A three dimensional kinematic method for determining the effectiveness of arm segment rotations in producing clubhead speed

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A Three Dimensional Kinematic Method for Determining the Effectiveness of Arm Segment Rotations in Producing ClubHead Speed

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A thesis submitted to the Nanyang Technological University
in partial fulfilment of the requirement for the degree of
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SUMMARY

Most golfers tend to associate the improvement of their games with the improvement of their golf swing. This set a trail of searches for the elusive 'perfect swing'. The aim of this research is to explore various methods to examine, quantify and analyse various aspects of the golf swing. One study involved utilizing electrogoniometer (Biometrics, UK) to obtain the joint angles and joint velocities of the left arm throughout the swing motion. Using algorithm developed with dual number, the dual Euler method, the locations and velocities of the measured segments or end-effectors were calculated. In this way, the swing can be examined, quantified and analysed. Verification of the results with video shows good agreement. The results were then animated to illustrate the arm movement and the swing path of the club. The goniometers and the dual Euler method together give plentiful of details and insights that conventional three-dimensional methods could not reveal. For example, segmental analysis reveals the importance of hand flexion and supination in contributing to high clubhead speed.

Another study presents a novel technique that directly estimates the axis of a screw motion as Plücker line. The Plücker line can be transformed via the dual-number coordinate transformation matrix. Developing the dual transformation matrix (DTM) from noisy data and determining the screw axis from a given DTM is done in a manner analogous to that for handling simple rotations. The aim is to facilitate more accurate computation of the unknown parameters. Simulations involving random errors and skin movement errors were first done. The simulation results showed that the algorithm perform well in the determination of the screw axis direction and

SUMMARY

position. This implies that the technique can be used to locate the axis of a screw motion under noisy environment. The algorithm was then applied on actual experimental data. The method was first applied the movement of the ankle joint which consisted of slow controlled movements. The algorithm produces pretty good results implying that it works well in normal clinical setting. The method was then applied to calculate the screw axis of the left arm with respect to the shoulder joint during a golf swing. The instantaneous screw axes of the arm with respect to the shoulder during a golf swing were determined for the first time. Though best result could not be obtained due to the high sampling frequency, the screw axes still offer golfers an alternative to interpret the swing in a different perspective.

Weight transfer aspect of the swing was also investigated. F-scan (Tekscan, Inc. USA) was used to record the foot pressure of the golfers. Golfer executed a series of swing with the COF (center of force) for each leg traced to determine the weight transfer path and the total force for both legs plotted to identify the weight transfer pattern. The study done on two golfers suggests that consistency, some swing characteristics and even errors could be identified through the Weight-Time graphs and COF graphs. The method should be able to help monitor certain mistakes that golfers make and guide the golfers into correcting it.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

The game of golf, in one form or another, has been around for over five hundred years. It can be assumed that even during the very early stages of the game, players were curious as to how the game can be improved. Over the years, changes to the golf swing have been based more on trial and error than on any application of scientific principles. It has only been over the last several decades that science, and physics in particular, has been used in a significant way to understand and improve the performance of golfers. In fact, it can be argued that the scientific study of the game undertaken by the Golf Society of Great Britain in 1968 in their book *Search for the Perfect Swing* is the starting point of the scientific development of the game of golf. Since then a myriad of papers in a wide range of journals have appeared on the subject. In 1990, the first World Scientific Congress of Golf was held at St Andrews, Scotland. This Congress has met every four years since. This Congress, together with many other conferences and journal articles on golf, reflects the continuous desire and keen interest for scientific research in golf community.

The game of golf has a simple objective of completing the round in the least possible shots. Most golfers tend to associate the improvement of their games with the improvement of their golf swing. This set a trail of searches for the elusive 'perfect swing'. Instead of looking beyond, one might as well examine his or her swing. Though human bodies may have similar features, the length and the portion of each

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segment differ. Additionally, the ability to generate speed may differ widely among individuals. Analysing individual swing may provide valuable insights. The ability to measure and quantify golf swing can reveals strength and weaknesses in the golfers and hence identify areas of improvement. It can also help in comparison with better golfers in a more scientific way.

1.2 OBJECTIVE

The aim of the study is to explore various methods to examine, quantify and analyse various aspects of the golf swing. Commercially available equipment would be used for the measurement and their suitability as a golf-coaching tool would be assessed. Using the measured data, algorithm would be developed for quantifying and calculating the golf swing when possible.

1.3 SCOPE

The scope of the study involves only the golf swing as it is a major aspect in the game of golf. The importance of it can be seen from the huge amount of studies done. The scope of this report involves:

- Quantifying and calculating the individual segmental rotation contribution to the final clubhead speed using the electrogoniometer.
- Calculating the screw axis of the shoulder during the swing using photogrammetric reconstruction method.
- Used of F-scan to analyse foot pressure during the swing and its suitability as a coaching tool.

CHAPTER TWO

LITERATURE REVIEW

2.1 MODELING AND STUDIES OF GOLF SWING

Several researchers have studied the golf swing using the two-link model, or double pendulum (Cochran and Stobbs, 1968; Jorgensen, 1970; Lamps, 1975; Budney and Bellow, 1982). The two links that are used in the model are included in Figure 2-1. The upper link represents the golfer's shoulders and arms, which rotate about a central hub corresponding roughly to a point between the golfer's shoulders. The lower link represents the golf club, which rotates about a point or hinge, located at the center of the hands and wrists of the golfer. The basis for the double pendulum model stems from the modern practice of keeping the left arm (for right-handed golfer) straight during the downswing.

Figure 2-1: The double pendulum model of the golf swing (a) at the beginning of the downswing; (b) at the release point; (c) after releasing the club and (d) at impact.

(Pictures adopted from Penner (2003))

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Budney and Bellow (1982)'s studies indicate that the wrist-cock angle stays approximately constant during the first half of the downswing. Therefore, the downswing in the double pendulum model has typically been divided into two stages. During the first stage, the wrist-cock angle is fixed and the double pendulum system rotates as one. The second stage of the downswing begins when the club swings outwards, either naturally or with a delayed release, and ends when contact with the ball is made. Equations of motions can be derived for the basic double pendulum model using the Langrangian approach (Jorgensen, 1970; Lampsas, 1975). Although, the two-stage double pendulum model is a good approximation to the downswing of skilled golfers, researchers have found that better agreement can be obtained if the constraint of a fixed hub is removed. Jorgensen (1994) did a detailed study on the swing of a professional golfer and found that including a lateral shift of the hub was necessary in order to get good agreement between modeled and experimental positions and speed of the club, obtained from a stroboscopic photograph during the downswing. The parameters used in Jorgensen's double swing model cannot be considered as unique, although the agreement that was obtained between the modeled clubhead speeds and the measured ones was found to be excellent. Reyes and Mittendorf (1999) also analyzed the golf swing of a long-driving champion using a two-dimensional mathematical swing model with the hub constraint removed.

Researchers have used the double pendulum model to determine the effect that the various swing parameters have on the impact speed of the clubhead. The clubhead speed at impact is the primary factor in determining the length of a drive. Increasing the backswing angle or the initial wrist-cock angle in the double pendulum model is found to lead to a greater clubhead speed at impact (Reyes and Mittendorf, 1999).

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Many researchers also considered the effect of delaying the release of the club on the double pendulum swing model and have found that, in general, this will result in a greater clubhead speed (Penner, 2003). While these studies may be interesting, the simple modeling renders the results too basic to be useful in understanding the golf swing. Turner and Hills (1999) have used a triple-link model in their analysis of the golf swing. As with the double pendulum, all three links are taken to be in the same plane. The upperlink represents the motion of the golfer's shoulder as it rotates about the central hub of the golfer. The middle and lower links correspond to the left arm, for a right-handed golfer, and the golf club respectively. However, modeling golf swing as a two-dimensional action may be a little simplistic to provide useful feedback to enhance performance.

Jorgensen (1994) used a two-degree of freedom swing model that constrained in a single plane to examine the values of the angles, angular velocities, and angular acceleration, of the wrist-cock angle. He has however not examined the angular motion beyond the wrist joint. Pickering and Vickers (1999) investigated the effect of positioning the ball so that contact between clubhead and ball takes place at the point in the downswing where the clubhead achieves its maximum horizontal component of velocity. This study provided useful information for golfers about positioning of ball in order to maximize clubhead speed's effect at impact.

2.2 MEASUREMENTS TAKEN ON GOLFERS

In order to measure the positions of the club and golfer during the downswing, researchers have taken high-speed films, videos or stroboscopic photographs of the

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swings of both skilled and unskilled golfers. These have then allowed for the determination of the linear and angular velocities and accelerations of both the club and the golfer. Using measurements taken from the photographed swings of skilled golfers, Milburn (1982) determined the time evolution of the angular velocities and accelerations of both the clubs and arms and Vaughan (1981) determined the time evolution of the speed of the golfer's hands and clubhead. Both found that the velocity of the arms and hands reached a maximum approximately halfway through the downswing and then decreased until impact. For Milburn's golfers, the angular velocity of the club peaked just prior to impact. Cooper and Mather (1994) also determined the time evolution of the angular velocity of the club in their analysis of swings of professional, low-handicap and high-handicap golfers. For professional golfers, the angular velocity of the club was found to peak exactly at ball impact. For low-handicap golfers the angular velocity curve peaked just before impact whereas for high-handicap golfers it was found that the angular velocity of the club increased very rapidly during the first half of the downswing but peaked well before impact and then decreased substantially before impact.

Several researchers have determined, from measured angular velocities and accelerations of the arms and club, the time history of the resultant couple that must have been exerted, in terms of the swing model, at the hub and at the hands. There appear to be fundamental differences in the ways golfers used their hand prior to impact. If golfers swing out the club too early in the downswing they will need to apply a negative couple at the hands later in the downswing in order to retard the swinging out of the club so as to have it in the proper position at impact. If the release of the club is delayed too much, a positive couple would need to be applied at the

golfers' hands in order to get the club around in time for impact. A benefit of this action is that applying a positive couple to the club just prior to impact would result in an increased clubhead speed.

2.3 THREE-DIMENSIONAL ANALYSIS

Although there are many studies done on golf swing, no study has effectively provided quantitative information about arm movement of golf swing. Sprigings and Mackenzie (2002) were noted to have ascertained the sources of segmental power that are responsible for increasing clubhead speed. The study was, however, based on a two-dimensional model that is inadequate in describing three-dimensional segmental movements, especially the rotational movement about the arm's own lengthwise axis which is relevant to golfers who are concerned about the technique of arm swing.

Three-dimensional analysis of end-effector (clubhead or racquet-head) velocity in human kinematics chain, including segmental rotations along their own longitudinal axes, was originated from the study of Sprigings et al. (1994), who investigated the contribution that a segment's anatomical rotation make to racquet head speed and found that the contribution depends on both the segment's angular velocity and the instantaneous position of the head. Their theoretical development was a three-dimensional kinematics method based on formulating vector equations from the results of the video image analysis. Sprigings et al. (1994) cautioned that there are certain inaccuracies in the components summation curve, originating from the inaccuracy of the measurements for pronation/supination of the forearm. The inaccuracies in the direction are directly attributed to the small diameter of the wrist

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segment because this rotation was not directly measured but instead calculated from digitized points surrounding the wrist segment. Although not mentioned, the elbow joint could face the same problem too. The problem is made worse by typical video shortcomings like occlusion and poor image quality. One fundamental problem which largely looms over the accuracy of Sprigling's approach is that the anatomical axes systems chosen for his study were reconstructed from body markers. When markers have errors like unwanted movement (Lu and O'Connor, 1999) and digitizing errors, the reconstructed axes might not truly reflect the joint rotations. Not only do the errors affect the accuracy of the reconstructed anatomical axes, they would further affect the accuracies of the computed vectors and angular velocities.

It is important that the chosen axes are consistent with the functional axes of the joint under study. When only rotation is considered in biomechanical communities, either the joint coordinate system (Grood and Suntay, 1983) or Euler angle convention (Chao, 1980) is commonly adopted as a de facto procedure by practitioners (i.e. coaches and athletes) to describe the joint movements in terms of the associated anatomical rotations (e.g. flexion/extension, pronation/supination, internal/external rotation). However, in order to consider kinematics variables incurred by spatial movements of kinematics chains comprising several body segments connected by joints, neither approach is able to handle the movements of the joints due to their inability in handling the length of the body segments. Euler angles can only describe the rotation of a segment and an additional three-dimensional position vector is required to describe the translation. The position vector is referred with respect to the coordinate system of the fixed segment while the Euler angles are usually referred with respect to the coordinate system of the moving segment. The difference of the

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reference systems for rotation and translation does not facilitate the interpretation of the parameters.

One common and conventional method to describe the kinematic chain is via a rotation matrix and a three-dimensional position vector. The orientation of the moving link with respect to the reference link at any joint configuration can be expressed as the direction cosines between two sets of axes (Zatsiorsky, 1998), and represented by an orthogonal rotation matrix. The combination of a rotation matrix and a position vector, as a 4x4 homogeneous transformation matrix, can fully describe the six-degree-of-freedom joint motion. However, there is no physical interpretation for the parameters in the rotation matrix. Furthermore, the transformation matrix does not handle velocity directly. When finding the velocity of the end-effector, the transformation matrix is used to locate the position vector of the end-effector ($\vec{P}$), which is then cross-producted with the angular velocity ($\vec{\omega}$) before using the transformation matrix again to express the velocity in terms of the end-effector coordinates (Craig, 1989). The angular velocity however cannot be calculated simultaneously with the linear velocity using the transformation matrix method.

The screw axis method could describe full six-degree-of-freedom spatial motions with parameters easy to interpret, however, the technical screw axis method may be considered inappropriate to describe a somatic or anatomic motion. Hence, the dual Euler angles method was preferred in the first part of this study. A modification of the Euler angle formulation which used dual angles has been applied to the study of clinical biomechanics by the authors (Ying and Kim, 2002; Ying et al., 2004; Ying and Kim, 2005; Wong et al., 2005). With the dual Euler angles, the gross motion at

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the joint is represented by three screw motions through the coordinate axes of the reference Cartesian coordinate system. In this way, the rotation and translation of a segment are combined and interpreted with respect to the same Cartesian coordinate system. The dual Euler angles method can be further applied to kinematic chain problem as the trigonometric functions are well defined and all identities of ordinary trigonometry hold true for dual angles (Fischer, 1999), rendering a straightforward formation of transformation matrix. In this way, three-dimensional movements can be described as a series of ordered screw motions about each orthogonal axis in a streamlined matrix form. In this thesis, the velocity of the clubhead was analyzed using the dual velocity, which specifies the velocity distribution of a rigid body in screw motion at any instant time as the dual vector. With the dual velocity formulation, linear and angular velocities of end-effector can be conveniently computed simultaneously because its angular and linear components are merged into dual numbers and treated as a single entity which conventional tools could not afford. Moreover, since the dual velocity is specified as the dual vector, its analysis can be done in a similar manner as displacement analysis via the same dual transformation matrix. This feature, especially not requiring a separate formulation for velocities, might facilitate a practical analysis for practitioners because velocity and displacement analysis can be handled under a single rule. Finally, the Jacobian of a spatial open chain concept (McCarthy, 2000) could be used to relate the contributions of each segmental rotation (joint rates) to clubhead velocity (its end-effector velocity).

2.4 SCREW AXIS ANALYSIS

As mentioned in the previous section, the screw axis method could describe full six-degree-of-freedom spatial motions with parameters easy to interpret. The screw axis method has been employed to describe joint kinematics mainly in clinical assessments, e.g. Engin (1980). Although screw axis is not easily understood by clinician or non-practitioner, nevertheless, it is still a relatively good tool in understanding joint kinematics. Coupled with good graphical representation, the screw axis method does provide a good visual understanding on how the joint moves. The position and orientation of the screw axis will generally change throughout the motion (Woltring et al. 1985). Woltring et al. (1994) used instantaneous screw axes estimated from low-pass smoothed video data. A more thorough description of these parameters has been offered in other investigations (Woltring et al., 1985; Bottlang et al., 1998). Previous reports have investigated screw axis accuracy combining motion analysis technique and smoothing procedure (Bottlang et al. 1998; Stokdijk et al. 1999). Bottlang et al., (2000) has also examined changes in the location of screw axis due to an applied valgu-varus stress in the intact elbow while Fuss (1993) examined changes in the location of screw axis in the knee. Reproducibility of the screw axis has also been investigated. In an assessment of shoulder kinematics done on 47 tested persons under load and unloaded situations, Fuss (1994) found that the curves shapes and helical axis migrations were highly reproducible in the single individual and within the entire test group. New methods have been proposed and some of them do not assume a rigid body motion (Cheze et al, 1998; Halvorsen et al., 1999; Sahan and Joan, 2002). Accuracy describing relative movements between adjacent bodies with an electromagnetic tracking system was also assessed (Duck et al., 2004). In applications of screw axis, usually simple and two-dimensional actions are done;

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even in the more complex shoulder joint, simply abduction of the arm (Fuss, 1994). In addition, the actions were controlled and executed at slow speed due to either the equipment limitation or sensitivity of screw axis calculations.

One of the ways to describe a screw axis is by using the dual vector. The dual angle represents the relative displacement and orientation along a line in space, which can be represented by the dual vector (Fischer, 1999). Chasles's theorem states that a rigid body motion can be specified as a *screw*, a translation parallel to a unit axis and a simultaneous rotation about the same axis. Therefore it is intuitive that the geometry of the screw axis of a spatial displacement can be fully and easily studied using Plücker's coordinates that define the line directly (McCarthy, 2000). The Plücker line (dual vector) can be transformed via the dual transformation matrix. These parameterizations may be manipulated in accordance with rules of the motor algebra (e.g. Fischer (1999)). The robotic community seems to be the first in modern era to have explored the large body of line transformation work established over a century ago, e.g. the position of a robot end-effector may be specified through describing the positions and orientations of its joint axis with respect to one another (Denavit and Hartenberg, 1955). However the biomechanics and photogrammetric communities have not really done so—all conventional techniques use point transformation. Only recently, Ying and Kim (2002) applied Plücker lines to derive dual Euler angles to quantify three-dimensional human joint motions.

The dual number transformation matrix has been shown to be effective means of three-dimensional *line* transformation in displacement analysis (Ying and Kim, 2002). Ying and Kim (2002) also addressed computation of the dual Euler angles from the

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dual transformation matrix. Dual Euler angles were applied to investigate the passive motion characteristics of the human ankle joint complex (Ying et al., 2004). The dual Euler angle method was chosen so as to facilitate description of the translational and rotational movement occurring at both the ankle and subtalar joints (Wong et al., 2005). The repeatability of the methodology was studied based on the inter-tester and intra-tester variability analysis (Ying and Kim, 2005). Dual number and dual transformation matrix have been highly successful in kinematics situations and this thesis demonstrates another of its application by using dual vector to describe the screw axis in the later section.

2.5 FOOT FORCES ANALYSIS

In addition to the analysis of golf swings based on high-speed photography, researchers have also directly measured the forces exerted by golfer. Ground reaction force is one popular force measurement done on golfers. Most studies in the lower limbs focus on the ground reaction force which gives the weight transfer pattern, as summarised by Penner (2003). For example, Koenig et al. (1994) found that, for the golfers they measured, approximately 65% of their weight rested on their back foot at the top of the backswing. The golfers then rapidly shifted their weight to their front foot during the downswing. He also found that skilled golfer transfer their weight faster than unskilled golfers. In addition, the actual weight transfer for both the backswing and the downswing is found to be much less for the less skilled player. The results also seem to indicate that skilled golfers generate more power than unskilled golfers through the lower limbs. Although there exist differences in weight shift between golfers of different skill level, identifying golfers' skill level based on it

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seems difficult. The importance of foot-to-ground interaction on the outcome of the swing was examined by Richards et al. (1985) where it was hypothesized that discrimination between lesser and more skilled players would be possible on the basis of a characteristic ground-force reaction pattern. However, it was concluded that transfer of vertical force from the back foot to the target foot does not appear to be strongly associated with the golfer's skill level. Wallace et al. (1990) agreed with Richards et al. (1985) that the transfer of vertical force from the back foot to the target foot does not appear to be strongly associated with the golfer's skill level. Robinson (1994) studied the correlation between some predefined swing characteristics and the clubhead velocity. Interestingly, he found no correlation between the clubhead velocity at impact and the weight on the left foot at impact, a swing characteristic often emphasized in golf instruction. Another form of analysis of the foot is the Center of Force (COF) pattern analysis which is not so commonly used in golf. Pforringer and Rosemeyer (1989) utilised the analysis in the design of the golf shoe.

2.6 RESEARCH UNDERTAKEN

As we can see from the researches done, there is no aspect of golf that has been discussed, analysed and advised upon more than the golf swing. Though widely researched, the three-dimensional analysis of golf swing lacks the due amount of research it deserves, probably due to equipment limitations and the fast speed nature of the actual swing. Three-dimensional analysis of the swing is necessary in addition to the well-established two-dimensional studies to give proper insights and understanding to the swing. In executing the golf swing, all the parts (body, hands and club) must be moving in proper sequence, a chain reaction that results in the clubhead

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being delivered powerfully and squarely onto the ball (Leadbetter, 2003). Also, temporal proximal-to-distal sequence has been reported in many human movements requiring maximal end-point velocity. Therefore details of how each segmental rotations in the left hand eventually leads to the final velocity and the sequences and coordination of each rotations in executing a swing should be valuable to golfers and coaches. Shoulder joint is the link between the torso and the arm. Therefore the study on the shoulder mechanism in golf swing might add valuable insights and significant information useful to golfers in compliment to the segmental rotations. As mentioned earlier, the screw axis is often used in analysis of joint and is reproducible, therefore a screw axis analysis on the shoulder joint during the swing could be a relevant analysing method. However, the fast nature of the swing would be the challenge in the success of the analysis. Also, in executing the golf swing, the weight shift of the player has always been considered as one of the important factors. Lower limbs have an important role in executing a golf swing as it acts as the base of the golfer. Analysis on foot was mainly on the weight shift, other analysis might prove to be insightful and useful as an analyzing and coaching tool. The following chapters of this thesis try to handle these issues. Left arm segmental rotations were analysed using a new algorithm, dual Euler analysis, with the measurement taken from goniometer. The sequence and coordination of each segment were analysed with the graphs from the dual Euler analysis. A novel technique that directly estimates the axis of a screw motion as Plücker line will be presented and the screw axes of the shoulder joint during a swing will be shown for the first time. F-scan, a popular foot pressure measurement system, was tested on golfers to check the capability of the measurement on golf swing and its suitability as an analysing and training tool.

CHAPTER THREE

DUAL EULER ANALYSIS

As mentioned in the 'LITERATURE REVIEW' chapter, conventional three-dimensional kinematics methods have not been satisfactory in studying golf swing. This chapter introduces the underlying methodology for the kinematics study of the left arm used in this project. It also set the basis for chapter 4 and 5 that follows. In this chapter, the dual number is first explained, follow by the modeling of the subject and golf club as linkages. The transformation matrices illustrated the result of combining the dual number with the modeling. Finally, the main components of the analysis, the position and velocity analysis are introduced at the end of this chapter.

3.1 DUAL NUMBER METHOD

3.1.1 Dual Angles

The concept of dual number was introduced by Clifford during 1873 and the name was given by Study in 1903.

The dual number ε is defined such that

$$\varepsilon \neq 0 \quad (3-1)$$

$$\varepsilon^2 = 0 \quad (3-2)$$

A dual number is written as

$$\hat{a} = a + \varepsilon a_0 \quad (3-3)$$

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Where symbol a represents the primary (or real) part of duplex (or dual) number $\hat{a}$ and symbol a_0 represents the dual component of dual number $\hat{a}$.

The dual angle represents the relative displacement and orientation between two lines in space (as illustrated in Figure 3-1). If lines A and B intersect, dual angle $\hat{\theta}$ is a pure primary number. If the lines are parallel, dual angle $\hat{\theta}$ is a pure dual number.

Figure 3-1: The dual angle $\hat{\theta} = \theta + \epsilon s$ express the relationship between lines in space A and B.

3.1.2 Trigonometric Functions, Kinematics and Dual Euler Angles

The trigonometric functions of a dual angle are obtained using the Taylor-series expansion. Consequently,

$$\sin \hat{\theta} = \sin \theta + \epsilon s \cos \theta \quad (3-4)$$

$$\cos \hat{\theta} = \cos \theta - \epsilon s \sin \theta \tag{3-5}$$

$$\tan \hat{\theta} = \tan \theta + \epsilon s \sec^2 \theta \tag{3-6}$$

All identities of ordinary trigonometry hold true for dual angles.

Figure 3-2: Screw motion through the X-axis.

Consider a rigid body moving from the initial position, where the local coordinate system M on the rigid body coincides initially with the global coordinate system G, to another position in space by rotating about and translating along the X-axis of G as

shown in Figure 3-2. The screw motion displacement of the rigid body through the X-axis can be expressed in the form as $\hat{\alpha} = \alpha + \varepsilon a$, in which α represents the rotation angle about the X-axis and a represents the translation distance along the X-axis.

At the initial position, a vector on the rigid body is expressed in the form $\hat{\mathbf{V}}_0 = \mathbf{V}_0 + \varepsilon \mathbf{W}_0$, where $\mathbf{V}_0$ represents the magnitude and direction of the vector with respect to G, and $\mathbf{W}_0 = \mathbf{r} \times \mathbf{V}_0$, where $\mathbf{r}$ is a vector connecting the origin of G to any point on the line, on which the vector lies. After the screw motion through the X-axis, the same vector moves to the position 2. At the final position, the vector can be represented similarly as $\hat{\mathbf{V}} = \mathbf{V} + \varepsilon \mathbf{W}$. Then the vector satisfies the following dual transformation relationship

$$\hat{\mathbf{V}} = [\hat{\mathbf{R}}_x(\hat{\alpha})] \hat{\mathbf{V}}_0 \quad (3-7)$$

where

$$[\hat{\mathbf{R}}_x(\hat{\alpha})] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \hat{\alpha} & -\sin \hat{\alpha} \\ 0 & \sin \hat{\alpha} & \cos \hat{\alpha} \end{bmatrix} \text{ is the dual transformation matrix.}$$

In the same manner, the dual transformation matrices for a screw motion with respect to the Y-axis with a dual angle $\hat{\beta} = \beta + \varepsilon b$ and with respect to the Z-axis with a dual angle $\hat{\gamma} = \gamma + \varepsilon c$ are

$$[\hat{\mathbf{R}}_y(\hat{\beta})] = \begin{bmatrix} \cos \hat{\beta} & 0 & \sin \hat{\beta} \\ 0 & 1 & 0 \\ -\sin \hat{\beta} & 0 & \cos \hat{\beta} \end{bmatrix} \text{ and}$$

$$[\hat{\mathbf{R}}_z(\hat{\gamma})] = \begin{bmatrix} \cos \hat{\gamma} & -\sin \hat{\gamma} & 0 \\ \sin \hat{\gamma} & \cos \hat{\gamma} & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ respectively.}$$

A general spatial motion of a rigid body moving in space can be considered as three successive screw motions about the axes of the coordinate system on the rigid body or the global coordinate system. Any sequence of screw motions about the axes can be chosen as long as the same axis is not repeated consecutively. The resultant dual transformation matrix $[\hat{R}]$ is the combination of all three dual transformation matrices. In this study, screw motions about the coordinate system which are fixed on the moving segment are used to describe joint motion, and the three dual angles expressing the screw motions about the axes of the moving segment coordinate system are called dual Euler angles.

Similar to Euler angles method, the sequence of screw motions is important in dual Euler angles method. For example, if the sequence of screw motions is chosen as first with respect to the z-axis, then with respect to the new y-axis, and finally with respect to the new x-axis, the resultant dual transformation matrix can be obtained by

$$[\hat{R}] = [\hat{R}_z(\hat{\gamma})][\hat{R}_y(\hat{\beta})][\hat{R}_x(\hat{\alpha})] \quad (3-8)$$

Similar to the ordinary transformation matrix in Euler angles method, the dual transformation matrix is orthogonal, that is $[\hat{R}][\hat{R}]^T = I$.

3.1.3 Origin-Displacement Equation

We have seen that algebra of dual vectors has all the properties of conventional vector algebra. The “origin-displacement equation” is a relationship in dual-number mathematics which allows the relative position of one coordinate frame with respect to the other to be recovered from the dual-number coordinate-transformation matrix between them (Fischer, 1999). It was developed by Hsia and Yang (1981) who

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referred to it as the “principle of transference”—the connection that it provides between the geometry of points and the geometry of lines.

Consider two frames $\{A\}$ and $\{B\}$, let

$R = 3 \times 3$ primary-number matrix describing change in orientation of frame $\{B\}$ relative to frame $\{A\}$.

$\vec{D} = 3 \times 1$ primary-number vector describing displacement of origin of frame $\{B\}$ relative to frame $\{A\}$.

$[D] = 3 \times 3$ cross-product matrix of translational vector $\vec{D}$

$[I] = 3 \times 3$ identity matrix

From the definition of a dual vector, the dual transformation matrix of frame $\{B\}$ relates to frame $\{A\}$ as follows (see Fischer (1999) for detail derivation):

$${}^B_A \hat{M} = ([I] + \varepsilon[D])R$$

which can also be decomposed into primary and dual component

$${}^B_A \hat{M} = V + \varepsilon W$$

Equating the primary and dual components, we obtain

$$V = [R]$$

$$W = [D][R]$$

Which implies

$$[D] = WV^T \tag{3-9}$$

Therefore, given the dual transformation matrix, $\hat{M} = V + \varepsilon W$ between two frames, we can recover a 3×3 cross-product matrix, which forms the displacement vector between the origins of two frames.

3.1.4 Dual Velocities

Let the speed for the screw motion be:

V = linear speed along the screw axis

Ω = angular speed about the screw axis

The direction and location of the screw axis can be specified by the unit screw vector

$$\hat{\mathbf{u}} = \vec{\mathbf{u}} + \epsilon \left(\vec{\mathbf{OP}} \times \vec{\mathbf{u}} \right) \quad (3-10)$$

Where symbol $\vec{\mathbf{u}}$ represents a unit vector and the vector $\vec{\mathbf{OP}}$ extends from the origin of the coordinate system to any point on the screw axis. These quantities can be combined into a “motor”, $\hat{V}$, the dual multiple of unit line vector

$$\hat{V} = (\Omega + \epsilon V) \hat{\mathbf{u}} \quad (3-11)$$

It is observed that $\hat{V}$ and $\hat{\mathbf{u}}$ are dual vectors while Ω and V are scalars. Eq.(3-11) provides the dual velocity of any point P on the screw axis (all points on the screw axis have the same dual velocity).

We can also use transformation matrices to determine the dual velocity at the screw axis. The matrix approach can take into account not only the translation between the frame with origin on the screw axis and frame with origin at the point of interest, but also any rotation between the frames. Therefore we adopt a notation for dual velocity such that:

${}^R \hat{V}_{ij}^P$ = dual velocity at a point P of body i relative to body j in terms of the unit vectors of frame $\{R\}$.

If we know the dual velocity $\hat{V}_{ij}^{S \wedge S}$ of a rigid body expressed in a frame $\{S\}$ with origin at a point S on the screw axis, then the dual velocity at any other point P in the body expressed in the unit vectors of a frame $\{P\}$ with origin at point P and possibly rotated relative to frame $\{S\}$ is

$$\hat{V}_{ij}^{P \wedge P} = {}_S^P \hat{T} \hat{V}_{ij}^{S \wedge S} \quad (3-12)$$

To express this velocity in the unit vectors of a frame $\{Q\}$ different from frame $\{P\}$, it is necessary to consider only the rotation between the two frames. Therefore

$$\hat{V}_{ij}^{Q \wedge P} = {}_P^Q \hat{T}_P \hat{V}_{ij}^{P \wedge P} \quad (3-13)$$

Where matrix ${}_P^Q \hat{T}_P$ is the primary component of the dual-number coordinate-transformation matrix ${}_P^Q \hat{T}$. Because dual velocity is not a line vector, the translation between different frames in which it is expressed is not a factor.

The relative-velocity equation of elementary kinematics can be extended to dual-number mechanics

$$\hat{V}_{ki} = \hat{V}_{ji} + \hat{V}_{kj} \quad (3-14)$$

Where $\hat{V}_{mn}$ denotes the dual velocity of link m relative to link n . When link n is the ground (fixed or inertial body) that subscript is often omitted in the elementary kinematics formulation.

In order to add these dual velocities, they must all represent motions at the same point and be expressed in the same frame. To expressed every dual velocity at the same

point and then the same unit vectors, we transform $\hat{V}_{ji}^A$ to $\hat{V}_{ji}^B$ by

$${}^B\hat{V}_{ji} = {}^B\hat{T}_A {}^A\hat{V}_{ji} \quad (3-15)$$

Substituting Eq.(3-15) into Eq.(3-14) we have

$${}^B\hat{V}_{ki} = {}^B\hat{T}_A {}^A\hat{V}_{ji} + {}^A\hat{V}_{kj} \quad (3-16)$$

This is the relative dual-velocity equation. If dual velocity ${}^B\hat{V}_{ki}$ is required in the unit vectors of some frame $\{C\}$, we simply rotate:

$${}^C\hat{V}_{ki} = {}^C\hat{T}_B {}^B\hat{V}_{ki}. \quad (3-17)$$

For more details on the dual velocity, refer to Fischer (1999).

3.2 MODELING

In this study, a golfer performing golf swings was depicted as a serially joined five segment model consisting of the torso, left upper arm, left forearm, left hand and club (Figure 3-3). The procedure for computing the anatomical rotations consisted of attaching an orthogonal Cartesian frame to each of the segments. The local frame $\{1\}$ was attached to the rotating torso (segment 1) at the glenohumeral (shoulder) joint, $\{2\}$ was attached to the upper arm (segment 2) at the elbow joint, $\{3\}$ was attached to the forearm (segment 3) at the wrist joint, $\{4\}$ was attached to the hand (segment 4) at the end of the hand grip and finally $\{5\}$ was attached to the center of the clubhead (segment 5 is the golf club). In addition, a fixed frame $\{0\}$, with axes coinciding with that of the other local frames at neutral position, was attached to the fixed lower extremity at the waist. The directions of the three axes of each frame were assigned so as to approximate the different anatomical axes of rotation for each segment. It is

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Figure 3-3. The anatomical position and positions of markers on the subject and the clubhead.

important that the chosen axes are consistent with the functional axes of the joint examined in the study. The origins of these frames are located at the center of each relevant joint. The global frame, $\{G\}$, was fixed to the laboratory floor. The neutral positions of all local frames are chosen to be the positions when a subject assumes the classical anatomical posture (Figure 3-3).

3.3 TRANSFORMATION MATRICES

In order to quantify the joint motions of the swing, the dual Euler angles method [proposed by Ying and W.Kim (2002)] was used. The use of the dual number allows considerably compact formulations facilitating the analysis of biomechanical system. Unlike Euler angles which describe movements with the three anatomical rotations at the human joints, dual Euler angle takes account of the length of arm segment which are essential for calculating the end point velocity under a multi joint motor task such as a golf swing.

Similar to the Euler angle method, the order of the sequence of screw motions is important in dual Euler angle method. The $Zy'x''$ sequence of three screw motions was used by adopting the so called Aerospace Sequence (Kuipers, 1999); screw motion with respect to the Z -axis, attached in the proximal segment, then with respect to the new y -axis (y' -axis) which is called the floating axis, and finally with respect to the new x -axis (x'' -axis), attached in the distal segment. These local axes are denoted by sign prime (') and double prime (") according to the number of preceding screw motions defining their positions. Each joint permits motion with respect to all three axes of a set of mutually perpendicular axes.

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A human body in action can be modeled as a multilink system comprising several body segment connected by joints (Vaughan et al, 1982). A linkage of rigid bodies is referred to as a kinematics chain. With this work, the golf swing is modeled as a five-link serially chained motion. The links are numbered from 1 to 5. Except for the last link, each link of the chain has two joints connecting it to adjacent bodies. Local coordinate system, x_i, y_i, z_i , are attached to each link i . The position of frame $\{j\}$

relative to the frame $\{i\}$ is described by the 3×3 dual transformation matrix ${}^i\hat{M}_j$. The

operator ${}^i\hat{M}_j$ allows any vector expressed in terms of the unit vectors of frame $\{j\}$ to be expressed in terms of the unit vectors of frame $\{i\}$. The dual transformation matrices of local frames are assumed to be known by goniometer readings and anthropometrics measurement of the subject model. Thus considering the Denavit-Hartenberg parameters (Denavit and Hartenberg, 1955) the individual joint-link transformation matrices based on dual Euler angles can be expressed by concatenating the matrices:

${}^G\hat{M}_0, {}^0\hat{M}_1, {}^1\hat{M}_2, {}^2\hat{M}_3, {}^3\hat{M}_4$ and ${}^4\hat{M}_5$ with respect to the global frame $\{G\}$.

The transformation matrix of frame $\{0\}$ with respect to the global frame is represented by ${}^G\hat{M}_0$, which can be calculated from G_0R , the rotation matrix of frame $\{0\}$ with respect to the frame $\{G\}$, and the displacement between frame $\{0\}$ and frame $\{G\}$.

The movement of torso, $\{1\}$, with respect to the fixed lower extremity, $\{0\}$, is represented by:

$${}^0\hat{M}_1 = {}^0\hat{M}_{0'} {}^{0'}\hat{M}_{0''} {}^{0''}\hat{M}_1$$

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$$= \left[\hat{R}_Z(\hat{\gamma}_0) \right] \left[\hat{R}_{y'}(\hat{\beta}_0) \right] \left\{ \left[\hat{R}_{x''}(\hat{\alpha}_0 + \varepsilon L_{0x}) \right] \left[\hat{R}_Z(\varepsilon L_{0z}) \right] \right\} \quad (3-18a)$$

where L_{0z} and L_{0x} are the respective lengths along the z and x-axis measured from frame {1} with respect to frame {0}. γ_i , β_i and α_i are the Euler angles for the Zy'x'' sequence taken with respect to the neutral position. Frame {0} was selected with the origin at the waist, with the x-axis pointing inferiorly, the y-axis pointing posteriorly and the z-axis pointing laterally (to the left of the subject) with respect to the lower extremity.

The movement of upper arm, {2}, with respect to the torso, {1}, at the shoulder joint is represented by:

$${}^1_2 \hat{M} = {}^1_{1'} \hat{M} {}^{1'}_{1''} \hat{M} {}^{1''}_2 \hat{M} \\ = \left[\hat{R}_Z(\hat{\gamma}_1) \right] \left[\hat{R}_{y'}(\hat{\beta}_1) \right] \left[\hat{R}_{x''}(\hat{\alpha}_1 + \varepsilon L_1) \right] \quad (3-18b)$$

where L_1 represents the upper arm segmental length, the distance between the glenohumeral joint and the elbow joint. Frame {1} was assigned to the torso with its origin at the glenohumeral joint, with the x-axis pointing inferiorly, the y-axis pointing posteriorly and the z-axis pointing laterally (to the left of the subject) with respect to the torso.

The movement of lower arm, {3}, with respect to the upper arm, {2}, at the elbow joint is represented by:

$${}^2_3 \hat{M} = {}^2_{2'} \hat{M} {}^{2'}_3 \hat{M} \\ = \left[\hat{R}_Z(\hat{\gamma}_2) \right] \left[\hat{R}_{x''}(\hat{\alpha}_2 + \varepsilon L_2) \right] \quad (3-18c)$$

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where L_2 represents the lower arm segmental length, which is defined as the distance between the axes of flexion-extension of the elbow and the wrist joints at {2} and {3} respectively. Frame {2} was constructed with its origin at the elbow joint, with the z-axis coinciding with the flexion-extension axis at the elbow and the x-axis defined as a common normal to the axes of retroversion-anteversion in the glenohumeral and the axes of flexion-extension in the elbow joint. The y-axis was constructed as orthogonal to both x and z-axis.

The movement of hand, {4}, with respect to the lower arm, {3}, at the wrist joint is represented by:

$${}^3\hat{M}_4 = {}^3\hat{M}_{3'} {}^{3'}\hat{M}_{3''} {}^{3''}\hat{M}_4$$

$$= \left[\hat{R}_z(\hat{\gamma}_3) \right] \left[\hat{R}_{y'}(\hat{\beta}_3 + 15^\circ) \right] \left[\hat{R}_{x''}(\epsilon L_3) \right] \quad (3-18d)$$

where L_3 represents the length between the wrist joint to the grip. The addition 15° at β_3 arises from the way the subject (golfer) held the club. Frame {3} was constructed with its origin at the wrist joint, with the z-axis coinciding with the flexion-extension axis at the wrist, the x-axis defined as common normal to the axes of flexion-extension in the elbow and the wrist joint. The y-axis was constructed as orthogonal to both x and z-axis.

The movement of clubhead, {5}, with respect to the hand, {4}, is represented by:

$${}^4\hat{M}_5 = {}^4\hat{M}_{4'} {}^{4'}\hat{M}_5$$

$$= \left[\hat{R}_z(\epsilon L_4) \right] \left[\hat{R}_{x''}(-\epsilon L_5) \right] \quad (3-18e)$$

where L_4 represents the total sum of the grip length, shaft length and hosel length of the club. L_5 represents the distance from the end of the hosel to the center of the clubhead. Frame {4} was constructed with its origin at the grip end, with the z-axis coinciding with the shaft of the club and the x-axis pointing in the opposite direction as the clubhead, perpendicular to the shaft. The y-axis was constructed as orthogonal to both x and z-axis. Frame {5} was constructed with its origin at the middle of the clubhead and its orientation same as Frame {4}.

For the algorithm presented, the following assumptions are made: (a) The constructed orthogonal axes for the three segments closely represent their anatomical axes; (b) The valgus/varus rotation at the elbow joint is assumed as zero; (c) The longitudinal rotation of the hand at the wrist joint is also assumed to be zero; (d) The hand and club are one single rigid body; (e) The torso's linear velocities are negligible to club's overall velocity; (f) The interjoint offsets, defined as the distance between the origins of frames { i } and { $i+1$ } measured along the joint axis, z_i , are assumed to be negligible; (g) The twisted angle, which differs from zero when the axes of flexion-extension in the two joints of the same link (for example the elbow and wrist) are not exactly in the same (frontal) plane, are small enough to be ignored.

The successive coordinate transforms are obtained beginning from the torso to the upper arm to the lower arm then to the hand and finally to the clubhead, thereby the position of the end-effector is given as the concatenation of each joint-link matrices. The resultant dual transformation matrix can be obtained by

$${}^G\hat{M}_5 = {}^G\hat{M}_0 {}^0\hat{M}_1 {}^1\hat{M}_2 {}^2\hat{M}_3 {}^3\hat{M}_4 {}^4\hat{M}_5 \quad (3-19)$$

3.4 DUAL POSITION ANALYSIS

Visual feedback is essential in many analysis systems. Visual feedback will help researchers and the users understand the situation better and accelerate the understanding and learning process. Playing back the pictures in slow motion or frame-by-frame is a popular form of analysis and provides valuable information. This advantage can be seen from the popularity of using video analysis. Using goniometer alone does not provide any visual feedback to the user or researcher. This thesis hereby demonstrates the availability of using dual Euler analysis in providing visual feedback. Although the motion is not capture in picture form like the video method, the data collected can be reconstructed in animation form to give visual feedback.

Using the “origin-displacement equation” method, the position of the elbow, wrist and clubhead can be located. We adopt the following notation for position vector in this paper:

iP_e = position vector of element e in terms of coordinate frame i .

Therefore, the position vectors of the various link-ends in terms of the global coordinate is:

$$\begin{aligned}
 \text{Elbow:} \quad & {}^G P_{\text{elbow}} = {}^G R^1 D_{\text{elbow}} + {}^G P_{\text{shoulder}} \\
 \text{Wrist:} \quad & {}^G P_{\text{wrist}} = {}^G R^1 D_{\text{wrist}} + {}^G P_{\text{shoulder}} \\
 \text{Grip:} \quad & {}^G P_{\text{grip}} = {}^G R^1 D_{\text{grip}} + {}^G P_{\text{shoulder}} \\
 \text{Clubhead:} \quad & {}^G P_{\text{club head}} = {}^G R^1 D_{\text{club head}} + {}^G P_{\text{shoulder}}
 \end{aligned} \tag{3-20}$$

Where

${}^G P_{\text{shoulder}}$ is taken from the camera system.

${}^G_1R = {}^G_0R {}^0_1R$, is calculated using the algorithm proposed by Umeyama (1991) using the camera data.

iD_j is the position of the origin of frame $\{j\}$ in terms of coordinate frame $\{i\}$ calculated from ${}^i\hat{M}_j$.

3.5 DUAL VELOCITY ANALYSIS

In golf swing, the goal is to attain the maximal velocity of the most distal segment, specifically its terminal point, the clubhead. When an activated muscle develops tension, the tensile forces pulls on the attached bones and creates torque at the joints crossed by the muscle. The torque at the joints develops the joint velocity, which can be viewed as the difference between the corresponding segment velocities. These quantities can be combined into a “motor” and expressed in dual velocity.

Since the actions at segmental level involve only rotational actions and not translational ones, the dual velocities are simply left with the primary component only. Accordingly, the velocities by the torso rotations and those at the shoulder, elbow and wrist joints were obtained by differentiating the angular displacement time curves of the respective rotations (γ_i , β_i and α_i) and their angular velocities are denoted as $\dot{\gamma}_i$, $\dot{\beta}_i$ and $\dot{\alpha}_i$ respectively. Therefore, the velocities at each segment are as follows:

$$\text{Torso: } {}^0\hat{V}_{0G} = \begin{Bmatrix} 0 \\ 0 \\ \dot{\gamma}_0 \end{Bmatrix}, \quad {}^{0'}\hat{V}_{0'0} = \begin{Bmatrix} 0 \\ \dot{\beta}_0 \\ 0 \end{Bmatrix}, \quad {}^{0''}\hat{V}_{0''0'} = \begin{Bmatrix} \dot{\alpha}_0 \\ 0 \\ 0 \end{Bmatrix} \quad (3-21a)$$

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where ${}^0\hat{V}_{0G}^{o_0}$ represents the dual velocity at origin, o_0 , of frame $\{0\}$ caused by trunk flexion/extension in terms of reference frame $\{0\}$, ${}^{0'}\hat{V}_{0'0}^{o_0}$ represents the dual velocity at origin, o_0 , of frame $\{0'\}$ caused by trunk side bending in terms of the rotated reference frame $\{0'\}$ and ${}^{0''}\hat{V}_{0''0'}^{o_0}$ represents the dual velocity at origin, o_0 , of frame $\{0''\}$ caused by trunk rotation in terms of the further rotated reference frame $\{0''\}$. Again, these local axes are denoted by sign prime (') and double prime (') according to the number of preceding screw motions defining their positions

$$\text{Shoulder joint: } {}^1\hat{V}_{10''}^{o_1} = \begin{Bmatrix} 0 \\ 0 \\ \dot{\gamma}_1 \end{Bmatrix}, \quad {}^{1'}\hat{V}_{1'1}^{o_1} = \begin{Bmatrix} 0 \\ \dot{\beta}_1 \\ 0 \end{Bmatrix}, \quad {}^{1''}\hat{V}_{1''1'}^{o_1} = \begin{Bmatrix} \dot{\alpha}_1 \\ 0 \\ 0 \end{Bmatrix} \quad (3-21b)$$

where ${}^1\hat{V}_{10''}^{o_1}$ represents the dual velocity at origin, o_1 , of frame $\{1\}$ caused by upper arm anteversion/retroversion in terms of reference frame $\{1\}$, ${}^{1'}\hat{V}_{1'1}^{o_1}$ represents the dual velocity at origin, o_1 , of frame $\{1'\}$ caused by upper arm abduction/adduction in terms of rotated reference frame $\{1'\}$ and ${}^{1''}\hat{V}_{1''1'}^{o_1}$ represents the dual velocity at origin, o_1 , of frame $\{1''\}$ caused by upper arm internal/external rotation in terms of the further rotated reference frame $\{1''\}$.

$$\text{Elbow joint: } {}^2\hat{V}_{21''}^{o_2} = \begin{Bmatrix} 0 \\ 0 \\ \dot{\gamma}_2 \end{Bmatrix}, \quad {}^{2'}\hat{V}_{2'2}^{o_2} = \begin{Bmatrix} \dot{\alpha}_2 \\ 0 \\ 0 \end{Bmatrix} \quad (3-21c)$$

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where ${}^2 \hat{\wedge}_{o_2}$ represents the dual velocity at origin, o_2 , of frame {2} caused by lower arm flexion/extension in terms of reference frame {2} and ${}^{2'} \hat{\wedge}_{o_2}$ represents the dual velocity at origin, o_2 , of frame {2'} caused by forearm pronation/supination in terms of rotated reference frame {2'}.

$$\text{Wrist joint: } {}^3 \hat{\wedge}_{o_3} = \begin{Bmatrix} 0 \\ 0 \\ \dot{\gamma}_3 \end{Bmatrix}, \quad {}^{3'} \hat{\wedge}_{o_3} = \begin{Bmatrix} 0 \\ \dot{\beta}_3 \\ 0 \end{Bmatrix} \quad (3-21d)$$

where ${}^3 \hat{\wedge}_{o_3}$ represents the dual velocity at origin, o_3 , of frame {3} caused by hand flexion/extension in terms of reference frame {3} and ${}^{3'} \hat{\wedge}_{o_3}$ represents the dual velocity at origin, o_3 , of frame {3'} caused by hand ulnar/radius abduction in terms of rotated reference frame {3'}.

The contribution of individual joint rotation to the clubhead velocity in biomechanics will be explained in the context of the Jacobian in robotic research (Craig, 1989). First we note that Jacobian matrix relates joint velocities to Cartesian velocities. For example:

$$\mathbf{V} = \mathbf{J}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}} \quad (3-22a)$$

where $\boldsymbol{\theta}$ is the vector of joint angles of the upper extremity segments, and $\mathbf{V}$ is a vector of Cartesian velocities of the end-effector. Elements of the Jacobian, $\mathbf{J}(\boldsymbol{\theta})$ are partial derivatives that relate the infinitesimal displacement of the end point to the infinitesimal joint displacement at the present joint configuration. Note that for any given configuration of the kinematics chain, joint rates are related to velocity of the end-effector in a linear fashion. Finding Jacobians is not a simple matter for robotic

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researcher as well as biomechanical researchers alike even it could be obtained by directly differentiating the kinematics equations of the given mechanism. Hence, in this work, a novel approach was introduced to derive Jacobian in human kinematics chains by using the dual velocities and dual transformation matrix that have previously derived in the multilink chain and transformation analysis.

The angular velocities of each segment relate to both rotational and linear velocity at the clubhead. In order to find out how each segmental rotation contributes to the clubhead velocity, the angular velocities of the segments about their respective axes were multiplied with the corresponding dual Euler transformation matrices to determine the dual velocity at the clubhead.

Using the wrist as an example, the two segmental actions that contribute to the clubhead movement are classified as the hand flexion/extension and ulnar/radius abduction respectively. To determine the contribution of these two actions on the clubhead velocity, the segmental velocities were multiplied with the corresponding dual transformation matrices from its corresponding frame to the clubhead frame:

Hand ulnar/radius abduction contribution: ${}^5_3 \hat{M} {}^{3'}_3 \hat{V}_{3'3}$ where the dual transformation matrix ${}^5_3 \hat{M}$ transforms ${}^{3'}_3 \hat{V}_{3'3}$ to the terminal point (clubhead frame). The matrix is referred to as the structural equation of the kinematics chain moving from the wrist to the end point.

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Hand flexion/extension contribution : ${}^5\hat{M}_3 \quad {}^3\hat{V}_{32}^{o_3}$ where ${}^5\hat{M}_3$ transforms ${}^3\hat{V}_{32}^{o_3}$ to the terminal point (clubhead frame).

The total wrist contribution to the clubhead is simply the sum of the two actions:

$$\text{wrist contribution : } {}^5\hat{M}_3 \quad {}^3\hat{V}_{32}^{o_3} + {}^5\hat{M}_{3'} \quad {}^{3'}\hat{V}_{3'3}^{o_3} \quad (3-22b)$$

Note that Eq.(3-22b) states that the first element of the Jacobian for the wrist joint equals the dual transformation matrix between frame {3} caused by hand flexion/extension and the clubhead frame. The second element of this Jacobian equals the dual transformation matrix between frame {3'} caused by ulnar/radius abduction and the clubhead frame. This example illustrates that the Jacobian is defined simply by writing the dual transformation matrix between the acting joint axis and clubhead frame.

Therefore the dual velocity of end point (i.e. the point on the center of the clubhead) is given by sum of the contribution of each segmental rotation such that:

$$\begin{aligned} {}^5\hat{V}_{50}^{o_5} = & {}^5\hat{M}_0 \quad {}^0\hat{V}_{0G}^{o_0} + {}^5\hat{M}_{0'} \quad {}^{0'}\hat{V}_{0'0}^{o_0} + {}^5\hat{M}_{0''} \quad {}^{0''}\hat{V}_{0''0}^{o_0} + {}^5\hat{M}_{10} \quad {}^{10}\hat{V}_{10''}^{o_1} + {}^5\hat{M}_{1'} \quad {}^{1'}\hat{V}_{1'1''}^{o_1} + {}^5\hat{M}_{1''} \quad {}^{1''}\hat{V}_{1''1''}^{o_1} + {}^5\hat{M}_2 \quad {}^2\hat{V}_{21''}^{o_2} \\ & + {}^5\hat{M}_{2'} \quad {}^{2'}\hat{V}_{2'2}^{o_2} + {}^5\hat{M}_{3} \quad {}^3\hat{V}_{32'}^{o_3} + {}^5\hat{M}_{3'} \quad {}^{3'}\hat{V}_{3'3}^{o_3} \end{aligned} \quad (3-22c)$$

where ${}^5\hat{V}_{50}^{o_5}$ represents the dual velocity at origin, o_5 , of frame {5} with respect to frame {0} in terms of reference frame {5}. The resultant velocity of the clubhead is defined as the dual component of ${}^5\hat{V}_{3'0}^{o_5}$ while the primary component is the angular velocity of the clubhead with reference to {5}. The clubhead velocity can be written in terms of the global frame {G} as shown in Eq.(3-23) to facilitate the comparisons between other systems.

$${}^G \hat{V}_{5G} = {}^G \hat{M}_p {}^5 \hat{V}_{50} \quad (3-23)$$

where matrix ${}^G \hat{M}_p$ is the primary component of the dual transformation matrix of ${}^G \hat{M}$, which equals to the rotation matrix. Because the dual velocity is not a line vector, the translation between different frames in which it is expressed is not a factor.

Since the directional contributions of individual segments relative to the clubhead velocity are different, the magnitudes of the vector sums of the dual vectors in Eq.(3-22c) do not immediately represent the contributions of each part. Hence, the projections of the dual vectors to the direction of the actual resultant velocity of the clubhead must be determined first before the percentage of the resultant velocity generated by individual joints are to be determined.

Equation (3-22c) indicates that to determine the end-point velocity from joint movement, the following information is needed: length of links involved, joint angular velocities and the angle of the distal joint in the transformation matrix. In this study of golf swing, the clubhead was considered the end effector. The dual Euler analysis procedure include, in short, three main steps:

- 1) The joint angular velocities for the torso, shoulder, elbow and wrist were represented as dual vectors or motors. The torso rotations: $\dot{\alpha}_0$, $\dot{\beta}$ and $\dot{\gamma}_0$, are calculated using the algorithm proposed by Umeyama (1991). From the coordinates of the 3 markers placed on the subject's body (see Figure 3-3), the rotation matrices of the torso from one frame to the next can be obtained; after

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which the angular velocity for the torso rotation can be derived. The rest of the angular velocities were obtained from the electrogoniometer data.

- 2) The dual vectors from individual joints to the center of the clubhead were determined as in the dual transformation matrix.
- 3) The dual velocities from a series of joint rotation were calculated and added in Eq.(3-22c).

CHAPTER FOUR

EXPERIMENT & RESULTS

After going through the underlying methodology in the previous chapter, we now apply the method to golfers. This chapter introduces the experimental aspect of the project. Verification of the method with conventional method and the results of the various analyses applicable are presented in this chapter too.

4.1 EQUIPMENT

The experiment uses the following equipment:

- Electrogoniometers (Biometrics, UK), maximum frequency 1000Hz.
- Biometrics Datalog v 2.0a software.
- Mathcad (Mathsoft Engineering & Education, Inc. Cambridge, USA).
- Matlab (The Math Works, Inc., Natick, USA).
- PEAK MOTUS (Peak Performance Technologies, Inc; USA).

4.2 EXPERIMENTAL PROCEDURE

There are three subjects involved for this study: subject 1 is a semiprofessional golfer with a single handicap while subject 2 and subject 3 are weekend golfers. The method involved utilizing electrogoniometer (Biometrics, UK) to obtain the joint angles throughout the motion and by using dual Euler position and velocity analysis, the locations and velocities of the measured segments can be estimated. The use of

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goniometer to measure joint rotations is widely accepted and tested (Chao, 1980; Lindsay and Horton, 2002). Flexible goniometers were used for this experiment as it is reliable (Shiratsu and Coury, 2003) and its small and light properties are highly suitable for motion analysis. Two electrogoniometer (EGMs) were used to record movement about the shoulder joint, one 2D-EGM (anterversion/retroversion, abduction/adduction) and one torsionmeter (external/internal rotation). The goniometers were attached to the acromion process and to the upper arm, just below the deltoid attachment. Another goniometer (2D-EGM, one motion only: extension/flexion) was attached to the dorsolateral side of upper arm and forearm to record the movement about the elbow. The goniometer for the wrist (2D-EGM, extension/flexion, radial/ulnar abduction) and the torsionmeter for the forearm (pronation/supination) were connected to the dorsal side of hand and forearm. Figure 4-1 shows the attachments of the EGMs.

Figure 4-1: Attachments of the EGMs and the markers.

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Additionally, an accelerometer was equipped with a 1D-fingergoniometer (8th channel). This accelerometer was attached to the side of the club-shaft, and consisted of a cantilever and a mass at the end thereof. Any acceleration of the club caused a deflection of the cantilever towards the club-shaft which will be measured by the goniometer. The only high deceleration occurs during the impact, when the club and ball are decelerated and accelerated respectively, due to momentum conservation. The first high deflection of the goniometer indicated the impact. The data sampling frequencies of the EGMs were set at the maximum of 1000Hz. The data was stored by a data logger and subsequently imported into a laptop computer. Eight channel of the data logger were used in this experiment. The files were opened with Biometrics Datalog v 2.0a software and converted into an ASCII-file which was then imported into Mathcad (Mathsoft Engineering & Education, Inc. Cambridge, USA). Savitzky-Golay smoothing filters (also called digital smoothing polynomial filters or least squares smoothing filters) were used to smooth the data and filter the noise, whose frequency span (without noise) is large (Savitzky and Golay, 1964). Savitzky-Golay smoothing filters perform better than standard averaging FIR filters, as it retain a significant portion of the signal's high frequency content while filtering out the noise—especially regions near the impact. Savitzky-Golay smoothing filters, with 99 points and order of 8, were first used to smooth the angular displacement time curves using a standard routine in Matlab (The Math Works, Inc., Natick, USA) before the curves were differentiated to get the velocity. The velocity curves were then smoothed again with Savitzky-Golay smoothing filters, with 99 points and order of 8.

The angular velocity for the torso rotation was obtained by using the three-dimensional photometric reconstruction method--PEAK MOTUS (Peak Performance

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Technologies, Inc; USA). Three high-speed video cameras operating at 200Hz were used. Three reflective markers were placed on the shoulders & sternum of the subjects (Figure 3-3). The markers on the images of video were digitized and reconstructed by the direct linear transformation method to obtain the angular velocity. The object space calibration errors were 0.1311%, 0.1161%, 0.2176% in X, Y, Z directions respectively.

4.3 VERIFICATION

In order to evaluate the validity of this proposed method, the computed clubhead linear velocity (using the dual Euler angles and dual velocities algorithm) was compared with measured values obtained from video analysis. This follows the verification method used by Springs et al. (1994). The computed and measured clubhead linear velocities are presented in Figure 4-2. Both the computed and measured velocities were resolved in negative x-direction of the laboratory frame, where the target was located. The computed clubhead velocity-time trace was in good agreement with the measured velocity-time curve. The closeness between the two traces indicates the viability of this algorithm in the study. The trend of the velocity curves also agrees well with those from previous studies (Cooper and Mather, 1994; Milburn, 1982). The comparison of the calculated clubhead displacement-time curve, in global frame, using dual position analysis with that of the video measured displacement-time curve (Figure 4-3) also indicated similarity in trend and magnitude. Fig 4.3 shows the clubhead path, i.e. the actual three-dimensional position of the clubhead. The apparent 2 paths for each measured and calculated graphs show the path for the upswing till the turning point (as indicated in the graphs) and the path for

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Figure 4-2: The comparison of calculated and measured clubhead speed in global frame {G} and the se

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Figure 4-3: The comparison of calculated and measured clubhead path in global frame {G}.

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the downswing till impact. An obvious difference between the measured and calculated velocities and displacements occurred at about 0.1 second before impact, i.e. the turning point of the swing path. The possible reasons for the difference will be discussed in Section 5.5.

4.4 RESULTS

4.4.1 Segmental Analysis

For simplicity sake, only subject 1's detailed results are presented in this section since the subjects shared similar trend. As for subject 2 and 3, only the angle-time graphs will be presented in this section (Refer to Appendix A for subjects' other results). Time histories of angular velocities of the different joints are presented in Figure 4-4. The computed rotational velocities for the upper arm, forearm and hand of subject 1 during the golf swing indicated that the highest overall angular velocity was generated by supination of the forearm (22 rad/s). This is followed by wrist ulnar deviation (17 rad/s), external rotation of the upper arm (10 rad/s), hand flexion (9 rad/s), elbow extension (6 rad/s), shoulder extension (6 rad/s) and shoulder abduction (5 rad/s). At the ball contact, the wrist extension has the highest angular velocity of 12 rad/s followed by the upperarm external rotation with the angular velocity of 10 rad/s. The forearm supination also had a distinguished angular velocity of 8 rad/s followed by wrist ulnar deviation (5 rad/s). Figure 4-5 shows the angular velocity of the clubhead in the Z_4 direction, i.e. the club shaft direction, indicating that the clubhead experienced high angular rotations with a peak of -26 rad/s just before the impact and -7 rad/s at the impact.

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The calculated results indicated that for subject 1, besides the hand extension with 9 m/s, the upper arm external rotation contributes most significantly to the final forward velocity of the clubhead speed at impact with 6 m/s (Figure 4-6 and table 4-1). This is followed by the upperarm abduction and forearm supination with 5 m/s each. These movements were the main contributions to the final clubhead speed at impact and together, they accounted for about 60% of the final velocity. Although the hand ulnar deviation did not contributed much to the forward velocity but together with the hand flexion/extension, it forms the wrist “cocking” mechanism. Similarly, other rotations that contributed insignificantly to the final velocity may help put other segments in the position for performing a rapid swing.

The calculated results indicated that for subject 2, the forearm supination and the hand extension contributes most significantly to the final forward velocity of the clubhead speed at impact with 10 m/s each (Table 4-1). This is followed by the upper arm external rotation with 4 m/s. These movements were the main contributions to the final clubhead speed at impact and together, they account for 70% of the final velocity. As for subject 3, the main contributors were forearm supination (8 m/s), upperarm abduction (5 m/s), hand extension (4 m/s) and shoulder retroversion (3 m/s). Together they made up almost 70% of the final velocity. Similar to subject 1, the results for other subjects also indicated that some arm rotations contributed insignificantly or even negatively to the final velocity as their function is to put other segments in the position for performing a rapid swing.

Table 4-1 shows the segmental contributions to the final forward velocity for the subjects. As mentioned earlier, wrist ulnar abduction, upper arm external rotation,

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upperarm abduction and forearm supination made up the main contributors in the swing velocity for subject 1. The same goes for subject 2. But for subject 2, the order of importance is slightly different. As for subject 3, it's almost the same except that shoulder retroversion replaced the upperarm external rotation as one of the main contributor. This shows the difference in emphasis of the swing for the subjects but neither the less the importance of the few main movements in contributing to the resultant speed remains true.

4.4.2 Sequence Analysis

In order to check the temporal sequence of release of the club, we look at the Angle-Time graphs. Figure 4-7 shows the Angle-Time graphs for the subjects. The release for a particular segment is the first significant change in direction of the segment from the upswing direction. The graphs are purposely scaled in such a way to facilitate the determination of the release moment for each segment. A detailed analysis of the graphs yield the following results: For subject 1, the order of release is in the sequence of wrist flexion followed by shoulder anteversion followed by upperarm external rotation, elbow extension and forearm supination almost together and lastly wrist ulnar abduction. For subject 2, the order of release is in the sequence of elbow extension followed by wrist flexion and upperarm external rotation almost together followed forearm supination, shoulder anteversion and lastly wrist ulnar abduction. For subject 3, the order of release is in the sequence of elbow extension followed by wrist flexion, followed by upperarm external rotation and shoulder anteversion almost together followed forearm supination and lastly wrist ulnar abduction. The upperarm abduction for the subjects is omitted as it could not give meaningful result due to reason that will be explained in Section 5.5.

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Figure 4-4: Angular velocity of different arm segment rotation.

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Figure 4-5: Angular velocity of the clubhead in the Z₄-axis direction.

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Figure 4-6: Contribution of different arm segment to the forward velocity of club head speed.

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subject		Resultant club-head velocity	Torso	Upper arm: retroversion+ / anteversion-	Upper arm: adduction+ / abduction-	Upper arm: internal rotation+ / external rotation-	Forearm: extension+ / flexion-	Forearm: pronation+ / supination-	Hand: extension+ / flexion-	Hand: ulnar abduction+ / radial abduction-
1	Angular velocity (rad/s)	39	12	-1	-4	-10	4	-8	12	5
	Individual contribution of Linear velocity on the club head forward velocity (m/s)(%)	43 (100%)	13 (30%)	0 (0%)	5 (12%)	6 (14%)	3 (7%)	5 (12%)	9 (21%)	2 (5%)
2	Angular velocity (rad/s)	26	13	-1	-2	-6	1	-14	16	-6
	Individual contribution of Linear velocity on the club head forward velocity (m/s)(%)	34 (100%)	8 (24%)	-1 (-3%)	2 (6%)	4 (12%)	1 (3%)	10 (29%)	10 (29%)	0 (0%)
3	Angular velocity (rad/s)	25	11	5	-4	-1	-4	-25	6	-4
	Individual contribution of Linear velocity on the club head forward velocity (m/s)(%)	29 (100%)	8 (28%)	3 (10%)	5 (17%)	0 (0%)	-2 (-7%)	8 (28%)	4 (14%)	3 (10%)

Table 4-1. Angular velocity and individual contributions to the resultant velocity at the impact.

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Figure 4-7: Angle-time graphs for subjects

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4.4.3 Coordination and Synchronisation Analysis

Angular Velocity-Time Graphs

Figure 4-4 and the graphs in Appendix A1 shows the Angular Velocity-Time graphs for the three subjects. The graphs show the swing actions starting from the later part of the backswing (-0.25s) through the start of the downswing (-0.1s) until the point of impact (0s). For the upperarm retroversion/anteversion plots at impact, subject 3 shows a positive average value while the other two golfers recorded values near to zero. This implies that subject 3 was moving his arms closer to his body at impact. The negative value at impact for adduction graphs shows that all three golfers made an abduction. The arm rightfully should have been abducting at impact and so we have good indication that the measurements were taken in the right direction.

The upperarm's external/internal rotation and the forearm's pronation/supination are often neglected in golf research as they are difficult to measure. However, they were measured in our study and are clearly reported in this thesis. These two actions were studied concurrently as their movements are closely related to one another.

As shown by the two curves which run almost parallel to each other, from the point - 0.05s onwards of the swing, subject 1's upperarm external rotation and forearm supination were fairly synchronized. The same synchronization could not be observed in the other two subjects. Subjects 1 and 2 registered positive values at impact for forearm extension/flexion, indicating that their left arms were extended. However, subject 3 registered a negative value, indicating the opposite. The extension/flexion of the hand and its ulnar/radial abduction are responsible for the cocking/uncocking action of the wrist. This cocking/uncocking action is an important aspect of the golf

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swing. All three subjects exhibited similar hand extensions/flexions near the impact. Subject 1's action was smooth and gradual as shown by the smooth, gentle slope of the curve while the extension for the other two subjects was rapid. The rapid extension created a steep slope. This indicates that subjects 2 and 3 may have been uncocking their wrists in the late part the swing. Judging from the ulnar/radial abduction and hand extension/flexion curves, subjects 1 and 2 showed relatively good coordination at the wrist, as both curves appear to move in sync. Subject 1's ulnar/radial abduction led to an extension/flexion movement in his hand. Subject 2 demonstrated the opposite actions initially but later reversed his lead nearer to the impact point. The same synchronization could not be observed in subject 3.

Segmental Velocity Contribution-Time Graphs

Figure 4-6 and the graphs in Appendix A2 shows the Segmental Velocity Contribution-Time graphs for the three subjects. The graphs also show the swing actions starting from the later part of the backswing (-0.25s) through the start of the downswing (-0.1s) until the point of impact (0s). The retroversion/anteversion curve is relatively flat for subject 1, indicating no significant contribution to the clubhead velocity. For the other two subjects, the curves fluctuate during the downswing until the impact position. This implies that subject 1 used this action for positioning rather than for speed generation. The abduction/adduction curve for subject 1 stays near zero for most part of the downswing with a slight peak just before the impact position, while for subject 2, it stays relatively near to zero throughout the swing. For subject 3, the curve fluctuates and in particular, there is a significant negative contribution to the resultant speed at -0.05s. This indicates a potential problem with subject 3's swing. The curve for subject 1's upperarm external/internal rotation and the curve for his

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forearm pronation/supination run almost parallel near the impact point, indicating that the movement was well-coordinated. The same trend could not be observed in the other two subjects. In fact, their curves show a disproportionately large contribution from the forearm supination around the impact region.

Subject 1 and 2's forearm extension/flexion contribution curves are rather smooth with a positive contribution at impact whereas subject 3's curve is not as smooth and ends with a negative contribution at impact. The wrist flexion/extension curve for subject 1 is rather flat before peaking at impact. The other two subjects' curves also show significant positive contributions at impact but there is a significant negative contribution prior to that too. This again indicates a potential problem in their swing. The wrist ulnar/radial abduction curve for subject 1 shows a significant contribution prior to the impact point. This is missing from the other two subjects' graphs. Table 4-1 shows each segmental contribution at impact.

CHAPTER FIVE

APPLICATIONS AND COMPARISON STUDIES

The use of goniometer together with dual Euler method offers a range of applications from motion capture and animation to various motion analyses covering details that are previously not available. This chapter presents the various applications with discussion on the merits and shortcomings of the method before drawing a conclusion for this important aspect of the project, the dual Euler analysis.

5.1 MOTION CAPTURE

Motion capture and analyses of human motion are of major interest in many fields such as sports, dances, movie industry and medical community. The traditional approach is to record human motion via cameras and analyze the pictures frame by frame. The camera method, although, widely used, has its own limitation, i.e. occlusion and tedious post-processing for digitization. Using the position analysis method in our study, the time-series of the elbow, wrist and hand can be calculated and plotted to give a visual feedback on the swing (Figure 5-1). These plots can also be animated to show the actual swing motion. When plotted in three-dimensional space and with the sides colour-coded, rotations like supination can be shown clearly. Figure 5-2 shows the animated three-dimensional plot of the swing. Though the qualities of the pictures are slightly distorted when taken from the real player, rotations are still visible. The arm and club are separated into four different sides

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when subject assumes its initial position (anatomical position, Figure 3-3) and each side is given a different colour.

An advantage of this method is that the angular displacement can be checked easily and directly right after swing, hence facilitating the ease of animating the motion captured from any angles. This avoids time-consuming three-dimensional reconstruction technique with video analysis. Also the maximum sampling frequency of 1000Hz for goniometers offers possibilities of capturing high-speed motions and in better resolution than the camera technique, which in standard format can offer maximum speed 200 Hz.

Figure 5-1: Swing action of golfer from downswing to impact (near frontal view, global coordinates frame).

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Figure 5-2: Three-dimensional animation of swing action of golfer (near frontal view, global coordinates frame).

5.2 SEGMENTAL ANALYSIS

From the results in chapter 4.4.1, we can conclude that forearm supination and hand extension contributed significantly to the final clubhead velocity. Since the hand extension together with ulnar deviation forms the hand “cocking” mechanism, the result shows good agreement with many simulations and experiments done by Springings and Neal (2000) about the importance of wrist “uncocking” in achieving a high clubhead speed. The study also uncovers the importance of external rotation of upper arm and supination of the forearm in golf swing. These two rotations along the longitudinal axes cannot be detected using traditional 2-dimensional approaches and are not often emphasized in conventional golf instruction. The importance of these two rotations should not be neglected. By concentrating on these two rotations together with the wrist “uncocking”, subject 2 was able to achieve reasonable impact speed of about 24 m/s (Table 4-1). As we can see from Table 4-1, subject 2’s impact speed was greatly generated by these actions (70%), the rest of the arm actions were not as significant which may indicate that they were used merely for positioning the club. For all subjects, the torso rotation did play a significant part in producing the speed at impact (at least 24%). Although all subjects used the torso significantly, subject 1 uses his torso better with the torso contributing about 5 m/s more than the other subjects. As the swing action is the summation of the various rotations, the clubhead naturally experiences high angular rotations. This can be seen from Table 4-1 and Figure 4-5. The high angular rotation means that golfers should never manipulate the club to square the clubface.

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5.3 SEQUENCE ANALYSIS

As mentioned earlier that the order of the temporal proximal-to-distal sequence is important in generating maximal end-point velocity, we could see how the triggering mechanism affect the overall impact speed in this study. One of the possible reason why subject 1 is able to generate higher clubhead speed at impact could be due to the fact that he has observed the proximal-to-distal sequence of release better than subject 2 and 3. Other than the wrist flexion, subject 1's order of release has followed the proximal-to-distal sequence of release: starting with upperarm movements followed by elbow and ending with the hand. On the other hand, subject 2 and 3's order of release may show the shortcoming of subjects' swing. Instead of shoulder anteversion triggering the movement, that particular movement was done almost last. Instead, subject 2 and 3 triggered their swing with the extension of the elbow first which might not be as effective. For all golfers, the wrist flexion was observed to be release quite early in the sequence, this might be used for positioning the club in a more comfortable position for the downswing. Also all golfers has the wrist ulnar abduction action last in the sequence indicating that the golfers have tried to maintain the wrist in the "cocked" position till the last moment before releasing it.

5.4 COORDINATION AND SYNCHRONISATION ANALYSIS

Synchronizing arm and body movements is very important in the execution of a good golf swing. In fact, one of the major concerns in golf training currently is the execution of the proper sequence of motion to get the body in sync during a golf swing (Leadbetter, 2003). Unfortunately, as the swing action is fast, it is not easy to capture and analyze. The use of goniometer in this study provides an easy and

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convenient alternative in capturing and studying the sequence of rotations involved in a typical golf swing.

5.4.1 Upperarm Retroversion/Anteversion

This range of motion is very important for the execution of a good golf swing as it determines the plane of the swing. The plane of the swing is simply formed by a circular disc with the spine at the center of rotation. All good golfers strive to swing the club on plane as this action is the easiest to repeat and allows them to strike the ball consistently well. As seen from the Segmental Velocity Contribution-Time graph, subject 1's swing demonstrates this idea well. He uses this action for positioning rather than for speed generation. Subject 2 did not control this movement well enough as seen from the fluctuations in the Segmental Velocity Contribution-Time graph midway through the downswing. As mentioned in chapter 4.4.3, subject 3's left arm moved closer to his body at impact. This was probably because his hips and shoulders moved too fast and threw his arm out of plane, hence making it necessary for him to execute a retroversive action to bring the arm back into plane at the point of impact. Instead of using a combination of torso rotation and abduction to hit the shot, subject 3 was also probably using this action more to direct the club to the ball. This can be seen from the unsteady angular velocity and the positive velocity contribution at impact. This in turn could have resulted in a bent left arm at impact. We will explore this possibility in a later section. Overall, subject 3's coordination of his torso and arm is poorer than that of the two other subjects.

5.4.2 Upperarm Adduction/Abduction

The abduction movement will contribute positively to the resultant velocity of the clubhead in downswing. The values only become negative when adduction occurs,

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and this happens when the torso is rotated too fast that the left arm lags behind. Subject 3's velocity contribution was negative at the middle of the downswing, showing that he was rotating his torso too fast. The abduction contribution to the final clubhead velocity was relatively high as compared to the other movements, indicating again that subject 3 was hitting the ball using the strength of his upperarm.

Subject 2's abduction velocity contribution stayed relatively flat throughout the swing. The relatively flat contribution at the beginning of the downswing shows that subject 2's torso and arm moved in pace, as they ideally should. However, near the impact, the contribution curve still remains relatively flat indicating that he was not using this action effectively. The abduction action should make a significant positive contribution at impact and subject 2's inability to use this action to generate a significant contribution makes his weakness clear. Subject 1's execution was similarly flawed but to a lesser degree, earlier in the downswing. His arm was able to catch up with his torso midway through the downswing and moved on to make a positive contribution until the impact. Not only was subject 1's abduction contribution at impact higher than subject 3's, its contribution by percentage was also more balanced than the subject 3's, indicating a more balanced swing.

5.4.3 Upperarm External/Internal Rotation and Forearm Pronation/Supination

In contrast to subject 1, the rotations of subject 2 and 3's upperarm and forearm (pronation/supination) made disproportional velocity contributions. Both their forearms played a much more significant role. The reason for this result is that subjects 2 and 3 were not rotating their left arm correctly at the impact position. They were bringing the clubhead to the square position by focusing on forearm supination

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only and neglecting the rotation of their upperarm. Ideally, the left arm should behave like one single rod and the correct movement would be the rotation of the upper portion of the rod (upperarm) in sync with the lower portion of the rod (forearm) to generate a consistent torque. This energy is then channeled towards the golf shot. As bigger muscles are located on the left shoulder than on the forearm, the use of upperarm rotation to bring the clubhead to the square position would probably result in a higher clubhead speed and accuracy than the use of forearm supination alone. This advantage was clearly demonstrated by subject 1.

5.4.4 Forearm Extension/Flexion

The graph for this action shows whether the left arm is kept straight throughout the downswing. As the club is pulled towards the impact position, the centrifugal force of the clubhead causes the left arm to straighten. Thus, the ideal downswing is one where the left arm has no change in flexion or extension. Furthermore, a straighten arm facilitates the synchronisation of the upper and lower arm better.

Subject 3's left arm was flexing at impact (mentioned in Section 4.4.3) and was thus not swinging the right way. This was probably because he was trying to bring his club back in plane by retroverting his arm (as mentioned in Section 5.4.1). However, as the centrifugal force threw the club outwards, this retroversion action did not propagate to the whole arm evenly. The upperarm retroverted more than the forearm and caused the forearm flexion. In a circular motion, tangential velocity depends on the radius. If the left arm is bent, the radius taken from left shoulder to clubhead becomes smaller, implying that clubhead speed at impact would be compromised. In golfing terms, it is

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said to be swinging with a very narrow arc. For subject 3 unfortunately, one mistake just led to another.

5.4.5 Hand Extension/Flexion and Ulnar/Radial Abduction

In a golf swing, these two actions together bring about the uncocking of the wrist that is important for the generation of high club head speed (Sprigings and Neal, 2000). The graphs indicated that all three subjects delayed the uncocking of their wrists until the later half of their downswing. However, the length of time taken for the whole wrist action to take place is very important. A smooth uncocking of the wrist (as executed by subject 1) should make it easier for the golfer to hit the ball more consistently than a hurried and unsteady motion (as executed by subjects 2 and 3).

Subjects 2 and 3 obviously had a mistake in their wrist uncocking. There were huge negative velocity contributions from the wrist actions prior to impact. This error could be due to both subjects delaying the uncocking of the wrist beyond the controllable point (both subjects are observed to have wrist uncocking movements later than subject 1 from the curves). The fact that high angular velocity of the wrist extension was observed just prior to impact strongly suggests that there is a very large inertial (centrifugal) force pulling the clubhead outwards – thus, straightening the wrist joint (Milne and Davis, 1992). Golfers were taught to delay the wrist uncocking, but due to the huge inertial force, there will be a point which the delay becomes uncontrollable. Attempting to delay the uncocking beyond that point could result in the unsteady velocity contribution and worst still, a huge negative contribution as demonstrated by subject 2 and 3. The huge inertial force probably pulled subject 1's wrist into ulnar abduction direction, thus initiating the uncocking of his wrist. This contributed

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significantly to the resultant velocity prior to impact. Near impact however, because of the orientation of the hand (caused by external rotation and supination of the arm), ulnar abduction could not contribute much, hence wrist extension took over as the major contributor of the two. Subject 1 demonstrated effective coordination between the two actions as evidenced from the velocity-contribution graph.

5.5 DISCUSSION

Motion capture and contribution of individual rotational velocities of each segment to the final velocity of clubhead can be ascertained using the dual Euler analysis. There are many advantages in using this approach as compared to the conventional image analysis method. The errors of joint angular velocities can be reduced significantly as only the first derivative of the angular displacement data is required; It is also easier for the coach/athlete to understand joint angular velocities as compared to acceleration; Subject's quantified swing can be compared to elite golfers' and thus enabling identification of areas for remedies. As the arm have 7 degrees of freedom, focusing on the few important actions helps to simplify the swing coordination and accelerate the learning process (subject 2 and 3). Once the golfer becomes more provision, he can then improve on other aspects of the swing (subject 1). The use of electrogoniometers also provides many advantages. For example, the electrogoniometers used in the study sample signals at 1000Hz that enable the removal of some aliasing errors compared with the used of video which maximally samples at the rate of 200Hz. Unlike image analysis approach which is time-consuming, data reduction from electrogoniometers can be made available almost immediately without much processing. With the use of electrogoniometer, the errors

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of digitizing the video image in data analysis approach can be eliminated. The present method is also simpler and generates results easier than the method for the arm segment rotations proposed by Sprigings et al. (1994). Therefore, the present method can serve as a viable method of ascertaining the contribution of segmental rotations to the clubhead velocity.

From processing the data in this project, the errors in using this approach as compared to the conventional image analysis method were clearly lesser. The project involves getting the torso rotation on top of the segmental rotations; however the error incurred by some of the data sets in calculating the torso rotation caused the final results to be meaningless. The error in torso rotation is due to the fact that video analysis was used to obtain its data. The common shortcomings of the video analysis were evident when processing the data. The errors were incurred in the torso velocity calculation as well as they were calculated from the position data. Therefore the best output data, which gives better and more meaningful result, were used in this report.

Although the method gives satisfactory results, there are slight differences between the measured clubhead velocity and the calculated one. The reason why a discrepancy occurs at 0.1s before the impact comes for the way that the goniometer was attached. For the measurement of the upper arm adduction/abduction, the goniometer was attached to the acromion process and to the upper arm, just below the deltoid attachment. This gives only the measurement of the upper arm adduction/abduction by deltoid and supraspinatus muscles but not the scapula rotation. Upper arm motion involves three joints: a) sternoclavicular, between the thorax and clavicle; b) the acromioclavicular, between the clavicle and scapular; and c) the glenohumeral,

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between the scapula and humerus. At about 0.1s before the impact, the outward rotation of scapula together with the deltoid and supraspinatus muscle helps put the arm in the appropriate position, above the shoulder joint, to execute the swing. Before that, when the arm was not high in the air, scapula rotation did not play a major role in positioning the arm. Hence we can see that, in Figure 4-2, the 2 curves match nicely initially. The missed portion of measurement of the scapula rotation accounts for the differences in the 2 lines. The problem can also be seen in Figure 4-3. The calculated path and the measured path match most of the time except at the top of the swing, which is where the scapular rotation becomes significant. To eliminate this problem, we suggest that the goniometer be attached at the lower side of the arm, from the medial side of the right upperarm to the breastbone, under the armpit. This way, the goniometer reading will give the measurement of the arm adduction/abduction generated by the deltoid and supraspinatus muscles as well as the scapula rotation. Another way will be to lengthen or build a small add-on attachment for the goniometer in order to attach it from the suprasternal notch to just below the deltoid attachment.

The other small differences may be due to the errors of the measured velocity that was processed with image analysis using cameras of 200 Hz. Also the shaft of the club was modeled as a rigid rod and the contribution of the torso rotation was also calculated using the camera technique, which has its own errors. Another limitation of the study is that the electrogoniometers are attached to the subject's skin. As Lu and O'Connor (1999) found, the skin movement may displace the goniometer and introduce errors to the data. Besides the skin movements, the muscles could also displace goniometers. Hence in order to get accurate results from the use of

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electrogoniometer, detail calibration and careful positioning of the goniometers are required.

From the comparison studies done, there seems to be some characteristics of a good swing. However as only three subjects of different skill levels were compared, those characteristics uncovered in this project cannot be said to be the rightful characteristics of other good golfers. Nevertheless there should be some general characteristic of a good swing and the results from the diagnosis of swing using the graphs presented in the earlier section should prove interesting and helpful to coaches and golfers. The following are a few general recommendations for golfers:

- Do observe the temporal proximal-to-distal sequence.
- Do not compensate for a mistake with another action. Compensating for one mistake causes the execution of another.
- Keep the upper arm anteversion angle constant as it determines the plane of the swing.
- At the beginning of the downswing, keep the upperarm adduction angle constant, i.e., torso and arm moved in paced.
- Near impact, use arm abduction to generate extra speed.
- External rotation and supination are important for generating speed.
- The arm should move as a rod with the upperarm and forearm rotating in sync.
- Wrist movements are also important for generating speed.
- Do not delay uncocking too much until it becomes an unsteady and uncoordinated action.

5.6 CONCLUSION FOR DUAL EULER ANALYSIS

In our study, the dual Euler analysis provides a convenient assessment of the effectiveness of the golf swing. It helps to quantify the golf swing and shows the detail swing actions at the level of segmental rotations. It has identifies and ascertains the importance of the arm rotations which other two-dimensional methods missed. The segmental velocity contribution and angular velocity graphs also give insights that conventional methods could not provide. They provide a new and easy method to analysis the sequence and coordination in the swing.

From the results, golfers should never neglect the arm rotations in executing the swing. Golfers should try to observe the proximal-to-distal sequence of release in order to get a faster swing. Not only do better golfers have a better sequence of release, they also show better coordination between different segments of the torso and hand. As the movements made in a golf swing are closely linked to one another, golfers should pay attention to their segmental coordination. Often, one wrong movement will bring about another to compensate for it.

In general, this method facilitates capturing and analyzing of the human motion with ease. The use of a biomechanical model in conjunction with the dual-number coordinate transformation for motion estimation was shown to provide accurate and reliable results for both simulated and real measurements. In particular, the advantage of using the Euler angles based on the dual-number coordinate transformation approach is that they allow for a complete three-dimensional motion representation using six parameters for each anatomical distinct. This method has proved to be an effective means to examine high-speed movement in three-dimensional space. It also

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provides an option in assessing the contributions of the individual segmental rotations in producing the overall motion speed. Other than successfully applying it to golf swing, this present method can also be applied to other sports to examine segmental rotations as well.

CHAPTER SIX

SCREW AXIS ANALYSIS

The previous chapters have present a detail and comprehensive analysis on the segmental rotations on the left arm during golf swing using dual numbers. However, the use of dual numbers in kinematics analysis does not end there. This chapter presents another application of dual numbers in joint kinematics. After going through the methodology, simulations results are presented. Experimental applications of the method are also done and the results presented. Study on the shoulder mechanism during golf swing using the method might add insights and information useful to golfers in compliment to the segmental rotations previously done. This marks another important aspect of the whole project.

6.1 EQUIPMENT

The experiment uses the following equipment:

- Mathcad (Mathsoft Engineering & Education, Inc. Cambridge, USA).
- Matlab (The Math Works, Inc., Natick, USA).
- PEAK MOTUS (Peak Performance Technologies, Inc; USA).
- 'Flock of Birds' (FOB) (Ascension Technology Inc., Burlington, Vermont, USA)

6.2 STUDY BACKGROUND

As mentioned in the LITERATURE REVIEW chapter, screw axis method has been employed to describe joint kinematics mainly in clinical assessments. Generally, points data are used to calculate the screw axis. The problem of estimating the motion and orientation parameters of a body segment from two n points set patterns is of significant importance in many areas, for example: biomechanics, motion synthesis and analysis, computer vision and photogrammetry. The position and orientation of one Cartesian coordinate system with respect to that of the previous moment are unknown. This is the problem of determining rigid-body motion. In most instances we have available two sets of noisy observations of n points of a rigid body and determination of the orientation and position are based on these points data. However this chapter addresses the case where they are two sets of *lines*, which are derived from the points data.

One of the fundamental problems in calculating screw axis, or any other kinematics problems, is noisy data. Since screw axes directions and positions are highly sensitive to data noise, its directions will deviate and its positions scatter in a seemingly random manner under noisy situation causing the result to be meaningless. Screw axis is also undefined with the normal set of equations when rotations angles equal zero, i.e. under a pure translation. Another set of equations is needed for this special case (Fischer, 1999). This caused additional difficulty in determining the screw axis under small rotations. Conventional algorithms generally used a set of point patterns with least square methods to solve the noise problem. Finding the axis of rotation usually involves mapping a set of data points to the corresponding data points after movement. Contrary to conventional algorithms, the new algorithm presented in this

Figure 6-1: Conventional algorithms usually involve mapping a set of rotated data points (Initial set of points after screw motion) to the corresponding set of final data points (Final set of points) using least square methods. The new algorithm involves mapping a set of rotated vectors ($\tilde{\mathbf{V}}_i$) to the corresponding set of final vectors ($\hat{\mathbf{V}}_i$) using least square methods.

chapter takes advantage of set of line patterns, which are derived from point data using dual number relationship. All conventional techniques use point transformation to calculate the screw axis. In this chapter, we present a novel technique that directly estimates the axis of a screw motion as Plücker line. The aim is to find algorithms less complex than those in conventional use, and thus facilitating more accurate computation of the unknown parameters. The method involves mapping a set of vectors to the corresponding vectors after movement (Figure 6-1). The mapping of vectors is made possible by the use of dual transformation matrix.

This chapter demonstrates the robustness and reliability of generating transformation results from the mapping of vectors. In addition, this chapter presents an alternative method, from Fischer (1999) and McCarthy (2000), in determining the dual vector from the dual transformation matrix based on eigenvector and the singular value decomposition (SVD)(Press et al., 1992).

6.3 ALGORITHM FOR COMPUTING DUAL-NUMBER TRANSFORMATION MATRIX FROM POINT COORDINATES

The method for calculating the screw axis in this thesis involves obtaining the dual transformation matrix (DTM) from coordinated points. The obtained DTM represents a screw motion through a given axis in space. A three-dimensional line is typically specified with 6 parameters (Plücker's coordinates), 3 for position and 3 for direction. The coordinates can be written as a dual vector. For applications of its use in robotics and kinematics, see McCarthy (2000). Because most measurement systems used in joint kinematics studies acquire the coordinates of points (markers) on a rigid body

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with respect to a pre-defined fixed coordinate system, the algorithm for calculating the DTM $\begin{bmatrix} \hat{R} \end{bmatrix}$ based on the three-dimensional coordinates of the points on the rigid body is developed as follows. It is well known that the coordinates of n ($n \geq 3$) non-collinear points on a rigid body can determine the position and orientation of the rigid body in space. Suppose that with respect to the pre-defined global coordinate system the coordinates of n ($n \geq 3$) points on the rigid body at the initial position and final position are measured as $\mathbf{r}_{0i}$ and $\mathbf{r}_i$ ($i = 1, 2, 3, \dots, n$) respectively. Thus, the centroids of the points at the initial and final position are given by $\mathbf{c}_0 = \frac{1}{n} \sum_{i=1}^n \mathbf{r}_{0i}$ and $\mathbf{c} = \frac{1}{n} \sum_{i=1}^n \mathbf{r}_i$ respectively. According to the dual transformation relationship, at the final position, the vector connecting the centroid to the i th point can be estimated by:

$$\tilde{\mathbf{V}}_i = \tilde{\mathbf{V}}_i + \varepsilon \tilde{\mathbf{W}}_i = \begin{bmatrix} \hat{R} \end{bmatrix} \hat{\mathbf{V}}_{0i} \quad (6-1)$$

where $\hat{\mathbf{V}}_{0i} = (\mathbf{r}_{0i} - \mathbf{c}_0) + \varepsilon \mathbf{c}_0 \times (\mathbf{r}_{0i} - \mathbf{c}_0)$ represents the vector connecting the centroid to the i th point when the rigid body is at the initial position. On the other hand, at the final position, the same vector can also be calculated from the measured data as:

$$\hat{\mathbf{V}}_i = \mathbf{V}_i + \varepsilon \mathbf{W}_i = (\mathbf{r}_i - \mathbf{c}) + \varepsilon \mathbf{c} \times (\mathbf{r}_i - \mathbf{c}) \quad (6-2)$$

Figure 6-1 shows the graphical representations of the points and vectors. Because of noise, there is difference between $\tilde{\mathbf{V}}_i$ and $\hat{\mathbf{V}}_i$. In the least-square error sense, the dual-number transformation matrix $\begin{bmatrix} \hat{R} \end{bmatrix}$ should minimize the following function:

$$J = \frac{1}{n} \sum_{i=1}^n (\|\mathbf{V}_i - \tilde{\mathbf{V}}_i\|^2 + \|\mathbf{W}_i - \tilde{\mathbf{W}}_i\|^2) \quad (6-3)$$

where $\| \cdot \|$ represents the norm of a three-dimensional vector. This optimization problem is subject to the orthogonal constraint $[\hat{R}][\hat{R}]^T = [I]$. The dual form constraint can be expanded into the ordinary matrix form as

$$\begin{aligned} [R][R]^T &= [I] \\ [R][S]^T + [S][R]^T &= [0] \end{aligned} \tag{6-4}$$

where $[R]$ and $[S]$ are the primary and dual components of the dual transformation matrix respectively. From Eq.(6-4) twelve ordinary constraint functions are obtained. The eighteen elements in the dual-number transformation matrix were determined by solving the constrained optimization problem using sequential quadratic programming (SQP) methods (Fletcher, 1980). Optimization toolbox in MATLAB (The Math Works Inc., Natick, MA, USA) was used as the computational tool. The solution provides the optimal estimate of the dual-number transformation matrix $[\hat{R}]$. This method has been mentioned in Ying and Kim (2002).

6.4 THE GEOMETRY OF SCREW AXES

With the dual transformation matrices $[\hat{R}]$ obtained, the screw axis positions and directions can then be determined. Other than the screw axis, the angle turned and distance moved along the axis can be determined as well although these were not addressed in this thesis.

We have seen that for every spatial displacement there is a fixed line, called its screw axis. Following the derivation by McCarthy (2000) and incorporating dual vector

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algebra in it, we show that the Plücker coordinates in dual vector representation, $\hat{V} = V + \varepsilon W$, of this screw axis satisfy the condition that it is invariant under the 3×3 dual transformation matrix $[\hat{R}]$.

$$\hat{V} = [\hat{R}]\hat{V}. \quad (6-5)$$

where

$$\begin{aligned} V + \varepsilon W &= ([R] + \varepsilon[S])(V + \varepsilon W) \\ &= [R]V + \varepsilon([S]V + [R]W) \end{aligned} \quad (6-6)$$

We rewrite Eq.(6-5) as

$$[I - \hat{R}]\hat{V} = 0, \quad (6-7)$$

and seek solution other than $\hat{V} = 0$. This is easily done if we separate it into the pair of vector equations compromising the primary and dual component respectively

$$\begin{aligned} [I - R]V &= 0 \\ [I - R]W &= [D]V \end{aligned} \quad (6-8)$$

Where $[D]$ is the skew-symmetric matrix defined by $[D]V = d \times V$ for any translation and we used the property of $[R]V = V$.

Now $[D]V = d \times V$ must be orthogonal to V . Therefore, it does not have a component in the direction of the null space of $[I - R]$, which is V . This means that we can solve this equation for V . Its algebraic derivation here is straightforward because it is direct consequence of applying the eigenvalue problem to three-dimensional lines. It is probable that specifying the axis of a screw motion as Plücker line is the most suitable linear representation because it minimizes the influence of non-linear transformations, frequently used as closed form solutions in conventional methods.

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Two different methods of calculating the screw axis from dual transformation matrix were addressed in Fischer (1998) and McCarthy (2000). In this thesis, a different approach is stated by using the eigenvector and the singular value decomposition (SVD) to solve for the dual vector, especially the dual component $\mathbf{W}$ in Eq.(6-8). It is adopted on the ground that it is more straightforward in implementing. Furthermore the other two methods could not produce the screw axis under a very small rotation or at near pure translation while the proposed method in the following section could still produce meaningful results.

The first equation in Eq.(6-8) means that the $\mathbf{V}$, also the direction of the screw axis, is simply the eigenvector of the primary component, $\mathbf{R}$, of the dual transformation matrix. Looking at the dual component $\mathbf{W}$ in Eq.(6-8), the matrix $[\mathbf{I} - \mathbf{R}]$ turned out to be ill-conditioned and this can be efficiently evaluated using the singular value decomposition (SVD) (Press et al., 1992):

$$(\mathbf{R} - \mathbf{I})^{-1} = \mathbf{V} \cdot [\text{diag}(1/w_j)] \cdot \mathbf{U}^T \quad (6-9)$$

where $\mathbf{V}$ and $\mathbf{U}$ are orthogonal matrices and $\mathbf{w}$ is a square diagonal matrix which satisfies the following equation:

$$(\mathbf{R} - \mathbf{I}) = \mathbf{U} \cdot \begin{pmatrix} w_1 & & & \\ & w_2 & & \\ & & \dots & \\ & & & w_j \end{pmatrix} \cdot \mathbf{V}^T \quad (6-10)$$

Therefore the dual part $\mathbf{W}$ in equation (6-8) can be found as follows:

$$\mathbf{W} = \mathbf{V} [\text{diag}(1/w_j)] \mathbf{U}^T (-[\mathbf{D}]\mathbf{V}) \quad (6-11)$$

Because $V \cdot W = 0$, we see that $\hat{V} = V + \varepsilon W$ are the Plücker coordinates of a line, which representing the screw axis. The reference point C for the screw axis $\hat{V}$ is determined by $C = V \times W$.

It should be noted that though the case $n=3$ represents the minimum number of non-collinear points necessary to estimate the dual-number transformation matrix of the rigid body, it is better that the coordinates of $n>3$ points are used in estimation to reduce errors (Pittman et al., 1991). For the proposed method, minimum smoothing regime is applied. The n points are only smoothed once with Savitzky-Golay smoothing filters (Savitzky and Golay, 1964) using standard routine in Matlab (The Math Works, Inc., Natick, USA). The computed results are then smoothed with Savitzky-Golay smoothing filters once more.

6.5 SIMULATIONS

The proposed method was tested on simulated data. The simulation follows closely to that of Halvorsen et al. (1999). Four markers at coordinates (0, 0, -5), (0, 0, -30), (0, -5, -15) and (0, 5, -15) were rotated 1° per frame for a total of 60° around an axis parallel to the y-axis and going through the point (0, 0, 5). The unit is in cm.

Two simulations were carried out. In the first simulation, random sequences corresponding to measurement errors were added to the data set. For each sequence, the signal-to-noise ratio (SNR), the ratio of the energy of the signal sequence to that of the random sequence, was calculated. The signal was defined by subtracting the mean from each of the marker paths, since the constant part does not carry

information on the movement (Halvorsen et al., 1999). The energy, E_x , of a sequence, $x(n)$, is:

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 \quad (6-12)$$

In the second simulation, skin movement position artefacts were added with reference to Cappozzo et al. (1996)'s result. The author had intended to follow Halvorsen's paper but was unsure of how the position artifacts were added. Hence the author implemented the following: The artefact movement at ten instances during the swing phase of a walking stride was estimated from Cappozzo et al. (1996)'s paper for four markers (HF, LM, m6 and m7) and expressed in the coordinate system used in this paper. The skin movements were then added to the simulated data. Simulations with each set of errors varied at 50%, 100%, 150% and 200% were run. No smoothing was performed for the second simulation.

6.6 RESULTS FOR SIMULATIONS

The simulation results in Halvorsen et al. (1999) (commented as "new" method and "Finite Helical Axis (FHA) method") were used as references for the proposed algorithm. The results of the "new" method from Halvorsen et al. (1999) were superimposed to our simulation results in Figure 6-2 and 6-3 as a comparison. The general trend of the error follows that of those reference methods. Figure 6-2 shows the results for the mean error in direction and position of the axis as a function of SNR in the random error simulation. Polynomial lines of order 2 were each fitted to the simulated results using least squares fitting. The error in direction was lower

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compared to the reference methods when there's significant noise. The error in position, like the "FHA" method, was kept much lower than that of "new" method. Figure 6-3 shows the results for the mean error in direction and position of the axis as a function of skin movement in the simulation. Polynomial lines of order 2 were also fitted to the simulated results using least squares fitting. The estimation of the position using the proposed method was better than both referenced methods. The error in direction is reasonably good and follows closely to that of the "FHA" method in terms of magnitude and trend.

The simulation results showed that the proposed method worked well for its applications, especially in estimating the direction of the screw axis when noise is present. This is due to the fundamental difference between the proposed algorithm and the conventional methods. The mapping of vectors gives a better estimate of the rotation than mapping of points data and hence better results could be derived from it. The results are highly significant as they demonstrate an advantage of using line instead of points. Together with the smoothing, the presented method is highly robust and provides reliable estimates of the screw axis, even under high noise situation. For the proposed method, the error tends to follow that of the FHA method. This is probably due to the rigid body assumption used in both methods. Therefore, as compared to the "new" method by Halvorsen et al. (1999), the proposed algorithm performs better when random noise is present. However, when there are significant skin artifacts, the proposed method is inferior to the "new" method in direction estimation.

Figure 6-2: Mean error in the (a) estimated direction and (b) estimated position of the axis plotted against the SNR. The thick solid lines correspond to the presented method and the thin lines to the "new" method.

(a)

(b)

■ “new” method
◆ Plücker line method

Figure 6-3: Mean error in the (a) estimated direction and (b) estimated position of the axis plotted against the magnitude of the skin position artefact. The thick solid lines correspond to the presented method and the thin lines to the “new” method.

6.7 APPLICATIONS AND DISCUSSION

The method was then applied to experimental data in an attempt to calculate the screw axis. The method was first applied the movement of the ankle joint which consisted of slow controlled movements. The method was also applied to calculate the screw axis of the left arm with respect to the shoulder joint during a golf swing.

6.7.1 Ankle Movement

For the ankle joint movements, the data collected by Ying et al. (2004) were used as the raw data. The data were collected from *in vitro* experiments. For details of the experimental setup, refer to Ying et al. (2004). The following gives a brief outline of the experiments. In the *in vitro* experiments, the passive motions at the ankle joint complex were measured by the 'Flock of Birds' electromagnetic tracking system. The tracking system, consisting of four wired receivers and one standard range transmitter, was set at its default configurations during the measurements. In the study, the passive motions during the dorsiflexion-plantarflexion and eversion-inversion were measured without considering weight and muscle forces. Ten below-knee amputation cadaver specimens were tested. Prior to the experiment, a specimen was thawed for 24 hours at the room temperature. The specimen was dissected carefully to remove soft tissues around the ankle joint complex while the ligamentous system remained intact. The soft tissues at the head of the fibula and the tibia tuberosity were also removed. Four landmarks for definition of the tibia-fibula anatomical coordinate system — MM, LM, HF and TT were identified and marked for digitization. The screws of the receiver fixtures were inserted into the tibia, the talus, and the calcaneus respectively.

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After the setup was adjusted to the appropriate position, the specimen with three receivers attached was mounted to the setup. At the neutral position, each landmark was digitized five times. At this joint configuration, the position and orientation of the receivers attached to the bones were also recorded. After collecting the required data at the neutral position, while the footplate was rotated manually and slowly from the maximum dorsiflexion of the specimen to the maximum plantarflexion, the position and orientation of the receivers were recorded continuously. The same procedure was repeated ten times. The average of the results obtained from the ten times measurements would be used as the motion pattern during the dorsiflexion-plantarflexion for analyzing the joint kinematics of the specimen. Once the kinematic data during the dorsiflexion-plantarflexion were collected, the foot plate was rotated and secured to the direction where its longitudinal axis parallel to the horizontal axis to simulate the movement of inversion-eversion. Similarly, while the footplate was rotated manually from the maximal inversion of the specimen to the maximal eversion, the kinematic data of the receivers on the bones were recorded continuously. Again, the motion of eversion-inversion was repeatedly recorded ten times.

Figure 6-4 shows the typical result of the screw axis movement of the ankle during the dorsiflexion-plantarflexion motion. The axes are clearly not fixed, which is in agreement with studies on various joints. In the graph, each instantaneous axis is plotted with a different colour, starting from blue at the start of the movement till red at the end of the movement. The migration trend was clearly shown with the colour-coded axes. The results are superimposed on anatomical drawings of the foot in three different views to show the position of the axes and magnitude of the migration in relation to the foot. As can be seen from the graphs, at dorsiflexion, i.e. the beginning

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Figure 6-4: (a) Top view, (b) side view and (c) posterior view of screw axes during dorsiflexion to plantarflexion, superimposed on the ankle joint. (Pictures adopted from Donatelli (1996) and Cynthia and Pamela (1992))

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Figure 6-5: (a) Top view, (b) side view and (c) posterior view of screw axes during inversion to eversion, superimposed on the ankle joint. (Pictures adopted from Donatelli (1996) and Cynthia and Pamela (1992))

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of the movement, the dorsiflexion is coupled with eversion as shown from the directions of the axes in Figure 6-4a (top view). From Figure 6-4c (posterior view), the dorsiflexion is shown to be coupled with abduction of the foot. At plantarflexion, i.e. the end of the movement, the plantarflexion is shown to be coupled with inversion (Figure 6-4a) and adduction (Figure 6-4c).

Figure 6-5 shows the typical result of the screw axis movement of the ankle during the inversion-eversion motion. Similar to Figure 6-4, the results are colour-coded and superimposed anatomical drawings to show three different views. Again, the migration trend was clearly shown in this case. As can be seen from the graphs, at inversion, i.e. the beginning of the movement, the inversion is coupled with plantarflexion as shown from the directions of the axes in Figure 6-5a (top view). From Figure 6-5b (side view) the inversion is shown to be coupled with adduction. At eversion, i.e. the end of the movement, the eversion is shown to be coupled with dorsiflexion (Figure 6-5a) and abduction (Figure 6-5b). The above experiments were done with sampling frequency of 50Hz. The low sampling rate and the controlled action produces pretty good results. This shows that the algorithm works well in clinical setting and can be applied to normal action where sampling frequency of 50Hz suffice.

6.7.2 Golf Swing

In usual clinical applications of screw axis, the required movements were usually performed slowly like above. To estimate the screw axis for faster action, higher sampling rate has to be used. Besides equipment limitation, with the increased in sampling rate, more noise are introduced in the data. With the proposed method

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working well with random errors, the application is tested in fast actions. To successfully estimate the screw axis for golf swing would be a good example. With the instantaneous screw axis calculated, golfer can then attempt to interpret the swing in a different perspective. The subject involved for this study is a semiprofessional golfer with a single handicap. The experiment was conducted at the Singapore Sports Council biomechanics laboratory.

Figure 6-6 shows the positions of markers on the subject's body and arm. Three reflective markers were placed on the shoulders and sternum of the subject to determine the torso rotation. The torso rotation has to be taken into account (or rather, taking out the effects) since the aim is to estimate the instantaneous screw axis with respect to the left shoulder instead of the full body action. Another four reflective markers were each attached on the medial and lateral side of the left elbow and left wrist. Together with the left shoulder marker, these five markers were used to represent the left arm. The global coordinate frame, G , was set up with the target in the negative x -direction (Figure 6-6). The data were obtained by using the three-dimensional photometric reconstruction method--PEAK MOTUS (Peak Performance Technologies, Inc; USA). Three high-speed video cameras operating at 200Hz were used. The object space calibration errors are 0.1311%, 0.1161%, 0.2176% in X , Y , Z directions respectively.

Figure 6-7 shows the results of the proposed method in estimating the instantaneous screw axis with respect to the shoulder during a golf swing. The swing was separated into different section and shown in four different views. The results were presented in

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Figure 6-6: The positions of markers on the subject.

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the global coordinate frame so that the left arm position is easily related to the actual swing action. The results can then be interpreted with reference to the instantaneous arm position. The axes were again colour-coded to show the migration trend. The left arm positions of the golfer were shown as black lines. The results show a smooth and clear migration trend. The results are presented in three orthogonal views and one near frontal three-dimensional view. Torso rotation was not presented, therefore the arm can be seen to rotate about a single point: the initial shoulder position during address where the subject faces front. If torso rotation were presented, the pivot point of the arm (shoulder) will shift and hence shifting the corresponding instantaneous axis. The shifting of the shoulder and axis would cause the presented result to look messier and harder to interpret. Hence the results in Figure 6-7 are shown with the shoulder fixed to improve the presentation and interpretation. Figure 6-8 shows another way of presenting the results. It shows the full results of the swing, in three views, combining all section of Figure 6-7 with a rectangular box added in to illustrate the subject's torso dimension and position. In this way, the position of the screws axes with respect to the torso can be easily identified. This gives a clearer picture, than Figure 6-7, of the axes migration magnitude in relation to the torso. Again, Figure 6-8 is presented with the shoulder fixed.

From Figure 6-7 and 6-8, the screw axis can be seen to migrate around the shoulder joint. The migration of the axes around the shoulder joint is expected. Although the migration is expected, the migration of the axes at some instances seems rather huge. Some axes positions were not kept within the body. This shows errors in the positions of the axes as intuitively, the migration should be kept within the body. The error in position could well be from the algorithm which transform the rotating body and arm

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Figure 6-7: Instantaneous screw axis during a golf swing from (a) address to start of backswing, (b) backswing, (c) top of backswing and (d) top of backswing till impact.

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Figure 6-8: (a) Top view, (b) front view and (c) side view of instantaneous screw axis during a golf swing. A rectangular box was added in to illustrate the subject's torso dimension and position.

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to one fixed frame. The algorithm might not fully neutralize the effect of the torso rotation which eventually affected the results. Looking back at the error in position for the simulation with random error, the maximum error was about 2cm. As mentioned, higher sampling rate introduces more noise and the error in position of the axes naturally increased. This increase, although not huge with respect to the subject, might be enough to bring the axes out of the shoulder joint. This is one shortfall of current algorithm. Despite of the error, the results were still not too huge to render them useless. They still give a rough picture of the migration and more importantly, the directions of the axes can still be used to interpret the main rotations.

Looking at Figure 6-7a, the initial screw axis direction from the address was kept near the X-Y plane. The results showed that initially anteverision moved the arm up a little before the axes migrated to about 45° line meaning that the subject moved the club off the ground initially before flexing diagonally upwards where adduction come in to move the arm together with anteversion. For the upswing (Figure 6-7b), initially the screw axis continued the diagonal flexing trend as before until arm rotation came in to skew the axes at the middle part. Towards the end of Figure 6-7b, the arm rotation eased and the axes were almost perpendicular to the arm at the given instant. The anteverision and adduction continues to bring the arm up till the top of backswing (start of Figure 6-7c) before the axes rapidly migrated to almost parallel to z-axis (middle of Figure 6-7c, green lines), meaning that abduction was the initial action for the downswing. The axes quickly migrated to almost perpendicular to the arm again (end of Figure 6-7c) meaning that retroversion comes into play and arm rotation was not the main focus there. The screw axes were still nearly perpendicular to the arm near the front part of the downswing (start of Figure 6-7d), showing directions that

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resulted from both arm abduction and retroversion. The axes directions then quickly migrated away from the perpendicular direction. Towards impact (end of Figure 6-7d), the axes were getting more parallel to the arm when seen from front view (X-Y plane). These directions of screw axes were results from combination of arm abduction and external rotation, with the rotation gaining more dominance.

The screw axis basically gives an indication of which action dominates, as the screw direction will tends to follow the axis of the main action. By studying the screw axes, they give an indication on how an individual golfer executes the swing. It is not easy to interpret every screw axis of the golfer; however there should be a general trend in the migration of the axis. Further studies could be carried out on golfers to determine if there are optimum migration trend or range of screw axes that novice golfers can try to follow in order to improve.

6.8 CONCLUSION FOR SCREW AXIS ANALYSIS

In our study, the estimation of screw axes from a set of Plücker lines clearly outperforms the conventional methods using points data. The presented method also showed that screw axis could be easily determined from the dual transformation matrix simply because the fact that the axis is invariant during the transformation. This opens the range of new practical applications of Plücker lines in bio-kinematics situation. The eigenvector and SVD method proposed also worked well and gives a simple alternative to the dual vector calculation. The method provides an alternative analysis tool for estimation of screw axes and can be applied to actions such as gait analysis in clinical setting to estimate the screw axis.

CHAPTER SEVEN

OTHER ANALYSIS

The previous chapters focus on the upper body, namely left arm and shoulder, of the golfer during the golf swing. This chapter shifts the focus down to the lower body, namely foot reaction forces, of the golfer during the swing. This chapter is done in order to give a more complete picture of the golf swing. Both upper and lower bodies are important in executing a good swing. Although the main focus of the project is the analysis of the left arm, understanding the foot reaction forces and its function would complement what we have analysed so far and enhance the execution of the golf swing.

7.1 EQUIPMENT

The experiment uses the following equipment:

- F-Scan (Tekscan, Inc. USA).
- Accelerometers (Kistler Instrument Corp. USA)
- Dynamic Signal Analyzer (Nicolet Technologies, USA).
- Matlab (The Math Works, Inc., Natick, USA).

7.2 EXPERIMENTAL BACKGROUND

In executing the golf swing, the weight shift of the player has always been considered as one of the important factors. As can be seen from the various studies in the

LITERATURE REVIEW chapter, weight transfer is the emphasis in golf teaching as effective weight transfer will add power to golf shots or ball driving distance. A common move that some golfers want to correct in golf swing is the “reverse-c” which occurs when the hips are dramatically thrust out toward the target and the head is leaning back with the spine also arched backward at finish position to the full swing. It produces inconsistent shots, but more importantly it causes lower back problems. The reverse-c might not be easily detected by visual observation if it is subtle. Another aim of golfers is to get consistent shots which mean consistent weight transfers as well since consistent weight transfer will increase the consistency in ball striking. This is a primary study to check for consistency in weight transfer for golfers between shots and to investigate whether some swing characteristics can be identified through analysis of the foot using F-scan.

7.3 EXPERIMENTAL PROCEDURE

Two subjects were involved for this study: subject 1 is a semiprofessional golfer with a single handicap and subject 2 is a weekend golfer. F-scan (Tekscan, Inc. USA) was used to record the foot pressure of the golfers. Each golfer executed a series of swing at the driving range. The F-scan sensors detects the plantar pressure and consists of individuals pressure transducers arranged in rows and columns. The sensors were inserted into the shoe of the golfers before they execute the swings. An accelerometer was attached near the clubhead on the shaft of the club to identify the impact point. The Weight-Time graphs and the Center of Force graphs were compared between the subjects.

7.4 RESULTS

Figure 7-1 shows the Weight-Time graphs for both subjects. Five trials were presented for each subject. The graphs for both subjects agree well with the general trend of other studies (Pforringer and Rosemeyer, 1989; Robinson, 1994). With time 0 indicating the impact point, the general trend can be seen as: before impact, majority of the weight rest on the right leg and the weight was then shifted to the left leg just before impact. Figure 7-1a and 7-1b shows the Weight on the right and left foot of subject 1 respectively. It is observed that the majority of the subjects weight was in the right leg before the impact and just before the impact, the weight was transferred rapidly to the left leg and after the impact, the weight remains at the left leg. Figure 7-1c and 7-1d shows the weight on the right and left foot of subject 2 respectively. It is observed that the majority of the subjects' weight was in the right leg before the impact and just before the impact, the weight was transferred rapidly to the left leg. At the impact, although most weight still remained at the left leg, but it can be seen that the weight was starting to shift back to the right leg. After the impact, the weight was shifted back to the right leg for a short period before finally ending the swing with the weight on the left leg. From the Weight-Time graph, we can see that both golfers display consistent weight shift pattern with the better golfer being more consistent.

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Figure 7-1: Weight-Time graphs for five trials each.

Figure 7-2: Ratio of weight on the left foot to the total weight on both feet (five trials each).

The data for the Weight-Time graphs can be presented in another way to show a different perspective. Figure 7-2 shows the ratio of weight on the left foot to the total weight on both feet. As before, we can see that both golfers display consistent weight shift pattern with the better golfer being more consistent. It can be observed from Figure 7-2 that at the top of backswing, when most weight rested on the right foot, both subject had about 20% of the weight on the left foot, i.e. 80% of the weight was on the right foot. The rapid increase in ratio just before impact indicates the downswing. From Figure 7-2a, Subject 1's weight on left foot peaked at impact with

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about 80%. After that the weight on left foot dropped to remain at 70%. Figure 7-2b shows the Ratio-Time graphs for subject 2. Subject 2's weight on left foot peaked slightly before impact at about 70%. After impact, the weight on left foot drop drastically to 40%, indicating a shift of weight back to the left foot. After that, the weight is transferred back to the left foot again the percentage on left foot remained at 60%.

Figure 7-3 shows the Center of force (COF) graphs for the golfers. Figure 7-3a and 7-3b shows the path of COF on the right and left foot of subject 1 respectively. It can be seen that for subject 1, the COF at the dressing was in front and the COF shifted back when the subject goes into a backswing phase and at the downswing, the COF moves forward again with impact point near the tip. The COF moved back again after the impact. The COF path for subject 1 can be observed to be moving back and forth in a more lateral path. Figure 7-3c and 7-3d shows the path of COF on the right and left foot of subject 2 respectively. The COF path for subject 2 was somewhat not similar to subject 1. For subject 2, the COF starts near the back for the right foot and near the front for the left foot. At the backswing the COF moved back slightly and at impact, the COF for the right foot moved front whereas the COF for the left foot moved towards the left. The COF ends near the tip for the right foot whereas the COF ends nears the rear for the left foot. From the COF graph, we can see that the better golfer has a more consistent COF path.

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Figure 7-3: COF graphs with five trials each.

7.5 DISCUSSION

Referring to the Weight-Time graphs, the total weight for the golfers during swing action does not always add up to the total weight at rest. This is due to the fact that during movement, the body is accelerated and hence the change in the total weight measured. Also, the F-scan only measures upward force, the forces in other directions are not recorded. From the results of the Weight-Time graphs, the pattern of the weight shift in the golfer can be seen clearly. Looking at the graphs for subject 2, after the impact, the weight actually shifted back to the right leg for a while, this suggest that the subject actually goes into a “reverse-c” position although he still ended with majority of the weight on his left foot. From the Weight-Time graphs, swing errors like reverse-pivot and the reverse-c can be deduced. Usually these mistakes are identified through visual observation which becomes difficult when the golfer become more proficient. Identification of these subtle mistakes is still important as they still affect the swing.

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The weight shift pattern of a golfer can also be clearly interpreted in Ratio-Time graphs like those of Figure 7-2. As mentioned, about 80% of the weight rested on the back foot at the top of the back swing for both subjects. This is higher than the percentage reported by Konig et al. (1994). This is probably due to the fact that F-scan only measures upward forces. The Ratio-time graphs should show a clear and smooth trend when the golfer executes the weight shift correctly (Figure 7-2a). In contrast, if the golfer make mistake like “reverse-c”, there would be abrupt changes in the graphs and these can be clearly seen (Figure 7-2b). In addition, the Ratio-Time graph offers an excellent way to check when and how much (with respect to the total weight on both feet) a golfer moves his weight onto the left leg. The nearer to the impact for this peak weight transfer and the higher the ratio of weight in the left foot, the more power it should have in delivering the shot. In this aspect, subject 1 has clearly demonstrated a better weight transfer pattern both in terms of timing and ratio of the peak weight transferred.

From the results of the COF graphs, one can study how the golfers moved his weight around. Subject 1 is observed to have a “rocking” motion going back and forth. This “rocking” motion suggest that Subject 1 uses his leg more as a means of transferring his power generated from his swing action and his body weight into the shot rather than a mean of power generation. Whereas for Subject 2, one can observe that the staring point for his left leg is at the front whereas that for his right leg is more towards the heel. During the downswing the COF moves in different direction. The COF for the right leg moves forward whereas the one for left leg moves backward towards the heel. This suggests that Subject 2 is trying to generate the power for his swing from the leg. Both legs are positioned and moved in such a way that they will

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create an anti-clockwise rotation during the downswing. Hence from observing the path of the COF, we can deduce how the golfer tries to generate power for his swing. The difficulty in coordinating both leg to successfully generate high power is probably why the COF graphs for Subject 2 is more inconsistent than Subject 1 who merely uses his leg as support and force transfer mechanism. It is also probably more difficult to execute and causes the golfer to be unstable which is why Subject 2 goes into a “reverse-c” position to stable himself. The way Subject 1 shift his weight also allows him to shift his body weight towards the ball during the downswing, which probably helps to add more power in his shots.

7.6 CONCLUSION FOR F-SCAN ANALYSIS

The studies of the two golfers suggested that consistency, some swing characteristic like power generations and even errors could be identified through the Weight-Time graphs, Ratio-Time graphs and COF graphs. The method should be able to help monitor certain mistakes that golfer might have and guide the golfer into correcting it. The study could be further expanded to collect more Weight-Time, Ratio-Time and COF graphs of good golfers and see if there is a certain ideal pattern that lesser skilled golfers can follow.

CHAPTER EIGHT

CONCLUSION

For the entire project, the left arm segmental rotations, screw axes for shoulder joint and foot pressure have been analysed. The emphasis of the project has been the left arm segmental rotation, and indeed, much has been done. Below is the summary of what has been done for that aspect of the project:

- Ascertain the possibility of goniometer as a measurement tool in golf swing.
- Developed a new algorithm, dual Euler method, for the calculation of segmental contribution.
- Verify the dual Euler method with present methodology.
- Done a comparison studies on different golfers using the dual Euler method.
- Quantified segmental rotations.
- Successfully calculated the individual segmental rotation's contribution to the clubhead speed.
- Discovered the importance of arm internal rotation and supination in generating the clubhead speed which will be missed by two-dimensional methodology.
- Verified the importance of wrist cock in golf swing.
- Provide way to check temporal to distal sequence of release.
- Provide way to check synchronisation of arm segment during the swing.
- Developed a motion capture methodology using the dual Euler method.

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The kinematics application of the dual number is then extended to the screw axis analysis. The application of screw axis to joint kinematics and golf swing forms another important aspect of the project and below summaries the work done in this area:

- Developed a new algorithm, based on plücker line method, for estimation of screw axis.
- Verified the effectiveness of the plücker line method with previous work using simulations.
- Applied the plücker line method to ankle joint and successfully calculated the screw axes during dorsiflexion-planterflexion and inversion-eversion movements.
- Applied the plücker line method to the shoulder joint during golf swing and calculated the screw axes during the swing for the first time.

In addition to the studies done on left arm and shoulder, the foot reaction forces are looked into to give a more complete study of the golf swing. Below summaries what has been done on this addition aspect of the project:

- Used the Weight-time graph to check for subtle mistakes.
- Used the Ratio-Time graphs to check the effectiveness of the weight transfer.
- Used the COF graph to analyses golfers power generation method.
- Ascertain the possibility of F-scan as a measurement and analysing tool in golf swing.

The project presented various golf analysis methods and its verification. Successful initial trials have demonstrated their potential applications and ability in analyzing

CONCLUSION

various aspect of the golf swing. The project has set the basis for future potential project on extensive research with the methods presented in this report. As the project focus on development and verification of the methods, limited number of subjects were involved. Despite the small number in the subjects, there seems to be reason to believe that there could be some characteristics in the golf swing of better golfers. Using this project as reference, future project could proceed on to collect more data from a larger number of subjects with different skills level. Statistical analysis could be done to identify important swing characteristics from better golfers and common swing errors from weaker golfers. Potentially generalised golf swing pattern from better golfers could be quantified or mapped out for lesser skilled golfers to follow. Other than applying to golf, the methods discussed in this thesis can also be applied to analyses other sports or actions.

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ANGULAR VELOCITIES AND SEGMENTAL CONTRIBUTIONS GRAPHS OF
OTHER SUBJECTS

APPENDIX A

**ANGULAR VELOCITIES AND SEGMENTAL
CONTRIBUTIONS GRAPHS OF OTHER SUBJECT**

ANGULAR VELOCITIES AND SEGMENTAL CONTRIBUTIONS GRAPHS OF
OTHER SUBJECTS

A.1 ANGULAR VELOCITY-TIME GRAPHS

ANGULAR VELOCITIES AND SEGMENTAL CONTRIBUTIONS GRAPHS OF
OTHER SUBJECTS

ANGULAR VELOCITIES AND SEGMENTAL CONTRIBUTIONS GRAPHS OF
OTHER SUBJECTS

A.2 SEGMENTAL CONTRIBUTION-TIME GRAPHS

ANGULAR VELOCITIES AND SEGMENTAL CONTRIBUTIONS GRAPHS OF
OTHER SUBJECTS