Fairness and the political economy of economic growth

Leong, Chee Kian

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FAIRNESS AND THE POLITICAL ECONOMY OF ECONOMIC GROWTH

LEONG CHEE KIAN

School of Humanities and Social Sciences

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The original idea for Chapter 6 came from a debate with my Spanish classmate while I was studying German in Berlin in 2002. Coming from a country which had experienced the excesses of the Franco authoritarianism, he could not understand why Asian countries accept authoritarian rules. I presented a skeleton of my arguments in Chapter 5 to my class, in

German minus the mathematics! I do not believe that I convinced my classmates completely, but we traded plenty of wits and humor throughout the debate. Vielen Dank!

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Summary

Fairness has emerged as a major issue in the political economy of economic growth but the lack of a unified formal framework of fairness has impeded analysis in the past.

This thesis formalizes a concept of fairness which is then applied to a neoclassical economic growth model, so as to analyze issues of fairness in economic growth. This concept of fairness is analytically and computationally simple to derive in dynamic games and endogenous in the sense that it results from the interactions between heterogeneous agents.

The concept of fairness developed in this thesis can be described as follows. A pre-game ultimatum game determines a fair rule for sharing the joint cooperative gains. To limit the set of fair rule, we apply a well-established result of the Ultimatum Game (Binmore, 2007) which indicates that players tend to play fair and the fair outcome is a fifty-fifty split. Hence, we define our fair imputation as one which includes their individual payoff from the Markovian Nash equilibrium (the threat point) and half of their cooperative gain. The rule applies once the game starts and the rational acceptability (RA) criterion decides whether the players will play cooperatively or not. Rationally acceptable cooperative strategies exist if a fair imputation of the cooperative solution in a dynamic cooperative game offers players a higher payoffs compared to those obtained in the Markovian Nash equilibrium. The set of fair equilibrium is then the set of rationally acceptable strategies. The cooperative game will be played if the fair imputation payoffs are greater than those derived from a unilateral defection, that is, if the fair imputations are rationally acceptable. Should this not be the case, the fair equilibrium degenerates into a non-cooperative Nash equilibrium in which both players will play the non-cooperative game. Thus, the fair equilibrium is a cooperative one if rational acceptable fair imputations are available; otherwise, it is a non-cooperative one. The proposed fair equilibrium possesses "nice" properties of time consistency, subgame perfectness and Pareto efficiency.

This fair equilibrium is then applied to analyze why capitalism has prevailed as an institution in promoting economic growth despite its apparent unfairness. Using a dynamic game with a vote-maximizing government and profit-maximizing representative firm op-

erating within a neoclassical framework, the thesis demonstrates that capitalism is fairer compared to collectivism due to the absence of a rationally acceptable cooperative solution.

Two extensions are then discussed.

Firstly, uncertainty is introduced via a stochastic dynamic game. With uncertainty, the cooperative solution is always non-inferior to the non-cooperative Markovian Nash equilibrium.

Secondly, the assumption of an alignment between the government and the citizens is questioned in a reputational model. A "good" reputation convinces voters that the government is competent and will exert high effort in economic performance. The key problem in maintaining such a reputation in the absence of replacements or political renewal is that the government could succeed in convincing the voters that it is competent but may go "bad" subsequently.

THESIS

Chapter 1

Introduction

Issues of fairness in economic growth are important. This importance is highlighted by the World Bank in the World Development Report 2006, which has *Equity and Development* as its theme. The report concludes that unfairness in access to opportunity, both within and among nations, sustains extreme deprivation, results in wasted human potential and often weakens prospects for economic growth.

Despite its importance, issues of fairness in growth and development have until recently been neglected in the mainstream theoretical literature. To date, economists studying the phenomenon of economic growth have focused almost exclusively on how economic growth can be promoted (Barro and Sala-i-Martin, 2004). The neoclassical growth models have emphasized technology change as the main driver. Given technological spillovers over time, countries are expected to converge to the steady state growth path. On the other hand, recent endogenous growth models (Lucas, 1988; Romer, 1986, 1990; Mankiw et al, 1992; Grossman and Helpman, 1991; Acemoglu and Ventura, 2002) have highlighted factors such as research and development (R&D), human capital investment, international trade, institutions, and geography amongst others. The underlying rationale for both the neoclassical and endogenous approach is that once these factors have been identified, they can be implemented in developing countries to achieve convergence. Increasingly, however, such a technocratic approach is not adequate.

An initial objection can be found in the empirical observation that large differences in in-

come continue to persist across countries and over time despite rapid technological progress. Figure 1 plots the natural logarithm of the per capita GDP (at constant price) in 2004 against its 1965 values for 199 countries, with the world average line drawn for comparison. It is interesting to note that despite rapid technological changes and liberalization over the same period, the dispersion of per capita income has not changed much since the 1965, apart from a few growth miracles (such as China[CN]) and disasters(such as Suriname[SUR]). In fact, many countries seem to cluster below the world average. This runs contrary to the technocratic story which would predict that a greater level of technological changes and liberalization will spur technological transfer and level up the economically backward countries. If the technocratic story is true, we would expect the countries to cluster above the world average rather than below. The lack of convergence does suggest that other factors, such as institutions and political economy, may be important as well.
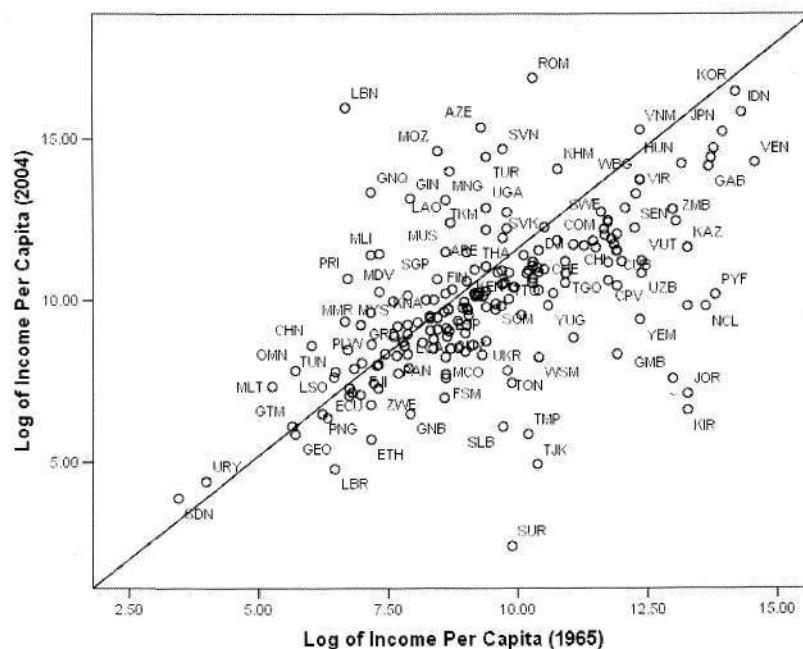
Figure 1.1: Log of Per Capita Income in 2004 and 1965 relative to World Average (Line) for 199 countries. Data Source: World Development Indicators 2006

More significantly, between 1965 and 2004, there was a structural break in economic history. By 1989, the fall of the Berlin Wall has resulted in almost all Communist countries abandoning central planning, with Cuba and North Korea being the few remaining exceptions. That such a historical turn occurred in the last decade of the last century is surprising

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since for more than half a century, the dominant economic ideology that held sway amongst intellectuals of different political persuasions was that a "fair" economic system could be achieved through some degree of central planning. Lancaster (1973) best exemplifies this line of thinking when he argued that capitalism as an economic system with its decentralization is dynamically inefficient, thus justifying the need for some cooperative solution, read central planning. Such a central planning ideology pervaded both economic methodology and economic policies for almost half of the last century. In economic methodology, the outcomes of decentralized decisions are always benchmarked against the outcome derived from a "benevolent" central planner in welfare analysis. The implicit belief is that this outcome is always socially optimal. Furthermore, Arrow (1963) demonstrates in his famous Arrow Impossibility Theorem that it is impossible to aggregate individual preferences unless this is achieved under an authoritarian system, thus giving further credence to the idea that central planning may be desirable. How central planning is manifested in actual policies differs across communist and socialist countries but a key theme appears to be the nationalization of industries. Yet this ideology collapsed towards the end of the 20th century when countries of different political persuasions rapidly abandoned central planning and embraced capitalism. The inherent puzzle is: why did these countries appear to forsake central planning for capitalism as an engine of economic growth? In other words, why did capitalism prevail despite its perceived "dynamic inconsistency" and "unfairness"?

Conventional arguments, such as those proposed by Hayek (1944) and his disciples, speculate that this could be due to the computational burden on the part of the central planner in achieving the efficient outcome made possible by the market economy in capitalist economies. Although this is a reasonable supposition, it is contentious empirically since there was a period in history when many centrally planned economies actually outperformed the capitalist economies (White, 1986). So the eventual return to a capitalist and decentralized economic system may have little to do with computational or performance inadequacy on the part of the central planner. Explanations have to be found elsewhere.

In this thesis, we propose an alternative explanation. We argue that however promising central planning may appear to be in rhetoric or as an ideology, the outcomes achieved under

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a collective optimization may not be fair to all the parties involved, namely the government, the workers and the firms. To achieve a fair outcome would involve finding a set of solutions that can be rationally accepted by all parties. But we demonstrate that doing so is almost impossible within a neoclassical growth framework.

The thesis contributes to the nascent literature on the issue of fairness in growth and development (Easterly, 2002; Sachs, 2005; Alesina and Angeletos, 2005; Benabou and Tirole, 2006). To date, the literature lacks a formal concept of fairness that can be applicable to analyze such macroeconomic issues. The seminal contribution in the literature of fairness is Rabin (1993) who emphasizes the importance of deriving a formal concept of fairness that emerges endogenously from microeconomic interactions. The downside to his behavioral approach is that the derivation of his fair equilibrium is computationally and analytically unwieldy in dynamic games. On the other hand, the growth theorists are contented with developing ad hoc and a priori notions of fairness in macroeconomic models. This is not entirely satisfactory since the selection of a fair outcome is arbitrary and often associated with some "median" or " mean" voter.

In part, this problem arises because the economic growth literature is commonly formulated using an optimal control framework, in which the decision maker's model of the state transition dynamics is exogenous. By adopting a multi-agent dynamic game framework in this thesis, parts of the decision maker's transition law governing endogenous state variables are affected by other agents' choices and therefore are equilibrium outcomes. In this way, a disciplined way to derive endogenously the fair equilibrium within a dynamic game framework can be attained. Hence, this thesis develops a formal concept of fairness in dynamic games. There are a number of significant advantages to this new formalization. Firstly, it does not rely on ad hoc or exogenous notions of fairness commonly found in the economic growth literature. Instead, we apply a well-established result in experimental economics (Binmore, 2007) to determine the fair imputations. We then proceed with a rational acceptabiility (RA) criterion, which allows the set of fair equilibrium to be determined easily in dynamic games. Thus, the new formalization preserves the key advantage of the Rabin's approach while providing an analytically precise and computationally simple way to derive

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the fair equilibrium in dynamic games.

In the main model, we assume that the government is a vote-maximizer who depends on the political support of the workers. This is distinct from current economic models in which the government is always represented as "benevolent". With such a characterization, however, we also make the important assumption that there exists an alignment of interest between the government and the workers. But this may not be always valid. One could cite numerous instances in which government (often authoritarian and communist) promises to fight for the interest of the workers and end up hurting them instead. Additionally, the political institutions may play a significant role. For example, in a large cross-country regression, Barro (1996) establishes a universal positive correlation between democracy and growth. However, when the effects of the rule of law, free markets, small government expenditure and high human capital were set aside along with the initial level of GDP, the overall effect of democracy on growth was weakly negative. Persson and Tabellini (2000, 2003, 2004) further this investigation by analyzing the impact of constitutions and electoral rules on economic policy and argued that "a parliamentary form of government is associated with better performance and better growth-promoting policies, measured by indexes for broad protection of property rights and of open borders" (Persson and Tabellini, 2004). An important short-coming of their analysis is that they studied 80 democracies, primarily Western and Latin American countries. In econometric terminology, they are committing the cardinal sin of self-selection. In fact, in our sample of 199 countries, there are many countries, such as China and Vietnam, which do not respect property rights and yet managed to improve the per capita income of their citizens above the world average. To add to the irony, these countries are authoritarian.

The typical characterization of authoritarian regimes in the political economy of economic growth is simplistic at best. For instance, Acemoglu and Robinson (2006) portray such regimes as follows:

> these nondemocratic regimes share one common element: instead of representing the wishes of the population at large, they represent the preferences of a subgroup of the population: the "elite". In China, it is mainly the wishes of the

Communist Party that matter.

Yet it is doubtful whether any authoritarian can afford to ignore the "wishes of the population at large" if its objective is to perpetuate its political survival. Furthermore, Keefer (2007) recently observes that the governance scores in countries with competitive elections differ little, on average from those in countries without. One could cite numerous examples. Between 1960 and 1985, the economies of Hong Kong, Indonesia, Japan, Taiwan, South Korea and Singapore were six of the fastest growing economies in the world. Between 1985 and 2004, the fastest growing country in the world is China, which has adopted capitalism since the early 1980s but continues to combine authoritarian rule with economic growth (Guo, 2006). Of course, one could also cite cases in which authoritarianism is also adopted in other countries with poor economic growth performance (Przeworski et al, 2000). Myanmar is a typical example. An important question then arises: in authoritarian economies, what incentives are there to ensure that the government will protect the interests of its citizens or workers?

To answer this question, we consider a model in which the government (whether authoritarian or democratic) has a reputation of "fairness" to uphold in order to sustain its political support. The game-theoretic literature on reputation effects, which is pioneered by Kreps and Wilson (1982), Milgrom and Roberts (1982) and Fudenberg and Levine (1989), has the general result that reputation enhances commitment power. Reputation effects find many recent applications in the theory of firms, such as Mailath and Samuelson (2001), Tadelis (2002) and Ely and Valimaki (2003). To the best of my knowledge, however, this thesis is the first application of reputation effects in analyzing the political economy of economic growth.

This thesis is organized as follows. Chapter 2 is a literature review on fairness and growth. We highlight the lack of a unified formal framework of fairness with which we can synthesize fairness models and economic growth models. In chapter 3, we propose such a framework by developing a fair equilibrium construct which emerges endogenously from the interaction of the players in dynamic games. We then apply this fair equilibrium to answer the question posed earlier: why does capitalism prevail over central planning as a

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system for generating economic growth? We extend the analysis in two directions. Firstly, chapter 5 considers a stochastic version of the game between the government and the firm in chapter 4. Secondly, chapter 6 analyzes the effects of reputation on government's effort to ensure fair outcomes for the workers under different types of political regimes. Finally, chapter 7 summarizes our main conclusions and proposes areas for further investigation.

Chapter 2

Fairness and Economic Growth: A Literature Review

2.1 Introduction

The field of fairness is a very vast one and it is not the objective of this chapter to provide an exhaustive survey (interested readers can consult Fehr and Schmidt, 2003 or Konow, 2003). Instead, this literature review focuses on key ideas and issues in the fairness literature which are pertinent to the development of a fairness construct for macroeconomic models. In particular, we highlight the importance of a unified formal framework of fairness in analyzing the political economy of economic growth models.

This literature review is organized as follows. Section 2.2 surveys the empirical evidence of fairness in economic experiments. Section 2.3 introduces attempts by economist to derive a theoretical construct of fairness. Section 2.4 surveys the nascent literature on fairness in economic growth. Finally, Section 2.5 concludes by pointing out potential directions for future research that could potentially overcome the shortcomings discussed in the earlier sections.

2.2 Experimental Evidence of Fairness

In many behavioral economics experiments, it has been discovered that in specific, identifiable situations, fairness concerns supersede self-interest (Kahneman, D, et al., 1986; Henrich et al. 2001; Carmerer 2003; Fehr and Schmidt 2003). Fairness concerns remain important even when the stakes are high (Fehr, Fischbacher et al. 2002). In this section, we survey the experimental evidence of the importance of fairness in economics.

The ultimatum game (Guth, et al, 1982) is one of the earliest experiments to suggest that fairness concerns may be more important to players than strategic concerns. In the ultimatum game, 2 players divide a sum of money (the "cake") using the procedure: player 1 makes a demand, which player 2 can accept or refuse. This concludes the game. If the demand is refused, both players receive nothing. A strategic solution would be that player 1 will demand and get all (or nearly all) of the cake. Thus, it is surprising that a much fairer division (frequently a "50-50" split) is the typical result in many replications and extensions of the original experimental game (Binmore, 2007).

Many other behavioral economists have noted that fairness preferences have less to do with mere economic consideration of distributional payoff and more to do with social considerations (Falk, Fehr et al. 1999; Falk, Fehr et al. 2000). Fairness preferences are often "shaped by the economic and social interactions of everyday life" (Henrich et al. 2001). For instance, contributions to the public good are influenced by social interactions with neighbors at least for 90% of subjects in a group (Falk, Fischbacher et al. 2002).

Other researchers have discovered that what is considered to be fair and unfair in market competition and society is not a universal standard but varies from person to person (Fischbacher, Fong et al. 2003) and is, in part, culturally biased. Experiments conducted in different countries and cultures document that fairness norms differ from country to country and from culture to culture (Henrich, Boyd et al. 2001). Studies comparing free-riding behavior among MBA students in China and the US found such behavior far less likely to occur in China where cultural norms discourage it (Latane, Williams et al. 1979; Earley 1989).

Despite the differences in fairness norms across countries and cultures, a consistent and

general result in many experiments on fairness (Fehr and Rockenbach 2003) is that people tend to behave pro-socially and punish antisocial behavior, at a cost to themselves, even when the probability of future interactions is extremely low, or zero. Some experiments pointed out that people are even willing to give up money to reduce perceived inequities.

Fairness can be enforced through a range of devices, processes and techniques – directly and indirectly (Masclet, Noussair et al. 2003) but the most common enforcement mechanism is reciprocity. Sanchez and Cuesta (2004) define strong reciprocity to be "the predisposition to cooperate with others and to punish non-cooperators at personal cost". Similarly, Fehr and Gachter (2002) note that "individuals punish, although the punishment is costly for them and yields no material gain". Interestingly, even third parties, whose economic payoff is unaffected by societal or group violations of equality distribution and cooperation norms are observed to punish violators (Fehr and Fischbacher 2004).

In a nutshell, experimental results consistently suggest that people have a sense of fairness and are prepared to punish unfair behavior. People give a small portion of their endowment to others, even though they can keep it all. They are prepared to suffer a monetary loss just to punish behavior that is deemed "unfair". Similarly, they are willing to suffer a loss in order to reward actions that they perceive as generous or fair. These conclusions are found to be robust to changes in the size of the monetary stakes or the profiles of the players.

2.3 Theoretical Constructs of Fairness

The experimental evidences in the last section suggest the importance of fairness in interactions between economic agents. In this section, we consider several theoretical constructs of fairness in welfare economics, bargaining theory and behavioral games.

Formal analysis of fairness in economics was originally confined to welfare economics, where the need to choose between different Pareto optimal outcomes involves choosing between alternative sets of ethical values. A Pareto optimal outcome merely asserts that, given the initial wealth distribution, competitive markets ensure the best possible deal for everyone given their resources. However, there is no judgment about the fairness of any

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wealth distribution. As Sen (1970) points out, "A society or an economy can be Pareto optimal and still be perfectly disgusting."

Binmore (1994, 1998) suggests that the two main contending conceptions which inform the choices between Pareto optimal outcomes are contractarian (John Locke and Jacques Rousseau) versus utilitarianism (David Hume, Adam Smith, Jeremy Bentham and J S Mill). Harsanyi (1975) and Rawls (1971) are representatives of these respective traditions. Both Harsanyi and Rawls share the same premise that a "fair" criterion of social welfare must be one that a rational person would choose if he or she were "fair-minded". To ensure this, each postulates an "original position", in which the individual contemplates this choice without knowing his or her personal social position. Although both imagine such a choice is made under uncertainty over who will end up as in the society of your choice, they differ in what they view as the "fair" decision rule to guide the choice in the original position.

Harsanyi's idea is to employ the von Neumann-Morgenstern (VNM) axiomatic description of rationality under conditions of uncertainty. Assume a finite number of outcomes or prospects $y_1, y_2, \cdots, y_m$, $m \geq 1$ with respective probabilities $\mathbf{p} : (p_1, p_2, \cdots, p_m)$. The set of all possible lotteries is given by

$$L = \left\{ \mathbf{p} \in \mathbb{R}^m \mid p_i \geq o \forall i \text{ and } \sum_{i=1}^{m} p_i = 1 \right\}$$

According to Bayesian theory, a rational decision maker will equate the utility of any lottery to its expected utility, hence, the cardinal utility of agent k will be given by $u_k(\mathbf{p}) = \sum_{i=1}^{m} p_i u_k(y_i)$, where $u_k(y_i)$ is a VNM utility function over social state y_i, which is unique to positive affine transforms. Acting under risks and uncertainty, the rational decision maker will maximize his or her expected utility. Two possible approaches are offered by him. The first is known as Harsanyi's **aggregation theorem**.

Theorem 2.3.1 *Let the agents' preferences and social preference relation satisfy the axioms of expected utility. Denote $u_k, j = 1, \cdots, n$ and u be the VNM utility representations of the individuals' preference and the social preference relation respectively. Given that Pareto indifference is satisfied, there exist numbers α_k, $k = 1, c \ldots, n$ and β such that for all elements*

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p from the set of lotteries L,

$$u(\boldsymbol{p}) = \sum_{k=1}^{n} \alpha_k u_k(\boldsymbol{p}) + \beta$$

Here, the theorem neither suggests that $(a_1, \cdots, a_n; b)$ is unique nor restricts the coefficients α_k to be positive or at least non-negative. A negative α_k implies that agent k's utility contributes negatively to social welfare and a zero value implies no contribution. Moreover, the linear aggregation rule does not allow for interpersonal comparisons of utilities.

Known as the **equiprobability model of the objective observer**, Harsanyi (1953,1955)'s second approach is closer to Rawls's approach. In this model, an objective observer (anyone in society) who is sympathetic to the interests of every member in society, imagines himself or herself in the position of person i, $i = 1, \cdots, n$ under diffferent social states $y_1, \cdots, y_m$. Since there are n agents in the society, then this probability is $1/n$. A rational evaluation of the social state y_i will, then depend on its expected utility $(1/n)\sum_k^n u_i(y_i)$. Between two social states y_a and y_b, $a \neq b$, the one with the higher utility (say y_a) must be preferred or

$$\sum_k^n u_i(y_a) > \sum_k^n u_i(y_b)$$

Unlike Harsanyi, Rawls (1971) approaches fairness based on two principles of justice which he regards as fundamental to the liberty and equality of citizens in any well-ordered society. Firstly, Rawls requires that each person has an equal right to the most extensive basic liberty compatible with the same liberty for others. These include but are not restricted to political liberty, freedom of expression and the freedom to own properties. The second principle, also known as the **difference principle**, posits an original position in which one acts under complete ignorance (behind what Rawls terms a "veil of ignorance"). In this case, choice cannot be guided by either expected utility or the assignment of probabilities as in Harsanyi's models. Instead, a risk-averse individual will rationally order social states according to what will happen if he or she ends up as society's worse-off member. In other words, such a *maximin* decision rule will dictate y_a will be preferred to y_b if

$$\min[u_1(y_a), ..., u_N(y_a)] > \min[u_1(y_b), ..., u_N(y_b)]$$

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In bargaining theory, the question is how should a number of players divide a pie fairly amongst themselves? An axiomatic approach is proposed by Nash(1950). Our discussion here follows Muthoo (1999).

Formally, suppose that two players $i, -i$ negotiate over the allocation of R units of resources. R is divisible, denote the allocations as r_i, r_{-i}, where $r_i + r_{-i} \leq R$. Both players receive the respective utility payoffs $u_i(r_i), u_{-i}(r_{-i})$, where the utility function $u_k(\cdot)$ is strictly increasing and concave for both players. In the absence of any agreement, each player has a status quo or disagreement utility, which is given by $\bar{u}_k$ for $k \in \{i, -i\}$.

Let Ω be the set of feasible utility levels (u_i, u_{-i}) that can be achieved through some allocation of R units of resources, that is,

$$\Omega = \{(u_i, u_{-i}) : u_i = u_i(r_i), u_{-i} = u_{-i}(r_{-i}), r_i + r_{-i} \leq R\}.$$

Given the assumption of the utility functions, the boundary of this feasible set is a locus of points given by the function $g(u_i) = u_{-i}\left(R - u_i^{-1}(r_i)\right)$, which for simplicity, shall be assumed to be twice-differentiable. The set of Pareto optimal allocations is

$$\Omega^e = \{w \in \Omega : u_i \geq \bar{u}_i, u_{-i} \geq \bar{u}_{-i}\}.$$

Generally, a bargaining solution is a pair $(\Omega, \bar{u})$, where $\bar{u}$ is the vector of disagreement values. The set of bargaining games is Ξ, and a bargaining solution is a function $F : \Xi \to \mathbb{R}^2$. Denote F_k as the bargaining allocation to player k.

Axiom 1. Invariance to Equivalent Utility Representation

Let $u_i' = \alpha_i u_i + \beta_i$ and $\bar{u}_i' = \alpha_i \bar{u}_i + \beta_i$ for $\alpha_i > 0$ and Ω' is defined accordingly. Then $F_i\left(\Omega', \bar{u}_i'\right) = \alpha_i F_i(\Omega, \bar{u}) + \beta_i$ for player $k = i, -i$. Thus, affine transformations of utility functions and disagreement utilities do not alter the bargaining outcomes.

Axiom 2. Pareto Efficiency

If $F(\Xi) = (u_i, u_{-i})$, there does not exist another allocation $\left(u_i', u_{-i}'\right) \in \Omega$ such that $u_i' > u_i$ for some i and $u_j' \geq u_j$ for some $j \neq i$.

Hence, the players are not able to improve on the bargaining solution by choosing an

allocation that makes one of the bargainers better off without reducing the utility of the other.

Axiom 3.Symmetry

Suppose $\bar{u}_i = \bar{u}_{-i}$ and assume that $(u_1, u_2) \in \Omega \iff (u_2, u_1) \in \Omega$. Then $F_i(\Omega, \bar{u}) = F_{-i}(\Omega, \bar{u})$.

As such, the allocation depends only on the player's preferences and disagreement values.

Axiom 4.Independence of irrelevant alternatives.

Given two bargaining solutions $(\Omega, \bar{u})$ and $\left(\Omega', \bar{u}\right)$ such that $\Omega' \subset \Omega$ and $F(\Omega, \bar{u}) \subset \Omega'$. Then $F(\Omega, \bar{u}) = F\left(\Omega', \bar{u}\right)$.

This axiom suggests that the bargaining solution is not affected if allocations other than the solution are eliminated.

Based on these axioms, Nash's bargaining solution is the utility allocation $(u_i, u_{-i}) \in \Omega$ that maximizes

$$(u_i - \bar{u}_i)(u_{-i} - \bar{u}_{-i})$$

subject to

$$u_i \geq \bar{u}_i \quad and \quad u_{-i} \geq \bar{u}_{-i}.$$

The solution to this constrained optimization is given by

$$-g'(u_i) = \frac{u_{-i} - \bar{u}_{-i}}{u_i - \bar{u}_i}$$

and

$$u_{-i} = g(u_i)$$

The Nash bargaining solution is

$$\frac{u_i(r_i) - \bar{u}_i}{u_i'(r_i)} = \frac{u_{-i}(R - r_i) - \bar{u}_{-i}}{u_{-i}'(R - r_i)}$$

If the disagreement or status quo values and the utility functions are the same for both players, the Nash bargaining shares are $r_i = r_{-i} = \frac{1}{2}R$.

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Although rigorous, a key limitation of such formalizations of fairness is that they are based on assumptions of what constitutes "fairness" which are not founded on more empirical micro-foundations. In fact, they ignore the large body of psychological evidence of "fair behavior" as detailed in the previous section.

Rabin (1993) attempts to address this limitation by adopting the psychological game framework defined by Geanakoplos et al (1989) and formalizing fairness based on three stylized facts which are derived from psychological evidence:

1. People are willing to sacrifice their own material well-being to help those who are being kind.

2. People are willing to sacrifice their own material well-being to punish those who are being unkind

3. Both motivations have a greater effect on behavior as the material cost of sacrificing becomes smaller.

Consider a two-player, normal form game with strategy set S_i, $i = 1, 2$ for player 1 and 2. Let $a_1 \in S_1$ and $a_2 \in S_{12}$ represent the strategies chosen by the two players while b_1 and b_2 represent player 2's beliefs of player 1's strategy choice and player 1's beliefs of player 2's strategy choice respectively. Rabin's first step in incorporating fairness into economic analysis is to define a kindness function $f_i(a_i, b_j)$, which measures how kind player i is to player j. If player j is choosing strategy, player i chooses among the payoff pair $\big(\pi_i(a_i, b_j), \pi_j(b_j, a_i)\big)$ from among the set of all payoffs feasible $\prod(b_j) \equiv \big\{ \big(\pi_i(a, b_j), \pi_j(b_j, a)\big) \big| a \in S \big\}$.

Let $\pi_j^h(b_j)$ be player j's highest payoff in $\pi(b_j)$ and $\pi_j^l(b_j)$ be the player's lowest payoff among points that are Pareto-efficient in $\prod(b_j)$. The "equitable payoff" is defined by $\pi_j^e(b_j) = 1/2 \left[\pi_j^h(b_j) + \pi_j^l(b_j) \right]$ while $\pi_j^{\min}(b_j)$ is the worst possible payoff for player j in the set $\prod(b_j)$

Definition Player i's kindness to player j is given by

$$f_i(a_i, b_j) \equiv \frac{\pi_j(b_j, a_i) - \pi_j^e(b_j)}{\pi_j^h(b_j) - \pi_j^{\min}(b_j)}$$

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If $\pi_j^h(b_j) - \pi_j^{\min}(b_j)$, then $f_i(a_i, b_i) = 0$.

Definition Player i's belief about how kind player j is to him is given by

$$\tilde{f}_j(b_j, c_i) \equiv \frac{\pi_j(c_i, b_j) - \pi_i^e(c_i)}{\pi_i^h(c_i) - \pi_i^{\min}(c_i)}$$

If $\pi_i^h(c_i) - \pi_i^{\min}(c_i) = 0$, then $\tilde{f}_j(b_j, c_i) = 0$

Each agent chooses a_i to maximize his expected utility

$$U_i(a_i, b_j, c_i) = \pi_i(a_i, b_j) + \tilde{f}_j(b_j, c_i) \cdot [1 + f_i(a_i, b_j)]$$

Definition The pair of strategies $(a_1, a_2) \in (S_1, S_2)$ is a *fairness equilibrium* if, for $i = 1, 2, j \neq i$

$$a_i \in \arg\max_{a \in S_i} U(a, b_j, c_i)$$

$$c_i = b_i = a_i$$

Definition A strategy pair $(a_1, a_2) \in (S_1, S_2)$ is a *mutual-max outcome* if, for $i = 1, 2, j \neq i$

$$a_i \in \arg\max_{a \in S_i} U(a, a_j)$$

Definition A strategy pair $(a_1, a_2) \in (S_1, S_2)$ is a *mutual-min outcome* if, for $i = 1, 2, j \neq i$

$$a_i \in \arg\min_{a \in S_i} U(a, a_j)$$

With this framework, Rabin concludes that

1. Any Nash equilibrium that is either a mutual-max outcome or mutual min outcome is also a fair equilibrium.

2. If material payoffs are small, then, roughly, an outcome is a fair equilibrium if and only if it is a mutual-max or a mutual-min outcome.

3. If material payoffs are small, then, roughly, an outcome is a fair equilibrium if and only if it is a Nash equilibrium.

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Rabin's contribution is important because it formalizes a fair equilibrium based on experimental and psychological insights on reciprocity. However, his approach entails quite complex "kindness accounting" and applies to simple two-person, normal-form, complete information one-off games. Even in such simple games, the existence of multiple fair equilibria is a common feature. Specifically, one equilibrium may result in both players being "kind" to each other while another may result in both players being "nasty" towards one another. In this case, it is difficult to determine which equilibrium will be played.

Extending his fairness model to more general settings involves further complications. Fehr and Schmidt (2003), for instance, note that in sequential prisoner's dilemma, unconditional cooperation of the second player is part of the fair equilibrium and this is not optimal for the player. Dufwenberg and Kirschsteiger (1998) and Falk and Fischbacher (1999) attempted to generalize Rabin(1993) to N-person extensive form games. Dufwenberg and Kirschsteiger introduce a "sequential reciprocity equilibrium" (SRE) which tracks beliefs about intentions as the game evolves. Based on this system of beliefs, strategies have to form a fair equilibrium in every proper subgame. Even in these very simple sequential games, the equilibrium analysis is highly complex and involves multiple equilibria. Falk and Fischbacher extend the Rabin model to sequential games with incomplete information and measure "kindness" in terms of inequity aversion. The resulting model is very complicated. At each node, player i must evaluate the kindness of player j that depends on the expected payoff difference between the two players and on what player j could have done about this difference. Next, this "kindness" term is multiplied by a "reciprocation term" which is positive if player i is kind to player j and negative if i is unkind. The product is then multiplied by an individual reciprocity parameter that measures the weight of player i's desire to reciprocate over the desire to obtain a higher material payoff. A subgame perfect psychological Nash equilibrium of this game is termed a "reciprocity equilibrium". The resulting model achieves a better fit with the empirical evidence on fairness and inequity aversion but at a considerable cost in terms of complexity.

In a more recent attempt to integrate social preferences with intention-based reciprocity, Charness and Rabin (2000) introduce the "reciprocal fairness equilibrium"(RFE) which is

both a strategy profile and a demerit profile such that each player is maximizing his or her utility function given other players's strategies and given the demerit profile that is itself consistent with the profile of strategies. The notion of RFE has several limitations. Firstly, since preferences are defined only in equilibrium, the model is incomplete and it is not clear how to evaluate non-equilibrium outcomes or multiple equilibria. Secondly, all players must have the same utility function and agree on a "quasi-maximin" social welfare function to determine the demerit profile. Since the model is very complex and has too many free parameters, an empirical test of its validity is almost impossible.

In view of such difficulties, it is not surprising that economists attempting to apply fairness in macroeconomic models resort to simpler, more ad-hoc formulations. Alesina and Angeletos (2005), for example, assume a static economy with a large number (a measure-one continuum) of agents, who live for two periods and has total pre-tax life-cycle income y_i, defined by

$$y_i = A_i[\alpha k_i + (1 - \alpha)e_i] + \eta_i$$

where A_i is the inherent ability or talent, k_i is investment in the first period of life, e_i is the effort in the second period of life, η_i is "noise" which can be interpreted as pure random luck or the effect of socially undesirable activities such as corruption, rent seeking and political subversion, and $\alpha \in (0, 1)$ is the share of income sunk when the tax rate is set.

They then apply the idea that people share a common conviction that one should get what one deserves and deserves what one gets to define the measure of social injustice as

$$\Omega = \int_i (u_i - \hat{u}_i)^2$$

where u_i denotes the actual level of utility and $\hat{u}_i$ denotes the "fair" level of utility. The latter is defined as the utility the agent deserves on the basis of his talent and effort, namely $\hat{u}_i = V_i(\hat{c}_i, k_i, e_i)$, where

$$\hat{c}_i = \hat{y}_i = A_i[\alpha k_i + (1 - \alpha)e_i]$$

and $\hat{c}_i, \hat{y}_i$ represent the fair levels of consumption and income respectively. The residual, $y_i - \hat{y}_i = \eta_i$ measures the unfair component of income.

Alesina and Angeletos's definition of fairness is embedded in individual preferences. However, it does not explain how such preferences originate and why particular sources of income are considered "fair" while others "unfair". For example, a median agent in the United States may believe that it is "fair" that the poor is poor just because the median tends to be white while the poor tends to be black.

Benabou and Tirole (2006) employ a different approach, starting from the premise that belief in fairness is a "fundamental delusion". Such a belief may have functional, affective and religious basis. Like Alesina and Angeletos (2005), they assume an economy populated by a continuum of agents whose actions take place according to the timeline in table 2.1.

Period 0		Period 1		Period 2	
Receive a signal about the long run return to effort, θ	Choose recall or awareness rate λ, for oneself or one's children	Vote on tax rate τ	Choose effort e^i.	Individual outcomes y^i realized.	Redistributive consumption

Table 2.1: Timing of Signals and Actions

Each agent produces period 2 output with the technology

$$y^i = \begin{cases} 1 & \text{with probability} \quad \pi^i + \theta e^i \\ 0 & \text{with probability} \quad 1 - (\pi^i + \theta e^i) \end{cases}$$

where e^i is the level of effort (or human capital investment) the agent chose in period 1, π^i reflects the agent's social background-resources or social capital inherited from parents, etc, and θ is a signal about the return to effort. The expected utility perceived by agent i at $t = 0, 1$ may take one of three possible specifications, depending on the source of fairness belief:

$$U_t^i = \begin{cases} E\left[(1-\tau)y^i + \tau\bar{y} - (e^i)^2 \big/ 2\beta_t \,\big|\, \Omega_t^i\right] & \text{functional} \\ E\left[(1-\tau)y^i + \tau\bar{y} - (e^i)^2 \big/ 2\beta_t + u\left((1-\tau)\theta(\mu^i)\right) \big|\, \Omega_t^i\right] & \text{affective} \\ E\left[(1-\tau)y^i + \tau\bar{y} - (e^i)^2 \big/ 2\beta_t + \theta(\mu^i) e^i \,\big|\, \Omega_t^i\right] & \text{religious} \end{cases}$$

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where τ is the tax rate faced by the agent in period 2, Ω_t^i is the date t information set and $\beta_1 \equiv \beta < 1 \equiv \beta_0$ represents the importance the present plays in affecting preferences when effort must be exerted. μ^i is the agent's posterior beliefs given by $\mu^i \equiv \Pr[\sigma^i = \Phi | \Omega_1^i] =$, where $\sigma^i = \Phi$ denotes that they receive no bad news. For pessimists, $\mu^i = 0$ while for optimists, $\mu^i = r$, where r indicates an assessment of the reliability of no bad news message.

If the source of fairness belief is affective, $\mu^i > 0$, meaning that people like to think that "effort pays" or have a high net return $(1 - \tau)\,\theta\,(\mu^i)$. In the case of religious beliefs, $\theta\,(\mu^i)$ reflects the strength of his religious faith. The various specifications are isomorphic with one another. Hence agents with different sources of fairness beliefs derive similar utility from their beliefs.

One key problem with Benabou and Tirole's model of fairness is their fundamental assumption that belief in fairness is a "fundamental delusion". It is questionable whether this assumption is true or reasonable. A test of a reasonable assumption is whether it contradicts any other important assumptions in the model. In this case, if fairness does not exist, then it is a fundamental delusion to believe in fairness. But it is arguably irrational that the agents in Benabou and Tirole's model persistently derive utility from this "delusion" over time and across generations.

From our preceding discussion, it is obvious that a formal and consistent definition of "fairness" is lacking. On the other hand, the fairness literature has focused too much on microeconomic behavior and largely ignored potential applications in macroeconomics. In the next section, we will review the nascent literature on fairness in economic growth literature where the use of different and ad hoc theoretical constructs of fairness often leads to different conclusions.

2.4 Fairness and Growth

The preceding section highlights the difficulties in formalizing a theoretical construct of fairness that can be employed in applied work. The lack of an applicable theory of fairness could possibly explain why economic growth models have largely neglected discussion of

fairness in growth models. Recently, however, such issues of fairness have re-emerged in theoretical and empirical literature in economic growth. In this section, we survey critically the current work of Alesina and Rodrik (1994), Persson and Tabellini(1994), Alesina and Angeletos (2005) and Benabou and Tirole (2006).

Alesina and Rodrik (1994) examine a model of endogenous growth in which distributional conflict is "endogenized" via the median voter theorem. They demonstrate that only when government "cares" solely about pure capitalist will there exists a tax rate that maximizes growth.

Persson and Tabellini (1994) obtain similar results with a simplified overlapping-generation model in which agents live for two periods and income is taxed for redistributive purposes.

A crucial assumption in their models is that the distribution of assets is predetermined and remains constant. Since voting decisions in any period affect growth and distribution in subsequent period, voters in their model therefore have to internalize the problem of time consistency in voting decisions. In reality, the informational constraint imposed on the voters would be considerable. Therefore, the "optimal" tax derived is time-inconsistent.

Both models lack a proper mechanism to study the endogenous evolution of a fair income distribution within a growth model. For instance, their result is contingent on the assumption that a majority voting rule is the "fair" political mechanism for policy making. Their basis for appealing to this rule as a basis for fairness is not cogent. A serious reservation is that fair mechanisms cannot be chosen a priori; they evolve. Subsequent generations of policy makers may have every incentive to renege on the commitments of early generation once society's conception of fairness evolves.

Alesina and Angeletos (2005) demonstrate that different beliefs about the fairness of social competition and what determines income inequality can influence the choice of a redistributive policy in a society. The interaction between social beliefs and welfare policies may lead to multiple equilibria. If a society believes that individual effort determines income, and that all have a right to enjoy the fruits of their effort, it will choose low redistribution and low taxes. Steady state effort will be high and the market outcomes will be relatively fair and social beliefs will be self-fulfilled. If instead, a society's belief that

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luck, birth, connections and/or corruption determines wealth, it will levy high taxes, thus distorting allocations and beliefs.

There are a number of contentious points with this model. Firstly, it is debatable whether such exogenously determined perceptions are dynamically consistent. The model fails to explain what will happen to growth and redistributive policies if an endogenous shift in the perceptions of society's belief occurs. Secondly, the relationship between perceptions and policy is tenuous. If Alesina and Angeletos's insight is true, any elected government can perpetuate a low-tax policy with minimal political cost by consistently socializing in its citizens the perception of equality and opportunity. Thirdly, in their model, fairness is simply defined as a demand for insurance. It is arguable whether this definition of fairness is adequate.

Moreover, their model is designed specifically to explain fairness of growth in the United States and developed European countries and may not generalize across countries. In developing countries, for instance, the low economic growth of the countries may affect the demand for redistribution while in developed countries, it is possible that low growth vis-à-vis other countries may swing the balance of perception and hence policies.

Benabou and Tirole (2006) argue that belief in a fair world is a "fundamental delusion" that is the result of collective beliefs that are functional, affective and religious in origin. People derive utility or satisfaction when they believe that they live in a fair world, where hard work and good behavior pay off. Their model attempts to explain why redistributive policies are more extensive and income tax structure more progressive in Europe than in the United States. According to Benabou and Tirole, this arises because the "fundamental delusion" of fairness is stronger in the United States than in Europe.

A fundamental point of contention is: if beliefs in fairness are "fundamental delusions" as claimed by Benabou and Tirole, why are such collective beliefs so persistent over time and across generations? Surely, such persistent "delusions" contradict all bounds of rationality.

The current literature on the political economy of growth thus relies on different ad hoc constructs of fairness. This poses quite a challenge to the comparability of results. Additionally, unlike the fairness construct of Rabin (1993), the ad hoc nature of the fairness

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constructs in these growth models lacks a micro-foundation.

2.5 Concluding Remarks

In this chapter, we have reviewed critically the existing theoretical and empirical literature on fairness and economic growth. Above all, we identify some of the limitations of existing research approaches and emphasize the lack of formal and consistent definition of "fairness" to analyze such issues in the political economy of economic growth. Existing literature on fairness and growth contains many useful insights but these insights are the result of authors adopting different and ad hoc definitions of fairness. Consequently, it is difficult to consolidate key conclusions, identify key hypothesis and focus future research. On the other hand, the fairness literature in behavioral economics emphasizes microeconomic behavior and could not be generalized and applied to macroeconomic growth models. Hence, developing such a unified theoretical construct of fairness would pave the way for clearer thinking on the issues of fairness in economic growth and possibly provide a common platform from which more robust conclusions can be distilled.

We conclude this chapter by considering what could potentially qualify as a way forward. One possibility is to employ a consistent formalization of fairness to analyze issues of fairness adequately. Instead of relying on some pre-conceived notions of what fairness entails, this concept of fairness should be endogenous in the sense that it results from the interactions between heterogeneous agents. The notion of fairness should be based on well-established experimental results in economics. Above all, the fairness concept should be analytically and computationally simple to derive.

The optimal control framework is commonly employed for analyzing both neoclassical and endogenous growth models. Since we wish to incorporate fairness in economic growth models with heterogenous agents, the appropriate approach would be to adopt a differential game framework. Differential games are continuous time dynamic games embedded within an optimal control framework in which players interact repeatedly through time and for which the state of the system changes according to one or more differential equations. How-

ever, their interaction is not a trivial repetition of the original game, as the initial conditions for each game differ through the continuous change in the state over time. The foundation of dynamic games originates with Rufus Isaacs (1965) in his analysis of missile versus enemy aircraft pursuit schemes. Over the years, dynamic games have found many applications in economics and management. Surveys of more recent development in the literature can be found in Dockner et al (2000) and Yeung and Petrosyan (2006).

Developing a theoretical construct of fairness within a differential game framework is what we set out to do in the next chapter.

Chapter 3

Rational Acceptability and Fair Equilibrium in Dynamic Games

3.1 Introduction

While economists would agree that fairness is important in issues of political economy and macroeconomics, such as growth and distribution, current literature (see for example, Alesina and Angeletos, 2005; Benabou and Tirole, 2006) incorporates the economists' judgment of fairness and equity and ignores the concerns for fairness and equity of the economic players under analysis. As noted in the last chapter, this is partly due to the lack of a conception of fair equilibrium that is suitable for macroeconomic analysis. Most of the fair equilibria proposed in the literature are invariably microeconomic in focus, static and complicated to derive, limiting their usefulness in macroeconomic applications and dynamic analysis.

To address these problems, this chapter proposes a construct of fair equilibrium, which can be applied to analyze issues of fairness in dynamic games and dynamic macroeconomic models with heterogeneous agents. Unlike most existing constructs, the proposed fair equilibrium emerges endogenously from the interaction of the players in dynamic games. As such, the present chapter adopts a game theoretic formulation similar to Rabin (1993). However, Rabin's fairness equilibrium is based on specific psychological assumptions of

kindness and applies to one-stage games. His approach also depends on the construct of a kindness function, which is complicated to compute in dynamic games. Moreover, it is unlikely that kindness consideration is as important in games between macroeconomic agents as in games between microeconomic agents. In contrast, the present construct does not depend on a kindness function.

For simplicity, we restrict our discussion to dynamic games with two players.

The first question deals with the issue on how to divide up the cooperative payoffs in a way that is acceptable to everyone. Various possible ways are proposed to pick a particular cooperative solution. One possible approach is proposed by Yeung and Petrosyan (2006). They design a solution imputation using a characteristic function framework which establishes a basis for formulating distribution schemes of the total cooperative payoffs that are acceptable to the participating players. They use $\Gamma_v(y_0, T - t_0)$ to denote a cooperative differential game in characteristic form. The optimality principle for a two-player game then requires

1. an agreement on a set of cooperative strategies or controls $u^* = \left\{ u_i^*, u_{-i}^* \right\}$ for $t \in [t_0, T]$

2. a mechanism to distribute the total payoffs among players. The share of player k from the cooperative payoffs is denoted by $\pi_k(y_0, T - t_0)$.

Similar schemes are considered in contributions by Petrosyan and his co-authors. Petrosyan and Danilov (1982) introduce the payoff distribution procedure (PDP) for cooperative solution. More recently, Yeung and Petrosyan (2001) develop time consistent solutions in dynamic games and identified the conditions which the allocation distribution procedure must satisfy. Petrosyan (2003) applies the regularization method to construct time consistent bargaining procedures and Petrosyan and Zaccour (2003) study the time consistent Shapley value allocation in a differential game of pollution cost reduction.

On the other hand, an alternative approach is to apply the axiomatic Nash bargaining approach which characterizes the problem as a bargaining problem and the resulting outcomes as that emerging from the bargaining problem. But the Nash bargaining solution is essentially a static solution which depends on the inital stage. The issue is actually more

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complicated in dynamic games which are continuous time games. At any point in time, players are selecting their respective strategies given the history of past moves. At some intermediate stage in the dynamic game, it may well be advantageous to switch to a different strategy and achieve a greater payoff than what was initially agreed on at the initial stage. For instance, a player can switch from a cooperative strategy to a non-cooperative one without there being any first point in time when someone is not cooperative. To have a well-defined game, it is crucial to restrict the set of possible strategies for each player to rule out such non-cooperative behavior from emerging or find a more sophisticated rule to define the realized outcome path for all pairs of possible strategies.

This issue is first raised by Haurie(1976) and is fundamentally an enforcement issue, which is raised in the second question. In other words, besides ensuring that the share of the cooperation is fairly distributed, the actual game must also be fair in process in that every player will play fairly by following the strategies to achieve the fair outcome and not cheat by deviating. Towards this end, Tolwinski et al (1986) suggest memory strategies to ensure cooperation. They apply Friedman (1971)'s δ-strategy approach, where a δ-strategy for player k is a mapping associating a control $u_i(t)$ with every $t \in [0, T)$ and every history of the game up to a certain moment $t' \in [0, t]$. Knowledge of the history up to t implies knowledge of the state $y(t)$ but not vice versa. As such, the resulting equilibrium strategies require access to information about the state $y(t)$, rather than the information about the other player's control function on the interval $[0, t)$.

The fair equilibrium construct in this chapter resolves this question as follows: a pre-game determines a fair rule for dividing *any gains* from cooperation to obtain the fair imputations. The rule states that the players should get at least their threat point payoffs, hence the problem is one of dividing up any gains. The two players can either play the cooperative dynamic game or the non-cooperative dynamic game. A rational acceptability criterion decides between cooperation and non-cooperation. The cooperative game will be played if the fair imputation payoffs are greater than those derived from a unilateral defection. Such fair imputations are said to be rationally acceptable. Otherwise, the fair equilibrium degenerates into a non-cooperative Nash equilibrium in which both players will play the non-cooperative

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game. Thus, the fair equilibrium is a cooperative one if rational acceptable fair imputations are available; otherwise, it is a non-cooperative one.

Three key differences of the current construct with existing constructs may be noted. Firstly, the fair equilibrium in this chapter involves Markovian strategies, whereas most constructs in the literature depend on non-Markovian strategies (Tolwinski et al, 1986). Secondly, unlike other constructs, it does not require a trigger strategy to enforce cooperation as the RA condition suffices to decide the fair imputations is rationally acceptable and whether cooperation is desirable or not. The players are thus not forced to cooperate if doing so is to their disadvantage. Thirdly, like Yeung and Petrosyan (2006), a pre-game bargaining determines the fair imputations. On the other hand, the process of determining the fair imputations is simplified here by applying a well-known result in experimental economics.

Further, we demonstrate the fair equilibrium is subgame perfect, time consistent and Pareto efficient. The last property is especially important as all current leading notions of fairness may perversely reduce welfare, including the possibility of reducing everyone's welfare (Kaplow and Shavell, 2002).

The chapter is organized as follows. Section 3.2 provides an intuitive motivations for the fair equilibrium. In section 3.3, the new concept of fair equilibrium and the idea of rational acceptability are introduced. The properties of this equilibrium, such as time consistency, subgame perfectness and Pareto efficiency, are then discussed. Section 3.4 presents the concluding remarks.

3.2 Intuitive Motivations

To motivate and interpret the key ideas in the chapter, this section provides the intuitions behind the concepts of rational acceptability and fair equilibrium.

To fix ideas, consider two players who are engaged in a game. Before the game begins, the players engage in a pre-game ultimatum game in which they explicitly decide on a fair rule for dividing *any gains* from cooperation, that is, the surplus of the total cooperative payoffs over the non-cooperative payoffs. Two important points must be emphasized here.

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Firstly, the players in the pre-game are not expressedly working out the stream of payoffs for the whole game. Instead, they are only agreeing on a fair imputation rule for sharing the joint cooperative gains. The rule applies once the game starts. Secondly, the players need not be optimizing some objective functionals. If they are, the measure of the fair imputation may be too large for practical consideration or the results may not be necessarily fair as one player may end up with the total surplus. To limit this measure, we rely on the results of numerous laboratory experiments of the Ultimatum Game (Binmore, 2007) which show that players tend to play fair and the most likely proposal is a fifty-fifty split. Hence, we define our fair imputation as one which includes their individual Nash payoff (the status quo or threat payoff) and half of their cooperative gain. It should be stressed here that the 50-50 split of the cooperative gains used in the imputation is neither a universal nor a unique rule. In societies where other split formula exists and are considered fair, another specification of the fair imputation is possible. For all intents and purposes, what matters is there is a fair imputation rule rather than the specific form of the rule. For our practical applications, we adopt the 50-50 split which is a "fair" social norm or outcome which is commonly observed in economic experiments of the Ultimatum Game.

Once the game begins, the two players can either play the cooperative dynamic game or the non-cooperative dynamic game. If the two players cooperate, they must abide by the sharing rule which both players have agreed upon in the pre-game. This sharing rule dictates the fair imputations in the game. Still this could be problematic in a dynamic game because it may be to the advantage of one player to switch to non-cooperation to get more than the fair imputation. However, it would not be rational to do so if the fair imputation payoffs are greater than those derived from a unilateral defection. If this is the case, the fair imputations are said to be *rationally acceptable* and the fair equilibrium will be the set of cooperative strategies for the players. In the absence of such rationally acceptable solutions, the fair equilibrium degenerates into a non-cooperative Nash equilibrium in which both players will not cooperate. In this case, non-cooperation is rationally acceptable than cooperation because cooperation generates payoffs that are Pareto dominated by the non-cooperative payoffs. As such, the fair equilibrium will be the set of non-cooperative strategies for the players.

Thus, the choice of cooperative or non-cooperative strategy for each player depends on the rationally acceptability criterion.

3.3   Fairness in Dynamic Game

Consider a meta-game $\Gamma(s,\gamma,p)$, starting at stage s with 2 players, i and $-i$, where γ is the strategy vector and p the payoff vector. $\Gamma(s,\gamma,\pi)$ may encompass either one of two types of games $\Gamma^D(s,\bar{u},\pi)$ and $\Gamma^C(s,\hat{u},\xi)$, where $\bar{u}$ and $\hat{u}$ are the respective equilibrium control set for all players and π and ξ are the respective payoff vectors for Γ^D and Γ^C.

$$\Gamma(s) = \begin{cases} \Gamma^C(s) & \text{if the players cooperate from stage } s \text{ onwards}, \\ \Gamma^D(s) & \text{if the players do not cooperate from stage } s \text{ onwards} \end{cases}$$

where s denotes the stage of the game. s is defined by a state-time tuple $(y(t),t)$, where $y(t)$ being the state at time t. For instance, s_0 is the initial stage defined by (y_0,t_0). If the underlying games are autonomous, then s depends purely on the state y. Additionally, the two types of games are not equivalent in the sense that the set $\{\Gamma(s)|\Gamma^C(s)=\Gamma^D(s)\}$ has Lebesgue measure 0. In other words, if the players are playing the cooperative game, they cannot be defecting and vice-versa.

Next, define the information structure of the game $\Gamma(s,\gamma,\pi)$. There are several ways to define the information structure of this game. If the information structure is Markovian or feedback, the strategy of the game depends on time t and the state of the system at time t, $y(t)$. On the other hand, if the equilibrium is non-Markovian or history dependent, whatever agreement on the cooperative outcomes agreed on prior to the game will be sustained by some mechanism such as a trigger strategy whereby any unilateral defection from the cooperative path will trigger off the non-cooperative game after some very small lag. This was the approach taken by Tolwinski et al (1986). In this case, however, the informational requirement may be quite immense for each player. In addition, a defection (whether deliberate or a mistake) will perpetuate the threat of non-cooperation indefinitely. In this chapter, we consider only Markovian strategy.

Definition A strategy set (γ_i, γ_{-i}) is the set of strategies of both players defined as the index function

$$
\gamma_k \;=\; \begin{cases} C(s, \hat{u}_k(s)) & \text{if } \Gamma(s) = \Gamma^C(s)\,, \\[2mm] D(s, \bar{u}_k(s)) & \text{if } \Gamma(s) = \Gamma^D(s) \end{cases}
$$

for $k = i, -i$

Each strategy has an associated set of control for given stage in the respective game, namely $\{\hat{u}_i(s), \hat{u}_{-i}(s)\}$ for the cooperative game $\Gamma^C(s)$ and $\{\bar{u}_i(s), \bar{u}_{-i}(s)\}$ for the noncooperative game $\Gamma^D(s)$. The strategy adopted by each player k leads to the payoff streams $\pi_k(s, \gamma_k)$.

The non-cooperative dynamic game $\Gamma^D(s, \gamma, \pi)$ has an infinite-horizon given by $[t_0, \infty)$. Each player optimizes an objective functional J_k, $k \in \{i, -i\}$. The objective functional is also known as **payoff functional** and is defined over the planning horizon. The game $\Gamma^D(s, \gamma, \pi)$ can be represented as

$$
\max_{u_k(t)} J_k(u_i, u_{-i}) = \int_{t_0}^{\infty} e^{-\rho t} F_k(y(t), u_i(t), u_{-i}(t), t)\, dt \tag{3.1}
$$

subject to the dynamical system

$$
\dot{y}(t) = f(y(t), u_i(t), u_{-i}(t), t), y(t_0) = y_0 \tag{3.2}
$$

where u_i and u_{-i} denote the control functions for player i and $-i$ respectively; $y(t)$ is the **state vector** which evolves dynamically from an initial state y_0. If the players acts independently, each player will optimize independently and derive the set of controls $(\bar{u}_i, \bar{u}_{-i})$.

Definition A strategy set $(\bar{\gamma}_i(s), \bar{\gamma}_{-i}(s))$ constitutes a Markovian or feedback Nash equilibrium solution for the game $\Gamma^D(s, \gamma, \pi)$ if and only if

$$
\bar{\gamma}_i(s) \;=\; D(s, \bar{u}_i, \pi_i) \tag{3.3}
$$

$$
\bar{\gamma}_{-i}(s) \;=\; D(s, \bar{u}_{-i}, \pi_{-i}) \tag{3.4}
$$

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where

$$\overline{u}_i \quad = \quad \arg\max_{u_i} \; J_i(u_i(s), \overline{u}_{-i}(s))$$

$$\overline{u}_{-i} \quad = \quad \arg\max_{u_{-i}} \; J_{-i}(\overline{u}_i(s), u_{-i}(s))$$

and

$$\pi_i \quad = \quad J_i(\overline{u}_i(s), \overline{u}_{-i}(s))$$

$$\pi_{-i} \quad = \quad J_{-i}(\overline{u}_i(s), \overline{u}_{-i}(s))$$

Hence, the strategy set is identified by its stage s, the control set $(\overline{u}_i, \overline{u}_{-i})$ and the associated payoffs (π_i, π_{-i}). We consider only Markovian trajectories which can be revised after the start of the game. This is in contrast with open loop trajectories which are completely pre-specified based on the initial state. Open loop controls are computationally easier to derive though it may not be time consistent, which may be a crucial feature in many macroeconomic games. As we are interested in a fair equilibrium in which the players can decide whether to sustain cooperation over defection at each stage, the Markovian solution is preferred as the reference point. In this case, this Markovian Nash equilibrium can be interpreted as its status quo or threat point *at the current stage of the game* if the players fail to agree on cooperating.

On the other hand, denote a cooperative game by $\Gamma^c(s, \gamma, \pi)$. In the cooperative game $\Gamma^c(y_0, t_0)$, the players jointly optimize subjected to the same constraint as $\Gamma(y)$, given some initial state y_0. Specifically, $\Gamma^c(s_0, \gamma, \pi)$ is given by

$$\max_{u_i, u_{-i}} J^c(u_i, u_{-i}) = \sum_{k \in \{i, -i\}} \left\{ \int_{t_0}^{\infty} e^{-\rho t} F_k(y(t), u_i(t), u_{-i}(t), t) dt \right\} \qquad (3.5)$$

subject to (3.2).

Pontryagin's Maximum Principle can be invoked to obtain a set of optimal controls that defines the set of cooperative control actions $(\hat{u}_i, \hat{u}_{-i})$.

The question of division of cooperative gains arises in this context. We proceed as

follows. Before the game begins, a pre-game bargaining in the form of a simple Ultimatum game is conducted to determine the division of any gain from cooperation. The measure of the bargaining set can be very large. To limit this measure, we apply a consistent result from numerous laboratory experiments of the Ultimatum Game (Binmore, 2007): players tend to play fair and the most likely proposal is a fifty-fifty split. Hence, we define our fair imputation as one which includes their Nash payoffs and half of their cooperative gain. This agreement on the share of the joint cooperative payoff is captured by a fair imputation rule.

Definition A *fair imputation* of the cooperative payoff for each player k at stage s, denoted by $\xi_k(s)$, is defined as one which includes individual Nash payoff $\pi_k(s)$ and half of their cooperative gains $\sum_k \xi_k(s) - \sum_k \pi_k(s)$, where $\sum_k \xi_k(s)$ is the total cooperative payoffs. In short,

$$\xi_k(s) = \pi_k(s) + \frac{1}{2}\sum_k (\xi_k(s) - \pi_k(s))$$

Definition A strategy set $(\hat{\gamma}_i(s), \hat{\gamma}_{-i}(s))$ constitutes a cooperative solution for the game $\Gamma^c(s, \gamma, \pi)$ if and only if

$$\hat{\gamma}_i(s) = C(s, \hat{u}_i, \xi_i) \tag{3.6}$$

$$\hat{\gamma}_{-i}(s) = C(s, \hat{u}_{-i}, \xi_{-i}) \tag{3.7}$$

where

$$\hat{u}_i = \arg\max_{u_i} J_i(u_i(s), u_{-i}(s))$$

$$\hat{u}_{-i} = \arg\max_{u_{-i}} J_{-i}(u_i(s), u_{-i}(s))$$

and

$$\sum_k \xi_k(s) = J_i(\hat{u}_i, \hat{u}_{-i})$$

$$\xi_k = \pi_k + \frac{1}{2}\sum_k (\xi_k - \pi_k)$$

for $k = i, -i$

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Rationally acceptable cooperative strategies are defined as the set of cooperative strategies with payoffs greater than or at least equal to the payoffs from pursuing non-cooperative strategies. More formally, acceptable strategies can be defined as follows.

Definition A cooperative strategies set $(\hat{\gamma}_i, \hat{\gamma}_{-i})$ is **rationally acceptable** (RA) to all players at stage s if and only if the continuation of this strategy set from s has a fair imputation that dominates the continuation of the Markovian Nash strategy set $(\bar{\gamma}_i, \bar{\gamma}_{-i})$, that is

$$\xi_k(\hat{\gamma}_i, \hat{\gamma}_{-i}, s) \geq \pi_k(\bar{\gamma}_i, \bar{\gamma}_{-i}, s) \tag{3.8}$$

for $k = i, -i$, with strict inequality for at least one player.

A fair equilibrium can then be defined as follows. So long as RA cooperative solutions exist, the fair equilibrium is the cooperative equilibrium sustained over the whole game. In the absence of rationally acceptable solution, each player will switch to non-cooperation. A formal definition is as follows.

Definition The fair equilibrium for the game $\Gamma(s, \gamma, \pi)$ is the strategy set $(\gamma_i^*, \gamma_{-i}^*)$ is defined as

$$\gamma_k^*(s) = \begin{cases} \hat{\gamma}_k(s) & \text{if RA condition is satisfied,} \\ \bar{\gamma}_k(s) & \text{otherwise} \end{cases}$$

Hence, the fair equilibrium involves a pre-game Ultimatum game in which a fair rule for dividing *any gains* from cooperation is determined to obtain the fair imputations. After the meta-game begins, the two players can either play the cooperative dynamic game or the non-cooperative dynamic game. They will play the cooperative game if the fair imputation payoffs are greater than those derived from a unilateral defection, that is, if the fair imputations are RA. Otherwise, the fair equilibrium degenerates into a non-cooperative Nash equilibrium in which both players will play the non-cooperative game.

It is worthwhile here to note three key differences of the current construct with existing constructs such as those in Tolwinski et al (1986). Firstly, in contrast to their non-Markovian

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strategies, the fair equilibrium in this chapter involves Markovian strategies. Secondly, most constructs require a trigger strategy to enforce cooperation. This is not necessary for the present case as the RA condition suffices to decide the fair imputations is rationally acceptable and whether cooperation is desirable or not. The players in our game are thus not forced against their will to cooperate which may be to their disadvantage. Thirdly, the pre-game bargaining determine the fair imputations. This is similar in spirit to Yeung and Petrosyan (2006) but we simplify the process by making use of a well-known result in experimental economics to determine the fair imputations.

Next, we consider the existence and properties of this fair equilibrium.

3.3.1 Existence of Fair Equilibrium

Theorem 3.3.1 *A fair equilibrium defined by the strategy set $(\gamma_i^*, \gamma_{-i}^*)$ exists for the dynamic game $\Gamma(y)$.*

Proof Suppose rationally acceptable solution does not exist. By definition, the fair equilibrium strategy set is given by $\{\gamma_k^* = \bar{\gamma}_k, k = i, -i\}$. This is the Markovian Nash equilibrium solution to the dynamic game $\Gamma(y)$, if it exists.

Consider the optimal control problem for player k with objective functional (3.1) and the constraint (4.3). Define a T-truncation of the objective functional, $J_T(u_k)$ as

$$J_T(\cdot) = \int_{t_0}^{T} e^{-\rho t} F(y(t), u_i(t), u_{-i}(t), t) dt$$

Consider the following conditions:

- there exists a unique continuous solution y of the initial value problem

$$\dot{y} = f(y(t), u_i(t), u_{-i}(t), t), y(t_0) = y_0$$

- for all $k = i, -i$, let V^k be a continuously differentiable function which satisfies the

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Hamilton-Jacobi-Bellman equation

$$\rho V^k(\cdot)(y,t) - V_t^k(\cdot)(y,t) = \max_{u_k} F^k(u_k(y), \bar{u}_{-k}(y), y) + V_y^k(\cdot)(y,t) f(u_k(y), \bar{u}_{-k}(y), y)$$

- for $T \to \infty$, either V^k is bounded function and $\rho > 0$ or V^k is bounded from below, $\rho > 0$ and $\limsup_{t \to \infty} e^{-\rho t} V^k(y(t), t)$

- when V^k is not bounded from above, $\limsup_{t \to \infty} e^{-\rho t} V^k(y_t, t) \leq 0, \; \forall (y_t, t)$

Given that these conditions are satisfied, $\{u_i, u_{-i}\}$ is the Markovian Nash equilibrium control set.

If rationally acceptable solution exists for $\Gamma(y)$, the existence of a fair equilibrium is trivial. ∎

Next, we consider three "desirable" properties of this fair equilibrium: time consistency, subgame perfection and Pareto efficiency.

3.3.2 Properties of the Fair Equilibrium

A strategy profile is a subgame perfect Nash equilibrium if a) it is a Nash equilibrium and b) its relevant action rules are a Nash equilibrium for every subgame. A subgame is a game consisting of a node which is a singleton in every player's information partition, that node's successors and the payoffs of the associated end nodes. A subgame can be infinite in length. Every subgame looks exactly like the original game but begins at a different point in time. On the other hand, time consistency (sequential rationality in macroeconomics) merely requires that strategies are best responses in subgames starting from nodes on equilibrium path, instead of all subgames. It is therefore less stringent than subgame perfection. Formally,

Definition A strategy set $\{\gamma_i^*, \gamma_{-i}^*\}$ is **time consistent** provided that the associated control set $\left(u_i^*(\gamma_i^*), u_{-i}^*(\gamma_{-i}^*)\right)$ constitutes a set of equilibrium controls for the truncated game $\Gamma(t, y^*(t))$, where $y^* = y(s_0, u_i^*, u_{-i}^*)$ for every $t \in (0, \infty)$

Unlike time consistency, subgame perfectness not only requires that the strategy is an equilibrium of the subgame $\Gamma(t, y^*(t))$, where $y^*(\cdot)$ is the state trajectory generated by the equilibrium controls, but is also an equilibrium for all subgames $\Gamma(t, y(t))$.

Definition A strategy set $\{\gamma_i^*, \gamma_{-i}^*\}$ is **subgame perfect** provided that the associated control set $\left(u_i^*(\gamma_i^*), u_{-i}^*(\gamma_{-i}^*)\right)$ constitutes a set of equilibrium controls for the truncated game $\Gamma(t, y(t))$, where $y(t)$ is an arbitrary chosen state reachable from some initial state y_0

Naturally, subgame perfectness implies time consistency but the reverse is not always true.

Theorem 3.3.2 *The fair equilibrium defined by the strategy set* $(\gamma_i^*, \gamma_{-i}^*)$ *is time consistent.*

Proof There are two cases to consider.

Case 1: Cooperative solution is rationally acceptable.

Substitute the set of cooperative controls $\{\hat{u}_i(y), \hat{u}_{-i}(y)\}$ into (4.3) to obtain the dynamics of the optimal cooperative trajectory

$$\dot{y} = f[y, \hat{u}_i(y), \hat{u}_{-i}(y)] \quad y(t_0) = y_0 \tag{3.9}$$

Let $\hat{y}$ denotes the cooperative solution. Denote $\xi_k(\cdot)$, $k \in \{i, -i\}$ to be an imputation of the payoffs of player k.

For the cooperative solution to be rationally acceptable, the following condition is required:

$$\xi_k(\cdot) \geq \pi_k(\cdot), \ \forall k \in \{i, -i\} \tag{3.10}$$

Denote $\xi(t) = \{\xi_i(t), \xi_{-i}(t)\}$ as the set of payoffs at time $t \in [0, \infty)$ for the cooperative game $\Gamma^C(\cdot)$. Specifically, along the cooperative trajectory, $\{\hat{y}(t)\}_{t \geq 0}$,

$$\xi_k(y_0) = \int_0^\infty e^{-\rho(t-\tau)} \xi_k(t) dt$$
$$\xi_k(y_\tau) = \int_\tau^\infty e^{-\rho(t-\tau)} \xi_k(t) dt,$$

for $k \in \{i, -i\}$ and $\tau \geq 0$.

Further,

$$\gamma^k(0;0,y_0) = \int_0^\infty e^{-\rho(t)}\xi_k^C(t)dt = \xi^k(\hat{y}_0)$$

$$\gamma^k(0;\tau,y_0) = \int_\tau^\infty e^{-\rho(t)}\xi_k^C(t)dt,$$

for $k \in \{i,-i\}$ and $\tau \in [0,\infty)$

Time consistency of the imputations is ensured by

$$\begin{aligned}
\gamma^k(0;\tau,\hat{y}_0) &= e^{-\rho(\tau)}\int_0^\infty e^{-\rho(t-\tau)}\xi_k^C(t)d\tau \\
&= e^{-\rho(\tau)}\xi^k(\hat{y}_\tau) \\
&= e^{-\rho(\tau)}\gamma^k(0;\tau,\hat{y}_0)
\end{aligned}$$

for $k \in \{i,-i\}$.

Intuitively, the extension of the solution to a situation with a later starting time and along the optimal trajectory remains optimal. Additionally, the cooperative solution must be rationally acceptable.

Case 2: Cooperative solution is not rationally acceptable. In this case, the Markovian Nash equilibrium applies. Let $(\bar{\gamma}_i,\bar{\gamma}_{-i})$ be the Markovian Nash equilibrium of the game $\Gamma(s,\gamma,\pi)$. The Markovian Nash equilibrium is defined by $y(\cdot)$ and the associated control set $\left(u_i^*(\gamma_i^*),u_{-i}^*(\gamma_{-i}^*)\right) = \left(\bar{u}_i(\gamma_i^*),\bar{u}_{-i}(\gamma_{-i}^*)\right)$.

Now suppose that the equilibrium is not time consistent. This implies that there must exist a time $\tau \in [0,\infty)$ such that player i can improve on her payoff given that the other player $-i$ stick to their strategy $\bar{\gamma}_{-i}$. Consider the strategy for player i defined as follows:

$$\tilde{\gamma}_i(t,k(t)) = \begin{cases} \bar{\gamma}_i(t,y_t) & \text{if } t \in [o,\tau), \\ \hat{\gamma}_i(t,y_t) & \text{if } t \in [\tau,\infty) \end{cases}$$

Since $\tilde{\gamma}_i$ is a better strategy than $\bar{\gamma}_i$, it must hold that $\pi(\tilde{\gamma}_i) > \pi(\bar{\gamma}_i)$ which contradicts that $(\bar{\gamma}_i,\bar{\gamma}_{-i})$ is the Markovian Nash equilibrium of the game $\Gamma(s,\gamma,\pi)$. ∎

Theorem 3.3.3 *The fair equilibrium defined by the strategy set $(\gamma_i^*,\gamma_{-i}^*)$ is subgame perfect.*

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Proof Two cases must be considered:

Case 1: RA solution does not exist

Thus, the fair equilibrium is the Markovian Nash equilibrium. Suppose the following conditions are satisfied:

1. for every (y_τ, τ), there exists a unique absolutely continuous solution $y_{(y_\tau, \tau)}$ of the initial value problem

$$\dot{y}(t) = f(y(t), u_i(t), u_{-i}(t), t), y(\tau) = y_\tau$$

2. for all $k = i, -i$ there exists a continuously differentiable function V^k such that the Hamiltonian-Jacobi-Bellman equation

$$\rho V^k(\cdot)(y,t) - V_t^k(\cdot)(y,t) = \max_{u_k} F^k(u_k(y), \bar{u}_{-k}(y), y) + V_y^k(\cdot)(y,t) f(u_k(y), \bar{u}_{-k}(y), y)$$

is satisfied for all (y_τ, τ).

3. either V^k is bounded function and $\rho > 0$ or V^k is bounded from below, $\rho > 0$ and $\limsup_{t \to \infty} e^{-\rho t} V^k(y(\tau), \tau)$

4. when V^k is not bounded from above, $\limsup_{t \to \infty} e^{-\rho t} V^k(y_\tau, \tau) \leq 0$, $\forall (y_\tau, \tau)$

then the resulting Markovian Nash equilibrium is subgame perfect for all truncated game $\Gamma(y_\tau, \tau)$, where y_τ is an arbitrary chosen state reachable from some initial state y_0.

Case 2: RA solution exists

In this case, the fair imputation set $\{\xi_i, \xi_{-i}\}$ is such that

$$\xi_k(\hat{\gamma}_i, \hat{\gamma}_{-i}, y(\tau), \tau) \geq \pi_k(\bar{\gamma}_i, \bar{\gamma}_{-i}, y(\tau), \tau)$$

for $k = i, -i$, with strict inequality for at least one player and for all possible $(\tau, y(\tau))$ such that $\tau \geq 0$ and $y(\tau_0) \geq y(\tau_0)$. This applies for all truncated game $\Gamma(t, y(t))$, where $y(t)$ is an arbitrary chosen state reachable from some initial state y_0 so long as there exists a

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continuously differentiable function H^C such that the conditions for case 1 are likewise satisfied for all (y,t). ∎

The next theorem establishes that the fair equilibrium is Pareto efficient. This is important because Kaplow and Shavell (2002) establish that all the leading notions of fairness (Kant's categorical imperative and Rawls veil-of-ignorance) may perversely reduce welfare, including the possibility of reducing everyone's welfare. Such possibility does not exist for the present construct of fairness.

Theorem 3.3.4 *The fair equilibrium defined by the strategy set* $(\gamma_i^*, \gamma_{-i}^*)$ *is Pareto efficient.*

Proof A set of strategies $\tilde{\gamma}$ is Pareto efficient if the set of inequalities

$$\pi_k(\gamma) \leq \pi_k(\tilde{\gamma})$$

is satisfied for $k \in \{i, -i\}$, with at least one of the inequalities strict. In other words, a Pareto solution is never dominated.

If a RA solution exists, the fair equilibrium is $\gamma_k^* = \hat{\gamma}_k(s_0)$, i.e. the whole game will be played cooperatively from s_0 and the following RA condition applies:

$$\xi_k(\hat{\gamma}) \geq \pi_k(\tilde{\gamma})$$

with strict inequality for at least one player. Since $\xi_k = \pi_k + \frac{1}{2}\sum_k (\xi_k - \pi_k)$, the RA condition can be simplified to

$$\sum_k \xi_k(\hat{\gamma}) > \sum_k \pi_k(\tilde{\gamma})$$

Assume that $\gamma_k^* = \hat{\gamma}_k$ is not Pareto efficient. Then there exists a tuple of non-cooperative strategies $\tilde{\gamma}$ such that

$$\xi_k(\gamma_k^*) \leq \pi_k(\tilde{\gamma})$$

with at least one inequalities strict. However, this implies that

$$\sum_k \xi_k(\gamma^*) < \sum_k \pi_k(\tilde{\gamma})$$

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which contradicts the RA condition.

If no RA solution exists, the fair equilibrium is the Markovian Nash equilibrium is Pareto optimal by definition,. ∎

3.4 Concluding Remarks

A key contribution of this chapter is to formalize a set of fair equilibrium in dynamic games with heterogeneous agents. Because heterogeneous agents have different beliefs about fairness, there is no unique system or rule that will be deemed fair by all parties. Even so, it is still possible to arrive at a fair outcome in a fair way that everyone, regardless or their conceptions of fairness, can rationally accept. In short, fairness beliefs are already embedded in the strategies chosen by the agents and therefore one needs only to examine the outcomes and process for fairness. Loosely defined, a fair outcome is regarded as fair in process for all agents if no player can legitimately protest the process and the result. When such rationally acceptable solutions exist, cooperation is possible; otherwise, the players will play non-cooperatively.

Unlike other constructs of fairness in the literature, the present fair equilibrium emerges endogenously from the interaction of the players in dynamic games. Rationally acceptable strategies exist when a fair imputation of the cooperative solution in a dynamic cooperative game offers players a higher payoffs compared to the stream of payoffs obtained in a Markovian Nash equilibrium. The set of fair equilibrium is then the set of rationally acceptable strategies. The fair equilibrium in this chapter involves Markovian strategies, which is distinct from non-Markovian strategies adopted by Tolwinski et al (1986). Many existing constructs assume implicitly that cooperation is desirable and must be enforced at all cost using trigger strategy or other enforcement mechanisms. In contrast, we do not assume that cooperation is necessarily desirable. The RA condition plays an important role in deciding whether cooperation is desirable or not. As such, the fair equilibrium here may be equivalent to the non-cooperative equilibrium if the latter is rationally acceptable. Above all, the players in the game are not forced against their will to cooperate even if doing so will be

to their disadvantage. Additionally, we make use of an important insight from experimental economics to simplify the determination of the fair imputation rule in pre-game bargaining. Further, this fair equilibrium is time consistent and subgame perfect and Pareto efficient. The last property is especially important as Kaplow and Shavell (2002) have noted that all current leading notions of fairness may perversely reduce welfare, including the possibility of reducing everyone's welfare.

Our analysis is restricted to games with two players. This is for analytical tractability though little qualitative insights may be lost in a more general model with many players. For instance, in dynamic games with n players, an important consideration is what the fair imputation should be. One simple possibility is to divide up the cooperative gains over the n players. However, this simple division does not take into consideration the possibility of different players forming different sets of coalitions throughout the duration of the games to improve on the simple division. Work in this area has begun for static games (Ray, 2007) and offers potential for future research in dynamic games.

For practical purposes, the resulting complexity in generalizing the results to n players may hardly be worth the trouble. The applied economists can easily restrict their dynamic game model to two representative players (for example, representative firm and consumer) and still derive rich dividends in insights from such simple models. In this way, the fair equilibrium derived in this chapter has many potential applications in the analysis of fairness in dynamic models of macroeconomics or political economy with heterogeneous agents. In the next chapter, we apply this concept of the fair equilibrium in a dynamic game of economic growth.

Chapter 4

Why Capitalism Prevails Despite its Dynamic Inefficiency: A Game-Theoretic Perspective

4.1 Introduction

Since the collapse of communism, it is widely accepted that capitalism is the key to prosperity. Even countries like China, which is communist in ideology, has adopted capitalism in practice. The raison d'être in China's economic policy since the 1980s can be regarded as one of maximum growth in the initial phase of development with postponed consumption for a later phase. Yet, such a strategy is not without its problems. Like most capitalist economies, such an intertemporal contract may be neither acceptable to the workers who form the core of political support for the party nor sustainable over the long term. According to World Bank estimates, while the real GDP grew at an annual average rate of 10 % in China during the last two decades of the last century, the income disparity has also widened. This has prompted many to argue for enforcing some collective solutions so as to curb the dynamic inefficiency of capitalism and bring about a "harmonious society".

This situation is not unique to China. Since Malthus, Ricardo and Marx, "the essence of capitalism (is perceived) to be centered on the problems of capital accumulation and the

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distribution of income between workers and capitalists" (Lancaster, 1973). Within such a paradigm, Lancaster (1973) shows that capitalism is dynamically inefficient when compared to a social optimum which can be achieved by both the workers and the capitalism cooperating together. The attractiveness of such cooperative or collective outcomes has achieved commanding heights in mid 20th century, with many countries adopting communism or some forms of socialism. Perhaps because of the scars from the Great Depression, capitalism was somewhat discredited and the idea of a "benevolent dictator" appealed to both influential economists and policymakers. In China, for instance, in the early years (1949-52) of Communist rule, private firms were allowed to continue their operations but beginning 1953, the capitalists were ordered to surrender their enterprises, "until they became only managers of the enterprise and had to follow government instructions if they were to remain part of it." (Chow 2002). The collapse of communism and the adoption of capitalism by Communist China, Russia and Vietnam attest that such collective or cooperative outcomes, while apparently attractive, are not sustainable. Countries which persist in collective solutions, such as North Korea and Myanmar(Burma), continue to suffer from dismal growth.

If capitalism is dynamically inefficient and apparently "unfair", why is it the choice of practically every economy in the world today? The answer offered here is that capitalism may be more rationally acceptable and fairer than collectivism. This is demonstrated using a government-firm (GF) dynamic game with a vote-maximizing government (G) and profit-maximizing representative firm (F). In this GF game, a fair imputation of cooperative or collective solutions which is rationally acceptable for all players does not exist. Rationally acceptable(RA) strategies are the set of cooperative strategies with payoffs greater than or at least equal to the payoffs from pursuing non-cooperative strategies. Regardless of the stages of development, the firm always finds it rationally unacceptable to cooperate because the profits earned by the firm under the Markovian Nash equilibrium always dominate the profits under cooperation. On the other hand, the government only finds cooperative solution to be rationally acceptable when the economy is above the steady state. Below the steady state, developing countries are trapped in low growth and political instability. Thus, while group rationality dictates cooperation, such cooperation is not realized because it is not

rationally acceptable for both the government and the firms. When the RA condition is not fulfiled, a fair equilibrium degenerates to the Markovian Nash equilibrium.

This chapter contributes to the game-theoretic literature on capitalism and economic growth. Phelps and Pollak (1968) are perhaps the first to consider a game-theoretic approach in economic growth. They model economic growth and distribution as an intergenerational conflict. In their model, the present generation derives its utility from the consumption pattern of infinitely many nonoverlapping generations but it can only control its own saving rate. As a result, the Nash equilibrium of this intergenerational game results in undersaving. Phelps and Riley (1976) extend the analysis by applying Rawlsian Maximin to achieve intergenerational justice. Contrary to conventional beliefs, "maximin" growth will not lead to zero growth if initial capital is sufficiently scarce and if each generation is altruistically interested in future utilities possibilities. However, both papers considere neither the issues of distributional conflict between different types of players nor the possibility of cooperation. These issues are explored by Lancaster (1973) who adopts a two player noncooperative dynamic game where the workers control the share of their consumption in total output while the capitalists control the share of investment in the surplus. Comparing the Markovian Nash equilibrium with the cooperative solution (from maximizing a weighted sum of worker and capitalist consumption), Lancaster demonstrates that both players obtain more consumption under cooperation, hence demonstrating the dynamic inefficiency of capitalism. This has been extended by others (see Dockner et al, 2000 for a survey) in various degrees of sophistication but the basic conclusion is fundamentally the same. For example, Kaitala and Pohjola(1990) consider a variation on the original Lancaster model in which the politically powerful group of workers controls redistribution while the economically powerful group of capitalists controls accumulation. Grim trigger strategies are employed by both groups to sustain cooperation as an equilibrium. In their model, the workers and capitalists receive returns equivalent to the labor and capital share respectively. In all these models, it is implicitly assumed that some binding agreement can be accepted by all and enforced rigidly, without worrying whether such binding agreement can be achieved in the first place.

The present model thus departs from the literature in two respects. Firstly, the govern-

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ment in this present model is a vote-maximizer while the firm receives a return equivalent to the marginal product of capital. The characterization of the government as a vote-maximizer is a distinctive feature of the present model. This is a significant departure from conventional economic models, in which the government is typically characterized as a benevolent dictator with the aim of maximizing social welfare. The idea of a vote-maximizing government follows from Nordhaus(1975). However, the government in Nordhaus' political business cycle model faces the short-run Phillips inflation-unemployment tradeoff. In contrast, the government in this model deals with the long-run political "tradeoff" between economic growth and distributional equity. Vote-maximization is not to be taken literally to imply a democracy. Instead, the vote function in this chapter can be interpreted as a function for political support. For instance, in the case of China, the Communist Party depends on the political support of the people, despite the absence of any democratic mechanisms. Hence, regardless of whether a government is democratic or authoritarian, we assume that its main objective is to maximize its political support or vote function.

Secondly, unlike Lancaster and his follower, we derive explicitly the set of fair imputations of the cooperative payoffs so as to determine whether the cooperative solution is rationally acceptable. Rationally acceptable strategies exist if a fair imputation of the cooperative solution offers players higher payoffs compared to those obtained in a Markovian Nash equilibrium. Where rationally acceptable cooperative solution exists, the fair equilibrium is simply the set of rationally acceptable strategies. Otherwise, the fair equilibrium is the set of Markovian Nash equilibrium. Our approach is distinct from those adopted in the recent literature on distributional fairness in growth and development, such as Alesina and Angeletos (2005) and Benabou and Tirole (2006). These papers incorporate the economists' judgment of fairness and equity and ignore the concerns for fairness and equity of the economic players under analysis. In contrast, the present chapter derives the fair equilibrium explicitly from the endogenous and strategic interactions between the players.

The rest of this chapter is organized as follows. Section 4.2 presents the GF game of economic growth. The Markovian Nash equilibrium and the cooperative solutions are derived and discussed in Section 4.3. Section 4.4 explores the characterization of a fair equilibrium

based on rational acceptability. Section 4.5 concludes.

4.2   Government-Firm Game of Economic Growth

Consider a dynamic game of economic growth with two players, a government(G) and a representative firm(F). Henceforth, we refer this game as the GF game. The economy has a neoclassical production function, which is represented in intensive form as $y = f(k), f'(k) > 0, f''(k) < 0, \lim_{k \to 0}[f'(k)] = \infty, \lim_{k \to \infty}[f'(k)] = 0$. The labor force receives an income equal to its marginal product $f(k) - kf'(k)$ while firm derives a rent equivalent to its marginal product $f'(k)$.

The firm in the model owns the capital and has to decide between retaining its capital earnings for investment and consuming the dividends payments. Its objective is to maximize the stream of dividends payments over time. The present game analysis suggests a more active role of the firm in the policy-making process. Not only will the investment strategy of the firms adjust dynamically to the tax policy of the government, the corporate tax strategy of the government will also change in response to changes in the firms' investment policy.

The government is a vote-maximizer: it will adopt policies that will best assure its continuation in power, increase its political support or improve its vote-getting power. This is represented using a vote function $v[k, x(\cdot), s(\cdot)]$, where $x(\cdot)$ represents the tax or social transfer within the government's control while $s(\cdot)$ represents the investment rate controlled by a representative firm.

The government's objective functional can thus be expressed as follows:

$$\max_{x(.)} J^G(k_0, x(\cdot), s(\cdot)) = \int_0^\infty e^{-\rho t} v[k, x(\cdot), s(\cdot)] dt.$$

It is worthwhile to post some brief comments about this vote function here. In general, the vote function may be interpreted as a function of political support. There is no need to assume a democracy here. On the other hand, the implicit assumption in such a specification is that the government's political support is aligned with the satisfaction of the workers, who form the electorate. This will be elaborated in the next chapter.

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Assume a balanced budget and that the policy instrument $x(t)$ used by the government to promote growth and effect redistribution is lump-sum. The government faces the long-run political tradeoff between long run economic growth and distributional equity through its control of the transfer x. A positive transfer will increase the government's political support (higher vote share) though this will also reduce the amount available to the firm for investment, thus resulting in low future economic growth. Conversely for a negative transfer. The conflict can be encapsulated in the vote function:

$$v[k,x(\cdot),s(\cdot)] = f(k) - kf'(k) + x.$$

The transfer or tax, $x(t)$, on the workers must be less than or equal to their incomes while redistribution disbursement cannot exceed the marginal product of workers. Similarly, the tax must be less than or equal to profits when it is imposed on the firm and the subsidy to the firm will not exceed its marginal product. Hence, the constraint $-f(k) + k.f'(k) \leq x \leq f(k) - k.f'(k)$ is binding.

Assume a representative firm which owns the capital in the production process and controls investment. The objective of the firm is to maximize the flow of dividend payment $\pi[x(\cdot),s(\cdot)]$ for the planning horizon. Since the government may tax or subsidize the firm, its after tax/subsidy profit is given by $f'(k) - x$. Out of this after-tax profit, the firm must decide how much to pay out as dividends to shareholders and how much to retain for investing in capital by adjusting $s(\cdot)$, the rate of capital investment. The firm's objective functional is given by

$$\max_{s(.)} J^F(k_0,x(\cdot),s(\cdot)) = \int_0^\infty e^{-\rho t} \pi[k,x(\cdot),s(\cdot)] \, dt,$$

where $\rho > 0$ is a positive discount rate ; $k(t_0) = k_0 > 0$ is an initial capital-labor ratio; $s(t)$ be the investment rate which is controlled by the firm and $\pi[k,x(\cdot),s(\cdot)]$ are the dividends payments, given by

$$\pi[k,x(\cdot),s(\cdot)] = (1-s)[f'(k) - x], 0 \leq s \leq 1.$$

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Assume that labor consumes fully its wage. As such, only the firm contributes to the accumulation of capital. Capital accumulation then follows the dynamics

$$\dot{k} = g\left(x(.),s(.),k\right) = s\left[f'(k) - x\right] - (n+\delta)k,$$

where δ denotes effective depreciation for the capital-labor ratio k, n is the population growth rate and $n+\delta > 0$; x is a per person lump sum which is controlled by the government, so that $f'(k) - x$ is the after tax/subsidy profit for the firm.

The GF game thus described involves the government and the private sector acting independently, affecting a common state variable and each other's payoffs through time and is hence a dynamic game. In this GF game, each player takes into account the other player's decision while making his or her own decision. Since the game is dynamic, each player will take into account not only the current but also future decisions of the other player.

The complete GF game, $\Gamma(k)$, can be characterized as follows:

Government

$$\max_{x(.)} J^G\left(k_0, x(\cdot), s(\cdot)\right) = \int_0^\infty e^{-\rho t}\left[f(k) - k.f'(k) + x\right] dt, \tag{4.1}$$

Firm

$$\max_{s(.)} J^F\left(k_0, x(\cdot), s(\cdot)\right) = \int_0^\infty e^{-\rho t}(1-s)\left[f'(k) - x\right] dt, \tag{4.2}$$

subject to

$$\dot{k} = s\left[f'(k) - x\right] - (n+\delta)k, \qquad k(0) = k_0, \tag{4.3}$$

$$0 \leq s \leq 1, \tag{4.4}$$

$$-f(k) + k.f'(k) \leq x \leq f'(k). \tag{4.5}$$

The control function pair $(x(\cdot), s(\cdot))$ is such that $x(\cdot) : [0,\infty) \to X$ and $s(\cdot) : [0,\infty) \to S$ where X and S are the control sets of the government and the firm respectively. Assume information is complete.

Alternatively, a cooperative GF game can be defined as $\Gamma^c(k)$, which denotes a coop-

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erative game between the government and the firm with the game structure of $\Gamma(k)$, given some initial state k_0. Group rationality dictates the joint maximization of the sum of payoffs for both players. Specifically,

$$\max_{x(.)} J^C(k_0, x(\cdot), s(\cdot)) = \int_0^\infty e^{-\rho t} \{v[k, x(\cdot), s(\cdot)] + \pi[k, x(\cdot), s(\cdot)]\} dt, \qquad (4.6)$$

subject to (4.3) through (4.5).

4.3 Markovian Nash Equilibrium and Cooperative Solutions

In this section, the Markovian Nash equilibrium to the GF game, $\Gamma(k)$, and the cooperative solutions to the cooperative GF game, $\Gamma^C(k)$, are derived. These solutions are then computed for a common specification of the neoclassical production function.

4.3.1 Markovian Nash Equilibrium

In the GF game, $\Gamma(k)$, the problem for the government is to take $s(\cdot)$ as given and choose a transfer/tax strategy, $x(\cdot)$, so as to maximize its political payoff (4.1). Taking $x(\cdot)$ as given, the firm chooses an investment strategy, $s(\cdot)$, so as to maximize its after tax profit flow (4.2). Both are subject to the constraints (4.3) through (4.5). This section discusses the Markovian Nash equilibrium to the GF game $\Gamma(k)$.

Theorem 4.3.1 *A set of strategies $\{\bar{x}(k), \bar{s}(k)\}$ constitutes a Markovian Nash equilibrium solution to the game $\Gamma(k)$ if there exists functionals, $J^G(k) : R^2 \to R$ and $J^F(k) : R^2 \to R$ satisfying the following set of partial differential equations:*

$$\rho J^G(k) = \max_x \left\{ v(k, x, \bar{s}) + J_k^G(k) g(k, x, \bar{s}) \right\} \qquad (4.7)$$

$$\rho J^F(k) = \max_s \left\{ \pi(k, \bar{x}, s) + J_k^F(k) g(k, \bar{x}, s) \right\} \qquad (4.8)$$

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where

$$J^G(k) = \int_\tau^\infty e^{-\rho(t-\tau)} \left[f(k) - k.f'(k) + x \right] dt \tag{4.9}$$

$$J^F(k) = \int_\tau^\infty e^{-\rho(t-\tau)} (1-s) \left[f'(k) - x \right] dt, \tag{4.10}$$

represents the current value payoffs of the government and the firm at time τ

Proposition 4.3.2 *The Markovian Nash equilibrium* $(\bar{x}(k), \bar{s}(k))$ *for* $\Gamma(k)$ *is given by*

$$(\bar{x}(k), \bar{s}(k)) = (f'(k), 0) \tag{4.11}$$

Proof A noncooperative Markovian Nash equilibrium solution to $\Gamma(k)$ is characterized by:

$$\rho J^G(k) = \max_x \left\{ f(k) - kf'(k) + x + J_k^G(k) \left(\bar{s} \left[f'(k) - x \right] - (n+\delta)k \right) \right\} \tag{4.12}$$

$$\rho J^F(k) = \max_s \left\{ (1-s) \left[f'(k) - \bar{x} \right] + J_k^F(k) \left(s \left[f'(k) - \bar{x} \right] - (n+\delta)k \right) \right\} \tag{4.13}$$

Performing the above maximizations yields:

$$\bar{x}(k) = f'(k), \bar{s}(k) = 0$$

If the tax is $\bar{x}(k) = f'(k)$, the whole rental income of the firm is effectively taxed away and the firm will have to stop investing, thus $\bar{s}(k) = 0$. The economic intuition is as follows. In the GF game, a government may postpone redistribution to later stages so as to facilitate the most rapid economic growth. But the firm predicts as much and being free to optimize on its investment decisions, will stop investing just before profits are being taxed. But the government is also aware of the firm's reaction: its best response is to impose taxes on capital income earlier. The process will then converge to the Markovian Nash equilibrium $(\bar{x}(k), \bar{s}(k)) = (f'(k), 0)$.

Substitution of the Markovian Nash equilibrium solutions into (4.12) and (4.13) to solve for the Markovian Nash payoffs of the government ($\bar{v}$) and the firm ($\bar{\pi}$) respectively:

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$$\bar{v} \equiv \rho J^G(k) \;=\; \{f(k) - kf'(k) + f'(k) - (n+\delta)k\} \tag{4.14}$$

$$\bar{\pi} \equiv \rho J^F(k) \;=\; 0 \tag{4.15}$$

Thus, the government wins positive "votes" or political support for its action but at the expense of the firm which will have to contend with normal profits. How will this affect the economic growth rate and profit growth rate?

The economic growth rate can be derived as follows:

$$\frac{\dot{y}}{y} = f'(k)\frac{k}{f(k)} = \left[kf'(k)/f(k)\right]\left(\frac{\dot{k}}{k}\right),$$

while the after-transfer profits is given by

$$\pi(x,s) = (1-s)\left[f'(k)-x\right]$$

For the Markovian Nash payoff for the firm, $\dot{k} = -(n+\delta)k$ and $\pi(\bar{x}(k),\bar{s}(k)) = 0$, hence $\frac{\dot{y}}{y} < 0$ and $\frac{\dot{\pi}}{\pi} = 0$.

Hence, in terms of economic growth rate, the profitability of the firm and capital accumulation, the consequences of the Markovian Nash equilibrium will be more adverse for the developing countries which have less initial capital stock compared to relatively more developed countries. The results so far are not controversial and are similar to Lancaster(1973) and Kaitala and Pohjola(1990). These authors went on to argue that cooperation between the government and the firm will be more beneficial compared to the dynamic inefficiency of capitalism. They assume implicitly that the cooperative solution is desirable and will be accepted by all. Hence, it must be enforced at all cost. In the case of Kaitala and Pohjola(1990), the cooperation is enforced by trigger threats that force everyone to cooperate but whether such a cooperation is fair or not is not considered. In the next section, the cooperative solution to the game and the set of fair imputations of the cooperative payoff are explicitly derived so as to determine whether cooperation is indeed rationally acceptable

compared to the Markovian Nash equilibrium.

4.3.2 Cooperative Solutions

Consider a cooperative GF game $\Gamma^c(k)$.

Theorem 4.3.3 *A set of strategies $\{\hat{x}(k), \hat{s}(k)\}$ constitutes a solution to the game $\Gamma^c(k)$ if there exists functionals, $W^G(k) : R^2 \to R$ satisfying the infinite horizon Hamilton-Jacobi-Bellman equation:*

$$\rho J^C(k) = \max_{x,s} \left\{ v(k,x,s) + \pi(k,x,s) + J_k^C(k) g(k,x,s) \right\}$$

where $J_k^C(k) g(k,x,s) = \int_\tau^\infty e^{-\rho(t-\tau)} [f(k) + (1-k)f'(k) - s(f'(k) - x)]dt.$

More specifically, the cooperative solution of the GF game $\Gamma^c(k)$ can be obtained by considering the optimization problem

$$\max_{x(.),s(.)} J^C(k_0, x(\cdot), s(\cdot)) = \int_0^\infty e^{-\rho t} \left[f(k) + (1-k)f'(k) - s(f'(k) - x) \right] dt, \qquad (4.16)$$

subject to (4.3) through (4.5).

Due to the simple linear structure of the model, the problem can be solved more directly by applying the Most Rapid Approach Path (MRAP) technique (Kamien and Schwartz, 1991). First, use the state equation (4.3) to obtain

$$J^C(k_0, x(\cdot), s(\cdot)) = \int_0^\infty e^{-\rho t} \left[f(k) + (1-k)f'(k) - \left(\dot{k} + (n+\delta)k \right) \right] dt$$

Next, integrate the term containing $\dot{k}$ and use the initial condition $k(0) = k_0$ to obtain

$$J^C(k_0, x(\cdot), s(\cdot)) = k_0 + \int_0^\infty e^{-\rho t} [h(k) - ((n+\delta+\rho)k)]dt.$$

where $h(k) = f(k) + (1-k)f'(k)$. The integrand in this representation of the objective functional J^C is strictly concave of the state variable k and attains its maximum at the unique

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steady state value $k = k^{SS}$ defined by the equation $h'(k^{SS}) = n + \delta + \rho$. It follows that to maximize J^C, the state trajectory must approach the steady state k^{SS} as fast as possible and remain there forever. It is trivial that this is the case if and only if the controls are selected as follows:

$$
\hat{s}[f'(k) - \hat{x}] =
\begin{cases}
f(k) + (1 - k) f'(k) & k < k^{SS} \\
(n + \delta)k & k = k^{SS} \\
0 & k > k^{SS}
\end{cases}
\tag{4.17}
$$

From this, it is straightforward to derive the set of cooperative strategies to the game.

Proposition 4.3.4 *The set of cooperative strategies $\{\hat{x}(k), \hat{s}(k)\}$ to the game $\Gamma^c(k)$ is given by*

$$
\hat{s}(k) =
\begin{cases}
1 & k < k^{SS} \\
u, u \in (0, 1] & k = k^{SS} \\
0 & k > k^{SS}
\end{cases}
\tag{4.18}
$$

$$
\hat{x}(k) =
\begin{cases}
kf'(k) - f(k) & k < k^{SS} \\
f'(k) - (n + \delta)u^{-1}k, u \in (0, 1] & k = k^{SS} \\
f(k) & k > k^{SS}
\end{cases}
\tag{4.19}
$$

Applying the results from Chapter 3, we define a *fair imputation* of the cooperative payoff for each player as one which includes their Nash payoffs and half of their cooperative gains. Hence, the imputed payoffs for each player in the cooperative game $J^C = \left\{ J^{CG}, J^{CF} \right\}$ are

$$
J^{CG}(k) = \frac{1}{\rho} \left\{ f(k) - kf'(k) + \frac{1}{2} \left[sf'(k) - (n + \delta)k \right] \right\}
\tag{4.20}
$$

$$
J^{CF}(k) = \frac{1}{\rho} \left\{ \frac{(s - 2)}{2} f'(k) - \left(\frac{n + \delta}{2} \right) k \right\}
\tag{4.21}
$$

Accordingly, denote $\hat{v} \equiv \rho J^{CG}$ and $\hat{\pi} \equiv \rho J^{CF}$, which are given respectively by

$$\hat{v}(k) = \begin{cases} f(k) - kf'(k) + \frac{1}{2}\left[f'(k) - (n+\delta)k\right] & k < k^{SS} \\ f(k) + \left(\frac{1}{2}u - k\right)f'(k) - \left(\frac{n+\delta}{2}\right)k & k = k^{SS}, 0 < u \le 1 \\ f(k) - kf'(k) - \frac{1}{2}(n+\delta)k & k > k^{SS} \end{cases} \qquad (4.22)$$

$$\hat{\pi}(k) = \begin{cases} -\frac{1}{2}f'(k) - \left(\frac{n+\delta}{2}\right)k & k < k^{SS} \\ \frac{(u-2)}{2}f'(k) - \left(\frac{n+\delta}{2}\right)k & k = k^{SS}, 0 < u \le 1 \\ -f'(k) - \left(\frac{n+\delta}{2}\right)k & k > k^{SS} \end{cases} \qquad (4.23)$$

Similarly, the growth rate and the rate of growth for the after tax profit can be computed respectively.

$$\frac{\dot{y}}{y} = \begin{cases} -\left[kf'(k)/f(k)\right]\left(\left[f'(k) - f(k)\right]/k - f'(k) - (n+\delta)\right)(n+\delta) & k < k^{SS} \\ 0 & k = k^{SS} \\ \left[kf'(k)/f(k)\right](n+\delta)^2 & k > k^{SS} \end{cases}$$

$$\frac{\dot{\pi}}{\pi} = \begin{cases} (f(k) + (1-k)f'(k))/k - (n+\delta) & k < k^{SS} \\ 0 & k = k^{SS} \\ (n+\delta) & k > k^{SS} \end{cases}$$

4.3.3 Solutions for Specific Neoclassical Production Function

To make concrete the solution concepts and allow easy comparison, it is useful to adopt a specific neoclassical function. The most common specification for this in the literature is the constant elasticity of substitution production function,given by:

$$y = f(k) = \tilde{A} \cdot \left[a \cdot (bk)^{\psi} + (1-a) \cdot (1-b)^{\psi}\right]^{\frac{1}{\psi}} \qquad (4.24)$$

where $0 < a < 1, o < b < 1$ and $\psi < 1$. The marginal product of capital is given by

$$f'(k) = \tilde{A}ab^{\psi}\left[a \cdot b^{\psi} + (1-a) \cdot (1-b)^{\psi} \cdot k^{-\psi}\right]^{\frac{1-\psi}{\psi}} \qquad (4.25)$$

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Without loss of generality, this can be simplified to a Cobb-Douglas form by letting $\psi < 1 \to 0$ and applying l'Hôpital's rule to obtain $f(k) = Ak^a$, where $A = \tilde{A}b^a(1-b)^{1-a}$ and $0 < a < 1$.

The results are summarized in the table 4.1.

From these, the economic growth rate and the rate of growth of after-tax profits for the firm can be derived for the cooperative solutions.

$$\frac{\dot{y}}{y} = \begin{cases} -a\left(Ak^{a-1}\left(k^{-1}-2\right)-(n+\delta)\right)(n+\delta) & k < k^{SS} \\ 0 & k = k^{SS} \\ a(n+\delta)^2 & k > k^{SS} \end{cases}$$

$$\frac{\dot{\pi}}{\pi} = \begin{cases} Ak^{a-1}\left[(1-a)+ak^{-1}\right] & k < k^{SS} \\ 0 & k = k^{SS} \\ (n+\delta) & k > k^{SS} \end{cases}$$

Table 4.2 summarizes the direction of change for both economic growth rates $\frac{\dot{y}}{y}$ and profit growth rates $\frac{\dot{\pi}}{\pi}$ under Markovian Nash equilibrium and cooperative solution.

From table 4.2, three key observations can be made:

1. Below the steady state, the economic growth rates for both the Markovian Nash equilibrium and cooperative solution are negative, whereas the profit growth rate is positive when there is cooperation and negative otherwise.

2. For the cooperative and Markovian Nash solution, the rates of economic growth and profit growth are both zero at the steady state.

3. Above the steady state, both the economic growth rate and the profit growth rate are positive under cooperation.

An interpretation for these observations follows. From an aggregate level, collectivism or cooperation appears to perform better than capitalism. Developed countries operating above the steady state will find the cooperative solution attractive since both economic growth and profit growth will be positive. Developing countries operating below the steady

Nash	$\bar{x}(.)$	$\bar{s}(.)$	$\bar{v}(.)$	$\bar{\pi}(.)$
$k < k^{ss}$	Aak^{a-1}	0	$Ak^a\left(1-a+ak^{-1}\right)-(n+\delta)k$	$(n+\delta)k$
$k = k^{ss}$	Aak^{a-1}	0	$Ak^a\left(1-a+ak^{-1}\right)-(n+\delta)k$	$(n+\delta)k$
$k > k^{ss}$	Aak^{a-1}	0	$Ak^a\left(1-a+ak^{-1}\right)-(n+\delta)k$	$(n+\delta)k$
Cooperative	$\hat{x}(.)$	$\hat{s}(.)$	$\hat{v}(.)$	$\hat{\pi}(.)$
$k < k^{ss}$	$(a-1)Ak^a$	1	$\bar{v}(k)-\frac{1}{2}\left[Aak^{a-1}+(n+\delta)k\right]$	$-\frac{1}{2}Aak^{a-1}-\left(\frac{n+\delta}{2}\right)k$
$k = k^{ss}$	$Aak^{a-1}-\frac{(n+\delta)}{u}k$	u	$\bar{v}(k)-\frac{1}{2}\left[(1-2u)Aak^{a-1}+(n+\delta)k\right]$	$\frac{(u-2)}{2}Aak^{a-1}-\left(\frac{n+\delta}{2}\right)k$
$k > k^{ss}$	Aak^{a-1}	0	$\bar{v}(k)-\frac{1}{2}\left[2Aak^{a-1}+(n+\delta)k\right]$	$-Aak^{a-1}-\left(\frac{n+\delta}{2}\right)k$

where $u \in (0,1]$

Table 4.1: Respective Strategies x, s and Payoffs v, π for the Government and the Firm under Cooperative and Markovian Nash Equilibria

	$sgn\left(\frac{\dot{y}}{y}\right)$		$sgn\left(\frac{\dot{\pi}}{\pi}\right)$	
	Markovian Nash	Cooperative	Markovian Nash	Cooperative
Below k^{ss}	-	-	0	+
At k^{ss}	0	0	0	0
Above k^{ss}	-	+	0	+

Table 4.2: Economic Growth Rates $\frac{\dot{y}}{y}$ and Profit Growth Rates $\frac{\dot{\pi}}{\pi}$ under Markovian Nash Equilibrium and Cooperative Solution

state may be better off cooperating as they will enjoy positive long term economic growth and profit growth once their capital stock exceeds the steady state level. But this requires the workers to sacrifice short term growth and suffer from possible inequity as the firm's profits grow. Will such a sacrifice be rationally acceptable? To answer this question, one must derive the fair imputations for each players and compare these to the Markovian Nash outcomes. This is what we set out to do in the next section.

4.4 Rational Acceptability and Fair Equilibrium

In this GF game of growth, the cooperative solution can be interpreted as a social contract between the voters, the government and the firm, whereby the voters or workers curtail their present consumption for economic growth with the expectation that at some point in the future, the government will ensure a transfer to them for the earlier sacrifice. On the other hand, in the context of a communist country, it can be interpreted as the firm subjecting itself to the collective will. In both cases, the idea is to prevent the dynamic inefficiency of capitalism. But is the cooperative solution, which is derived from a fair imputation of the collective maximization, necessarily better than the Markovian Nash equilibrium?

To answer this question, we apply the concepts of rational acceptability and fair equilibrium which were derived in chapter 3. As defined in chapter 3, rationally acceptable strategies can be defined as the set of cooperative strategies with payoffs greater than or at least equal to the payoffs from pursuing non-cooperative strategies. Assume the planning horizon can be divided into intervals of an arbitrary point τ and players utilize all information up to τ. More formally,

Definition A cooperative strategies pair $(\hat{x}, \hat{s})$ is *rationally acceptable* to both players at initial time t_0 and state k_0 if and only if the continuation of this strategy pair at $(\tau, k(\tau))$ yields payoffs that are greater than or at least equal to the payoffs obtained with the continuation of the Markovian Nash solutions at $(\tau, k(\tau))$ for all possible $(\tau, k(\tau))$ such that $\tau > 0$ and $k(\tau) \geq k_0$ *for all players.*

Substitute the set of cooperative controls $\{\hat{x}(k), \hat{s}(k)\}$ into (4.3) to obtain the dynamics of the optimal cooperative trajectory

$$\dot{k} = g[k, x(k), s(k)] = \hat{s}(k)\left[f'(k) - \hat{x}(k)\right] - (n+\delta)k, \; k(0) = k_0 \qquad (4.26)$$

Let $\hat{k}$ denote the solution to (4.26). Denote $\xi^i(\hat{k})$, $i \in \{G, F\}$ to be an imputation of the payoffs of the ith player.

For the cooperative solution to be rationally acceptable, the following condition is required:

$$\xi^i(\hat{k}) \geq J^i(\hat{k}), \; \forall i \in \{G, F\} \qquad (4.27)$$

Having considered the concept of rational acceptability in the context of the current model, the next step is to establish such a class of fair equilibria for the game, based on that developed in chapter 3.

Definition The fair equilibrium for the game is the pair of strategies $(x^*(\cdot), s^*(\cdot))$, described by:

$$x^*(t, k(t)) = \begin{cases} \hat{x}(t, k_0) & \text{if RA condition is satisfied,} \\ \bar{x}(t, k(t)) & \text{otherwise} \end{cases}$$

and

$$s^*(t,k(t)) = \begin{cases} \hat{s}(t,k_0) & \text{if RA condition is satisfied,} \\ \bar{s}(t,k(t)) & \text{otherwise} \end{cases}$$

Figure 4.1 and 4.2 plot the respective payoffs for the government and the firm under the cooperative and Markovian Nash equilibria. From figures 4.1 and 4.2, it is evident that in this GF game with a neoclassical production technology, there can be no rationally acceptable cooperative outcomes for countries in the initial stage of development or $k < k^{SS}$. For these countries, the cooperative payoff for both the government and the firm is worse than the Markovian Nash outcome. Accordingly, the cooperative solution is rationally unacceptable, hence the fair equilibrium will be equivalent to the Markovian Nash equilibrium. The consequences for the developing countries are vicious cycles of low growth and low capital accumulation and the government ends up with falling political support or political instability.
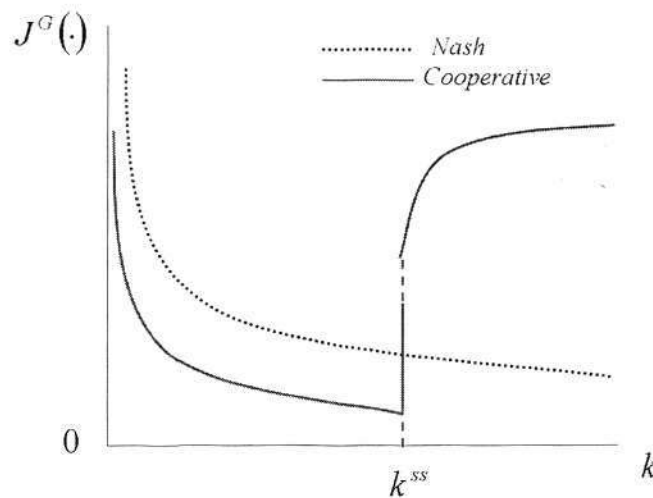

Figure 4.1: Payoffs for the Government under Cooperative and Markovian Nash Equilibria

At a very advanced stage of development,$k > k^{SS}$, the government will find the cooperative solution to be rationally acceptable as its vote payoff increases but the firm will continue

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$J^F(.)$

------ *Nash*
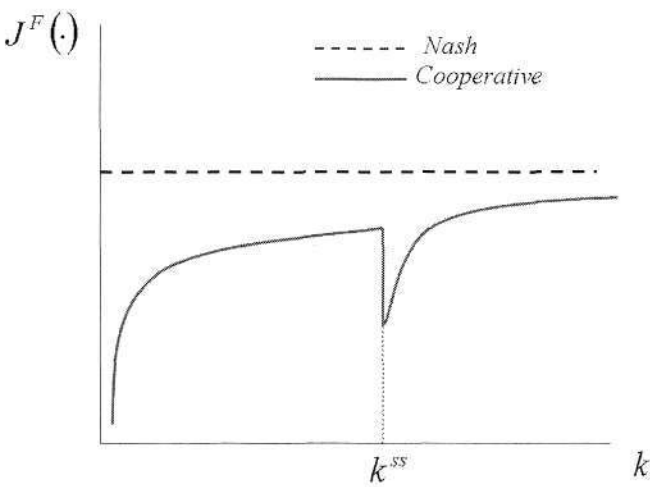
―――― *Cooperative*

k^{ss}

k

Figure 4.2: Payoffs for the Firm under Cooperative and Markovian Nash Equilibria

to find it rationally unacceptable to cooperate because the profits earned by the firm under the Markovian Nash equilibrium still dominate the profits under cooperation. Cooperation breaks down and the Markovian Nash equilibrium is adopted, but the consequence is not so dire as $k < k^{SS}$ because firm can realize positive profit growth which if invested can generate positive future economic growth.

At the baseline steady state k^{SS}, the firm will not cooperate because doing so puts it at a disadvantage compared to its Markovian Nash outcome. The government will find the cooperative solution to be rationally acceptable in some cases and unacceptable in others.

One plausible explanation for these observations is as follows. For the government in a developing country operating below the steady state level, there will always be the politically popular pressure of redistributing the national output, hence the Markovian Nash strategy yields a higher political payoff than the collective solution. On the other hand, for developed countries operating above the steady state level, the populace is fairly well-to-do and a cooperative strategy with liberal tax benefits to the firm is likely to have more political support.

The interesting result in Figure 4.2 indicates that for the firm, the Markovian Nash payoff always dominates the cooperative fair imputation. Intuitively, one would expect cooperation

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to lead to better payoffs than non-cooperation[1], hence the result is both interesting and surprising. In fact, the result is not as counter-intuitive as it appears and admits a plausible explanation. A firm acting independently in a non-cooperative capitalist economy has a strong incentive to produce efficiently to optimize its profits. Such motivations are absent in a cooperative solution where the firm is subsumed under a cooperative or collective arrangement. Moreover, as Hayek(1945) has pointed out, there is also the issue of a firm in a cooperative or nationalized economy being able to amass and process the vast amounts of decentralized information effectively to make the right kind of decisions. Historically, the dismal cooperative result is borne out by the abandonment of nationalized industries by the 1980s, long before the final collapse of communism as a political and an economic ideology in 1989.

Hence, cooperation cannot be attained because the fair equilibrium degenerates easily into the Markovian Nash equilibrium. While group rationality may dictate cooperation, such cooperation is not realized even with fair imputations of the cooperative rewards because these are not rationally acceptable for both the government and the firms. Regardless of the stages of development, the firm always finds it rationally unacceptable to cooperate while the the government only finds cooperative solution to be rationally acceptable when the economy is above the steady state. Below the steady state, developing countries are trapped in low growth and political instability.

These results depart from Lancaster (1973) and Kaitala and Pohjola(1990). Both these papers argue that cooperation can help resolve the dynamic inefficiency of capitalism. The results here demonstrate that such cooperation cannot be rationally acceptable even if fair imputations of the cooperative payoffs are awarded to each players. A fair equilibrium that

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[1] In most games, such as a two-player bargaining problem, the payoff from joint maximization of two players will never be below the sum of the payoffs when these players act non-cooperatively. In other words, cooperation always offers a better payoff than non-cooperation. This is, however, conditional on the symmetry of the payoffs and the convexity of the agreement set, say A. Suppose for two players, there is a status quo point $(d1, d2)$. Convexity assures that these exists some members $(a1, a2)$ in A, such that $a1 > d1$ and $a2 > d2$.

In many other cases such as the one discussed here, the situation is far more complex and cooperation may not necessarily lead to a better payoff than non-cooperation. For instance, suppose the payoffs for two players in a one-shot game are non-symmetric and given respectively by $p1(d1, d2) = g - f$ and $p2(d1, d2) = f - 2g$, where f and g are strictly greater than 0. In this case, the cooperative payoff is given by $-g$. In other words, cooperation can lead to a negative payoff which will result in a fair imputation that is lower than the non-cooperative payoff. The situation is even more complex in a dynamic game where the payoffs can change depending on the evolution of the states and strategies throughout the game.

is cooperative cannot even be attained because of the strong dominance of the Markovian Nash payoffs for the players in most cases. Consequently, in a neoclassical growth model, capitalism may be more rationally acceptable than collectivism or other forms of cooperative solutions.

4.5 Concluding Remarks

Capitalism has prevailed as an institution in promoting economic growth. This chapter argues that capitalism prevails as an institution as it is more rationally acceptable than collectivism.

The chapter develops a dynamic GF game with a vote-maximizing government and profit-maximizing representative firm. In the Markovian Nash equilibrium, a government may postpone redistribution to later stages so as to facilitate the most rapid economic growth. However, the firm predicts as much and being free to optimize on its investment decisions, will stop investing just before profits are being taxed. Since the government is also aware of the firm's reaction, its best response is to impose taxes on capital income earlier. The process will then converge to the Markovian Nash equilibrium, with the government taxing away all the profits of the firm and the firm will eventually stop investing altogether. The political support for the government falls. Thus, in the Markovian Nash equilibrium, the economy in a developing country below the steady state will stop growing eventually because the firms are getting subnormal profits and will not be motivated to invest. This in turn perpetuates a vicious cycle of low capital accumulation level, low growth and political instability. Developed countries which have achieved steady state growth and beyond enjoy normal or supernormal profits and enter into higher levels of investment.

The implication here appears to be that a cooperative or collective solution should be enforced through a "benevolent dictator". This is exactly what Lancaster(1973) and Kaitala and Pohjola(1990) concludes Unfortunately, it is demonstrated in this chapter that a rationally acceptable cooperative or collective solution may not exist in a neoclassical growth model. This is because the firm will always find it rationally unacceptable to cooperate

since the profits earned by the firm under the Markovian Nash equilibrium always dominate the profits under cooperation; the government only finds cooperative solution to be rationally acceptable when the economy is above the steady state. Developing countries are hence trapped in low growth and political instability. Generally, cooperation is not rationally acceptable for both the government and the firm. As a result, a fair equilibrium degenerates easily into the Markovian Nash equilibrium. The significant insight here is that capitalism may be more rationally acceptable and fairer compared to collectivism.

This model also serves to illustrate the importance of rational acceptability in obtaining fair equilibrium. Existing literature takes for granted that cooperative solutions are always preferred to non-cooperative solutions in a static context. In contrast, we use the rational acceptability criteria to demonstrate that the existence of rational acceptable cooperative equilibrium solutions in dynamic games is not trivial. As a result, even if cooperative solutions may trump the non-cooperative solutions in some truncated subgames, the failure of arriving at some rationally acceptable fair imputations of the cooperative outcomes for each players for the overall game may undermine cooperation.

The Markovian Nash solution in which the rent of the firm is totally taxed away is admittedly extreme and unrealistic. In practice, the political pressure to redistribute is always present for both developing and developed countries though it is unlikely that the government will completely tax away the rent of the firm. Our key insight on this issue is that the consequences for taxing the firm are less dire for developed countries than developing countries.

As a caveat, it should be emphasized that this should not be perceived as a carte blanche endorsement of capitalism. As a description of capitalism, the model here is a very simple one in the sense that the labor market is competitive, implying full employment at each instant of time. As a description of collective or cooperative planning, we assume implicitly that the social planner is fully cognizant of the "prices" of capital and labor, whereas such prices may not exist in the absence of a competitive market for a collective economic system. These assumptions are for analytical tractability and little qualitative insights are lost in a more general meodel.

Indeed, the unfairness of capitalism is a real one for developing countries so that it is always easy and tempting to argue for some collective solution. This is the case for China, where embracing capitalism has brought about phenomenal growth and widening disparity, prompting recent debates about bringing about a "harmonious society" through a return to some forms of collectivism. However, if capitalism is more rationally acceptable than collectivism, such collective solutions are bound to fail, as they had in the past. A policy implication would be that policymakers have to work harder to come up with more creative solutions to achieve a "harmonious society" within a capitalist framework rather than imposing rationally unacceptable collective solutions.

Finally, uncertainty does not play a role in the whole analysis. Current differential games of economic growth are all deterministic. Since we are dealing with an infinite-horizon game, the decisions of the individual players may be affected by uncertainty about the future. Moreover, the state variable may not be fully deterministic but may be subjected to stochastic disturbance. How will the results be affected once we introduce uncertainty into the dynamic game? In the next chapter, we explore this question by incorporating the role of uncertainty in the GF game.

Chapter 5

A Stochastic Dynamic Game of

Economic Growth

5.1 Introduction

The GF game in chapter 4 is deterministic. Over an infinite time horizon, however, it is inevitable that the decisions of the individual players may be affected by uncertainty about the future. In this chapter, we explore the consequences of introducing uncertainty into the GF game.

Current differential games of economic growth are all deterministic. Hence, a central contribution of this chapter is that we extend the existing literature to analyze the role of uncertainty in these games. Specifically, we develop a stochastic differential GF game by using methods in stochastic calculus, an area that has been predominantly studied in finance and mostly applied in stochastic control problems (Oksendal, 2003; Chang, 2004). We demonstrate that many conclusions in the deterministic setting can be undermined if stochastic elements are incorporated into the model. We draw on the stochastic Solow equation developed by Merton (1975) and elaborated in Chang and Malliaris (1987). Merton (1975) considers a one-sector neoclassical growth model of the Solow-type where the dynamics of the capital-labour ratio can be described by a diffusion-type stochastic process. The particular source of uncertainty chosen is the population size. Using the Reflection Principle,

Chang and Malliaris (1987) demonstrate the existence and uniqueness of the solution to the classic Solow equation under continuous time uncertainty for the class of strictly concave production functions which are continuously differentiable on the nonnegative real numbers. This class contains all CES functions with elasticity of substitution less than unity. A steady state distribution also exists for this class of production functions with a bounded slope at the origin. A condition on the drift-variance ratio of the stochastic differential equation alone, independent of technology and the savings ratio, is found to be necessary for the existence of a steady state. In contrast to both Merton(1975) and Chang and Malliaris (1987), we analyze a stochastic dynamic game in continuous time. The characterization of the solutions in such games is generally problematic, especially in the cooperative case. As noted by Jorgensen and Zaccour (2002), conditions for ensuring time consistency of stochastic cooperative solutions are generally strict and intractable. More recently, Yeung and Petrosyan (2004, 2006) develop a generalized theorem for the derivation of an analytically tractable payoff distribution procedure (PDP) that leads to subgame-consistent solutions in stochastic games, thus enabling the hitherto intractable problems in stochastic cooperative games to be fruitfully studied.

In this chapter, we derive the stochastic Markovian Nash equilibrium for the stochastic GF game and determine the conditions under which cooperative solutions can fulfill the RA condition. Under the stochastic Markovian Nash equilibrium, the government will tax less than the full amount of the rent accrued to the firm, which will post a positive rate of investment. The rate of investment depends on not only the capital-labor ratio but also the discount rate, the depreciation rate, the population growth rate and uncertainty. This is significantly different from the Markovian Nash equilibrium obtained under deterministic conditions in chapter 4. In the deterministic case, the rent of the firm is completely taxed away and the firm stops investing completely, which is a very extreme and unrealistic solution. Introducing uncertainty into the model thus produces a solution that is less extreme and hence more realistic.

Although it is not possible to determine the cooperative solutions from the model, we are able to prove that the cooperative solutions is always non-inferior to the Markovian feedback

Nash equilibrium or that the RA condition is strongly satisfied.

The rest of this chapter is organized as follows. In section 5.2, we derive the stochastic capital accumulation equation and formulate the stochastic GF game of economic growth. The stochastic Markovian Nash equilibrium is derived in Section 5.3. Section 5.4 discusses the rational acceptability of the stochastic cooperative solution. Finally, Section 5.4 presents the concluding remarks.

5.2 The Stochastic Government-Firm (GF) Dynamic Game

In a stochastic dynamic game, the state variable may not be fully deterministic. Instead, it may be subjected to stochastic disturbance. As a result, the optimal control must be stated in Markovian form, in terms of the state since the state obtained cannot be known in advance. Therefore, instead of the usual differential equations, we have the state trajectory represented by the following Ito stochastic differential equation:

$$dk = \left[s\left(f'(k) - x \right) - \left(\delta + n - \sigma^2 \right) k \right] dt + \sigma(k, x, s) dW,$$

where dW is the increment of a Wiener (white noise) process z while $\sigma(k, x, s) = \sigma k$ can be interpreted as the uncertainty associated with the state. The derivation of this stochastic capital accumulation equation is not so straightforward as the usual differential equation.

5.2.1 The Stochastic Capital Accumulation Equation

In the stochastic case, the capital accumulation equation is given by

$$dK = \left[s\left(f'(k)L^{-1} - X \right) - \delta K \right] dt,$$

where the production function $F(K, L)$ is homogenous of degree one in K and δ is the depreciation rate and X denote the transfer.

Assume population dynamics follows a geometric Brownian motion with expected rate

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n and instantaneous variance σ:

$$dL = nLdt + \sigma LdW.$$

If the initial condition $L(0) > 0$ is given, then

$$L(t) = L(0)exp\left\{(n - \sigma^2/2)t\right\} exp\left\{\sigma W(t).\right\}$$

The reason for modeling the population dynamics as a geometric Brownian motion is that in doing so, the size of the population is positive with probability 1 at all times (as should be the case) even though the Wiener process $W(t)$ assumes unbounded negative values with positive probability.

We can treat capital-labor ratio $k = K/L$ as a function of K and L, i.e. $k = h(K,L) = K/L$. Since

$$h_K = \frac{1}{L} \qquad h_{KK} = 0$$

$$h_L = -K\left(\frac{1}{L}\right)^2 \qquad h_{LL} = 2K\left(\frac{1}{L}\right)^3$$

$$h_{KL} = -\left(\frac{1}{L}\right)^2.$$

Applying the multivariate Ito's lemma (Oksendal, 2003) , we obtain

$$dk = h_K dK + h_L dL + \frac{1}{2}h_{KK}(dK)^2 + h_{KL}(dK)(dL) + \frac{1}{2}h_{LL}(dL)^2. \qquad (5.1)$$

Substituting the various terms obtained earlier into (5.1),

$$dk = \frac{1}{L}dK - \left(\frac{K}{L^2}\right)dL - \left(\frac{(dK)(dL)}{L^2}\right) + \left(\frac{K}{L^3}\right)(dL)^2.$$

Since $dK = \left[s\left(f'(k)L^{-1} - X\right) - \delta K\right]dt$ and $dL = nLdt + \sigma LdW$,

$$dk = \left[s\left(f'(k) - x\right) - \delta k\right]dt - \frac{K}{L}\frac{dL}{L} + \frac{K}{L}\left(\frac{dL}{L}\right)^2 - \left(\frac{(dK)(dL)}{L^2}\right).$$

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It can be easily shown that $\left(\frac{dL}{L}\right)^2 = \sigma^2 dt$. Further, as $(dK)(dL) = o(dt)$ and $f(k) = F(K/L, 1)$, the state trajectory can be simplified to the Ito stochastic differential equation

$$dk = \left[s\left(f'(k) - x\right) - \left(\delta + n - \sigma^2\right)k\right]dt - \sigma k dW.$$

Remark When $\sigma = 0$,

$$
\begin{aligned}
dk &= \left[s\left(f'(k) - x\right) - (\delta + n)k\right]dt, \\
\dot{k} &= s\left(f'(k) - x\right) - (\delta + n)k,
\end{aligned}
$$

which is similar to the deterministic state trajectory.

Next, we formulate the stochastic government-firm (GF) dynamic game.

5.2.2 The Stochastic Government-Firm (GF) Dynamic Game

The stochastic GF dynamic game, $\Gamma(k, \sigma)$, can be characterized as follows:

Government

$$J^G\left(k_0, x(\cdot), s(\cdot)\right) = \max_{x(.)} E \int_0^\infty e^{-\rho t}\left[f(k) - kf'(k) + x\right]dt, \tag{5.2}$$

Firm

$$J^F\left(k_0, x(\cdot), s(\cdot)\right) = \max_{s(.)} E \int_0^\infty e^{-\rho t}(1-s)\left[f'(k) - x\right]dt, \tag{5.3}$$

subject to

$$
\begin{aligned}
dk &= \left[s\left(f'(k) - x\right) - \left(\delta + n - \sigma^2\right)k\right]dt + \sigma(k, x, s)dW, & (5.4) \\
k(0) &= k_0, & (5.5) \\
\sigma(k, x, s) &= -\sigma k, & (5.6) \\
0 \leq s &\leq 1, & (5.7) \\
-f(k) + kf'(k) \leq x &\leq f'(k). & (5.8)
\end{aligned}
$$

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where dW is the increment of a Wiener (white noise) process z while $\sigma(k,x,s) = \sigma k$ can be interpreted as the uncertainty associated with the state.

5.3 Stochastic Markovian Nash Equilibrium (SMNE)

In this section, we define and derive the stochastic Markovian Nash equilibrium for the stochastic GF game $\Gamma(k,\sigma)$.

Definition A set of strategies $\{\bar{x}, \bar{s}\}$ constitutes a *stochastic Markovian Nash equilibrium* (SMNE) if there exist functionals $J^i(k)$, $i \in \{G, F\}$, which satisfy the following set of stochastic partial differential equations

$$-J_t^G(k) - \frac{1}{2}\sigma^2 k^2 J_{kk}^G(k) = \max_{x(.)} \left\{ e^{-\rho t} \left[f(k) - kf'(k) + x \right] + J_k^G \left[\bar{s}\left(f'(k) - x\right) - \left(\delta + n - \sigma^2\right)k\right] \right\},$$
$$(5.9)$$

$$-J_t^F(k) - \frac{1}{2}\sigma^2 k^2 J_{kk}^F(k) = \max_{s(.)} \left\{ e^{-\rho t} \left[(1-s)\left[f'(k) - \bar{x}\right] \right] + J_k^F \left[s\left(f'(k) - \bar{x}\right) - \left(\delta + n - \sigma^2\right)k\right] \right\}.$$
$$(5.10)$$

The controls $\{\bar{x}, \bar{s}\}$ in the SMNE are Markovian in the sense that they are functions of current time and current state.

Respective maximizations of the equation (5.9) with respect to $x(.)$ and the equation (5.10) with respect to $s(.)$ yield

$$J_k^G = e^{-\rho t}\frac{1}{\bar{s}},$$

and

$$J_k^F = e^{-\rho t}.$$

These values can be substituted back into the respective equations (5.9) and (5.10) to obtain

$$-J_t^G(k) - \frac{1}{2}\sigma^2 k^2 J_{kk}^G(k) = e^{-\rho t}\left\{ f(k) - kf'(k) + f'(k) - \frac{k}{\bar{s}}\left(\delta + n - \sigma^2\right)\right\},$$

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$$-J_t^F(k) - \frac{1}{2}\sigma^2 k^2 J_{kk}^F(k) = e^{-\rho t}\left\{f'(k) - \bar{x} - (\delta+n-\sigma^2)k\right\}.$$

We can try a solution to each partial differential equation.

For the government, let $J^G(k,t) = e^{-\rho t}Ak^2$. Hence, $J_t^G(k,t) = -\rho e^{-\rho t}Ak^2, J_k^G(k,t) = e^{-\rho t}2Ak$ and $J_{kk}^G(k,t) = e^{-\rho t}2A$

Substitute these into (5.9) and solve for A, which is given by

$$A = \frac{f(k) - kf'(k) + f'(k) - \frac{k}{\bar{s}}(\delta+n-\sigma^2)}{(\rho-\sigma^2)k^2}.$$

Thus,

$$J^G(k,t) = e^{-\rho t}\frac{f(k) - kf'(k) + f'(k) - \frac{k}{\bar{s}}(\delta+n-\sigma^2)}{\rho-\sigma^2}. \tag{5.11}$$

In the case of the firm, we try the solution $J^F(k,t) = e^{-\rho t}Bk^2$.

Thus, $J_t^F(k,t) = -\rho e^{-\rho t}Bk^2, J_k^F(k,t) = e^{-\rho t}2Bk$ and $J_{kk}^F(k,t) = e^{-\rho t}2B$. Substitute these back into the firm's partial differential equation (5.10) and solving for B,

$$B = \frac{f'(k) - \bar{x} - (\delta+n-\sigma^2)k}{(\rho-\sigma^2)k^2}.$$

It follows that

$$J^F(k,t) = e^{-\rho t}\frac{f'(k) - \bar{x} - (\delta+n-\sigma^2)k}{(\rho-\sigma^2)}. \tag{5.12}$$

Proposition 5.3.1 *The SMNE for the firm is given by*

$$\bar{s} = \frac{k(\rho+2\delta+2n-3\sigma^2)}{2\left[f(k) - kf'(k) + f'(k)\right]}. \tag{5.13}$$

s is non-negative and also depends on not only k but the discount rate ρ, depreciation rate δ, population growth rate n and uncertainty σ^2.

Proof Earlier we establish that $\bar{s} = e^{-\rho t}\frac{1}{J_k^G} = \frac{1}{2Ak}$. From this, obtaining the SMNE for the firm is straightforward. ∎

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Lemma 5.3.2 $\bar{s}$ *is non-negative if the following condition holds:*

$$2\sigma^2 - (\rho + \delta + n) \le \delta + n - \sigma^2 \le \frac{2}{k}\left[f(k) - kf'(k) + f'(k)\right] + 2\sigma^2 - (\rho + \delta + n). \quad (5.14)$$

Proof We know that the investment rate s must satisfy the constraint $0 \le \bar{s} \le 1$. Therefore, it follows from the SMNE for the firm and this constraint that

$$3\sigma^2 \le \rho + 2\delta + 2n \le \frac{2}{k}\left[f(k) - kf'(k) + f'(k)\right] + 3\sigma^2.$$

Rearranging this condition, we obtain (5.14).   ∎

$\delta + n - \sigma^2$ can be interpreted as the replacement rate for the breakeven investment $(\delta + n - \sigma^2)k$, the amount of investment must be done to keep k at its existing level. This arises because of the depreciation of existing capital δk, the need to keep up with the rate of growth of effective capital nk and the uncertainty associated with the breakeven investment $\sigma^2 k$. The following lemma follows easily.

Lemma 5.3.3 *If replacement rate $\delta + n - \sigma^2$ for the breakeven investment $(\delta + n - \sigma^2)k$ is strictly positive, then*

$$\sigma^2 - \rho > \delta + n - \sigma^2. \quad (5.15)$$

Proposition 5.3.4 *The SMNE for the government is*

$$\bar{x} = f'(k) - \left(\delta + n + \frac{1}{2}\rho - \frac{1}{2}\sigma^2\right)k. \quad (5.16)$$

Thus, with uncertainty, the SMNE solution for the government will be to tax less than the full amount of the rent accrued to the firm.

Proof Since $J_k^F = e^{-\rho t} = e^{-\rho t}2Bk$, $B = \frac{1}{2k}$. From $\rho Bk^2 - \sigma^2 k^2 B = f'(k) - \bar{x} - (\delta + n - \sigma^2)k$, we obtain the SMNE of the government.   ∎

Proposition 5.3.5 *If $\delta + n - \sigma^2 > -(\delta + n + \rho)$, the government will tax less than the full amount of the rent accrued to the firm.*

Proof Under the SMNE, the government will tax less than the full amount of the firm's rent $f'(k)$ if $2\delta + 2n + \rho > \sigma^2$. Rearranging this condition, $\delta + n - \sigma^2 > -(\delta + n + \rho)$. ∎

5.4 Stochastic Cooperative Game

Before we can consider the cooperative solution and compare this to the SMNE, we must formulate the cooperative analogue to the stochastic GF game $\Gamma(k, \sigma)$.

We denote the stochastic cooperative GF dynamic game as $\Gamma^c(k, \sigma)$, which is characterized as follows:

$$J^C(k_0, x(\cdot), s(\cdot)) = \max_{x(.),s(.)} E \int_0^\infty e^{-\rho t} \left[f(k) - kf'(k) + x + (1-s)\left(f'(k) - x\right) \right] dt, \quad (5.17)$$

subject to the constraints (5.4)-(5.8)

Definition A set of strategies $\{\hat{x}, \hat{s}\}$ constitutes a *stochastic Markovian cooperative solution*(SMCS) if there exists a functional $J^C(k)$ which satisfies the following stochastic partial differential equation

$$-J_t^C(k) - \frac{1}{2}\sigma^2 k^2 J_{kk}^C(k) =$$
$$\max_{s(.),x(.)} \left\{ e^{-\rho t}\left[f(k) - kf'(k) + x + (1-s)\left(f'(k) - x\right)\right] + J_k^C\left[s\left(f'(k) - x\right) - \left(\delta + n - \sigma^2\right)k\right] \right\}.$$
$$(5.18)$$

Maximization of this partial differential equation with respect to $\hat{x}$ or $\hat{s}$ will produce the result

$$J_k^C = e^{-\rho t}.$$

Substitute this back into (5.18) to obtain the partial differential equation

$$-J_t^C(k) - \frac{1}{2}\sigma^2 k^2 J_{kk}^C(k) = e^{-\rho t}\left\{f(k) - kf'(k) + f'(k) - \left(\delta + n - \sigma^2\right)k\right\} \quad (5.19)$$

We can try a solution $J^C(k,t) = e^{-\rho t} C k^2$ to (5.19). Hence, $J_t^C(k,t) = -\rho e^{-\rho t} C k^2$, $J_k^C(k,t) = e^{-\rho t} 2Ck$ and $J_{kk}^C(k,t) = e^{-\rho t} 2C$. Substituting these back into the partial differential equation

(5.19) and solving for C,

$$C = \frac{f(k) - kf'(k) + f'(k) - (\delta + n - \sigma^2)k}{(\rho - \sigma^2)k^2}.$$

Hence,

$$J^C(k,t) = e^{-\rho t}\frac{f(k) - kf'(k) + f'(k) - (\delta + n - \sigma^2)k}{(\rho - \sigma^2)}. \qquad (5.20)$$

It is not possible to determine $\{\hat{x}, \hat{s}\}$ from the set of conditions above. But we can consider whether the RA condition can be satisfied. Specifically, we wish to find out if $J^F(k,t) + J^G(k,t) < J^C(k,t)$? From (5.11) and (5.12),

$$J^F(k,t) + J^G(k,t) =$$
$$\frac{e^{-\rho t}}{(\rho - \sigma^2)}\left[\left(1 - \frac{2(\delta + n - \sigma^2)}{(\rho + 2\delta + 2n - 3\sigma^2)}\right)\left[f(k) - kf'(k) + f'(k)\right] - \left(2\delta + 2n + \frac{1}{2}\rho - \frac{3}{2}\sigma^2\right)k\right].$$

Lemma 5.4.1 *The RA condition is strongly satisfied for the stochastic GF game $\Gamma(k,\sigma)$.*

Proof Comparing both $J^F(k,t) + J^G(k,t)$ and $J^C(k,t)$, we note that the cooperative solution will dominate the stochastic Markovian Nash equilibrium if the following conditions are satisfied:

$$\left(1 - \frac{2(\delta + n - \sigma^2)}{(\rho + 2\delta + 2n - 3\sigma^2)}\right) < 1,$$

and

$$\delta + n - \sigma^2 < 2\delta + 2n + \frac{1}{2}\rho - \frac{3}{2}\sigma^2.$$

These conditions simplify to the following inequalities:

$$\delta + n - \sigma^2 > 0,$$
$$\delta + n - \sigma^2 > -\frac{1}{2}(\sigma^2 + \rho).$$

But $(\sigma^2 + \rho) > 0$, so that a strong RA condition would be given by $\delta + n - \sigma^2 > 0$.

Next, the condition $\delta + n - \sigma^2 > 0$ should be true for the stochastic GF game $\Gamma(k,\sigma)$. To

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see why this so, recall that the state trajectory is given by

$$dk = \left[s \left(f'(k) - x \right) - \left(\delta + n - \sigma^2 \right) k \right] dt + \sigma(k,x,s) dW.$$

If $\delta + n - \sigma^2 < 0$, the per-capita capital stock k will increase out-of-bound for any possible saving rate $s \in [0,1]$ as time $t \to \infty$ Hence, a stationary state does not exist. A solution to the infinite horizon autonomous optimization problem cannot be obtained if there does not exist a stationary state in the dynamics of the state variable.

Therefore, the RA condition is strongly satisfied for the for the stochastic GF game $\Gamma(k,\sigma)$. ∎

The economic intuition for this condition is that the rationally acceptable cooperative solution is always non-inferior to the non-cooperative stochastic Markovian Nash equilibrium.

5.5 Concluding Remarks

In this chapter, we extend the analysis in chapter 4 by studying the GF game in a stochastic setting. In the deterministic GF game of that chapter, the rent of the firm is completely taxed away and the firm stops investing completely, which is a very extreme and unrealistic solution. In contrast, in the stochastic Markovian Nash equilibrium, the government will tax less than the full amount of the rent accrued to the firm, which will post a positive rate of investment while the rate of investment depends on not only the capital-labor ratio but also the discount rate, the depreciation rate, the population growth rate and uncertainty. Although the cooperative solution is indeterminate from the model, we are able to prove that the RA condition is strongly satisfied. Hence, the rationally acceptable cooperative solution is always non-inferior to the non-cooperative stochastic Markovian Nash equilibrium.

A key assumption in chapter 4 and 5 is that both the government and the firm represent the interests of their respective principals effectively. The government in the model depends very much on voters (workers) for political support while the firm depends on their shareholders for support. As such, the government is an agent for the voters (workers) while the firm is an agent for their shareholders. How do the principals ensure that their respective

agents will exert efforts to represent their interests? In this principal-agent scenario, the rational acceptability of the outcome may be contingent on the performance of the government and the firm as perceived by their principals. Effectively, this would be a multi-principals-agents dynamic game. Establishing rationally acceptable outcomes for players with different objective functionals and possibly different state equations is challenging. The next chapter attempts to deal with the question of aligning the performance of the government with the interests of the workers under different types of political regimes.

Chapter 6

Reputational Effects in Fairness and Political Economy

6.1 Introduction

In the model of chapter 4, the government is assumed to be a vote-maximizer which depends on the political support of the workers. An important assumption in that model is that there exists an alignment of interest between the government and the workers. Such an alignment of interest may be valid in a democracy but may not be so in authoritarian societies. There are numerous cases in which governments (often authoritarian and communist) promise to fight for the interests of the workers and end up hurting them instead. In this chapter, we probe deeper into this relationship.

Representative governments, such as those found in democracy, serve the interests of their constituents. In contrast, authoritarian governments, such as those found in communist countries, may purport to fight for the interests of the workers and the people but have neither a mandate nor electoral accountability. In the latter case, the autocratic political structure does not confer any incentive for the ruler to act in the interests of the ruled. Hence, it is surprising therefore that many authoritarian government actually turn out stellar performances which often led to their eventual demise. Political economists, such as Przeworski et al (2000), have also noted that

1. Of regimes that grow at an average rate of 7% per year for at least 10 years between 1950 and 1990, all were authoritarian (except for the Bahamas). But 8 out of 10 countries with the lowest growth rates over a ten-year growth period were also authoritarian.

2. Of authoritarian countries that are economic successes, many consolidate their political authority through economic successes and remain authoritarian. Thus, authoritarian China remains authoritarian after decades of economic growth. China is the paradigm for countries like Vietnam which followed economic, legal and political reforms in China and modeled its foreign investment regime on China's. Laos, a single party socialist state has similarly pursued market reforms in the 1980s but remains authoritarian.

3. Of authoritarian countries that are economic disasters, many are still authoritarian. Ninety-six percent of countries with per capita income under $1000 are dictatorships.

In this chapter, we attempt to answer the interesting question: how can an authoritarian government, such as the Chinese Communist government, be relied on to exert a high level of effort to ensure good economic outcomes for its citizens or workers despite the lack of an inherent incentive to do so given the autocratic political system? The critical issue is how the same set of authoritarian institutions can shape different economic growth outcomes, with some authoritarians producing economic miracles while others war, misery and famine. This is an issue of considerable interests to economists concerned with how institutions influence economic growth (Olson, 1993; Niskanen, 1997, Acemoglu and Robinson, 2000). We argue that the concern for reputation is the key to understanding why some authoritarian governments manage to achieve stellar economic performance while others economic disasters.

The essence of our arguments is as follows: like democracy, the existence and survival of non-democracy depend on the political support of the citizens. The non-democratic regime may be competent or inept. The reputation of the government is defined as the citizens' posterior expectation that a government is competent. A competent non-democratic government may exert a high or low governance effort while inept non-democracy only exerts

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low governance effort. If an inept non-democratic regime can be replaced, competent government will exert high effort only if the cost of foregone rent is not too large. If an inept regime cannot be replaced, even competent government may end up exerting low effort.

Replacements are not the only mechanisms by which incentives for high effort by the government can be ensured and sustained. We also consider the case of democratic electoral competition. In these electoral competitions, voters can stop supporting a party and impose a high significant cost at the polls after any reduction in beliefs in the ability of the government to deliver high governance effort.

The analysis of authoritarianism in this chapter contributes to the literature on political economy and political institutions. To date, most of these analyses have focused exclusively on democratic regimes. However, based on the POLITY IV (2000) data, nodemocratic regimes rule the majority of countries and the majority of the world's population over the last two centuries until 1991 (Mulligan, Gil and Sala-i-Martin, 2004). Since 1991, more than 40 percent of countries and people were ruled by nondemocratic regimes . Hence, a political economy theory devoted exclusively to democratic institutions is not complete without a theory of authoritarianism.

Our chapter is related to and builds on the literature on game-theoretic literature on reputation effects as pioneered by Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Levine (1989) and Kreps (1990). The general result of this literature is that reputation enhances commitment power by leaving a long-lived agent at least as well-off as he would be in the complete absence of external incentives but typically raises long-run payoffs, often to the agent's first-best. Reputation effects find many recent applications in the theory of firms, such as Mailath and Samuelson (2001), Tadelis (2002) and Ely and Valimaki (2003) and in macroeconomic models of monetary policies (see for example, Drazen, 2000 and Persson and Tabellini, 2000). To the best of our knowledge, however, this is the first application of reputation effects in political economy of economic growth. Although the results here are related to this literature, it is important to point out a fundamental difference. Essentially, this difference arises from the tradeability of reputations for a firm and a government respectively. As noted in Kreps (1990), a firm's reputation is a tradeable asset

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and recent literature centers on the conditions that guarantee long term incentives through an active market for reputation. In contrast, it is assumed here that reputation is not tradeable in the case of government. This is a reasonable assumption since a political entity cannot be separate from its identity or political ideology.

The rest of this chapter is organized as follows. Section 5.2 presents the reputational model of an authoritarian government. Two cases are considered: the case in which an authoritarian government can be replaced and the contrasting case of an authoritarian government which cannot be replaced. Section 5.3 extends the discussion by presenting electoral competition as an alternative mechanism to replacement to ensure sustainability of high efforts by a government. Finally, section 5.4 concludes. Longer proofs of the propositions are collected in the appendix.

6.2 A Reputational Model of An Authoritarian Government

An authoritarian government is an agent for the citizens, who are the principals. The small and anonymous uninformed citizens receive idiosyncratic signals and respond continuously to changes in their beliefs. These signals are independent of their actions. In other words, the citizens or workers cannot vote out an authoritarian government, no matter how pessimistic they might be concerning the government's type and effort level.

Formally, consider an infinitely-lived authoritarian government facing a continuum of small, anonymous long-lived citizens, indexed by $i \in [0,1]$. At the beginning of every period t, voters assign a probability $\mu_{\bar{z},t}$ that the government is competent and derive a utility v_t from the governance outcome. The government chooses an effort level $x_t \in [L,H]$, where L and H denote low effort and high effort respectively. In return, it receives a political support payoff equivalent to the utility derived by the voter. Voters and the government then observe the realized political support level and update beliefs about the type of government and hence their expected political support. The government maximizes the discounted sum of expected political support or "vote", with discount factor, δ. There are two types of

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government, which for simplicity, can be termed as "good" and "bad" or synonymously, competent and inept. The quality of the authoritarian government is not immutably fixed but evolves according to a Markov process. Every government would like to avoid being labeled as a "bad" government. An inept government can only choose low effort. Each citizen observes a noisy signal of the government's performance with two possible values, $\bar{z}$(competent) and $\underline{z}$(inept), with marginal distribution

$$f^i(\bar{z}|x) = \begin{cases} \beta_H & x = H, \\ \beta_L & x = L, \end{cases}$$

where $0 < \beta_L < \beta_H < 1$.

The aggregate distribution of the signals received by voters in any period is perfectly informative about the government's effort choice in that period. Hence, citizens only need to observe the fraction of good signals to infer the government's effort though they observe neither the aggregate distribution nor the signal of any other citizen.

The government's payoff in terms of political support is the difference between its political support and its costs in the stage game. There is no cost in not making any effort but making an effort will involve a cost of c. This cost may be interpreted as the opportunity cost of effort expended or more specifically the rent foregone. Naturally, this makes our government a rent-seeker rather than a benevolent dictator, a characterization consistent with the earlier chapters. It follows that an inept government which expends low effort will retain the bulk of the political rent. The political support function or vote function will depend on citizens' expectation about the effort level. Voter expectations are given by a distribution F, with $F(p)$ being the proportion of citizens who expect the government to exert a high effort level with probability less than or equal to p. To simplify, assume that citizens receive a payoff of 1 from $\bar{z}$ and 0 from $\underline{z}$.

Suppose that $\mathcal{F}$ on $[0, 1]$ is the set of possible distribution functions describing a citizen's expectations. The government political support, as a function of $F \in \mathcal{F}$, is defined by $v : \mathcal{F} \to \mathbb{R}$.

Assume further that v is strictly increasing: higher expectations of high effort leads to

higher political support. More formally,

$$F' \succ F \Rightarrow v\left(F'\right) > v(F)$$

where $\succ$ denotes strict first-order stochastic dominance. Additionally, assume that $v\left(F^n\right) \to v(F)$ for all sequences $v(F)$ converging weakly to F.

Let $v(1)$ and $v(0)$ denote the net vote of the government in the special case in which every citizen expects high effort with probability 1 and 0 respectively. To ensure that high effort is the efficient choice, assume that $\beta_H - \beta_L > c$. Furthermore, $v(1) - v(0) > c$ will make H the pure Stackelberg strategy for the government.

Before the game begins, let nature determines the original type of government, with probability $\mu_{\bar{z},0}$ that the government is competent and probability $\mu_{\underline{z},0}$ that the government is inept. It is trivial that $\mu_{\underline{z},0} = 1 - \mu_{\bar{z},0}$. In each subsequent period, there is a probability λ that the government is replaced and a probability θ that the new government is good or competent.

λ can be interpreted as the survivor probability of the incumbent or the probability that an exogenous change in institutions, such as (informal) term limits or a takeover by a rival faction within the same authoritarian machinery or a revolution, occurs resulting in an existing government leaving the political scene, to be replaced by a new government. In this interpretation, the government's effective discount factor is $\delta(1 - \lambda)$ and the government is concerned only with payoffs in votes conditional on not being replaced at the hustings. Alternatively, one can interpret replacement as a change in characteristics of a continuing government in which case the appropriate discount factor is δ and the government's expected payoff would include flow payoffs received after having being replaced. However, since the government cannot affect the replacement probability, the two formulations should yield similar results.

Replacement introduces contestability into the authoritarian political system without electoral contest. This is analagous to contestable market. The latent possibility of replacement results in a competent authoritarian government performing very much like one with electoral competition, providing it sufficient motivation to expend effort for the interest

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of the citizens.

On the other hand, θ introduces the perpetual possibility that a competent government may be replaced by an inept government. This arises naturally in the course of time if a competent government becomes complacent and eventually corrupts itself.

At the start of period t, each citizen i has a posterior probability that the government is good and another posterior probability that the government will exert a high effort, denoted β. A good government will make its policy choice and gains votes which depend on the distribution F_t of consumers' beliefs about the government's effort, but not on the government's type or action in that period. Voters independently observes their signals and update their beliefs about the type of government.
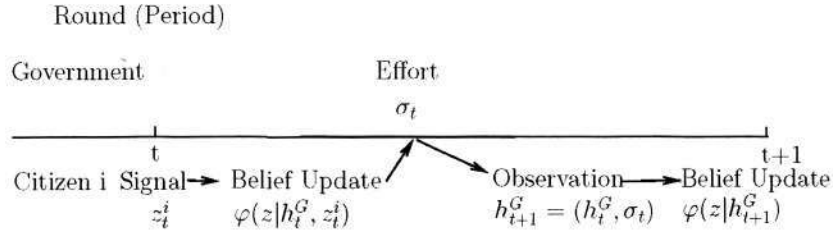
Round (Period)

Figure 6.1: Posterior Updating of Beliefs

Definition A period t history for citizens is a t-tuple of signals, $h_t^i \in [\bar{z}, \underline{z}] \equiv \mathcal{H}_t^i$, which describes the payoffs citizen i received from period 0 to $t-1$. The set of all citizen histories is $\mathcal{H}^i = \cup_t \mathcal{H}_t^i$.

Definition A *belief function* for citizen i is a function $\beta : \mathcal{H}^i \rightarrow [0,1]$, where $\beta^i\left(\mathcal{H}^i\right)$ is the probability assigned by citizen i to the government exercising high effort in period t, given history h_t^i.

Every history of signals has a positive probability under any sequence of strategy choices of the government. Moreover, assume that citizens apply the Bayes' rule and start with a common prior. Hence, any two citizens observing the same sequence of signals can be expected to entertain the same beliefs about the government's behavior. More specifically, $\beta_j\left(h_t^i\right) = \beta_k\left(h_t^i\right)$ for all $h_t^i \in \mathcal{H}^i$ and all $j, k \in [0,1]$. Accordingly, citizens' beliefs can be described by a single function $\beta : \mathcal{H}^i \rightarrow [0,1]$

Definition A period t history h^G for the government can be taken as the t-tuple of realized strategy choices, $h_t^G \in \{L,H\}_t \equiv \mathcal{H}^G$ describing the strategy choices made in periods 0 through $t-1$. The set of all possible government histories is given by $\mathcal{H}_G = \cup_t \mathcal{H}_t^G$.

Definition A pure strategy for a good government is a strategy, $\sigma : \mathcal{H} \to \{L,H\}$, giving the policy choice after the history h_t is observed.

If $\lambda > 0$, it is definite that there will be an infinite number of replacement events, infinitely many of which will introduce new government into the game. But the description of histories ignores such replacement events. By restricting attention to government histories in $\mathcal{H}$, a new good government, entering after the policy history, h_t, behaves exactly as an existing good government after the same history. Such restriction, naturally, rules out some equilibria. However, any equilibrium under this assumption will also be an equilibrium without it. Hence, a strategy, σ, is referred as the good government's strategy, although it describes the behavior of all new good governments as well.

Definition The pair (σ, β) is an equilibrium if $\sigma(h_t)$ maximizes the votes for competent governments after every policy history $h_t \in \mathcal{H}$ and citizens' beliefs about policy choice, β are correctly informed by Bayes' rule.

Generally, it is not possible to offer a precise and general definition of such an equilibrium because the government may choose a mixed strategy or the replacements to the government may be either competent or inept. In such cases, a random sequence of policy is generated. Because the government's strategy requires different policy choices after different policy histories, citizens must base their posterior over the government's policy histories based on their outcome histories. Naturally, such an updating process is very complicated. For instance, the posterior probability may be assigned by the citizens to the government being competent is not necessarily a sufficient statistic for their individual history of outcomes. The equilibrium in this definition, however, assumes the competent government employs the same equilibrium policy after any realized policy-level history, thus implying that the citizens' posterior belief of the government's efficacy is a sufficient statistic for their individual outcome history.

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6.2.1 With Government Replacement

This section examines a pure strategy equilibrium in which the competent government always opt for a policy of high effort.

Denote μ as the prior probability that the government is competent and that the competent government chooses a high effort policy. Let μ_z be the posterior probability that the government is competent or not, after the vote has received a single signal, $z \in \{\underline{z}, \bar{z}\}$ given :

$$\mu_{\bar{z}} \equiv \varphi(\mu|\bar{z}) = (1-\lambda)\frac{\beta_H\mu}{\beta_H\mu + \beta_L(1-\mu)} + \lambda\theta$$

and

$$\mu_{\underline{z}} \equiv \varphi(\mu|\underline{z}) = (1-\lambda)\frac{(1-\beta_H)\mu}{(1-\beta_H)\mu + (1-\beta_L)(1-\mu)} + \lambda\theta$$

Definition A strategy profile (σ, β) is a *high effort equilibrium profile* if

1. $\sigma\left(h_t^i\right) = H$ is vote maximising for the competent government, $\forall\ h_t^i \in \mathcal{H}^i$, given β, and

2. $\beta\left(h_t^G\right) = \varphi\left(\mu|h_t^G\right)\ \forall\ h_t^G \in \mathcal{H}^G$

Since the only off-the-equilibrium-path information sets are those of the competent government, a high-effort equilibrium is trivially sequential.

Definition A strategy profile (σ, β) is a *low effort equilibrium profile* if

1. $\sigma\left(h_t^i\right) = L$ is vote maximising for the competent government, $\forall\ h_t^i \in \mathcal{H}^i$, given β, and

2. $\beta\left(h_t^G\right) = 0\ \forall\ h_t^G \in \mathcal{H}^G$

In this profile, the citizens never expect high effort from the government. Hence, the signals are uninformative and the competent government has no incentive to perform transfers. The low-effort equilibrium profile is thus a sequential equilibrium for all costs of efforts and all discount factors.

Proposition 6.2.1 *Given $\lambda \in (0, 1)$, there exists a cost $\bar{c}$ such that a high effort equilibrium can be found for all costs $c \in [0, \bar{c})$.*

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Proof See appendix

It can be expected that there exists a high-effort equilibrium only if the cost of high effort, c, is not too large. The upper bound for c being not too large is nonzero only if there is a positive probability that an inept government will be replaced, or more formally, $\lambda(1-\theta)$. The value functions induced by high and low effort approach each other as the posterior probability of competent government $\mu_{\bar{z}}$ approaches 1. The values diverge only through the effect of current outcomes on future expectations and current outcomes have very marginal effect on future expectations if citizens are currently quite confident of the government's type. If there is a possibility of replacing an inept government, the posterior probability of $\mu_{\bar{z}}$ will be bounded away from 1, thus assuring that higher values for high effort than low effort. Hence, high effort is optimal for sufficiently small $c > 0$.

6.2.2 Without Government Replacement

Consider the case in which there is no replacement and assume that $\theta = 1$, that is, the original and only government is competent with certainty or complete information. It follows that the only pure-strategy equilibrium in repeated games is low effort exerted by the government. Although citizens receive the signal μ_z, they assume that the government exerts high effort since it is *supposed* to exert high effort and conclude that they have received just an unfortunate draw from the monitoring distribution. As the signals are idiosyncratic, such bad signals result in no punishments by citizens. This creates a powerful incentive for the government to exercise low effort.

Proposition 6.2.2 *If no replacement of the government is possible or $\lambda = 0$, a unique pure-strategy equilibrium exists in which the competent government exerts low effort in every period.*

Proof See appendix

The possibility of an inept government provides an incentive for the government to exert high effort, because a citizen receiving signal $\underline{z}$ punishes the government by increasing

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the probability with which the citizen believes that the government is inept. Ironically, a government that builds a reputation is too successful at building the reputation. Eventually, almost all the citizens become almost certain that the government is competent, in the sense that the posterior probability for a competent government approaches 1 for a critical mass of citizens. The incentives to exert high effort arise only out of the desire to affect citizens' beliefs about the government. Once the posterior probability of a competent government is close to unity, the effects of $\bar{z}$ and $\underline{z}$ become smaller. Eventually, the current signal will have such a small effect on the current belief that the cost c of high effort exceeds the very small differences in beliefs caused by $\bar{z}$ rather than $\underline{z}$ and the competent government then finds it optimal to succumb to low effort. Voters and the government can foresee this outcome, resulting in the unraveling of the equilibrium. Consequently, the only pure-strategy equilibrium requires the exertion of only low effort.

This holds also in cases when $\theta < 1$ or if there is incomplete information whether the government is competent in the first place. Suppose the competent government is following a pure strategy. The posterior probability that the government is competent is normal, given a prior probability of $\mu_{\bar{z}}$ and the signal $z \in \{\underline{z}, \bar{z}\}$ is

$$\varphi(\mu|\bar{z}) = \frac{[\alpha\beta_H + (1-\alpha)\beta_L]\mu}{[\alpha\beta_H + (1-\alpha)\beta_L]\mu + \beta_L(1-\mu)}$$

and

$$\varphi(\mu|\underline{z}) = \frac{[\alpha(1-\beta_H) + (1-\alpha)(1-\beta_L)]\mu}{[\alpha(1-\beta_H) + (1-\alpha)(1-\beta_L)]\mu + (1-\beta_L)(1-\mu)}$$

where $\alpha \in \{0,1\}$ is the probability of H.

It is then trivial that $\varphi\left(\mu|h_t^G\right)$ is the update from a prior μ after the history h_t^G. If the citizens believe that the competent government is following the pure strategy σ, they attach probability $\varphi(\mu_{\underline{z}}|h_t^G)$ to the government exerting high effort, after observing history h_t^G.

Both the propositions lead to the interesting and important conclusion that even in the absence of replacement, it is good for a dominant government to have citizens worry constantly that the government might go "bad". The purpose of a reputation is to convince

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citizens that the government is competent and will exert high effort. The problem in maintaining such a reputation in the absence of replacements is that the government essentially succeeds in convincing the citizens that it is competent, even though it may no longer be.

6.3 Electoral Competition

Replacements are not the only mechanisms by which incentives for high effort by authoritarian governments can be sustained: electoral competition is an alternative mechanism. As noted by Przeworski et al (2000), electoral contest is not exclusive to democracy and is present in many authoritarian states. Why do authoritarian governments go through the charade of electoral competition? One explanation is that authoritarian governments are concerned about their reputation and hold electoral contests to consolidate their authority and enhance their legitimacy as a "democracy", especially if they believe that they will win. However, electoral competition can occur in which voters can stop supporting a party and impose a high significant cost at the polls after any reduction in beliefs. In some cases, such "democratic" gestures may lead to electoral losses, resulting in a state of emergency or coup d'etat being declared and revision of the constitution to prevent further electoral defeats. Prominent examples include South Korea under Chung-Hee Park, the Phillipines under Ferdinand Marcos and Myanmar (Burma) under military rule.

This section extends the analysis to electoral competition and demonstrates that such electoral competition will ensure government to sustain reputations for high effort. However, such a "democracy" is fragile. We adapt Horner (2000) in developing a model of electoral parties with common voters and endogenous probability of inept type.

In period t, there is a set of possible political parties who may be competent or inept. Competent government can exert either high (H) or low (L) effort, while inept government only exert low effort. In every period, all voters of a government either receive a good governance outcome $\bar{y}$, with probability p_x, given effort level x from that government or receive a bad governance outcome $\underline{y}$ with probability $1 - p_x$. As before, L costs nothing for the government, but H involves an effort cost $c > 0$.

Assume a continuum of voters, of mass $I > 0$, with types indexed by i, uniformly distributed on $[0, I]$. I is large enough to ensure that there is an interior equilibrium. A voter receives a payoff of 1 from the good governance outcome $\bar{y}$ and payoff 0 from a bad governance outcome $\underline{y}$. A voter of type i will support a political party which exerts H with probability π at the support level v. In this case, voter i receives a payoff of

$$\pi p_H + (1 - \pi) p_L - v - i$$

The term $\beta p_H + (1 - \beta) p_L$ is the surplus that voter i obtains from participating in the electoral process while the last two terms $-v$ and $-i$ represent the opportunity cost of participation.

Denote $M_{\bar{z}}$ to be the mass of active competent parties that enter the election and mass $M_{\underline{z}}$ to be the mass of inept parties that enter the election. Allowing for free entry and exit in the election, the steady state $\left(M_{\bar{z}}^*, M_{\underline{z}}^*\right)$ can be derived, where $M_{\bar{z}}^*$ is the steady state mass of active competent parties while $M_{\underline{z}}^*$ is the steady state mass of active inept parties. These masses can be determined by the free-entry condition that the value of entering the election just compensates the party for its opportunity cost in participating.

The total mass of active parties in the election is $M^* = M_{\bar{z}}^* + M_{\underline{z}}^*$. In equilibrium, the active parties will represent mass M^* of voters. The opportunity cost of the marginal voter in an election serving M^* voters is M^*. As a result, each party must produce a surplus (the difference between its expected probability of outcome $\bar{y}$ and its support level) which is equivalent to M^*.
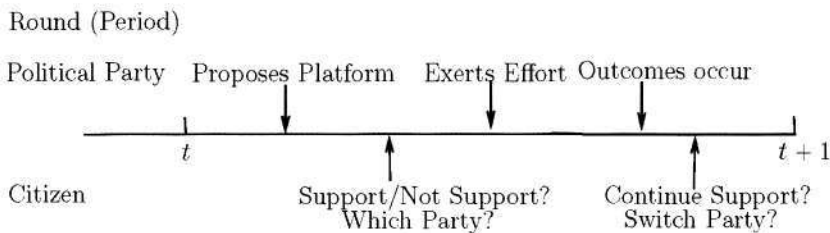

Figure 6.2: Electoral Competition

Voters observe the histories of past governance outcomes produced by each party in

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government. On first producing a governance failure, a party is forsaken by the voters and leaves the government. The composition of parties in the election is described by a pair of sequences $\left\{M_{\bar{z}}^t\right\}_{t=0}^{\infty}$ and $\left\{M_{\underline{z}}^t\right\}_{t=0}^{\infty}$ where $M_{\bar{z}}^t$ and $M_{\underline{z}}^t$ are respectively the mass of competent and inept parties in election and who have been in government for t periods, during which they have exhibited t consecutive realizations of $\bar{y}$. Thus, $M_{\bar{z}}^t = p_H M_{\bar{z}}^{t-1}$ and $M_{\underline{z}}^t = p_L M_{\underline{z}}^{t-1}$. It follows that

$$M_{\bar{z}} \equiv \sum_{t=0}^{\infty} M_{\bar{z}}^t = \frac{M_{\bar{z}}^0}{1 - p_H}$$

$$M_{\underline{z}} \equiv \sum_{t=0}^{\infty} M_{\underline{z}}^t = \frac{M_{\underline{z}}^0}{1 - p_L}$$

The posterior probability that a party can be a competent government after t consecutive realizations of $\bar{y}$ is given by

$$\begin{aligned}
\mu^t &= \frac{M_{\bar{z}}^t}{M_{\bar{z}}^t + M_{\underline{z}}^t} \\
&= \frac{p_H^t M_{\bar{z}}^0}{p_H^t M_{\bar{z}}^0 + p_L^t M_{\underline{z}}^0} \\
&= \left(1 + \left(\frac{p_L}{p_H}\right)^t \frac{M_{\underline{z}}^0}{M_{\bar{z}}^0}\right)^{-1}
\end{aligned}$$

Denote v^t to be the political support or votes commanded by a party after t consecutive realizations of $\bar{y}$.

$$v^t = \mu^t (p_H - p_L) + p_L - M$$

To make this an equilibrium for the party which has produced a failure to leave the government , assume a disequilibrium event of a lower vote level and political support or a continuation in government by the party after a governance failure gives rise to the voter expectation that the party in question in certainly inept. So long as $M > p_L$, such an event will result in a negative political support and hence make it optimal for the party to exit the

government.

The expected payoff to a party in this government can be derived once the respective measures of competent and inept entrants in the electoral competition in each period, $M_{\bar{z}}^0$ and $M_{\underline{z}}^0$, are established. Let $\Pi\left(M_{\bar{z}}^0, M_{\underline{z}}^0, \bar{z}\right)$ and $\Pi\left(M_{\bar{z}}^0, M_{\underline{z}}^0, \underline{z}\right)$ be the payoffs of the competent and inept party and close the model by requiring that entrants earn zero payoffs or

$$\Pi_{\bar{z}}\left(M_{\bar{z}}^0, M_{\underline{z}}^0\right) = (1 - \delta)\,\kappa_{\bar{z}}$$

$$\Pi_{\underline{z}}\left(M_{\bar{z}}^0, M_{\underline{z}}^0\right) = (1 - \delta)\,\kappa_{\underline{z}}$$

where $\kappa_{\bar{z}}$ and $\kappa_{\underline{z}}$ are the entry costs of a competent and inept party respectively.

It follows that

$$\Pi_{\bar{z}}\left(M_{\bar{z}}^0, M_{\underline{z}}^0\right) = (1 - \delta)\sum_{t=0}^{\infty}\rho_H^t\delta^t\left[\left(1 + \left(\frac{p_L}{p_H}\right)^t\frac{M_{\underline{z}}^0}{M_{\bar{z}}^0}\right)^{-1}(p_H - p_L) + p_L - M^* - c\right]$$

$$\Pi_{\underline{z}}\left(M_{\bar{z}}^0, M_{\underline{z}}^0\right) = (1 - \delta)\sum_{t=0}^{\infty}\rho_H^t\delta^t\left[\left(1 + \left(\frac{p_L}{p_H}\right)^t\frac{M_{\underline{z}}^0}{M_{\bar{z}}^0}\right)^{-1}(p_H - p_L) + p_L - M^*\right]$$

The entry conditions can be then derived as follows:

$$\sum_{t=0}^{\infty}\rho_H^t\delta^t\left[\left(1 + \left(\frac{p_L}{p_H}\right)^t\frac{M_{\underline{z}}^0}{M_{\bar{z}}^0}\right)^{-1}(p_H - p_L)\right] = \kappa_{\bar{z}} + \frac{M^* - p_L + c}{1 - \delta p_H}$$

$$\sum_{t=0}^{\infty}\rho_H^t\delta^t\left[\left(1 + \left(\frac{p_L}{p_H}\right)^t\frac{M_{\underline{z}}^0}{M_{\bar{z}}^0}\right)^{-1}(p_H - p_L)\right] = \kappa_{\underline{z}} + \frac{M^* - p_L}{1 - \delta p_L}$$

Last but not least, the steady-state values $\left(M_{\bar{z}}^*, M_{\underline{z}}^*\right)$, and equivalently $M^* = M_{\bar{z}}^* + M_{\underline{z}}^*$ and $\frac{M_{\underline{z}}^0}{M_{\bar{z}}^0}$, can be solved for by fixing a value for $\kappa_{\underline{z}}$ and for each M^* associate the value $\frac{M_{\underline{z}}^0}{M_{\bar{z}}^0} = g\left(M^*\right)$ that satisfies the second entry conditions. Then g is strictly decreasing with p_L being the minimum value of M^* consistent with equilibrium or ensuring the exit of parties which have been governance failures and $\overline{M^*}$ is the maximum value of M^* consistent with the second entry conditions. By fixing a value $M^* \in \left(p_L, \overline{M^*}\right)$ and using the first entry

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conditions, the locus of pairs $(\kappa_{\bar{z}}, c)$ satisfying the competent party's entry condition can be determined.

If $\kappa_{\bar{z}}$ and c are too small, there will be no inept party in the government. A competent political party will nonetheless exert effort, as a failure can prompt voters to stop supporting the party despite the lack of any revision in beliefs. Clearly, this is not a Markov equilibrium. The Markov equilibrium is given by the equilibrium for the inept party, given the assumption that voters expecting a party that remains in government after a governance failure to be certainly inept. Completing the equilibrium, it can be shown that the competent government finds it optimal to exert high effort. A governance failure leads to a continuation payoff of 0 while a governance success leads to a continuation payoff of at least $\kappa_{\bar{z}}$ so long as $c \in (0, \bar{c})$ and c is sufficiently small. The first entry condition will be preserved by increasing $\kappa_{\bar{z}}$ and decreasing c.

For simplicity, the analysis here assumes common voters. When voters are idiosyncratic, small changes in the voters's beliefs will lead to small changes in behavior. In such cases, replacement can ensure the changes in beliefs never become too small. In a richer model, even small changes in voter behavior may have magnified impact on the government and hence, there is no need to impose a lower bound on belief revision. As an example, consider the case of many parties, with some parties having similar ideological reputation. A small change in the voters's beliefs about a particular party will prompt a switch of support to another party with similar ideology, resulting in large political losses for the party in question.

6.4 Conclusion

We begin this chapter by posing the question: how can a government serving as an agent for the citizens or workers act in the interests of the principals?

This may be trivial in the case of a democracy, since competition for the electoral support of a fully enfranchised citizenry encourages political parties to pursue good governance outcomes. An influential literature argues precisely this: elections prevent elites from ex-

propriating non-elites, thereby encouraging non-elite investment and growth. Acemoglu and Robinson (2006) develop a framework for analyzing the creation and consolidation of such democracy. Different social groups prefer different political institutions because of the way they allocate political power and resources. Democracy is preferred by the majority of citizens but opposed by the elites. Dictatorship, however, is not stable when citizens can threaten social disorder and revolution. In response, when the costs of repression are sufficiently high and promises of concessions are not credible, elites may be forced to create democracy. By democratizing, elites credibly transfer power to the citizens, ensuring social stability. Democracy consolidates when elites do not have a strong incentive to overthrow it. This set of arguments is similar to those of Rustow (1970). Rustow argues that democracy originates from a bargain reached by conflicting groups which come to recognize the inevitability of power-sharing.

This may not be the case for an authoritarian government though it usually comes into power with promises of serving the workers. We present a reputational model which synthesize both the cases for both autocratic and democratic governments. In this model, an authoritarian government finds it good to have their citizens who constantly worry that the government might go "bad". The purpose of a reputation is to convince voters that the government is competent and will exert high effort. The problem in maintaining such a reputation in the absence of replacements is that the government essentially succeeds in convincing the voters that it is competent even though it may no longer be so. Replacements are not the only mechanisms by which incentives for high effort by the government can be sustained. Electoral competition is another mechanism in which voters can stop supporting a party and impose a high significant cost at the polls after any reduction in beliefs.

Our conclusion would suggest the quality of the political institutions is more important compared to the forms of the political institutions. This runs contrary to the implicit assumption in the literature (Acemoglu and Robinson, 2006) that democracy is always best. It is useful to observe that most nondemocratic regimes emerge from the failure of democratic regimes to represent the wishes of the population at large. History abounds with examples of transitions *from* democracy (Linz and Stepan, 1978). The classic examples of democratic

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breakdowns include Germany, Italy and Spain between the two World Wars. In Germany, the Weimar Republic was overthrown by Hitler through a democratic process. When democracies collapse, the underlying cause is often similar to those in authoritarian regimes. The crucial factor is the failure of the government to resolve critical problems, particularly to deliver good economic outcomes. Authoritarianism, like democracy, must represent the wishes of the population at large for its own political survival.

A more recent comparison can be made of China and India. Both are populous and agrarian economies which achieved political autonomy after the Second World War. The irony is evident. On the one hand, while India may be the world's largest democracy, its human development records are marred by massive poverty, illiteracy and inequality. On the other hand, while China may be the world's largest authoritarian state and its economic records are marred by the Great Leap Forward (in which around 40 million people died in five years), China's economy grew about twice as fast as India's in the forty years after the Second World War until 1980. Ultimately, to quote the late Chinese leader Deng Xiao-peng, "it does not matter whether the cat is white or black as long as it catches mice".

It is important to emphasize that the arguments in this chapter are not intended to prove the merits of authoritarianism over democracy. There are certainly much to be deplored about authoritarian governments. However, any attempt to understand the political economy in democratic regimes cannot be complete without an attempt to understand non-democratic regimes. Hopefully, this chapter represents a first step towards that understanding.

Chapter 7

Conclusions

In this thesis, we highlight the lack of a formal framework of fairness with which we can apply to deal with issues of fairness in the political economy of economic growth. In chapter 3, we propose such a framework by developing a fair equilibrium construct which emerges endogenously from the interaction of the players in dynamic games. A pre-game ultimatum game determines a fair rule for sharing the joint cooperative gains. To limit the set of fair rule, we apply a well-established result of the Ultimatum Game (Binmore, 2007) which indicates that players tend to play fair and the most likely outcome is a fifty-fifty split. Hence, we define our fair imputation as one which includes their individual Nash payoff (the threat point payoff) and half of their cooperative gain. The rule applies once the game starts and the rational acceptability (RA) criterion decides whether the players will play cooperatively or not. Rationally acceptable cooperative strategies exist if a fair imputation of the cooperative solution in a dynamic cooperative game offers players a higher payoffs compared to the stream of payoffs obtained in a Markovian Nash equilibrium. The set of fair equilibrium is then the set of rationally acceptable strategies. The cooperative game will be played if the fair imputation payoffs are greater than those derived from a unilateral defection, that is, if the fair imputations are rationally acceptable. If this is not the case, the fair equilibrium degenerates into a non-cooperative Nash equilibrium in which both players will play the non-cooperative game. Thus, the fair equilibrium is a cooperative one if rational acceptable fair imputations are available; otherwise, it is a non-cooperative one. The fair equilibrium in

this thesis involves Markovian strategies, which is distinct from non-Markovian strategies adopted by Tolwinski et al (1986). Many existing constructs assume implicitly that cooperation is desirable and must be enforced at all cost using trigger strategy or other enforcement mechanisms. In contrast, we do not assume that cooperation is necessarily desirable. The RA condition plays an important role in deciding whether cooperation is desirable or not. As such, the fair equilibrium here may be equivalent to the non-cooperative equilibrium if the latter is rationally acceptable. Above all, the players in the game are not forced against their will to cooperate even if doing so will be to their disadvantage. Additionally, we make use of an important insight from experimental economics to simplify the determination of the fair imputation rule in pre-game bargaining. Furthermore, the proposed fair equilibrium possesses "nice" properties of time consistency, subgame perfectness and Pareto efficiency. The last property is especially important as all current leading notions of fairness may perversely reduce welfare, including the possibility of reducing everyone's welfare (Kaplow and Shavell, 2002).

The fairness framework thus derived has many potential applications in analyzing fairness in macroeconomics or political economy. In this thesis, we apply it to answer a central question in economic history: why has capitalism prevailed as an institution in promoting economic growth despite its "dynamic inefficiency" ? The answer is that capitalism may be rationally acceptable and fairer compared to collectivism, as is demonstrated in a dynamic game with a vote-maximizing government(G) and profit-maximizing representative firm(F). In this GF game, a fair imputation of cooperative or collective solutions which is rationally acceptable for all players does not exist. In every stage of economic development, the firm always finds it rationally unacceptable to cooperate because the profits earned by the firm under the Markovian Nash equilibrium always dominate the profits under cooperation. On the other hand, the government only finds the cooperative solution to be rationally acceptable when the economy is above the steady state. Below the steady state, developing countries are trapped in low growth and political instability. Hence, collectivist cooperation between the government and the firm cannot be realized because they are not rationally acceptable for both and a fair equilibrium cannot be attained with collectivism.

We extend the analysis in two directions.

Firstly, existing differential games of economic growth are all deterministic. We contribute to the literature by analyzing the role of uncertainty in GF game. Specifically, we develop a stochastic differential GF game by using methods in stochastic calculus. In the deterministic GF game, the rent of the firm is completely taxed away and the firm stops investing completely, which is a very extreme and unrealistic solution. In contrast, in the stochastic Markovian Nash equilibrium, the government will tax less than the full amount of the rent accrued to the firm, which will post a positive rate of investment while the rate of investment depends not only on the capital-labor ratio but also the discount rate, the depreciation rate, the population growth rate and uncertainty. Although the cooperative solution is indeterminate from the model, we are able to prove that the RA condition is strongly satisfied. With uncertainty, it turns out that the cooperative solution is always non-inferior to the non-cooperative Markovian Nash equilibrium.

Secondly, we analyze the issue of aligning the performance of the government with the interests of the workers under different types of political regimes. In the GF games, the government is a vote-maximizer which depends on the political support of the workers. This distinguishes the thesis from current political economic models in which the government is always represented as "benevolent". A key assumption is that there exists an alignment of interest between the government and the workers. While this may be true for a democracy, where the votes reflect the wishes or the biases of the people, it may not be the case of an authoritarian government. We then pose an important question: in authoritarian economies, what incentives are there to ensure that the government will protect the interests of its citizens or workers?

We argue that the concern for reputation is the key to understanding why some authoritarian governments manage to achieve stellar economic performance for their citizens while others economic disasters. We present a model in which the authoritarian government has a reputation of "fairness" to uphold in order to sustain its political support. A "good" authoritarian government will allow their citizens to constantly worry that the government might go "bad". The purpose of a reputation is to convince voters that the government is competent

and will exert high effort in economic performance. The key problem in maintaining such a reputation in the absence of replacements or political renewal is that the government could succeed in convincing the voters that it is competent but may go "bad" subsequently. Replacements are not the only mechanisms in which incentives for high effort by authoritarian governments can be sustained. Electoral contest, which is not exclusive to democracy, is an alternative mechanism found in some authoritarian states. Such electoral contests imply that citizens can exercise their votes to stop supporting a party and impose a high significant cost at the polls after any reduction in beliefs in the competence of the government.

Although we have considered a number of critical assumptions, there are other assumptions which we have not discussed in this thesis.

The fair imputation rule derived in chapter 3 depends on the insights from experimental economics and assumes a 50-50 split following Binmore (2007). The question is whether the rule will be acceptable to both players if the relative sizes of the non-cooperative payoffs differ a lot, say in the proportion of 20:1. By the definition of fairness adopted in this thesis, the 50-50 split will still remain a fair rule even if the relative sizes of the non-cooperative payoffs are in the proportion of 20:1. However, it is by no means the only fair rule available. Could a proportional rule with a 20:1 split be a fair rule? It is possible though in the context of income distribution in economic growth models, this may be tantamount to "the rich getting richer and the poor getting poorer". In short, a proportional split could be a fair rule but it would be an unjust rule in a Rawlsian sense and hence may not be "reasonable". However, the thesis deals with fairness and not justice, which is a far more complex issue worth exploring in future research.

Another assumption is the neoclassical production framework which is adopted for analysis in chapter 4 and 5. *Quantitatively*, one would expect the results to be different depending on the specification of the endogenous production function, of which there are many (Barro and Sala-i-Martin, 2004). However, the key results of the analysis will not be affected *qualitatively* if an endogenous production framework is adopted. In short, the results are qualitatively robust to the specification of the production function.

In addition, our analysis has focused on long-run growth. Capitalism, on the other hand,

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is also characterized by bouts of booms and busts. The introduction of business cycles into the framework poses a big challenge. This is because in electoral competition, it is very likely that government will be tempted to "perform" only just before an election and the incumbent may have an advantage in adjusting the performance to the business cycle. The interface between short-run business cycle and long-run growth and the resulting political business cycles are interesting open problems which merit further research in the future.

Finally, although we have explored the alignment of interests between the government and the workers in chapter 6, we have not considered the alignment of interests between the firms and their stakeholders. It is reasonable to expect that a firm which represents the interests of its shareholders would seek rent by attempting to influence the policy decisions of the government, which may in turn depend on monetary support from the firms to fund its re-election campaign. Effectively, this would be a multi-principals-agents dynamic game. In this multiple principal-agent scenario, the government will be forced to choose an optimal balance in aligning with both the workers and the firm. Consequently, the rational accept-ability of the outcome may be contingent on the performance of the government and the firm as perceived by their principals. Establishing rationally acceptable outcomes for play-ers with different objective functionals and possibly different state equations is challenging. On the flip side, it presents rich opportunities for future research.

APPENDIX

Appendix A

Appendix

A.1 Proof of Proposition 6.2.1

Proof Let $\bar{\mu}$ solve $\varphi(\mu|\bar{z})$ and $\underline{\mu}$ solve $\varphi(\mu|\underline{z})$.

Assume that the competent government always exercise a high effort strategy. Thus, $\varphi(\mu|z) \in [\lambda\mu^0, (1-\lambda)+\lambda\mu^0]$ for all $\mu \in [0,1]$ $z \in \underline{z}, \bar{z}$. Moreover,

$$\lambda\mu^0 < \underline{\mu} < \mu^0 < \bar{\mu} < (1-\lambda)+\lambda\mu^0$$

and $\varphi(\mu|z) \in [\underline{\mu}, \bar{\mu}] \forall \mu \in [\underline{\mu}, \bar{\mu}]$ and $z \in \{\underline{z}, \bar{z}\}$.

Denote the inverse of μ under $\varphi(\cdot|z)$ as $\varphi^{-1}(\mu|z)$. Then set

$$\varphi^{-1}(\mu|z) = \begin{cases} 0 & \mu < \min\varphi(\mu|z), \\ 1 & \mu > \max\varphi(\mu|z), \end{cases}$$

As $\varphi^{-1}(\mu|\underline{z}) - \varphi^{-1}(\mu|\bar{z}) > 0$ for all $\mu \in (\lambda\mu^0, (1-\lambda)+\lambda\mu^0)$, there is a constant $\beta > 0$ such that $\varphi^{-1}(\mu|\underline{z}) - \varphi^{-1}(\mu|\bar{z}) \geq \beta$ for all $\mu \in [\underline{\mu}, \bar{\mu}]$

Let G_x to be the distribution over posteriors that the government is competent in period $t+1$ that results from the choice of effort x conditional on the distribution of voter posteriors, G. Then, $G_x(\mu|\bar{z}) = p_x G(\varphi^{-1}(\mu|\bar{z})) + (1-p_x) G(\varphi^{-1}(\mu|\underline{z}))$. Hence

$$G_L(\mu) - G_H(\mu) = (p_H - p_L) G(\varphi^{-1}(\mu|\underline{z})) - G(\varphi^{-1}(\mu|\bar{z})) \geq 0$$

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Since the average voter's posterior under G is given by $\int \mu \, dG(\mu) = 1 - \int G(\mu) \, d\mu$, it is possible to choose ε such that $0 < \varepsilon < \min\{\beta, \underline{\mu} - \varphi^{-1}(\underline{\mu}|\bar{z}), 1 - \bar{\mu}\}$. But since $\varepsilon < \beta$

$$\int_0^1 \left[G\left(\varphi^{-1}(\mu|\underline{z})\right) - G\left(\varphi^{-1}(\mu|\bar{z})\right) \right] d\mu \geq \int_{\underline{\mu}}^{\bar{\mu}} \left[G\left(\varphi^{-1}(\mu|\underline{z}) + \varepsilon\right) - G\left(\varphi^{-1}(\mu|\bar{z})\right) \right] d\mu$$

Let K be the largest integer k for which $\left(\varphi^{-1}(\underline{\mu}|\bar{z})\right) + k\varepsilon \leq \bar{\mu}$. As such, $\left(\varphi^{-1}(\underline{\mu}|\bar{z})\right) + (K+1)\varepsilon \leq 1$. It is then possible to construct an increasing sequence $\left\{\underline{\mu}_k\right\}_{k=0}^{K+1}$ by setting $\underline{\mu}_k = \varphi\left(\left(\varphi^{-1}(\underline{\mu}|\bar{z})\right) + k\varepsilon|\bar{z}\right)$ with $\underline{\mu}_0 = \underline{\mu}$.

For $k = 0, \cdots, K$, define $f_k : \left[\underline{\mu}_0, \underline{\mu}_1\right] \rightarrow \left[\underline{\mu}_k, \underline{\mu}_{k+1}\right]$ such that $f_k(\mu) = \varphi\left(\left(\varphi^{-1}(\underline{\mu}|\bar{z})\right) + k\varepsilon|\bar{z}\right)$. It follows that $f_k\left(\underline{\mu}_0\right) = \underline{\mu}_k$ and $f_k\left(\underline{\mu}_1\right) = \underline{\mu}_{k+1}$. Since f_k is continuous, it is onto.

Additionally, $f_{k+1}(\mu) = \varphi\left(\varphi^{-1}(f_k(\mu)|\bar{z}) + \varepsilon|\bar{z}\right)$. As $\varphi(\cdot|\bar{z})$ is concave,

$$\frac{df_{k+1}(\mu)}{d\mu} \leq \frac{df_k(\mu)}{d\mu}$$

Hence, for $k = 0, \cdots, K$,

$$\int_{\underline{\mu}_k}^{\underline{\mu}_{k+1}} G\left(\varphi^{-1}(\mu|\underline{z}) + \varepsilon\right) d\mu = \int_{\underline{\mu}_0}^{\underline{\mu}_1} G\left(\varphi^{-1}(f_k(\mu)|\underline{z}) + \varepsilon\right) \frac{df_k(\mu)}{d\mu} d\mu$$

$$= \int_{\underline{\mu}_0}^{\underline{\mu}_1} G\left(\varphi^{-1}(f_{k+1}(\mu)|\underline{z})\right) \frac{df_k(\mu)}{d\mu} d\mu$$

$$\geq \int_{\underline{\mu}_0}^{\underline{\mu}_1} G\left(\varphi^{-1}(f_{k+1}(\mu)|\underline{z})\right) \frac{df_{k+1}(\mu)}{d\mu} d\mu$$

Because $\underline{\mu}_0 = \underline{\mu}$, $\underline{\mu}_{K+1} > \bar{\mu}$ and the support of G is a subset of $\mu \in \left[\underline{\mu}, \bar{\mu}\right]$, it follows that

$$\int_{\underline{\mu}}^{\bar{\mu}} \left[G\left(\varphi^{-1}(\mu|\bar{z} + \varepsilon)\right) - G\left(\varphi^{-1}(\mu|\bar{z})\right) \right] d\mu$$

$$= \sum_{k=0}^{K} \int_{\underline{\mu}_k}^{\underline{\mu}_{k+1}} \left[G\left(\varphi^{-1}(\mu|\bar{z} + \varepsilon)\right) - G\left(\varphi^{-1}(\mu|\bar{z})\right) \right] d\mu$$

$$\geq \int_{\underline{\mu}_0}^{\underline{\mu}_1} \sum_{k=0}^{K-1} G\left(\varphi^{-1}(f_{k+1}(\mu)|\underline{z})\right) \frac{df_{k+1}(\mu)}{d\mu} - \sum_{k=0}^{K} G\left(\varphi^{-1}(f_k(\mu)|\underline{z})\right) \frac{df_k(\mu)}{d\mu} d\mu$$

$$+ \int_{\underline{\mu}_K}^{\underline{\mu}_{K+1}} G\left(\varphi^{-1}(\mu|\bar{z} + \varepsilon)\right) d\mu$$

$$= \int_{\underline{\mu}_K}^{\underline{\mu}_{K+1}} G\left(\varphi^{-1}(\mu|\bar{z} + \varepsilon)\right) d\mu - \int_{\underline{\mu}_0}^{\underline{\mu}_1} G\left(\varphi^{-1}(\mu|\bar{z})\right) d\mu$$

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$$\geq \quad G\left(\varphi^{-1}\left(\underline{\mu}_K|\bar{z}+\varepsilon\right)\right)\left(\underline{\mu}_{K+1}-\underline{\mu}_K\right)-G\left(\varphi^{-1}\left(\underline{\mu}_1|\bar{z}+\varepsilon\right)\right)\left(\underline{\mu}_1-\underline{\mu}_0\right)$$

$$\geq \quad \underline{\mu}_{K+1}-\underline{\mu}_K$$

The last inequality is true because $\varphi^{-1}\left(\underline{\mu}_1|\bar{z}\right)\leq\underline{\mu}$ and $\varphi^{-1}\left(\underline{\mu}_K|\bar{z}\right)\geq\bar{\mu}$.

Hence, $\int_0^1 [G_L(\mu)-G_H(\mu)]\,d\mu$ is bounded away from 0, the bound depending only on ε and not on the period t distribution G or on t. Since the competent government is always expected to choose high effort, $\int_0^1 [F_L(\mu)-F_H(\mu)]\,d\mu$, where F is the distribution in period $t+1$ of the citizen expectations of the probability that the government exerts high effort. This suggests that there is an autonomous constant Δ (i.e. independent of t) such that period $t+1$ vote returns exceed those after low effort in period t by at least Δ. Hence, a sufficient condition for an equilibrium with high effort is that the discounted value of this difference exceeds the cost, $\delta(1-\lambda)\Delta > c$.

A.2 Proof of Proposition 6.2.2

Proof Given an equilibrium (σ,β), where σ is a pure strategy that requires a competent government to sometimes exert high effort in equilibrium. Since no replacement is possible, the government history evolves in a deterministic manner. Thus, σ determines the periods in which the government will exert high effort. Evidently, σ requires the government to always exert high effort. Should this not be the case, there is a final period T in which the government exerts high effort. The political support in every subsquent period would be $v(0)$ which is independent of the outcome in period T. Hence, high effort is suboptimal in period T.

Let $s_\tau(H) \equiv \{t\geq\tau:x^t(\sigma)=H\}$ be those period large than τ in which high effort is exerted. Analagously, $s_\tau(L) \equiv \{t\geq\tau:x^t(\sigma)=L\}=\{\tau,\tau+1,\cdots\}-s_\tau(H)$.

For $t\in s_\tau(H)$, all voters expect the government to choose low effort with probability one, that is,

$$v\left(F^t\right)=v(0)$$

where F^t is the distribution of consumer expecations over the government's effort level in

period t.

Then for any history h_i^t,

$$\beta\left(h_i^t\right) = \varphi(\mu_{\underline{z}}|h_i^t),$$

that is, the probability distribution function F^t of voters' period t probabilities of high effort is the distribution function of voters' posterior beliefs that the government is competent, G^t. For $t \in s_\tau(L)$, $G^t = G^{t+1}$.

Conditional on the government being competent, $s_\tau(H)$ is non-empty for all τ. Hence, $G^t(x) \to 0$ for all $x < 1$ as $t \to \infty$. Intuitively, voters eventually become convinced that their government is a competent one.

Thus, for all $\varepsilon > 0$, there exists $t(\varepsilon)$ such that for all $t \geq t(\varepsilon)$, $G^t(1-\varepsilon) < \varepsilon$. In other words, at least a fraction $(1-\varepsilon)$ of voters have observed a private history h_i^t that leads to an update $\varphi\left(\mu_{\underline{z}}|h_i^t\right) > (1-\varepsilon)$.

For $\eta > 0$ and $k \in N$, there exists $\varepsilon(\eta, k) > 0$ such that $\mu > 1 - \varepsilon(\eta, k)$ implies $\varphi\left(\mu|\underline{z}^{(k)}\right) > 1 - \eta$, where $\underline{z}^{(k)}$ is the sequential realizations of bad outcome $\underline{z}$. Further, this also implies that for any k-period history h_i^k, $\varphi\left(\mu_{\underline{z}}|h_i^t\right) > (1-\eta)$.

Let G^η be the distribution function given by

$$G^\eta(x) = \begin{cases} \eta & x < 1-\eta, \\ 1 & x \geq 1-\eta, \end{cases}$$

Then G^t has a first order stochastic dominance over G^η for all and all. Omitting the normalization $(1-\delta)$, the lower bound for the continuation payoff from deviating in a period $t' \in s_{t(\varepsilon(\eta,k))}(H)$ is given by

$$v(G^\eta) + \sum_{t'+1 < t \leq t'+k} \delta^{t-t'}\left(v(G^\eta) - c\right) + \sum_{t'+1 < t \leq t'+k} \delta^{t-t'}v(0) + \frac{\delta^{K+1}}{(1-\delta)}\left(v(0) - c\right)$$

for any $k > 0$.

Since $v(G^\eta) \to v(1)$ as $\eta \to 0$, by choosing a large k and small η, the lower bound can

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be made arbitrarily close to

$$v(1) + \sum_{t \geq t'+1} \delta^{t-t'} (v(1) - c) + \sum_{t \geq t'+1} \delta^{t-t'} v(0)$$

The government has a profitable deviation because the lower bound for the continuation payoff for pursuing the pure strategy σ is only

$$\sum_{t \geq t'+1} \delta^{t-t'} (v(1) - c) + \sum_{t \geq t'+1} \delta^{t-t'} v(0)$$

In short, there does not exist any equilibrium in pure strategies in which the competent government exerts high effort.

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