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Unconventional Spiral Phase Elements: Design and Application

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SUMMARY

Optical vortex (OV), also known as helical beam and optical singularity, is a specific mode of laser beam having a phase term of $\exp(i\ell\varphi)$, here ℓ stands for the topological charge of the OV and φ is the azimuthal angle of the polar coordinates. This phase structure results in an intensity zero and a phase singularity at the center of the beam. Some previous researchers have been interested in the generation, analysis and application of OV, and the spiral phase optical components involved are computer generated holograms (CGH), spiral phase plates (SPP), spatial light modulators (SLM), et cetera.

Properties of conventional spiral phase elements have been well studied and various advanced applications have been introduced, which include laser trapping and manipulation of microscopic particles, the vortex coronagraph, spiral interferometry, spiral phase contrast, and free-space vortex communication. The focus of this thesis is to further extend the advantages as well as eliminate the drawbacks of ordinary spiral phase elements by modifying the conventional optical components. The unconventional designs of the spiral phase elements and the improvement of performance discussed in this thesis include coaxial hollow spiral phase plate (HSPP), raised-cosine-modulated spiral phase filter, Dammann vortex grating. Furthermore, analysis has been performed for fractional and quantized multilevel SPPs.

Spiral interferometry is an elegant technique to identify the phase structure without elevation/depression confusion. HSPP is implemented in a 4-f system to perform spiral interferometry coaxially. In the HSPP structure, the reference beam passes through the central hollow hole without any phase modulation, while the outer portion of the plate performs the same modulation function for the higher spatial frequency beam as an ordinary SPP. This provides a very compact optical component, which could be implemented in conventional microscopic systems. The interferogram of a thick phase

sample, i.e. a micron scale polystyrene bead and its reconstructed phase structure have been shown.

Unlike spiral interferometry, coherent beams carrying information from weak phase samples do not produce interferograms when passing through a spiral phase spatial filter. Spiral phase filters intrinsically extend 1-D Hilbert transform into a radial region, and the radial Hilbert transformed images are edge-enhanced homogenously in all radial directions. However, using conventional SPP to perform the filter function will result in radial Hilbert-transformed images with redundant noise adjacent to the enhanced edges. To address this problem, a complex modulation, namely raised-cosine modulation, has been incorporated with the SPP to optimize the edge enhancement effect. Edge-enhancement effect with improved Michelson contrast of 82.79% has been achieved.

In the analysis of fractional vortices, the theory that a fractional vortex could be decomposed into different integer charges was first reviewed. Analysing the Fourier expansion coefficients, a fractional vortex can be approximated using linear superposition of two adjacent integer-charged vortex beams and these two beams constitute at least 80% of the energy. The theoretical analysis is experimentally verified with a vortex sensing grating.

A further step is to analyse the multilevel SPP. Ideal SPP takes the continuous surface topology, but due to the actual fabrication complexity, usually a multilevel structure is used to approximate the surface topology, which degrades the designed vortex quality. Multilevel SPP produces OV with different topological charge spectra according to the number of phase levels. In order to generate intrinsic OV with 95% and 99% efficiency, the numbers of quantized levels needed are 8 and 16 respectively. A 1-D Dammann vortex grating has been used to justify the analysis experimentally.

A vortex-sensing grating is a brilliant tool to identify and analyse the sign and number of topological charges of OV. Moreover, it could be implemented in free-space optical communications as detector of incoming vortex beams. The original design is the well-known bifurcated grating used in generating OV, and the apparent drawback of such kind of grating is that the diffraction efficiencies at higher orders are significantly lower, which

makes this grating incapable to detect a large range of topological charges. By introducing a Dammann vortex grating which combines the Dammann grating design with an ordinary vortex grating, this problem is resolved. The Dammann design will produce equal energy distribution among the designed diffraction orders so higher orders become useful, and easily facilitate comparison among the orders. Topological charge detection from -12 to +12 is shown using a fabricated binary phase 5x5 Dammann vortex grating.

LIST OF ABBREVIATIONS AND SYMBOLS

OAM – Orbital Angular Momentum

SAM – Spin Angular Momentum

SLM – Spatial Light Modulator=

TM – Transverse Magnetic field

TE – Transverse Electric field

SPP – Spiral Phase Plate

HSPP – Hollow Spiral Phase Plate

CGH – Computer Generated Hologram

k – wavevector

λ – wavelength

r, φ – polar coordinates

I – intensity

ℓ – topological charge number of optical vortices

q – fractional topological charge number

$\hbar$ – Plank constant

CHAPTER I

INTRODUCTION

1-1 Background of optical vortices and spiral phase elements

The very first discussion about electro-magnetic wave singularities was published by P.A.M. Dirac in 1931 [1], following the upsurge in the atomic physics, optics and photonics research. Interestingly, after that was years of silence, maybe because of the rise of atomic science research attracted every resource, or because World War II interfered with the whole research society too much. The result we know today seems like that the optics people have forgotten this brilliant idea. 41 years later, in 1972, J. F. Nye and M. V. Berry found the starting point of optical vortex research and published the most influential work “dislocation of wavetrains” [2], again. Soon after the publication, researchers started to focus on this topic and many studies and applications began to come out.

The generation of optical vortices could be categorized into two broad techniques: internal modulation and external modulation of laser beams. In 1989, Couillet et al described a vortex solution of the laser equation and used a Maxwell-Bloch model to quantitatively analyze optical vortices numerically and analytically[3]. In addition, they proposed that laser cavities with large Fresnel number could be ideal candidates to isolate optical vortices. And Brambilla and co-workers have reported detailed analysis and experimental generation of several kinds of phase singularities formed by corresponding transverse cavity modes[4].

On the other hand, external modulation of fundamental modes of laser beams is an elegant way to generate optical vortices. In 1990, Bazhenov et al generated optical vortices using multimode fiber and analysed the vortices using a Mach-Zehnder interferometer, they even produced higher order optical vortices using

the “forked” amplitude hologram [5]. Heckenberg et al has proposed a CGH i.e. a Fresnel zone plate to generate optical vortices[6]. Soon after that, Beijersbergen et al reported optical vortices generated from an SPP in 1994[7]. Thereafter, spiral phase elements have become a uniquely new chapter of study.

Optical vortices generated using spiral phase elements are versatile tools and have been intensively used in various applications. The photons in optical vortices carry orbital angular momentums (OAM), and after the early work by Allen et al [8], the study of the mechanical nature of OAM has been ignited. The OAM possessed by optical vortices is capable to manipulate microscopic particles, and the optical singularity at the beam center is superior for some laser cooling application. In quantum entanglement and information, the SAM only offers “qubits” for information communication. Now with the OAM of optical vortices, theoretically an infinite number of high dimensional quantum states can be exploited and thus dramatically increase the communication channels.

Other than generation of optical vortices, spiral phase elements can be implemented for image processing purposes. The spiral phase profile offers uniquely radial Hilbert transform property when the spiral phase element is placed in the Fourier plane of a 4-f image processing system. The radial Hilbert transform produces the elegant result of an edge-enhanced version of the input object’s image, and it works for both amplitude and phase objects. Furthermore, if one uses the filtered image to interfere with another reference beam, spiral interference phenomenon will occur. Such an interference technique overcomes the ambiguity of phase elevation and depression experienced by conventional interference techniques.

These are just a small number of the applications of spiral phase elements, the application range includes astronomy [9], microscopy [10], surface plasmonics [11] and many others. Moreover, the research of spiral phase elements is ongoing today and the number of fields that utilize spiral phase elements is rapidly growing.

1-2 Motivation of this work

Spiral phase elements have been implemented in various applications, such as optical trapping and manipulation, image processing, optical cryptography, etc. And the demand induced by the research advancements put forth higher standards and more stringent requirements of optical vortices generated and spiral phase elements used. Conventional spiral phase elements have inherent drawbacks and the specific needs of each different application requires the element to be specially designed and tailored. Therefore, the need of application-oriented modification arises for spiral phase elements.

In optical trapping, SPPs have been extensively used because of their high efficiency in optical vortex generation. Unconventional spiral phase elements have also been employed in specific applications to trap and manipulate transparent microscopic particles. Elliptical optical vortices generated from irregular shaped SPPs were reported for optical manipulation [12]. And in 2005, Tao et al reported the use of fractional optical vortices' OAM and their radial opening to guide and transport colloidal micro particles. In 2007, Franke-Arnold and coworkers reported generation of bright and dark optical lattices by interfering two optical vortices. These optical lattices are ideal for trapping cold atoms, and they even showed that by shifting the laser frequency the ring lattices can be made to rotate. As the unconventional spiral phase elements play more important roles in many optics research fields, the higher the requirement for detailed study on the modified element itself.

In optical imaging, conventional spiral phase elements have been proven to perform well as a spatial filter, and radial Hilbert transform using SPP has been reported and analyzed [13]. On the other hand, unconventional spiral phase filters offer fractional Hilbert transform and can produce heterogeneous edge enhancement in fingerprints analysis [14]. Fürhapter et al then reported spiral

interferometry and spiral phase contrast works using specially designed SPPs encoded on SLMs [15, 16]. SPPs can also be implemented using other techniques, and bulky parts such as SLMs impede the miniaturization of the optical setup.

In 2006, aiming at the residual noise in the radial Hilbert transform realized by standard spiral phase filters, Guo et al reported their work to suppress the noise using a Laguerre-Gaussian function modulated filter thus achieving better edge enhancement results [17]. This opens a valuable aspect of spiral phase element research, other than the Laguerre-Gaussian modulation there could exist different modulation methods which can produce the optimized edge enhancement result.

Optical vortices also play a part in free-space optical communications owing to its special characteristics of OAM. An optical vortex communication system based on OAM could be at the quantum level or classical level. When the classical domain is considered, Gibson et al first demonstrated a complete system to transmit and receive optical vortices with different topological charges [18]. Later Lin et al reported transmitting of collinear multiplexed optical vortices in free-space systems [19]. These brilliant ideas have opened a new chapter of classical optical communication and the further study of high-density communication systems using optical vortices has good prospects.

1-3 Objectives and scope of the research

The major purpose of this thesis is to deliver a few modifications to conventional spiral phase elements for some specific applications, and also provide analysis of several special types of unconventional spiral phase elements. The objectives can be broken down into the following segments.

1-3-1 Design and implement unconventional (modified) spiral phase elements for optical image processing and arbitrary optical vortex detection

- Design a coaxial SPP, which is capable to facilitate spiral interferometry. The system should be compact and ready to incorporate into conventional microscopic systems.
- Design a complex modulated spiral phase filter, which is capable to optimize the radial Hilbert transform results produced by conventional SPP. The redundant noise adjacent to enhanced edges should be eliminated while preserving most of the energy.

1-3-2 Study the properties of and optimize unconventional spiral phase elements for applications

- Use Fourier expansion to evaluate fractional OV's composition and their influence. Theoretically model the linear superposition of two topological charge combination of a fractional vortex, and demonstrate experimentally the validity of the decomposition using a vortex sensing grating.
- Experimentally analyze the topological charge components of quantized multilevel SPP and develop a suitable model to interpret the mode purity resulting from different numbers of steps.

1-3-3 Improve vortex gratings for optical vortices sensing and communication systems

- Develop an improved grating which is capable to generate and detect large volume optical vortices simultaneously, and overcome the problem of higher order power inefficiency.

1-4 Significance and contributions of the thesis

This thesis produces several key results on the analysis, design and application of unconventional spiral phase elements. It is first demonstrated in this work that a fabricated HSPP can produce collinear spiral interference. By placing the modified phase plate in the filtering plane, and scrutinizing the parameters of the quantized multilevel spiral phase area and the central hollow hole, spiral interference result is successfully obtained. And the HSPP allows miniaturization of the optical setup and therefore is suitable for integration with standard microscope systems.

The raised-cosine modulation of conventional spiral phase filters are designed and implemented for optimized image edge enhancement. Although Laguerre-Gaussian modulated spiral phase filters generate good noise suppression results experienced by ordinary spiral phase filters, the function has only one variable parameter to tune for optimized results. The raised-cosine function is frequently used in image processing and offers two degree of freedom, the cutoff value and the roll-off factor. The filtered results indicate that compared to the Laguerre-Gaussian modulation the raised-cosine modulation delivers higher transmission efficiency while retaining similar noise suppression ability.

The intensive use of different kinds of unconventional spiral phase elements increases the demand for a comprehensive analysis of the elements, among which the fractional SPP and quantized multilevel topology are most common. It has been known that a fractional vortex can be expanded into combinations of infinite angular harmonics, and optical vortices generated by multilevel SPPs contain different topological charge components. In the thesis, the detailed analysis of the optical vortices composition is first shown and by using a vortex spectrum analyzer grating, experimental detection results are also provided. The accurate detection of fractional topological charge has been reported in the thesis within 10% error range.

The thesis also demonstrates that the Dammann vortex grating proposed as a vortex spectrum analyzer is an outstanding tool for vortex communication applications. The two dimensional binary phase Dammann design uniformly distributes energy among the diffraction orders and therefore eliminates the low efficiency problem at normal grating's higher orders. Hence the synthesized grating extends the optical vortices detection range significantly and works excellently as a high density vortex beam demultiplexer in free-space optical communication systems.

These contributions of the thesis have been recognized by the global research community. The research works have resulted in six publications in several internationally recognized scientific journals, and two reports presented in international conferences. The publication details can be found at the end of the thesis.

1-5 Thesis organization

Figure 1-1. Mindmap of this thesis.

This thesis is organized into the following chapters.

- Chapter 1 is the introduction part, which consists of background discussion on optical vortices and spiral phase elements, the motivation, objectives, and contributions of this work.
- Chapter 2 is the literature review part. The concepts and techniques of some relevant historical works are investigated in detail. And the present state of the advancement in these optics areas is appraised individually.
- Chapter 3 discusses the HSPP and provides the analysis and implementation in spiral interferometry. The interferogram of a colloidal micron-size bead is processed and its phase structure has been retrieved.
- Chapter 4 covers the optimization of spiral phase filtering using raised-cosine modulation. The design of the modulation is first discussed and then its advantages are evaluated and compared with standard and Laguerre-Gaussian modulated spiral phase filters.
- Chapter 5 demonstrates the experimental analysis of fractional SPP. The Fourier analysis theory for optical vortices with fractional topological charges is first reviewed, and then vortex sensing gratings are employed for the experimental identification of the integer charge components.
- Chapter 6 studies another type of unconventional spiral phase element, the quantized multilevel SPP. The multilevel topology is first analyzed with Fourier expansion. And Dammann vortex grating is developed for the experimental analysis of the charge composition.
- Chapter 7 introduces the 2-D binary phase Dammann vortex grating. The design of such a grating is first described and then the demonstration of extending optical vortices detection range is shown.

- Chapter 8 concludes the thesis, discusses the problems and difficulties faced during the research work, and finally provides some possible research directions for the future.

CHAPTER II

LITERATURE REVIEW

In this chapter, a literature review on the topic of spiral phase elements is provided. The historical researches on the characteristics of optical vortex beams and the related applications will be investigated with emphasis on spiral phase elements. After that, recent developments of unconventional spiral phase elements will be discussed.

2-1 Historical study and applications of optical vortices

2-1-1 Discovery, description and explanation

Followed by the optics and atomics age of 1930s, P. M. Dirac published the first academic paper discussing the phase singularity in electro-magnetic waves[1]. After a few decades, the most influential work of optical phase singularities was then published by Nye and Berry in 1974[2].

Optical vortices are commonly characterized by the helical wave fronts surrounding the phase singularities at the center. In laser physics, the Laguerre-Gaussian (LG) modes family is well known to have the azimuthally dependent phase term $\exp(i\ell\varphi)$, where ℓ is the topological charge and φ is the azimuthal angle in polar coordinate system.

(i) General Information

An optical vortex (also known as helical beam or phase singularity) is a zero of an optical field, a point of zero intensity. Besides the unique feature of a phase singularity, optical vortices pose some other special characteristics such as OAM [8]. A Laguerre-Gaussian (LG) laser beam has an optical vortex in its center. The

beam is a solution of the scalar wave equation in the paraxial approximation to Maxwell's equations consisting of the Laguerre polynomials which has the form

$$\Psi \propto r^{|m|} e^{-r^2} e^{i\ell\varphi}.$$

(i) Properties such as topological charges and OAM

An optical vortex can be characterized by its helical wave front along the beam's direction of propagation and an azimuthal component of the Poynting vector. The term topological charge of an OV beam corresponds to the number of revolution of the twisted phase front within one wavelength. And the twisting wave front gives each constituent photon of this kind of light beam an OAM of value $\ell\hbar$, where ℓ corresponds to the topological charge of the vortex beam[8]. Unlike SAM, which possesses only two states, OAM can take indefinitely many states theoretically, so can be considered as a fundamentally new spatial degree of freedom in optics. In general, the OAM cannot be considered individually because the total angular momentum of a photon contains spin and orbital contribution, but taking paraxial approximation into consideration, the two momentums can be examined independently. Marrucci et al reported some results that when interacting with inhomogeneous and anisotropic materials, SAM of circularly polarized light can be converted into OAM [5].

Photons in a Laguerre-Gaussian beam have $\ell\hbar$ of OAM. The integer ℓ (known as the topological charge) also gives the strength of the vortex at the beam's centre. The wave front of a vortex beam is twisted, as shown in Figure 2-1.

Figure 2-1. An optical vortex has helical phase front

2-1-2 Generation Techniques

The creation of optical vortices can be realized by a number of different techniques, including CGH[6], phase plates[7], Hermite-Gaussian mode conversion[3]. There are also some other variations of optical vortices generation techniques, such as using SLM[20], phase wedges[21].

Figure 2-2. (a) Phase gratings displayed on SLM to generate optical vortices. (b) Schematic drawing of the phase structure of an SPP.

The generation of optical vortices of topological charge 1 from SLM can use the phase grating of Figure 2-2(a), in which black portion stands for phase of zero and white portion stands for 2π phase. The static SPP as shown in Figure 2-2(b) is another spiral phase element for generating optical vortex. The higher topological charges can be designed to suit the wavelength of the laser in the same way to create desired vortices.

2-1-3 Applications

(i) Laser trapping and manipulation

In 1995, the first experimental observation of an OV beam's OAM transferred to microscopic object was reported by He et al. [22] They trapped absorptive microscopic particles using OV produced by CGH, and observed the rotation of the particles. Moreover, the rotation direction can be easily reversed by changing the sign of the OV beam.

After that, Gahagan et al demonstrated three dimensional trapping of low index micro sized particles, including hollow glass spheres and density-matched water droplet in acetophenone, using strongly focused optical vortices [23, 24]. And in 1999, they successfully trapped different sized low-index and high-index particles simultaneously with a single optical vortex trap[25]. Figure 2-3(a) shows the intensity profile of a vortex beam with a Gaussian waist w_0 and characteristic core of size w_v , and Figure 2-3(b) shows the stable trapping positions, z_{trap} and z'_{trap} for low- and high-index particles, respectively. The focal plane (f.p.) is assigned the position $z=0$. They also reported that the trapping of high-index particle has some stability problem when the depth of focus is increased.

Figure 2-3. Vortex beam for trapping. (a) Intensity profile of a vortex beam. (b) Stable trapping positions. [25]

While the trapping of high-index dielectric particles are resulted from the gradient force, the OAM of optical vortices takes the form of a tangential component to the beam's linear momentum density that can be transferred to illuminated objects[20, 26]. And the optical vortices will trap colloidal particles and transfer the OAM to the particles, resulting in the orbiting of the particles along the circumference of the beam. In Figure 2-4(a), the SLM imposes the phase $\Phi(\vec{r}) = \ell\phi \bmod(2\pi)$ on the incident TEM_{00} beam, converting it into a helical beam that is focused into an optical vortex. The inset phase mask encodes an $\ell = 40$ optical vortex. And Figure 2-4(b) shows the image of the resulting optical vortex obtained by placing a mirror in the focal plane. The central spot is the diffraction-limited focus of a separate coaxial TEM_{00} beam and coincides with the optical axis. In Figure 2-4(c), a time-lapse image of a single colloidal sphere traveling around the optical vortex is shown. In addition, this kind of motion of mesoscopic particles can be utilized to produce optomechanical pumps in micro-fluidic environment [27]. Figure 2-5 depicts the time-lapse composite of 16 images in half-second intervals of colloidal spheres in the holographic pump at $P = 2.4$ W. The red dots identify the trajectory of a single sphere as it moves $25\ \mu\text{m}$ to the left in 7 sec, with peak speed of $5\ \mu\text{m}/\text{sec}$.

Figure 2-4. (a) Schematic diagram of dynamic holographic optical tweezers creating an optical vortex. [20]

Figure 2-5. Time-lapse composite images of an optical pump. [27]

Implementing the OAM with optical tweezers, a range of prospective applications has been suggested, such as optical spanner and optical micro

motor.[28] Single photon's OAM can take multiples of $\ell\hbar$, which is superior than the SAM $\pm\hbar$. [29]

The dark focus is suitable for trapping reflecting[30], absorbing[31] or low-dielectric-constant objects[24] that would be damaged or repelled by conventional optical tweezers. Because such traps lack the radiation pressure from axial rays, they make more efficient traps for large dielectric objects than conventional optical tweezers do [32-34]. Smaller dielectric particles are drawn to the ring's circumference, as shown in Figure 2-5(c).

What really distinguishes optical vortices and other ring-like optical traps is the ability to exert torques as well as forces [22, 30]. In 1992, Allen et al demonstrated that in addition to intrinsic SAM, each photon in a helical mode carries an OAM $\ell\hbar$, where ℓ is the quantum unit of angular momentum [8]. A single colloidal microsphere is shown circulating around such a topological ring-trap under the influence of the optical angular momentum flux in the time-lapse photograph of Fig. 6c. Thus the name optical vortices or optical spanners have come to be known because of such toroidal torque-exerting traps, and they have potentially widespread technological applications [23, 35]. Valuable insights into the interplay of photon spin and OAM have been provided by studying the motion of objects in optical vortices, and this has been useful in interpreting the quantum mechanical nature of helical beams[4, 20, 22, 26, 30, 36].

An optical vortex's radius, R_ℓ , increases with its topological charge [20, 37]; therefore, the intensity pattern can be tailored to different applications. For example, a properly scaled ring of light can be projected onto the teeth of a micro fabricated gear, thereby creating a reliable micrometre-scale motor. The distributed drive made possible by projecting multiple optical vortices should also help to alleviate problems associated with friction in micromechanical systems. The SAM carried by circularly polarized optical tweezers similarly has been used to apply torques to birefringent components [38-40]. Using an SLM to control the polarization of multiple optical tweezers opens up even more

possibilities for developing extensive micro machines assembled and driven by light.

Some micromechanical applications may require no microfabrication at all. Rapidly circulating particles entrain flows that can mix and pump extremely small volumes of fluid. This solves a problem in microfluidic systems, whose laminar flows are ideal for transporting minuscule quantities of reagents but do not promote mixing when needed. Furthermore, the holographic optical tweezer technique can project multiple optical vortices, such as the 3x3 array in Figure 2-6(a), each with an individually specified intensity and topological charge [41]. This 3x3 array of $\ell = 30$ optical vortices matrix is shown trapping 800 nm diameter polystyrene spheres. Cooperative flows in such arrays can be reconfigured dynamically by modifying the trap-forming hologram, opening up the possibility of developing adaptive microfluidics on length scales ranging all the way down to tens of nanometres.

A family of distinct optical micromanipulators can be constructed using other variations on this theme, and each with its own applications. For instance, when a helical beam is superposed on a conventional beam, not only the helical wavefronts' structure will appear, as in Figure 2-6(b), an orientated intensity pattern will also be created that is useful for orientating asymmetric objects[42]. Superposing instead a helical mode on its mirror-image counterpart creates 3D arrays of discrete traps that can be rotated arbitrarily in three dimensions by varying the beams' relative phase[43]. Modulating the helical pitch of an optical vortex results in another class of optical rotators[44], an example of which appears in Figure 2-6(c). Further generalizations create intensity patterns related to the caustics seen at the bottom of swimming pools and which can move objects along complex trajectories transverse to the optical axis, all with static holograms and no moving parts. Other superpositions can focus on micrometre-scale dark regions surrounded by light on all sides known as optical bottles [45].

Figure 2-6. Generalizations of the optical vortex principle. (a), The 3x3 array of optical vortices created by holographic optical tweezer technique. (b), Interference pattern of an $\ell = 3$ helical mode with a plane wave. (c), An optical rotator created with a five-fold modulation of the helicity of an $\ell = 60$ optical vortex. [28]

Light has been a very important tool to trap and manipulate atoms. Instead of the Gaussian modes often employed, Laguerre-Gaussian beams have two advantages: firstly, the central singularity offers zero intensity, so the residual scattering is eliminated and results in no heating of atoms; secondly, the vortex line offers a quasi-one dimensional trapping area [46]. Some other superpositions can form a dark lattice, which has dark regions surrounded by bright areas. These dark lattices are useful for trapping very small dark-seeking objects, including clouds of ultracold atoms [45, 47]. Holographic arrays of optical bottles therefore should be useful for manipulating atoms[48], perhaps for quantum computing applications, and will help to extend pioneering efforts to apply optical tweezers to atomic physics[49, 50].

(ii) Optical image processing

Spiral phase filters can be implemented as radial Hilbert transformers for image processing purposes. Hilbert transform is a useful tool in image processing

because it can selectively enhance the input image's edge. Although spiral phase elements has been used in generating and recognizing OV beams for years [51], it was barely considered in any optical image processing applications. In 1999, Davis et al reported their analysis and experiments in which they use the spiral phase element to extend one dimensional Hilbert transform into radial Hilbert transform and successfully edge-enhanced the input image both homogenously and heterogeneously [13]. Figure 2-7(a) shows the one Dimensional Hilbert mask and Figure 2-7(b) is a radial Hilbert mask. And in Figure 2-8 the output are given when a circular aperture is used as the input object, the SLM is programmed with (a) no pattern, (b) the $P=1$, $Q=1$ two-dimensional Hilbert transform, (c) the $P=1$ radial Hilbert transform, (d) the $P=1/2$ radial Hilbert transform.

Figure 2-7. (a) One-dimensional Hilbert mask. Gray levels represent different phase values. (b) Radial Hilbert mask. Gray levels represent different phase values for $P=1$.

Figure 2-8. Hilbert filtering output when a circular aperture is used as the input object.[13]

Owing to the elegant property, spiral phase elements have been considered as a novel optical image processing tool in optical image processing and especially in microscopy area. The application in spiral phase optical image filtering could be classified into two main categories: spiral interferometry and spiral edge enhancement.

Spiral interferometry works in the same principle as conventional interferometry techniques in that the information carrying phase altered beam is interfered with another reference beam, and the interference fringe pattern depicts wanted information [15]. Spiral interferometry deals with thick phase samples, i.e. the phase difference is larger than 2π . The unique characteristic of spiral interferometry is that the interferogram appears in a spiral way instead of contour maps for the same phase value. And this resolves the difficulty to differentiate phase elevation and phase depression faced by other interferometry technique, moreover, with proper phase retrieval technique, the sample's phase information could be restored without confusion using only single interferogram.

In Figure 2-9, the experimental setup is depicted. A phase sample at a microscope sample stage is illuminated with light from a laser diode. The transmitted light is collected by a microscope objective. A relay lens L_1 is used to image the Fourier transform of the image wave at an SLM displaying a phase hologram. The light diffracted from the SLM into the first diffraction order is imaged through lens L_2 at a CCD camera, after the other diffraction orders are blocked. At the left the central section of a spiral phase hologram is sketched, which is displayed at the SLM (gray levels correspond to phase values). The modification necessary to produce the described spiral fringes is done by substituting a blazed grating (indicated above), which diffracts the zero order Fourier spot of the incident image wave, for a circular area about the central singularity. At the image detector this zero-order Fourier component evolves into a plane wave, which interferes with the remainder of the image wave, as required for spiral interferometry.

Figure 2-9. Sketch of the spiral interference experimental setup. [15]

Figure 2-10 shows experimentally obtained interferogram of an oil drop smeared on a glass coverslip. (a) Normal contour like interference fringes. (b) Spiral interferogram of the same sample region obtained after filtering with the modified spiral phase hologram (the blazed grating in a small central area). (c) Section of the spiral interferogram to be processed. (d) Single contour line of the

spiral interferogram. (e) Surface profile reconstructed by processing the contour line and fitting the surface at the obtained sampling points.

Figure 2-10. Experimentally obtained interferogram of an oil drop smeared on a glass coverslip and phase reconstruction with spiral interferometry. [15]

Spiral phase contrast

On the other hand, spiral phase contrast (or spiral edge enhancement) works primarily for thin phase samples (samples with phase difference less than 2π)[16]. The power of spiral edge enhancement is its high sensitivity of phase jumps, so it is suitable for analyzing microscopic imaging, especially for biological samples such as cells. Figure 2-11 illustrates the experimental setup, the displayed holograms A-D used for certain spatial filtering purposes in the Fourier plane are not to scale. In Figure 2-12, from the left: bright-field, spiral phase contrast and dark-field image of an absorptive sample (resolution target, size of the total image area is 400 x 300

microns), imaged in transmission geometry. The three images were taken under identical illumination and camera settings. And Figure 2-13 shows contrast enhancement of phase objects. Two pairs (A-B, and C-D) of bright-field images (A, C) and spiral phase filtered images (B, D) of the same sample areas of a phase object. In A-B the sample consisted of a scratch in a glass coverslip, coated with water, which shows part of an extended scan over a larger sample area, comparing the two imaging modes. In (C) and (D) another sample is imaged, consisting of a sharp oil-water boundary [16].

Figure 2-11. Sketch of experimental setup for spiral phase contrast. [16]

Figure 2-12. Bright-field, spiral phase contrast and dark-field image of an absorptive sample. [16]

Figure 2-13. Contrast enhancement of (A-B) a scratch in a glass coverslip coated with water and (C-D) a sharp oil-water boundary. [16]

In 2008, Maurer et al integrated the spiral phase element with a standard bright field microscope and performed phase contrast image. And a slight shift of the spiral phase element's position even results relief-like images similar to differential interference contrast microscopy [52].

In Figure 2-14, graphic A shows the set-up for transmissive probes, set-up B is for reflective probes with epi-illumination. The optical path after the tubus lens ($TL_f = 160$ mm) is the same for both set-ups. The SPP is inserted into the Fourier plane of the horizontal front port of the microscope. The distance from the tubus lens to the CCD is approximately 370 mm, and the lens L_1 has a focal length of 50 mm. In the case of a transmissive probe (set-up A) the illumination aperture has to be chosen rather small ($NA = 0.03$ in our experiments). In the case of a reflective set-up (B), a beam splitter and an additional lens L_2 ($f = 100$ mm) have to be placed into the optical path of the microscope. The lens L_2 focuses the input beam to the Fourier plane of the objective. The collimated light leaving the objective is either reflected by the sample or, in the case of a transmissive sample, reflected by an additional mirror. Lens L_2 focuses the illumination beam to a spot size of $100\mu\text{m}$ to the focal plane of the objective.

Figure 2-14. The light path modifications used for spiral filtering. [52]

In Figure 2-15, the images show fibroblast cells in line A as viewed with a 20 $\times$ air objective, in line B recorded with a 100 $\times$ oil immersions objective. The images at the left side show the bright-field images, whereas at the right are the corresponding spiral phase-filtered images. The centre of the SPP is placed 50 μm away from the optical axis. The recording time of the spiral phase-filtered images is only half as long as that of the bright-field images.

Figure 2-15. Microscopic images of fibroblast cells. (Left) bright-field images, (Right) corresponding spiral phase-filtered images. [52]

And Figure 2-16 depicts sections of the surface of a semi-conductor device. The images in line A and B are recorded with a 10 $\times$ objective, and the images in line C are viewed with a 63 $\times$ air objective. The left column shows the bright-field image, the spiral phase-filtered images are shown on the right side. The SPP is centred at the optical axis of the microscope.

Figure 2-16. Bright-field images and spiral phase filtered images of sections of the surface of a semi-conductor device. [52]

(iii) Free-space optical vortex communication

Spiral phase elements have been intensively used for generating optical vortices carrying OAM. In information communication, OAM could be used in both classical and quantum levels, and it serves as a better basis for higher dimensional qubit communication, this is because unlike SAM that can take only two states,

OAM can take an infinite number orthogonal basis states (in principle, if infinite aperture is available).

The first demonstration of quantum entanglement of OAM was reported in 2001. Mair et al conducted the experiment using two CGHs to generate Laguerre Gaussian modes [53]. Figure 2-17 shows the experimental set-up for single-photon mode detection. After parametric downconversion, each of the photons enters a mode detector consisting of a computer generated hologram and a mono-mode optical fibre. By diffraction at the hologram, the incoming mode undergoes a mode transformation in such a way that an LG mode can be transformed into a gaussian mode. As it has a smaller spatial extension than all LG modes, only the gaussian mode can be coupled into the mono-mode fibre. Thus observation of a click projects the mode incident on the fibre coupler into the gaussian mode. The hologram is a phase grating with Δm dislocations in the centre blazed for first-order diffraction. An incoming gaussian laser beam passing through the dislocation of the hologram is diffracted by the grating, and the n th diffraction order becomes an LG mode with an index $\ell = n\Delta m$ and vice versa. Intuitively speaking, the phase dislocation exerts a 'torque' onto the diffracted beam because of the difference of the local grating vectors in the upper and lower parts of the grating. This 'torque' depends on the diffraction order n and on Δm . Consequently the right and left diffraction orders gain different handedness. Reversing this process, a photon with angular momentum $\Delta m\hbar$ before the grating can be detected by the mono-mode fibre detector placed in the first diffraction order. A photon with zero angular momentum (gaussian mode) is detected by diffracting the beam at the border of the hologram far away from the dislocation. All measurements by Mair et al were performed in coincidence detection between the two down-converted photons.

Figure 2-17. Mair's experimental set-up for single-photon mode detection. [53]

Later in 2004, Oemrawsingh et al reported their research on entangled quantum states of OAM using non-integer SPPs [54]. The proposed experimental setup is shown in Figure 2-18. A nonlinear crystal (NLC) splits a pump photon to a signal photon and an idler photon by the process of spontaneous parametric down-conversion (SPDC). In each path, a SPP ($\text{SPP}_{s,i}$) is inserted with a single-mode fiber $F_{s,i}$, together forming the analyzer. The coincidence count rate of detectors $D_{s,i}$ is measured as a function of the SPP angular settings α_s and α_i .

Figure 2-18: Experimental setup proposed by Oemrawsingh. [54]

In classical optical communications, encoding OAM in free-space optical links offers two apparent advantages: higher information channels and improved security. The OAM encoding and decoding was first proposed by Gibson et al in 2004, and the authors demonstrated a 15m free-space optical communication channel using two SLMs performing encoding and decoding accordingly [18]. And they have also analyzed the security property and showed that the OAM encoded optical link is resistant to eavesdropping. The system configuration is shown in Figure 2-19.

Figure 2-19. Optical configuration of the free-space optics (FSO) demonstration system. Shown here is the case of a vortex beam with $\ell = 8$. [18]

(iv) Other applications

Apart from the applications discussed, spiral phase elements have been applied in a wide range of fields such as astronomy[9], generation of radially or azimuthally polarized laser beams [55], generating electron vortex beams[56], and surface plasmonic imaging[57], etc.

2-2 Recent developments in the researches of unconventional spiral phase elements**2-2-1 Spiral interferometry****Concept of spiral interference**

In optical image processing, it is shown that phase contrast can be enhanced by spatial filtering techniques in association with a vortex transform or two-dimensional isotropic Hilbert transform [13]. The spiral phase was also proposed for applications of the spiral-shaped interference fringes in optical interferometry [15]. Different from the conventional interference fringes, the spiral-shaped fringe obtained in an interferogram will not only show 2-D shape of a thick phase object, but also contains additional phase gradient information in terms of elevation or depression. This uniqueness will enable the overall phase profile to be reconstructed uniquely. It was reported that a modified off-axis CGH was implemented to generate both spiral beam in the first-order and a reference beam in the zero-order [15]. For this purpose, the central part of the CGH was replaced by a blazed grating which diffracts the zero-order to the same angle as the first-order for interference, the setup is shown in Figure 2-9. Hence the CGH ought to be realized by diffractive optical element for phase modulation with grey-scale. SLMs are typical commercial dynamic devices for grey-scale phase modulation for

example a Holoeye SLM was employed in Ref [15]. Figure 2-20 shows comparison between the normal interferogram and spiral interferogram of a simulated phase profile.

Figure 2-20. (a) A simulated phase profile showing phase elevations and depressions. (b) Normal interferogram. (c) Spiral interferogram

(ii) Phase retrieval mechanisms of spiral interferometry

The phase reconstruction of the input object is analogous to conventional interferometry. And specifically for spiral phase interferogram demodulation, Jesacher et al proposed three different methods, each of which has its own advantages, as listed below [58].

- Contour Line Demodulation
- Center Line Demodulation
- Multi-Image Demodulation

Contour line demodulation and center line demodulation both require only a single interferogram and the process can be very fast. While the multi-image demodulation is based on established multi-image demodulation techniques in conventional interferometry. And comparing to single interferogram method, it has higher resolution and noise reduction, however, the process requires more interferograms so the process is slower.

Figure 2-21 shows demodulation using contour lines. (a) “Classical” closed-fringe interferogram of a deformation in a plastic film, (b) corresponding spiral interferogram, (c) and (d) single contour line raw and after preprocessing, respectively. The reconstructed three-dimensional shape is shown in (e) and (f).

Figure 2-21. Contour line demodulation. [58]

Figure 2-22 shows the center line demodulation technique. (a) “Skeleton” of the spiral fringe pattern, which roughly consists of connected intensity maxima, (b) and (c) reconstructed three-dimensional shapes.

Figure 2-22. Demodulation based on center lines. [58]

Figure 2-23 illustrates the principle of multi-image demodulation. The mean value of three “complexified” images is proportional to the spiral filtered object field, but without its zeroth order. After restoration of the missing field amplitude, the spiral back transformation is accomplished. Finally, the original phase distribution is restored by using a standard phase unwrapping algorithm.

Figure 2-23. Principle of multi-image demodulation. [58]

2-2-2 Edge enhancement

(i) Basic concepts about the phase filtering function

For thick phase samples, spiral interferometry can be achieved using spiral phase elements as spatial filters. However for thin phase samples, such as cells, the refractive index difference with the surrounding media is only in the order of 2 decimal places (1.395 for typical yeast cells compared to 1.335 for nutrient medium), and the thickness of which are in microns, thus no inteferogram will form due to the retardation of phase is quite small. So there is a strong need for highly sensitive technique to enhance the phase features when imaging biological samples.

Hilbert transform has been a long-established method to achieve edge enhancement in image processing, which improves the apparent sharpness of amplitude and phase objects' edges. Using spiral phase element for filtering can be considered as image processing with the radial Hilbert transform.[13] When an input image $t_i(x, y)$ enters a 4-f optical correlator, the output $t_o(x, y)$ will be the convolution between the input function $t_i(x, y)$ and the Hilbert function $h(x, y)$,

$$t_o(x, y) = t_i(x, y) * h(x, y), \quad 2 - 1$$

where $h(x, y)$ is the Fourier transform of the Hilbert function $H(u, v)$.

The next step is to develop a radial Hilbert filter from a one dimensional Hilbert filter. With one dimensional Hilbert transform mask function:

$$H_p(u) = \exp\left(\frac{iP\pi}{2}\right)S(u) + \exp\left(-\frac{iP\pi}{2}\right)S(u) \quad 2 - 2$$

where P is the order of the Hilbert transform and $S(u)$ represents the step function.

If one dimensional Hilbert mask is used on the x axis, edge enhancement will only occur in the x direction, the other directions of the image possessing phase change will not be enhanced. From one dimensional Hilbert mask it is possible to develop a two dimensional Hilbert mask which has the function of $H_p(u)H_q(v)$, this filter will only give edge enhancement image with x and y symmetry.

In order to obtain a uniform Hilbert transform in any radial direction, radial Hilbert transform concept is introduced. On any radial lines of a radial Hilbert mask, each half has phase difference $P\pi$ with respect to the other half.

The function of a radial Hilbert transform $H_p(\rho, \theta)$ is

$$H_p(\rho, \theta) = \exp(iP\theta) \quad 2 - 3$$

where (ρ, θ) represents polar coordinates at the Fourier plane.

By spatial filtering, the output will be

$$t_0(x, y) = t_i(x, y) * h_p(r, \theta) \quad 2 - 4$$

where the Fourier transform of the radial Hilbert mask $H_p(\rho, \theta)$ is the Hankel transform function $h_p(r, \theta)$ of order P. The convolution result will enhance the input object's edge, which depends on the order of the Hankel transform function. The experiment is first confined within $P=1$, so the edge enhancement will be narrowest. Expressed in sum of Bessel functions[59], the Fourier transform is :

$$h_1(r, \theta) = xJ_0 + \left(\frac{\pi x}{2}\right) [J_1(x)H_0(x) - J_0(x)H_1(x)] \quad 2 - 5$$

where J_0 and J_1 are Bessel functions of zero and first orders and H_0 and H_1 are Struve functions of zero and first orders respectively. Here $x=kRr/f$, where R is the radius of the aperture, k is the wave vector of the light, and f is the focal length of the Fourier transform lens.

The spiral phase filtering setup is the same as can be done using a liquid crystal SLM, and the basic spatial filtering process is through a 4-f correlator. In the 4-f correlator SLM filtering system shown in Figure 2-24, the SLM is placed at the Fourier plane of L1 for filtering purpose. After the spiral filtering process, the zero diffraction order is blocked, first diffraction order is collected by L2 for imaging.

Figure 2-24. A 4-f correlator filtering system with an SLM.

For spatial filtering with SLM rather than SPP, the first diffracted order will be used. It can be considered as an off-axis hologram, which contains blazed phase hologram working as diffraction mask.

(ii) Complex modulation of the filter

In the research of modified amplitude modulation, C.-S. Guo et al analyze the point spread function (PSF) of the image processing system for radial Hilbert transform and propose a novel spiral phase filter, called the Laguerre–Gaussian spatial filter (LGSF) [17]. Using Laguerre–Gaussian spatial filter (LGSF) is to implement the radial Hilbert transform for the purpose of eliminating the diffraction noise and improving the resolution. And they have done some

theoretical analysis and real experiments, shown that the LGSF possesses some advantages in comparison with the conventional SPP.

In conventional image processing using SPP, there exists some smear around the enhanced edges, which is obviously useless and needs to be avoided at best, because it results in low contrast and disturbance of the output image. There may be two cause of this undesired effect, the first one is due to the sharp aperture of the filter, but this could not be the main factor, because the intensity distribution of the residual oscillations is very different from that appearing in an imaging system only with a sharp aperture. The second factor maybe the diffraction light from the part near the central singular point of the SPP and its interference with the light from the sharp aperture.

A new filter is designed to suppress the extra residuals based on the reasons listed above. The transmittance function is as follows:

$$H(\rho, \varphi) = \frac{\rho}{\omega} \exp \left[- \left(\frac{\rho}{\omega} \right)^2 \right] \text{circ} \left(\frac{\rho}{R} \right) \exp(-j\varphi) \quad 2 - 6$$

Where R is the radius of the filter aperture, $\text{circ}(\rho/R)$ is the circular aperture function describing the sharp aperture of radius R , and ω is a parameter controlling the position of the maximum amplitude. This function has a similar form as one of Laguerre-Gaussian modes.

The above function provides an amplitude modulation of the filter. When parameter ω is properly chosen, the suppression of the residuals can be realized for both the outer aperture and central singular region.

Figure 2-25. Amplitude profile in one radial section of the PSF for the radial Hilbert transform using (a) SPP, (b) LGSF with $w=0.74R$, and (c) LGSF with $w=0.64R$. [17]

In Figure 2-25(b) and (c), the PSF along one radial direction take parameter ω as $0.74R$ and $0.64R$ respectively. Obviously the side lobes have been suppressed. The optimal value found in the simulation is $\omega \approx 0.6R$.

The following figure shows some simulation results of the differences between LGSF and a normal spiral phase filter. For simplicity, a rectangular phase step is used as input and the cross-section of the results are examined.

The edge enhancement result is obviously better when LGSF is used, one can notice this merely from the image without looking through the cross-section. From the cross-section part, the LGSF was found to produce virtually perfect edge enhancement compared with the normal filter, the diffraction noise is clearly eliminated. However, when checking the cross-section carefully, it can be found that the intensity of the normal filtered image is higher than the intensity produced by the LGSF, this pitfall is reasonable as LGSF utilized amplitude modulation to block the center and outer area of the filter, and especially the center part has blocked much as the filter is placed at the Fourier plane (it is the focal plane here) in the filtering system.

Figure 2-26. (a) The input image of a rectangular phase step, different colors represent different phase values. (b) The edge enhanced image using a normal spiral phase filter. (c) The edge enhancement image using a LGSF. (d) The intensity distribution of the cross-section of (b). (e) The intensity distribution of the cross-section of (c).

The transmittance function is coded to a CGH, which appears as follows.

Figure 2-27. Examples of the CGHs designed for the radial Hilbert transform, where (a) is a conventional spiral phase mask, and (b) is based on equation of the LGSF. [17]

And the experimental results are shown as below,

Figure 2-28. A typical experimental result of the radial Hilbert transform using a SPP as the spatial filter, where (a) is the photo of the input object, (b) is the intensity distribution of the output image after the radial Hilbert transform, and (c) is the result from LGSF. [17]

2-2-3 Study of fractional SPPs and anisotropic edge enhancement

As discussed in Section 2-2-2, spiral phase filters with embedded integer topological charges perform isotropic optical edge enhancement, each edge of the input pattern is enhanced regardless of its orientation. However, in some special

situations the features of some specific edges are of more interest than those of others, so anisotropic edge enhancement technique is required. There are various different methods to perform anisotropic edge enhancement. Davis et al have experimentally demonstrated anisotropic filtering using fractional derivatives, and an acoustic-optical light modulator [60, 61]. In 2005, Greenberg et al has reported adaptive anisotropic filters for fingerprint analysis [14].

Other than these methods, fractional Hilbert transform can also perform anisotropic edge enhancement. A conventional Hilbert filter can be defined as

$$H_1(v) = \exp\left(+i\frac{\pi}{2}\right)S(v) + \exp\left(-i\frac{\pi}{2}\right)S(-v), \quad 2-7$$

where $S(v)$ is a step function.

In 1996, Lohmann et al first defined the fractional Hilbert transform operation thus generalized the Hilbert transform as

$$H_p(v) = \exp(+i\phi)S(v) + \exp(-i\phi)S(-v), \quad 2-8$$

where $\phi = P(\pi/2)$.

And they have also presented the computer simulation of the fractional Hilbert filtering [62]. One year later, they successfully realized optical fractional Hilbert transform using polarization separation technique and performed edge enhancement for one dimensional and two dimensional objects [63], as shown in Figure 2-29. And the computer simulated results for the fractional Hilbert transform of a square function input are depicted in Figure 2-30.

Figure 2-29. Optical setup for performing the conventional Hilbert transform and fractional Hilbert transform. [62]

Figure 2-30. Computer simulation of the fractional Hilbert transform for an input square function. [62]

One year later, Lohmann et al reported their extension of one dimensional generalized fractional Hilbert transform into two dimensional, and some optical processing results were provided [63]. Figure 2-31 shows the fractional Hilbert filtered images of a circular aperture.

Figure 2-31. Fractional Hilbert filtered images of a circular aperture shown in the upper left-hand corner of this figure. [63]

Later in 1998, Davis et al reported an in-depth analysis of the fractional Hilbert transform [64]. They used one dimensional rectangular function as input and then filtered by different fractional Hilbert filters, and the results showed that the enhanced edge can be selected by changing the parameter P of the Hilbert filter. Figure 2-32 shows that (a) Output intensity obtained with the $P=1/2$ Hilbert transform (the right-hand edge is emphasized). (b) Output intensity obtained with the $P=1$ Hilbert transform (both edges are emphasized). (c) Output intensity obtained with the $P=3/2$ Hilbert transform (the left-hand edge is emphasized).

Figure 2-32. The selection of enhanced edge of a rectangular function by choosing different P values of the fractional Hilbert transform [64]

For image processing of a two dimensional input object, simply perform the Hilbert transform in two orthogonal direction will not result in homogeneous edge enhancement in all directions. Khonina et al first proposed a phase roter filter, which is essentially a SPP, to produce equally edge enhanced images in two dimensions [65]. Davis et al then examined two dimensional Hilbert transform by forming the product of two Hilbert masks as $H_p(u)H_q(v)$, and found that this

only produce edge enhancement results with x and y symmetry [13]. In order to address this problem, one dimensional Hilbert transform is extended to all radial directions to form radial Hilbert transform and the radial Hilbert phase mask $H_p(\rho, \theta)$ can be written as

$$H_p(\rho, \theta) = \exp(iP\theta), \quad 2 - 9$$

where (ρ, θ) are the polar coordinates on the filter plane and P represents the order of Hilbert transform.

The integer Hilbert transform can homogeneously enhance the edges of the input image on all radial directions, and fractional Hilbert transform can selectively enhance the edges when different P values are selected, as shown in Figure 2-33.

Figure 2-33. Output when a circular aperture is used as the input object. Liquid crystal SLM programmed with (a) no pattern, (b) the $P = 1$, $Q = 1$ two-dimensional Hilbert transform, (c) the $P = 1$ radial Hilbert transform, (d) the $P = 1/2$ radial Hilbert transform. [13]

When spiral phase filters are employed for edge enhancement, the enhanced orientation can easily be selected by using fractional topological charge of the spiral phase filter and changing the direction of the discontinuity. However this is not the only way to enhance selected orientations, Situ et al has closely analyzed the fractional SPPs and found that shifting the filter laterally also produces anisotropic edge enhancement results [66]. And they experimentally examined their analysis using an SLM in a 4-f system. Figure 2-34 shows that different edges are enhanced when changing p value.

Figure 2-34. Density plots of $|u_p(r, \varphi, \vartheta_{=0})|$ with $p=(a) 0.2, (b) 0.5, (c) 0.8$.

On the upper-left side of these figures are the status of the SPF. [66]

And in Figure 2-35, the filter with $p=0.5$ is rotated clockwise for $\pi/2$ radian, and it is clear shown the enhanced edge is also rotated in the same direction by the same amount.

Figure 2-35. Density plot of $|u_p(r, \varphi, \vartheta_{=-\pi/2})|$ with $p=0.5$. [66]

They also reported another interesting finding that the enhanced edge can be selected by laterally shifting the spiral phase filter, and this is shown in Figure 2-36. The dark ring in the figure is the boundary of the filter. Note that in the calculation the area of the SPF outside the boundary is zero. It is shown here just for a clearer illustration of the state of the SPF.

Figure 2-36. Density plot of $|_s u_1|$ with (ρ_0, θ) equal to (a) $(0.2R, \pi/4)$, (b) $(0.55R, \pi/4)$, (c) $(0.85R, \pi/4)$, (d) $(0.55R, 3\pi/4)$. On the upper-left-side of these figures are the status of the SPF. [66]

Other than the fractional spiral phase filtering application, there have been several studies on fractional SPPs and the fractional optical vortices specifically. Berry analyzed the evolution of optical vortices from integer and fractional SPPs, starting from $z=0$, both exactly and paraxially [67]. For an integer topological charge $\ell = n$, perfect optical vortices are formed. When $n < \ell < (n + 1/2)$, the

interference between the phase discontinuity and the wave scattered from $r = 0$ forms a pattern of vortex line close to z-axis. This case is shown in Figure 2-37. And when $(n + 1/2) < \ell < (n + 1)$, another vortex is born. And Berry predicts that when $\ell = (n + 1/2)$, an infinite chain of alternating strength vortices will form.

Figure 2-37. A density plot of the paraxial wave $|\psi_{\alpha=2.2}|$ in the ξ, η plane.

[67]

Later in that year, Leach et al used an SLM to experimentally generate optical vortices with varying ℓ values and verified Berry's prediction [68]. They used an interference-based phase measurement method to examine the phase structures after propagation, and an alternating charged vortices line is observed near the phase discontinuity. Figure 2-38 shows the modeled and experimental results corresponding to non-integer l step heights. And they reported another finding and analysis that the angular momentum about the z-axis is only proportional to integer charged or half-integer charged values.

Figure 2-38. Modelled (top) and experimental (bottom) results corresponding to non-integer l step heights. The red and green circular arrows indicate the first two alternating sign vortices along the line of the initial radial discontinuity. [68]

In 2004, Basisty et al reported their research on generating fractional vortices using modulated contrast gratings [69]. The fundamental diffraction orders possess integer charged vortices while the subharmonic orders have fractional charged vortices. The analytical approach they presented for a $1/2$ charge optical vortex shows that the beam structure can be decomposed into a vortex-free component and a charge 1 vortex component. This is shown in Figure 2-39. However, they predicted the vortex component is a superposition of infinite integer azimuthal indices Laguerre-Gaussian modes therefore has anisotropic structure (axial insymmetry).

Figure 2-39. Calculated intensity distributions in the vortex-free component alone (a), in the vortex component alone (b), and in the superposition of the components (c). Phase map of the beam (equiphase line separation $\pi/25$): single-charge optical vortex and phase saddle are detected (d). [69]

2-2-4 Quantized multilevel SPPs

SPPs have been intensively used in generating optical vortex beams and various applications, as discussed in Section 2-1-2 and 2-1-3. SPPs are usually fabricated using a multilevel and segmented topology, as this structure reduces the fabrication cost in most of the circumstances, no matter the fabrication media is photoresist or glass or any other transparent materials.

In 1994, Beijersbergen et al used a fabricated SPP to produce Laguerre-Gaussian beams and analyzed their mode components [7]. The SPP was fabricated using acrylic machined with normal tools, and then immersed into index-matched liquid to work for a wavelength of 633nm. The SPP was machined repetitively from edge to center, each time the tool moves down 10 μ m and the plate is rotated

by 5 degree, so there are 72 steps in total. Table 2-1 depicts the mode contents of the beam generated from a LG_{00} mode and a charge 1 SPP.

Table 2-1. Mode contents of a LG_{00} mode that has acquired a phase $\exp(-i\theta)$ due to a SPP with $\Delta\ell = 1$. Indicated is the relative intensity in percentages of the LG_{st} modes; the mode index s is on the horizontal and t on the vertical axis. [7]

6							
5							0.79
4						1.17	
3					1.92		
2				3.68			
1			9.82				
0		78.5					
(00)	0	1	2	3	4	5	6

In 2004, Cheong et al reported fabrication of SPP using direct electron beam lithography technique [70]. The SPP has 24 levels and a diameter of 500 μm . The total height is 1.04 μm , which corresponds to 2π phase difference at 633nm wavelength. After fabrication, the SPP is experimentally verified in a trapping system. Figure 2-40(a) is the plan-view of fabricated 24 level SPP, 500 μm (in diameter) and (b) is the height profile of the fabricated SPP ($\sim 1.04\mu\text{m}$).

Figure 2-40. (a) Fabricated SPP. (b) Surface profile. [70]

Later in the same year, Sueda et al reported fabrication of 10cm diameter SPP containing 16 steps for generating ultrashort high intensity Laguerre-Gaussian laser pulse [71]. The multilevel SPP was fabricated using electron beam evaporation deposition of SiO_2 , the phase difference was controlled by the vapor deposition film thickness. The process is shown in Figure 2-41 and the fabricated SPP is shown in Figure 2-42. They also calculated the Laguerre-Gaussian beam produced by such SPP and showed that the conversion efficiency from LG_{00} mode to LG_{01} mode produced by a 16-step SPP is 77.5%, a very slight difference than 78.5% produced by continuous SPP, as shown in Table 2-2.

Figure 2-41. Multilevel SPP of 16 steps fabrication process. [71]

Figure 2-42. Image of the fabricated SPP. [71]

Table 2-2. Dependence of the number of discrete steps N on conversion efficiency

Discrete step N	∞	32	16	8	4
$LG_{00} \rightarrow LG_{10}$ conversion efficiency (%)	78.5	78.3	77.5	74.6	63.7

To complement the applications of stepped SPPs, Guo et al conducted the analysis of the relationships between the step numbers and the topological charges of SPPs [72]. For a multilevel SPP with topological number L and level number M , they found that the generated beam consists of vortex components with charges $L-kM$. And they also reported some specific relationships such as when $M/L=9$, the principal vortex component has 96% of the total energy; when $M/L=4$, the total OAM reaches its maximum; when $M/L \approx 2.77$, the principal vortex's OAM reaches its maximum; and when $M/L=2$, the total OAM is zero and the focused output beam has an intensity profile of a periodic lattice ring. These results are shown in Figure 2-43.

Figure 2-43. Examples of MLSPs with 36 levels (left column) and their corresponding intensity distributions of the focused output beams (right column), when the phase step height is taken as (from top to bottom) $\Delta\Phi = 2\pi/9$ (or $L = 4$), $\Delta\Phi = \pi/2$ (or $L = 9$), $\Delta\Phi = 13\pi/18$ (or $L = 13$) and $\Delta\Phi = \pi$ (or $L = 18$), respectively. [72]

Furthermore, Kotlyar et al approximated ideal SPPs using multilevel polygonal apertures and obtained the analytical solution of the Fraunhofer diffraction for such SPPs [73]. Figure 2-44 illustrates the polygon approximations and their amplitude and phase in the Fraunhofer region. In the paper they also calculated

the number of sectors required given the criteria that the RMS value of the Fraunhofer diffraction between ideal SPPs and the modeled phase plates do not exceed 2%, as in **Table 2-3**.

Figure 2-44. Fraunhofer diffraction pattern of the plane wave by the multilevel limited SPP ($n=6$): (a), (d), (g) DOE phase; (b), (e), (h) amplitude; and (c), (f), (i) phase in the Fraunhofer diffraction zone. The number of sectors: (a)–(c) 18, (d)–(f) 30, (g)–(i) 42.

Table 2-3. *Minimal Number of Sectors versus the Topological Charge of the SPP*

Topological Charge (n)	Minimal Number of Sectors (p)
2	19
4	25
6	29
8	35
10	39

2-2-5 Vortex beam identification and analysis

Given a phase pattern $V_\ell(\varphi) = \exp(i\ell\varphi)$, vortex beams with different topological charge values ℓ can be easily generated, and the intensity profiles of such beams are dark cores surrounded by bright rings, thus called doughnut beams. In many applications, the identification of the topological charge and measurement of the OAM are required. However, the helical beams having different chirality have the same intensity profile, and this results in identification difficulty from the intensity patterns. Moreover, as the absolute value of the topological charge number increases, the diameter of the dark core increases. In most circumstances, the optical vortex beams are collimated or focused. Consequently, it is very difficult to distinguish between the beams having different topological charges, especially when different focal lengths are used.

There have been several works covering the topic of detection of optical vortices. Mair et al reported an OAM detection method in their quantum entanglement experiment [53]. Photons carrying different OAM first pass through a blazed vortex grating, the LG mode is down-converted to Gaussian mode, and only the Gaussian mode can be coupled into the mono-mode fibre. The experimental setup is shown previously in Figure 2-17. And in Figure 2-45, the dislocation of the hologram in the beam of photon 1 is shifted out of the beam centre step by step (top, middle, bottom). In these positions, this hologram - together with the mono-mode fibre detector - projects the state of photon 1 into a coherent superposition of LG and Gaussian modes. The mode filter for photon 2 with the hologram taken out makes a scan of the second photon's intensity distribution (detector position) in order to identify the location of its singularity with respect to the beam centre. The coincidences show that the second photon is also detected in a superposition of LG and gaussian modes. Classical correlation would yield a coincidence picture which is just a mixture of gaussian and LG modes. In that case, the intensity minimum would remain in the beam centre but

would become washed out. In the experiment a hologram with two dislocations in the first diffraction order was used.

Figure 2-45. Experimental evidence (left; right, simulation) of entanglement of photon states with phase singularities. [53]

In 2004, Leach et al proposed an interferometric method to measure single photon's SAM, OAM and total angular momentum [74]. In their setup, a Mach-Zehnder interferometer is used, and in one of the arms there is a rotation device. The rotation device can contain Dove prisms or half-wave plates or both, and photons having OAM or SAM can be sorted. As shown in Figure 2-46, the optical path of a photon can be selected using a Mach Zehnder interferometer with a rotation device in one arm. Rotation of the phase and intensity (Dove prisms) will sort photons containing OAM (a); rotation of the polarization (half-wave plates) will sort photons containing SAM (b). Combining these two systems so that the

whole beam is rotated will sort photons containing TAM (c). Note that second stage sorting of OAM and TAM is shown, and idealized Dove prisms that do not change the polarization state are also shown.

Figure 2-46. A single stage of the general sorting scheme. [74]

Later in that year, Gibson et al reported a free-space OAM communication system, the receiver of which consists of an SLM [18]. The system is shown in Figure 2-19. The SLM displays an analyzing hologram. The hologram is essentially two orthogonal vortex gratings, which diffract the incoming beam into nine orders in a 3x3 grid, each order contains a different helicity. In Figure 2-47, the phase-hologram patterns are represented in grey-scale. (a) Horizontally shifted $\ell = 1$ beam. (b) Setting the phase at each point in the hologram pattern for the horizontally shifted $\ell = 1$ beam to either 0 or π (whichever is closest) gives three horizontally separated beams with $\ell = -1, 0$ and $+1$. (c) Similarly, three vertically separated beams with $\ell = -3, 0$ and $+3$ can be obtained. (d) The sum of the two phase patterns that create the horizontally and vertically separated beams gives a

3×3 array of beams with $\ell = -4, -3, \dots, +3, +4$. If the topological charge of the incoming beam is ℓ and the diffraction order has helicity of $-\ell$, the resulting beam has a planar phase front, then this beam can be focused through a pin hole and detected by a CCD array.

Figure 2-47. Some examples of phase-hologram designs (top) and schematic representation of the corresponding far-field diffraction patterns under plane wave illumination (bottom). [18]

There is an even more simple solution for the detection of helical wave fronts' chirality. In the experiment conducted by Niv et al, a ZnSe SPP was employed to identify the helical wave fronts with +1 or -1 chirality [75]. In the experimental setup shown in Figure 2-48, the SPP has topological charge 1, the incoming helical wave front carrying -1 chirality will produce a charge zero beam while the helical wave front with +1 chirality will result in a charge 2 beam, thus identifies the wave fronts.

Figure 2-48. Schematic illustration of the experiment. [75]

In 2009, Moreno et al reported vortex sensing diffraction gratings, which extended to arbitrary detection range of the topological charge numbers of optical vortices [76]. The vortex grating they proposed was encoded on an SLM and has a 0 and π rad binary phase. And the fill factor is carefully selected so that the desired diffraction orders are present and obvious, as shown in Figure 2-49. Using such grating, the sign and order of the topological charge of the incoming optical vortex beam can easily be detected. Moreover, they have demonstrated the detection of arbitrarily assigned fractional topological charged vortex beam using the proposed grating.

Figure 2-49. Vortex diffraction gratings having a period of 32 pixels for (a) $\ell=0$, (b) $\ell=1.5$, (c) $\ell=3$. Here black represents a phase of 0 rad, while white represents a phase of π rad. [76]

CHAPTER III

COAXIAL SPIRAL INTERFEROMETRY WITH FABRICATED HSPP

The concept of spiral interferometry is first discussed and then HSPP is proposed for coaxial spiral interferometry. Experimental results of the self-referenced spiral interference are shown using an HSPP fabricated by electron beam lithography. From the interferogram obtained, the phase structure of a polystyrene bead is successfully reconstructed by contour line phase retrieving technique. Comparing with conventional interference systems, the proposed phase element provides a simple and robust approach to obtaining spiral interference fringes in a self-referenced interferometric way. Experimental implementation confirms that the element can be employed as an external tool kit for simple modification of existing optical microscopes to interferometers.

3-1 Spiral phase elements for spiral interferometry

The use of spiral phase filter and its unique features: distinguish phase elevations and depressions from a single frame of image.

Laser beams with spiral phase i.e. vortex beams have attracted great interest owing to their applications in optical trapping [77], non-interference radially polarized beams [78], secure free-space optical communication [19] and phase contrast imaging in microscopy [16]. In the past decades, there existed various methods for generation of spiral wavefront experimentally, for example computer-generated holograms (CGHs) [6] and SPP (also known as vortex lens) [7] are the typical ones reported in literature. Comparing with CGHs, the SPP has unique advantages in terms of high power efficiency and can be directly used in femtosecond laser beams without additional dispersion compensation elements

for small bandwidth [79]. The SPP is well known for converting a Gaussian laser beam into a vortex beam with its helical phase profile in terms of $\exp(i\varphi)$, where φ is the azimuthal angle measured from the beam's centre in a plane perpendicular to the propagation direction. It can be realized by spiral phase modulation either in the form of refractive index modulation in liquid crystal [80] or 3D surface profile structure in photoresist [70].

In optical image processing, it is shown that phase contrast can be enhanced by spatial filtering techniques in association with a vortex transform or two-dimensional isotropic Hilbert transform [13]. The spiral phase was also proposed for applications of the spiral-shaped interference fringes in optical interferometry [15]. Different from the conventional interference fringes, the spiral-shaped fringe obtained in an interferogram will not only show 2-D shape of a thick phase object, but also contains additional phase gradient information in terms of elevation or depression. This uniqueness will enable the overall phase profile to be reconstructed uniquely. It was reported that a modified off-axis CGH was implemented to generate both spiral beam in the first-order and a reference beam in the zero-order [15]. For this purpose, the central part of the CGH was replaced by a blazed grating which diffracts the zero-order to the same angle as the first-order for interference. Hence the CGH ought to be realized by diffractive optical element for phase modulation with grey-scale. SLMs are typical commercial dynamic devices for grey-scale phase modulation for example a Holoeye SLM was employed in Ref [15].

As suggested in Ref [15], a better design for spiral interferometry is to insert a transparent spiral phase element into a Fourier plane of the beam path. This configuration would not only reduce the number of optical components required in the system but also the external vibrations because the interference would be based on a self-referenced interferometric setup. Here the objective is to implement the configuration by using a special design of the SPP with a hole of 80 μm diameter in the centre to realize the spiral interferometry in a collinear way.

The phase element which is referred to as HSPP is proposed and shown in *Figure 3-1(a)*.

Figure 3-1. (a) simulated phase structure of the HSPP with a centre-hole diameter of $80\mu\text{m}$; (b) fabricated HSPP.

3-2 Theory of phase retrieval techniques

Spiral fringes formed of thick phase samples and using the shape and number of the spiral fringes to retrieve the phase information.

The HSPP comprises two parts: the outer spiral phase structure with topological charge 1 and a hollow with constant phase in the middle of the plate. With the HSPP inserted in the Fourier transform plane, it will perform the same function as an off-axis CGH, with the zero-order and the first-order diffracted beams passing through the center part and the outer part respectively. The two beams will then interfere collinearly at the imaging plane without additional reference beam path.

Phase distribution of the HSPP is expressed as follows:

$$p(x, y) = \begin{cases} \exp(i\varphi); & (\sqrt{x^2 + y^2} > r_0) \\ c; & (\sqrt{x^2 + y^2} \leq r_0) \end{cases} \quad 3-1$$

where φ is the azimuthal angle revolving from 0 to 2π , i.e. $\varphi = \arg(x + iy)$ and r_0 the radius of the central hole, c represents the constant phase value in the hollow.

The phase retrieval mechanism of the system above can be expressed as follows [81]: the expression for the phase object is

$$p_{\text{obj}}(x, y) = \exp[i\delta(x, y)] \quad 3-2$$

The first lens will perform Fourier transformation, then the light at the focal plane of the first lens can be represented by

$$U(u, v) = U_0(u, v) + U_1(u, v) \quad 3-3$$

Where

$$\begin{aligned} U_0(u, v) &= F[c_0(x, y)] \\ U_1(u, v) &= F[p_{\text{obj}}(x, y) - c_0(x, y)] \\ c_0(x, y) &= 1 \end{aligned} \quad 3-4$$

Here U_0 represents the light diffracted in the zero-order and U_1 the higher diffraction orders in the focal plane. After the second lens, the expression of the light at the image plane can be written as

$$\begin{aligned} q_{\text{img}}(x, y) &= F\{U_0(u, v) \cdot \exp(ic) + U_1(u, v) \cdot \exp[i\theta(u, v)]\} \\ &= \exp(ic) + [p_{\text{obj}}(x, y) - c_0] * F\{\exp[i\theta(u, v)]\} \end{aligned} \quad 3-5$$

where $\exp(ic)$ is a constant term representing a plane wave, and φ is the spiral phase excluding the hollow part. It has been proved by Davis *et al* [10] that the convolution part in Eq. (3-5) is a radial Hilbert transform which is able to enhance

the edge of $P_{\text{obj}}(x, y) - c_0$ and Jesacher *et al* reported that the phase profile of the object can be retrieved by unwrapping the spiral fringes [58].

3-3 Fabrication of the HSPP

Based on the design, the proposed HSPP was fabricated by direct electron-beam lithography in SU-8 2005 photoresist with an overall size of 3 mm by 3 mm, and the central hole is 80 μm in diameter. The SU-8 photoresist was spun-coated on an indium-tin-oxide glass substrate, and then exposed using a LE0982 FESEM converted lithography tool. The exposure dosage was calibrated so that the 72 fan-shaped sections will form thickness difference of 900nm after development. For working wavelength of 532 nm, the HSPP was designed with the maximum height close to 900 nm, and SU-8 2005 photoresist has a refractive index of about 1.59, which corresponds to the 2π phase difference we need. Therefore the helical phase profile was formed that the entire phase ramp in azimuthal direction completes a 2π linear change. The centre portion was not exposed so the photoresist will be washed away after development, leaving the desired central hole.

The fabrication details using direct electron-beam lithography technique can be found in our research group's previous paper on conventional SPPs [70]. *Figure 3-1(b)* shows the microscopic image of the fabricated HSPP.

3-4 Experimental results and analysis

3-4-1 The optical setup

In the experiment, the fabricated HSPP was inserted into a microscopic spiral interferometric system for phase object measurement. A schematic diagram is

shown in Figure 3-2, where a laser beam (Quantum Ventus 532) is reflected by a dichroic mirror and then passes through the phase object (polystyrene microspheres immersed in water) on the sample stage. The beam is coupled into an objective lens (Olympus M Plan Apo 100) and then processed in the 4-f system, where the HSPP is located at the Fourier plane. Finally, a CCD camera (Watec LCL-211H) collects the light coming out from L3 and captures the interferogram.

Figure 3-2. schematic diagram of the microscopic spiral interferometric system using the fabricated HSPP.

In the interferometric system, the HSPP was inserted at the Fourier transform plane in a 4-f system for spatial filtering, as shown in Figure 3-3. In the configuration, a sample phase object is illuminated by a plane wave, the light emerges from the object is then passing through the 4-f system. The focused light is spatially filtered by the HSPP at the focal plane of the first lens, then passing through the second lens for inverse Fourier transform. Finally, a CCD camera is placed after the 4-f system to capture the interference image.

Figure 3-3. Configuration of the interference system. The 4- f system is formed by two convex lenses L1 and L2, and the HSPP is inserted into the Fourier plane of the 4- f system for spatial filtering.

3-4-2 Spiral interference result

The spiral interferometric experiment was carried out for measurement of polystyrene microspheres (mean diameter: 15.02 μm) immersed in water. The captured image shows the interferogram in the form of spiral fringes with phase gradient information (number of fringes) as well as elevation and depression of the phase (orientation of the spiral fringe). Arising from the spiral interferogram, the reconstruction of phase profile was carried out by interpreting the interferogram as shown in Figure 3-4(a). The same microsphere imaged with white light illumination is shown in Figure 3-4(b). The first step was to identify the spiral fringes from this image, namely a series of sampling discrete point array were manually located on the skeleton of the spiral fringe as shown in Figure 3-4(c). Diffraction of the microsphere has resulted in some additional circular rings beyond the outer most spiral fringe and these artifacts can be removed in the data process.

Figure 3-4. (a) spiral interferogram of the polystyrene microsphere; (b) the same microsphere imaged with white light illumination; (c) sampling points on the spiral fringe.

3-4-3 Retrieving phase information of polystyrene micro-spheres.

Based on the center line demodulation method described by Jesacher et al [58], the phase profile of a pure phase object can be obtained by processing the spiral from one end to the other, along a certain polar angle to the center of the microsphere, the adjacent maximum intensity fringe will have a difference to 2π phase change. The phase gradient g_{ph} at each point on the fringe is perpendicular to the local tangent; therefore, the phase value is determined by the angle of the local fringe's tangent. On the spiral fringe, each point will have a phase gradient value with varying tangent of the local spiral curvature as the phase gradient g_{ph} is always perpendicular to the local tangent of the fringe.

The height of the polystyrene microsphere at the specific fringe location is given by:

$$\Delta n \cdot H = \frac{-\alpha_{\tan}}{2\pi} \lambda \quad 3-6$$

where $\Delta n = 0.26$ is the refractive index difference between polystyrene and water, $-\alpha_{\tan}$ the local tangential angle of the fringe and up to multiples of 2π .

Experimental reconstruction of the embedded phase information of the polystyrene microsphere is done by a program developed using MATLAB®. The 3D surface profile in Figure 3-5(a) depicts the reconstructed height value unwrapped from the phase information. The projection of the surface profile to the bottom plane in part (a) gives a contour map, which is shown in Figure 3-5(b), where the microsphere's diameter is obtained as $15.64 \pm 0.51 \mu\text{m}$ comparing with the mean diameter of $15.02 \mu\text{m}$ provided by the manufacturer. The values in the contour map indicate the integrated optical path length (red dashed lines) in vertical direction as shown in Figure 3-5(c), the location of the focal plane is shown by the black dashed line. From Figure 3-5(c), the optical path length can be directly calculated as $2\Delta n\sqrt{r^2 - x^2}$, where Δn is the refractive index difference, r is the radius of the microsphere, and x is the horizontal position of the optical path.

Figure 3-5. (a) reconstructed phase profile; (b) contour map of the reconstructed surface profile; (c) schematic of optical path length in the vertical direction, the focal plane is shown as the black dashed line.

3-5 Chapter summary

In conclusion, a fabricated HSPP is used to perform the spatial filtering function in spiral microscopic interferometry. In addition to its unique advantages based on

the off-axis CGH setup, the HSPP provides an alternative to the spiral interferometry in terms of a self-referenced system with collinear interference. The proposed hollow structure and configuration simplified the optical-path setup for interference. The design concept is expected to be useful in other image processing applications.

CHAPTER IV

OPTIMIZED OPTICAL IMAGE PROCESSING WITH HYBRID RAISED-COSINE-MODULATED SPIRAL PHASE FILTER

The experimental results of optical edge enhancement using a modified filter, i.e. hybrid raised-cosine spiral phase filter (SPF), are reported. This filter is able to produce optimized optical image processing results. Comparing with a conventional SPF, the proposed filter is able to suppress redundant noise for better contrast and resolution of the edge-enhanced image with improved efficiency. The proposed filtering process is demonstrated using off-axis holograms displayed on an SLM and can be readily incorporated within conventional microscopic system.

4-1 Spiral edge enhancement and spiral interferometry

It is recently reported that optical elements with spiral phase profile are able to perform one dimensional Hilbert transform uniformly in all radial directions[13], resulting in a uniform edge enhancement effect in optical image processing applications [82], such as spiral interferometry and spiral phase contrast microscopy [15, 16]. In Chapter 3, a modified HSPP which is able to perform spiral interference in an optical spatial filtering system was proposed, and the resultant spiral fringes possess unique features for reconstruction of the phase profile of sample objects [83].

4-2 The properties of conventional spiral phase filter and other modifications implemented for optimization

4-2-1 The noise residuals in conventional spiral phase filtering

When imaging amplitude and phase samples, the abrupt phase change and the positions of the edges are of particular importance to many applications, such as imaging of biological samples. It is noted that spiral phase filter is very sensitive to the phase changes and hence it can be applied to edge enhancement [82]. When conventional spiral phase elements are implemented at the filter plane, however, there exist some residuals adjacent to the filter-enhanced image, and the noise results in low contrast and resolution of the output image, as shown in Figure 4-1(a).

4-2-2 Drawbacks of high-pass modulation

To overcome this problem, a high pass filter could be employed to block the central portion of the spiral phase element for better contrast of the edge enhancement [16]. It is also suggested that the residuals are due to the interference between light diffracted from the central area of the spiral phase element and the outer sharp aperture of the filter, so a Laguerre-Gaussian function is implemented [17]. The simulation shown in Figure 4-1(a) and (b) depict the edge enhanced images produce by conventional SPF and high-pass SPF respectively. As these optimization techniques have only one parameter to tune, and a modulation function with higher degree of freedom to optimize the SPF can be introduced.

Figure 4-1. Simulated results: Spiral phase filtered images of a circular binary input object using conventional SPF (a), high-pass modulation (b) and raised-cosine modulation (c), and their cross-sectional intensity profiles (d), (e) and (f) respectively.

4-3 Hybrid raised-cosine-modulated spiral phase filter design and parameter optimization

Scrutinizing the shape of residuals in the intensity profile, it is noted that the high-pass filter does provide suppression of the adjacent residuals of conventional filtered image. However, unnecessary diffracted light could be introduced due to the hard sharp edge of the high-pass filter and the quality of filtered image is degraded. Inspired by the hybrid high-pass SPF problem, a raised-cosine-modulated spiral phase element is proposed to suppress the residuals adjacent to the enhanced edges, as well as to improve contrast of the edge enhanced image. The raised-cosine filter is known for its smooth transition from pass-band to stop-band in image processing. Transmittance function of the raised-cosine-modulated SPF is given by:

$$t_p(r, \varphi) = H(r) \exp(i\ell \varphi) \quad 4-1$$

Where

$$H(r) = \begin{cases} 0, & r \leq \frac{1-\beta}{2k} \\ \frac{1}{2} \left\{ 1 - \cos \left[\frac{\pi k}{\beta} \left(r - \frac{1-\beta}{2k} \right) \right] \right\}, & \frac{1-\beta}{2k} < r < \frac{1+\beta}{2k} \\ 1, & \text{otherwise} \end{cases} \quad 4-2$$

Here r is the radial distance from the filter center, β is the roll-off factor with value from 0 to 1, and $\frac{1}{2k}$ determines the transition center from pass-band to stop-band. The line profile of the raised-cosine function is shown in *Figure 4-2*, and the high-pass modulation and the raised-cosine-modulations with different β values are shown in *Figure 4-3*.

Figure 4-2. Profile of a raised-cosine function, here $k=0.1$, and different β values are shown.

In the simulated result of *Figure 4-1(c)*, when raised-cosine-modulated filter is implemented, the suppression of adjacent residuals is nearly complete, and extra noise is virtually nonexistent. Even if the noise is not quantitatively zero at the

proximity of the edge, it brings no observable disturbance to the simulation results, as seen in *Figure 4-1(f)*.

Figure 4-3. The different modulations to be implemented to optimize the spiral phase filtered image. The simple high pass filter (a) represented by a center block; $\beta=0.5$ (b) and $\beta=1$ (c). Raised-cosine modulation is of the same cutoff value k .

In the process to get an optimized image, two parameters in the modulation function can be adjusted: the cutoff value k and the roll-off factor β . The choice of optimized k value is subject to the experimental setup and its effect is highly dependent on β value, as discussed later, so the effects of different roll-off factor β is of great interest.

The roll-off factor β controls the transition characteristics of the raised-cosine function. When β takes the value of 1, the transition from stop-band to pass band can be considered very smooth, and the existence of the cutoff edge is not observable. When the roll-off factor gets smaller, the cutoff position becomes clearer, and the raised-cosine function is closer to a high-pass function. This can be explained from Eq. (4-2), when $\beta=1$, r can take the value in the interval of $\left(1, \frac{1}{k}\right)$,

and the cosine term $\cos\left[\frac{\pi k}{\beta}\left(r - \frac{1-\beta}{2k}\right)\right]$ becomes $\cos[(\pi k r)]$, so the smooth

transition from 0 to 1 ranges from 1 to $1/k$. On the other hand, when $\beta=0$, the transition point is at $\frac{1}{2k}$, and the change is abrupt as there is no room for r .

Figure 4-4. Comparison of noise adjacent to edge enhanced images produced by raised-cosine-modulated spiral filters with different β numbers: (a) $\beta=0.3$; (b) $\beta=0.7$; (c) $\beta=1$.

In the enlarged portions in Figure 4-4, it is clearly shown that as the β value of the raised-cosine function varies, the noise intensity changes. Moreover, the noise tends to expand to positions which are further away from the image peak when β is smaller, i.e. more like a high-pass filtered case. And when β takes values of 1, the noise distribution is more like that of conventional SPF. The normalized peak values of noise when $\beta=0.3$, $\beta=0.7$ and $\beta=1$ are listed in Table 1.

Table 4-1. Comparison of noise values of hybrid raised-cosine filtered image with $\beta=0.3$, $\beta=0.7$ and $\beta=1$.

Normalized intensity of edge enhanced image = 100%			
Roll-off factor	$\beta=0.3$	$\beta=0.7$	$\beta=1$
Maximum noise value	3.89%	0.56%	2.57%

4-4 Complex modulation of the Filter with SLM

Although the technique described above has produced promising results, the drawbacks are obvious. It is difficult to encode the modulation function onto the phase element. To achieve the desired image processing, an SLM is used as a substitute for SPP. As the SLM used only perform phase modulation and unable to operate in phase mode and amplitude mode concurrently. Thus an alternative method must be adopted in order to encode amplitude information onto the SLM. One possible solution it to modify the diffraction efficiency of the blazed grating formed on the SLM [84].

Complex modulation consists of phase modulation and amplitude modulation together. And the complex modulation $F(u) = |M(u)|\exp[i\Phi(u)]$ is to be encoded onto a single phase-only SLM. Here $M(u)$ represents the amplitude of the function and $\Phi(u)$ represents the phase function. As in some other techniques introduced to amplitude modulate the phase-only filter, they used carrier frequency, which is much higher than the original frequency working as the amplitude modulation. However, when using SLM to perform filtering task, it is impossible to generate any frequency higher than the intrinsic frequency built-in the SLM, which is determined by the pixel size of the liquid crystal matrix.

Therefore, the possible approach is to spatially modulate the phase that is encoded onto the filter. As the depth of the phase distribution changes, the diffraction efficiency of the filter is spatially modified. Light that is not diffracted into the first order is sent into the zero order, effectively allowing for amplitude modulation of either the first-order or the zero-order diffraction efficiency. This technique has several applications in optical pattern recognition, including amplitude modulation, low-pass filtering, high-pass filtering, and inverse filters.

Owing to the complementary nature between the first and the zero diffracted orders, the zero-order information will be low-pass filtered if the first order is

high-pass filtered and vice versa. Therefore the technique can also be used for image processing.

Figure 4-5. Schematic drawing showing diffraction of light by a phase grating with (a) phase depth of 2π rad over each period, (b) phase depth of less than 2π rad over each period.

Figure 4-6. Schematic drawing showing diffraction of light by a phase grating where the phase depth varies spatially: (a) low-pass filter—first-order diffracted light is low-pass filtered, and zero-order light is high-pass filtered; (b) high-pass filter—first-order diffracted light is high-pass filtered, and zero-order light is low-pass filtered.

Assume one wants to encode the complex function mentioned above,

$$F(u) = |M(u)| \exp[i\Phi(u)] \quad 4-3$$

The proposition is to use the following function instead,

$$T(u) = \exp[iM(u)\Phi(u)] \quad 4-4$$

This means the phase transmission function $T(u)$ can be obtained by multiplying the amplitude $M(u)$ and the phase $\Phi(u)$. $T(u)$ can be seen as being formed of several diffraction orders and that one of them reproduces the desired complex function $F(u)$. The phase distribution given by Eq. (4-4) can be represented [32,33] by a mixed Fourier–Taylor series. The results show that the original transmission function can be rewritten as a Fourier series as:

$$T(u) = \sum_{n=-\infty}^{\infty} T_n(u) \exp[in\Phi(u)] \quad 4-5$$

Where the coefficient $T_n(u)$ are given by

$$T_n(u) = \exp\{i[n - M(u)]\pi\} \frac{\sin\{\pi[n - M(u)]\}}{\pi[n - M(u)]} \quad 4-6$$

From the above equation, the different diffraction orders are shown, with the phase distribution $\exp[in\Phi(u)]$ and amplitude information within coefficients $T_n(u)$. The first diffraction order ($n=1$) reproduces the original phase function $\Phi(u)$ with an amplitude modulation $T_1(u)$. As mentioned above, one can choose the value of $M(u)$ between 0 and 1, and because of the sinc function shown in Eq. (4-6) the value of $T_1(u)$ is determined by the value of $M(u)$. For example, $T_1(u)$ will be 0 when $M(u)$ is 0 and if $M(u)$ is selected to be 1, then the value of $T_1(u)$ will be 1.

Based on this concept, it is possible to produce any desired complex transmission function virtually, using the proposed technique. The zero diffraction order approximately reproduces the complement of the original function. For the $n=0$ term the sinc function of Eq. (4-6) will have values near 1 when $M(u)$ is near to zero and values near zero when $M(u)$ is near to 1. Therefore $T_0(u)$ approximately reproduces the complement of the original function given by $1-M(u)$. Although there exists some distortion due to the sinc function, the distortion can be removed by introducing two further optimizing steps [84].

Now it is ready to encode amplitude modulation to the radial Hilbert mask displayed on SLM and using it for optimized edge enhancement.

4-5 Experimental comparison of the filtering results

To experimentally verify the proposed hybrid raised-cosine-modulated SPF, a versatile experimental setup where the scale of Fourier transform can easily be adjusted is constructed to validate the proposed hybrid filter [85]. The testing input is a sharply-edged aperture with 10 leaf-segments, which has a dimension of 3mm. The spatial filtering process is realized by the SLM, the actual model used is Hamamatsu X8267-11 parallel-aligned liquid crystal SLM (PAL-LCSLM). The SLM is able to perform filtering tasks using blazed phase holograms. The gray scale image displayed on the computer's monitor is simultaneously displayed on the SLM as phase values accordingly. The dark to bright variation corresponds to the phase value of 0 to 2π on the SLM. The blazed holograms used for the filtering tasks are calculated as off-axis holograms, such that the light diffracted into the first order contains the desired image.

4-5-1 Comparison with conventional spiral phase filter: optimized depression of noise and increased contrast and resolution.

When a center block is introduced, the blocked portion does not display the grating, so there is no diffraction of light at the first diffraction order, which is analogous to a high-pass modulation as seen from the first order. In 1999, J. A. Davis et al proposed a method to encode amplitude information onto phase-only LCSLM filters, that is, by holding the grating pitch constant and varying the phase depth of individual pixel of the SLM, the diffraction efficiency of the first order diffracted light can be modulated, thus to achieve the desired amplitude modulation at the first diffraction order[86]. According to this method, the raised-cosine function is then encoded onto the SPF, giving a smooth transition from stop-band to pass-band. Experimental comparison of the edge enhanced image and the intensity profiles are shown as in Figure 4-7. In the experiment for edge

enhancement using the proposed raised-cosine-modulated SPF, and after optimizing the parameters of modulation, the raised-cosine modulation with cut-off value $1/2k = 781\mu m$ (equivalent to 30 pixels) and $\beta=0.83$ is used.

Figure 4-7. The edge enhancement results of the input object using different filtering techniques. The results of conventional SPF (a), high-pass SPF (b) and raised-cosine SPF (c) are shown as (d), (e), (f) respectively. The corresponding cross-sectional intensity profiles (g), (h) and (i) are depicted below the images.

Conventional SPF is known for producing edge enhanced image. However, as seen from the result, the residuals start with the image peak and decrease gradually, making the spatial resolution of the image edge very poor. With a high-

pass-modulation, the filter is able to suppress the residuals nested with the image peak and show the sharp edges, but there appears extra noise, even at positions far away from the actual edges, this phenomenon obviously needs to be avoided. When raised-cosine-modulated filter is used, besides uniformly enhancing edges in any radial directions, it is noted that the filter is able to suppress excessive residuals adjacent to the enhanced edges and improve the image's contrast and resolution.

To determine the quality of a signal relative to the noise level in imaging field, the Michelson contrast is typically used. As shown in Table 2, Michelson contrast is defined as the difference of image intensity and noise intensity divided by the sum of the two intensities, so the contrast can vary from 0 (i.e. noise equals signal, no contrast at all) to 1 (i.e. noise equals zero, best contrast available). The Michelson contrast ratios of the edge and noise intensity for the three filtering techniques are vastly different.

Table 4-2. Michelson contrast ratios for the edge enhancement images produced using conventional SPF, high-pass-modulated SPF and raised-cosine-modulated SPF

$Michelson \text{ Contrast} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$		
Conventional SPF	High-pass SPF	Raised-cosine SPF
24.76%	52.48%	82.79%

One possible concern related to the filter discussed above is that the intensity loss due to the implemented amplitude modulation. In fact, the illumination of the input image largely controls the intensity of the filtered results, so the image intensity is adjustable. And any conventional dark field filter (i.e. high-pass filter)

has the amplitude modulation and will result in even more intensity loss compared to the raised-cosine filter. As in most microscopy systems, the absolute intensity is not a significant parameter, once the more desirable sharper and higher contrast image is obtained; the sacrifice of some intensity is only a trivial problem.

4-6 Chapter summary

In conclusion, the proposed hybrid raised-cosine-modulated SPF has been closely investigated for optical image processing. Edge enhancement result is compared with those produced by conventional and high-pass modulated SPFs. The comparison showed pronounced advantages of the raised-cosine-modulated technique, which includes noise reduction and contrast improvement. In order to experimentally verify the proposed filter, a high resolution SLM is used for displaying the desired filter function, and the filtered images of a test aperture input are acquired using different filtering techniques. As expected, the hybrid raised-cosine-modulated SPF not only uniformly enhanced the aperture edges, but also suppressed the redundant noise exists in conventional and high-pass spiral phase filtered images. Moreover, the Michelson contrast of the edge enhanced image produced by our proposed filter is 82.79%, compared to 24.76% using conventional SPF and 52.48% of high-pass SPF. To sum up, having the improved edge enhancement function, the optimized raised-cosine-modulated filter provides an enriching tool for optical processing. The proposed filter can also be encoded as conventional CGH or fabricated as phase plates, and then incorporated with a conventional microscope system to fulfill versatile needs of microscopic optical image processing purposes.

CHAPTER V

FRACTIONAL VORTEX ANALYSIS USING VORTEX DETECTION GRATING

The Fourier analysis of an optical vortex with fractional topological charge was reviewed first and then a theory to approximate the fractional vortex as a linear combination of two adjacent integer topological charges was focused on. This theory is experimentally verified utilizing a vortex grating spectrum analyzer, i.e. a CGH with charge 1 vortex grating. By extracting the intensity value of the center portion of the target diffraction orders, both the number and sign of the topological charge of any input vortex can be identified and analyzed.

5-1 Characteristics of optical vortices with fractional topological charge and the applications associated

5-1-1 Study of fractional charged optical vortices.

A vortex beam is characterized by the transmission function $v(r, \varphi) = A(r) \exp(iq\varphi)$, where $A(r)$ is the amplitude profile of the incoming laser beam, q is the topological charge, and φ is the azimuth angle of the polar coordinates. For simplicity only uniform illumination where $A(r) = 1$ is considered, so only the phase $\exp(iq\varphi)$ is of interest. Usually, the output patterns are formed in the focal plane of a lens and represent the Fourier transform of the transmission function. When q takes an either positive or negative integer value, the vortex beam forms a conventional doughnut beam with a dark central core at the Fourier plane. The diameter of the doughnut beam increases as the absolute value of the

vortex charge increases. However, when q takes fractional values, the resultant fractional vortex beam at the Fourier plane will have a non-circularly symmetric distributed intensity. Crabtree et al [[82]] used computer simulations to approximate fractional beams where $0 \leq q \leq 1$ by a linear combination of a zero order vortex and a first order vortex beam. However no proof was given. Berry [67] and Situ et al [66] showed that the fractional beams could be written as a Fourier series of integer vortex beams.

Because the diameter of the vortex beam depends on both the topological charge and the focal length of the lens, it is very difficult to experimentally characterize optical vortices by looking only at the intensity profiles. It is also impossible to differentiate between two vortices with same magnitude but opposite topological charges. Recently, a vortex sensing diffraction grating encoded onto a liquid crystal display (LCD) was introduced [76] that allowed identification of both the sign and value for the topological charge of a vortex beam simultaneously. The detection is achieved by analyzing the diffraction pattern formed by the product of the incident beam and the grating, and then the input topological charge is identified as a well focused spot at the output plane. However, for the detection of optical vortices with fractional topological charges, the vortex sensing diffraction grating requires a highly programmable LCD to generate the embedded fractional spiral phase pattern of the exact opposite charge.

The objective of this chapter is to propose and experimentally demonstrate a technique capable to identify, in a single step, vortex beams with arbitrary fractional topological charge. It is first shown that an optical vortex with a fractional charge can be well approximated by a linear combination of the two nearest integer charge vortices. Then how to measure the coefficients of the Fourier series using a vortex grating spectrum analyser is elaborated. Experimental results match theoretical predictions.

5-2 Decomposition of fractional charged optical vortex

5-2-1 Fourier expansion of fractional vortex

As stated earlier, the vortex beam is characterized by the simplified transmission function $v_q(r, \varphi) = \exp[i(q\varphi + \alpha)]$, where q is the topological charge, φ is the azimuth angle of the polar coordinates, and the parameter α allows for a rotation of the axes of the system. When q takes an either positive or negative integer value, the phase rotates through an integral number of 2π radians. However for the fractional cases, there will be a phase discontinuity whose direction can be adjusted with the parameter α .

The Fourier transform of the vortex pattern is represented by $V_q(\rho, \Phi)$. For integer values, these are characterized by circular doughnut shaped intensity patterns whose diameter increases with the integer vortex charge. However the intensity pattern is more complicated for fractional vortex beams.

Then consider fractional topological charge q where $N \leq q \leq N + 1$ and N is an integer. These optical vortices with fractional topological charge q can be expanded in a Fourier series as [66, 67]:

$$v_q(r, \varphi) = \exp[i(q\varphi + \alpha)] = \sum_{n=-\infty}^{+\infty} c_n \exp(in\varphi) \exp(i\alpha). \quad 5-1$$

Here q stands for the fractional charge and n adopts integer values. Thus, the vortex beam with fractional topological charge can be viewed as the superposition of a series of vortex beams with integer charges, each one weighted with the Fourier coefficient c_n .

The Fourier coefficients c_n can be calculated as

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{iq\varphi} e^{-in\varphi} d\varphi = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(q-n)\varphi} d\varphi = \frac{\sin[\pi(q-n)]}{\pi[q-n]}. \quad 5-2$$

Therefore, the intensity of each integer charge vortex beam composing the fractional charge vortex beam is given by:

$$I_n = |c_n|^2 = \left| \frac{\sin(\pi(q-n))}{\pi(q-n)} \right|^2 = \text{sinc}^2(q-n). \quad 5-3$$

Figure 5-1(a) shows a plot of the intensity of the $n = 0$ component versus q for the case where $-1 \leq q \leq 1$. Figure 5-1(b) compares the intensities for $n = 0$ and $n = 1$ components in the region $0 \leq q \leq 1$. These curves show that these two components dominate the Fourier series and roughly resemble sine and cosine functions in qualitative agreement with the assumption made in Ref. [82]. The summation of the energies in these two vortices is shown in Figure 5-1(c). Please note that the combination of a charge $n = 0$ vortex and a charge $n = 1$ vortex constitutes over 80% of the intensity of any fractional vortex with topological charge between 0 and 1. The difference with respect to 100% is the residual intensity going to the other orders.

Figure 5-1. Intensity profiles of (a) intensity of the $n = 0$ component for $-1 \leq q \leq 1$, (b) intensities for $n = 0$ and $n = 1$ components with $0 \leq q \leq 1$, and (c) summation of $n = 0$ and $n = 1$ components.

Figure 5-2 shows the Fourier decomposition for different fractional vortex beams. Figure 5-2(a)-(c) show the intensity of each integer charge vortex beam composing the fractional charge vortex beam (Eq. (5-3)) as the fractional value q

increases from 0.25 to 0.5, to 0.75, showing how the energy shifts from the $n = 0$ vortex to the $n = 1$ vortex. There is some energy in the other orders. Then as the fractional value q increases from 1.25 to 1.5 to 1.75 in Figure 5-2(d)-(f), the energy is mainly distributed between $n = 1$ and $n = 2$ vortices. This trend continues for all fractional vortex values. This result indicates that any fractional vortex can be approximated as a linear combination of the two nearest integer vortices. This is a major conclusion of this work.

Figure 5-2. Fourier Decomposition For Different Fractional Vortex Beams

5-2-2 Representation of arbitrary optical vortex with two integer components

This expansion can be used to examine the intensity profile for these fractional beams. For simplicity, assume that the phase spiral mask that generates the fractional vortex can be written as the sum of the two strongest adjacent integer terms as

$$v_q(r, \varphi) = \exp(iq\varphi) \approx c_N(q) \exp(iN\varphi) + c_{N+1}(q) \exp[i(N+1)\varphi] \quad 5-4$$

with $N \leq q \leq N+1$.

Now the Fourier transform can be written as the superposition of the two vortex beams as

$$V_q(\rho, \Phi) = c_N(q) V_N(\rho, \Phi) + c_{N+1}(q) V_{N+1}(\rho, \Phi) \quad 5-5$$

where (ρ, Φ) denotes the spatial frequencies expressed in polar coordinates.

The case when $0 \leq q \leq 1$ is particularly simple. The intensity consists of three terms as

$$\begin{aligned} |V_q(\rho, \Phi)|^2 &= |c_0(q) V_0(\rho, \Phi)|^2 + |c_1(q) V_1(\rho, \Phi)|^2 \\ &+ (c_0^*(q) c_1(q) V_0^*(\rho, \Phi) V_1(\rho, \Phi) + c_0(q) c_1^*(q) V_0(\rho, \Phi) V_1^*(\rho, \Phi)) \end{aligned} \quad 5-6$$

where $*$ denotes the complex conjugated function.

The first term $V_0(\rho, \Phi)$ represents the zero order vortex beam. It is a real valued function which forms a circularly symmetric sharply focused peak. It corresponds to the Fourier transform of the amplitude $A(r)$ described at the introduction. If $A(r)$ is constant, it becomes a sharply focused function centered at the origin. In our case, it can be viewed as Fourier transform of the vortex mask aperture. The second and third terms are characterized by zeros at the origin since $V_1(\rho=0)=0$. The second term is circularly symmetric while the third

interference term reflects the phase of the first order vortex beam since the zero order vortex beam $V_0(\rho, \Phi)$ is real valued. Note that this cosine term depends on the orientation angle α of the coordinate system. Figure 3 shows computer simulations as a function of the fraction q . The phase distribution and Fourier transform of input vortices with charges ranging from $q=0$ to $q=+1$ are depicted in Figure 5-3(a)-(b). The patterns at low charge numbers are closer to the pattern for a charge 0. As the charge increases, the pattern appears more like the charge 1 vortex. Note that the intensity at the origin $|V_q(\rho=0)|^2 = I_0(q)|V_0(\rho=0)|^2$ changes with the charge directly proportional to $I_0(q)$, i.e., it can be used as a direct measure of the fractional charge. The sign of the charge affects only the direction of the intensity distribution. As shown in Figure 5-3(c), by rotating the orientation angle α of the SPP, the pattern of the fractional vortex can be rotated accordingly. This confirms the original hypothesis in Ref. [82]. It is this feature that was useful for anisotropic image processing [13, 66, 82].

Figure 5-3. Phase distribution and Fourier transform of input vortices with different fractional topological charges and rotation angles.

It is difficult to experimentally identify the contribution of each individual integer vortex. However this can be accomplished if a characteristic point is defined to be located at the center of the pattern because the only contribution is from the $n = 0$ term. The rest of the vortex components ($n \neq 0$) do not contribute since they all present a null intensity at the center. A numerical simulation is performed with the fractional vortices of charge $q = -1$ to $q = +1$, as already shown in Figure 5-1(a). When the intensity values at the center pixels are extracted from the simulations in Figure 5-3(a) and plotted against the input charge, the resultant curve is exactly the same as the curve of I_0 vs. q in Figure 5-1(a).

Next section is the discussion of the principle of this decomposition.

5-3 Principle of vortex detecting grating

Spatially separated diffraction orders used for detection of optical vortices of integer topological charges.

To experimentally confirm the predictions of the above analysis, a previously reported vortex grating spectrum analyzer will be used [76], which consists of an ordinary vortex producing diffraction grating. This is a modified CGH with a “fork” bifurcation at the center that has been widely used for generating vortex beams [6].

The vortex grating is represented by a Fourier series expansion as

$$g(x, y) = \sum_{m=-\infty}^{+\infty} g_m \exp(im\ell\varphi) \exp(im\gamma x). \quad 5-7$$

Here $\gamma = 2\pi/p$, where p is the period of the grating; the coefficients g_m are given by $g_m = [\sin(m\pi w/p)]/(m\pi)$, where w/p is the fill factor of the grating [76].

The diffraction pattern generated with such a grating under collimated plane wave illumination is expressed as:

$$G(\rho, \Phi) = \sum_{m=-\infty}^{+\infty} g_m V_{m\ell}(\rho, \Phi) \otimes \delta(p - m\gamma). \quad 5-8$$

Here $\otimes$ denotes the convolution operation. This shows that a vortex beam $V_{m\ell}(\rho, \Phi)$ is generated and centered at the location of the m diffraction order.

It is now assumed that the grating is illuminated with a fractional (q) vortex beam. Its transmission through the vortex grating is given by the product $\exp(iq\phi)g(x, y)$, and the corresponding diffraction pattern $W(\rho, \Phi)$ is given by the convolution $V_q(\rho, \Phi) \otimes G(\rho, \Phi)$. This can be expressed as

$$\begin{aligned} W(\rho, \Phi) &= \sum_{m=-\infty}^{+\infty} [g_m V_q(\rho, \Phi) \otimes V_{m\ell}(\rho, \Phi)] \otimes \delta(p - m\gamma) \\ &= \sum_{m=-\infty}^{+\infty} E_m(\rho, \Phi) \otimes \delta(p - m\gamma) \end{aligned} \quad 5-9$$

which shows that now the m diffraction order is characterized by a function $E_m(\rho, \Phi) = g_m V_q(\rho, \Phi) \otimes V_{m\ell}(\rho, \Phi)$.

Now consider that the incident fractional vortex beam can be represented as the sum of the two dominant terms as in Eq. (5-4). To make the analysis easier and to agree with the experimental situation, $\ell = 1$ is selected. Then, the product $\exp(iq\phi)g(x, y)$ can be approximated as

$$\begin{aligned} \exp(iq\phi)g(x, y) &\cong \left\{ \left(c_N(q) \exp(iN\phi) + c_{N+1}(q) \exp[i(N+1)\phi] \right) \right\} \times \\ &\quad \times \left\{ \sum_{m=-\infty}^{+\infty} g_m \exp(im\phi) \exp(im\gamma x) \right\} \end{aligned} \quad 5-10$$

One can note that the phases of the two main components of the incident vortex beam will be canceled by the $m = -N$ and $m = -(N+1)$ phase terms of the grating. Next these two terms of the grating are concentrated on to determine the interesting result that

$$\begin{aligned} \exp(iq\varphi)g(x, y) \cong & g_{m=-N} \{c_N(q) + c_{N+1}(q)\exp[-i\varphi]\} \exp(-iN\gamma x) \\ & + g_{m=-N-1} \{c_N(q)\exp[-i\varphi] + c_{N+1}(q)\} \exp(-i(N+1)\gamma x) \end{aligned} \quad 5-11$$

If the analysis is restricted to these two diffracted orders ($m = -N$ and $m = -(N+1)$), the diffraction pattern in Eq. (5-9) can be approximated as

$$W(\rho, \Phi) \cong E_{-N}(\rho, \Phi) \otimes \delta(p + N\gamma) + E_{-N-1}(\rho, \Phi) \otimes \delta[p + (N+1)\gamma] \quad 5-12$$

where the order located at $p = -N\gamma$ is characterized by the function

$$E_{-N}(\rho, \Phi) \cong g_{-N} [c_N(q)V_0(\rho, \Phi) + c_{N+1}(q)V_{-1}(\rho, \Phi)] \quad 5-13$$

while the order located at $p = -(N+1)\gamma$ is characterized by

$$E_{-N-1}(\rho, \Phi) \cong g_{-N-1} [c_N(q)V_{-1}(\rho, \Phi) + c_{N+1}(q)V_0(\rho, \Phi)] \quad 5-14$$

So one can see that these two diffracted orders are both characterized by the sum of two electric fields equivalent to the case considered earlier in Eq. (5-5).

To examine the results more easily, the location $p = m\gamma = -N\gamma$ is concentrated on (Eq. (5-13)). It is identical to Eq. (5-5) with two differences. First, the entire electric field strength is governed by the strength of the Fourier coefficient g_{-N} of the grating. Secondly, the component $c_N(q)$ is now multiplying the Fourier transform for a vortex corresponding to $N = 0$ while the component $c_{N+1}(q)$ controls the strength of a $N = 1$ vortex. This situation occurs for any value of N .

As before in Eq. (5-6), the intensity at this diffracted order consists of three terms as

$$\begin{aligned} |E_{-N}(\rho, \Phi)|^2 = & |g_{m=-N}|^2 \times \left\{ |c_N(q)V_0(\rho, \Phi)|^2 + |c_{N+1}(q)V_{-1}(\rho, \Phi)|^2 + \right. \\ & \left. + c_N^*(q)c_{N+1}(q)V_0^*(\rho, \Phi)V_{-1}(\rho, \Phi) + c_N(q)c_{N+1}^*(q)V_{-1}^*(\rho, \Phi)V_0(\rho, \Phi) \right\} \end{aligned} \quad 5-15$$

Eq. (5-15) is very similar to Eq. (5-6) and again shows three terms. The first yields a focused beam whose strength depends on the coefficient $c_N(q)$. The second term corresponds to a first order vortex beam regardless of the value of N . However this beam has no strength at the origin. As before, the third term shows an interference pattern, but again has no intensity at the origin. Consequently the intensity at the origin of this diffracted spot measures the strength of the coefficient $c_N(q)$, multiplied by the strength of the order from the grating. The diffracted peak at $p = m\gamma = -(N + 1)\gamma$ gives a similar result. The intensity at the origin of this diffracted spot measures the intensity of the $c_{N+1}(q)$ coefficient, again multiplied by the strength of the grating.

Please note that the simplifications in the Fourier series for the vortex beams do not affect this result. Each of the other terms in the Fourier series of Eq. (5-1) will yield a higher order vortex beam at the focused spot and will have no intensity at the origin.

5-4 Detection and analysis of arbitrary fractional vortices

5-4-1 Experiment setup and vortex spectrum analyzer

In the experiment, a 532 nm expanded and collimated laser beam generated from a Verdi V5 laser system is used. The beam then illuminates a Hamamatsu X8267-11 reflective type PAL-LCSLM. The SLM operates in pure phase modulation mode and acts as a secondary display controlled by a computer. When a grayscale image is displayed, the SLM will produce the same pattern and translate the gray level of the input image into corresponding phase values. Therefore, a grayscale spiral pattern is encoded onto the SLM to form a SPP for generating the input fractional vortex. For an 8-bit image displayed on the SLM, value 0 (darkest) denotes no phase change, where value 256 (brightest) denotes 2.4π phase change.

The gray level value of 200 indicates a 2π phase change, according to the characteristic curve of the SLM.

Figure 5-4. Experimental setup of the fractional vortex analyzing system.

The grating spectrum analyzer used is a charge $\ell = 1$ vortex CGH imprinted on a photographic film as described in Section 5-3. The period of the grating is designed to be $250\mu\text{m}$, and according to Ref. [76], the fill factor w/p of the grating is selected to be $1/4$, which allows the grating to be able to display up to ± 3 diffraction orders. The embedded topological charge of CGH is controlled only by the characteristics of the bifurcation and is insensitive to other properties such as the grating's period or the wavelength used, thus make the CGH a sound tool for analyzing the input vortex beam. The grating is aligned on the reflected optical path of the SLM and the spacing in-between is not critical, since the input beam is collimated. After that, a lens of focal length 1000mm is used to perform the Fourier transform and form the resultant image. The focused spots are relatively small, so a magnifying lens system is used to enlarge the image three times for a CCD camera to acquire. The schematic drawing of the experiment setup is depicted in Figure 5-4.

5-4-2 Extraction of central intensity value for determining the composition of one integer charge

First, the input optical vortices with topological charges between -1 and +1 are analysed. Figure 5-5(a) shows the +1, 0, and -1 diffracted orders in the focal plane of the lens when the grating is illuminated with various input vortex beams with charges -1 to +1. It is obvious that when the topological charge of the input beam is +1 or -1, the diffraction order -1 or +1 becomes a focused bright spot. To analyse the fractional input charges, the central intensity of the diffracted order $m=0$ is extracted. Once a series of values is obtained, the curve is plotted and then compared with the simulated value. As shown in Figure 5-5(b), the experimentally obtained curve conforms well to Eq. (5-15), which justifies the proposed model. This data indicates that the charge of an incident fractional vortex beam can be identified within a 10% error.

Figure 5-5. (a) Diffraction patterns on the focal plane on the +1, 0, and -1 orders with input vortices range from $-1 \leq q \leq 1$. (b) Intensity of the central pixel at diffraction order $m=0$.

Next higher order fractional vortices are analyzed. Figure 5-6(a) shows the +1, +2, and +3 diffracted orders from the grating as the input beam charge varies from -1 to -3. As expected, when the input beam has integer charges of -1, -2, and -3, the corresponding diffraction orders at +1, +2, and +3 become delta functions. For the fractional charges between the integer values, it is easy to characterize them by looking at the central intensity value of its two nearest neighbors. So the input charge between -1 and -2 can be characterized by the vortex patterns at diffraction order 1 and 2, and for input charge between -2 and -3, it can be characterized by order 2 and 3. So when the center intensity at order 2 is extracted for a series of input charge range from -1 to -3, the normalized curve shown in Figure 5-6(b) conforms well to the calculated curve. If the input charge value is unknown, it can be first located on the diffraction orders by looking for the smallest spots, and then extract the central intensity to identify the input vortex. Again, this data indicates that the charge of an incident fractional vortex beam can be identified within a 10% error.

Figure 5-6. (a) Diffraction patterns on the focal plane on the +1, +2, and +3 orders with input vortices range from $-3 \leq q \leq -1$. (b) Intensity of the central pixel at diffraction order $m=+2$.

5-5 Chapter summary

In conclusion, optical vortices with arbitrary fractional topological charges can be written as a Fourier series of integer vortex beams. This series can be approximated by the two neighboring integer vortices. Then a vortex grating spectrum analyzer is implemented to prove the proposal. By extracting the intensity value at the central characteristic region of the patterns at desired diffraction orders, the value and sign of the topological charge of arbitrary input

optical vortex can be determined. Since the spectrum analyzer only requires an ordinary vortex producing grating and a proper imaging system, it can be easily incorporated in any optical system with optical vortices which requires the identification and analysis of the topological charge.

CHAPTER VI

ANALYSIS OF QUANTIZED STEP SPP

Optical vortices can be easily generated using a multilevel SPP. However the quality of the generated vortex beam depends on the number of phase segments. The theory for this multilevel SPP is first reviewed using a Fourier expansion of integer topological charged vortices. Then the validity of this expansion is examined using a fabricated Dammann vortex diffraction grating spectrum analyzer. The Dammann vortex diffraction grating is fabricated using SU-8 photoresist on glass substrate and yields uniform diffraction efficiency across a desired number of diffracted orders. Experimental results show the extra angular harmonics of a multilevel SPP and agree with the theory.

6-1 Study of quantized form of SPP and applications

Optical vortices, i.e. laser beams having a helical wavefront with a phase term $\exp(iL\phi)$ (here L is used to replace ℓ to represent the topological charge), have been extensively studied owing to their various applications. The ideal SPP has a continuous surface thickness transition topology that imposes the desired azimuthal phase. However, due to the limitations of fabrication techniques, usually the SPP takes a multilevel quantized form where the spiral is divided into a number M of segments, each having a relative quantized phase level of $2\pi L / M$, to approximate the continuous transition of phase values. This kind of multilevel SPP can be fabricated using various methods, including direct electron-beam writing on negative photoresists [52] and multi-stage vapor deposition process [70].

However the quantized phase levels degrade the mode purity of the optical vortices. Guo et al. [72] examined the vortex properties of an optical beam transmitted by a multilevel SPP and obtained expressions for the intensities of the vortex components as a function of the size of the phase step between adjacent

spiral segments. Their results show that the generation of the desired vortex beam improves as the number of levels increases. Kotlyar et al. [73] examined the same problem. They approximated the diffraction efficiency of the quantized SPP using quantized polygon regions and derived somewhat similar results using Fraunhofer diffraction theory. However, neither group provided experimental results showing the validity of the theory nor the characteristics of the output topological charge composition of the quantized SPP.

There are three objectives in this work. First in Section 6-2, the topological charge composition and the efficiency of the charge components of the output from a quantized SPP are analyzed using the Fourier series approach of Ref. [72]. For convenience, charge $L=1$ is concentrated on. One finding is that the SPP primarily produces the desired vortex component and that the efficiency increases as the number of steps increases. For a smaller number of steps, the remaining energy is predicted to occur in neighboring vortex charge components. To demonstrate the accuracy of this model, the vortex content of the beam produced by these SPP designs is studied using a Dammann vortex grating spectrum analyzer approach that is well documented [76, 87, 88] and will be discussed in Section 6-3-1. In Section 6-3-2, the experimental detection of the charge composition of a quantized SPP using the Dammann vortex grating is presented.

6-2 Fourier expansion, spectrum analysis and efficiency of stepped SPP

6-2-1 Theoretical representation of quantized SPP into composition of different charges with different coefficients.

The simplified transmittance function of a conventional phase only SPP can be represented in polar coordinates as $u(r, \varphi) = \exp(iL\varphi)$, where L stands for the topological charge which can take any integer value. Note that the total angular

phase is given by $2\pi L$. It is assumed that the total number of phase steps in the multi-level SPP is given by M and that each step has a phase height of $\Delta\Phi = 2\pi L / M$. Consequently each region covering a phase of 2π has a total of M / L steps. This stepped or quantized SPP function can be described mathematically as the function $u_s(r, \varphi)$ obtained from $u(r, \varphi)$ as:

$$u_s(r, \varphi) = \exp \left[i\Delta\Phi \cdot \text{floor} \left(\frac{M\varphi}{2\pi} \right) \right], \quad 6-1$$

where $\text{floor}(x)$ is the function giving the largest integer not greater than x .

As this transmittance function can be considered as a periodic function of variable φ with period 2π , it is possible to expand the function into a Fourier series as [11]:

$$u_s(r, \varphi) = \sum_{m=-\infty}^{+\infty} C_m \exp(im\varphi). \quad 6-2$$

Here m is the topological charge component that takes integer values and the Fourier coefficient of each charge C_m is calculated as

$$C_m = \frac{1}{2\pi} \int_0^{2\pi} u_s(r, \varphi) \exp(-im\varphi) d\varphi. \quad 6-3$$

Equation (6-2) shows that the beam generated with the stepped SPP mask can be regarded as the superposition of perfect vortex beams $\exp(im\varphi)$ with integer charges m , each one with a weight C_m . Following the treatment in Ref. [72], the relative intensity of each component with topological charge of m is given by

$$I_m = |C_m|^2 = \begin{cases} \text{sinc}^2 \left(\frac{m}{M} \right), & \frac{m-L}{M} = 0, \pm 1, \pm 2, \dots \\ 0, & \text{otherwise} \end{cases}. \quad 6-4$$

6-2-2 Calculate the Fourier spectrum and the efficiency allocated to the desired order of different number of steps used.

The case where $L = 1$ and the SPP is quantized into M steps as shown in Figure 6-1(a-d) is first examined. Figure 6-2 shows the harmonic content intensity distribution for various values of M and several characteristics are easily discovered. First, it is observed that the intensity of the desired $m = L = 1$ harmonic increases as $\text{sinc}^2(1/M)$ as the number of steps increases. In addition, there are a number of orders where the intensity is zero. For example when the SPP is quantized into 2 steps, i.e. $M = 2$, the intensity has non-zero values at $m = +1, -1, +3, -3, \dots$ and the intensity of the desired component $m = 1$ is 40%. When $M = 3$, the intensity has non-zero values at $m = +1, -2, +4, -5, \dots$ and the intensity of the desired component is now 68%. Finally, Figure 6-3 shows how the efficiency of the $m = 1$ component increases as the number of steps increases, reaching 95% when $M = 8$ and 99% when $M = 16$.

Figure 6-1. (a) - (d): Sketches of quantized SPPs with topological charge $L=1$ and step number from $M=2$ to $M=5$, the grayscale represents 0 (black) to 2π (white) phase values. (e)-(h): the respective optical vortices generated experimentally by the quantized SPPs.

Figure 6-2. The intensity spectrum of topological charge components of an $L=1$ quantized SPP with step number (a) $M=2$; (b) $M=3$; (c) $M=4$; (d) $M=5$.

Figure 6-3. The intensity of topological charge 1 component $I_{m=1, L=1}$ versus number of steps M of a multi-level SPP.

In fact, these intensity distributions are exactly the same as those in the diffraction efficiency of a one-dimensional linear binary step grating [89, 90]. It is obvious that the important quantity regarding the efficiency of any multilevel SPP

is the number of phase steps per 2π section. This is depicted in Figure 6-4, where the Fourier spectra are shown for SPPs with $L=1$, $L=2$, and $L=3$ at a constant M/L ratio of 3. The designed vortex components with topological charges 1, 2 and 3 respectively all have the same efficiency of 68%. When this number becomes $M/L=8$, then that optical vortex will be generated with 95% efficiency in agreement with the predictions of both references [72, 73]. In fact, Figure 1 of Ref [72] agrees exactly with Figure 5 of Ref [90] as well as Figure 6-2 here. In next section an experimental technique for detecting this harmonic vortex intensity spectrum will be discussed.

Figure 6-4. The Fourier spectra of SPPs with (a) $M=3$, $L=1$; (b) $M=6$, $L=2$; and (c) $M=9$, $L=3$. The diffraction efficiencies are the same when the M/L value is constant ($M/L=3$).

6-3 Experimental verification of the composition of a stepped SPP

6-3-1 Dammann vortex grating spectrum analyzer

The Vortex Grating Spectrum Analyzer [76, 87, 88] is an excellent probe of the values and signs of the topological charge of an arbitrary input optical vortex beam. Consequently it is an excellent probe of the harmonic content of the beam transmitted by the segmented multilevel SPP. In this approach, the incident vortex beam produced by the SPP is transmitted through a specially designed binary phase vortex sensing diffraction grating. This binary phase grating combines a spiral phase pattern for a given vortex charge with a linear grating as

$g(x) = \exp[i(q\varphi + \gamma x)]$. Here q is the topological charge of the spiral phase and $\gamma = 2\pi/d$, where d is the period of the grating. The binarized version of this grating can be written as

$$g(x) = \sum_{n=-\infty}^{+\infty} c_n \exp[i(nq\varphi + n\gamma x)], \quad 6-5$$

and this produces a series of diffracted vortex beams having a charge of nq where n is the diffracted order.

Now the diffraction pattern from the product of the incident beam (from Eq. (6-2)) and the grating (from Eq. (6-5)) forms a delta function when the input beam has the vortex sign opposite to the vortex sign of the diffracted order (i.e. when $nq = -ML$).

In order to increase the diffraction efficiency of the vortex grating, a Dammann grating structure [91, 92] is encoded to produce equally distributed energy across the useful diffraction orders, allowing easy comparison of the efficiency values. These structures are formed by encoding phase values of 0 or π radians at selected transition points within each period. These transition points are chosen to maximize the total diffracted efficiency for a given number of orders and to provide equal energy in each diffracted order.

Nine diffracted orders are chosen to be generated using transition points listed in Ref. [93] over a half period of $x_1=0.099750$, $x_2=0.159152$, $x_3=0.369009$, and $x_4=0.491743$, for displaying the 9 diffraction orders from -4 to +4. Figure 6-5(a) shows the standard Dammann grating. Using the approach of Ref. [94], the spiral phase grating is encoded which combining the diffraction grating and the spiral phase. The resulting Dammann vortex grating is shown in Figure 6-5(b).

Figure 6-5. (a) 1-D Dammann grating and (b) Dammann vortex grating with embedded topological charge 1, with phase values shown as 0 (black) and π (gray). (c) and (d) are their diffraction patterns respectively.

6-3-2 Experimental results

In the experiment, a Quantum Ventus 532 laser system is used to generate an expanded and collimated laser beam having wavelength of 532nm . The desired incident vortex beam is formed by passing the laser beam through a Hamamatsu X8267-11 reflection type PAL-LCSLM which operates in a pure phase modulation mode and acts as a secondary display controlled by a computer. The dimension of the SLM is 20mm by 20mm and the pixel size is $26\mu\text{m}$. Therefore, the desired multilevel SPP phase functions can be encoded onto the SLM. The vortex patterns are shown in Figure 6-1(a-d) while the optical vortices generated by these fabricated SPPs are shown in Figure 6-1(e-h) for step numbers of 2, 3, 4, and 5.

The generated optical vortex beam is then passed through the fabricated Dammann vortex sensing grating as discussed earlier. In order to encode the π phase step, UV lithography technique is used to fabricate SU-8 photoresist medium coated on glass slide, which gives us a transmissive grating. The refractive index of the SU-8 photoresist is 1.59 at $532nm$, and the respective height for π phase difference is calculated to be $h=0.5\lambda/\Delta n=450.85nm$. To obtain this film thickness, the diluted SU-8 2005 photoresist was spin-coated onto a glass slide at a spin speed of $1500rpm$, and this produced an SU-8 coating with thickness around $450nm$. After a pre-exposure bake of 60s at $95^{\circ}C$, the photoresist was exposed using a Karl Suiss MJB3 Photomask Aligner, using a UV wavelength of $406nm$. The dose was $14mW/cm^2$, with an exposure time of 30s. After the post-exposure back of 60s at $95^{\circ}C$, the prepared phase mask pattern is successfully transferred to the photoresist. The surface thickness of the fabricated SPP is measured using a Deteck3 Surface Profiler, and the typical height value is $455nm$.

The transmitted beam is then focused by a $30mm$ focal length converging lens to the CCD camera. Figure 6-5(c) shows the diffraction pattern formed by the conventional 1-D Dammann grating of Figure 6-5(a) with a uniform beam incident onto it. The peak grayscale intensity values of the nine diffracted spots are 220, 248, 253, 254, 254, 252, 253, 252, 231, almost equal. Figure 6-5(d) shows the output using the Dammann vortex grating of Figure 6-5(b). Each order forms a vortex beam having a charge of nq where n is the diffracted order and q is the initial charge of the vortex beam used to generate the grating ($q=1$ in this case). All these vortex beams show a null intensity at the center of the corresponding diffraction order.

When the input vortex from a quantized SPP is incident on the grating, the detection pattern will occur at the diffraction order having opposite charge number, as explained in Ref. [76]. For example when $M/L=2$, the components of the input vortex beam are at $m=+1,-1,+3,-3,\dots$ from Figure 6-2. So the product of the incident beam from the multilevel SPP and the Dammann vortex grating will

yield delta function outputs at the diffracted orders corresponding to $n = -1, +1, -3, +3, \dots$. This is shown in Figure 6-6, first row, where now these orders show a spot on the center of the diffraction order, whose intensity is proportional to $|C_n|^2$ (in this case it is noticeable the two bright spots located at the $n = \pm 1$ orders). Those harmonic components that are absent are noticed as diffraction orders that keep the null intensity at the center ($n = \pm 2, \pm 4, \dots$ in this case). Similar results are shown for cases where $M/L = 3, 4, 5$ and 100. In each case, the strength of the delta function increases as M/L increases while the intensities of the other orders decrease and agree with all theoretical predictions.

Figure 6-6. Experimental detection results using the Dammann vortex sensing grating, the input vortex beam has topological charge $L=1$ and the number of steps are listed on the right, the diffracted order numbers are shown at the bottom. The arrows indicate the vortex components detected.

After obtaining the experimental results, the intensity values of the resultant delta functions are measured for comparison with theory. When a continuous SPP is used, the maximum intensity of the $n = -1$ diffraction order recorded at the CCD camera is 198, according to the 8-bit grayscale value. This value is used as a benchmark to examine the efficiency of the topological charge components. When the number of steps increases from 2 to 5, the recorded intensity values are 84, 135, 162, and 168, which translate to intensity values of 42.4%, 68.2%, 81.8% and 84.8%. Comparing these values with the calculated value using Eq. (6-5), the errors are below 5%, which supports the validity of the detection technique.

6-4 Chapter summary

In this work, the capability of a quantized SPP to generate an optical vortex beam is examined. The SPP is marked by an azimuthal phase shift of $2\pi L$ where L is the desired vortex beam charge. The transmission function of a quantized SPP is first expanded using a Fourier series approach, and then the Fourier spectra and diffraction efficiencies of the generated vortex beams are calculated. One finding is that the important quantity is the number of segments per 2π phase level with the requirement that 16 steps are required for 99% generation of the desired vortex beam.

To experimentally identify the vortex components, a vortex sensing Dammann grating approach is utilized to clearly reveal the vortex pattern of the incident beam. The Dammann vortex grating greatly increased the energies distributed into higher diffraction orders and clearly verifies the theoretical predictions for a vortex beam having a topological charge of $L = 1$. Experimental results agree with the theoretical model.

CHAPTER VII

STATIC DAMMANN VORTEX GRATING FOR OPTICAL VORTEX ANALYSIS

7-1 Upgrade conventional vortex grating with Dammann function

The Dammann vortex spectrum analyser discussed in the previous chapter is an elegant example of the applications of Dammann vortex gratings. While vortex beams have been implemented intensively in many fields, the requirement of proper and accurate identification and analysis also increases. Specifically, in free-space optical vortex communication systems, as the OV may be multiplexed when transmitted[19, 95], the detection of OAM at the receiver end is essential.

7-1-1 Detection capability and downsides of vortex sensing gratings

Recently, vortex sensing grating was introduced to identify OV encoding with arbitrary topological charges [76], which used a 1-D vortex sensing binary phase grating displayed on an SLM to analyze the incoming vortex beam. Although the filling factor of the grating has been altered to display a few higher diffraction orders, the ability of this kind of grating is still limited by two factors. The first factor is the filling factor of the grating, for instance, using a filling factor of $1/4$, and normalizing the 0th order intensity to 1, then the ± 1 st, ± 2 nd and ± 3 rd diffraction orders will have relative intensity approximately of 0.80, 0.35 and 0.07 respectively, as shown in Figure 7-1(a) and (b). When vortex grating is built on this kind of grating, the higher diffraction orders become unobservable, as shown in Figure 7-1(c) and (d). Although the filling factor of the binary grating can be further reduced to show more diffraction orders, however, there exists the second

limiting factor, which is the resolution of the SLM. Reducing the filling factor requires smaller feature size, and for SLM the smallest feature size is the pixel size. A filling factor of $1/4$ needs at least four pixels for one grating period, a filling factor of $1/10$ will need 10 pixels, and so on. Increasing the grating period size will diminish the order separation ability, which is undesirable for the detection applications.

Figure 7-1. The phase structure (black for 0 and white for π) and simulated diffraction pattern of the following gratings (filling factor of $1/4$) are shown. (a) and (b): 1-D binary phase diffraction grating; (c) and (d): conventional vortex grating formed using the binary phase

grating; (e) and (f): 2-D binary grating; (g) and (h): 2-D binary vortex grating.

The topological charge detection has also been used in free-space optical communication [19]. When conventional vortex gratings are used, the reduced intensity of higher order problem is extended to a 2-D scenario. A 2-D binary grating and its diffraction patterns are shown in Figure 7-1(e) and (f). At the outermost four corners, the diffraction spots are still observable, the intensity of the spots are 0.25 comparing to 1 at the zero order. However, when a vortex grating is encoded, as shown in Figure 7-1(g) and (h), the diffraction patterns at the outermost corners are unobservable, and only nine patterns at the center are clear and useful. The nine diffraction orders at the center are only capable to detect 9 topological charge numbers range from -4 to +4, when topological charge 1 is encoded in the horizontal direction and charge 3 is encoded in the vertical direction.

7-1-2 Incorporation of Dammann function

To address the problem of low intensity at higher diffraction orders, the 2-D Dammann vortex grating is proposed, which retains the advantage of 2-D diffraction grating. The grating is fabricated in SU-8 photoresists by optical lithography technology.

Dammann gratings have been well known for uniformly distributing energy among the designed diffraction orders and have been applied in many fields[91, 92]. The applications include array generators[96], multiple imaging[97], and coherent summation of laser beams[98], and there have been some variation developments recently [99, 100]. This concept provides a resolution for the disadvantage of conventional binary grating, i.e. low efficiency at higher diffraction

orders. Therefore, the features of conventional vortex grating can be combined with a Dammann grating, to produce equal intensity OV patterns at the desired diffraction orders and facilitate easy comparison among the diffraction orders.

The phase topology of Dammann grating could be in binary form or continuous format. However, to simplify the design procedure, only the binary case is discussed here. As shown in the previous section, 2-D vortex gratings have larger charge capacity and are more suitable for detection purpose. Incorporating the two orthogonal grating design, the 2-D Dammann vortex grating will have an extended detection capacity, as all the designed diffraction orders become useful, e.g. 5x5 grating is able to detect 25 topological charges from -12 to +12.

The procedure to produce the phase function of the desired Dammann grating could be subdivided into three steps. The first step is to create x and y dimensional blazed spiral phase grating using the grating function $g(x) = \exp[i(p\phi + \gamma x)]$ and $g(y) = \exp[i(q\phi + \gamma y)]$, here p and q are the intrinsic topological charge number of the grating on x and y direction respectively, and $\gamma = 2\pi/d$, where d is the period of the blazed grating, so the spiral phase function is blazed from 0 to 2π phase value on the x and y axis.

The next step is to binarize each of the blazed spiral phase grating into 0 and π phase values using Dammann parameters, as discussed in one previous work [94]. Dammann parameters are the position value of the transition points where the phase changes from either 0 to π or vice versa. In order to obtain the parameters, one can start from a symmetric Dammann grating function at interval $0 \leq x \leq \frac{1}{2}$, which has the transmission function of

$$t(x) = \sum_{n=0}^N (-1)^n \text{rect} \left[\frac{x - (x_{n+1} + x_n)/2}{x_{n+1} - x_n} \right], \quad 7-1$$

here $\text{rect}(x) \equiv 1$ when $|x| \leq \frac{1}{2}$ and $\text{rect}(x) \equiv 0$ when $|x| > \frac{1}{2}$. The negative portion of the function at $-\frac{1}{2} \leq x < 0$ is obtained symmetrically.

By Fourier transformation, the amplitudes of the diffraction orders are characterized by the Fourier coefficients:

$$T_m = \begin{cases} 2 \sum_{n=0}^N (x_{n+1} - x_n), & m = 0 \\ \frac{1}{m\pi} \sum_{n=0}^N (-1)^n [\sin(2\pi m x_{n+1}) - \sin(2\pi m x_n)], & m \neq 0 \end{cases} \quad 7-2$$

The unique feature of the Dammann grating is equal distribution of energy among the designed orders, which means to equate the energy term $|T_m|^2$, thus the partial solution of the transition points x_n for the equations could be found. This procedure has been well discussed and the optimization of the results can be done using various techniques [93, 101, 102]. For the 5x5 grating, the parameters used are the N=2 values in Ref. [92], which gives $x_1=0.132$ $x_2=0.480$, and an efficiency $\eta=77.4\%$. The construction of binary phase Dammann vortex grating is depicted in Figure 7-2(a) to (c). Where the intrinsic topological charge of the x axis grating is 1 and the intrinsic charge of y axis grating is 5, so the 5x5 Dammann vortex grating formed contains charges from -12 to +12.

Figure 7-2. (a) 1-D Horizontal Dammann vortex mask pattern with intrinsic topological charge 1, and (b) 1-D vertical mask pattern with intrinsic charge 5 are used to construct the 2-D 5x5 Dammann vortex phase mask (c). The center portion of the fabricated phase mask is shown in (d).

7-2 Fabrication process

Besides the increased detection range of topological charges, another advantage of 2-D Dammann grating is, to have the same detection power, the minimum feature size required is significantly larger than the 1-D Dammann grating, and this will greatly reduce the fabrication complexity. For example, according to Ref. [93], using the 1-D Dammann grating to generate 9 diffraction orders from -4 to +4 requires $N=4$, the minimum feature size needed is $\Delta_{\min} = 8.3 \times 10^{-3}$; however, if 2-D grating concept is employed, only $N=1$ is needed in both x and y directions to have the same number of orders. Therefore, the minimum feature size is increased

to $\Delta_{\min} = 1.3 \times 10^{-1}$, which is 2 orders of magnitude larger and it is much easier to fabricate the grating.

The phase mask pattern is first produced using printed transparency due to the easy and fast fabrication process. Figure 7-2(d) shows the central bifurcation area of the phase mask. A one-inch phase mask pattern is designed to have a period size of 427 μm . The resolution of this technique is about 20 μm , which is satisfactory for the experiment purpose. Higher resolution phase masks could easily be fabricated using other lithography techniques.

After that, UV lithography technique is implemented to transfer the phase mask pattern to the photoresist, and the photoresist used is SU-8 2005 manufactured by Microchem Corp. The process is shown in Figure 7-3. For better adhesion of the photoresist, a glass slide is first spin-coated with a thin layer of Omnicoat™ at 3000rpm for 30s. The refractive index of SU-8 photoresist is 1.59 at 532nm, so the respective height for π phase difference is calculated to be $h=0.5\lambda/\Delta n=450.85\text{nm}$. According to the calculated value, then the diluted SU-8 2005 photoresist is spin-coated onto the glass slide at spin speed 1500 rpm for 60s, and this produced an SU-8 coating with thickness around 450nm. After pre-exposure bake at 95°C for 60s, the photoresist is then exposed using an Karl Suiss MJB3 Photomask Aligner, the UV wavelength is 406nm and the dose is 14mW/cm², with exposure time 30s. After the post-exposure bake at 95°C for 60s, the prepared phase mask pattern is successfully transferred to the photoresist which possesses the designed phase structure. The photoresist height of the fabricated SPP is measured using a Deteck3 Surface Profiler, and the typical height value recorded is 465nm. The microscopic image of the fabricated grating is shown in Figure 7-4.

Figure 7-3. The process flow of UV lithography for fabricating the static grating using SU-8 photoresist.

Figure 7-4. Microscopic images of the fabricated binary phase 5x5 Dammann vortex grating using SU-8. (a): the phase pattern of the central bifurcation area; (b): the typical phase pattern of outer areas.

7-3 Demonstration of the improved experimental detection capability

To examine the detection capability of the fabricated Dammann vortex grating, OV is first generated from a Quantum Ventus 532 laser system and an SLM. The wavelength of the expanded and collimated laser beam is 532nm, and then a

Hamamatsu X8267-11 reflect type PAL-LCSLM is used to modulate the beam to form the desired OV. The SLM has a dimension of 20mm by 20mm with pixel size of 26 μ m, and is tuned to operate in pure phase modulation mode. A computer is used as a control system so that the SLM is acting as a secondary display of the computer. The phase modulation mechanism is: when a grayscale image is displayed on the primary monitor, the SLM will have the same pattern as the image displayed but the gray level of the image will be translated into corresponding phase level of the liquid crystal cells in the SLM. In this way, arbitrary phase functions could be encoded onto the SLM using a gray scale image as input. Therefore, OV with various topological charge numbers can be easily generated using the SLM.

The fabricated 2-D Dammann vortex grating is then placed in the reflected optical path after the SLM to generate the diffracted patterns. After passing through the grating, the beam is then focused onto the CCD camera by a 30mm focal length converging lens. When there is no input vortex present, the diffraction pattern of the Dammann vortex grating is shown in Figure 7-5(a). When the input vortex generated by the SLM is incident on the grating, the detection pattern will occur at the diffraction order having opposite charge number, as explicitly explained in Ref. [15]. E.g. When the input charge $l=+3$, the detection order will be at -3; when $l=-8$, the detection will appear at +8 order.

Figure 7-5. OV detection results using the fabricated Dammann vortex grating. The topological charges of the input OV are (a) 0, (b) -2, (c) -7 and (d) +12, and the labels show the detection orders.

As the Dammann vortex grating used here generates a 5x5 detection matrix, which is capable to detect OV with topological charges from -12 to +12, so input charge numbers of -2, -7, and +12 are chosen to test the fabricated grating.

The detection results are shown in Figure 7-5. When there is no OV input, the Dammann vortex grating generates OV patterns from the input plane wave, with improved efficiency at higher orders. This can be compared with Figure 7-1(h). When the input beam is OV with topological charges -2, -7 and +12, the focused spot can be easily located at the diffraction orders +2, +7 and -12 respectively, which correspond to the input vortex. After obtaining the experimental results, the intensity value of the resultant delta functions are measured, which could be used to examine the experimental efficiency of the proposed Dammann grating. The

Thorlabs DC310C CCD camera used has 8-bit depth, and the output intensity values range from 0 to 255. From the captured images, the maximum intensity value of the detected are 255, 246 and 232, the intensity variation is less than 10%, which justifies the Dammann vortex grating's detection capability at higher orders.

7-4 Chapter summary

In conclusion, to overcome the disadvantage of conventional vortex gratings, a 2-D Dammann vortex grating is proposed and fabricated using SU-8 photoresist by UV lithography technique. Experimental result has verified that this grating is able to distribute energy uniformly among all the designed diffraction orders, and with this feature, higher diffraction orders become observable and useful, so the OV detection range is successfully extended. This type of grating can be designed to have more diffraction orders and can easily be fabricated using various lithography techniques and materials with improved accuracy. Dammann vortex gratings can be implemented to generate and fan-out OV with large topological charge capacity as well as to detect large volume of OV simultaneously.

CHAPTER VIII

CONCLUSION

8-1 Review of the capabilities of unconventional spiral phase elements

To facilitate a comprehensive summary, the thesis can be classified into two main branches as follows. Chronicle and development sequence are not followed here.

- 1) The design and implementation of unconventional spiral phase elements for spiral interference, image processing with radial Hilbert transform, and optical vortices identification.
- 2) The analysis of the vortex component in fractional optical vortices and multilevel SPPs.

Chapter 3, 4 and 7 are devoted to the first area and each chapter studied the respective application. In Chapter 3, a fabricated HSPP is designed and implemented in a microscopic spiral interferometry system. The HSPP provides an alternative tool for spiral interferometry and has the unique characteristics in terms of a self-referenced system with collinear interference. In the experiment, the phase of a polystyrene bead is successfully reconstructed using its single interferogram. The hollow structure and configuration simplifies the optical-path setup for interference. And the design concept is expected to be useful in other image processing applications.

Chapter 4 follows the spiral phase filtering technique and covers the optimization of edge enhancement results. A hybrid raised-cosine-modulation function is proposed to modify conventional spiral phase filter. Then the modified

filter has been closely investigated for optical image processing. Edge enhancement results show excellent optimization results. Comparing with conventional and other modulation functions, the raised-cosine-modulated technique shows pronounced advantages, which include noise reduction, contrast improvement and low energy loss. In the experiment the hybrid raised-cosine-modulated SPF uniformly enhanced the aperture edges as expected, and also suppressed the redundant noise exists in conventional and high-pass spiral phase filtered images. The optimized raised-cosine-modulated filter provides an enriching tool for optical processing. The proposed filter can also be incorporated with a conventional microscope system to fulfill versatile needs of microscopic optical image processing.

Chapter 7 introduces a 2-D Dammann vortex grating for extending optical vortices detection range. The Dammann design is able to distribute energy uniformly among all the designed diffraction orders. With this feature, higher diffraction orders become observable and useful, so the OV detection range is successfully extended comparing with ordinary vortex gratings. A 5x5 binary phase 2-D Dammann vortex grating is fabricated using SU-8 photoresist by UV lithography technique, and then verified experimentally. The detection results indicate that this type of grating with improved accuracy can be easily designed to have more diffraction orders and fabricated using various lithography methodologies.

Chapter 5 and 6 focus on the analysis of two specific kinds of spiral phase elements. In Chapter 5, the theory to decompose optical vortices with arbitrary fractional topological charges is first reviewed using Fourier series. From the decomposition, arbitrary fraction vortex beam can be approximated by its two neighboring integer vortices. To prove the proposal, a vortex grating spectrum analyzer is implemented for the experimental verification. The value and sign of the topological charge of arbitrary input optical vortex can be determined by extracting the intensity value at the central characteristic region of the patterns at desired diffraction orders. Since fractional optical vortices have wide application

range and the spectrum analyzer only requires an ordinary vortex producing grating and a proper imaging system, this method can be easily employed in many optical systems which require the identification and analysis of the optical vortices.

In Chapter 6, a quantized SPP has been investigated. Firstly the transmission function of a quantized SPP is expanded using a Fourier series approach, and then calculated the Fourier spectra and diffraction efficiencies of the vortex beams generated. One finding is that the important quantity is the number of segments per 2π phase level with the requirement that 16 steps are required for 99% generation of the desired vortex beam. A vortex sensing Dammann grating approach was utilized to experimentally identify the vortex components. The Dammann vortex grating can distribute energy equally across a large number of orders and clearly verifies the theoretical predictions for a vortex beam having a topological charge of $L = 1$.

In general, two kinds of unconventional spiral phase elements, namely HSPP and raised-cosine modulated spiral phase filter, have been proposed and each gives outstanding performance in image processing area. Then fractional vortices and multilevel SPPs are closely investigated and their properties have been experimentally analyzed. During these researches, Dammann vortex grating is developed as a Fourier spectrum analyzer, then it is further elaborated for application in optical vortex communication area.

8-2 Prospects for future research

- a) Unconventional spiral phase elements application in STED microscopy.

Stimulated Emission Depletion (STED) microscopy is a ready established super resolution microscopy method [103, 104]. And optical vortices have been used in STED system [10, 105]. And the state of art resolution achievable by STED method

is astonishing 5.8nm [106]. As the system is sensitive to the quality of the depletion beam, the use of unconventional spiral phase element can control the depletion beam and offer a more flexible tool for the STED system.

b) Optical vortex communication system with wavelength division multiplexing (WDM).

Current optical communication systems often use WDM as a dense multiplexing technique. As optical vortices with different topological charges carry different OAM, the beam can be multiplexed with another dimension of quality, i.e. the topological charge, or OAM states. The transmitter end could use collinear multiplexing or beam combining to form charge-multiplexed beam. And the Dammann vortex grating designed in Chapter 7 could be one kind of receiver. The charge domain does not interfere with WDM therefore offers another degree of freedom. The system may take the following two forms,

1) Free-space based

The free-space optical communication system has many advantages such as high bit rates, low bit rate error rates, immunity to electromagnetic interference, and high security. Prototype of such communication system has been reported by our research group and further development is ongoing [107]. The topological charges are first collinearly multiplexed using a designed CGH encoded on an SLM. By dynamically modulating the CGH, vortex beams with different charge composition are transmitted. At the receiver end, a Dammann vortex grating is used for the detection of the charge composition.

2) Multimode Fiber based

Fiber based optical communication system has advantages like low loss, no atmospheric interference, and mature industry support. There have been some studies on the vortices in fiber systems [108]. However, this area still needs more fundamental and breakthrough research.

PUBLICATIONS

Journal publications:

1. N. Zhang, X. C. Yuan, and R. E. Burge, "Extending the detection range of optical vortices by Dammann vortex gratings," *Opt. Lett.* **35**, 3495-3497 (2010).
2. N. Zhang, J. A. Davis, I. Moreno, D. M. Cottrell, and X. C. Yuan, "Analysis of multilevel spiral phase plates using a Dammann vortex sensing grating," *Opt. Express* **18**, 25987-25992 (2010).
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6. P. S. Tan, G. H. Yuan, Q. Wang, N. Zhang, D. H. Zhang, and X.-C. Yuan, "Phase singularity of surface plasmon polaritons generated by optical vortices," *Opt. Lett.* **36**, 3287-3289 (2011)

International conferences:

1. N. Zhang, X. C. Yuan, and Z. X. Wang, "High Capacity OAM Multiplexing and De-Multiplexing for Free-Space Optical Vortex Communication," (OSA-IEEE-COS Topical Meeting on Advances in Optoelectronics and Micro/nano-optics, Guangzhou, China, 2010).
2. N. Zhang, and X. Yuan, "Hybrid raised-cosine spiral phase filter for optimized optical image processing with edge enhancement," A. A. S. Awwal, K. M. Iftekharruddin, and S. C. Burkhardt, eds. (SPIE, San Diego, California, USA, 2010), pp. 779717-779717.

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APPENDICES

Optical property of SU-8 2005 photoresist

(Information obtained from www.microchem.com)

Figure 1. Cauchy Coefficients

Figure 2. Optical Transmittance

SU-8 2000 is a high contrast, epoxy based photoresist designed for micromachining and other microelectronic applications, where a thick, chemically and thermally stable image is desired. SU-8 2000 is an improved formulation of SU-8, which has been widely used by MEMS producers for many years. The use of a faster drying, more polar solvent system results in improved coating quality and increases process throughput. SU-8 2000 is available in twelve standard viscosities. Film thicknesses of 0.5 to >200 microns can be achieved with a single coat process. The exposed and subsequently thermally cross-linked portions of the film are rendered insoluble to liquid developers. SU-8 2000 has excellent imaging characteristics and is capable of producing very high aspect ratio structures. SU-8 2000 has very high optical transmission above 360 nm, which makes it ideally suited for imaging near vertical sidewalls in very thick films. SU-8 2000 is best suited for permanent applications where it is imaged, cured and left on the device.

Specifications of Hamamatsu X8267 SLM

(Information obtained from www.hamamatsu.com)

The PPM, Programmable phase modulator, is an electrically-addressed phase modulator using an optically-addressed PAL-SLM, Parallel Aligned Nematic Liquid Crystal Spatial Light Modulator, optically coupled through a lens to an electrically addressed intensity modulator.

Conventional optically-addressed modulators have difficulty in a computer control, while electrically-addressed modulators have drawbacks that unwanted diffraction light is generated due to the pixel structure in the electric signal input section and that the light utilization efficiency is low due to its small fill factor. The PPM uses an LCD to enable the computer control easily and also employs a newly developed optical system to eliminate the unwanted diffraction light originating from the LCD pixel structure. Moreover the PPM offers large phase modulation characteristics, making it possible to develop adaptive optics that uses optical diffraction and spatial phase control.

Parameter	X8267 SERIES	Unit
Features	<ul style="list-style-type: none">· Uses an XGA LCD with a large number of control points.· Has a built-in LD module.· Has no LCD pixel structure in the output image.	—
Input signal (IBM PC/AT (Windows®/DOS))	XGA	—
Number of control pixels	Approx. 590 000	pixels
Effective image area	20 × 20 (Four corners rounded)	mm
Phase modulation level (readout light wavelength: 633 nm)(Min.)	More than 2π	radian
Maximum display spatial resolution	19	Lp/mm
Input voltage (AC)	100 to 240 (50 Hz / 60 Hz)	V
Power consumption	45	VA
Weight	Approx. 1600	g

Figure 3. diffraction efficiency characteristics (633 nm readout standard device, binary phase lattice)

Figure 4. Diffraction ability at different number of pixels

Figure 5. Phase modulations vs. input signal level

Figure 6. Diffraction efficiency vs. spatial frequency