Nonlinear control designs for hard disk drive servos

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NONLINEAR CONTROL DESIGNS FOR HARD DISK DRIVE SERVOS

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING
NANYANG TECHNOLOGICAL UNIVERSITY

2005

Nonlinear Control Designs for Hard Disk Drive Servos

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Summary

This thesis presents research results on control issue in head positioning mechatronics for hard disk drive (HDD) servo system. Specifically, nonlinear control algorithms are developed to compensate or overcome control limitations and design tradeoffs due to linear feedback control.

Followed by a brief overview of HDD head positioning servomechanism, the fundamentals of HDD servo system are introduced including control structure and design technology. The performance limitations related to both linear and nonlinear control are investigated.

Fast seek is always associated with large overshoot and long transient process in track-seeking control. Two kinds of nonlinear controllers are developed to improve the seek-settling performance in this thesis. Focused on the constraints of fixed closed-loop poles, Controller 1 proposes a nonlinear damping control to dynamically adjust the locations of closed-loop poles, and to take both advantages of small damping for large tracking errors and large damping for small tracking errors. Different from Controller 1, Controller 2 takes unknown bias/friction torque into the consideration, and develops a nonlinear PID (NPID) controller to improve the settling performance due to bias compensation when the servo modes hand off from

seeking to following.

To improve the seeking performance for both short and long seeking length due to performance requirement and limited actuator power, current estimator based input shaper is developed to generate desired reference profile. Due to the introduction of feedforward input shaper, both tracking error and error rate are limited to a small varied area. This will avoid actuator saturation especially for long track seeking. Two-degree-of-freedom (2DOF) control algorithm is proposed by combining both input shaper and nonlinear feedback controller. Therefore, the performances of fast and smooth settling and disturbance rejection are achieved by nonlinear feedback controller, and performance due to set point changes is improved in the 2DOF control.

Two challenging problems in the track following mode are discussed in the thesis. Narrow-band position error at mid-frequency (500 Hz – 2 kHz) is not effectively rejected in a typical HDD servo loop due to phase stabilization. A nonlinear mid-frequency narrow-band disturbance compensation scheme is proposed. By resetting pole of ordinal first order mode, it presents a 0 dB/decade gain slope while 51.9° phase lead in cascade with a lead filter from describing function analysis. Thus, it is designed to compensate for phase loss due to internal high gain control. This technology would help to cancel the position error induced by disk flutter.

The sensitivity limitation imposed by linear feedback control in the HDD is studied based on Bode's Integral Theorem. A composite nonlinear feedback controller is developed to suppress sensitivity hump in this thesis. Nonlinear PD control law is designed to produce an open-loop high gain in high frequency band. In the low

frequency band, the hard disk is driven by the nominal PID controller to achieve desired disturbance rejection and steady-state performance. As system responds differently to disturbance magnitude, sensitivity hump can be suppressed or even be removed with the increase of disturbance magnitude. As such track mis-registration (TMR) in track-following mode would be reduced and high track density can be achieved.

This thesis presents the fundamental analysis, limitation study and advanced technologies in HDD control design, which would contribute to the achievement of high bandwidth and low power servo system for the high-performance, high density, low cost and small size HDD.

Nomenclature

Unless otherwise specified, the following abbreviations and symbols are used throughout this thesis.

AFC	Adaptive Feedforward Cancellation
ARE	Algebraic Riccati Equation
BPI	Bits Per Inch
CAGR	Compound Annual Growth Rate
DAC	Digital-Analogue-Converter
DFT	Discrete Fourier Transform
DOB	Disturbance Observer
DSA	Dynamic Signal Analyzer
FFT	Fast Fourier Transform
FPC	Flexible Printed Circuit
HDA	Head Disk Assembly
HDD	Hard Disk Drive
IMP	Internal Model Principle
IVC	Initial Value Compensation

LDV	Laser Doppler Vibrometer
LTI	Linear Time-invariant
LQG	Linear Quadratic Gaussian
LTR	Loop Transfer Recovery
NRRO	Non-repeatable Runout
PA	Power Amplifier
PES	Position Error Signal
PID	Proportional-Integral-Derivative
PSD	Power Spectral Density
PTOS	Proximate Time Optimal System
RHP	Right Half Plane
RPM	Revolution Per Minute
RRO	Repeatable Runout
RPT	Robust and Perfect Tracking
TMR	Track Mis-registration
TPI	Tracks Per Inch
VCM	Voice Coil Motor
ZOH	Zero-order Holder

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Chapter 1

Introduction

This thesis deals with nonlinear control of mechatronics with particular attention to Hard Disk Drives (HDDs) head-positioning servomechanism. Theoretical and implemented improvements will be shown either to avoid or to complement linear control theory whenever “unpleasant” features and limitations of linear control prevent certain control objectives from being achieved. Although an emphasis is laid on control applications to the HDD servomechanism, the methodologies are broadly applicable to other systems with the same challenges.

1.1 Background

With the marvellous advancement of the mechatronic system, the demand for control designs involved in the mechatronics becomes higher and higher [144]. Modeling and identification of dynamic systems, simple yet reliable tuning of robust controllers, comparison of various control methods in terms of performance and implementation costs including complexity, robust control without high-gain nature for prolonged actuator life and minimized vibration, control with low-frequency output measurements, etc., all those listed above are real challenges for control engineering.

As a small, however not less important branch of the mechatronic systems, HDD servo systems are being rapidly improved with the latest technologies in control and mechanical fields. The HDD servo systems have never been so demanding for servo control, due to the insatiable thirst exhibited by complicated computing applications and huge online data storage. To meet the stringent requirements of ultra high data density, speed and reliability for hard disk drives, more powerful and practical controllers need to be explored.

1.1.1 HDD Servomechanism

A servomechanism is a control system with the specific task of controlling the position or velocity of some mechanical plant [158]. The overall system is usually regarded as electromechanical because electronic hardware is used to control and make measurements on the mechanical plant. The hardware which is responsible for the actuation is referred to as an actuator, which is what we normally think of as the plant of the control system.

Hard Disk Drives

HDDs are the most important permanent information storage devices used in computers nowadays. Components of a typical HDD are shown in Figure 1.1. Other storage devices include floppy disks, CD-ROMs, tape drives, removable mass storage such as ZIP drives and flash disks, and magneto-optical (MO) drives. HDDs differ from others primarily in three aspects: capacity, cost per megabytes (MB), and performance. During the past five decades, HDDs had made dramatic progress

in terms of capacity, speed, and price. The first hard disk drive, invented by International Business Machine (IBM) Company in 1956, had a capacity of 5 MB with the recording density of around 2 Kbits/in² and the data rate of 70 Kbits/s, costing over \$100 per MB [26]. But now, popular hard disks with capacity of tens or even hundreds of gigabytes (GB) cost less than a cent per MB [1]. Meanwhile, the performance, reliability, and data transfer-rate have been improved dramatically.

As shown in Figure 1.1, an HDD contains round, flat disks called platters, or disks. Information is stored on the magnetic coating of the disk surface. The disks have a hole in their center, and are stacked along a spindle. The stacked disks are rotated by an electric motor inside the spindle, called the spindle motor. Electromagnetic read/write (R/W) heads, which are often referred to as sliders, are used to either record information onto the disks or read information from them. The heads are mounted on suspensions through gimbals. Suspensions are mounted on a carriage, or E-block, whose side view has an “E” shape. The heads, the suspensions, and the E-block together form the Head Disk Assembly (HDA). The flexible structure of the gimbals and the suspensions help the heads maintain constant flying height about 10–20 nanometers (nm) on an air bearing over the rotating disks [56]. During the operation, the heads should never touch the rotating disks. Otherwise, the heads and the disk surface will be severely damaged. The R/W heads over the disk surface are positioned by an actuator, which controls the movement of HDA. Voice coil motor (VCM) is widely used as the actuator in hard disk drives. VCM controls the HDA and a selected head to follow a track or to switch from one track to another. It is also referred to as single-stage servo control, in contrast to the

dual-stage servo where two actuators (VCM and micro-actuator) combine to work for different range of movement [160].

Figure 1.1: Perspective view of a hard disk drive.

In an HDD, data are stored with bit cells in tightly-packed concentric rings on the magnetic disk surface where the cells are called tracks, and organized for fast access. Each track is further divided into smaller sectors including servo sectors for servo information and data sectors for user data. Basically, there are two methods to record servo information: dedicated record servo and embedded record servo (or sectored record servo). The dedicated record servo uses all tracks on one disk surface of the stacked disk arrays to store the servo information for head positioning, other disks are used for recording user data. This servo record method is obsolete because the effect of environment such as temperature on it is not matched with that on other user data disks, which would lead to disk arrays distortion and even misoperation. Virtually, most state of the art HDDs employ the embedded record servo. In this scheme, servo information are embedded in the servo sectors along the data tracks at equally spaced angles with user data sectors in between on each

Figure 1.2: Disk stacks for dedicated and embedded record servos.

disk surface. User data sectors and servo information sectors are interleaved in a track as if the servo sectors are embedded in the data sectors. As the drive spins, this spatial multiplexing becomes a temporal multiplexing [165]. The number and the width of the servo sectors should be minimized for data format efficiency of HDDs. Figure 1.2 illustrates the widely used embedded record servo and the former dedicated record servo schemes. Servo sectors are pre-written on the disk before the disk can be used. Servo writing is performed by precision servo writers. Because the servo system uses the heads and servo sectors as the position sensor, the servo writers must achieve much higher positioning accuracy than what the HDD servo system can do ([31], [146] and [171]).

The stored data in servo sectors can then be demodulated into the track number and the position error signal (PES). R/W head reads sector number, track ID, and the PES. The servo system calculates voltage command based on PES demodulated from servo sectors. The voltage command is then fed through an analog power amplifier (PA) via a digital-analogue-converter (DAC). The DAC acts as a zero-order holder (ZOH). The output current of PA passes through the coils of the VCM,

and the resultant torque drives the actuator for track following and seeking.

In an embedded servo system, the PES can only be obtained in discrete-time format. Thus, its servo system is discrete in nature. The product of the spindle speed and the number of sectors determines its sampling rate, i.e.

$$F_s = N_{\text{sector}} \frac{\text{RPM}}{60} \quad (1.1)$$

where F_s is the sampling rate in Hz, N_{sector} is the number of the servo sectors along one track, and RPM is the spindle speed in revolution per minute. The number of the sectors should be minimized so that a larger area can be used for recording data. Thus, it is a challenge to ensure acceptable performance with low sampling rates.

Increase of the HDD capacity is achieved by storing more information in the same physical disks, or increase of the areal density of the disks. Both bit density, also named as linear density of tracks, and track density of disks must be increased for higher areal density. The former is measured by bits per inch (BPI), and the latter by tracks per inch (TPI). Areal density, in bits per square inch, is the multiplication of BPI and TPI. The areal density of an HDD product is currently keeping at the 100% compound annual growth rate (CAGR) [57]. Increasing TPI is a relatively easier approach to increase the areal density compared with increasing BPI [35], but still a challenging task itself. Historically, it involves interdisciplinary technological advances in the magnetic media, actuators design, disk platter material and thickness, and servo control design. This thesis emphasizes the improvement of the HDD servo control system.

HDD Servo Systems

In general, an HDD servo control system includes two fundamental problems: the point-to-point control and the tracking control problems. The tracking control is to position the R/W heads over the desired track with minimum statistical deviation from the track center (also named as track-following control). Tracking control is also extremely important in many other mechanical systems such as automated mobile robot system [29] and manual flight control [119]. The point-to-point control in the HDDs is to re-position the heads from one track to another swiftly within the constraints imposed by the actuator and power amplifier, which is generally referred to as track-seeking control. Switching from track-seeking to track-following must be accomplished smoothly and fast during the transition phase. The settling process when moving from seeking mode to following mode is not unique to an HDD servo system, hence any improvement in the HDD settling control can also be applied to other similar physical control systems, for example, high precision table positioning system [84]. Figure. 1.3 shows a detailed description of servo modes in an HDD [25].

Figure 1.3: Servo modes in an HDD.

(1) Track-Seeking Control

The nature of track-seeking control is to force the actuator angular velocity to follow an ideal angular velocity profile that will guarantee the shortest possible seek time with minimum jerk. Fast seeking would reduce the time in preparation for data read/write; smooth seeking would yield less acoustic noise to make an HDD work quietly in consumer electronic appliances. The actuator angular velocity is digitally estimated by an algorithm in the microprocessor that uses position error information obtained from servo sectors during the process. Originally, the seek mode was implemented as a PD (Proportional plus Derivative) or PD type controller with a constant monitoring of the remaining distance to the target track [19]. Modern control theory such as optimal control has potentially made it possible to achieve better performances than that with classical PD control.

To minimize the access time, the saturation of the actuator must be taken into the consideration in the track-seeking control. For time-optimal control, bang-bang control is known to provide the time optimal solution. Unfortunately, time-optimal control cannot be implemented directly to many practical applications including the HDD. Recently, a novel nonlinear control algorithm, named as proximate time optimal servomechanism (PTOS) has been a popular track-seeking control law in an HDD servo system ([44], [55], [92] and [158]). In PTOS, it calculates the velocity profile from current position to the target track based on time optimization that boosts the velocity of actuator to the maximum; once the distance to the target track is under certain threshold, the actuator is decelerated and prepared for switching to track-following mode.

(2) Settling Control from Seeking to Following

When the actuator is less than one track pitch away from the target track, the settling mode takes over from the seeking to following mode. In the HDD servo design, two different types of controllers are used for track-seeking and track-following. And mode switching control is usually adopted in the transient process so that the center of the recording head is kept within a certain position error tolerance of the target track center line for a fast control mode switching. Usually a control system is designed under the assumption that the initial values are zero. But in this case, the initial states of track-following mode cannot be ignored since the “initial” means a time at mode switching and state variables such as velocity and position may have non-zero values. Those initial states may cause undesirable settling response if not compensated. Some designs to improve the settling performance have been proposed, such as initial value compensation (IVC) ([166], [167] and [168]) and optimal control based on IVC [77]. Some advanced control designs independent of the system initial states were developed too. Method named as robust and perfect tracking (RPT) was a case in point ([15] and [149]). Besides that, some nonlinear control methods were proposed for better settling response such as nonlinear feedback control law with time-varying damping factor [148].

(3) Track-Following Control

In the track-following mode, the servo objective is to stay as close to the track center line as possible for reading and writing information. It was mentioned before that the track-following control in an HDD is a tracking problem. From a measurement point of view, however, the HDD track-following control is different from the tracking control in other systems such as precision positioning problem. In the lat-

ter, the control objective is to track a desired trajectory, where the desired output can be treated as a known signal and the actual output can be measured, which makes it possible to use a feedforward control algorithm. On the other hand, the HDD track-following control must work on position error signal (PES), which is the difference between the desired track position and the actual head position. Thus, the principal element for controlling such system should be a feedback controller and therefore a regulation system, which makes the track-following control problem more challenging than the tracking control in that:

- steady state characteristic: In the HDDs, the tracking accuracy is strictly set beforehand by the index of track mis-registration (TMR).
- nominal stability: The closed-loop system must be stable.
- robust stability: The stability of the close-loop system must be maintained in the presence of parametric uncertainties and drifts as well as working conditions.

In track-following control, the actuator saturation is not explicitly considered, and the feedback controller is designed primarily based on linear control theory for stability, such as PID (Proportional plus Integral plus Derivative) action and lead-lag filters. Due to various disturbance sources and noises existing in the servo channel, and these feedback controllers are not enough for accurate positioning, various disturbance compensators are therefore usually inserted in the feedback loop. The performance of positioning is evaluated by TMR which is defined by the standard

deviation of PES ($\pm 3\sigma$) [107]:

$$\text{TMR} = 3\sigma = 3\sqrt{\int_0^\infty P(\omega)d\omega}, \quad (1.2)$$

where ω is radian frequency; $P(\omega)$ is the power spectral density of PES. Usually, it is acceptable that the TMR in track-following is about 10% of the track width.

Figure 1.4 is a schematic block diagram of a disk drive following control loop.

Figure 1.4: Generalized review of track-following control system.

Generally, the amount of TMR reduction caused by the increase of the servo bandwidth is mainly determined by 3 factors. They are:

1. Spectra of disturbance and measurement noise;
2. Plant resonance modes;
3. Limitations of sampling rate and control calculation delay.

To find the contributors and sources to TMR ([37], [60] and [62]), and make the corresponding improvement in mechanical and electronic designs [118] is the most preliminary investigation. But when the mechanical and electronic designs are fixed,

the servo control system has to be active to attenuate the vibrations via various loop shaping. The research and development work on HDD track-following technologies may involve the following topics:

- Advanced Optimal Control

To suppress on-track disturbances, the frequency response of an HDD servo loop can be shaped through classical or modern optimal control approaches, such as LQG (Linear Quadratic Gaussian) [3], LTR (Loop Transfer Recovery) ([6] and [13]), and H_2/H_∞ optimization with robust considerations ([49], [50], [87], [99] and [123]). LQG design optimizes the linear quadratic criterion and LTR technique generates the estimator for making the output feedback loop the same as the full state feedback system. H_2/H_∞ controllers are derived based on the minimization of H_2 -norm/ H_∞ -norm of certain system transfer functions, where the H_2 -norm minimization could be interpreted as TMR minimization [99] and H_∞ -norm minimization means the minimization of difference between the desired and designed closed-loop frequency responses [50]. While these advanced control methods are effective to attenuate effects of disturbance to some extent, they result in high-order controllers, which may increase the complexity of DSP implementation.

- Classical Disturbance Observer Compensation

Pivot friction forces, torques and tensions are important disturbance sources in HDDs [108]. Basically, they all belong to low frequency portions. The disturbance observer is a useful method for handling such kind of disturbances ([36], [75], [81] and [157]). It estimates an equivalent disturbance by taking

the difference between the actual control input and the predicted total input. The predicted control input is obtained by processing the plant output via an approximate inverse of the nominal plant dynamics. It includes an approximation because the actual inverse is not realizable. Furthermore, measurement noise in the HDDs limits the frequency range where inversion makes sense. Although disturbance observer has been found effective in dealing with low frequency disturbance in many control systems, it is still subject to limitation of sampling rate in HDDs.

- Periodical Disturbance Compensation

In the track-following mode, the control problem is a regulation problem for rejecting external disturbances with a constant reference input if each track is a perfect circle and there exists no any eccentricity between the center of each circle and the spindle axis. As a matter of fact, repeatable runout (RRO) caused by the imperfect/eccentric disks, offset of the track center with respect to the spindle center, bearing geometry and motor geometry exists indeed and is one of major disturbance sources. The PES may exhibit large tracking error at multiples of the spindle frequency if RRO is not compensated.

Repetitive control is a powerful method to deal with RRO compensation and was effectively applied to the HDDs ([22], [68] and [86]). The underlying property that is used is the fact that a linear feedback system has perfect disturbance rejection at some frequency if and only if the controller gain is infinite at that frequency. This is the same as the control based on internal model principle (IMP) [42], known as narrow-band filtering. The difference

between IMP and repetitive control is that the former has a pair of poles on the $j\omega$ -axis at a location corresponding to the frequency of the disturbance, while the latter results in the placement of an infinite number of poles at the disturbance frequency as well as all its multiples. Therefore, the stability problem of repetitive control becomes more difficult as more poles are added on the $j\omega$ -axis. More remarks on robustness of repetitive control can be found in [94].

Adaptive feedforward cancellation (AFC) is another approach to deal with periodic disturbances based on a totally different concept, where the disturbance is simply cancelled at the input of the plant by adding the negative of its value at all times ([10], [126], [127] and [174]). Since only frequency is known while magnitude and phase of disturbance are unknown, they are estimated with an adaptive algorithm. In spite of different control structures, the analysis shows that the AFC algorithm is equivalent in some sense to an IMP algorithm [126].

Previous studies have shown that besides RRO which is synchronous with the spindle revolution, disk flutter, suspension vibration and actuator windage are also important contributing factors to TMR. Such signals are not phase locked to the disk rotation, and are thus called non-repeatable runout (NRRO). Despite not synchronous with the disk rotation, periodic NRRO still can be solved by repetitive/narrow-band compensation methods mentioned above. Typically, narrow-band compensation can be effective to deal with low frequency NRRO ([61], [85] and [173]). However, for some of NRRO whose energy focus on the mid-frequency band, inserting a repetitive controller/narrow-band

filter will affect the stability of the closed-loop, which is subject to the fundamental limitation in minimum-phase linear control system, i.e., one-to-one magnitude-phase relationship [132]. Mid-frequency compensation is currently a rather challenging work for an HDD servo system [161].

Unexceptionally, all methods mentioned above are potentially useful to compensate periodic disturbances in other motion control applications.

- Resonance Compensation

The existence of plant resonance is one of the main obstacles for high servo bandwidth in an HDD servo. Modern hard disk drives employ the rotary actuator driven by the voice coil motor (VCM). The whole actuation mechanism including the VCM and suspension may have the resonance around 2–5 kHz. If not well compensated, the resonance would threaten the stability of servo system. Gain stabilization methods, such as notch filtering ([103] and [153]), have been widely used to compensate for the resonant modes based on the small gain theorem [165]. While its drawback is the phase shift that occurs around the cross-over frequency, which also restricts the servo bandwidth. The phase characteristic of the plant have been taken into account for the phase stabilization design [89], which would be more robust to the resonance shift than gain stabilization.

- Multi-rate Control

As mentioned before, sampling rate is limited by the fact that the disk can only sacrifice a small portion of its available capacity to store servo information,

and is one of the major obstacles in pushing bandwidth higher [156]. Under the constraint on the measurement sampling frequency, input updating rate may be set larger for performance improvement. That is referred to as multi-rate control. The multi-rate control designs were reported to improve the HDD servo performances ([20], [54], [69] and [131]), where the servo controller works with shorter sampling period than the PES. Multi-rate design can also recover phase loss caused by a ZOH [67]. With more phase margin, the servo bandwidth can be pushed higher than the single-rate design. The use of a first-order holder (FOH) in place of ZOH was proposed for smoothing the control input [46]. The efficiency of multi-rate technologies used at other information storage devices such as optical disk drive and tape drives was reported in [121].

- Dual-stage Servo Control

The performance of track-following servo system is limited by the dynamics of actuator. With the scaling-down of media grain and R/W head, the traditional actuator, VCM, seems to be too bulky to provide very small positioning resolution. Therefore, the idea that uses a secondary smaller actuator to increase the positioning accuracy of the HDD servo system has been studied extensively ([39], [73], [124], [128] and [129]). In such a dual-stage servo, the micro-actuator fulfills the fine positioning and works mainly for track-following operation; the primary actuator, VCM, works for the coarse movement and long-span track seeking. Furthermore, the micro-actuator may enable the servo loop to bypass the resonance of the VCM and help to improve the short-span seeking performance.

Table 1.1: Basic performance specifications for a commercial HDD.

Storage Capacity per HDA	12.3	Gigabytes (GB)
Track Density	51.2	kTPI
Bit Density	609.5	kBPI
Rotational speed	15,037	RPM
Latency average	1.99	ms
Data transfer rate	960	Mbits/sec
Average seek time	3.9	ms
Track to track seek time	0.4	ms
Full track seek time	7.2	ms
Approximate 3σ TMR	0.5–1	microns

1.1.2 High Performance HDD Servos

In order to put matters in perspective, a list of critical parameters for the IBM Ultrastar 15K73 disk drive will be discussed.

The list of performance specifications in Table 1.1 illustrate the challenge to the HDD servo system designer. The HDD is striving for larger data capacity and faster data rate. Areal density of a platter measures how many bits can be packed into each square inch. As mentioned before, areal density consists of two components: track density in TPI and bit density in BPI. The servo system is responsible for maintaining the heads on-track near the center of increasingly narrow tracks. 12.3 GB storage capacity per HDA necessitates 51.2 kTPI track density and maximal 609.5 kBPI bit density, which needs the actuator to position accurately with less

than $0.5\ \mu\text{m}$ track width. Higher revolution per minute (RPM) and more sectors per track provide higher sampling rate. In general, higher sampling rate results in higher HDD internal performance. IBM Ultrastar 15K73 HDD has exceeded over 15,000 RPM. However, high RPM HDDs will consume more power, generate more heat, and introduce more self-induced noises. High sampling rate servo system also demands faster DSP speed, and reduces the formatting efficiency. Therefore, trade-offs have to be made. Latency is the amount of time that the R/W heads must wait to reach the target sector. The average latency is defined to be half of the time of a full disk rotation. The worst case happens when the R/W head has to wait a full revolution to reach the target track. e.g., the average latency is 5.6 ms for 5400 RPM drive, 4.2 ms for 7200 RPM, 2.8 ms for 10,000 RPM, and 1.99 ms for 15,000 RPM disk drive. Increasing the spindle speed will reduce latency. Faster spindle speed also increases data transfer rate for a given BPI.

Seek time is the amount of time required for the R/W heads to move between tracks, normally in a order of milliseconds (ms). Average seek time is the average time from one random track (cylinder) to any other. Track-to-track seek time, or single track access time is the amount of time that is required to seek between adjacent tracks. This should be less than 1 ms for a current HDD. Full track seek time is the amount of time to seek the entire width of the disk, from the innermost disk (ID) to the outermost disk (OD). Track seeking is a mechanical process that involves using the actuator to physically move the R/W heads. Track to track seek time is very important because switches to adjacent tracks occur much more frequently than random seeks. With a fixed mechanical design, servo system is

responsible for reducing average seek time and track to track time, without increasing acoustic noises. The seek time specifications of Table 1.1 translate into information that advanced control strategies have to be used to achieve such objectives. The seek performance only pays off if the position regulation at the end of a seek does not suffer a large position error transient process. This fact implies smooth transfer from the minimum time controller to the minimum variance controller. At track-following, the position error of this HDD is maintained to be a statistical deviation of 0.5 micron, which is 3σ standard deviation of the servo head from its track center. Meeting the minimum time and the minimum variance objectives with a smooth transition is a tough challenge in the HDD servo.

1.2 Motivation

Currently, prevailing HDD controllers are linear. Previous study showed that there are various limitations and tradeoffs associated with linear control systems ([53], [110] and [141]). Typically, Bode gain-phase relation [9], Bode integral theorem [17], and time-domain constraint [113] are particularly restricting the performance of an HDD servo system. Much of the motivation for the research, which is summarized in this thesis comes from the application of nonlinear control theory to the head positioning servomechanism in HDD mechatronics, aims to develop nonlinear control techniques which are potential of breaking the performance limitations in linear control systems.

1.2.1 Linear and Nonlinear Control

Since James Watt's centrifugal governor for the speed control of a steam engine in the eighteenth century, the study of linear control has witnessed a long history and developed into a mature subject successfully applied in many industrial fields [116]. As the core of classical control theory, the frequency-response and root-locus methods lead to stable systems and to satisfy a set of more or less arbitrary performance requirements. Classical control theory, which deals only with single-input single-output (SISO) systems, however become inadequate as modern plants with many inputs and outputs becomes more and more complex. Modern control theory, based on time-domain analysis and synthesis using state variables, appeared in 1960's and has been developed to cope with the increased complexity of modern plants and the stringent requirements on accuracy, reliability and cost.

The highlights of the development of linear control are the optimization theories. They have been well developed as the cornerstones of modern control. Among them, H_∞ control theory has become a powerful tool to deal with optimal and robust control problems ([28], [43], [51], [162], [163] and [178]). It is also regarded as a representative of "post-modern control" [177]. One of the motivations for the original introduction of H_∞ methods was to deal with plant uncertainty, specified in the frequency domain [177]. From that point of view, it belongs to a class of robust control. The H_∞ norm was found to be appropriate for specifying both the level of plant uncertainty and the signal gain from disturbance inputs to error outputs in the controlled system. While the H_∞ norm alone can only give a conservative prediction of robust performance. The methods of μ -analysis and μ -synthesis were

thus introduced and developed, which give an effective analysis tool for assessing robust performance in the presence of structured uncertainty. And the synthesis of the controllers that satisfy such a μ value criterion can be approached iteratively with the H_∞ synthesis for a scaled system as an intermediate step ([40] and [70]).

The majority of the available theorems are concerned with linear, continuous or discrete systems. Unfortunately, no physical system strictly belongs to the class of linear systems. Virtually, all practical control systems include nonlinear elements or portions with nonlinear characteristics. Nonlinear control system analysis and designs are necessary to provide a sharper understanding of the real world. Different from linear control systems, a remarkable property in nonlinear control system is that different systems require very different techniques and the analysis thereof is distinctive case by case. There are a variety of approaches to control of nonlinear systems such as nonlinear optimal control [47], backstepping [176], learning-forwarding control [172], Nonlinear Lyapunov function [109] and conditioning control [66], however each with its own limitations in practice.

System nonlinearities are often classified as inherent and intentional nonlinearities [5]. Inherent nonlinearities are those which exist in the ‘presumably linear’ components within the system. Typically, such characteristics are saturation in amplifiers and motors, dead zones in valves and nonlinear friction and backlash in gears. The basic idea to deal with such nonlinearities is to linearize them by compensation or approximation and the resulting linear methods are used for determining the response of such systems. Generally, those nonlinearities whose variables undergo only relatively small changes in value are treated by approximate linearization based on

small-signal theory, which takes Taylor series approximations about the operating point up to first order derivative terms only. For those signals which cannot be considered so small that small-signal theory does not apply, the extension of small-signal theory makes possible piecewise linear representation. Two common linear segmented characteristics are used to approximate dead zone and saturation and usually referred to as ideal dead zone and saturation characteristics. For nonlinear friction forces which are common in control systems, three linear modes or their combination, i.e., static friction, Coulomb friction and viscous friction are used to approximate them [5]. In an HDD servo system, linear modeling of nonlinear pivot dynamics were used and thus compensated via various linear control methods ([52], [80], [81] and [169]).

On the other hand, intentional nonlinearities are those which are deliberately introduced into the system to perform a specific nonlinear function. In most cases, nonlinearities in control systems will degrade the system performance. Whereas in some cases, by inserting appropriate nonlinear element into the system, we may improve the system performance of linear control. It is feasible to apply them into real control systems with the development of powerful microprocessors, low cost dedicated digital signal processors and power electronics. It is the motivation of this thesis to take full advantages of nonlinear characteristics to compensate and improve the performance of the current linear HDD control system.

Since a nonlinear system does not obey the law of superposition, analysis and measurements carried out in linear control system are not appropriate any longer. Some specified methods were developed to apply for certain nonlinear systems, such

as phase-plane and describing function approaches [101]. Phase-plane is actually a graphical approach for nonlinear differential equations without recourse to any analytical solutions. It can be of assistance in design and for studies of systems with more than one nonlinear element. While it is only fit for lower order systems whose portrait descriptions are clear and obviously. Describing function is a kind of quasi-linear representation for a nonlinear element based on such assumption that the nonlinear element has a sinusoidal input [5]. It is an extension of frequency response approach in linear system to nonlinear system. Since most control systems contain low pass plant transfer functions and low pass filtering of any periodic signal tends to make it sinusoidal, periodic signals might be approximately replaced by sinusoidal at the nonlinearity input because the other harmonics are attenuated by the low-pass plants. It is thus possible to design gain and phase for each nonlinear section based upon fundamental components of its input and output.

A central problem in the design of any nonlinear system is that of stability. The work of Lyapunov can provide exact solution to this problem without solving the nonlinear differential equations [88]. All the stability verifications thereafter in this thesis are based on the theorem of Lyapunov stability.

1.2.2 Directions for Improvement

The limiting factors of performance in an HDD servo system, particularly those which are related to the current control strategy or structure, are the basic motivation for the research outlined in this thesis. Although a detailed investigation and further research work are carried specially on the HDD servo system in the thesis,

they can be broadly applied to many analogous motion control systems. Specifically, this thesis aims to develop some nonlinear control technologies, which can indeed overcome certain types of limitations which are a consequence of the plant acting in combination with linear time-invariant feedback control. Fundamental limitations and tradeoff in linear system, such as Bode gain-phase characteristics and Bode Integral Theorem, are key obstacles to improve servo bandwidth and the capability of disturbance attenuation. In the time domain, settling process are always prolonged due to undesired overshoot, undershoot and oscillation from internal or external sources. In the frequency domain, disturbance rejection at some frequency band would lead to disturbance amplification at other frequency bands. Further, Bode gain-phase relationship in the linear open-loop stable system makes mid-frequency NRRO compensation difficult or even impossible via linear control. Whereas all the constraints listed above are not necessarily always existent in nonlinear system, those limitations may be broken through with assistance of certain nonlinear characteristics.

It will be shown that through exploration and investigation of some nonlinear control techniques, the performance of HDD servo can be improved and the tradeoffs made in linear systems can be loosened to some extent.

1.3 Contributions

The work accomplished in this thesis would have not been possible without those published and unpublished references wherever results or ideas presented were learned or borrowed.

The contributions of this thesis are as follows:

- Development of a current-estimator based input shaper. The new reference profile is designed to remove the abrupt change of input signal, and makes tracking error and error rate vary in a relatively small range. It avoids effectively actuator saturation due to gains of nonlinear feedback controller.
- Development of composite nonlinear feedback control scheme to improve the seek-settling performance by dynamically adjusting the location of closed-loop poles. As the design tradeoff between rise time and overshoot in the response of set point changes is removed, nonlinear control is proposed to achieve better seek-settling performance for both short and long seeking length.
- Development of a reduced-order observer for implementation of the composite nonlinear feedback and PTOS controllers in an open HDD platform. Since only the PES was available, velocity observer which has an order less than that of the given plant is adequate for implementation.
- Development of nonlinear PID control to improve settling performance due to unknown bias. It is difficult to achieve desired settling performance for linear compensators without prior knowledge of bias force. Through the investigation and analysis, it is found that integral element is the main source that leads to large overshoot and thus long settling time. Nonlinear PID control is proposed to achieve fast settling performance in this thesis. Its robustness is also verified in the implementation.
- Development of two-degree-of-freedom (2DOF) control scheme by combining

input shaper and nonlinear feedback controller. Besides settling performance, servo bandwidth and disturbance compensation, set point responses for both short and long seeking length are improved as well with the 2DOF control.

- Development of nonlinear narrow-band mid-frequency compensation in track-following control. High gain filter is usually used to compensate for narrow-band disturbance, but is typically used in the low frequency band due to phase stabilization. Nonlinear control scheme is proposed to break the one-to-one Bode gain-phase relationship associated with linear systems, and thus makes mid-frequency narrow band compensation available and more effective.
- Development of nonlinear feedback control to suppress sensitivity hump and improvement of head positioning accuracy in the track-following mode. The proposed control scheme consists of two parts: linear PID control law and its associated nonlinear PD control law. Disturbance rejection at low frequency is governed by the linear controller, while disturbance beyond servo bandwidth will be dealt with both linear and nonlinear laws. Nonlinear describing function analysis shows that sensitivity “hump” can be suppressed greatly, and Bode’s Integral Theorem does not hold for the proposed nonlinear control systems.
- Proof of global asymptotic stability for the composite nonlinear control using a Lyapunov function, which reveals that the added nonlinear part does not affect the stability of the whole system if the linear system is stable under the saturation.

- Proof of global asymptotic stability of nonlinear PID settling control, which is based on Lyapunov functions of two stable linear systems.
- Experiment validation of the developed techniques and concepts contained in this thesis.

1.4 Thesis Outline

This chapter has described the HDD servo system, its related terminologies, and servo control issues. The motivations and contributions of this thesis have also been introduced. The rest of the thesis is organized as follows:

Chapter 2 reviews the fundamental limitations associated with both linear and nonlinear systems. Both limitations and their application to HDD servo systems are compared and analyzed. It is shown that nonlinear control technology has the potential to break some limitations existent in the linear control system, as well as to outperform linear control in some cases.

Chapter 3 presents the modeling of the HDD servo system. Both analytic derivation and system identification results are presented in this chapter. Analytic derivation needs much knowledge about the material and physics of the plant, furthermore, the resulting model is usually complicated. All the modelings in the thesis are obtained via system identification. A frequency response identification technique is presented and shows that the identified models can well match the measured ones.

Chapters 4 and 5 deal with seek-settling performance. In Chapter 4, two kinds of nonlinear feedback control theorems are developed to break through the time domain

performance constraints of the linear control systems. Through the dynamic pole-placement method, Controller 1 achieves fast rise time and small overshoot. The effect of unknown bias and friction torque is considered in the design of Controller 2. Settling performance is improved due to integral compensation.

In Chapter 5, a 2DOF control scheme is discussed by incorporating a current-estimator based input shaper and the proposed nonlinear feedback controller. Generally, actuator saturation will degrade system performance and even lead to system instability. To avoid saturation, reference command is re-shaped before entering the closed-loop system. With the 2DOF control, fast seek-settling performance is achieved without degrading other performance specifications such as disturbance rejection and non-zero initial state effect. This will be verified through the experimental results.

Leaving the seek-settling performance for a while, Chapters 6 and 7 deal with the special disturbance compensation in the track-following mode. Chapter 6 is concerned with mid-frequency NRRO compensation for smaller TMR. For the VCM actuator, servo bandwidth is limited below 2 kHz due to unmodeled high frequency resonances. Inserting a high gain filter around the mid-frequency (500 Hz – 2 kHz) would make system unstable. The key point is phase stabilization since Bode's gain is stabilized at all frequencies. Nonlinear mid-frequency compensation is proposed in this chapter, which can provide a lead phase without affecting gain characteristic within servo bandwidth from the describing function analysis. Therefore, phase stabilization is achieved and the capability of mid-frequency disturbance attenuation is verified to be improved through the simulation and experimental results.

Chapter 7 investigates the essence of sensitivity function constraint and develops a nonlinear feedback control scheme to suppress sensitivity hump due to linear feedback control law. The design is carried out in the frequency domain and is analyzed through nonlinear describing function. Simulation and experimental results show that sensitivity hump can be suppressed or even removed with the increase of disturbance magnitude.

Chapter 8 summarizes the findings and results of the thesis and presents some considerations for future research.

Chapter 2

Limitations in Control Design

2.1 Introduction

This chapter is concerned with fundamental limits in the design of feedback control systems. A theme throughout the study of fundamental limitations is that there exists unavoidable tradeoffs between the potential benefits of feedback, such as stabilization, disturbance attenuation and sensitivity reduction, and the potential costs of feedback, such as noise response, excessive control signals, and poor stability margins. These limits tell us what is feasible and, conversely, what is infeasible, in a given set of circumstances. Their significance arises from the fact that they include any particular solution to a problem by defining the characteristics of all possible solutions. And since knowledge of inherent design limitations is used early in the design circle, it is often possible to intentionally select sensors, actuators, and the control architecture so that the resulting feedback design problem is feasible.

Furthermore, this chapter shows that some of the limitations are inherently linked to the plant and thus hold irrespective of how the input is generated — be it via linear, nonlinear or time-varying feedback. Other limitations are a consequence of

the plant acting in combination with linear time-invariant feedback control which is generally adopted in most practical control engineering. This leaves unanswered issue of such limitations arising from the use of nonlinear control. And, it may be possible that these constraints can be relaxed to some extent or be broken thoroughly by using nonlinear or time-varying feedback.

The rest of this chapter is organized as follows. Section 2.2 investigates fundamental design limitations in linear SISO systems, while Section 2.3 presents the results on nonlinear systems. Finally, this chapter is summarized in Section 2.4.

2.2 Performance Limitations in Linear Systems

Consider the SISO linear feedback system depicted in Figure 2.1, where P and C are the transfer functions of the plant and compensator, r is a reference input, y is the system output, u is the control output, n represents measurement noise, and d_i and d_o are disturbances at the plant input and output. And the capital letters are used as their individual Laplace transforms, that is $R(s)$, $Y(s)$, $U(s)$, $N(s)$, $D_i(s)$ and $D_o(s)$.

Figure 2.1: One-degree-of-freedom feedback system.

Define the open-loop transfer function

$$L(s) = P(s)C(s), \quad (2.1)$$

sensitivity function

$$S(s) = \frac{1}{1 + L(s)}, \quad (2.2)$$

and complementary sensitivity function

$$T(s) = \frac{L(s)}{1 + L(s)}. \quad (2.3)$$

The response of this feedback system to exogenous inputs is governed by four closed-loop transfer functions, $S(s)$, $T(s)$, $S(s)P(s)$, and $C(s)S(s)$:

$$Y(s) = S(s)D_o(s) + T(s)(R(s) - N(s)) + S(s)P(s)D_i(s), \quad (2.4)$$

$$E(s) = S(s)(R(s) - D_o(s) - N(s) - P(s)D_i(s)), \quad (2.5)$$

$$U(s) = -T(s)D_i(s) + C(s)S(s)(R(s) - N(s) - D_o(s)). \quad (2.6)$$

The system will be stable if all four transfer functions have no poles in the Right Half Plane (RHP). This requires that all signals in the feedback system of Figure 2.1 remain bounded in response to all bounded inputs, and rules out the existence of unstable pole zero cancellation between the plant and the controller [132]. For example, cancelling an unstable plant pole with a controller zero will result in the transfer function $S(s)P(s)$ being unstable.

2.2.1 Frequency Domain Constraints

Because the closed-loop transfer functions describe so many important properties of a feedback system, one may consider design specifications stated in terms of fre-

quency dependent bounds upon their magnitudes. For example, the sensitivity function governs the effects of output disturbance upon the closed-loop response [141]. The steady-state frequency response of the system output to output disturbance $d_o = d_o e^{j\omega t}$, is given by

$$y_d(t) = S(j\omega) d_o e^{j\omega t}. \quad (2.7)$$

Note that for scalar system, $S(j\omega) = |S(j\omega)| e^{j\omega t}$ and the steady-state response to a complex sinusoid of frequency ω is given by the input scaled by a “complex gain” equal to $S(j\omega)$. The response to d_o can be made small by requiring $|S(j\omega)| \ll 1$.

Furthermore, large $|S(j\omega)|$ may also affect the system stability robustness. The effect can be seen from the Nyquist plot of the open-loop transfer function, as follows in Figure 2.2. Note that $S^{-1}(j\omega) = 1 + L(j\omega)$ is the vector from the -1 point to the point on the Nyquist plot corresponding to the frequency ω . Thus, there is a portion of the plot where $|S(j\omega)|$ is less than one and another portion where $|S(j\omega)|$ increases above one. From the Nyquist graphical interpretation of Figure 2.2, it is obvious that the stability is robust for $|S(j\omega)| < 1$, and conversely, not so robust for large $|S(j\omega)|$.

Figure 2.2: Nyquist graphical interpretation of the sensitivity function.

Likewise, a similar analysis for T shows that the response of the system to sensor noise of any frequency ω can be made small if $|T(j\omega)| \ll 1$.

Based on the closed-loop transfer functions described in (2.4), (2.5) and (2.6), the design specifications S , T , CS , and SP cannot be set arbitrarily, but have frequency dependent bounds upon their magnitudes. That is,

$$|S(j\omega)| < M_S(\omega) \quad (2.8)$$

$$|T(j\omega)| < M_T(\omega) \quad (2.9)$$

$$|C(j\omega)S(j\omega)| < M_{CS}(\omega) \quad (2.10)$$

$$|S(j\omega)P(j\omega)| < M_{SP}(\omega) \quad (2.11)$$

Although these bounds are not assigned exact values in the thesis, they are implicitly present in any feedback control design problem. For instance, the response to disturbance is always wanted to be small (2.8) and (2.11), the effect of noise cannot be severe (2.9), and the control signal not be overly aggressive (2.10), and so forth.

In the frequency domain, the theory of inherent design limitations presented shortly implies that certain algebraic relationships exist in the four transfer functions S , T , CS and SP , and thus not all specifications of the form (2.8)-(2.11) can be achieved.

“Algebraic” Frequency Tradeoffs

From the definitions in (2.2) and (2.3), an obvious relation is

$$S(j\omega) + T(j\omega) = 1. \quad (2.12)$$

This identity states that $|S(j\omega)|$ and $|T(j\omega)|$ cannot be both very small at the same frequency. Hence, at each frequency the desirable system properties of disturbance attenuation and noise rejection must be tradeoff against each other. Note that this tradeoff is really a consequence of the structure of a feedback loop. It will be shown shortly to hold for the nonlinear feedback control systems.

Some else algebraic tradeoffs are also present, such as a tradeoff between sensitivity reduction and control output response, whose severity depends on the magnitude of the plant gain,

$$C(s)S(s) = P^{-1}(s)T(s) \quad (2.13)$$

It implies that if $|S(j\omega)| \ll 1$, then $|C(j\omega)S(j\omega)| \approx |P^{-1}(j\omega)|$. It follows that $|S(j\omega)| \ll 1$ in a frequency range for which the plant gain is small requires a high controller gain; hence, the response of the control output to disturbances in this range will be very large, usually leading to actuator saturation. Thus, It can be concluded that the range of sensitivity reduction, i.e., the bandwidth of the closed-loop system, is limited by the natural bandwidth of the plant combined with limits of the actuator authority.

Other bandwidth limitations arise from the needs to reject high frequency noise and to maintain robustness against high frequency modeling uncertainty. The following relation is easily obtained from the definitions (2.1)-(2.3) that

$$\frac{\tilde{T} - T}{\tilde{T}} = S \frac{\tilde{L} - L}{\tilde{L}} \quad (2.14)$$

where $\tilde{L}$ is the actual open-loop transfer function, and L is the nominal open-loop transfer function. Likewise, $\tilde{T}$ is the actual complementary sensitivity function and T is the nominal one and the same meanings for sensitivity functions of $\tilde{S}$ and S .

As mentioned before, to reduce the effect of high frequency noise, it requires that $|T| \ll 1$, which means $|S|$ is large. (2.14) shows that the complementary sensitivity will be not robust with respect to changes of the loop gain in frequency ranges where the nominal sensitivity is large.

The Bode Gain-Phase Relation

A famous result of Bode gain-phase relation delineates some kinds of transfer functions whose phase of the frequency response is uniquely determined by the magnitude of the frequency response and vice versa.

Theorem 2.1 [9] *Assume that $L(s)$ is a transfer function with no poles or zeros in the open RHP. Assume further that the gain has been normalized so that $L(0^+) > 0$.*

Then at each frequency ω_0 , phase and gain are related by:

$$\theta(\omega_0) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{d \log |L(j\omega)|}{d\nu} \log(\coth \frac{|\nu|}{2}) d\nu \quad (2.15)$$

where $\nu = \log(\omega/\omega_0)$ and $\log$ means the denary logarithm.

It follows immediately from (2.15) that the phase is completely determined by gain's slope. Hence the two parameters (phase and gain) that describe a complex function yield only one degree-of-freedom in design. Furthermore, it follows that phase depends upon the slope of the Bode gain plot and a weighting function. More generally, the weighting function is

$$\log(\coth \frac{|\nu|}{2}) = \log \left| \frac{\omega + \omega_0}{\omega - \omega_0} \right|. \quad (2.16)$$

This function is plotted in Figure 2.3. The weighting function becomes logarithmic infinite at the point $\omega = \omega_0$, and phase depends more strongly upon the rate of

increase/decrease in $|L(j\omega)|$ at frequencies in a range about a decade in width centered at ω_0 . Thus, it can be conclude from (2.15) that the slope of the magnitude curve in the vicinity of ω_0 , say N ,

$$\frac{d\log|L|}{d\nu} = N, \quad (20N\text{dB/decade}). \quad (2.17)$$

determines the phase:

$$\theta(\omega_0) \approx \frac{N}{\pi} \int_{-\infty}^{+\infty} \log(\coth \frac{|\nu|}{2}) d\nu = \frac{N\pi}{2}, \quad (90N^\circ). \quad (2.18)$$

Hence, for stable and minimum phase transfer function, a slope of $20N$ dB/decade in the gain in the vicinity of ω_0 implies a phase of approximately $N\pi/2$ rad ($N = 0, \pm 1, 2, 3, \dots$).

It concludes that to achieve closed-loop stability and a reasonable phase margin, it is necessary that the open-loop gain roll off relatively slowly near gain crossover frequency, i.e., among mid-frequency band. This fact implies a tradeoff of loop design between high gain specification in low frequency and low gain portion in high frequency.

The Bode Sensitivity Integral

The Bode sensitivity integral theorem was initially derived for the linear system with relative degree of at least 2 in the open-loop transfer function. It addresses that in such system, the total area of the logarithmical sensitivity function will be a non-negative value so that it is impossible to achieve arbitrary sensitivity reduction (i.e., $|S(j\omega)| < 1$).

Theorem 2.2 [177] *Assume that $S(s)$ is stable and that $L(s)$ has at least two more*

Figure 2.3: Weighting function of the Bode gain-phase relationship.

poles than zeros. Then,

$$\int_0^\infty \log|S(j\omega)|d\omega = \pi \sum_{i=1}^{N_p} \text{Re}\{p_i\}. \quad (2.19)$$

where $p_i : i = 1, 2, \dots, N_p$ is the set of open RHP poles of the open-loop transfer function $L(s)$.

Suppose first that $L(s)$ has no open RHP poles. Then the integral (2.19) establishes that the net area subtended by the plot of $|S(j\omega)|$ on a logarithmic scale is zero. Hence, the contribution to this area of those frequency ranges where there is sensitivity attenuation ($|S(j\omega)| < 1$) must equal that of those ranges where there is sensitivity amplification ($|S(j\omega)| > 1$). This tradeoff is illustrated in Figure 2.4. And this phenomenon is known as the “waterbed” or “push-pop” effect. This demonstrates that there is a compromise between sensitivity magnitudes in different frequency ranges. Moreover, since the sensitivity function quantifies a system’s ability to reject disturbances, the area constraint establishes that it is impossible to achieve

arbitrary disturbance rejection at all frequencies. The source of the tradeoff implied by (2.19) may be seen from the Nyquist plot in Figure 2.2. If $L(s)$ is stable and has at least two more poles than zeros, then $L(j\omega)$ must penetrate the unit circle centered at the critical point, thus forcing $|S(j\omega)| > 1$.

Unfortunately, the single stage actuator in an HDD servo system employs a VCM having a slope of -40 dB/decade and hence a double integrator behavior. And thus it is subject to this constraint, i.e., increasing the disturbance rejection at one frequency means that you must amplify disturbances at another frequency.

Figure 2.4: Area balance of the sensitivity integral.

Suppose next that $L(s)$ has open RHP poles. Then the area of sensitivity increase must exceed that of sensitivity decrease by an amount proportional to the distance from those poles to the $j\omega$ axis. In principle, the area of sensitivity increase needed to balance that of sensitivity decrease may be obtained by letting $\log|S(j\omega)|$ exceed zero over an arbitrarily large frequency range by an arbitrarily small amount. Any practical design, however, is affected by bandwidth constraints. Indeed, several factors such as sensor noise and plant bandwidth lead to the desirability of decreasing the open-loop gain at high frequency, thus putting a limit on the bandwidth of the

closed loop. It is reasonable to assume that the open-loop gain satisfies a design specification of the type

$$|L(j\omega)| \leq \delta \left(\frac{\omega_c}{\omega}\right)^{1-k}, \quad \forall \omega \geq \omega_c, \quad (2.20)$$

Note that k, ω_c and δ can be adjusted to provide upper limits to the slope of magnitude roll-off, the frequency where roll-off starts and the gain at that frequency, respectively. If a special kind of multi-valued function [132] is used,

$$\log S(j\omega) = \log |S(j\omega)| + j\arg S(j\omega). \quad (2.21)$$

it is not difficult to get that

$$\left| \int_{\omega_c}^{\infty} \log |S(j\omega)| d\omega \right| \leq \int_{\omega_c}^{\infty} |\log |S(j\omega)|| d\omega \quad (2.22)$$

$$\leq \int_{\omega_c}^{\infty} |\log S(j\omega)| d\omega \quad (2.23)$$

$$= \int_{\omega_c}^{\infty} |\log [1 + L(j\omega)]| d\omega \quad (2.24)$$

$$\leq \frac{3\delta\omega_c^{1+k}}{2} \int_{\omega_c}^{\infty} \frac{1}{\omega^{1+k}} d\omega \quad (2.25)$$

$$= \frac{3\delta\omega_c}{2k}. \quad (2.26)$$

The above result shows that the area of the Bode sensitivity integral over the infinite frequency range $[\omega_c, \infty]$ is limited. Hence, any area of sensitivity reduction must be compensated by a finite area of sensitivity increase, which will necessarily lead to a large peak value in the sensitivity frequency response before ω_c . Hence, the Bode sensitivity integral (2.19) imposes a clear design tradeoff when natural bandwidth constraints are assumed for the closed-loop system. Whereas, this tradeoff may be alleviated if the closed-loop bandwidth is large, i.e., large ω_c in (2.20).

Notice, however, that if the relative degree of the open-loop transfer function $L(s)$ is less than 2, tradeoff may exist or not dependent on the proportional gain

in $L(s)$ [159]. And thus, the “waterbed” effect may be overcome by designing controllers to satisfy some conditions such as relative degree of less than 2. A recent paper provides one of such solutions in an HDD dual-stage servo system [120].

The Poisson Sensitivity Integral

In Poisson sensitivity integral theorem, it assumes that there are open RHP zeros in open-loop transfer function $L(s)$. It shows that the presence of open RHP zeros also imposes a “waterbed” effect and even more stringent constraint on the sensitivity function than the Bode integral theorem.

Theorem 2.3 [45] *Assume that $S(s)$ is stable. Let $z = x + jy, x > 0$, denote an open RHP zero of $L(s)$. Then*

$$\int_0^\infty \log|S(j\omega)|W(z, \omega)d\omega = \pi \log|B_p^{-1}(z)|. \quad (2.27)$$

where the weighting function

$$W(z, \omega) = \frac{x}{x^2 + (y - \omega)^2} + \frac{x}{x^2 + (y + \omega)^2}, \quad (2.28)$$

and $B_p(z)$ is the Blaschke products of the open RHP poles of L ,

$$B_p(z) = \prod_{i=1}^{N_p} \frac{p_i - z}{\bar{p}_i + z}. \quad (2.29)$$

Suppose first that $L(s)$ has no open RHP poles. Since $W(z, \omega) > 0, \forall \omega$, and $|B_p^{-1}(z)| = 1$, it follows that $|S(j\omega)| < 1$ over any frequency range, then necessarily $|S(j\omega)| > 1$ at other frequencies. Different from that of the Bode integral theorem, the presence of the weighting function in the Poisson integral precludes the possibility of compensating an area of sensitivity reduction over a finite range

of frequencies by an area where $|S|$ is allowed to be slightly greater than one over an arbitrarily large range of frequencies. This is because that the weighted area of the $j\omega$ -axis is finite and will be seen shortly that it is equal to π . Therefore, there exists a guaranteed peak in $|S(j\omega)|$ even without the assumption of an additional bandwidth constraint.

Suppose next that $L(s)$ does have open RHP poles. Then the integral (2.27) is greater than zero. Indeed, since

$$\int_0^\infty W(z, \omega) d\omega = - \lim_{\omega_0 \rightarrow \infty} \angle \left(\frac{z - j\omega_0}{z + j\omega_0} \right) = \pi, \quad (2.30)$$

it is easy to derive that

$$\|S\|_\infty \geq |B_p^{-1}(z)| = \prod_{i=1}^{N_p} \frac{\bar{p}_i + z}{p_i - z}, \quad (2.31)$$

where $\|S\|_\infty = \sup_\omega |S(j\omega)|$ is the peak value in the Bode gain plot of the sensitivity function. It follows from (2.31) that systems with approximate open RHP pole-zero cancellations will have very poor feedback properties. In particular, since large values $|S(j\omega)|$ imply that the Nyquist plot of $L(j\omega)$ is very close to the critical point, it follows that stability margins will be small.

Special attention is paid to that there are also counterparts for complementary sensitivity function $T(j\omega)$ although both the Bode integral theorem and the Poisson integral theorem aim at sensitivity function $S(j\omega)$. However, as both S and T are specifications used to describe the closed-loop characteristics and have one-by-one relationship, only limitations related to S are introduced in the thesis. The counterparts about T can be similarly concluded.

Recently, a new control technique named preview control was investigated to improve the performance limitations due to open RHP zeros of the plant transfer

function $P(s)$ [112]. It states that the performance constraints of the Poisson sensitivity integral (2.27) may be reduced with a finite ‘preview’ of future values of the reference trajectory.

Theorem 2.4 [111] *Assume that the closed-loop system is stable, with a reference trajectory preview of t_p . Let $z = x + jy, x > 0$, denote an open RHP zero of $P(s)$. Then the Poisson sensitivity integral of preview control can be established*

$$\int_0^\infty \log|S(j\omega)|W(z, \omega)d\omega = \pi(-xt_p + \log|B_p^{-1}(z)|) \geq -\pi xt_p. \quad (2.32)$$

where the same definitions of weighting function $W(z, \omega)$ and Blaschke products $B_p(z)$ in Theorem (2.3).

Theorem 2.4 states the performance constraints can be reduced by the term of $-xt_p$, particularly for a ‘large preview’ where $xt_p \gg 1$. Furthermore, the infimal achievable weighted H_∞ norm $\gamma_{\min}$ of sensitivity function is verified to be improved with preview control [112]. With the definition

$$\gamma_{\min} = \inf\{\|w(s)S(s)\|_\infty\}, \quad (2.33)$$

where $w(s)$ is a real rational stable minimum phase function. It addresses that for any open RHP zero z of $P(s)$

$$\gamma_{\min} \geq |w(z)| e^{-\text{Re}(z)t_p}, \quad (2.34)$$

and further, if there is only one open RHP zero z in $P(s)$,

$$\gamma_{\min} = |w(z)| e^{-\text{Re}(z)t_p}, \quad (2.35)$$

Thus, $\gamma_{\min}$ may still be kept small by selecting a large t_p and in the limit it approaches zero when $t_p \rightarrow \infty$.

2.2.2 Time Domain Constraints

Classical control theory discusses design specifications imposed upon such properties of the step response as rise time, settling time, overshoot and undershoot. For a second order system with no zeros, a well-known design tradeoff is that short rise time would lead to large overshoot in the step response. In this part, the impacts on the step response of the general closed-loop system with open-loop poles at the origin, open RHP poles and zeros will be analyzed. The following results quantify the performance limitations to be constraints on transient properties of the system. Furthermore, these results are found to differ from or even be opposite to those in an open-loop stable minimum-phase system.

Let's go back to see Figure 2.1. Assume that the feedback system is stable and consider its response to a unit step command $r(t) = 1, \forall t \geq 0$. Then, assuming that the measurement error is zero, the resulting error signal is given by $e(t) = 1 - y(t)$, and thus has the Laplace transform $E(s) = S(s)/s$. The fundamental constraints in such kind of feedback system are now described as follows.

Open-loop Integrators

Theorem 2.5 [132] (i) Suppose that $L(s)$ has a pole at $s = 0$, then

$$\begin{aligned} \lim_{s \rightarrow 0} sL(s) &= c_1, \quad 0 < |c_1| < \infty. \\ \int_0^\infty e(t)dt &= \frac{1}{c_1}. \end{aligned} \tag{2.36}$$

(ii) Suppose that $L(s)$ has two poles at $s = 0$, then

$$\int_0^\infty e(t)dt = 0. \tag{2.37}$$

In practice, $L(s)$ will be strictly proper and the initial value theorem states that the step response satisfies $y(0^+) = 0$. Hence, there will exist an initial time interval over which $e(t) > 0$. It follows from (2.37) that there must also exist a time interval over which $e(t) < 0$, and thus $y(t) = 1 - e(t) > 1$. Hence any system with a double integrator must necessarily exhibit overshoot in the step response. In fact, the same conclusions can be drawn for the open-loop with more than two integrators.

Likewise, the HDD servo system is constrained to this when the servo controller includes an integrator to deal with steady-state error. That is, overshoot is unavoidable in the step response in a linear one-degree-of-freedom HDD control structure. However, it may be circumvented by use of multiple degree-of-freedom control structure or nonlinear controllers, which will be discussed in the later chapters.

If $L(s)$ has but one integrator, i.e., Case (i), this constraint does not imply unavoidable overshoot, and also the tendency to overshoot may be reduced by using a small velocity constant. However, this comes at the expense of a large steady state tracking error in response to a ramp input; furthermore, it is found to inevitably overshoot for a relative slow rise time. To see this, consider first the notation of rise time t_r introduced in [110]:

$$t_r = \sup_T \left\{ T : y(t) \leq \frac{t}{T}, t \in [0, T] \right\}. \quad (2.38)$$

This following result is immediate. If $t_r > (2/c_1)$, i.e., the rise time is sufficiently slow, then

$$\frac{1}{c_1} = \int_0^\infty e(t) dt \quad (2.39)$$

$$\geq \int_0^{t_r} \left(1 - \frac{t}{t_r}\right) dt + \int_{t_r}^\infty e(t) dt \quad (2.40)$$

$$= \frac{t_r}{2} + \int_{t_r}^{\infty} e(t)dt. \quad (2.41)$$

Thus,

$$\int_{t_r}^{\infty} e(t)dt \leq \frac{1}{c_1} - \frac{t_r}{2} < 0. \quad (2.42)$$

And thus, $y(t) > 1$ for some $t \in (t_r, \infty)$, the unit step response $y(t)$ will overshoot.

RHP Open-loop Poles

Theorem 2.6 [132] *Suppose that the open-loop transfer function $L(s)$ has a pole at $s = p$ such that $\text{Re}(p) > 0$, then*

$$\int_0^{\infty} e^{-pt} e(t)dt = 0, \quad (2.43)$$

and

$$\int_0^{\infty} e^{-pt} y(t)dt = \frac{1}{p}. \quad (2.44)$$

Consider a real pole, $p > 0$. Then $e^{-pt} > 0$, and it follows, as in the discussion of (2.37), that the step response will necessarily exhibit overshoot.

Furthermore, there exists a guaranteed minimum overshoot whose size is proportional to the rise time of the system from the definition of rise time in (2.38). From the definition, $y(t) \leq t/t_r$ for $t \leq t_r$, i.e., $e(t) \geq 1 - t/t_r$. Using this, the integral equality (2.43) can be rewritten to be,

$$-\int_{t_r}^{\infty} e^{-pt} e(t)dt \geq \int_0^{t_r} e^{-pt} (1 - \frac{t}{t_r})dt. \quad (2.45)$$

suppose the overshoot y_{os} is the maximum value by which the output exceeds its final set point value, i.e.,

$$y_{os} = \sup_t -e(t), \quad (2.46)$$

immediately

$$y_{os} \frac{e^{-pt_r}}{p} = y_{os} \int_{t_r}^{\infty} e^{-pt} dt \quad (2.47)$$

$$\geq \int_0^{t_r} e^{-pt} \left(1 - \frac{t}{t_r}\right) dt \quad (2.48)$$

$$= \frac{(pt_r - 1 + e^{-pt_r})}{p^2 t_r}, \quad (2.49)$$

and further

$$y_{os} \geq \frac{pt_r}{2}. \quad (2.50)$$

This is seen that long rise time (i.e., “slow” closed-loop response) in a system with real open RHP poles implies large overshoot in a unity feedback configuration. This differs from the case of an open-loop stable system where large overshoots normally arise from short rise time. Moreover, as p becomes more unstable, the rise time of the closed loop system must decrease to maintain the desired overshoot, i.e., the farther from the $j\omega$ -axis the poles are, the more stringent this bandwidth demand will be.

RHP Open-loop Zeros

A result for plants with open RHP zeros symmetric to that of plants with open RHP poles holds, may also constrain the step response as follows.

Theorem 2.7 [132] *Suppose that $s = q$ is a zero of $L(s)$ with $\text{Re}(q) > 0$. Then,*

$$\int_0^{\infty} e^{-qt} e(t) dt = \frac{1}{q}, \quad (2.51)$$

$$\int_0^{\infty} e^{-qt} y(t) dt = 0. \quad (2.52)$$

Consider a real zero, $q > 0$. It follows from (2.52) and the fact that $e^{-qt} > 0$ that the system must necessarily exhibit undershoot in its step response, in the sense that $y(t) < 0$ over some time interval.

The definitions of settling time t_s and undershoot y_{us} in [132] is used,

$$t_s = \inf_T \left\{ T : |y(t) - 1| \leq \epsilon, \forall t \in [T, \infty) \right\}, \quad (2.53)$$

and the undershoot is the maximum negative peak of system output, i.e.,

$$y_{us} = \sup_t -y(t) \quad (2.54)$$

where ϵ is a given settling level, commonly between 1 – 10%, from (2.52) it is not difficult to get that

$$y_{us} \geq \frac{1 - \epsilon}{e^{qt_s} - 1}. \quad (2.55)$$

It is clear that for a given zero q , the undershoot must become large as t_s becomes small. This is quite opposite to that for open RHP poles where open RHP poles will demand a large closed-loop bandwidth (i.e. short rise time) for good performance.

Special attention is paid to that Theorem 2.7 implies that the actual plant, $P(s)$ in Figure 2.1, has an open RHP zero and the control signal is any bounded stable function. Thus, in particular, this constraint and all its extension such as relationship between settling time and undershoot in (2.56) are independent of the type of controller used, i.e., they hold for any linear and nonlinear control. However, it is not true for Theorem 2.6, i.e., its constraint is not necessarily existent in nonlinear control system, which will be discussed shortly in the next section.

Likewise, preview control was reported to reduce the time domain constraints induced by open RHP zeros [112]. It says that for preview control with preview of

$t_p < T$, the bound (2.55) becomes

$$y_{us} \geq \frac{1 - \epsilon}{e^{q(t_s+t_p)} - 1}. \quad (2.56)$$

Clearly, preview control reduces the undershoot bound induced by the open RHP zeros.

2.3 Performance Limitations in Nonlinear Systems

The limitations related to linear control systems have been investigated and analyzed in Section 2.2 . An interesting and valuable question is that whether they still hold in nonlinear systems or not. As mentioned before, some of them are inherently linked to the plant characteristics, such as time domain constraints with open RHP zeros, and thus hold irrespective of what kind of controllers are used; while others may be a consequence of the plant combined with linear feedback controller, such as the Bode sensitivity integral theorem. Since some of them may not exist necessarily in nonlinear systems, it is potentially possible to overcome or relax them to some extent by design of nonlinear feedback controller. In this section, some unavoidable limitations which also hold for nonlinear systems will be discussed.

2.3.1 Frequency Domain Constraints

Before investigating the “algebraic” relations, some preliminary notation and definitions of nonlinear operators will be introduced ([122] and [132]).

Let X be a linear space. Let H be a nonlinear operator on X , i.e., a mapping between its domain $D(H) \subset X$ into X . The domain and range of H are defined as

follows:

$$D(H) = \{x \in X : Hx \in X\},$$

$$R(H) = \{Hx : x \in D(H)\}.$$

If H_1 and H_2 are nonlinear operators on X and $D(H_1) \cap D(H_2) \neq \phi$, the addition $H_1 + H_2 : D(H_1 + H_2) \rightarrow X$ is defined by $(H_1 + H_2)x = H_1x + H_2x, \forall x \in D(H_1 + H_2) = D(H_1) \cap D(H_2)$. And further assume that X is a Banach space in the following, and say H is stable if its domain is X and say that H is unstable if its domain is a strict subset of X . Also, say that H is non-minimum phase if the closure of its range is a strict subset of X . It is denoted by Lipschitz operator $\text{Lip}(D, X)$ for the class of operators $H : D(H) = D \subset X \rightarrow X$ such that

$$\|H\|_L = \sup |Hx - Hy|/|x - y| : x, y \in D, x \neq y < \infty, \quad (2.57)$$

where $|\cdot|$ is the norm in X and $\|H\|_L$ is called Lipschitz constant, or incremental gain of H . In fact, the Lipschitz constant is the extension of linear ordinary norm to nonlinear case [133]. A member H of $\text{Lip}(X)$ is said to be invertible in $\text{Lip}(X)$ if there is a $H^{-1} \in \text{Lip}(X)$ such that $HH^{-1} = H^{-1}H = I$.

“Algebraic” Complementarity Constraint

Consider the feedback interconnection of plant P and controller C as displayed in Figure 2.1, where it is assumed that $r = d_i = 0$. Define the nonlinear sensitivity operator S , and the nonlinear complementary sensitivity operator T , as

$$S = H_{ydo}|_{(n=0)}, \quad (2.58)$$

$$T = -H_{yn}|_{(d_o=0)}, \quad (2.59)$$

where the notation $H_{ba}|_{c=0}$ stands for the total map from signal a to signal b when signal c is identically zero. In other words, S is the mapping between output disturbance and system output in the absence of the sensor noise, and T is the mapping between sensor noise and system output in the absence of output disturbances. From the definition in (2.58) and Figure 2.1, the sensitivity operator is given by [132]

$$S = (PC + I)^{-1}, \quad (2.60)$$

and the complementary sensitivity operator is given by

$$T = PC(PC + I)^{-1} = PCS. \quad (2.61)$$

Further, the following theorem can be derived.

Theorem 2.8 [133] *Consider the operators defined in (2.58) as nonlinear operators on a linear space. Then on $D(S) = D(T)$*

$$S + T = I. \quad (2.62)$$

The theorem clearly indicates that the “algebraic” tradeoff also holds for nonlinear system if and only if S and T have the same domain $D(S) = D(T)$ (It is naturally satisfied for stable closed-loop system), and further, that is only determined by the structure of the feedback control loop together with the additive nature of the disturbance inputs. Hence, the constraint is independent of whether the plant or controller are linear or nonlinear operators.

Sensitivity Limitations

The Poisson integral theorem states a performance limitation in sensitivity reduction that for non-minimum phase plant(i.e., with open RHP zeros), sensitivity function

cannot be made arbitrarily small over all frequency band as its open RHP zero will provide a lower bounds on its infinity norm. An analogous criterion is, however, proved to hold for nonlinear system, which actually extends the familiar “waterbed” effect in linear systems to nonlinear systems [132].

Before obtaining the main result on sensitivity lower bounds, a preliminary lemma is introduced first.

Lemma 2.1 [132] *Consider that the open-loop $L = PC$ is a nonlinear operator $L : D(L) \subset X \rightarrow X$, such that the operators S and T have a nonempty domain, then*

(i) If L is non-minimum phase, then T is non-minimum phase.

(ii) If L is unstable and $D(L)$ is closed, then S is non-minimum phase.

The following theorem gives lower bounds on the Lipschitz constants of the nonlinear sensitivity operators for open-loop non-minimum phase and unstable systems.

Theorem 2.9 [132] *Let $L : D(L) \subset X \rightarrow X$ be such that the sensitivity operator S is in $\text{Lip}(X)$. Then*

(i) If L is non-minimum phase, then $\|S\|_L \geq 1$.

(ii) If L is unstable, then $\|T\|_L \geq 1$.

where $\|\cdot\|_L$ is the Lipschitz constant defined in (2.57). Theorem 2.9 points out the incremental gain of nonlinear sensitivity operator S is larger than one for non-minimum phase open-loop system and that of nonlinear complementary sensitivity operator T is larger than one for unstable open-loop system. It is the nonlinear extension of Theorem 2.3 for the case of a closed-loop system belonging to the

class of Lipschitz operators. It does not assume sensitivity reduction and hence it represent limits for any possible control design.

2.3.2 Time Domain Constraints

Classical tradeoff in the step response due to different damping ratios of second-order systems has been proved to be not available through nonlinear control methods ([11], [95] and [105]). It may succeed overcoming it by design of system damping ratio varied with tracking error via appropriate nonlinear function. More complicated algorithm was also developed for higher order linear plant system ([14] and [151]). Obviously, the design tradeoff of overshoot and rise time due to the fixed closed-loop poles does not exist in the nonlinear systems. Compared to those constraints in linear systems, an interesting question is how matters stand for the limitations due to plant with open RHP poles and zeros in nonlinear systems?

A qualitative analysis about it is given as follows. From Lemma 2.1, case (i) is true for any system, while case (ii) may not hold always for any kind of nonlinear system since that $D(L)$ is closed may not apply for any system [132]. It can also be noticed that both Theorem 2.5 and Theorem 2.6 are existent in the condition that $S(s)$ is non-minimum phase since $L(s)$ is open unstable. And thus, it can be presumed that these two limitations do not necessarily hold on if nonlinear controller is used. As a matter of fact, some experimental results have shown that nonlinear design can indeed break through these limitations in time domain ([41] and [97]).

As case (i) in Lemma 2.1 holds for any bounded control system, i.e., independent of how it was produced via linear or nonlinear forms, open RHP zeros character-

istics which make linear system constrained by quantitative performance criterion in Theorem 2.7 can be extended to nonlinear operators. That is, undershoot is not avoidable in nonlinear system as long as there is no unstable pole zero cancellation.

2.4 Conclusions

The chapter discussed both frequency domain and time domain design constraints in the linear feedback control systems, and extended those results to the case of nonlinear systems.

Based on the theorems listed before, some of the limitations are, in fact, independent of the type of controller used, i.e. hold for any (linear, nonlinear) control signal generated in the same control structure. By extension of the known concept of sensitivity operators in linear feedback system to the case of nonlinear system, it has been shown that the nonlinear sensitivity and complementary sensitivity operators satisfy the “algebraic” constraint, the same as that in linear control system. Further, the extension of linear ordinary norm, Lipschitz constant of nonlinear sensitivity operator is proved to be bounded over 1 for non-minimum phase and open-loop unstable nonlinear systems.

However, some of them are restricted to the linear feedback loop alone, such as “Bode gain-phase relationship” in the frequency domain and unavoidable overshoot of double integrator in the time domain. A more detailed comparison of the limitations in both linear and nonlinear system are illustrated in Table 2.1, where $\checkmark$ means it exists and $\times$ means not, and the detailed conditions were given in the previous theorems and not listed in the table. Also notice that all these formulas

are represented with linear operators, their extensions to nonlinear operators have been defined before. These conclusions give us a hint that some of performance limitations have the potential to be broken through by use of nonlinear controller.

In an HDD control system, open-loop transfer function, which consists of a VCM actuator and the conventional linear controller, is minimum-phase and open-loop stable. Although performance limitations due to open RHP poles and zeros are not existent any longer, settling performance is still affected due to double integrator constraints in the HDD control system. This, however, can be potentially improved through nonlinear control design. More details will be described in Chapters 4 and 5. In the frequency domain, Bode's gain-phase relationship also holds for the HDD control system. This specifically limits the capability of mid-frequency disturbance rejection in the track-following mode. As nonlinear describing function breaks through the one-to-one relationship between gain and phase, nonlinear disturbance compensation at mid-frequency range is greatly improved, which will be discussed in Chapter 6.

Although an HDD control system is not constrained by Poisson integral theorem which holds for both linear and nonlinear system, however, Bode's sensitivity integral limitation still exists in the linear control HDD system. Specifically, due to the double integrator characteristic in the VCM actuator frequency response, sensitivity integral of the HDD control system establishes that the net area is zero. Hence, the contribution to the area of sensitivity attenuation ($|S(j\omega)| < 1$) must equal that of those ranges of sensitivity amplification ($|S(j\omega)| > 1$). Disturbance beyond cross-over frequency such as disk vibration and windage will be thus amplified instead

of rejection. In Chapter 7, nonlinear loop shaping method will be introduced to suppress sensitivity hump due to linear feedback controller.

Before discussing the control design techniques, let's introduce first the system modeling which includes actuator and disturbance modelings.

Table 2.1: Limitation comparisons in linear and nonlinear systems.

Control Schemes		Linear	Nonlinear
Frequency Domain	“Algebraic” relation: $S + T = 1$	✓	✓
	Bode gain-phase relation: (2.15)	✓	×
	Bode sensitivity Integral: (2.19)	✓	×
	Sensitivity bounds: (open RHP zeros) $\ S\ _{\infty} > 1$	✓	✓
Time Domain	Overshoot (double integrator)	✓	×
	Overshoot (open RHP poles)	✓	×
	Undershoot (open RHP zeros)	✓	✓

Chapter 3

System Modeling

This chapter describes the modeling of HDD systems. The design specifications for HDD servo systems are given in Section 3.1. Plant modeling is discussed in detail in Section 3.2, where the single-stage actuator is considered. TMR sources in which the servo system operates are described in Section 3.3, which includes repeatable components, such as servo writing errors and disk slip synchronized to spindle rotation; non-repeatable components, such as sensor noise, disk vibrations and friction torque. This chapter is summarized in Section 3.4.

3.1 Servo Design Specifications

Commonly used design specifications for HDD servo systems are listed as follows [4]:

1. Minimal 5 dB gain margin and 30° phase margin;
2. Asymptotic tracking of step commands;
3. Minimal transition time for track seeking to following;
4. Minimal TMR for track following.

Stability margins are covered in Criterion 1. Due to the mass production of disk drive components, variation and uncertainties from the plant model are expected and must be accounted for in the servo system design. The gain margin indicates how much loop gain variation can be tolerated before the system falls into instability. The phase margin determines the stability impact from phase loss caused by computation delay and high frequency actuator resonances. Criterion 2 concerns the ability to track step position command and to reject constant torque disturbances, such as the bias torque from the flexible printed circuit (FPC). The seeking and following performances are reflected in criteria 3 and 4, respectively.

3.2 Hard Disk Servo Plant Modeling

Traditional HDDs use single stage actuators with the VCM sitting at one end of actuator assembly which rotates about a pivot bearing, whereas the R/W head sits at the other end of the actuator assembly. The crossover frequency of the current single-stage HDDs is 1.0–1.5 kHz and has proved to limit the improvement of the positioning accuracy to keep up with increasing track density. The dual stage actuator servo system, with a PZT or a slider as a small secondary actuator attached to the suspension, has attracted extensive attention as the next generation HDD servo technology to achieve a higher bandwidth and better TMR performance. While its reliability, system integration cost, and added complexity are among the most concerned issues for its implementation. Currently, the single stage actuator servo is still dominating in the HDD industry. This thesis will focus on the research on control system design of the single-stage HDDs.

Generally, there are two ways to obtain plant models: analytical derivation and system identification in frequency/time domain. The analytical derivation needs much knowledge about the physics of the plant to work out the model. In system identification, the plant is taken as a black box to measure its input-output characteristics. Both of them will be presented in the following subsections.

3.2.1 Analytical Derivation

For a single-stage HDD, the plant consists of power amplifier (PA), voice coil motor (VCM) and head stack assembly (HSA). The servo system is usually implemented by a digital signal processor (DSP). The DSP processes PES, computes and generates voltage command via a DAC. With the command, the PA controls the coil current and torque to drive the HSA.

Figure 3.1: A plant model of HDDs.

Figure 3.1 shows a plant model including the PA, VCM and HSA. The PA itself is a closed-loop dynamics, where an analogue feedback loop controls the VCM current

to follow the voltage command. The PA op-amp is modeled as a first-order system with a cutoff frequency ω_{pa} with a very large gain G_{pa} . In practice, the dynamics of PA is often ignored and substituted with a constant gain, given as follows:

$$i = \frac{G_{pacl}G_{op}}{R_s}U \quad (3.1)$$

G_{pacl} and R_s are the determining parameters inside the feedback loop. G_{op} is the tuning parameter outside the loop. Ignoring the PA saturation and the back *e.m.f.* effect, the dynamics of the PA is described as:

$$\frac{i(s)}{U(s)} = \frac{G_{pa}\omega_{pa}G_{pacl}G_{op}}{(s + \omega_{pa})(Ls + R)G_{pacl} + G_{pa}\omega_{pa}R_s} \quad (3.2)$$

Using the final value theorem, it is easy to show that the DC gains from voltage command U to current i is:

$$K_{pau} = \frac{G_{pacl}G_{op}}{\frac{RG_{pacl}}{G_{pa}} + R_s} \approx \frac{G_{pacl}G_{op}}{R_s} \quad (3.3)$$

as G_{pa} is much larger than RG_{pacl} . In this case of ignoring the PA saturation and the back *e.m.f.* effect, the PA is modeled as a constant gain K_{pau} .

Figure 3.2: Plant model during PA saturation in HDDs.

Plant model during the power amplifier saturation is shown in Figure 3.2. The saturation commonly occurs during the acceleration phase of a long seek. In this

case, the maximal voltage is applied to the PA. $U_{\max}$ is the maximum op-amp voltage output. The plant is completely different from that with the op-amp operation in its linear range. In saturation mode, the PA current feedback loop has no effects on $U_{\max}$.

In track-following mode, the PA operates in its linear range without saturation. The PA bandwidth is usually ten times higher than servo bandwidth and the dynamics of PA is ignored. As a result, for servo design and TMR analysis, the PA dynamics is often simplified as a constant gain K_{pau} .

The VCM is basically a DC motor with restricted movement and can be modeled by $\frac{1}{Ls+R}$. L is the coil inductance. R is the resistance sum of coil and current sensor. Generally speaking, the value of R is much larger than inductance L and thus, the VCM itself is also modeled to be a pure gain $1/R$. Voltage u_2 across the VCM generates the coil current i . Torque $T = K_t i$ where K_t is the torque constant drives the HSA to move the R/W head. VCM voltage is also affected by the back *e.m.f.* voltage which is proportional to the coil velocity v . HSA is often modeled as a pure inertia with resonance modes. The resonances are mainly due to the flexibility of the pivot bearing, arm, suspension, etc., which can be modeled as second-order systems at different resonant frequencies. When the servo bandwidth is 5 – 10 times lower than the main resonant frequency, the resonances may not be a limiting factor to the servo design. In this case, the HSA model can be considered as the simplified one which is a double integrator. In practice, the response of the system is dominated by three main resonances: typically the first suspension torsion mode in the ranges of 1500 – 2500 Hz, also referred to as butterfly mode without phase loss; the second

torsion mode at 4000 – 8000 Hz and the first sway mode at 8000 – 1200 Hz [1], the phase loss caused by them imposes great challenge to increase the servo bandwidth.

Figure 3.3: Modeling pivot friction for HSA.

The pivot friction is a complicated nonlinear phenomenon. The pivot damping and stiffness are nonlinear but often approximated by constant average values, and thus it is often regarded as a linear resonance at low-frequency and simply modeled by a second-order system. Figure 3.3 shows the bode plot of HSA with and without the linear model of the pivot friction. The pivot friction limits the open-loop gain at low frequency, and makes the system more susceptible to disturbances [12]. While it does not affect the response of mid-frequency and high frequency. Therefore, its effect is often ignored when modeling for seeking control and is taken into consideration for the following controller design.

The plant model is therefore modeled as a fourth-order, sixth-order or eighth-order system up to how many resonances are considered. These resonant modes can be cascaded with the system of double-integrator (without pivot friction) or linear

second-order model (with pivot friction).

3.2.2 Frequency-Domain Identification Algorithm

In the laboratory study of HDD servos, a more popular method to obtain the plant modeling is the system identification. When identifying a plant model, persistent excitation is required. Typical excitation sources include white noise and swept sine signal which can be obtained from the Dynamic Signal Analyzer (DSA). In our identification setup, swept sine signal is used to measure the VCM frequency response. The current is about 1 mA to 20 mA depending on the frequency band. Generally, low current amplitude is used in the low frequency and high current amplitude for high frequency band. The Laser Doppler Vibrometer (LDV) can be used to measure the movement of an HDD R/W head, which allows non-contacting measurement of velocity and displacement. Since there is no additional sensor/transducer attached to actuators, the dynamics of the measured mechanical system is not changed. The input-output frequency characteristic of the actuator can then be easily obtained by FFT analysis from the above information. Figure 3.4 shows our identification setup.

With the measured data of frequency response, the transfer function of an actuator can be derived by optimization algorithm. Suppose that the transfer function of a plant with m th-order in s -domain is

$$G(s) = \frac{N(s)}{D(s)} = \frac{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_0}{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_0}, \quad (3.4)$$

where $[b_n \cdots b_0]$ and $[a_m \cdots a_0]$ are real numbers and $n \leq m$. At certain frequency ω_k , the input signal is $X(j\omega_k)$, then output data $Y(j\omega_k)$. The parameters $[b_n \cdots b_0]$ and

Figure 3.4: Model identification setup.

$[a_m \cdots a_0]$ can be obtained by minimizing the following estimated error equation:

$$\sum_{k=1}^i W_k \left\| \frac{N(j\omega_k)}{D(j\omega_k)} - \frac{Y(j\omega_k)}{X(j\omega_k)} \right\|^2, \quad (3.5)$$

where i is the number of sample data and W_k is a weighting function to put emphasis on certain frequency range.

The above minimization is a nonlinear least-square problem, which can be transferred to a linear optimization problem if $W_k = D(j\omega_k)$ and suppose $Y(j\omega_k)/X(j\omega_k) = R(\omega_k) + jI(\omega_k)$, so as to minimize the modified error norm:

$$J = \sum_{k=1}^i \|N(j\omega_k) - D(j\omega_k)(R(\omega_k) + jI(\omega_k))\|^2, \quad (3.6)$$

which can be solved by finding $\hat{a}_i, \hat{b}_j$ such that

$$\frac{\partial J}{\partial a_i} \Big|_{a_i=\hat{a}_i} = 0 \quad \frac{\partial J}{\partial b_j} \Big|_{b_j=\hat{b}_j} = 0 \quad (3.7)$$

The additional requirement that $\partial^2 J / \partial a_i^2, \partial^2 J / \partial b_j^2 > 0$ is tacitly assumed to be fulfilled [38]. More conveniently, one can obtain it by software routines such as the command `elis` in MATLAB.

Figure 3.5: VCM frequency responses: Seagate HDD model.

Figure 3.6: VCM frequency responses: IBM HDD model.

In this thesis, two industrial HDD models are tested on the experiment. One is Seagate SCSI HDD model ST31276A which we used for track-seeking mode in the thesis and another is IBM HDD project model used for track-following mode. The identification results are shown in Figures 3.5 and 3.6. In both figures, “Measured” means the measured results via LDV and DSA, and “Identified” is the system iden-

tification results by use of the above algorithm. As we see in Figure 3.5, the first resonance occurs around 1.7 kHz which is caused by the first suspension torsion mode; while it occurs around 5.5 kHz in IBM model because the suspension was originally detached from the actuator in this model, and thus shows the second torsion mode resonance due to arm cartridge. Further, low frequency pivot friction was also piecewise measured in the IBM HDD model for track-following control design. A linear second-order model was thus obtained to identify it, as shown in Figure 3.6. In the both two models, only one resonant mode is considered at high frequencies and other higher resonant modes are neglected since they are far away from the crossover frequency. Finally, two 4th-order systems for both models are obtained:

$$G_{\text{Seagate}}(s) = \frac{6.3648 \times 10^6}{s^2} \times \frac{s^2 + 477.5s + 1.425 \times 10^8}{s^2 + 332.5s + 1.228 \times 10^8}, \quad (3.8)$$

which includes a double integrator and the first torsion mode resonance.

$$G_{\text{IBM}}(s) = \frac{2.097 \times 10^7}{s^2 + 150.8s + 6.317 \times 10^4} \times \frac{1.238 \times 10^9}{s^2 + 7037s + 1.238 \times 10^9}, \quad (3.9)$$

which considers low-frequency pivot friction and one main resonance due to arm cartridge.

3.3 Disturbance Modeling in HDDs

As explained before, the nominal model of plant can be analytically obtained through the mechanical and electronic platform or online identification through experiment setup. However, the servo system consists of not only the plant, but also the distur-

bance and noise environment in which it operates. Without disturbance and noise, perfect track following would have been achieved. Position error signal (PES) is typically measured in terms of its standard deviation σ . It is usually required that 3σ be less than 10% of the track width under the normal operating condition [76]. The requirement is called TMR Budget. There are two approaches to achieve the TMR budget, either by reducing the TMR at their sources, or attenuating PES via the servo system. Reducing TMR at their sources often require costly and time-consuming re-designs of mechanical platforms and electronics. The servo design, usually involving only firmware changes of controller, is always given a high priority for meeting the TMR budget.

Figure 3.7: Disturbance and noise in an HDD servo loop.

Figure 3.7 shows the sources of disturbance and noise in an HDD servo loop. TMR components can be categorized according to three different criteria [37]: (1) RRO with a fundamental spindle frequency content and its harmonics, versus NRRO which is not synchronous with spindle revolution, (2) seek-TMR, which results from residual mechanical vibrations and servo errors that are induced by recently-

completed seeks, versus on-track TMR, which is due to steady-state sources, and (3) externally-induced TMR, which is caused by shocks and/or vibrations imposed upon the drive from outside, versus self-induced TMR which is caused by internal noise sources such as air turbulence, sensor noise, and mechanical vibrations of the disks. From control point of view, the process disturbance and noise are classified as follows (Figure 3.8):

1. Input disturbance including power-amplifier noise, D/A quantization error, pivot bearing friction and all torque vibrations induced by air flow;
2. Output disturbance including suspension vibration, disturbance caused by disk flutter, etc.;
3. Noise including PES demodulation noise and A/D quantization error.

Figure 3.8: Block diagram of an HDD servo loop.

As shown in Figure 3.8, the input disturbance affects PES through disturbance transfer function, output disturbance through sensitivity function, and noise through

complementary sensitivity function. Suppose

$$\begin{cases} d_i(s) &= D_i(s)\omega_1; \\ d_o(s) &= D_o(s)\omega_2; \\ n(s) &= N(s)\omega_3 \end{cases} \quad (3.10)$$

where ω_1 , ω_2 and ω_3 are uncorrelated white noise with power of 0 dB; the transfer functions of $D_i(s)$, $D_o(s)$ and $N(s)$ are the disturbance/noise models in HDD servo loop.

Suppose that $P(s)$, $T(s)$ and $S(s)$ are the actuator model, the complementary sensitivity transfer function and the sensitivity function and S_y , S_i , S_o , S_n are the power spectra of y , d_i , d_o , n , then

$$S_y = |P(s)S(s)|^2 S_i + |S(s)|^2 S_o + |T(s)|^2 S_n. \quad (3.11)$$

Therefore,

$$S_y = |P(s)S(s)|^2 |D_i(s)|^2 + |S(s)|^2 |D_o(s)|^2 + |T(s)|^2 |N(s)|^2. \quad (3.12)$$

So far, a lot of research work have been carried out to obtain the disturbance/noise models ([2], [37], [74] and [170]). An analysis of the vibrations induced by fluid bearing and ball bearing was reported in [30].

3.3.1 Process PES in Time Domain

In this section, the method to evaluate RRO and NRRO in both time domain and frequency domain with available measured PES will be discussed. RRO is a deterministic value at each servo sector, and can be estimated by the mean of the PES

read from the specific sector. NRRO can then be obtained by removing the RRO from the PES, and can be modelled as a zero mean Gaussian random process.

PES from MicroE System (MicroE Systems Inc., Natick, MA) [58] are collected to illustrate the evaluation procedures in this thesis. In this system, a 3.5-in glass disk of 1.27 mm thick is used. The spindle is a fluid dynamic bearing spindle motor rotating at 4800 RPM. The driver has 360 sectors per track, and sampling frequency of 80 Hz at each sector. 6320 points PES are collected three times with 20 revolutions per collection. Three collections of PES are superposed shown in Figure 3.9 with units of V from MicroE system ($1 \text{ V} = 0.11 \mu\text{m}$).

Figure 3.9: Three collections of PES are superposed.

As introduced before, PES contains deterministic RRO components and random NRRO components. Linear averaging method in time domain is used in order to separate PES into PES RRO and PES NRRO. The first step to process PES is to determine RRO. As PES is assumed to be ergodic and the average of NRRO converges to zero, RRO for one revolution can be calculated with M (number of

revolutions measured) rows of data composed of N (number of sectors per revolution) measured PES in every row:

$$\text{PES}(k)_{\text{RRO}} = \frac{1}{M} \sum_{i=1}^M \text{PES}_i(k), \quad k = 1, 2, \dots, N \quad (3.13)$$

where k means the index of PES sample, and i means the index of PES data row. $\text{PES}_i(k)$ is the k^{th} point in the i^{th} revolution PES collection. RRO of M revolutions can be obtained by duplicating one revolution RRO to all other revolutions. Figure 3.10 shows the calculated RRO components with $N = 6320$ and $M = 20$. Three sets of data are shown simultaneously and show that it exhibits a strong repetitive pattern.

NRRO is obtained by removing RRO from PES as follows:

$$\text{PES}_i(k)_{\text{NRRO}} = \text{PES}_i(k) - \text{PES}(k)_{\text{RRO}}, \quad k = 1, 2, \dots, N \quad (3.14)$$

Figure 3.11 gives the evaluated NRRO with three collections and shows that it is a colored random process. The histogram of the PES NRRO is shown in Figure 3.12 and further indicates that it is a zero-mean and Gaussian-distributed random process.

3.3.2 Process PES in Frequency Domain

In the HDD track following servos, RRO appears in harmonic of disk rotation frequency and NRRO appears overall frequency range with a colored random process. This thesis uses discrete Fourier transform (DFT) to analyze signals in frequency domain. In DFT, the signals are represented as a superposition of complex sinusoids. Suppose the collected PES is represented as a discrete sequence $y(k)$, it can

Figure 3.10: Evaluated PES RRO with three collections.

Figure 3.11: Evaluated PES NRRO with three collections.

be represented by a Fourier integral of the form

$$y(k) = \frac{1}{2\pi} \int_{-\pi}^{\pi} Y(e^{j\omega}) e^{j\omega k} d\omega \quad (3.15)$$

where $Y(e^{j\omega})$ is the Fourier transform of $y(k)$

$$Y(e^{j\omega}) = \sum_{k=-\infty}^{\infty} y(k) e^{-j\omega k} \quad (3.16)$$

Figure 3.12: The histogram of the evaluated PES NRRO.

The above two equations form a Fourier representation for $y(k)$. Since $y(k)$ is a finite-duration sequence, discrete Fourier transform (DFT) can be used to analyze its frequency components. DFT of a sequence can be regarded as a sampled Fourier transform of the sequence. The DFT $F_y(k)$ of a causal sequence $y(n)$ is given as

$$F_y(k) = \sum_{n=1}^N y(n) e^{-j2\pi \frac{(k-1)(n-1)}{N}}, \quad k = 1, 2, \dots, N \quad (3.17)$$

where N is the length of $y(n)$. $F_y(k)$ is a complex number sequence of length N , and represents sinusoids with magnitude $|F_y(k)|$ at frequency $\frac{k-1}{N} F_s$, where F_s is the sampling frequency. For real signals of PES, DFT $F_y(k)$ has a real and symmetric magnitude and asymmetric phase. Thus, only the first half of $F_y(k)$ needs to be kept and the normalized DFT $F_{ny}(k)$ is given by

$$F_{ny}(k) = \frac{2F_y(k)}{N} \quad (3.18)$$

In an HDD study, stand deviation and power spectral density (PSD) are usually adopted as evaluation indices in frequency domain. By definition, assuming $y(n)$

Figure 3.13: PSD of RRO PES (with a 20 revolutions measurement).

has a zero mean, its standard deviation is given as

$$\sigma_y = \sqrt{\frac{\sum_{n=1}^N y^2(n)}{N}} \quad (3.19)$$

Figure 3.14: PSD of NRRO PES (with a 20 revolutions measurement).

PSD can be estimated using Welch's method [155] which gives that

$$\text{PSD}_y(k) = \frac{F_y^2(k)}{N}, \quad k = 1, 2, \dots, N \quad (3.20)$$

According to the Parseval's relation,

$$\frac{\sum_{n=1}^N y^2(n)}{N} = \frac{\sum_{k=1}^N \text{PSD}_y(k)}{N} \quad (3.21)$$

the standard deviation of $y(n)$ can then be obtained from PSD that

$$\sigma_y = \sqrt{\frac{\sum_{k=1}^N \text{PSD}_y(k)}{N}} \quad (3.22)$$

Figures 3.13 and 3.14 show the PSD of RRO and NRRO of the above obtained PES from MicroE system with 4800 RPM disk rotation speed.

3.4 Conclusions

Servo design specifications were defined at the beginning of this chapter. HDD servo systems and TMR modelings were introduced. Both analytical deviation and system identification via experiment were carried out for an HDD plant modeling. The TMR sources are categorized into repeatable servo written-in errors, and non-repeatable sensor noise, torque disturbance, and disk modes. Based on the obtained measured results, practical procedures were developed to identify them using only the raw PES in both time domain and frequency domain.

In the next chapters, control techniques will be developed based on the identified plant and TMR source modelings, to improve the HDD servo system performances under different working status, that is, track seeking, settling and following.

Chapter 4

Nonlinear Feedback Control Design in Track-seeking

The aim of track-seeking control is to position the R/W head from current track to a destination track fast and smoothly. Fast seeking would reduce the time in preparation for data reading/writing; smooth seeking would yield less acoustic noise to make HDD work quietly in consumer electronic appliances. For crash protection of the R/W head in case of a power failure, there is a maximum coast velocity limit for the head. When the servo starts a long seek, the power amplifier (PA) is first saturated and the maximum voltage is applied to the VCM. When the head velocity reaches the maximum, the servo changes to the velocity deceleration control mode. Usually a proportional-differential (PD) type controller is used for this purpose [19], although it cannot provide good settling performance at the end of seeking mode, in that fast access will inevitably bring to large overshoot and long oscillation process. This is actually the fundamental performance limitation in linear control HDD servo systems. Usually, an additional settling control mode is added for PD seeking controllers. Theoretical conclusion shows that nonlinear control can break through the tradeoff between fast rise time and small overshoot ([95], [101] and [105]), which

motivates this chapter on dynamic nonlinear control scheme and its application in the HDD seek-settling servos.

4.1 Introduction

A case in point that nonlinear control can beat linear one is the time-optimal control based on a nonlinear signum function of the system states [91], which can provide the minimal access time response to set point changes compared with PD controller.

In this chapter, nonlinear control technique will be developed to improve both the seek and settling performance. Two kinds of nonlinear controllers will be described in details. Both of them are designed based on the linear PD controller for fast seeking phase, however, the design motivations of nonlinear parts are different. One comes from the internal relationship of closed-loop poles and overshoot in the step response; the other, nonlinear PID control, is from the effect of external disturbance such as unknown friction torque and bias.

In the following, the design techniques of two kinds of nonlinear feedback controllers will be presented individually. Their application to the hard disk drive and experimental results will be given in the next chapter.

4.2 Nonlinear Feedback Controller 1

As mentioned in the previous chapter that fast rise time is always associated with large overshoot and thus long settling process for a unity feedback system. However, if control input is changed to increase the system damping ratio before overshoot

occurs, settling performance can be improved to some extent by reducing the overshoot and shortening the oscillation process. The nonlinear feedback controller 1 proposed in this section is motivated by that idea.

The nonlinear controller consists of two parts. Linear one is designed to be a common HDD seeking controller with fast response. Nonlinear one is designed based on Lyapunov function of linear controller for stability requirement, at the same time to increase system damping ratio when the system response approaches the target value. That two controllers are combined to work was originally proposed by Gutman *et al.* to solve linear system stability with saturating actuator [63], where an additional linear control law is constructed and added to the first linear feedback law to increase the level of utilization of actuator magnitude capacity. Later on, more composite control techniques such as high/low gains design and semi-global approach were developed for control of linear systems with saturating actuators ([102], [125], [145] and [151]).

The design of the nonlinear feedback controller is sequential and divided into three steps. Steps 1 and 2 deal respectively with the design of the linear and the nonlinear control laws. In Step 3, these linear and nonlinear feedback laws are combined to form the desired composite nonlinear feedback law. As the design is made in the state space, all the equations use the state-space representations.

4.2.1 Linear Feedback Design

The linear one is designed as a PD type controller with state feedback for fast seeking performance. Suppose that a linear system with an amplitude constrained actuator

is considered

$$\begin{cases} \dot{x} = Ax + B\text{sat}(u), & x(0) = x_0 \\ y = C_1x \end{cases} \quad (4.1)$$

where “sat” represents actuator saturation, and without loss of generality, assume that the pair of (A, B) is controllable and (A, C_1) is observable. The objective is to design a linear feedback controller to stabilize the linear system with fast response.

In the HDD servos, as only position error signal (PES) is measurable, it is necessary to estimate the unmeasurable state (velocity) via state observer. As only two states (position and velocity) are considered in the design, assume that C_1 is with the form of

$$C_1 = [1 \quad 0]. \quad (4.2)$$

Then, the system (4.1) can be rewritten as

$$\begin{cases} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \text{sat}(u), & x_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix} \\ y = [1 \quad 0] \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \end{cases} \quad (4.3)$$

where the original state x is partitioned into two parts, x_1 and x_2 with $y \equiv x_1$. Thus only x_2 needs to be estimated in the reduced-order observer. Let K_R be weighting factor to be chosen such that $A_{22} + K_RA_{12}$ is asymptotically stable. Here note that it was shown in [116] that (A_{22}, A_{12}) is detectable if and only if (A, C_1) is detectable. Thus, there exists a stabilizing K_R .

For reduced-order observer with $\tilde{x}_2$ to be the estimated state of x_2 :

$$\dot{\tilde{x}}_2 = (A_{22} - K_RA_{12})\tilde{x}_2 + K_R\dot{x}_1 + (A_{21} - K_RA_{11})x_1 + (B_2 - K_RB_1)\text{sat}(u) \quad (4.4)$$

If new state $x_v = \tilde{x}_2 - K_Rx_1$ is constructed, then the above dynamic estimator

equation can be written as

$$\dot{x}_v = (A_{22} + K_R A_{12})x_v + (A_{21} + (A_{11} - A_{22} - K_R A_{12})K_R)y + (B_2 + K_R B_1)\text{sat}(u). \quad (4.5)$$

The linear feedback control law is written as:

$$u_L = F \begin{pmatrix} y \\ x_v + K_R y \end{pmatrix} + Gr \quad (4.6)$$

where F is chosen such that (1) $A + BF$ is asymptotically stable, and (2) the closed-loop system $C_1(sI - A - BF)^{-1}B$ has certain desired properties, e.g., having a small damping ratio. Such F and weighting factor K_R can be designed by using any of linear control techniques such as pole-placement method. F is then partitioned in conformity with x_1 and x_2 :

$$F = [F_1 \quad F_2]. \quad (4.7)$$

Furthermore, G is a scalar and is given by

$$G = -[C_1(A + BF)^{-1}B]^{-1}. \quad (4.8)$$

Here, G is well-defined because $A + BF$ is stable. The following lemma determines the amplitude of reference command r that can be tracked by such a dynamic control law without exceeding the control limits.

Lemma 4.1 *Given a positive definite matrix W_P , let $P > 0$ be the solution to the Lyapunov equation,*

$$(A + BF)'P + P(A + BF) = -W_P. \quad (4.9)$$

Given another positive definite matrix W_Q with

$$W_Q > F_2' B' P W_P^{-1} P B F_2, \quad (4.10)$$

let $Q > 0$ be the solution to the Lyapunov equation,

$$(A_{22} - K_R A_{12})'Q + Q(A_{22} - K_R A_{12}) = -W_Q. \quad (4.11)$$

Note that such P and Q exist as $A + BF$ and $A_{22} - K_R A_{12}$ are asymptotically stable.

For any $\delta \in (0, 1)$, let c_δ be the largest positive scalar such that for all

$$\begin{pmatrix} x \\ x_v \end{pmatrix} \in X_\delta := \left\{ \begin{pmatrix} x \\ x_v \end{pmatrix} : \begin{pmatrix} x \\ x_v \end{pmatrix}' \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix} \begin{pmatrix} x \\ x_v \end{pmatrix} \leq c_\delta \right\}, \quad (4.12)$$

it is not difficult to get,

$$\left| \begin{bmatrix} F & F_2 \end{bmatrix} \begin{pmatrix} x \\ x_v \end{pmatrix} \right| \leq u_{\max}(1 - \delta). \quad (4.13)$$

Also, let

$$H := [1 - F(A + BF)^{-1}B]G, \quad (4.14)$$

and

$$x_e := -(A + BF)^{-1}BGr. \quad (4.15)$$

The linear control law (4.6) will drive the system output $y(t)$ to asymptotically track the step input of amplitude r , provided that the initial state x_0 , x_{v0} and r satisfy:

$$\begin{pmatrix} x_0 - x_e \\ x_{v0} + K_R x_{10} - x_{20} \end{pmatrix} \in X_\delta \quad \text{and} \quad |Hr + Gm| \leq \delta \cdot u_{\max} \quad (4.16)$$

Proof. Let us define new states as following

$$\tilde{x} = x - x_e, \quad \text{and} \quad \tilde{x}_v = x_v - x. \quad (4.17)$$

It is simple to verify that the linear feedback control law of (4.6) can be rewritten

as

$$\begin{cases} \dot{\tilde{x}}_v &= (A_{22} - K_R A_{12})\tilde{x}_v \\ u_L &= \begin{bmatrix} F & F_2 \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} + Hr \end{cases} \quad (4.18)$$

Hence for all states

$$\begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} \in X_\delta \Rightarrow \left| \begin{bmatrix} F & F_2 \end{bmatrix} \begin{pmatrix} x \\ x_v \end{pmatrix} \right| \leq u_{\max}(1 - \delta) \quad (4.19)$$

and for any r satisfying

$$|Hr| \leq \delta \cdot u_{\max}, \quad (4.20)$$

it has

$$|u_L| = \left| \begin{bmatrix} F & F_2 \end{bmatrix} \begin{pmatrix} x \\ x_v \end{pmatrix} + Hr \right| \leq \left| \begin{bmatrix} F & F_2 \end{bmatrix} \begin{pmatrix} x \\ x_v \end{pmatrix} \right| + |Hr| = u_{\max}. \quad (4.21)$$

Thus, for all $\tilde{x}$ and $\tilde{x}_v$ satisfying the condition as given in (4.19), the closed-loop system comprising the given plant and the linear control law can be rewritten as

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\tilde{x}}_v \end{pmatrix} = \begin{bmatrix} A + BF & BF_2 \\ 0 & A_{22} - K_R A_{12} \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}. \quad (4.22)$$

Next, define a Lyapunov function for the closed-loop system (4.22),

$$V = \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}. \quad (4.23)$$

Along the trajectories given in system (4.22), the derivative of the Lyapunov function is given by

$$\begin{aligned} \dot{V} &= \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & PBF_2 \\ F_2' B' P & -W_Q \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} \\ &= \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & 0 \\ 0 & -\tilde{W}_Q \end{bmatrix} \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix} \end{aligned} \quad (4.24)$$

where

$$\hat{x} = \tilde{x} - W_P^{-1} PBF_2 \tilde{x}_v, \quad (4.25)$$

and

$$\tilde{W}_Q = W_Q - F_2' B' P W_P^{-1} PBF_2. \quad (4.26)$$

With the choice of W_Q satisfying (4.10), it is obvious that $\dot{V} \leq 0$. This shows that X_δ is an invariant set of the closed-loop system (4.22) and all the trajectories will converge asymptotically to the origin. Thus for the initial states x_0 and x_{v0} and tracking amplitude r that satisfy (4.16),

$$\lim_{t \rightarrow \infty} \tilde{x}_v = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} x(t) = x_e, \quad (4.27)$$

which imply

$$\lim_{t \rightarrow \infty} y = \lim_{t \rightarrow \infty} C_1 x(t) = r. \quad (4.28)$$

This completes the proof of Lemma 4.1.

4.2.2 Nonlinear Feedback Design

The nonlinear feedback control law is given by

$$u_N = \rho(r, y) B' P \left[\begin{pmatrix} y \\ x_v + K_R y \end{pmatrix} - x_e \right] \quad (4.29)$$

where ρ is any non-positive function which is used to increase the closed-loop system damping ratio as the output approaches the target value. P is the solution defined in the Lyapunov equation (4.9). When the feedback gain is chosen to be robust solution $B'P$, the Lyapunov equation defined in (4.9) converts to the algebraic Riccati equation (ARE) with positive definite matrix W_p :

$$(A + BF)'P + P(A + BF) - PBB'P + W_p = 0. \quad (4.30)$$

The choice of such a $\rho(r, y)$ will be discussed now.

Suppose a second-order linear system (4.1) is considered with

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ b \end{bmatrix} \quad (4.31)$$

and for a negative constant ρ , a linear feedback law be

$$u = \rho B' P (x - x_e) \quad (4.32)$$

The relationship between ρ and closed-loop poles can be obtained on the condition that symmetric matrix $P > 0$ be the unique solution to the Lyapunov equation

$$A'P + PA = -\bar{P} \quad (4.33)$$

where $\bar{P}$ is any positive definite matrix.

Supposed that let P be partitioned as

$$P = \begin{bmatrix} p_1 & p_2 \\ p_2 & p_3 \end{bmatrix} \quad (4.34)$$

Then, simple algebra shows that the two poles of the closed-loop systems are given by

$$\lambda_1(\rho) = \frac{a_2}{2} + \frac{\rho p_3 b^2}{2} - \frac{1}{2} \left[(a^2 + \rho p_3 b^2)^2 - 4(a_1 + \rho p_2 b^2) \right]^{\frac{1}{2}} \quad (4.35)$$

$$\lambda_2(\rho) = \frac{a_2}{2} + \frac{\rho p_3 b^2}{2} + \frac{1}{2} \left[(a^2 + \rho p_3 b^2)^2 - 4(a_1 + \rho p_2 b^2) \right]^{\frac{1}{2}} \quad (4.36)$$

It is now straightforward to verify that

$$\lim_{\rho \rightarrow -\infty} \lambda_1(\rho) = -\frac{2p_2}{p_3} \quad \lim_{\rho \rightarrow -\infty} \lambda_2(\rho) = -\infty \quad (4.37)$$

Guided by the above formula, to retain fast response of a system with a small damping ratio and at the same time, to increase the damping ratio as the output approaches the reference input, $|\rho(r, y)|$ can be chosen to be a decreasing function of the error signal $|y - r|$ such that

$$\lim_{|y-r| \rightarrow \infty} \rho(r, y) = 0 \quad \lim_{|y-r| \rightarrow 0} \rho(r, y) = -\beta \quad (4.38)$$

where β is a large positive constant.

4.2.3 Composition of Linear and Nonlinear Feedback

The final nonlinear feedback law is then composed as

$$\begin{cases} \dot{x}_v &= (A_{22} + K_R A_{12})x_v + (A_{21} + (A_{11} - A_{22} - K_R A_{12})K_R)y + (B_2 + K_R B_1)\text{sat}(u) \\ u &= F \left[\begin{pmatrix} y \\ x_v + K_R y \end{pmatrix} - x_e \right] + Hr + \rho(r, y)B'P \left[\begin{pmatrix} y \\ x_v + K_R y \end{pmatrix} - x_e \right] \end{cases} \quad (4.39)$$

With the nonlinear feedback law as given by (4.39), the stability condition is presented in the following theorem.

Theorem 4.1 *Consider the given system (4.1). Then there exists a scalar ρ^* such that for any non-positive function $\rho(r, y)$ and $|\rho(r, y)| \leq \rho^*$, the nonlinear control law (4.39) will drive the system to asymptotically track the reference command with an amplitude r from an initial state x_0 , provided that x_0 , x_{v0} and r satisfy (4.16).*

Proof. Again, let $\tilde{x} = x - x_e$, and $\tilde{x}_v = x_v - x$. In the new coordinates $(\tilde{x} \ \tilde{x}_v)'$, the closed-loop system comprising the given plant (4.1) and the control law (4.39) can be expressed as

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\tilde{x}}_v \end{pmatrix} = \begin{bmatrix} A + BF & BF_2 \\ 0 & A_{22} - K_R A_{12} \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \omega. \quad (4.40)$$

where

$$\omega = \text{sat}(u_L + u_N) - u_L, \quad (4.41)$$

and for the initial states $\tilde{x}(0)$ and $\tilde{x}_v(0)$ satisfying (4.16), they have

$$\begin{pmatrix} \tilde{x}(0) \\ \tilde{x}_v(0) \end{pmatrix} \in X_\delta. \quad (4.42)$$

Using the following Lyapunov function:

$$V = \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} P & 0 \\ 0 & Q \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}, \quad (4.43)$$

the derivative of V along the trajectories given in (4.40) is written to be

$$\dot{V} = \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & PBF_2 \\ F_2'B'P & -W_Q \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} + 2\tilde{x}'PB\omega \quad (4.44)$$

Note that for all

$$\begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} \in X_\delta \Rightarrow \left| \begin{bmatrix} F & F_2 \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} \right| \leq u_{\max}(1 - \delta). \quad (4.45)$$

Evaluation of the derivative of V along the trajectories of the closed-loop system (4.40) is shown for three different cases:

Case 1. If $|u_L + u_N| \leq u_{\max}$, then

$$\omega = u_N = \rho[B'P \quad B'P] \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}, \quad (4.46)$$

which implies

$$\begin{aligned} \dot{V} &= \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & PB(F_2 + \rho B'P) \\ (F_2 + \rho B'P)'B'P & -W_Q \end{bmatrix} \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix} + 2\rho\tilde{x}'PBB'P\tilde{x} \\ &\leq \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & 0 \\ 0 & -\tilde{W}_Q \end{bmatrix} \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix}, \end{aligned} \quad (4.47)$$

where

$$\begin{aligned} \hat{x} &= \tilde{x} - W_P^{-1}PB(F_2 + \rho B'P)\tilde{x}_v, \\ \tilde{W}_Q &= W_Q - (F_2 + \rho B'P)'B'PW_P^{-1}PB(F_2 + \rho B'P). \end{aligned} \quad (4.48)$$

Note (4.10), i.e., $W_Q > F_2'B'PW_P^{-1}PBF_2$, it is clear that there exists a $\rho_1^* > 0$ such that for any scalar function satisfying $|\rho(r, y)| \leq \rho_1^*$, $\tilde{W}_Q > 0$ and hence $\dot{V} \leq 0$.

Case 2. If $u_L + u_N > u_{\max}$, then for the trajectories inside X_δ ,

$$\left| \begin{bmatrix} F & F_2 \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} + Hr \right| \leq u_{\max}, \quad (4.49)$$

which implies

$$0 < \omega < \rho[B'P \quad B'P] \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}. \quad (4.50)$$

Let us express

$$\omega = q\rho \begin{bmatrix} B'P & B'P \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}, \quad (4.51)$$

for an appropriate positive piecewise continuous function $q(t)$ bounded by 1 for all t . In this case,

$$\begin{aligned} \dot{V} &= \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & PB(F_2 + q\rho B'P) \\ (F_2 + q\rho B'P)'B'P & -W_Q \end{bmatrix} \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix} + 2q\rho \tilde{x}'PBB'P\tilde{x} \\ &< \begin{pmatrix} \hat{x}_+ \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & 0 \\ 0 & -\tilde{W}_{Q+} \end{bmatrix} \begin{pmatrix} \hat{x}_+ \\ \tilde{x}_v \end{pmatrix}, \end{aligned} \quad (4.52)$$

where

$$\begin{aligned} \hat{x}_+ &= \tilde{x} - W_P^{-1}PB(F_2 + q\rho B'P)\tilde{x}_v, \\ \tilde{W}_{Q+} &= W_Q - (F_2 + q\rho B'P)'B'PW_P^{-1}PB(F_2 + q\rho B'P). \end{aligned} \quad (4.53)$$

Again, noting (4.10), it can be shown that there exists a $\rho_2^* > 0$ such that for any scalar function satisfying $|\rho(r, y)| \leq \rho_2^*$, $\tilde{W}_{Q+} > 0$ and hence $\dot{V} \leq 0$.

Case 3. If $u_L + u_N < -u_{\max}$, then

$$\rho \begin{bmatrix} B'P & B'P \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} < \omega = -u_{\max} - u_L < 0, \quad (4.54)$$

and suppose

$$\omega = q'\rho \begin{bmatrix} B'P & B'P \end{bmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix}, \quad (4.55)$$

with positive piecewise continuous function $q'(t)$ bounded by 1 for all t . The derivative of V becomes,

$$\begin{aligned} \dot{V} &= \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & PB(F_2 + q'\rho B'P) \\ (F_2 + q'\rho B'P)'B'P & -W_Q \end{bmatrix} \begin{pmatrix} \hat{x} \\ \tilde{x}_v \end{pmatrix} + 2q'\rho \tilde{x}'PBB'P\tilde{x} \\ &< \begin{pmatrix} \hat{x}_- \\ \tilde{x}_v \end{pmatrix}' \begin{bmatrix} -W_P & 0 \\ 0 & -\tilde{W}_{Q-} \end{bmatrix} \begin{pmatrix} \hat{x}_- \\ \tilde{x}_v \end{pmatrix}, \end{aligned} \quad (4.56)$$

where

$$\begin{aligned}\hat{x}_- &= \tilde{x} - W_P^{-1}PB(F_2 + q'\rho B'P)\tilde{x}_v, \\ \tilde{W}_{Q-} &= W_Q - (F_2 + q'\rho B'P)'B'PW_P^{-1}PB(F_2 + q'\rho B'P).\end{aligned}\tag{4.57}$$

Again, it can show that there exists a $\rho_3^* > 0$ such that for any scalar function satisfying $|\rho(r, y)| \leq \rho_3^*$, $\dot{V} \leq 0$ is easily obtained.

Finally, let $\rho^* = \min(\rho_1^*, \rho_2^*, \rho_3^*)$. Then we have for any non-positive scalar function $\rho(r, y)$ satisfying $|\rho(r, y)| \leq \rho^*$,

$$\dot{V} \leq 0 \quad \forall \begin{pmatrix} \tilde{x} \\ \tilde{x}_v \end{pmatrix} \in X_\delta.\tag{4.58}$$

This in turn implies that, for all initial states $\tilde{x}(0)$ and $\tilde{x}_v(0)$ and the reference command of amplitude r that satisfy (4.16),

$$\lim_{t \rightarrow \infty} \tilde{x}_v(t) = 0 \quad \text{and} \quad \lim_{t \rightarrow \infty} x(t) = x_e\tag{4.59}$$

and hence

$$\lim_{t \rightarrow \infty} y = \lim_{t \rightarrow \infty} C_1 x(t) = r.\tag{4.60}$$

This completes the design of nonlinear feedback controller 1 for fast seek-settling performance in HDDs.

Remark 4.1 From Theorem 4.1, the nonlinear part u_N given by (4.29) does not impair the asymptotical tracking ability of the nominal linear feedback control law u_L given by (4.6). Specifically, any reference command that can be tracked by the linear feedback law (4.6) without saturation can also be tracked by the composite nonlinear feedback control law (4.39) for any bounded non-positive $\rho(r, y)$.

As presented before, the design is carried out in the continuous-time domain. It will be transformed to the discrete-time domain by using bilinear transformation

with a certain sampling rate in the real application to the hard disk drive. The description of its implementation and simulation and experimental results will be presented in the next chapter.

In the following section, another kind of nonlinear feedback controller will be discussed. It is designed for the case when the R/W head is approaching to the track center and servo mode is handing over from seeking to following, the effect of disturbance on the settling phase cannot be neglected.

4.3 Nonlinear Feedback Controller 2

Bias, the average torque (or current) required to maintain a set actuator position, is one of significant disturbance sources in HDDs. It varies depending on several factors, including the actuator position, the length of time at the set position, the direction of the actuator motion, the distance travelled by the actuator, the proximity of the set position to locations at which the actuator changed direction, and the proximity of the set position to locations at which the actuator rested [34].

When a bias disturbance is present in a servo system, usually integral compensation is included in the closed-loop control. There are several ways to provide the integral action, of which the two most commonly used methods are state augmentation and bias estimation [77]. The aim of the integral control is to cancel the bias disturbance in the steady state so that the positioning error is minimized. However, the dynamics of the system with integral compensation may significantly increase the settling time in the transient response of the system, as described in performance limitations of Chapter 2.

Initial state compensation based on bias on the integral component was developed to improve settling performance. In [77], bias is assumed to be known or approximated by a constant within each zone. This is only effective when actuator is rest at track-following mode, and changes of bias are not obvious; unknown bias was predicted and calibrated in ([33] and [34]). Besides initial state compensation, mode switching control scheme was proposed in [41], where integrator was introduced to the servo loop when tracking error arrived at some threshold. However, it is not robust for parameter variation.

The goal of this section is to develop a kind of nonlinear PID (NPID) controller that improves settling performance due to unknown bias without degrading other performance specifications such as servo bandwidth and steady state performance. Nonlinear controller works continuously in the whole process, therefore makes the mode change unnecessary and further, to improve the robustness of the servo system.

PD controller was originally usually adopted for fast response, while it does not deal with steady state error. In most cases, integrator is usually introduced to the servo loop when servo mode is transferring to the following from seeking mode. This switching behavior will degrade settling performance due to initial state on the mode switching. Nonlinear PID controller, which consists of a PD controller and a nonlinear integrator shaped by a nonlinear Gaussian function of PES, is developed for fast response without steady state error. When the tracking error is large, the integrator is almost deactivated and PD controller works mainly for short rise time; with the output approaching target value, the gain of integrator increases gradually to a pre-defined value to deal with offset due to unknown bias in the HDD servo

loop.

Different from Controller 1 described in Section 4.2, all the designs in the section are made in the discrete-time domain.

4.3.1 Linear PID Design

A discrete linear PID controller is designed first according to the specifications such as servo bandwidth and stability margins, which is represented as

$$C_L(z) = K_p + K_d(1 - z^{-1}) + \frac{K_i}{1 - z^{-1}}. \quad (4.61)$$

If $K_i = 0$, it is PD controller.

Supposed the plant is modelled to be a second-order system with low frequency torque friction considered and with high frequency resonances removed:

$$P(s) = \frac{c}{s^2 + as + b} \quad (4.62)$$

Discretizing it with a sampling period of T and taking tracking error $e(k) = r(k) - y(k)$ and all its differences to be states, the closed-loop difference equations with discretized second-order plant (4.1) is obtained:

$$X(k+1) = A_L X(k), \quad (4.63)$$

with

$$A_L = \begin{bmatrix} A_1 & A_2 \\ m & A_3 \end{bmatrix} \quad (4.64)$$

where $X(k) = [x_1(k) \ x_2(k) \ x_3(k)]' = [e(k) \ \dot{e}(k) \ \ddot{e}(k)]'$; and

$$A_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad A_2 = \begin{bmatrix} T & 0 \\ 1 & T \end{bmatrix} \quad A_3 = [-(cK_p + b)T \quad -(cK_d + a)T + 1]. \quad (4.65)$$

Note that only $m = -cK_iT$ is related to integrator coefficient. For convenient analysis later, it is separated from other partitioned matrices. The following lemma may determine how the output can track r stably by such a linear control law.

Lemma 4.2 *Given a negative definite symmetric matrix W_L , let real symmetric matrix $H > 0$ be the solution of the following Lyapunov equation:*

$$W_L = A_L' H A_L - H. \quad (4.66)$$

Then, the control law in (4.61) will cause the system output y to track asymptotically a step command input r . It is easily verified by choosing the following Lyapunov function

$$V(k) = X'(k) H X(k). \quad (4.67)$$

Along the trajectories in (4.63), the increment of the Lyapunov function in (4.67) is given by:

$$\begin{aligned} \nabla V(k+1) &= X'(k+1) H X(k+1) - X'(k) H X(k) \\ &= (AX(k))' H (AX(k)) - X(k)' H X(k) \\ &= X'(k) W_L X(k) < 0 \end{aligned} \quad (4.68)$$

This shows that X in the closed-loop system (4.63) will converge to the origin, thus

$$\lim_{k \rightarrow \infty} y(k) = r. \quad (4.69)$$

4.3.2 Nonlinear PID Design

Let the PD part in (4.61) unchanged. A nonlinear Gaussian function of PES is developed to tune the integrator gain,

$$f(e) = k \exp(-ne^2) \quad (4.70)$$

where n is a tunable parameter to shape the integrator state, the selection of k is determined by the integrator gain K_i in (4.61), and “exp” means the common exponential function. The portrait illustration of Gaussian function is shown in Figure 4.1. When the PES is large, $f(e)$ would be so small that the integrator is almost deactivated, and the main control task is executed by an approximate PD type controller to achieve fast response. While PES is approximating to zero at the settling mode, $f(e)$ would increase exponentially so that the integrator strengthens its work gradually to deal with the system steady performance. Thus, the closed-loop performance is determined by a dynamic PID controller continuously varying with PES. By the appropriate selection of the parameters n and k , both the PD controller for the fast response and the PID controller for the steady-state performance can be taken advantages of while their respective adverse effects are alleviated.

Figure 4.1: The function of $f(e)$.

From (4.70), the parameter n will affect the shaping degree of Gaussian function and thus determine whether the NPID controller can work perfectly or not to coor-

dinate small rise time and improved settling process. It can be determined by the following equation,

$$n = \ln k / (\Delta r)^2 \quad (4.71)$$

where Δ is the percentage width of the settling belt in the step response, it is usually acceptable for 10% of target value in HDDs. The formula is selected such that the integrator gain at the settling boundary is just K_i in (4.61). Then, the NPID controller is formulated

$$C(z) = K_p + K_d(1 - z^{-1}) + \left(\frac{K_i}{1 - z^{-1}} \right) f(e). \quad (4.72)$$

Theorem 4.2 *Consider the discrete-time closed-loop system (4.63) with conditions of (4.65), for any non-negative function $f(e)$ which is less than unity, the NPID control law (4.72) will drive the system output to track asymptotically the step command input, provided that the linear PID and PD systems in (4.63) are asymptotically stable.*

Proof. Let $x_1(k) = r(k) - e(k)$ and $x_n(k)$ ($n = 2, 3$) is its n th differences, the nonlinear closed-loop system can be written as

$$X(k+1) = A_N X(k), \quad (4.73)$$

Note that A_N is almost the same as A_L in (4.63), except $mf(e)$ which is related to nonlinear integrator,

$$A_N = \begin{bmatrix} A_1 & A_2 \\ mf(e) & A_3 \end{bmatrix} \quad (4.74)$$

with the same definition of A_1, A_2 and A_3 . Define a Lyapunov function with the same positive symmetric matrix H in (4.66),

$$V(k) = X'(k) H X(k). \quad (4.75)$$

it can be evaluated that the increment of $V(k)$ along the trajectories of the closed-loop system in (4.73) as follows:

$$\begin{aligned}\nabla V(k+1) &= X'(k+1)HX(k+1) - X'(k)HX(k) \\ &= X'(k)(A'_N H A_N - H)X(k)\end{aligned}\quad (4.76)$$

If $W_N = A'_N H A_N - H$ is defined, and supposed H_{ij} is described to be the i th row j th column element of matrix H , then

$$W_N = \begin{bmatrix} 2mf(e)W_0 + (mf(e))^2W_1 & TC_1 + mf(e)W_{21} & TC_2 + mf(e)W_{22} \\ (TC_1 + mf(e)W_{21})' & W_{31} & W_{32} \\ (TC_2 + mf(e)W_{22})' & W_{32} & W_{33} \end{bmatrix} \quad (4.77)$$

with

$$C_1 = H_{11} + (cK_p + b)H_{13} \quad (4.78)$$

$$C_2 = H_{12} + (cK_d + a)H_{13} \quad (4.79)$$

Furthermore, we have $C_1 = C_2 = 0$ is the stability requirements for linear PD control system (4.63) with $m = 0$ in (4.64). Therefore, it can be further obtained

$$W_N = \begin{bmatrix} 2mf(e)W_0 + (mf(e))^2W_1 & mf(e)W_2 \\ (mf(e)W_2)' & W_3 \end{bmatrix} \quad (4.80)$$

where $W_0 = TH_{13}$ and W_1 are scalars, $W_2 = [W_{21} \ W_{22}] \in \mathbb{R}^{1 \times 2}$ and $W_3 \in \mathbb{R}^{2 \times 2}$ is symmetric matrix independent of $mf(e)$. Since $0 < f(e) < 1$, and $H_{13} > 0$ is the stability demand of PD system, and suppose that $W_{L(N)}(i)$ the i th-order principal minor determinant of $W_{L(N)}$, the following formulae can be derived:

$$\begin{aligned}W_N(1) &= 2mf(e)W_0 + (mf(e))^2W_1 \\ &< 2mf^2(e)W_0 + (mf(e))^2W_1 \\ &= f^2(e)W_L(1) < 0,\end{aligned}\quad (4.81)$$

From Lemma,

$$W_L(2) = W_L(1)W_3(1) - (mW_2(1))^2 > 0, \quad (4.82)$$

which means $W_3(1) < 0$. And thus,

$$\begin{aligned} W_N(2) &= W_N(1)W_3(1) - (mf(e)W_2(1))^2 \\ &= f^2(e)(2mW_0/f(e)W_3(1) + W_1W_3(1) - (mW_2(1))^2) \\ &> f^2(e)(2mW_0W_3(1) + W_1W_3(1) - (mW_2(1))^2) \\ &= f^2(e)W_L(2) > 0 \end{aligned} \quad (4.83)$$

And,

$$\begin{aligned} W_N(3) &= \begin{vmatrix} W_N(1) & mf(e)W_2 \\ mf(e)W_2' & W_3 \end{vmatrix} \\ &= f^2(e)(W_1/f(e)|W_3| + \Phi) \end{aligned} \quad (4.84)$$

where Φ is not related to nonlinear gain $f(e)$. And note that $|W_3| > 0$ is the stability result of PD system. Therefore,

$$\begin{aligned} W_N(3) &= f^2(e)(W_1/f(e)|W_3| + \Phi) \\ &< f^2(e)(W_1|W_3| + \Phi) \\ &= f^2(e)W_L(3) < 0 \end{aligned} \quad (4.85)$$

Therefore, symmetric matrix W_N is negative definite according to Hurwitz criterion [147]. X is an invariant set of the closed-loop system in (4.73), which in turn indicates X will converge to the origin, and

$$\lim_{k \rightarrow \infty} y(k) = r. \quad (4.86)$$

Remark 4.2 Theorem shows that nonlinear integrator gain does not affect the ability for the closed-loop system to track command input. Any command that can be tracked by the linear feedback law in (4.63) can also be tracked by the nonlinear

law in (4.73) for any non-negative function $f(e) < 1$. This freedom can be used to improve the performance with different choice of $f(e)$.

This completes the theorem of nonlinear feedback controller 2. Its efficacy with disturbance rejection, initial value effect, and robustness will be shown in the next Chapter.

4.4 Conclusions

In this chapter, two control theories were proposed for fast seek-settling performance. One is motivated by the design tradeoff of fixed closed-loop poles on the step response. That is, poles with small damping ratio would necessarily lead to large overshoot and long settling process. Variable pole-placement technique was proposed based on the Lyapunov function in this design. The other, considered the effect of disturbance on the settling phase when the following mode is taking over from seeking mode. Therefore, a nonlinear integrator was developed to deal with unknown bias with short settling time.

Both of the practical design procedures on the applications will be presented in the next chapter. Feedback control designs focus on the settling performance, servo bandwidth and disturbance rejection. However, for the effect of set point changes, the two-degree-of-freedom (2DOF) control which combines both feedforward and feedback designs is obviously more powerful. The details are described in Chapter 5.

Chapter 5

2DOF Control Design and Application Results

This chapter introduces the two-degree-of-freedom (2DOF) control for the HDDs. 2DOF control was extensively applied to motion control systems ([79], [83] and [100]). As shown in Figure 5.1, the 2DOF control structure consists of two major components: a nonlinear feedback controller and a feedforward reference generator, also named as input shaper. Two kinds of feedback controllers have been described in detail in Chapter 4. Input shaper will be described in Section 5.2. Both of two kinds of 2DOF control schemes will be applied in the real hard disk drive. The efficacy will be demonstrated experimentally.

5.1 Introduction

The primary objective of the 2DOF control is to improve the HDD response for set point changes, settling performance and capacities of disturbance rejection.

The motivation of 2DOF control 1 with nonlinear feedback controller 1 comes from long seek performance, which significantly leads to actuator saturation due to both linear and nonlinear high gains. 2DOF control 2 takes friction and bias distur-

bance into the consideration, and improves both seeking and following performance in one unified servo.

The 2DOF control has two main components: 1) nonlinear feedback controller; 2) reference generator. The feedback controller is to solve the settling problems, disturbance compensation for the pivot friction and the compliance bias from the flexible printed circuit (Figure 1.1). The reference generator provides the desired position profile for track seeking, avoids the actuator saturation, and further improves the closed-loop stability.

Figure 5.1: The 2DOF control structure.

The rest of this chapter is organized as follows. Section 5.2 describes the design of input shaper. Application details of two kinds of 2DOF control scheme to HDDs are presented in Section 5.3 and Section 5.4 respectively. Conclusions are given in Section 5.5.

5.2 Design of Input Shaping

It can be seen from Figure 5.1 that the 2DOF control system is a position tracking system. Performance requirements and actuator power limitation during the track seeking are all reflected in the position reference profile. Control saturation may

degrade system performance and even cause system instability. For example, Theorem 4.1 states that the composite nonlinear system stability is based on the stable linear system without saturation. This makes the reference profile a key element in the 2DOF control structure. The saturation issue was reported in [71] through anti-windup feedback control design in dual-stage HDDs. The technique of input shaping was also adopted to reduce vibrations for disk drive heads [106].

5.2.1 Reference Profiles Requirements

In the HDD servos, the requirements for the reference profiles of the 2DOF control structure includes the following:

1. It should generate aggressive profile faster than feedback servo so as not to increase the seek time;
2. The profiles must be achieved by the actuator, considering its limited bandwidth;
3. The profiles should be smooth enough not to excite high-frequency suspension and arm resonance modes;
4. The profile should have no overshoot at the end of a seek;
5. The implementation should not be too complicated, considering the computing power and memory constraints.

A well-known fact [16] in linear second-order system is the contradiction between fast response (requirement 1) and small overshoot (requirement 4). It is well-known

that the response speed is actually related to system bandwidth, limited bandwidth would constrain response speed (requirement 2). To meet all of the above requirements for the reference profiles, a parameterized second-order dynamic system is developed in this section. It was originally proposed by Han [65] to construct the differentiation of discontinuous signal for nonlinear systems, and was modified based on full-state estimator to produce reference profile in ([98] and [150]).

5.2.2 Predictive-estimator Based Input Shaper

The design is carried out in the continuous-time domain, and will be transformed to the discrete-time domain using bilinear transformation when implementing it in the hard disk drive.

Theorem 5.1 *For a second-order system*

$$\dot{z}(t) = Az(t), z(t) = (z_1(t) \quad \dot{z}_1(t))' \quad (5.1)$$

so that $A \in \mathbb{R}^{2 \times 2}$ is Hurwitz, the following system

$$\dot{x}_d = A_d x_d + B_d r, x_d(t) = (x_{d1}(t) \quad \dot{x}_{d1}(t))'. \quad (5.2)$$

satisfies

$$\lim_{R \rightarrow \infty} x_{d1}(t) = r, \forall t \in (0, T] \quad (5.3)$$

where r is reference command, $R > 0$ is tunable parameter, and if define

$$\hat{R} := \begin{bmatrix} 1 & 0 \\ 0 & R \end{bmatrix}, C := C_d' r = \begin{bmatrix} r \\ 0 \end{bmatrix} \quad (5.4)$$

Then,

$$A_d = R^2 \hat{R} A \hat{R}^{-1}, B_d = -A_d C_d'. \quad (5.5)$$

Proof. Let

$$x_{d1}(t) = z_1(Rt) + r \quad (5.6)$$

then,

$$x_{d2}(t) = \dot{x}_{d1}(t) = R\dot{z}_1(Rt) = Rz_2(Rt), \quad (5.7)$$

and

$$\dot{x}_{d2}(t) = R^2 z_2(Rt). \quad (5.8)$$

Replacing the system (5.1) with (5.6), (5.7) and (5.8), the new tracking system can be obtained, i. e. (5.2). Further,

$$\lim_{R \rightarrow \infty} x_{d1} = r \quad \forall t \in (0, T] \quad (5.9)$$

This completes the proof of Theorem 5.1.

Remark 5.1 Theorem 5.1 shows that for any asymptotically stable system (5.1) with infinite settling time, system (5.2) will approach the target value r in finite time without any other performance degradation as long as R is large enough. Specifically, the settling time would be decreased by times of $1/R$ after transformation in system (5.2).

With bandwidth about 800 Hz and acceleration factor $R = 5$, the reference trajectory with the proposed continuous-time input shaper is shown in Figure 5.2. With 1 μm track length, the profile arrives at the target value with less than 0.2 ms without overshoot.

From Theorem 5.1, the proposed input shaper is essentially an infinite impulse response (IIR) filter with specified acceleration factor R and bandwidth which is

determined by A_d and B_d . If converted into the discrete-time version, it will show the same characteristic as predictive estimator [113]. However, this will lead to response lag at the first sampling period, and will further increase unnecessary controller overhead (extra access time). The improvement will be shown shortly in the next subsection.

Figure 5.2: Reference trajectory with predictive estimator.

5.2.3 Improvement Based on Current-estimator

To solve the controller overhead at the first sampling period, the discrete-time input shaper is modified. At every sampling period, the response is updated again with the newest result. This modification is like the transformation of predictive estimator to current estimator [116], where the term “current” refers to time. With sampling

period T , the modified input shaper is written as:

$$\begin{cases} \hat{x}_{d1}(k+1) &= \hat{x}_{d1}(k) + T\hat{x}_{d2}(k) \\ \hat{x}_{d2}(k+1) &= R^2k_1(\hat{x}_{d1}(k) - r(k)) + (Rk_2 + 1)\hat{x}_{d2}(k) \\ x_{d1}(k+1) &= (1 - L_c)\hat{x}_{d1}(k+1) + L_cr(k). \end{cases} \quad (5.10)$$

Tracking bandwidth is determined by k_1 and k_2 (elements in the matrix A_d and B_d); r is the seeking length, *i.e.*, step size in micrometer. L_c is the weighting factor defined in current estimator, which can be obtained from pole-placement method in MATLAB. The poles selection is based on closed-loop dynamics that its response should be faster than the closed-loop feedback system.

Figure 5.3 shows the trajectory of the improved input shaper. Through bilinear transformation at sampling rate of 20 kHz, the result of predictive input shaper (5.2) is also plotted. The improvement of response at the first sampling period of 0.05 ms is obvious with current input shaper (5.10).

Figure 5.3: Reference trajectories with discretized input shaper and its improvement.

5.3 Application Performance Analysis with 2DOF

Scheme 1

Following the design techniques of nonlinear 2DOF controller 1 and current estimator-based input shaper, this section uses example of Seagate HDD model, mentioned in Chapter 3, i.e., Seagate SCSI HDD model ST31276A to illustrate the design process, and both simulation and experimental results are shown in this Section.

5.3.1 Control Design Specifications

The actual design is not so complicated as described in the previous section. The procedures can be simplified as following when implementing it in an HDD:

1. Determine a state feedback gain matrix F in (4.6) using any appropriate method such as pole-placement algorithm to have a fast response with limited control input;
2. Select an appropriate weighting matrix K_R in (4.5) so that the dynamics of reduced-order observer is faster than that of feedback controller;
3. Determine state matrices A_d and B_d in (5.1) to achieve desired response faster than feedback loop in reference generator, while can be achieved by the actuator;
4. Determine an appropriate weighting factor L_c in (5.10) and acceleration factor R in (5.4);
5. Compute H in (4.14), G in (4.8) and steady state x_e in (4.15);

6. Select an appropriate $W_P > 0$ and solve the Lyapunov equation (4.9) for $P > 0$, which would be used in nonlinear part design;
7. Select an appropriate nonlinear damping ratio $\rho(r, y)$ to achieve improved settling performance.

The identified 4th-order Seagate model in (3.8) of Chapter 3 was used for simulation. The HDD servomechanism considered here is a double integrator with a mechanical resonance around 1.8 kHz. However, it is safe to remove the resonance by notch filter when focusing on the track seek-settling performance due to its limited working bandwidth. Thus, only double integrator model is taken into the consideration during the controller design. The model used is:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} y(t) \\ v(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 6.3648 \times 10^6 \end{pmatrix} u(t) \quad (5.11)$$

For the given model, the following linear feedback gain

$$F = -\frac{1}{6.3648 \times 10^6} [4\pi^2 f^2 \quad 4\pi f \zeta] \quad (5.12)$$

can be derived. The eigenvalues of $A + BF$ are placed at $(-\zeta \pm j\sqrt{1 - \zeta^2})2\pi f$. The nonlinear damping function is chosen as follows,

$$\rho(r, y) = -\beta e^{-n \left| \frac{y - x_{d1}}{r} \right|} \quad (5.13)$$

which is chosen to decrease exponentially with the increase of tracking error $|y - x_{d1}|$.

To make the improvements of response for both small and large step inputs realized, command size r is introduced to have ρ independent of step amplitude. And further,

$$H = 0, \quad G = -\frac{4\pi^2 f^2}{6.3648 \times 10^6}, \quad \text{and} \quad x_e = \begin{pmatrix} r \\ 0 \end{pmatrix}. \quad (5.14)$$

If the system parameters are substituted into the dynamics of reduced-order observer (4.5), it is further obtained that

$$\dot{x}_v = -K_R x_v - K_R^2 y + 6.3648 \times 10^6 \text{sat}(u) \quad (5.15)$$

Solving the above equation gives the following result

$$x_v = e^{A(t-t_0)} x_v(t_0) + \int_{t_0}^t e^{A(t-\tau)} B u(\tau) d\tau \quad (5.16)$$

When implementing, the control algorithm is transformed into the discrete-time domain using the bilinear transformation with sampling period of T . The estimated solution (5.16) is then re-written as

$$x_v(k+1) = e^{-K_R T} x_v(k) + \left[\frac{6.3648 \times 10^6}{K_R} (1 - e^{-K_R T}) \quad -K_R (1 - e^{-K_R T}) \right] \begin{pmatrix} u \\ y \end{pmatrix} \quad (5.17)$$

and this results in the estimated state

$$\tilde{x}_2 = \frac{1 - e^{-K_R T}}{K_R (z - e^{-K_R T})} (6.3648 \times 10^6 u - K_R^2 y) + K_R y. \quad (5.18)$$

For reference generator, the bandwidth is chosen four times faster than the closed-loop servo. Specifically, with pole-placement technique the natural frequency of reference generator $f_d = 4f$ and $\zeta_d = \zeta$. Theoretically speaking, the larger the acceleration factor R is, the faster response can be achieved. While it cannot be set large arbitrarily due to effect of noise and actuator bandwidth limitation.

5.3.2 The Well-known PTOS Controller

As mentioned in Chapter 4, time optimal control can achieve the minimum access time for head tacking seeking. However, it is not practical for servo system to implement in that:

1. Even the smallest system operation or measurement noise will cause control “chatter” due to the signum function used in the controller. This will excite the high frequency modes.
2. Any error in the plant model will cause limit cycles to occur.

In practice, proximate time optimal servomechanism (PTOS) is used instead ([55] and [92]). PTOS essentially uses the maximal acceleration when it is practical to do so. It replaces the signum function in time-optimal control law by a saturation function. So it switches to a linear control law when the error is small, not like the time optimal control to switch to either the maximal or the minimum values. The PTOS control law is based on a linear plant model of a rigid body, and is given by

$$u = u_{\max} \cdot \text{sat} \left(\frac{k_2 [f(e) - v]}{u_{\max}} \right) \quad (5.19)$$

where e is the tracking error and the function $f(e)$ is defined as

$$f(e) = \begin{cases} \frac{k_1}{k_2} e & \text{for } |e| \leq y_\ell \\ \text{sgn}(e) \left[\sqrt{2au_{\max}\alpha} |e| - \frac{u_{\max}}{k_2} \right] & \text{for } |e| > y_\ell \end{cases} \quad (5.20)$$

Here k_1 and k_2 are respectively the feedback gains for the position and velocity. α is a constant between 0 and 1 and is referred to as the acceleration discount factor. $\sqrt{2au_{\max}} |e|$ in $f(e)$ is the maximal velocity calculated from time-optimal control law. y_ℓ is the size of linear region.

As a comparison, PTOS control system was constructed in this section. Simulation results (Figures 5.6 and 5.8) present a pictorial illustration of advantages of the proposed 2DOF controller and make the motivation clearer. Experimental results (Figure 5.7) in real HDD setup are shown the validity of simulation.

In the PTOS design, the linear control gains k_1 and k_2 are similar with linear gain (5.12) used in the 2DOF control. The acceleration discount factor $\alpha = 0.95$. And the maximal input voltage is set to 8 V as experiment hardware limits. Both controllers were designed in continuous-time domain and converted to discrete-time domain with 20 kHz sampling frequency before entering the DSP system.

5.3.3 Simulation and Experimental Results

To make the work more complete, the design techniques were implemented on actual hard disk drives with some highly advanced and accurate equipment. A brief description of the key software and hardware tools used throughout this thesis to obtain the simulation and implementation results is presented below.

1. MATLAB and SIMULINK. All off-line computation and simulation of the results in this thesis are done using the well known products from Math Work, MATLAB 6.5 with its simulation package SIMULINK 5.1.
2. The dSPACE DSP system. A dSPACE digital signal processor (DSP) system (dSPACE GmbH, Paderborn, Germany) is used in the actual implementation throughout the thesis. The system has the following main components:
 - The dSPACE Add-on Card. The main component of the dSPACE DSP system is its add-on card, DS1103, which is built upon a Texas Instruments floating-point DSP. The DSP has been supplemented by a set of analog to digital and digital to analog converters with input span of ± 10 V, a DSP-micro-controller based digital I/O subsystem and incremental

sensor interfaces.

- Real-Time Interface (RTI) and Real-Time Workshop (RTW). RTI acts as a link between SIMULINK and the dSPACE hardware. It has built-in hardware control functions and blocks for DS1103 add-on card based on SIMULINK. This together with RTW automatically generate real-time codes from SIMULINK off-line models and implement these codes on the dSPACE real-time hardware.
 - The dSPACE Control Desk. This is a software platform that combines all the above tools of dSPACE for controlling, monitoring, and automating the implementation process on the actual hard disk drives.
3. Polytec laser doppler vibrometer (LDV) (Polytec OFV 3001S, Polytec, Waldbronn, Germany). The Polytec LDV is an optical instrument for accurately measuring velocity and displacement of vibrating surfaces completely without contact. The LDV system consists of two main components: 1) An optical sensor head which measures the dynamic Doppler shift from the vibrating object, 2) A controller (processor) which provides power to the optics and demodulates the Doppler information using various types of Doppler signal decoder electronics, thereby producing an analog vibration signal (velocity and/or displacement). This instrument is used to measure the displacement and velocity of the R/W head of hard disk drives.
 4. Dynamic signal analyzer (DSA) (HP 35670A, Hewlett Packard Company, Washington). The HP dynamic signal analyzer is a dynamic monitoring and

Figure 5.4: Implementation setup for Hard Disk Drives.

measuring instrument which can be used for characterizing the performance and stability of a control system. Performance parameters, such as rise time, overshoot, and settling time, are generally specified in the time domain. Stability criteria, gain/phase margins, are generally specified in the frequency domain. The instrument can also be used for system identification which was mentioned in Chapter 3.

5. Vibration free table (Newport RS 3000TM, Newport Electronics, Holland). Since the success of the actual implementation largely depends on the accurate measurement of the very small displacements of less than $1\ \mu m$, there is a need to isolate the HDD implementation setup from the external vibrations.

The experimental setup used in this thesis is depicted in Figure 5.4. The HDD with open casing which was described and identified in Chapter 3 is used in the experiment. The head displacement is measured using the scanning LDV, and is then

fed to the dSPACE system through which the necessary control input is generated. A sampling frequency of 20 kHz is employed in the DSP-base system. The practically evaluated controller bandwidth will not exceed 2 kHz and the largest considered actuator resonance is 5.5 kHz. Hence, the sampling frequency should be sufficiently fast for implementation of the continuous control scheme. The VCM driver input voltage was kept in the range of $[-8 \text{ V}, 8 \text{ V}]$.

Simulation and actual implementation results of the proposed nonlinear controller, the optimal controller PTOS (5.19), as well as the common linear PD type controller are presented in the following. For short seeking length, the step response with $0.4 \mu\text{m}$ is tested, which corresponds to the single-track performance in an HDD with 64 kTPI track density. The access time, which is the time to arrive at the target value, is 0.7 ms in the 2DOF and 0.9 ms in the PTOS. The reason why the proposed 2DOF scheme runs faster is that small size of step command makes the PTOS work mostly in the linear status. Whereas, as mentioned before in the design of nonlinear damping factor, the step size was introduced to make ρ independent of the step amplitude.

Although it is somewhat slower than (but approximate to) the PTOS with the increase of tracking length (from $10 \mu\text{m}$ to $100 \mu\text{m}$ in the test), the tracking performances in the proposed control are still good since the PTOS works as a time optimal controller in these tracking regions, which indicates that the 2DOF control law can achieve fast enough response without any degradation of other performance specifications. If the tracking length is kept increasing ($300 \mu\text{m}$ and $500 \mu\text{m}$), the actuator would be saturated in the PTOS control system. However, due to the ref-

erence generator, the 2DOF still works normally in the nonlinear closed-loop system even for 500 μm tracking length.

Figure 5.6 shows the simulation step responses of 10 μm seeking length. It is noted that the access time in the 2DOF control is quite close to that in the PTOS which is a time-optimal control at this stage. With the increase of tracking length, experimental results for 300 μm seeking performance and its control signals are shown in Figure 5.7. The simulation responses of 500 μm are shown in Figure 5.8, which clearly indicates that the proposed control scheme can achieve better performance with short rise time while small control input, and hence the challenge of actuator saturation can be relaxed to some extent. To have a complete overview of the two controllers (the PTOS and the proposed 2DOF) comparison, Figure 5.5 shows the access time curves against different seeking lengths.

Besides the optimal controller, the commonly-used PD type controller is also tested as a performance comparison, as shown Figures 5.9 and 5.10. Without degrading the track speed, the proposed nonlinear control scheme can remove the overshoot and furthermore, the control signal is obviously reduced due to the novel reference profile. With the increase of tracking length, control voltage is possible to be saturated at the acceleration phase as shown in Figure 5.10, where the advantage of reference profile to remove the actuator saturation is obvious and therefore, the settling performance is improved greatly.

Figure 5.5: Access time curves against seeking lengths.

(a). Output responses.

(b). Control signals.

Figure 5.6: Simulation results: Responses and Control signals for $r=10 \mu\text{m}$ seeking length.

(a). Output responses.

(b). Control signals.

Figure 5.7: Experimental results: Responses and Control signals for $r=300 \mu\text{m}$ seeking length.

(a). Output responses.

(b). Control signals.

Figure 5.8: Simulation results: Responses and Control signals for $r=500 \mu\text{m}$ seeking length.

(a). Output responses.

(b). Control signals.

Figure 5.9: Experimental results: Responses and Control signals of the proposed and linear controllers.

(a). Output responses.

(b). Control signals.

Figure 5.10: Simulation results: Responses and Control signals of the proposed and linear controllers.

5.4 Application Performance Analysis with 2DOF

Scheme 2

This section will give the application example of 2DOF scheme 2 to an open IBM HDD servo system. Seek-settling performance will be analyzed via phase-plane simulation. Bias disturbance is inserted to the servo loop manually as the flexible printed circuit was removed from the open HDD. Robustness of the 2DOF control is verified against bias disturbance, set point changes, non-zero initial state and plant perturbations.

5.4.1 Design Parameters

The model of VCM used in this experiment is an IBM open HDD, whose identification result is described in Chapter 3:

$$P(s) = \frac{2.097 \times 10^7}{s^2 + 150.8s + 6.317 \times 10^4} \times \frac{1.238 \times 10^9}{s^2 + 7037s + 1.238 \times 10^9}, \quad (5.21)$$

which consists of a second order system due to the pivot friction whose frequency is around 40 Hz and a main mechanical resonance due to arm carriage with the frequency of about 5.5 kHz. As mechanical resonance about 5.5 kHz is far away from servo bandwidth, which is below 2 kHz. The resonance peak can be notched easily by notch filter without any adverse effect.

The nonlinear feedback control law is implemented in the setup show in Figure 5.4 with a sampling frequency of 20 kHz, where $K_p = 0.77$, $K_d = 6.952$, $K_i = 0.007724$ are selected in this paper. Figure 5.11 displays the open-loop transfer function of linear PID control system after removing the mechanical resonance by notch filter

at 5.5 kHz. The gain margin is 5.3 dB, phase margin is about 40° , and the crossover frequency is 1.55 kHz. The NPID controller is obtained by making $n = 200$ and $k = 1.2$,

$$C(z) = 0.77 + 6.952(1 - z^{-1}) + \exp^{-200e^2} \frac{0.0086}{1 - z^{-1}}. \quad (5.22)$$

Figure 5.11: Open-loop frequency response of linear control servo system.

The bandwidth of reference generator is selected to be 2 kHz and weighting factor L_c is determined via pole-placement method by four times faster than the closed-loop system. Similar with 2DOF scheme 1, acceleration factor $R = 5$.

5.4.2 Settling Performance Analysis with Phase-Plane

The phase-plane analysis technique is to show the system state trajectory accurately with time to be a parameter without recourse to analytical solutions of the system difference equations, and thus it is widely used in studying nonlinear system performance [101].

Since high frequency resonance can be attenuated by notch filter, a second-order

plant, i.e., low frequency part in (5.21) is used to obtain the system state difference equations. Discretizing the second-order plant with “ZOH”, state difference equations of linear and nonlinear feedback systems are obtained by combining control laws in (4.61) and (4.72) and plant model. The two-dimensional phase-plane trajectories then can be easily drawn in MATLAB package.

Figures 5.12 and 5.13 give the phase-plane plots for both linear and nonlinear control systems. Here, supposed that track error $e = r - y$ and error rate as states to be considered. For the step response of zero initial state system, initial track error should be the input size. With the increase of time, the trajectories move clockwise about the origin and reach P_1 , the maximal negative excursion beyond the final steady state of the origin, so that it indicates overshoot. With moving further, it arrives at the maximal positive excursion P_2 , which indicates the undershoot in step response. In linear control system, the overshoot is about 28% and undershoot is about 10% together with some oscillation. While nonlinear step response of 1.4% overshoot and zero undershoot can be regarded to have entered the settling belt of 10–15% directly, and naturally, the settling process is shortened greatly.

Figure 5.12: Phase-plane trajectory using linear PID control

Figure 5.13: Phase-plane trajectory using NPID control

5.4.3 Robustness of the 2DOF Controller

(1) Against set point changes

Previous studies have shown that with the increase of track density, measured by track-per-inch (TPI), the single-track access time, which is an important perfor-

mance index in HDDs, does not decrease much correspondingly over these years [27]. An important reason is due to relatively long settling time.

From time-optimal control theory in [91], the optimal single-track access time is proportional to the square root of tracking length, i.e., reciprocal of the track density TPI.

$$t_{\min} = \sqrt{\frac{4r}{au_{\max}}}, \quad (5.23)$$

where a is a constant related to the plant; $u_{\max}$ means the maximal available control power and is predefined based on actuator limitation; r is tracking length.

The simulation results of NPID controller and data of Seagate product [27] (after fitting with a second-order polynomial) are plotted in Figure 5.14. The step size is verified from $3.66 \mu\text{m}$ to $0.2 \mu\text{m}$, which corresponds to track density from 6.9 to 128 kTPI. In order to show the improvement of variation tendency of single-track access time in the proposed NPID control system, the optimal prediction curves of two control systems (“NPID” control system and Seagate product data) are also plotted respectively with a reference point of time value at 6.9 kTPI, since the parameters a and $u_{\max}$ are different in the compared control systems. The optimal time can then be calculated based on the reference point with being proportional to the square root of the reciprocal of TPI. It is obvious that the variation tendency of the proposed controller is more approximate to the optimal prediction one, compared with the Seagate product data. To have a clearer view of comparison for the time variation, Figure 5.15 shows the time varying percentage with the increase of track density. Likewise, take the values at 6.9 kTPI as the reference points, the percentage of time variation is obtained through the backward difference calculation. Normally, two

prediction curves for the compared systems can be obtained. However, they are almost overlapped since the variation tendency of optimal time is the same and only one is presented here. Obviously again, compared with product data, the proposed controller is more approximate to time optimal control.

Figure 5.14: Comparisons of single-track access time and its variation against TPI.

Figure 5.15: Comparisons of single track access time variation percentage against TPI.

(2) Against non-zero initial state

Control systems are designed under the assumption that the initial states are zero. In a mode switching system which is usually used from seeking to following in HDD, the initial values cannot be ignored since “initial” means an instant of mode-switching [166], and state such as velocity may have non-zero values. At this case, a technique called initial value compensation (IVC) was usually proposed to improve the settling performances [167] in HDDs.

In order to simulate the effect of non-zero initial position and velocity on the performance, Figure 5.16 constructs sudden negative switching of 0.5 ms and positive switching at 10 ms to generate a non-zero initial velocity, its corresponding velocity responses are shown in Figure 5.17. Generally, non-zero initial state will lead to undesired settling process in the step response. However, due to nonlinear integrator in the feedback controller that works selectively to deal with both disturbance and non-zero initial state, an improved seek-settling performance is achieved.

Figure 5.16: Simulation results: switching step responses without IVC.

Figure 5.17: Simulation results: velocity responses.

(3) Against bias disturbance

As the implemented HDD is an open one and major bias such as cable forces is removed. Bias is thus intentionally inserted into the servo loop from -10 mV to 10 mV for robustness tests. Table 5.1 lists the experimental results. Under different bias, the 2DOF control system can work robustly and maintain much faster seek-settling performance than linear PID control system.

Table 5.1: Seek-settling Time (ms) with different control schemes (step = $0.4 \mu\text{m}$).

Control Strategy	Bias in mV			
	-10	-5	5	10
PID	1.2	1.1	1.0	1.1
2DOF	0.3	0.25	0.21	0.2

Figure 5.18 and Figure 5.19 show one case of the experimental results with the step size of $0.4 \mu\text{m}$ and bias of 10 mV, where step signal source of 200 mV comes

from signal generator and the resolution in LDV is $2 \mu\text{m}/\text{V}$. There are 4 signals displayed in both figures, which represent step reference, step response, control input and average control input after moving channel noises, respectively. Although both controllers can remove the steady-state error, it takes smaller time of about 0.2 ms for the 2DOF control system to enter the settling belt without extra control power which is shown in “control”, much faster than that in the linear PID one.

Figure 5.18: Experimental results for linear PID control system.

Figure 5.19: Experimental results for the proposed control system.

(4) Against plant parameter variation

The nonlinear feedback servo is designed based on mathematical models identified from experimental measurement. As resonance may shift with operating environments, this approximation of true dynamics of the actual plant and model used results in modeling uncertainties. Plant uncertainties on the other hand arises from plant parameter perturbations under various manufacturing environments. The 10 μm seeking performance of the designed nonlinear servo will be analyzed with effects of $\pm 5\%$ resonant frequency shift in the VCM actuator, as can be seen in Figure 5.20. The 2DOF control system is still stable under the resonance frequency shifts. Furthermore, plant perturbation does not severely degrade the seek-settling performance.

Figure 5.20: Simulation results: step responses with nominal and perturbed models.

5.5 Conclusion

In this chapter, the 2DOF control scheme which includes an input shaper and non-linear feedback controller was described. A new input shaper was proposed for desired reference trajectory so as to avoid actuator saturation, and it was further modified to improve the response lag at the first sampling period due to discrete transformation.

The proposed 2DOF control 1 focused on long seek-settling performance, which significantly leads to actuator saturation. Compared with the well-known PTOS controller, the 2DOF control 1 was shown to improve long-disturbance seek-settling performance, while not degrading short seeking performance.

The 2DOF control 2 took the bias disturbance effect into the consideration. With nonlinear feedback controller, settling performance was improved without degrading steady-state performance. Input shaper provided desired reference profile, which

made fast seeking performance available. The comparison of single track access time with Seagate product indicated that the 2DOF control is more approximate to optimal value derived from optimal control theory. Its robustness was also verified against different set point changes, bias disturbance, non-zero initial state, and plant parameter variation.

When the head is stationary over a single track after settling process, track-following servo modes commence working to minimize the effects of internal and external disturbances on the position error signal (PES). Besides bias and friction torque, many other disturbances such as disk vibration, windage and air turbulence may significantly affect the track-following performance. From next chapter on, two challenging problems of head positioning accuracy in the track-following mode, *i.e.*, mid-frequency narrow band NRRO disturbance compensation and sensitivity function “waterbed” effect will be investigated.

Chapter 6

Nonlinear Mid-frequency Disturbance Compensation

Fast rotation speed induces a lot of vibrations of disk platters in HDDs, which will affect the head positioning accuracy considerably. These vibrations are certain spectrally rich, *i.e.*, typically presents narrow band characteristic. Material and mechanical solutions have been studied extensively to suppress disk vibrations by alternate disk substrate ([104] and [139]). When the material and mechanical designs are fixed, the control design is aimed at attenuating the vibrations via various types of loop shaping.

As disk vibration energy focuses on certain frequencies, a commonly used approach to suppress them is to increase the servo loop gain at the desired frequency by inserting a narrow band filter to the servo loop ([36] and [136]). However, as well known in [9], a slope of $-20N$ dB/decade of magnitude in a minimum-phase system is accompanied by a phase lag of $90N^\circ$ ($N = 0, \pm 1, 2, 3, \dots$). Narrow band filter will introduce a high peak at central frequency at the cost of phase loss after central frequency. And it is thus typically used at low frequency band due to phase stabilization [85]. For narrow band disturbances whose frequencies are far higher

than servo bandwidth, narrow band filter can be viewed as a high frequency resonance, where the phase around the peak frequency can be carefully designed to ensure phase stabilization [89]. Unfortunately, most of the disk flutter frequencies are in the range of mid-frequency band (500 – 2000 Hz). Inserting such a narrow band filter will thus affect the stability of the closed-loop system. Therefore, mid-frequency disturbance compensation is currently a challenging work in the HDDs. This chapter will investigate a nonlinear narrow band compensation that breaks through Bode’s gain-phase relationship and improves the stability of the closed-loop system, and thus makes mid-frequency narrow band compensation more effectively.

6.1 Introduction

Previous studies have shown that the major contributing factors to track misregistration (TMR) in an HDD servo system, are repeatable runout (RRO) and non-repeatable runout (NRRO). RRO is synchronous to disk rotation, *i.e.*, the frequencies of RRO components are integer multiples of the disk spinning frequency and the phase of RRO is steady. Many control methods have been proposed to compensate for the periodic runout, such as repetitive control [86], adaptive feedforward compensation [127] and iterative learning control ([164] and [82]), etc. NRRO is asynchronous to disk rotation, such as disk vibration and windage. While both of the disturbances are important, NRRO imposes fundamental limitations on the viability of very high track-per-inch (TPI) drives [37]. Furthermore, disk flutter is a dominant factor to make servo work harder ([4] and [58]).

Higher rotational speed will induce larger disk vibration that may aggravate

the head position error during track-following operation. Such position error concentrates at some frequencies of disk resonant modes and does not follow the disk rotation in phase. Figure. 6.1 plots a typical power spectral densities of NRRO at the rotation speed of 4800 RPM. It can be observed that disk flutter produces prominent narrow band NRRO around mid-frequency band.

Figure 6.1: Power spectral densities of NRRO at rotation speed of 4800 RPM.

Figure 6.2: Control schemes for disturbance rejection.

Figure 6.2 shows two control schemes for disturbance rejection, feedback and feedforward modes. Control with internal high gain filter (disturbance model), which is named as narrow band filter in this thesis, typically belongs to the feedback type.

Another scheme of disturbance rejection utilizes the control signal and the system output to construct a compensation signal for the output disturbance. As control information and disturbance control are in the same direction, it thus belongs to the feedforward type. The disturbance detector may be implemented by the “hard” way of using sensors [59] or “soft” way by using disturbance observer (DOB) [157]. The control schemes that place the disturbance compensators inside the feedback structure may have more robustness than the schemes of feedforward compensation, especially when the disturbance lacks steady phase.

6.1.1 Narrow Band Filter

The notch filter in servo loop is typically used to reduce the control signal power to avoid the excitation of resonances, while the narrow band filter is to increase the moment of actuator to counteract the extrinsic disturbance. The high gain produced by narrow band filter forms a deep notch in the sensitivity function, which will attenuate the disturbance at that frequency. High gain narrow band filter is typically with the following form in s -domain:

$$F(s) = \frac{s^2 + 4\pi\zeta_1 f_1 s + 4\pi^2 f_1^2}{s^2 + 4\pi\zeta_2 f_2 s + 4\pi^2 f_2^2} \quad (6.1)$$

where ζ_i ($i = 1, 2$) are the damping ratios, $\zeta_1 > \zeta_2$ is selected to produce a high gain peak value. Generally, the bigger the ratio ζ_1/ζ_2 is, the sharper and higher peak of narrow band filter will be introduced to the servo loop. However, in order to deal with several main disturbance modes with one narrow band filter, the ratio is suggested not to be set arbitrarily large. f_i ($i = 1, 2$) are the undamped natural

frequencies. Note that the frequency parameters f_i ($i = 1, 2$) are related to the desired narrow band (peak) frequency f . The simplest case is that natural frequencies for the numerator and denominator have been set to $f_1 = f_2 = f$.

Figure 6.3: Frequency responses of a narrow band filter with the central frequency of f .

Figure 6.3 plots the frequency responses of the narrow band filter with $f_1 = f_2 = f$. As stated in Bode's gain phase relation, $-20N$ dB/decade magnitude slope will lead to $90N^\circ$ phase loss. It has only a high gain peak value around the center frequency and is flat at other frequencies. Naturally, it provides phase lead before center frequency while the same phase loss after center frequency.

A narrow band filter may be designed according to Figure 6.3, in which M and N decide the gain peak height; Δ is the percentage of the frequency variation. The possible phase loss is estimated by θ . When $M \gg N$, ζ_1 and ζ_2 could be determined by:

$$\zeta_1 = \frac{\Delta^2 + 2\Delta}{2(1 + \Delta)} \sqrt{n^2 - 1}, \quad (6.2)$$

$$\zeta_2 = \frac{\zeta_1}{m}, \quad (6.3)$$

$$\theta = \arctan \frac{m-1}{2\sqrt{m}} \quad (6.4)$$

where $m = 10^{M/20}$, $n = 10^{N/20}$.

6.1.2 Disturbance Observer

Disturbance observer is a typical feedforward disturbance compensation scheme. As the name suggests, the disturbance observer creates an estimate of the disturbance and uses this value to compensate for its effects. The disturbance observer was originally proposed in [117], and was introduced to the HDD servos ([64] and [138]). The block diagram of disturbance observer for output disturbance is shown in Figure 6.4. More details for input disturbance observer can be found in ([8], [138] and [157]).

Figure 6.4: Disturbance observer block diagram.

In Figure 6.4, C is servo controller and P is actuator. The disturbance observer consists of the nominal dynamics of actuator P_n , the inverse plant P_n^{-1} and the Q filter. The low-pass filter Q is inserted to make QP_n^{-1} realizable. The transfer function G_{dy} from output disturbance to the PES is then given by

$$G_{dy} = \frac{1 - Q}{1 + PC + Q(P_n^{-1} - 1)} \quad (6.5)$$

If the Q filter is set to unity and the nominal plant P_n is accurate enough so that $PP_n^{-1} \approx 1$, then the disturbance is cancelled ($G_{dy} \approx 0$). Conversely, if $Q = 0$, then the disturbances directly sees output due to the nominal feedback ($G_{dy} = \frac{1}{1+PC}$). By designing Q to be a low pass filter with unity DC gain, it is possible to take the advantage of disturbance attenuation in the low-frequency range. Theoretically, the wider the bandwidth of Q filter, the more disturbance will be attenuated. However, it cannot be set arbitrarily large due to system stability and attenuation of measurement noise [134]. Therefore, the compensation scheme by use of disturbance observer is only effective for relatively low frequency disturbance.

6.2 Nonlinear Narrow Band Compensation

Mid-frequency narrow band disturbance compensation is a challenge in the HDD servo system. Based on the non-smooth control technique ([24] and [175]), this chapter develops a nonlinear mid-frequency disturbance compensation scheme by incorporating a reset controller and narrow band filter. The state-dependent reset controller is designed to reset its sub-state to zero whenever the input crosses zero, other states are kept to work in the linear operation environment. It is shown through the describing function analysis that the reset controller can provide the maximal phase lead with a 0 dB/dec gain characteristic in a large frequency range, which has the potential to balance phase loss due to narrow band filter at the desired frequency. Before introducing the nonlinear mid-frequency narrow band filter, the dynamics of reset control system is presented as follows.

6.2.1 The Dynamics of Reset Control System

A state-dependent reset controller is a linear time-invariant (LTI) system whose states, or subset of states, are forced to zero when the controller input is zero. A typical feedback control system with a reset controller is shown in Figure 6.5 where L includes all other linear elements such as linear controller and the plant. The linear compensator is used to stabilize and shape the closed-loop to satisfy certain performance specifications (for instance, servo bandwidth and stability margins). The reset controller is then designed to meet the constraints due to the common linear control. r is the reference input, e is the feedback error and u_r is the reset controller output. The reset controller is described by the impulsive differential equation

Figure 6.5: Block diagram of a reset control system.

$$\begin{aligned} \dot{\mathbf{x}}_r(t) &= \mathbf{A}_r \mathbf{x}_r(t) + \mathbf{B}_r e(t), & e(t) \neq 0 \\ \mathbf{x}_r(t^+) &= \mathbf{A}_{rs} \mathbf{x}_r(t), & e(t) = 0 \\ u_r(t) &= \mathbf{C}_r \mathbf{x}_r(t), \end{aligned} \tag{6.6}$$

where $\mathbf{x}_r(t)$ is the reset controller state, t^+ is the resetting time, $\mathbf{A}_r \in \mathbb{R}^{n_r \times n_r}$, $\mathbf{B}_r \in \mathbb{R}^{n_r \times 1}$ and $\mathbf{C}_r \in \mathbb{R}^{1 \times n_r}$. The matrix $\mathbf{A}_{rs} \in \mathbb{R}^{n_r \times n_r}$ selects the states to be reset. Without loss of generality, $\mathbf{A}_{rs} = \begin{bmatrix} \mathbf{I}_d & \mathbf{0} \\ \mathbf{0} & \mathbf{0}_d \end{bmatrix}$ is assumed where d (of the n_r controller states) states are reset.

Assuming that the linear part L is strictly proper and adopt the realization:

$$\begin{aligned}\dot{\mathbf{x}}_l(t) &= \mathbf{A}_l \mathbf{x}_l(t) + \mathbf{B}_l u_r(t), \\ y(t) &= \mathbf{C}_l \mathbf{x}_l(t),\end{aligned}\tag{6.7}$$

where $\mathbf{A}_l \in \mathbb{R}^{n_l \times n_l}$, $\mathbf{B}_l \in \mathbb{R}^{n_l \times 1}$ and $\mathbf{C}_l \in \mathbb{R}^{1 \times n_l}$. The closed-loop reset control system is

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A} \mathbf{x}(t) + \mathbf{B} r(t), \quad e(t) \neq 0 \\ \mathbf{x}(t^+) &= \mathbf{A}_S \mathbf{x}(t), \quad e(t) = 0 \\ y(t) &= \mathbf{C} \mathbf{x}(t), \\ e(t) &= r(t) - \mathbf{C} \mathbf{x}(t),\end{aligned}\tag{6.8}$$

where $\mathbf{x}(t) = [\mathbf{x}_l \quad \mathbf{x}_r]'$ and

$$\begin{aligned}\mathbf{A} &= \begin{bmatrix} \mathbf{A}_l & \mathbf{B}_l \mathbf{C}_r \\ -\mathbf{B}_r \mathbf{C}_l & \mathbf{A}_r \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{B}_r \end{bmatrix}, \\ \mathbf{A}_S &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{rs} \end{bmatrix}, \mathbf{C} = [\mathbf{C}_l \quad \mathbf{0}].\end{aligned}\tag{6.9}$$

In the absence of resetting, i.e., when $\mathbf{A}_S = \mathbf{I}$, the resulting closed-loop system $\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$ is called the base-linear system. The internal stability of the closed-loop system (6.8) was established in [7] by giving a necessary and sufficient condition (called the “ H_β -condition”) on the base-linear system: there exists a $\beta \in \mathbb{R}^d$ and a positive-definite $\mathbf{P}_d \in \mathbb{R}^{d \times d}$ such that

$$H_\beta(s) = [\beta \mathbf{C}_l \quad \mathbf{0}'_d \quad \mathbf{P}_d] (s\mathbf{I} - \mathbf{A})^{-1} \begin{bmatrix} \mathbf{0} \\ \mathbf{0}'_d \\ \mathbf{I}_d \end{bmatrix}\tag{6.10}$$

is strictly positive real (SPR). The steady-state performance of reset control system is also addressed in [7] that the reset action will not degrade or affect the asymptotic tracking capability of the base-linear system.

The above stability condition provides a theoretical solution of the input-output stability for a reset control system. In digital implementation, the resettling time may not be at the accurate moment of zero input crossing due to sampling and quantization. The robustness to implementation errors of a reset control system is reported in [18].

6.2.2 Describing Function Analysis of Reset Element

Frequency response is a useful tool to study the input/output relation of an LTI system, while, the describing function is the corresponding tool for the study of nonlinear system. The describing function of a nonlinear dynamics is its linearization by ignoring higher harmonics in its sinusoid input response [5]. A key assumption for use of describing function is that the nonlinear element's fundamental harmonic component must be dominant compared to all the higher harmonic components. As the double integrator characteristic in the VCM actuator of the HDD servos, it satisfies this assumption. Therefore, the describing function of the reset element is developed:

Theorem 6.1 *Assume that the LTI dynamics is strictly proper and has the following transfer function:*

$$T(s) = C(sI - A)^{-1}B \quad (6.11)$$

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times l}$, $C \in \mathbb{R}^{l \times n}$ are the state-space system matrix, input matrix and output matrix of $T(s)$ respectively. Then the reset element created from $T(s)$ by resetting all of its states to zero when its input crosses zero, has the following

describing function:

$$\begin{aligned} D_F &= \frac{Y_1(j\omega)}{X(j\omega)} \\ &= T(j\omega) - \frac{C\omega}{\pi} [(j\omega I + A)^{-1} + (j\omega I - A)^{-1}] (j\omega I - A)^{-1} (e^{A\pi/\omega} + I)B \end{aligned} \quad (6.12)$$

where $Y_1(j\omega)$ is the fundamental harmonic component of the output of reset element when the sinusoidal input is $X(j\omega)$.

Proof. The solution of the base linear system for the reset element is

$$y(t) = Ce^{At}y_0 + C \int_0^t e^{A(t-\tau)} Bx(\tau) d\tau. \quad (6.13)$$

Let $y_0 = 0$, then

$$y(t) = C \int_0^t e^{A(t-\tau)} Bx(\tau) d\tau. \quad (6.14)$$

Suppose the input of the reset element is a pure sinusoid wave with frequency of ω and magnitude of a :

$$x(t) = a \sin \omega t = a \frac{e^{j\omega t} - e^{-j\omega t}}{2j}. \quad (6.15)$$

Then

$$\begin{aligned} y(t) &= C \int_0^t e^{A(t-\tau)} a \sin \omega \tau d\tau B \\ &= C \int_0^t e^{At} e^{-A\tau} a \frac{e^{j\omega \tau} - e^{-j\omega \tau}}{2j} d\tau B \\ &= \frac{aC}{2j} \int_0^t e^{(j\omega I - A)\tau} - e^{-(j\omega I + A)\tau} d\tau e^{At} B \\ &= \frac{aC}{2j} [(j\omega I - A)^{-1} (e^{(j\omega I - A)t} - I) + (j\omega I + A)^{-1} (e^{-(j\omega I + A)t} - I)] B \\ &= \frac{aC}{2j} [(j\omega I - A)^{-1} (e^{j\omega t} I - e^{At}) + (j\omega I + A)^{-1} (e^{-j\omega t} I - e^{At})] B. \end{aligned} \quad (6.16)$$

When the input of reset element crosses zero, its states and output are set to zero.

Suppose the input is a pure sinusoid wave as (6.15), the output is obviously periodic

function, and have the property:

$$\int_0^{\pi/\omega} y(t)\omega dt = - \int_{\pi/\omega}^{2\pi/\omega} y(t)\omega dt. \quad (6.17)$$

The DC component of the reset output is

$$Y_0 = \frac{1}{2\pi} \int_0^{2\pi/\omega} y(t)\omega dt = 0. \quad (6.18)$$

The fundamental component is

$$\begin{aligned} Y_1 &= \frac{1}{2\pi} \int_0^{2\pi/\omega} y(t)e^{-j\omega t}\omega dt \\ &= \frac{1}{\pi} \int_0^{\pi/\omega} y(t)e^{-j\omega t}\omega dt \\ &= \frac{1}{\pi} \int_0^{\pi/\omega} \frac{aC}{2j} [(j\omega I - A)^{-1} (e^{j\omega t} I - e^{At}) + (j\omega I + A)^{-1} (e^{-j\omega t} I - e^{At})] B e^{-j\omega t}\omega dt \\ &= \frac{aC}{2\pi j} \int_0^{\pi/\omega} [(j\omega I - A)^{-1} (e^{j\omega t} I - e^{At}) + (j\omega I + A)^{-1} (e^{-j\omega t} I - e^{At})] e^{-j\omega t}\omega dt B \\ &= \frac{aC}{2\pi j} (j\omega I - A)^{-1} \int_0^{\pi/\omega} (e^{j\omega t} - e^{At}) e^{-j\omega t}\omega dt B \\ &\quad + \frac{aC}{2\pi j} (j\omega I + A)^{-1} \int_0^{\pi/\omega} (e^{-j\omega t} - e^{At}) e^{-j\omega t}\omega dt B. \end{aligned} \quad (6.19)$$

Furthermore, we have

$$\begin{aligned} Y_1 &= \frac{aC}{2\pi j} (j\omega I - A)^{-1} [\pi - \omega(A - j\omega I)^{-1} (e^{(A-j\omega I)\pi/\omega} - I)] B \\ &\quad - \frac{aC\omega}{2\pi j} (j\omega I + A)^{-1} (A - j\omega I)^{-1} (e^{(A-j\omega I)\pi/\omega} - I) B. \end{aligned} \quad (6.20)$$

The Fourier Transform of the input (6.15) is

$$X(j\omega) = \frac{1}{2\pi} \int_0^{2\pi} a \sin \omega t e^{-j\omega t} d\omega t = \frac{a}{2j}. \quad (6.21)$$

Therefore, the describing function for the reset element is

$$D_F = \frac{Y_1(j\omega)}{X(j\omega)} \quad (6.22)$$

$$\begin{aligned}
 &= \frac{C}{\pi} (j\omega I - A)^{-1} [\pi - \omega(A - j\omega I)^{-1} (e^{(A-j\omega I)\pi/\omega} - I)] B \\
 &\quad - \frac{C\omega}{\pi} (j\omega I + A)^{-1} (A - j\omega I)^{-1} (e^{(A-j\omega I)\pi/\omega} - I) B \\
 &= C(j\omega I - A)^{-1} B - \frac{C\omega}{\pi} (j\omega I - A)^{-2} (e^{A\pi/\omega} + I) B \\
 &\quad - \frac{C\omega}{\pi} (j\omega I + A)^{-1} (A - j\omega I)^{-1} (e^{A\pi/\omega} + I) B \\
 &= T(j\omega) - \frac{C\omega}{\pi} [(j\omega I + A)^{-1} + (j\omega I - A)^{-1}] (j\omega I - A)^{-1} (e^{A\pi/\omega} + I) B.
 \end{aligned}$$

6.2.3 Nonlinear Narrow Band Filter

The proposed nonlinear narrow band filter consists of two parts: reset controller and linear narrow band filter. The purpose of reset controller is to provide the maximal phase lead for the narrow band filter without affecting the gain characteristic. Therefore, the base-linear system of the reset controller is designed to be a lag filter with a large crossover frequency l_1 . It is further divided in to two parts: one is the reset first order mode with crossover frequency of $l_1 = \beta l_2 = 2\beta\pi f_p (\beta \gg 1)$ which is designed to reset the pole to zero whenever the input goes to zero; the other is a linear lead filter to balance the gain roll-off and certain phase loss due to first order mode. The reset controller can therefore be described by the impulsive differential equations:

$$\begin{aligned}
 \begin{bmatrix} \dot{x}_{r1} \\ \dot{x}_{r2} \end{bmatrix} &= \begin{bmatrix} -l_1 & 0 \\ l_1(l_2 - l_1) & -l_2 \end{bmatrix} \begin{bmatrix} x_{r1} \\ x_{r2} \end{bmatrix} + \begin{bmatrix} 1 \\ l_1 \end{bmatrix} e, \quad e(t) \neq 0 \\
 \mathbf{x}_r(t^+) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{x}_r(t), \quad e(t) = 0 \\
 u_r(t) &= [0 \quad 1] \mathbf{x}_r(t),
 \end{aligned} \tag{6.23}$$

where $\mathbf{x}_r(t) = [x_{r1} \ x_{r2}]'$ and only state x_{r2} is to be reset which corresponds to the pole of first order mode $1/(s + l_2)$.

The describing function of the reset first order mode is calculated based on the Theorem 1 with $A_r = -l_2$, $B_r = C_r = 1$ and $l_2 = 2\pi f_p$:

$$E = \frac{1}{j\omega + 2\pi f_p} \left(1 + \frac{j2\omega^2(1 + e^{-\frac{2\pi^2 f_p}{\omega}})}{\pi(\omega^2 + 4\pi^2 f_p^2)} \right). \quad (6.24)$$

Further, it is piecewise analyzed as following:

1. When $\omega \ll 2\pi f_p$,

$$1 + \frac{j2\omega^2}{\pi(\omega^2 + 4\pi^2 f_p^2)} \left(1 + e^{-\frac{2\pi^2 f_p}{\omega}} \right) \rightarrow 1 \quad (6.25)$$

The reset element behaves very much like a linear element over this part of frequency range, and responses of the nonlinearly designed system are essentially the same as those of a linear design at this low frequency band.

2. When $\omega \gg 2\pi f_p$,

$$1 + \frac{j2\omega^2}{\pi(\omega^2 + 4\pi^2 f_p^2)} \left(1 + e^{-\frac{2\pi^2 f_p}{\omega}} \right) \rightarrow 1 + j\frac{4}{\pi} \quad (6.26)$$

which has a gain of $20\log 1.62 = 4.2$ dB and phase of 51.9° . The reset behavior makes the first-order mode functions with gain of -20 dB/decade roll off but with phase loss of $-90^\circ + 51.9^\circ = -38.1^\circ$.

3. When ω is around $2\pi f_p$,

$$1 + \frac{j2\omega^2}{\pi(\omega^2 + 4\pi^2 f_p^2)} \left(1 + e^{-\frac{2\pi^2 f_p}{\omega}} \right) \quad (6.27)$$

is bounded by above two limitations.

A computed transfer function of the ordinary first-order mode and the simulated describing function of the reset one are shown in Figure 6.6. Swept sine signal from 0.1 Hz to 10 kHz was piecewise sampled and inserted into the simulated reset model. At each piecewise frequency range, the output was online analyzed by Fourier series analysis to retrieve its fundamental component. The describing function was then obtained as the ratio of the fundamental component to input sine signal. Compared with ordinary first-order mode, gain property is preserved and shows a -20 dB/decade roll off with an extra 4.2 dB gain beyond 10 Hz. However, it introduces less 51.9° phase loss than the nominal one.

Figure 6.6: Transfer functions of linear and nonlinear first-order mode.

The lead part in the reset controller is maintained to work in a linear status, which is designed to balance the gain for a 0 dB/decade roll-off at the same time to provide the maximal phase lead for compensating -38.1° phase loss of the first-order mode. This can be achieved by the following design.

Suppose the lead filter has the transfer function of:

$$G_l = \frac{1 + \beta\tau s}{1 + \tau s}, \quad \beta > 1 \quad (6.28)$$

and the magnitude-frequency and phase-frequency responses can be expressed to be:

$$\begin{aligned} |G_l(\omega)| &= \frac{\sqrt{1 + \beta^2\tau^2\omega^2}}{\sqrt{1 + \tau^2\omega^2}}, \\ \varphi(\omega) &= \tan^{-1}(\beta\tau\omega) - \tan^{-1}(\tau\omega). \end{aligned} \quad (6.29)$$

Suppose further that at $\omega = \omega_c$, the corresponding gain is $|G_l(\omega_c)| = C_{lead}$, and phase $\varphi(\omega_c) = P_{lead}$. The maximal phase lead at $\omega = \omega_c$ can be formulated as functions of ω_c and C_{lead} . Define $x = \tau\omega_c$,

$$\gamma = \tan P_{lead} = \frac{\beta x - x}{1 + \beta x^2} \quad (6.30)$$

The maximum γ can be calculated by solving

$$\frac{d\gamma}{dx} = 0 \quad \text{and} \quad \frac{d^2\gamma}{dx^2} < 0 \quad (6.31)$$

which results in

$$x = \sqrt{1/\beta} \quad (6.32)$$

Substituting (6.32) into (6.29), this lead to

$$\beta = C_{lead}^2 \quad \text{and} \quad \tau = \frac{1}{\sqrt{\beta}\omega_c^2} \quad (6.33)$$

and further

$$\gamma_{\max} = \frac{C_{lead}^2 - 1}{2C_{lead}} \quad (6.34)$$

With the known parameters C_{lead} and ω_c which are determined by the describing function of reset element, the frequency response of the reset controller is plotted

in Figure 6.7 to show the maximal phase lead about 52° with 0 dB/decade gain characteristic in a wide frequency range. This thus breaks through the Bode's gain-phase relationship of $-N$ dB/decade gain roll off introduces $-90N^\circ$ phase ($N = 0, \pm 1, 2, \dots$) at the same frequency band.

Figure 6.7: The simulated describing function of reset controller.

As analyzed in Section 6.2.2, the frequency response of the nonlinearly design system is much like a linear element at the very low frequency range. However, the frequency response in a wide range of 10 Hz – 3 kHz shows a large phase lead up to 51.9° with 0 dB/decade gain slope, which cannot be achieved with any linear design. The nonlinear characteristic is thus useful to work in series with narrow band filter to compensate for phase loss in the mid-frequency disturbance compensation. At very high frequency which is far away from the servo bandwidth, its describing function property will not affect the servo system performance considerably.

The nonlinear narrow band compensation is thus constructed by incorporating

Figure 6.8: The block diagram of nonlinear mid-frequency disturbance compensation.

the nonlinear phase stabilization mode and narrow band filter. The nonlinear narrow band compensator is inserted into the control path in an “add-on” fashion, in which the narrow band compensation is included without requiring changes to the original feedback controller, as shown in Figure 6.8. Because of the varying environmental conditions, limited computation power, variable structure of the typical disk drive controller and short product cycles, “add-on” controller is a desirable feature.

As the nonlinear phase stabilization mode breaks through Bode’s gain-phase relationship of linear system to present a 0 dB/decade gain characteristic with about 50° lead phase in a wide frequency range, the nonlinear narrow band compensator is thus more effective to suppress mid-frequency disturbance with reliable phase stabilization.

6.3 Simulation and Experimental Results

In this section, simulation and experimental results with the proposed nonlinear mid-frequency disturbance scheme and the nominal PI-lead feedback control scheme will be presented. In the simulation, the plant used is the IBM model identified in

Chapter 3. The transfer function of VCM model is

$$P(s) = \frac{2.6 \times 10^{16}}{(s^2 + 150.8s + 6.317 \times 10^4)(s^2 + 7037s + 1.238 \times 10^9)}. \quad (6.35)$$

In a conventional track following servo system, the PID controller is widely used to stabilize the track-following servo loop. For easier implementation, the derivative part is usually replaced by a lead filter. In a PI-lead control system, the frequency response at mid-frequencies reflects a typical lead characteristic which compensate for the double-integrator characteristic in VCM to make the open-loop crossover with a slope of about -20 dB/decade. The integral element in the control presents a desired low frequency frequency response, and to remove the bias disturbance at VCM input. The designed feedback controller is

$$C = \frac{12.2014(s + 2651)(s + 0.6283)}{s(s + 3.977 \times 10^4)}. \quad (6.36)$$

Application of the nominal controller to the plant resulted in an open-loop gain crossover frequency of 1.12 kHz with 4.04 dB gain margin and 58.2° phase margin, as shown in Figure 6.9.

The reset behavior in the proposed nonlinear phase stabilization mode is constructed with two enabled systems with inverse inputs in the SIMULINK package in the off-line simulation. When it was implemented to the experimental setup, the resetting time may not be at the accurate moment of zero input crossing due to sampling and quantization. However, it does not degrade the performance improvement over the conventional compensation scheme for a high sampling frequency, which will be shown shortly. Theoretical verification of the robustness of reset control to implementation errors is reported in [18].

Figure 6.9: Open-loop frequency response of an HDD servo system.

For nonlinear compensator, $\beta = 1000$ and $f_p = 10$ Hz are selected. In the design of narrow band filter, $f_1 = f_2 = f$ is the disturbance frequency to be rejected by the narrow band filter and $\zeta_1 = 10\zeta_2$ is selected. And the sampling frequency is 40 kHz when implemented to the experimental setup.

6.3.1 Time Traces of Mid-frequency Disturbance

All the tests were performed in the track following mode with reference signal is set to zero. With the plant model (6.35) and feedback controller (6.36), the nominal linear closed-loop system was constructed. First, 700 Hz Sine disturbance is inserted to the closed-loop system to test the effectiveness of the proposed scheme to compensate for mid-frequency disturbance. With the open-loop crossover frequency of 1.12 kHz and about 40° phase margin at 700 Hz, inserting a narrow band filter at the desired frequency is apt to make the system unstable (refer to Figure 6.9). However, as the additional 59.1° phase introduced due to the reset controller, the stability issue may

not exist any more in the proposed control scheme.

This is actually verified by the experimental results, shown in Figure 6.10. About 66.7% disturbances are attenuated through the nonlinear compensation, which is approximate to a contribution of 10 dB gain notch at 700 Hz in the sensitivity function frequency response. However, it is impossible for a common compensation to achieve as 10 dB gain notch means phase lag will be introduced, which would lead to system unstable taking computation delay into consideration. The corresponding control signals are presented in Figure 6.11. It is observed that the baseline of two control signals are the same except when the resetting action occurs, which cause some large spikes. It is necessary for a reset control system with a lead or differentiate mode and usually leads to the actuator saturation. We further observed that the induced spikes is related to the disturbance amplitude. Therefore, the problem is potential to be relaxed through the first compensation for low frequency RRO. More details about this issue will be presented in the future work of Chapter 8.

Figure 6.10: Experimental results: steady-state output responses with sine disturbance at 700 Hz. The suppression is about 16.7% with the nominal linear feedback control and about 66.7% with the proposed algorithm.

Figure 6.11: Experimental results: control signals for both control schemes.

6.3.2 NRRO PES Compensation

This part shows the simulation and experimental results of the proposed compensation scheme to NRRO disturbance.

Figure 6.12 gives the NRRO power spectral densities without compensation. Besides power spectra, the area beneath the power spectral density curve was computed to determine the cumulative 3σ TMR, as the relation between the variance and the power spectral density of PES is given as [93]:

$$\sigma^2(\text{PES}) \cong \frac{\sum_{i=1}^N \text{PSD}_i(\text{PES})}{N} \quad (6.37)$$

where N represents the total number of NRRO PES data points. Dominant mid-frequency modes can be seen clearly, especially for those at 648, 696 and 724 Hz which are introduced by the disk flutter. These peaks contribute to the jumps at modal frequencies in the spectrum cumulative sum.

Figure 6.13 shows NRRO power spectral densities after nonlinear narrow band compensation with one narrow band filter of central frequency at 700 Hz, and magnitudes reduction are given in Table 6.1. Obviously, power spectral densities at dominant mid-frequencies are greatly attenuated for only one narrow-band filter. Its cumulative sum of PES is decreased greatly in scale and also shows a flat gradual increase manner.

Figure 6.12: NRRO power spectral densities and cumulative sum without compensation.

Figure 6.13: Simulation results: NRRO power spectral densities and cumulative sum with nonlinear compensation.

Figure 6.14: Experimental results: the NRRO PES with PI-lead controller with $3\sigma = 16.38$ nm.

Figure 6.15: Experimental results: the corresponding control signal.

Figure 6.16: Experimental results: the NRRO PES with the proposed control scheme with $3\sigma = 13.52$ nm.

Figure 6.17: Experimental results: the corresponding control signal.

The time domain responses of the NRRO rejection and the corresponding control signals are shown in Figure 6.14 – Figure 6.17. In the implementation, the disturbance model which was introduced and obtained in Chapter 3 was inserted to the servo loop, plus the position signal from the LDV to be fed to the dSPACE

system. More NRRO disturbances at mid-frequency range are compensated, and thus the TMR value is approximately improved 18%. Also note that the control signals are almost the same except that zero-crossing points where large spikes are induced when compensating mid-frequency NRRO and thus present high frequency harmonics, which may excite the high frequency actuator resonances. This can be solved by modifying the reset control with longer duration and will be presented more in the future work.

Figure 6.18: NRRO power spectral densities and cumulative sum with PI-lead feedback controller.

NRRO power spectral densities and the corresponding cumulative sum of PI-lead feedback controller (6.36) is shown in Figure 6.18. Feedback servo can only suppress disturbances at low frequency band. For NRRO at the mid-frequency from 600–1000 Hz, feedback servo is not effective to deal with.

In Table 6.1, power spectral densities of NRRO before and after nonlinear compensation are compared. For sharp peaks around several main mid-frequency modes,

Table 6.1: NRRO power spectral densities

Frequency Hz	Magnitude (μm)		reduction %
	Before comp.	After comp.	
598	0.4E-5	0.9E-9	99
648	5.05E-5	5.8E-9	99
696	3.9E-5	1.25E-9	100
724	1.1E-5	3E-9	99
884	0.5E-5	6.5E-9	99
1018	0.28E-5	7.2E-9	99

they are almost removed completely via nonlinear narrow band compensation.

6.3.3 RRO PES Compensation

RRO, the repeatable periodic disturbance with its phase locked to the spindle rotation, has been dealt with either by internal model principle (IMP) or adaptive feedforward cancellation (AFC). The compensation based on the IMP is a extreme of narrow band filter, which constructs an infinity open-loop high gain at the disturbance frequency. Although the AFC scheme can effectively attenuate RRO and its harmonics to some extent ([86], [126], [127], [164] and [154]), convergence time of disturbance attenuation is a challenge due to the adaption gain. Furthermore, Both IMP and AFC schemes are typically used for RRO compensation with frequencies far lower than the open-loop crossover frequency due to the stability issue, and harmonics around the open-loop crossover frequency may be amplified instead.

The proposed narrow band compensation scheme is also applied to the RRO compensation. RRO disturbance with basic frequency of 80 Hz (described in Chapter 3) is inserted into the servo loop. Figure 6.19 plots the settling time (named as convergence time in the AFC) of the PES with the 5th harmonic RRO compensation. With the conventional IMP control scheme, the PES fast converges to zero within 4 ms, and the proposed nonlinear narrow band filter does not degrade the settling time, about 3 ms. Figure 6.20 shows the control signals for both of the control schemes.

The PES power spectral densities with RRO harmonics compensation around open-loop crossover frequency are plotted in Figure 6.21. The central disturbance of narrow band filter in the proposed scheme is selected to be 1040 Hz (13th harmonic frequency), and 80 Hz basic frequency in the conventional high gain compensator. Therefore, it can remove the fundamental component of RRO with the nominal high gain filter, but amplify the harmonics around the open-loop crossover frequency and higher order harmonics, caused by the Bode's sensitivity integral theorem. More detailed comparisons of RRO compensation with the nominal and the proposed high gain control schemes are shown in Figure 6.22. 13th harmonic is removed with the proposed mid-frequency narrow band compensation, and the harmonics near that frequency such as 11th and 15th harmonics are also reduced. However, lower order harmonics attenuation is governed by the feedback control, which performance is not so obvious as that in the nominal high gain compensation at the fundamental frequency.

Figure 6.19: Simulation results: the settling time of the PES signal with 5th harmonic RRO.

Figure 6.20: Simulation results: the control signals of 5th harmonic RRO compensation with both the proposed control scheme and the nominal high gain control.

Figure 6.21: The PES power spectral density with RRO and its harmonics in both the nominal and the proposed narrow band compensation with $f_p = 1040$ Hz.

Figure 6.22: RRO attenuation rates in the fundamental frequency high gain compensation and the proposed compensation scheme at the 13th harmonic frequency ($f_p = 1040$ Hz).

6.4 Conclusions

Since disk vibrations are more abundant around some frequency intervals than others, and more mid-frequency vibrations are induced with the increase of disk rotational speed, it is reasonable to enhance the disturbance suppression at these frequencies to reduce the overall TMR.

Narrow band filter design is an effective control method to suppress narrow band disturbance at the cost of phase loss. This chapter extends the linear narrow band filter design to the nonlinear design that is able to suppress mid-frequency disturbances reliably and thus to increase the track-following accuracy in HDDs. The nonlinear compensator can provide extra phase lead without degrading magnitude characteristic, and thus make mid-frequency narrow band filter more effectively.

Simulation and experimental tests considering both mid-frequency sine disturbance and NRRO signals showed its capability of mid-frequency disturbance suppression, and resulted in reduction of average NRRO power spectral density of 99.8%. RRO harmonics compensation was also tested in this chapter. The proposed narrow band compensation can effectively deal with RRO around the open-loop crossover frequency, which cannot be achieved with the conventional high gain IMP and AFC schemes. Compared with disturbance observer, nonlinear narrow band compensation provides better TMR performance.

It may also be noted that although mid-frequency disturbances are removed through mid-frequency narrow band compensation, disturbances outside servo bandwidth typically show large power spectral densities as shown in Figures 6.13 and 6.18. This is actually due to the sensitivity function constraint governed by Bode's inte-

gral theorem. In the next chapter, the essence of constraints is analyzed and a kind of composite nonlinear control is developed to suppress sensitivity “hump” at high frequency and to relax the effect of Bode’s integral theorem on the HDD servo loop.

Chapter 7

Suppressing Sensitivity Hump via Nonlinear Loop Shaping

Generally, the disturbance rejection capability in HDD servo loop is affected by both servo bandwidth and the sensitivity transfer function “hump” phenomenon. This chapter will study the sensitivity limitation in the HDD linear servo system and present a kind of nonlinear loop shaping method to suppress the sensitivity hump to as low as possible, and furthermore the Bode’s Integral Theorem on the sensitivity integral is shown to be broken through. As such, high frequency disturbances would not be amplified and TMR would be reduced.

7.1 Introduction

So far, the research in an HDD track-following servo has been focused on how to enlarge the servo bandwidth for the stronger disturbance rejection capability via various loop shaping. The disturbance within the servo bandwidth is suppressed by the servo loop, while the disturbance beyond the bandwidth could be amplified by the sensitivity hump. To minimize TMR, the height of sensitivity peak should be as low as possible. However, in an HDD linear servo system that uses the VCM

actuator and PID controller, the sensitivity hump is unavoidable due to Bode's Integral Theorem which results in amplification of disturbances after gain crossover frequency. This implies a "waterbed" phenomenon in the sensitivity transfer function of servo loop, *i.e.*, more reduction at low frequency band would cause more amplification at high frequency band [132]. As such, extending the servo bandwidth to certain value would not necessarily decrease TMR.

This chapter presents a new kind of nonlinear control technique to suppress sensitivity hump due to the conventional linear servo system. The rest of this chapter is organized as follows. The nonlinear feedback control scheme is described in Section 7.2. Section 7.3 gives the sensitivity analysis of the nonlinear feedback control system. Section 7.4 presents the HDD servo system design with the nonlinear control scheme. Simulation results show that low or no sensitivity hump can be achieved through nonlinear feedback control. Concluding remarks are made in Section 7.5.

7.2 Nonlinear Feedback Control Scheme

In this section, we will provide the algorithm of composite nonlinear feedback control to improve that capability of disturbance rejection by suppression of sensitivity hump. It consists of two parts, a conventional linear PID controller and its associated nonlinear PD controller.

Figure 7.1 shows the block diagram of track-following servo loop. As the positioning target is track center, it is essentially a regulation problem with $r = 0$. Major contributors to TMR come from three factors: input disturbance such as torque disturbance or bias, output disturbances due to disk flutter, disk-spindle RRO, windage

and so forth, and noise related to magnetic media and channel electronics.

The controller design is sequential and is divided into three steps. Steps 1 and 2 deal respectively with the design of linear and the nonlinear feedback laws. In Step 3, the linear and nonlinear feedback laws are combined to form a composite nonlinear feedback law.

Figure 7.1: The track-following servo loop.

7.2.1 Linear PID Controller Design

The linear PID controller is designed to satisfy the suggested shape of open-loop transfer function possessing (i) high gain at low frequencies to improve steady state performance (ii) low gain at high frequencies to attenuate the effect of measurement noise (iii) phase bounded away from $\pm 180^\circ$ to take advantage of feedback for stabilizing the closed-loop system.

In an HDD servo system, the actuator is often modelled as a pure inertia with resonance modes. The resonances are mainly due to the flexibility of the pivot bearing, arm, suspension, which can be respectively modelled as a second-order system with light damping ratio [167]. Since high frequency resonances can be removed by notch filters, the plant can be simplified to be a second-order system

due to low frequency pivot friction:

$$\begin{cases} \dot{x} &= Ax + Bu \\ y &= Cx \end{cases} \quad (7.1)$$

where

$$A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ b \end{bmatrix}, C = [1 \quad 0] \quad (7.2)$$

and the pair (A, B) is controllable, the pair (A, C) is observable. The output of plant is head position, also PES in track-following mode. Considering the integral operation in linear feedback controller, the system can be augmented as a third-order system such that:

$$\begin{cases} \dot{x}_a &= A_a x_a + B_a u \\ y &= C_a x_a \end{cases} \quad (7.3)$$

where

$$A_a = \begin{bmatrix} 0 & 1 & 0 \\ a_1 & a_2 & 0 \\ 1 & 0 & 0 \end{bmatrix}, B_a = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, C_a = [1 \quad 0 \quad 0] \quad (7.4)$$

where augmented state x_3 means integration of head position. Then a linear PID feedback law is designed as

$$u_L = F_a x_a \quad (7.5)$$

such that $A_a + B_a F_a$ is stable, where define

$$F_a = [f_1 \quad f_2 \quad -k_i] = [F \quad -k_i], x_a = [x_1 \quad x_2 \quad x_3]' = [x \quad x_3]'. \quad (7.6)$$

7.2.2 Nonlinear Feedback Law

Utilizing an appropriate algebraic Ricatti equation (ARE) for the closed-loop system under this linear feedback law, the nonlinear feedback control law is constructed

to cause the closed-loop system to be highly damped selectively so as to produce relatively low closed-loop magnitude at certain frequency range, and thus reduce sensitivity hump. Since the design is focused on high frequency where sensitivity hump occurs, design of nonlinear law is made based on linear PD part so that

$$u_N = -\rho B' P x \quad (7.7)$$

where P is the positive definite solution of the following algebraic Riccati equation with positive definite matrix Q :

$$(A + BF)'P + P(A + BF) - PBB'P + Q = 0 \quad (7.8)$$

and ρ is any non-negative function bounded in x . The role of the nonlinear part is to cause the closed-loop system to be highly damped around high frequency range so as to produce relatively low sensitivity magnitude, and thus to reduce sensitivity hump.

7.2.3 Composite Nonlinear Control Scheme

The final composite nonlinear feedback law is then composed as

$$u = u_L + u_N = F_a x_a - \rho B' P x \quad (7.9)$$

As the additional nonlinear part takes effect around high frequency band, performance in low frequency range will not be affected. The composite nonlinear feedback system will be shown to break through Bode's integral theorem with sensitivity areal integral relation.

The nonlinear controller is determined by both ARE-based high gain and nonlinear damping factor ρ . The following analysis about the closed-loop poles will

determine how to choose an appropriate ρ .

Figure 7.2: The function ρ .

Suppose feedback gain

$$-B'P = [k_1 \quad k_2] \quad (7.10)$$

Then, simple symbolic computation in MATLAB show that three poles of the augmented closed-loop systems are governed by the function ρ :

$$\lim_{\rho \rightarrow +\infty} \lambda_1(\rho) = -\infty, \lim_{\rho \rightarrow +\infty} \lambda_2(\rho) = -\infty, \lim_{\rho \rightarrow +\infty} \lambda_3(\rho) = -\infty \quad (7.11)$$

and

$$\lim_{\rho \rightarrow 0} \lambda_1(\rho) = \lambda_{10}, \lim_{\rho \rightarrow 0} \lambda_2(\rho) = \lambda_{20}, \lim_{\rho \rightarrow 0} \lambda_3(\rho) = \lambda_{30}. \quad (7.12)$$

where λ_{10} , λ_{20} and λ_{30} are the poles of linear PID feedback systems. Guided by the above equations, to generate high gain around high frequency and at the same time not to change the low frequency performance, ρ can be chosen to be an increasing function of the position error signal $|e|$ such that

$$\lim_{|e| \rightarrow +\infty} \rho = \alpha, \lim_{|e| \rightarrow 0} \rho = 0 \quad (7.13)$$

where α is a large positive constant. In the above analysis, it is assumed that the system response corresponds to high frequency band when the system is at initial

states and try to track command input, and will move gradually to low frequency band when the system approaches to the steady state. As such, the function ρ is chosen to be (Figure 7.2)

$$\rho = \begin{cases} 0 & |e| < e_1 \\ k|e| - ke_1 & e_1 \leq e \leq e_2 \\ ke_2 - ke_1 & |e| > e_2 \end{cases} \quad (7.14)$$

7.3 Sensitivity Describing Function Analysis

In a unity negative feedback configuration of the HDD servo system, sensitivity function governs the effects of output disturbance upon the closed-loop performance. This section discusses the performance constraints of sensitivity function and controller solutions with their effects on the sensitivity function.

As the VCM actuator in HDDs works as a double integrator and shows a low-pass characteristic in the frequency domain, it is feasible to analyze the nonlinear HDD servo system via describing function. This part will present describing function analysis of nonlinear control law and show how it can improve the capability of disturbance rejection by suppressing sensitivity hump.

The nonlinear law (7.7) returns to a linear law when tracking error e is beyond the range $[e_1, e_2]$. When tracking error $e \in [e_1, e_2]$, control output is obtained by combination of (7.14) and (7.1),

$$u = (k_1 e + k_2 \dot{e})k |e|. \quad (7.15)$$

Suppose the input to controller is $e = a \sin(\omega t)$, the fundamental wave is obtained

by Fourier series analysis:

$$u_1 = u_{1a} \sin(\omega t) + u_{1b} \cos(\omega t), \quad (7.16)$$

where

$$\begin{aligned} u_{1a} &= \frac{1}{\pi} \int_0^{2\pi} u(t) \sin(\omega t) d(\omega t) = \frac{8k_1 k a^2}{3\pi} \\ u_{1b} &= \frac{1}{\pi} \int_0^{2\pi} u(t) \cos(\omega t) d(\omega t) = \frac{4k_2 \omega k a^2}{3\pi}. \end{aligned} \quad (7.17)$$

And the resulting describing function:

$$N = \frac{u_1}{e} = M(a, \omega) e^{j\theta(\omega)}, \quad (7.18)$$

where

$$\begin{cases} M(a, \omega) &= \frac{4ka}{3\pi} \sqrt{4k_1^2 + k_2^2 \omega^2} \\ \theta(\omega) &= \arctan \frac{\omega k_2}{2k_1} \end{cases} \quad (7.19)$$

If the coefficient in magnitude response is not considered, the nonlinear describing function can be regarded as a high gain PD type controller functioning at high frequency. The coefficient indicates that it responds differently to disturbance magnitude. With the increase of disturbance magnitude, the equivalent open-loop gain becomes larger and since $S^{-1}(j\omega) = 1 + L(j\omega) = 1 + P(j\omega)C(j\omega)$, it further results in more suppression of sensitivity function.

The relation of sensitivity and complementary sensitivity function $S + T \equiv 1$ holds for both linear and nonlinear control system [132], which was discussed in Chapter 2. Therefore, the trade-off between desirable small S and small T simultaneously is unavoidable. As such, the main setback of such an improvement of sensitivity function is that the degradation of high frequency performance in complementary sensitivity function T . This is usually intolerable as measurement noise

in the PES demodulation channel and high frequency plant uncertainties will not be attenuated. However, PES sensor noise can be reduced using methods such as patterned media [48] and alternative PES generation. The robustness against high frequency plant uncertainties will be discussed. The efficacy with the proposed control scheme will be shown through the track-following tests in the next section.

7.4 HDDs Design Using Nonlinear Feedback Control

In this section, the proposed algorithm is applied to the HDD track-following servo with the simulation tests. Comparisons of the PES with the conventional and proposed servo were made. The composite nonlinear controller will provide a good low frequency and mid-frequency performance through its linear part and more accurate head positioning through its nonlinear component by suppression of high frequency disturbances.

7.4.1 Parameters Selection

Based on the control scheme description in Section 7.2, a simplified design procedure can be summarized as follows.

1. Determine parameters of linear PID feedback controller based on open-loop frequency response;
2. Select an appropriate positive definite matrix Q and solve the algebraic Riccati equation (7.8) and get an appropriate gain matrix P ;

3. Select nonlinear tuning function ρ parameters such that the nonlinear law focuses on high frequencies to suppress sensitivity “hump” while not to affect low frequency response.

The plant used in the simulation test was still the identified IBM HDD model described before. Two second-order systems are considered in the identified result. One is due to the rigid body rotation about the actuator pivot around 45 Hz. The other, around 5 kHz, resulted from the mechanical resonance of the actuator arm. Removing high frequency resonance by notch filter, the plant model can be written as

$$A = \begin{bmatrix} 0 & 1 \\ -6.32 \times 10^4 & -150.78 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 2.1 \times 10^7 \end{bmatrix} \quad (7.20)$$

The controller was designed in continuous time and then converted to discrete-time using the bilinear transformation with a sampling rate of 20 kHz when simulating it in the MATLAB. The parameters are selected as

$$F_a = [-1.2048 \quad -2.1572 \times 10^{-4} \quad -100], \quad (7.21)$$

Figure 7.3 shows the resulting open-loop frequency response of linear PID control system, with 15.6 dB gain margin and 56.5° phase margin at 1.11 kHz. Figure 7.4 shows the corresponding closed-loop sensitivity magnitude frequency response. With the conventional feedback controller, sensitivity hump is avoidable. How much the disturbance is amplified is dependent on the sensitivity hump peak value. To improve the accuracy of head positioning, sensitivity hump should be reduced as small as possible.

Figure 7.3: The open-loop frequency response of the conventional PID type control system.

Figure 7.4: The simulated sensitivity function magnitude frequency response with the conventional PID type controller.

Let $Q = I$ and solve (7.8) for P such that the nonlinear feedback gain

$$k_1 = -1.21 \quad k_2 = -4 \times 10^{-4}. \quad (7.22)$$

e_1 and e_2 are chosen to be 20% and 80% of the magnitude of input sinusoidal signal with $k = 5$. The simulated open-loop describing function of the proposed nonlinear algorithm is shown in Figure 7.5. Swept sine signal at intervals from 10 Hz to 100 kHz with magnitude of $1 \mu\text{m}$ was inserted into the simulated nonlinear HDD servo. At each piecewise frequency range, the output was retrieved to online analyze by Fourier series analysis. The describing function was then obtained to be the ratio of the fundamental component of output to input sine signal.

Figure 7.5: Simulated open-loop describing function of the proposed nonlinear control system.

Compared with the conventional servo in Figure 7.3, high frequency response beyond 1000 Hz is different due to the effect of nonlinear part. With higher magnitude response and phase lead, the sensitivity magnitude peak is assumed to be suppressed and the system stability is enhanced, either. Figure 7.6 plots the corresponding sensitivity magnitude describing function. The capacity of low frequency disturbance rejection of linear PID control is not degraded, however, the high fre-

Figure 7.6: Simulated sensitivity magnitude describing function.

quency sensitivity hump is obviously attenuated compared with the conventional one in Figure 7.4.

As previous mentioned, frequency response of the proposed nonlinear algorithm is not only related to the frequency, but also to the disturbance magnitude. Figure 7.7 plots the sensitivity magnitude describing function at 1000 Hz against different size of disturbance. With the increase of disturbance magnitude, sensitivity hump will be more suppressed .

7.4.2 Vibration Results

Vibration disturbances are significant problems in the HDD servos. With the increase of rotation speed of disk, more disk flutter and spindle vibration will be induced typically in the frequency range of 500 Hz – 2 kHz. Some of them within the servo bandwidth can be compensated through the feedback control and disturbance compensation, while others may be amplified due to sensitivity hump in the

Figure 7.7: Sensitivity magnitude response at 1000 Hz against disturbance magnitude.

conventional HDD servo system. To study the effects of vibration disturbance on the PES, the recent disturbance model obtained in DSI ([32]) is added into the servo loop.

The power spectral density of the PES with the conventional and proposed servos are shown in Figure 7.8. Without degrading the low frequency disturbance attenuation capabilities, the proposed servo can improve disturbance rejection at high frequencies, which resulted in the cumulative PES and TMR being improved, as shown in Figure 7.9. Experimental results of sine disturbance within servo bandwidth and beyond servo bandwidth are shown in Figure 7.10 and Figure 7.11. The proposed nonlinear feedback control law does not degrade the capability of low disturbance rejection within servo bandwidth of the original linear servo loop, however, reduces the disturbance amplification due to sensitivity hump beyond servo bandwidth.

Figure 7.8: Power spectral densities of the PES with the linear and the proposed nonlinear control.

Figure 7.9: Cumulative sum of the PES without control, with the linear and nonlinear control (raw PES: $3\sigma = 7.2\%$ of track, with linear control: $3\sigma = 3.44\%$ of track, with nonlinear control: $3\sigma = 3.2\%$ of track).

7.4.3 Noise Response

As mentioned in the previous section, the high frequency sensitivity hump is reduced to enable better disturbance capabilities. This in fact would affect the frequency

Figure 7.10: Experimental results of sine disturbance response within servo bandwidth: nonlinear feedback control law does not degrade the capability of low frequency disturbance rejection.

Figure 7.11: Experimental results of sine disturbance response beyond servo bandwidth: nonlinear feedback control law reduces the disturbance amplification due to sensitivity hump.

response of complementary sensitivity function and thus compromise noise at high frequencies. The noise model in [30] is added to the servo loop to test the effect of measurement noise on PES. The time trace of measurement noise and the PES with the conventional and proposed servos for such noise model are shown in Figures 7.12 and 7.13. Much of the measurement noise contaminating the PES with the proposed servo is still effectively removed (rejected by 70%), compared with the conventional servo system (rejected by 73%).

Figure 7.12: Simulation results: time trace of measurement noise.

7.4.4 Sensitivity Robustness

The proposed servo system is designed based on the mathematical model identified from an operating range of frequencies of the actuator. However, the actuator is known to contain plant parameters that are not predictable with high confidence. For example, resonant frequencies of the actuator do not keep constant under various operating environment and the frequency shift becomes a source of uncertainty. In

Figure 7.13: Simulation results: effects of measurement noise on PES with the conventional servo (rejected by 73%) and the proposed servo (rejected by 70%).

this subsection, the sensitivity robustness of the designed nonlinear servo will be discussed. Sensitivity describing functions of the proposed servo with effects of $\pm 10\%$ resonant frequency shifts in the VCM model are plotted in Figure 7.14. The generation of describing function curve is the same as that in Figure 7.6, which was described before in details. The shifts in resonant frequencies do not affect the sensitivity hump suppression and hence, disturbance rejection capabilities are still retained under the plant perturbations.

7.5 Conclusions

This chapter explored a nonlinear loop shaping method to suppress sensitivity hump in the HDD servo system. It was designed based on the conventional PID track-following servo, at the same time to shape the high frequency response with a nonlinear high gain PD control. It was able to increase the track-following accuracy

Figure 7.14: Simulation results: sensitivity describing functions of the proposed servo with the nominal and perturbed VCM actuators.

by compensating more disturbance, which would lead to increase of data density.

Vibration test showed that the proposed servo can improve the capability of high frequency disturbance compensation, without degrading low frequency response of the conventional servo. Noise response showed that although complementary sensitivity function would be affected due to sensitivity peak suppression, high frequency measurement noise contaminating the PES was still effectively removed with the proposed servo. Sensitivity function performances verify that it is robust with the high frequency plant uncertainties.

Because the nonlinear loop shaping is focused on the frequency range of sensitivity hump, the selection of ρ is empirical and is assumed that some of disturbance characteristic such as scale was known before. The approximation of true frequency range that hump occurs and the estimation results would decrease the algorithm

effectiveness. However, this problem may be solved with the development of nano-technology that low cost sensor will be applied to the HDDs and make some of real-time disturbance dynamics available [76].

Chapter 8

Conclusions and Suggestions on the Future Research

8.1 Conclusions

Current hard disk drive controllers are mainly linear. Previous studies have shown that there are a lot of control limitations associated with linear control system. This thesis has contributed some nonlinear control techniques to compensate or overcome performance limitation due to linear control for hard disk servo systems.

Fundamental performance limitations and their associated conditions in the design of feedback control system were first investigated in Chapter 2. By defining four different closed-loop transfer functions, inherent “algebraic” relationships of performance specifications in linear systems were derived. These relationships typically indicated design tradeoffs of different performance criteria. In frequency domain, well-known Bode’s gain-phase relationship, Bode’s integral theorem and Poisson’s sensitivity integral were described in detail. Control limitations of hard disk servo system were analyzed by employing their conditions. In time domain, three specific systems — with integrators, with open-loop RHP poles and with open-loop RHP

zeros were considered. Four performance specifications in step response, namely, rise time, overshoot, settling time and undershoot, were analyzed to show the unavoidable design constraints in these systems.

Besides linear systems, limits of nonlinear system in both frequency domain and time domain were either considered by defining nonlinear operators. In comparison with linear system, Bode's gain-phase relationship, Bode's integral theorem and time domain constraints with integrators were found not to be necessary to hold for nonlinear systems, which actually motivated the development of nonlinear control techniques for the HDDs in this thesis.

Servo design criteria were first defined before hard disk modelings were introduced in Chapter 3. Certain stability margins and TMR performance, the asymptotic tracking ability and the quick and quiet track seeking are desired for the overall servo system. Hard disk plant modeling was described through both analytical derivation and system identification in frequency domain. In theoretical derivation, the HDD actuator is treated as comprising known mechanical-electronic components to analyze their connections and relationships and to derive the transfer function. In system identification, the actuator is regarded as a black box to measure its input-output frequency response via experiment setup. Final results showed that theoretical derivation and system identification were matched very well.

The disturbance environment in the hard disk drive application was addressed in this thesis. The disturbances were made by various TMR sources, which are generally categorized into two kinds of major components: RRO with a fundamental spindle frequency component and its harmonics, NRRO which is not synchronous

with spindle revolution. Each of the components was individually analyzed. Based on their key characteristics in the time domain, simple procedures to identify them from raw PES were proposed and used. Normalized discrete-time Fourier transform (DFT) was used to analyze them in frequency domain. Standard derivation of the time domain sequence and power spectral density (PSD) in frequency domain were shown to be easily obtained from their DFT magnitude.

The two fundamental control problems — track seeking and track following control, were both investigated in this thesis. Chapters 4 and 5 dealt with the seek-settling performance and Chapters 6 and 7 considered the disturbance and vibration compensation in the track-following mode.

In Chapter 4, two kinds of nonlinear feedback control techniques were developed to break through the performance limitation of time domain in the conventional HDD servo system. The nonlinear system stability was verified through appropriate Lyapunov functions. To avoid actuator saturation especially for long seeking length, an estimator-based input shaper was proposed to generate a pre-defined reference trajectory in Chapter 5. By combining input shaper and feedback controller, two-degree-of-freedom (2DOF) control scheme was proposed. Simulation and experimental results demonstrated that the nonlinear 2DOF control 1 performs better than the well-known PTOS controller, an approximate time-optimal control for servomechanism, especially for long seeking span. And 2DOF control 2 with nonlinear PID (NPID) feedback controller improved the seek-settling process when servo modes transfer from seeking to following by reducing the overshoot from 28% in the conventional HDD servo system to 1.4%, undershoot from 10% to 0% without

degrading steady-state performance. Furthermore, the robustness of 2DOF control 2 was verified against different step sizes (tracking lengths), different bias force, plant parameter perturbation and non-zero initial state. Simulation and experimental tests showed that track-to-track access time was decreased averagely by 59% compared with the industry product (Seagate data) with track density from 6.9 to 64 kTPI.

Track-following control issues were investigated with two kinds of challenging disturbance compensation in Chapters 6 and 7. Specifically, mid-frequency narrow band disturbance was considered in Chapter 6. With the increase of rotational speed, larger disk vibration will be induced to aggravate the head positioning error during track-following operation. As the energy of disk vibration typically concentrates on certain frequency range and shows a sharp high peak around the central frequency in its power spectrum plot, disturbance rejection through feedback control is not effective. In this thesis, a nonlinear narrow band compensation scheme was proposed by combination of a nonlinear phase stabilization mode and a narrow band filter. The nonlinear mode is essentially a first order mode in cascade with lead filter except with ability to reset its state whenever input goes to zero, so that it shows a describing function characteristic of 51.9° phase lead while 0 dB/decade slope of magnitude response, which is potential to be used to compensate for the phase loss due to narrow band filter to stabilize the closed-loop system. Sine disturbance responses at 800 Hz with the proposed compensation scheme and the PI-lead feedback control law were compared to show that the capability of mid-frequency disturbance rejection was improved greatly. Both power spectral density (PSD) and

3σ TMR value of NRRO were evaluated to show that NRRO PSD was decreased and 3σ TMR value was improved greatly with the proposed control scheme.

Chapter 7 dealt with sensitivity hump phenomenon due to Bode's integral theorem. A composite nonlinear feedback control law was developed to suppress sensitivity hump to as low as possible. As mentioned before, Bode's integral theorem limits the disturbance rejection since the net area subtended by the plot of sensitivity function $|S(j\omega)|$ on a logarithmic scale is zero. The feedback controller will thus amplify disturbance beyond servo bandwidth. The proposed nonlinear control worked as a normal linear track-following servo at low frequency range, and generated an open-loop high gain at high frequency range by combinations of both linear and nonlinear laws to suppress sensitivity hump. Simulation and experimental results demonstrated that sensitivity hump can be suppressed or even be removed with the increase of disturbance magnitude. Therefore, more accurate head positioning in the track-following was achieved.

8.2 Suggestions for the Future Research

It is possible to make improvement to the ideas and techniques contained in this thesis, as well as extend to the other applications.

This thesis has been emphasized on the nonlinear control and implementation of hard disk servos, and successful experimental results have been obtained. When designing the control system, some factors were simplified or neglected in one way or another. With the increase of storage capacity and data transfer rate, higher performance servo system is necessary to achieve more precise positioning of the

R/W head (track-seeking) and faster transition from one track to another target track (track-following). All these factors would be better to be considered for the HDD servomechanism design. Specifically, the following aspects for investigation are suggested:

- Time-delay

Time-delay is a common phenomenon in servomechanism and other types of systems. There are much theoretical literature devoted to control with state time-delay ([21], [135] and [140]). The control time-delay problem was handled in ([23] and [96]). In an HDD servomechanism with high-speed data acquisition, time-delay is usually caused by the computations and the converters between the analog and digital signals. It will contribute additional phase lag to the system and further affect system stability.

- Friction torque

The VCM actuator in the HDD has several undesirable characteristics such as resonances and nonlinear friction torque, which limit the maximum achievable performance in terms of closed-loop bandwidth and, in turn, track density. Due to its discontinuity at zero velocity and the downward bend of the friction torque at low velocity, friction is responsible for positioning in accuracy and motion intermittence in high performance applications. Nonlinear friction was modelled as a linear second-order system with high damping in this thesis. However, such a simple model does not reproduce the stick-slip motion that is typical of a real system with nonlinear friction [115]. Hence, more accurate modeling and compensation of nonlinear friction would be an important goal

for future research. The basic modeling of friction can be found in ([78], [114] and [142]). The friction were usually dealt with complicated adaptive control. However, robust control, which can handle large uncertainties, is not fully discussed in current literature.

Besides the above aspects related to hard disk servomechanism which are simplified or neglected in this thesis, control methodologies development is still crucial and necessary in order to improve the performance of the HDDs. In an academic setting, research on control technologies may also need to be tied to that on the mechanical aspects of the HDDs, such as disk vibrations, media noise, etc. Specifically, the following control methodologies are recommended for further research in hard disk drive servo systems.

- Modified reset control for undesired abrupt change in control

Reset control was verified to be effective to deal with mid-frequency disturbance compensation. However, the discontinuity of state leads to the abrupt change and large spikes in the control signal. This may lead to the actuator saturation and excite high frequency actuator resonances. If the discontinuity is relaxed by increase the reset duration, this issue will be solved. Different from state-dependent reset control described in Chapter 6, resetting time should be used to described the reset action so that reset duration can be accurately set with the same reset energy achieved.

- TMR source identification

More accurate TMR source models are required for better servo system designs

and optimizations. In Chapter 3, PES model was measured from a special hard disk setup MicroE system ahead of the design of servo system. Although both time domain and frequency domain analysis can characterize the TMR environment, it may not accurately reflect the real-time characteristics of TMR sources in the designed servo system when implemented. The identification of their frequency contents may be best achieved with a theoretical analysis of windage, air fluid dynamics, disk vibrations and so on.

- Nonlinear “soft” sensor development

Disturbance observer or predictor is typically used for disturbance rejection by estimating the disturbance signal from the source, and cancelling it via feedforward control. The performance of linear disturbance observer is limited due to response speed, bandwidth, phase loss, etc. As nonlinear control responds differently to signals, it might be possible to show different amplitude and frequency characteristic with disturbance data such as NRRO. And thus, it is possible to achieve better “prediction” than linear one.

- NRRO phase compensation

Although periodic NRRO with known frequency can be dealt with narrow band compensation method, phase of such signals are usually time-variant. If phase identification is neglected, transient response would be degraded. Adaptive control has the potential to be adopted for phase recognition in real-time. More literature about adaptive control theory and application to different fields can be found in ([130], [137], [143] and [152]). And the implementation

of adaptive algorithm to hard disk servo system may be a challenging work.

- Servo loop optimization Although the performance improvement in this thesis was compared or predicted with optimal time control (in Chapter 5), the optimality consideration of performance specifications such as disturbance attenuation, control energy and positioning accuracy is significant. Furthermore, it is also constructive to take a global optimal design for the HDD head-positioning servomechanism.

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Curriculum Vitae

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1. Y. Li, G. Guo and Y. Wang. A nonlinear control scheme for fast settling in hard disk drives. *IEEE Transactions on Magnetics*, 40(4):2086–2088, July 2004.
2. Y. Li, V. Venkataramanan, G. Guo and Y. Wang. Improvement of seek-settling performances by dynamic nonlinear control in hard disk drive servo systems. Submitted to *IEEE Transactions on Industrial Electronics*.
3. Y. Li, G. Guo and Y. Wang. Nonlinear reset control for narrow-band disturbance compensation in hard disk drive servos. Submitted to *IEEE Transactions on Control System Technology*.

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