Integrated control chart systems

Md. Shamsuzzaman

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Integrated Control Chart Systems

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Abstract

A multi-stage and multi-stream manufacturing system has several process stages, each of which consists of one or more parallel streams (or machines). A number of methods have been developed for implementing Statistical Process Control (SPC) for multi-stream manufacturing processes, but none of them has designed an SPC scheme for a multi-stage and multi-stream manufacturing system in an integrative and optimal manner. This research presents a number of algorithms for the optimization designs of the integrated control chart systems. They consist of several control charts for monitoring the multi-stage and multi-stream manufacturing processes. It is believed that the studies on the SPC for multi-stage and multi-stream manufacturing systems, and the development of the integrated control chart systems are original contributions to the literature and practice of quality engineering.

The studies are conducted in two phases. The first phase (Chapter 3 and Chapter 4) presents the statistical and economical designs of the $\bar{X}$ control chart systems monitoring process mean. Chapter 3 presents two models for the statistical designs of the integrated $\bar{X}$ control chart systems. In Chapter 4, two economical models for single process are first designed based on random mean shifts. The models are implemented through a three-phase SPC operational scenario proposed in this thesis. The same concept is then applied to the design of the integrated $\bar{X}$ control chart system for a multi-stage and multi-stream manufacturing system.

The second phase (Chapter 5 and Chapter 6) develops the statistical and economical models for the designs of the $\bar{X}$ & S control chart systems for monitoring process shifts in both mean and variance. In Chapter 5, two optimization models for the statistical designs of the integrated $\bar{X}$ & S control chart systems are developed. In

Chapter 6, two economical models for a single process are first designed, and then the same concept is applied to developing the economical designs of the integrated $\bar{X}$ & S control chart system.

The performance of all the models are compared to that of the conventional approaches and also illustrated through examples. Software systems for automating the designs of the SPC systems are also developed.

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Chapter 1

Introduction

1.1 Preliminaries

For decades, quality has been established as a key competitive priority in the world of business. As a result, related academic research has been revived and is reported in increasing volumes in all aspects of Quality Management, and Statistical Quality Control (SQC) in particular. Although the need for maintaining and improving quality standard is growing with increasing competition, the idea of quality control is not a new one. What is new about quality control is the use of statistical techniques, which are helpful in maintaining and improving quality standards. The use of statistics to monitor a process was first presented by Dr. Walter A. Shewhart. The frequently cited terms in the use of statistical methods are *Statistical Process Control* (SPC), *quality control*, and *control charts*. The use of these terms sometimes creates some confusion because the term *control* is also used in *Engineering Process Control* (EPC) within the process industries. In SPC, the control means monitoring a process, and indicates when corrective action should be taken (Montgomery 2001). For EPC, control means adjusting a process to keep process output variables on target (Box *et al.* 1997, Janakiram and Keats 1998). In this study, the term control is used in the SPC context.

SPC is a collection of powerful problem-solving tools of SQC, which are useful in achieving process stability and improving process capability. The function of SPC is to detect changes in the probability distribution of critical measurement of quality as early as possible. The most widely used and probably the most technically sophisticated tool of SPC is the control chart which was developed in the 1920s by Shewhart (1926). The control charts are the on-line statistical tool of detecting out-of-

control conditions of a process, so that the corrective actions can be taken timely before a large amount of defective units are produced. The use of a control chart requires that the engineer or analyst select the sample size, the sampling interval, and the control limits for the chart. The determination of these parameters is usually called the *design* of the control chart.

The implementation of a control chart is accomplished through two steps, and is known as the *two-phase SPC scenario*. In the first phase, the process to be controlled is studied to determine its in-control characteristics. One or more process variables are identified and a measurement procedure is established. Data are collected and analyzed for some period of time during which the process is established. These studies test the distribution assumptions and estimate the in-control distribution parameters such as mean and variance. When this information has been obtained, one or more test methods and statistics are chosen and control limits are calculated. In the second phase, the process is tracked by taking periodic samples, measuring the characteristic(s) of interest, plotting the values of the associated statistics on a control chart, and then examining the plotted points against the control limits established in the first phase. A modern manufacturing process may adopt automated data measurement, computer analysis and computer display of control chart, and sample points. However, the same two-phase SPC scenario is still used in these processes.

If a quality characteristic can be measured and expressed as a number on some continuous scale of measurement (such as a dimension, weight or volume), it is usually called a *variable* and the corresponding control chart is the *variable control chart*. Shewhart $\bar{X}$ chart and S (or R) chart are the typical variable control charts. The $\bar{X}$ chart is used to monitor the process mean, whereas the joint $\bar{X}$ & S (or $\bar{X}$ & R) charts are used to monitor process mean and variance together. Although different alternative

approaches, such as Exponentially Weighted Moving-Average (EWMA) or CUMulative-SUM (CUSUM) charts, have been proposed, the Shewhart $\bar{X}$ chart as well as the $\bar{X}$ & S charts are still considered the most widely used variable control charts because of their simplicity for implementation and easy to understand.

1.2 Research motivation

The fabrication of a product usually includes many process stages. The integration of all these stages results in a *manufacturing system*. For example, in the manufacturing of a mechanical part, each stage may correspond to the machining of a dimension. Several process stages are usually required in order to fabricate the dimensions of a part (see Figure 1.1). Some of the dimensions are critical to the overall quality of the product, and the corresponding process stages have to be monitored by the control charts. A *control chart system* is the combination of all the charts that are used to monitor the critical stages of a manufacturing system. Owing to the differences in production rates and other factors, some stages (e.g. stages with lower production rates) may have more than one parallel stream (or machine). This type of manufacturing system can be referred to as a *multi-stage and multi-stream manufacturing system*. Examples of multi-stage and multi-stream manufacturing systems include: (i) the manufacturing and assembly of automotive, aerospace, or appliance products, in which parts are manufactured and assembled on multiple stations; (ii) the transfer line machining process which involves operations of a single part on multiple stations; and (iii) the progressive die stamping process which involves multiple stamping stations to form the parts. To develop the SPC schemes for the multi-stage and multi-stream manufacturing systems, the existing literatures suggest to using a group of control charts to monitor the quality characteristic at each stage.

Figure 1.1 (a) Dimensions of the part; (b) Process stages

In a survey paper, Woodall and Montgomery (1999) pointed out that the scope of SPC should be broadened to include a much wider variety of methods for understanding variation over time, not just traditional control charting. Multi-step production processes and measurement processes should be considered. Although different research journals have published many articles concerning various quality control procedures, only a few references (see Section 2.4.1 in Chapter 2) are directly or indirectly related to the multi-stage and multi-stream manufacturing processes. None of the articles considers the multi-stage and multi-stream manufacturing processes as a whole and designs the charting parameters in an integrative and optimal manner. Since the processes in different stages in a manufacturing system have different precision and other characteristics (e.g. process capability, magnitudes of the shifts), if all the charts in the chart system are designed in an integrative and optimal

manner, the performance of the chart system as a whole will be improved significantly. This research project studies the multi-stage and multi-stream manufacturing system and proposes several algorithms for the *optimization designs* of the integrated control chart systems. In this thesis, the optimization design refers to the minimization of the out-of-control average time to signal (*ATS*) or the total cost associated with the control chart system on condition that the false rate of the chart system is kept at a specified level. Some of the proposed design algorithms focus on tackling mean shifts, and the others monitor mean and variance shifts together. The latter take place most frequently in practice and attract most of the attentions of Quality Assurance (QA) personnel. The $\bar{X}$ chart as well as the $\bar{X}$ & S charts are selected to investigate and illustrate the proposed optimization design methodologies because they have widespread use in industry and can be easily understood and implemented. The general methodologies could be extended to other control charts.

1.3 Research objectives

The objectives of this research project are to develop the algorithms for the optimization designs of the integrated control chart systems used in the multi-stage and multi-stream manufacturing systems. The software systems for automating the design procedures are also developed in this project.

1.4 Scope of the research

The scope of the research project is two fold. In the first part (Chapter 3 and Chapter 4), both statistical and economical designs of the integrated $\bar{X}$ control chart systems are developed considering only mean shift. In Chapter 3, two optimization models are developed for the statistical designs of the integrated $\bar{X}$ control chart systems. The first

model only optimizes the control limits of each $\bar{X}$ chart in the chart system. The second model optimizes the sample size, sampling interval, and control limits together. In Chapter 4, two economical $\bar{X}$ chart models are first designed for a single process. Then, the same concept is applied to the economical design of the integrated $\bar{X}$ control chart system. The economical models consider the random mean shifts and employ a *three-phase SPC scenario*.

The second part (Chapter 5 and Chapter 6) of the project is to design the integrated $\bar{X}$ & S control chart system for monitoring process mean and variance together. In Chapter 5, two optimization models for the statistical designs of the integrated $\bar{X}$ & S chart systems are developed, where the first model only optimizes the control limits of each chart in the chart system and the second model optimizes the sample sizes, sampling intervals and control limits together. In Chapter 6, two economical models are first designed for a single process and, then, the same concept is applied to the economical design of the integrated $\bar{X}$ & S control chart system.

Table 1.1 Model code letters

1	2	3	4	5
Type of process shift	Type of design	Type of Process	Type of control chart scheme	Design variables
Mean ($\bar{X}$ chart) M	Statistical S	Independent processes I	Integrated $\bar{X}$ chart system or integrated $\bar{X}$ & S chart system S	Control limits only L
Mean and variance ($\bar{X}$ & S charts) V	Economical E	Interdependent processes D	Single $\bar{X}$ chart or single pair of $\bar{X}$ & S charts I	Sample size, sampling interval, and control limits N
				Sample size, sampling interval, control limits, and manpower M

The *model code letters* shown in Table 1.1 have been adopted for the easy identification of the different models developed in this thesis. The five-digit code letters are defined as follows.

1. **M:** Only the process mean is tackled and only the $\bar{X}$ chart is used to monitor the process
- V:** Process shifts both in mean and variance are tackled and the joint $\bar{X}$ & S charts are used to monitor the process
2. **S:** Represents the statistical design of the control chart system where the out-of-control Average Time to Signal ATS is minimized
- E:** Represents the economical designs of the control chart system where the quality losses or the total costs are minimized
3. **I:** Denotes that the process stages in the manufacturing system are independent of each other
- D:** Denotes that the process stages in the manufacturing system are interdependent
4. **S:** Represents the $\bar{X}$ chart system or $\bar{X}$ & S chart system monitoring the multi-stage and multi-stream manufacturing system
- I:** Represents a single $\bar{X}$ chart or a single pair of $\bar{X}$ & S charts monitoring a single process
5. **L:** Denotes that only the control limits of the $\bar{X}$ chart or the $\bar{X}$ & S charts are optimized
- N:** Denotes that the sample size, sampling interval, and control limits of the $\bar{X}$ chart or the $\bar{X}$ & S charts are optimized together
- M:** Denotes that the sample size, sampling interval, control limits of the $\bar{X}$

chart or the $\bar{X}$ & S charts and the required manpower are optimized together

As an example, the model **MSDSL** ((1) mean ($\bar{X}$ chart) / (2) statistical design / (3) interdependent processes / (4) control chart system / (5) only control limits are optimized) is used to denote a statistical model for the design of a dependent $\bar{X}$ control chart system in which control limits will be optimized. All of the charting models developed in this thesis are summarized below, and also displayed in Figure 1.2.

Model-1: **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) and **MSDSL** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / control limits). Statistical designs of the independent and interdependent $\bar{X}$ control chart systems in which control limits are optimized (Chapter 3, Section 3.2.1, Section 3.3).

Model-2: **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) and **MSDSN** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / sample size, sampling interval, and control limits). Statistical designs of the independent and interdependent $\bar{X}$ control chart systems in which sample sizes, sampling intervals, and control limits are optimized (Chapter 3, Section 3.2.2, Section 3.3).

Model-3: **MEIIN** ($\bar{X}$ chart / economical design / independent process / single chart / sample size, sampling interval, and control limits). Economical design of the single $\bar{X}$ control chart in which sample size, sampling interval, and control limits are optimized. It minimizes the quality loss (Chapter 4, Section 4.3).

Figure 1.2 Summary of the scopes of the research project

Model-4: **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower). Economical design of the single $\bar{X}$ control chart in which sample size, sampling interval, and control limits are optimized. It minimizes the expected total cost and optimally deploys manpower to the SPC scheme (Chapter 4, Section 4.4).

Model-5: **MEDSM** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower). Economical design of the integrated $\bar{X}$ control chart system in which sample sizes, sampling intervals, and control limits are optimized. It minimizes the expected total cost and optimally deploys manpower to the SPC system (Chapter 4, Section 4.5).

Model-6: **VSISL** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / control limits). Statistical design of the integrated $\bar{X}$ & S control chart system in which control limits are optimized (Chapter 5, Section 5.2).

Model-7: **VSISN** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits). Statistical design of the integrated $\bar{X}$ & S control chart system in which sample sizes, sampling intervals, and control limits are optimized (Chapter 5, Section 5.3).

Model-8: **VEIIN** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, and control limits). Economical design of the single pair of $\bar{X}$ & S control chart in which sample size, sampling interval, and control limits are optimized. It minimizes the

quality loss and focuses on the continuous improvement of the control chart during the use of the chart (Chapter 6, Section 6.3).

Model-9: VEIIM ($\bar{x}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower). Economical design of the single pair of $\bar{x}$ & S control chart in which sample size, sampling interval, and control limits are optimized. It minimizes the expected total cost and optimally deploys manpower to the SPC scheme (Chapter 6, Section 6.4).

Model-10: VEISM ($\bar{x}$ & S charts / economical design / independent processes / control chart system / sample size, sampling interval, control limits, and manpower). Economical design of the integrated $\bar{x}$ & S control chart system in which sample sizes, sampling intervals, and control limits are optimized. It minimizes the expected total cost and optimally deploys manpower to the SPC system (Chapter 6, Section 6.5).

1.5 Assumptions

The following assumptions and conventions have been adopted throughout the designs of the models discussed in this thesis.

- (1) The quality characteristic x (e.g. the dimension of a mechanical part) in each process is normally distributed with in-control mean μ_0 and standard deviation σ_0 . Moreover, μ_0 is equal to the central value between the lower and upper specification limits LSL and USL .
- (2) Since the processes will often operate in the in-control condition for most of the time, or for relatively long periods of time (Montgomery 2001), it is

assumed that only one process is out of control at any moment in a manufacturing system.

- (3) The g_i parallel streams (or machines) in the i th stage of a manufacturing system have the same mean, standard deviation and target and use the identical control charts (with same sample size, sampling interval, and control limits). This assumption is adopted by many researchers studying the multiple stream systems (Nelson 1986, Mortell and Runger 1995, Runger and Montgomery 1996).
- (4) If several operators are engaged in the inspection activities in a manufacturing system, they can share their inspection time in all the stages.
- (5) The time required to inspect a unit in a stage is substantially smaller than the sampling interval of that stage.

1.6 Input parameters

The parameters listed in Table 1.2 have been used in the designs of the models proposed in this thesis. Most of these parameters can be determined easily. In fact, many of them are required by the individual chart designs.

The parameters s , g_i , LSL_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, t_i , p_i , $c_{min,i}$, M , and τ are generally used by both the statistical and economical designs. The numbers s and g_i can be found from the process layout (e.g. Figure 3.1(b) in Chapter 3) of the manufacturing system. The values of LSL_i and USL_i can be found from the engineering drawings (e.g. Figure 3.1(a) in Chapter 3). The distribution parameters $\mu_{0,i}$ and $\sigma_{0,i}$ are usually estimated from the data observed in the pilot runs or the process capability study. The values of t_i and p_i can be estimated from historical data or from a field test.

Table 1.2 Parameters used in the designs of the models

Parameter	Definition
s	Number of stages in the control chart system
g_i	Number of streams in the i th stage
LSL_i	Lower specification limit in the i th stage
USL_i	Upper specification limit in the i th stage
$\mu_{0,i}$	In-control process mean in the i th stage
$\sigma_{0,i}$	Standard deviation of the process in the i th stage
t_i	Time required to inspect a unit in the i th stage
p_i	Probability of occurrence of out-of-control case in the i th stage
$c_{min,i}$	Out-of-control (or minimum allowable) process capability index (C_{pk}) in the i th stage
M	Available number of operators equipped with measuring instruments in the manufacturing system
τ	Minimum allowable in-control Average Time to Signal (ATS_0) of the whole system
$MTBO_i$	Mean time between out-of-control cases in the i th stage
$\mu_{\delta\mu,i}$	Mean value of the mean shifts $\delta\mu$ in the i th stage
Q	Number of units produced per unit time, or the production rate
$C_{k,i}$	Cost required to fix an out-of-specification unit (product) produced in the i th stage in the workshop
C_m	Manpower cost for an operator per unit time. It includes the depreciation cost of the measuring equipment
R	Maximum allowable inspection rate

The value of $c_{min,i}$ is usually decided in accordance with some considerations, for example, whether the process is an existing one or a new one, or whether the product is critical to safety (Montgomery 2001). A default value of one will be used for $c_{min,i}$ in this thesis. It results in a *PPM* (nonconforming Parts Per Million) value of 2700.

The value of M depends on the total time that the operators in the manufacturing system are engaged with the quality inspections. If a single operator spends his/her full time for the inspection activities, then the value of M is equal to one. However, in general, each operator in the manufacturing system will not spend his/her full time on the inspection activities, so the value of M may also be less than one. As an example, if each of three operators in a manufacturing system can spend one fifth of his/her time in the inspection activities, then, M is equal to $3/5$.

The variable τ is specified based on the trade off between the false alarm rate and detection power. If the cost of handling the false alarms is high, a larger τ should be used in order to reduce the false alarm frequency. However, a large τ may impair the effectiveness of the control chart. The resultant (or actual) in-control ATS_0 must be equal to or larger than the specified value τ .

The additional parameters ($MTBO_i$, $\mu_{\delta\mu,i}$, Q , $C_{k,i}$, C_m , and R) are used by the economical designs only. The values of $MTBO_i$, $\mu_{\delta\mu,i}$ and Q can be estimated from historical data or from a field test (the estimation of $MTBO_i$ and $\mu_{\delta\mu,i}$ will be detailed in Chapter 4). The information of the cost components $C_{k,i}$ and C_m can be acquired from the financial department of the manufacturing organization. The value of the maximum allowable inspection rate R can be decided based on the available inspection resources and the results of the field test.

Chapter 2

Literature Review

This chapter reviews the quality improvement and the SPC, the conventional and optimization designs of the control charts, the statistical and economical designs of the control charts, and the control charts for multi-stage and multi-stream manufacturing processes.

2.1 Quality and quality improvement

Quality has been an integral part of all products and services in the modern business world. The concept of *quality* has been defined in many ways. The traditional definition of quality is based on the viewpoint that *products and services must meet the requirements of those who use them* (Montgomery 2001). Juran (1974) defined quality as *fitness for use*. Another frequently cited definition provided by Crosby (1979) states that *quality is conformant to requirements or specifications*. The modern definition of quality referred in Montgomery's textbook (2001) states that, *Quality is inversely proportional to variability*. Garvin (1987) provided an excellent discussion of different dimensions of quality such as *performance, reliability, durability, serviceability, aesthetics, features, perceived quality* and *conformance to standards*.

In this era of ever-growing competition, it has become absolutely necessary for a businessman to keep a continuous watch over the quality of the goods produced. Because of increasing competition in the marketplace, companies are always required to produce products of high quality but low cost. A continuous quality improvement program is the only tool to achieve this goal. Quality improvement means the systematic elimination of waste (Montgomery 1994). A Successful quality

improvement effort may lead to lower costs, higher productivity, increased customer satisfaction, higher market share, and ultimately the higher profits for the company.

2.2 Statistical quality control and statistical process control

It is necessary for a company to keep the variation in the quality characteristics of its products to a minimum, and to make it as the main task in quality control. The need for quality control arises from the fact that even after the quality standards have been met, some variation in quality is unavoidable. The term of quality in Statistical Quality Control (SQC) is usually related to some measurement made on the produced items. A good quality item conforms to standards specified for measurement. SQC includes those statistical and engineering methods that are used in measuring, monitoring, controlling and improving quality. They include Acceptance Sampling, Design of Experiments (DOE), Statistical Process Control (SPC) and Process Capability Analysis. Shewhart, in 1924, presented the first sketch of the modern control chart. Dodge and Roming (1959) developed the statistical theory for the sampling inspection. These three men constituted much to the basis of the modern SQC.

As quality has become the key issue in sustaining in the modern competitive business market, the manufacturers are more and more conscious about the quality of their products. This has been accompanied by a significant increase in the industrial applications of statistical methods.

SPC is a collection of powerful techniques for achieving process stability and improving capability of operating around the target with little variability. In recent years, due to the quality revolution, research activities on SPC have been increased greatly. The basic tools which are also known as *the magnificent seven of SPC* are

- (1) Histogram or stem-and-leaf display
- (2) Check sheet
- (3) Pareto chart
- (4) Cause and effect diagram
- (5) Defect concentration diagram
- (6) Scatter diagram
- (7) Control chart

Although a control chart is used as an important tool for quality improvement in both manufacturing and non-manufacturing sectors, it could not provide the maximum benefits if used as a stand-alone tool (Gaafar and Keats 1990). It is suggested to use the other six SPC tools in conjunction with the control charts to solve the quality related problems in the organization. However, a control chart is the most widely used on-line SPC procedure.

2.3 General introduction to control charts

A control chart is a graphic device for presenting data so as to directly reveal the frequency and extent of variations from established standards or goals. The variation of a quality characteristic can be divided into *chance variation* (or *natural variation*) and *assignable variation* (or *special variation*). The chance variation results from many minor causes that behave in a random manner. This type of variation is permissible, and indeed inevitable. Assignable variation is attributed to special non-random causes and may be the result of several factors such as change in the raw material, presence of a new operator, improper machine setting, broken or worn-out parts, and mechanical faults in plant. Experience has shown that control charts are able

to efficiently warn the presence of assignable variation and also measure the extent of chance variation that must be reduced by management action. The control chart proposed by Shewhart is commonly known as the Shewhart control chart. Figure 2.1 shows a typical Shewhart control chart.

Figure 2.1 A typical Shewhart control chart

A Shewhart control chart typically consists of three horizontal lines. Central Line (CL) represents the mean or normal value of the quality characteristic corresponding to the in-control state. The other two horizontal lines are Lower Control Limit (LCL) and Upper Control Limit (UCL). The distances of the control limits from the center line are expressed in terms of the in-control standard deviation. The control limits are chosen so that if the process is in control, nearly all of the sample points will fall between them. Therefore, as long as the points plot within the control limits in a random manner, the process is assumed to be in control. On the other hand, if any of the sample points falls beyond LCL or UCL, the process is thought out-of-control. However, even if all the points plot inside the control limits but in a systematic or nonrandom manner, then this could be an indication that the process is out-of-control (Montgomery 2001).

The Shewhart $\bar{X}$ control chart is relatively insensitive to small shifts in the process. Nelson (1984) and Deming (1986) suggested applying some *runs rules* to increase the effectiveness of the Shewhart control chart. Some well-known supplementary runs rules have been suggested by Western Electric Handbook (1956). Another class of supplementary runs rules known as *trend rules* have also been proposed by Davis and Woodall (1988). Divoky and Taylor (1995) used simulation to examine the sensitivity of the trend rules used in conjunction with the standard supplementary runs rules. The use of different sensitizing rules however increases the false alarm rate substantially (Champ and woodall 1987). It also reduces the simplicity and ease of interpretation of the Shewhart control chart (Montgomery 2001). Thus, the sensitizing rules should be used with considerable caution to avoid the excessive number of false alarms and complexity in decision process.

Control charts may be classified into two general types as *variable control chart* and *attribute control chart*. Variables are those quality characteristics, which are measurable and can be expressed in specific units (e.g. diameter, length). The control chart for mean ($\bar{X}$ chart), range (R chart) and standard deviation (S chart) are the typical variable control charts. Attributes, on the other hand, are those product characteristics, which are not amenable to measurement. Such characteristics can only be identified by their presence or absence from the product. The control charts for fraction non-conforming (p chart), number of non-conforming units (np chart), number of non-conformities (c chart), and number of non-conformities per unit (u chart) are the widely used attribute control charts. In this research project, variable control charts, especially the $\bar{X}$ chart and the joint $\bar{X}$ & S charts are investigated in the control chart systems.

2.3.1 Traditional Shewhart control charts

Shewhart $\bar{X}$ control chart

The Shewhart $\bar{X}$ chart is the most widely used control chart in practice (Ryan 1989, Montgomery 2001). It shows the averages of the samples of the quality characteristic x drawn from a given process. The design of an $\bar{X}$ control chart requires the determination of control limits LCL and UCL , sample size n , and sampling interval h . Conventionally, LCL and UCL of an $\bar{X}$ control chart are calculated by

$$\begin{aligned} LCL &= \mu_0 - k \frac{\sigma_0}{\sqrt{n}} \\ UCL &= \mu_0 + k \frac{\sigma_0}{\sqrt{n}} \end{aligned} \quad (2.1)$$

Where μ_0 and σ_0 are the in-control process mean and standard deviation. The control limit coefficient k is usually taken as three or determined based on the type I error probability α .

$$k = \Phi^{-1}(1 - 0.5\alpha) \quad (2.2)$$

Where $\Phi^{-1}()$ is the inverse function of the cumulative probability function of the standard normal distribution. Conventional design methods typically suggest a sample size of 4, 5 or 6 and 3-sigma control limits (Shewhart 1931, Duncan 1986, Grant and Leavenworth 1988, Dooley and Anderson 1992, Montgomery 2001). These values of sample sizes and control limits are widely used and are considered as the norms. The concept of *rational subgrouping* is often recommended in collecting the sample data. According to this concept, sampling intervals are selected to minimize the opportunity that assignable variability occurs within the sample but to maximize the opportunity that assignable variability occurs between samples. Grant and Leavenworth (1988) discussed the rational subgrouping and the associated type I and type II error probabilities. Duncan (1986) provided more quantitative methods of selecting sample

sizes and evaluating control schemes. The decisions regarding sample size and sampling interval can also be made based on the Average Run Length (ARL) of the control chart. ARL is the average number of samples that must be inspected before a sample point indicates an out-of-control condition. When the process is in control, the average run length of the control chart is known as the in-control ARL_0 which is an indication of the false alarm rate. The value of out-of-control ARL can be used to evaluate the effectiveness or power of the control chart. The smaller the value of out-of-control ARL , the more effective the control chart is. Page (1954a) selected sample size and control limits to minimize the out-of-control ARL . Klein (2000) constructed two $\bar{X}$ control chart schemes based on runs rules and evaluated the performance of the schemes in terms of out-of-control ARL . Specific values for sampling intervals are rarely suggested and quantitative methods of selecting sampling intervals are seldom used (Keats *et al.* 1995). Usually, for selecting the sampling interval the practitioner is advised to consider factors such as the production rate, the expected frequency of shifts to an out-of-control state and the possible consequences of such process shifts (Montgomery 2001).

Shewhart $\bar{X}$ & S control chart

When dealing with a variable, it is usually necessary to monitor both mean and variability (Montgomery 2001). Shewhart 3-sigma $\bar{X}$ & S (or $\bar{X}$ & R) control charts have been used widely for this purpose. While the $\bar{X}$ chart monitors the process mean shifts, the S (or R) chart controls the standard deviation shifts. Usually, the sample size n is chosen as 4, 5 or 6. However, when monitoring critical processes or detecting small shifts, the sample size needs to be moderately large (possibly $n = 15$ to 25). For such cases, the R chart loses statistical efficiency and the S chart is preferable.

Conventionally, the lower control limit LCL_s and the upper control limit UCL_s of an S control chart are calculated by

$$\begin{aligned} LSL_s &= c_4 \sigma_0 - k_s \sigma_0 \sqrt{1 - c_4^2} \\ USL_s &= c_4 \sigma_0 + k_s \sigma_0 \sqrt{1 - c_4^2} \\ c_4 &= \sqrt{\frac{2}{n-1}} \cdot \frac{\Gamma\left(\frac{n-2}{2}\right)}{\Gamma\left(\frac{n-3}{2}\right)} \end{aligned} \quad (2.3)$$

Where, like k in Equation (2.1) for the $\bar{X}$ chart, k_s is the control limit coefficient of the S chart, usually taken as three, and c_4 is a constant that depends on the sample size n .

The gamma function $\Gamma(x)$ is defined to be an extension of the factorial to complex and real number arguments. For example, if the argument x is an integer, $\Gamma(x) = (x - 1)!$.

On the other hand, if the argument is a fraction, $\Gamma\left(x + \frac{1}{2}\right) = \frac{(2x-1)!!}{2^x} \sqrt{\pi}$. The control limits LCL_s and UCL_s can also be determined based on the type I error probability α_s of the S chart.

$$\begin{aligned} LCL_s &= \sigma_0 \sqrt{\frac{[\chi_{n-1}^2]^{-1}(\alpha_s / 2)}{n-1}} \\ UCL_s &= \sigma_0 \sqrt{\frac{[\chi_{n-1}^2]^{-1}(1 - \alpha_s / 2)}{n-1}} \end{aligned} \quad (2.4)$$

Where $[\chi_{n-1}^2]^{-1}$ is the inverse function of the cumulative probability function of the Chi-square distribution with degrees of freedom equal to $(n - 1)$.

2.3.2 Other control charts

The Shewhart control chart is relatively insensitive to small shifts in the process. Two important alternatives to the Shewhart control chart may be used when the shifts in the

process are small. They are the CUmulative-SUM (CUSUM) control chart and the Exponentially Weighted Moving-Average (EWMA) control chart.

CUmulative-SUM (CUSUM) chart

The CUSUM chart is a powerful charting technique in detecting small shifts and linear trends in process. Page (1954*b*) first proposed the CUSUM chart and a number of modifications have been proposed since then (Hawkins 1981, Nishina and Peng-Hsiung 1996, Koning and Does 2000). Though the CUSUM charts are very sensitive to small shifts, it is not as efficient as the Shewhart control charts to detect the large shifts. In order to make the CUSUM procedure competitive with the $\bar{X}$ chart for detecting large shifts, a combined Shewhart-CUSUM chart was introduced by Lucas (1982).

Exponentially Weighted Moving-Average (EWMA) chart

The EWMA chart is also a good alternative to the Shewhart control chart in detecting small shifts in process. Roberts (1959) first introduced the EWMA chart. Several theoretical studies of the out-of-control *ARL* properties of the EWMA control charts are also reported in literature (Lucas and Saccucci 1990, Gan 1998, Steiner 1999). The EWMA chart is used extensively in time series modelling and in forecasting (Montgomery *et al.* 1990, Box *et al.* 1994). Like the CUSUM charts, the EWMA charts perform well against small shifts but do not react to large shifts as quickly as the Shewhart chart. The combined Shewhart-EWMA control scheme is effective against both large and small shifts (Montgomery 1997).

2.3.3 Design of control chart

Design of control chart means the selection of control limits LCL and UCL , sample size n , and sampling interval h .

The control limits are used as the basis for judging the significance of the quality variations from sample to sample, lot to lot, or from time to time. Specifying the control limits is one of the most critical decisions in designing a control chart. Type I and type II errors are associated with the application of the control charts. Type I error α is the probability that a sample point falls beyond the control limits (indicating the out-of-control condition) when the process is in fact in control. Type II error β is the probability of a sample point falling between the control limits when the process is actually out of control. Therefore, the probability to detect a shift when the process is actually out-of-control is $(1 - \beta)$, which is known as the power of the control chart. Moving the control limits further from the central line decreases the type I error. However, it will increase the type II error. If the control limits move closer to the central line, the opposite effect is obtained. Regardless of the distribution of the quality characteristic, it is a standard practice to set the control limits as a multiple of the standard deviation of the statistic plotted on the chart.

The sample size n and the sampling interval h are also important factors in designing a control chart. Generally h depends on the rational subgrouping or the production shift and n is decided according to the available resources (e.g. operators and measuring instruments) and the expected detection power. Duncan (1986) emphasized that a sample should represent subgroups of output that are as homogeneous as possible. Conventionally, large samples are taken to detect small shifts in the process because the large samples make the control limits tighter and thus increase the probability of detecting a shift in the process. The most desirable situation

from the perspective of detecting a shift is to take large samples in short intervals (Montgomery 1997). However, this is not economically feasible, because the increase in sample size or sampling frequency requires the increased amount of manpower and measuring equipment for the quality control activities. Therefore, there should be a trade-off between the sample size and the sampling interval so that the effectiveness of a control chart system is maximized within the available resources. However, since on-line measurement and distributed computing systems are becoming the norm in today's SPC applications (Woodall and Montgomery 1999), the sampling interval may be much smaller than the working shift and the notation of rational subgrouping may not enforced.

2.3.4 Measure of control chart performance

Detecting assignable causes quickly and having few false alarms are the performance objectives of any control charts (Runger and Montgomery 1993). Two frequently used performance measures are *ARL* (Average Run length) and *ATS* (Average Time to Signal).

Average Run Length (*ARL*)

The random number *RL* (Run Length) is the number of samples inspected to signal an out-of-control condition after it has occurred. The mean value of *RL* is the Average Run Length (*ARL*). The out-of-control *ARL* is commonly used to indicate the effectiveness or power of the control charts and the in-control ARL_0 measures the false alarm rate. When the process is out of control the chart should signal as early as possible (i.e. smaller out-of-control *ARL*). On the other hand, when the process is in

control the chart should produce fewer false alarms (i.e. larger ARL_0). The in-control ARL_0 of the Shewhart control chart can be calculated by

$$ARL_0 = \frac{1}{\alpha} \quad (2.5)$$

Where α is the type I error probability. The out-of-control ARL of the Shewhart control chart can be calculated by

$$ARL = \frac{1}{1 - \beta} \quad (2.6)$$

Where β is the type II error probability. For an $\bar{x}$ chart, β can be calculated by

$$\beta = \Phi\left(\frac{UCL - (\mu_0 + \delta_\mu \sigma_0)}{\delta_\sigma \sigma_0 / \sqrt{n}}\right) - \Phi\left(\frac{LCL - (\mu_0 + \delta_\mu \sigma_0)}{\delta_\sigma \sigma_0 / \sqrt{n}}\right) \quad (2.7)$$

Where δ_μ and δ_σ are the mean shift and standard deviation shift in terms of σ_0 .

Average Time to Signal (ATS)

Average Time to Signal (ATS) may be more meaningful than ARL to indicate the performance of a control chart when sampling interval h is different. ATS is the average time required to signal an out-of-control state. If samples are taken at a fixed interval h , then the in-control ATS_0 and out-of-control ATS are computed by

$$ATS_0 = h \times ARL_0 \quad (2.8)$$

$$ATS = h \times ARL \quad (2.9)$$

The ATS computed in this way assumes the *zero-state* mode, in which the shift is present at the beginning when the process control is initiated. Practically, the shift may occur after the process has been running for some time, rather than at the start. A more realistic mode assumes that the control statistic has reached a steady-state or stationary distribution by the time a shift in process occurs. Moreover, when the shift occurs within a sampling interval, the time of shift is uniformly distributed over the

interval (Runger and Pignatiello 1991, Runger and Montgomery 1993). The *ATS* computed under this assumption is called the *steady-state ATS* (Reynolds *et al.* 1990, Reynolds and Stoumbos 2000) and is given by

$$ATS = (ARL - 0.5) \times h \quad (2.10)$$

2.4 Different research approaches

The SPC methods have been used extensively and many new methodologies have been developed in the recent years. This section presents the literatures related to the control charts for multi-stage and multi-stream manufacturing processes, and the statistical and economical designs of the control charts.

2.4.1 Control charts for multi-stage and multi-stream manufacturing processes

During the fabrication of a product, the raw material usually passes through different manufacturing stages and is finally converted into the finished product. Moreover, a stage may have more than one parallel stream or machine (or a machine with several heads) with each machine or head performing identical operation or producing identical product. However, even though most of the manufacturing systems consist of multiple process stages, the literature on the design of control chart system that monitors multi-stage and multi-stream manufacturing system is limited.

Ott and Snee (1973) studied filling weight data from a machine which had twenty-four heads. The focus of the article is to show a variety of methods to analyze the data and the different results. They used plots, residual analysis and ANOVA technique to determine the differences in heads, times, and groups of heads.

Woodall and Ncube (1985) investigated the multivariate CUSUM control procedures (MCUSUM) that can be used to monitor the quality variables in several

different stages. They compared the MCUSUM with the Hotelling's T^2 procedure in terms of the out-of-control ATS and in-control ATS_0 , and concluded that the MCUSUM is often preferable to the Hotelling's T^2 chart. They also discussed the definitions of the in-control and out-of-control status for a multi-stage system including a few variables.

Nelson (1986) described the design and operation of the group chart. He explained how this chart is able to distinguish between a shift in a single stream and a change common to all streams. The incorporation of runs rules into the group chart is also discussed. Nelson pointed out that the group chart can be adopted as long as each stream has the same target value and the same variation, and the distribution of the measurement is as close to normality as is required by the conventional $\bar{x}$ & R charts. Montgomery (1991) reviewed Nelson's (1986) approach for multi-stream process. He discussed the control scheme for detecting a shift in the mean of all of the product streams and also for detecting a shift in the mean of an individual product stream in the multiple stream process. To detect a shift in the mean of all of the streams he suggested taking a sample from each of the streams and then plotting the largest and smallest sample means. He also presented tables of out-of-control ARL for various numbers of streams and consecutive times that a particular stream has the largest (or smallest) value. Mortell and Runger (1995) proposed a modified model for the group control chart. Its most distinctive feature is to use the range, R_t , of the stream means to monitor the shifts in a single stream with respect to the others. The results of their numerical studies show that a CUSUM chart based on R_t can, in general, produce a smaller out-of-control ATS than the conventional runs control scheme, when the control limits of the charts are designed to meet a specified in-control ATS_0 . Runger *et al.* (1996) and Runger and Fowler (1998) considered the highly automated

environment with dozens to hundreds of streams. They showed how some additional control charts could be used to supplement the pair of charts recommended by Mortell and Runger (1995). Runger *et al.* (1996) used principal component analysis to develop the control charts that are able to detect the assignable causes that affect all streams and assignable causes that affect one or a few streams in multiple process streams.

Williams and Peters (1989) presented the economic design of an np-control scheme integrated within a multi-stage (single-stream) process. The design is to determine a set of sampling parameters including the sample sizes, sampling intervals and control limits. The objective of this system is to yield the minimum total expected cost, including the costs for false alarms and out-of-control cases. A combination of dynamic programming and direct search techniques is applied to identify the optimal design solution.

Hawkins (1993) developed several regression adjustment schemes for the multivariate (multi-stage) quality control. In particular, a regression of the variable in each stage for those variables in the upstream stages is proposed. His method makes the monitoring for the multiple-stage processes more effective than those based on variables individually.

Amin *et al.* (1999) proposed the MaxMin EWMA control chart based on the smallest (Min) and largest (Max) observations in each sample. When there is any change in the process, the chart shows which parameters have increased or decreased. They explained the design procedure and implemented the charting technique in a single stage multi-stream process.

Zhang (1989), Fong and Lawless (1998), Lawless *et al.* (1999), Ding *et al.* (2002a, 2002b), and Zantek *et al.* (2002) discussed the variation transmission through multiple process stages.

Nonetheless, in all of the previous studies, the multi-stage and multi-stream manufacturing processes have not been considered as a whole and the charting parameters (e.g. control limits, sample size or sampling interval) have not been designed in an integrative and optimal manner. Instead, identical parameter values have been used by each of the individual charts.

2.4.2 Statistical and economical designs of control charts

Research effort has been made to develop more advanced control charts in two main directions, namely the *statistical designs* and the *economical designs*. The statistical design is to minimize the out-of-control ARL (or the out-of-control ATS) at one or a few out-of-control states by optimizing n , h , LCL and UCL , on condition that the in-control ARL_0 (or the in-control ATS_0) is no smaller than a specified value. However, the statistical design does not directly measure the costs (or quality losses) resulting from the out-of-control cases. This section reviews some of the important statistical and economical designs of the control charts.

Statistical design of control chart

The statistical design usually involves selecting the sample size, sampling interval, and control limits for detecting a particular shift in quality characteristic. The focus is usually on the minimization of out-of-control ARL (Woodall 1997). Many researchers (Woodall 1985, English and Case 1988, Keats *et al.* 1995, Albin *et al.* 1997, Klein 2000, Zhang *et al.* 2001, Wu *et al.* 2002) designed and evaluated the performance of the control charts based on out-of-control ARL (or out-of-control ATS). Conventional SPC techniques use fixed sample size and sampling interval. However, to increase the statistical efficiency of the control charts, some statistical designs start using *adaptive*

(or *dynamic*) *control schemes*. A control chart is adaptive if either its sample size, or sampling interval or both are changed depending on the value of the sampling statistic. Such control schemes include the Shewhart charts with variable sample size (VSS) (Prabhu et al. 1993, Costa 1994) and variable sampling interval (VSI) (Reynolds *et al.* 1988, Runger and Pignatiello 1991). Prabhu *et al.* (1994) and Costa (1999) designed the VSSI $\bar{x}$ chart, where both the sample size and the sampling interval are varied with time. Though the control charts with adaptive control parameters may result in considerable improvement in performance, they are admittedly harder to administer, and their applications may run up against technical difficulties, communication problems, and inescapable resistance to change (Tagaras 1998).

Economical design of control chart

Various economic models (Duncan 1956, Montgomery 1986, Chung 1993, Collani *et al.* 1994, Castillo and Montgomery 1996, Das *et al.* 1997, Al-Oraini and Rahim 2002) have been proposed for economically choosing the control chart parameters. Most of the economical designs of control charts are developed based on the pioneer model proposed by Duncan (1956). The advantages of the economical design models over the conventional models are that they select the chart parameters in such a way that the total cost (including the cost of inspection, cost of producing non-conforming items, cost of detecting out-of-control signals, and other costs) is minimized. That means the economic models minimize the expected total cost per unit time associated with the SPC schemes. Montgomery (1980) and Ho and Case (1994) reviewed the available economic models up to 1991. Some researchers (Parkhideh and Case 1989, Chung and Lin 1993, Ohta and Rahim 1997, Park and Reynolds 1999) proposed adaptive economical control schemes. Tagaras (1998) presented a survey of research works in

statistical and economical designs of adaptive charts. Some criticism has been raised about the economic models. Woodall (1986, 1987) criticized that the economic models generally have poor statistical properties and produce excessive number of false alarms. Moreover, the complex mathematical models and optimization techniques and, especially, the difficulty in estimating costs and other model parameters result in the lack of practical implementation of the economic charts (Montgomery 1980, 2001). Saniga (1989) developed a statistical economic model that incorporates statistical constraints on the minimum out-of-control ATS and the maximum in-control ATS_0 with the economic model.

2.5 Optimization design algorithm

Many of the best-known connections between statistical quality control and optimization designs are in the area of control charts for process monitoring and control. The general optimization model used in SPC is to minimize the out-of-control ATS (or out-of-control ARL) or sample size n , keeping the in-control ATS_0 (or ARL_0) at a specified level. The minimization of out-of-control ATS is consistent with a management philosophy that encourages doing it right the first time (Lucas 1985). Several authors (Weiler 1952, Page 1954a, Keats *et al.* 1995) adopted this technique to optimize the chart parameters in designing an $\bar{x}$ control scheme. The constraint on in-control ATS_0 has been widely used in designing the adaptive control schemes (Reynolds 1989, Runger and Pignatiello 1991, Prabhu *et al.* 1995). Prabhu *et al.* (1997) minimized the out-of-control ATS of a CUSUM chart considering the relationship between sample sizes and sampling intervals. Many authors (Duncan 1956, Svoboda 1991, Gupta and Sachdev 1992, Ho and Case 1994) minimized the cost associated with the control chart scheme in economical designs. The out-of-control cases of a

process may be caused by mean shift, standard deviation shift, or both. In order to improve the effectiveness and economic results of the control charts, a few statistical and economic models for the optimization designs of the $\bar{X}$ & S or $\bar{X}$ & R charts have been proposed (Saniga 1979, Saniga and Montgomery 1981, Costa and Rahim 2000, Wu *et al.* 2002). Saniga (1979) and Saniga and Montgomery (1981) presented the optimization algorithms for the economical designs of $\bar{X}$ & S control charts considering a single assignable cause. They used exponential distribution with constant hazards rate and sampling interval to describe the time of occurrence of the assignable causes. Costa and Rahim (2000) proposed economic models for the design of $\bar{X}$ & S control charts for continuous production process considering two independent assignable causes. They assumed that the occurrence time of assignable causes has a Weibull distribution. Wu *et al.* (2002) optimized the sample size and determined the corresponding control limits of the $\bar{X}$ & S charts that provide an optimal balance between the error probabilities, detection power and sampling cost.

The optimization strategies for finding the global optimum solution are still very much the subject of research. An important issue related to global optimum is the convexity of the objective function. It is theoretically interesting to test the convexity or nonconvexity for a nonlinear program. However, it may be very difficult to carry out for an engineering optimization. As Reklaitis *et al.* (1983) discussed, it can be cumbersome to test the convexity of any nonlinear problem, and we must often be satisfied to identify local optima since the nonlinear objective will typically not be convex and will therefore be multimodal. Researchers usually make no attempt to secure a global optimum solution. Instead, they focus on deriving a convenient and systematic procedure to identify a satisfactory and workable solution that could be adopted in practice (Duncan 1956). The optimization design of the control charts may

involve a large number of design variables, some of them integers and others real. Therefore, some optimization search strategies search the optimum from point to point until the improvement is negligible (Carlyle *et al.* 2000).

2.6 Computer-aided design of control chart

Woodall and Montgomery (1999) mentioned that many researchers generally considered their work ended with the publication of their results but usually made little effort to implement them in practice. User-friendly computer applications are rarely developed. There is certainly a need for a stronger trend toward more relevance in the SPC research. They encouraged the researchers to develop computer programs for existing and newly proposed methods, because the newly proposed methods may not be implemented in practice if the computer programs are not available. Some research works that presented computer programs for SPC applications are shown in Table 2.1.

Table 2.1 Summary of research works on computer program

Researchers	Description
Montgomery (1982)	A FORTRAN program for the optimal economical design of $\bar{x}$ control chart based on the cost model of Duncan (1956). The program finds the optimal sample size, control limit coefficient and sampling interval in order to minimize the expected total costs per unit time.
Castillo (1996)	A C program for the computation of Run Length (<i>RL</i>) distribution and out-of-control <i>ARL</i> value of $\bar{x}$ charts with unknown process variance.
Pulak and Al-Sultan (1997)	A FORTRAN computer package for solving nine selected targeting models. The models search the optimal target value of a process under various conditions.
Davis (1999)	A BASIC program to determine the sample size and the upper control limit of an <i>S</i> -chart with two out-of-control <i>ARL</i> constraints proposed.

Table 2.1 Summary of research works on computer program (cont'd)

Researchers	Description
Wu and Spedding (2000)	A C program that computes upper and lower control limits and other chart parameters of the synthetic control charts
Gan and Chang (2000)	A FORTRAN program for computing both the in-control ARL_0 and the out-of-control ARL of the EWMA charts.
Williams <i>et al.</i> (2001)	A FORTRAN program that jointly determine the parameters of $\bar{X}$ charts used in combination with either R or S charts using economic, statistical, or economic-statistical design criteria.
Rahim (1993)	A FORTRAN program that determines the parameters of $\bar{X}$ charts by minimizing the expected cost per unit time where the amount of time the process remains in control has a Weibull distribution.
Linderman and Love (2000)	A C program for evaluating the ARL values for the MEWMA chart through simulation and then using these results to determine the optimal chart parameters given process cost and time information.

2.7 Concluding remarks

Two research issues are identified from the literature review.

- (1) In designing a control chart scheme for multi-stage and multi-stream manufacturing system, the researchers considered a group of control charts each for a single stage of the system. They did not, however, design the charting parameters in an integrative and optimal manner. As a result, the overall effectiveness of the chart system may be inadequate. More research should be conducted in the optimization designs of the integrated control chart systems for multi-stage and multi-stream manufacturing system by considering the system as a whole.
- (2) Many economical designs of control charts have been proposed. The economic models are able to minimize the expected total cost associated with the SPC

schemes. Unfortunately, these models find little application in practice because of great difficulty in cost specifications. The trend in the economic modelling of control charts is incorporating effective statistical constraints and simplifying the parameter specifications. The development of more realistic and applicable economical model with statistical constraint would be a major advance.

Chapter 3

Statistical Designs of Integrated $\bar{X}$ Control Chart Systems for Monitoring Process Mean

This chapter develops the algorithms for the optimization designs of the **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits), **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits), **MSDSL** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / control limits), and **MSDSN** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / sample size, sampling interval, and control limits) chart systems for multi-stage and multi-stream manufacturing system. The design algorithms consider all the $\bar{X}$ charts within a system in an integrative and optimal manner. The optimization design of the integrated control chart system can improve the overall performance of the chart system significantly and, consequently, the product quality will be further enhanced. Such improvement is achieved with minimal additional cost and effort for operation. Furthermore, the shop floor operators can utilize and understand the control chart system as easily as with the conventional system. In this chapter, the design algorithms consider only the mean shift in optimizing the charting parameters such as the sample sizes, the sampling intervals, and the control limits of the charts in a system. The results from the comparative study and the application examples show that the integrated $\bar{X}$ chart system is able to reduce the out-of-control *ATS* substantially compared to the conventional $\bar{X}$ chart system. Some useful guidelines are also provided to aid the users to adjust the sample sizes, sampling intervals, and control limits of the $\bar{X}$ charts in a system.

3.1 Introduction

Products are typically fabricated through various stages in a manufacturing system. For example, in the manufacturing of a mechanical part, each stage corresponds to the machining of a dimension. The functional dimensions are the critical dimensions that need to be controlled. Depending on the quality characteristics, the process stages may be independent or interdependent. A multi-stage and multi-stream manufacturing system is shown in Figure 1.1 in Chapter 1. In this manufacturing system, the process stages are independent of each other because each tolerance chain contains only one functional dimension. However, if more than one functional dimension resides in one tolerance chain, the process stages may be interdependent. The interdependent manufacturing system has to be handled by some special treatments and will be discussed in Section 3.3.

Because of the differences in production rate (e.g. bottleneck machine) and other factors, some stages may have more than one parallel stream or machine. In some applications, a single Shewhart chart or a group chart is used to monitor the outputs from all the streams of a stage. However, in our approach, a separate chart will be applied to the output of each individual stream. This scenario helps to detect and diagnose the out-of-control stream (Montgomery 2001). Usually, the parallel streams in a single stage have the same mean, standard deviation, and target (Runger *et al.* 1996) and, therefore, a group of identical control charts are used to monitor the quality characteristics of the parallel streams in a stage.

In this report, a process means one single-stream stage or one stream in a multi-stream stage. The total number G of the processes that are monitored by the control chart system is equal to

$$G = \sum_{i=1}^s g_i \quad (3.1)$$

Where s is the number of the functional stages and g_i is the number of streams in the i th stage ($g_i \geq 1$). However, since the g_i streams in the i th stage are monitored by identical control charts (with same sample size, sampling interval, and control limits), there are only s different charts in the control chart system.

If one process is out of control, the control chart system should produce a signal as quickly as possible; otherwise the overall quality of the product is likely to deteriorate. However, if all the charts in the system report in-control status, the whole manufacturing system for the product is assumed to be in control and the product quality should be satisfactory.

Conventionally, each individual chart in a system is designed in isolation. For example, the control limits LCL and UCL of an $\bar{x}$ chart are calculated as follows.

$$\begin{aligned} LCL &= \mu_0 - k \frac{\sigma_0}{\sqrt{n}} \\ UCL &= \mu_0 + k \frac{\sigma_0}{\sqrt{n}} \end{aligned} \quad (3.2)$$

In a traditional $\bar{x}$ chart system, n is usually set around 5, and k is made identical (typically taken as 3) for each chart. The sampling interval h is mainly decided by the working shifts (i.e. the rational subgrouping). However, since on-line measurement and distributed computing systems are becoming the norm in today's SPC applications, the sampling interval may be much smaller than the working shift and the notation of rational subgrouping may not be enforced.

If all of the charts in a system are designed in an integrative and optimal manner, the overall effectiveness of the system may be improved. The optimization design of the control chart system may result in different n , h and k values, or allocate different power to different charts in a system based on the values of certain

parameters (e.g. process capability, magnitudes of the mean shifts) that would affect the performance of the control chart system.

Similar to a single control chart, the power or effectiveness of the control chart system is measured by the out-of-control *ATS*. For a charting system, *ATS* is defined as the average time that one of the control charts in the chart system gives an out-of-control signal subsequent to any process in the manufacturing system going out of control (Woodall and Ncube 1985). When any process is out of control, the users want the control chart system to signal quickly, i.e. to have a small out-of-control *ATS*.

The remaining sections in this chapter are organized as follows.

- Section 3.2 develops two models assuming that the process stages of the manufacturing systems are independent. The first model (**MSISL** chart system) in Section 3.2.1 optimizes the control limits of each $\bar{X}$ chart in the chart system, whereas the second model (**MSISN** chart system) in Section 3.2.2 optimizes the sample sizes, the sampling intervals and the control limits all together.
- Section 3.3 demonstrates how to handle the interdependency between the process stages of a manufacturing system (**MSDSL** and **MSDSN** chart systems).
- Section 3.4 analyzes the performance of the models through comparative study.
- Section 3.5 demonstrates the performance of the proposed models through examples.
- Finally, Section 3.6 summarizes the models developed in this chapter.

3.2 Optimization design of the independent $\bar{X}$ control chart system

Consider the part shown in Figure 1.1(a) (in Chapter 1). Dimensions to be machined in stages 1, 2 and 3 are functional (critical) ones, which directly affect the function of the product. Therefore, three control charts as a system are required to carry out on-line monitoring of the manufacturing process. Furthermore, two identical machines are employed to turn dimension 1 (the outer surface). Thus, control chart one for the first stage has two duplicates; each controls one of the two parallel machines. A block diagram of this control chart system is displayed in Figure 1.1(b). All non-functional process stages are ignored in the block diagram and are not included in the design of the control chart system. It is assumed that the quality requirements on the non-functional dimensions can be easily met and on-line monitoring is unnecessary.

Two optimization models are proposed in this section. The first model **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) optimizes the control limits of each chart in a system with specified sample sizes and sampling intervals and the second model **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) optimizes the sample size, sampling interval, and control limits all together of each chart in the chart system.

3.2.1 Independent $\bar{X}$ chart system with optimal control limits

The design algorithm discussed in this section optimizes the control limits of each chart in the **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) chart system by considering the mean shift only; the sample sizes and the sampling intervals are specified (fixed) as in the conventional-system. Therefore, this model can be implemented in a manufacturing system where changes

of the sample size and sampling interval are not allowed because of fixed working shifts or fixed resource allocation or other difficulties. Under such circumstances, the control limits can be easily adjusted without changing the regular working schedule.

3.2.1.1 Input parameters

The parameters s (the number of stages in the control chart system), g_i (the number of streams in the i th stage), USL_i (the upper specification limit in the i th stage), h_i (the sampling interval of the control chart in the i th stage), n_i (the sample size of the control chart in the i th stage), $\mu_{0,i}$ (the in-control process mean in the i th stage), $\sigma_{0,i}$ (the standard deviation of the process in the i th stage), τ (the minimum allowable in-control ATS_0), $\delta_{\mu,i}$ (the maximum allowable mean shift in the i th stage), and p_i (the probability of out-of-control occurrence in the i th stage) are required in this design.

The maximum allowable mean shift $\delta_{\mu,i}$ indicates the rejectable quality level (the barely tolerated mean shift). It may be determined by the QA engineer directly based on factors such as quality, manufacturing cost, and customer's requirements. More preferably, $\delta_{\mu,i}$ is calculated from the specified tolerance and the allowable process capability ratio C_{pk} . Each process must have a minimum allowable value $c_{min,i}$ for the process capability ratio. The product quality will degrade significantly if C_{pk} is lower than $c_{min,i}$. For the i th process stage, if $c_{min,i}$ corresponds to $\delta_{\mu,i}$

$$c_{min,i} = \frac{USL_i - (\mu_{0,i} + \delta_{\mu,i}\sigma_{0,i})}{3\sigma_{0,i}} \quad (3.3)$$

Therefore, the maximum allowable mean shift $\delta_{\mu,i}$ for the i th process stage can be calculated as follows.

$$\delta_{\mu,i} = (USL_i - \mu_{0,i} - 3\sigma_{0,i}c_{min,i}) / \sigma_{0,i} \quad (3.4)$$

The probability p_i that an out-of-control case takes place in the i th stage may have to be estimated from the historical data of the out-of-control cases. If such historical data are not available, it may be reasonable to estimate p_i by the following formula.

$$p_i = \frac{g_i}{\sum_{j=1}^s g_j} \quad (3.5)$$

This means the probability that the out-of-control case happens in the i th stage is proportional to the number of parallel streams in this stage provided that the probability of out-of-control for each stream is the same. In Equation (3.5), the denominator denotes the total number of streams in the system and the numerator denotes the number of streams in stage i . Therefore, for any given number of streams in the system the probability p_i in stage i will be increased as the number of stream g_i is increased.

3.2.1.2 Optimization model

Based on the above parameters, the optimization design of the **MSISL** chart system can be conducted by using the following nonlinear optimization model.

$$\text{Objective function: } ATS = \text{minimum} \quad (3.6)$$

$$\text{Constraint function: } ATS_0 \geq \tau \quad (3.7)$$

$$\text{Design variables: } LCL_i, UCL_i \ (i = 1, \dots, s)$$

Where ATS is the average time to signal of the control chart system when any process in the manufacturing system is out of control (or more specifically, when any process mean has shifted by an amount of $\delta_{\mu,i}$ as determined by Equation (3.4)). The objective

function ATS is a function of $(s, g_i, USL_i, h_i, n_i, \mu_{0,i}, \sigma_{0,i}, \tau, \delta_{\mu,i}, p_i, LCL_i, \text{ and } UCL_i)$. ATS_0 is the average time to signal when the whole system is in control (or all process means remain in $\mu_{0,i}$). Since the process is in control, the signal is an unwanted false alarm. Therefore, the upper limit of the ATS_0 is kept open. In some cases, the user may put a constraint on the maximum value of the out-of-control ATS based on the actual situation of an application.

In the actual optimization search, the design variables LCL_i, UCL_i ($i = 1, \dots, s$) or equivalently the control limit coefficients $k_1, \dots, k_s$ are replaced by the corresponding type I error probabilities $\alpha_1, \dots, \alpha_s$ associated with the s control charts. α_i is the probability that the i th control chart produces an out-of-control signal when the i th process is in fact in control. Since

$$k_i = \Phi^{-1}(1 - 0.5\alpha_i) \quad (3.8)$$

Optimizing α_i is equivalent to optimizing k_i , but the former is easier to handle during the optimization search. It is noted that α_i has finite lower bound (0) and upper bound (1) for the search (conversely, the upper bound of k_i is infinite) and that a simple equation among $\alpha_1, \dots, \alpha_s$ (Equation (3.22)) can be established easily. The objective of the optimization model is to identify the optimal values for $(\alpha_1, \dots, \alpha_s)$ that will jointly minimize ATS (maximize effectiveness) with ATS_0 (the false alarm rate) equal to the specification. The underlying idea of the optimization design is to enhance the power of those charts which take a very long period to produce a signal when the corresponding processes become out of control. This procedure must sacrifice the power of some other charts in the system. However, if the α_i values of all of the charts are adjusted in an integrative and optimal manner, the overall (average) effectiveness of the control chart system will be improved.

The final outputs of the optimization design are the optimal control limits for each control chart. They can be derived from the optimal value of α_i through k_i (see Equation (3.8)).

$$\begin{aligned} LCL_i &= \mu_{0,i} - k_i \frac{\sigma_{0,i}}{\sqrt{n_i}} \\ UCL_i &= \mu_{0,i} + k_i \frac{\sigma_{0,i}}{\sqrt{n_i}} \end{aligned} \quad (3.9)$$

The major step in the optimization procedure is to express the objective function ATS (Equation (3.6)) and the in-control ATS_0 (Equation (3.7)) in terms of $\alpha_1, \dots, \alpha_s$.

3.2.1.3 Calculation of in-control ATS_0 of the chart system

In a single time unit, the probability that a control chart in the i th stage produces a false alarm is approximately equal to α_i / h_i , where, α_i is the false alarm probability in a single sample. For example, if the sampling interval h_i is equal to one hour, then the false alarm probability in a single sample or per hour is α_i . On the other hand, if h_i is equal to two hours, then the false alarm probability in a single sample is still equal to α_i , but equal to $(\alpha_i / 2)$ per hour. Generally, the probability of false alarm per unit time is inversely proportional to h_i . In this thesis, h_i is allowed to be less than one hour, which means a sample will be taken for each half an hour. The false alarm probability α_i is given by

$$\alpha_i = 2 \cdot \Phi \left(\frac{LCL_i - \mu_{0,i}}{\sigma_{0,i} / \sqrt{n_i}} \right) \quad (3.10)$$

Thus, the probability that a control chart in the i th stage does not produce a false alarm is equal to $(1 - \alpha_i / h_i)$. It is recalled that each of the g_i streams in the i th stage runs a

duplicate of the i th control chart. Therefore, the probability P_0 that the whole control chart system generates a false alarm in a time unit is

$$P_0 = 1 - \prod_{i=1}^s \left(\prod_{j=1}^{g_i} (1 - \alpha_i / h_i) \right) = 1 - \prod_{i=1}^s (1 - \alpha_i / h_i)^{g_i} \quad (3.11)$$

Finally,

$$ATS_0 = 1 / P_0 \quad (3.12)$$

3.2.1.4 Calculation of out-of-control ATS of the independent $\bar{X}$ chart system

Suppose that one of the g_i streams in the i th stage is out of control, i.e. its mean value has shifted to $(\mu_{0,i} + \delta_{\mu,i} \sigma_{0,i})$. The type II error probability β_i that the corresponding control chart fails to give a signal in a single sample is

$$\beta_i = \Phi \left(\frac{UCL_i - (\mu_{0,i} + \delta_{\mu,i} \sigma_{0,i})}{\sigma_{0,i} / \sqrt{n_i}} \right) - \Phi \left(\frac{LCL_i - (\mu_{0,i} + \delta_{\mu,i} \sigma_{0,i})}{\sigma_{0,i} / \sqrt{n_i}} \right) \quad (3.13)$$

Where the control limits LCL_i and UCL_i are dependent on α_i (Equations (3.9) and (3.10)). The probability that all other in-control streams in the i th stage do not give any signal (i.e. do not produce any false alarm) in a single sample is $(1 - \alpha_i)^{g_i - 1}$. Therefore, the probability that none of the charts in the i th stage gives a signal in a single sample on condition that exactly one of the g_i streams is out-of-control and all other streams is in-control is

$$w'_i = (1 - \alpha_i)^{g_i - 1} \beta_i \quad (3.14)$$

Next, for the duration of one sampling interval h_i of the out-of-control stage i , the probability that a chart in j th stage ($j \neq i$) gives a signal due to the type I error can be approximated by

$$\theta = \frac{h_i}{h_j} \alpha_j \quad (3.15)$$

The ratio (h_i / h_j) is used to match the sampling interval h_j in the j th stage to h_i in the i th stage. Equation (3.15) can be rearranged as, $\theta = (\alpha_j / h_j) \cdot h_i$, where (α_j / h_j) is the false alarm probability in the j th stage in a single time unit (see the discussion in Section 3.2.1.4). Now, the approximate value of the false alarm probability (θ) in j th stage during h_i is the product of (α_j / h_j) and h_i . Thus, the probability that none of the charts (or duplicates) in all stages (excluding the i th stage) gives a signal during h_i is

$$w_i'' = \prod_{j=1, j \neq i}^s (1 - \theta)^{g_j} \quad (3.16)$$

Since it is assumed that only one process (i.e. one stream in a stage) in a manufacturing system may go out of control at any moment, the probability that an out-of-control signal will be produced during h_i is calculated as follows.

$$z_i = 1 - w_i' w_i'' = 1 - (1 - \alpha_i)^{g_i - 1} \beta_i \prod_{j=1, j \neq i}^s \left(1 - \frac{h_i \alpha_j}{h_j} \right)^{g_j} \quad (3.17)$$

Where z_i is the power of the whole system when there is one and only one stream goes out-of-control in the i th stage. Therefore, given that one stream in the i th stage goes out of control, the value ats_i of the out-of-control ATS is

$$ats_i = \left(\frac{1}{z_i} - 0.5 \right) \times h_i \quad (3.18)$$

It is the steady-state formula (refer to Equation (2.10) in Chapter 2) which provides a more realistic evaluation of the out-of-control ATS than the zero-state counterpart. In Equation (3.18), $1/z_i$ is actually the out-of-control ARL . The power z_i only considers the mean shift $\delta_{\mu i}$ produced by one of the streams in stage i .

Since an out-of-control case may occur in any of the s stages, the final ATS for the manufacturing system is

$$ATS = \sum_{i=1}^s (ats_i \cdot p_i) \quad (3.19)$$

Where p_i is the probability that the out-of-control case takes place in the i th stage.

The formulae for calculating ATS assume that, in any time instance, only one process may go out of control. When more than one process are out of control simultaneously (for example, due to common assignable causes), the out-of-control ATS of the control chart system will become even smaller than that calculated by Equation (3.19).

Equations (3.14) and (3.16) imply that the out-of-control signal may be produced by a chart monitoring a process currently in control. Thus, it could provide a misleading signal. As type I errors of the charts cannot be set at zero, such misleading signals are inevitable and always exist in any control chart system. Fortunately, the probability of a misleading signal is much lower than that of the correct signal, as the power $(1 - \beta)$ is usually much larger than the type I error probability α .

A numerical experiment is conducted to study the accuracy of the formulae for calculating ATS_0 and ATS . Since the values of ATS_0 and ATS are affected by the parameters s , g_i , h_i , n_i , α_i , $\sigma_{0,i}$, and $\delta_{\mu,i}$, both ATS_0 and ATS are evaluated for 25 cases with different parameter values. The parameter values are randomly selected in this study to avoid the biasness of sampling. In each of the 25 cases, s , the number of stages will be first selected randomly between 2 and 8. Subsequently, other parameters for each of the s stages will be randomly decided from the ranges below.

$$g_i \quad [1, 3]$$

$$h_i \quad [50, 250]$$

$$n_i \quad [3, 7]$$

$$\alpha_i \quad [0.0005, 0.0045]$$

$$\sigma_{0,i} \quad [0.01, 0.03]$$

$$\delta_{\mu,i} \quad [0.2, 2.5]$$

The parameter p_i is estimated from g_i by Equation (3.5).

Obviously, these 25 cases represent the applications of notable varieties of the influential parameters. For each case, after the parameters are determined, ATS_0 and ATS are evaluated by both simulation (simulation sample size is 10000) and calculation, respectively. The relative discrepancies are then evaluated. For an in-control situation

$$e_0 = \left| \frac{ATS_0 - ATS_{0,simu}}{ATS_{0,simu}} \right| \quad (3.20)$$

For an out-of-control situation

$$e_1 = \left| \frac{ATS - ATS_{1,simu}}{ATS_{1,simu}} \right| \quad (3.21)$$

Where ATS_0 and ATS are the calculated values (by Equations (3.12) and (3.19)), and $ATS_{0,simu}$ and $ATS_{1,simu}$ are the simulated values.

For the 25 different cases, the average and maximum values are only 0.01402 and 0.02949 respectively for the in-control discrepancy e_0 , and only 0.02060 and 0.07005 respectively for the out-of-control discrepancy e_1 .

3.2.1.5 Optimization search algorithm

The optimization design can be carried out by using any software for nonlinear constrained programming. In this implementation, a *dynamic search algorithm* is employed. In this algorithm, the optimal values of α_i of the first $(s-1)$ control charts

are searched step by step in $(s - 1)$ levels, using the same step size $d\alpha$. The last α_s is finally determined so that the resultant ATS_0 is exactly equal to the specified τ (Equation (3.7)).

Combining Equations (3.11) and (3.12), and replacing ATS_0 by τ , the value of any individual α_i can be expressed in terms of other α_j ($j = 1, \dots, s; j \neq i$) and τ .

$$\alpha_i = h_i \left[1 - \left(\frac{1 - 1/\tau}{\prod_{j=1, j \neq i}^s (1 - \alpha_j / h_j)^{g_j}} \right)^{1/g_i} \right] \quad (3.22)$$

This equation can be used to determine the range of α_i in the i th level search.

As a simple example, let us consider a manufacturing system with three stages ($s = 3$), where $g_1 = g_2 = g_3 = 1$, $h_1 = 100$, $h_2 = 120$, $h_3 = 200$. There are two levels of searches. The first level is to search the optimal α_1 . The maximum possible value of α_1 can be found from Equation (3.22) by setting both α_2 and α_3 equal to zero (i.e. all the power is allocated to the first chart). That is

$$\alpha_{1,\max} = 100 / \tau \quad (3.23)$$

Therefore, α_1 will be searched in a range of $[0, 100/\tau]$ in the first level.

The second level is to search the optimal α_2 , while α_1 is given. The maximum possible value $\alpha_{2,\max}$ of α_2 should be determined dynamically because it is dependent on the current value of α_1 given in the first level. $\alpha_{2,\max}$ is also found from Equation (3.22) by setting α_3 equal to zero. That is

$$\alpha_{2,\max} = 120 \left(1 - \frac{1 - 1/\tau}{1 - \alpha_1 / 100} \right) \quad (3.24)$$

Therefore, α_2 will be searched between 0 and $\alpha_{2,\max}$ in the second level.

Finally, α_3 is determined directly by Equation (3.22) without search. That is, α_3 (or α_s in a general case) will take the maximum possible value in order to make full use of the power of the control chart system (determined by the specified τ). In this example,

$$\alpha_3 = 200 \left(1 - \frac{1 - 1/\tau}{(1 - \alpha_1/100)(1 - \alpha_2/120)} \right) \quad (3.25)$$

It is clear that the resultant design point $(\alpha_1, \alpha_2, \alpha_3)$ will ensure that ATS_0 is equal to τ .

The overall optimization search is outlined as follows.

- (1) Specify $s, g_i, USL_i, h_i, n_i, \mu_{0,i}, \sigma_{0,i}, \delta_{\mu,i}, p_i$, and τ .
- (2) Initialize ATS_{min} as a large number, say 10^7 (ATS_{min} is used to record the minimum ATS).
- (3) Carry out the searches in $(s - 1)$ levels.
 - (3.1) Each design point $(\alpha_1, \dots, \alpha_s)$ will automatically make ATS_0 equal to τ .
 - (3.2) Calculate ATS by Equation (3.19).
 - (3.3) If this ATS value is smaller than the current ATS_{min} , replace ATS_{min} by ATS and record the current design point $(\alpha_1, \dots, \alpha_s)$ as the temporary optimal solution.
- (4) When the whole search is completed, the optimal design point $(\alpha_1, \dots, \alpha_s)$ is finalized.
- (5) Calculate the control limits of the s control charts by Equations (3.8) and (3.9).

The algorithm searches all possible combinations of α_i values within the feasible region $(ATS_0(\alpha_1, \dots, \alpha_s) \geq \tau)$ in order to identify the optimum set of α_i . The

value of the step size $d\alpha$ is decided so that the total number of search points is always limited to 10^5 regardless of the number of α_i .

3.2.2 Independent $\bar{X}$ chart system with optimal sample sizes, sampling intervals and control limits

This model is an extension of the model **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) presented in the last section (Section 3.2.1) where the effectiveness of the chart system is improved by optimizing the control limits only. Here, the extended model **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) optimizes the sample size, the sampling interval, and the control limits all together. It further enhances the effectiveness of the chart system. The optimization design is carried out under resource constraints such as number of operators equipped with measuring instruments. The optimal chart system does not change the inspection rate or increase the required resources, but ensures the proper utilization of the available resources in the manufacturing system. This model can be implemented in a manufacturing system where sample sizes and sampling intervals of the charts can be adjusted. As with the model discussed in the last section, this model also only considers the mean shifts in the processes.

3.2.2.1 Input parameters

The parameters s , g_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, τ , $\delta_{\mu,i}$, p_i , M (the available number of operators equipped with measuring instruments in the manufacturing system), and t_i (the time required to inspect a unit in the i th stage) are required in this design (see Table 1.2 in Chapter 1 for the details of the specifications).

3.2.2.2 Optimization model

Based on the above parameters, the optimization design of the **MSISN** chart system can be conducted by using the following nonlinear optimization model.

$$\text{Objective function: } ATS = \text{minimum} \quad (3.26)$$

$$\text{Constraint functions: } ATS_0 \geq \tau \quad (3.27)$$

$$m \leq M \quad (3.28)$$

$$\text{Design variables: } n_i, h_i, LCL_i, UCL_i \ (i = 1, \dots, s)$$

Where the variable m is the actual (or required) number of operators equipped with measuring instruments. The objective function ATS is a function of $(s, g_i, USL_i, \mu_{0,i}, \sigma_{0,i}, \tau, \delta_{\mu,i}, p_i, M, t_i, n_i, h_i, LCL_i, \text{ and } UCL_i)$. As mentioned earlier, the user may put some more constraints on the maximum value of out-of-control ATS , sample size n_i , or sampling interval h_i based on the actual situation of an application.

3.2.2.3 Optimization search algorithm

The optimum values of the design variables n_i, h_i, LCL_i , and UCL_i are searched in a two level optimization procedure. In the high-level optimization, the sample size n_i and the sampling interval h_i at each stage of the control chart system are optimized. The purpose of the high-level optimization is to minimize ATS under the constraint of $(m \leq M)$. In the low-level optimization, the control limits LCL_i and UCL_i at each stage are optimized using the same procedure as in the last section where only the control limits are optimized. The purpose is also to minimize ATS , but under the constraint of $(ATS_0 \geq \tau)$.

High-level optimization

The purpose of the high-level optimization is to seek an optimum set of n_i and h_i , which minimizes the out-of-control ATS without requiring extra manpower (i.e. satisfying $(m \leq M)$). The first step in the high-level optimization is to decide the initial values of n_i and h_i , so that the condition $(m \leq M)$ is satisfied at the starting point. Then, the step sizes of n_i and h_i are determined. Finally, a *gradient based search algorithm* is employed to approach the optimal set of n_i and h_i step by step.

Determination of the initial values of n_i and h_i

It is a common practice to take a sample size around 5 for the $\bar{X}$ chart, so the initial value of the sample size at all the stages is set as

$$n_i = 5 \quad (3.29)$$

In general, small samples are taken at short intervals or large samples at long intervals. That means that the sampling intervals are generally proportional to the sample sizes. It can be approximately written as

$$h_i \propto n_i, \text{ or, } h_i = an_i \quad (3.30)$$

Where a ($a > 0$) is a proportionality constant. To determine a , it is noted that the time required to complete a sampling inspection is proportional to the sample size. Therefore, for a sample of size n_i , the time required to complete a sampling inspection in a stream of the i th stage is $n_i t_i$. Suppose T is an infinitely long time period. Then, the number of samples to be taken over the time period T in the i th stage is T / h_i . Therefore, for the whole s stages (each has g_i streams), the actual inspection time required is

$$I = \sum_{i=1}^s \frac{T}{h_i} n_i g_i t_i \quad (3.31)$$

$$\text{Or, } \frac{I}{T} = \sum_{i=1}^s \frac{n_i g_i t_i}{h_i} \quad (3.32)$$

Here, $\frac{I}{T}$ is the actual number m (in Equation (3.28)) of operators required to finish the inspection. Then from Equation (3.32)

$$m = \sum_{i=1}^s \frac{n_i g_i t_i}{h_i} = \sum_{i=1}^s \frac{n_i g_i t_i}{a n_i} = \frac{1}{a} \sum_{i=1}^s g_i t_i \quad (3.33)$$

In order to make m equal to M at the starting point,

$$m = \frac{1}{a} \sum_{i=1}^s g_i t_i = M$$

$$\text{Or, } a = \frac{1}{M} \sum_{i=1}^s g_i t_i \quad (3.34)$$

From the above discussion, it is clear that the initial values of n_i and h_i determined by Equations (3.29) and (3.30) ensure the satisfaction of the constraint ($m \leq M$).

Determination of step sizes of n_i and h_i

The minimum incremental value of a sample size is one. Therefore, the step size (Δn_i) of n_i for all stages is taken as one.

It is rational to make the step size Δh_i of h_i proportional to h_i itself, that is

$$\Delta h_i \propto h_i, \text{ or, } \Delta h_i = b h_i \quad (3.35)$$

Where b ($b > 0$) is a proportionality constant. From Equation (3.33)

$$\begin{aligned} \frac{\partial m}{\partial n_i} &= \frac{g_i t_i}{h_i} \\ \frac{\partial m}{\partial h_i} &= -\frac{n_i g_i t_i}{h_i^2} \end{aligned} \quad (3.36)$$

Now, for an incremental Δn_i ($\Delta n_i = 1$) of every sample size, the average number of additional operators required is

$$\overline{\Delta m_n} = \frac{\sum_{i=1}^s \frac{\partial m}{\partial n_i} \Delta n_i}{s} = \frac{\sum_{i=1}^s \frac{g_i t_i}{h_i}}{s} \quad (3.37)$$

Similarly, for a decrement of Δh_i ($\Delta h_i = b h_i$) of every sampling interval, the average number of additional operators required is

$$\overline{\Delta m_h} = \frac{\sum_{i=1}^s \frac{\partial m}{\partial h_i} (-\Delta h_i)}{s} = \frac{\sum_{i=1}^s \left(-\frac{n_i g_i t_i}{h_i^2} \right) (-b h_i)}{s} = \frac{b \sum_{i=1}^s \frac{n_i g_i t_i}{h_i}}{s} \quad (3.38)$$

By equating Equations (3.37) and (3.38)

$$b = \frac{\sum_{i=1}^s \frac{g_i t_i}{h_i}}{\sum_{i=1}^s \frac{n_i g_i t_i}{h_i}} \quad (3.39)$$

Obviously, the step sizes Δh_i calculated by constant b make $\overline{\Delta m_h}$ (average increment of manpower due to $(-\Delta h_i)$) equal to $\overline{\Delta m_n}$ (average increment of manpower due to Δn_i). Or, in other words, the decrease of h_i by step Δh_i has the similar effect on m as the increase of n_i by step Δn_i .

At this stage, the initial values and step sizes of the sample sizes and the sampling intervals are determined. Now, the sample sizes and the sampling intervals can be increased or decreased step by step until the out-of-control ATS cannot be reduced any more.

It is well known that increasing n_i by Δn_i results in the decrease of ATS (or gaining detection power) and decreasing n_i results in the increase of ATS (or losing detection power). Conversely, decreasing h_i by Δh_i means moving the sampling interval in the gaining direction and increasing h_i by Δh_i means moving the sampling interval in the losing direction.

If n_i and h_i are handled jointly by a general variable X_j ($j = 1, \dots, 2s$), the gaining factor G_j pertaining to X_j per unit increase of m is given by

$$G_j = \frac{|\Delta ATS_j|}{\Delta m_j} = \frac{-\Delta ATS_j}{\frac{\partial m}{\partial X_j} \Delta X_j} \quad (3.40)$$

Where ΔATS_j is the change of the out-of-control ATS when one X_j moves one step in the gaining direction (i.e. 1 for n_i , or $-\Delta h_i$ for h_i) and all other X_j s are kept unchanged. ΔATS_j is always no larger than zero. Specifically, if an X_j is a sample size,

$$G_j = \frac{-\Delta ATS_j}{\frac{\partial m}{\partial n_j} \Delta n_j} = \frac{-\Delta ATS_j}{\frac{g_i t_i}{h_i} (1)} = \frac{(-\Delta ATS_j) \cdot h_i}{g_i t_i} \quad (3.41)$$

And, if an X_j is a sampling interval,

$$G_j = \frac{-\Delta ATS_j}{\frac{\partial m}{\partial h_j} \Delta h_j} = \frac{-\Delta ATS_j}{(-\frac{n_i g_i t_i}{h_i^2})(-b h_i)} = \frac{(-\Delta ATS_j) \cdot h_i}{n_i g_i t_i b} \quad (3.42)$$

The gaining factor G_j of different X_j may vary. Larger G_j means that if the corresponding X_j moves in the gaining direction, a higher reduction in ATS will be obtained with a relatively smaller increase in m . Conversely, lower G_j means that if the corresponding X_j moves in the losing direction, a higher reduction in m will be obtained with a relatively smaller increase in ATS . Therefore, the general variables X_j are first arranged according to the descending order of G_j .

As an example, let us consider a three-stage manufacturing system ($s = 3$). As each stage consists of a sample size n_i and a sampling interval h_i , there will be six ($= 2s$) general variables X_j in total. Suppose each of them has a G_j value at the starting point (calculated by Equation (3.40)), as: 2.5, 3.7, 2.8 for n_1, n_2, n_3 , and 1.2, 2.6, 1.7 for h_1, h_2, h_3 , respectively. Then, these six general variables X_j are ranked all together in descending order according to their values of G_j . That is

$$X_1 = n_2, X_2 = n_3, X_3 = h_2, X_4 = n_1, X_5 = h_3, X_6 = h_1$$

At each search point (either the starting point or the subsequent points), we will move K general variables X_j with the largest G_j values one step in their gaining directions (that is, if a X_j is a sample size, increase it by Δn_i and if a X_j is a sampling interval, decrease it by Δh_i). This will lead to a reduction in ATS with a corresponding increase of m . In order to balance out the increase in m , the rest $(2s - K)$ variables X_j will be moved one step in their losing directions. This second movement may adversely increase ATS , but this increase in ATS must be smaller than the decrease of ATS in the first movement. A searching mechanism is employed to identify the maximum possible value for K (called the delimiting point), which ensures the satisfaction of the constraint ($m \leq M$) and, meanwhile, achieves the maximum reduction in ATS .

The steps of the search algorithm for the high-level optimization are summarized below.

- (1) Specify $s, g_i, \mu_{0,i}, \sigma_{0,i}, \delta_{\mu,i}, p_i, USL_i, \tau, t_i$, and M
- (2) Set $\Delta n_i = 1$, for $i = 1, \dots, s$.
- (3) Decide the starting point (i.e. the initial values of n_i and h_i) by Equations (3.29), (3.30), and (3.34).
- (4) Call the low-level optimization algorithm to determine $ATS(I)$, that is the out-of-control ATS value at the starting point (the low-level optimization will be discussed shortly).
- (5) Calculate Δh_i by Equations (3.35) and (3.39) at the current design point.
- (6) Calculate the gaining factor G_j by Equation (3.40) for each general variable X_j (n_i and h_i) at the current design point.
- (7) Rank X_j in the descending order according to the value of G_j .

- (8) Find the delimiting point K , and then move the first K general variables X_j one step in the gaining directions and move the other general variables one step in the losing directions. It results in a new search point.
- (9) Call the low-level optimization algorithm to determine $ATS(II)$, that is the ATS values in the new search point. The control limits are determined in the meantime.
- (10) If $(ATS(I) - ATS(II))$ is substantial with reference to some predetermined criterion, the current search point is replaced by the new point and $ATS(I)$ is replaced by $ATS(II)$. Then, go back to step (5) for a new search.
- (11) Otherwise (i.e. if $(ATS(I) - ATS(II))$ is negligible), the high-level optimization is terminated.

In this high-level optimization, a set of optimum n_i and h_i is obtained, which reduces the out-of-control ATS of the chart system and ensures $(m \leq M)$. Further reduction in ATS of the chart system is obtained by the low-level optimization, where the control limits of the charts are optimized.

Low-level optimization

The purpose of the low-level optimization is, for a given set of (n_i, h_i) , to determine the optimal values of the LCL_i and UCL_i of the charts in the control chart system, so that the out-of-control ATS is minimized and the in-control ATS_0 remains constant as the specification τ .

The detailed formulae and optimization search algorithm used for the low-level optimization have been described in the last section (Section 3.2.1) where only LCL_i and UCL_i are optimized.

The final outputs of the optimization design (both high and low level optimization) of the control chart system are the optimal sample size, sampling interval, and control limits for each control chart in the system.

3.3 Optimization design of the interdependent $\bar{X}$ control chart system

This section develops **MSDSL** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / control limits) and **MSDSN** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / sample size, sampling interval, and control limits) chart systems. These models are the extensions of the models **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) and **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) discussed in Section 3.2 where the dependency between the process stages has not been considered. Here, the extended models **MSDSL** and **MSDSN** consider the dependency between the process stages. All other formulations and design procedures remain the same as the **MSISL** and **MSISN** chart systems (discussed in Sections 3.2.1 and 3.2.2).

Consider a manufacturing system as shown in Figure 3.1, where the quality characteristic in one stage are influenced by the previous stages. Four critical dimensions x_i ($i = 1, 2, 3, 4$) are formed in four stages. Dimension x_1 is formed by facing surface 1 using surface 0 as the datum. Dimension x_2 is associated with surface 2, which is machined with reference to surface 1. Then, using surface 2 as the datum, hole 3 and surface 4 are drilled and milled, respectively, and dimensions x_3 and x_4 are determined. It is noteworthy that x_2 depends directly on x_1 , and both x_3 and x_4 are affected directly by x_2 .

Four control charts as a system are used to carry out the on-line monitoring of the processes of the four stages machining x_1 to x_4 . Furthermore, since two identical machines are used side by side to mill dimension x_4 , control chart four has two duplicates, each controlling one of the two parallel machines. A block diagram of this

control chart system is displayed in Figure 3.1(b), and the quality linkages between the process stages are displayed in Figure 3.1(c).

Figure 3.1 (a) Dimensions of the part; (b) Process stages; (c) Quality linkages

3.3.1 Special features of the interdependent systems

The designs of the **MSDSL** and **MSDSN** chart systems need to consider two more specifications in order to identify the effects between stages and to determine the induced mean shift due to the mean shifts of the influencing stages. The calculations of the out-of-control *ATS* (Section 3.2.1.4) of the **MSISL** and **MSISN** chart systems only handle the mean shift δ_{μ_i} produced by the i th stage itself. However, the induced mean shifts because of the interdependency of the process stages may also affect the out-of-control *ATS*.

3.3.1.1 Additional specifications required for handling the interdependency between the process stages

The following two additional design specifications are required.

V_i a vector $[v_1, v_2, \dots]^T$ indicating the cause stage numbers or upstream influencing stage numbers. The dimension x_i produced in the i th stage is directly dependent on the dimensions produced in all of these cause stages.

Δ_i the induced mean shift. It is the mean shift undergone by the i th stage due to the mean shifts of the cause stages. Δ_i is expressed as a function $f_i(\delta_{v_1}, \delta_{v_2}, \dots)$ of the output mean shifts of all cause stages in vector V_i .

The vectors V_i and the formulae for Δ_i can be determined with the help of the quality linkages (Zantek *et al.* 2002). For example, the cascade quality linkages in Figure 3.1(c) indicate that x_2 is directly affected by x_1 , and both x_3 and x_4 are directly affected by x_2 . Correspondingly,

$$V_2 = [1]^T, \quad V_3 = [2]^T, \quad V_4 = [2]^T \quad (3.43)$$

The formulae for Δ_i can take many forms and may be determined by many different methods. For part machining, these formulae may be determined based on the information of design dimensioning and process planning. From the example in Figure 3.1,

$$\Delta_2 = f_2(\delta_{\mu,1}) = \delta_{\mu,1}, \quad \Delta_3 = f_3(\delta_{\mu,2}) = \delta_{\mu,2}, \quad \Delta_4 = f_4(\delta_{\mu,2}) = \delta_{\mu,2} \quad (3.44)$$

For example, $(\Delta_2 = \delta_{\mu,1})$ indicates that stage 2 will have an induced mean shift of Δ_2 if stage 1 has an output mean shift of $\delta_{\mu,1}$, and Δ_2 is equal to $\delta_{\mu,1}$. The induced mean shift may be a very complicated function in some applications.

3.3.1.2 Calculation of out-of-control *ATS* of the interdependent $\bar{X}$ chart system

Suppose that one of the g_k streams in the k th stage ($1 \leq k \leq s$) is out of control (i.e. its mean shift is equal to the maximum allowable mean shift $\delta_{\mu,k}$ as determined by Equation (3.4)). The probability w_i that the control charts in the i th stage do not give a signal is calculated differently for the following three cases.

Case 1 ($i = k$, the out-of-control stage):

The probability β_k of the type II error that the control chart monitoring the out-of-control stream fails to give a signal in a single sample is

$$\beta_k = \Phi\left(\frac{UCL_k - (\mu_{0,k} + \delta_{\mu,k}\sigma_{0,k})}{\sigma_{0,k}/\sqrt{n_k}}\right) - \Phi\left(\frac{LCL_k - (\mu_{0,k} + \delta_{\mu,k}\sigma_{0,k})}{\sigma_{0,k}/\sqrt{n_k}}\right) \quad (3.45)$$

The probability that none of the charts in the k th stage gives a signal in a single sample on condition that exactly one of the g_k streams is out-of-control and all other streams is in-control is (referring to discussion in Section 3.2.1.4)

$$w_k = (1 - \alpha_k)^{g_k - 1} \beta_k \quad (3.46)$$

Case 2 ($i \neq k$, and stage i is independent of stage k):

For the duration of one sampling interval h_k of the out-of-control stage k , the probability that a duplicate of the i th chart gives a signal due to the type I error can be approximated by

$$\theta = \frac{h_k}{h_i} \alpha_i \quad (3.47)$$

The ratio h_k / h_i is used to match the sampling interval h_i in the i th stage to h_k in the k th stage. Thus, the probability that none of the charts in the i th stage gives a signal during a time period of h_k is

$$w_i = (1 - \theta)^{g_i} \quad (3.48)$$

Case 3 ($i \neq k$, but stage i depends directly or indirectly on stage k):

Each of the g_i streams of stage i will undergo an induced mean shift as

$$\delta_{\mu,i} = \Delta_i = f_i(\delta_{v1}, \delta_{v2}, \dots) \quad (3.49)$$

Where $(\delta_{v1}, \delta_{v2}, \dots)$ are the output mean shifts of the cause stages (or the immediate upstream stages having impact on the i th stage). The cause stages can be found from the dependent relationship vector V_i (or the quality linkages). Sometimes, a cause stage of the i th stage is, by itself, influenced by another stage. Consequently, $\delta_{\mu,i}$ has to be determined by a recursive procedure. Moreover, if a cause stage is the out-of-control stage k , the output mean shift from this cause stage is equal to $(\delta_{\mu,k} / g_k)$. This is because that one, and only one, of the g_k streams in the k th stage goes out of control.

For the duration of one sampling interval h_k of the out-of-control stage k , the probability that a duplicate of the i th chart gives a signal is calculated by

$$\theta = \frac{h_k}{h_i} (1 - \beta_i) \quad (3.50)$$

It is similar to Equation (3.47) except that α_i is replaced by $(1 - \beta_i)$ (β_i is calculated by Equation (3.45) as β_k). Then, the probability w_i for Case 3 can be evaluated by Equation (3.48).

Combining all of the three cases, the probability that an out-of-control signal is produced during a time period of h_k is calculated as follows.

$$z_k = 1 - \prod_{i=1}^s (w_i \mid \text{stage } k \text{ is out of control}) \quad (3.51)$$

Therefore, given that one process in the k th stage goes out of control, the value of the out-of-control ATS is

$$ats_k = \left(\frac{1}{z_k} - 0.5 \right) \times h_k \quad (3.52)$$

In Equation (3.52), $1/z_k$ is the Average Run Length (ARL). The power z_k handles both the mean shift $\delta_{\mu,i}$ produced by the i th stage itself and the induced mean shifts $(\delta_{v1}, \delta_{v2}, \dots)$ received by the dependent downstream stages.

Since an out-of-control case may occur in any of the s stages, the final ATS for the control chart system is

$$ATS = \sum_{k=1}^s (ats_k \cdot p_k) \quad (3.53)$$

Where p_k is the probability that the out-of-control case takes place in the k th stage.

Admittedly, in any control chart system, a chart monitoring a process that is currently in control may produce misleading out-of-control signal. For the interdependent system, it is mainly because of the induced mean shifts discussed in Case 3. Therefore, whenever a control chart produces an out-of-control signal, not only the process being monitored by this chart, but also all upstream processes having direct or indirect impacts, must be investigated. More advanced diagnostic methods

can be found in many references (Ding *et al.* 2002 *a, b*, Zantek *et al.* 2002). The type I error may also generate a misleading signal, but it is negligible.

Following the experimentation in Section 3.2.1.4, the *ATS* of the interdependent chart system is also evaluated for 25 cases with different parameter values of s , g_i , h_i , n_i , α_i , $\sigma_{0,i}$, and $\delta_{\mu,i}$. In each case, the parameter values are randomly determined from the same ranges used in Section 3.2.1.4 and the relative discrepancy e_1 is calculated (Equation (3.21)). For the 25 different cases, the average and maximum values of the discrepancy e_1 for the interdependent system are only 0.02322 and 0.06427, respectively.

3.4 Comparative study

This section studies the effects of five parameters (g_i , $\sigma_{0,i}$, $\delta_{\mu,i}$, p_i , t_i) and the interdependence of the process stages on the performance of the **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) and **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) chart systems for a two-stage manufacturing system. This section also compares the performance of the **MSISN** chart system to the traditional Shewhart $\bar{X}$ chart system as well as the **MSISL** chart system.

- (1) Conventional 3-sigma $\bar{X}$ chart system. In this chart system, $n_1 = n_2 = 5$ and both h_1 and h_2 are equal to an_i (Equation (3.30)). The control limit coefficients for the two control charts are equal to three.

- (2) **MSISL** chart system. This chart system has optimal control limits LCL_i and UCL_i determined by the optimization design discussed in Section 3.2.1. However, the values of n_i and h_i of the two control charts are determined as for the conventional $\bar{X}$ chart system.
- (3) **MSISN** chart system. In this chart system, the values of n_i , h_i , LCL_i , and UCL_i of the two control charts are determined by the optimization design presented in Section 3.2.2.

Each of the five parameters (g_i , $\sigma_{0,i}$, $\delta_{\mu,i}$, p_i , t_i) has a low value, a nominal value, and a high value, as shown in Table 3.1. The nominal value denotes the value under normal circumstances. The nominal value of g_i is equal to its low value, because it is assumed that there is usually only one stream in a stage, unless the production rate in this stage is much lower than that in other stages.

Without loss of generality, $\mu_{0,i}$ is set at zero. Consequently, $LSL_i = (-USL_i)$. The specification τ for the in-control ATS_0 of the integrated chart systems (**MSISL** and **MSISN** chart system) is set equal to the ATS_0 value produced by the conventional 3-sigma $\bar{X}$ chart system (i.e. all the three chart systems always have the identical ATS_0 values).

Table 3.1 Parameters values for comparative study

Parameter	Low	Nominal	High
g_i	1	1	4
$\sigma_{0,i}$	0.01	0.025	0.04
$\delta_{\mu,i}$	1.00	1.125	1.25
p_i	0.1	0.5	0.9
t_i	1	2	3

It is also assumed that each of two operators in the manufacturing system spends, on average, 10% of his/her time for the inspection of the quality characteristics. Then the value of M is equal to 0.20 for the system.

The performance of the two-stage manufacturing system is first studied for five different cases, where the two stages are assumed independent of each other. In each of these five cases, all the parameters in both stages take their nominal values, except that one selected parameter (called the *active parameter*) takes its low value in stage one and high value in stage two. For example, in case one, the first parameter g_i is designated as the active parameter. Therefore,

$$g_1 = 1, \quad g_2 = 4$$

$$\sigma_{0,1} = \sigma_{0,2} = 0.025, \quad \delta_{\mu,1} = \delta_{\mu,2} = 1.125, \quad p_1 = p_2 = 0.5, \quad t_1 = t_2 = 2$$

Finally, Case 6 studies the impact of inter-stage dependence on system performance. In this case, all the parameters in both stages take their nominal values. But, the mean value of the second stage depends on the mean shift of the first stage (i.e. $V_2 = [1]^T$ and $\Delta_2 = \delta_{\mu,1}$).

Each case is investigated for the three control chart systems (i.e. the conventional, **MSISL** and **MSISN** chart systems). All three chart systems always result in the same $ATS_0 (= \tau)$ and $m (= M)$.

To facilitate the comparison, a ratio $B_{\bar{x}}$ of the out-of-control ATS values resulting from each control chart system is calculated as

$$B_{\bar{x}} = \frac{ATS}{ATS_{\bar{x}}} \quad (3.54)$$

Where $ATS_{\bar{x}}$ is obtained from the conventional $\bar{X}$ chart system and ATS is obtained from each particular chart system. Obviously, if the $B_{\bar{x}}$ value of a chart system in a case is less than one, this chart system will outperform the conventional $\bar{X}$ chart system when the corresponding active parameter has different values in the two stages.

The $B_{\bar{x}}$ values for all six cases are listed in Table 3.2 under columns (7) and (8). It is observed that in all of the six cases, the **MSISL** chart system usually substantially outperforms the conventional $\bar{X}$ chart system; however, the **MSISN** chart system can always achieve a much more significant improvement in effectiveness compared to the **MSISL** chart system. Among all, the difference in parameter $\sigma_{0,i}$ (the in-control process standard deviation) and the dependence between stages promise the greatest potential of using the **MSISN** chart system. For example, a reduction of ATS by about 72% compared to the conventional $\bar{X}$ chart system and 60% compared to the **MSISL** chart system can be achieved when $(\sigma_{0,1} = 0.01, \sigma_{0,2} = 0.04)$ and all other parameters are fixed. In the same case, however, the **MSISL** chart system reduces the ATS by about 29% compared to the conventional $\bar{X}$ chart system. It is also noted that the **MSISL** chart system cannot make any improvement when t_1 differs from t_2 , because the inspection time t_i has no impact on the control limits of the charts.

The actual $B_{\bar{x}}$ value would differ for different systems and circumstances. However, it is believed that the idea of the integrated control chart system would be effective in general and beneficial to many real systems. The reason is that in any manufacturing system the influential parameters would be different across several stages.

Table 3.2 Comparative study

(1) Case No.	(2) Active parameter		(3) Optimal n_i		(4) Optimal h_i		(5) Optimal k_i		(6) $A_{\bar{x},i}$		Ratio $B_{\bar{x}}$	
											(7) MSISL chart system	(8) MSISN chart system
1	$g_1 = 1$	$g_2 = 4$	9	10	294.80	575.78	2.738	2.823	0.0112	0.0062	0.946	0.583
2	$\sigma_{0,1} = 0.01$	$\sigma_{0,2} = 0.04$	1	23	42.86	299.98	3.274	2.621	0.0071	0.0293	0.711	0.283
3	$\delta_{\mu,1} = 1.0$	$\delta_{\mu,2} = 1.25$	13	3	157.13	173.77	2.661	3.364	0.0311	0.0051	0.698	0.316
4	$p_1 = 0.1$	$p_2 = 0.9$	4	8	155.20	107.78	2.912	2.936	0.0089	0.0253	0.899	0.639
5	$t_1 = 1$	$t_2 = 3$	11	8	155.00	186.00	2.996	2.709	0.0237	0.0159	1.000	0.607
6	$\mathbf{V}_2 = [1]^T$ and $\Delta_2 = \delta_{\mu,1}$		1	8	217.36	83.86	4.999	2.839	0.0009	0.0336	0.909	0.260

The optimal values of n_i , h_i , and k_i (control limit coefficient) of the optimal control chart system are also enumerated in Table 3.2 (columns (3), (4) and (5)). A parameter $A_{\bar{x},i}$ is calculated for each stage and listed in column (6).

$$A_{\bar{x},i} = \frac{n_i}{k_i h_i} \quad (i = 1, 2) \quad (3.55)$$

If $A_{\bar{x},1}$ is larger than $A_{\bar{x},2}$, that means more detection power should be allocated to the chart in the first stage in order to increase the overall effectiveness of the system. That is, n_1 is larger than n_2 and/or h_1 is smaller than h_2 and/or LCL_1 and UCL_1 are tighter than LCL_2 and UCL_2 . On the other hand, if $A_{\bar{x},1}$ is smaller than $A_{\bar{x},2}$, the chart in stage two should be made more powerful.

As the general guidelines drawn from Table 3.2, the following control charts should be made more powerful.

- (1) The control chart that is used in a stage with smaller number of streams; and/or
- (2) The control chart that is used to monitor a process with larger variance; and/or
- (3) The control chart that is used to monitor a process with smaller allowable mean shift; and/or
- (4) The control chart that is used to monitor a process stage where an out-of-control case is more likely to occur; and/or
- (5) The control chart that is used to monitor a process stage where the time required to inspect a unit is smaller; and/or
- (6) The control chart that is used to monitor a stage the quality characteristic of which depends on that of other stages.

With these guidelines in mind, the users can adjust the sample size, sampling interval, and control limits of each chart in the control chart system rationally and effectively, even if more complicated computation has not been carried out.

3.5 Examples

The performances of the **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits), **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits), **MSDSL** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / control limits) and **MSDSN** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / sample size, sampling interval, and control limits) chart systems developed in Sections 3.2 and 3.3 are demonstrated by two examples. The first example in Section 3.5.1 considers a manufacturing system where the process stages are independent of each other (for **MSISL** and **MSISN** chart systems), whereas the second example in Section 3.5.2 considers a manufacturing system that has interdependent process stages (for **MSDSL** and **MSDSN** chart systems).

3.5.1 Example for independent $\bar{X}$ chart systems

A mechanical part as shown in Figure 1.1(a) (in Chapter 1) is to be manufactured. Among all process stages, only three of them are functional (or critical to the overall quality of the part) and have to be closely monitored by the $\bar{X}$ charts. The design engineers indicate the dimensions and design tolerances on the drawing. The specifications of the system are listed below.

Number of stages in the system: $s = 3$

Number of streams: $g_1 = 2, g_2 = g_3 = 1$

Upper specification limit (mm): $USL_1 = 16.065, USL_2 = 4.09, USL_3 = 8.13$

In-control process means (mm): $\mu_{0,1} = 16.00, \mu_{0,2} = 4.00, \mu_{0,3} = 8.00$

Process standard deviations (mm): $\sigma_{0,1} = 0.012, \sigma_{0,2} = 0.023, \sigma_{0,3} = 0.038$

Originally, the system uses 3-sigma $\bar{X}$ charts for all the processes, i.e. all charts have the same type I error probability of 0.0027. The values of n_i , h_i , LCL_i , UCL_i and k_i (control limit coefficient) of the control chart system are as follows.

Stage 1: $n = 5, h = 60$ min, $LCL = 15.984\text{mm}, UCL = 16.016\text{mm}, (k = 3.0)$

Stage 2: $n = 5, h = 60$ min, $LCL = 3.969\text{mm}, UCL = 4.031\text{mm}, (k = 3.0)$

Stage 3: $n = 5, h = 60$ min, $LCL = 7.949\text{mm}, UCL = 8.051\text{mm}, (k = 3.0)$

To improve the effectiveness of the control chart system, optimization designs are carried out by using the algorithms discussed in Section 3.2.1 and Section 3.2.2. For this purpose, some more specifications for t_i (the time required to inspect a unit in the i th stage), M (the available number of operators equipped with measuring instruments in the manufacturing system), τ (the minimum allowable in-control ATS_0), $\delta_{\mu,i}$ (the maximum allowable mean shift in the i th stage), and p_i (the probability of out-of-control occurrence in the i th stage) are required. From the historical data, the values of t_i are

$$t_1 = 0.40 \text{ min}, t_2 = 0.60 \text{ min}, t_3 = 0.75 \text{ min}$$

For this set of t_i values, the in-control ATS_0 of the 3-sigma $\bar{X}$ chart system is equal to 5531 min (calculated by Equation (3.12)).

Since an operator spends 18% of his/her time (per shift per day) for the inspection activities, the value of M is equal to 0.18.

These τ and M values are used as the specifications for the optimization designs of the integrated control chart systems, with the aim of maintaining the false alarm rate and required manpower at the same level as in the 3-sigma $\bar{X}$ chart system.

The values of $\delta_{\mu,i}$ are calculated from the upper specification limits USL_i and the minimum allowable process capability ratio $c_{min,i}$. In this example, $c_{min,i}$ is specified as one for the whole manufacturing system. Thus, when PPM is equal to 2700 in any process, the control chart system is required to provide a signal as soon as possible.

The maximum allowable mean shifts are (Equation (3.4))

$$\delta_{\mu,1} = (16.065 - 16 - 3 \times 0.012 \times 1) / 0.012 = 2.42mm$$

$$\delta_{\mu,2} = (4.09 - 4 - 3 \times 0.023 \times 1) / 0.023 = 0.91mm$$

$$\delta_{\mu,3} = (8.13 - 8 - 3 \times 0.038 \times 1) / 0.038 = 0.42mm$$

For this set of mean shifts, the out-of-control ATS of the 3-sigma $\bar{X}$ chart system is equal to 589 minutes (Equation (3.19)).

The probabilities p_i can be estimated from historical records, which show that the numbers of out-of-control cases occurring in each of the three stages are 8, 6 and 4, respectively. Therefore

$$p_1 = 8 / (8 + 6 + 4) = 0.445$$

$$p_2 = 6 / (8 + 6 + 4) = 0.334$$

$$p_3 = 4 / (8 + 6 + 4) = 0.223$$

Based on the above specifications, the computer program *msisn.c* designs the **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) and the **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) chart systems

in 14 CPU seconds in a personal computer (Pentium IV 2.4GHz). The results are listed below.

MSISL chart system:

Stage 1: $n = 5$, $h = 60$ min, $LCL = 15.977\text{mm}$, $UCL = 16.023\text{mm}$, ($k = 4.197$)

Stage 2: $n = 5$, $h = 60$ min, $LCL = 3.969\text{mm}$, $UCL = 4.031\text{mm}$, ($k = 3.003$)

Stage 3: $n = 5$, $h = 60$ min, $LCL = 7.955\text{mm}$, $UCL = 8.045\text{mm}$, ($k = 2.649$)

$ATS_0 = 5531$ min, $m = 0.18$, $ATS = 398$ min

MSISN chart system:

Stage 1: $n = 4$, $h = 66$ min, $LCL = 15.978\text{mm}$, $UCL = 16.022\text{mm}$, ($k = 3.709$)

Stage 2: $n = 16$, $h = 177$ min, $LCL = 3.984\text{mm}$, $UCL = 4.016\text{mm}$, ($k = 2.806$)

Stage 3: $n = 19$, $h = 185$ min, $LCL = 7.981\text{mm}$, $UCL = 8.019\text{mm}$, ($k = 2.211$)

$ATS_0 = 5531$ min, $m = 0.18$, $ATS = 157$ min

It is observed that all the chart systems have the same $ATS_0 (= 5531 = \tau)$ and same m values ($= 0.18 = M$). However, the values of out-of-control ATS generated by the three chart systems are quite different. They are 589, 398, and 157 minutes for the 3-sigma $\bar{X}$ chart system, **MSISL** chart system and **MSISN** chart system, respectively. Or in other words, the **MSISL** chart system reduces ATS by about 32% and the **MSISN** chart system reduces ATS by an even high percentage of 73%, compared to the 3-sigma $\bar{X}$ chart system. The further reduction in ATS achieved by the **MSISN** chart system is attributable to the optimization of the sample sizes and sampling intervals in addition to the optimization of the control limits. In the optimal solution, majority of the detection power of the **MSISN** chart system is allocated to stage 3 (i.e.

n_3 is large and h_3 and k_3 are small). The reason is that stage 3 has relatively large variance, small allowable mean shift, and high probability of out-of-control occurrence.

3.5.2 Example for interdependent $\bar{X}$ chart systems

A mechanical part shown in Figure 3.1(a) is to be machined in a manufacturing system. The dimensions and tolerances on the drawing are determined by the design engineers. Among all the dimensions, only four stages (x_1, x_2, x_3, x_4) are functional (or critical to the overall quality of the part) and have to be closely monitored by the $\bar{X}$ charts. The block diagram of the control chart system is displayed in Figure 3.1(b) and the quality linkages in Figure 3.1(c). The specifications of the system are listed below.

Number of stages in the system: $s = 4$

Number of streams: $g_1 = g_2 = g_3 = 1, g_4 = 2$

Upper specification limit (mm): $USL_1 = 16.09, USL_2 = 4.07, USL_3 = 8.13, USL_4 = 11.08$

In-control process means (mm): $\mu_{0,1} = 16.00, \mu_{0,2} = 13.00, \mu_{0,3} = 8.00, \mu_{0,4} = 11.00$

Process standard deviations (mm): $\sigma_{0,1} = 0.015, \sigma_{0,2} = 0.012, \sigma_{0,3} = 0.039, \sigma_{0,4} = 0.018$

Originally, the system uses 3-sigma $\bar{X}$ charts for all the processes. The values of n_i, h_i, LCL_i, UCL_i and k_i of the system are as follows.

Stage 1: $n = 5, h = 60 \text{ min}, LCL = 15.980\text{mm}, UCL = 16.020\text{mm}, (k = 3.0)$

Stage 2: $n = 5, h = 60 \text{ min}, LCL = 12.984\text{mm}, UCL = 13.016\text{mm}, (k = 3.0)$

Stage 3: $n = 5, h = 60 \text{ min}, LCL = 7.948\text{mm}, UCL = 8.052\text{mm}, (k = 3.0)$

Stage 4: $n = 5, h = 60 \text{ min}, LCL = 10.976\text{mm}, UCL = 11.024\text{mm}, (k = 3.0)$

In order to improve the effectiveness of the control chart system, optimization design is carried out using the algorithms proposed in Section 3.3. Thus, some more specifications for t_i , M , τ , $\delta_{\mu,i}$, p_i , V_i (the vector indicating the upstream influencing stage numbers), and Δ_i (the induced mean shift) are required.

From the historical data, the values of the inspection time t_i are

$$t_1 = 0.25 \text{ min}, t_2 = 0.40 \text{ min}, t_3 = 0.50 \text{ min}, t_4 = 0.45 \text{ min}$$

For this set of t_i values, the in-control ATS_0 of the 3-sigma $\bar{X}$ chart system is equal to 4445 minutes (calculated by Equation (3.12)).

Since an operator spends 20% of his/her time (per shift per day) for the inspection activities, the value of M is equal to 0.20.

The τ and M values of the integrated control chart systems are also specified as 4445 min and 0.20, respectively, so that the false alarm rate and the required manpower are the same for all the chart system.

Similar to the previous example, the values of the maximum allowable mean shift $\delta_{\mu,i}$ are calculated from the upper specification limits USL_i and the minimum allowable process capability ratio $c_{min,i}$ (Equation (3.4)). In this example, $c_{min,i}$ is specified as one for the whole manufacturing system.

$$\delta_{\mu,1} = (16.09 - 16.00 - 3 \times 0.020 \times 1) / 0.015 = 2.00(mm)$$

$$\delta_{\mu,2} = (13.07 - 13.00 - 3 \times 0.019 \times 1) / 0.012 = 1.08(mm)$$

$$\delta_{\mu,3} = (8.13 - 8.00 - 3 \times 0.039 \times 1) / 0.039 = 0.34(mm)$$

$$\delta_{\mu,4} = (11.08 - 11.00 - 3 \times 0.018 \times 1) / 0.018 = 1.45(mm)$$

For this set of mean shifts, the out-of-control ATS of the traditional 3-sigma $\bar{X}$ chart system is equal to 1022 minutes (Equation (3.53)).

The probabilities p_i of out-of-control occurrences can be estimated from historical records, which show that the numbers of out-of-control cases occurring in each of the four stages are 4, 3, 8, and 6, respectively. Therefore

$$p_1 = 4 / (4 + 3 + 8 + 6) = 0.190$$

$$p_2 = 3 / (4 + 3 + 8 + 6) = 0.143$$

$$p_3 = 8 / (4 + 3 + 8 + 6) = 0.381$$

$$p_4 = 6 / (4 + 3 + 8 + 6) = 0.286$$

The dependent relationship vectors V_i and the formulae for Δ_i have been shown in Equations (3.43) and (3.44). That is

$$V_2 = [1]^T, \quad V_3 = [2]^T, \quad V_4 = [2]^T$$

$$\Delta_2 = \delta_{\mu,1}, \quad \Delta_3 = \delta_{\mu,2}, \quad \Delta_4 = \delta_{\mu,2}$$

Thus, if stage 1 goes out of control, stage 2 will be affected directly. From the discussion in Case 3 in Section 3.3.1.2

$$\delta_{\mu,2} = \Delta_2 = \delta_{\mu,1} / g_1 = 2.00 / 1 = 2.00$$

Meanwhile, stages 3 and 4 will be influenced indirectly through the change in stage 2.

$$\delta_{\mu,3} = \Delta_3 = \delta_{\mu,2} = 2.00$$

$$\delta_{\mu,4} = \Delta_4 = \delta_{\mu,2} = 2.00$$

On the other hand, if stage 2 is out of control, only stages 3 and 4 will be affected.

$$\delta_{\mu,3} = \Delta_3 = \delta_{\mu,2} / g_2 = 1.08 / 1 = 1.08$$

$$\delta_{\mu,4} = \Delta_4 = \delta_{\mu,2} / g_2 = 1.08 / 1 = 1.08$$

Based on the above specifications, the computer program *msdsn.c* designs the **MSDSL** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / control limits) and the **MSDSN** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / sample size, sampling interval, and control limits) chart systems in 18 CPU seconds in a personal computer (Pentium IV 2.4GHz). The results are listed below.

MSDSL chart system:

Stage 1: $n = 5$, $h = 60$ min, $LCL = 15.976\text{mm}$, $UCL = 16.024\text{mm}$, ($k = 3.604$)

Stage 2: $n = 5$, $h = 60$ min, $LCL = 12.981\text{mm}$, $UCL = 13.019\text{mm}$, ($k = 3.604$)

Stage 3: $n = 5$, $h = 60$ min, $LCL = 7.956\text{mm}$, $UCL = 8.044\text{mm}$, ($k = 2.506$)

Stage 4: $n = 5$, $h = 60$ min, $LCL = 10.971\text{mm}$, $UCL = 11.029\text{mm}$, ($k = 3.593$)

$ATS_0 = 4445$ min, $m = 0.20$, $ATS = 595$ min

MSDSN chart system:

Stage 1: $n = 1$, $h = 34$ min, $LCL = 15.944\text{mm}$, $UCL = 16.056\text{mm}$, ($k = 3.747$)

Stage 2: $n = 1$, $h = 77$ min, $LCL = 12.955\text{mm}$, $UCL = 13.045\text{mm}$, ($k = 3.747$)

Stage 3: $n = 18$, $h = 87$ min, $LCL = 7.978\text{mm}$, $UCL = 8.022\text{mm}$, ($k = 2.379$)

Stage 4: $n = 7$, $h = 145$ min, $LCL = 10.978\text{mm}$, $UCL = 11.022\text{mm}$, ($k = 3.211$)

$ATS_0 = 4445$ min, $m = 0.20$, $ATS = 224$ min

It is observed that all the chart systems have the same $ATS_0 (= 4445 = \tau)$ and same m values ($= 0.20 = M$). However, the values of out-of-control ATS generated by the three chart systems are quite different. They are 1022, 595, and 224 minutes for the 3-sigma $\bar{X}$ chart system, **MSDSL** chart system and **MSDSN** chart system, respectively. In other words, the **MSDSL** chart system reduces ATS by 42% and the **MSDSN** chart system reduces ATS by an even higher percentage of 78%, compared to the 3-sigma $\bar{X}$ chart system. In the optimal solution, the majority of the detection power of the **MSDSN** chart system is allocated to stage 3 (i.e. n_3 is large and h_3 and k_3 are small). The reason is that stage 3 has relatively large variance, small allowable mean shift, and high probability of out-of-control occurrence. In addition x_3 is dependent on x_1 and x_2 directly or indirectly.

3.6 Summary

This chapter develops the **MSISL** ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits), **MSISN** ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits), **MSDSL** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / control limits) and **MSDSN** ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / sample size, sampling interval, and control limits) chart systems for multi-stage and multi-stream manufacturing systems. The design algorithms consider only the mean shifts in optimizing the charting parameters n_i , h_i , LCL_i , and UCL_i . It is found that by properly allocating the power among the individual charts based on the values of the influential parameters, the effectiveness of the system as a whole can be significantly improved and, consequently, the product quality is further guaranteed. It is noted that the values of the design parameters n_i , h_i , LCL_i , and UCL_i are obtained based on the current values of the in-control process mean μ_0 and standard deviation σ_0 . If the values of μ_0 and σ_0 of the application are changed due to the change of production conditions, then the chart system should be redesigned using the latest values of the μ_0 and σ_0 .

In Section 3.2.1, only the control limits of each chart in the chart system are optimized; the sample size and the sampling interval are fixed as in the conventional system. In Section 3.2.2, the optimization design is further extended to optimize the sample size and sampling interval together with the control limits. Both optimization designs are able to handle independent systems as well as interdependent system. The optimization design is carried out under resource constraint (operators equipped with measuring instruments).

Most of the specifications for the optimization design are similar to that required by the conventional chart system designs, or available from manufacturing records. Furthermore, the use of the integrated control chart system will not in any sense increase the difficulties for the shop floor operators to run and understand the control charts. The design algorithm of the control chart system can be easily computerized. The design of a system has to be conducted only once, and the resultant optimal system can be used continuously until the process parameters have changed.

It is also noted that the actual model, layout, and scenario of a particular system may be somewhat different from the model discussed in this chapter, but the general algorithm proposed will be applicable. The formulae can be easily modified to cope with different models.

Finally, the general guidelines for adjusting the sample size, sampling interval, and control limits of each $\bar{X}$ chart in a system are particularly useful and universally applicable.

Chapter 4

Economical Designs of Integrated $\bar{X}$ Control Chart Systems for Monitoring Process Mean

This chapter designs the integrated control chart systems from an economic viewpoint. As the first part of this chapter, two economic models for a single process are first designed based on random mean shifts. The first model **MEIIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) minimizes the quality losses measured by the overall mean value of the loss function per out-of-control case. The second model **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) proposes an algorithm for the optimal deployment of manpower to the SPC for a process so that the resultant control chart minimizes the expected total cost incurred in the implement of SPC. The concept of the **MEIIN** and **MEIIM** charts is then applied to the design of the integrated control chart system **MEDSM** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower) for monitoring a multi-stage and multi-stream manufacturing system. A three-phase operational scenario for SPC is proposed for the first time in these approaches to design and operate the **MEIIN** chart, the **MEIIM** chart, as well as the **MEDSM** chart system. The probability distribution of the random mean shifts can be modeled by a Rayleigh distribution based on the sample data of the mean shifts acquired in the three-phase scenario. Unlike in the traditional economical control chart designs, the design algorithms discussed in this chapter only require limited number of specifications that can be easily determined. The results of the comparative studies

show that the proposed models are significantly superior to the Shewhart control chart in view of the overall performance. The algorithms are also illustrated through examples. Some useful guidelines are provided to aid users in deciding the appropriate amount of manpower for a particular application. The quality costs discussed in this chapter is modeled by Taguchi's loss function. However, an additional section outlines how the quality costs can also be measured by the defective rate of the process.

4.1 Introduction

Most of the researchers (see Chapter 2) have designed the control charts either in a statistical or in economical framework. While the statistical designs do not directly measure the cost associated with the SPC scheme, the economical designs also suffer from several weaknesses, including poor statistical properties, complex mathematical modelling and, especially, the difficulties in estimating costs and other model parameters (Woodall 1986, Montgomery 2001).

Another problem for both statistical and economical designs stems from the fact that only one or a few values of the mean shift δ_μ are taken into consideration during the optimization design (Page 1954a, Wheeler 1983, Saniga 1989), and these δ_μ values are usually decided subjectively. As a result, the design procedure may be inadequate to reflect the real working conditions. In other words, the resultant control chart may have optimal performance for the particular value(s) of δ_μ , but may work unsatisfactorily for other values of δ_μ . The mean shift δ_μ is a random variable and has different probability distributions for different processes. If the data of δ_μ can be collected during the field operation of the control chart, the optimization design can be carried out based on these data (or the corresponding probability distribution).

Consequently, the performance of the control chart will be improved over the whole range of δ_μ .

A main objective of implementing SPC is to minimize the total relevant cost (C_{total}), including quality cost ($C_{quality}$) due to poor quality (Besterfield *et al.* 1995) and manpower cost (C_{man}). The amount of manpower M is measured by the equivalent number of operators engaged with the SPC activities (see the discussion in Chapter 3). It is a fraction if an operator spends only part of his time in SPC, and it is larger than one if more than one operator is deployed to the SPC system. Despite the importance of manpower to the total cost, little research effort has been made to study its deployment to the SPC scheme.

The loss function is used broadly in industry to measure the cost due to poor quality (Spring and Yeung 1998, Reynolds and Glosch 1981). The loss ℓ associated with a particular value x of the quality characteristic is proportional to the square of the difference between x and the target value T .

$$\ell = K(x - T)^2 \quad (4.1)$$

Where K is a constant depending on the cost associated with the specification limits. It is clear that, the smaller the loss, the more x is close to T and the better the product quality. Therefore, reducing ℓ is consistent with one's effort for continuous quality improvement. The expected value, L , of ℓ over the distribution of x is equal to (Ross 1989)

$$L = K[\sigma^2 + (\mu - T)^2] \quad (4.2)$$

Where μ and σ are the mean value and standard deviation of x .

The remaining sections in this chapter are organized as follows.

- Section 4.2 discusses the three-phase SPC operational scenario by which the probability distribution of the random mean shift can be modeled.
- Section 4.3 develops the design algorithm of the **MEIIN** chart that minimizes the overall mean value of the loss function per out-of-control case.
- Section 4.4 designs the **MEIIM** chart that optimally deploys the manpower to SPC monitoring a single process.
- Section 4.5 develops the integrated **MEDSM** chart system that optimally deploys the manpower to SPC in a multi-stage and multi-stream manufacturing system.
- Section 4.6 discusses how the defective rate of the process rather than Taguchi's loss function can be used to measure the quality costs in designing the economic models.
- Finally, Section 4.7 summarizes all of the models developed in this chapter.

4.2 The three-phase SPC scenario and the probability distribution of the mean shift

The probability distribution of the mean shift δ_μ can be established through a three-phase SPC scenario. In the first phase, the data of the quality characteristics x are collected in order to estimate the in-control μ_0 and σ_0 and to determine the control limits of a workable control chart (Montgomery 2001). The workable control chart is then used to monitor the process in the second phase and to acquire N sample values of δ_μ (denoted by $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N}$). Suppose an out-of-control condition is signaled at the j_2 th sample and the follow-up investigation discovers that the responsible assignable cause occurs between the $(j_1 - 1)$ th and the j_1 th samples ($j_1 \leq j_2$). It means that the j_1 th to j_2 th samples are taken under the out-of-control condition (Wu *et al.* 2002). As a

result, the grand average $\bar{\bar{X}}$ of the sample means $\bar{X}_J$ ($J = j_1, j_1+1, \dots, j_2$) of these samples can be used as the estimate of the mean shift $\hat{\delta}_{\mu,i}$ (the sample value of δ_μ for this particular out-of-control case).

$$\bar{\bar{X}} = \frac{\sum_{J=j_1}^{j_2} \bar{X}_J}{j_2 - j_1 + 1}, \quad \hat{\delta}_{\mu,i} = \frac{(\bar{\bar{X}} - \mu_0)}{\sigma_0} \quad (4.3)$$

When N sample values, $\hat{\delta}_{\mu,i}$, are available, the control chart can be modified by using one of the following two approaches.

(1) The non-parametric approach

The sample values of δ_μ (i.e. $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N}$) are used directly without being fitted into a theoretical probability distribution.

(2) The parametric approach

The sample values, $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N}$, are first fitted into a theoretical probability distribution. The Rayleigh distribution may be an appropriate candidate (Wu *et al.* 2002). This probability distribution is often used to model the positional deviation from a target in geometrical tolerancing. The probability density function of the Rayleigh distribution is given by

$$f_{\delta_\mu}(\delta_\mu) = \frac{\pi \delta_\mu}{2 \mu_{\delta_\mu}^2} \exp\left(-\frac{\pi \delta_\mu^2}{4 \mu_{\delta_\mu}^2}\right) \quad (4.4)$$

Which is characterized by a single parameter—the mean value μ_{δ_μ} of δ_μ .

Figure 4.1 shows the density function of the Rayleigh distribution with μ_{δ_μ} equal to one. The parameter μ_{δ_μ} can be estimated from $(\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N})$.

$$\mu_{\delta\mu} \approx \frac{\sum_{i=1}^N \hat{\delta}_{\mu,i}}{N} \quad (4.5)$$

Figure 4.1 Rayleigh distribution for mean shift ($\mu_{\delta\mu} = 1.0$)

Now, the modified control chart can be used to monitor the forthcoming process in phase three. Moreover, when new data of δ_μ have come to light, the control chart can be re-designed using the latest information. Consequently, the three-phase SPC scenario is able to continuously update the charts and maintain it as an optimal one. The three-phase SPC scenario is obviously superior to the current two-phase SPC procedure (in which phase one is used to estimate μ_0 and σ_0 in order to design the workable control chart and, then, the workable chart is used to monitor the process in phase two). While the two-phase SPC scenario results in just a workable chart with average performance, the three-phase SPC scenario produces an optimal chart that is designed according to the actual working conditions of a particular process and will have the first-rate performance for that process.

Whether the parametric or non-parametric approach is employed, the accuracy of the model depends on the number N of the sample values of the mean shifts δ_μ . The larger is the N , the smaller is the discrepancy. As an example, the **MEIN** chart minimizes the overall mean value ML of the loss function per out-of-control case. The discrepancy e_{ML} related to the estimated ML is calculated as

$$e_{ML} = \left| \frac{ML_{act} - ML_{est}}{ML_{act}} \right| \quad (4.6)$$

Where ML_{act} is the accurate values of ML (assuming that the probability distribution of δ_μ is exactly known) and ML_{est} is the estimated values (i.e. the probability distribution is approximated based on the N sample values of δ_μ). A simulation test shows that if δ_μ follows a Rayleigh distribution, the average value of e_{ML} produced by the **MEIN** chart is smaller than 10% for $N \geq 11$ and smaller than 5% for $N \geq 44$.

4.3 Optimization design of the single $\bar{X}$ chart that minimizes the quality loss

This section develops an algorithm for the optimization design of the **MEIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart based on the data of the mean shifts δ_μ . These data are acquired from the observations of the out-of-control cases within the three-phase SPC scenario. The design algorithm optimizes n , h , LCL and UCL in order to minimize ML (the overall mean value of the Taguchi's loss function per out-of-control case) on condition that the false alarm rate and the inspection rate are not increased.

4.3.1 Input parameters

The parameters USL , τ , μ_0 , σ_0 , R (the maximum allowable inspection rate), and μ_{δ_μ} (the mean value of the mean shifts δ_μ) are required in designing this chart.

According to the three-phase SPC scenario the value of μ_{δ_μ} can be estimated from the N sample values of δ_μ (i.e. $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N}$) that are obtained from the observations of N out-of-control cases during the operation of the control chart. In fact, the specification of μ_{δ_μ} can be substituted by the N discrete values $\hat{\delta}_{\mu,i}$.

4.3.2 Optimization model

The design algorithm of the **MEIN** chart can be described by the following optimization model.

$$\text{Objective function: } ML = \text{minimum} \quad (4.7)$$

$$\text{Constraint functions: } ATS_0 \geq \tau \quad (4.8)$$

$$r \leq R \quad (4.9)$$

$$\text{Design variables: } n, h, LCL, UCL$$

Where r is the actual (or required) inspection rate. The optimization model optimizes n , h , LCL , and UCL in order to minimize ML (the overall mean value of the loss function per out-of-control case, over the probability distribution of the random mean shift δ_μ), on condition that the constraints on ATS_0 and r are all satisfied. The objective function ML is a function of (USL , τ , μ_0 , σ_0 , R , μ_{δ_μ} , n , h , LCL , and UCL). The minimization of ML will reduce the loss in quality (or the cost, or the damage) incurred by the out-of-control cases.

Among the four design variables n , h , LCL , and UCL , the sample size n is the only independent variable. Other three variables can be determined as follows.

(1) Sampling interval h

In order to satisfy constraint (4.9) on the inspection rate r and to make full use of the available resource, it is desired to have

$$R = r = n / h \quad (4.10)$$

Therefore,

$$h = n / R \quad (4.11)$$

(2) Upper control limit UCL

In order to satisfy constraint (4.8) on ATS_0 and to make the control chart most powerful, it is ideal to have

$$ATS_0 = \tau \quad (4.12)$$

The corresponding type I error probability is

$$\alpha = h / ATS_0 = h / \tau \quad (4.13)$$

Therefore, UCL can be calculated as follows.

$$UCL = \mu_0 + \frac{\Phi^{-1}(1 - 0.5\alpha)\sigma_0}{\sqrt{n}} \quad (4.14)$$

(3) Lower control limit LCL

Since the control limits of the $\bar{X}$ chart are symmetric about μ_0 , the lower control limit LCL can be determined easily.

$$LCL = 2\mu_0 - UCL \quad (4.15)$$

After n , h , LCL , and UCL have all been determined, the objective function ML can be calculated by

$$ML = \int_0^{\infty} [Q \cdot ATS(\delta_{\mu}) \cdot L(\delta_{\mu}) \cdot f_{\delta_{\mu}}(\delta_{\mu})] d\delta_{\mu} \quad (4.16)$$

Where $L(\delta_{\mu})$ and $ATS(\delta_{\mu})$ are the expected values of the loss function and the out-of-control ATS , respectively, for a given value of the mean shift δ_{μ} . The variable Q is the number of units produced per unit time. The product of Q and ATS is the average number of units produced under an out-of-control case. The probability density function $f_{\delta_{\mu}}(\delta_{\mu})$ of the Rayleigh distribution is given by Equation (4.4) (parametric approach). In fact, if N sample values of δ_{μ} ($\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N}$) are available, they may be used directly (non-parametric approach) to design the proposed charts, which would give the following expression of ML

$$ML = \frac{1}{N} \sum_{i=1}^N [Q \cdot ATS(\hat{\delta}_{\mu,i}) \cdot L(\hat{\delta}_{\mu,i})] \quad (4.17)$$

It is noted that a small mean shift δ_{μ} may contribute a substantial amount to ML because small δ_{μ} results in large ATS . On the other hand, a large δ_{μ} may also significantly increase the value of ML because large δ_{μ} makes the expected loss $L(\delta_{\mu})$ great. Integration (4.16) takes into account various mean shifts with different sizes and probabilities, and will provide a more comprehensive measure of the overall loss. For a given value of δ_{μ} the expected loss function $L(\delta_{\mu})$ over the distribution of x is equal to (Ross 1989) (see Equation (4.2) and note $T = \mu_0$).

$$L = K[\sigma_0^2 + \delta_{\mu}^2 \sigma_0^2] \quad (4.18)$$

Therefore, the objective function can be rewritten as

$$ML = KQ \int_0^{\infty} [ATS(\delta_{\mu}) \cdot (\sigma_0^2 + \delta_{\mu}^2 \sigma_0^2) \cdot f_{\delta_{\mu}}(\delta_{\mu})] d\delta_{\mu} \quad (4.19)$$

Since the product of K and Q is a constant and has no effect on the optimal values of n , h , LCL , and UCL , it will be neglected in the subsequent discussion for simplicity.

The value of $ATS(\delta_\mu)$ is calculated by

$$ATS(\delta_\mu) = (ARL(\delta_\mu) - 0.5)h \quad (4.20)$$

Where,

$$ARL(\delta_\mu) = \frac{1}{1 - \Phi\left(\frac{UCL - \mu}{\sigma_0/\sqrt{n}}\right) + \Phi\left(\frac{LCL - \mu}{\sigma_0/\sqrt{n}}\right)} \quad (4.21)$$

Where $\mu = \mu_0 + \delta_\mu\sigma_0$. It is well known that ARL is a decreasing function of the sample size n , and h is an increasing function of n (because of constraint (4.8)). Namely, if a small n is used, ARL will be large and h will be small. Conversely, if a large n is used, ARL will be small and h will be large. As a result, the change of the independent parameter n in either direction will engender ARL and h to compete with each other. Since both ARL and h are the factors of ATS (Equation (4.20)), there must be an n value that will make ATS small for a given value of δ_μ . Furthermore, from an overall viewpoint, there must be an optimal value of n that will make the objective function ML minimum. The final expression of ML is

$$ML = \int_0^\infty \left[\frac{1}{1 - \Phi\left(\frac{UCL - \mu}{\sigma_0/\sqrt{n}}\right) + \Phi\left(\frac{LCL - \mu}{\sigma_0/\sqrt{n}}\right)} - 0.5 \right] h \cdot (\sigma_0^2 + \delta_\mu^2 \sigma_0^2) \cdot f_{\delta_\mu}(\delta_\mu) d\delta_\mu \quad (4.22)$$

The integration (4.22) can be evaluated by using the *Gauss-Legendre translation rule* on a few integration points.

4.3.3 Optimization search algorithm

The entire optimization design is implemented as a *single-variable search*. The single variable n is increased from one in step of size one. The procedures of the search algorithm are outlined below.

- (1) Specify USL , τ , μ_0 , σ_0 , R , and $\mu_{\delta\mu}$.
- (2) Initialize ML_{min} as a very larger number, say 10^7 (ML_{min} is used to store the minimum value of ML).
- (3) Search n from one in step of one until ML cannot be further reduced. For a given value of n ,
 - (3.1) Calculate h by using Equation (4.11).
 - (3.2) Calculate UCL by using Equations (4.13) and (4.14).
 - (3.3) Calculate LCL by using Equation (4.15).
 - (3.4) Calculate ML by using Equation (4.22).
 - (3.5) If the calculated ML is smaller than the current ML_{min} , replace the latter by the former, and the current values of n , h , LCL , and UCL are stored as the temporary optimal design.
- (4) At the end of the entire search, the ML_{min} is the minimum ML and the corresponding optimal values of n , h , LCL , and UCL are also determined.

4.3.4 Comparative study

The performance of the **MEIIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart and other three charts is compared in this section, in which the mean shift δ_μ is assumed to follow a Rayleigh distribution. The in-control μ_0 and σ_0 are fixed at zero and one, respectively. Consequently, $LSL = (-USL)$. The four control charts are as follows.

- (1) A Shewhart $\bar{X}$ chart that uses a fixed sample size of five.
- (2) An $\bar{X}_{1.0}$ chart. The design of this chart searches the optimal values of n , h , LCL , and UCL in order to minimize ML . But it only considers a fixed mean shift

value ($\delta_\mu = 1.0$) rather than the whole probability distribution of δ_μ . Referring to Equation (4.17) (neglecting Q)

$$ML = ATS(1.0) \cdot (\sigma_0^2 + 1.0^2 \cdot \sigma_0^2) \quad (4.23)$$

The fixed value ($\delta_\mu = 1.0$) is adopted in many previous optimization designs of the control charts (Prabhu *et al.* 1997, Das and Jain 1997).

- (3) An $\bar{X}_{\mu\delta_\mu}$ chart. It is almost the same as the $\bar{X}_{1.0}$ chart, except that δ_μ is fixed at μ_{δ_μ} rather than 1.0. It implies that μ_{δ_μ} is known.
- (4) A **MEIIN** chart that is proposed in this section.

To facilitate the comparison, a normalized ML_{normal} value for each chart is calculated by using the ML value of the **MEIIN** chart as the norm.

$$ML_{normal} = \frac{ML}{ML_{ML}} \quad (4.24)$$

Obviously, if the value of ML_{normal} of a chart is larger than one, the performance of this chart is inferior to that of the **MEIIN** chart, and *vice versa*.

Study one

The comparison is first conducted under the following general conditions.

$$\tau = 400, R = 4, \mu_{\delta_\mu} = 0.8$$

The charting parameters, the ML value, and the normalized ML_{normal} value for each of the four control charts are worked out and listed below.

Shewhart $\bar{X}$ chart: $n = 5, h = 1.25, LCL = -1.322, UCL = 1.322,$

$$ML = 45.568, ML_{normal} = 2.437$$

$\bar{X}_{1.0}$ chart: $n = 14, h = 3.50, LCL = -0.701, UCL = 0.701,$

$$ML = 24.492, ML_{normal} = 1.310$$

$\bar{X}_{\mu_{\delta\mu}}$ chart: $n = 20, h = 5.00, LCL = -0.559, UCL = 0.559,$

$$ML = 20.932, ML_{normal} = 1.119$$

MEIIN chart: $n = 36, h = 9.00, LCL = -0.380, UCL = 0.380,$

$$ML = 18.702, ML_{normal} = 1.000$$

It is noted that all of the above charts have satisfied the constraint functions on both ATS_0 and r (Equations (4.8) and (4.9)).

The values of ML_{normal} of the four charts clearly indicate that the **MEIIN** chart outperforms all other charts. The **MEIIN** chart is more effective (in terms of ML) than the Shewhart $\bar{X}$ chart, the $\bar{X}_{1.0}$ chart, and the $\bar{X}_{\mu_{\delta\mu}}$ chart by 143.7, 31.0, and 11.9%, respectively, for this particular case. It is worthy to note that both the **MEIIN** chart and the $\bar{X}_{\mu_{\delta\mu}}$ chart are designed based on the information of $\mu_{\delta\mu}$. However, since the **MEIIN** chart is designed by taking into account the entire probability distribution of δ_μ , it is considerably more effective than the $\bar{X}_{\mu_{\delta\mu}}$ chart.

It is also found that the $\bar{X}_{\mu_{\delta\mu}}$ chart substantially outperforms the Shewhart $\bar{X}$ chart and the $\bar{X}_{1.0}$ chart. It is because the $\bar{X}_{\mu_{\delta\mu}}$ chart uses the mean value $\mu_{\delta\mu}$ of δ_μ rather than a subjectively determined value of δ_μ . This finding highlights the importance and usefulness of the three-phase SPC scenario. Thanks to this scenario, the sample values of δ_μ can be acquired and used to estimate $\mu_{\delta\mu}$, which can be, in turn, used to design the $\bar{X}_{\mu_{\delta\mu}}$ chart or **MEIIN** chart.

Study two

Next, the performance of the four charts is further studied by a 2^3 factorial experiment (Montgomery 2001). The three parameters τ , R , and $\mu_{\delta\mu}$ are used as the input factors and, ML_{normal} is taken as the response. Each of the three factors τ , R , and $\mu_{\delta\mu}$ varies at two levels, resulting in 8 runs (i.e. 8 combinations of the values of the three factors). The low and high levels for each factor are decided below.

- τ [100, 600]
- R [2, 10]
- $\mu_{\delta\mu}$ [0.7, 1.8]

The ML_{normal} value of the control charts under each of the eight runs are calculated and enumerated in Table 4.1. It can be seen that the **MEIIN** chart is always the most effective chart in all of the runs.

Table 4.1 Comparative study

Run	Factor			ML_{normal}		
	τ	R	$\mu_{\delta\mu}$	Shewhart	$\bar{X}_{1.0}$	$\bar{X}_{\mu_{\delta\mu}}$
1	100.0	2.0	0.7	1.421	1.122	1.005
2	100.0	2.0	1.8	1.022	1.314	1.000
3	100.0	10.0	0.7	2.285	1.333	1.077
4	100.0	10.0	1.8	1.069	1.135	1.069
5	600.0	2.0	0.7	2.431	1.394	1.090
6	600.0	2.0	1.8	1.099	1.103	1.099
7	600.0	10.0	0.7	4.414	1.884	1.315
8	600.0	10.0	1.8	1.591	1.003	1.406
$\overline{ML}_{normal}$				1.917	1.286	1.133

The average $\overline{ML}_{normal}$ of the ML_{normal} values for each chart over the eight runs is calculated and listed at the bottom of Table 4.1. The values of $\overline{ML}_{normal}$ indicate that, from an overall viewpoint (over different combinations of the values of τ , R and $\mu_{\delta\mu}$), the **MEIIN** chart is more effective than the Shewhart $\bar{X}$ chart, the $\bar{X}_{1.0}$ chart and the $\bar{X}_{\mu\delta\mu}$ chart by 91.7, 28.6, and 13.3 percentage, respectively.

Finally, the effects of the three factors (τ , R , $\mu_{\delta\mu}$) on the response (ML_{normal}) are calculated and listed in Table 4.2.

Table 4.2 Effects of three factors

No.	Factors	Effects on ML_{normal}		
		Shewhart	$\bar{X}_{1.0}$	$\bar{X}_{\mu\delta\mu}$
1	τ minimum allowable in-control ATS_0	0.9345	0.8467	-1.4427
2	R the maximum allowable inspection rate	0.1197	0.1058	-0.2946
3	$\mu_{\delta\mu}$ the mean value of the mean shifts δ_μ	0.1897	0.1682	0.0219

A positive impact on ML_{normal} means that the **MEIIN** chart may achieve a more significant loss reduction when the corresponding factor is smaller and *vice versa*. For example, factor τ has the most significant impact on ML_{normal} for each of the three cases. However, the effect of R and $\mu_{\delta\mu}$ on ML_{normal} is relatively small.

4.3.5 Example

A manufacturing factory produces a special type of bearing. The number Q of units produced per hour is equal to 45. The diameter x of the bearing is a key dimension and specified as $80\pm0.008mm$. The process mean can be easily adjusted to the nominal value (80mm) at the center between the lower and upper specification limits. The QA

engineer specifies the minimum allowable in-control ATS_0 as 400 hours and the maximum allowable inspection rate as 4 units per hour.

In the phase one operation, the pilot runs are conducted. It is found that the distribution of the diameter can be well approximated by a normal distribution and the standard deviation σ_0 is very close to $0.002mm$. From these data, a workable Shewhart $\bar{X}$ chart is designed in which the sample size n is set as five. Other parameters are also worked out as follows.

$$n = 5, h = 1.25 \text{ hr}, LCL = 79.99736mm, UCL = 80.00264mm$$

Then, in the phase two operation, the Shewhart $\bar{X}$ chart starts to monitor the process mean. Meanwhile, 15 sample values of δ_μ (i.e. $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,15}$) are observed and recorded (see Table 4.3, $\mu_{\delta_\mu} \approx 0.774$ by Equation (4.5)). At the end of phase two, these $\hat{\delta}_{\mu,i}$ values are used to design the **MEHN** chart directly (using non-parametric procedure).

Table 4.3 Sample values of mean shift

No.	$\hat{\delta}_{\mu,i}$	No.	$\hat{\delta}_{\mu,i}$
1	0.566	9	0.432
2	0.619	10	0.406
3	0.462	11	0.402
4	0.600	12	0.575
5	1.697	13	0.507
6	0.226	14	1.275
7	1.865	15	0.465
8	1.065		

The computer program *meiin.c* designs the **MEIIN** control chart following the proposed algorithms in almost no time in a personal computer. The parameters of the chart are listed below.

$$n = 36, h = 9.00 \text{ hr}, LCL = 79.99924\text{mm}, UCL = 80.00076\text{mm}$$

Both the Shewhart $\bar{X}$ chart and the **MEIIN** chart produce the same in-control ATS_0 and inspection rate r that are equal to the specified values. However, the ML values resulting from the Shewhart $\bar{X}$ chart and the **MEIIN** chart are 2057 and 618, respectively. It indicates a significant improvement achieved by the **MEIIN** chart. It reduces the loss incurred per out-of-control case by 232% on average, compared to the Shewhart $\bar{X}$ chart.

4.3.6 Discussions

The model presented in this section develops the **MEIIN** ($\bar{X}$ chart-economical design-independent process-single control chart-sample size, sampling interval, and control limits) chart, which minimizes the overall mean value ML of the loss function per out-of-control case by optimizing the sample size, sampling interval, and control limits of the chart. The comparative studies show that the **MEIIN** chart outperforms, to a significant degree and from an overall viewpoint, the Shewhart $\bar{X}$ chart as well as some other charts that are designed based on the fixed mean shift values. The design and operation of the **MEIIN** chart is implemented through a three-phase SPC scenario, in which the data for the random mean shifts δ_μ can be collected and the probability distribution model can be built. The concept of the three-phase SPC scenario paves the

way for converting the workable control chart with average performance to the optimal chart.

4.4 Optimization design of the single $\bar{X}$ chart that minimizes the total cost

This section proposes an algorithm for the optimization design of the **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart. The model **MEIIM** is an extension of the model **MEIIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) discussed in Section 4.3. The model **MEIIN** only minimizes the quality losses $C_{quality}$ (or the costs) incurred by the out-of-control cases. Here, the extended model **MEIIM** minimizes the expected total cost C_{total} including the manpower cost C_{man} and the quality cost $C_{quality}$ by optimizing the amount of allocated manpower M in the SPC scheme. In the mean time, the sample size, sampling interval, and control limits of the control chart are also optimized. The design algorithm also takes into account the probability distribution of the random mean shifts δ_μ that is modeled by the Rayleigh distribution based on the sample data of the mean shifts acquired in the three-phase SPC scenario. Some general guidelines for deploying manpower to an SPC scheme are also provided.

4.4.1 Input parameters

The parameters USL , τ , μ_0 , σ_0 , $\mu_{\delta\mu}$, Q (number of units produced per unit time), C_m (manpower cost for an operator per unit time), C_k (cost required to fix an out-of-specification product in the workshop), $MTBO$ (mean time between out-of-control cases), and w (number of units that can be inspected per unit time by an operator) are required in designing this model.

The value of $\mu_{\delta\mu}$ and $MTBO$ can be estimated from the N sample values of $\delta\mu$ (i.e. $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N}$) and the entire time span of the N out-of-control cases acquired in the three-phase SPC scenario. The estimation of $\mu_{\delta\mu}$ has been detailed in Section 4.2. The estimation of $MTBO$ will be described in the section of example (Section 4.4.5).

4.4.2 Optimization model

The design algorithm of the **MEIIM** chart is implemented by the following optimization model.

$$\text{Objective function: } C_{total} = \text{minimum} \quad (4.25)$$

$$\text{Constraint function: } ATS_0 \geq \tau \quad (4.26)$$

$$\text{Design variables: } M, n, h, LCL, UCL$$

The optimization model optimizes M, n, h, LCL , and UCL in order to minimize C_{total} , on condition that the resultant (or actual) in-control ATS_0 is not less than the specified value τ . The objective function C_{total} is a function of ($USL, \tau, \mu_0, \sigma_0, \mu_{\delta\mu}, Q, C_m, C_k, MTBO, w, n, h, LCL$, and UCL).

Among the five design variables, only M and n are independent. The other three can be determined by M and n . Firstly, since the number of units that can be inspected per unit time is equal to either $(M \times w)$ ($(M \times w)$ is the maximum inspection rate R used in the last section) or (n / h) , the sampling interval h can be determined by (see Equation (4.11))

$$h = \frac{n}{R} = \frac{n}{M \times w} \quad (4.27)$$

It indicates that h is an increasing function of n when the amount of manpower M is given. In other words, sampling should be infrequent if a large sample size is adopted, and *vice versa*.

Next, the control limits LCL and UCL can be determined by following step 2 and step 3 in Section 4.3.2 (using Equations (4.13), (4.14) and (4.15)).

When M , n , h , LCL , and UCL are all determined, the objective function (the total cost C_{total} per unit time) can be calculated as the sum of the two cost components.

$$C_{total} = C_{quality} + C_{man} \quad (4.28)$$

It is noted that the total relevant cost may also include other fixed costs, such as overhead costs as well as the loss function when the process is in control (depending on σ_0). However, since such cost components are constant, they are omitted from the objective function for model simplicity.

Both $C_{quality}$ and C_{man} are direct or implicit functions of manpower M . If M is small, manpower cost C_{man} is small but manpower shortage may result in a large out-of-control ATS and, hence, a large quality cost $C_{quality}$. Conversely, if M is large, C_{man} is large but $C_{quality}$ may be substantially reduced, as increased manpower allows the use of large sample size and/or frequent sampling resulting in the prompt signal of out-of-control cases. Therefore, a change of M in either direction will engender C_{man} and $C_{quality}$ to compete with each other. Since these two cost components of C_{total} change in opposite directions, logically, there is an optimal value of M that minimizes C_{total} .

The manpower cost C_{man} per unit time is calculated by

$$C_{man} = C_m \times M \quad (4.29)$$

The quality cost $C_{quality}$ is the cost incurred by the out-of-control cases. It depends on the frequency ($1/MTBO$) of the out-of-control cases, the average number $q(\delta_\mu)$ of units produced after the occurrence of a mean shift of size δ_μ , the expected loss function

$L(\delta_\mu)$ per product for a given δ_μ and, finally, the distribution of the mean shifts δ_μ . The quality cost $C_{quality}$ per unit time is

$$C_{quality} = \int_0^\infty \left[\frac{1}{MTBO} \cdot q(\delta_\mu) \cdot L(\delta_\mu) \cdot f_{\delta_\mu}(\delta_\mu) \right] d\delta_\mu \quad (4.30)$$

Where the probability density function $f_{\delta_\mu}(\delta_\mu)$ of the Rayleigh distribution is given by Equation (4.4) (parametric approach). If the N sample values of δ_μ (i.e. $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,N}$) are used directly (non-parametric approach), the quality cost $C_{quality}$ is evaluated by

$$C_{quality} = \frac{1}{N} \sum_{i=1}^N \left[\frac{1}{MTBO} \cdot q(\hat{\delta}_{\mu,i}) \cdot L(\hat{\delta}_{\mu,i}) \right] \quad (4.31)$$

It is noted that only positive mean shifts need to be considered because of the symmetry of the normal distribution of x . It is also noted that $q(\delta_\mu)$ is a function of n and h , both of which depend on the manpower M . The average number $q(\delta_\mu)$ is equal to the product of Q and $ATS(\delta_\mu)$. Therefore, from Equation (4.30)

$$C_{quality} = \int_0^\infty \left[\frac{1}{MTBO} \cdot Q \cdot ATS(\delta_\mu) \cdot L(\delta_\mu) \cdot f_{\delta_\mu}(\delta_\mu) \right] d\delta_\mu \quad (4.32)$$

The $C_{quality}$ in Equation (4.32) is the same as ML in Equation (4.16) except for a factor $1/MTBO$. The final expression of $C_{quality}$ can be obtained by using Equations (4.18), (4.20) and (4.21).

$$C_{quality} = \frac{Q \cdot h \cdot K \cdot \sigma_0^2}{MTBO} \int_0^\infty \left[\left(\frac{1}{1 - \Phi\left(\frac{UCL - \mu}{\sigma_0/\sqrt{n}}\right) + \Phi\left(\frac{LCL - \mu}{\sigma_0/\sqrt{n}}\right)} - 0.5 \right) \cdot (1 + \delta_\mu^2) f_{\delta_\mu}(\delta_\mu) \right] d\delta_\mu \quad (4.33)$$

The coefficient K can be calculated from the cost component C_k associated with the specification limits (see Equation (4.1)).

$$K = \frac{C_k}{(USL - \mu_0)^2} \quad (4.34)$$

4.4.3 Optimization search algorithm

The optimum values of the two independent variables (the amount of manpower M and the sample size n) are sought in a two-level optimization search. The high-level optimization is a single variable search for the optimal value of M . The search can be carried out by using a nonlinear programming model. In our implementation, a *Successive Quadratic Estimation Method* (Powell 1964) is employed. Then, for a given value of M determined in the high level, an *exhaustive low-level search* is evoked to find the optimal value of n . While the high-level optimization makes a trade-off between the quality cost $C_{quality}$ and the manpower cost C_{man} , the low level optimization trades off between the sample size n and the sampling interval h (i.e. whether using large but sparse samples, or small but frequent samples). Both levels aim at minimizing the total cost C_{total} . The overall design procedures are outlined below.

- (1) Initialize C_{min} as a very large number, say 10^7 (C_{min} is used to store the minimum value of C_{total}).
- (2) Calculate K by Equation (4.34).
- (3) At the high level, the Successive Quadratic Estimation method is used to search the optimal value of M .
- (4) At the low level, for a given value of M determined in step (3), the sample size n is increased from one in step of one until C_{total} cannot be further reduced. Now, for the given pair of M and n ,
 - (4.1) Calculate h by Equation (4.27).

- (4.2) Calculate UCL and LCL by Equations (4.13), (4.14) and (4.15).
 - (4.3) Calculate the manpower cost C_{man} and the quality cost $C_{quality}$ by Equations (4.29) and (4.33).
 - (4.4) Calculate the total cost C_{total} by Equation (4.28).
 - (4.5) If the calculated C_{total} is smaller than the current C_{min} , replace the latter by the former, and the current values of M , n , h , LCL , and UCL are stored as the temporary optimal design.
- (5) When the entire search is completed, the final C_{min} is the minimum C_{total} and the corresponding optimal values of M , n , h , LCL , and UCL are also determined.

4.4.4 Comparative study

A 2^k factorial experiment is carried out to compare the performance (measured by the total cost C_{total}) of the **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart with that of a conventional $\bar{X}$ chart as well as a **MEIIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart (discussed in Section 4.3). Like the **MEIIM** chart, the **MEIIN** chart is designed to minimize the total cost C_{total} , but it only has its charting parameters n , h , LCL and UCL optimized (namely, only the low-level optimization is conducted) and uses a fixed amount of manpower M_0 equal to that used by the conventional $\bar{X}$ chart (that is, the high-level optimization is omitted). It is also assumed that the conventional $\bar{X}$ chart uses a sample size of five and the 3-sigma control limits.

Eight factors τ , USL , Q , w , $MTBO$, $\mu_{\delta\mu}$, C_m , and C_k are studied. Each of them varies at two levels as shown below.

τ	[200, 600]
USL	[3, 6]
Q	[100, 600]
w	[0.3 Q , 3 Q]
$MTBO$	[100, 600]
$\mu_{\delta\mu}$	[0.5, 1.5]
C_m	[5, 20]
C_k	[0.5, 10]

Noteworthy, the inspection rate w is expressed as a function of the production rate Q . In some automatic processes, the inspection rate w is lower than the production rate Q . Nevertheless, for the machining of some components where inspection is much simpler than fabrication, w could be larger than Q .

As in the previous study (Section 4.3.4), the in-control μ_0 and σ_0 are fixed at zero and one, respectively. It is also assumed that the mean shift δ_μ follows a Rayleigh distribution with a mean value of $\mu_{\delta\mu}$ as given above.

The fixed amount of manpower M_0 used by both the conventional $\bar{X}$ chart and the **MEIN** chart is determined by (Equation (4.27))

$$M_0 = \frac{n_0}{h \cdot w} \quad (4.35)$$

Where ($n_0 \equiv 5$) is the sample size used by the conventional $\bar{X}$ chart. Further, by referring to Equation (4.9)

$$M_0 = \frac{n_0}{\alpha \cdot \tau \cdot w} \quad (4.36)$$

For a 3-sigma $\bar{X}$ chart

$$\alpha = 2 \cdot \Phi(-3) \quad (4.37)$$

The total number of runs (different combinations of the values of the eight factors) of the whole factorial experiment is equal to 256. For each run, the conventional $\bar{X}$ chart, the **MEIIN** chart and the **MEIIM** chart are designed and all of them always generate the same in-control ATS_0 (equal to the specified value of τ). The following three responses are investigated.

R_{M-C} the ratio between the total cost of the **MEIIM** chart and that of the conventional $\bar{X}$ chart.

R_{S-C} the ratio between the total cost of the **MEIIN** chart and that of the conventional $\bar{X}$ chart.

R_M the ratio between the optimal manpower M of the **MEIIM** chart and the manpower M_0 for the conventional and **MEIIN** charts.

Obviously, if the value of R_{M-C} (or R_{S-C}) is less than one, the performance of the **MEIIM** chart (or the **MEIIN** chart) is superior to that of the conventional $\bar{X}$ chart in a particular run in terms of the total cost, and *vice versa*.

The grand averages, $\bar{R}_{M-C}$, $\bar{R}_{S-C}$ and $\bar{R}_M$, of the three responses over the 256 runs are 0.3538, 0.5901 and 19.15, respectively. The value of $\bar{R}_{S-C}$ indicates that, from an overall viewpoint (considering different combinations of τ , USL , Q , w , $MTBO$, $\mu_{\delta_{\mu}}$, C_m and C_k), the **MEIIN** chart reduces the total cost C_{total} by about 40% compared to the conventional $\bar{X}$ chart. This improvement is achieved by optimizing the charting

parameters n and h . Whereas, the value of $\bar{R}_{M-C}$ shows that the **MEIIM** chart is able to further reduce C_{total} by 24% (the C_{total} of the **MEIIM** chart is equal to about one third of that of the conventional $\bar{X}$ chart). This further reduction in total cost is completely attributable to the optimal deployment of manpower in the SPC scheme, suggesting that the optimal deployment of manpower is an effective method to achieve better economic results in SPC.

Finally, the value of the grand average $\bar{R}_M$ ($= 19.15$) of the third response shows that the conventional $\bar{X}$ chart usually uses far less manpower than needed for the minimization of the total cost. The deployment of more manpower leads to a significant reduction in quality cost that outweighs the increase in the manpower cost. As a result, the total cost is substantially reduced. It is noted that the optimal amount of the manpower M for a particular SPC scheme has to be determined based on the process specifications.

The effects of the eight factors (τ , USL , Q , w , $MTBO$, $\mu_{\delta\mu}$, C_m , C_k) on the two responses ($\bar{R}_{M-C}$ and $\bar{R}_M$) are also calculated and listed in Table 4.4. A positive effect on R_{M-C} means that the **MEIIM** chart may achieve a more significant cost reduction when the corresponding factor is smaller and *vice versa*. For example, factor $\mu_{\delta\mu}$ has the most significant positive impact (0.2518) on R_{M-C} . Thus, when $\mu_{\delta\mu}$ is smaller, the **MEIIM** chart will reduce the total cost to a greater degree. The effects (on R_{M-C}) of the factors indicate that, the superiority of the **MEIIM** chart over the conventional $\bar{X}$ is more notable when τ , Q , w , C_k are high and/or USL , $MTBO$, $\mu_{\delta\mu}$, C_m are low.

On the other hand, a positive effect on R_M means that the amount of manpower should be increased when the corresponding factor is large and *vice versa*. For example, the production rate Q has a positive effect on R_M . If it is high, more manpower should be allocated to increase the sample size and/or to reduce the

Table 4.4 Effects of eight factors

No.	Factors		Effect	
			On R_{M-C}	On R_M
1	τ	minimum allowable in-control ATS_0	-0.0897	23.4586
2	USL	upper specification limit	0.0529	-14.9967
3	Q	production rate	-0.1634	31.5887
4	w	inspection rate per operator	-0.0857	25.1358
5	$MTBO$	mean time between out-of-control cases	0.0529	-14.9967
6	$\mu_{\delta\mu}$	mean value of the mean shifts δ_μ	0.2518	-8.3443
7	C_m	manpower cost per inspector per unit time	0.0529	-14.9967
8	C_k	cost required to fix an out-of-specification unit	-0.1245	27.6343

sampling interval, so that the out-of-control cases can be signaled as quickly as possible and fewer defective units have to be fixed. As a general guideline, a relatively large amount of manpower M should be deployed if (the following points are listed in descending order of their effects on R_M)

- (1) The production rate (Q) is high; and/or
- (2) The cost for fixing an out-of-specification product (C_k) is high; and/or
- (3) The inspection rate (w) of an inspector is high; and/or
- (4) The required in-control ATS_0 (τ) is high; and/or
- (5) The upper specification limit (USL) is small; and/or
- (6) The mean time between out-of-control cases ($MTBO$) is small; and/or
- (7) The manpower cost (C_m) is low; and/or
- (8) The mean value ($\mu_{\delta\mu}$) of the mean shifts is low.

With these guidelines in mind, users can deploy manpower to an $\bar{X}$ chart rationally and effectively, even when more complicated computation has not been carried out.

4.4.5 Example

A manufacturing factory produces a type of part by an injection molding process. The thickness x of the part is a key dimension and specified as $80 \pm 0.006mm$. The mean of x can be easily adjusted to the nominal value ($80mm$) at the center between the lower and upper specification limits. During a pilot run, it is found that the distribution of x can be well approximated by a normal distribution and the standard deviation σ_0 is very close to $0.00152mm$. The process produces 300 parts per hour and a full-time inspector is able to inspect 150 parts per hour. The cost for fixing an out-of-specification part is 5.8 dollars and the hourly salary for an inspector is 8 dollars (the cost pertaining to the measuring equipment is negligible). The control limits should make the in-control ATS_0 equal to about 370 hours. The above specifications are summarized as follows.

USL	$= 80.006mm$, the upper specification limit.
τ	$= 370$ hours, the minimum allowable in-control ATS_0 .
μ_0	$= 80.000mm$, the lower specification limit.
σ_0	$= 0.00152mm$, the in-control standard deviation.
Q	$= 300$, the number of units produced per hour.
C_m	$= \$8.00$, the manpower cost per inspector per hour.
C_k	$= \$5.80$, the cost to fix an out-of-specification unit in the workshop.
w	$= 150$, the number of units inspected by an inspector per hour.

Originally, a conventional 3-sigma $\bar{X}$ chart is used to carry out SPC. An inspector takes a sample of size five for every one hour (the resultant ATS_0 is approximately equal to 370 hours). The amount of manpower used is equal to (Equation (4.35))

$$M_0 = n / (h \times w) = 5 / (1 \times 150) = 0.033$$

That is, the inspector spends about two minutes per hour on SPC inspection.

After running the traditional $\bar{X}$ chart for 1650 hours (excluding the shutdown time), ten out-of-control cases are observed and ten sample values of mean shifts $\hat{\delta}_{\mu,i}$ (i.e. $\hat{\delta}_{\mu,1}, \dots, \hat{\delta}_{\mu,10}$) are collected. The mean $\mu_{\delta\mu}$ of the ten $\hat{\delta}_{\mu,i}$ is equal to 1.389 (Equation (4.5)) and is used as an estimate of the mean $\mu_{\delta\mu}$ of the mean shifts (parametric approach). Meanwhile, the mean time between two consecutive out-of-control cases can be estimated by

$$MTBO \approx \frac{1650}{10} = 165$$

More accurate estimates of $\mu_{\delta\mu}$ and $MTBO$ can be acquired subsequently when more out-of-control cases are detected and dissected.

Now, the QA engineer decides to make use of the estimated $\mu_{\delta\mu}$ and $MTBO$ to design a **MEIIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart and a **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart, in order to reduce the total cost C_{total} . The computer program *meiim.c* completes the design of the **MEIIM** chart in 0.719 second of CPU time in a personal computer (Pentium IV 2.4GHz).

While the **MEIIN** chart is designed by using the same manpower ($M_0 = 0.033$) as the conventional chart, the design of the **MEIIM** chart searches for an optimal

amount of manpower that minimizes the total cost. The charting parameters and the resultant costs of all three charts are listed in Table 4.5. The control limits of the charts are determined so that all three charts generate the same in-control ATS_0 equal to the specified value of 370.

As shown in Table 4.5, the total cost C_{total} of the conventional $\bar{X}$ chart is high. This is because the deployed manpower is inadequate and the resultant quality cost is high. The **MEIIN** chart has substantially reduced the total cost even though it employs the same amount of manpower as the conventional $\bar{X}$ chart. The improvement is attributable to the optimization of the sample size n and the sampling interval h (or the optimal trade-off between a large n and a short h). Finally, the **MEIIM** chart has further cut down the total cost to a significant degree. The design of the **MEIIM** chart includes not only the optimization of n and h , but also the search for the optimal amount of manpower M . The optimal value of M in this example is 0.206, about six times as large as the manpower used by both the conventional and **MEIIN** charts. As such, in order to implement the **MEIIM** chart, some managerial arrangement must be made to increase the operator's time to be spent on SPC in this process. Even though the increase of manpower raises the manpower cost C_{man} , it allows the **MEIIM** chart to use a large sample size and a small sampling interval. As a result, the **MEIIM** chart is very effective for detecting out-of-control cases and significantly reduces the quality cost $C_{quality}$ as well as the total cost C_{total} . In this example, the total cost C_{total} of the **MEIIM** chart is only equal to 0.361 of that of the conventional $\bar{X}$ chart and 0.542 of that of the **MEIIN** chart.

Table 4.5 The three control charts in the example

Chart	Sample size n	Sampling interval, hr h	Control limits		Manpower M	Cost, \$		
			LCL	UCL		C_{man}	$C_{quality}$	C_{total}
Conventional $\bar{X}$ chart	5	1.000	79.99796	80.00204	0.033	0.27	12.10	12.37
MEIN chart	15	2.997	79.99896	80.00104	0.033	0.27	7.96	8.23
MEIM chart	33	1.069	79.99921	80.00079	0.206	1.65	2.82	4.47

4.4.6 Discussions

The model discussed in this section develops the **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart to minimize the expected total cost associated with an SPC scheme by optimizing the deployed manpower. Meanwhile, the sample size, sampling interval, and control limits of the chart are also optimized. The comparative study shows that the **MEIIM** chart is able to reduce the total cost C_{total} by nearly 65% on average, and the optimal deployment of manpower alone may reduce C_{total} by 24%. With such significant improvements, the proposed control chart obviously provides a useful and effective tool for the industry to achieve better quality with limited resource. The design and operation of the **MEIIM** chart is implemented through the three-phase SPC scenario, in which the data for the random mean shifts δ_μ can be collected and the probability distribution model can be built. Some general guidelines provided for deploying manpower to an $\bar{X}$ control chart are also useful to practitioners.

4.5 Optimization design of the integrated $\bar{X}$ control chart system that minimizes the total cost

This section designs the **MEDSM** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system based on the concept developed in Section 4.4. The model **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) discussed in Section 4.4 optimally deploys the manpower to an SPC scheme monitoring a single process. The model **MEDSM** discussed in this section proposes an algorithm for deploying manpower to an SPC system that monitors a multi-stage and multi-stream manufacturing system. The algorithm minimizes the expected total cost by optimizing the amount of allocated manpower in the SPC system. In the mean time, the sample sizes, sampling intervals and control limits of the control charts are also optimized. Similar to the models discussed in Sections 4.3 and 4.4, this design also takes into account the random mean shifts at each process stage. It is modeled by the Rayleigh distribution based on the sample data of the mean shifts acquired in the three-phase SPC scenario. The Taguchi's loss function discussed in Sections 4.3 and 4.4 are used to model the quality costs $C_{quality}$ incurred during the out-of-control cases.

Consider the example in Figure 4.2(a) (similar example has been used in Chapter 3). Four critical dimensions x_i ($i = 1, 2, 3, 4$) are machined in four stages in a manufacturing system. Dimension x_1 is formed by facing (datum: surface 0). Dimension x_2 is machined (datum: surface 0). Dimension x_3 and dimension x_4 are drilled and milled (datum: surface 2). Four control charts as an SPC system are used to carry out the on-line monitoring of the processes. A block diagram of this SPC system is displayed in Figure 4.2(b). The quality linkages are displayed in Figure 4.2(c).

Figure 4.2 (a) Dimensions of the part; (b) Process stages; (c) Quality linkages

4.5.1 Input parameters

The parameters, s , g_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, t_i , Q , $\mu_{\delta\mu,i}$ (mean value of the mean shift $\delta_{\mu,i}$ in the i th stage), $MTBO_i$ (mean time between the out-of-control cases in the i th stage), $C_{k,i}$

(cost required to fix an out-of-specification unit in the workshop in the i th stage), C_m , τ , V_i (vector $[v_1, v_2, \dots]^T$ indicating the cause stage or upstream influencing stage numbers), and Δ_i (induced mean shift. It is the mean shift undergone by the i th stage due to the mean shifts of the cause stages) are required in designing this model (see Table 1.2 for the details of the specifications).

Among the above parameters, s , g_i , USL_i , $\sigma_{0,i}$, and t_i have been discussed and used in Chapter 3 and, the parameters, $\mu_{\delta\mu,i}$, $MTBO_i$, τ , C_m , and $C_{k,i}$ have been discussed for a single process in Section 4.4.

Following the discussion in Section 3.3.1 in Chapter 3, the vectors V_i and the formulae for Δ_i are determined by the quality linkages. The quality linkages in Figure 4.2(c) indicate that both x_3 and x_4 are directly affected by x_2 , and, consequently,

$$V_3 = [2]^T, \quad V_4 = [2]^T \quad (4.38)$$

The formulae for Δ_i can be determined based on the information of design dimensioning and process planning. In Figure 4.2, it is easy to find

$$\Delta_3 = f_3(\delta_{\mu,2}) = \delta_{\mu,2}, \quad \Delta_4 = f_4(\delta_{\mu,2}) = \delta_{\mu,2} \quad (4.39)$$

4.5.2 Optimization model

Based on the above parameters, the design of the **MEDSM** chart system can be conducted by using the following nonlinear optimization model.

$$\text{Minimize:} \quad C_{total} = C_{quality} + C_{man} \quad (4.40)$$

$$\text{Subject to:} \quad ATS_0 \geq \tau \quad (4.41)$$

$$\text{Design variables:} \quad M, n_i, h_i, LCL_i, UCL_i \quad (i = 1, 2, \dots, s)$$

Where the objective function C_{total} is a function of $(s, g_i, USL_i, \mu_{0,i}, \sigma_{0,i}, t_i, Q, \mu_{\delta\mu,i}, MTBO_i, C_{k,i}, C_m, \tau, V_i, \Delta_i, n_i, h_i, LCL_i, \text{ and } UCL_i)$. The parameters n_i, h_i, LCL_i and UCL_i are the sample size, sampling interval, and controls limits in the i th stage. In fact, the charting parameters n_i, h_i, LCL_i , and UCL_i are the same as n, h, LCL , and UCL of a single chart (discussed in Section 4.4) except that all the charting parameters here are designed in an integrative manner. The in-control ATS_0 is calculated by Equation (3.12) derived in Section 3.2.1.3 in Chapter 3.

Both $C_{quality}$ and C_{man} are implicit or explicit functions of manpower M . The manpower cost C_{man} per unit time is the product of M and C_m (see Equation (4.29)).

$$C_{man} = C_m \times M \quad (4.42)$$

The quality cost $C_{quality}$ per unit time is the sum of the costs incurred by the out-of-control cases occurring at each process stage.

$$C_{quality} = \sum_{k=1}^s C_{q,k} \quad (4.43)$$

Where $C_{q,k}$ is the quality cost due to the mean shifts $\delta_{\mu,k}$ at the k th stage. It depends on the occurrence frequency ($1/MTBO_k$) of out-of-control cases, the average number $q_k(\delta_{\mu,k})$ of units produced after a mean shift of size $\delta_{\mu,k}$, the expected loss function $L_k(\delta_{\mu,k})$ per product for a given $\delta_{\mu,k}$, and finally, the distribution of the mean shifts $\delta_{\mu,k}$.

$$\begin{aligned} C_{q,k} &= \int_0^{\infty} \left[\frac{1}{MTBO_k} \cdot q_k(\delta_{\mu,k}) \cdot L_k(\delta_{\mu,k}) \cdot f_{\delta_{\mu}}(\delta_{\mu,k}) \right] d\delta_{\mu,k} \\ &= \int_0^{\infty} \left[\frac{1}{MTBO_k} \cdot Q \cdot ats_k(\delta_{\mu,k}) \cdot L_k(\delta_{\mu,k}) \cdot f_{\delta_{\mu}}(\delta_{\mu,k}) \right] d\delta_{\mu,k} \end{aligned} \quad (4.44)$$

Where $q_k(\delta_{\mu,k}) = Q \cdot ats_k(\delta_{\mu,k})$. The out-of-control $ats_k(\delta_{\mu,k})$ is defined as the average time that any of the control charts in the chart system gives an out-of-control signal after a stream in stage k goes out of control (Woodall and Ncube 1985). $ats_k(\delta_{\mu,k})$ is calculated by Equation (3.52) derived in Section 3.3.1.2 in Chapter 3. The expected

loss function $L_k(\delta_{\mu,k})$ is the sum of the losses incurred in each stage and can be calculated by using Taguchi's loss function (see the discussions in Section 4.3 and Section 4.4).

$$L_k(\delta_{\mu,k}) = \sum_{i=1}^s l_{k,i} \quad (4.45)$$

$$l_{k,i} = \begin{cases} K_i [\sigma_{0,i}^2 + \delta_{\mu,i}^2 \sigma_{0,i}^2] & i \neq k \\ K_k \left[\sigma_{0,k}^2 + \left(\frac{\delta_{\mu,k}}{g_k} \right)^2 \sigma_{0,k}^2 \right] & i = k \end{cases} \quad (4.46)$$

Where $l_{k,i}$ is the contribution from the i th stage when a mean shift $\delta_{\mu,k}$ occurs in a stream of stage k , and K_i is the loss function coefficient for stage i (see Equation (4.47)). It is noted that, when an out-of-control case occurs in stage k , mean shifts will be incurred not only in stage k , but also in some other stages that are directly or indirectly affected by stage k . In Equation (4.46), $\delta_{\mu,i}$ (when $i \neq k$) is the mean shift induced by the out-of-control case in stage k . If stage i is independent of stage k , $\delta_{\mu,i} = 0$. The determination of the induced mean shifts $\delta_{\mu,i}$ is detailed in Section 3.3.1.2 in Chapter 3. It is also noted that, in stage k , only one out of the g_k streams goes out of control. Therefore, the equivalent mean shift of the entire stage k is equal to $(\delta_{\mu,k} / g_k)$.

The loss function coefficient K_i in the i th stage (in Equation (4.46)) that is used to calculate the loss function can be determined from the cost component $C_{k,i}$ associated with the specification limits (referring to Equation (4.34)).

$$K_i = \frac{C_{k,i}}{(\text{USL}_i - \mu_{0,i})^2} \quad (4.47)$$

Finally, the random mean shifts $\delta_{\mu,k}$ in stage k can be modeled by the Rayleigh distribution based on the sample data of the mean shifts acquired in the three-phase

scenario (parametric approach). According to Equation (4.4), the probability density function $f_{\delta\mu}(\delta_{\mu,k})$ of the Rayleigh distribution at stage k is given by

$$f_{\delta\mu}(\delta_{\mu,k}) = \frac{\pi\delta_{\mu,k}}{2\mu_{\delta\mu,k}^2} \exp\left(-\frac{\pi\delta_{\mu,k}^2}{4\mu_{\delta\mu,k}^2}\right) \quad (4.48)$$

Which is characterized by a single parameter $\mu_{\delta\mu,k}$, that is, the mean value of the mean shifts $\delta_{\mu,k}$. Non-parametric approach can also be used to evaluate the quality cost $C_{q,k}$.

$$C_{q,k} = \frac{1}{N} \sum_{k=1}^N \left[\frac{1}{MTBO_k} \cdot q_k(\hat{\delta}_{\mu,k}) \cdot L_k(\hat{\delta}_{\mu,k}) \right] \quad (4.49)$$

The calculation of C_{total} can be summarized as follows.

- (1) Calculate manpower cost C_{man} by Equation (4.42).
- (2) Calculate quality cost $C_{quality}$ by summing up the contribution ($C_{q,k}$) from each stage using Equation (4.43). For a particular stage k , the contributed $C_{q,k}$ (in Equation (4.44)) can be evaluated by applying the Gauss-Legendre translation rule to a few integration points. At each of these points
 - (2.1) Calculate $ats_k(\delta_{\mu,k})$ by Equation (3.52) derived in Section 3.3.1.2 in Chapter 3.
 - (2.2) Calculate the loss function $L_k(\delta_{\mu,k})$ by Equations (4.45) and (4.46) (the values of $\delta_{\mu,i}$ are determined according to the discussion in Section 3.3.1.2 in Chapter 3).
 - (2.3) Calculate the density function $f_{\delta\mu}(\delta_{\mu,k})$ by Equation (4.48).
- (3) Finally, calculate the total cost C_{total} by Equation (4.40).

The above formulae for calculating C_{total} has been verified by 300 different cases using Monte Carlo simulation. The average relative discrepancy is only 0.0316.

4.5.3 Optimization search algorithm

The optimal values of M , n_i , h_i , LCL_i , and UCL_i are sought in a three-level optimization search. The high-level optimization is a single-variable search for the optimal value of M using the Successive Quadratic Estimation method (Powell 1964). Then, for a value of M determined in the high level, a gradient based mid-level search is employed to find the optimal values of n_i and h_i (see the gradient based search algorithm discussed in Section 3.2.2 in Chapter 3 and note that ΔATS_j in Equation (3.40) will be replaced by $\Delta C_{total,j}$ in this case). Finally, a dynamic low-level search (see the dynamic search algorithm discussed in Section 3.2.1 in Chapter 3) is invoked to find the optimal LCL_i and UCL_i . They jointly make the resultant ATS_0 exactly equal to the specified value τ and, in the mean time, minimize the total cost C_{total} for a set of values of (M, n_i, h_i) that are determined in the high and middle levels. While the high-level optimization makes a trade-off between the quality cost $C_{quality}$ and the manpower cost C_{man} , the mid-level optimization trades off between the sample size n_i and the sampling interval h_i , and the low-level optimization tightens the control limits of some charts and loosens that of others. Ultimately, all the three levels aim at minimizing the total cost C_{total} .

The overall design procedures are outlined below.

- (1) Initialize C_{min} as a very large number, say 10^7 (C_{min} is used to store the minimum value of C_{total}).
- (2) At the high level, the optimal value of M is searched.
- (3) At the mid-level, for a value of M that is determined in step (2), the optimal values of n_i and h_i are searched.

- (4) The low-level search optimizes LCL_i and UCL_i that ensure ATS_0 equal to τ . For each given set of $(M, n_i, h_i, LCL_i, UCL_i)$ ($i = 1, \dots, s$)
 - (4.1) Calculate the total cost C_{total} by using the aforementioned procedure (discussed in Section 4.5.2).
 - (4.2) If the calculated C_{total} is smaller than the current C_{min} , replace the latter by the former. The current values of M, n_i, h_i, LCL_i , and UCL_i are stored as the temporary optimal solution.
- (5) When the entire search is completed, the final C_{min} is the minimum C_{total} and the corresponding optimal M, n_i, h_i, LCL_i , and UCL_i are also finalized.

4.5.4 Comparative study

Similar to the study in Section 4.4.4, this section also compares the performance (measured by the total cost C_{total}) of the **MEDSM** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system with that of a traditional $\bar{X}$ chart system as well as a **MEDSN** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits) chart system. Like the **MEDSM** chart system, the **MEDSN** chart system is designed to minimize the total cost C_{total} , but it only has its charting parameters n_i, h_i, LCL_i , and UCL_i optimized (namely, only the middle and low level optimization searches are conducted) and uses a fixed amount of manpower M_0 as used by the traditional $\bar{X}$ chart system (that is, the high-level optimization is omitted). It is also assumed that each chart in the traditional $\bar{X}$ chart system uses a sample size of five, an identical sampling interval, and the 3-sigma control limits.

The performance of the chart systems is examined over 100 runs (or cases). For each of the 100 runs, the values of the four system parameters s , τ , Q and C_m are first decided randomly between the lower and upper bounds below.

s	[2, 10]
τ	[50, 300]
Q	[50, 300]
C_m	[5, 20]

The values of other seven parameters at each of the s stages are generated, also randomly, within the following ranges.

g_i	[1, 3]
$\sigma_{0,i}$	[1, 3]
$\mu_{\delta\mu,i}$	[0.5, 1.5]
USL_i	$[2.5\sigma_{0,i}, 6\sigma_{0,i}]$
$MTBO_i$	$[0.25\tau, \tau]$
$C_{k,i}$	$[0.05C_m, C_m]$

Noteworthily, some parameters are expressed as a function of other parameters. The dependence between the stages (i.e. the vectors V_i and the induced mean shift functions Δ_i) is also randomly decided. Each stage may have up to two dependent stages and Δ_i is equal to either $(+\delta_{vi})$ or $(-\delta_{vi})$. The in-control $\mu_{0,i}$ is fixed at zero.

The fixed amount of manpower M_0 used by both the traditional and **MEDSN** chart systems is determined by Equation (3.33) derived in Chapter 3.

$$M_0 = \sum_{i=1}^s \frac{n_i g_i t_i}{h_i} \quad (4.50)$$

Where $(n_i \equiv 5)$ is the sample size used by each chart of the traditional $\bar{X}$ chart system. The identical sampling interval h_i at i th stage can be determined by combining Equations (3.11) and (3.12) (derived in Section 3.2.1.3 in Chapter 3) and replacing ATS_0 by τ .

$$h_i = \frac{\alpha}{1 - (1 - 1/\tau)^{1/G}} \quad (4.51)$$

Where $\alpha = 2 \cdot \Phi(-3)$ is the type I error probability for 3-sigma control limits and G is the total number of processes in the manufacturing system as defined in Equation (3.1) in Chapter 3. This value of h_i ensures that the in-control ATS_0 of the traditional $\bar{X}$ chart system is equal to τ .

For each of the 100 runs, the traditional $\bar{X}$ chart system, the **MEDSN** chart system and the **MEDSM** chart system are designed. All of them always generate the same in-control ATS_0 (equal to τ). The following three responses are investigated for each run.

- R_{M-C} the ratio between the total cost of the **MEDSM** chart system and that of the traditional $\bar{X}$ chart system.
- R_{S-C} the ratio between the total cost of the **MEDSN** chart system and that of the traditional $\bar{X}$ chart system.
- R_M the ratio between the optimal manpower M of the **MEDSM** chart system and the manpower M_0 for the **MEDSN** and traditional $\bar{X}$ chart systems.

The grand averages, $\bar{R}_{M-C}$, $\bar{R}_{S-C}$ and $\bar{R}_M$ of the three responses over the 100 runs are 0.246, 0.745 and 37.35, respectively. The value of $\bar{R}_{S-C}$ indicates that, from an overall viewpoint (considering different combinations of s , τ , Q , C_m , g_i , $\sigma_{0,i}$, $\mu_{\delta\mu,i}$, USL_i , t_i , $MTBO_i$, and $C_{k,i}$), the **MEDSN** chart system reduces the total cost C_{total} by about 25% compared to the traditional $\bar{X}$ chart system. This improvement is achieved by the optimization of the charting parameters n_i , h_i , LCL_i , and UCL_i . The value of $\bar{R}_{M-C}$ shows that the **MEDSM** chart system further reduces C_{total} substantially (the total cost C_{total} of the **MEDSM** chart system is, on average, equal to about one quarter of that of the traditional $\bar{X}$ chart system). This further reduction in C_{total} is completely attributable to the optimal deployment of manpower in the **MEDSM** chart system.

Next, the value of the grand average $\bar{R}_M$ ($= 37.35$) of the third response shows that the traditional $\bar{X}$ chart system usually uses a far less amount of manpower than needed for the minimization of the total cost. The deployment of more manpower leads to a significant reduction in quality cost that outweighs the increase in manpower cost.

The effects of the eleven parameters (s , τ , Q , C_m , g_i , $\sigma_{0,i}$, $\mu_{\delta\mu,i}$, USL_i , t_i , $MTBO_i$, $C_{k,i}$) on the two responses $\bar{R}_{M-C}$ and $\bar{R}_M$ are also calculated and listed in Table 4.6. The effect of a parameter on a response is defined as the difference between R_u and R_l , which are the average values of the response when the parameter takes its upper and lower bound values, respectively. A positive effect on R_{M-C} means that the **MEDSM** chart system will achieve a more significant cost reduction when the corresponding parameter is smaller and *vice versa*. For example, parameter $\mu_{\delta\mu,i}$ has a significant positive impact (0.2514) on R_{M-C} . Thus, when $\mu_{\delta\mu,i}$ is smaller, the **MEDSM** chart system will reduce the total cost to a greater degree. The effects of the parameters on

Table 4.6 Effects of eleven factors

No.	Factors	Effect	
		On R_{M-C}	On R_M
1	τ minimum allowable in-control ATS_0	-0.1213	19.01
2	Q number of units produced per unit time	-0.3407	63.31
3	C_m manpower cost per inspector per unit time	-0.0580	-6.52
4	s number of stages in a manufacturing system	-0.3087	50.28
5	g_i number of streams in the i th stage	-0.0657	11.49
6	$\sigma_{0,i}$ in-control process standard deviation in the i th stage	-0.0286	8.22
7	$\mu_{\delta\mu,i}$ mean value of the mean shifts δ_i in the i th stage	0.2514	-8.70
8	USL_i upper specification limit in the i th stage	0.0482	-15.64
9	t_i time required to inspect a unit in the i th stage	0.1975	-24.70
10	$MTBO_i$ mean time between out-of-control cases in the i th stage	-0.0030	-2.33
11	$C_{k,i}$ cost required to fix an out-of-specification unit in the i th stage	-0.0926	15.34

R_{M-C} indicate that the superiority of the **MEDSM** chart system over the traditional $\bar{X}$ chart system is more notable when τ , Q , C_m , s , g_i , $\sigma_{0,i}$, $MTBO_i$, $C_{k,i}$ are high and/or $\mu_{\delta\mu,i}$, USL_i , t_i are low.

On the other hand, a positive effect on R_M means that the amount of manpower should be increased more when the corresponding parameter is large and *vice versa*. For example, the production rate Q has the most positive effect (63.31) on R_M . Therefore, more manpower should be allocated in order to increase the sample size and/or reduce the sampling interval when the production rate Q is high. In sum, a relatively large amount of manpower M should be deployed if (the parameters are listed in a descending order according to their effects on R_M):

- (1) The production rate (Q) is high; and/or
- (2) The number of stages of the manufacturing system (s) is high; and/or
- (3) The time required to inspect a unit (t_i) is small; and/or
- (4) The required in-control ATS_0 (τ) is high; and/or
- (5) The upper specification limit (USL_i) is small; and/or
- (6) The cost for fixing an out-of-specification product ($C_{k,i}$) is high; and/or
- (7) The number of streams (g_i) is high; and/or
- (8) The mean value ($\mu_{\delta\mu,i}$) of the mean shifts is low; and/or
- (9) The in-control process standard deviation ($\sigma_{0,i}$) is high; and/or
- (10) The manpower cost (C_m) is low; and/or
- (11) The mean time between out-of-control cases ($MTBO_i$) is low.

4.5.5 Example

Consider the mechanical part shown in Figure 4.2(a). Stage 1 and stage 2 turn surface 1 and surface 2 (datum: surface zero), respectively; stage 3 drills the hole (datum: surface 2); and stage 4 mills the slot (datum: surface 2). The block diagram of the SPC system is displayed in Figure 4.2(b) and the quality linkages in Figure 4.2(c). The specifications of the system are listed below.

Number of stages:	$s = 4$
Number of streams:	$g_1 = g_2 = g_3 = 1, g_4 = 2$
Lower specification limit (mm):	$LSL_1 = 15.91, LSL_2 = 12.93, LSL_3 = 7.87, LSL_4 = 10.92$
Upper specification limit (mm):	$USL_1 = 16.09, USL_2 = 13.07, USL_3 = 8.13, USL_4 = 11.08$
Standard deviation (mm):	$\sigma_{0,1} = 0.025, \sigma_{0,2} = 0.022, \sigma_{0,3} = 0.049, \sigma_{0,4} = 0.028$
Production rate (per hour):	$Q = 120$
Fixing cost (\$):	$C_{k,1} = 4.2, C_{k,2} = 5.3, C_{k,3} = 6.0, C_{k,4} = 1.5$
Manpower cost (\$):	$C_m = 10.0$
Inspection time (min):	$t_1 = 0.30, t_2 = 0.50, t_3 = 0.60, t_4 = 0.45$

The dependent relationship vectors V_i and the formulae for Δ_i can be determined by simply checking the drawing and quality linkages in Figure 4.2.

$$V_3 = [2]^T, \quad V_4 = [2]^T, \quad \Delta_3 = \delta_{\mu,2}, \quad \Delta_4 = \delta_{\mu,2}$$

Originally, the system uses the traditional 3-sigma $\bar{X}$ charts for all the processes. Operators take a sample of size five for every two hours for each chart. The amount of manpower M_0 that is used for the system is equal to 0.096 (Equation (4.50)). That is, on average, about 6 minutes are spent on SPC inspection per hour. The in-

control ATS_0 of the traditional 3-sigma $\bar{X}$ chart system is equal to 149 hours (Equation (3.12) in Chapter 3).

During the operation of this traditional $\bar{X}$ chart system, ten sample values $\hat{\delta}_{\mu,i}$ (calculated by Equation (4.3)) of the mean shift $\delta_{\mu,1}$ at stage 1 are observed and recorded following the three-phase SPC scenario discussed in Section 4.2. The mean $\mu_{\delta_{\mu,1}}$ of these ten sample values $\hat{\delta}_{\mu,i}$ is 1.45 (Equation (4.5)) and the mean time between two consecutive out-of-control cases at stage 1 is,

$$MTBO_1 \approx \frac{1000}{10} = 100$$

Similarly, $\mu_{\delta_{\mu,i}}$ and $MTBO_i$ are determined for other stages.

$$\mu_{\delta_{\mu,2}} = 0.93, \mu_{\delta_{\mu,3}} = 1.35, \mu_{\delta_{\mu,4}} = 1.51$$

$$MTBO_2 = 122 \text{ hr}, MTBO_3 = 87 \text{ hr}, MTBO_4 = 94 \text{ hr}$$

Now, the QA engineer decides to design a **MEDSN** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits) chart system and a **MEDSM** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system in order to reduce the total cost C_{total} . The computer program *medsm.c* completes the design of the **MEDSM** chart system in 6.093 seconds of CPU time in a personal computer (Pentium IV 2.4GHz).

While the **MEDSN** chart system is designed by using the same manpower ($M_0 = 0.096$) as the traditional chart system, the design of the **MEDSM** chart system searches for an optimal amount of manpower that minimizes the total cost. The charting parameters and the resultant costs of the three chart systems are listed in Table 4.7. The control limits of the charts are determined so that all three chart systems generate the same in-control ATS_0 ($= 149$).

Table 4.7 The three control chart systems in the example

System	Stage	Sample size n	Sampling interval (hr) h	Control limits (mm)		Manpower M	Cost (\$)		
				LCL	UCL		C_{man}	$C_{quality}$	C_{total}
Traditional $\bar{X}$ chart system	1	5	2.00	15.967	16.033	0.096	0.96	147.83	148.79
	2	5	2.00	12.969	13.031				
	3	5	2.00	7.937	8.063				
	4	5	2.00	10.962	11.038				
MEDSN chart system	1	9	1.82	15.974	16.026	0.096	0.96	112.43	113.39
	2	3	3.19	12.957	13.043				
	3	7	2.67	7.949	8.051				
	4	9	3.65	10.974	11.026				
MEDSM chart system	1	67	1.96	15.991	16.009	1.266	12.66	29.20	41.86
	2	65	2.02	12.990	13.010				
	3	67	2.02	7.983	8.017				
	4	67	2.02	10.990	11.010				

As shown in Table 4.7, the total cost C_{total} of the traditional $\bar{X}$ chart system is high. This is because that the manpower deployed to the traditional system is inadequate and the resultant quality cost is very high. The **MEDSN** chart system has reduced the total cost to some degree even though it employs the same amount of manpower as the traditional $\bar{X}$ chart system. The improvement is attributable to the optimization of the sample size n_i , the sampling interval h_i , and the control limits LCL_i and UCL_i . Finally, the **MEDSM** chart system has further cut down the total cost to a significant degree. The design of the **MEDSM** chart system includes not only the optimization of n_i , h_i , LCL_i , and UCL_i , but also the search for the optimal amount of manpower M . The optimal value of M in this example is 1.266, about thirteen times as large as the manpower used by both the traditional and **MEDSN** chart systems. Even though the increase of manpower raises the manpower cost C_{man} , it allows the **MEDSM** chart system to use large sample sizes and small sampling intervals. As a result, the **MEDSM** chart system is very effective for detecting out-of-control cases and significantly reduces the quality cost $C_{quality}$ as well as the total cost C_{total} . In this example, the total cost C_{total} of the **MEDSM** chart system is equal to only 0.281 of that of the traditional $\bar{X}$ chart system and 0.369 of that of the **MEDSN** chart system.

4.5.6 Discussions

This section develops the **MEDSM** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system from an economic viewpoint. The design algorithm optimizes the deployed manpower in order to minimize the expected total cost associated with the SPC system monitoring the multi-stage and multi-stream manufacturing system. The sample sizes, sampling intervals and control limits of the charts in the chart system are also optimized in the mean time. The numerical studies show that the

MEDSM chart system is able to reduce the total cost C_{total} by nearly 75% on average, and the optimal deployment of manpower alone may reduce C_{total} by about 50%. Some general guidelines are provided for the deployment of manpower to an $\bar{X}$ control chart system monitoring a multi-stage and multi-stream manufacturing system.

4.6 Calculation of the quality costs based on defective rate

A successful control chart will signal the out-of-control cases promptly, so that the number of defectives (scraps or/and reworks) can be reduced. Defectives are the primary productivity-killers in any operation. If scrap and rework are reduced, then the productivity increases, cost decreases, and production capacity (measured in the number of good parts per hour) increases (Montgomery 2001). In manufacturing, loss function expresses the economic consequences associated with the deviation from the target value. However, it may be inappropriate to use the simple quadratic loss function to describe the relationship between the quality cost and the deviation from the target for some real applications. Moreover, the current industrial practice still prefers to use the design specifications to measure the conformance of the products. In other words, quality characteristics are often evaluated against specifications (Montgomery 2001). For such circumstances the defective rate $D(\delta_\mu)$ instead of Taguchi's loss function $L(\delta_\mu)$ can be adopted to measure the quality costs and used as the objective function in Sections 4.3, 4.4 and 4.5. All other formulation and designs procedures remain the same.

The defective rate $D(\delta_\mu)$ for a given mean shift value δ_μ is the probability that the quality characteristics x falls beyond the lower specification limit LSL or the upper specification limit USL .

$$D(\delta_\mu) = 1 - \Phi\left(\frac{USL - (\mu_0 + \delta_\mu \sigma_0)}{\sigma}\right) + \Phi\left(\frac{LSL - (\mu_0 + \delta_\mu \sigma_0)}{\sigma}\right) \quad (4.52)$$

4.7 Summary

This chapter designs the **MEIIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart, the **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart and the **MEDSM** ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system from an economic viewpoint. The first two models are designed for monitoring a single process, whereas the third model presents the economical designs of the integrated $\bar{X}$ control chart systems for monitoring a multi-stage and multi-stream manufacturing system. Unlike the statistical designs, the design algorithms of the proposed models directly link the operating characteristics of the chart systems with the economic criterion. However, different from the conventional economical designs (in which many input costs have to be determined), the design specifications of the proposed models are relatively easy to handle. Most of the specifications that are used in the designs of the proposed models are also required by the traditional system or can be obtained from design and manufacturing information and records. Furthermore, the design algorithms take into consideration the whole probability distribution of the mean shifts that are modeled through the three-phase SPC scenario. The quality costs are estimated using Taguchi's loss function. However, some discussions are also presented for using the defective rate to measure the quality costs. The last two models (**MEIIM** chart and **MEDSM** chart system) minimize the expected total SPC cost via

the optimal deployment of manpower. It is first time that the manpower has been included in the designs. It is found that much more manpower should be deployed to most SPC schemes from the viewpoint of total cost reduction.

The proposed design algorithms can be easily computerized. More importantly, the proposed models do not bring forth any additional difficulty for operators to run and understand the chart systems.

Finally, the general guidelines for allocating manpower to a particular application are useful and universally applicable.

Chapter 5

Statistical Designs of Integrated $\bar{X}$ & S Control Chart Systems for Monitoring Process Mean and Variance

This chapter develops the **VSISL** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / control limits) and **VSISN** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) chart systems for monitoring process shifts in both mean and variance in a multi-stage and multi-stream manufacturing system. These chart systems consist of several pairs of $\bar{X}$ & S charts, each of which monitors one of the critical quality characteristics (e.g. dimensions) of a product. The design algorithm optimally allocates the detection power of the system among different stages as well as between the $\bar{X}$ chart and S chart within each stage based on the values of certain parameters (e.g. process capability, magnitudes of process shifts) that would affect the performance of the chart system. Consequently, the performance characteristics of the system as a whole can be considerably improved and the product quality will be further enhanced. Such improvement is achieved without requiring additional cost and effort for inspection. Furthermore, the floor operators can implement and understand the proposed chart system as easily as for the conventional system. The results from the comparative study show that the integrated **VSISL** and **VSISN** chart systems are significantly superior to the conventional $\bar{X}$ & S chart systems. Some useful guidelines are also brought forth to aid the users to adjust the sample sizes, sampling intervals and control limits of the $\bar{X}$ & S charts in a system.

5.1 Introduction

Shewhart 3-sigma $\bar{X}$ & S (or $\bar{X}$ & R) control charts is widely used in industry to monitor process shifts in SPC. Usually, the sample size n is chosen as 4, 5, or 6. However, when monitoring critical processes or detecting small shifts, the value of n needs to be moderately large. For such cases, the R chart loses statistical efficiency and the S chart is preferable.

There are a few statistical and economic models (see Chapter 2) for the optimization designs of the $\bar{X}$ & S or $\bar{X}$ & R charts including Costa's (1999) optimization design that considered only one pair of mean shift δ_μ and standard deviation shift δ_σ , and Saniga's (1989) method that allowed the specification of process shifts at one expected level and several supplementary levels. However, it may be difficult for the users to specify critical pairs of δ_μ and δ_σ at multiple levels.

The models proposed in this chapter develop the integrated $\bar{X}$ & S control chart system (**VSISL** and **VSISN** chart systems) to monitor the mean and variance of the processes in different stages in a multi-stage and multi-stream manufacturing system. The in-control and out-of-control system conditions are related to the process capability index C_{pk} (Wu *et al.* 2002). The index C_{pk} contains more managerial meaning than the process shifts δ_μ and δ_σ . The QA engineers, especially, as well as the operators and managers at different levels, are more familiar with the process capability index C_{pk} and feel more comfortable to specify and manipulate it.

The optimization design algorithms developed in this chapter are similar to that of the chart systems monitoring only mean shifts δ_μ (discussed in Chapter 3). The difference here is in the calculation of the power z_i of the chart system that handle process shifts in both mean δ_μ and variance δ_σ and decomposes the type I error probability α to $\alpha_{\bar{x}}$ (for the $\bar{X}$ chart) and α_s (for the S chart).

The remaining sections in this chapter are organized as follows.

- Section 5.2 develops the **VSISL** chart system that only optimizes the control limits of each chart in the chart system.
- Section 5.3 presents the **VSISN** chart system that optimizes the sample size, sampling interval, and control limits all together.
- Section 5.4 analyzes the performance of the models through comparative study.
- Section 5.5 showcases the performance of the proposed models through an example.
- Finally, Section 5.6 presents the summary of the models that are developed in this chapter.

5.2 Integrated $\bar{X}$ &S chart system with optimal control limits

The section designs the **VSISL** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / control limits) chart system that optimizes the control limits of each chart in the chart system by considering the process shifts both in mean and variance. The sample sizes and the sampling intervals are fixed. The objective is to speed up the signalling of the out-of-control cases, on condition that the false alarm rate of the system is not greater than a specified value.

5.2.1 Input parameters

The parameters s , g_i , n_i , h_i , LSL_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, τ , p_i (probability of occurrence of the out-of-control cases in the i th stage), and $c_{min,i}$ (out-of-control (or critical) value of the process capability index in the i th stage) are required in this design.

Among these parameters, s , g_i , h_i , n_i , LSL_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, τ , and p_i have been discussed and used in Chapter 3.

In this model, the process status is related to a single capability index C_{pk} instead of two parameters δ_μ and δ_σ (Wu *et al.* 2002)

$$C_{pk} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right) \quad (5.1)$$

A single C_{pk} can oversee both the centering and spread of a quality characteristic with reference to specification limits. When process shifts (δ_μ and/or δ_σ) take place due to some assignable causes, C_{pk} will decrease. As C_{pk} decreases to a critical value c_{min} , the product quality will degrade significantly and the *PPM* value of the product will increase to an intolerable level. Thus, the process is considered out of control and a signal should be produced quickly when $C_{pk} = c_{min}$.

5.2.2 Optimization model

Based on the above parameters, the design of the **VSISL** chart system can be conducted by using the following optimization model.

$$\text{Objective function: } T = \text{minimum} \quad (5.2)$$

$$\text{Constraint function: } ATS_0 \geq \tau \quad (5.3)$$

$$\text{Design variables: } LCL_{\bar{x},i}, UCL_{\bar{x},i}, UCL_{s,i} \ (i = 1, \dots, s)$$

Where the objective function T is a function of $(s, g_i, n_i, h_i, LSL_i, USL_i, \mu_{0,i}, \sigma_{0,i}, \tau, p_i, c_{min}, LCL_{\bar{x},i}, UCL_{\bar{x},i}, \text{ and } UCL_{s,i})$. $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$ are the lower and upper control limits of the $\bar{X}$ charts and $UCL_{s,i}$ is the upper control limit of the S charts in the i th stage. Since control charts are most often used to detect a deterioration in process

quality (Reynolds and Glosch 1981), only the increasing standard deviation shift $\delta_{\sigma,i}$ is handled in this model and the lower control limit $LCL_{s,i}$ of the S chart is fixed as zero. It will enhance the power of the S chart for detecting the increasing hazardous $\delta_{\sigma,i}$ shifts. The in-control ATS_0 of the chart system in constraint (5.3) is calculated by Equation (3.12) derived in Section 3.2.1.3 in Chapter 3. The objective function T is called the *average informative time*. Like the out-of-control ATS , the smaller is T , the more effective is the chart system in detecting out-of-control cases.

$$T = \sum_{i=1}^s \left(\frac{1}{z_i} - 0.5 \right) h_i \cdot p_i \quad (5.4)$$

Where z_i is called the *informative power* of stage i , that is, the power generated by the $\bar{X}$ & S charts in a stream (or machine) in stage i when an out-of-control case happens in this stream (or more specifically, when $C_{pk,i}$ in this stream decreases to $c_{min,i}$). The average informative time T is equal to the out-of-control ATS of the system if the informative power z_i is replaced by the overall power Z_i . When an out-of-control case occurs in a particular stream (the *out-of-control stream*) in stage i , the overall power Z_i of the chart system is the union of z_i and $z_{i,mislead}$.

$$Z_i = z_i \cup z_{i,mislead} \quad (5.5)$$

Where z_i is the power generated by the out-of-control stream where mean shift $\delta_{\mu,i}$ and/or standard deviation shift $\delta_{\sigma,i}$ takes place. The misleading power $z_{i,mislead}$ is the union of the power generated by the $\bar{X}$ & S charts in other streams and/or other stages rather than the out-of-control stream. The misleading power $z_{i,mislead}$ mainly results from the dependency between the quality characteristics x_i . The consideration of both the induced mean shifts and the standard deviation shifts ($\delta_{\mu,i}$ and $\delta_{\sigma,i}$) pertaining to the interdependency of the process stages makes the formulation very complicated. The design procedure also becomes too complex that may discourage the QA engineers to

use the model. Furthermore, the general influence of $z_{i,mislead}$ on the performance of the chart system may be negative because it may adversely mislead the follow-up investigation of the assignable causes and increase the Mean Time To Repair (MTTR). In view of this, the optimization model proposed in this section will make no effort to increase $z_{i,mislead}$ and the objective function is the informative time T (considering only the informative power z_i) rather than the Average Time to Signal ATS (considering both the informative power and the misleading power).

5.2.3 Calculation of the informative power z_i of a stream in stage i

For a pair of mean shift $\delta_{\mu,i}$ and standard deviation shift $\delta_{\sigma,i}$ (expressed in terms of $\sigma_{0,i}$)

$$\mu_i = \mu_{0,i} + \delta_{\mu,i}\sigma_{0,i}, \quad \sigma_i = \delta_{\sigma,i}\sigma_{0,i} \quad (5.6)$$

When the process is in control, $\delta_{\mu,i} = 0$ and $\delta_{\sigma,i} = 1$.

There are infinite number of pairs of $(\delta_{\mu,i}, \delta_{\sigma,i})$ that jointly reduce the process capability index $C_{pk,i}$ from the in-control value to the out-of-control value $c_{min,i}$. In other words, a given $c_{min,i}$ may have resulted from infinite number of different combinations of $\delta_{\mu,i}$ and $\delta_{\sigma,i}$ (e.g. large $\delta_{\mu,i}$ and small $\delta_{\sigma,i}$, small $\delta_{\mu,i}$ and large $\delta_{\sigma,i}$, or moderate $\delta_{\mu,i}$ and $\delta_{\sigma,i}$). To evaluate the informative power z_i , we cannot arbitrary adopt a single pair of $\delta_{\mu,i}$ and $\delta_{\sigma,i}$, instead, we must take all possible combination of $(\delta_{\mu,i}, \delta_{\sigma,i})$ into consideration. A factor λ ($0 \leq \lambda \leq 1$) is used to take all of these combinations into account. When $\lambda = 0$, $\delta_{\sigma,i}$ is equal to its smallest value one and $\delta_{\mu,i}$ gets its maximum value $\delta_{\mu,i,max}$.

$$\delta_{\mu,i,max} = \frac{USL_i - \mu_{0,i} - 3c_{min,i}\sigma_{0,i}}{\sigma_{0,i}} \quad (5.7)$$

On the other hand, when $\lambda = 1$, $\delta_{\mu,i}$ is equal to zero and $\delta_{\sigma,i}$ gets its maximum value $\delta_{\sigma,i,max}$.

$$\delta_{\sigma,i,\max} = \frac{USL_i - \mu_{0,i}}{3c_{\min,i}\sigma_{0,i}} \quad (5.8)$$

Apart from above two extreme cases, $\delta_{\mu,i}$ and $\delta_{\sigma,i}$ are determined by a linear interpolation when $0 < \lambda < 1$.

$$\begin{aligned} \delta_{\mu,i}^2(\lambda) &= (1-\lambda)C\delta_{\mu,i,\max}^2 \\ \delta_{\sigma,i}^2(\lambda) &= \lambda C\delta_{\sigma,i,\max}^2 \end{aligned} \quad (5.9)$$

$$C = \left[\frac{USL_i - \mu_{0,i}}{\sigma_{0,i}(3c_{\min,i}\sqrt{\lambda}\delta_{\sigma,i,\max} + \sqrt{1-\lambda}\delta_{\mu,i,\max})} \right]^2 \quad (5.10)$$

Where the common parameter C ensures that $\delta_{\mu,i}$ and $\delta_{\sigma,i}$ jointly make $C_{pk,i}$ exactly equal to $c_{\min,i}$. The interpolation is applied to the squares of $\delta_{\mu,i}$ and $\delta_{\sigma,i}$ (rather than $\delta_{\mu,i}$ and $\delta_{\sigma,i}$ themselves), because the sum of $\delta_{\mu,i}^2$ and $\delta_{\sigma,i}^2$ is equal to the mean value of the loss function, except for a constant (Cyrus 1997). For a given value of λ , the power $z_{\bar{x},i}(\lambda)$ of the $\bar{X}$ chart in the i th stage is

$$z_{\bar{x},i}(\lambda) = 1 - \left\{ \Phi \left[\frac{UCL_{\bar{x},i} - (\mu_{0,i} + \delta_{\mu,i}(\lambda)\sigma_{0,i})}{\delta_{\sigma,i}(\lambda)\sigma_{0,i}/\sqrt{n_i}} \right] - \Phi \left[\frac{LCL_{\bar{x},i} - (\mu_{0,i} + \delta_{\mu,i}(\lambda)\sigma_{0,i})}{\delta_{\sigma,i}(\lambda)\sigma_{0,i}/\sqrt{n_i}} \right] \right\} \quad (5.11)$$

The power $z_{s,i}(\lambda)$ of the corresponding S chart is

$$z_{s,i}(\lambda) = 1 - \chi_{n_i-1}^2 \left[\frac{n_i-1}{(\delta_{\sigma,i}(\lambda)\sigma_{0,i})^2} \times UCL_{s,i}^2 \right] \quad (5.12)$$

The power $z_{\bar{x}\&s,i}(\lambda)$ of the joint $\bar{X}$ and S charts is

$$z_{\bar{x}\&s,i}(\lambda) = z_{\bar{x},i}(\lambda) + z_{s,i}(\lambda) - z_{\bar{x},i}(\lambda)z_{s,i}(\lambda) \quad (5.13)$$

Finally, the informative power of stage i is equal to the following integration over the whole range of λ .

$$z_i = \int_0^1 z_{\bar{x}\&s,i}(\lambda) f(\lambda) d\lambda \quad (5.14)$$

Where $f(\lambda)$ is the probability density function of λ and is equal to one (assuming a uniform distribution). Equation (5.14) takes into account of all the possible combinations of $\delta_{\mu,i}$ and $\delta_{\sigma,i}$ that correspond to the out-of-control condition ($C_{pk,i} = c_{min,i}$).

5.2.4 Optimization search algorithm

The optimization search is carried out in two levels. The high-level search is to distribute the detection power of the chart system among the s stages and the low-level search is to allocate the power between the $\bar{X}$ chart and S chart in a stream within each stage.

The high-level search is carried out by using the dynamic search algorithm discussed in Section 3.2.1 in Chapter 3. In this algorithm, the optimal values of α_i (α_i is the type I error probability of a stream in the i th stage) of the first $(s-1)$ control charts are searched step by step in $(s-1)$ levels. The last α_s is finally determined so that the resultant ATS_0 is exactly equal to the specified τ . The purpose is to identify the optimal set of $(\alpha_1, \dots, \alpha_s)$ so that the average informative time T will be minimized and ATS_0 will equal to the specification τ . The underlying idea is to enhance the power (or increase α_i) of the $\bar{X}$ & S charts in some stages and sacrifice the power of the $\bar{X}$ & S charts in other stages in order to improve the overall (average) effectiveness of the control chart system.

In the low-level, for a set of $(\alpha_1, \dots, \alpha_s)$ determined in the high level, the control limits of the $\bar{X}$ & S charts of each stream will be optimized. The low level search loops all s stages. For each of them, an allocating factor ω_i ($0 \leq \omega_i \leq 1$) is used to allocate the type I error α_i to $\alpha_{\bar{X},i}$ (for the $\bar{X}$ chart) and $\alpha_{S,i}$ (for the S chart).

$$\begin{aligned}\alpha_{\bar{x},i} &= \omega_i \alpha_i \\ \alpha_{s,i} &= \frac{(1 - \omega_i) \alpha_i}{1 - \omega_i \alpha_i}\end{aligned}\quad (5.15)$$

Equation (5.15) ensures that the union of $\alpha_{\bar{x},i}$ and $\alpha_{s,i}$ is always equal to α_i for whatever value of ω_i . Then, the control limits can be determined.

$$\begin{aligned}k_{\bar{x},i} &= \Phi^{-1}(1 - 0.5\alpha_{\bar{x},i}) \\ LCL_{\bar{x},i} &= \mu_{0,i} - k_{\bar{x},i} \times \frac{\sigma_{0,i}}{\sqrt{n_i}}, \quad UCL_{\bar{x},i} = \mu_{0,i} + k_{\bar{x},i} \times \frac{\sigma_{0,i}}{\sqrt{n_i}} \\ UCL_{s,i} &= \sigma_{0,i} \sqrt{\frac{[\chi_{n_i-1}^2]^{-1}(1 - \alpha_{s,i})}{n_i - 1}}\end{aligned}\quad (5.16)$$

Next, based on the values of $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$ and $UCL_{s,i}$, the informative power z_i of stage i can be evaluated (see Section 5.2.3). The low-level search will seek the optimal value of ω_i (and the corresponding optimal set of control limits) for each stage, so that the informative power z_i will be maximized. Further, it will in turn lead to the minimization of the objective function T (Equation (5.4)). The overall optimization search is outlined as follows.

- (1) Specify s , g_i , n_i , h_i , LSL_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, τ , p_i , and $c_{min,i}$.
- (2) Initialize T_{min} as a large number, say 10^6 . T_{min} is used to record the minimum T .
- (3) Carry out the high-level search to investigate different combinations of $(\alpha_1, \alpha_2, \dots, \alpha_s)$. Each set of $(\alpha_1, \dots, \alpha_s)$ will automatically make ATS_0 equal to τ .
 - (3.1) For a given set of $(\alpha_1, \dots, \alpha_s)$, loop all s stages. For each stage, initialize $z_{max,i}$ as a very small number, say 10^{-6} ($z_{max,i}$ is used to record the maximum z_i) and, then, carry out the low-level search on ω_i ($0 \leq \omega_i \leq 1$). For a given value of ω_i ,

- (3.1.1) Determine the control limits $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$, and $UCL_{s,i}$ by Equations (5.15) and (5.16).
- (3.1.2) Calculate z_i by using Equations (5.6) to (5.14).
- (3.1.3) If this z_i value is larger than the current $z_{max,i}$, replace $z_{max,i}$ by z_i and record the control limits $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$, and $UCL_{s,i}$ as the temporary low-level optimal control limits for stage i .
- (3.2) When the loop over s stages is completed, calculate T by Equation (5.4).
- (3.3) If this T value is smaller than the current T_{min} , replace T_{min} by T and store the low-level optimal control limits of all s stages as the temporary high-level optimal control limits.
- (4) When the whole search is completed, the optimal set of $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$, and $UCL_{s,i}$ ($i = 1, \dots, s$) is finalized.

5.3 Integrated $\bar{X}$ & S chart system with optimal sample sizes, sampling intervals and control limits

This section develops the **VSISN** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) chart system for monitoring process shifts in mean and variance in different stages in a multi-stage and multi-stream manufacturing system. The model **VSISN** is an extension of the model **VSISL** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / control limits) discussed in Section 5.2. The model **VSISL** only optimizes the control limits of the charts in a chart system. Here, the extended model **VSISN** designs the sample sizes, the sampling intervals and the control limits of all the $\bar{X}$ and S charts in a system in an integrative and optimal manner. Similar to the model **VSISL**, the in-control and out-of-control system conditions are again

related to the process capability index C_{pk} . The optimization design is carried out under resource constraint (operators equipped with measuring instruments).

5.3.1 Input parameters

The parameters $s, g_i, LSL_i, USL_i, \mu_{0,i}, \sigma_{0,i}, \tau, p_i, c_{min,i}, t_i$ (time required to inspect a unit in the i th stage), and M (available number of operators) are required in this design (see Table 1.2 for the details of the specifications).

5.3.2 Optimization model

Based on the above parameters, the design of the VSISN chart system can be conducted by using the following optimization model.

$$\text{Objective function: } T = \text{minimum} \quad (5.17)$$

$$\text{Constraint function: } ATS_0 \geq \tau \quad (5.18)$$

$$m \leq M \quad (5.19)$$

$$\text{Design variables: } n_i, h_i, LCL_{\bar{x},i}, UCL_{\bar{x},i}, UCL_{s,i} \ (i = 1, \dots, s)$$

Where the objective function T is a function of $(s, g_i, n_i, h_i, t_i, LSL_i, USL_i, \mu_{0,i}, \sigma_{0,i}, \tau, p_i, c_{min}, M, LCL_{\bar{x},i}, UCL_{\bar{x},i}, \text{ and } UCL_{s,i})$. The in-control ATS_0 of the chart system in constraint (5.18) is calculated by Equation (3.12) derived in Section 3.2.1.3 in Chapter 3. The actual (or required) inspection resources m of the system in constraint (5.19) is calculated by Equation (3.33) derived in Section 3.2.2.3 in Chapter 3. The objective function T is again the average informative time (discussed in Section 5.2) and is given by Equation (5.4).

5.3.3 Optimization search algorithm

The optimal values of n_i , h_i , $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$ and $UCL_{s,i}$ are sought in a three level optimization search. The high-level search optimizes the n_i and h_i . The mid-level search distributes the detection power (or type I error probability α_i) of the chart system among the s stages and the low-level search allocates the power between the $\bar{X}$ chart and S chart in a stream within each stage (i.e. optimize the control limits $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$ and $UCL_{s,i}$).

The high-level search is carried out by using the gradient based search algorithm discussed in Section 3.2.2 in Chapter 3 (note that ΔATS_j in Equation (3.40) in Chapter 3 will be replaced by ΔT_j in this case). The purpose is to determine the optimum values of n_i and h_i so that the required inspection resources (m) is not greater than the specified value (M) of the manufacturing system (i.e. satisfy constraint (5.19)). For any given values of (n_i, h_i) determined in the high-level, the mid and low level searches are carried out by following the algorithms discussed in the last section (i.e. the high-level and low-level search algorithms discussed in Section 5.2.4).

While the high-level optimization trades off between the sample size n_i and the sampling interval h_i , the mid-level optimization enhances the power (or increase α_i) of the $\bar{X}$ & S charts in some stages and sacrifice the power of the $\bar{X}$ & S charts in other stages, and the low-level search seeks the optimal set of control limits for each stage so that the informative power z_i is maximized. Ultimately, all three levels aim at minimizing the objective function T (Equation (5.4)). The overall optimization search is outlined as follows.

- (1) Specify s , g_i , LSL_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, τ , p_i , $c_{min,i}$, t_i , and M .

- (2) Initialize T_{min} as a large number, say 10^6 . T_{min} is used to record the minimum T .
- (3) Carry out the high-level search to determine the optimal values of (n_i, h_i) which ensures that m is equal to M .
- (4) For a given set of (n_i, h_i) , carry out the mid-level search to investigate different combinations of $(\alpha_1, \dots, \alpha_s)$. Each set of $(\alpha_1, \dots, \alpha_s)$ will automatically make ATS_0 equal to τ .
- (5) For a given set of (n_i, h_i, α_i) , carry out the low-level search to loop all s stages. For each stage, initialize $z_{max,i}$ as a very small number, say 10^{-6} ($z_{max,i}$ is used to record the maximum z_i) and, then, carry out the low level search on ω_i ($0 \leq \omega_i \leq 1$). For a given value of ω_i ,
 - (5.1) Determine the control limits $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$, and $UCL_{s,i}$ by Equations (5.15) and (5.16).
 - (5.2) Calculate z_i by using Equations (5.6) to (5.14). If this z_i value is larger than the current $z_{max,i}$, replace $z_{max,i}$ by z_i and record the control limits $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$ and $UCL_{s,i}$.
 - (5.3) When the loop over s stages is completed, calculate T by Equation (5.4).
 - (5.4) If this T value is smaller than the current T_{min} , replace T_{min} by T and store the optimal values of $n_i, h_i, LCL_{\bar{x},i}, UCL_{\bar{x},i}$, and $UCL_{s,i}$ of all s stages as the temporary optimal solution.
- (6) When the entire search is completed, the final T_{min} is the minimum T and the corresponding optimal set of $n_i, h_i, LCL_{\bar{x},i}, UCL_{\bar{x},i}$, and $UCL_{s,i}$ ($i = 1, \dots, s$) are also finalized.

5.4 Comparative study

This section compares the performance (measured by the average informative time T) of the **VSISN** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) chart system (discussed in Section 5.3) with the traditional 3-sigma $\bar{X}$ & S chart system as well as a **VSISL** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / control limits) chart system (discussed in Section 5.2). Both the **VSISN** and **VSISL** chart systems are designed to minimize the average informative time T , but the latter only has the control limits $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$ and $UCL_{s,i}$ optimized (namely, only the middle and low level optimization searches are conducted) and, n_i and h_i are fixed (that is, the high-level optimization is omitted). In this comparative study it is assumed that each chart in the traditional $\bar{X}$ & S chart system uses a sample size of five, identical sampling interval, and 3-sigma control limits. This section also studies the effects of six parameters (g_i , t_i , p_i , $\sigma_{0,i}$, USL_i , $c_{min,i}$) on the performance of the **VSISN** chart system as well as the **VSISL** chart system. Each of the six parameters has a low value, a nominal value and a high value, as shown below (see Table 5.1).

Table 5.1 Parameter values for comparative study

Parameter	Low	Nominal	High
g_i	1	1	4
t_i	0.02	0.04	0.06
p_i	0.1	0.5	0.9
$\sigma_{0,i}$	0.01	0.025	0.04
USL_i	$3.2\sigma_{0,i}$	$4.6\sigma_{0,i}$	$6\sigma_{0,i}$
$c_{min,i}$	1	1.25	1.5

The nominal value denotes the value under normal circumstances. It is also noted that USL_i is expressed as a function of $\sigma_{0,i}$.

The specifications M and τ for the available resources M and the in-control ATS_0 of the **VSISN** chart system and the **VSISL** chart system are set equal to the M and ATS_0 values of the traditional 3-sigma $\bar{X}$ & S chart system (i.e. all systems always have the identical M and ATS_0 values).

The performance of a two-stage manufacturing system is studied for six different cases. In each case, all the parameters take their nominal values in both stages, except that one selected *active parameter* takes its low value in stage 1 and high value in stage 2. For example, in case one, the first parameter g_i is designated as the active parameter. Therefore

$$g_1 = 1, \quad g_2 = 4$$

$$t_1 = t_2 = 0.04, \quad p_1 = p_2 = 0.5, \quad \sigma_{0,1} = \sigma_{0,2} = 0.025, \quad USL_1 = USL_2 = 4.6\sigma_{0,i}, \quad c_{min,1} = c_{min,2} = 1.25$$

To facilitate the comparison, a ratio $B_{\bar{X}\&S}$ of the average informative time T values resulting from each control chart system is calculated as

$$B_{\bar{X}\&S} = \frac{T}{T_{\bar{X}\&S}} \quad (5.20)$$

Where $T_{\bar{X}\&S}$ is produced by the traditional $\bar{X}$ & S chart system and T is by each particular chart system. Obviously, if the $B_{\bar{X}\&S}$ value of a chart system in a case is less than one, this chart system will outperform the traditional $\bar{X}$ & S chart system when the corresponding active parameter has different values in the two stages.

The $B_{\bar{x}\&S}$ values for all six cases are listed in Table 5.2 under column (9). It is observed that in all of the six cases, the **VSISL** chart system usually substantially outperforms the traditional $\bar{X}$ & S chart system; however, the **VSISN** chart system can always achieve a more significant improvement in effectiveness compared to the **VSISL** chart system. Among all, the difference in parameter USL_i (the upper specification limits in terms of in-control process standard deviation) and $c_{min,i}$ (the out-of-control process capability index) promise the greatest potential of using the **VSISN** chart system. For example, a reduction of T by about 65% compared to the traditional $\bar{X}$ & S chart system and 37% compared to the **VSISL** chart system can be achieved when USL_i has different values ($USL_1 = 3.2\sigma_{0,i}$, $USL_2 = 6\sigma_{0,i}$ in case five) and all other parameters are fixed. In the same case, however, the **VSISL** chart system reduces the T by about 44% compared to the traditional $\bar{X}$ & S chart system.

The optimal values of n_i , h_i and α_i of the **VSISN** chart system are also enumerated in Table 5.2 (columns (4), (5) and (6)). A parameter $A_{\bar{x}\&S,i}$ is calculated for each stage and listed in column (7)

$$A_{\bar{x}\&S,i} = \frac{n_i \alpha_i}{h_i} \quad (i = 1, 2) \quad (5.21)$$

These $A_{\bar{x}\&S,i}$ values provide the users with the general guidelines for adjusting the sample size, sampling intervals, and control limits for different stages in a system. For example, if $A_{\bar{x}\&S,1}$ is larger than $A_{\bar{x}\&S,2}$, it means that more detection power should be allocated to stage 1. That is, n_1 is larger than n_2 , h_1 is smaller than h_2 , and/or the control limits $LCL_{\bar{x},1}$, $UCL_{\bar{x},1}$ and $UCL_{s,1}$ of the $\bar{X}$ and S charts in stage 1 should be made relatively tighter than that in stage 2. As the general guidelines drawn from Table 5.2, the following stage should be made more powerful where

Table 5.2 Comparative study

(1) Case	(2) Stage	(3) Active parameter	(4) Optimal n	(5) Optimal h	(6) Optimal α	(7) $A_{\bar{X}\&S}$	(8) Optimal k		(9) Ratio $B_{\bar{X}\&S}$	
							$k_{\bar{X}}$	k_S	VSISL chart system	VSISN chart system
1	1	$g_1 = 1$	15	1.773	0.017555	0.1485	2.530	2.655	0.8074	0.5330
	2	$g_2 = 4$	11	2.660	0.015318	0.0634	2.527	2.873		
2	1	$t_1 = 0.02$	18	2.000	0.009994	0.0900	2.722	2.861	0.8758	0.5322
	2	$t_2 = 0.06$	11	3.000	0.024530	0.0899	2.358	2.682		
3	1	$p_1 = 0.1$	9	3.180	0.013009	0.0368	2.584	2.964	0.7444	0.4388
	2	$p_2 = 0.9$	19	2.650	0.024060	0.1725	2.417	2.508		
4	1	$\sigma_{0,1} = 0.01$	16	3.300	0.022255	0.1079	2.445	2.553	0.8758	0.5313
	2	$\sigma_{0,2} = 0.04$	17	3.300	0.021228	0.1094	2.462	2.568		
5	1	$USL_1 = 3.2\sigma_0$	25	3.800	0.045189	0.2973	2.070	2.569	0.5590	0.3517
	2	$USL_2 = 6.0\sigma_0$	13	3.800	0.004773	0.0163	2.957	3.169		
6	1	$c_{min,1} = 1.0$	8	2.400	0.001466	0.0049	3.304	3.664	0.5221	0.4281
	2	$c_{min,2} = 1.5$	16	2.400	0.030072	0.2005	2.189	3.173		

- (1) The number of streams is smaller; and/or
- (2) The time required to inspect a unit is smaller; and/or
- (3) The out-of-control case is more likely to occur; and/or
- (4) The process has larger variance; and/or
- (5) The specification limits are tighter with respect to the in-control standard deviation; and/or
- (6) The out-of-control process capability index is larger.

The control limits coefficients $k_{\bar{x}}$ for the $\bar{X}$ chart and k_s for the S chart of the VSISN chart system are also listed in Table 5.2 under column (8). It is found that $k_{\bar{x}}$ is usually smaller than k_s (for the conventional system, $k_{\bar{x}} \equiv k_s \equiv 3$). It indicates that the control limits of the $\bar{X}$ chart should be, in general, made tighter than that of the S chart.

5.5 Example

A mechanical part is machined in a manufacturing system (see Figure 3.2(a) in Chapter 3). The specifications of the system are listed below.

Number of stages in the system: $s = 4$

Number of streams: $g_1 = g_2 = g_3 = 1, g_4 = 2$

Lower specification limit (mm): $LSL_1 = 15.91, LSL_2 = 12.90, LSL_3 = 7.84,$
 $LSL_4 = 10.95$

Upper specification limit (mm): $USL_1 = 16.09, USL_2 = 13.10, USL_3 = 8.16,$
 $USL_4 = 11.05$

Process standard deviations (mm): $\sigma_{0,1} = 0.018, \sigma_{0,2} = 0.027, \sigma_{0,3} = 0.020, \sigma_{0,4} = 0.013$

Inspection time (min): $t_1 = 0.40, t_2 = 0.35, t_3 = 0.65, t_4 = 0.50$

Out-of-control process capability: $c_{min,1} = c_{min,2} = c_{min,3} = c_{min,4} = 1$

The probabilities p_i of the out-of-control occurrences are estimated from historical records as follows

$$p_1 = 0.286, p_2 = 0.381, p_3 = 0.190, p_4 = 0.143$$

Originally, the system uses the traditional 3-sigma $\bar{X}$ & S charts for all the processes. Operators take a sample of size five for every two hours for each chart. As a result the amount of manpower M used in the traditional system is equal to 0.10 (Equation (3.33) in Chapter 3). The in-control ATS_0 of the traditional $\bar{X}$ & S chart system is equal to 61 hours (Equation (3.12) in Chapter 3).

Now, the QA engineer decides to design a **VSISL** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / control limits) chart system and a **VSISN** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) chart system in order to improve the effectiveness of the chart system. He decides to maintain the false alarm rate and the required manpower at the same level as in the traditional chart system. Thus, the τ and M values of the **VSISL** and **VSISN** chart systems are specified as 61 hours and 0.10, respectively. The computer program *vsisn.c* designs the **VSISN** chart system in 58.781 CPU seconds in a personal computer (Pentium IV 2.4GHz). The results are listed in Table 5.3.

As shown in Table 5.3, the values of the average informative time T generated by the three chart systems are quite different. They are 15.5, 9.6 and 5.7 hours for the traditional $\bar{X}$ & S chart system, **VSISL** chart system and **VSISN** chart system, respectively. In other words, the **VSISL** chart system reduces T by 38% and the **VSISN** chart system reduces T by an even high percentage of 63%, compared to the

Table 5.3 The three control chart systems in the example

System	Stage	Sample size n	Sampling interval (hr) h	Control limits (mm)			Type I error probability α	The average informative time (hr)	
				$LCL_{\bar{x}}$	$UCL_{\bar{x}}$	UCL_s		T	
Traditional $\bar{X}$ &S chart system	1	5	2.00	15.976	16.024	0.3535	0.006588	15.5	
	2	5	2.00	12.964	13.036	0.0530	0.006588		
	3	5	2.00	7.973	8.027	0.0393	0.006588		
	4	5	2.00	10.983	11.017	0.0255	0.006588		
VSISL chart system	1	5	2.00	15.974	16.026	0.0407	0.001636	9.6	
	2	5	2.00	12.971	13.029	0.0544	0.017999		
	3	5	2.00	7.970	8.030	0.0438	0.001636		
	4	5	2.00	10.984	11.016	0.0282	0.005812		
VSISN chart system	1	9	2.70	15.981	16.019	0.0325	0.002209	5.7	
	2	17	3.29	12.985	13.015	0.0381	0.028712		
	3	2	2.09	7.956	8.044	0.0691	0.002209		
	4	13	5.81	10.991	11.009	0.0198	0.017006		

traditional $\bar{X}$ & S chart system. The further reduction in T achieved by the **VSISN** chart system is attributable to the optimization of the sample sizes and sampling intervals in addition to the optimization of the control limits. In the optimal solution, the majority of the detection power of the **VSISN** chart system is allocated to stage 2 (i.e. n_2 and α_2 are large and h_2 is small). The reason is that stage 2 has only one stream, large variance, and high probability of out-of-control occurrences.

5.6 Summary

This chapter develops the **VSISL** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / control limits) and **VSISN** ($\bar{X}$ & S charts / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) chart systems for multi-stage and multi-stream manufacturing system. The design algorithm considers the $\bar{X}$ charts and the S charts in all stages integratively and handles mean and standard deviation shifts simultaneously. It is found that, by determining sample sizes, sampling intervals and control limits of the individual charts of a system in an integrative and optimal manner, the effectiveness of the system as a whole can be improved significantly, and therefore, the product quality is further guaranteed. The optimization design also ensures that no extra resources are required and the false alarm rate is not increased.

Most of the specifications for the integrated design are similar to that required by the traditional system design or available from manufacturing information and records. The process conditions are defined in terms of process capability index C_{pk} that is more understandable and manageable for the QA engineers, the managers, as well as the floor supervisors.

Finally, the general guidelines for adjusting the sample size, sampling interval, and control limits of the $\bar{X}$ & S charts in a system are particularly useful and universally applicable.

Chapter 6

Economical Designs of Integrated $\bar{X}$ &S Control Chart Systems for Monitoring Process Mean and Variance

This chapter designs the integrated control chart system for monitoring process shifts in mean and variance in a multi-stage and multi-stream manufacturing system from an economic viewpoint. Similar to Chapter 4, two economic models for a single process are first designed based on random process shifts (i.e. mean and standard deviation shifts). The first model **VEIIN** ($\bar{X}$ &S charts / economical design / independent process / single control chart / sample size, sampling interval, and control limits) focuses on the continuous improvement of the control chart during the use of the chart. The sample data of the mean shifts and the standard deviation shifts are estimated through the three-phase SPC scenario (see the extended discussion in Section 6.2), and these sample data are then used to update the charting parameters (i.e. the sample size, sampling interval, and control limits of the charts) step by step. The second model **VEIIM** ($\bar{X}$ &S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) focuses on the optimal deployment of manpower to a single pair of $\bar{X}$ &S charts so that the expected total cost incurred in the implement of SPC is minimized. The design algorithm of the **VEIIM** chart does not only optimize the deployed manpower, but also determines the sample size, sampling interval, and control limits of the charts in an optimal manner. The results of the comparative studies show that the **VEIIN** and **VEIIM** charts are significantly superior to the conventional $\bar{X}$ &S charts in view of the overall performance. Some useful guidelines are also provided to aid users in deciding the appropriate amount of manpower for the $\bar{X}$ &S charts monitoring a single process.

Finally, the concept of the **VEIIN** and **VEIIM** charts is applied to the economical designs of the integrated **VEISM** ($\bar{X}$ & S charts / economical design / independent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system for monitoring process shifts in mean and variance in a multi-stage and multi-stream manufacturing system. Similar to the models discussed in Chapter 4, the models discussed here uses Taguchi's loss function to measure the quality losses (or the costs) incurred in the out-of-control cases.

6.1 Introduction

As aforementioned, the deployment of manpower to an SPC scheme is an important factor in achieving the greatest gain from the implement of SPC. However, the optimal deployment of manpower to an $\bar{X}$ & S control chart scheme has not been studied in the current literature.

The out-of-control conditions of a process may be caused by mean shift δ_μ , standard deviation shift δ_σ , or both. Ideally, the charting parameters (i.e. sample size, sampling interval, and control limits) of the control charts are determined so that the charts work excellently in in-control condition (i.e. having low false alarm rate), as well as in out-of-control conditions (i.e. producing signal promptly). To ensure the in-control performance, the charting parameters of the $\bar{X}$ & S charts should be determined based on the in-control process mean μ_0 and standard deviation σ_0 that may be estimated from the historical data. For the out-of-control cases, the charting parameters should be adjusted based on the probability distribution of the random process shifts δ_μ and δ_σ . This probability distribution is different from process to process. Currently, when designing the control charts, the QA engineers usually take the in-control data (e.g. μ_0 and σ_0) into consideration, but seldom look into the characteristics of the out-

of-control cases (the probability distribution of δ_μ and δ_σ). They collect data to estimate μ_0 and σ_0 , but make little effort to acquire the data of δ_μ and δ_σ . It is believed that if people pay attention to observing, collecting and analyzing the data of the out-of-control cases and make use of them to design and modify the control charts, the performance of the charts will be improved to a significant degree. For examples, considering the joint $\bar{X}$ & S chart.

- (1) If mean shift δ_μ always prevails in a particular process, one can tighten the control limits of the $\bar{X}$ chart and relax the control limits of the S chart so that the overall detection effectiveness is enhanced and the false alarm rate is maintained in the same level.
- (2) If the increasing δ_μ and/or δ_σ are often dominant in a particular process, one can tighten the upper control limits of the $\bar{X}$ and S charts and relax their lower control limits.
- (3) If the magnitudes of δ_μ and/or δ_σ are usually small in a particular process, one can use large sample size and infrequent sampling so that the overall detection effectiveness is increased and the inspection rate is unchanged in the meantime.

Since continuous improvement of product quality is a key element for the success of any quality program, the continuous improvement of the control tools, such as the control charts, must be important too.

The remaining sections in this chapter are organized as follows.

- Section 6.2 discusses the estimation of the process shifts (i.e. mean shift δ_μ and standard deviation shift δ_σ).
- Section 6.3 develops the **VEIIN** chart that minimizes the overall mean value of the loss function per out-of-control case. It focuses on how the control charts can be updated during the implementation.
- Section 6.4 designs the **VEIIM** chart that minimizes the expected total SPC cost by optimally deploying the manpower to the control chart scheme.
- Section 6.5 develops the **VEISM** chart system for monitoring process shifts in mean and variance in a multi-stage and multi-stream manufacturing system. The design algorithm minimizes the expected total cost by optimizing the amount allocated manpower in the SPC system.
- Finally, Section 6.6 summarizes the models discussed in this chapter.

6.2 Estimation of the process shifts δ_μ and δ_σ from the data of the out-of-control cases

Suppose an out-of-control condition is signaled at the j_2 th sample and the follow-up investigation discovers that the responsible assignable cause occurs between the $(j_1 - 1)$ th and the j_1 th samples ($j_1 \leq j_2$) (see the discussion in Section 4.2 in Chapter 4). It means that the j_1 th to j_2 th samples are taken under the out-of-control condition. As a result, the mean shift $\hat{\delta}_\mu$ (the sample value of δ_μ for this particular out-of-control case) can be estimated from the grand average $\bar{\bar{X}}$ of the sample means $\bar{X}_j$ ($J = j_1, j_1+1, \dots, j_2$) and the standard deviation shift $\hat{\delta}_\sigma$ (the sample value of δ_σ for this particular out-of-control case) from the average $\bar{S}$ of the sample standard deviation S_j (Montgomery 2001).

$$\hat{\delta}_\mu = \frac{\bar{\bar{X}} - \mu_0}{\sigma_0} = \frac{\left(\sum_{j=j_1}^{j_2} \bar{X}_j \right) / (j_2 - j_1 + 1) - \mu_0}{\sigma_0} \quad (6.1)$$

$$\hat{\delta}_\sigma = \frac{\bar{S}}{\sigma_0} = \frac{\sqrt{\left(\sum_{j=j_1}^{j_2} S_j^2 \right) / (j_2 - j_1 + 1)}}{\sigma_0} \quad (6.2)$$

6.3 Optimization design of the single pair of $\bar{X}$ & S charts that minimizes the quality loss

This section develops the algorithm for the optimization design of the **VEIIN** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart. The algorithm is able to continuously improve the $\bar{X}$ & S charts in a single process along with the acquisition of the data from each detected out-of-control case. Specifically, the design algorithm will modify or update the sample size n , sampling interval h , and control limits $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s , and UCL_s of the $\bar{X}$ & S charts whenever the new data of δ_μ and δ_σ from an out-of-control case is available. By this way, the charting parameters are adjusted gradually to the real distribution of δ_μ and δ_σ of the process, and the detection effectiveness of the $\bar{X}$ & S charts is improved continuously. It is a further development of the three-phase SPC scenario. In Chapter 5, the lower control limit LCL_s of the S charts has not been considered. However, in some application, the QA engineers may prefer to use the LCL_s that can signal to improve the product quality and helps reducing the malfunction of the equipments and operators. Similar to the model **MEIIN** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits) discussed in Section 4.3 in Chapter 4, the objective here is to minimize the overall mean value of the loss function per out-of-control case.

6.3.1 Input parameters

The parameters USL , τ , μ_0 , σ_0 and R (maximum allowable inspection rate) are required in designing this model.

6.3.2 Optimization model

Initial design of the VEIIN chart

In the commencing stage, one may have little or no idea about the out-of-control cases of a process. Therefore, the VEIIN chart may be designed as the Shewhart $\bar{X}$ & S chart based on μ_0 and σ_0 . Usually, the sample size n is first set around five. The sampling interval h is then determined based on n so that the actual inspection rate r is equal to the specified R . Finally, identical control limit coefficients $k_{\bar{x}}$ and k_s are adopted for the two sides of both $\bar{X}$ chart and S chart so that the control limits $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s jointly make the actual in-control ATS_0 equal to the specified τ .

Updating of the VEIIN chart

During the monitoring of the process, whenever an out-of-control case is detected, the process mean shift δ_μ and standard deviation shift δ_σ are estimated (the estimation of $\hat{\delta}_\mu$ and $\hat{\delta}_\sigma$ is discussed in Section 6.2). The estimated $\hat{\delta}_\mu$ and $\hat{\delta}_\sigma$ are then used to update the charting parameters n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s using the following optimization model. The objective is to minimize ML (the overall mean of the loss function per out-of-control case) under the two constraints on ATS_0 and r (r is the actual or required inspection rate).

$$\text{Objective function: } ML = \text{minimum} \quad (6.3)$$

$$\text{Constraint functions: } ATS_0 \geq \tau \quad (6.4)$$

$$r \leq R \quad (6.5)$$

$$\text{Design variables: } n, h, LCL_{\bar{x}}, UCL_{\bar{x}}, LCL_s, UCL_s$$

Where the objective function ML is a function of $(USL, \tau, \mu_0, \sigma_0, R, n, h, LCL_{\bar{x}}, UCL_{\bar{x}}, LCL_s, \text{ and } UCL_s)$. For a given set of $n, h, LCL_{\bar{x}}, UCL_{\bar{x}}, LCL_s$ and UCL_s , the objective function ML is calculated by

$$ML = \int_0^{\infty} \int_{-\infty}^{\infty} L(\delta_{\mu}, \delta_{\sigma}) \cdot q(\delta_{\mu}, \delta_{\sigma}) \cdot f(\delta_{\mu}, \delta_{\sigma}) \cdot d\delta_{\mu} d\delta_{\sigma} \quad (6.6)$$

Where $f(\delta_{\mu}, \delta_{\sigma})$ is the joint density function of δ_{μ} and δ_{σ} ; L is the expected value of the Taguchi's loss function (see the discussion in Chapter 4); and $q(\delta_{\mu}, \delta_{\sigma})$ is the average number of units produced during an out-of-control case and is a function of the out-of-control ATS (the calculation of ATS is detailed in Section 6.3.3).

$$q(\delta_{\mu}, \delta_{\sigma}) = Q \cdot ATS(\delta_{\mu}, \delta_{\sigma}) \quad (6.7)$$

Where Q is the number of units produced in a time unit. Equation (6.6) can be rewritten as (see Equation (4.2) in Chapter 4)

$$ML = \int_0^{\infty} \int_{-\infty}^{\infty} KQ\sigma_0^2 \cdot (\delta_{\sigma}^2 + \delta_{\mu}^2) \cdot ATS(\delta_{\mu}, \delta_{\sigma}) \cdot f(\delta_{\mu}, \delta_{\sigma}) \cdot d\delta_{\mu} d\delta_{\sigma} \quad (6.8)$$

It is noted that a large shift may significantly increase the value of ML , because large shift makes the expected loss L great. On the other hand, a small shift may also contribute a substantial amount to ML , because small shift results in a large ATS .

In Equation (6.8), since the constants K, Q and σ_0 have no effect on the optimal values of the charting parameters, they can be neglected for the sake of simplicity.

$$ML = \int_0^{\infty} \int_{-\infty}^{\infty} (\delta_{\sigma}^2 + \delta_{\mu}^2) \cdot ATS(\delta_{\mu}, \delta_{\sigma}) \cdot f(\delta_{\mu}, \delta_{\sigma}) \cdot d\delta_{\mu} d\delta_{\sigma} \quad (6.9)$$

The density function $f(\delta_{\mu}, \delta_{\sigma})$ is rarely known, but it may be approximated by a group of the values of $\hat{\delta}_{\mu}$ and $\hat{\delta}_{\sigma}$ estimated from a collection of out-of-control cases. Suppose N pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ ($i = 1, \dots, N$) have been obtained from N out-of-control cases. Then, the integration (6.9) can be approximated by the following summation (non-parametric approach).

$$ML = \frac{1}{N} \sum_{i=1}^N [(\hat{\delta}_{\sigma,i}^2 + \hat{\delta}_{\mu,i}^2) \cdot ATS(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})] \quad (6.10)$$

When a new out-of-control case is detected and $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ are obtained, N will be increased by one. The larger is N , the more closely does the summation formula (Equation (6.10)) approximate Equation (6.9).

Sometimes, the probability density function $f(\delta_{\mu}, \delta_{\sigma})$ is a function of time (i.e. varying with time). For such cases, Equation (6.10) may be modified by either of the following two methods.

- (1) Only use the latest γ pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ rather than all of the N available pairs ($\gamma \leq N$).

$$ML = \frac{1}{\gamma} \sum_{i=N-\gamma+1}^N [(\hat{\delta}_{\sigma,i}^2 + \hat{\delta}_{\mu,i}^2) \cdot ATS(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})] \quad (6.11)$$

- (2) Use an EWMA scheme to fade out the influence of the remote out-of-control cases.

$$ML = \frac{1-\theta}{\theta^{N-1}(1-\theta^N)} \cdot \sum_{i=1}^N [\theta^{N-i} \cdot (\hat{\delta}_{\sigma,i}^2 + \hat{\delta}_{\mu,i}^2) \cdot ATS(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})] \quad (6.12)$$

The smoothing factor θ may be set as 0.9.

6.3.3 Calculation of out-of-control ATS of the $\bar{X}$ & S charts

The out-of-control $ATS(\delta_\mu, \delta_\sigma)$ is calculated by

$$\begin{aligned} ATS(\delta_\mu, \delta_\sigma) &= [ARL(\delta_\mu, \delta_\sigma) - 0.5] \cdot h \\ ARL(\delta_\mu, \delta_\sigma) &= \frac{1}{Z_{\bar{X}\&S}(\delta_\mu, \delta_\sigma)} \end{aligned} \quad (6.13)$$

Where $ARL(\delta_\mu, \delta_\sigma)$ is the Average Run Length and $Z_{\bar{X}\&S}(\delta_\mu, \delta_\sigma)$ is the detecting probability of the joint $\bar{X}$ & S charts. $Z_{\bar{X}\&S}(\delta_\mu, \delta_\sigma)$ is equal to the union of the detecting probabilities $z_{\bar{X}}(\delta_\mu, \delta_\sigma)$ and $z_S(\delta_\mu, \delta_\sigma)$ contributed by the $\bar{X}$ chart and S chart, respectively.

$$Z_{\bar{X}\&S}(\delta_\mu, \delta_\sigma) = z_{\bar{X}}(\delta_\mu, \delta_\sigma) + z_S(\delta_\mu, \delta_\sigma) - z_{\bar{X}}(\delta_\mu, \delta_\sigma)z_S(\delta_\mu, \delta_\sigma) \quad (6.14)$$

Where

$$\begin{aligned} z_{\bar{X}}(\delta_\mu, \delta_\sigma) &= 1 - \left\{ \Phi \left[\frac{UCL_{\bar{X}} - (\mu_0 + \delta_\mu \sigma_0)}{\delta_\sigma \sigma_0 / \sqrt{n}} \right] - \Phi \left[\frac{LCL_{\bar{X}} - (\mu_0 + \delta_\mu \sigma_0)}{\delta_\sigma \sigma_0 / \sqrt{n}} \right] \right\} \\ z_S(\delta_\mu, \delta_\sigma) &= 1 - \left\{ \chi^2_{n-1} \left[\frac{UCL_s^2 \cdot (n-1)}{(\delta_\sigma \sigma_0)^2} \right] - \chi^2_{n-1} \left[\frac{LCL_s^2 \cdot (n-1)}{(\delta_\sigma \sigma_0)^2} \right] \right\} \end{aligned} \quad (6.15)$$

6.3.4 Updating algorithm

The actual modification of the **VEIIN** chart (the search for the solution of the optimization model (Equations (6.3) to (6.5))), based on the N pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$, can be carried out via a two-level search. At the high level, the optimal value of sample size n is searched (the sampling interval h is dependent on n). Usually, the optimal sample size is large if the magnitudes of the process shifts (δ_μ and/or δ_σ) are small and *vice versa*. At the low level, the optimal values of the type I errors $\alpha_{\bar{X}l}$, $\alpha_{\bar{X}u}$, α_{Sl} and α_{Su} at the two sides of the $\bar{X}$ chart and S chart are sought. It is equivalent to optimizing the control limits $LCL_{\bar{X}}$, $UCL_{\bar{X}}$, LCL_s and UCL_s . The ultimate objective of both high

and low level searches is to minimize the overall mean value ML of the loss function (Equation (6.3)). The search procedures are outlined below.

- (1) Initialize ML_{min} as a very larger number, say 10^7 (ML_{min} is used to store the minimum value of ML).
- (2) At the high level, increase n from two in step of one until ML cannot be further reduced. For a given value of n , calculate h by

$$h = n / R \quad (6.16)$$

It ensures the satisfaction of constraint (6.5) (i.e. the actual inspection rate is equal to the specified R).

- (3) At the low level, search α_{sl} , α_{su} , $\alpha_{\bar{x}l}$ and $\alpha_{\bar{x}u}$ for given n and h (the detailed procedure is given in Section 6.3.5). This procedure ensures the satisfaction of constraint (6.4). For a tentatively determined set of the type I error, the control limits can be calculated as follows.

$$\begin{aligned} LCL_{\bar{x}} &= \mu_0 + \frac{\Phi^{-1}(\alpha_{\bar{x}l})\sigma_0}{\sqrt{n}}, \quad UCL_{\bar{x}} = \mu_0 + \frac{\Phi^{-1}(1-\alpha_{\bar{x}u})\sigma_0}{\sqrt{n}} \\ LCL_s &= \sigma_0 \sqrt{\frac{[\chi^2_{n-1}]^{-1}(\alpha_{sl})}{n-1}}, \quad UCL_s = \sigma_0 \sqrt{\frac{[\chi^2_{n-1}]^{-1}(1-\alpha_{su})}{n-1}} \end{aligned} \quad (6.17)$$

- (4) Now, for a tentatively determined set of n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s calculate ATS by Equations (6.13) to (6.15) and then ML by Equation (6.10). If the calculated ML is smaller than the current ML_{min} , replace the latter by the former and the current values of n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s are stored as the temporary optimal design.
- (5) At the end of the entire two-level search, the minimum ML can be identified and the optimal values of the charting parameters n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s are also determined. In other words, the **VEIIN** chart have been

modified by using the N pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ acquired from the N out-of-control cases.

6.3.5 Algorithm for searching the type I errors $\alpha_{\bar{x}l}$, $\alpha_{\bar{x}u}$, α_{sl} and α_{su}

The low-level search for α_{sl} , α_{su} , $\alpha_{\bar{x}l}$ and $\alpha_{\bar{x}u}$ mentioned in step (3) above are detailed here. In order to satisfy constraint (4) on ATS_0 and to make the control chart most effective, it is ideal to have

$$ATS_0 = \tau \quad (6.18)$$

Correspondingly, the overall type I errors of the $\bar{X}$ & S charts is

$$\alpha = h / \tau \quad (6.19)$$

It is composed of four elements $\alpha_{\bar{x}l}$, $\alpha_{\bar{x}u}$, α_{sl} and α_{su} .

$$\alpha = \alpha_{sl} + \alpha_{su} + \alpha_{\bar{x}l} + \alpha_{\bar{x}u} - (\alpha_{sl} + \alpha_{su}) \times (\alpha_{\bar{x}l} + \alpha_{\bar{x}u}) \quad (6.20)$$

The four elements can be searched in three levels (see the dynamic search algorithm discussed in Section 3.2.1 in Chapter 3).

- (1) Level one searches α_{sl} between zero and $\alpha_{sl,max}$, where

$$\alpha_{sl,max} = \alpha \quad (6.21)$$

- (2) Level two searches α_{su} between zero and $\alpha_{su,max}$, where

$$\alpha_{su,max} = \alpha - \alpha_{sl} \quad (6.22)$$

The value of α_{sl} has been tentatively determined in level one.

- (3) Level three searches $\alpha_{\bar{x}l}$ between zero and $\alpha_{\bar{x}l,max}$, where

$$\alpha_{\bar{x}l,max} = \frac{\alpha - (\alpha_{sl} + \alpha_{su})}{1 - (\alpha_{sl} + \alpha_{su})} \quad (6.23)$$

The values of α_{sl} and α_{su} have been tentatively determined in levels one and two, respectively.

- (4) Finally, $\alpha_{\bar{x}u}$ is calculated, without search, based on the given set of the values of α_{sl} , α_{su} and $\alpha_{\bar{x}l}$.

$$\alpha_{\bar{x}u} = \frac{\alpha - (\alpha_{sl} + \alpha_{su})}{1 - (\alpha_{sl} + \alpha_{su})} - \alpha_{\bar{x}l} \quad (6.24)$$

This equation ensures the satisfaction of constraint (6.4) on ATS_0 .

6.3.6 The overall scenario of the proposed chart

The modification and operation of the **VEIIN** chart can be illustrated by Figure (6.1).

Figure 6.1 Overall scenario of the proposed chart

- (1) In the commencing stage, design the Shewhart $\bar{X}$ & S chart and use them as the initial **VEIIN** chart. At this stage, there is no data about process shift ($\delta_\mu, \delta_\sigma$) (i.e. $N = 0$).
- (2) Use the current **VEIIN** chart to monitor the process.
- (3) When a new out-of-control case is detected and the data are analyzed, one more pair of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ is acquired (i.e. N is increased by one).
- (4) Use the current N pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ to modify the **VEIIN** chart by following the procedures discussed in the previous sections. Then, go back to step (2) to continue the process monitoring.

The distinctive feature of the **VEIIN** chart is the continuous improvement in performance as the out-of-control data have come to light one by one. While the Shewhart $\bar{X}$ & S chart can be considered as the workable chart with average performance, the **VEIIN** chart is the improvable chart that become increasingly more suitable to the real working conditions of a particular process (extension of the three-phase SPC scenario) and will have the first-rate performance for that process.

6.3.7 Example

A manufacturing factory produces a special type of bearing. The diameter x of the bearing is a key dimension and specified as $80 \pm 0.008 \text{ mm}$. The process mean μ_0 can be easily adjusted to the nominal value (80 mm) at the center between the lower and upper specification limits. The number Q of units produced per hour is equal to 60. The cost incurred by an out-of-specification unit is equal to 12 dollars. It is found from the pilot runs that the distribution of the diameter x can be well approximated by a normal distribution and the standard deviation σ_0 of x is very close to 0.004 mm .

The Shewhart $\bar{X}$ & S chart is first designed. They use a sample size of five and a sampling interval of one hour. That is, five units will be inspected at the end of every one hour. Consequently, the inspection rate is five per hour. Furthermore, 3-sigma control limits are adopted. As a result

$$LCL_{\bar{X}} = 79.99463, UCL_{\bar{X}} = 80.00537, LCL_S = 0, UCL_S = 0.00785$$

These control limits results in an in-control ATS_0 equal to 151 hours.

In order to enhance the detection effectiveness, the QA manager decides to turn to the **VEIIN** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart, but still maintain the same false alarm rate (or ATS_0) and inspection rate. The above Shewhart $\bar{X}$ & S chart is used as the initial design. The specifications are summarized as follows.

τ = 151 hours, the minimum allowable in-control ATS_0

USL = 80.008 mm, the upper specification limit

R = 5 / hour, the maximum allowable inspection rate

Q = 60 / hour, the production rate

C_k = 12 dollars, the cost incurred by an out-of-specification unit

μ_0 = 80 mm, the in-control process mean

σ_0 = 0.004 mm, the in-control process standard deviation

Therefore, the cost constant K of the loss function (see Equation (4.1) in Chapter 4) is

$$K = \frac{C_k}{(USL - \mu_0)^2} = \frac{12}{(80.008 - 80)^2} = 187500 \quad (6.25)$$

In addition to the **VEIIN** chart, another scheme, called the **VEIIL** ($\bar{X}$ &S charts / economical design / independent process / single control chart / control limits) chart, is also explored. For the **VEIIL** chart, only the control limits $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s are allowed to be adjusted. The sample size n and sampling interval h are fixed. The **VEIIL** chart can be designed similarly as the **VEIIN** charts except that the high level search for n is omitted (only searching the optimal values of $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s via $\alpha_{\bar{x}l}$, $\alpha_{\bar{x}u}$, α_{sl} and α_{su}). The **VEIIL** chart is useful when n and h are not allowed to be changed because of some managerial reasons.

It is assumed that both the mean shift δ_μ and the standard deviation shift δ_σ of this process follow the normal distributions, $\delta_\mu \sim N(0.5, 0.2^2)$ and $\delta_\sigma \sim N(1.3, 0.2^2)$. Therefore, the joint density function (used in Equation (6.9)) for this example is expressed as follows.

$$f(\delta_\mu, \delta_\sigma) = \frac{1}{2 \times 0.2 \times 0.2 \times \pi} \exp \left\{ -0.5 \times \left[\left(\frac{\delta_\mu - 0.5}{0.2} \right)^2 + \left(\frac{\delta_\sigma - 1.3}{0.2} \right)^2 \right] \right\} \quad (6.26)$$

Even though this density function is unknown to the QA manager or anyone else, one can follow the procedure described in Section 6.2 to obtain the estimate $(\hat{\delta}_\mu, \hat{\delta}_\sigma)$ whenever an out-of-control case is detected. The N pairs of $(\hat{\delta}_\mu, \hat{\delta}_\sigma)$ that are currently available are used to update the charting parameters of the **VEIIN** chart, as well as the parameters of the **VEIIL** chart. Figure 6.2(a) displays 30 sample points of $(\delta_\mu, \delta_\sigma)$ from the distribution described by Equation (6.26). The assembly of these points is taken as an approximate of the distribution of $(\delta_\mu, \delta_\sigma)$. Figure 6.2(b) shows the first five points of $(\delta_\mu, \delta_\sigma)$ and the corresponding estimate $(\hat{\delta}_\mu, \hat{\delta}_\sigma)$.

Figure 6.2 (a) 30 actual shift points; (b) First five pairs of actual and estimated shift points

Figure 6.3 displays the ML values of the three schemes of the control charts (the Shewhart, **VEIIL** and **VEIIN** charts) in 30 runs. All these ML values are calculated by Equation (6.9) using the density function (6.26). It is noted that while the charting parameters of the **VEIIL** and **VEIIL** charts are updated by using the

estimated pairs of $(\hat{\delta}_\mu, \hat{\delta}_\sigma)$, the actual ML value should be evaluated based on the real distribution of $(\delta_\mu, \delta_\sigma)$.

Figure 6.3 Values of ML and n of different charts

In Figure 6.3, Run 0 represents the time period when no out-of-control information is available, or the time period from the beginning to the detection of the first out-of-control case. In Run 0, both the **VEIIL** and **VEIIN** charts are the same as the Shewhart $\bar{X}$ & S charts because the initial designs of these two schemes are just the Shewhart charts. Run 1 indicates the period between the detection of the first and the second out-of-control cases. The **VEIIL** and **VEIIN** charts used in Run 1 differ from the Shewhart charts, because both have been modified by the first pair of $(\hat{\delta}_{\mu,1}, \hat{\delta}_{\sigma,1})$. Similarly, the N th Run covers the period between the detection of the N th and the

$(N+1)$ th out-of-control cases. Both the **VEIIL** and **VEIIN** charts used in the N th Run are modified by the first N pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$.

The Shewhart $\bar{X}$ & S chart always has the same ML value because their charting parameters have never been updated during the 30 runs. The ML values of all three schemes are identical in Run 0, because all of the charts are the same Shewhart $\bar{X}$ & S chart. However, from Run 1 on, the ML value of the **VEIIL** chart is always considerably smaller than that of the Shewhart $\bar{X}$ & S chart. The ML value of the **VEOCN** chart is even smaller. It is because the **VEIIL** and **VEIIN** charts have been improved by using the estimated pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ that come to light one by one. In general, the ML value can be reduced about 45% by the **VEIIL** chart and 65% by the **VEIIN** chart, compared to the Shewhart $\bar{X}$ & S chart. It indicates a significant improvement in performance achieved by the **VEIIL** or **VEIIN** chart. The ML value of the **VEIIN** chart is even smaller than that of the **VEIIL** chart, because more parameters of the former have been optimized. It is noted that all of above three schemes (the Shewhart, **VEIIL** and **VEIIN** charts) always have the identical in-control ATS_0 (false alarm rate) and inspection rate r .

In Figure 6.3, it can be seen that by just making use of several pairs of $(\hat{\delta}_{\mu}, \hat{\delta}_{\sigma})$, one can substantially reduce the ML values of the **VEIIL** and **VEIIN** charts. The change of the sample size n of the **VEIIN** charts is also demonstrated in Figure 6.3. It seems that the ML values as well as the charting parameters of the **VEIIL** and **VEIIN** charts can be stabilized within a few runs. At the end of the 30 runs, the charting parameters of the **VEIIL** and **VEIIN** charts have evolved into the follows.

The **VEIIL** chart:

$$n = 5, h = 1.0$$

$$LCL_{\bar{x}} = 79.99067, UCL_{\bar{x}} = 80.00457, LCL_s = 0.00006, UCL_s = 0.00845$$

The **VEIIN** chart:

$$n = 20, h = 4.0$$

$$LCL_{\bar{x}} = 79.99534, UCL_{\bar{x}} = 80.00186, LCL_s = 0.00121, UCL_s = 0.00558$$

Compared to the parameters of the Shewhart $\bar{X}$ &S chart, the upper control limits $UCL_{\bar{x}}$ and UCL_s of the **VEIIL** and **VEIIN** charts are tightened and the sample size n of the **VEIIN** charts is increased. These adjustments are made according to the approximated probability distribution of $(\delta_\mu, \delta_\sigma)$ of the process (Figure 6.2(a)). In this example, both δ_μ and δ_σ are likely to be increasing and their magnitudes are relatively small.

It is noted that the probability distributions of $(\delta_\mu, \delta_\sigma)$ of other processes must be different from the distribution in this example. But, the proposed charts will be generally applicable and effective. The reason is that the special probability distribution for every individual process can be learnt step by step by the proposed algorithm, and the acquired information can be used to modify the **VEIIN** control chart and improve the effectiveness.

6.3.8 Discussions

In this section, the **VEIIN** ($\bar{X}$ &S charts / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart has been proposed. This scheme makes use of the acquired out-of-control information to

continuously modify the control charts, namely, updating their sample size, sampling interval, and control limits. The objective is to minimize the overall mean loss ML of a process. The results from the example show that the **VEIIN** chart is able to considerably reduce the ML value compared to the Shewhart $\bar{X}$ & S chart, on condition that both the in-control ATS_0 and inspection rate are maintained unchanged. The actual improvement that can be achieved by the **VEIIN** chart depends on the distribution of $(\delta_\mu, \delta_\sigma)$ of individual process. Many times the information contained in just a few estimated points of $(\hat{\delta}_\mu, \hat{\delta}_\sigma)$ may help to substantially improve the performance of the proposed charts. The **VEIIL** ($\bar{X}$ & S charts / economical design / independent process / single control chart / control limits) chart is also discussed in this section. They can be applied to the situations where the sample size and sampling interval are preferred to be fixed. However, a computer must be available to carryout the modification and updating of the proposed chart.

6.4 Optimization design of the single pair of $\bar{X}$ & S charts that minimizes the total cost

This section designs the **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart that optimize the amount of manpower M in order to minimize the expected total cost C_{total} for the SPC operation in a single process. This model is an extension of the model **VEIIN** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits) discussed in Section 6.3. The model **VEIIN** only minimizes the quality losses $C_{quality}$ (or the costs) incurred by the out-of-control cases. Here, the extended model **VEIIM** minimizes the expected total cost C_{total} (including the manpower cost C_{man} and the quality cost $C_{quality}$)

by optimizing the amount of allocated manpower M in the SPC scheme. In the mean time, the sample size, sampling interval, and control limits of the control charts are also optimized. Some general guidelines for deploying manpower to an $\bar{X}$ &S control chart scheme are also provided.

6.4.1 Input parameters

The parameters USL , τ , σ_0 , μ_0 , Q , C_m , C_k , $MTBO$ and w (number of units that can be inspected per unit time by an operator, or the inspection rate per operator) are required in designing this model.

All of these parameters have been discussed and used in building the models in previous chapters, especially in Chapter 4.

6.4.2 Optimization model

The design algorithm of the **VEIIM** chart is implemented by the following optimization model.

$$\text{Objective function: } C_{total} = \text{minimum} \quad (6.27)$$

$$\text{Constraint functions: } ATS_0 \geq \tau \quad (6.28)$$

$$\text{Design variables: } M, n, h, LCL_{\bar{x}}, UCL_{\bar{x}}, LCL_s, UCL_s$$

Where the objective function C_{total} is a function of (USL , τ , σ_0 , μ_0 , Q , C_m , C_k , $MTBO$, w , M , n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s , and UCL_s). The design algorithm of the **VEIIM** chart is similar to the **MEIIM** ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart discussed in Section 4.4 in Chapter 4. While the **MEIIM** chart

considers only mean shift δ_μ , the **VEIIM** chart considers both the mean shift δ_μ and standard deviation shift δ_σ . The main differences between the formulation for **MEIIM** and **VEIIM** charts are in the calculation of the out-of-control *ATS* and the modelling of the probability distribution of the process shifts.

Among the seven design variables, only M and n are independent. The other five can be determined by M and n . Following the Equation (4.27) in Chapter 4,

$$h = \frac{n}{M \times w} \quad (6.29)$$

Next, the control limits $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s can be determined by following the discussion in Sections 6.3.4 and 6.3.5 (using Equations (6.17) to (6.24)).

After M , n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s , have all been determined, the objective function C_{total} is calculated by

$$C_{total} = C_{quality} + C_{man} \quad (6.30)$$

The manpower cost C_{man} and the quality cost $C_{quality}$ per unit time is calculated by following the Equations (4.29) and (4.30) derived in Chapter 4.

$$C_{man} = C_m \times M \quad (6.31)$$

$$C_{quality} = \int_0^\infty \int_{-\infty}^\infty \left[\frac{1}{MTBO} \cdot q(\delta_\mu, \delta_\sigma) \cdot L(\delta_\mu, \delta_\sigma) \cdot f(\delta_\mu, \delta_\sigma) \right] d\delta_\mu d\delta_\sigma \quad (6.32)$$

Where $q(\delta_\mu, \delta_\sigma) = Q \cdot ATS(\delta_\mu, \delta_\sigma)$ and, $L(\delta_\mu, \delta_\sigma)$ is the expected loss associated with an out-of-control case. Finally,

$$C_{quality} = \frac{K \cdot Q \cdot \sigma_0^2}{MTBO} \int_0^\infty \int_{-\infty}^\infty [(\delta_\sigma^2 + \delta_\mu^2) \cdot ATS(\delta_\mu, \delta_\sigma) \cdot f(\delta_\mu, \delta_\sigma)] d\delta_\mu d\delta_\sigma \quad (6.33)$$

Where $ATS(\delta_\mu, \delta_\sigma)$ is calculated by using Equations (6.13) to (6.15). The coefficient K can be calculated from the cost component C_k associated with the specification limits (see Equation (6.25)).

The integration (6.33) can be approximated by the summation based on the N pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ (non-parametric approach).

$$C_{quality} = \frac{1}{N} \sum_{i=1}^N \frac{K \cdot Q \cdot \sigma_0^2}{MTBO} [(\hat{\delta}_{\sigma,i}^2 + \hat{\delta}_{\mu,i}^2) \cdot ATS(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})] \quad (6.34)$$

6.4.3 Optimization search algorithm

The design of the **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart, based on the N pairs of $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ ($i = 1, \dots, N$), can be carried out via a three-level optimization search. The high-level optimization uses the Successive Quadratic Estimation method (Powell 1964) to find the optimal value of manpower M . Then, for a given value of M , an exhaustive mid-level search is evoked to find the optimal value of sample size n (the sampling interval h is dependent on n). Finally, for a given value of M , n and h , the optimal values of type I error probabilities $\alpha_{\bar{x}l}$, $\alpha_{\bar{x}u}$, α_{sl} , and α_{su} at the two sides of the $\bar{X}$ chart and S chart are sought (it is equivalent to optimizing the control limits $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s , and UCL_s) in a low-level search. While the high-level optimization makes a trade-off between the quality cost $C_{quality}$ and the manpower cost C_{man} , the mid-level optimization trades off between the sample size n and the sampling interval h , and the low-level search tighten some control limits and relax the others according to the types and magnitudes of the process shifts in order to achieve higher detection effectiveness and to satisfy the constraint on ATS_0 (Equation (6.28)). Ultimately, all the three levels aim at minimizing the objective function C_{total} (Equation (6.27)). The search procedures are outlined below.

- (1) Initialize C_{min} as a very large number, say 10^7 (C_{min} is used to store the minimum value of C_{total}).
- (2) Calculate K by Equation (6.25).
- (3) At the high-level, the Successive Quadratic Estimation method is used to search the optimal value of M .
- (4) At the mid-level, for a given value of M determined in step (3), the sample size n is increased from two in steps of one until C_{total} cannot be further reduced. The sampling interval h is determined in the mean time (by Equation (6.29)).
- (5) At the low-level, search $\alpha_{\bar{x}l}$, $\alpha_{\bar{x}u}$, α_{sl} and α_{su} for given value of M , n and h (the details are given in Section 6.3.5). It ensures the satisfaction of constraint on ATS_0 . For any given set of $(\alpha_{\bar{x}l}, \alpha_{\bar{x}u}, \alpha_{sl}, \alpha_{su})$,
 - (5.1) Calculate $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s by Equation (6.17).
 - (5.2) Calculate the manpower cost C_{man} and the quality cost $C_{quality}$ by Equations (6.31) and (6.34).
 - (5.3) Calculate the total cost C_{total} by Equation (6.30).
 - (5.4) If the calculated C_{total} is smaller than the current C_{min} , replace the latter by the former, and the current values of M , n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s are stored as the temporary optimal design.
- (6) When the entire search is completed, the final C_{min} is the minimum C_{total} and the corresponding optimal values of M , n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s are also determined.

6.4.4 Comparative study

Similar to the study discussed in Section 4.4.4 in Chapter 4, a 2^k factorial experiment (Montgomery 2001) is carried out in this section to compare the performance

(measured by the total cost C_{total}) of the **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart with that of a conventional $\bar{X}$ & S chart as well as a **VEIIN** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits) chart. Like the **VEIIM** chart, the **VEIIN** chart is designed to minimize the total cost C_{total} , but it only has its charting parameters n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s optimized (namely, only the middle and low level optimization is conducted) and uses a fixed amount of manpower M_0 as used by the conventional $\bar{X}$ & S chart (that is, the high-level optimization is omitted). It is assumed that the conventional $\bar{X}$ & S chart uses a sample size of five. The control limits of the three charts (the conventional, **VEIIN** and **VEIIM** charts) are adjusted so that the resultant ATS_0 for each case is always equal to the specified value τ .

Seven factors, τ , USL , Q , w , $MTBO$, C_m and C_k are studied. Each of them varies at two levels as shown below.

τ	[200, 600]
USL	[3, 6]
Q	[100, 600]
w	[0.3 Q , 3 Q]
$MTBO$	[100, 400]
C_m	[5, 15]
C_k	[0.5, 10]

Similar to the previous study (Section 4.4.4 in Chapter 4), the inspection rate w is expressed as a function of the production rate Q and the in-control μ_0 and σ_0 are fixed at zero and one, respectively.

The fixed amount of manpower M_0 used by both the conventional and **VEIIN** charts are determined by (see Equation (6.29))

$$M_0 = \frac{n_0}{h \cdot w} \quad (6.35)$$

Where ($n_0 \equiv 5$) is the sample size used by the conventional $\bar{X}$ & S chart. Further, by referring to Equation (6.19),

$$M_0 = \frac{n_0}{\alpha \cdot \tau \cdot w} \quad (6.36)$$

Where α is the joint type I error probability of a 3-sigma $\bar{X}$ & S chart,

It is assumed that both the mean shift δ_μ and the standard deviation shift δ_σ of the process follow the normal distributions, $\delta_\mu \sim N(0.8, 0.2^2)$ and $\delta_\sigma \sim N(1.2, 0.1^2)$. Therefore, the joint density function $f(\delta_\mu, \delta_\sigma)$ used in Equation (6.33) for this study can be determined by (referring to Equation (6.26))

$$f(\delta_\mu, \delta_\sigma) = \frac{1}{2 \times 0.2 \times 0.1 \times \pi} \exp \left\{ -0.5 \times \left[\left(\frac{\delta_\mu - 0.8}{0.2} \right)^2 + \left(\frac{\delta_\sigma - 1.2}{0.1} \right)^2 \right] \right\} \quad (6.37)$$

The total number of runs (different combinations of the values of the seven factors) of the whole factorial experiment is equal to 128. For each of the 128 runs, the conventional $\bar{X}$ & S chart, the **VEIIN** chart, and the **VEIIM** chart are designed and all of them always generate the same in-control ATS_0 (equal to the specified value of τ). The following three responses are investigated.

R_{M-C} the ratio between the total cost of the **VEIIM** chart and that of the conventional $\bar{X}$ &S chart.

R_{S-C} the ratio between the total cost of the **VEIIN** chart and that of the conventional $\bar{X}$ &S chart.

R_M the ratio between the optimal manpower M of the **VEIIM** chart and the manpower M_0 for the conventional and **VEIIN** charts.

The grand averages, $\bar{R}_{S-C}$, $\bar{R}_{M-C}$ and $\bar{R}_M$, of the three responses over the 128 runs are 0.6673, 0.3941 and 9.24, respectively. The value of $\bar{R}_{S-C}$ indicates that, from an overall viewpoint (considering different combinations of τ , USL , Q , w , $MTBO$, C_m and C_k), the **VEIIN** chart reduces the total cost C_{total} by about 33% compared to the conventional $\bar{X}$ &S chart. This improvement is achieved by optimizing the charting parameters n , h , $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s , and UCL_s . The value of $\bar{R}_{M-C}$ shows that the **VEIIM** chart is able to further reduce C_{total} by about 27% (the C_{total} of the **VEIIM** chart is less than half of that of the conventional $\bar{X}$ &S chart). This further reduction in total cost is completely attributable to the optimal deployment of manpower in the SPC scheme, suggesting that the optimal deployment of manpower is an effective method to achieve better economic results in SPC.

Finally, the value of the grand average $\bar{R}_M$ ($= 9.24$) of the third response shows that the conventional $\bar{X}$ &S chart usually uses far less amount of manpower than needed for the minimization of the total cost. The deployment of more manpower leads to a significant reduction in quality cost that outweighs the increase in the manpower cost.

The effects of the seven factors (τ , USL , Q , w , $MTBO$, C_m , C_k) on the two responses ($\bar{R}_{M-C}$ and $\bar{R}_M$) are also calculated and listed in Table 6.1. A positive effect on R_{M-C} means that the **VEIIM** chart may achieve a more significant cost reduction when the corresponding factor is smaller and *vice versa*. For example, factor $MTBO$ has the most significant positive impact (0.1171) on R_{M-C} . Thus, when $MTBO$ is smaller, the **VEIIM** chart will reduce the total cost to a greater degree. The effects (on R_{M-C}) of the factors indicate that, the superiority of the **VEIIM** chart over the conventional $\bar{X}$ & S is more notable when τ , Q , w , C_k are high and/or USL , $MTBO$, C_m are low.

Table 6.1 Effects of seven factors

No.	Factors		Effect	
			On R_{M-C}	On R_M
1	τ	minimum allowable in-control ATS	-0.1145	4.0889
2	USL	upper specification limit	0.0892	-7.8066
3	Q	no. of units produced per unit time	-0.1630	8.3698
4	w	inspection rate per operator	-0.1193	6.8887
5	$MTBO$	mean time between out-of-control cases	0.1171	-9.5132
6	C_m	manpower cost per inspector per unit time	0.0678	-3.3373
7	C_k	cost required to fix an out-of-specification unit	-0.2318	13.3896

On the other hand, a positive effect on R_M means that the amount of manpower should be increased when the corresponding factor is large and *vice versa*. For example, the production rate Q has a positive effect on R_M . If it is high, more manpower should be allocated to increase the sample size and/or reduce the sampling interval, so that the out-of-control cases can be signaled as quickly as possible and fewer defective units will be produced. As a general guideline, a relatively large

amount of manpower M should be deployed if (the following points are listed in descending order of their effects on R_M)

- (1) The cost for fixing an out-of-specification product (C_k) is high; and/or
- (2) The production rate (Q) is high; and/or
- (3) The inspection rate (w) per operator is high; and/or
- (4) The required in-control ATS_0 (τ) is high; and/or
- (5) The mean time between out-of-control cases ($MTBO$) is small; and/or
- (6) The upper specification limit (USL) is small; and/or
- (7) The manpower cost (C_m) is low.

6.4.5 Example

A manufacturing factory produces a type of part by an injection molding process. The thickness x of the part is a key dimension and specified as $80 \pm 0.006mm$. The mean of x can be easily adjusted to the nominal value ($80mm$) at the center between the lower and upper specification limits. During a pilot run, it is found that the distribution of x can be well approximated by a normal distribution and the standard deviation σ_0 is very close to $0.00142mm$. The control limits should make the in-control ATS_0 equal to about 370 hours. The above and some other specifications are summarized as follows.

τ = 370 hours, the minimum allowable in-control ATS_0 .

USL = $80.006mm$, the upper specification limit.

Q = 300, the number of units produced per hour.

w = 150, the number of units inspected by an inspector per hour.

C_m = \$8.00, the manpower cost per inspector per hour.

- C_k = \$5.80, the cost to fix an out-of-specification unit in the workshop.
- σ_0 = 0.00142mm, the in-control standard deviation.

Originally, a conventional $\bar{X}$ & S chart is used to carry out SPC. An operator takes a sample of size five for every one hour (control limits are adjusted so that the resultant ATS_0 is approximately equal to 370 hours). The amount of manpower used is equal to (Equation (6.35))

$$M_0 = n_0 / (h \times w) = 5 / (1 \times 150) = 0.033$$

That is, the operator spends about two minutes per hour on SPC inspection.

During the operation of this conventional $\bar{X}$ & S chart, the sample values ($\hat{\delta}_{\mu,i}$, $\hat{\delta}_{\sigma,i}$) (calculated by Equations (6.1) and (6.2)) of the process shifts ($\delta_{\mu,i}$, $\delta_{\sigma,i}$) are observed and recorded following the three-phase SPC scenario discussed in Section 4.2 in Chapter 4. After running the conventional $\bar{X}$ & S chart for 2475 hours (excluding the shutdown time), fifteen out-of-control cases are observed and the fifteen pairs of the process shifts ($\hat{\delta}_{\mu,i}$, $\hat{\delta}_{\sigma,i}$) ($i = 1, \dots, 15$) are collected (see Table 6.2). Meanwhile, the mean time between two consecutive out-of-control cases can be estimated by

$$MTBO \approx \frac{2475}{15} = 165$$

Now, the QA engineer decides to make use of the estimated ($\delta_{\mu,i}$, $\delta_{\sigma,i}$) and $MTBO$ to design a **VEIIN** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits) chart and a **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart, in order to reduce the total cost C_{total} (non-parametric approach). The computer program **veiim.c** completes

the design of the **VEIIM** chart in 4.359 seconds of CPU time in a personal computer (Pentium IV 2.4GHz).

Table 6.2 Sample values of the process shifts

No.	$\hat{\delta}_{\mu,i}$	$\hat{\delta}_{\sigma,i}$	No.	$\hat{\delta}_{\mu,i}$	$\hat{\delta}_{\sigma,i}$
1	0.770	1.187	9	0.771	1.269
2	0.737	1.178	10	0.958	1.184
3	0.706	1.238	11	0.973	1.211
4	0.926	1.073	12	1.002	1.260
5	0.452	1.295	13	0.675	0.991
6	0.281	1.138	14	0.723	1.386
7	0.740	1.021	15	0.866	1.207
8	0.777	1.175			

While the **VEIIN** chart is designed by using the same manpower ($M_0 = 0.033$) as the conventional chart, the design of the **VEIIM** chart searches for an optimal amount of manpower that minimizes the total cost. The charting parameters and the resultant costs of all three charts are listed in Table 6.3. The control limits of the charts are determined so that all three charts generate the same in-control ATS_0 equal to the specified value of 370.

As shown in Table 6.3, the total cost C_{total} of the conventional $\bar{x}$ & S chart is high. This is because the deployed manpower is inadequate and the resultant quality cost is high. The **VEIIN** chart has substantially reduced the total cost even though it employs the same amount of manpower as the conventional $\bar{x}$ & S chart. The improvement is attributable to the optimization of the sample size n , sampling interval h and the control limits $LCL_{\bar{x}}$, $UCL_{\bar{x}}$, LCL_s and UCL_s . Finally, the **VEIIM** chart has further cut down the total cost to a significant degree. The design of the **VEIIM** chart

Table 6.3 The three control charts in the example

Chart	Sample size n	Sampling interval, hr h	Control limits				Manpower M	Cost, \$		
			$LCL_{\bar{x}}$	$UCL_{\bar{x}}$	LCL_s	UCL_s		C_{man}	$C_{quality}$	C_{total}
Traditional $\bar{X}$ &S chart	5	1.000	79.99805	80.00195	0.000095	0.002966	0.033	0.27	9.36	9.63
VEIIN chart	20	4.000	79.99865	80.00074	0.000562	0.002150	0.033	0.27	4.34	4.61
VEIIM chart	30	1.387	79.99889	80.00070	0.000701	0.00207	0.144	1.15	1.44	2.59

includes not only the optimization of n , h , $LCL_{\bar{X}}$, $UCL_{\bar{X}}$, LCL_s , UCL_s , but also the search for the optimal amount of manpower M . The optimal value of M in this example is 0.1442, about five times as large as the manpower used by both the conventional and **VEIIN** charts. In this example, the total cost C_{total} of the **VEIIM** chart is only equal to 0.269 of that of the conventional $\bar{X}$ & S chart and 0.562 of that of the **VEIIN** chart.

6.4.6 Discussions

This section designs the **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart that minimize the expected total cost associated with an $\bar{X}$ & S control chart scheme by optimizing the deployed manpower. Meanwhile, the sample size, sampling interval, and control limits of the charts are also optimized. The comparative study shows that the proposed chart is able to reduce the total cost C_{total} by nearly 61% on average, and the optimal deployment of manpower alone may reduce C_{total} by 27%. As in the previous studies, it is also found that, much more manpower should be deployed for most SPC schemes from the viewpoint of total cost reduction. Some general guidelines are also provided in this design for deploying manpower to an $\bar{X}$ & S control chart monitoring a single process.

6.5 Optimization design of the integrated $\bar{X}$ & S control chart system that minimizes the total cost

This section develops the **VEISM** ($\bar{X}$ & S charts / economical design / independent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system for monitoring process shifts in mean and variance in a multi-stage and multi-stream manufacturing system. The design procedure of the **VEISM** chart system is similar to the **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart discussed in Section 6.4. While the model **VEIIM** is for a single pair of $\bar{X}$ & S charts monitoring a single process, the integrated design **VEISM** consists of several pairs of $\bar{X}$ & S charts, each of which is used to monitor a critical process stage in a manufacturing system.

6.5.1 Input parameters

The parameters s , g_i , USL_i , $\mu_{0,i}$, $\sigma_{0,i}$, t_i , Q , $MTBO_i$, $C_{k,i}$, C_m , τ and $c_{min,i}$ (out-of-control (or critical) value of the process capability index in the i th stage) are required in designing this control chart system where the process stages are independent.

6.5.2 Optimization model

Based on the above parameters, the design of the **VEISM** chart system can be conducted by using the following nonlinear optimization model.

$$\text{Minimize:} \quad C_{total} = C_{quality} + C_{man} \quad (6.38)$$

$$\text{Subject to:} \quad ATS_0 \geq \tau \quad (6.39)$$

$$\text{Design variables:} \quad M, n_i, h_i, LCL_{\bar{X},i}, UCL_{\bar{X},i}, LCL_{S,i}, UCL_{S,i} \quad (i = 1, \dots, s)$$

Where the objective function C_{total} is a function of $(s, g_i, USL_i, \mu_{0,i}, \sigma_{0,i}, t_i, Q, MTBO_i, C_{k,i}, C_m, \tau, c_{min,i}, M, n_i, h_i, LCL_{\bar{x},i}, UCL_{\bar{x},i}, LCL_{s,i}$ and $UCL_{s,i})$. The parameters $n_i, h_i, LCL_{\bar{x},i}, UCL_{\bar{x},i}, LCL_{s,i}$ and $UCL_{s,i}$ are the sample size, sampling interval, and control limits of the $\bar{X}$ charts and S charts in the i th stage. The charting parameters $n_i, h_i, LCL_{\bar{x},i}, UCL_{\bar{x},i}, LCL_{s,i}$ and $UCL_{s,i}$ are the same as $n, h, LCL_{\bar{x}}, UCL_{\bar{x}}, LCL_s$ and UCL_s of the single process chart system (discussed in Section 6.4) except that all the charting parameters here are designed in an integrative manner. The in-control ATS_0 can be calculated by Equation (3.12) derived in Section 3.2.1.3 in Chapter 3.

The manpower cost C_{man} per unit time is given by

$$C_{man} = C_m \times M \quad (6.40)$$

The quality cost $C_{quality}$ per unit time is the sum of the costs incurred by the out-of-control cases occurring at each process stage (see Equation (4.43) in Chapter 4).

$$C_{quality} = \sum_{i=1}^s C_{q,i} \quad (6.41)$$

Where $C_{q,i}$ is the quality cost due to the process shifts $(\delta_{\mu,i}, \delta_{\sigma,i})$ at the i th stage.

According to Equation (6.32)

$$C_{q,i} = \int_0^\infty \int_{-\infty}^\infty \left[\frac{1}{MTBO_i} \cdot q_i(\delta_{\mu,i}, \delta_{\sigma,i}) \cdot L_i(\delta_{\mu,i}, \delta_{\sigma,i}) \cdot f(\delta_{\mu,i}, \delta_{\sigma,i}) \right] d\delta_{\mu,i} d\delta_{\sigma,i} \quad (6.42)$$

Where

$$q_i(\delta_{\mu,i}, \delta_{\sigma,i}) = Q \cdot ats_i(\delta_{\mu,i}, \delta_{\sigma,i}) \quad (6.43)$$

$$ats_i(\delta_{\mu,i}, \delta_{\sigma,i}) = \left(\frac{1}{z_i(\delta_{\mu,i}, \delta_{\sigma,i})} - 0.5 \right) \times h_i \quad (6.44)$$

The power $z_i(\delta_{\mu,i}, \delta_{\sigma,i})$ is the power of the out-of-control stage i and can be calculated by Equation (5.14) derived in Section 5.2.3 in Chapter 5. The power $z_i(\delta_{\mu,i}, \delta_{\sigma,i})$ does not consider the false alarm probability of the parallel streams as it has negligible

effect on the system ATS . The expected loss function $L_i(\delta_{\mu,i}, \delta_{\sigma,i})$ is the loss incurred in stage i and can be calculated by using Taguchi's loss function (see the discussions in Sections 4.3 and 4.4 in Chapter 4).

$$L_i(\delta_{\mu,i}, \delta_{\sigma,i}) = K_i(\delta_{\sigma,i}^2 \sigma_{0,i}^2 + \delta_{\mu,i}^2 \sigma_{0,i}^2) \quad (6.45)$$

Finally, Equation (6.42) can be rewritten as (see Equation (6.33))

$$C_{q,i} = \frac{K_i \cdot Q \cdot \sigma_{0,i}^2}{MTBO_i} \int_0^\infty \int_{-\infty}^\infty [(\delta_{\sigma,i}^2 + \delta_{\mu,i}^2) \cdot ats_i(\delta_{\mu,i}, \delta_{\sigma,i}) \cdot f(\delta_{\mu,i}, \delta_{\sigma,i})] d\delta_{\mu,i} d\delta_{\sigma,i} \quad (6.46)$$

The cost coefficient K_i can be calculated from the cost component $C_{k,i}$ associated with the specification limits (see Equation (6.25)).

$$K_i = \frac{C_{k,i}}{(USL_i - \mu_{0,i})^2} \quad (6.47)$$

If both the mean shift $\delta_{\mu,i}$ and the standard deviation shift $\delta_{\sigma,i}$ in stage i are assumed to follow the normal distributions, $\delta_{\mu,i} \sim N(\mu_{\delta\mu,i}, \sigma_{\delta\mu,i}^2)$ and $\delta_{\sigma,i} \sim N(\mu_{\delta\sigma,i}, \sigma_{\delta\sigma,i}^2)$. Then, the joint density function $f(\delta_{\mu,i}, \delta_{\sigma,i})$ used in Equation (6.46) can be determined by (referring to Equation (6.26)).

$$f(\delta_{\mu,i}, \delta_{\sigma,i}) = \frac{1}{2 \times \sigma_{\delta\mu,i} \times \sigma_{\delta\sigma,i} \times \pi} \exp \left\{ -0.5 \times \left[\left(\frac{\delta_{\mu,i} - \mu_{\delta\mu,i}}{\sigma_{\delta\mu,i}} \right)^2 + \left(\frac{\delta_{\sigma,i} - \mu_{\delta\sigma,i}}{\sigma_{\delta\sigma,i}} \right)^2 \right] \right\} \quad (6.48)$$

The integration (6.42) can also be approximated by the summation based on N pairs of process shifts $(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i})$ (non-parametric approach).

$$C_{q,i} = \frac{1}{N} \sum_{i=1}^N \frac{1}{MTBO_i} \cdot q_i(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i}) \cdot L_i(\hat{\delta}_{\mu,i}, \hat{\delta}_{\sigma,i}) \quad (6.49)$$

The calculation of C_{total} can be summarized as follows.

- (1) Calculate manpower cost C_{man} by Equation (6.40).

- (2) Calculate quality cost $C_{quality}$ by summing up the contribution ($C_{q,i}$) from each stage of the chart system using Equation (6.41). For a particular stage i , $C_{q,i}$ (in Equation (4.42)) can be evaluated by applying the Gauss-Legendre translation rule to a few integration points. At each of these points
 - (2.1) Calculate $ats_i(\delta_{\mu,i}, \delta_{\sigma,i})$ by Equation (6.44) and Equation (5.14) derived in Section 5.2.3 in Chapter 5.
 - (2.2) Calculate the loss function $L_i(\delta_{\mu,i}, \delta_{\sigma,i})$ by Equation (6.45).
 - (2.3) Calculate the density function $f(\delta_{\mu,i}, \delta_{\sigma,i})$ by Equation (6.48).
- (3) Finally, calculate the total cost C_{total} by Equation (6.38).

6.5.3 Optimization search algorithm

The optimal values of M , n_i , h_i , $LCL_{\bar{x},j}$, $UCL_{\bar{x},j}$, $LCL_{s,i}$, and $UCL_{s,i}$ can be searched in a four-level optimization process. The first-level search uses the Successive Quadratic Estimation method (Powell 1964) to find the optimal value of M . For a given value of M , a gradient based second-level search (discussed in Section 3.2.2 in Chapter 3, note that ΔATS_j in Equation (3.40) will be replaced by $\Delta C_{total,j}$ in this case) is evoked to find the optimal values of n_i and h_i . Then, for a given set of (M, n_i, h_i) , a third-level dynamic search (discussed in Section 3.2.1 in Chapter 3) distributes the detection power (or type I error α_i) of the chart system among the s stages and ensures the satisfaction of the constraint on ATS_0 (Equation (6.39)). Finally, the fourth-level search allocates the power between the $\bar{X}$ chart and S chart (i.e. searches the optimal values of type I errors $\alpha_{\bar{x}l,j}$, $\alpha_{\bar{x}u,j}$, $\alpha_{sl,i}$, and $\alpha_{su,i}$ at the two sides of the $\bar{X}$ chart and S chart) within each stage (i.e. optimizing the control limits $LCL_{\bar{x},j}$, $UCL_{\bar{x},j}$, $LCL_{s,i}$, and $UCL_{s,i}$).

The fourth-level search algorithm has been discussed for a single process in Section 6.3.5. The overall search procedures are outlined below.

- (1) Initialize C_{min} as a very large number, say 10^7 (C_{min} is used to store the minimum value of C_{total}).
- (2) Calculate K_i by Equation (6.47).
- (3) At the first-level, the optimal value of M is searched.
- (4) At the second-level, for a given value of M , the optimal values of n_i and h_i are searched.
- (5) For a given set of (M, n_i, h_i) , carry out the third-level search to investigate different combinations of $(\alpha_1, \dots, \alpha_s)$. Each set of $(\alpha_1, \dots, \alpha_s)$ will automatically make ATS_0 equal to τ .
- (6) For a given set of (M, n_i, h_i, α_i) , the fourth-level search finds the optimal values of $\alpha_{\bar{x}l,i}$, $\alpha_{\bar{x}u,i}$, $\alpha_{sl,i}$, and $\alpha_{su,i}$ (by using Equations (6.20) to (6.23) derived in Section 6.3.5) for each stage. For any given set of $(\alpha_{\bar{x}l,i}, \alpha_{\bar{x}u,i}, \alpha_{sl,i}, \text{ and } \alpha_{su,i})$ ($i = 1, \dots, s$),
 - (6.1) Calculate $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$, $LCL_{s,i}$, and $UCL_{s,i}$ by Equation (6.17).
 - (6.2) Calculate the total cost C_{total} by the aforementioned procedure (described in Section 6.5.2).
 - (6.3) If the calculated C_{total} is smaller than the current C_{min} , replace the latter by the former. And the current values of M , n_i , h_i , $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$, $LCL_{s,i}$, $UCL_{s,i}$ are stored as the temporary optimal solution.

- (7) When the entire search is completed, the final C_{min} is the minimum C_{total} and the corresponding optimal M , n_i , h_i , $LCL_{\bar{x},i}$, $UCL_{\bar{x},i}$, $LCL_{s,i}$, $UCL_{s,i}$ are also finalized.

The model discussed in this section is applicable to the manufacturing systems where the process stages are independent (see Chapter 4 for detail discussion). The consideration of dependency between the process stages will need two more specifications, that is V_i (vector $[v_1, v_2, \dots]^T$ indicating the cause stage or upstream influencing stage numbers) and Δ_i (induced process shifts, i.e. the mean and/or standard deviation shifts undergone by the i th stage due to the mean and/or standard deviation shifts of the cause stages) in order to identify the effects between stages and to determine the induced process shifts due to the process shifts of the influencing stages. The calculations of $ats_i(\delta_{\mu,i}, \delta_{\sigma,i})$ (Equation (6.44)) and $L_i(\delta_{\mu,i}, \delta_{\sigma,i})$ (Equation (6.45)) must also be modified to handle both the process shifts in the out-of-control stage and the induced process shifts due to the dependency between the process stages.

6.5.4 Discussions

This section designs the **VEISM** ($\bar{X}$ &S charts / economical design / independent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system for monitoring process shifts both in mean and variance in a multi-stage and multi-stream manufacturing system. The optimization model and the design algorithms have been discussed in details. The performance of the **VEISM** chart system has not been analyzed through a numerical study. It can be carried out by straightforwardly following the numerical study of the **VEIIM** ($\bar{X}$ &S charts / economical design / independent process / single control chart / sample size, sampling

interval, control limits, and manpower) chart for single process discussed in Section 6.4.4.

6.7 Summary

This chapter develops the **VEIIN** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, and control limits) chart, **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower) chart and **VEISM** ($\bar{X}$ & S charts / economical design / independent processes / control chart system / sample size, sampling interval, control limits, and manpower) chart system. The first two models are designed for monitoring a single process, whereas the third model presents the economical design of the integrated control chart system. The model **VEIIN** illustrates how the $\bar{X}$ & S charts can be continuously improved during the use of the chart, and the model **VEIIM** optimally deploys the manpower to the $\bar{X}$ & S charts in order to minimize the expected total cost associated with the implement of SPC. Finally, the concepts of the models **VEIIN** and **VEIIM** are applied to developing the **VEISM** chart system for multi-stage and multi-stream manufacturing system. The quality costs are estimated using Taguchi's loss function.

The design algorithm of the proposed models can be easily computerized. More importantly, these models will not in any sense increase the difficulties for operators to run and understand the control charts.

Chapter 7

Conclusions and Recommendations

This chapter draws the conclusions and summarizes the contributions of the whole research project. It also provides the recommendation for the future research.

7.1 Conclusions

This thesis is to study the optimization designs of the integrated control chart systems for multi-stage and multi-stream manufacturing systems. The whole project is comprised of two phases.

The first phase (Chapter 3 and Chapter 4) presents the optimization designs of the integrated $\bar{X}$ control chart systems for monitoring process mean. In this phase, both statistical and economical designs of the $\bar{X}$ control chart systems are presented. In Chapter 3, two statistical design models of the $\bar{X}$ control chart schemes are presented, where the performance of the $\bar{X}$ chart systems is evaluated by the statistical measure, i.e. the out-of-control *ATS*. The first model optimizes the control limits of the $\bar{X}$ charts at each stage of the manufacturing processes. It increases the effectiveness of the $\bar{X}$ chart system significantly. This model can be applied to the situations where the sample sizes and sampling intervals are preferred to be fixed. In the second model, the design algorithm optimizes the sample sizes, sampling intervals, and control limits together. It further enhances the effectiveness of the $\bar{X}$ control chart system. The models are discussed for both independent and interdependent manufacturing systems. In Chapter 4, economical designs of the $\bar{X}$ control chart systems are presented, where the performance of the $\bar{X}$ chart systems is evaluated in terms of monetary value. It considers random mean shifts and adopts a three-phase SPC scenario. The losses in

quality (or the costs) are measured by Taguchi's loss function. However, an extended section outlines how the defective rate of a process can also be used to measure the quality losses. The economical designs mainly focus on the optimal deployment of manpower to SPC so that the expected total cost incurred in the implement of SPC is minimized.

The second phase (Chapter 5 and Chapter 6) of the thesis presents the optimization designs of the integrated $\bar{X}$ & S control chart systems for monitoring process shifts in both mean and variance. In Chapter 5, two statistical models of the $\bar{X}$ & S control chart systems are presented. The first model optimizes the control limits of the $\bar{X}$ & S charts, whereas the second model optimizes the sample sizes, sampling intervals, and control limits together so that the detection effectiveness of the $\bar{X}$ & S chart systems is enhanced. In Chapter 6, optimization design algorithms for the economical designs of the integrated $\bar{X}$ & S control chart systems are presented. Similar to the models presented in Chapter 4, the models in Chapter 6 also focus on the optimal deployment of manpower to SPC so that the expected total cost is minimized. The models also consider random process shifts (i.e. mean and standard deviation shifts) and adopt a three-phase SPC scenario.

Extensive numerical studies are presented in each chapter that analyze the performance of the proposed models and compare it with the performance of the conventional Shewhart chart systems. All of the design algorithms have been implemented in the computer program.

A total of ten models have been developed in this research project. Out of the ten models, six models are for the designs of the integrated control chart system for multi-stage and multi-stream manufacturing systems. The other four models are for a single process, and are served as the basis for the economical designs of the integrated

control chart systems. The implementations of the designed models will be beneficial to many manufacturing and non-manufacturing sectors. The following table (Table 7.1) summarizes the designed models and their applications.

Table 7.1 The designed models and their applications

No.	Model	Application
1	MSISL ($\bar{X}$ chart / statistical design / independent processes / control chart system / control limits) or MSDSL ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / control limits)	<ul style="list-style-type: none"> • Multi-stage independent or interdependent processes • Mean shift is dominant • Sample size and sampling interval are not allowed to be adjusted • Purpose is to reduce out-of-control average time to signal
2	MSISN ($\bar{X}$ chart / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits) or MSDSN ($\bar{X}$ chart / statistical design / interdependent processes / control chart system / sample size, sampling interval, and control limits)	<ul style="list-style-type: none"> • Multi-stage independent or interdependent processes • Mean shift is dominant • Sample size, sampling interval, and control limits all are allowed to be adjusted • Purpose is to reduce out-of-control average time to signal
3	MEIIN ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, and control limits)	<ul style="list-style-type: none"> • Single process • Mean shift is dominant • Sample size, sampling interval, and control limits all are allowed to be adjusted • Purpose is to reduce quality loss
4	MEIIM ($\bar{X}$ chart / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower)	<ul style="list-style-type: none"> • Single process • Mean shift is dominant • Sample size sampling interval and control limits all are allowed to be adjusted • Purpose is to reduce total cost
5	MEDSM ($\bar{X}$ chart / economical design / interdependent processes / control chart system / sample size, sampling interval, control limits, and manpower)	<ul style="list-style-type: none"> • Multi-stage interdependent processes • Mean shift is dominant • Sample size sampling interval and control limits all are allowed to be adjusted • Purpose is to reduce total cost

Table 7.1 The designed models and their applications (cont'd)

No.	Model	Application
6	VSISL ($\bar{x}$ & S charts / statistical design / independent processes / control chart system / control limits)	<ul style="list-style-type: none"> • Multi-stage independent processes • Both mean and standard deviation shifts are need to be monitored • Sample size and sampling interval are not allowed to be adjusted • Purpose is to reduce out-of-control average time to signal
7	VSISN ($\bar{x}$ & S charts / statistical design / independent processes / control chart system / sample size, sampling interval, and control limits)	<ul style="list-style-type: none"> • Multi-stage independent processes • Both mean and standard deviation shifts are need to be monitored • Sample size, sampling interval, and control limits all are allowed to be adjusted • Purpose is to reduce out-of-control average time to signal
8	VEIIN ($\bar{x}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, and control limits)	<ul style="list-style-type: none"> • Single process • Both mean and standard deviation shifts are need to be monitored • Sample size, sampling interval, and control limits all are allowed to be adjusted • Purpose is to reduce quality loss
9	VEIIM ($\bar{x}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower)	<ul style="list-style-type: none"> • Single process • Both mean and standard deviation shifts are need to be monitored • Sample size, sampling interval, and control limits all are allowed to be adjusted • Purpose is to reduce total cost
10	VEISM ($\bar{x}$ & S charts / economical design / independent processes / control chart system / sample size, sampling interval, control limits, and manpower)	<ul style="list-style-type: none"> • Multi-stage independent processes • Both mean and standard deviation shifts are need to be monitored • Sample size, sampling interval, and control limits all are allowed to be adjusted • Purpose is to reduce total cost

7.2 Summary of the contributions

The algorithms developed in this research project have several advantages over the conventional chart schemes. The original contributions made in this research are summarized below.

Statistical designs of the integrated $\bar{X}$ control chart systems for monitoring process mean

It is the first time that the parameters of the $\bar{X}$ control charts used in a multi-stage and multi-stream manufacturing system are designed in an integrative and optimal manner. The optimization designs substantially improve the effectiveness of the $\bar{X}$ control chart schemes as a whole. Such improvement is achieved by optimizing the sample sizes, sampling intervals and control limits. The optimization designs also ensure that no extra resources (operators and measuring instruments) are required to maintain the quality control activities and the false alarm rate is not increased. The floor operators can utilize and understand the integrated $\bar{X}$ control chart systems as easily as for the conventional system. The mathematical models are well developed and the input parameters can be specified relatively easily. Specifically,

- (1) The formula has been developed to calculate the in-control ATS_0 of an $\bar{X}$ control chart system used in a multi-stage and multi-stream manufacturing system.
- (2) The formula has been developed to calculate the out-of-control ATS value of an $\bar{X}$ control chart system used in the independent or interdependent multi-stage and multi-stream manufacturing processes.
- (3) Techniques to handle the dependency between the process stages in a multi-stage and multi-stream manufacturing system have been developed.
- (4) A new gradient based search algorithm has been developed. It is very efficient to search the optimal sample sizes and sampling intervals of the stages in a system.
- (5) A dynamic search algorithm has been proposed to determine the optimal values for the control limits.

- (6) Comparative studies of the influence of different parameters on the performance of the $\bar{X}$ chart system have been carried out. Based on the results of the studies, some useful guidelines have been provided to aid the users to adjust the sample size, sampling interval, and control limits of each $\bar{X}$ chart in a system.

Economical designs of the integrated $\bar{X}$ control chart systems for monitoring process mean

It is the first time that the deployment of manpower to an $\bar{X}$ chart or an $\bar{X}$ chart system has been studied. Unlike in the traditional economical chart designs, most input specifications required by these designs can be determined relatively easily by the practitioners or from engineering drawings. The optimization designs significantly reduce the expected total cost incurred in the implement of SPC. These designs will aid the management in deciding the appropriate amount of manpower required to obtain the greatest gain from the implement of SPC. It is also found that, in most SPC schemes, the allocated amount of manpower is far less than needed and, consequently, the total cost can be significantly reduced by making some managerial arrangement to increase SPC manpower. Particularly,

- (1) The design algorithms optimize the deployed manpower as well as the sample size, sampling interval, and control limits of the $\bar{X}$ control charts using in-control ATS_0 as the constraint.
- (2) The optimization designs consider the whole range of the random mean shifts, rather than just one or a few fixed values. It substantially increases the effectiveness of the $\bar{X}$ control chart system and the validity of the optimal solution compared to the conventional approaches.

- (3) Taguchi's loss function has been used to measure the quality losses (or the costs) incurred in the out-of-control cases. The minimization of the Taguchi's loss function is consistent with one's effort for continuous quality improvement. However, an additional section outlines an approach of measuring the quality costs by defective rate of the process.
- (4) Comparative studies of the influence of different parameters on the total cost C_{total} have been carried out. Based on the results of the studies, some useful and universally applicable guidelines have been brought forth to aid the users in deciding the appropriate amount of manpower for a particular application.
- (5) The economical design models are implemented through the newly proposed three-phase SPC operational scenario rather than the conventional two-phase SPC scenario. This new scenario enables one to design the control charts according to the actual working conditions of a particular process, and to update a workable chart with an average performance to an optimal chart with a first-rate functioning. The optimization designs also propose a scenario for continuously improving the control charts during the use of the charts.

Statistical designs of the integrated $\bar{X}$ & S control chart systems for monitoring process mean and variance

It is the first time that the algorithms have been developed for the optimization designs of the integrated $\bar{X}$ & S control chart systems for monitoring process shifts in both mean and variance in different process stages in a manufacturing system. The optimization algorithms design the charting parameters in an integrative and optimal manner. Specifically,

- (1) The design algorithms optimize the sample size, sampling interval, and control limits in order to speed up the signalling of the out-of-control cases. The optimization design ensures that no extra resources for conducting the quality control activities are required and the false alarm rate is not increased.
- (2) These design algorithms relate the in-control and out-of-control system conditions to the process capability index which is more understandable and manageable by the QA engineers, as well as the operators and managers at different levels.
- (3) Based on the results of the comparative studies, some useful and universally applicable guidelines have been brought forth to aid the users to adjust the sample size, sampling interval, and control limits of each $\bar{X}$ & S chart in a system.

Economical designs of the integrated $\bar{X}$ & S control chart systems for monitoring process mean and variance

Algorithms have been developed for the economical designs of the integrated $\bar{X}$ & S control chart systems for monitoring process shifts in both mean and variance in a multi-stage and multi-stream manufacturing system. The design algorithms consider the random mean and standard deviation shifts and adopt the newly proposed three-phase SPC scenario in estimating the sample data of the process shifts (i.e. mean shifts and standard deviation shifts). Similar to the previous economical models for monitoring mean shifts, the models for monitoring process shifts in mean and variance developed here also minimize the expected total cost associated with the implement of SPC based on the optimal deployment of manpower. The sample size, sampling

interval, and control limits of each chart in a system are also optimized in the mean time. The Taguchi's loss function is used to measure the quality losses.

Software systems

The software systems for automating the design procedures of the nine models presented in this thesis have been developed using C computer programming language. The computer program for the model **VEISM** ($\bar{X}$ & S charts / economical design / independent processes / control chart system / sample size, sampling interval, control limits, and manpower) has not been developed. However, it can be developed easily based on the computer program of the model **VEIIM** ($\bar{X}$ & S charts / economical design / independent process / single control chart / sample size, sampling interval, control limits, and manpower). The following table (Table 7.2) lists the models and the corresponding computer programs that have been built up in this research project. The programs have been copied in a CD (Compact Disk) and attached with the thesis.

Table 7.2 The designed models and corresponding computer programs

No.	Model	Computer program
1	MSISL and MSDSL	<i>msisl.c</i> and <i>msdsl.c</i>
2	MSISN and MSDSN	<i>msisn.c</i> and <i>msdsn.c</i>
3	MEIIN	<i>meiin.c</i>
4	MEIIM	<i>meiim.c</i>
5	MEDSM	<i>medsm.c</i>
6	VSISL	<i>vsisl.c</i>
7	VSISN	<i>vsisn.c</i>
8	VEIIN	<i>veiin.c</i>
9	VEIIM	<i>veiim.c</i>

7.3 Recommendations for further extension of the models proposed in this thesis

Each of the ten models proposed in this thesis have been developed considering different practical situations (see Table 7.1), however, they can be further extended in several directions. Some guidelines have been provided below.

- (1) The algorithms for the economical designs of the control chart systems discussed in this thesis (Chapter 4 and Chapter 6) assume that the manufacturing system produces only one type of product. This assumption simplifies the model and helps to focus the discussion on the main topic of the integrated chart systems. However, more than one type of products can be fabricated in a manufacturing system and their relevant costs may be different. Therefore, the proposed design algorithms can be further extended to handle more than one type of products fabricated in a manufacturing system.
- (2) The proposed economical designs algorithms (Chapter 4 and Chapter 6) minimize the expected total cost including manpower cost and quality cost. These models can be further extended to include the trade-off between capital investment for reducing process variability (increase C_{pk}) and reduction in quality cost.
- (3) The algorithm for the deployment of manpower to $\bar{X}$ & S control chart system monitoring process shifts in mean and variance in a manufacturing system has been developed in this thesis (Chapter 6). The proposed model does not consider the dependency between the process stages because the consideration of both the induced mean shift and standard deviation shift pertaining to the interdependency of the process stages makes the formulation very complicated.

The design procedure also becomes too complex that may discourage the QA engineers to use the model. However, the model can be further extended for the deployment of manpower to an $\bar{X}$ & S chart system in an interdependent manufacturing system. The major task is to develop a method to evaluate the quality losses due to the interdependency between the process stages.

- (4) For model simplicity, many researchers (Nelson 1986, Mortell and Runger 1995, Runger and Montgomery 1996) who studied the multiple stream systems use the assumption that parallel streams have the same values for mean, standard deviation and target. The same assumption has been adopted in this thesis. However, different machines may be used in the same stage and their characteristics may be different. So, the algorithm proposed in this thesis can be further extended to consider different machines and different types of charts in a stage of a manufacturing system.
- (5) The performance of the models proposed in this thesis has been studied and compared to other existing models by using either the one-factor-at-a-time method or the factorial experiment. However, further research can be conducted using ANOVA (ANalysis Of VAriance) to check the degree of significance of the relative performance of the models proposed in this thesis
- (6) The proposed models in this thesis assume that the probability distribution of the output from a stage is normal even when one stream goes out of control. The proposed formulae can be modified for considering the actual output distribution.

7.4 Recommendations for future research

The literature on control chart design is quite large and still growing fast. Some recommendations for the future research are given below.

- (1) The Shewhart $\bar{X}$ chart as well as the $\bar{X}$ & S charts are considered as the most widely used variable control charts because of their simplicity for implementation and easy to understand. However, the Shewhart chart is relatively insensitive to small shifts in the process (Montgomery 2001). Two important alternatives to the Shewhart control chart may be used when the shifts in the process are small. They are the CUSUM control chart and the EWMA control chart. Future research can develop the integrated control chart system considering the CUSUM chart and the EWMA chart
- (2) Attribute charts are generally not as informative as variable charts because there is typically more information in a numerical measurement than in a mere classification of the conformity or the nonconformity (Montgomery 2001). However, the attribute charts do have important applications. They can help move the firm's processes toward a zero percentage defective rate (Gitlow *et al.* 1995). They are particularly useful and necessary in service industries, healthcare, and other non-manufacturing sections, since many quality characteristics found in these sections are not easily measured on a numerical scale. Therefore, future research can develop the integrated control chart system considering the attribute control charts such as the p charts and the np charts. The integrated design of the p chart or the np chart will further improve the performance of the chart system.

- (3) Some researchers proposed the algorithms for the designs of the adaptive (or dynamic) control chart schemes, in which either the sample size or sampling interval or both are changed depending on the value of the sampling statistic. Further research can develop the adaptive integrated control chart system for a manufacturing system.

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