Study of energy storage systems based on axial-flux flywheel-rotor electric machines

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Study of Energy Storage Systems Based on Axial-Flux Flywheel-Rotor Electric Machines

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School of Electrical and Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfilment of the requirement for the degree of
Doctor of Philosophy

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Summary

This thesis has proposed a compact and efficient Flywheel Energy Storage System (FESS) based on axial-flux partially-self-bearing permanent magnet (PM) machine. Five aspects on the design, analysis, control and implementation of the proposed FESS have been covered in the following sequence: (1) Design of the proposed FESS; (2) Modeling of axial-flux synchronous PM machine; (3) FEM analysis and model verification for the axial-flux PM machine; (4) Control design and experimental implementation of the proposed FESS; (5) Stabilizing synchronization control for the radial magnetic bearings.

The conceptual FESS has been presented whereby the implementation of the axial-flux partially-self-bearing PM machine that provides dual functions of rotating the flywheel-rotor and levitating the flywheel-rotor in axial direction. Its design procedures have been described and the performance of a high-speed FESS employing such PM machine has been analyzed.

This thesis has developed a multi-variable non-linear mathematical model of the dual-stator axial-flux PM synchronous machine. A linear model has been achieved after linearization. The three-dimensional finite element analyses have been implemented to examine the feasibility of the proposed FESS, to test the accuracy of the motor design and to verify the appropriateness of the mathematical model.

Model-based predictive control (MPC) and proportional-integral-derivative (PID) control algorithms have both been employed to operate the axial-flux PM machine in the simulations. The PID control strategy for the proposed FESS has been implemented experimentally into a DSP-based drive control system whilst the hardware prototype of the FESS has been constructed. The experimental results have validated that the proposed FESS could be well droved.

A stabilizing synchronization design has also been proposed by incorporating cross-coupling technology into the optimal control architecture in the control of radial

magnetic bearings for FESS. The simulation results has revealed that the resulting control system could be effective in recovering the unstable rotor and in suppressing the coupling effects of the gyroscopic rotation at high spinning speed in the presence of disturbances.

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List of Abbreviations

2D	Two-dimensional
3D	Three-dimensional
AC	Alternating Current
ADC	Analogue to Digital Conversion
AMB	Active Magnetic Bearing
CAES	Compressed Air Energy Storage
CPU	Central Processing Unit
DAC	Digital to Analogue Conversion
DC	Direct Current
DOD	Depth of Discharge
DOF	Degree of Freedom
DSP	Digital Signal Processor
EMALS	Electromagnetic Aircraft Launch System
EMF	Electromotive-force
FEA	Finite Element Analysis
FEM	Finite Element Method
FESS	Flywheel Energy Storage System
FESSs	Flywheel Energy Storage Systems
FET	Field Effect Transistor
IGBT	Insulated Gate Bipolar Transistor
LEO	Low Earth Orbiting
Li-ion	Lithium-ion
LQG	Linear Quadratic Gaussian
MMF	Magnetomotive Force
MPC	Model Predictive Control
NASA	National Aeronautics Space Administration
NdFeB	Neodymium-iron-boron

NiCad	Nickel Cadmium
NiMH	Nickel Metal Hydride
PCB	Printed Circuit Board
PD	Proportional-Derivative
PI	Proportional-Integral
PID	Proportional-Integral-Derivative
PM	Permanent Magnet
PMSM	Permanent Magnet Synchronous Machine
PWM	Pulse Width Modulation
QP	Quadratic Programming
RDE	Riccati Difference Equation
RMS	Root Mean Square
rpm	Revolution per minute
SPWM	Sinusoidal Pulse Width Modulation
SMES	Super-conducting Magnetic Energy Storage
SVPWM	Space-vector Pulse Width Modulation
UPS	Uninterruptible Power Supply
VSI	Voltage-source Inverter

CHAPTER 1

INTRODUCTION

1.1 Motivation

With the advances in high strength and light weight composite material, high performance magnetic bearings, and power electronics technology in recent years, Flywheel Energy Storage Systems (FESS) constitute a viable alternative to traditional battery storage systems [1-3]. Their growing energy storage density can be partly attributed to improvements in material engineering, composite manufacturing techniques, and manufacturing control [4]. It is now possible to construct flywheels that are strong enough to be operated reliably at very high speed, say, above 20,000 rpm. On the other hand, steel materials are still dominant in the applications of lower speed flywheels with the advantage of lower cost. Magnetic bearings are ideally suited for high-speed and vacuum-housed flywheels due to their contact-free operation, low friction losses, adjustable damping, good stiffness characteristics, and also the fact that no lubricants are required [5, 6]. Progress in power electronics, particularly in high-power IGBTs and FETs, underlies higher-power flywheel operation. With the higher-power devices, fewer individual components are needed, and the application of power electronics packaging technology can make the whole system size smaller [4].

Flywheel energy storage devices offer improvement over electrochemical batteries with regard to energy storage density, cycle life, and toxicity issues. The flywheel battery whose speed exceeds 60,000 rpm can generate more than 20 Whr/lbm energy with a depth-of-discharge (DOD) of up to 75%. The energy storage density of a typical

nickel-hydrogen battery is only 5-6 Whr/lbm, and the DOD cannot exceed 30% to 40%. The life of the flywheel battery can often reach up to 20 years, which is about 4-5 times that of the life of chemical batteries [7]. The flywheel batteries do little harm to the environment and are therefore clean and green batteries.

FESS is now considered as an important technology for many applications in spaceflight, transportation, power industry, military equipment, etc [1, 2, 5, 8-11]. In transportation, hybrid vehicles have been gradually put into applications. Energy is required to be stored in hybrid vehicles; flywheels can therefore find use in hybrid gas-turbine/electric trains, where chemical battery banks could be too large or too heavy. Space vehicles, especially those in orbit around the earth, can also make good use of FESSs. In earth orbit, the sun is the primary energy source, so energy must be stored for the utility when the satellite is in eclipse. In military applications, there is growing trend to depend more heavily on electricity for the propulsion of both ships and manned as well as unmanned ground vehicles. Electrical energy is also required for the weapon, navigation, communication, and intelligence systems they carry. This growing reliance on the use of electric energy has to be met by higher energy density storage [11, 12].

However, almost all FESS modules in present usage are made up of a separate motor/generator, a flywheel, two sets of radial magnetic bearing, at least one set of axial magnetic bearings and two sets of touchdown bearings. The structure is fairly complicated that it brings many difficulties in the manufacturing and makes the system large in size and weight. Furthermore, all the magnetic bearings and motor/generator should be precisely controlled, which introduce complexity arising from the installation of a number of sensors and increases challenges on control performance.

Considering the disadvantages mentioned above, it would be timely to design a compact and efficient FESS which has a simpler and more integrated structure, which is easier to be controlled, and possesses better performances such as high torque, energy and power density. This research is motivated by the quest for a novel design of FESS and the modeling, analysis and control of its components.

1.2 Objectives

The objectives of this research project are listed below:

Objective 1: Development of a new concept of FESS with superior performance

As described in Section 1.1, the FESS has many potential advantages over traditional chemical batteries and has wide applications in numerous fields. However, in most of the current FESS devices, a separate motor/generator, two sets of radial magnetic bearings and at least one set of axial magnetic bearings are implemented [1, 4, 10, 13]. An encoder to detect the velocity of the flywheel and at least five position sensors to measure the displacements in horizontal and vertical directions are hence required. Also, the corresponding actuators and controllers to rotate the flywheel and keep the flywheel balanced in these directions would be required. All of these will increase the complexities in structure and the difficulties in control system design and implementation.

The FESS of the future should have the following features [3, 13]: (i) compact structure and integrated configuration, (ii) high power density and energy density, (iii) high efficiency, (iv) easily operated and controlled.

Considering all the above-mentioned issues, the main objective of this research is to develop a novel conceptual FESS with improved performances.

Objective 2: Modeling and control of an axial-flux partially-self-bearing PM machine and its radial magnetic bearings

As part of the conceptual FESS, a specially designed axial-flux partially-self-bearing PM motor/generator is proposed to drive the flywheel and keep it in levitation. The unconventional structure of the motor/generator causes it to have a different mathematical model compared to that of the traditional electrical machines. So the modeling of the motor/generator is necessary for subsequent control system design.

Mechanical bearings can be used to constrain the degrees of freedom in radial directions in this research. However radial magnetic bearings would be a good substitute, especially in very high speed system with the requirement of long lifespan. Many control approaches have been proposed for the control of the magnetic bearings. Among these control methods, the synchronization control has proved to be a capable technique in dealing with nonlinear uncertain models, and is very effective in disturbance rejection for systems subjected to synchronous motion. So it is studied for the control of the radial magnetic bearings for the proposed axial-flux PM machine.

Objective 3: Test of the designed motor/generator, and verification of the derived mathematical model of the axial flux PM machine before experimentation.

The finite element method (FEM) is particularly flexible, reliable and effective in the analysis of electrical machines. Therefore finite element analyses (FEA) have been conducted to test the feasibility of the design, to validate the dimensions and parameters of the motor/generator, to calculate the desired electrical parameters, and furthermore, to verify the accuracy of the derived mathematical model for this machine.

Objective 4: Investigation of the feasibility and performance of the proposed FESS by experiments

Though difficult and costly, experiment is the most direct, effective and accurate approach to test the feasibility and performances of the proposed system. The objective is to build up a prototype of the FESS, set up the control system with the controller and processor, and implement the proposed control strategy to operate the system.

1.3 Major Contributions of the Thesis

The major contributions documented in this thesis are given below.

(1) A novel flywheel energy storage system based on partially-self-bearing axial-flux PM motor/generator has been proposed in this thesis.

There are numerous novel features in the proposed FESS. The motor and generator are combined to be a single machine in order to save space. Axial-flux disk-type synchronous PM machine is employed with the advantages of high efficiency, high power and torque densities, low rotor losses and small magnetic thickness. The stator and rotor both have disk-type geometries, the surface mounted PM rotor together with composite rim functions as the flywheel and is sandwiched between two disk-type stators to minimize the space. Mechanical or active magnetic bearings are used to restrict the displacement and rotation along radial directions, while the displacement and rotation along axial direction are controlled by the axial-flux PM machine itself. The combined use of mechanical or magnetic bearings and self-bearing features can cut down the complexity of control and make the system more stable, viable and cost-effective.

A high speed 2-pole machine with a maximum speed of 60,000 rpm and a low speed 4-pole prototype machine with a rated speed of 1500 rpm which is more amenable for initial experiments have been designed. The mechanical performances such as power density, energy density and the efficiency of the high speed machine were analyzed to verify its feasibility and performances.

(2) Modeling and control of the axial-flux partially-self-bearing PM machine

The proposed machine combines axial magnetic bearing and motoring functionality into a single magnetic actuator, which not only spins the rotor but also generates a levitation force to overcome the gravity. And the axial-flux machine which has pancake geometry makes it quite different from the conventional cylindrical machines. So the mathematical model of this machine differs greatly from those of the traditional ones. The basic electromagnetic theory is the basis in the modeling, and the force and torque equations were derived based on co-energy method. The equations of voltages, speed and displacement of the rotor in the axial direction have been obtained accordingly. The model and the modeling approach can serve as good examples for the modeling of other axial-flux electric machines.

Model predictive control (MPC) method with the advantages of pre-calculating the system

behavior, providing satisfactory tracking response and being robust against model-mismatch and unexpected dynamics was employed in the operation of the machine. Proportional-Integral-Derivative (PID) control was also implemented to be compared with the MPC approach. Hysteresis current control scheme and space-vector pulse width modulation (SVPWM) control method were used to operate the voltage source inverter.

(3) Finite element analysis which tested the design of the axial-flux PM motor, and verified the accuracy of the motor's mathematical model.

After the design and modeling of the axial-flux PM motor, FEA has been utilized to test the feasibility of the motor design and to validate its dimensions and parameters. Furthermore, FEA verified the accuracy of the mathematical model by calculating the variables such as torque and axial forces and comparing them with those which are calculated by the model. It is a valuable attempt and has proven to be a good approach for the verification of the motors' mathematical models.

(4) Development and experimental implementation for a prototype of the proposed FESS

The prototype of the low-speed version of the proposed FESS and the control system with inverters and controllers have been set up. Dspace-1103 DSP control board was employed to provide the analog-digital interface and to handle the proposed control strategies. The experimental results indicate that the system can be effectively operated, and prove that the system is well designed and the mathematical model is accurate.

(5) Stabilizing synchronization control for the radial magnetic bearings

The problem of rotor-magnetic bearing system has been formulated, and a novel stabilizing synchronization design has been proposed by incorporating cross-coupling technology into the optimal control architecture. The simulation results revealed that with adequately adjusted control parameters, the resulting control system can be very effective in recovering the unstable rotor and suppressing the coupling effects of gyroscopic rotation at high spinning speeds with the presence of disturbances.

1.4 Organization of the Thesis

Given below is a brief introduction of what is covered in each chapter:

Chapter 2 gives the literature review of the FESS and its related components. Its basic components, advantages compared with chemical batteries and its applications are introduced. The basic knowledge of active magnetic bearings, axial flux PM motors, vector control of the synchronous motor and active magnetic bearing are described as well.

Chapter 3 proposes a novel FESS based on axial-flux partially-self-bearing PM motor/generator and presents the design of a high-speed version and a low-speed version. The entire configuration with mechanical bearings and magnetic bearings is introduced. Machine design approach is described for a high-speed 2-pole version, and also a low-speed 4-pole version used for experimentation.

Chapter 4 presents the modeling approach for the axial-flux partially-self-bearing PM machine. Co-energy and decoupling methods have been employed to derive the mathematical model of the machine. The nonlinear model is linearized for the simplified implementation in the control system.

Chapter 5 describes the finite element analyses used to test the feasibility of the designed axial flux PM machines, and to verify the accuracy of the mathematical model of the motor/generator. The basic theory of 3D FEM is presented, and the solution process of FEA is introduced.

Chapter 6 deals with control system design and experimental setup implementation for the proposed FESS. MPC and PID control strategies are employed in the motor drive system. Control schemes are designed based on these two approaches and simulation results have been obtained and compared. The hardware prototype of the system, control board and DSP-based processor are described. Lastly, the experimental results obtained are analyzed and compared with the simulation counterparts.

Chapter 7 presents the problem of radial-magnetic bearing system in the FESS and a novel stabilizing synchronization control strategy is proposed to stabilize it. Synchronization design and its stability properties with the proposition and proof are described. Simulation results are presented to show the effective control performance.

Chapter 8 provides the concluding remarks of this thesis together with the recommendations for future research.

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Flywheel energy storage is now considered as an important technology for many applications in space satellites, hybrid electric vehicles and certain UPS related applications. They offer significant advantages over other kinds of energy storage systems. For instance, the efficiency of FESS is projected to exceed 95%, which is considerably better than any electrochemical battery. The power densities can soar to 5 – 10 kW/kg, which is several times that of a typical gasoline-powered engine and up to 100 times that of typical electrochemical batteries [3, 4, 13].

Sufficient background information has provided the motivation to carry out the research in this field, to define the research objectives and to set up goals. However, in order to proceed with the research, it is required to have a thorough understanding of the components of the FESS, as well as the configuration and operation of such components.

This review not only introduces the fundamental and origins of the flywheel systems, but also takes other alternative energy storage types into consideration and compares them with the FESS. Moreover, the components and their principle and functions are described in detail. The description of the structure and the basic parts of a typical FESS present a clear overview of the whole system. Finally, the applications of the FES systems are discussed with examples, and the development of future FESSs is examined.

2.2 FESS Fundamental and Origins

2.2.1 Fundamental of the FESS

Flywheel energy storage system is a form of energy storage device. It is basically a rechargeable battery. The system is used to absorb electric energy from a source, store it as kinetic energy of rotation, and then deliver it to a load at the appropriate time, in the form that meets the load needs[4]. As shown in Fig. 2.1, a typical system consists of a flywheel, a motor/generator, and control electronics for connecting to a larger electric power system.

Fig. 2. 1 Basic components and principle of FESS

The input power may differ from the output power in its temporal profile, frequency, or other attributes. It is converted by the input electronics into a form that is appropriate for efficiently driving a variable-speed motor. The motor spins the flywheel which stores energy mechanically. The flywheel slows down as it delivers energy to a load. The decreased mechanical energy is then converted into electrical form by the generator. The output electronics convert and deliver the electrical energy to the load. Since the input and output are usually separated, many approaches combine the motor and the generator into a single machine, and place the input and output electronics into a single module so as to reduce weight and costs.

Modern high-speed flywheels differ from their forebears in being lighter and spinning much faster. The energy stored in a flywheel increases linearly as its moment of inertia or as the square of its rotational speed goes up. It is noted that the moment of inertia is proportional to the square of the radius of a cylindrical flywheel. The flywheel's radius however cannot be too large because the centrifugal force acting on the flywheel increases with the increase in radius; there is a risk that the flywheel may be destroyed when it spins

at a very high speed. In this case, tradeoff is a solution. To prevent from flying apart, modern high-speed flywheels have complex structures with the use of extremely strong materials like carbon fibers. Air friction losses are addressed by housing the flywheel in evacuated chambers; the level of vacuum being determined by whatever value of air friction loss is tolerable in a particular application [13].

2.2.2 Flywheel origins

The origins and use of flywheel technology for mechanical energy storage began several hundred years ago and was developed during the Industrial Revolution. One of the first modern dissertations on the theoretical stress limitations of rotational disks is the seminal work by Dr. A. Stodola. Development of advanced flywheel began in the 1970s following the oil embargo and gas crisis. Interest in flywheel system development was generated after the publication of several papers that quantified the benefits of an advanced system which utilize composite materials and high speed bearings to build a system named a “super-flywheel”.

The Energy Research and Development Agency (ERDA) of America, and later the Department of Energy (DOE), funded flywheel system development for a number of applications including a “super-flywheel” for an electric vehicle. The Lewis Research Centre was involved in the development of mechanical bearings to be used in a vacuum, and transmissions for flywheel systems to be used in hybrid vehicle. NASA also provided funds for research work in the area of magnetic bearings for spacecraft momentum wheels at the Goddard Space Flight Center (GSFC). During the 1980s, funding for flywheel energy storage was curtailed at the DOE, but further work by NASA continued in support of satellite flywheel systems. Work on magnetic bearings was carried on at GSFC and work on flywheel momentum configurations and combined energy storage and attitude control was conducted at Langley Research Center and Marshall Space Flight Center [3].

2.3 Comparisons of Alternative Forms of Energy Storage with FESS

2.3.1 Category of energy storage forms

There are many energy storage methods, which can be classified in Table 2.1:

Table 2. 1 Category of energy storage forms

Energy Storage Category	Energy Storage Forms
Chemical Energy Storage	Gasoline/Petrol; Bio-diesel; Methane; Alcohol; Charcoal
Electrochemical Energy Storage	Batteries; Fuel Cells
Electrical Energy Storage	Capacitor/Super-Capacitor; Super-Conducting Magnetic Energy Storage (SMES)
Mechanical Energy Storage	Compressed-Air Energy Storage; FESS; Hydroelectric Energy Storage

2.3.2 Introduction of major energy storage systems

In recent years, several energy storage approaches are studied intensely. These include electrochemical battery, super-capacitor, compressed air energy storage, superconducting magnetic energy storage and flywheel energy storage.

1. Electrochemical battery

An electrochemical battery is a device that stores potential energy through the chemical reaction of its components. The reaction creates free electrons that, when connected to a circuit, will move through the circuit and drive a load (a motor or light-bulb, for example). This reaction will continue until the chemical reactants are depleted [14, 15].

There are many types of the rechargeable electrochemical batteries that can function as energy storage, among which Lead-acid battery, Nickel Cadmium (NiCad) battery, Nickel Metal Hydride (NiMH) battery and Lithium-ion (Li-ion) have been intensively researched or applied.

Lead-acid batteries are the most economically viable batteries for electrical vehicle

applications [16, 17]. They can provide 200-500 recharge cycles depending on the usage. They are more sensitive to temperature compared with other batteries. However, Lead-acid batteries do not support deep discharge or overcharge. The depth-of-discharge (DOD) is about 30%-50%, the energy density is 35-40 Whr/kg, the power density is 100-150 W/kg, and the cost is 100-150 \$/kWh [18, 19].

NiCad batteries offer a relatively long life, high discharge rate and low prices [20]. Given equal power output and run time, NiCad batteries are about 25% lighter than Lead-acid batteries. They have approximately 1500 or more recharge cycles whilst supporting deep discharge and performing better than Lead-acid batteries at low temperature [21]. However, they must be completely discharged before they can be ready for recharge; otherwise their performance per charge will be diminished. Another drawback is that NiCad batteries contain toxic metals and are not environmental friendly [22].

The performance of NiMH batteries is similar to that of NiCad except that NiMH batteries provide a power-to-weight saving of 25%-40%. Their energy density is about 50-60 Whr/kg whereas the power density is around 200 W/kg. NiMH batteries have long life up to 600-1000 charge cycles. They are more robust and can be recharged at any point of the discharge cycle. Moreover, they are less toxic during the recycling process compared with NiCad batteries. However NiMH batteries are fairly expensive, their cost is about 300-400 \$/kWh [18, 19, 23]. They also lose their charge faster than other types of batteries regardless of being in use or left idle [24]. The NiMH batteries should be topped up prior to use if they have been stored for more than a week without use [25, 26].

Li-ion batteries have long shelf life and high energy density [27, 28]. Lithium is the lightest metal, with a high electrochemical potential which provides it with a very large theoretical specific energy capability. The energy density and power density of Li-ion batteries can reach about 100 Whr/kg and 1000 W/kg respectively. Li-ion batteries can accommodate a few thousand charge cycles when the DOD is 60%-65% [18, 19, 23, 24]. However, the batteries are not environmental friendly and require special disposal. Furthermore, Li-ion batteries require tight thermal control, and taper charge profile is needed [29].

Considering all the above-mentioned electrochemical batteries, it is observed that there are a number of drawbacks of these batteries[11]:

- Narrow operational temperature range. The performance of the chemical battery will be deteriorated sharply at high or low temperature.
- Capacity decreases over time. The capacity of the chemical battery cannot be maintained at a high level throughout its life; it will drop as time goes by.
- Difficulty in estimating charge status. The degree of the charge of the chemical battery is difficult to be determined precisely as the chemical reaction in the battery is not easily measured and monitored.
- Overcharge and over-discharge. Chemical battery should neither be over-discharged nor be over-charged; otherwise its life span will be shortened sharply.
- Environmental concerns. Many elements of the chemical battery are poisonous; they can harm the environment and the health of human population.

2. Fuel cells

A fuel cell is an electrochemical device similar to a battery, but differs from the latter in that it is designed for continuous replenishment of the reactants consumed; i.e. it produces electricity from an external fuel supply [30, 31]. It is more of an energy conversion system than an energy storage system.

Typical reactants used in a fuel cell are hydrogen on the anode side and oxygen on the cathode side (a hydrogen cell). In contrast, conventional batteries consume solid reactants and once these reactants are depleted, they must be discarded or recharged with electricity by running the chemical reaction backwards, or, at least in theory, have their electrodes replaced. Typically in fuel cells, reactants flow in whereas reaction products flow out, and continuous long-term operation is virtually feasible as long as these flows are maintained.

Fuel cells are attractive in some applications because of their high efficiency and low level of pollution. These applications include base load utility power plants, emergency backup

power, off-grid power storage, portable electronics, and electrically-powered vehicles [32, 33]. However, fuel cells have the disadvantages of long start-up times, etc.

3. Capacitors/supercapacitors

Capacitors store their energy in an electrostatic field rather than in chemical form. They basically consist of two electrodes (plates) of opposite polarities separated by an electrolyte. The capacitor is charged by applying a voltage across the terminals which causes the charge to migrate to the surface of the electrode of opposite polarities [34].

The energy stored is related to the charge at each interface, q (Coulombs), and potential difference, V (Volts), between the electrodes. The energy, E (Joules), stored in a capacitor with capacitance C (Farads) is expressed by the formula [34]

$$E = 0.5qV = 0.5CV^2 \tag{2.1}$$

Supercapacitors are simply capacitors employing plates with extremely high surface areas providing a high storage capacity [35]. Maximizing the surface area of the electrodes within the available space means the thickness of the dielectric must be minimized. This in turn limits the maximum working voltage of the capacitor.

Table 2. 2 Comparisons between batteries and capacitors or supercapacitors

Device	Energy density (Whr/lbm)	Power density (W/lbm)	Cycle life (Cycles)	Discharge time (Seconds)
Batteries	50-250	150	$1 - 10^3$	>1000
Capacitors	0.05-5	$10^5 - 10^8$	$10^5 - 10^6$	<1

Since capacitors store charge only on the surface of the electrode, rather than within the entire electrode, they tend to have low energy storage capability and low energy densities[36]. The charge/discharge reaction is not limited by ionic conduction into the electrode bulk, so capacitors can be running at high rates and they therefore produce very high specific powers in a short period. Since there are no chemical reactions involved, the

charge/discharge reactions can typically be cycled many more times than that of chemical batteries. For the same reason, capacitors do not require any special charging circuits.

4. Compressed air energy storage

Compressed Air Energy Storage (CAES) uses pressurized air as the energy storage medium. An electric motor-driven compressor is utilized to pressurize the storage reservoir using off-peak energy and air is released from the reservoir through a turbine during peak hours to produce electrical energy. Ideal locations for large compressed air energy storage reservoirs include aquifers (water bearing rock formations), conventional mines in hard rock, and hydraulically mined salt caverns. Air can be stored in pressurized tanks for small systems [37, 38].

5. Pumped storage - The Hydraulic Battery

Pumped storage hydroelectricity is another relatively simple method of storing and releasing large amount of electricity to supply high peak demands. At times of low electrical demand, excess electrical capacity is used to pump water into an elevated reservoir. When there is higher demand, water is released back into the lower reservoir through a turbine to generate electricity [39, 40]. About 70% of the electrical energy used to pump the water into the elevated reservoir can be regained in this process. Some facilities use abandoned mines as the lower reservoir, but many use the natural height difference between two natural bodies of water or artificial reservoirs. Many pumped storage plants have been installed all over the world.

6. Superconducting magnetic energy storage

Superconducting magnetic energy storage (SMES) systems store energy in the field of a large magnetic coil with direct current flowing inside [41-43]. The direct current can be converted back to AC electric current as needed. The basic structure of SMES system is shown in Fig. 2.2. Low temperature SMES cooled by liquid helium is commercially available [44]. When cooled to cryogenic temperatures, superconducting wires exhibit zero

resistance, which means that a current circulating in such a coil can persist for a very long time without any loss. High temperature SMES cooled by liquid nitrogen is still in the development stage and may become a viable commercial energy storage system in the future [45].

Fig. 2. 2 Basic structure of SMES system

2.3.3 Comparisons between energy storage systems

From the review above, it is obvious that the presence of the shortcomings of the chemical batteries makes them not-so-appealing for many applications. Instead, FESS and SMES have become potential alternative forms of energy storage.

Table 2.3 shows the comparison among chemical battery, FESS and SMES systems. Given the state of development of both flywheel batteries and SMES systems, it is expected that the costs of these two types of energy storage systems can be lowered with further technical development. On the other hand, electrochemical batteries already have a tremendous economy of scale that has driven costs down.

Compared to flywheel batteries, SMES systems consume energy in order to keep the coil at cryogenic temperature, which is a reason why SMES systems are favorable for long-term energy storage. The fact that SMES system has larger size and lower power density in comparison with FESS systems lowers its competitiveness in some applications, such as space station and electrical vehicles. However in applications where the weight and size do not count heavily in the total cost, SMES systems may still be attractive, for example in

utility applications [46, 47].

Table 2. 3 Comparison among three energy storage systems [4]

Comparison Items	Lead-acid	FESS	SMES System
Storage mechanism	Chemical	Mechanical	Electromagnetic
Life(years in service)	3-5	>20	~20
Technology	Proven	Promising	Promising
Number of manufacturers	~700	~10	~1
Annual sales(US \$millions)	~7000	~2	A few
Temperature range	Limited	Less Limited	Controlled
Energy density (Whr/kg)	~30	~100	A few
Power density (W/kg)	~100	~10,000	~10,000
Environmental concerns	Disposal issues	Slight	Slight
Relative size	Larger	Smallest	In between
Price, per kilowatt	\$50-\$100	\$400-\$800	>\$300

Besides what have been given in Table 2.3, there are also some other potential advantages that an FESS has over a chemical battery [4, 12, 48]:

- Higher energy storage density and power density.

The flywheel battery whose speed exceeds 60,000rpm can generate more than 80Whr/kg energy and 4 kW/kg power under the depth-of-discharge (DOD) of 75% nowadays [49, 50], and projection values for the specific energy and power is 200Wh/kg and 30 kW/kg respectively [3].

- No capacity decreases over time.

The life of the flywheel battery depends mainly on the life of power electronic devices and can reach up to 20 years.

- No overcharge and over-discharge.

The performance of the flywheel battery is not affected when it is discharged heavily, and the overcharge can be avoided with the assistance of power electronic devices.

- Charge and discharge status is measurable and the charging and discharging speed is much higher than that of electrochemical batteries.

Since mechanical energy is proportional to the square of the flywheel speed, the stored energy level indicator is a simple speed measurement. In addition, the charge of the flywheel battery can be restored in a few minutes; however, it takes several hours for chemical battery to complete its charging.

Fig. 2.3 shows the comparison of several energy storage forms in terms of energy and power densities. The advanced flywheel systems have higher power density compared to electrochemical energy storage types, and higher energy density compared to supercapacitors and electrochemical batteries. With the application of composite material and high power motor/generator, future flywheel systems are projected to have a energy density over 200 Wh/kg and a power density over 30 kW/kg [3].

Fig. 2. 3 Comparison of energy storage devices in terms of power and energy densities

2.4 Configuration and Components of FESS

2.4.1 Configuration of FESS

1. Conventional FESS

A conventional flywheel energy storage system consists of a rotating flywheel assembly

and a set of controlled electronics [7]. The energy medium is a high speed rotating mass contained in the flywheel assembly.

Each flywheel assembly consists of:

- A high speed rotor to store the energy;
- Magnetic bearings for minimizing frictional losses;
- A motor/generator that convert electrical energy to mechanical energy and vice versa;
- Mechanical touchdown bearings to support start-up and shut-down, and to protect against magnetic bearing anomalies;
- A hermetic housing to provide structural support for the bearings and vacuum enclosure.

The power control electronics regulate the rate at which the energy is extracted or delivered.

The primary electronics assemblies within the power control electronics are:

- A motor/generator and power electronics devices to control the current into and out of the motor windings;
- A processing unit to provide voltage regulation, motor winding current control, magnetic bearing control, health monitoring, and serial communication for command and telemetry;
- A secondary power supply to furnish low voltage power for the electronics;
- Magnetic bearings drive amplifiers for driving the magnetic bearing coils;
- An EMI/EMC filter to limit conducted electromagnetic interference;
- Auxiliary bearing actuator electronics

Fig. 2.4 shows the functional block diagram of a typical FESS. Axial and radial magnetic bearings are utilized to constrain the degrees of freedom of the rotor/flywheel in axial and radial directions respectively. The motor/generator is employed to drive the flywheel. The controllers are designed to operate the motor and magnetic bearings through the power electronics and drive amplifier devices. Auxiliary bearings are implemented to support the

flywheel in the course of start-up, shutdown and emergency situations.

Fig. 2. 4 Functional Block Diagram of Conventional FESS [7]

Most of the commercial FESSs have such typical configuration [2, 11, 51-54]. However the separate motor/generator together with two sets of magnetic bearings increase the complexity of the whole system, and increase the volume and mass. Combinations of two or more components can reduce the system complexity and therefore increase the power and energy density of the FESS. Integrated FESS has thus been recently proposed.

2. Integrated FESS

An integrated FESS is one in which the energy storage accumulator and the electromagnetic rotor are combined into a solid rotor [55, 56]. The rotor and flywheel are combined together, such that the rotor is used for both motor/generator and energy storage functions [57, 58]. The advantages of integrated design include reduced system weight, lower component count, reduced material cost, lower mechanical complexity, and reduced manufacturing cost.

An integrated FESS with homopolar inductor motor/generator is presented in [56]. A single piece of solid steel flywheel functions as the rotor. The slotless homopolar motor is

employed to drive the flywheel/rotor. An axial-field PM motor has been implemented in another study. In this system, the rotor/flywheel is sandwiched between two stators to minimize the space [55]. However in these integrated FESSs, mechanical bearings are applied to support the rotor against gravity and to constrain the movements in other degrees of freedom. This increases the mechanical loss, reduces the efficiency and shortens the life-span of the FESS.

2.4.2 Components of FESS

In order to better understand the operation of the FESS, its components are described in this section.

1. Composite rotor

One way for the flywheel battery to store more energy is to use composite flywheel rotors that have high tensile strength. The application of advanced composite structures in flywheel can contribute significantly to better results such as increasing energy density, reducing structural weight, facilitating manufacturing and effectively containing failures[59-61].

The energy stored in the flywheel is proportional to the inertia and the square of its rotational speed. The spin speed and the configuration dimension of the flywheel are restricted by the material strength, especially the tensile strength. For a thin rim type flywheel, the relationship between the energies it stored and the tensile force permitted by the material is expressed by the following formula [61]:

$$E = \frac{1}{2}mr^2\omega^2 = \frac{1}{2}I\omega^2 \leq m\sigma_h / \rho \quad (2.2)$$

where m is the mass of a disk, r is the rotational radius, I is the moment of inertia, ω is the angular velocity, ρ is the material density, σ_h is the safety stress of the material, and E is the kinetic energy of the flywheel. From (2.2), it is observed that the material of high

stress ratio (σ_h / ρ) is essential to acquire maximal stored energy.

Fig. 2. 5 Energy storage density for different materials [62]

Fig. 2.5 shows the energy densities of several materials which can be used in fabrication of flywheel. The tensile strength of the graphite fibers is much greater than steels. Their energy density could exceed 500 Wh/kg, which is about 10 times larger than that of alloy steels [62].

2. Magnetic bearings

Magnetic bearing system represents a completely different approach to support the rotating equipment. It is a non-contacting technology, which implies negligible frictional loss, no wear and high reliability. It also enables previous unachievable surface speeds to be attained, and the use of lubrication is eliminated [5, 54].

The application of magnetic bearings is based upon the principle that an electromagnet attracts ferromagnetic materials. As shown in Fig. 2.6, magnetic bearings provide attractive electromagnetic suspension through applying electric current to ferromagnetic materials used in both the stationary and rotating parts (the stator and rotor, respectively) of

the magnetic bearing. The current creates a flux path that includes both parts with the air gap separating them, through which non-contact operation is made possible. A ferromagnetic rotor can thus be supported in a magnetic field generated by the bearing electromagnetic stator.

Fig. 2. 6 Sketch of magnetic bearing

Active magnetic bearings and passive magnetic bearings have been used in flywheel energy storage systems[5, 63, 64]. Active magnetic bearing has advantages such as simple construction and capability of supporting large loads, but the complexity of the control system is daunting. The typical design requires a minimum of five sensors and three sets of actively controlled electromagnets to support a single rotor. The cost and size of the control system are thus both significant obstacles to implementation of magnetic bearings into practical systems. Passive magnetic bearings employing permanent magnets can be smaller in size and are more efficient, but they are more difficult and expensive to be constructed and generally have the same control problems as conventional attractive magnetic bearings.

3. Motor/ Generator

The motors and generators used in most flywheel systems are AC rotating machines [65]. Integrated systems have always been the goal of the modern flywheel systems. The generator and motor share common components to make it compact [66]. When the FESS absorbs energy from the source, by speeding up the flywheel to transform the electrical

energy into kinetic energy, the motor/generator is working in motoring mode. When the flywheel slows down, the motor/generator is working in generating mode to transform the kinetic energy back into electrical energy. For the purpose of increasing energy density, the motor/generator is required to operate at high rotational speed.

Many types of electrical machines such as switched reluctance motor, synchronous reluctance motor, induction motor and permanent magnet motor have been applied in FESSs [67-72]. Out of all these machines, PM motors have generated great popularity, especially in space and electrical vehicles applications.

In typical FESS, the flywheel is coupled to the shaft of the motor/generator. The motor/generator drives the rotor and in turn makes the flywheel rotate. Integrated FESS combines the flywheel and the rotor to reduce the size and mass of the system. For the purpose of storing more energy, a rotor-flywheel with large radius should be designed to achieve a large inertia of the moment. However in traditional radial-flux motor, the increase of rotor size will add the dimension of the stator, which will increase the whole volume and mass of the system thereby.

Axial-flux motor presents an ideal structure for the integrated FESS. Axial-flux motors have an axially directed air-gap flux distribution. The radial length from the stator's inner radius to its outer radius is the motor active part that produces the torque [73]. The structure of the axial-flux machines include double sided motor with internal disk rotor sandwiched between two pancake stators, double-sized motor with one stator installed between two rotating rotors, single-sided motors with one stator and one rotor, and multi-disk motors [73-75]. Because the rotor is not encompassed by the stator in axial-flux machines, the rotor can be constructed with a large diameter or with a composite material rim mounted on its fringe to achieve a high inertia. This is an advantage of integrated FESS in which the rotor and flywheel are combined.

Axial flux machines have been studied with increasing interest during last two decades as a result of the advantages such as flexible shape, compact structure and high power density

compared with the cylindrical radial-flux motors [76, 77]. Researches on the axial flux motor are mostly focused on permanent magnet (PM) motor and induction motor, especially axial flux PM motors which are more popular because of simpler structure, high torque density and high power density [75, 78, 79]. These motors have been widely used in electrical vehicles, pumps, centrifuges, machines tools, robots and industrial equipments [74, 80, 81].

Self-bearing motors (also called bearingless motors) are getting increasing research interests since 1990's [82-86]. For this type of motor, the functions of magnetic bearings and motor are combined into a single actuator. There are two sets of windings with different pole numbers installed in the stator, one of which is used to produce torque through revolving magnetic field. This magnetic field is intentionally unbalanced by the currents of another set of windings to produce radial forces acting on the rotor [87]. Self-bearing motor has the advantages of compact structure and high speed. However the requirement of two sets of windings make the structure complicated, the control of rotation and four degrees of freedom simultaneously is difficult to realize.

Self-bearing concept can also be applied in axial-flux motor without the additional control windings [88, 89]. The axial-flux self-bearing motor produces rotating magnetic flux in the air gap to generate the motoring torque and axial force. The movement and rotation along and about radial directions cannot be actively controlled in the motor, but the gravity on the rotor, the main contributor of the mechanical losses, can be overcome by the axial force. This is an attractive feature for application in FESSs.

4. Power Electronics

The function of the power electronics is to control the rate at which the energy is delivered or extracted from the flywheel. Fig. 2.7 shows the power electronic devices to manage the energy in dual directions. Two converters are used to inter-transform kinetic energy to electrical energy. When the motor/generator is working in motoring mode, i.e., the speed of the flywheel is increasing, converter 1 works as a rectifier to absorb sinusoidal input

current and boost the DC link voltage, while converter 2 works as an inverter to drive the motor to reach the aimed velocity. In contrast, when the motor/generator is working in generator mode, converter 2 functions as a rectifier to transform AC power into DC power, and converter 1 functions as an inverter to transform DC power into AC power, which is fed back to the power line.

Fig. 2. 7 Power electronic devices for the electric machine of FESS

With the long life expected of the mechanical components of flywheels, the lifespan of the power electronics would be the limiting factor. However the efficiency and reliability of power electronics have improved steadily in the past two decades. One of the major challenges in FESS is to improve the power electronics design and to successfully integrate the FESS into the power system.

5. Auxiliary Components

The pole faces of the magnetic bearing stators, together with the outside of the rotors, are precision machined surfaces. In order to protect these surfaces in the event of a loss of power to the bearings, an auxiliary bearing system and an alternate source of power are provided. Auxiliary bearings typically make use of either conventional rolling element bearings or passive landing sleeves. In the event of power loss, the rotating assembly is caught by the auxiliary bearings. These bearings are sometimes known as backup bearings,

catcher bearings, or touch-down bearings.

Touchdown Bearing

The touchdown bearing is one of the most important elements in any high speed magnetic bearing system. The touchdown bearing functions to fully off-load or load share with the magnetic bearing and to prevent catastrophic system failures [90, 91]. The proper design of this component is a vital step in the design of the entire system and should not be treated as a stand-alone sub-component. The impact load that these bearings have to endure is a challenge to the designer. The high temperature environment of this bearing adds additional constraints to the design options.

Fig. 2. 8 The torque-limited touchdown bearing system [92]

Fig. 2.8 shows a torque-limited touchdown bearing system (TLTBS). It is a backup mechanical-bearing system for a high-speed rotary machine in which the rotor shaft is supported by magnetic bearings at steady-state normal operation. The TLTBS provides ball-bearing support to augment or supplant the magnetic bearings during startup,

shutdown, or failure of the magnetic bearings. The TLTBBS also provides support in the presence of conditions (in particular, rotational acceleration) that make it difficult or impossible to control the magnetic bearings or when the magnetic bearings are not strong enough (e.g., when the side load against the rotor exceeds the available lateral magnetic force).

Containment and Security

Using high speed flywheel for energy storage always raises the question of safety. Safety is to be handled with a multi-tiered system. Each flywheel system contains an on-board health monitoring system that minimizes rotor imbalance detected by the magnetic bearings, or rim geometry detected by proximity sensors [7, 49].

In addition, high speed rotating FESS may require safety devices to limit collateral damage during rotor bursts (i.e. rapid failure and disintegration of substantial portions of a rotating flywheel that occurs when centrifugal forces generate stresses in excess of rotor material ultimate stress) [50]. Such a failure may occur during an over-speed or may be caused by manufacturing defects. Loading on the containment structure during a burst event is a function of rotor design and containment geometry. Graphite-reinforced composite cylinders may be employed to dissipate radial kinetic energy from the rotor debris and reduce torque loads transmitted to the rotor housing and mounting hardware.

2.5 Control of FESS

2.5.1 Vector control of PM synchronous machine

Permanent magnet synchronous machine (PMSM) has been widely used in FESS to rotate the flywheel with the advantages of high power quality, high efficiency, good dynamic performance, simple structure, and ease in control. The rotational speed of a PMSM operated from the AC line is directly proportional to the frequency of the applied voltage. The use of a variable frequency inverter to supply the PMSM allows a wide

range of speed to be obtained.

The principle of controlling the PMSM is known as vector control, where the rotor position serves as the angle of reference. This method is analogous to the control of a separately excited DC machine by field and armature currents. According to this principle, the stator currents are transformed into a frame of reference that moves with the rotor flux. Therefore, a direct and a quadrature component of the stator current vector can be utilized as non-interacting components for controlling the magnitude of the flux and torque respectively.

The expression for the electromagnetic torque can be derived as [93]

$$T_{em} = \frac{3}{2} P \lambda_f i_q \quad (2.3)$$

where, P is the number of pole pairs, λ_f is the magnitude of the air-gap flux, i_q is the quadrature axis current. This equation shows that the torque depends only on the quadrature axis current. Therefore, if the PM machine is employed in FESS, when $i_q > 0$, the torque will be of positive value and the flywheel velocity will hence increase, in this scenario the machine works as a motor. However, when $i_q < 0$, the flywheel speed will decrease and the machine is acting as a generator. The airgap flux λ_f provided by the PMs is constant as the length of the air-gap is fixed. If the air-gap flux needs to be boosted or weakened, the required direct current component i_d can be fed to the machine. Therefore, in axial flux PM machine, the air-gap flux together with the air-gap length can be controlled by adjusting the value of i_d . The independent torque and air-gap flux control is just like that of a separately excited DC machine.

To implement the vector control strategy, the instantaneous rotor position must be sensed. This may be achieved by having rotary transducers which are connected to the shaft to monitor the shaft rotation and transform angular movements into electrical signals. The

widely used sensors are the resolver and the incremental encoder. A resolver can provide the absolute value of the shaft position even at power switch-off and standstill position, but it needs a more complex electronic circuit to process the resolver signals. The incremental encoder generates digital pulses in response to a given angular increment of the shaft rotation. This enables it to be directly interfaced to digital controllers. The information is available in two output signals with 90 degrees phase shift. Digital counters are then used for the determination of the actual position. By using the information from the two outputs channels, the direction of rotation can be determined. The major drawback of the incremental encoders is the initialization needed at power switch-on to define the home position.

Sensorless control has the advantages of reducing the complexity of the system by eliminating a hardware component and its associated failure modes, volume and mass. Back Electromotive-force (EMF) technique is most frequently used in sensorless control systems. The back EMF technique is based on the principle that the spinning rotor magnets will generate a back EMF at the terminals of the machine. This back EMF can be integrated to calculate the stator flux. From the stator flux vector and an estimate of the torque angle, the rotor flux position can be estimated.

2.5.2 Current regulation control for the voltage-source inverter

In DC/AC conversion, voltage-source inverters (VSI) have become a simple and reliable solution [94, 95]. In today's practice, voltage-source inverters with current regulation are employed in many applications where fast response, high accuracy and high-level performance are required. Two main current regulation categories can be considered: Nonlinear current control and linear current control [96].

1. Nonlinear current controller

The current-regulator function is to force the instantaneous phase currents to follow their reference values at all times by means of pulse-width-modulation. Hysteresis controller is

a nonlinear PWM current controller which has been widely used.

A simplified diagram of a typical three-phase hysteresis current controller is shown in Fig. 2.9. The actual load currents i_a , i_b and i_c are detected and compared with the respective current commands i_a^* , i_b^* and i_c^* using three independent hysteresis comparators with a hysteresis band H . The outputs of the comparators are sent to the distributing circuit where the signals generated are used to control the inverter switches.

Fig. 2. 9 Nonlinear current control scheme

The advantage of the hysteresis current controller lies in its simplicity and its provision of excellent dynamic performance. On the other hand, the disadvantage is that the switching frequency varies during the fundamental period, resulting in irregular operation of the inverter at times. The switching losses are increased thereafter. The resulting inverter output harmonic spectrum distributed characteristic depends largely on the variations of load and the input voltage.

2. Linear current controller

There are two different schemes of linear current controllers depending on the reference frame used to perform the current regulation. Linear current control uses PI error amplifiers to produce voltage references for PWM modulation.

A schematic of the current regulator in the rotating reference frames is shown in Fig. 2.10.

The characteristic of this regulation scheme is that it operates on DC quantities, and therefore has the advantage of achieving zero steady state error in the regulated currents. It is well suited to drives and other applications where sinusoidal current and voltage waveforms are required. An additional benefit of the regulator is that, being sensitive to the fundamental component of current, it ensures the correct phase relationship of the load current with its reference. The feature is of particular importance for drive applications. However, it requires two current transformations which add to the complexity of the implementation of the drive system [95].

Fig. 2.11 shows the schematic of the current regulator in the stator reference frame. This method is simpler in terms of implementation. However, since the regulator performs its regulation on AC quantities at the operating frequency, it is not capable of producing zero steady-state errors.

Fig. 2. 10 Current regulator in the rotor reference frame

Fig. 2. 11 Current regulator in the stator reference frame

3. PWM modulation schemes

In VSI, the conversion of DC power to three-phase AC power is exclusively performed in

the switched mode. The actual power flow in each motor phase is controlled by the duty cycle of the respective switches. The desired sinusoidal waveform of the currents is achieved by varying the duty cycles sinusoidally with time, employing techniques of pulsewidth modulation (PWM).

PWM techniques have been the subject of intensive research during the last few decades [97]. A large variety of methods, different in concept and performance, have been developed and documented. Among all these proposed PWM methods, Sinusoidal PWM (SPWM) and Space Vector PWM (SVPWM) are the two options which have been most frequently used. PWM approaches have been described in many literatures, so the introduction of these methods will not be presented in this review.

2.5.3 Control of active magnetic bearings

Active magnetic bearing (AMB) system has unique features such as no mechanical contact and active control compared with the traditional bearings. An AMB system is inherently nonlinear as a result of the nonlinearities of its electromagnetic field and as well as the fact that it is an open-loop unstable system [98]. Furthermore, the AMB system involves many uncertain factors, including the mass imbalance of the rotor, gyroscopic effects, the flexibility effect of the shaft, the electrical dynamics of the actuator, and the variable whirling frequency of the rotor.

Five degrees of freedom (DOFs) should be controlled in the magnetic bearing systems. The interactions among these DOFs can be ignored when the rotor is rotating at the equilibrium point, the system can hence be decoupled to five subsystems and the distribution control may be used to reduce the complication of the control system. However the coupling effects among the DOFs will increase when the rotor is inclined, and the couplings will become intensive when heavy axial load is imposed. The fully distribution control is therefore not feasible.

Many control strategies including the classic PID control and those based on modern

control theory have been implemented in the control systems for the magnetic bearings. PD and PID controllers have simple structure and robust control performance; they are hence commonly used in the control applications of magnetic bearing system. However outside disturbance and plant uncertainties make the classical PID feedback control not entirely effective and satisfactory, especially in super-high speed operation (in the range of 60-80 krpm) [99]. For this reason, many modern control approaches based on state space equations have been attempted in the control of magnetic bearings.

Linear control theories have been applied to AMB systems extensively. H_∞ approach was adopted to eliminate the unbalance vibration in a repulsive type magnetic bearing system [100]. The μ -synthesis control system was used to provide robustness against mass variations in a flexible rotor-magnetic bearing system [101]. Sliding mode control was used to suppress the nonlinearities generated by rotor mass imbalances and the dynamic characteristics of the electrical actuator [102, 103]. The model used in these linear designs is derived from linearizing the dynamic equations about the equilibrium point. Consequently, their performance may suffer rapid deterioration when the operation deviates from the equilibrium point. That will hence influence the control performance because of the nonlinearity and uncertainty of the plant model.

In modern industrial engineering, model-based predictive control (MPC) has become a popular strategy, and in the past two decades, the theory of predictive control has been well developed in various aspects such as stability, nonlinearity, and robustness [104-107]. A MPC controller uses a mathematical model of the system to predict the future effect of current control actions. It has the capability of online optimization to determine the best control action; it can deal with multivariable, multi-input multi-output systems; it is capable of controlling open-loop unstable systems. So MPC has the potential to overcome the difficulties of nonlinearity and model uncertainty in AMB control system. The challenge of the MPC approach is the heavy computations required by the on-line optimization strategy. However the problem can be solved with the newer generation of advanced higher-speed processors.

In the AMB system, a spinning rotor under magnetic suspension experiences two kinds of whirl modes. Although gyroscopic forces in the conical whirl model have a stabilizing effect on the rotor system [98, 108], they may also twist the rotor, thereby affecting the stability of the rotor if it is not properly controlled [109]. The translatory whirl mode constrains the rotor to synchronous motion in the radial directions so as to suppress the gyroscopic rotation. Such a translatory mode can be achieved by the synchronization control that has been extensively used in the industry [110, 111]. The synchronization control has been shown to be capable in dealing with nonlinear uncertain models, and effective in disturbance rejection for systems subject to synchronous motion [112].

2.6 Applications of FESS

FESSs have functions of energy storage, attitude control, pulse power provision, uninterruptible power supply, and voltage control. Energy storage and attitude control are mainly used in space applications; pulse power provision function focuses on aircraft launch, combat vehicles, hybrid and electric vehicles; the functions of uninterruptible power supply and voltage control are commonly used to retain the power quality, such as voltage sag correction, and instantaneous interruption compensation.

2.6.1 Applications in space

The FESS has attributes that make it attractive on the space station [3, 7, 52, 113]. It fits in roughly the same space and has similar weights of the chemical batteries it would replace, and is likely to last three to ten times longer. Potential space applications fall primarily into the energy storage category. They encompass the range of vehicles from small satellites and probes to manned space stations. Low earth orbiting satellites, in particular, can benefit from this technology because of the large number of charge/discharge cycles they experience. In spacecraft, a significant benefit can be gained by combining energy storage with attitude control functions [29]. In this approach the flywheel momentum and torque characteristics are used as an attitude control resource rather than viewed as a potential disturbance source. This concept reduces the space vehicle bus mass, volume, cost, and

maintenance requirements while maintaining or improving the space vehicle performance [114].

2.6.2 Applications in transportations

Hybrid electric vehicles use both internal combustion engine and electric machine for propulsion. The average power needed to propel the vehicle should be supplied by the engine. Short bursts of power, for climbing hill and acceleration, are taken from an energy store, which is replenished directly by the engine or by regenerative braking when the vehicle is slowed down. A FESS is a more attractive energy store than a chemical battery in the application of electric vehicles [55, 115]. Not only it is smaller and lighter than the battery it replaces, but it is also expected to have three to five times the lifespan compared to the chemical battery pack [8, 115].

Other than being developed for road transport applications, flywheel batteries are also suitable for high-speed rail application [4, 116-118]. A hybrid approach, in which a gas turbine driving a high-speed generator supplies the average power while a FESS handles power leveling and storage applications. The energy and power stored by FESSs would be mostly used in stopping and starting the train set at railway stations [12].

2.6.3 Applications in power industry

As seasonal loads and shock loads increase, the power consumption is concentrated at a certain time zone of a day. The imbalance in the use of power makes the power system unstable and results in the power quality problems such as instantaneous interruption, voltage sag, lack of reactive power or active power. To retain the quality of power system, uninterruptible power supply (UPS) system is usually considered as one of the best solutions. [9]. However the conventional UPS systems use the lead-acid battery as the energy storage element, whose drawbacks have been discussed earlier in this chapter. Many researches have been conducted by using FESS in UPS systems [9, 119], dynamic voltage compensation [120, 121], transient stability enhancement [122], and so forth. The

applications of FESS in the power system can greatly improve the quality and the reliability of power utility.

2.6.4 Applications in combat vehicles

Hybrid electric power is an essential technology for many future combat vehicles, given the number of electrically powered subsystems planned for them. The hybrid electric power systems will have some power-averaging energy storage components, like the flywheel and super-capacitors, which are aimed to provide both continuous and pulsed power for the various systems of the vehicle. For those combat systems that must be provided with power in less than about $10\mu\text{s}$, the flywheel batteries would first charge a bank of super-capacitors, which would then power the high-speed systems [4, 91].

Another likely application of flywheel battery is in the launching of aircraft from carriers. Today, launch catapults are driven by steam systems, which use steam accumulators to store enough energy to perform the job [4]. Electromagnetic aircraft launch system (EMALS) has the potential for reduced weight and volume compared to the large and heavy steam catapults currently being used. Flywheel technology has been considered a potential energy storage choice due to its potential for reduced weight and volume, high duty-cycle tolerance, and low maintenance requirements [123]. The flywheel based EMALS technology could deliver higher launch energy capability and provide improved controllability.

2.6.5 Prospect for FESS

Flywheel energy storage system has the potential to be a promising technology, and the practical application in the market has demonstrated an increasing growth. Fig. 2.12 shows the average annual growth on sales of FESS from 1996 to 2000 is 17% [124].

The FESS is a technology that is still undergoing development. Many improvements can be fulfilled to make the whole system more robust, the controls of the motor and magnetic

bearings more stable and viable, the performances of the energy density, power density and efficiency more attractive. According to the current technology projections on material, control and magnetic bearing systems, it is not unreasonable to expect a higher power density up to 30 kW/kg, and a higher energy density exceeding 200 Whr/kg. By reaching such performance characteristics, the FESS will definitely have a brilliant prospect.

Fig. 2. 12 Revenue on sales of FESS from 1996 to 2000 [124]

2.7 Conclusions

This chapter has provided a general review of the FESS. In Section 2.2, the fundamentals and the origin of FESS were introduced, which gave an overall view of the nature of FESS and its research history. In Section 2.3, different energy storage systems were discussed and compared with the FESS. The advantages of the FESS are sufficient motivations for further research in them. Section 2.4 has described the configuration and components of the FESSs, which provide a clear overview of their structures and principles. In Section 2.5, the control of FESS has been reviewed, taking various aspects of motor/generator, power electronics and magnetic bearings into consideration. The applications of the FESS have also been introduced in Section 2.6. All the materials show that FESS is an appealing technology that will definitely have wide applications and optimistic prospects. However, there is need to improve the energy density and power density, to reduce the complexity of their design and controls, and to lower the costs of production.

CHAPTER 3

DESIGN OF FLYWHEEL ENERGY STORAGE SYSTEM BASED ON AXIAL-FLUX PM MACHINES

3.1 Introduction

Designing a flywheel energy storage system is a multi-disciplinary project. Disciplines of Electrics, Electronics, Magnetics, Mechanics, Material and Control are involved in this research. Accordingly, in the design of FESS, numerous issues arising from these disciplines should be taken into consideration. In this project, the design is mostly focused on the electromagnetic aspect, especially the design of a partially-self-bearing motor/generator.

A compact and efficient FESS based on partially-self-bearing dual air-gap axial-flux synchronous PM machine is proposed in this work [57, 58]. The disk-type PM rotor functions as the flywheel and is sandwiched between two disk-type stators. The rotation torque and axial levitation force can be generated by the interaction of the magnetic fields produced by the windings and the PMs. This design allows the axial flux PM motor/generator to achieve self magnetic bearing in the axial direction. The other four degrees of freedom can either be actively controlled by two sets of radial magnetic bearings or they can be restricted by mechanical bearings.

In this study, two versions of the proposed FESS are designed: one for normal high speed operation, and one for scaled-down low speed operation. The high speed system is designed for studying its performance, and for verifying its feasibility by mathematical and

FEM analyses. The scaled-down low speed system, however, is designed because it is more amenable and less costly for preliminary laboratory prototyping. Active magnetic bearings or mechanical bearings are to be employed separately to constrain the degrees of freedom (DOFs) in radial directions. The latter were eventually implemented in the laboratory prototype for simple operation. The design, modeling and control of the former have also been done to study the feasibility and performances of active magnetic bearing system for radial constraints.

This chapter is organized as follows: The design philosophy is described in Section 3.2, the overall structure and the basic features of the components are described in Section 3.3, and the magnetic analysis for improving machine design is presented in Section 3.4. In Section 3.5, the design procedures of the axial-flux synchronous PM machine are presented and the performances of the high-speed FESS are analyzed in Section 3.6. In Section 3.7, the design of the scaled-down low-speed axial-flux machine which is to be used for experiments is also presented. The concluding remarks are given at the end of the chapter.

3.2 Design Philosophy

3.2.1 Design requirements for FESS

High efficiency is required for the design of FESS so that the flywheel can be an effective energy storage medium. To achieve the goal, the motor/generator has to be of high efficiency with windage losses and frictional losses being minimal. Vacuum housing is a good accommodation to reduce windage losses and the implementation of magnetic bearings can further eliminates the influence of frictional loss during the operation.

High energy density is expected in FESSs, especially for applications in space station and vehicles. To achieve high energy density, the flywheel has to spin at a very high speed of above 50,000 rpm. The weight of the flywheel should not be excessive to reduce overall system weight. This is the reason of selecting high strength and light weight composite material to be the rim of the flywheel.

High power density is also required for FESSs. It is almost wholly dependent upon the motor/generator. Based on current technology projections, it is possible to expect a power density up to 30 kW/kg [3]. This can be achieved with higher rotational speed and better heat exchangers.

High torque density is another requirement of the FESS. In space applications, the charge and discharge durations are the periods of non-eclipse and eclipse respectively, which will last about 35 minutes for LEO satellites. As a result, the torque should be large enough to increase the speed to 50,000 rpm or more during the short charging time. And it is more demanding for the combat vehicles: the energy has to be discharged in a very short time to gain a high enough power to launch shells or missiles.

3.2.2 Motor/generator selection

High efficiency and low rotor losses are the key requirements for the motor/generator. High-speed flywheels operate in vacuum to reduce windage losses, the main paths for heat transfer from the rotor are through radiation and the bearings (if the mechanical bearings are used). Therefore, controlling the rotor losses is crucial to prevent overheating of the rotor.

The possible types of machines considered are as follow:

Switched Reluctance Machine

The switched reluctance machine has a high torque density [125]. The switched reluctance generator (SRG) can be considered the dual of the switched reluctance motor (SRM) except there are differences in control objectives and control implementations. However, a laminated rotor is essential in the switched reluctance machine. The use of laminations requires a separate central shaft and a hole in the center of each lamination that will at least double the stress at the inner radius and hence decrease the peak speed [126]. Furthermore, lamination tensile strength is typically 500 Mpa, which is only half of the high-strength steel. To store the same amount of energy, the laminated rotor would need at least four

times the mass of a solid rotor. Considering all the above-mentioned conditions, the switched reluctance machine is not suitable for FESSs.

Synchronous Reluctance Machine

Synchronous reluctance machine has two features which make them attractive for this application [70]. First, they are excited in singles, so that there are negligible iron losses when the flywheel is not generating power. Second, in the ideal scenario, there are no rotor losses. However, it is difficult to construct a synchronous reluctance rotor with high L_d / L_q ratio while maintaining a robust rotor structure. This leads to a low power factor and hence increases the required VA rating of the drive and can add significantly to the cost of the system. According to these considerations, the synchronous reluctance machine is not attractive for application in FESSs.

Induction Machine

A solid rotor induction machine provides an attractive rotor configuration, as the simple disk geometry can be employed. However, the power factor is generally poor and the rotor losses are high. Therefore, the induction machines are almost ruled out in high speed FESSs. However in low-speed flywheel systems, they are still favorable options. Homopolar inductor motor/generator is an option with its advantages like compact structure, high power and high efficiency. However, the energy capacity and energy density of such motor/generator are very low [56].

Permanent Magnet Machine

Permanent magnet machines offer high torque densities, high efficiency and low rotor losses. Moreover, the PM machine is easier to be controlled with the absence of electrical excitation. And the PMs in the rotor offer large magnetic flux which can generate large torque. In spite of having its drawbacks such as low strength, high cost and temperature sensitivity of the PMs, PM machine is still more competitive compared with other

machines when they are to be implemented in FESSs. For this reason, PM synchronous machine is employed in flywheel systems.

Axial Flux Motor

The basic knowledge of axial flux motors has been introduced in Chapter 2. It has many advantages such as higher power density, higher torque density, higher efficiency over radial flux machine, which make it fairly attractive in the applications for FESS. Dual-stator axial-flux PM motor was therefore considered in this study.

However, one disadvantage is that using distributed windings in pancake-type of stator may result in end-windings of greater length compared to the active part of the coil conductors. This obviously results in poor machine performance, as a significant part of the machine copper (eg: more than 50% of the total in some machine designs) is producing heat but no torque [77]. Concentrated windings can solve this problem as they have simple design, easy arrangement and high efficiency.

3.2.3 Concept of the partially-self-bearing machine with flywheel-rotor

Generally, the flywheel is independent from the motor/generator. Radial flux electric machines are often employed in conventional FESS modules, and their rotors are mounted on the shafts of the flywheels to inter-convert between the mechanical energy and the electrical energy. However with this arrangement, at least one set of axial magnetic bearings are required to levitate the flywheel in axial direction. This separate driving motor-generator in addition to the magnetic bearings makes the system larger and the shaft of the rotor becomes longer. This will then increase the size of the whole system and is apt to generate bending vibrations and hence makes the system unstable. The large motor-bearing system adds difficulties in miniaturization [127]. To solve these problems, a partially-self-bearing permanent magnet machine is proposed here.

Self-bearing motor is an electric machine which combines magnetic bearing and motoring functionality to perform both bearing force and motor torque production. In previous

works [83, 87, 128], two sets of stator windings for producing rotation and levitation force are used, which makes construction of the motor and control system complicated. To overcome this difficulty, a novel partially-self-bearing machine is proposed in this study, of which the stator has only one set of winding but provides dual functions as rotary motor and axial magnetic bearings. Such design can reduce the complexity of the stator structure and the control system, and the implementation into the axial-flux machine can reduce the overall length of the machine because less mechanical bearings are required, thus increasing power density, reducing weight, and lowering susceptibility of rotor dynamic vibrations [129].

According to this idea, axial flux synchronous PM machine with dual-stator and double-sided single-rotor is suitable for the system. The rotor also functions as the flywheel and is sandwiched between two disk-type stators. PMs are mounted on the surfaces of the steel rotor. Composite material ring can be mounted on the rim of the steel rotor. The high-strength light-weight material can store more energy when spinning at a very high speed without being destroyed by the centrifugal force.

Furthermore, the number of poles determines the synchronous drive frequency at full speed. A large number of poles will result in a high frequency and hence increase the core loss in the stator and the switching loss in the inverters. Especially when the flywheel spins in high speed, the frequency will tend to be large. For instance, when the full speed is 60,000rpm, the synchronous drive frequency will be 2 kHz for a 4-pole motor, so when the pole number decreases to 2, the frequency will decrease to 1 kHz, reducing the iron losses and switching losses. However, a 4-pole motor can be more stable in $x-y$ plane, especially when it rotates in low speed. So in this study, a 2-pole machine is adopted for the study of high-speed flywheel system, while a 4-pole design is used in the scaled-down low-speed laboratory prototype.

3.2.4 Constraints for five degrees-of-freedom of the flywheel

As shown in Fig. 3.1, there are three directions along x , y and z axes within the flywheel, such that five degrees-of-freedom (DOF) which are the displacement and rotation of x - and y - axes and the displacement of the z -axis should be controlled with the help of mechanical or magnetic bearings.

Active magnetic bearings have been widely implemented in FESSs due to its many advantages as mentioned in Chapter 2. Most of them require at least one set of axial magnetic bearing and two sets of radial magnetic bearing to manage the 5 DOFs. Since the control of each DOF requires a sensor, an actuator and a controller, the entire system becomes complex in terms of the design of its mechanical/electrical parts and the control system [130].

Fig. 3. 1 Three motion directions of flywheel.

Mechanical bearings have the advantages of simple structure and easy operation, but the frictional loss and therefore, the use of lubrication should always be taken into consideration. Especially, the friction occurring on the bearing which is along the direction of the gravity, i.e., the direction along z axis in Fig. 3.1, is much greater than those in the other directions. For this reason, it is not practical to use mechanical bearings along this axis, while for the other axes, they may still be tolerated.

By considering these points described above, this study has considered both types of bearings. The translation and rotation along and about axial directions are actively

controlled by the partially-self-bearing machine, but the motions in other directions are restricted by two types of bearings, i.e., active radial magnetic bearings or mechanical bearings. Radial magnetic bearings are not implemented in the laboratory prototype, but the modeling and control method are presented with simulation results. In the experimental prototype, mechanical bearings are utilized to constrain the DOFs of the flywheel in radial directions. The combined use of self magnetic bearing and external bearings can cut down the complexity of control and make the system more stable, viable and cost-effective.

3.3 Overall Structure of the Flywheel System

This section introduces the conceptual structure of the proposed FESS. The linear motions and rotations along and about the radial directions are to be constrained by mechanical bearings or by magnetic bearings. The motor/generator and the flywheel are structurally integrated. For high and low speed flywheel systems, the stator and rotor constructions vary with the different pole numbers.

3.3.1 Structure of Partially-Self-Bearing Axial-Flux PM Machine

Fig. 3.2 shows the components of the main sub-system of the proposed FESS, which is termed as a partially-self-bearing axial-flux machine. As shown in Fig. 3.2 (a), the rotor with a carbon composite rim doubles as the flywheel and is sandwiched between two disk-type stators; two sets of mechanical bearings are installed at the ends of the shaft. This design maximizes the torque production area of the disk-type rotor.

As shown in Fig. 3.2 (b) and (c), each of the upper and lower stators carries a set of three-phase copper windings to be fed with sinusoidal currents; concentrated windings are implemented to reduce the power loss. If distributed windings are used, the winding-ends will span half the circumference of the rotor. The ends are much longer compared to the effective parts of coil conductors, and the copper loss of the windings will thus be larger. In this particular design, there are 3 coils for a 2-pole stator and 6 coils for a 4-pole stator; each of the coils surrounds a stator tooth. The distribution of the three phases and directions

of the three-phase currents at a particular instance are shown in Fig. 3.3. Besides improved efficiency, simple structure and easy installation of the stator winding can also be realized in this design.

Fig. 3. 2 Basic Parts of Proposed Flywheel System

The geometry of the flywheel-rotor is depicted in Fig. 3.2 (d) and (e). The disk-type steel rotor is installed on the nonmagnetic steel shaft. Permanent magnets are mounted on both surfaces of the rotor. The arrangement of these PMs and the magnetic flux flowing in the

motor are shown in Fig. 3.3. PMs are settled in opposite directions of upper and lower rotor faces, so that they would attract each other and increase the total flux in the magnetic circuits. The carbon composite rim is molded outside the steel rotor. The light weight and high tensile strength make it possible to spin at a high speed over 60,000 rpm. A guard ring made of high strength non-magnetic material is used to assist the PMs in resisting the centrifugal force, as shown in Fig. 3.2 (f).

Fig. 3. 3 Motor development structure and 2D flux pattern

3.3.2 Overall structure of the FESS based on radial mechanical bearings

The structure of the mechanical bearing is shown in Fig. 3.4 (a). The two mechanical bearings with 2 holes in their ears are fixed at the ends of the shaft. Two smooth sliding bars are installed on 2 salient bases and go through the holes, as shown in Fig. 3.4 (b). Then the flywheel together with the two mechanical bearings can slide up and down along the bars. The dimension between the surfaces of the mechanical bearing and the salient base is only 0.5 mm, which is half of the air-gap length between the stator and the rotor. These settings can avoid the flywheel touching the stator in the case of starting up or emergency failure. The salient base can also support the rotor-shaft assembly when the FESS is at standstill. In this design, touchdown bearings are not required, making the system more compact and lighter.

Fig. 3.5 shows the cross-sectional diagram of the flywheel energy storage system assisted by mechanical ball bearings. Its components are listed in Table 3.1. Items 1 and 5 are the upper and lower stators fixed on the system housing, which are designed to dissipate radial kinetic energy from any rotor debris and ensure safety in the event of mechanical failure. The flywheel spins in vacuum environment to reduce windage losses. Axial flux permanent magnet synchronous machine is implemented to drive the flywheel, which is also functioned as a rotor with composite material rim mounted.

Fig. 3. 4 Configuration of mechanical bearings

Fig. 3. 5 Cross-sectional diagram of the proposed system with radial mechanical bearings

Mechanical rotational bearings are mounted at the two ends of the shaft to constrain its radial motion and assist the rotation of the flywheel-rotor. The axial motion can be realized with the aid of the holes in the mechanical bearing ear and the smooth sliding bars.

Non-contact eddy current displacement sensor and photo electrical sensor are placed on the top and bottom of the housing to detect the displacement and angular position along z-axis when the rotor spins, (items #13 and #7 in Fig. 3.5). Touchdown bearings are no longer needed with the use of mechanical bearings.

Table 3. 1 Components of the Proposed FESS with radial mechanical bearings

Item Number	Item Name
1	Upper stator
2	Stator windings
3	System vacuum housing
4	Steel rotor
5	Lower stator
6	Shaft
7	Rotation sensors
8	Rotor permanent magnets
9	Non-magnetic material guard ring
10	Composite rim
11	Smooth sliding bar
12	Solid resistive base
13	Displacement sensor
14	Mechanical ball bearing

3.3.3 Overall structure of the FESS based on radial magnetic bearings

Instead of using mechanical bearings, it is also possible to use radial magnetic bearings. The structure and material of the electromagnetic iron for the radial magnetic bearings in practical applications is much like that of radial flux stators. As shown in Fig. 3.6, there are 16 slots in the inner circumstance of the stator, and each coil surrounds two teeth. When DC current flows though the coil, a pair of poles (NS) will be generated. Each pair of poles will generate a force acting on the rotor (it can be mounted on the shaft of flywheel), then

eight forces as depicted in Fig. 3.6 can be obtained with these eight pairs of electromagnetic poles. By controlling the currents, the total forces in x and y directions can be adjusted to zero so as to keep the rotor balanced.

Fig. 3. 6 Structure of the magnetic bearing

Fig. 3. 7 Basic parts of a magnetic bearing-based flywheel energy storage system

Fig. 3.7 shows the cross-sectional view of another version of the FESS which is assisted by radial magnetic bearings. The components of the FESS are listed in Table 3.2. The functions of the components are similar to those in Table 3.1. As shown in Fig. 3.7, the middle parts of the system are all identical, the differences are the upper and lower magnetic bearings (items #11 and #7 in Fig. 3.7) which replace the mechanical bearings. In this case, the touchdown bearings (items #8 and #12) are necessary to assist the flywheel during starting-up, shutting-down and in case of magnetic bearing failures. None of the sensors is depicted in the diagram.

Table 3. 2 Components of the Proposed FESS with Radial Magnetic Bearings

Item Number	Item Name
1	Upper stator
2	Stator windings
3	System vacuum housing
4	Steel rotor
5	Lower stator
6	Shaft
7	Lower radial magnetic bearing
8	Lower touchdown bearing
9	Permanents magnets
10	Composite rim
11	Upper radial magnetic bearing
12	Upper touchdown bearing

3.4 Magnetic analysis

Magnetic circuit analysis plays an important role in PM machine design. The air-gap flux density can be calculated by employing open-circuit analysis, and the relationship between the thickness of the PMs and the air-gap length may be deduced. These will provide valuable guidance in the machine design. The analysis of the equivalent Magnetomotive Force (MMF) of the PMs will also give a good reference in the design.

3.4.1 Magnetic circuit analysis on open-circuit

Fig. 3.8 shows the equivalent magnetic circuit of a flux path which is depicted in Fig. 3.3. In the following, the steel core of the stator and rotor are assumed to be infinitely permeable.

Fig. 3. 8 Equivalent magnetic circuit of one flux path [131]

Each magnet can be represented by a ‘Norton’ equivalent circuit consisting of a flux generator in parallel with an internal leakage permeance

$$\Phi_r = B_r A_m \quad (3.1)$$

$$P_{m0} = \frac{\mu_r \mu_0 A_m}{l_m} \quad (3.2)$$

where A_m is the pole area of the magnet; l_m is the magnet length in the direction of magnetization; B_r is the remanent flux density and μ_r is the relative permeability.

The remaining permeance in the magnetic circuit is the rotor leakage permeance P_{r1} , which represents the path of magnet flux component that fails to cross the airgap. The rotor leakage permeance is always small, typically 5-20 percent of the magnet internal permeance, and it is convenient to include it in a modified magnet internal permeance by

writing [131]

$$P_m = P_{m0} + P_{r1} = P_{m0}(1 + p_{r1}) \quad (3.3)$$

$$R_m = 1 / P_m \quad (3.4)$$

where p_{r1} is the normalized rotor leakage permeance, i.e., normalized to P_{m0} . In the design of electrical machines with rated power in the range 1-10 kW, the rule of thumb value of p_{r1} is 0.1 [79, 87, 131-133]; it is therefore adopted in this work.

The air-gap reluctance R_{g1} and R_{g2} can be represented by

$$R_{g1} = \frac{l_g' + \Delta g}{\mu_0 A_g} \quad (3.5)$$

$$R_{g2} = \frac{l_g' - \Delta g}{\mu_0 A_g} \quad (3.6)$$

where l_g' is the equivalent air-gap length allowing for slotting when rotor is in the middle position between these two stators. $l_g' = K_c l_g$, here, $K_c = 1.05$, which is the Carter's factor.

Δg is the displacement of the rotor in the direction of axial axis. A_g is the area through which the flux passes it as it crosses the gap.

According to Fig. 3.8, the flux flowing in the magnetic circuit can be calculated as

$$\frac{1}{2} \Phi_g = \frac{4\Phi_r R_m}{8R_m + 4R_{g1} + 4R_{g2}} \quad (3.7)$$

The air-gap flux density can be extracted as

$$B_g = \frac{\Phi_g}{A_g} = \frac{l_m A_m}{l_m A_g + \mu_r (1 + p_{r1}) l_g' A_m} B_r \quad (3.8)$$

A well-known approach to understand magnet characteristics is the B-H loop (or called hysteresis loop). The hysteresis loop of a hard PM material is a straight line throughout the

second quadrant, which is called the demagnetization curve as shown in Fig. 3.9. The slope of the demagnetization curve is the relative permeability, μ_r . The line drawn from the origin through the magnet's operating point is called the load line and the absolute value of its slope is the permeance coefficient (PC) [131].

Fig. 3. 9 Magnet demagnetization curve [131]

From the figure, we can obtain the equations below

$$B_m = B_r + \mu_r \mu_0 H_m \quad (3.9)$$

$$PC = \frac{B_m}{-\mu_0 H_m} \quad (3.10)$$

From Fig. 3.8, the magnetic field intensity in the magnet H_m can be derived as

$$H_m = -\frac{l_{g1} H_{g1} + l_{g2} H_{g2}}{2l_m} = -\frac{l_g}{l_m} \frac{B_g}{\mu_0} \quad (3.11)$$

The magnet flux density B_m can be derived by substituting (3.8) and (3.11) into (3.9)

$$B_m = \frac{1 + \mu_r p_{r1} \frac{l_g'}{l_m} \frac{A_m}{A_g}}{1 + \mu_r (1 + p_{r1}) \frac{l_g'}{l_m} \frac{A_m}{A_g}} B_r \quad (3.12)$$

Substituting (3.8), (3.9) and (3.12) into (3.11), the equation can be changed to

$$PC = \frac{1 + p_{r1} \mu_r \frac{A_m}{A_g} \frac{l_g'}{l_m}}{\frac{A_m}{A_g} \frac{l_g'}{l_m}} \quad (3.13)$$

In the surface mounted PM motors, p_{r1} is very small, if $A_m / A_g \approx 1$, then PC is roughly equal to l_m / l_g' .

From (3.8) and (3.13), it is clear that the value of p_{r1} has very slight influence on the calculation of air-gap flux density B_g and the ratio between magnet thickness and air-gap length l_m / l_g' .

Minimum length of air-gap is set by mechanical constraints and is unlikely to be less than 0.3 mm. Magnets depth should generally be minimized so as to reduce the cost of the magnets. Manufacturing restrictions make it difficult to have magnets less than 2.0mm deep. In this design, l_g is selected as 1 mm, and l_m is set to be 3 mm.

Substituting (3.13) into (3.12), (3.12) can be simplified as

$$B_m = \frac{PC}{PC + \mu_r} B_r \quad (3.14)$$

3.4.2 The equivalent MMF of the PMs

PMs can generate the magnetic field without external excitation. However the equivalent MMF generated by the magnet should be given so that it could be used in the modeling and calculations.

As shown in Fig. 3.3, the PMs mounted on the surfaces of the rotor are shorted by highly permeable iron. In order to describe it clearly, Fig. 3.10 is employed to describe the situation. As shown in Fig. 3.10, a magnet is shorted by highly permeable iron. When the magnet is shorted, there is no H field, and the air-gap length $l_g' = 0$. From (3.12), the magnet flux density then equals to the remanent flux density

$$B_m = B_r \quad (3.15)$$

From Fig. 3.9, the magnetic field intensity in the magnet equals to H_c

$$H_c = \frac{B_r}{\mu_r \mu_0} \quad (3.16)$$

Suppose that the magnet is no longer magnetized. The flux density B_m will be a stable value. The equivalent MMF F needed to generate the same B_m can be calculated as a constant value, which is described as

$$F = H_c l_m + H_{iron} l_{iron} = H_c l_m \quad (3.17)$$

Fig. 3. 10 Diagram of a magnet shorted by highly permeable iron

In the FESS designed in this thesis, 3 mm deep NdFeB magnets with the remanent flux

density $B_r = 1.23$ are employed. The equivalent MMF F generated by the magnet can be derived according to (3.16) and (3.17):

$$F = \frac{B_r l_m}{\mu_r \mu_0} = \frac{1.23 \times 0.003}{1.09 \times 4\pi \times 10^{-7}} \approx 2694 \text{ Amps} \tag{3.18}$$

Therefore, for small machines, PM rotor is much more attractive than wound rotor.

3.5 Design of the partially-self-bearing axial-flux PM machine

As described above, the bearing support along axial direction is to be generated by the motor/generator itself, but the support in radial directions can be contributed by external magnetic bearings or mechanical bearings. The geometries of the stators and the rotor are of the pancake-type rather than of cylindrical shape, and the flux is in axial direction instead of in radial direction. The design of this machine is therefore different from that of the conventional cylindrical machine.

3.5.1 Design specifications of the high speed 2-pole machine

Table 3. 3 Specifications of the high speed 2-pole machine

Maximum power	4 kW
Rated voltage	220 V
Energy stored	350 Whr
Power factor	0.9
Efficiency	0.9
Maximum speed	60,000 rpm
Frequency at full speed	1000 Hz
Slot fill factor	0.4
Remanent flux density	1.23 T
Magnet recoil permeability	1.1
Carter’s factor	1.05

The specifications and other design data of the high speed 2-pole machine are listed in Table 3.3. According to the FESS developed by NASA [49], the maximum flywheel

velocity is selected as 60,000 rpm. The maximum power of the machine is 4 kW, and the rated voltage is 220 V. The expected power factor and efficiency are both set to 0.9. A stored energy of 350 Whr is acquired when the flywheel spins at maximum speed.

The remanent flux density of the permanent magnet comes from the data sheet provided by the PM manufacture. The slot fill factor, the magnet recoil permeability and the Carter's factor are common values in electric machine design [132,133]. The axial-flux PM machine was designed as described in the following sections in accordance with the specifications and design data.

3.5.2 Dimension relationship

The traditional electric machines have radial-flux cylindrical structures; the electric machine design methods are hence based on this type of structure. To employ the machine design approaches, it is necessary to transform the dimensions of the axial flux machine to an equivalent sized radial machine.

As shown in Fig. 3.11, the axial-flux machine with the outer and inner diameters D_o and D_i can be represented in terms of an equivalent sized radial machine by the following simple transformation [76].

$$D = \frac{D_o + D_i}{2} \quad (3.19)$$

$$L = \frac{D_o - D_i}{2} \quad (3.20)$$

where, D_o and D_i are the outer and inner diameters of the axial-flux disk-type machine, D and L are the inner diameter and length of the outer rotor of the equivalent radial-flux machine respectively.

After the transformation, the radial-flux machine design procedures can be applied for designing the proposed axial-flux machine, adopting the dimensions of an equivalent radial-flux machine.

The pole pitch can be expressed as

$$\tau = \frac{\pi D}{2P} \quad (3.21)$$

where, P is the number of pole pairs.

(a) Axial flux machine shape and dimension (b) Radial machine shape and dimension

Fig. 3. 11 The dimension transformation between axial and radial machines

The no-load rms voltage induced in one phase of the stator winding is [131]

$$E_f = \sqrt{2}\pi f N_1 K_w \Phi_f \quad (3.22)$$

where N_1 is the number of the stator turns per phase, K_w is the winding factor, Φ_f is the magnetic excitation flux, and f is the input frequency. The fundamental component of Φ_f can be calculated as

$$\Phi_{f1} = L \int_0^\tau B_{g1} \sin\left(\frac{\pi}{\tau}x\right) dx = \frac{2}{\pi} \tau L B_{g1} \quad (3.23)$$

where, B_{g1} is the maximum value of the fundamental component of air-gap flux density B_g which can be derived from (3.8).

The peak value of electric loading can be defined as [132]

$$A_m = \frac{\sqrt{2}mN_1I_a}{\pi D} \quad (3.24)$$

where, I_a is the phase current flowing into the winding. The electrical rating can be defined as [131]

$$S = mE_fI_a \quad (3.25)$$

By substituting Equations (3.21)-(3.24) into Equation (3.25), the following equation is formed

$$S = \pi^2 K_w n_s B_{g1} A_m D^2 L \quad (3.26)$$

Defining

$$K_R = D_o / D_i \quad (3.27)$$

Maximum torque is generated when $K_R = \sqrt{3}$ [80].

After substituting Equations (3.19), (3.20) and (3.27) into Equation (3.26), the outer diameter of the axial-flux machine can be written as

$$D_o = \sqrt[3]{\frac{8SK_R^3}{(K_R + 1)^2 (K_R - 1)C_0 n_s}} \quad (3.28)$$

Where C_0 is the output coefficient, n_s is the rated speed in r.p.s

$$C_0 = \pi^2 B_{g1} A_m K_w \times 10^{-3} \quad (3.29)$$

From Equation (3.27), the inner diameter of the axial-flux machine can be obtained as

$$D_i = \frac{D_o}{K_R} \quad (3.30)$$

3.5.3 Stator windings and core design

A. Pole and slot pitches

The number of slots is defined as [132]

$$Z = 2mpq \quad (3.31)$$

where, m is the phase number of the AC machine, p is the pole pair number, and q is the slot number per pole per phase.

The inner and outer pole pitch of the axial-flux machine are written as

$$\tau_i = \frac{\pi D_i}{2p} \quad (3.32)$$

$$\tau_o = \frac{\pi D_o}{2p} \quad (3.33)$$

Then the average pole pitch can be obtained as follows

$$\tau_{av} = \frac{\tau_o + \tau_i}{2} \quad (3.34)$$

Likewise, the inner, outer and average slot pitches are indicated as below

$$s_i = \frac{\pi D_i}{Z} \quad (3.35)$$

$$s_o = \frac{\pi D_o}{Z} \quad (3.36)$$

$$s_{av} = \frac{s_o + s_i}{2} \quad (3.37)$$

B. Stator winding design

As described in Sections 3.2 and 3.3, the concentrated windings are employed in axial-flux machines. Two-pole and four-pole stator windings are depicted in Fig. 3.12. The three different colors represent 3 phases of the machines. In this study, 3 coils are installed in

2-pole machine whilst 6 coils are employed in 4-pole machine.

Fig. 3. 12 Concentrated short pitch stator windings for the axial-flux machine

As shown in Fig. 3.2 (b) and (c), due to the implementation of concentrated windings, the left and right effective conductor parts of two different phase windings are inserted into one slot. The 2-pole machine thus has 3 slots while the 4-pole machine has 6 slots. However, in this machine design, it is assumed that a winding occupies 2 slots. Therefore, the slot numbers per pole per phase of the two machines are both 1, i.e., $q = 1$.

The electrical angle between two slots is

$$\alpha = \frac{p \times 2\pi}{Z_1} \quad (3. 38)$$

where, Z_1 is the slot number used in machine design, which is double of the actual slot number of the machine, i.e., $Z_1 = 2Z$.

The distribution factor of the windings can be calculated as [132]

$$K_p = \frac{\sin(q\alpha / 2)}{q \times \sin(\alpha / 2)} \quad (3. 39)$$

The winding factor is defined as [131]

$$K_w = K_p \times K_q \quad (3. 40)$$

where, K_q is the pitch factor, which is 1 in these two designs.

The flux per pole can be calculated as below

$$\Phi_{gp} = B_{gav} \tau_{av} L = K_g B_g \tau_{av} L \quad (3.41)$$

where, B_{gav} is the average air-gap flux density, K_g is the air-gap flux shape factor, L is the equivalent length as described in (3.20).

The machine windings are in Y connection, so the stator voltage per phase can be calculated as

$$V_{N\Phi} = \frac{V_N}{\sqrt{3}} \quad (3.42)$$

where, V_N is the rated voltage of the machine.

The turn number of the winding can be obtained as below [132]

$$N_{ph} = \frac{V_{N\Phi}}{4.44 f \Phi_{gp} K_w} \quad (3.43)$$

The phase current of the stator winding can be described as [131]

$$I_s = \frac{P_N}{m V_{N\Phi} \eta_N \cos \Phi_N} \quad (3.44)$$

where, P_N is the rated power, η_N is the efficiency, and $\cos \Phi_N$ is the power factor of the electric machine. Given the current density J of the winding conductor, the area of the winding conductor may be calculated as [132]

$$A_c = \frac{I_s}{J} \quad (3.45)$$

Then the diameter of the conductor can be derived

$$D_c = \sqrt{\frac{4A_c}{\pi}} \quad (3.46)$$

According to this diameter, the specific wire used for the conductors of the windings can be selected. Taking safety into consideration, the diameter of the wire D_{cw} should be selected greater than D_c .

C. Stator design

The stator structures of the 2-pole and 4-pole axial-flux machines are shown in Fig. 3.13. Assuming that the flux density in the stator teeth is B_t , the average width of the stator teeth can be described as

$$t_{av} = \frac{s_{av} B_{gav}}{B_t} \quad (3.47)$$

Then the width of the slot can be expressed as

$$b_s = s_{av} - t_{av} \quad (3.48)$$

Fig. 3. 13 Stator structure of the 2-pole and 4-pole axial flux machines

The required area for bare conductors in a slot is

$$A_{sw} = N_s A_{cw} \quad (3.49)$$

Supposing the space factor of the conductors filled in the slot is K_s , which is selected to be

0.4 in this design, then the area of each slot becomes

$$A_s = \frac{A_{sw}}{K_s} \quad (3.50)$$

The effective slot length to install the conductors can then be obtained

$$l_{sc} = \frac{A_s}{b_s} \quad (3.51)$$

Fig. 3. 14 Slot and stator core shapes and dimensions of the axial-flux machine

The slot and stator core shapes with the dimensions are shown in Fig. 3.14.

The flux in stator yoke is

$$\Phi_{yk} = \frac{\Phi_{gp}}{2} \quad (3.52)$$

Assuming the flux density in the stator yoke is B_{yk} , the area of the space where the flux flows through the stator yoke can be derived as

$$A_{yk} = \frac{\Phi_{yk}}{B_{yk}} \quad (3.53)$$

Then the depth of the stator yoke

$$d_{yk} = \frac{A_{yk}}{L} \quad (3.54)$$

3.5.4 Rotor design

As the machine is axial-flux PM type, the selection and arrangement of permanent magnets should also be taken into careful consideration.

A. Selection and arrangement of permanent magnets

Alnicos, Ceramics and Rare-earth Materials are the three classes of PMs which are currently used for electric motors [133]. The main advantages of Alnico include its high magnetic remanent flux density and low temperature coefficients. Unfortunately, its coercive force is relatively low and the demagnetization curve is extremely non-linear. Therefore, both magnetization and demagnetization can occur easily. A ferrite has a higher coercive force but lower remanent magnetic flux density compared with Alnico. Samarium-cobalt (SmCo) and neodymium-iron-boron (NdFeB) are the primary materials for rare-earth PMs. SmCo₅ has the advantage of high remanent flux density, high coercive force, high energy product, linear demagnetization curve and low temperature coefficient. However, Sm and Co are both expensive due to their supply restrictions. Neodymium (Nd) is a much more abundant rare-earth element in comparison with Sm. NdFeB magnets, which are now produced in increasing quantities, have better magnetic properties than those of SmCo, however only at room temperature. The demagnetization curves, especially the coercive force, are strongly temperature dependent. NdFeB magnets have great potential for considerably improving the performance-to-cost ratio for many applications. By considering all the pros and cons of these magnets, NdFeB is selected to be used in this study.

There are two main configurations for axial-flux PM rotors, which are surface-mounted type and inset-type. For surface-mounted type, the permanent magnets are mounted on the surface of the rotor, but for inset type, the PMs are totally inserted into the rotor. Surface-mounted PM rotor has the advantages of simple and low-cost construction, and easy to be controlled with the same reactances in d and q axes. Its disadvantage of

mechanical insecurity can be overcome by using a guard-ring. In this study, the PMs are mounted on the surface of the iron rotor, and inset into the high strength non-magnetic material guard-ring, as shown in Fig. 3.15.

Fig. 3. 15 Surface-mounted PM rotor and the guard ring

B. Dimensions of the rotor

As analyzed in Section 3.4, the air-gap length is selected to be 1 mm, and the depth of the PMs is 3 mm.

As shown in Fig. 3.16, the inner and outer diameters of the PMs are the same as those of the stator. The PM is designed to be 120° -segment for the 2-pole PM machine, or 60° for the 4-pole machine. Considering the arrangement of the guard ring, the diameter of the steel rotor D_r is designed to be slightly larger than D_o . This is not including the composite material flywheel rim.

Fig. 3. 16 The shape and dimensions of the PMs mounted on the rotor

Supposing the outer diameter of the composite material rim is D_{rm} , the inertia of the total flywheel-rotor is

$$J = \frac{1}{8} M_{rim} (D_{rm}^2 + D_{rt}^2) + \frac{1}{8} M_{rotor} (D_{rt}^2 + D_{sh}^2) \quad (3.55)$$

The mass of the rim and the steel rotor: M_{rim} and M_{rotor} can be calculated as

$$M_{rim} = \frac{1}{4} \rho_{rim} \pi (D_{rm}^2 - D_{rt}^2) h \quad (3.56)$$

$$M_{rotor} = \frac{1}{4} \rho_{steel} \pi (D_{rt}^2 - D_{sh}^2) h \quad (3.57)$$

where, ρ_{rim} and ρ_{steel} are the density of the composite material and the steel material, h is the height of the flywheel-rotor.

The energy stored in the flywheel-rotor can be expressed as [7]

$$E = \frac{1}{2} J \omega^2 \quad (3.58)$$

Therefore, the outer diameter of the rim D_{rm} can be calculated according to Equations (3.55) to (3.58).

3.5.6 Design results of the 2-pole axial-flux machine

Based on the design requirement data shown in Table 3.3, and followed by the design procedures shown above, the 2-pole axial flux PM machine design results were obtained as in Table 3.4.

The stator iron is built up of amorphous iron ribbons to minimum eddy current loss [8]. The material for the rotor is high strength steel. High stiffness non-magnetic stainless steel is adopted as the shaft material. Carbon composite material has higher tensile strength combined with lower density compared to steel, it can resist the huge centrifugal force when spinning in a high speed, that make it suitable to be the material of the rim. The

housing is to be made up of light weight high strength material; aluminum alloy can be selected in this case. High speed angular contact bearings with silicone nitride balls are implemented in this design, vacuum grease lubrication is used in such bearings [134, 135].

Table 3. 4 Main design geometrical data of 2-pole axial-flux PM machine

No. of pole pairs	P	1
No. of slots	Z	3
Outer diameter of stator	D_o	62 mm
Inner diameter of stator	D_i	36 mm
Outer diameter of the rim	D_{rm}	260 mm
Length of the shaft	l_{sh}	220 mm
Diameter of the shaft	D_{sh}	15 mm
Permanent magnets length	l_m	3 mm
Air gap length	l_g	1 mm
Slot width	b_s	20 mm
Slot height	l_s	34 mm
Stator yoke thickness	d_{yk}	18 mm
Rotor total thickness	h	60 mm
No. of turns per phase	N_{ph}	60

3.6 Performance analysis of the designed high-speed FESS

After the design of the proposed high-speed FESS, it is necessary to analyze the performances of the designed FESS to investigate whether the design satisfies the desired requirement or not.

3.6.1 Mechanical performance analysis

According to the designed geometries and the properties of materials selected, the mass of the components of the FESS can be calculated, which is shown in Table 3.4.

The machine can be optimized to have a smaller size by operating in the state approaching saturation. And the total mass of the system can also be decreased after optimizing for all the components.

Table 3. 5 Mass of the FESS components

Components	Mass (kg)
Steel rotor	1.26
Rim	4.78
Shaft	0.24
Stators	2*0.61
Stator windings	2*0.23
Mechanical bearings	2*0.28
Housing	0.72
Other components	0.58
Total mass expected	10

The energy stored in the flywheel when it is operating at maximum speed is

$$E_{\max} = 1/2J\omega_{\max}^2 \tag{3.59}$$

Letting the speed decrease to 20,000 rpm during discharge period, the depth of discharge (DOD) is

$$DOD = [1 - (1/3)^2] \cdot 100\% \approx 88.9\% \tag{3.60}$$

So the usable energy is

$$E = E_{\max} \cdot DOD \tag{3.61}$$

Then the estimated performance of the system can be listed in Table 3.6.

Table 3. 6 Predicted performance of the designed FESS with 2-pole PM machine

Maximum power	4 kW
Maximum speed	60,000 rpm
Mass of the system	10 kg
Moment of inertia for the flywheel	0.0659 kg.m ²
Depth of discharge	88.9%
Maximum energy stored	361.3 Whr
Usable energy stored	321.2 Whr
Power density	400 W/kg
Energy density	32.1 Whr/kg

The power density can be improved by redesigning a higher power motor. And the energy density can be enhanced by increasing the rotating speed and the moment of inertia of the flywheel. The latter can be realized by increasing the outer diameter and the height of the rim.

3.6.2 Efficiency analysis of the high-speed FESS

There are various power losses in this flywheel system. The copper loss of the stator windings and the mechanical losses due to friction in the mechanical bearings are well known. The loss due to windage can be neglected with the use of vacuum housing. The remaining loss is the stator core loss, which can be significant during high speed operation.

A. Copper Loss

The copper loss is easy to estimate after knowing the total length and the diameter of the wire for the 3-phase windings in the dual-stator [132].

$$p_{Cu} = 2 \times 3 \times I^2 R \quad (3.62)$$

$$R = \rho_w \frac{N_{ph} l_{coil}}{A_{wire}} \quad (3.63)$$

where, p_{Cu} is the copper loss in the windings, I is the rms phase current flowing in the windings, R is the resistance per phase, ρ_w is the resistance ratio, N_{ph} is the turn number of the windings per phase, l_{coil} is the length of the coil per turn, and A_{wire} is the area of the copper wire. Using Equation (3.62) and (3.63), the copper loss in the entire FESS can be obtained. When the maximum current with rms value of 5.25 A is assigned, the loss is only 12.73 W (Refer to Appendix A for the detailed calculation).

B. Core Loss

Core losses in a magnetic material occur when the material is subjected to a time varying magnetic flux density $B(t)$. The nature of core losses can be attributed to two different

mechanisms: hysteresis loss and eddy current loss. Hysteresis loss is due to the energy expended in the redirection of the magnetic domains of the material during every flux direction reversal. Eddy current is the circulating current induced in the material by changing magnetic flux as a consequence of electromagnetic induction. It is dependent on the resistivity and geometry of the material and the rate of change of flux. Both hysteresis effect and eddy currents result in heating losses known collectively as core loss [136].

According to the conventional Steinmetz model, when a piece of core material is subjected to sinusoidal flux density of varying magnitude and frequency, the core loss density (in W/kg) is given by a two-term function [136].

$$\bar{P}_{core} = \bar{P}_{hysteresis} + \bar{P}_{eddy} = k_h f \hat{B}^n + k_e f^2 \hat{B}^2 \quad (3.64)$$

where, f is the frequency, $\hat{B}$ is the peak flux density in Tesla, k_h , k_e and n are the constants of the core material. This formula has been applied to estimate core losses in various parts of a motor and summing these losses would give the total core losses of the machine. This technique is however not straightforward for PM motors as the flux densities in critical regions are usually non-sinusoidal.

The hysteresis loss depends mainly on the magnitude of peak-to-peak flux density. The fact the flux density variation may not be sinusoidal is not critical, as long as the flux density variation is periodic. It is the eddy current loss that is greatly affected by harmonics in the flux density waveform, as induced circulating currents are functions of dB/dt . One approach is to represent the flux density as a Fourier series and sum the losses up according to the series of harmonic components. It is of course necessary to know the wave-shape of $B(t)$ in order to determine the harmonic components.

Alternatively, the eddy current loss can be expressed in terms of dB/dt . If the flux is sinusoidal, then

$$B = \hat{B} \sin(2\pi ft) \quad (3.65)$$

After differentiating,

$$\frac{dB}{dt} = 2\pi f \hat{B} \cos(2\pi f) \quad (3.66)$$

Then (3.64) can be transformed as

$$\bar{P}_{eddy} = k_e f^2 \hat{B}^2 = \frac{k_e}{2\pi^2 T} \int_0^T \left(\frac{dB}{dt} \right)^2 dt \quad (3.67)$$

It can be seen that the instantaneous eddy current loss is thus

$$P_{eddy}(t) = \frac{k_e}{2\pi^2} \left(\frac{dB}{dt} \right)^2 \quad (3.68)$$

The main issue in the application of this equation is the formulation of the dB/dt function.

An empirical formula can be used to approximate the $B(t)$ wave-shape from which the average eddy current core loss is computed. In the simplest form, a trapezoid can be assumed for the $B(t)$ in the teeth and yoke of the PM motor stator. As the rotor rotates, the flux density in the stator tooth at the leading edge of the rotor magnet rises from zero to B_T and remains constant while the magnet passes. The time Δt required for the magnet edge to transverse the average tooth width t_{av} is

$$\Delta t = \frac{2t_{av}}{(r_o + r_i)\omega_m} \quad (3.69)$$

where ω_m is the mechanical angular velocity, r_o and r_i are the outer and inner radius of the stator. The rate of change of flux density in the tooth during this period is

$$\frac{dB_T}{dt} = \frac{\hat{B}_T}{\Delta t} = \frac{(r_o + r_i)\omega_m \hat{B}_T}{2l_T} \quad (3.70)$$

This dB_T/dt occurs four times in each tooth during each electrical cycle of the period $T = 2\pi / P\omega_m$. Therefore the average eddy current loss density in the tooth (in W/kg) can be expressed by

$$\bar{P}_{eddy(T)} = \frac{k_e}{2\pi^2 T} \int_0^T \left(\frac{dB_T}{dt} \right)^2 dt = \frac{k_e}{2\pi^2} \frac{4\Delta t}{T} \left(\frac{(r_o + r_i)\omega_m \hat{B}_T}{2t_{av}} \right)^2 = \frac{P(r_o + r_i)}{2\pi^3 t_{av}} k_e \omega_m^2 \hat{B}_T^2 \quad (3.71)$$

where, P is the number of pole pairs. The flux density at any point in the stator yoke changes linearly from $-B_Y$ to B_Y in the time interval Δt for the effective magnet pole arc α_m to transverse that point. This occurs twice in every electrical cycle. Therefore the average eddy current loss density in the yoke is

$$\bar{P}_{eddy(Y)} = \frac{k_e}{2\pi^2 T} \int_0^T \left(\frac{dB_Y}{dt} \right)^2 dt = \frac{k_e}{2\pi^2} \frac{2\Delta t}{T} \left(\frac{2\omega_m \hat{B}_Y}{\alpha_m} \right)^2 = \frac{2P}{\pi^3 \alpha_m} k_e \omega_m^2 \hat{B}_Y^2 \quad (3.72)$$

With known stator teeth volume V_T , yoke volume V_Y and core material density δ , the eddy current loss in the stator is

$$\bar{P}_{eddy} = \delta (V_T \bar{P}_{eddy(T)} + V_Y \bar{P}_{eddy(Y)}) = \frac{Pk_e \omega_m^2 \delta}{\pi^3} \left(V_T \frac{r_o + r_i}{2t_{av}} \hat{B}_T^2 + V_Y \frac{2}{\alpha_m} \hat{B}_Y^2 \right) \quad (3.73)$$

The hysteresis loss in the stator is given by

$$\bar{P}_{hysteresis} = \frac{Pk_h \omega_m \delta}{2\pi} (V_T \hat{B}_T^\beta + V_Y \hat{B}_Y^\beta) \quad (3.74)$$

The peak flux densities in the tooth and the yoke can be estimated from the average air-gap flux density, B_{gav} by

$$\hat{B}_T = \frac{B_g(t_{av} + b_s)}{t_{av}} \quad (3.75)$$

$$\hat{B}_y = \frac{(r_o + r_i) \alpha_m B_g}{4d_{yk}} \quad (3.76)$$

where b_s and d_{yk} are the widths of the slot and the yoke respectively, as shown in (3.48) and (3.74).

In the calculation and later FEM analysis, M19 non-oriented steel is selected as the stator core material, the motor dimensions and material data used in the core loss prediction are given in Table 3.7.

Table 3.7 Stator core dimensions and material data

Hysteresis loss coefficient	k_h	0.0234
Eddy-current loss coefficient	k_e	7.08×10^{-5}
Constant of core material	β	1.793
Stator teeth volume	V_T	$4.27 \times 10^{-5} \text{ m}^3$
Stator yoke volume	V_Y	$3.60 \times 10^{-5} \text{ m}^3$
Core material mass density	δ	$7.7 \times 10^3 \text{ kg/m}^3$

The core loss calculated in the two stators is 74.35 W (Refer to Appendix A for the detailed calculation) when the motor is operated at 60,000 rpm. The core loss will be decreased greatly if amorphous iron ribbons are used in the stators. The core loss can also be analyzed by FEM approach, which will be discussed in Chapter 4.

C. Friction Loss

For the ball bearing, its frictional moment M consists of load-dependent moment M_p resulting from rolling and sliding friction in the loaded rolling contacts, lubricant friction moment M_L [137]

$$M = M_p + M_L \quad (3.77)$$

These moment components can be calculated as [137]

$$M_p = 0.5\mu P_L d \tag{3.78}$$

$$M_L = 10^{-7} f_L (\nu n)^{2/3} [(d + D)/2]^3 \tag{3.79}$$

where μ is friction coefficient, P_L is the bearing load, d and D are the bearing bore diameter and outer diameter respectively, f_L is lubrication friction factor, ν is kinematic viscosity and n is the bearing speed in rpm. Then the friction loss can be obtained as [137]

$$p_f = 2\pi(n/60)M \tag{3.80}$$

For this system, the axial load is zero because the flywheel is supported by magnetic bearings, and the radial load is also zero if the unbalance of the flywheel is neglected. So M_p is infinitesimal that it can be ignored.

The parameters and constant of the mechanical bearing are listed in Table 3.8. According to the equations and datum, the friction loss is calculated to be 73.85 W (Refer to Appendix A for the detailed calculation) for these two mechanical ball bearings.

Though the estimated friction loss here is not large, in practice, the friction loss will increase considering the unbalance of the flywheel, temperature increase, and some other factors. Furthermore, the maintenance of the bearing and lubricant should also be taken in consideration. For all these reasons, radial magnetic bearing may be used as an alternative substitute.

Table 3. 8 Mechanical ball bearing dimensions and friction parameters

Bearing bore diameter, d	15 mm
Bearing outer diameter, D	40 mm
Friction coefficient, μ	0.002
Lubrication friction factor, f_L	0.8
Kinematic viscosity, ν	3.5

Based on the losses calculated above, the efficiency of the high-speed FESS can be

obtained easily. When the system is operating at the maximum speed, the input power is 4 kW, so the efficiency is 95.98% (Refer to Appendix A for the detailed calculation). It shows that the high speed flywheel system has a high efficiency with the combined use of magnetic bearing and mechanical bearings.

3. 7 Design specifications and results of the 4-pole axial-flux PM machine

High technical requirements and many security concerns should be considered in the design and construction of the high speed flywheel system. High strength steel and high stiffness stainless steel should be used as the material of the rotor and shaft. High speed angular contact bearings with silicone nitride balls are suggested to be implemented, special lubrication with low friction factor is required to decrease the friction loss. All these will greatly increase the cost of the whole system. Furthermore, the structure of the system, especially the rotor-shaft-bearing subsystem should be verified and optimized to ensure the security when the flywheel spins in high speed. The vacuum housing is necessary for the safety in case of rotor burst.

With the consideration of cost, duration and technique for the construction of the flywheel system, and for the sake of security and convenience in the experiments, a low speed FESS instead of the conceptual high speed one is designed and constructed. Four-pole axial-flux synchronous PM machine is employed in this system, the stator and rotor are shown in Fig. 3.4 (c) and (d), but the composite material rim is not included in the flywheel for simple construction. This is just a test equipment to verify the feasibility of the system structure and the accuracy of the mathematical model. So the rated speed is only selected to be 1500 rpm.

The specification data of the motor/generator is listed in Table 3.9. By following the design method and procedure described in the former sections, the design results can be obtained as shown in Table 3.10. The mechanical performance and efficiency can also be calculated and analyzed according to the method described in Section 3.6.

Table 3. 9 Design specification of the 4-pole axial-flux PM machine

Rated power	1 kVA
Phase current, rms	1.85 A
Power factor	0.9
Efficiency	0.9
Rated speed	1500 rpm
Frequency	50 Hz
Slot fill factor	0.4
Remanent flux density	1.23 T
Magnet recoil permeability	1.05
Carter’s factor	1.05

Table 3. 10 Design geometrical data of the 4-pole axial-flux PM machine

No. of pole pairs	2
No. of slots	6
Outer diameter of stator	130 mm
Inner diameter of stator	76 mm
Permanent magnets length	3 mm
Air gap length	1 mm
Slot width	28 mm
Slot depth	22 mm
Stator yoke thickness	18 mm
Rotor core thickness	60 mm
Air gap flux density	0.805 T
No. of turns per phase	416

3.8 Conclusions

This chapter has presented the design of the proposed FESS. Many novel concepts and techniques were applied in this design. The axial flux synchronous PM machine with dual-stator was implemented in the system. The flywheel is to be sandwiched between two pancake-shape stators, functioning as the rotor. Self-bearing motor concept was employed in this FESS. The axial-flux PM machine of the system has dual functions of motoring functionality and acting as axial magnetic bearings; it is therefore named here as partially-self-bearing machine. With such arrangements, mechanical ball bearings or active magnetic bearings can be used to constrain the rotations and translations in radial

directions. The existing FESSs often have at least two sets of radial magnetic bearings, one set of axial magnetic bearings and two sets of touch-down bearings to support the flywheel; a separate motor/generator is employed to rotate the flywheel. Compared to the traditional FESSs, the proposed system has the advantages of simpler structure, easier operation and lower losses. Furthermore, the energy density, power density and efficiency of the proposed high speed FESS are comparable to those of the existing FESSs such as the FESSs developed by NASA [49].

In this chapter, the magnetic analysis has been carried out to assist the machine design. The specification of a high-speed axial-flux PM machine was proposed and the machine was designed based on the specific procedures. The dimensions of the axial-flux machine were transformed into the dimensions of the equivalent radial-flux machine, and the design approach of the conventional radial-flux machines was employed to design the axial-flux machine. The performance and efficiency analysis of the designed high-speed FESS has shown the design to be practical. A 4-pole low-speed axial-flux PM machine has also been designed to be used in FEM analysis, mathematic model verification, control system design and experimental implementation in the later chapters.

CHAPTER 4

MODELING OF AXIAL FLUX PARTIALLY-SELF-BEARING PM MACHINES

4.1 Introduction

The motor/generator is the most important component of a flywheel system. Especially for the proposed FESS in this study, the partially-self-bearing PM machine not only drives the rotor-flywheel in rotation but also lifts it up and keeps it in levitation. Hence the control of the machine is a critical issue, to make sure the rotor-flywheel operates in proper status. The accurate modeling of this machine is therefore important for its control.

Permanent magnet synchronous motors (PMSM) can be treated as a conventional synchronous motor except that the field windings are replaced by permanent magnets. It can be readily analyzed by assuming that the machine is excited by a field current of constant value, and that the various machine inductances are calculated based on the effective permeability of the permanent magnet rotor [93].

The modeling of axial-flux PM motor is quite similar to that of radial-flux PM motor; the only difference is that the flux and the air-gap are in axial direction. The basic principle of AC machine can thus be used to derive its mathematical model. However this motor is a special machine which has dual air-gaps and the air-gaps are variable. Hence the inductances are variable instead of being constant. The torques and forces, which are functions of air-gap length, are thus variable. Co-energy method can be used to derive the torques and forces generated by the two stators, and the total force and torque can hence be

obtained.

4.2 Modeling of Dual-stator Axial-flux Synchronous PM machine

As described in Section 4.1, the inductances of the stator are very important parameters, from which we can derive the flux linkage and co-energy of the motor. Then the forces and torques can be obtained along with the voltage equations. Using reference-frame transformation, all these equations based on stator-reference frame can be transformed to equations in the rotor-reference frame.

4.2.1 Derivation of winding inductances

The model of the proposed flywheel system shown in Fig. 3.4 is presented here. The stator has three-phase windings denoted as a, b and c with the same winding number N_s . Permanent magnets are mounted on the surface of the disk-type rotor, hence, it is a non-salient rotor. The rotor has two degrees of freedom in the z and ϕ_z directions, while the DOFs in directions x , y , ϕ_x and ϕ_y are to be constrained by mechanical or active magnetic bearings. Here, the rotor's permanent magnets have been replaced by an equivalent rotor current i_f with the winding number N_f .

It is convenient to employ the development diagram of the machine shown in Fig. 4.1. The waveform of the MMFs produced by the stator phase windings and the equivalent rotor current i_f may be considered as coarse approximation of sinusoidal functions of ϕ_s and ϕ_r , the same as the distribution of the windings [93, 138]. Their functions are listed as follows [138],

$$MMF_{as} = -\frac{N_s}{2P} i_{as} \cos P\phi_s \quad (4.1)$$

$$N_{as} = \frac{N_s}{2} \sin P\phi_s \quad (4.2)$$

$$MMF_{bs} = -\frac{N_s}{2P} i_{bs} \cos\left(P\phi_s - \frac{2}{3}\pi\right) \quad (4.3)$$

$$N_{bs} = \frac{N_s}{2} \sin\left(P\phi_s - \frac{2}{3}\pi\right) \quad (4.4)$$

$$MMF_{cs} = -\frac{N_s}{2P} i_{cs} \cos\left(P\phi_s + \frac{2}{3}\pi\right) \quad (4.5)$$

$$N_{cs} = \frac{N_s}{2} \sin\left(P\phi_s + \frac{2}{3}\pi\right) \quad (4.6)$$

$$MMF_f = -\frac{N_f}{2P} i_f \cos P\phi_r \quad (4.7)$$

$$N_{rf} = \frac{N_f}{2} \sin P\phi_r \quad (4.8)$$

where, P is the number of pole pairs.

Fig. 4. 1 The development diagram of the proposed machine

By integrating N_{as} over an electrical pole-pitch, i.e., from 0 to π/P , and taking into account the number of pole pairs P , we can get

$$P \int_0^{\pi/P} \frac{N_s}{2} \sin P\theta d\theta = N_s \quad (4.9)$$

In the equation, N_s is the number of turns per phase in series in the equivalent sinusoidally

distributed windings of the stator. Here, we assume that each of the three phase windings has the same turn number.

By using Fourier analysis method, the rectangular MMF wave of the concentrated full-pitch coil can be resolved into a Fourier series comprising a series of odd harmonics and a fundamental component which equals to $4/\pi$ times the peak flat-topped value. The number of turns for the fundamental components of a distributed winding can be derived as [131]

$$N_s = \frac{4}{\pi} k_w N_{ph} \quad (4.10)$$

where, N_{ph} is the actual number of turns in series per phase.

Fig. 4. 2 The stator with short-pitched concentrated windings

As shown in Fig. 4.2 (a) and (b), the windings are short-pitched concentrated types. The pitch factor can be calculated as

$$k_{p1} = \cos\left(\frac{\beta}{2}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} \quad (4.11)$$

where, β is the electrical angle difference between the short-pitched winding and full-pitched winding. For concentrated winding, the distribution factor is

$$k_{d1} = \frac{\sin \frac{q\alpha}{2}}{q \sin \frac{\alpha}{2}} = 1 \quad (4.12)$$

where, q is the number of slots per pole per phase and α is the slot-pitch in electrical degrees. So the winding factor is

$$k_w = k_{p1} \times k_{d1} = \sqrt{3}/2 \quad (4.13)$$

The maximum value of the equivalent MMF produced by the PMs, MMF_m , is calculated according to (3.17)

$$MMF_m = \frac{N_f i_f}{2P} = H_m l_m \quad (4.14)$$

where, l_m and H_m denote the magnet length and the magnetic field intensity when the magnet is shorted by permeable iron. Then the value of $N_f i_f$ can be derived as

$$N_f i_f = 2PH_m l_m = 2P \frac{B_r l_m}{\mu_r \mu_0} \quad (4.15)$$

where, B_r is the remanent flux density of the PMs, μ_r is the relative permeability, and μ_0 is the magnetic permeability of the air with the value $4\pi \times 10^{-7}$ H/m.

As axial forces are produced by the derivatives of inductance variations with respect to the rotor radial displacement, inductance functions must be derived considering the variations of flux distribution due to the rotor displacements. If the air gap length between the surfaces of stator and the rotor itself is defined as g , then the magnetic flux density B and magnetic flux are respectively [138]:

$$B = \mu_0 \frac{MMF}{g} \quad (4.16)$$

$$\phi = \int B ds \quad (4.17)$$

As an example, let us determine the total flux linkages of the winding due to current flowing only in a winding. Here

$$\begin{aligned} \lambda_{aa} &= L_{ls} i_{as} + \int N_{as}(\phi_s) \phi_{as}(\phi_s) d\phi_s \\ &= L_{ls} i_{as} + P \int_0^{\pi/P} \frac{N_s}{2} \sin P\phi_s \left[\int_{\phi_s}^{\phi_s + \pi/P} \left[-\frac{\mu_0 N_s (R_o^2 - R_i^2)}{4Pg} \right] i_{as} \cos P\xi d\xi \right] d\phi_s \quad (4.18) \\ &= L_{ls} i_{as} + \frac{\mu_0 \pi (R_o^2 - R_i^2) N_s^2}{8P^2 g} i_{as} \end{aligned}$$

where R_o and R_i are the outer and inner radius of the stator respectively.

Recalling the definition of self inductance of a magnetically linear system as the ratio of flux linked by a winding to the current flowing in that winding without the presence of all other winding currents, the self inductance of a winding is given as

$$L_{aa} = \frac{\lambda_{aas}}{i_{as}} = L_{ls} + \frac{\mu_0 \pi (R_o^2 - R_i^2) N_s^2}{8P^2 g} = L_{ls} + L_s \quad (4.19)$$

where L_{ls} is the stator leakage inductance due primarily to leakage flux at the end turns.

Generally, the inductance accounts for 5% to 10% of the maximum self-inductance.

Similarly, we can get

$$L_{aa} = L_{bb} = L_{cc} = L_{ls} + L_s \quad (4.20)$$

$$L_{ff} = \frac{\lambda_f}{i_f} = \frac{\mu_0 \pi (R_o^2 - R_i^2) N_f^2}{8P^2 g} = L_f \quad (4.21)$$

The mutual flux linkage in phase a winding due to current in phase b winding is found in a similar manner, in this case, the flux linkage can be written as

$$\begin{aligned}
\lambda_{ab} &= \int N_{as}(\phi_s) \phi_{bs}(\phi_s) d\phi_s \\
&= P \int_0^{\pi/P} \frac{N_s}{2} \sin P\phi_s \int_{\phi_s}^{\phi_s + \pi/P} \left(-\frac{\mu_0 N_s (R_o^2 - R_i^2)}{4Pg} \right) i_{as} \cos P(\xi - 2/3\pi) d\xi d\phi_s \quad (4.22) \\
&= -\frac{\mu_0 \pi (R_o^2 - R_i^2) N_s^2}{16P^2 g} i_{bs}
\end{aligned}$$

Thus the mutual inductance M_{ab} can be found using

$$M_{ab} = \frac{\lambda_{ab}}{i_{bs}} = -\frac{\mu_0 \pi (R_o^2 - R_i^2) N_s^2}{16P^2 g} \quad (4.23)$$

Similarly, the other mutual inductances can be obtained as

$$M_{bc} = M_{ca} = M_{ab} = -\frac{\mu_0 \pi (R_o^2 - R_i^2) N_s^2}{16P^2 g} \quad (4.24)$$

The mutual inductance between the a and f windings is determined by

$$\begin{aligned}
\lambda_{af} &= \int N_{as}(\phi_s) \phi_f(\phi_s) d\phi_s \\
&= P \int_0^{\pi/P} \frac{N_s}{2} \sin P\phi_s \int_{\phi_s}^{\phi_s + \pi/P} \left[-\frac{\mu_0 N_f (R_o^2 - R_i^2)}{4Pg} \right] i_f \cos P(\xi - \theta) d\xi d\phi_s \quad (4.25)
\end{aligned}$$

In the same way mentioned above, M_{af} , M_{bf} , M_{cf} can be expressed as

$$M_{af} = \frac{\mu_0 \pi (R_o^2 - R_i^2) N_s N_f}{8P^2 g} \cos P\theta = L_m \cos P\theta \quad (4.26)$$

$$M_{bf} = \frac{\mu_0 \pi (R_o^2 - R_i^2) N_s N_f}{8P^2 g} \cos(P\theta - 2/3\pi) \quad (4.27)$$

$$M_{cf} = \frac{\mu_0 \pi (R_o^2 - R_i^2) N_s N_f}{8P^2 g} \cos(P\theta + 2/3\pi) \quad (4.28)$$

Then the inductances can be written in matrix form as:

$$\mathbf{L} = \begin{pmatrix} L_{ff} & M_{fa} & M_{fb} & M_{fc} \\ M_{af} & L_{aa} & M_{ab} & M_{ac} \\ M_{bf} & M_{ba} & L_{bb} & M_{bc} \\ M_{cf} & M_{ca} & M_{cb} & L_{cc} \end{pmatrix} \quad (4.29)$$

Here, $M_{ab} = M_{ba}$, $M_{bc} = M_{cb}$, $M_{ac} = M_{ca}$, $M_{af} = M_{fa}$, $M_{bf} = M_{fb}$, $M_{cf} = M_{fc}$. The values of inductances are determined by the calculations above

4.2.2 Voltage equations

The flux linkage in any winding is the summation of its self inductances and mutual inductances caused by other currents. Hence the flux linkages in phase a , b & c and the field winding are expressed as

$$\boldsymbol{\lambda} = (\lambda_f \quad \lambda_a \quad \lambda_b \quad \lambda_c)^T \quad (4.30)$$

$$\mathbf{i} = (i_f \quad i_a \quad i_b \quad i_c)^T \quad (4.31)$$

$$\boldsymbol{\lambda} = \mathbf{L}\mathbf{i} \quad (4.32)$$

Then the voltage equations for the axial-flux machine can be written as [93,138]:

$$v_a = R_s i_{as} + p\lambda_a \quad (4.33)$$

$$v_b = R_s i_{bs} + p\lambda_b \quad (4.34)$$

$$v_c = R_s i_{cs} + p\lambda_c \quad (4.35)$$

where $p = d/dt$, and R_s is the resistance of the stator winding.

4.2.3 Transformation of machine equations

In general, 3-phase variables in any reference frame can be transformed to the orthogonal

components with the following relationship [93,138]:

$$\begin{bmatrix} x_{dq0} \end{bmatrix} = \mathbf{C} \begin{bmatrix} x_{abc} \end{bmatrix} \quad (4.36)$$

In the equation above, x represents any variable such as voltage, current or flux linkage.

The matrix of transformation $\mathbf{C}$ is [93]:

$$\mathbf{C} = \frac{2}{3} \begin{pmatrix} \cos \theta & \cos(\theta - 2/3\pi) & \cos(\theta + 2/3\pi) \\ -\sin \theta & -\sin(\theta - 2/3\pi) & -\sin(\theta + 2/3\pi) \\ 1/2 & 1/2 & 1/2 \end{pmatrix} \quad (4.37)$$

The inverse of $\mathbf{C}$ is expressed as [93]:

$$\mathbf{C}^{-1} = \begin{pmatrix} \cos \theta & -\sin \theta & 1 \\ \cos(\theta - 2/3\pi) & -\sin(\theta - 2/3\pi) & 1 \\ \cos(\theta + 2/3\pi) & -\sin(\theta + 2/3\pi) & 1 \end{pmatrix} \quad (4.38)$$

After the transformation on both the flux linkages and the currents, the inductance expression of Equation (4.32) becomes much simpler when they are expressed in terms of dq0 variables. The manipulations are rather laborious and are omitted here. The results are

$$\begin{bmatrix} \lambda_f \\ \lambda_d \\ \lambda_q \end{bmatrix} = \begin{pmatrix} L_f & 3/2L_m & 0 \\ L_m & 3/2L_s + L_{al} & 0 \\ 0 & 0 & 3/2L_s + L_{al} \end{pmatrix} \begin{bmatrix} i_f \\ i_d \\ i_q \end{bmatrix} \quad (4.39)$$

where, i_d and i_q are the direct and quadrature currents in the dq reference frame.

Ignoring the leakage inductance, the following equation is derived

$$L_d = L_q = 3/2L_s = \frac{3\mu_0\pi(R_o^2 - R_i^2)N_s^2}{16P^2g} \quad (4.40)$$

The amplitude of the flux induced by the permanent magnets of the rotor in the stator phases,

$$\lambda_m = L_m i_f = \frac{\mu_0 \pi (R_o^2 - R_i^2) N_s N_f}{8P^2 g} i_f \quad (4.41)$$

The voltage equations can be transformed as [138]

$$v_d = R_s i_d + p \lambda_d - \varpi \lambda_q \quad (4.42)$$

$$v_q = R_s i_q + p \lambda_q + \varpi \lambda_d \quad (4.43)$$

where $\varpi = p\theta = d\theta/dt$ is the rotor electrical angular velocity.

Substituting (4.39) into (4.42) and (4.43), they can be transformed into

$$\frac{d}{dt} i_d = \frac{1}{L_d} v_d - \frac{R_s}{L_d} i_d + \frac{L_q}{L_d} (P\varpi_r) i_q \quad (4.44)$$

$$\frac{d}{dt} i_q = \frac{1}{L_q} v_q - \frac{R_s}{L_q} i_q - \frac{L_d}{L_q} (P\varpi_r) i_d - \frac{\lambda_m}{L_q} (P\varpi_r) \quad (4.45)$$

4.2.4 Equations of position and velocity

The torque and axial force can be derived by the co-energy approach. The magnetic energy stored may be calculated as [138]

$$W = \frac{1}{2} \lambda_f i_f + \frac{1}{2} \lambda_d i_d + \frac{1}{2} \lambda_q i_q \quad (4.46)$$

As a result, the attractive force F can be obtained as

$$F = -\frac{\partial W}{\partial g} = \frac{\mu_0 \pi (R_o^2 - R_i^2)}{16P^2 g^2} \left[N_f^2 i_f^2 + \frac{5}{2} N_s N_f i_d i_f + \frac{3}{2} N_s^2 (i_d^2 + i_q^2) \right] \quad (4.47)$$

Applying the Fleming's left-hand rule, the rotating torque T can be written as

$$T = \frac{3}{2} \times P \times (\lambda_d i_q - \lambda_q i_d) = \frac{3\mu_0 \pi (R_o^2 - R_i^2) N_s N_f}{16Pg} i_f i_q \quad (4.48)$$

Here, the air gap between the surfaces of the stator and the PMs at the equilibrium point is defined as l_g , so the effective air gap between the stator and rotor at the equilibrium point can be obtained as

$$g_0 = K_c (l_g + l_m / \mu_r) \quad (4.49)$$

where K_c is the Carter's coefficient, which is approximately equal to 1. Then the force and torque F_1 and T_1 which are generated by the upper stator can be calculated by substituting $g = g_0 + z$, $i_d = i_{d1}$ and $i_q = i_{q1}$ into (4.47) and (4.48); the force and torque generated by the lower stator, F_2 and T_2 , can be calculated by substituting $g = g_0 - z$, $i_d = i_{d2}$ and $i_q = i_{q2}$ into these equations as well. i_{d1} and i_{q1} , i_{d2} and i_{q2} are the direct and quadrature currents of the upper and lower stators respectively. Therefore the total levitation force F_e and rotating torque T_e can be respectively expressed as

$$F_e = F_2 - F_1 = \frac{\mu_0 \pi (R_o^2 - R_i^2)}{16P^2} \left\{ \frac{1}{(g_0 - z)^2} [N_f^2 i_f^2 + \frac{5}{2} N_s N_f i_{d2} i_f + \frac{3}{2} N_s^2 (i_{d2}^2 + i_{q2}^2)] - \frac{1}{(g_0 + z)^2} [N_f^2 i_f^2 + \frac{5}{2} N_s N_f i_{d1} i_f + \frac{3}{2} N_s^2 (i_{d1}^2 + i_{q1}^2)] \right\} \quad (4.50)$$

$$T_e = T_1 + T_2 = \frac{3\mu_0 \pi (R_o^2 - R_i^2) N_s N_f i_f}{16P(g_0^2 - z^2)} [g_0 (i_{q2} + i_{q1}) + z (i_{q2} - i_{q1})] \quad (4.51)$$

From the functions above, it can be seen that the torque is independent of the directional currents i_{d1} , i_{d2} , supposing that i_{q1} and i_{q2} are equally controlled

$$i_{q1} = i_{q2} = i_{cq} \quad (4.52)$$

In order to reduce the control variables and simplify (4.50), the equations are defined as shown below

$$i_{d1} = I_{d0} - i_{cd} \quad (4.53)$$

$$i_{d2} = I_{d0} + i_{cd} \quad (4.54)$$

where the offset current I_{d0} is a constant, control current i_{cd} is a variable.

Without considering the gravity and other external disturbances, the levitation force F_e is 0 when the flywheel is at the equilibrium point, i.e., $z = 0$. Then $i_{d1} = i_{d2} = I_{d0}$, i.e., $i_{cd} = 0$. Linearizing (4.50) at $z = 0$ and $i_{cd} = 0$, expanding it into Taylor series and taking the first order term

$$F_e \approx \frac{5K}{g_0^2} N_s (N_f i_f) i_{cd} + \frac{4K}{g_0^3} \left[(N_f i_f)^2 + \frac{5}{2} N_s (N_f i_f) I_{d0} + \frac{3}{2} N_s^2 (I_{d0}^2 + i_{cq}^2) \right] z = K_1 z + K_2 i_{cd} \quad (4.55)$$

where,

$$K = \frac{\mu_0 \pi (R_o^2 - R_i^2)}{16P^2} \quad (4.56)$$

$$K_1 = \frac{4K}{g_0^3} \left[(N_f i_f)^2 + \frac{5}{2} N_s (N_f i_f) I_{d0} + \frac{3}{2} N_s^2 (I_{d0}^2 + i_{cq}^2) \right] \quad (4.57)$$

$$K_2 = \frac{5K}{g_0^2} N_s (N_f i_f) \quad (4.58)$$

The radial motion of the rotor is assumed to be restricted by passive mechanical bearings. Therefore, the axial motion of the rotor is independent from radial motion. The dynamic equation of the axial motion of the rotor is

$$mz'' = F_e + G = K_1 z + K_2 i_{cd} + G \quad (4.59)$$

where, G is the gravity of the flywheel. Let $I_{cd0} = G / K_2$, (4.59) can be changed to be

$$mz'' = K_1 z + K_2 (i_{cd} + I_{cd0}) \quad (4.60)$$

Let

$$\tilde{i}_{cd} = i_{cd} + I_{cd0} \quad (4.61)$$

Equation (4.60) can be modified to be

$$mz'' = K_1 z + K_2 \tilde{i}_{cd} \quad (4.62)$$

The prototype machine parameters are shown in Table 4.1. By substituting these parameters into (4.56) and (4.57), K_1 can be calculated as

$$K_1 = 3.228 \times 10^{-2} \times (89677260 + 314646 \times i_{cq}^2) \quad (4.63)$$

Similarly, $K_2 = 455.44$ was calculated by substituting the parameters of Table 4.1 into (4.56) and (4.58).

As the rated root-mean-square (RMS) value of phase current is 1.85 A, then the maximum value of the current is $1.85 \times \sqrt{2} = 2.62$ A, i.e. $|i_{cq}| \leq 2.62$. The range of K_1 is

$$2.89 \times 10^6 \leq K_1 \leq 2.96 \times 10^6 \quad (4.64)$$

The changes of i_{cq} only cause 2.36% difference in the value of K_1 . The control current i_{cq} can hence be ignored in the position control, and K_1 is regarded as a constant. By applying this method, (4.62) is linearized to be a simpler equation, where the position z is controlled only by $\tilde{i}_{cd}$.

As z is far smaller than g_0 , (4.51) can be simplified to

$$T_e = \frac{3\mu_0\pi(R_o^2 - R_i^2)N_sN_fi_f}{16P(g_o^2 - z^2)}[2g_o(i_{q2} + i_{q1})] = \frac{3\mu_0\pi(R_o^2 - R_i^2)N_sN_fi_f}{8Pg_o}i_{cq} = K_Ti_{cq} \quad (4.65)$$

where, K_T is torque constant defined as the torque per Ampere generated by the current i_{cq} . The value can be calculated as 3.028 based on the parameters shown in Table 4.1.

Table 4.1 Prototype motor parameters in the design

Parameter	Value
N_s	458 turns
N_fi_f	8898 ampere-turns
J	0.0158 kg.m ²
m	4.09 kg
g_o	0.00277 m
R_i	0.038m
R_o	0.065m
P	2
μ_0	$4\pi \times 10^{-7}$
I_{d0}	1

Then the speed equation can be written as $J \frac{d\varpi_r}{dt} = T_e - F_f\varpi_r - T_m$, i.e [93,138].

$$\frac{d\varpi_r}{dt} = \frac{1}{J}(T_e - F_f\varpi_r - T_m) \quad (4.66)$$

$$\frac{d\theta}{dt} = \varpi_r \quad (4.67)$$

Here, J is the combined inertia of the rotor-flywheel and support shaft, F_f is combined viscous friction of rotor-flywheel and radial bearings, θ is rotor mechanical angular position, and T_m is any external disturbance mechanical torque acting on the rotor-flywheel. After applying magnetic levitation in axial direction, the friction is very small and F_f can hence be ignored. There is no applied mechanical load for the FESS, T_m

is thus regarded to be zero. In this case, (4.66) can be rewritten as

$$\frac{d\varpi_r}{dt} = \frac{1}{J} T_e = \frac{K_T}{J} i_{cq} \quad (4.68)$$

When the machine is in generation mode, T_e is negative. In other words, by controlling i_{cq} to be negative, energy would be extracted from the FESS.

Though i_{cq} has slight influence on the position z and thus can be ignored in position control, it dominates the velocity control as a control input. From the equations (4.62) and (4.68), it is clear that the position and velocity has been decoupled and controlled by $\tilde{i}_{cd}$ and i_{cq} separately.

4.2.5 The entire mathematic model

The proposed axial-flux electric machine can be looked upon as two motor/generator combined together. These two motor/generators share a common rotor and hence have the same mechanical characteristics such as load, the moment of inertia and viscous friction. As the electrical features of these two machines are identical with the same stator structure and PM arrangement, the voltage equations are therefore almost identical except that different currents may be fed. From the derivations shown above, the entire mathematic model can be written as follow:

Torque and force equations

$$T_{el} = \frac{3}{2} \times P (\lambda_{d1} i_{q1} - \lambda_{q1} i_{d1}) = \frac{3\mu_0 \pi (R_o^2 - R_i^2) N_{s1} M m f_1}{16P(g_0 - z)} i_{q1} \quad (4.69)$$

$$F_{s1} = \frac{\mu_0 \pi (R_o^2 - R_i^2)}{16P^2 (g_0 - z)^2} \left[M m f_1^2 + \frac{5}{2} N_s M m f_1 i_{d1} + \frac{3}{2} N_{s1}^2 (i_{d1}^2 + i_{q1}^2) \right] \quad (4.70)$$

$$T_{e2} = \frac{3}{2} \times P(\lambda_{d2} i_{q2} - \lambda_{q2} i_{d2}) = \frac{3\mu_0 \pi (R_o^2 - R_i^2) N_{s2} M m f_2}{16P(g_0 + z)} i_{q2} \quad (4.71)$$

$$F_{s2} = \frac{\mu_0 \pi (R_o^2 - R_i^2)}{16P^2(g_0 + z)^2} \left[M m f_2^2 + \frac{5}{2} N_{s2} M m f_2 i_{d2} + \frac{3}{2} N_s^2 (i_{d2}^2 + i_{q2}^2) \right] \quad (4.72)$$

$$T = T_{e1} + T_{e2} \quad (4.73)$$

$$F = F_{s1} - F_{s2} \quad (4.74)$$

Velocity and position equations

$$\frac{d}{dt} \varpi_r = \frac{1}{J} T_e - \frac{1}{J} F_f \varpi_r - \frac{1}{J} T_m \quad (4.75)$$

$$\frac{d}{dt} \theta = \varpi_r \quad (4.76)$$

$$\frac{d}{dt} z = \dot{z} \quad (4.77)$$

$$\frac{d}{dt} \dot{z} = \frac{1}{m} F + \frac{1}{m} f \quad (4.78)$$

Voltage equations

$$\begin{aligned} \frac{d}{dt} i_{d1} &= \frac{1}{L_{d1}} v_{d1} - \frac{R_{s1}}{L_{d1}} i_{d1} + \frac{L_{q1}}{L_{d1}} (P \varpi_r) i_{q1} \\ &= \frac{16P^2(g_0 - z)}{3\mu_0 \pi (R_o^2 - R_i^2) N_{s1}^2} v_{d1} - \frac{16P^2(g_0 - z) R_{s1}}{3\mu_0 \pi (R_o^2 - R_i^2) N_{s1}^2} i_{d1} + (P \varpi_r) i_{q1} \end{aligned} \quad (4.79)$$

$$\begin{aligned} \frac{d}{dt} i_{q1} &= \frac{1}{L_{q1}} v_{q1} - \frac{R_{s1}}{L_{q1}} i_{q1} - \frac{1}{L_{q1}} (P \varpi_r) (L_{d1} i_{d1} + \lambda_{m1}) \\ &= \frac{16P^2(g_0 - z)}{3\mu_0 \pi (R_o^2 - R_i^2) N_{s1}^2} v_{q1} - \frac{16P^2(g_0 - z) R_{s1}}{3\mu_0 \pi (R_o^2 - R_i^2) N_{s1}^2} i_{q1} - (P \varpi_r) i_{d1} - \frac{2M m f_1}{3N_s} (P \varpi_r) \end{aligned} \quad (4.80)$$

$$\begin{aligned}\frac{d}{dt}i_{d2} &= \frac{1}{L_{d2}}v_{d2} - \frac{R_{s2}}{L_{d2}}i_{d1} + \frac{L_{q2}}{L_{d2}}(P\varpi_r)i_{q2} \\ &= \frac{16P^2(g_0+z)}{3\mu_0\pi(R_o^2-R_i^2)N_{s2}^2}v_{d2} - \frac{16P^2(g_0+z)R_{s2}}{3\mu_0\pi(R_o^2-R_i^2)N_{s2}^2}i_{d2} + (P\varpi_r)i_{q2}\end{aligned}\quad (4.81)$$

$$\begin{aligned}\frac{d}{dt}i_{q2} &= \frac{1}{L_{q2}}v_{q2} - \frac{R_{s2}}{L_{q2}}i_{q2} - \frac{1}{L_{q2}}(P\varpi_r)(L_{d2}i_{d2} + \lambda_{m2}) \\ &= \frac{16P^2(g_0+z)}{3\mu_0\pi(R_o^2-R_i^2)N_{s2}^2}v_{q2} - \frac{16P^2(g_0+z)R_{s2}}{3\mu_0\pi(R_o^2-R_i^2)N_{s2}^2}i_{q2} - (P\varpi_r)i_{d2} - \frac{2Mmf_2}{3N_s}(P\varpi_r)\end{aligned}\quad (4.82)$$

where,

R_{s1} : Resistance of the stator windings of machine 1,

i_{d1}, i_{q1} : d and q axis current of machine

u_{d1}, u_{q1} : d and q axis voltage of machine1

Mmf_1 : $N_{f1}i_{f1}$, magnetomotive induced by the upper PMs on the rotor

T_{e1} : Electromagnetic torque mechanical system machine1

F_{s1} : Electromagnetic Force generated by machine 1

R_{s2} : Resistance of the stator windings of machine 2

N_{s2} : Stator windings numbers of machine 2

i_{d2}, i_{q2} : d and q axis current of machine 2

u_{d2}, u_{q2} : d and q axis voltage of machine 2

Mmf_2 : $N_{f2}i_{f2}$, magnetomotive induced by the lower PMs on the rotor

T_{e2} : Electromagnetic torque mechanical system of machine 2

F_{s2} : Electromagnetic Force generated by machine 2

P : Numbers of pole pairs, (same for machine1 and machine2)

ϖ_r : Angular velocity of the rotor (mechanical)

$\varpi_e = p\varpi_r$: Angular velocity of the rotor (electrical)

T : Total electromagnetic torque mechanical system

F : Total electromagnetic force

J : Combined inertia of the rotor and load

F_f : Combined viscous friction of rotor and load

f : Adscititious force

θ : Rotor mechanical angular position

T_m : External mechanical torque acting on rotor-flywheel

z : Position displacement

The torque, force, position and velocity equations (4.69)-(4.78) show the relationship among the dq currents and the position and velocity. The voltage equations (4.79)-(4.82) expound the relationship between the dq currents and voltages, which are the inner characteristics of the upper and lower motor.

The model of the axial-flux partially-self-bearing PM machine is nonlinear, especially for the force and torque equations. These equations become quite simple after the linearization conducted in Section 4.2.4. Because the position displacement z is very small compared with the air-gap length, its influences on the voltage equations may be neglected. The current loop control can hence follow the control approach of the traditional PM motors.

4.2.6 State-space equations

For the convenience of the implementation of advanced modern control theory, i.e. the MPC control method which will be introduced in next chapter, the state-space equation form of the mathematic model is developed.

Equations (4.62), (4.67) and (4.68) can be rewritten as

$$\ddot{z} = \frac{K_1}{m} z + \frac{K_2}{m} \tilde{i}_{cd} \quad (4.83)$$

$$\frac{d\theta}{dt} = \varpi_r \quad (4.84)$$

$$\frac{d\varpi_r}{dt} = \frac{K_T}{J} i_{cq} \quad (4.85)$$

By taking the state vector $\mathbf{x} = [\theta, \omega_r, z, \dot{z}]^T$, the control input vector $\mathbf{u} = [i_{cq}, \tilde{i}_{cd}]^T$, the output vector $\mathbf{y} = [\omega_r, z]^T$, (4.82)-(4.84) can be changed to

$$\begin{cases} \dot{\mathbf{x}} = \mathbf{A}_e \mathbf{x} + \mathbf{B}_e \mathbf{u} \\ \mathbf{y} = \mathbf{C}_e \mathbf{x} \end{cases} \quad (4.86)$$

where $\mathbf{A}_e$, $\mathbf{B}_e$ and $\mathbf{C}_e$ are system matrices given by

$$\mathbf{A}_e = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & K_1/m & 0 \end{bmatrix}, \quad \mathbf{B}_e = \begin{bmatrix} 0 & 0 \\ K_T/J & 0 \\ 0 & 0 \\ 0 & K_2/m \end{bmatrix}, \quad \mathbf{C}_e = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

4.3 Conclusions

In this chapter, a mathematical model of the axial-flux partially-self-bearing PM synchronous machine has been developed. The basic theory of AC electrical machine has been implemented in the modeling of this special machine. The equations of the inductances have been derived after the calculation of the flux linkages. Co-energy method was applied to derive the equations of the torque and axial force acting on the rotor. The equations of speed and axial displacement of the rotor were obtained thereafter. As this model is a non-linear multi-variable system, linearization has been used to ease the implementation of the model for control applications. The entire mathematic model has been developed. This model is applicable for both motoring and generation modes.

CHAPTER 5

FEM ANALYSIS AND MODEL VERIFICATION FOR THE AXIAL-FLUX PARTIALLY-SELF-BEARING PM MACHINE

5.1 Introduction

The finite element method (FEM) is a numerical technique for obtaining approximate solutions to boundary-value problems of mathematical physics. It was first proposed in the 1940s and its use began in the 1950s for aircraft design. Thereafter, the method has been developed and applied very extensively to problems of structural analysis and other fields. Today, FEM has been recognized as a general method of wide applicability to engineering and mathematical problems [139].

The problem of electromagnetic analysis is actually a problem of solving Maxwell's equations subject to given boundary conditions. Maxwell's equations are a set of fundamental equations governing all macroscopic electromagnetic phenomena, which will be introduced in the next section.

The finite element method has proved to be particularly flexible, reliable and effective in the analysis and synthesis of power-frequency electromagnetic and electromechanical devices [140, 141]. Even in the hands of non-specialists, modern FEM packages are user friendly and allow for calculating the electromagnetic field distribution and integral parameters without detailed knowledge of applied mathematics. The FEM can analyze magnetic circuits of any shape and material. There is no need to calculate reluctances,

leakage factors or the operating point on the magnetic recoil line. The magnetic magnetization curve is input into the finite element program which can then calculate the variation of the magnetic flux density throughout the magnetic system. A remarkable advantage of FEM analysis over other approaches to analyze electric machines is the inherent ability to calculate accurately armature reaction effects, electromagnetic force and torque [74, 142].

This chapter presents the three-dimensional (3D) FEM analysis of electromagnetic characteristics of the axial-flux partially-self-bearing PM machine to verify its feasibility, to test the accuracy of the machine design. Both the high-speed and low-speed versions of the PM machine described in Chapter 3 have been subjected to the FEM analysis. The FEM analysis is also used in the verification of the proposed mathematic model presented in Chapter 4. The forces, torques and speeds obtained from the calculation of the mathematic model and the finite element analysis are compared to determine whether they are in agreement.

5.2 Theory of 3D Computational Electromagnetics

Conventional two-dimensional (2D) FEA used for cylindrical radial flux machines is not suitable for the proposed axial flux PM motor/generator because of its special pancake-type geometry. In this case, 3D FEM has to be utilized in the electromagnetic analyses.

5.2.1 Maxwell's equations

Maxwell's equations were derived from the earlier Biot-Savart Law, Faraday's Law and Gauss's Law. These equations were formulated to describe the theory of electromagnetic field.

First Maxwell's equation

The differential form of the first Maxwell's equation is [133, 140]

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} + \nabla \times (\vec{D} \times \vec{v}) + \vec{v} \nabla \cdot \vec{D} \quad (5.1)$$

where, $\vec{H}$ denotes the magnetic field density, $\vec{J}$ is the electric current density, $\partial \vec{D} / \partial t$ is the density of the displacement current, $\nabla \times (\vec{D} \times \vec{v})$ is the density of the current due to the motion of a polarized dielectric material, and $\vec{v} \nabla \cdot \vec{D}$ is the density of the convection current. For $\vec{v} = 0$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \quad (5.2)$$

The current density, J , is proportional to the electric field that is established due to the potential difference [140]

$$J = \sigma E = -\sigma \nabla \Phi \quad (5.3)$$

where, E is the electric field, σ is the conductivity of the material, and Φ is the electric potential.

By expanding (5.2) in 3D vectors, it has the following scalar form:

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x + \frac{\partial D_x}{\partial t} \quad (5.4)$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y + \frac{\partial D_y}{\partial t} \quad (5.5)$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z + \frac{\partial D_z}{\partial t} \quad (5.6)$$

Second Maxwell's equation

The second Maxwell's equation in the differential form is [133, 140]

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \nabla \times (\vec{B} \times \vec{v}) \quad (5.7)$$

For $\vec{v} = 0$

$$\nabla \times \vec{E} = -\partial \vec{B} / \partial t \quad (5.8)$$

where, $\vec{E}$ is the electric field intensity, $\vec{B}$ is the magnetic field density. The scalar form of the last equation is

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\frac{\partial B_x}{\partial t} \quad (5.9)$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \quad (5.10)$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \quad (5.11)$$

For magnetically isotropic bodies

$$\vec{B} = \mu_0 \mu_r \vec{H} \quad (5.12)$$

where $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic permeability of free space, and μ_r is the relative magnetic permeability.

Third Maxwell's equation

The third Maxwell's equation in differential form is [133, 140]

$$\nabla \cdot \vec{B} = 0 \quad (5.13)$$

In scalar form it becomes

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \quad (5.14)$$

5.2.2 Forces and torques obtained by co-energy method

The FEM is based on the conservation of energy. The law of conservation of energy in electrical machines can be derived from Maxwell's equations. The net electrical input active power is [139]

$$P = \int_V \sigma \vec{E}^2 dV = \int_V \vec{E} \cdot \vec{J} dV = \int_V \left(\int_S (\vec{J} \cdot d\vec{S}) \vec{E} \cdot d\vec{l} \right) \quad (5.15)$$

The EMF can be written as

$$e = \int_V \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S} \quad (5.16)$$

and the current enclosed from Ampere's circuital law is

$$I = \int_S \vec{J} \cdot d\vec{S} = \oint_V \vec{H} \cdot d\vec{l} \quad (5.17)$$

Substituting (5.16) and (5.17) into (5.15), the net electrical input power is derived as

$$P = \oint_V \vec{H} \cdot d\vec{l} \left[-\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{S} \right] = -\int_V \vec{H} \cdot \frac{\partial \vec{B}}{\partial t} dV = -\frac{\partial}{\partial t} \int_V \left[\int_0^B \vec{H} \cdot d\vec{B} \right] dV \quad (5.18)$$

Then the stored magnetic co-energy is obtained

$$W' = \int_V \left[\int_0^B \vec{H} \cdot d\vec{B} \right] dV \quad (5.19)$$

The force and torque can be calculated as the derivative of the stored magnetic co-energy W' with respect to a small displacement. Then the component of instantaneous force F_s in the direction of the displacement s is [133,142]

$$F_s = \frac{dW'}{ds} \quad (5.20)$$

The instantaneous torque T with a small angular displacement θ is represented by [133,142]

$$T = \frac{\partial W'(i, \theta)}{\partial \theta} / i = \text{const} \quad (5.21)$$

5.3 Electromagnetic FEM Analysis and Mathematical Model Verification

The 4-pole low-speed prototype machine described in Chapter 3 was analyzed using a time-stepping three-dimensional finite element simulator [143]. Besides analyzing the electromagnetic characteristics of the motor, FEM is also used to verify the mathematic model derived in Chapter 4.

5.3.1 FEM analysis technique

Mesh of the motor

To represent an electric or magnetic field over a large, irregularly-shaped region, we can divide the region into many tetrahedra (pyramid-shaped sub-regions). The field in each tetrahedron (element) is represented by a separate polynomial. This collection of tetrahedra is referred to as the finite element mesh or simply, the mesh. A mesh is generated for each model prior to computing a field solution.

Fig. 5. 1 A single tetrahedron in the mesh

A single tetrahedron is shown in Fig. 5.1. To obtain a precise description of the field, we must size each tetrahedron so that it is small enough for the field to be adequately interpolated from the nodal values. The value of a vector field at a location within a

tetrahedron can be interpolated from the field nodal values.

The magnetic field solver divides the H-field into a homogenous and a particular solution. For the homogenous solution of H, the system stores a scalar potential at each node. For the particular solution, the system stores the components of H that are tangential to the tetrahedral edges.

There is a trade-off between the size of mesh, the desired level of accuracy, and the amount of available computing resources. The accuracy of the solution depends on how small each of the individual elements (tetrahedra) is. Solutions based on meshes using thousands of elements are more accurate than solutions based on coarse meshes using relatively few elements. To generate a precise description of a field quantity in higher field-gradient regions with the problem space, each element must occupy a region that is small enough for the field to be adequately interpolated from the nodal values. However, generating a field solution involves inverting a matrix with approximately as many elements as the tetrahedra nodes. For meshes with a large number of elements, such an inversion requires a significant amount of computing power and memory. Therefore, it is desirable to use a mesh that is fine enough to obtain an accurate field solution but not yet so fine that it overwhelms the available computer memory and processing power.

According to the principles described above, the mesh shape of the analysis model was obtained as shown in Fig. 5.2. The tetrahedron numbers of the machine components after meshing are shown in Table 5.1. Only one stator and rotor were implemented in the FEM analysis in order to save computational time, yet it should be sufficient to describe the performances of the entire system.

Table 5. 1 Tetrahedron numbers of the machine components

Items	Stator	Rotor	Coils	PMs
Mesh Numbers	31868	12910	36071	8210

Fig. 5.2 Mesh shape of analysis model

Solution process

The system computes the magnetic field in two steps:

- Firstly, the system performs a conduction current solution. To simulate the model current flow, it computes the current density, J , arising from the currents assigned inside the conductors. Current can be defined by specifying the voltage across a conductor or by specifying the current flowing through a conductor.
- Next, the system performs a magnetic field solution. It computes the model H-field using the current density as a source.

Fig. 5. 3 Solution process of FEM analysis

The solution process of the analysis is shown in Fig. 5.3. The solver first simulates the conduction current, J , in all conductors, computes the solution error, and compares it to the conduction percentage error. If the solution error is greater, it refines the mesh in the tetrahedral to reduce the error, and starts another conduction solution using the refined mesh. The conduction percent error can be specified according to the accuracy requirement.

The solver then computes the magnetic field, H , at the vertices and midpoints of the edges of each tetrahedron in the finite element mesh, using the conduction current as input. If nonlinear materials are present, it computes the field using the Newton-Raphson method, which uses the slope of the BH curve to compute a linear approximation of the nonlinear solution. The solver writes the completed solution to a file and performs an error analysis. In an adaptive analysis, it refines the tetrahedral mesh, and continues solving until the stopping criterion is met. After getting the magnetic field intensity H , the solver can calculate the magnetic flux density B , the Co-energy of the magnetic field, and the force and torque can then be derived.

Time stepping electromagnetic computation is employed in the 3D FEM analysis. In the time stepping FEM model, the stator circuit equation and the motion equation are coupled with the electromagnetic field equations and solved simultaneously at each time step. The solution can take into account the saturation effect, eddy current effect, rotor movement and non-sinusoidal quantities that are very difficult to consider by using the analytical approach. Many empirical factors then become unnecessary.

Preliminary Simulation Results

Fig. 5.4 shows the time-stepping FEA results when the stator windings are fed with three-phase 50 Hz sinusoidal currents under open-loop condition with the rotor having an initial speed of 1500 rpm. Fig. 5.4 (a) and (b) shows the three-phase flux linkages and the induced voltages of the motor. It is obvious that both are quasi-sinusoidal. The rotor velocity and position are shown in Fig. 5.4 (c) and (d) respectively. The rotor velocity oscillates around the synchronous speed and settles down to this speed eventually. The analysis results show that the proposed machine can be analyzed as a sinewave motor, and the mathematical analysis in chapter four is tenable.

Fig. 5.5 (a) and (b) show the magnetic flux density in the stator and rotor at a particular time-step. It is obvious that there are 4 regions having higher flux density in the surfaces of the stator and rotor. They indicate the instantaneous positions of the 4 poles in the motor.

Permanent magnets of material type NdFeB35 with the remanent flux density $B_r = 1.23\text{ T}$ are mounted on the surface of the rotor, so in the region under the PMs, the flux density is definitely higher than those at other places. Fig. 5.5 (c) and (d) show the magnetic flux vectors in the stator and rotor. The arrows represent the directions of the magnetic flux, the different colors indicates the values of the flux density. These magnetostatic analyses help to check for unexpected magnetic saturation in various parts of the stator and the rotor.

Fig. 5. 4 Time-stepping FEM simulation results when stator is fed with 50 Hz sinusoidal current.

Fig. 5.5 Magnetic flux distribution of the stator and rotor

5.3.2 Verification of the mathematical model

Three phase currents can be decoupled into rotor reference-frame currents, as shown below

$$i_a = i_d \cos(\theta) - i_q \sin(\theta) \quad (5.22)$$

$$i_b = i_d \cos(\theta - 120^\circ) - i_q \sin(\theta - 120^\circ) \quad (5.23)$$

$$i_c = i_d \cos(\theta - 240^\circ) - i_q \sin(\theta - 240^\circ) \quad (5.24)$$

where θ is the electrical angle of the rotor.

Making $i_q = 0$, we obtain average zero torque as shown in Fig. 5.6 (a). It is evident that the torque has no relation to i_d . Similarly, assigning $i_d = 0$ and $i_q = 1$, the force, torque and speed can also be obtained as shown in Fig. 5.6 (b)-(d). The force and torque are approximately constant, and the speed increases linearly. It proves that the torque is

proportional to i_q .

In this way, by assigning i_q or i_d to zero, and then changing the values of i_d or i_q accordingly, the variation of axial magnetic force and torque with i_d or i_q were obtained, as shown in Fig. 5.7. Solid lines represent the calculation results using analytical model Equations (4.47) to (4.48) directly, and the asterisk data points are the FEM analysis results. The parameters used in the analytical model are listed in Table 4.1.

Fig. 5.6 FEM analysis results when either i_q or i_d equals zero

Fig. 5.7 Comparison between FEM simulation results and analytical calculation results

Fig. 5.7 (a) and (b) show the force and torque when $i_d = 0$ and i_q is changing, Fig. 5.7 (c)

and (d) are the cases when $i_q = 0$ and i_d is changing. When $i_q = 0$, the torque is small enough to be regarded as zero. From Fig. 5.7 (b) and (c), we can see that the FEM simulation results correspond to analytical calculation results when magnitudes of the currents are low. When they are higher than the maximum rated current which is $1.85 \times \sqrt{2} = 2.62$ A in this design, the force and torque computed by FEM deviate from the analytical model curves. This is caused by the magnetic saturation when high currents are present. In Fig. 5.7 (a), the errors between FEM and mathematical model for small currents still exist. That is because when $i_d = 0$, $N_f^2 i_f^2$ dominates the value of the force given by Equation (4.47). A very small error between the $N_f i_f$ values calculated by using Equation (4.15) and FEM analysis software will cause a large difference in the force values. Fig. 5.7 (e) and (f) show the changes in the torque and force when different air-gap lengths are assigned. The results obtained by the two approaches are almost the same. From all the figures, we can draw the conclusion that the results calculated from the mathematical model correspond with the FEA results very well.

In order to further verify the accuracy of the mathematical model, Matlab/Simulink has been employed to simulate the dynamic performances of the machine. The simulation diagram is shown in Fig. 5.8. S-function was utilized to construct the machine's simulation module according to its entire mathematical model described by Equations (4.69) to (4.82). The derived simulation results were then compared with the FEM analysis data.

With the currents $i_d = 0$ and $i_q = 1$ assigned to the stator windings, the simulation results by Simulink and FEA are compared in Fig. 5.9. The parameters of the motor in the simulation are listed in Table 3.7. By comparing the force and torque curves obtained by these two methods, we can see that their average values are quite similar, despite some fluctuations in FEM analysis results.

Fig. 5. 8 Simulation diagram by assigning dq currents to the motor

Fig. 5. 9 Comparisons between Simulink simulation results and FEM analysis results when the same currents are assigned

Similarly, we have also applied voltages which are the functions of electrical angle to the motor models in both approaches mentioned above; their results obtained are shown in Fig. 5.10. The corresponding curves of force, torque are similar in shapes and values. The

results are the evidence that support the accuracy of the mathematical model.

From the FEA results and the comparisons between FEA and simulation results, it is clear that the proposed system is feasible, and the derived mathematical model is accurate and can be used to design the axial-flux PM machine. The mathematical model can also used for real-time control of the FESS based on this machine, which is covered in detail in Chapter 6.

(a) Axial magnetic force by Matlab simulation.

(b) Axial magnetic force by FEA.

(c) Torque by Matlab simulation.

(d) Torque by FEA.

(e) Speed by Matlab simulation

(f) Speed by FEA.

Fig. 5.10 Comparison between Matlab simulation and FEM analysis results

5.4 FEM Analysis for High Speed Axial Flux PM Motor

The high speed 2-pole version of the axial flux PM synchronous machine designed in Chapter 3 was also analyzed by FEM to verify its feasibility and validate its dimensions and parameters. For simple operation, the shaft and outer rim are not engaged, but they do not affect the analysis results. The mesh shape of analysis model is shown in Fig. 5.11. Both stators are included in the simulations.

Fig. 5.11 Mesh shape of analysis model

Fig. 5.12 FEM simulation results when stator fed with 1000Hz sinusoidal current

Fig. 5.13 Magnetic flux distribution of the stator and rotor.

When the stators are fed with 1000-Hz sinusoidal currents under open-loop condition, and given an initial speed of 60,000 rpm, both the flux linkages and induced voltages are

quasi-sinusoidal. The maximum value of the induced voltage is 180 V, which accords with the design requirement. FEM analysis results are shown in Fig. 5.12. It proves that the design requirement can be realized.

Fig. 5.13 (a) shows the magnetic flux density on the surface of the stator. There are 2 regions having higher flux density on their surfaces. These are the 2 poles of the motor. Fig. 5.13 (b) shows the flux vectors in the volume of the rotor. Permanent Magnets of material NdFeB35 with the remanent flux density $B_r = 1.23$ T are mounted on the surface of the rotor with the arrangement shown in Fig. 3.5 (b), so the flux vectors move from one PM to another on the same surface and the opposite PM mounted on another surface respectively.

Similarly, decoupling the 3-phase currents into dq currents using Equations (5.22)–(5.24), making $i_q = 0$ and $i_d = 7.42$, we can get the analysis results shown in Fig. 5.14. As shown in Fig. 5.14 (a), the starting torque is small enough to be regarded as zero and the average value tends to be zero as well.

Similarly, assigning $i_d = 0$ and $i_q = 7.42$ A, the analysis results can then be obtained as shown in Fig. 5.14 (b)–(f). Fig. 5.14(b) shows the axial forces acting on two stators and rotor respectively. The forces acting on the stators are both constant with the same value but opposite directions, so the resultant force on the rotor equals their summation and is approximately zero. The radial forces on the rotor in x and y directions are shown in Fig. 5.14 (c). They are very small and can be ignored. Fig. 5.14 (d) shows the torque acting on the rotor, which has a nearly constant average value despite having certain vibrations. Fig. 5.14 (e) shows the rotational velocity of the rotor, the speed increases linearly from zero. The analysis was terminated when the speed reached 6,000 rpm to save computational time. With the increase of rotational velocity, the current frequency will rise accordingly, and the sampling rate in the FEM simulator will be higher and higher, which will thus make the computation burden much heavy. It will take months if the FEM analysis were to terminate at the maximum speed of 60,000 rpm. Fig. 5.14 (f) shows the induced voltage in

the 3 phases of both stators. The induced voltages are all approximately sinusoidal with their amplitudes increasing linearly and proportional to the value of speed. When the speed reaches 6,000 rpm, the magnitudes are about 18 V. Therefore, when the velocity reaches 60,000 rpm, the induced voltage will be around 180 V, which is in agreement with the design requirements.

Fig. 5.14 (g) shows the transient core loss in the two stators when the speed of rotor arises from 0 to 6,000 rpm. The core loss increases corresponding with the change of the current frequency, i.e., the rotational velocity of the rotor. This calculation should be ended at the speed of 60,000 rpm to obtain the core loss when the rotor keeps rotation in this velocity. However it was terminated at 6,000 rpm with the consideration of the calculation duration. A substitute approach is setting the initial speed to 60,000 rpm, the core loss curve around this velocity can be obtained as shown in Fig. 5.14 (h), the average core loss in these two stators is 100 W.

From the FEM analysis results, it is clear that the rotor can spin at 60,000 rpm with the given design data, and the electro-magnetic performances fit the design requirements. It indicates that the proposed system is feasible in the electromagnetic aspect, and the machine has been appropriately designed.

(a) Torque when $i_q = 0$

(b) Forces in z direction when $i_d = 0$

Fig. 5.14 FEM analysis results when either i_q or i_d equals zero

5.5 Conclusions

This chapter has presented the finite element analyses of the axial-flux PM machines for

both the low-speed FESS prototype and the conceptual high-speed FESS. The 3D FEM has been implemented considering the special geometry of the pancake-type motor, and the basic theory of the 3D FEM has been introduced in the beginning of this chapter. The mesh method and solution process have been described based on this theory. The FEA results have justified that both the low-speed and high-speed versions are well designed and the design requirements can be realized. The results have also verified the accuracy of the mathematical model of the motors, which contributes to the correctness of the design of the control system. FEM analysis has proven to be an excellent approach to assist the design of the motors and a powerful tool that verifies the feasibility of the proposed machines as well as the accuracy of their mathematical models.

CHAPTER 6

CONTROL SYSTEM DESIGN & IMPLEMENTATION FOR THE AXIAL-FLUX PARTIALLY-SELF-BEARING PM MACHINE

6.1 Introduction

For the proposed axial-flux partially-self-bearing machine, the rotor-flywheel of the machine is to be levitated by axial magnetic forces, so the rotor rotation and the axial displacement should both be controlled at the same time. However, the design of an active control system for magnetic bearings is complicated even for one DOF, and constitutes a challenging task due to the highly nonlinear and inherently unstable open-loop electromagnetic dynamics. Also, the control performance of active magnetic bearing is very sensitive to outside disturbance and plant uncertainties, hence the classical PID feedback control is not entirely effective and satisfactory under such situations, especially in super-high speed operation (at the order of 60-80 krpm) [99, 112, 144]. Thus, many control methods have been proposed to solve these problems, such as gain schedule H_∞ robust control [145], nonlinear control [146, 147], adaptive auto-centering control [148], and sliding mode control [149]. Unfortunately, all these approaches need a fairly precise mathematical model of the active magnetic bearing system [150]. However the precise mathematical model can not be easily obtained for the proposed self-bearing PM motor with consideration of magnetic saturation, demagnetization, mass unbalance and disturbance.

Model-based predictive control approach has the advantage of control acumen to deal with the multivariable constrained control problems, as the actual control objectives and the operating constraints can be represented explicitly in the optimization function. Furthermore, it can be used to control a wide variety of processes such as non-minimum phase and unstable processes, without requiring too much special precaution.

The space state model shown in (4.86) is a linearized model without considering the mass unbalance and uncertain disturbances. MPC control method has advantages of pre-calculating the system behavior, providing satisfactory tracking response and being robust against model-mismatch and unexpected dynamics. Due to this reason, both MPC and PID control algorithms are implemented in this Chapter for the control of the axial-flux PM machine. The controllers applying these two approaches were designed separately. The drive simulation systems employing MPC and PID methods were implemented in the MATLAB/SIMULINK simulation environment. The simulation results have been obtained and compared. Experimental results on the PID controller are also presented in this Chapter.

6.2 Control Strategy

The entire model of the system shown in (4.69)-(4.82) is the nonlinear model with the original force, toque, velocity, position and voltage equations. The space state model shown in (4.86) is a linearized velocity and position model derived from (4.69)-(4.78). The voltage equations shown in (4.79)-(4.82) expound the relationships between the currents and voltages, which are the inner characteristics of the upper and lower motors.

Fig. 6. 1 Two step control diagram of the proposed FESS

According to the analyses in Chapter 4, two-step control shown in Fig. 6.1 can be implemented in the control system. The first step is the position and velocity control by using the equations (4.83)-(4.86). In this step, in order to let the velocity and position, ω_r and z track the commands ω_r^* and z^* , the control currents $\tilde{i}_{cd}^*$ and i_{cq}^* are needed. And the currents track i_{d1}^* , i_{q1}^* ; i_{d2}^* and i_{q2}^* are required to be generated in the upper and lower windings of the FESS. The second step is the current control. It is used to control the currents i_{d1} , i_{q1} ; i_{d2} and i_{q2} to track the commands i_{d1}^* , i_{q1}^* ; i_{d2}^* and i_{q2}^* . In this step, the voltage equations (4.79)-(4.82) are related. The current hysteresis control approach shown in Fig. 2.9 and linear current control methods shown in Fig. 2.10 and Fig. 2.11 can be implemented to control the dq currents. In these methods, current feedback control is employed to make the currents tracking the commands. These approaches are well known and widely used in electrical machine control [83, 89, 94, 95]. Therefore, this work will not conduct further study on them.

6.3 MPC Control for the System

In PID control, the bearing stiffness generally increases with the increase in proportional gain; hence the PID controller must use high feedback gain to reduce seismic response of the rotor. However, the control is not satisfying because the high gain may cause instability and reduction in various control performances. The MPC method has advantages of pre-calculating the system behavior, providing satisfactory tracking response and being robust against model-mismatch and unexpected dynamics. Furthermore, it is easy to deal with multivariable constrained control problems, and explicitly incorporate plant model uncertainties to provide satisfactory tracking response. So MPC approach suits the proposed system very well and hence was investigated for the control system.

6.3.1 MPC controller design

The linearized, continuous-time equation of (4.86) can be transformed into the

discrete-time, state-space model

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \end{cases} \quad (6.1)$$

where k is the normalized discrete time, $\mathbf{x}$ is an n -dimensional state vector, $\mathbf{u}$ is an ℓ -dimensional control input vector, $\mathbf{y}$ is an m -dimensional vector of outputs, $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are discrete-time matrices transformed from $\mathbf{A}_c$, $\mathbf{B}_c$ and $\mathbf{C}_c$ with appropriate dimensions. In this control system, $\mathbf{x} = [\theta, \omega, z, z']^T$, the control input vector $\mathbf{u} = [i_{cq}, \tilde{i}_{cd}]^T$, the output vector $\mathbf{y} = [\omega, z]^T$, $n = 4$, $m = 2$, and $\ell = 2$.

As the goal of the controller design is to achieve satisfactory tracking accuracy, we can define the cost function to quantify the deviations of the predicted outputs $\hat{\mathbf{y}}(k+i|k)$ from a (vector) reference trajectory $\mathbf{r}(k+i|k)$ as [104, 107, 151]

$$V(k) = \sum_{i=1}^{N_p} \|\mathbf{r}(k+i|k) - \hat{\mathbf{y}}(k+i|k)\|_{\mathbf{Q}(i)}^2 + \sum_{i=1}^{N_u} \|\hat{\mathbf{u}}(k+i-1|k)\|_{\mathbf{R}(i)}^2 \quad (6.2)$$

where $\hat{\mathbf{u}}(k+i-1|k)$ is the predicted control input vector, N_p and N_u are the prediction horizon and control horizon satisfying $N_u \leq N_p$, $\mathbf{Q}(i)$ and $\mathbf{R}(i)$ are time-varying weighting matrices for the tracking errors and the control inputs respectively, and $\mathbf{Q}(i) = \mathbf{I}$, $\mathbf{R}(i) = \lambda(i)\mathbf{I}$, where, $\lambda(i)$ is the positive weighting factor use to adjust the control action.

Minimization of the cost function (6.2) with respect to the future control input $\hat{\mathbf{u}}(k+i-1|k)$ results in the optimal algorithm which would provide good tracking response. For this, rewrite (6.2) as

$$\begin{aligned}
V(k) = & \| \mathbf{r}(k + N_p) - \hat{\mathbf{y}}(k + N_p) \|^2_{\mathbf{Q}(N_p)} \\
& + \sum_{i=1}^{N_p-1} \{ \| \mathbf{r}(k + N_p - 1 - i | k) - \hat{\mathbf{y}}(k + N_p - 1 - i | k) \|^2_{\mathbf{Q}_i} \\
& + \sum_{i=1}^{N_u} \| \hat{\mathbf{u}}(k + N_p - 1 - i | k) \|^2_{\mathbf{R}_i} \}
\end{aligned} \quad (6.3)$$

where

$$\mathbf{Q}_i = \begin{cases} \mathbf{Q}(N_p - i - 1) & \text{if } i = 0, 1, \dots, N_p - 2 \\ 0 & \text{if } i = N_p - 1 \end{cases} \quad (6.4)$$

$$\mathbf{R}_i = \begin{cases} \infty \mathbf{I}_n & \text{if } i = 0, 1, \dots, N_p - N_u - 1 \\ \mathbf{R}(N_p - i) & \text{if } i = N_p - N_u, \dots, N_p - 1 \end{cases} \quad (6.5)$$

As $\mathbf{r}(k + i | k)$ is an externally prescribed signal, an artificial state model $\mathbf{r}(k + i | k) = \mathbf{A}^d \mathbf{r}(k + i - 1 | k)$ can be incorporated for it, for tracking of constant reference signals, $\mathbf{A}^d = \mathbf{I}_m$ may be chosen. In practice, according to the receding horizon strategy, only the control action $k = N_p - 1$ is applied. Hence, the optimal solution for the Linear Quadratic Gaussian (LQG) problem is given by [107,159]

$$\mathbf{u}(k)_{opt} = - \left(\mathbf{B}^T \mathbf{P}_{N_p-1} \mathbf{B} + \mathbf{R}_{N_p-1} \right)^{-1} \mathbf{B}^T \left(\mathbf{P}_{N_p-1} \mathbf{A} \mathbf{x}(k) + \boldsymbol{\Pi}_{N_p-1} \mathbf{A}^d \mathbf{r}(k) \right) \quad (6.6)$$

where $\mathbf{P}_{N_p-1}$ is obtained from the regulation Riccati Difference Equation (RDE) [158],

$$\mathbf{P}_{i+1} = \mathbf{A}^T \mathbf{P}_i \mathbf{A} - \mathbf{A}^T \mathbf{P}_i \mathbf{B} (\mathbf{B}^T \mathbf{P}_i \mathbf{B} + \mathbf{R}_i)^{-1} \mathbf{B}^T \mathbf{P}_i \mathbf{A} + \mathbf{C}^T \mathbf{Q}_i \mathbf{C} \quad (6.7)$$

with the initial condition $\mathbf{P}_0 = \mathbf{C}^T \mathbf{Q}(N_p) \mathbf{C}$, and $\boldsymbol{\Pi}_{N_p-1}$ is obtained from a coupled Lyapunov equation [159],

$$\boldsymbol{\Pi}_{i+1} = \left(\mathbf{A} - \mathbf{B} (\mathbf{B}^T \mathbf{P}_i \mathbf{B} + \mathbf{R}_i)^{-1} \mathbf{B}^T \mathbf{P}_i \mathbf{A} \right)^T \boldsymbol{\Pi}_i \mathbf{A}^d - \mathbf{C}^T \mathbf{Q}_i \quad (6.8)$$

with the initial condition $\Pi_0 = -C^T Q(N_p)$.

Theorem

Considering equations (6.7) and (6.8) with constant weighting matrices Q and R , and infinite prediction and control horizons. If

- P_0 is nonnegative definite,
- R is positive definite,
- $[A, B]$ is controllable,
- $[A, Q^{1/2}C]$ is observable,

then, the sequences P_{i+1} and Π_{i+1} will converge to steady-state values P_∞ and Π_∞ .

This theorem indicates that the horizons N_p and N_u and the weighting factor λ are key parameters for guaranteeing the closed loop stability. In practice, in the case of infinite prediction horizon, the system will definitely be stable. It means that by selecting a prediction horizon which is large enough, both a reduced computation and a stable controller can be obtained. Furthermore, in the derivation of the control law, it is assumed that the parameters of the process are known and fixed. Otherwise, an identification algorithm such as the Recursive Least Squares would be necessary to estimate system parameters on-line and then, an adaptive version of the control algorithm can be generated.

6.3.2 Control scheme based on MPC

In the project, vector control method is used to decouple the torque and the flux; the position displacement along the z axis and the rotational speed of the flywheel are controlled by the currents $\tilde{i}_{cd}$ and i_{cq} respectively. Current regulation control is very simple because the position displacement and the speed are only relational to $\tilde{i}_{cd}$ and i_{cq} . Furthermore, better performance and faster response can be achieved if current-fed control

is used. In AC machines, current control reduces the dependence on stator parameters and allows an immediate action on the torque delivered or absorbed by the machine.

Fig. 6.2 shows the detailed control scheme based on MPC with current regulation control for voltage-source inverter. The rotor angle θ and the rotational angle speed ω are detected by the encoder, and the displacement from the equilibrium point along the z axis direction, z , can be detected by non-contact eddy current displacement sensor. They together with axial position command z^* and the speed command ω^* are sent to MPC controller. In MPC control system, $[\omega, z]^T$ is the output vector $y(k)$ given in (6.1); $[\omega^*, z^*]^T$ is the reference vector given in (6.2). Position command z^* is always set to 0 to make sure the flywheel is right in the midpoint between the two stators. By applying MPC control method, the control input vector $u(k) = [i_{cq}^*, \tilde{i}_{cd}^*]^T$ is generated. The objective of MPC control is to minimize the cost function $V(k)$ and make it satisfy the constraint equation (6.2). The detailed realization of the MPC control by Matlab programming is given in Appendix B.

The direct currents of the stator windings i_{d1}^* and i_{d2}^* can be generated by substituting $\tilde{i}_{cd}^*$ and the offset current I_{d0} and I_{cd0} to equations (4.53), (4.54) and (4.61). When the rotor spins in constant speed, the current i_{cq}^* is 0; $\tilde{i}_{cd}^*$ always has a small value regardless the rotor-flywheel status, which will cause difficulties in the current regulation control. The problem may be solved by introducing an offset current I_{d0} . The value of the offset current I_{d0} is set to 1 in this simulation.

Three-phase currents in the output side of the two inverters are detected and transformed into the respective d-q currents in rotor coordinates. Linear current regulation method introduced in Chapter 2 is used to generate PWM modulated signals to control the switches of the two inverters. The currents from the outputs of the inverters flow into the windings

of the stators and generate the magnetic forces and torques, which levitate the flywheel and control the speed of rotation to vary the stored kinetic energy.

Fig. 6.2 Control system block diagram based on MPC

6.3.3 Simulation results based on MPC

The simulation is based on the low-speed prototype design by employing MPC method and the control scheme shown in Fig. 6.2. The horizons $H_p = 6$, $H_c = 3$, $\lambda = 0.03$ and the sampling period $T_s = 0.0008$ are selected to solve the RDE (6.7). The conditions in the theorem can be satisfied with these parameters, and the system can hence be stable. The parameters of the flywheel system used for simulation are shown in Table 4.1.

Fig. 6.3 (a) shows the z-axis position displacement of the rotor. It fluctuates in the range of 1.7×10^{-6} m, which is much smaller than the air gap length, 5×10^{-4} m. Fig. 6.3 (b) shows the angular velocity of the flywheel, which increased almost linearly after starting and is kept constant after the speed reached the reference speed ω^* , and this is the period of energy charging and storage. When speed reference was changed to 31.4 rad/s, 10% of the maximum speed, the speed slowed down linearly to the reference and stayed constant; that means the mechanical energy is discharged during this period. When the reference arose

again, the speed increased too. It indicates that the charge and discharge process can be successfully realized.

The energy stored in the flywheel when it is operating at the maximum reference speed 314.2 rad/s is

$$W = 1/2 J \omega_r^2 = 1/2 J (\omega_e / P)^2 = 194.98 \text{ J} \quad (6.9)$$

The change in stored energy when the speed decreases to 31.4 rad/s is

$$W = 1/2 J (\omega_{r1}^2 - \omega_{r2}^2) = 193.03 \text{ J} \quad (6.10)$$

The time spent during the discharge period is 0.11 s, so the generated power is

$$P = W / t = 1754.82 \text{ W} \quad (6.11)$$

The low-speed prototype machine was designed to spin at a rated speed of only 1500 rpm; the stored energy is therefore very low. However, this does not affect the study on the feasibility of the proposed conceptual FESS design and the validity of the derived mathematic model.

Fig. 6.3 (c) and (d) are the simulation diagrams of levitation force and rotational torque respectively. The force is almost equal to that due to the gravity. When the speed remains constant, the torque drops to zero suddenly. When the speed is decreasing, the value of torque becomes negative. It reflects the motoring and generating states of the motor. So the proposed FESS is capable of responding according to the request of the controller.

The control currents $\tilde{i}_{cd}$ and i_{cq} are shown in Fig. 6.3 (e) and (f). The current i_{cq} is proportional to the torque, so the curve of i_{cq} is similar to that of the torque. Because $\tilde{i}_{cd}$ and i_{cq} are used to control the position displacement and the torque respectively, when the torque decreases to zero and the speed becomes constant, $\tilde{i}_{cd}$ is close to constant and

i_{cq} becomes zero. The diagrams of the dq currents of the two stators are shown in Fig. 6.3 (g) and (h). It is obvious that i_{q1} and i_{q2} are nearly equal to i_{cq} , and i_{d1} is always larger than i_{d2} . This is in accordance to our analysis of the mathematical model.

Fig. 6.3 (i) shows the 3 phase currents of stator 1. The currents are less than 0.4 A under the steady state, which are very low so that the energy loss can be kept small. The voltage of phase a in stator 1 is shown in Fig. 6.3 (j).

From the simulation results, it is obvious that the displacement and rotation along axial axis of the system can be controlled well, so the concept of partial self bearing can be used in the compact and efficient flywheel energy storage system to get good performance.

(a) Displacement position in axial direction

(b) Rotational velocity

(c) Levitation force in axial direction

(d) Torque acting on the rotor

Fig. 6.3 Simulation Results Based on MPC

6.4 PID Control for the System

Though the control system employing MPC approach displayed good performances, PID control method still has the advantages of simple algorithm, stable properties and convenience in implementation. The study on PID control for use in the proposed FESS is hence meaningful and important.

6.4.1 Axial position displacement control

Using the Laplace transformation, we can transform (4.59) to

$$ms^2 z(s) = K_1 \cdot z(s) + K_2 \cdot i_{cd}(s) + G \quad (6.12)$$

$$z(s) = \frac{K_2}{ms^2 - K_1} i_{cd}(s) + \frac{G}{ms^2 - K_1} \quad (6.13)$$

Since K_1 is positive, it is obviously that the system is unstable. To stabilize the system, a proportional derivative (PD) controller can be used. The transfer function of a PD controller can be written as:

$$G_c(s) = k_p + k_d s \quad (6.14)$$

Where k_p is the proportional coefficient, k_d is the differential coefficient.

The control structure is drawn as shown in Fig. 6.4. The characteristic polynomial of the system can be written as

$$ms^2 + K_2 k_d s + (K_2 k_p - K_1) = 0 \quad (6.15)$$

From Routh's criteria, when $k_d > 0$, $k_p > K_1 / K_2$, the system is stable. Where, K_1 , K_2 are determined by (4.57) and (4.58). When K_1 is given the maximum value, if parameters k_p and k_d can make the system stable, then the system will be stable when K_1 changes to

other values. According to the calculations shown in Section 4.2.4, $K_1 = 2.96 \times 10^6$ and $K_2 = 455.44$ are determined.

Fig. 6. 4 Close-loop control structure with PD controller

Selection of k_p and k_d

As mentioned above, when $k_d > 0$ and $k_p > K_1 / K_2 \approx 6.5 \times 10^3$, the system is stable. Fig. 6.5 shows the root locus of the close loop system when $k_p = 7 \times 10^3, 1.5 \times 10^4, 5 \times 10^4$, and $k_d = 0 \rightarrow \infty$. It is seen that when k_p increases, the system trends to more stable.

Fig. 6. 5 The root loci of the close-loop system when $k_p = 7 \times 10^3, 1.5 \times 10^4$ and 5×10^4

Fig. 6.6 and Fig. 6.7 show the step response and bode diagrams of the close-loop system when $k_p = 1.5 \times 10^4$ and $k_p = 5 \times 10^4$. From the bode diagrams, we can see that when k_p increases, the system has a larger band width. However, the increasing of the gain k_p will result in the enlargement of the high frequency noise. In practical application, k_p can not be too large, it can be tuned according to the experiment performances.

Fig. 6.6 The Step responses of the close-loop system when $k_p = 1.5 \times 10^4$ and $k_p = 5 \times 10^4$

Fig. 6.7 The bode diagram of the close-loop system when $k_p = 1.5 \times 10^4$ and $k_p = 5 \times 10^4$

The parameter k_d is selected based on the step response and bode diagrams shown in Fig. 6.6 and Fig. 6.7. For the step response, to have better performance, the overshoot can not be large, the rise time and settling time should be small. For the bode diagram, the magnitude spectrum should have no overshoot. Suitable values of k_p and k_d can be determined by considering the performance requirements mentioned above. In the simulation, $k_p = 1.5 \times 10^4$, $k_d = 15$ are selected.

6.4.2 Rotational velocity control

By using the Laplace transformation, (4.67) and (4.68) can be transformed to

$$\theta(s) = \frac{1}{s} \omega(s) \quad (6.16)$$

$$\omega(s) = \frac{K_T}{J s} i_{cq}(s) \quad (6.17)$$

So the angle velocity can be controlled by a PI controller.

Fig. 6.8 Diagram of speed control

As shown in Fig. 6.8, a PI controller is used to control the velocity of the machine, and the feedback control is employed in the velocity control system. The controller is expressed as

$$G_c(s) = k_{p1} + \frac{k_{i1}}{s} \quad (6.18)$$

The transfer function of the close-loop system is

$$\frac{Y(s)}{U(s)} = \frac{K_T k_{p1} s + K_T k_{i1}}{Js^2 + K_T k_{p1} s + K_T k_{i1}} \quad (6.19)$$

Because J , K_T , k_{p1} , k_{i1} are all larger than 0, the system will be stable with the PI controller.

Fig. 6.9 shows the bode diagram and step response of the close-loop system. It is obvious that k_{i1} has almost no influences on the performances. The bandwidth of the rise time have no changes when k_{i1} was assigned different values. However, k_{i1} is always applied to reduce the velocity static error. So it can be tuned in simulations and experiments to achieve the desired control precision. In practical system, pure PI controller is not enough because there is always high frequency noise. The velocity spectrums can be obtained by using pseudo-random binary sequence (PRBS) measurement method, which will be introduced in next section.

Fig. 6.9 Bode diagram & step response of close-loop system when $k_{p1}=30$ and k_{i1} changing

6.4.3 Control scheme based on PID approach

The system block diagram based on PID control strategy is shown in Fig. 6.10. Most parts of the diagram are similar to which of MPC method. The difference is that PD and PI

controllers are used to control the displacement and speed respectively.

The rotational angle speed ω are detected and then compared with the speed command ω^* . The speed error signal $\omega^* - \omega$ is amplified by PI controller to generate current command i_{cq}^* in the rotational coordinate. Similarly, the error between z and z^* are amplified by the PD controllers to obtain the current command $\tilde{i}_{cd}^*$. The d -axis current commands of the two stators i_{d1}^* and i_{d2}^* can hence be generated by applying $I_{d0} + I_{cd0} - \tilde{i}_{cd}^*$ and $I_{d0} - I_{cd0} + \tilde{i}_{cd}^*$.

Fig. 6.10 System Block Diagram Based on PID Control

6.4.4 Simulation results based on PID control

Fig. 6.11 shows the simulation results based on PID control strategy. In the simulation, $k_p = 1.5 \times 10^4$ and $k_d = 15$ are used in the PD controller; $k_{p1} = 30$ and $k_{i1} = 0.5$ are applied in the PI controller. From these results, we can see there are larger overshoots in the q-axis currents when the value of q-axis currents have sharp changes, as shown in (e) and (f). As the simulation results are similar to which of MPC method, detailed descriptions of the

figures are not provided.

(a) Position displacement in axial direction

(b) Levitation force acted on the rotor in axial direction

(c) DQ currents of the upper stator

(d) DQ currents of the lower stator

(e) Three phase current of the upper stator

(f) Voltage of phase A in upper stator

Fig. 6.11 Simulation Results Based on PID Control

The simulation results based on MPC and PID approaches show that the MPC control system can pre-calculate the system behavior and provide more satisfactory tracking response; it therefore has better control performance: there are no overshoots when the velocity changes and the tracking responses are smoother. However the results also show that the PID method is a feasible and simple method with reasonable performance, and which can be readily implemented in the proposed FESS.

6.5 Implementation and Experiment for the FESS Prototype

The hardware setup was built up based on the designs of the low-speed prototype structure and drive system of the proposed FESS. For convenience and simplicity, the composite material rim which is to be mounted outside the iron rotor and the system housing are not included in this prototype. DSP-1103 control card is used as the controller of the FESS. A PCB control board which consists of two inverters together with their drive cards and protection circuits was built. This control board which connects the DSP and FESS prototype transfers the current, voltage, displacement and angle position signals to the DSP control card after detecting them. The PCB control board can also receive the control signals from the DSP and provide the appropriate control currents to operate the FESS prototype after regulation and amplification.

6.5.1 Construction of the hardware system

The hardware system consists of three parts: CPU joint with DSP control card, PCB control board and FESS prototype. Fig. 6.12 shows the block diagram of the hardware system. The connections of various parts are indicated in this diagram.

1. DSP 1103 card

The DSP-1103 PPC controller card is employed as the controller to conduct A/D and D/A conversion, calculation, PWM signal generation, and so forth. It consists of Motorola Power PC 640e/333 MHz main processor and Texas instruments' TMS320F240/40 MHz

slave subsystem with powerful features. 16 Channels of 16-bit ADC, 4 channels of 12-bit ADC, 8 channels of 14-bit DAC, 6 channels of digital input incremental encoder subsystem and 4 channels of 8-bit digital I/O ports are available on this board. The DSP controller card is installed on a Pentium III host computer, and it has real-time interface (RTI) with MATLAB/SIMULINK. The simulation diagram and control algorithm can be downloaded to the Dspace Control Desk Software to implement the controller in real time.

Fig. 6. 12 Block diagram of the hardware system

2. PCB control board

PCB control board functions as a bridge between DSP and FESS prototype. It consists of two set of inverters together with their driver cards for the upper and lower windings of the PM machine, protection circuit for the inverters and their driver cards, and the current & voltage detection circuits.

A. Inverter and its driver board

One integrated power converter module (SEMIKRON SKiiP 10 NEC 063) that is made up of a single-phase bridge rectifier and a three-phase bridge inverter has been used in the hardware system to drive the FESS prototype. The inverter can be switched up to a

switching frequency of 20 kHz. SKHIBS 01 IGBT driver board is used to drive the inverter and the rectifier on the integrated power module. In SKHIBS 01 driver kit, driver card together with the transformer form the minimal configuration which drives 6 IGBT switches. Six opto-couplers are used for driving these IGBTs in the inverter. The transformer provides an isolated supply voltage. The 6 IGBTs of the inverter are supplied with regulated +15 V for switching on and -8 V for switching off. The 3 top-IGBTs have separate supply voltage whereas the bottom-IGBTs have a common supply voltage.

B. Current and voltage detection

In current hysteresis control, three phase currents flowing in the windings of the motor should be detected and compared with the current commands to generate the PWM signals. In this design, only two phase currents are measured. The currents pass through the current transducers before being transformed to voltage analog signals by the transducers. These analog signals are then converted to digital signals by the A/D converter of DSP-1103 controller. The third phase current can be calculated according to the values of these two currents. Over-current (OC) and Over-voltage (OV) are very likely to destroy the inverters. However, they could be avoided if the current and voltage of the DC link bus are measured and fed to the DSP to process A/D conversion and comparison. If the measured values exceed the maximum values expected, the DSP generates OV and OC signals and transfers them to the protection circuit.

LEM current transducers that use Hall Effect principle to measure instantaneous value of current waveforms are utilized in the hardware setup, while PCB-mounted voltage transducers LEM LV25-P are used for measuring the voltages of DC links of the two inverters.

C. Interlock and protection circuit

The protection of the system is crucial in the power electronic circuit. Reliable protection is indispensable for the debugging and stable operation of the system. As there is no

protection functions in the driver board of the inverter, a separate interlock and protection circuit is necessary to be designed.

In order to protect the IGBTs in the inverter, the upper and lower IGBTs of the same arm cannot be in on-state simultaneously, or the DC link will be shorted. Though the dead-times are imposed on the PWM signals to compensate the time-delay when the IGBTs are in the status of switching on or off, there are still possibilities of direct-through for the IGBTs of the same arm, which may be caused by the external interfaces or software malfunctions in the DSP.

Fig. 6.13 Inter-lock and protection circuit for the inverter

Fig. 6.13 shows the interlock and protection circuit of the inverters. When the input control pulses for the IGBTs of the same bridge are both in high levels, the output signals will switch to low level, which will then be used to turn off the upper and lower IGBTs. Once over-current, over-voltage of the DC bus or over-temperature of the driver cards is detected, all the power devices are to be shut down. As shown in Fig. 6.13, all the fault signals are connected to OR-NOT gate circuit. Once a fault signal is detected, the output of the OR-NOT gate will be set to low-level. All six PWM output signals will thus be changed to

low-level as well, and the six IGBTs in the inverter would then be turned off.

6.5.2 Configuration design of the DSP based hysteresis current control system

Fig. 6.14 shows the configuration design of the DSP-based hysteresis current control system for the FESS prototype.

Fig. 6.14 Configuration design of the DSP based hysteresis current control system

The phase currents of upper and lower stators, bus currents and voltages of two inverters are detected by the current and the voltage transducers. The displacement of the rotor along axial direction is measured by the eddy current displacement sensor. All these analogue signals are sent to the A/D converters in the DSP-1103 board. After A/D conversion, the displacement digitized value will be compared with the displacement command. The difference between the two displacement values is then be sent to displacement PID regulator. The command value i_d^* is generated after this regulation. The optical encoder is used to detect the angular position of the rotor. Pulse signals are generated by the encoder and then transferred to encoder position counter in the DSP board, and the rotational angle of the rotor can hence be calculated. The speed is obtained after differential calculation of angle position. It is compared with the command value and the difference is amplified to i_q^* , the desired value of i_q , by PI speed regulator. The dq current commands for the upper

and lower stators, i_{dq1}^* and i_{dq2}^* , are obtained by applying Equations (4.52-4.54).

Three-phase current commands i_a^* , i_b^* and i_c^* are obtained from dq-to-abc transformation.

These commands and the three-phase current-feedback signals are then input into the current hysteresis controllers to generate the PWM signals.

The DSP control software is designed with two levels of timer-driven interruption. The speed estimation and speed regulation are put into a lower priority interruption level with sampling frequency set at 2 kHz. The phase commutation signal generation is put into a higher priority interruption level with a sampling frequency of 10 kHz. The reason of using two levels of interruption service is that the electrical time constant and mechanical time constant are vastly different. Higher sampling frequency is needed to get sufficient phase current information to determine the required switching status of the IGBTs in the inverters. The use of high sampling frequency can reduce the harmonics of the currents, and thus improve the system performance.

6.5.3 Experimental setup of the FESS prototype

The experimental setup has been constructed based on the system design, FEM analysis and simulations of the low speed FESS prototype whose maximum velocity is only 1500 rpm. The specifications of the experimental setup are shown in Table 3.10 and Table 3.11 in Section 3.7. As shown in Fig. 6.15, the hardware system consists of a PC with DSP-1103 board, FESS prototype together with encoder and displacement sensor, and PCB control board.

As shown in Fig. 6.16, the FESS prototype is built up without the vacuum housing and the composite material rim for the purpose of simplicity in construction. The rotor can move along the sliding bar, but its movement is restricted within the range of half of the air-gap length, 0.5 mm deviation from the middle point between two stators. The optical encoder that has a resolution of 500 pulses per revolution is installed on the upper end of the shaft.

Fig. 6. 15 Experimental setup of the FESS prototype and its drive system

Fig. 6. 16 Prototype of the developed FESS

The eddy current displacement sensor (multiNCDT 100) is set up on top of a metal board which is mounted on the sliding part. This displacement measuring system operates in a non-contact environment by using eddy current technology. The sensor is used for

measuring the displacement between targets made of either ferromagnetic or non-ferromagnetic electrically conductive materials. High-frequency alternating current flows through a coil in the sensor casing. The electromagnetic coil field induces eddy currents in the conductive target, thus changing the AC resistance of the coil. This variance in impedance is interpreted by demodulation electronics which generate an electrical signal proportional to the distance of the target from the sensor.

6.5.4 Experiment based spectrum analysis and controller design

In the experimental implementation, MPC method was not employed. It is because the computation burden is heavy with numerous matrix calculations. Furthermore, all the I/O and ADC/DAC processing, and the driver control would have also to be realized in the same DSP. DSP-1103 can not finish all the work within the given sampling period. The objective of the experiment is mainly to verify the feasibility of the proposed conceptual FESS. Therefore, only PID control was implemented to evaluate the designed prototype.

Section 6.4 has described the PID control design for the proposed system. The control parameters were selected based on the simplified linear model. However, the practical systems are always different from the model with the injection of disturbances and noises. An effective way to get more real information of the system is obtaining its spectrum. One approach is importing PRBS to the input side of the system and getting the responses from the output. For instance, as shown in Fig. 6.14, in order to get the velocity spectrum of the upper motor, we let $i_{d1}^* = 0$, and assign the PRBS signals which range from $-K_g$ to K_g as the current command i_{q1}^* , where K_g is selected to be 20% of the maximum i_q . The velocity can be calculated after recording the encoder positions. By applying Discrete Fourier Transformation (DFT) to the velocity sequences, the open-loop velocity spectrum can be obtained.

1. Velocity spectrums and controller design

The open-loop velocity spectrums of the upper and lower motors are shown in Fig 6.17. It is observed that there are two resonances at 250 Hz and 400 Hz respectively. Though the resonances are not very high, it is better to reduce those using filters to avoid vibrations in close-loop. The notch filters to reduce the 250 Hz and 400 Hz resonances are designed as

$$F_{\omega 1}(z) = \frac{0.9604 - 1.3189z^{-1} + 0.9076z^{-2}}{1 - 1.3189z^{-1} + 0.8679z^{-2}} \quad (6.20)$$

$$F_{\omega 2}(z) = \frac{0.9392 - 0.5639z^{-1} + 0.8871z^{-2}}{1 - 0.5639z^{-1} + 0.8263z^{-2}} \quad (6.21)$$

Fig. 6. 17 The open-loop spectrums of the upper and lower motor

Bode diagrams of these two filters are shown in Fig. 6.18.

The PI controller designed in Section 6.4 together with the notch filters can be used to control the flywheel velocity. Based on the pre-selected k_{p1} and k_{i1} , after try and trial, $k_{p1} = 24.5$ and $k_{i1} = 0.01$ were determined finally. Note that the integrator was employed

after the velocity settling to the velocity command ω^* . When the velocity is increasing, the velocity error will be amplified largely by the integrator, which will make the calculation overflow. The employment of the integrator when the velocity is around constant speed can reduce the velocity static error.

Fig. 6. 18 Bode diagrams of notch filters at 250 Hz and 400 Hz

Fig. 6. 19 Velocity close-loop of the upper motor

The velocity close-loop spectrum of the upper motor is shown in Fig. 6.19. The bandwidth

of the spectrum is about 160 Hz. The resonances at 250 and 400 Hz were depressed. Because the open-loop spectrums of upper and lower motor are very similar, only the close-loop spectrum of upper motor was tested.

2. Position controller design and spectrum analysis

The flywheel can not keep balance in axial direction when the system is running at open-loop. The open-loop spectrum hence can not be obtained. In this case, the designed PD controller parameters and the simulation results in Section 6.4 provide great helps in the experimental implementation.

Fig. 6. 20 Axial position close-loop spectrum only with PD controller

Firstly, the PD controller with the parameters $k_p = 1.5 \times 10^4$ and $k_d = 15$ was employed. Then we can keep velocity control loop in open-loop, close the position loop with the PD controller, and input PRBS signals into the position command z^* side. The position close-loop spectrum can be obtained as shown in Fig. 6.20. There is a high resonance at about 300 Hz and the amplitude when the frequency above 550 Hz is high. In order to reduce the resonance and high frequency noise, a notch filter and a low path filter was designed. Their discrete transfer functions are given as below

$$F_{p1}(z) = \frac{0.9299 - 1.0602z^{-1} + 0.8759z^{-2}}{1 - 1.0602z^{-1} + 0.8053z^{-2}} \quad (6.22)$$

$$F_{p2}(z) = \frac{0.3404 + 0.6809z^{-1} + 0.3404z^{-2}}{1 + 0.1842z^{-1} + 0.1776z^{-2}} \quad (6.23)$$

Fig. 6. 21 Bode diagrams of the notch filter and low-pass filter

Fig. 6. 22 Position close-loop spectrum after employing filters

The bode diagrams of the notch filter and low pass filter are shown in Fig. 6.21. The DC

gains of these two filters are 1.087 and 0.5216 respectively, then the value of k_p was increase to 2.9×10^4 to get enough bandwidth. Similarly, in order to reduce the static error, an integrator with the parameter k_i was parallelized to PD controller when the velocity trend to constant. The parameter k_i was selected to be 0.0015 after tuning. In this case, a full PID controller was implemented in the experiments. After the redesigned controller, the position close-loop spectrum was shown in Fig. 6.22. The resonance and the high frequency noise were reduced greatly.

6.5.5 Experimental results of the FESS drive system

As shown in Fig. 6.14, in the experiment, the position command z^* was set to 0 and velocity command ω^* was assigned a pulse signal with amplitude 314 rad/s at first 0.6 second and 31.4 rad/s at the next 0.6 second. The velocity controller is the series of the PI controller and two notch filters whose discrete transfer functions are $F_{\omega 1}(z)$ and $F_{\omega 2}(z)$. The position controller is composed of the PID controller, the notch filter and low-pass filter with the transfer functions $F_{p1}(z)$ and $F_{p2}(z)$. In the experiment, the current commands i_{d1}^* , i_{q1}^* , i_{d2}^* and i_{q2}^* were constrained not to exceed 2 A. This is because their values are likely to be large after the calculation of controller, and the currents input to the motor will hence be large and may destroy the coils.

Fig. 6.23-6.26 show a set of experimental results of the FESS drive system. As an oscilloscope can only record current and voltage signals and has limited storage, in order to obtain complete information of the velocity, currents and voltages synchronously from the startup state to the steady state operation of the FESS prototype, the capture scope function of the DSP-1103 is employed to capture the long term information.

Fig. 6.23 (a) shows the rotational speed of the rotor, which increased up to the rated value of 1500 rpm within 0.5 second and remained in the steady state, after that the speed

decreased to 150 rpm to discharge. The displacement of the rotor is shown in Fig. 6.23 (b); it fluctuated around 1.1×10^{-5} m. There are large overshoots when the velocity changed from dynamic states to steady states.

Fig. 6.24 (a) and (b) show the dq currents of the upper and lower stators respectively. The q -axis currents of the two stators i_{q1} and i_{q2} also had overshoots when the velocity changed from increasing/decreasing to constant.

(a) Rotational speed of the flywheel/rotor

(b) Position displacement of the flywheel-rotor

Fig. 6. 23 Rotational speed and the axial displacement of the flywheel-rotor

(a) dq currents of upper stator

(b) dq currents of lower stator

Fig. 6. 24 dq currents of the upper and lower stators

(a) Phase a current i_a of the upper stator

(b) Phase b current i_b of the upper stator

(c) Phase c current i_c of the upper stator

(d) Three phase currents of the upper stator at the constant speed 314.2 rad/s

Fig. 6. 25 Three-phase currents of the flywheel system

Fig. 6.25 (a)-(c) shows the three-phase currents i_a , i_b and i_c of the upper stator respectively during the whole charge and discharge period. The frequency of the currents changed based on the velocity. In charge period, the velocity increased and the currents frequency increased accordingly; in discharge period, the velocity and currents frequency both dropped down. Three phase currents when the flywheel-rotor rotated at a the synchronous speed of 314.2 rad/s (1500 rpm) are shown in Fig. 6.25 (d), the currents frequency kept at 50 Hz in this speed.

(a) Switch pulse of one IGBT

(b) Line voltage u_{ab}

Fig. 6. 26 Switch pulse and line voltages

Fig. 6.26 shows the switch pulse of an IGBT and the line voltages of the upper stator. The line voltage waveforms are all PWM pulse with the hysteresis current control.

The experimental results shown in Fig. 6.23-6.26 coincide the simulation results based on PID control method. This proposed conceptual FESS is hence validated to be feasible in experimental implementation.

6.6 Conclusions

This chapter has presented the design, the simulation studies and the experimental implementation of the control system for the proposed FESS based on axial-flux partially-self-bearing PM machine

Model-based predictive control has many advantages over other control methods, particularly in dealing with multivariable constrained control problems. On the other hand, PID control is simple, stable and easily implementable. Hence controllers that were based on MPC and PID have both been designed. The control schemes by applying these two approaches have been formulated accordingly. The simulation results obtained have shown that the proposed FESS can be properly controlled and the control performances are

satisfactory. The simulation results have also shown that the control performance of MPC is better than that of PID, but the PID approach is simpler and easier to be implemented.

A low speed experimental setup has been constructed according to the designed control system. The FESS prototype has a simple structure in which motor and generator are combined together and the rotor functions as flywheel. The axial magnetic bearing and radial rotation functions are realized in one electric machine without using separate axial magnetic bearings. The mechanical bearings are employed in the radial directions instead of magnetic bearings to reduce the complexity of structure and control. The prototype setup is only used to verify the feasibility of the proposed conceptual FESS. Therefore, the stators are made up of solid iron to save cost and the maximum speed is only 1500 rpm; the energy density, power density and efficiency are hence not optimized and were therefore not analyzed. The high speed FESS with a maximum speed of 60,000 rpm described in Chapter 3 was not constructed after considering the cost, manufacturing and security issues. The performances of the high speed FESS has been analyzed in Section 3.6, these performances are attractive compared with the current FESS Systems. The construction and experiments on the high speed FESS are recommended as future work.

The PID control algorithm has been implemented in a DSP-based control platform. The experimental results have been obtained and have shown that the proposed FESS can be well controlled. The experimental results have close correspondences with the simulation results. The proposed FESS is therefore validated to be feasible in experimental implementation.

CHAPTER 7

STABILIZING SYNCHRONIZATION CONTROL OF RADIAL MAGNETIC BEARINGS FOR FESS

7.1 Introduction

As mentioned in Chapter 2, studies on active magnetic bearings and their applications are becoming increasingly important as a result of the non-contact and low loss attributes of the bearings. Especially in high-speed applications, magnetic bearings exhibit promising advantages compared with conventional mechanical bearings. In most of the developed FESS modules, radial magnetic bearings have been implemented to constrain the DOFs in radial directions. Chapter 3 introduced the partially-self-bearing FESS which can be assisted by radial magnetic bearings. Hence it would be timely to carry out some studies on the behaviors and control of these bearings.

The conical and translatory whirl modes of the radial magnetic bearings are described in Section 7.2, together with the synchronization control that overcomes the conical whirl mode. In Section 7.3, the dynamic equations of the radial magnetic bearings are presented, and the state-space forms of the mathematic model for the synchronization control design are shown. The synchronization design and the stability properties are described in detail in Section 7.4. Finally computer simulations on this rotor-flywheel assisted by magnetic bearings to verify the performances of the proposed synchronization control are discussed in Section 7.5.

7.2 Conical and translatory mode & synchronization control

Referring to Fig. 3.7, radial magnetic bearings restrict the translations and rotations along and about radial directions. The spinning rotor-flywheel under magnetic constraint experiences two kinds of whirl modes, namely, conical mode and translatory mode as shown in Fig. 7.1. Although the conical whirl mode has a stabilizing effect on the rotor system [98, 108], it may also twist the rotor, thereby affecting the stability of the rotor if it is not properly controlled [109]. Furthermore, the surfaces of the rotor are not horizontal in conical mode, which will make air-gaps between the stators and rotor of the proposed partially-self-bearing axial-flux PM machine non-uniform. This will affect the control of this machine and generate overall instability of the control system. In translatory whirl mode, the rotor-flywheel is constrained to synchronous motion in the radial directions so as to suppress the gyroscopic rotation; it hence helps the stability of the rotor-flywheel system.

Fig. 7. 1 Translatory and conical whirl modes of the rotor-flywheel system assisted by radial magnetic bearings

The translatory mode can be achieved by the synchronization control that has extensively

been used in the industry [110, 111]. The synchronization control is capable in dealing with nonlinear uncertain models, and effective in disturbance rejection for systems subject to synchronous motion. The prevalent use of the synchronization controller has been restricted to the stable mechanical systems with the advent of optimal control. Therefore the controller is not readily applicable to the magnetic systems which are unstable in nature and highly nonlinear in mechatronic mechanisms [152-154]. An adaptive synchronization controller for magnetically suspended rotor system was developed in [155], in which the gyroscopic effects are eliminated by enabling a translatory mode for the whirling rotor along the bearing axes. However, this design seems rather complicated, because a pole placement gain matrix and cross-coupling controllers for different pairs of axes are required, for the purpose of enhancing the stability of the rotor within the magnetic bearings and suppressing the coupling effects of the gyroscopic rotation of the rotor when it spins at high speeds.

Linear Quadratic Gaussian (LQG) optimal control is an effective strategy to address such a synchronization problem. It has the control acumen to deal with the multivariable constrained control problems and to control a wide variety of processes without having to take much special precautions. In the past three decades, the theory of optimal control has been well developed in almost every aspect such as stability, nonlinearity, and robustness [151, 156]. Applications of the optimal control to synchronize multiple motion axes have been reported in [157], where cross-coupling designs of generalized predictive control were presented by compensating both the tracking error and the synchronous error between the process and the reference model. In this chapter, a new cross-coupling control algorithm for synchronization control of rotor-magnetic bearing systems is developed. The controller is developed through optimization of a new cost function in which both the synchronization errors and their differences are embedded. In this case, the synchronization errors can converge to zero and the potentially detrimental effects of digital control on the gyro-dynamics can be effectively avoided. Furthermore, by solving the corresponding Riccati Difference Equation (RDE), the stability of the control system can be easily investigated.

7.3 Dynamics of the Radial Magnetic Bearing System

In this section, the dynamics of the proposed axial-flux rotor-flywheel with radial magnetic bearings are examined as shown in Fig. 7.2 (a). The rotor-flywheel and radial magnetic bearings are depicted in Fig. 3.7. They provide a clear view of their attributes and principles. To begin with, the magnetic-bearing dynamics will first be described to serve as the background information of the system.

7.3.1 Force equations

Fig. 7.2 Radial magnetic bearing system for proposed FESS

The sketch of the rotor-flywheel is shown in Fig. 7.2 (a), where the two sets of magnetic bearings *a* and *b* are located at the distances l_a and l_b from the mass center of the rotor respectively. The dual axial-flux stators figured in Fig. 3.7 are not shown in this sketch. The electromagnets are capable of inducing magnetic forces in the radial directions when they are charged. Fig. 7.2 (b) shows one of the magnetic bearings, where I_0 is the magnitude of the bias currents of the eight pairs of electromagnets, and the corresponding

magnetic forces are denoted as $f_1, f_2, \dots, f_8$. The control currents of the electromagnets near x -axis and y -axis are indicated by i_x and i_y .

Based on the Maxwell's law, the magnetic forces along the radial directions as a result of the electromagnets can be derived as below [98, 108]

$$f_1 = G \left(\frac{I_0 + i_y}{h_0 - x \cos \alpha - y \sin \alpha} \right)^2, f_2 = G \left(\frac{I_0 + i_x}{h_0 - x \sin \alpha - y \cos \alpha} \right)^2 \quad (7.1)$$

$$f_3 = G \left(\frac{I_0 + i_x}{h_0 - x \sin \alpha + y \cos \alpha} \right)^2, f_4 = G \left(\frac{I_0 - i_y}{h_0 - x \cos \alpha + y \sin \alpha} \right)^2 \quad (7.2)$$

$$f_5 = G \left(\frac{I_0 - i_y}{h_0 + x \cos \alpha + y \sin \alpha} \right)^2, f_6 = G \left(\frac{I_0 - i_x}{h_0 + x \sin \alpha + y \cos \alpha} \right)^2 \quad (7.3)$$

$$f_7 = G \left(\frac{I_0 - i_x}{h_0 + x \sin \alpha - y \cos \alpha} \right)^2, f_8 = G \left(\frac{I_0 + i_y}{h_0 + x \cos \alpha - y \sin \alpha} \right)^2 \quad (7.4)$$

where x and y are the displacements of the rotor along x -axis and y -axis from the bearing center, α is the acute angle between x -axis and f_1 , in the design shown in Fig. 7.2 (b), $\alpha = 67.5^\circ$; h_0 is the nominal air gap at equilibrium, and G is the electromagnet constant given by $G = \frac{1}{4} \mu_0 A_g N^2$ in which μ_0 denotes the air permeability, A_g is the cross sectional area of the air gap, and N is the number of turns of the winding circuit.

f_x and f_y , the total forces along x -axis and y -axis, can be derived as follows

$$f_x = (f_2 + f_3 - f_6 - f_7) \sin \alpha + (f_1 + f_4 - f_5 - f_8) \cos \alpha \quad (7.5)$$

$$f_y = (f_1 + f_8 - f_4 - f_5) \sin \alpha + (f_2 + f_7 - f_3 - f_6) \cos \alpha \quad (7.6)$$

From (7.1) to (7.4), it can be seen that the magnetic forces vary nonlinearly with respect to

the control currents and displacements. However, at equilibrium point $x = 0$ and $y = 0$, f_x and f_y are both 0 when $i_x = 0$ and $i_y = 0$. The control objective is to adjust the rotor center to be the equilibrium point. Therefore, (7.5) and (7.6) can be linearized as

$$f_x = K_p x + K_i i_x \quad (7.7)$$

$$f_y = K_p y + K_i i_y \quad (7.8)$$

where $K_p = \frac{8GI_0^2}{h_0^3}$ and $K_i = \frac{8GI_0 \sin \alpha}{h_0^2}$. So, for the rotor with magnetic bearings a and b , the force equations can be simplified as

$$\begin{bmatrix} f_{xa} \\ f_{xb} \\ f_{ya} \\ f_{yb} \end{bmatrix} \approx K_p \begin{bmatrix} x_a \\ x_b \\ y_a \\ y_b \end{bmatrix} + K_i \begin{bmatrix} i_{xa} \\ i_{xb} \\ i_{ya} \\ i_{yb} \end{bmatrix} \quad (7.9)$$

7.3.2 Position equations

Let x_c and y_c denote the displacements of the mass center of the rotor, and θ and φ be the roll angles of rotation about x -axis and y -axis. The following equations are generated [98, 108],

$$x_c = \frac{l_b}{l_a + l_b} x_a + \frac{l_a}{l_a + l_b} x_b \quad (7.10)$$

$$y_c = \frac{l_b}{l_a + l_b} y_a + \frac{l_a}{l_a + l_b} y_b \quad (7.11)$$

$$\theta = \frac{1}{l_a + l_b} (y_b - y_a) \quad (7.12)$$

$$\varphi = \frac{1}{l_a + l_b} (x_a - x_b) \quad (7.13)$$

It is observed that θ and φ are normally assumed to be small as the air gap is very narrow within the magnetic bearings. It is also assumed that the rotor is rigid with its inertia perfectly balanced about z -axis, thus the flexibility and eccentricity of the rotor are not considered herein. As a result, the effects of variation of inertia tensor due to the roll motions of the rotor are negligible. Hence, the motion of the rotor is described as [98]

$$\begin{cases} m\ddot{x}_c = f_{xa} + f_{xb} \\ m\ddot{y}_c = f_{ya} + f_{yb} \\ J_x\ddot{\theta} = f_{yb}l_b - f_{ya}l_a - J_z\omega\dot{\varphi} \\ J_y\ddot{\varphi} = f_{xa}l_a - f_{xb}l_b + J_z\omega\dot{\theta} \end{cases} \quad (7.14)$$

where m is the rotor mass, J_x , J_y and J_z are the moments of inertia of the rotor about x -axis, y -axis and z -axis respectively, ω is the spinning rate of the rotor about z -axis.

When the state variable $\chi = [x_a, \dot{x}_a, x_b, \dot{x}_b, y_a, \dot{y}_a, y_b, \dot{y}_b]^T$, the forcing vector $u = [i_{xa}, i_{xb}, i_{ya}, i_{yb}]^T$ and the output vector $\zeta = [x_a, x_b, y_a, y_b]^T$ are given, a linearized version of model (7.14) is obtained as

$$\begin{cases} \dot{\chi} = \mathbf{A}_c\chi + \mathbf{B}_cu \\ \zeta = \mathbf{C}_c\chi \end{cases} \quad (7.15)$$

where

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \left(\frac{1}{m} + \frac{l_a^2}{J_y}\right)K_p & 0 & \left(\frac{1}{m} - \frac{l_al_b}{J_y}\right)K_p & 0 & 0 & -\frac{J_z\omega l_a}{J_y(l_a+l_b)} & 0 & \frac{J_z\omega l_a}{J_y(l_a+l_b)} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \left(\frac{1}{m} - \frac{l_al_b}{J_y}\right)K_p & 0 & \left(\frac{1}{m} + \frac{l_b^2}{J_y}\right)K_p & 0 & 0 & \frac{J_z\omega l_b}{J_y(l_a+l_b)} & 0 & -\frac{J_z\omega l_b}{J_y(l_a+l_b)} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{J_z\omega l_a}{J_x(l_a+l_b)} & 0 & -\frac{J_z\omega l_a}{J_x(l_a+l_b)}\left(\frac{1}{m} + \frac{l_a^2}{J_x}\right)K_p & 0 & \left(\frac{1}{m} - \frac{l_al_b}{J_x}\right)K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -\frac{J_z\omega l_b}{J_x(l_a+l_b)} & 0 & \frac{J_z\omega l_b}{J_x(l_a+l_b)}\left(\frac{1}{m} - \frac{l_al_b}{J_x}\right)K_p & 0 & \left(\frac{1}{m} + \frac{l_b^2}{J_x}\right)K_p & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \left(\frac{1}{m} + \frac{l_a^2}{J_y}\right)K_i & \left(\frac{1}{m} - \frac{l_a l_b}{J_y}\right)K_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \left(\frac{1}{m} - \frac{l_a l_b}{J_y}\right)K_i & \left(\frac{1}{m} + \frac{l_b^2}{J_y}\right)K_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{m} + \frac{l_a^2}{J_x}\right)K_i & \left(\frac{1}{m} - \frac{l_a l_b}{J_x}\right)K_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{1}{m} - \frac{l_a l_b}{J_x}\right)K_i & \left(\frac{1}{m} + \frac{l_b^2}{J_x}\right)K_i \end{bmatrix} \quad \mathbf{C}_c = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

Model (7.15) is capable of simulating the rotor dynamics controlled by the active radial magnetic bearings. During a control process, the position and rotation velocity of the shaft are constantly monitored by contactless sensors, and transferred to a micro-controller to process. Therefore a control current fed to the coils of the electromagnets, which attract or repel the shaft, is amplified and then fed back. The overall control system produces active forces that can be changed over a wide range in response to the signals from the sensors.

Due to the nature of machining of the rotor and its components, unbalance always exists, though in small quantities. The amount of unbalance can be reduced to a tolerable level by certain unbalance compensation techniques, such as mechanical compensation and notch filter. Nevertheless, the residual unbalance acting as a forcing function on the rotor and tending to “pull” the rotor away from the undeformed rotor centerline during operation is always present and measurable in the rotor-flywheel system. In this situation, a feedback and/or feedforward compensation could effectively cancel the residual unbalance. Therefore, the following controller is developed with an assumption of perfect compensation of the unbalance.

7.4 Synchronization Design and Stability Properties

7.4.1 Synchronization design

Equation (7.15) can be transformed into the corresponding discrete-time model as described by

$$\begin{cases} \chi(k+1) = \mathbf{A}\chi(k) + \mathbf{B}u(k) \\ \zeta(k) = \mathbf{C}\chi(k) \end{cases} \quad (7.16)$$

where k denotes the discrete-time.

In order to synchronize the motions of the rotor-flywheel in the radial directions, the synchronization errors are defined as

$$\delta_{xx} = x_a - x_b, \delta_{yy} = y_a - y_b \quad (7.17)$$

Note that even if $\delta_{..}(k) = 0$ may be achieved at an instant time, the overall system may not be synchronized perfectly due to $\Delta\delta_{..}(k+1) \neq 0$, where $\Delta\delta_{..}(k+1) = \delta_{..}(k+1) - \delta_{..}(k)$. For this reason, the difference of the consecutive synchronization errors should also be considered. That is

$$\varepsilon_{xx}(k) = \delta_{xx}(k) - \delta_{xx}(k-1), \quad \varepsilon_{yy}(k) = \delta_{yy}(k) - \delta_{yy}(k-1) \quad (7.18)$$

which can be grouped as

$$\delta(k) = \mathbf{L}\mathbf{C}\chi(k) \quad (7.19)$$

$$\varepsilon(k) = \delta(k) - \delta(k-1) \quad (7.20)$$

where

$$\delta = \begin{bmatrix} \delta_{xx} \\ \delta_{yy} \end{bmatrix}, \quad \varepsilon = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \end{bmatrix}, \quad \mathbf{L} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}.$$

Introducing the following cost function [107],

$$\begin{aligned} V(k) = & \sum_{i=1}^{H_p} \zeta^T(k+i) \zeta(k+i) + \sum_{i=1}^{H_c} \lambda_i u^T(k+i-1) u(k+i-1) \\ & + \nu \sum_{i=1}^{H_p} \delta^T(k+i) \delta(k+i) + \eta \sum_{i=1}^{H_p} \varepsilon^T(k+i) \varepsilon(k+i) \end{aligned} \quad (7.21)$$

where $\zeta(k+i)$ is the future output vector, $\mathbf{u}(k+i-1)$ is the future control input vector, $\delta(k+i)$ and $\varepsilon(k+i)$ are the future synchronization errors, H_p is the prediction horizon, H_c is the control horizon, λ_i is the positive weighting factor used to adjust the control action, ν and η are the non-negative weighting factors for the synchronization errors.

Then, the cost function (7.21) can be rewritten as

$$V(k) = \sum_{j=0}^{H_p-1} \ell(\chi(k+H_p-1+j), \mathbf{u}(k+H_p-1+j)) + \|\chi(k+H_p)\|_{\mathbf{P}_0}^2 \quad (7.22)$$

where

$$\begin{aligned} \ell(\chi(k+H_p-1+j), \mathbf{u}(k+H_p-1+j)) &= \|\chi(k+H_p-1+j)\|_{\mathbf{Q}_j}^2 \\ &+ \mathbf{u}(k+H_p-1+j)^T \mathbf{M}_j \chi(k+H_p-1+j) + \|\mathbf{u}(k+H_p-1+j)\|_{\mathbf{R}_j}^2 \end{aligned} \quad (7.23)$$

$$\mathbf{P}_0 = \mathbf{C}^T (\mathbf{I} + \nu \mathbf{L}^T \mathbf{L}) \mathbf{C} \quad (7.24)$$

$\mathbf{I}$ denotes the unit matrix

$$\mathbf{Q}_j = \begin{cases} \mathbf{C}^T (\mathbf{I} + \nu \mathbf{L}^T \mathbf{L}) \mathbf{C} + \eta (\mathbf{A} - \mathbf{I})^T \mathbf{C}^T \mathbf{L}^T \mathbf{L} \mathbf{C} (\mathbf{A} - \mathbf{I}), & \text{if } j = 0, 1, \dots, H_p - 2 \\ \mathbf{0}, & \text{if } j = H_p - 1 \end{cases} \quad (7.25)$$

$$\mathbf{R}_j = \begin{cases} \infty \mathbf{I}, & \text{if } j = 0, 1, \dots, H_p - H_c - 1 \\ \lambda_j \mathbf{I} + \eta \mathbf{B}^T \mathbf{C}^T \mathbf{L}^T \mathbf{L} \mathbf{C} \mathbf{B}, & \text{if } j = H_p - H_c, \dots, H_p - 2 \\ \lambda_j \mathbf{I}, & \text{if } j = H_p - 1 \end{cases} \quad (7.26)$$

$$\mathbf{M}_j = \begin{cases} \eta (\mathbf{A} - \mathbf{I})^T \mathbf{C}^T \mathbf{L}^T \mathbf{L} \mathbf{C} \mathbf{B}, & \text{if } j = 0, 1, \dots, H_p - 2 \\ \mathbf{0} & \text{if } j = H_p - 1 \end{cases}. \quad (7.27)$$

The control constraints $\mathbf{u}(k+H_p-1-j)$, for $j = 0, 1, \dots, H_p - H_c - 1$, can be realized by defining the weighting factors λ_j as infinite beyond the control horizon. Hence, based on

the receding horizon strategy, minimization of the performance index (7.22) results in the following optimal control input vector,

$$u(k) = -K_{H_p-1} \chi(k) \quad (7.28)$$

where

$$K_{H_p-1} = \left(B^T P_{H_p-1} B + R_{H_p-1} \right)^{-1} \left(B^T P_{H_p-1} A + M_{H_p-1}^T \right) \quad (7.29)$$

Here, P_{H_p-1} is obtained by iterating the following Riccati Difference Equation (RDE) [158]

$$P_{j+1} = A^T P_j A - \left(B^T P_j A + M_j^T \right)^T \left(B^T P_j B + R_j \right)^{-1} \left(B^T P_j A + M_j^T \right) + Q_j \quad (7.30)$$

from the initial condition P_0 . In the next section, stability properties of the proposed design are discussed.

7.4.2 Stability properties

In order to investigate the stability properties, some propositions are proposed and proved as shown below.

Proposition 1 [107, 159]:

Considering P_j , the solution of the RDE (7.30), with P_0 , Q_j , R_j and M_j given in (7.24)-(7.27). Then, for all $j \geq 0$, P_j is a symmetric nonnegative definite matrix.

In particular, when $R_j = \infty$, Q_j becomes a constant matrix, denoted as Q , and the following equation is obtained,

$$P_{j+1} = A^T P_j A + Q \quad (7.31)$$

and

$$\mathbf{P}_j = (\mathbf{A}^j)^T \mathbf{P}_0 \mathbf{A}^j + \sum_{i=0}^{j-1} (\mathbf{A}^i)^T \mathbf{Q} \mathbf{A}^i \quad (7.32)$$

It can be derived that $\mathbf{P}_j$ is monotonically nondecreasing. But for stable $\mathbf{A}$, the solution $\mathbf{P}_j$ converges as $j \rightarrow \infty$, which indicates that with infinite $\mathbf{R}_j$ and finite $\mathbf{Q}$, the control inputs will always be zeroes, also, questions of stabilizability are not sensible. However, this may yield a useful procedure to develop a monotonically nonincreasing $\mathbf{P}_j$ with finite $\mathbf{R}_j$, which will be described in Proposition 2.

Proposition 2 [107, 160]:

If $\mathbf{P}_j$ converges beyond the control horizon, then for all $j \geq H_p - H_c$, $\mathbf{P}_j$ is always monotonically nonincreasing, i.e.

$$\mathbf{P}_{j+1} \leq \mathbf{P}_j, \text{ for } j \geq H_p - H_c. \quad (7.33)$$

Referring to the receding horizon strategy, only the control action at the $(H_p - 1)$ th step is applied. Therefore, substitute $j = H_p - 1$ in (7.30) will result in

$$\mathbf{P}_{H_p-1} = \mathbf{A}^T \mathbf{P}_{H_p-1} \mathbf{A} - \mathbf{A}^T \mathbf{P}_{H_p-1} \mathbf{B} (\mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{B} + \lambda_0 \mathbf{I})^{-1} \mathbf{B}^T \mathbf{P}_{H_p-1} \mathbf{A} + \bar{\mathbf{Q}}_{H_p-1} \quad (7.34)$$

where

$$\bar{\mathbf{Q}}_{H_p-1} = \mathbf{P}_{H_p-1} - \mathbf{P}_{H_p} \quad (7.35)$$

The stability of the control scheme will be given in Proposition 3.

Proposition 3 [160]:

Considering equation (7.34) with the weighting matrix $\bar{\mathbf{Q}}_{H_p-1}$ given in (7.35).

If

- $[\mathbf{A}, \mathbf{B}]$ is stabilizable,
- $[\mathbf{A}, \bar{\mathbf{Q}}_{H_p-1}^{1/2}]$ is detectable,
- $\mathbf{P}_{H_p-1}$ is noincreasing, $\lambda_0 > 0$,

then the proposed control scheme is stabilizing, i.e. the closed-loop transition matrix

$$\bar{\mathbf{A}}_{H_p-1} = \mathbf{A} - \mathbf{B}(\mathbf{B}^\top \mathbf{P}_{H_p-1} \mathbf{B} + \lambda_0 \mathbf{I})^{-1} \mathbf{B}^\top \mathbf{P}_{H_p-1} \mathbf{A} \quad (7.36)$$

has all its eigenvalues strictly within the unit circle.

Obviously, this proposition indicates that the prediction horizon H_p is a key parameter that guarantees the closed loop stability, increasing H_p is favorable to satisfy the conditions in Proposition 3. In other words, a H_p that is large enough would guarantee system stability. In fact, this is the main motivation for the replacement of the one-step-ahead control to the long-range predictive control.

7.5 Simulation Study

The high-speed FESS designed in Chapter 3 is implemented in this simulation study. The rotor-flywheel is actively constrained by the radial magnetic bearings described in Section 7.3. The parameters used in the synchronization control of the rotor-flywheel are summarized in Table 7.1.

As the eigenvalues of $\mathbf{A}_c$ are: $\pm 2.0353i$, $\pm 10.4i$ and ± 149.3 respectively, the open-loop system is unstable. The parameters H_p , H_c , λ , ν and η are to be selected to make the

close-loop system stable.

The prediction and control horizons are closely related to the stability of the closed-loop system [153, 155]. A larger H_p results in a smaller gain, the stability robustness of the control can hence be improved by choosing a large H_p . In practice, if the closed-loop system is stable, the prediction horizon can be chosen equal to the integer of t_{st}/T_s , where t_{st} is the settling time of the step response of the closed-loop system for $H_p \rightarrow \infty$. For the control horizon H_c , in a simple system which open-loop is stable but has possible dead-time and non-minimum phase, the value 1 gives generally acceptable control. Increasing H_c makes the control and the output response more active until a stage is reached, and further increase in H_c makes little difference. For complex system, an increase value of H_c is more appropriate as good control can be achieved if H_c is at least equal to the number of unstable or badly damped poles. $H_c = 3$ is thus be selected according to the eigenvalues of $\mathbf{A}_c$. The use of $H_c < H_p$ can significantly reduce the computational burden. In practice, H_p and H_c cannot be too large, which will increase the calculation burdens. The criterion is selecting the relatively smaller H_p and H_c but making sure the closed-loop system is stable.

The weighting factors ν and η are used to compensate the synchronization error. Increasing ν can improve the synchronization performance, but after a certain value, the synchronization performance will remain unchanged [157]. With an adequate ν and increased η , the synchronization error can be effectively reduced [159]. λ is the control input weighting factor. It depends on the system and must usually be determined by simulations in combination with trial-and-error method.

If $H_p = 6, H_c = 3, \lambda = 0.01, \nu = 10, \eta = 5$ and the sampling period $T_s = 0.008$ second are

selected to solve the RDE (7.30) with $\mathbf{P}_0 = \mathbf{C}^T (\mathbf{I} + \nu \mathbf{L}^T \mathbf{L}) \mathbf{C}$, all of the eigenvalues of the closed-loop transition matrix $\overline{\mathbf{A}}_{H_p-1}$ will then be limited within the unit circle, which are: $-0.7824 \pm 0.5554i$, -0.3791 , 0.4812 , 0.3784 , $0.1257 \pm 0.1954i$ and -0.0266 respectively. In other words, $H_p = 6$ and $H_c = 3$ are large enough to ensure the nominal stability of the control scheme for this flywheel system.

Table 7. 1 Parameters of the radial magnetic bearing system

Disk:		
Mass	$m = 7.28$	kg
Radial Moment-of-Inertia	$J_x = J_y = 0.16272$	kg-m2
Axial Moment-of-Inertia	$J_z = 0.0659$	kg-m2
Spinning Speed	$\omega = 30,000$	rpm
Location to Bearings a	$l_a = 0.1$	m
Location to Bearings b	$l_b = 0.1$	m
Angle between axis x and f_1	$\alpha = 67.5$	deg
First flexural natural frequency	630	Hz
Magnetic Bearings:		
Magnetic Constant	$G = 2.879 \times 10^{-7}$	N-m2/Amp2
Air Gap	$h_0 = 0.00025$	m
Nominal Current	$I_0 = 1.33$	A

Fig. 7.3 shows the detailed evolutions of $\mathbf{P}_j$ and the control gain $\mathbf{K}_j$ with respect to the iteration step, where (1,1) represents the indexes of the corresponding elements. It is seen that the convergence of $\mathbf{P}_j$ to the steady-state value has occurred and therefore the stability of the control system is guaranteed.

Fig. 7.3 Evolutions of P_j and K_j

Fig. 7.4 The close-loop structure of the control system with disturbance injection

In this simulation, some force disturbances affecting the magnetic bearings are considered which act on axes x_a , x_b , y_a and y_b . The close-loop structure of the control system with disturbance injections is shown in Fig. 7.4. The equation (7.16) can hence be rewritten as

$$\begin{cases} \chi(k+1) = A\chi(k) + Bu(k) + B_d d_x(k) \\ \zeta(k) = C\chi(k) \end{cases} \quad (7.37)$$

where $d_x(k)$ is the system disturbance, B_d is the disturbance matrix,

$$d_x(k) = [f_{xad}, f_{xbd}, f_{yad}, f_{ybd}]^T, \quad B_d(k) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1/m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1/m & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/m & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/m \end{bmatrix}$$

Fig. 7.5 Displacements of the rotor in the radial directions

Fig. 7. 6 Control currents used to reject the outside disturbances

Fig. 7. 7 Mass center of the rotor

In order to evaluate the stability of the control system, the force disturbances $f_{xad}, f_{xbd}, f_{yad}, f_{ybd}$ with the values 1N, -1N, 1N and -1N are injected on axes x_a, x_b, y_a and y_b , at time = 0.2, 0.6, 1.0, 1.4 seconds, with the duration of 0.2 seconds each and every time.

Fig. 7.5- Fig. 7.7 show the numerical results of the control algorithm of the two cases: with $\nu = 0, \eta = 0$; and $\nu = 10, \eta = 5$. Clearly, when no cross-coupling control action due to $\nu = 0, \eta = 0$ is present, evident synchronization errors and a conical whirl mode during the transient responses are produced.

However, when $\nu = 10$ and $\eta = 5$ are selected, the synchronization performance can be improved significantly, especially in terms of the roll angles as shown in Fig. 7.7. Hence, with control parameters selected adequately, the improved synchronization performance as well as the guaranteed stability of the flywheel system can be achieved. As a result, the whirling rotor would be more easily stabilized near the nominal position when disturbances are present.

7.6 Conclusions

In this chapter, the problem of radial-magnetic bearing system for high-speed FESS has been formulated, and a stabilizing synchronization design has been presented by incorporating cross-coupling technology into the optimal control architecture. The basic idea of the control strategy is to minimize a new quadratic performance index in which the synchronization errors are embedded, so that the potentially detrimental effects of digital control on the gyro-dynamics can be effectively avoided. On the other hand, because optimal control that uses receding horizon idea is a feedback control, there is a risk that the resulting closed-loop system might be unstable. For this reason, stability of the control system based on the solution of the Riccati Difference Equation has also been investigated with various given properties. The illustrative example reveals that with appropriate

control parameters, the resulting control system may be very effective in recovering the unstable rotor and suppressing the coupling effects of the gyroscopic rotation at high spinning speeds in the presence of disturbances.

The prototype of these radial magnetic bearings was not constructed for the reason of complexity of structure after employing the axial-flux partially-self-bearing PM motor. The control scheme was hence not implemented in the experiments. From the analyses shown in Section 3.6, the friction loss of the high speed FESS occupies a large portion in the total loss. The employment of the radial magnetic bearings can greatly reduce the loss. Therefore, the study of the synchronization control of the radial magnetic bearings for FESS is still meaningful. The implementation of the control strategy and scheme is recommended as future work.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 Conclusions

This thesis has reported a research project conducted in the area of electrical energy storage system. The research includes a review of energy storage systems, particularly the flywheel energy storage systems; the proposal and design of a conceptual FESS based on axial-flux partially-self-bearing PM machine; the modeling of the axial-flux PM machine; the FEM analysis and model verification of this machine; the control design and experimental implementation for the proposed FESS; and the stabilizing synchronization control of the radial magnetic bearings for the FESS.

Many novel concepts are involved in the design of the compact and efficient FESS. The axial-flux PM machine is adopted with the advantages of high torque density, compact volume and a geometry which is compliant for the flywheel system. The motor and generator are combined into one single actuator in the FESS because the functions of energy charging and discharging do not occur at the same time; the rotor functions as flywheel and is sandwiched between two pancake-type stators to minimize the space. The PM machine provides dual functions of rotating and levitating the flywheel/rotor about and along the axial direction and hence is termed as a partially-self-bearing PM machine. The mechanical or magnetic bearings can be implemented to constrain the other 4 DOFs in radial directions.

The axial-flux PM machine have been designed by transforming the dimensions to those of an equivalent radial-flux machine, and then following the design procedures of the

radial-flux machines. Magnetic analysis has been employed to assist the motor design. The design process has been described in detail. The design results of a low speed prototype machine and a conceptual high speed machine have been presented. The performance analysis results show the high speed machine can rotate at a velocity of 60,000 rpm with an efficiency of 96%, an energy storage of 360Whr, a peak power of 4 kW, a power density of 400 W/kg and an energy density of 32 Whr/kg. All these data show that the proposed FESS has satisfactory performance in principle.

The mathematical model of the proposed axial-flux partially-self-bearing PM machine has been developed from the basic principle of the AC electrical machine. The distribution of the windings and the waveform of the MMFs have all been assumed to be sinusoidal. After the calculation of the flux linkages, the equations of the inductances which are variable according to the change of the air-gap length have been derived. Co-energy method has been adopted to derive the equations of the torque and force acting on the rotor about and along the axial axis. The velocity and displacement equations have been obtained accordingly. The developed model is a non-linear multi-variable system; linearization has thus been carried out to make the system simple and amenable to be controlled.

Three dimensional FEM has been used in the electromagnetic analyses for the designed axial-flux partially-self-bearing PM machine. The FEM analysis results have verified that the low-speed prototype machine and the high-speed machine both have been properly designed and the design requirements have been satisfied. The results have also validated the accuracy of the derived mathematical model of the axial-flux PM machine.

The MPC method has been implemented in the control system design with the advantages of pre-calculating the system behavior, providing satisfactory tracking response and being robust against model-mismatch and unexpected dynamics. Conventional PID control has also been employed to provide a comparison with the MPC approach. The simulation results have shown that the system can be well controlled by both the PID and the MPC approaches. However, it is observed that the control performance based on MPC method is superior to that of PID, but the PID approach is simpler and easier to be employed in the

control system. The experimental prototype has been constructed according to the designed FESS and its control system. Experimental verification of the proposed FESS has been conducted based on the PID control strategy for the purpose of simple implementation. The experimental results reveal that the proposed FESS can be well operated. A good match has been found between the experimental results and the simulation predictions.

The dynamics of the radial magnetic bearings for high-speed FESS have been formulated. The synchronization design has been presented together with the stability properties of the proposed design by incorporating cross-coupling technology into the optimal control architecture. Computer simulations on the radial magnetic bearing system show that with appropriate control parameters, the resulting control system can be effective to recover the instability of the flywheel-rotor and to reduce the coupling effects of the gyroscopic rotation at high spinning speeds even with the presence of disturbances.

8.2 Recommendations

Based on the contributions of the completed project, further researches are recommended in following areas.

1. Experimental verification based on MPC control strategy

As a result of the consideration of convenience and simplicity, the MPC approach has not been implemented experimentally. Further experiments are suggested to be implemented by applying this control strategy. One problem need to be considered is that there will be a number of matrix computations when employing the MPC method; the calculation burden will hence be heavy. Simplifying the algorithm to reduce the computational complexity is the main challenge prior to the application of this method.

2. Experimental implementation of the high speed FESS

A conceptual high speed FESS that spins at 60,000 rpm has been designed and analyzed in this research. The analysis results show that the system has satisfactory performances in terms of power density, energy density and overall efficiency. It is thus suggested to construct such a high speed FESS prototype based on the design. Taking safety into consideration, a solid housing is required to dissipate radial kinetic energy from the rotor debris in the event of rotor malfunction. Vacuum environment is needed to reduce the windage loss and hence increase the system efficiency. Other security options such as on-line health monitoring system that supervises rotor imbalance or rim geometry as detected by proximity sensors can also be considered. Further study in this aspect is necessary for a full functioning high speed FESS.

3. The implementation of slotless PM motor concept in the proposed FESS

Slotless motor has a distinct structure compared to slotted motor, in which the windings are usually printed circuit coils that are placed in the air-gap [161, 162]. The special construction of slotless motor eliminates the magnetic saturation and iron loss in the stator teeth; higher power density and efficiency can thus be expected [163]. Moreover, slotless motor has uniform air-gap and thus uniform reluctance round the periphery of the air-gap, which enables it to produce extremely low cogging torques. The disadvantages of the slotless motor are that the air-gap needs to be large enough to accommodate the windings and the arrangement of the windings will be more difficult compared with the slotted motor [162, 164].

4. Sensorless control for the proposed FESS

In this project, the PM machine is operated by using the vector control approach which requires the rotor position information be provided by using either an encoder or a resolver. The sensorless approach can be employed with the advantages of reducing the complexity of the system through eliminating the encoder/resolver and its associated failure modes, volume and mass.

Back Electromotive-force (EMF) technique and self-sensing method are the widely used approaches in sensorless control. The back EMF technique follows the principle that the spinning rotor magnets will generate a voltage (the back EMF) which can be integrated to estimate the stator flux. The rotor flux position can be estimated from the stator flux vector and an estimate of the torque angle. However, since back EMF is zero at standstill and it is proportional to speed, the measured terminal voltage has large signal-to-noise ratio that it cannot detect zero crossing at low speeds. Therefore, this technique is normally used in the situations when the speed is above 1000 rpm [165-167]. The self-sensing method relies on a magnetic saliency present in the machine to provide position-dependent information which can be extracted with the appropriate signal processing. Unlike the back EMF methods, this technique does not depend on fundamental excitation of the machine and position estimation is thus possible at low or zero speeds [51, 52, 168].

In the FESS, the self-sensing method can be used to determine the rotor position at zero and low speeds. At higher speeds, back EMF technique can be applied to estimate the position of the rotor. It is recommended to carry out further study on these sensorless control techniques for use in FESS.

Author's Publications

1. **C. Zhang**, K.J. Tseng, "A Novel Flywheel Energy Storage System with Partially-Self-Bearing Flywheel-Rotor", *IEEE Transactions on Energy Conversion*, (In print).
2. Y. Xiao, K.Y. Zhu, **C. Zhang**, K.J. Tseng, "Stabilizing Synchronization control of magnetic bearing-based flywheel energy storage systems", *IME: Part I, Journal of Systems and Control Engineering*, vol. 219 (7), pp. 499-510, 2005.
3. **C. Zhang**, K.J. Tseng, "Design and Analysis of a Novel Flywheel Energy Storage System with Partially-Self-Bearing Flywheel-Rotor", *IEEE Transactions on Magnetics*, (In Revision).
4. **C. Zhang**, P. Wu, K.J. Tseng, "FEM Analyses for the Design and Modeling of a Novel Flywheel Energy Storage System Assisted by Integrated Magnetic Bearing", *The IEEE International Electric Machines and Drives Conference*, San Antonio, TX, May. 2005.
5. P. Wu, **C. Zhang**, K.J. Tseng, "New Concept of Multi-Phase Inductively-Driven Shaft-less Centrifugal Pumps", *The IEEE International Electric Machines and Drives Conference*, San Antonio, TX, May. 2005.
6. Y. Xiao, K.Y. Zhu, **C. Zhang**, K.J. Tseng, "Stabilizing Synchronization Design of the Flywheel Energy Storage Systems", *Proceedings of the 8th International Conference on Control, Automation, Robotics and Vision*, Kunming, China. Dec. 2004.
7. **C. Zhang**, K.J. Tseng, "Design and FEM Analysis of a Flywheel Energy Storage System Assisted by Integrated Magnetic Bearings", *Proceedings of the 30th Annual Conference of IEEE Industrial Electronics*, Busan, Korea, Nov. 2004.

8. **C. Zhang**, K.J. Tseng and J. Zhou, "A compact and efficient flywheel energy storage system with integrated magnetic bearings", *Proceedings of the 4th International Power Electronics & Motion Control Conference*, Xi'an China, Aug.2004.
9. **C. Zhang**, K.J. Tseng, Y. Xiao and K.V. Zhu, "Model-Based Predictive Control for a Compact and Efficient Flywheel Energy Storage System with Magnetically Assisted Bearings", *Proceedings of the 35th IEEE Power Electronics Specialists Conference*, Aachen, Germany, Jun. 2004.
10. **C. Zhang**, K.J. Tseng and J. Zhou, "Modeling and simulation of a disk-type permanent magnet bearingless motor", *Proceedings of the 5th International Conference on Power Electronics and Drive Systems*, Singapore, Nov. 2003.

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APPENDICES

Appendix A: Efficiency Calculation of the High Speed Motor

A.1. Core Loss of the Stators

Fig. A. 1 Stator and transverse section of the slot

The stator overview picture and the transverse section of the slot are shown in Fig. A.1. In Fig. A.1 (b), ABCDEF is the transverse section of the slot, the dimensions of the lines in the figure can be calculated as below

$$OA = D_i / 2 = 18 \quad (\text{A. 1})$$

$$AG = 1/2 AE = 1/2 b_s = 10 \quad (\text{A. 2})$$

$$\alpha + \beta = \arcsin(AG / OA) = 33.75^\circ \quad (\text{A. 3})$$

$$OB = D_o / 2 = 31 \quad (\text{A. 4})$$

$$BH = AG = 10 \quad (\text{A. 5})$$

$$\beta = \arcsin(BH / OB) = 18.82^\circ \quad (\text{A. 6})$$

$$AB = \sqrt{OA^2 + OB^2 - 2OAOB \cos \alpha} = 14.38 \tag{A. 7}$$

$$FC = OC - OF = (D_o - D_i) / 2 = 13 \tag{A. 8}$$

The average length of the slot section

$$\overline{l_{slc}} = (AB + FC) / 2 = 13.69 \tag{A. 9}$$

Redraw Fig. 3. 14 (a) as Fig. A. 2.

Fig. A. 2 The section plan of the slot

The dimensions shown in Fig. A. 2 were listed in Table A. 1

Table A. 1 Dimensions of the slot

b_o	4 mm
l_{sl}	2 mm
l_{sw}	4 mm
l_{sc}	28 mm
b_s	20 mm
d_{yk}	18 mm

From Fig .A. 2 and Table A. 1, the area of the section plan of the slot can be calculated as

$$A_{sl} = b_o \times l_{sl} + 1/2 \times (b_o + b_s) \times l_{sw} + l_{sc} \times b_s = 616 \text{ mm}^2 \tag{A. 10}$$

The volume of the slot is

$$V_{sl} = A_{sl} \times \overline{l_{slc}} = 8433.04 \text{ mm}^3 \quad (\text{A. 11})$$

The total height of the slot is calculated as

$$l_s = l_{sl} + l_{sw} + l_{sc} = 34 \quad (\text{A. 12})$$

The volume of the stator teeth is written as

$$V_T = \pi(D_o^2 - D_i^2) \times l_s / 4 - 3V_{sl} = 42707 \text{ mm}^3 \quad (\text{A. 13})$$

The volume of the yoke is

$$V_Y = \pi(D_o^2 - D_i^2) \times d_{yk} / 4 = 36000 \text{ mm}^3 \quad (\text{A. 14})$$

Rewriting (3.73)-(3.76) to be (A. 15)-(A. 18)

$$\overline{P}_{eddy} = \delta \left(V_T \overline{P}_{eddy(T)} + V_Y \overline{P}_{eddy(Y)} \right) = \frac{Pk_e \omega_m^2 \delta}{\pi^3} \left(V_T \frac{r_o + r_i}{2t_{av}} \hat{B}_T^2 + V_Y \frac{2}{\alpha_m} \hat{B}_Y^2 \right) \quad (\text{A. 15})$$

$$\overline{P}_{hysteresis} = \frac{Pk_h \omega_m \delta}{2\pi} (V_T \hat{B}_T^\beta + V_Y \hat{B}_Y^\beta) \quad (\text{A. 16})$$

$$\hat{B}_T = \frac{B_g (t_{av} + b_s)}{t_{av}} \quad (\text{A. 17})$$

$$\hat{B}_Y = \frac{(r_o + r_i) \alpha_m B_g}{4d_{yk}} \quad (\text{A. 18})$$

The eddy current loss density and hysteresis current loss density can be calculated with equations (A. 15)-(A. 18). The values of the parameters used in these equations are listed in Table A. 2.

Table A. 2 The parameters used in core loss calculation

Hysteresis loss coefficient	k_h	0.0234
Eddy-current loss coefficient	k_e	7.08×10^{-5}
Constant of core material	β	1.793
Core material mass density	δ	$7.7 \times 10^3 \text{ kg/m}^3$
Mechanical angular velocity	ω_m	1000 rad/s
Magnet pole arc angle	α_m	$2/3\pi$
Pole pair number	P	1

The average tooth width t_{av} can be calculated as

$$t_{av} = \frac{\pi(D_o + D_i)}{2Z} - b_s = 31.29 \tag{A. 19}$$

Recall the Equation (3.8), the flux density of the air-gap is

$$B_g = \frac{\Phi_g}{A_g} = \frac{l_m A_m}{l_m A_g + \mu_r (1 + p_{r1}) l_g ' A_m} B_r \tag{A. 20}$$

And as shown in Fig. 3.16, $A_m = 2/3 A_g$

The parameters in the Equation (3. 8) are shown in Table A. 3

Table A. 3 Parameters used in air-gap flux density calculation

Permanent magnets length	l_m	3
Equivalent air-gap length	$l_g '$	1.05
Relative permeability	μ_r	1.15
Leakage permeance	p_{r1}	0.1
Remanent flux density	B_r	1.23

Then the air-gap flux density can be calculated as

$$B_g = \frac{3 \times 2/3 A_g}{3 A_g + 1.15 \times 1.1 \times 1.05 \times 2/3 A_g} \times 1.23 = 0.65 \tag{A. 21}$$

The flux density in the stator teeth can be written as

$$\hat{B}_T = \frac{B_g (t_{av} + b_s)}{t_{av}} = 1.07 \quad (\text{A. 22})$$

The flux density in the stator yoke is calculated as

$$\hat{B}_Y = \frac{(r_o + r_i) \alpha_m B_g}{4d_{yk}} = 0.93 \quad (\text{A. 23})$$

Substituting the parameters shown in Table A. 3 and the other values calculated, the eddy current loss can be calculated as

$$\bar{P}_{eddy} = \frac{Pk_e \omega_m^2 \delta}{\pi^3} (V_T \frac{r_o + r_i}{2t_{av}} \hat{B}_T^2 + V_Y \frac{2}{\alpha_m} \hat{B}_Y^2) = 46.46 \quad (\text{A. 24})$$

The hysteresis current loss is calculated as

$$\bar{P}_{hysteresis} = \frac{Pk_h \omega_m \delta}{2\pi} (V_T \hat{B}_T^\beta + V_Y \hat{B}_Y^\beta) = 14.48 \quad (\text{A. 25})$$

The total core loss density is

$$\bar{P}_{core} = \bar{P}_{hysteresis} + \bar{P}_{eddy} = 60.94 \text{ W/kg} \quad (\text{A. 26})$$

The mass of the stator can be calculated as

$$m = \delta(V_T + V_Y) = 0.61 \text{ kg} \quad (\text{A. 27})$$

The core loss of the two stators is hence

$$P_{core} = 2 \times \bar{P}_{core} \times m = 74.35 \text{ W} \quad (\text{A. 28})$$

A. 2. Copper Loss

Recall the equations of (3.44)-(3.46) with the parameters used in the equations shown in Table A. 4. The current and dimensions can be calculated

$$I_s = \frac{P_N}{mV_{N\Phi}\eta_N \cos \Phi_N} = 7.29 \quad (\text{A. 29})$$

$$A_c = \frac{I_s}{J} = 1.823 \quad (\text{A. 30})$$

$$D_c = \sqrt{\frac{4A_c}{\pi}} = 1.523 \quad (\text{A. 31})$$

Then the diameter of the wire is selected to be $D_w=1.6\text{mm}$. The actual area of the wire can be calculated

$$A_{\text{wire}} = 1/4\pi D_w^2 = 2.01 \text{ mm}^2 \quad (\text{A. 32})$$

The average length of the coil per turn can be evaluated as

$$l_{\text{coil}} = 2(l_{\text{slc}} + 1/2b_s) + 1/3\pi(D_o + D_i) = 149.95 \quad (\text{A. 33})$$

Table A. 4 Parameters used in copper loss calculation

Rated Power (one motor)	P_N	2000
Phase number	m	3
Stator voltage per phase	$V_{N\Phi}$	$220/\sqrt{3}$
Efficiency	η_n	0.9
Power factor	$\cos \Phi_n$	1.23
Current density	J	0.4
Copper resistance ratio	ρ_w	1.72×10^{-8}
Coil turn number per phase	N_{ph}	60

The resistance of the coil per phase is written as

$$R = \rho_w \frac{N_{ph} l_{coil}}{A_{wire}} = 0.077 \Omega \quad (\text{A. 34})$$

The copper loss of the two stators are calculated as below

$$p_{Cu} = 2 \times 3 \times I^2 R = 12.73 \text{ W} \quad (\text{A. 35})$$

A. 3. Friction Loss

Recall Equations (3.77), (3.79) and (3.80) with the parameters shown in Table A. 5, the friction loss can be calculated.

Table A. 5 Parameters used in friction loss calculation

Bearing bore diameter, d	15 mm
Bearing outer diameter, D	40 mm
Bearing speed in rpm, n	60,000
Lubrication friction factor, f_L	0.8
Kinematic viscosity, ν	3.5

The lubricant friction moment M_L is calculated as

$$M_L = 10^{-7} f_L (\nu n)^{2/3} [(d + D)/2]^3 = 5.88 \times 10^{-3} \text{ N.m} \quad (\text{A. 36})$$

The load-dependent moment M_p is ignored; the friction loss is hence calculated to be

$$p_f = 2 \times 2\pi (n/60) M_L = 73.85 \text{ W} \quad (\text{A. 37})$$

A. 4 Efficiency Calculation

The total loss equals

$$p_{total} = p_{core} + p_{cu} + p_f = 160.93 \text{ W} \quad (\text{A. 38})$$

The efficiency of the motor can be calculated

$$\eta = \frac{4000 - 160.93}{4000} \times 100\% = 95.98\% \quad (\text{A. 39})$$

Appendix B: Realization of MPC approach in Matlab programming

```

/*****
Model-based Predictive Controller in State-space for FESS
u(t)=-Kx x(t)-Kd w(t)
where Kx=(B^T P11 B + Lambda I)^{-1} B^T P11 A
      Kd=(B^T P11 B + Lambda I)^{-1} B^T P11 A^d
*****/

#define S_FUNCTION_NAME sfun_SSMPC
#define S_FUNCTION_LEVEL 2
#include "simstruc.h"
#include <math.h>
/*****
Controller parameters
*****/

#define Np 6 // prediction horizon
#define Nu 3 // control horizon
#define lambda 0.03 //weighting factor for control input
/*****
System parameters
*****/

/* % flywheel control system
%% Model definition
%SS model
% x_i(t+1)=A_ix_i(t)+B_iu_i(t)
% y_i(t)=T_ix_i(t)      i=1,2    r=2
% parameters definition*/

real_T m, k1, k2, kt, J;
/*m=9.83;
k1=7.78e7;
k2=876.42;
kt=0.526
J=0.049;*/

#define nSystem 2 // number of the subsystems
#define nOrder 2 // order of each subsystem
#define Ts 8e-4 //sampling interval
#define nAxis 2 //motor number
real_T A1[nSystem][nOrder]=[0, 1.0, 0, 0];

```

```

real_T B1[nOrder 2][1]=[0, kt/J];
real_T C1[1][nOrder 2]=[0,1];
real_T D1=0;

real_T A2[nSystem][nOrder]=[0, 1.0, k1/m, 0];
real_T B2[nOrder 2][1]=[0, k2/m];
real_T C2[1][nOrder 2]=[1,0];
real_T D2=0;

/*Ts=8e-4; /*% sampling interval */
nAxis=2; /*% motor number */
nOrder=2; /*% subsystem order */

/*% convert to discrete model*/
real_T Ad1[nSystem][nOrder]=C2DM(A1,Ts,'zoh');
real_T Bd1[nOrder 2][1]=C2DM(B1,Ts,'zoh');
real_T Cd1[1][nOrder 2]=C2DM(C1,Ts,'zoh');

real_T Ad2[nSystem][nOrder]=C2DM(A2,Ts,'zoh');
real_T Bd2[nOrder 2][1]=C2DM(B2,Ts,'zoh');
real_T Cd2[1][nOrder 2]=C2DM(C2,Ts,'zoh');

/*[Ad1,Bd1,Cd1,Dd1] = C2DM(A1,B1,C1,D1,Ts,'zoh');
[Ad2,Bd2,Cd2,Dd2] = C2DM(A2,B2,C2,D2,Ts,'zoh'); */

/*% integrate the two model*/
real_T A[nSystem*nOrder][nSystem*nOrder]=[Ad1, zeros(nOrder,nOrder),
zeros(nOrder,nOrder), Ad2];
real_T B[nSystem*nOrder][nSystem]=[Bd1, zeros(nOrder, 1); zeros(nOrder, 1),
Bd2];
real_T T1=Cd1;
real_T T2=Cd2;

/*#define nSystem 2 // number of the subsystems
#define nOrder 2 // order of each subsystem

real_T A[nSystem*nOrder][nSystem*nOrder] = {1, 0.0008, 0, 0,
0, 1, 0, 0,
0, 0, 4.7995, 0.0016686,
0, 0, 13206, 4.7995};
real_T B[nSystem*nOrder][nSystem]={0.00000343510204, 0,
0.00858775510204, 0,
0, 0.00004280148832,
0, 0.14876611010043}; // system matrices

```

```

real_T T[nSystem][nOrder]={0, 1.0,
                           1.0, 0}; // C=T: output matrix

real_T Ad[nSystem][nSystem]={1.0, 0,
                              0, 1.0}; // reference matrix */
/*****
Matrix definition for Reccati Difference Equation
*****/
real_T P11[nSystem*nOrder][nSystem*nOrder],
QP11[nSystem*nOrder][nSystem*nOrder];
real_T Pi11[nSystem*nOrder][nSystem], QPi11[nSystem*nOrder][nSystem];
real_T invBXB[nSystem][nSystem];
/*****
Control gains
*****/
real_T Kx[nSystem][nSystem*nOrder];
real_T Kd[nSystem][nSystem];
/*****
temporary matrix
*****/

real_T P11B[nSystem*nOrder][nSystem];

/*****
definition of subroutines
*****/

void initial_PP11(void);
void cal_invBXB(int);
void rec_P11(void);
void rec_Pi11(void);
void cal_Kx(void);
void cal_Kd(void);

/*****
* Function: mdlInitializeSizes                                     *
* Abstract:                                                         *
*     Setup Sizes of the various vectors                             *
*****/
static void mdlInitializeSizes(SimStruct *S)
{
    ssSetNumSFcnParams(S, 0);
    if (ssGetNumSFcnParams(S) != ssGetSFcnParamsCount(S)) {
        return; /* Parameter mismatch will be reported by Simulink */
    }
}

```

```

    }

    ssSetNumContStates(S, 0);
    ssSetNumDiscStates(S, 2);
    if (!ssSetNumInputPorts(S,1)) return;
    ssSetInputPortWidth(S, 0, 6);
    ssSetInputPortDirectFeedThrough(S, 0, 1);
    if (!ssSetNumOutputPorts(S,1)) return;
    ssSetOutputPortWidth(S, 0, 2);
    ssSetNumSampleTimes(S,1);

    /* Take care when specifying exception free code - see sfuntmpl_doc.c */
    /* ssSetOptions(S, SS_OPTION_EXCEPTION_FREE_CODE |
SS_OPTION_USE_TLC_WITH_ACCELERATOR); DYNAMICALLY_SIZED */

}

/*****
* Function: mdlInitializeSampleTimes
* Abstract:
*   Specify that we inherit our sample time from the driving block.
*
*****/
static void mdlInitializeSampleTimes(SimStruct *S)
{
    ssSetSampleTime(S, 0, INHERITED_SAMPLE_TIME);
    ssSetOffsetTime(S, 0, 0.0);
}

/*****
* Function: mdlInitializeConditions
* Abstract:
*   Evaluate variables Kx and Kd
*
*****/
#define MDL_INITIALIZE_CONDITIONS
static void mdlInitializeConditions(SimStruct *S)
{
    real_T *x0 = ssGetRealDiscStates(S);
    int k;
    // initialize P11, Pi11, QP11, QPi11
    initial_PP11();

/*****
Finish initialization and start recursion
Define invBXB as inverse of (B^T*P11*B+lambda I)

```

```

*****/

for (k=0; k<Np-1; k++) {
    // calculate inverse of (B^T*P11*B+lambda I)
    cal_invBxB(k);
    // recursion of P11
    rec_P11();
    // recursion of Pi11
    rec_Pi11();
} // end of k

// for k=Np-1
// calculate inverse of (B^T*P11*B+lambda I)
cal_invBxB(Np-1);
// calculate control gain: Kx
cal_Kx();
// calculate control gain: Kd
cal_Kd();}

/*****
* Function: mdlOutputs                                     *
* Abstract:                                               *
*           Computation of Optimum Input                  *
*****/

static void mdlOutputs(SimStruct *S, int_T tid)
{
    InputRealPtrsType uPtrs = ssGetInputPortRealSignalPtrs(S,0);
    real_T *y = ssGetOutputPortRealSignal(S,0);
    int_T width = ssGetOutputPortWidth(S,0);
    real_T *x = ssGetRealDiscStates(S);
    real_T t = ssGetT(S);
    int_T i,j;
    real_T temp;
    real_T Kxx[nSystem], Kdw[nSystem];
    real_T X[nSystem*nOrder], W[nSystem], opt[nSystem];

    X[0] = *uPtrs[0];
    X[1] = *uPtrs[1];
    X[2] = *uPtrs[2];
    X[3] = *uPtrs[3];
    W[0] = *uPtrs[4];
    W[1] = *uPtrs[5];

    if(t==0) {
        for (i=0;i<nSystem;i++) {

```



```

        y[i]=0;
    }
}

else {
    /* computation of - Kx times x(t) */

    for (i=0;i<nSystem;i++) {
        temp = 0;
        for (j=0;j<nSystem*nOrder;j++) {
            temp += Kx[i][j] * X[j];
        }
        Kxx[i] = -temp;
    }
    /* computation of - Kd times w(t) */

    for (i=0;i<2;i++)
    {
        temp = 0;
        for (j=0;j<2;j++)
        {
            temp += Kd[i][j] * W[j];
        }
        Kdw[i] = -temp;
    }

    /* computation of u(t)opt and evaluation of eqn (62) */

    for (i=0;i<nSystem;i++)
    {
        opt[i]=Kxx[i] + Kdw[i];  /* have to declare u(t) at the beginning */
    }
    y[0]=opt[0];
    y[1]=opt[1];
}
}

/*****
* Function: mdlTerminate
* Abstract:
*   No termination needed, but we are required to have this routine.
*
*****/
static void mdlTerminate(SimStruct *S)

```

```

{
}
// initialize P11, Pi11, QP11, QPi11
void initial_PP11(void)
{
    int_T i,j,m,n;
    /*****
Initial conditions for recursion
P11=QP11=diag(T1^T*T1, ..., Tn^T*Tn),
P12=0,
P22=QP22=mu(Np)I,
P33=eta I
*****/

    for (i=0; i<nSystem; i++) {
        for (j=0; j<nSystem; j++) {
            for (m=0; m<nOrder; m++) {
                for (n=0; n<nOrder; n++) {
                    if (i==j) {
                        QP11[i*nOrder+m][j*nOrder+n]=T[i][m]*T[i][n];
                        P11[j*nOrder+m][j*nOrder+n]=QP11[j*nOrder+m][j*nOrder+n];
                    }
                    else {
                        QP11[i*nOrder+m][j*nOrder+n]=0;
                        P11[j*nOrder+m][j*nOrder+n]=0;
                    }
                } // end of n
            } // end of m

            if (i==j) {
                for (m=0; m<nOrder; m++) {
                    QPi11[i*nOrder+m][j]=-T[i][m];
                    Pi11[j*nOrder+m][j]=QPi11[j*nOrder+m][i];
                }
            }
            else {
                for (m=0; m<nOrder; m++) {
                    QPi11[i*nOrder+m][j]=0;
                    Pi11[j*nOrder+m][j]=0;
                }
            }
        } // end of j
    } // end of i
}

```

```

// invBxB = inverse of (B^T*P11*B+lambda I)
void cal_invBxB(int_T k)
{
    int_T i,j,m;
    real_T temp,temp1;
    if (k<=Np-1-Nu) {
        for (i=0; i<nSystem; i++) {
            for (j=0; j<nSystem; j++) {
                invBxB[i][j]=0;
            }
        }
    } // infinite weighting factor for control input

    else {
        // calculate P11B=P11*B
        for (i=0; i<nSystem*nOrder; i++) {
            for (j=0; j<nSystem; j++) {
                temp=0;
                for (m=0; m<nSystem*nOrder; m++) {
                    temp+=P11[i][m]*B[m][j];
                }
                P11B[i][j]=temp;
            }
        }

        // calculate invBxB=B^T*P11B+lambda I
        for (i=0; i<nSystem; i++) {
            for (j=0; j<nSystem; j++) {
                temp=0;
                for (m=0; m<nSystem*nOrder; m++) {
                    temp+=B[m][i]*P11B[m][j];
                }
                if (i==j) {
                    invBxB[i][j]=temp+lambda;
                }
                else {
                    invBxB[i][j]=temp;
                }
            }
        }

        // calculate invBxB=invBxB^{-1}, only for 2 by 2 matrix
        temp1=invBxB[0][0]*invBxB[1][1]-invBxB[0][1]*invBxB[1][0];
        temp=invBxB[0][0];
        invBxB[0][0]=invBxB[1][1]/temp1;
    }
}

```

```

        invBxB[1][1]=temp/temp1;
        temp=invBxB[0][1];
        invBxB[0][1]=-invBxB[1][0]/temp1;
        invBxB[1][0]=-temp/temp1;
    }
}

// recursion of P11
void rec_P11(void)
{
    int_T i,j,m;
    real_T temp;
    real_T P11A[nSystem*nOrder][nSystem*nOrder],
    ATP11A[nSystem*nOrder][nSystem*nOrder];
    real_T ATP11B[nSystem*nOrder][nSystem];
    real_T ATP11Bi[nSystem*nOrder][nSystem],
    ATP11BiBTP11A[nSystem*nOrder][nSystem*nOrder];

    // calculate P11*A
    for (i=0; i<nSystem*nOrder; i++) {
        for (j=0; j<nSystem*nOrder; j++) {
            temp=0;
            for (m=0; m<nSystem*nOrder; m++) {
                temp+=P11[i][m]*A[m][j];
            }
            P11A[i][j]=temp;
        }
    }

    // calculate A^T*P11*A
    for (i=0; i<nSystem*nOrder; i++) {
        for (j=0; j<nSystem*nOrder; j++) {
            temp=0;
            for (m=0; m<nSystem*nOrder; m++) {
                temp+=A[m][i]*P11A[m][j];
            }
            ATP11A[i][j]=temp;
        }
    }

    // calculate A^T*P11*B=A^T*P11B
    for (i=0; i<nSystem*nOrder; i++) {
        for (j=0; j<nSystem; j++) {
            temp=0;

```

```

        for (m=0; m<nSystem*nOrder; m++) {
            temp+=A[m][i]*P11B[m][j];
        }
        ATP11B[i][j]=temp;
    }
}

// calculate  $A^T * P11 * B * \text{inv}BXB = ATP11B * \text{inv}BXB$ 
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;
        for (m=0; m<nSystem; m++) {
            temp+=ATP11B[i][m]*invBXB[m][j];
        }
        ATP11Bi[i][j]=temp;
    }
}

// calculate  $A^T * P11 * B * \text{inv}BXB * B^T * P11 * A = ATP11Bi * ATP11B^T$ 
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem*nOrder; j++) {
        temp=0;
        for (m=0; m<nSystem; m++) {
            temp+=ATP11Bi[i][m]*ATP11B[j][m];
        }
        ATP11BiBTP11A[i][j]=temp;
    }
}

// calculate  $P11 = ATP11A - ATP11BiBTP11A + QP11$ ;
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem*nOrder; j++) {
        P11[i][j]=ATP11A[i][j]-ATP11BiBTP11A[i][j]+QP11[i][j];
    }
}

}

// recursion of P11
void rec_P11(void)
{
    int_T i,j,m;
    real_T temp;
    real_T ATP11[nSystem*nOrder][nSystem];

```

```

real_T ATPi11Ad[nSystem*nOrder][nSystem];
real_T ATP11B[nSystem*nOrder][nSystem];
real_T ATP11Bi[nSystem*nOrder][nSystem];
real_T BTPi11[nSystem][nSystem];
real_T BTPi11Ad[nSystem][nSystem];
real_T ATP11BiBTPi11Ad[nSystem*nOrder][nSystem];

// calculate ATPi11=A^T*Pi11
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;
        for (m=0; m<nSystem*nOrder; m++) {
            temp+=A[m][i]*Pi11[m][j];
        }
        ATPi11[i][j]=temp;
    }
}

// calculate ATPi11Ad=A^T*Pi11*A^d
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;
        for (m=0; m<nSystem; m++) {
            temp+=ATPi11[i][m]*Ad[m][j];
        }
        ATPi11Ad[i][j]=temp;
    }
}

// calculate A^T*P11*B=A^T*P11B
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;
        for (m=0; m<nSystem*nOrder; m++) {
            temp+=A[m][i]*P11B[m][j];
        }
        ATP11B[i][j]=temp;
    }
}

// calculate A^T*P11*B*invBxB=ATP11B*invBxB
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;

```



```

        for (m=0; m<nSystem; m++) {
            temp+=ATP11B[i][m]*invBXB[m][j];
        }
        ATP11Bi[i][j]=temp;
    }
}

// calculate BTPi11=B^T*Pi11
for (i=0; i<nSystem; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;
        for (m=0; m<nSystem*nOrder; m++) {
            temp+=B[m][i]*Pi11[m][j];
        }
        BTPi11[i][j]=temp;
    }
}

// calculate BTPi11Ad=B^T*Pi11*A^d
for (i=0; i<nSystem; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;
        for (m=0; m<nSystem; m++) {
            temp+=BTPi11[i][m]*Ad[m][j];
        }
        BTPi11Ad[i][j]=temp;
    }
}

// calculate ATP11BiBTPi11Ad=A^T*P11*B*invBXB*B^T*Pi11*A^d
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem; j++) {
        temp=0;
        for (m=0; m<nSystem; m++) {
            temp+=ATP11Bi[i][m]*BTPi11Ad[m][j];
        }
        ATP11BiBTPi11Ad[i][j]=temp;
    }
}

// calculate Pi11=ATPi11Ad-ATP11BiBTPi11Ad+QPi11
for (i=0; i<nSystem*nOrder; i++) {
    for (j=0; j<nSystem; j++) {
        Pi11[i][j]=ATPi11Ad[i][j]-ATP11BiBTPi11Ad[i][j]+QPi11[i][j];
    }
}

```

```

    }
}

// calculate control gain: Kx
void cal_Kx(void)
{
    int_T i,j,m;
    real_T temp;
    real_T BTP11A[nSystem][nSystem*nOrder];

    // calculate  $B^T P_{11} A = (P_{11} B)^T A$ 
    for (i=0; i<nSystem; i++) {
        for (j=0; j<nSystem*nOrder; j++) {
            temp=0;
            for (m=0; m<nSystem*nOrder; m++) {
                temp+=P11B[m][i]*A[m][j];
            }
            BTP11A[i][j]=temp;
        }
    }

    // calculate  $K_x = \text{inv}B X B^T P_{11} A$ 
    for (i=0; i<nSystem; i++) {
        for (j=0; j<nSystem*nOrder; j++) {
            temp=0;
            for (m=0; m<nSystem; m++) {
                temp+=invBXB[i][m]*BTP11A[m][j];
            }
            Kx[i][j]=temp;
        }
    }
}

// calculate control gain: Kd
void cal_Kd(void)
{
    int_T i,j,m;
    real_T temp;
    real_T BTPi11[nSystem][nSystem];
    double BTPi11Ad[nSystem][nSystem];

    // calculate  $B^T P_{i11} = B^T P_{11}$ 

```

```

    for (i=0; i<nSystem; i++) {
        for (j=0; j<nSystem; j++) {
            temp=0;
            for (m=0; m<nSystem*nOrder; m++) {
                temp+=B[m][i]*Pi11[m][j];
            }
            BTPi11[i][j]=temp;
        }
    }

    // calculate BTPi11Ad=B^T*Pi11*A^d
    for (i=0; i<nSystem; i++) {
        for (j=0; j<nSystem; j++) {
            temp=0;
            for (m=0; m<nSystem; m++) {
                temp+=BTPi11[i][m]*Ad[m][j];
            }
            BTPi11Ad[i][j]=temp;
        }
    }

    // calculate Kd=invBXB*BTPi11*A^d
    for (i=0; i<nSystem; i++) {
        for (j=0; j<nSystem; j++) {
            temp=0;
            for (m=0; m<nSystem; m++) {
                temp+=invBXB[i][m]*BTPi11Ad[m][j];
            }
            Kd[i][j]=temp;
        }
    }
}

#ifdef  MATLAB_MEX_FILE    /* Is this file being compiled as a MEX-file? */
#include "simulink.c"      /* MEX-file interface mechanism */
#else
#include "cg_sfun.h"       /* Code generation registration function */
#endif

```