Analysis and modeling of laser diodes for WDM system

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Analysis and Modeling of Laser Diodes for WDM System

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School of Electrical & Electronics Engineering

A thesis submitted to Nanyang Technological University
in fulfillment of the requirement for the degree of
Doctor of Philosophy

2005

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Statement of Originality

I hereby certify that the content of this thesis is the results of work done by me and has not been submitted for a higher degree to any other university or institution.

Aug 18, 2005
Date

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Table of Contents

Acknowledgments..... I

Table of Contents..... III

Summary VII

List of TablesXI

List of Figures XII

Chapter 1 Introduction..... 1

1.1 Motivation.....5

1.2 Objectives7

1.3 Major contribution of the thesis 8

1.4 Organization of the thesis 10

Chapter 2 Design and Modeling of Multiple Phase Shift Distributed Feedback Laser diodes 12

2.1 Introduction..... 12

2.2 Developed transmission line laser model (TLLM)..... 14

2.2.1 Scattering matrix 16

2.2.2 Connecting matrix..... 17

2.2.3 Phase shift model 18

2.2.4 The carrier rate equation20

2.3 Analysis of multiple phase shift DFB laser diodes21

2.3.1 Dynamic characteristics23

2.3.2 Longitudinal Distribution.....	25
2.3.3 Effects of Phase Shift Position and Value on the 3PS DFB LDs Characteristics.....	29
2.3.4 Optimum designs of the MPS DFB LDs based on flatness and side mode suppression ratio	32
2.4 Summary	35
Chapter 3 Polarization Investigation of Vertical Cavity Surface Emitting Lasers.....	37
3.1 Introduction of spin-flip model (SFM)	37
3.2 Polarization stability analysis of VCSEL	42
3.3 DBR effects on the polarization switching	47
3.3.1 Improve transmission line laser model by incorporating into spin-flip model (SFM).....	47
3.3.2 Scattering process in the gain section	49
3.3.3 Connecting process	49
3.3.4 Carrier rate equations	50
3.3.5 Simulation results.....	51
3.4 Summary	52
Chapter 4 Design of Two-Dimension VCSEL-Based Wavelength Converter	53
4.1 Introduction of the wavelength converters	53
4.2 Design of the two dimension VCSEL wavelength converters.....	54
4.3 Developed two-dimension transmission line laser model	56
4.4 Analysis of the proposed structure.....	60
4.4.1 Extinction ratio and peak chirp character studies	60

4.4.2 Wavelength optimization	65
4.4.3 Reflectivity optimization	68
4.5 Summary	72
Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL	74
5.1 Introduction of anti-resonant reflecting optical waveguide VCSEL	74
5.2 Scalar transfer matrix method used to analyze ARROW VCSEL.....	75
5.2.1 Scalar transfer matrix method.....	75
5.2.2 Transversal mode of ARROW VCSEL using scalar transfer matrix method.....	78
5.3 Vector transfer matrix method used to analyze ARROW VCSEL	83
5.3.1 Vector transfer matrix method	83
5.3.2 Modal loss property anslysis by using vector transfer matrix method ..	87
5.3 Conclusion	92
Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL...94	
6.1 Introduction.....	94
6.2 Design of anti-resonant reflecting PC-VCSEL.....	96
6.3 Performance analysis of ARPC VCSELs by multipole method.....	98
6.4 Conclusion	106
Chapter 7 Conclusions and Recommendations.....	108
7.1 Conclusions.....	108
7.2 Recommendations for further research.....	112
Appendix A.....	115
A.1 Formulation of the field Identites.....	115

A.2 Boundary conditions and field coupling 122

A.3 Generalized Rayleigh identity 124

A.4 Solutions of the field coefficients 125

Bibliography 132

Author’s Publications 142

Summary

Semiconductor lasers are key devices for wide range of applications (e.g. fiber-optical communications, optical disk memory and laser printer) due to various attractive features such as small size, low power consumption, fast response etc. However, design and fabrication of laser diodes involve various complicated procedures. The process is expensive and time consuming. Efficient and accurate simulation tools are in great demand for the laser design and optimization. Moreover, the design considerations vary from each other greatly for different types of laser diodes.

In this thesis, two types of laser diodes, Le., Distributed Feedback Laser Diodes (DFB LDs) and Vertical Cavity Surface Emitting Lasers (VCSELs) are studied and analyzed. The development and the challenges of DFB LDs and VCSELs are introduced in Chapter 1. In Chapter 2 we discuss the characteristics of the multiple phase shift distributed feedback laser diodes (MPS DFB LDs). In chapters 3 to 6, the design of VCSEL and VCSEL-based devices and the analysis of their characteristics are presented.

Distributed feedback laser diodes (DFB LDs) play an important role in the long-haul and high-bit-rate optical-fiber transmission systems. However, the single-mode operation is difficult to be achieved with the uniform DFB LDs or the quarter-wave-shift DFB LDs. Multiple phase shift DFB LD is a promising device to provide single-mode operation with a high coupling coefficient. In the design of multiple-phase-shift DFB LDs, two important criteria were considered, Le., the requirement of a high longitudinal side-mode suppression ratio and the necessary

flatness of the longitudinal field distribution. A transmission-line laser model has been developed to model the phase shift effects. We have successfully optimized the design of the phase shift location and parameter values by using our transmission-line laser model.

The optical cavities of the VCSELs are orthogonal to those of the conventional edge-emitting lasers. The arrangement in the orientation of cavity significantly improves the output performance and the flexibility of fabrication. In Chapter 3, the intrinsic optical anisotropic VCSEL is investigated. A four-level spin-flip model (SFM) which takes into account the phase information of the two orthogonal polarizations is described and used to analyze the stability of the VCSEL. The SFM model is further developed to take into account the effects of DBR (Distributed Bragg Reflector) mirrors in the VCSEL. The effects of the reflectivity of Bragg reflectors on the bistable characteristics of the VCSEL are discussed.

VCSELs can be designed and used as amplifiers. In Chapter 4, we have proposed a two-dimensional VCSEL-based wavelength converter, where VCSEL works as an amplifier as well as a laser source. In the configuration of the converter, the active region of VCSEL is extended in the horizontal direction. The signal wave is incident at the horizontal direction and the output is in the vertical direction, thereby a two-dimensional wavelength converter is realized. The extended active region is used to amplify the input signal wave. With this, a strong cross-gain modulation will be guaranteed and high extinction ratio of the probe wave can be achieved. A two-dimensional transmission-line laser model is developed for the

analysis of the proposed two-dimensional VCSEL and the performance of the proposed wavelength converter is also discussed.

For the design of high-power VCSELs, large aperture size is usually required. However, higher-order transverse modes will also be supported in such VCSELs. To obtain single-mode operation while maintaining a large cavity size, a new class of VCSELs, namely the anti-resonant reflecting-optical-waveguide (ARROW) VCSELs is designed. An ARROW VCSEL is characterized by a core with a low refractive index surrounded by alternating high-index and low-index cladding layers. In Chapter 5, both scalar transfer matrix method and vector transfer matrix methods (TMMs) are introduced and applied to investigate the characteristics of ARROW VCSELs. Optimized designs of the structure dimension are carried out by applying the analytical methods.

In chapter 6, anti-resonant reflecting photonic crystal (ARPC) structure is introduced in VCSELs to achieve single-mode operation with large aperture. Multipole method is derived to analyze the transverse mode performance of the proposed APRC-VCSELs. The modal fields are calculated and analyzed by using the above developed method. Furthermore, the modal loss properties with the changes of the wavelength are analyzed. And the optimization designs of the photonic crystal structure are discussed.

In conclusion, we have developed comprehensive laser models to investigate the characteristics of MPS DFB LDs as well as VCSELs. Our achievements include:

(1) stable single longitudinal mode operation is obtained by the optimum designs

of MPS DFB LDs; (2) the effects of the DBR on the polarization stability of VCSELs have been evaluated; (3) two-dimension VCSEL based wavelength converters are proposed and demonstrated for a wide converting range; (4) the modal performance of ARROW VCSELs has been analyzed and the structural parameter of ARROW VCSEL is optimized; (5) ARPC-VCSEL devices are proposed and investigated. Large modal discrimination is achieved with such a cavity design.

List of Tables

Table 2.1 Parameters for QWS DFB LDs and 3PS DFB LDs..... 22

Table 4.1 Parameters for VCSELs..... 62

Table 5.1 Effective modal index calculated by vector and scalar methods 91

Table 6.1 Mode effective refractive index for the first four modes..... 99

List of Figures

Fig. 1.1	The schematic diagram of distributed feedback laser diodes.	2
Fig. 1.2	The schematic diagram of vertical-cavity surface emitting lasers.....	4
Fig. 2.1	Schematic diagram of 3PS DFB LD	21
Fig. 2.2	Transient response of QWS DFB LD using developed TLLM MPS model and Lowery's model in reference [22].....	23
Fig. 2.3	Transient response of 3PS DFB LD by using developed TLLM MPS Model and Lowery's phase stub model in reference [24].....	24
Fig. 2.4	Transient longitudinal carrier density of QWS DFB LD.....	24
Fig. 2.5	Transient longitudinal carrier density of 3PS DFB LD.	25
Fig. 2.6	Longitudinal carrier density distribution of QWS DFB LD calculated by different models.	25
Fig. 2.7	Longitudinal intensity distribution of QWS DFB LD, uniform DFB and 3PS DFB LD.....	27
Fig. 2.8	Longitudinal photon density distribution of QWS LD (right) 3PS DFB LD (left) under different injection currents.	28
Fig. 2.9	Longitudinal carrier density distribution of QWS DFB LD (left) and 3PS DFB LD (right) under different injection currents.	28
Fig. 2.10	Longitudinal refractive index distribution of QWS DFB LD (left) 3PS DFB LD (right) under different injection currents.	28
Fig. 2.11	Normalized intensity distribution of QWS DFB LD (left) amd 3PS DFB LD (right) under different injection currents.	29
Fig. 2.12	Variation in flatness versus ψ for different values of coupling coefficient κ	30
Fig. 2.13	Variation in flatness versus ψ for different phase shift.	30

Fig. 2.14	Flatness contour map with the phase shifts of 3PS DFB.	31
Fig. 2.15	Spectrum of 3PS DFB.....	32
Fig. 2.16	Spectrum of uniform DFB.	33
Fig. 2.17	SMSR contour map with the phase shifts of 3PS DFB.	34
Fig. 2.18	Variations in the SMSR and flatness with respect to the phase shifts. The shaded area covers all phase combinations that satisfy both $SMSR > 50$ and $F < 0.05$ selection criteria for stable mode operation.....	35
Fig. 3.1	Four-level model for polarization dynamics in quantum well vertical cavity surface emitting lasers.....	40
Fig. 3.2	Stability diagram for $\gamma_a / \gamma = -0.2$, $\hat{x}$ – polarized emission favored under threshold.....	44
Fig. 3.3	Stability diagram for $\gamma_a / \gamma = 0.2$, $\hat{y}$ – polarized emission favored under threshold current.	45
Fig. 3.4	Polarization state switch from $\hat{x}$ – polarized emission to $\hat{y}$ – polarized emission when $\gamma_a / \gamma = -0.2$	46
Fig. 3.5	Polarization state switch from $\hat{y}$ – polarized emission to $\hat{x}$ – polarized emission when $\gamma_a / \gamma = 0.2$	47
Fig. 3.6	The steady state characteristics of VCSEL, X – and Y – polarized light outputs versus Y – polarized light input. The numbers of layers of Bragg reflectors are $A=48$, $B=53$ and $C=58$	51
Fig. 4.1	Co-propagating and counter-propagating wavelength converters.	54
Fig. 4.2	Schematic of two-dimension wavelength converter.	55
Fig. 4.3	VCSEL based two-dimension wavelength converter.	55
Fig. 4.4	Schematic of the two-dimension TLLM.....	57

Fig. 4.5	Pulse pattern of signal wave (upper), probe wave (middle) and frequency chirp (lower) at 2.5Gbps($R_1=97.4\%$, $R_2=55\%$, $\lambda_{signal} = 1565nm$ and $\lambda_{probe} = 1525nm$, the ER of input signal=10dB, Input power=25mW).	61
Fig. 4.6	The evolution of peak chirp frequency and extinction ratio in the transverse direction of the probe wave at different facet reflectivity ($R_1= 97.4\%$).63	
Fig. 4.7	Negative peak chirp frequency and extinction ratio of the probe wave as a function of input signal power at with different wavelength locations ($R_1=97.4\%$, $R_2=55\%$, $\lambda_{signal} = 1565nm$).	64
Fig. 4.8	Contour plot of the extinction ratio with respect to signal wavelength and probe wavelength ($R_1=97.4\%$, $R_2=55\%$, the ER of input signal=10dB, Input power=25mW).	65
Fig. 4.9	Contour plot of the positive peak chirp with respect to signal wavelength and probe wavelength.....	67
Fig. 4.10	Optimization of the signal wavelength and the probe wavelength (shaded region) with respect to both of the extinction ratio and peak chirp frequency.....	67
Fig. 4.11	Extinction ratio of the probe wave as a function of facet reflectivity for different input powers. ($R_1= 97.4\%$).	68
Fig. 4.12	Negative frequency peak chirp of the probe wave as a function of facet reflectivity for different input powers. ($R_1= 97.4\%$).	69
Fig. 4.13	Extinction ratio of the probe wave as a function of facet reflectivity in different locations in transverse directions. (input power 25mW).	72
Fig. 5.1	Schematic of ARROW VCSELs and the refractive index profile.....	76

Fig. 5.2	Transverse field amplitude of LP_{01} mode along the radial direction, where the solid line represents the real part of the field and the dashed line is the imaginary part of the field.....	79
Fig. 5.3	Field intensity distribution of LP_{01} mode.....	80
Fig. 5.4	Transverse field amplitude of LP_{11} mode along the radial direction, where the solid line represents the real part of the field and the dashed line is the imaginary part of the field.....	80
Fig. 5.5	Field intensity distribution of LP_{11} mode.....	81
Fig. 5.6	Effective modal index n_{eff} of LP_{01} mode with the variation of parameter d_2 . (a) Real part; (b) Imaginary part.	82
Fig. 5.7	Effective modal index of LP_{11} mode with the variation of parameter d_2 . (a) Real part; (b) Imaginary part.	82
Fig. 5.8	Modal losses scan with the changes of the parameter d_2	83
Fig. 5.9	E_z field of HE_{11} mode along the radial direction, where the solid line represents the real part of the electrical field and the dashed line is the imaginary part of the electrical field.	88
Fig. 5.10	Transverse electric field of HE_{11} mode.....	88
Fig. 5.11	H_z field of TE_{01} mode along the radial direction, where the solid line represents the real part of the magnetic field and the dashed line is the imaginary part of the magnetic field.	89
Fig. 5.12	Transverse electric field of TE_{01} mode.	89
Fig. 5.13	E_z field of TM_{01} mode along the radial direction, where the solid line represents the real part of the electrical field and the dashed line is the imaginary part of the electrical field.	90

Fig. 5.14	Transverse electric field of TM_{01} mode.....	90
Fig.5.15	Modal loss figures of HE_{11} mode by vector method and LP_{01} mode by scalar method with the variation of parameter d_2	91
Fig.5.16	Modal loss figures of TE_{01} and TM_{01} mode by vector method and LP_{11} mode by Scalar method with the variation of parameter d_2	92
Fig. 6.1	(a) Schematic of anti-resonant reflecting photonic crystal VCSEL. (b) Waveguide incorporated in ARRP-VCSEL after Hardley model. (c) Cross section of the ARRP-VCSEL, the black regions represent high index cylinders.	97
Fig. 6.2	Determinant scans of the structure with just one ring at the wavelength $\lambda = 0.98\mu m$	100
Fig. 6.3	Transverse electric field of HE_{11}^y mode.....	100
Fig. 6.4	Transverse field of TE_{01} mode.	101
Fig. 6.5	Transverse field of TM_{01} mode.	101
Fig. 6.6	Transverse field of HE_{21} mode.	102
Fig. 6.7	E_y field of HE_{11}^y mode.....	102
Fig. 6.9	The modal loss of HE_{11} (dotted line) and TE_{01} (solid line) modes with the change of the wavelength.....	105
Fig. 6.10	Map of photonics bandgaps found for cladding photonics crystal structure.....	106
Fig. A.1	Schematic diagram of the microstructued optical waveguide.	115

CHAPTER 1

Chapter 1 Introduction

Semiconductor laser diodes play a crucial role in the success of optical communication systems due to the fact that they are coherent light sources. There are several types of laser diodes such as Fabry-Perot laser diodes, distributed feedback laser diodes and vertical cavity surface emitting lasers, etc.

In optical fiber communication systems, one of the most important characteristics required for a semiconductor laser is the stable mode operation featured by high side-mode-suppression-ratio (SMSR) and narrow spectral linewidth. In conventional Fabry-Perot lasers, the feedback of radiation into the active medium is accomplished by the reflective facets. Without any frequency discrimination, several longitudinal oscillation modes will be observed. To obtain stable single-mode operation, the distributed feed back laser diodes (DFB LDs) were proposed and developed. The working principle of a uniform-grating DFB laser with perfectly anti-reflective facet coatings (a conventional DFB laser) was first explained by Kogelnik and Shank in 1971 using couple wave theory. But it was not until 1974 that the first DFB laser operating at room temperature under continuous wave operation was built [1-2]. Fig. 1.1 shows the schematic of the configuration of DFB LDs, in which etched diffraction grating structures are introduced in the active region and utilized as wavelength selective elements. However when the coupling coefficient of gratings in the DFB LDs is large the

single mode operation will be threatened. One of the challenges for the design of DFB LDs is to get the stable single longitudinal mode operation.

Fig. 1.1 The schematic diagram of distributed feedback laser diodes.

Vertical-cavity surface-emitting laser is another kind of semiconductor laser which is received rapid development recently. VCSELs are made by sandwiching a light-emitting layer between two highly reflective mirrors. Light is emitted from the surface of the mirrors and in the direction normal to the surface. The schematic of a VCSLE is shown in Fig. 1.2. The structure characteristics of VCSELs allow on-wafer testing rather than dicing and packaging in the facet emitting lasers, which makes the test procedure simplified.

Early work on the vertical-cavity surface-emitting laser (VCSEL) can be traced chronologically to the late 1970s [5]. The first VCSEL was proposed and fabricated by K. Iga and his colleagues, with the first demonstrated electrically-pumped InGaAsP/InP VCSEL under pulse operation at 77K in 1979 [6]. While the cavity design was novel as compared to the established edge-emitting lasers, the

CHAPTER 1 INTRODUCTION

initial device performance was so poor in terms of the high threshold current density and low output power. Continued work by Iga and various coworkers, however, showed continues improvements in performance, eventually leading to the first continuous wave VCSEL operation reported in 1988 [7-9]. One of important innovations which brought growing attention to VCSELS is the development of the high reflectivity semiconductor distributed Bragg reflectors (DBRs) with the advancing crystal growth techniques of molecular beam epitaxy and metal organic chemical vapor deposition [10-11]. The other promising properties for the rapid progress in the development of GaAs-based VCSELS are the optical materials. The successful developments of GaAs/AlGaAs quantum-well (QW) materials for the facet emitting lasers in the later 1980s provide the solution for VCSELS [12]. The rapid development of GaAs-based laser led to the commercialization of short-wavelength (*i.e.* 850-980nm) VCSELS in 1990s.

As the rapid development of VCSELS, some challenges still exist. One challenge for the VCSEL design is to obtain the high output power and keep the VCSEL work at single transverse mode. This is because the output power is scaled with the aperture area of the active layer, and high-order transverse modes are excited when the aperture is large (for example to get 40mW output power, the aperture diameter can be as large as $16\text{ }\mu\text{m}$).

Fig. 1.2 The schematic diagram of vertical-cavity surface emitting lasers.

As laser fabrication processes are very complicated and expensive, efficient and relatively accurate simulation tools are in great demand for the laser design and optimization. Simple rate equation model was used in preliminary investigation of the dynamic response of laser diodes. Hence the time variation of carrier concentration and photon density inside the active layer can be calculated. However, the rate equation model performs poorly in the analysis of complicated structure such as distributed-feedback lasers, distributed Bragg reflector laser diodes and VCSELs. To analyze the structures and devices mentioned above, laser models are in demand to accurately capture the various characteristics of the structures.

1.1 Motivation

As described already, distributed-feedback semiconductor lasers have greater mode selectivity than Fabry-Perot devices, and are preferred as sources for long-haul high-capacity fiber system. However, dynamic single-mode operation is still difficult to achieve due to the existence of two-mode degeneracy in uniform DFB laser diodes. Quarter-wave-shift (QWS) DFB lasers have been proposed [3] to get single-longitudinal-mode operation. Unfortunately, in such QWS DFB lasers, spatial hole burning (SHB) effects [4] will enhance the side modes when the coupling coefficient is large. The spatial hole-burning effects have been known to limit the performance of DFB LDs. As the biasing current of a single quarterly-wavelength-shifted (QWS) DFB LD increases, the gain margin reduces. Therefore, the maximum single-mode output power of the QWS DFB LD restricts it to a relatively low power operation. The SHB phenomenon caused by the intense electric field leads to a local carrier depletion at the center of the cavity. Such a change in carrier distribution alters the refractive index along the laser cavity and ultimately affects the lasing characteristics. By changing the structural parameters inside the DFB LDs, the effect of the SHB in DFB LDs can be reduced. By introducing more phase shifts along the laser cavity, SHB can be suppressed and as a result, a large single-mode power and consequently a narrower spectral linewidth may be obtained. To optimize the locations and the values of the phase shifts in a multiple-phase-shift DFB, extensive investigation is necessary.

VCSELs show special optical and electrical properties (single-longitudinal-mode emission, low-threshold current, low-divergence circular output beam, simplicity for building two-dimensional arrays, etc) which make these devices extremely attractive for several applications. Novel devices such as using VCSEL as a semiconductor amplifier could be a promising topic.

The polarization state is usually well stabilized in conventional edge-emitting laser diodes in which cavity effects dominate. However, the special geometry of vertical-cavity surface-emitting lasers leads to a number of polarization instabilities in which nonlinear gain properties are expected to play a more important role. Thus polarization study of VCSEL becomes crucial. Polarization control of VCSELs is required for their applications in polarization-sensitive equipment and coherent detection systems. It is worthwhile to investigate the factors which dominantly control the polarization properties.

The advance in fabrication technologies has significantly improved the performance of VCSELs such as low threshold, high power. However, multi-transverse-mode operation in large-aperture VCSELs is a serious problem and has to be solved in order to replace facet-emitting lasers. Therefore novel VCSEL structures for single-transverse-mode operation at high power (10mW) are needed in order to satisfy the requirement of high-speed long-haul single-mode fiber communication systems. This poses the challenge of designing VCSEL structures for single-transverse-mode operation with large aperture size. One method can be used to maintain single-transverse-mode operation in oxide aperture VCSELs with large cavity size antiguiding effects in epitaxially regrown materials. The basic

CHAPTER 1 INTRODUCTION

idea of this method is to introduce a spatial filter to eliminate the excitation of higher-order transverse modes. The mechanism of the antiguiding effects and its application in novel VCSEL designs will be an interesting research topic.

1.2 Objectives

This thesis focuses on the design and the analysis of laser diodes for WDM system. Firstly, the characteristics and optimization design for multiple-phase-shift DFB LDs will be studied. After that, we will investigate the characteristics and performances of VCSELs. In particular, we will carry out the analysis and design research in the following aspects:

- (1) To modify the transmission-line laser model and apply it in studying the performance of multiple-phase-shift DFB laser diodes
- (2) To investigate the polarization characteristics of VCSELs including polarization stability, polarization switching and to study the effects of the DBR on the polarization switching properties of the VCSEL.
- (3) To design VCSEL-based two-dimensional wavelength converters and carry out theoretical performance studies on the proposed structure.
- (4) To optimize the structure parameters of anti-resonant reflectivity optical waveguide (ARROW) VCSELs.
- (5) To design anti-resonant photonic crystal VCSEL for single-mode operation with large aperture.

1.3 Major contribution of the thesis

In this thesis, we focus on the design and the analysis of the laser diodes for WDM system. We have done extensively research study on the characteristics of VCSELs and several VCSEL-based devices have been proposed to take advantage of the VCSEL structures. Till now we have the following achievements:

At the first stage, our primary effort was to study the characteristics of multiple-phase-shift DFB laser diodes. In order to achieve this, we have modified transmission-line laser model and applied it to the analysis of a multiple-phase-shift DFB laser diode. The optimization of the multiple-phase-shift DFB LD has also been carried out by considering the locations and the values of the phase shifts in order to get single-longitudinal-mode operation.

Secondly, the polarization characteristics of a VCSEL have been investigated. The polarization stability has been calculated and the regions of mono-stability, bistability, and instability are obtained. The stability map is useful in choosing the parameters of the VCSEL to achieve different polarization state. In order to study the DBR effects on the polarization switching performance, transmission-line laser model has been developed and incorporated into the SFM polarization model. The DBR effects on the width of the bistable hysteresis loops of VCSELs have been investigated with the developed model.

Thirdly, two-dimensional VCSEL-based wavelength converters have been designed. In this configuration, a VCSEL is used both as the laser source for the

CHAPTER 1 INTRODUCTION

probe wave and as the semiconductor optical amplifier (SOA) for performing cross-gain modulation. Two-dimensional transmission-line laser model has been developed for the proposed structure. The characteristics of the proposed two-dimensional wavelength converters have been investigated by applying the developed model. We have demonstrated that by employing the proposed structure, the wideband wavelength converter can be achieved.

Fourthly, the modal field characteristics of anti-resonant reflecting optical waveguide (ARROW) VCSELs have been explored. Two methods (scalar transfer matrix method and vector transfer matrix method) are presented to analyze the performance of ARROW VCSELs. The optimization studies on the ARROW VCSEL structure parameters for the best transverse-mode discrimination have been done using both of analytical methods.

The last contribution of this thesis is the design of the anti-resonant photonic crystal (ARPC) VCSELs. In the proposed structure anti-resonant reflecting photonic crystal structure is employed and introduced in vertical-cavity surface-emitting lasers to achieve large aperture single-mode emission. In this configuration, a hexagonal array of high-index cylinders, which runs vertically in the cladding region, is introduced in the top of the VCSEL p-type DBR mirror regions. The transverse modal properties of the proposed structure, especially leakage loss, have been investigated. It was found that the loss properties of the modes are varied periodically with the change of the lasing wavelength. Our theoretical studies suggest that the proposed ARPC VCSELs can work at single-mode condition even at the large mode field diameter around $10\mu\text{m}$.

1.4 Organization of the thesis

The rest of the thesis is organized as follows. Chapter 2 gives the brief introduction of the development of the laser models. The transmission line laser model is described and developed to take into account the phase effects. Furthermore, uniform DFB LDs, Quarter wave shift DFB LDs as well as multiple phase shift DFB LDs have been analyzed and compared by the developed laser model. The optimum designs of the phase shift locations and values of three-phase-shift DFB LDs have been discussed.

Chapter 3 addresses theoretical investigation of the polarization stability and the influence of the Bragg reflectivity on the polarization bistability of a VCSEL. A spin flip model, which takes into account the phase information of two orthogonal polarizations through saturable dispersion is deduced to analyze the polarization stability of VCSELs. In addition, by combination of spin flip model and transmission line laser model together, a four level laser model is developed to investigate the effects of the Bragg reflectivity on the bistability of VCSELs.

In Chapter 4, a novel two-dimensional VCSEL-based wavelength converter is reported, in which the VCSEL is used both as a laser source for the probe wave and as a SOA for performing cross-gain modulation. A complete two-dimensional transmission-line laser model (TLLM) is presented to analyze the proposed structure. The optimizations with respect to wavelength, input power level as well as the reflectivity of the VCSEL facet have been done to get the optimum extinction ratio and peak chirp frequency.

CHAPTER 1 INTRODUCTION

Cylindrical anti-resonant reflecting optical waveguide (ARROW) type VCSELs are analyzed and discussed in Chapter 5. Scalar and vector TMMs are presented to study the modal characteristics of cylindrical ARROW VCSELs. The optimum designs of the structure parameters have been done with the aim to achieve single-mode operation with large modal discrimination.

In Chapter 6, anti-resonant reflecting photonic crystal VCSELs is proposed to achieve large aperture single-mode operation. The modal field properties of the proposed structures with one ring of high index cylinders are investigated by the developed multipole method. The loss property of several lowest-order modes under a range of lasing emission wavelength is investigated and analyzed.

Finally, in Chapter 7, we summarize the achievements of our research, and present the future research work based on what we have accomplished in this thesis.

CHAPTER 2

Chapter 2 Design and Modeling of Multiple Phase Shift Distributed Feedback Laser diodes

2.1 Introduction

In recent years, distributed feedback laser diodes (DFB LDs) play important roles in the long-haul and high bit-rate optical fiber transmission systems because of their single longitudinal mode (SLM) operation. To overcome the two modes degeneracy and achieve a pure single-mode operation in uniform DFB LDs, quarter wave shift (QWS) DFB lasers have been proposed [3]. However, in such QWS DFB lasers, spatial hole burning (SHB) effects will enhance the side modes when the coupling coefficient is large. In order to achieve SLM operation as well as eliminate SHB effects, multiple shift DFB lasers have been proposed [13]. It has been shown that side modes can be effectively suppressed and a stable and pure single-mode operation can be realized.

With the development of laser structures, efficient and relatively accurate simulation models are becoming increasingly important for laser designs and operation optimization due to the complicated and expensive laser fabrication processes. Some new dynamic models for analysis of laser diodes have been developed in [14]-[16]. Whiteaway, Zhang and Carroll [17] have used frequency-

domain analysis methods based on the concatenation of transfer matrices. In these models, the effect of longitudinal spatial-hole burning is usually calculated by using the intensity profile of the dominant mode, which can be found by searching in the frequency domain. However, these models suffer from convergence problems when more than one mode exist [18]. Moreover, it is computationally inefficient to simulate the dynamics of lasers by these models, as they have to extend frequency-domain techniques to the time domain. In comparison, models by time-domain methods, such as transmission line laser model, are easier to be extended for efficient simulation of laser dynamics.

Up to now, several time-domain methods have been reported in the literature. One method developed by John Carroll et al [1], which was based on low-order finite-difference method and spectral filtering, had been applied to analyze several structures. The other frequently used method is transmission line laser model (TLLM), which was developed by Lowery [19]-[20]. In this method, time-domain numerical algorithms were employed for laser simulation. TLLM gives a clear physical interpretation of the laser's internal processes such as reflections, gain/loss, filtering, noise, and coupling, in terms of circuit theory. Although TLLM has been used previously to analyze microwave devices however, recently it has also opened up a fresh new perspective [21] in optical communications field.

TLLM model splits the laser cavity longitudinally into a number of sections. In each section, it uses a scattering matrix to represent the optical process, such as stimulation emission, spontaneous emission and attenuation. The matrices of these sections are then connected by transmission lines, which account for the

propagation delays of the waves. From the iterations of scattering and connecting processes, the output electric field in the time domain can be obtained. Then by applying Fourier transform we can easily obtain the laser output spectra. Large signal dynamic with nonlinear effects such as the changes of electron and photon densities along the length of the laser and spatial hole burning are key issues in the analysis of DFB laser diodes. These dynamic effects can be investigated easily by using transmission line laser model.

In Ref. [22], TLLM models have been used to analyze QWS DFB LDs. With the insertion of a zero-reflection interface (identify matrix) at half way along the cavity, the effects of QWS on laser operation have been simulated successfully. However, using this method we can only analyze DFB laser structures with one $\pi/2$ phase shift at the center of the cavity. We cannot use this technique to analyze other phase shift values and also multiphase shift (MPS) DFB lasers.

In this chapter, we have developed phase shift model and incorporated it into TLLM. The developed TLLM can easily model any arbitrary phase shift value and the characteristics of MPS DFB LDs have been investigated by the developed method.

2.2 Developed transmission line laser model (TLLM)

Transmission line laser model divides the longitudinal propagation path of the laser cavity into a number of sections where the electric field distribution is represented by a series of pulses along the sections. The model for each section

Chapter 2 Design and Modeling of Multiple Phase Shift Distributed Feedback Laser diodes

consists of: (i) a scattering process which represents the stimulated and spontaneous emissions as well as loss; (ii) a connecting process which allows wave propagation with the appropriate delay (one iteration-time step) between the sections; and (iii) a carrier rate-equation model which provides the local electron density within the conduction band and controls the scattering process of each section. TLLM is such a model that we can convert a physical problem into a computer algorithm with physical meaning.

In TLLM, two adjacent scattering sections are connected by transmission lines. In each step of the iteration, the incident traveling wave pulses at each scattering section incident port are scattered according to scattering matrix and become reflected pulses at the output port. These pulses then travel along the transmission lines to the adjacent sections, ready to be scattered again at the next iteration. During the iteration process, by using the value of the photon density, the carrier density in each section can be calculated by using the carrier rate equations. Consequently, any change in local carrier density affects the optical gain of each section and hence controls the scattering and connecting processes at next iteration. Both the scattering and connecting processes are represented by scattering and connecting matrices, which involve simple numerical operations. In the following, we introduce the scattering and connecting matrices as well as the carrier density rate equation, separately.

2.2.1 Scattering matrix

The scattering matrix S describes the propagation procedures of the traveling waves in each section (from the input port to the output port), which may be expressed as [23]

$${}_k V(n)^r = S \cdot {}_k V(n)^i + {}_k V(n)^s \quad (2.1)$$

where ${}_k V(n)^r$ denotes the reflected wave output from the n th scattering section at the k th time step, ${}_k V(n)^i$ denotes the incident wave at the input of that section, and ${}_k V(n)^s$ is the wave due to the spontaneous emission. S is the scattering matrix that takes account of the frequency-dependent gain and attenuation as well as the spontaneous emission. In the above equations, i , r and s stand for “incident”, “reflected” and “spontaneous”, respectively. It should be noted that the traveling waves are composed of the backward and the forward waves. The scattering matrix can be expressed as

$$S = \frac{1}{y} \begin{bmatrix} t(g+y) & 2tY_C & 2tY_L \\ g & 2Y_C - y & 2Y_L \\ g & 2Y_C & 2Y_L - y \end{bmatrix} \quad (2.2)$$

where g is the gain, t is the attenuation, Y_C is the capacitive reactance, Y_L is inductive reactance and $y = 1 + Y_L + Y_C$ is the total admittance of the RLC stub filter [20].

2.2.2 Connecting matrix

Connecting process is used to connect two adjacent scattering sections together. It provides incident wave of the scattering section from the reflected wave of the previous section. That is

$$V_{k+1}(n)^i = C \cdot V_k(n)^r \quad (2.3)$$

where C is the connecting matrix. The connecting processes for the inductive and capacitive stubs can be expressed as

$${}_{k+1} \begin{bmatrix} F_L(n) \\ B_L(n) \end{bmatrix}^i = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} {}_k \begin{bmatrix} F_L(n) \\ B_L(n) \end{bmatrix}^r \quad (2.4)$$

and

$${}_{k+1} \begin{bmatrix} F_C(n) \\ B_C(n) \end{bmatrix}^i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} {}_k \begin{bmatrix} F_C(n) \\ B_C(n) \end{bmatrix}^r \quad (2.5)$$

where $F_C(n)$ and $B_C(n)$ represent the forward and backward waves in the capacitive stubs of the n th section, whereas $F_L(n)$ and $B_L(n)$ are those of the inductive stub as well as the phase shift stubs.

In DFB LDs, the forward and backward waves couple to each other due to the modulation of the refractive index. Correspondingly, in the TLLM model, following two different matrices are given to represent this coupling effect: (i) transition from a high-impedance to low-impedance (i.e. from section n to section $n+1$, where $n = 1, 3, 5, \dots$) where in this case the matrix can be expressed as [22]

$${}_{k+1} \begin{bmatrix} F(n+1) \\ B(n) \end{bmatrix}^i = \begin{bmatrix} 1 + \kappa \Delta L & -\kappa \Delta L \\ \kappa \Delta L & 1 - \kappa \Delta L \end{bmatrix} {}_k \begin{bmatrix} F(n) \\ B(n+1) \end{bmatrix}^r \quad (2.6)$$

and (ii) transition from a low-impedance to high-impedance (i.e. from section $n+1$ to $n+2$). The matrix in this case becomes

$${}_{k+1} \begin{bmatrix} F(n+2) \\ B(n+1) \end{bmatrix}^i = \begin{bmatrix} 1 - \kappa \Delta L & \kappa \Delta L \\ -\kappa \Delta L & 1 + \kappa \Delta L \end{bmatrix} {}_k \begin{bmatrix} F(n+1) \\ B(n+2) \end{bmatrix}^r \quad (2.7)$$

where F and B denote the forward and backward waves, respectively, κ is the coupling per unit length and ΔL is the length of arbitrary section.

In a standard DFB device, Eqs.(2.6) and (2.7) are applied alternately along the device longitudinal cavity. For quarter-wave-shifted (QWS) grating devices a zero-reflection interface (identify matrix) is inserted half-way along the cavity [22].

2.2.3 Phase shift model

Lowery [22] has used TLLM to model the QWS DFB by inserting a zero-reflection interface half way along cavity. However, this approach cannot be applied to model other single- and/or multi-phase shift structures (i.e. it is only applicable to $\pi/2$ single-phase shift structure). To overcome this problem, in previous TLLM's [24][25], the phase shifts of the optical fields were modeled by using phase stubs. In that proposed model, a transmission line stub is introduced

Chapter 2 Design and Modeling of Multiple Phase Shift Distributed Feedback Laser diodes

which connects the output of the scattering section and the connecting transmission lines. Due to the impedance mismatching between the phase stub and the main transmission line, phase shift can be modeled. However, due to impossibility to produce same phase shift over an infinite bandwidth by the same phase stub, the simulation bandwidth will be limited to narrow band to get accurate phase shifts.

In this chapter, the broad band phase shifters are implemented by multiplying the phase term $\exp(-i \cdot \theta)$ with the optical fields which connect the two adjacent scattering nodes together, where θ is the phase advance in radian. Within the two optical fields one is the reflected wave from the scattering node and the other one is the incident wave to the next scattering node. Same value phase shift will be applied to within the simulation bandwidth for any lasing wavelength. By employing this method, current laser model can be operated anywhere in the simulation bandwidth.

By using such phase term model, in the sections which have phase shifts, the scattering process will remain the same as described in section 2.2.1 and the connecting process can be written as

$${}_{n+1} \begin{bmatrix} F(s+1) \\ B(s+1) \end{bmatrix}^I = \begin{bmatrix} \exp(-i \cdot \theta) & 0 \\ 0 & \exp(-i \cdot \theta) \end{bmatrix} \cdot C \cdot {}_n \begin{bmatrix} F(s) \\ B(s) \end{bmatrix}^R \quad (2.8)$$

2.2.4 The carrier rate equation

The carrier rate-equation model can provide the local electron density within the conduction band and control the scattering and connecting processes of each section. The rate equation for carrier density may be expressed as [20]:

$$\frac{dN}{dt} = -\frac{N}{\tau} - \frac{ac\Gamma}{n_g}(N - N_0)S + \frac{I}{wdlq} \quad (2.9)$$

where N is carrier density, a is the spatial gain per unit carrier density, c is the light velocity in free space, τ is the lifetime of the carrier, N_0 is the transparent carrier density, Γ is the optical confinement factor, n_g is the group refractive index, S is the photon density, I is the injection current and q is the charge of an electron. w , d and l denote width, depth and length of the laser's active region, respectively. It should be noted that in the above carrier rate equation, the carrier diffusion along the cavity is neglected. This is because the longitudinal diffusion is small (typically $2\mu\text{m}$) and if enough number of sections is used, the length of each section would be not longer than the diffusion length.

The photon density at each section can be obtained from the incident voltage pulses which represent the optical fields. The average photon density, S , in a section is related to the two traveling waves as

$$S = n_g (|F|^2 + |B|^2) / (Z_c h f c) \quad (2.10)$$

Where Z_c is the impedance of the waveguide, h is the Planck's constant and f is the frequency of the laser.

Once the photon density is calculated, the carrier density of each section can be updated for next iteration using Eq. (2.9) where it controls the scattering process

by changing the gain of each section. To measure the uniformity of the light intensity distribution we define a flatness factor F as follows [2]

$$F = \frac{1}{L} \int_0^L (P(z) - \bar{P})^2 dz \quad (2.11)$$

where $P(z)$ is the normalized longitudinal field intensity distribution along the laser cavity with length L and $\bar{P}$ is the average of $P(z)$. To minimize the longitudinal spatial hole burning effect and thus get single longitudinal mode oscillation, it has been proposed that the flatness F should be below 0.05 in DFB laser diodes [16].

2.3 Analysis of multiple phase shift DFB laser diodes

Based on the TLLM MPS DFB model introduced in Section 2.2, we have analyzed the operation of QWS DFB LDs as well as 3PS DFB LDs. Parameters used in our analysis are given in Table 2.1. The reflectivity at both facets is assumed to be 0. To make it easy to be understood, a schematic diagram of the 3PS DFB LD structure is shown in Fig. 2.1.

Fig. 2.1 Schematic diagram of 3PS DFB LD .

The section number of sections used in our simulation is 100 which corresponds to the sampling rate of around 4.05×10^{13} Hz which is nearly 20 times larger than the twice of the laser's bandwidth $\Delta f = 1.0 \times 10^{12}$ Hz. This ensures that no information will be lost [20].

Table 2.1 Parameters for QWS DFB LDs and 3PS DFB LDs

Physical Parameter	Value
Operating wavelength	$\lambda_0 = 1.55 \mu m$
Cavity Length	$L = 500 \mu m$
Active layer thickness	$d = 0.15 \mu m$
Active layer width	$w = 2 \mu m$
Spatial gain constant	$a = 0.5 \times 10^{-16} \text{ cm}^{-2}$
Optical confinement factor	$\Gamma = 0.3$
Transparency carrier density	$N_0 = 9.0 \times 10^{17} \text{ cm}^{-3}$
Carrier lifetime	$\tau = 4.0 \text{ ns}$
Spontaneous emission coupling factor	$\beta = 10^{-4}$
Internal attenuation	$t = 0$
Laser facet reflectivities	$R = 0.0$
Monomolecular recombination coefficient	$A = 1.0 \times 10^8 \text{ s}^{-1}$
Bimolecular recombination coefficient	$B = 8.6 \times 10^{-11} \text{ cm}^3 \text{ s}^{-1}$
Auger recombination coefficient	$C = 4.0 \times 10^{-29} \text{ cm}^6 \text{ s}^{-1}$
Number of model sections	$S = 100$
Grating coupling per unit length	$\kappa = 160 \text{ cm}^{-1}$
Initial carrier density	$N_i = 1.5 \times 10^{18} \text{ cm}^{-3}$

2.3.1 Dynamic characteristics

To verify the validity of TLLM MPS DFB model, QWS DFB LDs have been simulated and the result is compared with that of Lowery's QWS model [22]. It should be noted that Ref. [22] has inserted a zero-reflection interface halfway along the cavity whereas we have used transmission line stub to represent quarter wave phase shift. Figures 2.2 shows the transient response QWS DFB using our proposed and Ref. [22] models under 100mA injection current. Figure 2.2 clearly indicates that our results and those of Ref. [22] are in good agreement.

Fig. 2.2 Transient response of QWS DFB LD using developed TLLM MPS model and Lowery's model in reference [22].

The dynamic characteristics of 3PS DFB lasers have also been analyzed using TLLM MPS DFB model. In the simulation, we set $L_1 = L_2 = L_3 = L_4$, $\theta_1 = \theta_2 = \theta_3 = \pi/3$. The transient response of the 3PS DFB laser diode by using developed TLLM MPS Model and Lowery's phase stub model in Ref. [24] is shown in Fig.2.3. From Fig. 2.2 and Fig. 2.3, we can conclude that the new

proposed model is accurate enough to be used for analysis of both single- and multi-phase shifts in DFB LDs.

Fig. 2.3 Transient response of 3PS DFB LD by using developed TLLM MPS Model and Lowery's phase stub model in reference [24].

Fig. 2.4 Transient longitudinal carrier density of QWS DFB LD.

Figures 2.4 and 2.5 illustrate the evolution of longitudinal carrier density profile of QWS DFB LDs and the 3PS DFB LDs. The carrier densities in these two structures are initially uniform, but spatial-hole burning causes local minima at the

phase shift positions on the longitudinal carrier density profiles. Compared with the QWS structure, the 3PS structure produces a more uniform carrier density distribution at the steady states.

Fig. 2.5 Transient longitudinal carrier density of 3PS DFB LD.

2.3.2 Longitudinal distribution

Fig. 2.6 Longitudinal carrier density distribution of QWS DFB LD calculated by different models.

The DFB laser carrier density, photon density, refractive index and field intensity are different in each section and they play an important role in the laser operation. Therefore, it is necessary to study the longitudinal distributions of these physical parameters in DFB lasers. Figure 2.6 shows variations of the longitudinal carrier density distribution for the QWS DFB in static operation. Simulations have been performed by using three different models namely Lowery's QWS model [22] where QWS phase shift is modeled by inserting a zero-reflection interface half way along cavity as stated before, our proposed TLLM MPS model and the Advanced Laser Diode Simulator (ALDS) model.

The ALDS model is commercial software provided by Apollo Photonics Company, which is three-dimensional static model [15]. In this model, the 3-D structure is first subdivided into a number of sections along the laser cavity. For each section, the optical field equation, the carrier transport equations are solved by the finite difference method. From the transverse carrier in each longitudinal section, the optical gain, the index change, and the optical loss are calculated accordingly. Using weighted integrals of these parameters by the transverse optical field intensity, the corresponding modal parameters are obtained. And the complex gain and complex coupling coefficient are then calculated from these modal parameters. By substituting the coefficients into the coupled equations, the longitudinal field distribution and the eigen value can be obtained by using transfer matrix method. Figure 2.6 clearly shows that our results agree well with the other two models.

Fig. 2.7 Longitudinal intensity distribution of QWS DFB LD, uniform DFB and 3PS DFB LD.

The internal electric field intensity distributions of uniform DFB, QWS DFB and 3PS DFB LDs are shown in Fig. 2.7. In the simulation we have assumed $L_1 = L_2 = L_3 = L_4$ and $\theta_1 = \theta_2 = \theta_3 = \pi/3$. All the electric field distributions have been normalized so that the intensity at the laser facets becomes unity. It can be seen that the internal electric field distribution of the QWS DFB LDs is highly non-uniform. This induces the spatial hole burning effects and thus affects the single mode stability of the laser. With three phase shifts incorporated into the cavity, the intensity distribution spreads out and the overall electric field distribution becomes more uniform and hence single mode operation can be realized even under high coupling coefficient.

Fig. 2.8 Longitudinal photon density distribution of QWS LD (right) 3PS DFB LD (left) under different injection currents.

Fig. 2.9 Longitudinal carrier density distribution of QWS DFB LD (left) and 3PS DFB LD (right) under different injection currents.

Fig. 2.10 Longitudinal refractive index distribution of QWS DFB LD (left) 3PS DFB LD (right) under different injection currents.

Fig. 2.11 Normalized intensity distribution of QWS DFB LD (left) and 3PS DFB LD (right) under different injection currents.

Figures 2.8 to 2.11 show the longitudinal distributions of the (i) photon density, (ii) carrier density, (iii) refractive index and (iv) internal intensity of QWS DFB LD and 3PS DFB LD under different injection currents. In these calculations the threshold current is set to 20mA. It has been shown that the introduction of phase shifts can flatten the longitudinal distributions of carrier density, photon density, refractive index and internal field intensity.

2.3.3 Effects of Phase Shift Position and Value on the 3PS DFB LDs Characteristics

The effects of the phase shift positions on the field intensity distribution have been investigated for the 3PS DFB LDs. In the analysis, we set the position of one of the phase shifts at the center of the cavity while the positions of the other two phase shifts are set symmetrically with respect to the laser cavity center point

(i.e. $L_1 = L_4$). To indicate the position of these two-phase shifts more accurately we define a parameter called normalized location as $\psi = L_1 / (L_1 + L_2)$.

Fig. 2.12 Variation in flatness versus ψ for different values of coupling coefficient κ .

Fig. 2.13 Variation in flatness versus ψ for different phase shift.

Figure 2.12 shows the effects of variation ψ on flatness for different coupling coefficient κ . When small values of κ are used, the field intensity distribution becomes less uniform when the phase shifts θ_1 and θ_3 shift towards the laser

facets. As the optical feedback becomes stronger with increasing κ , the field intensity distribution becomes more intense near the center of the laser cavity.

Fig. 2.14 Flatness contour map with the phase shifts of 3PS DFB.

The effects of variation ψ on flatness is shown in Fig. 2.13 for three different phase shift values of $\theta_1 = \theta_2 = \theta_3 = \pi/2, \pi/3$ and $\pi/4$. It can be seen that when the normalized location changes from $\psi = 0.1$ to 0.6 , for all three cases the parameter flatness F is less than 0.05 which satisfied the necessary condition for the single mode operation.

We have also investigated the effects of the phase shifts on the intensity flatness of the 3PS DFB LDs for the case when the normalized position $\psi = 0.5$ (that is $L_1 = L_2 = L_3 = L_4$). The contour map of the flatness for various phase values is shown in Fig. 2.14. In the contour map, x-axis represents the central phase shift θ_2 whereas y-axis represents the other phase shifts (i.e. $\theta_1 = \theta_3$). This figure clearly shows that to meet the requirement of $F < 0.05$, $\theta_1 = \theta_3 = \theta_{side}$ must lie in the range of $65^\circ < \theta_{side} < 110^\circ$.

2.3.4 Optimum designs of the MPS DFB LDs based on flatness and side mode suppression ratio

The emission spectrums of the laser diodes can be obtained by performing Fourier transform on the dynamic output fields. We have calculated the field density versus spectrum of the uniform DFB and 3PS DFB LDs. The spectrum of 3PS DFB LDs in Fig.2.15 shows this structure operation at single mode with high side-mode-suppression-ratio. However the spectrum of the uniform DFB in Fig. 2.16 shows multiple mode operation with side mode suppression ratio as low as 20dB. Traced back to Fig. 2.7 we can find that the flatness of uniform DFB is lower than 0.05 which showed good flat field distribution. However, Fig. 2.16 shows that multiple operations still exist in such structures. These results demonstrate that the flatness less than 0.05 is necessary but not enough to guarantee single mode operation.

Fig. 2.15 Spectrum of 3PS DFB.

Fig. 2.16 Spectrum of uniform DFB.

As already known, the spatial hole burning will cause the local variations in the carrier density, and hence the variations in the real refractive index and gain. This gives rise to changes in the magnitude and phase of the feedback from the each grating sections. All of this changes the longitudinal-mode intensity distribution and at the same time alters the gain suppression ratio of the side modes relating the lasing modes. Thus to get a stable single mode operation, it is necessary to obtain a more uniform field distribution and while maintaining a high side-mode-suppression ratio. In the following optimization, field flatness as well as the side-mode-suppression ratio will simultaneously be examined. In current communication systems, side-mode suppression ratio as high as 50dB is sufficient. Here side mode suppression ratio (SMSR) larger than 50dB will be assumed as another optimization design criteria of DFB LDs.

The contour map of the SMSR with variation of the phase shifts is shown in Fig. 2.17. To satisfy the requirement of SMSR larger than 50dB, it is shown that

$\theta_{side} > 105^\circ$ or $\theta_{side} < 80^\circ$. Comparing Fig. 2.17 and Fig. 2.14, it can be seen that a tradeoff exists in selecting the optimum value of phase shift values based on the SMSR and flatness.

As a result, a compromise has to be made in selecting the phase shifts in 3PS DFB lasers to get stable single mode operation such that the SMSR larger than 50dB and flatness F less than 0.05. Figure 2.18 is the combination of the region $F < 0.05$ from Fig. 2.14 and $\text{SMSR} > 50\text{dB}$ from Fig. 2.17. The phase combination in the shaded area will satisfy the design criteria of both SMSR and flatness and thus give rise to stable single mode operation.

Fig. 2.17 SMSR contour map with the phase shifts of 3PS DFB.

Fig. 2.18 Variations in the SMSR and flatness with respect to the phase shifts. The shaded area covers all phase combinations that satisfy both $\text{SMSR} > 50$ and $F < 0.05$ selection criteria for stable mode operation.

2.4 Summary

In this chapter, Phase shift model for any arbitrary phase shift value is proposed and incorporated into the general transmission lines laser models for MPS DFB LDs. In particular, we have analyzed the characteristic performance of a three-phase-shift (3PS) distributed feedback (DFB) laser diode based on the developed model (TLLM). Effects of phase shift position and phase values on the laser performance have also been analyzed. As expected, the simulation results indicate that the longitudinal field distributions of three phase shift (3PS) DFB are more uniform than QWS DFB and hence the spatial hole burning effect is reduced significantly in this structure. Furthermore optimum designs for the three-phase-

shift DFB laser diode have been studied based on the parameter flatness of the field distribution and the parameter side mode suppression ratios.

CHAPTER 3

Chapter 3 Polarization Investigation of Vertical Cavity Surface Emitting Lasers

3.1 Introduction of spin-flip model (SFM)

Vertical-cavity surface emitting lasers offer a number of opportunities to study fundamental aspects of semiconductor laser dynamics. In this chapter, the issue of laser polarization state selection and the competition of the polarization state are particularly investigated. Polarization selection in conventional edge-emitting lasers is mainly determined by the geometry for any polarization direction in the plane of the wafer. However, due to the cylindrical symmetry of the VCSEL, in principle, the two orthogonal polarizations can be located at any azimuthal directions. Both of the polarization states have the same gain and can be excited simultaneously above threshold. However, it has been shown that the growth of quantum wells (QW's) epitaxial layers on the non-(001) substrates gives in-plane gain anisotropy. In addition, the discrimination of optical gain and frequency of the two orthogonal polarizations are observed in VCSELs due to the electro-optically induced birefringence [12]. These intrinsic optical anisotropies have a significant influence on the polarization properties in VCSLEs. Polarization control requires a better fundamental understanding of the physical mechanism of polarization selection [26].

Light emitted from VCSELs is typically linearly polarized with the vector field oriented along one of two orthogonal crystal axes which are perpendicular to the emission direction. The two linearly polarized modes also have slightly different emission frequencies due to the birefringence of the crystal [27]. Even though several VCSELs display stable linear polarization emission at any value of the applied current, many of these devices show a polarization instability relatively abrupt switch in the orientation of the optical field known as polarization switching (PS), which occurs as the injection current increased further above threshold [28-29].

One explanation offered for polarization state selection in VCSELs is based on the different rates of the thermal wavelength shift of the gain curve and the cavity resonances combined with the uncontrolled strain present in the growth process which may cause VCSEL anisotropies (birefringence and dichroism) [30]. However, the above explanation cannot explain the fact that in gain-guided VCSELs polarization switching is still observed when the active region temperature is kept constant by using fast-pulse low-duty-cycle operation [31].

Spin-flip model (SFM) model was developed by M. San Miguel in 1995 [32]. SFM for the polarization dynamics of quantum-well (QW) VCSELs has been a useful instrument to describe a number of polarization properties of VCSELs. Saturable dispersion associated with the linewidth enhancement factor and the carrier dynamics with the different magnetic sublevels of the conduction and heavy hole valence bands are taken into account in the SFM model.

Chapter 3 Polarization Investigation of Vertical Cavity Surface Emitting Lasers

A basic feature of the SFM is that the dominant electron-hole recombination in the QW between conduction band and heavy-hole bands occurs through two distinct carrier densities. The two carrier densities are associated with spin-up and spin-down and they are coupled by spin-flip processes which tend to equalize the two carrier densities. Recombination of electrons with spin-up or spin-down gives rise to two lasing transitions associated with right- or left-handed circularly polarization emission, which are coupled through birefringence [33].

Fig. 3.1 depicts the main ingredients of the model, which features two coupled lasing transitions associated with right (E_+) and left (E_-) circularly polarized output fields [34]. The electron states of the conduction band have a total angular momentum quantum number $J = 1/2$. The upper valence bands are commonly known as heavy hole and light hole. For bulk material, the heavy hole and light hole bands are degenerate at the center of the band gap with a total angular momentum quantum number $J = 3/2$. For quantum wells, the quantum confinement removes the degeneracy. In the case of unstrained quantum wells, the heavy hole band, which is associated with $J_z = \pm 3/2$, has a higher energy. The transitions from the conduction to the light hole bands are neglected. The quantum allowed transitions are those in $\Delta J_z = \pm 1$. Thus two transitions are allowed between the conduction band and the heavy hole band. The transition from $J_z = -1/2$ to $J_z = -3/2$ is associated with right circularly polarized light and transition from $J_z = 1/2$ to $J_z = 3/2$ is associated with left circularly polarized light.

Fig. 3.1 Four-level model for polarization dynamics in quantum well vertical cavity surface emitting lasers.

The four level model describe the transitions can be written as [32]

$$\frac{dE_{\pm}}{dt} = -\kappa E_{\pm} - i\omega_0 E_{\pm} + \kappa(1 + i\alpha)(N \pm n)E_{\pm} \quad (3.1)$$

$$\frac{dN}{dt} = -\gamma(N - \mu) - \gamma(N + n)|E_+|^2 - \gamma(N - n)|E_-|^2 \quad (3.2)$$

$$\frac{dn}{dt} = -\gamma_s n - \gamma(N + n)|E_+|^2 + \gamma(N - n)|E_-|^2 \quad (3.3)$$

The variable N represents the total carrier number in excess of its value at transparency, normalized to the value of that excess at the lasing threshold. The variable n represents the difference in the carrier numbers of the two magnetic sublevels normalized in the same way as N . μ is the normalized injection current, κ is the field decay rate, γ is the decay rate of the total population inversion, γ_s is the spin-flip rate that accounts for the mixing of the population with opposite spin.

Incorporating the linear phase anisotropy and the linear amplitude anisotropy in (3.1), then the Eq. (3.1) can be rewritten as [32]

Chapter 3 Polarization Investigation of Vertical Cavity Surface Emitting Lasers

$$\frac{dE_{\pm}}{dt} = -\kappa(1+i\alpha)(N \pm n-1)E_{\pm} - i\gamma_p E_{\mp} - \gamma_a E_{\mp} \quad (3.4)$$

where parameter γ_a represents the linear amplitude anisotropy, γ_p represents the linear phase anisotropy.

The meaning of the linear phase anisotropy γ_p and the linear amplitude anisotropy γ_a can be clearly displayed if these equations are rewritten in terms of the orthogonal linear components of the electric field:

$$E_x = \frac{E_+ + E_-}{\sqrt{2}} \quad (3.5)$$

$$E_y = -i \frac{E_+ - E_-}{\sqrt{2}} \quad (3.6)$$

then for the $\hat{x}$ – and $\hat{y}$ – polarized components the complete model becomes [32]

$$\frac{dE_x}{dt} = -(k + \gamma_a)E_x - i(k\alpha + \gamma_p)E_x + k(1+i\alpha)(NE_x + inE_y) \quad (3.7)$$

$$\frac{dE_y}{dt} = -(k - \gamma_a)E_y - i(k\alpha - \gamma_p)E_y + k(1+i\alpha)(NE_y - inE_x) \quad (3.8)$$

$$\frac{dN}{dt} = -\gamma[n(1 + |E_x|^2 + |E_y|^2) - \mu + in(E_y E_x^* - E_x E_y^*)] \quad (3.9)$$

$$\frac{dn}{dt} = -\gamma_s n - \gamma[n(|E_x|^2 + |E_y|^2) + iN(E_y E_x^* - E_x E_y^*)] \quad (3.10)$$

where it is clear that γ_p leads to a frequency difference of $2\gamma_p$ between the $\hat{x}$ – and $\hat{y}$ – polarized solutions and that γ_a leads to different thresholds for the linearly polarized solutions.

3.2 Polarization stability analysis of VCSEL

The stability of a particular solution is studied by writing the solution of (3.2-3.4) as [32]

$$E_{\pm} = (A_{\pm} + a_{\pm}) e^{i(\omega_{\pm} t \pm \psi)} \quad (3.11)$$

$$N = N_0 + \Delta \quad (3.12)$$

$$n = n_0 + \delta \quad (3.13)$$

where $A_{\pm}$ is the amplitude, $a_{\pm}$ is a complex perturbation of the field amplitude, Δ and δ are perturbations of carrier variable of N and n .

After substituting the solution (3.11-3.13) into (3.2-3.4), we can get the following linear coupled differential equations for $a_{\pm}$, Δ , and δ as [32]

$$\frac{da}{dt} = k(1+i\alpha)(N_0 \pm n_0 - 1)a_{\pm} - i\alpha k a_{\pm} + \kappa(1+i\alpha)(\Delta \pm \delta)A_{\pm} - i\gamma_p a_{\pm} e^{\mp i2\psi} - \gamma_a a_{\pm} e^{\mp i2\psi} \quad (3.14)$$

$$\frac{d\Delta}{dt} = -\gamma(N_0 + n_0)A_+(a_+ + a_+^*) - \gamma(N_0 - n_0)A_-(a_- + a_-^*) - \gamma(1 + A_+^2 + A_-^2)\Delta - \gamma(A_+^2 - A_-^2)\delta \quad (3.15)$$

$$\frac{d\delta}{dt} = -\delta(N_0 + n_0)A_+(a_+ + a_+^*) + \gamma(N_0 - n_0)A_-(a_- + a_-^*) - \gamma(A_+^2 - A_-^2)\Delta - [\gamma_s + \gamma(A_+^2 + A_-^2)]\delta \quad (3.16)$$

Eqs of (3.14-3.16) can be written in a vectorial form as

$$\partial_t \vec{X} = M \vec{X} \quad (3.17)$$

where $\vec{X} = (a_+, a_+^*, a_-, a_-^*, \Delta, \delta)$, and M is a 6×6 matrix derived from the Eqs (3.14-3.16). The linear stability of a steady state solution is given by the real parts

Chapter 3 Polarization Investigation of Vertical Cavity Surface Emitting Lasers

of the eigenvalues λ which indicate if the solution is stable (when $\text{Re}(\lambda) < 0$ for all λ) or unstable (when $\text{Re}(\lambda) > 0$ for at least one λ).

According the deduced procedure proposed in [34], the stability of the solutions can be determined by the following polynomial for λ [32]

$$P(\lambda) = \lambda^3 + \left(\frac{\gamma_s}{\gamma} + 2A^2 \mp 4 \frac{\gamma_a}{\gamma} \right) \lambda^2 + 4 \left[\frac{\kappa}{\gamma} A^2 N_0 + \left(\frac{r_a}{\gamma} \right)^2 + \left(\frac{r_p}{\gamma} \right)^2 \mp \frac{\gamma_a}{\gamma} \left(\frac{\gamma_s}{\gamma} + 2A^2 \right) \right] \lambda + 4 \left[\left(\frac{r_a}{\gamma} \right)^2 + \left(\frac{r_p}{\gamma} \right)^2 \right] \left(\frac{\gamma_s}{\gamma} + 2A^2 \right) \mp 8 \frac{\kappa}{r^2} A^2 N_0 \left(\frac{\gamma_a}{\gamma} + \alpha \frac{\gamma_p}{\gamma} \right) \quad (3.18)$$

where the upper sign holds for the stability of the linear $\hat{y}$ – polarized solution, while the lower sign holds for the stability of the linear $\hat{x}$ – polarized solution. The stability of the linearly polarized solutions is determined by the zeroth-order term of (3.18). where for the $\hat{x}$ – polarized solution

$$A^2 = \frac{1}{2} \frac{\mu - N_0}{N_0}, \quad N_0 = 1 + \frac{\gamma_a}{\kappa} \quad (3.19)$$

and for the $\hat{y}$ – polarized solution

$$A^2 = \frac{1}{2} \frac{\mu - N_0}{N_0}, \quad N_0 = 1 - \frac{\gamma_a}{\kappa} \quad (3.20)$$

In the following calculation, two cases are considered. The first case is when $\gamma_a / \gamma = -0.2$, the second case is $\gamma_a / \gamma = 0.2$. In the calculation, $\gamma = 10^9$, $\kappa = 300 \times 10^9$, $\gamma_s = 50 \times 10^9$, $\alpha = 3$. Figure 3.2 shows the stability diagram when γ_a is negative and $\gamma_a / \gamma = -0.2$, the stability for the $\hat{x}$ – polarized and $\hat{y}$ – polarized solution is indicated in the figure. We can see there are zones in

which only one mode is stable, zones of bistability and zones in which neither linearly polarized mode is stable. The $\hat{x}$ -polarized emission is stable below the dotted curve and $\hat{y}$ -polarized emission is stable between solid curves. It can be figured out that the $\hat{x}$ -polarized emission is favored in this case for the fact that its lasing threshold current is lower than the threshold of the $\hat{y}$ -polarized emission. On the other hand we observe at the domain $0.05\gamma < \gamma_p < 6.0\gamma$, when $\mu > 1.1$ the $\hat{y}$ -polarized state is stable, however $\hat{x}$ -polarized state is unstable. This characteristic indicates that the emission will switch from $\hat{x}$ -polarized state to $\hat{y}$ -polarized state as the current is increased.

Fig. 3.2 Stability diagram for $\gamma_a/\gamma = -0.2$, $\hat{x}$ -polarized emission favored under threshold.

Figure 3.3 shows the stability diagram when γ_a is positive and $\gamma_a/\gamma = 0.2$, the figure shows the $\hat{x}$ -polarized emission is stable between dotted curve, $\hat{y}$ -polarized emission is stable below and to the left the solid curve. We can

Chapter 3 Polarization Investigation of Vertical Cavity Surface Emitting Lasers

observe from the figure that the $\hat{y}$ – polarized emission is favored because its lasing threshold is lower than $\hat{x}$ – polarized emission. And when $\gamma_p > 10.0\gamma$ the emission will switch from $\hat{y}$ – polarized state to $\hat{x}$ – polarized as the current increases.

Fig. 3.3 Stability diagram for $\gamma_a/\gamma = 0.2$, $\hat{y}$ – polarized emission favored under threshold current.

Figure 3.4 shows the polarization state switching evolution from $\hat{x}$ – polarized emission to $\hat{y}$ – polarized emission. In the simulation, $\mu = 1.5$ and $\hat{y}$ – polarized is set at dominant at the beginning. We can see due to the anisotropies in both amplitude and phase, $\hat{x}$ – polarized emission will be dominant and the $\hat{y}$ – polarized emission is suppressed. The parameters for the happening of the switching locate at the zone in which only one mode is stable in Fig. 3.2.

Fig. 3.4 Polarization state switch from $\hat{x}$ – polarized emission to $\hat{y}$ – polarized emission when $\gamma_a / \gamma = -0.2$.

when $\gamma_a / \gamma = 0.2$, as shown in Fig. 3.5, switching evolution from $\hat{x}$ – polarized emission to $\hat{y}$ – polarized emission is observed. The parameters for the happening of the switching locate at the zone in which only one mode is stable in Fig. 3.3.

Fig. 3.5 Polarization state switch from $\hat{y}$ – polarized emission to $\hat{x}$ – polarized emission when $\gamma_a / \gamma = 0.2$.

3.3 DBR effects on the polarization switching

3.3.1 Improve transmission line laser model by incorporating into spin-flip model (SFM)

The polarization control of VCSELs is crucial for the applications in polarization-sensitive equipment such as magneto-optic disks and coherent detection systems [35]. Several methods have been proposed to stabilize the polarization of VCSELs such as with anisotropic transverse cavity geometries [36] or anisotropic stress to the gain layer [37]. However these methods required re-engineering of the devices structure. External optical injection is an alternative method to stabilize the polarization of lasers. It is believed that polarization switching can be obtained by injecting into the laser an optical signal whose polarization is orthogonal to that

emitted by the lasers [28]. Two different situations need to be considered. If the system is bistable, once switch of polarization state happens due to a sufficiently strong pulse injected signal, the laser will remain on the new state. On the other hand, if the system is monostable, the laser will go back to the initial state after the injected signal is removed.

In this section, we will consider a nonresonant optical pump, so that by changing the pump ellipticity one can selectively favor one of the two lasing transitions. We consider the polarization dynamics of optically pumped vertical-cavity surface-emitting lasers with a pump light and the VCSEL works on bistable condition.

As described in Chapter 2, we have applied transmission line laser model (TLLM) to study DFB LDs. To model the passive DBR parts we need make some modification on the existed TLLM. The main modification is on the scattering matrix of the DBR parts. For those DBR sections we assume the parameter $g = 0$ in equation (2.2) of scattering matrix which indicates there is no gain in these DBR sections. To take the effects of the DBR into consideration, we have developed transmission line laser model by incorporating the SFM model into the TLLM. That is the laser model is composed by scattering process and connecting process. In the scattering process, the gain section will be described with SFM model.

The field propagation along the laser cavity can be composed by scattering process and connecting process in the developed TLLM model as described in the following subsections.

3.3.2 Scattering process in the gain section

In the m th iteration, the scattering process which takes into account the gain and the interference of $\hat{x}$ - and $\hat{y}$ - polarized S section can be represented using following time-stepping algorithm:

$${}_m \begin{bmatrix} E_x(S) \\ E_y(S) \end{bmatrix}^R = S_A^* {}_m \begin{bmatrix} E_x(S) \\ E_y(S) \end{bmatrix}^I \quad (3.21)$$

$${}_m \begin{bmatrix} B_x(S) \\ B_y(S) \end{bmatrix}^R = S_A^* {}_m \begin{bmatrix} B_x(S) \\ B_y(S) \end{bmatrix}^I \quad (3.22)$$

$$S_A = \begin{bmatrix} [-(k+\gamma_a)-i(k\alpha+\gamma_p)+k(1+i\alpha)N(S)]\Delta t+1 & ik(1+i\alpha)n(S)\Delta t \\ -ik(1+i\alpha)n(S)\Delta t & [-(k-\gamma_a)-i(k\alpha-\gamma_p)+k(1+i\alpha)N(S)]\Delta t+1 \end{bmatrix} \quad (3.23)$$

where R is the reflected wave from the S section, and I is the incident wave to that section. In the m th iteration with time step Δt , $N(S)$ and $n(S)$ are updated with the information of field of $E_x(S)$ and $E_y(S)$, S_A is the scattering matrix for gain sections. It should be noted that the electric forward wave $E_x(S)$ and $E_y(S)$ coexist with backward waves $B_x(S)$ and $B_y(S)$.

3.3.3 Connecting process

The connecting process which connecting the previous section with the next section can be expressed as

$${}_{m+1} \begin{bmatrix} E_x(S+1) \\ B_x(S) \end{bmatrix}^I = C_P^* {}_m \begin{bmatrix} E_x(S) \\ B_x(S+1) \end{bmatrix}^R \quad (3.24)$$

$$\begin{bmatrix} E_y(S+1) \\ B_y(S) \end{bmatrix}_{m+1}^T = C_p^* \begin{bmatrix} E_y(S) \\ B_y(S+1) \end{bmatrix}_m^R \quad (3.25)$$

It should be noted that in the gain sections, the connecting matrix C_p is an identity matrix. However, in the DBR sections, coupling exists between the forward wave and the backward wave, C_p should be able to represent this coupling effect. To demonstrate the wave changes at the DBR reflectors, we employ the connecting matrices using effective index method [38]:

$$C_p = \begin{bmatrix} 2\sqrt{n_l n_h} / (n_l + n_h) & -(n_l - n_h) / (n_l + n_h) \\ (n_l - n_h) / (n_l + n_h) & 2\sqrt{n_l n_h} / (n_l + n_h) \end{bmatrix} \quad (3.26)$$

which represents the connecting matrix when the wave travel from low-index layer to high-index layer and

$$C_p = \begin{bmatrix} 2\sqrt{n_l n_h} / (n_l + n_h) & -(n_h - n_l) / (n_l + n_h) \\ (n_h - n_l) / (n_l + n_h) & 2\sqrt{n_l n_h} / (n_l + n_h) \end{bmatrix} \quad (3.27)$$

which describes the wave traveling from the high-index to low-index layer. Here n_l is the effective index of the low index layer and n_h is the effective index of the high index layer.

3.3.4 Carrier rate equations

By taking the backward wave into consideration the carrier rate equation (3.2) and (3.3) is rewritten as

$$\frac{dN}{dt} = -\gamma[n(1 + |E_x|^2 + |E_y|^2 + |B_x|^2 + |B_y|^2) - \mu + in(E_y E_x^* - E_x E_y^* + B_y B_x^* - B_x B_y^*)] \quad (3.28)$$

$$\frac{dn}{dt} = -\gamma_s n - \gamma[n(|E_x|^2 + |E_y|^2 + |B_x|^2 + |B_y|^2) + iN(E_y E_x^* - E_x E_y^* + B_y B_x^* - B_x B_y^*)] \quad (3.29)$$

3.3.5 Simulation results

The longitudinal distribution of optical field inside the VCSEL is dependent on the design of Bragg reflectors. Bragg reflectors have great effects on the coupling strength between the external injection light and active layer which will play important roles in changing the bistability state. The influence of Bragg reflectivity on the bistable mechanism of VCSELs is studied in this section.

Fig. 3.6 The steady state characteristics of VCSEL, X – and Y – polarized light outputs versus Y – polarized light input. The numbers of layers of Bragg reflectors are A=48, B=53 and C=58.

We have simulated the steady state characteristics of VCSEL by the external injection light with the number of Bragg reflectors for $A = 48$, $B = 53$ and $C = 58$ respectively. As we can see from Fig. 3.6 when the number of the DBR layers is increased, larger width of the hysteresis loop is observed and on the other hand high input optical switching power is also required. This can be explained by the fact that with the number of DBR layer is increased, the coupling strength of the injected optical power and active layer is also increased. Due to the increasing coupling strength, the input power can vary in large region, thus wide hysteresis loops will be obtained. On the other hand, due to increasing the number of DBR layer, the VCSEL output power will be increased thus higher injected optical power is required to carry out the polarization switching.

3.4 Summary

Theoretical studies on the polarization stability and the influence of the Bragg reflectivity on the polarization switching of VCSEL have been discussed in this chapter. It is found that the reflectivity of Bragg reflectors affects the bistable characteristics of VCSEL's. Our results show that for devices with low reflectivity (small number of dielectric layers of Bragg reflectors), low input power is required to achieve bistable mechanism but the coupling strength between the injected light and DBR layers is reduced (narrow hysteresis loops). On the other hand, for VCSELs with high reflectivity, coupling strength is improved but at the expense of high external injection power. Thus reflectivity of Bragg reflectors can be optimized to provide suitable hysteresis loops width which indicates the tolerance of the input power.

CHAPTER 4

Chapter 4 Design of Two-Dimension VCSEL-Based Wavelength Converter

4.1 Introduction of the wavelength converters

Semiconductor optical amplifier (SOA) based wavelength converters have the potential to support wavelength switching in WDM optical networks [39,40]. In the wavelength converter intensity modulated input signal at the wavelength λ_{Signal} (called signal) is used to modulate the carrier density of the SOA. With the modulation of the λ_{Signal} , the (inverted) data can be copied onto another continuous wave on the wavelength λ_{Probe} (called probe). The signal wave and the probe wave can be injected to the SOA either co- or counter-propagating (Fig. 4.1). However shortcomings existed in these two configurations. For the co-propagating injection, an optical filter is needed to suppress the signal wave. For the counter-propagating method, though optical filter is avoided, additional isolator at the probe input is needed to suppress the strongly amplified input signal. Moreover, in the latter configuration smaller bandwidth as well as an enhanced amplified spontaneous emission (ASE) noise level is suffered [41].

Two dimension side-injection light-controlled bistable laser diodes (SILC-BLD) had been reported by K. Nonaka [42, 43]. These reports revealed that SOA based

2-D devices were quite promising for optical signal processing. In this chapter a novel two-dimension VCSEL based wavelength converter is proposed. In the configuration, the VCSEL acts both as the laser source for the probe wave and as the SOA to perform cross-gain modulation effect. The probe wave and the signal wave will propagate along the orthogonal directions with the advantage of simplifying the structure. We develop a modified transmission line laser model (TLLM) to describe the VCSEL based wavelength converter. The dynamic characteristics of the proposed wavelength converter had been investigated.

Fig. 4.1 Co-propagating and counter-propagating wavelength converters.

4.2 Design of the two dimension VCSEL wavelength converters

To remove the optical filter or the optical isolator from the wavelength converters, the probe wave and the signal wave can be incident on the SOA along orthogonal directions (Fig. 4.2). Owing to the limitation of the incident wave intensity, the length of the SOA should be very short. However, the cavity as short as $10\ \mu\text{m}$ is very difficult to fabricate [44], and it is not easy to get stable operation condition for such short cavity laser. To overcome this difficulty, VCSEL can be employed for the advantage of its short cavity [45,46]. A novel VCSEL based wavelength

Chapter 4 Design of Two Dimension VCSEL Based Wavelength Converter

converter is proposed as shown in Fig. 4.3. Here we adopted the VCSEL structure [38][47] and extended the active region at the horizontal direction. In the configuration, the signal wave will incident at the horizontal direction and the probe wave will output at the vertical direction, thereby a two-dimension wavelength converter is realized. In this structure, the extended active region was used to amplify the input signal wave, which guarantees a strong cross-gain modulation effect. By this means, high extinction ratio of the probe wave can be achieved.

Fig. 4.2 Schematic of two-dimension wavelength converter.

Fig. 4.3 VCSEL based two-dimension wavelength converter.

In the proposed configuration, the VCSEL is used both as the laser source for the probe wave and as SOA for performing cross-gain modulation. By employing this structure, two main advantages will be provided. First, additional devices such as optical filter or optical isolator are not required anymore. Second, additional laser source for the probe wave can be saved because VCSEL works as a laser source as well as an amplifier.

4.3 Developed two-dimension transmission line laser model

To analyze the characteristics of the proposed structure, a two-dimension TLLM is required. Details of one-dimension TLLM has been given in [20]. The schematic of the modified TLLM model for the two-dimension VCSEL based wavelength converter is shown in Fig.4.4. In the developed model the forward and backward probe waves are named as F_p and B_p , and those of the signal wave are named as F_s and B_s , respectively. We describe the scattering process at time t of these four waves in the active region as:

$${}_t[F_p(n, m)]^r = S_p * {}_t[F_p(n, m)]^i + {}_t[I_{sp}(n, m)Z_p T / 2] \quad (4.1)$$

$${}_t[B_p(n, m)]^r = S_p * {}_t[B_p(n, m)]^i + {}_t[I_{sp}(n, m)Z_p T / 2] \quad (4.2)$$

$${}_t[F_s(n, m)]^r = S_s * {}_t[F_s(n, m)]^i + {}_t[I_{sp}(n, m)Z_p T / 2] \quad (4.3)$$

$${}_t[B_s(n, m)]^r = S_s * {}_t[B_p(n, m)]^i + {}_t[I_{sp}(n, m)Z_p T / 2] \quad (4.4)$$

where i denotes the incident wave, r denotes the reflective wave, S_s is the scattering matrix for the signal wave, S_p is the scattering matrix for the probe wave, I_{sp} is the spontaneous noise current, T is the attenuation factor, n and m are the section numbers in the traveling direction of the probe wave and the signal

Chapter 4 Design of Two Dimension VCSEL Based Wavelength Converter

wave, respectively. Z_p is the cavity wave impedance given as $Z_p = \overline{n_e} 120\pi / n_e$, where $\overline{n_e}$ is the effective group index, n_e is the effective index, Scattering matrices S_s and S_p can be found in [20]. The ASE noise is modeled with noise current I_{sp} , which is represented by three statistically independent Gaussian distributed random processes that satisfy the following correlation [20], [48]:

$$\langle I_{sp}(z, x, t) I_{sp}^*(z, x, t) \rangle = 2\beta L R_{sp} \delta(z - z') \delta(x - x') \delta(t - t') \times hf / Z_p \quad (4.5)$$

where β is the spontaneous coupling factor, R_{sp} is the spontaneous emission rate which is assumed as bimolecular recombination (BN^2), $\delta(x)$ is the Delta-function, hf is the photon energy and L is the laser cavity length.

Fig. 4.4 Schematic of the two-dimension TLLM.

The information of the material gain is required in the scattering process. In our model material gain depends on both the carrier density and the wavelength, given as [48]

$$g(N, \lambda_k) = \frac{a_0 \ln(N / N_0) - a_1 (\lambda_k - \lambda_p)^2 + a_2 (\lambda_k - \lambda_p)^3}{1 + \varepsilon(P_s + P_p)} \quad (4.6)$$

where a_0, a_1 , and a_2 are gain constants, λ_p is the gain-peak wavelength, ε is the gain compression factor, N is the local carrier density and N_0 is the transparent carrier density. The gain for the probe wave or the signal wave can be obtained when λ_k is set as λ_{Probe} or λ_{Signal} .

The connecting process which connects two scattering nodes together can be written as [20]:

$${}_{t+1}[F_p(n+1, m)]^i = C_p * {}_t[F_p(n, m)]^r \quad (4.7)$$

$${}_{t+1}[B_p(n, m)]^i = C_p * {}_t[B_p(n+1, m)]^r \quad (4.8)$$

$${}_{t+1}[F_s(n, m+1)]^i = C_s * {}_t[F_s(n, m)]^r \quad (4.9)$$

$${}_{t+1}[B_s(n, m)]^i = C_s * {}_t[B_s(n, m+1)]^r \quad (4.10)$$

where C_p is the connecting matrix for the probe wave and C_s is the connecting matrix for the signal wave. It should be noted that C_p and C_s are identity matrices for those sections where there are no distributed Bragg reflectors (DBR) such that no reflections happen between forward waves and backward waves.

However, out of the cross-gain modulation area, DBR exists in the vertical direction. In these sections C_p is not identity matrix anymore and should be modified to reflect the wave coupling and reflection in DBR. The connecting process in these sections is described as

$${}_{t+1} \begin{bmatrix} F_p(n+1, m) \\ B_p(n, m) \end{bmatrix}^i = C_p * {}_t \begin{bmatrix} F_p(n, m) \\ B_p(n+1, m) \end{bmatrix}^r \quad (4.11)$$

In the previous model, gratings can be modeled by connecting two adjacent scattering matrices with impedance unmatched transmission lines [22]. However, the coupling coefficient is difficult to calculate and cannot well reflect the effects

Chapter 4 Design of Two Dimension VCSEL Based Wavelength Converter

of the DBR in VCSEL. To represent wave changes at the Bragg reflectors, we employ the connecting matrices using effective index method [38]:

$$C_P = \begin{bmatrix} 2\sqrt{n_l n_h} / (n_l + n_h) & -(n_l - n_h) / (n_l + n_h) \\ (n_l - n_h) / (n_l + n_h) & 2\sqrt{n_l n_h} / (n_l + n_h) \end{bmatrix} \quad (4.12)$$

which represents the connecting matrix when the wave travel from low-index layer to high-index layer and

$$C_P = \begin{bmatrix} 2\sqrt{n_l n_h} / (n_l + n_h) & -(n_h - n_l) / (n_l + n_h) \\ (n_h - n_l) / (n_l + n_h) & 2\sqrt{n_l n_h} / (n_l + n_h) \end{bmatrix} \quad (4.13)$$

which describes the wave traveling from the high-index to low-index layer. Here n_l is the effective index of the low index layer and n_h is the effective index of the high index layer.

After the scattering and connecting processes, local photon density can be obtained from the local field intensity [20]. Carrier density will be updated with the information of the local photon density. The carrier density will be determined by the photon density of both probe wave and signal wave, given as [49,50]

$$\frac{dN_{MQW}}{dt} = \frac{N_{SCH}}{\tau_r} \cdot \frac{d_{SCH}}{d_{MQW}} - \frac{N_{MQW}}{\tau_e} - \frac{N_{MQW}}{\tau_{rec(MQW)}} - v_g (\Gamma_P g_P P_P + \Gamma_S g_S P_S) \quad (4.14)$$

$$\frac{dN_{SCH}}{dt} = \frac{J}{e \cdot d_{SCH}} - \frac{N_{SCH}}{\tau_r} + \frac{N_{MQW}}{\tau_e} \cdot \frac{d_{MQW}}{d_{SCH}} - \frac{N_{SCH}}{\tau_{rec(SCH)}} \quad (4.15)$$

where N_{MQW} is the carrier density in the multiple quantum well active layer, N_{SCH} is the carrier density in the separate confinement layer, d_{MQW} and d_{SCH} are the thickness of the active layer and the separate confinement layer, respectively; v_g is the group velocity, τ_e is the carrier emission constant, τ_{rec} is the recombination time and τ_r is the carrier diffusion time, e is the charge of the electron, J is the injection current density, P_s and P_p are the photon densities, g_s and g_p are the material gains of the signal wave and the probe wave, respectively.

The frequency chirp of the converted signals is calculated by differentiating its phase with respect to time $\{1/(2\pi)(d\Phi/dt)\}$ [48]. And the phase differentiation due to the change of the refractive index can be calculated through the variation of the carrier density with time [24].

4.4 Analysis of the proposed structure

4.4.1 Extinction ratio and peak chirp character studies

We have analyzed the proposed structure by applying the proposed TLLM [51] [52]. Simulation parameters are given in Table 4.1. Time domain response has been investigated at the bit rate of 2.5Gbits/s. Figure 4.5 shows the pulse patterns of signal wave (upper) and probe wave (middle) as well as frequency chirp (lower) at the bit rate of 2.5Gbits/s. In the simulation the signal wave is located at the gain-peak and the probe wavelength was at -40nm shift from the gain-peak, i.e. $\lambda_{signal} = 1565\text{nm}$ and $\lambda_{probe} = 1525\text{nm}$. We observe that the probe signal undergoes overshoot on the rising edge. This can be explained by the fact that the carriers are greatly consumed by the signal wave when it is at high state ('1' state), thus the

front end of the probe wave experiences overshoot in the gain recovery process when the signal wave at the low state ('0' state). The result also reveals that the '1' state power of the signal wave is as high as 25mw to get an extinction ratio of the probe wave larger than 10dB. This is the reason why we have to amplify the signal wave first before cross-gain modulation. In the simulation the input signal powers refer to the power after amplification. Moreover the effects of ASE noise on the probe wave can be seen in Fig.4.5. It is shown that ASE noise at low state is smaller than at high state, since gain saturation may reduce the ASE noise at the high input signal power.

Fig. 4.5 Pulse pattern of signal wave (upper), probe wave (middle) and frequency chirp (lower) at 2.5Gbps ($R_1=97.4\%$, $R_2=55\%$, $\lambda_{signal} = 1565nm$ and $\lambda_{probe} = 1525nm$, the ER of input signal=10dB, Input power=25mW).

Table 4.1 Parameters for VCSELs

Parameter	Symbol	Value
Gain peak wavelength	λ	1565 nm
Active layer length	d	0.75 μm
Radius of core region	r	1.125 μm
Facet reflectivity *	R_1	0.974
Facet reflectivity *	R_2	0.55
Attenuation	T	0.998
Wave impedance	Z_p	123.1 Ω
Transparency carrier concentration *	N_0	$2.0 \times 10^{24} \text{ m}^{-3}$
Spontaneous coefficient	β	5×10^{-4}
Bimolecular recombination	B	$1.4 \times 10^{-16} \text{ m}^3 / \text{s}$
	a_0	$1.5 \times 10^5 \text{ m}^{-1}$
	a_1	$0.074 \times 10^{20} \text{ m}^{-3}$
Material gain constant	a_2	$3.155 \times 10^{25} \text{ m}^{-4}$
	ϵ	$1.3 \times 10^{-23} \text{ m}^3$
Nonlinear gain compression factor	ϵ	$1.3 \times 10^{-23} \text{ m}^3$
Confinement factor *	Γ	0.158
Effective index of high-index layer of Bragg reflector *	n_h	3.504
Effective index of low-index layer of Bragg reflector *	n_l	2.952

* Parameters are obtained from [47]

Chapter 4 Design of Two Dimension VCSEL Based Wavelength Converter

The evolution of extinction ratio and peak chirp (peak of the frequency chirp) of the probe wave in the transverse direction at different facet reflectivity is shown in Fig.4.6. Here extinction ratio is defined as $10 \log[Power(1)/Power(0)] dB$. Where $Power(1)$ represents the power of state “1” and $Power(0)$ represents the power of state “0”. From the definition of extinction ratio, it is obvious that the higher the extinction ratio it is easier to distinguish state “1” from state “0”. We observe that ER increases along the transverse direction because of the amplification of the signal power in this direction. And the lower the reflectivity, the higher ER is obtained due to the stronger amplification effect. On the other hand, the peak chirp is also affected by the reflectivity. With high reflectivity, the amplification of the signal wave will be reduced as more carriers are consumed by the probe wave. Therefore, the cross gain modulation will be weakened which results in longer rise time and fall time for the probe wave. Fig.4.6 reveals that high reflectivity leads to a lower peak chirp frequency because of the longer rise time and fall time.

Fig. 4.6 The evolution of peak chirp frequency and extinction ratio in the transverse direction of the probe wave at different facet reflectivity ($R_1 = 97.4\%$).

Fig. 4.7 Negative peak chirp frequency and extinction ratio of the probe wave as a function of input signal power at with different wavelength locations ($R_1=97.4\%$, $R_2=55\%$, $\lambda_{\text{signal}} = 1565\text{nm}$).

We have investigated the effects of input power on the extinction ratio and chirping in Fig. 4.7. In the calculation, wavelength difference of signal wave and probe wave $\Delta\lambda$ equal to -40nm , -20nm and 40nm respectively is analyzed where signal wavelength is fixed at the peak of the gain spectrum. In the simulation, the input ER is kept at 10dB as the power of input signal varies. Higher ER and peak chirp have been observed when $\Delta\lambda$ is -40nm rather than 40nm . This is due to the asymmetrical gain profile. It showed that the up conversion (probe wavelength is longer than signal wavelength) can provide large ER in our simulated device. From the results we see that the ER and the negative peak chirp frequency increase with the input power, however the ER saturates when the input power is above 80mW. The observed ER saturation can be explained as following. The ER of the probe wave depends on the “1” state level of the signal wave. As

“1” state of input signal increases to a specified level, it will consume almost all the carriers and make the probe wave work under lasing threshold. After this critical point, even the input power increased, the “0” state of the probe wave will keep at a constant. On the other hand, the “0” state of the input signal has little effect on the cross-gain modulation. Therefore the output of the probe wave will mainly depend on the “1” state of the probe wave.

4.4.2 Wavelength optimization

As we have described in equation (4.6), the material gain is asymmetrical with wavelength locations. In this section we will study the effect of the wavelength on the conversion performance. Furthermore optimization of the wavelength location will be investigated.

Fig. 4.8 Contour plot of the extinction ratio with respect to signal wavelength and probe wavelength ($R_1=97.4\%$, $R_2=55\%$, the ER of input signal=10dB, Input power=25mW).

Figure 4.8 shows the contour plot of the output extinction ratio (ER) for the signal wavelength and the probe wavelength in the region of 1515~1615nm. Owing to the asymmetric gain profile, we observe that down-conversion (λ_{probe} is shorter than λ_{Signal}) leads to a higher extinction ratio than that of up-conversion (λ_{probe} is longer than λ_{Signal}). The peak chirp contour map with respect to the signal and the probe wavelengths can also be calculated. Figure 4.9 is the contour map of the positive peak chirp. The negative peak chirp map is similar to the peak chirp except the negative peak chirp is larger than the positive peak chirp. We observe that at the wavelength where high extinction ratio is obtained, large peak chirp frequencies also appear due to shorter rise time and fall time of the converted wave. Therefore optimum wavelength region of signal wave and probe wave can be found by considering both of the extinction ratio and peak chirp frequency. In Fig.4.10, the shaded region indicates the optimum wavelength region, where extinction ratio is higher than 10dB, and peak chirp frequency is between -3.5GHz and 3GHz.

Fig. 4.9 Contour plot of the positive peak chirp with respect to signal wavelength and probe wavelength.

Fig. 4.10 Optimization of the signal wavelength and the probe wavelength (shaded region) with respect to both of the extinction ratio and peak chirp frequency.

4.4.3 Reflectivity optimization

In this section we will focus on investigating the effects of the facet reflectivity on the extinction ratio (ER) and frequency peak chirp in the proposed structure. To investigate the effect of the reflectivity in the wider range, the modulation bit rate is set at 1Gbits/s other than 2.5Gbits/s used in previous sections. This is due to the transient period of the probe wave is at small ratio of the “1” or “0” state hold time at 1Gbits/s other than 2.5Gbits/s. Thus it is possible to get the exactly cross gain modulation even at low facet reflectivity.

Fig. 4.11 Extinction ratio of the probe wave as a function of facet reflectivity for different input powers. ($R_1=97.4\%$).

The following investigation is focus on the effects of facet reflectivity on the ER and peak chirp of the probe wave. In our analysis R_1 is fixed at 97.4%, and R_2 varies from 15%~65% with signal wave at different input powers. Figure 4.11

shows the variation of ER of the probe wave as a function of facet reflectivity at different input levels. It was observed that in Fig. 4.11 that ER showed saturation characteristics with the changes of the reflectivity R_2 . The appearance of peak values of the ER can be explained by the following two reasons. (1) When signal wave at “0” state, the “1” state output power of the probe wave will increase with the increase of the reflectivity. However at certain high reflectivity, the output probe wave will be fixed at one value due to the gain saturation effect. (2) When signal wave at “1” state, the probe wave will be depressed totally at low reflectivity. At this case, stimulation emission of the probe wave is possible, which means the probe wave cannot be depressed totally. Thus the output power of the probe wave will become obviously increased. With the combination of the facts (1) and (2), a maximum value will be produced at certain reflectivity.

Fig. 4.12 Negative frequency peak chirp of the probe wave as a function of facet reflectivity for different input powers. ($R_1=97.4\%$).

The effects of the input power of the signal wave on the ER of the probe wave can also be observed in Fig.4.11. It is shown that at high reflectivity, the ER of the probe wave will increase with the input power. However, at low reflectivity, the ER will decrease with the increase of the input power. These results can be explained as following: at high reflectivity, the ER mainly depends on the “1” state of the signal wave. At high input power, the emission of the probe wave will be highly depressed, which will produce high extinction ratio. On the other hand, when the reflectivity is low, the ER mainly depends on the “0” state of the signal wave. At high “0” state input signal power, the probe output “1” will be low which results the low extinction ratio.

Negative peak frequency chirp of the probe wave as a function of facet reflectivity at different input powers is shown in Fig.4.12. The method to calculate the peak chirp has already been given in [48]. It is observed that the peak chirp is high when the reflectivity is low or the input power is high. This is due to the strong cross gain modulation produced at low reflectivity and high input signal power.

The optimization of the facet reflectivity regarding both ER and peak chirp can be done now. From Fig4.11 and Fig 4.12, we can get the conclusion high reflectivity will be recommend when the same ER can be achieved with both of high and low reflectivity due to small peak chirp will be brought with in the high reflectivity case. On the other hand, from Fig 4.11 we can get optimum reflectivity to achieve maximum ER.

We also analyzed the effects of the facet reflectivity on the extinction ratio of the probe wave along the transverse directions. Figure 4.13 shows the extinction ratio at transverse direction of $\frac{L}{3}$, $\frac{2L}{3}$ and L with the variation of facet reflectivity R_2 in the range of 0.15~0.65. The results show us that along transverse direction peak ER values are obtained with the changing of reflectivity as observed in Fig.4.11. Moreover, at high reflectivity region the ER increased along the transverse direction and at low reflectivity area the ER decreased on the other hand. This is due to the fact that with the change of the reflectivity, the dominant factors to determine the ER are changed. At high reflectivity, ER of the probe wave is determined by the “1” state of the signal. Along the transverse direction with the input power “1” state amplified higher depression of the probe wave will produced at location L , thus the high ER of the probe wave will be obtained along the transverse direction. However, for the case of the low reflectivity, the ER of the probe wave is determined by the “0” state of the signal wave. With the “0” state of the signal wave amplified along the transverse direction, low power of the probe wave at “1” state will result. Thus low ER will be produced along the transverse direction.

Fig. 4.13 Extinction ratio of the probe wave as a function of facet reflectivity in different locations in transverse directions. (input power 25mW).

4.5 Summary

We have proposed a novel two-dimension VCSEL based wavelength converter and analyzed it using our developed two dimensions TLLM. The proposed VCSEL is used both as the laser source for the probe wave and as SOA for performing cross-gain modulation. The developed TLLM model provides an accurate tool to analyze the dynamic characteristics of the VCSEL based wavelength converter. In our analysis, extinction ratio, facet reflectivity, wavelength location as well as the variation of the field along the VCSEL diameter are taken into account. We show that wideband wavelength converter can be designed by employing the proposed structure.

Chapter 4 Design of Two Dimension VCSEL Based Wavelength Converter

The effects of the reflectivity on the extinction ratio and frequency peak chirp of the proposed VCSEL based two-dimension wavelength converter have been investigated in this chapter. In the analysis the reflectivity is changed in the ranger between 0.15~0.65. The results show that optimal reflectivity exists to achieve maximum extinction ratio.

CHAPTER 5

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

5.1 Introduction of anti-resonant reflecting optical waveguide VCSEL

In chapter 2, we have shown the efforts spent to get single-longitudinal-mode operation in multiple phase shift (MPS) DFB LDs. Different from MPS DFB LDs, VCSELs can always perform single longitudinal mode operation. However, in VCSELs, when the emission aperture becomes large, single transverse mode operation will be challenged. In this and the following chapters, we will discuss the VCSEL designs to achieve single transverse mode operation.

Antiresonant reflecting optical waveguides (ARROW) featured with single-mode have been developed recently [53-54]. The ARROW waveguides consist of core and interference cladding which is sandwiched by the core and the semiconductor substrate with the index of the core lower than the cladding layer. The antiresonant reflection corresponds with the situation that the thickness of interference cladding satisfies the antiresonant condition. And theoretical models have been developed to analyze these planar ARROW structures [55-57].

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

Recently, cylinder ARROW such as ARROW VCSEL has been reported [58-61]. In order to reduce the edge radiation losses for the fundamental mode of an antiguided structure, antiresonant reflecting optical waveguides (ARROW) have been employed for lateral waveguiding in facet emitting lasers. The lateral distribution of the effective index of ARROW structure is composed by a low-index core which is surrounded by a pair of reflectors with high-index. The reflector regions are designed to be antiresonant for the fundamental lateral mode. Higher-order lateral modes which do not meet the antiresonant conditions suffer radiation losses greater than those of fundamental mode. Thus this ARROW structure favor the fundamental mode and has better modal discrimination than normal VCSELs.

5.2 Scalar transfer matrix method used to analyze ARROW VCSEL

5.2.1 Scalar transfer matrix method

In this section, a field-transfer matrix model will be used to study the modal characteristics of the cylindrical multilayered waveguide. Figure 5.1 shows the schematic of the cylindrical multilayered waveguide to be investigated.

Fig. 5.1 Schematic of ARROW VCSELs and the refractive index profile.

In Fig 5.1, ARROW VCSEL is constructed by four layers of concentric rings. The optical fields satisfy the scalar Helmholtz equation in cylindrical coordinate. It can be written as [62]

$$E_i = E_i^+ H_v^{(1)}(\beta_i r) + E_i^- H_v^{(2)}(\beta_i r) \quad (5.1)$$

where E_i represents the field profile in the i th con-centric ring, E_i^+ and E_i^- are the corresponding inward- and outward- traveling field amplitude, respectively.

$H_v^{(1)}$ and $H_v^{(2)}$ are the first kind and the second kind Hankel functions of order v ,

respectively. $\beta_i = (\frac{2\pi}{\lambda} \sqrt{n_i^2 - n_{eff}^2})$ is the lateral propagation coefficient, λ is the

vacuum wavelength, n_i is the refractive index for i th layer, and n_{eff} is the effective

refractive index which will be deduced. According the boundary condition where the field and the field derivative are continuous across the adjacent con-centric ring

layers, the electric field E_i and its derivative E_i' can be related to E_{i+1} and E_{i+1}' by

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

$$\begin{bmatrix} E_i \\ E'_i \end{bmatrix}_{r=r_i} = T_i \cdot \begin{bmatrix} E_{i+1} \\ E'_{i+1} \end{bmatrix}_{r=r_{i+1}} \quad (5.2)$$

$$T_i = \begin{bmatrix} H_v^{(1)}(\beta_i r_i) & H_v^{(2)}(\beta_i r_i) \\ H_v'^{(1)}(\beta_i r_i) & H_v'^{(2)}(\beta_i r_i) \end{bmatrix} \cdot \begin{bmatrix} H_v^{(1)}(\beta_i r_{i+1}) & H_v^{(2)}(\beta_i r_{i+1}) \\ H_v'^{(1)}(\beta_i r_{i+1}) & H_v'^{(2)}(\beta_i r_{i+1}) \end{bmatrix}^{-1} \quad (5.3)$$

T_i is the matrix which connect to fields in adjacent tow layers. It should be noted that the above boundary condition is valid only if the index contrast is small. Under this assumption the EM wave can be approximated by a scalar wave-a wave with a constant polarization, only its amplitude varying with space. And this approximation is also called weakly guiding approximation.

The total transfer matrix is $T = \prod_{i=1}^3 T_i$. Boundary conditions at $r = r_1$ can be described as following (1) for fundamental mode (i.e. $v = 0$) the field derivative goes to zero (i.e. $E'_1|_{r=r_1} = 0$) due to the symmetric property of the electric fields; (2) For high order mode (i.e. $v > 0$) the field at $r = r_1$ goes to zero (i.e. $E_1|_{r=r_1} = 0$). For the outer con-centric ring (i.e. $r > r_4$) since there has no in-coming wave (i.e., $E_4^- = 0$), we can get the following equation

$$\begin{bmatrix} E_4 \\ E'_4 \end{bmatrix} = \begin{bmatrix} H_v^{(1)}(\beta_4 r) & H_v^{(2)}(\beta_4 r) \\ H_v'^{(1)}(\beta_4 r) & H_v'^{(2)}(\beta_4 r) \end{bmatrix} \begin{bmatrix} E_4^+ \\ 0 \end{bmatrix} \quad (5.4)$$

To relate the electric field and its derivative by a constant (i.e. the constant has no dependence on r), the above equation should be further simplified to remove the components that depend on r . The following approximation have been made to the Hankel functions [62]:

$$H_v^1(\beta_4 r) \approx \sqrt{\frac{2}{\pi \beta_4 r}} \left[\cos\left(\beta_4 r - \frac{\pi}{4} - \frac{1}{2} v \pi\right) + j \sin\left(\beta_4 r - \frac{\pi}{4} - \frac{1}{2} v \pi\right) \right] \quad (5.5)$$

and hence we can approximate $H_v^{(1)}(\beta_4 r) \approx j\beta_4 H_v^{(1)}(\beta_4 r)$. Similar approximation can be applied for $H_v^{(2)}(\beta_4 r)$. This approximation is satisfied because $\beta_4 r \gg v$ (usually we consider several lowest order modes i.e. $v = 0, 1$ etc.), and $r > r_4$. Hence we have for the fundamental mode (i.e. $v = 0$)

$$\begin{bmatrix} E_1 \\ 0 \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} E_4 \\ E_4' \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} 1 \\ j\beta_4 \end{bmatrix} \cdot E_4^+ \cdot H_v^1(\beta_4 r) \quad (5.6)$$

the eigen equation for the radiation modes in cylinder ARROW can be reduced to

$$f(n_{eff}) = t_{21} + j\beta_4 t_{22} = 0 \quad (5.7)$$

for the high order mode (i.e. $v > 0$), equation (5.6) can be written as

$$\begin{bmatrix} 0 \\ E_1' \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} E_4 \\ E_4' \end{bmatrix} = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix} \begin{bmatrix} 1 \\ j\beta_4 \end{bmatrix} \cdot E_4^+ \cdot H_v^1(\beta_4 r) \quad (5.8)$$

and then eigen equation is expressed as

$$f(n_{eff}) = t_{11} + j\beta_4 t_{12} = 0 \quad (5.9)$$

Effective index n_{eff} can be obtained by searching for roots of equation (5.7) or equation (5.9). The mode fields and loss property can then be calculated based on the n_{eff} .

5.2.2 Transversal mode of ARROW VCSEL using scalar transfer matrix method

We have calculated the amplitude of the transverse mode LP_{01} along the radial distance (shown in Fig. 5.2) using the scalar transfer matrix described in the previous section. The two dimensional normalized intensity distributions are

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

shown in Fig. 5.3. In the calculation, we set the wavelength at $0.98 \mu\text{m}$, $d_1 = 4 \mu\text{m}$, $d_2 = 1.3 \mu\text{m}$ and $d_3 = 4 \mu\text{m}$. It is noted that the effective refractive index n_{eff} is a complex number which indicates that the mode calculated is a leaky mode. The leakage loss can be obtained from the imaginary part of the effective index. The modal loss α is calculated by using $\alpha = \frac{\pi \Im(n_{\text{eff}})}{25\lambda}$.

Fig. 5.2 Transverse field amplitude of LP_{01} mode along the radial direction, where the solid line represents the real part of the field and the dashed line is the imaginary part of the field.

Fig. 5.3 Field intensity distribution of LP₀₁ mode.

The field amplitude along the radial distance of LP₁₁ mode is shown in Fig. 5.4 and the intensity distribution is shown in Fig. 5.5. The imaginary part of n_{eff} for LP₀₁ mode is less than that for LP₁₁ mode. This shows that lower order mode has lower radiation loss compared with high order modes.

Fig. 5.4 Transverse field amplitude of LP₁₁ mode along the radial direction, where the solid line represents the real part of the field and the dashed line is the imaginary part of the field.

Fig. 5.5 Field intensity distribution of LP₁₁ mode.

We have calculated the effective refractive index n_{eff} of LP₀₁ mode and LP₁₁ mode by changing d_2 in the region of $0 \sim 10 \mu\text{m}$, and the results are shown in Fig. 5.6 and Fig. 5.7 respectively. It is observed that n_{eff} (both real part, $\Re(n_{eff})$, and imaginary part, $\Im(n_{eff})$) for the LP₀₁ and LP₁₁ modes change with d_2 periodically. As we have known, ARROW VCSELs confine the light in the low refractive index core when the reflected lights by the surrounding high- and low- periodic cladding layers are constructive [63]. And these constructive conditions are greatly affected by the structure parameter d_2 .

Fig. 5.8 shows us the loss shape of LP₀₁ mode and LP₁₁ mode with the change of parameter d_2 . We can get the optimum design of the ARROW VCSEL structure by choosing a certain d_2 where leakage difference between LP₀₁ mode and LP₁₁ mode is kept the largest. For the structure we investigated, from Fig. 5.8, the parameter d_2 can be design as around $0.45 \mu\text{m}$, $1.3 \mu\text{m}$, $2.15 \mu\text{m}$ and $2.95 \mu\text{m}$ etc.

With large modal loss discrimination, we can get the single mode operation because high order modes have been leaked due to their high leakage losses.

Fig. 5.6 Effective modal index n_{eff} of LP_{01} mode with the variation of parameter d_2 . (a) Real part; (b) Imaginary part.

Fig. 5.7 Effective modal index of LP_{11} mode with the variation of parameter d_2 . (a) Real part; (b) Imaginary part.

Fig. 5.8 Modal losses scan with the changes of the parameter d_2

5.3 Vector transfer matrix method used to analyze ARROW VCSEL

5.3.1 Vector transfer matrix method

As we described in the section 5.2.1 already scalar TMM is limited to analyze the weakly guiding waveguide. The more general solution can be obtained from [64] which can analyze the waveguide with large index contrast. We name the method as vector transfer matrix method (TMM). For the less assumption in the analysis, vector TMM is more exact than scalar TMM.

In vector TMM we take the z axis as the direction of propagation, so that the every field component has the form

$$\Phi(r, \theta, z, t) = \Phi(r, \theta) e^{i(\beta z - \omega t)} \quad (5.10)$$

where Φ can be $E_z, E_r, E_\theta, H_z, H_r, H_\theta$. ω is the angular frequency and β is the propagation constant.

From electromagnetic theory we known that the transverse field components can be expressed in terms of E_z and H_z ,

$$E_r = \frac{i\beta}{\omega^2 \mu \epsilon - \beta^2} \left(\frac{\partial}{\partial r} E_z + \frac{\omega \mu}{\beta} \frac{\partial}{r \partial \theta} H_z \right) \quad (5.11)$$

$$E_\theta = \frac{i\beta}{\omega^2 \mu \epsilon - \beta^2} \left(\frac{\partial}{r \partial \theta} E_z - \frac{\omega \mu}{\beta} \frac{\partial}{\partial r} H_z \right) \quad (5.12)$$

$$H_r = \frac{i\beta}{\omega^2 \mu \epsilon - \beta^2} \left(\frac{\partial}{\partial r} H_z - \frac{\omega \mu}{\beta} \frac{\partial}{r \partial \theta} E_z \right) \quad (5.13)$$

$$H_\theta = \frac{i\beta}{\omega^2 \mu \epsilon - \beta^2} \left(\frac{\partial}{r \partial \theta} H_z + \frac{\omega \mu}{\beta} \frac{\partial}{\partial r} E_z \right) \quad (5.14)$$

Hence we can solve E_z and H_z first. E_r, E_θ, H_r and H_θ can be derived using (5.11-1.14). $E_z(r, \theta)$ and $H_z(r, \theta)$ satisfy the wave equation

$$\left[\nabla_t^2 + (\omega^2 \mu \epsilon - \beta^2) \right] \begin{pmatrix} E_z \\ H_z \end{pmatrix} = 0 \quad (5.15)$$

where $\nabla_t^2 = \nabla^2 - \partial^2 / \partial z^2$ is the transverse Laplacian operator.

The general solutions can be written as

$$E_z = [A_i J_l(k_i r) + B_i Y_l(k_i r)] \cos(l\theta + \phi) \quad (5.16)$$

$$H_z = [C_i J_l(k_i r) + D_i Y_l(k_i r)] \sin(l\theta + \mathcal{G}) \quad (5.17)$$

where A, B, C, D, ϕ and $\mathcal{G}$ are constants, l is an integer, J_l is the l^{th} order Bessel function of the first kind, Y_l is the l^{th} order Bessel function of the second kind.

$$k_i = \frac{2\pi}{\lambda} \sqrt{n_i^2 - n_{\text{eff}}^2} \quad i = 1, 2, \dots \quad (5.18)$$

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

where $n_i = \mu_i \varepsilon_i$, and $\beta = \frac{2\pi}{\lambda} n_{eff}$, n_{eff} is the effective modal index.

we can simplify the waves by setting $\phi = \vartheta = 0$ as,

$$E_z = [A_l J_l(k_l r) + B_l Y_l(k_l r)] \cos l\theta \quad (5.19)$$

$$H_z = [C_l J_l(k_l r) + D_l Y_l(k_l r)] \sin l\theta \quad (5.20)$$

Using the boundary condition, where E_z, H_z, E_θ and H_θ are continuous at the interface $r = \rho$, a 4×4 matrix can be found which relate $A_{i+1}, B_{i+1}, C_{i+1}, D_{i+1}$ to A_i, B_i, C_i, D_i

$$M(i+1, \rho) \begin{pmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{pmatrix} = M(i, \rho) \cdot \begin{pmatrix} A_i \\ B_i \\ C_i \\ D_i \end{pmatrix} \quad (5.21)$$

with

$$M(i, \rho) = \begin{bmatrix} J_l(k_i \rho) & Y_l(k_i \rho) & 0 & 0 \\ \frac{\omega \varepsilon_i}{\beta k_i} J'_l(k_i \rho) & \frac{\omega \varepsilon_i}{\beta k_i} Y'_l(k_i \rho) & \frac{\omega \varepsilon_i}{k_i^2 \rho} J_l(k_i \rho) & \frac{\omega \varepsilon_i}{k_i^2 \rho} Y_l(k_i \rho) \\ 0 & 0 & J_l(k_i \rho) & Y_l(k_i \rho) \\ \frac{\omega \varepsilon_i}{k_i^2 \rho} J_l(k_i \rho) & \frac{\omega \varepsilon_i}{k_i^2 \rho} Y_l(k_i \rho) & \frac{\omega \varepsilon_i}{\beta k_i} J'_l(k_i \rho) & \frac{\omega \varepsilon_i}{\beta k_i} Y'_l(k_i \rho) \end{bmatrix} \quad (5.22)$$

we can get

$$\begin{pmatrix} A_{i+1} \\ B_{i+1} \\ C_{i+1} \\ D_{i+1} \end{pmatrix} = M_i \cdot \begin{pmatrix} A_i \\ B_i \\ C_i \\ D_i \end{pmatrix} \quad (5.23)$$

where $M_i = M(i+1, \rho)^{-1} \cdot M(i, \rho)$.

The field at outermost cladding can be represented by Hankel functions. Which can be written as

$$E_z = [A_i H_i^{(1)}(k_i r) + B_i H_i^{(2)}(k_i r)] \cos l \theta \quad (5.24)$$

$$H_z = [C_i H_i^{(1)}(k_i r) + D_i H_i^{(2)}(k_i r)] \sin l \theta \quad (5.25)$$

where $H_i^{(1)}(k_i \rho) = J_i(k_i \rho) + iY_i(k_i \rho)$ is Hankel function of the first kind and $H_i^{(2)}(k_i \rho) = J_i(k_i \rho) - iY_i(k_i \rho)$ is Hankel function of the second kind. Physically speaking, $H_i^{(1)}$ denotes outgoing field, $H_i^{(2)}$ denotes incoming field. Then the transfer matrix is written as:

$$N(i, \rho) = \begin{bmatrix} H_i^{(1)}(k_i \rho) & H_i^{(2)}(k_i \rho) & 0 & 0 \\ \frac{\omega \varepsilon_i}{\beta k_i} H_i^{(1)}(k_i \rho) & \frac{\omega \varepsilon_i}{\beta k_i} H_i^{(2)}(k_i \rho) & \frac{\omega \varepsilon_i}{k_i^2 \rho} H_i^{(1)}(k_i \rho) & \frac{\omega \varepsilon_i}{k_i^2 \rho} H_i^{(2)}(k_i \rho) \\ 0 & 0 & H_i^{(1)}(k_i \rho) & H_i^{(2)}(k_i \rho) \\ \frac{\omega \varepsilon_i}{k_i^2 \rho} H_i^{(1)}(k_i \rho) & \frac{\omega \varepsilon_i}{k_i^2 \rho} H_i^{(2)}(k_i \rho) & \frac{\omega \varepsilon_i}{\beta k_i} H_i^{(1)}(k_i \rho) & \frac{\omega \varepsilon_i}{\beta k_i} H_i^{(2)}(k_i \rho) \end{bmatrix} \quad (5.26)$$

Thus for the outermost layer, the matrix is $N_n = N(n+1, \rho)^{-1} \cdot M(n, \rho)$.

Now we can get the global matrix as

$$\begin{pmatrix} A_{n+1} \\ B_{n+1} \\ C_{n+1} \\ D_{n+1} \end{pmatrix} = M_G \cdot \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} \quad (5.27)$$

where $M_G = N_n \cdot M_{n-1} \cdots M_1$.

It should be noted that there is no incoming field in the outermost layer since there is no reflection happening beyond the last interface. Therefore we can set $B_{n+1} = D_{n+1} = 0$. As we already knew, Bessel function of the second kind will be

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

infinite at the $\rho = 0$ (i.e., $Y_l(0) \rightarrow \infty$), thus to get the right expression of the field at the first layer we can set $B_1 = D_1 = 0$. Finally we can get an eigen-equation as

$$\begin{bmatrix} m_{21} & m_{23} \\ m_{41} & m_{43} \end{bmatrix} \begin{bmatrix} A_1 \\ C_1 \end{bmatrix} = 0 \quad (5.28)$$

To get the nonzero field coefficients, the determinant of the 2×2 matrix in (5.28) should be zero, *i.e.*

$$F(n_{eff}) = M_G(2,1) \times M_G(4,3) - M_G(4,1) \times M_G(2,3) = 0 \quad (5.29)$$

The effective index n_{eff} has been obtained by solving equation (5.29).

5.3.2 Modal loss property analysis by using vector transfer matrix method

In Section 5.2.2 we have investigated the transverse mode property by scalar analysis method. In this section we will apply the vector method mentioned in the Section 5.3.1 to study the modal loss property. At the same time we will compare the results obtained by both scalar method and vector method.

To facilitate comparison, we still analyze the ARROW VCSEL structure shown in Fig. 5.1 in this section by using the vector transfer matrix method. Figure 5.9 shows Z-component of the electric field E_z of HE_{11} mode and Fig. 5.10 shows its transverse electric field E_r . The calculated magnetic field in the z-axis H_z of TE_{01} mode along the radial direction and the transverse electric field of TE_{01} mode are shown in Fig. 5.11 and Fig. 5.12 separately. The E_z and H_z of TM_{01} mode are shown in Fig. 5.13 and Fig. 5.14, respectively

Comparing the n_{eff} values of TE_{01} and TM_{01} modes, we found that these two modes nearly have the same effective refractive index. Thus the TE_{01} and TM_{01} modes can be degenerated to one mode. We can imagine that the TE_{01} and TM_{01} modes will have the same loss properties with LP_{11} mode calculated by the scalar methods discussed in section 5.2.2.

Fig. 5.9 E_z field of HE_{11} mode along the radial direction, where the solid line represents the real part of the electrical field and the dashed line is the imaginary part of the electrical field.

Fig. 5.10 Transverse electric field of HE_{11} mode.

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

Fig. 5.11 H_z field of TE_{01} mode along the radial direction, where the solid line represents the real part of the magnetic field and the dashed line is the imaginary part of the magnetic field.

Fig. 5.12 Transverse electric field of TE_{01} mode.

Fig. 5.13 E_z field of of TM_{01} mode along the radial direction, where the solid line represents the real part of the electrical field and the dashed line is the imaginary part of the electrical field.

Fig. 5.14 Transverse electric field of TM_{01} mode.

Table 5.1 shows us the effective modal index which are obtained by scalar TMM method and vector TMM separately at $d_2 = 2.15\mu m$.

Table 5.1 Effective modal index calculated by vector and scalar methods

Vector TMM			Scalar TMM		
$\Re(n_{eff})$		$\Im(n_{eff})$	$\Re(n_{eff})$		$\Im(n_{eff})$
HE_{11}	3.2987	8.8081×10^{-6}	LP_{01}	3.2987	8.8081×10^{-6}
TE_{01}	3.2966	5.4618×10^{-6}	LP_{11}	3.2966	5.4618×10^{-6}
TM_{01}	3.2966	5.4618×10^{-6}

Fig.5.15 Modal loss figures of HE_{11} mode by vector method and LP_{01} mode by scalar method with the variation of parameter d_2 .

We have calculated the confinement loss of HE_{11} , TE_{01} and TM_{01} modes with changing d_2 in the region of $0 \sim 4 \mu m$. Fig. 5.15 shows the loss figure of HE_{11} mode. Fig. 5.16 shows the loss of TE_{01} mode and TM_{01} mode. We also compared these vector modal losses with the linear polarization modal losses shown in Fig. 5.8 with the scalar transfer matrix method. It is found that the loss figure of LP_{01} agrees very well with the loss of HE_{11} mode, and the loss of LP_{11} mode agrees well with the loss of TE_{01} and TM_{01} modes when the confinement loss is low. However the agreement is not that well between these modes when the confinement loss is

high. The mismatch in the case of high loss can be tracked back to Equation (5.1) and Equation (5.16) and (5.17). The reason lies in that these equations are not accurate enough to describe these seriously leaky modes which demonstrate high modal loss.

Fig.5.16 Modal loss figures of TE_{01} and TM_{01} mode by vector method and LP_{11} mode by Scalar method with the variation of parameter d_2 .

5.3 Conclusion

In this Chapter, cylindrical anti-resonant reflecting optical waveguide (ARROW) type VCSEL is introduced. Such VCSEL favors the fundamental mode and has better modal discrimination than conventional index-guiding VCSEL. Two types of field transfer matrices methods (scalar transfer matrix method and vector transfer matrix method) have been developed to analyze the modal characteristics of cylindrical ARROW VCSELs. Both methods were applied to calculate the modal field and modal loss. It is found that scalar and vector methods have good

Chapter 5 Cylindrical Anti-Resonant Reflecting Optical Waveguide (ARROW) VCSEL

agreements with each other. The optimization with parameter d_2 has also been suggested to obtain large modal discrimination.

CHAPTER 6

Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL

6.1 Introduction

High power, single mode vertical-cavity surface-emitting lasers (VCSELs) have attracted significant interest in the past few years [65-66]. The conventional VCSEL structure incorporates a built-in positive-index waveguide, designed to support a single fundamental mode. However single mode operation for such a VCSEL is normally obtained with small emission apertures, which will limit the maximum output power. There are increasing demands for the larger aperture and higher continuous output powers. Due to the fact that multiple modes will exist in larger aperture waveguide, one of the solutions is to increase the modal discrimination and make the devices favor fundamental mode only [67]. As described in Chapter 5, ring-type anti-resonant-reflecting optical waveguide (ARROW) VCSELs [60] have been reported to operate at single mode with emitting aperture size as large as $8 \sim 12 \mu m$. The cladding of the ring-type ARROW VCSEL can be approximately regarded as a stack of high- and low-index Bragg reflectors. Average refractive index of the stack is higher than core index. Such waveguide's strong leakage discrimination on high-order modes is responsible for achieving single mode operation with large aperture.

Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL

Index guiding photonic-crystal structure, a waveguide whose cladding is made of holey photonic crystal, has recently been applied in VCSEL design to get single-fundamental-mode operation by Dae-Sung Song *et al.* [68]. This kind of photonic-crystal VCSEL is expected to provide endlessly single-mode operation. However, the endlessly single mode operation can be provided only if the ratio of the hole diameter to the hole-to-hole distance stays smaller than a certain value. This threshold value could be very small if index contrast involved is relatively high, which would make fabrication a difficult process. Another limitation to fabricate hollow etched holes comes from the different thermal resistivity of air and semiconductor material. The heat dissipation will become serious in such kind of devices

In this chapter, we propose an alternative design [69]. On the top of the p-type DBR mirror structure, hexagonal arrays of high-index cylinders are introduced in the cladding region. These high-index cylinders are tuned in dimension to strongly reflect light back to the core region. We refer to this design as anti-resonant reflecting photonic crystal vertical cavity surface emitting lasers (ARPC-VCSEL). In contrast to a positive index waveguide, the ARPC-VCSEL can operate with only a fundamental mode even cylinder dimension is relatively big as compared to cylinder-to-cylinder distance since high order modes suffer excessively large loss in this structure.

6.2 Design of anti-resonant reflecting PC-VCSEL

The proposed ARPC-VCSEL is shown in Fig. 6.1(a). As stated in reference (Fig. 2 in Ref. 60), one proper way to get the cylindrical ARROW structure is by chemically etching a thin GaAs-GaInP spacer layer. Following by regrowth process, the high- and low-index ring reflectors can be defined. Our proposed APRC-VCSEL can be realized exactly in the same way. That is, the GaAs-GaInP spacer layer is selectively etched with a photomask whose pattern matches the array of high index cylinders. The places where the spacer layer remains (an array of circular regions in this case) are responsible for the formation of high index cylinders after Hadley's effective index modeling [70]. After our proposed structure is converted to Hardley's effective index model, the waveguide adopted in our proposed VCSEL can be schematically shown in Fig.6 1(b). Background region is of lower index. Black regions represent high-index rods that run along the waveguide's axial direction. As the average index of cladding is higher than that of core, index-guiding is dismissed in such waveguide. In fact, PC formed by the hexagonal array of rods in cladding region reflects light strongly at certain wavelength ranges. Such light confinement in low-index core region is attributed to Bragg reflections at cylinder boundaries along any propagation direction in the entire transverse plane [71]. Photonic bandgap (PBG) is another explanation for the Bragg reflection mentioned here [72].

The cross section of the ARPC-VCSEL is shown in the Fig.6.1(c). d is the diameter of the high index cylinders and Λ is the pitch of the cladding PC. In our design, the effective refractive indices of cylinder $n_{cylinder}$ (black areas) and core (or

Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL

the background region, white areas) n_{core} are assumed to be 3.35 and 3.3, respectively. Λ is chosen at $7.36\mu m$, and diameter of the cylinder $d = 3.31\mu m$. The emission wavelength is $0.98\mu m$. The core diameter is roughly $2\Lambda = 14.7\mu m$, which is considered very big.

Fig. 6.1 (a) Schematic of anti-resonant reflecting photonic crystal VCSEL. (b) Waveguide incorporated in ARRP-VCSEL after Hardley model. (c) Cross section of the ARRP-VCSEL, the black regions represent high index cylinders.

Such waveguide with ARPC structure is relatively complex and it is difficult to get the modal effective index n_{eff} , especially the imaginary n_{eff} part. Hence we need some accurate modeling tools to analyze such structures. In the next section, we will describe the generation of multipole method and use that method to investigate the proposed ARPC VCSEL.

6.3 Performance analysis of ARPC VCSELs by multipole method

Multipole method was proposed and developed in [73-74] to analysis microstructured optical fibers (MOFs). Such fibers have multiple cylindrical inclusions arranged around a core. A key aspect of multipole method is that it makes use of the circularity of the inclusions of the micro structured waveguide [75]. By taking into account the symmetry properties of modes in MOF with regularly arranged inclusions multipole method is sufficiently converging. The detail of the development of multipole method is given in Appendix A. In this section we will apply the method to analyze our proposed structure.

In this section we will investigate the structure proposed in the Section 6.2 (Fig. 6.1) by the method described in the last section. The parameters are chosen as, in the Fig. 6.1 pitch $\Lambda = 7.36\mu m$, and diameter of the cylinder $d = 3.31\mu m$. $n_{cladding} = 3.35$ is the refractive index for high-index cylinders. $n_{core} = 3.30$ is the refractive index for the host material.

We scan determinant of M given by equation (A-66) at $\lambda = 0.98\mu m$ and the scanning results are shown in Fig.6.2. This scan has been done by varying the real part of the refractive index n_{eff} in a certain range. Each dip in the scan suggests a mode existence. And to make the problem simpler we assume the imaginary part of the n_{eff} is zero. After the determinant scan, we can have a rough estimate of the real part of the n_{eff} . Combination with the initial guess of the imaginary part of the n_{eff} , we could use the function *fsolve* to get the exactly solution of the effective

Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL

modal index n_{eff} . The n_{eff} of different modes and the results are obtained by the above processes and the effective refractive index of the first four modes are given in the following Table 6.1.

Table 6.1 Mode effective refractive index for the first four modes

	$\Re(n_{eff})$	$\Im(n_{eff})$	Mode description
1	3.299472078	1.34037e-005	HE_{11} mode
2	3.298710904	5.76512e-005	TE_{01} mode
3	3.298710374	5.76629e-005	TM_{01} mode
4	3.298711106	5.76648e-005	HE_{21} mode

From Table 6.1, we note that $n_{eff} < n_{core} < n_{cladding}$, the results indicate that the effective index is outside the range of bound modes in the individual cylinders.

From the values of the effective refractive index, we can observe that TE_{01} , TM_{01} and HE_{21} modes nearly have the same real parts and imaginary parts, which shows that these three modes have roughly the same propagation constant and also have nearly the same mode losses.

Fig. 6.2 Determinant scans of the structure with just one ring at the wavelength $\lambda = 0.98 \mu\text{m}$.

Fig. 6.3 Transverse electric field of HE_{11}^y mode.

Since we have already obtained the solution of n_{eff} . Now the field of the all kinds of modes can be calculated with the expression described in the section 6.3. The transverse modal E_t ($E_t = E_x + iE_y$) field of HE_{11} , TE_{01} , TM_{01} and HE_{21} modes

Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL

are shown in Fig. 6.3 to Fig. 6.6. The intensity of E_y field for HE_{11} and TE_{01} modes are shown in Fig. 6.7 and Fig. 6.8 separately. From the field distribution in Fig. 6.3 to 6.6 and the intensity profile in Fig. 6.7 and Fig. 6.8, we can observe that the field can be well confined in the core though the refractive index of the core is lower than its cladding. This can be explained with photonic bandgap mechanism existing in this structure.

Fig. 6.4 Transverse field of TE_{01} mode.

Fig. 6.5 Transverse field of TM_{01} mode.

Fig. 6.6 Transverse field of HE_{21} mode.

Fig. 6.7 E_y field of HE_{11}^y mode.

Fig. 6.8 E_y field of TE_{01} mode.

We also investigated the modal effective index with the change of the wavelength. The modal confinement loss α can be obtained from the imaginary part of the effective mode index $\Im(n_{eff})$ by using $\alpha = \frac{\pi \Im(n_{eff})}{25\lambda}$. We have investigated the modal confinement losses with the change of lasing wavelength. The modal losses of HE_{11} and TE_{01} modes are calculated over a range of wavelengths from $0.65 \sim 1.3 \mu m$ and the results are shown in Fig. 6.9.

From Fig. 6.9, it is noticed that the confinement losses for both modes consists of a number of lower regions separated by narrow regions where the loss increases rapidly. This behavior is fundamentally different from the index guiding VCSELs for which the loss increases monotonically with the wavelength. And the explanation of the characteristics was given by T.P.White *et al.* in [76-77]. Generally speaking, when the resonant condition of the photonic crystal cladding is met, field tends to be confined in the cladding cylinders, and core mode will hence suffer from high loss. Good confinement of the core mode will happen when the

photonic crystal cladding is highly anti-resonant. The resonant condition of the photonic crystal cladding (where the large modal loss located) can be written as

$$J_l(k_{ex}d/2) = 0 \quad (6.1)$$

where k_{ex} is the transverse component of the wavevector $\mathbf{k}$ in high index regions.

If we assume $n_{eff} \approx n_{core}$, the peak-loss wavelength points can then be predicted by the following expression

$$d \cdot \frac{\pi}{\lambda} \sqrt{n_{cylinder}^2 - n_{core}^2} = roots(J_l) \quad (6.2)$$

where $roots(J_l)$ means the roots of Bessel function of order l

In Fig. 6.9, loss peaks appear around $\lambda = 0.695 \mu m$, $\lambda = 0.85 \mu m$ and $\lambda = 1.05 \mu m$. These values are in good agreement with the calculated ones. For example, $\lambda = 0.695 \mu m$ corresponds to $k_{ex}d/2 = 8.63$ in equation (6.1), which is quite near to the third root of Bessel function J_0 . Loss rate for fundamental mode is always below that for high-order modes. At $\lambda = 0.98 \mu m$ (lasing wavelength), we can get large modal discrimination as well as keep the fundamental modal loss relatively small. The modal loss difference between HE_{11} mode and TE_{01} mode at that wavelength approaches to maximum, which is about $7.5 cm^{-1}$. Hence, our proposed VCSEL is supposed to favor the fundamental model, and the high-order modes are greatly depressed.

Another method for loss peaks prediction is to employ the PBG theory. PBG-map for photonic crystal cladding depicted in Fig. 6.1 has been calculated by using the plane wave expansion (PWE) method. Fig. 6.10 shows the gap-map obtained in the

Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL

coordinate of n_{eff} versus wavelength λ . Shaded regions are PBG gap regions, within which no propagating mode is supported by the cladding structure. As a result the light is rejected back to the core region.

In order to make the z-propagating light to be confined in the core region, the modal effective index n_{eff} should be less than but very close to 3.30. As an approximation, we draw a level line at this $n_{eff} = 3.3$ in Fig. 6.10. From the figure, we can see $n_{eff} = 3.3$ line crosses several gap regions. Light can be confined in the core region if the level line stays within the gap region. Light will experience large leakage loss and cannot be confined in the core region if the level line is outside the gap regions. We notice in Fig. 6.10, the level line stay outside gap regions around $0.71\mu\text{m}$, $0.87\mu\text{m}$ and $1.1\mu\text{m}$ wavelength points, which agree with loss peak positions in Fig. 6.9 very well.

Fig. 6.9 The modal loss of HE_{11} (dotted line) and TE_{01} (solid line) modes with the change of the wavelength

Fig. 6.10 Map of photonics bandgaps found for cladding photonics crystal structure

Our results and analysis show that we can design the lasing wavelength for the ARPC VCSELs with the structure parameters fixed to get the best mode discrimination. It also shows that if the lasing wavelength is fixed we can choose a suitable diameter of the cylinder to obtain good single mode operation.

6.4 Conclusion

Anti-resonant reflecting photonic crystal structure is employed in vertical cavity surface emitting lasers (VCSELs) to achieve photon confinement in lateral direction. Such VCSEL design is promising in supporting large aperture single-mode emission. In our proposal configuration, a hexagonal array of high-index

Chapter 6 Anti-Resonant Reflecting Photonic Crystal VCSEL

cylinders which run vertically in the cladding region are introduced in the top of the VCSEL p-type DBR mirror region. Multipole methods have been described in detail in the chapter. The transverse modal property of the proposed structure, especially the characteristics with the change of the wavelength, has been investigated by the multipole methods. Optimum designs for the minimum radiation loss while maintaining single-mode operation have been discussed in this chapter.

CHAPTER 7

Chapter 7 Conclusions and Recommendations

7.1 Conclusions

In this thesis the performance characteristics of semiconductor laser diodes (i.e. multiple phase shift distributed feedback laser diodes (MPS DFB LDs) and vertical cavity surface-emitting lasers (VCSELs)) have been investigated and analyzed.

Firstly, we focused on the analysis and optimization of multiple phase shift DFB LDs. Transmission line laser model (TLLM) was developed to investigate such kind of laser diodes. In order to minimize the spatial hole-burning effect and hence to improve the maximum available single mode output power, we developed a stable single longitudinal model (SLM) laser diodes which show uniform field intensity and high side mode suppression ratio (SMSR). On the basis of lasing above threshold, selection criteria were set at flatness $F < 0.05$ and the side mode suppression ratio $SMSR > 50dB$ for DFB laser diodes. In the analysis of the quarter wave shift QWS DFB laser structure, which is characterized by its non-uniform field distribution, a large dynamic range of spatially distributed refractive index was observed. Along the carrier concentration profile, a dip was shown at the center of the cavity where the largest stimulated photon density was found. This highly non-uniform distribution will cause the single mode operation unstable. In

Chapter 7 Conclusiosn and Recommendations

order to achieve stable single mode operation, three phase shift (3PS) DFB LDs have been introduced which will produce more uniform field distribution. In our simulation results, more uniform distributions were observed in the carrier density, photon density, the refractive index as well as field intensity in 3PS DFB LDs. Optimizations of the phase shift values of the 3PS DFB LDs have been done with the optimization region fulfilling the selection criteria.

We have investigated the polarization characteristics of VCSELs. The spin-flip model (SFM) can take into account the phase information between the two orthogonal polarizations of field. Moreover, both saturable dispersion associated with the linewidth enhancement factor and the carrier dynamics with the different magnetic sublevels of the conduction and heavy hole valence bands are considered in this model. The spin-flip model for the polarization dynamics of quantum-well VCSELs has been proved to be a useful instrument for describing a number of polarization properties of the VCSELs. We have applied the SFM model to investigate the polarization stability of QW VCSELs. With the stability map obtained, injection current region can be suggested for the different polarization status in the VCSELs. The SFM model has also been refined to take into account the effects of the DBR sections on the polarization bistability. Our results show that for devices with low reflectivity, low input power is required to achieve bistable operation. On the other hand, due to the reduced coupling strength between the injected light and the active layers, narrow hysteresis loops are resulted. For the case of VCSELs with high reflectivity, wide hysteresis loops are obtained at the expense of high external injection power.

A novel VCSEL-based two-dimensional wavelength converter was proposed and explored in this thesis. The basic idea is for the probe wave and the signal wave to the SOA-based wavelength converter along orthogonal directions, thus the optical filter or the optical isolator in the conventional wavelength converter design can be removed. In order to achieve this objective, we adopted the VCSEL structure and extended the active region in the horizontal direction. In the proposed structure, known as the VCSEL-based two dimensional wavelength converter, the signal wave enters the VCSEL's active region in the horizontal direction and the outgoing probe wave is in the vertical direction, where the VCSEL is used both as the laser source for the probe wave and as the non-linear medium for inducing strong cross-gain modulation necessary for wavelength conversion. There are two main advantages in this structure: (1) Devices such as optical filter or optical isolator which are necessary in the traditional wavelength converters are not required; (2) a separate laser source for the probe wave can be avoided because the VCSEL serves as an embedded laser source as well as an amplifier. Two-dimensional TLLM model has been developed to provide an accurate tool to analyze the dynamic characteristics of the VCSEL-based wavelength converter. The detailed modeling of the VCSEL-based wavelength converter allows us to examine the effects of various parameters, e.g., extinction ratio, facet reflectivity, wavelength location and field variation, on the performance of the wavelength converter and to determine the optimal configuration. It is shown that the proposed structure is well suited for wideband wavelength conversion.

Another design that we have investigated is cylindrical anti-resonant reflecting optical waveguides (ARROW) types VCSELs. ARROW VCSELs are proposed to

Chapter 7 Conclusiosn and Recommendations

reduce the edge radiation losses for the fundamental mode. In the ARROW VCSELs, lateral distribution of the effective index of ARROW structure are composed of a low-index core surrounded by pairs of reflectors with high-index. The reflector regions are designed to be antiresonant for the fundamental lateral mode. Higher-order lateral modes which do not meet the antiresonant conditions suffer radiation losses greater than those of the fundamental mode. Thus ARROW structure favors the fundamental mode and has better modal discrimination than normal VCSELs. Two kinds of methods (scalar transfer matrix method and vector transfer matrix method) have been described and applied in analyzing ARROW VCSEL structures. Transverse modal behaviors of ARROW VCSELs have been investigated. The results obtained by these two methods show very good agreement with each other. It was found that the loss of a transverse mode greatly depends on the structure parameters. One of the structure parameters is optimized to obtain the largest transverse modal discriminations.

Finally, other than using cylindrical ARROW VCSELs to obtain the large aperture VCSEL and achieve good photon confinement in lateral direction, anti-resonant reflecting photonic crystal (ARPC) VCSEL is designed and analyzed. In the configuration, a hexagonal array of high-index cylinders which run vertically in the cladding region are introduced in the top of the VCSEL p-type DBR mirror region. Such designs possess the advantages of supporting large aperture single-mode emission. Multipole method has been introduced to analyze the transverse mode properties. The transverse modal properties of the proposed structure have been investigated by applying multipole methods. It is found that the transverse mode loss changes periodically with the lasing wavelength. The mechanisms for the

appearance of the loss peaks are discussed. The resonant condition of the photonic crystal cladding will cause the field be confined in the cladding cylinders, which in turn leads to high loss for the core mode. To obtain good confinement for the core mode, ARPC VCSELs are required to operate at a wavelength at which the photonic crystal cladding is highly antiresonant.

7.2 Recommendations for further research

There are a number of research directions which worth further investigation based on the work presented in this thesis.

My first recommendation is the investigation and fabrication of the two-dimensional VCSEL based wavelength converters as well as anti-resonant reflecting photonic crystal (ARPC) VCSEL. In the case of two-dimensional VCSEL based wavelength converters, the fabrication difficulties lie in two aspects. One problem is how to make the active layer on the heterojunction as well as N-DBR layer. The other one is how to keep the lattice match at the adjacent of heterojunction and those DBR mirrors. The fabrication quality will great affect the performance of the VCSEL based wavelength converters. For the case of the fabrication of the APRC VCSEL device, realization of the micro structure in the space layer is the key point. The major fabrication difficulty lies in how to grow the GaAs (high index cylinders) into the etching hollow cylinders with the background of GaInP material. As the diameter of the etching holes are quite small (around $2\ \mu\text{m}$), difficulties arises to make regrowth process in these micro structures. One of the solutions considered by the author is to fill the material into

Chapter 7 Conclusiosn and Recommendations

these hollow cylinders other than by regrowth process. As a future work, experimental test should be carried out to find the best technical solutions. All in all, on the standpoint of fabrication process, it is believed by the author that definitely there is a long way to go to make these two proposed structures be realized and commercialized thereafter.

In Chapter 5 and Chapter 6 we have described the possible methods to achieve single transverse mode operation with large aperture for VCSELs. In the analysis, the transverse modal fields are obtained by assuming the structure passive waveguide. However, for VCSELs operating at high power, the performance of VCSELs will be influenced by the spatial hole burning (SHB). It is believed that SHB is excited by stimulated recombination of carrier concentration inside the active layer, and the shape of SHB follows the profile of transverse fundamental mode [12]. This SHB will reduce the modal gain of the fundamental mode so that higher-order transverse mode can be excited. This is due to the reduction of overlap between the transverse distribution of optical gain and the fundamental mode. Therefore fundamental mode stability will be reduced. It is worthwhile to investigate the spatial hole burning effect on the operation stability of transverse-mode profiles of ARROW VCSELs as well as those of APRC VCSELs. One approach to study such effects can start from the analysis of the carrier profile in the active region. As we know, on the one side SHB arises from carrier induced index change. And on the other side, carrier distribution will affect the modal gain of each transverse mode. Designs of the different waveguide regarding the consideration of the stability of the transverse modes are required to achieve high output power.

The other extension of research work presented in this thesis can be design and fabricate array anti-resonant reflecting photonic crystal (ARPC) VCSELs. This is advantageous especially in those VCSELs with structures of photonic crystals. It is very convenient for us to obtain array VCSELs with design of a number of apertures in photonic crystal structures. However, this special extension could not be achieved through ARROW structure described in chapter 5. It is necessary to make it clear that the array VCSELs we described here are different from those traditional VCSEL arrays. In the traditional VCSEL arrays, each VCSEL is an independent device. However, in our extended research, each VCSEL is driven by the same current, which means they will be modulated by the same electrical signal. This type of VCSELs arrays can find their application in the information broadcast for example. Further investigation could start from study of coupling effects of each adjacent VCSEL in array structures which will help to design appropriate array photonics crystal structures. Fabrication of these kinds of devices would also be an interesting challenge.

Appendix A

A.1 Formulation of the field Identities

The schematic diagram for a homogeneous microstructured optical waveguide is shown in Fig. A.1. where the region of the largest circle indicate the jacket boundary. And the inner circles indicate the cylindrical inclusions. The refractive index of the inner circle denotes by n_- . The host material is n_+ , and index beyond jacket is n_0 .

Fig. A.1 Schematic diagram of the microstructured optical waveguide.

The longitudinal field components, electric field $E_z^{(l)}$ and magnetic field $H_z^{(l)}$ in and outside the l th cylinder satisfy the Helmholtz equations

$$(\nabla_{\perp}^2 + k^2 n_l^2 - \beta^2) \begin{Bmatrix} E_z^{(l)} \\ K_z^{(l)} \end{Bmatrix} = 0 \quad (\text{A-1})$$

where $K = ZH$, and Z is free space impedance $\sqrt{\mu_0/\epsilon_0}$, $\nabla_{\perp}^2$ is the transverse

Laplacian. β is the propagation constant.

Local cylindrical coordinates (r_l, θ_l, z) are employed for the l th cylinder. The Helmholtz equations are written as in this coordinate system as

$$\left(\frac{\partial^2}{\partial r_l^2} + \frac{1}{r_l} \frac{\partial}{\partial r_l} + \frac{1}{r_l^2} \frac{\partial^2}{\partial \theta_l^2} + k^2 n_l^2 - \beta^2 \right) \begin{Bmatrix} E_z^{(l)} \\ K_z^{(l)} \end{Bmatrix} = 0 \quad (\text{A-2})$$

The other four field components can be obtained from the $E_z^{(l)}$ and $K_z^{(l)}$ through the following relations:

$$E_r^{(l)} = \frac{i}{k^2 n_l^2 - \beta^2} \left[\beta \frac{\partial E_z^{(l)}}{\partial r_l} + \left(\frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} \frac{k}{r_l} \frac{\partial K_z^{(l)}}{\partial \theta_l} \right] \quad (\text{A-3})$$

$$E_\theta^{(l)} = \frac{i}{k^2 n_l^2 - \beta^2} \left[\frac{\beta}{r_l} \frac{\partial E_z^{(l)}}{\partial \theta_l} - \left(\frac{\mu_0}{\varepsilon_0} \right)^{\frac{1}{2}} k \frac{\partial K_z^{(l)}}{\partial r_l} \right] \quad (\text{A-4})$$

$$K_r^{(l)} = \frac{i}{k^2 n_l^2 - \beta^2} \left[- \left(\frac{\varepsilon_0}{\mu_0} \right)^{\frac{1}{2}} \frac{kn^2}{r_l} \frac{\partial E_z^{(l)}}{\partial \theta_l} + \beta \frac{\partial K_z^{(l)}}{\partial r_l} \right] \quad (\text{A-5})$$

$$K_\theta^{(l)} = \frac{i}{k^2 n_l^2 - \beta^2} \left[\left(\frac{\varepsilon_0}{\mu_0} \right)^{\frac{1}{2}} kn^2 \frac{\partial E_z^{(l)}}{\partial \theta_l} + \frac{\beta}{r_l} \frac{\partial K_z^{(l)}}{\partial r_l} \right] \quad (\text{A-6})$$

The field within the l th cylinder respectively can be expressed in terms of Fourier Bessel series as,

$$E_z^-(r_l, \theta_l) = \sum_m \left[A_m^{E-} J_m(k_\perp^- r_l) + B_m^{E-} H_m^{(1)}(k_\perp^- r_l) \right] \cdot e^{im\theta} \quad (\text{A-7})$$

$$K_z^-(r_l, \theta_l) = \sum_m \left[A_m^{K-} J_m(k_\perp^- r_l) + B_m^{K-} H_m^{(1)}(k_\perp^- r_l) \right] \cdot e^{im\theta} \quad (\text{A-8})$$

where $k_\perp^- = (\beta^2 - k^2 n_l^2)^{\frac{1}{2}}$ is the transverse wave-number in the l th cylinder, A_m^{E-} ,

B_m^{E-} , A_m^{K-} , B_m^{K-} are coefficients to be determined. J_m is the m th order Bessel

function of the first kind and $H_m^{(1)}$ is the m th order Hankel function of the first

Appendix A

kind. Physically speaking, Bessel function is for incoming wave and Hankel function is for outgoing wave.

In the neighborhood around the l th cylinder, the field is

$$E_z^+(r_l, \theta_l) = \sum_m \left[A_m^{El} J_m(k_\perp^+ r_l) + B_m^{El} H_m^{(1)}(k_\perp^+ r_l) \right] \cdot e^{im\theta} \quad (\text{A-9})$$

$$K_z^+(r_l, \theta_l) = \sum_m \left[A_m^{Kl} J_m(k_\perp^+ r_l) + B_m^{Kl} H_m^{(1)}(k_\perp^+ r_l) \right] \cdot e^{im\theta} \quad (\text{A-10})$$

where $k_\perp^+ = (k^2 n_c^2 - \beta^2)^{\frac{1}{2}}$ is the transverse propagation constant for the cladding near the l th cylinder. And A_m^{E+} , B_m^{E+} , A_m^{K+} , B_m^{K+} are coefficients to be determined.

Another description of the fields is originally due to Wijngaard [78]. He reasoned that the a field in a region can be written as a superposition of outgoing waves from all source bodies in the region. This can be proven using Green's function.

The wijngaard expansion takes the form

$$E_z = \sum_{l=1}^{N_c} \sum_m B_m^{El} H_m^{(1)}(k_\perp^+ |r_l|) e^{im\theta} + \sum_m A_m^{E0} J_m(k_\perp^+ r) e^{im\theta} \quad (\text{A-11})$$

Each term of the series is an outgoing wave field with a source at cylinder l whereas the final term, indexed by 0 is the regular field originating at the jacket boundary.

Equating Eqs. (A-9) and (A-11), we can get the first identity equation

$$\sum_m A_m^{El} J_m(k_\perp^+ r_l) e^{im\theta_l} = \sum_{\substack{j=1 \\ j \neq l}}^{N_c} \sum_m B_m^{Ej} H_m^{(1)}(k_\perp^+ r_l) e^{im\theta_j} + \sum_m A_m^{E0} J_m(k_\perp^+ r_l) e^{im\theta} \quad (\text{A-12})$$

From above equation, we notice that there are $N_c + 1$ series expansions. Each expansion has $2m + 1$ terms, in a polar coordinate corresponding to one cylinder or to the jacket. Now we need to transform all expansions into l th cylinder's coordinate, which is called coordinate projection [74].

Appendix A

For example one H_m expansion in equation (A-10) (in j th cylinder's coordinate) can be transformed into cylinder l 's coordinate as

$$\sum_m B_m^{Ej} H_m(k_\perp^+ r_j) e^{im\theta_j} = \sum_n A_n^{Elj} J_n(k_\perp^+ |r_l|) e^{in \arg(r_l)} \quad (\text{A-13})$$

where

$$A_n^{Elj} = \sum_m P_{nm}^{lj} B_m^{Ej} \quad (\text{A-14})$$

$$P_{nm}^{lj} = H_{n-m}(k_\perp^+ |r_l|) e^{-i(n-m) \arg(r_l)} \quad (\text{A-15})$$

Similarly, the final expansion in equation (A-11) which is in jacket's coordinates can be transformed into cylinder l 's coordinate as

$$\sum_m A_m^{E0} J_m(k_\perp^+ r_j) e^{im\theta} = \sum_n A_n^{El0} J_n(k_\perp^+ |r_l|) e^{in \arg(r_l)} \quad (\text{A-16})$$

where

$$A_n^{El0} = \sum_m Q_{nm}^{l0} A_m^{E0} \quad (\text{A-17})$$

$$Q_{nm}^{l0} = (-1)^{n-m} J_{n-m}(k_\perp^+ |r_l|) e^{-i(n-m) \arg(r_l)} \quad (\text{A-18})$$

we substitute (A-13) and (A-16) into (A-12), then equality (A-12) becomes

$$\sum_m A_m^{El} J_m(k_\perp^+ |r_l|) e^{im \arg(r_l)} = \sum_{\substack{j=1 \\ j \neq l}}^{N_c} \sum_n A_n^{Elj} J_n(k_\perp^+ |r_l|) e^{in \arg(r_l)} + \sum_n A_n^{El0} J_n(k_\perp^+ |r_l|) e^{in \arg(r_l)} \quad (\text{A-19})$$

Note that in order to keep the matrices square m should be equal to n . We can get a system of linear equations as

$$\begin{bmatrix} A_{-m}^{El} \\ A_{-m+1}^{El} \\ \vdots \\ A_{m-1}^{El} \\ A_m^{El} \end{bmatrix} = \begin{bmatrix} A_{-n}^{El1} \\ A_{-n+1}^{El1} \\ \vdots \\ A_{n-1}^{El1} \\ A_n^{El1} \end{bmatrix} + \begin{bmatrix} A_{-n}^{El2} \\ A_{-n+1}^{El2} \\ \vdots \\ A_{n-1}^{El2} \\ A_n^{El2} \end{bmatrix} + \dots + \begin{bmatrix} A_{-n}^{ElN_c} \\ A_{-n+1}^{ElN_c} \\ \vdots \\ A_{n-1}^{ElN_c} \\ A_n^{ElN_c} \end{bmatrix} + \begin{bmatrix} A_{-n}^{El0} \\ A_{-n+1}^{El0} \\ \vdots \\ A_{n-1}^{El0} \\ A_n^{El0} \end{bmatrix} \quad (\text{A-20})$$

We can write the above matrix in short as

Appendix A

$$A^{El} = A^{El1} + A^{El2} + \dots + A^{ElN_c} + A^{El0} \quad (A-21)$$

Notice there is $j \neq l$ condition on the equation (A-19), thus when $l = j$ one of the single-column matrix will be zero. For example, if $l = 2$, then the second column at the right hand of equation (A-20) is null. Equation (A-14) can be written in the matrix form as

$$\begin{bmatrix} A_{-n}^{El1} \\ A_{-n+1}^{El1} \\ \vdots \\ A_{n-1}^{El1} \\ A_n^{El1} \end{bmatrix} = \begin{bmatrix} P_{(-n)(-m)}^{l1} & P_{(-n)(-m+1)}^{l1} & \dots & P_{(-n)(m-1)}^{l1} & P_{(-n)(m)}^{l1} \\ P_{(-n+1)(-m)}^{l1} & P_{(-n+1)(-m+1)}^{l1} & \dots & P_{(-n+1)(m-1)}^{l1} & P_{(-n+1)(m)}^{l1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ P_{(n-1)(-m)}^{l1} & P_{(n-1)(-m+1)}^{l1} & \dots & P_{(n-1)(m-1)}^{l1} & P_{(n-1)(m)}^{l1} \\ P_{(n)(-m)}^{l1} & P_{(n)(-m+1)}^{l1} & \dots & P_{(n)(m-1)}^{l1} & P_{(n)(m)}^{l1} \end{bmatrix} \times \begin{bmatrix} B_{-m}^{E1} \\ B_{-m+1}^{E1} \\ \vdots \\ B_{m-1}^{E1} \\ B_m^{E1} \end{bmatrix} \quad (A-22)$$

Thus we have

$$A^{Elj} = P^{lj} B^{Ej} \quad \text{where } j=1,2,\dots,N_c \quad (A-23)$$

Also for the final single-column vector at the right side of equation of (A-20) we have

$$A^{El0} = Q^{l0} A^{E0} \quad (A-24)$$

From equations of (A-23) and (A-24) we can rewrite the equality (A-21) as

$$A^{El} = \sum_{j=1, j \neq l}^{N_c} P^{lj} B^{Ej} + Q^{l0} A^{E0} \quad (A-25)$$

we can derive the same equality for the magnetic field as

$$A^{Kl} = \sum_{j=1, j \neq l}^{N_c} P^{lj} B^{Kj} + Q^{l0} A^{K0} \quad (A-26)$$

According (A-25) and (A-26) we can get the following expression

$$\begin{bmatrix} A^{El} \\ A^{Kl} \end{bmatrix} = \begin{bmatrix} P^{l1} & 0 \\ 0 & P^{l1} \end{bmatrix} \begin{bmatrix} B^{E1} \\ B^{K1} \end{bmatrix} + \begin{bmatrix} P^{l2} & 0 \\ 0 & P^{l2} \end{bmatrix} \begin{bmatrix} B^{E2} \\ B^{K2} \end{bmatrix} + \dots + \begin{bmatrix} P^{lN_c} & 0 \\ 0 & P^{lN_c} \end{bmatrix} \begin{bmatrix} B^{EN_c} \\ B^{KN_c} \end{bmatrix} + \begin{bmatrix} Q^{l0} & 0 \\ 0 & Q^{l0} \end{bmatrix} \begin{bmatrix} A^{E0} \\ A^{K0} \end{bmatrix} \quad (A-27)$$

Generalize this matrix we have

Appendix A

$$\tilde{A}^I = \tilde{P}^{I1} \tilde{B}^1 + \tilde{P}^{I2} \tilde{B}^2 + \dots + \tilde{P}^{I1} \tilde{B}^1 + \tilde{Q}^{I0} \tilde{A}^0 \quad (A-28)$$

From equation (A-28) we now can get the matrix about the coefficients A and B in a short form as

$$A = PB + Q^{B0} \tilde{A}^0 \quad (A-29)$$

$$\text{Where } A = \begin{bmatrix} A^{E1} \\ A^{K1} \\ A^{E2} \\ A^{E2} \\ \vdots \\ A^{EN_c} \\ A^{EN_c} \end{bmatrix}, \quad A^{EI} = \begin{bmatrix} A_{-m}^{EI} \\ A_{-m+}^{EI} \\ \vdots \\ A_{m-1}^{EI} \\ A_m^{EI} \end{bmatrix}, \quad A^{KI} = \begin{bmatrix} A_{-m}^{KI} \\ A_{-m+}^{KI} \\ \vdots \\ A_{m-1}^{KI} \\ A_m^{KI} \end{bmatrix}, \quad B = \begin{bmatrix} B^{E1} \\ B^{K1} \\ B^{E2} \\ B^{E2} \\ \vdots \\ B^{EN_c} \\ B^{EN_c} \end{bmatrix},$$

$$B^{EI} = \begin{bmatrix} B_{-m}^{EI} \\ B_{-m+}^{EI} \\ \vdots \\ B_{m-1}^{EI} \\ B_m^{EI} \end{bmatrix}, \quad B^{KI} = \begin{bmatrix} B_{-m}^{KI} \\ B_{-m+}^{KI} \\ \vdots \\ B_{m-1}^{KI} \\ B_m^{KI} \end{bmatrix}, \quad \text{And } \tilde{A}^0 = \begin{bmatrix} A^{E0} \\ A^{K0} \end{bmatrix}, \quad A^{E0} = \begin{bmatrix} A_{-m}^{E0} \\ A_{-m+}^{E0} \\ \vdots \\ A_{m-1}^{E0} \\ A_m^{E0} \end{bmatrix}, \quad A^{K0} = \begin{bmatrix} A_{-m}^{K0} \\ A_{-m+}^{K0} \\ \vdots \\ A_{m-1}^{K0} \\ A_m^{K0} \end{bmatrix}$$

The dimension of matrix P is $N_c(4m+2) \times N_c(4m+2)$ and the dimension of matrix Q^{B0} is $(4m+2) \times (4m+2)$. And we can get the every element of P and Q^{B0} from the above derivations.

So far the first equality (A-12) have be transformed to equation (A-29). Now we will move to the second identity.

The second equality states that outgoing field in the vicinity of the jacket boundary is due to outgoing field from all cylinders in the host material. It can be written as

$$\sum_m B_m^{E0} H_m(k_{\perp}^+ r) e^{im\theta} = \sum_{j=1}^{N_c} \sum_m B_m^{Ej} H_m(k_{\perp}^+ |r_j|) e^{im \arg(r_j)} \quad (A-30)$$

Appendix A

Again, by using the coordinate projection, we transform a Hankel function in cylinder j th coordinate in to a series of Hankel Functions in the jacket cylinder's coordinate. The transformation can be expressed as

$$\sum_m B_m^{Ej} H_m(k_{\perp}^+ | r_j) e^{im \arg(r_j)} = \sum_n B_n^{E0j} H_n(k_{\perp}^+ | r) e^{im \arg(r)} \quad (\text{A-31})$$

where

$$B_n^{E0j} = \sum_m Q_{nm}^{0j} B_m^{Ej} \quad (\text{A-32})$$

$$Q_{nm}^{0j} = J_{n-m}(k_{\perp}^+ | r_{j0}) e^{-i(n-m) \arg(r_{j0})} \quad (\text{A-33})$$

Substitute (A-31) into (A-30) we have

$$B_m^{E0} H_m(k_{\perp}^+ r) e^{im\theta} = \sum_{j=1}^{N_c} \sum_n B_n^{E0j} H_n(k_{\perp}^+ | r) e^{im \arg(r)} \quad (\text{A-34})$$

We set $m = n$, then the matrix form of equation (A-34) is

$$\begin{bmatrix} B_{-m}^{E0} \\ B_{-m+1}^{E0} \\ \vdots \\ B_m^{E0} \end{bmatrix} = \begin{bmatrix} B_{-n}^{E01} \\ B_{-n+1}^{E01} \\ \vdots \\ B_n^{E01} \end{bmatrix} + \begin{bmatrix} B_{-n}^{E02} \\ B_{-n+1}^{E02} \\ \vdots \\ B_n^{E02} \end{bmatrix} + \dots + \begin{bmatrix} B_{-n}^{E0N_c} \\ B_{-n+1}^{E0N_c} \\ \vdots \\ B_n^{E0N_c} \end{bmatrix} \quad (\text{A-35})$$

or in brief

$$\mathbf{B}^{E0} = \mathbf{B}^{E01} + \mathbf{B}^{E02} + \dots + \mathbf{B}^{E0N_c} \quad (\text{A-36})$$

And

$$\begin{bmatrix} B_{-n}^{E01} \\ B_{-n+1}^{E01} \\ \vdots \\ B_{n-1}^{E01} \\ B_n^{E01} \end{bmatrix} = \begin{bmatrix} Q_{(-n)(-m)}^{01} & Q_{(-n)(-m+1)}^{01} & \dots & Q_{(-n)(m-1)}^{01} & Q_{(-n)(m)}^{01} \\ Q_{(-n+1)(-m)}^{01} & Q_{(-n+1)(-m+1)}^{01} & \dots & Q_{(-n+1)(m-1)}^{01} & Q_{(-n+1)(m)}^{01} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ Q_{(n-1)(-m)}^{01} & Q_{(n-1)(-m+1)}^{01} & \dots & Q_{(n-1)(m-1)}^{01} & Q_{(n-1)(m)}^{01} \\ Q_{(n)(-m)}^{01} & Q_{(n)(-m+1)}^{01} & \dots & Q_{(n)(m-1)}^{01} & Q_{(n)(m)}^{01} \end{bmatrix} \times \begin{bmatrix} B_{-m}^{E1} \\ B_{-m+1}^{E1} \\ \vdots \\ B_{m-1}^{E1} \\ B_m^{E1} \end{bmatrix} \quad (\text{A-37})$$

Thus we have

$$\mathbf{B}^{E0j} = \mathbf{Q}^{0j} \mathbf{B}^{Ej} \quad \text{where } j = 1, 2, \dots, N_c \quad (\text{A-38})$$

Appendix A

Equation (A-36) then becomes

$$\mathbf{B}^{E0} = \mathbf{Q}^{01} \mathbf{B}^{E1} + \mathbf{Q}^{02} \mathbf{B}^{E2} + \dots + \mathbf{Q}^{0N_c} \mathbf{B}^{EN_c} \quad (\text{A-39})$$

For the magnetic field, we will have the same equation as (A-39)

$$\mathbf{B}^{K0} = \mathbf{Q}^{01} \mathbf{B}^{K1} + \mathbf{Q}^{02} \mathbf{B}^{K2} + \dots + \mathbf{Q}^{0N_c} \mathbf{B}^{KN_c} \quad (\text{A-40})$$

Combine equations of (A-39) and (A-40) we have

$$\begin{bmatrix} \mathbf{B}^{E0} \\ \mathbf{B}^{K0} \end{bmatrix} = \begin{bmatrix} \mathbf{Q}^{01} & 0 & \mathbf{Q}^{02} & 0 & \dots & \mathbf{Q}^{0N_c} & 0 \\ 0 & \mathbf{Q}^{01} & 0 & \mathbf{Q}^{02} & \dots & 0 & \mathbf{Q}^{0N_c} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{E1} \\ \mathbf{B}^{K1} \\ \mathbf{B}^{E2} \\ \mathbf{B}^{K2} \\ \vdots \\ \mathbf{B}^{EN_c} \\ \mathbf{B}^{KN_c} \end{bmatrix} \quad (\text{A-41})$$

we brief equation (A-41) as

$$\tilde{\mathbf{B}}^0 = \mathbf{Q}^{0B} \mathbf{B} \quad (\text{A-42})$$

A.2 Boundary conditions and field coupling

The expression for E_z^+ , H_z^+ , E_z^- and H_z^- (A-7), (A-8) and (A-9), (A-10) are used in (A-3) to (A-6) to generate the azimuthal components of electric and magnetic field inside and outside cylinder l . These are then employed in the boundary conditions to generate linear equations linking the coefficients A_m^{E-} , B_m^{E-} , A_m^{K-} , B_m^{K-} , A_m^{E+} , B_m^{E+} , A_m^{K+} , B_m^{K+} .

The four continuity equations can be expressed:

$$E_z^+ = E_z^- \quad (\text{A-43})$$

$$H_z^+ = H_z^- \quad (\text{A-44})$$

Appendix A

$$E_{\theta}^{+} = E_{\theta}^{-} \quad (\text{A-45})$$

$$H_{\theta}^{+} = H_{\theta}^{-} \quad (\text{A-46})$$

Now we begin to derive exterior reflection matrix which represents outgoing electrical field (noted by B_m^{+} coefficients) in terms of incoming optical fields (noted by A_m^{+} coefficients). From Eqs. (A-43) to (A-46), we can get the reflection matrix as

$$B_m^{E+} = R_m^{EE+} A_m^{E+} + R_m^{EK+} A_m^{K+} \quad (\text{A-47})$$

$$B_m^{K+} = R_m^{KE+} A_m^{E+} + R_m^{KK+} A_m^{K+} \quad (\text{A-48})$$

where we can get the matrix expression as

$$\begin{bmatrix} B^{E+} \\ B^{K+} \end{bmatrix} = \begin{bmatrix} R^{EE+} & R^{EK+} \\ R^{KE+} & R^{KK+} \end{bmatrix} \begin{bmatrix} A^{E+} \\ A^{K+} \end{bmatrix} \quad (\text{A-49})$$

or

$$\tilde{B}^{+} = \tilde{R}^{+} \tilde{A}^{+} \quad (\text{A-50})$$

for the coefficients for the field in the cylinder, we can get the same internal reflection matrix as

$$A_m^{E-} = R_m^{EE-} B_m^{E-} + R_m^{EK-} B_m^{K-} \quad (\text{A-51})$$

$$A_m^{K-} = R_m^{KE-} B_m^{E-} + R_m^{KK-} B_m^{K-} \quad (\text{A-52})$$

the matrix expression is

$$\begin{bmatrix} A^{E-} \\ A^{K-} \end{bmatrix} = \begin{bmatrix} R^{EE-} & R^{EK-} \\ R^{KE-} & R^{KK-} \end{bmatrix} \begin{bmatrix} B^{E-} \\ B^{K-} \end{bmatrix} \quad (\text{A-53})$$

or

$$\tilde{A}^{-} = \tilde{R}^{-} \tilde{B}^{-} \quad (\text{A-54})$$

The reflection coefficients of R_m^{EE+} , R_m^{EK+} , R_m^{KE+} , R_m^{KK+} , R_m^{EE-} , R_m^{EK-} , R_m^{KE-} and R_m^{KK-} are given in the Appendix A.

A.3 Generalized Rayleigh identity

Untill now we have obtained two identities equations (A-29) and (A-42) as well as two reflection matrices (A-50) and (A-54). We now restate these four key equations. We will link these equation with each other and get a single matrix equation which is named as *the generalized Rayleigh identity*.

Two identities

$$A = PB + Q^{B0} \tilde{A}^0 \quad (A-55)$$

$$\tilde{B}^0 = Q^{0B} B \quad (A-56)$$

and two reflection matrices

$$\tilde{B}^+ = \tilde{R}^+ \tilde{A}^+ \quad (A-57)$$

$$\tilde{A}^- = \tilde{R}^- \tilde{B}^- \quad (A-58)$$

In equation (A-57) and (A-58), we use superscripts *+* and *−* to describe the exterior and interior reflection. We note that the exterior reflection can be applied for all cylinder inclusions while the interior reflection can be applied to the jacket.

Thus equations (A-57) and (A-58) can be written as

$$\tilde{B}^I = \tilde{R}^I \tilde{A}^I \quad (A-59)$$

$$\tilde{A}^0 = \tilde{R}^0 \tilde{B}^0 \quad (A-60)$$

From (A-59) we can get the relationship of A and B as

$$B = RA \quad (A-61)$$

where

$$R = \text{diag}[\tilde{R}^I] \quad (A-62)$$

We multiply *R* to the equation of (A-55) and use equation (A-61) and it is easy to get

Appendix A

$$B = RPB + RQ^{B0} \tilde{A}^0 \quad (\text{A-63})$$

then we substitute equations (A-60) and (A-56) into (A-63), finally we get the Rayleigh identity

$$B = RPB + RQ^{B0} \tilde{R}^0 Q^{0B} B \quad (\text{A-64})$$

or

$$[I - R(P + Q^{B0} \tilde{R}^0 Q^{0B})]B = 0 \quad (\text{A-65})$$

where we define matrix M

$$M = I - R(P + Q^{B0} \tilde{R}^0 Q^{0B}) \quad (\text{A-66})$$

the dimension of M is $(4m+2)N_c \times (4m+2)N_c$. To get Non-trivial solution for the co-efficient vector B , it is required that the determinant of matrix M should be zero. As we know the determinant of matrix M is the function of propagation constant β or effective index n_{eff} , thus we can get certain β or n_{eff} values that make $\det(M) = 0$.

A.4 Solutions of the field coefficients

After we get the n_{eff} value corresponding to a particular mode, we can substitute it back to matrix M [75], and equation (A-65) becomes

$$\begin{bmatrix} m_{11} & \cdots & m_{1(N_c-1)} & m_{1N_c} \\ \vdots & \ddots & \vdots & \vdots \\ m_{(N_c-1)} & \cdots & m_{(N_c-1)(N_c-1)} & m_{(N_c-1)N_c} \\ m_{N_c} & \cdots & m_{N_c(N_c-1)} & m_{N_c N_c} \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_{(N_c-1)} \\ B_{N_c} \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix} \quad (\text{A-67})$$

Appendix A

We need to note that $B_{N_c} = \begin{bmatrix} B^{EN_c} \\ B^{KN_c} \end{bmatrix}$, and $B^{EN_c} = \begin{bmatrix} B_{-m}^{EN_c} \\ B_{-m}^{EN_c} \\ \vdots \\ B_{m-1}^{EN_c} \\ B_m^{EN_c} \end{bmatrix}$, $B^{KN_c} = \begin{bmatrix} B_{-m}^{KN_c} \\ B_{-m}^{KN_c} \\ \vdots \\ B_{m-1}^{KN_c} \\ B_m^{KN_c} \end{bmatrix}$, and to get

the solution of the vector B , we can set B_{N_c} to 1. We then can get $N_c - 1$ linear equations

$$\begin{bmatrix} m_{11} & \cdots & m_{1(N_c-1)} \\ \vdots & \ddots & \vdots \\ m_{(N_c-1)1} & \cdots & m_{(N_c-1)(N_c-1)} \end{bmatrix} \begin{bmatrix} B_1 \\ \vdots \\ B_{(N_c-1)} \end{bmatrix} = \begin{bmatrix} -m_{N_c1} \\ \vdots \\ -m_{N_c(N_c-1)} \end{bmatrix} \quad (\text{A-68})$$

From equation (A-68) we can get the solution of vector B . From the deduced equation (A-61), (A-56) and (A-60) we can get the solution of vector A , $\tilde{B}^0$ and $\tilde{A}^0$. The fields E_z and K_z outside the cylinder can be calculated using the Wijngaard's expansion in the equation (A-9) and (A-10).

$$E_z = \sum_{l=1}^{N_c} \sum_m B_m^{E+} H_m^{(1)}(k_{\perp}^+ |r_l|) e^{im\theta} + \sum_m A_m^{E0} J_m(k_{\perp}^+ r) e^{im\theta} \quad (\text{A-69})$$

$$K_z = \sum_{l=1}^{N_c} \sum_m B_m^{K+} H_m^{(1)}(k_{\perp}^+ |r_l|) e^{im\theta} + \sum_m A_m^{K0} J_m(k_{\perp}^+ r) e^{im\theta} \quad (\text{A-70})$$

The E_z and K_z field in the cylinder inclusion can be written as

$$E_z(r_l, \theta_l) = \sum_m A_m^{El-} J_m(k_{\perp}^- r_l) e^{im\theta_l} \quad (\text{A-71})$$

$$K_z(r_l, \theta_l) = \sum_m A_m^{Kl-} J_m(k_{\perp}^- r_l) e^{im\theta_l} \quad (\text{A-72})$$

It should be noted that there is no outgoing terms for field inside a cylinder. This is due to the fact that there is no source residing inside a cylindrical inclusion.

In the jacket, the E_z and E_z field can be written as

Appendix A

$$E_z^+(r_0, \theta) = \sum_m B_m^{E0+} H_m(k_\perp^{0+} r_0) e^{im\theta_0} \quad (\text{A-73})$$

$$K_z^+(r_0, \theta) = \sum_m B_m^{K0+} H_m(k_\perp^{0+} r_0) e^{im\theta_0} \quad (\text{A-74})$$

The methods to obtain coefficients A_m^{El-} , A_m^{Kl-} , B_m^{E0+} and B_m^{K0+} can be derived in the following deduction .

The four continuity equations can be expressed:

$$E_z^+ = E_z^- \quad (\text{A-75})$$

$$H_z^+ = H_z^- \quad (\text{A-76})$$

$$E_\theta^+ = E_\theta^- \quad (\text{A-77})$$

$$H_\theta^+ = H_\theta^- \quad (\text{A-78})$$

from the continuity of (A-75) to (A-78), we have

$$\sum_m [A_m^{E+} J_m(k_\perp^+ r_i) + B_m^{E+} H_m(k_\perp^+ r_i)] e^{im\theta} = \sum_m [A_m^{E-} J_m(k_\perp^- r_i) + B_m^{E-} H_m(k_\perp^- r_i)] e^{im\theta} \quad (\text{A-79})$$

here we abbreviate $J_m(k_\perp^- r_i)$ as J_m^- , and $J_m(k_\perp^+ r_i)$ as J_m^+ . Hankel terms are also abbreviated at the same way. Series expansion coefficients corresponding to the same order equals on the both sides. We hence have

$$A_m^{E+} J_m^+ + B_m^{E+} H_m^+ = A_m^{E-} J_m^- + B_m^{E-} H_m^- \quad (\text{A-80})$$

$$A_m^{K+} J_m^+ + B_m^{K+} H_m^+ = A_m^{K-} J_m^- + B_m^{K-} H_m^- \quad (\text{A-81})$$

$$\frac{m\beta}{(k_\perp^+)^2 r_i} (A_m^{E+} J_m^+ + B_m^{E+} H_m^+) + \frac{ik}{k_\perp^+} (A_m^{K+} J_m^+ + B_m^{K+} H_m^+) = \frac{m\beta}{(k_\perp^-)^2 r_i} (A_m^{E-} J_m^- + B_m^{E-} H_m^-) + \frac{ik}{k_\perp^-} (A_m^{K-} J_m^- + B_m^{K-} H_m^-) \quad (\text{A-82})$$

$$\frac{m\beta}{(k_\perp^+)^2 r_i} (A_m^{K+} J_m^+ + B_m^{K+} H_m^+) - \frac{ik r_i^2}{k_\perp^+} (A_m^{E+} J_m^+ + B_m^{E+} H_m^+) = \frac{m\beta}{(k_\perp^-)^2 r_i} (A_m^{K-} J_m^- + B_m^{K-} H_m^-) + \frac{ik}{k_\perp^-} (A_m^{E-} J_m^- + B_m^{E-} H_m^-) \quad (\text{A-83})$$

External Reflection

Appendix A

Now we begin to derive exterior reflection matrix which expresses outgoing fields in terms of incoming optical fields. It is need to note that within a single cylinder, there is not any source present, so there should not have any outgoing field in existence within a single cylinder. Hence it is reasonable to let all $B_m^- = 0$ during the analysis.

From the (A-80) to (A-83) we can obtain

$$B_m^{E+} = R_m^{EE+} A_m^{E+} + R_m^{EK+} A_m^{K+} \quad (\text{A-84})$$

$$B_m^{K+} = R_m^{KE+} A_m^{E+} + R_m^{KK+} A_m^{K+} \quad (\text{A-85})$$

where

$$R_m^{EE+} = \frac{1}{\delta_m} \left[(\alpha_{J^-H^+}^+ - \alpha_{H^+J^-}^-) (n_-^2 \alpha_{J^-J^+}^+ - n_+^2 \alpha_{J^+J^-}^-) - m^2 J_m^+ H_m^+ J_m^{-2} \tau^2 \right] \quad (\text{A-86})$$

$$R_m^{EK+} = \frac{1}{\delta_m} \left[\frac{2m\tau}{\pi k r_i} \frac{k_{\perp}^-}{k_{\perp}^+} J_m^{-2} \right] \quad (\text{A-87})$$

$$R_m^{KE+} = -n_+^2 R_m^{EK+} \quad (\text{A-88})$$

$$R_m^{KK+} = \frac{1}{\delta_m} \left[(\alpha_{J^-J^+}^+ - \alpha_{J^+J^-}^-) (n_-^2 \alpha_{J^-H^+}^+ - n_+^2 \alpha_{H^+J^-}^-) - m^2 J_m^+ H_m^+ H_m^{-2} \tau^2 \right] \quad (\text{A-89})$$

with

$$\delta_m = (\alpha_{H^+J^-}^- - \alpha_{J^-H^+}^+) (n_-^2 \alpha_{J^-H^+}^+ - n_+^2 \alpha_{H^+J^-}^-) + (m J_m^- H_m^+ \tau)^2 \quad (\text{A-90})$$

$$\tau = \frac{\beta}{r k_{\perp}^- k_{\perp}^+} (n_+^2 - n_-^2) \quad (\text{A-91})$$

$$\alpha_{J^-H^+}^+ = \frac{k_{\perp}^+}{k} J_m^+ H_m^+ \quad (\text{A-92})$$

and

$$\alpha_{H^+J^-}^- = \frac{k_{\perp}^-}{k} H_m^+ J_m^- \quad (\text{A-93})$$

Appendix A

Internal Reflection

We need to note the internal reflection refer to the reflection between the interface of host material and the Jacket which surrounding the cylinders. The derivation is quite similar to the way we get the exterior reflection matrix. We set $A_m^+ = 0$, as there is no source existing outside the jacket boundary. Thus we have

$$A_m^{E-} = R_m^{EE-} B_m^{E-} + R_m^{EK-} B_m^{K-} \quad (\text{A-94})$$

$$A_m^{K-} = R_m^{KE-} B_m^{E-} + R_m^{KK-} B_m^{K-} \quad (\text{A-95})$$

reflection coefficient can be written as

$$R_m^{EE-} = \frac{1}{\delta_m} \left[(\alpha_{J^-H^+}^+ - \alpha_{H^+J^-}^-) (n_-^2 \alpha_{H^-H^+}^+ - n_+^2 \alpha_{H^+H^-}^-) - m^2 J_m^- H_m^- H_m^{+2} \tau^2 \right] \quad (\text{A-96})$$

$$R_m^{EK-} = \frac{1}{\delta_m} \left[\frac{2m\tau}{\pi k r_i} \frac{k_{\perp}^+}{k_{\perp}^-} H_m^{+2} \right] \quad (\text{A-97})$$

$$R_m^{KE-} = -n_-^2 R_m^{EK-} \quad (\text{A-98})$$

$$R_m^{KK-} = \frac{1}{\delta_m} \left[(\alpha_{H^-H^+}^+ - \alpha_{H^+H^-}^-) (n_-^2 \alpha_{J^-H^+}^+ - n_+^2 \alpha_{H^+J^-}^-) - m^2 J_m^- H_m^- H_m^{+2} \tau^2 \right] \quad (\text{A-99})$$

External to internal transmission

To get the transmission between the external and internal of an isolated cylinder, we still start from the boundary continuities expressed in the equations (A-75) to (A-78). For the reason there is no source existing inside a cylinder, B_m^{E-} and B_m^{K-} will be set as zeros. We can have

$$\begin{bmatrix} A^{E-} \\ A^{K-} \end{bmatrix} = \begin{bmatrix} T^{EE+-} & T^{EK+-} \\ T^{KE+-} & T^{KK+-} \end{bmatrix} \begin{bmatrix} A^{E+} \\ A^{K+} \end{bmatrix} \quad (\text{A-100})$$

or

Appendix A

$$\tilde{A}^- = \tilde{T}^{+-} \tilde{A}^+ \quad (\text{A-101})$$

we can calculate regular field coefficients in a cylinder l using

$$\tilde{A}^{l-} = \tilde{T}^{l+-} \tilde{A}^{l+} \quad (\text{A-102})$$

with

$$T_m^{EE+-} = \text{diag}[T_m^{EE+-}] \quad (\text{A-103})$$

$$T_m^{EK+-} = \text{diag}[T_m^{EK+-}] \quad (\text{A-104})$$

$$T_m^{KE+-} = \text{diag}[T_m^{KE+-}] \quad (\text{A-105})$$

$$T_m^{KK+-} = \text{diag}[T_m^{KK+-}] \quad (\text{A-106})$$

Transmission coefficient for each order of expansion can be written as

$$T_m^{EE+-} = \frac{1}{\delta_m} [n_+^2 (\alpha_{J^+H^+}^- - \alpha_{H^+J^+}^-) (\alpha_{H^+J^-}^- - \alpha_{J^-H^+}^-)] \quad (\text{A-107})$$

$$T_m^{EK+-} = \frac{1}{\delta_m} [im \omega J_m^- H_m^+ (\alpha_{H^+J^+}^- - \alpha_{J^+H^+}^-)] \quad (\text{A-108})$$

$$T_m^{KE+-} = n_+^2 T_m^{EK+-} \quad (\text{A-109})$$

$$T_m^{KK+-} = -\frac{1}{\delta_m} [(\alpha_{J^+H^+}^- - \alpha_{H^+J^+}^-) (n_-^2 \alpha_{H^+J^-}^- - n_+^2 \alpha_{J^-H^+}^-)] \quad (\text{A-110})$$

Internal to External transmission

$$\begin{bmatrix} B^{E+} \\ B^{K+} \end{bmatrix} = \begin{bmatrix} T^{EE-+} & T^{EK-+} \\ T^{KE-+} & T^{KK-+} \end{bmatrix} \begin{bmatrix} B^{E-} \\ B^{K-} \end{bmatrix} \quad (\text{A-111})$$

or

$$\tilde{B}^+ = \tilde{T}^{-+} \tilde{B}^- \quad (\text{A-112})$$

In the jacket, outgoing field expansion coefficient can be written as

$$\tilde{B}^{0+} = \tilde{T}^{0-+} \tilde{B}^{0-} \quad (\text{A-113})$$

Appendix A

with

$$T_m^{EE-+} = \text{diag}[T_m^{EE-+}] \quad (\text{A-114})$$

$$T_m^{EK-+} = \text{diag}[T_m^{EK-+}] \quad (\text{A-115})$$

$$T_m^{KE-+} = \text{diag}[T_m^{KE-+}] \quad (\text{A-116})$$

$$T_m^{KK-+} = \text{diag}[T_m^{KK-+}] \quad (\text{A-117})$$

and the transmission coefficients are

$$T_m^{EE-+} = \frac{1}{\delta_m} \left[n_-^2 (\alpha_{J^-H^-}^+ - \alpha_{H^-J^-}^+) (\alpha_{H^+J^-}^+ - \alpha_{J^-H^+}^+) \right] \quad (\text{A-118})$$

$$T_m^{EK-+} = \frac{1}{\delta_m} \left[im \tau J_m^- H_m^+ (\alpha_{H^-J^-}^+ - \alpha_{J^-H^-}^+) \right] \quad (\text{A-119})$$

$$T_m^{KE-+} = -n_-^2 T_m^{EK-+} \quad (\text{A-120})$$

$$T_m^{KK-+} = -\frac{1}{\delta_m} \left[(\alpha_{H^-J^-}^+ - \alpha_{J^-H^-}^+) (n_-^2 \alpha_{J^-H^+}^+ - n_+^2 \alpha_{H^+J^-}^+) \right] \quad (\text{A-121})$$

As we stated before E_θ, K_θ, E_r and K_r can be calculated by through equations (A-3) to (A-6), thus we have obtained all the fields in the whole transverse domain.

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