Global control design and its applications to power systems

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Global Control Design and Its Applications to Power Systems

Zhang Guohua

School of Electrical and Electronics Engineering

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Summary

The thesis concerns the applications of global control in particular consideration of high degree of nonlinearities, uncertainties and interconnections. The desire to coordinate various control actions and deal with wide ranges of operating regions in real world systems has raised great interests in global control of complex systems. Although the global control structure have given general guide of controller design and the global control strategies, such as heterogeneous control and fuzzy dynamic control, have been developed largely, there still lacks of a systematic global control method which can conveniently take into accounts dynamic behaviors of complex systems. This research aims to take advantages of modern control techniques and global control structure to propose a global control design framework for the practical applications of complex system control.

Based on the global control structure, the unified global control design for complex systems begins with the application of multiple nonlinear partial models. These models can describe typical phenomena of practical complex systems, such as different operating states, nonlinearities, abrupt dynamics and different control objectives. Since the complex system is represented by multiple partial models and their coordination rules, the global controller is obtained by synthesizing the partial controllers upon their operating regions, where membership functions provide proper weighting and smooth transitions. Consequently, with the closed-loop system analysis based on Lyapunov stability, global control design is gradually extended from a simpler situation to the final complex systems: uncertain systems, a class of nonlinear systems and large-scale interconnected system.

The applications of global control are demonstrated in power system for transient stability enhancement. Global control of uncertain system and large-scale interconnected system are applied to SMIB and multi-machine power systems respectively. Multiple partial models are used to represent the power system under different states: the pre-fault state, faulty state, fault clearing and post fault state. Then, the global controllers for generator excitation in SMIB or multi-machine power system are obtained by coordinating four partial controllers from different states. System parameters, interconnections and fault

locations are all treated as the uncertainties and handled by the robust control approach in each partial model.

Global control for a class of nonlinear systems is employed to two benchmark control systems: the Ball and Beam (BB) system and the Cart and Pole (CP) system. The proposed global control design is employed to solve these typical nonlinear problems which cannot be systematically handled by existing control approaches. Global control design of the BB system improves controller performance by decreasing the number of partial controllers in comparison with the fuzzy dynamic control strategies. For the global control of the CP system, two control objectives: only regulate the displacement of pole and regulate displacement of cart and pole simultaneously, are respectively implemented.

Global control scheme is extended to uncertain power systems to coordinate different, sometimes inherently conflicting, control objectives and decentralized controllers within wide operating regions. The SMIB and multi-machine power system with SVC are utilized as example systems for global control scheme to maintain the transient stability and achieve proper post-fault voltage. In the global control scheme, decentralized partial controllers are respectively designed for generator excitation and SVC to coordinate two conflicting control objectives. System uncertainties and controller interactions in power systems are taken into account in the design of robust partial controllers.

The applications of the proposed global control design will obtain the following specific advantages in the large.

1. Different control objectives and actions over wide operating regions can be coordinated in terms of proposed global control design.
2. All the controllers designed in the thesis need only local measurement to implement feedback controls. However, the interactions between those partial controllers can be taken into accounts.
3. Global control systems are robust and stable irrespective of the system parameter uncertainties, nonlinearities and interactions.
4. The stabilization of each decentralized system can guarantee the one of the global complex systems in terms of Lyapunov stability.

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Multi-Machine Power System

$\delta_i(t)$	the power angle the i th generator
δ_{0i}	the power angle of the i th generator at the operating point
$\omega_i(t)$	the rotor speed of the i th generator
ω_0	the synchronous machine speed
$P_{ei}(t)$	the active electrical power delivered by the i th generator
P_{mi}	the mechanical input power of the i th generator
$Q_{ei}(t)$	the reactive power of the i th generator
D_i	the damping constant of the i th generator
H_i	the inertia constant of the i th generator
T'_{doi}	the direct axis transient open circuit time constant of the i th generator
$E_{qi}(t)$	the EMF in the quadrature axis of the i th generator
$E_{di}(t)$	the EMF in the direct axis of the i th generator
$E'_{qi}(t)$	the transient EMF in the quadrature axis of the i th generator
$E'_{di}(t)$	the transient EMF in the direct axis of the i th generator
$E_{fi}(t)$	the equivalent EMF in the excitation coil of the i th generator
$u_{fi}(t)$	the input of the SCR amplifier of the i th generator
k_{ci}	the gain of the excitation amplifier of the i th generator
$I_{fi}(t)$	the excitation current of the i th generator

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$I_{qi}(t)$	the quadrature axis current of the i th generator
$I_{di}(t)$	the direct axis current of the i th generator
X_{di}	the direct axis reactance of the i th generator
X'_{di}	the direct axis transient reactance of the i th generator
X_{adi}	the mutual reactance between the excitation coil and the stator coil of the i th generator
$V_i(t)$	the i th generator terminal voltage
B_{ij}	the i th row and j th column element of nodal susceptance matrix
$\delta_{ij}(t)$	$= \delta_i(t) - \delta_j(t)$
$\omega_{ij}(t)$	$= \omega_i(t) - \omega_j(t)$
SVC	
$B_L(t)$	the susceptance of the TCR of the SVC
T_R	the time constant of the control system of the TCR of the SVC
k_B	the gain of the control system of the TCR of the SVC
u_B	the input of the control system of the TCR of the SVC
B_{L0}	the susceptance of the TCR of the SVC at the operating point
B_{C0}	the susceptance of the FC of the SVC
$B_{SVC}(t)$	the susceptance of SVC
$\Delta B_L(t)$	$= B_L(t) - B_{L0}$
$\alpha(t)$	the firing angle
V_B	the RMS value of SVC bus voltage

B_s the full susceptance of the TCR of SVC

SMIB Power System

$\delta(t)$ the power angle of the generator

δ_0 the power angle of the generator at the operating point

$\omega(t)$ the rotor speed of the generator

ω_0 the synchronous machine speed

$P_e(t)$ the active electrical power delivered by the generator

P_m the mechanical input power of the generator

$Q_e(t)$ the reactive power of the generator

D the damping constant of the generator

H the inertia constant of the generator

T'_{do} the direct axis transient open circuit time constant

$E_q(t)$ the EMF in the quadrature axis of the generator

$E_d(t)$ the EMF in the direct axis of the generator

$E'_q(t)$ the transient EMF in the quadrature axis of the generator

$E'_d(t)$ the transient EMF in the direct axis of the generator

$E_f(t)$ the equivalent EMF in the excitation coil of the generator

$u_f(t)$ the input of the SCR amplifier of the generator

k_c the gain of the excitation amplifier of the generator

$I_f(t)$ the excitation current of the generator

$I_q(t)$ the quadrature axis current of the generator

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$I_d(t)$	the direct axis current of the generator
$V_t(t)$	the generator terminal voltage
V_s	the infinite bus voltage
X_d	the direct axis reactance of the generator
X'_d	the direct axis transient reactance of the generator
X_{ad}	the mutual reactance between the excitation coil and the stator coil
X_l	the reactance of the transmission line
R_l	the resistance of the transmission line
X_T	the reactance of the transformer
X_E	the Thevenin's equivalent reactance of the network to the right of bus 1 as shown in Figure 2.6
X_{ds}	$= X_d + X_T + X_E$
X'_{ds}	$= X'_d + X_T + X_E$
X_s	$= X_T + X_E$

The Global Control Law

$\mu_l(t)$ The normalized membership function of l th partial model and satisfy membership functions of heterogeneous control law $\sum_{l=1}^m \mu_l(t) = 1$

Acronyms:

AC	Alternating Current
BB	Ball and Beam
CP	Cart and Pole

List of Tables

DFL	Direct Feedback Linearization
EMF	Electromotive-force
FACTS	Flexible Alternating Current Transmission Systems
FC	Fixed Capacitor
PSCAD	Power Systems Computer-Aided Design
PSS	Power System Stabilizer
RMS	Root Mean Square
SCR	Silicon Controlled Rectifier
SMIB	Single Machine Infinite Bus
SVC	Static VAr Compensator
TCR	Thyristor Controlled Reactor
UPFC	Unified Power Flow Controller
VAr	Volt-Ampere reactive

Chapter 1 Introduction

1.1 Motivation

The thesis concerns the applications of global control in consideration of high degree of nonlinearities, uncertainties and interconnections. Global control aims to coordinate various control actions and objectives over wide operating conditions by combining the qualitative and quantitative knowledge of entire operating conditions of complex system [1]. The general global control design procedure will be proposed based on the global control structure [1] and modern control techniques. Then theoretical proofs of global control strategies are gradually provided from a simpler situation to the final complex systems: the uncertain system, a class of nonlinear systems and the large-scale interconnected system. Finally, the proposed global control strategies will be applied to power systems and two classical control examples.

Complex systems are generally large-scale systems and characterized by substantial nonlinearities, uncertain models, high noise levels, abrupt changes in dynamics and strict performance requirements. A great number of today's problems are highly complex, "large" in dimension and uncertain by nature. A system can be called a "large-scale system" if it consists of a number of independent subsystems which serve particular functions, share resources, and are connected by a set of interconnections ([2] and [3]). In real world systems, the unmanned aerial vehicles, aerospace system, automated highways, process control systems, power systems and communication networks, etc., all are complex systems. Systematic control of these dynamical systems has provided great challenges to control engineers.

Over the past decade and before, various control theories have been advanced to reach the gradually increasing control requirements. At the earlier stage, control engineer began with the desire of human to harness the materials and forces of nature to their advantages. For examples, the feedback control was applied to clock regulating systems and the mechanism to keep the windmills pointed to the wind [4]. Then the problem of design accurate control system in presence of uncertainties was classical for manufacturing.

Stability criterion and sensitivity function were proposed to provide a measure for system accuracy improvement, at least for the sufficiently small plant variations [5]. It was not until the state variable concepts presented that above results of single-input single-output system were extended to the multi-input multi-output systems. Ultimately, modern robust control theories were systematically developed [6]. Except for the accuracy problem due to uncertainties, the wide operating regions of control systems are another key problem to be handled in practical applications. Since much control theory is based on the linear models which usually work well for steady state regulation at a fixed operating point, it is necessary to introduce nonlinearities. Feedback linearization, adaptive control, passivity based approach, multi-models based switching control, fuzzy control and heterogeneous control, are popular ways to solve nonlinearity problems [6]-[9]. Moreover, the control problems of large-scale system have attracted a great deal of attentions, such as power systems and information systems. These control problems involve hierarchical control, decentralized control, estimation and filtering, modeling of large-scale system, robust control, fuzzy decentralized control etc.

In control of complex systems, there are three main difficulties, i.e. nonlinearities uncertainties and interconnections. Physical systems in general, and technical systems in particular, are as a rule of nonlinear, i.e. they comprise nonlinearities, which include any deflections from linear characteristics or from linear equation of the system's dynamics [10]. Unlike the linear systems which are valid on the local regions, complex system may have different control objectives and consequently control actions possible in wide regions of operation. The presence of uncertainties in the complex systems has been the rule not the exception mainly due to parameter variation, nonlinear elements, input disturbances and measurement noise. On the other hand, those systems under consideration of analysis and control design are often not perfectly known because their modeling based on many assumptions, which made the behavior of any identified model different from that of the original physical system. Consequently, the behavior of original complex systems is only partially known systems, also called uncertain systems [6]. For the large-scale interconnected systems, the sources of uncertainties appear to be within the individual subsystems and through the coupling links among subsystems. In some systems, strong interconnections among distributed subsystems can make a major contribution to the instability of whole complex system. The above difficulties have

called for the need of a highly intelligent control system and modern control techniques to design a controller from global operations.

To take power systems as a typical example complex system, applications of global control can be investigated from the viewpoint of global operations.

To begin with, the wide operating states are one of key issues in control of power systems. Power system is a highly nonlinear uncertain system by nature and operates over wide operating states. Electric power system is a complex network of individual components, including generators, transformers, buses, switching devices, transmission lines, load and Flexible Alternating Current Transmission Systems (FACTS) devices. Since all those components are inherently nonlinear and can operate from a normal condition to an abnormal one, power system could operate over wide regions. Except the usual unmodeled dynamics and parameter uncertainty, load and network structures are two major sources of uncertainty. Load level and dynamic behaviors change continuously due to small load events or abruptly due to large event [11]. The area of load modeling is certainly the weak point in power system modeling [12]. The structure of electric power network and its parameters are also susceptible to change from time to time by severe disturbance, repair and failure and so on. Therefore, the controller design for modern power system to achieve good transient performance should consider various nonlinearities and uncertainties over wide operating states.

Then, different control objectives and actions must be employed under wide operating state of power systems. Two of most important control problems of power system are real power/frequency regulation and reactive power/voltage regulation. However, those control objectives sometimes are conflicting with each other. One of most typical examples is the inherent conflict between voltage regulation and damping improvement for synchronous generators [13]-[17]. The coordination of these two functions is based on a sequential design consisting of two operating stages: one stage is to meet the required voltage regulation performance and another one is to meet the required damping performance. When power system is subjected to a large disturbance, the dynamic phenomena of system can be classified into transient and long time frames, and there is almost always a clear separation between two time frames [18]. On these two qualitative distinct stages, the generator excitation controller is required to maintain the transient

stability and achieve satisfactory voltage respectively [16]. In Reference [1], those different control objectives had been elaborated in details and a global controller to coordinate those control objectives was proposed for a single machine infinite bus (SMIB) power system. More complicated power systems are under investigation for this global control strategy.

Figure 1.1 Power system controls related with transient stability

Furthermore, multiple controllers are required to coordinate to contribute to the satisfactory operation in the power systems. Power system consists of various subsystem associated with control, including generating unit controls, system generation control and transmission controls. In overall structure of power systems, there are controllers operating directly on individual system elements. For the transient stability study, the power system components and associated control are depicted in Figure 1.1. Undesirable interactions among excitation controllers and FACTS-based stabilizers may have detrimental effects to the overall performance of the power system ([19] and [20]). Therefore, the interactions and coordination of multiple controllers within the large scale power systems is a global control problem of practical importance.

Finally, the development of power systems has increasingly inspired the strict controller performance from a global operation. As the deregulation of power industry and environment concerns advance recently, the transmission lines are expected to operate at maximum capacity close to thermal limits without expansion of the existing transmission

and generation infrastructure. Consequently, system behavior will be more dependent on nonlinear characteristics and thus more complex. Ultimately, control based capability and controller interactions play an important role for reliable and economical operation of power system, especially when it is subjected to disturbances. Global control of power systems is challenging and important for this stressed situation.

Figure 1.2 Typical structure of complex system ($i=1, \dots, N$)

In the light of the characteristics of power systems, complex system generally features high degree of nonlinearities, uncertainties and interconnections. The typical structure of complex system can be described with N interconnected subsystems in Figure 1.2. Various control objectives and multiple decentralized controllers over wide operating regions call upon the global control design for complex system. All above brief overview made up the motivation of this research towards global control application. A unified framework for application of global control method will resolve system modeling identification and control design for complex systems. Moreover, coordination of various control actions and control objectives over wide operating state are of practical importance for global control of complex system.

1.2 Literature review

In this section, the control techniques related with global control of complex system are reviewed.

1.2.1 Multi-model based controller design

Multi-model based controller design have been proved to be a popular choice to control of complex systems for their special characteristics, such as, substantial nonlinearities, large scale and variable uncertainties. Up till now, there are a lot of modern control techniques to deal with the dynamic behavior of complex systems. For example, the nonlinearities and uncertainties of complex systems can be solved by nonlinear control techniques ([10], [6]-[23]) and robust control theory ([6], [24]-[26]) respectively. Decentralized control [27]-[30] and decentralized adaptive control [31]-[33] have been proposed to resolve the large-scale interconnection problems. However, models that are constructed within the bounds of a single paradigm are insufficient for modeling all aspects of complex systems [34]. One example is that it usually appealed to different models to describe different control actions and objectives of complex systems. Therefore, even though modeling and simulation systems that utilize a single modeling paradigm are the current norm, a multi-model approach [35] is widely explored in control of complex system.

A multi-model approach is defined as one in which more than one model each derived from a different perspective, and employed correspondingly distinct control strategies [34]. Because the local models can be simple and intuitive, and global behavior can be analyzed in terms of transitions among local operating regions, multi-model approach is an appropriate way to describe qualitatively different behaviors of complex system over whole operating regions. While a multi-model approach can provide different perspectives on a problem and facilitate addressing the problem from different levels of abstraction, properly integrating different models and designing their controllers are also concerned with.

Various multi-model based approaches, such as gain scheduling, switch control [36], fuzzy control, heterogeneous control, have been applied to complex systems. In those control approaches, complex system was linearized at several equilibrium points, and then classical control theory can analyze these multiple local linear models, provide a linear feedback controller at each point and implement the resulting family of linear controllers as a single controller to complex system. Although it is convenient to design multiple linear controllers, it relies on the assumptions of small range operation for the

linear model to be valid. For complex systems, the large range of operation cannot avoid and specific control performances are required to take nonlinear dynamics into accounts. As a result, the major problem of those approaches remains that as the complexity of nonlinear system increases, ability to make precise and yet significant statements about its local behavior diminishes. Intuitively, the number of local controllers increases and the switching of various controllers for different operating regions will be become complicated. At the worst, the smoothness of the transition between local models and stability of the global system are degraded. The models involving nonlinear dynamics and adapting to wide range of operating region are called upon for complex system control in recent literatures.

Stability is another important issue of complex systems control since unstable systems have no practical significance. For the model-based control, the global stability criteria are usually established based on Lyapunov stability [21]. Consequently, the design of local feedback controllers and their quadratic Lyapunov functions to guarantee the stability of complex systems is a necessary and difficult task in multi-model based control.

1.2.2 Fuzzy dynamic control

Fuzzy dynamic control has been emerged as one of most active and fruitful approaches to control complex system. Fuzzy dynamic control, i.e. fuzzy model based control, includes both fuzzy IF-THEN rules and local dynamic models to deal with the uncertainties in the control process. It provides a way to combine both qualitative and quantitative knowledge into a united framework. Fuzzy dynamic control has found wide applications in control of complex systems, such as, in control of robot manipulators, power system and complex processes. For example, an adaptive fuzzy-logic SVC damping controller was designed controllers for transient stability improvement [36]. The robust decentralized technique was also combined with fuzzy technique to design proportional and proportional-plus-integral (PI) controllers for multi-machine power system [38].

Takagi and Sugeno had systematically developed model-based analysis methods into fuzzy control [39]. The basic idea of this approach is to decompose the complicated input space into subspaces and then approximate the system in each subspace by a simple linear

dynamic model. Hence, the T-S fuzzy model is considered as a combination of interconnected subsystems with simpler models. Since the consequent part of the rules is not defined by a membership function but by crisp analytical functions and the application of fuzzy control to complex system is not a trivial task. It become evident that many basic issues remain to be further addressed. Stability analysis and systematic design are certain among the most important issues for fuzzy control systems.

Since the T-S fuzzy model is deterministic in the sense that there is no uncertainty expressed in the local models, in order to consider the uncertainties in the practical systems, a lot of research work extends the T-S fuzzy model to uncertain systems and derives stability conditions. Tanaka et al. [40]-[42] discussed the stability and the design of robust fuzzy control systems. They present some checking conditions for stability, which can be used to design fuzzy control laws, and rule reductions were provided to reduce the number of IF-THEN rules as well. Unfortunately, the stability conditions require that for all the local linear models a common positive-definite matrix P must be existed to satisfy the Lyapunov equation. However, some research work, such as ones proposed by Cao and Feng [43]-[48] can get around the problem of determining the common P matrix by applying piecewise Lyapunov function. The design of a fuzzy control system converted to the design of a set of linear time-invariant systems. This allows the designer to take advantage of conventional linear system theory to analyze and design fuzzy control systems. But these results may tend to give a conservative controller design due to the discontinuity of multi-models across the boundaries of the different subspaces.

There has been rapidly growing interest in combining fuzzy control with decentralized control technique to get a system design with the guarantee of stability and control performance for large-scale interconnected systems [49]-[51]. Owing to physical configuration and high dimensionality of interconnected systems, the T-S fuzzy model combined with decentralized control can simplify the controller design. The solution of a large problem was obtained by decomposing the large one into a number of smaller sub-problems and then combining their solutions in a suitable ways. In Reference [50], a fuzzy decentralized control strategy was constructed through overlapping decomposition and the stability conditions of composite fuzzy system were derived in terms of Lyapunov functions and M matrices. Fuzzy decentralized variable structure tracking control with

optimal was proposed to improve robustness designs for nonlinear interconnected systems [51].

In order to guarantee the stability condition of fuzzy control for complex system, various control techniques had incorporated with fuzzy models, such as quadratic stabilizability, H_∞ control theory, linear matrix inequalities (LMI), adaptive control, neural networks and genetic algorithm (GA) [52]. Robust stability of fuzzy control system had been analyzed via quadratic stabilizability, H_∞ control theory and LMI ([40], [48] and [52]). Adaptive and robust fuzzy control systems were proposed to guarantee stability of system with external disturbances ([54] and [55]). The strict matching conditions of uncertainties and interconnections have been relaxed by considering geometric conditions or combining with neural networks. The adaptive fuzzy controllers were optimized to robustly stabilize systems with bounded perturbations and disturbances by using genetic algorithm ([56] and [57]).

Although lots of achievements have been made for complex system control by taking advantage of fuzzy dynamic control and various modern techniques, there are several difficulties to be overcome. The most important aspect is that building a Takagi-Sugeno model is not an easy exercise in practice. Specially, the procedure of finding the optimum premise structure is technically involved and may lead to a nonlinear programming problem. Then, stability conditions cannot be easily guaranteed in fuzzy dynamic control of complex systems. For example, boundary conditions for stability of global system behavior in the fuzzy control were still very restrictive due to the discontinuity of the function across the boundaries of the different subspaces. Finally, the unrealistically high order of a model due to nonlinearities of practical problems is still a challenging issue although adaptive control can take some types of high order nonlinearities into account.

1.2.3 Heterogeneous control

Heterogeneous control is another attractive global control strategy for the complex dynamical systems. Heterogeneous control was presented as an alternate formulation and extension of fuzzy control to overcome the difficulties of fuzzy control applications in industrial practice [58]. Thus, it shares many goals with and draws much inspiration from fuzzy control. Like fuzzy logic control, these methods apply even in the presence of

incomplete knowledge of the structure of the system, the boundaries of the operating regions, or even the control action to take. Unlike fuzzy logic control, these methods can be analyzed by a combination of classical and qualitative methods.

Heterogeneous control was first presented to combine local control laws appropriate to different operating regions [58]. This approach has several practical and theoretical benefits, such as it provided high resolution control and can be analyzed by classical methods; the global heterogeneous control law is designed as the weighted average of the local control laws and provides smooth transitions between local regions; it allows to be analyzed by qualitative simulation even in the face of incomplete knowledge of the underlying system or the operating region membership functions. Two example of heterogeneous control were given for level control of a water tank and reaction control of chemical reactor.

References [59]-[61] then argued that the use of qualitative models further improves the strengths of the multi-model approach by allowing each local model to describe a large class of useful non-linear dynamical systems. A methodology called qualitative heterogeneous control was proposed to synthesize robust and optimal versions of global behaviors in complex systems. Reasoning with qualitative models naturally identifies qualitative constraints on the controller designs, sufficient to guarantee the desired local properties and to determine the possible transitions between local regions. The global control of an inverted pendulum system was used to demonstrate this process of controller synthesis.

In view of practical benefits, heterogeneous control has been widely applied and developed recently. Reference [62] used qualitative reasoning to write expert controllers. The advantages for each element of a heterogeneous control law are shown in Reference [63]. An integrated real-time control and simulation environment were presented to support the tasks of heterogeneous controllers [64]. Reference [65] applied a heterogeneous control law to the start-up and disturbance recovery of a distillation column. In addition, heterogeneous control employed to coordinate different control objectives in power systems. The coordination of transient stability and voltage regulation was investigated according to different operating conditions ([1] and [16]). The

coordinated control scheme for a unified power flow was designed to restore line voltage, improve transient stability and enhance the dampening of inter-area oscillations [66].

1.2.4 Global control

Global control is a relatively new concept ([67] and [68]) that has evolved from techniques like multi-model control towards the capability to handle complex system. It combines the qualitative and quantitative knowledge through some hierarchy to coordinate various control actions, which deal with all the anticipated problems and operating conditions.

Figure 1.3 Global control structure [1]

Reference [67] first proposed global control for stressed power system. A multilevel control scheme was established over wide ranges of operating conditions by bifurcation analysis. Then a global control structure was presented in the large [1]. The design of global control structure [1] shown in Figure 1.3 can be decomposed into three stages. In the first stage of identification, coordination rules, membership functions and local models are identified according to qualitative distinct operating conditions and global control objectives. Then local controllers can be designed based on the local models and local performance requirements in the stage of local controller design. After the local controllers are available, the next stage is controller synthesis. The final global controller

is obtained as a weighted average of local controllers, which can provide smooth transitions upon local operating regions. The global control achieves the interpolation rules to combine the local model and local controllers in a higher level of supervision. Several examples strategies, such as heterogeneous control, fuzzy dynamic control and global dissipativity control, are utilized in the application of global control to mechanical systems and power systems. Global control was used to coordinate transient stability and voltage regulation according to different operating conditions in a single machine infinite bus (SMIB) power system ([1] and [16]). The global control scheme was also designed for a unified power flow controller (UPFC) to coordinated different control actions: restore line voltage, improve transient stability and enhance the dampening of inter-area oscillations [66].

1.3 Objectives

The objective of the thesis mainly concerns the application of global control for complex systems. Complex systems are characterized by substantial nonlinearities, uncertain models, high order interconnections, high noise levels, abrupt changes in dynamics and strict performance requirements. Hence, control of complex systems remains a challenge for its qualitatively different behaviors over wide operating regions. Mathematical models, stability analysis and systematic design are certainly among the most important issue for complex systems control.

One typical example complex system, power systems, is a complex network of system components, which are inherently nonlinear and can operate from a normal condition to an abnormal one. Thus, control of power system is required to be valid over wide operating states. For some of components in power systems, such as the generators and FACTS devices, frequency and voltage regulation are two necessary control objectives in their operations. However, those control objectives are conflicting with each other sometimes. One of most typical examples is the inherent conflict between voltage regulation and damping improvement for synchronous generators [13]-[17]. Another example is revealed by analysis that the SVC voltage and damping control, in fact, conflict with each other and one function performed always at the price of sacrificing the performance of another function [69]. Furthermore, since power systems expand a vast

geographical domain, they naturally lend themselves to decentralized control. Undesirable interactions among those decentralized excitation controllers and FACTS-based stabilizers may have detrimental effects to the overall performance of the power system ([19] and [20]). Finally, with the deregulation of power industry and environment concerns, power system are expected to operate under heavily stressed conditions and the stability margin is reduced considerably. Therefore, it is necessary to require the control system to maintain the stability of this stressed system under a severe disturbance regardless of the system parameter uncertainties, nonlinearities and interactions. Global control of power systems has been increasingly inspired to coordinate various control actions and objectives over wide ranges of operating states.

Although the global control structure had provided the general guide for global controller design, the performance of controller is determined by the existing global control strategies. From previous literature reviews on fuzzy dynamic control and heterogeneous control, those two control techniques have their own advantages in control of complex systems. Heterogeneous control mainly employed the conventional control design and fuzzy dynamic control have incorporated with modern control techniques, such as the nonlinear control, robust control and decentralized control. Notwithstanding, fuzzy dynamic control can take into accounts more dynamic behavior of complex systems, heterogeneous control are prevailing for its simplification in industry practices. Based on the global control structure, the thesis aims to take advantages of these global control strategies and propose a unified global control design for control of complex systems. The application of proposed global control design will obtain the following specific objectives:

1. Global system models can represent the dynamic behavior of complex system over wide ranges of operating states. The nonlinearities, uncertainties, interconnections and abrupt dynamics of real world system can be represented by these models. The models of different control objectives and actions can also be covered if it is necessary.
2. Different control objectives and actions over wide operating regions can be coordinated in term of coordination rules of global control.

3. All the local controllers are designed based on local control signals only. However, the interactions between those local controllers can be taken into accounts and the soft switching of local controllers is realized by the membership functions.
4. Global control systems are robust and stable irrespective of the system parameter uncertainties, nonlinearities and interactions.

The proposed procedure of global control design will be applied to power systems and two classical control examples. The global control design will be used to enhance transient stability of multi-machine and SMIB power systems. In addition, the global control is about to be utilized to two benchmark examples, the ball and beam (BB) system and the cart and pole (CP) system. Finally, global control approach will be investigated to solve inherently conflicting control goals and coordinate multiple controllers in power systems.

1.4 Major contributions of the thesis

The research focuses on the application of global control for complex systems with high degree of nonlinearities, uncertainties and interconnections. The major contributions of the thesis are summarized as follows:

1. The global control design to complex system is presented based on the global control structure and modern control techniques. Multiple partial models are proposed to handle practical model problems of complex systems which cannot be easily dealt with by existing control techniques, such as different operating states, nonlinearities and different control objectives. Since the complex system is represented by multiple partial models and their coordination rules, the objective of global control becomes to coordinate partial controllers designed for partial models and simultaneously guarantee the stability of control system. Accordingly, global control in application to multiple partial models can take advantages of modern control techniques and global control structure to meet the complicated requirements in system modeling, control and stabilities conditions of complex systems. Theoretical basis for global control design is extended to uncertain systems, a class of nonlinear systems and large-scale interconnected system.

2. Global control of uncertain system and large-scale interconnected system are respectively applied to an SMIB and multi-machine power systems to enhance transient stability. Multiple partial models are used to represent the power system under different states, the pre-fault state, faulty state, fault clearing and post fault state. Then, the global controllers for generator excitation in the SMIB or multi-machine power system are obtained by coordinating the partial controllers from different states. System parameters, interconnections and fault locations are all treated as the uncertainties and handled by the robust control approach in each partial model. Thus, the global control scheme can damp the oscillations regardless of the system operating points and fault locations. The stabilization of each robust partial controller for generator excitations can guarantee the overall stability of the SMIB or multi-machine power system irrespective of network parameters, fault locations and the system operating conditions.
3. Global control for a class of nonlinear systems is employed to two benchmark control systems: the BB system and the CP system. In the global control of the BB system, the BB system is represented by three partial models in view of the nonlinearities and uncertainties. Then three partial controllers are respectively designed for each partial model and the global controller are constructed by combining three partial control actions through heterogeneous control membership functions. The global control scheme improves system performance by decreasing the number of partial controllers in comparison with the fuzzy dynamic control strategies. For the global control of the CP system, two control objectives: only regulate the displacement of the pole and control the displacement of both the cart and pole, are respectively designed for according to the same design procedure as the BB system. The global controllers obtain good dynamic performance irrespective of initial operating points for both of control objectives.
4. Global control scheme is extended to uncertain power systems to coordinate different, sometimes inherently conflicting, control objectives and decentralized controllers within wide operating regions. The SMIB and multi-machine power system with SVC are utilized as example systems for global control scheme to maintain the transient stability and achieve proper post-fault voltage. In the global

control scheme, decentralized partial controllers are respectively designed for generator excitation and SVC to coordinate two conflicting control objectives: transient stabilization and voltage regulation. Power system uncertainties and controller interactions are taken into account in the design of robust partial controllers. Therefore, the global control scheme for power system with SVC coordinate decentralized SVC and generator excitation controllers, maintain the transient stability and achieve proper post-fault voltage irrespective of system parameters and fault locations.

1.5 Organization of the thesis

The thesis is organized as follows:

Chapter 1 introduces the motivation of research topic, a literature review about related control theories and applications, the objectives and major contributions of the thesis.

Chapter 2 presents multiple partial models to handle practical problems of complex systems, such as different operating states, nonlinearities, different control objectives. According to characteristics of complex system, several typical problems which cannot be easily dealt with by existing control techniques were classified with solutions of multiple partial system models. The mathematical models of example control systems and their partial system models were provided in this chapter to demonstrate the validity of the proposed global control strategies and compared with other control approaches.

Chapter 3 proposes the global control design procedure to consider complicated characteristics of complex systems. The related control techniques are reviewed to solve the nonlinearities, uncertainties and interconnection problems and coordinate various control actions in complex system. Therefore, based on the provided theoretical basis for controller design, the global control can be extended to uncertain systems, a class of nonlinear systems and large-scale interconnected system.

In Chapter 4, the proposed global control of uncertain systems is applied to the SMIB power system to enhance transient stability. The results of simulation demonstrate that global controller designed greatly enhance transient stability of SMIB system in comparison of the classical linear controller given in Chapter 2. The global control

scheme can damp the oscillations regardless of the system operating points and fault locations.

Chapter 5 utilizes the global control for a class of nonlinear systems to two benchmark control examples: the BB system and the CP system. Simulation results show that the global control of BB system could obtain good dynamic performance irrespective of various parameters and initial operating points. In comparison with the fuzzy dynamic control strategies, global control scheme improves system performance by decreasing the number of partial controllers, which greatly simplify the switching of controllers for different operating regions. For the global control of the CP system, two control objectives: only regulate the displacement of pole and both control displacement of cart and pole, are respectively designed for the CP system according to the same design procedure. Simulation results show that the CP control system could obtain good dynamic performance over wide operating regions irrespective of initial operating points for both of control objectives.

In Chapter 6, the global control of large-scale interconnected systems is applied to the multi-machine power system to enhance transient stability. Simulation results demonstrate that global controller designed greatly enhance transient stability of multi-machine power system in comparison with the classical linear controllers given in Chapter 2. The global control scheme can damp the oscillations irrespective of the system operating points and fault locations.

Chapter 7 extends the global control scheme to uncertain power systems to coordinate decentralized controllers and inherently conflicting control objectives within wide operating regions. The SMIB and three machine power system with SVC are utilized as example systems for global control scheme to maintain the transient stability and achieve proper post-fault voltage. In the systematical global control scheme, decentralized partial controllers are respectively designed for generator excitation and SVC to coordinate two conflicting control objectives: transient stabilization and voltage regulation. Power system uncertainties and partial controller interactions are taken into account in the design of robust partial controllers.

Chapter 8 concludes the thesis and provides suggestions for future research.

Chapter 2 System Modeling of Global Control

2.1 Introduction

The proper system models and systematical control design which can be adapted to practical systems are attractive issues in control of complex system. In the analysis, design and synthesis of real life problems, the first step is the development of mathematical model which maybe a substitute for real problem. As the characteristics of real life are time-varying and diverse, it is unlikely to use only one mathematical model to describe these systems.

Multiple local models have been widely applied to approximate the dynamic of actual system in many control methods for complex systems, such as heterogeneous control, fuzzy control, switch control and their derivation, global control. The advantage of this approach is that the overall complexity of the collection of local models is often significantly less than that of the corresponding global model necessary to describe the dynamics over the entire operating range. However, these local linear models are only valid in a neighborhood of an operating point, and hence, only describe the behavior of the actual system in vicinity. As the complexity of system increases, it is required plenty of local linear models to describe a complex system. In addition, some system phenomena cannot be captured by a set of linear equations. For example, some natural nonlinear effects are discontinuous and the response from these effects cannot be approximated by a set of linear functions [6].

To avoid these inherent defects of local linearized models, **multiple partial models**, a set of nonlinear mathematical models, are proposed to describe complex systems in the global control in the thesis. Note that those models can be applied to the large nonlinear system generally. In comparison with the local linear models, these partial models are valid over a wide operating region and represent much richer dynamics in complex system. Consequently, various methodologies and tools for nonlinear control design and analysis can be employed to those partial nonlinear models. In this thesis, multiple partial

system models are applied to solve some practical control problems of complex systems in Section 2.2.

Moreover, this chapter covers all mathematical models used to demonstrate the validity of the proposed global control strategies in the following chapter. The typical complex system, power system, is considered as a challenging test system. As mechanism of the FACTS provide flexible and multiple compensations in power systems, the most popular FACTS device, Static Var Compensation (SVC), is investigated to coordinate with other controllers in the design of global control system. Further, an SMIB system model is also presented to deal with preliminary investigation on global controller design for simplicity. Finally, two nonlinear benchmark control systems, the Ball and Beam (BB) system and the Cart and Pole (CP) system, are represented by state space system models with rich nonlinearities. Those models are employed to demonstrate the effectiveness of proposed multiple partial models for complex systems.

The remainder of this chapter is organized as follows: some typical dynamics of complex system and corresponding global control models are proposed in Section 2.2. The mathematical models of multi-machine and an SMIB power system based on the work ([13], [19], [70] and [71]) are presented in Section 2.3. The conventional linear excitation controller is also reviewed as the comparison of proposed global control schemes introduced in the next chapter. The dynamic models and basic control of SVC are discussed as well. Finally, Section 2.4 provides two benchmark system models, the BB system and the CP system.

2.2 Complex system and global control models

Complex systems are characterized by uncertain models in wide operating regions, high degree of nonlinearities, various interconnections from geographically distributed subsystems, and abrupt changes in dynamics. Some of these nonlinear phenomena can take place only in presence of nonlinearities. Hence, they cannot be described or predicted by linear models. In this thesis, multiple partial models are applied to represent some of nonlinear phenomena of complex systems.

Unlike the linear systems which have one isolated equilibrium state, complex system may possess more than one stable equilibrium states. A possible equilibrium state is

determined by a system's parameter, initial conditions and magnitudes and forms of external excitations [6]. As the system's parameter, initial condition or excitation is changed, the trajectory of the systems may diverge to other equilibrium states. Thus, it is not unusual that the complex system exhibit the transformation of two or more equilibrium states. For example, a typical real life complex system, power systems operate over wide range of states due to dynamics of system components. An increase in load or severe disturbances can bring about onset of dramatically changing the structures of power systems and end up with different operating states. Power systems are conceptually classified the system operating conditions into five states: normal, alert emergency, in extremis, and restorative [72]. Figure 2.1 depicts these operating states and the ways in which transition can take place from one state to another.

Figure 2.1 Power system operating states

Multiple partial models can be applied to represent above five operating states of N machine power systems:

$$\dot{x}_i(t) = (A_{i,l} + \Delta A_{i,l}(t))x_i(t) + (B_{i,l} + \Delta B_{i,l}(t))u_{i,l}(t) + \sum_{j=1}^N \sum_{k=1}^{T_{kj,l}} \{[G_{kij,l} + \Delta G_{kij,l}(t)]g_{kij,l}(x_i, x_j)\} \quad (2.1)$$

where l ($l=1,2..5$) is the number of partial models for the i th ($i=1, 2, \dots, N$) machine in power systems; $x_i(t) \in \mathbb{R}^n$ and $u_{i,l}(t) \in \mathbb{R}^n$ is the state variables and input variables of the i th machine; the matrices $A_{i,l}$, $B_{i,l}$ and $G_{kij,l}$ are known real constant matrices of

Chapter 2 System modeling of Global Control

appropriate dimension that describe the nominal model of the l th partial models, $\Delta A_{i,l}$, $\Delta B_{i,l}$, and $\Delta G_{kij,l}$ are real time varying parameter uncertainties of the l th partial models; k is the number of the nonlinearities and interconnection in i th machine ($k = 1, 2, \dots, T_{jk}$); $g_{kij,l}(x_i, x_j)$ are unknown nonlinear vector functions that represent nonlinearities of the i th machine and interactions between the i th machine and j th machine in the l th partial models. These multiple partial models for multi-machine and SMIB power system will be shown in Section 2.3.

Except for advantage of representing different operating states of complex systems, multiple partial models are utilized to solve certain nonlinearity classes as well. Consider the following complex systems:

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) + f_1(x, t, \Delta) + f_2(x_i, x_j, t, \Delta) \quad (2.2)$$

where Δ represents the model uncertainties; $f_1(x, t, \Delta)$ is bounded time varying nonlinear functions and there exist a known constant matrix W_1 such that the nonlinear function $f_1(x, t, \Delta)$ satisfies that $\|f_1(x, t, \Delta)\| \leq \|W_1\| \|x\|$. $f_2(x_i, x_j, t, \Delta)$ is also a nonlinear function satisfying $\|f_2(x_i, x_j, t, \Delta)\| \leq \|W_2\| \|x_i\|^m \|x_j\|^n$. where m and n are integers, W_2 is a constant matrix, x_i and x_j are i th and j th components of state vector x . Therefore, $f_2(x_i, x_j, t, \Delta)$ can represent the nonlinearities related with any state variables in complex systems.

For the system with bounded nonlinearities $f_1(x, t, \Delta)$, various nonlinear control theory and tools have been developed to handle it with precise performance requirements. For example, robust control theory ([6], [24]-[26]), H_∞ ([24], [25] and [52]), LMI ([40], [48] and [52]) and adaptive ([31], [33], [54] and [55]) control techniques has successfully employed to design a stabilizing feedback controller for the nonlinear system with bounded time varying nonlinearities and uncertainties. For all these design of feedback controllers, the stability analysis of systems are based on Lyapunov's methods and bounded parameters in the system models are necessary presumptions to achieve stabilization of whole systems.

It should be noted there lack of systematical methods to deal with nonlinear function $f_2(x_i, x_j, t, \Delta)$ in (2.2) compared with these bounded nonlinearities $f_1(x, t, \Delta)$. The common approach to handle nonlinear function $f_2(x_i, x_j, t, \Delta)$ is to approximate the system models over certain stable operating points, such that the classical control theory can analyze the segmental dynamics of the system and provide high resolution controls. As a result, it is difficult to describe precise local behaviors of complex systems if the complexity of systems increases. In the global control of complex system (2.2), the nonlinear function $f_2(x_i, x_j, t, \Delta)$ is supposed to be locally bounded by decomposing system into several partial models according to different characteristics. Therefore, Lyapunov based nonlinear tools can be employed to design feedback controllers for these partial models with bounded parameters. As an illustration for multiple partial models in global control design, the typical nonlinear functions $f_1(x, t, \Delta)$ and $f_2(x_i, x_j, t, \Delta)$ are considered in benchmark control examples in Section 2.4.

Multiple partial models can avoid the abrupt dynamics and instant noises of complex systems in the global control design. There are many abrupt changes of system dynamics in real life systems, such as relay switches and the power system structures changes for the sake of break operations. These discontinuities in real system are difficult to be taken into account if the complex systems are considered over whole operating regions. Fortunately, above problems can be solved by multiple partial models in global control design for complex system.

From the viewpoint of global control objectives in practical systems, multiple partial models can also be applied to represent system models for different control objectives based on the work from ([1] and [16]). References [1] and [16] proposed a preliminary implementation of global control for SMIB power system, where the system models for transient stability enhancement differ from the one for voltage regulation. In the design of transient controller, $[\Delta\delta(t) \ \Delta\omega(t) \ \Delta P_e(t)]^T$ are chosen as states of the system model; on the other hand, $[\Delta V_i(t) \ \Delta\omega(t) \ \Delta P_e(t)]^T$ are states of system model for voltage controller. As a result, different models are adopted to coordinate conflicting control objectives in the global control design. In this thesis, existing controllers designed for different control objectives are coordinated and system uncertainties and controller

interactions are taken into account as well. The proposed global control scheme is applicable in practice.

In the following Sections, a typical real complex system, large-scale power systems, is used as an example to investigate different nonlinearity and uncertainty ranges over wide operating regions and dynamic discontinuities in complex system. Moreover, two benchmark control examples, the BB system and the CP system, are utilized to demonstrate the global control design to deal with complicated uncertainties and nonlinearities in complex system models.

2.3 Modeling and classical control schemes in power systems

The practical power systems are considered as challenging test systems since they provide computational and analytical difficulties in modeling, control and stability analysis for their high dimensionalities, nonlinearities and complexities. Mathematical models of a power system consist of differential and algebraic equations representing the models of system components, including generators, transformers, buses, lines, load and FACTS devices. Since all components in power system presents a rich source of nonlinearities and experience dynamic variations by nature, there is no a unified model to represent dynamic characteristic of power system over whole operating regions and time scales. Therefore, it is necessary to choose proper system models for different operating states and control objectives of various time frames in the control of power systems.

In this section, the multi-machine system model is utilized to represent the dynamics of large scale power system. The generator excitation control and power system stabilizers (PSS) used in the practical power systems are presented as well. The control of FACTS devices, which provide flexible and multiple compensations in power systems, are also taken into account to coordinate with synchronous machines from a viewpoint of global control. Dynamic model of Static Var Compensation (SVC) is investigated in details.

2.3.1 Multi-machine power system

Multi-machine power system is a typical large-scale interconnected nonlinear system, where an N machine power system consists of N independent dynamic subsystems interconnected together. Thus the N machine system is reduced to an electrical network

with N internal machine buses. The classical synchronous model is used in the multi-machine power system ([13], [19] and [71]).

The important equations in the analysis of synchronous machine are the rotational inertia equations describing the effect of unbalance between the electromagnetic torque and the mechanical torque of the machine. The motion of the rotor of the i th ($i = 1, 2, \dots, n$) machine is given by the swing equation:

Mechanical equation:

$$\Delta \dot{\delta}_i(t) = \Delta \omega_i(t) \quad (2.3)$$

$$\Delta \dot{\omega}_i = -\frac{D_i}{2H_i} \Delta \omega_i(t) - \frac{\omega_0}{2H_i} (P_{ei}(t) - P_{mi}) \quad (2.4)$$

To obtain a time solution for the rotor angle, the expression of the mechanical and the electrical power is needed to develop. However, the complete electrical dynamics of generator is very complicated, including the dynamics of stator windings, field windings, damping winding, exciter and governor. So the most controller designs use an approximate linearized one-axis generator model. This simplification requires the following assumptions ([13] and [20]):

1. The mechanical power input remains constant during the period of the transient.
2. Damping or asynchronous power is negligible.
3. The synchronous machine can be represented by (electrically) by a voltage source behind a transient reactance.
4. The mechanical angle of the synchronous machine rotor coincides with the electrical phase angle of the voltage behind transient reactance.
5. The only electrical transient considered in the study is the varying $E'_q(t)$ and $E'_d = 0$.
6. The transient saliency is ignored, i.e., $X'_d = X'_q$.
7. The static exciter is simply modeled by a constant gain and its excitation limits as the time-constant of the exciter is relatively short.

8. Assume the exciter is to be supplied from a compensated supply so that its excitation function is not impaired during the fault period.

9. Load are replaced by constant shunt admittances

10. Network is reduced to the machine internal buses by eliminating all physical buses of the system

Under these assumptions, the electrical dynamics of synchronous machine with generator excitation and multi-machine network are represented as:

Electrical dynamics of synchronous machine:

$$\dot{E}'_{qi} = (E_{fi}(t) - E_{qi}(t)) \frac{1}{T'_{doi}} \quad (2.5)$$

Electrical equation:

$$E_{qi}(t) = E'_{qi}(t) - (X_{di} - X'_{di})I_{di}(t) \quad (2.6)$$

$$E_{fi}(t) = k_{ci}u_{fi}(t) \quad (2.7)$$

$$P_{ei}(t) = \sum_{j=1}^n E'_{qi}(t) E'_{qj}(t) B_{ij} \sin(\delta_{ij}(t)) \quad (2.8)$$

$$Q_{ei}(t) = -\sum_{j=1}^n E'_{qi}(t) E'_{qj}(t) B_{ij} \cos(\delta_{ij}(t)) \quad (2.9)$$

$$I_{qi}(t) = \sum_{j=1}^n E'_{qj}(t) B_{ij} \sin(\delta_{ij}(t)) \quad (2.10)$$

$$I_{di}(t) = -\sum_{j=1}^n E'_{qj}(t) B_{ij} \cos(\delta_{ij}(t)) \quad (2.11)$$

$$E_{qi} = X_{adi} I_{fi}(t) \quad (2.12)$$

Note that the changes of electric power network structure during different operating states and the presence of FACTS device will make the nodal susceptance B_{ij} a variable parameter. The dynamics of FACTS controller are viewed as part of susceptance change in the analysis of power system network.

Remark 2.3.1: From above equation (2.3)-(2.11), it can be derived that the system model of i th machine in N machine power system can be represented by a set of nonlinear mathematical models. When the multi-machine power system is subjected to a severe disturbance, it will experience at least three different operating states: the pre-fault steady state, faulty state, fault clearing steady state and post fault steady state. The multiple partial models in Equation (2.1) can be used to describe those different operating states. The l th partial model respectively represent the pre-fault, faulty, fault clearing and post fault state of multi-machine power systems as l was set to 1, 2, 3 and 4.

Multiple partial models for different control objectives are adopted in order to achieve global control objectives. For example, the state space power system model for transient stability study differs from the one for voltage regulation. In the design of transient controller, $[\Delta\delta_i(t) \ \Delta\omega_i(t) \ \Delta P_{ei}(t)]^T$ are chosen as states of the system model; on the other hand, $[\Delta V_i(t) \ \Delta\omega_i(t) \ \Delta P_{ei}(t)]^T$ are states of system model for voltage controller.

2.3.2 Generator excitation control and power system stabilizer (PSS)

The excitation system is one of the control element which helps provide synchronous operation of the generators. Excitation control helps the generator hold the flux level and increase its output power under certain disturbances. In response to the large disturbances, the automatic voltage regulator (AVR), the core of the excitation system, is actually destabilizing the generators [13]. This drawback is circumvented with the use of PSS, which adds damping to the generator oscillation by controlling its excitation using auxiliary stabilizing signal(s). The effect of PSS is quite pronounced on the oscillations after the first swing caused by large disturbances. The function block diagram of IEEE type ST1A exciter and PSS shown in Figure 2.4 ([72] and [73]) are used as a linear control scheme as a comparison of proposed global control schemes in the thesis. The wind or non-windup limits and gating function in the diagram represent the limiting and gating or auctioneering action in the physical device. Parameters of the linear excitation controller used in the simulation are given in Appendix A.1 ([19] and [20]).

Figure 2.2 The excitation system with ST1A and PSS

As seen from Figure 2.2, a typical excitation system is a cascade of simple linear filters and protective functions to ensure that the capability limits of the exciter are not exceeded. The design of excitation system is based on a linear approximation of a nonlinear power plant at a specific operating point. However, the operating point can drift because of changes in the system, such as load changes to the system or disturbances. For the large disturbance, such as a short circuit fault, the response of exciter builds up from normal excitation to ceiling excitation. The limiting and gating circuits of the excitation system will come into play. Since those physical limits and saturation are usually ignored for simplification in the design of excitation control system. Those nonlinear problems make the operating point of the system vary considerably beyond the equilibrium point of the linear excitation controllers. The resulting excitation system may produce undesirable behavior, such as integrator wind-up, large overshoots, long setting times, and even instability when system experiences a large disturbance. Thus the linear excitation controller will not stabilize the system under such severe disturbance.

2.3.3 Static Var Compensator (SVC)

The configuration of SVC

The SVC is a shunt FACTS that provides controllable shunt capacitive or inductive reactive power using conventional thyristor control. Many hundreds of SVCs are currently in service throughout the world. The configuration of a one-line diagram of a typical SVC is shown in Figure 2.3. The components of SVC usually include the following elements ([73] and [75]):

Figure 2.3 Typical SVC configuration [73]

1. The step-up transformer connected to a high voltage (HV) AC transmission line.
2. Thyristor controlled reactor (TCR), which consists of a fixed reactor of inductance and a bi-directional thyristor valve.
3. Thyristor switched capacitor (TSC), which consists of a capacitor, a bi-directional thyristor valve and a relatively small surge current limiting reactor.
4. The fixed capacitor (FC), which provides a permanently connected source of reactive power.
5. Harmonic filters used to remove harmonic currents caused by thyristor control.
6. Control system, which generates the firing angle for TSC and TCR.

Mathematical model of SVC in power system

Because the function of TSC is to switch capacitors and minimize transient, SVC in effect is a combination of FC and TCR, which is equivalent to a variable shunt susceptance:

$$B_{svc}(t) = B_C - B_L(t) \quad (2.13)$$

where the susceptance of TCR $B_L(t)$ is a time varying reactor controlled by the firing angle α of the thyristor. The steady state relationship between them is as follows:

$$B_L(t) = \frac{B_s(2\pi + \sin 2\alpha - 2\alpha)}{\pi} \quad (2.14)$$

Since SVC are usually designed for two control objectives, voltage regulation and transient stability enhancement, which are inherent conflicting. The different mathematical models are applied for these two control objectives.

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In order to derive the general voltage relationship between SVC and power system, a three-bus system shown in Figure 2.4 is considered, where a FC-TCR model represents SVC for simplification. It can resemble any case where power is transmitted through a transmission line and SVC is located in the middle of the line. X_S and X_R represent the reactance of the transmission lines, B_S is the full susceptance of the TCR of SVC, $V_S \angle \delta_1$, $V_B \angle \delta_B$ and $V_R \angle \delta_2$ represent bus voltage.

Figure 2.4 Three bus SVC system

Considering the voltage phase angle of SVC bus δ_B as the reference and neglecting the voltage phase angle variations, the flowing equations can be obtained to describe this three-bus system ([20], [76] and [77]):

$$l_1 \frac{di_1(t)}{dt} = V_S(t) \cos(\delta_1(t) - \delta_B(t)) - V_B(t) \quad (2.15)$$

$$l_2 \frac{di_2(t)}{dt} = V_R(t) \cos(\delta_2(t) - \delta_B(t)) - V_B(t) \quad (2.16)$$

$$C \frac{dV_B(t)}{dt} = i_C(t) \quad (2.17)$$

$$L(t) \frac{di_L(t)}{dt} + i_L(t) \frac{dL(t)}{dt} = V_B(t) \quad (2.18)$$

$$i_1(t) + i_2(t) = i_C(t) + i_L(t) \quad (2.19)$$

where L_1 and L_2 are line inductances of X_S and X_R ; C is the capacitance of SVC; $L(t)$ is the inductance of SVC; $i_1(t)$, $i_2(t)$, $i_C(t)$ and $i_L(t)$ are currents through X_S , X_R , B_C and $B_L(t)$, respectively. Differentiate Equations (2.17) and (2.19) and get

$$C \frac{d^2 V_B(t)}{dt^2} = \frac{di_C(t)}{dt} \quad (2.20)$$

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di_C(t)}{dt} + \frac{di_L(t)}{dt} \quad (2.21)$$

In order to maintain the voltage V_B at the SVC bus, the following equation is established from above equation (2.15) - (2.21):

$$\frac{d^2 \Delta V_B(t)}{dt^2} = -\frac{1}{B_C} \left(\frac{1}{X_S} + \frac{1}{X_R} + B_L(t) \right) \Delta V_B(t) - \frac{i_L(t) \dot{B}_L(t)}{B_C B_L(t)} + d(t) \quad (2.22)$$

where $\Delta V_B(t) = V_B(t) - V_{B0}$, $\dot{B}_L(t) = (i_C(t) - i_{C0}) / B_C$, V_{B0} is the SVC bus voltage at the operating point; $i_L(t)$ is the current through the inductor in SVC; $i_C(t)$ and i_{C0} are the capacitor current and the one at the operating point in the SVC respectively which are readily measurable in practice, and $d(t)$ can be treated as an external disturbance which can be described by the following equation:

$$d(t) = \frac{V_S(t) \cos(\delta_1(t) - \delta_B(t)) - V_{S0} \cos(\delta_{10} - \delta_{B0})}{B_C X_S} + \frac{V_R(t) \cos(\delta_2(t) - \delta_B(t)) - V_{R0} \cos(\delta_{20} - \delta_{B0})}{B_C X_R} \quad (2.23)$$

where δ_{10} , δ_{20} , δ_{B0} are the phase angles of bus 1, bus 2 and the SVC bus at the operating point. In above simple three-bus SVC system, all the parameters related with the transmission line are known, so $B_L(t)$ is the only variable to control.

For another function of SVC, transient stability improvement, the relative speed at the SVC bus $\Delta \omega_L$ instead of bus voltage V_B is considered as control signal of SVC mathematical model.

SVC control scheme

The SVC dynamic model can be described by ([20], [76] and [77]).

$$\dot{B}_L(t) = \frac{1}{T_R} (-B_L(t) + B_{L0} + k_B u_B(t)) \quad (2.24)$$

The output of the SVC is physically limited by its rating as $B_{L\min} \leq B_L(t) \leq B_{L\max}$ where $B_{L\min}$ and $B_{L\max}$ are the minimum and maximum value of the reactor susceptance, respectively. The parameters and physical limits of SVC used in the thesis are given in Appendix A.2.

The control of TCR in the FC-TCR type SVC needs to provide following basic functions, as shown in Figure 2.5.

Figure 2.5 Functional control scheme for FC-TCR type SVC

One important function is generating the firing angle from SVC controller. The design of a control method, such as PID or nonlinear control, provides a relationship between the control signals and the firing angle α of TCR. There are two factors need to be considered in the controller application:

1. The dynamic characteristic of the susceptance $B_L(t)$ of the TCR. The time delays T_R and its regulator gain k_B should be combined with the control law to embody the dynamics of the TCR.

2. The physical limits of the susceptance of $B_L(t)$. The current methodologies for design control system usually ignore those limits in the first stage of the control design process whenever to design a PID or nonlinear controller.

The second function is synchronism timing. A phase locked loop (PLL) circuit that runs in synchronous with the ac system voltage and generates appropriate timing pulses with respect to the peak of that voltage is usually used to provide this function. When system experience faults or severe disturbance in transient stability study, voltage exhibits wild

fluctuations and large distortion, which make PLL become necessary to provide synchronous time for generating firing angle.

The firing pulse generators provide the necessary gate current pulse for the TCR according to the firing angle α provided by SVC controller and synchronous timing pulse. Measuring and processing circuit and reference signals vary with different control strategies and SVC controller designs.

In the general control scheme for FC-TCR type SVC, it is necessary to design different control laws for corresponding control objectives. It was demonstrated that SVC voltage control and damping control, in fact, conflict with each other and one function performed always at the price of sacrificing the performance of another function [18]. The design of controller that can realize both of functions is a practical issue.

2.3.4 Single machine infinite bus (SMIB) power system

For simplicity of study of power system, an SMIB system model is widely used to deal with preliminary investigation on transient stability problems as well as the validity of controller design. Because the relative size of the system with which the machine dynamics associated is far larger than that of machine there will be no change in the voltage and frequency of the large system caused by the machine. Usually, an infinite bus represents the large system in the transient stability study. The dynamic behavior of the synchronous machines in the power system can be equivalent to a machine connected to a large system through transmission lines. The assumption of an SMIB system may usually be valid for faults around low-inertia system that are connected to a high inertia external system by long tie lines.

Figure 2.6 The SMIB system with SVC for transient stability enhancement

An SMIB system model with SVC is shown in Figure 2.6, where SVC is located at the mid-point of a transmission line to maximize its impact on the transmission line impedance and voltage control, and consequently, obtain the maximum adjustable range of the transmitted power. The parameters of the SMIB power system used in the thesis are given in Appendix A.3.

In the simulation there are two symmetrical three phase short circuit faults are considered Fault 1 occurs at the terminal of generator bus.

Fault 2 occurs at the midpoint of the transmission line between the generator bus and SVC bus.

Modeling of Synchronous Machine:

The electrical dynamics of synchronous machine in SMIB system are as same as ones in the multi-machine system and can be represented as ([13], [19] and [70]):

Mechanical Equation:

$$\Delta \dot{\delta}(t) = \Delta \omega(t) \quad (2.25)$$

$$\Delta \dot{\omega} = -\frac{D}{2H} \Delta \omega(t) - \frac{\omega_0}{2H} (P_e(t) - P_m) \quad (2.26)$$

Electrical Dynamics:

$$\dot{E}'_q = (E_f(t) - E_q(t)) \frac{1}{T_{do}} \quad (2.27)$$

Power system model with SVC

The steady state equations for power system shown in Figure 2.6 with SVC are as follows:

$$E_q(t) = \frac{X_{ds} E'_q(t) - (X_d - X'_d) V_s \cos(\delta(t))}{X'_{ds}} \quad (2.28)$$

$$E_f(t) = k_c u_f(t) \quad (2.29)$$

$$P_e(t) = \frac{V_s E_q(t)}{X_{ds}} \sin(\delta(t)) \quad (2.30)$$

$$I_q(t) = \frac{V_s}{X_{ds}} \sin(\delta(t)) = \frac{P_e(t)}{X_{ad} I_f(t)} \quad (2.31)$$

$$I_d(t) = \frac{E'_q(t) - V_s \cos(\delta(t))}{X_{ds}} \quad (2.32)$$

$$Q_e(t) = \frac{V_s E'_q}{X_{ds}} \cos(\delta(t)) - \frac{V_s^2}{X_{ds}} \quad (2.33)$$

$$E_q = X_{ad} I_f(t) \quad (2.34)$$

$$V_t(t) = \frac{\sqrt{X_s^2 E_q^2 + V_s^2 X_d^2 + 2 X_s X_d X_{ds} P_e(t) \cos(\delta(t))}}{X_{ds}} \quad (2.35)$$

$$X_E(t) = \frac{2(X_{l1} + X_{l2}) - X_{l1} X_{l2} B_{SVC}(t)}{4} \quad (2.36)$$

where $X_{ds} = X_d + X_T + X_E$; $X'_{ds} = X'_d + X_T + X_E$ and X_E is the Thevenin's equivalent reactance of the network to the right of bus 1 as shown in Figure 2.6. X_E is a time varying parameter because of the change of the susceptance of SVC and the structure change of power systems. $B_{SVC}(t) = 0$ while SVC do not provide injected power.

Remark 2.3.2: According to the above models, SMIB power system is actually a special case of multi-machine power system where $i=1$. Similar with multi-machine power systems, the multiple partial models in Equation (2.1) can be used to describe the SMIB power system over different operating states and designed for different control objectives.

2.4 Control systems with complicated nonlinearities

Two nonlinear benchmark control systems, the BB system and the CP system, are investigated as two example complex systems in (2.2).

2.4.1 The Ball and Beam (BB) system

The BB system is one of classical benchmark examples which evoke high interest in creative nonlinear control designs. The familiar BB experiment is depicted in Figure 2.7. The beam is made to rotate in a vertical plan by applying a torque at the center of rotation

and the ball is free to roll along the beam. It required that the ball remains in contact with the beam and that the rolling occurs without slipping [25].

Figure 2.7 The ball and beam system

Let the moment of inertia of the beam be J , the mass and moment of inertia of the ball be M and J_b , respectively, the radius of the ball be R , and the acceleration of gravity be G . Define $B := M / (J_b / R^2 + M)$.

Choosing the beam angle θ and the ball position r as generalized position coordinates for the system, the torque applied to the beam τ as a new input u and the state variables $x = [x_1 \ x_2 \ x_3 \ x_4]^T := [r \ \dot{r} \ \theta \ \dot{\theta}]^T$, the BB system can be written in state space form as [25]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B(x_1 x_4^2 - G \sin x_3) \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \quad (2.37)$$

$y = x_1$ and $y := r$.

The system parameters used for the simulation are $M = 0.05\text{kg}$, $R = 0.01\text{m}$, $J = 0.02\text{kg/m}^2$, $J_b = 2 \times 10^{-6}\text{kg/m}^2$, $G = 9.81\text{m/s}^2$ (thus $B = 0.7143$).

The control goal the BB system is to design the feedback control law $u(x)$ such that the closed-loop system output y will converge to zero from certain initial conditions.

Figure 2.8 Nonlinear function $f_1(x_1, x_4) = x_1 x_4^2$

In the model of BB system, there are two critical nonlinearities $\sin x_3$ and $x_1 x_4^2$. It is obvious that $\|\sin x_3\| \leq \|x_3\| = \|W_1\| \|x\|$ while $W_1 = [0 \ 0 \ 1 \ 0]$ thus it belongs to the nonlinearity $f_1(x, t, \Delta)$ of the complex system (2.2). Regarding the nonlinearity $f_1(x_1, x_4) = x_1 x_4^2$ in the BB system, it satisfies that $\|f_1(x_1, x_4)\| \leq \|W_2\| \|x_i\|^m \|x_j\|^n$ where $i=1$, $j=4$, $m=1$ and $n=2$ in the complex system (2.2). The 3D graph of the nonlinearity $f_1(x_1, x_4)$ shown in Figure 2.8 illustrates the relationship of nonlinearities $f_1(x_1, x_4)$ with respect to the variables x_1 and x_4 . It is shown that $x_1 x_4^2$ cannot be bounded in the whole region, but if the BB system is decomposed into the following three spaces:

$$S_1 = \{x \mid |x_4| \leq 0.4\};$$

$$S_2 = \{x \mid x_1 x_4 > 0, |x_4| > 0.4\};$$

$$S_3 = \{x \mid x_1 x_4 < 0, |x_4| > 0.4\};$$

The nonlinearity $x_1 x_4^2$ is bounded in S_1 , and it is treated as bounded uncertainty in partial model. In the operating regions S_2 and S_3 , nonlinearity $x_1 x_4^2$ can be linearized since nonlinearity $x_1 x_4^2$ are monotone in both regions.

2.4.2 The Cart and Pole (CP) System

The CP system shown in Figure 2.9 is a common benchmark problem in the control systems literature. It consists of a cart that must be moved using the force $u(t)$ so that the pole remains in the upright position and the cart returns to the origin as $t \rightarrow \infty$. The pole is represented by a point mass attached to the end of a massless thin rod of length L that is attached to the cart at a pivot capable of unconstrained rotation. The differential equations describing the motion of the system may be obtained by writing the sum of forces in the horizontal direction and the sum of the moments about the pivot point. The effect of friction is neglected in the analysis.

Figure 2.9 The cart and pole system

The dynamics of the cart and pole system [78] can be obtained either by applying Newton's second law or by Euler-Lagrange formulation. The sum of the forces in the horizontal direction is:

$$(M + m)\ddot{y}(t) - mL \sin(\theta(t))\dot{\theta}^2(t) + mL \cos(\theta(t))\ddot{\theta}(t) = u(t) \quad (2.38)$$

where θ is the angular displacement from the equilibrium position, m is the mass of the pendulum, M is the mass of the vehicle, y is the translation of the vehicle, $u(t)$ equals the force on the cart and L is the distance from the mass m to the pivot point. g is the gravity constant. The sum of torques about the pivot point is:

$$mL \cos(\theta(t))\ddot{y}(t) + mL^2\ddot{\theta}(t) - mgL \sin(\theta(t)) = 0 \quad (2.39)$$

The following can be drawn from above two equations:

$$\ddot{y}(t) = \frac{u(t) + mL\dot{\theta}^2(t) \sin(\theta(t)) - mg \sin(\theta(t)) \cos(\theta(t))}{M + m \sin^2(\theta(t))}$$

$$\ddot{\theta}(t) = \frac{(M + m)g \sin(\theta(t)) - u(t) \cos(\theta(t)) - mL\dot{\theta}^2(t) \sin(\theta(t)) \cos(\theta(t))}{[M + m \sin^2(\theta(t))]L}$$

In the simulations, the parameters of cart and pole system are chosen as $M = 1.0 \text{ kg}$, $m = 0.1 \text{ kg}$, $L = 1.0 \text{ m}$, and $g = 9.8 \text{ m/s}^2$.

Choose the displacement of the cart y , the velocity of the cart $\dot{y}$, the angular displacement θ , and the angular velocity $\dot{\theta}$ as the state vectors, $x = [y \ \dot{y} \ \theta \ \dot{\theta}]^T$, the cart and pole system can be represented as the following state space form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \frac{mLx_4^2 \sin x_3 - mg \sin x_3 \cos x_3}{M + m \sin^2 x_3} \\ x_4 \\ \frac{(M + m)g \sin x_3 - mLx_4^2 \sin x_3 \cos x_3}{[M + m \sin^2 x_3]L} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M + m \sin^2 x_3} \\ 0 \\ -\frac{\cos x_3}{[M + m \sin^2 x_3]L} \end{bmatrix} u(t) \quad (2.40)$$

The following two control objectives are considered in the global controller design:

1. The cart is required to return to the origin and the pole remains in the upright position from initial conditions when the force $u(t)$ is applied on the cart. From the viewpoint of state space, it means that state trajectories of the CP system will be stable on the origin $x = [0 \ 0 \ 0 \ 0]$.

2. The pole is required to remain in the upright position from initial conditions as the force $u(t)$ is applied on the cart.

In the model of the CP system (2.40), there are several nonlinearities:

$\frac{mLx_4^2 \sin x_3}{M + m \sin^2 x_3}$, $\frac{mg \sin x_3 \cos x_3}{M + m \sin^2 x_3}$, $\frac{(M + m)g \sin x_3}{[M + m \sin^2 x_3]L}$, $\frac{mLx_4^2 \sin x_3 \cos x_3}{[M + m \sin^2 x_3]L}$, $\frac{1}{M + m \sin^2 x_3}$ and $\frac{\cos x_3}{[M + m \sin^2 x_3]L}$. It is obvious that nonlinearities $\frac{mg \sin x_3 \cos x_3}{M + m \sin^2 x_3}$, $\frac{(M + m)g \sin x_3}{[M + m \sin^2 x_3]L}$, $\frac{1}{M + m \sin^2 x_3}$ and $\frac{\cos x_3}{[M + m \sin^2 x_3]L}$ are bounded and belong to $f_1(x, t, \Delta)$ of complex

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system (2.2). Regarding the nonlinearities $f_2(x_3, x_4) = \frac{mLx_4^2 \sin x_3}{M + m \sin^2 x_3}$ and

$f_3(x_3, x_4) = \frac{mLx_4^2 \sin 2x_3}{2[M + m \sin^2 x_3]L}$, they satisfy $\|f_2(x_i, x_j, t, \Delta)\| \leq \|W_2\| \|x_i\|^m \|x_j\|^n$ where $i=4$,

$j=3$, $m=2$ and $n=1$ in the complex system (2.2). The 3D graph of the nonlinearities $f_2(x_3, x_4)$ and $f_3(x_3, x_4)$ shown in Figures 2.10 and 2.11 respectively illustrate the relationship of nonlinearities with respect to the variables x_3 and x_4 . It is shown that $f_2(x_3, x_4)$ and $f_3(x_3, x_4)$ cannot be bounded in the whole region, but if the CP system is decomposed into the following two subspaces:

$$S_1 = \{x \mid |x_4| \leq 1\};$$

$$S_2 = \{x \mid |x_4| > 1\};$$

Figure 2.10 Nonlinear function $f_2(x_3, x_4) = \frac{mLx_4^2 \sin x_3}{M + m \sin^2 x_3}$

The nonlinearities $f_2(x_3, x_4)$ and $f_3(x_3, x_4)$ are bounded in S_1 , and it can be treated as bounded uncertainties in partial model. In the operating region S_2 , nonlinearities $f_2(x_3, x_4)$ and $f_3(x_3, x_4)$ can be linearized at the certain operating point since nonlinearities $f_2(x_3, x_4)$ and $f_3(x_3, x_4)$ are monotone with respect to the variables x_4 .

Figure 2.11 Nonlinear function $f_3(x_3, x_4) = \frac{mLx_4^2 \sin 2x_3}{2[M + m \sin^2 x_3]L}$

2.5 Concluding remarks

This chapter presented multiple partial models to handle practical problems of complex systems, such as different operating states, nonlinearities, different control objectives. According to characteristics of complex system, several typical problems which cannot be easily dealt with by existing control techniques were classified with solutions of multiple partial system models in Section 2.2. Then mathematical models of example control systems and their partial system models were provided in this chapter to demonstrate the validity of the proposed global control strategies and compared with other control approaches in the following chapters.

The power systems models were chosen in consideration of controller design and model accuracy. The dynamic model of generators was simplified as a third order system under certain assumptions, and the classical generator excitation controller was explained to compare with proposed global controllers in the following chapters. Except for the multi-machine power systems, the model of Static Var Compensation (SVC), was also investigated to coordinate different controllers with various control objectives in power systems in Chapter 7. Furthermore, the mathematical models of the SMIB system was provided to deal with preliminary investigation of global controller design.

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In additions, two nonlinear benchmark control systems, the BB system and the CP system, were given to compare with fuzzy control strategies in Chapters 5. The control objectives and partial system model of those two systems were given in state space forms.

Chapter 3 Global Control Design

3.1 Global control design procedure

Global control refers to the control of complex system over a wide range of operating conditions to achieve a set of control objectives [1]. It aims to combine the qualitative and quantitative knowledge of complex system through some hierarchy and coordinate various control actions, which deal with all the anticipated problems and operating conditions. The global control structure had been proposed in [1], where control strategies, such as heterogeneous control, fuzzy dynamic control and global dissipativity control, are considered as global control strategies. However, as the complexity of systems increases, those existing global control strategies become insufficient in system modeling and stability conditions of controller design. As a result, the multiple partial models were proposed in Chapter 2 to extend the application of global control. Global control in application to multiple partial models can take advantages of modern control techniques and global control structure to meet the complicated requirements in system modeling, control and stability conditions of complex systems. The design of global control can follow the three stages in global control structure: identification, partial controller design and controller synthesis. Figure 3.1 depicts the detailed procedures of three stages.

Complex systems are very complicated high order nonlinear systems and their operating condition varying in wide ranges as mentioned in Chapter 2. Global control of complex system involves coordinating various control objectives over wide operating conditions and controllers of distributed subsystems. In the stage of identification, the coordination rules and nonlinear partial models are identified according to qualitative distinct operating condition and global control objectives. The state space partition is implemented to decompose complex system.

The partial controllers are respectively designed for nonlinear partial models in second stage of global controller design. The conventional or nonlinear feedback linearization technique is first applied to linearize the partial models. Then uncertainties of those

linearized models were structured and robust feedback controllers are designed for uncertain models. The interactions among different subsystems were considered as uncertainties by robust decentralized control techniques as well. Moreover, based on the coordination rules identified in the first design stage, the effective operating regions of partial controllers can be characterized by membership functions of heterogeneous control.

Figure 3.1 Global controller design procedures of complex systems

After the partial controllers and their corresponding membership functions are available, the next stage is the controller synthesis. The global controller is obtained by synthesizing the partial controllers upon their operating regions, where membership functions provide proper weighting and smooth transitions. The synthesized global controller feed into complex system to form the closed loop system. Fortunately, the global stability of this close-loop system can be guaranteed in terms of Lyapunov stability. Thus, this global controller can be utilized in the simulations of complex system.

The modern control techniques involved in above procedures are to be discussed in the following subsections. These control techniques include feedback linearization, robust control theory, robust decentralized control and heterogeneous control. In addition, the Lyapunov stability theory is provided to analyze the stability conditions of closed-loop system in the proposed global control.

3.1.1 Feedback linearization

Feedback linearization ([8] and [9]) is a popular approach used to design a control law for a system to add a term compensating for the nonlinearities in the system, making the sum linear and therefore suitable for well-understood control methods. Unlike the conventional approximately linearized techniques, i.e. Jacobian linearization, feedback linearization is achieved by exact state transformations and feedback. However, the key problem of nonlinear feedback linearization is that this approach usually demands precise knowledge of the nonlinear system.

The direct feedback linearization (DFL) is the feedback linearization techniques used in the thesis to cancel the nonlinearities of the system models. The central idea of this approach is to employ nonlinear feedback compensating law and algebraically transform a nonlinear system dynamics into a fully or partly linear one, so that linear control techniques can be applied. The fundamental application of the DFL technique for nonlinear system $\dot{x} = f(x, u)$ is explained in Figure 3.2, where x is the state vector of the controlled system, u is the original input vector and v is the new input vector to the feedback linearized system.

Figure 3.2 Fundamental of DFL technique in nonlinear controller design

DFL technique differs entirely from conventional linearization. A nonlinear system can be directly transformed to a system whose closed-loop dynamics are linear over a very wide range. This transformation is achieved by exact state transformations and feedback, rather than by linear approximations of dynamics. By comparison, the approximately linearized model developed using the Taylor expansion technique is only valid within a very small region. Linear control theories can be applied to this feedback linearized model to design an effective feedback control law. Therefore, the DFL technique can solve the control problem of nonlinear systems with linear control theories.

3.1.2 Robust control theory

Almost all the dynamic systems are uncertain and experience parameter variations by nature. Robust control theory is widely applied to eliminate those uncertainties in the design of controllers. The uncertain system models and its robust stability conditions are discussed in this section.

Consider the following parameter uncertain system:

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)u(t) \quad (3.1)$$

where $x(t) \in \mathbb{R}^n$ are state variables; $u(t) \in \mathbb{R}^m$ is the input variables of the system. (A, B) are the nominal system matrices, $\Delta A(t)$ and $\Delta B(t)$ represent uncertainties in the system matrices which are time varying.

The parametric uncertainties in ΔA and ΔB can be decomposed as follows:

$$\begin{bmatrix} \Delta A & \Delta B \end{bmatrix} = LF(t) \begin{bmatrix} E_1 & E_2 \end{bmatrix} \quad (3.2)$$

where L, E_1 and E_2 are known constant matrices which characterize the structure of the uncertainty. The uncertainty $F(t)$ satisfies $F(t)^T F(t) \leq \alpha^2 I$ ($\alpha > 0$); α is a constant, I is a unitary matrix.

The following results on the robust controller design of uncertain systems (3.1) are obtained ([24] and [26]).

Theorem 3.1.1: The uncertain system (3.1) is quadratically stable via linear control if there exists a positive definite symmetric solution matrix $P \geq 0$ for the following algebraic Riccati equation:

$$(A - BR^{-1}E_2^T E_1)^T P + P(A - BR^{-1}E_2^T E_1) + P(\alpha^2 LL^T - BR^{-1}B^T)P + E_1^T (I - E_2 R^{-1} E_2^T) E_1 + Q = 0 \quad (3.3)$$

where Q is a positive defined matrix, which can be chosen by the designer, and $R = E_2^T E_2 > 0$. In this case, a stabilizing state-feedback control law is given by

$$u(t) = -R^{-1}(B^T P + E_2^T E_1)x(t) \quad (3.4)$$

Remark 3.1.1: The above stability condition (3.3) and feedback control law (3.4) can be successfully employed to eliminate the uncertainties of above linear system (3.1). However, this approach cannot make it proper for complex system as the complicity of system increases, the bounds of uncertainty range become large such that the unified system model for the control design does not necessarily exist. For instance, a Riccati equation is not always solvable for the large uncertainty range. Thus, robust control methods may not guarantee a stabilizing solution for the large uncertainty bound.

3.1.3 Robust decentralized control

In practical complex system, decentralized control design instead of centralized one has been taken for granted for the following reasons:

1. For complex system, centralized controller is bounded to have a large number of states, inputs and outputs, which is precisely the situation classical control design algorithm cannot handle.

2. Using centralized control scheme typically requires modifying the original topology of the complex system [79]. The undesirable difficulties in complexity of design, debugging, communication network and reliabilities will make it less practical and more expensive than decentralized control design.

Therefore, robust decentralized control, decentralized adaptive control, decentralized fuzzy control and decentralized sliding mode control are presented in the recent literatures. Robust decentralized control technique is well developed in solving the uncertainty problem of complex system.

Consider complex systems described by the interconnection of n subsystems as follows:

$$\begin{aligned} \dot{x}_i(t) = & (A_i + \Delta A_i(t))x_i(t) + (B_i + \Delta B_i(t))u_{\pi}(t) \\ & + \sum_{j=1}^n \{p_{1ij}[G_{1ij} + \Delta G_{1ij}]g_{1ij}(x_i, x_j)\} + \sum_{j=1}^n \{p_{2ij}[G_{2ij} + \Delta G_{2ij}]g_{2ij}(x_i, x_j)\} \end{aligned} \quad (3.5)$$

where for the i th ($i=1, 2, \dots, n$) subsystem; x_i is the state; u_{π} is the input, the matrices A_i , B_i , G_{1ij} and G_{2ij} are known real constant matrices of appropriate dimension that describe the nominal model, ΔA_i , ΔB_i , ΔG_{1ij} and ΔG_{2ij} are real time varying parameter uncertainties; g_{1ij} and g_{2ij} are unknown nonlinear vector functions that represent nonlinearities in the subsystem and the interactions with other subsystems; and the parameter p_{1ij} and p_{2ij} are constants of either 1 or 0 (if they are 0 means that the i th subsystem has no connection with the j th subsystem).

The uncertain matrices ΔA_i , ΔB_i , ΔG_{1ij} and ΔG_{2ij} are assumed to be of the following structure:

$$[\Delta A_i(t) \quad \Delta B_i(t)] = L_i F_i(t) [E_{1i} \quad E_{2i}] \quad (3.6)$$

$$\Delta G_{1ij} = L_{1ij} F_{1ij}(t) E_{1ij} \quad (3.7)$$

$$\Delta G_{2ij} = L_{2ij} F_{2ij}(t) E_{2ij} \quad (3.8)$$

where L_i , E_{1i} , E_{2i} , L_{1ij} , E_{1ij} , L_{2ij} and E_{2ij} are constant matrices; $F_i(t)$, $F_{1ij}(t)$ and $F_{2ij}(t)$ are unknown matrix function with Lebesgue measurable elements and satisfying $F_i(t)^T F_i(t) \leq I_i$; $F_{1ij}(t) F_{1ij}(t)^T \leq I_{1ij}$ and $F_{2ij}(t) F_{2ij}(t)^T \leq I_{2ij}$.

The following assumption is made, suppose there exist known matrices $\overline{W}_{1i}$, $\overline{W}_{2i}$, W_{1ij} and W_{2ij} such that g_{1ij} and g_{2ij} are bounded as follows:

$$\|g_{1ij}(x_i, x_j)\| \leq \|\overline{W}_{1i} x_i(t)\| + \|W_{1ij} x_j(t)\| \quad (3.9)$$

$$\|g_{2ij}(x_i, x_j)\| \leq \|\overline{W}_{2i} x_i(t)\| + \|W_{2ij} x_j(t)\| \quad (3.10)$$

The following results on the robust decentralized controller of uncertain complex systems (3.5) are obtained ([19] and [20]).

Theorem 3.1.2: To solve the robust decentralized control of a class of nonlinear system model (3.5) depends on solving the following algebraic Riccati equation:

$$A_i^T P_i + P_i A_i + \overline{P_i B_i B_i^T} P_i - v_i^{-2} B_{Pi}^T R_i^{-1} B_{Pi} + v_i^2 E_{li}^T E_{li} + \sum_{j=1}^n p_{1ij} (\overline{W_{1i}}^T \overline{W_{1i}} + W_{1ij}^T W_{1ij}) + \sum_{j=1}^n p_{2ij} (\overline{W_{2i}}^T \overline{W_{2i}} + W_{2ij}^T W_{2ij}) + Q = 0 \quad (3.11)$$

where $B_{Pi} = B_i^T P_i + v_i^2 E_{2i}^T E_{2i}$; $R_i = E_{2i}^T E_{2i} > 0$; Q is a positive defined matrix, which can

be chosen by the designer; $\overline{B_i B_i^T} = v_i^{-2} L_i L_i^T + \sum_{j=1}^n p_{1ij} [G_{1ij} (I - \lambda_{1ij}^2 E_{1ij}^T E_{1ij})^{-1} G_{1ij}^T + \lambda_{1ij}^{-2} L_{1ij} L_{1ij}^T]$

$+ \sum_{j=1}^n p_{2ij} [G_{2ij} (I - \lambda_{2ij}^2 E_{2ij}^T E_{2ij})^{-1} G_{2ij}^T + \lambda_{2ij}^{-2} L_{2ij} L_{2ij}^T]$; $v_i > 0$; $\lambda_{1ij} > 0$ and $\lambda_{2ij} > 0$ ($i, j = 1, 2, \dots$

n) are scaling parameters to be chosen, with λ_{1ij} and λ_{2ij} satisfying $\lambda_{1ij}^2 E_{1ij}^T E_{1ij} < I$ and $\lambda_{2ij}^2 E_{2ij}^T E_{2ij} < I$, $\forall i, j = 1, 2, \dots n$.

If and only if there exists a stabilizing solution matrix $P_i \geq 0$ for the above Riccati equation (3.11), the complex system (3.5) is stable for all admissible uncertainties, satisfying (3.6)-(3.10), via the following decentralized feedback control law:

$$\overline{v_{fi}}(t) = -v_i^{-2} R_i^{-1} (B_i^T P_i + v_i^2 E_{2i}^T E_{2i}) x_i(t) \quad (3.12)$$

Remark 3.1.2: The above stability condition (3.11) and feedback control law (3.12) give a conservative condition to eliminate the uncertainties of above interconnected system (3.5). However, this approach can meet the same problem with robust control technique that sometimes Riccati equations are not always solvable for the large uncertainty range.

3.1.4 Heterogeneous control

Heterogeneous control to the complex dynamical systems is attractive because the application of multiple qualitative local models can make the controller design simple and intuitive, and the global behavior can be analyzed in terms of transitions among local operating regions. Although heterogeneous control shares many goals with and draw inspiration from fuzzy logic control [58], there are the following important differences between heterogeneous and fuzzy logic control.

1. Within frame work of fuzzy logic control, it is difficult to exploit, or even to relate to, the methods or results of traditional control theory. Heterogeneous control use landmark-based qualitative reasoning to combine the benefits of fuzzy control with the analysis methods of traditional control theory [58].
2. Like fuzzy logic control, heterogeneous control applies even in presence of incomplete knowledge of the structure of the system, the boundaries of the operating regions or even the control action to take. Unlike fuzzy logic control, heterogeneous control can be analyzed by an algebraic combination of classical and qualitative control laws [58]. They provide real, not fuzzy outputs, so “defuzzification” is not necessary.
3. Heterogeneous control is convenient to have local models and corresponding regions that overlap, or have gradual rather than sharp boundaries. Such regions can be described by fuzzy set membership functions [59]. The dynamical system in an overlap region is the weighted average of the overlapping local models, weighted by the values of the membership functions. However, the design philosophy behind fuzzy control typically treats overlap regions as transitions; in some case discontinuity occurs across the boundaries of different subspaces.

The heterogeneous control approach decomposed the design of a controller into two relatively independent decisions. The first is to define the specification of natural, qualitatively distinct operating regions. Then to specify a control law for each region and provide the weighted sum combination method to facilitate smooth transition from one region to another and study of global dynamic behavior. In general, the heterogeneous control law of the complex system v can be described as follows:

$$v = \sum \mu_i v_i \quad (3.13)$$

where $\sum \mu_i = 1$ and μ_i is the weighting membership function of i th controller.

v_i is the input of i th controller.

Remark 3.1.3: The heterogeneous control law thus takes the average of control laws, weighted by the operating region membership functions. Any type of controller from classical PID to modern nonlinear controllers which may involve in any nonlinear control

techniques, such as nonlinear feedback and model based control design, could be used under heterogeneous control law.

3.1.5 Lyapunov stability

Stability is the most important qualitative property of complex systems since unstable systems have no practical significance. From the beginning of the theory of differential equations, the key problem of stability becomes to obtain the system's behavior without solving differential equations. The main viewpoint of stability is to observe the asymptotic behavior of the state of the system in vicinity of the equilibrium state or oscillatory states, which is usually referred to as the Lyapunov stability [6]. In order to analyze system stability through this method, we have to construct a proper Lyapunov function candidate V and determining the conditions required to make it indeed a Lyapunov function of the closed-loop system.

The stability of a dynamical complex system is difficult to handle with in that the complexity of the analysis grows rapidly as the order of the system increases. Usually, if the complex system can be modeled as an interconnection of lower order subsystems, then the stability analysis can be deduced into two steps [8]. In the first step, the systems were decomposed into smaller isolated subsystem by ignoring interconnections, and the stability of each subsystem was analyzed through Lyapunov function respectively. In the second step, all conclusions from the first step with information about the interconnections were combined to draw conclusions about the stability of whole complex system. Therefore, the controller design and stability analysis of complex system can be solved by this classical Lyapunov synthesis approach.

For the model-based control, such as fuzzy logic control and decentralized control, the global stability criteria are established based on this classical Lyapunov synthesis approach as well. In the design of global control, complex systems are represented by multiple partial models. Consequently, global control of complex systems becomes the design of local feedback controllers and their quadratic Lyapunov functions. The stability analysis of complex system ends up with the stability of partial systems. Stability disturbance compensation, steady state performance and any other performance index guarantee the stability of the whole system.

Reviewing of above control techniques and stability analysis has basically provided the theoretical basis on global control of complex systems. Mentioned control techniques have been respectively developed to solve the nonlinearities, uncertainties and interconnection problems and coordinate various control actions in complex system. Therefore, the global control can take advantages of all those methods to meet the complicated requirements in system modeling, control and stabilities conditions of complex systems. Consequently, the systematical design and stability analysis of global control approach are gradually investigated from a simpler situation to the final complex systems: the uncertain system, a class of nonlinear systems and the large-scale interconnected system, in Sections 3.2-3.5. The global system models are presented according to different characteristics of complex systems and the corresponding design procedure of global controllers are proposed to guarantee the global system stabilization.

3.2 Global control of uncertain systems

3.2.1 Problem statement for uncertain systems

The nonlinear system is usually modeled by a finite number of first-order ordinary differential equations [10]. Thus, it can be represented as:

$$\dot{x}(t) = f(x, u, t) \quad (3.14)$$

where $x(t) \in \mathbb{R}^n$ is the vector of state variables; $u(t) \in \mathbb{R}^m$ is the vector of input variables of the system; f is a nonlinear function of (x, u, t) representing the models of the system.

In most practical situations, nonlinear systems operate in wide operating conditions. Therefore, the basic idea of global control is to decompose nonlinear systems into several partial systems according to the control objectives and characteristics of systems. We can adopt the control objectives, system uncertainty ranges and the regions of operating etc. as a criterion to decompose the whole system.

After system decompositions, nonlinear feedback linearization technique is usually applied to linearize the partial nonlinear models so that it can be algebraically transformed into a fully or partly linear one. Then the nonlinear system can be investigated by

aggregating a set of linearized partial models, which can be represented by the following l th partial models to describe the input-output relations with uncertainties:

$$\dot{x}(t) = (A_l + \Delta A_l(t))x(t) + (B_l + \Delta B_l(t))u(t) \quad l = 1, 2, \dots, m. \quad (3.15)$$

where m is the number of partial system models; $x(t) \in \mathbb{R}^n$ is state variables; $u(t) \in \mathbb{R}^m$ is the input variables of the system. A_l and B_l represent the l th linearized partial model of system, and $\Delta A_l(t)$ and $\Delta B_l(t)$ represent uncertainties and can be decomposed as the following assumption.

Assumption 3.2:

The parametric uncertainties in ΔA_l and ΔB_l can be decomposed as follows:

$$\begin{bmatrix} \Delta A_l & \Delta B_l \end{bmatrix} = L_l F_l(t) \begin{bmatrix} E_{1l} & E_{2l} \end{bmatrix} \quad (3.16)$$

where L_l , E_{1l} and E_{2l} are known constant matrices which characterize the structure of the uncertainty. The uncertainty $F_l(t)$ satisfies $F_l^T(t)F_l(t) \leq I$; I is a unitary matrix.

Control objectives:

Determine the robust feedback controllers for all the partial models (3.16) such that the closed-loop partial systems are stable and the stability of the nonlinear systems with uncertainties is guaranteed.

3.2.2 Systematical design and stability analysis of global control for uncertain system

The design of global control for uncertain systems involves several specific features, such as nonlinearities, variable uncertainties, different control objectives, and consequently control actions possible varying in different regions of operation. In order to take those aspects into account, the global control approach follows the global controller design procedure in Figure 3.1. The uncertain systems are decomposed according to the control objectives and characteristics of uncertainties. Then robust controllers are designed for different partial models respectively. Finally, the global control law is obtained by aggregating the partial controllers with heterogeneous control membership functions. The

stability conditions of the robust controller design are derived to guarantee the global stabilization of uncertain systems.

Since the global uncertain system behavior is described by aggregating the partial models with membership functions, the global control law is the weighted average of the partial control laws, where weights are provided by operating region membership functions. The proposed global control scheme for uncertain systems is illustrated in Figure 3.3.

Figure 3.3 Global control scheme for uncertain systems

It is salient that the robust controllers can be respectively designed for partial models of each subsystem based on the known results of robust control theory. However, the stability of the global controller for the uncertain system is to be discussed.

For the l th linearized partial models (3.15), the following robust controllers can be designed to guarantee the stability of close loop partial systems

$$u_l(t) = -K_l x(t) \quad (3.17)$$

Then the global control law is obtained by aggregating the partial controllers with membership functions.

$$u(t) = -\sum_{l=1}^m \mu_l(t) K_l x(t) \quad (3.18)$$

where $\mu_l(t)$ is the normalized membership function of the l th partial model and satisfy membership functions of heterogeneous control law $\sum_{l=1}^m \mu_l(t) = 1$.

Since the partial models are characterized by membership functions, it is convenient to decompose their corresponding operating regions overlapped or have gradual rather than sharp boundaries. The dynamics of the global uncertain system in an overlap region is the

weighted average of overlapping partial models, weighted by the values of the membership functions. The global closed loop uncertain system is then described by aggregating the partial models with membership functions.

$$\dot{x}(t) = \sum_{l=1}^m \mu_l(t) \{A_l + \Delta A_l(t) + (B_l + \Delta B_l(t))K_l\}x(t) \quad (3.19)$$

The stability of this global closed loop system (3.19) is then studied by using quadratic Lyapunov stability theory.

Theorem 3.2:

If the l th linearized partial model described in equation (3.15) is quadratically stable for all admissible uncertainties, satisfying Assumption 3.2, via the partial control law if there exists positive definite symmetric matrices P_l for the following algebraic Riccati equations

$$(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) + P_l (L_l L_l^T - B_l R_l^{-1} B_l^T) P_l + E_{1l}^T (I - E_{2l} R_l^{-1} E_{2l}^T) E_{1l} + Q_l = 0 \quad (3.20)$$

where Q_l are symmetric positive defined matrices, which can be chosen by the designer, $R_l = E_{2l}^T E_{2l} > 0$, In this case, the l th stabilizing state-feedback control laws are given by

$$u_l(t) = -R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) x_l(t) \quad (3.21)$$

If the linearized partial model described in (3.15) is controllable, using the corresponding state feedback control law described in (3.21), and then the global uncertain system is asymptotically stable if switching time of different partial controllers is finite and the derivative of membership function $\dot{\mu}_l(t)$ is bounded.

Proof of Theorem 3.2:

We define an energy function for l th linearized partial models:

$$V_l = x^T P_l x \quad (3.22)$$

$P_l (l = 1, 2, \dots, m)$ are positive-definite symmetric matrices, which make the l th closed loop partial model asymptotically stable. The following Lyapunov candidate is used to analyze the stability of the global closed-loop system (3.19):

$$V = \sum_{l=1}^m \mu_l x^T P_l x \quad (3.23)$$

It is obvious that $V > 0, \forall x \neq 0$ from above construction of V . The derivative of the energy function of the l th linearized partial model for uncertain systems can be represented as the following form:

$$\begin{aligned} \dot{V}_l &= \dot{x}^T P_l x + x^T P_l \dot{x} \\ &= x^T [(A_l + \Delta A_l)^T P_l + P_l (A_l + \Delta A_l)]x - x^T K_l^T (B_l + \Delta B_l)^T P_l x - x^T P_l (B_l + \Delta B_l) K_l x \\ &= x^T [(A_l + \Delta A_l)^T P_l + P_l (A_l + \Delta A_l)]x - 2x^T P_l (B_l + \Delta B_l)^T K_l x \end{aligned} \quad (3.24)$$

Substituting feedback control laws (3.21) and Assumption 3.2 yields

$$\begin{aligned} \dot{V}_l &= x^T [(A_l + L_l F_l(t) E_{1l})^T P_l + P_l (A_l + L_l F_l(t) E_{1l})]x \\ &\quad - 2x^T P_l (B_l + L_l F_l(t) E_{2l})^T R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l})x \\ &= x^T [A_l P_l + P_l A_l - 2P_l B_l R_l^{-1} B_l^T P_l - 2P_l B_l R_l^{-1} E_{2l}^T E_{1l}]x \\ &\quad + x^T [(L_l F_l(t) E_{1l})^T P_l + P_l L_l F_l(t) E_{1l} - 2P_l L_l F_l(t) E_{2l} R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l})]x \\ &= x^T [A_l P_l + P_l A_l - 2P_l B_l R_l^{-1} B_l^T P_l - P_l B_l R_l^{-1} E_{2l}^T E_{1l} - E_{1l}^T E_{2l} R_l^{-1} B_l^T P_l]x \\ &\quad + 2x^T P_l L_l F_l(t) [E_{1l} - E_{2l} R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l})]x \\ &\leq x^T [(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) - 2P_l B_l R_l^{-1} B_l^T P_l]x + \\ &\quad x^T P_l L_l F_l(t) F_l(t)^T L_l^T P_l x + x^T [E_{1l} - E_{2l} R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l})]^T [E_{1l} - E_{2l} R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l})]x \\ &\leq x^T [(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) - P_l B_l R_l^{-1} B_l^T P_l \\ &\quad + P_l L_l L_l^T P_l + E_{1l}^T E_{1l} - E_{1l}^T E_{2l} R_l^{-1} E_{2l}^T E_{1l}]x \\ &= -x^T Q_l x \leq 0 \end{aligned} \quad (3.25)$$

where $Q_l = (A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) +$

$$P_l (L_l L_l^T - B_l R_l^{-1} B_l^T) P_l + E_{1l}^T (I - E_{2l} R_l^{-1} E_{2l}^T) E_{1l}$$

Hence, the derivative of energy function $\dot{V}_l < 0$. The derivative of energy function for global uncertain system $\dot{V}$ can be given by:

$$\frac{dV}{dt} = \sum_{l=1}^m \dot{\mu}_l x^T P_l x + \frac{\partial V}{\partial x} \frac{dx}{dt} = \sum_{l=1}^m \dot{\mu}_l x^T P_l x + \sum_{l=1}^m \mu_l \dot{V}_l \quad (3.26)$$

From the definition of membership function $\mu_l(t)$, it can be obtained that $\|\dot{\mu}_l(t)\|$ are bounded and $\dot{\mu}_l(t) \rightarrow 0$ within a finite time interval because the switching time of different partial controllers are finite. It follows immediately that $\dot{V}(t) < 0, \forall x \neq 0$. The equilibrium state $x(t) = 0$ of the system is globally uniformly asymptotically stable for all admissible uncertainties.

Remark 3.2: The global stabilization of uncertain systems can be guaranteed with the assumptions in Theorem 3.2 and proper partial robust controllers. The most obvious advantage in the global controller design is that the stabilization of each partial system (3.15) can guarantee the stabilization of the global uncertain system (3.19).

3.3 Global control of a class of nonlinear system

3.3.1 Problem statement for a class of nonlinear systems

The nonlinear system with uncertainties is usually represented by the following equation:

$$\dot{x}(t) = f(x, u, t, \Delta) \quad (3.27)$$

where $x(t) \in \mathbb{R}^n$ is the vector of state variables; $u(t) \in \mathbb{R}^m$ is the vector of input variables of the system; Δ represents the model uncertainties. f , nonlinear function of (x, u, t, Δ) representing the nominal model of the system, is continuously differentiable with respect to (x, u, t, Δ) .

On the basis that the qualitative knowledge of a system can be represented in a nonlinear functional form over different operating regions, global control method decompose a

nonlinear system into several nonlinear parts, which can be transformed into the following partial models with the nonlinearities and uncertainties by nonlinear feedback technique.

$$\dot{x}(t) = (A_l + \Delta A_l(t))x(t) + (B_l + \Delta B_l(t))u(t) + \sum_{k=1}^{N_l} [G_{k,l} + \Delta G_{k,l}(t)]g_{k,l}(x) \quad l = 1, 2, \dots, m \quad (3.28)$$

where m is the number of partial system models. N_l is the number of nonlinearities variables in l th partial model. A_l and B_l represent the l th linearized model of partial system. A_l , B_l and $G_{k,l}$ are known parameter matrices. $\Delta A_l(t)$, $\Delta B_l(t)$ and $\Delta G_{k,l}(t)$ representing parameter uncertainties are continuous matrix function of time t . $g_{k,l}(x)$ stands for the k th nonlinearities in the l th partial model. The uncertainties and nonlinearities in l th partial model satisfy the following bounded conditions.

Assumption 3.3.1:

The parametric uncertainties in ΔA_l and ΔB_l can be decomposed as follows:

$$\begin{bmatrix} \Delta A_l & \Delta B_l \end{bmatrix} = L_l F_l(t) \begin{bmatrix} E_{1l} & E_{2l} \end{bmatrix} \quad (3.29)$$

where L_l , E_{1l} and E_{2l} are known constant matrices which characterize the structure of the uncertainty. The uncertainty $F_l(t)$ satisfies $F_l^T(t)F_l(t) \leq I$; I is a unitary matrix.

The uncertainties $\Delta G_{k,l}$ can be decomposed as follows:

$$\Delta G_{k,l} = L_{kg,l} F_{kg,l}(t) E_{kg,l} \quad (3.30)$$

where $F_{kg,l}(t)$ are satisfying $F_{kg,l}(t)F_{kg,l}^T(t) \leq I$

Assumption 3.3.2:

There exist known constant matrices $W_{k,l}$ such that for all the nonlinearity term $g_{k,l}(x)$ in the l th partial model belongs to a sector:

$$\|g_{k,l}(x)\| \leq \|W_{k,l}x\| \quad (3.31)$$

Control objectives:

Determine the robust feedback controllers for all the partial models (3.28) such that the closed-loop partial systems are stable and the stability of the nonlinear systems with uncertainties is guaranteed.

3.3.2 Systematical design and stability analysis of global control for a class of nonlinear systems

Control of a class of nonlinear systems remains a challenge in the modern control for its qualitatively different behaviors over wide operating regions. Mathematical models, stability analysis and systematic design are certainly among the most important issue for nonlinear systems control.

This section is aimed to propose a systematical global control approach to further improves the feasibility of nonlinear system control. The global control scheme for a class of nonlinear systems shown in Figure 3.4 conceptually follows the global controller design procedure in Figure 3.1. First nonlinear systems are decomposed into partial models and corresponding coordinated rules which can be characterized by partial membership functions. The partial models are utilized to describe the decomposed systems in consideration of the nonlinearities and uncertainties and partial controllers are respectively designed for each partial models based on the feedback linearization and robust control theory. The global control law is then constructed by combining all the control actions through the partial membership functions.

Figure 3.4 Global control scheme for a class of nonlinear systems

It is salient that the partial controllers can be respectively designed for each partial models based on the feedback linearization and robust control theory. However, the stability of the global control system which is supposed to be constructed by combining all the control actions through the partial membership functions needs to be guaranteed.

For the l th linearized partial models (3.28), the following robust controllers can be designed to guarantee the stability of close loop partial systems

$$u_l(t) = -K_l x(t) \quad (3.32)$$

Then the global control law is obtained by aggregating the partial controllers with membership functions.

$$u(t) = -\sum_{l=1}^m \mu_l(t) K_l x(t) \quad (3.33)$$

where $\mu_l(t)$ is the normalized membership function of the l th partial model and satisfy membership functions of heterogeneous control law $\sum_{l=1}^m \mu_l(t) = 1$. It is noted that the global stabilizing feedback law is in general a nonlinear function of the system state $x(t)$.

Since the partial models are characterized by membership functions, it is convenient to decompose their corresponding operating regions overlapped or have gradual rather than sharp boundaries. The dynamics of the global uncertain system in an overlap region is the weighted average of overlapping partial models, weighted by the values of the membership functions. The global closed loop nonlinear system is then described by aggregating the partial models with membership functions.

$$\dot{x}(t) = \sum_{l=1}^m \mu_l(t) \{ [A_l + \Delta A_l(t)] + (B_l + \Delta B_l(t)) K_l \} x(t) + \sum_{k=1}^{N_l} [G_{k,l} + \Delta G_{k,l}(t)] g_{k,l}(x) \quad (3.34)$$

The stability of this global closed loop system (3.34) is then studied by using quadratic Lyapunov stability theory.

Theorem 3.3:

If the l th linearized partial model described in equation (3.28) is quadratically stable for all admissible uncertainties and nonlinearities, satisfying Assumptions 3.3.1 and 3.3.2, via

the partial control law if there exists positive definite symmetric matrices P_l for the following algebraic Riccati equations

$$(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) + P_l [-B_l R_l^{-1} B_l^T + L_l L_l^T + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T]] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} + Q_l = 0 \quad (3.35)$$

where Q_l is a symmetric positive defined matrix, which can be chosen by the designer, $R_l = E_{2l}^T E_{2l} > 0$, $\varepsilon_{kg,l}$ is scaling parameter to be chosen, with $\varepsilon_{kg,l}$ satisfying $\varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l} < I$. In this case, a stabilizing state-feedback control law is given by:

$$u_l(t) = -K_l x(t) = -R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) x_l(t) \quad (3.36)$$

If the linearized partial model described in (3.28) is controllable, using the corresponding state feedback control law described in (3.36), and then the global nonlinear system is asymptotically stable if switching time of different partial controllers is finite and the derivative of membership function $\dot{\mu}_l(t)$ is bounded.

Proof of Theorem 3.3

To prove the theorem 3.3, the following preliminary results are introduced first [70].

Lemma 3.1:

for any matrices X and Y with appropriate dimensions we have

$$X^T Y + Y^T X \leq \beta X^T X + \frac{1}{\beta} Y^T Y, \text{ for any } \beta > 0.$$

Lemma 3.2:

Let G, L, E and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ being a matrix function. Then we have for any $\varepsilon > 0$ such that $\varepsilon^2 E^T E \leq I$, and $F(t)^T F(t) \leq I$,

$$(G + LF(t)E)[(G + LF(t)E)]^T \leq G(I - \varepsilon^2 E^T E)^{-1} G^T + \frac{1}{\varepsilon^2} LL^T$$

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Now in order to prove the asymptotic stability of closed loop system for a class of nonlinear systems (3.34), we define an energy function for the l th linearized partial model as follows:

$$V_l(x, t) = x^T(t)P_l x(t) + \sum_{k_l=1}^{N_l} \int_0^t [W_{k,l} x(s)]^T [W_{k,l} x(s)] ds - \sum_{k_l=1}^{N_l} \int_0^t g_{k,l}^T [x(s)] g_{k,l} [x(s)] ds \quad (3.37)$$

where $P_l (l = 1, 2, \dots, m)$ are positive-definite symmetric matrices, which make the l th closed loop partial model asymptotically stable. In view of Assumptions 3.3.1 and 3.3.2, it is obvious that $V_l > 0, \forall x \neq 0$ from above construction of V_l . The following Lyapunov candidate is used to analyze the stability of the global closed-loop system (3.34):

$$\begin{aligned} V &= \sum_{l=1}^m \mu_l(t) V_l(x, t) \\ &= \sum_{l=1}^m \mu_l(t) \left\{ x^T(t) P_l x(t) + \sum_{k_l=1}^{N_l} \int_0^t [W_{k,l} x(s)]^T [W_{k,l} x(s)] ds \right. \\ &\quad \left. - \sum_{k_l=1}^{N_l} \int_0^t g_{k,l}^T [x(s)] g_{k,l} [x(s)] ds \right\} \end{aligned} \quad (3.38)$$

Since $V_l > 0, \forall x \neq 0$ that $V > 0, \forall x \neq 0$ from above construction of V .

The partial model (3.28) with the feed back control law (3.32) can be represented as the following form:

$$\begin{aligned} \dot{x}(t) &= (A_l + \Delta A_l)x(t) + (B_l + \Delta B_l)u(t) + \sum_{k=1}^N [G_{k,l} + \Delta G_{k,l}(t)]g_{k,l}(x) \\ &= (A_l - B_l K_l + L_l F_l(t)(E_{1l} - E_{2l} K_l))x(t) + \sum_{k=1}^N [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}]g_{k,l}(x) \\ &= (\bar{A}_l + L_l F_l(t) \bar{E}_l)x(t) + \sum_{k=1}^N \tilde{G}_{kg,l} g_{k,l}(x) \end{aligned}$$

where $\bar{A}_l = A_l - B_l K_l$, $\bar{E}_l = E_{1l} - E_{2l} K_l$, $\tilde{G}_{kg,l} = G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}$.

The derivative of the energy function of the l th linearized partial model for nonlinear systems can be represented as the following form:

$$\begin{aligned}
L(x,t) &= \dot{V}_l \\
&= \dot{x}^T(t)P_l x(t) + x^T(t)P_l \dot{x}(t) + \sum_{k=1}^N x^T W_{k,l}^T W_{k,l} x - \sum_{k=1}^N g_{k,l}^T(x) g_{k,l}(x) \\
&= x^T ((\bar{A}_l + L_l F_l(t) \bar{E}_l)^T P_l + P_l (\bar{A}_l + L_l F_l(t) \bar{E}_l)) x + \sum_{k=1}^N \{ [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}] g_k(x) \}^T P_l x \\
&\quad + \sum_{k=1}^N x^T P_l \{ [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}] g_k(x) \} + \sum_{k=1}^N x^T W_{k,l}^T W_{k,l} x - \sum_{k=1}^N g_{k,l}^T(x) g_{k,l}(x) \\
&= x^T (\bar{A}_l^T P_l + P_l \bar{A}_l + (L_l F_l(t) \bar{E}_l)^T P_l + P_l (L_l F_l(t) \bar{E}_l)) x \\
&\quad + \sum_{k=1}^N \{ [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}] g_k(x) \}^T P_l x + \sum_{k=1}^N x^T P_l \{ [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}] g_k(x) \} \\
&\quad + \sum_{k=1}^N x^T W_{k,l}^T W_{k,l} x - \sum_{k=1}^N g_{k,l}^T(x) g_{k,l}(x)
\end{aligned}$$

Using Lemma 3.1 with $\beta = 1$, we obtain

$$\begin{aligned}
L(x,t) &\leq x^T (\bar{A}_l^T P_l + P_l \bar{A}_l + \bar{E}_l^T F_l(t)^T F_l(t) \bar{E}_l + P_l L_l L_l^T P_l) x + \sum_{k=1}^N x^T W_{k,l}^T W_{k,l} x - \sum_{k=1}^N g_{k,l}^T(x) g_{k,l}(x) \\
&\quad + x^T P_l \left\{ \sum_{k=1}^N [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}] [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}]^T \right\} P_l x + \sum_{k=1}^N g_{k,l}^T(x) g_{k,l}(x) \\
&= x^T \{ \bar{A}_l^T P_l + P_l \bar{A}_l + \bar{E}_l^T F_l(t)^T F_l(t) \bar{E}_l + P_l L_l L_l^T P_l \\
&\quad + P_l \left\{ \sum_{k=1}^N [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}] [G_{k,l} + L_{kg,l} F_{kg,l}(t) E_{kg,l}]^T \right\} P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} \} x
\end{aligned}$$

Now employing Lemma 3.2 and the assumption $F^T(t)F(t) \leq I$, it follows that

$$\begin{aligned}
L(x,t) &\leq x^T \{ \bar{A}_l^T P_l + P_l \bar{A}_l + \bar{E}_l^T \bar{E}_l + P_l L_l L_l^T P_l \\
&\quad + P_l \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} \} x
\end{aligned}$$

Substituting the feed back control law (3.36) and Assumption 3.3.1, we obtain,

$$L(x,t) = x^T \{ (A_l - B_l R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}))^T P_l + P_l (A_l - B_l R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}))$$

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$$\begin{aligned}
& +(E_{1l} - E_{2l}R_l^{-1}(B_l^T P_l + E_{2l}^T E_{1l}))^T (E_{1l} - E_{2l}R_l^{-1}(B_l^T P_l + E_{2l}^T E_{1l})) \\
& + P_l [L_l L_l^T + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T]] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} \} x \\
& = x^T \{ (A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) - P_l^T B_l (R_l^{-1})^T B_l^T P_l - P_l B_l R_l^{-1} B_l^T P_l \\
& \quad + E_{1l}^T E_{1l} - E_{1l}^T E_{2l} R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) - ((B_l^T P_l + E_{2l}^T E_{1l})^T (R_l^{-1})^T E_{2l}^T E_{1l} \\
& \quad + (B_l^T P_l + E_{2l}^T E_{1l})^T (R_l^{-1})^T E_{2l}^T E_{2l} R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) \\
& \quad + P_l [L_l L_l^T + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T]] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} \} x \\
& = x^T \{ (A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) - P_l^T B_l (R_l^{-1})^T B_l^T P_l - P_l B_l R_l^{-1} B_l^T P_l \\
& \quad + E_{1l}^T E_{1l} - E_{1l}^T E_{2l} R_l^{-1} B_l^T P_l - E_{1l}^T E_{2l} R_l^{-1} E_{2l}^T E_{1l} - P_l B_l (R_l^{-1})^T E_{2l}^T E_{1l} - E_{1l}^T E_{2l} (R_l^{-1})^T E_{2l}^T E_{1l} \\
& \quad + E_{1l}^T E_{2l} (R_l^{-1})^T B_l^T P_l + E_{1l}^T E_{2l} (R_l^{-1})^T E_{2l}^T E_{1l} + P_l B_l (R_l^{-1})^T B_l^T P_l + P_l B_l (R_l^{-1})^T E_{2l}^T E_{1l} \\
& \quad + P_l [L_l L_l^T + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T]] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} \} x
\end{aligned}$$

Noting $R_l^{-1} = E_{2l}^{-1} (E_{2l}^T)^{-1} = (R_l^{-1})^{-T}$, it follows that

$$\begin{aligned}
L(x, t) & = x^T \{ (A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) + P_l [-B_l R_l^{-1} B_l^T + L_l L_l^T \\
& \quad + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T]] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} \} x \\
& = -x^T Q_l x
\end{aligned}$$

where $Q_l = (A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) + P_l [-B_l R_l^{-1} B_l^T + L_l L_l^T$

$$+ \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T]] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} \} x$$

Hence, the derivative of energy function $\dot{V}_l < 0$. The derivative of energy function for global uncertain system $\dot{V}(t)$ can be given by:

$$\frac{dV}{dt} = \sum_{l=1}^m \dot{\mu}_l(t) V_l + \frac{\partial V(t)}{\partial x} \frac{dx}{dt} = \sum_{l=1}^m \dot{\mu}_l V_l + \sum_{l=1}^m \mu_l \dot{V}_l \quad (3.39)$$

Since operating regions are defined gradually by membership functions $\mu_l(t)$, it is obvious that $\|\dot{\mu}_l(t)\|$ is bounded and $\dot{\mu}_l(t) \rightarrow 0$ within a finite time interval as the switching time of different partial controllers are finite. It follows immediately that $\dot{V}(t) < 0, \forall x \neq 0$. The equilibrium state $x(t) = 0$ of the system is globally uniformly asymptotically stable for all admissible uncertainties.

Remark 3.3: The global stabilization of a class of nonlinear systems can be guaranteed with the assumptions in Theorem 3.3 and proper partial controllers. The most obvious advantage in the global controller design is that the stabilization of each partial system (3.28) can guarantee the stabilization of the global nonlinear systems (3.34).

3.4 Global control of large-scale interconnected system

3.4.1 Problem statement for large-scale interconnected systems

Consider the large-scale nonlinear system composed of N interconnected subsystems, where i th subsystem is described as:

$$\dot{x}_i(t) = f_i(x_i, u_i, t) + \Delta f_i(x_i, u_i, t) + \sum_{j=1}^N g_{ij}(x_i, x_j) \quad (3.40)$$

where for the i th ($i=1, 2, \dots, N$) subsystem; $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^m$ are the vector of state variables and input variables of i th subsystem; f_i , nonlinear function of (x_i, u_i, t) representing the nominal model of the i th subsystem, is continuously differentiable with respect to (x_i, u_i, t) , Δf_i represents the model uncertainties of the i th subsystem, $g_{ij}(x_i, x_j)$ are unknown nonlinear vector functions that represent the nonlinearities of the i th subsystem and the interactions between the i th subsystem and j th subsystem.

The N interconnected subsystems in the large-scale nonlinear system shown in Figure 3.5 can be described by the following state space form:

$$S_i: \dot{x}_i(t) = (A_i + \Delta A_i(t))x_i(t) + (B_i + \Delta B_i(t))u_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \{[G_{kij} + \Delta G_{kij}(t)]g_{kij}(x_i, x_j)\} \quad (3.41)$$

where k is the number of the nonlinearities and interconnection in i th subsystem ($k = 1, 2, \dots, T_{jk}$); the matrices A_i , B_i and G_{ij} are known real constant matrices of appropriate dimension that describe the nominal model; ΔA_i , ΔB_i , and ΔG_{ij} are real time varying parameter uncertainties; $g_{kij}(x_i, x_j)$ are unknown nonlinear vector functions that represent nonlinearities in the subsystem and the interactions with other subsystems.

Figure 3.5 The N interconnected subsystems in the large-scale nonlinear system

On the basis that the nonlinear subsystem is qualitatively complicated, the subsystem in (3.41) can be decomposed into several nonlinear parts, which can be transformed into the linearized partial models by nonlinear feedback technique. These partial models represent input-output relations of the subsystems with the nonlinearities, uncertainties and interconnections. The l th linearized partial models of the i th subsystem can be described in the following form:

$$\dot{x}_i(t) = (A_{i,l} + \Delta A_{i,l}(t))x_i(t) + (B_{i,l} + \Delta B_{i,l}(t))u_{i,l}(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk,l}} \{[G_{kij,l} + \Delta G_{kij,l}(t)]g_{kij,l}(x_i, x_j)\} \quad (3.42)$$

$$k = 1, 2, \dots, T_{jk,l} \quad l = 1, 2, \dots, m_i$$

where m_i is the number of partial models for the i th ($i = 1, 2, \dots, N$) subsystem; $u_{i,l}(t) \in \mathbb{R}^n$ is the input variables of the i th subsystem; the matrices $A_{i,l}$, $B_{i,l}$ and $G_{kij,l}$ are known real constant matrices of appropriate dimension that describe the nominal model of the l th linearized partial models, $\Delta A_{i,l}$, $\Delta B_{i,l}$, and $\Delta G_{kij,l}$ are real time varying parameter uncertainties of the l th linearized partial models; $g_{kij,l}(x_i, x_j)$ are unknown nonlinear

vector functions that represent nonlinearities of the i th subsystem and interactions between the i th subsystem and j th subsystem in the l th linearized partial models.

Assumption 3.4.1:

The uncertain matrices $A_{i,l}$, $B_{i,l}$, $G_{kij,l}$ are assumed to be decomposed of the following structure:

$$\begin{bmatrix} \Delta A_{i,l}(t) & \Delta B_{i,l}(t) \end{bmatrix} = L_{i,l} F_{i,l}(t) \begin{bmatrix} E_{1i,l} & E_{2i,l} \end{bmatrix} \quad (3.43)$$

$$\Delta G_{kij,l} = L_{kij,l} F_{kij,l}(t) E_{kij,l} \quad (3.44)$$

where $L_{i,l}$, $E_{1i,l}$, $E_{2i,l}$, $L_{kij,l}$, and $E_{kij,l}$ are constant matrices; $F_{i,l}(t)$ and $F_{kij,l}(t)$ are unknown matrix function with Lebesgue measurable elements and satisfying $F_{i,l}^T(t)F_{i,l}(t) \leq I_i$; $F_{kij,l}(t)F_{kij,l}^T(t) \leq I_{kij,l}$.

Assumption 3.4.2:

There exist known matrices $W_{ki,l}$ and $W_{kij,l}$ such that nonlinear terms $g_{kij,l}(x_i, x_j)$ satisfy:

$$\|g_{kij,l}(x_i, x_j)\| \leq \|W_{ki,l}x_i(t)\| + \|W_{kij,l}x_j(t)\| \quad (3.45)$$

Control objectives:

For each subsystem, determine the decentralized feedback controllers for all the partial models (3.42) such that the closed-loop partial systems are stable and the stability of the interconnected nonlinear systems is guaranteed.

3.4.2 Systematical design and stability analysis of global control for large-scale interconnected system

Global control of the interconnected nonlinear system follows the global controller design procedure in Figure 3.1. The basic idea of the global control scheme is straightforward as shown in the Figure 3.6. The interconnected nonlinear systems are first represented by the subsystems with interconnections. Then each subsystem is decomposed into several nonlinear parts described by partial models. The partial feedback controllers are respectively designed for partial models and the decentralized controllers for each subsystem are obtained with membership functions. Finally, the global controller is the

summation of all the control actions from every subsystem to guarantee the stabilization of the interconnected nonlinear systems.

Figure 3.6 Global control scheme for the large-scale interconnected system

It is salient that the decentralized partial controllers can be respectively designed for partial models of each subsystem based on the known results of decentralized control. However, the stability of the global controller for the interconnected system is to be discussed.

For the above partial models (3.42), the following robust controllers can be designed to guarantee the stability of close loop partial systems

$$u_{i,l}(t) = -K_{i,l}x_i(t) \quad (3.46)$$

Then the control law for the i th subsystem is obtained by aggregating the partial controllers with membership functions.

$$u_i(t) = -\sum_{l=1}^{m_i} \mu_{i,l}(t) K_{i,l} x_i(t) \quad (3.47)$$

where $\mu_{i,l}(t)$ is the normalized membership function of the l th linearized partial models and satisfy membership functions of heterogeneous control law $\sum_{l=1}^{m_i} \mu_{i,l}(t) = 1$.

Since the partial models are characterized by membership functions, the dynamics of the global large-scale interconnected system is weighted by the values of the membership functions. The global closed loop system for the i th subsystem is then described by the following equation:

$$\begin{aligned} \dot{x}_i(t) = & \sum_{l=1}^{m_i} \mu_{i,l}(t) \{ (A_{i,l} + \Delta A_{i,l}(t) + (B_{i,l} + \Delta B_{i,l}(t)K_{i,l})x_i(t) \\ & + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \{ [G_{kij,l} + \Delta G_{kij,l}(t)]g_{kij,l}(x_i, x_j) \} \} \end{aligned} \quad (3.48)$$

The stability of the N interconnected subsystems (3.48) is then studied by using quadratic Lyapunov stability theory.

Theorem 3.4:

If the interconnected uncertain systems in equation (3.42) are quadratically stable for all admissible uncertainties, satisfying Assumptions 3.4.1 and 3.4.2, via the decentralized control laws and if there exists positive definite symmetric matrices $P_{i,l}$ for the following algebraic Riccati equations

$$\begin{aligned} & (A_{i,l} - B_{i,l}R_{i,l}^{-1}E_{2i,l}^T E_{1i,l})^T P_{i,l} + P_{i,l}(A_{i,l} - B_{i,l}R_{i,l}^{-1}E_{2i,l}^T E_{1i,l}) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} (W_{kji,l}^T W_{kji,l} + W_{ki,l}^T W_{ki,l}) \\ & + P_{i,l} \{ L_{i,l} L_{i,l}^T - B_{i,l} R_{i,l}^{-1} B_{i,l}^T + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} [G_{kij,l} (I - \varepsilon_{kij,l}^2 E_{kij,l}^T E_{kij,l}) G_{kij,l}^T + \frac{1}{\varepsilon_{kij,l}^2} L_{kij,l} L_{kij,l}^T] \} P_{i,l} + Q_{i,l} = 0 \end{aligned} \quad (3.49)$$

where $Q_{i,l}$ are symmetric positive defined matrices, which can be chosen by the designer, $R_{i,l} = E_{2i,l}^T E_{2i,l} > 0$, $\varepsilon_{kij,l}$ are scaling parameters to be chosen, with $\varepsilon_{kij,l}$ satisfying $\varepsilon_{kij,l}^2 E_{kij,l}^T E_{kij,l} < I$, $\forall i, j = 1, 2, \dots, N$. In this case, the stabilizing state-feedback control laws are given by

$$u_{i,l}(t) = -R_{i,l}^{-1} (B_{i,l}^T P_{i,l} + E_{2i,l}^T E_{1i,l}) x_i(t) \quad (3.50)$$

If the linearized partial model described in (3.42) is controllable, using the corresponding state feedback control law described in (3.50), and then the global interconnected

nonlinear system is asymptotically stable if switching time of different partial controllers is finite and the derivative of membership function $\dot{\mu}_{i,l}(t)$ is bounded.

Proof of Theorem 3.4:

Now in order to prove the asymptotic stability of closed loop system for the interconnected uncertain systems (3.50), we define an energy function of the l th linearized partial model of the i th subsystem (3.42):

$$V_{i,l}(x_i, t) = x_i^T(t)P_{i,l}x_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \int_0^{T_{jk}} \{ [W_{kij,l}x_j(s)]^T [W_{kij,l}x_j(s)] + [W_{ki,l}x_i(s)]^T [W_{ki,l}x_i(s)] - g_{kij,l}^T[x_i(s), x_j(s)]g_{kij,l}[x_i(s), x_j(s)] \} ds \quad (3.51)$$

where $P_{i,l}$ ($l = 1, 2, \dots, m_i$) are positive-definite symmetric matrices, which make the l th closed loop partial model asymptotically stable. In view of Assumptions 3.4.1 and 3.4.2, it is obvious that $V_{i,l} > 0, \forall x \neq 0$ from above construction of $V_{i,l}$. Thus the Lyapunov function candidate for the i th subsystem (3.41) is as follows:

$$\begin{aligned} V_i(x_i, t) &= \sum_{l=1}^{m_i} \mu_{i,l} V_{i,l}(x_i, t) \\ &= \sum_{l=1}^{m_i} \mu_{i,l} \{ x_i^T(t)P_{i,l}x_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \int_0^{T_{jk}} \{ [W_{kij,l}x_j(s)]^T [W_{kij,l}x_j(s)] \\ &\quad - g_{kij,l}^T[x_i(s), x_j(s)]g_{kij,l}[x_i(s), x_j(s)] \} ds \} \end{aligned} \quad (3.52)$$

The Lyapunov function candidate for the interconnected uncertain systems (3.50) is as follows:

$$\begin{aligned} V(x, t) &= \sum_{i=1}^N V_i(x_i, t) \quad (3.53) \\ &= \sum_{i=1}^N \{ \sum_{l=1}^{m_i} \mu_{i,l} \{ x_i^T(t)P_{i,l}x_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \int_0^{T_{jk}} \{ [W_{kij,l}x_j(s)]^T [W_{kij,l}x_j(s)] \\ &\quad + [W_{ki,l}x_i(s)]^T [W_{ki,l}x_i(s)] - g_{kij,l}^T[x_i(s), x_j(s)]g_{kij,l}[x_i(s), x_j(s)] \} ds \} \} \\ &= \sum_{l=1}^{m_i} \mu_{i,l} \{ \sum_{i=1}^N \{ x_i^T(t)P_{i,l}x_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \int_0^{T_{jk}} \{ [W_{kij,l}x_j(s)]^T [W_{kij,l}x_j(s)] \} \} \} \end{aligned}$$

$$\begin{aligned}
& +[W_{ki,l}x_i(s)]^T[W_{ki,l}x_i(s)] - g_{kij,l}^T[x_i(s), x_j(s)]g_{kij,l}[x_i(s), x_j(s)]\}ds\} \\
\text{Let } V_{\Sigma,l}(x,t) &= \sum_{i=1}^N \{x_i^T(t)P_{i,l}x_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \int_0^{T_{jk}} \{[W_{kij,l}x_j(s)]^T[W_{kij,l}x_j(s)] \\
& +[W_{ki,l}x_i(s)]^T[W_{ki,l}x_i(s)] - g_{kij,l}^T[x_i(s), x_j(s)]g_{kij,l}[x_i(s), x_j(s)]\}ds\} \\
\text{then } V(x,t) &= \sum_{l=1}^{m_l} \mu_{i,l} V_{\Sigma,l}(x,t) \tag{3.54}
\end{aligned}$$

In view of Assumptions 3.4.1 and 3.4.2, it is obvious that $V > 0, \forall x \neq 0$ from above construction of V (3.54).

The l th linearized partial model of the i th subsystem (3.42) with the feed back control law (3.50) can be represented as the following form:

$$\begin{aligned}
\dot{x}_i(t) &= (A_{i,l} + \Delta A_{i,l}(t))x_i(t) + (B_{i,l} + \Delta B_{i,l}(t))u_{i,l}(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \{[G_{kij,l} + \Delta G_{kij,l}(t)]g_{kij,l}(x_i, x_j)\} \\
&= (\bar{A}_{i,l} + L_{i,l}F_{i,l}(t)\bar{E}_{i,l})x_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \{[G_{kij,l} + L_{kij,l}F_{kij,l}(t)E_{kij,l}]g_{kij,l}(x_i, x_j)\}
\end{aligned}$$

where $\bar{A}_{i,l} = A_{i,l} - B_{i,l}K_{i,l}$, $\bar{E}_{i,l} = E_{1i,l} - E_{2i,l}K_{i,l}$.

The derivative of the energy function of the l th linearized partial model of the i th subsystem for interconnected systems can be represented as follows:

$$\begin{aligned}
L(x,t) &= \dot{V}_{\Sigma,l} \\
&= \sum_{i=1}^N \{\dot{x}_i^T(t)P_{i,l}x_i(t) + x_i^T(t)P_{i,l}\dot{x}_i(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \{[W_{kij,l}x_j]^T[W_{kij,l}x_j] \\
&\quad +[W_{ki,l}x_i]^T[W_{ki,l}x_i] - g_{kij,l}^T(x_i, x_j)g_{kij,l}(x_i, x_j)\}\} \\
&= \sum_{i=1}^N \{x_i^T(t)[(\bar{A}_{i,l} + L_{i,l}F_{i,l}(t)\bar{E}_{i,l})^T P_{i,l} + P_{i,l}(\bar{A}_{i,l} + L_{i,l}F_{i,l}(t)\bar{E}_{i,l})]x_i(t) \\
&\quad + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} \{[G_{kij,l} + L_{kij,l}F_{kij,l}(t)E_{kij,l}]g_{kij,l}(x_i, x_j)\}^T P_{i,l}x_i(t)
\end{aligned}$$

$$\begin{aligned}
& +x_i^T(t)P_{i,l}[G_{kij,l}+L_{kij,l}F_{kij,l}(t)E_{kij,l}]g_{kij,l}(x_i,x_j)\} \\
& +\sum_{j=1}^N\sum_{k=1}^{T_{jk}}\{[W_{kij,l}x_j]^T[W_{kij,l}x_j]+[W_{ki,l}x_i]^T[W_{ki,l}x_i]-g_{kij,l}^T(x_i,x_j)g_{kij,l}(x_i,x_j)\}\} \\
& =\sum_{i=1}^N\{x_i^T(t)[\bar{A}_{i,l}^TP_{i,l}+(L_{i,l}F_{i,l}(t)\bar{E}_{i,l})^TP_{i,l}+P_{i,l}\bar{A}_{i,l}+P_{i,l}L_{i,l}F_{i,l}(t)\bar{E}_{i,l}])x_i(t) \\
& +\sum_{j=1}^N\sum_{k=1}^{T_{jk}}\{g_{kij,l}^T(x_i,x_j)[G_{kij,l}+L_{kij,l}F_{kij,l}(t)E_{kij,l}]^TP_{i,l}x_i(t) \\
& +x_i^T(t)P_{i,l}[G_{kij,l}+L_{kij,l}F_{kij,l}(t)E_{kij,l}]g_{kij,l}(x_i,x_j)\} \\
& +\sum_{j=1}^N\sum_{k=1}^{T_{jk}}\{[W_{kij,l}x_j]^T[W_{kij,l}x_j]+[W_{ki,l}x_i]^T[W_{ki,l}x_i]-g_{kij,l}^T(x_i,x_j)g_{kij,l}(x_i,x_j)\}\}
\end{aligned}$$

Using Lemma 3.1 in Section 3.3.2 with $\beta = 1$, we obtain

$$\begin{aligned}
L(x,t) & \leq \sum_{i=1}^N\{x_i^T(t)[\bar{A}_{i,l}^TP_{i,l}+P_{i,l}\bar{A}_{i,l}+\bar{E}_{i,l}^TF_{i,l}^T(t)F_{i,l}(t)\bar{E}_{i,l}+P_{i,l}L_{i,l}L_{i,l}^TP_{i,l}])x_i(t) \\
& +\sum_{j=1}^N\sum_{k=1}^{T_{jk}}\{g_{kij,l}^T(x_i,x_j)g_{kij,l}(x_i,x_j) \\
& +x_i^T(t)P_{i,l}[G_{kij,l}+L_{kij,l}F_{kij,l}(t)E_{kij,l}][G_{kij,l}+L_{kij,l}F_{kij,l}(t)E_{kij,l}]^TP_{i,l}x_i(t) \\
& +\sum_{j=1}^N\sum_{k=1}^{T_{jk}}\{[W_{kij,l}x_j]^T[W_{kij,l}x_j]+[W_{ki,l}x_i]^T[W_{ki,l}x_i]-g_{kij,l}^T(x_i,x_j)g_{kij,l}(x_i,x_j)\}\}
\end{aligned}$$

Since $\sum_{i=1}^N\sum_{j=1}^N\sum_{k=1}^{T_{jk}}[W_{kij,l}x_j]^T[W_{kij,l}x_j]=\sum_{i=1}^N\sum_{j=1}^N\sum_{k=1}^{T_{jk}}x_i^TW_{kij,l}^TW_{kij,l}x_i$

$$\begin{aligned}
L(x,t) & = \sum_{i=1}^N\{x_i^T(t)[\bar{A}_{i,l}^TP_{i,l}+P_{i,l}\bar{A}_{i,l}+\bar{E}_{i,l}^TF_{i,l}^T(t)F_{i,l}(t)\bar{E}_{i,l}+P_{i,l}L_{i,l}L_{i,l}^TP_{i,l}])x_i(t) \\
& +\sum_{j=1}^N\sum_{k=1}^{T_{jk}}x_i^T(t)P_{i,l}[G_{kij,l}+L_{kij,l}F_{kij,l}(t)E_{kij,l}][G_{kij,l}+L_{kij,l}F_{kij,l}(t)E_{kij,l}]^TP_{i,l}x_i(t) \\
& +\sum_{j=1}^N\sum_{k=1}^{T_{jk}}x_i^T(W_{kij,l}^TW_{kij,l}+W_{ki,l}^TW_{ki,l})x_i\}
\end{aligned}$$

Now employing Lemma 3.2 in Section 3.3.2 and the assumption $F_l^T(t)F_l(t) \leq I$, it follows that

$$\begin{aligned} L(x, t) \leq & \sum_{i=1}^N \{x_i^T(t) [\bar{A}_{i,l}^T P_{i,l} + P_{i,l} \bar{A}_{i,l} + \bar{E}_{i,l}^T \bar{E}_{i,l} + P_{i,l} L_{i,l} L_{i,l}^T P_{i,l}] x_i(t) \\ & + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} x_i^T(t) P_{i,l} [G_{kij,l} (I - \varepsilon_{kij,l}^2 E_{kij,l}^T E_{kij,l}) G_{kij,l}^T + \frac{1}{\varepsilon_{kij,l}^2} L_{kij,l} L_{kij,l}^T] P_{i,l} x_i(t) \\ & + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} x_i^T (W_{kij,l}^T W_{kij,l} + W_{ki,l}^T W_{ki,l}) x_i\} \end{aligned}$$

Substituting the $\bar{A}_{i,l}$, $\bar{E}_{i,l}$ and feed back control law (3.50) $K_{i,l} = -R_{i,l}^{-1}(B_{i,l}^T P_{i,l} + E_{2i,l}^T E_{1i,l})$,

$$\begin{aligned} L(x, t) = & \sum_{i=1}^N x_i^T(t) \{ (A_{i,l} - B_{i,l} R_{i,l}^{-1} (B_{i,l}^T P_{i,l} + E_{2i,l}^T E_{1i,l}))^T P_{i,l} \\ & + P_{i,l} (A_{i,l} - B_{i,l} R_{i,l}^{-1} (B_{i,l}^T P_{i,l} + E_{2i,l}^T E_{1i,l})) \\ & + (E_{1i,l} - E_{2i,l} R_{i,l}^{-1} (B_{i,l}^T P_{i,l} + E_{2i,l}^T E_{1i,l}))^T (E_{1i,l} - E_{2i,l} R_{i,l}^{-1} (B_{i,l}^T P_{i,l} + E_{2i,l}^T E_{1i,l})) \\ & + P_{i,l} \{ \sum_{j=1}^N \sum_{k=1}^{T_{jk}} [G_{kij,l} (I - \varepsilon_{kij,l}^2 E_{kij,l}^T E_{kij,l}) G_{kij,l}^T + \frac{1}{\varepsilon_{kij,l}^2} L_{kij,l} L_{kij,l}^T] + L_{i,l} L_{i,l}^T \} P_{i,l} \\ & + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} (W_{kij,l}^T W_{kij,l} + W_{ki,l}^T W_{ki,l}) \} x_i(t) \\ = & \sum_{i=1}^N x_i^T(t) \{ (A_{i,l} - B_{i,l} R_{i,l}^{-1} E_{2i,l}^T E_{1i,l})^T P_{i,l} + P_{i,l} (A_{i,l} - B_{i,l} R_{i,l}^{-1} E_{2i,l}^T E_{1i,l}) - P_{i,l} B_{i,l} (R_{i,l}^{-1})^T B_{i,l}^T P_{i,l} \\ & - P_{i,l} B_{i,l} R_{i,l}^{-1} B_{i,l}^T P_{i,l} + E_{1i,l}^T E_{1i,l} - E_{1i,l}^T E_{2i,l} R_{i,l}^{-1} B_{i,l}^T P_{i,l} - E_{1i,l}^T E_{2i,l} R_{i,l}^{-1} E_{2i,l}^T E_{1i,l} \\ & - P_{i,l} B_{i,l} (R_{i,l}^{-1})^T E_{2i,l}^T E_{1i,l} - E_{1i,l}^T E_{2i,l} (R_{i,l}^{-1})^T E_{2i,l}^T E_{1i,l} + E_{1i,l}^T E_{2i,l} R_{i,l}^{-1} B_{i,l}^T P_{i,l} \\ & + E_{1i,l}^T E_{2i,l} (R_{i,l}^{-1})^T E_{2i,l}^T E_{1i,l} + P_{i,l} B_{i,l} (R_{i,l}^{-1})^T B_{i,l}^T P_{i,l} + P_{i,l} B_{i,l} (R_{i,l}^{-1})^T E_{2i,l}^T E_{1i,l} \\ & + P_{i,l} \{ \sum_{j=1}^N \sum_{k=1}^{T_{jk}} [G_{kij,l} (I - \varepsilon_{kij,l}^2 E_{kij,l}^T E_{kij,l}) G_{kij,l}^T + \frac{1}{\varepsilon_{kij,l}^2} L_{kij,l} L_{kij,l}^T] + L_{i,l} L_{i,l}^T \} P_{i,l} \} \end{aligned}$$

$$+ \sum_{j=1}^N \sum_{k=1}^{T_{jk}} (W_{kij,l}^T W_{kij,l} + W_{ki,l}^T W_{ki,l}) \} x_i(t)$$

Note: $R_i^{-1} = E_{2i}^{-1} (E_{2i}^T)^{-1} = (R_i^{-1})^{-T}$

$$\begin{aligned} L(x,t) &= \sum_{i=1}^N x_i^T(t) \{ (A_{i,l} - B_{i,l} R_{i,l}^{-1} E_{2i,l}^T E_{1i,l})^T P_{i,l} + P_{i,l} (A_{i,l} - B_{i,l} R_{i,l}^{-1} E_{2i,l}^T E_{1i,l}) - P_{i,l} B_{i,l} R_{i,l}^{-1} B_{i,l}^T P_{i,l} \\ &\quad + P_{i,l} \{ \sum_{j=1}^N \sum_{k=1}^{T_{jk}} [G_{kij,l} (I - \varepsilon_{kij,l}^2 E_{kij,l}^T E_{kij,l}) G_{kij,l}^T + \frac{1}{\varepsilon_{kij,l}^2} L_{kij,l} L_{kij,l}^T] + L_{i,l} L_{i,l}^T \} P_{i,l} \\ &\quad + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} (W_{kij,l}^T W_{kij,l} + W_{ki,l}^T W_{ki,l}) \} x_i(t) \\ &= -x^T(t) Q x(t) \end{aligned}$$

where $Q_i = (A_{i,l} - B_{i,l} R_{i,l}^{-1} E_{2i,l}^T E_{1i,l})^T P_{i,l} + P_{i,l} (A_{i,l} - B_{i,l} R_{i,l}^{-1} E_{2i,l}^T E_{1i,l})$

$$\begin{aligned} &+ P_{i,l} \{ L_{i,l} L_{i,l}^T - B_{i,l} R_{i,l}^{-1} B_{i,l}^T + \sum_{j=1}^N \sum_{k=1}^{T_{jk}} [G_{kij,l} (I - \varepsilon_{kij,l}^2 E_{kij,l}^T E_{kij,l}) G_{kij,l}^T + \frac{1}{\varepsilon_{kij,l}^2} L_{kij,l} L_{kij,l}^T] \} P_{i,l} \\ &+ \sum_{j=1}^N \sum_{k=1}^{T_{jk}} (W_{kij,l}^T W_{kij,l} + W_{ki,l}^T W_{ki,l}) \end{aligned}$$

Hence, the derivative of energy function $\dot{V}_{\Sigma,l} < 0$. The derivative of energy function $\dot{V}$ can be given by:

$$\frac{dV}{dt} = \sum_{l=1}^{m_l} \dot{\mu}_{i,l} V_{\Sigma,l} + \frac{\partial V_{\Sigma}}{\partial x} \frac{dx}{dt} = \sum_{l=1}^{m_l} \dot{\mu}_{i,l} V_{\Sigma,l} + \sum_{l=1}^{m_l} \mu_{i,l} \dot{V}_{\Sigma,l} \quad (3.55)$$

From the definition of membership function $\mu_{i,l}(t)$, it can be obtained that $\|\dot{\mu}_{i,l}(t)\|$ are bounded and $\dot{\mu}_{i,l}(t) \rightarrow 0$ within a finite time interval because the switching time of different partial controllers are finite. It follows immediately that $\dot{V}(t) < 0, \forall x \neq 0$. The equilibrium state $x(t) = 0$ of the system is globally uniformly asymptotically stable for all admissible uncertainties.

Remark 3.4: The global stabilization of large-scale interconnected systems can be guaranteed with the assumptions in Theorem 3.4 and proper partial controllers. The most

obvious advantage in the global controller design is that the stabilization of each partial system (3.42) can guarantee the stabilization of the global large-scale nonlinear systems (3.48). The design of partial decentralized controller only use the local control signals

3.5 Concluding remarks

In this chapter, the global control design procedure was presented to consider complicated characteristics of complex systems. The design of global control was decomposed into three stages: identification, partial controller design and controller synthesis. The related control techniques utilized in global control design were reviewed and the stability issues of controller design were discussed. These mentioned control techniques had been respectively developed to solve the nonlinearities, uncertainties and interconnection problems and coordinate various control actions in complex system to some extent. Therefore, the global control can take advantage of all those methods to meet the complicated requirements in system modeling, control and stabilities conditions of complex systems.

The global control approach was gradually extended a simpler situation to the final complex systems: uncertain systems, a class of nonlinear systems and large-scale interconnected system. Global system models and systematical design procedures for these systems were respectively presented in Sections 3.2 to 3.4. Thus, the final result, Theorem 3.4 is one of the main theories results of the thesis. The proposed global controller design can deal with nonlinearities, uncertainties and interconnections in practical systems. The stabilization of each partial system can guarantee the one of the global complex systems, and all partial controllers are designed using the local control signals. All those advantages of global control design make it distinguish from the other global control strategies.

In next chapter, global control of uncertain system will applied to an SMIB power system to enhance transient stability. The global control of a class of nonlinear systems is about to be utilized to two benchmark examples, the BB system and the CP system in Chapter 5. The multi-machine power systems will be used as an example to demonstrate validity of global control for the large-scale interconnected system in Chapter 6.

Chapter 3 Global Control Design

Except for the simplicity of global control design in handling with the various nonlinearities, uncertainties and interconnections in complex system, it can also be employed to coordinate different control objectives. In Chapter 7, global control approach will be investigated to solve inherently conflicting control goals and coordinate multiple controllers in uncertain power systems.

Chapter 4 Global Control of an SMIB Power System for Transient Stability Enhancement

4.1 Introduction

The issue for designing the global controller for uncertain systems has been systematically proposed to deal with various uncertainties of complex systems in previous chapter. An SMIB power system in Figure 4.1 is used as an example to demonstrate validity of the global control strategy for uncertain systems in this chapter. The SMIB power system is widely used to deal with preliminary investigation on transient stability problems as well as the validity of controller design.

Figure 4.1 The single machine infinite bus (SMIB) power system

This chapter is organized as follows: Section 4.2 review the basic concepts of transient stability enhancement and generator excitation controller design in power systems. The global control of uncertain systems is applied to the SMIB system to enhance transient stability in Section 4.3. Simulation results of global control scheme are shown in Section 4.4 and a conclusion is given at the end of this chapter.

4.2 Transient stability analysis and controller design for an SMIB power system

In order to keep up with worldwide power demand for electricity, power system has become a complicated and interconnected system. The solution for increasing load has typically engaged in building larger generating units and requiring the development of

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higher capacity transmission lines at first. As the deregulation of power industry and environment concerns advance recently, the transmission lines are expected to operate at maximum capacity close to thermal limits without expansion of the existing transmission and generation infrastructure. Therefore, improving transient stability becomes increasingly important for reliable and economical operation of power system.

Transient stability is the ability of the power system to maintain synchronism when subjected to a severe transient disturbance [72]. It evaluates the power system's ability to withstand the large disturbance, usually short-circuits, and survive transition to a normal or new operating condition. The main criterion for transient stability is that the synchronous machines maintain synchronism at the post-disturbance steady state operation. So the generator rotor angle response to a transient disturbance within 3 to 5 seconds is always used to analysis transient stability although the study period may extend to 10-20 seconds for very large system with dominant inter-area modes of oscillation ([72], [80] and [81]).

Transient stabilization is one of most challenging problems in the control of power systems. Since the problems arise of how to prevent generators from losing synchronism and dampening the subsequent oscillation quickly as the system is subjected to the severe disturbances, by improving the generator excitation control the transient stability can be greatly enhanced. As a result, the performance of control system plays an important role in the power system transient stability. The models of control objects, control strategy and control signal have significant affects on the dynamic characteristics of system.

In the design of conventional control systems, approximately linearized power system models are employed. However, power systems are inherently nonlinear. Linear controllers do not guarantee the system stability when a severe fault occurs. It is necessary to consider the nonlinearity in order to understand its behaviors and get better control performance. In order to realize the nonlinear control, the nonlinear feedback linearization techniques [9] are employed to linearize the system models. Nonlinear feedback technique was introduced to design nonlinear regulators for synchronous generator exciter and governor [80], nonlinear observer for power system stabilization, and a fast valve control of an interconnected power system ([83] and [84]). The direct feedback linearization (DFL) technique was used to design generator excitation controller

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as well as coordinated excitation, fast valving and FACTS controllers for transient stability enhancement ([17], [70] and [84]).

Although nonlinear controllers can improve the dynamic performance of power system, they are of more complicated structure and harder to be implemented in practice compared with linear controllers. In addition, feedback linearization schemes need exact plant parameters to cancel the inherent system nonlinearities as explained in Section 3.2.1. However, the power system is an uncertain system and experience parameter variations by nature. The structure of electric power network is susceptible to change from time to time by severe disturbance, repair and failure and so on. The uncertainties also exist as a result of modeling errors, such as, unmodeled dynamics, unknown time delays, noise from measurement instrument and unpredicted disturbance inputs. Therefore, the controller design for modern power system applications to achieve good transient performance should consider uncertainty of power system.

Robust control theory [6] is widely applied for its effectiveness in solving the uncertainty problem of the system model. A lot of literature applied this technique to solve the practical robustness issues in stability control. The application of multivariable robust control techniques in the design of robust power system excitation control was investigated in [85]. The robust excitation control algorithms, together with power system stabilizer (PSS) were studied and compared for transient stability by direct methods [86]. Some of research extended robust control design using H_∞ methods. Suboptimal robust excitation controllers are designed based on H_∞ control theory in consideration of additional constraints [87]. A novel controller for the generator excitation system based on a combination of H_∞ control theory and the geometric approach was presented [88]. In addition, in order to consider the uncertainties of power systems, some research works apply adaptive nonlinear technique, neural network and fuzzy logic to design controllers for transient stability improvement ([38], [70], [89], [90] and [91]).

From above literatures on transient stabilization, the power system must be modeled in consideration of nonlinearities and uncertainties to guarantee performance of excitation controller and the advanced control techniques have to be investigated to deal with those complicated models as well. As the power system is a real life system and operates in wide operating region, the dynamic characteristic of systems may not be accurately

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described by a single nonlinear model when it is affected by the disturbance, especially the derivatives of disturbance. Global control scheme for uncertain systems is expected to solve those practical problems.

4.3 Design of global controller on transient stability enhancement for an SMIB power system

In this section, an SMIB power system in Figure 4.1 is considered as an example to demonstrate the effectiveness of global control approach. The key idea of global control is that the nonlinear power system model is first decomposed into four nonlinear subsystems according to switching actions of breakers under a severe disturbance. Four nonlinear subsystems represent pre-fault steady state, faulty state, fault clearing steady state and post-fault steady state of the SMIB power system respectively. Then the direct feedback linearization (DFL) technique is used to linearize four partial nonlinear models and robust controllers are designed for the linearized partial models for different states. Finally, the global control law is obtained by integrating the partial controllers with membership functions of heterogeneous control law. The configuration of the proposed global control scheme for SMIB power system is depicted in Figure 4.2.

Figure 4.2 Global control of the SMIB power system

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4.3.1 Applications of DFL technique to an SMIB power system

The power system model is crucial for the performance of controllers especially under severe disturbances. It is required to take into account the important nonlinearities and be independent of the equilibrium point. By employing the direct feedback compensating law (DFL) in Section 3.1.1, the nonlinear model of SMIB power systems can be transformed into a linear one with parameter uncertainties, so that the global robust control technique discussed above can be applied.

Considering the SMIB power system model (2.25)-(2.35) in Section 2.3.4, we first eliminate physically immeasurable $E_q'(t)$ by differentiating the active electrical power equation (2.30) and using (2.25)-(2.35) to obtain:

$$\dot{P}_e(t) = V_s \dot{E}_q(t) B(t) \sin(\delta(t)) + V_s E_q(t) \dot{B}(t) \sin(\delta(t)) + V_s E_q(t) B(t) \cos(\delta(t)) \Delta\omega(t) \quad (4.1)$$

$$\text{and } \dot{E}_q(t) = (k_c u_f(t) - E_q(t)) \frac{1}{T'} - \frac{(X_d - X_d') V_s \sin(\delta(t))}{X_{ds}'} \Delta\omega(t)$$

Let $\Delta P_e(t) = P_e(t) - P_m$, and use the equation (2.25)-(2.35) then (4.1) becomes

$$\begin{aligned} \Delta \dot{P}_e(t) = & -\frac{1}{T'} \Delta P_e(t) + \frac{1}{T'} \left\{ \frac{P_e(t)}{X_{ad} I_f(t)} k_c u_f(t) + T_{do}' (X_d - X_d') \left[\frac{P_e(t)}{X_{ad} I_f(t)} \right]^2 \Delta\omega(t) \right. \\ & \left. + T' [Q_e(t) + V_s^2 B(t)] \Delta\omega(t) - P_m \right\} + V_s E_q(t) \dot{B}(t) \sin(\delta(t)) \end{aligned} \quad (4.2)$$

where $T' = T_{do}' X_{ds}' / X_{ds}$, Let

$$v_f(t) = \frac{P_e(t)}{X_{ad} I_f(t)} k_c u_f(t) + T_{do}' (X_d - X_d') \left[\frac{P_e(t)}{X_{ad} I_f(t)} \right]^2 \Delta\omega(t) + T' [Q_e(t) + V_s^2 B(t)] \Delta\omega(t) - P_m \quad (4.3)$$

$$\text{then, } \Delta \dot{P}_e(t) = -\frac{1}{T'} \Delta P_e(t) + \frac{1}{T'} v_f(t) + V_s E_q(t) \dot{B}(t) \sin(\delta(t)) \quad (4.4)$$

If the system parameters are known exactly, the nonlinear DFL compensating law can be obtained from (4.3) as,

$$u_f(t) = \frac{X_{ad} I_f(t)}{k_c P_e(t)} \{v_f(t) + P_m - T' [Q_e(t) + V_s^2 B(t)] \Delta\omega(t)\} - \frac{(X_d - X_d') P_e(t)}{k_c X_{ad} I_f(t)} T_{do}' \Delta\omega(t) \quad (4.5)$$

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Remark 4.3.1: The mapping (4.5) from $v_f(t)$ to $u_f(t)$ is invertible except when $I_f(t) = 0$, i.e. $\sin(\delta(t)) = 0$, which is not in the normal working region for a generator. When the compensating law (4.5) is employed, the linearized model of the generator is independent of the operating point of the system. Since $\Delta\omega(t)$ is readily available, and $Q_e(t)$, $P_e(t)$ and $I_f(t)$ are readily measurable variable, the DFL compensating law above is practically realizable. This is of great importance in power system design.

Remark 4.3.2: After the DFL compensation, the generator model is linear with respect to the new input $v_f(t)$ in steady state of SMIB power system. However, the DFL compensating law still contains nonlinearities due to the switching of transmission lines when a fault occurs in the SMIB system, the system parameter, X_{ds} and X'_{ds} will change considerably, so will the parameter T' , $B(t)$ and $\dot{B}(t)$. In moment of switching breakers under large disturbances, the derivative of susceptance $\dot{B}(t)$ changes dramatically and it cannot be bounded. However, it remains zero when structure of transmission lines does not change.

In the global controller design, the switching moments are not taken into accounts since the switching time can be neglected during the transient analysis. The SMIB system is decomposed into pre-fault, faulty, fault clearing and post-fault state. Choosing the state as $x^T(t) = [\Delta\delta(t) \quad \Delta\omega(t) \quad \Delta P_e(t)]$, the characteristic of the generator in four states after linearized by DFL technique is given by the following equations:

$$\begin{cases} \Delta\dot{\delta}(t) = \Delta\omega(t) \\ \Delta\dot{\omega} = -\frac{D}{2H}\Delta\omega(t) - \frac{\omega_0}{2H}(P_e(t) - P_m) \\ \Delta\dot{P}_e(t) = -\frac{1}{T'}\Delta P_e(t) + \frac{1}{T'}v_f(t) \end{cases} \quad (4.6)$$

In order to design a feedback controller to guarantee the overall stability of SMIB power system irrespective of uncertainty parameters X_{ds} , X'_{ds} , and T' , the robust technique is employed for the DFL compensated system of four states.

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4.3.2 Robust controllers design for uncertain partial models

To overcome the uncertainty problem, such as the structure change of transmission parameters X_{ds} and X'_{ds} under the faults, the uncertainties result of modeling errors, the generator model with uncertainty is established.

Considering the parameter changes in T' and $B(t)$, the compensating law (4.5) can be rewritten as follows:

$$u_f(t) = \frac{X_{ad}I_f(t)}{k_c P_e(t)} \{v_f(t) + P_m - \bar{T}'[Q_e(t) + V_s^2 \bar{B}(t)]\Delta\omega(t)\} - \frac{(X_d - X'_d)P_e(t)}{k_c X_{ad}I_f(t)} T'_{do} \Delta\omega(t) \quad (4.7)$$

Remark 4.3.3: The difference between (4.5) and (4.7) is that the uncertain parameters T' and $B(t)$ in (4.3) are replaced respectively by the known parameters $\bar{T}'$ and $\bar{B}(t)$ in (4.7). $\bar{T}'$ and $\bar{B}(t)$ are the average value of T' and $B(t)$, respectively. The equation (4.6) can be rewritten as follows:

$$\Delta\dot{P}_e(t) = -\left[\frac{1}{\bar{T}'} + \mu(t)\right]\Delta P_e(t) + \left[\frac{1}{\bar{T}'} + \mu(t)\right]v_f(t) + \psi(t)\Delta\omega(t) \quad (4.8)$$

where $\psi(t) = \left[\frac{1}{\bar{T}'} + \mu(t)\right]\{\Delta T' Q_e(t) + \frac{1}{X_{ds} + \Delta X} \Delta T' V_s^2\}$; $\mu(t) = \frac{1}{\bar{T}' + \Delta T'} - \frac{1}{\bar{T}'}$;

$\Delta T' = T'_{do} \frac{X'_{ds} + \Delta X}{X_{ds} + \Delta X} - \bar{T}'$. ΔX denotes the uncertainty in reactance X_{ds} and X'_{ds} . $\psi(t)$, $\mu(t)$ and $\Delta T'$ are bounded.

Choosing the state as $x^T(t) = [\Delta\delta(t) \quad \Delta\omega(t) \quad \Delta P_e(t)]$, the linearized generator models in consideration of uncertainties can be given in a compact form:

$$\dot{x}(t) = (A_l + \Delta A_l(t))x(t) + (B_l + \Delta B_l(t))v_f(t) \quad (4.9)$$

where $A_l = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D}{2H} & -\frac{\omega_0}{2H} \\ 0 & 0 & -\frac{1}{\bar{T}'} \end{bmatrix}$ and $B_l = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\bar{T}'} \end{bmatrix}$ represent the real known constant

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matrices of the nominal model:

$$\Delta A_l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \psi_l(t) & -\mu_l(t) \end{bmatrix} \text{ and } \Delta B_l = \begin{bmatrix} 0 \\ 0 \\ \mu_l(t) \end{bmatrix} \text{ are the matrices representing uncertainties;}$$

$\psi_l(t)$ and $\mu_l(t)$ are bounded uncertain parameters of different partial models.

The parametric uncertainties in ΔA_l and ΔB_l can be decomposed as Assumption 3.2:

$$L_l(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; E_{1l}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_l(t)|_{\max} & 0 \\ 0 & 0 & |\mu_l(t)|_{\max} \end{bmatrix}; E_{2l}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_l(t)|_{\max} \end{bmatrix}; \text{ and}$$

$$F_l(t) = \begin{bmatrix} 0 & -\frac{\psi_l(t)}{|\psi_l(t)|_{\max}} & -\frac{\mu_l(t)}{|\mu_l(t)|_{\max}} \end{bmatrix} \text{ satisfies } F_l^T(t)F_l(t) \leq I, \text{ } I \text{ is a unitary matrix.}$$

The ranges of uncertainties in power system model are various with different operating conditions. For example, in the steady states, the uncertainties of the power system only take into account model errors, which is a relatively smaller range in comparison with the structure change of power system under a severe disturbance.

To solve the global robust control problem for the DFL compensated generator model (4.9) involves solving the algebraic Riccati equation (3.20) in Section 3.2.

$$(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) +$$

$$P_l (L_l L_l^T - B_l R_l^{-1} B_l^T) P_l + E_{1l}^T (I - E_{2l} R_l^{-1} E_{2l}^T) E_{1l} + Q_l = 0$$

where Q_l are symmetric positive defined matrices, which can be chosen by the designer, $R_l = E_{2l}^T E_{2l} > 0$, In this case, the l th stabilizing state-feedback control laws are given by

$$v_{\beta l}(t) = -R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) x_l(t)$$

Remark 4.3.4: The Riccati equation may not always be solvable for uncertainty bounds over the whole operating conditions that the closed loop linearized system may not be stable for all admissible uncertainties and the power system may lose synchronism when a fault occurs. However, when we neglect the switching moment of breakers during the transient analysis and consider four linearized partial models in different states: pre-fault

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steady state, faulty state, fault clearing steady state and post-fault steady state, uncertainty bounds of each partial model become smaller than the one over the whole operating region because of ignoring the moment of transmission line switching. In partial controller design, only bounds of the generator parameters and transmission line reactance have to be known, which makes it easier to obtain robust partial control laws for the SMIB power system.

4.3.3 Global control using membership functions

Four partial controllers are separately designed by robust control technique for corresponding partial models, pre-fault, during fault, fault clearing and post-fault system models. Since we consider the fault are clear and the transmission line are restore after disturbances, the pre-fault and the post-fault partial models are same. In order to realize a soft switching type mechanism for changing of different operating conditions, the heterogeneous control membership function (3.18), is used to aggregate all the partial models and construct a global nonlinear model.

In the transient stability study, the rotor speed of the generator, $\Delta\omega(t)$, the active electrical power of the generator, $\Delta P_e(t)$, and switching time of the breakers z , are used to construct the heterogeneous control membership functions of the partial models $\mu_i(t)$, where $x = \sqrt{\Delta\omega^2 + \Delta P_e^2}$ and z are considered as state variables. The membership functions of state variables shown in Figure 4.3 are as follow:

$$\begin{cases} mf_1(x) = \frac{1}{1+e^{-120(x+0.4)}} - \frac{1}{1+e^{-120(x-0.4)}} & |x| \leq 0.4 \\ mf_2(x) = 1 - mf_1(x) & |x| > 0.4 \end{cases} \quad (4.10)$$

$$\begin{cases} mf_3(z) = \frac{1}{1+e^{-200(z-t_1)}} - \frac{1}{1+e^{-200(z-t_2)}} & t_1 \leq z \leq t_2 \\ mf_4(z) = 1 - mf_3(z) & z < t_1 \text{ or } z > t_2 \end{cases} \quad (4.11)$$

where t_1 and t_2 are respectively the opening and closing time of breakers and $t_1 < t_2$. The membership functions of the states variables can be selected as shown in Figure. 4.3, Then the heterogeneous control membership functions of the partial models

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$$\mu_l(t) = \frac{y_l(t)}{\sum_{l=1}^m y_l(t)}, \quad \text{and where } y_1(t) = mf_1(x)mf_4(z) ; \quad y_2(t) = mf_2(x)mf_4(z) ;$$

$y_3(t) = mf_3(z)$ and $\sum_{l=1}^m \mu_l(t) = 1$. Therefore, heterogeneous control membership functions for partial models are time-varying according to state variables, $\Delta\omega(t)$ and z .

Figure 4.3 The membership functions of state variables x and z

Then the global control law is obtained by aggregating the partial controllers with heterogeneous control membership functions.

$$v_f(t) = \sum_{l=1}^m \mu_l(t) v_{fl}(t) \quad (4.12)$$

Remark 4.3.5: Four robust controllers are designed for the partial systems with different operating conditions, pre-fault steady state, faulty state, fault clearing steady state and post-fault steady state. System parameters, fault location and system operating points are all treated as the uncertainties and handled by the robust control approach in each partial model. The global control law is obtained by aggregating the partial controllers with heterogeneous control membership functions. As shown in the stability conditions of Theorem 3.2, the stabilization of each robust partial controller for generator excitation can guaranteed the overall stability of the SMIB power system irrespective of network parameters, fault locations and the system operating conditions.

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4.4 Case study

To evaluate the above global control scheme for generator excitation to enhance transient stability, the example SMIB system in Figure 4.1 is used in the simulation study. The system parameters and physical limits are given in Appendix A.3.

The fault sequences in the simulation are given as follows:

Stage 1: The system is in a pre-fault steady state.

Stage 2: A three phase short circuit fault occurs at $t = 0.1$ second at the terminal of generator bus.

Stage 3: The fault is removed by opening the breaker of the fault line at t_1 second.

Stage 4: The transmission line is restored with the fault clear at $t=1.4$ second.

Stage 5: The system is in a post-fault state.

The SMIB power system is decomposed into four partial models according to the above fault sequences. Because the SMIB system operates at the original steady state after the fault, the partial models in the pre-fault and post-fault steady state are same. Only three partial control laws are to be designed to form a global control law.

4.4.1 Global controller design

In the simulation, four partial system models can be written in the form of (4.9) that is representative of the generator in the example system.

$$\dot{x}(t) = (A_l + \Delta A_l(t))x(t) + (B_l + \Delta B_l(t))v_f(t)$$

$$\text{where } A_l = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D}{2H} & -\frac{\omega_0}{2H} \\ 0 & 0 & -\frac{1}{T_l} \end{bmatrix}, B_l = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_l} \end{bmatrix}; \Delta A_l = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \psi_l(t) & -\mu_l(t) \end{bmatrix} \text{ and } \Delta B_l = \begin{bmatrix} 0 \\ 0 \\ \mu_l(t) \end{bmatrix}$$

For the first and fourth partial models, the parameters and uncertainty bounds are as follows:

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$$A_1 = A_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.625 & -39.27 \\ 0 & 0 & -0.31518 \end{bmatrix}; B_1 = B_4 = \begin{bmatrix} 0 \\ 0 \\ 0.31518 \end{bmatrix}; \Delta B_1 = \Delta B_4 = \begin{bmatrix} 0 \\ 0 \\ \mu_1(t) \end{bmatrix};$$

$$\Delta A_1 = \Delta A_4 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_1(t) & -\mu_1(t) \end{bmatrix};$$

$$X_{ds1} = X_d + X_T + X_l = 1.863 + 0.127 + 0.2426 = 2.2326;$$

$$X'_{ds1} = X'_d + X_T + X_l = 0.657 + 0.127 + 0.2426 = 1.0266;$$

$$\bar{T}'_1 = T'_{do} X'_{ds1} / X_{ds1} = 6.9 * 1.0266 / 2.2326 = 3.1728;$$

Since both the pre-fault and post-fault states are steady states of power system, only the parameter uncertainties of X_{ds1} and X'_{ds1} , such as the transmission line reactance variations due to different weather and measurements, are considered. Thus we assume uncertainty bound in reactance $|\Delta X_1|_{\max} = 0.1X_{ds1}$ and $|Q_{e1}(t)|_{\max} \leq 0.6$ in the steady states, then,

$$|\Delta T'_1|_{\max} = \left| T'_{do} \frac{X'_{ds} + \Delta X}{X_{ds} + \Delta X} - \bar{T}' \right|_{\max} = 0.3388$$

$$|\mu_1(t)|_{\max} = \left| \frac{1}{\bar{T}'_1 + \Delta T'_1} - \frac{1}{\bar{T}'_1} \right|_{\max} = \left| \frac{1}{(\bar{T}'_1 / \Delta T'_1 + 1) \bar{T}'_1} \right|_{\max} = 0.0377;$$

$$|\psi_1(t)|_{\max} = \left| \left[\frac{1}{\bar{T}'_1} + \mu_1(t) \right] \{ \Delta T'_1 Q_{e1}(t) + \frac{1}{X_{ds1} + \Delta X_1} \Delta T'_1 V_s^2 \} \right|_{\max} = 0.1204;$$

For the second partial model, the parameters and uncertainty bounds are as follows:

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.625 & -39.27 \\ 0 & 0 & -0.338 \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ 0 \\ -0.338 \end{bmatrix}; \Delta B_2 = \begin{bmatrix} 0 \\ 0 \\ \mu_2(t) \end{bmatrix},$$

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$$\Delta A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_2(t) & -\mu_2(t) \end{bmatrix};$$

When a fault occurs, X_{ds} and X'_{ds} change drastically. The lower bound for X_{ds} and X'_{ds} are $X_d + X_T$ and $X'_d + X_T$ respectively. The upper bound are the following X_{ds2} and X'_{ds2} respectively.

$$X_{ds2} = X_d + X_T + X_l = 1.863 + 0.127 + 0.2426 = 2.2326;$$

$$X'_{ds2} = X'_d + X_T + X_l = 0.657 + 0.127 + 0.2426 = 1.0266;$$

Thus, $\bar{X}_{ds2} = 2.1113$ and $\bar{X}'_{ds2} = 0.9053$. Then, we assume the uncertainty bound

$$|\Delta X_2|_{\max} = 0.5X_l + 0.1\bar{X}_{ds2} \text{ and } |Q_{e2}(t)|_{\max} \leq 1.5 \text{ then,}$$

$$|\Delta T_2'|_{\max} = \left| T_{do}' \frac{\bar{X}'_{ds2} + \Delta X_2}{\bar{X}_{ds2} + \Delta X_2} - \bar{T}_2' \right|_{\max} = 0.536;$$

$$|\mu_2(t)|_{\max} = \left| \frac{1}{\bar{T}_2' + \Delta T_2'} - \frac{1}{\bar{T}_2'} \right|_{\max} = \left| \frac{1}{(\bar{T}_2' / \Delta T_2' + 1)\bar{T}_2'} \right|_{\max} = 0.075;$$

$$|\psi_2(t)|_{\max} = \left| \left[\frac{1}{\bar{T}_2'} + \mu_2(t) \right] \{ \Delta T_2' Q_{e2}(t) + \frac{1}{\bar{X}_{ds2} + \Delta X_2} \Delta T_2' V_s^2 \} \right|_{\max} = 0.423;$$

For the third partial model, the transmission line with fault is cut by breakers; the structure and parameters of transmission line are changed. The system parameters and uncertainty bounds are as follows:

$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.625 & -39.27 \\ 0 & 0 & -0.29719 \end{bmatrix}; B_3 = \begin{bmatrix} 0 \\ 0 \\ 0.29719 \end{bmatrix}; \Delta B_3 = \begin{bmatrix} 0 \\ 0 \\ \mu_3(t) \end{bmatrix},$$

$$\Delta A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_3(t) & -\mu_3(t) \end{bmatrix};$$

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$$X_{ds3} = X_d + X_T + 1.5 X_l = 1.863 + 0.127 + 1.5 * 0.2426 = 2.3539;$$

$$X'_{ds3} = X'_d + X_T + 1.5 X_l = 0.657 + 0.127 + 1.5 * 0.2426 = 1.1479;$$

$$\bar{T}'_2 = T'_{do} X'_{ds3} / X_{ds3} = 6.9 * 1.0266 / 2.2326 = 3.3648;$$

Choose the parameter uncertainties of X_{ds3} and X'_{ds3} as 10% of X_{ds3} and X'_{ds3} . Thus, we assume $|\Delta X_3|_{\max} = 0.1 X_{ds3}$ and $|Q_{e3}(t)|_{\max} \leq 1.5$ then,

$$|\Delta T'_3|_{\max} = \left| T'_{do} \frac{X'_{ds3} + \Delta X_3}{X_{ds3} + \Delta X_3} - \bar{T}'_3 \right|_{\max} = 0.322;$$

$$|\mu_3(t)|_{\max} = \left| \frac{1}{\bar{T}'_3 + \Delta T'_3} - \frac{1}{\bar{T}'_3} \right|_{\max} = \left| \frac{1}{(\bar{T}'_3 / \Delta T'_3 + 1) \bar{T}'_3} \right|_{\max} = 0.031;$$

$$|\psi_3(t)|_{\max} = \left| \left[\frac{1}{\bar{T}'_3} + \mu_3(t) \right] \{ \Delta T'_3 Q_{e3}(t) + \frac{1}{X_{ds3} + \Delta X_3} \Delta T'_3 V_s^2 \} \right|_{\max} = 0.199;$$

The structure of the uncertainties for partial models of the generator can be expressed as follows:

$$L_1(t) = L_2(t) = L_3(t) = L_4(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \quad E_{11}(t) = E_{14}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_1(t)|_{\max} & 0 \\ 0 & 0 & |\mu_1(t)|_{\max} \end{bmatrix};$$

$$E_{12}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_2(t)|_{\max} & 0 \\ 0 & 0 & |\mu_2(t)|_{\max} \end{bmatrix}; \quad E_{13}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_3(t)|_{\max} & 0 \\ 0 & 0 & |\mu_3(t)|_{\max} \end{bmatrix};$$

$$E_{21}(t) = E_{24}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_1(t)|_{\max} \end{bmatrix}; \quad E_{22}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_2(t)|_{\max} \end{bmatrix}; \quad E_{23}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_3(t)|_{\max} \end{bmatrix};$$

$$F_1(t) = F_4(t) = \begin{bmatrix} 0 & -\frac{\psi_1(t)}{|\psi_1(t)|_{\max}} & -\frac{\mu_1(t)}{|\mu_1(t)|_{\max}} \end{bmatrix}; \quad F_2(t) = \begin{bmatrix} 0 & -\frac{\psi_2(t)}{|\psi_2(t)|_{\max}} & -\frac{\mu_2(t)}{|\mu_2(t)|_{\max}} \end{bmatrix};$$

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$$F_3(t) = \begin{bmatrix} 0 & -\frac{\psi_3(t)}{|\psi_3(t)|_{\max}} & -\frac{\mu_3(t)}{|\mu_3(t)|_{\max}} \end{bmatrix}.$$

Remark 4.4.1: The uncertainty bound of each partial model is given in consideration of different transmission structures. The maximum uncertainty bound of equivalent reactance X_{Σ} is $[0, X_l]$. But if we take the moment of structure changes into account, extra $0.5 X_l$ have to be regarded as uncertainties in the controller design. Therefore, the uncertainty bounds for each partial model are smaller than the one over the whole operating region. Note that the uncertainty bounds and system parameters of partial models are different from each other.

Following the design procedure in Section 4.3 and solving the Riccati equation (3.20), the partial control laws for four partial models are obtained:

$$v_{f1}(t) = v_{f4}(t) = 68.461 \Delta\delta + 7.72 \Delta\omega - 80.83 \Delta P_e;$$

$$v_{f2}(t) = 12.12 \Delta\delta + 6.34 \Delta\omega - 20.39 \Delta P_e;$$

$$v_{f3}(t) = 98.79 \Delta\delta + 15.55 \Delta\omega - 75.03 \Delta P_e;$$

Then the global control law is obtained by aggregating the partial controllers with heterogeneous control membership functions.

$$v_f(t) = \sum_{l=1}^{m=4} \mu_l(t) v_{fl}(t)$$

4.4.2 Simulation results

The performance of global controller for generator excitation is tested by comparing with the classic linear control scheme mentioned in Section 2.3.2 [73]. The parameters and physical limits of the linear controller are given in Appendix A.1.

The example SMIB power system operates at two different operating conditions under a symmetrical three-phase short circuit fault as shown in Figure 4.1. Two different operating points are considered as: the low operating point ($\delta_{10} = 42.0^\circ$; $P_{e10} = 0.8$ p.u.; $V_{r10} = 1.03$ p.u., $Q_{e10} = 0.13$ p.u.) and the high operating point ($\delta_{20} = 71.36^\circ$, $P_{e20} = 1.0$

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p.u.; $V_{t20} = 1.05$ p.u., $Q_{e20} = 0.24$ p.u.). In the simulation, two symmetrical three phase short circuit faults are considered

Fault 1 occurs at the terminal of generator bus.

Fault 2 occurs at the midpoint of the transmission line.

Three different cases are studied to demonstrate the feasibility of the global control strategies in transient stability enhancement.

Case 4.1 Global control scheme at the high operating point

The global control scheme is compared with the classic linear generator excitation controller given in Section 2.3.2 to improve transient stability, where the SMIB power system operates at the high operating point, Fault 1 and fault clearing time $t_1 = 0.22$ sec.. The comparisons of the power angle of the generator, $\delta(t)$, the generator bus terminal voltage, $V_t(t)$, the real power of the generator, $P_e(t)$, the relative speed of the generator, $\omega(t)$, control outputs of the generator excitation, $E_f(t)$, are shown in Figure 4.4 (a)–(e) respectively.

(a) $\delta(t)$

(b) $V_t(t)$

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(c) $P_e(t)$

(d) $\omega(t)$

(e) $E_f(t)$

Figure 4.4 Case 4.1 ($t_1 = 0.22$ sec.), Responses of $\delta(t)$, $V_t(t)$, $P_e(t)$, $\omega(t)$ and $E_f(t)$ at the high operating point and Fault 1

To compare the performance of the global control scheme with the linear classic generator excitation given in Section 2.3.2, a longer fault clearing time was adopted in the simulation, $t_1 = 0.32$ sec.. The comparisons of the power angles of the machines, $\delta(t)$, the relative speeds of the machines, $\omega(t)$, and excitation output of the generators $E_{fi}(t)$ are shown in Figure 4.5(a)-(b) respectively. In addition, the critical fault clearing times of these two control schemes are compared when the example SMIB power system operates upon the high operating point and Fault 1 occurs. The SMIB power system with classical linear control system can not bring the example system back to the original steady state when the critical fault clearing time is between 0.30 sec. and 0.31sec. However, the

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critical fault clearing time for the global control system is between 0.32 sec. and 0.33 sec. Therefore, the global control scheme can improve the transient stability in comparison with the classical linear controller.

Figure 4.5 Case 4.1 ($t_1 = 0.32$ sec.), Responses of $\delta(t)$ and $\omega(t)$ at the high operating point and Fault 1

Case 4.2 Global control scheme at the low operating point

Similar with the simulation at the high operating point, the global control scheme at the low operating point is compared with the classic linear one given in Section 2.3.2, and the fault clearing time of Fault 1 $t_1 = 0.28$ sec.. The comparisons of the power angle of the generator, $\delta(t)$, the generator bus terminal voltage, $V_t(t)$, the real power of the generator, $P_e(t)$, the relative speed of the generator, $\omega(t)$, control outputs of the generator excitation, $E_f(t)$, are shown in Figure 4.6 (a)–(e) respectively.

(a) $\delta(t)$

(b) $V_t(t)$

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(c) $P_e(t)$

(d) $\omega(t)$

(e) $E_f(t)$

Figure 4.6 Responses of $\delta(t)$, $V_i(t)$, $P_e(t)$, $\omega(t)$ and $E_f(t)$ at the low operating point and Fault 1

Case 4.3: Different fault locations with the low operating point 1

To verify the robustness of global control scheme for different fault locations, the SMIB system operate at the low operating point with two different faults. The comparisons of the power angle of the generator, $\delta(t)$, with different faults, are shown in Figure 4.6 (a) and (b).

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Figure 4.7 Responses of $\delta(t)$ at the low operating point with different faults

From above three simulation cases for SMIB power system, it can be concluded that:

1. The global control scheme can damp the oscillations and enhance the transient stability regardless of the system operating points and fault locations. It is obvious for all three cases that the global control scheme can damp the oscillations and bring the system back to the steady state operating point faster than the classic linear generator excitation controller given in Section 2.3.2. The proposed controller can provide satisfactory global performance in power system transient stability, even under the most severe fault occurred at the terminal of the generator.
2. The global controller for the SMIB power system is designed by using available local control signals, which are conveniently implemented to enhance the transient stability in practice.
3. The SMIB power systems either with the classic linear controller or with global controller are subject to transient dynamics when the breakers switch due to a severe disturbance.

4.5 Concluding remarks

The proposed global control of uncertain systems in Section 3.2 was applied to the SMIB power system to enhance transient stability in this chapter. As an illustration of global control strategy for uncertain systems, the SMIB power system model was first decomposed into four nonlinear subsystems according to switching actions of breakers

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under a severe disturbance. Four nonlinear subsystems represented pre-fault steady state, faulty state, fault clearing steady state and post-fault steady state of the SMIB power system respectively. Then the DFL technique was employed to linearize four partial nonlinear models and robust controllers were designed for the linearized partial models for different states. Finally, the global control law was obtained by integrating the partial controllers with membership functions of heterogeneous control law.

The results of simulation cases demonstrate that global controller designed greatly enhance transient stability of SMIB system in comparison of the classical linear controller given in Section 2.3.2. The global control scheme can damp the oscillations regardless of the system operating points and fault locations. The global control of SMIB system clearly supports the objectives of proposed global control of uncertain systems. The attributes of the proposed global controller are as follows:

- (1) Global system model can represent the SMIB power system over a wide operating region in consideration of system nonlinearities and uncertainties.
- (2) The global control scheme can damp the oscillations and enhance the transient stability regardless of the system operating point and fault locations.
- (3) The global controller is designed by using available local control signals.
- (4) The stabilization of each robust partial controller can guaranteed the overall stability of SMIB power system, which greatly simplifies global controller design.

The application of global control for a class of nonlinear systems will be provided in next chapter for the benchmark BB and CP control systems.

Chapter 5 Global Control of Two Benchmark Control Systems

5.1 Introduction

In this chapter, the application of global control for a class of nonlinear systems proposed in Section 3.3 is provided to two benchmark control systems: the BB and the CP systems. The design of global controller for the BB system is presented to show the validation of global control for a class of nonlinear systems in Section 5.2. The concepts of global control theory are substantiated by the simulation results in comparison with fuzzy dynamic control. Global control for a class of nonlinear systems is also utilized to another benchmark control system, the CP system shown in Section 2.4.2. Two control objectives: only regulate the displacement of pole and control displacement of both cart and pole, are respectively designed for CP system in Section 5.3. The proposed global control schemes are supported by simulation results as well. Finally, the effectiveness and benefits of the proposed global control schemes are concluded in Section 5.4.

5.2 Global control of the Ball and Beam System

5.2.1 Controller designs for the BB system

The BB system is an inherently unstable system, in which the ball tends to move to one of the ends of the beam. The task is thus to apply a sequence of upward and downward forces to control the system so that the ball can be dynamically balanced at the origin of the beam within a short period of time and the beam can be kept at a horizontal position.

The representative of the difficulties for the BB system model in Section 2.4.1 is introduced by rapidly growing nonlinearities. The critical nonlinearity is the centrifugal force $x_1 x_4^2$, where x_1 is the ball's displacement from its equilibrium and x_4 is the beam's angular velocity. This force is destabilizing when it opposes the controlling gravitational force $-\sin x_3$, where x_3 is the beam's angle.

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How to design a practical controller for a complicated nonlinear control object like the well-known BB system is an important issue. The BB system is usually approximate to multiple local linear models over various operating regions, where classical control theory can analyze the segmental dynamics of the system and provide high resolution controls. Switching and fuzzy controllers were widely employed to solve the nonlinearity problems in BB system. Those classical approaches have undoubtedly the advantage of designing analytical controllers and evaluating quantitatively their stability bounds. However, there are inherent weaknesses by only using those approximate linearized techniques to solve nonlinearities problems since linear control methods rely on the assumption of a small range operation for the linear model to be valid. The major problem of those approaches remains that as the complexity of nonlinear system increases, ability to make precise and yet significant statements about its local behavior diminishes. Intuitively, the number of local controllers increases and the switching of various controllers for different operating regions will be become complicated. At the worst, the smoothness of the transition between local models and stability of the global system are degraded. For the fuzzy dynamic control, constructing an adequate fuzzy controller becomes difficult because of many input items and tuned parameters.

To overcome the inherent disadvantages above, a single nonlinear system that is input-output linearizable was proposed to track output signals of the BB system in Reference [90]. Although the controller has good behavior in a small region, it fails in a larger region. The input-output linearization of a nonlinear system is based on the invertibility of the input-output map, and it therefore relies on the exact cancellation of nonlinear term. This presents a limitation of the theory: in the presence of modeling errors exact cancellation is not possible. For this reason, the robust problem of nonlinear system was investigated in depth.

Recently, the Takagi and Sugeno (TS) fuzzy dynamic model of a BB system has extended the linear models to the nonlinear model form ([1], [41] and [93]). Several systematic theories have been developed to consider the uncertain systems by incorporating Takagi-Sugeno fuzzy model with robust control technique. However, boundary conditions for stability of global system behavior in the fuzzy control were still very restrictive due to the discontinuity of the function across the boundaries of the different subspaces. For example, a sufficient condition for the system stability is obtained by finding a common

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Lyapunov function for all the fuzzy subsystems. For a fuzzy plant model with P rules, a fuzzy controller with P rules is required to close the feedback loop, and at least P Lyapunov conditions will come out. On the other hand, a number of IF-THEN rules are needed to construct a fuzzy model for a complicated system and rule reduction are required to reducing the number of rules as well as the computation effort.

In addition, the neural network [94] and genetic algorithms [57] were also applied to a BB system to solve the nonlinear problems, where the complex controllers were designed and parameters of controller were needed to optimize

In order to solve the problems mentioned above, such as nonlinearities, uncertainties of the BB system model and its consequence of multiple local controllers and complex controller design, we apply the systematical global control theory proposed in Section 3.3 to the BB system in the following section.

5.2.2 Design of global controller for the BB system

To demonstrate feasibility of global control for a class of nonlinear systems, this section illustrates the application of the proposed global control to the BB system. Base on the above physical characteristics of BB system and the proposed global control approach, the BB system is first decomposed into three nonlinear parts, which can be described by the partial models valid over wider operating region than local linear ones in consideration of the nonlinearities and uncertainties. Then three partial controllers are respectively designed for each partial model according to Theorems 3.3. Finally, the global control law is constructed by combining three partial control actions through heterogeneous control membership functions. The configuration of the proposed global control scheme for the BB system is depicted in Figure 5.1.

Figure 5.1 Global control of the BB system

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Decompose the BB system

As discussed in Chapter 2, the model of the BB control system has the typical nonlinear functions $f_1(x, t, \Delta)$ and $f_2(x_i, x_j, t, \Delta)$ in Equation (2.2), which cannot be systematically handled by existing control approaches. Therefore, the proposed global control for a class of nonlinear systems is employed to solve the nonlinear problems in the BB control system. The BB system model presented in Section 2.4.1 is in the following state space form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B(x_1 x_4^2 - G \sin x_3) \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \quad (5.1)$$

There are two critical nonlinear terms $\sin x_3$ and $x_1 x_4^2$ in the $\dot{x}_2$ -equation. The term $\sin x_3$ can be considered as a bounded nonlinearity in the controller design. The term $x_1 x_4^2$ grows with the ball position r and the square of the beam angular velocity $x_4 = \dot{\theta}$. The worse condition is that the large ball position r combining with the peaking of x_4 will result in a strong positive feedback, which leads to instability and makes the ball fly off the beam. Thus, by applying the multiple partial models in Chapter 2, the BB system can be decomposed into the following three subspaces:

$$S_1 = \{x \mid |x_4| \leq 0.4\};$$

$$S_2 = \{x \mid x_1 x_4 > 0, |x_4| > 0.4\};$$

$$S_3 = \{x \mid x_1 x_4 < 0, |x_4| > 0.4\};$$

Then the ball and beam system can be transformed as the following form in the first subspace:

$$\dot{x} = A_1 x + B_1 u + \sum_{k=1}^2 G_{k,1} g_{k,1}(x), \quad x \in S_1 \quad (5.2)$$

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$$\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.25B & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ Bx_1(x_4^2 + 0.25) \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ BG \sin x_3 \\ 0 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.25B & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ B \\ 0 \\ 0 \end{bmatrix} g_{1,1}(x) - \begin{bmatrix} 0 \\ BG \\ 0 \\ 0 \end{bmatrix} g_{2,1}(x)
\end{aligned}$$

where $g_{1,1}(x) = x_1(x_4^2 + 0.25)$, $g_{2,1}(x) = \sin x_3$.

Since $\|x_1(x_4^2 + 0.25)\| \leq \|(x_4^2 + 0.25)\|_{\max} \| [1 \ 0 \ 0 \ 0] \| \|x\|$ and

$\|\sin(x_3)\| \leq \| [0 \ 0 \ 1 \ 0] \| \|x\|$, $W_{1,1} = 0.41 [1 \ 0 \ 0 \ 0]$ and $W_{2,1} = [0 \ 0 \ 1 \ 0]$.

For the second and third subspaces, the ball and beam system can be linearized at the certain operating point, such as:

$\dot{x} = A_2x + B_2u + G_{1,2}g_{1,2}(x)$, $x \in S_2$, operating point: $x_1x_4 = 1.2, |x_4| = 0.4$

$\dot{x} = A_3x + B_3u + G_{1,3}g_{1,3}(x)$, $x \in S_3$, operating point: $x_1x_4 = -1.2, |x_4| = 0.4$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ Bx_4^2/2 & 0 & 0 & Bx_1x_4/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u - \begin{bmatrix} 0 \\ BG \\ 0 \\ 0 \end{bmatrix} \sin x_3 \quad (5.3)$$

$$\text{where } A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ Bx_4^2/2 & 0 & 0 & Bx_1x_4/2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, G = \begin{bmatrix} 0 \\ -BG \\ 0 \\ 0 \end{bmatrix}, g(x) = \sin x_3$$

and $W = [0 \ 0 \ 1 \ 0]$.

Remark 5.2.1: It should be noted that only three partial models are used to describe the BB system in the global control approach. Comparatively, 16 linearized T-S fuzzy models

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were used to represent the BB system by Taylor expansion in [41], and 6 fuzzy rules and local models was employed to describe 15 fuzzy subspaces in [93]. Therefore, the global control approach can decompose the nonlinear system with fewer partial models than classical linearization techniques, and fewer partial controllers need designing.

According to the decomposition of BB system, the heterogeneous control membership functions of the partial models $\mu_i(t)$ can be obtained from the membership functions of state variable as follows.

$$\begin{cases} mf_1(x_4) = \frac{1}{1 + e^{-20(x_4+0.4)}} - \frac{1}{1 + e^{-20(x_4-0.4)}} & |x_4| \leq 0.4 \\ mf_2(x_4) = 1 - mf_1(x_4) & |x_4| > 0.4 \end{cases} \quad (5.4)$$

$$\begin{cases} mf_3(x_1x_4) = \frac{1}{1 + e^{-20x_1x_4}} & x_1x_4 \geq 0 \\ mf_4(x_1x_4) = 1 - mf_3(x_1x_4) & x_1x_4 < 0 \end{cases} \quad (5.5)$$

Figure 5.2 Membership functions of state variables

The membership functions of the states variables can be selected as shown in Figure. 5.2, Then the heterogeneous control membership functions of the partial

models $\mu_i(t) = \frac{\omega_i(t)}{\sum_{l=1}^m \omega_l(t)}$, and where $\omega_1(t) = mf_1(x_4)$; $\omega_2(t) = mf_2(x_4)mf_3(x_1x_4)$;

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$\omega_3(t) = mf_2(x_4)mf_4(x_1x_4)$ and $\sum_{l=1}^m \mu_l(t) = 1$. Therefore, heterogeneous control membership functions for partial models are time-varying according to state variables.

The BB system model with uncertainties

Consider the uncertainties of the ball and beam systems, in which $\bar{G} = 9.81m/s^2$, $\bar{B} := M / (J_b / R^2 + M) = 0.7143$, assume $|\Delta G| \leq 0.1\bar{G} = 0.981m/s^2$, $|\Delta B| \leq 0.2\bar{B} = 0.14286$. The normalized state form of BB system can be draw from the equations (5.2) and (5.3) according to the above considerations.

$$\dot{x}(t) = (A_l + \Delta A_l)x(t) + (B_l + \Delta B_l)u(t) + \sum_{k=1}^{N_l} [G_{k,l} + \Delta G_{k,l}(t)]g_{k,l}(x) \quad (l=1,2,3) \quad (5.6)$$

$$\text{where } A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.17858 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Delta A_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -0.25\Delta B(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.0571 & 0 & 0 & 0.4286 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Delta A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.08\Delta B(t) & 0 & 0 & 0.6\Delta B(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.0571 & 0 & 0 & -0.4286 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Delta A_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0.08\Delta B(t) & 0 & 0 & -0.6\Delta B(t) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_1 = B_2 = B_3 = [0 \ 0 \ 0 \ 1]^T, \Delta B_1 = \Delta B_2 = \Delta B_3 = 0,$$

$$G_{1,1} = \begin{bmatrix} 0 \\ \bar{B} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.7143 \\ 0 \\ 0 \end{bmatrix}, \Delta G_{1,1} = \begin{bmatrix} 0 \\ \Delta B(t) \\ 0 \\ 0 \end{bmatrix}, G_{2,1} = G_{1,2} = G_{1,3} = \begin{bmatrix} 0 \\ -\bar{B}\bar{G} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -7.0073 \\ 0 \\ 0 \end{bmatrix},$$

$$\Delta G_{2,1} = \Delta G_{1,2} = \Delta G_{1,3} = \begin{bmatrix} 0 \\ -(\Delta B(t)\Delta G(t) + 0.7143\Delta G(t) + 9.81\Delta B(t)) \\ 0 \\ 0 \end{bmatrix}.$$

Uncertainties and nonlinearities decomposition

Since the parametric uncertainties in ΔA_l , ΔB_l and $\Delta G_{k,l}$ are required to satisfy Assumption 3.3.1, the parametric uncertainties in partial models can be decomposed as (3.29) and (3.30):

$$[\Delta A_l \quad \Delta B_l] = L_l F_l(t) [E_{1l} \quad E_{2l}]$$

$$\Delta G_{k,l} = L_{kg,l} F_{kg,l}(t) E_{kg,l}$$

$$\text{where } L_1 = L_2 = L_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, E_{1,1} = \begin{bmatrix} 0.035715 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$E_{1,2} = E_{1,3} = \begin{bmatrix} 0.0114 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.0857 \end{bmatrix}, E_{2,1} = E_{2,2} = E_{2,3} = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix},$$

$$F_1(t) = \begin{bmatrix} \Delta B(t)/0.14286 & 0 & 0 & 0 \end{bmatrix}, F_2(t) = \begin{bmatrix} \Delta B(t)/0.14286 & 0 & 0 & \Delta B(t)/0.14286 \end{bmatrix},$$

$$F_3(t) = \begin{bmatrix} \Delta B(t)/0.14286 & 0 & 0 & -\Delta B(t)/0.14286 \end{bmatrix}.$$

$$L_{1g,1} = L_{2g,1} = L_{1g,2} = L_{1g,3} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, E_{1g,1} = 0.14286, F_{1g,1}(t) = \Delta B(t)/0.14286,$$

$$E_{2g,1} = E_{1g,2} = E_{1g,3} = 2.2423,$$

$$F_{2g,1}(t) = F_{1g,2}(t) = F_{1g,3}(t) = \begin{bmatrix} -(\Delta B(t)\Delta G(t) + 0.7143\Delta G(t) + 9.81\Delta B(t)) \\ 2.2423 \end{bmatrix}.$$

Control laws design for partial models

According to Theorem 3.3 of proposed global control approach, the stabilizing state-feedback control laws for the partial models of BB system can be obtained by solving the algebraic Riccati equation (3.35) in Section 3.3.

$$(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) + P_l [-B_l R_l^{-1} B_l^T + L_l L_l^T + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T]] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} + Q_l = 0$$

where Q_l is a symmetric positive defined matrix, which can be chosen by the designer, $R_l = E_{2l}^T E_{2l} > 0$, $\varepsilon_{kg,l}$ is scaling parameter to be chosen, with $\varepsilon_{kg,l}$ satisfying $\varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l} < I$. In this case, a stabilizing state-feedback control law is given by (3.36):

$$u_l(t) = -K_l x(t) = -R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) x_l(t)$$

For the first partial model, let $Q_1 = \text{diag} [100, 500, 1000, 100]$, $\varepsilon_{1g,1} = 5.431$ and $\varepsilon_{2g,1} = 0.184$, solving the algebraic Riccati equation gives $K_1 = [-1.896 \ -4.225 \ 22.572 \ 9.764]$.

Similarly, for second partial model, let $Q_2 = \text{diag} [100, 1000, 1000, 100]$ and $\varepsilon_{1g,2} = 0.298$, it gives that $K_2 = [0.001 \ 0.014 \ 22.383 \ 9.684]$.

For third partial model, let $Q_3 = \text{diag} [10, 10, 10, 10]$ and $\varepsilon_{1g,3} = 0.409$, it gives that $K_3 = [-0.030 \ 0.088 \ 2.438 \ 3.003]$.

The following global control law is then obtained according to (3.33).

$$u(t) = -\sum_{l=1}^3 \mu_l(t) K_l x(t) = -(\mu_1(t) K_1 + \mu_2(t) K_2 + \mu_3(t) K_3) x(t)$$

Remark 5.2.2: It is noted that only three partial controllers are designed to obtain the global control law for the BB system. Comparatively, 16 local controllers were used to control the BB system before fuzzy rules reduction in [41], and 6 local controllers was designed for 15 fuzzy subspaces in [93]. Therefore, the global control approach can decrease the number of partial controllers, which greatly simplify the switching of

controllers for different operating regions and coordinate different controllers effectively and smoothly by membership functions.

5.2.3 Simulation results

To demonstrate the excellent performance of global control approach, the BB system has been operated with various parameters and initial conditions in the following simulations.

As $|\Delta G| \leq 0.981m/s^2$, $|\Delta B| \leq 0.14286$ are considered in the global control design, three sets of parameters in the BB system are utilized to verify the robustness of proposed global controller:

The first parameter set is the average value, where $G = 9.81m/s^2$, $B = 0.7143$;

The second parameter set is $G = 8.829m/s^2$, $B = 0.5714$ and

The third parameter set is $G = 10.791m/s^2$, $B = 0.8572$.

Two sets of four initial conditions are mainly considered in the simulations.

The first set is same as the ones in [93],

$$x(0) = [2.4 \ 0.1 \ 0.6 \ 0.1], [1.6 \ 0.05 \ -0.6 \ -0.05], [-1.6 \ -0.05 \ 0.6 \ 0.05]$$

and $[-2.4 \ 0.1 \ -0.6 \ -0.1]$.

The second set is a special set for the wide operating regions of global control.

$$x(0) = [32 \ -0.1 \ 0.6 \ 0.1], [9.6 \ 0.05 \ -0.6 \ -0.05], [-9.6 \ -0.05 \ 0.6 \ 0.05]$$

and $[-32 \ 0.1 \ -0.6 \ -0.1]$.

Three different cases with above parameters set and initial condition sets are studied to demonstrate the performance of global control scheme proposed for the BB system. In additions, other initial conditions in Case 5.1 are used to compare the performance of global control and fuzzy dynamic control.

Case 5.1. The first parameter set ($G = 9.81m/s^2$ and $B = 0.7143$)

The outputs of closed-loop BB system with above two sets of initial conditions are shown in Figure 5.3(a) and (b). As shown in Figure 5.3 (a), the proposed global controller for BB system has achieved as good dynamic performance as fuzzy controller in [93], which

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have utilized 15 fuzzy subspaces and 6 linearized local controllers with same initial conditions, the first set $x(0)$. Figure 5.3 (b) demonstrates the global control of BB system could smoothly regulate the ball to the origin from second set of initial conditions $x(0)$, which start with far larger ball position r than the first set initial condition.

Figure 5.3 Global control of the BB system with $G = 9.81m/s^2$ and $B = 0.7143$

To compare those two control methods further in the operating ranges, the output of the BB system with other four initial conditions are shown in Figure 5.4 (a)-(d) respectively. In comparison with fuzzy control system, the global control system take shorter time to converge to zero and the amplitude of the transient process is smaller. Thus, the global control is superior to fuzzy control [93], especially in the wider operating regions.

Figure 5.4 Comparative results with $G = 9.81m/s^2$ and $B = 0.7143$

As shown in Figures 5.5 and 5.6, the simulation results for Case 5.2 and Case 5.3 indicate that the BB system with global controller can obtain good dynamic performance in presence of various parameters and initial operating points. Therefore, the global controller for the BB system is robust and stable within certain parameter uncertainties ranges.

Case 5.2. The second parameter set ($G = 8.829m/s^2$ and $B = 0.5714$)

Figure 5.5 Global control of the BB system with $G = 8.829m/s^2$ and $B = 0.5714$ and

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Case 5.3. The third parameter set is ($G = 10.791 \text{ m/s}^2$ and $B = 0.8572$)

Figure 5.6 Global control of the BB system with $G = 10.791 \text{ m/s}^2$ and $B = 0.8572$.

From above three simulation cases for the BB system, it can be concluded that:

1. Global control scheme for the BB system can obtain better dynamic performance over wide operating regions due to nonlinear partial models.
2. Global controller proposed for the BB system is robust regardless the uncertainties of system parameters and different initial conditions.
3. Switching the different partial controllers of the BB system is smoothly realized by heterogeneous control membership functions, which somewhat take advantages of fuzzy membership functions and are obtained from the membership function of state variables. However, the membership functions in global control are simple and continuous, which can guarantee the stability of the system over all operating region.

5.2.4 Concluding remarks

The global control for a class of nonlinear systems proposed in Section 3.3 was applied to the BB system. The BB system is first decomposed into three nonlinear parts, which can be described by the partial models valid over wider operating region than local linear ones in view of the nonlinearities and uncertainties. Then three partial controllers are respectively designed for each partial model based on robust control theory. Finally, the

global control law is constructed by combining three partial control actions through heterogeneous control membership functions. The simulation results showed that the BB system could obtain good dynamic performance over wide operating regions irrespective of various parameters and initial operating points. In comparison with the fuzzy dynamic control strategies, the number of controllers reduced greatly by global control method. The switching of partial controllers was automatically coordinated by heterogeneous control membership functions, which are time-varying and correspond to the decomposition of the BB system.

5.3 Global Control of the Cart and Pole System

Similar with the BB control system, the model of the CP control system shown in Section 2.4.2 also has the typical nonlinear functions $f_1(x, t, \Delta)$ and $f_2(x_i, x_j, t, \Delta)$ in Equation (2.2), which cannot be systematically handled by existing control approaches. In this section, global control for a class of nonlinear systems is utilized to the CP system. Two control objectives: only regulate the displacement of pole and regulate displacement of cart and pole simultaneously, are designed by proposed global control approach.

5.3.1 Global control of the displacement of the pole

In this section, the objective of global control is only displacement of the pole, which means the pole is required to remain in the upright position from initial conditions as the force $u(t)$ is applied on the cart; and the cart can move freely without any constraints. As only the displacement of the pole is regulated, the angular displacement θ , and the angular velocity $\dot{\theta}$ are chosen as the state vectors, $x = [\theta \ \dot{\theta}]^T$, the CP system in Section 2.4.2 can be represented as the following state space form:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} x_2 \\ \frac{(M+m)g \sin x_1 - mLx_2^2 \sin x_1 \cos x_1}{[M+m \sin^2 x_1]L} \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\cos x_1}{[M+m \sin^2 x_1]L} \end{bmatrix} u(t) \\ &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{\cos x_1}{[M+m \sin^2 x_1]L} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ \frac{-mLx_2^2 \sin x_1 \cos x_1 + (M+m)g \sin x_1}{[M+m \sin^2 x_1]L} \end{bmatrix} \end{aligned} \quad (5.7)$$

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$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\Delta B_2 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\Delta G_{21} \end{bmatrix} \sin 2x_1 + \begin{bmatrix} 0 \\ \frac{(M+m)g}{ML} + \Delta G_{22} \end{bmatrix} \sin x_1$$

$$\text{where } \Delta B_2 = \frac{\cos x_3}{ML + m \sin^2 x_3 L} \leq \frac{1}{ML}$$

$$g_1(x) = \sin 2x_1; \|\sin(2x_1)\| \leq 2\| [1 \ 0] \| \|x\| \text{ and } W_1 = [2 \ 0];$$

$$\Delta G_{21} = \frac{mx_2^2}{2[M + m \sin^2 x_1]} \leq \frac{mx_2^2}{2M};$$

$$g_2(x) = \sin x_1; \|\sin(x_1)\| \leq \| [1 \ 0] \| \|x\| \text{ and } W_2 = [1 \ 0];$$

$$\Delta G_{22} = \frac{(M+m)g}{[M + m \sin^2 x_1]L} - \frac{(M+m)g}{ML} \leq \frac{(M+m)g}{ML} - \frac{g}{L} = \frac{mg}{ML}.$$

Decompose the CP system

Similar with the global control of BB system, the CP system was decomposed into the following two subspaces according to Section 2.4.2:

$$S_1 = \{x \mid |x_2| \leq 1\};$$

$$S_2 = \{x \mid |x_2| > 1\};$$

The first partial model

Then the CP system can be transformed as the following form in the first subspace:

$$\dot{x} = (A_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u + (G_{1,1} + \Delta G_{1,1}(t))g_{1,1}(x), \quad x \in S_1$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\Delta B_2 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\Delta G_{21,1} \end{bmatrix} \sin 2x_1 + \begin{bmatrix} 0 \\ \frac{(M+m)g}{ML} + \Delta G_{22,1} \end{bmatrix} \sin x_1$$

$$\text{where } A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \Delta A_1 = 0; B_1 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; \Delta B_1 = \begin{bmatrix} 0 \\ -\Delta B_{2,1} + 1 \end{bmatrix}; \Delta B_{2,1} \leq \frac{1}{ML};$$

$$g_{1,1}(x) = \sin 2x_1; G_{1,1} = \begin{bmatrix} 0 \\ -0.1 \end{bmatrix}; \Delta G_{1,1} = \begin{bmatrix} 0 \\ \Delta G_{21,1} + 0.1 \end{bmatrix}; \Delta G_{21} \leq \frac{m}{2M};$$

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$$g_{2,1}(x) = \sin x_1; \quad G_{2,1} = \begin{bmatrix} 0 \\ \frac{(M+m)g}{ML} \end{bmatrix}; \quad \Delta G_{2,1} = \begin{bmatrix} 0 \\ \Delta G_{22,1} \end{bmatrix}; \quad \Delta G_{22} \leq \frac{mg}{ML}.$$

The second partial model

For the second subspaces, the CP system can be linearized at the certain operating point, such as:

$$\dot{x} = (A_2 + \Delta A_2)x + (B_2 + \Delta B_2)u + (G_{1,2} + \Delta G_{1,2}(t))g_{1,2}(x), \quad x \in S_2, \text{ operating point: } |x_2| = 2$$

$$= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -\Delta B_{2,2} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\Delta G_{21,2} \end{bmatrix} \sin 2x_1 + \begin{bmatrix} 0 \\ \frac{(M+m)g}{ML} + \Delta G_{22,2} \end{bmatrix} \sin x_1$$

$$\text{where } \Delta G_{21,2} = \frac{mx_2^2}{2[M+m\sin^2 x_1]} \leq \frac{mx_2^2}{2M}. \text{ Since } |x_2| > 1, \quad \Delta G_{21,2} = \frac{m}{2M} + \frac{m(x_2^2 - 1)}{2M}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \quad \Delta A_2 = 0; \quad B_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}; \quad \Delta B_2 = \begin{bmatrix} 0 \\ -\Delta B_{2,2} + 1 \end{bmatrix}; \quad \Delta B_{2,2} \leq \frac{1}{ML};$$

$$g_{1,2}(x) = \sin 2x_1; \quad G_{1,2} = \begin{bmatrix} 0 \\ -\frac{m}{2M} \end{bmatrix}; \quad \Delta G_{1,2} = \begin{bmatrix} 0 \\ \Delta G'_{21,2} \end{bmatrix}; \quad \Delta G'_{21,2} = \frac{m(x_2^2 - 1)}{2M} \leq \frac{3m}{2M};$$

$$g_{2,2}(x) = \sin x_1; \quad G_{2,2} = \begin{bmatrix} 0 \\ \frac{(M+m)g}{ML} \end{bmatrix}; \quad \Delta G_{2,2} = \begin{bmatrix} 0 \\ \Delta G_{22,2} \end{bmatrix}; \quad \Delta G_{22,2} \leq \frac{mg}{ML}.$$

Membership functions

According to the decomposition of the CP system, the membership functions of the partial models $\mu_i(t)$ can be obtained from the membership functions of state variable as follows.

$$\begin{cases} mf_1(x_2) = \frac{1}{1+e^{-20(x_2+1)}} - \frac{1}{1+e^{-20(x_2-1)}} & |x_2| \leq 1 \\ mf_2(x_2) = 1 - mf_1(x_2) & |x_2| > 1 \end{cases} \quad (5.8)$$

The membership functions of the states variables are shown in Figure 5.7. Then the heterogeneous control membership functions of the partial models $\mu_i(t) = mf_i$ and

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$\sum_{l=1}^m \mu_l(t) = 1$. Therefore, heterogeneous control membership functions for partial models are time-varying according to state variables.

Figure 5.7 Membership functions of x_2

Uncertainties and nonlinearities decomposition

Since the parametric uncertainties in ΔA_l , ΔB_l and $\Delta G_{k,l}$ are required to satisfy Assumption 3.3.1, the parametric uncertainties in partial models can be decomposed as (3.29) and (3.30):

$$[\Delta A_l \quad \Delta B_l] = L_l F_l(t) [E_{1l} \quad E_{2l}]$$

$$\Delta G_{k,l} = L_{kg,l} F_{kg,l}(t) E_{kg,l}$$

$$\text{where } L_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; E_{1,1} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}; E_{2,2} = \begin{bmatrix} 1 \\ |\Delta B_{2,1}|_{\max} + 1 \end{bmatrix}; F_2(t) = \begin{bmatrix} 0 & \frac{\Delta B_{2,1} + 1}{|\Delta B_{2,1}|_{\max} + 1} \end{bmatrix};$$

$$L_{1g,1} = L_{2g,1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; F_{1g,1}(t) = \begin{bmatrix} \frac{\Delta G_{21,1} + 0.1}{|\Delta G_{21,1}|_{\max} + 0.1} \end{bmatrix}; E_{1g,1} = |\Delta G_{21,1}|_{\max} + 0.1; F_{2g,1}(t) = \begin{bmatrix} \frac{\Delta G_{22,1}}{|\Delta G_{22,1}|_{\max}} \end{bmatrix};$$

$$E_{2g,1} = |\Delta G_{22,1}|_{\max};$$

$$L_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; E_{1,2} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}; E_{2,1} = \begin{bmatrix} 1 \\ |\Delta B_{2,2}|_{\max} + 1 \end{bmatrix}; F_1(t) = \begin{bmatrix} 0 & \frac{\Delta B_{2,2} + 1}{|\Delta B_{2,2}|_{\max} + 1} \end{bmatrix};$$

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$$L_{1g,2} = L_{2g,2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} ; \quad F_{1g,2}(t) = \begin{bmatrix} \Delta G'_{21,2} \\ \left| \Delta G'_{21,2} \right|_{\max} \end{bmatrix} ; \quad E_{1g,2} = \left| \Delta G'_{21,2} \right|_{\max} ; \quad F_{2g,2}(t) = \begin{bmatrix} \Delta G_{22,2} \\ \left| \Delta G_{22,2} \right|_{\max} \end{bmatrix} ;$$

$$E_{2g,2} = \left| \Delta G_{22,2} \right|_{\max} ;$$

Control laws design for partial models

According to Theorem 3.3 of proposed global control approach, the stabilizing state-feedback control laws for the partial models of the CP system can be obtained by solving the algebraic Riccati equation (3.35) in Section 3.3.

$$(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) + P_l [-B_l R_l^{-1} B_l^T + L_l L_l^T] \\ + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} + Q_l = 0$$

where Q_l is a symmetric positive defined matrix, which can be chosen by the designer, $R_l = E_{2l}^T E_{2l} > 0$, $\varepsilon_{kg,l}$ is scaling parameter to be chosen, with $\varepsilon_{kg,l}$ satisfying $\varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l} < I$. In this case, a stabilizing state-feedback control law is given by (3.36):

$$u_l(t) = -K_l x(t) = -R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) x_l(t)$$

For the first partial model, let $Q_1 = \text{diag} [1E+4, 1]$, $\varepsilon_{1g,1} = 6.438$ and $\varepsilon_{2g,1} = 0.496$, solving the algebraic Riccati equation gives $K_1 = [-464.6969 \ -46.4112]$.

Similarly, for second partial model, let $Q_2 = \text{diag} [1E+6, 1]$, $\varepsilon_{1g,2} = 0.667$ and $\varepsilon_{1g,3} = 0.732$, it gives that $K_2 = [-804.144 \ -80.228]$.

The following global control law is then obtained according to (3.33).

$$u(t) = -\sum_{l=1}^2 \mu_l(t) K_l x(t) = -(\mu_1(t) K_1 + \mu_2(t) K_2) x(t)$$

Simulation results

To demonstrate the effectiveness of global controller, the CP system are required that the pole remains in the upright position from two initial conditions $x_1(0) = [89^0, 0, 0, 0]$,

$x_2(0) = [20^\circ, 0, 0, 0]$. The simulation results of the CP system are shown in Figures 5.8 and 5.9, where solid lines represent for the results from $x_1(0)$ and dot lines for $x_2(0)$.

Figure 5.8 Responses of the angular displacement θ from two initial conditions (global control of the displacement of the pole)

Figure 5.9 Responses of the displacement of the cart y from two initial conditions (global control of the displacement of the pole)

In comparison with other control strategies [47], the global control of the CP system take shorter time to converge to zero and the amplitude of the transient process is smaller. Thus, the global control is superior to other control schemes.

5.3.2 Global control of the displacement of both the cart and the pole

As described in equation (2.40) of Section 2.4.2, the dynamics of the CP system to regulate both displacement of the cart and the pole is given:

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} &= \begin{bmatrix} x_2 \\ \frac{mLx_4^2 \sin x_3 - mg \sin x_3 \cos x_3}{M + m \sin^2 x_3} \\ x_4 \\ \frac{(M + m)g \sin x_3 - mLx_4^2 \sin x_3 \cos x_3}{[M + m \sin^2 x_3]L} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M + m \sin^2 x_3} \\ 0 \\ -\frac{\cos x_3}{[M + m \sin^2 x_3]L} \end{bmatrix} u(t) \quad (5.9) \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} + \Delta B_2 \\ 0 \\ -\Delta B_4 \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\frac{mg}{2M} + \Delta G_{21} \\ 0 \\ 0 \end{bmatrix} g_1(x) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{(M + m)g}{ML} + \Delta G_{42} \end{bmatrix} g_2(x) \\ &\quad + \begin{bmatrix} 0 \\ \frac{mLx_4^2 \sin x_3}{M + m \sin^2 x_3} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{mLx_4^2 \sin 2x_3}{2[M + m \sin^2 x_3]L} \end{bmatrix} \end{aligned}$$

$$\Delta B_2 = \frac{1}{M + m \sin^2 x_3} - \frac{1}{M} \leq \frac{m}{M(M + m)}; \quad \Delta B_4 = \frac{\cos x_3}{ML + m \sin^2 x_3 L} \leq \frac{1}{ML}$$

$$g_1(x) = \sin 2x_3; \quad \|\sin(2x_3)\| \leq 2\|[0 \ 0 \ 1 \ 0]\| \|x\| \text{ and } W_{1,1} = [0 \ 0 \ 2 \ 0]$$

$$\Delta G_{21} = \frac{mg}{2M} - \frac{mg}{2(M + m \sin^2 x_3)} \leq \frac{mg}{2M} - \frac{mg}{2(M + m)} = \frac{m^2 g}{2M(M + m)}$$

$$g_2(x) = \sin x_3; \quad \|\sin(x_3)\| \leq \|[0 \ 0 \ 1 \ 0]\| \|x\| \text{ and } W_{2,1} = [0 \ 0 \ 1 \ 0]$$

$$\Delta G_{42} = \frac{(M + m)g}{[M + m \sin^2 x_3]L} - \frac{(M + m)g}{ML} \leq \frac{(M + m)g}{ML} - \frac{g}{L} = \frac{mg}{ML}.$$

The control objective is to drive its state trajectories to the origin $x = [0 \ 0 \ 0 \ 0]$.

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Decompose the CP system

Since the model of CP control system has the typical nonlinear functions $f_1(x, t, \Delta)$ and $f_2(x_i, x_j, t, \Delta)$ in Equation (2.2), the above CP system is decomposed into the following two subspaces by applying multiple partial models in Section 2.4.2:

$$S_1 = \{x \mid |x_4| \leq 1\};$$

$$S_2 = \{x \mid |x_4| > 1\};$$

The first partial model

Then the CP system can be transformed as the following form in the first subspace:

$$\begin{aligned} \dot{x} &= (A_1 + \Delta A_1)x(t) + (B_1 + \Delta B_1)u + \sum_{k=1}^2 [G_{k,1} + \Delta G_{k,1}(t)]g_{k,1}(x), \quad x \in S_1 \\ &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} + \Delta B_{2,1} \\ 0 \\ -\Delta B_{4,1} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\frac{mg}{2M} + \Delta G_{21,1} \\ 0 \\ -\Delta G_{41,1} \end{bmatrix} g_{1,1}(x) + \begin{bmatrix} 0 \\ \Delta G_{22,1} \\ 0 \\ \frac{(M+m)g}{ML} + \Delta G_{42,1} \end{bmatrix} g_{2,1}(x) \end{aligned}$$

$$\text{where } A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \Delta A_1 = 0; B_1 = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix}; \Delta B_1 = \begin{bmatrix} 0 \\ \Delta B_{2,1} \\ 0 \\ -\Delta B_{4,1} \end{bmatrix}; g_{1,1}(x) = \sin 2x_3;$$

$$G_{1,1} = \begin{bmatrix} 0 \\ -\frac{mg}{2M} \\ 0 \\ 0 \end{bmatrix}; \Delta G_{1,1} = \begin{bmatrix} 0 \\ \Delta G_{21,1} \\ 0 \\ -\Delta G_{41,1} \end{bmatrix}; G_{2,1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{(M+m)g}{ML} \end{bmatrix}; \Delta G_{2,1} = \begin{bmatrix} 0 \\ \Delta G_{22,1} \\ 0 \\ \Delta G_{42,1} \end{bmatrix};$$

$$g_{2,1}(x) = \sin x_3; \quad \Delta B_{2,1} \leq \frac{m}{M(M+m)}; \quad \Delta B_{4,1} \leq \frac{1}{ML}; \quad \Delta G_{21,1} \leq \frac{m^2 g}{2M(M+m)};$$

$$\Delta G_{41,1} = \frac{mx_4^2}{2[M+m \sin^2 x_3]} \leq \frac{m}{2M}; \quad \Delta G_{22,1} = \frac{mLx_4^2}{M+m \sin^2 x_3} \leq \frac{mL}{M}; \quad \Delta G_{42,1} \leq \frac{mg}{ML}$$

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The second partial model

For the second subspaces, the CP system can be linearized at the certain operating point, such as:

$$\dot{x} = (A_2 + \Delta A_2)x + (B_2 + \Delta B_2)u + \sum_{k_1=1}^2 [G_{k,2} + \Delta G_{k,2}(t)]g_{k,2}(x), \quad x \in S_2, \quad \text{operating point:}$$

$$|x_4| = 2$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \Delta A_{24,2} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \Delta A_{44,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} + \Delta B_{2,2} \\ 0 \\ -\Delta B_{4,2} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -\frac{mg}{2M} + \Delta G_{21,2} \\ 0 \\ 0 \end{bmatrix} g_{1,2}(x) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{(M+m)g}{ML} + \Delta G_{42,2} \end{bmatrix} g_{2,2}(x)$$

$$\text{where } \Delta A_{24,2} = \frac{mLx_4 \sin x_3}{M + m \sin^2 x_3} \leq \frac{\sqrt{m}Lx_4}{2\sqrt{M}}$$

$$\Delta A_{44,2} = \frac{mx_4 \sin 2x_3}{2(M + m \sin^2 x_3)} \leq \frac{mx_4}{2M}$$

$$\text{where } A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \Delta A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta A_{24,2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Delta A_{44,2} \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ \frac{1}{M} \\ 0 \\ 0 \end{bmatrix};$$

$$\Delta B_2 = \begin{bmatrix} 0 \\ \Delta B_{2,2} \\ 0 \\ -\Delta B_{4,2} \end{bmatrix}; \quad G_{1,2} = \begin{bmatrix} 0 \\ -\frac{mg}{2M} \\ 0 \\ 0 \end{bmatrix}; \quad \Delta G_{1,2} = \begin{bmatrix} 0 \\ \Delta G_{21,2} \\ 0 \\ 0 \end{bmatrix}; \quad G_{2,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{(M+m)g}{ML} \end{bmatrix};$$

$$\Delta B_{2,2} \leq \frac{m}{M(M+m)}; \Delta B_{4,2} \leq \frac{1}{ML}; \Delta G_{21,2} \leq \frac{m^2 g}{2M(M+m)}; g_{1,2}(x) = \sin 2x_3;$$

$$g_{2,2}(x) = \sin x_3; \Delta G_{2,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Delta G_{42,2} \end{bmatrix}; \Delta G_{42,2} \leq \frac{mg}{ML};$$

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Membership functions

According to the decomposition of the cart and pole system, the membership functions of the partial models $\mu_i(t)$ can be obtained from the membership functions of state variable as follows.

$$\begin{cases} mf_1(x_4) = \frac{1}{1+e^{-20(x_4+1)}} - \frac{1}{1+e^{-20(x_4-1)}} & |x_4| \leq 1 \\ mf_2(x_4) = 1 - mf_1(x_4) & |x_4| > 1 \end{cases} \quad (5.10)$$

The membership functions of the states variables are shown in Figure 5.10. Then the heterogeneous control membership functions of the partial models $\mu_i(t) = mf_i$ and

$\sum_{i=1}^m \mu_i(t) = 1$. Therefore, heterogeneous control membership functions for partial models are time-varying according to state variables.

Figure 5.10 Membership functions of x_4

Uncertainties and nonlinearities decomposition

Since the parametric uncertainties in ΔA_i , ΔB_i and $\Delta G_{k,i}$ are required to satisfy Assumption 3.3.1, the parametric uncertainties in partial models can be decomposed as (3.29) and (3.30):

$$[\Delta A_i \quad \Delta B_i] = L_i F_i(t) [E_{1i} \quad E_{2i}]$$

$$\Delta G_{k,i} = L_{kg,i} F_{kg,i}(t) E_{kg,i}$$

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$$L_1 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; E_{1,1} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; E_{2,1} = \begin{bmatrix} 1 \\ |\Delta B_{2,1}|_{\max} \\ |\Delta B_{4,1}|_{\max} \end{bmatrix}; F_1(t) = \begin{bmatrix} 0 & \frac{\Delta B_{2,1}}{|\Delta B_{2,1}|_{\max}} & 0 \\ 0 & 0 & \frac{-\Delta B_{4,1}}{|\Delta B_{4,1}|_{\max}} \end{bmatrix};$$

$$L_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; E_{1,2} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & |\Delta A_{24,2}|_{\max} \\ 0 & 0 & 0 & |\Delta A_{44,2}|_{\max} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; E_{2,2} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ |\Delta B_{2,1}|_{\max} \\ |\Delta B_{4,1}|_{\max} \end{bmatrix};$$

$$F_2(t) = \begin{bmatrix} 0 & \frac{\Delta A_{24,2}}{|\Delta A_{24,2}|_{\max}} & 0 & \frac{\Delta B_{2,1}}{|\Delta B_{2,1}|_{\max}} & 0 \\ 0 & 0 & \frac{\Delta A_{44,2}}{|\Delta A_{44,2}|_{\max}} & 0 & \frac{-\Delta B_{4,1}}{|\Delta B_{4,1}|_{\max}} \end{bmatrix};$$

$$L_{1g,1} = L_{2g,1} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}; F_{1g,1}(t) = \begin{bmatrix} 0 & \frac{\Delta G_{21,1}}{|\Delta G_{21,1}|_{\max}} & 0 & 0 \\ 0 & 0 & 0 & \frac{-\Delta G_{41,1}}{|\Delta G_{41,1}|_{\max}} \end{bmatrix};$$

$$E_{1g,1} = \begin{bmatrix} 0 \\ |\Delta G_{21,1}|_{\max} \\ 0 \\ |\Delta G_{41,1}|_{\max} \end{bmatrix}; F_{2g,1}(t) = \begin{bmatrix} 0 & \frac{\Delta G_{22,1}}{|\Delta G_{22,1}|_{\max}} & 0 & 0 \\ 0 & 0 & 0 & \frac{\Delta G_{42,1}}{|\Delta G_{42,1}|_{\max}} \end{bmatrix}; E_{2g,1} = \begin{bmatrix} 0 \\ |\Delta G_{22,1}|_{\max} \\ 0 \\ |\Delta G_{42,1}|_{\max} \end{bmatrix};$$

$$L_{1g,2} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}; F_{1g,2}(t) = \begin{bmatrix} \frac{\Delta G_{21,2}}{|\Delta G_{21,2}|_{\max}} \end{bmatrix}; E_{1g,2} = |\Delta G_{21,2}|_{\max}; L_{2g,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}; F_{2g,2}(t) = \begin{bmatrix} \frac{\Delta G_{42,2}}{|\Delta G_{42,2}|_{\max}} \end{bmatrix};$$

$$E_{1g,2} = |\Delta G_{42,2}|_{\max};$$

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Control laws design for partial models

According to Theorem 3.3 of proposed global control approach, the stabilizing state-feedback control laws for the partial models of CP system can be obtained by solving the algebraic Riccati equation (3.35) in Section 3.3.

$$(A_l - B_l R_l^{-1} E_{2l}^T E_{1l})^T P_l + P_l (A_l - B_l R_l^{-1} E_{2l}^T E_{1l}) + P_l [-B_l R_l^{-1} B_l^T + L_l L_l^T] \\ + \sum_{k=1}^N [G_{k,l} (I - \varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l})^{-1} G_{k,l}^T + \frac{1}{\varepsilon_{kg,l}^2} L_{kg,l} L_{kg,l}^T] P_l + \sum_{k=1}^N W_{k,l}^T W_{k,l} + Q_l = 0$$

where Q_l is a symmetric positive defined matrix, which can be chosen by the designer, $R_l = E_{2l}^T E_{2l} > 0$, $\varepsilon_{kg,l}$ is scaling parameter to be chosen, with $\varepsilon_{kg,l}$ satisfying $\varepsilon_{kg,l}^2 E_{kg,l}^T E_{kg,l} < I$. In this case, a stabilizing state-feedback control law is given by (3.36):

$$u_l(t) = -K_l x(t) = -R_l^{-1} (B_l^T P_l + E_{2l}^T E_{1l}) x_l(t)$$

For the first partial model, let $Q_1 = \text{diag} [1, 1, 70, 5]$, $\varepsilon_{1g,1} = 4.919$ and $\varepsilon_{2g,1} = 0.987$, solving the algebraic Riccati equation gives $K_1 = [-5.117 \ -7.7104 \ -46.690 \ -12.422]$.

Similarly, for second partial model, let $Q_2 = \text{diag} [1, 1, 70, 5]$, $\varepsilon_{1g,2} = 10.028$ and $\varepsilon_{1g,3} = 0.718$, it gives that $K_2 = [-18.669 \ -21.547 \ -302.392 \ -81.185]$.

The following global control law is then obtained according to (3.33).

$$u(t) = -\sum_{l=1}^2 \mu_l(t) K_l x(t) = -(\mu_1(t) K_1 + \mu_2(t) K_2) x(t)$$

Simulation results

To demonstrate the effectiveness of global controller, the CP system is required that the pole remains in the upright position and the cart returns to the origin from two initial conditions $x_1(0) = [55^\circ, 0, 0, 0]$, $x_2(0) = [20^\circ, 0, 0, 0]$. That means the global controller needs to regulate the cart and pole at the same time in the following simulations.

The simulation results of the Cart and Pole system are shown in Figures 5.11 – 5.13, where solid lines represent for the results from $x_1(0)$ and dots line for $x_2(0)$. The proposed

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global controller can regulates the cart and pole at the same time in the simulations irrespective of initial operating points. The switching of partial controllers was automatically and smoothly coordinated by time-varying partial membership functions.

Figure 5.11 Responses of the angular displacement θ from two initial conditions

Figure 5.12 Responses of the displacement of the cart y from two initial conditions

Figure 5.13 Control signals of the CP system with two initial conditions

5.3.3 Concluding remarks

The global control for a class of nonlinear systems proposed in Section 3.3 was applied to the CP system. Two control objectives: only regulate the displacement of pole and control displacement of both cart and pole, are respectively designed for the CP system according to the same design procedure. The CP system is first decomposed into two nonlinear parts, which can be described by the partial models valid over wider operating region than local linear ones in view of the nonlinearities and uncertainties. Then two partial controllers are respectively designed for each partial model based on robust control theory. Finally, the global control law is constructed by combining two partial control actions through heterogeneous control membership functions.

The simulation results show that the CP system could obtain good dynamic performance over wide operating regions irrespective of initial operating points for both of control objectives. The switching of partial controllers was automatically and smoothly coordinated by time-varying partial membership functions.

5.4 Conclusions

The proposed global control for a class of nonlinear systems has been successfully applied to two benchmark control examples: the BB system and the CP system. The attributes of the global control design are as follows:

- (1) Global system model for a class of nonlinear systems can represent the BB and the CP system over a wide operating region in consideration of system nonlinearities and uncertainties.
- (2) The global control scheme improves system performance by decreasing the number of partial controllers, which greatly simplify the switching of controllers for different operating regions and coordinate different controllers effectively and smoothly by membership functions.
- (3) Global controller is robust regardless the uncertainties of system parameters and the initial operating points.
- (4) The stabilization of each robust partial controller can guaranteed the overall stability of the nonlinear systems, which greatly predigests global controller design.

Global control for the large-scale interconnected systems will be applied to the multi-machine power systems to enhance transient stability in next chapter.

Chapter 6 Global Control of Multi-Machine Power System for Transient Stability Enhancement

6.1 Introduction

The strategy of global control for large-scale interconnected systems has been elaborated in Section 3.4. In this chapter, the strategy applies to real life large-scale interconnected systems, multi-machine power system, to enhance transient stability. The inherent characteristic and special control objectives of multi-machine power system are reviewed in Section 6.2. The control techniques employed to the multi-machine power systems in recent literature are provided as well. In Section 6.3, the design of global controller for multi-machine power system to improve transient stability is given in detail. A three-machine nine-bus power system is used as an example system to demonstrate the effectiveness and feasibility of proposed global controller by comparing with the classic linear controllers in Section 6.4. The simulation results substantiate the advantages of proposed global control theory.

6.2 Controller design on transient stability enhancement for multi-machine power systems

Multi-machine power system consists of several independent dynamic subsystems interconnected together. Those subsystems are large, geographically distributed and highly nonlinear system, power system naturally lend themselves to decentralized control. The objective of decentralized control is to maintain a stable system with varying interconnections due to system faults and subsequent line-switching, with the controller for each subsystem utilizing information only on states local to that subsystem. However, multiple decentralized controllers applied to control-station plant may result in undesirable interaction with detrimental effects to the overall performance of the power system. Therefore, besides the nonlinearity and uncertainty problems as ones mentioned

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in controller design of the SMIB in Section 4.2, plenty of literatures investigate proper coordination of versatile controllers in multi-machine power system.

Proper coordination of multiple controllers to avoid detrimental interactions has received much attention on transient stability enhancement. Earlier research works are mainly concerned with the design of a decentralized state feedback controller for PSS units, excitation and fast valve controllers to enhance transient stability and ensure a guaranteed performance ([71], [88] [95] and [96]). The decentralized adaptive feedback control law was then applied to multi-machine power systems by designing a high-gain observer to estimate system perturbation, which includes all subsystem nonlinearities and interactions between subsystems [97]. The robust decentralized technique was also combined with fuzzy technique to design of proportional and proportional-plus-integral (PI) controllers for multi-machine power system [38]. Moreover, the genetic algorithm, neural network and Hamiltonian control based control scheme are also studied for simultaneous coordination of multiple power system damping controllers [98]-[100].

For above controller designs, the multi-machine power system was approximated by linear models with certain uncertainties and nonlinearities were bounded by polynomials or represented by neural network. Most of controller designs ended up with the linear controllers and the quadratic Lyapunov functions. As the complexity of system increases, the dynamic characteristic of multi-machine power systems may not be accurately described by those linear models when it is subjected to large disturbances. Therefore, the controller designed for multi-machine power system to enhance transient stability is expected to satisfy the following conditions:

1. The need to use of nonlinear system models which are independent of the equilibrium points and take into account the important nonlinearities of the power system's model is crucial for the performance of controllers under severe disturbances.
2. It is not realistic to assume all the parameters involved in the power system are exactly known and system is not affected at all by disturbances. System parameter uncertainties should be counted on in the controller design.

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3. When the controllers in different places are responsible for a part of the overall power system, control signals are locally available for convenience, which in a control system design is often referred as the decentralized control.
4. The decentralized controllers may unknowingly interact and result in detrimental effects. So it is needed to ensure proper coordination of those geographically distributed controllers to avoid the undesirable interactions.

Global control scheme for multi-machine power system is proposed to satisfy the above practical design requirements in the next section.

6.3 Global control of multi-machine power systems

Figure 6.1 Global control scheme for multi-machine power system

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In this section, the global controller is designed for multi-machine power system to enhance transient stability. Global control system models are first proposed to represent the multi-machine power systems by interconnected subsystems with uncertainties and nonlinearities. Then each subsystem is decomposed into four nonlinear partial systems according to switching actions of breakers under a severe disturbance. Four partial systems represent pre-fault steady state, faulty state, fault clearing steady state and post-fault steady state of the subsystem respectively. Then the DFL technique is used to linearize four partial nonlinear models and decentralized robust controllers are designed for the linearized partial models for different states. Finally, the global control law is obtained by integrating the partial controllers from every subsystem with heterogeneous control membership functions. The configuration of the proposed global control scheme for multi-machine power system is depicted in Figure 6.1.

6.3.1 Applications of DFL technique to Multi-machine power system

Multi-machine power system is a typical large-scale interconnected nonlinear system. An N machine power system consists of N independent dynamic subsystems interconnected together. The classical multi-machine power system model is given in Section 2.3.1.

To alleviate the nonlinearities and interconnections between different generators, DFL technique is employed. The immeasurable variable $\dot{E}'_{qi}(t)$ is eliminated by differentiating equation (2.8):

$$\begin{aligned} \dot{P}_{ei}(t) = & \sum_{j=1}^n \dot{E}'_{qi}(t) E'_{qj}(t) B_{ij} \sin(\delta_{ij}(t)) + \sum_{j=1}^n E'_{qi}(t) \dot{E}'_{qj}(t) B_{ij} \sin(\delta_{ij}(t)) \\ & + \sum_{j=1}^n E'_{qi}(t) E'_{qj}(t) \dot{B}_{ij} \sin(\delta_{ij}(t)) + \sum_{j=1}^n E'_{qi}(t) E'_{qj}(t) B_{ij} \cos(\delta_{ij}(t)) \Delta\omega_{ij}(t) \end{aligned} \quad (6.1)$$

Let $\Delta P_{ei}(t) = P_{ei}(t) - P_{mi0}$, and use the equation (2.3)-(2.12) then (6.1) becomes

$$\begin{aligned} \Delta \dot{P}_{ei}(t) = & -\frac{1}{T_{d0i}} \Delta P_{ei}(t) + \frac{1}{T_{d0i}} [k_{ci} I_{qi}(t) u_{fi}(t) + (X_{di} - X'_{di}) I_{di}(t) I_{qi}(t) - T'_{doi} Q_{ei}(t) \Delta\omega_i(t) - P_{mi0}] \\ & + \sum_{j=1}^n E'_{qi}(t) [\dot{E}'_{qj}(t) B_{ij} + E'_{qj}(t) \dot{B}_{ij}] \sin(\delta_{ij}(t)) \end{aligned}$$

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$$-\sum_{j=1}^n E'_{qi}(t)E'_{qj}(t)B_{ij} \cos(\delta_{ij}(t))\Delta\omega_{ij}(t) \quad (6.2)$$

Let $\Delta P_{ei}(t) = P_{ei}(t) - P_{mi}$, and use the equation (2.5)-(2.12) then (6.2) becomes

$$\begin{aligned} \Delta P_{ei}(t) = & -\frac{1}{T'_{doi}} \Delta P_{ei}(t) + \frac{1}{T'_{doi}} v_{fi}(t) + \sum_{j=1}^n E'_{qi}(t)[\dot{E}'_{qj}(t)B_{ij} + E'_{qj}(t)\dot{B}_{ij}] \sin(\delta_{ij}(t)) \\ & - E'_{qi}(t) \sum_{j=1}^n E'_{qj}(t)B_{ij} \cos(\delta_{ij}(t))\Delta\omega_{ij}(t) \end{aligned} \quad (6.3)$$

$$\text{By letting } v_{fi}(t) = k_{ci}I_{qi}(t)u_{fi}(t) + (X_{di} - X'_{di})I_{di}(t)I_{qi}(t) - T'_{doi}Q_{ei}(t)\Delta\omega_i(t) - P_{mi} \quad (6.4)$$

Remark 6.3.1: The mapping (6.4) from $v_{fi}(t)$ to $u_{fi}(t)$ is invertible except when $I_{qi}(t) = 0$, which is not in the normal working region for a generator. When the compensating law (6.4) is employed, the linearized model of the generator is independent of the operating point of the system. Since $\Delta\omega_i(t)$ and $Q_{ei}(t)$ are locally measurable, $I_{di}(t)$ and $I_{qi}(t)$ can be calculated from the three phase current of the i th generator, I_{Ai} , I_{Bi} and I_{Ci} . Thus the DFL compensating law above is practically realizable.

Remark 6.3.2: After the DFL compensation, the generator model is linear with respect to the new input $v_{fi}(t)$ in steady state of multi-machine power system. However, the DFL compensating law still contains nonlinearities due to the effects of remote dynamics of the j th generator on the i th generator, such as switching of transmission lines when a fault occurs in the power system, the system parameter, $B_{ij}(t)$ will change considerably. In moment of switching breakers under large disturbances, the derivative of susceptance $\dot{B}_{ij}(t)$ changes dramatically and it cannot be bounded. However, it remains zero when structure of transmission lines does not change.

In the global controller design, we do not take the switching moment into account since the switching time can be neglected during the transient analysis. The multi-machine power system is decomposed into pre-fault steady state, faulty state, fault clearing steady states and post-fault steady state. Choosing the state as $x_i^T(t) = [\Delta\delta_i(t) \quad \Delta\omega_i(t) \quad \Delta P_{ei}(t)]$,

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the characteristic of the generator in four states after linearized by DFL technique is given by the following equations:

$$\begin{cases} \Delta \dot{\delta}_i(t) = \Delta \omega_i(t) \\ \Delta \dot{\omega}_i = -\frac{D_i}{2H_i} \Delta \omega_i(t) - \frac{\omega_0}{2H_i} (P_{ei}(t) - P_{mi}) \\ \Delta P_{ei}(t) = -\frac{1}{T'_{doi}} \Delta P_{ei}(t) + \frac{1}{T'_{doi}} v_{fi}(t) + \sum_{j=1}^n E'_{qi}(t) [\dot{E}'_{qj}(t) B_{ij} + E'_{qj}(t) \dot{B}_{ij}] \sin(\delta_{ij}(t)) \\ \quad - E'_{qi}(t) \sum_{j=1}^n E'_{qj}(t) B_{ij} \cos(\delta_{ij}(t)) \Delta \omega_{ij}(t) \end{cases} \quad (6.5)$$

In order to design a feedback controller to guarantee the overall stability of multi-machine power system irrespective of uncertainty parameters T'_{d0i} and $B_{ij}(t)$, the decentralized robust technique is employed design feedback controller for the DFL compensated system of four states.

6.3.2 Decentralized robust controllers for partial models

The N -machine system also contain the uncertainty in the generator parameter T'_{d0i} and $T'_{d0i} = \bar{T}'_{d0i} + \Delta T'_{d0i}$. the compensating law (6.4) can be rewritten as follows:

$$v_{fi}(t) = k_{ci} I_{qi}(t) u_{fi}(t) + (X_{di} - X'_{di}) I_{di}(t) I_{qi}(t) - \bar{T}'_{doi} Q_{ei}(t) \Delta \omega_i(t) - P_{mi} \quad (6.6)$$

Remark 6.3.3: The difference between (6.4) and (6.6) is that the uncertain parameters T'_{d0i} in (6.4) are replaced by the known parameters $\bar{T}'_{d0i}$ in (6.6). $\bar{T}'_{d0i}$ is the average value. Then (6.3) can be rewritten as follows:

$$\begin{aligned} \Delta \dot{P}_{ei}(t) = & -\left[\frac{1}{\bar{T}'_{d0i}} + \mu_i(t)\right] \Delta P_{ei}(t) + \left[\frac{1}{\bar{T}'_{d0i}} + \mu_i(t)\right] v_{fi}(t) - \psi_i(t) \Delta \omega_i(t) \\ & + \sum_{j=1}^n E'_{qi}(t) [\dot{E}'_{qj}(t) B_{ij} + E'_{qj}(t) \dot{B}_{ij}] \sin(\delta_{ij}(t)) - \sum_{j=1}^n E'_{qi}(t) E'_{qj}(t) B_{ij} \cos(\delta_{ij}(t)) \Delta \omega_{ij}(t) \end{aligned} \quad (6.7)$$

where $\mu_i(t) = \frac{1}{\bar{T}'_{d0i} + \Delta T'_{d0i}} - \frac{1}{\bar{T}'_{d0i}}$ and $\psi_i(t) = \left[\frac{1}{\bar{T}'_{d0i}} + \mu_i(t)\right] \Delta T'_{doi} Q_{ei}(t)$.

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The l th linearized partial models of the i th subsystem with parameter uncertainties, nonlinearities and interconnections become the following state space form:

$$\dot{x}_i(t) = (A_{i,l} + \Delta A_{i,l}(t))x_i(t) + (B_{i,l} + \Delta B_{i,l}(t))u_{i,l}(t) + \sum_{j=1}^N \sum_{k=1}^{T_{jk,l}} \{[G_{kij,l} + \Delta G_{kij,l}(t)]g_{kij,l}(x_i, x_j)\}$$

$$i = 1, 2, \dots, N, T_{jk} = 2, k = 1, 2, \dots, T_{jk,l}, l = 1, 2, 3, 4 \quad (6.8)$$

Choose $x_i^T(t) = [\Delta\delta_i(t) \quad \Delta\omega_i(t) \quad \Delta P_{ei}(t)]$ as the state variables; and $v_{fi}(t)$ as the input variables, the following can be obtained:

$$A_{i,l} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{D_i}{2H_i} & -\frac{\omega_0}{2H_i} \\ 0 & 0 & -\frac{1}{T'_{doi}} \end{bmatrix}; B_{i,l} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T'_{doi}} \end{bmatrix}; \Delta A_{i,l} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_{i,l}(t) & -\mu_{i,l}(t) \end{bmatrix};$$

$$\Delta B_{i,l} = \begin{bmatrix} 0 \\ 0 \\ \mu_{i,l}(t) \end{bmatrix}; G_{1ij,l} = G_{2ij,l} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \Delta G_{1ij,l} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{1ij,l}(t) \end{bmatrix}; \Delta G_{2ij,l} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{2ij,l}(t) \end{bmatrix};$$

$$g_{1ij,l}(x_i, x_j) = \sin(\delta_i(t) - \delta_j(t)); g_{2ij,l}(x_i, x_j) = \Delta\omega_j(t);$$

$$\gamma_{1ij,l}(t) = E'_{qi,l}(t)[\dot{E}'_{qj,l}(t)B_{ij,l} + E'_{qj,l}(t)\dot{B}_{ij,l}]; \gamma_{2ij,l}(t) = -E'_{qi,l}(t)E'_{qj,l}(t)B_{ij,l}\cos(\delta_{ij}(t)).$$

$$\mu_{i,l}(t) = \frac{1}{T'_{doi} + \Delta T'_{doi}} - \frac{1}{T'_{doi}} \text{ and } \psi_{i,l}(t) = [\frac{1}{T'_{doi}} + \mu_{i,l}(t)]\Delta T'_{doi}Q_{ei,l}(t)$$

The uncertain matrices $A_{i,l}$, $B_{i,l}$ and $G_{kij,l}$ are required to satisfy Assumption 3.4.1, the parametric uncertainties in partial models can be decomposed as (3.43) and (3.44):

$$[\Delta A_{i,l}(t) \quad \Delta B_{i,l}(t)] = L_{i,l}F_{i,l}(t)[E_{1i,l} \quad E_{2i,l}]$$

$$\Delta G_{kij,l} = L_{kij,l}F_{kij,l}(t)E_{kij,l}$$

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$$\text{where } L_{i,l}(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; E_{1i,l}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_{i,l}(t)|_{\max} & 0 \\ 0 & 0 & |\mu_{i,l}(t)|_{\max} \end{bmatrix}; E_{2i,l}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_{i,l}(t)|_{\max} \end{bmatrix};$$

$$F_{i,l}(t) = \begin{bmatrix} 0 & -\frac{\psi_{i,l}(t)}{|\psi_{i,l}(t)|_{\max}} & -\frac{\mu_{i,l}(t)}{|\mu_{i,l}(t)|_{\max}} \end{bmatrix}; E_{kij,l}(t) = 1; L_{kij,l}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{kij,l}(t)|_{\max} \end{bmatrix}^T;$$

$$\text{and } F_{kij,l}(t) = \begin{bmatrix} \frac{\gamma_{kij,l}(t)}{|\gamma_{kij,l}(t)|_{\max}} \end{bmatrix};$$

In the multi-machine cases, the electrical power of each generator and the electrical power flow through each transmission line are all bounded notice that:

$$P_{ei}(t) = \sum_{j=1}^n E'_{qi}(t) E'_{qj}(t) B_{ij} \sin(\delta_{ij}(t))$$

Thus $E'_{qi}(t) E'_{qj}(t) B_{ij}$ ($i = 1, 2, \dots, N$) are also bounded and

$$|\gamma_{2ij,l}(t)| \leq |E'_{qi,l}(t) E'_{qj,l}(t) B_{ij,l}| \leq |P_{ei,l}(t)|_{\max}$$

Notice also that the excitation voltage E_f and E_{ff} may rise by 6 times of the $E_{qj}(t)$ when there is no load in the system, then,

$$|\dot{E}'_{qj}| = \left| (E_{ff}(t) - E_{qj}(t)) \frac{1}{T'_{d0j}} \right| \leq 5 |E_{qj}(t)|_{\max} \frac{1}{|T'_{d0j}|_{\min}}$$

The dynamic changes of transmission parameter are considered as $\dot{B}_{ij} \neq 0$ only at the moment when the electrical network changes its structure. Thus, it follows:

$$|\gamma_{1ij,l}(t)| = |E'_{qi,l}(t) [\dot{E}'_{qj,l}(t) B_{ij,l} + E'_{qj,l}(t) \dot{B}_{ij,l}]| \leq 5 |P_{ei,l}(t)|_{\max} \frac{1}{|T'_{d0j}|_{\min}}$$

Remark 6.3.4: The ranges of uncertainties in power system model are various with different operating conditions. For example, in the steady states, the uncertainties of the power system only take into account model errors, which is a relatively smaller range in comparison with the structure change of power system under a severe disturbance. It is

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obvious that the bounds of $\mu_{i,l}(t)$, $\psi_{i,l}(t)$, $\gamma_{1ij,l}(t)$ and $\gamma_{2ij,l}(t)$ in the power system model only depend on generator parameters $\bar{T}'_{d0i}$, $\Delta T'_{d0i}$, $Q_{ei,l}(t)$ and $|P_{ei,l}(t)|_{\max}$.

The nonlinear terms $g_{kij,l}(x_i, x_j)$ are required to satisfy Assumption 3.4.2. Thus the following known matrices $W_{ki,l}$ and $W_{kij,l}$ exist that the nonlinear terms $g_{kij,l}(x_i, x_j)$ in the multi-machine system are satisfy the equation (3.45).

$$\begin{aligned} \|g_{kij,l}(x_i, x_j)\| &\leq \|W_{ki,l}x_i(t)\| + \|W_{kij,l}x_j(t)\| \\ \|g_{1ij,l}(x_i, x_j)\| &= \|\sin(\delta_i(t) - \delta_j(t))\| \leq \|\delta_i(t) - \delta_j(t)\| \\ &= [1 \ 0 \ 0][\Delta\delta_i(t) \ \Delta\omega_i(t) \ \Delta P_{ei}(t)]^T - [1 \ 0 \ 0][\Delta\delta_j(t) \ \Delta\omega_j(t) \ \Delta P_{ej}(t)]^T \\ \|g_{2ij,l}(x_i, x_j)\| &= \|\Delta\omega_j(t)\| = [0 \ 1 \ 0][\Delta\delta_j(t) \ \Delta\omega_j(t) \ \Delta P_{ej}(t)]^T \\ W_{1i,l} &= [1 \ 0 \ 0]; W_{1ij,l} = [1 \ 0 \ 0]; W_{2i,l} = [0 \ 0 \ 0]; W_{1ij,l} = [0 \ 1 \ 0]; \end{aligned}$$

To solve the global robust control problem for DFL compensated generator model (6.8) involves solving the algebraic Riccati equation (3.49) in Section 3.4.

$$\begin{aligned} (A_{i,l} - B_{i,l}R_{i,l}^{-1}E_{2i,l}^TE_{1i,l})^TP_{i,l} + P_{i,l}(A_{i,l} - B_{i,l}R_{i,l}^{-1}E_{2i,l}^TE_{1i,l}) + \sum_{j=1}^N \sum_{k=1}^{T_k} (W_{kji,l}^TW_{kji,l} + W_{ki,l}^TW_{ki,l}) \\ + P_{i,l}\{L_{i,l}L_{i,l}^T - B_{i,l}R_{i,l}^{-1}B_{i,l}^T + \sum_{j=1}^N \sum_{k=1}^{T_k} [G_{kij,l}(I - \varepsilon_{kij,l}^2E_{kij,l}^TE_{kij,l})G_{kij,l}^T \\ + \frac{1}{\varepsilon_{kij,l}^2}L_{kij,l}L_{kij,l}^T]\}P_{i,l} + Q_{i,l} = 0 \end{aligned}$$

where $Q_{i,l}$ are symmetric positive defined matrices, which can be chosen by the designer, $R_{i,l} = E_{2i,l}^TE_{2i,l} > 0$, $\varepsilon_{kij,l}$ are scaling parameters to be chosen, with $\varepsilon_{kij,l}$ satisfying $\varepsilon_{kij,l}^2E_{kij,l}^TE_{kij,l} < I$, $\forall i, j = 1, 2, \dots, N$. In this case, the stabilizing state-feedback control laws are given by

$$u_{i,l}(t) = -R_{i,l}^{-1}(B_{i,l}^TP_{i,l} + E_{2i,l}^TE_{1i,l})x_i(t)$$

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Remark 6.3.5: The Riccati equation may not always be solvable for uncertainty bounds of the subsystems over the whole operating conditions that the closed loop linearized system may not be stable for all admissible uncertainties and the power system may lose synchronism when a fault occurs. However, when we neglect the switching moment of breakers during the transient analysis and consider every subsystem as four linearized partial models in different states: pre-fault steady state, faulty state, fault clearing steady state and post-fault steady state, uncertainty bounds of each partial model become smaller than the one over the whole operating region because of ignoring the moment of transmission line switching. In partial controller design, only bounds of the generator parameters have to be known, which makes it easier to obtain robust partial control laws for each subsystem of multi-machine power system.

6.3.3 Global control using membership functions

Four partial controllers are separately designed by robust control technique for corresponding partial models, pre-fault, during fault, fault clearing and post-fault system models. Since we consider faults are clear and transmission lines are restored after disturbances, the pre-fault and the post-fault partial models are same. Therefore, three partial models are combined to represent every subsystem in multi-machine systems over the whole operating condition. In order to realize a soft switching type mechanism for changing of different operating conditions, the heterogeneous control membership function (3.18), is used to aggregate all the partial models and construct a global nonlinear model.

Similar with the global control design of the SMIB power system for transient stability enhancement, the rotor speed of the generator, $\Delta\omega_i(t)$, the active electrical power of the generator, $\Delta P_{ei}(t)$, and switching time of the breakers z , are used to construct the heterogeneous control membership functions of the partial models $\mu_{i,l}(t)$, where $x_i = \sqrt{\Delta\omega_i^2 + \Delta P_{ei}^2}$ and z are considered as state variables. The membership functions of state variables and heterogeneous control law are same with the ones in Section 4.3.3. Then the global control law is obtained by aggregating the partial controllers with heterogeneous control membership functions.

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$$v_{fi}(t) = \sum_{l=1}^m \mu_{i,l}(t) v_{fi,l}(t) \quad (6.9)$$

Remark 6.3.6: Three robust controllers are designed for the partial subsystems with different operating conditions: pre-fault steady state, faulty state, fault clearing and post-fault steady state. System parameters, fault location and system operating points are all treated as the uncertainties and handled by the decentralized robust control approach in each partial model of subsystems. The global control law is obtained by aggregating the partial controllers with heterogeneous control membership functions. As shown in the stability conditions of Theorem 3.4, the stabilization of each decentralized robust partial controller for generator excitation can guaranteed the overall stability of multi-machine power system irrespective of network parameters, fault locations and the system operating conditions.

6.4 Case study: A three-machine nine-bus power system

6.4.1 The three-machine nine-bus power system

Figure 6.2 The 3-machine 9-bus power system

In this section, the global control scheme which is proposed in Section 6.3 is tested on the following power system model shown in Figure 6.2 (where $R_1 = R_2 = 2R_{78} = 0.017$, and $X_1 = X_2 = 2X_{78} = 0.144$). This system consists of 3 machines, 9 buses and 3 constant

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loads [13]. The generator data and the pre-fault network parameters are shown in Table 6.1, Table 6.2 and Table 6.3 respectively.

Generator Data

Table 6.1 Generator Data

Generator	1	2	3
Rated MVA	247.5	192.0	128.0
KV	16.5	18.0	13.8
Power factor	1.0	0.85	0.85
Type	Hydro	Steam	Steam
Speed	180r/min	3600r/min	3600r/min
x_d	0.1460	0.8958	1.3125
x'_d	0.0608	0.1198	0.1813
x_q	0.0969	0.8645	1.2578
x'_q	0.0969	0.1969	0.25
x_l (leakage)	0.0336	0.0521	0.0742
T'_{d0}	8.96	6.00	5.89
T'_{q0}	0	0.535	0.600
Stored energy at rated speed	2364MW.s	640MW.s	301MW.s

In the transient stability study, power systems are usually operating at four operating stages: pre-fault, faulty, fault clearing and post-fault stages. The fault sequences used in the simulation are given as follows:

Stage 1: The system is in a pre-fault steady state.

Stage 2: A three phase short circuit fault occurs at $t = 0.1$ second.

Stage 3: The fault is removed by opening the breaker of the fault line at t_1 second.

Stage 4: The transmission line is restored with the fault clear at $t=1.4$ second.

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Stage 5: The system is in a post-fault state.

Network parameters

Table 6.2 Transmission parameters

Transmission lines Bus No.	Impedance		Shunt admittances
	R	X	G
1-4	0	0.0576	0
2-7	0	0.0625	0
3-9	0	0.0586	0
4-5	0.0100	0.0850	0.088
4-6	0.0170	0.0920	0.079
5-7	0.0320	0.1610	0.153
6-9	0.0390	0.1700	0.179
7-8*	0.0085	0.0720	0.0745
8-9	0.0119	0.1008	0.1045

*But substitute the transmission line between bus No. 7 and No. 8 by parallel lines with the reactance of each line twice the original value.

Load

Table 6.3 Fixed load

Load	Bus No.	Active power(MW)	Reactive power(MVar)
Load A	5-0	125	50
Load B	6-0	90	30
Load C	8-0	100	35

Remark 6.4.1: Since the faults are restored at the post-fault state, system structures are same at the pre-fault and post fault stages. Three linearized partial models can be used to represent four operating stages of each generator in the power system.

6.4.2 Global controller design for three-machine nine-bus power system

To design the global controller for three-machine nine-bus power system, global control scheme for multi-machine power system in Figure 6.1 is applied to the test system. The procedure of the global controller design is as follow:

Check the number of generators in the test system ($i = 3$) and divide the whole system into three different subsystems.

According to switching actions of breakers under a severe disturbance, three partial systems are used to represent pre-fault steady state, faulty state, fault clearing steady state and post-fault steady state of the subsystem respectively. Each partial model can be written in the form of (6.8).

Uncertainties and interconnections between each subsystem of partial models are decomposed as (3.43), (3.44) and (3.45) respectively. The DFL compensation laws for each generator are set up according to the compensating law (6.6).

Utilize Matlab program from the Mathwork, Inc. to solve the Riccati equation (3.49), thus the partial control laws for three partial models of three generators are obtained.

Design the heterogeneous control membership functions to characterize the operating region of each partial model.

Finally, the global controller is obtained by aggregating the partial controllers with heterogeneous control membership functions via equation (6.9).

Remark 6.4.2: The decentralized partial controllers are designed separately and based on the local measurement only in the interconnected systems. The decentralized feedback laws guarantee the overall stability of multi-machine systems with uncertainties and interconnections.

In the simulation, consider the uncertainties $\Delta T'_{d0i} = 0.1\bar{T}'_{d0i}$. The power system model can be written in the form of (6.8), where i th (1, 2 and 3) subsystems are representative of the generators (1, 2 and 3) in the example system.

For the generator 1, the partial models in the example system is as follow:

$$\dot{x}_1(t) = (A_{1,i} + \Delta A_{1,i}(t))x_1(t) + (B_{1,i} + \Delta B_{1,i}(t))u_{1,i}(t) + \Delta G_{112,i} \sin(\delta_1(t) - \delta_2(t))$$

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$$+\Delta G_{113,l} \sin(\delta_1(t) - \delta_3(t)) + \Delta G_{211,l}(t)\Delta\omega_1(t) + \Delta G_{212,l}(t)\Delta\omega_2(t) + \Delta G_{213,l}(t)\Delta\omega_3(t) \quad l=1,2,3$$

where $l=1$ represent for the partial model of pre-fault and post-fault steady state. $l=2$ is for faulty state, $l=3$ is for fault clearing steady state. Since the uncertainty bounds only depend on generator parameters $\bar{T}'_{d0i}$, $\Delta T'_{d0i}$, $Q_{ei,l}(t)$ and $|P_{ei,l}(t)|_{\max}$, these bounds are very similar for the faulty states and fault clearing steady state. If we assume the same bounds for second and third partial model, the parameters of the second and third partial model are same. Consequently, the same partial controller is designed for the second and third partial controller. We only present the first and second partial controller design here.

$$A_{1,1} = A_{1,2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.0212 & -6.6447 \\ 0 & 0 & -0.1116 \end{bmatrix}; B_{1,1} = B_{1,2} = \begin{bmatrix} 0 \\ 0 \\ 0.1116 \end{bmatrix}; \Delta B_{1,1} = \Delta B_{1,2} = \begin{bmatrix} 0 \\ 0 \\ \mu_1(t) \end{bmatrix};$$

$$\Delta A_{1,1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_{1,1}(t) & -\mu_1(t) \end{bmatrix}; \Delta G_{112,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \Delta G_{113,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\Delta G_{211,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \Delta G_{212,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \Delta G_{213,1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$G_{112,1} = G_{112,2} = G_{113,1} = G_{113,2} = G_{212,1} = G_{212,2} = G_{213,1} = G_{213,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};$$

$$\Delta A_{1,2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_{1,2}(t) & -\mu_1(t) \end{bmatrix}; \Delta G_{112,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \Delta G_{113,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$\Delta G_{211,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \Delta G_{212,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \Delta G_{213,2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix};$$

$$|\mu_1(t)|_{\max} = \left| \frac{1}{\bar{T}'_{d01} + \Delta T'_{d01}} - \frac{1}{\bar{T}'_{d01}} \right|_{\max} \leq 0.0101.$$

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By choosing $|Q_{e1,1}(t)|_{\max} \leq 1$, $|P_{e1,1}(t)|_{\max} \leq 3$, $|Q_{e1,2}(t)|_{\max} \leq 3$ and $|P_{e1,2}(t)|_{\max} \leq 5$ then

$$|\psi_{1,1}(t)|_{\max} = \left| \left[\frac{1}{T'_{d01}} + \mu_1(t) \right] \Delta T'_{d01} Q_{e1,1}(t) \right|_{\max}$$

$$|\gamma_{112,1}(t)|_{\max} \leq 5 |P_{e1,1}(t)|_{\max} \frac{1}{|T'_{d02}|_{\min}}; |\gamma_{113,1}(t)|_{\max} \leq 5 |P_{e1,1}(t)|_{\max} \frac{1}{|T'_{d03}|_{\min}}$$

$$|\gamma_{211,1}(t)|_{\max} = |\gamma_{212,1}(t)|_{\max} = |\gamma_{213,1}(t)|_{\max} \leq |P_{e1,1}(t)|_{\max}$$

$$|\psi_{1,2}(t)|_{\max} = \left| \left[\frac{1}{T'_{d01}} + \mu_1(t) \right] \Delta T'_{d01} Q_{e1,2}(t) \right|_{\max}$$

$$|\gamma_{112,2}(t)|_{\max} \leq 5 |P_{e1,2}(t)|_{\max} \frac{1}{|T'_{d02}|_{\min}}; |\gamma_{113,2}(t)|_{\max} \leq 5 |P_{e1,2}(t)|_{\max} \frac{1}{|T'_{d02}|_{\min}}$$

$$|\gamma_{211,2}(t)|_{\max} = |\gamma_{212,2}(t)|_{\max} = |\gamma_{213,1}(t)|_{\max} \leq |P_{e1,2}(t)|_{\max}$$

The structure of the uncertainties for partial models of the generator 1 can be expressed as follows:

$$L_{1,1}(t) = L_{1,2}(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; E_{11,1}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_{1,1}(t)|_{\max} & 0 \\ 0 & 0 & |\mu_1(t)|_{\max} \end{bmatrix};$$

$$E_{11,2}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_{1,2}(t)|_{\max} & 0 \\ 0 & 0 & |\mu_1(t)|_{\max} \end{bmatrix}; E_{21,1}(t) = E_{21,2}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_1(t)|_{\max} \end{bmatrix};$$

$$F_{1,1}(t) = \begin{bmatrix} 0 & -\frac{\psi_{1,1}(t)}{|\psi_{1,1}(t)|_{\max}} & -\frac{\mu_1(t)}{|\mu_1(t)|_{\max}} \end{bmatrix}; F_{1,2}(t) = \begin{bmatrix} 0 & -\frac{\psi_{1,2}(t)}{|\psi_{1,2}(t)|_{\max}} & -\frac{\mu_1(t)}{|\mu_1(t)|_{\max}} \end{bmatrix};$$

$$L_{112,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{112,1}(t)|_{\max} \end{bmatrix}^T; F_{112,1}(t) = \begin{bmatrix} \gamma_{112,1}(t) \\ |\gamma_{112,1}(t)|_{\max} \end{bmatrix};$$

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$$L_{113,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{113,1}(t)|_{\max} \end{bmatrix}^T; F_{113,1}(t) = \begin{bmatrix} \frac{\gamma_{113,1}(t)}{|\gamma_{113,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{211,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{211,1}(t)|_{\max} \end{bmatrix}^T; F_{211,1}(t) = \begin{bmatrix} \frac{\gamma_{211,1}(t)}{|\gamma_{211,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{212,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{212,1}(t)|_{\max} \end{bmatrix}^T; F_{212,1}(t) = \begin{bmatrix} \frac{\gamma_{212,1}(t)}{|\gamma_{212,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{213,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{213,1}(t)|_{\max} \end{bmatrix}^T; F_{213,1}(t) = \begin{bmatrix} \frac{\gamma_{213,1}(t)}{|\gamma_{213,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{112,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{112,2}(t)|_{\max} \end{bmatrix}^T; F_{112,2}(t) = \begin{bmatrix} \frac{\gamma_{112,2}(t)}{|\gamma_{112,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{113,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{113,2}(t)|_{\max} \end{bmatrix}^T; F_{113,2}(t) = \begin{bmatrix} \frac{\gamma_{113,2}(t)}{|\gamma_{113,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{211,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{211,2}(t)|_{\max} \end{bmatrix}^T; F_{211,2}(t) = \begin{bmatrix} \frac{\gamma_{211,2}(t)}{|\gamma_{211,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{212,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{212,2}(t)|_{\max} \end{bmatrix}^T; F_{212,2}(t) = \begin{bmatrix} \frac{\gamma_{212,2}(t)}{|\gamma_{212,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{213,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{213,2}(t)|_{\max} \end{bmatrix}^T; F_{213,2}(t) = \begin{bmatrix} \frac{\gamma_{213,2}(t)}{|\gamma_{213,2}(t)|_{\max}} \end{bmatrix};$$

and $E_{112,1}(t) = E_{113,1}(t) = E_{211,1}(t) = E_{212,1}(t) = E_{213,1}(t) = 1$;

$$E_{112,2}(t) = E_{113,2}(t) = E_{211,2}(t) = E_{212,2}(t) = E_{213,2}(t) = 1$$

For the generator 2, the partial models in the example system is as follows:

$$\dot{x}_2(t) = (A_{2,l} + \Delta A_{2,l}(t))x_2(t) + (B_{2,l} + \Delta B_{2,l}(t))u_{2,l}(t) + \Delta G_{121,l} \sin(\delta_2(t) - \delta_1(t))$$

$$+ \Delta G_{123,l} \sin(\delta_2(t) - \delta_3(t)) + \Delta G_{221,l}(t)\Delta\omega_1(t) + \Delta G_{222,l}(t)\Delta\omega_2(t) + \Delta G_{223,l}(t)\Delta\omega_3(t) \quad l = 1, 2, 3$$

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where $l=1$ represent for the partial model of pre-fault and post-fault steady state. $l=2$ is for faulty state, $l=3$ is for fault clearing steady state. Similar with generator 1, we assume the same bounds for second and third partial model, and the same partial controller is suitable for these two partial models. Only the first and second partial controller designs are presented.

$$A_{2,1} = A_{2,2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.0781 & -24.544 \\ 0 & 0 & -0.167 \end{bmatrix}; B_{2,1} = B_{2,2} = \begin{bmatrix} 0 \\ 0 \\ 0.167 \end{bmatrix}; \Delta B_{2,1} = \Delta B_{2,2} = \begin{bmatrix} 0 \\ 0 \\ \mu_2(t) \end{bmatrix};$$

$$\Delta A_{2,1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_{2,1}(t) & -\mu_2(t) \end{bmatrix}; \Delta G_{121,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{121,1}(t) \end{bmatrix}; \Delta G_{123,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{123,1}(t) \end{bmatrix};$$

$$\Delta G_{221,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{221,1}(t) \end{bmatrix}; \Delta G_{222,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{222,1}(t) \end{bmatrix}; \Delta G_{223,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{223,1}(t) \end{bmatrix};$$

$$G_{121,1} = G_{123,1} = G_{221,1} = G_{222,1} = G_{223,1} = G_{121,2} = G_{123,2} = G_{221,2} = G_{222,1} = G_{223,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};$$

$$\Delta A_{2,2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_{2,2}(t) & -\mu_2(t) \end{bmatrix}; \Delta G_{121,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{121,2}(t) \end{bmatrix}; \Delta G_{123,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{123,2}(t) \end{bmatrix}$$

$$\Delta G_{221,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{221,2}(t) \end{bmatrix}; \Delta G_{222,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{222,2}(t) \end{bmatrix}; \Delta G_{223,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{223,2}(t) \end{bmatrix};$$

$$|\mu_2(t)|_{\max} = \left| \frac{1}{\bar{T}'_{d02} + \Delta T'_{d02}} - \frac{1}{\bar{T}'_{d02}} \right|_{\max} \leq 0.0152.$$

By choosing $|Q_{e2,1}(t)|_{\max} \leq 1$, $|P_{e2,1}(t)|_{\max} \leq 2$, $|Q_{e2,2}(t)|_{\max} \leq 2$ and $|P_{e2,2}(t)|_{\max} \leq 4.0$ then

$$|\psi_{2,1}(t)|_{\max} = \left| \left[\frac{1}{\bar{T}'_{d02}} + \mu_2(t) \right] \Delta T'_{d02} Q_{e2,1}(t) \right|_{\max}$$

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$$|\gamma_{121,1}(t)|_{\max} \leq 5|P_{e2,1}(t)|_{\max} \frac{1}{|T'_{d01}|_{\min}}; |\gamma_{123,1}(t)|_{\max} \leq 5|P_{e2,1}(t)|_{\max} \frac{1}{|T'_{d03}|_{\min}};$$

$$|\gamma_{221,1}(t)|_{\max} = |\gamma_{222,1}(t)|_{\max} = |\gamma_{223,1}(t)|_{\max} \leq |P_{e2,1}(t)|_{\max}$$

$$|\psi_{2,2}(t)|_{\max} = \left| \left[\frac{1}{T'_{d02}} + \mu_2(t) \right] \Delta T'_{d02} Q_{e2,2}(t) \right|_{\max}$$

$$|\gamma_{121,2}(t)|_{\max} \leq 5|P_{e2,2}(t)|_{\max} \frac{1}{|T'_{d01}|_{\min}}; |\gamma_{123,2}(t)|_{\max} \leq 5|P_{e2,2}(t)|_{\max} \frac{1}{|T'_{d03}|_{\min}};$$

$$|\gamma_{221,2}(t)|_{\max} = |\gamma_{222,2}(t)|_{\max} = |\gamma_{223,2}(t)|_{\max} \leq |P_{e2,2}(t)|_{\max}$$

The structure of the uncertainties for partial models of the generator 2 can be expressed as follows:

$$L_{2,1}(t) = L_{2,2}(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; E_{12,1}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_{2,1}(t)|_{\max} & 0 \\ 0 & 0 & |\mu_2(t)|_{\max} \end{bmatrix};$$

$$E_{12,2}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_{2,2}(t)|_{\max} & 0 \\ 0 & 0 & |\mu_2(t)|_{\max} \end{bmatrix}; E_{22,1}(t) = E_{22,2}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_2(t)|_{\max} \end{bmatrix};$$

$$F_{2,1}(t) = \begin{bmatrix} 0 & -\frac{\psi_{2,1}(t)}{|\psi_{2,1}(t)|_{\max}} & -\frac{\mu_2(t)}{|\mu_2(t)|_{\max}} \end{bmatrix}; F_{2,2}(t) = \begin{bmatrix} 0 & -\frac{\psi_{2,2}(t)}{|\psi_{2,2}(t)|_{\max}} & -\frac{\mu_2(t)}{|\mu_2(t)|_{\max}} \end{bmatrix};$$

$$L_{121,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{121,1}(t)|_{\max} \end{bmatrix}^T; F_{121,1}(t) = \begin{bmatrix} \frac{\gamma_{121,1}(t)}{|\gamma_{121,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{123,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{123,1}(t)|_{\max} \end{bmatrix}^T; F_{123,1}(t) = \begin{bmatrix} \frac{\gamma_{123,1}(t)}{|\gamma_{123,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{221,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{221,1}(t)|_{\max} \end{bmatrix}^T; F_{221,1}(t) = \begin{bmatrix} \frac{\gamma_{221,1}(t)}{|\gamma_{221,1}(t)|_{\max}} \end{bmatrix};$$

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$$L_{222,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{222,1}(t)|_{\max} \end{bmatrix}^T; F_{222,1}(t) = \begin{bmatrix} \frac{\gamma_{222,1}(t)}{|\gamma_{222,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{223,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{223,1}(t)|_{\max} \end{bmatrix}^T; F_{223,1}(t) = \begin{bmatrix} \frac{\gamma_{223,1}(t)}{|\gamma_{223,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{121,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{121,2}(t)|_{\max} \end{bmatrix}^T; F_{121,2}(t) = \begin{bmatrix} \frac{\gamma_{121,2}(t)}{|\gamma_{121,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{123,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{123,2}(t)|_{\max} \end{bmatrix}^T; F_{123,2}(t) = \begin{bmatrix} \frac{\gamma_{123,2}(t)}{|\gamma_{123,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{221,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{221,2}(t)|_{\max} \end{bmatrix}^T; F_{221,2}(t) = \begin{bmatrix} \frac{\gamma_{221,2}(t)}{|\gamma_{221,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{222,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{222,2}(t)|_{\max} \end{bmatrix}^T; F_{222,2}(t) = \begin{bmatrix} \frac{\gamma_{222,2}(t)}{|\gamma_{222,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{223,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{223,2}(t)|_{\max} \end{bmatrix}^T; F_{223,2}(t) = \begin{bmatrix} \frac{\gamma_{223,2}(t)}{|\gamma_{223,2}(t)|_{\max}} \end{bmatrix};$$

$$\text{and } E_{121,1}(t) = E_{123,1}(t) = E_{221,1}(t) = E_{221,1}(t) = E_{223,1}(t) =$$

$$= E_{121,2}(t) = E_{123,2}(t) = E_{221,2}(t) = E_{221,2}(t) = E_{223,2}(t) = 1$$

For the generator 3, the partial models in the example system is as follows:

$$\dot{x}_3(t) = (A_{3,l} + \Delta A_{3,l}(t))x_3(t) + (B_{3,l} + \Delta B_{3,l}(t))u_{3,l}(t) + \Delta G_{131,l} \sin(\delta_3(t) - \delta_1(t))$$

$$+ \Delta G_{132,l} \sin(\delta_3(t) - \delta_2(t)) + \Delta G_{231,l}(t)\Delta\omega_1(t) + \Delta G_{232,l}(t)\Delta\omega_2(t) + \Delta G_{233,l}(t)\Delta\omega_3(t) \quad l = 1, 2, 3$$

where $l=1$ represent for the partial model of pre-fault and post-fault steady state. $l=2$ is for faulty state, $l=3$ is for fault clearing steady state. Similar with generator 1, we assume the same bounds for second and third partial model, and the same partial controller is suitable for these two partial models. Only the first and second partial controller designs are presented.

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$$A_{3,1} = A_{3,2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -0.166 & -52.19 \\ 0 & 0 & -0.17 \end{bmatrix}; B_{3,1} = B_{3,2} = \begin{bmatrix} 0 \\ 0 \\ 0.167 \end{bmatrix}; \Delta B_{3,1} = \Delta B_{3,2} = \begin{bmatrix} 0 \\ 0 \\ \mu_3(t) \end{bmatrix};$$

$$\Delta A_{3,1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_{3,1}(t) & -\mu_3(t) \end{bmatrix}; \Delta G_{131,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{131,1}(t) \end{bmatrix}; \Delta G_{132,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{132,1}(t) \end{bmatrix};$$

$$\Delta G_{231,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{231,1}(t) \end{bmatrix}; \Delta G_{232,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{232,1}(t) \end{bmatrix}; \Delta G_{233,1} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{233,1}(t) \end{bmatrix};$$

$$G_{131,1} = G_{132,1} = G_{231,1} = G_{232,1} = G_{233,1} = G_{131,2} = G_{132,2} = G_{231,2} = G_{232,2} = G_{233,2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix};$$

$$\Delta A_{3,2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -\psi_{3,2}(t) & -\mu_3(t) \end{bmatrix}; \Delta G_{131,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{131,2}(t) \end{bmatrix}; \Delta G_{132,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{132,2}(t) \end{bmatrix};$$

$$\Delta G_{231,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{231,2}(t) \end{bmatrix}; \Delta G_{232,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{232,2}(t) \end{bmatrix}; \Delta G_{233,2} = \begin{bmatrix} 0 \\ 0 \\ \gamma_{233,2}(t) \end{bmatrix};$$

$$|\mu_3(t)|_{\max} = \left| \frac{1}{\overline{T}_{d03} + \Delta T'_{d03}} - \frac{1}{\overline{T}_{d03}} \right|_{\max} \leq 0.0154.$$

By choosing $|Q_{e3,1}(t)|_{\max} \leq 1$, $|P_{e3,1}(t)|_{\max} \leq 2$, $|Q_{e3,2}(t)|_{\max} \leq 2$ and $|P_{e3,2}(t)|_{\max} \leq 4.0$ then

$$|\psi_{3,1}(t)|_{\max} = \left| \left[\frac{1}{\overline{T}_{d03}} + \mu_3(t) \right] \Delta T'_{d03} Q_{e3,1}(t) \right|_{\max};$$

$$|\gamma_{131,1}(t)|_{\max} \leq 5 |P_{e3,1}(t)|_{\max} \frac{1}{|T'_{d01}|_{\min}}; |\gamma_{132,1}(t)|_{\max} \leq 5 |P_{e3,1}(t)|_{\max} \frac{1}{|T'_{d02}|_{\min}};$$

$$|\gamma_{231,1}(t)|_{\max} = |\gamma_{232,1}(t)|_{\max} = |\gamma_{233,1}(t)|_{\max} \leq |P_{e3,1}(t)|_{\max};$$

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$$|\psi_{3,2}(t)|_{\max} = \left| \left[\frac{1}{T'_{d03}} + \mu_3(t) \right] \Delta T'_{d03} Q_{e3,2}(t) \right|_{\max};$$

$$|\gamma_{131,2}(t)|_{\max} \leq 5 |P_{e3,2}(t)|_{\max} \frac{1}{|T'_{d01}|_{\min}}; |\gamma_{132,2}(t)|_{\max} \leq 5 |P_{e3,2}(t)|_{\max} \frac{1}{|T'_{d02}|_{\min}};$$

$$|\gamma_{231,2}(t)|_{\max} = |\gamma_{232,2}(t)|_{\max} = |\gamma_{233,2}(t)|_{\max} \leq |P_{e3,2}(t)|_{\max};$$

The structure of the uncertainties for partial models of the generator 3 can be expressed as follows:

$$L_{3,1}(t) = L_{3,2}(t) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; E_{13,1}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_{3,1}(t)|_{\max} & 0 \\ 0 & 0 & |\mu_3(t)|_{\max} \end{bmatrix};$$

$$E_{13,2}(t) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & |\psi_{3,2}(t)|_{\max} & 0 \\ 0 & 0 & |\mu_3(t)|_{\max} \end{bmatrix}; E_{23,1}(t) = E_{23,2}(t) = \begin{bmatrix} 0 \\ 0 \\ -|\mu_3(t)|_{\max} \end{bmatrix};$$

$$F_{3,1}(t) = \begin{bmatrix} 0 & -\frac{\psi_{3,1}(t)}{|\psi_{3,1}(t)|_{\max}} & -\frac{\mu_3(t)}{|\mu_3(t)|_{\max}} \end{bmatrix}; F_{3,2}(t) = \begin{bmatrix} 0 & -\frac{\psi_{3,2}(t)}{|\psi_{3,2}(t)|_{\max}} & -\frac{\mu_3(t)}{|\mu_3(t)|_{\max}} \end{bmatrix};$$

$$L_{131,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{131,1}(t)|_{\max} \end{bmatrix}^T; F_{131,1}(t) = \begin{bmatrix} \frac{\gamma_{131,1}(t)}{|\gamma_{131,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{132,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{132,1}(t)|_{\max} \end{bmatrix}^T; F_{123,1}(t) = \begin{bmatrix} \frac{\gamma_{123,1}(t)}{|\gamma_{123,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{231,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{231,1}(t)|_{\max} \end{bmatrix}^T; F_{231,1}(t) = \begin{bmatrix} \frac{\gamma_{231,1}(t)}{|\gamma_{231,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{232,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{232,1}(t)|_{\max} \end{bmatrix}^T; F_{232,1}(t) = \begin{bmatrix} \frac{\gamma_{232,1}(t)}{|\gamma_{232,1}(t)|_{\max}} \end{bmatrix};$$

$$L_{233,1}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{233,1}(t)|_{\max} \end{bmatrix}^T; F_{233,1}(t) = \begin{bmatrix} \frac{\gamma_{233,1}(t)}{|\gamma_{233,1}(t)|_{\max}} \end{bmatrix};$$

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$$L_{131,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{131,2}(t)|_{\max} \end{bmatrix}^T; F_{131,2}(t) = \begin{bmatrix} \frac{\gamma_{131,2}(t)}{|\gamma_{131,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{132,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{132,2}(t)|_{\max} \end{bmatrix}^T; F_{132,2}(t) = \begin{bmatrix} \frac{\gamma_{132,2}(t)}{|\gamma_{132,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{231,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{231,2}(t)|_{\max} \end{bmatrix}^T; F_{231,2}(t) = \begin{bmatrix} \frac{\gamma_{231,2}(t)}{|\gamma_{231,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{232,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{232,2}(t)|_{\max} \end{bmatrix}^T; F_{232,2}(t) = \begin{bmatrix} \frac{\gamma_{232,2}(t)}{|\gamma_{232,2}(t)|_{\max}} \end{bmatrix};$$

$$L_{233,2}(t) = \begin{bmatrix} 0 & 0 & |\gamma_{233,2}(t)|_{\max} \end{bmatrix}^T; F_{233,2}(t) = \begin{bmatrix} \frac{\gamma_{233,2}(t)}{|\gamma_{233,2}(t)|_{\max}} \end{bmatrix};$$

$$\text{and } E_{131,1}(t) = E_{132,1}(t) = E_{231,1}(t) = E_{231,1}(t) = E_{233,1}(t) \\ = E_{131,2}(t) = E_{132,2}(t) = E_{231,2}(t) = E_{231,2}(t) = E_{233,2}(t) = 1$$

Following the design procedure in Section 6.3.2 and solving the Riccati equation (3.49), the partial control laws for three partial models of three generators are obtained:

The partial controllers for generator 1 are as follow:

$$v_{f1,1}(t) = v_{f1,4}(t) = 146.3 \Delta \delta_1 + 102.18 \Delta \omega_1 - 287.43 \Delta P_{e1};$$

$$v_{f1,2}(t) = v_{f1,3}(t) = 279.71 \Delta \delta_1 + 159.62 \Delta \omega_1 - 603.27 \Delta P_{e1};$$

The partial controllers for generator 2 are as follow:

$$v_{f2,1}(t) = v_{f2,4}(t) = 223.13 \Delta \delta_2 + 89.19 \Delta \omega_2 - 297.61 \Delta P_{e2};$$

$$v_{f2,2}(t) = v_{f2,3}(t) = 713.44 \Delta \delta_2 + 189.57 \Delta \omega_2 - 813.78 \Delta P_{e2};$$

The partial controllers for generator 3 are as follow:

$$v_{f3,1}(t) = v_{f3,4}(t) = 156.58 \Delta \delta_3 + 129.18 \Delta \omega_3 - 297.64 \Delta P_{e3};$$

$$v_{f3,2}(t) = v_{f3,3}(t) = 168.71 \Delta \delta_3 + 130.53 \Delta \omega_3 - 531.56 \Delta P_{e3};$$

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Then the global control law of each generator ($i=1, 2$ and 3) is obtained by aggregating the partial controllers with heterogeneous control membership functions.

$$v_{fi}(t) = \sum_{l=1}^m \mu_{i,l}(t) v_{fi,l}(t)$$

6.4.3 Simulation results

To evaluate the global control scheme for transient stability enhancement, the three-machine power system shown in Figure 6.2 is utilized as a test system. Numerical simulations of the three-machine nine-bus power system were implemented in PSCAD (Power Systems Computer-Aided Design) simulator from Manitoba HVDC Research Centre.

Figure 6.3 illustrate the block diagram of the PSCAD simulations. Based on the generator data, network parameters and load information shown in Section 6.4.1, the initial conditions of the example power system are calculated by Newton-Raphson power flow method. The system base is chosen to be 100MVA and the values for the direct axis reactance of the generators X_{di} are utilized to calculate the initial power angle of the generators. Therefore, the initial operating points of the three-machine nine-bus power system are as presented in Table 6.4.

Table 6.4 The operating point of three-machine nine-bus power system

	1#	2#	3#
δ_0 (degree)	5.33°	62.03°	55.52°
P_{m0} (p.u.)	0.716	1.63	0.85
Q_{m0} (p.u.)	0.27	0.067	-0.109
V_{t0} (p.u.)	1.04	1.025	1.0251

A symmetrical three-phase short circuit fault is considered at two typical fault locations in the simulation: Fault 1 occurs near bus 8 and Fault 2 occurs near bus 7 on the lower transmission line between bus 7 and bus 8. Therefore, two different cases are applied to test the performance of the control system under different fault locations and transmission line parameters. The performance of global controller for generator excitation is tested by

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comparing with classical linear controller in [13]. The parameters and physical limits of linear controller are given in Appendix A4.

Figure 6.3 The block diagram of the PSCAD simulations

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Case 6.1: Global control scheme with Fault 1

The global control scheme is compared with the classical linear generator excitation controller in [13] to improve transient stability, where the fault clearing time $t_1 = 0.20$ sec.. The responses of global control scheme are shown in Figures 6.4 (a)-(e), where the real power output of machines, $P_{ei}(t)$, the relative speeds of the machines, $\omega_i(t)$, excitation output of the generators $E_{fi}(t)$, the power angles of the machines, $\delta_i(t)$ and the power angle differences of machines $\delta_{ij}(t)$ are given respectively.

(a) $P_{ei}(t)$ (b) $\omega_i(t)$ (c) $E_{fi}(t)$ (d) $\delta_i(t)$

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(e) $\delta_{ij}(t)$

Figure 6.4 Responses of machines with global control schemes with Fault 1

Figures 6.4 (a)-(c) illustrate that when the three-machine nine-bus power system is subjected to a disturbance near bus 8, the system variables $P_{ei}(t)$ and $\omega_i(t)$, and control signal $E_f(t)$ will come back to the initial steady states within two seconds. Consequently, the power angles $\delta_i(t)$ and the power angle differences $\delta_{ij}(t)$ converge to the initial steady state as well.

The responses of classical linear control scheme are shown in Figure 6.5(a)-(g), where the relative speeds of the machines, $\omega_i(t)$, the real power output of machines, $P_{ei}(t)$, the power angles of the machines, $\delta_i(t)$ and the power angle differences of machines $\delta_{ij}(t)$ are given respectively.

Comparing Figure 6.4 (a) with Figure 6.5 (a) and Figure 6.4 (b) with Figure 6.5 (b) show that classical linear control system takes much longer time than the proposed global control scheme to damp oscillations resulted from the same fault. The system responses of $P_{ei}(t)$ and $\omega_i(t)$ of the classical linear controllers were damped to the initial steady states after eight seconds instead of two seconds used by the proposed global control scheme. In addition, Figures 6.5 (c)-(g) compares the responses of the power angles, $\delta_i(t)$, and the power angle differences $\delta_{ij}(t)$ between the classical linear control system and the global control schemes. It is shown that the responses of power angles $\delta_i(t)$ and power angle difference $\delta_{ij}(t)$ for both control systems can return to the initial steady states. However, the response from classical linear control system is quite slower than the one from the global control system. Note that the responses of power angle difference

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$\delta_{21}(t)$ and $\delta_{31}(t)$ in Figures 6.5 (h)-(g) demonstrate that the classical linear controllers, particularly PSS, can quickly damp the local mode oscillations in the earlier swings, but they have limited capacity to damp later oscillations which may resulted from the inter machine power flow exchange.

The critical fault clearing times of these two control schemes are compared when the example power system operates upon the steady states and Fault 1 occurs. The three-machine nine-bus power system with classical linear controllers can not bring the system back to the original steady state when the critical fault clearing time is between 0.23 sec. and 0.24 sec. However, the critical fault clearing time for the global control system is between 0.34 sec. and 0.35 sec. Therefore, the global control scheme can obviously improve the transient stability in comparison with the classical linear controllers.

(a) $\omega_i(t)$

(b) $P_{ei}(t)$

(c) $\delta_1(t)$

(d) $\delta_2(t)$

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Figure 6.5 Responses of machines with classical linear control schemes and Fault 1

Case 6.2: Global control scheme with Fault 2

To verify the robustness of the global control scheme for the different fault locations, the most sensitive Fault 2 was chosen and the clearing time t_1 was extended to 0.22 sec.. The responses of global control scheme are shown in shown in Figures 6.6(a)-(e), where the real power output of machines , $P_{ei}(t)$, the relative speeds of the machines, $\omega_i(t)$, excitation output of the generators $E_{fi}(t)$, the power angles of the machines, $\delta_i(t)$ and the power angle differences of machines $\delta_{ij}(t)$ are given respectively.

Although the sever fault occurs at the most sensitive fault location, near bus 7 of the example system, Figures 6.6 (a)-(b) demonstrate that the global control system can bring

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the system variables $P_{ei}(t)$ and $\omega_i(t)$ to the initial steady states within three seconds. Simultaneously, the control signal $E_{fi}(t)$, the power angles $\delta_i(t)$ and the power angle differences $\delta_{ij}(t)$ converge to the steady state as well.

Figure 6.6 Responses of machines with global control schemes with Fault 2

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From above simulation cases for the three-machine nine-bus power system, it can be concluded that:

1. Global system model for a large-scale interconnected system can represent multi-machine power system over a wide operating region in consideration of system nonlinearities, uncertainties and interconnections.
2. The global control scheme can damp the oscillations and enhance the transient stability regardless of fault locations. It is obvious that the global control scheme can damp the oscillations and bring the system back to the steady state operating point faster than the classic linear generator excitation controller. The proposed controller can provide satisfactory global performance in power system transient stability, even under the most severe fault occurred at the terminal of the generator. Note that the most sensitive fault locations Fault 2 are chosen in the simulations. The simulation results with Fault 2 s are obviously more persuasive than the ones with the faults chosen randomly.
3. The global controller for the multi-machine power system is designed by using available local control signals, which are conveniently implemented to enhance the transient stability in practice.

6.5 Concluding remarks

The proposed global control of large-scale interconnected systems in Section 3.4 was applied to the multi-machine power system to enhance transient stability in this chapter. As an illustration of global control strategy for large-scale interconnected systems, the multi-machine power system was represented by interconnected subsystems with uncertainties and nonlinearities. Then each subsystem is decomposed into four nonlinear partial systems according to switching actions of breakers under a severe disturbance. Four partial systems represent pre-fault steady state, faulty state, fault clearing and post-fault steady state of the subsystem respectively. After that, the direct feedback linearization (DFL) technique is used to linearize four partial nonlinear models and decentralized robust controllers are designed for the linearized partial models for different states. Finally, the global control law is obtained by integrating the partial controllers from every subsystem with heterogeneous control membership functions.

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The results of simulation cases demonstrate that global controller designed greatly enhance transient stability of multi-machine power system in comparison of the classical linear controller. The global control scheme can damp the oscillations regardless of the system operating point and fault locations. The global control of multi-machine system clearly supports the objectives of proposed global control of large-scale interconnected systems. The attributes of the proposed global controller are as follows:

- (1) Global system model are successfully introduced to represent the multi-machine power system over a wide operating region in consideration of system nonlinearities, uncertainties and interconnections.
- (2) The global control scheme can damp the oscillations and enhance the transient stability regardless of the system operating points and fault locations.
- (3) The global controller is designed by using available local control signals.
- (4) The decentralized partial controllers are coordinated effectively and smoothly by membership functions of the heterogeneous control law.
- (5) The stabilization of each decentralized robust partial controller can guaranteed the overall stability of multi-machine power system, which greatly simplifies global controller design.

In next chapter, global control approach will be applied to uncertain power systems to solve inherent conflicting control goals.

Chapter 7 Global Control of Power System for Transient Stability Enhancement and Voltage regulation

7.1 Introduction

In previous chapters, nonlinearities, uncertainties and controller interactions of complex systems have been considered in design of global controller. In this chapter, the global control is extended to coordinate different, sometimes inherently conflicting, control objectives within varying operating regions. This global strategy is then investigated to maintain the transient stability and achieve proper post-fault voltage of a power system in consideration of uncertainties and controller interactions.

Power systems are highly nonlinear structured systems with multiple control objectives, which mainly include regulation of voltages and frequency, damping oscillations adequately and preserving synchronism in face of large disturbances. However, traditional power system controllers are installed in power system to serve a single objective. There exists the lack of systematic coordination for two of the most important problem of the power system: real power/frequency regulation and reactive power/voltage regulation. Thus, power systems operating in several different conditions may need to coordinate different control goals, which may have inherent conflicts and need to find a trade-off between these objectives. Two control systems are evidently demonstrated the conflicting control objectives during different control stages. One of most typical examples is the interaction between AVR and PSS [13]-[17].

The conventional approach to AVR and PSS design can basically be classified as a sequential design for voltage regulation and stabilization since these two control objectives cannot be achieved simultaneously [13]. At first, attempts have been made to coordinate those two goals in the design of linear controllers. A coordinated approach to the design was warranted, with due regard for priorities for relative performance under different operating conditions [14]. A practical scheme used switching of a transient stability control module to argument the usual PSS [13]. All those linear controllers are

designed based on the local linearized system model, which cannot guarantee stabilization of power system subjected to a severe disturbance.

Advanced nonlinear control techniques have been applied to design generator excitation controllers to maintain the transient stability and achieve satisfactory post-fault voltage. A new DFL excitation controller was proposed through switching DFL-LQ optimal controller and DFL voltage regulator, which are designed based on nonlinear feedback linearization techniques ([15] and [17]). Afterwards, automatically shifting these nonlinear transient and voltage controllers was successfully realized by the global control law in ([1] and [16]). The global controller is weighted average of the controllers for different control objectives, weighted by the value of heterogeneous control membership functions. Although these nonlinear feedback linearization controllers can improve the dynamic performance of power system, they need exact plant parameters to cancel the inherent system nonlinearities, which is practical for the real life power system with parameter variations by nature.

Another example is revealed by analysis that the SVC voltage and damping control, in fact, conflict with each other and one function performed always at the price of sacrificing the performance of another function ([69], [101] and [102]). Based on the optimal search in the nonlinear programming, a coordination scheme was proposed to tackle the counteraction between two different control function of SVC [69]. A Linear Matrix Inequality (LMI) based H_∞ controller was designed to regulate voltage and active power flow [102]. A μ -synthesis SVC control design was presented for damping the inter-area oscillation and providing network voltage control [104]. Although above controllers have been applied modern control techniques, they are all designed based on the linear SVC model to coordinate two conflicting control objectives.

Except for the trade-off control objectives of generator excitation and SVC, the problem of interactions among multiple power system controllers has found and solutions to the problem by coordination have been under active investigation. One of the most typical examples is the interactions and coordination among multiple damping controllers including PSS and FACTS-based stabilizers [101]. For coordination of generator excitation and SVC controllers in SMIB system, a system in which SVC was located at the terminal of the generator terminal investigated in Reference [105]. That means control

signal are locally available, which is not practical because most of FACTS are geographically distributed; Reference [106] supposed SVC at the middle of the transmission line but assumed the control signal could be exchanged; The nonlinear coordinated control of SVC and generator excitation was presented for transient stability enhancement ([19] and [107]). The control laws of controllers were based on only local measurement SVC was modeled as a linear first-order adjustable reactance and controlled by a PI controller. A nonlinear third-order SVC model was employed to coordinate with nonlinear robust generator excitation controller for transient stability improvement and voltage regulation [20].

From above literature review, the design of global coordinated control scheme for generator excitation and SVC is a valid approach to improve transient stability and regulate voltage as well. Nonlinear feedback linearization techniques could linearize the power system models; thereby alleviate the operating point dependent nature of the linear designs. Robust decentralized control approach would guarantee the asymptotic stability of the system considering the effects of parametric uncertainties and remaining dynamic interconnections between generator and SVC. Further, the membership functions of heterogeneous control can characterize and automatically shift different operating regions of controllers for various control objectives.

The rest of the chapter is organized as follows; Section 7.2 discusses the global control of the uncertain power systems generally. The SMIB and three-machine power system with the SVC are utilized as example systems to demonstrate the effectiveness of proposed global control scheme for transient stability enhancement and voltage regulation in Section 7.3 and Section 7.4 respectively.

7.2 Global control of uncertain power systems

Global control was first employed to a multi-machine stressed power system over wide ranges of operating regions by multilevel control scheme using bifurcation analysis [67], and then transient stability and voltage regulation was coordinated by global control law with operating region membership functions [1]. Coordinated global control scheme are also assigned to the unified power flow controller (UPFC), PSS and generator excitation to maintain transient stability and voltage stability [66]. However, system uncertainties

and interaction of multiple controllers were not taken into account in these controller designs. This section extends the global control to uncertain power systems with utilization of modern nonlinear control methodologies.

Figure 7.1 Global control scheme for uncertain power systems

The global control scheme applied to the uncertain power systems is shown in Figure 7.1. The uncertain power system is first decomposed into several partial models with coordination rules according to different control objectives, operating stages, controller interactions and global control objectives. Then the DFL technique is applied to linearize the partial nonlinear models and robust partial controllers are designed to consider the system uncertainties and controller interactions for different partial models. Finally, the global control law is obtained by aggregating the partial controllers with membership functions of heterogeneous control.

The global control objectives are coordinated by the heterogeneous control law described in (3.13) of Section 3.1.4.

$$v = \sum \mu_i v_i \quad (7.1)$$

where $\sum \mu_i = 1$ and μ_i is the weighting membership function which define the operating regions of controllers for different objectives. v_i is controllers for different control objectives.

7.3 Global control of the SMIB system with SVC

Figure 7.2 Systematical global control scheme for the SMIB system with SVC

This section focuses on the global control design of an SMIB system with the SVC shown in Figure 2.6. Systematical global control scheme for the example system shown in Figure 7.2 is proposed to maintain the transient stability and achieve proper post-fault voltage. Decentralized partial controllers are respectively designed for generator excitation and SVC to realize corresponding control objectives: transient stabilization and voltage regulation. Power system uncertainties and partial controller interactions are taken into account in the design of robust partial controllers. Intuitively, the global controller is the weighted average of these partial controllers and the counteractions between two different control objectives, transient stability and voltage regulation, are tackled by the following coordinated control rules [1].

$$u = \mu_v(t)u_v(t) + \mu_T(t)u_t(t) \quad (7.2)$$

where $\mu_v(t)$ and $\mu_T(t)$ are the operating region membership function for voltage controller and transient controller respectively. $u_v(t)$ and $u_t(t)$ are the voltage and transient controller

Transient controller of generator excitation

The characteristic of uncertain generator model in the SMIB system with SVC are same as in Section 4.3, after linearized by DFL technique is given by the following equations.

$$\begin{cases} \Delta \dot{\delta}(t) = \Delta \omega(t) \\ \Delta \dot{\omega} = -\frac{D}{2H} \Delta \omega(t) - \frac{\omega_0}{2H} (P_e(t) - P_m) \\ \Delta \dot{P}_e(t) = -[\frac{1}{T} + \mu(t)] \Delta P_e(t) + [\frac{1}{T} + \mu(t)] v_{fi}(t) + \psi(t) \Delta \omega(t) \end{cases} \quad (7.3)$$

and the compensating law can be written as follows:

$$u_{fi}(t) = \frac{X_{ad} I_f(t)}{k_c P_e(t)} \{v_{fi}(t) + P_m - \bar{T}' [Q_e(t) + V_s^2 \bar{B}(t)] \Delta \omega(t)\} - \frac{(X_d - X_d') P_e(t)}{k_c X_{ad} I_f(t)} T_{do}' \Delta \omega(t) \quad (7.4)$$

Remark 7.3_1: The generator model with uncertainty is established including structure change of transmission parameters X_{ds} and X_{ds}' under the faults, the varying reactance of SVC, the uncertainties result of modeling errors

To solve DFL compensated generator model with uncertainties (7.3) involves solving the algebraic Riccati equation in [107], where details to decompose the uncertainties and obtain the robust feedback control law can be found. Therefore, the partial controller to guarantee transient stability of uncertain generator models can be obtained as

$$v_{fi}(t) = -K_\delta \Delta \delta - K_\omega \Delta \omega - K_P \Delta P_e \quad (7.5)$$

Since $[\Delta \delta(t) \quad \Delta \omega(t) \quad \Delta P_e(t)]^T$ are chosen as states in transient controller, transient stability of the SMIB system with SVC can be guaranteed. However, the generator terminal voltage $V_i(t)$ may stay at a different post-fault state if the system structure changed after the fault.

Voltage controller of generator excitation

To avoid the undesirable $V_i(t)$ variations with the change of power system structures, the nonlinear voltage controller choosing $[\Delta V_i(t) \quad \Delta \omega(t) \quad \Delta P_e(t)]^T$ as the states have been proposed in [108].

Since $V_i(t)$ is a nonlinear function of $\delta(t)$, $\omega(t)$ and $P_e(t)$, differential equation is given in [108].

$$\dot{V}_i(t) = f_1(t)\omega(t) - \frac{f_2(t)}{T'} \Delta P_e(t) + \frac{f_2(t)}{T'} v_{fv}(t) \quad (7.6)$$

where $f_1(t)$ and $f_2(t)$ are highly nonlinear functions of $\delta(t)$, $P_e(t)$ and $V_i(t)$. Since they are dependent on the operating region, their bounds are regarded as the uncertainties in the controller design. The uncertainties of T' and $B(t)$ can be considered same as in the partial model of transient controller.

Choosing $x^T(t) = [\Delta V_i(t) \quad \Delta\omega(t) \quad \Delta P_e(t)]$ as the states, the linearized uncertainties generator models can be represented in the following compact form:

$$\dot{x}(t) = (A + \Delta A(t))x(t) + (B + \Delta B(t))v_{fv}(t) \quad (7.7)$$

To solve DFL compensated generator model with uncertainties (7.7) involves solving the algebraic Riccati equation in [108], where details to decompose the uncertainties and obtain the robust feedback control law can be found. Therefore, the partial controller to regulate voltage of uncertain generator model can be obtained as

$$v_{fv}(t) = -K_v V_i(t) - K_\omega \omega(t) - K_p \Delta P_e(t) \quad (7.8)$$

Since $[\Delta V_i(t) \quad \Delta\omega(t) \quad \Delta P_e(t)]^T$ are chosen as states in partial controller, the generator terminal voltage $V_i(t)$ can be regulated around certain operating conditions. However, rotor angle of generator may not maintain its nominal value in post-fault state if the system structure changed. It has been observed that the operating changes caused by the voltage regulation will degrade the transient stability of the system [1].

Global controller design for generator excitation

To solve the inherent conflicting between transient stability enhancement and voltage regulation, the membership functions have been successfully applied to characterize the operating regions and realize the global control objectives. The membership functions [1] shown in Figure 7.3 are used in this chapter:

$$\mu_V(z_G) = \left(1 - \frac{1}{1 + \exp(-120(z_G - 1))}\right) \times \frac{1}{\exp(-120(z_G + 1))} \quad (7.9)$$

$$\mu_\delta(z_G) = 1 - \mu_V$$

$$\text{where } z_G = \sqrt{\alpha_1(\Delta\omega)^2 + \alpha_2(\Delta V_t)^2} \quad (7.10)$$

and α_1 and α_2 are positive design constants providing appropriate scaling according to the sensitive requirement of power frequency and voltage. Since voltage controller is effective while z_G is smaller than 1, the parameter α_2 should be small enough to reach this requirement. On the other side, the parameter α_1 is expected to be large enough to make z_G larger than 1.

Figure 7.3 Membership functions for coordinating different control objectives

In the global control of generator excitation, membership functions (7.9) dynamically partition the whole operating region into two subspaces, where partial controllers for transient stability enhancement and voltage regulation function respectively. In operating region of transient stability, the real excitation control of generator is $u_{ft}(t)$ obtained from the feed back control law (7.5) through the compensating law (7.4). On the other hand, the real excitation control in operating region of voltage regulation is $u_{fv}(t)$ obtained from the feed back control law (7.8) through the same compensating law (7.4). Therefore, the global controller is the weighted average of the partial controllers according to the coordinated control rule (7.2), and the excitation control input v_f takes the form:

$$v_f = \mu_V(z_G)u_{fv}(t) + \mu_\delta(z_G)u_{ft}(t) \quad (7.11)$$

The most appealing ability of membership functions is that they automatically and smoothly interpolate the two partial controllers irrespective of fault sequences and locations. It is quite suitable for the practical requirements of global control objectives.

7.3.1 Global control of the SVC

The control of SVC for voltage regulation and damping is a similar conflict happened in the controller design of the generator excitation. The global control approach is applied to the SVC to improve transient stability and achieve proper voltage level in the post-fault state.

SVC transient controller

Since the dynamic model of SVC in Section 2.3.3 is a first order linear equation, the following feedback control law in [107] is chosen to maintain stability:

$$u_{Bt}(t) = -K_{BL}\Delta B_L(t) - K_{B\omega}\Delta\omega_L(t) \quad (7.12)$$

where $\Delta\omega_L$ is the relative speed at the SVC bus. K_{BL} and $K_{B\omega}$ can be designed by pole placement method.

SVC voltage controller

The dynamic behavior of the simple three-bus SVC system described in Section 2.3.3 can be linearized by the DFL technique in a state space form [76]:

$$\begin{bmatrix} \Delta\dot{V}_B \\ \Delta\ddot{V}_B \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{B_C}(\frac{1}{X_S} + \frac{1}{X_R}) & 0 \end{bmatrix} \begin{bmatrix} \Delta V_B \\ \Delta\dot{V}_B \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} v_L(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t) \quad (7.13)$$

where X_S and X_R represent the reactance of the transmission lines, V_B is the SVC bus voltage, $v_L(t)$ is the new control input. The nonlinear feedback control law can be obtained:

$$u_B(t) = -\frac{B_C B_L(t) T_R}{k_B i_L(t)} v_L(t) + \frac{1}{k_B} (B_L(t) - B_{L0}) - \frac{B_L^2(t) T_R}{k_B i_L(t)} \Delta V_B(t) \quad (7.14)$$

Considering the parameter changes in T_R , X_{ds} and X'_{ds} , the SVC model (7.13) becomes as follows:

$$\dot{x}(t) = (A + \Delta A)x(t) + (B + \Delta B)v_L(t) \quad (7.15)$$

To solve DFL compensated SVC model with uncertainties (7.15) involves solving the algebraic Riccati equation in [107], where details to decompose the uncertainties and obtain the robust feedback control law can be found. Therefore, the partial controller to regulate voltage of uncertain SVC model can be obtained as

$$v_L(t) = -K_{v1}\Delta V_B(t) - K_{v2}\Delta \dot{V}_B(t) \quad (7.16)$$

Only the SVC parameters and the bounds of system parameters have to be known in the design of the nonlinear SVC voltage controller. Exact parameters of transmission lines are not required. System parameters and system operating points are treated as the uncertainties and handled by the robust control approach.

Global controller design for the SVC

Similar with the global control of generator excitation, the membership functions (7.9) are used to partition the whole operating regions of SVC into two parts. The voltage V_B and the relative speed $\Delta\omega_L$ at the SVC bus are chosen as the input of membership functions and

$$z_{SVC} = \sqrt{\alpha_1(\Delta\omega_L)^2 + \alpha_2(\Delta V_B)^2} \quad (7.17)$$

In operating region of transient stability, the control input of is $u_{Bt}(t)$ obtained from the feed back control law (7.12). On the other hand, the control input in operating region of voltage regulation is $u_B(t)$ obtained from the feed back control law (7.16) through compensating law (7.14). Therefore, the global controller is weighted average of the partial controllers and the control input u_{SVC} takes the form:

$$u_{SVC} = \mu_V(z_{SVC})u_B(t) + \mu_\delta(z_{SVC})u_{Bt}(t) \quad (7.18)$$

Membership functions will automatically and smoothly interpolate the two partial controllers for transient stability enhancement and voltage regulation irrespective of fault sequences and locations. Thus, it is convenient for global control scheme to realize the two practical functions of SVC, damping oscillation in face of large disturbances and providing the voltage support.

7.3.2 Global control scheme for the SMIB system with SVC

Figure 7.4 Global control of the SMIB system with SVC

To this point, four partial controllers for different control objectives have been designed separately. Only local measurements are used for each partial controller. From above design procedures, the partial controllers for generator and the SVC are independent of each other since the interconnections between them are treated as the parameter uncertainties by the robust control approach. Therefore, the global control of the SMIB system with SVC for transient stability enhancement and voltage regulation can be demonstrated in Figure 7.4, where four parts in dash rectangle represented four partial controllers.

7.3.3 Simulation results

To evaluate the above global control schemes for transient stability enhancement and voltage regulation, the example SMIB system in Figure 2.6 is utilized in the simulation study. Since the global control of generator excitation for transient stability and voltage regulation has been elaborated in ([1] and [16]), we only present the simulation results relative with the global control of SVC and the global control of the SMIB system with SVC in this section.

The system parameters and physical limits of the generator and the linear excitation system are given in Appendices A.3 and A.1, and the parameters and physical limits of SVC are given in Appendix A.2. The operating point of the system considered in the simulation is $\delta_0 = 72.0^\circ$, $P_{e0} = 1.0$ p.u., $V_{t0} = 1.05$ p.u., $Q_{e10} = 0.24$ p.u.

A symmetrical three-phase short circuit fault occurs at the terminal of the generator bus. Two fault sequences in the simulation are described as follows:

Case 7.1. Temporary Fault:

Stage 1: The system is in a pre-fault steady state.

Stage 2: A three phase short circuit fault occurs at $t = 0.1$ second.

Stage 3: The fault is removed by opening the breaker of the fault line at 0.22 second.

Stage 4: The transmission line is restored with the fault clear at $t = 1.4$ second.

Stage 5: The system is in a post-fault state.

Case 7.2. Permanent Fault:

Stage 1: The system is in a pre-fault steady state.

Stage 2: A three phase short circuit fault occurs at $t = 0.1$ second.

Stage 3: The fault is removed by opening the breakers of the fault lines at $t = 0.22$ second.

Stage 4: the system is in the post-fault state.

Using the system parameters stated above, four partial controllers can be obtained.

The transient controller of generator excitation:

$$v_{fi}(t) = 33.48 \Delta\delta + 10.65 \Delta\omega - 84.16 \Delta P_e$$

The voltage controller of generator excitation:

$$v_{fv}(t) = -29.76 V_t(t) + 6.66 \omega(t) - 11.96 \Delta P_e(t)$$

The SVC transient controller:

$$u_{Bt}(t) = -10 \Delta B_L(t) - 4.5 \Delta\omega_L(t)$$

The SVC voltage controller:

$$v_L = -13.75 \Delta V_B - 6.56 \dot{\Delta V}_B$$

The constant α_1 and α_2 are chosen 10 and 0.5 respectively in the simulation.

Global control of the SVC

In order to demonstrate the effectiveness of the global control of the SVC clearly, the conventional linear excitation controller is used with the global controller of SVC.

The power system responses with different SVC controllers subjected to different faults are shown in Figures 7.5 and 7.6. It can be observed that the global SVC controller can achieve satisfied performance compared with the controllers designed for single objective. The global SVC controller can help restore the steady pre-fault voltage value and damp the oscillation when the system is subjected to the temporary or permanently faults.

(a) $\delta(t)$

(b) $V_t(t)$

(c) $B_{svc}(t)$

Figure 7.5 Responses with different SVC controllers (Case 7.2)

Figure 7.6 Responses with global SVC controller (Cases 7.1 and Case 7.2)

Global control of the SMIB system with SVC

(c) $E_f(t)$

(d) $\omega(t)$

(e) $V_B(t)$

Figure 7.7 Responses with global control scheme (Case 7.2)

In this subsection, the performances of the global control scheme shown in Figure 7.4 are illustrated in Figures 7.7 and 7.8. Figure 7.7 exhibits good transient performance and restoration voltage of global control scheme compared with the system with only global control of generator excitation. The power system responses shown in Figure 7.8 demonstrate that the global control objectives can be realized in the presence of different fault cases by the proposed global control scheme. Thus, the trade-off problems can be successfully solved by this control scheme.

Figure 7.8 Responses with different fault cases.

From above three simulation cases for the SMIB power system, it can be concluded that:

1. The global SVC controller can effectively improve transient stability and regulate voltage irrespective of fault locations. It can achieve satisfied performance compared with the controllers designed for single objective.
2. The conflicting control objectives in the SMIB system with SVC are easily coordinated by the heterogeneous control law. The global control scheme takes advantage of the existing single objective controllers and makes smooth transitions from controllers for different objectives.
3. The global control scheme for the SMIB system with SVC can enhance the transient stability and regulate voltage regardless of the system operating points and fault locations. The proposed controller took into account power system

nonlinearities, uncertainties and controller interactions and coordinated the different control objectives over wide ranges of operating regions.

4. In the global control scheme design for the SMIB system with SVC, all the partial controllers are designed by using available local control signals, thus the global control scheme is conveniently implemented in practice.

7.4 Global control of multi-machine power system with SVC

In this section, global control scheme will extend to multi-machine power system with SVC based on the preliminary result in the former section and the works in ([1], [71] and [109]).

7.4.1 Global control scheme for multi-machine power system

Similar with the global control scheme for the SMIB power system with SVC, global control objective for multi-machine power system is also to maintain the transient stability and achieve proper post-fault voltage. Since different partial models can be used to describe those two operating stages, different partial controllers can be designed for specific control objectives and global control objective can be realized by coordinated control rules (7.2). Therefore, global control scheme can take advantage of the existing controller design and realize particular global control objectives. In this section, two existing controller designs for transient stability and voltage regulation are adopted for the generator excitation of multi-machine power system.

Transient controller of generator excitation

For the multi-machine power systems, the characteristic of uncertain generator model are same as in Section 6.3, after linearized by DFL technique is given by the following equations.

$$\begin{cases} \Delta \dot{\delta}_i(t) = \Delta \omega_i(t) \\ \Delta \dot{\omega}_i = -\frac{D_i}{2H_i} \Delta \omega_i(t) - \frac{\omega_0}{2H_i} (P_{ei}(t) - P_{mi}) \\ \Delta P_{ei}(t) = -\frac{1}{T_{doi}} \Delta P_{ei}(t) + \frac{1}{T_{doi}} v_{fi}(t) + \sum_{j=1}^n E'_{qi}(t) \dot{E}'_{qj}(t) B_{ij} \sin(\delta_{ij}(t)) \\ \quad - E'_{qi}(t) \sum_{j=1}^n E'_{qj}(t) B_{ij} \cos(\delta_{ij}(t)) \Delta \omega_j(t) \end{cases} \quad (7.19)$$

and the compensating law can be written as follows:

$$v_{fi}(t) = k_{ci} I_{qi}(t) u_{fi}(t) + (X_{di} - X'_{di}) I_{di}(t) I_{qi}(t) - \bar{T}_{doi} Q_{ei}(t) \Delta \omega_i(t) - P_{mi} \quad (7.20)$$

To solve DFL compensated generator model with uncertainties (7.19) involves solving the algebraic Riccati equation in [71], where details to decompose the uncertainties and obtain the robust feedback control law can be found. Therefore, the partial controller to guarantee transient stability of uncertain generator models (7.19) can be obtained as

$$v_{fi}(t) = -K_{\delta i} \Delta \delta_i(t) - K_{\omega i} \Delta \omega_i(t) - K_{P_i} \Delta P_{ei}(t) \quad (7.21)$$

Since $[\Delta \delta_i(t) \quad \Delta \omega_i(t) \quad \Delta P_{ei}(t)]^T$ are chosen as states in transient controller, transient stability of the power system can be guaranteed. However, the generator terminal voltage $V_{ti}(t)$ may stay at a different post-fault state if the system structure changed after the fault.

Voltage controller of generator excitation

To avoid the undesirable $V_i(t)$ variations with the change of power system structures, the nonlinear voltage controller choosing $[\Delta V_{ti}(t) \quad \Delta \omega_i(t) \quad \Delta P_{ei}(t)]^T$ as the states have been proposed in [109], this extended the voltage controller of generator excitation to multi-machine power systems.

Choosing $[\Delta V_{ti}(t) \quad \Delta \omega_i(t) \quad \Delta P_{ei}(t)]^T$ as the states, the partial controller to regulate voltage of uncertain generator model can be obtained as

$$v_{fi}(t) = -K_v V_i(t) - K_{\omega} \omega(t) - K_p \Delta P_e(t) \quad (7.22)$$

Since $[\Delta V_{ti}(t) \quad \Delta \omega_i(t) \quad \Delta P_{ei}(t)]^T$ are chosen as states in partial controller, the generator terminal voltage $V_{ti}(t)$ can be regulated around certain operating conditions. However, rotor angle of generator may not maintain its nominal value in post-fault state if the system structure changed. It has been observed that the operating changes caused by the voltage regulation will degrade the transient stability of the system [1].

To solve the inherent conflicting between transient stability enhancement and voltage regulation, the coordinated control rules in (7.2) and membership function (7.12) are applied as in the multi-machine power system with SVC. Power system uncertainties and partial controller interactions are taken into account in the design of robust partial controllers.

7.4.2 Global control of multi-machine power system SVC

To demonstrate the application of global control scheme to multi-machine power system, the three-machine nine-bus power system with SVC shown in Figure 6.8 are used as an example for global control design. The generator data, the pre-fault network parameters system operating point and fault sequences are as same as ones in Section 6.4.

Figure 7.9 The 3-machine 9-bus power system with SVC

By employing the controller design procedures mentioned above, the partial controllers for generator excitations and SVC are designed as follow:

The transient controller of generator excitation 1:

$$v_{f1}(t) = 279.71 \Delta\delta_1 + 159.62 \Delta\omega_1 - 603.27 \Delta P_{e1};$$

The voltage controller of generator excitation:

$$v_{fv1}(t) = -35.37 V_{t1}(t) + 8.64 \omega_1(t) - 20.87 \Delta P_{e1}(t);$$

The transient controller of generator excitation 2:

$$v_{f2}(t) = 713.44 \Delta\delta_2 + 189.57 \Delta\omega_2 - 813.78 \Delta P_{e2}(t);$$

The voltage controller of generator excitation 2:

$$v_{fv2}(t) = -39.76 V_{t2}(t) + 7.88 \omega_2(t) - 17.96 \Delta P_{e2}(t);$$

The transient controller of generator excitation 3:

$$v_{f3}(t) = 168.71 \Delta\delta_3 + 130.53 \Delta\omega_3 - 531.56 \Delta P_{e3};$$

The voltage controller of generator excitation 3:

$$v_{fv3}(t) = -29.37 V_{t3}(t) + 8.64 \omega_3(t) - 49.87 \Delta P_{e3}(t);$$

The constant α_1 and α_2 in coordinated control rules of generator excitation are chosen 200 and 0.5 respectively.

The SVC transient controller:

$$u_{Bt}(t) = -10 \Delta B_L(t) - 4.5 \Delta\omega_L(t);$$

The SVC voltage controller:

$$v_L = -12.83 \Delta V_B - 12.24 \Delta \dot{V}_B;$$

The constant α_1 and α_2 in coordinated control rules of SVC are chosen 5 and 0.5 respectively.

7.4.3 Simulation results

In order to evaluate the above global control scheme for multi-machine power system to maintain transient stability and regulate post-fault voltage, the example three machine power system with SVC in Figure 7.9 is utilized in the simulation study. A symmetrical

three-phase short circuit fault is considered in the simulation: Fault 1 occurs near bus 7 on the lower transmission line between bus 7 and bus 8. Two fault types, temporary and permanent fault are considered. Thus, two cases are designed to test the validity of global control scheme.

Case 7.1: Global control scheme with Fault 1

To demonstrate the effectiveness of the global control scheme for this three-machine nine-bus power system with SVC clearly, where the fault 1 clearing time $t_1 = 0.22$ sec., the responses of global control scheme are compared with the control systems with single objective. The power system responses with different control objectives are shown in Figures 7.10 (a)-(p), where the real power output of machines, $P_{ei}(t)$, the relative speeds of the machines, $\omega_i(t)$, the terminal voltage of the machines, $V_{ti}(t)$, the voltage of SVC bus, $V_b(t)$, the power angles of the machines, $\delta_i(t)$, the power angle differences of machines, $\delta_{ij}(t)$, and the susceptance of SVC, $B_{SVC}(t)$, are given respectively.

Figures 7.10 (a)-(f) illustrate that three of control systems, transient control system, voltage control system and the global control scheme, can effectively bring the real power output $P_{ei}(t)$ and the relative speeds $\omega_i(t)$ to their steady states within three seconds when three-machine nine-bus power system are subjected to a sever fault, near the terminal of machine 2. However, among these three control systems, the voltage controllers can obtain the best responses of the terminal voltage $V_{ti}(t)$ and the voltage of SVC bus $V_b(t)$ as shown in Figures 7.10 (g)-(j); while the transient controllers can obtain the best responses of the power angles $\delta_i(t)$ and the power angle differences $\delta_{ij}(t)$ as shown in Figures 7.10 (k)-(o).

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(a) $P_{e1}(t)$

(b) $P_{e2}(t)$

(c) $P_{e3}(t)$

(d) $\omega_1(t)$

(e) $\omega_2(t)$

(f) $\omega_3(t)$

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(g) $V_{t1}(t)$

(h) $V_{t2}(t)$

(i) $V_{t3}(t)$

(j) $V_B(t)$

(k) $\delta_1(t)$

(l) $\delta_2(t)$

Figure 7.10 Responses of global control scheme with fault 1

Note that transient controllers and voltage controllers are designed for regulating power angles and voltages to the original steady states respectively. Although the responses of the power angle differences between machines remain the same with voltage controllers in Figures 7.10 (n) and (o), voltage controllers can not damp the responses of the power angles to the original steady state, but other relatively high steady states as shown in Figures 7.10 (k)-(m). For the practical power systems, especially geographically distributed large scale power systems, when the power system are subjected to the large disturbances, it is impossible for all the power angles of the generators changed to such high new steady states due to the large scale of power systems. The power systems are

most likely equivalent to a generator connected with infinite bus. The responses of power angles of the generators near the disturbance are required to return to the steady states near original one in order to keep synchronous with the rest generators. Thus, it is necessary to both regulate the post-fault voltages and maintain power angles of generators to the reasonable steady states in the practical applications. In order to take advantages of transient controllers and voltage controllers, and overcome the fatal shortcoming of voltage controllers in practical power systems, which enable the power angles $\delta_i(t)$ to converge to relatively high steady states shown in Figures (k)-(m), the proposed global control can obtain an trade-off controller performance shown in Figures (g)-(o). It can be observed that the global controller can achieve satisfied performance compared with the controllers designed for the single objective. The global controller can help restore the steady pre-fault voltage value and damp the oscillation when the system is subjected to faults.

Case 7.2: Global control scheme with different fault types

To verify the robustness of the global control scheme for the different fault types, permanent Fault 1 occurs with clearing time $t_1 = 0.22$ sec.. The response of global control scheme are shown in shown in Figure 7.11 (a)-(h), where the power angles of the machines, $\delta_i(t)$, the terminal voltage of the machines, $V_{ti}(t)$, the voltage of SVC bus, $V_B(t)$, and the susceptance of SVC, $B_{SVC}(t)$, are given respectively.

(a) $\delta_1(t)$

(b) $\delta_2(t)$

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(c) $\delta_3(t)$

(d) $\delta_{21}(t)$

(e) $\delta_{31}(t)$

(f) $V_{t1}(t)$

(g) $V_{t2}(t)$

(h) $V_{t3}(t)$

Figure 7.11 Responses of global control scheme with different fault types

The power system responses shown in Figure 7.11 demonstrate that the global control objectives can be realized in the presence of different fault types by the proposed global control scheme. The power angles $\delta_i(t)$, the terminal voltage $V_{ti}(t)$, the voltage of SVC bus $V_B(t)$, and the susceptance of SVC $B_{SVC}(t)$ came back to the original steady state when the example power system is subjected to a temporary fault. By contrast, these system variables can operate upon a new steady state under a permanent fault. Thus, the trade-off problems can be successfully solved by this control scheme.

From above simulation cases for the three-machine power system, it can be concluded that:

1. The conflicting control objectives in the three-machine nine-bus power system with SVC are easily coordinated by the heterogeneous control law. The global control scheme takes advantage of the existing single objective controllers and makes smooth transitions from controllers for different objectives.
2. The global control scheme for the three-machine power system with SVC can enhance the transient stability and regulate voltage. The proposed controller took into account power system nonlinearities, uncertainties and controller interactions and coordinated the different control objectives over wide ranges of operating regions.

3. In the global control scheme design for the multi-machine system with SVC, all the partial controllers are designed by using available local control signals, thus the global control scheme is conveniently implemented in practice.

7.5 Concluding remarks

In this chapter, the global control is extended to uncertain power systems to coordinate decentralized controllers and inherently conflicting control objectives within wide operating regions. Two control systems, interaction of PSS and AVR, and the SVC voltage and damping control, have been evidently demonstrated the conflicting control objectives during different control stages of power systems. Therefore, the SMIB power system with SVC and three machine power system with SVC are utilized as an example system for global control scheme to maintain the transient stability and achieve proper post-fault voltage.

In the systematical global control scheme, the decentralized partial controllers are respectively designed for generator excitation and SVC to coordinate two conflicting control objectives: transient stabilization and voltage regulation. Power system uncertainties and partial controller interactions are taken into account in the design of robust partial controllers. Intuitively, the global controller is the weighted average of these partial controllers and the counteractions between two different control objectives, transient stability and voltage regulation, are tackled by the coordinated control rules. The attributes of the proposed global control scheme are as follows:

- (1) The conflicting control objectives can be easily coordinated by the heterogeneous control law. The global control scheme takes advantage of the existing single objective controllers and makes smooth transitions from controllers of different objectives
- (2) The global SVC controller can effectively improve transient stability and regulate voltage irrespective of fault locations.
- (3) The decentralized partial controllers for single control objective are designed by using available local control signals, and then these controllers coordinated effectively and smoothly by membership functions of heterogeneous control law. Thus the global control scheme is conveniently implemented in practice.

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(4) The global control scheme for both the SMIB and multi-machine power system with SVC can maintain transient stability and regulate voltage regardless of the system operating points and fault locations. The proposed controller took into account power system nonlinearities, uncertainties and controller interactions and coordinated the different control objectives over wide ranges of operating regions.

Chapter 8 Conclusions and Recommendations

This chapter concludes with a summary of research work done so far and recommends tasks for future research.

8.1 Conclusions

The desire to coordinate various control actions and deal with wide ranges of operating regions in real world systems has raised great interests in global control of complex systems. Since complex systems are generally large-scale systems and characterized by substantial nonlinearities, uncertain models, high noise levels, abrupt changes in dynamics and strict performance requirements, there are three main difficulties in global control of complex systems, i.e. nonlinearities uncertainties and interconnections. Although the global control structure have given general guide of controller design and the global control strategies, such as heterogeneous control and fuzzy dynamic control, have been developed abundantly, there still lacks of a systematic global control method which can conveniently take into accounts dynamic behaviors of complex systems. This research aims to take advantages of modern control techniques and the global control structure to propose a global control design framework for the practical applications of complex system control.

The unified global control design for complex systems begins with the application of multiple nonlinear models. To avoid the local linear models used in existing global control strategies, multiple partial models, a set of nonlinear mathematical models, are presented in Chapter 2. These models can describe typical phenomena of practical complex systems, such as different operating states, nonlinearities, abrupt dynamics and different control objectives.

Based on the global control structure, the design of global control was decomposed into three stages: identification, partial controller design and controller synthesis. In the stage of identification, the coordination rules and partial models are identified according to qualitative distinct operating condition and global control objectives. Heterogeneous

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control can be utilized to design coordination rules and membership functions. In the design of partial controllers, the application of feedback linearization, robust control theory and robust decentralized control can respectively cancel the nonlinearities uncertainties and interconnections of partial models. Since the complex system is represented by multiple partial models and their coordination rules, the global controller is obtained by synthesizing the partial controllers upon their operating regions, where membership functions provide proper weighting and smooth transitions. The last but most importantly, the global stability of this close-loop complex system can be guaranteed in terms of Lyapunov stability.

Following the above procedure, the global control design is gradually extended from a simpler situation to the final complex systems: uncertain systems, a class of nonlinear systems and large-scale interconnected system. Systematical design and stability analysis of global control design are provided. What make this global control design distinguish from other global control strategies lies in the feasibility of controller design. The stabilization of each partial system can guarantee the one of the global complex systems, and all partial controllers are designed using the local control signals.

The applications of global control have been made in power system for transient stability enhancement. Global control of uncertain system and large-scale interconnected system are applied to an SMIB and multi-machine power systems respectively. When power systems are subjected to a severe disturbance and the structure of transmission network changed dramatically, power systems operate over four different states: the pre-fault state, faulty state, fault clearing and post fault state. Multiple partial models are used to represent different states of power systems. Then, the global controllers for generator excitation in SMIB or multi-machine power system are obtained by coordinating partial controllers from different states. In the partial controller design, the DFL technique is used to linearize four partial models; robust controllers are designed for the linearized partial models of SMIB power system and decentralized robust controllers for models of multi-machine power system. In the global control design, all controllers are designed using available local control signals. The stabilization of each robust partial controller for generator excitations can guarantee the overall stability of the SMIB or multi-machine power system. System parameters, interconnections and fault locations are all treated as the uncertainties and handled by the robust control approach in each partial model. Thus,

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the global control scheme can damp the oscillations regardless of the system operating points and fault locations.

Global control for a class of nonlinear systems is employed to two benchmark control systems: the BB system and the CP system. As discussed in Chapter 2, the models of these benchmark control systems have some typical nonlinear functions $f_1(x, t, \Delta)$ and $f_2(x_i, x_j, t, \Delta)$ in Equation (2.2), which cannot be systematically handled by existing control approaches. The proposed global control design is employed to solve these typical nonlinear problems. In the global control of the BB system, the BB system is represented by three partial models in view of the nonlinearities and uncertainties. Then three partial controllers are respectively designed for each partial model and the global controller are constructed by combining three partial control actions through heterogeneous control membership functions. The global control scheme improves system performance by decreasing the number of partial controllers in comparison with the fuzzy dynamic control strategies. For the global control of the CP system, two control objectives: only regulate the displacement of pole and regulate displacement of cart and pole simultaneously, are respectively implemented according to the same design procedure as the BB system. The global controllers obtain good dynamic performance irrespective of initial operating points for both of control objectives.

Global control scheme is extended to uncertain power systems to coordinate different, sometimes inherently conflicting, control objectives and decentralized controllers within wide operating regions. The SMIB and three-machine power system with SVC are utilized as example systems for global control scheme to maintain the transient stability and achieve proper post-fault voltage. It has been proved that both synchronous generators and SVC have inherent conflicting control objectives: voltage regulation and transient stabilization. That means one function performed always at the price of sacrificing the performance of another function. Furthermore, undesirable interactions among the decentralized excitation controllers and SVC controllers may have detrimental effects to the overall performance of the power system. In the global control scheme, decentralized partial controllers are respectively designed for generator excitation and SVC to coordinate two conflicting control objectives. Power system uncertainties and controller interactions are taken into account in the design of robust partial controllers.

Therefore, the global control scheme for the SMIB and multi-machine power system with SVC coordinate the decentralized SVC and the generator excitation controllers, maintain the transient stability and achieve proper post-fault voltage irrespective of system parameters and fault locations.

To sum up, global control coordinate various control actions and objectives by combining the qualitative and quantitative knowledge of entire operating conditions of complex system. The proposed global controller design can deal with nonlinearities, uncertainties and interconnections in practical systems. It should be pointed out that all the controllers designed in the thesis need only local measurement to implement feedback controls. The stabilization of each partial system can guarantee the one of the global complex systems. All those advantages make it distinguish from the other global control strategies.

8.2 Recommendation for future work

The thesis investigates the applications of global control in special consideration of nonlinearities, uncertainties and interconnections of complex systems. However, control of complex systems reveals more aspects beyond this category. To take power systems as the example, further research of global control is given in details.

Figure 8.1 Power system controls [72]

Global control of power system involves in many system components and multiple controllers. Power system is a network of system components, including generators, transformers, buses, lines, load and FACTS devices. It usually consists of various

subsystem associated with control, which can be depicted in Figure 8.1 [72]. Multiple controllers operate directly on individual system elements to implement different functions. For example, the excitation control is to regulate generators voltage and reactive power output; the steam valve control is concern with rotor speed regulation and control of energy supply system variables; the primary purpose of load frequency control is to balance the total system generation against system load and losses so that power interchange with neighboring systems is maintained; FACTS control can provide power and voltage support in transmission networks. All those system components and their associated control have propounded effects on dynamic performance of power system.

Global control of power system also spans long time frame of controls. The dynamic phenomena of power system control [18] can be described in the time response charts in Figure 8.2. It is obvious that most of control objectives have clear time response frame and control objectives are dependent on the operating states of systems. Thus, the control of power system is a global issue involving different control objectives of a long time frame.

Figure 8.2 Time responses of power system control [18]

8.2.1 Multi-machine power system with FACTS devices

In Chapter 7 of the thesis, the global control to uncertain power systems is presented to coordinate decentralized controllers and inherently conflicting control objectives. However, only the SMIB and three-machine power system with SVC are utilized to present some preliminary results. As those power systems are only simplified models of practical systems, this global control scheme is expected to extend to the multi-machine systems with FACTS devices. Similar with the SMIB power system, the dynamic phenomena of multi-machine power system can be classified into two different stages for transient stabilization and voltage regulation. Moreover, in recent literatures, the decentralized control techniques have been successfully employed to coordinate the synchronous machine and FACTS devices [109]-[111]. Therefore, in global control design of multi-machine power systems with FACTS devices, heterogeneous control law can be used to coordinate transient stabilization and voltage regulation over different operating stages, and robust decentralized control techniques can be applied to consider the interactions of decentralized controllers.

8.2.2 Coordination of different control objectives

In power systems, control objects dependent on the operating states and time frames of power system control. For reliable operations of power system, frequency and voltage are two of most important indices. Under normal conditions, the control objectives are to maintain these two indices to normal values. When an abnormal condition develops, new objectives must be met to restore two indices to values. For major systems failures, power system cannot be restored to normal operating states, may result in collapse of a secure systems. For examples, to several power system blackouts happened in US and Europe, the cascading outages lasted for long-term and involved various aspects in power system control. The studies have shown that fast voltage collapse play an important roles in the event of overall event. Consequently, the global control of different control objectives over long term frame is practically important for power system control.

The thesis has investigated coordination of two different control objectives in short term. And global control design of power system has also been provided over wide operating regions in the thesis and Reference [1]. Further research can be coordination of control

objectives in long term. For example, when global control of power system aims to maintain voltage stability, short term control objectives have undeniable impacts on long term control objectives and the coordination of control objectives in different frame is necessary.

8.2.3 Multilevel global control design

Figure 8.3 Power system control hierarchy [112]

Power system control has been based on a hierarchical structure [112] where control tasks are shared by different hierarchical levels. A schematic representation of all major system control functions is shown in Figure 8.3. This figure indicates the typical temporal and spatial separation of systems control in a horizontally structured interconnected system. Primary controllers on individual generating units are at present decentralized in that they respond to deviation of local outputs from the steady state set values assigned from higher levels. The steady state values of primary controllers are determined at a regional (secondary) level, assuming weak interconnections among the regions. The regional controllers are, however, not systematically coordinated at present, leading to deviations from optimal system wide performance and possibility of system wide instability.

Consequently, the global control taking the structure of power systems into accounts can be a topic of future research. It can exploit the advantages of multiple qualitative models, system constraints and reasoning techniques within a single system.

8.2.4 Controller design considering time delay

The concept of time delay arises in many practical engineering applications such as signal processing, fiber optics research, and very large scale integrated circuit design. The subject of delay and its analysis has already received attention in these areas. For wide area hierarchy power systems, the time delay problem have been prominent in the inter area mode ([113] and [114]), Thus, it cannot be ignored when power system considered from global concepts. Global control is expected to take into account the time delay problem due to long time delay compared to that in local systems.

8.2.5 Network based global control design

There is wide variety of research investigations on the nonlinearities as mentioned in Chapter 2 and 5. In the thesis, global controllers were designed for two benchmarked control systems based on model analysis and system stabilization. The critical nonlinearities of models can be analyzed from the models of two control examples. An interesting further consideration is for the global control design of large scale systems, where many subsystems have complicated nonlinearities and some of critical nonlinearities may inter-react with other subsystems. As a result, the network structures may have important impact on the controller design. The investigation of controller design combining with network analysis is promising in dealing with such complex systems.

Author's Publications

Portions of Research work from this thesis have been reported in the following publications:

1. G. H. Zhang, Y. Wang and D. J. Hill, "Global Control of a Class of Nonlinear Systems", *Systems & Control Letters* (submitted)
2. G. H. Zhang and Y. Wang, "Global Control of Power Systems for Transient Stability and Voltage Regulation", *The 15th Power System Computation Conference*, August 22-26, 2005, Liege, Belgium.
3. G. H. Zhang, Y. Wang and D. J. Hill, "Systematical Design and Stability Analysis of Global Control for a Class of Nonlinear Systems", *the 5th International Conference on Control and Automation*, June 26 – 29, 2005, Budapest, Hungary.
4. D. J. Hill, G. H. Zhang and Y. Wang, "Global Robust Control of Uncertain Systems and Its Application to Power Systems", *The 23rd Chinese Control Conference*, August 10-13, 2004, Wuxi, China.
5. Y. Wang, G. H. Zhang and D. J. Hill, "Robust power system transient stability enhancement: a global control approach", *the 6th IFAC symposium on Nonlinear Control Systems*, September 1-3, 2004, Stuttgart, Germany.

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Appendices

A.1 Parameters of the linear excitation controller

AVR (IEEE ST1A excitation system):

$$K_A=60; T_A=0.05\text{sec.}; K_f=0.05; T_f=1.0\text{sec.}; I_{LR}=4.4 \text{ p.u.}; K_{LR}=4.54 \text{ p.u.}; |V_A|<999.0 \text{ p.u.}; |E_f|<6.0 \text{ p.u.}$$

PSS:

$$\text{The transfer function of PSS can be represented as: } G_{PSS} = K_{PSS} \frac{T_r s}{1 + T_r s} \frac{(1 + T_1 s)^2}{(1 + T_2 s)^2}.$$

$$K_{PSS}=15; T_r=3 \text{ sec.}; T_1=0.25 \text{ sec.}; T_2=0.05\text{sec.}; |V_{PSS}|<0.05 \text{ p.u.}$$

A.2 Parameters and physical limits of SVC

$$0.02 \leq T_R \leq 0.1 \text{ sec.}; B_{L0}=1.0 \text{ p.u.}; B_{C0}=1.0 \text{ p.u.}; k_B=1; 0.2 \leq B_L(t) \leq 1.2 \text{ p.u.}$$

A.3 Parameters of the SMIB power system

Synchronous machine:

$$\omega_0 = 314.159 \text{ rad/sec.}; D=5.0 \text{ p.u.}; H=4.0 \text{ sec.}; T'_{d0}=6.9 \text{ sec.}; T'_{q0}=0.06 \text{ sec.}; k_c=1; \\ X_d=1.863\text{p.u.}; X'_d=0.657 \text{ p.u.}; X_{ad}=1.712 \text{ p.u.}; X_q=0.657 \text{ p.u.}; X''_q=0.27 \text{ p.u.};$$

The physical limits of the excitation voltage is $\max|k_c u_f| = 6.0 \text{ p.u.}$

$$\text{Transmission lines: } X_l = \frac{X_{l1} + X_{l2}}{2} = 0.2426 \text{ p.u.}; R_l = 0.03 \text{ p.u. } f_0 = 50\text{Hz}.$$

$$\text{Transformer: } X_T = 0.127 \text{ p.u.}$$

$$\text{Infinite bus: } V_s = 1 \angle 0 \text{ p.u.}$$

A.4 Parameters of classical linear controller for three-machine nine-bus power system

AVR: (IEEE Type 3 SCPT fast exciter, 2.0 p.u. response):

$K_A=120$; $K_E=1.0$; $K_f=0.02$; $K_R=1.0$; $K_p=1.19$; $K_l=2.62$; $T_A=0.15\text{sec.}$; $T_E=0.05\text{sec.}$;

$T_f=0.6\text{sec.}$; $T_R=0\text{ sec.}$; $|V_R|<1.2\text{ p.u.}$; $V_{b\max}=2.78$; $|E_f|<6.0\text{ p.u.}$

PSS:

The transfer function of PSS can be represented as: $G_{PSS} = \frac{K_{PSS}s (1+T_1s)^2}{1+T_rs (1+T_2s)^2}$.

$K_{PSS}=25$; $T_r=3\text{ sec.}$; $T_1=0.2\text{ sec.}$; $T_2=0.05\text{sec.}$; $|V_{PSS}|<0.1\text{ p.u.}$