Flexure-based electromagnetic parallel-kinematics manipulator system

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FLEXURE-BASED ELECTROMAGNETIC PARALLEL-KINEMATICS MANIPULATOR SYSTEM

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Abstract

Manipulation between the nano and meso scales at elevated bandwidth with large continuous output force remains as a technological gap in the field of ultra-high precision manipulation over the past decades. Current ultra-high precision manipulators are incompetent of bridging this gap due to their failure in recognizing the inter-dependency between each subsystem, e.g., the positioning mechanisms, the nano-positioning actuators and the control systems etc. This research focuses on the development of a multiple degrees-of-freedom Flexure-Based Electromagnetic Parallel-Kinematics Manipulator (FEPK) targeted with several capabilities, i.e., nanometric resolution, large continuous output force, fast actuating speed, high system stiffness and a large workspace of a few millimeters and degrees, so as to bridge the existing technological gap. Each subsystem is investigated with the inter-dependency between these subsystems being taken into consideration. Consequently, these subsystems are treated as one unified mechatronics system with a common goal, i.e., to realize those targeted capabilities. In this research, a new analytical model, termed semi-analytic model, is formulated to predict the nonlinear deflection of a beam-based flexure joint coupled with various rigid-link lengths. Experimental investigations have shown that this analytical model provides a highly accurate solution with an average deviation of 2.3% when compared to the experimental results. A new magnetic circuit is introduced to address the low output force limitation of a Lorentz-force actuation. Termed Dual-Magnet (DM) configuration, it offers a large effective air gap and increases the magnetic flux density by 40%. Hence, an Electromagnetic Driving Module (EDM) that adopts the DM configurations can achieve a large continuous output force through small input current and a compact-sized module. Based on this enhanced Lorentz-force EDM and beam-based flexure supporting bearings, a

novel nano-positioning linear actuator is developed. Termed Flexure-Based Electromagnetic Linear Actuator (FELA), it achieves a positioning accuracy of ± 10 nm and a continuous output force of 60 N/Amp throughout a translational motion of 4 mm. In addition, a new 5-DOF compliant joint, which offers larger deflections and orientations with lower driving stiffness as compared to an elementary rod flexure joint, is proposed. Such a 5-DOF compliant joint will play an important role in the development of a class of spatial-motion compliant manipulators that target for large workspace of few millimeters and degrees. With the proposed compliant joints and a 3-limbs Prismatic-Prismatic-Spherical (3PPS) parallel-kinematics configuration, a 3-DOF 3PPS FEPM, which offers a θ_x - θ_y - Z motion, is developed. This 3-DOF 3PPS FEPM has achieved an open-loop positioning and orientation resolution of ± 10 nm and $0.05''$ respectively, a large continuous output force of 160 N/Amp throughout a workspace of $\pm 2.5^\circ \times \pm 2.5^\circ \times \pm 2.5$ mm, and a fast traveling speed of 250 mm/sec. With these capabilities, the 3-DOF 3PPS FEPM becomes a promising solution in bridging the existing technological gap in the field of ultra-high precision manipulation.

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“All innovations are changes, but not all changes are innovative.”

– Teo Tat Joo

Contents

Abstract	i
Acknowledgement	iii
Contents	vi
List of Figures	xiii
List of Tables	xxiii
List of Abbreviations	xxvii
List of Notations	xxviii
Chapter 1 Introduction	1
1.1 Motivation	2
1.1.1 Proficiency of Compliant Joints	3
1.1.2 Modeling of Compliant Joints and Mechanisms	3
1.1.3 Capabilities of Nano-Positioning System Drivers	4
1.2 Objectives	5

<i>Contents</i>	<i>vii</i>
1.3 Thesis Overview	6
1.4 Organization of the Thesis	8
Chapter 2 Literature Review	10
2.1 Compliant joints	10
2.1.1 Leaf-Spring	11
2.1.2 Elementary Notch Hinge	13
2.1.3 Serially-Connected Flexure Joints	13
2.1.4 Parallel-Kinematics Architecture	16
2.1.5 Planar-Motion Flexure-Based Parallel-Kinematics Manipulators . . .	17
2.1.6 Spatial-Motion Flexure-Based Parallel-Kinematics Manipulators . . .	18
2.2 Electromagnetic Driving Scheme	21
2.2.1 Electromagnetic Propulsion	21
2.2.2 Magnetic Levitation	24
2.2.3 Lorentz-Force Actuation	27
2.3 Towards A Large Workspace of Few Millimeters and Degrees	28
Chapter 3 Large Deflection Beam-Based Flexure Modeling	32
3.1 Background	32
3.1.1 Small Deflection Theorem	33
3.1.2 Previous Works	34
3.2 A Semi-Analytic Model	39
3.2.1 Bernoulli-Euler Law	39
3.2.2 Shifting of Pivot Point During Large Deflection	42

3.2.3	Large Deflection Analysis of A Beam-Based Flexure with Moment End Load	44
3.2.4	Large Deflection Analysis of A Beam-Based Flexure Joint Coupled with Rigid-Link	46
3.2.5	Initial Pivot Point Location of A Beam-Based Flexure Joint Coupled with A Rigid-Link	47
3.3	Theoretical Comparisons	48
3.3.1	Approximations of Pivot Point Shifting	48
3.3.2	Semi-Analytic Model V.S. Nonlinear Model	50
3.3.3	Approximation of Initial Location of Pivot Point	51
3.3.4	Semi-Analytic Model V.S. PRB Model	53
3.4	Experimental Investigations	56
3.4.1	Experimental Setup	56
3.4.2	Results	60
3.5	Case Study: Stiffness Analysis of A Double Compound Linear Spring	64
3.6	Summary	67
Chapter 4 A Dual-Magnet Configuration		69
4.1	Background	69
4.1.1	Lorentz-Force Theorem	70
4.1.2	Drawbacks of Lorentz-Force Actuation	71
4.1.3	Previous Works	71
4.2	A Dual-Magnet Configuration	75
4.3	Magnetic Field Modeling of A Dual-Magnet Configuration	76

<i>Contents</i>	<i>ix</i>
4.3.1 Magnetic Scalar Potential Theory	77
4.3.2 Boundary-Value Problem	79
4.3.3 Superposition of Boundary Conditions	81
4.3.4 Magnetic Field Solution	82
4.4 Prototypes and Experiments	95
4.4.1 Prototypes	95
4.4.2 Experimental Setup	95
4.5 Results	96
4.5.1 Magnitude of Magnetization	96
4.5.2 Effect of Iron Separating the Symmetrical DM Configuration	97
4.5.3 DM Configuration V.S. Conventional Magnetic Configuration	98
4.5.4 Analytical Results V.S. Experimental Data	100
4.6 Summary	105
Chapter 5 Flexure-Based Electromagnetic Linear Actuator	107
5.1 Background	107
5.2 Design and Analyses	109
5.2.1 Electromagnetic Driving Module	110
5.2.2 Flexure Bearing Mechanism	115
5.3 Prototype	119
5.3.1 Current-Force Relationship	120
5.3.2 Force-Displacement Relationship	121
5.4 Position Control Design and Implementation	123

5.4.1	System Dynamics Modeling	123
5.4.2	Controller Design	124
5.4.3	Results	126
5.5	Direct-Force Control Design and Implementation	131
5.5.1	System Dynamics Modeling	131
5.5.2	Controller Design	132
5.5.3	Results	133
5.6	Imprinting of Micro-Sized Channels Through A Hot-Embossing Process . . .	135
5.6.1	Hybrid Position/Force Control	135
5.6.2	Hot-Embossing Process	138
5.6.3	Imprinted Micro-Sized Features	139
5.7	Summary	141
Chapter 6 Design of A Co-Planar Nano-Alignment Manipulator		143
6.1	Background	143
6.2	Kinematic Design of A 3-Limbs Prismatic-Prismatic-Spherical Parallel-Kinematics Configuration	145
6.2.1	Type Synthesis	145
6.2.2	Forward Kinematic Analysis	147
6.2.3	Initial Parametric Analysis	152
6.3	Conceptual Design of the Proposed 3PPS FPM	153
6.4	Active Compliant Prismatic Joint	154
6.4.1	Displacement Stiffness Modeling	155

Contents	xi
6.4.2 Parameters Analyses and Selections	156
6.5 Passive Compliant Prismatic Joint	157
6.5.1 Stiffness Modeling	157
6.5.2 Parameters Analyses and Selections	158
6.6 Passive 5-DOF Compliant Joint	159
6.6.1 Design Concept of Segment 1 and 2	160
6.6.2 Design Concept of Segment 3	161
6.6.3 Complete Module	162
6.6.4 Adopting the 5-DOF Compliant Joint into the 3PPS FPM	163
6.6.5 Modeling of Deflection Stiffness Along X- and Y-Axes	164
6.6.6 Modeling of Stiffness Along Z-Axis and Critical Loading	165
6.6.7 Modeling of Angular Stiffness About X- and Y-Axes	167
6.6.8 Modeling of Angular Stiffness About Z-Axis	168
6.6.9 Deflection Along X- or Y-Axis Due to A Translational Force	175
6.6.10 Angular Motion About X- or Y-Axis Due to An External Moment	178
6.6.11 Critical Loading on the Beam-Based Flexure Joints	180
6.6.12 Angular Motion About Z-Axis of Proposed 5-DOF Compliant Joint Due to An External Moment	181
6.7 Stiffness Modeling of the Proposed 3PPS FPM	182
6.7.1 Compliance Matrix of Each Parallel Limb	184
6.7.2 Stiffness Matrix of the Proposed FPM	189
6.8 Final Design of the Proposed 3PPS FPM	192
6.8.1 Removal of Proposed Passive Compliant Prismatic Joint	192

6.8.2	Removal of Segment 3 of the Passive 5-DOF Compliant Joint	193
6.8.3	Final Design of the Proposed 3PPS FPM	196
6.9	Summary	197
Chapter 7 Prototype of A Co-Planar Nano-Alignment Manipulator		198
7.1	Prototype Development	198
7.1.1	Materials Selection	199
7.1.2	Wire-Cut EDM	201
7.2	Active Compliant Prismatic Joint	202
7.2.1	Stiffness Evaluation	202
7.3	Passive 5-DOF Compliant Joint	205
7.3.1	Deflection Stiffness Along X- or Y-Axis Evaluation	206
7.3.2	Angular Stiffness About Z-Axis Evaluation	207
7.4	Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator	209
7.4.1	Workspace Evaluation	210
7.4.2	Active Joint-Level Position Control	212
7.4.3	Large Continuous Output Force	220
7.5	Summary	221
Chapter 8 Conclusions		223
8.1	Contributions	227
8.2	Limitations and Future Works	228

<i>Contents</i>	<i>xiii</i>
Appendix A Magnetic Field Modeling	231
A.1 Magnetostatic Energy	231
A.2 Surface Charge Model	232
A.3 Additional for Magnetic Field Solution	235
A.4 Parameters Used In the Proposed 2D Mathematical Model for Magnetic Field Modeling	237
A.5 Parameters Used In the Finite Element Analysis	238
Appendix B Flexure Modeling	239
B.1 Classical Nonlinear Solution for A Cantilever Beam Subjected to Moment End Load	239
B.2 Choosing the Maximum Bending Moment of Small Deflection Theory	241
Appendix C Accuracy of Finite Element Analysis on Elastic Deflection	242
List of Author’s Publications and Awarded Patents	245
Bibliography	247

List of Figures

1.1	An overview of the design concepts and solutions used in realizing a multi-DOF flexure-based electromagnetic parallel-kinematic manipulator	6
2.1	H. A. Roland with his ruling engine for diffraction gratings [42].	11
2.2	(a) A leaf-spring hinge and (b) a double cross-strip hinge [43].	12
2.3	(a) An optical slit mechanism, (b) a parallel spring mechanism for positioning the Michelson interferometer mirror and (c) a gravimeter-vertical seismograph [45].	12
2.4	A leaf-spring compound linear spring mechanism V.S. a monolithic compound linear spring mechanism [42].	13
2.5	Three types of notch flexure joints; (a) a circular shape, (b) an elliptical shape and (c) a small-radius shape.	13
2.6	(a) Serially-connected flexure joints, (b) a linear spring stage and (c) a compliant prismatic joint.	14
2.7	(a) A compound linear spring mechanism and (b) a symmetrical compound linear spring mechanism.	14
2.8	(a) A stacked serial-kinematics approach used in (b) a X-Y nanopositioning stage from PI [55].	15
2.9	(a) A nested serial-kinematics approach used in (b) a biaxial micro-positioning stage from NIST [59].	16

List of Figures	xv
2.10 (a) A double compound flexure pivoted mechanism lever as one of the parallel limb used in (b) a X-Y- θ z wafer positioning stage [60].	17
2.11 (a) A X-Y translational serially-connected flexure joint used in (b) a 3-DOF wafer alignment stage [61].	18
2.12 (a) A 3RRR compliant mechanism used in (b) a 3-DOF parallel micro-mechanism [62].	18
2.13 (a) A classical universal flexure joint without intersecting axes, (b) a classical universal flexure joint with orthogonal intersecting axes and (c) a modern universal flexure joint with a common rotational center.	19
2.14 (a) Delta-Cube and (b) one of its parallel limb based based on (c) a parallel-kinematics delta robot concept [71].	19
2.15 (a) A 3SPS FPM [72] and (b) a 3RPPR FPM [73]	20
2.16 (a) A Steward platform FPM [76] and (b) a hybrid 6-DOF FPM [77]	20
2.17 A schematic representation of a conventional electromagnetic propulsion technique [78].	22
2.18 (a) Electromagnetically driven X-Y- θ z stage and (b) schematic plot of all three motions [79].	22
2.19 (a) A HexFlex and (b) its schematic breakdown [4].	23
2.20 (a) Working principle of a Sawyer motor and (b) the arrangement of stator and rotor of a Sawyer motor [87].	24
2.21 (a) Angstrom machine [88] and (b) Sub-Atomic Measuring Machine [92]. . .	25
2.22 (a) A 6-DOF high-precision magnetic levitation stage [93] and (b) a 6-DOF magnetic levitation actuator [94].	25
2.23 (a) Δ -stage [96] and (b) a Y-stage [97].	26
2.24 (a) A moving coil configuration and (b) a moving magnet configuration [98].	27
2.25 (a) A Lorentz-force positioner (b) a 3-DOF highly-precise direct drive positioner.	28

2.26 (a) A planar 3-DOF FPM [102] and (b) Tribias 6-DOF robot [103].	29
2.27 (a) Delta ³ II [104] , (b) Sigma-6 [105] and (c) beam-based flexure joints. . . .	29
2.28 Various types of X-Y FPMs constructed by the beam-based flexure joints. . .	30
3.1 A cantilever beam subjected to a vertical force at free end.	35
3.2 A cantilever beam subjected to a moment end load.	35
3.3 (a) A flexure joint coupled with a rigid-link that is represented in (b) a PRB small-length flexure pivot [116].	36
3.4 Schematic diagram of a PRB model on a flexure joint coupled with a rigid-link.	37
3.5 Other PRB models for (a) a fixed-pinned cantilever beam, (b) a fixed-guided cantilever beam and (c) a cantilever beam subjected to moment end loading [116].	38
3.6 Shifting of pivot point during large deflection of beam.	42
3.7 Comparison between $\frac{1-\cos\theta}{\theta}$ and $\frac{\sin\theta}{2}$	43
3.8 (a) A beam-based flexure joint coupled with a rigid-link subjected to a point load treated as (b) a beam-based flexure joint subjected to a moment end load.	44
3.9 Comparison between $\frac{1-\cos\theta}{\theta}$ and $\frac{\sin\theta}{2\varpi}$	45
3.10 Schematic diagram of a beam-based flexure subjected to a moment end load.	45
3.11 Semi-analytic schematic diagram of a flexure joint coupled with rigid link. .	48
3.12 ‘Pivot’ point shift of a deflected cantilever beam.	49
3.13 Comparison between the nonlinear and semi-analytic models.	51
3.14 Beam-based flexure joint coupled with various rigid-links lengths.	54
3.15 Steps for obtaining a set of deflections from both semi-analytic and PRB models based on the initial desired deflection.	54

3.16	Specimens of the beam-based flexure joint coupled with various rigid-link lengths.	57
3.17	Experimental platform.	57
3.18	ATOS 3D measuring device for capturing the deflection of each specimen. . .	58
3.19	(a) Two spherical balls attached to the rigid-link for the measuring device to capture (b) the image of the actual coordinates of deflection.	59
3.20	(a) Triangular coordinates determined through the center of the spherical balls from each deflection were used to (b) calculate the actual deflections in the y- and x-axes.	59
3.21	(a) A 3DOF 3PPR flexure stage and (b) its active compliant prismatic mechanism.	64
3.22	Schematic diagram of a compound linear spring.	65
3.23	Stiffness error between the experimental data and the semi-analytic model, and PRB model respectively.	66
4.1	Various types of magnetic circuits found in (a) a conventional VC actuator, (b) a H2W VC actuator and (c) a BEI VC actuator.	72
4.2	(a) Flux focus and (b) conventional effective air gaps	73
4.3	(a) A symmetrical DM configuration in (b) an electromagnetic module. . . .	75
4.4	A 2D geometry reduced from half of a DM configuration.	79
4.5	Prototypes of (a) a DM configuration and (b) a conventional magnetic configuration.	95
4.6	Experimental setup for flux measurement.	96
4.7	Flux measurement.	96
4.8	Effect of the iron thickness between two PMs on the magnitude of magnetic field.	98

4.9	Magnetic flux density measured from (a) a conventional magnetic configuration and (b) a DM configuration.	99
4.10	Magnetic flux density distribution in X-Z plane measured from (a) a conventional magnetic configuration and (b) a DM configuration at $Y = 2$ mm and $Y = 7$ mm respectively.	100
4.11	Magnetic flux density distribution in X-Y plane at $Z = 25$ mm: (a) experimental data and (b) results from the proposed mathematical model and (c) from the numerical analysis.	101
4.12	Difference in magnetic flux density between the results obtained from the proposed mathematical model and the experiments (X-Y plane at $Z = 25$ mm).	102
4.13	Magnetic flux density distribution in Y-Z plane at $X = 27$ mm: (a) experimental data and (b) results from the proposed mathematical model and (c) from the surface charge model.	103
4.14	Difference in magnetic flux density between the results obtained from the proposed mathematical model and the experiments (Y-Z plane at $X = 27$ mm).	104
5.1	(a) A fine-orientation stage and (b) a complete SFIL setup [137].	108
5.2	(a) An EDM with symmetrical DM configuration with (b) the moving air-coil supported by flexure bearings to form a FBM.	110
5.3	A coil holder within the effective air gap of a DM configuration in X-Y plane.	111
5.4	Frame of the coil holder slotted within the center of the stator in Y-Z plane.	113
5.5	(a) FBM-1: A symmetrical double-compound linear spring design with its schematic diagram and (b) FBM-2: a bi-stable design and its schematic diagram.	115
5.6	(a) A bi-stable FBM is represented as (b) a symmetrical four-bar linkage. (c) A schematic diagram of a four-bar linkage.	118
5.7	A FELA prototype.	119
5.8	The experiment setup for the current-force evaluation on the EDM.	120

List of Figures	xix
5.9 Predicted current-force relationship from the proposed analytical model plot against the actual current-force experimental data.	121
5.10 Experimental setup for investigating the stiffness of the bi-stable FBM. . . .	121
5.11 A comparison between the predicted stiffness of FBM from proposed analytical model and actual stiffness obtained from experiments.	122
5.12 (a) Block diagram of the proposed positioning control system with FELA represented as (b) a mass-spring-damper system.	123
5.13 Hardware used to realize the position control loop.	126
5.14 Laser interferometer setup on FELA.	127
5.15 Position accuracy of the end-effector at neutral position.	128
5.16 Repetitive 20 nm steps generated from FELA with positioning control. . . .	128
5.17 FELA performs a ± 2 mm of large displacement stroke with high accuracy demonstrated at both ends of the stroke.	129
5.18 Position accuracy of end-effector at 2 mm.	129
5.19 Position repeatability at targeted position of 5 μm	130
5.20 Position repeatability at targeted position of 2 mm.	130
5.21 (a) Block diagram of a proposed direct-force control system. (b) Schematic diagram of a mass-spring-damper system with an additional pair of spring-damper representing the force sensor.	131
5.22 A 10-N force profile generated by FELA with ± 0.15 N of accuracy.	134
5.23 A 60-N force profile generated by FELA with ± 0.15 N of accuracy.	134
5.24 Changes in temperature during a 60-N force profiling.	135
5.25 Block diagram of the proposed mode-switching control scheme.	136
5.26 Block diagram of the proposed impedance control scheme.	136

5.27 Force, position and command signal obtained from a mode-switching control. 137

5.28 Force, position and command signal obtained from an impedance control. . . 137

5.29 Hot embossing process graph. 138

5.30 (a) FELA setup for a hot-embossing process to imprint micro-sized features
in (b) the template onto (c) the PMMA substrate. 139

5.31 (a) The imprinted micro-sized features on the substrate from (b) the micro-
sized features on the template. 139

5.32 Height measurements of the imprinted micro-sized features. 141

6.1 A group of $Z-\theta_x-\theta_y$ ultra-high precision manipulators. 144

6.2 Schematic diagram of the proposed PPS manipulator. 147

6.3 Top projection of manipulated spherical joints with respect to the base frame. 149

6.4 Proposed 3PPS FPM. 154

6.5 An active compliant prismatic joint driven by an electromagnetic driving scheme. 155

6.6 A monolithic compound linear spring with beam-based flexure joints. 155

6.7 (a) A passive compliant prismatic joint, which is formed by four parallel (b)
beam-based flexure joints, based on (c) a bi-stable design. 157

6.8 (a) An elementary rod flexure joint, (b) a compliant spherical joint and (c) an
elementary beam-based flexure joint. 160

6.9 Design concept of the proposed 5-DOF compliant joint as compared to an
elementary rod flexure. 161

6.10 Design concept of segment 3 of the proposed 5-DOF compliant joint. 162

6.11 The proposed 5-DOF compliant joint. 162

6.12 (a) The deflection path of each passive 5-DOF compliant joint from top
projection and (b) the working plane of each parallel limb. 163

List of Figures	xxi
6.13 Beam-based flexure joint coupled with a rigid-link subjected to a horizontal force.	165
6.14 Beam-based flexure joint coupled with a rigid-link subjected to a axial compressive force.	166
6.15 Beam-based flexure joint coupled with a rigid-link subjected to external moment.	167
6.16 Torsion and shearing found within a pair of beam-based flexure with a space in-between subjected to a external moment.	169
6.17 (a) A compound linear spring formed by (b) a pair of linear springs connected in series.	174
6.18 Proposed 5-DOF compliant joint subjected to an external moment about z-axis.	181
6.19 FEA modeling of the proposed 5-DOF compliant joint for external moment about the z-axis analysis.	181
6.20 (a) A limb, which is constructed from a serially-connected compliant joints, is used to (b) form each symmetrical limb of a parallel-kinematics manipulator.	183
6.21 Local coordinate frame attached to individual compliant joint and the displacement with respect to the local coordinate frame attached to the tip of each limb.	184
6.22 Local coordinate frame at the tip of each limb with respect to the reference coordinate frame at the center of the mobile platform.	190
6.23 FEA simulation of the proposed 3-PPS with the variation of limb 1 and 2 set at -3 mm and +3 mm respectively.	193
6.24 Stress concentration of the proposed 5-DOF compliant joint of limb 2 during the simulation with the displacement of limb 1 and 2 set at -3 mm and +3 mm respectively.	194
6.25 Final design of the proposed 3PPS FPM.	196
7.1 Wire-cut EDM machining.	201

7.2	Prototype of the compound linear spring.	202
7.3	Displacement stiffness evaluation of an active compliant prismatic joint. . . .	203
7.4	Experimental and the semi-analytic modeling results on the displacement stiffness of the active compliant prismatic joint.	203
7.5	Errors between the experimental and analytical results on the stiffness of the active compliant prismatic joint.	204
7.6	(a) Prototype of a passive 5-DOF compliant joint and (b) a bird-eye view of two pairs of beam-based flexure joints through the center hole of the prototype.	205
7.7	Experimental setup for investigating the deflection stiffness of the passive 5-DOF compliant joint.	206
7.8	Deflection stiffness of the 5-DOF compliant joint obtained from the experimental and analytical analyses.	207
7.9	Setup for evaluating the angular stiffness of the 5-DOF compliant joint. . . .	208
7.10	Experimental results on the angular stiffness of the 5-DOF compliant joint. .	208
7.11	A 3-DOF 3PPS FEPM prototype.	209
7.12	A 6-DOF laser tracking robot.	210
7.13	A laser interferometry setup for evaluating the positioning resolutions of the developed prototype at the end-effector.	213
7.14	The positioning resolutions at neutral position obtained from the end-effector.	214
7.15	The positioning resolutions at 2.5 mm obtained from the end-effector.	214
7.16	An increment of 20 nm steps obtained from the end-effector.	215
7.17	The photoelectric autocollimator setup for evaluating the orientation resolution of the prototype.	216
7.18	The consistency of fine orientation verse the changes in height displacement along the z-axis at the end-effector of the prototype.	219

List of Figures	xxiii
7.19 The step response of the prototype from 0 to 2.5 mm.	220
7.20 Prototype maneuvering a 15 Kg-load in an orientation of $\pm 2.5^\circ$ about the x- and y-axes.	221
A.1 Illustration of a DM configuration for surface charge model analysis.	236
C.1 ANSYS element and mesh selection for (a) SOLID45 and (b) SOLID92. . .	243
C.2 (a) ANSYS modeling of a 3D beam-based flexure joint coupled with a rigid- link and (b) the deflection obtained when subjected to a vertical force. . . .	243

List of Tables

2.1	Characteristics of existing electromagnetic-driven FPMs	31
3.1	List of slope values corresponds to deflection angles and $f(x)$	34
3.2	Parameters used in the FE analysis	54
3.3	Results obtained from the proposed semi-analytic model and PRB model on beam-based flexure joint coupled with rigid-link of various lengths	55
3.4	List of samples used for experimental investigation	56
3.5	Deflections obtained from specimen 1 through experiment, semi-analytic model and conventional PRB model.	62
3.6	Deflections obtained from specimen 2 through experiment, semi-analytic model and conventional PRB model.	62
3.7	Deflections obtained from specimen 3 through experiment, semi-analytic model and conventional PRB model.	63
3.8	Deflections obtained from specimen 4 through experiment, semi-analytic model and conventional PRB model.	63
3.9	Specifications of beam-based flexure joint used in the compound prismatic joint.	64
5.1	Selection of conducting wire based on coiling optimization.	114
5.2	Results obtained from mobility analyses on the proposed FBM designs. . . .	116

List of Tables	xxv
5.3 FE results on the proposed FBM designs for analyses 1 and 2.	117
5.4 FE results on the proposed FBM designs for analyses 3 and 4.	117
5.5 Measurements obtained from the imprinted features on the PMMA substrates and the features from the silicon templates.	141
6.1 List of maximum θ_x and θ_y values obtained from given actuator displacements	153
6.2 List of required driving force and maximum bending stress values correspond- ing to various beam-based flexure joint-thicknesses	156
6.3 List of required driving force and off-axis stiffness obtained from various beam- based flexure joint-widths	159
6.4 Analytical and FEA results for Case A1.	177
6.5 Analytical and FEA results for Case A2.	177
6.6 Analytical and FEA results for Case A3.	177
6.7 Analytical and FEA results for Case A4.	178
6.8 Analytical and FEA results for Case B1.	179
6.9 Analytical and FEA results for Case B2.	179
6.10 Analytical and FEA results for Case B3.	179
6.11 Analytical and FEA results for Case B4.	180
6.12 List of results obtained from flexure thickness selection.	180
6.13 Resultant displacements obtained from the analytical and FEA analyses on the 5-DOF compliant joint subjected to external moment about the z-axis. .	182
6.14 Maximum stress obtained from the analytical and FEA analyses on the 5-DOF compliant joint subjected to external moment about the z-axis.	182
6.15 Angular motion about the z-axis obtained from segments 1 and 2, and from segment 3 of the 5-DOF compliant joint.	195

6.16 Angular motion obtained from segments 1 and 2, and from segment 3 of the 5-DOF compliant joint. 196

7.1 Properties of several metals 200

7.2 List of the positions and orientations achieved by the prototype 211

7.3 List of the desired and actual fine orientations about the x-axis achieved by the prototype 217

7.4 List of the desired and actual fine orientations about the y-axis achieved by the prototype 218

C.1 Deflections of a beam-based flexure joint coupled with a rigid-link subjected to a point force through experimental investigations and FEA simulator with various mesh sizes. 244

List of Abbreviations

AWG	American Wire Gauge
DM	Dual-Magnet
DOF	Degrees Of Freedom
EDM	Electromagnetic Driving Module
EM	Electromagnetic
FBM	Flexure Bearing Mechanism
FE	Finite Element
FEA	Finite Element Analysis
FELA	Flexure-Based Electromagnetic Linear Actuator
FEPM	Flexure-Based Electromagnetic Parallel-Kinematics Manipulator
FPM	Flexure-Based Parallel-Kinematics Manipulator
MEMS	Micro-Electro Mechanical System
ODE	Ordinary Differentiation Equation
PM	Permanent Magnet
PMMA	Poly-Methyl-Methacrylate-Acrylic
RMS	Root-Mean-Square
UV	Ultra-Violet

List of Notations

M	external moment loading
E	Young's modulus of the material
I	second moment of area
θ	deflection angle
Θ	PRB modeling deflection angle
θ_f	classical nonlinear modeling deflection angle
ϖ	semi-analytic modeling factor for parasitic shift approximation
ρ	semi-analytic modeling factor for 'pivot' point location approximation
δ_y	deflection in the y-axis
δ_x	deflection in the x-axis
l	beam-based flexure length
t	beam-based flexure thickness
w	beam-based flexure width
L	rigid-link length
F	input force
K	stiffness of flexure joint or mechanism
q	an electric charge particle
$\mathbf{v}$	instantaneous velocity of particle
$\mathbf{E}$	electric field
$\mathbf{B}$	magnetic flux density
$\mathbf{J}$	free current density
i	input current
μ	permeability

μ_0	permeability of free space
$\mathbf{H}$	magnetic field strength of a permanent magnet
$\mathbf{M}$	magnetization of a permanent magnet
$\mathbf{D}$	electric flux density
Φ	magnetic scalar potential
$\mathbf{A}$	magnetic vector potential
B_r	remanence magnetic flux density of permanent magnet
H_c	coercivity field strength of permanent magnet
R_{coil}	internal coil resistance
m	mass
b	damping
k	spring stiffness
K_p	proportional gain
T_i	integral time
T_d	derivative time
K_c	compensation gain
ω_n	undamped natural frequency of the system
ζ	damping ratio of the system
γ	additional pole placement
T_r	rise time
b_s	damping of force sensor
k_s	spring stiffness of force sensor
σ	bending stress
A	cross-sectional area
G	modulus of rigidity
τ	shear stress
ε	shear displacement
β	twisting angle
$\mathbf{C}$	compliance matrix
$\mathbf{K}$	stiffness matrix
$\mathbf{J}$	Jacobian matrix

Chapter 1

Introduction

“Anyone who has never made a mistake has never tried anything new.”

– Albert Einstein (1879 – 1955).

Ultra-high precision manipulators are key apparatus in many nano-scale manipulation processes such as scanning microscopy and optical-lithography, etc. Success of these manipulators depends on the competency of their positioning mechanisms in achieving high repeatable motions and the capabilities of the system drivers in offering high positioning resolutions. Over the past two decades, implementations of the compliant members have become the most popular approach to achieve high repeatability motions in the ultra-high precision manipulators. Taking the advantages of elastic deformation, compliant joints can overcome the limitations of the conventional bearing-based joints such as dry friction, mechanical play, backlash and wear-and-tear etc [1]. Consequently, a compliant mechanism can offer a frictionless and highly repeatable motion making it suitable for micro/nano-scale applications [2].

Various literatures have reported numerous prototype systems achieving micro/nano manipulation with the complaint mechanisms. Jywe et al. [3] developed a five Degrees-Of-Freedom (DOF) co-planar nanometer-scale stage that achieves a workspace of $9.11 \mu\text{m} \times 9.71 \mu\text{m} \times 5.33 \mu\text{m}$ for the X - Y - Z axes with a step error of 10 nm and a rotational angle control error of 0.004°. Culpepper and Anderson [4] designed a low-cost nano-manipulator, termed HexFlex, that achieves a workspace of $100 \mu\text{m}^3$ with a positioning resolution of 5 nm.

Chen and Culpepper [5] later presented a 6-axes micro-scale nano-positioner that registered a positioning error of ± 10 nm over a workspace volume of $8.4 \mu\text{m} \times 12.8 \mu\text{m} \times 8.8 \mu\text{m}$ and $19.2 \text{ mrad} \times 17.5 \text{ mrad} \times 33.2 \text{ mrad}$ for the X - Y - Z axes, and the θ_X - θ_Y - θ_Z respectively. Choi and Kim [6] presented a 2-DOF manipulator that achieves a translational motion of $90 \mu\text{m}$ with a positioning accuracy of ± 1.5 nm. Smith et al. [7] developed a monolithic single-axis stage that demonstrated a positioning resolution of 50 nm over a traveling range of $300 \mu\text{m}$. Similarly, such compliant-based manipulators are also developed by the industries. One example includes a NanoCube by Physik-Instrumente, which has a positioning resolution of 1 nm within a workspace of $100 \mu\text{m}^3$ [8]. All above mentioned manipulators are usually driven by system drivers utilizing piezoelectric [9, 10, 11, 12], magnetostrictive [13], or friction micro-feed devices [14] to provide the necessary positioning resolutions.

1.1 Motivation

Future development of the compliant manipulators will be focusing on their competency of manufacturing miniature products with multi-scale dimensions. In Micro-Electro-Mechanical-System (MEMS) fabrication, such devices require the integration of physical and chemical processes that operate between micro- and meso-scale levels [15]. Fabrications of the optical-lithography masks [16], the micro-fluidic devices and the bio-medical scaffolds [17], which are millimeters in sizes and packed with micro-scale features, require manipulations that cover from sub-micrometers to millimeters range. In the semiconductor industry, an increasing demand for the measurement systems to orientate within larger workspace also suggests a need for nano-to-meso manipulations [18]. Most importantly, the emerging of nanoimprint lithography, i.e., Ultra Violet (UV) nanoimprint lithography, micro-contact printing, and hot-embossing, further highlight the importance of an integration between nano, micro and meso manipulations as these processes play a crucial role in massive and effective fabrications of bit pattern media, multi-layer storage media, optical components, high density inter-connectors, polymer electronics, and bio-medical apparatuses etc.

These requirements lead to the recent development in the large displacement compliant stages, which include Awtar and Slocum [19] who presented an X-Y flexure-based manipulator with a motion range of $5 \text{ mm} \times 5 \text{ mm}$, Bacher [20] who developed an ultra-high precision electro-discharge machine using an X-Y-Z compliant mechanism with a positioning repeatability of 10 nm over a workspace of 2 mm^3 , and an X-Y-Z decoupled flexure-based manipulator, which has a workspace of 1 mm^3 , developed by Tang et al. [21]. Although recent research efforts have shown promising results in manipulations between nano- and meso-scales, achieving large continuous forces throughout such motion ranges remains a major challenge. With nanoimprint lithography processes becoming future solutions for high volume meso/nano-scale product fabrications, the demands for nanometric positioning accuracy with large continuous output force throughout a workspace of a few millimeters and degrees remains a technological gap in the field of ultra-high precision manipulation. The incompetence of current ultra-high precision manipulators in bridging this gap is mainly influenced by the following issues:

1.1.1 Proficiency of Compliant Joints

Compliant joints can be classified into two varieties, a notch type and a beam-based type. First analyzed by Paras and Weisbord [22], a notch flexure joint has since being widely used in today's compliant mechanisms. Yet, it possesses high actuating stiffness and high stress concentration resulting in limited motion range. On the other hand, a beam-based flexure joint offer a larger motion range along the actuating direction as compared to the notch flexure joints. However, it suffers from poor rotational and translational stiffness in other non-actuating directions. Though recent research efforts by, e.g., Goldfarb et al. [23] and Moon et al. [24] etc., have attempted to reduce such limitations of a beam-based flexure joint with various proposed joint designs, the modeling of such joints remain highly complicated.

1.1.2 Modeling of Compliant Joints and Mechanisms

A compliant mechanism is mainly classified into two types; a fully-compliant and a partially-compliant [25]. A fully-compliant mechanism represents that the entire mechanism is flexible

with unconstraint motions. Modeling of such mechanisms is usually theoretically complex and requires the finite element approach to conduct stiffness and stress analyses. On the other hand, a partially-compliant mechanism consists of rigid links with compliant joints that provide necessary degree of motions. Such mechanisms are more predictable by analyzing the deflection of the compliant joints. Conventionally, small deflection theorem has always been employed in analyzing the behavior of these joints since they are usually notch type that possesses small deflecting ranges. However, this theorem runs into its limitations in analyzing a beam-based flexure joint that produces large but nonlinear deflection. Though various methods have been proposed to predict the nonlinearity of a beam-based joint, each of these methods still has its own limitations in providing a simple and quick solution for analyzing such flexure joints.

1.1.3 Capabilities of Nano-Positioning System Drivers

Most conventional compliant manipulators are driven through a piezoelectric actuator due to its nanometric resolution and large actuating force [26, 27, 28, 29, 30, 31, 32]. However, these actuators are not suitable for future compliant manipulators that target to achieve large traveling range, i.e., a few millimeters, because their limited stroke displacements [33, 34, 35]. Though some commercially available nano-positioning actuators are able to eliminate such limitations, the displacement amplification techniques used in these actuators have inherited other drawbacks. Some nano-positioning actuators that use piezoelectric-driven high-pitch screw to achieve millimeters of travel have poor repeatabilities due to backlash and Coulomb frictions [36, 37, 38]. Others that use piezoelectric or magnetostrictive clamping method to drive an internal shaft for achieving millimeters of strokes have low payload capacities [39, 40]. In addition, all these actuators possess a slow response speed that makes them unsuitable for high bandwidth dynamic control of a compliant manipulator.

1.2 Objectives

This thesis focuses on the investigation of a multi-DOF compliant manipulator targeted to achieve nanometric positioning accuracy with large continuous output force throughout a large workspace. To achieve such performances, this research will be mainly focusing on:

1. Establishing a new analytical model that predicts the nonlinear behavior exhibited by the beam-based flexure joints during large deflection. This new model will provide a simple, fast and accurate approximation that is essential for quick parametric studies and understandings on the beam-based flexure modules.
2. Introducing a new magnetic circuit that will increase the continuous output force of a Lorentz-force electromagnetic driving scheme, which will be employed for driving the proposed compliant manipulator. For accurate parametric studies and analytical analyses, a new mathematical model that provides a closed-form solution for predicting the magnetic field behavior within the new magnetic circuit, will be formulated.
3. Developing a novel nano-positioning system driver with large continuous output force throughout a few millimeters of traveling range. With such capabilities, this single-axis nano-positioning actuator will provide a new solution in the field of ultra-high precision manipulation.
4. Developing a new 3-DOF compliant manipulator to achieve a θ_x - θ_y - Z motion. To achieve a large workspace, a newly proposed compliant joint with multi-DOF and large displacement beam-based flexure modules will be presented and developed. Together with an enhanced Lorentz-force electromagnetic driving scheme, this compliant manipulator will achieve nanometric resolution with large continuous output force throughout a few millimeters and degrees workspace.

Consequently, these presented works will form a comprehensive and promising solution in bridging the current technological gap in the field of ultra-high precision manipulation.

1.3 Thesis Overview

This thesis presents a new class of compliant manipulators based on the amalgamation of the compliant bearings and the electromagnetic driving scheme. Termed flexure-based electromagnetic parallel-kinematics manipulators, it is applicable to any nano-scale process that requires nano-to-meso manipulations. Most importantly, it is very suitable for enhancing the nanoimprint lithography processes by realizing a workspace of few millimeters and degrees with nanometric resolutions, and providing high-precision imprinting through its direct-force control capability.

Figure 1.1: An overview of the design concepts and solutions used in realizing a multi-DOF flexure-based electromagnetic parallel-kinematic manipulator

To realize nanometric resolutions over a large workspace of a few millimeters and degrees, the beam-based flexure joints will be used as the frictionless bearings to support the non-contact actuation of an electromagnetic driving scheme. As a beam-based flexure joint offers a large but nonlinear deflection, a semi-analytic model is presented to provide a simple and quick solution for analyzing such deflections. As a result, an accurate prediction on the displacement stiffness of the flexure supporting bearings can be obtained. A Lorentz-force actuation will be employed to form the electromagnetic driving module due to its

direct non-commutation control and its linearity between the input current, and the output force. However, this form of actuation has poor current-force sensitivity that leads to low force generation. Thus, a novel Dual-Magnet (DM) configuration is proposed to enhance the strength of the magnetic field within the electromagnetic driving module so as to increase the current-force sensitivity and subsequently to achieve large force generation. A mathematical model of the proposed DM configuration is formulated for a complete and accurate magnetic field analysis. With the proposed flexure supporting bearings and the electromagnetic driving module, a single-axis nano-positioning linear actuator is developed. Termed Flexure-Based Electromagnetic Linear Actuator (FELA), it achieves a traveling range of 4 mm, a positioning accuracy of ± 10 nm (restricted by the resolution of existing encoder) and a continuous output force of 60 N/Amp. Subsequently, FELA is used to fabricate micro-channels through a hot-embossing process, which mechanically imprints the micro-scaled features from a silicon template onto the Poly-Methyl-Methacrylate-Acrylic substrates. On the other hand, FELA also serves as a platform for evaluating the accuracy of the proposed analytical models and the performance of the enhanced electromagnetic driving module that incorporated the proposed DM configurations. A 3-limbs Prismatic-Prismatic-Spherical (3PPS) parallel-kinematics configuration is introduced to realize a θ_x - θ_y - Z motion. For parametric studies and analyses, a direct-kinematic analysis of this 3PPS parallel-kinematics configuration is presented. Based on the proposed configuration, a 3-DOF compliant manipulator, termed Flexure-Based Electromagnetic Parallel-Kinematics Manipulator (FEPM), is developed. Here, a 5-DOF compliant joint with large deflections and orientations is presented and employed by the developed compliant manipulator. A complete set of analytical models is derived for analyzing the stiffness in all driving and non-driving directions of the proposed 5-DOF compliant joint. Driven by the enhanced electromagnetic driving modules, this 3-DOF 3PPS FEPM achieves a workspace of $\pm 2.5^\circ \times \pm 2.5^\circ \times \pm 2.5$ mm ($\theta_x \times \theta_y \times Z$), an open-loop positioning resolution of ± 10 nm, an open-loop orientation resolution of $0.05''$, and a continuous output force of 160 N/Amp. With such capabilities, this 3-DOF 3PPS FEPM is a promising solution for realizing the active co-planar nano-alignment between the template and the substrates, while enhancing the imprinting process in the nanoimprint lithography processes.

1.4 Organization of the Thesis

The remaining chapters of this thesis are organized as follows. Chapter two reviews the past development of the ultra-high precision manipulators in relation to the various types of compliant joints and the electromagnetic driving schemes. It also covers the latest research efforts where the beam-based flexure joints and electromagnetic driving schemes are adopted to develop future ultra-high precision manipulators targeted for larger workspace. Chapter three presents the derivations of a semi-analytic model for analyzing the nonlinear deflection of the beam-based flexure joints. Theoretical validations between the conventional analytical models and the semi-analytic model will be discussed. This chapter also covers the experimental investigations for evaluating the accuracy of the semi-analytic model and discusses on the evaluation results in details. Chapter four presents a DM configuration adopted by the electromagnetic driving modules that are used to drive the proposed nano-positioning linear actuator and compliant manipulator. The design concept and advantages of this DM configuration are discussed. This chapter covers the derivation of a mathematical model that offers high accuracy in predicting the magnetic field behavior within a DM configuration. Experimental investigations for evaluating the accuracy of the proposed mathematical model and the derived current-force model for the proposed electromagnetic driving module are documented in this chapter. Results from these investigations are also discussed in details. Chapter five describes the development of FELA through the integration of the beam-based flexure supporting mechanism and the enhanced electromagnetic driving module. Here, the displacement stiffness model of the beam-based flexure supporting mechanism derived through the proposed semi-analytic modeling is evaluated using the developed prototype. Implementations of the position, direct-force, and impedance control schemes on FELA prototype are discussed in details. This chapter also covers the implementation of the FELA prototype in fabricating micro-sized features through a hot-embossing process. Experimental setup and investigation results are documented in details. Chapter six presents the design concept of a 3-DOF FEPM in which a 3PPS parallel-kinematics configuration is proposed and forward kinematic analysis of this configuration is derived. The stiffness modeling of all proposed compliant joints that forms

each symmetrical $\underline{\text{PPS}}$ limb is documented. In this chapter, a stiffness model of the 3-DOF $3\underline{\text{PPS}}$ FEPM is established and the design refinements of this compliant manipulator are discussed. Chapter seven presents the development and evaluations of the 3-DOF $3\underline{\text{PPS}}$ FEPM prototype. In this chapter, the selections of the materials and machining processes to fabricate each compliant joint are discussed in details. In addition, the derived analytical stiffness models for each compliant joint are evaluated through the developed prototypes and all experimental setups, and evaluation results are documented. Most importantly, all various experimental setups used to evaluate the capabilities of the developed prototype and the investigation results will be discussed in details. Chapter eight concludes the work done in this research and highlights the contributions, and the future work of this research.

Chapter 2

Literature Review

“The fundamental qualities for good execution of a plan is first; intelligence; then discernment and judgment, which enable one to recognize the best method as to attain it.”

– Ferdinand Foch (1851 - 1929)

A marriage between the compliant joints and electromagnetic driving schemes forms the fundamental methodology in developing the proposed compliant manipulator. This chapter reviews the contributions of these two areas in the field of ultra-high precision manipulations.

2.1 Compliant joints

The use of a compliant joint can be dated as early as 1826, when Ohm first replaced the original torsional members of a classical torsion balance with metal strips to increase its precision of measuring fine torque when subjected to the mechanical or electrical loads [41]. In 1902, H. A. Roland also adopted such slender strips in supporting his ruling engine for grating diffraction (Fig. 2.1) so as to avoid the effects of friction [42]. By the dawn of World War II, these shock-proof torsional leaf-springs have been increasingly used in electrical instruments to replace jeweled pivots [41]. Subsequently, the compliant joints became widely used in developing highly sensitive measurement instruments, highly accurate load cells for force measurement and the pendulum pivots of miniaturized force-balance accelerometers etc [43, 44].

Over the last 50 years, compliant joints have edged out among the competitors, such as the hydrostatic, hydrodynamic, air and magnetic bearings etc, by offering a simple, low cost, maintenance free, and non-powered solution for achieving frictionless and non-contact support. Taking the advantages of their inherited elastic behavior, the compliant joints become the cheapest yet most effective kinematic joints to achieve nanometric and repeatable positioning by avoiding the coulomb frictions, and backlash exhibited in the conventional ball-bearing joints. The compliant joints are known as flexures, which are mainly classified into two types; a notch hinge and a leaf-spring. The used of a leaf-spring is the pre-war (before World War II) efforts of achieving high-precision motions through the elastic movements while a notch hinge is the post-war effort, which plays a primary role in the modern history of compliant mechanism.

Figure 2.1: H. A. Roland with his ruling engine for diffraction gratings [42].

2.1.1 Leaf-Spring

The earliest form of a compliant joint is a leaf-spring, which can be easily formed by a metal slender strip (Fig. 2.2a). The earliest attempted to use a leaf-spring for high-precision motions can be found in a torsion balance when Ohm employed the leaf-springs as the torsional members. Absence of 'sticking' effects makes it possible to register very small changes in torque with meaningful observations of 10^{-9} radian change in orientations [41]. A cross-strip hinge, which was later introduced to increase the stiffness of non-actuating directions except the rotational direction (Fig. 2.2b), was well-adopted by subsequent torsion balances. In 1950, Jones [45] realized the advantages of the elastic motions and replaced the

Figure 2.2: (a) A leaf-spring hinge and (b) a double cross-strip hinge [43].

dovetail or ball-bearing slides found in the optical slit mechanism of an infrared spectrometer with leaf-springs to support the slit jaws (Fig. 2.3a). As a result, he found no backlash between the micrometer and the jaws, and could translate the resolution of the micrometer directly onto the optical slit mechanism. Leaf-springs were also used in other measuring devices, e.g., Michelson interferometer mirror positioner (Fig. 2.3b) and the highly sensitive seismograph for measuring earthquakes (Fig. 2.3c) etc.

Figure 2.3: (a) An optical slit mechanism, (b) a parallel spring mechanism for positioning the Michelson interferometer mirror and (c) a gravimeter-vertical seismograph [45].

The use of leaf-springs requires additional assemblies that generally affect the precision of a mechanism. In addition, the leaf-springs have poor stiffness in other non-driving directions, which will further deteriorate the precision when subjected to off-axis external loading. To avoid these issues, the use of the notch flexure joints (or notch hinges) becomes a promising solution to replace these traditional leaf-springs. Using a monolithic-cut approach to produce these notch hinges, the entire mechanism can be fabricated from a single piece of workpiece to form a monolithic flexure mechanism with no assembly will be involved (Fig. 2.4).

Figure 2.4: A leaf-spring compound linear spring mechanism V.S. a monolithic compound linear spring mechanism [42].

2.1.2 Elementary Notch Hinge

The simplest form of a notch hinge (Fig. 2.5a) is made from a circular shape, which incorporates a circular cutout on both side of a blank to form a necked-down section. This necked-down section, which serves as a fixed center of rotation, exhibits an almost pure rotational motion within a dedicated range. In 1965, Paras and Weisbord [22] first presented a complete analysis of such a notch hinge, which soon gained tremendous popularities among researchers in developing their compliant mechanisms. Yet a circular shape leads to high stress concentrations during operations and researchers are forced to look into the optimizations of these notch hinges. Consequently, the elliptical (Fig. 2.5b) and small-radius (Fig. 2.5c) shapes are introduced to avoid such high bending stresses and have been well-studied by Xu [46], Smith et. al [47], Tseytlin [48], and Lobontiu [49, 50].

Figure 2.5: Three types of notch flexure joints; (a) a circular shape, (b) an elliptical shape and (c) a small-radius shape.

2.1.3 Serially-Connected Flexure Joints

Elementary notch hinge alone does not offer dexterous motions that are much needed in a multi-DOF compliant mechanism. The fundamental approach to realize a higher dimensional

workspace is usually achieved by connecting these elementary notch hinges in series. One of the earliest forms of a serially-connected flexure joint (Fig. 2.6a) was found in a linear spring mechanism (Fig. 2.6b), which was suggested by Thompson in 1955 [42]. This mechanism employs serially-connected flexure joints to form the parallel limbs between the fixed member and the moving member. Such a mechanism operates as a compliant prismatic joint, which is used in micro-motion stages, to achieve highly repeatable translational motions [51, 52].

Figure 2.6: (a) Serially-connected flexure joints, (b) a linear spring stage and (c) a compliant prismatic joint.

However, a parasitic motion perpendicular to the actuation direction exists in these linear spring mechanisms. To avoid this parasitic motion, a common approach is to combine two linear spring mechanisms to form another configuration of single-axis translational compliant prismatic joint (Fig. 2.6c). This combination also formed another variation of compliant prismatic joint known as a compound linear spring mechanism (Fig. 2.7a), which is a popular choice for realizing a rectilinear translational motion.

Figure 2.7: (a) A compound linear spring mechanism and (b) a symmetrical compound linear spring mechanism.

To enhance the insensitivity of this translational mechanism towards external disturbances, a symmetrical design is adopted. Consequently, a symmetric compound linear spring mechanism (Fig. 2.7b) has very low susceptibility to position error caused by the thermal expansion [53]. One typical example of this mechanism is presented by Ho et al. who developed a single-axis flexure-based positioner for the wafer-bumps inspections with a positioning resolution of 50 nm over 100 μm of travel [54].

To achieve higher degrees of motion, the simplest approach is by stacking one flexure mechanism onto another in a series configuration (Fig. 2.8a). This stacked serial-kinematics architecture is well adopted by several compliant stages, including an X-Y nanopositioning stage (Fig. 2.8b) from Physik Instrumente (PI) GmbH & Co. [55]. This stage claims a positioning resolution less than 1 nm over a workspace of 100 μm^2 .

Figure 2.8: (a) A stacked serial-kinematics approach used in (b) a X-Y nanopositioning stage from PI [55].

Another common approach is to nest a single-axis compliant mechanism into another similar compliant mechanism (Fig. 2.9a). Such a nested serial-kinematics architecture has been used to develop several multi-DOF nano-positioning stages [56, 57, 58]. One particular example includes the 2-DOF translational optical lens steering stage (Fig. 2.9b) developed by U.S. National Institute of Standard and Technology (NIST) for space communication purposes [59].

Figure 2.9: (a) A nested serial-kinematics approach used in (b) a biaxial micro-positioning stage from NIST [59].

Unfortunately, the precisions of the end-effectors in these stacked or nested compliant stages are usually affected by the accumulated position errors. Behavior of these stages at elevated speed becomes unpredictable due to the external loading factor. In addition, these stages have non-symmetric natural frequencies, i.e. the top mechanism has a higher resonant frequency than the bottom mechanism for stacked approach, while the embedded mechanism has a higher resonant frequency than the outer mechanism for nested approach.

2.1.4 Parallel-Kinematics Architecture

To obtain a higher precision and better performance in multi-DOF manipulations, a parallel-kinematics architecture design is widely adopted by today's multi-DOF compliant mechanisms. This architecture plays an important role in the success of these compliant mechanisms due to its advantages of a lower inertia, programmable centers of rotations, superior dynamic behavior and insensitivity to external disturbances, e.g. thermal expansion. In addition, parallel-kinematics architecture is realized by a number of parallel limbs, where each limb is formed by a series of joints. Consequently, the concept of serially-connected flexure joints can be adopted to form the parallel limbs. Most importantly, the limited displacement of a serially-connected flexure joints suits the nature of a parallel-kinematics architecture, which is known for its limited workspace. As a result, the fusion between the serially-connected flexure joints and parallel-kinematics architecture becomes an ideal solution for developing ultra-high precision multi-DOF compliant manipulators.

In this thesis, such a compliant manipulator is termed Flexure-Based Parallel-Kinematics Manipulator (FPM).

2.1.5 Planar-Motion Flexure-Based Parallel-Kinematics Manipulators

Planar-motion FPMs constructed using the serially-connected flexure joints are well demonstrated by several authors. Ryu et al. [60] developed an X - Y - θ_z FPM based on three parallel double compound flexure pivoted mechanical lever formed by a set of serially-connected flexure joints (Fig. 2.10a). This planar-motion FPM (Fig. 2.10b) achieves a positioning resolution of 8 nm along the X - and Y -axes, and a rotational resolution of $0.057''$ about the Z -axis. This stage has a total traveling range of $41.5\ \mu\text{m}$ and $47.8\ \mu\text{m}$ along X - and Y -axes respectively, with a total rotation range of $322.8''$ ($\sim 1.565\ \text{mrad}$).

Figure 2.10: (a) A double compound flexure pivoted mechanism lever as one of the parallel limb used in (b) a X - Y - θ_z wafer positioning stage [60].

Lee and Kim [61] employed several serially-connected flexure joints to form a parallel limb that provides two translational motions in X - and Y -directions (Fig. 2.11a). Such parallel limbs are subsequently used to construct an X - Y - θ_z ultra-precision stage (Fig. 2.11b) for aligning a wafer size of 200 mm and achieves a resolution of 10 nm and $0.2''$ in the translational and rotational axes respectively.

Figure 2.11: (a) A X-Y translational serially-connected flexure joint used in (b) a 3-DOF wafer alignment stage [61].

Yi et al. [62] replaced the bearing-based Revolute (R) hinges of a conventional 3RRR planar parallel mechanism with three serially-connected flexure joints (Fig. 2.12a) to develop a 3-DOF parallel micro-mechanism (Fig. 2.12b). This high-precision stage achieves an X-Y translational workspace of $100\ \mu\text{m}^2$ with a rotational range of 0.1° .

Figure 2.12: (a) A 3RRR compliant mechanism used in (b) a 3-DOF parallel micro-mechanism [62].

2.1.6 Spatial-Motion Flexure-Based Parallel-Kinematics Manipulators

Other than the planar motions, orthogonal motions can also be realized through serially-connected flexure joints. Figure 2.13a illustrates a classical 2-DOF (or universal) flexure joint without intersecting axes, which is made by machining two elementary flexure hinges at 90° to each other. Another classical universal flexure joint with orthogonal intersecting

axes can be realized by combining several elementary flexure joints with an intermediate member (Fig. 2.13b). With the advancement in electro-discharge machining in the 90s', the modern universal flexure joints are more compact and offer common rotational center axes (Fig. 2.13c). Most importantly, these universal joints have played an important role in realizing various types of spatial-motion FPMs [63, 64, 65, 66, 67, 68, 69, 70].

Figure 2.13: (a) A classical universal flexure joint without intersecting axes, (b) a classical universal flexure joint with orthogonal intersecting axes and (c) a modern universal flexure joint with a common rotational center.

A well-known example of a spatial-motion FPM is presented by Henein [71] in Ecole Polytechnique Fédérale de Lausanne (EPFL). Termed Delta-Cube (Fig. 2.14a), it is constructed based on Prof. Clavel's parallel-kinematics delta robot (Fig. 2.14c), where each limb is formed by three rigid-links coupled together through the universal flexure joints (Fig. 2.14b). This FPM has a size of 100 mm^3 and provides an X-Y-Z translational motion with a positioning repeatability of 100 nm.

Figure 2.14: (a) Delta-Cube and (b) one of its parallel limb based based on (c) a parallel-kinematics delta robot concept [71].

Figure 2.15: (a) A 3SPS FPM [72] and (b) a 3RPPR FPM [73]

Other spatial-motion FPMs made from a series of Prismatic (P) and Spherical (S) flexure joints can also be found in past literature. Examples include an X-Y-Z FPM proposed by Hara and Sugimoto [72], which was constructed based on three sets of S-P-S serially-connected flexure joints (Fig. 2.15a). Tanikawa et al. [73, 74, 75] developed another form of spatial 3-DOF translational FPM (Fig. 2.15b) with three sets of R-P-P-R serially-connected flexure joints. This FPM has a workspace of $100\ \mu\text{m} \times 100\ \mu\text{m} \times 20\ \mu\text{m}$ and a positioning accuracy of less than 100 nm for all three axes.

Figure 2.16: (a) A Steward platform FPM [76] and (b) a hybrid 6-DOF FPM [77]

Using six sets of S-P-S serially-connected flexure joints, Oiwa et al presented a 6-DOF compliant Steward-Platform manipulator (Fig. 2.16a), which has a translational accuracy of 160 nm and rotational accuracy of $2\ \mu\text{rad}$ [76]. Recently, Zuo et al. [77] from Massachusetts Institute of Technology (MIT) constructed a hybrid 6-DOF FPM (Fig. 2.16b) by combining

an X - Y - θ_z FPM and a Z - θ_x - θ_y FPM together to achieve a workspace of $140\ \mu\text{m} \times 140\ \mu\text{m} \times 5\ \text{mm} \times 2.4^\circ \times 2.4^\circ \times 7.6^\circ$.

A noticeable characteristic of these FPMs is that they have very limited workspace (in hundreds of micrometers) and demands large actuating forces to generate any motion. This is mainly due to the limited elastic deformation and high deflection stiffness of a notch hinge. In addition, a notch hinge is constantly exposing to high-level of stress concentration during operations, which directly increases the fatigue of hinges. Most importantly, the solid-state piezoelectric actuators, which are commonly used to drive these FPMs, have limited displacement strokes of up to several hundreds of micrometers. Consequently, other driving schemes are explored by researchers in search for a better solution to achieve nanometric resolutions with large traveling range of at least a few millimeters.

2.2 Electromagnetic Driving Scheme

Electromagnetic driving scheme is perceived to have the potential for developing ultra-high precision systems with large traveling range. Such a driving scheme offers a non-contact nature and frictionless drive that is much desired in ultra-high precision manipulations. Over the past two decades, ultra-high precision positioning stages based on this driving scheme are heavily investigated. These research efforts have demonstrated the competency of this driving scheme in providing millimeters of displacement, positioning resolutions in sub-nanometers with fast accelerations and speed responses. To realize an electromagnetic driving scheme, three different types of techniques are employed separately by the existing electromagnetically driven stages. These three techniques are the electromagnetic propulsion, the magnetic suspension, and the Lorentz-force actuation.

2.2.1 Electromagnetic Propulsion

An electromagnetic propulsion technique is simply the attraction or repulsion of a ferromagnetic moving part. To initiate a motion, the fixed stator either generates an electromagnetic attractive force to propel the ferromagnetic moving part towards it or generates a repulsive

force to drive the moving part away from it (Fig. 2.17). As a result a translation motion is realized. This technique generates higher actuating force as compared to the other electromagnetic driving techniques. Hence, it is employed if the moving stages with good force-to-size ratio or when maneuver of high payloads are necessary.

Figure 2.17: A schematic representation of a conventional electromagnetic propulsion technique [78].

Figure 2.18: (a) Electromagnetically driven X-Y-θz stage and (b) schematic plot of all three motions [79].

A few authors have utilized such a technique for ultra-high precision positioning. This include Higuchi and Yamaguchi [80] who used electromagnetic attractive forces to actuate position servomechanism, and Chen et al. [79, 81] who developed an X-Y-θz stage (Fig. 2.18) through an electromagnetic propulsion scheme. The latter gives a good demonstration on the utilization of an electromagnetic propulsion technique for the development of an ultra-high precision stage. This stage produces a minimum positioning resolution of 50 nm with

a translational range of $\pm 80 \mu\text{m}$ and a rotational range of $\pm 3.52 \text{ mrad}$. In addition, the electromagnetic members generate a driving force of 50 N with an input current of 0.5 Amp under a air-gap separation of $250 \mu\text{m}$.

Recently, Culpepper and Anderson [4] from MIT presented a 6-DOF fully-compliant manipulator driven by the electromagnetic propulsion technique. Measurement over a workspace volume of 100 nm^3 show a positioning resolution better than 5 nm, while achieving an open-loop error of $20 \mu\text{m}$ over a workspace volume of $100 \mu\text{m}^3$. Termed HexFlex, this manipulator, which is built under USD \$2000, is used as an ultra-precision fiber-optic aligning stage (Fig. 2.19). Chen and Culpepper [5] later developed a smaller version, termed $\mu\text{HexFlex}$, which achieves a positioning resolution of 8 nm with a maximum range of $8.4 \mu\text{m} \times 12.8 \mu\text{m} \times 8.8 \mu\text{m}$ in the X-Y-Z translations and $19.2 \text{ mrad} \times 17.5 \text{ mrad} \times 33.2 \text{ mrad}$ in the X-Y-Z rotations.

Figure 2.19: (a) A HexFlex and (b) its schematic breakdown [4].

However, all electromagnetic propulsion stages have very small traveling range because such a technique requires a very-small air-gap between the Permanent Magnet (PM) moving stage and the fixed solenoid core (in the order of several hundred micrometers) to optimize its perform. Thus, it limits the traveling range of such stages. In addition, the magnetic hysteresis and Eddy-current effect are introduced due to the electro-magnetization of the ferromagnetic stators. Consequently, these stages inherit the nonlinearities of such effects, which increase the complexity of the control schemes.

2.2.2 Magnetic Levitation

A magnetic levitation (or suspension) technique for ultra-high precision positioning has been the most popular approach for non-contact multi-DOF manipulations. Similar to a stepper motor working principal, the variable-reluctance type of stator polarization attracts and repels the PM-rotor to generate a desirable motion. This technique requires no mechanical bearings to support the moving stages. Hence, the absence of mechanical restrictions allows higher DOF with higher speed manipulations to be achieved. These properties are necessary for the development of a ultra-high precision manipulator with large workspace, high positioning resolution and high control bandwidth.

An early example of a magnetic levitated planar stage is the Sawyer motor (Fig. 2.20) that is invented in 1968. This X-Y translational motor, which is variable reluctance type and used without any positioning feedback, achieves a step resolution of $250\ \mu\text{m}$. The original version of the Sawyer motor was then improved for high-precision positioning applications with a particular emphasis on the semiconductor wafer alignment [82, 83]. With the advancement of PM material, PM-structured moving stage was used to enhance the performance of the Sawyer motors [84, 85, 86].

Figure 2.20: (a) Working principle of a Sawyer motor and (b) the arrangement of stator and rotor of a Sawyer motor [87].

A good example of utilizing magnetic levitation for ultra-high precision manipulation is none other than Prof. Hocken's Angstrom machine (Fig. 2.21a) from University of Carolina,

Charlotte (UNCC). Holmes [88] constructed this Angstrom machine using twelve magnetic bearings for actuation and achieves a positioning noise of 0.2 nm peak-to-peak measurements throughout a workspace volume of $100 \mu\text{m}^3$ [89, 90, 91]. Another prominent work in this area was presented by Prof. Trumper [78] of MIT who developed a magnetic suspension system using seven magnetic bearings to manipulate an object with 5-DOF of motions and achieves a linear travel of 50 mm with a position stability of 5 nm peak-to-peak. Later, Hocken and Trumper [92] presented a Sub-Atomic Measuring Machine (SAMM) as shown in Fig. 2.21b. SAMM demonstrated a Root-Mean-Square (RMS) positioning noise of 0.12 nm in the x-axis, 0.082 nm in the y-axis, 1.45 nm in the z-axis, 22 nrad about the x-axis, 18 nrad in about the y-axis, and 2.7 nrad about the z-axis, and a workspace of $25 \text{ mm} \times 25 \text{ mm} \times 100 \mu\text{m}$.

Figure 2.21: (a) Angstrom machine [88] and (b) Sub-Atomic Measuring Machine [92].

Figure 2.22: (a) A 6-DOF high-precision magnetic levitation stage [93] and (b) a 6-DOF magnetic levitation actuator [94].

Kim and Trumper [93, 95] later developed another 6-DOF high-precision magnetic levitation stage (Fig. 2.22a), which achieves a translational workspace of $50 \text{ mm} \times 50 \text{ mm} \times 400 \mu\text{m}$ with a positioning resolution of 5 nm RMS and an acceleration of 1 g . Kim and Masheshwari [94] extended this technique to develop an 6-DOFs actuator (Fig. 2.22b) with a translational range of $\pm 500 \mu\text{m}$ and a step response in the order of 50 nm . The authors presented a novel configuration that offers a very lightweight and compact actuator, which lead to high bandwidth (on the order of 100 Hz) and high acceleration of 3 g .

Subsequently, Verma et al. [96] demonstrated Kim's concept with another type of six-axes nano-positioning stage, termed Δ -stage (Fig. 2.23a). This stage has a positioning resolution of 5 nm RMS and covers a traveling range of $300 \mu\text{m}$ in the X-Y-Z translations, and a rotation range of 3.5 mrad about the X-Y-Z axes. Recently, Kim et al. [97] further improved this magnetic levitation stage by presenting the Y-stage (Fig. 2.23b), which achieves a higher positioning resolution of 3 nm RMS over a traveling range of $5 \text{ mm} \times 5 \text{ mm}$.

Figure 2.23: (a) Δ -stage [96] and (b) a Y-stage [97].

Unfortunately, all magnetic levitation stages have unstable behavior and are expressed with highly nonlinear differential equations. Though linear modeling about a nominal operating point is commonly used to design these magnetic levitation stages, performance deteriorates rapidly with deviation from the nominal operating point. Such deviations are caused by the change in suspending masses and the variation of resistance or induction from electromagnet heating, etc. Nevertheless, large displacements with high positioning resolutions can only be ensured when only a nonlinear model is considered. This resulted

in the utilizations of highly costly and complex control systems to facilitate these stages in achieving the desired performances.

2.2.3 Lorentz-Force Actuation

A Lorentz-force actuation provides another form of frictionless drive through a direct non-commutation control. This technique offers a constant output force with positioning resolution throughout the entire travel without any complex algorithm and costly systems. Its working principal can be found in a conventional Voice-Coil (VC) actuator, where the magnetic field from a PM and the applied current through the conducting coils are used to generate a force. Subsequently, this generated force drives the moving member of a VC actuator and generates a translational motion. The moving member is usually an air-core coil with a fixed PM stator (Fig. 2.24a). This technique offers numerous advantages. First, it delivers infinite positioning sensitivity, limited only by the encoder used for the feedback. Second, a constant output force throughout the entire traveling ranges. Third, the light-weight moving-coil generates extremely high acceleration.

Figure 2.24: (a) A moving coil configuration and (b) a moving magnet configuration [98].

Another configuration of this technique is to fix the coil while moving the magnet instead (Fig. 2.24b). This configuration offers several advantages over a moving-coil configuration. First, the wound coils around the stationary core are connected to the housing structure so that the housing can be used as a heat sink to effectively dissipate heat. Second, more windings could be used to increase the output force without changing the moving mass. Lastly, the moving PM has higher positioning accuracy without the lead wires.

With such advantages, a few high-precision stages using a Lorentz-force actuation can be found in recent literature. Mori et al. [99] demonstrated a highly precise tracking positioner using such a technique. With air bearing as a linear guide, this single-axis positioner achieves a positioning resolution of 0.5 nm through a simple controller (Fig. 2.25a).

Figure 2.25: (a) A Lorentz-force positioner (b) a 3-DOF highly-precise direct drive positioner.

Another good example of utilizing a Lorentz-force actuation to realize large displacements, high positioning resolutions and high bandwidth actuation is demonstrated by Sprenger et al. [100, 101]. The authors developed a 3-DOF positioner using four linear-drive actuators with a planar air bearing (Fig. 2.25b). This positioner has an X-Y translational workspace of 60 mm² together with a rotational range of ± 1.5 mrad. It offers a positioning resolution of 4 nm (restricted by sensor resolutions) and an acceleration of 3 g limited to the used of power amplifiers. Experiment results have also shown that a 30 mm translational stroke can be achieved within 90 msec with a transient overshoot smaller than 900 nm. In addition, the stationary positioning noise is smaller than 30 nm, which is dependent to the stiffness of the used supporting structure.

2.3 Towards A Large Workspace of Few Millimeters and Degrees

Developments of electromagnetically driven stages in recent years have inspired researchers to explore such driving schemes to assist the FPMs in achieving a large workspace of few millimeters and degrees. Increase in traveling range hence requires the compliant joints to

exhibit higher flexibility. Consequently, a leaf-spring or beam-based flexure joint is commonly employed due to its ability of achieving large deflections. One simple example of such a combination is a 3-DOF planar FPM presented by Fukada and Nishimura [102]. Constructed by leaf-springs and driving by VC actuators (via Lorentz-force actuation), this X-Y- θ_z stage (Fig. 2.26a) achieves a travel range of 1 mm with a resolution of 2 nm in the x-y translations and 0.01° about the z-axis.

Figure 2.26: (a) A planar 3-DOF FPM [102] and (b) Tribias 6-DOF robot [103].

Figure 2.27: (a) Delta³ II [104], (b) Sigma-6 [105] and (c) beam-based flexure joints.

Researchers from EPFL have made tremendous efforts in developing electromagnetic driven spatial FPMs. Pernette [103] developed a 6-DOF FPM, termed Tribias (Fig. 2.26b), which achieves ± 10 mm in the x- and y-axes, ± 5 mm in the z-axis, $\pm 2.5^\circ$ in the x-y-z rotations

with repeatabilities of 100 nm. Bacher [104] presented an improved version of Delta-Cube, termed Delta-Cube II (Fig. 2.27a), which achieves ± 2 mm in the x-y-z translations with a positioning repeatability of 10 nm. Helmer [105] constructed a 6-DOF FPM, termed Sigma-6 (Fig. 2.27b), which achieves a workspace of ± 5 mm in all translations and $\pm 5^\circ$ in all rotations with repeatabilities of 10 nm and 100 nrad. These FPMs are constructed with parallel limbs made of serially-connected beam-based flexure joints (Fig. 2.27c).

With these beam-based flexure joints beginning to look promising for achieving large displacement motions, recent development of the compliant mechanisms have also adopted such compliant joints. Choi and Kim [6] developed a planar 2-DOF FPM, which is constructed by the monolithic-cut beam-based flexure joints (Fig. 2.28a). This FPM achieves a positioning accuracy of 3 nm in the x-y axes yet has a traveling range of 90 μm due to the limited stroke of the piezoelectric actuators. Awtar and Slocum [19] presented a 2-DOF FPM that is also constructed based on the beam-based flexure joints (Fig. 2.28b). This 300 mm $\times$ 300 mm FPM achieves a motion range of 5 mm $\times$ 5 mm.

Figure 2.28: Various types of X-Y FPMs constructed by the beam-based flexure joints.

A chart in Table 2.1 summarized the characteristics of the electromagnetic-driven FPMs mentioned in this section. It shows that the combination of beam-based flexure joints and electromagnetic driving schemes has delivered promising results in achieving Macro-to-Nano scale manipulation. Nevertheless, all these electromagnetic-driven FPMs face two common issues that are yet to be resolve. First, a beam-based flexure joint exhibits nonlinear deflection that requires complex modeling even for quick parametric studies and analyses.

Second, the electromagnetic driving schemes used by these FPMs either possess inconsistent actuating force and highly complex control system or low output force and poor force-to-size ratio. Consequently, these electromagnetic-driven FPMs could only be used to manipulate non-contact processes, e.g., micro-electro-discharge-machining and optics manipulations etc. With the rise of nano-scale fabrications through mechanical imprinting techniques, both issues must be addressed in order to deliver a complete solution for driving towards a large workspace of few millimeters and degrees.

Table 2.1: Characteristics of existing electromagnetic-driven FPMs

System	DOF	Total Dimension	Workspace	Resolution
Planar FPM	3	$14.4 \times 10^6 \text{ mm}^3$	$\pm 1 \text{ mm (x, y)}$ $100 \text{ arcsec } (\theta_z)$	2 nm 0.01 arcsec
Tribias	6	$10 \times 10^6 \text{ mm}^3$	$\pm 10 \text{ mm (x, y)}$ $\pm 5 \text{ mm (z)}$ $\pm 2.5^\circ (\theta_x, \theta_y)$	100 nm 500 μm -
Delta ³ II	3	$8 \times 10^6 \text{ mm}^3$	$\pm 2 \text{ mm (x, y, z)}$	10 nm
Sigma-6	6	$6 \times 10^6 \text{ mm}^3$	$\pm 4 \text{ mm (x, y, z)}$ $\pm 4^\circ (\theta_x, \theta_y, \theta_z)$	10 nm 0.054 arcsec
X-Y FPM	2	$300 \times 300 \text{ mm}^2$	$\pm 2.5 \text{ mm (x, y)}$	-

Chapter 3

Large Deflection Beam-Based Flexure Modeling

“I keep the subject of my inquiry constantly before me, and wait till the first dawning opens gradually, by little and little, into a full and clear light.”

– Sir Isaac Newton (1642 – 1727).

3.1 Background

Implementation of the compliant joints on the positioning mechanisms has been the most popular approach to achieve ultra-high precision motions. Taking the advantages of elastic deformation, such compliant joints can overcome the limitations of the conventional bearing-based joints such as coulomb friction, mechanical play, backlash and wear-and-tear [87]. Consequently, these compliant-joint mechanisms offer a frictionless and highly repeatable motion with nanometric resolutions, making them suitable for high precision positioning applications [53]. As mentioned in Chapter 2, a compliant joint can be classified into two categories, a notch hinge and a beam-based flexure joint. First analyzed by Paras and Weisbord [22], a notch flexure joint has since being widely used in today's ultra-high precision mechanisms. Yet, it possesses high deflection stiffness and high stress concentration that result in limited motion range. On the other hand, a beam-based flexure joint offers a larger deflection due to its higher flexibility. Consequently, it becomes a promising

solution for developing cheap kinematic bearings with millimeters of displacements and nanometric positioning resolutions for next generation of ultra-high precision mechanisms [6, 19]. Unfortunately, a beam-based flexure joint, which is commonly treated as a cantilever beam, exhibits nonlinear deflection behavior due to a parasitic shift of the ‘pivot’ point that causes the deflection to be greater than the length of the beam [106].

3.1.1 Small Deflection Theorem

The Bernoulli-Euler law states that the bending moment at any point of the bar is proportional to the change in the curvature caused by the action of load

$$\frac{M}{EI} = \frac{d\theta}{ds} = \frac{d^2y/dx^2}{[1 + (dy/dx)^2]^{\frac{3}{2}}} \quad (3.1)$$

where M is the moment, E is the Young’s modulus of the material, I is the beam second moment of area, $d\theta/ds$ is the rate of change in angular deflection along the beam, and can be expressed in rectangle coordinates which forms a nonlinear ordinary differential equation due to the square of the slope, $(dy/dx)^2$. As bending moment is a function of x , Eqn. (3.1) can be re-expressed as

$$\frac{M}{EI} = \frac{d^2y/dx^2}{f(x)} \quad (3.2)$$

where $f(x) = [1 + (dy/dx)^2]^{\frac{3}{2}}$.

For many years, the square of slope, $(dy/dx)^2$, has been approximate to zero based on small deflection assumption. This assumption allows that $f(x) = 1$ and leads to the classical beam-moment-curvature equation

$$M = EI \frac{d^2y}{dx^2} \quad (3.3)$$

Table 3.1: List of slope values corresponds to deflection angles and $f(x)$

dy/dx	deg θ	$f(x)$
0.01	0.6	1.0001
0.05	2.9	1.0037
0.10	5.7	1.0150
0.25	14.0	1.0952
0.50	26.6	1.3976
1.00	45.0	2.8281
2.00	63.4	11.1857

Table 3.1 shows a list of value of the slope corresponding to the deflection angle and $f(x)$ obtained based on Eqn. (3.2). When value of slope is very small, the value of $f(x)$ is indeed close to 1. However, it is noticeable that as value of the slope increases, $f(x)$ increases. Hence, this comparison shows that $(dy/dx)^2 = 0$ is justified provided the deflections are very small as compared to the length of the cantilever beam. Yet as deflection increases, $f(x)$ slowly becomes a major contribution to the solution of large deflections. This suggests that the classical beam-moment-curvature equation becomes less applicable in predicting such situations. Consequently, $(dy/dx)^2$ must be considered during large deflection analysis.

3.1.2 Previous Works

Throughout the past five decades, tremendous amount of works have been done in modeling the large deflection cantilever beam. These modeling approaches can be classified into three main types; elliptic integrals method, the finite-element-type and Pseudo-Rigid-Body (PRB) model, which gives a closed-form analytical solution, a numerical solution, and an approximation solution of large deflection cantilever beam respectively.

3.1.2.1 Closed-Form Analytical Solution

In the literature, the exact curvature of a deflected beam is expressed by the classical Bernoulli-Euler beam theorem, which leads to a second order nonlinear differential equation, i.e., Eqn. (3.2). Using elliptic integrals of the first and second kinds, Bisshopp and Drucker [106] first derived the classical closed-form solution for this second order nonlinear differential

equation, which forms the foundation in analyzing the large deflection of a cantilever beam subjected to a vertical point load at free end (Fig. 3.1). This method is subsequently adopted by other authors [107, 108, 109, 110, 111] to establish a closed-form analytical solution for a large deflection cantilever beam. Nevertheless, the flexure joints used in most flexure-based modules are usually coupled with rigid-links to increase the stiffness in other non-actuating directions. Hence, this model is not effective in analyzing such flexure-based modules.

Figure 3.1: A cantilever beam subjected to a vertical force at free end.

Figure 3.2: A cantilever beam subjected to a moment end load.

For a flexure joint coupled with a rigid link (Fig. 3.3a), an applied force at the free end of the rigid link can be seen as a moment applied at the end of a cantilever beam (Fig. 3.2). Based on Eqn. (3.1) (refer to Appendix B.1 for details), the nonlinear deflections of the cantilever beam subjected to an moment end load in the y- and x-axes are given as

$$\delta_{y,nonlinear} = l \left(\frac{1 - \cos \theta_f}{\theta_f} \right) \quad (3.4)$$

$$\delta_{x,nonlinear} = l \left(\frac{\theta_f - \sin \theta_f}{\theta_f} \right) \quad (3.5)$$

where θ_f represents the deflection angle used in these nonlinear models. Unfortunately, these nonlinear models that are complex and incommodious even for solving the beam-based flexure modules, which are formed by several beam-based flexure joints coupled with rigid-links.

3.1.2.2 Numerical Solution

The finite-element-type proposed by another group of researchers [112, 113, 114], are capable of solving beam-based flexure modules with much more complex geometries and loadings. However, these models using Galerkin finite-element, boundary-element and iterative methods etc., are too cumbersome for quick parametric studies on those beam-based flexure modules during the initial design and analysis stage.

3.1.2.3 Approximation Solution

The most simplified modeling of a flexure-based module was the PRB model, which represents a flexure joint as a rotational joint with a torsional spring. Howell and Midha [115] first uses this PRB technique to model small-length flexure pivots, i.e., a flexure joint coupled with a long rigid-link as shown in Fig. 3.3. According to the authors, the deflection path can be approximated by two rigid links that are joined at a pivot point, termed *characteristic pivot*. This characteristic pivot is located at the center of the flexure joint. A torsional spring at the characteristic pivot represents the stiffness of the flexure joint.

Figure 3.3: (a) A flexure joint coupled with a rigid-link that is represented in (b) a PRB small-length flexure pivot [116].

According to the authors, the deflections in the x- and the y-axes (or a and b referring to Fig. 3.3b) can be approximated as

$$a = \frac{l}{2} + \left(L + \frac{l}{2}\right) \cos \Theta \quad (3.6)$$

$$b = (L + \frac{l}{2}) \sin \Theta \quad (3.7)$$

and the force-deflection relationship is given as

$$F = \frac{K\Theta}{F(L + \frac{l}{2}) \sin(\phi - \Theta)} \quad (3.8)$$

where $K = \frac{(EI)l}{I}$ is the stiffness of the torsional spring (or the flexure joint), ϕ is the initial deflection angle, and Θ is the PRB approximated deflection angle.

PRB approximation has been widely recognized as a tool for quick approximation and evaluation of a beam-based flexure module [117, 118, 119]. However, there is a limitation to this approximation solution.

Figure 3.4: Schematic diagram of a PRB model on a flexure joint coupled with a rigid-link.

Based on the PRB model of a small length flexure pivot, Fig. 3.4 shows that the reaction moments at the ends of the flexible segment are $M_1 = F(L + l)$ and $M_2 = FL$. For pure moment loading,

$$\frac{M_1}{M_2} = 1 + \frac{l}{L} \Rightarrow \frac{M_1}{M_2} = 1 \quad (3.9)$$

Equation (3.9) shows that L must be at least ten times greater than l , for M_1 and M_2 to be equal. Hence, the PRB model of a small-length flexure pivot is suitable for approximating short flexure joints coupled with long rigid-links. In addition, this PRB modeling is effective for analyzing the notch hinges because it is capable of analyzing small deflection flexure

joints accurately. Nevertheless, if these conditions are not met, other forms of PRB models will be used depending on the configurations and designs of the flexure-based modules, and types of loading (Fig. 3.5).

Figure 3.5: Other PRB models for (a) a fixed-pinned cantilever beam, (b) a fixed-guided cantilever beam and (c) a cantilever beam subjected to moment end loading [116].

Consequently, selection between these PRB models and applying them required experiences and well-versed knowledge on various beam-based flexure modules and PRB models. At the initial design and analysis stage, alteration of the length of flexure and rigid-link to achieve the desired stiffness of the beam-based flexure modules often changes the configurations of each flexure joint. As each PRB model is targeted to a specific flexure joint configuration, utilizing the correct PRB model is restrictive in the design stage and often provides inaccurate stiffness analysis due to misjudgment and inappropriate application of these PRB models on unsuitable flexure joint configurations. Most importantly, such inaccuracies become more significant during large deflection analyses. For large deflection analyses of a beam-based flexure joint, careful selections of those PRB models become crucial as each model requires different representations of the flexure joint configurations and in turn formulates different forms of solutions. Thus, a PRB approximation technique cannot provide a single, simple and common solution for various flexure-based modules configurations.

3.2 A Semi-Analytic Model

A semi-analytic model is introduced to address all the drawbacks of current analytical models by providing a single yet simple solution for analyzing the large deflection beam-based flexure modules of various configurations. Two important factors are considered during the formulation of the proposed semi-analytic model. First, a factor that enables the proposed model to approximate the shifting of the ‘pivot’ point location during large deflection. Second, a factor that identifies the initial location of the ‘pivot’ point according to various types of flexure-based joint configurations. The shifting of the ‘pivot’ point is essential as it is one of the main factors that contribute to the nonlinear behavior of large deflection beams. As mentioned in the previous section, the square of slope in Bernoulli-Euler law plays an important role in nonlinear large deflection analysis. In this section, the proposed factor is derived based on the classical Bernoulli-Euler law.

3.2.1 Bernoulli-Euler Law

Based on the classical Bernoulli-Euler law, its nonlinear ordinary differential form is expressed as

$$\frac{M}{EI} = \frac{d^2y/dx^2}{[1 + (dy/dx)^2]^{\frac{3}{2}}} \quad (3.10)$$

Let $\frac{dy}{dx} = \lambda$, Eqn. (3.10) is re-expressed as

$$\frac{M}{EI} = \frac{d\lambda/dx}{[1 + (\lambda)^2]^{\frac{3}{2}}} \quad (3.11)$$

Integrate Eqn. (3.11) with respect to x yields

$$\frac{Mx}{EI} = \frac{\lambda}{\sqrt{1 + \lambda^2}} + C \quad (3.12)$$

Since $\lambda = \frac{dy}{dx}$, Eqn. (3.12) can be re-expressed as

$$\frac{Mx}{EI} = \frac{dy/dx}{\sqrt{1 + (dy/dx)^2}} + C \quad (3.13)$$

The constant, C , can be determined from the boundary condition, whereby at the fixed end, i.e., $x = 0$, the slope will be zero, i.e., $dy/dx = 0$. Hence, this leads to $C = 0$.

Given that dy , dx and ds are infinitesimal, it is true that [116]

$$\frac{dy}{ds} = \sin \theta \quad (3.14)$$

and

$$\frac{dx}{ds} = \cos \theta \quad (3.15)$$

Using the chain-rule

$$\frac{dy}{dx} = \frac{dy}{ds} \cdot \frac{ds}{dx} \quad (3.16)$$

Substituting Eqn. (3.15) and (3.16) into Eqn. (3.16) derives

$$\frac{dy}{dx} = \tan \theta \quad (3.17)$$

Substituting Eqn. (3.17) into Eqn. (3.13) yields

$$\frac{Mx}{EI} = \frac{\tan \theta}{\sqrt{1 + (\tan \theta)^2}} \quad (3.18)$$

Further simplified Eqn. (3.18) gives

$$\frac{Mx}{EI} = \sin \theta \quad (3.19)$$

3.2.1.1 For Small Deflection

When small deflection theorem is applied to Eqn. 3.19, i.e., $\theta \rightarrow 0$, $\sin(\theta) = \theta$, an expression to determine the deflection angle due to a moment at the free end is obtained as

$$\frac{Mx}{EI} = \theta \quad (3.20)$$

3.2.1.2 For Large Deflection

Using Taylor's series expansion, Eqn. (3.19) can be re-expressed as

$$\frac{Mx}{EI} = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \dots$$

Thus, the large deflection caused by a moment at free end is expressed as

$$\frac{Mx}{EI} = \theta \varpi \quad (3.21)$$

where

$$\varpi = 1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} \quad (\text{only use up to } 2^{\text{nd}} \text{ term})$$

In this work, the derived factor, ϖ , will be used to approximate the shifting of 'pivot' point on the beam-based flexure joint.

3.2.2 Shifting of Pivot Point During Large Deflection

Based on observations and past investigations, the ‘pivot’ point on a cantilever beam shifts to different locations due to the parasitic shift (Fig. 3.6). Parasitic shift is less significant in a conventional notch hinge due to the geometry shape of this flexure joint. However, a parasitic shift becomes significant in a beam-based flexure joint. For the semi-analytic model to be effective in analyzing large deflection beam-based flexure joint, the shifting of the ‘pivot’ point location must be identified.

Figure 3.6: Shifting of pivot point during large deflection of beam.

Considering the deflection along the y-axis of the semi-analytic model to be expressed as

$$\delta_y = \eta l \sin \theta \quad (3.22)$$

where η represents the ‘pivot’ point of the flexure joint as shown in Fig. 3.4.

With maximum bending moment occurs at $x = l$, Eqn. (3.21) becomes

$$\frac{Ml}{EI} = \theta \varpi \quad (3.23)$$

To determine η of the semi-analytic model, first equate Eqn. (3.22) and (3.4), resulting in

$$\frac{EI}{M}(1 - \cos \theta) = \eta l \sin \theta \quad (3.24)$$

Substituting Eqn. (3.23) into (3.24) yields

$$\frac{l}{\varpi} \left(\frac{1 - \cos \theta}{\theta} \right) = \eta l \sin \theta \quad (3.25)$$

Judging from Fig. 3.7, $(1 - \cos \theta)/\theta$ can be approximated as

$$\frac{1 - \cos \theta}{\theta} \approx \frac{\sin \theta}{2} \quad (3.26)$$

with an accuracy valid up till 40° .

Figure 3.7: Comparison between $\frac{1 - \cos \theta}{\theta}$ and $\frac{\sin \theta}{2}$.

Substituting Eqn. (3.26) into Eqn. (3.25) yields

$$\eta = \frac{1}{2\varpi} \quad (3.27)$$

Here, Eqn. (3.27) represents the ‘pivot’ point location of a beam-based flexure joint with ϖ providing the shifting in the ‘pivot’ point in large deflection analysis (refer to Section 3.3.1 for more details).

3.2.3 Large Deflection Analysis of A Beam-Based Flexure with Moment End Load

A beam-based flexure joint coupled with a rigid-link (Fig. 3.8a) can be treated as a beam-based flexure joint subjected to a moment end load. This is because the force, F , applied to the free end of the rigid-link can be converted as a moment, i.e., $M = F \times L$, at the free end of the beam as shown in Fig. 3.8b.

Figure 3.8: (a) A beam-based flexure joint coupled with a rigid-link subjected to a point load treated as (b) a beam-based flexure joint subjected to a moment end load.

3.2.3.1 Deflection in the Y-Axis

By substituting Eqn. (3.27) into Eqn. (3.22) yields

$$\delta_y = l \left(\frac{\sin \theta}{2\varpi} \right) \quad (3.28)$$

A comparison between Eqn. (3.28) with Eqn. (3.4) gives

$$\frac{\delta_y}{\delta_{y, \text{nonlinear}}} = \frac{\sin \theta / 2\varpi}{(1 - \cos \theta) / \theta} \quad (3.29)$$

Figure 3.9 shows that the results obtained from both $\sin \theta / 2\varpi$ and $(1 - \cos \theta) / \theta$ are almost identical up till the deflection angle of 60° . Hence, Eqn. (3.29) is valid from 0 to 60° and this indicates that the proposed approximated solution is equivalent to the classical nonlinear model for up to 60° of deflection angle.

Figure 3.9: Comparison between $\frac{1 - \cos \theta}{\theta}$ and $\frac{\sin \theta}{2\varpi}$.

Multiplying both sides of Eqn. (3.28) by θ gives the semi-analytic solution for deflection in the y-axis

$$\delta_{y,semi-analytic} = \frac{l}{2}\theta \quad (3.30)$$

3.2.3.2 Deflection in the X-Axis

Expressing Eqn. (3.28) in trigonometry form yields

$$\sin \theta = \frac{\delta_y}{(l/2\varpi)} \quad (3.31)$$

Figure 3.10: Schematic diagram of a beam-based flexure subjected to a moment end load.

From Fig. 3.10, $a = \frac{l}{2\varpi} \cos \theta$. Hence, the semi-analytic solution for deflection in the x-axis can be expressed as

$$\delta_{x,semi-analytic} = \frac{l}{2} \left(1 - \frac{\cos \theta}{\varpi} \right) \quad (3.32)$$

3.2.4 Large Deflection Analysis of A Beam-Based Flexure Joint Coupled with Rigid-Link

For quick parametric studies of the beam-based flexure modules, large deflection analysis of a beam-based flexure joint coupled with rigid-link of various lengths is essential as the stiffness of such beam-based flexure modules can be swiftly determined by breaking down to individual flexure joint coupled with the rigid-links.

3.2.4.1 Deflection in the Y-Axis

Based on Eqn. (3.28) and through approximation, the semi-analytic model for deflection in the y-axis of a beam-based flexure joint coupled with a rigid-link can be expressed as

$$\sin \theta = \frac{\Delta_y}{L + \frac{l}{2\varpi}}$$

$$\Delta_y = \left(L + \frac{l}{2\varpi} \right) \sin \theta \quad (3.33)$$

Similarly from Eqn. (3.32) and through approximation, the semi-analytic model for the deflection in the x-axis can be expressed as

$$\begin{aligned} \cos \theta &= \frac{a}{L + \frac{l}{2\varpi}} \\ a &= \left(L + \frac{l}{2\varpi} \right) \cos \theta \end{aligned}$$

$$\Delta_x = \left(L + \frac{l}{2} \right) - \left(L + \frac{l}{2\varpi} \right) \cos \theta \quad (3.34)$$

3.2.5 Initial Pivot Point Location of A Beam-Based Flexure Joint Coupled with A Rigid-Link

The stiffness analysis of a beam-based flexure joint coupled with a rigid-link of various lengths requires the identification of the initial ‘pivot’ point location on the beam-based flexure joint. This initial location will determine the length of the moment arm, which is one of the main factors in analyzing stiffness of the flexure joint. As the initial location of a ‘pivot’ point is dependent on the length of the beam-based flexure and the rigid-link, identifying it becomes critical in the stiffness analysis of a beam-based flexure joint coupled with a rigid-link. Here, a new factor, ρ , is introduced to form a generalize solution for all types of configurations, i.e., $L = 0$, $L < l$, $L = l$, and $L > l$ (details are presented in the following section).

$$\rho = \frac{l\sqrt{1.8} + L}{l + L} \quad (3.35)$$

The moment end load is recognized as a tangential force, F_t , applied to the moment arm described from the ‘pivot’ point to the beam end (Fig. 3.11), which gives

$$T = M = F_t(L + \frac{l}{2\varpi}) \quad (3.36)$$

Substituting Eqn. (3.35) into Eqn. (3.36), the changing moment arm is expressed as

$$M = F_t(L + \frac{\rho l}{2\varpi}) \quad (3.37)$$

Given that the tangential force as

$$F_t = F \sin(\frac{\pi}{2} - \theta) \quad (3.38)$$

Substituting Eqn. (3.20) with $x \rightarrow l$ for the maximum bending moment (refer to Appendix B.2 for details), and Eqn. (3.38) into Eqn. (3.37), the semi-analytic model for the force-deflection can be expressed as

$$F = \frac{EI\theta}{l(L + \frac{\rho l}{2\varpi}) \sin(\frac{\pi}{2} - \theta)} \quad (3.39)$$

Figure 3.11: Semi-analytic schematic diagram of a flexure joint coupled with rigid link.

3.3 Theoretical Comparisons

3.3.1 Approximations of Pivot Point Shifting

The derived factor that accounts for the shifting of ‘pivot’ point on the cantilever beam in large deflection analysis is expressed as

$$\varpi = 1 - \frac{\theta^2}{3!} + \frac{\theta^4}{5!} \quad (3.40)$$

Based on Eqn. (3.27) and with $\theta = 0$, the original location of ‘pivot’ point is given as

$$\delta_o = \frac{1}{2} \quad (3.41)$$

Consequently, based on Eqn. (3.27) and (3.41), the dimensionless magnitude of the shifting of the ‘pivot’ point is expressed as

$$\frac{\delta_{p,semi-analytic}}{l} = \frac{1}{2\varpi} - \frac{1}{2} \quad (3.42)$$

To investigate whether the derived factor, ϖ , accounts for the ‘pivot’ point shifts, Eqn. (3.42) is compared with Haringx’s model of ‘pivot’ point shifting [120]

$$\frac{\delta_{p,Haringx}}{l} = \frac{2 \sin(\theta/2)}{\theta} - \cos(\theta/2) \quad (3.43)$$

In the past literature [1], Haringx’s model was compared with Young’s experimental data [121] and the results obtained from the model were found to be consistent with the experimental results. Hence, Haringx’s model is used as a benchmark for investigating the effectiveness of the derived factor, ϖ , in determining the shifting of the ‘pivot’ point. As the comparisons between Haringx and Young were conducted up to 40° of deflection angle [1], the theoretical comparisons between Eqn. (3.42) and (3.43) are also conducted up to the similar deflection angle and plotted in Fig. 3.12.

Figure 3.12: ‘Pivot’ point shift of a deflected cantilever beam.

Figure 3.12 shows that the factor, ϖ , does provide a good approximation of the shifting of ‘pivot’ point when compared with Haringx’s model. As Haringx’s model has been validated through experimental results in the past, it is reasonable to conclude that the factor, ϖ , which is adopted by the proposed semi-analytic model, does offers a good approximation on the shifting of ‘pivot’ point up to 40° of deflection angle.

3.3.2 Semi-Analytic Model V.S. Nonlinear Model

With the derived factor, ϖ , being proven to be effective in accounting for the shifting of ‘pivot’ point, the semi-analytic models derived from it could enhance the accuracy of the approximation approach. Based on the proposed semi-analytic models from Eqn. (3.30) and (3.32), the non-dimensional deflection ratio in the y- and x-axes are given as

$$\frac{\delta_{y,semi-analytic}}{l} = \frac{\theta}{2} \quad (3.44)$$

$$\frac{\delta_{x,semi-analytic}}{l} = \frac{1 - (\cos \theta / \varpi)}{2} \quad (3.45)$$

The non-dimensional deflection ratio in the y- and x-axes from the nonlinear models, i.e., Eqn. (3.4) and (3.5), are given as

$$\frac{\delta_{y,nonlinear}}{l} = \left(\frac{1 - \cos \theta_f}{\theta_f} \right) \quad (3.46)$$

$$\frac{\delta_{x,nonlinear}}{l} = \left(\frac{\theta_f - \sin \theta_f}{\theta_f} \right) \quad (3.47)$$

For comparisons, Eqn. (3.44), (3.45), (3.46) and (3.47) are plotted in Fig. 3.13.

From Fig. 3.13, the semi-analytic models have shown a high-degree of accuracy in approximating the large deflection of a beam-based flexure due to an external moment end

Figure 3.13: Comparison between the nonlinear and semi-analytic models.

load. The approximation through the proposed models is consistent with the nonlinear models in the deflection ratio ranging from 0 to 0.55 in between 0 to 70° of deflection angle. In theory, this comparison has concluded that the semi-analytic model, e.g., Eqn. (3.44), provides a simple approximation to the nonlinear model, e.g., Eqn. (3.46), with a high-degree of accuracy up to a relatively large angular deflection of 70°.

3.3.3 Approximation of Initial Location of Pivot Point

To address the initial location of the ‘pivot’ point, another new factor, ρ , is introduced. For various types of beam-based flexure joint coupled with rigid-link configurations, i.e. $L = 0$, $L < 1$, $L = 1$, and $L > 1$, the initial location of ‘pivot’ point is different and, in turn gives a different moment arm during the stiffness analysis. From Eqn. (3.39), the length of the changing moment arm is given as

$$L + \frac{\rho l}{2\omega} \quad (3.48)$$

By excluding the constant, L , and the shifting factor, ϖ , the initial length of the deflected flexure joint, which is measured from the initial location of the ‘pivot’ point to the location of force input, is expressed as

$$S = \frac{l}{2} \left(\frac{l\sqrt{1.8} + L}{l + L} \right) \quad (3.49)$$

3.3.3.1 Case 1

For $L = 0$, the vertical force applied to the rigid link free end is transfer to the free end of the flexure joint. Hence, based on Eqn. (3.49), the length of the deflected beam-based flexure joint when $L = 0$ is given as

$$S = 0.67l \quad (3.50)$$

Equation (3.50) is consistent to the past literature [1] and a widely recognized fact that the length of the deflected cantilever beam subjected to a vertical force (Fig. 3.1) is

$$S = \frac{2}{3}l \quad (3.51)$$

3.3.3.2 Case 2

For $L \gg l$, L becomes dominating over l and hence $\rho \rightarrow 1$. This becomes pure bending moment expressed in Eqn. (3.9). Consequently, based on Eqn. (3.49), the length of the deflected flexure joint with $L \gg l$ is given as

$$S = \frac{l}{2} \quad (3.52)$$

Equation (3.52) is consistent with the conventional PRB model for small-length flexure pivot that assumes that length of the deflected flexure joint is based on the mid-point of the flexure joint.

3.3.3.3 Case 3

For other mixture of various flexure and rigid-link lengths, Thorpe in 1953 has proposed from experimental analyses that the length of the deflected flexure joint for a beam-based flexure joint coupled with a rigid-link is given as [122]

$$S_{Thorpe} = \frac{lL + (2/3)l^2}{l + 2L} \quad (3.53)$$

Comparison between Eqn. (3.49) and (3.53) leads to

$$\frac{S}{S_{Thorpe}} = 1 \quad (3.54)$$

Equation (3.52) shows that Eqn. (3.49) is consistent with the Thorpe's model [1]. Therefore, this theoretical comparison can conclude that the proposed factor, ρ , is effective in identifying the initial location of a 'pivot' point and in turn gives the moment arm length, which is essential in analyzing the stiffness of a beam-based flexure joint coupled with rigid-link of various lengths.

3.3.4 Semi-Analytic Model V.S. PRB Model

With the two factors ρ and ϖ shown to be effective in identifying the initial location of the 'pivot' point and its the parasitic shift of the 'pivot' point during large deflection, the semi-analytic model could provide a quick stiffness analysis on a beam-based flexure joint coupled with various rigid-link lengths. In this section, the results obtained from the Finite-Element (FE) analyses of a beam-based flexure with various rigid-link lengths are used to validate this claim. These configurations are illustrated in Fig. 3.14.

Figure 3.14: Beam-based flexure joint coupled with various rigid-links lengths.

Figure 3.15: Steps for obtaining a set of deflections from both semi-analytic and PRB models based on the initial desired deflection.

Table 3.2: Parameters used in the FE analysis

Material	Alu 7075-T6
Young's Modulus	71GPa
Poisson ratio	0.33
Beam thickness	0.2 mm
Rigid-link thickness	20 mm
Element type	BEAM3 (2D elastic)
Nonlinear large displacement	5 substeps, 1000 maximum substeps, 1 minimum sbusteps

The parameters used in the FE analysis is listed in Table 3.2 and the analysis was carried out in the following manner. Based on a set of desired deflections, the force required for achieving these desired deflection angles are obtained from the semi-analytic model, i.e., Eqn. (3.33) and (3.39), and the conventional PRB model, i.e., Eqn. (3.7) and (3.8). Subsequently, each set of obtained forces was input to the FE simulator, i.e., ANSYS10, which in turn computes the deflections of each beam-based flexure joint configuration. Figure 3.15 shows a diagram on the steps to obtain the deflections from both semi-analytic and PRB models and results are listed in Table 3.3.

Table 3.3: Results obtained from the proposed semi-analytic model and PRB model on beam-based flexure joint coupled with rigid-link of various lengths

Desired deflection (m)	Ratio ($\frac{\delta_y}{l}$)	Conventional PRB model		Semi-analytic model	
		Output deflection (m)	Error (%)	Output deflection (m)	Error (%)
Rigid-link length, $L = 1\text{mm}$ and flexure joint length, $l = 3\text{mm}$					
2.5000×10^{-4}	0.0625	2.8066×10^{-4}	12.2640	2.4301×10^{-4}	2.7960
5.0000×10^{-4}	0.1250	5.6385×10^{-4}	12.7700	4.8704×10^{-4}	2.5920
1.0000×10^{-3}	0.2500	1.1509×10^{-3}	15.0900	9.8384×10^{-4}	1.6160
2.0000×10^{-3}	0.5000	2.5013×10^{-3}	25.0650	2.0389×10^{-3}	1.9450
Rigid-link length, $L = 3\text{mm}$ and flexure joint length, $l = 3\text{mm}$					
1.0000×10^{-3}	0.1667	1.0425×10^{-3}	4.2500	9.8350×10^{-4}	1.6500
2.0000×10^{-3}	0.3333	2.1172×10^{-3}	5.8600	1.9802×10^{-3}	0.9900
3.0000×10^{-3}	0.5000	3.2560×10^{-3}	8.5333	2.9995×10^{-3}	0.0167
Rigid-link length, $L = 5\text{mm}$ and flexure joint length, $l = 3\text{mm}$					
1.0000×10^{-3}	0.1250	1.0196×10^{-3}	1.9600	9.8920×10^{-4}	1.0800
2.0000×10^{-3}	0.2500	2.0499×10^{-3}	2.4950	1.9823×10^{-3}	0.8850
3.0000×10^{-3}	0.3750	3.1012×10^{-3}	3.3733	2.9823×10^{-3}	0.5900
4.0000×10^{-3}	0.5000	4.1829×10^{-3}	4.5725	3.9911×10^{-3}	0.2225
Rigid-link length, $L = 10\text{mm}$ and flexure joint length, $l = 3\text{mm}$					
1.0000×10^{-3}	0.0769	1.0067×10^{-3}	0.6700	9.9621×10^{-4}	0.3790
2.0000×10^{-3}	0.1538	2.0140×10^{-3}	0.7000	1.9916×10^{-3}	0.4200
3.0000×10^{-3}	0.2308	3.0259×10^{-3}	0.8633	2.9886×10^{-3}	0.3800
4.0000×10^{-3}	0.3077	4.0434×10^{-3}	1.0850	3.9868×10^{-3}	0.3300
5.0000×10^{-3}	0.3846	5.0681×10^{-3}	1.3620	4.9866×10^{-3}	0.2680
6.0000×10^{-3}	0.4615	6.1017×10^{-3}	1.6950	5.9879×10^{-3}	0.2017
7.0000×10^{-3}	0.5385	7.1455×10^{-3}	2.0786	6.9905×10^{-3}	0.1357
Rigid-link length, $L = 20\text{mm}$ and flexure joint length, $l = 3\text{mm}$					
1.0000×10^{-3}	0.0435	1.0019×10^{-3}	0.1900	9.9880×10^{-4}	0.1200
2.0000×10^{-3}	0.0870	2.0054×10^{-3}	0.2700	1.9989×10^{-3}	0.0550
3.0000×10^{-3}	0.1304	3.0064×10^{-3}	0.2133	2.9960×10^{-3}	0.1333
4.0000×10^{-3}	0.1739	4.0100×10^{-3}	0.2500	3.9948×10^{-3}	0.1300
5.0000×10^{-3}	0.2174	5.0147×10^{-3}	0.2940	4.9939×10^{-3}	0.1220
6.0000×10^{-3}	0.2609	6.0208×10^{-3}	0.3467	5.9930×10^{-3}	0.1167
7.0000×10^{-3}	0.3043	7.0286×10^{-3}	0.4086	6.9924×10^{-3}	0.1086

In the first case, when the rigid-link is much smaller than the flexure joint, i.e., $L \ll l$, the deflections obtained from the semi-analytic model deviate less than 3 % when compared with the initial desired deflections. Deflections obtained from the PRB model have a maximum deviation of 25 % when compared with the initial desired deflections. When $L \leq l$, the deflections obtained from the semi-analytic model deviate less than 2 % when compared with the initial desired deflections. On the other hand, the deflections obtained from the

PRB model have a deviation up to 8 %. As L becomes larger than l , the deviations between the PRB approximated deflection and desired deflection slowly reduce to less than 0.5 %. These results shows that the PRB model is indeed very accurate when $L \gg l$. For this similar case, the deviations between the approximated deflection from the semi-analytic model and desired deflection not only just reduce to 0.1 % but show constant differences throughout the incrementing deflections. Assuming that the FE simulator produces accurate analyses, these results have validated that the semi-analytic model offers a high-degree of accuracy and robustness in approximating the deflection of beam-based flexural joints coupled with various rigid-link lengths.

3.4 Experimental Investigations

Besides numerical comparisons, an experimental investigation has been conducted to validate the accuracy of the proposed semi-analytic model for beam-based flexure joint coupled with rigid-link. This experiment uses the simplest approach to investigate the large deflection of beam-based flexure samples, which is by adding weighs on the free end of the rigid-link and detecting the deflection through a high-resolution measurement device.

3.4.1 Experimental Setup

A total of four specimens of beam-based flexure joint coupled with rigid-link are prepared for the experiment. The dimensions of each specimen are calculated to ensure that a large deflection ratio of 0.5 is achievable within its elastic limit. The specifications of the four specimens are listed in Table 3.4 and Fig. 3.16 shows the actual experimental specimens used in this investigation.

Table 3.4: List of samples used for experimental investigation

Specimens	Beam-based flexure joint length, l (mm)	Rigid-link length, L (mm)	Ratio ($l : L$)	Material
1	150	75	2 : 1	SUS304
2	20	20	1 : 1	ALU7075-T6
3	20	40	1 : 2	ALU7075-T6
4	20	100	1 : 5	ALU7075-T6

Figure 3.16: Specimens of the beam-based flexure joint coupled with various rigid-link lengths.

Figure 3.17: Experimental platform.

In this experiment, specimen 1 is designed with a long beam-based flexure joint coupled to a shorter rigid-link, while the beam-based flexure joints of specimen 2 to 4 are similar in length but each coupled with different rigid-link lengths. In addition, these specimens are fabricated in larger dimensions so as to obtain the magnified magnitude of the deflections from the experiment and both analytical models. An experimental platform is constructed

for this investigation (Fig. 3.17), where each specimen is fixed on a mounting (1) that is adjustable in the z-axis. Specimens are mounted in this condition to avoid the mass of the rigid-link to affect the actual loadings that deflect the specimens. A nylon string (2), which can withstand 5 Kg of load, connects the end of the rigid-link with a container (3). This container is used to put pre-calculated weights, which act as the load for deflecting the specimens. The nylon string is rested on a bearing (4), to ensure that the nylon string is horizontal with the specimen deflection paths. This bearing is held in place by a rotating shaft (5), which is connected to an adjustable mounting (6). This mounting can oriented about θ_x and θ_y so as to ensure that the rotating shaft is parallel to the specimens.

In this experiment, a 3-dimensional (3D) digitizing measuring device from ATOS (model: standard) is used to capture the deflection of the beam-based flexure specimens as shown in Fig. 3.18. This device has a resolution of $3\ \mu\text{m}$, an accuracy of $30\ \mu\text{m}$ and a measurement volume of $150 \times 250 \times 200\ \text{mm}^3$. As all specimens are fabricated in larger dimensions, the differences between the experimental data and the results obtained both models are in few hundreds of micrometers. Hence, an accuracy of $30\ \mu\text{m}$ is sufficient for this evaluation.

Figure 3.18: ATOS 3D measuring device for capturing the deflection of each specimen.

Figure 3.19: (a) Two spherical balls attached to the rigid-link for the measuring device to capture (b) the image of the actual coordinates of deflection.

To capture the precise coordinates of the deflection, two spherical stainless steel balls of diameter 5.4 mm were attached onto the rigid-link of each specimen as shown in Fig. 3.19a. One of them was in line with the nylon string, which carried the load, while the other acts as a reference. Beginning from the initial non-deflected position, five set of pre-calculates weights are loaded into the container and caused the specimen to deflect. Each deflection caused by each set of weights were capture by the measuring device. Subsequently, all five images of the sequenced deflections were combined together through the referencing markers and formed a complete image of these deflections as shown in Fig. 3.19b. Consequently, the center of each spherical ball was processed by the measurement device, which determined the coordinates of each spherical ball.

Figure 3.20: (a) Triangular coordinates determined through the center of the spherical balls from each deflection were used to (b) calculate the actual deflections in the y- and x-axes.

For each deflection, the distances from the coordinates of the center of the spherical balls with respected to their initial non-deflected position were measured (Fig. 3.20a). These distances formed a triangle where an angle, β , can be determined from

$$\beta = \cos^{-1}\left(\frac{A^2 + C^2 - B^2}{2AC}\right) \quad (3.55)$$

Subsequently, this angle is used to determine the deflection in the y-axis

$$\Delta_y = C \sin \beta \quad (3.56)$$

From Eqn. (3.56), the deflection in the x-axis expressed as

$$\Delta_x = \sqrt{B^2 - \Delta_y^2} \quad (3.57)$$

3.4.2 Results

For the experimental results, the deflections in the y- and x-axes are calculated using Eqn. (3.56) and (3.57), whereby the values of A, B and C are obtained from the coordinates of the spherical balls via the capture images. By converting each load to an input force, these force values are used to obtain the deflection angles based on the semi-analytic model, i.e., Eqn. (3.39), and the PRB model, i.e., Eqn. (3.8). Consequently, the deflection angles obtained from both models are used to determine the respective deflection in the y- and x-axes based on Eqn. (3.33) and (3.34) of the semi-analytic model and Eqn. (3.6) and (3.7) of the PRB model. All deflection results obtained from the four specimens through the experimental, the semi-analytic model and the PRB model are listed in Table 3.5 to 3.8.

From Table 3.5, the results obtained from specimen 1 (beam-based flexure = 150 mm and rigid-link = 75 mm) have shown that the semi-analytic model has higher accuracy in predicting the deflection in both y- and x-axes. The errors found between the experiment and semi-analytic model in the y-axis deflection range from 0.4 % to 4 %. On the other hand,

the errors found between the experiment and PRB model in the y-axis deflection range from 6 % to 13 %. The errors found between the experiment and semi-analytic model in the x-axis deflection range from 0.1 % to 6.5 %. These errors are smaller than the errors found between the experiment and PRB model in the y-axis deflection, which ranges from 1.6 % to 8 %.

From Table 3.6, the results obtained from specimen 2 (beam-based flexure = 20 mm and rigid-link = 20 mm) show that the predictions made by the semi-analytic model on the deflection in the y-axis are far more accurate than the predictions from the PRB model. Similarly, the predictions from the semi-analytic model on the deflection in the x-axis are closer to the actual results as compared to the predictions made by the PRB model. From Table 3.7, the results obtained from specimen 3 (beam-based flexure = 20 mm and rigid-link = 40 mm) shows that the errors between the experiment and semi-analytic model in the y-axis deflection are getting smaller with four sets of values being equal or less than 1 %. In addition, the predictions from the semi-analytic model on the deflection in the x-axis are much closer to the actual deflections as compared to the predictions made by the PRB model. As for Table 3.8, the results obtained from specimen 4 (beam-based flexure = 20 mm and rigid-link = 100 mm) show that the predictions from the semi-analytic model on the deflection in the y-axis are more accurate although the predictions made by the PRB model also become closer to the actual deflection when the length of rigid-link increases. In addition, it is observed that the predictions of the PRB model on the deflection in the x-axis are also closer to the actual deflection. Nevertheless, the experimental data collected from all four specimens have shown that the approximations made by the semi-analytic model are closer and more consistences with the actual deflections measured. In addition, it also show the robustness of the semi-analytic model in predicting the nonlinear deflection of beam-based flexure joint coupled with various rigid-link lengths.

Table 3.5: Deflections obtained from specimen 1 through experiment, semi-analytic model and conventional PRB model.

Specimen 1: $L = 75\text{mm}$, $l = 150\text{mm}$					
	Experimental results (m)	Semi-analytic model		PRB model	
		Results (m)	Error (%)	Results (m)	Error (%)
	Under 7.65 gram loading				
Δ_y	1.413025×10^{-2}	1.386221×10^{-2}	1.8969	1.244720×10^{-2}	11.9109
Δ_x	5.436734×10^{-4}	5.344350×10^{-4}	1.6992	5.173348×10^{-4}	4.8446
	Under 16.55 gram loading				
Δ_y	2.841336×10^{-2}	2.954022×10^{-2}	3.9660	2.654037×10^{-2}	6.5919
Δ_x	2.441983×10^{-3}	2.437569×10^{-3}	0.1808	2.366641×10^{-3}	3.0853
	Under 25.52 gram loading				
Δ_y	4.257364×10^{-2}	4.442323×10^{-2}	4.3444	3.994316×10^{-2}	6.1787
Δ_x	5.330071×10^{-3}	5.552246×10^{-3}	4.1683	5.415963×10^{-3}	1.6115
	Under 35.49 gram loading				
Δ_y	6.135437×10^{-2}	5.952120×10^{-2}	2.9878	5.355894×10^{-2}	12.7056
Δ_x	1.078057×10^{-2}	1.007261×10^{-2}	6.5670	9.887761×10^{-3}	8.2817
	Under 47.45 gram loading				
Δ_y	7.389608×10^{-2}	7.540205×10^{-2}	2.0379	6.787162×10^{-2}	8.1526
Δ_x	1.688400×10^{-2}	1.640540×10^{-2}	2.8347	1.623363×10^{-2}	3.8520

Table 3.6: Deflections obtained from specimen 2 through experiment, semi-analytic model and conventional PRB model.

Specimen 2: $L = 20\text{mm}$, $l = 20\text{mm}$					
	Experimental results (m)	Semi-analytic model		PRB model	
		Results (m)	Error (%)	Results (m)	Error (%)
	Under 122.42 gram loading				
Δ_y	3.177983×10^{-3}	3.000276×10^{-3}	5.5918	2.838706×10^{-3}	10.6759
Δ_x	1.305699×10^{-4}	1.335940×10^{-4}	2.3161	1.346062×10^{-4}	3.0913
	Under 248.32 gram loading				
Δ_y	6.159923×10^{-3}	5.985160×10^{-3}	2.8371	5.663002×10^{-3}	8.0670
Δ_x	5.050740×10^{-4}	5.344745×10^{-4}	5.8210	5.393413×10^{-4}	6.7846
	Under 385.14 gram loading				
Δ_y	9.222142×10^{-3}	9.016148×10^{-3}	2.2337	8.530095×10^{-3}	7.5042
Δ_x	1.149437×10^{-3}	1.224097×10^{-3}	6.4953	1.238264×10^{-3}	7.7278
	Under 535.34 gram loading				
Δ_y	1.257617×10^{-2}	1.201248×10^{-2}	4.4822	1.136030×10^{-2}	9.6680
Δ_x	2.100025×10^{-3}	2.201686×10^{-3}	4.8409	2.234130×10^{-3}	6.3859
	Under 709.02 gram loading				
Δ_y	1.521955×10^{-2}	1.500982×10^{-2}	1.3780	1.418049×10^{-2}	6.8271
Δ_x	3.281051×10^{-3}	3.498777×10^{-3}	6.6358	3.563024×10^{-3}	8.5940

Table 3.7: Deflections obtained from specimen 3 through experiment, semi-analytic model and conventional PRB model.

Specimen 3: $L = 40\text{mm}$, $l = 20\text{mm}$					
	Experimental results (m)	Semi-analytic model		PRB model	
		Results (m)	Error (%)	Results (m)	Error (%)
	Under 76.15 gram loading				
Δ_y	5.238885×10^{-3}	5.014000×10^{-3}	4.2926	4.901920×10^{-3}	6.4320
Δ_x	2.338048×10^{-4}	2.351269×10^{-4}	0.5655	2.408683×10^{-4}	3.0211
	Under 155.15 gram loading				
Δ_y	9.922219×10^{-3}	1.002947×10^{-2}	1.0810	9.802496×10^{-3}	1.2066
Δ_x	8.860251×10^{-4}	9.467045×10^{-4}	6.8485	9.703042×10^{-4}	9.5121
	Under 240.15 gram loading				
Δ_y	1.497597×10^{-2}	1.504013×10^{-2}	0.4284	1.469161×10^{-2}	1.8987
Δ_x	2.059379×10^{-3}	2.151842×10^{-3}	4.4898	2.207150×10^{-3}	7.1755
	Under 335.15 gram loading				
Δ_y	2.020208×10^{-2}	2.003451×10^{-2}	0.8295	1.955171×10^{-2}	3.2193
Δ_x	3.815851×10^{-3}	3.877934×10^{-3}	1.6270	3.981194×10^{-3}	4.3331
	Under 445.95 gram loading				
Δ_y	2.522960×10^{-2}	2.500202×10^{-2}	0.9020	2.436239×10^{-2}	3.4373
Δ_x	6.525764×10^{-3}	6.166943×10^{-3}	5.4985	6.336811×10^{-3}	2.8955

Table 3.8: Deflections obtained from specimen 4 through experiment, semi-analytic model and conventional PRB model.

Specimen 4: $L = 100\text{mm}$, $l = 20\text{mm}$					
	Experimental results (m)	Semi-analytic model		PRB model	
		Results (m)	Error (%)	Results (m)	Error (%)
	Under 35.35 gram loading				
Δ_y	1.137355×10^{-2}	1.106681×10^{-2}	2.6969	1.100878×10^{-2}	3.2072
Δ_x	6.823270×10^{-4}	5.410894×10^{-4}	20.6994	5.522651×10^{-4}	19.0615
	Under 72.15 gram loading				
Δ_y	2.311437×10^{-2}	2.214547×10^{-2}	4.1918	2.202288×10^{-2}	4.7221
Δ_x	2.506762×10^{-3}	2.182085×10^{-3}	12.9521	2.227124×10^{-3}	11.1554
	Under 111.95 gram loading				
Δ_y	3.318850×10^{-2}	3.320817×10^{-2}	0.0593	3.300704×10^{-2}	0.5467
Δ_x	5.066524×10^{-3}	4.966646×10^{-3}	1.9713	5.068903×10^{-3}	0.0470
	Under 156.05 gram loading				
Δ_y	4.473701×10^{-2}	4.405417×10^{-2}	1.5263	4.375271×10^{-2}	2.2002
Δ_x	9.536236×10^{-3}	8.893797×10^{-3}	6.7368	9.075771×10^{-3}	4.8286
	Under 208.95 gram loading				
Δ_y	5.545359×10^{-2}	5.501776×10^{-2}	0.7859	5.457757×10^{-2}	1.5797
Δ_x	1.460879×10^{-2}	1.420769×10^{-2}	2.7457	1.449456×10^{-2}	0.7820

3.5 Case Study: Stiffness Analysis of A Double Compound Linear Spring

The semi-analytic models are applied to analyze the displacement stiffness of the active compliant prismatic mechanism from an X - Y - θ_z flexure stage as shown in Fig. 3.21a. Due to large displacements requirements, this active compliant prismatic mechanism is formed by a double compound linear spring, which consists of eight pairs of beam-based flexure joints coupled with rigid-links (Fig. 3.21b). Each pair of beam-based joints is formed by a SUS-304 stainless steel shim, which is being clamped by two solid blocks that forms the rigid-link between the joints. For a compact-size design, each rigid-link is only 1.5 times longer than the beam-based flexure joint. Table 3.9 contains the specifications of each beam-based flexure joint used to form each compound linear spring.

Figure 3.21: (a) A 3DOF 3PPR flexure stage and (b) its active compliant prismatic mechanism.

Table 3.9: Specifications of beam-based flexure joint used in the compound prismatic joint.

Beam-based flexure length, l	5 mm
Beam-based flexure thickness, t	0.1 mm
Young's Modulus of flexure, E	$\sim 190\text{GPa}$ (SUS-304)
Rigid link length, L	15 mm
Rigid link thickness	10 mm
Width of flexures and rigid-links, b	15 mm

In this analysis, each compound linear spring mechanism is treated as two sets of parallel flexible bearings supporting a moving platform. Each flexible bearing comprises of a pair of

limbs, with each limb consists of two beam-based flexure joints connected by a rigid-link. Hence, the deflection of each limb, δ_x , is given as

$$\delta_x = \frac{disp_x}{2} \quad (3.58)$$

where $disp_x$ represents the desired displacement of the entire mechanism (Fig. 3.22).

By substituting the deflection of each limb obtained from Eqn. 3.58 into Eqn. (3.33), the desired deflection angle, θ , is obtained. Hence, the driving force, F , to achieve this desired deflection angle is determined from Eqn. (3.39). Thus, the stiffness of each limb, K_{limb} , is expressed as

$$K_{limb} = \frac{F}{\delta_x} \quad (3.59)$$

Figure 3.22: Schematic diagram of a compound linear spring.

As each pair of limbs forms a single linear spring mechanism, the stiffness of such a mechanism will be double the stiffness of a limb. On the other hand, the stiffness of a compound linear spring mechanism is only half of a single linear spring since it forms a serially-connected linear spring (Fig. 3.22). With the active compliant prismatic mechanism formed by a double compound linear spring mechanism, its stiffness is given as

$$K_{SYS} = 2K_{limb} \quad (3.60)$$

The derived stiffness model from the semi-analytic approach is used to predict the actuating stiffness of the developed active compliant prismatic mechanism. In addition, stiffness analysis through a PRB approach is also conducted. On the other hand, the experimental results are measured from the prototype of the compound linear spring mechanism as shown in Fig. 3.21b. Using a VC actuator, its moving air-core coil is attached to the moving platform to provide a non-contact actuation. By energizing the coil, the generated Lorentz-force will drive the moving platform of the mechanism. With each input current given, a Force/Torque (F/T) sensor from ATI (model: MINI-36) is used to record the force generated while the displacement generated by the mechanism is measured using a Micro-E linear optical encoder (resolution of 20 nm/count).

Figure 3.23: Stiffness error between the experimental data and the semi-analytic model, and PRB model respectively.

The results plotted in Fig. 3.23 have shown that the errors between the semi-analytic model and the obtained experimental data are smaller as compared to the errors between the PRB model and the experimental data. Based on the PRB model, the obtained displacements have an average error of about 7 % when compared to the experimental data. On the other hand, the semi-analytic model has higher accuracy with an average error of about 3 % being registered when compared to the experimental data. Hence, this analysis not only shows that the semi-analytic model can be used for quick parametric studies and

understandings on the beam-based flexure modules, it also provides much higher accuracy and robustness as compared to conventional approximation approaches.

3.6 Summary

A semi-analytic model for predicting the large deflection of beam-based flexure joints is introduced. The accuracy of this model is not limited by any specific configuration of the beam-based flexure joint. A new factor, ϖ , is introduced to determine the parasitic shifting of the ‘pivot’ point during the large deflection of a beam-based flexure joint. Comparisons with previous theoretical model and experimental results have shown that this derived factor is effective in predicting the parasitic shift of the ‘pivot’ point. Using this factor, a simple and straight-forward semi-analytic model is derived to predict the nonlinear large deflection of a beam-based flexure joint subjected to an external moment end load. This model is compared with the classical nonlinear model and has proven its accuracy in approximating the nonlinear behavior of a large deflection beam-based flexure joint. Using the approximation technique, this semi-analytic model is extended to solve the beam-based flexure coupled with rigid-link of various lengths. Another factor, ρ , is introduced to identify the initial pivot point of a beam-based flexure joint coupled with various rigid-link lengths. Theoretical comparisons with past models and observations have shown that this factor can effectively predict the initial pivot point, which is used to determine the moment arm of the deflecting beam-based flexure. With this factor, the force-deflection relationship of a beam-based flexure joint coupled with any rigid-link lengths can be derived. An experiment has been conducted to evaluate the accuracy of this approximated solution. Experimental results show that the semi-analytic model has higher accuracy in approximating the large deflection beam-based flexure coupled with various rigid-link lengths as compared to the PRB model. Predictions of the semi-analytic model have been consistent with an average deviation of 2.3 % as compared to the actual deflection measured from the beam-based flexure specimens. Subsequently, the semi-analytic model is used to approximate the displacement stiffness of the beam-based flexure joint modules. Experimental investigations conducted on an actual beam-based flexure module have shown that the semi-analytic model offers high accuracy and

robustness in approximating the displacement stiffness of such modules. Based on all the experimental results, the proposed semi-analytic model has proven to be a simple, fast and accurate analytical tool that is essential for quick parametric studies and understandings on any beam-based flexure modules.

Chapter 4

A Dual-Magnet Configuration

“The only laws of matter are those that our minds must fabricate and the only laws of mind are fabricated for it by matter.”

– James Clerk Maxwell (1831 – 1879).

4.1 Background

Electromagnetic (EM) actuation has recently become a promising contender in the area of ultra-high precision manipulation due to its capability of providing millimeters of displacement with infinite positioning resolutions. As mentioned in Chapter 2, the three main types of EM techniques currently used to realize the ultra-high precision manipulations are EM propulsion, magnetic levitation (or Maglev) and Lorentz-force actuation. An EM propulsion is achieved by the attraction and repulsion of a ferromagnetic moving part using the EM field generated from a solenoid. Yet, it possesses inconsistent actuating forces and hysteresis due to the electro-magnetization of the ferromagnetic stators. While Maglev has been the most popular approach for developing support-less and non-contact multi-DOF stages, it exhibits nonlinear characteristics and requires complex algorithms and costly control systems. Among these techniques, Lorentz-force actuation is a direct non-commutation drive, which provides a constant output force with infinite positioning

resolution throughout the entire traveling range without any complex control algorithm or system.

4.1.1 Lorentz-Force Theorem

The current-force relationship of a Lorentz-force actuation is govern by

$$F = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (4.1)$$

where F is the force generated, q is an electric charge particle, $\mathbf{v}$ is the instantaneous velocity of the electric charge, $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the external magnetic flux density. In the continuum limit, Eqn. (4.1) can be generalized to

$$f = \rho\mathbf{E} + \mathbf{J} \times \mathbf{B} \quad (4.2)$$

where f is the force density, ρ is the free charge density and $\mathbf{J}$ is the free current density.

For a VC actuator, the electric field is absence, i.e. $\mathbf{E} \rightarrow 0$, due to the used of an air-core coil. Hence from Eqn. (4.2), the force imparted to the current-carrying wire within the external magnetic field region is expressed as

$$F = i \int dL \times \mathbf{B} \quad (4.3)$$

where i is input current and L is the length of coil within the effective air gap.

Equation (4.3) suggests that the generated force is linearly proportional to the input current, the magnitude of magnetic flux density and the total coil length within the effective air gap. Although this relationship plays an important role for the linearity and constancy between input current and generated force, it is also the major contributor to the limitations of a Lorentz-force actuation, such as small output force and poor force-to-size ratio.

4.1.2 Drawbacks of Lorentz-Force Actuation

To increase the generated force, all three variables, i.e. the input current, the length of coil and the magnetic flux density, in Eqn. (4.3) play a vital role. However, an increase in input current will elevate the heat generation and affect the precision of the motions. Hence, this approach has always been the last option to be considered. On the other hand, increment of the coil length requires the effective air gap to be larger. However, the magnitude of magnetic flux density reduces gradually as it propagates away from the surface of a PM. As a result, coils further away from the PM pole surface experience much lower magnetic flux density and hence contribute limited increment to the generated force. Thus, this approach is the least efficient as it has less significance in increasing the force and yet it increases the size tremendously. Last but at least, the presence of magnetic flux density within the effective air gap is usually smaller by an order of 10^{-1} as compared to the other two variables. As a result, the magnitude of magnetic flux density becomes a reduction ratio in Eqn. (4.3) that lowers the effectiveness of incrementing the input current or the coil length. Consequently, increasing the magnetic flux density within the effective air gap becomes the best option among these variables.

4.1.3 Previous Works

4.1.3.1 Magnetic Circuit

Based on a conventional VC actuator, an effective air gap is usually forms in between the housing and the PM, where the air-core coil is operating (Fig. 4.1a). In this air gap, the magnetic flux density is well orientated and concentrated. Hence, an introduction of current perpendicular across the flux direction generates a force that propels the moving member. By natural phenomenon, the magnetic flux density within an effective air gap emanates from a PM. Due to the low permeability of air, the magnitude of this magnetic flux density is usually about 35 % of the remanence magnetic flux density of a PM. For example, a PM with 1 Tesla (T) of remanence magnetic flux density can only register 0.35 T within the effective air gap.

Figure 4.1: Various types of magnetic circuits found in (a) a conventional VC actuator, (b) a H2W VC actuator and (c) a BEI VC actuator.

An effective approach of increasing the magnetic flux density is to increase the size of a PM. This approach however requires other forms of magnetic circuit designed. Several good examples can be found in the VC actuators from H2W Technologies [98] (Fig. 4.1b), and BEI Technologies [123] (Fig. 4.1c). These modified magnetic circuit designs increased the magnetic flux density through large PMs. Nevertheless, these circuits still require a very small effective air gap since the magnetic flux density varies with respect to the distance from the magnet-polarized surface. Most importantly, these circuits have smaller effective air gaps as compared to a conventional VC actuator. As a result, force generated varies according to the position of the air-core coil during operations.

4.1.3.2 Magnetic Field Modeling of Magnetic Circuits

After synthesizing these magnetic circuits, it will be essential to analyze the magnitude of the magnetic flux density obtained within the effective air gap. Based on the current developments of the magnetic circuits in the VC actuators, the effective air gaps can be separated to two forms as illustrated in Fig. 4.2.

In these magnetic circuits, one common characteristic is that a single PM is the only magnetic source. Assuming that the PM is ideal, the field relationship can be described by the linear second quadrant of a PM de-magnetization curve. Consequently, a constitutive relation for a PM is expressed as

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \quad (4.4)$$

Figure 4.2: (a) Flux focus and (b) conventional effective air gaps

where μ_0 is the permeability of free space, $\mathbf{H}$ is the magnetic field strength and $\mathbf{M}$ is the magnetization of a PM.

On the other hand, assuming air to be a linear and homogeneous media with the absence of magnetization, i.e., $\mathbf{M} \rightarrow 0$, the field relation within the air can be described by simplifying Eqn. (4.4) into

$$\mathbf{B} = \mu_0 \mathbf{H} \quad (4.5)$$

A. Magnetostatic Energy

For very small effective air gaps, such as the flux focus form (Fig. 4.2a) found in a H2W or BEI VC actuator, the magnetostatic energy method has been an effective analytical model for predicting the magnetic flux density within the effective air gaps. Assuming the core has an infinite permeability, i.e., $\mu \approx \infty$, and without flux leakage, the magnetic flux density within the effective air gap is given as

$$B_g = \frac{\mu_0 B_r}{(l_g/l_m)(B_r/H_c) + \mu_0(A_g/A_m)} \quad (4.6)$$

where B_r and H_c are the remanence magnetic flux density and the coercivity field strength of a PM respectively, while l_m and l_g are the path lengths of the PM and the effective air

gap respectively, and A_g and A_m are the cross-sectional area of the effective air gap and the PM respectively (see Appendix A.1 for more details).

Equation (4.6) provides a quick estimation of the magnetic flux density within a very small effective air gap. Results obtained are fairly accurate though more precise approximations require the accounting of flux leakages and actual permeability of the core. Unfortunately, the magnetostatic energy method only gives an estimated value of the magnetic flux density within the effective air gap. Hence, it is only suitable for very small effective air gaps analyses since it is assume that the magnetic flux density is constant within such small gaps. For larger effective air gaps, this method fails as the magnetic flux density varies with respect to the distance from the PM pole surface.

B. Surface Charge Model

The surface charge model is a better method for analyzing the deviation of magnetic flux density within the effective air gap. In this method, a PM is treated as an element with a distribution of magnetic dipole, where the positive monopole represents the North pole surface of a PM and the negative monopole represents the south pole surface of a PM. Although magnetic monopoles are not found in nature, theory allows their existence because the propagation of magnetic field through free space can be treated as a point-to-point motion of electric charge. Assuming that the effective air gap is a current-free region, this model proposed that the magnetic flux density within this gap is expressed as

$$B(D) = \frac{\mu_0 M}{\pi} \left\{ \tan^{-1} \left[\frac{(D+t)\sqrt{a^2 + b^2 + (D+t)^2}}{ab} \right] - \tan^{-1} \left(\frac{D\sqrt{a+b+D}}{ab} \right) \right\} \quad (4.7)$$

where $2a$, $2b$ and t are the length, width and thickness of a PM, and D represents the distance from the polarized surface of a PM (refer to Fig. 4.2) (see Appendix A.2 for more details).

Equation (4.7) provides a good estimation of the magnetic flux density with respect to the distance away from the polarized surface of a PM in one-dimensional (1D) space. This surface charge model is very suitable for modeling the magnetic flux density within the effective air gap of a conventional VC actuator. Previous efforts have demonstrated this

model provide accurate prediction of magnetic flux density within free air space emanates from a PM [124, 125].

4.2 A Dual-Magnet Configuration

In this thesis, a new magnetic circuit is introduced to enhance and improve the capabilities of a Lorentz-force actuation. Termed Dual-Magnet (DM) configuration (Fig. 4.3), it consists of a ferromagnetic stator and two pairs of rare-earth PMs, as shown in Fig. 4.3a. Each pair of PMs will be placed in the attracting position within the gaps of the ferromagnetic stator and forms an effective air gap between them. The moving member with conducting coil provides a translational motion within the effective air gap as shown in Fig. 4.3b. Unlike a conventional magnetic circuit, a DM configuration offers:

Figure 4.3: (a) A symmetrical DM configuration in (b) an electromagnetic module.

Large effective air gap: In a conventional magnetic circuit, the magnetic flux density within the effective air gap varies with respect to the distance from the PM polarized surface. On the other hand, a DM configuration ensures an evenly distributed magnetic flux density throughout the air gap. Thus, magnetic flux density remains constant even at tens of millimeters away from the PM polarized surface.

Good force-to-size ratio: To achieve a larger output force, a bigger size PM or longer coil wire is required in a conventional magnetic circuit. Yet any of these approaches will increase the size of a VC actuator. A DM configuration offers higher and evenly distributed

magnetic flux density within the effective air gap as compared to the conventional magnetic circuit. This allows larger force generation with a more compact configuration.

Low heat generation: Generating large continuous output force will elevate the heat from the conducting coil and causes thermal expansion in any material. Such expansions, which can be in nanometric or micrometers, will affect the positioning accuracy. A DM configuration allows the electromagnetic driving module to generate large continuous output force with small amount of input current so as to ensures low heat generation and minimize the thermal expansion.

4.3 Magnetic Field Modeling of A Dual-Magnet Configuration

An accurate magnetic field model of a DM configuration is essential for the establishment of an accurate current-force model for the proposed electromagnetic driving module. In addition, a precise prediction of the magnetic field gives a fundamental understanding of the magnetic field strength and magnetic flux distribution in the design optimization stage of a DM configuration. These parameters also contribute to the complexity in control implementation to achieve the required performance specifications. Based on Eqn. (4.3), the current-force relationship is associated by the total coil length, $Coil$, and the magnetic flux density within the effective air gap. In this analysis, the coil length is a fixed constant and the magnetic flux density is assumed to vary within the effective air gap. Hence, the force generated in the x-axis (refer to Fig. 4.3) by the current flowing within the coil along the z-axis and magnetic flux density along the y-axis, $B\hat{y}$, can be expressed as

$$F = i \int_0^{-Coil} (-\hat{z}dz) \times (B\hat{y})\hat{x} = iNBCoil_z \quad (4.8)$$

where i is the amount of input current, N is the number of coil turn, B is the magnitude of the magnetic flux density within the effective air gap, and $Coil_z$ is the coil length of each turn along the z-axis.

Equation (4.8) indicates that an accurate current-force model requires a good prediction of the magnetic flux density in two-dimensional (2D) space because the number of coil turn is mapped in a cross-sectional area in the x - y plane (Fig. 4.3b). Hence, an accurate prediction of the magnetic flux density at each coil location, (x,y) , will result in an accurate current-force model. Initially, the study of the magnetic flux density within the effective air gap of a DM configuration is conducted in one-dimension (1D) using the surface charge model. In the 1D theoretical analysis, Eqn. (4.7) proves to be an accurate model in predicting the magnetic flux density based on experimental verifications. Unfortunately, the same principle becomes less effective in describing the magnetic field within the effective air gap of a DM configuration in higher dimensions. In this chapter, a 2D mathematical model using the magnetic scalar potential theorem is presented for modeling the magnetic field within the effective air gap of the proposed DM configuration efficiently. Unlike previous efforts of using this theorem to represent magnetic field through boundary-value approach [68, 126, 127], superposition of boundary conditions is proposed to obtain the closed-form solution of the magnetic field in the effective air gap between two magnetic sources.

4.3.1 Magnetic Scalar Potential Theory

Prior to applying the magnetic scalar potential theorem, this section shows how this theorem is derived since past literatures provide less in-depth studies on its derivation. Here, the magnetic scalar potential theorem is derived mainly through *Hamiltonian* principle and *Helmholtz's* theorem.

According to Maxwell's equations [128], the curl of magnetic field strength arises from charge and current sources,

$$\nabla \cdot \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad (4.9)$$

where $\mathbf{D}$ is the electric flux density. In this analysis, magnetic field modeling of the effective air gap within a DM configuration is based on the assumption that the effective air gap is

a current-free region, i.e., $\mathbf{J} \rightarrow 0$. Hence, it becomes a magnetostatic analysis, where there is no displacement of current density, i.e., $\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \rightarrow 0$. Consequently, Eqn. (4.9) can be simplified as,

$$\nabla \cdot \mathbf{H} = 0 \quad (4.10)$$

Based on *Hamiltonian* principle, the total energy of a physical system, which is independent to time, can be sum of potential energy and kinetic energy. In this case, potential and kinetic energy is the field from PM and electric current respectively. This principle can be realized through the use of *Helmholtz's* theorem [129] that allows both potential and kinetic energy to be directly separated into two portions

$$\mathbf{V} = -\nabla\Phi + \nabla \times \mathbf{A} \quad (4.11)$$

whereby sum of the divergence of scalar potential, $\nabla\Phi$, and the curl of vector potential, $\nabla \times \mathbf{A}$, results in a vector field, $\mathbf{V}$. Due to the absence of vector potential, i.e., $\mathbf{A} \rightarrow 0$, vector field is said to be an irrotational field, i.e., $\nabla \times \mathbf{A} = 0$, and together with Eqn. (4.11) yields

$$\mathbf{H} = -\nabla\Phi \quad (4.12)$$

Consequently, combining Eqn. (4.10) and (4.12) suggest that a magnetic field in a current-free environment can be expressed as

$$\nabla^2\Phi = 0 \quad (4.13)$$

whereby the magnetic field strength being describe by a scalar potential with Laplace's operator, also known as the magnetic scalar potential theorem.

4.3.2 Boundary-Value Problem

The proposed stator of the electromagnetic module is designed in a rectangular form with rectangular shape NdFeB PMs (Fig. 4.4). Assuming that these rectangular PMs are uniformly magnetized, the magnetic field along the z-axis will be similar to that along the x-axis (Fig. 4.4). Thus, a 3D problem can be reduced to a 2D problem. With its symmetrical design, a DM configuration can be further simplified into five regions:

- Region I: half of the effective air gap of a DM configuration.
- Region II: air breach between the PM-1 and the stator.
- Region III: half of the PM-1 of a DM configuration.
- Region IV: air breach between the PM-2 and the stator.
- Region V: half of the PM-2 of a DM configuration.

Figure 4.4: A 2D geometry reduced from half of a DM configuration.

In this analysis, the method of separation of variables is used to obtain the solution of the scalar potential since Eqn. (4.13) is a linear homogeneous partial differentiation equation. By letting the scalar potential to be a product of single variable function, this yields

$$\Phi_i(x, y) = X_i(x)Y_i(y) \quad (4.14)$$

where $i=I,II,III,IV,V$ representing region I to region V respectively (Fig. 4.4).

Based on Eqn. (4.14), Eqn. (4.13) is further expanded into two independent terms and yields

$$\frac{\ddot{X}_i(x)}{X_i(x)} + \frac{Y_i''(y)}{Y_i(y)} = 0 \quad (4.15)$$

where each term is a function of a single variable that can be represented with an arbitrary constant, i.e. $k_{i,x}^2 + k_{i,y}^2 = 0$ with $\ddot{X}_i(x) = \partial^2 X_i(x)/\partial x^2$ and $Y_i''(y) = \partial^2 Y_i(y)/\partial y^2$.

In this analysis, $k_{i,x}^2$ is chosen to be a negative value and thus leading $k_{i,y}^2$ to being a positive value as $k_{i,x}$ has to be imaginary since the field behavior in the x-direction is unpredictable. Consequently, $k_{i,y}$ can be a real as the value increases or decreases according to the distance from the PM polarized surface in the y-direction. Therefore, Eqn. (4.15) can be separated into two independent Ordinary-Differentiation-Equation (ODE). Solving those ODEs yields,

$$X_i^n(x) = S_{1,i}^n \cos(k_i^n x) + S_{2,i}^n \sin(k_i^n x) \quad (4.16)$$

and

$$Y_i''(y) = S_{3,i}^n e^{k_i^n y} + S_{4,i}^n e^{-k_i^n y} \quad (4.17)$$

where $S_{i,j}^n$ ($j = 1, 2, 3, 4$) are constants.

Substituting Eqn. (4.16) and (4.17) into (Eqn. 4.14), and applying the superposition principle yields the general form of scalar potential solution for each region

$$\Phi_i(x, y) = \sum_{n=1}^{\infty} [S_{1,i}^n \cos(k_i^n x) + S_{2,i}^n \sin(k_i^n x)] (S_{3,i}^n e^{k_i^n y} + S_{4,i}^n e^{-k_i^n y}) \quad (4.18)$$

Thus, Eqn. (4.13) becomes a 2D Dirichlet boundary-value problem expressed in Eqn. (4.18), whereby each constant can be determined by imposing appropriate boundary conditions to each region.

4.3.3 Superposition of Boundary Conditions

Here, superposition of boundary conditions is proposed to obtain a closed-form solution for the scalar potential of the effective air gap between two magnetic sources, PM-1 and PM-2. The scalar potential of the effective air gap will be solved separately under the influence of each PM and subsequently superimposed together. Thus, the total scalar potential, $\Phi_{I,Total}(x, y)$, of the effective air gap can be expressed as

$$\Phi_{I,Total}(x, y) = \Phi_{I,PM-1}(x, y) + \Phi_{I,PM-2}(x, y) \quad (4.19)$$

where $\Phi_{I,PM-1}(x, y)$ is the scalar potential of effective air gap under the influence of PM-1 and $\Phi_{I,PM-2}(x, y)$ is that under the influence of PM-2.

Generally, the proposed boundary conditions are made under the following hypotheses:

1. The permeability of the iron stator is infinite ($\mu = 0$).
2. When analyzing the effective air gap under the influence of PM-1, PM-2 is treated as a free-air region. Thus, the tangential component vanishes when approaching c along the y -axis (refer to Fig. 4.4).
3. When analyzing the effective air gap under the influence of PM-2, PM-1 is treated as a free-air region. Thus, the tangential component vanishes when approaching 0 along the y -axis (refer to Fig. 4.4).
4. The normal component of the magnetic flux density at the center of the effective air gap and the PMs equals to zero (refer to Fig. 4.4 at $x = 1$).
5. The tangential component of the magnetic field strength along the closed-loop path is equal to zero.

6. Magnetization within the PM is only orientating normal to the polarization surface.

Thus, the magnetization for these regions is defined as:

$$M = \begin{cases} M\hat{y} & \text{Region III,V} \\ 0 & \text{Region I,II,IV} \end{cases} \quad (4.20)$$

4.3.4 Magnetic Field Solution

4.3.4.1 Air Gap Influence Under PM-1

1. Scalar potential of region I (effective air gap)

From Eqn. (4.18), the general solution of Region I in 2D Cartesian space is given as

$$\Phi_{I,PM-1}(x, y) = \sum_{n=1}^{\infty} [S_{1,I}^n \cos(k_I^n x) + S_{2,I}^n \sin(k_I^n x)] (S_{3,I}^n e^{k_I^n y} + S_{4,I}^n e^{-k_I^n y}) \quad (4.21)$$

A. Based on boundary condition stated in Section 4.3.3, postulation no. 5 and Eqn. (4.12) gives

$$H_{Iy,PM-1}^n(0, y) = -\frac{\partial \Phi_{I,PM-1}^n(x, y)}{\partial y} \Big|_{x=0} = 0 \quad (a \leq y \leq b) \quad (4.22)$$

Applying Eqn. (4.22) to Eqn. (4.21) yields

$$S_{1,I}^n = 0 \quad (4.23)$$

Substituting Eqn. (4.23) into Eqn. (4.21), the scalar potential of Region I is re-expressed as

$$\Phi_{I,PM-1}^n(x, y) = (C_{1,I}^n e^{k_I^n y} + C_{2,I}^n e^{-k_I^n y}) \sin(k_I^n x) \quad (4.24)$$

where $C_{1,I}^n = S_{2,I}^n S_{3,I}^n$ and $C_{2,I}^n = S_{2,I}^n S_{4,I}^n$ are constants.

B. Based on boundary condition stated in Section 4.3.3, postulation no. 4, and Eqn. (4.5) and (4.12) gives

$$B_{I_x, PM-1}^n(l, y) = \mu_0 \left(-\frac{\partial \Phi_{I, PM-1}^n(x, y)}{\partial x} \Big|_{x=l} \right) = 0 \quad (a \leq y \leq b) \quad (4.25)$$

Applying Eqn. (4.25) to Eqn. (4.24) yields

$$\cos(k_I^n l) = 0 \quad (4.26)$$

Consequently, Eqn. (4.26) suggests that

$$k_I^n = \frac{(2n-1)\pi}{2l} \quad (n = 1, 2, 3, \dots) \quad (4.27)$$

C. Based on boundary condition stated in Section 4.3.3, postulation no. 2 and Eqn. (4.12) gives

$$H_{I_x, PM-1}^n(x, c) = -\frac{\partial \Phi_{I, PM-1}^n(x, y)}{\partial x} \Big|_{y=c} = 0 \quad (g \leq x \leq l) \quad (4.28)$$

Applying Eqn. (4.28) to (4.24) yields

$$C_{2,I}^n = -C_{1,I}^n e^{2k_I^n c} \quad (4.29)$$

Substituting Eqn. (4.29) into Eqn. (4.24), the scalar potential of region I is re-expressed as

$$\begin{aligned} \Phi_{I, PM-1}(x, y) &= [C_{1,I}^n e^{k_I^n y} + (-C_{1,I}^n e^{2k_I^n c}) e^{-k_I^n y}] \sin(k_I^n x) \\ &= C_{1,I}^n [e^{k_I^n y} - e^{-k_I^n (y-2c)}] \sin(k_I^n x) \end{aligned} \quad (4.30)$$

Based on hyperbolic trigonometry identities

$$\sinh(u) = \frac{1}{2}(e^u - e^{-u})$$

let $k_I(y - y') = u$

$$\begin{aligned} \sinh(k_I(y - y')) &= \frac{1}{2}(e^{k_I(y-y')} - e^{-k_I(y-y')}) \\ 2 \sinh[k_I(y - y')]e^{k_I y'} &= (e^{k_I y} \cdot e^{-k_I y'} - e^{-k_I y} \cdot e^{k_I y'})e^{k_I y'} \\ &= e^{k_I y} - e^{-k_I(y-2y')} \end{aligned} \quad (4.31)$$

Consequently, substituting Eqn. (4.27) and (4.31) (where $y' = c$) into Eqn. (4.30), along with the principle of superposition, the scalar potential of Region I under the influence of PM-1 is re-expressed as

$$\Phi_{I,PM-1}(x, y) = \sum_{n=1,2,3,\dots}^{\infty} C_{Air-gap,PM-1}^n \sin\left[\frac{(2n-1)\pi}{2l}x\right] \sinh\left[\frac{(2n-1)\pi}{2l}(y-c)\right] \quad (4.32)$$

where $C_{Air-gap,PM-1}^n = 2C_{1,I}^n \exp\left[\frac{(2n-1)\pi}{2l}c\right]$ is constant.

2. Scalar potential of region II (air breach 1)

From Eqn. (4.18), the general solution of Region II in 2D Cartesian space is given as

$$\Phi_{II}(x, y) = \sum_{n=1}^{\infty} [S_{1,II}^n \cos(k_{II}^n x) + S_{2,II}^n \sin(k_{II}^n x)] (S_{3,II}^n e^{k_{II}^n y} + S_{4,II}^n e^{-k_{II}^n y}) \quad (4.33)$$

A. Based on boundary condition stated in Section 4.3.3, postulation no. 5 and Eqn. (4.12) gives

$$H_{IIx}^n(x, 0) = -\frac{\partial \Phi_{II}^n(x, y)}{\partial x} \Big|_{y=0} = 0 \quad (0 \leq x \leq g) \quad (4.34)$$

Applying Eqn. (4.34) to Eqn. (4.33) yields

$$S_{4,II}^n = -S_{3,II}^n \quad (4.35)$$

Substituting (4.35) into Eqn. (4.33), the scalar potential of region II is re-expressed as

$$\Phi_{II}^n(x, y) = [C_{1,II}^n \cos(k_{II}^n x) + C_{2,II}^n \sin(k_{II}^n x)] \sinh(k_{II}^n y) \quad (4.36)$$

where $C_{1,II}^n = 2S_{3,II}^n S_{1,II}^n$ and $C_{2,II}^n = 2S_{3,II}^n S_{2,II}^n$ are constants.

B. Based on boundary condition stated in Section 4.3.3, postulation no. 5 and Eqn. (4.12) gives

$$H_{II,y}^n(0, y) = -\frac{\partial \Phi_{II}^n(x, y)}{\partial y} \Big|_{x=0} = 0 \quad (0 \leq y \leq a) \quad (4.37)$$

Applying Eqn. (4.37) to Eqn. (4.36) yields

$$C_{1,II}^n = 0 \quad (4.38)$$

Consequently, substituting Eqn. (4.38) into Eqn. (4.36), along with the principle of superposition, the scalar potential of Region II is re-expressed as

$$\Phi_{II}(x, y) = \sum_{n=1,2,3,\dots}^{\infty} C_{Air-breach1}^n \sin(k_{II}^n x) \sinh(k_{II}^n y) \quad (4.39)$$

where $C_{Air-breach1}^n = C_{II}^n$ is constant and k_{II}^n will be determined in the following section.

3. Scalar potential of region III (PM-1)

From Eqn. (4.18), the general solution of Region III in 2D Cartesian space is given as

$$\Phi_{III}(x, y) = \sum_{n=1}^{\infty} [S_{1,III}^n \cos(k_{III}^n x) + S_{2,III}^n \sin(k_{III}^n x)] (S_{3,III}^n e^{k_{III}^n y} + S_{4,III}^n e^{-k_{III}^n y}) \quad (4.40)$$

A. Based on boundary condition stated in Section 4.3.3, postulation no. 5 and Eqn. (4.12) gives

$$H_{III_x}^n(x, 0) = -\frac{\partial \Phi_{III}^n(x, y)}{\partial x} \Big|_{y=0} = 0 \quad (g \leq x \leq l) \quad (4.41)$$

Applying Eqn. (4.41) to Eqn. (4.40) yields

$$S_{4,III}^n = -S_{3,III}^n \quad (4.42)$$

Substituting Eqn. (4.42) into Eqn. (4.40), the scalar potential of Region III is re-expressed as

$$\Phi_{III}^n(x, y) = [C_{1,III}^n \cos(k_{III}^n x) + C_{2,III}^n \sin(k_{III}^n x)] \sinh(k_{III}^n y) \quad (4.43)$$

where $C_{1,III}^n = 2S_{3,III}^n S_{1,III}^n$ and $C_{2,III}^n = 2S_{3,III}^n S_{2,III}^n$ are constants.

B. The tangential component of magnetic field strength at the boundary between air breach 1 and PM-1 must be continuous. This condition gives

$$H_{II_y}^n(g, y) = H_{III_y}^n(g, y) \quad (0 \leq y \leq a) \quad (4.44)$$

Substituting Eqn. (4.36) and (4.43) into Eqn. (4.44) yields

$$k_{II}^n = k_{III}^n \quad (4.45)$$

$$C_{1,II}^n = C_{1,III}^n \quad (4.46)$$

Substituting Eqn. (4.38) and (4.46) into Eqn. (4.43) yields

$$\Phi_{III}^n(x, y) = C_{Magnet1}^n \sin(k_{III}^n x) \sinh(k_{III}^n y) \quad (4.47)$$

where $C_{Magnet1}^n = C_{2,III}^n$ is a constant.

C. Based on boundary condition stated in Section 4.3.3, postulation no. 4, and Eqn. (4.5) and (4.12) gives

$$B_{III_x}^n(l, y) = \mu_0 \left(-\frac{\partial \Phi_{III}^n(x, y)}{\partial x} \right) \Big|_{x=l} = 0 \quad (0 \leq y \leq a) \quad (4.48)$$

Applying Eqn. (4.48) to Eqn. (4.47) yields

$$\cos(k_{III}^n l) = 0 \quad (4.49)$$

Consequently, Eqn. (4.49) suggest that

$$k_{III}^n = \frac{(2n-1)\pi}{2l} \quad (n = 1, 2, 3, \dots) \quad (4.50)$$

Substituting Eqn. (4.50) into Eqn. (4.47), along with the principle of superposition, the scalar potential of Region III is re-expressed as

$$\Phi_{III}(x, y) = C_{Magnet1}^n \sum_{n=1,2,3,\dots}^{\infty} \sin\left[\frac{(2n-1)\pi}{2l}x\right] \sinh\left[\frac{(2n-1)\pi}{2l}y\right] \quad (4.51)$$

4. Scalar potential of Region I under influence of PM-1

A. The tangential component of magnetic field strength at the boundary between the effective air gap and PM-1 must be continuous. This condition gives

$$H_{I_x, PM-1}(x, a) = H_{III_x}(x, a) \quad (g \leq x \leq l) \quad (4.52)$$

Substituting Eqn. (4.32) and (4.51) into (4.52) yields

$$C_{Magnet1}^n = C_{Air-gap, PM-1}^n \frac{\sinh\left[\frac{(2n-1)\pi}{2l}(a-c)\right]}{\sinh\left[\frac{(2n-1)\pi}{2l}a\right]} \quad (4.53)$$

B. The tangential component of magnetic flux density at the boundary between the effective air gap and PM-1 must be continuous. This condition gives

$$B_{I_y, PM-1}(x, a) = B_{III_y}(x, a) \quad (g \leq x \leq l) \quad (4.54)$$

Substituting Eqn. (4.32) and (4.51) into (4.54) yields

$$\sum_{n=1,2,3,\dots}^{\infty} C_{Air-gap, PM-1}^n \frac{(2n-1)\pi}{2l} \sin\left[\frac{(2n-1)\pi}{2l}x\right] U_I = M \quad (4.55)$$

where

$$U_I = \sinh\left[\frac{(2n-1)\pi}{2l}(a-c)\right] \coth\left[\frac{(2n-1)\pi}{2l}a\right] - \cosh\left[\frac{(2n-1)\pi}{2l}(a-c)\right] \quad (4.56)$$

Multiplying both sides of Eqn. (4.55) by $\sin\left[\frac{(2n-1)\pi x}{2l}\right]$ and integrating with respect to x yields (refer to Appendix A.3 for more details)

$$C_{Air-gap, PM-1}^n = \frac{8Ml(1)^n}{U_I[(2n-1)\pi]^2} \quad (4.57)$$

Consequently, substituting Eqn. (4.57) into (4.32) forms the complete solution for the scalar potential of the effective air gap (region I) under the influence of PM-1 that is expressed as

$$\Phi_{I,PM-1}(x, y) = \frac{8Ml}{\pi^2} \sum_{n=1,2,3,\dots}^{\infty} \frac{1}{U_I(2n-1)^2} \sin\left[\frac{(2n-1)\pi}{2l}x\right] \sinh\left[\frac{(2n-1)\pi}{2l}(y-c)\right] \quad (4.58)$$

4.3.4.2 Air Gap Influence Under PM-2

1. Scalar potential of region I (effective air gap)

A. Based on boundary condition stated in Section 4.3.3, postulation no. 3 and Eqn. (4.12) gives

$$H_{Ix,PM-2}^n(x, 0) = -\frac{\partial \Phi_{I,PM-2}^n(x, y)}{\partial x} \Big|_{y=0} = 0 \quad (g \leq x \leq l) \quad (4.59)$$

Applying Eqn. (4.59) to Eqn. (4.24) yields

$$C_{1,I}^n = -C_{2,I}^n \quad (4.60)$$

Substituting Eqn. (4.27) and (4.60) into Eqn. (4.24), along with the principle of superposition, the scalar potential of Region I under the influence of PM-2 can be re-expressed as

$$\Phi_{I,PM-2}(x, y) = \sum_{n=1,2,3,\dots}^{\infty} C_{Air-gap,PM-2}^n \sin\left[\frac{(2n-1)\pi}{2l}x\right] \sinh\left[\frac{(2n-1)\pi}{2l}y\right] \quad (4.61)$$

where $C_{Air-gap,PM-2}^n = 2C_{1,I}^n$ is a constant.

2. Scalar potential of region IV (air breach 2)

From Eqn. (4.18), the general solution of Region IV in 2D Cartesian space is given as

$$\Phi_{IV}(x, y) = \sum_{n=1}^{\infty} [S_{1,IV}^n \cos(k_{IV}^n x) + S_{2,IV}^n \sin(k_{IV}^n x)] (S_{3,IV}^n e^{k_{IV}^n y} + S_{4,IV}^n e^{-k_{IV}^n y}) \quad (4.62)$$

A. Based on boundary condition stated in Section 4.3.3, postulation no. 5 and Eqn. (4.12) gives

$$H_{IV_x}^n(x, c) = -\frac{\partial \Phi_{IV}^n(x, y)}{\partial x} \Big|_{y=c} = 0 \quad (0 \leq x \leq g) \quad (4.63)$$

Applying Eqn. (4.63) to Eqn. (4.62) yields

$$S_{4,IV}^n = -S_{2,IV}^n \exp(2k_{IV}^n c) \quad (4.64)$$

Consequently, substituting Eqn. (4.64) and (4.31) (where $y' = c$) into Eqn. (4.62), the scalar potential of Region IV is re-expressed as

$$\Phi_{IV}^n(x, y) = [C_{1,IV}^n \cos(k_{IV}^n x) + C_{2,IV}^n \sin(k_{IV}^n x)] \sinh[k_{IV}^n (y - c)] \quad (4.65)$$

where $C_{1,IV}^n = 2S_{3,IV}^n S_{1,IV}^n \exp(k_{IV}^n c)$ and $C_{2,IV}^n = 2S_{3,IV}^n S_{2,IV}^n \exp(k_{IV}^n c)$ are constants.

B. Based on boundary condition stated in Section 4.3.3, postulation no. 5 and Eqn. (4.12) gives

$$H_{IV_y}^n(0, y) = -\frac{\partial \Phi_{IV}^n(x, y)}{\partial y} \Big|_{x=0} = 0 \quad (b \leq y \leq c) \quad (4.66)$$

Applying Eqn. (4.66) to Eqn. (4.65) yields

$$C_{1,IV}^n = 0 \quad (4.67)$$

Consequently, substituting Eqn. (4.67) into Eqn. (4.65), along with the principle of superposition, the scalar potential of Region IV is re-expressed as

$$\Phi_{IV}(x, y) = \sum_{n=1,2,3,\dots}^{\infty} C_{Air-breach2}^n \sin(k_{IV}^n x) \sinh(k_{IV}^n (y - c)) \quad (4.68)$$

where $C_{Air-breach2}^n = C_{2,IV}^n$ is a constant and k_{IV}^n will be determined in the following section.

3. Scalar potential of region V (PM-2)

From Eqn. (4.18), the general solution of Region V in 2D Cartesian space is given as

$$\Phi_V(x, y) = \sum_{n=1}^{\infty} [S_{1,V}^n \cos(k_V^n x) + S_{2,V}^n \sin(k_V^n x)] (S_{3,V}^n e^{k_V^n y} + S_{4,V}^n e^{-k_V^n y}) \quad (4.69)$$

A. Based on boundary condition stated in Section 4.3.3, postulation no. 5 and Eqn. (4.12) gives

$$H_{V_x}^n(x, c) = -\frac{\partial \Phi_V^n(x, y)}{\partial x} \Big|_{y=c} = 0 \quad (g \leq x \leq l) \quad (4.70)$$

Applying Eqn. (4.70) to Eqn. (4.69) yields

$$S_{4,V}^n = -S_{3,V}^n \exp(2k_V^n c) \quad (4.71)$$

Consequently, substituting Eqn. (4.64) and (4.31) (where $y' = c$) into Eqn. (4.62), the scalar potential of Region IV is re-expressed as

$$\Phi_V^n(x, y) = [C_{1,V}^n \cos(k_V^n x) + C_{2,V}^n \sin(k_V^n x)] \sinh[k_V^n (y - c)] \quad (4.72)$$

where $C_{1,V}^n = 2S_{3,V}^n S_{1,V}^n \exp(k_V^n c)$ and $C_{2,V}^n = 2S_{3,V}^n S_{2,V}^n \exp(k_V^n c)$ are constants.

B. The tangential component of magnetic field strength at the boundary between the air breach 2 and PM-2 must be continuous. This condition gives

$$H_{IV_y}^n(g, y) = H_{V_y}^n(g, y) \quad (b \leq y \leq c) \quad (4.73)$$

Substituting Eqn. (4.65) and (4.72) into (4.73) yields

$$C_{1,IV}^n = -C_{1,V}^n \quad (4.74)$$

$$k_{IV}^n = k_V^n \quad (4.75)$$

Substituting Eqn. (4.74) into Eqn. (4.72)

$$\Phi_V^n(x, y) = C_{Magnet2}^n \sin(k_V^n x) \sinh[k_V^n (y - c)] \quad (4.76)$$

where $C_{Magnet2}^n = C_{2,V}^n$ is a constant.

C. Based on boundary condition stated in Section 4.3.3, postulation no. 4

$$B_{V_x}^n(l, y) = \mu_0 \left(-\frac{\partial \Phi_V^n(x, y)}{\partial x} \Big|_{x=l} \right) = 0 \quad (0 \leq y \leq a) \quad (4.77)$$

Applying Eqn. (4.77) to Eqn. (4.76) yields

$$\cos(k_V^n l) = 0 \quad (4.78)$$

Consequently, Eqn. (4.78) suggest that

$$k_V^n = \frac{(2n-1)\pi}{2l} \quad (n = 1, 2, 3, \dots) \quad (4.79)$$

Substituting Eqn. (4.79) into Eqn. (4.76), along with the principle of superposition, the scalar potential of Region V is re-expressed as

$$\Phi_V(x, y) = \sum_{n=1,2,3,\dots}^{\infty} C_{Magnet2}^n \sin\left[\frac{(2n-1)\pi}{2l}x\right] \sinh\left[\frac{(2n-1)\pi}{2l}(y-c)\right] \quad (4.80)$$

4. Scalar potential of Region I under influence of PM-2

A. The tangential component of magnetic field strength at the boundary between the effective air gap and PM-2 must be continuous. This condition gives

$$H_{I_x, PM-2}(x, b) = H_{V_x}(x, b) \quad (g \leq x \leq l) \quad (4.81)$$

Substituting Eqn. 4.61 and 4.80 into 4.81 yields

$$C_{Magnet2}^n = C_{Air-gap, PM-2}^n \frac{\sinh\left[\frac{(2n-1)\pi}{2l}b\right]}{\sinh\left[\frac{(2n-1)\pi}{2l}(b-c)\right]} \quad (4.82)$$

B. The tangential component of magnetic flux density at the boundary between the effective air gap and PM-2 must be continuous. This condition gives

$$B_{I_y, PM-2}(x, b) = B_{V_y}(x, b) \quad (g \leq x \leq l) \quad (4.83)$$

Substituting Eqn. (4.61) and (4.80) into (4.83) yields

$$\sum_{n=1,2,3,\dots}^{\infty} C_{Air-gap, PM-2}^n \frac{(2n-1)\pi}{2l} \sin\left[\frac{(2n-1)\pi}{2l}x\right] U_{II} = M \quad (4.84)$$

where

$$U_{II} = \sinh\left[\frac{(2n-1)\pi}{2l}b\right] \coth\left[\frac{(2n-1)\pi}{2l}(b-c)\right] - \cosh\left[\frac{(2n-1)\pi}{2l}b\right] \quad (4.85)$$

Multiplying both sides of Eqn. (4.84) by $\sin\left[\frac{(2n-1)\pi x}{2l}\right]$ and integrating with respect to x yields (refer to Appendix A.3 for more details)

$$C_{Air-gap, PM-2}^n = \frac{8Ml(1)^n}{U_{II}[(2n-1)\pi]^2} \quad (4.86)$$

Consequently, substituting Eqn. (4.86) into (4.61) forms the complete solution for the scalar potential of the effective air gap (region I) under the influence of PM-2 that is expressed as

$$\Phi_{I, PM-2}(x, y) = \frac{8Ml}{\pi^2} \sum_{n=1,2,3,\dots}^{\infty} \frac{1}{U_{II}(2n-1)^2} \sin\left[\frac{(2n-1)\pi}{2l}x\right] \sinh\left[\frac{(2n-1)\pi}{2l}y\right] \quad (4.87)$$

4.3.4.3 Proposed 2D Analytical Model

Based on Eqn. (4.5), (4.12) and (4.19), the magnetic flux density within the effective air gap of a DM configuration is expressed as

$$\mathbf{B}_{I, Total} = -\mu_0 \nabla [\Phi_{I, PM-1}(x, y) + \Phi_{I, PM-2}(x, y)] \quad (4.88)$$

Lastly, by substituting Eqn. (4.58) and (4.87) into Eqn. (4.88), the tangential component of the magnetic flux density within the effective air gap of a DM configuration is described as

$$B_{I_y, Total}(x, y) = -\frac{4\mu_0 M}{\pi} \sum_{n=1,2,3,\dots}^{\infty} \frac{1}{(2n-1)} \sin\left[\frac{(2n-1)\pi}{2l}x\right] \left\{ \frac{1}{U_I} \cosh\left[\frac{(2n-1)\pi}{2l}(y-c)\right] + \frac{1}{U_{II}} \cosh\left[\frac{(2n-1)\pi}{2l}y\right] \right\} \quad (4.89)$$

4.4 Prototypes and Experiments

4.4.1 Prototypes

Two types of magnetic circuit prototypes are developed for evaluation and comparison. One is the proposed DM configuration (Fig. 4.5a), the other is a conventional magnetic configuration, using a single PM and a closed-loop path (Fig. 4.5b). Both prototypes employ 54 mm × 50 mm × 7.5 mm NdFeB PMs (Type N45M) with remanence magnetic flux density of 1.33 T and a maximum operating temperature of 120°C. Both prototypes have an effective air gap height of 11 mm.

Figure 4.5: Prototypes of (a) a DM configuration and (b) a conventional magnetic configuration.

4.4.2 Experimental Setup

A LAKESHORE Hall-sensor probe and Gauss meter are the measurement tools employed in the experimental investigations (Fig. 4.6). The Hall-sensor probe is a single-axis magnetic flux density measuring device attached to a three-axis translational manipulator, which is programmed to position the probe at an incremental step of 0.5 mm in the X-, Y- and Z-directions within the effective air gap. The Gauss meter measures the amount of magnetic flux density instantaneously with noise resolution of 0.001 T and accuracy at ± 0.005 T, and records the data via a personal computer. Based on the assumption that the magnetic flux density distribution is symmetrical for both halves of the effective air gap volume, half of the air gap volume (from the middle to the edge of a PM) has been measured for both prototypes (Fig. 4.7). Due to the size of the Hall-sensor probe (~ 1.75 mm in diameter) the allowable height for measurement is restricted to 7 mm, i.e., between 2 mm to 9 mm of the effective

air gap along the z-axis (Fig. 4.7). Thus the measured volume within the air gap is 27 mm $\times$ 50 mm $\times$ 7 mm (Fig. 4.7).

Figure 4.6: Experimental setup for flux measurement.

Figure 4.7: Flux measurement.

4.5 Results

4.5.1 Magnitude of Magnetization

To predict the magnetic flux density within the effective air gap of the DM configuration, the magnetization of the PM is required as shown in the proposed 2D mathematical model, i.e., Eqn. (4.89). In the literatures, the magnetization of the PM is usually assumed

to be proportional to the remanence magnetic flux density of the PM, i.e., $\mathbf{B}_r = \mu_0 \mathbf{M}$, or to have the equivalent magnitude as the coercive force, $\mathbf{H}_c$ [130]. Some authors also suggested that the magnetization lies between a range from $\mathbf{H}_c$ to $\mathbf{B}_r$ for each particular PM [131]. Nevertheless, none of the existing literatures have provided an analytical model that accurately determines the magnetization of the PM from these magnetic parameters. In this work, an effective empirical approach, which is well-demonstrated by Lee [132], will be used to estimate the magnetization of a PM. Based on this approach, the magnetic flux density that emanates from the middle of a PM sample (used in the prototypes) is measured at 3.5 mm, 7 mm and 10.5 mm along the y-axis from the PM surface using the Gauss meter. The measured values are substituted into Eqn. (4.7) with parameters, $a = 25$ mm, $b = 27.5$ mm and $L = 7.5$ mm for D at 3.5 mm, 7 mm, and 10.5 mm respectively to estimate the actual magnetization of the PM sample. As a result, the calculated magnitude of the magnetization is obtained as 899.23×10^3 A/m.

4.5.2 Effect of Iron Separating the Symmetrical DM Configuration

At the center of the proposed electromagnetic driving module stator (Fig. 4.3), two symmetrical DM configurations are separated by an 8-mm thick iron, which forms a closed-loop path for the magnetic flux flow. Unfortunately, the experiments conducted in this work have shown otherwise based on the developed prototype (Fig. 4.8). Using the conventional magnetic configuration prototype, the magnetic flux density within the effective air gap that emanates from PM-A is measured (Fig. 4.8a). Next, PM-B is added beneath PM-A, where both PMs facing each other with the same pole and separated by an 8-mm thick iron path. Subsequently, the magnetic flux density within the effective air gap is re-measured (Fig. 4.8b). A comparison between both measurements shows a drop of the magnetic flux density that emanates from PM-A after PM-B is added. It shows that the thickness of the iron is ineffective in guiding the magnetic flux back to each respective DM configuration. Such ineffectiveness, which leads to the drop of magnetic flux density within the effective air gap, need to be considered in the prediction of the magnetic field behavior using the proposed analytical model. Consequently, the differences between both measurements are used to

back-calculate the magnetization using Eqn. (4.5), (4.12) and (4.58) with $a = 7.5 \times 10^{-3}$ m, $c = 18.5 \times 10^{-3}$ m, $l = 32 \times 10^{-3}$ m and $\mu_0 = 4\pi \times 10^{-7}$, which are the parameters obtained from the actual prototype. This approach returns a magnetization of 103.45×10^3 A/m, which indicating loss of magnetic flux density due to the ineffectiveness of the thickness of separating iron. In addition, it is assumed that the calculated magnetization will be doubled in magnitude due to a DM configuration.

Figure 4.8: Effect of the iron thickness between two PMs on the magnitude of magnetic field.

4.5.3 DM Configuration V.S. Conventional Magnetic Configuration

Magnetic flux densities within the effective air gap of the conventional magnetic configuration and the DM configuration are measured experimentally along the X-Y plane at $Z = 25$ mm (refer to Fig. 4.7, at the Z-axis), and are plotted in Fig. 4.9. In the DM configuration, the magnetic flux density within the effective air gap is 40 % higher (Fig. 4.9a) as compared to a conventional magnetic configuration (Fig. 4.9b). Based on Fig. 4.9, the magnetic flux leakage only occurs between 0 to 5 mm of the effective air gap within the DM configuration,

Figure 4.9: Magnetic flux density measured from (a) a conventional magnetic configuration and (b) a DM configuration.

while such leakages occur between 0 to 15 mm of the effective air gap within the conventional magnetic configuration. This shows that the magnetic flux density is uniformly distributed between 5 mm to 27 mm of the effective air gap within the DM configuration. In addition, the magnetic flux density within the effective air gap of a conventional magnetic configuration reduces as it moves away from the PM face, while magnetic flux density within the effective air gap of the DM configuration remains unchanged.

Figure 4.10 plots the magnetic flux density within the effective air gap of a conventional magnetic configuration and a DM configuration, which is experimentally measured along the X-Z plane at $Y = 2\text{ mm}$ and $Y = 7\text{ mm}$ (refer to Fig. 4.7, at Y-axis). For a conventional magnetic configuration, Fig. 4.10a shows that magnetic flux density of 0.35 T is registered evenly across the effective air gap at $Y = 2\text{ mm}$. Yet at $Y = 7\text{ mm}$, the same amount of magnetic flux density can only be recorded near the middle portion across the effective air

Figure 4.10: Magnetic flux density distribution in X-Z plane measured from (a) a conventional magnetic configuration and (b) a DM configuration at $Y = 2\text{ mm}$ and $Y = 7\text{ mm}$ respectively.

gap. On the other hand, Fig. 4.10b shows that magnetic flux density of 0.52 T is registered evenly across the effective air gap at $Y = 2\text{ mm}$ and $Y = 7\text{ mm}$. Thus, a DM configuration offers a constant distribution of magnetic flux density across the effective air gap regardless of the distance from the PM surface. These experimental results have shown that a DM configuration is able to maintain an evenly distributed magnetic field throughout a large effective air gap, which cannot be achieved using the conventional magnetic configuration.

4.5.4 Analytical Results V.S. Experimental Data

In Fig. 4.11, the magnetic flux density measured experimentally along the X-Y plane at $Z = 25\text{ mm}$ (refer to Fig. 4.7, at Z-axis) is plotted (Fig. 4.11a) against the results obtained from the proposed 2D mathematical model, i.e., Eqn. (4.89), (Fig. 4.11b) and the numerical analysis (Fig. 4.11c) respectively (see Appendix A.4 and A.5 for the parameters used). A comparison between Fig. 4.11a and 4.11b shows that the 2D mathematical model has made

an accurate prediction on the magnitude of the magnetic flux density and the magnetic field behavior between 5 mm to 27 mm of the 2D plane of the effective air gap. Magnetic flux leakage between 0 to 5 mm of the 2D plane of the effective air gap is also well-predicted by the 2D analytical model. Comparatively, Fig. 4.11c shows that the numerical analysis predicts a slightly lower magnetic flux density, i.e., ~ 0.46 T, between 10 mm to 27 mm of the 2D plane of the effective air gap. A larger area of magnetic flux leakage derived from the numerical field solution also suggests that the 2D analytical model offers a better prediction on the magnetic flux distribution for a DM configuration. This is because the numerical solution provided by ANSYS is formulated based on magnetic vector potential theorem. Consequently, this comparison shows that the magnetic scalar potential theorem is a better approach.

Figure 4.11: Magnetic flux density distribution in X-Y plane at $Z = 25$ mm: (a) experimental data and (b) results from the proposed mathematical model and (c) from the numerical analysis.

The difference in the magnetic flux density between the analytical and the experimental results is plotted in Fig. 4.12. It shows that the 2D mathematical model accurately predicts the magnitude of the magnetic flux density throughout the 2D plane of the effective air gap with a deviation of ± 0.02 T. Based on Fig. 4.12, a slight inaccuracy is found at the two corners of the effective air gap, i.e., from 0 to 1 mm. Such inaccuracies are accountable for as the actual PMs used in the prototypes have fillet edges (refer to Fig. 4.5) instead of sharp 90° edges, which are assumed during the formulation of the 2D mathematical model. As a result, more magnetic flux leakage is registered as projected at the two corner edges of the effective air gap in Fig. 4.11a.

Figure 4.12: Difference in magnetic flux density between the results obtained from the proposed mathematical model and the experiments (X-Y plane at $Z = 25$ mm).

The magnetic flux density measured experimentally along the Y-Z plane at $X = 27$ mm (refer to Fig. 4.7, at the X-axis) is plotted in Fig. 4.13a. For comparison, the same analytical result obtained from the 2D analytical model is plotted in Fig. 4.13b. It has shown that the 2D mathematical model is also accurate in predicting the magnitude of the magnetic flux density along the Y-Z plane. On the other hand, the result obtained from the conventional

surface charge model is plotted in Fig. 4.13c (refer to Appendix A.2). With a difference between 0.04 T to 0.23 T, it shows that the conventional model has underestimates the magnetic flux density within the effective air gap of a DM configuration. In addition, the conventional model is ineffective in predicting the flux distribution and leakages within air gap of a DM configuration. These differences are mainly due to surface charge model derived from Eqn. (A.10) only considered the surface that confines the magnetic field. As a result, the magnitude of the magnetic flux density within a volume is obtained from layer-by-layer representation of magnetic flux density. By comparing all the three plots, it is clear that the 2D mathematical model provides a more accurate prediction of the magnetic flux density and field behavior within the effective air gap of a DM configuration as compared to the conventional surface charge model.

Figure 4.13: Magnetic flux density distribution in Y-Z plane at $X = 27$ mm: (a) experimental data and (b) results from the proposed mathematical model and (c) from the surface charge model.

Figure 4.14 plots the difference in the magnetic flux density between the analytical and the experimental results. It shows that the 2D mathematical model can accurately predict the magnitude of the magnetic flux density throughout the 2D plane of the effective air gap with a deviation of ± 0.02 T. Similarly, slight inaccurate predictions occur at the two corners of the effective air gap region due to the manufacturing defects of the actual PMs used in the prototypes. Nevertheless, all these results have shown the effectiveness of the 2D mathematical model in predicting the magnetic field behavior within the effective air gap of a DM configuration in both X-Y and Y-Z planes.

Figure 4.14: Difference in magnetic flux density between the results obtained from the proposed mathematical model and the experiments (Y-Z plane at $X = 27$ mm).

From Section 4.3.2, the 3D effective air gap of a rectangular form has been simplified into a 2D Cartesian boundary-value problem by assuming that the rectangular PMs are uniformly magnetized and the magnetic field along the z-axis is similar to that along the x-axis to form a symmetrical magnetic flux distribution. This assumption has proven to be valid based on the experimental results plotted in Fig. 4.10b. Here, the magnetic flux density along the X-Z plane from $Z = 5$ mm to 25 mm (middle of the PM) has almost similar magnitude.

Likewise, the magnetic flux density along the X-Z plane from $X = 5$ mm to 27 mm (middle of the PM) also registered nearly similar magnitude. Figure 4.10b also shows that such evenly distributed magnetic flux density is consistent along the y-axis of the effective air gap. Here, the magnetic flux density from 0 to 5 mm along the x-axis and z-axis can be ignored since magnetic leakages are expected, and the moving air-core coils will avoid operating in these regions. Nevertheless, the magnitude and distribution of the magnetic flux density along the X-Y plane at other Z values (except from 0 to 5 mm) is almost identical to the case of $Z = 25$ mm as shown in Fig. 4.11. In addition, the magnitude and distribution of the magnetic flux density along the Y-Z plane at other X values (except from 0 to 5 mm) is also identical to the case of $X = 27$ mm as shown in Fig. 4.13. Thus, these investigations have shown that the proposed 2D mathematical model is sufficient for analyzing the magnetic field within a 3D effective air gap of a DM configuration where the moving air-core coil will be operating.

4.6 Summary

A DM configuration is presented in this thesis to enhance the performance of a Lorentz-force electromagnetic driving module. Experimental data obtained from the prototype has shown that a DM configuration can provides 40% increase in the magnetic flux density within the effective air gap. It also shows that a DM configuration offers an evenly distributed magnetic flux density through the entire air gap of 11mm. Consequently, this DM configuration becomes a promising solution for developing an enhanced Lorentz-force electromagnetic driving module that offers large continuous output force from small input current, good force-to-size ratio and low heat generation. In addition, a 2D mathematical model that predicts the magnetic field behavior within the effective air gap of a DM configuration is formulated. The magnetic field within the effective air gap is mathematically represented in the scalar potential form and solved as a boundary-value problem. A superposition of boundary conditions approach is proposed to solve this boundary-value problem and formulated a closed-form solution that describe the magnetic field within a region under the influence of two PM sources. A comparison between the analytical and experimental results have shown that this 2D mathematical model offers an accurate prediction of the

magnitude of the magnetic flux density across the 2D plane of the effective air gap with a deviation of ± 0.02 T. Hence, it is useful for rapid evaluation and parametric design of a DM configuration. Most importantly, it allows the derivation of an accurate current-force model of the proposed electromagnetic driving module, which is essential for high precision motion and direct-force control.

Chapter 5

Flexure-Based Electromagnetic Linear Actuator

“All matter originates and exists only by virtue of a force... We must assume behind this force the existence of a conscious and intelligent Mind. This Mind is the matrix of all matter.”

– Max Planck (1858 – 1947).

5.1 Background

Step-and-flash imprint lithography (SFIL) or UV nanoimprint lithography, is a type of nanoimprint lithography that has successfully demonstrated multi-layer-interconnections fabrications through a mechanical imprinting process [133]. Unlike a conventional hot-embossing lithography [134, 135], SFIL is a room-temperature and low pressure process that relies on chemical and mechanical steps to transfer high resolution patterns from the templates (or molds) onto the substrates [136]. The key differences between SFIL and hot-embossing lithography are resulted from a liquid etch barrier used by the SFIL that eliminates high temperatures and imprinting forces. This is crucial because high temperatures and imprinting forces are undesirable as they cause major technical issues in accurate overlaying of multiple layers. Consequently, SFIL becomes one of the highly potential processes that could replace the conventional optical lithography, which runs into technical limits when

producing transistors and resistors of sizes below 45 nm due to the wavelength limitation and diffraction effects of the photons.

A SFIL system [137] mainly comprises of a fine-orientation stage, a micro-resolution Z-axis actuator, an X-Y step-and-repeat positioning mechanism, a wafer orientation unit and other etching, and UV light exposure components (Fig. 5.1). The fine-orientation stage is driven by three high-resolution PZT actuators for active orientation about the x-axis, θ_x , the y-axis θ_y , and active translation in the z-axis. This stage is used to realize co-planer nano-alignment between the template and substrate. However, their limited displacement strokes require an additional Z-axis actuator to control the displacement of a few millimeters between the template and substrate. Few millimeters of displacements are essential so as to perform other chemical and UV exposure steps during the process, de-molding steps and overlaying of multiple layers. As the PZT actuators have a maximum stroke of a few hundreds micrometers, the Z-axis actuator must achieved micro-resolutions, which is realized by a stepper motor and a micro-resolution linear ball-screw.

Figure 5.1: (a) A fine-orientation stage and (b) a complete SFIL setup [137].

Unfortunately, the used of stepper motor and micro-resolution ball-screws require complex control systems, limit the overlay alignment resolution [133], and hinder the implementation of a direct-force control for precise imprinting. One major reason for these limitations is the lack of an appropriate actuator suitable for driving the SFIL system. The actuator for SFIL must have positioning accuracy in nanometers for performing ultra-high alignment and several millimeters of travel for layer-over-layer fabrications. It must also provide a continuous imprinting force of about 100 N that is required in the SFIL process and most

importantly allow direct-force control implementation. However, initial studies recognized that existing actuators have limitations in meeting these requirements. In particular:

1. Piezoelectric (PZT) actuators, which are commonly used for nano-scale positioning, have limited strokes of up to several hundreds microns [33, 138, 139].
2. PZT-driven actuators that use high-pitch screw actuating-shaft to achieve millimeters of displacement have poor repeatability due to backlash and Coulomb frictions. Others that use the magnetostrictive clamping method [38], the inchworm [40] or the impact driving technique [140] have low payload capacities less than 50N.
3. Conventional voice-coil linear actuators that offer frictionless actuation only deliver small current-force ratio at about 10 N/Amp [101].
4. Solenoid actuators offer a large but inconsistent output force throughout its allowable traveling range [79].

5.2 Design and Analyses

The drawbacks of existing nano-positioning actuators motivate the development of a Flexure-based Electromagnetic Linear Actuator (FELA). This novel nano-positioning actuator is designed to achieve a positioning accuracy of ± 20 nm, a minimum displacement stroke of ± 2 mm, a current-force ratio of about 50 N/Amp, an actuating speed of greater than 100 mm/s and high stiffness at all non-actuating axes. A FELA mainly comprises of a Lorentz-force electromagnetic driving module and flexure-supporting bearings as shown in Fig. 5.2. The Lorentz-force actuation is realized by an Electromagnetic Driving Module (EDM) with a fixed PM stator and a moving air-core coil configuration (Fig. 5.2a). Most importantly, this stator employs the newly proposed DM configuration, which can produce a uniform magnetic flux distribution within a large air effective gap where the moving air-core coil is operating. This DM configuration allows a large output force to be obtained from small amount of input current. In addition, the moving air-core coil enables a linear current-force relationship, which is a merit for effective direct-force control implementation.

Figure 5.2: (a) An EDM with symmetrical DM configuration with (b) the moving air-coil supported by flexure bearings to form a FBM.

The holder, which the conducting coil is wound, is supported by the beam-based flexure joints that forms a Flexure Bearing Mechanism (FBM) as illustrated in Fig. 5.2b. Such a FBM provide a frictionless support, which retains the infinite positioning resolution of the FELA driven by the electromagnetic driving scheme. The proposed FBM must ensure high stiffness in all non-actuating directions and low stiffness in the actuating direction within its allowable travel range. Subsequently, the proposed EDM is embedded inside the proposed FBM to form a complete FELA assembly.

5.2.1 Electromagnetic Driving Module

To drive this proposed FELA, a Lorentz-force actuation is employed for its linearity between input current and output force. Its competency of achieving infinite positioning resolution and constant output force throughout the entire range of travel also makes it a promising approach. From the predetermined capabilities of a FELA, it must have the ability to generate a large continuous output force of at least 50 N with an input current of 1 Amp. Considering that a certain amount of force is needed to drive the moving air-coil due to the stiffness of the FBM, the proposed EDM is required to produce a current-force sensitivity of more than 50 N/Amp. In Chapter 4, Eqn. (4.8) suggested that the peak force of a Lorentz-force actuation is governed by the amount of input current, the total length of coil and the magnitude of magnetic flux density within the effective air gap of a DM configuration. Considering that the magnitude of the magnetic flux density plays an important factor,

which should be as close to 1, a design optimization is done through the parametric studies of DM configuration using the derived Eqn. (4.89). As a result, a 54 mm $\times$ 50 mm $\times$ 7.5 mm NdFeB PM (Type N45M) with a remanence magnetic flux density of 1.33 T and a maximum operating temperature of 120°C is employed. This PM is used to construct a symmetrical DM configuration stator with each effective air gap being optimized at 11 mm. Although the DM configuration produces a magnetic flux density of 0.52 T, previous attempts during the optimization stage in obtaining higher magnetic flux density only resulted in PMs that are too large to be accepted due to constraints such as the compactness and weight of the proposed FELA etc. Hence, 0.52 T is considered as the optimized magnetic flux density.

Figure 5.3: A coil holder within the effective air gap of a DM configuration in X-Y plane.

In this section, the other parameters to be considered are the total length of coil within the effective air gap of a DM configuration when the input current is fixed. Based on the dimensions of the selected PMs, each effective air gap is dimensioned as 54 mm $\times$ 50 mm $\times$ 11 mm. As results obtained from Eqn. (4.89) shows slight magnetic leakages that stretch for 3 mm at both ends of the effective air gap, the coil holder is dimensioned as 44 mm $\times$ 50 mm $\times$ 9 mm (Fig. 5.3). This design consideration is based on an assumption that the moving air-core coil can only operated within the region where no magnetic leakage is observed so as to ensure a constant force output and high positioning accuracy is achievable within the targeted traveling range of ± 2 mm. The total coil length within the effective air gap is mainly determined by the outer diameter of the conducting wire with the assumption that the coil winding is perfect. Based on Fig. 5.3, the number of layers of wire within the coil holder can be expressed as

$$N_{layer} = \frac{9}{OD} \quad (5.1)$$

where OD represents the outer diameter of conducting wire. And the number of turns of wires per layer can be expressed as

$$N_{turns} = \frac{44}{OD} \quad (5.2)$$

Consequently, the total length of coil within the effective air is given as

$$Coil_{eff} = L_{eff} N_{layer} N_{turns} \quad (5.3)$$

where L_{eff} represents the effective coil length for each turn, which is fixed at 100×10^{-3} m. As continuous output force of a Lorentz-force actuation is about 70 % of the peak force, substituting Eqn. (5.3) into Eqn. (4.8) yields

$$F_{continuous} = 0.7iBCoil_{eff} \quad (5.4)$$

where B is the magnetic flux density, i is the input current and $F_{continuous}$ will be the actual operational force generate by the proposed EDM.

Here, $F_{continuous}$ must be greater than 50 N to meet the proposed requirements. Unfortunately, the diameter of conducting wire not only determine the total length coil within the effective air gap, it also associated with the heat generation of the proposed EDM. With a FELA being proposed as a nano-positioning actuator with large continuous output force, heat generation must be minimized. Consequently, other than meeting the force requirement, power generation must be considered in the optimization of the conducting coil.

The total coil length to be wounded onto the coil holder is determined by

$$Coil_{total} = N_{turns}[(158 \times 10^{-3})(N_{layer}) + 2(OD)(N_{layer}^2 + N_{layer})] \quad (5.5)$$

where 158×10^{-3} m is the initial frame circumference of the coil holder as shown in Fig. 5.4.

Figure 5.4: Frame of the coil holder slotted within the center of the stator in Y-Z plane.

With the coil length derived from Eqn. (5.5), the total internal coil resistance is given as

$$R_{coil} = \frac{\chi \text{Coil}_{total}}{\pi(ID/2)^2} \quad (5.6)$$

where χ represents the resistivity of the conducting material, which is $1.7 \times 10^{-8} \Omega\text{m}$ for copper, and ID represents the actual conducting wire diameter (excluding the insulator), which is $\sim 0.8 \mu\text{m}$ smaller than the OD .

With the maximum voltage supply being fixed 48 VDC, the maximum current that can be supplied to the proposed EDM is expressed as

$$i_{max} = \frac{48}{R_{coil}} \quad (5.7)$$

Hence, the maximum continuous force generated, F_{max} , can be derived from Eqn. (5.4).

Consequently, the current required to generate the desired force, $F_{desired}$, is given as

$$i_{required} = i_{max} \frac{F_{desired}}{F_{max}} \quad (5.8)$$

Thus, the power generated is expressed as

$$W = i_{required}^2 R_{coil} \quad (5.9)$$

In addition, the total internal coil resistance can also affect the open-loop positioning resolution of the EDM. Based on a open-loop voltage per command count, ν , the force generated from 1 Amp, F_{1amp} , and the stiffness of the FBM, K_{FBM} , the open-loop positioning resolution per count, ε , is expressed as

$$\varepsilon = \frac{F_{1amp}(\nu/R_{coil})}{K_{FBM}} \quad (5.10)$$

Based on American Wire Gauge (AWG) standards, six classifications of copper conducting wire are used in the optimization of coiling in the proposed EDM. In this optimization phase, the primary consideration is the force generation, followed by the power generation and the open-loop positioning resolution per count. In addition, the stiffness of the FBM is approximated as 1000 N/m while the open-loop voltage per command count obtained from the controller is 0.46×10^{-3} V/count. For each wire classification, results obtained from the above equations are listed in Table 5.1.

Table 5.1: Selection of conducting wire based on coiling optimization.

Type AWG	Diameter (m)	R_{coil} (Ω)	ε (m/count)	Power generated at 60 N (watt)	Current to achieve 60 N (Amp)
27	0.36×10^{-3}	58.9494	543.6×10^{-9}	43.3098	0.8571
26	0.40×10^{-3}	41.9029	668.3×10^{-9}	40.3021	0.9807
25	0.45×10^{-3}	26.7073	846.5×10^{-9}	39.4224	1.2149
24	0.50×10^{-3}	18.7107	1.041×10^{-6}	37.1867	1.4098
23	0.57×10^{-3}	11.2576	1.355×10^{-6}	36.5037	1.8007
22	0.64×10^{-3}	7.45690	1.704×10^{-6}	34.8901	2.1618
21	0.72×10^{-3}	4.47624	2.167×10^{-6}	35.9392	2.8335
20	0.81×10^{-3}	2.87064	2.671×10^{-6}	35.1361	3.4985
19	0.91×10^{-3}	1.84036	3.376×10^{-6}	34.4889	4.3290
18	1.02×10^{-3}	1.18370	4.172×10^{-6}	33.9709	5.3571

Results have shown that the selected these wires, i.e., 27AWG, 26AWG and 25AWG, allow the proposed EDM to produce a continuous output force of 60 N from an input current of about 1 Amp. All three wires allow the smallest allowable positioning resolution of the proposed EDM to be in the region of several hundreds of nanometers. Although the coil resistance varies largely between all three types of wires, the power generated for achieving 60 N is close comparatively. Consequently, 26AWG, which falls in between these three types of wires, is chosen as the conducting wire for the proposed EDM.

5.2.2 Flexure Bearing Mechanism

In FELA, the beam-based flexure joints are employed to support the moving air-core coil via the coil-holder of the EDM. This complete mechanism, termed FBM, is used to retain the capabilities of an electromagnetic driving scheme, e.g., frictionless and large motions etc. Most importantly, it provides necessary stiffness to eliminate the stationary positioning noise exhibited by this driving scheme. To facilitate a FELA in achieving its predetermined capabilities, the proposed FBM must meet three main requirements. First, low stiffness in driving direction to achieve ± 2 mm of displacement from small amount of force. Second, the parasitic errors at the non-actuating directions must be minimal, e.g., less than $0.1 \mu\text{m}$. Third, the stiffness at non-actuating directions must be a few hundreds times more than the actuating direction. Consequently, two promising designs have been proposed in the development stage of a FBM as shown in Fig. 5.5.

Figure 5.5: (a) FBM-1: A symmetrical double-compound linear spring design with its schematic diagram and (b) FBM-2: a bi-stable design and its schematic diagram.

In FBM-1, a double-compound linear spring with beam-based flexure joints are used to support the moving coil-holder as shown in Fig. 5.5a. A symmetrical design is adopted to minimize the parasitic errors at non-actuating directions. Such a configuration allows a large displacement using relatively small driving force. In FBM-2, a pair of beam-based flexure joints is used to support the moving coil-holder to form a linear spring mechanism

(5.5b). Similarly, a symmetrical design is adopted to minimize the parasitic errors at the non-actuating directions. This leads to a bi-stable design that offers small driving force within its permissible travel range.

5.2.2.1 Mobility Analyses

In the mobility analysis, each compliant joint within the proposed FBM designs can be treated as a kinematics joint with the equivalent DOF. Hence, this approximation approach allows the mobility analyses of the proposed FBM designs to be conducted using Gruebler's equation, which is expressed as

$$DOF = 3(n - 1) - 2J_1 - J_2 \quad (5.11)$$

where n is the number of links, while J_1 and J_2 are the number of lower, i.e., 1-DOF, and higher joints, i.e., 2-DOF, respectively. Table 5.2 listed the results obtained from the mobility analysis. Based on the results, FBM-1 design provides a 1-DOF analytically while the mobility analysis for FBM-2 shows that this design is an over-constraint. However, FBM-2 is formed by a symmetrical linear spring design that offers a single degree of motion based on Eqn. (5.11) when $n = 4$, $J_1 = 4$, and $J_2 = 0$. Hence, the symmetrical design of FBM-2 can definitely provide a single but small motion.

Table 5.2: Results obtained from mobility analyses on the proposed FBM designs.

Design	n	J_1	J_2	DOF
FBM-1 (symmetrical double compound linear spring)	12	16	0	1
FBM-2 (bi-stable)	6	8	0	-1
FBM-2 (in single linear spring form)	4	4	0	1

5.2.2.2 Finite Element Stiffness Analyses

Using ANSYS 8.1, all proposed FBM designs underwent four stages of FE analyses:

1. To analyze the driving force require to achieve 2 mm in the actuating direction, i.e., in the x-axis.
2. To analyze the parasitic errors in other two non-actuating directions during the actuation, i.e., in the y- and z-axes.

- 3. To analyze the stiffness in non-actuating direction in the y-axis when an external force, which is similar to the driving force in the x-axis, is applied to the y-axis of the actuating body at actuated (+2 mm), neutral and retracted (-2 mm) position respectively.
- 4. To analyze the stiffness in non-actuating direction in the z-axis when an external force, which is similar to the driving force in the x-axis, is applied to the z-axis of the actuating body at actuated (+2 mm), neutral and retracted (-2 mm) position respectively.

In the analyses, the element type is chosen as SOLID 45 and the material selected for all proposed FBM designs is stainless steel. This material is assumed to be linearly elastic property with Young’s modulus = 190 GPa, and Poisson’s ratio = 0.33. Tables 5.3 and 5.4 summarize the results obtained from these FE analyses.

Table 5.3: FE results on the proposed FBM designs for analyses 1 and 2.

Proposed design	Analysis 1	Analysis 2	
	Force to achieve 2 mm (N)	Parasitic error in y-axis (m)	Parasitic error in z-axis (m)
FBM-1	10	0.01×10^{-6}	0.25×10^{-9}
FBM-2	11.5	0	2.2×10^{-9}

Table 5.4: FE results on the proposed FBM designs for analyses 3 and 4.

Proposed design	Analysis 3		Analysis 4	
	x:y axis stiffness ratio (Extend/retract)	x:y axis stiffness ratio (Neutral)	x:z axis stiffness ratio (Extend/retract)	x:z axis stiffness ratio (Neutral)
FBM-1	$1:2.5 \times 10^3$	1:429	1:542	1:499
FBM-2	1:INF	$1:1.7 \times 10^3$	$1:3.1 \times 10^3$	$1:3 \times 10^3$

Based on these finite element results, FBM-2 requires slightly more driving force to achieve the desired displacement as compared to FBM-1. However, FBM-2 offers the much higher stiffness at all non-actuating directions as compared to FBM-1. This is a very much-desired characteristic for an actuator. Hence, the proposed FBM will be developed based on the design of FBM-2.

Figure 5.6: (a) A bi-stable FBM is represented as (b) a symmetrical four-bar linkage. (c) A schematic diagram of a four-bar linkage.

5.2.2.3 Analytical Stiffness Analysis

As mentioned earlier, an FBM, which adopts the bi-stable mechanism design, can be described as a pair of symmetrical linear spring mechanisms with beam-based flexure joints (Fig. 5.6). Each of these linear spring configurations can be represented by a planar four-bar mechanism. Here, the desired displacement, Δ , will be used to determine the deflection angle through Eqn. (3.33), whereby $L = r_2/2 = r_4/2$, with r_1, r_2, r_3 and r_4 represent the length of each rigid link. The obtained deflection angle will be substituted into Eqn. (3.39) to determine the force, F_{in} , required to achieve the desired displacement. Subsequently, the stiffness of a beam-based flexure joint coupled with half of a rigid-link is expressed as

$$K = \frac{F_{in}}{\Delta} \quad (5.12)$$

A single linear spring (or a four-bar linkage) has two parallel limbs, which comprised of two flexure joints connected by a rigid-link. The stiffness of each parallel limb is expressed as

$$K_{limb} = \frac{K}{2} \quad (5.13)$$

As the entire FBM has a total of 8 limbs, the stiffness of this bi-stable FBM is given as

$$K_{FBM} = 8K_{limb} \quad (5.14)$$

Thus, the stiffness of FBM in relation to the desired displacement and force required is

$$F_{in} = K_{FBM}\Delta \quad (5.15)$$

5.3 Prototype

A FELA prototype of dimensions 100 mm × 70 mm × 70 mm (length × width × height) is developed as shown in Fig. 5.7. For the EDM, the AWG26 wire that was selected during the coil optimization phase is used to form the moving air-core coil. This moving air-core coil and a fixed iron stator, which adopts the DM configurations, will form the entire EDM. Here, the FBM consists of four symmetrical linear springs. which are formed by 80 μ m thick stainless steel shims. The middle section of each shim is clamped with a pair of clamping blocks with a length of 21 mm. This leaves 3 mm of the unclamped shim at both ends to form the beam-based flexure joints. Subsequently, the entire FBM encloses the EDM and its external frame (Fig. 5.6a) is connected with the fixed stator of the embedded EDM to form the main frame of FELA. The main frame also serves as a fixed rigid-body that supports the translational motion of the air-core coil via the beam-based flexure joints.

Figure 5.7: A FELA prototype.

5.3.1 Current-Force Relationship

Force measurement was performed on the EDM prototype using a DC linear amplifier (PMDI, model: BTA-28V-6A) and a six-axes force/torque (F/T) sensor (ATI, model: Mini-40, max: 240 N, resolution: 0.01 N) as shown in Fig. 5.8. The experimental results plotted in Fig. 5.9 reflect a linear relationship between the input current and the output force. The EDM prototype generates an output force 6.1 N with an input current of 0.1 Amp. Based on such a linear relationship, the EDM prototype is capable of generating an output force of greater than 100 N at 2 Amp. On the other hand, the analytical results obtained from Eqn. (5.4), which based on the selected PMs and conducting wire, are plotted against the experimental results in Fig. 5.9. It shows that the analytical model predicts a current-force ratio of 6.118 N per 0.1 Amp to be generated from the EDM. At 0.5 Amp, the analytical model predicts 30.59 N while the F/T sensor on the EDM prototype registered 31 N. With slight derivations between these analytical results and experimental data, it shows that the proposed current-force model gives a good prediction on the current-force relationship and will be useful for rapid parametric analysis, and design of the proposed EDM.

Figure 5.8: The experiment setup for the current-force evaluation on the EDM.

Figure 5.9: Predicted current-force relationship from the proposed analytical model plot against the actual current-force experimental data.

5.3.2 Force-Displacement Relationship

The FBM was used to validate the accuracy of the established force-displacement analytical model. The experiment setup as shown in Fig. 5.10 consists of a THRUST DC linear amplifier (model: TA-115, max: 48VDC, 8Amp), an ATI F/T sensor (model: Mini-36, max: 36N, resolution: 0.01N), a MicroE-Systems optical linear encoder (model: M3000Si, resolution: 20 nm/count) and an external VC actuator that is attached to the FBM.

Figure 5.10: Experimental setup for investigating the stiffness of the bi-stable FBM.

To measure the stiffness of the FBM experimentally, a set of open-loop command signals was input into the VC actuator to drive the FBM. For each set of command signal, the

position encoder registered the displacements of the FBM while the F/T sensor measured the driving force require to achieve the displacements. Consequently, the force-displacement relationship of the FBM is obtained experimentally through this setup. On the other hand, the stiffness of the FBM is also obtained analytical through Eqn. (5.15) and is plotted against the experimental results as shown in Fig. 5.11.

Figure 5.11: A comparison between the predicted stiffness of FBM from proposed analytical model and actual stiffness obtained from experiments.

The analytical results indicate a displacement stiffness of 1.96 N/mm for the FBM while a displacement stiffness of 1.9 N/mm was obtained from the actual FBM experimentally. In addition, the analytical results are consistent with the experimental results through the entire displacement range. From this investigation, the established analytical model has shown to be suitable for predicting the force-displacement relationship of such a beam-based flexure module. Furthermore, it is interesting to note that the analytical results are more accurate as compared to the numerical analyses conducted through the finite element simulations. In Section 5.2.2, the numerical simulator calculated a stiffness of 5.75 N/mm for the FBM. This numerical result is almost three times the amount of the actual stiffness. Such an error may due to incorrect element-type or size of elements selected. (For issues regarding in the accuracy of the FEM simulations, further analyses have been conducted and discussed in Appendix C in details).

5.4 Position Control Design and Implementation

A position control scheme is implemented on the FELA to validate its performances in terms of positioning accuracy, displacement range and actuating speed. This closed-loop control is mainly govern by a Proportional, Integral and Differential (PID) controller, and a position feedback as shown in Fig. 5.12a.

Figure 5.12: (a) Block diagram of the proposed positioning control system with FELA represented as (b) a mass-spring-damper system.

5.4.1 System Dynamics Modeling

A FELA is treated as a linear mass-spring-damper system as illustrated in Figure 5.12b. In this system, the mass, m , represents the weight of moving air-core coil and the stiffness of the FBM is represented by the spring stiffness, k . Unlike conventional flexure mechanisms, the damping of this system is caused by a force generated through the induced eddy current from the relative motion of the moving core and the PMs. Hence, the transfer function of the FELA, $G_F(s)$, is

$$G_F(s) = \frac{X_a(s)}{F_c(s)} = \frac{1}{ms^2 + bs + k} \quad (5.16)$$

where $F_c(s)$ is the command force and $X_a(s)$ is the actual position. On the other hand, the transfer function of a conventional PID controller, $G_c(s)$, is

$$G_c(s) = \frac{F_c(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (5.17)$$

where $E(s)$ is the error between the desired and actual values, K_p is the proportional gain, T_i is the integral time and T_d is the derivative time. Consequently, the transfer function of the entire closed-loop control system is

$$\frac{X_a(s)}{X_d(s)} = \frac{K_c K_p (T_d s^2 + s + T_i^{-1})}{m s^3 + (b + K_c K_p T_d) s^2 + (k + K_c K_p) s + K_c K_p T_i^{-1}} \quad (5.18)$$

where $X_d(s)$ is the desired position and K_c is the compensation gain, which accounts for the quantization, and amplifier gain of the open-loop system.

5.4.2 Controller Design

To determine the control parameters, Eqn. (5.18) is compared against the transfer function of a standard third-order system so as to formulate the relevant equations that represent K_p , T_i and T_d . In this work, the transfer function of a standard third-order system is given as

$$G(s) = \frac{\omega_n^2}{(s + \gamma)(s^2 + 2\zeta\omega_n s + \omega_n^2)} \quad (5.19)$$

where ω_n represents the undamped natural frequency of the system and ζ represents the damping ratio of the system. The transfer function of a standard third-order system is derived by adding a additional real pole, γ , to the transfer function of a standard second-order system. The value of additional pole must be at least 5 times the value of undamped natural frequency to ensure that it has minimum effect on the system characteristic. Hence, a comparison between the coefficients of equations Eqn. (5.18) and (5.19) yields

$$K_p = \frac{m(\omega_n^2 + 2\zeta\omega_n\gamma) - k}{K_c} \quad (5.20)$$

$$T_i = \frac{K_c K_p}{m\gamma\omega_n^2} \quad (5.21)$$

$$T_d = \frac{m(\gamma + 2\zeta\omega_n) - b}{K_c K_p} \quad (5.22)$$

Here, the rise time, T_r , and damping ratio are used to determine the system performance. Using these two parameters, the undamped natural frequency is obtained using

$$\omega_n = \frac{\tan^{-1}[\zeta^{-1}(\sqrt{1 - \zeta^2})]}{T_r \sqrt{1 - \zeta^2}} \quad (5.23)$$

Predictions of the PID control parameters through Eqn. (5.20) to (5.22) require the values of mass, spring and damper. MATLAB *System Identification Toolbox* is used to identify these values based on the output step response of the FELA. Initially, a series of step input was first given to the FELA to obtain the output step response of the open-loop system. These step inputs are command forces, which include $1.98623 \times 10^{-7}\text{N}$, $1.98623 \times 10^{-6}\text{N}$ and $9.93116 \times 10^{-6}\text{N}$. Subsequently, the output response obtained from each step input is recorded at a sampling time of 1 msec. Both input and output data are fed into the MATLAB toolbox to estimate a transfer function of the open-loop system. As a result, the estimated transfer function, $G_{est}(s)$, is given as

$$G_{est}(s) = \frac{279.9}{(1 \times 10^{-6})s^2 + (2.254 \times 10^{-3})s + 1} \quad (5.24)$$

where the values of 279.9, 1×10^{-6} , 2.254×10^{-3} and 1 represent the ratio between the compensation gain, mass, damping and spring stiffness respectively.

Figure 5.13: Hardware used to realize the position control loop.

5.4.3 Results

For position control, a rise time of 0.4 msec and damping ratio of 0.6 is used to determine the value of the natural frequency, which in turn used for estimating the PID control parameters in Eqn. (5.20) to (5.23). As a result, $K_p = 0.2064$, $T_i = 0.47491$ and $T_d = 0.27193$ are obtained. At this stage, it is noticeable that K_p of 0.2064 is lower than the K_p values used on other conventional electromechanical modules. This is because the moving air-core coil has low moment of initial, which requires low K_p values to avoid large overshoot in the transient response or uncontrollable oscillations. The derived PID control parameter values were subsequently input to the PID controller, which is written in the FPGA environment via a National Instrument (NI) FPGA controller card (model: PCI-7833R, max: 3 M Gates, processing speed: 25 ns/command). This FPGA controller allow the PID servo-loop to run at 10 kHz, while the trajectory generator runs on NI LabVIEW environment with a control frequency of 1 kHz. Other hardware include an industrial PC with P4 processor, a TRUST 48 VDC DC linear amplifier and a MicroE-Systems optical linear encoder with a resolution of 5 nm/count as shown in Fig. 5.13.

A RENISHAW laser interferometer with a resolution of 10 nm/count (model: RLE10) is employed to verify the finest and largest achievable steps, accuracy, and repeatability of the FELA (Fig. 5.14). With the PID servo-control, a positioning accuracy of ± 10 nm is obtained at the end-effector of the FELA (Fig. 5.15). A 20-nm repetitive step is performed

by the FELA and plotted in Fig. 5.16. It shows a positioning accuracy of ± 10 nm at every step and validates that the smallest achievable step from FELA to be 20 nm. The maximum stroke of FELA is performed and plotted in Fig. 5.17. It shows that the FELA achieves a ± 2 mm step and settles quickly after 0.7 sec. With a constant error of 50 nm, the positioning accuracy at 2 mm is ± 10 nm peak-to-peak (Fig. 5.18). To verify the repeatability of FELA, five repetitive runs to each targeted position, i.e. 5 μ m and 2 mm, are conducted. For the targeted position at 5 μ m, all five runs performed by FELA fall within $\pm 1.5 \sigma$ (Fig. 5.19). With a constant error of 24 nm, it shows a positioning repeatability of ± 10 nm. As for the targeted position at 2 mm, all five runs performed by FELA fall within $\pm 1.5 \sigma$ and indicate a positioning repeatability of ± 20 nm (Fig. 5.20) with a constant error of 72 nm. Each constant error is the deviation between the encoder reading and the actual end-effector position obtained from the laser interferometer. Nevertheless, these errors can be used to form an error map to compensate for such deviations during actual operations. Most importantly, the positioning accuracy of FELA is very much limited by the encoder resolution of 5 nm/count. Based on Fig. 5.15, an accuracy of ± 2 count suggests that an encoder with higher resolution will further improve the positioning accuracy of FELA.

Figure 5.14: Laser interferometer setup on FELA.

Figure 5.15: Position accuracy of the end-effector at neutral position.

Figure 5.16: Repetitive 20 nm steps generated from FELA with positioning control.

Figure 5.17: FELA performs a ± 2 mm of large displacement stroke with high accuracy demonstrated at both ends of the stroke.

Figure 5.18: Position accuracy of end-effector at 2 mm.

Figure 5.19: Position repeatability at targeted position of 5 μm .

Figure 5.20: Position repeatability at targeted position of 2 mm.

5.5 Direct-Force Control Design and Implementation

One of the most important features of a FELA is its ability to achieve a direct-force control for imprinting tasks. Due to the nature of the direct non-commutation Lorentz force driving scheme, the relationship between input current and output force is linear and proportional as shown in Fig. 5.9. Consequently, this characteristic allows a direct control on the output force of the actuator. Using a PID control scheme, the ATI F/T sensor becomes the main feedback source for the PID controller when the FELA operates under the direct-force control mode (Fig. 5.21a). In this mode, the analog signal from the F/T sensor is directly feed into the FPGA controller to achieve a signal-processing frequency of 100kHz.

Figure 5.21: (a) Block diagram of a proposed direct-force control system. (b) Schematic diagram of a mass-spring-damper system with an additional pair of spring-damper representing the force sensor.

5.5.1 System Dynamics Modeling

In a direct force control mode, the FELA and the F/T sensor become an integrated system as shown in Fig. 5.21b. Here, the F/T sensor is treated as a linear spring-damper where the spring stiffness, k_s , represents the stiffness of the strain gauge within the F/T sensor and the damper, b_s , represents the friction between the F/T sensor, and the workpiece. Hence, the transfer function of the F/T sensor is

$$X_a = \frac{F_a(s)}{k_s} \quad (5.25)$$

where $F_a(s)$ is the actual force detected from the end-point of the F/T sensor. Consequently, the transfer function the integrated system is

$$\frac{F_a(s)}{F_c(s)} = \frac{k_s}{ms^2 + (b + b_s)s + k + k_s} \quad (5.26)$$

In the case of direct force control, the transfer function of the entire closed-loop control system becomes

$$\frac{F_a(s)}{F_d(s)} = \frac{K_c K_p k_s (T_d s^2 + s + T_i^{-1})}{ms^3 + (b_t + K_c K_p T_d)s^2 + (k_t + K_c K_p)s + K_c K_p T_i^{-1}} \quad (5.27)$$

where $F_d(s)$ is the desired force, $b_t = b + b_s$ and $k_t = k + k_s$.

5.5.2 Controller Design

A comparison between the coefficients of Eqn. (5.19) and (5.27) yields

$$K_p = \frac{m(\omega_n^2 + 2\zeta\omega_n\gamma) - k_t}{K_c} \quad (5.28)$$

$$T_i = \frac{K_c K_p}{m\gamma\omega_n^2} \quad (5.29)$$

$$T_d = \frac{m(\gamma + 2\zeta\omega_n) - b_t}{K_c K_p} \quad (5.30)$$

A set of command forces, which includes 9.93116×10^{-6} , 1.98623×10^{-5} and 1.39036×10^{-4} , is given to the integrated system and a set of output forces is recorded. These sets of data are fed into the MATLAB *System Identification Toolbox* and the estimated transfer function is

$$G_{est}(s) = \frac{4.145 \times 10^5}{(2.9591 \times 10^{-5})s^2 + (89.426 \times 10^{-3})s + 1} \quad (5.31)$$

where the values of 4.145×10^5 , 2.9591×10^{-5} , 89.426×10^{-3} and 1 represent the ratio between compensation gain, m , b_s and k_s respectively.

5.5.3 Results

For direct-force control, the settling time of the transient response is more crucial than the rise time because the time to reach steady-state is more important in a nanoimprint process. Hence, a settling time of 0.5 msec and damping ratio of 0.6 is used to estimate the PID control parameters using Eqn. (5.28) to (5.30). As a result, $K_p = 0.2686$, $T_i = 0.06039$ and $T_d = 0.03740$ are obtained from the analyses. These parameters are used by the PID controller when performing a direct-force control. A 10-N force profile generated by the FELA is plotted in Fig. 5.22. It shows that FELA holds at 10 N for about 40 sec with an accuracy of ± 0.15 N. A 60-N force profile generated by the FELA is plotted in Fig. 5.23 with an accuracy of ± 0.15 N. In both cases, a simple PID scheme can be employed due to the linearity between the input current and the output force. With a DM configuration, a current of less than 1 Amp of current is used to generate the 60-N profile.

A thermal sensor from PICO Technology is used to record the heat generated from the air-core coil for every second during the force control operation. Fig. 5.24 plots the measured temperature when FELA is generating a continuous thrust force of 60 N for 15 mins. It shows that the temperature rises from 22 °C to 32 °C at a rate of 0.0173 °C/sec and saturated at 32°C after 11 mins. Hence, a 60N-profile force control has a maximum temperature increase of 10 °C. With the air-coil holder length of 44 mm and the coefficient of thermal expansion of $24 \times 10^{-6}/^\circ\text{C}$, any 60-N profiling operation that is below 11min requires compensation of 18.2688 nm/sec in the actuating direction (in the x-axis). For 60-N profiling operations that require more than 11 mins, a maximum length extension of 10.56 μm in the actuating direction (in the x-axis) must be compensated. Such temperature

compensations are additional means for enhancing the accuracy of the imprinting process during a direct-force control operation.

Figure 5.22: A 10-N force profile generated by FELA with ± 0.15 N of accuracy.

Figure 5.23: A 60-N force profile generated by FELA with ± 0.15 N of accuracy.

5.6.1.3 Results

In the mode-switching control, the direct-force control loop took over the position control loop immediately when a contact force is detected as shown in Fig. 5.27. However, the sudden surge in the command signal causes a sudden pressing effect that is reflected in the force and position profiles at about 140 msec. In the impedance control, Fig. 5.28 suggests that the output force slowly approaches the desired force without any sudden surge. However, the initial stage of inheriting the direct-force control command signal causes a slight dip, which is reflected in the force and the position plots. Mode-switching control offers a fast response but sudden surge in the pressing force. On the other hand, an impedance control offers a smooth approach to desired force, which is essential in the imprinting processes to prevent any sudden surge that may damage the templates or substrates. Consequently, the impedance method is used to realize the hybrid position/force control of a FELA for a smooth transition between a nano-positioning control and a direct-force control.

Figure 5.27: Force, position and command signal obtained from a mode-switching control.

Figure 5.28: Force, position and command signal obtained from an impedance control.

5.6.2 Hot-Embossing Process

A hot-embossing process requires heating of the silicon template and the Poly-Methyl-Methacrylate-Acrylic (PMMA) substrate to about 160°C before imprinting the template on the substrate. This imprinting force is kept constant for about five minutes before reducing to zero. Subsequently, the template will be de-molded from the substrate leaving the imprinted micro-sized features on the substrate (Fig. 5.29).

Figure 5.29: Hot embossing process graph.

Figure 5.30a shows the entire setup of a FELA in realizing this hot-embossing process. The FELA is fixed vertically downwards with an embossing head mounted on the actuator end-effector. This embossing head, which carries the template (Fig. 5.30b), has two cartridge heaters for heating up the template. Through the embossing head, the FELA presses the template onto the substrate (Fig. 5.30c) that is placed on an embossing base. This embossing base has three cartridge heaters for heating up the substrate. All cartridge heaters are calculated and sized to heat up the template, and the substrate to 200°C within a minute. Temperatures of these cartridge heaters are controlled by the temperature controllers with the thermocouple being the feedback to the control loop.

Figure 5.30: (a) FELA setup for a hot-embossing process to imprint micro-sized features in (b) the template onto (c) the PMMA substrate.

Figure 5.31: (a) The imprinted micro-sized features on the substrate from (b) the micro-sized features on the template.

5.6.3 Imprinted Micro-Sized Features

The imprinted features on the substrate and the micro-features from the template are taken from a high-resolution optical microscope, and are shown in Fig. 5.31. The actual features

from the template (Fig. 5.31b) has a width of $33.335\ \mu\text{m}$. The imprinted features on the substrate has a measured width of $33.680\ \mu\text{m}$. In addition, the width of a pair of micro-channels from the template is $82.655\ \mu\text{m}$, while the width of a pair of micro-channels on the substrate is $82.319\ \mu\text{m}$. Both measurements have shown that the imprinted micro-sized features on the substrate are closed to the actual feature from the template with an accuracy of $\pm 0.35\ \mu\text{m}$.

After the initial attempt, another template with different feature patterns is attached to the embossing head for more in-depth evaluations. The main differences between this template and the previous template is that the current template has five micro-sized channels that varies from $502\ \mu\text{m}$ to $28\ \mu\text{m}$ in their respective widths. Using a high-resolution optical microscope, the template features and the imprinted features are measured and listed in Table 5.5. According to the measured data, the width of the micro-sized features on the substrates and templates for all five samples are very close. The imprinted feature heights for sample 1 and 2, which has a width of $500\ \mu\text{m}$ and $257\ \mu\text{m}$ respectively, are very close to the actual sample heights. However, it is interested to note that the imprinted feature heights for sample 3, 4 and 5 are much shallower than the template feature heights. Figure 5.32 plots the actual height measurements of another set of imprinted features. Similarly, it shows that the imprinted features height are much lower as compared to the desired height (or the template feature height) when the width of the imprinted features is $75.5\ \mu\text{m}$. All these results have shown a trend that when the width of the template feature reduces, the transferring of the template feature height onto the substrate becomes more difficult. This can be due to the air trapped between the template and the substrate, which hinders the transferring of these features. Nevertheless, this effort has helps to identify some issues of the hot-embossing process when imprinting features sizes of less than $100\ \mu\text{m}$. Most importantly, both attempts have shown that the FELA has successfully realized the imprinting of micro-sized features using a hot-embossing process.

Table 5.5: Measurements obtained from the imprinted features on the PMMA substrates and the features from the silicon templates.

Feature		Silicon template	PMMA
1	Width (μm)	502.0	501.0
	Height (μm)	79.0	78.5
2	Width (μm)	256.5	254.0
	Height (μm)	78.5	76.5
3	Width (μm)	103.5	104.0
	Height (μm)	75.5	33.0
4	Width (μm)	55.5	56.5
	Height (μm)	75.5	13.5
5	Width (μm)	27.5	25.0
	Height (μm)	68.5	10.0

Figure 5.32: Height measurements of the imprinted micro-sized features.

5.7 Summary

A novel nano-positioning actuator, termed FELA, which mainly comprised of an electromagnetic moving air-core coil and supported by the flexure bearings is presented. Analytical modeling of the static current-force relationship of the electromagnetic driving module is discussed. Experimental investigations are conducted and validated the accuracy static current-force analytical model. In addition, a static force-displacement analytical model of the flexure bearings is derived and its accuracy is also verified experimentally. A system dynamics analysis of the FELA is conducted analytically and experimentally for the purpose of designing a PID controller to realize both position and direct-force controls. Detailed

control modeling is discussed to estimate the appropriate PID control parameters for both control schemes. Based on the position servo-control, FELA achieves a positioning accuracy of ± 10 nm (limited by the encoder resolutions) over a displacement of 4 mm. It achieves a smallest output step of 20 nm and a positioning repeatability of $\pm 1.5 \sigma$. With a direct force-control, this compact-sized FELA is capable of generating any force profile with a continuous output force of 60 N/Amp and an accuracy of ± 0.15 N. An impedance control is implemented on the FELA to allow a smooth transition between the position and direct-force control schemes. Subsequently, the FELA is used to realize the imprinting of micro-sized features through a hot-embossing process. Experimental results have shown that the imprinted features on the PMMA substrate have high aspect ratios with a maximum deviation of $\pm 0.35 \mu\text{m}$ as compared to the actual feature from the silicon template. Consequently, these results have proved that the FELA has successfully realized the imprinting of micro-sized features using a hot-embossing process with high degree of accuracy.

Chapter 6

Design of A Co-Planar Nano-Alignment Manipulator

“Simplicity is the ultimate sophistication.”

– Leonardo da Vinci (1452 – 1519).

6.1 Background

The co-planar nano-alignment manipulator of a SFIL system is mechanically complex and bulky due to the combinations of a fine-orientation stage and a coarse-positioning stage [137] as mentioned in Chapter 5. Such complexities are extended to the control systems and schemes required to operate the entire mechatronics system. In addition, implementations of a direct-force control for achieving a precise imprinting process are impossible due to the presences of a stepper motor coupled with a linear ball-screw that forms the coarse-positioning stage. In Chapter 5, the promising performances obtained from a FELA, i.e., ± 10 nm of positioning accuracy and a continuous output force of 60 N/Amp throughout a traveling range of ± 2 mm, have shown the merits of the integration between the beam-based flexure bearings and the enhanced Lorentz-force EDM, which incorporated the DM configurations. Based on this similar integration concept, a new co-planar nano-alignment manipulator, which has a θ_x - θ_y - Z motion, is proposed to enhance the SFIL systems.

Figure 6.1: A group of Z - θ_x - θ_y ultra-high precision manipulators.

In recent literatures, only a few θ_x - θ_y - Z ultra-high precision manipulator were developed. Examples include the well-known micro-motion Flexure-Based Parallel-Kinematics Manipulator (FPM) presented by Lee [141] (Fig. 6.1a). This FPM uses a 3-limbs Revolute-Prismatic-Spherical (3RPS) parallel-kinematics configuration to realize a θ_x - θ_y - Z motion. However, it is driven by PZT actuators that limit the workspace to tens of micrometers. Using the same 3RPS parallel-kinematics configuration, a θ_x - θ_y - Z FPM, termed Orion (Fig. 6.1b), is developed by Besson [142]. Although Orion displays a large workspace of $\pm 2.5^\circ \times \pm 2.5^\circ \times \pm 5$ mm, it does not have large output force and its end-effector does not have sufficient stiffness to withstand interference from external forces. Another large workspace θ_x - θ_y - Z manipulator is the Nasmyth-Adaptive-Optics-System (NAOS) developed by Spanoudakis et al [143] (Fig. 6.1c). NAOS, which is an electromagnetically-driven manipulator, also has poor output force since it is designed for space telescope mirror alignment. Lastly, Lee and Gweon [144] presented another form of θ_x - θ_y - Z manipulator (Fig. 6.1d). This manipulator is electromagnetically-driven and suspended by an air-bearing system. Although this manipulator achieves a positioning and angular resolutions of 25 nm

6.2. Kinematic Design of A 3-Limbs Prismatic-Prismatic-Spherical Parallel-Kinematics Configurat.

and $0.29 \mu\text{rad}$ respectively, it only achieves a translational range of $40 \mu\text{m}$ and an orientation range of $460 \mu\text{rad}$. As none of these manipulators can provide a large workspace of few millimeters and degrees, nanometric positioning resolution, arc-second orientation resolution, large continuous output force, with simple position, and direct-force controls capabilities, the proposed manipulator will meet such requirements that are essential in enhancing the co-planar nano-alignment and imprinting process of various nanoimprint lithography techniques, i.e., SFIL, micro-contact printing, and hot-embossing.

6.2 Kinematic Design of A 3-Limbs Prismatic-Prismatic-Spherical Parallel-Kinematics Configuration

The proposed 3-DOF co-planar nano-alignment manipulator will be developed through an electromagnetically-driven FPM, which adopts a spatial-motion parallel-kinematics manipulator to achieve a θ_x - θ_y - Z motion. In this section, the kinematics design and analysis of a spatial-motion parallel-kinematics manipulator will be discussed in details.

6.2.1 Type Synthesis

According to the literatures [145, 146], a general criteria for designing a 3-DOF spatial-motion parallel-kinematic manipulator with symmetric geometry is having three limbs with identical configurations to support a mobile platform, while each limb must have five degrees of motion. Other design considerations include:

1. Types of joints - Four types of joints are commonly considered in developing a spatial mechanism, i.e., 1-DOF Revolute (R) joint, 1-DOF Prismatic (P) joint, 2-DOF universal joint and 3-DOF Spherical (S) joint.
2. To achieve a more compact design, a spherical joint can be used in each limb to reduce the number of passive joints and it should be connected the mobile platform.

3. For a θ_x - θ_y - Z motion, the center of the spherical joint in each limb must be operating within a vertical plane. All three vertical planes, which are placed 120° apart, must intersect at the Z -axis of the base platform.
4. For each limb, the trajectory path of the remaining passive joints must also operate within the respective vertical plane.
5. As the enhanced Lorentz-force EDM will be used to drive the proposed manipulator, each limb will have an active prismatic joint.
6. Active prismatic joints should be mounted onto the base to reduce the moment of inertia, increase loading capacity and motion acceleration due to the mass of the EDM.

Based on the general criteria and design considerations 1 to 4, eight various parallel-kinematics configurations are iterated, i.e., 3RRS , 3RRS , 3RPS , 3RPS , 3PRS , 3PRS , 3PPS , and 3PPS . From past literatures, several of these configurations, i.e., 3RRS , 3RRS , 3RPS , 3RPS , 3PRS , 3PRS , have been introduced and used to develop various θ_x - θ_y - Z parallel-kinematics manipulators and FPMs [145, 146, 147, 148]. However due to design considerations 5 and 6, only the 3PRS and 3PPS configurations are feasible solutions for the development of the proposed FPM. Unfortunately, the presence of a passive revolute joint raises other issues in the FPM context. A passive compliant revolute joint, which is targeted for larger displacements, requires a thin slender design that is unsuitable for withstanding an imprinting force of 100 N. In addition, a passive revolute joint with slender design is sensitive to external disturbances, e.g., vibrations etc, and it has poor off-axis stiffness, which will affect the precision of the motion. On the other hand, a passive prismatic joint can offer a deterministic and precise motion, and high off-axis stiffness to withstand the imprinting force through symmetrical designs and parallel configurations. Hence, the 3PPS parallel-kinematics configuration is selected as the solution for developing the proposed 3-DOF FPM. With each compliant joint being treated as a kinematics joint with the equivalent degree of freedom, the mobility of the proposed 3PPS FPM can be determined by

6.2. Kinematic Design of A 3-Limbs Prismatic-Prismatic-Spherical Parallel-Kinematics Configurat.

$$m = 6(n - g - 1) - \sum_{k=1}^g f_k \quad (6.1)$$

where m represents the DOF of the manipulator, n is the number of rigid member, g is the number of joints and f_i represents the number of DOF of the k -th joint. For the proposed 3PPS parallel-kinematics FPM, $n = 8$, $g = 9$ and $f_k = 3$ for each of the passive spherical, $f_k = 1$ for each of the passive prismatic and active prismatic joints. Based on Eqn. (6.1), substituting these values yields $m = 3$, which shows that the proposed FPM offers three degrees of motion. In addition, as the center of the passive spherical joint in each limb is always operating within the vertical plane, the proposed 3PPS FPM will achieve the targeted θ_x - θ_y - Z motion.

Figure 6.2: Schematic diagram of the proposed PPS manipulator.

6.2.2 Forward Kinematic Analysis

The schematic diagram of the proposed 3PPS parallel-kinematics manipulator is illustrated in Fig. 6.2. It consists of a mobile platform that has three spherical joints, which are placed at 120° apart at a radius r from the center of the mobile platform. Each of these spherical

joints is connected to a prismatic joint where its translational path is formed towards the center of the base platform. Subsequently, each prismatic joint is connected to an extensible limb, which is located at each corner of the mobile platform to a base platform. Each limb is placed in a vertical position and is perpendicular to the base platform. All three limbs are extensible due to the active prismatic joints located at the end, which are placed 120° apart at a radius R from the center of the base platform.

A Cartesian coordinate frame xyz is fixed at the center of the mobile platform with the z -axis perpendicular to the mobile platform plane and pointing vertically upward, and the y -axis pointing towards the spherical joint 3, P_3 as shown in Fig. 6.2. Similarly, a coordinate frame XYZ is attached to the center of the mobile platform with the Z -axis normal to the base platform and the X -axis pointing towards the base of active prismatic joint, B_3 . Hence, the coordinates of the base of the active prismatic joints in the XYZ -frame are

$$B_1 = [-\frac{R\sqrt{3}}{2}, -\frac{R}{2}, 0] \quad B_2 = [\frac{R\sqrt{3}}{2}, -\frac{R}{2}, 0] \quad B_3 = [0, R, 0] \quad (6.2)$$

while the the coordinates of the spherical joints in the xyz -frame are

$$P_1 = [-\frac{r\sqrt{3}}{2}, -\frac{r}{2}, 0] \quad P_2 = [\frac{r\sqrt{3}}{2}, -\frac{r}{2}, 0] \quad P_3 = [0, r, 0] \quad (6.3)$$

Here, the rotation matrix of the proposed 3PPS parallel-kinematics manipulator is defined by the following rotational angles. Assuming that the initial location of the mobile platform frame coincides with the fixed base platform frame and all rotations take place about the axis in the fixed base frame, the final orientation is obtained by a rotation about the X -axis, θ_x , and followed by a rotation about the Y -axis, θ_y . Premultiplying the rotation matrix about X -axis, $R_x(\theta_x)$, and the rotation matrix about Y -axis, $R_y(\theta_y)$, yields a 3×3 rotation matrix, ROT , which is expressed as

6.2. Kinematic Design of A 3-Limbs Prismatic-Prismatic-Spherical Parallel-Kinematics Configurati

$$\begin{aligned}
 ROT &= R_y(\theta_y)R_x(\theta_x) \\
 &= \begin{bmatrix} c\theta_y & s\theta_x s\theta_y & c\theta_x s\theta_y \\ 0 & c\theta_x & -s\theta_x \\ -s\theta_y & c\theta_y s\theta_x & c\theta_x c\theta_y \end{bmatrix}
 \end{aligned} \tag{6.4}$$

where c and s are the shorthand notations for cosine and sine respectively.

Figure 6.3: Top projection of manipulated spherical joints with respect to the base frame.

For this proposed 3PPS parallel-kinematics manipulator, the active prismatic joints vary the limbs' length to manipulate the mobile platform with respect to the base platform. Figure 6.3 illustrates the top projection of each spherical joint with respect to the base coordinate frame in the X-Y plane. It shows that each spherical joint moves along a trajectory path of its respective connected passive prismatic joint, where all three trajectory paths intersect at the center of the base coordinate frame. Consequently, the location of each spherical joint in X-Y plane due to the variation of each limb length can be expressed as

$$(P_{2,x} - P_{1,x})^2 + (P_{2,y} - P_{1,y})^2 = L_{12}'^2 = L_{12}^2 - (P_{2,z} - P_{1,z})^2 \tag{6.5}$$

$$(P_{3,x} - P_{2,x})^2 + (P_{3,y} - P_{2,y})^2 = L'_{23} = L_{23} - (P_{3,z} - P_{2,z})^2 \quad (6.6)$$

$$(P_{1,x} - P_{3,x})^2 + (P_{1,y} - P_{3,y})^2 = L'_{13} = L_{13} - (P_{1,z} - P_{3,z})^2 \quad (6.7)$$

where $P_{i,j}$ represents the x , y or z component value of the i -th spherical joint denotes by j .

Due to the limited motions of the compliant joints, the 2D coordinates of the spherical joints as projected onto the XY plane of the base frame are

$$P_1 = \left[-\frac{R_1\sqrt{3}}{2}, -\frac{R_1}{2}\right] \quad P_2 = \left[\frac{R_2\sqrt{3}}{2}, -\frac{R_2}{2}\right] \quad P_3 = [0, R_3] \quad (6.8)$$

where R_1 , R_2 and R_3 represent the distances from the origin O to P_1 , P_2 and P_3 respectively (refer to Fig. 6.3). Hence, substituting Eqn. (6.8) into Eqn. (6.5) to (6.7) respectively yields

$$R_2^2 + R_2R_1 + R_1^2 = L_{12} - (P_{2,z} - P_{1,z})^2 \quad (6.9)$$

$$R_3^2 + R_3R_2 + R_2^2 = L_{23} - (P_{3,z} - P_{2,z})^2 \quad (6.10)$$

$$R_1^2 + R_1R_3 + R_3^2 = L_{13} - (P_{1,z} - P_{3,z})^2 \quad (6.11)$$

In forward kinematic analysis, given the values of $P_{1,z}$, $P_{2,z}$ and $P_{3,z}$, the three unknowns, i.e., R_1 , R_2 and R_3 , can be found through Eqn. (6.9) to (6.11). With these three simultaneous equations, four sets of solutions for those unknowns are obtained. Out of these four sets, only one set of solution, which gives positive values for R_1 , R_2 and R_3 , will be used. Subsequently, the X- and Y-coordinates of each spherical joint in Eqn. (6.8) is determined from the

6.2. Kinematic Design of A 3-Limbs Prismatic-Prismatic-Spherical Parallel-Kinematics Configurat

obtained values of R_1 , R_2 and R_3 . Consequently, a 3×3 orientation matrix, M , based on the coordinates of all three spherical joints is derived as

$$M = \begin{bmatrix} \vec{X} & \vec{Y} & \vec{Z} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \quad (6.12)$$

where

$$\vec{X} = \vec{P}_{12} \quad (6.13)$$

$$\vec{Z} = \vec{P}_{12} \times \vec{P}_{23} \quad (6.14)$$

$$\vec{Y} = \vec{Z} \times \vec{X} \quad (6.15)$$

Hence, the position of the end effector, which is at the center of mobile platform, due to the variation of limb lengths is given as

$$\begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix} = \begin{bmatrix} \frac{P_{1,x} + P_{2,x} + P_{3,x}}{3} \\ \frac{P_{1,y} + P_{2,y} + P_{3,y}}{3} \\ \frac{P_{1,z} + P_{2,z} + P_{3,z}}{3} \end{bmatrix} \quad (6.16)$$

As for the orientation angles, a comparison between the rotation matrix in Eqn. (6.4) and the orientation matrix in Eqn. (6.12) shows that θ_x and θ_y are obtained by

$$\theta_x = \arctan 2(r_{33}, r_{32}) \quad (6.17)$$

$$\theta_y = \arctan 2(r_{11}, -r_{31}) \quad (6.18)$$

where $\arctan 2(x, y)$ computes as $\tan^{-1}(\frac{y}{x})$ but uses both signs of x and y to determine the quadrant where the obtained angles lie. For example, $\arctan 2(-2.0, -2.0) = -135^\circ$ and $\arctan 2(2.0, 2.0) = 45^\circ$ etc.

In this section, the forward kinematics of this 3PPS parallel-kinematics configuration is established to facilitate the design of each respective compliant joint. As this work does not cover the closed-loop control of the proposed FPM, the inverse kinematics will not be covered at this stage of research.

6.2.3 Initial Parametric Analysis

The proposed 3PPS FPM will be targeted to achieve an orientation of at least $\pm 2^\circ$ about the x - and y -axes, and a displacement of ± 2.5 mm along the z -axis. From Eqn. (6.9) to (6.11), the variations of limb lengths and the distance between each spherical joint have direct influence on the orientation of the mobile platform. As the targeted displacement of the proposed FPM is ± 2.5 mm, the maximum variation of each limb length must have similar magnitude. The distance between each spherical joint, L , is fixed and can be determined based on the dimension of the mobile platform. As mentioned earlier, each of three spherical joints is placed at 120° apart at a radius r from the center of the mobile platform. Hence, this arrangement forms a triangular-shaped mobile platform and the distance of the spherical joint from the center can be given as

$$r = \frac{2}{3}D \quad (6.19)$$

where D is the height of the triangular-shape mobile platform. Thus, the distance between each spherical joint is expressed as

$$L = r\sqrt{3} \quad (6.20)$$

Based on forward kinematic analysis, three various heights of the triangular-shaped mobile platform, i.e., $L = 75$ mm, $L = 100$ mm and $L = 125$ mm, are used for initial parametric analysis of the proposed 3PPS parallel-kinematics manipulator. Two analyses are conducted based on each given height of the triangular-shaped mobile platform. In the first analysis, $P_{1,z} = 2.5$ mm, $P_{2,z} = 2.5$ mm and $P_{3,z} = -2.5$ mm are fixed to obtain the maximum achievable angular deflection about x-axis. In the second analysis, $P_{1,z} = -2.5$ mm, $P_{2,z} = 2.5$ mm and $P_{3,z} = 0$ mm are fixed to obtain the maximum achievable angular deflection about y-axis. Table 6.1 listed the results obtained from both analyses.

Table 6.1: List of maximum θ_x and θ_y values obtained from given actuator displacements

D (mm)	L (mm)	θ_x ($^\circ$)	θ_y ($^\circ$)
75	86.60	3.82267	2.86726
100	115.47	2.86599	2.14960
125	144.34	2.2924	1.71936

The analytical results suggest that when the mobile platform becomes bigger in dimensions, the targeted orientations become less achievable through the maximum displacement of 2.5 mm provided by the linear actuators. For example, when $D = 125$ mm, the maximum achievable orientation about the y-axis is less than $\pm 2^\circ$. Hence, larger strokes are required from the linear actuators to achieve the targeted orientations. In this analysis, it has also suggest that a displacement of ± 2.5 mm from the linear actuator is sufficient to achieve a workspace of $\pm 2^\circ \times \pm 2^\circ \times \pm 2.5$ mm when $D = 100$ mm. Although the obtained workspace is larger when $D = 75$, the dimension of the triangular-shaped mobile platform based on $D = 75$ may not be sufficient to accommodate three sets of EDMs and active compliant prismatic joints. Consequently, the height of the triangular-shaped mobile platform will be selected as 100 mm.

6.3 Conceptual Design of the Proposed 3PPS FPM

The proposed 3PPS FPM consists of three identical compliant limbs, where each limb comprises of an active compliant prismatic joint, a passive compliant prismatic joint and a

compliant spherical joint. Each limb supports the mobile platform with the active compliant prismatic joint fixed vertically and perpendicular to the base platform. Subsequently, each active compliant prismatic joint carries a passive compliant prismatic joint at its center of the actuating direction. The center of the moving platform of the passive compliant prismatic joint carries a passive compliant spherical joint, which is connected to the edge of the mobile platform. Consequently, this arrangement forms the 3PPS FPM as illustrated in Fig. 6.4.

Figure 6.4: Proposed 3PPS FPM.

6.4 Active Compliant Prismatic Joint

Each active compliant prismatic joint comprises of a flexure bearing support, i.e., FBM, driven by the moving air-core coil of an EDM attached to it (Fig. 6.5). This arrangement is similar to the proposed FELA except the FBM is formed by a pair of single piece monolithic-cut compound linear spring. Such a compound linear spring configuration helps to eliminate the undesired motions in the non-actuating directions and provides a rectilinear translational

motion. In addition, the monolithic-cut allows higher precision motion due to the absence of assembly errors.

Figure 6.5: An active compliant prismatic joint driven by an electromagnetic driving scheme.

Figure 6.6: A monolithic compound linear spring with beam-based flexure joints.

6.4.1 Displacement Stiffness Modeling

The displacement stiffness model of the proposed FBM in the actuating direction is similar to the displacement stiffness model established in Section 3.5 of Chapter 3. Hence, the displacement of each limb, δ_z , is given as

$$\delta_z = \frac{disp_z}{2} \quad (6.21)$$

where $disp_z$ represents the targeted displacement of the entire FBM. The displacement stiffness of the entire FBM, K_{FBM} is given as

$$K_{FBM}^z = 2 \frac{F_d}{\delta_z} \quad (6.22)$$

where the driving force, F_d , is obtained from Eqn. (3.39) with a given deflection angle, θ_d . This deflection angle is determined from the targeted displacement using Eqn. (3.33). The maximum bending stress experienced by each beam-based flexure joint due to the desired displacement is thus

$$\sigma_{max} = \frac{F_d[l/2 + (L + l/2\varpi) \cos \theta_d](t/2)}{I} \quad (6.23)$$

6.4.2 Parameters Analyses and Selections

The initial parametric analyses conducted in previous section have suggested that the linear actuator must have at least ± 2.5 mm of displacement. Hence, this requirement suggests that the FBM must also provide the same magnitude of displacement. With safety factor being considered, the stiffness analysis will be conducted to ensure that the maximum bending stress of the beam-based flexure joints is within the material yield strength at a maximum displacement limit of ± 3 mm. Based on the derived displacement stiffness of the FBM, a quick parametric analysis was conducted. From Eqn. (3.39) and (6.23), changing the thickness of beam-based flexure will have significant influence on the amount of driving force required to achieve 3 mm and the maximum bending stress experienced by the flexures. Although other parameters, i.e., the length of rigid-link and flexures, can influence the driving force and stress of the flexures, increasing the length of rigid-link will increase the size of the entire FBM while increase in flexure length will reduce the stiffness in non-actuating directions. Hence, the length of the rigid-link is fixed at 35 mm, the length and width of the beam-based flexures are fixed at 3 mm and 20 mm respectively. Using aluminum 7075-T6 material, the maximum yield strength of this material is 503 MPa. Thus, 250 MPa will be the maximum allowable yield stress. With the maximum displacement limit fixed at 3 mm, a set of flexure thicknesses is used to determine the required driving force and stress.

Table 6.2: List of required driving force and maximum bending stress values corresponding to various beam-based flexure joint-thicknesses

Flexure thickness (mm)	F_d (N)	σ_{max} (MPa)
0.1	0.16	50.58
0.2	1.31	101.18
0.3	4.41	151.76
0.4	10.46	202.35
0.5	20.43	252.94

The results listed in Table 6.2 has shown that the flexure thicknesses of 0.1 mm and 0.2 mm require lower driving forces and experience lower bending stresses among the rest. However, such thicknesses with high tolerances are difficult to achieve from local manufacturers. The best thickness among the remaining achievable dimensions is 0.3 mm as the FBM achieves a

required driving force of less than 5 N and stress of about 152 MPa. Based on this analysis, the thickness of the beam-based flexure joint employ by the proposed FBM is chosen to be 0.3 mm.

6.5 Passive Compliant Prismatic Joint

The moving air-core coil, which is supported by the FBM, is connected in series to a passive compliant prismatic joint. With respect to its local coordinate frame, this prismatic joint must have a low actuating stiffness in the x-axis and high stiffness in the z-axis (Fig. 6.7a). In addition, it must be compact in size and light weight to reduce the inertia effect. To achieve such stringent requirements, a bi-stable design is adopted as it offers low actuating stiffness within an allowable range but high stiffness in the non-actuating directions. To further reduce the actuating stiffness, beam-based flexure joints are proposed as the supporting compliant joints (Fig. 6.7b).

Figure 6.7: (a) A passive compliant prismatic joint, which is formed by four parallel (b) beam-based flexure joints, based on (c) a bi-stable design.

6.5.1 Stiffness Modeling

The bi-stable design is a symmetrical linear spring mechanism (Fig. 6.7c) and the displacement stiffness of a single linear spring mechanism is given as [1]

$$K_{linear-spring}^x = \frac{F_d}{\delta_x} = \frac{24EI}{l^3} \quad (6.24)$$

where δ_x is the desired displacement, l represents the length of the beam-based flexure joint.

Therefore, the displacement stiffness of the passive compliant prismatic joint is expressed as

$$K_{prismatic}^x = 2K_{linear-spring}^x \quad (6.25)$$

The non-actuating direction stiffness in the z-axis for the passive prismatic joint is double the stiffness given in [1] and is expressed as

$$K_{linear-spring}^z = \frac{Etb^3}{l^3} \quad (6.26)$$

Last but not least, the maximum bending stress experience by each beam-based flexure joint due to the desired displacement is given as [1]

$$\sigma_{max} = \delta_x \frac{3Et}{l^2} \quad (6.27)$$

6.5.2 Parameters Analyses and Selections

In the forward kinematic analysis, a set of limb length variations can be used to determine the distance of each spherical joint with respect to the center of the base platform, i.e., R_1 , R_2 and R_3 . Judging from Fig. 6.3, the difference between R and R_i (where $i = 1, 2, 3$) represents the displacement of each passive compliant prismatic joint. Based on the initial parametric analysis conducted in previous section, a maximum displacement of $70.6 \mu\text{m}$ is required from the passive compliant prismatic joint when the proposed 3PPS parallel-kinematics manipulator achieves $\pm 2^\circ$ about the x- and y-axes (note: results are obtained based on $D = 100 \text{ mm}$ with limb length variations of $\pm 2.5 \text{ mm}$). In this analysis, Eqn. (6.24) and (6.26) suggest that the width of the beam-based flexures plays an important role in the displacement stiffness and off-axis stiffness. With the desired displacement being fixed at $71 \mu\text{m}$, the thickness of the flexures fixed at 0.3 mm due to the limitations of manufacturers and the length fixed at 15 mm due to space constraint, various flexure width dimensions are

used to conduct a parametric analysis of the proposed prismatic joint. The material used in the analysis is aluminum 7075-T6 with Young’s Modulus is 71 GPa.

Table 6.3: List of required driving force and off-axis stiffness obtained from various beam-based flexure joint-widths

Flexure width (mm)	F_d (N)	$K_{linear-spring}^z$ (N/mm)
5	0.81	78.89×10^5
10	1.61	63.11×10^{13}
15	2.42	213.00×10^{13}
20	3.23	504.89×10^{13}
25	4.03	986.11×10^{13}

Results listed in Table 6.3 show that all five width dimensions allow the proposed passive compliant prismatic joint to achieve low displacement stiffness. However, the flexure widths of 5 mm and 10 mm offer very low off-axis stiffness. Although a flexure width of 25 mm may offer the highest off-axis stiffness, it may also affect the overall mass and height of the proposed joint. Among the remaining flexure widths of 10 mm and 20 mm, 20 mm is a better choice due to its higher off-axis stiffness. Consequently, the width of the beam-based flexure joints of the proposed passive compliant prismatic joint is chosen as 20 mm.

6.6 Passive 5-DOF Compliant Joint

In past developed θ_x - θ_y -Z FPMs, an elementary rod flexure joint has been a popular approach in realizing a conventional spherical joint motion [141, 144]. Although the elementary rod flexure joint has five degrees of motion (Fig. 6.8a), keeping the length short often allows researchers to treat it as a compliant spherical joint with three degrees of motion (Fig. 6.8b). Unfortunately, an elementary rod flexure joint runs into its elastic deformation limit when large deflection in multi-DOF is required. Although new forms of compliant joints with larger deflection or rotation motion have been presented in recent years [23, 24, 149], these compliant joints have less than 2-DOF and possess high driving stiffness. Here, a new 5-DOF compliant joint, which offers larger deflections and orientations with lower driving stiffness, is introduced. This 5-DOF compliant joint will play an important role in the development of spatial-motion FPMs that target large workspace of few millimeters and degrees.

Figure 6.8: (a) An elementary rod flexure joint, (b) a compliant spherical joint and (c) an elementary beam-based flexure joint.

The design concept of this 5-DOF compliant joint relies very much on the basic motions of an elementary beam-based flexure joint as shown in Fig. 6.8c. In most literatures, an elementary beam-based flexure joint has three degrees of motion, which include a deflection, an angular rotation and a torsional motion. Here, all these basic motions will be considered during the development of this 5-DOF compliant joint, which will comprise of three segments. Segment 1 and 2 will each consist of a pair of beam-based flexure joints, while segment 3 will be made up of four sets of equally spaced compound linear spring in a circular arrangement. The following sections will discuss on each segment in details.

6.6.1 Design Concept of Segment 1 and 2

An elementary rod flexure joint consists of five degrees of motion, e.g., the deflection in the x-axis, X , the deflection in the y-axis, Y , the angular motion about the x-axis, θ_x , the angular motion about the y-axis, θ_y , and the torsional motion about the z-axis, θ_z , as shown in Fig. 6.9. To design a 5-DOF compliant joint with larger deflections and rotations, a pair of elementary beam-based flexure joints is employed in the segment 1 of this proposed 5-DOF compliant joint as shown in Fig. 6.9. This pair of beam-based flexure joints is used to realize the deflection along the x-axis, X , the angular motion about the y-axis, θ_y , and the angular motion about the z-axis, θ_z .

Next, segment 1 is connected in series with segment 2, which also comprises of a pair of elementary beam-based flexure joints. However, this pair is placed orthogonal to segment 1, which is 90° interesting the x-axis. As a result, this pair of beam-based flexure joints is used

Figure 6.9: Design concept of the proposed 5-DOF compliant joint as compared to an elementary rod flexure.

to realize the deflection in the y-axis, Y , the angular motion about the x-axis, θ_x , and the angular motion about the z-axis, θ_z . In segment 1, both beam-based flexure joints are placed apart to increase the stiffness in the y-axis. This is to prevent undesired motion occurring in segment 1 when a force is acting along the y-axis. Similarly, both beam-based flexure joints in segment 2 are placed apart to increase the stiffness in the x-axis. This prevents undesired motion occurring in segment 2 when a force is acting along the x-axis. Nevertheless, this arrangement also increases the angular stiffness about the z-axis of the proposed 5-DOF compliant joint. In some cases, the desired angular motion about the z-axis may not be achievable by the combination of angular motion about the z-axis from segment 1 and 2 since the deflections in the x- and y-axes are the primary objective.

6.6.2 Design Concept of Segment 3

Consequently, four sets of compound linear springs will be incorporated into the proposed 5-DOF compliant joint (Fig. 6.10). A compound linear spring design provides low displacement stiffness and avoids parasitic motion. All four sets of compound linear springs are equally placed in a circular arrangement and in turn form the segment 3 of the spatial joint. Thus segment 3 will be used to realize the additional angular motion about the z-axis is required when the angular motion about the z-axis from segment 1 and 2 is insufficient.

Figure 6.10: Design concept of segment 3 of the proposed 5-DOF compliant joint.

6.6.3 Complete Module

Adding all three segments together forms the complete 5-DOF compliant joint, which offers five degrees of motion with large deflections and low driving stiffness, as shown in Fig. 6.11.

Figure 6.11: The proposed 5-DOF compliant joint.

The segment 1 is connected in series with segment 2 where the base of the beam-based flexures in segment 1 being fixed to the deflecting portion of the beam-based flexures in segment 2. This arrangement allows the rotation axis of both segments to intersect, hence the angular motion about the x-axis, θ_x , and the angular motion about the y-axis, θ_y , will be independent to each other. Subsequently, the base of the beam-based flexures in segment 2 is connected in series with the segment 3. As segment 3 serves as part of the effort in

realizing the angular motion, the actual torsional motion, θ_z , for the 5-DOF compliant joint will be the combination of all three angular motions obtained from segment 1, 2, and 3.

6.6.4 Adopting the 5-DOF Compliant Joint into the 3PPS FPM

To adopt this 5-DOF compliant joint into the 3PPS FPM, each joint is connected to the corner of the mobile platform with the deflection along the x-axis (from segment 1) falls within the vertical working plane of each limb (Fig. 6.12). Consequently, the deflection along the y-axis (from segment 2), which is perpendicular to the deflection along the x-axis, will be out of the working plane (Fig. 6.12a). This arrangement eliminates 1-DOF from each 5-DOF compliant joint due to the physical constraint of the mobile platform, which is connected to the fixed base platform. In addition, as deflection along the x-axis coincides with the displacement path of each passive compliant prismatic joint, such a deflection can be treated as part of the passive translation motion. With the remaining three orientations, i.e., θ_x , θ_y and θ_z , the 5-DOF compliant joint can be treated as a compliant spherical joint required to form each PPS limb.

Figure 6.12: (a) The deflection path of each passive 5-DOF compliant joint from top projection and (b) the working plane of each parallel limb.

6.6.5 Modeling of Deflection Stiffness Along X- and Y-Axes

Segment 1 or 2 is made up by a pair of beam-based flexures connected by a rigid member for mounting purpose. Hence, the deflection along the x- and y-axes from segment 1 and the segment 2 respectively can be determined by the semi-analytic model of a beam-based flexure joint coupled with a rigid-link subjected to a horizontal force. Based on Eqn. (3.39), the relationship between the horizontal force and the deflection angle can be expressed as

$$F = \frac{EI\alpha}{l(L + \frac{\rho l}{2\omega}) \sin(\frac{\pi}{2} - \alpha)} \quad (6.28)$$

Due to a horizontal force, the derived deflection angle from Eqn. (6.28) can be used to determine the deflection along the x- or y-axis. From Eqn. (3.33), the deflection along the x- or y-axis of the proposed 5-DOF compliant joint is expressed as

$$\Delta_i = (L + \frac{l}{2\omega}) \sin \alpha \quad (6.29)$$

where $i = 1, 2$ represents segment 1 or 2, L is the length of rigid-link, l is the length of beam-based flexure joint and α is the deflection angle as illustrated in Fig. 6.13.

Subsequently, the deflection stiffness of the proposed 5-DOF compliant joint is given as

$$K_{F,\Delta_i} = \frac{2F}{\Delta_i} \quad (6.30)$$

In addition, Fig. 6.13 suggests that the deflection angle can be used to determine the deflection along the z-axis of the proposed 5-DOF compliant joint, Δ_z , in the form of

$$\Delta_z = R \sin \alpha \quad (6.31)$$

Figure 6.13: Beam-based flexure joint coupled with a rigid-link subjected to a horizontal force.

where R is the radius of the proposed 5-DOF compliant joint. While from Eqn. (3.34), the deflection of a beam-based flexure joint coupled with a rigid-link along the z -axis thus can be expressed as

$$\delta_z = \left(L + \frac{l}{2}\right) - \left(L + \frac{l}{2\varpi}\right) \cos \alpha \quad (6.32)$$

Last, the maximum bending stress on the beam-based flexure joint can be expressed as

$$\sigma_{max} = \frac{F[l/2 + (L + l/2\varpi) \cos \alpha](t/2)}{I} \quad (6.33)$$

6.6.6 Modeling of Stiffness Along Z-Axis and Critical Loading

For beam-based flexure joints that subjected to an axial force, P , the compression or extension of these flexure joints can be expressed as [1]

$$\delta_z = \frac{Pl}{EA} \quad (6.34)$$

where A is the cross-sectional area of the beam-based flexure as illustrated in Fig. 6.14.

The stress due to the axial force can be expressed as

$$\sigma_{axial} = \frac{P}{A} \quad (6.35)$$

Subsequently, the axial stiffness of the proposed 5-DOF compliant joint is expressed as

$$K_{P,\delta_z} = 2\frac{EA_{1,2}}{l} + 16\frac{EA_3}{l} \quad (6.36)$$

where $A_{1,2}$ is the cross-sectional area of the beam-based flexure used in segment 1 and 2, A_3 is the cross-sectional area of the beam-based flexure used in segment 3, and l_3 is the flexure length in segment 3.

Figure 6.14: Beam-based flexure joint coupled with a rigid-link subjected to a axial compressive force.

Other than the axial stiffness, it is important to identify the critical load that these beam-based flexure joints can take to avoid premature failing. In the literature [150], the critical load is derived as

$$P_{cr} = \frac{\pi EI}{(Kl)^2} \quad (6.37)$$

where in this work, $K = 2$ assuming the beam-based flexure joints are in guided-free, guided-hinge or clamped-free condition.

6.6.7 Modeling of Angular Stiffness About X- and Y-Axes

The angular motion about the x- or y-axis from segment 1 and 2 respectively can be determined by the elementary bending moment equation (Fig. 6.15). Hence, the relationship between the external moment and angular motion of the 5-DOF compliant joint is expressed as

$$\theta = \frac{Ml}{EI} \quad (6.38)$$

where moment, M , is given as

$$M = F_v \times R \quad (6.39)$$

In addition, the maximum bending stress is given as

$$\sigma = \frac{M(t/2)}{I} \quad (6.40)$$

Figure 6.15: Beam-based flexure joint coupled with a rigid-link subjected to external moment.

The non-linear deflection of the beam-based flexure joint along the x- or y-axis is expressed as [116]

$$\delta = l \left(\frac{1 - \cos \theta}{\theta} \right) \quad (6.41)$$

and the amplified deflection due to the rigid-link is expressed as

$$\Delta_L = L \sin \theta \quad (6.42)$$

The deflection of the proposed compliant spatial along the x- or y-axis due to the external moment thus can be written as

$$\Delta_i = \delta + \Delta_L \quad (6.43)$$

In addition, the non-linear deflection of the beam-based flexure joint along the z-axis is given as

$$\delta_z = l \left(1 - \frac{\sin \theta}{\theta} \right) \quad (6.44)$$

Referring to Fig. 6.15, it also suggests that the deflection angle derived from Eqn. (6.38) can be used to determine the deflection along the z-axis of the proposed 5-DOF compliant joint as

$$\Delta_z = R \sin \theta \quad (6.45)$$

6.6.8 Modeling of Angular Stiffness About Z-Axis

The angular motion about the z-axis of the proposed 5-DOF compliant joint is contributed by the angular motions from all three segments. The angular stiffness of segment 1 and 2 will be similar since both segments comprise of a pair of beam-based flexures joints with a

space in-between. On the other hand, segment 3, which comprised of four sets of compound linear springs, will have a different angular stiffness.

6.6.8.1 Angular stiffness of segment 1 or 2

The angular motion of a pair of beam-based flexure joints with an in-between space is very complex to derive despite of such simple arrangements. The angular motion caused by an external moment can be considered as a force couple acting on the beam-based flexure joints (Fig. 6.16). In such condition, the angular stiffness is affected by the internal torsion of each beam-based flexure and the shearing within the internal wall of each beam-based flexure. Thus, the combination of the torsional stiffness and the shear stiffness will be the angular stiffness of segment 1 or 2.

Figure 6.16: Torsion and shearing found within a pair of beam-based flexure with a space in-between subjected to an external moment.

In the literatures, the torsional stiffness of a rectangular-shape object subjected to a torque, T , is given as [150]

$$K_{T,\theta} = \kappa \frac{Gt^3w}{3l} \quad (6.46)$$

where κ is a factor that varies according to the width and thickness of the beam-based flexure, G is the modulus of rigidity while t , w and l are the thickness, width and length of the beam-based flexure respectively. The external moment, T , is given as

$$T = F \times R \quad (6.47)$$

where $R = D + W$ (refer to Fig. 6.16).

To find κ , first look at the maximum shear stress, τ_{max} , as

$$\tau_{max} = \frac{T}{Q} \quad (6.48)$$

and [150]

$$Q = \frac{8a^2b^2}{3a + 1.8b} \quad (6.49)$$

where $w = 2a$ and $t = 2b$.

It is also known that the angular twist, θ , due to the moment can also be expressed as [150]

$$\theta = \frac{Tl}{\gamma G} \quad (6.50)$$

where

$$\gamma = ab^3 \left\{ \frac{16}{3} - \frac{3.36b[1 - (b^4/12a^4)]}{a} \right\} \quad (6.51)$$

From Eqn. (6.48) and (6.50), it yields

$$\tau_{max} = \frac{G\theta}{l} \left(\frac{\gamma}{Q} \right) \quad (6.52)$$

Given that the maximum shear stress can be expressed as [1],

$$\tau_{max} = \kappa \frac{G\theta t}{l} \quad (6.53)$$

Equating Eqn. (6.52) and (6.53) yields

$$\kappa = \frac{\gamma}{Qt} \quad (6.54)$$

where γ is expressed in Eqn. (6.51) and Q is expressed in Eqn. (6.49).

With the torsional stiffness derived, the shearing within the internal wall of each beam-based flexure will be focused. In the literature, the shear stiffness of a pair of beam-based flexure joints with an in-between space is given as [1]

$$\frac{F}{\varepsilon} = \frac{6EI}{l^3} \quad (6.55)$$

where ε is the shear displacement due to the applied force.

From Fig. 6.16, the twisting angle, β , can be used to determine the shear displacement in the form of

$$\varepsilon = \frac{D}{2} \sin \beta \quad (6.56)$$

Based on small deflection theory, Eqn. (6.56) becomes

$$\varepsilon = \frac{D}{2} \beta \quad (6.57)$$

Substituting Eqn. (6.57) into Eqn. (6.55) yields

$$\frac{2F}{D\beta} = \frac{6EI}{l^3} \quad (6.58)$$

Meanwhile, the end force causing deflections can be interpreted as couples on the spacing of D and $D\theta$ respectively. This yields,

$$F = \frac{M}{2D} \quad (6.59)$$

Substituting Eqn. (6.59) into Eqn. (6.58) yields

$$\frac{M}{\beta} = \frac{6ED^2}{l^3} I \quad (6.60)$$

With $I = \frac{wt^3}{12}$, the shear stiffness of a pair of beam-based flexure joints with an in-between space is given as

$$\frac{M}{\beta} = \frac{EDwt^3}{2l^3} \quad (6.61)$$

The total angular stiffness for a pair of beam-based flexure joints with a space in-between found segment 1 or 2 thus can be expressed as

$$K_{M,\theta_z} = \frac{M}{\theta_z} = 2\left(\kappa \frac{Gt^3w}{3l}\right) + \frac{EDwt^3}{2l^3} \quad (6.62)$$

where $M = F \times R$.

Based on the derived deflection angle, θ_z , from Eqn. (6.62), the planar angular motion about the z-axis in segment 1 or 2 is given as

$$\Delta_i = R\theta_z \quad (6.63)$$

The stress concentration in the beam-based flexure joints due to this angular motion is complex. This stress is form by the combination of the bending stress and the shear stress.

Based on Eqn. (6.40), given that $M = F \times l$, yields

$$\sigma_{bending} = \frac{Fl(t/2)}{I} \quad (6.64)$$

Given that the force-deflection stiffness of a beam-based flexure is [1]

$$k_{F,\Delta_i} = \frac{3EI}{l^3} \quad (6.65)$$

Substituting Eqn. (6.65) into Eqn. (6.64), the bending stress experience by the beam-based flexure can be expressed as

$$\sigma_{bending} = \frac{3\Delta_i E(t/2)}{l^2} \quad (6.66)$$

The twist angle, θ , required by Eqn. (6.52) to calculate the shear stress experiences by the beam-based flexure is given as

$$\theta = \frac{\Delta_i}{(w/2)} \quad (6.67)$$

In the 2D plane, the shear stress is equivalent to the bending stress [1]

$$\sigma_{shear} = \tau \quad (6.68)$$

Here, the total stress experience by the beam-based flexure in segment 1 or 2 is written as

$$\sigma_i = \frac{3\Delta_i E(t/2)}{l^2} + \kappa \frac{Gt}{l} \left[\frac{\Delta_i}{(w/2)} \right] \quad (6.69)$$

where $i = 1$ and 2 .

6.6.8.2 Angular Stiffness of Segment 3

The segment 3 of the proposed 5-DOF compliant joint comprises of four sets of compound linear springs arranged in parallel. Hence, the angular stiffness of segment 3 will be based on the displacement stiffness of each compound linear spring. The angular stiffness of a beam-based flexure joint due to external moment can be expressed as

$$K_{M,\theta} = \frac{EI}{l} \quad (6.70)$$

and the stiffness of a single linear spring system is given as

$$K_{F,\Delta_3} = \frac{4K_{M,\theta}}{(L^*)^2} \quad (6.71)$$

where $L^* = L + l$ as shown in Fig. 6.17.

Figure 6.17: (a) A compound linear spring formed by (b) a pair of linear springs connected in series.

As a compound linear spring is formed by a pair of linear springs connected in series, its stiffness becomes half of a single linear spring. Based on Eqn. (6.70) and (6.71) the displacement stiffness of each compound linear spring is expressed as

$$K_{f,\Delta_3} = \frac{f}{\Delta_3} = \frac{2EI}{l(L^*)^2} \quad (6.72)$$

where $f = F/2$. This is because the 5-DOF compliant joint has two compound linear springs at each half, the input force, f , for each compound linear spring is half of the force, F , that is obtained from the external moment, i.e., $M = F \times R$. In addition, the stress experience in each flexure joint of the compound linear spring is expressed as

$$\sigma_3 = \frac{Et}{2l} \left(\frac{\Delta_3}{2} \right) \quad (6.73)$$

6.6.8.3 Total Angular Motion

The total angular motion about the z-axis of the proposed 5-DOF compliant joint is given as

$$\Delta_{total} = \Delta_1 + \Delta_2 + \Delta_3 \quad (6.74)$$

The angle, θ_z , due to this angular motion is given as

$$\theta_z = \sin^{-1} \left(\frac{\Delta_{total}}{R} \right) \quad (6.75)$$

The maximum stress can be determine by either Eqn. (6.69) or (6.73) depending on which is more dominant.

6.6.9 Deflection Along X- or Y-Axis Due to A Translational Force

Based on the derived stiffness models, each parameter, i.e., the radius of the joint, R , the length of rigid members, L , the length of flexure, l , the width of flexure, w , and the thickness of flexure, H , has direct effect on the deflection stiffness of the proposed 5-DOF compliant joint. In this section, four combinations of parameters are used in the parametric analysis:

Case A1: $R = 17.5 \text{ mm}$, $L = 15 \text{ mm}$, $l = 5 \text{ mm}$, $H = 0.5 \text{ mm}$, width = 10 mm

Case A2: $R = 17.5 \text{ mm}$, $L = 15 \text{ mm}$, $l = 5 \text{ mm}$, $H = 0.5 \text{ mm}$, width = **25 mm**

Case A3: $R = 17.5 \text{ mm}$, $L = \underline{5 \text{ mm}}$, $l = 5 \text{ mm}$, $H = 0.5 \text{ mm}$, width = 10 mm

Case A4: $R = 17.5 \text{ mm}$, $L = 15 \text{ mm}$, $l = 5 \text{ mm}$, $H = \underline{0.3 \text{ mm}}$, width = 10 mm

For each case, a set of desired deflections along the x- or y-axis is input into Eqn. (6.29) to obtain a set of deflection angles. This set of deflection angles is input into Eqn. (6.28) to obtain a set of translational forces required. In addition, the deflection along the z-axis and the maximum stress experience by the flexures can also be obtained from those deflection angles based on Eqn. (6.31) and (6.33) respectively. Numerical simulations through ANSYS 10 simulator are also used in this parametric analysis. The element used is SHELL63, $E = 71 \text{ GPa}$ for the aluminum material and poisson ratio = 0.33. A large displacement analysis is used for a non-linear analysis with mesh size = 1 (finest). Using a set of translational forces obtained from Eqn. (6.28), the deflections along the x- or y-axis, the deflections along the z-axis and the maximum stress experience by the flexure obtained from the Finite-Element-Analysis (FEA) are recorded.

The analytical and FEA results for all four cases are listed from Table 6.4 to 6.7. A quick comparison shows that the results obtained from the analytical and FEA analyses are consistent for all cases. Assuming that the FEA simulations are accurate, this comparison shows that the derived stiffness models are accurate. The parameters used in Case A1 give a decent deflection stiffness as less than 10 N of force is required to achieve a deflection of $900 \mu\text{m}$ with stress well below 250 MPa. In Case A2, an increase in width only increases the deflection stiffness. Similarly, Case A3, which has a reduced rigid member, produces high deflection stiffness. As high stiffness is undesirable, the parameters of both case A2 and A3 will not be used. In Case A4, a decrease of thickness from 0.5 mm to 0.3 mm has shown significant reduction in the deflection stiffness. However, such a thin beam-based flexure may not be able to withstand an external imprinting force of approximately 100 N. Hence, further analysis will need to be conducted for the selection between Case A1 and A4.

Table 6.4: Analytical and FEA results for Case A1.

Force (N)	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
0.954	1.00×10^{-4}	1.000×10^{-4}	22.9	1.040×10^{-4}	1.030×10^{-4}	22.5
2.863	3.00×10^{-4}	3.000×10^{-4}	68.7	3.120×10^{-4}	3.100×10^{-4}	67.6
4.774	5.00×10^{-4}	5.000×10^{-4}	114.5	5.200×10^{-4}	5.160×10^{-4}	113.0
6.687	7.00×10^{-4}	7.000×10^{-4}	160.4	7.290×10^{-4}	7.220×10^{-4}	158.0
8.603	9.00×10^{-4}	9.000×10^{-4}	206.2	9.370×10^{-4}	9.280×10^{-4}	203.0

Table 6.5: Analytical and FEA results for Case A2.

Force (N)	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
2.386	1.00×10^{-4}	1.000×10^{-4}	22.9	1.040×10^{-4}	1.030×10^{-4}	22.5
7.159	3.00×10^{-4}	3.000×10^{-4}	68.7	3.120×10^{-4}	3.100×10^{-4}	67.6
11.935	5.00×10^{-4}	5.000×10^{-4}	114.5	5.200×10^{-4}	5.160×10^{-4}	113.0
16.718	7.00×10^{-4}	7.000×10^{-4}	160.4	7.280×10^{-4}	7.220×10^{-4}	158.0
21.508	9.00×10^{-4}	9.000×10^{-4}	206.2	9.360×10^{-4}	9.280×10^{-4}	203.0

Table 6.6: Analytical and FEA results for Case A3.

Force (N)	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
4.976	1.00×10^{-4}	2.333×10^{-4}	59.7	1.038×10^{-4}	2.334×10^{-4}	57.8
9.956	2.00×10^{-4}	4.666×10^{-4}	119.4	2.077×10^{-4}	4.680×10^{-4}	116.0
12.447	2.50×10^{-4}	5.833×10^{-4}	149.3	2.596×10^{-4}	5.858×10^{-4}	145.0
14.941	3.00×10^{-4}	6.999×10^{-4}	179.2	3.116×10^{-4}	7.039×10^{-4}	173.0
17.437	3.50×10^{-4}	8.166×10^{-4}	209.1	3.637×10^{-4}	8.223×10^{-4}	202.0
19.935	4.00×10^{-4}	9.332×10^{-4}	239.0	4.158×10^{-4}	9.409×10^{-4}	231.0

Table 6.7: Analytical and FEA results for Case A4.

Force (N)	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
1.031	5.00×10^{-4}	5.000×10^{-4}	68.7	5.110×10^{-4}	5.070×10^{-4}	67.3
1.548	7.50×10^{-4}	7.500×10^{-4}	103.1	7.680×10^{-4}	7.600×10^{-4}	101.0
2.066	1.00×10^{-3}	1.000×10^{-3}	137.5	1.020×10^{-3}	1.013×10^{-3}	135.0
2.585	1.25×10^{-3}	1.250×10^{-3}	171.9	1.280×10^{-3}	1.265×10^{-3}	168.0
3.106	1.50×10^{-3}	1.500×10^{-3}	206.4	1.536×10^{-3}	1.517×10^{-3}	202.0

6.6.10 Angular Motion About X- or Y-Axis Due to An External Moment

An external moment, M , is applied to the proposed 5-DOF compliant joint that can be expressed as a force with a moment arm, which in this case

$$M = F_v \times R \tag{6.76}$$

where F_v represents the vertical input force.

Based on this derived moment, the deflection angle about the x- or y-axis, i.e., θ_x or θ_y , is obtained from Eqn. (6.38). This deflection angle is also useful for identifying the deflection along the x- or y-axis, Δ_i , the deflection along the z-axis, Δ_z , and the maximum bending stress, σ , from Eqn. (6.43), (6.45) and (6.40) respectively. Similarly, four combinations of parameters are used in this analysis:

Case B1: $R = 17.5$ mm, $L = 15$ mm, $l = 5$ mm, $H = 0.5$ mm, width = 10 mm

Case B2: $R = 17.5$ mm, $L = \underline{5 \text{ mm}}$, $l = 5$ mm, $H = 0.5$ mm, width = 10 mm

Case B3: $R = \underline{12.5 \text{ mm}}$, $L = 15$ mm, $l = 5$ mm, $H = 0.5$ mm, width = 10 mm

Case B4: $R = 17.5$ mm, $L = 15$ mm, $l = 5$ mm, $H = \underline{0.3 \text{ mm}}$, width = 10 mm

For each combination, a set of vertical forces are used to obtain the deflections along all axes and maximum bending stress from both the analytical and FEA analyses. All these results obtained from both analyses on all combinations are listed from Table 6.8 to 6.11.

Based on the comparisons between both analytical and FEA results, the derived angular stiffness models have produced good approximations on both the angular motions and stress of the proposed 5-DOF compliant joint. In a similar behavior observed in previous analysis, the parameters used in Case B2 and B3 have produce high stiffness, which is undesirable for the proposed joint. Once again, the parameters used in both Case B1 and B4 have offer a decent angular stiffness, which is desirable for the proposed 3-DOF FPM to realize large θ_x - θ_y - Z motions.

Table 6.8: Analytical and FEA results for Case B1.

F_v	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
2.0	2.070×10^{-4}	2.070×10^{-4}	42.0	2.160×10^{-4}	2.160×10^{-4}	42.5
4.0	4.140×10^{-4}	4.140×10^{-4}	84.0	4.310×10^{-4}	4.310×10^{-4}	85.0
6.0	6.210×10^{-4}	6.210×10^{-4}	126.0	6.470×10^{-4}	6.470×10^{-4}	128.0
8.0	8.280×10^{-4}	8.280×10^{-4}	168.0	8.620×10^{-4}	8.620×10^{-4}	170.0
10.0	1.035×10^{-3}	1.035×10^{-3}	210.0	1.078×10^{-3}	1.078×10^{-3}	213.0

Table 6.9: Analytical and FEA results for Case B2.

F_v	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
2.0	8.873×10^{-5}	2.070×10^{-4}	42.0	9.346×10^{-5}	2.182×10^{-4}	42.3
4.0	1.775×10^{-4}	4.140×10^{-4}	84.0	1.869×10^{-4}	4.365×10^{-4}	84.7
6.0	2.662×10^{-4}	6.210×10^{-4}	126.0	2.804×10^{-4}	6.547×10^{-4}	127.0
8.0	3.548×10^{-4}	8.279×10^{-4}	168.0	3.739×10^{-4}	8.730×10^{-4}	169.0
10.0	4.434×10^{-4}	1.035×10^{-3}	210.0	4.673×10^{-4}	1.091×10^{-3}	212.0

Table 6.10: Analytical and FEA results for Case B3.

F_v	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
5.0	3.697×10^{-4}	2.641×10^{-4}	75.0	3.869×10^{-4}	2.765×10^{-4}	75.6
7.0	5.175×10^{-4}	3.697×10^{-4}	105.0	5.417×10^{-4}	3.871×10^{-4}	106.0
9.0	6.653×10^{-4}	4.752×10^{-4}	135.0	6.965×10^{-4}	4.976×10^{-4}	136.0
11.0	8.131×10^{-4}	5.808×10^{-4}	165.0	8.513×10^{-4}	6.082×10^{-4}	166.0
13.0	9.608×10^{-4}	6.863×10^{-4}	195.0	1.006×10^{-3}	7.188×10^{-4}	196.0

Table 6.11: Analytical and FEA results for Case B4.

F_v	Analytical results			FEA results		
	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)	x- or y-deflection Δ (m)	z-axis deflection Δ_z (m)	Stress σ (MPa)
0.2	9.585×10^{-5}	9.585×10^{-5}	11.7	9.681×10^{-5}	9.681×10^{-5}	11.5
1.0	4.792×10^{-4}	4.792×10^{-4}	58.3	4.840×10^{-4}	4.841×10^{-4}	57.5
1.8	8.624×10^{-4}	8.623×10^{-4}	105.0	8.712×10^{-4}	8.713×10^{-4}	104.0
2.6	1.245×10^{-3}	1.245×10^{-3}	151.7	1.259×10^{-3}	1.259×10^{-3}	150.0
3.4	1.627×10^{-3}	1.627×10^{-3}	198.3	1.646×10^{-3}	1.646×10^{-3}	196.0

6.6.11 Critical Loading on the Beam-Based Flexure Joints

Based on the previous two parametric analyses, both flexure thicknesses of 0.3 mm and 0.5 mm produce a decent deflection and angular stiffness for the proposed 5-DOF compliant joint. In this section, two beam-based flexure joints with 0.3 mm and 0.5 mm of thickness respectively underwent a critical loading analysis based on Eqn. (6.37), a compressive stress analysis based on Eqn. (6.35) and a displacement in the z-axis analysis based on Eqn. (6.34). Based on Eqn. (6.37) with $l = 5$ mm and $w = 10$ mm, the results obtained are listed in Table 6.12.

Table 6.12: List of results obtained from flexure thickness selection.

Flexure thickness (mm)	Critical loading (N)	Compressive stress subjected to 100N (MPa)	Displacement in z-axis when subjected to 100N (m)
0.3	157.667	33.33	0.704×10^{-6}
0.5	729.939	20.00	0.117×10^{-6}

The results have shown that both beam-based flexure thicknesses can withstand an axial loading of 100 N as their critical loadings are higher than 100 N. When subjected to an axial loading of 100 N, the axial stress concentration for both flexures is very low too. However, the beam-based flexure with 0.5 mm of thickness has smaller change in the axial displacement as compared to the beam-based flexure with thickness of 0.3 mm. As all three analyses have shown that the beam-based flexure joint with a thickness of 0.5 mm has higher performance, this thickness is selected for developing the beam-based flexure joints in segment 1 and 2 along with a length of 5 mm and a width of 10 mm.

6.6.12 Angular Motion About Z-Axis of Proposed 5-DOF Compliant Joint Due to An External Moment

An external moment that causes an angular motion about the z-axis of the proposed 5-DOF compliant joint can be seen as a force-couple given in Eqn. (6.47) (Fig. 6.18). In this section, the resultant displacement in the x- and y-axes will be used to evaluate the angular stiffness about the z-axis because the resultant displacement can be used to determine the deflection angle, θ_z , based on Eqn. (6.75). Consequently, the resultant displacements in the x- and y-axes are a combination of the displacements from all three segments and can be expressed in Eqn. (6.74). It is also assumed that the maximum stress, expressed in Eqn. (6.69), will be experienced by the beam-based flexure joints in segment 1 or 2.

Figure 6.18: Proposed 5-DOF compliant joint subjected to an external moment about z-axis.

Figure 6.19: FEA modeling of the proposed 5-DOF compliant joint for external moment about the z-axis analysis.

Using a 3D FEA model of a 5-DOF compliant joint (Fig. 6.19), an evaluation of the accuracy of the analytical model is also conducted. For the FEA model, the rigid member that connects the beam-based flexure joints has a length of 15 mm and a diameter of 35 mm. The beam-based flexure length, width and thickness are 5 mm, 10 mm and 0.5 mm respectively. The element used is SOLID92, $E = 71 \text{ GPa}$ and poisson ratio = 0.33. A large displacement analysis is used for a non-linear analysis with mesh size = 5. In this evaluation, six different forces are used to generate the desired torque, while the resultant displacements and the maximum stress generated from all these forces are listed in Table 6.13 to 6.14.

Table 6.13: Resultant displacements obtained from the analytical and FEA analyses on the 5-DOF compliant joint subjected to external moment about the z-axis.

Force (N)	Analytical results: Deflection along z-axis (m)	FEA results: Deflection along z-axis (m)	Error (%)
5	$4.68789938 \times 10^{-5}$	$4.74050000 \times 10^{-5}$	1.12
10	$9.37579877 \times 10^{-5}$	$9.48100000 \times 10^{-5}$	1.12
15	$1.40636982 \times 10^{-4}$	$1.42210000 \times 10^{-4}$	1.12
20	$1.87515975 \times 10^{-4}$	$1.89620000 \times 10^{-4}$	1.12
25	$2.34394969 \times 10^{-4}$	$2.37020000 \times 10^{-4}$	1.12
30	$2.81273963 \times 10^{-4}$	$2.84430000 \times 10^{-4}$	1.12

Table 6.14: Maximum stress obtained from the analytical and FEA analyses on the 5-DOF compliant joint subjected to external moment about the z-axis.

Force (N)	Analytical results: Maximum stress (MPa)	FEA results: Maximum stress (MPa)	Error (%)
5	50.0	50.3	0.60
10	100.0	101.0	1.00
15	150.0	151.0	0.67
20	200.0	201.0	0.50
25	250.0	251.0	0.40
30	300.0	302.0	0.67

The FEA results show that the proposed 5-DOF compliant joint produces a compliance of 0.173 rad/N-m. Such compliance is sufficient in achieving large displacements. An average error of 1.12 % is obtained when compared between the analytical and FEA results on the resultant displacements in the x- and y-axes. In addition, the errors between the analytical and FEA results on the maximum stress are less than 1 %. The low percentage errors have indicated that the analytical models can predict the displacements and stress with high accuracy. Subsequently, substituting these derived displacements to Eqn. (6.75) will result in an accurate prediction of the deflection angle.

6.7 Stiffness Modeling of the Proposed 3PPS FPM

With all the stiffness formulated and the parameters selected for each compliant joint of the proposed PPS limb, a stiffness model of the proposed 3PPS FPM can be derived. A FPM comprises of a mobile platform supported by j number of parallel and symmetrical limbs.

Figure 6.20: (a) A limb, which is constructed from a serially-connected compliant joints, is used to (b) form each symmetrical limb of a parallel-kinematics manipulator.

Each limb is formed by a group of flexure joint connected in series by the links (Fig. 6.20a). Here, only the flexure joint is assumed to be a compliant member with a compliant matrix, C_i , established at the local coordinate frame attached to it. To establish the compliant matrix of each limb, C_{limb} , a Jacobian matrix, J_i , that maps the local coordinate frame of each compliant joint to the local coordinate frame of the tip of each limb is required. Each Jacobian matrix is determined by the exact change in displacements and orientations due to the elastic deformation of each compliant joint, and not through the conventional kinematic assumptions that the links and joints are just the projection of the end-effector motion from the actuator. Consequently, the compliance matrix of each limb is given as

$$C_{limb} = \sum_{i=1}^n J_i C_i J_i^T \quad (6.77)$$

On the other hand, the mobile platform, which is supported by the j number of limbs, is considered to be infinitely rigid. However, the local coordinate frame of the tip of each limb may have a linear displacement and orientation with respect to the reference coordinate frame established at the end-effector of the parallel manipulator, which is located at the center of the mobile platform (Fig. 6.20b). Hence, a Jacobian matrix, J_j , that maps the local coordinate frame of each limb, which is attached to the tip, to this reference coordinate

frame is required. The stiffness matrix of the parallel-kinematics manipulator is thus given as

$$\mathbf{K} = \sum_{j=1}^n \mathbf{J}_j^{-T} \mathbf{K}_{limb,j} \mathbf{J}_j^{-1} \quad (6.78)$$

where $\mathbf{K}_{limb,j} = \mathbf{C}_{limb,j}^{-1}$

6.7.1 Compliance Matrix of Each Parallel Limb

Prior to the establishment of the compliance matrix of each limb, let index A, B, and C, be the representations of the active compliant prismatic joint, the passive compliant prismatic joint, and the 5-DOF compliant joint respectively. The local coordinate frame is attached to the center of each compliant joint and all local coordinate frames are inline along the z-axis with an offset with respect to the local coordinate frame at the tip (Fig. 6.21).

Figure 6.21: Local coordinate frame attached to individual compliant joint and the displacement with respect to the local coordinate frame attached to the tip of each limb.

6.7.1.1 Active Compliant Prismatic Joint

From Eqn. (6.22), the compliance of the active compliant prismatic joint in the z-axis is expressed as

$$C_A^z = \frac{1}{K_{FBM}^z} \quad (6.79)$$

Assuming that other axes have high stiffness and can be ignored, the 6×6 compliance matrix of the active compliant prismatic joint, $\mathbf{C}_A$, is given as

$$\mathbf{C}_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_A^z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6.80)$$

6.7.1.2 Passive Compliant Prismatic Joint

From Eqn. (6.25), the compliance of the passive compliant prismatic joint in the z-axis is expressed as

$$C_B^x = \frac{1}{K_{prismatic}^x} \quad (6.81)$$

and from Eqn. (6.26), the compliance of the passive compliant prismatic joint in the z-axis can be expressed as

$$C_B^z = \frac{1}{K_{prismatic}^z} \quad (6.82)$$

In this analysis, the stiffness in the x- and z-axes will be the main consideration while other axes are assumed to have high stiffness and can be ignored. Hence, the 6×6 compliance matrix of the passive compliant prismatic joint, $\mathbf{C}_B$, is given as

$$\mathbf{C}_B = \begin{bmatrix} C_B^x & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_B^z & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (6.83)$$

6.7.1.3 Passive 5-DOF Compliant Joint

In this analysis, the stiffness of the passive 5-DOF compliant joint in all axes will be considered and the 6×6 compliance matrix of this joint, $\mathbf{C}_C$, is given as

$$\mathbf{C}_C = \begin{bmatrix} C_C^x & 0 & 0 & 0 & 0 & 0 \\ 0 & C_C^y & 0 & 0 & 0 & 0 \\ C_C^{Fx,z} & C_C^{Fy,z} & C_C^z & C_C^{Mx,z} & C_C^{My,z} & 0 \\ 0 & 0 & 0 & C_C^{\theta x} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_C^{\theta y} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_C^{\theta z} \end{bmatrix} \quad (6.84)$$

Within this matrix, C_C^x and C_C^y are denoted as the compliance in the x- and y-axes. Based on Eqn. (6.30), both representations are given as

$$C_C^m = \frac{1}{K_{F,\Delta_n}} \quad (6.85)$$

where n is either x or y . In addition, C_C^z is denote as the compliance in the z-axis. Based on Eqn. (6.36), it is given as

$$C_C^z = \frac{1}{K_{P,\delta_z}} \quad (6.86)$$

Within this matrix, $C_C^{\theta x}$ and $C_C^{\theta y}$ are denoted as the rotational compliance about the x- and y-axes. Based on Eqn. (6.38), both representations are given as

$$C_C^{\theta n} = \frac{l}{EI} \quad (6.87)$$

where n is either x or y .

On the other hand, $C_C^{\theta z}$ is denoted as the rotational compliance about the z-axis. Based on Eqn. (6.75), it is given as

$$C_C^{\theta z} = \frac{\theta z}{M} \quad (6.88)$$

The coupling effects of the passive 5-DOF compliant joint can be identified within this compliance matrix. Here, the translation force in the x-axis not only causes a deflection along the x-axis, it also causes a displacement change along the z-axis. Similarly, Eqn. (6.84) also shows that a translation force in the y-axis will cause a deflection along both the y-axis and a displacement change along the z-axis. Such two degrees of motions are also exhibited when the compliant joint is subjected to an external moment about the x- and y-axes.

However, Eqn. (6.84) is unlike the compliance matrix, which represents the conventional notch-type flexure joints [74]. For conventional notch-type flexure joints with limited deflections, the angular motions caused by the translation forces and the external moments are assumed to be similar based on the small deflection theorem. Yet for the beam-based flexure joints, the angular motions caused by the translation forces are different to those caused by the external moments during large deflection (refer to Section 3.2.1 for more details). Hence for better representation on the compliance of the passive 5-DOF compliant joint, each displacement change along the z-axis represents the additional degree of motion resulted from each translation force and moment.

Here, $C_C^{Fx,z}$ and $C_C^{Fy,z}$ are denoted as the compliance in the z-axis due to the translation force in the x- and y-axes respectively. Based on Eqn. (6.31), both representations are given as

$$C_C^{m,z} = \frac{R \sin \alpha}{2i} \quad (6.89)$$

where i is either Fx or Fy .

In addition, $C_C^{Mx,z}$ and $C_C^{My,z}$ are denoted as the compliance in the z-axis due to the moment about the x- and y-axes respectively. Based on Eqn. (6.45), both representations are given as

$$C_C^{j,z} = \frac{R \sin \theta}{2j} \quad (6.90)$$

where j is either Mx or My .

6.7.1.4 Jacobian Matrix

The local coordinate frame attached to each compliant joint has the same orientation but has an offset with respect to the local coordinate frame attached to the tip of the limb. Hence, the Jacobian matrix for each compliant joint, $\mathbf{J}_m$ is given as

$$\mathbf{J}_m = \begin{bmatrix} 1 & 0 & 0 & 0 & h_m & 0 \\ 0 & 1 & 0 & -h_m & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (6.91)$$

where m represents A, B or C for respective compliant joint.

6.7.1.5 Compliance Matrix of Each Limb

The compliance matrices $\mathbf{C}_A$, $\mathbf{C}_B$ and $\mathbf{C}_C$ of the active compliant prismatic joint, passive compliant prismatic joint and 5-DOF compliant joint respectively can be assembled to form an 18×18 matrix as

$$\mathbf{C}_{\text{total}} = \text{diag}(\mathbf{C}_A, \mathbf{C}_B, \mathbf{C}_C) \quad (6.92)$$

On the other hand, the Jacobian matrices, $\mathbf{J}_A$, $\mathbf{J}_B$ and $\mathbf{J}_C$, can be assembled as a 6×18 matrix as

$$\mathbf{J}_{\text{limb}} = [\mathbf{J}_A \quad \mathbf{J}_B \quad \mathbf{J}_C] \quad (6.93)$$

Based on Eqn. (6.77), the compliance matrix of each limb is determined as

$$\begin{aligned} \mathbf{C}_{\text{limb}} &= \mathbf{J}_{\text{limb}} \mathbf{C}_{\text{total}} \mathbf{J}_{\text{limb}}^T \\ &= \begin{bmatrix} C_B^x + C_C^x + C_C^{\theta y}(h_C)^2 & 0 & 0 & 0 & C_C^{\theta y}(h_C) & 0 \\ 0 & C_C^y + C_C^{\theta x}(h_C)^2 & 0 & -C_C^{\theta x}(h_C) & 0 & 0 \\ C_C^{F_x, z} + C_C^{M_y, z}(h_C) & C_C^{F_y, z} - C_C^{M_x, z}(h_C) & C_A^z + C_B^z + C_C^z & C_C^{M_x, z} & C_C^{M_y, z} & 0 \\ 0 & -C_C^{\theta y}(h_C) & 0 & C_C^{\theta x} & 0 & 0 \\ C_C^{\theta y}(h_C) & 0 & 0 & 0 & C_C^{\theta y} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_C^{\theta z} \end{bmatrix} \end{aligned} \quad (6.94)$$

6.7.2 Stiffness Matrix of the Proposed FPM

Prior to the derivation of stiffness matrix, the Jacobian matrix, $\mathbf{J}_j$, that maps the local coordinate frame of each limb, to this reference coordinate frame must first be established.

Figure 6.22: Local coordinate frame at the tip of each limb with respect to the reference coordinate frame at the center of the mobile platform.

Figure 6.22 illustrates that the local coordinate frame attached to the tip of each limb has different orientations and a linear displacement with respect to the reference coordinate frame attached to the center of the mobile platform. Here the reference coordinate frame attached to the center of the mobile platform falls on the same plane as the local coordinate frame at the tip of each limb. Hence, there is no variation in the z-axis between both coordinate frames. The orientation of the local coordinate frame at the tip of each limb with respect to the reference coordinate frame can be expressed as

$$\begin{aligned} X &= x_j \cos \psi_j + y_j \cos(\psi_j + 90^\circ) \\ Y &= x_j \sin \psi_j + y_j \sin(\psi_j + 90^\circ) \end{aligned} \quad (6.95)$$

where index $j = 1, 2$ or 3 .

By representing Eqn. (6.95) in matrix form, the rotation matrix is given as

$$\begin{bmatrix} \cos \psi_j & \cos(\psi_j + 90^\circ) \\ \sin \psi_j & \sin(\psi_j + 90^\circ) \end{bmatrix} \quad (6.96)$$

The relationship between the angular displacements at the end-effector of the FPM and the changes in displacements at the tip of each limb is

$$\begin{aligned} \begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} &= \begin{bmatrix} \delta \theta_x \\ \delta \theta_y \\ \delta \theta_z \end{bmatrix} \times \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \\ &= \begin{bmatrix} 0 & r_z & -r_y \\ -r_z & 0 & r_x \\ r_y & -r_x & 0 \end{bmatrix} \begin{bmatrix} \delta \theta_x \\ \delta \theta_y \\ \delta \theta_z \end{bmatrix} \end{aligned} \quad (6.97)$$

where $[r_x, r_y, r_z]$ is the displacement vector between the local coordinate frame and the reference coordinate frame.

Based on Eqn. (6.96) and (6.97), the Jacobian matrix that maps the local coordinate frame of each limb to the reference coordinate frame is given as

$$\mathbf{J}_j = \begin{bmatrix} \cos \psi_j & \cos(\psi_j + 90^\circ) & 0 & 0 & 0 & -r \sin \theta_j \\ \sin \psi_j & \sin(\psi_j + 90^\circ) & 0 & 0 & 0 & r \cos \theta_j \\ 0 & 0 & 1 & r \sin \theta_j & -r \cos \theta_j & 0 \\ 0 & 0 & 0 & \cos \psi_j & \cos(\psi_j + 90^\circ) & 0 \\ 0 & 0 & 0 & \sin \psi_j & \sin(\psi_j + 90^\circ) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (6.98)$$

Consequently, based on Eqn. (6.78), (6.95) and (6.98), the stiffness matrix of the proposed 3PPS FPM can be written as

$$\mathbf{K} = \sum_{j=1}^3 \mathbf{J}_j^{-T} \mathbf{K}_{limb} \mathbf{J}_j^{-1} \quad (6.99)$$

where $\mathbf{K}_{limb} = \mathbf{C}_{limb}^{-1}$.

6.8 Final Design of the Proposed 3PPS FPM

6.8.1 Removal of Proposed Passive Compliant Prismatic Joint

The stiffness analysis conducted in previous section, gives an overall assessment of the compliance of each limb. Based on the compliance matrix in Eqn. (6.95), the first element that is given as

$$C_B^x + C_C^x + C_C^{\theta y}(h_C)^2 \quad (6.100)$$

Equation (6.100) shows that the main compliant joints that cause a displacement in the x-axis is the passive compliant prismatic and 5-DOF joints. Based on the selected beam-based flexure joint dimensions for each joint and a desired displacement of 71 μm , the passive compliant prismatic joint requires 3.23 N of force while the passive 5-DOF compliant joint only require 0.954 N of force. These results conducted in earlier sections show that the 5-DOF compliant joint has higher translational compliance. When connected in series, higher compliance will dominate the motion along the axis. Thus, the proposed passive compliant prismatic joint is taken out while allowing the deflection along the x-axis of each passive 5-DOF compliant joint to realize the desired passive prismatic motion (note: one degree of motion is eliminated by the constraint arrangement of the 3PPS FPM). Hence, the new compliance matrix of each limb is given as

$$\mathbf{C}_{limb} = \begin{bmatrix} C_C^x + C_C^{\theta y}(h_C)^2 & 0 & 0 & 0 & C_C^{\theta y}(h_C) & 0 \\ 0 & C_C^y + C_C^{\theta x}(h_C)^2 & 0 & -C_C^{\theta x}(h_C) & 0 & 0 \\ C_C^{F_{x,z}} + C_C^{M_{y,z}}(h_C) & C_C^{F_{y,z}} - C_C^{M_{x,z}}(h_C) & C_A^z + C_C^z & C_C^{M_{x,z}} & C_C^{M_{y,z}} & 0 \\ 0 & -C_C^{\theta y}(h_C) & 0 & C_C^{\theta x} & 0 & 0 \\ C_C^{\theta y}(h_C) & 0 & 0 & 0 & C_C^{\theta y} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_C^{\theta z} \end{bmatrix} \quad (6.101)$$

Here, Eqn. (6.101) shows that the compliance in the z-axis is currently represented by $C_A^z + C_C^z$. Hence, a lower compliance in the z-axis leads to an increase in stiffness in the z-axis, which is desirable for withstanding the imprinting load.

6.8.2 Removal of Segment 3 of the Passive 5-DOF Compliant Joint

The proposed 3PPS FPM is modeled in the FEA simulator (ANSYS 10) for final evaluation of the selected parameters for all the proposed active and passive compliant joints. The full-scale FEA model is formed by SOLID92 element type with the mesh-size selected as 1 (finest), Young's Modules set as 71GPa and Poisson ratio chosen as 0.33. Based on the forward kinematics analysis, the end-effector reaches 2.9° about the x-axis with limb 1 = 2.5 mm, limb 2 = 2.5 mm and limb 3 = -2.5 mm, while it only reaches 2.1° about the y-axis with limb 1 = 2.5 mm, limb 2 = -2.5 mm, and limb 3 = 0 mm. As the proposed active compliant joints are designed with a maximum limit of ±3 mm, the end-effector can achieves a higher orientation about the y-axis. By simulating the FEA model under this condition, i.e., limb 1 = 3 mm, limb 2 = -3 mm, and limb 3 = 0 mm, all proposed compliant joints may experience the maximum deflections and stresses.

Figure 6.23: FEA simulation of the proposed 3PPS with the variation of limb 1 and 2 set at -3 mm and +3 mm respectively.

Figure 6.23 shows the FEA simulation of the proposed 3PPS FPM with the displacement of limb 1 and 2 set at -3 mm, and +3 mm respectively. It shows that the maximum and minimum displacements of the mobile platform in the z-axis are ±3.197 mm and thus an orientation greater than 2.1° about the y-axis can be achieved. The additional displacement

of 0.197 mm is contributed by the rotation of proposed 5-DOF compliant joint. Based on the results listed in Table 6.8, the deflections along the x- or y-axis and the z-axis are determined from such rotations. From Eqn. (6.45), the spatial joint is experiencing a rotation of 0.678° under the simulated condition that produces a deflection about 0.2 mm along the z-axis.

Figure 6.24: Stress concentration of the proposed 5-DOF compliant joint of limb 2 during the simulation with the displacement of limb 1 and 2 set at -3 mm and +3 mm respectively.

Figure 6.24 shows the stress concentration of the 5-DOF compliant joint, where the blue color represents the minimum stress concentration while the red color represents the maximum stress concentration. A closer examination shows that no stress is registered in the flexure joints in segment 3. This suggests that the segment 3 has no deflection and does not contribute to the angular motion about the z-axis for the proposed compliant joint. It appears that the required angular motion about the z-axis for the proposed compliant joint can be achieved from segment 1 and 2 alone. This observation is validated by the stress concentration registered in the flexure joints in segment 2, i.e., 60 MPa. Based on a deflection of about 0.2 mm in the z-axis, the flexure joints in segment 1 should be experiencing a stress of 42 MPa. With the stress concentration registered in the flexure joints in segment 1 at about 100 MPa, this shows that segment 1 contributes to the angular motion about the z-axis due to the additional 60 MPa. Based on Eqn. (6.74), the total angular motion about the z-axis is contributed by all three segments of the proposed 5-DOF compliant joint. From Eqn.

(6.63) and (6.72), the angular motion about the z-axis, θ_z , from each individual segment is obtained and listed in Table 6.15.

Table 6.15: Angular motion about the z-axis obtained from segments 1 and 2, and from segment 3 of the 5-DOF compliant joint.

Angular motion about the z-axis from segments 1 and 2 (m)	Angular motion about the z-axis from segment 3 (m)	Total angular motion about the z-axis (m)	Ratio (Top:Bottom)
3.756×10^{-5}	9.317×10^{-6}	4.688×10^{-5}	1 : 0.25

Results listed in Table 6.15 show that the angular motion about the z-axis from segment 1 and 2 is four times larger than the angular motion about the z-axis from segment 3. To reduce the angular stiffness about the z-axis of segment 3, a parametric analysis is conducted by changing the dimensions of the flexure joint width w , thickness, t , length, l , or the rigid-link length, L . For each configuration, Eqn. (6.72) is used to obtain the angular motion about the z-axis from segment 3 and compared against the angular motions about the z-axis from segment 1 and 2. These results are listed in Table 6.16 and the various configurations used in this analysis are listed below:

Case C1: $w = \underline{5 \text{ mm}}$, $t = \underline{0.3 \text{ mm}}$, $l = 0.5 \text{ mm}$, and $L = 10 \text{ mm}$

Case C2: $w = 10 \text{ mm}$, $t = 0.5 \text{ mm}$, $l = \underline{2 \text{ mm}}$, and $L = \underline{20 \text{ mm}}$

Case C3: $w = 10 \text{ mm}$, $t = \underline{0.3 \text{ mm}}$, $l = \underline{2 \text{ mm}}$, and $L = 10 \text{ mm}$

Case C4: $w = 10 \text{ mm}$, $t = \underline{0.3 \text{ mm}}$, $l = \underline{2 \text{ mm}}$, and $L = \underline{15 \text{ mm}}$

The configurations used in all cases have shown significant effects in reducing the angular stiffness about the z-axis of segment 3. For Case C2 to C4, the angular motion about the z-axis of segment 3 is at least four times larger than the angular displacement from segment 1 and 2. Such performances are essential for segment 3 since its main function is to provide the desired angular motion about the z-axis when the angular stiffness about the z-axis of segment 1 and 2 is too high to achieve it. Nevertheless, current investigations have shown that segment 3 of the passive 5-DOF compliant joint is redundant in achieving the targeted

Table 6.16: Angular motion obtained from segments 1 and 2, and from segment 3 of the 5-DOF compliant joint.

Case	Angular motion about the z-axis from segments 1 and 2 (m)	Angular motion about the z-axis from segment 3 (m)	Total angular motion about the z-axis (m)	Ratio (Top:Bottom)
C1	3.756×10^{-5}	8.627×10^{-5}	1.238×10^{-4}	1 : 2.30
C2	3.756×10^{-5}	1.636×10^{-4}	2.012×10^{-4}	1 : 4.36
C3	3.756×10^{-5}	2.254×10^{-4}	2.629×10^{-4}	1 : 6.00
C4	3.756×10^{-5}	4.523×10^{-4}	4.898×10^{-4}	1 : 12.04

workspace. Analytical and FEA results have verified that the angular motion about the z-axis from segment 1 and 2 is sufficient for the proposed 3PPS FPM to achieve the desired orientation. For this particular 3PPS FPM and its targeted workspace, segment 3 of passive 5-DOF compliant joint can be removed.

6.8.3 Final Design of the Proposed 3PPS FPM

Figure 6.25: Final design of the proposed 3PPS FPM.

With the passive compliant prismatic joint removed, each moving air-core coil, which is supported by the active compliant prismatic joint, will be connected to a passive 5-DOF

compliant joint that only consists segment 1 and 2. As a constraint arrangement is imposed to eliminate one degree of motion from each 5-DOF compliant joint, each limb will have one active and four passive motions while three symmetrical limbs in parallel arrangement will give three degrees of controllable motions to the mobile platform as shown in Fig. 6.25.

6.9 Summary

A new 3PPS FPM is introduced in this chapter. This 3PPS FPM is proposed to realize a θ_x - θ_y - Z motion with a targeted orientation and displacement of $\pm 2^\circ$ and ± 2.5 mm respectively. A complete forward kinematic analysis on the 3PPS parallel-kinematics configuration is established and is useful for workspace analysis. Here, each symmetrical PPS limb is formed by serially-connected compliant joints, which are proposed, designed and analyzed. In addition, a new form of 5-DOF compliant joint is proposed. Such a 5-DOF compliant joint offers larger deflections and orientations with lower driving stiffness as compared to a conventional elementary rod flexure joint. This 5-DOF compliant joint is essential in developing a class of spatial-motion FPMs that targets large workspace of few millimeters and degrees. In this chapter, the analytical modeling on the stiffness of each proposed compliant joint and the stiffness modeling of the proposed 3PPS FPM are also conducted. Through the stiffness and FEA analyses, the redundant portions from the initial proposed FPM have been systematically removed and thus reducing it to a more compact design. The final form of the proposed 3PPS FPM will be used as an active co-planar nano-alignment manipulator that will be discussed in the following chapter.

Chapter 7

Prototype of A Co-Planar Nano-Alignment Manipulator

“The only justification for our concepts and systems of concepts is that they serve to represent the complexity of our experiences; beyond this they have no legitimacy.”

– Albert Einstein (1879 – 1955).

The 3PPS FPM proposed in Chapter 6 is employed to develop a 3-DOF electromagnetically-driven FPM. Termed Flexure-Based Electromagnetic Parallel-Kinematics Manipulator (FEPM), the development of the FEPM prototype and the evaluations of its capabilities will be discussed in this chapter.

7.1 Prototype Development

The proposed 3-DOF 3PPS FEPM comprises of three symmetric parallel limbs, where each limb consists of an active compliant prismatic joint and a passive 5-DOF compliant joint. As a constraint arrangement is imposed to eliminate one degree of motion from each 5-DOF compliant joint, each limb will have one active and four passive motions while three symmetrical limbs in parallel arrangement will give three degrees of controllable motions to the mobile platform. Here, both compliant joints are monolithically-cut to minimize the assembly errors. As these compliant joints form the main frame of the proposed FEPM and integrate with the electromagnetic driving scheme, the selection of material for these joints becomes essential as it may affect the performances and capabilities of the FEPM.

7.1.1 Materials Selection

7.1.1.1 Mechanical Properties

As all compliant joints are designed to achieve large deflections, a high yield strength, S_y , becomes the most important selection factor. These joints are also required to operate within the elastic region to produce highly repeatable and predictable motions. Hence, a good Young's Modulus, E , is necessary. In addition, it is often desirable for the material to permit the largest deflection within the limits of its yield strength. Consequently, S_y/E must also be high. However, high S_y/E will lead to low stiffness and subsequently low natural frequency. To increase the natural frequency, these compliant joints must also be light and thus material density, ρ , needs to be low.

7.1.1.2 Thermal Properties

Thermal disturbance can be contributed either by environmental changes or by localized internal sources such as electronic motors etc. Assumption that a body with moderate size will remain in thermal equilibrium with its surroundings, the moving air-core coil, which is a heat dissipative device, can be conducting an approximately steady heat flow. Under this condition, the linear expansion of a homogenous and isotropic material can be summarized as

$$\varepsilon = \frac{qL}{2A} \left(\frac{\alpha}{k} \right) \quad (7.1)$$

where q is a steady heat flow, L is the length of the shim, A is the cross-sectional area of the shim, α is the thermal expansion coefficient and k is the thermal conductivity. As the active compliant prismatic joints are supporting the moving air-core coil, the material must have low α/k to minimize any expansion caused by heat.

7.1.1.3 Magnetic Properties

Each pair of active compliant prismatic joint has an EDM embedded in between and each EDM, which consists of a DM configuration, will introduce magnetic flux to its surrounding.

To prevent this magnetic leakage from affecting the functionality of these compliant joints, the material use must have no response to magnetization. In theory, the relationship between the magnetization, M and the applied field, H , is given as

$$M = \chi_{mag}H.$$

(7.2)

The magnetic susceptibility, χ_{mag} , is usually used to determine the amount of magnetic effect on the non-ferrous materials. Here, these materials must have very low magnetic susceptibility in order for the compliant joints to have no response to magnetization.

Table 7.1: Properties of several metals

Materials	E (GPa)	S_y (MPa)	S_y/E ($\times 10^{-3}$)	ρ (Kg/m ³)	α/k (W/m) ($\times 10^6$)	χ_{mag} ($\times 10^{-5}$)
Aluminium (6061)	71	276	3.9	2700	45	2.2
Aluminium (7075-T6)		503	7.1			
Spring steel	210	1200	5.7	7850	75	-
Stainless steel	190	540	2.8			
Titanium (Ti-6Al-4V)	114	885	7.8	4400	87	0.4
Titanium (Ti-5Al-2Sn)		1130	9.9			

Metals listed in Table. 7.1 are used during the selection of a material that is suitable for developing the compliant joints. The titanium is a clear winner among the listed materials because of high yield strength, highest value for S_y/E and lowest magnetic susceptibility. Unfortunately, it is very expensive and possesses high values for α/k . Among the remaining two materials, steel is one the most common material used to develop the compliant joints due to its high machinability and yield strength. However, it is a ferrous magnetic material that makes it unsuitable for this work because the magnetic flux from the surroundings of the EDM will have magnetic effects on the proposed compliant joints, which may in turn affect the efficiency of these joints or the EDM. Examples include, increase in the stiffness of the active prismatic compliant joints due to magnetic attractions or interference in the magnetic distribution within the EDM resulted from additional ferrous materials, etc. On the other hand, the aluminum 7075-T6 has fulfilled all criterions due to its high yield strength, low magnetic susceptibility and lightest in weight. The S_y/E of this aluminum material is also high as compared to the steel. Most important, it has the lowest α/k , which is essential as

heat generated from the coil will have the least effect on this material. Hence, the aluminum 7075-T6 is selected for developing the proposed compliant joints.

7.1.2 Wire-Cut EDM

All proposed compliant joints are based on a single monolithic-cut design to avoid unnecessary assembly errors that may reduce the precision of these joints. To form a monolithic-cut compliant joint from a single workpiece, a wire-cut EDM process will be used. In this process, a thin single strand brass wire is fed through a workpiece, which is usually submerged in a tank of dielectric fluid. The cutting is realized by introducing electrons through the wire, which forms an electrical discharged arc between the wire and the workpiece that erodes the materials in the path of the wire (Fig. 7.1a). Advantages of the wire-cut EDM include its capability of machining complex shapes, insensitive to material hardness and offers very close tolerances cut with good surface finish. In addition, the local machining companies, which use $\varnothing 0.2\text{ mm}$ wire, are able to produce an average cutting path of $33.5\text{ }\mu\text{m}$ as the wire is usually accurately controlled with a positioning resolution of up to $4\text{ }\mu\text{m}$. Consequently, these advantages and performances suggest that this process is very suitable for fabricating the proposed monolithic-cut compliant joints.

Figure 7.1: Wire-cut EDM machining.

7.2 Active Compliant Prismatic Joint

The proposed active compliant prismatic joint is formed by a pair of symmetrical compound linear spring with the beam-based flexures joints to achieve large displacement and high repeatable motion. Using the selected material, i.e., aluminum 7075-T6, a prototype of the compound linear spring was fabricated through the wire-cut EDM process as shown in Fig. 7.2. The entire prototype is developed based on the parameters selected from the analyses conducted in Chapter 6, except that round fillets of 0.1 mm are formed at the edges of each beam-based flexure joint due to the wire, which has a diameter of 0.2 mm. Although these fillets are not considered in the design stage, their presences are essential for reducing the stress concentration at the edges of the flexure joints during actual deflection.

Figure 7.2: Prototype of the compound linear spring.

7.2.1 Stiffness Evaluation

A complete prototype of the proposed active compliant prismatic joint is formed using a pair of developed compound linear springs, which are used to support a translational mobile coil-holder. Prior to the use of these active compliant prismatic joints to form part of the proposed 3-DOF 3PPS FEPM, a displacement stiffness evaluation is conducted on the prototype. In this evaluation, an external VC actuator, which is attached with an ATI 6-DOF F/T sensor, is connected to the coil-holder as shown in Fig. 7.3. By energizing the VC actuator, a force is generated to drive the coil-holder while the displacement of the active compliant prismatic

joint is registered by a MicroE-Systems linear encoder and the generated force is measured by the F/T sensor. Most importantly, the moving air-core coil, which is attached to coil-holder via the F/T sensor, provides a non-contact actuation that increases the accuracy of this experimental setup. With a resolution of 0.01 N for the F/T sensor and a resolution of 20 nm for the encoder, the obtained measurements are sufficient for analyzing the stiffness of the prototype active compliant prismatic joint.

Figure 7.3: Displacement stiffness evaluation of an active compliant prismatic joint.

Figure 7.4: Experimental and the semi-analytic modeling results on the displacement stiffness of the active compliant prismatic joint.

Experimental results obtained from this evaluation are plotted against the analytical results obtained from the semi-analytic modeling in Fig. 7.4. It shows that the active compliant prismatic joint has a linear displacement stiffness of 1.52 N/mm and is well-predicted by the displacement stiffness model derived from the semi-analytic modeling in Section 6.4 of Chapter 6, which computes a linear displacement stiffness of 1.48 N/mm for the active compliant prismatic joint. A deviation of 2.6% shows that the semi-analytic model is accurate in predicting the displacement stiffness of such beam-based flexure joint modules.

Figure 7.5: Errors between the experimental and analytical results on the stiffness of the active compliant prismatic joint.

In addition, the deviations between the experimental and analytical results obtained from both semi-analytic and PRB models are plotted in Fig. 7.5. A comparison between the experimental and semi-analytic modeling results shows a maximum deviation of 13 % occurring at the maximum displacement limit of 3 mm. On the other hand, a maximum deviation of 23.5 % occurring at 3 mm is obtained when comparing the experimental results against the PRB modeling results. The increasing deviations between the experimental and both analytical results are mainly because both models are formulated based on the concept of separating and converting the input force as moment end load subjected to individual beam-based flexure joint. As a result, this concept may become inaccurate when

the displacement increases as shown in Fig. 7.5. Although it is mentioned in Chapter 3 that the PRB model will be effective when the length of rigid-link is ten times longer than the length of compliant joint (in this case the length of rigid-link is 35 mm and the length of beam-based flexure joint is 3 mm), it is interesting to note that the semi-analytic model is able to reduce those inherited inaccuracies tremendously. From Fig. 7.5, the deviations between the experimental and semi-analytic modeling results are approximately 10 % lower as compared to the deviations between the experimental and PRB modeling results. Thus, this evaluation shows that the semi-analytical model offers high accuracy in predicting the stiffness of such a beam-based flexure joint module. Most importantly, this evaluation has also shown that the developed active compliant prismatic joint has achieved the desired stiffness and displacement, which is essential for achieving the targeted performances of the proposed 3-DOF 3PPS FEPM.

7.3 Passive 5-DOF Compliant Joint

The passive 5-DOF compliant joints were also made of the same aluminum 7076-T6 material and fabricated by a wire-cut EDM process. Figure 7.6a shows one of the developed passive 5-DOF compliant joints.

Figure 7.6: (a) Prototype of a passive 5-DOF compliant joint and (b) a bird-eye view of two pairs of beam-based flexure joints through the center hole of the prototype.

Based on a single cylindrical shaped workpiece with specific diameter, a through hole with controlled diameter was first drilled from the top of the workpiece. The wire-cut EDM process was used to cut through the sides of the workpiece to machine a pair of beam-based flexure

joints. Subsequently, another symmetrical pair of beam-based flexure joints is fabricated orthogonally to the first pair (Fig. 7.6b) to form the passive 5-DOF compliant joint, which will be used to provide the remaining degrees of motion to each limb of the proposed FEPM.

7.3.1 Deflection Stiffness Along X- or Y-Axis Evaluation

The deflection of the 5-DOF compliant joint is one of the important motions as it will be used to realize the passive prismatic motion of the proposed 3PPS configuration. Hence, the deflection stiffness of such a compliant joint has been investigated. The entire experimental setup for the deflection stiffness evaluation is shown in Fig. 7.7. In this investigation, segment 1 of the 5-DOF compliant joint is tied to a nylon string, which is used to carry the weight masses for generating the deflection. The height displacement of segment 1, which are proportional to the deflection, is measured by a laser displacement sensor (KEYENCE, model: LJ-G030, resolution: $1\text{ }\mu\text{m}$) that is mounted above the 5-DOF compliant joint. During the experiment, by increasing the weight masses will increase the height displacement of the 5-DOF compliant joint.

Figure 7.7: Experimental setup for investigating the deflection stiffness of the passive 5-DOF compliant joint.

The results obtained from this evaluation are plotted in Fig. 7.8. It shows that the developed 5-DOF compliant joint has a linear height displacement stiffness of 8 N/mm . The stiffness

predicted using the semi-analytic model, i.e., Eqn. (6.31), is also plotted in Fig. 7.8. It shows that the prediction of the semi-analytic model is consistent to the experimental results as the deviation between the experimental and analytical results at each load is about 2 %. Due to the design of the current prototype (refer to Table 6.4 in Chapter 6), the height displacement is equivalent to the deflection along the x-axis (from segment 1). This shows that the semi-analytic model offers accurate approximations on the deflection stiffness along the x-axis of the developed 5-DOF compliant joint. As the beam-based flexure joints of segment 1 and 2 have similar dimensions, it is reasonable to assume that the deflection stiffness along the y-axis (from segment 2) has also been well-predicted by the semi-analytic model.

Figure 7.8: Deflection stiffness of the 5-DOF compliant joint obtained from the experimental and analytical analyses.

7.3.2 Angular Stiffness About Z-Axis Evaluation

The angular stiffness about the z-axis of the 5-DOF compliant joint was also evaluated and the entire experimental setup is shown in Fig. 7.9. A micro-stepping motor was connected to the 5-DOF compliant joint via an F/T sensor. Each micro-step generated by the motor will cause an angular motion about the z-axis of the 5-DOF compliant joint. Each motion was measured by a high-precision measuring device (Mahr, model: Millitron 1240, resolution: 1 μm) where its measurement probe was placed at the dowel pin hole, which is located at

12.5 mm from the center of the 5-DOF compliant joint. Based on this fixed distance of 12.5 mm, each measured motion is converted to a deflection angle value while the F/T sensor registered the amount of applied moment that caused such an angular motion.

Figure 7.9: Setup for evaluating the angular stiffness of the 5-DOF compliant joint.

Figure 7.10: Experimental results on the angular stiffness of the 5-DOF compliant joint.

The experimental results are plotted in Fig. 7.10, which shows that the developed 5-DOF compliant joint has a linear relationship between the deflection angle and the applied moment. In addition, the analytical results obtained from Section 6.6.8 are also plotted

7.4. Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator209

against the experimental results. It shows that the angular stiffness about the z-axis of the developed 5-DOF compliant joint is well-predicted by the proposed analytical model. Hence, these results suggest that the analytical model can be used for quick and accurate predictions on the angular stiffness about the z-axis of the 5-DOF compliant joint during the design, and analysis stage.

7.4 Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator

Using the developed active compliant prismatic joints and the passive 5-DOF compliant joints, a 3-DOF 3PPS FEPM prototype with a targeted motion of θ_x - θ_y - Z has been developed (Fig. 7.11). For each limb, the active compliant prismatic joint supports a translational mobile coil-holder, which is wound by the AWG26 wire. Hence, the coil-holder and a stator consisting of symmetrical DM configurations forms an EDM to drive each mobile coil-holder, which is connected to the mobile platform via a 5-DOF compliant joint.

Figure 7.11: A 3-DOF 3PPS FEPM prototype.

7.4.1 Workspace Evaluation

As mentioned in Chapter 6, this 3-DOF 3PPS FEPM prototype is designed and developed to achieve a minimum workspace of $\pm 2^\circ \times \pm 2^\circ \times \pm 2.5 \text{ mm}$. To evaluate the workspace of the prototype, a laser tracking robot (LEICA, model: Absolute Tracker, measuring volume: 18 m^3 , resolution: $60 \mu\text{m}$) was used. The methodology of this robot is to transmit a laser beam to a reflector, which is installed in a target device (LEICA, model: T-MAC 6-DOF Tracking Device). This reflector then reflects the laser beam back to the robot so that it can accurately determine the all three position coordinates and all three orientations of the target device. During the evaluation, the target device first guided the laser tracking head of the robot towards the position of the prototype before being fixed onto the mobile platform as shown in Fig. 7.12. Once the robot had locked on the target device, any changes in the positions and orientations of the mobile platform will be registered by the tracking robot.

Figure 7.12: A 6-DOF laser tracking robot.

In this evaluation, an open-loop control was employed to vary the displacement of each active compliant prismatic joint, which will be indicated by a MicroE-Systems optical linear encoder (5 nm resolutions) located at each of these joints. By varying the displacement of each active compliant prismatic joint, the end-effector that is located at the center of the mobile platform of the prototype can be manipulated. Denote Disp 1, 2 and 3 as

7.4. *Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator*211

the displacement of active compliant prismatic joint 1, 2, and 3 respectively, the targeted workspace of the end-effector can be obtained from:

- 1. +2.5 mm along z-axis: Disp 1 = 2.5 mm, Disp 2 = 2.5 mm, Disp 3 = 2.5 mm
- 2. -2.5 mm along z-axis: Disp 1 = -2.5 mm, Disp 2 = -2.5 mm, Disp 3 = -2.5 mm
- 3. +2.5 ° about x-axis: Disp 1 = 2.5 mm, Disp 2 = 2.5 mm, Disp 3 = -2.5 mm
- 4. -2.5 ° about x-axis: Disp 1 = -2.5 mm, Disp 2 = -2.5 mm, Disp 3 = 2.5 mm
- 5. +2.5 ° about y-axis: Disp 1 = 2.5 mm, Disp 2 = -2.5 mm, Disp 3 = 0
- 6. -2.5 ° about y-axis: Disp 1 = -2.5 mm, Disp 2 = 2.5 mm, Disp 3 = 0

Each achieved position or orientation of the end-effector is registered by the laser tracking robot and is listed in Table 7.2. The mobile platform of the prototype has $D = 107.5$ mm, which is the height of the triangular mobile platform formed by the locations of the three passive 5-DOF compliant joints. Based on Table 6.1 found in Chapter 6, the orientation about the y-axis obtained from the experimental investigations turn out to be at least 0.4° higher than the theoretical predictions. This can be explained by the rotation from the 5-DOF compliant joint of limb 1 and 2 respectively, which causes an additional displacement along the z-axis in either positive or negative direction.

Table 7.2: List of the positions and orientations achieved by the prototype

Achieve by prototype	Positive	Negative
Position at the z-axis	2.514 mm	-2.500 mm
Orientation about the x-axis	2.981°	-2.868°
Orientation about the y-axis	2.578°	-2.556°

A simple verification can justify this assumption. Based on the displacement stiffness of the active compliant prismatic joint obtained experimentally, i.e., 1.52 N/mm, a displacement of 2.5 mm will require a driving force of 3.8 N. The deflection stiffness of the passive 5-DOF compliant joint along the z-axis obtained experimentally, i.e., 8 N/mm, suggests that a driving force of 3.8 N will cause a displacement of 0.475 mm along the z-axis. With

$D = 107.5$ mm, Eqn. (6.20) gives $L = 124.13$ mm. Using half of L , $\arcsin(0.475/62.065)$ yields 0.44° . Hence, this shows that the additional displacement of 0.475 mm along the z -axis will cause an additional orientation of 0.4° about the y -axis. On the other hand, the theoretical predictions on the orientation about the x -axis are closer to the experimental results because the displacement along the z -axis for the passive 5-DOF compliant joint of limb 1 is minimal in these conditions (refer to Table 6.1). For the passive 5-DOF compliant joint of limb 2 and 3, the resultant rotation from segment 1 and 2 has also caused an additional displacement along the z -axis. However, both additional displacements from limb 2 and 3 are equal but in different directions. Hence, the displacements of segment 1 and 2 cancel each other, and there are no additional displacements along the z -axis for the 5-DOF compliant joint of limb 2 and 3 to cause additional orientation about the x -axis. From this evaluation, the developed prototype has not only achieved the targeted workspace but has achieved a larger controllable workspace of $\pm 2.5^\circ \times \pm 2.5^\circ \times \pm 2.5$ mm.

7.4.2 Active Joint-Level Position Control

A position control scheme is implemented on each active compliant prismatic joint to validate the open-loop performances of the prototype. This scheme is similar to the PID servo-control that is used to control the FELA in Chapter 5. A MicroE-System optical linear encoder (5 nm resolution), which is attached to each active compliant prismatic joint, serves as the feedback sensor for each individual PID servo-loop. All three PID controllers are written in the FPGA environment via a NI FPGA controller card, whereby each PID servo-control loop runs at 10 KHz with a control frequency of 1 KHz. The predictions of PID control parameters are done through MATLAB *System Identification Toolbox*, which is used to identify these values based on the open-loop step response obtained from each active compliant prismatic joint (note: the steps to obtain the PID parameters are documented in details in Section 5.4 of Chapter 5). Consequently, each PID servo-control allows high precision position control on each active compliant prismatic joint, which will directly manipulate the end-effector of the prototype. Current implemented scheme can only be considered as an active joint-level positioning control as there is no feedback control loop implemented on the end-effector. As a result, the implemented joint-level position control scheme can only validate the open-loop

7.4. *Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator*213

performances of the prototype in terms of positioning resolutions, orientation resolutions and actuating speed.

7.4.2.1 *Nanometric Positioning Resolutions*

A RENISHAW laser interferometer (model: RLE10) is employed to monitor the finest resolutions and the smallest steps that the prototype can deliver at the end-effector. This interferometer, which has a resolution of 10 nm/count, was placed above the prototype with its laser pointing towards the end-effector as shown in Fig. 7.13a. A metrology mirror was mounted at the end-effector of the prototype (Fig. 7.13b) to receive and transmit the laser back to the interferometer unit for accurate measurement of the displacement at the end-effector.

Figure 7.13: A laser interferometry setup for evaluating the positioning resolutions of the developed prototype at the end-effector.

Figure 7.14 plots the laser interferometer readings of the end-effector at neutral position when the servo-control of each active compliant prismatic joint is activated. It shows that an average positioning resolution of ± 10 nm is registered with a maximum peak-to-peak resolution of ± 20 nm being traced occasionally. The laser interferometer readings of the end-effector at 2.5 mm, which are plotted in Fig. 7.15, also shows similar characteristics. Although both plots have shown minor drifts at the end-effector, it can be eliminated once

a closed-loop positioning control is implemented to the prototype. This closed-loop control can also eliminate the occasional peak-to-peak resolution of ± 20 nm. Nevertheless, the current joint-level control scheme has allow the prototype to achieve an open-loop positioning resolution of ± 10 nm at the end-effector throughout the controllable workspace.

Figure 7.14: The positioning resolutions at neutral position obtained from the end-effector.

Figure 7.15: The positioning resolutions at 2.5 mm obtained from the end-effector.

7.4. *Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator*215

Using the joint-level position control scheme, the prototype was manipulated to move from neutral position to 60 nm and back to neutral with an incremental step of 20 nm. Figure 7.16 plots the laser interferometer readings obtained at the end-effector during this operation. It shows that the prototype has achieved the smallest achievable steps of 20 nm. Similarly, an occasional peak-to-peak resolution of ± 20 nm is registered. At 60 nm, the prototype is overshoot by 10 nm but settled back to 60 nm. Other than this slight error, this evaluation has shown that the end-effector can achieves an open-loop displacement through 20 nm incremental steps.

Figure 7.16: An increment of 20 nm steps obtained from the end-effector.

7.4.2.2 *Fine Orientation Resolutions*

The orientation resolution of the prototype was evaluated as well. In this evaluation, a photoelectric autocollimator (NIKON, model: Two-axes photoelectric autocollimator, resolution: 0.05") was used to validate this performance (Fig. 7.17). A metrology mirror was mounted on the end-effector to reflect the light transmitted from the photoelectric autocollimator. With a slight tilt at the end-effector, it changes the intensity of the reflected light back to the photoelectric autocollimator. Subsequently, this change in light intensity can be used to accurately calculate the orientation angle.

Figure 7.17: The photoelectric autocollimator setup for evaluating the orientation resolution of the prototype.

Based on unit conversion, $0.05''$ is approximately $0.2424 \mu\text{rad}$. As mentioned in Section 7.4.1, the height of the triangular mobile platform of the prototype is given as $D = 107.5 \text{ mm}$. Hence, the orientation of the end-effector about the x-axis via the mobile platform, θ_x , due to the disp 1 is given by

$$\theta_x = \arcsin\left(\frac{\text{disp } 1}{2D/3}\right) \quad (7.3)$$

Simultaneously, the orientation about the x-axis due to the disp 2 and 3, which are in the opposite direction of disp 1, is given as

$$\theta_x = \arcsin\left(\frac{\text{disp } i}{D/3}\right) \quad (7.4)$$

where $i = 2$ or 3 .

In this evaluation, the values of disp 1, 2 and 3 forms a combination to achieve the orientation of the end-effector about the x-axis. For example, given that $\text{disp } 1 = 20 \text{ nm}$,

7.4. *Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator*217

disp 2 = -10 nm and disp 3 = -10 nm, Eqn. (7.3), and (7.4) will predict an orientation about the x-axis of 0.279 μ rad. The photoelectric autocollimator should detect such an orientation since its resolution is 0.05'' per count. Subsequently, a set of combinations for disp 1, 2 and 3 was given to the prototype to orientate the end-effector about the x-axis to the desired orientations while the autocollimator measured the actual orientations. Both desired and actual orientations about the x-axis are listed in Table 7.3. The experimental results have verified that the prototype has achieved these desired orientations about the x-axis. The only inconsistency is found at combination no. 6, where the autocollimator detected 1 count higher than the desired orientation. This is possibly due to the drifting of the end-effector, which has no closed-loop control. Nevertheless, the detected readings are consistent with the desired orientations for all remaining combinations. Most importantly, this evaluation shows that the prototype has an orientation resolution of 0.05'' about the x-axis.

Table 7.3: List of the desired and actual fine orientations about the x-axis achieved by the prototype

Combination	Disp 1 (nm)	Disp 2 (nm)	Disp 3 (nm)	Desired orientation (")	Actual orientation (")
1	20	-10	-10	-0.05	-0.05
2	-20	10	10	0.05	0.05
3	40	-20	-20	-0.10	-0.10
4	-40	20	20	0.10	0.10
5	60	-30	-30	-0.15	-0.15
6	-60	30	30	0.15	0.20
7	80	-40	-40	-0.20	-0.20
8	-80	40	40	0.20	0.20
7	100	-50	-50	-0.25	-0.25
8	-100	50	50	0.25	0.25
9	120	-60	-60	-0.30	-0.30
10	-120	60	60	0.30	0.30

As for the orientation of the end-effector about the y-axis, another set of parameters and calculations are used to determine the required combinations for disp 1, 2 and 3. Based on $D = 107.5$ mm, the distance between two passive 5-DOF compliant joints is given as $L = 124.13$ mm based on Eqn. (6.19) and (6.20). For the orientation about the y-axis, disp 3 will be kept at neutral position while varying disp 1 and 2 to achieve the desired

orientation. Hence, the orientation of the end-effector about the y-axis, θ_y , due to the disp 1 and 2 is given as

$$\theta_y = \arcsin\left(\frac{\text{disp } j}{L/2}\right) \tag{7.5}$$

where $j = 1$ or 3 and the directions of both displacements will be opposite one another.

Based on Eqn. (7.5), given that disp 1 = 15 nm and disp 2 = -15 nm, the expected orientation about the y-axis is 0.242 μrad or 0.05". Through this concept, a set of combinations for disp 1 and 2 with disp 3 = 0 was used to orientate the end-effector about the y-axis to the desired orientations while the autocollimator monitored the actual orientations. Both the desired and actual orientations about the y-axis are listed in Table 7.4. In this evaluation, the actual orientations obtained from the autocollimator are consistent with the desired orientations for all given combinations. It has also shown that the prototype has an orientation resolution of 0.05" about the y-axis.

Table 7.4: List of the desired and actual fine orientations about the y-axis achieved by the prototype

Combination	Disp 1 (nm)	Disp 2 (nm)	Disp 3 (nm)	Desired orientation (")	Actual orientation (")
1	15	-15	0	0.05	0.05
2	-15	15	0	-0.05	-0.05
3	30	-30	0	0.10	0.10
4	-30	30	0	-0.10	-0.10
5	45	-45	0	0.15	0.15
6	-45	45	0	-0.15	-0.15
7	60	-60	0	0.20	0.20
8	-60	60	0	-0.20	-0.20
7	75	-75	0	0.25	0.25
8	-75	75	0	-0.25	-0.25
9	90	-90	0	0.30	0.30
10	-90	90	0	-0.30	-0.30

As previous two evaluations have shown that the prototype can achieve a fine orientation resolution of 0.05" or 0.242 μrad about the x- and y-axes, another evaluation was conducted to examine on the consistency of this fine orientation resolution verse the changes in height

7.4. *Prototype of A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator*219

along the z-axis. First, the end-effector was set at 0.10'' about x-axis through disp 1 = -40 nm, disp 2 = 20 nm and disp 3 = 20 nm. Subsequently, the end-effector was moved up to 5 μm, 50 μm and 0.5 mm while maintaining the orientation of 0.10'' about the x-axis. During this evaluation, the autocollimator monitored the orientation reading from neutral position to 0.5 mm and these readings are plotted in Fig. 7.18. The results have clearly shown that the fine orientation at the end-effector can be maintained throughout the changes in height displacements along the z-axis. The evaluation had to stop at 0.5 mm because the autocollimator will be out-of-range at 1 mm. Nevertheless, this evaluation has shown the consistency of the prototype in achieving such fine orientations throughout the deviations in height displacement along the z-axis.

Figure 7.18: The consistency of fine orientation verse the changes in height displacement along the z-axis at the end-effector of the prototype.

7.4.2.3 *Fast Traveling Speed*

The active joint-level control scheme is used to move the end-effector of the prototype from 0 to 2.5 mm. Using the same RENISHAW laser interferometer, which was mounted above the end-effector (Fig. 7.13a), this large displacement motion was monitored and plotted in Fig. 7.19. The step response registered by the laser interferometer has shown that the end-effector

moved from neutral position to about 2.5 mm within 10 msec before settling at 2.5 mm at approximately 100 msec. Based on this step response, the prototype demonstrated a fast traveling speed of 250 mm/sec. In this evaluation, the velocity of each compliant prismatic joint driven by the EDM can still be adjusted to higher value. However, this traveling speed is sufficient since the prototype is targeted to achieve 100 mm/sec of traveling speed during the design stage. Yet, this evaluation is conducted without external loading at the end-effector. Hence, further evaluations will be necessary in this aspect in future. Nevertheless, it is expected that similar traveling speed can still be achieved even with external loadings so long as sufficient current is supply to the EDMs during these operations.

Figure 7.19: The step response of the prototype from 0 to 2.5 mm.

7.4.3 Large Continuous Output Force

In this investigation, the prototype is targeted to achieve a continuous output force of 100 N, which is required in the nano-imprint processes. Although a closed-loop force control scheme has yet to be implemented in the current stage of research, an open-loop control is used to evaluate the loading capacity of the prototype. By maneuvering the load, which was placed on the mobile platform of the prototype, this evaluation can prove that the prototype has produces sufficient output force required to move the load. Based on calculations, the

prototype is capable of generating a continuous output force of 160 N/Amp. Hence, in this evaluation a load of 15 Kg was placed on the mobile platform of the prototype. As a result, the prototype will require to generate at least 147.15 N to maneuver the load. Based on the preset open-loop command signals and displacement sequences implemented on each active compliant prismatic joint, the prototype was able to maneuver the 15 Kg-load within its maximum achievable orientations about the x- and y-axes of $\pm 2.5^\circ$, as shown in Fig. 7.20. Through this evaluation, the prototype has successfully maneuvered the load in a continuous θ_x - θ_y -Z motion. Most importantly, this evaluation has shown that the prototype has achieved more than 100 N of continuous output force. It has also shows that the prototype has a force-current sensitivity of 160 N/Amp. Consequently, the prototype is capable of generating a continuous output force of at least 300 N of with 2 Amp of input current.

Figure 7.20: Prototype maneuvering a 15 Kg-load in an orientation of $\pm 2.5^\circ$ about the x- and y-axes.

7.5 Summary

A prototype of the proposed 3-DOF 3PPS FEPM is developed and presented in this chapter. Developed based on a 3PPS configuration, this prototype has three symmetrical limbs with each limb comprising of a passive 5-DOF compliant joint and an active compliant prismatic joint. As a constraint arrangement is imposed to eliminate one degree of motion from each

5-DOF compliant joint, each limb will have one active and four passive motions while three symmetrical limbs in parallel arrangement will give three degrees of controllable motions to the mobile platform. Subsequently, each compliant joint is fabricated based on the aluminum 7076-T6 material and the wire-EDM cut process. An evaluation on the displacement stiffness of the active compliant prismatic joint was conducted and shows the analytical predictions are closed to the experimental results with a maximum deviation of 13 % occurring at the maximum displacement limit of 3 mm. In addition, the experimental investigations on the deflection and angular stiffness of the passive 5-DOF compliant joint were also conducted. The experimental results have shown that the proposed analytical models are accurate with a maximum deviation of approximately 2 % in predicting the deflection stiffness and a maximum deviation of about 6 % in predicting the angular stiffness. After the stiffness evaluations, these compliant joints are put together to form a complete FEPM prototype with each active compliant prismatic joint driven by an individual EDM. The workspace of the prototype has been evaluated by a laser tracking robot and results have shown that the prototype has achieved a controllable workspace of $\pm 2.5^\circ \times \pm 2.5^\circ \times \pm 2.5$ mm. In addition, an external laser interferometer was used to evaluate the open-loop positioning resolution at the end-effector of the prototype. The laser interferometer readings have shown that the prototype has achieved an open-loop positioning resolution of ± 10 nm and a smallest achievable step of 20 nm. A photoelectric collimator, which was used to measure the orientation resolution at the end-effector, has shown that the prototype has achieved an open-loop orientation resolution of $0.05''$ about the x- and y-axes. In addition, a step response from neutral position to 2.5 mm was performed by the prototype and shows that the prototype has achieved a fast traveling speed of 250 mm/sec. Last but not least, the prototype was tasked to maneuver a 15 Kg-load and this evaluation has shown that it has the capability of generating a continuous output force of 160 N/Amp. All these experimental results have shown that the prototype has achieved a large displacement, nanometric resolutions, fast traveling speed and large continuous output force. With these capabilities, this prototype becomes a promising solution for facilitating the nanoimprint lithography processes in the nano-scale fabrications.

Chapter 8

Conclusions

“If I have a thousand ideas and only one turns out to be good, I am satisfied.”

– Alfred Bernhard Nobel (1833 - 1896).

In this thesis, a new class of compliant manipulator, which is targeted for ultra-high precision positioning and large continuous output force throughout a few millimeters, and degrees of displacement and orientation respectively, has been investigated. Termed flexure-based electromagnetic parallel-kinematics manipulator, it is realized by a marriage between two elements, i.e., the beam-based flexure bearings and a Lorentz-force actuation. Both elements are essential for achieving a large workspace with the Lorentz-force actuation providing a non-contact actuation while the beam-based flexure bearings providing a frictionless support for this actuating scheme. However, each of these elements has certain flaws that need to be addressed prior to the realization of the proposed compliant manipulator.

A beam-based flexure joint, which is usually treated as a cantilever beam, exhibits a nonlinear deflection due to a parasitic shift of the ‘pivot’ point, which causes a deflection to be greater than the length of the beam [106]. For an accurate approximation of this nonlinear deflection, a semi-analytic model is presented. In this model, a factor, ϖ , is introduced to determine the parasitic shifting of the ‘pivot’ point during the large deflection while a factor, ρ , is introduced to predict the initial location of the ‘pivot’ point due to different configurations of the beam-based flexure. To evaluate the accuracy of the semi-

analytic model, a set of specimens consisting of beam-based flexure joints coupled with different rigid-link lengths was fabricated. The deflection stiffness of each specimen was obtained experimentally and compared against the approximated deflection stiffness obtained from the semi-analytic model and the conventional PRB model. An average deviation of 2.3 % is observed from the comparison between the experimental and the semi-analytic model results. This shows that the semi-analytic model offers an accurate prediction on the nonlinear deflection of the beam-based flexure joints. In addition, this average error remains consistent for all the specimens of various beam-based flexure configurations. On the other hand, for configurations with shorter rigid-link lengths, an average deviation of 8.5 % is observed from the comparison between the experimental and the conventional PRB model results. As the length of rigid-link increases, the average deviation between the experimental and the conventional PRB model results reduces to 3 %. Hence, this shows that the semi-analytic model is more accurate and robust than the conventional approximation model in predicting the nonlinear deflection of a beam-based flexure joint coupled with various rigid-link lengths. Furthermore, the semi-analytic model was also employed to analysis the displacement stiffness of a double compound linear spring mechanism. Similarly, an average deviation of less than 3 % is observed from the comparison between the experimental and the semi-analytic model results. Thus, the proposed semi-analytic model has shown to provide a simple, fast and accurate approximation that will be essential for quick parametric studies and understandings on the beam-based flexure joint modules. Most importantly, this semi-analytic model becomes an effective and efficient analytical tool for the design, and development of the beam-based flexure bearings, which will be used to form the proposed compliant manipulator.

Generating a small output force has been a major limitation of the Lorentz-force actuation. To address this common issue, a new magnetic circuit is introduced to enhance and improve the capabilities of a Lorentz-force actuation. Termed a Dual-Magnet (DM) configuration, it is used to strengthen the magnetic flux density within the effective air gap of the stator, which the moving air-core coil is operating. With higher magnetic flux density, this allows a larger force generation without increasing the size of the stator or increasing the amount of

input current. Other than increasing the magnetic flux density, a DM configuration ensures an evenly distributed magnetic flux density throughout the air gap. Thus, the magnetic flux density remains constant even at tens of millimeters away from the PM polarized surface and a large effective air gap can be realized. Two prototypes have been developed to validate the claimed capabilities. The first prototype is an iron stator comprising of a DM configuration and the second prototype is an iron stator with a conventional magnetic circuit. The magnetic flux density within the effective air gap of each prototype was measured through a hall-effect sensor. Experimental results obtained from both prototypes show that a DM configuration provides 40 % increase in the magnetic flux density within the effective air gap as compared to the conventional magnetic circuit. It also shows that a dual-magnet configuration offers an evenly distributed magnetic flux density through the entire effective air gap of 11 mm. Consequently, this DM configuration offers a large effective air gap and high magnetic flux density that provide a promising solution for developing a Lorentz-force actuation module with large continuous output force from small amount of input current, good force-to-size ratio and low heat generation characteristics. In addition, a two-dimensional (2D) mathematical model that accurately predicts the magnetic field behavior within the effective air gap of a DM configuration is formulated. A comparison between the analytical and experimental results have shown that the 2D mathematical model offers an accurate prediction of the magnetic flux density across the 2D plane of the effective air gap with a deviation of ± 0.02 Tesla. Hence, this 2D mathematical model becomes an effective and efficient analytical tool for the design, and development of the stator with a pair of DM configurations that will be used to enhance the force generation of the proposed compliant manipulator.

The first attempt to realize the proposed concept of integrating both beam-based flexure bearings and a Lorentz-force actuation is the development of a novel nano-positioning linear actuator. Termed Flexure-based Electromagnetic Linear Actuator (FELA), it mainly comprised of a flexure bearing mechanism and an electromagnetic driving module. The electromagnetic driving module is formed by a fixed iron stator, which adopts the DM configuration, and a mobile air-core coil. The translating mobile air-core coil is then

supported by the flexure bearing mechanism, which is designed based on a bi-stable configuration with beam-based flexure joints. A closed-loop position control through a PID servo-controller is implemented on the FELA. To evaluate the performances of the FELA, a laser interferometer was used to measure the end-effector of FELA. In the evaluation, the FELA achieves a positioning accuracy of ± 10 nm over a displacement of 4 mm. It also performs the smallest achievable output step of 20 nm and a positioning repeatability of $\pm 1.5 \sigma$. Using a force-torque sensor as a feedback, a direct-force control through a PID servo-controller is implemented on the FELA. Through this direct-force control, the FELA is able to generate any force profile with a continuous output force of 60 N/Amp. The 10-N and 60-N force profiling, which are generated by the FELA separately, have shown that an accuracy of ± 0.15 N is achieved through the implemented control scheme. For a smooth transition between the position and direct-force control schemes, an impedance control is implemented on the FELA so that it can be used for automating the transferring of micro-sized channels onto the PMMA substrate via a hot-embossing process. Experimental results have shown that the imprinted features have high aspect ratios with a maximum deviation of $\pm 0.35 \mu\text{m}$ as compared to the actual feature from the silicon template. However when the features get below $100 \mu\text{m}$, the air trapped between the template and substrate has hinders the fabrication process. Hence, this implementation also helps to identify issues of the imprinting processes when the width of the template features reduces in sizes. All these results have shown that the proposed concept has allow the developed single-axis FELA to achieve nanometric positioning resolution and large output force throughout a few millimeters of displacement. The presence of an enhanced Lorentz-force actuation through the DM configuration, allow the FELA to be used for imprinting of micro-sized features through a hot-embossing process. Most importantly, the imprinted features can achieve high-aspect ratio due to the direct-force control capability of the FELA.

Finally, a 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator (FEPM) is developed. It provides a θ_x - θ_y - Z motion and is targeted as a co-planar nano-alignment manipulator that can be used in the nanoimprint lithography processes, i.e., UV nano-imprint lithography, micro-contact imprinting and hot embossing, for ultra-high precision

co-planarity alignment and direct-force control imprinting. To realize the θ_x - θ_y - Z motion, a new 3-limbs Prismatic-Prismatic-Spherical (3PPS) parallel-kinematics configuration is introduced. The forward kinematics of this 3PPS configuration has been established to facilitate the design and developed of the compliant joints. Based on the 3PPS configuration, each parallel limb consists of an active compliant prismatic joint and a newly proposed passive 5-DOF compliant joint, which offers five degrees of motion with large deflection and low stiffness characteristics. The workspace of the developed 3-DOF 3PPS FEPM was evaluated by a laser tracking robot and the experimental results have shown that this compliant manipulator has achieved a controllable workspace of $\pm 2.5^\circ \times \pm 2.5^\circ \times \pm 2.5$ mm. A laser interferometer was used measured the end-effector of the 3-DOF 3PPS FEPM and has shown that the end-effector has achieved an open-loop positioning resolution of ± 10 nm. In addition, the results obtained from a photoelectric collimator have shown that the 3-DOF 3PPS FEPM has achieved an orientation resolution of $0.05''$ about the x- and y-axes. Evaluated by the laser interferometer, the 3-DOF 3PPS FEPM achieves a smallest achievable step of 20 nm and a step response from 0 to 2.5 mm shows that it has a fast traveling speed of 250 mm/sec. By maneuvering a 15 Kg-load, the 3-DOF 3PPS FEPM has also shown that it has the capability of generating a large continuous output force of 160 N/Amp. These capabilities allow the 3-DOF 3PPS FEPM to be a promising solution for realizing ultra-high precision co-planarity alignment and direct-force control imprinting.

8.1 Contributions

The contributions from this investigation are listed as follow:

- A semi-analytic model is formulated to accurately approximate the nonlinear deflection of a beam-based flexure joint with any rigid-link length. This model can provide a simple, fast and accurate approximation that is essential for quick parametric studies and understandings on the beam-based flexure joint modules.
- A dual-magnet configuration is introduced to address the low output force generation of a Lorentz-force actuation. With this dual-magnet configuration, the enhanced Lorentz-

force actuation module can generate large continuous output force with small amount of input current.

- A novel nano-positioning linear actuator is developed. Termed Flexure-based Electromagnetic Linear Actuator (FELA), it has a positioning accuracy of ± 10 nm, a displacement of 4 mm and a continuous output force of 60 N/Amp. With these capabilities, the FELA offers a new solution in the field of ultra-high precision manipulation.
- A 5-DOF compliant joint is developed. It offers larger displacements and orientations with lower driving stiffness as compared to an elementary rod flexure joint. This 5-DOF compliant joint can be used to develop a class of spatial FPMs that are targeting for large displacement and orientation workspace.
- A 3-DOF Flexure-Based Electromagnetic Parallel-Kinematics Manipulator (FEPM) is developed based on a 3PPS parallel-kinematic configuration, which provides an alternate solution in achieving a θ_x - θ_y - Z motion. This 3-DOF 3PPS FEPM has achieved a workspace of $\pm 2.5^\circ \times \pm 2.5^\circ \times \pm 2.5$ mm, an open-loop positioning and orientation resolution of ± 10 nm and $0.05''$ respectively, a fast traveling speed of 250 mm/sec and a continuous output force of 160 N/Amp. These capabilities make this 3-DOF 3PPS FEPM a promising solution for realizing an ultra-high precision co-planarity alignment and a direct-force control imprinting.

8.2 Limitations and Future Works

The proposed semi-analytic model is accurate in predicting the nonlinear deflection of a beam-based flexure joint coupled with a rigid-link, where such a configuration is treated as a beam-based flexure joint subjected to a moment end load. The limitation of the proposed semi-analytic model is the inefficiency in predicting a beam-based flexure joint subjected to a translation force. Based on the current investigations, the prediction of the proposed semi-analytic model on the deflection of a beam-based flexure joint subjected to a translation force is only accurate up to the deflection angle of 20° when compared to the nonlinear

model, i.e., the elliptic integral method. Consequently, the semi-analytic model is inefficient in modeling the large deflection fully-compliant modules where the loadings are usually modeled as translational forces acting on the beam-based flexure joints.

A flexure joint either beam-based or notch is a spring element that provides frictionless motion through the elastic deformation. Hence, a FEPM can be treated as a spring-mass system where the stiffness of the spring and moving mass determines the natural frequency of the system. For the developed 3-DOF 3PPS FEPM, the beam-based flexure joints have much lower driving stiffness as compared to the traditional notch flexure joints. Thus, the stiffness of the entire system will be low and this limits the natural frequency of the entire system. Consequently, the limitation of current 3-DOF 3PPS FEPM is the inefficiency of performing high-bandwidth tasks due to its low natural frequency.

In this work, a moving air-core coil configuration has been adopted to reduce the moving mass of the 3-DOF 3PPS FEPM. Hence, the conducting coil is wound on the aluminum moving components. However, the heat generated from the coil during the high force imprinting operations will cause these aluminum components to experience thermal expansion and affects the accuracy of the manipulator. If this thermal expansion and heat is linear, an active compensation method can be implemented. Unfortunately, heat generation has nonlinear relationship with respect to the input current that will contribute to the complexity of the compensation method. Consequently, the thermal expansion of those aluminum components limit the current 3-DOF 3PPS FEPM to only short durations during the imprinting operation.

For future works, further modifications will be made to the proposed semi-analytic model to enhance its accuracy and robustness in accurately predicting the nonlinear deflection of a beam-based flexure joint subjected to a translation force. Subsequently, the enhanced version can be directly applied to approximate the nonlinear deflection of the beam-based flexure joints that are used in other flexure modules such as leaf-spring parallel mechanisms, etc. Most importantly, the enhanced semi-analytic model will become a generic solution for modeling any type of flexure joints, i.e., notch or beam-based, in any form of

configurations, i.e., with or without coupling a rigid-link. For the thermal expansion of aluminum components, more detail analyses will be conducted to determine the magnitude of such expansions with respect to the heat generated from the coil. If heat generation becomes nonlinear, those aluminum components that hold the conducting coil will need to be replaced by components made from Zerodur material, which has near zero thermal expansion.

Currently, the natural frequency of the 3-DOF 3PPS FEPM has not been investigated. As the developed manipulator is targeted for ultra-high precision positioning and imprinting applications that require slow-speed operations, its system dynamics, which is a crucial factor for high bandwidth operations, was not considered. In future, it will be necessary to investigate the system dynamics of the manipulator to ensure that it can operate within its allowable bandwidth for any given task. At this point of research, the 3-DOF 3PPS FEPM is manipulated based on an active joint-level positioning control as no feedback control is implemented on the end-effector of the manipulator. To implement a closed-loop control on the 3-DOF 3PPS FEPM for task-based manipulation, inverse kinematics of the proposed 3PPS parallel-kinematics configuration will be established. Based on this derived inverse kinematics, a closed-loop position and direct-force control on the 3-DOF 3PPS FEPM can be implemented. Consequently, a precise positioning control of the end-effector allows the 3-DOF 3PPS FEPM to realize an active co-planar nano-alignment and direct-force control imprinting.

Appendix A

Magnetic Field Modeling

A.1 Magnetostatic Energy

Based on Maxwell's equation in integral form, the surface integral of current density and the displacement of charge density, $\partial \mathbf{D} / \partial t$, forms the magnetic field strength within a close-loop path

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S (\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}) \cdot d\mathbf{s} \quad (\text{A.1})$$

Apply Eqn. (A.1) to model the magnetic circuit path illustrated in Fig. 4.2a yields

$$\oint_C H \cdot dl = H_m l_m + H_{core} l_c + H_g l_g = 0 \quad (\text{A.2})$$

where H_m , H_{core} , H_g are the magnetic field strength in the PM, core and effective air gap respectively while l_m , l_g , l_c are the lengths of the path in respective sections. Here, $\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \rightarrow 0$ since all the flux is only supplied by the PM.

Assuming the core has an infinite permeability, i.e. $\mu \approx \infty$, with no leakage of flux, substituting Eqn. (4.6) into Eqn. (A.2) yields

$$B_g = \frac{\mu_0 B_r}{(l_g/l_m)(B_r/H_c) + \mu_0(A_g/A_m)} \quad (\text{A.3})$$

where B_r and H_c are the remanence magnetic flux density and the coercivity field strength of a PM while A_g and A_m are the cross-sectional area of the effective air gap and the PM respectively.

A.2 Surface Charge Model

Currently, magnetic field of a PM is commonly analyzed using an analytical model that describes a PM with a distribution of dipole charge.

For magnetostatic analysis with current-free region

$$\mathbf{H} = -\nabla\Phi \quad (\text{A.4})$$

$$\nabla \cdot \mathbf{B} = 0 \quad (\text{A.5})$$

Substituting Eqn. (A.4) and (A.5) into Eqn. (4.4) yields

$$\nabla^2\Phi = \nabla M \quad (\text{A.6})$$

Green's function is applied to solve Eqn. (A.6), which is a linear non-homogenous Partial-Differential-Equation (PDE). Based on Green's second identity,

$$\int \int \int_D (u \nabla^2 v - v \nabla^2 u) d\mathbb{X} = \int \int_S (u \frac{\partial v}{\partial \eta} - v \frac{\partial u}{\partial \eta}) ds$$

Denote $G(\mathbb{X}, \mathbb{X}') = v$ and $\Phi = u$

$$\int \int \int_{V'} (\Phi \nabla^2 G(\mathbb{X}, \mathbb{X}') - G(\mathbb{X}, \mathbb{X}') \nabla^2 \Phi) dV' = \int \int_S (\Phi \frac{\partial}{\partial \eta} G(\mathbb{X}, \mathbb{X}') - G(\mathbb{X}, \mathbb{X}') \frac{\partial \Phi}{\partial \eta}) dS'$$

considering only volume, surface integral is ignored

$$\begin{aligned} \int \int \int_{V'} \Phi \nabla^2 G(\mathbb{X}, \mathbb{X}') dV' &= \int \int \int_{V'} G(\mathbb{X}, \mathbb{X}') \nabla^2 \Phi dV' \\ \Phi(\mathbb{X}) \int \int \int_{V'} \delta(\mathbb{X} - \mathbb{X}') dV' &= \int \int \int_{V'} G(\mathbb{X}, \mathbb{X}') \nabla^2 \Phi dV' \end{aligned}$$

substituting (A.6) with $\int \int \int_V \delta(\mathbb{X} - \mathbb{X}') dV' = 1$

$$\Phi(\mathbb{X}) = \int \int \int_V G(\mathbb{X}, \mathbb{X}') (\nabla M) dV' \quad (\text{A.7})$$

For 3D problems, it is well-known that the green kernel function for free space is expressed as

$$G(\mathbb{X} - \mathbb{X}') = -\frac{1}{4\pi} \frac{1}{|\mathbb{X} - \mathbb{X}'|} \quad (\text{A.8})$$

Consequently, substitute Eqn. (A.8) into Eqn. (A.7) yields

$$\Phi(\mathbb{X}) = -\frac{1}{4\pi} \int_{V'} \frac{\nabla M(\mathbb{X}')}{|\mathbb{X} - \mathbb{X}'|} dV' \quad (\text{A.9})$$

Given that

$$\begin{aligned} \nabla(M(\mathbb{X}') dV') &= \nabla \cdot M(\mathbb{X}') dV' + M(\mathbb{X}') \cdot \nabla dV' \\ \nabla \cdot M(\mathbb{X}') dV' &= \nabla M(\mathbb{X}') dV' - M(\mathbb{X}') \cdot \nabla dV' \\ \int_{V'} \nabla \cdot M(\mathbb{X}') dV' &= \int_{V'} \nabla M(\mathbb{X}') dV' - \int_{V'} M(\mathbb{X}') \cdot \nabla dV' \end{aligned}$$

Substituting Eqn. (A.9) yields

$$\Phi(\mathbb{X}') = -\frac{1}{4\pi} \int_{V'} \frac{\nabla \cdot \mathbf{M}(\mathbb{X}')}{|\mathbb{X} - \mathbb{X}'|} dV' + \frac{1}{4\pi} \oint_{S'} \frac{M(\mathbb{X}')}{|\mathbb{X} - \mathbb{X}'|} \hat{n} dS' \quad (\text{A.10})$$

where the solution gives a sum of magnetic volume, V , and the surface, S , confining it. In addition, $\mathbb{X}$ is the vector of observation point, $\mathbb{X}'$ is the vector of source point, ∇ operates on the primed coordinates, and $\hat{n}$ is the outward unit normal to the surface.

For 1D analysis of the magnetic flux density above a PM, it is assumed that the magnetization, $\mathbf{M} = My \rightarrow -\nabla \cdot \mathbf{M} = 0$. Using Eqn. (4.5), (A.10), $\mathbb{X} = y'y$, and $\mathbb{X}' = x'x + z'z$, the magnetic flux density along y-axis for a DM configuration is expressed as,

$$B_y^{Total}(D) = B_y^{PM-1}(D) + B_y^{PM-2}(D - G) \quad (\text{A.11})$$

where

$$B(\zeta) = \frac{\mu_0 M}{\pi} \left\{ \tan^{-1} \left[\frac{(\zeta + t) \sqrt{a^2 + b^2 + (\zeta + t)^2}}{ab} \right] - \tan^{-1} \left(\frac{\zeta \sqrt{a + b + \zeta}}{ab} \right) \right\} \quad (\text{A.12})$$

and a, b, t , are the dimension variables describing the PM, and ζ represents the distance from the polarized surface of PM-1 or PM-2 (refer to Fig. 4.2b).

As for 3D analysis of the magnetic flux density on the surface above a PM, the same assumption $\mathbf{M} = My \rightarrow -\nabla \cdot \mathbf{M} = 0$ is used and together with Eqn. (A.10), the magnetic flux density on the X-Z plane along Y-axis is expressed as,

$$B_y(x, y, z) = \frac{\mu_0 M}{4\pi} \sum_{k=1}^2 \sum_{p=1}^2 \sum_{m=1}^2 (-1)^{k+p+m} \times \tan^{-1} \left\{ \frac{(x - x_p)(z - z_m)}{(y - y_k)[(x - x_p)^2 + (y - y_k)^2 + (z - z_m)^2]^{\frac{1}{2}}} \right\} \quad (\text{A.13})$$

where x, z , are variables along x -axis and z -axis respectively. While $y = D$, is the distance from the polarized surface of a PM, and $X_1, Y_1, Z_1, X_2, Y_2, Z_2$ are dimension positions describe in Fig. A.1. Consequently, using Eqn. (A.13) the magnetic flux density within the effective air gap influence by two PMs can be expressed as,

$$B_y^{Total}(x, y, z) = B_y^{PM-1}(x, D, z) + B_y^{PM-2}(x, D - G, z) \quad (A.14)$$

The parameters require for obtained a magnetic field behavior from the developed analytical model are as follow:

- $M = 692.33 \times 10^3$ (A/m) Note: Obtained from $899.23 \times 10^3 - 2 \times (103.45 \times 10^3)$
- $\mu_0 = 4\pi \times 10^{-7}$ (Wb/Am)
- $X_1 = 0$ (m)
- $X_2 = 54 \times 10^{-3}$ (m)
- $Y_1 = 0$ (m)
- $Y_2 = 50 \times 10^{-3}$ (m)
- $Z_1 = -15 \times 10^{-3}$ (m)
- $Z_2 = 0$ (m)

A.3 Additional for Magnetic Field Solution

A. To solve for $\int_0^l \sin^2\left[\frac{(2n-1)\pi}{2l}x\right]dx$, first denote $z = \frac{(2n-1)\pi}{2l}x$ and hence

$$\frac{dz}{dx} = \frac{(2n-1)\pi}{2l}$$

For upper limit, $z_{upper} = \frac{(2n-1)\pi}{2l} = \eta$, and for lower limit, $z_{lower} = 0$

Figure A.1: Illustration of a DM configuration for surface charge model analysis.

Consequently,

$$\begin{aligned} \int_0^l \sin^2\left[\frac{(2n-1)\pi}{2l}x\right]dx &= \frac{2l}{(2n-1)\pi} \int_0^\eta \sin^2(z)dz \\ &= \frac{2l}{(2n-1)\pi} \int_0^\eta \left[\frac{1}{2} - \frac{1}{2}\cos(2z)\right]dz \quad \cos(2z) = 1 - 2\sin^2(z) \\ &= \frac{l}{(2n-1)\pi} \left[\int_0^\eta dz - \int_0^\eta \cos(2z)dz\right] \end{aligned}$$

Denote $\zeta = 2z$, hence $\frac{d\zeta}{dz} = 2$

For upper limit, $\zeta_{upper} = 2\eta = (2n-1)\pi$, and for lower limit, $\zeta_{lower} = 0$

Consequently,

$$\begin{aligned} \int_0^l \sin^2\left[\frac{(2n-1)\pi}{2l}x\right]dx &= \frac{l}{(2n-1)\pi} \left[\int_0^\eta dz - \int_0^\eta \cos(2z)dz\right] \\ &= \frac{l}{(2n-1)\pi} \left[\int_0^\eta dz - \int_0^{2\eta} \cos\zeta \frac{1}{2}d\zeta\right] \\ &= \frac{l}{2} \end{aligned}$$

B. To solve for $\int_0^l \sin\left[\frac{(2n-1)\pi}{2l}x\right]dx$, first denote $z = \frac{(2n-1)\pi}{2l}x$ and hence

A.4. Parameters Used In the Proposed 2D Mathematical Model for Magnetic Field Modeling 237

$$\frac{dz}{dx} = \frac{(2n-1)\pi}{2l}$$

For upper limit, $z_{upper} = \frac{(2n-1)\pi}{2l} = \eta$, and for lower limit, $z_{lower} = 0$

Consequently,

$$\begin{aligned} \int_0^l \sin\left[\frac{(2n-1)\pi}{2l}x\right]dx &= \frac{2l}{(2n-1)\pi} \int_0^\eta \sin(z)dz \\ &= \frac{2l}{(2n-1)\pi} \left\{1 - \cos\left[\frac{(2n-1)\pi}{2}\right]\right\} \end{aligned}$$

and since

$$1 - \cos\left[\frac{(2n-1)\pi}{2}\right] = 1 - 0 = 1 \quad n = 1, 2, 3, \dots$$

hence

$$\int_0^l \sin\left[\frac{(2n-1)\pi}{2l}x\right]dx = \frac{2l(1)^n}{(2n-1)\pi}$$

A.4 Parameters Used In the Proposed 2D Mathematical Model for Magnetic Field Modeling

The parameters required for obtaining a magnetic field behavior from the proposed 2D mathematical model are:

- $M = 692.33 \times 10^3$ (A/m) Note: Obtained from $899.23 \times 10^3 - 2 \times (103.45 \times 10^3)$
- $\mu_0 = 4\pi \times 10^{-7}$ (Wb/Am)
- $a = 7.5 \times 10^{-3}$ (m)
- $b = 18.5 \times 10^{-3}$ (m)

- $c = 26 \times 10^{-3}$ (m)
- $g = 3 \times 10^{-3}$ (m)
- $l = 30 \times 10^{-3}$ (m)

A.5 Parameters Used In the Finite Element Analysis

In this work, ANSYS 10 was used to conduct finite element analyses on a 2D model of a DM configuration. The parameters used to obtain the field solution within the effective air gap of a DM configuration are as follow:

- Element: PLANE 53, 8 nodes per element.
- Air regions: relative permeability, $\mu_r = 1$.
- NdFeB PMs: Coercive force, 692.33×10^3 (A/m).
- Irons: relative permeability, $\mu_r = 200$.
- Permeability in space: $4\pi \times 10^{-7}$ (Wb/Am).
- Boundary of the model: VECTOR POTENTIAL, FLUX PARALLEL ON LINE
- Solver: STATIC ANALYSIS, MVP FORMULATION

The 2D model was dimensioned according to the DM configuration prototype. For the effective air gap, each element, which is formed by 8 nodes, is sized as a 0.5 mm square. The magnetic flux density, B_y , of each element is obtained by averaging the values of all the nodes that form the element.

Appendix B

Flexure Modeling

B.1 Classical Nonlinear Solution for A Cantilever Beam Subjected to Moment End Load

The Bernoulli-Euler law states that the bending moment at any point of the bar is proportional to the change in the curvature, which gives

$$\frac{M}{EI} = \frac{d\theta}{ds} \quad (\text{B.1})$$

Hence, the deflection angle at the deflection beam end, θ_f , can be determined by integrating both sides of Eqn. (B.1) and yields

$$\frac{M}{EI} \int_0^l ds = \int_0^{\theta_f} d\theta$$

$$\theta_f = \frac{Ml}{EI} \quad (\text{B.2})$$

Using the chain rule on Eqn. (B.1) with dy , dx and ds been infinitesimal yields

$$\frac{M}{EI} = \frac{d\theta}{dy} \cdot \frac{dy}{ds} = \frac{d\theta}{dy} \cdot \sin \theta \quad (\text{B.3})$$

and

$$\frac{M}{EI} = \frac{d\theta}{dx} \cdot \frac{dx}{ds} = \frac{d\theta}{dx} \cdot \cos \theta \quad (\text{B.4})$$

Integrating both sides of Eqn. (B.3) yields

$$\int_0^{\delta_y} \frac{M}{EI} dy = \int_0^{\theta_f} \sin \theta d\theta$$

$$\frac{\delta_{y,nonlinear}}{l} = \frac{EI}{Ml} (1 - \cos \theta_f) \quad (\text{B.5})$$

Deflection along y-axis is derived by substituting Eqn. (B.2) into Eqn. (B.5) and expressed as

$$\frac{\delta_{y,nonlinear}}{l} = \frac{1 - \cos \theta_f}{\theta_f} \quad (\text{B.6})$$

Integrating both sides of Eqn. (B.4) yields

$$\int_0^{\delta_x} \frac{M}{EI} dx = \int_0^{\theta_f} \cos \theta d\theta$$

$$\frac{M}{EI} \delta_{x,nonlinear} = \sin \theta_f \quad (\text{B.7})$$

Deflection along x-axis is derived by substituting Eqn. (B.2) into Eqn. (B.7) and expressed as

$$\frac{l - \delta_{x,nonlinear}}{l} = \frac{\sin \theta_f}{\theta_f} \quad (\text{B.8})$$

B.2 Choosing the Maximum Bending Moment of Small Deflection Theory

If $\frac{Mx}{EI} = \theta\varpi$ is substituted into Eqn. (3.36), this yields

$$\frac{\theta\varpi EI}{x} = F_T(L + \frac{\rho l}{2\varpi}) \quad (\text{B.9})$$

For maximum bending moment, $L = 0$ and $x = l$ leads to ϖ been cancel out. This shows that with the presents of $\rho l/\varpi$, the factor, ϖ , has already been a contributing factor in Eqn. (3.36). Therefore, the use of $\frac{Mx}{EI} = \theta$ is adequate.

Appendix C

Accuracy of Finite Element Analysis on Elastic Deflection

The finite element modeling through a FEA simulator, e.g., ANSYS etc, has been playing an important role in analyzing the compliant joints and mechanisms. However, it is necessary to understand the element type and mesh size selected during such numerical analyses. In chapter 5, SOLID45 element type provided by ANSYS was used to analyze the proposed compliant mechanism designs. Yet, it is interested to find that the FEA results are almost twice the amount of the actual stiffness of the developed compliant mechanism. Hence, this large difference suggested that the SOLID45, which uses a brick meshing approach, is identified in this work to be unsuitable for large deflection analyses through elastic deformation. Such an element type even with the finest mesh selection is rather suitable for structure analyses (Fig. C.1a).

The incompetence of SOLID45 has made way for SOLID92 element type provided by ANSYS to be used for 3D compliant joints and mechanisms analyses in this chapter and later part of this work. This element uses a tetrahedral meshing approach to mesh a given compliant object (Fig. C.1b). As each element is mesh in a triangular shape, the nodes formed are much closer to each other. Such formations of nodes seems to be more suitable for analyzing elastic deflections of a compliant object.

To evaluate the accuracy of this element type, a 3D beam-based flexure joint coupled with a rigid-link is modelled in the FEA simulator (Fig. C.2). The beam-based flexure length,

Figure C.1: ANSYS element and mesh selection for (a) SOLID45 and (b) SOLID92.

Figure C.2: (a) ANSYS modeling of a 3D beam-based flexure joint coupled with a rigid-link and (b) the deflection obtained when subjected to a vertical force.

width and thickness is 20 mm, 20 mm and 0.4 mm respectively, while the rigid-link length is 20 mm. For this evaluation, E is selected as 71 GPa and poisson ratio is selected as 0.33 while a large displacement analysis is used for a non-linear analysis on this 3D model. By fixing the input force, a set of mesh size is selected to mesh the 3D model. Based on each mesh size, the deflections obtained due to the input force are recorded. The experimental results from

a similar dimensioned beam-based flexure joint coupled with a rigid-link sample (see chapter 3) are used to evaluate the results obtained from this FEA analysis. Both experimental and FEA results are listed in Table. C.1.

Table C.1: Deflections of a beam-based flexure joint coupled with a rigid-link subjected to a point force through experimental investigations and FEA simulator with various mesh sizes.

Mesh size	Deflection in Y-axis			Deflection in X-axis			Time taken
	Experimental (m)	FEA (m)	Error (%)	Experimental (m)	FEA (m)	Error (%)	
10	1.522×10^{-2}	1.388×10^{-2}	8.78	3.281×10^{-3}	2.765×10^{-3}	15.72	10 sec
8	1.522×10^{-2}	1.444×10^{-2}	5.12	3.281×10^{-3}	3.000×10^{-3}	8.57	3 mins
6	1.522×10^{-2}	1.449×10^{-2}	4.78	3.281×10^{-3}	3.022×10^{-3}	7.90	6 mins
4	1.522×10^{-2}	1.449×10^{-2}	4.78	3.281×10^{-3}	3.023×10^{-3}	7.87	7 mins
2	1.522×10^{-2}	1.451×10^{-2}	4.66	3.281×10^{-3}	3.031×10^{-3}	7.62	20 mins
1	1.522×10^{-2}	1.452×10^{-2}	4.61	3.281×10^{-3}	3.068×10^{-3}	6.51	31 mins

Results from Table. C.1 have shown that SOLID92 element with mesh size of 8 and below have good predictions on the deflections of the compliant joint. An average error of 5 % is obtained for predicting the deflection in Y-axis and an average error of 8 % is obtained for predicting the deflection in X-axis. As the mesh size decreases, the errors decreases. However, the reduction of errors are not significant from mesh size 8 onwards. In addition, the time taken for each FEA analysis becomes longer with a reduction in mesh size. These observations suggest that mesh size 8 will be sufficient for compliant object analyses. Based on this evaluation, SOLID92 element type with at least mesh size of 8 have shown to be suitable and accurate in predicting the elastic deflection of compliant objects. Judging from the low percentage of errors when compared with experimental results, the FEA analyses, which used SOLID92 element with mesh size of 8 and below, conducted in this chapter is considered to be accurate. Consequently, the derived analytical models for predicting the deflection and angular stiffness of the proposed 5-DOF compliant joint are considered to be accurate due to the low percentage of errors obtained when compared against the results from FEA analyses that used SOLID92 element with mesh size of 8 and below.

List of Author's Publications and Awarded Patents

Journal Publications

1. T. J. Teo, I.-M. Chen, G. L. Yang, and W. Lin, "A flexure-based electromagnetic linear actuator", *Nanotechnology*, vol. 19, pp. 315501, 2008.
2. T. J. Teo, I.-M. Chen, G. L. Yang, and W. Lin, "Magnetic field modeling of a dual-magnet configuration," *Journal of Applied Physics*, vol. 102, pp. 074924-12, 2007

Conference Publications

1. T. J. Teo, I.-M. Chen, G. L. Yang, and W. Lin, "A semi-analytic model for large deflection beam-based flexure joints," in *2008 IEEE/ASME International Conference on Advanced Intelligent Mechatronics*, (Xi'an, China), pp. 78-83, July 2008.
2. T. J. Teo, C. M. Kiew, G. L. Yang, I. M. Chen, and W. Lin, "Hybrid position/force control of a flexural-based electromagnetic linear actuator", in *10th Anniversary of European Society for Precision Engineering and Nanotechnology International Conference*, (Zurich, Switzerland), vol. 1, pp. 198-210, May 2008.
3. T. J. Teo, G. L. Yang, I.-M. Chen, W. Lin and H. Z. Li, "A novel actuator with nanometers resolution and high force-current ratio throughout millimeters of stroke", in *7th European Society for Precision Engineering and Nanotechnology International Conference*, (Bremen, Germany), vol. 1, pp. 208-211, May 2007.

4. T. J. Teo, I.-M. Chen, G. L. Yang and W. Lin, "A novel actuator for high-precision alignment in a nanoimprint multi-layers-interconnection fabrication", in *IEEE International Conference on Robotics and Automation*, (Rome, Italy), vol. 1, pp. 1149-1154, April 2007.
5. T. J. Teo, I.-M. Chen, G. L. Yang, and W. Lin, "A flexure-based electromagnetic linear actuator for nano-positioning," in *16th CISM-IFTOMM Symposium on Robot Design, Dynamics, and Control*, (Warsaw, Poland), pp. 371-378, June 2006.
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1. T. J. Teo and I.-M. Chen, "Metrology crisis in nano-manufacturing", *Robotic Research Center of Nanyang Technological University report*, 2006.
2. T. J. Teo, G. L. Yang, and W. Lin, "A flexure-based linear actuator for nano-positioning", *Singapore Institute of Manufacturing Technology annual technical report*, 2005.
3. H. L. Ho, G. L. Yang, W. Lin, W. H. Chen and T. J. Teo, "Development of a 1-DOF flexure-based positioning stage for wafer-bumps inspection", *Singapore Institute of Manufacturing Technology annual technical report*, 2004.

Patents

1. T. J. Teo and G. L. Yang, "Nano-positioning electromagnetic linear actuator", in *World Intellectual Property Organization*, Publication No.: WO/2007/055656, International Application No.: PCT/SG2006/000311.

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