Multi-aided inertial based localization for outdoor ground vehicles

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Multi-Aided Inertial Based Localization for Outdoor Ground Vehicles

Liu Bingbing

School of Electrical & Electronic Engineering

A Thesis Submitted to the Nanyang Technological University
in Fulfillment of the Requirement for the Degree of
Doctor of Philosophy

2007

To my parents.

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Abstract

The thesis focuses on the study of multi-aided inertial based localization for ground vehicles in outdoor, uneven environments. The objectives are to propose solutions that reduce the effects of noise, biases and other error sources that exist when low-cost inertial measurement devices are used for the estimation of the pose of the vehicle. Initially, a noise analysis of the measured vehicle dynamics is made for the data output from an inertial measurement unit (IMU) in the form of raw inertial signals measured when the vehicle is stationary and in motion, in order to reduce the noise that corrupts the raw inertial signals. The resulting signals are used as inputs to a novel multi-aided inertial navigation algorithm that has been developed as part of this research. For this purpose, other complementary sensors are used, namely vehicle odometers, a single-axis gyroscope as well as knowledge of the vehicle kinematic constraints. It should be noted that the proposed algorithm does not use GPS to provide absolute position constraints. A lateral velocity estimator is proposed to model the vehicle constraints more accurately, especially when the hosted vehicle negotiates sharp turns. The proposed algorithms have been verified via simulations, which have been made based on principle inertial algorithms, given

any smooth curve as the vehicle's trajectory.

Further investigation has been conducted into the cross correlation between the prediction and observation noise of the aided inertial navigation system. It is noted that there is cross correlation due to the transformation of the body frame velocity of the vehicle into the navigation frame, which violates the generally used assumption that the estimated system states are corrupted with white noise. The violation of this assumption in turn causes the estimation filter to diverge. A solution to this issue is formulated by including the cross-correlation term as part of the estimation filter.

Experimental results have been presented for the multi-aided inertial navigation system (INS), which shows that even without the aiding of GPS, the proposed multi-aided INS algorithm can operate correctly for extended periods in a stand alone manner.

Based on the accurate and reliable localization ability of the multi-aided INS, a 3D range data representation of the vehicle's outdoor uneven environments has been implemented by adopting a laser range scanner. The successful representation of the range data in a 3D space as the vehicle moves also demonstrates the reliability of the proposed localization solution.

Finally, in order to interpret the LADAR's data correctly, in particular for applications such as feature detection, data association and data representation, one of the major sources of error has been studied, namely occlusion. For this purpose, a model to represent occlusions existing in non-coaxial LADARs has been developed.

Chapter 1

Introduction

1.1 Rationale and Objectives

This thesis addresses the issue of providing a multi-aided inertial based localization and 3D data acquisition system for ground vehicles in outdoor environments.

A low-cost inertial measurement unit (IMU) is the main sensor adopted in this work. The term “low cost” is used to symbolize a “strapdown” IMU built with low grade accelerometers and gyroscopes which can be implemented cost effectively in civilian applications. An inertial navigation system (INS) is an algorithm which uses an IMU to compute the position, velocity and attitude information of a vehicle which carries the IMU. Inertial navigation suffers from the problem of unbounded error growth since the sensor errors accumulate over time. In order to constrain the errors, external aiding is necessary. In this thesis, multi-aiding information from vehicle constraints, wheel encoders and an additional single-axis gyroscope will be used in the INS algorithm.

Additional to the inertial localization issue for ground vehicles in outdoor environments, a 3D data representation system is also implemented. Based on the

vehicle's localization ability from the multi-aided INS, 3D geometric data of the environments can be acquired using a 2D laser range finder, combined with the estimated motion of the vehicle. In conjunction with the use of the laser range finder, a sensor error model, to overcome range measurement errors for the type of range finder used is also developed.

The purpose of this thesis is to provide such a multi-aided inertial based localization and 3D data representation system. In order to fulfil this purpose, the goal objectives are:

- To understand the principles and implications of implementing low-cost IMUs for ground localization applications.
- To develop appropriate error models as process models for low-cost IMUs which determine the accuracy of the inertial navigation system.
- To understand the error sources of the INS algorithm and determine the main error components.
- To develop a new method, which determines the local noise power and reduces it in the raw data of IMUs so as to improve the accuracy of the INS.
- To develop a multi-aiding system for the IMU, without the aiding of GPS, but still achieve an accurate and reliable localization ability for IMUs.
- To address the cross correlation problem between the process and observation noise of the multi-aided INS, which is caused by the transformation of the body frame velocity observations into the navigation frame. This cross correlation

violates the assumption that the estimated INS states are corrupted with white noise.

- To simulate raw inertial signals from theoretically given vehicle trajectories so that the multi-aided INS algorithm can be tested and analyzed using raw inertial data with controlled biases and noise, as well as knowledge of ground truth.
- To develop a 3D data representation system with the adoption of a 2D laser range finder.
- To evaluate the errors associated with the laser sensors and to model the occlusion effect, which is a dominant error source in non-coaxial laser sensors.

1.2 Autonomous Navigation Systems

In this section, a literature survey is presented in the area of autonomous navigation systems. In this thesis, the term “navigation” is defined in the sense of localization. This localization problem is encountered by all autonomous vehicles, no matter whether in the air, on the ground or under water. Hence, the autonomous navigation problem has been the subject of extensive research in the mobile robotics community.

Much effort has concentrated on solving the famous simultaneous localization and map building (SLAM) problem for mobile robots exploring indoor environments [1, 2, 3, 4, 5, 6]. Odometry (usually wheel and steering encoders) play a very important role for indoor mobile robotics especially in the case of wheeled vehicles.

This dead reckoning method is able to provide position estimates for short distances only. Odometric sensors are subject to unbounded, accumulated, systematic and non-systematic errors. Some researchers have focused on improving the odometric sensing accuracy. For example, Borenstein et al proposed methods to measure and reduce systematic errors for odometry used in mobile robots, [7, 8, 9]. In [9], a technique named the UMBmark test is presented, which calibrates the systematic errors of odometry for a two wheel mobile robot. In [10], Chong et al developed a new error model for the non-systematic errors of odometry. In this approach, the entire vehicle path is divided into small segments, which allows efficient covariance matrix updating after each segment and hence avoids the necessity of incrementally updating the covariance matrix.

Contrary to fairly flat and structured indoor environments, it is much more difficult for mobile robots to localize in general outdoor locations. Firstly, wheel encoders suffer much larger systematic errors for heading estimation when the vehicles run on uneven surfaces. Secondly, outdoor environments are often semi-structured or even totally unstructured, which causes feature detection and data association methods ([11, 12, 13, 14, 15, 16]), widely used in indoor cases, to fail.

A straightforward solution to the problem of localization in outdoor environments is the use of the Global Positioning System (GPS). However the application of GPS is subject to multi-path errors and the problem of limited view of enough satellites, especially in urban or forest areas where high buildings or tall trees exist. Further, GPS can be easily jammed and it maybe totally unavailable, for example, in

planetary exploration applications. Research towards solving localization based on GPS data only, when a reduced number of satellite signals can be received, has been carried out. In [17], the minimum number of satellites necessary for GPS has been reduced by using a constraint method by modeling the path of the vehicle in urban canyon environments as segments of lines. It is reported that with this constraint, the minimum number of necessary satellites can be reduced to two, which is satisfied in most urban canyon environments.

Due to the fragile nature of GPS data, more robust methods of localization need to be found for ground vehicles in outdoor environments. Inertial navigation systems are non-jammable and self-contained and can provide pose estimation in three dimensions due to a triad of orthogonal accelerometers and gyroscopes. Low-cost inertial measurement units (IMUs) are increasingly becoming made commercially available and the use of INS in automotive applications has continuously increased in the past decade. Since rate information has to be integrated to produce velocity, position and attitude, the small errors in the rate measurements will cause accumulated, unbounded errors in the integrated measurements. Hence, usually IMUs are combined with external sensors to produce effective vehicle pose information.

It is typical to use GPS as the absolute data source to bound INS errors and many INS/GPS navigation systems for autonomous ground applications have been developed successfully [18, 19, 20, 21]. In recent years, more and more research has been focused on the error analysis and performance evaluation of GPS/INS navigation systems, [22, 23]. In [24], an INS/GPS data fusion algorithm has been

utilized for autonomous airborne navigation, which the authors refer to as six degree of freedom SLAM.

When GPS is unavailable, other methods towards solving the problem of errors in INS exist in the literature. Barshan *et al* modeled the biases and drifts of inertial sensors as exponential functions of time, and estimated an augmented mobile robot pose state, which contained these bias terms [25, 26]. In [27] and [28], data from odometry and gyroscopes have been fused together for improved localization ability and better accuracy. In [29], a method was presented for combining odometry and inertial information to provide a location estimate in 6 degrees of freedom of a rough terrain rover. In [30], an aircraft dynamic model was used as a means for aiding inertial navigation systems in estimating the navigation unknowns, namely, position, velocity and orientation. This idea is of great interest in conjunction with low-cost IMUs since it was demonstrated by simulations that without any external sensors, the use of aircraft dynamics for aiding a low-grade INS yields considerably better performance than a pure INS. Inspired by the work of [30], Gamini et al [31] put forward an on-the-fly alignment method by using the constraints that govern the motion of a vehicle to improve the accuracy of low-cost INS. They made the INS velocities observable by using these constraints and odometric information. Limitations of their method are that the position and attitude are not directly observable. The constraints presented in [31] are often violated in actual applications, especially if the vehicle negotiates a bend. In this case side slip, caused by the centripetal forces, is not negligible, and modeling the lateral velocity as Gaussian white noise

becomes inadequate. As an extension of the work of [31], Ma et al [32] used not only the vehicle constraints that govern the motion of a land vehicle, but also virtual position measurement from a vehicle kinematic model, to provide aiding for a low-grade INS. It is claimed that with virtual position and velocity observations from the vehicle kinematic model and constraints, the accuracy of the low-grade INS could be considerably improved. However only simulation results have been presented in [32].

Besides improving the performance of INS by external aiding methods (from GPS, odometry or vehicle constraints), little work exists on achieving better performance by directly modeling the errors associated with INS, [26, 33]. In [33], a nonlinear angular approach is presented to solve the problem of the assumption of small angular errors (i.e., < 5 degrees) which are generally used in INS. This new angular approach is a general angular error model that can be used for both small and large angular errors and hence, with this approach, good quality gyros and external tilt sensors are not required for initial alignment, since the assumption of small orientation errors is not necessary. However, this angular error model is nonlinear and subsequently increases the computational load of the whole navigation system. With small angular error assumptions, the system is capable of functioning in a linear fashion and thus reduces the problem of linearization. That is the reason that the nonlinear angular error model has to be converted back to small angular error model whenever the condition becomes satisfied (angular errors < 5 degrees). As technology advances, the small angular errors assumption is easy to be satisfied for

commercially available IMUs, for example, the Inertial Science IMUs, [34]. Hence, there is no absolute necessity to adopt the nonlinear angle error model for these IMUs.

As well as INS, other 3D pose estimation or positioning algorithms exist, [35, 36]. In [36], an impressive, automatic implementation of 3D SLAM using cameras has demonstrated advances in the range and agility of motions which can be tracked in three dimensions. However, limited to the range ability of cameras and the weak motion assumption of cameras (that camera motion is sufficiently modeled by a constant, low speed model), this vision based 3D SLAM algorithm is only confined to short distance and low speed applications.

Based on the above literature survey, it is clear that an inertial navigation system is needed for autonomous ground vehicles operating in uneven outdoor environments. Since GPS could be unavailable, it is necessary to propose a set of alternative aiding sensors to help bound the accumulated navigation errors of the inertial navigation system during GPS outages. The idea of modeling vehicle constraints that govern the motion of a vehicle as “virtual sensors” was presented in [31] and [32]. Together with wheel encoders to measure the forward velocity of the vehicle, this “virtual sensor + encoder” sensors suit may be used as an effective substitute of GPS. However, the problems associated with the methods proposed in [31] and [32] have been discussed already. There is a need to put forward a novel lateral velocity estimator to improve the performance of the “virtual sensors” formed by the vehicle constraints and experiments need to be carried out to prove the algorithm.

The velocity observations from the “virtual sensor + encoder” sensors suit are in the vehicle’s body frame and need to be transformed into the navigation frame, which involves the vehicle’s orientation estimations (the Euler angles, yaw, pitch and roll). Because the Euler angles are part of the vehicle’s system vector, the use of them in the transformation of the velocity observations cause a problem of cross correlation between the system observation errors and the system errors, which violates the assumption of the Kalman filter and could lead to filter divergence. This problem inspires a formulation of the cross correlation term to be added into the Kalman filter. By running the Kalman filter with and without the cross-correlation term and comparing the error covariance of each individual state variable, a conclusion will be made on the affects of the the cross-correlation term on the system’s performance.

What’s more, it is also necessary to model the noise and biases, which are the major error sources of inertial signals. In the literature, little work exists on achieving better performance by directly modeling the errors associated with INS, [26, 33]. There is a high necessity to study the major error sources, noise and/or biases in order to reduce their affect.

Hence, in this thesis, a multi-aided inertial based navigation system will be proposed to contribute to the research needs in the area of autonomous navigation in uneven, outdoor environments. A novel lateral velocity estimator will be proposed with a kinematic model to improve the performance of this “virtual sensor + encoder” sensors suit. Together with an extra single-axis gyroscope to produce heading observations, this multi-aided inertial navigation system is aimed to extend the INS

performance for a much longer period during GPS outages. Meanwhile, noise estimation and reduction and other necessary techniques such as the formulation of the cross correlation term are to be developed to ensure its successful implementation.

1.3 Thesis Contributions

The main contributions of this thesis are:

1. The understanding and formulation of appropriate error models for low-cost inertial navigation systems for ground localization applications. The quantification and analysis of errors associated with low-cost IMUs and the determination of the dominant error sources as from noise and biases for low-cost inertial navigation systems.
2. The application of a minimum controlled recursive averaging method to estimate and reduce the noise level in the raw IMU data.
3. The development of a reliable and accurate multi-aided inertial based localization solution for outdoor ground vehicles, which uses a kinematic model comprised of odometric information and vehicle constraints with a novel lateral velocity estimator, as well as an external gyroscope.
4. The simulation of raw inertial signals from theoretically given vehicle trajectories, which enables the performance evaluation of the multi-aided INS algorithm with the generated raw inertial signals and ground truth.

5. The theoretical investigation on the cross correlation between the process and observation noise of the multi-aided INS with simulated results. The cross correlation is introduced by transformation of the body frame velocity observations into the navigation frame, which results in the violation of the assumption that the estimated INS states are corrupted with white noise and hence possible divergence of the estimation filter. The purposive introduction of the additional cross-correlation term into the estimation filter can solve this problem, which is approved by simulations.
6. The 3D data representation of outdoor uneven environments based on the 3D localization ability of the multi-aided INS as well as the modeling of the range errors caused by occlusion in non-coaxial laser range finders for scene interpretation, for the improved 3D data representation.

1.4 Thesis Organization

This section describes the organization of this thesis.

- In Chapter 2, the principles of inertial navigation systems are introduced. Error equations associated with the inertial navigation system states are derived. The Pinson error model is derived under the assumption that the Coriolis terms due to the rotation of the Earth are too small for the low grade gyros, for the analysis of the propagation of errors in the INS states. Sensor errors and their effects are investigated quantitatively in a case study of the IMU model used in this work, which helps the decision of the dominant error sources for IMUs

and hence the correct selection of an IMU for a specific application. A method for the initial calibration and alignment of IMUs is also presented. In this method, tilt sensors are used to determine the initial biases in the accelerations when the vehicle is stationary. Experimental results have demonstrated that with this method adopted, the free running INS can function accurately for an extended period of approximately 50 seconds.

- In Chapter 3, a noise analysis method, named minima controlled recursive averaging (MCRA) is introduced for noise estimation and reduction. The MCRA method estimates the noise in IMU signals by recursively averaging the noise power spectrum under the control of the minimum values of a smoothed periodogram of the noisy IMU data. Both simulation and experimental results are presented to demonstrate the effectiveness of this noise estimation method.
- Chapter 4 presents the development of the multi-aided inertial navigation system. The Pinson error model is adopted as the process model of this system and the observation model is formulated with multi-aiding information from odometry, vehicle constraints and an additional single-axis gyro. A novel lateral velocity estimator is proposed to model the vehicle constraints more accurately.
- Given theoretical vehicle trajectories, simulations of raw IMU signals have been developed and presented in Chapter 5. This has been carried out so that the multi-aided inertial navigation system can be tested in the presence of artificially injected biases and noise whilst still having knowledge of theoretically

simulated ground truth.

- Chapter 6 discusses the problem of the cross correlation between the process and observation noise of the multi-aided inertial navigation system. This cross correlation results from transforming the velocity observations into the navigation frame, which involves the estimation of the Euler angles. Subsequently, the cross correlation violates the assumption that the predictions and observations are independent and can cause the estimation filter to diverge. An approach is proposed towards addressing this problem by adding the modeled cross-correlation term in the estimation filter. Simulations are provided to show the improved filter performance when including this term.
- In Chapter 7, the experimental setup and results of the multi-aided INS is presented. Experimental results reveal that the multi-aided INS is reliable and accurate for the localization of ground vehicles in uneven outdoor environments for extended periods of time. Even without the aiding of GPS, the results show that the multi-aided INS can provide correct estimate of the navigation unknowns of the vehicle, that is, position, velocity and attitude angles.
- Chapter 8 introduces the 3D data representation system based on the inertial localization ability. Laser range scanners or laser detection and ranging sensors (LADARs) are commonly utilized for data representation systems. This chapter further demonstrates that non-coaxial LADARs suffer from incorrect range readings due to occlusion, which is a serious systematic error. It is further demonstrated that a correct understanding of such systematic errors is

essential for the correct further processing of the laser data.

- Finally, Chapter 9 provides a conclusion of this thesis.

Chapter 2

Inertial Navigation Systems

2.1 Introduction

In this chapter, the principles of inertial navigation systems are introduced. Firstly the principles with respect to different frames of navigation will be presented. Subsequently, error equations associated with the inertial navigation system states, that is, the positions, velocities and attitudes, will be derived and the Pinson error model will be given. The Pinson error model provides the mathematical basis for the analysis of the propagation of errors in the INS states. When certain assumptions, such as the Coriolis terms due to the rotation of the Earth, are safely ignored, this error model is especially useful for low-cost IMUs. It is therefore used in this thesis since it is our aim is to build a low-cost inertial navigation system.

In order to show how inertial sensor errors affect the system states, sensor errors and their effects are investigated quantitatively in a case study of the IMU model used in this work. Finally, a calibration and alignment method will be introduced to address the problems with inertial navigation systems caused by the dominant sensor errors, which are biases associated with the IMUs.

2.2 Principles of Inertial Navigation Systems

2.2.1 Definitions

A full INS usually consists of three orthogonal accelerometers and three orthogonal gyroscopes to measure the acceleration in three dimensions and the angular rates about these axes. Accelerometers measure acceleration with respect to an inertial frame, while gyroscopes measure rate of rotation. Theoretically speaking, with single and double integration of the accelerometer and gyroscopic measurements, velocity, position and attitude information can be generated. However, in practical applications, biases associated with the accelerometers, drifts associated with the gyros, scale factor errors and noise in the raw data of these sensors accumulate without bound and soon render the navigation information invalid, [37], [38], [19] and [39]. Hence, calibration and alignment is needed to keep the measurement errors bounded and make the navigation information valid for extended periods. Not only is an initial calibration and alignment necessary, but an in-flight one is also important for an INS to function for long periods of time, accurately and reliably.

2.2.2 Frames of Reference

It is important to define a number of Cartesian co-ordinate reference frames for inertial navigation. All the frames are orthogonal and right handed coordinate frames or axis sets. The derivation of inertial navigation equations in this section follows that of [37]. The common frames used in inertial navigation are:

1. **The inertial frame** (i -frame) is with its origin at the center of the Earth and its axes non-rotating with respect to the fixed stars. The axes are defined as O_{x_i} , O_{y_i} and O_{z_i} with O_{z_i} aligned with the Earth's polar axis.
2. **The Earth frame** (e -frame) is also with its origin at the center of the Earth and axes being fixed with respect to the Earth, defined as the axes O_{x_e} , O_{y_e} and O_{z_e} with O_{z_e} coincident with the Earth's polar axis. The axis O_{x_e} lies along the intersection of the Earth's equatorial plane and the Greenwich meridian plane. Hence the earth frame rotates at a rate $\boldsymbol{\Omega}$ about the axis O_{z_i} with respect to the inertial frame.
3. **The navigation frame** is also called the local geographic frame. The origin of this frame is at the starting location of the navigation system, point $\boldsymbol{P}$ (Figure 2.1) and the axes are aligned with the directions of north, east and the local vertically down direction (pointing to the center of the Earth), $(N, E, D)^n$, or $(North, East, Down)^n$. The navigation frame rotates with respect to the Earth fixed frame at a rate $\boldsymbol{\omega}_{en}$, which is governed by the motion of the point $\boldsymbol{P}$ with respect to the Earth.
4. **The body frame** initially has its origin also at the location of the navigation system, point $\boldsymbol{P}$ and axes $(X, Y, Z)^b$ that are aligned with the roll, pitch and yaw axes of the vehicle. The relationship between the Earth frame, the navigation frame and the body frame is presented in Figure 2.1.

Figure 2.1: *Navigation frames.*

2.2.3 Euler Representation Approach for Attitude Propagation

The transformation matrix C_b^n is required to evaluate the acceleration vector in the navigation frame given the acceleration vector in the body frame. The transformation matrix consists of the roll, pitch and yaw angles required to rotate the body frame to the navigation frame and hence is updated continuously since the body frame is always rotating with respect to the navigation frame. There are a number of algorithms available for this kind of attitude propagation: Euler, direction cosine and quaternion, [37, 40]. When an algorithm is chosen, it is necessary to consider the computational load and the errors caused due to simplifying the algorithm for easy computations. In real applications, the direction cosine and the quaternion approaches are used more often for their lower computational requirements. The Euler representation is the easiest form to understand among these algorithms and it is introduced here in detail to explain the formation of the transformation matrix.

It is the mathematical analog form of a gimbal system. The roll θ , pitch β and yaw γ angles used to represent the rotations from the body frame to the navigation frame are as follows:

1. rotate through angle γ about the z axis in the body frame
2. rotate through angle β about the new body frame y axis
3. rotate through angle θ about the new body frame x axis

Hence, a transformation matrix $\mathbf{C}_b^n$ is formed by multiplying the consecutive rotation matrices representing a roll, followed by a pitch and then a yaw,

$$\begin{aligned} \mathbf{C}_b^n &= \begin{bmatrix} \gamma_c & -\gamma_s & 0 \\ -\gamma_s & \gamma_c & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \beta_c & 0 & \beta_s \\ 0 & 1 & 0 \\ -\beta_s & 0 & \beta_c \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta_c & -\theta_s \\ 0 & \theta_s & \theta_c \end{bmatrix} \\ &= \begin{bmatrix} \beta_c \gamma_c & -\theta_c \gamma_s + \theta_s \beta_s \gamma_c & \theta_s \gamma_s + \theta_c \beta_s \gamma_c \\ \beta_c \gamma_s & \theta_c \gamma_c + \theta_s \beta_s \gamma_s & -\theta_s \gamma_c + \theta_c \beta_s \gamma_s \\ -\beta_s & \theta_s \beta_c & \theta_c \beta_c \end{bmatrix} \end{aligned} \quad (2.1)$$

where subscripts s and c represent sine and cosine of the angle. As the body frame rotates, the update of the Euler angles can be obtained through integration by

$$\begin{aligned} \gamma(k+1) &= \gamma(k) + \int \dot{\gamma} dt \\ \beta(k+1) &= \beta(k) + \int \dot{\beta} dt \\ \theta(k+1) &= \theta(k) + \int \dot{\theta} dt \end{aligned} \quad (2.2)$$

with $k + 1$ and k being two consecutive time tags. The derivatives of the Euler angles $\dot{\gamma}$, $\dot{\beta}$ and $\dot{\theta}$ (refer to [37]) used in equation 2.2 are obtained from

$$\begin{aligned}\dot{\gamma} &= \frac{w_y \sin \theta + w_z \cos \theta}{\cos \beta} \\ \dot{\beta} &= w_y \cos \theta - w_z \sin \theta \\ \dot{\theta} &= w_x + (w_y \sin \theta + w_z \cos \theta) \tan \beta\end{aligned}\tag{2.3}$$

where $[w_x, w_y, w_z]^T$ are the rotation rates measured by the gyros about the x, y and z axes.

2.2.4 Navigation with Respect to Different Frames

2.2.4.1 Navigation with Respect to the Inertial Frame

First let's consider the case where a vehicle is to navigate with respect to the inertial frame. The sum of the specific forces acting on the vehicle and the gravitational field component determine the acceleration components of the vehicle with respect to this space fixed reference frame. These quantities can be integrated to produce estimates of velocity and position in this frame. Let $\mathbf{p}$ represent the position vector of the point $\mathbf{A}$ with respect to O , the origin of the inertial frame shown in Figure 2.2. The acceleration of $\mathbf{A}$ with respect to the inertial frame (i -frame, denoted by the subscript i), is defined by:

$$\mathbf{a}_i = \left. \frac{d^2}{dt^2} \mathbf{p} \right|_i\tag{2.4}$$

Figure 2.2: *Position vector with respect to the inertial frame.*

The specific force acting at point **A** can be measured by a set of orthogonal, perfect accelerometers as:

$$\mathbf{f}_s = \frac{d^2}{dt^2} \mathbf{p} \Big|_i - \mathbf{g} \quad (2.5)$$

where **g** is the gravitation vector. Rearranging equation 2.5 gives:

$$\frac{d^2}{dt^2} \mathbf{p} \Big|_i = \mathbf{f}_s + \mathbf{g} \quad (2.6)$$

This is the navigation equation in the inertial frame (*i*-frame). A first integration of it will yield the velocity vector of point **A** with respect to the *i*-frame:

$$\mathbf{v}_i = \frac{d}{dt} \mathbf{p} \Big|_i \quad (2.7)$$

and a second integration will produce its position in that frame.

2.2.4.2 Navigation with Respect to the Earth Frame

In reality, when navigating in the vicinity of the Earth, it is often necessary to derive estimates of a vehicle's velocity and position with respect to a rotating frame, for example, the Earth frame. In this case, additional forces will be acting which are due to motion of the reference frame. Thus, the ground speed of the vehicle, $\mathbf{v}_e$, may be computed from the inertial velocity, $\mathbf{v}_i$, using the Coriolis theorem:

$$\mathbf{v}_e = \mathbf{v}_i - \boldsymbol{\omega}_{ie} \times \mathbf{p} \quad (2.8)$$

where $\boldsymbol{\omega}_{ie} = [0 \ 0 \ \Omega]^T$ is the rotational rate of the Earth frame with respect to the i-frame and $\times$ denotes a vector cross product.

2.2.4.3 Navigation with Respect to the Navigation Frame

In order to derive the navigation equations in the navigation frame, one needs to calculate a vehicle's speed with respect to the Earth (i.e. the ground speed, in the inertial frame, denoted by $\mathbf{v}_e^i$) first, and then transform it to the speed expressed in the navigation frame after reference frame transformation.

The inertial velocity expressed in terms of the ground speed using the Coriolis equation is:

$$\left. \frac{d}{dt} \mathbf{p} \right|_i = \left. \frac{d}{dt} \mathbf{p} \right|_e + \boldsymbol{\omega}_{ie} \times \mathbf{p} \quad (2.9)$$

Differentiating this and writing $(d/dt)\mathbf{p}|_e = \mathbf{v}_e$,

$$\frac{d^2}{dt^2}\mathbf{p}|_i = \frac{d}{dt}\mathbf{v}_e|_i + \frac{d}{dt}[\boldsymbol{\omega}_{ie} \times \mathbf{p}]|_i \quad (2.10)$$

Since it can be assumed that the rotational rate of the Earth is constant, that is,

$$(d/dt)\boldsymbol{\omega}_{ie} = 0,$$

$$\begin{aligned} \frac{d}{dt}[\boldsymbol{\omega}_{ie} \times \mathbf{p}]|_i &= \left[\frac{d}{dt}\boldsymbol{\omega}_{ie}|_i \times \mathbf{p}\right] + [\boldsymbol{\omega}_{ie} \times \frac{d}{dt}\mathbf{p}|_i] \\ &= 0 + \boldsymbol{\omega}_{ie} \times \left(\frac{d}{dt}\mathbf{p}|_e + \boldsymbol{\omega}_{ie} \times \mathbf{p}\right) \\ &= \boldsymbol{\omega}_{ie} \times \mathbf{v}_e + \boldsymbol{\omega}_{ie} \times [\boldsymbol{\omega}_{ie} \times \mathbf{p}] \end{aligned} \quad (2.11)$$

substituting the left side of equation 2.11 into equation 2.10 gives:

$$\frac{d^2}{dt^2}\mathbf{p}|_i = \frac{d}{dt}\mathbf{v}_e|_i + \boldsymbol{\omega}_{ie} \times \mathbf{v}_e + \boldsymbol{\omega}_{ie} \times [\boldsymbol{\omega}_{ie} \times \mathbf{p}] \quad (2.12)$$

Combining equations 2.6 and 2.12 gives:

$$\frac{d}{dt}\mathbf{v}_e|_i = \mathbf{f}_s - \boldsymbol{\omega}_{ie} \times \mathbf{v}_e - \boldsymbol{\omega}_{ie} \times [\boldsymbol{\omega}_{ie} \times \mathbf{p}] + \mathbf{g} \quad (2.13)$$

In this equation, $\mathbf{f}_s$ is the specific force acceleration to which the vehicle is subjected and $\boldsymbol{\omega}_{ie} \times \mathbf{p}$ is the acceleration caused by its velocity over the surface of the rotating Earth, usually referred to as the Coriolis acceleration. The term $\boldsymbol{\omega}_{ie} \times [\boldsymbol{\omega}_{ie} \times \mathbf{p}]$ defines the centripetal acceleration experienced by the vehicle due to the rotation of the Earth. This term is not separately distinguishable from the mass attraction

gravitational acceleration, $\mathbf{g}$. The sum of these two terms constitutes the commonly known local gravity vector, to which a plumb bob would align itself if held above the Earth [37]. The local gravity vector is denoted as $\mathbf{g}_l$:

$$\mathbf{g}_l = \mathbf{g} - \boldsymbol{\omega}_{ie} \times [\boldsymbol{\omega}_{ie} \times \mathbf{p}] \quad (2.14)$$

Now equation 2.13 can be written as:

$$\left. \frac{d}{dt} \mathbf{v}_e \right|_i = \mathbf{f}_s - \boldsymbol{\omega}_{ie} \times \mathbf{v}_e + \mathbf{g}_l \quad (2.15)$$

If the ground speed of the vehicle is expressed in an Earth fixed coordinate frame to give $\mathbf{v}_e^e$, it follows the Coriolis equation and may be expressed in terms of its rate of change in the inertial frame:

$$\left. \frac{d}{dt} \mathbf{v}_e \right|_e = \left. \frac{d}{dt} \mathbf{v}_e \right|_i - \boldsymbol{\omega}_{ie} \times \mathbf{v}_e \quad (2.16)$$

Substituting equation 2.15 into equation 2.16 gives:

$$\left. \frac{d}{dt} \mathbf{v}_e \right|_e = \mathbf{f}_s - 2\boldsymbol{\omega}_{ie} \times \mathbf{v}_e + \mathbf{g}_l \quad (2.17)$$

In order to navigate on the Earth's surface, navigation information is most often required in the navigational or local geographic axis set, i.e. in terms of north, east and vertical velocity and position components. Such information can be computed by using the position estimates provided by the inertial or Earth frame expressions

described above. However, this kind of computation involves a further transformation of the vector quantities $\mathbf{v}_e^i$ or $\mathbf{v}_e^e$, which was derived in equations 2.15 and 2.17 respectively. Hence, the navigation equations in the navigation frame are derived, which are often used when navigating on the Earth. The ground speed of the vehicle is expressed with respect to the navigation axis set to produce $\mathbf{v}_e^n$. The rate of change of $\mathbf{v}_e^n$ with respect to the navigation axes may be expressed in terms of its rate of change in the inertial coordinates as the following:

$$\left. \frac{d}{dt} \mathbf{v}_e \right|_n = \left. \frac{d}{dt} \mathbf{v}_e \right|_i - (\boldsymbol{\omega}_{ie} + \boldsymbol{\omega}_{en}) \times \mathbf{v}_e \quad (2.18)$$

Substituting from equation 2.15 for $(d/dt)\mathbf{v}_e|_i$ we get:

$$\left. \frac{d}{dt} \mathbf{v}_e \right|_n = \mathbf{f}_s - (2\boldsymbol{\omega}_{ie} + \boldsymbol{\omega}_{en}) \times \mathbf{v}_e + \mathbf{g}_l \quad (2.19)$$

This is the navigation equation in the navigation or local geographic frame and can be further expressed in navigation coordinates as:

$$\dot{\mathbf{v}}_e^n = \mathbf{C}_b^n \mathbf{f}_s^b - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \mathbf{v}_e^n + \mathbf{g}_l^n \quad (2.20)$$

where $\mathbf{C}_b^n$ is a direction cosine matrix used to transform the measured specific force acceleration vector $\mathbf{f}_s^b$ into the navigation frame. This matrix changes according to the following equation:

$$\dot{\mathbf{C}}_b^n = \mathbf{C}_b^n \boldsymbol{\Omega}_{nb}^b \quad (2.21)$$

where Ω_{nb}^b is the skew symmetric form of ω_{nb}^b , the body angular rates with respect to the navigation frame. This is obtained by subtracting the measured body rates, ω_{ib}^b , from estimates of the components of the navigation frame rates, ω_{in} . The navigation frame rates can be derived by summing the Earth's rates with respect to the inertial frame and the turn rates of the navigation frame with respect to the Earth, i.e. $\omega_{in} = \omega_{ie} + \omega_{en}$. Hence:

$$\omega_{nb}^b = \omega_{ib}^b - C_n^b(\omega_{ie} + \omega_{en}) \quad (2.22)$$

Equation 2.20 is the navigation equation in the navigation or local geographic frame. The first integral of this equation will yield the velocity vector with respect to the navigation frame and the second integral will produce the position in that frame. Hence this equation with its first and second integrals can be applied if a vehicle navigates on the Earth's surface, if an IMU sensor is used on the vehicle. However, from the discussion of the next section it will be seen that it is computationally more convenient to use the equations of the *errors* associated with the position, velocity and attitude components of the INS in real applications.

2.3 Error Equations of Position, Velocity and Attitude

The theoretical navigation equations have been derived in section 2.2.4 with respect to different frames of reference. However, no errors have ever been considered

in these equations. In real applications, inertial navigation systems are subject to many sources of error and thus an error analysis is important for realistic inertial navigation. It has been shown, in many navigation applications, that the use of the errors associated with the navigational measurements (position, velocity and attitude), as system states, makes the inertial navigation formulation more computationally convenient, [37] and [41]. Hence, in this section, the error equations of position, velocity and attitude will be derived and in particular the Pinson error model will be given for low-cost INS.

2.3.1 Attitude errors

The orientation of the triad of gyroscopes in a strapdown IMU system with respect to the navigation frame may be expressed in terms of the direction cosine matrix, $\mathbf{C}_b^n$, [37]. The estimated attitude, denoted by $\tilde{\mathbf{C}}_b^n$, may be written in terms of the true direction cosine matrix, $\mathbf{C}_b^n$, as follows:

$$\tilde{\mathbf{C}}_b^n = \mathbf{T}\mathbf{C}_b^n \quad (2.23)$$

where $\mathbf{T}$ represents the transformation of the true reference axes to the estimated ones. For small angles of misalignment, the matrix $\mathbf{T}$ may be approximated as a skew symmetric matrix which may be written as:

$$\mathbf{T} = [\mathbf{I} - \boldsymbol{\Psi}] \quad (2.24)$$

where $\mathbf{I}$ is a 3×3 identity matrix and Ψ is a skew-symmetric matrix:

$$\Psi = \begin{bmatrix} 0 & -\delta_\gamma & \delta_\beta \\ \delta_\gamma & 0 & -\delta_\theta \\ -\delta_\beta & \delta_\theta & 0 \end{bmatrix} \quad (2.25)$$

The elements δ_θ , δ_β and δ_γ correspond to the attitude errors with respect to the tilt errors and the heading error respectively. These terms may be equated to the Euler angular (roll, pitch and yaw) errors for small angle misalignments.

The estimated direction cosine matrix can now be written as:

$$\tilde{\mathbf{C}}_b^n = [\mathbf{I} - \Psi] \mathbf{C}_b^n \quad (2.26)$$

Since $\mathbf{C}_b^n$ is orthogonal,

$$(\mathbf{C}_b^n)^{-1} = (\mathbf{C}_b^n)^T \quad (2.27)$$

equation 2.26 can be rearranged as:

$$\Psi = \mathbf{I} - \tilde{\mathbf{C}}_b^n (\mathbf{C}_b^n)^T \quad (2.28)$$

Differentiating this equation yields:

$$\dot{\Psi} = -\dot{\tilde{\mathbf{C}}}_b^n (\mathbf{C}_b^n)^T - \tilde{\mathbf{C}}_b^n (\dot{\mathbf{C}}_b^n)^T \quad (2.29)$$

As shown in the analysis of Section 2.2.4, the direction cosine matrix, $\mathbf{C}_b^n$, propagates

as a function of the absolute body rate, Ω_{ib}^b and the navigation frame rate, Ω_{in}^n as follows:

$$\begin{aligned}\dot{\mathbf{C}}_b^n &= \mathbf{C}_b^n \Omega_{nb}^b \\ &= \mathbf{C}_b^n \Omega_{ib}^b - \mathbf{C}_b^n \Omega_{in}^b\end{aligned}\tag{2.30}$$

According to the similarity transformation for matrices (see [42]):

$$\Omega_{in}^b = \mathbf{C}_b^n \Omega_{in}^n \mathbf{C}_b^n\tag{2.31}$$

Hence, equation 2.30 can be written as:

$$\dot{\mathbf{C}}_b^n = \mathbf{C}_b^n \Omega_{ib}^b - \Omega_{in}^n \mathbf{C}_b^n\tag{2.32}$$

Similarly, the time differential of the estimated matrix, $\tilde{\mathbf{C}}_b^n$, is given by:

$$\dot{\tilde{\mathbf{C}}}_b^n = \tilde{\mathbf{C}}_b^n \tilde{\Omega}_{ib}^b - \tilde{\Omega}_{in}^n \tilde{\mathbf{C}}_b^n\tag{2.33}$$

where $\tilde{\Omega}_{ib}^b$ and $\tilde{\Omega}_{in}^n$ represent the measured body rate and the estimated turn rate of the navigation reference frame respectively.

Substituting for $\dot{\mathbf{C}}_b^n$ and $\tilde{\mathbf{C}}_b^n$ in equation 2.29 yields:

$$\begin{aligned}\dot{\Psi} &= -\tilde{\mathbf{C}}_b^n \tilde{\Omega}_{ib}^b (\mathbf{C}_b^n)^T + \tilde{\Omega}_{in}^n \tilde{\mathbf{C}}_b^n (\mathbf{C}_b^n)^T + \tilde{\mathbf{C}}_b^n \Omega_{ib}^b (\mathbf{C}_b^n)^T - \tilde{\mathbf{C}}_b^n (\mathbf{C}_b^n)^T \Omega_{in}^n \\ &= -\tilde{\mathbf{C}}_b^n [\tilde{\Omega}_{ib}^b - \Omega_{ib}^b] + \tilde{\Omega}_{in}^n \tilde{\mathbf{C}}_b^n (\mathbf{C}_b^n)^T - \tilde{\mathbf{C}}_b^n (\mathbf{C}_b^n)^T \Omega_{in}^n\end{aligned}\tag{2.34}$$

Substituting for $\tilde{\mathbf{C}}_b^n$ from equation 2.26 yields:

$$\dot{\Psi} = -[\mathbf{I} - \Psi] \mathbf{C}_b^n [\tilde{\Omega}_{ib}^b - \Omega_{ib}^b] + \tilde{\Omega}_{in}^n [\mathbf{I} - \Psi] \mathbf{C}_b^n (\mathbf{C}_b^n)^T - [\mathbf{I} - \Psi] \mathbf{C}_b^n (\mathbf{C}_b^n)^T \Omega_{in}^n \quad (2.35)$$

Defining $\delta\Omega_{in} = \tilde{\Omega}_{in} - \Omega_{in}$ and $\delta\Omega_{ib} = \tilde{\Omega}_{ib} - \Omega_{ib}$, we get:

$$\dot{\Psi} = \Psi \Omega_{in}^n - \Omega_{in}^n \Psi + \delta\Omega_{in}^n - \mathbf{C}_b^n \delta\Omega_{ib}^b (\mathbf{C}_b^n)^T \quad (2.36)$$

and the above equation may be expressed in vector form as:

$$\dot{\psi} = -\omega_{in}^n \times \psi + \delta\omega_{in}^n - \mathbf{C}_b^n \delta\omega_{ib}^b \quad (2.37)$$

where the misalignment vector $\psi = [\delta\theta \ \delta\beta \ \delta\gamma]^T$, and $\psi \times = \Psi$, $\omega_{in}^n \times = \Omega_{in}^n$, $\delta\omega_{in}^n \times = \delta\Omega_{in}^n$, $\delta\omega_{ib}^b \times = \delta\Omega_{ib}^b$. For low-cost IMUs, the Coriolis terms and the errors in them are small enough to be safely ignored and equation 2.37 can be simplified as:

$$\dot{\psi} = -\mathbf{C}_b^n \delta\omega_{ib}^b \quad (2.38)$$

Hence, the attitude errors can be estimated using the direction cosine matrix, $\mathbf{C}_b^n$ and the erroneous measurements of the gyroscopes, $\delta\omega_{ib}^b$.

2.3.2 Velocity and position errors

In this section, the velocity and position error equations will be derived. The velocity equation in the navigation frame can be written as (see equation 2.19):

$$\dot{\mathbf{v}} = \mathbf{C}_b^n \mathbf{f}_s^b - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \mathbf{v} + \mathbf{g}_l \quad (2.39)$$

where $\mathbf{f}_s^b$ represents the specific force acceleration in the body frame.

Similarly, it can be assumed that the estimated velocity propagates according to the following equation, with estimated quantities being again denoted by a tilde:

$$\dot{\tilde{\mathbf{v}}} = \tilde{\mathbf{C}}_b^n \tilde{\mathbf{f}}_s^b - (2\tilde{\boldsymbol{\omega}}_{ie}^n + \tilde{\boldsymbol{\omega}}_{en}^n) \times \tilde{\mathbf{v}} + \tilde{\mathbf{g}}_l \quad (2.40)$$

Subtracting equation 2.40 from 2.39 we get an expression for the velocity error:

$$\begin{aligned} \delta \dot{\mathbf{v}} &= \dot{\tilde{\mathbf{v}}} - \dot{\mathbf{v}} \\ &= \tilde{\mathbf{C}}_b^n \tilde{\mathbf{f}}_s^b - \mathbf{C}_b^n \mathbf{f}_s^b - (2\tilde{\boldsymbol{\omega}}_{ie}^n + \tilde{\boldsymbol{\omega}}_{en}^n) \times \tilde{\mathbf{v}} + (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \mathbf{v} + \tilde{\mathbf{g}}_l - \mathbf{g}_l \end{aligned} \quad (2.41)$$

Substituting for $\tilde{\mathbf{C}}_b^n$ from equation 2.26 and writing $\delta \mathbf{f}_s^b = \tilde{\mathbf{f}}_s^b - \mathbf{f}_s^b$, $\delta \mathbf{v} = \tilde{\mathbf{v}} - \mathbf{v}$, $\delta \boldsymbol{\omega}_{ie}^n = \tilde{\boldsymbol{\omega}}_{ie}^n - \boldsymbol{\omega}_{ie}^n$, $\delta \boldsymbol{\omega}_{en}^n = \tilde{\boldsymbol{\omega}}_{en}^n - \boldsymbol{\omega}_{en}^n$, and $\delta \mathbf{g} = \tilde{\mathbf{g}}_l - \mathbf{g}_l$ and ignoring the product of two error terms such as $\boldsymbol{\Psi} \mathbf{C}_b^n \delta \mathbf{f}_s^b$, gives:

$$\delta \dot{\mathbf{v}} = -\boldsymbol{\Psi} \mathbf{C}_b^n \mathbf{f}_s^b + \mathbf{C}_b^n \delta \mathbf{f}_s^b - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \delta \mathbf{v} - (2\delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n) \times \mathbf{v} - \delta \mathbf{g} \quad (2.42)$$

writing $\mathbf{C}_b^n \mathbf{f}_s^b = \mathbf{f}_s^n$ and rearranging yields:

$$\delta \dot{\mathbf{v}} = [\mathbf{f}_s^n \times] \boldsymbol{\psi} + \mathbf{C}_b^n \delta \mathbf{f}_s^b - (2\boldsymbol{\omega}_{ie}^n + \boldsymbol{\omega}_{en}^n) \times \delta \mathbf{v} - (2\delta \boldsymbol{\omega}_{ie}^n + \delta \boldsymbol{\omega}_{en}^n) \times \mathbf{v} - \delta \mathbf{g} \quad (2.43)$$

Again, ignoring errors in the Coriolis terms and in the gravity vector, the above equation reduces to:

$$\delta \dot{\mathbf{v}} = [\mathbf{f}_s^n \times] \boldsymbol{\psi} + \mathbf{C}_b^n \delta \mathbf{f}_s^b \quad (2.44)$$

Finally, the position errors, $\delta \mathbf{p}$, may be written as:

$$\delta \dot{\mathbf{p}} = \delta \mathbf{v} \quad (2.45)$$

It can be seen from the above equations that the velocity and position errors are mainly functions of the specific force $\mathbf{f}_s^b$, the direction cosine matrix $\mathbf{C}_b^n$, the attitude errors $\boldsymbol{\psi}$ and the accelerometer errors $\delta \mathbf{f}_s^b$. The propagation of the attitude errors $\boldsymbol{\psi}$ is rather simple, and mainly associated with the erroneous measurements of the angular rate gyroscopes, $\delta \boldsymbol{\omega}_{ib}^b$, as well as the direction cosine matrix $\mathbf{C}_b^n$. Hence, under the conditions that the raw sensor readings of a trajectory are given, the attitude errors and the velocity and position errors can be determined. The only terms needed in advance are the errors in the accelerometers $\delta \mathbf{f}_s^b$ and the errors with the gyroscopes $\delta \boldsymbol{\omega}_{ib}^b$, which can be evaluated via experiments. Equations 2.38, 2.44 and 2.45 are known as the Pinson error model [43], and have been adopted as the dynamic error propagation model in [20], and have also been used in the work presented in this thesis.

2.4 IMU Sensor Choice and Error Analysis

It is important, but often difficult for users of inertial technology, to decide which particular IMU will meet the application requirements best. The reason is that there are many principal measurement errors associated with inertial sensors and these errors control the accuracy of these sensors and hence the choice of these sensors. In this section, a case study of the IMU used in this work is given and all these principal measurement errors are introduced and evaluated quantitatively.

2.4.1 Inertial Science IMU DMARS-I-N

Figure 2.3 shows the inertial sensor used in this work, which is a DMARS-I-N model from Inertial Science, [34].

Figure 2.3: *The DMARS-I-N model IMU from Inertial Science.*

The specifications of this inertial sensor are shown in table 2.1. Each of these

Table 2.1: *Specifications of the DMARS-I-N model.*

No	Parameter	Value	Unit
I	Linear		
1	Bias	1,000	μg
2	Scale factor	300	ppm
3	Axis alignment	300	ppm
4	Random noise	30	$\mu\text{g}/\text{sqrt}(\text{hr})$
II	Angular		
5	Non g -sensitive bias	5	deg/hr
6	Random noise	<0.02	deg/sqrt(hr)
7	g sensitive drift	0.3	deg/hr/g
8	Scale factor error	300	ppm
9	Axis alignment	300	ppm

specifications and their effect on the sensor will be explained in Section 2.4.2.

2.4.2 Explanation and investigation of the inertial sensor errors

Table 2.1 lists almost all the principal specifications of the IMU model DMARS-I-N. The effect of these specifications on the IMU performance is investigated in this section. Generally, the error equation used for the gyroscopic accuracy, for example, along the x axis, can be written as,

$$\delta\omega_x = B + \mathbf{B}_g \mathbf{A}_b^T + S_f \omega_x + M_{isa}^y \omega_y + M_{isa}^z \omega_z + \nu_{\omega x} \quad (2.46)$$

where,

- B is the gyroscopic non- g sensitive bias, the value of which for this DMARS-I-N model is 5 deg/hr, from table 2.1.

- $\mathbf{B}_g$ is a 1×3 vector representing the g sensitive drift, the value of which for this sensor is 0.3 deg/hr/g.
- $\mathbf{A}_b = [a_x, a_y, a_z]$ is the acceleration vector in the body frame.
- S_f is the gyroscope scale factor, having the value of 300 ppm for this IMU.
- M_{isa}^y and M_{isa}^z represent the misalignment errors and both have the value 300 ppm for this IMU.
- $\nu_{\omega x}$ represents random noise, the value of which is < 0.02 deg/sqrt(Hz), for this inertial sensor.
- ω_x , ω_y and ω_z are the angular rates in the body frame.

Now it is possible to investigate the effect of each item in equation 2.46 on the gyroscopic performance of this IMU, based on the specifications given in table 2.1.

- Gyroscopic non- g sensitive bias indicates the increase of angular error over time. 5 deg/hr causes a 5-degree deviation per hour for this inertial sensor, along the x axis. This item is the main error source for the gyroscopes.
- g sensitive drifts are biases proportional to the magnitude of the applied acceleration. These errors arise in the spinning gyros as a result of mass imbalance in the rotor suspension, [37]. The magnitudes of these errors depend on both this g sensitive drift item itself, and on the body accelerations. For example, if a body acceleration vector is assumed to be, $\mathbf{A}_b = [1g, 0.01g, 0.01g]$, at some instants during the vehicle's accelerating period, the g sensitive error has the

value of 0.306 deg/hr at this time instant. Most of the time, the vehicle will run at approximately constant speed so that the accelerations will be small and the errors caused by this effect will become much smaller.

- The gyroscopic scale factor is an indication of the angular error which occurs during rotation. With 300 ppm scale factor error ($= 0.03\%$), the angular error is approximately 0.1 deg/hr with the assumption that the angular rate ω_x is 360 deg/hr.
- A misalignment between the gyroscope axes causes a cross-coupling between the measurement axes. These non-orthogonality caused errors may also be expressed in ppm. With the same value, 300, as the gyroscope scale factor and the same assumption of the angular rates ω_y or ω_z being 360 deg/hr, the angular error thus caused is also approximately 0.1 degs/hr.
- Random noise or random walk shows the noise level of the gyroscopes. The higher the noise level, the more noise encountered which affects the angular rates and thus the angular estimates. $0.02 \text{ deg}/\sqrt{hr}$ indicates that the angular error due to random walk is approximately 0.02 degrees after 1 hour or 0.06 degrees after 6 minutes. However, this noise level represents only the internal noise of the sensor, which originates from thermal noise. In real applications, the external noise would be the dominant noise source for the inertial sensor. The external noise, for example, caused by vibration when the host vehicle's motor is switched on, could generate high signal peaks above the average random noise level. Hence, the real noise level could be much larger than the

specified value and should also be seen as a significant error source for the IMUs. Another important point to be highlighted about the angular random walk is that, the random walk level is essential for the accuracy of “north seeking”. “North seeking” is an ability of high cost and high accuracy gyroscopes. By a special setup, [37], the gyroscope can measure two horizontal components of the Earth’s rotating rate with its two axes at the same time and by canceling the latitude dependent terms involved, the heading angle of the gyroscope can be determined with respect to the true north.

The error analysis with the accelerometers is similar. The error equation used for the x -axis accelerometer, for example, can be written as,

$$\delta f_x = B + S_f a_x + M_{isa}^y a_y + M_{isa}^z a_z + \nu_{ax} \quad (2.47)$$

In equation 2.47, the error items have similar meanings as those in equation 2.46. The error analysis is also similar and hence omitted here.

According to the qualitative case study on the errors of IMUs, the conclusion is that, for inertial sensors, biases are dominant in the error sources for general navigation applications. The other error sources, for example, the scale factor error and the misalignment error, are much smaller. This will also be demonstrated throughout this thesis. In real applications the noise level of IMUs is not trivial enough to be negligible. The angular random walk is especially important for some specific applications, for instance, “north seeking” for high grade gyroscopes. Hence, in this thesis, efforts are concentrated on dealing with the biases and noise. A new

noise estimation and reduction method will be introduced and applied to the raw data of IMUs. Furthermore, a multi-aiding method will be shown to reduce the effect of the biases on the navigation of the host vehicle.

2.5 Initial Calibration and Alignment

2.5.1 Faults with IMUs

The conclusion of the last section was that the dominant source of errors, for low-cost IMUs, are biases and noise. Considering these to be the most dominant terms, the acceleration and rotation rates, read by the accelerometers and gyroscopes respectively, can be simplified as (again, using the x axis as an example),

$$a_x^r = a_x + \delta a_x = a_x + b_{ax} + \nu_{ax} \quad (2.48)$$

$$\omega_x^r = \omega_x + \delta \omega_x = \omega_x + b_{\omega x} + \nu_{\omega x}$$

where a_x^r is the acceleration reading from the x axis accelerometer and a_x is the true acceleration. b and ν represent the bias and noise associated with the readings respectively. For short periods of time, the bias term can be approximated as a constant. The noise can be approximated as following a Normal distribution, as shown by the histograms of actually recorded noise associated with the acceleration and angular rate readings in Figures 2.4 and 2.5. In order to plot these two histograms, the IMU was put on a flat surface and switched on for a short period of time, 4000 samples of the x axis accelerometer and gyroscope readings were recorded and plot-

ted in the histograms. In the histograms, the x axis denotes the sensor readings, acceleration (Figure 2.4) or angular rate (Figure 2.5) and the y axis denotes the sample numbers.

Figure 2.4: *Histogram of noise associated with the x -axis accelerometer readings in an experiment.*

Figure 2.5: *Histogram of noise associated with the x -axis gyroscope readings in an experiment.*

The actual velocity, position and attitude are then obtained by the integration of equations 2.48.

$$\begin{aligned} v_x^r &= v_x + b_{ax}t + \int \nu_{ax}dt \\ p_x^r &= p_x + \frac{1}{2}b_{ax}t^2 + \int \int \nu_{ax}dt \\ \theta_x^r &= \theta_x + b_{\omega x}t + \int \nu_{\omega x}dt \end{aligned} \tag{2.49}$$

It can be shown from these equations that bias terms in the inertial sensors are the main fault source for the determination of the velocity, position and attitude information. The error in the body frame velocities and attitudes grow linearly with time, proportional to the bias terms, while the body frame position errors grow quadratically with time due to the bias in the accelerometers. However, the bias in the accelerometers is of course not the only source for the position error. The bias in the gyros greatly increases the problem.

Since gyroscopic data is used to update the transformation matrix $\mathbf{C}_b^n$, erroneous computation of the acceleration in the navigation frame is caused by the existence of drift in the gyroscopes, which couples the acceleration errors from one axis to another. Figure 2.6 presents an example, in which, due to any constant bias $b_{\omega z}$ in the z axis gyro measurement, a constant acceleration a_x along the x axis will introduce errors in the y -axis acceleration, and hence the velocity and position along that axis too.

Figure 2.6: The accelerometer bias along the y axis caused by the bias in the z -axis gyroscope .

Under the assumption of small angular increments, these errors are evaluated as,

$$e_{ay} \simeq a_x b_{\omega z} t \quad (2.50)$$

$$e_{vy} \simeq \frac{1}{2} a_x b_{\omega z} t^2$$

$$e_{py} \simeq \frac{1}{6} a_x b_{\omega z} t^3$$

Hence it can be seen that a constant gyro drift will introduce errors in position proportional to time cubed. An immediate conclusion is that the overall performance of an inertial sensor depends on the quality of the gyroscopes. As seen from equations 2.49 and 2.50, biases in the acceleration and angular rate readings greatly affect the evaluation of the inertial velocity, position and attitude and must be identified and reduced as far as possible during an initialization stage.

To demonstrate and quantify this, Figure 2.7 presents the accelerometer bias

of the IMU used in this work. Over a period of 1.5 hours, while the sensor was stationary, considerable bias variations were measured along each axis due to the internal temperature change of the sensor. Different profiles along different axes show the intrinsic characteristics of the **low-cost** IMU. Hence biases should be determined each time the vehicle is stationary in order to obtain good results. This is called the “initial calibration and alignment” of low-cost IMUs.

Figure 2.7: IMU accelerometer biases. The change in bias values is caused by the internal temperature changes of the sensor. Different profiles along different axes show the intrinsic characteristics of the low-cost IMU.

From equations 2.49 the integration of white noise is also a major error source for the inertial measurements. Figure 2.8 shows the effect of integrating the x axis gyroscope measurements 30 times. Since the sensor was stationary during these experiments and the bias effect was not obvious for such a short period, the measurements denote zero-mean “white” characteristics and the integration results are known as the random walk effect. Random walk generates an error proportional

Figure 2.8: *The random walk effect of the x axis gyroscope.*

to the standard deviation of the noise and the square root of time, which is actually called “Wiener process”. The higher the noise level, the larger the variance of the caused error. On the other hand, without any external aiding method, the error caused by white noise will become unbounded with respect to time, with a value lying within the bounds $\pm 2\sqrt{t}$ with a probability of 95%.

2.5.2 Alignment and calibration algorithms

The term “calibration” means the determination of biases in the accelerometers and gyroscopes. Alignment is the process of determining the initial orientation of the platform, on which the IMU is mounted. Alignment results in the initialization of the transformation matrix $\mathbf{C}_b^n$. The concept of initial calibration and alignment for IMUs first appeared in [43]. Subsequently Nebot and Sukkarieh extended the design and implementation of this idea, [41], [44].

Typically, low-cost IMUs use tilt sensors that provide measurements of the bank

and elevation of the inertial platform when stationary so as to provide the initial alignment information. These tilt sensors become ineffective during vehicle motion. A positive bank and elevation angle will cause the accelerometers in the x and y directions to measure components of the gravity vector as,

$$a_x = g \sin(\text{elevation}) \quad (2.51)$$

$$a_y = -g \sin(\text{bank})$$

which is illustrated in Figure 2.9.

Figure 2.9: *Bank and elevation angles measured by the tilt sensors.*

If the association between the bank and elevation angles, and the Euler angles is known, the initial alignment of the Euler angles can be solved by these tilt sensor readings. During the initial alignment and calibration process, the host vehicle must be stationary. Hence the measured acceleration $\mathbf{a}_n$ in the navigation frame should

of course be gravity along the vertical axis,

$$\mathbf{a}_n = \begin{bmatrix} 0 & 0 & -g \end{bmatrix}^T \quad (2.52)$$

The measured acceleration in the body frame is related to that in the navigation frame by the inverse of the direction cosine matrix as,

$$\mathbf{a}_b = (\mathbf{C}_b^n)^{-1} \mathbf{a}_n \quad (2.53)$$

Since $(\mathbf{C}_b^n)^{-1} = (\mathbf{C}_b^n)^T$, substituting for $\mathbf{C}_b^n$ from equation 2.1, equation 2.53 can be written as,

$$\begin{aligned} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} &= \begin{bmatrix} \beta_c \gamma_c & \beta_c \gamma_s & -\beta_s \\ -\theta_c \gamma_s + \theta_s \beta_s \gamma_c & \theta_c \gamma_c + \theta_s \beta_s \gamma_s & \theta_s \beta_c \\ \theta_s \gamma_s + \theta_c \beta_s \gamma_c & -\theta_s \gamma_c + \theta_c \beta_s \gamma_s & \theta_c \beta_c \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \\ &= \begin{bmatrix} g \sin \beta \\ -g \sin \theta \cos \beta \\ -g \cos \theta \cos \beta \end{bmatrix} \end{aligned} \quad (2.54)$$

where subscripts s and c represent the sine and cosine of the angle respectively.

Comparing equations 2.54 with 2.51, the pitch and roll Euler angles can be

expressed using the bank and elevation as,

$$\begin{aligned}\beta &= \textit{elevation} \\ \theta &= \arcsin\left(\frac{\sin(\textit{bank})}{\cos(\textit{elevation})}\right)\end{aligned}\tag{2.55}$$

Together with an external compass providing the heading angle, the initial alignment of the platform is completed and an initial value for $\mathbf{C}_b^n$ is determined.

During the initial calibration and alignment process, the host vehicle must be stationary and the theoretical accelerations in the body frame are presented in equation 2.54. After alignment is carried out, the Euler angles are solved and expressed in terms of the initial bank and elevation angles (equation 2.55). Consequently the theoretical accelerations in the body frame can also be determined from the bank and elevation angles. The calibration process can thus be completed by determining the biases associated with the body accelerations, which is simply defined as the difference between the measured accelerations and the theoretical ones.

$$\begin{aligned}b_{ax} &= a_{meas}^x - g \sin(\textit{elevation}) \\ b_{ay} &= a_{meas}^y + g \sin(\textit{bank}) \\ b_{az} &= a_{meas}^z + g \cos(\textit{bank}) \cos(\textit{elevation})\end{aligned}\tag{2.56}$$

where a_{meas} is the acceleration reading from each accelerometer.

2.5.3 Experimental results

Some experimental results will be shown in this section demonstrating the initial calibration and alignment method. The testing vehicle used in the experiments was a utility pickup truck (see Figure 2.10 (a)). The inertial sensor used in this work was a low-cost IMU from Inertial Science, DMARS-I-N (see Figure 2.3). The digital compass together with tilt sensors was a Honeywell module, HMR 3000 (see Figure 2.10 (b)). The specifications of the IMU are listed in table 2.1 and the specifications of the digital compass and tilt sensors are shown in table 2.2.

The compass and the IMU were mounted on a rigid platform, which was attached onto the top of the pickup truck. During the experiment, the testing vehicle was initially stationary for a few seconds, for the purpose of the initial calibration and alignment. The vehicle was driven along various trajectories within NTU.

Table 2.2: *Specifications of the Honeywell HMR 3000.*

No	Parameter	Value	Comments
I	Heading		
1	Accuracy	$< 0.5^\circ$ RMS	Dip $< 50^\circ$, Tilt $< 20^\circ$
2	Repeatability	$\pm 0.3^\circ$	
3	Resolution	0.1°	
II	Pitch and Roll		
5	Range	$\pm 40^\circ$	
6	Accuracy	$\pm 0.4^\circ$	Tilt $<20^\circ$
7	Repeatability	$\pm 0.2^\circ$	
8	Resolution	0.1°	

Figure 2.11 illustrates the raw body frame accelerations. From Figure 2.11 the bias is clearly seen in the x -axis acceleration since the measured accelerations of the initial stationary period were not zero, which can be calculated from equations 2.56 and removed from the raw accelerations.

(a) The utility pickup truck.

(b) The Honeywell HMR3000 digital compass and tilt sensors.

Figure 2.10: *The utility pickup and the tilt sensors used in the experiments.* Figure 2.11: *The raw body frame accelerations.*

Figure 2.12 shows the transformed raw and calibrated body frame accelerations (with biases removed) in the navigation frame. Here it is confirmed that the raw transformed accelerations initially show a bias in the East direction, while the transformed acceleration from the calibrated ones shows that the bias was removed. In order to demonstrate this effect more clearly, the view of the transformed raw and

calibrated body frame accelerations during the initial period are enlarged in Figure 2.13.

Figure 2.12: *Navigation frame accelerations comparison.*

Figure 2.13: *Enlarged view of the navigation frame accelerations during the initial period in Figure 2.12.*

The velocities (Figure 2.14 and 2.15) and positions (Figure 2.16) are computed from the raw and calibrated navigation frame accelerations. Here, the velocities and positions from an INS/GPS data fusion algorithm ([20]) were used as ground

truth. Figure 2.15 gives an enlarged view of the initial period of the navigation frame velocities shown in Figure 2.14.

Figure 2.14: *Navigation frame velocities comparison.*

Figure 2.15: *Enlarged view of the navigation frame velocities comparison from Figure 2.14.*

From the results in Figures 2.14 and 2.15, it is seen that, with the biases removed by the initial calibration, the velocities of the free INS can be estimated accurately for an extended period of time compared to the free INS using the raw IMU data.

In this example, an extend period of approximately 50 seconds has been achieved, during which the free INS, with initially calibrated data, functions accurately.

Figure 2.16: *Navigation frame positions comparison.*

(a) Enlarged view of the dashed black circle area (b) Enlarged view of the dashed black circle area of (a). of Figure 2.16.

Figure 2.17: *Enlarged view of the navigation frame positions comparison.*

From Figures 2.16 and 2.17, similar conclusions can be drawn to those from the velocity estimates, that with the initial calibration, the position estimates of the free INS are accurate for a longer time than those of the free INS using raw data. However, from Figure 2.17 (b), it is seen that, compared with ground truth, even the

position estimates of the free INS, using initially calibrated data, become incorrect quickly after the calibration. This shows the necessity of an on-the-fly calibration of the inertial navigation system and in the later part of this thesis, a multi-aiding method will be developed for this purpose.

2.6 Conclusions

In this chapter, the principles of inertial navigation systems have been introduced.

Different navigation frames used for INS have been presented and it has been further shown how to represent IMU data in different frames. The accelerometers measure accelerations in the inertial frame. However, the navigation frame or local geographic frame has been chosen and used for small area navigation purposes, for example, for autonomous vehicle applications. The navigation equations have been derived for each frame.

Error equations have been derived, including attitude, velocity and position. Using these equations, the attitude, velocity and position errors can be determined for low-cost IMUs. These error equations lead to the Pinson error model, which will be applied extensively in this thesis as the error propagation model, in later chapters.

IMUs are subject to many kinds of errors, which prevent accurate navigation taking place over extended periods of time by simple integration and transformation operations. Hence, a case study has been used to investigate how different errors affect the INS in a quantitative sense. According to the case study on the

qualitative error analysis of IMUs, the conclusion is that, biases dominate the errors for general navigation applications. The noise level in the accelerometers of IMUs is not negligible. Angular random walk is especially important for some specific applications, for instance, “north seeking” for high level gyroscopes. In the next chapter, a noise estimation and reduction method will be applied to the raw data of an IMU, to reduce this effect.

Finally it was shown that the position errors due to accelerometer biases grow with time squared, while the same errors caused by gyro biases grow with the cube of time. In order to limit the effects of these errors, calibration and alignment is necessary. An initial calibration and alignment method has been introduced to deal with the problems caused by the biases. Tilt sensors are used to provide initial calibration. Together with the help of an external compass and tilt sensors, initial alignment is achieved. Experimental results have demonstrated that with the initial alignment and calibration, the free INS runs for an extended period (50 seconds) accurately. However, in the long term, an on-the-fly calibration method is necessary to keep the errors of the free INS bounded. This will be the task of Chapter 4 of this thesis.

Chapter 3

Minima Controlled Recursive Averaging Noise Reduction for IMUs

3.1 Introduction

Typical INS sensors contain a triad of orthogonal accelerometers (translatory rate sensors) as well as gyroscopes (angular rate sensors). Using the acceleration and angular rate readings from IMUs, the host vehicle's pose, that is, the attitude, velocity and position can be computed. Since rate information has to be integrated (and then transformed) to produce velocity, position and attitude measurements, any small errors in the rates will cause accumulated unbounded errors in the integrated measurements. As stated in Chapter 2, noise and biases are the dominant error sources for IMUs in real applications. However, little work has been found in the literature, that is focused on analysis, modeling and subsequent reduction of these errors.

In this chapter, a noise analysis during signal presence and absence is carried out to reduce the noise, which corrupts the IMU data. It will be shown that commonly adopted noise filtering methods such as low pass filters or matched filters are inappropriate in this application. Hence, a noise analysis method, named minima controlled recursive averaging (MCRA), which was originally developed by Cohen *et al* for robust speech enhancement [45], and has also been utilized by Jose *et al* to reduce the noise level for their radar data successfully [46], is adopted. The MCRA algorithm estimates noise by averaging past spectral power values and using a smoothing parameter that is adjusted by the effective signal presence probability. In the application here, the effective signal is the output from the accelerometers. After the noise has been estimated in the frequency domain, it is subtracted from the original noisy power spectrum and transformed back to the time domain to obtain the reduced noise data.

In the literature related to speech enhancement, except for MCRA, other advanced noise estimation methods are also found. For example, Martin proposed a noise estimation approach, which is based on tracking the minima of a smoothed spectrum estimate of the noisy speech signals and known as the method of “Minimum statistics”, [47]. In [48], Cohen proposed an improved minima controlled recursive averaging (IMCRA) method and claimed improvement in the aspects of signal minima tracking, speech presence probability estimation and derivation of an additional bias compensation factor. In [49], Rangachari *et al* put forward a noise estimation algorithm that can handle the common problem with the approaches

proposed in [45], [47] and [48], which is that they are all relatively slow in adapting to any abrupt increase of the noise spectrum.

However, considering the case of our INS application, the major noise sources are already known (the electrical interference and vibration noise from generators etc). Abrupt changes of the noise spectrum are rare (as explained in detail in the next section), so that the relatively simple, but effective MCRA approach can be applied to the noise estimation and reduction in the proposed system. There is no necessity to apply the other advanced but more complicated noise estimation methods.

3.2 Noise Estimation of the IMU Data

For typical outdoor environment localization applications, low-cost IMUs are subject to errors from diverse sources such as biases, drifts, nonlinearities, scale factors and noise [37]. The noise is produced by various sources, such as vibrational and thermal noise. For low speed applications such as ground vehicle localization, the smaller accelerations and angular rates measured by low-cost IMUs can be indistinguishable from the noise. Figure 3.1 shows the frequency components of the Inertial Science DMARS-I IMU x -axis acceleration signal from one test when the vehicle was stationary but with the vehicle's motor switched on. Since the vehicle was not in motion, these spikes in the figure denote noise and need to be filtered before further processing to obtain accurate vehicle pose information. It can be seen from Figure 3.1 that the spikes in the frequency spectrum of the acceleration occur at specific frequencies and are probably caused by the vibrations from the vehicle's motor and

the onboard generator's motor.

Figure 3.1: *Frequency components of the acceleration in the Inertial Science DMARS-I IMU x-axis from a test with the vehicle stationary and the vehicle's motor switched on.*

Figure 3.2 illustrates the frequency components of the x -axis acceleration signal from a similar test as the one in Figure 3.1, when the vehicle was stationary but with the vehicle's motor switched off. It is seen that the noise spike at approximately 77 Hz in the frequency spectrum of Figure 3.1 decreases significantly as shown in Figure 3.2, when the vehicle's motor switched off. This probably explains that one of the main noise sources of the raw IMU data is from the vehicle's motor. The other sources are probably caused by the vibration of the onboard generator's motor. After checking the revolutions per minute (rpm) values of the host vehicles' motor and the onboard generator's motor, which correspond to the frequencies of the vibrations caused by these motors, it is found that the noise spikes occurring at approximately 25Hz and 77Hz were in the same frequency ranges. The noise spike at approximately 50Hz was possibly caused by the electrical interference because the frequency of the alternating current used is 50Hz. Although the noise sources

are now clear for the IMU data, it is almost impossible to remove them at the source and hence, a signal processing method to reduce the noise is necessary.

Figure 3.2: *Frequency components of the acceleration in the Inertial Science DMARS-I IMU x-axis from a test with the vehicle being stationary and the vehicle's motor switched off.*

For the signals shown in Figures 3.1 and 3.2, a low pass filter ([50]) would not suppress the noise, as it also corrupts the low frequency accelerations. Matched filters are commonly adopted in digital signal processing and communications [51, 50]. The purpose of the matched filter is to maximize the signal to noise ratio and to minimize the probability of undetected errors received from a signal. A matched filter functions by correlating a known signal with an unknown signal band to detect the presence of the known one in the unknown band. Hence, the exact frequency spectrum of the noisy signal is needed (which is unknown for the measured acceleration of the moving vehicle), so in general, for IMU noise reduction, this is also not a viable solution.

3.2.1 Minima Controlled Recursive Averaging

Due to the above shortcomings of standard noise reduction techniques, a method called minima controlled recursive averaging (MCRA) [45] is now introduced to process the noisy acceleration signals. This method avoids the necessity of assuming that the noise affects certain frequency bands only within the acceleration spectrum, and allows the noise reduction process to adapt to the most recent estimate of the noise power at individual frequencies. In particular, the method adaptively allows a unique estimate of the noise power to be made at each frequency. This can then be subtracted from the noisy power spectra, resulting in individually estimated spectral power values. Lower noise acceleration estimates can then be obtained from the inverse FFT of the reduced noise power spectra. The noise power estimate is performed by averaging past signal (for example, acceleration) spectral amplitudes and using a smoothing parameter. This smoothing parameter is adjusted by the estimated signal presence probability in the signal profile. The signal presence probability is obtained by taking the ratio between the amplitude of the local, noisy, signal spectra and its minimum value.

Let $a(n)$ denote a set of observed accelerations by one of the accelerometers in the IMU, where n is a discrete-time index and a is a function of n . It is now divided into overlapping frames by a w -point window function $b(n)$ whose length is $2w + 1 = N$ and analyzed using the discrete time Fourier transform. In the frequency domain,

the amplitude of the noisy signal spectra is given by,

$$\hat{A}(k, l) = \sum_{n=0}^{N-1} a(n + lM)b(n) e^{-j(\frac{2\pi}{N})nk} \quad (3.1)$$

where $\hat{A}(k, l)$ is the k -th amplitude value of l -th signal spectra, k is the frequency index, l is the frame index and M is the frame update step in time. For example, in one data set of our accelerations, the x -axis acceleration is $a(n)$, with the discrete-time index $n \in (1, 42000)$, $a(n)$ is equally divided into 60 frames with 700 acceleration values in each frame. The sampling frequency of the accelerometers is 200 Hz and hence, in equation 3.1, the frequency index $k \in (1, 200)$, the frame index $l \in (1, 60)$ and the frame update step $M = 350$ (50% overlapping windows).

Smoothing is then performed by a first order recursive averaging technique:

$$\hat{A}(k, l) = \alpha_s \hat{A}(k, l-1) + (1 - \alpha_s) \hat{A}(k, l) \quad (3.2)$$

where α_s is a weighting parameter ($0 \leq \alpha_s \leq 1$) and has the value of 0.8 in our case. This parameter α_s makes sure that the smoothing procedure is performed between the signals of two consecutive frames while applying larger weight to the earlier one.

First a minimum and then a temporary value of the local acceleration amplitude is initialized to $A_{min}(k, 0) = A_{tmp}(k, 0) = \hat{A}(k, 0)$. Then a signal wise comparison is

performed with the present data l and the previous data $l - 1$.

$$A_{min}(k, l) = \min \{A_{min}(k, l - 1), \hat{A}(k, l)\} \quad (3.3)$$

$$A_{tmp}(k, l) = \min \{A_{tmp}(k, l - 1), \hat{A}(k, l)\} \quad (3.4)$$

When L frames have been recorded, the temporary variable, A_{tmp} is initialized by

$$A_{min}(k, l) = \min \{A_{tmp}(k, l - 1), \hat{A}(k, l)\} \quad (3.5)$$

$$A_{tmp}(k, l) = \hat{A}(k, l) \quad (3.6)$$

The resolution of the local minima search is determined by the frame parameter L . For example, in our case, $L = 15$, which means that all 60 frames are divided into 4 local frequency bands to search for the local minima.

Let the signal-to-noise ratio, $A_{SNR}(k, l) = \frac{\hat{A}(k, l)}{A_{min}(k, l)}$ be the ratio between the local noisy signal amplitude and its derived minimum. In the Neyman-Pearson test [52], the optimal decision (*i.e.* whether a signal is present or absent) is made by minimizing the probability of the type II error (the false detection error), subject to a maximum probability of type I error (the missing detection error). The test, based on the *likelihood ratio*, is

$$\frac{p(A_{SNR} | H_1)}{p(A_{SNR} | H_0)} \underset{H_0}{\overset{H_1}{\gtrless}} \delta \quad (3.7)$$

where δ is a threshold and H_0 and H_1 designate hypothetical signal absence and

presence respectively. $p(A_{SNR} | H_0)$ and $p(A_{SNR} | H_1)$ are the conditional probability density functions that designate the probability density of the occurrence of A_{SNR} for signal absence and presence respectively. The decision rule of equation 3.7 can be expressed as

$$A_{SNR}(k, l) \underset{H_0}{\overset{H_1}{\geq}} \delta \quad (3.8)$$

An indicator function, $I(k, l)$ is defined where, $I(k, l) = 1$ for $A_{SNR} > \delta$ and $I(k, l) = 0$ otherwise.

The estimate of the conditional signal presence probability, $\hat{p}(k, l)$ is

$$\hat{p}(k, l) = \alpha_p \hat{p}(k, l-1) + (1 - \alpha_p) I(k, l) \quad (3.9)$$

where α_p is a smoothing parameter ($0 \leq \alpha_p \leq 1$). The value of α_p is chosen in such a way that the probability of signal presence in the previous frame has very small correlation with the next frame and $\alpha_p = 0.1$ in our application. The advantage of this choice of α_p is that it ensures that only a large probability of signal presence in the previous frame can affect the probability of signal presence in the current frame.

The variance of the noise, $\hat{\lambda}(k, l+1)$ at the k -th signal frequency is then denoted by

$$\hat{\lambda}(k, l+1) = \tilde{\alpha}_d(k, l) \hat{\lambda}(k, l) + [(1 - \tilde{\alpha}_d(k, l))] \hat{A}(k, l) \quad (3.10)$$

where

$$\tilde{\alpha}_d(k, l) = \alpha_d + (1 - \alpha_d)\hat{p}(k, l) \quad (3.11)$$

α_d is a smoothing parameter ($0 \leq \alpha_d \leq 1$) and $\alpha_d = 0.95$ in our case. A large value of α_d is chosen to put larger weight on the previous noise variance and smaller weight on the current noisy signal power. This choice ensures that the variances of the noise between consecutive frames are consistent. Subtracting the estimated, individual noise power values from the noisy spectra will give a reduced noise signal spectra.

3.3 Simulation and Experimental Results

In order to show the effect of the MCRA method, both simulation and experimental results are provided in this section. The simulation is carried out based on the raw signals generated for the general INS algorithms, which is presented in chapter 5.

3.3.1 Simulation of MCRA on Random Walk

Based on the generated raw signals of the circular arc trajectory in Chapter 5, the effect of random walk can be simulated and it can be further shown that, the MCRA method can reduce the noise in the raw angular rates and reduce the random walk. In the upper graph of Figure 3.3, 200 superimposed, angular yaw rate signals, each corrupted with zero mean Gaussian noise are shown. After integration with respect

to time, the results demonstrate the random walk effect, which is shown in the upper graphs of Figure 3.4 (a) and (b).

Figure 3.3: Comparison of simulated raw angular rates and the reduced-noise angular rates using MCRA.

After the MCRA method has been applied to each of these noisy angular rates individually, the noise variance is reduced by 20 to 25 percent, as shown in the lower graph of Figure 3.3. Hence, after integrating the yaw rates with respect to time, the reduced-noise angular rates show an alleviated random walk effect, since the noise level in the heading angles are also reduced, as shown in the lower graphs of Figure 3.4 (a) and (b).

To show the effect of the achieved noise reduction more clearly, in Figure 3.5, the magnitude of the errors of the raw headings (repeated 200 times) from Figure 3.4 is compared with the MCRA filtered one. The apparent decrease in the magnitude of the simulated heading errors shows the effect of this method. The heading errors from the raw angular rates (Figure 3.5, upper graph) show an obvious random walk effect. After the noise reduction processing, the random walk is reduced (Figure 3.5

Figure 3.4: Comparison of simulated raw headings and the headings from reduced-noise angular rates.

lower graph).

Figure 3.5: Comparison of the error of simulated raw headings and the headings from reduced-noise angular rates.

Figure 3.6 compares some of the frequency components of one simulated angular rate (upper graph) and its reduced-noise counter part after MCRA is applied (lower

graph). It can be seen that noise is present at almost all frequencies and after MCRA noise reduction, the noise level is slightly reduced.

Figure 3.6: Comparison of the frequency spectra of the simulated angular rates and the reduced-noise angular rates using MCRA.

3.3.2 Real Experimental Results

The upper graph of Figure 3.7 shows the IMU x-axis acceleration recorded with the IMU mounted on a pickup truck in one experiment with the initial calibration carried out to remove the bias (the experiment and the initial calibration have been introduced in the Section 2.5.3). After the MCRA noise estimation and reduction technique and inverse FFT are applied to the signal spectra, a reduced noise acceleration estimate is obtained, which is shown in the lower graph. In order to compare the noise levels before and after the MCRA technique is applied, the mean and standard deviation of the acceleration signals in both the upper and the lower graphs of Figure 3.7 have been calculated. This was carried out for the first 40 sec-

onds when the vehicle was stationary and the acceleration signals were pure noise. Although the mean values of both acceleration signals were almost the same, the standard deviation value of the acceleration signal before the MCRA was 0.33 m/s^2 and reduced to 0.25 m/s^2 after the MCRA had been applied.

Figure 3.7: Accelerations in the IMU x-axis, raw data (upper graph) and after noise reduction (lower graph).

Figure 3.8: Comparison of the frequency components of the acceleration and the reduced-noise values using MCRA.

In Figure 3.8, the frequency components of the raw accelerations in the upper graph of Figure 3.7 is compared with the reduced-noise, filtered values by applying

MCRA. It is seen from the lower graph that the overall noise is reduced, especially for the noise spikes at approximately 25Hz, 50Hz and 77Hz. Being able to drastically reduce these noise spikes across the entire IMU frequency spectrum shows that this MCRA method is useful for noise estimation and reduction in IMUs.

Figure 3.9 is the plot of the velocities in the North and East directions from a free running INS using the acceleration data before the MCRA algorithm is applied. The velocities from both a free running INS using reduced noise data by the MCRA and an INS/GPS data fusion algorithm ([20]) (taken as ground truth) are also portrayed in the same figure for comparison. The INS/GPS data fusion method, corrects both low frequency faults in the IMU caused by bias in the sensor readings and the misalignment of the unit, and high frequency faults from the GPS receiver caused by multi-path errors. Throughout this thesis, this INS/GPS fusion algorithm is assumed to provide the ground truth for our non-GPS aided navigation algorithms. The velocities from these two free INS methods are similar and deviate significantly from the ground truth, in both directions. Although the proposed MCRA method is effective in reducing the noise level of the raw IMU data (as shown in Figure 3.7), it only slightly contributes in reducing the errors in the velocity (the error reduction after applying MCRA is only approximately 5%) or the position estimates. The reason is that in the raw inertial data, *biases* rather than noise are the dominant errors. Hence initial alignment and calibration or in-flight alignment is fundamental in correctly using the IMUs to provide a vehicle with navigational information ([44], [31]). However the MCRA method still helps in terms of reducing the noise and

hence it is still useful when the accelerations are used as direct inputs to vehicle navigation algorithms.

Figure 3.9: *The comparison of velocities in the North and East directions from the free running INS and the free running INS using noise reduced data, with INS/GPS data taken as the ground truth.*

3.4 Conclusions

This chapter has presented a minima controlled recursive averaging (MCRA) approach for noise estimation and reduction for IMUs. This MCRA method was initially developed for speech enhancement and has been utilized here to estimate and reduce the noise level in the IMU data.

Noise and biases are the two main error sources for IMU data. However, the commonly used noise reduction methods, for example, the low pass filter or the matched filter, are unable to reduce the noise of the IMU data at all frequencies. The MCRA method approaches the problem by recursively averaging the noise power spectrum under the control of the minima values of a smoothed periodogram of

the noisy IMU data. Because the smoothing parameters in time and frequency are adapted according to the signal presence probability, MCRA has the ability to quickly follow abrupt changes in the noise spectrum and hence estimates noise quickly and accurately. After the noise has been estimated at each frequency, it is subtracted from the original noisy power spectrum and transformed back to the time domain to produce the reduced noise temporal data.

Both simulation and experimental results have been presented to demonstrate the effectiveness of this noise estimation method. Noisy angular rates have been simulated to show the random walk effect. After the MCRA has been applied to the simulated noisy angular rates, the noise variance has been decreased by 20 to 25 percent. Hence, the noise in the angles, integrated from the angular rates, has also been reduced as has therefore, the random walk effect. MCRA has also been applied to raw IMU data recorded in a real outdoor experiment. The results illustrate that after this noise estimation and reduction technique was applied, raw IMU data with less noise has been obtained, as shown both in the time and frequency domains.

Hence, the noise in the IMU's raw data (both in the accelerations and angular rates) can be reduced by MCRA, which will be helpful in estimating the vehicle's pose more accurately. The biases in the inertial data that are dominant in the inertial sensor errors, however, are not estimated and the accuracy of the vehicle's pose cannot be guaranteed. Hence, external information is still needed to aid the localization of the vehicle. The next chapter will present a multi-aiding observation method used for reliable and accurate INS.

Chapter 4

Multi-aided Inertial Navigation System

4.1 Introduction

In Chapter 3, it was shown that biases dominate the IMU sensor errors and hence external information is needed to “aid the INS” in localizing the vehicle, despite the MCRA noise-reduction technique. GPS has been used extensively as the external measurements to aid the INS. However, when GPS is unavailable, other methods towards solving the problem of errors in INS exist in the literature, as is mentioned already in Chapter 1.

Here, some closely related previous work is reviewed again in comparison with our proposed multi-aided INS algorithm. Barshan *et al* modeled the biases and drifts of inertial sensors as exponential functions of time, and estimated an augmented mobile robot pose state, which contained these bias terms [25, 26]. In [30], an aircraft dynamic model was used as a means for aiding inertial navigation systems in estimating the unknown navigation variables. This idea is of great interest

in conjunction with low-cost IMUs since it was demonstrated by simulations that without any external sensors, the use of aircraft dynamics for aiding a low-grade INS yields considerably better performance than a pure INS. Inspired by the work of [30], Gamini et al [31] put forward an on-the-fly alignment method by using the constraints that govern the motion of a vehicle to improve the accuracy of low-cost INS. They made the INS velocities observable by using these constraints and odometric information. Limitations of their method are that the position and attitude are not directly observable. The constraints presented in [31] are often violated in actual applications, especially if the vehicle negotiates a bend. In this case side slip, caused by the centripetal forces, is not negligible, and modeling the lateral velocity as Gaussian white noise becomes inadequate. As an extension of the work of [31], Ma *et al* [32] used not only the vehicle constraints that govern the motion of a land vehicle, but also virtual position measurements from a vehicle kinematic model, to provide aiding for a low-grade INS. It is claimed that with virtual position and velocity observations from the vehicle kinematic model and constraints, the accuracy of the low-grade INS could be considerably improved. However only simulation results have been presented in [32].

In this chapter, on the basis of the previous work, we propose a multi-aiding method to bound the errors in the estimated INS states, without the help of GPS, which is normally used to aid IMUs. There are some situations that GPS is unavailable, for example, if it is switched off or jammed or the radio signal is obstructed in the presence of heavy foliage or high buildings, which occurs regularly in certain en-

vironments. The multi-aiding concept in this thesis contains two aspects. The first is to use the vehicle constraints as “virtual sensors”, together with wheel encoders to provide direct velocity information. The second is to use an additional single-axis gyroscope to generate independent heading observations for the INS. The additional gyroscope is more accurate than the existing gyros in the common low-cost IMUs (and the one used in this work). A digital compass and a two-axes tilt sensor are also adopted to provide the heading information as well as the initial calibration and alignment information at the vehicle’s starting period. This multi-aided INS can be used a backup navigation solution during GPS outages for an extended period of time with navigation errors noticeably reduced. In the following sections, the details of implementing the multi-aided inertial navigation system will be presented.

4.2 INS Prediction Model

A standard Kalman filter is used to combine the inertial information and the multi-aiding data. The state vector $\mathbf{X}$ of the Kalman filter is:

$$\mathbf{X} = [\delta\mathbf{P}_n^T, \delta\mathbf{V}_n^T, \delta\mathbf{\Psi}_n^T]^T \quad (4.1)$$

where, $\delta\mathbf{P}_n = [\delta P_n^N, \delta P_n^E, \delta P_n^D]^T$, $\delta\mathbf{V}_n = [\delta V_n^N, \delta V_n^E, \delta V_n^D]^T$ and $\delta\mathbf{\Psi}_n = [\delta\Psi_n^\gamma, \delta\Psi_n^\beta, \delta\Psi_n^\theta]^T$ are position, velocity and attitude *error* vectors of the INS in the navigation frame (denoted by subscript $\mathbf{n}$) respectively and $\mathbf{\Psi}_n$ consists of γ, β, θ , which are yaw,

pitch and roll angles in the Euler representation. The state transition equation is:

$$\dot{\mathbf{X}} = \mathbf{f}(\mathbf{X}, \mathbf{u}) \quad (4.2)$$

where, $\mathbf{f}$ is the processing function, which will be defined in equation 4.4. The input $\mathbf{u}$ is:

$$\mathbf{u} = [\mathbf{A}_b^T, \boldsymbol{\omega}_b^T]^T \quad (4.3)$$

where, $\mathbf{A}_b = [A_b^x, A_b^y, A_b^z]^T$ and $\boldsymbol{\omega}_b = [\omega_b^x, \omega_b^y, \omega_b^z]^T$ are the acceleration vector and angular rate vector in the body frame¹ (denoted by subscript $\mathbf{b}$).

The Pinson error model, which has been derived in section 2.3, is used as the dynamic error propagation model of the INS. That is, the position, velocity and attitude error propagation equations can be written as

$$\delta \dot{\mathbf{P}}_n = \delta \mathbf{V}_n \quad (4.4)$$

$$\delta \dot{\mathbf{V}}_n = [\mathbf{A}_n \times] \delta \boldsymbol{\Psi}_n + \mathbf{C}_b^n \delta \mathbf{A}_b$$

$$\delta \dot{\boldsymbol{\Psi}}_n = -\mathbf{C}_b^n \delta \boldsymbol{\omega}_b$$

where it should be noted that $\mathbf{A}_n = [A_n^N, A_n^E, A_n^D]^T$ and $\mathbf{C}_b^n$ are functions of the input vector $\mathbf{u}$. $[\mathbf{A}_n \times]$ is the acceleration in the navigation frame represented in a

¹Please refer to Section 2.2.2 for details on different frames of INS.

skew-symmetric form as,

$$[\mathbf{A}_n \times] = \begin{bmatrix} 0 & -A_n^D & A_n^E \\ A_n^D & 0 & -A_n^N \\ -A_n^E & A_n^N & 0 \end{bmatrix}.$$

In equation 4.4, $\delta \mathbf{A}_b = [\delta A_b^x, \delta A_b^y, \delta A_b^z]^T$ and $\delta \boldsymbol{\omega}_b = [\delta \omega_b^x, \delta \omega_b^y, \delta \omega_b^z]^T$ are the uncertainty vectors in the accelerometers and gyroscopes in the body frame. These errors can be evaluated accurately when the vehicle is stationary and an initial alignment and calibration has been carried out with the IMU [20], [44]. Hence the state model in equation 4.4 can be rewritten as:

$$\begin{bmatrix} \delta \dot{\mathbf{P}}_n \\ \delta \dot{\mathbf{V}}_n \\ \delta \dot{\boldsymbol{\Psi}}_n \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} & 0 \\ 0 & 0 & [\mathbf{A}_n \times] \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \mathbf{P}_n \\ \delta \mathbf{V}_n \\ \delta \boldsymbol{\Psi}_n \end{bmatrix} = \mathbf{F} \begin{bmatrix} \delta \mathbf{P}_n \\ \delta \mathbf{V}_n \\ \delta \boldsymbol{\Psi}_n \end{bmatrix} \quad (4.5)$$

In equation 4.5, the bias components of the $\delta \mathbf{A}_b$ and $\delta \boldsymbol{\omega}_b$ are assumed to be removed from equation 4.4 after the initial alignment and calibration and it is further assumed that the remaining components in these error terms can be ignored. The effectiveness of the initial alignment and calibration is shown in Section 2.5, where the bias components of the inertial sensors are determined by tilt sensors when the vehicle is stationary, giving good pose estimation over an extended time.

Equation 4.5 is the fundamental equation that enables the computation of the state $\mathbf{X}$ of the vehicle from an initial state $\mathbf{X}(0)$ and the inputs $\mathbf{A}_b$ and $\boldsymbol{\omega}_b$. The

process model $\mathbf{F}$ comprises time-varying terms, $[\mathbf{A}_n \times]$. Thus, numerical methods are used to determine it. Because the update frequency of $\mathbf{F}$ is much larger than the frequency of the ground vehicle's dynamics during the sampling interval Δt , equation 4.5 can be discretised using the discrete transition matrix $\mathbf{F}(k)$

$$\begin{aligned}\mathbf{F}(k) &= \exp(\mathbf{F}\Delta t) \\ &= \mathbf{I} + \mathbf{F}\Delta t + \frac{(\mathbf{F}\Delta t)^2}{2!} + \dots\end{aligned}\tag{4.6}$$

The discretisation is only taken to the first order term since any higher order terms are assumed negligible for small Δt .

The discretized transition equation is

$$\mathbf{X}(k) = \mathbf{F}(k)\mathbf{X}(k-1) + \boldsymbol{\omega}(k)\tag{4.7}$$

where $\boldsymbol{\omega}(k)$ represents a Gaussian white noise term at time k with covariance $\mathbf{Q}(k)$.

That is,

$$E[\boldsymbol{\omega}(k)\boldsymbol{\omega}(j)^T] = \mathbf{Q}\delta_{kj}$$

with δ being the Kronecker delta function.

Now the discretisation of the state prediction equation can obtained as

$$\hat{\mathbf{X}}(k|k-1) = \mathbf{F}(k)\hat{\mathbf{X}}(k-1|k-1)\tag{4.8}$$

Since the acceleration and angular rate input vector $\mathbf{u}$ affects $\mathbf{A}_n$ in the process

model directly, there is no control vector in the state update equation. The resulting system is linear but $\mathbf{F}(k)$ must be updated with the input vector $\mathbf{u}$ at each time step.

The IMU is initially aligned and calibrated by adopting Nebot's initial alignment and calibration algorithm [44] and all the biases (initially assumed constant) are assumed to have been removed, thus $\hat{\mathbf{X}}(1|0)$ is set to zero. However, a corresponding growth in uncertainty in the states due to the drift in the IMU, should be evaluated by the predicted covariance matrix

$$\begin{aligned}
 & \mathbf{P}(k|k-1) \\
 &= E \left[(\mathbf{X}(k) - \hat{\mathbf{X}}(k|k-1))(\mathbf{X}(k) - \hat{\mathbf{X}}(k|k-1))^T | \mathbf{Z}^{k-1} \right] \\
 &= \mathbf{F}(k)\mathbf{P}(k-1|k-1)\mathbf{F}(k)^T + \mathbf{Q}(k)
 \end{aligned} \tag{4.9}$$

This 9×9 matrix represents the uncertainty in the IMU predicted errors, in which $\mathbf{Z}^{k-1}$ represents all the observation information up to time step $k-1$. That is,

$$\mathbf{Z}^{k-1} \triangleq \{\mathbf{z}(i), i \leq k-1\}$$

The observation information will be described in the following section.

4.3 Multi-Aiding Information for INS

The aim of this section is to make a robust INS based navigation system. The vehicle constraints, used as a “virtual sensor”, together with the optical encoders and a single-axis gyroscope, will be implemented and this subsystem will provide not only velocity observations from the encoders and the “virtual sensor”, but also attitude observations from the gyroscope. Hence this multi-aided INS should provide accurate pose predictions for autonomous vehicles. To illustrate these theoretical concepts in a more concrete way, in Figure 4.1 a block diagram of the major steps in the implementation of this multi-aided INS is shown. From the block diagram it can be seen that, initially, tilt sensors are used to calibrate and align the IMU according to the algorithms introduced in Section 2.5. A digital compass (with tilt sensors built in, see table 2.2) provides the initial heading estimation of the vehicle. Then, the raw data of the IMU will be passed to the MCRA filter to reduce the noise level, before they are passed to the inertial computer. The inertial computer computes predictions of the vehicle’s states, that is, positions, velocities and angles in all three axes and the error with each item. These predictions are corrected by the data from the aiding sensors within the structure of a Kalman filter, which produces final optimal estimates of the vehicle’s states.

In the following part of this section it will be shown in detail how the multi-aiding information can be used so that the INS accumulated errors can be constrained efficiently.

Figure 4.1: Block diagram showing a multi-aided inertial navigation system. Initially data from the tilt sensors are used to calibrate and align the IMU and a digital compass provides the starting absolute heading of the vehicle. The MCRA method is used to reduce the noise associated with the raw data of the IMU. Then the multi-aiding sensors provide observations for the inertial navigation system states. Finally a Kalman filter fuses the INS predictions and observations and produces optimal estimates of the localization information of the vehicle.

4.3.1 Formulation of vehicle constraints

Under ideal conditions, when the vehicle moves on a surface, it does not slide or leave the ground, which means there is negligible side slip along the lateral direction of the vehicle (y -axis in the body frame) and no motion normal to the road surface (z -axis in the body frame) [31].

Thus, reasonable constraints for the vehicle's motion are:

$$\begin{aligned} v_y^G &= 0 \\ v_z^G &= 0 \end{aligned} \tag{4.10}$$

where, v_y^G and v_z^G are velocities along the y and z axes in the body frame respectively.

In practical situations, an approximation can be made to model the constraint violations due to side slip and vibrations caused by the vehicle and road imperfections as follows:

$$v_y^G = \nu_y^G \tag{4.11}$$

$$v_z^G = \nu_z^G \tag{4.12}$$

where, ν_y^G and ν_z^G are Gaussian white noise sources with zero mean and variances σ_y^2 and σ_z^2 respectively.

In Figure 4.2, the forward velocity v_x^G of the IMU along the x axis in the body frame can also be calculated using the forward velocity of the vehicle V_r , which can be calculated from the wheel incremental encoders directly. Figure 4.2 presents how

the forward velocity v_x^G of the IMU is related to the vehicle's forward velocity V_r .

The vehicle constraints can, however, often be violated, meaning that this model would fail when the vehicle turns, as the side slip is not negligible in actual applications. The vehicle would then suffer a considerable lateral velocity and it would therefore not be appropriate to model the side velocity v_y^G in the body frame as a white Gaussian noise source. This is especially important when the vehicle has to run in actual urban environments where many sharp turns have to be made. In this section a kinematic model solves this problem by using odometric information to estimate the IMU's body velocities as part of the IMU's kinematic model.

Figure 4.2: *Vehicle kinematics for a commonly used vehicle with encoders and an IMU.*

Ideally, an IMU should be mounted exactly at the center of gravity of the vehicle. However, there is usually some offset in actual experiments. In Figure 4.2, P and Q define the offset distances of the IMU to the center of the vehicle's rear axle. L is the length of the vehicle's wheelbase and it is assumed that the actual radii of the

vehicle's rear wheels are the same, R . When the vehicle is performing a turn, R_{rr} and R_i are the radii of curvature of the path taken by the rear right wheel and the IMU respectively about the instantaneous center of rotation (ICR, denoted by “ O ” in the figure). According to the assumption that the vehicle is rigid, the angular rate at which any point on the vehicle rotates around “ O ” is the same, ω_v . The forward velocity of the vehicle is v_r and the velocities in the IMU's body frame along the x and y axes are v_x^G and v_y^G respectively. It is possible to combine the pair of wheels on the rear axle and replace them with a single virtual wheel which lies at the center of the rear axle. The rear left, right and virtual wheels have angular velocities ω_{rl} , ω_{rr} and ω_v respectively and the first two angular velocities can be calculated from the encoders mounted on the pair of rear wheels directly. The angular velocity of the virtual wheel can simply be calculated as,

$$\omega_r = \frac{1}{2}(\omega_{rl} + \omega_{rr}) \quad (4.13)$$

By geometry,

$$R_{rr} = \frac{\omega_{rr}L}{\omega_{rl} - \omega_{rr}} \quad (4.14)$$

Since,

$$V_r = \omega_r R = \omega_v \left(R_{rr} + \frac{L}{2} \right)$$

$$\omega_v = \frac{\omega_r R}{R_{rr} + \frac{L}{2}}$$

The velocity of the IMU, $\mathbf{V}_i$ is,

$$\mathbf{V}_i = \omega_v R_i = \frac{\omega_r R}{R_{rr} + \frac{L}{2}} \times \frac{R_{rr} + \frac{L}{2} - Q}{\cos \theta}$$

While θ is then given by,

$$\tan \theta = \frac{P}{R_{rr} + \frac{L}{2} - Q}$$

Hence, the velocities in the IMU's body frame along the x and y axes, v_x^G and v_y^G are,

$$v_x^G = V_i \cos \theta = \frac{\omega_r R}{R_{rr} + \frac{L}{2}} \times (R_{rr} + \frac{L}{2} - Q) \quad (4.15)$$

$$v_y^G = V_i \sin \theta = \frac{\omega_r P R}{R_{rr} + \frac{L}{2}} \quad (4.16)$$

where, ω_r and R_{rr} are defined in equations 4.13 and 4.14. Hence the observed velocities v_x^G and v_y^G of the IMU in the body frame, using the information from two encoders mounted on the rear wheels of the vehicle, can be estimated.

4.3.2 INS observation model

Now it is possible to obtain a set of velocities in the body frame by combining v_z^G from the “virtual sensor” (equation 4.12) and v_x^G and v_y^G from the kinematic model (equations 4.15 and 4.16) to calculate the corresponding velocities in the navigation frame. It can be seen that only the velocity sub-state of the state vector

$$\mathbf{X} = [\delta \mathbf{P}_n^T, \delta \mathbf{V}_n^T, \delta \mathbf{\Psi}_n^T]^T$$

is currently observable. In practical operation, when the odometry information is available, v_x^G and v_y^G are obtained by using equations 4.15 and 4.16. While at the same time, $v_z^G = \nu_z^G$ is provided by the “virtual sensor”. Hence the velocity observation can be made,

$$\begin{aligned} \mathbf{z}_V^{aiding}(k) &= \mathbf{C}_b^n(k) \begin{bmatrix} v_x^G(k) \\ v_y^G(k) \\ v_z^G(k) \end{bmatrix} \\ &= \begin{bmatrix} v_x^G(k) \cos \beta \cos \gamma + v_y^G(k) (-\cos \theta \sin \gamma + \sin \theta \sin \beta \cos \gamma) \\ v_x^G(k) \cos \beta \sin \gamma + v_y^G(k) (\cos \theta \cos \gamma + \sin \theta \sin \beta \sin \gamma) \\ -v_x^G(k) \sin \beta + v_y^G(k) \sin \theta \cos \beta \end{bmatrix}. \end{aligned} \quad (4.17)$$

where $\mathbf{z}_V^{aiding}(k)$ represents the velocity observations from the aiding method.

Thus the observation vector of the KF is

$$\mathbf{z}(k) = \mathbf{z}_V^{inertial}(k) - \mathbf{z}_V^{aiding}(k) \quad (4.18)$$

where $\mathbf{z}_V^{inertial}(k)$ represents the velocity prediction from the inertial sensor.

In order to make the attitude also observable so that the error growth of the INS can be further reduced, a relatively accurate, single-axis gyroscope is mounted, aligned with the center line of the vehicle, to measure its heading angle. The observation model for the extra gyro is simple, as:

$$\dot{\gamma} = \dot{\Gamma} \quad (4.19)$$

where, $\dot{\Gamma}$ is the angular rate reading directly from the gyroscope. It is beneficial to use this accurate, single-axis gyroscope to provide the heading observation for the INS if the gyroscope is more accurate than the yaw axis gyro within the INS. As is the case here, it is now possible to find commercially available low-cost accurate gyroscopes and the one adopted in our work is a model from KVH, the DSP-5000 fiber optic gyro. The bias level of this model ($1^\circ/hr$) is much smaller than the gyros contained in the IMU ($5^\circ/hr$). Now, based on all the observations, the best estimate for the state vector $\mathbf{X}$ can be obtained.

When an observation from an aiding sensor, that is the encoders, the “virtual sensor” or the gyroscope, is available, the observation vector is

$$\begin{aligned} \mathbf{z}(k) &= \begin{bmatrix} \mathbf{z}_V^{inertial}(k) - \mathbf{z}_V^{aiding}(k) \\ \mathbf{z}_\gamma^{inertial}(k) - \mathbf{z}_\gamma^{gyro}(k) \end{bmatrix} \\ &= \begin{bmatrix} (\mathbf{V}_T(k) + \boldsymbol{\delta}_V(k)) - (\mathbf{V}_T(k) - \boldsymbol{\nu}_V(k)) \\ (\gamma_T(k) + \delta_\gamma(k)) - (\gamma_T(k) - \nu_\gamma(k)) \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{\delta}_V(k) \\ \delta_\gamma(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\nu}_V(k) \\ \nu_\gamma(k) \end{bmatrix}, \end{aligned} \quad (4.20)$$

where, $\mathbf{V}_T(k)$ and $\gamma_T(k)$ are the true values of the velocities and yaw angle and $\boldsymbol{\nu}_V(k)$ and $\nu_\gamma(k)$ are the noise sources of the aiding sensors at time k respectively. The observation is thus the error between the velocities and yaw angle from the INS and those of the aiding sensors, and the uncertainty in this observation is reflected by the noise of the aiding observation.

Hence the observation matrix is

$$\mathbf{H}(k) = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \quad (4.21)$$

Now the observation equation is

$$\mathbf{z}(k) = \mathbf{H}(k)\mathbf{X}(k) + \boldsymbol{\nu}(k), \quad (4.22)$$

where $\boldsymbol{\nu}(k)$ is the observation noise vector at time k . As with the process noise $\boldsymbol{\omega}$, the observation noise is also assumed to be a zero mean Gaussian white noise with $E[\boldsymbol{\nu}(k)\boldsymbol{\nu}(j)^T] = \mathbf{R}\delta_{kj}$. It is further assumed that the process noise and observation noise are uncorrelated. That is,

$$E[\boldsymbol{\omega}(k)\boldsymbol{\nu}(j)^T] = \mathbf{0} \quad \forall k, j.$$

However this assumption will in general be violated because of the fact that the transformation matrix $\mathbf{C}_b^n$ is used to transform the body frame velocity observations into the navigation frame. Since $\mathbf{C}_b^n$ consists of the Euler angles γ, β, θ , the system predictions and observations are correlated. An approach to address this problem will be presented in Chapter 6.

When the prediction is available (equation 4.8), the predicted measurement fol-

lows as,

$$E[\mathbf{z}(k)] = E[\mathbf{H}(k)\hat{\mathbf{X}}(k) + \boldsymbol{\nu}(k)], \quad (4.23)$$

Since the measurement noise $\boldsymbol{\nu}(k)$ is zero mean and white, this becomes

$$\hat{\mathbf{z}}(k|k-1) = \mathbf{H}(k)\hat{\mathbf{X}}(k|k-1) \quad (4.24)$$

When the real observation measurement is available, together with the predicted measurement (equation 4.24), the measurement prediction error is produced as,

$$\tilde{\mathbf{z}}(k|k-1) = \mathbf{z}(k) - \hat{\mathbf{z}}(k|k-1) = \mathbf{H}(k)\tilde{\mathbf{X}}(k|k-1) + \boldsymbol{\nu}(k) \quad (4.25)$$

Now both prediction and observation equations have been obtained, the whole navigation loop can be completed. At each time sample from the IMU data, the system state and the associated uncertainty of those states are predicted by equations 4.8 and 4.9. When an observation from the external aiding sensors becomes available, an estimate of the system state can be computed as

$$\hat{\mathbf{x}}(k|k) = \hat{\mathbf{x}}(k|k-1) + \mathbf{W}(k)\mathbf{I}(k), \quad (4.26)$$

where $\mathbf{W}(k)$ is the Kalman gain and $\mathbf{I}(k)$ is the innovation vector, the same as the predicted measurement error as in equation 4.25.

The updated estimate defined in equation 4.26, is the sum of the current prediction of the state and a weighted innovation. The Kalman gain, which functions as

a weighting factor in the estimate update equation (4.26), is found by minimizing the mean squared error of the estimate. That is,

$$\mathbf{W}(k) = \mathbf{P}(k|k-1)\mathbf{H}^T(k)\mathbf{S}^{-1}(k), \quad (4.27)$$

where $\mathbf{S}(k)$ is the innovation covariance matrix and defined as,

$$\mathbf{S}(k) = \mathbf{R}(k) + \mathbf{H}(k)\mathbf{P}(k|k-1)\mathbf{H}^T(k), \quad (4.28)$$

Finally, the error covariance matrix can be updated. By taking the expectation of the estimated error and given all observations up to this time step k , the covariance matrix is computed as

$$\begin{aligned} & \mathbf{P}(k|k) \\ &= E \left[(\mathbf{X}(k) - \hat{\mathbf{X}}(k|k))(\mathbf{X}(k) - \hat{\mathbf{X}}(k|k))^T | \mathbf{Z}^k \right] \\ &= (\mathbf{I} - \mathbf{W}(k)\mathbf{H}(k))\mathbf{P}(k|k-1)(\mathbf{I} - \mathbf{W}(k)\mathbf{H}(k))^T + \\ & \quad \mathbf{W}(k)\mathbf{R}(k)\mathbf{W}^T(k) \end{aligned} \quad (4.29)$$

or formed symmetrically as,

$$\mathbf{P}(k|k) = \mathbf{P}(k|k-1) - \mathbf{W}(k)\mathbf{S}(k)\mathbf{W}^T(k) \quad (4.30)$$

The filter implementation as just discussed can provide optimal estimates of the vehicle's states in theory. However, without ground truth of the vehicle's states in the

real world, it is impossible to tell whether the filter runs properly and the estimated states generated are correct or not. Simulations can help solve this problem out since everything in need, including the raw inertial signals and the ground truth, can be generated perfectly. Hence in the next chapter, simulation of the inertial navigation system will be given.

In the real world, however, there is some information available which can be used, the observations. Hence, the only means to determine if the filter behaves correctly or not is the difference between the real observation and the predicted one, or the innovation in equation 4.25. The innovation is both unbiased and white and has covariance $\mathbf{S}(k)$ (equation 4.28) if the filter runs correctly. In order to use this property of the innovation, usually it is normalized first,

$$\boldsymbol{\xi} = \mathbf{I}^T(k)\mathbf{S}^{-1}(k)\mathbf{I}(k) \quad (4.31)$$

It is known that $\boldsymbol{\xi}$ follows an χ^2 distribution with m degrees of freedom with m being the dimension of the observation vector, if the filter runs correctly. Hence, this can be used to detect any observation outliers. When an observation becomes available, $\boldsymbol{\xi}$ is computed and compared with a preset threshold. If $\boldsymbol{\xi}$ is less than the threshold, the observation is accepted. Otherwise the observation is not trusted and will not be used for updating the filter. The threshold is set according to a standard χ^2 table based on the number of degrees of freedom of the observation vector and the confidence level. For the case of our application, the navigation frame velocities and the yaw angle are directly observable and hence $m = 4$. A threshold is chosen

as 9.5 at the confidence level of 95%.

4.4 Conclusions

This chapter has provided the derivation of the multi-aided inertial navigation system that is implemented in this thesis. The main contribution of this chapter is the detailed implementation of this navigation system by employing a standard Kalman filter. This multi-aided inertial navigation system can be deployed in ground vehicles and can theoretically localize these vehicles accurately in outdoor, uneven environments.

In specific scenarios, GPS signals are unavailable. Therefore a multi-aiding method, employing other sensors (a gyroscope and odometry), has been designed and presented for the purpose of the inertial navigation system. The multi-aiding information is from odometry, a relatively accurate single-axis gyroscope and use of the vehicle constraints. The essence of the vehicle constraints is in the exploitation of constraints on the motion of a ground vehicle. In more detail, the body velocity of the vehicle in the lateral direction as well as in the direction normal to the forward direction is expected to be zero, under ideal conditions. However, this constraint is violated when cornering. Under these conditions the lateral velocity is no longer accurately modeled by white noise, which accounts for practical slip. In order to compensate this violation, a kinematic model has been developed to estimate the inertial sensor's lateral velocity using an odometer's readings. Consequently, the IMU's forward and lateral velocities in the body frame can be observed by this com-

combination of constraints and odometric information. A relatively accurate single-axis gyroscope is also used to make the heading estimate observable and hence a more accurate heading estimate from low-cost IMUs can be achieved. In order to make this INS system more robust and accurate, it will be interesting to add additional aiding information whenever it is available. For example, an altimeter can be considered in the multi-aiding system so that the “down” direction position of the INS can also become observable and the inertial navigation method could improve its accuracy in all directions.

Since the Pinson error model is adopted to provide the IMU’s error propagation, the prediction and observation models are linear and a standard Kalman filter can be used as the optimized filtering tools. The process model has been derived on some basic assumptions and errors associated with positions, velocities and attitudes of the vehicle have been predicted. Whenever one of the multi-aiding sensors produces a measurement, the body velocity observations are transformed into the navigation frame, that is, the frame represented by *North*, *East* and *Down*, and the system states are estimated consequently, through the fully developed Kalman filter model. Hence, this navigation system allows for multiple aiding for the inertial sensors and in turn generates greater accuracy for ground vehicles running in outdoor, uneven environments.

Chapter 5

Simulation of the Inertial Navigation System

The aim of this thesis is to generate vehicle trajectories from raw inertial navigation data, and ultimately from aided inertial navigation data. Using real data is the ultimate aim. However, real data is corrupted with many sources of unknown errors, in which biases and noise are most crucial as denoted in chapter 2. When using real, raw data, in general no ground truth is available. Therefore with real data, erroneous results are extremely difficult to quantify. Hence, in this chapter, the aim is to initially generate simulated raw IMU data (body frame accelerations and angular rates) from theoretically given vehicle trajectories. These simulated raw signals will then be artificially corrupted with noise, and the multi-aided INS algorithms will be tested using these signals in order to attempt to re-generate the vehicle trajectories. This then offers the advantages of

1. Complete ground truth knowledge is available.
2. The error sources can be completely controlled and analyzed individually.

5.1 Simulation of the Raw Inertial Data

In this section, it will be shown that the raw signals can be simulated from basic principles. A vehicle with a perfect IMU mounted at a general point (offset from the vehicle's center), is assumed to execute any smooth trajectory, which can be generated and then used for raw inertial signal generation. This will be demonstrated for the cases of a straight line, a circular arc and then a general curved trajectory. The aim is to generate simulated raw data and ground truth and then test the basic inertial navigation algorithms. Given only a vehicle's trajectory in the navigation frame, the raw inertial data, that is, the body accelerations along "x" and "y" axes and angular rates about the "z" axis, in the body frame, are generated. The "z" direction acceleration and "x" and "y" angular rates are set to zero on the assumption that the vehicle runs on a flat surface to release the complexity of generating the coupled signals in these three orthogonal dimensions.

5.1.1 Raw data simulation for a straight line

In general, the vehicle's "North" position can be considered to be a function of its "East" position. Here, for illustration purposes, a *3rd* order polynomial with arbitrarily set coefficients is used as such a function to generate a curve as the vehicle's trajectory. It is assumed that the "East" displacement increases linearly

with time. The functions of the “East” and “North” positions are:

$$E_{pos} = Kt \quad (5.1)$$

$$N_{pos} = AE_{pos}^3 + BE_{pos}^2 + CE_{pos} + E$$

where, E_{pos} and N_{pos} denote the “East” and “North” positions, t is the time and K, A, B, C and E are coefficients that can be arbitrarily set. For a vehicle, a straight line is probably the simplest trajectory. Before dealing with any more complicated situations, the case of a straight line will be generated and studied first. With set the three parameters A, B, E to zero, equation 5.1 turns into a function of a straight line:

$$E_{pos} = Kt \quad (5.2)$$

$$N_{pos} = CE_{pos}$$

where K and C are coefficients that can be arbitrarily set.

In one simulation, an arbitrary end position in the “North” and “East” directions can be set in the navigation frame, with the beginning point of $(0, 0)$. Now an end point is chosen as $(300, 1000)$, that is, the vehicle runs 1000 meters in the “North” direction and 300 meters to the “East”. The yaw angle of the vehicle, is generated as

$$\gamma = \frac{\partial E_{pos}}{\partial N_{pos}} \quad (5.3)$$

where γ denotes the yaw angle. Following the inertial navigation convention, the yaw angle is defined as the angle from the positive “North” axis to the vehicle’s motion direction. In the case of the straight line trajectory, the vehicle’s yaw angle is obviously a constant value. The simulated trajectory and yaw angle are shown in Figure 5.1.

Figure 5.1: *The position and yaw angle of a simulated straight line trajectory.*

In the case of simulating a vehicle running in a straight line, it is common to assume that the vehicle initially runs at a constant acceleration for a certain period, then runs at a constant velocity and finally decelerates to stationary state at the same magnitude of deceleration. In this example, the maximum velocity of the vehicle is chosen to be 10 m/s, which approximately mimics the actual values of our experiments. The accelerating and hence decelerating period is set arbitrarily at 1/5 length of the whole motion period of the vehicle. Hence, the running time, the accelerations, the velocities and positions in both “North” and “East” directions are determined by these settings and the intrinsic physical principles. For example,

the “North” velocities are calculated by a numerical differentiation of the “North” positions. These values are illustrated in Figure 5.2.

Figure 5.2: The positions, velocities and accelerations in the “North” and “East” directions of a simulated straight line trajectory.

The aim of the simulation in this section is to generate the raw inertial signals that would be read by the inertial sensors, which are the accelerations and angular rates in the body frame. The body frame accelerations can be computed by transforming the navigation accelerations into the “body” coordinates. During the transformation, the navigation accelerations and the transformation matrix are used. The transformation matrix $\mathbf{C}_b^n$ (equation 2.1) is repeated here as,

$$\mathbf{C}_b^n = \begin{bmatrix} \beta_c \gamma_c & -\theta_c \gamma_s + \theta_s \beta_s \gamma_c & \theta_s \gamma_s + \theta_c \beta_s \gamma_c \\ \beta_c \gamma_s & \theta_c \gamma_c + \theta_s \beta_s \gamma_s & -\theta_s \gamma_c + \theta_c \beta_s \gamma_s \\ -\beta_s & \theta_s \beta_c & \theta_c \beta_c \end{bmatrix} \quad (5.4)$$

where subscripts s and c represent the sine and cosine of the angle, and θ , β and γ are the roll, pitch and yaw angles respectively.

The north and east accelerations in the navigation frame and the yaw angle have now been obtained. The roll and pitch angles are assumed to be 0° on the assumption of running the vehicle in a perfect horizontal plane. Hence, the transformation matrix $\mathbf{C}_b^n$ can be calculated using $\theta = \beta = 0$ and γ , at each time increment. $\mathbf{C}_n^b$, which is the navigation frame to body frame transformation matrix, can be obtained by just transposing $\mathbf{C}_b^n$ since $\mathbf{C}_b^n$ is orthogonal. Hence, the accelerations in the body frame can be calculated by

$$[\mathbf{A}_{bx}, \mathbf{A}_{by}, \mathbf{A}_{bz}]^T = \mathbf{C}_n^b \{[\mathbf{A}_{nn}, \mathbf{A}_{ne}, \mathbf{A}_{nd}]^T + \mathbf{G}\}$$

where $\mathbf{A}_{bx}$ represents the acceleration in the x direction with respect to the body frame, etc. and $\mathbf{G}$ is the gravity vector $[0, 0, g]^T$. Since the downward acceleration in the navigation frame, $\mathbf{A}_{nd}$, is assumed to counteract g perfectly, $\mathbf{A}_{bz}$ is 0 (since roll and pitch are also set to 0). The obtained body frame accelerations are shown in Figure 5.3.

5.1.2 Raw data simulation for a circular arc

A circular arc is the other common trajectory for a vehicle. Although under realistic conditions, a perfect, circular arc can never be executed by a vehicle, any curve can be seen as a composite of different circular sections. In this section, the case of a circular arc that is assumed as the trajectory of the vehicle will be simulated, in

Figure 5.3: The accelerations in the “ x ” and “ y ” directions of a simulated straight line trajectory, in the body frame.

which the vehicle’s yaw angle keeps changing all the time.

Using a circular arc with radius 200 meters, and an IMU sampling frequency still set to 100Hz, a quarter of the circle’s circumference is simulated as the vehicle’s trajectory, as shown in Figure 5.4. The yaw angle of the vehicle is computed from

Figure 5.4: The positions and yaw angles of a simulated circular arc.

the gradient of the trajectory curve, shown in the lower graph of Figure 5.4.

It is assumed that the trajectory in Figure 5.4 is the path of the center of the vehicle's rear axel P_c . Since the yaw angle of the vehicle keeps changing, the offset distances P and Q , of the IMU from P_c , must be considered to simulate the body velocities as in equations 4.15 and 4.16 in Section 4.3. If the values of the offset distances are chosen as $P = 0.5$ meters and $Q = 0.2$ meters (which are the values used in the real experiments to be explained later), the trajectory of the actual IMU will be the red profile in Figure 5.5, assuming the trajectory of P_c as the upper curve in Figure 5.4.

(a) Superimposed trajectories of the rear wheels center and the IMU

(b) Enlarged view of the initial trajectories.

Figure 5.5: *Superimposed trajectories of the IMU and P_c assuming the trajectory of P_c to be that of Figure 5.4.*

Unlike the straight line case, the accelerations at the IMU are of interest since the accelerations at the IMU and P_c will differ. The accelerations in the “North” and “East” directions at the IMU can be computed by differentiation of the velocities in these directions respectively, which are determined by the positions accordingly.

All these values are illustrated in figure 5.6.

Figure 5.6: The position, velocity and acceleration in the “North” and “East” directions of a simulated circular arc trajectory, at the position of the IMU.

Now the accelerations in the body frame can be computed, again, by the transformation matrix $\mathbf{C}_b^n$ and the navigation accelerations. Figure 5.7 illustrates the body accelerations.

The yaw angular rates are computed by differentiation of the yaw angles. Both the yaw angles and angular rates are shown in figure 5.8

5.1.3 Raw data simulation for a general trajectory

Any type of smooth trajectory can be simulated theoretically. In this section, a more general case will be studied. Actually, equation 5.1 is designed to generate a generally smooth curve. Here, a set of coefficients are set for K, A, B, C and D and the “North” position versus “East” position is shown in Figure 5.9 (a). In this

(a) Acceleration x (b) Acceleration y

Figure 5.7: The IMU accelerations in the “ x ” and “ y ” directions along a simulated circular arc trajectory in the body frame.

(a) Yaw angle

(b) Yaw angular rate

Figure 5.8: The yaw angles and yaw angular rates of the IMU along the simulated circular arc trajectory.

trajectory plot, the arrow denotes the vehicle's direction of motion. It is seen from the figure that the start point of the curve is (0,0) and the end point is (200,600), and the time is set as 20 seconds. The sampling frequency of the simulated inertial

(a) “North” Vs “East” positions.

(b) Yaw angle.

Figure 5.9: *The position and yaw angle of a simulated general curve trajectory.*

sensor is set at 100Hz, which is common for real low-cost IMUs. Now the aim is to compute the accelerations and angular rates in the body frame, given the generated “North”, “East” positions, yaw angle, time and frequency.

With positions in both “North” and “East” directions and the running time obtained, the navigation frame velocities are computed by numerical differentiation. With further numerical differentiation, the velocities and accelerations along these two directions are also computed. Positions, velocities and accelerations in both directions are plotted with respect to time, in Figure 5.10.

By numerical differentiation of the yaw angle, simulated yaw angular rate can be computed. Figure 5.11 shows both the yaw angle and the yaw angular rate, with

Figure 5.10: The positions, velocities and accelerations with respect to time in the “North” and “East” directions of the simulated general curve trajectory.

respect to time, for the curved trajectory of Figure 5.9.

Figure 5.11: The yaw angles and angular rates of the simulated general curve trajectory.

Again, the accelerations in the body frame can be computed, by the transformation matrix $\mathbf{C}_b^n$ and the navigation frame accelerations. Hence the raw inertial

signals for the general trajectory have been generated following the fundamental inertial principles, given the trajectory only, based on the above assumptions. All these inertial signals are illustrated in figure 5.12 and 5.13.

Figure 5.12: The accelerations in the “ x ”, “ y ” and “ z ” directions of the simulated general curve, in the body frame.

Figure 5.13: The roll, pitch and yaw angular rates of the simulated general curve, in the body frame.

Hence, the raw inertial signals can be simulated given that the trajectory of the hosted vehicle is a straight line, a circular arc or a really general curve. These simulated IMU signals will now be used as a basis for testing our multi-aided INS algorithm.

5.2 Simulation of the Multi-aided Inertial Navigation System, with Noisy and Biased External Sensors

In the last section, given the trajectories the testing vehicle follows and several necessary assumptions, raw inertial signals have been simulated. These simulated data are reasonable and thus useful for testing the aiding inertial navigation algorithms. In this section, these generated raw inertial signals, for the case of both a straight line and a circular arc, will be applied in the multi-aided inertial navigation algorithm, which has been presented in Chapter 4. It will be shown that the multi-aided INS algorithm works well with the simulated data, with estimated vehicle navigation information comparing well with ground truth, in the presence of biases and noise. In the simulations, the multi-aiding sensors, namely, the “virtual sensors” (constraints), the encoders and the gyroscope, will be assumed to be corrupted with zero mean Gaussian noise.

5.2.1 Multi-aided INS simulation for the straight line trajectory

According to the multi-aiding method in Chapter 4, the two encoders, which are connected to the two rear wheels of the hosted vehicle, can aid the IMU providing the vehicle’s velocities in the body frame. In the simulation of a straight line, the forward velocity of the vehicle measured by the encoders is obtained using the

simulated ground truth value corrupted with zero-mean white Gaussian noise. The variance of the noise at every time instant is set at 0.001 (m/s)^2 , a value of the variance of the noise measured from the pickup truck's actual encoders' data, when the hosted vehicle ran at controlled, different constant speeds in different runs. The lateral velocity is set at the value of the noise only, so that the simulation can mimic the actual case more realistically. The comparison of the simulated forward velocity and the ground truth is shown in Figure 5.14.

Figure 5.14: *The forward velocity of the vehicle, computed from odometry and the ground truth.*

The simulation of the single-axis gyroscope is similar and achieved by corrupting the true value of the yaw angle (relative to North) with a zero-mean, additive, white, Gaussian noise. The noise variance is set at a value that takes into consideration of the specifications of the actual gyroscope used in this work and the variance of the angles measured by the gyroscope (through many tests). Now with the simulated raw inertial signals and the odometry and gyroscopic data, the multi-aided inertial navigation system from Chapter 4 can be simulated. The raw accelerations and

angular rates in the body frame are assumed to be corrupted with bias signals and zero mean random noise. In order to mimic the increasing biases the IMU suffers in the real experiments, the biases of the linear accelerations are simulated with a linear function of time. By transforming the body accelerations via the transformation matrix $\mathbf{C}_b^n$, the navigation frame accelerations corrupted with biases and noise are generated. Figure 5.15 (a) illustrates the comparison of the “pure” IMU body frame accelerations calculated for the straight line trajectory (also shown in Figure 5.3 in Section 5.1), and the accelerations corrupted with biases and noise. The comparison of the navigation frame accelerations is shown in Figure 5.15 (b).

Figure 5.15: *The IMU accelerations and true accelerations when the vehicle executes the simulated straight line trajectory, both in the body and the navigation frames.*

The velocities from the free running INS and the multi-aided INS can be compared with ground truth taken from figure 5.2. Figures 5.16 and 5.17 show the comparison of the velocities in the navigation frame. The differences between the multi-aided INS velocities and the true velocities are much smaller than those be-

tween the free running INS versus the ground truth.

(a) “North” velocities

(b) Enlarged view of the dashed circle area of (a)

Figure 5.16: The free INS, multi-aided INS and true “North” velocities in the navigation frame for the simulated straight line trajectory.

(a) “East” velocities

(b) Enlarged view of the dashed circle area of (a)

Figure 5.17: The free INS, multi-aided INS and true “East” velocities in the navigation frame for the simulated straight line trajectory.

Figure 5.18 shows the comparison of the free running and multi-aided INS estimated positions with ground truth in the navigation frame.

(a) “North” against “East” positions

(b) Enlarged view of the dashed circle area of (a)

Figure 5.18: *The free INS, multi-aided INS and true positions in the navigation frame for the simulated straight line trajectory.*

From the above simulation results, it can be seen that,

1. The IMU alone subjected to biases and drifts cannot provide accurate velocity and position estimates in the long term. The errors in the resulting pose estimates are unbounded and divergent.
2. In the multi-aiding method presented in chapter 4, the aided INS can produce pose estimates accurately and reliably (close to the ground truth) in the case of a straight line trajectory.

5.2.2 Multi-aided INS simulation for the circular arc trajectory

The implementation of the multi-aided INS in the case of the circular arc trajectory is similar to the straight line case in the previous section. However, it is compli-

cated by the fact that not only are the yaw angular rates not zero when the hosted vehicle's running path is circular, but also for the fact that the lateral velocity in the body frame is also non-zero. Hence, the offset distances P and Q , to the center of the vehicle's rear axel P_c , must be taken into consideration to simulate the body velocities as in equations 4.15 and 4.16 in Section 4.3. Given the signals generated in the previous section and according to equation 4.15, the body frame velocities estimated from the encoders can be simulated with zero-mean white noise added as carried out in the case of the straight line trajectory, as shown in Figure 5.19.

Figure 5.19: *Velocities at the IMU in the body frame estimated by the encoders compared with ground truth.*

The single-axis gyroscopic data can be simulated in a similar manner. Again, the raw accelerations and angular rates in the body frame are assumed to be corrupted with biases and Gaussian white noise to produce the raw IMU data. The linearly increasing biases, similar to the biases used in the straight line trajectory case, are used here. Figure 5.20 shows the comparison of the generated IMU accelerations in

the body frame and the transformed accelerations in the navigation frame with the true values.

Figure 5.20: *The IMU measured and true accelerations of the simulated circular arc trajectory.*

From Figure 5.20 (b), it is worthy of noticing that, the corrupted navigation accelerations are considerably different from ground truth. It is due to the fact that in the simulation of the circular arc trajectory, the yaw angular rates are non-zero and assumed corrupted with noise too. Hence the transformation process adds more errors to the resulting accelerations in the navigation frame.

Results from the free running INS, the multi-aided INS are compared with the ground truth taken from Figure 5.6 in Section 5.1. Figure 5.21 shows the comparison of the estimated and true velocities in the navigation frame. Figure 5.22 shows the comparison of the estimated and true positions in the navigation frame.

(a) “North” velocities

(b) “East” velocities

Figure 5.21: The free INS, multi-aided INS and true velocities in the navigation frame of the simulated arc trajectory.

(a) “North” against “East” positions

(b) Enlarged view of the dashed circle area of (a)

Figure 5.22: The free INS, multi-aided INS and true positions in the navigation frame for the simulated circular arc trajectory.

5.3 Discussions

Based on the simulation results for the case of the straight line and the circular arc trajectories, the following conclusions can be drawn:

1. Based on the basic INS algorithms, that is, the transformations based on the Euler angle representations and the integrations from acceleration to velocity and velocity to position, inertial signals can be simulated correctly given any form of smooth trajectory, for further testing of the multi-aided INS, with knowledge of perfect ground truth.
2. With the simulated data, it is demonstrated that the IMU alone, subjected to biases and noise, cannot provide accurate velocity and position estimates in the long term. The errors in these pose estimates are unbounded and divergent. Hence, external aiding information is needed to make accurate and reliable inertial based navigation estimates.
3. The multi-aided inertial navigation system presented in Chapter 4 is proved to be able to produce accurate and reliable pose information, with the simulated inertial signals. The simulation results show that in both straight line and circular trajectory cases, the aiding information from external sensors, that is, the “virtual sensors” (vehicle constraints), the encoders and the gyroscope, can provide reliable aiding information for the unbounded raw inertial signals.

Chapter 6

Analysis of Cross Correlation between Prediction and Observation Errors

6.1 Introduction

In Chapter 4, it is the observations from the vehicle constraints together with encoder information and additional gyroscope data, that have been coupled with the inertial sensor measurements, to provide an accurate INS solution. This principle initially makes sense since the INS errors caused mainly by the biases in the inertial sensors, will be corrected by these observations, which keeps the INS solution from diverging.

However, there is a problem with the system which can be seen if the observation equation 4.20 is investigated. From equation 4.20, it can be seen that the noise vector $\boldsymbol{\nu}$ consists of two components, one of which is the noise of the aiding gyroscope, ν_γ , the statistical characteristics of which have been determined experimentally. The determination of the other components of the noise vector $\boldsymbol{\nu}$, which is the noise

vector associated with the aiding velocities, $\boldsymbol{\nu}_v$, is much more complicated.

It is seen from Figure 4.1 that, in order to make the velocity errors in the navigation frame $\delta\boldsymbol{v}_n$ observable, the body velocity $\boldsymbol{v}_b$ has to be pre-multiplied with the frame transformation matrix $\boldsymbol{C}_b^n$. Since $\boldsymbol{C}_b^n$ evolves with the three Euler angles γ, β, θ , the errors of which are part of the state vector $\boldsymbol{X}$ (equation 4.1), the error in the observation $\boldsymbol{v}_n$ is *NOT* strictly independent of the errors in the state vector. This means that even if the errors of the body velocity observation $\boldsymbol{v}_b$ can be assumed to be zero mean white Gaussian random sequences, the errors of the navigation velocity observation $\boldsymbol{v}_n$ are not white. This cross correlation between the observation errors and the system errors violate the assumption of the Kalman filter and could lead to filter divergence.

This cross correlation problem is often solved by the adoption of a shaping filter ([53], [54]), which is a linear time-invariant system driven by white Gaussian noise to duplicate virtually time-correlated noise. However, in the case of the proposed multi-aided INS application, it is impossible to properly fit the power spectral density of the colored noise associated with the navigation velocity in any existent time-invariant shaping filters. The shaping filter approach, thus, cannot be adopted here for this problem. In this chapter, another method will be adopted to solve this problem by adding the cross correlation term into the Kalman filter. By running the Kalman filter with and without the cross-correlation term and comparing the error covariance of each individual state variable, a conclusion will be made as to whether this term can be reasonably ignored. If the difference between the error

covariances of these two cases is small, which means this cross-correlation term has almost no effect on the error covariances of the system, it can be safely ignored. Otherwise, it is necessary to account for this term in the Kalman filter derivation, which will result in a heavier computational load for the system, but will keep the estimation filter convergent for a longer period of time.

6.2 Cross Correlation between Prediction and Observation Errors

In this section, we will try to evaluate the effect of the cross correlation between the system noise and the observation noise on the estimation of the state vector. In the literature, similar cross correlation cases have been addressed. As early as 1971, Brandenburg et al presented the problem of state estimation with colored measurement noise and a shaping filter representation solution was put forward, [53]. Bar-shalom et al investigated several cases in which the major assumptions of a Kalman filter are not satisfied. Typical cases occur when the process noise sequence is not white or the measurement noise is not white or both noise sources are correlated, [55]. In each case, a theoretical solution is given to handle the violation of these assumptions. More recently, Kuhlmann adopted a shaping filter for his Kalman-filter algorithm with colored measurement noise, which is applied to GPS measurements, [54].

In our case, due to the fact that the transformation matrix $\mathbf{C}_b^n$ is used both in the

process loop and in the observation loop, the process noise and the measurement noise are correlated. The cross correlation is introduced by using the $\mathbf{C}_b^n$ matrix to transform the velocity observation in the body frame to the navigation frame, as shown in equation 4.17. Further $\mathbf{C}_b^n$ is a function of the time-variant Euler angles, θ , β and γ . This cross correlation problem is often solved by the adoption of a shaping filter. However, this linear time-invariant system, which is driven by white Gaussian noise to duplicate virtually time-correlated noise, cannot be used in our case. Because the power spectral density of the colored noise corrupting the navigation velocity is difficult to correctly duplicate with any existent time-invariant shaping filters. Hence, the shaping filter is not able to help in solving this problem. In order to make the problem tractable, a simplifying assumption is made that the cross correlation between the process and observation noise occurs just one time step apart. This assumption is reasonable because $\mathbf{C}_b^n(k)$ is part of the estimation result at time instant k , and at the next time instant $k + 1$, it is used to transform the velocity observation from the body frame to the navigation frame.

In the following model, the cross-correlation term between the process noise and observation noise will be introduced in the estimation filter and thus, its effect will be investigated. Since in the proposed multi-aided INS application (refer to chapter 4), the process model is time-variant (equation 4.6) and the observation model is time-invariant (equation 4.21), the model here adopts a time-variant process model

and a time-invariant observation model for consistency.

$$\mathbf{X}(k) = \mathbf{F}(k)\mathbf{X}(k-1) + \boldsymbol{\omega}(k) \quad (6.1)$$

$$\mathbf{z}(k) = \mathbf{H}\mathbf{X}(k) + \boldsymbol{\nu}(k) \quad (6.2)$$

$$E[\boldsymbol{\omega}(k)\boldsymbol{\omega}(j)^T] = \mathbf{Q}\delta_{kj} \quad (6.3)$$

$$E[\boldsymbol{\nu}(k)\boldsymbol{\nu}(j)^T] = \mathbf{R}\delta_{kj} \quad (6.4)$$

$$E[\boldsymbol{\omega}(k)\boldsymbol{\nu}(j)^T] = \mathbf{U}\delta_{k,j-1} \quad (6.5)$$

with δ being the Kronecker delta function. Equations 6.1 to 6.4 represent a system with an assumed time-variant process model and an assumed time-invariant observation model with independent zero mean, white Gaussian noise, which is consistent with the proposed multi-aided inertial navigation system. Equation 6.5 denotes that the cross correlation occurs only between $\boldsymbol{\omega}(k)$ and $\boldsymbol{\nu}(k+1)$.

This is not a standard problem but the standard filter can be modified accordingly to obtain the filter for this case. It means that, in order to examine and include this correlation effect into the Kalman filter stage, the relevant state update equation and the error covariance update equations must be derived to take this effect into account.

The state prediction error is

$$\tilde{\mathbf{X}}(k|k-1) \triangleq \mathbf{X}(k) - \hat{\mathbf{X}}(k|k-1) = \mathbf{F}(k)\tilde{\mathbf{X}}(k-1|k-1) + \boldsymbol{\omega}(k-1) \quad (6.6)$$

The covariance between the state prediction and measurement noise is

$$E[\tilde{\mathbf{X}}(k|k-1)\boldsymbol{\nu}(k)^T] = E\left[[\mathbf{F}(k)\tilde{\mathbf{X}}(k-1|k-1) + \boldsymbol{\omega}(k-1)]\boldsymbol{\nu}(k)^T\right] = \mathbf{U} \quad (6.7)$$

The covariance between the state prediction and the measurement becomes

$$\begin{aligned} E[\tilde{\mathbf{X}}(k|k-1)\tilde{\mathbf{z}}(k|k-1)^T|\mathbf{Z}^{k-1}] &= E[\tilde{\mathbf{X}}(k|k-1)[\mathbf{H}\tilde{\mathbf{X}}(k|k-1) + \boldsymbol{\nu}(k)]^T|\mathbf{Z}^{k-1}] \\ &= \mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{U} \end{aligned}$$

The observation innovation covariance then becomes

$$\begin{aligned} \mathbf{S}(k) &= E[\tilde{\mathbf{z}}(k|k-1)\tilde{\mathbf{z}}(k|k-1)^T] \\ &= E[[\mathbf{H}\tilde{\mathbf{X}}(k|k-1) + \boldsymbol{\nu}(k)][\mathbf{H}\tilde{\mathbf{X}}(k|k-1) + \boldsymbol{\nu}(k)]^T] \\ &= \mathbf{H}\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{R} + \mathbf{H}\mathbf{U} + \mathbf{U}^T\mathbf{H}^T \end{aligned} \quad (6.8)$$

and the modified filter gain then becomes

$$\mathbf{W}(k) = [\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{U}][\mathbf{H}\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{R} + \mathbf{H}\mathbf{U} + \mathbf{U}^T\mathbf{H}^T]^{-1} \quad (6.9)$$

It is interesting to note that the correlation between the process and measurement noise has no effect on the estimated state equation itself (although the Kalman gain has changed according to equation 6.9). However, the updated error covariance of

the state at k is

$$\begin{aligned}
& \mathbf{P}(k|k) \tag{6.10} \\
&= \mathbf{P}(k|k-1) - [\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{U}]\mathbf{S}(k)^{-1}[\mathbf{P}(k|k-1)\mathbf{H}^T + \mathbf{U}]^T \\
&= \mathbf{P}(k|k-1) - \mathbf{P}(k|k-1)\mathbf{H}^T\mathbf{S}(k)^{-1}\mathbf{H}\mathbf{P}(k|k-1)^T \\
&+ \mathbf{P}(k|k-1)\mathbf{H}^T\mathbf{S}(k)^{-1}\mathbf{U}^T + \mathbf{U}\mathbf{S}(k)^{-1}\mathbf{H}\mathbf{P}(k|k-1)^T + \mathbf{U}\mathbf{S}(k)^{-1}\mathbf{U}^T \\
&= \mathbf{P}(k|k-1) - \mathbf{W}(k)\mathbf{S}(k)\mathbf{W}(k) \\
&+ \mathbf{P}(k|k-1)\mathbf{H}^T\mathbf{S}(k)^{-1}\mathbf{U}^T + \mathbf{U}\mathbf{S}(k)^{-1}\mathbf{H}\mathbf{P}(k|k-1)^T + \mathbf{U}\mathbf{S}(k)^{-1}\mathbf{U}^T
\end{aligned}$$

Comparing with equation 4.29, which is the normal error covariance matrix without considering any correlation, it is seen that, there are three more terms in equation 6.10 because of the introduction of $\mathbf{U}$. This means that certain elements of the covariance matrix $\mathbf{P}(k|k)$ will be increased. If a symbolic solution of this 9×9 matrix could be obtained, it would be possible to evaluate the effect of this additional cross-correlation term on the error covariance matrix and further on the whole performance of the navigation filter. However, the symbolic solution of each element of this matrix is rather complex to be used directly. Hence, in the next section, simulations will be utilized to show the effect of this correlation using realistic numerical examples.

6.3 The Effect of Cross Correlation between Prediction and Observation Errors

In this section, the simulated data which is generated to validate and explore both the basic and the multi-aided inertial navigation algorithms, presented in Chapter 5, will be used to demonstrate the effect of the cross correlation term on the state estimate. The simulated data set used here is from the simulation of the circular arc as the vehicle's trajectory. In this simulation, the vehicle's yaw angle is continuously changing, which satisfies the condition that the transformation matrix $\mathbf{C}_b^n(k)$ is not constant and hence the cross correlation exists.

In Section 6.2, it was seen that the cross-correlation term $\mathbf{U}$ affects the modified updated covariance matrix $\mathbf{P}(k|k)$. Without introducing $\mathbf{U}$, $\mathbf{P}(k|k)$ will be an over confident measure of the pose uncertainty, meaning that the true variance in certain states is higher than that estimated. Intuitively, the effect of $\mathbf{U}$ is that the value of some entries of the covariance matrix $\mathbf{P}(k|k)$ should be increased. It is necessary to know which elements of $\mathbf{P}(k|k)$ are affected, and by how much so as to study the effect of $\mathbf{U}$. In order to obtain this information, firstly we need work out how this cross-correlation term $\mathbf{U}$ can be introduced.

According to equation 6.5,

$$E[\boldsymbol{\omega}(k)\boldsymbol{\nu}(j)^T] = \mathbf{U}\delta_{k,j-1},$$

$\mathbf{U}$ will be a 9×4 matrix, since in our case, $\boldsymbol{\omega}$ is a 9×1 , zero mean white Gaussian

noise vector that associates with the state vector $\mathbf{X}$ (equation 4.1) at each time instant and $\boldsymbol{\nu}$ is a 4×1 , zero mean white Gaussian noise vector with the observation vector $\mathbf{z}$ (equation 4.20). We are going to study the effect of the incorporation of $\mathbf{U}$ on the covariance matrix $\mathbf{P}(k|k)$, using the simulated data set with a circular arc as the vehicle's trajectory, in which it is assumed that the vehicle runs on a perfect flat plan. Hence, the "z" direction acceleration, velocity and position and the roll and pitch angles are set to zero. Accordingly, in the 9×4 matrix of $\mathbf{U}$, these entries associated with items are also set to zero. Because the cross correlation item is introduced by the transformation of the body frame velocity to the navigation frame, we can further assume that the nonzero entries in the matrix of $\mathbf{U}$ are those introduced by the product of the "x" and "y" velocity components of $\boldsymbol{\nu}$ with the "x" and "y" position and velocity components of $\boldsymbol{\omega}$. Hence the nonzero entries of the matrix of $\mathbf{U}$ are U_{11} , U_{22} , U_{41} and U_{52} . Here we will use U_{11} to show how these entries can be computed. Obviously the entry U_{11} is the product of first component of $\boldsymbol{\omega}$, ω_1 and the first component of $\boldsymbol{\nu}$, ν_1 . In the simulation, ω_1 and ν_1 are generated as two zero mean white Gaussian processes, for a sufficiently large number of times, say 4000. At each time instant, both ω_1 and ν_1 are Gaussian variables with zero mean and preset standard deviations. For example, at a certain time instant, the histogram of ω_1 presents a standard distribution of a Gaussian variable, as shown in Figure 6.1.

The histogram of the generated U_{11} at a certain time instant is shown in Figure 6.2.

Figure 6.1: *Histogram of ω_1 at a certain time instant.*

Figure 6.2: *Histogram of U_{11} at a certain time instant.*

Similarly, the other nonzero entries of the matrix of $\mathbf{U}$ are also generated. With the generation of matrix $\mathbf{U}$, it is now possible to compare each element of the covariance matrix estimated with and without term $\mathbf{U}$ and the difference between these two cases. These are shown in Figures 6.3, 6.4 and 6.5, in the format of an intensity image.

Figure 6.3: Each element of the covariance matrix $\mathbf{P}(k|k)$ with the term $\mathbf{U}$, (a)-(i) at 9 different time instants. Black indicates zero variance, and white is the highest value, indicated by the scale bars for each sub-figure.

In Figure 6.3, each sub-figure (a) to (i) is an intensity image whose value directly comes from the 9×9 covariance matrix exactly at the time instant as denoted in each sub-figure's title. For example, sub-figure (a) actually consists of 9×9 blocks,

each representing its corresponding value from the 9×9 $\mathbf{P}(k|k)$ matrix and the grey level of each block corresponds to the grey level bar on the right side of the figure. The values in Figure 6.3 (a) occur at the time of $1/9$ of the whole time length, which was 40 seconds in this simulation. The difference of the three Figures 6.3, 6.4 and 6.5 is that the first two figures describe each element of the covariance matrix $\mathbf{P}(k|k)$ with and without the term $\mathbf{U}$ in equation 6.10 at exactly 9 time instants and the third shows the difference of each element of the first two at these time instants.

Figure 6.4: Each element of the covariance matrix $\mathbf{P}(k|k)$ without the term $\mathbf{U}$, (a)-(i) at 9 different time instants.

It is seen from the nine sub-figures in Figure 6.5 that, except the first and second

diagonal elements, the other ones of the covariance matrix are much smaller and hence much less affected by the cross-correlation term $\mathbf{U}$. The first and second diagonal elements of the covariance matrix correspond to the error covariances for the *North* and *East* positions. Hence, the existence of the cross-correlation term causes the positional error covariances to increase. Another interesting fact is that as seen from the value of each grey level bar, the maximum value keeps increasing as time passes, which means the brightest block of the intensity images at a later time instant has a larger value than the earlier one. This means the positional elements in the covariance matrix are increasingly more heavily affected as time increases, which implies the importance of introducing the cross-correlation term $\mathbf{U}$ for tests with longer periods of time.

The difficulty in viewing Figure 6.5 is due to the fact that a few elements (the first and second diagonal ones) are much larger than the others. In order to illustrate the results more clearly, Figure 6.5 is shown again in figure 6.6, in the format of 3D bar diagrams. In these bar diagrams, the two horizontal axes define each element of the matrix and the vertical axis shows how large the element is.

From Figure 6.6, it is seen much more clearly that the first and second diagonal elements of the covariance matrix, which correspond to the error covariances of the *North* and *East* positions respectively, are much larger than other elements.

From Figure 6.3 to 6.6, each of the $9 \times 9 = 81$ elements of the covariance matrix $\mathbf{P}(k|k)$ with and without taking into account of the cross-correlation term $\mathbf{U}$ and their differences have been plotted and compared. However, in order to display all

Figure 6.5: The differences between each element of the covariance matrix $P(k|k)$ with and without the term U , (a)-(i) at 9 different time instants.

Figure 6.6: 3D bar diagram of the differences between each element of the covariance matrix $P(k|k)$ with and without the term U , (a)-(i) at 9 different time instants.

81 elements of each covariance matrix, the values at only 9 time instants are shown. In order to check the results at all time instants, some elements have been chosen and shown in the ensuing figures. Since this chapter is based on the simulated data presented in chapter 5, we know that the position and velocity elements in the “North” and “East” directions are more important since the vertical position and velocity are assumed to be zero. Therefore, Figures 6.7 and 6.8 illustrate the comparison of the “North” and “East” position and velocity elements of the covariance matrix $\mathbf{P}(k|k)$ with and without the cross-correlation term $\mathbf{U}$ and their differences, at all time instants. In order to avoid physically meaningless units such as m^2 and $(m/s)^2$, the square root of each element is plotted. Now it is clear that at all time instants, the value of positional elements of the error covariance matrix, especially in the “North” direction are much larger than others. Hence only positional variances are clearly seen in the intensity images (Figure 6.3 to 6.6). It is also seen that, the value of difference of the “North” positional standard deviation with and without $\mathbf{U}$ are also larger than others, which means that the “North” positional error covariances are increased considerably due to the introduction of the cross correlation term. The “East” positional error covariances are also increased. The velocity error covariances are slightly increased by the term $\mathbf{U}$.

Figure 6.9 shows the errors (the differences between the estimated values and the ground truth) of the *North* and *East* positions with 2σ error bounds. The 2σ error bounds are calculated from the corresponding elements in the error covariance matrix $\mathbf{P}(k|k)$, again with and without the additional cross-correlation term $\mathbf{U}$. It is

(a) North positional variances

(b) East positional variances

Figure 6.7: The square root of the position variance of the covariance matrix $\mathbf{P}(k|k)$ with and without the cross-correlation term $\mathbf{U}$ (upper graphs) and their differences (lower graphs), in the “North” and “East” directions.

(a) North velocity variances

(b) East velocity variances

Figure 6.8: The square root of the velocity variance of the covariance matrix $\mathbf{P}(k|k)$ with and without the cross-correlation term $\mathbf{U}$ (upper graphs) and their differences (lower graphs), in the “North” and “East” directions.

seen from Figure 6.9 that because of the existence of the additional cross-correlation term, the positional error bounds were increased and hence filter consistency results for a longer time period, after which the positional errors grew beyond the 2σ error bounds. It is seen in Figure 6.10 that the differences in the 2σ error bounds with and without $\mathbf{U}$ are smaller because the velocity variances in the error covariance matrix are much less affected by the additional cross-correlation term.

Figure 6.9: The positional errors with 2σ error bounds, with and without $\mathbf{U}$ in the North (upper graph) and East (lower graph) directions. In both graphs, the red curves represent 2σ error bounds with $\mathbf{U}$ and the magenta ones represent those without $\mathbf{U}$.

6.4 Conclusions

In this chapter the cross correlation between the process and observation noise of the multi-aided INS has been presented. The cross-correlation term was introduced

Figure 6.10: The velocity errors with 2σ error bounds, with and without U in the North (upper graph) and East (lower graph) directions. In both graphs, the red curves represent 2σ error bounds with U and the magenta ones represent those without U .

because of the use of the transformation matrix $\mathbf{C}_b^n$ in the observation model to transform the body velocity observations to the navigation frame. Since $\mathbf{C}_b^n$ is a function of the Euler angles γ, β, θ , the errors of which are part of the system state vector, the error of the observation is *NOT* independent of the errors of the state vector and hence this cross-correlation term exists between the process and observation uncertainties.

The effect of the cross-correlation term on the error covariance matrix has been evaluated in theory. Based on the assumption that the process and observation noise is correlated only one time step apart, this cross correlation problem becomes mathematically tractable. This assumption is reasonable since $\mathbf{C}_b^n(k)$ is part of the estimation result at time instant k , and at the next time instant $k + 1$, it is used to transform the velocity observation from the body frame to the navigation frame and hence causes the cross correlation. With the correlation term being formulated theoretically, it has been incorporated into the linear Kalman filter and each relevant filter equation has been updated. By comparing the error covariance matrix of the estimated state with and without the inclusion of the cross-correlation term, an intuitive conclusion can be made that some entries of the error covariance matrix should be increased for correct filter behavior.

Because the symbolic solution to the updated error covariance matrix is intractable, a simulation has been produced using numerical signals to evaluate the cross-correlation term on the error covariance matrix and further on the performance of the filter. The simulated results have shown that the position error elements in

the error covariance matrix have been increased since the cross-correlation term was added into the estimation process. This means that the error bounds of the position estimates have been increased due to the consideration of the additional cross-correlation term in the filter, which could postpone the divergence of the filter. Without consideration of this term, the error covariance associated with the position estimates are smaller and the filter is “over confident”. Simulation results further reveal that only the positional error covariances are mainly affected and other elements such as the velocity and Euler angles are significantly less affected.

Chapter 7

Experimental Results with the Multi-aided Inertial Navigation System

7.1 Introduction

In this chapter, the experimental results obtained from the multi-aided inertial navigation system presented in chapter 4, will be shown. Actual data from the experiments have been collected and processed to implement the detailed INS algorithm depicted in Figure 4.1. This means, at the beginning of each run, the initial calibration and alignment for the IMU is carried out with the information of the tilt sensors. Then the raw IMU accelerations and angular rates are processed with the MCRA noise filter. Subsequently, the reduced noise data are input into the INS computer to generate the predicted positions, velocities and attitudes and the errors associated with them, in the navigation frame. The vehicle's initial heading is produced from a digital compass. These predictions and the observations from the

body velocity estimator (the encoders and the “virtual sensor”) and the single-axis gyroscope, are input to the Kalman filter (chapter 4) and the final pose estimates are obtained. In order to take into account the effect of the cross-correlation term between the process noise and observation errors on the final estimates of the states and the associated error covariance matrices, the Kalman filter algorithm is adjusted according to the method described in Chapter 6.

7.2 Test Equipment and Experimental Set-up

The testing vehicle used in the experiments was a utility pickup truck, shown in Figure 7.1, with all the mounted sensors. The inertial sensor used in this work was a low-cost IMU from Inertial Science, DMARS-I-N. The IMU together with the tilt sensors (for initial calibration) and the single-axis gyroscope were mounted on a rigid platform, which was attached to the top of the pickup. A digital Honeywell compass was also used to provide the initial heading of the vehicle. The optical encoders were mounted on the rear wheels as depicted in the left figure. A Trimble DGPS set was also used in the experiments and the INS/GPS data was fused using the algorithm from [20]. The INS/GPS fusion result was the best ground truth available, since GPS alone produced large jumps in the estimated vehicle path.

Results from three different kinds of inertial navigation methods, namely the free running INS; the MCRA reduced noise, multi-aided INS that has been presented throughout chapters 3 to 6 and the INS/GPS method from [20], are used for comparison. Both of the first two INS methods adopted the initial alignment and

calibration algorithm from [44].

Figure 7.1: *The testing pickup truck with mounted sensors.*

7.3 Experimental Results

7.3.1 First Loop

In the experiments, the testing vehicle was first driven in an outdoor and undulating environment on the campus of Nanyang Technological University. The altitude of this environment ranges from 20 to 50 meters above sea level. The whole path is approximately 1.1 km in length and the vehicle ran at a speed of around 25 km/h and it took around 3 minutes for the vehicle to complete one loop. The map of the environment and the vehicle's running path is shown in Figure 7.2.

In Figure 7.2, the red line shows the path generated from the INS/GPS integration technique, while the cyan line is the path from the multi-aided INS. Compared to the INS/GPS result, the proposed multi-aided INS path was somehow offset

Figure 7.2: The red curve is the path from INS/GPS taken as the ground truth. The cyan curve is the MCRA aided INS path.

from the ground truth. However, in the INS/GPS path (which is assumed to be the ground truth), there were some discontinuities due to the GPS errors. Figure 7.3 shows the path from the free running INS, compared with the two paths in Figure 7.2. As seen from the figure, even with the initial calibration, without other aiding techniques, the free INS can function accurately only for a limited period of time (approximately 50 seconds) as expected. The mean values of positional errors of the free INS result are 345 m and 106 m in the *North* and *East* directions respectively, which are much larger than those of the aided INS, 3.7 m and 7.5 m, respectively.

Figure 7.4 shows a comparison of the velocities from the three different INS methods. The velocities, integrated from the navigation frame accelerations in the north and east directions, have been plotted according to each method. It is clear that after initial calibration, the estimates of the velocities from the free running

Figure 7.3: The generated paths from the three localization methods. The red and cyan curves are from the INS/GPS and MCRA aided INS respectively. The large blue curve is the result of the free running INS.

INS soon diverge while the multi-aiding technique followed the ground truth well throughout the entire path. The mean values of the velocity errors of the free INS result are 11.6 m/s and 6.4 m/s in the *North* and *East* directions respectively, while those of the aided INS are reduced to 0.31 m/s and 0.32 m/s, respectively.

Figure 7.4: The comparison of the velocities in the North and East directions from the three localization methods.

In Figures 7.5 and 7.6, the positional and velocity errors from the multi-aided

INS method in the *North*, *East* and *Down* directions have been shown respectively. In these two figures, the blue curves in each sub figure are the differences between the aided INS results and those from the ground truth, while the red profiles are the 2σ error bounds, which are computed from the appropriate positional or velocity error terms in the estimated covariance matrix $\mathbf{P}(k | k)$ (equation 4.30). Notice that the errors are within the error bounds with around 95 percent of the time, as expected.

Figure 7.5: The positional errors in the *North*, *East* and *Down* directions from the MCRA aided INS. The blue curves are the errors and the red curves are the 2σ error bounds.

Figure 7.7 shows the velocities as well as positions in the “Down” direction, estimated using the three different INS methods. Since the vehicle ran in a 3D environment (with changes in the Down direction as well as North and East), the displacement estimate in this “Down” direction is also important. It is seen that the velocity and position estimation from the multi-aiding method follows the INS/GPS estimates much better than those estimated in the free running INS. Figure 7.8 illustrates the comparison of velocity and positional errors from the free running

Figure 7.6: The velocity errors in the North, East and Down directions from the MCRA aided INS. The blue curves are the errors and the red curves are the 2σ error bounds.

Figure 7.7: Comparison of velocities and positions from the three methods, in the "Down" direction.

INS and multi-aided INS, in the “Down” direction. Again the errors are computed using the velocity and position estimates from the INS/GPS algorithm.

Figure 7.8: Comparison of velocity and positional errors from the free running INS and multi-aided INS, in the “Down” direction.

In Figure 7.9 the yaw angles of the vehicle from the INS/GPS and the multi-aided method are plotted. The yaw angle estimate from the multi-aided method is relatively accurate and the mean value of the yaw error is 1.4° . In the figure, the two curves are too close to distinguish but the difference of these two curves is plotted in the lowest graph of Figure 7.11.

Figure 7.10 illustrates the three Euler angles of the MCRA aided INS. It is seen from Figure 7.10 that the roll and pitch angles are not zero when the vehicle was moving. The changing amplitude of the roll and pitch angles can be several degrees due to the fact that the driving path of the vehicle is uneven and bumpy.

Figure 7.11 shows the Euler angle errors of the MCRA aided INS, using INS/GPS integration attitudes as the ground truth. The blue curves are the errors and the red ones, again, are the 2σ error bounds. The errors are within the error bounds

Figure 7.9: Comparison of the yaw angles of the vehicle from the INS/GPS integration result and the MCRA aided INS.

Figure 7.10: The three Euler angles of the MCRA aided INS.

with approximately 95 percent of the time, as expected, except for times greater than 180 seconds.

Figure 7.11: The 3 Euler angle errors of the MCRA aided INS. The blue curves are the errors with the red curves being the 2σ error bounds.

7.3.2 Second Loop

In another experiment, the testing vehicle ran in an outdoor bumpy environment with an even longer path. The whole length of this path is approximately 1.6 km and it took the vehicle for approximately 4 minutes to complete one loop. In the following part the results will be shown and again, the results from the free running INS, the MCRA aided INS and the INS/GPS integration method are produced for comparison.

The map of the environment and the path is shown in figure 7.12. In Figure 7.12, the red curve shows the path generated from the INS/GPS integration technique, while the cyan curve is the path from the multi-aiding method using noise reduced inertial data by applying the MCRA method. Compared to the INS/GPS result,

the MCRA aided INS path was slightly offset from ground truth.

Figure 7.12: The red curve is the path from INS/GPS taken as the ground truth. The cyan curve is the MCRA aided INS path.

Figure 7.13 shows the path from the free running INS, compared with the two paths in Figure 7.12. Again, the free INS is shown to be able to function accurately only for a short period of time. The mean values of position errors of the free INS result are 461 m and 814 m in the *North* and *East* directions respectively, which are much larger than those of the MCRA aided INS, 7.3 m and 5.5 m.

Figure 7.14 shows a comparison of velocities from the three different INS methods. The velocities in the north and east directions have been plotted from each localization method. It is clear that after initial calibration, estimation of the velocities from the free running INS soon diverges while the MCRA aided method followed the ground truth well throughout the process. The mean values of velocity errors of the free INS result are 7.9 m/s and 9.4 m/s in the *North* and *East* directions respectively, while those of the aided INS are reduced to 0.29 m/s and 0.21 m/s.

Figure 7.13: The generated paths from the three localization methods. The red and cyan curves are from the INS/GPS and MCRA aided INS respectively. The large curve is the result of the free running INS.

Figure 7.14: Comparison of velocities in the North and East directions from the three localization methods.

Figure 7.15 shows the velocities as well as the positions in the “Down” direction estimated using the 3 different INS methods. It can be seen that the velocity and positional estimates from the MCRA aided INS follows the INS/GPS estimates much better than those estimated in the free running INS. Figure 7.16 illustrates the comparison of velocity and positional errors from the free running INS and multi-aided INS, in the “Down” direction. From the error plots it is seen that the free running INS suffers from a large error in its velocity and position estimates in the “Down” direction. Even without any direct aiding in the “Down” direction, the multi-aided INS provides a much more accurate estimate in this direction, benefitting from the multi-aided sensor system.

Figure 7.15: Comparison of velocities and positions from three methods, in the “Down” direction. The red and cyan curves are from the INS/GPS and MCRA aided INS respectively. The blue curves are from the free running INS.

In Figure 7.17 the yaw angles of the vehicle from the INS/GPS and the multi-aiding method are plotted. The mean value of heading error is approximately 1° for this experiment.

Figure 7.16: Comparison of velocity and position errors from the free INS and multi-aided INS, in the “Down” direction.

Figure 7.17: The upper graph is the comparison of the yaw angles of the vehicle from the INS/GPS (red curves) and the MCRA aided INS (cyan curves) and the lower graph shows the yaw errors of the MCRA aided INS.

7.3.3 Multi-aided INS + GPS Plots

This section describes the result of the multi-aided INS together with GPS as additional observations if GPS readings are available. In order to incorporate GPS into the existing navigation system, just use position estimates of the north, east and down direction from the GPS as observations and extend the observation model (equations 4.20 and 4.21 in chapter 4) accordingly. The new observation vector becomes,

$$\begin{aligned} \mathbf{z}(k) &= \begin{bmatrix} \mathbf{z}_P^{inertial}(k) - \mathbf{z}_P^{aiding}(k) \\ \mathbf{z}_V^{inertial}(k) - \mathbf{z}_V^{aiding}(k) \\ \mathbf{z}_\gamma^{inertial}(k) - \mathbf{z}_\gamma^{gyro}(k) \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{\delta}_P(k) \\ \boldsymbol{\delta}_V(k) \\ \delta_\gamma(k) \end{bmatrix} + \begin{bmatrix} \boldsymbol{\nu}_P(k) \\ \boldsymbol{\nu}_V(k) \\ \nu_\gamma(k) \end{bmatrix}, \end{aligned} \tag{7.1}$$

where, the newly added items $\mathbf{z}_P^{aiding}(k)$ and $\boldsymbol{\nu}_P(k)$ denote the position estimates from GPS and their associated noise sources at time k respectively.

The new observation matrix becomes,

$$\mathbf{H}(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \quad (7.2)$$

The experimental data of the first loop in Section 7.3.1 was processed with the additional GPS observations and the plots are shown. Only the estimates from the original multi-aided INS and the new aided INS plus GPS algorithm are plotted, with the original INS/GPS result as ground truth. The trajectories estimated by these three INS algorithms are shown in Figure 7.18.

It is seen clearly from Figure 7.18 that with GPS fixes available, the aided INS plus GPS algorithm closely follows the INS/GPS path because GPS provides additional position observations which are absent for the multi-aided INS. This is further shown in Figure 7.19, where the position errors of the multi-aided INS and aided INS + GPS algorithms (with INS/GPS results as ground truth), in north, east and down directions, are plotted.

In Figure 7.20, the velocity errors of the multi-aided INS and aided INS + GPS algorithms (with INS/GPS results as ground truth), in north, east and down di-

Figure 7.18: The red curve is the path from INS/GPS taken as the ground truth. The cyan curve is the multi-aided INS path. The blue curve is the aided INS + GPS path.

Figure 7.19: Plots of position errors of the multi-aided INS and aided INS + GPS algorithms, in north, east and down directions. Since position is not observable for the multi-aided INS, additional GPS provides position observations and dramatically reduces the errors.

rections, are plotted and compared. Since velocity is directly observable in the multi-aided INS, the extra information from GPS benefits the velocity estimates only slightly. GPS positions can only slightly benefit the Euler angle estimates as well, as shown in Figure 7.21, where the yaw angle errors of the multi-aided INS and aided INS + GPS algorithms are compared. Hence, it is clearly seen from the results that, when additional GPS positional observations are available, the position estimates of the multi-aided INS are noticeably improved, when using INS/GPS results as ground truth. However, the estimates of the velocity and Euler angles are only slightly improved since the additional GPS position observations do not produce direct observations for these vehicle states estimates.

Figure 7.20: Plots of velocity errors of the multi-aided INS and aided INS + GPS algorithms, in north, east and down directions. Because velocity is observable for the multi-aided INS, the extra information from GPS benefits the velocity estimates slightly.

Figure 7.21: *Plots of the yaw angle errors of the multi-aided INS and aided INS + GPS algorithms. Because the yaw angle is also observable for the multi-aided INS, the extra GPS data benefits the yaw angle estimates slightly.*

7.4 Conclusions

This chapter has presented experimental results on the implementation of the multi-aided inertial navigation system developed in Chapter 4. The results of the free running inertial navigation and INS/GPS fusion navigation methods have also been presented for comparison.

The developed multi-aided inertial navigation system provides accurate and reliable results. The results illustrate the following arguments:

1. Firstly, the multi-aided navigation system corrects the errors in position, velocity and attitude of the free inertial navigation solution in three dimensions. Consequently, accurate aided navigation information can be provided for autonomous vehicles running in outdoor, uneven environments, for a noticeable

extended period of time when GPS is unavailable.

2. Secondly, the concept of utilizing the vehicle constraints as a “virtual sensor” as part of the aiding system has proved successful throughout these experimental results. Together with wheel encoders, this virtual sensor can be used with a kinematic model to provide velocity observations in the vehicle’s body frame. With the heading observation from the additional single-axis gyroscope, the observations from all the aiding sensors keep the errors in position, velocity and attitude estimations bounded, to an acceptable level for navigation purposes, as shown in the results, for a noticeable extension period during GPS outages. Within the experimental time, the multi-aided INS technique performances reliably and accurately. This aiding method hence proves to be an effective alternative algorithm, which does not require GPS, as opposed to INS/GPS fusion methods, for limited time periods. Under consideration that GPS is unavailable, this multi-aided inertial navigation system offers potential use in extensive areas. However, in comparison with GPS as an aided system, a disadvantage of this multi aiding technique is the lack of observations in the “Down” direction. In the future, an altimeter should be considered to produce this information.
3. Thirdly, this multi-aided navigation system was implemented on a pickup truck, in different outdoor uneven environments. The low-cost IMU has proved to be a very versatile dead reckoning sensor for ground vehicles. Provided information is continuously available from external sensors, large errors in position,

velocity and attitude estimates due to the biases or drifts present in the accelerometers and gyroscopes, can be prevented. Hence, the proposed inertial navigation system can function successfully on ground vehicles.

4. Finally, if GPS information is available, it is easily incorporated into the multi-aided INS approach and GPS positional observations provide external positional benefits.

Chapter 8

3D Range Data Representation in Uneven Environments and Range Error Modeling for Non-coaxial LADARs

8.1 Introduction

In the previous chapters, efforts have been made to solve the localization problem for mobile robots in outdoor, uneven environments. Another critical ability for mobile robots to perform autonomous navigation tasks is mapping. This partially explains why SLAM (Simultaneous Localization and Map Building) has been the “Holy Grail” of the mobile robotics research community in the past two decades, [3], [56], and [57]. However, the map definition of SLAM is only useful for the robot, which means the map consists of groups of points or lines as features, recorded from a moving robot and for the robot to navigate itself. In order to make 3D maps useful

and meaningful to both robots and humans in outdoor environments, a solution is proposed in this chapter. Building 3D maps of the environment is also referred to as developing 3D models or building 3D representations of the environments in the literature, [58], [59], [60], and [61]. In this work, we name it as building 3D range data representation.

The significance of building 3D representations of actual outdoor environments is not confined to mobile robot applications only. It has numerous other applications such as 3D urban database generation, environmental reconstruction, virtual reality creation and computer games. Generally two major approaches have been proposed for building 3D representations. One approach is based on using intensity information, e.g. produced by CCD cameras, as carried out in [62] and [63]. The other approach is using range information, e.g. given by LAser Detection And Ranging (LADAR) sensors, as carried out in [58], [59], [60], [61] and [64]. The key advantage of LADARs over cameras is that the range information is given directly and no computation is needed to extract range. In this chapter, our approach is also based on LADARs.

When processing LADAR sensor data for range data representation or scene interpretation, most previous work models such devices as processing range data which follows a Gaussian distribution which can be represented by a mean and variance. In this chapter, it is demonstrated that commonly used LADARs suffer from incorrect range readings at changes in surface reflectivity and/or range discontinuities, which can have a much more detrimental effect on such algorithms than random

noise. This chapter will further demonstrate that a correct understanding of such systematic errors is essential for the correct further processing of the data.

8.2 3D Range Data Representation for Outdoor Uneven Environments

8.2.1 Outline of this work

As mentioned in the introduction, much work towards 3D representation in indoor and outdoor environments, using laser range finders, has been accomplished [64], [58], [59], [60], and [61]. Because these laser range finders, or LADARs, are mounted on mobile robots, the accuracy of range data representation directly depends on the localization ability of those robots, especially in large scale outdoor environments. In the work of [58], a localization method was utilized based on scan matching and the localization accuracy in the horizontal plane was claimed to be within 1 meter. However, an assumption has been made in [58] that the testing vehicle ran on a flat road and that there is no topographic changes of the environment (called *flat terrain* assumption in this work). The same assumption can be found in [61]. Obviously this assumption is easily violated in real outdoor, uneven environments. In the work of [59] and [60], an IMU/GPS sensor set has been adopted towards solving the localization problem in uneven environments. Again, the common problems associated with GPS, such as occlusions from high buildings or foliage and multi-path errors, occur and affect the accuracy of the localization. In the work of [59],

additional odometry information is used and the localization ability is improved.

In this thesis, since the localization capability of the multi-aided INS has been shown to be capable of 3D localization, even without the help of GPS, a 3D range data representation of the environment is sought with a laser range finder. This sensor is mounted on the same utility vehicle as the navigation sensor set, as shown in Figure 8.1. Although the localization errors in the horizontal plane of our multi-aided INS are slightly larger than those of scan-matching based localization methods, it is definitely more reliable to utilize our inertial localization approach in outdoor *uneven* environments, because of the violation of the *flat terrain* assumption that occurs in the scan-matching based localization methods in such environments.

Figure 8.1: *The testing pickup truck with the INS suite and a laser range finder mounted on top of the vehicle.*

From Figure 8.1, it can be seen that a 2D SICK LADAR is mounted on the top of the testing vehicle, pointing upwards, [65]. As the vehicle moves forward in an uneven environment, the LADAR continuously makes vertical scans, building a

three dimensional range data representation of the environment. The SICK LADAR used in this work is a PLS 200 model, with the maximum detection range of 50m and approximate range sampling frequency of 35Hz. The scanning angle is 180° and its angular step is 0.5° , [65].

8.2.2 Range data processing

In order to associate the localization information achieved by our multi-aided inertial navigation system (Chapter 4), with the range data collected by the LADAR, in this section, the LADAR data is registered according to the estimated vehicle pose from the MCRA aided INS.

Firstly, the data collected by the LADAR needs to be aligned in the LADAR's coordinate system. Without loss of generality, assume a point target P is detected by the LADAR (see figure 8.2). The laser sensor measures the distance to the point r at scanning angle θ . Taking the vehicle's forward direction as the positive x -axis and the positive z axis as the up direction, the LADAR's frame is defined. The point target's coordinates in the sensor frame can be readily computed as,

$$\mathbf{P}_l = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ r \cos(\theta) \\ r \sin(\theta) \end{pmatrix}. \quad (8.1)$$

where l represents the LADAR's frame.

Now the point target needs to be transformed into the IMU's body frame. Since

Figure 8.2: Schematic diagram of the LADAR's coordinate system.

Figure 8.3: Schematic diagram of the translational offsets between the location of the IMU and the LADAR on the vehicle.

both the IMU and the LADAR are mounted on a rigid platform on the top of the vehicle, this transform involves translation only. The schematic diagram of the relative positions of these two sensors is shown in Figure 8.3. The offsets of these two sensors are m and n , and according to the definition of the inertial body frame (Refer to figure 2.1 in Chapter 2), the point target's coordinates in the body frame can be easily obtained as

$$\mathbf{P}_b = \begin{pmatrix} -n \\ r\cos(\theta) - m \\ r\sin(\theta) \end{pmatrix}. \quad (8.2)$$

Finally, the point target's coordinates in the navigation frame are computed using the transformation matrix $\mathbf{C}_b^n$ (Refer to equation 2.1 in Section 2.2.3),

$$\mathbf{P}_n = \mathbf{C}_b^n \begin{pmatrix} -n \\ r\cos(\theta) - m \\ r\sin(\theta) \end{pmatrix}. \quad (8.3)$$

Using equations 8.1, 8.2 and 8.3, the raw measurements r and θ are registered to the vehicle's navigation frame and hence, the 3D range data can be represented in the navigation frame, based on the localization information. The advantages of this 3D data representation are twofold. Firstly, the laser data is registered in the navigation frame, which can be a global system with the adoption of the GPS information, the 3D data is actually geo-referenced and globally consistent. The

other advantage is that, based on the 3D localization data generated from the inertial navigation system, this 3D point set is represented accurately in three dimensions. By transforming the LADAR data into the navigation frame, the estimated pitch and roll angles are utilized, which help to keep the data correctly aligned in a global coordinate system.

8.2.3 Results and discussions

In this section, results will show the 3D laser range data representation based on the multi-aided inertial navigation system. During the experiments, the vehicle was driven in outdoor, uneven environments in NTU. Some of the localization-only results of these experiments have been shown in Chapter 7.

Firstly, the vehicle was driven on a straight road around the N3 wing of the north spine of NTU. This environment is full of trees and low rise buildings. The front view and the top view of the generated 3D range data are shown in Figures 8.4 and 8.5.

In Figures 8.4 and 8.5, the profiles of trees and buildings in the environments are clearly seen. The reality of the environment around the N3 wing is approximately reconstructed in these 3D LADAR point clouds, represented in the navigation frame.

Further experimental results are shown here which are based on the data collected as the testing vehicle was driven around the larger loop in Figure 7.12 (Chapter 7). As can be seen from the localization result in the “Down” direction in Figure 7.15, clearly, the terrain is not flat. The assumption of flat terrain used by previous

Figure 8.4: *Front view of the 3D range data representation of the N3 wing of NTU.*

Figure 8.5: *Top view of the 3D range data representation of the N3 wing of NTU.*

researchers is violated here and the range data representation result will be seriously impaired if such an assumption is still adopted. Here, the results based on the 3D localization information are displayed. Since the whole loop is large, two specific positions are selected and shown. The first one is an overhead bridge and a bus stop near canteen A, as seen in the lower right corner of Figure 7.12. The processed range data is shown in Figure 8.6. A photo taken on the same site as comparison

Figure 8.6: *3D range data of an overhead bridge and a bus stop in NTU.*

is shown in figure 8.7 (a). The profiles of the overhead bridge and the bus stop are clearly displayed in Figure 8.6. However, the bus stop looks lower than it really is because the range LADAR was mounted on the top of the testing pickup truck, which is around 1.8 meters high.

In order to evaluate the range data based on the multi-aided INS, the range data has been represented again based on the localization result from the INS/GPS method, which is utilized as ground truth. The result is illustrated in Figure 8.8.

(a)

(b)

Figure 8.7: (a) The photo of the overhead bridge and bus stop in figure 8.6. (b) The photo of the SIMTech building in figure 8.9.

Figure 8.8: 3D range data from the scene in Figure 8.6, based on the INS/GPS localization data.

From a qualitative point of view, it is difficult to see any difference in the range data represented in figure 8.6 and 8.8, which proves that, the localization result of the multi-aided INS compares well with that of the fused INS/GPS system.

(a)

(b)

Figure 8.9: (a) The 3D range data from the SIMTech building based on the 3D multi-aided inertial localization data. (b) The range data of the same scene in (a) based on 2D localization data (assuming zero roll and pitch angles and zero “Down” position).

The second scene taken from the loop is the SIMTech building, as shown in Figure 8.7 (b). The 3D range data from this site is shown in Figure 8.9 (a). It is seen that the tree on the left of the road and the building on the right side are clearly displayed. However, the straight lines formed by the windows in the building are not transformed correctly, and seen as curves in Figure 8.9. This is due to the limitation of the localization accuracy.

The advantage of the multi-aided INS is that it not only provides the “Down” position of the range data, which makes sure the items to be placed at proper vertical positions, but also the “Roll” and “Pitch” angles of the perception sensor, which restrict the range data in correct positions. In order to show the importance of estimating the “Roll” and “Pitch” angles as well as the “Down” position, another

transformation is carried out without the information. Based on the same horizontal localization information from the multi-aided INS but using zero for the “Roll” and “Pitch” angles and the “Down” position, the same LADAR data is shown in Figure 8.9 (b). It is obvious that the LADAR data cannot be placed correctly without the “Roll” and “Pitch” angles and the “Down” position in an uneven environment.

Again, the range data of the scene in Figure 8.9 (a) was represented based on the localization result from the INS/GPS method and the result is illustrated in Figure 8.10 for comparison.

Figure 8.10: *The 3D range data of the scene in figure 8.9 (a), based on the INS/GPS localization data.*

Discussion:

1. 3D range points can be correctly represented in a global navigation frame based on the localization from the multi-aided inertial navigation system. The advantage of using INS localization is to take into account the topographical

information of uneven environments. The additional roll and pitch angles help map the LADAR point cloud to the correct positions in the navigation frame. With the adoption of GPS, geo-referencing could then be carried out for the 3D range data.

2. The horizontal localization accuracy of the multi-aided INS, which is several meters, is a limitation for more accurate representation of the 3D range points. The limitation of the localization accuracy impairs the reconstruction accuracy and the realism of the environments reconstructed. A more accurate 3D localization method is needed to improve the accuracy of the 3D range data representation.

8.3 Range Error Modeling for Non-coaxial LADARs

8.3.1 Introduction

Since 3D point cloud data is sought using scanning LADARs and the proposed INS algorithm, an analysis of the errors in laser sensors has also been carried out and is presented here. Most previous work models such devices as processing range data which follows a Gaussian distribution. In this section, it is demonstrated that commonly used LADARs suffer from incorrect range readings at changes in surface reflectivity and/or range discontinuities, which can have a much more detrimental effect on such algorithms than random noise.

Most LADARs fall into two categories – coaxial and separated transmitter and

receiver configurations. The latter offer the advantage that optical crosstalk is eliminated, since it can be guaranteed that all of the transmitted light leaves the LADAR and is not in any way partially reflected within it due to the beam-splitting techniques necessary in coaxial LADARs. However, they can introduce a significant disparity effect, as the reflected laser energy from the target can be partially occluded from the receiver. False range values can result due to this occlusion effect from scanned LADARs. This section will demonstrate that a correct understanding of such systematic errors is essential for the correct further processing of the data. A useful design criterion for the optical separation of the receiver and transmitter is also derived for non-coaxial LADARs, based on the minimum detectable signal amplitude of a LADAR and environmental edge constraints. By investigating the effects of various sensor and environmental parameters on occlusion, some advice is given on how to make use of non-coaxial LADARs correctly so as to avoid range errors when scanning environmental discontinuities.

In mobile robotics, range sensing is often a crucial component of navigational and localization tasks [11, 66, 67, 68]. LADARs, with range and bearing information, have become an integral component of many simultaneous localization and map building systems, due to their accuracy and relatively low-cost. Feature detection algorithms are often based on probabilistic methods which attempt to extract information from the range data in an optimal manner, on the assumption that the range data is corrupted with Gaussian noise [13, 69, 70]. For example Guivant *et al.* have made an observation model for their Kalman filter based outdoor naviga-

tion system, assuming Gaussian noise [11]. Arras formulated the detection of line segments in Hough space and was able to generate line segment estimates and their associated covariances, based on the simple model that each LADAR range value was accompanied by assumed known, Gaussian noise [71]. Adams was able to detect range/reflectance discontinuities, by optimally weighting new range data with simple recursive line and constant curvature models – again based on range data with Gaussian noise assumptions [69, 72].

In our experience, such feature extraction algorithms often fail due to other systematic, unmodelled errors from LADARs. While it is acknowledged that an estimate of range variance has a large effect on, for example, Kalman Filter based scale space detection algorithms [13, 73], such methods can fail catastrophically when range errors due to *cross-talk* [74], *disparity* [75] or *multi-path effects* [69] occur. Removing such “outliers” from range data can be achieved with limited success with standard techniques such as median filters, based on certain assumptions about the erroneous range data [76]. It is argued here, however, that range errors should, and can, be estimated and eliminated or replaced, based on the physics of the LADAR and its scanning technique.

This section will demonstrate that, besides the cross-talk and multi-path effects¹ that have been documented in the literature [74, 77, 78, 79], significant range errors can also occur due to disparity caused by a combination of transmitter and receiver separation and the scanning reflection principle, to the author’s knowledge, an effect previously not examined in the literature for LADARs. It is noticed that unlike the

¹sometimes referred to as *mixed pixels* [74].

mixed pixels and cross-talk effects, this effect is actually *independent* of the range detection method² of the LADAR. The theoretical analysis applies to all detection methods, since the range errors occur due to a significant drop in received signal amplitude.

8.3.2 3D scanning LADAR systems

In this section, the LADAR scanning systems used in this work are introduced. The main LADAR is based on a 1D Riegl LD90-3300EHS-FLP model, as shown in figure 8.11(a). This LADAR is a time-of-flight (TOF) based measurement system [8], with a reported maximum range measurement capability of 400m. The data acquisition rate of the sensor can be configured between a *single shot mode* of 12kHz, to an *averaging mode* of 2Hz, in which several samples are averaged before being output from the device [69].

Figure 8.11: (a) 1D Riegl LADAR LD90-3300EHS-FLP model, (b) in-house developed 3D scanning mechanism and (c) 3D Riegl LADAR LMS-Z210i. The 3D mechanism in (b) and the 3D LADAR in (c) allow continuous rotation of the scanning mirror about the vertical axis and simultaneous control of the mirror's elevation about a horizontal axis.

²Time-of-flight (TOF), Amplitude Modulated Continuous Wave (AMCW), Frequency Modulated Continuous Wave (FMCW), etc.

For robot navigation, a desirable feature of any ranging system is that it provides full 360° coverage around the robot in bearing, so that all objects of interest, within the field of view of the sensor can be “seen” from any vehicle orientation. Surprisingly, to the knowledge of the author, few such affordable systems exist, possibly due to the difficulty in powering and controlling the elevation of the scanning mirror, while allowing continuous rotation in bearing [80]. Figure 8.11(b) shows a 3D scanning system developed at Nanyang Technological University (NTU) for this purpose [81].

The scanning LADAR is to be used on board an outdoor terrain vehicle, which can traverse fields, hills and small trenches. As a consequence, the complete system can undergo significant changes in its roll and pitch angles as the vehicle moves. To take full advantage of the axis of elevation, twin axis rate gyros are used to create a quasi-gimbaled scanning system [81]. The aim of this system is to monitor the rate gyros and estimate the roll and pitch of the system, so that the elevation angle of the scanning mirror can be controlled as a function of its bearing angle. This is done to ensure that, as far as possible, the LADAR scans in the same plane³ as before any changes in the vehicle’s roll and pitch angles occurred. The associated problems of gyro drift are discussed in [81].

The 3D LADAR shown in Figure 8.11(c) is a LMS-Z210i model from Riegl [82]. Its rather slow rotating rate⁴ limits its use in mobile robotic applications. However, in this work, this kind of 3D LADAR is still used for comparison with the LADAR

³thus allowing the same potential objects to be sensed.

⁴At the highest rate of rotation, it takes around 20 seconds for the LADAR to fulfil a single 2D scan.

of figure 8.11(a) housed within the scanning mechanism in figure 8.11(b).

8.3.3 Range errors due to occlusion

In order to place this work into perspective, the occlusion effect will be presented in this section, along with an example of how these errors affect range data scans. The range errors caused by random noise will also be presented with an example for easy comparison.

Figure 8.12 shows an intensity image⁵ recorded from the 3D scanning Riegl LADAR (Figure 8.11(c)) in a laboratory environment. Each pixel row corresponds to

Figure 8.12: *Received signal amplitude image recorded in a laboratory environment. Each pixel value is proportional to the received signal amplitude. The small circle (labelled "A") shows a zero-signal amplitude point.*

a 330° section sweep of the environment⁶. Between each row of pixels, the elevation of the scanning mirror is changed by 0.4° . Figure 8.13 shows the corresponding range image where each pixel intensity value is proportional to range (darker pixels correspond to closer objects). Such images contain extremely useful information for robot navigation and offer the fundamental advantage over vision systems, in that

⁵received signal amplitude. In the chapter, the two words “intensity” and “signal amplitude” or “amplitude” will be used interchangeably. The intensity is a ratio, which is a dimensionless quantity that ranges from 0 (least reflective) to 255 (most reflective) which is based on the strength of the returned signal.

⁶This angle can be configured and for the LADAR’s designing constraint, it can provide a 360° complete sweep only at a low rate of rotating.

range information is directly available.

Figure 8.13: The corresponding range image of the scan in Figure 8.12. Each pixel value is proportional to the estimated range. Darker pixels show closer objects.

In Figure 8.12, the small blue circle at A is shown to denote zero, or extremely low received signal amplitude values. These can be seen more clearly in figure 8.14(a) where the amplitude of the received signal is plotted versus the scanning mirror's bearing angle, at a constant angle of elevation. This is taken from the

(a) Received signal amplitude versus scanning bearing angle θ ($135^\circ < \theta < 150^\circ$) for the middle pixel row of Figure 8.12 (elevation= 40°).

(b) Range versus scanning bearing angle θ ($135^\circ < \theta < 150^\circ$) for the middle pixel row of Figure 8.13.

Figure 8.14: Received signal amplitude (a) and range (b) versus bearing corresponding to Figure 8.15. Plots are shown here with bearing ranging only from 135° to 150° to show the zero amplitude/range readings.

middle row of pixels (elevation angle = 40°)⁷ of Figure 8.12. Figure 8.14(b) shows

⁷The elevation angle ranges from 0° to 80° and hence 40° corresponds to the case for this sensor to scan within a plane.

the corresponding range versus bearing angle. When the amplitude reaches zero (or extremely low values) it can be seen that false ranges (range estimate = 0m) occur. To elaborate further from exactly which artifacts within the environment these false range values occur, Figure 8.15 shows the planar range scan (looking from above) of the middle row pixels (LADAR mirror elevation = 40°) in Figure 8.13.

Figure 8.15: *Plan view of the range data recorded at 40° angle of elevation. The small triangle shows the position of the LADAR system. In this environment each labelled object is, **A** a person in the right edge of figure 8.12, **B** cubicles, **C** cabinets, **D** a door, **E** chairs and computers and **F** a black container.*

In Figure 8.15, the environment is labelled with capital letters to denote the range readings corresponding to real objects in Figure 8.12. At the edge formed by “F”—the black container and “E”—the background chairs and computers, the range readings are zero. This could, of course easily be detected as being false if its value were always zero. In reality, LADARs such as those from Riegl [82], Acuity Research [83] and SICK⁸ [65] respond in different ways to such range discontinuities. It will be proved in section 8.3.4 that *all* scanning LADARs with separated transmitter and receiver configurations will suffer a minimum in received signal amplitude due to the

⁸LADARs commonly used in mobile robotics research.

disparity between the transmitter and receiver aperture *irrespective of the technique used to estimate range*. The resulting effect of this reduced signal amplitude on the range estimate is then dependent on the measurement technique and is difficult to generalize to all LADARs. However, this section will present a method to reliably predict such range failures based on other measurable quantities.

The effect of disparity or the “missing parts” problem in triangulation systems has of course been well documented [75], however its subtle effect during the scanning of a LADAR and the resulting received amplitude profile does not appear to have been reported. This effect will be quantified in section 8.3.4.

Most researchers model the range data from LADAR sensors as an estimated value with Gaussian noise [71]. In figure 8.16, the histograms show the range data from the LADAR in Figure 8.11(a) at a constant target distance while the target reflectivity and illumination conditions were varied. The range data in each case approximately follows Gaussian distribution. Different target reflectivity and illumination conditions will affect the received signal amplitude according to Lambert’s cosine law [69] and then the variance of the LADAR range data may also be affected. Hence, target reflectivity and illumination conditions with strong contrast, that is, black and white targets, day time and night time are chosen for this standard deviation test. However, it can be seen that for this LADAR, the range standard deviations are almost independent of these changing conditions and hence received signal strength. The range of the standard deviation is from 1.5 mm to 3.5 mm. That is to say, the random range errors are much smaller than the range errors

caused by the occlusion effect. This is true in many tested commercially available LADARs such as Riegl [82], SICK [65], and IBEO [84].

Figure 8.16: Range distributions showing the resulting range variance for varying target reflectivities and day/night conditions. In each case 4,000 independent range readings were recorded from a fixed target.

8.3.4 Occlusion due to transmitter/receiver separation

In order to predict what sensor and environmental parameters will cause range errors due to occlusion, the physics causing occlusion will be studied in this section and a theoretical model will be derived.

To study the occlusion effect, an experimental setup was made as shown in Figure 8.17. The 1D LADAR in Figure 8.11(a) was mounted on the stacker of a milling machine again and was translated to scan the edge of a target and its background as shown in figure 8.17(a). The range and amplitude information during this process were recorded and for one particular experiment are shown in Figure 8.18. A minimum in the amplitude profile occurs because of the separated transmit-

ter and receiver configuration of the sensor, as will be proved later in this section. There is a loss in received energy due to the non-coaxiality of the sensor. When the amplitude drops below the minimum working threshold of the LADAR, the range reading cannot be trusted (it often reads zero for the two LADARs in Figure 8.11 but in general it may read any arbitrary value and hence cannot be compensated by a simple low-pass filter). This is the occlusion effect.

Most LADAR sensors fall into two categories: coaxial and separated transmitter and receiver configurations. These configurations determine whether or not occlusion of reflected laser energy from targets occurs. The Riegl LADAR used in the analysis here has a separated transmitter and receiver configuration as shown in Figure 8.11 (a). For the 3D Riegl LADAR as shown in figure 8.11 (c), the transceiver also has separated configuration.

The following analysis will mathematically derive the profile of the received signal power from non-coaxial LADARs, as their scanning mirrors rotate. It will then be shown how this profile can be used to predict when range errors, such as those shown in Figure 8.15 will occur, and how to detect them. In the analysis, the following assumptions are made:

1. The power in the transmitted and reflected light beams is uniformly distributed over the circular, cross-sectional area of the laser.⁹
2. The received signal amplitudes from each individually illuminated target are known (i.e. they can be measured by the LADAR – P_1 and P_2).

⁹Note that the following analysis could be easily extended to other non-uniform optical power distributions.

Figure 8.17: (a) The LADAR is displaced past an edge formed by a target and a background. (b) The sensor is rotated 90° so that the transmitter and the receiver are displaced in a “coaxial” manner relative to the vertical edge.

Figure 8.18: Range and amplitude when the 1D LADAR was displaced to scan the edge as shown in figure 8.17(a).

3. Assumptions 1 and 2 allow that, without loss of generality, the scanning procedure can be modelled as with targets 1 and 2 parallel to the motion of the LADAR, irrespective of their true orientation. This is because the total received power is derived from the normal projections of the elliptical footprints, which are, of course, circular again, and independent of the orientation of the sensed target.
4. Due to assumption 3, the rotational, scanning motion of the LADAR's mirror in Figure 8.11 (b) can be modelled as a linear displacement of the LADAR's optical footprint past the edge. This assumption is valid, since, the true rotational motion would simply make the targets relative orientation change slightly, relative to the LADAR, and since the normal projections of the elliptical footprints are considered, these orientations make no difference.

Initially, before the transmitted optical footprint intersects the edge, it will fully illuminate target 1 at range d_1 , as shown in Figure 8.19(a). In this case, the received signal strength is P_1 watts, which can be measured if the amplitude of the received signal is available as in the case of the Riegl LADARs (and some SICK devices [65]). As the mirror scans, the projected optical footprint will eventually completely traverse the edge, so that it fully illuminates target 2 at range d_2 . In this case it is assumed that the received signal strength is P_2 watts (figure 8.19(f)).

In Figures 8.19(a) and (f), the received power *densities* of these *end conditions* are defined as:

$$\mathcal{P}_{(x \leq 0)} = \frac{P_1}{\pi r_1^2} \quad \mathcal{P}_{(x \geq X_2)} = \frac{P_2}{\pi r_2^2} \quad (8.4)$$

Figure 8.19: (a) Only target 1 is illuminated. (b) Target 2 is completely occluded due to the transmitter – receiver separation s . The footprint's area on target 1 is reduced. (c) Point Q on target 2 just enters the field of view of the receiver aperture. (d) Light now received from both targets 1 and 2. (e) Target 1 is no longer illuminated. Target 2 is partially occluded by target 1, due to transmitter – receiver separation s . (f) Light is now received from target 2 only, with no occlusion from target 1.

where x is the motion of the LADAR measured from the edge of target 1 and r_1 and r_2 are the radii of the normal components of the optical footprints on targets 1 and 2 respectively. From Figures 8.19(a) and (f):

$$r_1 \approx \frac{d_1 \alpha}{2} \quad r_2 \approx \frac{d_2 \alpha}{2} \quad (8.5)$$

where α is the beamwidth of the transmitted laser.

The point $x = 0$ is defined when the transmitted light beam just reaches the edge of target 1, as shown in figure 8.19(a). The scanning of the LADAR's mirror is then considered to be equivalent to displacing the LADAR in the direction shown in Figures 8.19(a) to (f).

1. **LADAR displacement $x \leq 0$ (figure 8.19a).** The received signal power is given by:

$$P_{(x \leq 0)} = \mathcal{P}_{(x \leq 0)} \pi r_1^2 = P_1 = \text{constant} \quad (8.6)$$

Define X_1 as being the value of x at which target 2 just becomes visible to the receiver aperture. This scenario is depicted in Figure 8.19(c):

$$X_1 = \frac{(s - q)(d_2 - d_1)}{d_2} \quad (8.7)$$

where s is the transmitter – receiver aperture separation and q is the effective radius of the receiver lens.

2. **LADAR displacement $0 \leq x < X_1$ (Figure 8.19(b)).** The received power

in this interval is:

$$P_{(0 \leq x < X_1)} = \frac{A_1}{\pi r_1^2} P_1 \quad (8.8)$$

where it can be shown from Figure 8.19(b) that the illuminated area A_1 is:

$$A_1 = r_1^2 \left(\pi - \theta_1 + \frac{\sin 2\theta_1}{2} \right) \quad (8.9)$$

so that:

$$\begin{aligned} P_{(0 \leq x < X_1)} &= P_1 \left(1 - \frac{\theta_1}{\pi} + \frac{\sin 2\theta_1}{2\pi} \right) \\ \theta_1 &= \cos^{-1} \left(1 - \frac{2x}{d_1\alpha} \right) \end{aligned} \quad (8.10)$$

Equations 8.10 define the received power amplitude profile, which is expected when traversing the edge for $0 \leq x < X_1$.

3. LADAR displacement $X_1 \leq x < 2r_1$ (figure 8.19(d)).

The power received in the displacement interval $X_1 \leq x < 2r_1$ results from the *two* footprint sections shown in Figure 8.19(d).

$$P_{(X_1 \leq x < 2r_1)} = \frac{A_1}{\pi r_1^2} P_1 + \frac{A_2}{\pi r_2^2} P_2 \quad (8.11)$$

where it can be shown from Figure 8.19(d):

$$A_1 = r_1^2 \left(\pi - \theta_1 + \frac{\sin 2\theta_1}{2} \right) \quad \theta_1 = \cos^{-1} \left(1 - \frac{2x}{d_1\alpha} \right) \quad (8.12)$$

as before, and

$$A_2 = r_2^2 \left(\theta_2 - \frac{\sin 2\theta_2}{2} \right) \quad \theta_2 = \cos^{-1} \left[1 - \frac{2(x - X_1)}{d_2\alpha} \right] \quad (8.13)$$

so the received signal power in this interval is

$$P_{(X_1 \leq x < 2r_1)} = P_1 \left(1 - \frac{\theta_1}{\pi} + \frac{\sin 2\theta_1}{2\pi} \right) + P_2 \left(\frac{\theta_2}{\pi} - \frac{\sin 2\theta_2}{2\pi} \right) \quad (8.14)$$

Let $x = X_2$ be the displacement of the LADAR at which target 1 just fails to occlude target 2. In this case, geometrical considerations give

$$X_2 = \frac{(s - q)(d_2 - d_1)}{d_2} + d_1\alpha \quad (8.15)$$

4. LADAR displacement $2r_1 \leq x < X_2$ (Figure 8.19(e)).

The power received in the displacement interval $2r_1 \leq x < X_2$ results from the single footprint shown in figure 8.19(e):

$$P_{(2r_1 \leq x < X_2)} = \frac{A_2}{\pi r_2^2} P_2 \quad (8.16)$$

where it can be shown from Figure 8.19(e):

$$A_2 = r_2^2 \left(\theta_2 - \frac{\sin 2\theta_2}{2} \right) \quad \theta_2 = \cos^{-1} \left[1 - \frac{2(x - X_1)}{d_2\alpha} \right] \quad (8.17)$$

as before, so that the received signal power in this interval is

$$\begin{aligned} P_{(2r_1 \leq x < X_2)} &= P_2 \left(\frac{\theta_2}{\pi} - \frac{\sin 2\theta_2}{2\pi} \right) \\ \theta_2 &= \cos^{-1} \left[1 - \frac{2(x - X_1)}{d_2\alpha} \right] \end{aligned} \quad (8.18)$$

5. LADAR displacement $x \geq X_2$ (figure 8.19(f)).

Finally target 2 is fully illuminated and the LADAR has been displaced enough, so that no part of the footprint is occluded from the LADAR's receiver aperture. In this case

$$P_{(x \geq X_2)} = p_{(x \geq X_2)} \pi r_2^2 = P_2 = \text{constant} \quad (8.19)$$

Equations 8.6, 8.10, 8.14, 8.18, and 8.19 between them, when θ_1 and θ_2 are replaced by their respective functions of x , describe the complete expected received amplitude profile as the LADAR's transmitted laser is scanned past a range discontinuity. In figure 8.20(a) (the red dotted profile), the amplitude profile is plotted versus the illuminated footprint displacement. Compared with the measured am-

Figure 8.20: (a) Estimated amplitude profiles of separated transmitter and receiver configuration and "coaxial" configuration LADARs. In this case, the values of involving parameters are: $d_1 = 5.8\text{m}$, $d_2 = 12.6\text{m}$, $P_1 = 115$, $P_2 = 50$ and $s - q = 2.9\text{cm}$. (b) Actual amplitude and range profiles corresponding to the red dotted one in (a).

plitude in Figure 8.18 (repeated in Figure 8.20(b) for comparison), the estimated profile in Figure 8.20(a) (the red dotted profile) is similar and the minimum value in the amplitude is almost the same. However the width of the change in the amplitude

curve in the actual case (Figure 8.18) is slightly smaller than that in the simulated case (Figure 8.20(a), the red dotted profile). The possible reason of the difference is due to the assumption that the laser energy is uniformly distributed over the beam cross section, which probably is not true. In reality, the laser energy concentrates around the center of the beam cross section, rather than being uniformly distributed, which causes the effective value of α , the beamwidth of the transmitted laser, to be smaller than the assumed value and then causes the effective footprint radii r_1 and r_2 to be smaller than the theoretical values (equation 8.5). Hence the actual LADAR displacement regions $X_1 \leq x < 2r_1$ and $2r_1 \leq x < X_2$ are smaller, which causes the width of the actual change in the amplitude curves to be smaller.

From equations 8.6, 8.10, 8.14, 8.18, and 8.19, the complete expected received amplitude profile can be plotted versus LADAR beam displacement x . If the minimum value of the amplitude could be determined as a general function of the LADAR beam displacement x and the other factors (LADAR to target distances d_1 and d_2 , received end condition powers P_1 and P_2 , LADAR parameters s, q and α), then the precise conditions which cause this minimum to be below the minimum working amplitude of the LADAR could be examined. During LADAR design, the values of s, q and α could be selected which minimize the chances of the minimum amplitude being below this minimum detection threshold. During operation, recorded data from the LADAR (range/amplitude) could be used to reliably predict when the received amplitude will drop below the detection threshold.

In Figure 8.20(a), individual estimated amplitude profiles corresponding to a

certain transceiver separation angle of the LADAR relative to the sensed target are shown for particular values of $d1$, $d2$, $P1$, $P2$, α and $s - q$. Here, the transceiver separation angle θ is defined as the angle between the direction of the transmitted laser beam and the center line of the transceiver of the LADAR when the LADAR's scanning mirror rotates. In Figure 8.22(a), the transceiver separation angle 0° corresponds to the separated transceiver configuration, equivalent to the experimental set up of figure 8.17(a) while in Figure 8.22(c) the transceiver separation angle is 90° , corresponding to the “coaxial”¹⁰ type configuration equivalent to the set up in Figure 8.17(b). Figure 8.22(b) shows the general case with transceiver separation angle θ . As the transceiver separation angle increases, the minimum value of the amplitude profiles increases which means potential elimination of the measurement error at a certain angle. If the transceiver separation angle reaches 90° (as in figure 8.17(b)), the transceiver configuration effectively becomes “coaxial” and the minimum in the amplitude profile disappears (as shown in Figure 8.20(a), the blue solid profile). This is shown in Figure 8.21, which is the experimental result by using the setup in figure 8.17(b). In Figure 8.21, it can be seen that the minimum in amplitude no longer exists as it was recorded with $\theta = 90^\circ$ (Figure 8.22(c)). Note that the amplitude curve in this real case resembles the simulated one in Figure 8.20(a) (the blue solid profile, when $\theta = 90^\circ$).

The above analysis was carried out with the 3D LADAR scanner in Figure 8.11(b). For the 3D LADAR in figure 8.11(c), the rotating mirror rotates in unison with the transceiver and their relative position is constant. That means, for this

¹⁰ “Coaxial” only in the sense of scanning past *vertical* edges.

Figure 8.21: *Range and amplitude when the LADAR was translated past the same vertical edge in Figure 8.17(a) in a “coaxial” manner.*

Figure 8.22: *Top view of the scanning mirror in figure 8.11(b). ‘T,R’ denote the LADAR’s transmitter and receiver and the red arrows denotes the direction of transmitted laser beam. (a) $\theta = 0^\circ$, (c) $\theta = 90^\circ$ and (b) general case.*

kind of LADAR scanner, the transceiver is always “separated” (as the configuration in Figure 8.22(a)) no matter what the LADAR scanning mirror’s real bearing angle is. When this LADAR scans past a vertical edge as in Figure 8.17, the estimated profile will always have the form of the red dotted curve in Figure 8.20. As shown in figure 8.23, for this 3D LADAR, since the transceiver separation angle θ is effectively always 0° , it corresponds to the separated transmitter/receiver configuration in the 1D case.

Figure 8.23: *The scanning system of the Riegl LADAR LMS-Z210i (Figure 8.11 (c)) is designed so as to maintain full separation between the transmitter and receiver with respect to vertical edges, at all mirror bearing angles.*

8.3.5 Causes of range errors—applying the “occlusion” model

The theoretical model derived in the previous section (Equations 8.6, 8.10, 8.14, 8.18, and 8.19) can be used to estimate the received signal amplitude profile and hence predict and detect the range errors caused by this occlusion effect. However, it is more important for LADAR users or designers to know how to predict the range errors in real applications or possibly how to derive the necessary LADAR parameters in advance so as to avoid or minimize the chances of such errors. To avoid range errors the minimum amplitude in the profile must be increased to a

value above the minimum detectable amplitude of the sensor itself. A solution to this problem is, of course, by decreasing the minimum detectable amplitude of the sensor¹¹. In this section, the theoretical model will be analyzed to determine the effect of different LADAR design parameters and environmental parameters which can cause the received amplitude to fall below a predefined minimum value.

For LADARs of the type shown in Figure 8.11(b) or equivalently Figure 8.22, the fundamental parameter is the transceiver separation angle θ of the LADAR. For the 3D LADAR in Figure 8.11(c) or equivalently figure 8.23, it is not affected by this parameter at all since the transceiver is always “separated” with respect to a vertical edge. In Figure 8.20, by rotating the scanning mirror of the LADAR from 0° to 90° equivalent to turning the mirror from the case in Figure 8.22(a) to 8.22(c), the minimum amplitude keeps increasing.

Figure 8.24: Signal amplitude versus the sliding distance x and the transceiver separation angle θ . The other parameters used were $d1 = 5.8\text{m}$, $d2 = 12.6\text{m}$, $P1 = 115$, $P2 = 50$, $s - q = 2.9\text{cm}$ and $\alpha = 3\text{mrad}$. The minimum detectable amplitude of the LADAR is plotted as the plane with *amplitude* = 18.

In order to see what the amplitude profile will be at each transceiver separation angle, the estimated signal amplitude is shown versus the sliding distance (x in mm)

¹¹This is, of course, already an optimization criterion for most LADAR manufacturers.

and the transceiver separation angle (in degrees) in Figure 8.24, which corresponds to different transceiver configurations in figure 8.22 (the whole estimation is equivalent to setting the 1D sensor at a transceiver separation angle θ and allowing it to repeat the scan in Figure 8.17). The minimum detectable amplitude of this Riegl LADAR is assumed to be 18 (which is found by experiment), and as the transceiver separation angle θ reaches $(n + 1/2)\pi \text{ rads}$ (for integral values of n), there will be no minimum in the amplitude as the transceiver separation approaches the case of Figure 8.22(c) and then no range error will occur. In Figure 8.25, the minimum amplitude in each amplitude profile corresponding to a transceiver separation angle in Figure 8.24 is plotted versus the angle. Compared to the minimum detectable amplitude (the dotted line with amplitude=18), it can be seen that, the minimum will be larger than 18 and no range error will occur if $-160^\circ \leq \theta \leq -20^\circ$ and $20^\circ \leq \theta \leq 160^\circ$. Since the minimum detectable amplitude of each LADAR is different, the particular transceiver separation angular range in which the range error will disappear depends on this value for the given device. For LADARs which maintain a constant separation between the transmitter and receiver (figure 8.11(c)), the range errors can always occur.

The second parameter to be analyzed is a sensor parameter, the transceiver separation, $s - q$ in equation 8.7 with s being the transmitter – receiver aperture separation and q the effective radius of the receiver lens. As expected (see figure 8.26), by decreasing $s - q$, the minimum amplitude value is also increased, thus decreasing the chances of false range values. During LADAR design, this parameter

Figure 8.25: Minimum amplitude versus the transceiver separation angle θ . The minimum detectable amplitude of the LADAR is plotted as the line with amplitude = 18.

Figure 8.26: Estimated signal amplitude versus the sliding distance x and the transceiver separation $s - q$. The minimum detectable amplitude of the LADAR is plotted as the plane with amplitude = 18.

should be kept as small as possible while keeping the transmitter and receiver still separated to guarantee that all of the transmitted light leaves the LADAR.

Further parameters which affect the minimum value of the received amplitude are environmental ones, namely the distances and the reflectivities of the targets which form the scanned edge. In Figure 8.27, it can be seen that if the distance of the background target is decreased so that the two targets are closer to each other, the minimum amplitude value is also be increased. This makes sense since if the two targets are put at the same distance, the front target cannot occlude the reflected light from the background target.

Figure 8.27: Estimated signal amplitude versus the sliding distance x and the distance of the background target d_2 . The minimum detectable amplitude of the LADAR is plotted as the plane with amplitude = 18.

In Figure 8.28, the received power P_2 from the background target is increased (corresponding to a higher reflectivity of that target), the minimum amplitude value is then also increased. *It is interesting to note, that the minimum in the amplitude profile is hardly affected by the power which is received by any one of the surfaces either side of the edge, when fully illuminated, i.e. the reflectivity of each individual surface.* For almost all combinations of edge separation $d_2 - d_1$, increasing the reflectivity of any one of the surfaces does not significantly increase the minimum in amplitude. This probably reveals the truth that the dependency of the minimum amplitude is much more on the sensor parameters, such as the transceiver separation angle θ and transceiver separation $s - q$, than the environmental parameters. When the transceiver separation angle θ is already set as zero and $s - q$ is already set at a certain value, the minimum of the received amplitude is hardly increased by the change of the reflectivity of the targets.

Figure 8.28: *Simulated signal amplitude versus the sliding distance x and the amplitude of the signal from the background target. The minimum detectable amplitude of the LADAR is plotted as the plane with amplitude = 18.*

8.3.6 Experimental results

Finally, some experimental results are presented to show the effect of these different parameters on the minimum amplitude and the occurrence of range errors. It will also be demonstrated how range errors can be avoided by selecting various parameters. Firstly, a semi-structured, outdoors environment (a car park area within the NTU campus) is used to show how to predict and detect the range errors due to this occlusion effect in LADAR scans by using the model derived in Section 8.3.4. Figures 8.29 and 8.30 show received amplitude and range images respectively recorded from the 3D LADAR scanner in figure 8.11(b) in this environment.

Figure 8.29: *An intensity image of an semi-outdoors environment from a 3D LADAR. 'A' in the figure denotes a container.*

It can be seen from the right side of Figure 8.29 that there was a black container

Figure 8.30: *The corresponding range image of the scan in Figure 8.29. Darker pixels are closer to the sensor.*

(positioned at ‘A’) in front of a white wall. The container and the wall form an edge where the occlusion effect could possibly occur in a LADAR scan. The distances of the container and the wall to the LADAR’s position were recorded and so were the received signal amplitudes. With this information, as shown in Figure 8.31, an amplitude profile (the red amplitude profile denoted as “Estimated amplitude” in the legend) similar to those in Figure 8.20 by using the theoretical model was produced. It can be seen that the minimum expected amplitude value in this case is lower than the LADAR’s minimum detectable amplitude. Hence, in this case, range errors are expected at the edge. Compared with the actual amplitude data (the blue dotted profile) in Figure 8.31, it can be seen that the shape and minimum of the amplitude in both actual and estimated profiles are almost the same except that the width of the change in the estimated amplitude is larger than that in the actual one. The probable reason for this is that the effective laser beamwidth is smaller than the theoretical one used in the estimation since its energy is more concentrated at the center of the beam, as mentioned earlier.

Figure 8.32 shows the plan view of the range data from the middle row (elevation angle 40°) of figure 8.29. According to Figure 8.25, false range values may occur at certain transceiver separation angle ranges, which are $-180^\circ < \theta < -160^\circ$,

Figure 8.31: *Estimated amplitude profile from the middle row (elevation angle is 40°) of the scan in figure 8.29 compared with the actual amplitude profile.*

Figure 8.32: *Plan view of the range data from the middle row (elevation angle is 40°) of Figure 8.29. The small blue triangle denotes the position of the LADAR. The red lines are range data. Capital letters A to J denotes objects in the environment, in which A denotes a person, B, H, I are pillars, C is a container, D is a background wall, F denotes bicycles, E, G represents a corridor and J is another wall. The blue lines marked -20° , 0° etc are transceiver separation angle bounds defining areas where occlusion may occur (See figure 8.25).*

$-20^\circ < \theta < 20^\circ$ and $160^\circ < \theta < 180^\circ$ etc and these regions are marked on figure 8.32. The edge formed by the container and the wall falls in the transceiver separation angle range $-20^\circ < \theta < 20^\circ$ and then false range values are expected. Figure 8.33 shows the signal amplitude and the range data from the same row versus the LADAR mirror's transceiver separation angle to depict the false range values for the edge $C - D$ in figure 8.32. It can be seen that at the edge, a minimum in amplitude

Figure 8.33: *Range data and the signal amplitude data versus bearing from the same row (elevation angle 40°) of Figure 8.32.*

occurs and its value is below 18, the minimum detectable amplitude of the LADAR, producing false range estimates as expected. At this point, the range value should be replaced with its prediction¹² if the range data is to be used in feature detection methods [13].

¹²If this is available as is the case in certain edge detection filtering algorithms such as [13].

8.4 Conclusions

In this chapter, firstly a 3D range data representation based on the previously presented multi-aided inertial navigation system has been introduced. It is shown that by correctly registering the LADAR data into the navigation frame, taking into account of the “Roll” and “Pitch” angles and the “Down” position, the 3D range data representation can be achieved even in uneven outdoor environments. Experimental results have shown that outdoor environments, such as buildings, bridges, bus stops and trees can be reconstructed with acceptable realism. Without the “Roll” and “Pitch” angles and the “Down” position, even if the horizontal localization accuracy can be improved by adopting other methods, the data representation cannot be obtained properly for uneven terrains.

The applications of LADARs in mobile robotics have increased rapidly. Before LADARs are used in applications such as, map building, feature extraction, or data association, it is necessary to evaluate the effects of the systematic and non-systematic errors on the LADAR measurements. It has been demonstrated that commonly used LADARs suffer from incorrect range readings at changes in surface reflectivity and/or range discontinuities, which have a much more detrimental effect on such algorithms than random noise. It has been shown analytically and experimentally that partial occlusion of the received laser light can occur in these LADARs when scanning edges (discontinuities) between neighboring targets at different ranges. A minimum in the received signal power will occur, which can be below the minimum operating signal power necessary for correct operation

and hence cause erroneous range readings. For applications such as map building, feature detection and data association, this effect will introduce unwanted measurement errors. However, by adopting the proposed received signal power model, such erroneous range values can be predicted, detected and removed before further processing.

Also, from this model, it can be seen that the minimum received signal power depends on sensor parameters such as the transmitter-receiver aperture separation and the transceiver separation angle, as well as environmental parameters such as the reflectivity and distance/separation of the targets. This dependency information is important for LADAR users as well as LADAR designers and allows them to avoid or, at least predict false range data caused by occlusion. This chapter has shown that the chances of occlusion errors can be minimized by minimizing the separation of the LADAR's transmitter and receiver at the design stage. Further, it has been shown that a large separation of the targets being sensed is much more likely to cause false estimation than the reflective qualities of the targets themselves. Hence the derived models can be used to guarantee that all range values at edges will be sensed correctly within certain target separation bounds.

Chapter 9

Conclusions

The conclusions, contributions and some recommendations for future work of the thesis will be presented in this chapter.

Conclusions and Contributions

The conclusions and contributions of each chapter are reviewed individually.

- Chapter 2 introduced the principles of inertial navigation systems and presented the development of inertial equations required for low-cost inertial navigation systems for ground vehicles. Error equations have been derived, including attitude, velocity and position error equations. Using these equations, the attitude, velocity and position errors can be determined for low-cost IMUs. With assumptions such as, the Coriolis terms and their associated errors are safely ignored for low-cost INS, a Pinson error model has been formulated as the error propagation equations for low-cost IMUs. This especially developed error model for low-cost IMUs has been utilized extensively in the thesis.

This chapter also provided an error analysis for IMUs in a quantitative sense.

This error analysis is useful for the inertial sensor users to determine the dominant error sources of the IMUs. It was shown via a case study that, in practical applications, mainly biases and to a lesser extent, noise, are the main error sources for low-cost INS. Positional errors due to accelerometer biases grow with time squared, while the same errors caused by gyro biases grow with the cube of time. Hence calibration and alignment are necessary for correct inertial navigation.

Chapter 2 presented an initial calibration and alignment algorithm for the stationary states of the hosting vehicle which carries the IMU, especially in the vehicle's starting period. With the tilt sensors, biases in the accelerometers are estimated and calibrated. Together with an external compass, alignment is also made possible. Experimental results have been shown that the free INS runs for a period of approximately 50 seconds accurately, compared with the results from an INS/GPS data fusion algorithm as ground truth.

- Chapter 3 described a novel noise estimation and reduction algorithm for IMUs. This minimum controlled recursive averaging (MCRA) method recursively averages the noise power spectrum under the control of the minima values of a smoothed periodogram of the noisy IMU data. With the smoothing parameters in time and frequency adapted according to the signal presence probability, MCRA has the ability to estimate noise accurately. Simulation and experimental results have been presented to prove the effectiveness of this noise estimation method. Simulation results have been given to show that the

noise level of the raw gyroscopic data can be reduced by 20 to 25 percent and consequently the random walk effect has been reduced. Experimental results reveal that after this noise estimation and reduction technique was applied, the raw IMU data with less noise has been obtained, as shown in both time and frequency domains.

The main contribution of this chapter is the introduction and application of this MCRA noise estimation algorithm on noisy inertial signals to improve the performance of INS. The introduction of this MCRA algorithm addresses the problem that the commonly used noise reduction methods, such as the low pass filters and the matched filters are not able to filter the noise of the inertial signals.

- Chapters 4 and 5 detailed the development of the multi-aided inertial navigation system and the simulation results of this INS. The simulation has been carried out to make up for the lack of ground truth when evaluating the performance of the proposed INS algorithm.

Under consideration of the fragile nature of GPS data (i.e. it is easily jammed and subject to the multi-path errors and the problem of limited view of sufficient number of satellites), a multi-aiding method has been developed to bound the accumulated errors in the estimated INS states, without the help of GPS, which is commonly used to aid IMUs. The idea of using the vehicle's constraints as a way of aiding INS has been adopted here as a novel lateral velocity estimator. Together with wheel encoders, the vehicle constraints have

been used as “virtual sensors” to measure direct velocity information in the body frame. The multi-aiding concept also includes a single-axis gyroscope, which has a higher accuracy than the IMU’s yaw axis gyro, to generate heading observations for the INS. A standard Kalman filter has been adopted as the state estimator of this multi-aided INS.

In Chapter 5, raw INS signals have been simulated from theoretically given vehicle trajectories. These simulated raw signals have been artificially corrupted with noise and then used to test the multi-aided INS algorithm developed in Chapter 4. With this simulated data, the ground truth of the testing INS algorithm has been obtained and the performance of this aided INS has been evaluated. This multi-aided INS algorithm has also been tested with real data and the real experimental results have been shown in chapter 7.

The main contributions of these chapters are:

1. The development of an inertial navigation system aided by multi-aiding sources.
 2. The formulation of a novel lateral velocity estimator as an aiding source for the INS.
 3. The simulation of the raw INS signals and the performance examination of the multi-aided INS algorithm with simulated ground truth.
- Chapter 6 described an approach to quantify the problem of cross correlation between the prediction and observation noise of the multi-aided INS. Cross

correlation has been shown to be introduced by the use of the transformation matrix $\mathbf{C}_b^n$. The effect of the cross-correlation term on the error covariance matrix has been evaluated theoretically. Based on the assumption that the prediction and observation noise is correlated only one time step apart, this cross correlation problem becomes mathematically tractable. With the cross-correlation term being formulated from the prediction and observation noise, it has been incorporated into the normal Kalman filter and the relevant filter equations have been updated accordingly.

Because the symbolic solution to the updated error covariance matrix is too complicated, a simulation has been produced to evaluate the effect of the cross-correlation term on the error covariance matrix and further on the performance of the filter. The simulated results have shown that the positional variance elements in the error covariance matrix have been increased since the cross-correlation term was added to the estimated process. This means that the error bounds of the positional estimates should be increased due to the consideration of the additional cross-correlation term in the filter, which should postpone the divergence of the filter. If this term is ignored, the error covariance estimates associated with the positional estimates would be too small and the filter would be “over confident”.

The main contribution of this chapter is the introduction of the cross-correlation term into the estimation filter for the multi-aided inertial navigation system, which utilizes the transformation matrix to transform the body frame observa-

tions. The theoretical formulation and numerical simulations have shown the importance of incorporating this term into the filter and navigation system.

- Chapter 7 has presented experimental results on the implementation of the multi-aided inertial navigation system developed in Chapter 4. It has been shown that the multi-aided navigation system is able to provide reliable and accurate estimates for position, velocity and attitude of the navigation system in three dimensions, for extended periods of time, when compared to the free INS algorithm, in the absence of GPS. The concept of utilizing the vehicle constraints as a “virtual sensor” as part of the aiding system has proved successful throughout these experiments. This multi-aided INS has proved to be an effective alternative algorithm to INS/GPS fusion methods. With the application of the aided INS algorithm, low-cost IMUs have been shown to be a possibly, very versatile dead reckoning sensor for ground vehicles.
- Chapter 8 presented a 3D data representation approach using laser range finders and the modeling of occlusion as a serious systematic error for commonly used laser sensors.

It has been shown that by correctly registering the laser sensor data into the navigation frame, taking into account of the “Roll” and “Pitch” angles and the “Down” position, which are from the multi-aided inertial navigation system, 3D data representation, can be correctly achieved even in uneven outdoor environments. Experimental results have shown that, data recorded in outdoor environments from undulating roads, containing buildings, bridges,

bus stops and trees can be reconstructed with acceptable accuracy.

It has been shown that before the laser sensors or LADARs are employed in the applications such as, data representation, feature extraction, or data association, it is necessary to evaluate the effects of the systematic and non-systematic errors on the LADAR measurements. It has also been demonstrated that commonly used laser sensors suffer from incorrect range readings at changes in surface reflectivity and/or range discontinuities, which has a much more detrimental effect on such algorithms than random noise. It has been shown analytically and experimentally that partial occlusion of the received laser light can occur in these LADARs when scanning edges (discontinuities) between neighboring targets at different ranges. For applications such as data representation, feature detection and data association, this effect will introduce unwanted measurement errors. However, by adopting the proposed received signal power model, such erroneous range values can be predicted, detected and removed before further processing.

The main contributions of this chapter are:

1. A 3D data representation using laser range finders has been implemented on the basis of the localization information from the multi-aided INS.
2. Systematic errors caused by occlusion of LADARs have been modeled and a detection process has been formulated to detect such errors.

Recommendations for Future Work

When using low-cost IMUs for navigation purposes for ground vehicles, the issues that are of most concern relate to increasing the accuracy, which can be an interesting direction to pursue. To further increase the accuracy, the development of more accurate aiding methods for the inertial sensors and/or a better understanding of the errors of inertial signals is necessary. A full observability analysis for the aided inertial navigation system would be useful to decide which system states are unobservable and subsequently which extra aiding information should be added accordingly. Thereby the accuracy of the entire navigation system can be increased.

Another interesting research topic of inertial navigation systems for ground vehicles would be the incorporation of inertial sensors into the well known Simultaneous Localization and Mapping (SLAM) problem in mobile robotics. The SLAM problem has been extensively studied in both indoor and outdoor ground vehicle applications in the past decade. Almost all these applications use an assumption that the vehicle runs in a 2D plane, which is a serious limitation for many bumpy outdoor environments. SLAM can then be formulated in a 3D framework. In this case, inertial sensors are required to produce predictions of the full 3D system states, as well as 3D feature pose estimates. Inertial sensor based SLAM would enhance the applications of ground vehicles in many outdoor environments.

Author's Publications

Journal Paper

1. Bingbing Liu, Martin Adams, W.S. Wijesoma and Javier Ibanez-Guzman, "Range Error Detection Caused by Occlusion in Non-Coaxial LADARs for Scene Interpretation," *Journal of Robotic System*, Vol. 22, No. 10, Oct. 2005.

Conference Papers

1. Bingbing Liu, Martin Adams and Javier Ibanez-Guzman, "Minima Controlled Recursive Averaging Noise Reduction for Multi-aided Inertial Navigation of Ground Vehicles," Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS), Alberta, Canada, 2-6 Aug. 2005, page(s):3408 - 3414.
2. Bingbing Liu, Martin Adams and Javier Ibanez-Guzman, "Multi-aided Inertial Navigation for Ground Vehicles in Outdoor Uneven Environments," Proceedings of the IEEE International Conference on Robotics and Automation (ICRA), Barcelona, Spain, 18-22 April, 2005, page(s):4703 - 4708.

3. Bingbing Liu, Martin Adams, Javier Ibanez-Guzman and W.S. Wijesoma,
“Detection of range errors due to occlusion in separated transceiver LADARs,”
Proceedings of the International Conference on Control, Automation, Robotics
and Vision (ICARCV), Kunming, China, 6-9 Dec, 2004, Volume: 1, page(s):
443-448.
4. Bingbing Liu, Martin Adams, Javier Ibanez-Guzman and W.S. Wijesoma,
“Range Errors due to Occlusion in Non-Coaxial LADARs,” Proceedings of
the IEEE/RSJ International Conference on Intelligent Robots and Systems
(IROS), Sendai, Japan, Sep 28 - Oct 2, 2004, Volume.3, page(s): 2173 - 2178.
5. Bingbing Liu, Martin Adams, Javier Ibanez-Guzman and W.S. Wijesoma,
“False Range Data due to Occlusion in Separated Transmitter/Receiver LADARs,”
The Second International Conference on Computational Intelligence, Robotics
and Autonomous Systems (CIRAS), 15-18 Dec 2003, Singapore.

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