed, which will evidently give rise to a new generation of optical communication systems.

There are basically three factors limiting the progress of upgrading WDM systems with high speed and high channel counts. The first limiting factor is attributed to the optical-electrical-optical (OEO) conversion mechanisms [3,15] that are employed in WDM systems to conduct signal switching, routing, and regenerating. To overcome the bottleneck caused by the OEO devices, WDM systems are evolving to all optical networks by pushing as much as possible all the OEO devices to the ends of the network. In their places, all-optical filters are used as switches, routers, and all the other signal exchanging devices at each network nodes. This all-optical structure creates the opportunity to fully utilize the inherent bandwidth of the optical fiber and dramatically increases the transmission speed.

Figure 1.1: General applications of optical filter bank in a simple WDM system.

The second factor limiting the increase of transmission capacity comes from the available spectrum that optical fiber can support. Due to the existence of two strong absorption peaks at around 1380 nm and 1630 nm respectively, only discrete spectrum windows usually denoted by S-band, C-band and L-band are available for optical communications [3]. The currently employed WDM systems are mainly operating in the C-band. Clearly, narrowing the channel spacing so as to increase the number of channels is the only solution in this case to increase the bandwidth of the whole communication system. This results in the *dense* wavelength division multiplexing (DWDM) [32, 33]. However, narrowing channel spacing also puts a very stringent bandwidth requirement on all the optical components and devices used in the DWDM system. As a result, the implementation of DWDM system is very costly. With the recent advent of full-spectrum or All-wavesTM optical fiber technology, like SMF-28e fiber (Corning Product), the current WDM technology can be extended to cover a broader spectrum from 1285 nm to 1625 nm. The broadening of communication spectrum gives an economic solution to increase the transmission capacity without narrowing the bandwidth of each components and devices in optical communication system. As a result, *coarse* wavelength division multiplexing (CWDM) technology [34–36] is proposed and has found applications in metropolitan communication. In order to fully exploit the bandwidth brought by the all-spectrum fiber technology, it has been a concern in optical research community to combine the strengths of DWDM and CWDM to give higher channel counts. *Interleaver/de-interleaver*, which functions as an optical filter bank capable of splitting a set of densely spaced channels into several sets of channels with wide channel spacings, or doing the reverse, provides a cost effective means to use wide bandwidth components to upgrade the existing network with a broadband and high channel count WDM technology. Thus, it has received a lot of attention recently in the optical community [14, 20, 37–40].

As for the third factor, the uniformity of the light spectra generated either from

a broadband light source or an amplifier is not smooth, which has been recognized as one of the barriers to upgrade WDM communication system [41, 42]. The light spectra emitted from the broadband light sources, either the light emitting diode (LED) category [43] or amplified spontaneous emission (ASE) category, usually have Gaussian-like shapes [3]. For high precision optical signal processing and high speed WDM communication, the output spectrum of a broadband light source should ideally be as flat as possible over a wide range of wavelengths. Another non-uniformity of the light spectra results from the highly nonlinear gains of all optical amplifiers [44]. In the currently used erbium-doped fiber amplifier (EDFA), the gain exhibits a sharp peak gain at around 1530 nm and its profile depends upon the input optical signal power of EDFAs and the pump powers [45, 46]. In order to maintain a balanced signal-to-noise (SNR) ratios among all channels, spectrum equalization is a must in optical amplification. The spectrum gain compensation is also desired to achieve power balancing among channels in the presence of frequent channel add/drop in the next generation of all-optical WDM systems.

Since the optical filter banks are able to overcome the above mentioned three limitations and have the advantages of efficiency and simplicity, they have received a lot of attention recently in the optical community [14, 20, 37–40]. Moreover, in addition to the applications in optical communications, they can also be used in many other complex structures [6–11]. However, the design difficulties exist because of the requirement on achieving simultaneously the desired responses in each of the output channels. Hence, the design and implementation of optical filter banks with good performance are of great importance.

As the counterpart to the optical filter banks, digital filter bank (FB) systems have been extensively studied over the past twenty years [47–49], because they have vast applications in multi-rate signal processing and related fields, such as video and audio compression, subband coding, equalization and multiplexing/de-multiplexing

in telecommunication. The main idea in such systems is to decompose signals into different frequency bands so that they can be easily transmitted, analyzed or processed.

However, the theory, the design, and the implementation of digital filter bank system cannot be applied directly for the design of optical filter banks because of the specific physical principles used to construct the optical filters. There essentially four physical constraints on optical filter banks. First, the positive quantity of the input and output optical signals requires that the power spectra of the optical filter banks are to be designed so as to have the desired power responses. Second, the optical signal must be coherent in order to generate an interference phenomenon which is the physical principle used to develop and realize the optical filters. To yield coherent optical signals, the physical lengths of the proposed structures must be carefully chosen. Third, there are some restrictions in the entries of transfer matrix of the physically realizable design element are correlated. For example, the transfer matrix of an optical coupler which is a very common and basic element in optical devices and instruments is Hermitian. To overcome these restrictions on transfer matrices of the optical elements, the structure of the optical filter bank must be carefully designed. Fourth, as the level of precision and accuracy that can be achieved in the design parameters of optical filters is not as high as that of the digital filters, the designed optical filter bank must be more robust to variation and noise, as well as the fabrication errors of the designed parameters. To achieve this, difficulties in the structure design are clearly highlighted so that requirements for filters to be more robust to the design parameter deviations can be addressed more readily. Moreover, the high speed and high channel count WDM systems usually impose strict requirements on dynamic performance and tolerance in the presence of uncertainties [3, 15, 42]. This creates difficulties when designing optical filters using the available digital filter design theory, particularly in the design of optical filter banks that are multi-input multi-output (MIMO) nonlinear systems, where more

constraints are usually imposed for achieving the desired responses to meet certain specifications. Thus, to overcome the above mentioned constraints, the structure design of an optical filter bank is an important consideration in order to achieve desired filter bank responses, unlike the digital filter bank where more efforts are imposed on the filter performance design. With an efficient optical structure design, a better performance design of optical filter bank can be achieved, which further leads to a more reliable implementation design.

Therefore, the structure design, performance design and implementation design are critical aspects when designing optical filter banks, with the structure design being the most important.

1.2 Objectives

The thesis aims to theoretically and experimentally study the design of optical filter banks to overcome the three factors limiting the progress of upgrading WDM systems. The studies on the design of optical filter banks are carried out from the three critical aspects, namely, the *structural design*, the *performance design* and the *implementation design*, with more efforts imposed on the structural design. Firstly, optical filter bank should have an efficient and implementable structure. Secondly, based on the proposed structure, the performance of optical filter bank should be designed such that it can achieve the desired specifications, such as all the output transmissions shall have identical passband/stopband shape but with certain phase shift between them in frequency and each of the output transmissions shall have the specified characteristics. Thirdly, feasible and reliable implementation technologies need to be developed to realize optical filter banks in different plat-forms, such as all fiber, waveguide, thin film, and micro-electro-mechanical (MEM) systems [2,3].

From the structural design point of view, an optical filter bank obviously has mul-

multiple input and output ports since it is a MIMO system. Consequently, the optical filter bank deals with the models consisting of $M \times M$ matrices when the number of input/output ports is M , in contrast to a simple transfer function for a single-input single-output (SISO) system. This would obviously create the difficulties in designing the structure and the performance of filter banks. Clearly, a two-input-two-output structure, the system model of which is described by 2×2 matrices, is the simplest example of the filter bank. The resultant two-port filter banks can then be cascaded to create logarithmic filter banks. However, such a structure is complex and inefficient for designing multi-port optical filter banks. In practice, more elements and stages are needed, which would increase the insertion losses and connection losses. Furthermore, only configurations of power of two is possible with two-port filter banks and this would limit the capability of the optical filter bank in multiplexing more complicatedly coded signals that are needed in optical communications. Therefore, it is of great interest to design filter banks using higher order multi-port structures, especially the 3-port ones, in order to efficiently design the filter banks with less stages and to provide more flexibility for optical signal multiplexing. Evidently, a simple and efficient structure can greatly simplify the performance analysis and the implementation. Hence, the structural design of optical filter bank is of primary importance in the whole design procedure.

From the performance design perspective, two schemes can be used to design filter bank systems, namely finite impulse response (FIR) filter banks and infinite impulse response (IIR) filter banks [50–52]. Theoretically, the FIR and IIR concepts for optical filter banks are the same as their digital counterparts. FIR represents Finite Impulse Response with only zeros in the FIR filter transfer function and IIR denotes Infinite Impulse Response that has poles in the IIR filter transfer function. Physically, FIR optical filter banks have only finite optical interference circles (*e.g.* Mach-Zehnder interferometer) to produce finite impulse responses in each output channel, while IIR optical filter banks have infinite optical interference circles (*e.g.*

ring resonators or cavities) which generate infinite impulse responses in each output. FIR filters are commonly used in filter banks, due to their ability of providing linear phase response, while IIR filters provide good alternative as they offer very attractive advantages such as less complexity, compact size, and much lower filter order for achieving the same frequency-domain filter response. In an attempt to extensively explore the filter bank design theory that will meet the specifications of applications, design schemes based on both FIR and IIR configurations are examined.

From the perspective of implementation design, all-fiber implementation has the advantages in low connection loss, low cost, and good compatibility with the fiber optics transmission links in WDM communications. Thus, all-fiber implementation of optical components is highly preferred.

Therefore, this thesis aims to develop simple but efficient two-port and higher order multi-port structures for optical filter banks. It is intended to provide a systematic and theoretical investigation into the performance design of both two-port and higher multi-port optical filter banks to meet certain application specifications. It is also highly desired that they can be implemented in both FIR and IIR configurations. The implementation of such optical filter banks is preferred to be in an all-fiber structure for compatibility with the new generation optical communication systems. Based on the proposed design scheme, optical filter banks are further employed in the current optical networks to establish the key enabling technologies, such as the *optical interleaver* technology and the *spectrum equalization* technology, to revolutionize the new generation of optical communication systems.

1.3 Major Contribution of the Thesis

From the viewpoint of system theory, an optical filter bank can be treated as an MIMO system, which has a bank of filters to be simultaneously designed and pro-

cessed, as shown in Fig. 1.2. The inputs and outputs are light signals expressed either in time domain or in wavelength domain depending upon the application. For example, the signal is expressed in wavelength domain in the design of filters for spectrum equalization of EDFAs [3], or expressed in time domain in the design of modulators [3], or can be expressed both in time and wavelength domains in the design of filters for dispersion compensation [3].

Figure 1.2: Optical filter bank system from the system design theory point of view.

The input signal is not limited to be deterministic, and sometimes it is described as a stochastic signal. For example, the polarization mode dispersion (PMD) can be represented by a differential group delay (DGD) which has a Maxwellian probability density function [41]. Such problems are still open, and can be solved by statistical or adaptive filter design method.

The uncertainty in the system in Fig. 1.2 takes into account the system modelling errors, the distortion of the incident light signal, and the measurement noises. The modelling errors include fabrication errors and physical modelling errors due to the impurity of the material. In order to make the filter responses insensitive to

the above modelling errors, considerations must be given in both the design and fabrication stages so as to yield a robust filter design [53].

If the system is considered to be linear time-invariant and independent of the input signal, the optical filter bank can be designed in the frequency domain by a similar approach as used in traditional design of electronic analog and digital filters [51,52]. In such a case, the resultant optical filter banks can suitably approximate the static model of the real world.

In this thesis, the enabling optical filter bank technologies will be explored more from the system theory perspective to accommodate the new generation of all-optical WDM systems with high transmission speed and high channel counts. To this end, a formalized scheme of *optical filter bank design scheme on structure, design and implementation* is proposed. In this design scheme, the main principle in the design of a MIMO optical filter bank system is to transform it into a set of SISO systems. By adopting this design strategy, the design problem is converted into SISO filter designs. To this end, since only one port of transfer function is involved, the available filter design methods can then be readily employed. Thus, the computational complexity is greatly reduced and the performance of the filter banks can be specified. The design scheme is further demonstrated in the design of optical interleavers, spectrum equalization filters, and multi-wavelength fiber ring lasers. The original contributions of this thesis are established through the studies on the following aspects:

- *Mathematical modelling of the physical constraints of optical filter banks.*

Difficulties exist when employing the digital filter bank design approaches in the optical filter bank design. For example, the inherent constraints, such as only positive quantities of optical signals can be measured in passive optical filters and the physical structures restrict the transfer functions used to approximate the desired responses, must be taken into account in the filter

design.

- *Theoretical study of the two-port and higher order multi-port optical filter banks.*

So far, most of the conventional optical filter banks proposed have been limited to only two-port structures. As a follow-on characteristic, only configurations of power-of-two are possible, and this limits the capability of such optical devices. In this thesis, both two-port and higher order multi-port optical filter banks are theoretically studied.

- *Propose innovative structures of both FIR and IIR configurations for optical filter bank designs.*

Another contribution of the thesis is the realizations of both FIR and IIR efficient structures for optical filter bank designs. By properly selecting certain design parameters in the proposed structures, the design of MIMO systems is successfully simplified into SISO system design. Then, the rest of the design parameters are designed to achieve a filter response that will meet the required specifications.

- *Develop a systematic and quantitative design approach for optical filter banks with specified performance.*

Currently, most of the optical filters are designed by numerically optimizing certain integration criteria. These designs yield filters with acceptable performance. However, with the integration design criteria, the performance of the filters cannot be foreseen quantitatively before the design. It is of both practical and theoretical importance to design filters with a desired performance through optimally designed filter parameters, so that quantitative results of the filter parameters can be expected. This thesis addresses this problem.

-
- *Implement the all-fiber structure of the optical filter banks via innovative fabrication methods.*

One common drawback of the available implementation methods for optical filter banks is that they cannot be implemented as an all-fibre structure. In optical communication systems, using all-fibre instruments can decrease the insertion and connection losses where optical devices are connected serially. In addition, all-fibre implementations can provide further advantages such as uniform response over a wide wavelength range, very low dispersion and minimum polarization dependence. Thus, innovative fabrication technologies are developed in this thesis for the implementations of optical filter banks.

- *Study the robustness of the proposed design scheme.*

The level of precision and accuracy that can be achieved in the design parameters of optical filters is not as high as that of the digital filters [2], thus, the effects of uncertainty of a filter (sensitivity to variation and noise), as well as the deviations of the design parameters during the fabrication process, must be examined carefully to ascertain the robustness property.

1.4 Organization of the Thesis

The rest of the thesis is divided into 5 chapters. Chapter 2 provides the literature review of two enabling technologies and several common techniques for the design of optical filters. Chapters 3 and 4 deal with the design of two-port and three-port optical filter banks, respectively. By using a two-port optical lattice structure, Chapter 5 studies one of the important applications of optical filter banks for spectrum equalization. Chapter 6 presents the conclusion of this thesis, together with several future research recommendations. The details of each chapter contents are as follows:

Chapter 2. Literature reviews of optical interleaver/de-interleaver and spectrum equalization technologies are given in this chapter, together with a brief introduction of some basic techniques that are used in this thesis to implement optical filter banks.

Chapter 3. Two-port optical interleavers, *i.e.* two-port optical filter banks, are investigated in this chapter. An FIR optical filter bank is first studied. A quantitative design method is proposed for the design of FIR two-port filter bank under a given passband/stopband bandwidth. The basic idea is to minimize the worst-case performance in terms of insertion loss and isolation over the given bandwidth. Experimental results are given to demonstrate the effectiveness of the proposed scheme.

Next, a new efficient IIR structure of fiber optical interleaver is presented and demonstrated by using a 2×2 fiber coupler, with one of its output arms ended by a chirped Fiber Bragg Grating (FBG) and the other output arm ended by superimposing two chirped FBGs. It is shown that by using this structure, periodic spectra obtained at the two input ports of the fiber coupler can be controlled to have an identical shape but with a π -phase shift between them in frequency. An all-fiber 25G/50G optical interleaver is fabricated successfully via experiment.

Subsequently, a high performance IIR structure of fiber optical interleaver is proposed by using two fiber GTEs in a fiber Michelson interferometer. The amplitude of the periodic spectra can be designed to meet desired specifications such as flat-top passband, high isolation, and low insertion loss. To demonstrate the effectiveness of the proposed structure and design scheme, we develop an all-fiber 25G/50G optical interleaving filter successfully via experiment.

Chapter 4. Three-port optical filter banks are investigated in this chapter. A novel 3×3 lattice structure of an optical delay-line circuit is proposed to implement FIR three-port optical filter banks. The proposed structure consists of two parts of cascading lattice structures. Starting from the output ports, the first part of

lattice structure is designed to enable a $\frac{2\pi}{3}$ phase shift among the three channel transmissions of the whole filter. In the second part, N stages of the 3×3 couplers are cascaded port-to-port by differential delay lines. By adopting this structure, the design of three-port interleaver is simplified. A general synthesis approach is also derived to obtain the optimal design parameters of the interleaver. Several simulation and experimental examples are given to support the effectiveness of the proposed design approach.

As an alternative architecture, efficient IIR structures for three-port optical interleaver design are then examined. The structures require the length of ring resonators in the differential delay lines to be three times that of the differential delay of the 3×3 Mach Zehnder Interferometer (MZI), which greatly simplifies the design. It is shown that with the proposed structure, interleavers with desired specifications can be designed in two separate steps. When compared with the three-port FIR interleaver, the proposed IIR interleaver has a better spectrum performance in terms of wider passband bandwidth, higher channel isolation and more efficient structure. Simulation and experimental results are given to confirm the validity of the proposed IIR design scheme.

Chapter 5. In this chapter, spectrum equalization is studied in depth by using two-port optical filter banks. As a basic and natural building block, Gaussian-like spectrum is studied in this chapter. First, a minimax approach and a window integration approach are proposed to design spectrum equalization filters with pre-specified performance indicators. To present the proposed design approaches, a single stage MZI is used to implement a spectrum equalization filter for the equalization of a Super-luminescent Light Emitting Diode (SLED), which naturally has a Gaussian-like spectrum. Both the simulation and the experimental results show that, with the proposed design scheme, the output spectrum of SLED can be equalized over a wide spectral range.

An FIR filter bank design scheme that can equalize Gaussian-like spectra in the maximally flat sense is then presented. With a two-step design strategy, the spectral ripples can be assessed quantitatively. The equalization performance is also analyzed theoretically. It is shown that the equalization performance can only be improved with increased odd order of the lattice filter. As an illustrative example, the proposed scheme is applied to the design of an equalization filter to flatten the output spectrum of an SLED.

Lastly, a novel structure that consists of two optical lattice filters in an IIR structure is proposed. With the proposed IIR structure, the two critical specifications, *i.e.* the flattest output spectrum and the highest output power, of designing an equalizer can be satisfied. Using the two-step design scheme, the Gaussian-like spectrum is flattened to the maximal extension with improved performance and the equalization ripples can be quantitatively assessed by evaluating the least-squares approximation error.

Chapter 6. The conclusions of the thesis are given in this chapter. Some future research directions are also highlighted.

Chapter 2

Literature Review

2.1 Introduction

In this chapter, we will first provide a thorough literature review on the state-of-art optical interleaver/de-interleaver and spectrum equalization technologies. The drawbacks of the current approaches in the literature for the implementation of such technologies are discussed. In the next section, a brief introduction on some basic techniques, such as Mach-Zehnder interferometry (MZI) technique [2], Fabry-Perot interferometry (FPI) technique [54] and Fiber Bragg Grating (FBG) technique [4,9], is provided. The basic knowledge of these techniques will be used in the later chapters for the design of optical filter banks.

2.2 Optical Interleaver/De-interleaver Technology

An optical interleaver/de-interleaver is essentially an optical filter bank that allows existing DWDM components designed for wide channel spacing to be extended to narrow channel spacing, or vice versa [37–40]. The general principal behind inter-

leavers regardless of implementation methods is an interferometric overlap of two or more beams. The interference creates a periodic output as different integral multiples of wavelengths passing through the device. Controlling the fringe pattern sets the desired channel spacing of the device. Today's DWDM systems with interleavers can be upgraded with much narrower channel spacing and more channel counts for very large capacity applications [39, 40].

So far, several techniques have already been applied for the interleaver design. These include Mach-Zehnder (MZ) based lattice interleaver [14, 19, 37–39, 55], Michelson-Gires-Tournois (MGT) interleaver [56–61], Birefringent Gires-Tournois (BGT) interleaver [62–64], compound Fabry-Perot (FP) interferometer [65–67], Etalon interleaver [68, 69], array-waveguide grating (AWG) router [13, 18, 70, 71], and so on. The Mach-Zehnder based lattice interleaver are typically used for two-port interleavers and can be cascaded to realize logarithmic interleavers. However, such a structure requires $2^N - 1$ parts to realize an N -port interleaver which means it is a more complex and less economical design. The MGT and BGT interleavers are generally implemented in free-space by using bulk-optic technologies. It is suitable for two-port interleaver and might be used to realize asymmetric interleavers, however, it is difficult to realize multi-port design and the cost of such device is relatively high. The AWG router interleavers are suited for N -port multiplexing/de-multiplexing, but each port usually processes only one channel of optical signal. Besides, AWG filters cannot be used for lower order ports multiplexing/de-multiplexing due to high losses and difficulties in the implementation. Among these different technical approaches to interleavers, there is no clear technical winner as yet, but the preferred design in terms of cost, material, compatibility with the communication links, and connection loss is an all-fiber structure interleaver, such as all fiber MZ based interleaver [37, 38]. It is because of these advantages that the all-fiber based interleaver has attracted intensive interest in the optical communication community [37–40, 72, 73]. This thesis aims to investigate optical interleavers with all-fiber

structures.

Currently, one of the difficulties exists in the interleaver design is that, until now, most of the optical interleavers and the associated design approaches have been limited to only two-port structures [20]. The resultant optical interleavers have only two ports. In order to achieve high channel counts for a communication system, a number of interleaving filters have to be cascaded. In practice, the insertion loss will increase with the number of interleaving filters being used. Furthermore, the capability of the optical interleavers in multiplexing more complicatedly coded signals is limited. Therefore, it is of great interest to design filters using multi-port optical filter bank circuits. Since an $M \times M$ coupler implemented either in a fiber or in a waveguide platform has a more complicated power transfer matrix than those of 2×2 couplers, the transmission matrix of the filter implemented by a multi-port optical circuit has a complex structure and in turn the convenient structural properties that were exploited in the development of synthesis methods for two-port optical circuits [72, 74–79] are not available. This creates difficulties when designing optical interleavers using multi-port optical circuits. One of the major objectives of this thesis is to address this difficulty theoretically and experimentally.

Furthermore, although the available interleavers make use of different principles, they share a common feature. The design parameters of the interleaver are obtained by numerically optimizing certain criteria [20]. This burdens the filter design with a high computational complexity, particularly when the number of parameters involved is large. In addition, it is difficult to specify an explicit relationship between the design parameters and the performance of the designed interleaver. Thus, tedious trial-and-error steps have to be performed in order to obtain an interleaver with the desired specifications. It is therefore of practical and theoretical interest to develop a systematic and quantitative design approach capable of specifying the key performance parameters in the design stage. From the signal processing perspective,

theoretical designs of both FIR and IIR optical filter banks are highly desired.

2.3 Spectrum Equalization Technology

In long-haul networks with erbium-doped fiber amplification and optical engineering related fields, many optical components/devices have non-uniform spectral responses [41,44]. However, in many of these applications, a flatter spectral response is desired [3]. Thus, spectrum equalization in such cases is needed to improve channel count, provide uniform output, tighten optical signal-to-noise ratios and improve the ability to add and drop wavelengths. In various applications, these components/devices have a spectral response that can be approximated over a band by a Gaussian function. The typical scenarios include the use of light-emitting diodes (LED) to make broadband light sources and the applications of photo detectors to receive a broadband optical signals [3,15]. In the first scenario, one would like to obtain a light output with a flat spectrum over a certain band of interest to maintain an equal optical power distribution across the multiple wavelengths. This is extremely important in several applications such as the power control of optical signals in optical communications [80–82]; the generation of multi-wavelength spectrum-sliced sources for Time-Division-Multiplexing and Wavelength-Division-Multiplexing Telecommunication systems where the equalized spectra should be very flat so that the spectrum splicing technique [83–85] can be applied; the measurements of spectral characteristics of photonic components and devices [3,86–88]; Optical Low Coherence Reflectometry for test and measurement [89]; and optical coherent tomography imaging in bioscience [90–92]. LED, which has a Gaussian-like output spectrum with low coherence property [5,89], is an economical and effective solution to provide light sources for these applications [86,87,93]. In the second scenario, it is desired to measure the light power at multiple wavelengths by removing the distortions resulted from the Gaussian-like spectral response of photo

detectors [3]. The Gaussian approximation methodology can also be used to approximate much more complex spectra. It is therefore of great practical importance and theoretical interest to develop an equalization filter to equalize Gaussian-like spectra. The filter should have two critical specifications. First, it should be able to equalize the spectrum with a given level of flatness. Second, it should have as minimal an attenuation as possible in order to obtain the maximal power output/input.

Currently, there are two basic approaches for spectrum equalization, namely, the intrinsic and extrinsic approaches. Intrinsic methods consist of modifying the spectroscopic properties of stimulated materials. For example, Alumino-silicate [94] or fluoride based [95] glasses are the host matrices that improve the flatness of EDFA gain profile. Although the intrinsically equalized approaches demonstrate their effectiveness, they are accurate over only a limited bandwidth. Extrinsic methods equalize the spectrum by making use of a filtering device. Several approaches have been studied to fabricate gain equalization filters, including thin film filters [96,97], long-period gratings [98,99], liquid crystals [100], FBG [101–104], silica wave-guide structure [22, 23, 105], Mach-Zehnder based filters [77, 106], and acousto-optical filters [107–109]. Among these different technologies for spectrum equalization, the lattice filter has the advantage including the possibility of implementation in all-fiber structure, the available systematic design approach, and the potential of simultaneous multi-port spectrum equalizations. One of the objectives of this thesis is to investigate the spectrum equalization through the use of optical filter banks.

2.4 Techniques for Optical Filter Design

In this section, the basic concepts of three techniques, which can be used for optical filter design and analysis, are presented. The techniques for optical filter design come in three general classes, although there are many variations of each. The first class is

simply illustrated by the Mach-Zehnder interferometer (MZI) technology [2, 3], the second class is illustrated by the Fabry-Perot interferometer (FPI) technology [54], while the third class comes with Bragg grating technology [4, 9]. In the following subsections, we will give brief introductions about these techniques.

2.4.1 Mach-Zehnder Interferometry Technique

MZI is a basic passive interference device. Fig. 2.1 shows the layout of the first order MZI. It is a two-input two-output optical system where E_1 and E_2 represent the two input electrical fields, E_3 and E_4 represent the two output electrical fields, respectively. It consists of two couplers, denoted respectively as k_1 and k_2 , connected by a differential delay ΔL . An incoming signal is split equally into two paths at the first coupler k_1 . One path is delayed with respect to the other by a time $T = \Delta L n / c$ where n is the refractive index of the paths, c is the velocity of light in vacuum, and ΔL is the path length difference. The signals in the two paths are then recombined at the second coupler k_2 . Coherent interference at the combiner leads to a sinusoidal frequency response whose period is inversely proportional to the path length difference. The distinguishing feature of this general class of interferometers is that there are a finite number of delays and no feedback delay paths, the interfering paths are always feeding forward. The signal processing term used to designate this filter type is Finite Impulse Response (FIR) or Moving Average (MA) filter [52].

The normalized electric fields transfer function from the two inputs to the two outputs of each directional coupler can be expressed by the following transfer matrix [2, 3]:

$$M_i = \begin{pmatrix} c_i & -js_i \\ -js_i & c_i \end{pmatrix} \quad (2.1)$$

where $c_i = \sqrt{k_i}$ and $s_i = \sqrt{1 - k_i}$, $i = 1, 2$, with k_i being the bar-state intensity coupling ratio of the directional coupler.

Figure 2.1: The layout of Mach-Zehnder interferometer of the first general class.

Similarly, the transfer matrix of the differential delay can be written as

$$M^\phi = \begin{pmatrix} e^{-j\phi} & 0 \\ 0 & e^{j\phi} \end{pmatrix} \quad (2.2)$$

where $\phi = \Delta L \pi n / \lambda$, λ is the wavelength of the wave propagating through the fiber.

A multi-stage filter can be realized by concatenating N stages of MZI in a lattice architecture [110, 111]. The major advantages of this architecture are that a very low loss passband can be achieved [75] and only $N + 1$ couplers are needed for an N -stage filter. If the N -stage lattice-form optical circuit is lossless, its transfer matrix can be expressed by a unitary matrix M , which satisfies the following unimodulus property:

$$\det(M) = 1 \quad (2.3)$$

Furthermore, the unitary matrix M bounded by the unimodulus condition can be expressed in the following simple form:

$$M = \begin{pmatrix} A(z) & -B_*(z) \\ B(z) & A_*(z) \end{pmatrix} \quad (2.4)$$

where $A(z)$ and $B(z)$ indicate the bar-state transfer function and the cross-state function of the device, respectively. From (2.3) and (2.4), we have $A(z)A_*(z) + B(z)B_*(z) = 1$. $A(z)$ and $B(z)$ are expressed in the z -plane, $z = e^{jwT}$ where w is the relative optical angular frequency, and T is the unit time delay. The subscript $*$ indicates the following para-Hermitian conjugate $X_*(z) \equiv X^*(\frac{1}{z^*})$, and the superscript $*$ represents a complex conjugate.

From the structure of the N stages of MZI, the total transfer function M can thus be expressed by the product of $2N + 1$ basic component transfer matrices:

$$M = M_N M^\phi M_{N-1} \cdots M_2 M^\phi M_1 M^\phi M_0 \quad (2.5)$$

From Eqns.(2.1), (2.2), (2.4) and (2.5), we can have the recursive expression of $A(z)$ and $B(z)$, respectively, that can be further tailored to have the desired bar-state/cross-state filter responses. A lot of work can be found in the literature on the design of MZI [75, 77, 110–112].

2.4.2 Fabry-Perot Interferometry Technique

The FPI consists of a cavity surrounded by two partial reflectors that are parallel to each other [54, 65] as shown in Fig. 2.2. The frequency response comes from the interference of multiple reflections from the mirrors. The output is the infinite sum of delayed versions of the input signal weighted by the round trip cavity transmission. The signal processing term used to designate this kind of filter is Infinite Impulse Response (IIR) or Autoregressive (AR) filters [52].

As shown in Fig. 2.2, electrical field E_i incident perpendicularly to the FPI will generate a transmitted field E_t and a reflected field E_r , respectively [54]. We will consider the reflected field first. The overall reflected electrical field E_r can be

Figure 2.2: Illustration of Fabry-Perot interferometer of the second general class.

expressed as:

$$\begin{aligned} E_r &= E_{1r} + E_{2r} + E_{3r} + \dots \\ &= E_i \left[r_1 + \frac{t_1 r'_2 t'_1 \exp(-i\delta)}{1 - r'_1 r'_2 \exp(-i\delta)} \right] \end{aligned} \quad (2.6)$$

where E_{ir} represents the reflected electric field through i -th round in the FP cavity; $\delta = 4\pi nl$ is the cavity round-trip phase delay with l being the cavity length; t_1 , r_1 , t_2 and r_2 are field transmission and reflection coefficients incident from outside the cavity; t'_1 , r'_1 , t'_2 and r'_2 are field transmission and reflection coefficients incident from inside the cavity. Under the condition of lossless mirrors, $t_1 t'_1 = 1 - r_1^2$, $r_1 = -r'_1$, $r_2 = -r'_2$, thus we have from (2.6)

$$\frac{E_r}{E_i} = \frac{r_1 - r_2 \exp(-i\delta)}{1 - r_1 r_2 \exp(-i\delta)} \quad (2.7)$$

Similarly, for the transmitted field,

$$\frac{E_t}{E_i} = \frac{(1 - r_1 r_2) \exp(-i\delta/2)}{1 - r_1 r_2 \exp(-i\delta)} \quad (2.8)$$

Thus, the design parameters r_1 , r_2 and δ can be designed to have the desired filter responses of FPIs. Thin-film interference filters are the cascaded structures of coupled cavity FPI's [113]. The transmission and reflection of the FPI have an AR response and an ARMA response [52], respectively. The design of FPI and thin film

filter can be found widely in the literature [54, 65, 113, 114].

2.4.3 Fiber Bragg Grating Technique

A Fiber Bragg grating (FBG) is an important element in WDM optical communication systems for combining or separating individual wavelength. Basically, a grating is a periodic structure or perturbation in material as shown in Fig. 2.3. This variation in the material has the property of reflecting or transmitting light in a certain direction depending on the wavelength [3, 4, 9]. Since the interference mechanism is similar to that of the thin film filter, we will not derive the transfer functions of the reflection and transmission electric fields of FBGs as those of the thin film filters shown in the last subsection, however, we shall adopt the widely used coupled-mode theory [4, 9, 115–117] to obtain the filter characteristics of the Fiber Bragg gratings.

Figure 2.3: Configuration of Fiber Bragg grating of the third general class.

Consider a uniform Bragg grating formed within the core of an optical fiber with an average refractive index n_0 . The index of refractive profile can be expressed as

$$n(z) = n_0 + \Delta n \cos\left(\frac{2\pi z}{\Lambda}\right) \quad (2.9)$$

where Δn is the amplitude of the induced refractive index perturbation, and z is the distance along the fiber longitudinal axis. Using the coupled mode theory [4, 9], the reflectivity of a grating with constant modulation amplitude and period is given

by the following expression:

$$R(l, \lambda) = \frac{\Omega^2 \sinh^2(sl)}{\Delta k^2 \sinh^2(sl) + s^2 \cosh^2(sl)} \quad (2.10)$$

where $R(l, \lambda)$ is the reflectivity that is a function of the grating length l and wavelength λ . Ω is the coupling coefficient, $\Delta k = k - \pi/\lambda$ is the tuning wave vector, $k = 2\pi n_0/\lambda$ is the propagation constant, and $s^2 = \Omega^2 - \Delta k^2$.

Thus, we have the reflection transfer function that can be designed with desired specifications, such as the central reflected wavelength, the bandwidth of the pass-band transmission responses, and the channel isolation of the notch filter [118, 119]. It can be clearly seen that the reflectivity increases as the induced index of refraction change increases. Similarly, as the length of the grating increases so does the resultant reflectivity [4, 9].

2.5 Conclusion

In this chapter, we have reviewed two key enabling technologies, namely the optical interleaver/de-interleaver technology and spectrum equalization technology, for upgrading optical communication systems. The current limitations of these two technologies, such as the interleaver/de-interleaver design is only limited to two-port structures and the spectrum equalization filter designs have to satisfy certain specifications to meet the next generation optical communications, are examined. Brief reviews of the three basic filter design techniques, namely MZI technique, FPI technique and FBG technique, that can be used to build and design optical filter banks to solve the mentioned limitations are also studied. Although there are many other optical filter design techniques, such as acoustic-optical, bulk optics and array waveguide grating filtering technologies, we focus more on MZI, FPI and FBG

techniques and they will be used in the next three chapters.

Chapter 3

Two-port Optical Filter Banks

3.1 Introduction

To meet the explosive bandwidth demand in optical communications, DWDM systems must offer increasingly higher channel counts and channel speeds. From the perspective of cost effectiveness, the solution to achieve narrower channel spacings in the currently used bands supported by EDFAs through using interleaver and other technologies [14, 20, 37–39, 55–58, 62, 64–66, 68, 69] is preferred, because the existing optical components designed for wide channel spacing can still be used. In this solution, the optical filter bank technology (generally denoted as optical interleaver technology) plays a very critical role. In addition to this, it can also be employed in, for example wavelength coding/decoding, to enhance the information transmission efficiency and capacity for optical code division multiple access (OCDMA) [120, 121]. Today's DWDM systems with interleavers can be upgraded with much narrower channel spacings and more channel counts for very large capacity applications [20, 37–39, 55].

From the structure complexity perspective, an optical filter bank obviously has

multiple-input and multiple-output ports. Clearly, the simplest case of an optical filter bank is the two-port one. Since the design of two-port optical filter banks only involves the manipulations of 2×2 matrices, less constraints are imposed on achieving simultaneously the desired responses to meet certain specifications. Therefore, two-port filter banks are first examined in this chapter. We will extend this idea into the designs of a higher order multi-port optical filter bank in the later chapters.

Among the various implementation technologies listed in [20] for implementing two-port optical filter banks, all-fiber based optical interleavers have unique features in terms of low connection losses and good compatibility with the fiber optics transmission links in DWDM communications. Therefore, we aim to realize the two-port interleavers in all-fiber structures in this chapter.

Regardless of implementation technologies, the primary design goal for two-port optical filter banks is to obtain two periodic passband transmissions of which the amplitude spectrum are π phase-shifted but with identical spectral shapes. The amplitude spectrum of a passband should have a wide flat-top shape with minimum insertion loss and rapid roll-off on the band edges. In addition, the isolation between two adjacent channels should be as high as possible. Two means can be used to implement two-port optical interleavers. The first is to use a number of optical elements with finite impulse responses [37–39]. For example, the Fourier Filter Flat-Top (F^3T) interleavers in [37, 122], using several cascaded modified MZIs, basically have FIR responses. In these designs, by carefully controlling the path length difference, the channel spacing can be set to the desired value and matched very well to the ITU grid [123]. The second approach makes use of optical elements, like FP cavities, with infinite impulse responses (IIRs) [56–58, 62, 64, 124, 125]. The FIR optical interleavers have the characteristics of a possible linear phase response, which is highly desired in high speed DWDM communication systems [25, 32]. On the other hand, the optical interleavers based on IIR elements have advantages over

their FIR counterparts in that they can provide an efficient means to achieve a better performance such as flatter and wider passband bandwidth, higher channel isolation, and faster roll-off on band edges [72].

In this chapter, we will study both the FIR and IIR two-port optical interleavers. In the first section, an F^3T FIR interleaver based on MZI technique is investigated. Due to the proposed structure, the design of the interleaver is simplified to designing only one of the output ports by eliminating the phase shift requirement. Then, the current design approaches for F^3T interleavers use numerically optimizing certain integration criteria [126]. These designs do yield interleavers with acceptable performance. However, with the integration design criteria, the performance of the F^3T interleaver cannot be foreseen quantitatively before the design. In practice, it is preferable to design interleavers with a desired performance by specifying passband/stopband width. This section addresses this design problem. A minimax filter design method is proposed to design the F^3T interleavers under a given passband/stopband width. The basic idea is to minimize the worse case performance in terms of insertion loss and isolation over the given bandwidth. When describing the insertion loss and isolation as passband/stopband ripple, the interleaver performance is given by a kind of H_∞ norm [127, 128] of the power transfer function of the interleaver. Then, the interleaver design is treated as a minimax filter design problem and all design parameters are obtained by resolving the H_∞ norm. Thus, the proposed minimax filter scheme gives a quantitative approach to design interleavers under given specifications. An novel fabrication method is also proposed to realize all-fiber FIR 25G/50G two-port interleavers. Simulation and experimental results verify the proposed design and implementation schemes.

As an alternative architecture, an IIR two-port optical interleaver based on Superimposed Chirped Fiber Bragg Gratings (SCFBGs) is also developed. The essence of the available methods for producing all-fiber IIR optical interleavers is to construct

cascaded optical cavities on fiber by using the fiber Bragg grating (FBG) technology [124, 125] or bulk optic Gires-Tournois Etalons (GTEs) [60, 61]. However, difficulties exist in practical fabrication, especially in maintaining the amplitude spectra of two output transmissions of the interleaver a π phase-shift between them, and achieving compatibility with the DWDM communication links. To achieve the desired performance, a new efficient IIR structure of fiber optical interleaver is demonstrated by superimposing two chirped FBGs. The proposed optical interleaver consists of a 2×2 fiber coupler, with one of its output arms ended by a chirped FBG and the other output arm ended by superimposing two chirped FBGs. The displacement of the superimposed chirped FBGs is selected as twice that of the length difference of the output arms with FBGs. This structure helps the periodic spectra obtained at the two output ports of the fiber coupler to be controlled to have an identical shape but with a π -phase shift between them in frequency. The idea of constructing an MIMO system into a set of SISO systems is also realized. Using the proposed structure, an all-fiber 25G/50G optical interleaver is fabricated successfully by experiment. The interleaver responses recorded in the experiments have shown the effectiveness of the proposed interleaver structure and the design scheme.

An even more efficient IIR structure for designing the optical interleaver is then devised and demonstrated. The proposed optical interleaver is implemented by using two fiber GTEs in a fiber Michelson interferometer [129]. By using the proposed structure, the phase shift requirement of the interleaver design is also satisfied. While, the spectral performance of the interleaver is improved. To demonstrate the effectiveness of the proposed design, we develop an all-fiber 25G/50G optical interleaving filter via experiment.

3.2 Finite Impulse Response Two-Port Optical Filter Banks

3.2.1 Fourier Filter Flat-Top (F^3T) Interleaver

As shown in Fig. 3.1, an F^3T interleaver device is a modified unbalanced MZI. It consists of three cascaded couplers linked by two differential delays. The second differential delay line, seen from the left input port, is selected twice that of the first one. This is to add odd Harmonics of the Fourier series for simplifying the design of two-port optical filter banks, which are basically optical half band filters. It further aims to use fewer optical elements in the structure to realize a flat passband and high-isolation rejection band. The advantages of this structure for simplifying the design will be discussed in the later part of this section.

From Eqns.(2.1) and (2.2), the normalized electric fields transfer function from the two inputs to the two outputs can be expressed by the following matrix transfer function:

$$\begin{aligned} \begin{pmatrix} E_3 \\ E_4 \end{pmatrix} &= M_3 M_2^\phi M_2 M_1^\phi M_1 \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \\ &= \begin{pmatrix} c_3 & js_3 \\ js_3 & c_3 \end{pmatrix} \begin{pmatrix} e^{-2j\phi} & 0 \\ 0 & e^{2j\phi} \end{pmatrix} \begin{pmatrix} c_2 & js_2 \\ js_2 & c_2 \end{pmatrix} \begin{pmatrix} e^{-j\phi} & 0 \\ 0 & e^{j\phi} \end{pmatrix} \begin{pmatrix} c_1 & js_1 \\ js_1 & c_1 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \end{aligned} \quad (3.1)$$

where E_1 and E_2 denotes the electric fields of the two inputs, and E_3 and E_4 represent the electric fields of the two output ports; $c_i = \sqrt{k_i}$ and $s_i = \sqrt{1 - k_i}$, $i = 1, 2, 3$, with k_i being the bar-state intensity coupling ratio of the directional coupler.

Hence, the transfer function with respect to bar-state T_3 (from port E_1 to port E_3)

and with respect to cross-state T_4 (from port E_1 to port E_4) can be written in the following two equations, respectively:

$$T_3(\phi) = c_1 c_2 c_3 e^{j3\phi} - s_1 s_2 c_3 e^{j\phi} - c_1 s_2 s_3 e^{-j\phi} - s_1 c_2 s_3 e^{-j3\phi} \quad (3.2)$$

$$T_4(\phi) = j(c_1 c_2 s_3 e^{j3\phi} - s_1 s_2 s_3 e^{j\phi} + c_1 s_2 c_3 e^{-j\phi} + s_1 c_2 c_3 e^{-j3\phi}) \quad (3.3)$$

Three parameters are usually used to describe the performance of the interleaver. They are passband/stopband shape, isolation and insertion loss. Given a passband range, the task of interleaver design is aimed at achieving the minimal insertion loss and isolation. The next subsection will provide the details of the proposed design method.

Figure 3.1: Configuration of the Fourier Filter Flat-top interleaver which represents a two-port optical filter bank.

3.2.2 F^3T Interleaver Design by Minimax Approach

There are two basic requirements of the F^3T interleaver. First, the interleaver should possess identical filtering performance for both the bar-state and the cross-state outputs [73]. Second, the sum of the output powers should equal to the sum of the input powers. The proposed design method is therefore carried out based on these two basic demands.

As the F^3T interleaver should possess identical filtering performance for both the

bar-state and the cross-state output, it is required that

$$|T_3(\phi)|^2 = |T_4(\phi + \pi/2)|^2 \quad (3.4)$$

where $|T_3(\phi)|^2 = T_3(\phi)T_3^*(\phi)$ is the bar-state power transfer function and $|T_4(\phi + \pi/2)|^2 = T_4(\phi + \pi/2)T_4^*(\phi + \pi/2)$ is the cross-state power transfer function with a phase shift of $\pi/2$. Here, $T_4(\phi + \pi/2)$ is expressed as follows:

$$\begin{aligned} T_4(\phi + \pi/2) &= j(-jc_1c_2s_3e^{j3\phi} - js_1s_2s_3e^{j\phi} - jc_1s_2c_3e^{-j\phi} + js_1c_2c_3e^{-j3\phi}) \\ &= c_1c_2s_3e^{j3\phi} + s_1s_2s_3e^{j\phi} + c_1s_2c_3e^{-j\phi} - s_1c_2c_3e^{-j3\phi} \end{aligned} \quad (3.5)$$

From equations (3.2) , (3.4) and (3.5), it can be derived that

$$c_1 = s_1 \quad (3.6)$$

It can be seen clearly from (2.1) and the unitary property of the coupler matrix that $c_i (i = 1, 2, 3)$ can be written as $\cos \theta_i$ and s_i can be written as $\sin \theta_i$, where θ_i is in the interval $[0, \pi/2]$. Thus

$$c_1 = s_1 = \sqrt{2}/2 \quad (3.7)$$

Thanks to the proposed structure, an immediate conclusion of Eqn.(3.7) is that when $c_1 = s_1$, Eqn.(3.4) is satisfied automatically, which means that the requirement of the two output ports have an identical but π phase shift in frequency is satisfied. Then, the design of the two-port interleaver is reduced to meeting the other requirement such that the powers of the two output ports are conserved.

Applying (3.7) to (3.2) and (3.3), (3.2) and (3.3) can be rewritten in terms of θ_i

($i=1,2,3$) in the frequency spectrum as

$$T_3(w) = \frac{\sqrt{2}}{2}(\cos \theta_2 \cos \theta_3 e^{j3w} - \sin \theta_2 \cos \theta_3 e^{jw} - \sin \theta_2 \sin \theta_3 e^{-jw} - \cos \theta_2 \sin \theta_3 e^{-j3w}) \quad (3.8)$$

$$T_4(w) = j\frac{\sqrt{2}}{2}(\cos \theta_2 \sin \theta_3 e^{j3w} - \sin \theta_2 \sin \theta_3 e^{jw} + \sin \theta_2 \cos \theta_3 e^{-jw} + \cos \theta_2 \cos \theta_3 e^{-j3w}) \quad (3.9)$$

where $w = \phi/T$, T is the unit delay time corresponding to half of the first phase delay of the interleaver.

From energy conservation, we have

$$|H_0(w)|^2 + |H_1(w)|^2 = 1 \quad (3.10)$$

where $H_0(w) = T_3(w)$ and $H_1(w) = T_4(w)$ is applied for the general denotation.

From (3.10), it can be derived that

$$\delta_{p1} = 1 - \sqrt{1 - \delta_{s0}^2} \quad (3.11)$$

and

$$W_s(0) = W_p(1) \quad (3.12)$$

where δ_{s0} denotes the stopband ripple of $H_0(w)$, δ_{p1} the passband ripple of $H_1(w)$, $W_s(0)$ the stopband width of $H_0(w)$, and $W_p(1)$ the passband width of $H_1(w)$, respectively. As the bar-state and cross-state power transfer functions are identical, the passband ripple and stopband ripple are identical for $H_0(w)$ and $H_1(w)$, and so are the passband width and stopband width. From (3.11) and (3.12), we have

$$\delta_{p0} = \delta_{p1} \quad \text{and} \quad \delta_{s0} = \delta_{s1} \quad (3.13)$$

$$W_s(0) = W_s(1) = W_p(0) = W_p(1) \quad (3.14)$$

where δ_{p0} denotes the passband ripple of $H_0(w)$, δ_{s1} the stopband ripple of $H_1(w)$, $W_p(0)$ the passband width of $H_0(w)$, and $W_s(1)$ the stopband width of $H_1(w)$, respectively.

Hence, from Eqns.(3.11) and (3.13),

$$\delta_p = 1 - \sqrt{1 - \delta_s^2} \quad (3.15)$$

Figure 3.2: Bar-state and cross-state intensities of the two-port F^3T optical interleaver.

where δ_s corresponds to the isolation of the interleaver and δ_p is correlated to the maximal insertion loss in the passband, as seen in Fig. 3.2. Equation (3.15) gives the relation between δ_s and δ_p , from which it can be seen that if δ_s is minimized, then δ_p also achieves its minimum.

So far, the design objective can be stated as follows:

Given the stopband/passband width W_s/W_p for the interleaver, find the coupling ratios of those fiber couplers in the interleaver such that the stopband ripple and

insertion loss of the interleaver are minimal over the stopband and the passband.

As the passband/stopband shape of bar-state and cross-state are identical, it suffices to consider one channel. Without loss of generality, we select $H_0(w)$. From the discussion above, the problem for the design of such an interleaver can be specified as the optimization problem of finding a set of θ_i , $i = 2, 3$, such that

$$\min_{(\theta_2, \theta_3)} \max_{w_{s1} < w < w_{s2}} |H_0(w)| \quad (3.16)$$

where w_{s1} and w_{s2} are the lower and upper boundaries of the stopband of $H_0(w)$, respectively, (as shown in Fig. 3.2) and

$$\delta_s = \max_{w_{s1} < w < w_{s2}} |H_0(w)|^2 \quad (3.17)$$

Clearly from Eqn.(3.14), we have

$$w_{p1} = w_{s2} - \pi/2, \quad w_{p2} = w_{s1} + \pi/2 \quad (3.18)$$

where w_{p1} and w_{p2} are the lower and upper boundaries of the passband of $H_0(w)$, respectively.

Let

$$E = |H_0(w)|^2 = \frac{1}{2} + \frac{1}{2}(\sin^2 \theta_2 \sin 2\theta_3 - \sin 2\theta_2 \cos 2\theta_3) \cos 2w - \frac{1}{2} \cos^2 \theta_2 \sin 2\theta_3 \cos 6w \quad (3.19)$$

The stationary points are given by $\partial E / \partial w = 0$. Applying Eqn.(3.19) to this results in

$$-(\sin^2 \theta_2 \sin 2\theta_3 - \sin 2\theta_2 \cos 2\theta_3) \sin 2\hat{w} + 3 \cos^2 \theta_2 \sin 2\theta_3 \sin 6\hat{w} = 0 \quad (3.20)$$

where $\hat{w}$ is in the interval $[\pi/4, 3\pi/4]$.

If the stationary point corresponds to a maximum point, then $\partial^2 E / \partial^2 w < 0$, thus

$$-2(\sin^2 \theta_2 \sin 2\theta_3 - \sin 2\theta_2 \cos 2\theta_3) \cos 2\hat{w} + 18 \cos^2 \theta_2 \sin 2\theta_3 \cos 6\hat{w} < 0 \quad (3.21)$$

Let

$$a_2 = -(\sin^2 \theta_2 \sin 2\theta_3 - \sin 2\theta_2 \cos 2\theta_3) \quad (3.22)$$

and

$$a_3 = 3 \cos^2 \theta_2 \sin 2\theta_3 \quad (3.23)$$

From Eqn.(3.20), the stationary point can be derived from

$$\sin^2 2\hat{w} = \frac{a_2 + 3a_3}{4a_3} \quad \text{or} \quad \hat{w} = \pi/2 \quad (3.24)$$

From the first case of (3.24), we have

$$a_2 + 3a_3 > 0 \quad (3.25)$$

because $a_3 \geq 0$ for $\theta_3 \in [0, \pi/2]$.

From Eqn.(3.21), we have

$$(a_2 - 9a_3) \cos 2\hat{w} + 12a_3 \cos^3 2\hat{w} < 0 \quad (3.26)$$

Now, we consider the following two cases:

- If $\sin^2 2\hat{w} = (a_2 + 3a_3)/4a_3$, applying this equation into (3.26), we have

$$a_2 + 3a_3 < 0 \quad (3.27)$$

This equation contradicts (3.25), which shows that the curve near these stationary points is always concave. The function values $|H_0(\hat{w})|^2$ on these points are therefore the minimal values.

- If $\hat{w} = \pi/2$, (3.26) could be satisfied.

Thus, only when $\hat{w} = \pi/2$, the maximum could be reached, and the maximum value in the stopband is written as:

$$\begin{aligned}\hat{E} &= \delta_s = \max_{w_{s1} < w < w_{s2}} |H_0(w)|^2 = |H_0(\frac{\pi}{2})|^2 \\ &= \frac{1}{2} - \frac{1}{2}(\sin^2 \theta_2 \sin 2\theta_3 - \sin 2\theta_2 \cos 2\theta_3) + \frac{1}{2} \cos^2 \theta_2 \sin 2\theta_3 \\ &= \frac{1}{2} + \frac{1}{2} \sin 2(\theta_2 + \theta_3)\end{aligned}\tag{3.28}$$

It can be seen from Eqn.(3.19) that the frequency response is symmetrical with respect to $\pi/2$. Thus, at the stopband boundaries w_{s1} and w_{s2} , we have

$$E(w_{s1}) = E(w_{s2})\tag{3.29}$$

So we only need to calculate w_{s1} , where $\frac{\pi}{4} \leq w_{s1} \leq \frac{\pi}{2}$.

At w_{s1} , the boundary condition can be expressed as

$$E(w_{s1}) = \hat{E} = \frac{1}{2} + \frac{1}{2} \sin 2(\theta_2 + \theta_3)\tag{3.30}$$

Thus with (3.30), the problem of (3.16) converts into:

$$\text{Minimize } \hat{E}_{(\theta_2, \theta_3)} = \frac{1}{2} + \frac{1}{2} \sin 2(\theta_2 + \theta_3)\tag{3.31}$$

subject to

$$\frac{1}{2}(\sin^2 \theta_2 \sin 2\theta_3 - \sin 2\theta_2 \cos 2\theta_3)(\cos 2w_{s1} + 1) - \frac{1}{2} \cos^2 \theta_2 \sin 2\theta_3 (\cos 6w_{s1} + 1) = 0 \quad (3.32)$$

The above problem is a constrained optimization problem. By using the Lagrange method, analytical solutions can be obtained. As θ_i are limited to the boundary $[0, \pi/2]$, we have

$$\theta_2 = \arctan\left(-\frac{\sqrt{3}(2 \cos 2w_{s1} - 1)}{3}\right) \quad (3.33)$$

$$\theta_3 = \frac{\pi}{2} + \frac{1}{2} \arctan\left(\frac{\sqrt{3}}{2 \cos 2w_{s1} - 1}\right) \quad (3.34)$$

Thus, up to now, the design procedure is complete. Given a passband range, the optimal parameters θ_i , $i = 1, 2, 3$, are derived. Note that from Eqn.(3.7), $\theta_1 = \pi/4$. With these optimal parameters θ_i , the insertion loss and isolation of the interleaver reach their minimums, respectively.

It is interesting to note that the top of $|H_0(\hat{w})|^2$ is always tangential to gain of 1 when the optimization value is achieved. Here we give the proof. We rewrite the first case of (3.24) as

$$\sin^2 2\hat{w} = \frac{a_2 + 3a_3}{4a_3} \quad (3.35)$$

Using the solutions of θ_2 and θ_3 , and applying (3.33), (3.34) and (3.35) to (3.19), it can be derived that

$$\begin{aligned} E|_{w=\hat{w}} &= |H_0(\hat{w})|^2 \\ &= \frac{1}{2} + \frac{1}{2}(\sin^2 \theta_2 \sin 2\theta_3 - \sin 2\theta_2 \cos 2\theta_3) \cos 2\hat{w} - \frac{1}{2} \cos^2 \theta_2 \sin 2\theta_3 \cos 6\hat{w} = 1 \end{aligned} \quad (3.36)$$

which shows that at the stationary points, except for $\hat{w} = \pi/2$, the curve is always tangential to the unity-gain boundary when the optimal solutions are obtained.

In Fig.3.3, the three curves possess the same bandwidth, while the solid curve is

Figure 3.3: Comparison of the proposed method with the integration method for an F^3T interleaver design with the same bandwidth, only the solid curve derived by the proposed method is tangential to the boundary. *Solid line*: $\theta_2=0.9863$, $\theta_3=1.2786$; *dotted line*: $\theta_2=0.8527$, $\theta_3=1.3750$; *dashdot line*: $\theta_2=0.7527$, $\theta_3=1.4189$.

depicted by the optimal results of the proposed design method. It can be seen that only the solid curve touches the boundary.

3.2.3 The Simulation Study

In this section, simulation studies are carried out to verify the performance of interleavers designed by the proposed approach in the subsection 3.2.2.

3.2.3.1 Comparison of performance with different passband bandwidths

When the values of w_{s1} are selected as 0.9032, 1.2566 and 1.4530, the corresponding optical spectral responses derived by the proposed method are shown in Figs. 3.4 and 3.5.

It can be clearly seen that when $w_{s1} = 0.9032$, the passband/stopband width is the widest, however the insertion loss in the passband and the isolation are also the

Figure 3.4: Comparison of different passband bandwidth specification of an F^3T interleaver designed by using the proposed method. Each line represents the spectral characteristics of designed interleaver with a different passband bandwidth. *Solid* line: $w_{s1}=1.2566$; *dotted* line: $w_{s1}=1.4530$; *dashdot* line: $w_{s1}=0.9032$.

Figure 3.5: Magnified view of figure 3.4. Each line represents the spectral characteristics of designed interleaver with a different passband bandwidth. *Solid* line: $w_{s1} = 1.2566$; *dotted* line: $w_{s1} = 1.4530$; *dashdot* line: $w_{s1} = 0.9032$.

worst. On the other hand, when $w_{s1} = 1.4530$, the interleaver possesses the best isolation and insertion loss but the bandpass width is the thinnest. A compromise is achieved with $w_{s1} = 1.2566$. Table 3.1 tabulates the values of w_{s1} and their corresponding optimal solutions of θ_2 and θ_3 . The ratio of passband width to channel spacing is also displayed.

Table 3.1: Optimal Design Parameters for F^3T Interleavers with the proposed method.

w_{s1}	θ_1	θ_2	θ_3	W_s to Channel spacing
0.7854	0.7854	0.5236	1.0472	100%
0.8247	0.7854	0.5889	1.0798	95%
0.8639	0.7854	0.6486	1.1097	90%
0.9032	0.7854	0.7027	1.1367	85%
0.9425	0.7854	0.7514	1.1611	80%
0.9817	0.7854	0.7949	1.1829	75%
1.0210	0.7854	0.8337	1.2022	70%
1.0603	0.7854	0.8681	1.2194	65%
1.0996	0.7854	0.8984	1.2346	60%
1.1388	0.7854	0.9251	1.2480	55%
1.1781	0.7854	0.9485	1.2596	50%
1.2174	0.7854	0.9688	1.2698	45%
1.2566	0.7854	0.9863	1.2786	*40%
1.2959	0.7854	1.0013	1.2861	35%
1.3352	0.7854	1.0139	1.2924	*30%
1.3744	0.7854	1.0244	1.2976	25%
1.4137	0.7854	1.0327	1.3018	20%
1.4530	0.7854	1.0391	1.3050	15%
1.4923	0.7854	1.0436	1.3072	10%
1.5315	0.7854	1.0463	1.3086	5%
1.5708	0.7854	1.0472	1.3090	0%

At 40% of W_s to channel spacing, the isolation is -20.8 dB. At 30% of W_s to channel spacing, the isolation is -26.0 dB. In practical applications, the isolation is usually required to be at least -20 dB. The above table can be used for reference.

3.2.3.2 Comparison with the existing design approach

The initial attempt at designing the interleaver uses the integration method [126] by optimization. The problem is formulated as minimizing an objective function to solve for θ_2 and θ_3 , *i.e.* minimizing

$$E_{(\theta_2, \theta_3)}(w) = \int_{w \in W_s} z(w)(|H_1(w)| - 0)^2 dw \quad (3.37)$$

where $z(w)$ is a positive weighting function. If the passband bandwidth of $H_1(w)$ is selected from $2\pi/5$ to $3\pi/5$, when using the integration optimization method, the bar-state and cross-state intensity transfer responses according to frequency (phase) are depicted in Fig. 3.6 where $z(w)$ is selected as 1 in the passband.

When the weighting function $z(w)$ is well selected, the curve will change but not much. The plot is nearly the same as that when $z(w)$ is equal to a constant. When $z(w)$ in the passband is selected as follows:

$$z(w) = \begin{cases} 100(w - \frac{2\pi}{5}) & \frac{2\pi}{5} < w < \frac{9\pi}{20}, \\ 1 & \frac{9\pi}{20} < w < \frac{11\pi}{20}, \\ 100(\frac{3\pi}{5} - w) & \frac{11\pi}{20} < w < \frac{3\pi}{5}. \end{cases} \quad (3.38)$$

the bar-state and cross-state intensity transfer responses are depicted in Fig. 3.7.

Using our design method, the intensity transfer spectrum is depicted in Fig. 3.8. It can be clearly seen from Fig. 3.8 that the isolation and the insertion loss derived by the proposed minimax method are lower than that derived by using the integration method of [126].

Figure 3.6: Spectral responses of the two outputs of the interleaver designed by using the existing optimization method. *Dotted* line: cross-state output intensity; *solid* line: bar-state output intensity.

3.2.4 Fabrication and Experimental Results

In this section, the characteristics of the interleaver are experimentally investigated. A novel fabrication method for all-fiber cascaded Mach-Zehnder interferometer is proposed [122]. To the best of our knowledge, it is the first demonstration for such kind of fabrication in the literature.

3.2.4.1 Mach-Zehnder Interferometer Fabrication

When fabricating MZI by splicing two individual couplers together with a required length difference between the two arms, the major difficulty to general users is that

Figure 3.7: Spectral responses of the two outputs of the interleaver designed by using the existing optimization method when $z(w)$ is not a constant. *Dotted line*: cross-state output intensity *solid line*: bar-state output intensity.

the length difference must be accurately achieved before splicing. The other problem is that the two arms between the couplers have to be long enough for the splicing purpose. All the above difficulties make the MZI very difficult to achieve in the all-fiber sense. Fabricating consecutively two couplers using the coupling machine can avoid the problems caused by splicing, however, this is generally not achievable because when coupling begins for the second coupler, the output intensities detected by the photodetectors become interference signals and change randomly.

We suggest a new fabrication technique to overcome this problem. With this method, one coupler (in our case Coupler II as shown in Fig. 3.9) is fabricated as usual using the coupling machine. Then another coupler (Coupler I in Fig. 3.9)

Figure 3.8: Spectral responses of the two outputs of the interleaver designed by using the proposed minimax method. Dotted line: cross-state output intensity solid line: bar-state output intensity.

Figure 3.9: The developed novel experiment setup for the fabrication of Mach-Zehnder Interferometer.

is fabricated using a broadband LED as light source and an Optical Spectrum Analyzer (OSA) for spectrum response monitoring. When coupling begins for Coupler

I, the spectrum on the OSA becomes an interference pattern for the bar- and cross-state respectively. For a given c_1 , one can determine c_2 based on the interference pattern of the bar-state. For example, for the MZ interleaver which requires that the coupling ratios of both couplers are 50:50 is desired, then the expected splitting ratio of Coupler I would be 50:50 when the maximum interference pattern has been reached. Fig. 3.10 shows such a maximum interference pattern in the case of the coupling ratio of coupler II equals to 49.6:50.4. A later examination shows that the splitting ratio of Coupler I is 49.5:50.5. After this, the required length difference between the two arms is achieved by stretching one of the fibers between the two couplers through thermal heating during the stressing.

Figure 3.10: Experimental interference pattern of the bar-state output of the fabricated Mach-Zehnder Interferometer on an optical spectrum analyzer.

3.2.4.2 F^3T All-Fiber Interleaver Fabrication

Successful experimental results are obtained by using the proposed method stated in the subsection 3.2.4.1 and splicing the third coupler with the former two couplers. Figs. 3.11 (a) and (b) show the responses of bar- and cross-state of the fabricated F^3T interleaver in narrower range and wider range (can be expanded to the whole C band), respectively. This interleaver is fabricated for 25G/50G which means the

channel spacing is 0.2 nm. The isolation is more than 20 dB, and it can be measured from Figs. 3.11 (c) and (d) that the passbandwidth is around 52% in the sense of passband ripple lower than 0.3 dB.

Figure 3.11: Spectral responses of F^3T Interleaver with the proposed fabrication technique: (a) responses of bar- and cross-state outputs in a narrower spectral period; (b) responses of bar- and cross-state outputs in a wider spectral period; (c) the examination of passband bandwidth of the interleaver; (d) the examination of passband ripple of the interleaver.

3.2.5 Sensitivity to Fabrication Variations

Successful fabrication of an F^3T interleaver depends on the sensitivity of the important filter characteristics to manufacturing variations. The factors of concern are the coupling ratios and effective path length difference of the arms. They are evaluated separately in the following two subsections.

3.2.5.1 Sensitivity of the coupling ratios

We shall first examine the effects of coupling ratio variations. The impact of the coupling ratio variations are evaluated by taking the derivatives with respect to the coupling ratios of the output transfer function. Denote the power transfer function of $|H_0(w)|^2$ and $|H_1(w)|^2$ as E_{bar} and E_{cross} , respectively. From the relationship $k_i = \cos^2(\theta_i)$ ($i=1,2,3$), we have

$$d\theta_i = -\frac{1}{\sin 2\theta_i} dk_i \quad (3.39)$$

Taking derivatives of $E_{bar} = |H_0(w)|^2$ with respect to the coupling ratios k_1 , k_2 and k_3 , and applying (3.39), we have

$$dE_{bar} = \frac{\partial E_{bar}}{\partial \theta_1} d\theta_1 + \frac{\partial E_{bar}}{\partial \theta_2} d\theta_2 + \frac{\partial E_{bar}}{\partial \theta_3} d\theta_3 \quad (3.40)$$

Similarly, for the cross-state intensity variation, we get

$$\begin{aligned} dE_{cross} &= \frac{\partial E_{cross}}{\partial \theta_1} d\theta_1 + \frac{\partial E_{cross}}{\partial \theta_2} d\theta_2 + \frac{\partial E_{cross}}{\partial \theta_3} d\theta_3 \\ &= -\frac{1}{\sin 2\theta_1} \frac{\partial E_{cross}}{\partial \theta_1} dk_1 - \frac{1}{\sin 2\theta_2} \frac{\partial E_{cross}}{\partial \theta_2} dk_2 - \frac{1}{\sin 2\theta_3} \frac{\partial E_{cross}}{\partial \theta_3} dk_3 \end{aligned} \quad (3.41)$$

A standard deviation of k_i ($i=1,2,3$) is 2%. This level of variation is consistent with typical fabrication variations for the usual coupling machine. From Eqns.(3.40) and (3.41), we show in Fig. 3.12 all the responses of bar-state and cross-state intensities with the deviations dE_{bar} and dE_{cross} , where θ_1 , θ_2 and θ_3 are selected as 0.7854, 1.0139 and 1.2924, respectively. This level of variation can change filter characteristics such as passband width as well as the stopband isolation; however, the general filter shape remains unaltered. Here, it should be noted that the deviation considered above only take into count the systematic variations. The random variation is rather small as compared to the systematic variation, it can be put into the systematic variation and the operations are similar.

Figure 3.12: Sensitivity study of the proposed method for F^3T interleaver on the deviations with respect to the coupling ratios.

3.2.5.2 Sensitivity of the phase

A systematic variation in the path lengths changes the phase property. From $\phi = \Delta L \pi n / \lambda$, it can be derived that,

$$\Delta\phi = -\frac{1}{\lambda^2} \Delta L \pi n \Delta\lambda \quad (3.42)$$

Here, we select the central wavelength λ as 1550 nm, the channel spacing is 50 GHz which means that $\Delta\lambda=0.4$ nm and the refractive index n for single mode fiber as 1.476, then the physical length difference of the first delay is $\Delta L = 4.07$ mm. Thus the frequency shift can be expressed as:

$$\begin{aligned} dw = dwT = \Delta\phi &= \frac{\Delta L \pi n}{\lambda} dL \\ &= 1.218 \times 10^4 dL \end{aligned} \quad (3.43)$$

The changes of the bar-state response with respect to the length difference can be expressed as

$$\begin{aligned} dE_{\text{bar-length}} &= \partial E_{\text{bar}} / (\partial w) dw \\ &= 1.218 \times 10^4 \left(\sin 2\theta_1 \sin 2\theta_3 (3 \cos^2 \theta_2 \sin 6w - \sin^2 \theta_2 \sin 2w) \right. \\ &\quad \left. + \sin 2\theta_1 \sin 2\theta_2 \cos 2\theta_3 \sin 2w + 2 \cos 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \sin 4w \right) dL \end{aligned} \quad (3.44)$$

With the coupling machine stressing the fiber, the errors of the length difference are in the range of ± 0.02 mm, thus when applying (3.44) to $E_{\text{bar}} = |H_0(w)|^2$, the responses of the bar-state with the deviation $dE_{\text{bar-length}}$ produced by the length error are as shown in Fig. 3.13, where θ_1 , θ_2 and θ_3 are selected as 0.7854, 1.0139 and 1.2924, respectively.

Compared with Figs. 3.12 and 3.13, it can be seen that the responses generated by the deviation of coupling ratios shift upwards which means that it has more effect

Figure 3.13: Sensitivity study of the proposed method for F^3T interleaver on the deviations with respect to the length difference.

on the isolation and passband ripple, while in Fig. 3.13, the deviations of the length difference greatly affect the passbandwidth and stopbandwidth. Since the above two deviations are independent, the overall effects on the interleaver performance are the combinations of these two.

3.3 Infinite Impulse Response Two-port Optical Filter Banks

In Section 3.2, an FIR two-port filter bank is successfully developed and demonstrated through simulation and experimental studies. However, from the signal processing point of view, an IIR two-port filter bank is preferred in certain applications because it can provide infinite Fourier harmonics. In this section, an IIR optical interleaver is designed and demonstrated by using Superimposed Chirped Fiber Bragg Gratings (SCFBGs) and a fiber coupler. The main idea is also to find

a suitable structure to reduce the design of two-port IIR optical filter bank into designing only one port of transfer function. Compared with the FIR two-port optical interleaver presented in Section 3.2, IIR structure is more efficient for the design of two-port filter bank in terms of using less optical elements and having a better spectral performance.

3.3.1 IIR Optical Filter Banks with One Superimposed Chirped FBG

3.3.1.1 Proposed Scheme and Design Approach

The proposed optical interleaver with IIR structure is shown in Fig. 3.14. It consists of one 2×2 fiber coupler with one of its output arms (denoted by number 4) terminated by a linearly chirped FBG denoted by CFBG3 and arm 3 terminated by two linearly chirped FBGs denoted respectively by CFBG1 and CFBG2. All the chirped FBGs have different index modulation depths denoted respectively by Δn_1 , Δn_2 , and Δn_3 . But, from the point of view of fabrication, these chirped FBGs have the same length L , as they are written with an identical phase mask and setup. The choice of L determines the FBG transmission bandwidth, in which the interleaving spectra take effect. The grating CFBG1 is shifted with respect to CFBG2 by a distance twice that of the difference (ΔL) between the lengths of coupler's arms 3 and 4. Here, the arm length is calculated from the coupler to the starting position of the leftmost chirped FBG on the arm. The choice of the arm length difference ΔL determines the free spectral range (FSR) of the interleaver, *i.e.* a length difference ΔL of about 2 mm corresponds to a 50 GHz FSR given that the refractive index of the fiber n is 1.476.

As shown in [130], the transmission of a single linearly chirped FBG is an almost flat-bottom stopband transmission, like the one depicted in Fig. 3.15. The stopband

Figure 3.14: Proposed configuration of the two-port interleaver with an Infinite Impulse Response by using one superimposed chirped FBG.

attenuation is determined by the index modulation depth. In what follows, the average stopband attenuation of a single linearly chirped FBG is referred to as the effective power reflectance which is denoted by ρ_1 for CFBG1, ρ_2 for CFBG2, and ρ_3 for CFBG3. In the superimposed FBGs on coupler's arm 3, the grating CFBG2 is designed to have an effective power reflectance of approximate unity, *i.e.* $\rho_2 \cong 1$, so as to obtain an all-pass filter response. Similarly, the grating CFBG3 on coupler's arm 4 also yields $\rho_3 \cong 1$. The effective power reflectance of the chirped FBGs are chosen as design parameters to obtain flat-top passband spectra for the interleaver.

We denote the input and output fields at the m -th port of the coupler as E_m^i and E_m^o where m denotes the arm number, $m = 1, 2, 3, 4$. With the above configuration, two channels of interleaving passband transmissions can be obtained respectively from E_1^i to E_1^o and to E_2^o . To see this, we derive an approximate model for these transmissions. The approximation comes from expressing the reflection transmissions of chirped FBGs as an ideal square-like passband transmissions.

We denote z as a unit time delay given by $z^{-1} = e^{-j2\Delta L\pi n/\lambda}$ where λ is the wavelength

of wave propagating through free space and n is the refractive index of the fiber. For a directional coupler, its transmission can be expressed as

$$\begin{pmatrix} E_3^o \\ E_4^o \end{pmatrix} = \begin{pmatrix} z^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c & js \\ js & c \end{pmatrix} \begin{pmatrix} E_1^i \\ E_2^i \end{pmatrix} \quad (3.45)$$

with $c = \sqrt{\kappa}$ and $s = \sqrt{1 - \kappa}$ where κ is referred as the coupling ratio of the coupler.

As a single linearly chirped FBG has an almost flat-bottom transmission, the superimposed chirped FBGs on arm 3 can be understood as a FP cavity (usually refer to GTE [60] if the back mirror has a unit power reflectance) within the transmission bandwidth with two reflecting mirrors that have effective power reflectance ρ_1 and ρ_2 . Such an approximation will be justified later through simulation studies by using the transmission matrix method (TMM) [4] to simulate the responses of the two superimposed FBGs. Thus, the transmission from E_3^o to E_3^i can be approximated by an FP transfer function within the transmission bandwidth of the chirped grating CFBG1 or CFBG2. Similarly, the transmission from E_4^o to E_4^i can be approximated by a constant within the transmission band of the chirped grating CFBG3.

Therefore, the transmissions from E_3^o and E_4^o to E_3^i and E_4^i can be approximately expressed as

$$\begin{pmatrix} E_3^i \\ E_4^i \end{pmatrix} = \begin{pmatrix} H(z^{-2}) & 0 \\ 0 & \sqrt{\rho_3} \end{pmatrix} \begin{pmatrix} E_3^o \\ E_4^o \end{pmatrix} \quad (3.46)$$

where $H(z^{-2})$ denoted the transmission from E_3^o to E_3^i . Due to the existence of the difference between arms 3 and 4, the transmissions from E_3^i and E_4^i to E_1^o and E_2^o are given by

$$\begin{pmatrix} E_1^o \\ E_2^o \end{pmatrix} = \begin{pmatrix} c & js \\ js & c \end{pmatrix} \begin{pmatrix} z^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} E_3^i \\ E_4^i \end{pmatrix} \quad (3.47)$$

Let $H_1(z^{-1})$ and $H_2(z^{-1})$ represent the transmissions respectively from E_1^i to E_1^o

and to E_2^o . By selecting $\kappa = 0.5$, it can be shown from (3.45) to (3.47) that

$$|H_1(z^{-1})|^2 = |H_2(z^{-1}e^{j\pi})|^2 \quad (3.48)$$

which means that the proposed structure guarantees that the two output transmissions have power spectra with an identical passband/stopband shape but the spectra are phase-shifted by π between them when the coupling ratio of the coupler is 50:50. Then, the interleaver design problem is reduced to the design of the effective power reflectances ρ_1 , ρ_2 , and ρ_3 such that the passbands and stopbands of either $|H_1(z^{-1})|^2$ or $|H_2(z^{-1})|^2$ are isolated as much as possible, and more importantly, their passband shapes should be as flat as possible in order to reduce the effects of power variations resulted from channel wavelength shifts. Various design methods including those for designing Butterworth, Chebyshev and Elliptic band-pass filters [50, 51] can be employed to obtain the design parameters. As discussed above, the values of ρ_1 , ρ_2 , and ρ_3 are determined by the index modulation depths Δn_1 , Δn_2 , and Δn_3 . Therefore, after obtaining values of the effective power reflectance, they can be achieved by controlling the intensity of the UV laser when the chirped FBGs are being written.

As a numerical example, the proposed IIR structure is used to design a two-port 25G/50G optical interleaver with its 0.5-dB passband bandwidth specified as 80% of the channel spacing. The obtained parameters are also used in the next section to fabricate the optical interleaver by experiments. To achieve the FSR at 50 GHz, we choose $\Delta L=2$ mm. Thus, the two superimposed chirped FBGs, denoted by CFBG1 and CFBG2 in Fig. 3.14, are shifted by 4 mm. The chirped FBGs considered in the design are written with a 40 mm chirp phase mask and a chirp $C_h=6.12$ nm/cm. A hydrogen-loaded fiber with a core diameter of $4\mu\text{m}$ and a refractive index contrast of 0.01 is used. Assume that the effective power reflectance ρ_2 and ρ_3 take the value of 99%. The amplitude spectra of the transmissions of chirped gratings CFBG2 and

CFBG3 are simulated by using the TMM method and they are shown in Fig. 3.15 by the solid line. Using the least-squares (LS) method [51], the desired interleaver transmission $H_1(z)$ can be obtained by choosing $\rho_1 = 18.5\%$. Correspondingly, the transmission of CFBG1 take an amplitude spectrum marked as the dotted line in Fig. 3.15, and thus the superimposed chirped FBGs have an amplitude spectrum as shown by the dashed line in Fig. 3.15. We also simulate the amplitude spectra of the superimposed chirped FBGs by using an FP cavity with mirror amplitude reflectances respectively given by $\sqrt{\rho_1}$ and $\sqrt{\rho_2}$. The obtained amplitude responses are almost the same as the one obtained by the TMM method. This justifies the FP model that is used above to approximate the superimposed chirped FBG.

Figure 3.15: Transmissions of chirped FBGs and the superimposed chirped FBG cavity.

For the purpose of illustrating the efficiency of the proposed structure, we also design the interleaver with an F^3T structure. Simulation studies show that the F^3T interleavers should be formed by five fiber couplers in order to achieve almost the same specifications. The amplitude spectra of the F^3T interleaver and the IIR interleaver are shown in Fig. 3.17. Comparing the corresponding responses, it is clear that with the proposed IIR structure, fewer number of optical elements are

Figure 3.16: Interleaver responses with the design parameters bearing 5% fabrication variations.

required to deliver the desired interleaving passband transmissions.

Figure 3.17: Comparison of spectral responses between the proposed IIR interleaver and a five-stage F^3T FIR interleaver.

3.3.1.2 Experimental Results

An optical 25G/50G interleaver was successfully fabricated on a home-made FBG writing system with an excimer laser (240 nm and 100-mW) as a UV light source. In the experiment, hydrogen-loaded single mode fibers are used to fabricate FBGs. The fabrication process includes the following several phases. In the first phase, the superimposed chirped FBGs are fabricated. To this end, we first obtain the desired CFBG2 by using a scanning method with a chirp phase mask (6.12 nm/cm, StockYale Inc.). As mentioned in subsection 3.3.1.1, it is not possible to achieve 100% effective power reflectances for CFBG2 as desired. In the experiment, -22 dB FBG transmission is obtained with an index modulation depth of 14×10^{-4} . After fabricating CFBG2, we move the phase mask to the left by 4 mm with respect to the starting position of CFBG2, and then proceed to fabricate CFBG1. In the design, the effective power reflectances of CFBG1 is chosen as 18.5% which corresponds to a -0.88 dB FBG transmission with an index modulation depth of 1.2×10^{-4} . This index modulation depth can be achieved by controlling the scanning speed and the output power of the UV laser. In the second phase, we fabricate the chirped grating CFBG3 on a new fiber such that CFBG3 has the same spectral response as CFBG2. Finally in the third phase, the two pieces of fibers with chirped FBGs are spliced with a 50:50 fiber coupler. The length difference of fiber coupler arms connecting to FBGs is controlled at 2 mm. This is accomplished by first setting the length difference roughly as 2 mm in splicing, and then stretching one of the fiber arms by thermal heating [122].

The amplitude spectrum of the fabricated interleaver is measured at port 2 of the coupler with a light source input at port 1. The experimental result recorded on an optical spectrum analyzer is depicted in Fig. 3.18 together with the simulation result. It can be seen that the experimental result agrees well with the simulation result. Due to its all fiber structure, the insertion loss of the proposed interleaver

is lower 1.0 dB which is clearly seen in Fig. 3.18. The group delay of the interleaver measured with an optical dispersion analyzer (Agilent 86038A) is shown in Fig. 3.19 together with the simulated group delay. The results have shown that measured group delay variation of the designed interleaver has a similar shape with the simulated one but with about 10 ps group delay in contrast to 15 ps in theory in the 0.5-dB-passband bandwidth. The measured group delay is smaller than the theoretical design mainly because the IIR resonance response of the superimposed gratings has a fast decay. This may result from the fabrication errors, such as the grating strength of the chirped gratings is not uniform, and the length and position of the chirped gratings are not well controlled. It should be noted that, because of the dispersive property of CFBG, it causes a linear group delay slope within the whole operation range. This is the fundamental difference between the proposed structure and the free space implementation [60] where all wavelengths resonate at the same mirror position. The same interleaver responses can also be observed at port 1 through a circulator. Further observations have shown that the interleaving spectra and group delay characteristics can be obtained over a range of 16 nm. Within this operation range, the designed interleaver has an FSR varying from 45 GHz to 53.7 GHz, a 0.5-dB passband bandwidth from 16.7 GHz to 17.5 GHz, and a channel isolation changing from 18 dB to 22 dB. This result has demonstrated the effectiveness of the proposed interleaver structure and design scheme.

Unlike the two-port interleaver that is formed by embedding ring waveguide resonators in a 2×2 planar MZI [131], the proposed IIR cavity structure provides a more efficient configuration employing only one 2×2 fiber coupler that reduces the complexity of the interleaving filter. The cavity SCFBG reflectors structure is able to easily remove the channel spacing limitation as is the usual case with all-pass ring resonators, so broadband designs can be easily realized [2]. Besides, the implementation is also simplified without the need of fabricating waveguide ring resonators which are difficult to achieve in practice. In addition, the proposed optical inter-

Figure 3.18: Transmission responses of the designed 25G/50G IIR interleaver. Measured output transmission of the interleaver with IIR responses together with the simulation result.

leaver is fabricated with an all-fiber structure, thus the insertion and connection loss would be quite small.

We would like to make several comments. First, considering the index modulation depth for CFBG2, the total index modulation depth in the superimposed section amounts to 15.2×10^{-4} which is still within the range of the current FBG writing technology using hydrogen-loaded fibers. Secondly, in the fabrication of the superimposed chirped FBGs, writing CFBG1 on top of the written CFBG2 will cause the transmission of CFBG2 to shift to the longer wavelength side due to an increase in the average refractive index of the superimposed grating. This is a common problem in writing all kinds of superimposed FBGs [132]. The shift will yield a non-uniform transmission for the superimposed FBG, particularly at the edge of the transmission band. Consequently, this will produce a channel spacing of the interleaver that is not strictly uniform. Thirdly, considering the unavoidable deviations of the design parameters in practical fabrication, 5% tolerant fabrication errors are con-

Figure 3.19: Group delay of the designed 25G/50G interleaver. Enlarged top portion of one period of passband transmission (theoretical amplitude, simulated group delay in the 0.5 dB passband bandwidth and measured group delay); solid line represents the simulated amplitude response; dotted line represents the simulated group delay and circle-dotted line represents the measured group delay.

sidered for the design parameters ΔL , κ , ρ_1 , ρ_2 , and ρ_3 . The results indicate that the performance of the interleaver is acceptable when the design parameters can be controlled to within 5% deviations during the fabrication process. Lastly, it should be mentioned that the uniformity of the chirped grating strengths [130] is another important factor that affects the performance of the interleaver in the sense that the FSR, the passband bandwidth and the channel isolation of the interleaver will not be uniform.

3.3.2 IIR Optical Filter Banks with two Gires-Tournois Etalons

Gires-Tournois etalons (GTEs) [133] are periodic filters that have been widely used in optical fiber communication systems, such as in the compensation of chromatic dispersion [26, 134], polarization mode dispersion [135], compressing ultrashort pulses,

and building optical filters [136]. From the signal processing perspective, GTE functions as an all-pass infinite impulse response (IIR) filter. Thus it has the advantages in that they can provide an efficient means to achieve flatter and wider passband bandwidth, higher channel isolation, and faster roll-off on band edges. So far, several approaches have been proposed to design two channels of interleaving transmissions by using bulk-optics GTEs in a Michelson interferometer [56] or building GTEs in a birefringence interferometer [64]. Recently in [60], Hsieh and co-workers demonstrated an optical interleaving filter by using two bulk-optics GTEs. Although this scheme does demonstrate an optical interleaving filter with a channel isolation as high as 30 dB, its free-space nature determines that it is not compatible with the DWDM communication links, thus it would increase the connection loss. In addition, this scheme also increases the manufacturing cost. More recently, a fiber distributed GTE was demonstrated by using superimposed CFBGs [137]. It is shown that such grating based GTEs not only have similar periodical dispersion properties as the traditional GTEs, but also have the attractive advantages of full compatibility with optical fiber, low loss, and low cost when compared with the bulk-optics GTEs or thin film GTEs.

In this subsection, an even more efficient IIR structure of optical interleaving filter is proposed by using two fiber GTEs in a fiber Michelson interferometer. It is shown that the phase shift property of the interleaver is kept as that of the one shown in section 3.2 and subsection 3.3.1. In addition, the amplitude of the periodic spectra of the interleaver can be designed to meet desired specifications. To demonstrate the effectiveness of the proposed structure and design scheme, we develop an all-fiber 25G/50G optical interleaving filter successfully via experiment.

3.3.2.1 Improved Scheme and Design Approach

The proposed optical interleaving filter with GTEs is shown in Fig. 3.20. It consists of one fiber Michelson interferometer, formed by one 2×2 fiber coupler, with one of its output arms (denoted by number 3) terminated by GTE1, and arm 4 terminated by GTE2. The corresponding linearly chirped FBGs that composing GTE1 and GTE2 are denoted respectively by CFBG1, CFBG2, CFBG3, and CFBG4. The coupling ratio of the 2×2 fiber coupler is chosen as 50:50 for the same reason that it can eliminate the phase shift requirement of the interleaver as shown in the section 3.2. The grating CFBG1 is shifted with respect to CFBG2 by a distance twice that of the difference (ΔL) between the lengths of coupler's arms 3 and 4. The gratings CFBG3 and CFBG4 are also placed with the same length difference as that of the gratings CFBG1 and CFBG2. In what follows, the average stopband attenuation of a single linearly chirped FBG is referred to as the effective power reflectance which is denoted by $\rho_{1,1}$ for CFBG1, $\rho_{0,1}$ for CFBG2, $\rho_{1,2}$ for CFBG3, and $\rho_{0,2}$ for CFBG4. The grating CFBG2 is designed to have an effective power reflectance of approximate unity so as to obtain an all-pass GTE response, *i.e.* $\rho_{0,1} \cong 1$. This is also the case with the grating CFBG4.

Denote $H_1(z^{-1})$ and $H_2(z^{-1})$ as the transmissions respectively from input E_1^i to outputs E_1^o and E_2^o . By selecting $\kappa = 0.5$, we have $|H_1(z^{-1})|^2 = |H_2(z^{-1}e^{j\pi})|^2$. This ensures the two-port IIR interleaver is converted to a simple transfer function design. Then, an optimal interleaving filter transmissions $H_1(z)$ and $H_2(z)$ with a 0.5-dB passband bandwidth specified as 92.5% of the channel spacing can be obtained by choosing $\rho_{1,1} = 46.4\%$ and $\rho_{1,2} = 4.4\%$. These designed parameters are to be used in fabrication of the IIR interleaving filter.

To illustrate the effectiveness of the proposed structure, we also design the interleaving filter with an F^3T structure. The F^3T interleaving filter formed by six couplers are designed to have similar spectral characteristics as the IIR interleaving filter, as

Figure 3.20: Improved configuration of the two-port interleaver with an Infinite Impulse Response by using two superimposed chirped FBG.

shown in Fig. 3.21. It can be seen clearly in Fig. 3.21 that the roll-off of the proposed interleaving filter is much sharper than that of the F^3T one. The FOM is an important index to examine the roll-off of the interleaving filter, which is defined by the ratio between 0.5 dB bandwidth and 20 dB bandwidth in the passband operating wavelength. The proposed IIR interleaving filter has an FOM of as high as 62.5% in contrast to 38.8% for the six-stage F^3T interleaving filter. Thus, the proposed IIR structure is effective in achieving two-port IIR interleaving transmissions.

3.3.2.2 Experimental Results

An optical 25G/50G interleaver was successfully fabricated on a home-made FBG writing system with an excimer laser (240 nm and 100-mW) similar to the one shown in the Section 3.3.1. The amplitude spectrum of the fabricated interleaving filter is measured at port 2 of the coupler with an Erbium-doped fiber (EDF) broadband light source input at port 1. The spectrum recorded on an OSA is shown in Figs. 3.22 and 3.23. It is further examined that the proposed interleaving filter can deliver

Figure 3.21: Comparison of spectral responses between the proposed IIR interleaver and a six-stage F^3T FIR interleaver.

a periodic passband transmission with an FSR varying from 49.1 GHz to 50.7 GHz, a 0.5-dB passband bandwidth changing from 18.0 GHz to 18.8 GHz, and a channel isolation from 23 dB to 26 dB at the central stopband wavelength. The same spectrum with π -phase shift can also be observed at port 1 through a circulator. This result has demonstrated the effectiveness of the proposed interleaving filter structure and design scheme.

It is noted that although the SCFBG1 and SCFBG2 are fabricated under the same strain imposed on FBGs and the same laser output during inscription, the fabrication still suffers from a phase mask displacement error when moving the phase mask. Therefore, the cavity length of the two superimposed chirped FBGs is not exactly twice that of the length difference of the coupler's fiber arms. This would result in the variation of the FOM of the interleaving filter. In order to diminish this error, a phase mask with high displacement accuracy is highly desired.

Figure 3.22: Measured output transmission of a typical output of the improved IIR interleaving filter together with the simulated result.

Figure 3.23: Measured group delay of a typical output of the improved IIR interleaving filter together with the theoretical result.

3.4 Discussion

In this section, some comments on the development of two-port optical filter banks are presented. First, a comparison of the characteristics of the proposed two-port

optical filter banks to the existing technologies is made in order to highlight the advantages and key contributions of the proposed schemes. Second, some key ideas and novelties on the structure, the design, and the fabrication aspects of two-port optical filter banks are examined. Remarks on how to handle the optical constraints of the optical filter bank designs over their digital counterparts are also mentioned in this section.

First, the critical specifications of the proposed two-port FIR and IIR interleavers and those of the commercial products from industry companies, such as JDS Uniphase, OpLink, and Chorum, are compared in Table 3.2. It is clear from Table 3.2 that the existing FIR two-port interleaver is unfavorable because of its limited channel spacing of 100 GHz, high insertion loss of more than 5 dB, and high cost. The table also shows that the existing IIR two-port interleaver has poor performance as compared with the proposed IIR interleaver in that the loss is higher due to the coupling from bulk optics to fiber, and the cost is more expensive. Thus, in general, the proposed schemes for two-port optical interleavers have better specifications in low insertion loss, narrow channel spacing, all-fiber structure, low cost and simple quantitative design tools.

Next, we would like to make several comments on the design of two-port optical filter banks in terms of the structural design, the performance design and the implementation design:

- **Structure:** Clearly, an efficient structure plays a critical role in designing the optical filter banks. Although other structures exist for designing two port optical filter banks, the proposed methods have unique advantages. Firstly, the FIR two-port optical filter bank is implemented in a lattice structure which greatly minimizes the number of the design parameters [48, 138] and results in the simplification of the design procedure and the fabrication of such filters. The lattice structure also makes the optical filter relatively more robust to the

Table 3.2: Comparison of the proposed two-port optical filter bank schemes with the existing technology.

Specifications	Proposed FIR two-port interleaver	Existing FIR two-port interleaver	Proposed IIR two-port interleaver	Existing IIR two-port interleaver
Operating Wavelength Range	C band L band	C band L band	C band L band	C band L band
Channel Spacing (GHz)	12.5 - 100	100	< 25	< 25
Material Used	fiber	waveguide	fiber	bulk optics thin film
Insertion Loss (dB)	< 1	> 5	< 2	> 5
Cost (US dollar)	tens	thousands	tens	thousand
Passband to Channel Spacing (%)	~ 50	~ 50	~ 80	~ 80

design parameters deviations [138, 139]. Therefore, the optical constraints of optical filter bank over the digital counterpart such as the Hermitian structure of the filter and the sensitivity of the structure are overcome by using the proposed lattice structure. On the other hand, for the IIR two-port optical filter bank, since it is realized with all-pass optical elements, the filter performance is also insensitive to the design parameter variations [79, 136]. This property on the robustness of the designed filter bank is of great importance for the design of optical filters where the accuracy of filter coefficients cannot be exactly controlled in the fabrication in contrast to its digital filter counterparts. For both FIR and IIR two-port optical filter banks, to make the optical signals coherent so as to generate interference phenomena, the physical length differences of the proposed structures are chosen as several centimeters.

Secondly, the purpose of selecting a proper structure, *e.g.* the second differential arm is twice that of the first one in the case of FIR filter bank design or the ring path length is twice that of the arm difference of the coupler in the case of IIR filter bank design, serves to provide higher orders of the Fourier series

of the designed filter with fewer optical elements. Therefore, the proposed structure is more efficient.

Thirdly, by properly selecting certain design parameters of the proposed structure, the two-input two-output design problems can be converted into one of designing just one channel of transfer function, which greatly simplifies the design and fabrication of optical filter bank. This idea is one of the main contributions of the thesis and can be further extended to higher order multi-port filter bank designs.

- **Design:** Based on the obtained structure, we have developed a systematic scheme for the design of both FIR and IIR two port optical filter banks. Through a proper structural design, the optical filter bank design is converted into a single-input single-output filter design problem that can be tailored to meet the desired filter specifications using available approaches. This idea will also be employed and demonstrated in the subsequent chapters for designing higher order multi-port optical filter banks.
- **Implementation:** For the fabrication of the proposed two-port optical filter banks in both FIR and IIR structures, since fewer design parameters are required and the performance of the filter bank is robust to the design parameter deviations, which are guaranteed from the structure design, it efficiently relaxes the critical requirements on the fabrication of optical filters.

All-fiber implementations for both FIR and IIR two-port optical filter banks are also realized based on a novel and simple fabrication method proposed in this chapter. Although waveguide technology is suitable to fabricate two-port optical filter banks with higher filter lengths due to its maturity and compact size, it incurs much more insertion loss and connection loss. With the progress of waveguide technology, high performance of higher order two-port optical filter banks could be fabricated in the near future.

3.5 Conclusion

There important aspects of two-port optical filter bank designs are studied in this chapter. First, a new approach for designing a two-port FIR optical filter bank, *i.e.* two-port FIR optical interleaver, with guaranteed performance is described. Through describing the performance of the interleaver by an H_∞ norm of the power transfer function of the interleaver, a quantitative approach to interleaver design is developed. A novel fabrication method is also proposed to fabricate the all-fiber FIR interleaver without any splicing between the fiber couplers. Due to its all fiber structure, the loss caused by this process is minimized, the cost of the devices is reduced and the obtained optical device is more compatible with the communication systems. Second, with regards to the design of IIR two-port optical filter banks, two novel IIR design structures are proposed. The proposed interleaver is implemented by using a 50:50 fiber coupler with its output arms terminated respectively by one or two pairs of superimposed chirped FBGs. Relevant experimental results are obtained to illustrate the effectiveness of the proposed scheme. Compared with the commercially available technologies, the proposed schemes for designing either FIR or IIR interleavers have the advantageous features such as low insertion loss, wide operation wavelength range, all-fiber structure, low cost and simple quantitative design tools.

Chapter 4

Three-port Optical Filter Banks

4.1 Introduction

In the last chapter, we have presented new approaches for the design of a two-port optical filter bank by using the two-port optical circuits. Here, the associated design approaches are limited to a 2×2 structure. In order to achieve high channel counts for a communication system, a number of interleaving filters have to be cascaded. But this in practice will result in an increase in the insertion loss with the increasing number of interleaving filters being used. Furthermore, only logarithmic configurations are possible [140, 141] with two-port interleavers. Hence, it is important to design filters using multi-port optical delay-line circuits, especially the three-port ones, in order to efficiently increase the channel counts with less stages of interleaving filters and to provide more flexibility for optical signal multiplexing. In [142], Kishioka made the first attempt to design N -order optical filters using 3-port optical delay-line circuits. A synthesis design algorithm is proposed to design a filter with a wide wavelength-flattened response. However, as the 3×3 optical couplers in the optical delay-line circuit considered in [142] are made of three waveguides laid out in a plane, the transmission matrix structure of the resultant filter prevents one

from achieving $2\pi/3$ phase shift among the three channels of transmissions of the filter. Thus, only one channel of transmission can be designed to meet the desired response, while the transmissions of the other two channels have to be neglected.

In this chapter, a novel 3×3 lattice structure of optical delay-line circuit will be proposed to implement three-port FIR optical interleavers. A general synthesis approach is also devised to design optical interleavers using an N -order optical delay-line circuit with the proposed structure. Unlike [142], the 3×3 coupler in the proposed optical delay-line circuit has a triangular configuration seen from the cross section of the three waveguides. Thus each pair of input-output channels of the coupler has an identical cross power coupling coefficient. The proposed optical delay-line circuit consists of two parts of cascading lattice structures. Starting from the output ports, the first part of lattice structure is designed to enable a $2\pi/3$ phase shift among the three channels of transmissions of the whole filter. It is composed of two 3×3 couplers linked port-to-port by three differential delay lines. With one of the delay lines denoted as the reference, one of the other two is designed with one unit leading and the other one unit lagging with respect to the reference line, respectively. In the second part, N stages of the 3×3 couplers are cascaded port-to-port by differential delay lines. For the three differential delay lines linking any two couplers in this part of the lattice structure, one of them is chosen as the reference while the other two are designed respectively to have three-units leading and three-units lagging with respect to the reference line. It is shown that with the proposed lattice structure, if the coupling ratios of the two couplers in the first lattice part are chosen with appropriate values, the transmissions from one input port to the respective three outputs will have an identical power spectral shape but they are phase shifted by $2\pi/3$ in frequency. Thus, the design of the 3-port optical interleaver is reduced to one of designing a channel of power transfer function to meet other desired specifications. To this end, a recursive synthesis algorithm is developed in the thesis. It is shown that given a desired response, all the design parameters can be obtained recursively

by using the proposed algorithm. As illustrative examples, the maximally flat 3-port optical interleaver and an elliptic 3-port optical interleaver are designed by using the proposed lattice optical delay-line circuits and the proposed design approach. The simulation results verify the effectiveness of the proposed 3-port optical interleaver design. To further demonstrate the proposed design approach. A 100G/300G three-port all-fiber interleaver with a passband bandwidth of 40 GHz at 0.5 dB is demonstrated by experiment.

As an alternative approach to achieve an enhanced performance of the interleaver in delivering three channels of $2\pi/3$ shifted passband transmissions, an IIR architecture of three-port interleavers is also presented. The structure makes the length of ring resonators in the differential delay lines three times that of the differential delay of the 3×3 MZI, which greatly simplifies the design procedure. If only considering the structure by cascading several 3×3 IIR elements, the design problem to be solved would be a very complicated optimization problem. The filter stages will increase accordingly and this further complicates the filter structure. It is shown that by using the proposed structure, interleavers with desired specifications can be designed in two separate steps. In the first step, three channels of transmissions which are identical but $2\pi/3$ phase-shifted are obtained. In the second step, one channel of transmission is designed by choosing the parameters of ring resonators to satisfy the desired spectral specifications on passband and isolation. When compared with the three-port FIR interleaver, it is found the fabricated IIR interleaver can achieve a better spectral performance that proves the theoretical analysis.

Finally, an even more efficient IIR three-port optical interleaver is proposed. The structure makes use of three ring resonators or cavity etalons in the differential delay lines of a MZI. It is shown that, with more all-pass ring resonators or cavities, the performance of the IIR three-port optical interleaver can be improved.

4.2 Finite Impulse Response Three-port Optical Filter Banks

This section presents a novel lattice optical delay-line circuit using 3×3 directional couplers to implement three-port optical filter banks. The structure is motivated by the one presented in designing two-port FIR filter banks in the section 3.2 where the latter differential delay lines are selected as twice that of the first one so as to provide odd harmonics of the Fourier series for the design. The proposed 3×3 circuit also adopts this design strategy by selecting the latter differential delay lines three times that of the first one. This serves to provide multiple Fourier harmonics of power of 3 for the three-port interleaver design. As illustrative examples, interleavers with maximally flat-top passband transmissions and with given transmission performance on ripples and passband bandwidth, respectively, are designed to verify the effectiveness of the proposed design scheme.

4.2.1 General Issues of 3-port Optical Interleaver Design

Generally, a 3-port optical delay-line circuit consists of 3×3 directional couplers linked by optical delay lines and optical phase shifters, as shown in Fig. 4.1. The general operation principal behind optical delay-line circuit is an interferometric overlap of several beams. The design parameters of such circuit include the directional couplers, the optical delay-lines and the optical phase shifters, represented respectively by κ_i , M^{ϕ_i} and $\phi_{i,j}$ ($i = 1, 2, \dots, N$; $j = 1, 2$), as shown in Fig. 4.1.

Using an optical delay-line circuit, the design of a 3-port optical interleaver is to select appropriate design parameters such that one output port of the circuit has a filter response as shown in Fig. 4.2. Clearly, the design of a 3-port optical interleaver consists of two aspects. The first one is to guarantee that the output spectra of the

Figure 4.1: The general configuration of a 3-port delay-line circuit which consists of 3×3 directional couplers linked by optical delay lines and optical phase shifters.

three output channels have a phase shift of $\frac{2\pi}{3}$ between them in frequency. This is denoted as *phase shift* requirement. The other one is to guarantee that the filter response of one output transmission shall satisfy a desired filter performance such as wider passband bandwidth, lower insertion loss, higher channel isolation, faster roll-off and smaller passband ripples. These spectral characteristics constitute the second requirement of the interleaver, denoted as *spectral performance* requirement. More specifically, we denote $H_1(z)$, $H_2(z)$, and $H_3(z)$ as the transmissions from one input port to the three output ports, respectively, and z^{-1} as a unit time delay given by $z^{-1} = e^{-j2\Delta L\pi n/\lambda}$ where λ is the wavelength of wave propagating through free space, n is the refractive index and ΔL represents the unit length difference corresponding to a delay time difference of $\Delta\tau$. To this end, the first requirement can be described as:

$$|H_1(z)|^2 = |H_2(ze^{-j\frac{2\pi}{3}})|^2 = |H_3(ze^{-j\frac{4\pi}{3}})|^2 \quad (4.1)$$

where $|H_i(z)|^2 = H_i(z)H_i^*(z)$ ($i = 1, 2, 3$) is the power transfer function and the superscript $*$ denotes a complex conjugate. Simultaneously, the second requirement can be described such that the passbands and stopbands of the three output transmissions $|H_1(z)|^2$, $|H_2(z)|^2$ and $|H_3(z)|^2$ should be as well isolated as possible to achieve good channel isolation, and more importantly, they should be as flat as possible in order to reduce power variations resulting from channel wavelength shifts.

Figure 4.2: Typical spectral response of one channel of a desired 3-port interleaver.

From the signal processing perspective, to meet the requirements of a 3-port interleaver, the design of the 3-port interleaver is a multiple objective optimization problem that can be described as:

$$\min_{\text{design parameters}} \{\Phi_1, \Phi_2, \Phi_3\} \quad (4.2)$$

where the objectives are defined as

$$\begin{aligned}
 \Phi_1 &= \int_{\lambda \in \Omega_p, \Omega_s} \left| |f_1^*(z)|^2 - |H_1(z)|^2 \right| d\lambda \\
 \Phi_2 &= \int_{\lambda \in \Omega_p, \Omega_s} \left| |H_1(z)|^2 - |H_2(ze^{-j\frac{2\pi}{3}})|^2 \right| d\lambda \\
 \Phi_3 &= \int_{\lambda \in \Omega_p, \Omega_s} \left| |H_1(z)|^2 - |H_3(ze^{-j\frac{4\pi}{3}})|^2 \right| d\lambda
 \end{aligned} \tag{4.3}$$

where $f_1^*(z)$ denote the desired filter transfer functions from one input port to one output of the interleaver, $\Phi = \Phi_1 + \Phi_2 + \Phi_3$ measures the overall passband and stopband errors of the three outputs in L_2 norm that examines the energy, and the frequency regions Ω_p and Ω_s denote the passband and stopband, respectively. The above problem is an $5N + 1$ -dimensional constrained optimization problem. It can be solved by using commercially available optimization toolbox, such as Matlab Optimization Toolbox, however, the design of interleaver using optimization may have the following difficulties. First, optimizing these equations simultaneously is quite complex and tedious in that each of the equations requires a high computational cost of $O((5N + 1)^3/\varepsilon)$ [143] where ε represents the numerical approximation error. Second, the performance of the interleaver cannot be easily specified with desired spectral characteristics such as passband/stopband bandwidth, channel isolation, and passband ripples. Third, the optimal design solutions on the delay-line circuit structure obtained from the optimization cannot be easily implemented in practice since the design parameters such as differential arm lengths are obtained with arbitrary non-integer values that would raise difficulties in the fabrication.

To demonstrate the above analysis, we consider a design example by using eight 3×3 directional couplers in an optical delay-line lattice structure. The spectral performance of the resultant interleaver, shown in Fig. 4.3, is obtained by applying the corresponding transfer functions of the three output transmissions into (4.3). It can be clearly seen in Fig. 4.3 that the three output channels do not have the same

phase-shifted passband/stopband shape to satisfy the first *phase shift* requirement and the obtained performance of each output transmission cannot be predicted with desired specifications to meet the second *spectral performance* requirement. Further, the average computational cost of this method is about one hour by using a common personal computer. It should be noted that the computational complexity is affected by the initial design parameters, the number of design parameters, as well as the approximation errors. Thus, the general optimization design method for 3-port optical interleaver is quite complicated and inefficient. It is of great importance to take more considerations on the structural design of the lattice filter so as to simplify the design procedure, save the computational cost, improve the filter spectral performance, and ease the fabrication of the interleaver.

Figure 4.3: Power spectral responses of a three-port interleaver obtained by using the optimization method as outlined in (4.3) with 8 couplers in a lattice structure.

4.2.2 3×3 Optical-Fiber Directional Couplers

As shown in the last subsection, if the structural design of three-port interleaver using optical delay-line circuit is not considered, the general optimization design approaches cannot exactly satisfy the two design requirements. Thus, we proposed a

novel optical delay-line circuit structure to overcome this problem by cascading 3×3 triangular directional couplers linked with specially designed differential delay lines. It is shown that the transmissions from one input port to the respective three outputs can be selected to have an identical power spectral shape but with a $\frac{2\pi}{3}$ phase shift between them in frequency, if the coupling ratios of the first two couplers are chosen with appropriate values. Thus, the design problem of obtaining simultaneously three passband transmissions of the 3-port interleaver is greatly simplified by eliminating the first *phase shift* requirement as shown in Eqn.(4.1). Since only one channel of power transfer function is to be designed, the available filter design methods including those for the design of Butterworth, Chebyshev and Elliptic bandpass filters can then be employed. Thus, the computational complexity is greatly reduced and the performance of the 3-port interleaver can be pre-specified. In addition, as the differential arms are selected as integer multiples of the unit time delay line, it would be easier to tune the design parameters precisely during the fabrication process. Furthermore, the developed digital signal processing techniques, such as z -transform, can be readily adopted to facilitate the design.

First, we will investigate the 3×3 optical directional couplers. A 3×3 optical directional coupler is a passive component capable of splitting light power. It is made of three waveguides which can be either optical fibers or waveguides on dielectric substrate. Without loss of generality, fiber based couplers are used to illustrate the proposed design. Clearly, there are many possible configurations of three fibers when they are fused to form a 3×3 directional coupler. The most popular and convenient configuration may be the triangular structure as shown in Fig. 4.4(a), the cross section of which is shown in Fig. 4.4(b). By using the mode coupling theory, the power transfer matrix of such three port couplers can be obtained as given below. For detailed derivations, one can refer to [144] and [145].

$$\begin{pmatrix} E_1^O \\ E_2^O \\ E_3^O \end{pmatrix} = \begin{pmatrix} \gamma & \delta & \delta \\ \delta & \gamma & \delta \\ \delta & \delta & \gamma \end{pmatrix} \begin{pmatrix} E_1^I \\ E_2^I \\ E_3^I \end{pmatrix} = M(\kappa) \begin{pmatrix} E_1^I \\ E_2^I \\ E_3^I \end{pmatrix} \quad (4.4)$$

with

$$\gamma = (e^{j2\kappa} + 2e^{-j\kappa})/3 \quad (4.5)$$

$$\delta = (e^{j2\kappa} - e^{-j\kappa})/3 \quad (4.6)$$

where j stands for $\sqrt{-1}$, κ in the expressions of γ and δ is a multiplier of the coupling coefficient c and the length of the coupling region L , *i.e.* $\kappa = c \times L$. The coupling coefficient c between any pair of optical fibers is assumed to be equal, E_m^O and E_m^I respectively represent the input field and the output field at the m -th port for $m = 1, 2, 3$. The through- and cross-port power transmissions are denoted by $|\gamma|^2$ and $|\delta|^2$ respectively. As the three optical fibers are laid out symmetrically, the coupling coefficients between any pair of optical fibers are assumed to be equal.

Figure 4.4: 3×3 directional coupler. (a) Schematic diagram of a three port directional coupler with a triangular structure. (b) The cross section of the coupler.

4.2.3 Proposed Design Configuration and Design Problem

Fig. 4.5 shows the circuit configuration that we propose for the design of three port optical interleaving filter. The optical delay-line circuit is composed of two

parts of lattice structures denoted respectively by S_{out} and S^N . Starting from the output ports, the first part of the lattice is formed by two cascading 3×3 directional couplers linked by three differential delay lines. Each adjacent two lines have length differences of $-\Delta L$ from upper to lower. In the second part of the lattice denoted by S^N , N three-port directional couplers are cascaded by differential delay lines with adjacent path length difference of $-3\Delta L$ from upper to lower. On the differential delay lines in this part of the lattice structure, $2(N-1)$ phase shifters are introduced on the upper and lower differential delay line arms to control the precise phase differences among the three differential arms. The two parts of the lattice are connected port-to-port by three differential delay lines. As will be shown later, the differential delays between S_{out} and S^N can be selected arbitrarily. For simplicity of illustration, we assume the three delay lines have an identical length.

Figure 4.5: Proposed configuration which is composed of two parts for the design of 3×3 FIR interleaver.

If ΔL represents the unit length difference corresponding to a delay time difference of $\Delta\tau$, the transfer function of an optical delay-line with respect to a delay time $\Delta\tau$ can be written as $e^{-jw\Delta\tau}$ where w represents the relative optical frequency. With the z transform expression of the frequency characteristics, which was first developed for optic filters by Moslehi et.al. [110], the transfer function of an optical delay-line can also be written as z^{-1} . This means that optical delay-line circuits have periodic filter characteristics with a frequency period defined as $f_p = 1/\Delta\tau$. Similarly, the differential delay-lines with $3\Delta L$ shorter than the reference line can be specified by a transfer function of z^3 , while the ones with $3\Delta L$ longer than the reference line are specified by z^{-3} . For each phase shifter with a phase shift of ϕ , its transfer function can be expressed by $e^{j\phi}$.

With these denotations, we can obtain the transfer matrix for each part of the optical delay-lines circuit proposed in Fig. 4.5. For the output lattice S_{out} , its transfer matrix can be obtained as

$$S_{out} = M(\kappa_{o,2})M^{\phi_0}M(\kappa_{o,1}) \quad (4.7)$$

where $\kappa_{o,2}$ and $\kappa_{o,1}$ denote the coupling ratios of the couplers $S_{o,2}$ and $S_{o,1}$ in the first lattice structure S_{out} , and M^{ϕ_0} denotes the transfer function of the differential delay lines in S_{out} . See Fig. 4.5 for the illustration. Similarly, the transfer matrix of the second lattice S^N can be obtained by

$$S^N = \left(\prod_{i=N-1}^1 M(\kappa_{i+1})M^{\phi_i} \right) M(\kappa_1) \quad (4.8)$$

where $M(\kappa_{i+1})$ denotes the transfer function of the $(i+1)$ th coupler which is given in (4.4), and M^{ϕ_i} is the transfer matrix of the differential delay-lines with phase shifters. It can be represented by $M^{\phi_i} = \text{diag}\{z^3e^{j\phi_{i,1}}, 1, z^{-3}e^{j\phi_{i,2}}\}$.

Thus, if we choose $\kappa_{o,1} = \frac{2\pi}{9}$ and $\kappa_{o,2} = \frac{4\pi}{9}$, the transfer matrix of the output lattice

S_{out} takes the following form:

$$S_{out} = z^1 \begin{pmatrix} H_{11}^0(z) & H_{12}^0(z) & H_{13}^0(z) \\ H_{21}^0(z) & H_{22}^0(z) & H_{23}^0(z) \\ H_{31}^0(z) & H_{32}^0(z) & H_{33}^0(z) \end{pmatrix} \quad (4.9)$$

Let S_{01} , S_{02} , S_{03} represent each row of S_{out} respectively. From (4.9), it can be verified that the following equations are satisfied automatically:

$$S_{01}(z) = S_{02}(ze^{-j\frac{2\pi}{3}})e^{-j\frac{2\pi}{3}} \quad (4.10)$$

$$S_{01}(z) = S_{03}(ze^{-j\frac{4\pi}{3}}) \quad (4.11)$$

On the other hand, since all the differential delay lines in the lattice of S^N take transfer functions of the form $diag\{z^3 e^{j\phi_{i,1}}, 1, z^{-3} e^{j\phi_{i,2}}\}$, the transfer matrix of the lattice S^N given in (4.8) can be expressed into the following form:

$$S^N(z^3) = z^{3(N-1)} \begin{pmatrix} H_{11}^N(z^3) & H_{12}^N(z^3) & H_{13}^N(z^3) \\ H_{21}^N(z^3) & H_{22}^N(z^3) & H_{23}^N(z^3) \\ H_{31}^N(z^3) & H_{32}^N(z^3) & H_{33}^N(z^3) \end{pmatrix} \quad (4.12)$$

where $H_{pq}^N(z^3)$, $p, q = 1, 2, 3$, is a polynomial of z^{-3} with the highest order $z^{-2 \times 3(N-1)}$.

The attracting property of $S^N(z^3)$ is that the frequency response of $S^N(z^3)$ will not change with a shift of $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ in frequency, which can be seen directly from $S^N(z^3) = S^N\left((ze^{j\frac{2\pi}{3}})^3\right) = S^N\left((ze^{j\frac{4\pi}{3}})^3\right)$.

Now we consider the responses at the three output ports of the proposed optical delay-line circuit when to an input specified by $[1, 0, 0]^T$ is applied. In this case, only the first column of the overall transfer matrix of the proposed circuit configuration needs to be considered. Denote this column by $[\tilde{H}_{11}; \tilde{H}_{21}; \tilde{H}_{31}]^T$. Using (4.9) and

(4.12), it can be expressed by

$$\begin{aligned} \begin{pmatrix} \tilde{H}_{11}(z) \\ \tilde{H}_{21}(z) \\ \tilde{H}_{31}(z) \end{pmatrix} &= z^{-3N+2} S_{out} S^N(z^3) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} H_{11}^0(z)H_{11}^N(z^3) + H_{12}^0(z)H_{21}^N(z^3) + H_{13}^0(z)H_{31}^N(z^3) \\ H_{21}^0(z)H_{11}^N(z^3) + H_{22}^0(z)H_{21}^N(z^3) + H_{23}^0(z)H_{31}^N(z^3) \\ H_{31}^0(z)H_{11}^N(z^3) + H_{32}^0(z)H_{21}^N(z^3) + H_{33}^0(z)H_{31}^N(z^3) \end{pmatrix} \end{aligned} \quad (4.13)$$

From Eqns.(4.10),(4.11), and (4.13), it immediately follows that

$$\tilde{H}_{11}(z) = e^{-j\frac{2\pi}{3}} \tilde{H}_{21}(ze^{-j\frac{2\pi}{3}}) = \tilde{H}_{31}(ze^{-j\frac{4\pi}{3}}) \quad (4.14)$$

Therefore, we have

$$\|\tilde{H}_{11}(z)\|^2 = \|\tilde{H}_{21}(ze^{-j\frac{2\pi}{3}})\|^2 = \|\tilde{H}_{31}(ze^{-j\frac{4\pi}{3}})\|^2 \quad (4.15)$$

which means that the three channel transmissions from the port E_1^I respectively to the ports E_1^O , E_2^O , and E_3^O would have identical spectral shape and they are shifted by $2\pi/3$ in frequency. In other words, the proposed lattice structure can perform as an interleaving filter if the coupling coefficients of the two couplers in S_{out} is selected with values of $\kappa_{o,1} = \frac{2\pi}{9}$ and $\kappa_{o,2} = \frac{4\pi}{9}$.

Another immediate conclusion of the above analysis is that the design of a three-port optical interleaver with desired specifications can be expressed by designing only one channel of transfer function to meet the specifications. For instance, given a desired spectrum $\|\tilde{H}_{11}(z)\|^2$, the corresponding transfer function $\tilde{H}_{11}(z)$ can be obtained by factorizing $\|\tilde{H}_{11}(z)\|^2 = \tilde{H}_{11}(z)\tilde{H}_{11,*}(z)$, where $\tilde{H}_{11,*}(z)$ represents the para-Hermitian conjugate of $\tilde{H}_{11}(z)$, i.e. $\tilde{H}_{11,*}(z) \equiv \tilde{H}_{11}^*(1/z^*)$, and the superscript $*$ indicates the complex conjugate. Then the transfer functions $\tilde{H}_{21}(z)$ and $\tilde{H}_{31}(z)$

can be obtained as follows:

$$\begin{aligned}\tilde{H}_{21}(z) &= e^{j\frac{2\pi}{3}} \tilde{H}_{11}(ze^{j\frac{2\pi}{3}}) \\ \tilde{H}_{31}(z) &= \tilde{H}_{11}(ze^{j\frac{4\pi}{3}})\end{aligned}\quad (4.16)$$

The basic requirements of $\tilde{H}_{11}(z)$ are that the filter lengths must be with order of $6M + 3$ where M is an integer, and the power conservation property needs to be satisfied, all of which are shown in Appendix A. The detailed synthesis method for designing the three-port interleaving filter will be explored in the next subsection when given the desired specifications of the 3-port interleaver, such as the filter order, passband bandwidth, passband ripple, channel isolation, channel spacing, and so on.

From the above derivations of the filter responses, it is noted that the differential delay lines linking the lattices S_{out} and S^N may take different lengths. In this case, it is sufficient to include a diagonal transfer matrix with the form of $diag\{e^{j\phi_1}, 1, e^{j\phi_2}\}$ where ϕ_1 and ϕ_2 represent the induced phase changes. As a result, the first column of the transmission matrix in (4.13) can be expressed by

$$\begin{pmatrix} \tilde{H}_{11}(z) \\ \tilde{H}_{21}(z) \\ \tilde{H}_{31}(z) \end{pmatrix} = \begin{pmatrix} e^{j\phi_{k_1}} H_{11}^0(z) H_{11}^N(z^3) + H_{12}^0(z) H_{21}^N(z^3) + e^{j\phi_{k_2}} H_{13}^0(z) H_{31}^N(z^3) \\ e^{j\phi_{k_1}} H_{21}^0(z) H_{11}^N(z^3) + H_{22}^0(z) H_{21}^N(z^3) + e^{j\phi_{k_2}} H_{23}^0(z) H_{31}^N(z^3) \\ e^{j\phi_{k_1}} H_{31}^0(z) H_{11}^N(z^3) + H_{32}^0(z) H_{21}^N(z^3) + e^{j\phi_{k_2}} H_{33}^0(z) H_{31}^N(z^3) \end{pmatrix} \quad (4.17)$$

and it is easy to verify that Eqn.(4.15) is still satisfied. Hence, the above analysis does not depend upon whether the delay lines linking the two parts of the lattice structure S^N and S_{out} have different lengths. However, the assumption of an identical length will simplify the presentation of the synthesis algorithm developed in the following sections.

4.2.4 Synthesis Design Algorithm

As shown in the last subsection, the structure of the proposed optical delay-line lattice circuit has guaranteed that the obtained 3-port interleaving filter yields three channels of $2\pi/3$ interleaving transmissions. This meets the basic requirement of an optical interleaver. In this section, we will present a synthesis approach to design the 3-port optical interleaver so that its transmission spectrum can meet other specifications on passband ripples, passband shape, passband transient, channel isolation, and so on. This gives rise to an important issue of filter synthesis which is targeted to find all unknown parameters of the proposed delay line circuits when the desired performance of the filter is specified by a given transmission spectrum $|\tilde{H}_{11}(z)|^2$. The design parameters in the interleaving filter include the unit delay time of $\Delta\tau$, the coupling coefficients of κ_i ($i = 1 \dots N$) of N directional couplers, and the $2 \times (N - 1)$ phase shifters $\phi_{i,1}, \phi_{i,2}$ ($i = 1, \dots, N - 1$) in the S^N lattice part.

The proposed synthesis algorithm consists of two main stages as illustrated in the flow chart of Fig. 4.6. The first stage is to calculate the transfer function of $\tilde{H}_{11}(z)$ by considering the constraints as shown in Appendix A and the power conservation property, with given spectral specifications such as passband bandwidth, passband ripples, channel isolation, filter order and channel spacing of the desired interleaver. This stage consists of *step 1* and *step 2*. Using the obtained transfer function of $\tilde{H}_{11}(z)$, the purpose of the second stage is to determine proper circuit design parameters such as the coupling ratios of those fiber couplers and the phase shifters with a recursive design algorithm. This stage constitutes the rest of the steps in the proposed design algorithm.

In detail, the purpose of *step 1* is to calculate the coefficients of $\tilde{H}_{11}(z)$ from the

Figure 4.6: The flow chart illustrating the design algorithm for the design of 3×3 FIR interleaver.

desired power transfer function $|\tilde{H}_{11}(z)|^2$ which can be expressed by

$$\tilde{H}_{11}(z)\tilde{H}_{11,*}(z) = a_0 a_0^* \prod_{i=1}^{2(3N+1)} (z - \alpha_i) \left(\frac{1}{z} - \alpha_i^*\right) \quad (4.18)$$

where the subscript $*$ indicates the para-Hermitian conjugate. As all the zeros of $\tilde{H}_{11}(z)\tilde{H}_{11,*}(z)$ appear in reciprocal pairs given by $\{\alpha_i, 1/\alpha_i^*\}$ ($i = 1, \dots, 2(3N+1)$), $\tilde{H}_{11}(z)$ can be constructed by $\tilde{H}_{11}(z) = \prod_{i=1}^{2(3N+1)} (1/z - \alpha_i^*)$. The basic requirements on the transfer function of the implementable $\tilde{H}_{11}(z)$ are shown in Appendix A.

Since the optical delay-line circuits are passive filters, the maximum transmission cannot exceed 100%. Thus, in *step 2* the complex expansion coefficients of $\tilde{H}_{11}(z)$ are normalized so that

$$\begin{cases} \max_{|z|=1} |\tilde{H}_{11}(z)| \leq 1 \\ \text{s.t. } |\tilde{H}_{11}(z)|^2 + |\tilde{H}_{11}(ze^{\frac{2\pi}{3}})|^2 + |\tilde{H}_{11}(ze^{\frac{4\pi}{3}})|^2 = 1 \end{cases} \quad (4.19)$$

In *step 3*, the complex coefficients of $\tilde{H}_{11}(z)$, $\tilde{H}_{21}(z)$, and $\tilde{H}_{31}(z)$ are determined. From the analysis of subsection 4.2.3, it can be seen that given the transfer function $\tilde{H}_{11}(z)$, the transfer functions $\tilde{H}_{21}(z)$ and $\tilde{H}_{31}(z)$ can be determined by Eqn.(4.16).

Next, in *step 4* the coefficients of $H_{11}^N(z^3)$, $H_{21}^N(z^3)$ and $H_{31}^N(z^3)$ in (4.12) are calculated. Since S_{out} has been available in (4.7) with $\kappa_{o,1} = 2\pi/9$ and $\kappa_{o,2} = 4\pi/9$, it follows from (4.13) that the first column of S^N can be expressed by $[H_{11}^N(z^3) \ H_{21}^N(z^3) \ H_{31}^N(z^3)]^T = S_{out}^{-1}[\tilde{H}_{11}(z) \ \tilde{H}_{21}(z) \ \tilde{H}_{31}(z)]^T$, where S_{out}^{-1} is the reverse matrix of S_{out} . With the obtained $H_{11}^N(z^3)$, $H_{21}^N(z^3)$ and $H_{31}^N(z^3)$, the design parameters including the coupling ratios κ_{i+1} of the couplers, the phase shifters $\phi_{i,1}$ and $\phi_{i,2}$ ($i = 1, \dots, N-1$) are determined in the following steps. From (4.8), we can express $H_{11}^N(z^3)$, $H_{21}^N(z^3)$

and $H_{31}^N(z^3)$, in terms of the design parameters, by

$$z^{3(N-1)}H_{11}^N(z^3) = a_{1,2N-1}^N z^{3(N-1)} + a_{2,2N-1}^N z^{3(N-2)} + \dots + a_{2N-1,2N-1}^N z^{-3(N-1)} \quad (4.20)$$

$$z^{3(N-1)}H_{21}^N(z^3) = b_{1,2N-1}^N z^{3(N-1)} + b_{2,2N-1}^N z^{3(N-2)} + \dots + b_{2N-1,2N-1}^N z^{-3(N-1)} \quad (4.21)$$

$$z^{3(N-1)}H_{31}^N(z^3) = c_{1,2N-1}^N z^{3(N-1)} + c_{2,2N-1}^N z^{3(N-2)} + \dots + c_{2N-1,2N-1}^N z^{-3(N-1)} \quad (4.22)$$

where

$$a_{1,2N-1}^N = \prod_{i=1}^N \gamma_i e^{j\sum_{k=1}^{N-1} \phi_{k,1}} \quad (4.23)$$

$$a_{2N-1,2N-1}^N = \delta_1 \delta_N \prod_{i=2}^{N-1} \gamma_i e^{j\sum_{k=1}^{N-1} \phi_{k,2}} \quad (4.24)$$

$$b_{1,2N-1}^N = \delta_N \prod_{i=1}^{N-1} \gamma_i e^{j\sum_{k=1}^{N-1} \phi_{k,1}} \quad (4.25)$$

$$b_{2N-1,2N-1}^N = \delta_1 \delta_N \prod_{i=2}^{N-1} \gamma_i e^{j\sum_{k=1}^{N-1} \phi_{k,2}} \quad (4.26)$$

$$c_{1,2N-1}^N = \delta_N \prod_{i=1}^{N-1} \gamma_i e^{j\sum_{k=1}^{N-1} \phi_{k,1}} \quad (4.27)$$

$$c_{2N-1,2N-1}^N = \delta_1 \prod_{i=2}^N \gamma_i e^{j\sum_{k=1}^{N-1} \phi_{k,2}} \quad (4.28)$$

From Eqns.(4.23), (4.25), (4.26) and (4.28), we have

$$\frac{c_{2N-1,2N-1}^N}{b_{2N-1,2N-1}^N} = \frac{a_{1,2N-1}^N}{b_{1,2N-1}^N} = \frac{\gamma_N}{\delta_N} \quad (4.29)$$

Since both γ_N and δ_N are functions of the coupling coefficient κ_N of the N -th

directional coupler as defined in (4.5) and (4.6), the equation (4.29) actually gives an equation of κ_N . Thus, γ_N and δ_N can be obtained by solving (4.29).

It is noted from (4.23) to (4.28) that the proposed lattice structure has an inherent characteristic that

$$b_{1,2N-1}^N = c_{1,2N-1}^N \quad \text{and} \quad a_{2N-1,2N-1}^N = b_{2N-1,2N-1}^N \quad (4.30)$$

To proceed, the phase shifts $\phi_{N-1,1}$ and $\phi_{N-1,2}$ are to be determined. To this end, the first column of S^N is expressed as follows:

$$\begin{pmatrix} z^{3(N-1)} H_{11}^N(z^3) \\ z^{3(N-1)} H_{21}^N(z^3) \\ z^{3(N-1)} H_{31}^N(z^3) \end{pmatrix} = \begin{pmatrix} \gamma_N e^{j\phi_{N-1,1}} z^3 & \delta_N & \delta_N e^{j\phi_{N-1,2}} z^{-3} \\ \delta_N e^{j\phi_{N-1,1}} z^3 & \gamma_N & \delta_N e^{j\phi_{N-1,2}} z^{-3} \\ \delta_N e^{j\phi_{N-1,1}} z^3 & \delta_N & \gamma_N e^{j\phi_{N-1,2}} z^{-3} \end{pmatrix} \begin{pmatrix} z^{3(N-2)} H_{11}^{N-1}(z^3) \\ z^{3(N-2)} H_{21}^{N-1}(z^3) \\ z^{3(N-2)} H_{31}^{N-1}(z^3) \end{pmatrix} \quad (4.31)$$

Then, $H_{11}^{N-1}(z^3)$, $H_{21}^{N-1}(z^3)$ and $H_{31}^{N-1}(z^3)$ can be expressed by

$$\begin{pmatrix} z^{3(N-2)} H_{11}^{N-1}(z^3) \\ z^{3(N-2)} H_{21}^{N-1}(z^3) \\ z^{3(N-2)} H_{31}^{N-1}(z^3) \end{pmatrix} = \frac{1}{(2\delta_N^2 - \delta_N \gamma_N - \gamma_N^2)} \times \begin{pmatrix} -(\delta_N + \gamma_N) z^{-3} e^{-j\phi_{N-1,1}} & \delta_N z^{-3} e^{-j\phi_{N-1,1}} & \delta_N z^{-3} e^{-j\phi_{N-1,1}} \\ \delta_N & -(\delta_N + \gamma_N) & \delta_N \\ \delta_N z^3 e^{-j\phi_{N-1,2}} & \delta_N z^3 e^{-j\phi_{N-1,2}} & -(\delta_N + \gamma_N) z^3 e^{-j\phi_{N-1,2}} \end{pmatrix} \times \begin{pmatrix} z^{3(N-1)} H_{11}^N(z^3) \\ z^{3(N-1)} H_{21}^N(z^3) \\ z^{3(N-1)} H_{31}^N(z^3) \end{pmatrix} \quad (4.32)$$

Applying (4.20)-(4.22), it can be obtained that

$$\begin{aligned}
 z^{3(N-2)} H_{11}^{N-1}(z^3) = & \frac{e^{-j\phi_{N-1,1}}}{(2\delta_N^2 - \delta_N\gamma_N - \gamma_N^2)} \left((-\delta_N + \gamma_N)a_{1,2N-1}^N + \delta_N b_{1,2N-1}^N + \delta_N c_{1,2N-1}^N \right) z^{3(N-2)} \\
 & + \cdots + \left((-\delta_N + \gamma_N)a_{2N-3,2N-1}^N + \delta_N b_{2N-3,2N-1}^N + \delta_N c_{2N-3,2N-1}^N \right) z^{-3(N-2)} \\
 & + \left((-\delta_N + \gamma_N)a_{2N-2,2N-1}^N + \delta_N b_{2N-2,2N-1}^N + \delta_N c_{2N-2,2N-1}^N \right) z^{-3(N-1)} \\
 & + \left((-\delta_N + \gamma_N)a_{2N-1,2N-1}^N + \delta_N b_{2N-1,2N-1}^N + \delta_N c_{2N-1,2N-1}^N \right) z^{-3N} \quad (4.33)
 \end{aligned}$$

In the above expression, the coefficient of the term z^{-3N} is zero. It can be seen by substituting equations from (4.23) to (4.28). As shown in Appendix B, the coefficient of the term $z^{-3(N-1)}$ is also zero. Thus, $z^{3(N-2)} H_{11}^{N-1}(z^3)$ is a polynomial of z and z^{-1} with the highest order $3(N-2)$. Comparing the coefficients of the same order in polynomials on both sides of (4.33), the i -th ($1 \leq i \leq 2N-3$) coefficient of $H_{11}^{N-1}(z^3)$ can be obtained as

$$a_{i,2N-3}^{N-1} = \frac{e^{-j\phi_{N-1,1}}}{(2\delta_N^2 - \delta_N\gamma_N - \gamma_N^2)} \left((-\delta_N + \gamma_N)a_{i,2N-1}^N + \delta_N b_{i,2N-1}^N + \delta_N c_{i,2N-1}^N \right) \quad (4.34)$$

Similarly, we expand $z^{3(N-2)}H_{21}^{N-1}(z)$ and $z^{3(N-2)}H_{31}^{N-1}(z)$ in (4.32) as

$$\begin{aligned}
 & z^{3(N-2)}H_{21}^{N-1}(z) \\
 &= \frac{1}{2\delta_N^2 - \delta_N\gamma_N - \gamma_N^2} \left((\delta_N a_{1,2N-1}^N - (\delta_N + \gamma_N)b_{1,2N-1}^N + \delta_N c_{1,2N-1}^N) z^{3(N-1)} \right. \\
 &+ (\delta_N a_{2,2N-1}^N - (\delta_N + \gamma_N)b_{2,2N-1}^N + \delta_N c_{2,2N-1}^N) z^{3(N-2)} + \dots \\
 &+ (\delta_N a_{2N-2,2N-1}^N - (\delta_N + \gamma_N)b_{2N-2,2N-1}^N + \delta_N c_{2N-2,2N-1}^N) z^{-3(N-2)} \\
 &\left. + (\delta_N a_{2N-1,2N-1}^N - (\delta_N + \gamma_N)b_{2N-1,2N-1}^N + \delta_N c_{2N-1,2N-1}^N) z^{-3(N-1)} \right) \quad (4.35)
 \end{aligned}$$

$$\begin{aligned}
 & z^{3(N-2)}H_{31}^{N-1}(z) - (\delta_N + \gamma_N)c_{1,2N-1}^N z^{3(N-1)} \\
 &= \frac{e^{-j\phi_{N-1,2}}}{(2\delta_N^2 - \delta_N\gamma_N - \gamma_N^2)} \left((\delta_N a_{1,2N-1}^N + \delta_N b_{1,2N-1}^N - (\delta_N + \gamma_N)c_{1,2N-1}^N) z^{3N} \right. \\
 &+ (\delta_N a_{2,2N-1}^N + \delta_N b_{2,2N-1}^N - (\delta_N + \gamma_N)c_{2,2N-1}^N) z^{3(N-1)} \\
 &+ (\delta_N a_{3,2N-1}^N + \delta_N b_{3,2N-1}^N - (\delta_N + \gamma_N)c_{3,2N-1}^N) z^{3(N-2)} + \dots \\
 &\left. + (\delta_N a_{2N-1,2N-1}^N + \delta_N b_{2N-1,2N-1}^N - (\delta_N + \gamma_N)c_{2N-1,2N-1}^N) z^{-3(N-2)} \right) \quad (4.36)
 \end{aligned}$$

The first and the last items in (4.35) are zeros after substituting Eqns.(4.23)-(4.28), hence,

$$b_{i,2N-3}^{N-1} = \frac{1}{(2\delta_N^2 - \delta_N\gamma_N - \gamma_N^2)} (\delta_N a_{i+1,2N-1}^N - (\delta_N + \gamma_N)b_{i+1,2N-1}^N + \delta_N c_{i+1,2N-1}^N) \quad (4.37)$$

As shown in Appendix C, the first two items in (4.36) are zeros as well, and in turn,

$$c_{i,2N-3}^{N-1} = \frac{e^{-j\phi_{N-1,2}}}{(2\delta_N^2 - \delta_N\gamma_N - \gamma_N^2)} (\delta_N a_{i+2,2N-1}^N + \delta_N b_{i+2,2N-1}^N - (\delta_N + \gamma_N)c_{i+2,2N-1}^N) \quad (4.38)$$

Using the requirement on the coefficients of $H_{11}^{N-1}(z)$, $H_{21}^{N-1}(z)$ and $H_{31}^{N-1}(z)$ as

shown in Eqn.(4.30), and from Eqn.(4.34) and (4.37), we have

$$\frac{a_{2N-3,2N-3}^{N-1}}{b_{2N-3,2N-3}^{N-1}} = e^{-j\phi_{N-1,1}} \frac{(-(\delta_N + \gamma_N)a_{2N-3,2N-1}^N + \delta_N b_{2N-3,2N-1}^N + \delta_N c_{2N-3,2N-1}^N)}{(\delta_N a_{2N-2,2N-1}^N - (\delta_N + \gamma_N)b_{2N-2,2N-1}^N + \delta_N c_{2N-2,2N-1}^N)} = 1 \quad (4.39)$$

Therefore, the phase shift $\phi_{N-1,1}$ is obtained by

$$\phi_{N-1,1} = -\arg \frac{(\delta_N a_{2N-2,2N-1}^N - (\delta_N + \gamma_N)b_{2N-2,2N-1}^N + \delta_N c_{2N-2,2N-1}^N)}{(-(\delta_N + \gamma_N)a_{2N-3,2N-1}^N + \delta_N b_{2N-3,2N-1}^N + \delta_N c_{2N-3,2N-1}^N)} \quad (4.40)$$

Similarly, using (4.37) and (4.38), it follows that

$$\frac{c_{1,2N-3}^{N-1}}{b_{1,2N-3}^{N-1}} = e^{-j\phi_{N-1,2}} \frac{(\delta_N a_{3,2N-1}^N + \delta_N b_{3,2N-1}^N - (\delta_N + \gamma_N)c_{3,2N-1}^N)}{(\delta_N a_{2,2N-1}^N - (\delta_N + \gamma_N)b_{2,2N-1}^N + \delta_N c_{2,2N-1}^N)} = 1 \quad (4.41)$$

which gives the phase shift $\phi_{N-1,2}$ as

$$\phi_{N-1,2} = -\arg \frac{(\delta_N a_{2,2N-1}^N - (\delta_N + \gamma_N)b_{2,2N-1}^N + \delta_N c_{2,2N-1}^N)}{(\delta_N a_{3,2N-1}^N + \delta_N b_{3,2N-1}^N - (\delta_N + \gamma_N)c_{3,2N-1}^N)} \quad (4.42)$$

So far, the parameters of the N -th coupler and the $(N-1)$ -th phase shifts in S^N have been obtained from the desired transmission spectrum $\|H_{11}(z)\|^2$. Denote the transfer matrix from the first coupler to the $(N-1)$ -th coupler as S^{N-1} in the S^N lattice part. It has the same structure as S^N . Hence, the parameters of the $(N-1)$ -th coupler and the $(N-2)$ -th phase shift can also be determined by following the same procedures as in (4.20) to (4.42). Repeating the procedure, all the design

parameters in the lattice S^N can be obtained via the following recursive formulae:

$$\begin{cases} \frac{a_{1,2n-1}^n}{b_{1,2n-1}^n} = \frac{\gamma_n}{\delta_n} \Rightarrow \gamma_n \text{ and } \delta_n \\ \phi_{n-1,1} = -\arg \frac{(\delta_n a_{2n-2,2n-1}^n - (\delta_n + \gamma_n) b_{2n-2,2n-1}^n + \delta_n c_{2n-2,2n-1}^n)}{-(\delta_n + \gamma_n) a_{2n-3,2n-1}^n + \delta_n b_{2n-3,2n-1}^n + \delta_n c_{2n-3,2n-1}^n} \\ \phi_{n-1,2} = -\arg \frac{(\delta_n a_{2,2n-1}^n - (\delta_n + \gamma_n) b_{2,2n-1}^n + \delta_n c_{2,2n-1}^n)}{(\delta_n a_{3,2n-1}^n + \delta_n b_{3,2n-1}^n - (\delta_n + \gamma_n) c_{3,2n-1}^n)} \end{cases} \quad (4.43)$$

$$\begin{cases} a_{i,2n-3}^{n-1} = \frac{e^{-j\phi_{n-1,1}}}{(2\delta_n^2 - \delta_n \gamma_n - \gamma_n^2)} \left(-(\delta_n + \gamma_n) a_{i,2n-1}^n + \delta_n b_{i,2n-1}^n + \delta_n c_{i,2n-1}^n \right) \\ b_{i,2n-3}^{n-1} = \frac{1}{(2\delta_n^2 - \delta_n \gamma_n - \gamma_n^2)} \left(\delta_n a_{i+1,2n-1}^n - (\delta_n + \gamma_n) b_{i+1,2n-1}^n + \delta_n c_{i+1,2n-1}^n \right) \\ c_{i,2n-3}^{n-1} = \frac{e^{-j\phi_{n-1,2}}}{(2\delta_n^2 - \delta_n \gamma_n - \gamma_n^2)} \left(\delta_n a_{i+2,2n-1}^n + \delta_n b_{i+2,2n-1}^n - (\delta_n + \gamma_n) c_{i+2,2n-1}^n \right) \end{cases} \quad (4.44)$$

where $a_{i,2n-3}^{n-1}$, $b_{i,2n-3}^{n-1}$ and $c_{i,2n-3}^{n-1}$ represent the i -th ($i = 1, \dots, 2n-3$) expansion coefficients of $H_{11}^{n-1}(z)$, $H_{21}^{n-1}(z)$ and $H_{31}^{n-1}(z)$, respectively. The initial value of n is N .

Summarizing all the above procedures, we obtain a recursive synthesis algorithm for the design of 3-port optical interleaver with given specifications. The algorithm is illustrated in a flow chart in Fig. 4.6.

4.2.5 Simulation Studies

To demonstrate the effectiveness of the proposed design scheme, simulation studies are carried out in this subsection by considering different design specifications and the effect of the design parameters on the performance of the interleaver. Firstly, the design of a maximally flat passband 3-port interleaver is demonstrated. Secondly, a lower order elliptic 3-port interleaver is designed to obtain equi-ripple passband transmission to further widen the passband bandwidth and enhance the passband isolation. In this part, the effect of the order of the optical delay-line circuit is also investigated. This demonstrates that, when higher order optical delay-line circuit is used, the obtained interleaver is able to have a better performance than the

designed interleaver. All together, these two examples show that with the proposed design method the interleaver can be designed to meet the pre-specified spectral performance. Thirdly, a higher order elliptic 3-port interleaver is demonstrated to show the proposed scheme can be used to design the interleaver with a better spectral performance in terms of lower passband ripple, higher channel isolation and wider passband bandwidth than the existing interleaver while the computational cost is much lower than that using the general optimization method shown in (4.2) and (4.3). Lastly, the sensitivity of the design parameters on the performance of the 3-port interleaver is investigated to verify that the proposed structure can be practically realized in that it is tolerant to the design parameter deviations within certain tolerant ranges.

4.2.5.1 Design of an optical interleaver with a maximally flat-top passband

In applications where the power variation caused by the channel wavelength drifting is of concern, it is required that each passband transmission spectra of the interleaver shall have a square-like top. Thus, the maximally flat (Daubechies) [48, 52] optical interleaver is expected. In this example, we develop the design of a higher order optical interleaver with maximally flat-top passband. For each channel, the spectrum design can be described as one of designing the maximally flat filter in the area of signal processing [146] where some digital filters have already been developed. However, the digital filter design approaches cannot be adopted directly since there are constraints, as presented in Appendix A, on the design of 3-port interleaver.

In this example, the available maximally flat filter design principle in [146] is used to design the optical interleavers with the lattice structure proposed in this thesis. The filter length is chosen as $L = 9$, without loss of generality. In this case, the

power transmission characteristics of the desired response $\tilde{H}_{11}(z)$ is described as $|\tilde{H}_{11}(z)|^2 = \sum_{k=1}^9 A_k(z^{-(k-1)} + z^{(k-1)})/2$, where A_k are the power transmission coefficients. First, only the $\frac{2\pi}{3}$ phase shift in frequency condition (4.16) and the power conservation condition (4.19) have to be considered when designing the optical third-band low pass filters. Using the expression of $|\tilde{H}_{11}(z)|^2$, it can be easily verified that when $\kappa_2 = \frac{2\pi}{9}$ and $\kappa_3 = \frac{4\pi}{9}$, the first to third order derivatives of $|\tilde{H}_{11}(z)|^2$ cannot be zeros simultaneously except for a trivial case where all three channel transmissions are truly flat without any isolation between the channels. Based on this observation, the passband/stopband transmission of $|\tilde{H}_{11}(z)|^2$ is at most of second order flatness in the maximally flat sense. Thus, to have a maximally flat top, it suffices to satisfy $\frac{\partial\{|\tilde{H}_{11}(z)|^2\}}{\partial\lambda} \Big|_{\lambda=\lambda_c^*} = 0$, and $\frac{\partial^2\{|\tilde{H}_{11}(z)|^2\}}{\partial^2\lambda} \Big|_{\lambda=\lambda_c^*} = 0$, where λ_c^* is a small deviation to the channel central wavelength. The λ_c^* is added as a design parameter due to the consideration that the transmission spectrum of the interleaver could be asymmetric with respect to the points $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. On the other hand, in order to ensure that the insertion loss is as small as possible, the value of $|\tilde{H}_{11}(z)|^2$ at λ_c^* should be maximized. Thus, the maximally flat three-port interleaver design problem can be formulated as:

$$\max \left(|\tilde{H}_{11}(z)|_{\lambda=\lambda_c^*}^2 \right) \quad (4.45)$$

subject to :

$$\frac{\partial |\tilde{H}_{11}(z)|^2}{\partial \lambda} \Big|_{\lambda=\lambda_c^*} = 0 \quad (4.46)$$

$$\frac{\partial^2 |\tilde{H}_{11}(z)|^2}{\partial^2 \lambda} \Big|_{\lambda=\lambda_c^*} = 0 \quad (4.47)$$

This is a constrained optimization problem that can be solved by using commercially available toolbox, *e.g.* Matlab Optimization Toolbox. The constraints (4.46) and (4.47) are simple equations that can be easily solved. The computational cost is mainly determined by optimizing the target function (4.45) that involves only

the transfer function of $\tilde{H}_{11}(z)$. Thus, this greatly reduces the computational cost compared with the general optimization method (4.3). The power expansion coefficients of A_k ($k = 1, \dots, 9$) simultaneously satisfying both (4.19) and constraints in Appendix A can then be obtained, which are summarized in the second column in Table 4.1. The circuit parameters are thus obtained and shown in the last column of Table 4.1. Fig. 4.7 shows the power spectra of the three channels of the designed filter, where the frequency period is taken as 100-GHz which corresponds to a 0.8-nm channel spacing. It should be noted that the resultant maximally flat interleaver does not exactly satisfy the common maximally-flat properties as the maximal pass-band transmission does not reach 1 so that the zeros of the corresponding transfer function was not placed on the unit circle and the resultant insertion loss is about 0.01-dB in the passband. Besides, the coefficients associated with powers of 3 in $|\tilde{H}_{11}(z)|^2$ are very small and are rounded to zeros to satisfy the third band filter requirement [147].

This example demonstrates that the proposed structure and design scheme is effective for designing flat-top 3-port optical interleavers. With the proposed design scheme, the interleaver can be designed with desired specifications and the performance can be controlled in contrast to the general optimization method where the performance of the interleaver can only be obtained with try-and-error. In addition, the computational cost is reduced by removing the first *phase shift* requirement. To design the 3-port optical interleaver meeting different design specifications, it would be highly desirable to have a certain tolerance on the passband ripples of the interleaver which will be discussed in the next examples.

Figure 4.7: Power spectral responses of a three-port FIR interleaver with maximally flat passband designed with the proposed scheme.

Table 4.1: Power Expansion Coefficients, Amplitude Expansion Coefficients, and Circuit Parameters for FIR Maximally Flat Three-Port Interleaver.

k	power expansion coefficients A_k	amplitude expansion coefficients of $\tilde{H}_{11}(z^{-(k-1)})$	circuit parameters
1	0.333333	$-0.181836 + 0.277719i$	$\kappa_{01} = \frac{2\pi}{9}$
2	0.450163	$-0.181837 + 0.277720i$	$\kappa_{02} = \frac{4\pi}{9}$
3	0.232874	$-0.181836 + 0.277719i$	$\kappa_1 = 0.0644$
4	0	$-0.000690 + 0.021422i$	$\kappa_2 = 6.2813$
5	0.000976	$-0.018207 - 0.011309i$	$\phi_{1,1} = -2.0382$
6	-0.000971	$0.018897 - 0.010114i$	$\phi_{1,2} = 1.0872$
7	0	$-0.019287 + 0.009350i$	$\phi_{0,1} = 0$
8	-0.012207	$0.017740 + 0.012028i$	$\phi_{0,2} = 0$
9	-0.012436	$0.001546 - 0.021377i$	$\lambda_c^* = 0.02$

4.2.6 Design of an optical interleaver with specified passband ripples

In some applications, one needs to further widen the passband bandwidth with certain tolerance on passband flatness [48, 52]. In this case, one hopes to design the interleaver with given specifications on passband ripples, passband bandwidth, and channel isolation. Elliptic filters have the advantages that one can easily specify the performance parameters such as transmission passband ripples, passband shape, and channel isolation for the designed filters [51]. The design methods for digital third-band elliptic (equiripple) FIR filters have been reported, one of which is based on the Remez exchange algorithm [48], to design elliptic filters. In our example, a filter with a filter length of $L = 9$ is first chosen to design the interleaver with passband bandwidth 0.22-nm to 0.8-nm channel spacing and the passband ripple is specified as 0.02-dB. When using the Remez exchange algorithm, and by considering the constraints in (4.19) and Appendix A, one can easily obtain the power expansion coefficients of the third-band interleaver that are shown in Table 4.2. The amplitude expansion coefficients and the circuit parameters obtained by the proposed synthesis design method are also listed respectively in the last two columns. The three channel transmissions of the designed interleaver is depicted in Fig. 4.8.

To further improve the performance of the interleaver, we conduct a simulation study to examine the performance of the interleaver with respect to the filter orders. Fig. 4.9 shows the performance of channel isolation with respect to the passband bandwidth factor of 3-port interleavers when increasing the the number of directional couplers N from 3 to 8. Here, the ripple is defined as the amplitude difference within the passband (or stopband); the passband (or stopband) bandwidth is defined as the range between the two points where a line tangential to the ripple bottom (or the ripple peak for stopband) cuts the boundary of the spectrum curve [50] clearly illustrated in Fig. 4.2; and the bandwidth factor is defined as the ratio of

Figure 4.8: Power spectral responses of a three-port FIR interleaver with filter length $L = 9$ by using Remez exchange Method.

Table 4.2: Power Expansion Coefficients, Amplitude Expansion Coefficients, and Circuit Parameters for FIR Elliptic Three-Port Interleaver.

k	power expansion coefficients A_k	amplitude expansion coefficients of $\tilde{H}_{11}(z^{-(k-1)})$	circuit parameters
1	0.333333	$0.160742 - 0.289039i$	$\kappa_{01} = \frac{2\pi}{9}$
2	0.448903	$0.160743 - 0.289040i$	$\kappa_{02} = \frac{4\pi}{9}$
3	0.235875	$0.160742 - 0.289039i$	$\kappa_1 = 6.1947$
4	0	$-0.001300 - 0.029376i$	$\kappa_2 = 6.2832$
5	0.002300	$0.026090 + 0.013562i$	$\phi_{1,1} = 1.0309$
6	-0.002303	$-0.024790 + 0.015814i$	$\phi_{1,2} = 1.0447$
7	0	$0.024751 - 0.015875i$	$\phi_{0,1} = 0$
8	-0.017134	$-0.026124 - 0.013497i$	$\phi_{0,2} = 0$
9	-0.016538	$0.001373 + 0.029373i$	-

passband bandwidth to the channel spacing of the interleaver. It can be seen that the channel isolation drops as the passband bandwidth factor increases for the same filter length. Simultaneously, the interleaver with higher filter length has a better spectral performance, *i.e.* a higher channel distinction within the same passband width. Fig. 4.10 shows the passband ripple performance of the interleavers with respect to the passband bandwidth factor as the number of couplers N increases from 3 to 8. It is shown that the passband ripple increases as the passband bandwidth increases for the same filter length, while, interleavers with higher filter lengths have a better passband ripple performance.

Figure 4.9: Channel isolation performance of the designed three-port FIR interleaver with the number of couplers N increases from 3 to 8.

4.2.6.1 Design of an elliptic interleaver with higher order filter length

To show a better spectral performance of the interleaver, we plot a 3-port interleaver with the number of couplers $N = 8$, corresponding to a filter with a filter length $L = 45$. The passband bandwidth of the interleaver is selected as 85% of the channel spacing. Thus, the power expansion coefficients of A_k ($k = 1, \dots, 45$) simul-

Figure 4.10: Passband ripple performance of the designed three-port FIR interleaver with the number of couplers N increases from 3 to 8.

taneously satisfying both (4.19) and constraints in Appendix A can be obtained by using conventional Remez exchange method in [48] which are summarized in Table 4.3. Here the obtained amplitude expansion coefficients and the circuit parameters are also listed respectively in the columns. The computational cost of the proposed design scheme is mainly determined by the derivation of the amplitude expansion coefficients. The average computational cost is about several minutes using a common personal computer compared to one hour of the general optimization method when designing 3-port optical interleaver with similar filter length as discussed in subsection 4.2.1. The derivation of the circuit parameters obtained from the amplitude expansion coefficients using the proposed synthesis algorithm is very fast and the computational cost of which can be neglected. Thus, the proposed structure and design scheme provide a more efficient and faster design approach for the design of 3-port optical interleaver. The spectral characteristics of the interleaver are shown in Fig. 4.11 where the passband bandwidth is as wide as 85% of the channel spacing, channel isolation is as high as 23-dB, and a very fast roll-off with an FOM

of 63.3% in contrast to 15.9% of the interleaving filter with filter length $L = 9$. More importantly, there is no side-lobe between the passband and stopband in one channel of transmission in contrast to the previous examples where big side-lobes exist in the transition band which would degrade the performance of the interleaver. It is noted that the stopband ripples are not exactly equivalent to each other which is caused by the round-off errors of the finite length of the filter coefficients.

This example demonstrates that the proposed design scheme is able to produce a 3-port optical interleaver with a better spectral performance, and the computational cost through using the synthesis algorithm is greatly reduced as compared with the general optimization approaches. It demonstrates that the proposed structure and design algorithm provide a more efficient design scheme.

Figure 4.11: Power spectral responses of a higher order three-port FIR interleaver with filter length $L = 45$ by using Remez exchange Method.

Table 4.3: Power Expansion Coefficients, Amplitude Expansion Coefficients, and Circuit Parameters for Higher Order FIR Elliptic Three-Port Interleaver.

k	power expansion coefficients A_k	amplitude coefficients of $\hat{H}_{11}(z^{-(k-1)})$	k	power expansion coefficients A_k	amplitude coefficients of $\hat{H}_{11}(z^{-(k-1)})$	circuit parameters
1	0.333333	$0.001553 + 0.012234i$	25	0.000000	$0.002247 + 0.025284i$	$\kappa_{01} = \frac{2\pi}{9}$
2	0.541212	$-0.012064 - 0.002561i$	26	0.000960	$0.001365 + 0.004534i$	$\kappa_{02} = \frac{4\pi}{9}$
3	0.268473	$-0.012064 - 0.002561i$	27	0.000923	$-0.011976 - 0.038613i$	$\kappa_1 = 0.0469$
4	0.000003	$-0.006226 - 0.013753i$	28	0.000000	$0.015366 - 0.003900i$	$\kappa_2 = 0.0968$
5	-0.127454	$-0.006466 - 0.012255i$	29	0.000025	$-0.001803 + 0.005800i$	$\kappa_3 = 1.8507$
6	-0.098055	$-0.007271 - 0.016249i$	30	0.000077	$0.000220 + 0.010064i$	$\kappa_4 = 0.9153$
7	0.000003	$0.009872 - 0.004135i$	31	0.000000	$-0.004896 + 0.016293i$	$\kappa_5 = 3.4105$
8	0.063567	$0.003977 + 0.044006i$	32	-0.000756	$-0.009068 - 0.009249i$	$\kappa_6 = 1.2380$
9	0.052022	$0.018990 + 0.066434i$	33	-0.000205	$0.007886 - 0.008234i$	$\kappa_7 = 1.3084$
10	0.000000	$0.032767 + 0.111595i$	34	0.000000	$0.004280 + 0.001225i$	$\kappa_8 = 6.2819$
11	-0.035794	$0.049103 + 0.089402i$	35	-0.000142	$-0.002652 - 0.000157i$	$\phi_{1,1} = 2.7434$
12	-0.028869	$0.032733 - 0.029356i$	36	0.000081	$-0.001298 + 0.001415i$	$\phi_{1,2} = 3.0497$
13	0.000000	$0.024084 - 0.201842i$	37	0.000000	$-0.000560 + 0.004306i$	$\phi_{2,1} = 0.5561$
14	0.020518	$0.026280 - 0.312778i$	38	0.000137	$-0.005685 - 0.001631i$	$\phi_{2,2} = -1.4971$
15	0.017546	$0.029688 - 0.298251i$	39	-0.000043	$0.005142 - 0.005122i$	$\phi_{3,1} = 0.3708$
16	0.000000	$0.045250 - 0.193201i$	40	0.000000	$0.001232 + 0.001199i$	$\phi_{3,2} = -1.2035$
17	-0.010923	$0.045248 - 0.045198i$	41	0.000052	$-0.001827 + 0.002174i$	$\phi_{4,1} = 2.7934$
18	-0.008545	$0.055363 + 0.040733i$	42	-0.000043	$-0.001156 - 0.001668i$	$\phi_{4,2} = -0.9699$
19	0.000000	$0.041911 + 0.013501i$	43	0.000000	$-0.000040 + 0.000069i$	$\phi_{5,1} = -0.2542$
20	0.004326	$0.040773 - 0.047804i$	44	-0.000003	$0.000497 + 0.000298i$	$\phi_{5,2} = -1.6073$
21	0.002726	$0.008844 - 0.086097i$	45	0.000003	$-0.000542 + 0.000204i$	$\phi_{6,1} = 1.8293$
22	0.000000	$0.016958 - 0.040905i$	-	-	-	$\phi_{6,2} = 0.7157$
23	-0.002472	$0.016022 + 0.012566i$	-	-	-	$\phi_{7,1} = 3.1303$
24	0.000054	$0.010053 + 0.048037i$	-	-	-	$\phi_{7,2} = 0.0455$
-	-	-	-	-	-	$\phi_{0,1} = 2.4296$
-	-	-	-	-	-	$\phi_{0,2} = -1.9473$

4.2.7 Sensitivity Study of The Proposed 3-port Optical Filter Bank

In practice, the design parameters are subject to deviations during fabrication and the fiber coupling may suffer from certain loss [148]. It is therefore necessary to conduct simulation studies to investigate the sensitivity of the interleaver performance with respect to the design parameters. In the simulation, $\pm 0.5\%$ and $\pm 2.5\%$ tolerant deviations on the design parameters, *e.g.* κ_i , $\phi_{i,1}$ and $\phi_{i,2}$ (i represents the i -th directional coupler in S^N), are respectively considered for the previous example

on the elliptic interleaver with a filter length $L = 45$. Figs.4.12 and 4.13 shows the spectral responses of the interleaver with the design parameters varying randomly within the $\pm 0.5\%$ and $\pm 2.5\%$ tolerant ranges, respectively. The black line represents the spectrum of the nominal interleaver and the many gray lines indicate the interleaver characteristic deviations. The results clearly show that the performance of the proposed FIR interleaver almost has very little variation with $\pm 0.5\%$ deviations of the design parameters. Besides, even though the attenuation and bandwidth characteristics of the interleaver do vary with $\pm 2.5\%$ random deviation ranges, it is still sufficiently practical if the designed nominal interleaver has a better spectral performance such as having a higher channel isolation.

Figure 4.12: Sensitivity study on the spectra of the designed FIR interleaver with $\pm 0.5\%$ variations of the design parameters.

4.2.8 Experimental Results

A 100G/300G, or equivalently, a 0.8 nm channel spacing, three-port optical interleaver with a passband bandwidth of 40 GHz at 0.5 dB is successfully demonstrated by experiment. The interleaver is fabricated by fusing three pieces of single mode

Figure 4.13: Sensitivity study on the spectra of the designed FIR interleaver with $\pm 2.5\%$ variations of the design parameters.

fibers (SMF). The fabrication method is similar to the one proposed in the subsection 3.2.4 where the fiber interleaver is fabricated without any splicing. First, the 3×3 fiber coupler with $\kappa_1 = 0.0644$ is fabricated by monitoring the coupling ratio between the outputs E_1^O , E_2^O and E_3^O when a laser beam with a central wavelength at 1550 nm is sent from the port E_1^I . In the course of fusion, the power coupling ratio changes periodically when stretching the fibers. The fabrication of the first coupler is complete when we observe the power coupling ratio $|\gamma|^2 : |\delta|^2 : |\delta|^2$ are 99 : 0.5 : 0.5 in the beginning of the first cycle of the power plot. The desired coupler with $\kappa_1 = 0.0644$ is then obtained. By the similar procedures, the second and the third couplers with $\kappa_2 = 2\pi/9$ and $\kappa_3 = 4\pi/9$ are fabricated. The fabrication of the second coupler is stopped when the power coupling ratio of the three output channels reaches 1 : 1 : 1 in the first cycle, and for the third coupler, the fabrication is stopped when the power coupling ratio of 1 : 1 : 1 is observed in the second cycle.

In the fusion, the fibers linking the couplers are initially obtained with roughly the estimated lengths to meet the differential delay requirement. After the fabrication of the couplers, all the three fibers on one arm are then stretched to achieve the precise

length difference by monitoring the interference spectra with an OSA (*i.e.* 0.8 nm channel spacing corresponding to a length difference of 2 mm given a refractive index $n=1.476$). The same process is repeated for the other arm. After obtaining all the desired differential delays, the phase shifts can be obtained by using heaters on the sections of the 3θ and -3θ differential arms until the desired interleaving pattern is observed.

To evaluate the performance of the obtained fiber interleaver, the three output spectra in response to the broadband light sent at the input port E_1^I are observed using an OSA. The obtained responses are shown in Fig. 4.14. It is clear that the designed interleaver has a good flat-top passband of about 40 GHz at 0.5 dB, a channel spacing of around 100 GHz, and the passband isolation is about 19 dB. The insertion loss of the proposed component is estimated to be about 3 dB. One may reduce this value by using high resolution fiber coupler workstation. This result demonstrates that the proposed configuration and design approach is effective for designing all-fiber flat-top optical interleavers. Waveguide technology also has its advantages for making 3-port interleavers because of its maturity, dense integration, mass producibility, and higher precision control when fabricating higher order lattice structure filter.

Figure 4.14: Measured three output spectral responses of the designed FIR three-port interleaver on an optical spectrum analyzer.

4.3 Infinite Impulse Response Three-port Optical Filter Banks

4.3.1 Three-port Optical Filter Banks with IIR Architectures

As an alternative to the one demonstrated in the last section, an IIR design scheme for designing a three-port optical interleaver is presented in this subsection. The interleaver can be implemented either in a ring structure or a cavity structure. With the proposed IIR scheme, the design of an IIR interleaver is greatly simplified and it can achieve simultaneously three channels of $2\pi/3$ shifted passband transmissions. Simulation and experimental results are given to demonstrate the effectiveness of the proposed IIR interleaver design scheme.

4.3.1.1 Proposed Structure and Design Approach

The proposed three-channel IIR type interleaver is formed by embedding two ring resonators in a 3×3 MZI, as shown in Fig. 4.15. The MZI consists of two 3×3 directional couplers linked port-to-port by three differential delay paths. The three delays are chosen as ΔL , 0 and $-\Delta L$, respectively. The length difference ΔL determines the channel spacing of the interleaver. In the paths with two differential delays of ΔL and $-\Delta L$ in the MZI, all-pass ring resonators with a path length of $3\Delta L$ are embedded, respectively. Each resonator couples into the differential delay path with a coupling ratio of a_i . In each of the resonators, a phase shift $\phi_{1,i}$, $i = 1, 2$, is introduced.

Figure 4.15: Proposed configuration of the IIR three-port interleaver with two ring resonators.

With this configuration, the three channels of transmissions can be obtained from one input port to three output ports. Without loss of generality, the transmissions from the input port E_1^I to the three output ports E_1^O , E_2^O , and E_3^O are considered. To obtain three channels of identical but $2\pi/3$ phase-shifted transmissions satisfying

desired specifications, the interleaver is designed by choosing appropriate parameters including the coupling ratios κ_i , $i = 1, 2$, of the two 3×3 fiber couplers, the phase shifts $\phi_{1,i}$, $i = 1, 2$, in ring resonators, and the ring-to-path coupling ratios a_i , $i = 1, 2$. To this end, the three channels of transmissions are expressed as functions of the design parameters.

Next, we derive the transfer function of the differential lines with ring resonators. Denote z^{-1} as a unit time delay given by $z^{-1} = e^{-j\Delta L\pi n/\lambda}$ where λ is the wavelength of wave propagating through free space and n is the refractive index. For convenience of derivation, we represent the transmissions of two ring resonators by $S_1(z)$ and $S_2(z)$, respectively. The lossless ring resonator with a single coupler and a phase shifter is an all-pass filter. Its transfer function can be written as [2, 136]:

$$S_i(z^3) = e^{j\phi_{1,i}} \frac{t_i e^{-j\phi_{1,i}} - z^{-3}}{1 - t_i e^{j\phi_{1,i}} z^{-3}} \quad (4.48)$$

where $t_i = \sqrt{1 - a_i}$ and a_i is the bar-state coupling ratio of the single coupler on the i -th ($i = 1, 2$) ring resonator.

Considering the differential delay caused by delay lines themselves, the transmission of the differential delay lines with ring resonators is obtained as

$$D(z) = \begin{pmatrix} z^1 S_1(z^3) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & z^{-1} S_2(z^3) \end{pmatrix} \quad (4.49)$$

Therefore, the normalized electric fields transfer functions from the three inputs to the three outputs can be expressed by the following transfer matrix:

$$\begin{pmatrix} E_1^O \\ E_2^O \\ E_3^O \end{pmatrix} = M(\kappa_2) D(z) M(\kappa_1) \begin{pmatrix} E_1^I \\ E_2^I \\ E_3^I \end{pmatrix} \quad (4.50)$$

Denote $H_1(z)$, $H_2(z)$, and $H_3(z)$ as the transmissions from the input port E_1^I to the three output ports E_1^O , E_2^O , and E_3^O , respectively. They can be expressed as the following equations:

$$H_1(z) = \gamma_1\gamma_2z^1S_1(z^3) + \delta_1\delta_2 + \delta_1\delta_2z^{-1}S_2(z^3) \quad (4.51)$$

$$H_2(z) = \gamma_1\delta_2z^1S_1(z^3) + \delta_1\gamma_2 + \delta_1\delta_2z^{-1}S_2(z^3) \quad (4.52)$$

$$H_3(z) = \gamma_1\delta_2z^1S_1(z^3) + \delta_1\delta_2 + \delta_1\gamma_2z^{-1}S_2(z^3) \quad (4.53)$$

Then, the 3×3 optical interleaver design problem can be described as one of determining appropriate design parameters such that the power spectra of $|H_1|^2$, $|H_2|^2$, and $|H_3|^2$ have a shift of $\frac{2\pi}{3}$ in frequency, but with identical spectral shape, where $|H_i|^2 = H_iH_i^*$ ($i = 1, 2, 3$) is the power transfer function in the proposed configuration. In addition, it is highly desired that the passbands and stopbands of $|H_1|^2$, $|H_2|^2$, and $|H_3|^2$ are as much isolated as possible to achieve good channel isolation, and more importantly, they should be as flat as possible in order to reduce power variations resulting from channel wavelength shifts.

To meet the first requirement to have identical passband/stopband spectral shape for all the three channels as discussed in the subsection 4.2.1, the desired design parameters shall satisfy the following equations

$$|H_1(z)|^2 = |H_2(ze^{-\frac{2\pi}{3}})|^2 = |H_3(ze^{-\frac{4\pi}{3}})|^2 \quad (4.54)$$

Clearly, there is a great deal of difficulty in solving these equations due to the complicated expressions of $S_i(z)$ in (4.48) and $H_i(z)$ in (4.51)-(4.53). However, with the proposed structures of $S_i(z^3)$, we can solve (4.54) in an efficient way. It is seen from Fig. 4.15 that the path length of all-pass ring resonators are chosen as three times that of the unit differential length in the proposed interleaver design structure. Such a structure implies that the frequency response of the ring resonators $S_i(z^3)$

($i = 1, 2$) will not change with a shift of $\frac{2\pi}{3}$ or $\frac{4\pi}{3}$ in frequency. This can be seen directly from

$$S_i(z^3) = S_i\left((ze^{-j\frac{2\pi}{3}})^3\right) = S_i\left((ze^{-j\frac{4\pi}{3}})^3\right) \quad \text{for } i = 1, 2 \quad (4.55)$$

Using this property, the first requirement of an interleaver can be easily fulfilled by choosing appropriate values for the design parameters κ_i ($i = 1, 2$). To see this, we substitute Eqns.(4.51)-(4.53) into Eqn.(4.54), respectively, and then compare the coefficients of items consisting of $S_i(z)$. As a result, $\kappa_1 = \frac{2\pi}{9}$ and $\kappa_2 = \frac{4\pi}{9}$ can be obtained, and regardless of $S_i(z)$, we have

$$H_1(z)|_{\kappa_1=\frac{2\pi}{9}, \kappa_2=\frac{4\pi}{9}} = e^{-j\frac{2\pi}{3}} H_2(ze^{-j\frac{2\pi}{3}})|_{\kappa_1=\frac{2\pi}{9}, \kappa_2=\frac{4\pi}{9}} = H_3(ze^{-j\frac{4\pi}{3}})|_{\kappa_1=\frac{2\pi}{9}, \kappa_2=\frac{4\pi}{9}} \quad (4.56)$$

An immediate conclusion from this equation is that $|H_1|^2$, $|H_2|^2$, and $|H_3|^2$ have identical shapes with only a phase shift of $2\pi/3$ in frequency between each of them. Therefore, the problem of the three-port optical IIR interleaver design is converted into one of designing $S_i(z)$ such that any one of the channel transfer functions, *e.g.* $|H_1|^2$, satisfies the desired spectrum response. The filter design methods including those for the design of Butterworth, Chebyshev and Elliptic bandpass filters can be employed to determine the remaining design parameters such that the interleaving transmissions have desired spectral responses such as high channel isolation, flat passband, and wide bandwidth.

The proposed 3-port channel interleaver can also be implemented in more efficient way with a cavity structure. In the cavity structure, the ring resonators are substituted with reflectors, and the amplitude reflectance ρ_n replaces the coupler transmittance, t_n . Similarly, in order to avoid the problem of solving a set of complicated equations, the last one of which must have an amplitude reflectance of unity so as to obtain an all-pass transfer function. Thin films or Bragg gratings can be

used to realize the partial reflectors, but the index difference must be large enough to satisfy the wavelength independence constraint [2].

4.3.1.2 Numerical Performance of IIR Three-port Interleaver

As a design example, the proposed IIR structure of the optical interleaver is designed to deliver three channels of transmissions which are interleaved by $2\pi/3$ with each other, and each passband bandwidth is chosen as 65% of the channel spacing at -0.5 dB. Following the design scheme developed in the last section, the coupler ratios of the two 3×3 couplers are readily obtained as $\kappa_1 = 2\pi/9$ and $\kappa_2 = 4\pi/9$, respectively, so that the three channels of transmissions have identical spectral shapes but phase shifted by $2\pi/3$. Then, one of the transmissions, *e.g.*, $H_1(z)$, is designed to satisfy the passband requirement. It is known from Eqn.(4.51) that $H_1(z)$ is a sixth order IIR filter. Using the least-squares (LS) method [51], the desired transfer function of $H_1(z)$ can be obtained, and its numerator and denominator power expansion coefficients are listed in Table 4.4.

After obtaining the power expansion coefficients of $H_1(z)$, we determine the parameters of the ring resonators. The coupling ratios and phases for each ring resonator are easily calculated from the all-pass functions. For example, to produce a pole at z_n requires a coupling ratio of $a_n = 1 - |z_n^3|^2$ and a phase of $\arg(z_n^3)$. Thus, by using Eqns.(4.48) and (4.51), the design parameters of t_i and $\phi_{1,i}$ ($i = 1, 2$) are obtained and shown in Table 4.4. With these design parameters, an IIR three-port interleaver is obtained with the specified performance. Fig. 4.16 shows the interleaving transmission spectra of the designed IIR three-channel interleaver. To compare the performance quantitatively with the FIR three-channel interleaver proposed in [149], FOM of the transmission is calculated. A simple numerical calculation shows that the FOM of the IIR interleaver is 34.32%. It is increased by more than 100% when compared to 15.9% FOM of the FIR interleaver designed in the section 4.2 with

three directional couplers. Furthermore, only two 3×3 couplers are required in the IIR optical interleaver, while three are used in the FIR interleaver. Therefore, it can be concluded that the proposed three-port IIR interleaver can efficiently achieve better performance than the FIR three-port interleavers.

Table 4.4: Order of z^{-k} , Numerator Amplitude Expansion Coefficients, Denominator Amplitude Expansion Coefficients, and Circuit Parameters for Six Order IIR Three-Port Interleaver.

$k \ (z^{-k})$	numerator power expansion coefficients	denominator power expansion coefficients	circuit parameters
0	0.3651	1	$t_1 = 0.3651$
1	1	0	$t_2 = 1.0000$
2	1	0	$\phi_{1,1} = 3.1269$
3	1	0.3651	$\phi_{1,2} = 2.5209$
4	0.3651	0	$\kappa_1 = 2\pi/9$
5	0.3651	0	$\kappa_2 = 4\pi/9$

Figure 4.16: Power spectra of three outputs of the designed IIR three-port interleaver.

4.3.1.3 Experimental Results

The proposed design scheme is successfully demonstrated by fabricating a 100G/300G IIR optical interleaver by using the fabrication method similar to that in subsections 3.3.1 and 3.3.2. The optical interleaver is implemented in a cavity structure, which will be discussed in detail in the next subsection, where the ring resonators are implemented with SCFBGs. The obtained amplitude responses of the proposed interleaver is shown in Fig. 4.17. It can be seen that the designed interleaver can have a channel isolation of as high as 20 dB, the passband bandwidth is around 68% to the channel spacing which effectively demonstrate the developed design scheme.

Simulation studies are also carried out to investigate the sensitivity of the interleaver performance with respect to the design parameters and the insertion loss of the couplers. In the simulation, 5% tolerant deviations in the design parameters κ_i ($i = 1, 2$), t_i and $\phi_{1,i}$ ($i = 1, 2, 3$), respectively, are considered. The coupling loss is considered by substituting ηz^{-3} for z^{-3} , where loss in decibels for one feedback path is $-20\log_{10}(\eta)$. Fig. 4.18 shows the spectral responses of the interleaver with the design parameters and the coupling loss varying within the tolerant range. The results clearly showed that the performance of the proposed IIR interleaver is tolerant to the design parameters variations.

4.3.2 Efficient Structures with Improved Performance

In this subsection, we shall present a more efficient IIR structure for an optical interleaver design as compared with that presented in the subsection 4.3.1. The obtained interleaver can achieve a better performance than the available interleavers in the subsection 4.3.1 in terms of wider passband bandwidth, higher channel isolation, and lower insertion loss. A numerical design example will be given to demonstrate the effectiveness of the proposed IIR interleaver design scheme.

Figure 4.17: The measured amplitude responses of the proposed interleaver implemented with two superimposed chirped FBGs.

Figure 4.18: Sensitivity study on the proposed IIR interleaver responses with the design parameters bearing $\pm 5\%$ fabrication deviations.

4.3.2.1 Improved Structure and Design Approach

The improved three-channel IIR type interleaver is formed by embedding three ring resonators in a 3×3 MZI, as shown in Fig. 4.19. Since more Fourier harmonics

present when increasing the number of all-pass ring resonators, we add one more ring resonator into the differential arm where there is no ring resonator, as shown in the subsection 4.3.1. Then, in all the three differential arms of the MZI, all-pass ring resonators with a path length of $3\Delta L$ are embedded, respectively. Each resonator couples into the differential delay path with a coupling ratio a_i ($i = 1, 2, 3$). In each of the resonators, a phase shift $\phi_{1,i}$ ($i = 1, 2, 3$) is introduced.

Figure 4.19: Improved configuration of the IIR three-port interleaver with three ring resonators on each of the fiber arms of a three-port MZI.

For this configuration, the three channels of transmissions can be obtained from one input port to the three output ports. The derivation is similar to that presented in the subsection 4.3.1. We represent the transmissions of the three ring resonators by $S_1(z)$, $S_2(z)$, and $S_3(z)$ respectively. Their transfer functions can be obtained as shown in the subsection 4.3.1. We denote $H_1(z)$, $H_2(z)$, and $H_3(z)$ as the transmissions from the input port E_1^I to the three output ports E_1^O , E_2^O , and E_3^O , respectively. It also requires that the desired design parameters shall satisfy the following equations:

$$|H_1(z)|^2 = |H_2(ze^{-j\frac{2\pi}{3}})|^2 = |H_3(ze^{-j\frac{4\pi}{3}})|^2 \quad (4.57)$$

We select $\kappa_1 = \frac{2\pi}{9}$ and $\kappa_2 = \frac{4\pi}{9}$ to meet the phase shift requirement of the interleaver design. Then, equation (4.57) is satisfied regardless of $S_i(z)$. Therefore, the problem of the three-port optical IIR interleaver design is converted into one of designing $S_i(z)$ such that any one of the channel transfer functions, *e.g.* $|H_1|^2$, satisfies the desired spectral response.

We also propose a novel cavity structure to implement the 3-port IIR interleaver in a more efficient way, as shown in Fig. 4.20. In the cavity structure, the ring resonators are substituted with superimposed chirped FBGs.

Figure 4.20: Cavity implementation of the improved configuration of IIR three-port interleaver with three superimposed chirped FBGs.

4.3.2.2 Numerical Performance of IIR Three-port Interleaver

As a design example, the proposed IIR structure of the optical interleaver is designed to have a higher passband bandwidth of 85% of the channel spacing at -0.5 dB. The coupling ratios of the two 3×3 couplers are readily obtained as $\kappa_1 = 2\pi/9$ and $\kappa_2 = 4\pi/9$, respectively, so that the three channels of transmissions have identical spectral shapes but phase shifted by $2\pi/3$. Then, one of the transmissions, say, $H_1(z)$, is designed to satisfy the passband requirement. Using the least-squares

(LS) method [51], the desired transfer function of $H_1(z)$, which is a 12-*th* order filter, can be obtained, and its numerator and denominator power expansion coefficients are listed in Table 4.5. By using Eqns.(4.48), the design parameters of t_i and $\phi_{1,i}$, $i = 1, 2, 3$, are obtained and shown in Table 4.5.

Table 4.5: Order of z^{-k} , Numerator Amplitude Expansion Coefficients, Denominator Amplitude Expansion Coefficients, and Circuit Parameters for 12-*th* Order IIR Three-Port Interleaver.

$k \ (z^{-k})$	numerator amplitude expansion coefficients	denominator amplitude expansion coefficients	circuit parameters
0	0.0567	1	$t_1 = 0.1700$
1	0.1674	0	$t_2 = 0.5021$
2	0.2701	0	$t_3 = 0.8104$
3	0.4077	1.4830	$\phi_{1,1} = \pi$
4	0.4973	0	$\phi_{1,2} = \pi$
5	0.5173	0	$\phi_{1,3} = \pi$
6	0.4603	0.6301	$\kappa_1 = 2\pi/9$
7	0.3497	0	$\kappa_2 = 4\pi/9$
8	0.2502	0	
9	0.1356	0.0692	
10	0.0459	0	
11	0.0289	0	

Bandwidth and Band-ripple Performance

With these design parameters, a three-port IIR interleaver is obtained with the specified performance. Fig. 4.21 shows the interleaving transmission spectra of the designed three-port IIR interleaver. Its phase response and group delay response in one typical transmission channel are shown in Fig. 4.22 in which the top-portion of the passband is amplified. It is seen from Figs.4.21 and 4.22 that for the proposed IIR interleaver, the passband bandwidth is about 85 GHz, the channel isolation being around 31 dB within the passband, and the group delay is about 60 ps in

the passband. To compare the performance quantitatively with the FIR three-port interleaver proposed in the Section 4.2, FOM of the transmission is calculated. It is shown that the FOM of the IIR interleaver is 92.4%. It is increased by more than 65% when compared to the 56.2% FOM for the FIR interleaver with 8 directional couplers. For a clearer comparison, Fig. 4.23 displays one channel of transmission of the proposed IIR three-port interleaver and the corresponding FIR interleaver proposed in the section 4.2. It can be clearly seen that the passband transmission bandwidth and the channel isolation of the IIR interleaver are improved greatly. Furthermore, only two 3×3 couplers are required in the IIR optical interleaver, while three are used in the FIR interleaver. Therefore, it can be concluded that the proposed three-port IIR interleaver can achieve superior performance when compared to the FIR three-port interleavers.

Figure 4.21: Power spectra of three outputs of the improved IIR interleaver with three ring resonators or three optical cavities.

Crosstalk Performance

It can be seen from Fig. 4.21 that the channel isolation of the three-port interleaver is more than 30 dB within the adjacent passband. However, in the transition band of

Figure 4.22: Group delay in one typical channel of the improved IIR interleaver with three ring resonators.

Figure 4.23: Comparison between one channel power spectrum of the improved IIR three-port interleaver and that of the FIR three-port interleaver.

the three-port interleaver, there is a high side-lobe from an adjacent channel which has to be controlled. It can be proven that the side-lobe peak cannot be totally

eliminated by choosing the design parameters in the proposed structure, however, one can efficiently reduce the area of the side-lobes and squeeze them into a thinner side-lobe in the transition band by choosing appropriate design parameters. It can be seen that the isolation from the side-lobe peak in the transition band to the maximal transmission in the passband of one of the channels, *e.g.* $|H_1|^2$, is at most $-10\log(1/9)$ dB. The proof is shown below.

Without loss of generality, we examine one of the transmissions, *e.g.* $|H_1|^2$. To maximize the difference between the maximal transmission in the passband and the side-lobe peak in the transition band, we consider the following problem,

$$\max (|H_1(e^{jw})|_{w=0}^2 - |H_1(e^{jw})|_{w=\pi}^2) \quad (4.58)$$

We will first examine the stationary points of formula $|H_1(e^{jw})|_{w=\pi}^2$. Later study shows that the value of $|H_1(e^{jw})|_{w=0}^2$ will reach its maximum simultaneously when $|H_1(e^{jw})|_{w=\pi}^2$ achieves its minimal value by selecting optimal design parameters, such as $\phi_{1,i}$ and t_i ($i = 1, 2, 3$). Thus, the maximum value of Eqn.(4.58) can be obtained accordingly.

All the stationary points of $|H_1(e^{jw})|_{w=\pi}^2$ with respect to the design parameters can be obtained by solving the following equations:

$$\begin{cases} \frac{\partial |H_1(e^{jw})|_{w=\pi}^2}{\partial \phi_{1,1}} = \frac{\partial |H_1(e^{jw})|_{w=\pi}^2}{\partial \phi_{1,2}} = \frac{\partial |H_1(e^{jw})|_{w=\pi}^2}{\partial \phi_{1,3}} = 0 \\ \frac{\partial |H_1(e^{jw})|_{w=\pi}^2}{\partial t_1} = \frac{\partial |H_1(e^{jw})|_{w=\pi}^2}{\partial t_2} = \frac{\partial |H_1(e^{jw})|_{w=\pi}^2}{\partial t_3} = 0 \end{cases} \quad (4.59)$$

By applying Eqns.(4.48) and (4.51) into Eqn.(4.59), we have

$$A_1 = A_1^* \quad A_2 = A_2^* \quad A_3 = A_3^* \quad (4.60)$$

$$B_1 = -B_1^* \quad B_2 = -B_2^* \quad B_3 = -B_3^* \quad (4.61)$$

where $A_i = \frac{e^{j\phi_{1,i}}}{(1+t_i e^{j\phi_{1,i}})^2} \left(\frac{t_1 + e^{-j\phi_{1,1}}}{1+t_1 e^{-j\phi_{1,1}}} - \frac{t_2 + e^{-j\phi_{1,2}}}{1+t_2 e^{-j\phi_{1,2}}} + \frac{t_3 + e^{-j\phi_{1,3}}}{1+t_3 e^{-j\phi_{1,3}}} \right)$ and $B_i = \frac{1 - e^{2j\phi_{1,i}}}{(1+t_i e^{j\phi_{1,i}})^2} \left(\frac{t_1 + e^{-j\phi_{1,1}}}{1+t_1 e^{-j\phi_{1,1}}} - \frac{t_2 + e^{-j\phi_{1,2}}}{1+t_2 e^{-j\phi_{1,2}}} + \frac{t_3 + e^{-j\phi_{1,3}}}{1+t_3 e^{-j\phi_{1,3}}} \right)$, A_i^* and B_i^* are the conjugates of A_i and B_i ($i = 1, 2, 3$), respectively. In the three equations of (4.60), the first item on the left hand side is the conjugate to the item on the right hand side of the same equation. Thus, A_i ($i = 1, 2, 3$) should be a real number. In contrast, in the three equations of (4.61), the first item on the left hand side is the negative conjugate to the item on the right hand side of the same equation. Thus, B_i ($i = 1, 2, 3$) should be an imaginary number.

We denote $\left(\frac{t_1 + e^{-j\phi_{1,1}}}{1+t_1 e^{-j\phi_{1,1}}} - \frac{t_2 + e^{-j\phi_{1,2}}}{1+t_2 e^{-j\phi_{1,2}}} + \frac{t_3 + e^{-j\phi_{1,3}}}{1+t_3 e^{-j\phi_{1,3}}} \right) = \alpha e^{j\beta}$, then the angle of $\left(\frac{e^{j\phi_{1,i}}}{(1+t_i e^{j\phi_{1,i}})^2} \right)$ ($i = 1, 2, 3$) should be equal to $-\beta + 2k\pi$ to meet the requirement that A_i ($i = 1, 2, 3$) should be a real number, where k is an integer. Similarly, the angle of $\left(\frac{1 - e^{2j\phi_{1,i}}}{(1+t_i e^{j\phi_{1,i}})^2} \right)$ ($i = 1, 2, 3$) should be equal to $-\beta + 2k\pi + \pi/2$ to meet the requirement that B_i ($i = 1, 2, 3$) should be an imaginary number. Thus, from Eqns.(4.60) and (4.61), we have

$$\arg \left(\frac{e^{j\phi_{1,i}}}{(1+t_i e^{j\phi_{1,i}})^2} \right) = \phi_{1,i} - 2 \arctan \frac{t_i \sin \phi_{1,i}}{1+t_i \sin \phi_{1,i}} = -\beta + 2k\pi \quad (4.62)$$

$$\arg \left(\frac{1 - e^{2j\phi_{1,i}}}{(1+t_i e^{j\phi_{1,i}})^2} \right) = -\phi_{1,i} + \frac{\pi}{2} - 2 \arctan \frac{t_i \sin \phi_{1,i}}{1+t_i \sin \phi_{1,i}} = -\beta + 2k\pi + \frac{\pi}{2} \quad (4.63)$$

From Eqns.(4.62) and (4.63), we can obtain that $\phi_{1,i}$ shall take the values of $\phi_{1,i} = p\pi$ ($i = 1, 2, 3$ and p is an integer). That means no matter what value t_i , $i = 1, 2, 3$, takes, as long as $\phi_{1,i} = p\pi$, the corresponding point is always the stationary point with respect to the design parameter. It can be further examined by calculating the associated Hessian matrix that the stationary point of $|H_1(e^{jw})|_{w=\pi}^2$ obtained by choosing $\phi_{1,i} = p\pi$ is the minimal value in the transition band.

By applying the optimal design parameters $\phi_{1,i} = p\pi$ and Eqn.(4.48), it is verified that $|H_1|^2$, at $w = 0$ and $\phi_{1,i} = p\pi$, takes the value of 1, which is the maximal

transmission that the passive interleaver can achieve. Thus, the design target (4.58) is achieved at $\phi_{1,i} = p\pi$ where the maximal difference between the transmission at the central of passband and the side-lobe peak in the transition band is $\frac{8}{9}$. An intermediate result is that no matter what values of the design parameters are selected, the side-lobe in the transition band exists in the proposed designed structure. However, by proper selection of the design parameters of t_i , $i = 1, 2, 3$, we can efficiently squeeze the area of the side-lobe in the transition band as shown in Fig. 4.21. In addition, by adding more ring resonators on the differential arms of the MZI, the area of the side-lobe of the interleaver in the transition band can be further minimized.

Performance in the Presence of Parameter Variations

The simulation studies are conducted to investigate the sensitivity of the interleaver performance with respect to the design parameters and the insertion loss of the couplers. In the simulation, 5% tolerant deviations in the design parameters κ_i ($i = 1, 2$), t_i and $\phi_{1,i}$ ($i = 1, 2, 3$) are considered, respectively. Fig. 4.24 shows the spectral responses of the interleaver with the design parameters and the coupling loss varying within the tolerant range. The results clearly show that the performance of the proposed IIR interleaver is tolerant to deviations in both the design parameters and the coupling loss.

4.4 Conclusion

It is noted from the design of two-port optical interleavers, as shown in Chapter 3, that choosing a proper structure of the filter helps to simplify the design of the phase requirement and the spectral requirement of the interleaver. Motivated by this, we propose efficient structures of three-port optical interleavers in this chapter, based on which the design of three-port interleaver is reduced to one of designing only one channel of transmission of the interleaver. Therefore, all the advantages

Figure 4.24: Sensitivity study on the improved three-port IIR interleaver spectral responses with the design parameters bearing $\pm 5\%$ fabrication deviations.

of the structure design of two-port optical filter banks are retained. Then, with the structure, the computational complexity of the interleaver performance design is greatly reduced. Finally, although more constraints and difficulties exist in the fabrication of three-port optical filter banks when compared with two-port ones, they are successfully realized by using the proposed fabrication methods of subsection 3.2.4.1. The study, on the sensitivity of the interleaver performance to the design parameters deviations, proves that the proposed structures are tolerant to fabrication errors and variations. In conclusion, we have successfully studied the structure, the design and the implementation of three-port optical filter banks in this chapter. The developed three-port optical interleavers preserve the advantages of those two-port ones in chapter 3, such as low insertion loss, narrow channel spacing, all-fiber structure, low cost, and quantitative design tools provided.

Essentially, we have provided a methodology for designing a general optical delay-line circuit to implement three-port FIR optical filter banks. Since the proposed circuit is designed to have two parts of lattice structures which respectively consist

of N and 2 stages of 3×3 directional couplers, the resultant filter is capable of producing the same but $2\pi/3$ shifted three ports of interleaving transmissions. To meet further design requirements of the transmission, a recursive synthesis approach is developed. Therefore, the proposed optical delay-line circuit and the recursive synthesis approach constitute a systematic design scheme for three-port FIR optical filter bank design.

We have also presented an IIR structure using all-pass ring resonators in a 3×3 MZI. Using the proposed structure, the design of the desired three-port IIR optical filter banks can be simplified efficiently by designing the phase-shifts and the spectral responses, separately. Finally, from the signal processing point of view, since higher order Fourier harmonics of the interleaver can be provided by using more all-pass ring resonators or GTEs in a Michelson interferometer, we have also devised a more efficient structure through employing one more ring resonator (or cavity) for three-port IIR optical filter banks. With the structure, the interleaver can be designed to deliver much better filter responses in comparison with the one proposed in the subsection 4.3.1, in terms of wider passband bandwidth, higher channel isolation, and narrower side-lobe.

Up to now, we have successfully developed the structure, the performance and the implementation designs of both two-port and three-port optical filter banks. In the next chapter, we will employ the proposed filter bank design scheme, presented in the Chapters 3 and 4, for spectrum equalization.

Chapter 5

Application of Optical Filter Banks for Spectrum Equalization

5.1 Introduction

The optical filter banks designed in chapters 3 and 4 can be used for spectrum equalization. In this chapter, we will present the details by using two-port optical filter banks. Generally, each port of an optical filter bank can be used to equalize an optical spectrum. However, due to the spectral symmetry property of the optical filter bank, the original incident light spectra will have the spectral symmetry property. Thus, in this thesis we will focus on just one port of the optical filter bank for the design of a spectrum equalization filter. The extension to all other ports of the optical filter bank for spectrum equalization is straightforward.

In optical engineering applications, many optical components/devices have spectral responses that can be approximated by an analytical Gaussian function over a certain band [3]. Moreover, as basic building blocks in optical fields, Gaussian spectral components can be combined to approximate any complex spectrum. However,

flatter spectral responses are usually preferred in many of these applications so as to improve channel count, provide uniform output, tighten optical signal-to-noise ratios and improve the ability to add and drop wavelengths [3, 15]. Several applications of the equalization of Gaussian spectra were elaborated on in section 2.3 of the literature review chapter. Therefore, it is of theoretical importance and practical interest to develop an equalization filter to equalize Gaussian-like spectra.

Over the last few decades, much focus of the spectrum equalization has been given to the gain equalization of Erbium-Doped Fiber Amplifiers (EDFAs) [98, 103, 104, 108, 150, 151], while little is reported in the literature on the equalization of Gaussian-like spectra. In this chapter, the numerical design of Gaussian-like spectrum equalizer is first studied with two-port filter bank. To obtain the design parameters that correspond to a certain desired spectral response, two kinds of design approaches, namely a *minimax* design approach and a window *integration* design approach, are proposed. The basic idea of the minimax equalization approach is to equalize the output spectrum by minimizing the worst case variation of either the total intensity change or the passband ripple within a band of interest. Thus, the resultant output spectrum is “equalized” as much as possible with the ripples evenly distributed around the central wavelength. In contrast, the window integration approach is to flatten the spectrum in a least-squares sense. Therefore, it yields an as-smooth-as-possible type of output spectrum, and in turn it gives small ripples in the equalized spectrum. A Super-luminescent Light Emitted Diode (SLED), which has a Gaussian-like spectrum, is used as an illustrative example to verify the effectiveness of the proposed methods. Through simulations and experiments, it is shown that both the proposed approaches can numerically produce spectrum equalization with acceptable performance.

To further study the spectrum equalization in a more systematic and quantitative means, we theoretically investigate the Gaussian-like spectrum equalization in the

maximally flat sense. This study is motivated by noticing that although the available equalization filters make use of different principles [98, 103, 104, 108, 150, 151], they share a common feature. The design parameters of the filter are obtained by solving a multi-dimensional numerical optimization problem. This burdens the filter design with a high computational complexity, particularly when the number of parameters involved is large. In addition, the desired specifications cannot be easily incorporated into the filter design. To overcome this problem, a recursive approach was proposed in [75] to design an optical lattice filter with a given transmission spectrum. However, this approach cannot be used to design a Gaussian-like spectrum equalizer, because it does not provide a method to choose the transmission spectrum which satisfies the two critical specifications for a Gaussian spectrum equalizer.

- First, the filter should be able to equalize the spectrum with a given level of flatness.
- Second, it should have as minimal an attenuation as possible in order to obtain the maximal power output/input.

In addition to the above limitations, the phase shifters, which are difficult to realize in practice, are required in the lattice filter design in [75].

To overcome such difficulties, our contribution is the proposal of a formalized approach to design Gaussian-like spectrum equalizers by a two-port optical filter bank which is formed by cascading directional couplers with differential delay lines. The use of optical lattice filter is motivated by two considerations. On one hand, an optical lattice filter can be easily implemented by using fibre optics which will greatly reduce the insertion loss of the obtained equalization filter. On the other hand, the parameters of an optical lattice filter can be recursively obtained. In order to obtain an equalizer to meet the two critical specifications, the proposed filter design approach is established in two steps. In the first step, an analytic Gaussian func-

tion is obtained by using least-squares fitting approach to approximate the original Gaussian-like spectrum. In the second step, an M -th order lattice filter is designed to equalize the analytic Gaussian spectrum. The design is derived by seeking zero derivatives, for as high as possible order, of the overall transmission spectrum and at the same time achieving a minimal filter attenuation. Then, a synthesis algorithm is performed to calculate the parameters of the lattice filter. In this design scheme, numerical optimization is only employed in the first step where an analytic Gaussian function is used to approximate the Gaussian-like spectrum to be equalized. As only three parameters of an analytic Gaussian function are involved, the computational complexity of the optimization is greatly reduced. This, together with the recursive calculation of the parameters of the lattice filter in the second step, has made the whole equalizer design more efficient. Thus, the proposed equalization filter can flatten the Gaussian-like spectrum to the maximally flat extend and the equalization ripples can be quantitatively assessed by evaluating the least-squares approximation error in the first step. Moreover, since the equalizer is obtained without using phase shifters in the differential delay lines in the lattice filter, the equalizer is easy to be implemented by using fiber optics. To verify the effectiveness of the proposed design approach, equalization of an SLED spectrum is considered as an example.

Although the proposed quantitative approach can flatten the Gaussian-like spectra in the maximally flat sense, certain power is unavoidably lost from the other output port of the lattice filter. That is, the approach does not provide a means to choose the transmission spectrum with the highest output power which exactly satisfies the mentioned two critical specifications. An equalization filter which can flatten the Gaussian-like spectra with the highest output power and a high level of flatness is thus highly desired. To achieve this desirable objective, we shall propose a novel symmetric feedback structure which consists of two optical lattice filters. The two incorporated lattice filters are designed to have the same design parameters, *i.e.* the coupling ratios of the directional couplers and the differential delay lines, but

they are connected into a loop in a reverse manner. The purpose of choosing two symmetric lattice filters in the proposed feedback structure lies in two parts. First, by using the symmetric structure, one can greatly reduce the number of the design parameters of the equalization filter. Second, selecting the same lattice filters in a feedback structure serves to facilitate the design by simplifying the overall transfer function of the designed filter which further enables an analytical design of such equalization filter to meet the two critical specifications.

With the proposed feedback structure, the design of the equalization filter is also in the maximally flat sense and the design parameters are obtained quantitatively. Comparisons between this method, the numerical mini-max method, and the maximally flat design method without feedback structure are conducted through simulation and experiments. These results verify the effectiveness of proposed design scheme with feedback structure.

5.2 Numerical Design of Spectrum Equalizers for Gaussian-like Spectrum

In this section, we first consider the use of all-fiber MZI to numerically equalize the output spectrum of an SLED, which has a Gaussian-like spectrum. A mini-max approach and a window integration approach are proposed to design spectrum equalization filters with pre-specified performance indicators.

5.2.1 Filter Design by the Minimax Method

Generally, two performance indices are used to evaluate the quality of the equalized SLED output spectrum, depending upon the intended applications. The first index is concerned with the total intensity change, which is defined as the difference

between the maximal and minimal intensity values over a band of interest. It is intended to yield an output spectrum that has an evenly distributed variation over the band of interest. For example, when fabricating optical interleavers, the intensity of the light source's output is required to have an evenly distributed variation within the whole band of interest, as the characteristics of interleavers span over the whole band. The second index is specified by the passband ripple, which is defined as the difference between intensity values of the ripple peak and the ripple valley. Alternatively, this index can be specified by the passband width, which is the range between the two intersection points of the line tangential to the ripple bottom and the spectrum curve. The second index is more conventional and is used in applications where the passband ripple over the passband is more important, like in characterizing the performance of an AOTF that usually exhibits a notch-like spectral response with a line width of about 10 nm.

As an illustration example, a single stage unbalanced fiber MZI is considered in the development of the filter design approaches. As shown in the subsection 2.4.1, the unbalanced fiber MZI is a 2×2 passive component which is constructed from the connection of two couplers with two differential delay lines. The bar-state transmission can be obtained as follows:

$$T_{bar}(\lambda) = c_1 c_2 e^{j\phi} - s_1 s_2 e^{-j\phi} \quad (5.1)$$

where $c_i = \sqrt{\kappa_i}$, $s_i = \sqrt{1 - \kappa_i}$, $i = 1, 2$, κ_i is the bar-state coupling ratio of the i -th directional coupler, and $\phi = \frac{\Delta L \pi n}{\lambda}$ with ΔL being half the physical length difference of the two differential delays, λ the wavelength of the light wave propagating in free space, and n the refractive index of the optical fiber. It is known from the definitions of c_i and s_i that $T_{bar}(\lambda)$ is a function of κ_1 and κ_2 . For a clear denotation, we write it as $T_{bar}(\lambda, \kappa_1, \kappa_2)$ in the remaining parts of the section.

A typical spectrum of an SLED is shown in Fig. 5.1. Its intensity change (the

intensity difference between the maximal and minimal intensity values within the band) is roughly 5 dB within the band of 1500 nm - 1580 nm. To achieve a flatter spectrum in the band 1500 nm - 1580 nm, the desired spectral response of the equalization filter should take the form shown in Fig. 5.2. Since the spectral response of the actual equalization filter is in the form of (5.1), it has a symmetrical response. However, the response of the desired filter is not symmetrical. Thus, the equalized spectrum cannot be expected to be totally flat in the desired band. However, the error can be reduced by using multi-stage MZI interferometers.

Figure 5.1: The original Super-luminescent Light Emitted Diode spectrum output without equalization, note that the spectrum is expressed in dB so that it does not look exactly like Gaussian.

The equalization filter is to be designed in two steps. First, the length difference of the differential delay is designed. Suppose we want to obtain a flat spectrum in a range $[\lambda_l, \lambda_h]$ and the central wavelength is chosen at λ_c . For example, we take $\lambda_l=1500$ nm, $\lambda_h=1600$ nm, and $\lambda_c=1550$ nm for the equalization of the spectrum shown in Fig. 5.1. The choice of the central wavelength at 1550 nm is based on the center wavelength of the C-band and that it is also a good estimate to compensate for the asymmetry of the equalized spectrum. The response period of the MZI shall

Figure 5.2: The desired spectrum response of the equalization filter which has an inverse response to the original SLED spectrum.

cover the range 1500 nm - 1600 nm. From the phase property of MZI, we can choose ΔL such that $\frac{\pi \Delta L n}{1550} = m\pi$ and $\frac{\pi \Delta L n}{1600} = (m - \frac{1}{2})\pi$, where m represents an integer, n is the refractive index of the fiber. For the standard single-mode fiber, $n = 1.476$. From these two expressions, we obtain $\Delta L = 16.8\mu m$.

In the second step, the minimax design method is used to optimize the two coupling ratios by examining the following criterion:

$$\min_{\{\kappa_1, \kappa_2\}} \max_{\{\lambda_l < \lambda < \lambda_h\}} |T_{bar}(\lambda, \kappa_1, \kappa_2) - T_{desire}(\lambda)| \quad (5.2)$$

where $T_{desire}(\lambda)$ represents the desired spectral response as shown in Fig. 5.2. The objective of Eqn.(5.2) is to optimize κ_1 and κ_2 so that it minimizes the maximal difference within an interested band.

5.2.2 Design by the Window Integration Method

As an alternative design scheme, a window integration approach is proposed. This method is motivated by the research done in [126]. It is to optimize the following criterion

$$\min_{\{\kappa_1, \kappa_2\}} \int_{\lambda_l}^{\lambda_h} W(\lambda) |T_{bar}(\lambda, \kappa_1, \kappa_2) - T_{desired}(\lambda)| d\lambda \quad (5.3)$$

where λ_l and λ_h specify, respectively, the left and the right margins of the band of interest. $W(\lambda)$ is a window function which can implicitly specify the shape of the spectrum after equalization. For simplicity, it is chosen as an “ M ” shape piecewise function as follows:

$$W(\lambda) = \begin{cases} 1 - \frac{2\delta}{\lambda_h - \lambda_l}(\lambda - \lambda_l) & \text{if } \lambda_l < \lambda < \frac{\lambda_l + \lambda_h}{2}, \\ 1 + \frac{2\delta}{\lambda_h - \lambda_l}(\lambda - \lambda_h) & \text{if } \frac{\lambda_l + \lambda_h}{2} < \lambda < \lambda_h \end{cases} \quad (5.4)$$

where δ is a small constant.

So far, we have suggested two approaches to design spectrum equalization filter using unbalanced fiber MZI. Clearly, the proposed design approaches do not depend on the number of the stages of MZI that are used in the filter. If a multi-stage fiber MZIs is employed, it is sufficient to extend the optimization to include all design parameters $\kappa_1, \kappa_2, \dots, \kappa_{2n-1}, \kappa_{2n}$. Thus, the proposed approaches are general schemes which can be extended to the design of equalization filters using multiple stages of MZIs.

5.2.3 Case Studies via Simulation

5.2.3.1 Total Intensity Change over a Band of Interest

In the first part, we examine the first index - the “intensity change” property within the band (1500 nm - 1580 nm) by the proposed minimax approach. The optimal values of κ_1 and κ_2 obtained from the Eqn.(5.2) are $\kappa_1 = 48.5\%$ and $\kappa_2 = 8.2\%$ respectively. Using this filter, the intensity change within the band 1500 nm - 1580 nm is around 1.5 dB, while using the window integration approach in Eqn.(5.3) with $\delta = 0.01$, the values of the coupling ratios κ_1 and κ_2 are 52.5% and 5.2% respectively, the intensity change is around 1.8 dB within the band 1500 nm - 1580 nm. Here, it should be noted that the intensity changes obtained by the minimax approach and the window integration approach are comparable over the whole band (1500 nm - 1580 nm). Fig. 5.3 shows the spectra of the SLED after equalization using the minimax approach and the window integration approach. It can easily be calculated that the power penalties of the designed filters with the minimax approach and the window integration approach are around 4.30 dB and 4.26 dB, respectively, at the central wavelength. It also can be seen from Fig. 5.3 that the window integration approach gives a smoother spectral response than the minimax approach, but its total intensity change in the band of interest is bigger. This conclusion has been reflected in the choice of the criteria (5.2) and (5.3). However, both approaches still have comparable results in terms of the first index - the “intensity change”.

5.2.3.2 Passband Ripple over a Band of Interest

Now, we examine the second index - the “passband ripple” property after equalization by the proposed approaches. As shown earlier in Fig. 5.3, the spectral passband that is obtained by the minimax approach is equal to the whole band of interest (1500 nm - 1580 nm) as specified, while by the window integration approach, the

Figure 5.3: Comparison between the proposed minimax method and the window integration method of equalizing the SLED spectrum by evaluating the total intensity change over a band of interest (*solid* line: the spectrum obtained by the proposed minimax approach; *dashed* line: the spectrum obtained by the window integration approach).

passband is roughly 1508 nm - 1575 nm that cannot be foreseen before the design. To compare the passband ripples in the same passband for the two approaches, one can re-design the filter by the minimax approach through taking the minimax operation over the passband 1508 nm - 1575 nm. Unlike the window integration approach, the re-design is convenient with the minimax approach because it provides a quantitative way to specify the passband bandwidth without any trial-and-error used in the window integration approach. Under this passband, an equalization filter with parameters $\kappa_1 = 98.3\%$, $\kappa_2 = 19.0\%$ are obtained. Fig. 5.4 displays the spectra equalized respectively by the minimax approach and the window integration approach. It can easily calculated that the power penalties of the designed filters with the minimax approach and the window integration approach are both around 4.26 dB at the central wavelength. The curves in Fig. 5.4 clearly show that the minimax approach can achieve the specified passband ripple and the ripple peaks are as evenly

equalized as possible, while the window integration approach tends to smoothen the spectral response. Similar to the last case, both the minimax approach and the window integration approach yield quite comparable passband ripples in the specified passband. Again, it confirms that the passband can be specified by using the minimax approach, but the passband can only be obtained through trial-and-error in the window integration approach.

Figure 5.4: Comparison between the proposed minimax method and the window integration method of equalizing SLED spectrum by evaluating passband ripples over a band of interest within the same passband (*solid* line: the spectrum obtained by the proposed minimax approach; *dashed* line: the spectrum obtained by the window integration approach).

5.2.4 Fabrication and Experimental Result

The experimental setup for fabricating the MZI optical filter has been discussed in the subsection 3.2.4.1. Here, it should be pointed out that it is difficult to obtain the coupling ratio of the second coupler exactly at 8.2%. The uncertainties on the fiber

coupler ratios will contribute bias to the designed equalization spectrum. However, this error can be minimized. When fusing the second coupler, we stop when the isolation of the filter reaches 5 dB, then we jog the stretching fiber step-by-step (about 0.1 dB deepness of the patten per jog). In this way, we can increase the isolation gradually and thus minimize the error.

Another error is generated in the course of stretching one of the two fibers between the two couplers to achieve a precise length difference to $16.8\mu\text{m}$. Actually, the length difference between the two couplers cannot be exactly controlled at $16.8\mu\text{m}$ because the refractive index n of the fiber changes during stretching. However, since the proper length difference is obtained from observing the period of the interference pattern, the effect of the change of the refractive index can be overcome.

Figure 5.5: Comparison of the experimental results of the equalized spectrum of SLED with the equalization filter within the band 1500 nm - 1580 nm (*dotted* line: the response obtained by the window integration approach; *solid* line: the response obtained by the proposed minimax approach).

Fig. 5.5 shows the respective spectra equalized by filters designed using the window integration approach and the minimax approach within the band 1500 nm - 1580

nm. Due to the fabrication errors as stated above, the achieved intensity change by the window integration method is around 2 dB, while by the minimax method it is less than 1.8 dB (instead of the ideal value of 1.5 dB). Using the equalization filters designed in passband 1511 nm - 1575 nm, the output spectra of the SLED are displayed in Fig. 5.6, from which it can be seen clearly that both the minimax approach and the window integration approach can equalize the SLED spectrum with a 0.5 dB passband ripple.

Figure 5.6: Comparison of the experimental results of the equalized spectrum of SLED with the equalization filter when the minimax operation is applied over the band 1511 nm - 1575 nm (*dotted* line: the response obtained by the window integration approach; *solid* line: the response obtained by the proposed minimax approach).

5.3 Maximally Flat Equalization of Gaussian-like Spectra with FIR Structure

In the last section, we have discussed the Gaussian-like spectrum equalization by using numerical approaches. Although those methods are able to provide the equalization filter with acceptable performance, the overall performance of the equalized spectra cannot be pre-specified. Trial-and-error design must be done in such cases. To address this issue, a systematic design approach is highly desired. Thus, in this section, we demonstrate a filter design scheme that uses an all-fibre lattice filter to equalize Gaussian-like spectra in a more quantitative sense. With the proposed two-step design strategy, the designed equalization filter can flatten the Gaussian-like spectra to the maximally flat extent and the spectrum ripples can be assessed quantitatively. In addition, the equalization performance is analyzed theoretically. It is also shown that the equalization performance can only be improved with increased odd order of the lattice filter. As an illustrative example, the proposed scheme is applied to the design of an equalization filter to flatten the output spectrum of an SLED.

5.3.1 Equalizing Gaussian-like Spectrum via Lattice Filter

The output spectrum of a super-luminescence light emitted diode (SLED) is Gaussian-like as shown in Fig. 5.7. When an SLED is used to provide a broadband light source, its spectrum usually needs to be equalized by cascading a spectrum equalization filter. To equalize the Gaussian-like spectrum, an equalization filter is constructed in this chapter in two steps. In the first step, an analytic Gaussian function is used to approximate the original Gaussian-like spectrum. The Gaussian function used is expressed as $F = \beta e^{-(w-w_0)^2/2\delta^2}$, where w_0 corresponds to the central wavelength, β the amplitude, and δ the derivation of the Gaussian function. The approxima-

Figure 5.7: Gaussian-like spectrum equalization: The original Gaussian-like SLED spectrum, its Gaussian representation, and the equalized spectrum.

tion can be done in a least-squares sense when the measurement of the original Gaussian-like spectrum is available.

In the second step, an optical lattice filter is designed as an equalization filter with a transmission spectrum denoted by P as shown in Fig. 5.7. Applying the equalization filter to the spectrum F , the equalized spectrum can be expressed as PF . The objective of spectrum equalization can thus be stated as designing the filter with a transmission spectrum P satisfying two specifications. First, the overall spectrum PF is as flat as possible over a desired bandwidth. Second, the resultant attenuation of PF as shown in Fig. 5.7 should also be as small as possible.

To obtain the equalization filter, an M -th order optical lattice filter is designed. The filter is formed by linking $(M + 1)$ 2×2 directional couplers with differential time-delay lines, as shown in Fig. 5.8. The power coupling ratios of the directional couplers are denoted by κ_i ($i = 1, 2, \dots, M + 1$). All the differential delay lines are

Figure 5.8: Configuration of an M -order cascaded optical lattice filter that consists of directional couplers and differential delays.

assumed to have a differential length ΔL which can be chosen according to the bandwidth to be designed.

Denote the electric field at the i -th port as E_i ($i = 1, 2, 3, 4$), the normalized electric fields transfer function from the two inputs to the two outputs can then be expressed by the following matrix transfer function:

$$\begin{pmatrix} E_3 \\ E_4 \end{pmatrix} = \mathbf{S}_{M+1} \prod_{i=M}^1 (\mathbf{S}_i^\phi \mathbf{S}_i) \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \quad (5.5)$$

where $\mathbf{S}_i$ and $\mathbf{S}_i^\phi$ denote respectively the field transfer function of the i -th fibre coupler and the i -th differential time-delay line.

To proceed, we further denote the field transfer functions with respect to the bar-state (from E_1 to E_3) and cross-state (from E_1 to E_4) as T_{bar}^M and T_{cross}^M , respectively. The bar-state transmission spectrum P_{bar}^M is given by $T_{bar}^M T_{bar}^{M*}$, where T_{bar}^{M*} is the complex conjugate of T_{bar}^M . Then, the design of the equalization filter is to design

the bar-state transmission spectrum P_{bar}^M by choosing appropriate parameters c_i and s_i ($i = 1, 2, \dots, M + 1$) such that the overall spectrum $P_{bar}^M F$ is as flat as possible in its central wavelength while the magnitude of $P_{bar}^M F$ is as big as possible.

5.3.2 Derivation of the Equalization Filter

In this subsection, a design scheme is presented to obtain the equalization filter to flatten the Gaussian spectrum with the desired performance as stated above. The design includes several steps. First, the desired transmission spectrum P_{bar}^M is obtained by applying the maximally flat principle of analytic functions on the spectrum $P_{bar}^M(w)F(w)$ and meanwhile seeking a minimal filter attenuation. As a result, the coefficients of the desired P_{bar}^M is obtained explicitly by solving a constrained optimization problem. Then, the field transfer function T_{bar}^M is obtained by normalizing and factoring the spectrum P_{bar}^M . Lastly, the coupling ratio of each fiber coupler is calculated from T_{bar}^M by a recursive algorithm.

The bar-state field transfer function of T_{bar}^M can be expressed as follows:

$$T_{bar}^M = e^{jMw} \left(\sum_{k=1}^{M+1} b_{k,M+1} e^{-2j(k-1)w} \right) \quad (5.6)$$

where $b_{k,M+1}$ ($k = 1, \dots, M + 1$) are the amplitude expansion coefficients. From the structure of the proposed optical lattice filter, all the coefficients of T_{bar}^M are real numbers. Expanding the transfer function of $T_{bar}^M T_{bar}^{M*}$, the bar-state transmission spectrum P_{bar}^M can be expressed by a combination of $\cos(2(k-1)w)$ as follows:

$$P_{bar}^M = \left(\sum_{k=1}^{M+1} b_{k,M+1} e^{-2j(k-1)w} \right) \times \left(\sum_{k=1}^{M+1} b_{k,M+1} e^{2j(k-1)w} \right) = \sum_{k=1}^{M+1} a_{k,M+1} \cos 2(k-1)w \quad (5.7)$$

where $a_{k,M+1}$ ($k = 1, \dots, M + 1$) are the power expansion coefficients that are the combinations of the design parameters c_i and s_i , $i = 1, 2, \dots, M + 1$.

In order to obtain a flat spectral response, we examine the first $2M + 2$ derivatives of the spectrum $P_{bar}^M F$ with respect to w at $w = 0$. Using the chain rule, the q -th order derivative of $P_{bar}^M F$ can be obtained as:

$$\left. \frac{d^q(P_{bar}^M F)}{d^q w} \right|_{w=0} = \sum_{l=0}^q \binom{q}{l} \left. \frac{d^{q-l}(P_{bar}^M)}{d^{q-l} w} \frac{d^l(F)}{d^l w} \right|_{w=0} \quad (5.8)$$

for $q = 1, 2, \dots, 2M + 2$.

It is noted that all the odd order derivatives of $P_{bar}^M F$ are zeros automatically when evaluated at $w = 0$. This can be verified as follows. Denote $P_{bar}^M F$ by $f(w)$. As both P_{bar}^M and F are even functions, so is $f(w)$, thus we have

$$f^{(2p)}(w) = f^{(2p)}(-w) \quad ; \quad p=1, 2, \dots, M+1 \quad (5.9)$$

Then all its odd order derivatives with respect to w have the relationship

$$f^{(2p-1)}(w) = -f^{(2p-1)}(-w) \quad ; \quad p=1, 2, \dots, M+1 \quad (5.10)$$

That is, all the odd order derivatives are odd functions. Therefore, their values at $w = 0$ are zeros. To examine the maximally flatness of the equalized spectrum, it is sufficient to evaluate the even order derivatives of $P_{bar}^M F$. Then, we have the following set of $M + 1$ linear equations,

$$\begin{aligned} \left. \frac{d^{2p}(P_{bar}^M F)}{d^{2p} w} \right|_{w=0} &= \sum_{l=0}^p \binom{2p}{2l} \left. \frac{d^{2p-2l}(P_{bar}^M)}{d^{2p-2l} w} \frac{d^{2l}(F)}{d^{2l} w} \right|_{w=0} \\ &= \sum_{k=1}^{M+1} \left[\sum_{l=0}^p \binom{2p}{2l} (-1)^p (2k-2)^{(2p-2l)} \frac{\prod_{i=1}^l (2i-1)}{\delta^{2l}} \right] a_{k,M+1} = 0 \end{aligned} \quad (5.11)$$

with $p = 1, 2, \dots, M + 1$.

Introducing a vector $\mathbf{v} = [a_{1,M+1}, a_{2,M+1}, \dots, a_{M+1,M+1}]^T$, equations in (5.11) can be expressed in a matrix form as:

$$\begin{pmatrix} \frac{1}{\delta^2} & 2^2 + \frac{1}{\delta^2} & \cdots & (2M)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & 2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (2M)^4 + 6(2M)^2\frac{1}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^{M+1} (2i-1)}{\delta^{2M+2}}, & \sum_{l=0}^{M+1} \frac{C_{2M+2}^{2l} 2^{(2M+2-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \cdots & \sum_{l=0}^{M+1} \frac{C_{2M+2}^{2l} (2M)^{(2M+2-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \times \mathbf{v} = 0 \quad (5.12)$$

Denote the matrix in the left hand side of the above equation as $\mathbf{A}_0$. It is clearly a full rank matrix of $R^{(M+1) \times (M+1)}$. Therefore, an M -th order optical lattice filter cannot achieve a flat spectrum in a sense of $(M+1)$ -th order maximal flatness. Otherwise, the matrix $\mathbf{A}_0$ would be singular. In other words, only a flat spectrum in the sense of M -th order maximal flatness can be achieved by using the maximally principle. In order to determine the $(M+1)$ -dimensional unknown vector $\mathbf{v}$, one more constraint is needed. It can be sought from the second specification for an equalization filter to have a small attenuation in order to have a stronger output power. Thus, the overall attenuation resulted from the equalization filter shall be made as small as possible. This is equivalent to maximizing the value of $P_{bar}^M F$ at $w = 0$, *i.e.*,

$$\max_{\{a_{1,M+1}, a_{2,M+1}, \dots, a_{M+1,M+1}\}} \{(P_{bar}^M F)|_{w=0}\} \quad (5.13)$$

After applying Eqn.(5.7) and using $F = \beta e^{-(w-w_0)^2/2\delta^2}$, Eqn.(5.13) becomes

$$\max_{\{a_{1,M+1}, a_{2,M+1}, \dots, a_{M+1,M+1}\}} \left\{ \sum_{k=1}^{M+1} a_{k,M+1} \right\} \quad (5.14)$$

Now, the equalization filter can be designed such that the equalized spectrum is of M -th order maximally flatness and the overall attenuation is minimal. Such a filter

can be determined uniquely and explicitly by solving an optimization problem in Eqn.(5.13) with constraints specified by the first (M) linear equations in Eqn.(5.12). Thus, the desired coefficients vector v shall satisfy

$$\begin{cases} \max\{g(\mathbf{v})\} = \max \left\{ \sum_{k=1}^{M+1} a_{k,M+1} \right\} \\ \text{s.t. } \mathbf{A}\mathbf{v} = 0 \end{cases} \quad (5.15)$$

where $g(\mathbf{v}) = \sum_{k=1}^{M+1} a_{k,M+1}$ and the matrix $\mathbf{A}$ is given by the first M rows of $\mathbf{A}_0$, i.e.

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\delta^2} & 2^2 + \frac{1}{\delta^2} & \cdots & (2M)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & 2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (2M)^4 + 6(2M)^2\frac{1}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}}, & \sum_{l=0}^M C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \cdots & \sum_{l=0}^M C_{2M}^{2l} \frac{(2M)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \quad (5.16)$$

To proceed, the matrix $\mathbf{A}$ is partitioned into $\mathbf{A} = [\mathbf{A}_1, \mathbf{a}_1]$ where $\mathbf{A}_1$ is an $M \times M$ matrix formed by the first M columns of $\mathbf{A}$ and $\mathbf{a}_1$ is the last column of $\mathbf{A}$. Similarly, the vector $\mathbf{v}$ is partitioned into $\mathbf{v} = [\mathbf{v}_1^T, v_2]$ where $\mathbf{v}_1$ is formed by the first M elements of $\mathbf{v}$ and v_2 denotes the last element of $\mathbf{v}$. Using these denotations, the constraint in Eqn.(5.15) can be expressed by

$$\mathbf{A}_1 \mathbf{v}_1 + \mathbf{a}_1 v_2 = 0 \quad (5.17)$$

It is from Eqn.(5.7) that the element v_2 , the coefficient of the last item of P_{bar}^M , is determined from the first and the last items of T_{bar}^M . By examining the light paths of the optical lattice filter circuit, the last item of P_{bar}^M can be shown as $v_2 = -1/2 \sin 2\theta_1 \sin 2\theta_{M+1} \prod_{i=2}^M \cos \theta_i^2$. Hence, the absolute value of the last item cannot exceed $1/2$. At $v_2 = 0$, it corresponds to the coupling ratio of the last coupler being zero. Using Eqn.(5.17), $\mathbf{v}_1$ can be expressed in terms of v_2 as $\mathbf{v}_1 = -\mathbf{A}_1^{-1} \mathbf{a}_1 v_2$.

Substituting it into Eqn.(5.15), the filter design is converted into one of solving the following constrained optimization:

$$\begin{cases} \max \{ \mathbf{e}^T \mathbf{v}_1 + v_2 \} = \max \{ (1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1) v_2 \} \\ s.t. \quad -1/2 \leq v_2 \leq 0 \end{cases} \quad (5.18)$$

where $\mathbf{e}$ stands for an M dimensional vector $(1, 1, \dots, 1)^T$.

To solve Eqn.(5.18), we examine the value of $1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1$. It is easy to verify the following matrix identity

$$\begin{pmatrix} \mathbf{I} & 0 \\ -\mathbf{e}^T \mathbf{A}_1^{-1} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{I} & -\mathbf{A}_1^{-1} \mathbf{a}_1 \\ 0 & \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1 & 0 \\ 0 & 1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1 \end{pmatrix} \quad (5.19)$$

Taking determinants on both sides, we have

$$\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix} = \det(\mathbf{A}_1) \det(1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1) \quad (5.20)$$

Thus, we have

$$1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1 = \frac{\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}}{\det(\mathbf{A}_1)} \quad (5.21)$$

As shown in Appendix D, the sign of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is determined by the parity of M , which is also the number of differential delay lines. If M is an even number, the value of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is positive; otherwise, it is negative. It can also be proven that the determinant $\det(\mathbf{A}_1)$ is always positive (see the proof in Appendix E). Therefore, the sign of $1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1$ depends on the parity of M . As v_2 is negative, in order to make the value of $\mathbf{e}^T \mathbf{v}_1 + v_2$ bigger than zero, M has to be an odd number.

Thus, $(1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1) v_2$ will reach its maximal value by selecting v_2 as $-1/2$. Then, with Eqn.(5.17), the desired value of the vector v_1 can be obtained by

$$\mathbf{v}_1 = \frac{1}{2} \mathbf{A}_1^{-1} \mathbf{a}_1 \quad \text{with } M \text{ an odd number} \quad (5.22)$$

Now we have obtained the coefficients of the desired transmission P_{bar}^M that enables an M -order maximally flatness spectrum with minimal attenuation. Since the optical lattice filter is a passive filter, the maximum transmission cannot exceed 100%. Thus, the coefficients of P_{bar}^M must be normalized in order to obtain the corresponding values of the fibre couplers in the lattice filter. Here, the normalization is done by $\hat{a}_{i,M+1} = \frac{a_{i,M+1}}{(\sum_{i=1}^{M+1} |a_{i,M+1}|)}$, and the normalized transmission spectrum is denoted by $\hat{P}_{bar}^M$.

With $\hat{P}_{bar}^M$, a synthesis algorithm will be employed to calculate the design parameters c_i and s_i ($i = 1, 2, \dots, M+1$). Denote c_i ($i = 1, 2, \dots, M+1$) by $\cos \theta_i$ and s_i by $\sin \theta_i$ where θ_i is in the interval $[0, \pi/2]$. The design parameters are therefore θ_i . The synthesis algorithm can be established by following similar procedures as in [75], which include calculating the coefficients of the bar-state field transmission T_{bar}^M from the desired transmission spectrum $\hat{P}_{bar}^M$ via spectral factorization, and obtaining the parameters θ_i recursively as follows:

$$\theta_n = -\arctan \left(\frac{c_{n,n}}{b_{n,n}} \right) \quad (5.23)$$

$$b_{k,n-1} = (b_{k+1,n} \cos \theta_n - c_{k+1,n} \sin \theta_n) \quad (5.24)$$

$$c_{k,n-1} = (b_{k,n} \sin \theta_n + c_{k,n} \cos \theta_n) \quad (5.25)$$

for $k = 1, 2, \dots, n$, and $b_{k,n}$ and $c_{k,n}$ are the k -th coefficients of the bar-state and cross-state amplitude transmission in the n -th recursive steps, respectively. The initial value of n starts from $M+1$ to 1, where $b_{k,M+1}$ and $c_{k,M+1}$ are obtained from the first four steps in the synthesis algorithm as shown in the flow chart of Fig. 5.9.

Figure 5.9: The flow chart illustrating the proposed design algorithm for Gaussian spectrum equalization with the optical lattice filter.

So far, we have obtained a general formalized approach to the design of a spectrum equalizer which can maximally flatten the Gaussian spectrum while yielding a minimal attenuation. The approach is summarized as an algorithm as shown in the flow chart of Fig. 5.9. It is worth noting that the approximation in the second stage is in a maximally flat sense and as a result the overall equalization ripples is mainly determined by the approximation error resulted in the first step.

5.3.3 Design Issues of Gaussian-like Spectrum Equalization

There are two practical issues in design of the desired Gaussian-like spectrum equalization filters that need to be addressed. First, we need to analyze the bandwidth of the equalized spectrum that is obtained by the proposed scheme. An approximate expression of the bandwidth is derived in terms of the design parameters. Second, the tunability of the proposed equalization method has to be addressed.

5.3.3.1 Bandwidth of the Equalized Spectrum

As shown in the last subsection, the equalized spectrum is actually obtained by applying the maximally flat principle on the overall transfer function $P_{bar}^M F$ around the central wavelength characterized by $w = 0$. The first $(2M+1)$ -th derivatives of the overall transfer function are all zeros. Thus, the equalized spectrum can be approximated by a polynomial function which is characterized by $P_{bar}^M F \approx c_1 + c_{2M+2} w^{2M+2}$. Clearly, the two coefficients are determined by $c_1 = \max\{P_{bar}^M F|_{w=0}\}$ and $c_{2M+2} = (P_{bar}^M F)^{(2M+2)} / (2M+2)!|_{w=0}$ where the superscript $(2M+2)$ denotes the $(2M+2)$ -th derivative. Similar to (5.11), the $(2M+2)$ -th derivative of $P_{bar}^M F$ at

$w = 0$ can be also expressed in terms of the design parameters as follows:

$$(P_{bar}^M F)^{(2M+2)}|_{w=0} = \left(\frac{\prod_{i=1}^{M+1} (2i-1)}{\delta^{2M+2}} \quad \dots \quad \sum_{l=0}^{M+1} C_{2M+2}^{2l} \frac{(2M)^{(2M+2-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \right) \mathbf{v} \quad (5.26)$$

Denote the δ dependent vector in above equation as $\mathbf{u}(\delta)^T$. Using (5.13), (5.14), and the definition of $\mathbf{v}$, the equalized spectrum $P_{bar}^M F$ can be approximated by

$$P_{bar}^M F \approx \|\mathbf{v}\|_1 + \frac{\mathbf{u}(\delta)^T \mathbf{v}}{(2M+2)!} w^{2M+2} \quad (5.27)$$

where $\|\mathbf{v}\|_1 \triangleq \sum_{l=1}^{M+1} a_{k,M+1}$ and $\mathbf{v}$ is the design result obtained in the last section. Then, the bandwidth of the equalized spectrum can be evaluated by the ϵ -dB bandwidth. It is defined as the distance between two points where the equalized spectrum intersects the line with a power level less than the maximal output power by ϵ -dB. Here, ϵ is a small value that can be specified by the users, e.g. $\epsilon = 0.1$ represents a 0.1 dB loss of the overall equalization spectrum.

It can be easily seen from Eqn.(5.27) that

$$\epsilon = -10 \log \left(\frac{\|\hat{\mathbf{v}}\|_1 + \frac{\mathbf{u}(\delta)^T \hat{\mathbf{v}}}{(2M+2)!} w_b^{2M+2}}{\|\hat{\mathbf{v}}\|_1} \right) \quad (5.28)$$

where w_b is the right bound of the ϵ -dB bandwidth in the normalized w domain. Thus, the ϵ -dB bandwidth of the overall equalization spectrum in the wavelength domain can be assessed by

$$BW_\epsilon = 2(\lambda_b - \lambda_c) = 2 \left(\frac{\lambda_c \Delta L \pi n_{eff}}{\Delta L \pi n_{eff} - 2w_b \lambda_c} - \lambda_c \right) \quad (5.29)$$

where λ_b is the right-bound wavelength of the ϵ -dB bandwidth. From Eqn.(5.28),

w_b can be expressed by

$$w_b = \left[\frac{(2M + 2)! \|\hat{\mathbf{v}}\|_1 (1 - 10^{-\epsilon/10})}{\mathbf{u}(\delta)^T \hat{\mathbf{v}}} \right]^{\frac{1}{2M+2}} \tag{5.30}$$

Note that the spectrum given in (5.27) is just an approximation of the equalized spectrum around the central wavelength. The approximation of the ϵ -dB bandwidth has a good accuracy when ϵ take small values. Table.5.1 compares several typical ϵ -dB bandwidths calculated using (5.29) to those measured from real spectrum. It is shown that ϵ -dB bandwidth obtained by (5.29) has a good accuracy when it is used to evaluate up to 0.5-dB bandwidth which is sufficient for the assessment of equalized spectra.

Table 5.1: ϵ -dB Bandwidth of the Equalized Spectrum Using the Proposed Equalization Scheme ($\Delta L = 27.6\mu m$ and the Lattice Filter Order $M = 1$).

ϵ -dB	bandwidth calculated using (5.29) (nm)	bandwidth measured (nm)
0.01dB	18.11	18.20
0.05dB	24.95	25.63
0.1dB	29.67	30.83
0.2dB	35.25	37.26
0.5dB	44.07	48.31

5.3.3.2 Tunability of the Gaussian-like Spectrum Equalization

As the Gaussian-like spectrum of a light source depends on operation conditions such as pumping condition and operation temperature as well as aging [3, 54], it is desired in practice that the equalization filter is tunable and the filter design scheme shall support the design of tunable equalization filters, like those equalization filters for EDFAs [21, 152]. This tunability issue of the proposed equalization scheme and filters are addressed from two aspects.

Figure 5.10: Flow chart illustrating the tunability of the proposed scheme for Gaussian spectrum equalization implemented by using waveguide couplers.

Firstly, the proposed equalization filter design scheme can be easily extended to design a tunable equalization filter which is able to flatten a varying Gaussian-like spectrum in a sense of the maximally flat and minimum attenuation. This is useful if the optical lattice equalization filter is implemented by using the planar waveguide technology [153] where the design parameters such as the coupling ratios of those fiber couplers can be easily tuned by using the readily available tuning mechanisms

such as thermal or electro-optic tuning. In this case, the adaptive algorithm of the proposed equalization scheme operates in the flow chart diagram described in Fig. 5.10 and Block diagram in Fig. 5.11. An OSA is used to collect the input Gaussian spectrum. This spectrum is sent to a computer where the desired design parameters such as the coupling ratios of those waveguide couplers are calculated by following the five steps in the algorithm of Fig. 5.9 that was proposed in the last subsection. The obtained parameters are then outputted through A/D to the thermal or electro-optic tuning actuators to tune the coupling ratios in parallel. In what follows, the flattened spectrum is compared with the desired specifications and the above data collection and calculations are repeated until the difference error between the equalized spectrum and the specified spectrum is within the specified error range.

Figure 5.11: Block diagram of the adaptive tuning scheme for Gaussian spectrum equalization.

Secondly, the tunability of the proposed equalization filter is examined by consid-

ering the robustness of the filter with respect to small variations in the Gaussian spectrum to be equalized. These small variations may result from aging variations and small changes in operation temperature. In this case, an equalization filter is designed by applying the non-tune algorithm in Fig. 5.9 on a nominal Gaussian spectrum. Then, simulations are carried out to examine the performance of the designed equalization filter when the real Gaussian spectrum has $\pm 5\%$ tolerant deviations in width and amplitude with respect to the nominal Gaussian spectrum used in the design. Fig. 5.12 gives the equalized spectra corresponding to $\pm 5\%$ variations with a step size of 1% in amplitudes and widths of the input Gaussian spectrum, respectively. The results clearly show that the 0.5-dB bandwidth of the equalized spectra are acceptable in the presence of small amount of variations in the input Gaussian spectrum to be equalized.

Figure 5.12: The performance of the overall spectrum responses with respect to the parameters of the input Gaussian spectrum bearing $\pm 5\%$ variations where the designed equalization lattice filter is fixed.

5.3.4 Simulation Studies

In this subsection, simulation studies are conducted to verify the effectiveness of the proposed spectrum equalization scheme by applying it to the design of an equalization filter to flatten the Gaussian-like spectrum of an SLED light source. The original SLED spectrum is shown in Fig. 5.13. It looks like a Gaussian spectrum. Following the procedures of the proposed design scheme, the SLED spectrum is first approximated by an analytic Gaussian function. Using a least-squares approximation method, the optimal Gaussian function is obtained with the central wavelength at 1544.2 nm, the amplitude β at $\beta = 0.99$, and the derivation δ at $\delta = 0.68$. The resultant approximation error is within the range of 0.01 dB.

In the following, the performance of the proposed design scheme will be studied by considering three cases where a first order optical lattice filter, an even order optical lattice filter, and a higher odd order optical lattice filter are respective used to design the filter to flatten the SLED spectrum.

5.3.4.1 First order optical lattice filter (*i.e.* with two couplers)

First, we consider using an optical lattice filter of the first order as the equalization filter. The 1-st order optical lattice filter is formed by linking two couplers through a unit differential time delay line. The design problem in this case is summarized as follows:

$$\max \{P_{bar}^1 F|_{w=0}\} \quad (5.31)$$

subject to

$$\left. \frac{d(P_{bar}^1 F)}{dw} \right|_{w=0} = \left. \frac{d^2(P_{bar}^1 F)}{dw^2} \right|_{w=0} = \left. \frac{d^3(P_{bar}^1 F)}{dw^3} \right|_{w=0} = 0 \quad (5.32)$$

Figure 5.13: The original spectrum of the SLED.

Using the maximally flat conditions gives

$$\begin{pmatrix} \frac{1}{\delta^2}, & 2^2 + \frac{1}{\delta^2} \end{pmatrix} \begin{pmatrix} a_{1,2} \\ a_{2,2} \end{pmatrix} = 0 \quad (5.33)$$

Since $\mathbf{A}_1^1 = 1/\delta^2 > 0$, the last element v_2 will take the value $a_{2,2} = -1/2$ in order to achieve the maximum in (5.13). The power expansion coefficients of the bar-state transmission spectrum P_{bar}^1 can be obtained from Eqn.(5.22) with $a_{1,2} = 1.4248$, and $a_{2,2} = -0.5$. As the optical lattice filter is basically a passive filter, the maximal transmission of which cannot exceed 100%. To guarantee this, P_{bar}^1 is normalized by using $\hat{a}_{i,M+1} = \frac{a_{i,M+1}}{(\sum_{i=1}^{M+1} |a_{i,M+1}|)}$. The resultant coefficients are $\hat{a}_{1,2} = 0.7402$, and $\hat{a}_{2,2} = -0.2598$, which correspond to $\theta_1 = \theta_2 = 0.4025$. This means that the coupling ratios of the two couplers in the optical lattice filter are 84.66% and 84.66%, respectively. The result of applying the obtained equalization filter to the SLED spectrum is shown in Fig. 5.14. It is shown that the SLED spectrum has been

flattened in a maximally flat sense at the central wavelength and the power penalty of the designed filter is about 2.8 dB.

Figure 5.14: Overall SLED spectrum responses with the maximally flat transmission designed with the proposed scheme.

5.3.4.2 Even order optical lattice filters

Now we consider even orders of optical lattice filters, *i.e.*, $M = 2p$. Since $(1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1)$ is positive for $M = 2p$, as shown in Appendix D, thus the value of $(\mathbf{e}^T \mathbf{v}_1 + v_2) = (1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1)v_2$ cannot be positive. However, the physical realization requires that $(\mathbf{e}^T \mathbf{v}_1 + v_2) \geq 0$. In this case, it is not possible in optical implementation. Hence, $v_2 = 0$, and so is the coupling ratio of the last coupler. Consequentially, the maximally flat filter design using a $2p$ -th order optical lattice filter is exactly the same as the one using a $(2p - 1)$ -th order optical lattice filter. To see this, a simulation study of using a second order optical lattice equalization filter is carried out and compared with the result obtained using the first order optical lattice filter.

Using a second order optical lattice filter, it can be obtained from Eqn.(5.16) that

$$\begin{pmatrix} \frac{1}{\delta^2}, & 2^2 + \frac{1}{\delta^2}, & 4^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4}, & 2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4}, & 4^4 + 6\frac{4^2}{\delta^2} + \frac{3}{\delta^4} \end{pmatrix} \cdot \begin{pmatrix} a_{1,3} \\ a_{2,3} \\ a_{3,3} \end{pmatrix} = 0 \quad (5.34)$$

From Eqn.(5.18), we have $e^T v_1 + v_2 = 8.48 \times v_2 \leq 0$, thus, v_2 has to be zero and the design problem is reduced to first order optical lattice filter design. In order to justify that this solution is unique, we consider using an exhaustive search method to look for the desired design parameters θ_1 , θ_2 , and θ_3 such that the equalized spectrum is maximally flat with a minimal attenuation. In this method, the optimization index is defined as $\|P_{bar}^2 F - F_{SLED}\|$ where F_{SLED} denotes the original SLED spectrum to be equalized. The search is executed in the whole space given by $[0, \pi/2] \times [0, \pi/2] \times [0, \pi/2]$. The simulation result shows that when using a second order optical lattice filter as the equalization filter the coupling ratio of the last coupler will be zero. This justifies the conclusion that using a $2p$ -th order optical lattice filter will yield the same equalization results as the $(2p - 1)$ -th order optical lattice filter.

5.3.4.3 Higher odd order optical lattice filters

Lastly, we consider the effect of increasing the order of the optical lattice filter on the equalization performance. Intuitively, by increasing the order of the optical lattice filter, the equalized SLED spectrum will be flatter. However, the insertion loss brought by the equalization filter will be bigger. In other words, the value of $e^T v_1 + v_2$ would be smaller. Simulation studies are carried out to examine the equalized SLED spectra with the order of the optical lattice filters increased from 1 to 6. The amplitude expansion coefficients of the bar-state field transfer function obtained before and after normalization and the corresponding circuit design parameters are summarized in Table 5.2. The spectrum performance of SLEDs designed with vari-

ous orders of optical lattice filters are compared in Fig. 5.15. It is shown that, if the order of the optical lattice filters is increased, the bandwidth of the maximally flat spectrum will be increased. However, much bigger attenuation occurs. The results in Fig. 5.15 show that the maximal spectrum decreases from 0.5 to 0.2 when the order of the optical lattice filter is increased from 1 to 6.

All the above simulation results demonstrate that the proposed design scheme can be used to design an equalizer filter to flatten the SLED output spectrum with the maximally flat spectrum, and at the same time minimizes the attenuation of the overall spectrum.

Figure 5.15: Comparisons of the overall spectra of SLED designed by using forward lattice filters with different orders.

5.3.5 Experimental Results

To demonstrate the effectiveness of the proposed spectrum equalization design scheme, an all-fibre optical MZI was built to implement the optical lattice filter to equalize the SLED spectrum. It has been shown in the theoretical analysis that using a

Table 5.2: Design Parameters of Optical Lattice Filters in Forward Structure with Different Orders for the Maximally Flat Spectrum of SLED

Number of differential delay lines, M	Power expansion coefficients before norm $a_{i,M+1}$	Power expansion coefficients after norm $\hat{a}_{i,M+1}$	Circuit parameters θ_{M+1}	Results by the exhaustive searching method
1	$a_{1,2}=1.4248$ $a_{2,2}=-0.5$	$a_{1,2}^*=0.7402$ $a_{2,2}^*=-0.2598$	$\theta_1 = 0.4025$ $\theta_2 = 0.4025$	N/A
2	N/A	N/A	N/A	$\theta_1 = 0.3923$ $\theta_2 = 0.3923,$ $\theta_3 = 0.0221$
3	$a_{1,4}=43.30$ $a_{2,4}=-27.96$ $a_{3,4}=5.38$ $a_{4,4}=-0.50$	$a_{1,4}^*=0.5613,$ $a_{2,4}^*=-0.3624$ $a_{3,4}^*=0.0698$ $a_{4,4}^*=-0.0065$	$\theta_1 = 0.7731$ $\theta_2 = 1.0228$ $\theta_3 = 1.4866$ $\theta_4 = 0.8806$	N/A
4	N/A	N/A	N/A	$\theta_1 = 0.7671$ $\theta_2 = 1.0875,$ $\theta_3 = 1.4920$ $\theta_4 = 0.8611$ $\theta_5 = 0.0363$
5	$a_{1,6}=1127.7$ $a_{2,6}=-871.8$ $a_{3,6}=248.5$ $a_{4,6}=-52.8$ $a_{5,6}=7.4$ $a_{6,6}=-0.5$	$a_{1,6}^*=0.4885$ $a_{2,6}^*=-0.3776$ $a_{3,6}^*=0.1076$ $a_{4,6}^*=-0.0229$ $a_{5,6}^*=0.0032$ $a_{6,6}^*=-0.0002$	$\theta_1 = 0.0660$ $\theta_2 = 0.4903$ $\theta_3 = 1.4973$ $\theta_4 = 0.8234$ $\theta_5 = 0.8295$ $\theta_5 = 1.4768$	N/A

higher odd order optical lattice filter can yield flatter equalized spectrum. However, from the practical point of view, using a higher order optical lattice filter will increase the filter connection loss as well as the bigger attenuation. As shown in the previous subsection, a first order MZI was able to yield a flat equalized spectrum over 34 nm of wavelength range. Thus, we shall implement a first order MZI to examine the experimental behavior of the equalization filter designed by the proposed design scheme. According to the coupling ratios obtained in Table 5.2, the

MZI equalization filter is made by fabricating two fibre couplers successively on two piece fibres, as shown in the subsection 3.2.4. The first coupler is fabricated by a fibre coupling fusion workstation as usual with the coupling ratio being set at 84.66%, while the second coupler is fabricated with the help of a broadband light source and an OSA. When fusing the second coupler, the fibre lengths between the couplers are made roughly the same. After the fabrication of the second coupler, one of the two fibres is then stretched to achieve the desired length difference under the monitoring of the OSA.

Connecting the filter to the output of the SLED where the Gaussian-like spectrum is measured, the equalized spectrum is observed on the OSA and it is shown in Fig. 5.16. It is shown that, with the maximally flat equalization filter, 0.5-dB flat passband bandwidth increased from 25 nm to 47 nm and the 3-dB passband bandwidth of SLED increased from 62 nm to 81 nm. This demonstrates the effectiveness of the proposed design approach. It is also noted that a 3.4 dB attenuation of the equalized spectrum is observed at the central wavelength. This is mainly due to the power penalty of the filter, and the losses caused by stretching the fibre arms in the fabrication process. The fabrication losses can be reduced by using a fibre coupling fusion workstation with a more precise motion system.

5.4 Maximally Flat Equalization of Gaussian-like Spectra with IIR Structure

To further improve the output power performance of the equalized spectra, in this section, we present an improved method for equalizing Gaussian-like spectra by using all-fibre lattice filters with symmetric feedback structure, *i.e.* IIR structure. This structure is motivated from the observation that one output power of the lattice structure proposed in the section 5.3 is totally lost. Connecting this output port

Figure 5.16: Experimental SLED spectrum responses with the maximally flat transmission, where the original and equalized spectra are both displayed for comparison.

back to the other input port obviously will increase the total output power, but this would result in an all-pass filter that cannot be used to design the equalization filter. Thus, adding certain filter functions in the feedback loop would be useful to increase the output power as well as making the equalization filter feasible. The proposed feedback structure is designed to have two identical lattice filters but connected in a reverse manner. With the structure, the output power of Gaussian-like spectra is increased as compared with the one proposed in the section 5.3. Simulation and experimental results verify the theoretical analysis.

5.4.1 Proposed Structure and Design Scheme

To equalize the Gaussian-like spectrum, an equalization filter is also constructed in two steps like the method developed in the section 5.3. The objective of spectrum

equalization can thus be stated as designing the filter with a transmission spectrum P satisfying two critical specifications as shown in the section 5.3, *i.e.* to achieve the maximal output power and as-flat-as-possible the output spectrum.

The proposed feedback structure consists of two identical M -th order optical lattice filters, denoted respectively by lattice filter 1 and lattice filter 2, connected in a loop structure as shown in Fig. 5.17. Each M -th order optical lattice filter is formed by linking $(M + 1)$ 2×2 directional couplers with differential time-delay lines, as shown in Fig. 5.8. Denote the electric field at the i -th port of lattice filter 1 and lattice filter 2 as E'_i and E''_i , $i = 1, 2, 3, 4$, respectively. Two M -th order lattice filters are connected in a reverse manner as shown in Fig. 5.17. That is, the output port E'_4 of the lattice filter 1 is connected to the input port E''_1 of the lattice filter 2. Conversely, the output port E''_4 of the lattice filter 2 is connected to the input port E'_1 of the lattice filter 1.

Figure 5.17: The proposed equalization filter with symmetric feedback structure for Gaussian spectrum equalization.

As the two lattice filters are connected by fiber lines with a long length, much longer than the wavelength of the wave propagating through the equalization filter, the circular optical signals that pass through lattice filter 2 are incoherent with that transmitted directly through lattice filter 1. Thus, the power transfer function P_{fb}^M from input E'_1 to output E'_3 via the feedback loop can be easily expressed as

$$P_{fb}^M = \frac{2P_{bar}^M}{1 + P_{bar}^M} \quad (5.35)$$

where P_{bar}^M is the bar-state transmission spectrum that can be expressed by a combination of $\cos(2(k-1)w)$ like that of Eqn.5.7. Then, the design problem of the equalization filter is to obtain a transmission spectrum with the feedback loop P_{fb}^M by choosing appropriate parameters c_i and s_i , $i = 1, 2, \dots, M+1$, such that the overall spectrum $P_{fb}^M F$ is as flat as possible in its central wavelength while the magnitude of $P_{fb}^M F$ is as big as possible.

5.4.2 Design of the Equalization Filter

A design scheme is presented here to obtain the equalization filter to flatten the Gaussian spectrum with the desired performance as stated above. The design includes several steps. First, as it can be verified through Eqn.(5.35) that maximizing the value of $P_{fb}^M F$ equals to maximizing the value of $P_{bar}^M F$, the desired transmission spectrum P_{bar}^M is thus obtained by applying the maximally flat principle of analytic functions on the spectrum $P_{bar}^M(w)F(w)$ and meanwhile seeking a minimal filter attenuation. As a result, the coefficients of the desired P_{bar}^M are obtained by numerically solving a constrained optimization problem. As the constraints correspond to a set of M -dimensional nonlinear equations with respect to the design parameters, the optimization method in the last section 5.3 cannot be applied directly to obtain the optimal design parameters where linear property exists among those parameters.

Thus, a recursive design procedure is proposed in this section to achieve the optimal coefficients of design filter P_{bar}^M satisfying the two basic requirements. Then, the field transfer function T_{bar}^M is obtained by normalizing and factoring the spectrum P_{bar}^M . Lastly, the coupling ratio of each fibre coupler is calculated from T_{bar}^M by a recursive algorithm.

As the desired power output transmission after equalization is designed to have a spectrum as flat as possible, it can be predicated that the transmission value around its central wavelength within the flattest transmission bandwidth should be a constant. This constant value denoted as C can be expressed as

$$C = \frac{2P_{bar}^M}{1 + P_{bar}^M} e^{-w^2/2\delta^2} \Big|_{w=0} = \frac{2 \sum_{k=1}^{M+1} a_{k,M+1}}{1 + \sum_{k=1}^{M+1} a_{k,M+1}} \quad (5.36)$$

From Eqn.(5.36), we have

$$1 + P_{bar}^M = \frac{2}{C} P_{bar}^M F \quad (5.37)$$

In order to obtain a flat spectral response, we examine the first $2M+2$ derivatives on both sides of the Eqn.(5.37) with respect to w at $w = 0$. Using the chain rule, the q -th order derivative can be obtained as

$$\begin{aligned} \frac{d^q(1 + P_{bar}^M)}{d^q w} \Big|_{w=0} &= \frac{d^q(\frac{2}{C} P_{bar}^M F)}{d^q w} \Big|_{w=0} \\ &= \frac{2}{C} \sum_{l=0}^q \binom{q}{l} \frac{d^{q-l}(P_{bar}^M)}{d^l w^{q-l}} \frac{d^l(F)}{d^l w} \Big|_{w=0} \end{aligned} \quad (5.38)$$

for $q = 1, 2, \dots, 2M + 2$.

It is clear that the odd order derivatives of P_{bar}^M on the left side of the Eqns.(5.38) are zeros automatically when evaluated at $w = 0$. It is also noted that all the odd order derivatives of $P_{bar}^M F$ on right side of the Eqns.(5.38) are zeros when evaluated at $w = 0$. This can be verified as follows. Denote $P_{bar}^M F$ by $f(w)$. As both P_{bar}^M

and F are even functions, so is $f(w)$, thus we have $f^{(2p)}(w) = f^{(2p)}(-w)$, $p = 1, 2, \dots, M+1$. Then all its odd order derivatives with respect to w have the relationship $f^{(2p-1)}(w) = -f^{(2p-1)}(-w)$, ($p = 1, 2, \dots, M+1$). That is, all the odd order derivatives are odd functions. Therefore, their values at $w = 0$ are zeros. To examine the maximal flatness of the equalized spectrum, it is sufficient to evaluate the even order derivatives of P_{bar}^M and $P_{bar}^M F$. Then, we have the following set of $M+1$ equations from Eqns.(5.38),

$$\sum_{k=1}^{M+1} \left[(-1)^p (2k-2)^{(2p)} - \frac{2}{C} \sum_{l=0}^p \binom{2p}{2l} (-1)^p (2k-2)^{(2p-2l)} \frac{\prod_{i=1}^l (2i-1)}{\delta^{2l}} \right] a_{k,M+1} = 0 \quad (5.39)$$

with $p = 1, 2, \dots, M+1$.

Representing the unknown coefficients of $a_{k,M+1}$ ($k = 1, \dots, M+1$) to form a vector $\mathbf{v} = [a_{1,M+1}, a_{2,M+1}, \dots, a_{M+1,M+1}]^T$, equations in (5.39) can be expressed in a matrix form as:

$$\frac{2}{C} \begin{pmatrix} \frac{1}{\delta^2} & (1 - \frac{C}{2})2^2 + \frac{1}{\delta^2} & \dots & (1 - \frac{C}{2})(2M)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & \dots & \dots & (1 - \frac{C}{2})(2M)^4 + C_4^2 \frac{(2M)^2}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & a_{0,p,k} & \vdots \\ \frac{\prod_{i=1}^{M+1} (2i-1)}{\delta^{2M+2}} & \dots & \dots & (1 - \frac{C}{2})(2M)^{2M+2} + \sum_{l=1}^{M+1} C_{2M+2}^{2l} \frac{(2M)^{(2M+2-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \times \mathbf{v} = 0 \quad (5.40)$$

where $a_{0,p,k} = (1 - \frac{C}{2})(2k-2)^{2p} + \sum_{l=1}^p C_{2p}^{2l} \frac{(2k-2)^{(2p-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}$ with $p = 1, 2, \dots, M+1$ and $k = 1, 2, \dots, M+1$.

Denote the matrix in the left hand side of the above equation as $\mathbf{A}_0$. It is clearly a full rank matrix of $R^{(M+1) \times (M+1)}$. This matrix is obtained from the first specification for an equalization filter to have the maximally flat transmission. It is verified in a similar way, as shown in the Section 5.3, that only a flat spectrum in the sense of $(2M+1)$ -th order maximally flatness can be achieved by using this principle, *i.e.* an

M -th order nonlinear equations corresponding to even order derivatives of $P_{bar}^M F$ are need to be solved. In order to determine the $(M+1)$ -dimensional unknown vector $\mathbf{v}$, the second specification for an equalization filter to have a small attenuation in order to have a strong output power can be selected as one more constraint. Thus, the overall attenuation resulted from the equalization filter shall be made as small as possible. This is equivalent to maximizing the value of $P_{fb}^M F$ at $w = 0$, *i.e.*,

$$\max_{\{a_{1,M+1}, a_{2,M+1}, \dots, a_{M+1,M+1}\}} \{(P_{fb}^M F)|_{w=0}\} \quad (5.41)$$

From Eqn.(5.35), it can be verified that the maximization of the value of $P_{fb}^M F$ equals to maximize the value of $P_{bar}^M F$. After applying Eqn.(5.7) and using $F = \beta e^{-(w-w_0)^2/2\delta^2}$, Eqn.(5.41) becomes

$$\max_{\{a_{1,M+1}, a_{2,M+1}, \dots, a_{M+1,M+1}\}} \left\{ \sum_{k=1}^{M+1} a_{k,M+1} \right\} \quad (5.42)$$

Now, the design of equalization filter can be described by solving an optimization problem in Eqn.(5.42) with constraints specified by the first M linear equations in Eqn.(5.40). Thus, the desired coefficients vector $\mathbf{v}$ shall satisfy

$$\begin{cases} \max\{g(\mathbf{v})\} \\ s.t. \quad \mathbf{A}\mathbf{v} = 0 \end{cases} \quad (5.43)$$

where $g(\mathbf{v}) = \sum_{k=1}^{M+1} a_{k,M+1}$ and the matrix $\mathbf{A}$ is given by the first M rows of $\mathbf{A}_0$, i.e.

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\delta^2} & (1 - \frac{C}{2})2^2 + \frac{1}{\delta^2} & \cdots & (1 - \frac{C}{2})(2M)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & (1 - \frac{C}{2})2^4 + C_4^2 \frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (1 - \frac{C}{2})(2M)^4 + C_4^2 \frac{(2M)^2}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & a_{0,p,k} & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}} & \cdots & \cdots & (1 - \frac{C}{2})(2M)^{2M} + \sum_{l=1}^M C_{2M}^{2l} \frac{(2M)^{(2M-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \quad (5.44)$$

To proceed, similar to the derivation in the Section 5.3, the constraint in (5.43) can be expressed as

$$\mathbf{A}_1 \mathbf{v}_1 + \mathbf{a}_1 v_2 = 0 \quad (5.45)$$

where the matrix $\mathbf{A}$ is partitioned into $\mathbf{A} = [\mathbf{A}_1, \mathbf{a}_1]$; $\mathbf{a}_1$ being the last column of $\mathbf{A}$ and v_2 being the last element of $\mathbf{v}$.

It is from Eqn.(5.7) that the last item of $P_{bar}^M(w)$ can be shown as $v_2 = -1/2 \sin 2\theta_1 \sin 2\theta_{M+1} \prod_{i=2}^M \cos \theta_i^2$. Hence, the absolute value of the last item cannot exceed $1/2$. Using (5.45), $\mathbf{v}_1$ can be expressed in terms of v_2 as $\mathbf{v}_1 = -\mathbf{A}_1^{-1} \mathbf{a}_1 v_2$. Substituting it into (5.43), the filter design is converted to solve the following constraint optimization:

$$\begin{cases} \max\{\mathbf{e}^T \mathbf{v}_1 + v_2\} = \max\{(1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1) v_2\} \\ s.t. \quad -1/2 \leq v_2 \leq 0 \end{cases} \quad (5.46)$$

where $\mathbf{e}$ stands for an $(M-1)$ dimensional vector $(1, 1, \dots, 1)^T$.

To solve (5.46), we examine the value of $1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1$. It is easy to verify the following matrix identity

$$1 - \mathbf{e}^T \mathbf{A}_1^{-1} \mathbf{a}_1 = \frac{\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}}{\det(\mathbf{A}_1)} \quad (5.47)$$

As shown in Appendix F (the derivation of which is similar to Appendix D), the determinant of $\begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is determined by the parity of M that corresponds to the number of couplers $M+1$. If M is an odd number, the value of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is negative; else, it is positive. It is also proven in Appendix G (the derivation of which is similar to Appendix E) that the determinant of $\mathbf{A}_1$ is always positive. Therefore, the sign of $1 - \mathbf{e}^T \mathbf{A}_1 \mathbf{a}_1$ depends on the parity of M . As v_2 is negative, in order to make the value of $\mathbf{e}^T \mathbf{v}_1 + v_2$ bigger than zero, M has to be an even number. Then, using Eqn.(5.45), the desired value of the vector $\mathbf{v}_1$ can be obtained by

$$\mathbf{v}_1 = \frac{1}{2} \mathbf{A}_1^{-1} \mathbf{a}_1 \quad \text{with } M \text{ an even number} \quad (5.48)$$

Thus, from the expression of $g(\mathbf{v})$ and using (5.47), we have

$$g(\mathbf{v}) = \frac{\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}}{\det(\mathbf{A}_1)} v_2 \quad (5.49)$$

The above equation can be transformed into an $(M+1)$ -th order linear equation with respect to $g(\mathbf{v})$ with real constant coefficients. To solve the maximal value of $g(\mathbf{v})$, since it is a nonlinear equation with respect to the design parameters $\mathbf{v} = [\mathbf{v}_1^T \ v_2]^T$, the normalization can not be done in a linear way as shown in Section 5.3 where the values between $\mathbf{v}_1^T$ and v_2 are linear dependant. Thus, we conduct an optimization approach by using a recursive procedure to obtain the optimal value of $g(\mathbf{v})$. The recursive procedure is outlined in Fig. 5.18 and presented as follows:

Step 1, solving the equation (5.49) with the initial value $v_2 = -\frac{1}{2}$, we can obtain the initial maximal value of $g(\mathbf{v})$. Denote these values as $v_2^{(0)} = -\frac{1}{2}$ (for M is an odd number) and $g^{(0)}(\mathbf{v})$, respectively.

Step 2, from Eqn.(5.48) and initial values of $g^{(r)}(\mathbf{v})$ and $v_2^{(r)}$ obtained in the last step, we have the value of $\mathbf{v}_1^{(r)}$ as

$$\mathbf{v}_1^{(r)} = - \left(\mathbf{A}_1^{(r)} \right)^{-1} \mathbf{a}_1^{(r)} v_2^{(r)} \quad (5.50)$$

where r denotes the $(r+1)$ -th recursive round in the design procedure. It is noted that $\mathbf{A}_1^{(r)}$ and $\mathbf{a}_1^{(r)}$ are obtained by applying $g^{(r)}(\mathbf{v})$ into the expressions of $\mathbf{A}_1$ and $\mathbf{a}_1$ as shown in Eqn.(5.44). This also demonstrates the nonlinear dependant property between $\mathbf{v}_1^{(r)}$ and $v_2^{(r)}$.

Step 3, to this end, we have the vector $\mathbf{v}^{(r)} = [(\mathbf{v}_1^{(r)})^T v_2^{(r)}]^T$, however, the summation of the absolute value of each element of $\mathbf{v}^{(r)}$ may not be 1. From the expression of the maximal value of P_{fb}^M , we have $P_{fb,max}^M = 2 \sum_{i=1}^{M+1} |v_{i,bef}^{(n)}| / (1 + \sum_{i=1}^{M+1} |v_{i,bef}^{(n)}|)$. We know that the optimal value of $P_{fb,max}^M$ shall be $P_{fb,opt}^M = 1$ so as to give the maximum passband transmission of the overall transfer function, where $v_{i,bef}^{(r)}$ represents the i -th element of $\mathbf{v}^{(r)}$ before normalization. Thus, the summation of the absolute value of each element of $\mathbf{v}^{(r)}$ should be $\sum_{i=1}^{M+1} |v_{i,bef}^{(n)}| = 1$, however, this value cannot exceed 1 so as to make the proposed passive lattice filter realizable. Therefore, to this end, we conduct a normalization to make sure $\sum_{i=1}^{M+1} |v_{i,bef}^{(n)}| = 1$ at certain integer n . In this case, the designed filter spectrum touches 1 to give the maximal output power which also yields a maximally flat passband transmission simultaneously. The normalization is formulated as:

$$\mathbf{v}_{Norm}^{(r)} = \frac{\mathbf{v}^{(r)}}{\sum_{i=1}^{M+1} |v_{i,bef}^{(r)}|} \quad (5.51)$$

and $g(\mathbf{v})^{(r+1)}$ and $v_2^{(r+1)}$ are updated by

$$g^{(r+1)}(\mathbf{v}) = \sum_{i=1}^{M+1} (v_{Norm,i}^{(r)}) \quad (5.52)$$

$$v_2^{(r+1)} = v_{Norm,M+1}^{(r)} \quad (5.53)$$

where $v_{Norm,i}^{(r)}$ represents the i -th element of $\mathbf{v}_{Norm}^{(r)}$.

Step 4, conducting the recursive procedure from *Step 2* to *Step 3* using Eqn.(5.50)-(5.53) with the updated $g(\mathbf{v})^{(r+1)}$ and $v_2^{(r+1)}$, until $\sum_{i=1}^{M+1} |v_{i,bef}^{(n)}|$ approaches 1 from the left hand side with some given specified error, e.g. 10^{-5} , at some integer n . Then, we obtain the optimal values of $\hat{\mathbf{v}}$ and $\hat{g}(\mathbf{v})$.

So far, we have obtained the coefficients $\hat{\mathbf{v}}$ of the desired transmission P_{bar}^M that enables an $(2M+1)$ -th order maximally flatness spectrum with minimal attenuation. Since the optical cascaded MZIs are passive filters, the maximal transmission cannot exceed 100%. This requirement is guaranteed in the last design procedure in *Step 3*. Denote this optimal normalized transmission spectrum by $\hat{P}_{bar}^M$.

With $\hat{P}_{bar}^M$, a synthesis algorithm will be employed to calculate the design parameters c_i and s_i ($i = 1, 2, \dots, M+1$). Denote c_i ($i = 1, 2, \dots, M+1$) by $\cos \theta_i$ and s_i by $\sin \theta_i$ where θ_i is in the interval $[0, \pi/2]$. The design parameters are therefore θ_i . The design parameters of θ_i can thus be solved by using Eqns.(5.23)-(5.25). Thus, the whole synthesis algorithm for derivation of equalization filter with IIR structure can be shown in the flow chart of 5.19.

Up to now, we have obtained a general quantitative approach to the design of a spectrum equalizer which can maximally flatten the Gaussian spectrum while yielding a minimal attenuation. It is worth noting that the approximation in the second stage is in the maximally flat sense and as a result the overall equalization ripples is mainly determined by the approximation error resulted in the first step.

Figure 5.18: The flow chart determining the optimal filter coefficient $\hat{\mathbf{v}}$ of the proposed equalization filter with feedback structure.

Figure 5.19: The flow chart illustrating the design algorithm for Gaussian spectrum equalization with the proposed equalization filter with feedback structure.

5.4.3 Design Issues of the Spectrum Equalization

Similar to the studies in the subsection 5.3.3 for the forward structure, we will study the two design issues of the proposed feedback structure. First, without loss of generality, the tunable scheme of the proposed equalization filters proposed in the last section 5.3 can still be used as shown in Figs.5.10 and 5.11.

Second, the approximate expressions of the bandwidth obtained in the last section is still applicable. As shown in the last section, the ϵ -dB bandwidth of the overall equalization spectrum in the wavelength domain can be assessed by

$$BW_{\epsilon} = 2 \left(\frac{\lambda_c \Delta L \pi n_{eff}}{\Delta L \pi n_{eff} - 2w_b \lambda_c} - \lambda_c \right) \quad (5.54)$$

where λ_c is the central wavelength of the overall spectrum and w_b can be obtained by

$$w_b = \left[\frac{(2M+2)! C(1 - 10^{-\epsilon/10})}{\mathbf{u}(\delta)^T \hat{\mathbf{v}}} \right]^{\frac{1}{2M+2}} \quad (5.55)$$

The approximation of the ϵ -dB bandwidth thus has a good accuracy when ϵ take small values. Table.5.3 compares several typical ϵ -dB bandwidths calculated using (5.54) to those measured from real spectrum. It is shown that ϵ -dB bandwidth obtained by (5.54) has a good accuracy when it is used to evaluate up to 0.5-dB bandwidth which is sufficient to the assessment of equalized spectra.

5.4.4 Simulation Studies

In this section, we provide some simulation studies to verify the effectiveness of the proposed spectrum equalization scheme by applying it to the design of an equalization filter to flatten the Gaussian-like spectrum of an SLED light source. Using the least-squares approximation method, the optimal Gaussian function is obtained

Table 5.3: ϵ -dB Bandwidth of The Equalized Spectrum Using The Proposed Equalization Scheme ($\Delta L = 21.3\mu m$ and The Lattice Filter Order $M = 1$)

ϵ -dB	bandwidth calculated using (5.54) (nm)	bandwidth measured (nm)
0.01dB	15.17	15.24
0.02dB	18.06	18.25
0.05dB	22.72	23.08
0.1dB	27.02	27.85
0.2dB	32.09	33.68
0.5dB	40.11	43.86

with the central wavelength at 1544.2 nm, the amplitude β at $\beta = 0.98$, and the derivation δ at $\delta = 0.66$. The resultant approximation error is within the range of 0.015 dB.

5.4.4.1 First order optical lattice filter (*i.e.* with two couplers)

First, we consider using the first order optical lattice filter as the equalization filter. The first order optical lattice filter is formed by linking two couplers through a unit differential time delay line. The design problem in this case is summarized as follows:

$$\max \{P_{fb}^1 F|_{w=0}\} \tag{5.56}$$

subject to

$$\frac{d(P_{fb}^1 F)}{dw} \Big|_{w=0} = \frac{d^2(P_{fb}^1 F)}{dw^2} \Big|_{w=0} = \frac{d^3(P_{fb}^1 F)}{dw^3} \Big|_{w=0} = 0 \tag{5.57}$$

Using the maximally flat conditions gives

$$\left(-\frac{1}{\delta^2}, \left(1 - \frac{c}{2}\right)2^2 + \frac{1}{\delta^2}\right) \begin{pmatrix} a_{1,2} \\ a_{2,2} \end{pmatrix} = 0 \tag{5.58}$$

The bar-stage transmission coefficients of P_{bar}^1 can be obtained by maximizing the value of $a_{1,1} + a_{1,2}$ with the constraint in Eqn.(5.58). Following the recursive design procedure (5.50)-(5.53), the resultant coefficients are obtained as $\hat{a}_{1,2} = 0.69298$, and $\hat{a}_{2,2} = -0.30701$, which correspond to $\theta_1 = 1.1239$ and $\theta_2 = 1.1172$. This means that the coupling ratios of the two couplers in the optical lattice filter are 18.68% and 19.20%, respectively. The result of applying the obtained equalization filter to the SLED spectrum is similar to that shown in Fig. 5.14. It is shown that the SLED spectrum has been flattened in a maximally flat sense at the central wavelength.

5.4.4.2 Higher order optical lattice filters

Now, we consider the effect of increasing the order of the optical lattice filter on the equalization performance. Simulation studies are carried out to examine the equalized SLED spectra with the order of the optical lattice filters increased from 1 to 5. The spectrum performance of SLEDs designed with various orders of optical lattice filters are compared in Fig. 5.20. Besides, the optimal amplitude expansion coefficients of the bar-state field transfer function and the corresponding circuit design parameters are summarized in Table 5.4. It is shown that, if the order of the optical lattice filters is increased, the bandwidth of the maximally flat spectrum will be increased. However, bigger attenuation occurs. The results in Fig. 5.20 show that the maximal output power decreases from 0.57 to 0.13 when the order of the optical lattice filter is increased from 1 to 5.

5.4.4.3 Comparisons with the other design methods

Lastly, we shall compare the performance of the proposed feedback approach with other design methods for flattening the Gaussian spectra. The first order MZI is used as the equalization filter to flatten the Gaussian spectra by using the proposed method, the mini-max method in the section 5.2, and the maximally flat design

Table 5.4: Design Parameters of Optical Lattice Filters with Different Orders for The Maximally Flat Spectrum of SLED

Number of differential delay lines, M	Power expansion coefficients before normalization $a_{i,M+1}$	Power expansion coefficients after normalization $\hat{a}_{i,M+1}$	Circuit parameters θ_{M+1}
1	$a_{1,2}=1.0589$ $a_{2,2}=-0.50$	$a_{1,2}^*=0.69298$ $a_{2,2}^*=-0.30701$	$\theta_1 = 1.1239$ $\theta_2 = 1.1172$
2	N/A	N/A	N/A
3	$a_{1,4}=10.38$ $a_{2,4}=-12.55$ $a_{3,4}=3.88$ $a_{4,4}=-0.50$	$a_{1,4}^*=0.50091,$ $a_{2,4}^*=-0.39984$ $a_{3,4}^*=0.08998$ $a_{4,4}^*=-0.00926$	$\theta_1 = 1.5582$ $\theta_2 = 0.1260$ $\theta_3 = \pi - 0.5251$ $\theta_4 = 0.8517$
4	N/A	N/A	N/A
5	$a_{1,6}=108.45$ $a_{2,6}=-171.40$ $a_{3,6}=88.79$ $a_{4,6}=-29.49$ $a_{5,6}=5.74$ $a_{6,6}=-0.50$	$a_{1,6}^*=0.42321$ $a_{2,6}^*=-0.40347$ $a_{3,6}^*=0.13569$ $a_{4,6}^*=-0.03237$ $a_{5,6}^*=0.00489$ $a_{6,6}^*=-0.000350$	$\theta_1 = 1.1398$ $\theta_2 = 1.3479$ $\theta_3 = \pi - 0.6154$ $\theta_4 = 1.1056$ $\theta_5 = \pi - 1.3197$ $\theta_6 = 0.2912$

method with forward structure in the section 5.3, respectively. The equalized spectra are respectively shown in Fig. 5.21, where the solid line represents the proposed method, the dashed line represents the maximally flat design method with forward structure and the dotted line represents the mini-max method. It can be seen clearly from Fig. 5.21, compared with the mini-max method, the proposed method has a much flatter passband transmission. The output spectrum equalized by using the mini-max method has an unavoidable passband ripple. This is due to the reason that the ripple variations of the output spectrum obtained by using the mini-max method is evenly distributed within the transmission bandwidth, thus, higher pass-

Figure 5.20: Comparisons of the overall spectra of SLED designed by using lattice filters with different orders in feedback structure.

band ripple is unavoidable. Compared with the maximally flat design method with forward structure, the output spectrum flatten by using the proposed scheme has a higher output power at the central wavelength of around 0.57 than that of 0.45. In conclusion, all the above simulation results demonstrate that the proposed design scheme can be used to design an equalization filter to flatten the SLED output spectrum in the maximally flat sense, and simultaneously minimizes the attenuation of the overall spectrum.

5.4.5 Experimental Results

To illustrate the effectiveness of the proposed design scheme, all fibre MZI is built to implement the optical lattice filter to equalize the SLED spectrum. The equalized spectrum is observed on an OSA and is shown in Fig. 5.22. For the purpose of comparison, an equalization filter with forward structure fabricated in the subsection 5.3.5 is used. The two flatten spectra, by using feedback structure and forward

Figure 5.21: The flattened spectra of SLEDs by using the proposed feedback method for Gaussian spectrum equalization in comparison with other design methods.

structure, respectively, are shown in Fig. 5.22. It can be clearly seen that, with the proposed equalization filter, 0.5-dB and 3-dB flat passband bandwidths are roughly the same for the two methods. However, the output power increases about 2 dB by using the proposed feedback method. This demonstrates the effectiveness of the proposed design approach.

5.5 Discussion

In this section, a comparison of the characteristics of the proposed equalization filter to the existing technologies is presented to highlight the efficiency of the proposed method, as shown in Table 5.5. It can be seen clearly from table 5.5 that the proposed method has the advantages of low loss, low dispersion, wide equalization range and low cost as compared with the commercial technologies, including multiple side-tap Bragg gratings, long-period gratings, and acousto-optical tunable filers. In

Figure 5.22: Comparison of the experimental spectral responses of the overall SLED spectra designed with the two schemes of with and without feedback structures.

addition, the other specifications are as follows: the loss penalty for the maximally flat designed filter is 3.4 dB as shown in Fig. 5.16, simple calculation on the filter group delay within the passband is about 6 ps, wavelength drift is within the range of ± 3 nm with respect to the original central wavelength, and active tuning is achieved by two means in the subsection 5.3.3.2 but at the expense of a relatively complex tuning diagnostics. These results verify that the proposed scheme has superior performance over the existing equalization methods in wide wavelength equalization, such as in broadband light sources.

5.6 Conclusion

In this chapter, we have examined the application of optical filter bank for Gaussian-like spectrum equalization by using a two-port optical filter bank. The main idea is

Table 5.5: Comparison of the proposed equalization filter with the existing technologies.

Specifications	Proposed equalizer	Multiple side-tap FBG	Long Period FBG	Acousto-optical tunable filter
Equalization Range (nm)	> 60	~ 10	~ 30	~ 1 or < 30
Dispersion	low	high	low	low
Material Used	fiber waveguide	Bragg grating	Bragg grating	fiber waveguide
Insertion Loss (dB)	< 1	< 3	< 2	< 3
Cost (US dollar)	tens	tens	tens	hundred
Active tuning	Yes	Yes	Yes	Yes
Wavelength drift	small	large	small	small

to realize the inverse of the input spectrum by using the equalization filter. Another critical specification is to achieve as small as possible attenuation for the overall spectrum. Because of the importance of Gaussian-like spectra, we focused more on the equalization of Gaussian-like spectrum in this thesis. Due to the spectral symmetry property of optical filter bank, we only need to illustrate the use of one of the ports of optical filter bank used for spectrum equalization.

In this chapter, the efforts are put on the *performance design* of the equalization filter. Initially, two approaches namely the *minimax* and *integration* design methods with complementary merits are proposed to design an SLED spectrum equalizer using an all-fiber MZI. It is shown that the *minimax* approach is more useful for equalizing the unevenness in the band of interest while the window *integration* approach is more useful for reducing the ripples. The minimax approach offers an advantage in that one can design an equalizer with a pre-specified bandwidth or total intensity change. Then, a systematic approach to the design of an optical lattice filter is relaxed to equalize *Gaussian-like* spectra. Through the use of a two-step design procedure, it has been shown that the proposed design scheme can be used to design an equalization filter to flatten the Gaussian-like spectrum to the maximal extent. Since the design only involves a simple numerical optimization and a recur-

sive calculation of the lattice filter parameters, the equalizer design is much more efficient than the existing schemes. Finally, an optical lattice filter with feedback structure to equalize Gaussian-like spectra is proposed. With the proposed feedback structure, the total output power is increased. Since the feedback structure is symmetric to the forward one, it greatly reduces the number of design parameters, simplifies the design procedure and eases the fabrication of such equalization filters. All these results demonstrate that the proposed filter bank design approach is effective for the design of spectrum equalization filters.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

This thesis is dedicated to the study of the structure, the design and the implementation of optical filter bank systems. From the structure design point of view, both two-port and higher order multi-port structures are examined. From the signal processing perspective, both FIR and IIR optical filters are developed for the optical filter bank design. Then, the developed schemes are successfully applied in designing optical interleavers, spectrum equalization filters and multi-wavelength fiber ring lasers for applications in optical communications. To summarize the main contributions of the thesis, we emphasize the following:

1. A mathematical model of the physical constraints on optical periodic filter banks is established. The constraints come from the inherent properties such as the positivity of optical signal and the limited number of basic optical components that are available for constructing more complex devices.

2. In order to develop the design theory of optical filter bank, both two-port and more efficient three-port optical filter banks are systematically investigated and suitable implementations are proposed.
3. Most of the current available optical filters are designed by numerically optimizing certain integration criteria. In practice, it is preferred to design filters with a desired performance through optimally designed filter parameters, and quantitative results of the filter parameters are thus expected. Therefore, in this thesis, we have presented systematic and quantitative design approaches for optical filter bank designs with guaranteed performance for two-port and multi-port structures.
4. We have studied the FIR and IIR efficient structures for optical filter bank designs in details. We have shown that by selecting some of the design parameters in the structures properly, the design of MIMO systems can be successfully simplified into SISO system design. The rest of the design parameters can be used to design the filter responses to meet the required specifications. This greatly reduces the computational cost and simplifies the fabrication of such optical devices.
5. To realize an all-fibre implementation, we have developed an innovative fabrication technology to implement the designed optical filter banks. As we only fuse several pieces of single mode fibers by using this technology, the cost for fabrication of all-fiber devices is quite low.
6. The effects of designed filter's uncertainties (*i.e.* sensitivity to variations and noise) are investigated through simulations. It is shown that the proposed design scheme is robust to the variations of design parameters and these confirm the effectiveness of the proposed method.
7. The proposed design schemes have been successfully verified in the design of

optical interleavers and spectrum equalizers as application examples.

6.2 Recommendations for Future Research

While several results on optical filter banks have been derived, there are many other design approaches and concepts that can be investigated. A list of the recommendations for future research is compiled below.

1. As discussed in the thesis, the proposed structures for both two-port and three-port optical filter banks are efficient and they make less use optical elements as compared with the existing technologies. However, it is still unknown as to whether it is the *minimal* number of parameters required to build an MIMO optical filter bank system. It is expected that more efficient structures can be developed. This should be explored more from the structure design point of view. The construction of physically implementable optical building blocks that have efficient and useful transfer matrices would be a key factor.
2. It is of interest to examine whether the optical filter bank theory can be further applied to higher multi-band operations. Theoretically, there is no significant obstacle in achieving this target, however, the difficulty lies in that there is currently no appropriate technology for implementing such physical principle. The design and development of novel low-loss multi-mode interference (MMI) couplers or star couplers with more flexibility on their transfer matrices would help to simplify the design of multi-band optical filter banks. Alternatively, the decomposition of high dimensional matrices into simple lower dimensional matrices that can be easily implemented by standard optical elements is a viable option.

Although array waveguide gratings (AWGs) or waveguide grating routers (WGRs)

can be used in the multiplexing/de-multiplexing of multi-port transmissions, they are basically operating in only one channel for each output port. In addition, such technologies are developed by using the waveguide technology. It is still of great concerns as to whether they can be implemented in all-fiber structures.

3. The design of spectrum equalization filter bank has been illustrated for Gaussian-like spectrum. A future research direction would be to examine the design of filter bank for the equalization of a set of arbitrary input spectra. This could be explored from the structure design point of view by equalizing the arbitrary input spectra directly. This could also be developed by representing the arbitrary input spectra with several Gaussian functions, with each of the Gaussian functions equalized separately before combining them together to produce an equalized spectra of the initial arbitrary input.
4. Another new research area is photonic bandgap grating (PBG) engineering, which employs periodic structures with sub-micron element sizes to make devices. The PBGs are created by fabricating a periodic structure with a high index contrast so that it reflects over a large wavelength range, the photonic bandgap. One dimensional PBGs, which are high index contrast gratings, are FBGs. The PBG structures are expected to be more robust to random fabrication variations since the field can be localized in air regions away from the high dielectric contrast interfaces. Due to its intrinsic periodic structures of PBGs, PBGs are more suitable for optical periodic filter bank design. However, many details need to be explored, such as establishing efficient models and producing practical fabrication procedures. It is noted that the structure design of the cross section of the PBG is also of great importance in order to produce a desired bandgap filter response.
5. In many applications in the optical communication field, both the input signal

and the system usually exhibit strong stochastic properties. In addition, it is highly desired to actively tune or adjust the optical filter to accommodate dynamic variations of the optical communication system. Thus, adaptive filtering approaches should be explored to achieve adaptive optical filters, such as gain equalizer and dispersion compensator. These results can then be extended to the design of other optical filters such as multiuser detectors and optical add/drop multiplexer. In fact, in current Erbium-Doped Fiber Amplifier (EDFA) gain clamping system design, a feedback loop has been used to actively tune the parameters of the EDFA system to maintain the input and output power levels.

APPENDIX A.

Design Conditions on the Desired Response $\tilde{H}_{11}(z)$

The purpose of this appendix is to address several issues on the coefficients of $\tilde{H}_{11}(z)$. We will show that the following equation (A.1) will be satisfied if the number of coefficients of $\tilde{H}_{11}(z)$ is $(6M + 3)$ (M is an integer):

$$b_{1,2M-1}^M = c_{1,2M-1}^M \quad \text{and} \quad a_{2M-1,2M-1}^M = b_{2M-1,2M-1}^M \quad (\text{A.1})$$

We have, from the matrix expression S_{out} in Eqn.(4.9),

$$S_{out}^{-1} = \begin{pmatrix} \hat{H}_{11}^0(z) & \hat{H}_{12}^0(z) & \hat{H}_{13}^0(z) \\ \hat{H}_{21}^0(z) & \hat{H}_{22}^0(z) & \hat{H}_{23}^0(z) \\ \hat{H}_{31}^0(z) & \hat{H}_{32}^0(z) & \hat{H}_{33}^0(z) \end{pmatrix} \quad (\text{A.2})$$

where S_{out}^{-1} is the inverse matrix of S_{out} .

If the highest order of $z^{3M+1}\tilde{H}_{11}(z)$ is $3M + 1$, then the filter length of $\tilde{H}_{11}(z)$ is $2(3M + 1) + 1$. $\tilde{H}_{11}(z)$ can be expressed as

$$\begin{aligned} z^{3M+1}\tilde{H}_{11} &= \tilde{a}_{1,2(3M+1)+1}z^{3M+1} + \tilde{a}_{2,2(3M+1)+1}z^{3M} + \cdots + \tilde{a}_{2(3M+1)+1,2(3M+1)+1}z^{-(3M+1)} \\ &= (\tilde{a}_{1,2(3M+1)+1}z^{3M+1} + \tilde{a}_{4,2(3M+1)+1}z^{3M-2} + \cdots + \tilde{a}_{2(3M+1)-1,2(3M+1)+1}z^{-3M+1}) \\ &\quad + (\tilde{a}_{2,2(3M+1)+1}z^{3M} + \tilde{a}_{5,2(3M+1)+1}z^{3(M-1)} + \cdots + \tilde{a}_{2(3M+1),2(3M+1)+1}z^{-3M}) \\ &\quad + (\tilde{a}_{3,2(3M+1)+1}z^{3M-1} + \tilde{a}_{6,2(3M+1)+1}z^{3(M-1)-1} + \cdots + \tilde{a}_{2(3M+1)+1,2(3M+1)+1}z^{-3M-1}) \end{aligned} \quad (\text{A.3})$$

From Eqn.(4.16), $\tilde{H}_{21}$ and $\tilde{H}_{31}$ can be obtained from Eqn.(A.3):

$$\begin{aligned}
 z^{3M+1} \tilde{H}_{21} &= e^{j\frac{2\pi}{3}} \tilde{H}_{11}(ze^{j\frac{2\pi}{3}}) \\
 &= e^{j\frac{4\pi}{3}} (\tilde{a}_{1,2(3M+1)+1} z^{3M+1} + \tilde{a}_{4,2(3M+1)+1} z^{3M-2} + \dots + \tilde{a}_{2(3M+1)-1,2(3M+1)+1} z^{-3M+1}) \\
 &\quad + e^{j\frac{2\pi}{3}} (\tilde{a}_{2,2(3M+1)+1} z^{3M} + \tilde{a}_{5,2(3M+1)+1} z^{3(M-1)} + \dots + \tilde{a}_{2(3M+1),2(3M+1)+1} z^{-3M}) \\
 &\quad + (\tilde{a}_{3,2(3M+1)+1} z^{3M-1} + \tilde{a}_{6,2(3M+1)+1} z^{3(M-1)-1} + \dots + \tilde{a}_{2(3M+1)+1,2(3M+1)+1} z^{-3M-1})
 \end{aligned} \tag{A.4}$$

$$\begin{aligned}
 z^{3M+1} \tilde{H}_{31} &= \tilde{H}_{11}(ze^{j\frac{4\pi}{3}}) \\
 &= e^{j\frac{4\pi}{3}} (\tilde{a}_{1,2(3M+1)+1} z^{3M+1} + \tilde{a}_{4,2(3M+1)+1} z^{3M-2} + \dots + \tilde{a}_{2(3M+1)-1,2(3M+1)+1} z^{-3M+1}) \\
 &\quad + (\tilde{a}_{2,2(3M+1)+1} z^{3M} + \tilde{a}_{5,2(3M+1)+1} z^{3(M-1)} + \dots + \tilde{a}_{2(3M+1),2(3M+1)+1} z^{-3M}) \\
 &\quad + e^{j\frac{2\pi}{3}} (\tilde{a}_{3,2(3M+1)+1} z^{3M-1} + \tilde{a}_{6,2(3M+1)+1} z^{3(M-1)-1} + \dots + \tilde{a}_{2(3M+1)+1,2(3M+1)+1} z^{-3M-1})
 \end{aligned} \tag{A.5}$$

Since S_{out} is invertible, the first column of S^N can be expressed as $[H_{11}^M(z^3) \ H_{21}^M(z^3) \ H_{31}^M(z^3)]^T = S_{out}^{-1}[\tilde{H}_{11}(z) \ \tilde{H}_{21}(z) \ \tilde{H}_{31}(z)]^T$. Thus, we can have all the coefficients of $H_{11}^N(z^3)$, $H_{21}^M(z^3)$, and $H_{31}^M(z^3)$. By examining the first coefficient of the expressions $H_{21}^M(z^3)$ and $H_{31}^M(z^3)$ respectively, we have the following equations,

$$\begin{aligned}
 b_{1,2M-1}^M &= \tilde{a}_{1,2(3M+1)+1} + e^{j\frac{4\pi}{3}} \tilde{a}_{2,2(3M+1)+1} + e^{j\frac{2\pi}{3}} \tilde{a}_{3,2(3M+1)+1} \\
 c_{1,2M-1}^M &= \tilde{a}_{1,2(3M+1)+1} + e^{j\frac{2\pi}{3}} \tilde{a}_{2,2(3M+1)+1} + e^{j\frac{4\pi}{3}} \tilde{a}_{3,2(3M+1)+1}
 \end{aligned} \tag{A.6}$$

If $b_{1,2M-1}^M = c_{1,2M-1}^M$ is satisfied, we have $\tilde{a}_{2,2(3M+1)+1} = \tilde{a}_{3,2(3M+1)+1}$.

By examining the last coefficient of the expressions $H_{11}^M(z^3)$ and $H_{21}^M(z^3)$ respectively, we have the following equations,

$$\begin{aligned}
 a_{2M-1,2M-1}^M &= e^{j\frac{2\pi}{3}} \tilde{a}_{2(3M+1)-1,2(3M+1)+1} + e^{j\frac{2\pi}{3}} \tilde{a}_{2(3M+1),2(3M+1)+1} \\
 &\quad + e^{j\frac{2\pi}{3}} \tilde{a}_{2(3M+1)+1,2(3M+1)+1} \\
 b_{2M-1,2M-1}^M &= \tilde{a}_{2(3M+1)-1,2(3M+1)+1} + e^{j\frac{4\pi}{3}} \tilde{a}_{2(3M+1),2(3M+1)+1} \\
 &\quad + e^{j\frac{2\pi}{3}} \tilde{a}_{2(3M+1)+1,2(3M+1)+1}
 \end{aligned} \tag{A.7}$$

If $\tilde{a}_{2M-1,2M-1}^M = b_{2M-1,2M-1}^M$ is satisfied, we have $e^{j\frac{2\pi}{3}} \tilde{a}_{2(3M+1),2(3M+1)+1} = \tilde{a}_{2(3M+1)-1,2(3M+1)+1}$.

Besides, from Eqn.(4.29) we have $\frac{\tilde{a}_{2(3M+1)+1,2(3M+1)+1}}{\tilde{a}_{1,2(3M+1)+1}} = \frac{\tilde{a}_{2(3M+1),2(3M+1)+1}}{\tilde{a}_{2,2(3M+1)+1}}$. It is easy to verify that if the highest order of $\tilde{H}_{11}(z)$ is $3M$ or $3M + 2$, Eqn.(A.1) cannot be satisfied.

APPENDIX B

Proof That the Second Last Item of (4.33) is Zero

In this appendix, the objective is to verify that $-(\delta_N + \gamma_N)a_{2N-2,2N-1}^N + \delta_N b_{2N-2,2N-1}^N + \delta_N c_{2N-2,2N-1}^N$ is equal to zero automatically. By examining the optical light paths, we have

$$\begin{aligned}
 a_{2N-2,2N-1}^N &= \sum_{i=2}^{N-1} \delta_1 \delta_N \delta_i \delta_{i+1} \prod_{m \neq i, i+1}^{N-1} \gamma_m e^{j \sum_{k=1, k \neq \phi_{k,2}}^{N-1} \phi_{k,2}} \\
 &\quad + (\delta_1 \gamma_2 \delta_{N-1} \delta_N e^{j \sum_{k=1}^{N-2} \phi_{k,2}} + \delta_1 \delta_2 \gamma_{N-1} \delta_N e^{j \sum_{k=2}^{N-1} \phi_{k,2}}) \prod_{m=3}^{N-2} \gamma_m \\
 b_{2N-2,2N-1}^N &= \sum_{i=2}^{N-1} \delta_1 \delta_N \delta_i \delta_{i+1} \prod_{m \neq i, i+1}^{N-1} \gamma_m e^{j \sum_{k=1, k \neq \phi_{k,2}}^{N-1} \phi_{k,2}} \\
 &\quad + (\delta_1 \gamma_2 \delta_{N-1} \gamma_N e^{j \sum_{k=1}^{N-2} \phi_{k,2}} + \delta_1 \delta_2 \gamma_{N-1} \delta_N e^{j \sum_{k=2}^{N-1} \phi_{k,2}}) \prod_{m=3}^{N-2} \gamma_m \\
 c_{2N-2,2N-1}^N &= \sum_{i=2}^{N-1} \delta_1 \delta_N \delta_i \delta_{i+1} \prod_{m \neq i, i+1}^{N-1} \gamma_m e^{j \sum_{k=1, k \neq \phi_{k,2}}^{N-1} \phi_{k,2}} \\
 &\quad + (\delta_1 \gamma_2 \delta_{N-1} \delta_N e^{j \sum_{k=1}^{N-2} \phi_{k,2}} + \delta_1 \delta_2 \gamma_{N-1} \gamma_N e^{j \sum_{k=2}^{N-1} \phi_{k,2}}) \prod_{m=3}^{N-2} \gamma_m
 \end{aligned} \tag{B.1}$$

where $a_{2N-2,2N-1}^N$, $b_{2N-2,2N-1}^N$ and $c_{2N-2,2N-1}^N$ represent the second lowest order coefficients of

$z^{3(N-1)} H_{11}^N(z^3)$, $z^{3(N-1)} H_{21}^N(z^3)$ and $z^{3(N-1)} H_{31}^N(z^3)$ respectively, thus we have

$$\begin{aligned}
 &-(\delta_N + \gamma_N)a_{2N-2,2N-1}^N + \delta_N b_{2N-2,2N-1}^N + \delta_N c_{2N-2,2N-1}^N \\
 = &-(\delta_N + \gamma_N) \left(\sum_{i=2}^{N-1} \delta_1 \delta_N \delta_i \delta_{i+1} \prod_{m \neq i, i+1}^{N-1} \gamma_m \right. \\
 &+ (\delta_1 \delta_2 \gamma_{N-1} \delta_N + \delta_1 \gamma_2 \delta_{N-1} \delta_N) \prod_{m=3}^{N-2} \gamma_m \\
 &+ \delta_N \left(\sum_{i=2}^{N-1} \delta_1 \delta_N \delta_i \delta_{i+1} \prod_{m \neq i, i+1}^{N-1} \gamma_m + (\delta_1 \gamma_2 \delta_{N-1} \gamma_N + \delta_1 \delta_2 \gamma_{N-1} \delta_N) \prod_{m=3}^{N-2} \gamma_m \right) \\
 &+ \delta_N \left(\sum_{i=2}^{N-1} \delta_1 \delta_N \delta_i \delta_{i+1} \prod_{m \neq i, i+1}^{N-1} \gamma_m + (\delta_1 \gamma_2 \delta_{N-1} \delta_N + \delta_1 \delta_2 \gamma_{N-1} \gamma_N) \prod_{m=3}^{N-2} \gamma_m \right) \\
 = &0
 \end{aligned} \tag{B.2}$$

APPENDIX C

Proof That the Second Item of (4.36) is Zero

The purpose of this section is to verify that $\delta_N a_{2,2N-1}^N + \delta_N b_{2,2N-1}^N - (\delta_N + \gamma_N) c_{2,2N-1}^N$ is equal to zero automatically. By examining the optical light paths, we have

$$\begin{aligned}
 a_{2,2N-1}^N &= \sum_{i=1}^{N-1} \delta_i \delta_{i+1} \prod_{m \neq i, i+1} \gamma_m e^{j \sum_{k=1, k \neq i}^{N-1} \phi_{k,1}} \\
 b_{2,2N-1}^N &= \sum_{i=1}^{N-2} \delta_i \delta_{i+1} \prod_{m \neq i, i+1} \gamma_m e^{j \sum_{k=1, k \neq i}^{N-2} \phi_{k,1}} + \delta_{N-1} \gamma_N \prod_{m=1}^{N-2} \gamma_m e^{j \sum_{k=1}^{N-2} \phi_{k,1}} \\
 c_{2,2N-1}^N &= \sum_{i=1}^{N-2} \delta_i \delta_{i+1} \prod_{m \neq i, i+1} \gamma_m e^{j \sum_{k=1, k \neq i}^{N-2} \phi_{k,1}} + \delta_{N-1} \delta_N \prod_{m=1}^{N-2} \gamma_m e^{j \sum_{k=1}^{N-2} \phi_{k,1}}
 \end{aligned} \tag{C.1}$$

where $a_{2,2N-1}^N$, $b_{2,2N-1}^N$ and $c_{2,2N-1}^N$ represent the second highest order coefficients of $z^{3(N-1)} H_{11}^N(z^3)$, $z^{3(N-1)} H_{21}^N(z^3)$ and $z^{3(N-1)} H_{31}^N(z^3)$ respectively, thus we have

$$\begin{aligned}
 &\delta_N a_{2,2N-1}^N + \delta_N b_{2,2N-1}^N - (\delta_N + \gamma_N) c_{2,2N-1}^N \\
 &= \delta_N \left(\sum_{i=1}^{N-1} \delta_i \delta_{i+1} \prod_{m \neq i, i+1} \gamma_m \right) \\
 &\quad + \delta_N \left(\sum_{i=1}^{N-2} \delta_i \delta_{i+1} \prod_{m \neq i, i+1} \gamma_m + \delta_{N-1} \gamma_N \prod_{m=1}^{N-2} \gamma_m \right) \\
 &\quad - (\delta_N + \gamma_N) \left(\sum_{i=1}^{N-2} \delta_i \delta_{i+1} \prod_{m \neq i, i+1} \gamma_m + \delta_{N-1} \delta_N \prod_{m=1}^{N-2} \gamma_m \right) \\
 &= 0
 \end{aligned} \tag{C.2}$$

APPENDIX D.

The Sign of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is Determined by the Parity of M

The purpose of this appendix is to prove that the sign of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is determined by the parity of the order of the optical lattice filter M . It is shown that if M is an odd number, $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is negative; otherwise, $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is positive.

The proof can be established by performing several simple manipulations on the $(M+1) \times (M+1)$ matrix $(\mathbf{A}_1 \ \mathbf{a}_1; \mathbf{e}^T \ 1)$. Multiplying the last row with $-\prod_{i=1}^M (2i-1)/\delta^{2M}$, adding it respectively to the first M rows, and using Eqn.(5.16), the matrix $\begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ can be expressed as follows:

$$\begin{aligned}
& \begin{pmatrix} 1 & 0 & \cdots & 0 & -\frac{1}{\delta^2} \\ 0 & 1 & \cdots & 0 & -\frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \\
& \begin{pmatrix} \frac{1}{\delta^2} & 2^2 + \frac{1}{\delta^2} & \cdots & (2M)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & 2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (2M)^4 + 6(2M)^2 \frac{1}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}}, & \sum_{l=0}^M C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \cdots, & \sum_{l=0}^M C_{2M}^{2l} \frac{(2M)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \\ 1, & 1, & \cdots, & 1 \end{pmatrix} \\
& = \begin{pmatrix} 0 & 2^2 & \cdots & (2M)^2 \\ 0 & 2^4 + 6\frac{2^2}{\delta^2} & \cdots & (2M)^4 + 6(2M)^2 \frac{1}{\delta^2} \\ \vdots & \vdots & \ddots & \vdots \\ 0, & \sum_{l=0}^{M-1} C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \cdots, & \sum_{l=0}^{M-1} C_{2M}^{2l} \frac{(2M)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \\ 1, & 1, & \cdots, & 1 \end{pmatrix} \quad (D.1)
\end{aligned}$$

From this expression in (D.1), we have

$$\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix} = (-1)^{M+2} \det(\mathbf{B}_1) \quad (D.2)$$

where $\mathbf{B}_1$ denotes the upper right M by M sub-matrix of the matrix in the right hand side of Eqn.(D.1). Multiplying the first row of $\mathbf{B}_1$ with appropriate coefficients and adding it respectively to 2-nd - M -th rows of $\mathbf{B}_1$ (*i.e.* performing elementary row operations), it can be obtained that the determinant of $\mathbf{B}_1$ is equivalent to the

determinant of the following matrix:

$$\begin{pmatrix} 2^2 & 4^2 & \cdots & (2M)^2 \\ 2^4 & 4^4 & \cdots & (2M)^4 \\ \vdots & \vdots & \ddots & \vdots \\ 2^{2(M-1)} & 4^{2(M-1)} & \cdots & (2M)^{2(M-1)} \\ 2^{2M} & 4^{2M} & \cdots & (2M)^{2M} \end{pmatrix}$$

This matrix is a Vandermonde matrix. From the properties of Vandermonde matrix, we know that the determinant of the above matrix is always positive. Thus, from Eqn.(D.2), the sign of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is determined by the parity of M . If M is an odd number, $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is negative; else, it is positive.

APPENDIX E.

The Value of $\det(A_1)$ is Always Positive

In this section, we will prove that the determinant of matrix $\mathbf{A}_1$ is always positive.

By definition, $\mathbf{A}_1$ is the first M columns of the $M \times (M+1)$ matrix $\mathbf{A}$ defined in (5.16). Thus, the matrix $\mathbf{A}_1$ has the following expression:

$$\mathbf{A}_1 = \begin{pmatrix} \frac{1}{\delta^2} & 2^2 + \frac{1}{\delta^2} & \cdots & (2M-2)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & 2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (2M-2)^4 + 6(2M-2)^2\frac{1}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}}, & \sum_{l=0}^{M-1} C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \cdots & \sum_{l=0}^{M-1} C_{2M}^{2l} \frac{(2M-2)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix}$$

Before proving the positivity of $\det(\mathbf{A}_1)$, we first show that the matrix $\mathbf{A}_1$ can be transformed by elementary matrix transformations to the following matrix

$$\begin{pmatrix} \frac{1}{\delta^2} & 2^2 & \cdots & (2M-2)^2 \\ -\frac{3}{\delta^4} & 2^4 & \cdots & (2M-2)^4 \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{p-1} \frac{\prod_{i=1}^p (2i-1)}{\delta^{2p}} & 2^{2p} & \cdots & (2M-2)^{2p} \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{M-1} \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}}, & 2^{2M} & \cdots & (2M-2)^{2M} \end{pmatrix}$$

To show this assertion, an induction method is used to prove that the first p (for $p = 1, \dots, M$) rows of $\mathbf{A}_1$ can be transformed into the form given by

$$\begin{pmatrix} \frac{1}{\delta^2} & 2^2 & \cdots & (2M-2)^2 \\ -\frac{3}{\delta^4} & 2^4 & \cdots & (2M-2)^4 \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{p-1} \frac{\prod_{i=1}^p (2i-1)}{\delta^{2p}}, & 2^{2p} & \cdots & (2M-2)^{2p} \end{pmatrix} \quad (\text{E.1})$$

For ease of reference, we introduce a new notation $\mathbf{A}_1(p)$ as a matrix that is formed by the first p rows of $\mathbf{A}_1$. Clearly, the matrices $\mathbf{A}_1(1)$ and $\mathbf{A}_1(2)$ can be expressed into a form represented by (E.1). Now we assume that $\mathbf{A}_1(k-1)$ can be transformed into a form in (E.1) with $p = k-1$. That is, there are two elementary transformations denoted respectively by $\mathbf{E}_1$ and $\mathbf{E}_2$ satisfying

$$\mathbf{E}_1 \mathbf{A}_1(k) \mathbf{E}_2 = \begin{pmatrix} \frac{1}{\delta^2} & 2^2 & \cdots & (2M-2)^2 \\ -\frac{3}{\delta^4} & 2^4 & \cdots & (2M-2)^4 \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{k-2} \frac{\prod_{i=1}^{k-1} (2i-1)}{\delta^{2(k-1)}}, & 2^{2(k-1)} & \cdots & (2M-2)^{2(k-1)} \end{pmatrix} \quad (\text{E.2})$$

By induction, it is sufficient to prove that $\mathbf{A}_1(k)$ can be transformed into a form in (E.1) by elementary transformations. It is noted from the formation of $\mathbf{A}_1(k)$ that $\mathbf{A}_1(k)$ can be expressed in terms of $\mathbf{A}_1(k-1)$ by

$$\mathbf{A}_1(k) = \begin{pmatrix} \mathbf{A}_1(k-1) \\ \frac{\prod_{i=1}^k (2i-1)}{\delta^{2k}}, \sum_{l=0}^{k-1} C_{2k}^{2l} \frac{2^{(2k-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, \dots, \sum_{l=0}^{k-1} C_{2k}^{2l} \frac{(2M-2)^{(2k-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \quad (\text{E.3})$$

Based on the induction assumption, we have

$$\begin{aligned}
 & \begin{pmatrix} \mathbf{E}_1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{A}_1(k+1) \begin{pmatrix} \mathbf{E}_2 & 0 \\ 0 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} \mathbf{E}_1 \mathbf{A}_1(k) \mathbf{E}_2 \\ \frac{\prod_{i=1}^k (2i-1)}{\delta^{2k}}, \sum_{l=0}^{k-1} C_{2k}^{2l} \frac{2^{2(k-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, \dots, \sum_{l=0}^{k-1} C_{2k}^{2l} \frac{(2M-2)^{2(k-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \\
 &= \begin{pmatrix} \frac{1}{\delta^2} & 2^2 & \dots & (2M-2)^2 \\ -\frac{3}{\delta^4} & 2^4 & \dots & (2M-2)^4 \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{k-2} \frac{\prod_{i=1}^{k-1} (2i-1)}{\delta^{2k}}, & 2^{2(k-1)} & \dots & (2M-2)^{2(k-1)} \\ \frac{\prod_{i=1}^k (2i-1)}{\delta^{2k}}, & \sum_{l=0}^{k-1} \frac{C_{2k}^{2l} 2^{2(k-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \dots, & \sum_{l=0}^{k-1} \frac{C_{2k}^{2l} (2M-2)^{2(k-2l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \quad (\text{E.4})
 \end{aligned}$$

Multiplying the first $k-1$ rows in the above matrix with appropriate coefficients and adding them to the last row, the matrix (6.2) can be transformed into the form:

$$\begin{pmatrix} \frac{1}{\delta^2} & 2^2 & \dots & (2M-2)^2 \\ -\frac{3}{\delta^4} & 2^4 & \dots & (2M-2)^4 \\ \vdots & \vdots & \ddots & \vdots \\ (-1)^{(k-1)-1} \frac{\prod_{i=1}^{k-1} (2i-1)}{\delta^{2(k-1)}}, & 2^{2(k-1)} & \dots & (2M-2)^{2(k-1)} \\ h & 2^{2k} & \dots & (2M-2)^{2k} \end{pmatrix}$$

where the element h in the matrix is expressed as

$$\begin{aligned}
 h &= \left[-\frac{1}{\delta^2} C_{2k}^{2(k-1)} \frac{(2k-3)!}{2^{k-2}(k-2)!\delta^{2(k-1)}} + \frac{3}{\delta^4} C_{2k}^{2(k-2)} \frac{(2k-5)!}{2^{k-3}(k-3)!\delta^{2(k-2)}} + \dots \right. \\
 &\quad \left. + \frac{(-1)^{k-1} (2k-3)!}{2^{k-2}(k-2)!\delta^{2(k-1)}} C_{2k}^2 \frac{1}{\delta^2} \right] + \frac{(2k-1)!}{2^{k-1}(k-1)!\delta^{2k}} \quad (\text{E.5})
 \end{aligned}$$

If k is an odd number, it is trivial to verify that there are even items in the bracket in the above Eqn.(E.5) and their sum is zero. Thus, h can be simplified as

$$h = \frac{\prod_{i=1}^k (2i-1)}{\delta^{2k}} \quad \text{if } k \text{ is an odd number} \quad (\text{E.6})$$

If k is an even number, *i.e.* $k = 2n$ (n is a integer), there are $2n - 1$ items in the bracket in the Eqn.(E.5). Then, h can be expressed as

$$\begin{aligned} h &= \frac{2 \times 4n(4n-1)!}{2^{2n-1}(2n-1)!\delta^{2k}} \left\{ \sum_{i=1}^{2n-1} \frac{(-1)^i (2i-1)! 2^{2n-1} (2n-1)!}{2^{i-1} (i-1)! 2^{2n-i} (2n-i)! (2i)!} \right\} + \frac{\prod_{i=1}^k (2i-1)}{\delta^{2k}} \\ &= \frac{2 \times 4n(4n-1)!}{2^{2n-1}(2n-1)!\delta^{2k}} \left\{ \frac{-2}{8n} + \frac{1}{8n} \sum_{i=0}^{2n} (-1)^i C_{2n}^i \right\} + \frac{\prod_{i=1}^k (2i-1)}{\delta^{2k}} \\ &= -\frac{\prod_{i=1}^k (2i-1)}{\delta^{2k}} \quad \text{if } k \text{ is an even number} \end{aligned} \quad (\text{E.7})$$

Combining Eqns.(E.6) and (E.7) verifies the assertion that $\mathbf{A}_1(k)$ can be transformed into the form in Eqn.(E.1) by elementary transformations. This completes the induction.

Since all the involved transformations are elementary transformations, the determinant is not changed. Thus, the matrix $\mathbf{A}_1$ has the same determinant as the matrix in Eqn.(E.1) where the sign of the first column items are interlaced and the value of each item has the form shown in matrix (E.1). Then the determinant of matrix $\mathbf{A}_1$ in Eqn.(E.1) is obtained as

$$\det(\mathbf{A}_1) = \sum_{p=1}^M (-1)^{p+1} (-1)^{p-1} \frac{\prod_{i=1}^p (2i-1)}{\delta^{2p}} \det(\mathbf{A}_{p,1}) > 0 \quad (\text{E.8})$$

where $\mathbf{A}_{p,1}$ is an $(M-1) \times (M-1)$ matrix obtained after deleting the p -th row and the first column of $\mathbf{A}_1$. As $\mathbf{A}_{p,1}$ is a quasi-Vandermonde matrix, its determinant is positive. Thus, $\det(\mathbf{A}_1)$ is always positive, which confirms the conclusion.

APPENDIX F.

The Sign of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is Determined by the Parity of M

The purpose of this appendix is to prove that the sign of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is determined by the parity of the order of the optical lattice filter M . It is shown that if M is an odd number, $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is negative; otherwise, $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is positive.

The proof can be established by performing several simple manipulations on the $(M+1) \times (M+1)$ matrix $(\mathbf{A}_1 \ \mathbf{a}_1; \mathbf{e}^T \ 1)$. Multiplying the last row with $-\prod_{i=1}^M (2i-1)/\delta^{2M}$, adding it respectively to the first M rows, and using Eqn.(5.44), the matrix $\begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ can be expressed as follows:

$$\begin{aligned}
 & \begin{pmatrix} 1 & 0 & \cdots & 0 & -\frac{1}{\delta^2} \\ 0 & 1 & \cdots & 0 & -\frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \\
 & \begin{pmatrix} \frac{1}{\delta^2} & (1-\frac{C}{2})2^2 + \frac{1}{\delta^2} & \cdots & (1-\frac{C}{2})(2M)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & (1-\frac{C}{2})2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (1-\frac{C}{2})(2M)^4 + 6(2M)^2\frac{1}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}}, & (1-\frac{C}{2})2^{2M} + \sum_{l=1}^M C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \cdots, & (1-\frac{C}{2})(2M)^{2M} + \sum_{l=1}^M C_{2M}^{2l} \frac{(2M)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \\ 1, & 1, & \cdots, & 1 \end{pmatrix} \\
 & = \begin{pmatrix} 0 & (1-\frac{C}{2})2^2 & \cdots & (1-\frac{C}{2})(2M)^2 \\ 0 & (1-\frac{C}{2})2^4 + 6\frac{2^2}{\delta^2} & \cdots & (1-\frac{C}{2})(2M)^4 + 6(2M)^2\frac{1}{\delta^2} \\ \vdots & \vdots & \ddots & \vdots \\ 0, & (1-\frac{C}{2})2^{2M} + \sum_{l=1}^{M-1} C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}}, & \cdots, & (1-\frac{C}{2})(2M)^{2M} + \sum_{l=1}^{M-1} C_{2M}^{2l} \frac{(2M)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \\ 1, & 1, & \cdots, & 1 \end{pmatrix} \quad (\text{F.1})
 \end{aligned}$$

From this expression in Eqn.(F.1), we have

$$\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix} = (-1)^{M+2} \det(\mathbf{B}_1) \quad (\text{F.2})$$

where $\mathbf{B}_1$ denotes the upper right M by M sub-matrix of the matrix in the right hand side of Eqn.(F.1).

Multiplying the first row of $\mathbf{B}_1$ with appropriate coefficients and adding it respectively to from 2-nd - M -th rows of $\mathbf{B}_1$, it can be obtained that the determinant of $\mathbf{B}_1$ is equivalent to the determinant of the following matrix:

$$\begin{pmatrix} (1 - \frac{C}{2})2^2 & (1 - \frac{C}{2})4^2 & \cdots & (1 - \frac{C}{2})(2M)^2 \\ (1 - \frac{C}{2})2^4 & (1 - \frac{C}{2})4^4 & \cdots & (1 - \frac{C}{2})(2M)^4 \\ \vdots & \vdots & \ddots & \vdots \\ (1 - \frac{C}{2})2^{2(M-1)} & (1 - \frac{C}{2})4^{2(M-1)} & \cdots & (1 - \frac{C}{2})(2M)^{2(M-1)} \\ (1 - \frac{C}{2})2^{2M} & (1 - \frac{C}{2})4^{2M} & \cdots & (1 - \frac{C}{2})(2M)^{2M} \end{pmatrix}$$

The determinant of this matrix can be expressed further as

$$\det(\mathbf{B}_1) = (1 - \frac{C}{2})^M \begin{pmatrix} 2^2 & 4^2 & \cdots & (2M)^2 \\ 2^4 & 4^4 & \cdots & (2M)^4 \\ \vdots & \vdots & \ddots & \vdots \\ 2^{2(M-1)} & 4^{2(M-1)} & \cdots & (2M)^{2(M-1)} \\ 2^{2M} & 4^{2M} & \cdots & (2M)^{2M} \end{pmatrix}$$

Denote the right matrix in the above equation as $\mathbf{B}_2$. Since it is a Vandermonde matrix, from the properties of Vandermonde matrix, we know that the determinant of the above matrix is always positive. Thus, the determinant of $\begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ can be

expressed as:

$$\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix} = \left(\frac{C}{2} - 1\right)^M \det(\mathbf{B}_2) \quad (\text{F.3})$$

Considering $\left(\frac{C}{2} - 1\right)$ is negative, we can easily conclude from Eqn.(F.3) that the sign of $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is determined by the parity of M . If M is an add number, $\det \begin{pmatrix} \mathbf{A}_1 & \mathbf{a}_1 \\ \mathbf{e}^T & 1 \end{pmatrix}$ is negative; else, it is positive.

APPENDIX G.

The Value of $\det(\mathbf{A}_1)$ is Always Positive

In this section, we will prove that the determinant of matrix $\mathbf{A}_1$ is always positive.

By the definition, $\mathbf{A}_1$ is the first M columns of the $M \times (M+1)$ matrix $\mathbf{A}$ defined in (5.44). Thus, the matrix $\mathbf{A}_1$ has the following expression:

$$\mathbf{A}_1 = \begin{pmatrix} \frac{1}{\delta^2} & (1 - \frac{C}{2})2^2 + \frac{1}{\delta^2} & \cdots & (1 - \frac{C}{2})(2M-2)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & (1 - \frac{C}{2})2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (1 - \frac{C}{2})(2M-2)^4 + 6(2M-2)^2\frac{1}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}} & (1 - \frac{C}{2})2^{2M} + \sum_{l=1}^M C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} & \cdots & (1 - \frac{C}{2})(2M-2)^{2M} + \sum_{l=1}^M C_{2M}^{2l} \frac{(2M-2)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \quad (\text{G.1})$$

In order to obtain the determinant of $\mathbf{A}_1$, we perform several simple manipulations on the $(M) \times (M)$ matrix $(\mathbf{A}_1)$. Multiplying the first column with -1 , adding it respectively to the last $(M-1)$ rows, the matrix of $\mathbf{A}_1$ can be expressed as follows:

$$\begin{aligned} & \begin{pmatrix} \frac{1}{\delta^2} & (1 - \frac{C}{2})2^2 + \frac{1}{\delta^2} & \cdots & (1 - \frac{C}{2})(2M-2)^2 + \frac{1}{\delta^2} \\ \frac{3}{\delta^4} & (1 - \frac{C}{2})2^4 + 6\frac{2^2}{\delta^2} + \frac{3}{\delta^4} & \cdots & (1 - \frac{C}{2})(2M-2)^4 + 6(2M-2)^2\frac{1}{\delta^2} + \frac{3}{\delta^4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}} & (1 - \frac{C}{2})2^{2M} + \sum_{l=1}^M C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} & \cdots & (1 - \frac{C}{2})(2M-2)^{2M} + \sum_{l=1}^M C_{2M}^{2l} \frac{(2M-2)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \\ & \begin{pmatrix} 1 & -1 & \cdots & -1 & -1 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \\ & = \begin{pmatrix} \frac{1}{\delta^2} & (1 - \frac{C}{2})2^2 & \cdots & (1 - \frac{C}{2})(2M-2)^2 \\ \frac{3}{\delta^4} & (1 - \frac{C}{2})2^4 + 6\frac{2^2}{\delta^2} & \cdots & (1 - \frac{C}{2})(2M-2)^4 + 6(2M-2)^2\frac{1}{\delta^2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\prod_{i=1}^M (2i-1)}{\delta^{2M}} & (1 - \frac{C}{2})2^{2M} + \sum_{l=1}^{M-1} C_{2M}^{2l} \frac{2^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} & \cdots & (1 - \frac{C}{2})(2M-2)^{2M} + \sum_{l=1}^{M-1} C_{2M}^{2l} \frac{(2M-2)^{2(M-l)} \prod_{i=1}^l (2i-1)}{\delta^{2l}} \end{pmatrix} \end{aligned} \quad (\text{G.2})$$

By extracting $(1 - \frac{C}{2})$ out of the obtained matrix $\mathbf{A}_1$ from the first row, and multiplying the resulting first row with appropriate coefficients and adding it to the second

row to eliminate the terms with respect to power of 2. Then, extracting $(1 - \frac{C}{2})$ from the second row and multiplying the obtained second row appropriate coefficients and adding it to the next row to eliminate the terms responding to power of 4. Continue this procedure, the matrix $\mathbf{A}_1$ can be transformed by elementary matrix transformations to the following matrix

$$(1 - \frac{C}{2})^M \begin{pmatrix} h_1 & 2^2 & \cdots & (2M-2)^2 \\ h_2 & 2^4 & \cdots & (2M-2)^4 \\ \vdots & \vdots & \ddots & \vdots \\ h_p & 2^{2p} & \cdots & (2M-2)^{2p} \\ \vdots & \vdots & \ddots & \vdots \\ h_M & 2^{2M} & \cdots & (2M-2)^{2M} \end{pmatrix} \quad (\text{G.3})$$

where h_i ($i = 1, 2, \dots, M$) can be expressed in a recursive equation by using the values of h_p ($p = 1, 2, \dots, i-1$), i.e.

$$h_p = \frac{1}{(1 - \frac{C}{2})} \left(\frac{\prod_{i=1}^p (2i-1)}{\delta^{2p}} - \sum_{l=1}^{p-1} C_{2p}^{2l} \frac{\prod_{i=1}^l (2i-1)}{\delta^{2l}} h_{p-l} \right) \quad (\text{G.4})$$

and the first value of h_1 takes $h_1 = \frac{1}{\delta^2} / (1 - \frac{C}{2})$.

It can be easily verified from the expressions of h_p ($i = 1, 2, \dots, M$) that when p is odd number h_p is positive, else it is negative. The parity of h_p with respect to value of p can be expressed as:

$$h_p = \begin{cases} > 0 & p \text{ is odd} \\ < 0 & p \text{ is even} \end{cases} \quad (\text{G.5})$$

Since all the involved transformations on $\mathbf{A}_1$ are elementary transformations, the determinant is not changed. Thus, the matrix $\mathbf{A}_1$ has the same determinant as the

matrix in (G.3) where the sign of the first column items are interlaced and the value of each item has the form shown in matrix (G.4). Then the determinant of matrix $\mathbf{A}_1$ in (G.1) is obtained as

$$\det(\mathbf{A}_1) = \sum_{p=1}^M (-1)^{p+1} (-1)^{p+1} |h_p| \frac{\prod_{i=1}^p (2i-1)}{\delta^{2p}} \det(\mathbf{A}_{p,1}) > 0 \quad (\text{G.6})$$

where $\mathbf{A}_{p,1}$ is an $(M-1) \times (M-1)$ matrix obtained after deleting the p -th row and the first column of $\mathbf{A}_1$. As $\mathbf{A}_{p,1}$ is a quasi-Vandermonde matrix, its determinant is positive. Thus, $\det(\mathbf{A}_1)$ is always positive, which confirms the conclusion.

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