Longitudinal dynamics, optimization and control of aircraft transition maneuver using aerodynamic vectoring

Adnan Maqsood

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**LONGITUDINAL DYNAMICS, OPTIMIZATION AND
CONTROL OF AIRCRAFT TRANSITION MANEUVER
USING AERODYNAMIC VECTORING**

ADNAN MAQSOOD

**SCHOOL OF MECHANICAL AND AEROSPACE
ENGINEERING**

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LONGITUDINAL DYNAMICS, OPTIMIZATION AND CONTROL OF AIRCRAFT TRANSITION MANEUVER USING AERODYNAMIC VECTURING

ADNAN MAQSOOD

School of Mechanical & Aerospace Engineering

A thesis submitted to the Nanyang Technological University
in partial fulfillment of the requirement for the degree of
Doctor of Philosophy

2012

STATEMENT OF ORIGINALITY

I hereby certify that the work embodied in this thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

Date

Adnan Maqsood

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Summary

In order to enhance the flight envelope of Unmanned Air Vehicles (UAVs), there have been efforts to combine the excellent hover capabilities of rotorcraft with the endurance and speed performance of fixed-wing aircraft. Such efforts lead to a type of agile aircraft that can perform hover coupled with efficient flight during forward cruise. An inherent problem for such vehicles is the transition maneuver between forward flight and hover, which usually exhibits significant altitude variation, long transition time, large control effort, high thrust-to-weight ratio and loss of partial control. These characteristics are undesirable and reduce the maneuver potential of the vehicle in tight spaces. Moreover, the underlying flight dynamic characteristics of convertible platforms are still not fully explored.

To mitigate the problems associated with the transition maneuvers above, this work proposes the use of variable-incidence wing, where the angle of incidence of the wing with reference to the fuselage of the aircraft can be controlled during flight. This phenomenon is also referred as “aerodynamic vectoring” as the changing of the wing-incidence changes significantly the direction of the resultant aerodynamic force. A feasibility study of the usefulness of the variable-incidence wing scheme is conducted in comparison to a conventional fixed-wing platform. The study is focusing longitudinal

motion only. The lateral-directional motion is considered to be de-coupled and is out of the scope of the study.

Two kind of transition maneuvers are discussed in the thesis: Steady and Unsteady transition maneuvers. Steady transition maneuvers refer to the category in which the aircraft can sustain flight with certain pitch attitude between hover and cruise. In such transitions the aircraft can be considered to be at trim during the maneuver. Unsteady transition maneuvers refer to fast aerobatic/agile transition maneuvers between hover and cruise. In such maneuvers, the aircraft may not be in trimmed conditions between its initial and final desired states.

For the analysis, the aerodynamic forces and moments database is developed over the whole maneuver range using wind-tunnel-testing. For the steady transition case, the advantages of the variable-incidence wing feature are found, however eigenvalue analysis reveals that the dynamics may have some peculiarities. For this reason, further nonlinear dynamic analysis is carried out. Specifically, Multiple Time Scales (MTS) method in conjunction with bifurcation theory is used to uncover the peculiar system behavior and to understand the steady transition dynamics further.

For the unsteady transition maneuvers, a nonlinear constrained optimization problem is formulated for parametric analyses on the effects of the thrust, pitch angle, and wing incidence on the maneuvers. Both two and three degree-of-freedom nonlinear longitudinal dynamic models are

considered in the optimization study. The three-degree-of-freedom formulation gives further insight about the effects of elevator effectiveness, terminal velocity and unsteady aerodynamic phenomena on the transition maneuver. The stability of the optimal solutions is then analyzed using contraction theory. The analysis reveals that a closed-loop control is necessary for successful transitions.

Strategies on controlling the transition maneuvers are discussed in the last section. Results of the contraction analysis are used to devise a simple controller to achieve stability during the maneuver. Subsequently, an improved control approach using feedback linearization is carried out. The proposed aerodynamic vectoring feature avoids possible singularities in the control architecture and is shown to improve the maneuver performance. In the end, conclusions are laid down followed by the recommendations for future research.

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CHAPTER 1

Introduction

1.1 Background

The revolutionary powered flight at Kitty Hawk, North Carolina, on December 17, 1903 opened the new vistas of technology and innovation. The incremental research focus for the 20th century was primarily manned flight. The research focus was centered on achieving the trivial goals like high altitude, high maneuverability and high speed. Based on the research heritage from previous century pioneers, we have become successful to bring pilot out of the cockpit and enhance the versatility of flying titans. A peculiar feature of research during the last century was a high degree of tradeoff based on human performance limitations inside the cockpit. When the human factor requirements of a pilot and flight crew are no longer required in the onboard equation, substantial increase in the airplane performance can be achieved.

The design thrust for the 21st century can be characterized by the emergence of autonomous computer-controlled uninhabited flight [1]. There is an increasing requirement for multi-dimensional mission profiles including reconnaissance in the cluttered/urban terrains and tight space environments. Such technological advancements are pushed by development of avionics systems and autonomous capabilities. These developments have paved the way

for versatile small-scaled unmanned aerial vehicles (UAVs), which is also often referred as unmanned aerial system (UAS).

The UAV market is expected to grow dramatically by 2020 with potential applications encompassing military, civil and commercial scenarios. The utilization of UAVs in defense can be exemplified by the fact that Predators have been flying since 1994. They now have been involved in combat/surveillance operations since that time. Predators have flown on six continents over some 30 countries till now [2].

UAVs come in all sizes and shapes, from micro-size to large jet powered high altitude aircraft. This research is more closely related to the earlier one, specifically on the UAV that is desired to be capable of executing agile maneuvers in confined spaces and cluttered terrains as well as vertical takeoff and landing (VTOL), hover, fast forward cruise flight and also ‘perch and stare’. Small UAVs are emerging as a vital part of soldier’s back-pack for instantaneous and immediate intelligence data gathering from surroundings. Development of highly capable small UAV system presents unique challenges for technology protagonists [3]. Such challenges are mostly driven by the need to expand the capability envelope of the UAVs.

1.2 Area of Research

In order to enhance the flight envelope of the UAVs, there have been efforts to combine the excellent hover capabilities of rotorcraft with the endurance and speed performance of fixed-wing aircraft [1]. Such efforts lead to the so-called “convertible” aircraft that can exhibit hover mission segments coupled with efficient flight during forward cruise. An inherent problem for such vehicles is the

transition maneuver between forward flight (primary flight modality) and hover (secondary flight modality), which usually exhibits significant altitude variation and loss of partial control. These characteristics are undesirable and reduce the maneuver potential of the vehicle in tight spaces.

There have been substantial efforts in the past to develop convertible aircraft concept, mostly for piloted platform. The concept has been developed and revisited again and again for over half a century. Likely, the most famous of seminal aircraft are the Lockheed XFV-1, Convair XFY-1, and the Ryan X-13 Vertijet [4].

Several alternatives of the convertible configurations have been proposed in industry. The categorization of these vehicles at the highest level of hierarchy can be described as follows [4]:

- a) Tail Sitter
- b) Jet Thrust
- c) Tilt Fan
- d) Tilt Wing
- e) Tilt-prop
- f) Tilt-rotor

Figure 1-1 shows an example of the design associated with each category above. Tail Sitter configurations are designed with the rationale that the whole vehicle takes off vertically and transitions from 90° to 0° in cruise. The transitions between hover and forward flight modalities for tail-sitters were a daunting aspect of the design. However, the design was not abandoned because of its complexity, but rather due to the difficulty faced by pilots in taking off and landing vertically.

The Jet Thrust designs have come to some reality in the form of Harrier and Joint Strike Fighter (JSF) but a powerful propulsion system is needed as the backbone of such design. Moreover, extremely high fuel consumption in hover demands a high degree of trade-off with flight time and payload capacity. Tilt Fan in a way is a remedy to the fuel consumption problem of jet-thrust. Bell Aerospace developed X-22A using this tilt-fan feature and it remained operational for more than twenty years. The four ducted fans showed substantial improvement in less fuel consumption during hover.

Tilt Wing configurations are thought to be the 'next to best' solutions for convertible configurations as the downward force produced by prop-stream in hover on the wings is diminished. Tilt-props appeared on the horizon for relatively shorter time. X-19 was considered as the first tilt-prop design with four propellers. The design was abandoned because of a high disk loading and fuel consumption in hover [4]. Tilt-rotor technology was initiated in early 1950s and remained the most active area of research during these fifty years. Several aircrafts utilizing this feature remained in useful inventory like Boeing V-22 and Bell Agusta BA609.

Figure 1-1 (Top Row, Left to Right) Tail-sitter configuration *Lockheed XFV-1*—Earlier Jet Thrust configuration *Yakovlev Yak-38*;(Middle Row, Left to Right) Tilt Fan configuration *Bell X-22A*--- Tilt Wing Configuration *LTV XC-142A*; (Bottom Row, Left to Right) Tilt-prop design *Curtiss-Wright X-19*---Seminal Tilt-rotor design *Transcendental 1-G*

The next generation of convertible aircraft research will primarily encompass unmanned convertibles as the situation with the pilot out of the aircraft will enable to enhance the versatility of these platforms and extend their flight envelopes. This type of UAV is the focus of this work. Specifically, the investigations encompassed in this thesis address following areas:

1.2.1 Transition Dynamics

The dynamic behavior of the aircraft undergoes significant variations while transitioning between modes. Understanding such dynamics, especially in

low Reynolds' number regime for small vehicles, is crucial for improving the maneuver and for its control development.

1.2.2 Optimized Transition Maneuvers

Transition maneuvers that are optimal based on certain criteria are also investigated in this report. The parametric analysis of the maneuver and related performance evaluation are carried out. Moreover the associated control efforts are also analyzed.

1.2.3 Control Strategy

The development of strategies for transition control is also considered in the dissertation. The strategies are based on the results of dynamic analysis and optimal solutions. The advantages and disadvantages of these strategies are discussed.

1.2.4 Aerodynamic Vectoring

The unique feature of the UAV that is proposed and considered here to assist in the transition is 'aerodynamic vectoring'. In this work, this aerodynamic vectoring is achieved using 'variable-incidence wing', where in this case the angle of incidence of the wings with respect to the fuselage is decoupled and dealt as an independent control variable. The results with aerodynamic vectoring are compared with its fixed-incidence wing counterpart for the dynamics, optimal trajectories and control law design. The peculiarity of the aerodynamic vectoring is such that the angle of attack of the lifting surfaces can be adjusted freely so that more desirable forces can be harnessed from the resultant aerodynamic vector. An illustration of this concept is shown in Figure 1-2. It can be

understood that the aerodynamic vectoring scheme (right) is different from conventional scheme (left) in that its resultant aerodynamic force can be tailored to be more favorable for the flight maneuver being performed.

For the practical realization of this concept, a rotatable rod can be attached and passed through the quarter-chord point of the wing in span-wise direction to rotate the wings. Subsequently, high rate servos combined with gear mechanism can be used for actuation of the outer-wing incidence. A prototype version of such arrangement is implemented and shown in Figure 7-1.

Figure 1-2 Schematic of the concept of aerodynamic vectoring

1.3 Research Objectives and Scope

Typical mission profile for a small convertible UAV operating in cluttered terrains including indoor and caves is shown in Figure 1-3. The mission segments en-circled as blue are the regions of interest in this research. Typical mission profile includes vertical takeoff from ground and then subsequently transition to cruise. For the retrieval of information from the sight of interest, the

UAV hovers over the region until the time information acquisition is completed. Then it transitions to cruise/forward flight mode again. These transitions can occur multiple times in the mission profile as per the mission requirements.

Figure 1-3 A conceptual mission profile for the small UAV involving multiple transitions

There have been significant studies in the past for convertible UAVs. The contribution from the academia and industry over the transition maneuver is discussed in detail in Chapter 2. There are several problems associated with the transition maneuver like

- Significant altitude loss during hover-to-cruise transition,
- Longer transition times,
- High thrust-to-weight ratio designs,
- Poor understanding of dynamics because of nonlinear regime
- Higher control contributions during transitions and
- Loss of partial control.

Most of the past-work contributions (as discussed in Chapter 2) are related to design efficient control algorithms. Few studies have focused on the mitigation of

the transition problems by enhancing platform versatility. Therefore, in this research, a novel scheme using variable-incidence wing as an aerodynamic-vectoring device is proposed to assist in the transition maneuver. The goals expected to be achieved with the proposed transition methodology are:

- a) To extract maximum advantage from aerodynamic properties in terms of higher lift and reduced drag for the transition maneuver.
- b) To provide better control during the transition by the presence of additional control feature from the variable-incidence wing.
- c) To reduce the T/W requirement of 'convertible' UAVs for performing transition, which in turn allows the aircraft to maximize payload capacity.
- d) To reduce the time needed for the transitions without penalty on other requirements.

Associated with those goals, the objectives of the research are:

- a) To study the possible advantages in the maneuver control with the inclusion of the aerodynamic vectoring feature.
- b) To examine the dynamic characteristics of the transition maneuvers over the complete transition envelope, especially with the inclusion of the variable-incidence wing. The detailed focus of this dissertation is to understand the dynamics associated with transition maneuvers and flight performance characteristics. The investigation of dynamic characteristics delineates the effect of design parameters on flight characteristics of convertible platforms.

The transition maneuvers considered in this work can be classified into two distinct types, which are discussed below.

1.3.1 Steady Transition

In steady transition maneuver, the aircraft is in trimmed condition across the complete transition envelope. This represents a scenario where the aircraft flies in equilibrium between the two primary flight modalities, for example in low-speed sustained flight scenario where the aircraft has to maintain a certain pitch angle to achieve trim. Such maneuver can also be assumed as a slow transition between hover and cruise flight, where the aircraft can be assumed to be in steady equilibrium at any point during the transition.

1.3.2 Unsteady Transition

In unsteady transition, the time between the initial and final desired states is relatively short and thus the aircraft performs the transition without achieving trimmed condition in between. It can be understood that for such a transition, analysis approach about an equilibrium flight condition, which can be used for steady transition, does not work and will require a different approach that will consider the whole transition maneuver in its entirety.

1.4 Methodology

In this work, a systematic evaluation of the proposed transition scheme is carried out. Initially, the aerodynamic properties of the UAV are evaluated experimentally. For this purpose, wind tunnel testing of the scaled model is carried out across the complete flight envelope. The contribution from slip-stream flow field is significant at low velocities or near hover flight and is the primary source of generating control authority. Slip-stream modeling is carried out based on standard momentum theory approach. Subsequently, some

unsteady aerodynamic features are incorporated using some theoretical relationships such as Wagner function. After developing the aerodynamic model for analysis, the two classes of transitions (steady and unsteady) as discussed earlier are analyzed explicitly.

For steady state transition analysis, the trim states across a broad velocity spectrum between hover and cruise are evaluated using a nonlinear constrained optimization scheme based on sequential quadratic programming. Then linearized dynamic analysis around trim states is carried out in order to compare the characteristics of the conventional platform with the modified platform incorporating aerodynamic vectoring. After getting the notion of improvement in the performance with the proposed modification, stability properties are further investigated. For this purpose, nonlinear longitudinal dynamics of the UAV is modeled. Specifically, multiple time scales method in conjunction with bifurcation theory is used to obtain approximate solutions to the dynamics explicitly, from which the key parameters can be identified.

For unsteady transition analysis, a nonlinear constrained optimization scheme based on Sequential Quadratic Programming (SQP) is used to generate the optimal transition maneuvers. An implicit problem is formulated to study the optimal transition maneuvers with and without aerodynamic vectoring. First, the aircraft dynamics is modeled as a point mass and effect of different parameters like mass of aircraft and thrust-to-weight ratio are studied. The associated control histories are also analyzed. Subsequently, the vehicle dynamics is modeled as three degree of freedom motion in the longitudinal plane. Further parameters like effect of unsteady aerodynamic phenomena, terminal velocity

and elevator effectiveness are also studied. The stability analysis of these optimal solutions is carried out using nonlinear tool – “contraction theory”.

Finally, the appropriate control strategy for the proposed variable-incidence wing is discussed. For this purpose, nonlinear control synthesis is carried out using feedback linearization for both configurations (fixed and variable incidence). In the end, conclusive remarks are drawn as well as future directions emerging from the current research are explored.

1.5 Contributions

This work extends the research in dynamics and control of transition maneuvers between hover and cruise for small UAVs. More specifically, it includes the following contributions:

- Improvement in transition maneuver performance against various problem parameters (e.g. mass, thrust-to-weight ratio, time required for the maneuver, terminal velocity requirement and altitude variations) by using “aerodynamic-vectoring” phenomena.
- Numerical and analytical approaches in analyzing aircraft dynamic characteristics involving aerodynamic-vectoring feature. Specific dynamic attributes are delineated across the complete transition envelope under ‘steady’ and ‘unsteady’ transitions.
- The use of Multiple Time Scales (MTS) method and bifurcation theory to obtain approximate solutions of the dynamics for the steady transitions near stall. This results in the identification of the key parameters that contribute to the dynamic behavior specific to this class of aircraft.

- Development of the framework to obtain and analyze optimal solutions of transition maneuvers under specific constraints and benchmark the results of the aerodynamic-vectoring scheme with the fixed-wing conventional design ones. The contribution also includes the application of ‘contraction theory’ for analyzing the stability of solutions/trajectories. This approach is more general than conventional stability analysis as it discusses the stability of a solution instead of a fixed point.
- Development of transition maneuver control strategies based on the results of the dynamics and stability analyses above. Specifically, feedback linearization control architecture is examined to achieve the purpose and the control characteristics differences with and without aerodynamic-vectoring are studied.

1.6 Organization of the Thesis

This thesis comprises of seven chapters. The brief outline of each chapter is discussed as follow:

Chapter 1 --- Introduction

In this chapter, the background and the area of research are classified. The objectives, scopes and methodology used in the research are also briefly described in this chapter. A systematic outline of the report is given at the end of the chapter.

Chapter 2 --- Challenges and Related Work

A comprehensive summary of the literature study is given in this chapter. Identification of challenges from the literature survey is also presented as the driver for the research.

Chapter 3 --- Aerodynamic Modeling and Formulation

In this chapter, the UAV platform used in the study is described. Procedure used to obtain the platform aerodynamic characteristics using wind-tunnel testing is discussed. Moreover, the details of slip-stream modeling and incorporation of unsteady aerodynamic phenomena for further analysis are also delineated.

Chapter 4 --- Steady Transition Maneuver

In this chapter, the steady transition maneuver is described first. The trim analysis is carried out for conventional and aerodynamic vectoring configuration. It is followed by the linear analysis of both configurations, which specifically shows dynamic peculiarities that warrant further investigation. The subsequent dynamic analysis is marched into nonlinear domain by studying single and multiple degrees of freedom using Multiple Time Scales (MTS) method.

Chapter 5 --- Unsteady Transition Maneuver

In this chapter 'unsteady' optimal maneuver analysis for both configurations is carried out. Initially a two-degree-of-freedom point-mass model with pitching constraints is used for the modeling of the aircraft dynamics. Subsequently, the vehicle dynamics is modeled as three degree of freedom motion in the longitudinal plane. The discussion encompasses parametric

comparisons. The stability of optimal solutions is investigated at the end using 'contraction theory'.

Chapter 6 --- On Closing the Loop of Transition Maneuver

Based on the contraction analysis, a class of single degree-of-freedom pitch controller is first studied. Subsequently, to gain a better maneuver response feedback linearization control synthesis is carried out for both fixed-wing and variable-incidence wing configurations. The discussion encompasses performance analysis and singularity avoidance phenomena.

Chapter 7 --- Conclusions and Future Work

Conclusions from the current research presented in dissertation are derived and recommendations for future line of action are laid down in this chapter.

CHAPTER 2

Challenges and Related Work

2.1 Background

In this chapter, a profound effort is made to gather the resources deployed around the globe in the past to rationalize the underlying in-flight transition phenomena and plausible avenues that can be marched on from this point. As mentioned briefly in the previous chapter that ample amount of research has been carried out related to transition phenomena for manned platforms.

The present subject matter consists of class of vehicles known as mini UAVs or broadly speaking micro air vehicles (MAVs). They are often classified based on their size and weight as shown in Figure 2-1. The present research is based on the vehicles comprising of maximum span of around 1 meter.

Figure 2-1 Scale of Unmanned Air Vehicles - From Global Hawk to DARPA MUAV [1]

2.2 Related Work

The seminal work for the transition phenomena on UAV platforms can be attributed to Nieuwstadt and Murray [5]. The focus of the study is on numerical simulations of transition trajectories. The configuration considered consists of a simple ducted fan with wings. The use of differential flatness for the computation of a nominal trajectory for a fast transition between flight modes is investigated. The aerodynamic data base is developed through wind-tunnel tests. The investigation encompasses hover-to-cruise transition for a 4.6 N vehicle in 6 s. The authors report of the altitude loss during the transition but do not quantify the relevant performance parameters for this observation. During hover, the ducted fan and wings expose a large frontal area to any gust disturbance. Their control architecture does not cater the strategy for disturbance rejection during transition.

Okan et. al. [6-8] has proposed a tilt-rotor UAV with shrouded rotors at the wing-tips and an additional embedded rotor in fuselage near empennage for pitch and yaw control during hover and transition. The vehicle has a span of approximately 4.9 m and a gross takeoff weight of 1008 N. The investigations include preliminary design study, aerodynamic modeling based on empirical techniques, flight dynamic modeling and stability evaluations across multiple trim states during transition. Longitudinal equations are used to investigate the transition behavior of the vehicle. Several trim states are evaluated across the transition scheme and the basic control variables optimized are thrust as a function of propellers, nacelle and exit guide vane angles. From the stability analysis, the vehicle appears to be stable beyond 12 m/s and is unstable during

hover and slow forward flight. Linear Quadratic Regulator (LQR) controller is proposed for the transition maneuver control.

Stone et. al. [9-15] has carried out substantial investigations in design, simulation and testing of convertible tail-sitter platform. The concept demonstrator of the ‘T-Wing’ is a twin-engine; tail-sitter vehicle that derives control in low-speed vertical flight via wing and fin mounted control surfaces immersed in wash of its two propellers. The wing span of the vehicle is 2.18 m and weighs 29.5 kg. In the work, the aerodynamic modeling is primarily based on numerical panel methods. Stone et. al. [10, 14, 15] proposes a ‘stall-tumble’ transition maneuver for the tail-sitter aircraft. A typical mission trajectory followed during flight tests [15] is shown in the Figure 2-2. The flight test shows a significant *altitude drop* during the transition from slow helicopter mode forward flight to conventional cruise. Similarly, for cruise-to-hover transitions, the aircraft experiences a significant *altitude gain* which could be unacceptable in some applications.

Figure 2-2 Typical mission trajectory of T-wing UAV; multiple vehicle pictures in some frames are from successive images from a still camera [15]

During the study [14] for the optimization of transition maneuvers for T-wing UAV, numerical optimization coupled with six degree of freedom non-linear model is used to minimize the altitude loss during 'stall-tumble' and altitude gain during transition back to vertical flight. Non-linear constrained optimization routine implemented in MATLAB is used in the study. The investigations for 'stall-tumble' maneuvers are simplified for the fact that the vehicle always has some initial velocity to start the maneuver. Moreover, the investigations revealed that with the same thrust setting, the altitude loss is more as the mass of the vehicle is increased. For the mass of 31.75 kg, the altitude loss is about 6 m. The study did not cover for the scaling up of the thrust with the increase in the weight which might be useful to replicate on several scaled models.

SkyTote (see Figure 2-3) is an unmanned precision cargo delivery system. The work by Taylor et al. [16] encompasses investigations from conceptual to final flight testing phase. The vehicle is supposed to pick up the cargo, do vertical takeoff, transition to wing-borne flight, travel to the landing area, transition back to helicopter mode and land vertically. It has a co-axial, counter-rotating rotor system with a disc-loading comparable to the helicopter. The vehicle has a high similarity with the conventional tail-sitter design. Autonomous hover capabilities have been successfully carried out to-date [17]. The vehicle is used primarily as a concept demonstrator.

Figure 2-3 SkyTote - A new concept for cargo delivering UAV [16]

Aurora Flight Sciences have come up with the unique design of the clandestine UAV called Goldeneye [18-20]. It is a ducted fan configuration with the control surfaces submerged in the prop-stream. The vehicle is claimed to have a good hover gust rejection capability, efficient cruise performance and controlled transitions between hover and cruise as a result of its unique torsionally-decoupled outer wing panels. The vehicle has a span of approximately 3 m and hence it flies in high Reynolds number flow regime. Moreover, because of the proprietary nature of the work, no technical details have been published and no engineering studies are available for public release to-date. A still-camera view [20] of the Goldeneye during transition is shown in Figure 2-4.

Figure 2-4 Still-Camera images of transitions carried out by Goldeneye 50 clandestine systems[21]

In recent studies, Green and Oh [22-24] discuss the concept of ‘prop-hanging’ for fixed-wing UAVs and analyze the hover-to-forward-flight transitions. The investigations are of experimental in nature. With excessively high thrust-to-weight ratios, the aircraft, which can be classified as small agile UAV with wing-span of 0.9 m, can presumably ‘bully’ through the transition regime. The thrust-to-weight ratio for the vehicle is quoted as high as 1.67 in order to attain the successful transition maneuver in minimum time and space. The hover-to-cruise transitions are completed in about 2 s (Figure 2-5). The study lacks the investigations on the altitude variation vs. thrust-to-weight ratio tradeoff. Such a high thrust-to-weight ratio can be regarded as over-designed and generally penalizes the vehicle’s payload capacity.

Figure 2-5 Stills of MAV Prototype with 90 cm wingspan during transition maneuver [25]

High-Speed Autonomous Rotorcraft Vehicle (HARVee) is a tilt-wing experimental UAV under-development in University of Arizona. The study on HARVee reported in [26] covers its conceptual design and simulations. The vehicle consists of four propellers installed on the wings, two on each side of the fuselage. The vehicle has a wing span of 1.74 m and a fuselage length of 1.37 m. The aerodynamics of the UAV is modeled based on empirical techniques. The flight dynamic analysis for the transition from hover-to-cruise flight with the aid of H^∞ control approach is formulated for the vehicle. The study is slightly controversial as the whole wing is assumed stall beyond a certain angle of attack,

ignoring the fact that parts of the wing submerged in the slip-stream of the propellers are less likely to stall. Moreover, the investigation presumes that there is sufficient thrust available at all instants to cater for any aerodynamic lift force deficiency.

Korea Aerospace Research Institute has also initiated a tilt-rotor UAV program [26, 27] which includes designing of the platform, control architecture and simulating collision avoidance maneuvers. The vehicle has much similarity with Bell V-22 Osprey.

Revisiting the tail-sitter technology in a miniaturized form at Brigham Young University [28-30], Knoebel [28] has explored the plausible utilization of XFY-1 Pogo design for indoor applications. The wing span of the UAV is approximately 0.6 m. The aerodynamic model used in the analysis is based on empirical estimations. Quaternion Feedback Control approached is used for hover. Osborne [29] has studied the transition phenomenon between hover and level flight for a tail-sitter configuration. During the 10 s hovering test, the aircraft drifts back by 30 m because of the exposure of the large wing area to the head-wind. During the transition maneuvers, several control schemes are compared, including Proportional-Integral-Derivative (PID), feedback linearization and adaptive controllers. The altitude gain for level-to-hover flight transitions is approximately 22 times the span of the aircraft with the best control scheme.

Johnson et. al. [31] have studied the transition phenomena for a fixed wing aerobatic UAV of approximately 2.7 m span. The work encompasses the dynamic inversion with neural network adaptation to provide an adaptive controller capable of transitioning a fixed-wing UAV to and from hovering flight. The transitions are executed in the open space with fewer requirements for leveled

transition trajectories. The altitude variation during the transitions to and from hovering flight is reported to be approximately eight times the span of the aircraft.

The work on the transition maneuver of a Tail-Sitter UAV designed at the University of Tokyo [32, 33] shares some similarity with a part of this research effort. The investigations are of conceptual in nature and use numerical approach. The UAV consists of a twin-boom counter-rotating propellers. The ailerons, rudders and elevators of the UAV are immersed in the propeller slip stream for slow-speed controllability. The span of the vehicle is one meter. Optimal transition flight trajectories are generated numerically to estimate the altitude variations. For hover to cruise transitions the altitude variations are minimal while for cruise to hover transitions a significant altitude gain of 13 m is observed (Figure 2-6).

Figure 2-6 Trajectories for transitions between hover and cruise [33]

The utilization of flaps and slats is shown to improve the transition performance by reducing altitude gain but with penalty on the transition time (7.5 s). The studies, however, have some limitations. First, the aerodynamic estimation is based on totally empirical techniques. Second, the thrust-to-weight ratio is assumed to be excessively high. Besides these assumptions, the level flight to hover conversion is simulated with flaps and slats. The aerodynamic

estimation with flaps and slats deployed is again based on high Reynolds number empirical relationships. The mechanism to install movable flaps and slats is difficult to realize because of the small Mean Aerodynamic Chord (MAC) of just 0.15 m. The small MAC restrains the installation of movable mechanisms for flaps and slats.

Frank et. al. [34] uses the facility called 'Real-time indoor Autonomous Vehicle test Environment' (RAVEN) at MIT to implement the quaternion feedback control scheme on a small conventional aerobatic plane. The vehicle is observed to experience altitude loss and gain during hover to cruise and back to hover transitions about twice the span of the aircraft (Figure 2-7). Moreover, the study examines the tradeoff between pitch rate and thrust-to-weight ratio but does not quantify the effects. With excessively high thrust-to-weight ratios, the performance of the controller seems adequate but encounters disturbances particularly during the transition from level flight to hover.

Figure 2-7 Autonomous aircraft hover, transition to level flight and back to hover [34]

During the wind-tunnel testing by Moschetta et. al. [35] at Toulouse, tilt-wing concept is tested for micro air vehicles with a span of 0.2 m. The study reveals that there is not much significant difference between the tilt-wing and standard (tilt-body) configurations. The investigation reveals little difference on the results between the two configurations for a fact that wings are completely submerged in the slipstream.

2.3 Missing Links in Literature

The brief review of the related work points out several missing links in literature. Till today, existence of a unified design philosophy is scarce in literature. Most work refers to the coupling of helicopter mode with fixed-wing mode on a conventional configuration but lacks about the discussion on the platform versatility. Transition centric design philosophy is still pretty premature.

The dynamic behavior of the transition dynamics is still less understood and minimum efforts have been poured in for investigations of flight dynamic characteristics. Most of the work jumps from conceptual design directly to the closed-loop control synthesis without going through the detail of flight dynamic analysis. The nonlinear flight regime of transition maneuver is the major challenge which has slowed the efforts to understand this issue.

The work presented in this dissertation provides framework for assessing the transition maneuver capability on any generic platform. Key performance parameters required to fully assess the maneuver potential of an aircraft are identified and investigated. Moreover, detailed flight dynamic aspects across the maneuver encompassing both linear and nonlinear issues are studied. Based on

the investigations, the dynamic characteristics of the aircraft and the development of an appropriate control strategy based on the uncovered dynamics are documented in this dissertation.

CHAPTER 3

Aerodynamic Modeling and Formulation

3.1 Background

For the purpose of the aerodynamic estimation, it is assumed a priori that the aircraft motion is restricted in the longitudinal plane only. The lateral-directional components of the dynamics are not considered, which greatly reduces the number of parameters in the study. Even in the case where motion is confined to the longitudinal plane, lateral-directional parameters can still enter into the dynamics model. The lateral-directional coupling with the longitudinal dynamics has the following characteristics:

- The coupling is only significant at high angles of attack, due to the following peculiarities:
 - Asymmetric vortex shedding from the wings
 - Complex flow structures in the post-stall regime such as flow-separation and re-attachment
 - High degree of nonlinearity in the aircraft stability derivatives.
 - Inertial coupling of the longitudinal mode with lateral-directional plane.
- With the type of aircraft under study, the propeller rotation induces an additional rolling moment that increases with the increase in propeller RPM.

The rationale behind the consideration of longitudinal dynamics alone in this study is based on following facts:

- The purpose of aerodynamic-vectoring is to retain the main aerodynamic surface (wing) in the pre-stall regime and avoid departure to post-stall regime. Therefore, the lateral-dynamics coupling for the aerodynamic-vectoring transition will be less than the conventional transitions. The fixed-wing case will encounter the lateral-directional effects to a greater extent, however this case is not the focus of the study and used as a benchmark case only.

The propeller rotations induce rolling moment for both configurations at all angles of attack. The effect is obvious for propeller-driven aircraft. However, during practical implementation, the rolling moment can be cancelled by using either permanent deflection of ailerons OR contra-rotating propellers. The mitigation/compensation of rolling moment is envisioned as part of the future work.

By making these mild assumptions, the study is simplified yet rich enough to capture the dominant behavior of aerodynamic-vectoring phenomena.

In this chapter, the aircraft platform used as a case study for the transition maneuvers is first described in detail. The aerodynamic characteristics of the aircraft are evaluated by means of wind-tunnel testing. The details associated with the experimental setup and procedures involved are presented later. For transition dynamics, propeller slipstream is an important aspect affecting control effectiveness from elevators in the near hover flight regime. Hence, slipstream and unsteady aerodynamic effects are delineated in the later part of the chapter.

3.2 Description of the Platform

The UAV platform selected for the study consists of a conventional Radio Controlled (RC) aircraft model available commercially. It has a standard wing-tail configuration with tractor-type propulsion system. Its airframe consists of extended polypropylene particle (EPP) foam construction with composite landing gears. The geometric configuration of the UAV used in our study is shown in Figure 3-1.

Figure 3-1 Views of the UAV with aerodynamic vectoring feature

The model has a fuselage length as well as a wing span of 1 meter. The aspect ratio of the wings is 4.31. The recommended all up weight (AUW) for enhanced performance is about 400 grams but the vehicle can fly with an AUW of approximately 700-800 grams. Typical dimensional attributes of the model are shown in Table 3-1. The wings are divided into two sections: inboard and outboard sections. The inboard section (10 inch span) is fixed with the fuselage as it will be submerged in the slipstream of the propeller. The rest of the wings have an additional degree of freedom of rotation about their axis at the quarter-

chord axis. This feature acts as an aerodynamic vectoring device and is called variable-incidence wing.

Table 3-1 Geometric Properties of the UAV used in the study

Geometric Attribute	Dimensions
Fuselage Length	1 m
Span	1 m
Wing Airfoil	NACA 0012
Tail Airfoil	NACA 0012
Propeller Diameter	10 inch
Wing LE position	(0.14 ,0,0) m
Aspect Ratio	4.31
Movable Wing Span	0.74 m
MAC	0.2379
Centre of Gravity	(0.2,0,0) m

3.3 Aerodynamics Estimation

Past investigations associated with trajectory optimization and stability analysis [33, 36, 37] utilize aerodynamics information obtained from empirical or low fidelity computational fluid dynamic algorithms (e.g. potential flows). In this work, however, an experimental approach is adopted to investigate the aerodynamic properties of the UAV under study. The purpose is to accurately estimate the aerodynamic characteristics of the UAV at various aircraft flying configurations and conditions. The testing is performed in the Nanyang Technological University (NTU) low-speed closed-circuit wind-tunnel. The equipment used during the testing and step-by-step procedures followed to extract the aerodynamic data is documented next.

3.3.1 Wind Tunnel Facility

The facility belongs to the class of closed-circuit, low turbulence, subsonic wind tunnel. It is assembled horizontally. The closed-circuit configuration helps to reduce the total required power supply and to avoid the speed variations due to possible interferences in the air flow. It is provided with a closed type test-section and continuous speed variation system.

3.3.1.1 Operating Characteristics

The air velocity inside the wind-tunnel can be varied from 6 m/s to 90 m/s with continuous adjustment and nearly uniform distribution. The corresponding Mach number ranges from 0.029 to 0.26. The air speed distribution inside the test section can be considered constant and free from boundary layer in within 80% of the area of the section itself. The wind tunnel has a very good turbulence level because of the high contraction ratio and suitable number of anti-turbulence screens. The contraction ratio of the wind-tunnel is 9 and it has 3 anti-turbulence screens with different meshes to ensure low turbulence levels.

3.3.1.2 Test Section

The test section of the wind tunnel consists of the structural frame, side windows, and top and bottom wall. The structural frame is made of steel, while the side windows consist of plexy-glass framed in aluminum. The top and bottom walls are made of wood with aluminum frame.

The dimensions of the wind tunnel test section are tabulated in Table 3-2 and the wind tunnel test section is shown in Figure 3-2.

Table 3-2 Dimensions of the wind tunnel test section

Dimensions	Meter
Height	0.72
Width	0.78
Length	2.00

Figure 3-2 View of the wind tunnel test section from rear

The test section is equipped with a 6-component internal balance. The most important specification of the internal balance would be the range of load testing it provides, which is shown in Table 3-3.

Table 3-3 Load Range of internal balance

Range of internal balance	
Normal force (lift) (N)	1000
Axial force (drag) (N)	500
Side force (N)	800
Rolling moment (Nm)	30
Pitching moment (Nm)	75
Yawing moment (Nm)	60

3.3.1.3 Model Positioning Mechanism

The model positioning mechanism is equipped with a sting support. It is capable of rotating the model in three axes, namely roll, pitch and yaw. The model positioning mechanism is controlled by a dedicated computer and is able

to perform a pre-programmed motion. The range and accuracy of the positioning mechanism are the most important specification and these are listed in Table 3-4.

Table 3-4 Range and accuracy of the model positioning mechanism

	Min Deflection (°)	Max Deflection (°)	Resolution (°)	Accuracy (°)
Roll	-45	45	0.5	0.5
Pitch	-10	30	0.1	0.1
Yaw	-40	40	0.5	0.5

3.3.1.4 Data Acquisition, Reduction and Control System

The main functions of the data acquisition, reduction and control system (DARCS) are to provide data acquisition and data reduction from the internal balance and to control the model positioning system. Other important functions of DARCS include pre-testing data input, data storage and post-test data visualization, and user access management. DARCS is equipped with a personal computer (PC) with Windows XP operating system, a video camera to monitor the inside of the test section, as well as a base data acquisition (16 bits analog-to-digital converter (ADC)) and motion controller.

The main features of DARCS include its control system, data acquisition and reduction system, and database system. The control system allows for the manual and/or automatic setting of the angular and traversing position. On the other hand, the data acquisition and reduction system monitors speed, forces and moments. It also allows for the setting of sample and filtering, further converting the data to aerodynamic coefficients. The data acquisition and reduction system also records all data required for further analysis, and displays the data recorded in the form of chart and/or worksheet.

3.3.2 Load Cell Calibration

Before carrying out aerodynamic testing of a model, the first step is to calibrate the internal balance/load cell and examine its accuracy. The schematic of the internal balance is shown in Figure 3-3. The calibration is carried out at a specific point, that is, model interface shown on the extreme left in Figure 3-3.

Figure 3-3 Schematic of the internal balance/load cell (units in mm)

The load cell is calibrated across a broad spectrum of loads at several angles of attack. The forces measured by the load cell are axial, F_x and vertical, F_z with reference to its centre line as shown in Figure 3-6. The side-force component is not calibrated as it is not needed in this study. Initially, the balance is set at zero degree angle of attack and axial and vertical forces are calibrated. The difference in the actual and measured readings for loading in both directions can be expressed in the following form.

$$F_{x_{actual}} = F_{x_{measured}} + \Delta F_x \quad (3.1)$$

$$F_{z_{actual}} = F_{z_{measured}} + \Delta F_z \quad (3.2)$$

where $F_{x_{actual}}, F_{z_{actual}}$ are the actual/original loads used for the calibration, $F_{x_{measured}}, F_{z_{measured}}$ are the readings obtained from the DARCS and $\Delta F_x, \Delta F_z$ are the difference in the readings to be used for correction purposes. For calibration of axial axis, the load range is varied from 0.1 kgf to 3.0 kgf and corresponding effect on $F_{x_{measured}}$ and $F_{z_{measured}}$ is recorded. The setup to apply loading in axial

direction is shown in Figure 3-4. A nylon string is tied to the load cell and is passed over to the pulley. It can be observed in Figure 3-5 that the axial force $F_{x_{actual}}$ has little effect on the difference values ΔF_z and ΔF_x across the complete load range.

Figure 3-4 Experimental setup for axial loading

Figure 3-5 Error margin in longitudinal plane because of axial loading

For calibration of normal axis, the load range is varied from 0.4 kgf to 5.0 kgf and the corresponding effect on $F_{x_{measured}}$ and $F_{z_{measured}}$ is recorded. The setup to apply loading in axial direction is shown in Figure 3-6. A nylon string is tied to the load cell and weight pan is directly hanged with it to put several dummy loads instead of using a pulley like for the axial loading case.

Figure 3-6 Experimental setup for vertical loading

The difference between the actual and measured readings is plotted in Figure 3-7. The error accumulation with the increase in weight along vertical axis is observed but at higher load values. A significant amount of error accumulation along the axial direction because of the loading in vertical axis is observed. The trend is primarily linear and therefore can be easily corrected during the data correction phase. The identification of the cause of error in axial measurements because of the vertical loading is beyond the scope of the study. Based on the calibration curves, appropriate corrections are made in the data obtained from the wind tunnel to estimate aerodynamics of the UAV.

Figure 3-7 Error margin in longitudinal plane because of vertical loading

The load cell is also calibrated across the angles of attack range and at several loads. For this purpose, the calibration setup is similar to Figure 3-6 and the model positioning mechanism is used to tilt the sting balance to arrive at various angles of attack. The effect of loading on the vertical measurements is shown in Figure 3-8. It can be observed that slight errors are observed in the values but they are independent of the angle of attack.

Figure 3-8 Difference in vertical measurements at several angles of attack

The effect of loading on axial measurements at various angles of attack is plotted in Figure 3-9. The error bias because of the vertical loading is independent of the range of angles of attack. Up to this point, the calibration of

the load cell is completed and pertinent corrections required in the measured values in the longitudinal plane are identified and applied in the later tests.

Figure 3-9 Difference in axial measurements at several angles of attack

3.3.3 Procedural Validation

The flow conditions in a wind tunnel are not completely the same as unbounded stream most of the times. The flow is disturbed by various kinds of blockages, flow distortions, flow angularity, buoyancy and boundary layer interactions of walls with the flow over the aircraft [38]. In addition, the contributions to measurements due to tare and interference are also involved. Experimental results of the flat-plate of low aspect ratio at low Reynolds number (approximately 0.1 million) are already published by Torres and Mueller [39]. In this section, the same experiment is repeated in our wind-tunnel to verify the appropriateness of experimental procedures. The flat plate has same thickness-to-chord ratio of 2% and an aspect ratio of 1.00 to the benchmark case. The Reynolds number is approximately 0.16 million which is close to the benchmark case of 0.1 million. Forces and moments are evaluated and corrected for wind-tunnel blockage (solid blockage and wake blockage) according the

techniques presented in Barlow [38], which are discussed below. The mounted flat-plate model inside the wind-tunnel is shown in Figure 3-10.

Figure 3-10 Flat plate model inside the wind tunnel (bubble-leveler shown as well)

3.3.3.1 Blockage Corrections

Solid blockage refers to the ratio of the “frontal area” of the model to the stream cross-sectional area, and this value is effectively zero in atmospheric flights. In contrast, for wind tunnel tests, this ratio cannot be assumed zero and reflects the relative size of the test model and the test section. Hence solid blockage correction is required for wind tunnel tests.

Wake blockage effect is a result of the finite size of a body wake and is somewhat similar to solid blockage. In a closed test section, wake blockage increases the measured drag and cannot be neglected. The total solid and wake blockage corrections may be summed according to

$$\varepsilon_t = \varepsilon_{sb} + \varepsilon_{wb,t} \quad (3.3)$$

where ε_{sb} and $\varepsilon_{wb,t}$ are the solid blockage and wake blockage corrections respectively. The approximation for the total blockage correction factor can be approximated as[38]

$$\varepsilon_t = \frac{1}{4} \frac{\text{model frontal area}}{\text{test section area}} \quad (3.4)$$

The blockage corrections are required to produce the correct dynamic pressure that is used to calculate all coefficients, including pressure and hinge moment. The corrected dynamic pressure q_c is given as

$$q_c = q_A(1 + \varepsilon_T)^2 \quad (3.5)$$

while the corrected velocity is given as

$$V_c = V_A(1 + \varepsilon_T) \quad (3.6)$$

where q_A and V_A are the actual/experimental dynamic pressure and velocity respectively. The aerodynamic investigation for the flat plate is carried out at 21 m/s approximately. The comparison between the actual velocities experienced by the model and the flow velocities is shown in Figure 3-11.

Figure 3-11 Comparison between actual and corrected velocities

3.3.3.2 Tare and Interference Corrections

Any conventional wind-tunnel setup requires that the model be supported in some manner. In response, the supports will both affect the air flow about the model and contribute to the overall drag. The effect of the supports on the free air flow is called interference. The drag contribution of the supports is referred as tare. Evaluation of tare and interference is a complex job but needs to be

properly addressed because of high error contribution to the aerodynamic data estimates. Moreover, the tare and interference forces vary with angle of attack. Besides the tare drag and interference considerations, there is another type known as weight tare. The weight tare is a result of the model centre of gravity for not being on the balance moment centre. In DARCS, before capturing the certain data point, the option is available to delete the weight tare at that particular instant. However, during dynamic testing, the model is forced to undergo oscillatory/linear motion to evaluate the dynamic derivatives of the aircraft. Thus when the model is pitched, there will be pitching moment versus angle of attack contribution due to weight.

In our setup, the models are mounted with a sting in the trailing edge. Support interference studies have revealed that they influence the overall aerodynamic forces slightly, and therefore, proper corrective procedures are adopted. The setup is designed in such a way that the leading edge of the wings be placed far enough from the side walls of the tunnel even at high angles of attack.

3.3.3.3 Moment Transformation

In the experiment, the leading edge of the flat-plate is different from the sensor position. Therefore, the moment values need to be transformed to the desired location on the aircraft. These calculations generally affect only pitch, yaw and roll moments. It should be noted that all tare values are applied before the balance data is transformed from the moment center to the desired location. In our case, these tares involve lift and drag that are used in the moment transformation.

3.3.3.4 Reduction to Non-dimensional Coefficients

The forces and moments at this stage are reduced to non-dimensional coefficient forms using the corrected dynamic pressure q_c . The lift coefficient has no further corrections applied and thus the result obtained represents its final corrected value. The drag, pitching moment and angle of attack need further corrections because of the wall induced effects that will be discussed next. It should be noted that the coefficients are in body-axes of the wind-tunnel and they need to be transformed into stability axes at the end of corrections.

3.3.3.5 Wall Corrections

The effect of walls is another phenomenon that is present in the wind-tunnel and absent in the free-air. The interference of walls requires the application of the wall corrections into the results. The corrections are generally based on the lift generated by the model. For an airplane, this means that the wall corrections are based on the wing lift only. For some models, this is not possible because the wing is inseparable from the fuselage and thus they should be considered as a unit with the wing. In such situations, runs are made with the horizontal tail off (tail-off runs) to obtain the required data for wall corrections.

The spanwise distribution either can be assumed as uniform, elliptical or some custom loading type. The usage of type of spanwise distribution is based on the geometrical features of the wing. In the case of our studies, we have to make choice between uniform and elliptical distribution as it is readily available in literature and used quite frequently. We will approximate the distribution parameter, boundary correction factor δ , for both types and will use the conservative one in our subsequent calculations. Moreover, these corrections are

based on conventional, or Glauert-type, corrections, where the assumed vortex wake trails straight aft of the wing.

Figure 3-12 gives the values for the boundary correction factor δ for uniform and elliptic loading on the wings in a closed rectangular jet respectively [38]. The curves indicate results for various values of tunnel aspect ratio, and the important parameters are as follows:

$$\lambda = \frac{\text{tunnel height}}{\text{tunnel width}} \quad (3.7)$$

Figure 3-12 Values of δ for a wing with rectangular (right) and elliptic (left) loading in a closed rectangular jet [38]

The downwash correction factor for the wind tunnel test section used in this investigation is as shown in Table 3-5.

Table 3-5 Boundary correction factor for flat plate

Tunnel height (mm)	720
Tunnel width (mm)	780
$\lambda = \text{height/width}$	0.9231
Span (mm)	150
$k = \text{span/jet width}$	0.1923
When $\lambda=0.9$; $k=0.1923$,	
δ , wing with uniform loading	0.131
δ , wing with elliptical loading	0.135

The angle of attack after applying wall corrections can be represented as:

$$\alpha_c = \alpha_g + \Delta\alpha_{up} + \Delta\alpha_w \quad (3.8)$$

where α_c is the corrected angle of attack, α_g the geometric angle of attack and $\Delta\alpha_{up}$ the tunnel up-flow which is negligible in our case. Also,

$$\Delta\alpha_w = \frac{\delta S}{CC_{LW}} \quad (3.9)$$

where δ is the wall correction factor as computed above, S is the reference surface area, C is the test-cross-sectional area and C_{LW} is the wing or tail-off lift coefficient for the model wing configuration used in the run. It is observed that the correction required for angle of attack is significantly less than one degree as shown in Figure 3-13.

Figure 3-13 Comparison between geometric and corrected angle of attack

Similar to angle of attack, the coefficient of drag after applying wall corrections can be represented as:

$$C_{DC} = C_{Du} + \Delta C_{D,up} + \Delta C_{DW} \quad (3.10)$$

where C_{Du} is uncorrected drag coefficient.

$$C_{D,up} = C_{LW} \Delta\alpha_{up} \quad (3.11)$$

where $\Delta\alpha_{up}$ is in radians and in our case is negligible.

$$\Delta C_{DW} = \frac{\delta S}{CC_{LW}^2} \quad (3.12)$$

The wall effects on the model are minimum because of the sufficient clearance between the walls and the model.

3.3.3.6 Axes Transformation

At this point, the data have been corrected for angle of attack, forces and moments coefficients. Now the forces are transformed to the stability axes with respect to the free-stream flow.

$$\begin{aligned} L &= F_{Z_{guage}} \cos \alpha - F_{X_{guage}} \sin \alpha \\ D &= F_{Z_{guage}} \sin \alpha + F_{X_{guage}} \cos \alpha \end{aligned} \quad (3.13)$$

Or

$$\begin{aligned} C_L &= C_{F_{Z_{guage}}} \cos \alpha - C_{F_{X_{guage}}} \sin \alpha \\ C_D &= C_{F_{Z_{guage}}} \sin \alpha + C_{F_{X_{guage}}} \cos \alpha \end{aligned} \quad (3.14)$$

With this final transformation, the data is ready to be analyzed and compared.

3.3.3.7 Flat Plate Data Validation

The geometric properties of the square flat plate used for validation consist of an aspect ratio of 1.00, a thickness-to-chord ratio of 2% and is the side length of 0.15 m. The Reynolds number at which the flat plate aerodynamic data is extracted is approximately 0.16 million. The aerodynamic properties attained from our experiment are compared with the results of Torres and Mueller[39]. The model used by Torres and Mueller also consists of a square flat plate with an aspect ratio of 1.00 and a thickness-to-chord ratio of 2%. Their aerodynamic data

is extracted at Reynolds number of 0.1 million which is pretty close to our experimental setup.

There are numerous data-sets recorded. The measurements recorded are saved as data storage and transfer (DST) file. In order to manipulate the data recorded for analysis, the DST file first is exported to a text file, which is subsequently opened in Microsoft Office Excel 2007.

The angle of attack of the wing model was varied from 0 deg to 16 deg. Hysteresis was examined by bringing the wing back to 0 deg. No hysteresis effect was found in the testing. Forces and moment obtained from the internal balance readings are then corrected for various corrections as mentioned earlier. The repeatability test revealed that the aerodynamic data is within the 5% of the uncertainty bounds. The coefficient of lift is plotted in Figure 3-14 and compared with the benchmark data of Torres and Mueller. An excellent agreement is found between coefficients of lift evaluated with the benchmark data.

Figure 3-14 Comparison of coefficient of lift with experimental data

The coefficient of drag is plotted along with the benchmark data in Figure 3-15. Reasonable agreement is evident between both results. Our setup has shown signs of slightly higher drag measurement at high angles of attack but is still within the error margin of the readings.

Figure 3-15 Comparison of coefficient of drag with experimental data

The coefficient of pitching moment is compared with the benchmark data in Figure 3-16. It should be noted that the moment coefficient is transferred to the leading edge of the wing by addition of the contributions of the lift and drag coefficients. The coefficient of pitching moment is also showing a good agreement with the benchmark readings.

Figure 3-16 Comparison of coefficient of pitching moment by experimental data

3.3.4 UAV Wind Tunnel Testing

After ensuring the calibration and proper corrections for the NTU wind tunnel as well as its data validation, the evaluation of the UAV shown in Figure

3-1 is carried out. An important feature of the model is aerodynamic vectoring, i.e. the variable-incidence wing.

In order to simplify the problem, a minor assumption is made. The fuselage, vertical fin, inboard wing section and horizontal tail are assumed to be submerged in the propeller slip-stream. It is assumed that these components will always remain parallel to the prop-stream and bending effects of the prop-stream are neglected. The outboard section of the wings will experience the free-stream flow (outside of the prop-stream) and the unsteady aerodynamic phenomena will be considered on the surfaces submerged in free-stream only. The data presented here are therefore separated into two parts: in free-stream and inside slip-stream. During testing, the interference between the outboard wing and rest of the aircraft are recorded and are included implicitly in the force and moment curves. Finally the data are plotted and discussed in detail.

3.3.4.1 Model Fabrication

The wind-tunnel model is primarily made of Acrylonitrile Butadiene Styrene (ABS) commonly known to us as lego blocks material. The fabricated model is geometrically scaled down (50%) of the original aircraft size. However, during initial testing, it was found that the fuselage of the wind-tunnel model was not strong enough. As a next iteration, T6061 aluminum is selected as the material for the flat fuselage of the model. Therefore, the final model has an aluminum fuselage with other surfaces made of ABS honeycomb structure as shown in Figure 3-17.

Figure 3-17 Scaled model of the UAV for wind-tunnel testing

The inboard and outboard wing arrangement is shown in below. A freely rotatable rod is passed through the quarter-chord point of the wing in spanwise direction to rotate the wings.

Figure 3-18 Wind-tunnel model with specifications of inboard and outboard wings

An aluminium fuselage also allows the tapping of bolts into it. The angles of incidence of the outboard wings and elevators can be fixed using these bolts during the wind-tunnel testing. When required, loosening the bolts allows for ready adjustment of the angles of incidence for both outboard wings, reducing the time and effort required for such adjustments during wind-tunnel testing.

Figure 3-19 Final wind-tunnel model with the location of bolts highlighted

3.3.4.2 Experimental Procedures

The experimental procedures for correction of the raw aerodynamic data obtained from testing are similar as mentioned in Section 3.3.3. Force and moment coefficients presented in this report have all been corrected for wind-tunnel blockage (solid and wake blockage) according the techniques presented by Barlow [38]. The magnitude of the blockage effects varies with the change in angle of attack of the outboard wing. The higher the angle of attack, the higher the blockage effects will be.

The approximate blockage correction for this UAV model is tabulated below. Moreover, with the change in the outboard wing angle of attack, the model frontal area will vary. The blockage correction factors, calculated using Figure 3-12, are shown in the following table.

Table 3-6 Boundary correction factor for UAV

Tunnel height (mm)	720
Tunnel width (mm)	780
$\lambda = \text{height/width}$	0.9231
Span (mm)	500
$k = \text{span/jet width}$	0.641
When $\lambda=0.9$; $k=0.3$,	
δ, wing with uniform loading	0.143
δ, wing with elliptical loading	0.143

The corresponding blockage correction factor as a function of the angle of attack of the wing is calculated and plotted in Figure 3-20.

Figure 3-20 Blockage correction factor as a function of angle of attack of outboard wing

Overall, the effect of velocity corrections is not significant and is generally less than 5% even at high angles of attack as shown in Figure 3-21.

The angle of attack of the outboard wing is varied from 0 to 90 degrees angle of attack. The inboard fuselage is fixed at zero degree angle of attack throughout the data collection activity. The outboard wings are then brought back to zero degree angle of attack to determine whether hysteresis is present. No hysteresis is observed in any of the measurements. The Reynolds number at which testing is carried out is approximately 0.15 million.

Figure 3-21 Effect of blockage correction on velocity as a function of angle of attack of outboard wing

3.3.4.3 Discussion of the Results

The lift is generally 5 to 25 times larger than the drag, and it is usually sufficient to align the model so that no lift appears in the drag-reading apparatus. For balance alignment with respect to tunnel flow and accuracy purposes, the test is carried out by running the wing both normal and inverted from zero lift to stall.

Lift and drag are non-dimensionalized by the area of the wing and the measured dynamic pressure at each angle of attack. Pitching moment is determined from the sting balance measurements and is reported at the quarter-chord location of the mean aerodynamic chord of each wing. It is non-dimensionalized by the wing area, the dynamic pressure and the mean aerodynamic chord of the wing.

As mentioned earlier, the experiments in the wind tunnel are conducted with the model mounted in the upright as well as inverted position. The comparison between coefficients of lift in these two configurations is shown in Figure 3-22. The average of both data-sets is well in the error margin range of the experimental uncertainties. The data presented ahead in this report are based on average (true) loads from upright and inverted readings.

Figure 3-22 Upright, inverted and true coefficient of lift versus outboard-wing angle-of-attack curves

As mentioned earlier, it is assumed that the fuselage, vertical fin, inboard wing section and horizontal tail are submerged in the propeller slip-stream, whereas the outboard section of the wing experience the free-stream effects. In Figure 3-23, the aerodynamic characteristics are plotted for the varying outboard wing angles of attack. The stall angle of attack is found to be about 14-15° and the pre-stall data predict a fairly linear lift-curve slope.

Figure 3-23 Coefficient of lift for outboard wings

The conventional behavior of an increase in drag due to the increase in velocity and wing angle of attack is observed in Figure 3-24.

Figure 3-24 Coefficient of drag for outboard wings

The pitching moment is greatly governed by the center of pressure over the lifting surfaces. The center of gravity is assumed to be fixed at 20 cm aft of the nose. The centre of pressure moves back with the increase in angle of attack. As the angle of attack increases, the wing tip vortices move the center of pressure downstream. The change in the coefficient of pitching moment with free-stream angle of attack predicts satisfactory static longitudinal stability behavior as indicated in Figure 3-25.

Figure 3-25 Coefficient of pitching moment for outboard wings

The aerodynamic characteristics of the aircraft submerged under the propeller slipstream (fuselage, tail and inboard wings) under varying elevator deflections are shown in Figure 3-26 to Figure 3-28. Figure 3-26 shows the magnitude of the pitching moment coefficients generated at different elevator deflection angles. The response is primarily linear with the range of elevator deflection angles and no control reversal phenomenon is observed within the deflection range tested.

Figure 3-26 Contribution of the pitching moment coefficient from elevator+fuselage+inboard wing with elevator deflection (Aircraft Angle of attack = 0°)

The coefficient of drag is plotted as a function of elevator deflection angle in Figure 3-27. It can be observed that the deflection causes unsustantial rise in drag values

Figure 3-27 Contribution of the drag coefficient from elevator+fuselage+inboard wing with elevator deflection (Aircraft Angle of attack = 0°)

The coefficient of lift is plotted as a function of elevator deflection angle in Figure 3-28. There is a slight asymmetry observed in the trend and this can be attributed to interference between inboard wing and elevator deflections. These mild interference effects between the inboard wings and elevators can be safely neglected.

Figure 3-28 Contribution of the lift coefficient from elevator+fuselage+inboard wing with elevator deflection (Aircraft Angle of attack = 0°)

3.4 Slipstream Modeling and Estimation

In hover and low speed mode, the control surfaces are basically in-effective if relying on the freestream flow. A nice technique to increase the controllability

is to submerge the control surfaces in the slipstream of the propellers. Hence, the slipstream modeling is the key in analyzing the flight dynamics of convertible aircrafts. The slipstream modeling is taken from the theory proposed by McCormick [40]. It has been used recently by Stone [41] in the design of a Tail-Sitter UAV where he has coupled the solution of slipstream with a standard low-order panel method. For the slip-stream evaluation in this study, McCormick[40] formulation using momentum theory is used.

Slip-stream effects from propeller are assumed conserved inside a stream-tube such that the panels submerged under its effects are isolated from the neighboring panels. The stream-tube is approximately of the same diameter to that of the propeller. Moreover, it is assumed that there is no contraction of the stream tube to maintain the flow momentum and therefore, the diameter of the stream-tube remains constant throughout the slipstream region of the aircraft. Another assumption made in the subsequent analysis is that there is no bending of the slip-stream under the influence of free-stream air at high angles of attack.

Figure 3-29 Calculation of slip-stream velocity from classical momentum theory [42]

If V_s is the slipstream velocity, w_i is the induced velocity aft of the propeller and V_o is the free-stream velocity component parallel to the propeller (Figure 3-29), then

$$V_o = V_\infty \cos \alpha \quad (3.15)$$

and

$$w_i = \frac{1}{2} \left[-V_o + \sqrt{V_o^2 + (k_1 T)} \right] \quad (3.16)$$

where $k_1 = \frac{2}{\rho A}$. If we assume that there is no contraction of the slipstream to

retain the momentum, then slip-stream velocity can be expressed as

$$V_s = V_o + 2w_i \quad (3.17)$$

Putting the parallel free-stream velocity component (Equation 3.15) and induced velocity component (Equation 3.16) in Equation 3.17 reveals

$$V_s^2 = V_o^2 + k_1 T \quad (3.18)$$

In Figure 3-30, the slipstream velocities are plotted across thrust for several values of the free-stream parallel component. The thrust required to achieve the perfect hover condition for an aircraft is approximately 6.87 N. It can be seen that at the slipstream velocity at hover condition is approximately 12 m/s and it is the primary source of generating control efforts to maintain aircraft attitude and reject any disturbances. The slipstream velocity increases with the parallel component of free-stream velocity for a certain thrust value.

Figure 3-30 Slipstream velocity as a function of thrust and velocity parallel to the propeller

3.5 Unsteady Aerodynamic Estimation

The aerodynamic properties discussed up to this point are steady or quasi-steady in nature and has been used for evaluating agile maneuvers in most investigations[5, 29, 33, 43]. In a quasi-steady approach, any change in angle of attack of the aircraft results in instantaneous change in aerodynamic properties. The quasi-steady assumption, while attractive in its simplicity, is not sufficiently accurate and more advanced unsteady aerodynamic techniques must be used to predict accurately the dependency of aerodynamic force and moments on the dynamic motions. Any instantaneous change in attitude of the aerodynamic surface induces the flow-field change and resultant effective angle of attack is different from geometric angle of attack of the aircraft. The delay in achieving the new steady aerodynamic response occurs due to the time taken for the circulation around the surface to change to that to the new steady flow condition[44, 45].

In the quasi-steady approach, any change in effective angle of attack corresponds to instantaneous change in lift coefficient. In reality, it takes some

time for the lift coefficient to increase as a result of the increase in the effective angle of attack. Wagner function can be used to describe the change in lift coefficient with time as a result of angle of attack change. For example, airfoil is subjected to an instantaneous increase in angle of attack $\alpha = 0.5\alpha_o$. For quasi-steady model, there will be an instantaneous 50% increase in lift coefficient. For Wagner function model, the increase in the lift coefficient will be a function of time as illustrated in Figure 3-31.

Figure 3-31 Effect of a sudden change in the angle of incidence on lift

Wagner function is used to model the instantaneous change effects in the time domain. Before using Wagner function to evaluate the time-dependent aerodynamic responses, the procedure to find effective angle of attack is laid down first. Some assumptions made are as follows: the unsteady aerodynamic loads are span wise independent and can be treated on a two-dimensional basis; the mean flow speed is uniform over the two-dimensional wing and the instantaneous changes are small. Moreover, the unsteady effects are specifically modeled on the outboard wings as the aerodynamic vectoring is the primary area of investigation in this paper.

The aircraft pitch rate modifies the effective angle of attack on the wing, denoted as $\alpha_{wing,q}$ from here onwards. Another change in effective angle of attack is due to the instantaneous change in aircraft height, which will induce change in effective angle of attack, denoted as $\alpha_{wing,z}$. The angle of attack change is dealt for fuselage and outboard wing separately. The effective angle of attack is the sum of geometric angle of attack, induced angle of attack due to altitude change and induced angle of attack due to pitch rate.

$$\alpha_{wing,e} = \alpha_{wing} + \alpha_{wing,z} + \alpha_{wing,q} \quad (3.19)$$

The altitude change effect is evaluated by the ratio of the vertical velocity (downwash) to the free-stream velocity.

$$\alpha_{wing,z} = \frac{\dot{y}_F}{V_\infty} \quad (3.20)$$

The pitch rate induced angle of attack can be calculated as

$$\alpha_{wing,q} = \left(\frac{\frac{3}{4}\bar{c} - p_o}{V_\infty} \right) \dot{\theta}_{wing} \quad (3.21)$$

where p_o is the hinge point of the aerodynamic surface and $\dot{\theta}_{wing}$ is the pitch rate in Equation 3.21.

The time-lag in the buildup of aerodynamic forces is modeled using Wagner function. Wagner function gives the growth of the circulation about the two-dimensional wing due to a sudden increase of downwash. An approximate expression over the entire range $0 < \tau < \infty$ is given as [44]:

$$\Phi(\tau) = 1 - \frac{2}{\tau + 4} \quad (3.22)$$

where τ is the normalized time given by

$$\tau = \frac{V_{\infty} t}{\bar{c} / 2} \quad (3.23)$$

For a particular instant change, the aerodynamic properties are multiplied by the time-dependent Wagner function.

$$\left. \begin{aligned} C_{L_{eff}} &= C_L \times \Phi(\tau) \\ C_{D_{eff}} &= C_D \times \Phi(\tau) \\ C_{M_{eff}} &= C_M \times \Phi(\tau) \end{aligned} \right\} \quad (3.24)$$

It should be noted from Equation 3.24 that if the vehicle is constantly changing its attitude, the actual aerodynamic parameter will be 50 % of the static value.

CHAPTER 4

Steady Transition Maneuver

4.1 Background

The study presented in this chapter deals with the flight dynamics modeling and comparative analysis of conventional scheme with aerodynamic vectoring modification. This chapter deals with the steady transition analysis; during which the aircraft is under the trim state across the complete transition envelope. The analysis does not cater an aerobatic/agile maneuver, but it's a kind of pitch hold scenario at specific airspeed. The aircraft can have sustained flight with any desired velocity between hover and cruise. The aircraft is in helicopter mode pitched at high angle of attack during low velocities specifically below stall. The trim pitch angle of the aircraft decreases as the speed increases eventually following conventional forward flight conditions. The phenomenon is demonstrated in Figure 4-1. This analysis is useful for better understanding of flight characteristics as well as configuration and control design of UAV systems.

Figure 4-1 Longitudinal equilibrium attitudes in various flying conditions: at hover, low speeds under stall and fully developed forward flight

In this chapter, the stability characteristics are investigated for the UAV longitudinal dynamics. The trim analysis for conventional and aerodynamic-vectoring UAV across the complete flight envelope between hover and cruise conditions is investigated. The linear stability analysis around these trim points is carried out. Subsequently, eigenvalue migration for phugoid and short period modes are observed.

To get the complete picture of the dynamics, the analysis is extended to nonlinear domain. Specifically, Multiple Time Scales (MTS) method is used to approximate solutions in parametric form. For further investigations, bifurcation theory is used to study the effect of the nonlinear dynamics across the broad velocity envelope. The analytical derivation leads us to the closed form solution of the longitudinal dynamic phenomena. The solution offers an advantage over numerical approach in that the interdependence of the important parameters affecting the dynamic properties of the system can be easily seen.

4.2 Longitudinal Equations of Motion

In this section, the nonlinear equations of motion in longitudinal mode are discussed. The process of linearization of the longitudinal dynamics using Taylor expansion is also delineated. Moreover, the transformation between different longitudinal set of equations is also presented.

4.2.1 Nonlinear Equations of Motion

Since the aircraft has a symmetric configuration about its vertical plane and is assumed to perform only symmetric flight, it is reasonable to assess its longitudinal dynamics separately. Therefore coupling inertial terms are

neglected in the following equations. Moreover, only rectilinear motion in the vehicle's plane of symmetry is considered and the effects of elastic deformation are assumed to be negligible, that is, the UAV is considered to be of rigid platform. Under these usual assumptions, the longitudinal equations of motion are nonlinear and non-autonomous in general. The followings are the longitudinal equations of motion of the aircraft in body axes, where the axes notation is shown in Figure 4-2.

$$\dot{u} = X/m - g \sin \theta - qw \quad (4.1a)$$

$$\dot{w} = Z/m + g \cos \theta + qu \quad (4.1b)$$

$$\dot{q} = M/I_{yy} \quad (4.1c)$$

$$\dot{\theta} = q \quad (4.1d)$$

where u, w are horizontal and vertical velocities respectively; X and Z are the horizontal and vertical force vectors; M is the pitching moment; g is the acceleration due to gravity; q is the pitch rate; m is the mass of the aircraft and I_{yy} is the moment of inertia in the longitudinal mode. The forces and moment involved may be represented in the following manner:

$$X = X_{Aero} + X_{Thrust} + X_{Other} \quad (4.2a)$$

$$Z = Z_{Aero} + Z_{Thrust} + Z_{Other} \quad (4.2b)$$

$$M = M_{Aero} + M_{Thrust} + M_{Other} \quad (4.2c)$$

where $[]_{Aero}, []_{Thrust}$ and $[]_{Other}$ represents aerodynamic, propulsive and miscellaneous forces respectively.

Figure 4-2 Longitudinal Aircraft Dynamics Axes Transformation

The kinematic relationships from body to inertial axes are as follows:

$$\left. \begin{aligned} \ddot{x}_F &= \ddot{x}_B \cos \theta + \ddot{z}_B \sin \theta \\ \ddot{z}_F &= -\ddot{x}_B \sin \theta + \ddot{z}_B \cos \theta \\ \dot{x}_F &= \dot{x}_B \cos \theta + \dot{z}_B \sin \theta \\ \dot{z}_F &= -\dot{x}_B \sin \theta + \dot{z}_B \cos \theta \end{aligned} \right\} \quad (4.3)$$

where the subscript F indicates quantities expressed in inertial frame.

4.2.2 Linearized Longitudinal Equations of Motion

The longitudinal equations mentioned above are non-linear in nature. A common analysis practice is to linearize them around a specific trim point using small disturbance theory. In applying the small-disturbance theory, we assume that the motion of the airplane consists of small deviations about a steady flight condition (see Nelson [46]). All the variables in the equations of motion are replaced by a steady trimmed value plus a perturbation or disturbance as shown in Equation 4.4.

$$\begin{aligned} u &= u_o + \Delta u & w &= w_o + \Delta w & q &= q_o + \Delta q \\ X &= X_o + \Delta X & Z &= Z_o + \Delta Z & M &= M_o + \Delta M \end{aligned} \quad (4.4)$$

where $(.)_o$ indicates the trim state and $\Delta(.)$ indicates the perturbation or disturbance. Based on the derivation using small disturbance theory about a specific trim point, the longitudinal equations of motion are linearized in stability axis system. Nelson [46] has presented these equations in state-space form by neglecting several stability derivatives like $M_{\dot{w}}, Z_{\dot{w}}$ and Z_q . For the dynamics at hover and slow forward flight at high angles of attack, such simplification may not be appropriate and some other stability derivatives (usually neglected for small trim angles) must be considered. In the present analysis, a more detailed longitudinal dynamic model is considered such that the stability derivatives $M_{\dot{w}}, Z_{\dot{w}}$ and Z_q are not neglected. Go [47] has derived the state space representation with less neglected stability derivatives from the linearized equations of motion as shown in Equation 4.5 in state-space form.

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m - \hat{Z}_{\dot{w}} & 0 & 0 \\ 0 & -\hat{M}_{\dot{w}} & I_{yy} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \hat{X}_u & \hat{X}_w & 0 & -mg \cos \theta_o \\ \hat{Z}_u & \hat{Z}_w & \hat{Z}_q + mU_o & -mg \sin \theta_o \\ \hat{M}_u & \hat{M}_w & \hat{M}_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} \hat{X}_{\delta_e} & \hat{X}_{\delta_T} & \hat{X}_{\delta_i} \\ \hat{Z}_{\delta_e} & \hat{Z}_{\delta_T} & \hat{Z}_{\delta_i} \\ \hat{M}_{\delta_e} + \hat{M}_{\dot{w}}\hat{Z}_{\delta_e} & \hat{M}_{\delta_T} + \hat{M}_{\dot{w}}\hat{Z}_{\delta_T} & \hat{M}_{\delta_i} + \hat{M}_{\dot{w}}\hat{Z}_{\delta_i} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_T \\ \delta_i \end{bmatrix} \quad (4.5)$$

The Δ symbol with all the stability derivatives has been omitted in Equation 4.5 for brevity purposes. Equation 4.5 can be compared with the following state-space form

$$E\dot{\mathbf{x}} = \bar{A}\mathbf{x} + \bar{B}\mathbf{u} \quad (4.6)$$

$$\dot{\mathbf{x}} = E^{-1}(\bar{A}\mathbf{x} + \bar{B}\mathbf{u}) = A\mathbf{x} + B\mathbf{u} \quad (4.7)$$

where $\mathbf{x} = (u \ w \ q \ \theta)^T$, $\mathbf{u} = (\delta_e \ \delta_T \ \delta_i)^T$ and the matrices E , $\bar{A}$ and $\bar{B}$ are obvious through comparison of Equation 4.5 and Equation 4.6. If we present stability derivatives by dividing them with its moment of inertias or mass, such that $X_u = \hat{X}_u / m$, $Z_u = \hat{Z}_u / m$ and $M_q = \hat{M}_q / I_{yy}$, the comprehensive form of matrix A can be presented as:

$$A = \begin{bmatrix} X_u & X_w & 0 & -g \cos \Theta_0 \\ \frac{Z_u}{1 - Z_{\dot{w}}} & \frac{Z_w}{1 - Z_{\dot{w}}} & \frac{Z_q + U_0}{1 - Z_{\dot{w}}} & \frac{-g \sin \Theta_0}{1 - Z_{\dot{w}}} \\ M_u + Z_u \Gamma & M_w + Z_w \Gamma & M_q + (Z_q + U_0) \Gamma & -g \sin \Theta_0 \Gamma \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (4.8)$$

where $\Gamma = \frac{M_{\dot{w}}}{1 - Z_{\dot{w}}}$.

4.3 Linear Dynamic Analysis

The stability issues of the UAV across a broad velocity spectrum between hover and cruise are investigated here. The trim states are evaluated for the fixed and variable incidence wing configuration and subsequently the dynamic stability approximations are carried out to evaluate the UAV dynamics by computing stability derivatives.

4.3.1 Trim Analysis

In order to obtain the trim flight conditions, i.e. $\dot{u}$, $\dot{w}$ and $\dot{\theta}$ equal to zero, a numerical approach is used. The problem is formulated as a nonlinear constrained optimization problem and the MATLAB routine, *fmincon*, is used to find the trimmed states. *Fmincon* is based on hybrid Sequential Quadratic Programming (SQP), which represents state-of-the-art in nonlinear

programming methods, and Quasi-Newton methods. The method allows to closely mimic Newton's method for constrained optimization just as is done for unconstrained optimization. *Fmincon* finds the constrained minimum of scalar function of several variables starting with an initial estimate. This is generally referred to as constrained nonlinear optimization or nonlinear programming. The input is the initial guess of the variable/s to be optimized. At each iteration, the scalar objective function is evaluated subjected to the constraints posed to the problem. The output of each iteration is the input for the next iteration. In order to increase the convergence rate; the initial guess should be realistic and near to the optimal output. If the initial guess is remote from optimal values, then the convergence will be very slow and sometimes can be completely divergent.

For the case here, the motion is three degree of freedom and with the addition of the wing incidence as a control variable, there are overall four parameters to be optimized. The control parameters to be optimized for the trim states from 0 to 15 m/s velocity range are:

$$\bar{\mathbf{c}} = [\alpha_{fus}; \alpha_{wing}; T/W; \delta_{elev}]^T \quad (4.9)$$

where δ_{elev} is the elevator angle with respect to the fuselage. The cost function to be minimized in the optimization is as follows:

$$J = X^2 + Z^2 + M^2 \quad (4.10)$$

which corresponds to the trim flight conditions, where the components of the resultant forces and moments X , Z and M as shown in Equation 4.2 are in equilibrium state. The constraints posed to the state variables are shown below.

$$\left\{ \begin{array}{l} 0^0 \leq \alpha_{fus} \leq 90^0 \\ 0^0 \leq \alpha_{wing} \leq 30^0 (Variable-Incidence) \\ \alpha_{fus} = \alpha_{wing} (Fixed-Incidence) \\ 0 \leq T/W \leq 1.00 \\ -20^0 \leq \delta_{elev} \leq 20^0 \end{array} \right. \quad (4.11)$$

From several initial guesses, evaluated trim states across different velocities are shown from Figure 4-3 to Figure 4-6 for the conventional and aerodynamic vectoring cases.

Figure 4-3 Comparison of trimmed thrust-to-weight ratio

As shown in Figure 4-3, the T/W gradually decreases from the perfect hover condition at 0 m/s to the cruise conditions at 15 m/s for both cases. The T/W requirement for the variable-incidence wing scheme is substantially lower than the fixed-wing scheme until about 12 m/s. In this regime, the propulsive forces are augmented by the additional lift due to the variable incidence wing, which is always in pre-stall regime.

Figure 4-4 Comparison of trimmed pitch angle

The pitch angle of the aircraft reduces from 90° in hover to the cruise pitch angle as the velocity of the aircraft increases for both schemes as shown in Figure 4-4. For the variable-incidence wing case, an interesting sharp reduction in pitch is observed between 7 and 8 m/s whereas, for the fixed-wing case, the sharp reduction in pitch is not observed until about 9 m/s. This is due to the shift in the flight condition from thrust borne to aerodynamic borne in these regimes. This shift occurs at lower speeds for the variable-incidence wing case because of the improved aerodynamic efficiency due to effective wing angle in the pre-stall regime.

Figure 4-5 Comparison of trimmed wing angle of attack

From Figure 4-5, it can be observed that the wings remain at the Cl_{max} state until about 7-8 m/s in the variable-incidence case. Note that for the fixed-wing case, α_{wing} remains aligned with α_{fus} .

Figure 4-6 Comparison of trimmed elevator deflection angles

An advantage of the variable-incidence wing in the elevator control effort is also observed in Figure 4-6. The elevator deflection for the fixed-wing case is higher than that of the variable-incidence wing one at low speeds, thereby reducing the elevator control authority to counter disturbances. Ideally, reduced control efforts are desirable during slow speeds to have enough margins to counter any disturbances. From the discussion above, the variable-incidence wing/aerodynamic vectoring has shown advantages in terms of reduced T/W requirement and elevator deflection to achieve the trimmed low speed flight.

4.3.2 Stability Derivatives Evaluation

The stability derivatives are evaluated from the numerical aerodynamic data/empirical methods by assuming linearity in the aerodynamics about the trimmed flight states (Appendix A). The equations used for the evaluation of the stability derivatives are shown below. The u and w derivatives are calculated from Equations 4.12 to 4.16 based on the aerodynamic data obtained earlier.

$$X_u = \frac{-(C_{D_u} + 2C_{D_0})QS}{mu_0} \quad (4.12)$$

$$Z_u = \frac{-(C_{L_u} + 2C_{L_0})QS}{mu_0} \quad (4.13)$$

$$X_w = \frac{-(C_{D_\alpha} - C_{L_0})QS}{mu_0} \quad (4.14)$$

$$Z_w = \frac{-(C_{L_\alpha} + C_{D_0})QS}{mu_0} \quad (4.15)$$

$$M_u = C_{m_u} \frac{(QS\bar{c})}{u_0 I_{yy}} \quad (4.16)$$

$$M_w = C_{m_\alpha} \frac{(QS\bar{c})}{u_0 I_{yy}} \quad (4.17)$$

where $C_{D_u} = \frac{\partial C_D}{\partial u}$, $C_{L_u} = \frac{\partial C_L}{\partial u}$, $X_u = \frac{\partial X}{\partial u}$ and so on. Moreover, Q is dynamic pressure; S is surface area of the wing and $\bar{c}$ is the mean aerodynamic chord.

The downwash effect approximation from Phillips [48] is used in order to evaluate remaining stability derivatives. The downwash varies along the span of the horizontal tail and is affected by the planform shape of the wing as well as the presence of fuselage and nacelles. The downwash angle can be approximated by:

$$\varepsilon_d = \frac{4\kappa_v C_{L_w}}{\pi^2 \kappa_b AR_w} \quad (4.17)$$

where C_{L_w} =coefficient of the lift of the wing

AR_w =aspect ratio of the wing.

κ_v =wing-tip vortex strength factor from Prandtl's lifting-line theory

κ_b =wing-tip vortex span factor from Prandtl's lifting-line theory

The coefficient of lift as well as the aspect ratio of the aircraft is known. The wing-tip vortex strength and span factors can be evaluated from the Figure 4-7.

Figure 4-7 Prandtl's lifting line theory: Wing-tip vortex strength factor (left); Wing-tip vortex span factor (right) [48]

The rest of the stability derivatives are calculated using the approximations given in Equations 4.18 to 4.25 [46].

$$C_{Z_{\dot{\alpha}}} = -2\eta C_{L_{\alpha_t}} V_H \frac{d\varepsilon}{d\alpha} \quad (4.18)$$

$$Z_{\dot{w}} = C_{Z_{\dot{\alpha}}} \frac{\bar{c}}{2u_0} QS / (u_0 m) \quad (4.19)$$

$$C_{m_{\dot{\alpha}}} = -2\eta C_{L_{\alpha_t}} V_H \frac{l_t}{\bar{c}} \quad (4.20)$$

$$M_{\dot{w}} = C_{m_{\dot{\alpha}}} \frac{\bar{c}}{2u_0} \frac{QS\bar{c}}{u_0 I_{yy}} \quad (4.21)$$

$$C_{Z_q} = -2\eta C_{L_{\alpha_t}} V_H \quad (4.22)$$

$$Z_q = C_{Z_q} \frac{\bar{c}}{2u_0} QS / m \quad (4.23)$$

$$C_{m_q} = -2\eta C_{L_{\alpha_t}} V_H \frac{l_t}{\bar{c}} \quad (4.24)$$

$$M_q = C_{m_q} \frac{\bar{c}}{2u_0} (QS\bar{c}) / I_{yy} \quad (4.25)$$

where $Z_{\dot{w}} = \frac{\partial Z}{\partial \dot{w}}$, $C_{Z\alpha} = \frac{\partial C_Z}{\partial \alpha}$ and so on.

Moreover, η = horizontal tail efficiency factor

V_H = Horizontal tail volume ratio

$\frac{d\varepsilon}{d\alpha}$ = Change in downwash due to change in angle of attack

The stability derivatives discussed above are evaluated across the trim conditions specified in previous section.

4.3.3 Analysis of Dynamic Characteristics

The dynamic stability characteristics are evaluated using the model discussed in Section 4.2.2. The eigenvalues of the matrix A of Equation 4.8 are calculated to evaluate the open-loop stick-fixed stability of the UAV across the broad envelope. The dynamic stability considerations of the aerodynamic vectoring UAV are documented in comparison with the conventional fixed-wing UAV. It should be noted that the present linear stability analysis is based on the trim states presented in Section 4.3.1.

In Figure 4-8 and Figure 4-9, the eigenvalues associated with the short period mode for various trim airspeeds are plotted for the fixed and variable-incidence wing cases. At higher speeds (beyond 8 m/s), the short-period mode of both cases is stable and its damping increases with the increase in airspeed. This mode is also stable at the low speeds.

Figure 4-8 Variation of short-period eigenvalues with airspeed for the fixed-wing aircraft

For the variable-incidence wing case, this eigenvalue analysis suggests that the short period mode becomes unstable for the airspeed between 7 and 8 m/s. Unlike the variable-incidence wing case, in the fixed-wing configuration, the aircraft exhibits stable short period mode over the whole airspeed range.

Figure 4-9 Variation of short-period eigenvalues with airspeed for the variable-incidence wing aircraft

The eigenvalue comparison also suggests that the short-period natural frequency for the fixed-wing case is substantially higher than for the variable-incidence wing case. This is due to the fact that the frequency of short-period

mode is influenced mainly by M_α as can be seen from the approximation below [46].

$$\omega_{sp} = [M_q Z_w - M_\alpha]^{1/2} \quad (4.26)$$

The higher the magnitude of M_α , the higher the short-period natural frequency will be and vice-versa. The M_α comparison between the fixed and variable-incidence wing cases for the complete speed envelope is given in Figure 4-10.

Figure 4-10 Comparison of M_α for the fixed and variable-incidence wing cases

It can be seen that the magnitude of M_α for the fixed-wing case is substantially higher in most velocity regime compared to that of the variable-incidence wing case. The damping of the short period mode can be approximated using [46]

$$2\omega_{sp}\zeta_{sp} = -[M_q + u_o M_{\dot{w}} + Z_w] \quad (4.27)$$

which is a function of M_q , $M_{\dot{w}}$ and Z_w . These parameters are plotted in Figure 4-11 to Figure 4-13 as functions of airspeed. From these figures, it can be deduced that the main difference in the short period damping is due to the difference in Z_w between the two cases, which has a positive value for the

variable-incidence wing case at 7-8 m/s airspeed range, while it is negative for the fixed-wing case.

Figure 4-11 Comparison of M_q for the fixed and variable-incidence cases

Figure 4-12 Comparison of Z_w for the fixed and variable-incidence wing cases

The build-up of aerodynamic forces in this flow regime starts playing an important role and the primary difference is that the net C_{L_α} slope is negative (post-stall regime) for the fixed-wing case and is positive (pre-stall regime) for the variable-incidence one. This makes the total magnitude of the right hand side of Equation 4.27 negative and thereby contributes to the migration of the eigenvalues to the right-half of the complex plane for the variable-incidence wing case.

Figure 4-13 Comparison of $M_{\dot{w}}$ for the fixed and variable-incidence wing cases

It should be noted that the current analysis is based on linearization about steady trim points. Hence, even though the current analysis indicates short period instability in the 7-8 m/s speed regimes, the nature of the departure from the trim point is not necessarily exponential. The crossing of the eigenvalues from the left-half of the complex plane to the right –half plane often indicates the presence of Hopf bifurcation in the associated nonlinear system, where limit cycle type of oscillation appears instead of exponential instability. Indeed, that is the case here. Through numerical simulation of nonlinear longitudinal dynamic equations around these trim points, it is observed that limit cycles appear in this speed regime. Figure 4-14 shows an example of the aircraft response when the trim point associated with variable-incidence wing case at 8 m/s airspeed is perturbed.

Figure 4-14 Nonlinear response of velocity (left) and pitch rate(right) to perturbation for trim point of 8 m/s

In Figure 4-15 and Figure 4-16, the variation of the phugoid eigenvalues with airspeed is plotted for the fixed-wing and the variable-incidence wing cases. For the variable-incidence wing case, the aircraft shows an unstable phugoid behavior below 13 m/s. In this speed region, the fuselage angle of attack is higher than 10°.

Figure 4-15 Variation of phugoid eigenvalues with airspeed for the variable-incidence wing aircraft

For the fixed-wing case, the aircraft maneuver is unstable in two velocity regions: 10-12 m/s and 0-7 m/s. Comparison of the phugoid mode between the fixed and variable-incidence wing cases indicates that the variable-incidence wing aircraft has a reduced phugoid damping compared to the fixed-wing. The phugoid damping is affected by the lift to drag ratio as can be seen from the phugoid damping approximations below [46].

$$\zeta_{ph} = \frac{1}{\sqrt{2}} \frac{1}{L/D} \quad (4.28)$$

The higher the lift to drag ratio, the lower the damping will be as shown in Equation (4.28). For the variable incidence scheme, L/D is substantially higher because the wing remains in the pre-stall regime across all trimmed states, as shown in Figure 4-5, whereas for the fixed-wing case, the wing is in post-

stall regime for the most trimmed conditions, especially at low speeds. This causes low L/D value for the fixed-wing case and therefore, its phugoid damping is relatively higher to that of the variable-incidence one. In general, phugoid instability is less of a concern most flight regime compared to the short period one due to its relatively low frequency. For this reason, the focus here is more on the short-period instability.

Figure 4-16 Variation of phugoid eigenvalues with airspeed for the fixed-wing aircraft

4.4 Nonlinear Analysis

In this section, nonlinear analysis of the longitudinal dynamics is carried out. Based on the linear comparative analysis, between conventional and aerodynamic vectoring configurations, limit cycle phenomenon associated with aerodynamic vectoring configuration was discovered. Linear systems theory is limited and cannot decipher the limit cycle characteristics in relation to the aircraft parameters.

To overcome this limitation, nonlinear analysis is performed. Specifically, Multiple Time Scales (MTS) method is used to reduce the equations of motion into a set of first-order equations for the amplitude and phase of the dynamics.

Initially single degree of freedom involving only pitch dynamics is studied. Based on the limitation of the single DOF analysis, multiple degree of freedom analysis is then carried out. Bifurcation theory is used to assess the properties of the solution. Analytical solutions are derived and relevant stability parameters causing instability and limit-cycle behavior of the UAV is identified.

4.4.1 Multiple Time Scale Methods

4.4.1.1 Concept

The nonlinear analytical technique used here is known as Multiple Time Scales (MTS) Method. The development of this technique is based on the work of Ramnath [49, 50]. MTS is a well-established technique and ample amount of literature is available on its mathematical formulation and limitations, therefore readers are referred to these references for further details.

Complex dynamic systems generally exhibit a mixture of fast and slow response. Some parameters of a certain system may govern the fast dynamic behavior while other may affect the slow dynamic response of the system. In order to fully understand the system so that desired alterations during design and development cycle can be made, understanding the response due to relevant parameters on the slow and fast system behavior can be vital. Generally, instabilities in slow behavior are less threatening than in the fast dynamics. The MTS approach separates these slow and fast manifolds of the system explicitly and is based exactly on this separation idea.

The MTS method belongs to the family of perturbation methods. It is an asymptotic approach to approximate the physical problems that involve perturbations about nominal states specifically in limiting cases. Specifically,

MTS works on the idea of extension. The idea of the extension is to transform the existing dimension of time to a multiple dimensional space. Since many physical systems of interest exhibit multiple natural time scales, the MTS method is applicable to a wide range of problems. One example is the separation of phugoid mode (slow varying manifold) and short-period mode (fast varying manifold) in aircraft longitudinal dynamics.

The MTS method is extensively used by Go [51-54] for studying wing rock problem. The schematic for the extension of time scales is shown in Figure 4-17. Although the ordinary differential equations are transformed to partial differential equations in the multiple-scaling process, the resulting partial differential equations are usually in simpler form and more readily solvable than the original ordinary differential equations. Each time scale captures the certain response of the dynamic system. For example, the slow time scale will capture the slow manifold of the dynamic system. The extension of time to a higher dimensional space can be represented as:

$$t \rightarrow \{\tau_o, \tau_1, \dots, \tau_n\} \quad (4.29)$$

where $\tau_o, \tau_1, \dots, \tau_n$ are different time scales. These time scales are a function to time t and small perturbation parameter ε such that

$$\tau_i = t_i(t, \varepsilon) \quad (4.30)$$

The value of ε tunes a certain time scale to be slow or fast. For an ordinary differential equation, as the consequence of the extension of t , the dependent variable $y(t, \varepsilon)$ is also extended as

$$y(t, \varepsilon) \rightarrow Y(\tau_o, \tau_1, \dots, \tau_n, \varepsilon) \quad (4.31)$$

Once Y is solved, the time scales can be substituted back to approximate the solution of the original ordinary differential equation in time t :

$$Y(\tau_o(t, \varepsilon), \tau_1(t, \varepsilon), \dots, \tau_n(t, \varepsilon), \varepsilon) = y(t, \varepsilon) \quad (4.32)$$

Figure 4-17 shows a schematic illustration of the concept.

Figure 4-17 Illustration of multiple time scales concept

4.4.2 Mathematical Modeling of Longitudinal Dynamics

One set of nonlinear longitudinal equations of motion has already presented in Equation 4.1. Here we consider another set of equations, known as hybrid longitudinal equations of motion, frequently used in literature to model problems associated with longitudinal dynamics. .

$$\dot{V} = \frac{1}{m} [T \cos(\alpha + i) - D - mg \sin \gamma] \quad (4.33a)$$

$$\dot{\gamma} = \frac{1}{mV} [T \sin(\alpha + i) + L - mg \cos \gamma] \quad (4.33b)$$

$$\dot{q} = M / I_{yy} \quad (4.33c)$$

$$\dot{\alpha} = \dot{\theta} - \dot{\gamma} = q - \frac{1}{mV} [T \sin(\alpha + i) + L - mg \cos \gamma] \quad (4.33d)$$

where V is the velocity; γ is the flight path angle; i is the wing incidence angle and I_{yy} is the moment of inertia in the longitudinal mode. Equation 4.1 can be obtained from Equation 4.33 using following transformation:

$$\begin{bmatrix} V \\ \gamma \end{bmatrix} = \begin{bmatrix} \sqrt{u^2 + w^2} \\ \sin^{-1}(w/V) \end{bmatrix} \quad (4.34)$$

The inverse transformation from velocity and flight path angle to body axes velocity components is written as:

$$\begin{bmatrix} u \\ w \end{bmatrix} = \begin{bmatrix} V \cos \gamma \\ V \sin \gamma \end{bmatrix} \quad (4.35)$$

The aerodynamic lift and moment coefficients over a range of wing angle of attack are fitted in a cubic polynomial. The curve-fitting of outboard wing aerodynamic data with cubic-polynomials of lift and moment coefficient is shown in Figure 4-18.

Figure 4-18 Aerodynamic data versus cubic approximation for coefficients of lift and pitching moment

Then the lift and moment coefficients are plugged in the rigid body nonlinear equations of motion. The longitudinal dynamics is considered in vertical plane of symmetry and its coupling with lateral-directional motions is neglected. The coefficient of lift can be expressed as:

$$C_L = e_o + e_1 \alpha_{we} + e_2 \alpha_{we}^2 + e_3 \alpha_{we}^3 + C_{L_q} q + C_{L_{\dot{\alpha}}} \dot{\alpha} \quad (4.36)$$

where $e_{(\cdot)}$ represents the constant coefficients; C_{L_q} and $C_{L_{\dot{\alpha}}}$ are the dynamic stability derivatives with respect to pitch rate and rate of change in angle of attack and α_{we} is the effective wing angle of attack. It is observed that the main contribution of lift is from the wing and thus, the contributions from fuselage and elevator are neglected for simplicity. Similarly, the coefficient of pitching moment can be expressed as

$$C_M = c_3 \alpha_{we}^3 + c_2 \alpha_{we}^2 + c_1 \alpha_{we} + c_o + g_3 \alpha_{fee}^3 + g_2 \alpha_{fee}^2 + g_1 \alpha_{fee} + g_o + C_{m_q} q + C_{m_{\dot{\alpha}}} \dot{\alpha} \quad (4.37)$$

where $c_{(\cdot)}$ and $g_{(\cdot)}$ are constant coefficients; C_{m_q} and $C_{m_{\dot{\alpha}}}$ are the pitching moment derivatives with respect to pitch rate and rate of change in angle of attack and α_{eff} is the effective elevator angle of attack. The pitching coefficient is expressed as a function of α_{we} , α_{fee} , pitch rate q and rate of change of angle of attack $\dot{\alpha}$.

The kinematic relationships for the fuselage, wing and elevator angles can be represented as

$$\alpha_{fuse} = m_0 + \alpha, \alpha_{we} = m_1 + \alpha, \alpha_{fee} = m_2 + \alpha \quad (4.38)$$

where $m_0 = \alpha_{fuso}$, $m_1 = \alpha_{wo}$, $m_2 = \delta_{eo} + \alpha_{fuso}$ are constants for a certain trim state and α is the perturbation parameter. In addition to the above relationships,

$$\alpha = \theta - \gamma, \dot{\alpha} = \dot{\theta} - \dot{\gamma}, \ddot{\alpha} = \ddot{\theta}, \dot{\theta} = q \quad (4.39)$$

By substituting the above kinematic relationships into the pitching moment coefficient (Equation 4-37), the variables can be separated into two distinctive groups of nominal and perturbation expressions,

$$\begin{aligned}
C_M = & C_{m_q} q + C_{m_{\dot{\alpha}}} \dot{\alpha} + (c_3 + g_3) \alpha^3 + (3m_1 c_3 + c_2 + 3m_2 g_3 + g_2) \alpha^2 \\
& + (3m_1^2 c_3 + 2m_1 c_2 + c_1 + 3m_2^2 g_3 + 2m_2 g_2 + g_1) \alpha \\
& + (m_1^3 c_3 + m_1^2 c_2 + m_1 c_1 + c_o + m_2^3 g_3 + m_2^2 g_2 + m_2 g_1 + g_o)
\end{aligned} \tag{4.40}$$

Multiplication of $z_1 = \frac{\rho V^2 S \bar{c}}{2}$ on both sides of Equation 4.40

dimensionalizes the non-dimensional pitching moment coefficient into

$$M = \hat{d}_3 \alpha^3 + \hat{d}_2 \alpha^2 + \hat{d}_1 \alpha + \hat{d}_4 \dot{\theta} + \hat{d}_5 \dot{\alpha} + \hat{M}_o \tag{4.41}$$

where $\hat{d}_1 = z_1 (3m_1^2 c_3 + 2m_1 c_2 + c_1 + 3m_2^2 g_3 + 2m_2 c_2 + g_1)$,

$$\hat{d}_2 = z_1 (3m_1 c_3 + c_2 + 3m_2 g_3 + g_2),$$

$$\hat{d}_3 = z_1 (c_3 + g_3),$$

$$\hat{d}_4 = z_1 C_{m_q},$$

$$\hat{d}_5 = z_1 C_{m_{\dot{\alpha}}}, \text{ and}$$

$$\hat{M}_o = z_1 (m_1^3 c_3 + m_1^2 c_2 + m_1 c_1 + c_o + m_2^3 g_3 + m_2^2 g_2 + m_2 g_1 + g_o).$$

Similarly the kinematic relationships in Equations 4.38 and 4.39 are substituted into Equation 4.36 to yield

$$\begin{aligned}
C_L = & e_3 \alpha^3 + (e_2 + 3m_1 e_3) \alpha^2 + (e_1 + 2m_1 e_2 + 3m_1^2 e_3) \alpha \\
& + (e_o + m_1 e_1 + m_1^2 e_2 + m_1^3 e_3) + C_{L_q} q + C_{L_{\dot{\alpha}}} \dot{\alpha}
\end{aligned} \tag{4.42}$$

Multiplying both sides of Eq. (4.42) by $z_2 = \frac{\rho V^2 S}{2}$ gives us

dimensionalized lift relationship as

$$\hat{L} = \hat{b}_3 \alpha^3 + \hat{b}_2 \alpha^2 + \hat{b}_1 \alpha + \hat{b}_4 \dot{\theta} + \hat{b}_5 \dot{\alpha} + \hat{b}_6 \tag{4.43}$$

where $\hat{L}$ is lift, $\hat{b}_1 = z_2(e_1 + 2m_1e_2 + 3m_1^2e_3)$, $\hat{b}_2 = z_2(e_2 + 3m_1e_3)$, $\hat{b}_3 = z_2e_3$,
 $\hat{b}_4 = z_2C_{L_q}$, $\hat{b}_5 = z_2C_{L_{\dot{\alpha}}}$ and $\hat{b}_6 = z_2(e_o + m_1e_1 + m_1^2e_2 + m_1^3e_3)$. The equation for
the pitch motion can be expressed as:

$$\dot{q} = \ddot{\theta} = \frac{M}{I_{yy}} \quad (4.44)$$

Substitution of the pitching moment from Equation 4.41 into Equation 4.44 leads
to

$$\ddot{\theta} = d_3\alpha^3 + d_2\alpha^2 + d_1\alpha + d_4\dot{\theta} + d_5\dot{\alpha} \quad (4.45)$$

where $d(\bullet) = \frac{\hat{d}(\bullet)}{I_{yy}}$. It can be observed that in the Equation 4.45, M_o is zero

because the aircraft is under trim in nominal condition. The flight path equation
of the aircraft:

$$\dot{\gamma} = \frac{1}{mV} [T \sin \alpha_{fuse} + \hat{L} - W \cos \gamma_e] \quad (4.46)$$

where T is the trim thrust value of the aircraft. Also, $\gamma_e = \gamma_o + \Delta\gamma$ such that γ_e is
the effective flight path angle of the aircraft, γ_o is the trim flight path angle and
 $\Delta\gamma$ is the perturbed flight path. For straight and level trim flight conditions,
 $\dot{\gamma}_o = \gamma_o = 0$ which leaves $\Delta\gamma = \gamma$ as well as $\Delta\dot{\gamma} = \dot{\gamma}$. Before proceeding, Equation
4.43 is divided by mV on both sides so that

$$L = \frac{\hat{L}}{mV} = b_3\alpha^3 + b_2\alpha^2 + b_1\alpha + b_4\dot{\theta} + b_5\dot{\alpha} + b_6 \quad (4.47)$$

where $b(\bullet) = \frac{\hat{b}(\bullet)}{mV}$. Substituting Equation 4.46 into Equation 4.47 and

approximating $\sin \alpha_{fuse} \cong \alpha_{fuse} - \frac{\alpha_{fuse}^3}{6}$ and $\cos \gamma \cong 1 - \frac{\gamma^2}{2}$ gives us the following relationship.

$$(1 - b_4)\dot{\gamma} = \hat{k}_3\alpha^3 + \hat{k}_2\alpha^2 + \hat{k}_1\alpha + \hat{k}_4\gamma^2 + \hat{k}_5\dot{\alpha} + \hat{\Gamma}_o \quad (4.48)$$

where $\hat{k}_3 = b_3 - \frac{T}{6mV}$, $\hat{k}_2 = b_2 - \frac{Tm_o}{2mV}$, $\hat{k}_1 = b_1 + \frac{T}{mV}\left(1 - \frac{m_o^2}{2}\right)$, $\hat{k}_4 = \frac{W}{2mV}$,

$\hat{k}_5 = b_4 + b_5$ and $\hat{\Gamma}_o = b_6 + \frac{T}{mV}\left(m_o - \frac{m_o^3}{6}\right) - \frac{W}{mV}$. Equation 4.48 is transformed

into the standard form below by dividing both sides with $(1 - b_4)$

$$\dot{\gamma} = k_3\alpha^3 + k_2\alpha^2 + k_1\alpha + k_4\gamma^2 + k_5\dot{\alpha} + \Gamma_o \quad (4.49)$$

where $k(\bullet) = \frac{\hat{k}(\bullet)}{(1 - b_4)}$ and $\Gamma_o = \frac{\hat{\Gamma}_o}{(1 - b_4)}$. From the above equation, it can be

observed that if the aircraft is in level trim state, then $\Gamma_o = 0$. It implies that the trim flight path is horizontal to the flat earth. Therefore, the flight path equation reduces to

$$\dot{\gamma} = k_3\alpha^3 + k_2\alpha^2 + k_1\alpha + k_4\gamma^2 + k_5\dot{\alpha} \quad (4.50)$$

The angle of attack at any instant, as mentioned earlier, can be represented by the kinematic relationship $\alpha = \theta - \gamma$. Differentiating it with respect to time and substituting Equation 4.50 results in

$$\dot{\alpha} = \dot{\theta} - k_3\alpha^3 - k_2\alpha^2 - k_1\alpha - k_4\gamma^2 - k_5\dot{\alpha} \quad (4.51)$$

Rearranging Equation 4.51 such that $\dot{\alpha}$ terms are on one side yields

$$(1 + k_5)\dot{\alpha} = \dot{\theta} - k_3\alpha^3 - k_2\alpha^2 - k_1\alpha - k_4\gamma^2 \quad (4.52)$$

Differentiating Equation 4.52 with time leads to

$$(1 + k_5)\ddot{\alpha} = \ddot{\theta} - 3k_3\alpha^2\dot{\alpha} - 2k_2\alpha\dot{\alpha} - k_1\dot{\alpha} - 2k_4\dot{\gamma} \quad (4.53)$$

Substituting Equations 4.45 and 4.50 into Equation 4.53 and rearranging results in

$$\begin{aligned} \ddot{\alpha} = & \left(\frac{d_3}{1+k_5} \right) \alpha^3 + \left(\frac{d_2}{1+k_5} \right) \alpha^2 + \left(\frac{d_1}{1+k_5} \right) \alpha + \left(\frac{d_4}{1+k_5} \right) \dot{\theta} \\ & + \left(\frac{d_5 - k_1}{1+k_5} \right) \dot{\alpha} + \left(\frac{-3k_3}{1+k_5} \right) \alpha^2 \dot{\alpha} + \left(\frac{-2k_4 - 2k_2}{1+k_5} \right) \alpha \dot{\alpha} \\ & + \left(\frac{2k_4}{1+k_5} \right) \alpha \dot{\theta} + \left(\frac{2k_4}{1+k_5} \right) \dot{\alpha} \theta + \left(\frac{-2k_4}{1+k_5} \right) \theta \dot{\theta} \end{aligned} \quad (4.54)$$

Equation 4.54 can also be written in the following simplified form

$$\begin{aligned} \ddot{\alpha} - \omega^2 \alpha = & \mu \dot{\alpha} + p_1 \alpha^3 + p_2 \alpha^2 + p_3 \alpha \dot{\alpha} + p_4 \alpha^2 \dot{\alpha} \\ & + p_5 \alpha \dot{\theta} + p_6 \dot{\alpha} \theta - p_6 \theta \dot{\theta} + p_7 \dot{\theta} \end{aligned} \quad (4.55)$$

where the definitions of the coefficients ω^2 , μ and p_i are obvious from the comparison of Equations 4.54 and 4.55.

4.5 Single Degree-of-Freedom MTS Analysis

The MTS method is first applied to the simplified case where only single degree of freedom in pitching is considered. In such a case, the kinematic relations in Equation 4.39 becomes

$$\alpha = \theta, \dot{\alpha} = \dot{\theta}, \ddot{\alpha} = \ddot{\theta} \quad (4.56)$$

Equation 4.56 are then substituted in Equation 4.45 to yield

$$\ddot{\theta} = d_3 \theta^3 + d_2 \theta^2 + d_1 \theta + d_4 \dot{\theta} + d_5 \ddot{\theta} \quad (4.57)$$

Moreover, by letting $\mu = d_4 + d_5$ and $d_1 = -\omega^2$, this equation can be expressed as a second order ordinary differential equation as follows.

$$\ddot{\theta} + \omega^2 \theta = \mu \dot{\theta} + d_3 \theta^3 + d_2 \theta^2 \quad (4.58)$$

The focus of the analysis is on small perturbations about the equilibrium conditions of the aircraft where the damping term and the magnitude nonlinearities are small, i.e. the conditions corresponding to linear eigenvalues close to the imaginary axis of the complex plane. In such conditions, Equation 4.58 can be parameterized as follows.

$$\ddot{\theta} + \omega^2 \theta = \varepsilon (\mu \dot{\theta} + d_3 \theta^3 + d_2 \theta^2) \quad (4.59)$$

where $0 < \varepsilon \ll 1$ indicates a small positive non-dimensional parameter that serves as a bookkeeping device and will be cancelled out in the final result. For asymptotic approximation, MTS method is now invoked. Two time scales are selected in this analysis and therefore, the independent and dependent variables are expanded in this form:

$$t \rightarrow \{\tau_o, \tau_1\}, \quad \tau_o = t, \quad \tau_1 = \varepsilon t \quad (4.60)$$

It can be seen that τ_o represents fast time scale and τ_1 represents slow time scale. Moreover, θ can also be extended in terms of these selected time scales as

$$\theta(t) \rightarrow \theta_o(\tau_o, \tau_1) + \varepsilon \theta_1(\tau_o, \tau_1) + O(\varepsilon^2) \quad (4.61)$$

By neglecting the terms associated with $O(\varepsilon^2)$, the second order derivatives of Equation 4.61 can be written as follows:

$$\frac{d\theta}{dt} = \frac{\partial \theta_o}{\partial \tau_o} + \varepsilon \left(\frac{\partial \theta_o}{\partial \tau_1} + \frac{\partial \theta_1}{\partial \tau_o} \right), \quad \frac{d^2 \theta}{dt^2} = \frac{\partial^2 \theta_o}{\partial \tau_o^2} + \varepsilon \left(\frac{\partial^2 \theta_1}{\partial \tau_o^2} + 2 \frac{\partial^2 \theta_o}{\partial \tau_o \partial \tau_1} \right) \quad (4.62)$$

By substituting Equation 4.61 and Equation 4.62 into Equation 4.59, we get the following form

$$\frac{\partial^2 \theta_o}{\partial \tau_o^2} + \omega^2 \theta_o + \varepsilon \left(\frac{\partial^2 \theta_1}{\partial \tau_o^2} + \omega^2 \theta_1 + 2 \frac{\partial^2 \theta_o}{\partial \tau_o \partial \tau_1} \right) = \varepsilon \left(\mu \frac{\partial \theta_o}{\partial \tau_o} + d_3 \theta_o^3 + d_2 \theta_o^2 \right) \quad (4.63)$$

Only terms up to $O(\varepsilon)$ are shown in the above equation, as these are sufficient to obtain the zeroth and first order approximation of the solution. Equating like powers of ε on both sides of Equation 4.63 reveals two equations of $O(1)$ and $O(\varepsilon)$:

$$O(1) \quad \frac{\partial^2 \theta_o}{\partial \tau_o^2} + \omega^2 \theta_o = 0 \quad (4.64)$$

$$O(\varepsilon) \quad \frac{\partial^2 \theta_1}{\partial \tau_o^2} + \omega^2 \theta_1 = \mu \frac{\partial \theta_o}{\partial \tau_o} + d_3 \theta_o^3 + d_2 \theta_o^2 - 2 \frac{\partial^2 \theta_o}{\partial \tau_o \partial \tau_1} \quad (4.65)$$

The solution of the $O(1)$ equation is:

$$\theta_o = A(\tau_1) \sin \psi(\tau_1); \quad \psi = \omega \tau_o + B(\tau_1) \quad (4.66)$$

where ψ is the phase angle, A is the amplitude and B is the phase-correction of the solution. It can be noticed that the amplitude and phase-correction of the solution vary with the slow time scale τ_1 . In order to complete the approximation of the zeroth order solution, the solution of these variations is required. These can be found by evaluating the solution of $O(\varepsilon)$ - equation (Equation 4.65). By substituting Equation 4.66 into Equation 4.65 we get:

$$\begin{aligned} \frac{\partial^2 \theta_1}{\partial \tau_o^2} + \omega^2 \theta_1 = & \left(\mu \omega A - 2 \omega \frac{dA}{d\tau_1} \right) \cos \psi + \left(\frac{3}{4} d_3 A^3 + 2 \omega A \frac{dB}{d\tau_1} \right) \sin \psi \\ & - \frac{d_3 A^3}{4} \sin 3\psi + \frac{d_2 A^2}{2} - \frac{d_2 A^2}{2} \cos 2\psi \end{aligned} \quad (4.67)$$

The terms associated with $\cos \psi$ and $\sin \psi$ destroy the uniformity of the solution of Equation 4.67. These terms will contribute to the solution in terms of

$\tau_o \cos \psi$ and $\tau_o \sin \psi$, which are known as secular terms that will grow without bound with time. Therefore, to keep the approximation uniform and avoid secular terms, the coefficients of $\cos \psi$ and $\sin \psi$ are set to zero. This results in the following equations:

$$\frac{dA}{d\tau_1} = \frac{\mu}{2} A; \quad \frac{dB}{d\tau_1} = -\frac{3}{8\omega} d_3 A^2 \quad (4.68)$$

Equation 4.68 represents the amplitude and phase corrections that vary with the slow time scale. The amplitude equation determines the growth or decay of the solution over time thereby predicting the stability of the motion.

The amplitude equation can be solved independently. The solution can then be substituted into the phase-correction equation to obtain its solution. The process is relatively straight forward and thus not elaborated here. The solutions can be written as:

$$A = C_1 \exp\left(\frac{\mu}{2} \tau_1\right), \quad B = \frac{3d_3 C_1^2}{8\mu\omega} \exp(\mu\tau_1) + C_2 \quad (4.69)$$

where C_1 and C_2 are constants depending upon initial conditions. In this case, A diverges when $\mu > 0$ and decays to zero when $\mu < 0$. Therefore, the condition for asymptotic stability is $\mu < 0$, which corresponds to

$$M_q + M_{\dot{\alpha}} < 0 \quad (4.70)$$

However, this single degree-of-freedom MTS analysis indicates that the system does not give rise to the limit cycle type of motion. This is not as observed from the numerical simulation given in Figure 4-14. This shows that the simplification into single DOF formulation is not adequate to capture the true

behavior of the dynamics. Hence the analysis is further refined below to include the interaction among the multiple degrees of freedom of the motion.

4.6 Multiple Degree-of-Freedom MTS Analysis

The three degree-of-freedom dynamics involving angle of attack of aircraft (Equation 4.55), flight path angle (Equation 4.50) and pitch angle (Equation 4.45) are considered in this section. The kinematic relationships based from the formulation described earlier are considered valid and the velocity of the aircraft is assumed to be constant during the motion. As will be seen later, MTS method separates the fast dynamic variables (α, θ) and slow dynamic variable (γ) systematically, leading to better insight into this complex dynamics of the aircraft. As the focus of the analysis is on the vicinity of the eigenvalue-crossing region, the damping terms as well as the nonlinearities can be considered small in magnitude, and therefore the equations of motion can be parameterized as follows.

$$\begin{aligned}\ddot{\alpha} - \omega^2 \alpha &= \varepsilon (\mu \dot{\alpha} + p_1 \alpha^3 + p_2 \alpha^2 + p_3 \alpha \dot{\alpha} + p_4 \alpha^2 \dot{\alpha} + p_5 \alpha \dot{\theta} + p_6 \dot{\alpha} \theta - p_6 \theta \dot{\theta} + p_7 \dot{\theta}) \\ \dot{\gamma} &= \varepsilon (k_3 \alpha^3 + k_2 \alpha^2 + k_1 \alpha + k_4 \gamma^2 + k_5 \dot{\alpha}) \\ \ddot{\theta} - d_1 \theta &= -d_1 \gamma + \varepsilon (d_3 \alpha^3 + d_2 \alpha^2 + d_4 \dot{\theta} + d_5 \dot{\alpha})\end{aligned}\tag{4.71}$$

Similar to the single degree of freedom case, the MTS method is invoked and two time scales are used for the analysis. The independent variable time is expanded similarly as in Equation 4.60. The dynamic variables α , θ and γ are extended with respect to the multiple time scales in the following manner:

$$\alpha(t) = \alpha_o(\tau_o, \tau_1) + \varepsilon \alpha_1(\tau_o, \tau_1) + O(\varepsilon^2)$$

$$\gamma(t) = \gamma_o(\tau_o, \tau_1) + \varepsilon \gamma_1(\tau_o, \tau_1) + O(\varepsilon^2) \quad (4.72)$$

$$\theta(t) = \theta_o(\tau_o, \tau_1) + \varepsilon \theta_1(\tau_o, \tau_1) + O(\varepsilon^2)$$

By neglecting the terms associated with $O(\varepsilon^2)$, the first order derivatives of Equation 4.72 can be written as

$$\begin{aligned} \frac{d\alpha(t)}{dt} &= \frac{\partial \alpha_o(\tau_o, \tau_1)}{\partial \tau_o} + \varepsilon \left(\frac{\partial \alpha_o(\tau_o, \tau_1)}{\partial \tau_1} + \frac{\partial \alpha_1(\tau_o, \tau_1)}{\partial \tau_o} \right) \\ \frac{d\gamma(t)}{dt} &= \frac{\partial \gamma_o(\tau_o, \tau_1)}{\partial \tau_o} + \varepsilon \left(\frac{\partial \gamma_o(\tau_o, \tau_1)}{\partial \tau_1} + \frac{\partial \gamma_1(\tau_o, \tau_1)}{\partial \tau_o} \right) \\ \frac{d\theta(t)}{dt} &= \frac{\partial \theta_o(\tau_o, \tau_1)}{\partial \tau_o} + \varepsilon \left(\frac{\partial \theta_o(\tau_o, \tau_1)}{\partial \tau_1} + \frac{\partial \theta_1(\tau_o, \tau_1)}{\partial \tau_o} \right) \end{aligned} \quad (4.73)$$

Similarly,

$$\begin{aligned} \frac{d^2\alpha(t)}{dt^2} &= \frac{\partial^2 \alpha_o(\tau_o, \tau_1)}{\partial \tau_o^2} + \varepsilon \left(\frac{\partial^2 \alpha_1(\tau_o, \tau_1)}{\partial \tau_o^2} + 2 \frac{\partial^2 \alpha_o(\tau_o, \tau_1)}{\partial \tau_o \partial \tau_1} \right) \\ \frac{d^2\theta(t)}{dt^2} &= \frac{\partial^2 \theta_o(\tau_o, \tau_1)}{\partial \tau_o^2} + \varepsilon \left(\frac{\partial^2 \theta_1(\tau_o, \tau_1)}{\partial \tau_o^2} + 2 \frac{\partial^2 \theta_o(\tau_o, \tau_1)}{\partial \tau_o \partial \tau_1} \right) \end{aligned} \quad (4.74)$$

Substitution of Equations 4.73 and 4.74 into Equation 4.71 obtains

$$\frac{\partial^2 \alpha_o}{\partial \tau_o^2} + \omega^2 \alpha_o + \varepsilon \left(\frac{\partial^2 \alpha_1}{\partial \tau_o^2} + 2 \frac{\partial^2 \alpha_o}{\partial \tau_o \partial \tau_1} + \omega^2 \alpha_1 \right) = \varepsilon \begin{pmatrix} \mu \frac{\partial \alpha_o}{\partial \tau_o} + p_1 \alpha_o^3 + p_2 \alpha_o^2 \\ + p_3 \alpha_o \frac{\partial \alpha_o}{\partial \tau_o} + p_4 \alpha_o^2 \frac{\partial \alpha_o}{\partial \tau_o} \\ + p_5 \alpha_o \frac{\partial \theta_o}{\partial \tau_o} + p_6 \frac{\partial \alpha_o}{\partial \tau_o} \theta_o \\ - p_6 \theta_o \frac{\partial \theta_o}{\partial \tau_o} + p_7 \frac{\partial \theta_o}{\partial \tau_o} \end{pmatrix} \quad (4.75)$$

By equating terms of the same order on both sides of Equation 4.75, the zeroth order approximation can be written as:

$$O(1) \quad \frac{\partial^2 \alpha_o}{\partial \tau_o^2} + \omega^2 \alpha_o = 0 \quad (4.76)$$

The solution of Equation 4.76 is

$$\alpha_o = A_1(\tau_1) \sin \psi_1; \psi_1 = \omega \tau_o + B_1(\tau_1) \quad (4.77)$$

where ψ_1 is the phase angle, A_1 is the amplitude and B_1 is the phase-correction of the solution. As in the single degree of freedom case, the amplitude and phase correction of the solution vary with the slow time scale τ_1 , which can be obtained from analysis of $O(\varepsilon)$ set of equation.

The derivatives of the leading order approximate solution required in analysis ahead can be written as

$$\frac{\partial \alpha_o}{\partial \tau_o} = \omega A_1 \cos \psi_1; \quad \frac{\partial^2 \alpha_o}{\partial \tau_o \partial \tau_1} = \omega \frac{dA_1}{d\tau_1} \cos \psi_1 - \omega A_1 \frac{dB_1}{d\tau_1} \sin \psi_1 \quad (4.78)$$

Adopting the strategy similar to the angle-of-attack equation, the flight path equation becomes

$$\frac{\partial \gamma_o(\tau_o, \tau_1)}{\partial \tau_o} + \varepsilon \left(\frac{\partial \gamma_o(\tau_o, \tau_1)}{\partial \tau_1} + \frac{\partial \gamma_1(\tau_o, \tau_1)}{\partial \tau_o} \right) = \varepsilon \left(\begin{aligned} &k_3 \alpha_o^3 + k_2 \alpha_o^2 + k_1 \alpha_o \\ &+ k_4 \gamma_o^2 + k_5 \frac{\partial \alpha_o}{\partial \tau_o} \end{aligned} \right) \quad (4.79)$$

Order by order analysis of Equation 4.79 leads to the following:

$$O(1) \quad \frac{\partial \gamma_o(\tau_o, \tau_1)}{\partial \tau_o} = 0;$$

$$O(\varepsilon) \quad \frac{\partial \gamma_o}{\partial \tau_1} + \frac{\partial \gamma_1}{\partial \tau_o} = k_3 \alpha_o^3 + k_2 \alpha_o^2 + k_1 \alpha_o + k_4 \gamma_o^2 + k_5 \frac{\partial \alpha_o}{\partial \tau_o} \quad (4.80)$$

It can be observed that the leading order flight path approximation is straightforward to solve, yielding:

$$\gamma_o = C(\tau_1) \quad (4.81)$$

where C varies with the slower time-scale τ_1 . Moreover, the derivative of Equation 4.81 with slower time scale can be represented as:

$$\frac{\partial \gamma_o}{\partial \tau_1} = \frac{dC(\tau_1)}{d\tau_1} \quad (4.82)$$

Now plugging Equations 4.77, 4.78, 4.81 and 4.82 into the $O(\varepsilon)$ terms in Equation 4.80 reveals

$$\begin{aligned} \frac{\partial \gamma_1}{\partial \tau_o} = & \left[-\frac{dC}{d\tau_1} + k_4 C^2 + \frac{k_2 A_1^2}{2} \right] + \frac{3k_3 A_1^3}{4} \sin \psi_1 - \frac{k_3 A_1^3}{4} \sin 3\psi_1 \\ & - \frac{k_2 A_1^2}{2} \cos 2\psi_1 + k_1 A_1 \sin \psi_1 + k_5 \omega A_1 \cos \psi_1 \end{aligned} \quad (4.83)$$

If the first term on right hand side of Equation 4.83 is non-zero, secular terms will appear in the solution of γ_1 . Therefore, to keep the approximation uniform, these terms are set to zero, which obtains

$$\frac{dC}{d\tau_1} = \frac{k_2 A_1^2}{2} + k_4 C^2 \quad (4.84)$$

This is a nonlinear ordinary differential equation with quadratic nonlinear term. This special class of equation is generally referred to as Riccati equation[55], which can be reduced to a second order linear ordinary differential equation by letting $q_o = \frac{k_2 A_1^2}{2}$ and $q_2 = k_4$. Note that k_4 is a function of g/V , which is not zero, and therefore $q_2 = f(g/V)$. Equation 4.84 can now be written in an alternate form as:

$$\dot{C} = q_o + q_2 C^2 \quad (4.85)$$

Since the value of $q_2 \neq 0$ and is constant and the derivative of q_2 is equal to zero therefore, substituting the newly defined variable $v = Cq_2$ in Equation 4.85 reveals the new form as:

$$\dot{v} = v^2 + s^2 \quad (4.86)$$

where $s^2 = q_2 q_o$. q_o is a function of amplitude A_1 , which varies with the slower time scale. Hence, Equation 4.86 is of the form linear ordinary differential equation with variable coefficients. If the amplitude varies slowly with respect to time, it is reasonable to assume s^2 to be constant. This assumption simplifies the solution of Equation 4.86 as the new form is linear ordinary differential equation with constant coefficients. By doing another substitution using $v = -\frac{\dot{\lambda}}{\lambda}$, the original Riccati equation (Equation 4.84) is transformed into the second order linear ordinary differential equation below.

$$\frac{d^2 \lambda}{d\tau_1^2} - \Omega^2 \lambda = 0 \quad (4.87)$$

where $\Omega^2 = -s^2$. The general solution to this equation is

$$\lambda = R_1 \exp(\Omega \tau_1) + R_2 \exp(-\Omega \tau_1) \quad (4.88)$$

where R_1 and R_2 are constants depending on initial conditions. The derivative of Equation 4.88 is

$$\dot{\lambda} = \Omega R_1 \exp(\Omega \tau_1) - \Omega R_2 \exp(-\Omega \tau_1) \quad (4.89)$$

The transformed solution, Equation 4.88, will lead to the original solution by following the inverse substitution:

$$\gamma_o = C = -\frac{\dot{\lambda}}{q_2 \lambda} \quad (4.90)$$

Plugging Equations 4.88 and 4.89 into Equation 4.90 gives us

$$\gamma_o = -R_o \left[\frac{R_1 \exp(\Omega \tau_1) - R_2 \exp(-\Omega \tau_1)}{R_1 \exp(\Omega \tau_1) + R_2 \exp(-\Omega \tau_1)} \right] \quad (4.91)$$

where $R_o = \sqrt{-\frac{k_2}{2k_4}} A_1$ is the steady state value of γ_o as $t \rightarrow \infty$. Another important observation about R_o is that its value is dependent on the amplitude of the angle of attack response. This means that the flight path angle only converges to its initial nominal value if the amplitude of the angle-of-attack response is zero. In the presence of limit cycles on the angle-of-attack response, the flight path angle will go to another equilibrium value. Simple kinematic relationship $\theta_o = \alpha_o - \gamma_o$ can then be used to approximate θ_o :

$$\theta_o = A_1(\tau_1) \sin \psi_1 + R_o \left[\frac{R_1 \exp(\Omega \tau_1) - R_2 \exp(-\Omega \tau_1)}{R_1 \exp(\Omega \tau_1) + R_2 \exp(-\Omega \tau_1)} \right] \quad (4.92)$$

In order to solve for A_1 and B_1 , $O(\varepsilon)$ terms of Equation 4.75 is used, which leads to

$$\begin{aligned} \frac{\partial^2 \alpha_1}{\partial \tau_o^2} + 2 \frac{\partial^2 \alpha_o}{\partial \tau_o \partial \tau_1} + \omega^2 \alpha_1 = & \mu \frac{\partial \alpha_o}{\partial \tau_o} + p_1 \alpha_o^3 + p_2 \alpha_o^2 + p_3 \alpha_o \frac{\partial \alpha_o}{\partial \tau_o} + p_4 \alpha_o^2 \frac{\partial \alpha_o}{\partial \tau_o} \\ & + p_5 \alpha_o \frac{\partial \theta_o}{\partial \tau_o} + p_6 \frac{\partial \alpha_o}{\partial \tau_o} \theta_o - p_6 \theta_o \frac{\partial \theta_o}{\partial \tau_o} + p_7 \frac{\partial \theta_o}{\partial \tau_o} \end{aligned} \quad (4.93)$$

By substituting α_o and θ_o and their partial derivatives into Equation 4.93, we get

$$\begin{aligned} \frac{\partial^2 \alpha_1}{\partial \tau_o^2} + \omega^2 \alpha_1 = & \left(\mu \omega A_1 + p_7 \omega A_1 + \frac{p_4 \omega A_1^3}{4} - 2 \omega \frac{dA_1}{d\tau_1} \right) \cos \psi_1 \\ & + \left(\frac{3p_1 A_1^3}{4} + 2 \omega A_1 \frac{dB_1}{d\tau_1} \right) \sin \psi_1 + .. \end{aligned} \quad (4.94)$$

Just like before, the coefficients of the first harmonic terms of Equation 4.94 must be set to zero so that no secular terms appear in the solution. By doing so, we get the following set of first order ordinary differential equations for the amplitude and phase correction:

$$\frac{dA_1}{d\tau_1} = \frac{\sigma}{2} A_1 + \xi_1 A_1^3; \quad \frac{dB_1}{d\tau_1} = \xi_2 A_1^2 \quad (4.95)$$

where $\sigma = (\mu + p_7)$, $\xi_1 = \frac{p_4}{8}$ and $\xi_2 = -\frac{3p_1}{8\omega}$.

4.6.1 Bifurcation Analysis

Bifurcation analysis help to understand the properties of the ordinary differential equation as numerical values of certain parameters are varied. Specifically, it can describe the stability characteristics of Equation 4.95 and changes in the topological properties of the solutions.

The equilibria of amplitude equation (Equation 4.95), are $A_1 = 0$ and $A_1 = \sqrt{-\frac{\sigma}{2\xi_1}}$. Plotted in $A_1 - \sigma$ diagram, the equilibria consist of the σ axis and the parabola $\sigma = -2\xi_1 A_1^2$. The stability of these equilibria can be determined by examining the eigenvalues of the linearized systems around the equilibria of interest. The first equilibrium of interest in this case is at σ -axis i.e. $A_1 = 0$. The linearization around this equilibrium reveals

$$\frac{dA}{d\tau_1} = \frac{\sigma A}{2} \quad (4.96)$$

The eigenvalue of the Equation 4.96 is $\frac{\sigma}{2}$. The sign of σ governs the stability properties. Similarly the stability properties can be studied for the

second equilibrium of interest which in this case is about the parabola

$$\sigma = -2\xi_1 A_1^2 \text{ i.e. } A_1 = \sqrt{-\frac{\sigma}{2\xi_1}} . \text{ These equilibria together with their stability}$$

properties for $\xi_1 > 0$ and $\xi_1 < 0$ are shown in Figure 4-19 and Figure 4-20.

Figure 4-19 Bifurcation diagram for $\xi_1 > 0$

These diagrams imply that there occur finite amplitude oscillations (limit cycle) appearing and disappearing in the system as σ is varied across $\sigma = 0$. This phenomenon is known as Hopf bifurcation. For $\xi_1 > 0$, the Hopf bifurcation is subcritical, since the new branch of equilibria appear for the values of σ below the onset of bifurcation. For $\xi_1 < 0$, the Hopf bifurcation is supercritical, as the new branch of equilibria exist only for values of σ larger than the bifurcation onset.

Figure 4-20 Bifurcation diagram for $\zeta_1 < 0$

It can also be seen from the diagram, that the stable limit cycle is only possible when $\xi_1 < 0$. This implies that sustained oscillatory motion can only occur in this situation and the limit cycle amplitude is given by

$$A_1 = \sqrt{-\frac{\sigma}{2\xi_1}} \quad (4.97)$$

The amplitude is affected by parameters σ and ξ_1 in Equation 4.97. The physical interpretation of these parameters is explained ahead in next section.

4.6.2 Analytical Approximation of the Solution

A closed-form approximation of the system response, which includes transient motion, will now be developed by solving the amplitude and phase-correction equations (Equation 4.96). First we consider the amplitude differential equation. By doing the separation of variables, the amplitude equation can be written as:

$$d\tau_1 = \frac{dA_1}{A_1 \left(\xi_1 A_1^2 + \frac{\sigma}{2} \right)} \quad (4.98)$$

Carrying out partial fraction expansion, Equation 4.98 becomes:

$$d\tau_1 = \frac{2}{\mu A_1} dA_1 - \frac{2\xi_1 A_1}{\left(\mu \xi_1 A_1^2 + \frac{\sigma^2}{2}\right)} dA_1 \quad (4.99)$$

Integrating both sides of Equation 4.99 results in,

$$\tau_1 = \frac{2}{\sigma} \ln|A_1| - \frac{1}{\sigma} \ln \left| \mu \xi_1 A_1^2 + \frac{\sigma^2}{2} \right| + S_o \quad (4.100)$$

Then by taking exponential and simplifying Equation 4.100, we get

$$\exp(\sigma \tau_1) = \frac{A_1^2 \exp(S_o)}{\mu \xi_1 A_1^2 + \frac{\sigma^2}{2}} \quad (4.101)$$

By rearranging, Equation 4.101 can be expressed as

$$A_1^2 = \frac{\frac{\sigma^2}{2} \exp(\sigma \tau_1)}{\exp(S_o) - \mu \xi_1 \exp(\sigma \tau_1)} \quad (4.102)$$

By letting $S_1 = \sqrt{\frac{\sigma}{\exp(S_o)}}$. Equation 4.102 can be expressed as:

$$A_1 = \frac{\sqrt{\frac{S_1 \sigma}{2}} \exp\left(\frac{\sigma \tau_1}{2}\right)}{\sqrt{1 - S_1 \xi_1 \exp(\sigma \tau_1)}} \quad (4.103)$$

The constant S_1 in Equation 4.103 can be evaluated from the initial conditions. Moreover, the steady state value of Equation 4.103 matches with Equation 4.97. Then, substitution of Equation 4.103 into phase-correction equation (Equation 4.96) leads to

$$dB_1 = \left(\frac{\frac{S_1 \sigma}{2} \xi_2 \exp(\sigma \tau_1)}{1 - S_1 \xi_1 \exp(\sigma \tau_1)} \right) d\tau_1 \quad (4.104)$$

Integrating both sides of Equation 4.104 yields:

$$B_1 = -\frac{\xi_2}{2\xi_1} \ln|1 - S_1 \xi_1 \exp(\sigma \tau_1)| + S_2^* \quad (4.105)$$

where $S_2^* = \ln|S_2|$ is a constant. By letting $S_3 = \frac{\ln|S_2|}{-\frac{\xi_2}{2\xi_1}}$, Equation (4.105) becomes

$$B_1 = -\frac{\xi_2}{2\xi_1} \ln|S_3(1 - S_1 \xi_1 \exp(\sigma \tau_1))| \quad (4.106)$$

where S_3 is a constant and can be determined based on the initial condition. A closer look into the key parameters σ and ξ_1 is performed next. In terms of physical entities σ may be written as:

$$\sigma = \frac{(M_q + M_{\dot{\alpha}})(mV - L_q) - I_{yy} \left(L_\alpha + T \left(1 - \frac{m_o^2}{2} \right) \right)}{mV + L_{\dot{\alpha}}} \quad (4.107)$$

where $L_\alpha = \frac{1}{2} \rho V^2 S_{ref} (e_1 + 2m_1 e_2 + 3m_1^2 e_3)$. In the vicinity of stall conditions, the

first group of terms in the numerator, which is affected by the pitch damping derivatives, is usually small. Therefore the sign of σ is likely affected by the second group of terms in the numerator, which is affected by L_α , T and α_{fuso} . The

factor $T \left(1 - \frac{\alpha_{fuso}^2}{2} \right)$ is usually positive for most maneuvers, except at the very

high angle of attack (above 81 deg), and its magnitude depends on the thrust value required for the equilibrium. In the flight regime after stall, L_α becomes negative, and if the trim condition does not rely much on thrust (most of the aircraft weight is supported by aerodynamic lift), the value of σ can become positive, which enables the occurrence of limit cycles.

In terms of physical entities ξ_1 can be written as:

$$\xi_1 = \frac{-3\rho V^2 S_{ref} e_3 + T}{16(mV + L\dot{\alpha})} \quad (4.108)$$

It should be noted that the magnitude of ξ_1 will decrease with the increase in mass, m , or velocity, V , of the aircraft. However, the sign of ξ_1 is governed by the cubic nonlinearity coefficient of the lift curve, e_3 , velocity of the aircraft V and magnitude of the thrust, T , of the UAV. When the nonlinearity is strong and the thrust required for the maneuver is small, ξ_1 can become negative. In the present case of aerodynamic vectoring, the lift from the wings contributes to aircraft upward forces and reduces the demand on thrust to support the aircraft weight during flight. This scenario may drive ξ_1 to become negative. Coupled with the positive σ value, the negative sign on ξ_1 will give rise to the supercritical Hopf bifurcation in the vicinity of the stall, which leads to the appearance of sustained limit cycles. The reliance on the small thrust indicates that such limit-cycle oscillations are somehow unique to a small aircraft equipped with some aerodynamic-vectoring feature.

4.6.3 Comparison with Numerical Simulations

A generic UAV model as mentioned earlier is used to illustrate and validate our analytical representation of the dynamic of the aircraft motion in the vicinity of limit cycle. The accuracy of the analytical model is examined by simulating the aircraft response slightly above and below the stall point, which corresponds to the eigenvalues located slightly on the left and right-hand side of the imaginary axis in Figure 4-8. A simulation of the aircraft response for the damped case is shown first (Figure 4-21 to Figure 4-23). The results are compared with the

analytical solution developed earlier. A very good agreement is found between the analytical and numerical results for angle-of-attack response. For the pitch angle response, there is some discrepancy in the amplitude of the oscillations; however, this difference is relatively small, which is less than 5 percent of magnitude for this particular example. For the flight path angle, the transient response from the analytical result is slightly off but the trend is correctly predicted and both the numerical and analytical solutions reach the same steady state values. Moreover, the transient difference can be considered insignificant, which is approximately 1.14 deg in this case.

Figure 4-21 Angle of attack response for the damped case

Figure 4-22 Pitch angle response for the damped case

Figure 4-23 Flight path angle response for the damped case

A simulation of aircraft response for the case giving rise to limit cycles is shown next (Figure 4-24 to Figure 4-26). It can be observed that the limit cycle amplitude and frequency predicted by the analytical method are in excellent agreement with the numerical integration results. The new equilibrium conditions for the pitch and flight path angles are also accurately predicted by the analytical results. There are small amplitude oscillations in flight path angle response that are not captured by the analytical solutions. However, the amplitude of these oscillations is very small (less than 1 deg) for the majority of the cases tried, as well as for this particular case. The preceding validation demonstrates that the analytical solutions developed predict accurately the complicated aircraft dynamics as well as the interaction between the various degrees of freedom.

Figure 4-24 Angle of attack response for the limit cycle case

Figure 4-25 Pitch angle response for the limit cycle case

Figure 4-26 Flight path angle response for the limit cycle case

4.7 Concluding Remarks

The longitudinal dynamics of the small aircraft equipped with aerodynamic vectoring feature with multiple degrees of freedom under a broad envelope of hover and cruise flight conditions have been considered. The analysis technique using the linear dynamics, MTS method and bifurcation theory describes the system dynamics successfully leading to the solutions in closed parametric form. The limitation of linear analysis is observed in prediction of the aircraft stability behavior in the vicinity of the stall. Subsequently, it is shown that MTS analysis on simplified single degree-of-freedom nonlinear model fails to predict the dynamics of the system accurately. An interesting aspect of the dynamics that is not captured by the single degree of freedom analysis is the sustained limit-cycle oscillations in the vicinity of the stall regime. This aspect is captured successfully in the multiple degrees-of-freedom MTS analysis, which leads to the conclusion that the limit cycle in this case is the result of the interaction from the various degrees of freedom and cubic nonlinearity of the lift-curve slope. The bifurcation analysis suggests that the limit-cycle is primarily caused by the loss of damping and the changing sign of the lift slope coupled with the less reliance to thrust during the flight. This result has not been reported in literature before.

CHAPTER 5

Unsteady Transition Maneuver

5.1 Background

Contrary to the previous chapter, this chapter deals with the unsteady transition maneuver analysis, where the aircraft is not under trim state across the complete maneuver envelope. The aircraft is in continuous accelerating or decelerating phase during the whole maneuver. This analysis applies to the type of aerobatic/agile maneuver between hover and cruise.

Optimization of this unsteady transition maneuver is considered in this chapter. The effects of nonlinearities as well as aerodynamic vectoring feature on the transition performance are examined. The comparison is carried out between the optimal solutions of the fixed-wing conventional and aerodynamic vectoring platforms across various performance parameters. A nonlinear constrained optimization scheme based on Sequential Quadratic Programming (SQP) is used here.

To get the complete picture of the optimal maneuvers, stability properties of the solutions are also examined. Specifically, contraction theory is used to identify the stability characteristics of the optimal solutions.

The objective of this study is to achieve a transition scheme with minimal variation in altitude, reduced transition time, reasonable thrust-to-weight ratio and analyze associated stability characteristics. The analysis is initiated from a simple case and then extended to a more complex scenario in order to attain an

in-depth view of the unsteady transition maneuver. Initially, a two-degree-of-freedom point-mass model with pitching constraints is used for the modeling of the aircraft dynamics. Subsequently, the vehicle dynamics is modeled as a three degree of freedom dynamics in the longitudinal plane to give better insight about the effects of rotational dynamics on the maneuver. The results obtained are compared between the fixed-wing and the variable-incidence wing cases.

5.2 Optimization Method

It is not surprising that the development of the numerical methods for optimization have closely paralleled to the exploration of space and the development of the digital computer [56]. The topics of mathematical optimization are broad and the related literature is immense. The review presented in this section is restricted to the theoretical perspective of the algorithms used in the present work.

All optimization problems with explicit objectives can in general be expressed as nonlinearly constrained optimization problems in the following generic form

$$\begin{aligned}
 & \max/\min_{\mathbf{x} \in \mathbb{R}^n} f(\mathbf{x}), \mathbf{x} = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n, \\
 & \text{subject to } \varphi_j(\vec{x}) = 0, (j = 1, 2, \dots, M), \\
 & \varphi_k(\mathbf{x}) \geq 0, (k = 1, 2, \dots, N),
 \end{aligned} \tag{5.1}$$

where $f(\mathbf{x})$, $\phi_i(\mathbf{x})$ and $\varphi_j(\mathbf{x})$ are scalar functions of the real column vector $\mathbf{x}$. Here the components x_i of $\mathbf{x} = (x_1, \dots, x_n)^T$ are called design variables or decision variables. The vector $\mathbf{x}$ is often called a decision vector which varies in an n-

dimensional space $\mathfrak{R}^n$. The function $f(\mathbf{x})$ is called the objective function or cost function. In addition, $\phi_i(\mathbf{x})$ are constraints in terms of M equalities, and $\phi_j(\mathbf{x})$ are constraints written as N inequalities. So there are $M + N$ constraints in total. The optimization problem formulated here is a nonlinear constrained problem [57].

The space spanned by the decision variables is called the search space $\mathfrak{R}^n$, while the space formed by the objective function values is called the solution space. The optimization problem essentially maps the $\mathfrak{R}^n$ domain or space of decision variables into a solution space $\mathfrak{R}$.

5.3 Nonlinear Constrained Problems

As most of the real world problems are nonlinear, nonlinear constrained mathematical programming thus forms an important part of mathematical optimization methods. An interesting property of a nonlinear convex function f is that the vanishing of the gradient $\nabla f(\mathbf{x}_k) = 0$ guarantees that the iterated vector $\mathbf{x}_k$ is a global minimum or maximum of f . Several conditions are associated with the nonlinear constrained optimization problems. They are commonly known as ‘first order optimality conditions’ from the fact that the highest order of matrices involved in optimization problem is of first order. They are also named after the mathematicians Karush, Kuhn and Tucker as KKT conditions.

Let $\mathbf{x}_k$ be a local solution to the following problem.

$$\begin{cases} \min_x f(\mathbf{x}) \\ c_E(\mathbf{x}) = 0 \\ c_I(\mathbf{x}) \leq 0 \\ \mathbf{x} \in \mathfrak{R} \end{cases} \quad (5.2)$$

The KKT conditions to be satisfied at each iteration are:

$$\begin{cases} (a) \nabla f(\mathbf{x}_k) + A(\mathbf{x}_k)^T \boldsymbol{\lambda}_k = 0 \\ (b) c_E(\mathbf{x}_k) = 0, c_I(\mathbf{x}_k) \leq 0 \\ (c) (\boldsymbol{\lambda}_k)_I \geq 0 \\ (d) (\boldsymbol{\lambda}_k)_I^T c_I(\mathbf{x}_k) = 0 \end{cases} \quad (5.3)$$

The notation ∇ is used for a gradient with respect to the Euclidean scalar product (vector of partial derivatives). The above optimality conditions are called “first order”, for they only involve first-order derivatives of f and c .

Identity (a) is the optimality equation itself. The notation $A(\mathbf{x}_k)$ is used for the $m \times n$ Jacobian of the constraints at $\mathbf{x}_k$: $A(\mathbf{x}) = \nabla c(\mathbf{x})^T$, so that its (i, j) th element is the partial derivative $\partial c_i / \partial x_j$ evaluated at x . This equation can also be written as

$$\nabla_x l(\mathbf{x}_k, \boldsymbol{\lambda}_k) = 0 \quad (5.4)$$

where l is the *Lagrangian* of the problem:

$$l(\mathbf{x}, \boldsymbol{\lambda}) = f(\mathbf{x}) + \boldsymbol{\lambda}^T c(\mathbf{x}) \quad (5.5)$$

The vector $\boldsymbol{\lambda}_k$ is called the *Lagrange multiplier*. The name multiplier comes from the fact that it multiplies the constraint vector in the Lagrangian. The vector has as many components as there are constraints.

In (b), the feasibility of iterated value $\mathbf{x}_k$ is analyzed. In (c), the corresponding multipliers have a definite sign, depending on how the problem is formulated. Identity (d) is called *complimentarity conditions*. As $(\boldsymbol{\lambda}_k)_I \geq 0$ and

$c_I(\mathbf{x}_k) \leq 0$, this leads to the conclusion that the multipliers corresponding to inactive constraints are zero:

$$c_i(\mathbf{x}_k) < 0 \Rightarrow (\lambda_k)_i = 0 \quad (5.6)$$

When a pair $(\mathbf{x}_k, \lambda_k)$ satisfies the KKT conditions, then it is called a primal-dual solution to the problem, and $\mathbf{x}_k$ is said to be stationary.

5.3.1 Sequential Quadratic Programming

The Sequential Quadratic Programming (SQP) algorithm is a form of Newton's method to solve problem (Equation 5.2) that is well adapted to computation. The KKT conditions are nonlinear in nature. They can be linearized for a current iteration values $(\mathbf{x}_k, \lambda_k)$. The change in variables is denoted by (d_k, μ_k) . This solves the following system of equalities and inequalities in the unknown (d, μ) :

$$\begin{cases} \mathbf{L}_k \mathbf{d} + A_k^T \boldsymbol{\mu} = -\nabla_x l_k \\ (c_k + A_k \mathbf{d})^\# = 0 \\ (\lambda_k + \boldsymbol{\mu})_I \geq 0 \\ (\lambda_k + \boldsymbol{\mu})_I^T (c_k)_I + (\lambda_k)_I^T (A_k \mathbf{d})_I = 0 \end{cases} \quad (5.7)$$

where $c_k = c(\mathbf{x}_k)$, $A_k = A(\mathbf{x}_k) = c'(\mathbf{x}_k)$, $\nabla_x l_k = \nabla_x l(\mathbf{x}_k, \lambda_k)$ and $L_k = \nabla_{xx}^2 l(\mathbf{x}_k, \lambda_k)$.

Also $(c_k + A_k d)^\# = 0$ if and only if $c_E(\mathbf{x}_k) = 0$ and $c_I(\mathbf{x}_k) \leq 0$. The key observation is that a good interpretation can be obtained if we add to the last equation the term $(\boldsymbol{\mu})_I^T (A_k \mathbf{d})_I$. Compared with the others, this term is negligible when the steps μ_k and d_k are small, which should be the case when the values are close to a solution to the problem (Equation 5.2). Introducing the unknown $\lambda_{QP} = \lambda_k + \boldsymbol{\mu}$, the modified system (Equation 5.7) can be written as

$$\begin{cases} \mathbf{L}_k \mathbf{d} + A_k^T \lambda_{QP} = -\nabla f_k \\ (c_k + A_k \mathbf{d})^\# = 0 \\ (\lambda_{QP})_I \geq 0 \\ (\lambda_{QP})_I^T (c_k + A_k \mathbf{d})_I = 0 \end{cases} \quad (5.8)$$

A remarkable fact is that Equation 5.8 is the optimality system of the following quadratic problem (QP)

$$\begin{cases} \min_d \nabla f(\mathbf{x}_k)^T \mathbf{d} + \frac{1}{2} \mathbf{d}^T \mathbf{L}_k \mathbf{d} \\ c_E(\mathbf{x}_k) + A_E(\mathbf{x}_k) \mathbf{d} = 0 \\ c_I(\mathbf{x}_k) + A_I(\mathbf{x}_k) \mathbf{d} \leq 0 \end{cases} \quad (5.9)$$

This QP can be obtained from Equation 5.2. Its constraints are those of Equation 5.2, linearized at $\mathbf{x}_k$. Its objective function is hybrid, with $\nabla f(\mathbf{x}_k)$ in the linear part and the Hessian of the Lagrangian in its quadratic part. The schematic algorithm [58] is described below:

Step 0: An initial guess $(\mathbf{x}_1, \lambda_1)$ is given. Compute $c(\mathbf{x}_1), \nabla f(\mathbf{x}_1)$ and $A(\mathbf{x}_1)$. Set $k = 1$.

Step 1: Stop if the KKT conditions (Equation 5.12) holds

Step 2: Compute $L(\mathbf{x}_k, \lambda_k)$ and find a primal-dual stationary point, i.e., a solution $(\mathbf{d}_k, \lambda_k^{\text{QP}})$.

Step 3: Set $\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{d}_k$ and $\lambda_{k+1} = \lambda_k^{\text{QP}}$

Step 4: Compute $c(\mathbf{x}_{k+1}), \nabla f(\mathbf{x}_{k+1})$ and $A(\mathbf{x}_{k+1})$.

Step 5: Increase k by 1 and go to Step 1.

5.3.2 Quasi-Newton SQP Hybrid Implementation

SQP computes a displacement $\mathbf{d}_k$ at $\mathbf{x}_k$ by solving the quadratic problem for d whereas quasi-Newton accelerates the iterations by doing approximations for the Hessian instead of actually computing it. During recent years, Quasi-Newton SQP technique has been used tremendously to evaluate nonlinear constrained problems. In Quasi-Newton SQP, the basic problem (Equation 5.8) is converted to the following problem

$$\begin{cases} \min_d \nabla f(\mathbf{x}_k)^T \mathbf{d} + \frac{1}{2} \mathbf{d}^T \mathbf{M}_k \mathbf{d} \\ c_E(\mathbf{x}_k) + A_E(\mathbf{x}_k) \mathbf{d} = 0 \\ c_I(\mathbf{x}_k) + A_I(\mathbf{x}_k) \mathbf{d} \leq 0 \end{cases} \quad (5.10)$$

In the quasi-Newton version here, $\mathbf{M}_k$ becomes a symmetric positive definite matrix, updated at each iteration by the BFGS formula using two vectors $\mathbf{y}_k$ and $\mathbf{s}_k$. As we know from the previous section on SQP, $\mathbf{M}_k$ should approximate the Hessian of the Lagrangian. It therefore appears to be reasonable to take $\mathbf{y}_k = \mathbf{y}_k^1$, the variation of the gradient of the Lagrangian when x varies by $\mathbf{s}_k$.

$$\mathbf{y}_k^1 = \nabla_x l(\mathbf{x}_{k+1}, \boldsymbol{\lambda}_{k+1}) - \nabla_x l(\mathbf{x}_k, \boldsymbol{\lambda}_{k+1}) \quad (5.11)$$

$$\mathbf{s}_k = \mathbf{x}_{k+1} - \mathbf{x}_k \quad (5.12)$$

An important point to note is that even with the use of this algorithm, global convergence may be difficult to attain and is subjected to initial estimate unless we are assuming convexity[58].

The optimization algorithm used in the present research is a Quasi-Newton SQP hybrid optimization algorithm implemented as a built-in routine in Optimization Toolbox of MATLAB.

5.4 Two DOF Transition Maneuver Optimization

The study reported in this section is part of an effort to find an efficient transition technique for small UAVs to achieve minimal variation in altitude using reasonable T/W requirements. A two degree-of-freedom point-mass model with pitching constraints is used for the modeling of the aircraft dynamics. The

aerodynamic-force-and-moment database, which is needed for the optimization study is developed through wind-tunnel experimentation. An implicit problem is formulated to study the optimal transition maneuvers with and without aerodynamic vectoring.

5.4.1 Problem Formulation

The transition maneuver discussed here is restricted in the longitudinal plane and is assumed to occur in still atmosphere. These assumptions simplify the aerodynamics and vehicle dynamics substantially while still providing qualitative, as well as quantitative, insight to the transition properties. To analyze the variety of flight conditions ranging from hover to forward flight state, the following two-degree-of-freedom point mass model is used. It is similar to the one used in earlier pilot study [59].

$$m\ddot{x} = T \cos \alpha_{fus} - D - W \sin \gamma \quad (5.13)$$

$$m\ddot{z} = L + T \sin \alpha_{fus} - W \cos \gamma \quad (5.14)$$

where m is the mass of the aircraft, $\ddot{x}$ is the absolute acceleration aligned with horizontal direction, $\ddot{z}$ is the absolute acceleration aligned with vertical direction, $W = mg$ is the weight of the aircraft and T is the thrust. L and D represent aerodynamic lift and drag respectively. γ is the flight path angle of the aircraft.

The slipstream effects are modeled based on the formulation by McCormick[42] and discussed earlier in section 3.4. Moreover, in the present trajectory analysis, unsteady aerodynamic effects are not considered and the trajectories are computed solely on the steady aerodynamic phenomena.

The optimal trajectory evaluation is based on a fixed-time two-point boundary value problem between the hover and the forward flight states. The optimal trajectories are evaluated using commercial nonlinear constrained programming algorithm *fmincon* available in MATLAB optimization toolbox. The algorithm is based on sequential quadratic programming state of the art techniques coupled with Quasi-Newton methods for better efficiency. The algorithm is discussed in detail above as well as implemented in chapter four for trim flight problem. The sampling time between two consecutive control inputs is 0.2 sec. the classical Runge-Kutta fourth-order method is used for the shooting purpose. The control variables for the optimization problem are the angle of attack of the aircraft (α_{fus}), angle of attack of the wing (α_{wing}) and thrust (T), as follows:

$$\vec{u} = [\alpha_{fus} \quad \alpha_{wing} \quad T]^T \quad (5.15)$$

where $\vec{u}$ is the control variable vector. For the fixed-wing case, the same control vector can be used by imposing an additional equality constraint that, at any instant,

$$\alpha_{fus} = \alpha_{wing} \quad (5.16)$$

The objective function is formulated based on the derivation from mechanical energy formulation[59] as follows:

$$J = w_1 \cdot \frac{1}{2} m |(u_t)^2 - (u_{tgt})^2| + w_2 \cdot \frac{1}{2} m v_t^2 + w_3 \cdot \left(\sum_{i=1}^t |y_i| \right) \quad (5.17)$$

Optimal trajectories are evaluated from the objective function as shown in Equation 5.17. The first term in the objective function in Equation 5.17 indicates the difference in the kinetic energy due to the terminal horizontal velocity and

the target horizontal cruise velocity u_{tgt} , which are 15 m/s. The second term in the objective function represents the kinetic energy due to terminal vertical velocity, and the third term represents the potential energy. The minimization of the objective function indicates the desire to achieve the terminal horizontal target velocity of 15 m/s, with minimum altitude variation during the transition. The weighting factors (w_1 , w_2 and w_3) can be adjusted to achieve desired performance. The desired performance is defined in a way that the aircraft is able to achieve the target horizontal cruise velocity with minimum altitude changes within the specified time. As an example, if weighting parameter w_1 is increased then velocity is achieved but with the penalty of altitude loss. Based on iterative weight-tuning in this study, the weighting factors used are $w_1 = w_2 = w_3 = 1$. The objective function used is the same for both fixed and variable incidence wing cases. The common constraints applied to the dynamics of the vehicle during hover-to-cruise optimization for both the fixed-wing and variable-incidence wing cases are shown in Table 5-1.

Table 5-1 Constraints posed to the UAV dynamics for hover-to-cruise optimization

$V \geq 0$	$u_i \geq 0$	$u_i \leq 15m/sec$
$\alpha_{fus} \geq 0$	$\alpha_{fus} \leq \frac{\pi}{2}$	$\Delta\alpha_{fus} \leq 10^\circ / step$
$\alpha_{wing} \geq 0$	$\alpha_{wing} \leq \frac{\pi}{2}$	$\Delta\alpha_{wing} \leq 10^\circ / step$
$T_i \geq 0.5$	$T_i \leq T_{max}$	$\Delta T \leq 2N / step$
$\dot{\alpha}_{fus} \leq 50^\circ / sec$	$\dot{\alpha}_{wing} \leq 50^\circ / sec$	$\dot{T} \leq 4N / sec$

where V is the freestream velocity, u_i is the horizontal velocity at i^{th} instant, α_{fus} is the angle of attack of the aircraft, α_{wing} is the angle of attack of the outboard wing section, T_i is the thrust produced by the propeller at i^{th} instant.

The angular rate constraints are included in order to capture the pitching rate limitations so that the real aircraft dynamics are better represented.

5.4.2 Altitude Variations

To assess the influence of the aerodynamic vectoring in terms of the altitude variation during the transition, the trajectory properties for a specific value of maximum thrust to weight ratio $((T/W)_{max})$ are examined. In this study, the UAV with variable-incidence wing feature is compared with the fixed-wing one for various mass values from 0.7 kg to 2.5 kg while The $(T/W)_{max}$ is kept at 1.10.

Figure 5-1 Trajectory plot for conventional and aerodynamic vectoring cases for $(T/W)_{max}=1.10$

The resulting trajectories are plotted in Figure 5-1. If we increase the weighting factors w_2 and w_3 in Equation 5.26 from unity to higher values, the final velocity at the end of the transition maneuver will be less than the target value of 15 m/s. On the other hand, if we decrease the weight factors w_2 and w_3 , the resulting transition maneuvers will be subjected to more altitude loss.

Figure 5-2 Altitude loss for 2 sec optimized transitions for conventional and aerodynamic vectoring cases

Figure 5-2 shows the time history of the differential altitude loss for the transition time of 2 s. Differential altitude loss is the difference between the altitude of the variable-incidence and fixed wing cases. The figure clearly depicts that as the mass of the vehicle increases, the variable-incidence wing scheme becomes more and more effective to alleviate the altitude loss problem for a specific $(T/W)_{max}$. The final altitude-loss alleviation varies from 0.35 m to 0.7 m across the mass variation from 0.7 kg to 2.5 kg. An interesting phenomenon is observed at approximately 1.5 s until which, the velocity of the aircraft is low and the flight is primarily sustained by the propulsive force. As the velocity increases, the aerodynamic forces start to become dominant. During this regime, the altitude loss alleviation achieved using this proposed aerodynamic assisted transition control technique is more pronounced.

5.4.3 Effect of Mass on $(T/W)_{max}$ Requirements

In this part, the effect of the variable-incidence wing on the required $(T/W)_{max}$ to perform a hover-to-cruise transition with no loss of altitude is analyzed. In Figure 5-3, the required $(T/W)_{max}$ for the constant altitude transitions is plotted across several mass for both fixed and variable-incidence wing cases. Several observations can be made from the Figure 5-3. Firstly, with the increase in mass of the vehicle, the required $(T/W)_{max}$ increases for a particular transition time for both cases. This can be explained from the point of view that the transition maneuver is essentially a shift from propulsive-borne flight to aerodynamic-borne flight. In order to maintain constant altitude the following relationship must hold during the maneuver at all instants:

$$\frac{T \sin \alpha_{fus}}{W} + \frac{L}{W} = 1 \quad (5.18)$$

As the aerodynamic contribution for L remains unchanged, with the increase in W , L/W decreases and therefore T/W has to increase to satisfy Equation 5.18.

Figure 5-3 also indicates that, with a longer specified transition time, the required $(T/W)_{max}$ decreases. The trend is similar for different mass values. Moreover, with the use of the variable-incidence wing, the required $(T/W)_{max}$ can be significantly reduced (more so for the shorter transition times). As longer transition time is allocated, the advantage of the variable-incidence wing becomes less obvious such that the $(T/W)_{max}$ value approaches hover thrust. For indoor autonomous UAV application in which such agile maneuvers are carried out under space restrictions, a shorter time is very desirable, and with the

substantial decrease in $(T/W)_{max}$ requirement for a particular mass, the aerodynamic vectoring feature offers a significant advantage.

Figure 5-3 Effect of mass on $(T/W)_{max}$ for optimized hover-to-cruise transition for fixed and variable-incidence wing configurations

5.4.4 Control Variations

The control histories for several constant-altitude transition maneuvers at different transition time are plotted in Figure 5-4 and Figure 5-5 for the fixed and variable-incidence wings, respectively. It can be observed that α_{fus} has an almost linear trend for most of the transition time in both cases. Near the end of transition, when the aircraft flight path angle is small, α_{fus} is almost constant at the cruise value.

For the fixed-wing case, there is a significant dip in the thrust history near the end of the maneuver (Figure 5-4). This trend is observed for all the three transition times simulated. This phenomenon is associated with the vehicle's stall properties. As the vehicle starts this maneuver from a trimmed hover condition, which is in the post-stall regime, and transitions to lower α_{fus} , the

vehicle enters into pre-stall regime and results in a significant rise in lift. This sudden increase leads to the immediate decrease in the thrust needed to sustain the horizontal flight path. The thrust needed increases again to achieve the specified terminal flight velocity at the end of the transition.

Figure 5-4 Optimized conventional transitions: Angle of attack (left); thrust histories (right)

It can be observed from Figure 5-5 that for the variable-incidence wing case, α_{wing} remains in the pre-stall regime such that the outboard wing section poses less drag during the transition and more lift as the airspeed of the aircraft increases. α_{fus} behaves in almost a similar manner to that of the conventional case. As the vehicle picks up the speed, α_{wing} reduces to its cruise value near the end of the transition maneuver. As can be observed from the thrust history in Figure 5-5, the dip phenomenon as in the conventional transition case does not appear here because of the sustained aerodynamic contribution of the variable-incidence wing. The outboard wing is in the pre-stall flow regime for all time during the transition and does not cross the stall point. This reduces the variation in thrust, which will potentially reduce the appearance of unwanted dynamics due to the abrupt thrust variation.

Figure 5-5 Optimized aerodynamic-vectoring transitions: Fuselage and wing angle of attack (left); thrust histories (right)

5.4.5 Concluding Remarks

The optimization of the transition maneuver using point mass modeling of the dynamics indicate improvement in the transition performance using variable-incidence wing as compared to using fixed-wing over a variety of parameters like $(T/W)_{max}$, allocated transition time and altitude loss. The cruise-to-hover transitions are not explored at this point and will be part of the investigations using more detailed longitudinal dynamic model in the next section.

5.5 Three DOF Transition Maneuver Optimization

After getting the preliminary notion of the improvement in performance using aerodynamic vectoring feature of the UAV in hover-to-cruise transitions, the longitudinal dynamic model is changed from point mass model to a more elaborate three-degree-of-freedom longitudinal model. However, it is still assumed that the roll, yaw and sideslip dynamics will have no effect on the transition dynamics and the aircraft motion is restricted in the plane of symmetry only. In this section optimal trajectories are analyzed from some other

aspects such as elevator effectiveness and terminal velocity as well. Moreover, unsteady aerodynamic effects are also incorporated in order to understand their contribution on the maneuver. The transition maneuver performances are compared between the conventional fixed-wing configuration (two control variables: thrust and elevator deflection) and the proposed aerodynamic vectoring configuration (three control variables: thrust, elevator deflection and angle of incidence of the outboard wing).

5.5.1 Governing Equations of Motion

The generic governing equations for the three-degree-of-freedom longitudinal dynamics have been presented earlier in section 4.2. The equations of motion are presented here in a more descriptive form. .

$$\left. \begin{aligned} \dot{u} &= \frac{T}{m} + \frac{1}{2} \rho \frac{(u^2 + w^2) S_{ref}}{m} (C_L \sin(\alpha_{fus}) - C_D \cos(\alpha_{fus})) - \frac{W \sin \theta}{m} - qw \\ \dot{w} &= -\frac{1}{2} \rho \frac{(u^2 + w^2) S_{ref}}{m} (C_L \cos(\alpha_{fus}) + C_D \sin(\alpha_{fus})) + \frac{W \cos \theta}{m} + qu \\ \dot{q} &= \frac{1}{2} \rho \frac{(u^2 + w^2) S_{ref} \bar{c}}{I_{yy}} C_M \end{aligned} \right\} \quad (5.19)$$

The kinematic relationships from body to inertial axes are as follows:

$$\left. \begin{aligned} \dot{u}_F &= \dot{u} \cos \theta + \dot{w} \sin \theta \\ \dot{w}_F &= -\dot{u} \sin \theta + \dot{w} \cos \theta \\ \dot{s}_F &= u \cos \theta + w \sin \theta \\ \dot{y}_F &= -u \sin \theta + w \cos \theta \end{aligned} \right\} \quad (5.20)$$

5.5.2 Problem Formulation

The optimization formulation for the three-degree-of-freedom case is similar to the point-mass model case unless specified otherwise. The optimal maneuver evaluation is based on a fixed-time two point boundary value problem

between the near-hover and forward flight states. Like before, the optimal solutions are obtained using commercial nonlinear constrained programming algorithm *fmincon* available in MATLAB optimization toolbox. In this case, it is assumed that the UAV has a slight initial velocity of 1 m/s. This mild assumption is used for better scaling effects. The control variables for the optimization of conventional fixed-wing aircraft scheme are δ_e and T as follows:

$$\mathbf{u} = [\delta_e \quad T]^T \quad (5.21)$$

whereas, for the aerodynamic-vectoring transition scheme, the control variables are α_{wing} , δ_e and T such that:

$$\mathbf{u} = [\alpha_{wing} \quad \delta_e \quad T]^T \quad (5.22)$$

The objective function for the optimization of both schemes is as follows:

$$J = c_1 |V_{\infty N} - (V_T)| + c_2 \sum_{i=1}^N |y_i| \quad (5.23)$$

The first term in the objective function (Equation 5.23) indicates the difference in the terminal velocity and target terminal velocity; and the second term in Equation 5.23 represents the summation of absolute altitude changes. The minimization of the objective function (Equation 5.23) indicates the desire to achieve the terminal horizontal target velocity with minimum altitude variation during the transition. The weighting factors (c_1 and c_2) can be adjusted to achieve desired performance. In this study, the weighting factors used are: $c_1 = c_2 = 1$. The selection of these weighting factors is based on iterative adjustment. If parameter c_1 is increased, the emphasis on achieving terminal velocity within specified time is more than the altitude variations. As a consequence, the aircraft altitude hold performance is compromised during

transitions. Similarly, if parameter c_2 is increased, the emphasis on aircraft altitude hold is higher than the terminal velocity. Therefore, the objective function forces the aircraft to strictly maintain the altitude and less emphasis in achieving the terminal velocity. The objective function used is the same for both fixed and variable-incidence wing cases for fair comparison. The constraints applied to the dynamics of the vehicle during hover to cruise optimization for fixed-wing and variable-incidence wing cases are shown in Table 5-2 and Table 5-3 respectively.

Table 5-2 Constraints posed to the dynamics of conventional scheme for hover-to-cruise optimization

$V_\infty \geq 0$	$u_{F_i} \geq 0$	$u_{F_i} = V_T$
$\alpha_{fus} \geq 0$	$\alpha_{fus} \leq \frac{\pi}{2}$	$\Delta w_{F_i} \leq 0.05m / sec$
$T_i \geq 0.5$	$T_i \leq T_{\max}$	$\Delta T \leq 2.5N / step$
$\delta_{elev} \leq 25^\circ$	$\delta_{elev} \geq 25^\circ$	$ q_N - q_{N-1} \leq 0.001rad / s$

Table 5-3 Constraints posed to the dynamics of the aerodynamic-vectoring scheme for hover-to-cruise optimization

$V_\infty \geq 0$	$u_{F_i} \geq 0$	$u_{F_i} = V_T$
$\alpha_{fus} \geq 0$	$\alpha_{fus} \leq \frac{\pi}{2}$	$\Delta w_{F_i} \leq 0.05m / sec$
$\alpha_{wing} \geq 0$	$\alpha_{wing} \leq \frac{\pi}{2}$	$\Delta \alpha_{wing} \leq 20^\circ / step$
$T_i \geq 0.5$	$T_i \leq T_{\max}$	$\Delta T \leq 2.5N / step$
$\delta_{elev} \leq 25^\circ$	$\delta_{elev} \geq 25^\circ$	$ q_N - q_{N-1} \leq 0.001rad / s$

5.5.3 Optimized Transition Maneuvers

A representative case of the optimized transition maneuver is given in Figure 5-6 and Figure 5-7, which shows the conventional and aerodynamic vectoring cases for the transition time of 2 s. The aircraft position together with its orientation and the incidence angle of the wing are plotted at several indicated time instances and airspeeds. It is interesting to note and also as illustrated in Figure 5-7 that during the optimized transition, the angle of incidence of the wing remains in the pre-stall regime. A closer observation indicates that the wing incidence stays close to the value that yields maximum lift at the early transition phase and eventually reaches the necessary cruise incidence towards the end. This shows the importance of the variable-incidence wing in assisting the transition aerodynamically to achieve the optimized maneuver. Also note that the optimization scheme leads to the execution of the transition maneuver at practically constant altitude. For the fixed-incidence case, as depicted (Figure 5-6), constant altitude transitions is also feasible to achieve but with higher T/W and less favorable control histories as will be discussed in the next sections. The stalled wing of the fixed-wing configuration poses higher drag values in the initial stage of the transition. This results in a slightly delayed response to pick up the acceleration in the beginning of the maneuver.

Figure 5-6 Optimized transition trajectory for the fixed-wing UAV

Figure 5-7 Optimized transition trajectory for the aerodynamic vectoring UAV

5.5.4 Effect of Unsteady Aerodynamics on $(T/W)_{max}$ Requirements

During previous studies [5, 33, 59, 60], the effect of unsteady aerodynamics is not incorporated. Here, such effect is included in these simulations based on the formulation given in section 3.5. To enlighten the readers about the effects of the unsteady phenomena, the maximum thrust-to-weight ratio required for a transition maneuver from 1 m/s to 15 m/s is studied and the results are presented in Figure 5-8. The $(T/W)_{max}$ is plotted for both fixed and variable-incidence wing cases. Several observations can be made. First the effect of the unsteady aerodynamics is more significant for shorter transition times. As the time allocated to execute the transition is increased, the required $(T/W)_{max}$ becomes lower and eventually approaches the steady case. Second, the unsteady aerodynamics has more pronounced effect on $(T/W)_{max}$ needed for fixed wing transition maneuvers than for the variable-incidence wing ones. For

the fixed-wing case, the wing is initially at high angle of attack in post-stall regime. The fixed-wing will go under huge rotational motion to come in pre-stall regime. In case of variable-incidence wing, the wing angle-of-attack always remains in pre-stall regime and therefore, the rate of change of angle of attack is minimum. Therefore, the unsteady phenomenon is more dominant for fixed-wing case than variable-incidence wing.

Figure 5-8 Comparative $(T/W)_{max}$ requirements between steady and unsteady aerodynamics

5.5.5 Effect of Velocity on $(T/W)_{max}$ Requirements

The performance of the transition maneuvers is greatly affected by the targeted terminal velocity, an important parameter in the objection function (Equation 5.23). In Figure 5-9, the required $(T/W)_{max}$ for the optimal transitions is plotted across several terminal velocities for both fixed and variable-incidence wings. With the increase in requirement of terminal velocity of the vehicle, the required $(T/W)_{max}$ increases for a particular transition time for both cases. As the aircraft pitches down, it requires high value of thrust to attain the terminal velocity. Moreover, the advantage of aerodynamic vectoring in terms of smaller $(T/W)_{max}$ is obvious. The drag posed by the wings in the post-stall regime is

higher than in pre-stall regime. For the variable-incidence wing cases, the wings remain in the pre-stall regime thereby posing smaller drag and easing the aircraft to accelerate enough until the desired terminal velocity is reached. For indoor autonomous UAV application in which such agile maneuvers are carried out under space restrictions, a shorter transition time with the substantial decrease in $(T/W)_{\max}$ requirement is very desirable.

Figure 5-9 Effect of terminal velocity on maximumm thrust-to-weight ratio

5.5.6 Control Variations

The phenomenon of aerodynamic vectoring is discussed by plotting transition state histories for conventional and aerodynamic-vectoring cases. For this purpose, a sample case is selected such that the terminal velocity of 18 m/s is achieved in 2 s for both cases. In Figure 5-10, the thrust histories are compared. The thrust required to perform the optimal transition maneuver is generally higher for the fixed-wing case as compared to that of the variable-incidence wing one.

Figure 5-10 Thrust history for the optimized transitions

Wing and elevator angle histories are plotted in Figure 5-11 and Figure 5-12 for both cases. For the fixed-wing case, the whole aircraft has to transition from post-stall regime to pre-stall regime quickly thereby generating high pitch rate whereas, for the variable-wing case, the wing angle of attack always remain in the pre-stall regime thereby posing significantly less drag during the transition and generating more lift as the speed of the aircraft increases. Since the wing always remain in pre-stall regime, the pitch rate induced effect is also minimized for this case. The reduction of drag and rapid increase in lift effect for the variable-incidence wing case can therefore be observed in Figure 5-10 in terms of less thrust required than the fixed-wing case over the transition history.

Figure 5-11 Wing and elevator angle histories of the conventional configuration

Figure 5-12 Wing and elevator angle histories of the aerodynamic vectoring configuration

5.5.7 Effect of Elevator Effectiveness on $(T/W)_{max}$ Requirements

The key to such rapid transitions is to pitch the nose of the aircraft down as quickly as possible while maintaining the altitude. The elevator plays an important role at near-hover flying conditions in generating rotational moments. Therefore, the effect of elevator effectiveness on the transition maneuvers is also studied for the cases with and without aerodynamic vectoring.

Figure 5-13 Effect of elevator effectiveness on $(T/W)_{max}$

A comparative analysis for transitioning to 18 m/s in 1.8 s is plotted from Figure 5-13 to Figure 5-15. The elevator effectiveness of the original system is taken as 100 percent. In the study, the coefficient of pitching moment for the elevator contribution is systematically scaled down from 100 percent to 5 percent. The requirement of $(T/W)_{max}$ to maintain the altitude is computed for both fixed and variable-incidence wing cases and shown in Figure 5-13. It is interesting to observe that the $(T/W)_{max}$ requirement increases gradually for both cases with the decrease in elevator effectiveness. The increase in $(T/W)_{max}$ for the variable-incidence wing case is higher than the fixed-wing case. This can be interpreted that the aerodynamic vectoring configuration loses its edge over fixed-wing with the decrease in elevator effectiveness. The rise in $(T/W)_{max}$ values can be explained by plotting the pitch angle and elevator histories in Figure 5-14 and Figure 5-15.

Figure 5-14 Fixed-wing configuration history for elevator effectiveness: pitch angle (left) and elevator deflection (right)

Figure 5-15 Aerodynamic vectoring configuration history for elevator effectiveness: pitch angle (left) and elevator deflection (right)

Figure 5-14 shows that with the decrease in elevator effectiveness, the pitch down occurs at slower rates but achieves the same attitude at the end of the maneuver. The gaining of cruise attitude for the aircraft is delayed and therefore results in high $(T/W)_{max}$ requirement to maintain the altitude as well as accelerating at the end of the maneuver to achieve terminal velocity. Also with the decrease of elevator effectiveness, higher elevator control demands are generated to attain the cruise attitude. Moreover, the elevator demand for the variable-incidence wing case is slightly higher than the fixed-incidence case. This can be attributed to the fact that for the fixed-wing case, the wing in post-stall regime produces higher pitching down moment thereby posing fewer requirements on the elevator to generate additional pitching moments. On the other hand, for the variable-wing case, the angle of attack of the wing remains in pre-stall regime which consequently generates smaller pitching moments (Figure 5-15).

5.5.8 Cruise-to-Hover Transitions

We would like to remind the readers that in order to have improved cruise-to-hover transitions, the objectives are to achieve them with following attributes

- Lower altitude gain/fluctuations
- Lower horizontal distance travelled
- Reduced transition time

5.5.8.1 Problem Formulation

The objective function for cruise-to-hover transitions used here is the same as Equation 5.23, however the weighting factors are adjusted accordingly. After some iterations, the suitable weighting factors found are: $c_1 = 1, c_2 = 0.1$. In order to reduce velocity, we have to slightly penalize altitude deviation. On the other hand, the penalty should not be too much such that the altitude variations become significant. The criterion adopted here is that the altitude variations should be less than one span-length of the aircraft. The aircraft is set to decelerate from the initial velocity of 10 m/s. Moreover, the terminal velocity is set to be less than 3 m/s i.e. the vehicle is assumed to enter hover phase once its velocity reaches 3 m/s. This constraint relaxation is done for the reason that the altitude gain as well as time required for the final adjustment may become large if perfect hover condition is enforced as the target terminal states.

In addition to the general constraints discussed in section 3.1, α_{fus} is also constrained to be greater than 80° at the end of the maneuver.

5.5.8.2 Results and Discussion

Multiple optimal cruise-to-hover transition maneuvers are evaluated for the fixed- as well as the variable-incidence wing cases. In this case the aircraft must produce higher values of drag so that it decelerates fast entering hover phase. An inherent way of transitioning is to gain altitude (potential energy) to reduce velocity (kinetic energy), which is reported in literature [22, 33, 61]. With the usage of aerodynamic-vectoring, the wing is generally in pre-stall regime and thus its contribution to generate the drag for cruise-to-hover transitions is lower than the fixed-wing configuration. It can then be deduced that the advantage of the variable-incidence wing case for hover-to-cruise transitions will not be as significant as in hover-to-cruise transitions.

A sample of optimal cruise-to-hover trajectory is shown in Figure 5-16 for both fixed and variable-incidence wing cases. The transition time of the sample trajectory is 3 s. Both trajectories show some variations in altitude during deceleration. In terms of the horizontal distance travelled, the variable-incidence wing case travels a bit longer than the fixed-wing case and the situation intensify if the initial velocity is higher. Generally the altitude variations are reduced more as the time allocated for the transition maneuver is increased.

Figure 5-16 Optimized cruise-to-hover trajectory in inertial space for 3 s transition time

The associated histories of wing and fuselage angle of attack are shown in Figure 5-17. The aircraft pitches up in both cases and eventually reaches hover attitude. For the fixed-wing case, the aircraft reaches hover attitude earlier than the variable-incidence wing case. Because of the earlier entrance of the fixed-wing case into the post-stall regime, it is able to harness the drag more to decelerate over shorter horizontal distance (Figure 5-16).

Figure 5-17 Wing angle and Fuselage angle history for fixed and variable incidence wing cases

The corresponding velocity profile for both scenarios is plotted in Figure 5-18. It can be seen that there is a slight increase in the velocity at the beginning of the maneuver for both cases and it eventually reduces to reach hover. Based on the evidence collected from transition analysis, the usage of variable-incidence wing to assist cruise-to-hover transitions shows no obvious advantage.

Figure 5-18 Velocity profile for 3 s cruise-to-hover transition maneuver

5.6 Stability of Optimal Solutions

In this section, the stability of the optimal solutions obtained previously is examined. As these solutions in general do not represent equilibrium condition at each solution point, a general technique for evaluating the stability of equilibrium is not applicable. For this reason, a contraction theory based stability analysis is used here. Contraction theory is a newly coined nonlinear theory which establishes the stability criteria of the solution instead of a single point in trajectory. Based on this theory, the stability of the optimal transition maneuvers above can be analyzed.

5.6.1 Contraction Theory

Contraction theory is formulated on the fundamentals of fluid mechanics and differential geometry. It can be used in conjunction with Lyapunov theory to study the stability of nonlinear systems. Contraction theory assess the convergence of all neighboring trajectories to one another, therefore, it is a stricter stability condition than Lyapunov convergence, which only considers convergence to an equilibrium point. Moreover, contraction convergence results

are typically exponential, and thus stronger than those based on most Lyapunov-like methods. A brief review of the contraction theory is presented here and readers are referred to the references [62-64] for detailed descriptions and proofs.

Let the system be defined by the set of equations such that

$$\dot{\mathbf{x}} = f(\mathbf{x}, t) \quad (5.24)$$

where f is an $n \times 1$ nonlinear vector function and $\mathbf{x}$ is the $n \times 1$ state vector. All quantities are assumed to be real and smooth, by which it is meant that any required derivative or partial derivative exists and is continuous (Lipschitz condition). The system equation can be thought of as an n -dimensional fluid flow where $\dot{\mathbf{x}}$ is the n -dimensional “velocity” vector at the n -dimensional position $\mathbf{x}$ and time t . Based on this, the virtual displacement $\delta\mathbf{x}$ is defined as an infinitesimal quantity at certain time, which is a well-defined mathematical object in physics and calculus of variations.

$$\delta\dot{\mathbf{x}} = \frac{\partial f}{\partial \mathbf{x}}(\mathbf{x}, t) \delta\mathbf{x} \quad (5.25)$$

Equation 5.25 can be further described as

$$\frac{d}{dt}(\delta\mathbf{x}^T \delta\mathbf{x}) = 2\delta\mathbf{x}^T \mathbf{J}_s \delta\mathbf{x} \leq 2\lambda_m(\mathbf{x}, t) \delta\mathbf{x}^T \delta\mathbf{x} \quad (5.26)$$

where $\mathbf{J}_s = \frac{1}{2} \left(\frac{\partial f^T}{\partial \mathbf{x}} + \frac{\partial f}{\partial \mathbf{x}} \right)$ is the symmetric Jacobian. The largest eigenvalue of

the symmetric part of Jacobian is represented by $\lambda_m(\mathbf{x}, t)$ and $\delta\mathbf{x}^T \delta\mathbf{x}$ represents the squared distance between the neighboring trajectories. If this real eigenvalue $\lambda_m(\mathbf{x}, t)$ is strictly uniformly negative, then any infinitesimal length $\|\delta\mathbf{x}\|$ converges exponentially to zero.

Definition. Given the system of equations $\dot{\mathbf{x}} = f(\mathbf{x}, t)$, a region of the state space is called a contraction region if the Jacobian $\partial f / \partial \mathbf{x}$ is uniformly negative definite in that region[63].

The $\partial f / \partial \mathbf{x}$ uniformly negative definite Jacobean, J , means the evaluation of symmetric form of the above Jacobian reveals:

$$J_s = \frac{1}{2} (J + J^T) \quad (5.27)$$

where the subscript s stands for symmetric. Now, the virtual displacement vector $\delta \mathbf{x}$ can be expressed using the differential co-ordinate transformation

$$\delta \mathbf{z} = \Theta \delta \mathbf{x} \quad (5.28)$$

where $\Theta(x, t)$ is a square matrix. The time derivative of the above equation can be computed as follows

$$\frac{d}{dt} \delta \mathbf{z} = \dot{\Theta} \delta \mathbf{x} + \Theta \dot{\delta \mathbf{x}} \quad (5.29)$$

$$\frac{d}{dt} \delta \mathbf{z} = \left(\dot{\Theta} + \Theta \frac{\partial f}{\partial \mathbf{x}} \right) \Theta^{-1} \delta \mathbf{z} = F \delta \mathbf{z} \quad (5.30)$$

where the generalized Jacobian for a continuous time system is defined as

$$F = \left(\dot{\Theta} + \Theta \frac{\partial f}{\partial \mathbf{x}} \right) \Theta^{-1} \quad (5.31)$$

Now, Equation 5.29 can be written in form

$$\Theta^T \frac{d}{dt} \delta \mathbf{z} = M \dot{\delta \mathbf{x}} + \Theta^T \dot{\Theta} \delta \mathbf{x} \quad (5.32)$$

$$\Theta^T \frac{d}{dt} \delta \mathbf{z} = \left(M \frac{\partial f}{\partial \mathbf{x}} + \Theta^T \dot{\Theta} \right) \delta \mathbf{x} \quad (5.33)$$

where $M(\mathbf{x}, t) = \Theta^T \Theta$ represents a symmetric matrix. The matrix M should be taken as uniformly positive definite, so that the exponential convergence of $\delta \mathbf{z}$ to

0 also implies exponential convergence of $\delta \mathbf{x}$ to 0. Depending on the application, M may be identity matrix or obtained from geometric features (e.g. inertia matrix of mechanical system). It can also be the combination of simple contracting subsystems, semi-definite programming or sums-of-squares programming.

5.6.1.1 Generalized Linear Eigenvalue Analysis

Contraction analysis can be considered as a generalization of linear eigenvalue analysis based on the following points:

- Convergence of a dynamic system is treated separately from limit behavior leading to conceptual simplifications [62].
- Eigenvalue analysis performed using Contraction analysis can be made co-ordinate invariant [62].

The linear eigenvalue analysis is applied to linearized problems around a specific equilibrium point. However, contraction analysis can be applied to nonlinear systems directly [64]. A suitable differential co-ordinate transformation can be selected such that the generalized Jacobian is co-ordinate invariant. The usage of contraction theory for the stability analysis here is based on the fact that the solutions (trajectories) are not obtained through linearization around specific equilibrium points. An alternate stability analysis can be carried out using Lyapunov approach, however it is generally more tedious than the contraction analysis as it involves finding a suitable Lyapunov function in an ad hoc manner.

5.6.2 Stability of Open-Loop Dynamics

The longitudinal dynamics comprises of translational and rotational dynamics (Equation 5.34). The first two equations describe the translational dynamics of the aircraft whereas; the next two equations govern the rotational dynamics. In this section, aerodynamic contribution to the open-loop stability of the longitudinal dynamics is evaluated. Since the aerodynamic vectoring is used to improve the transition maneuver, its effect on the overall dynamics must be considered. The translational equations in longitudinal dynamics are given in the body axis as follows.

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \frac{T_c}{m} + \frac{\rho S_{ref} (u^2 + w^2) (C_L \sin \alpha_{fus} - C_D \cos \alpha_{fus})}{2m} - \frac{W \sin \theta}{m} - qw \\ - \frac{\rho S_{ref} (u^2 + w^2) (C_L \cos \alpha_{fus} + C_D \sin \alpha_{fus})}{2m} + \frac{W \cos \theta}{m} + qu \\ \frac{q}{2I_{yy}} \\ \frac{\rho S_{ref} \bar{c} C_M}{2I_{yy}} \end{bmatrix} \quad (5.34)$$

where $T = \frac{T_c}{m}$; $k = \frac{\rho S_{ref}}{2m}$; $p = \frac{\rho S_{ref} \bar{c}}{2I_{yy}}$ and $V = \sqrt{u^2 + w^2}$. The trigonometric

relationships of the angle of attack with the local velocities can be written as

$\sin \alpha(.) = \frac{w}{V}$; $\cos \alpha(.) = \frac{u}{V}$ and $\tan \alpha(.) = \frac{w}{u}$. Writing the longitudinal dynamics

based on these kinematic and trigonometric relationships result as

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} T + k C_L \sqrt{u^2 w^2 + w^4} - k C_D \sqrt{u^4 + u^2 w^2} - g \sin \theta - qw \\ - k C_L \sqrt{u^4 + u^2 w^2} - k C_D \sqrt{u^2 w^2 + w^4} + g \cos \theta + qu \\ q \\ p C_M \end{bmatrix} \quad (5.35)$$

To simplify the analysis, let the aerodynamic coefficients be approximated in terms of trigonometric functions of angle of attack.

$$\begin{bmatrix} C_L \\ C_D \\ C_M \end{bmatrix} = \begin{bmatrix} 1.75 \sin \alpha_w \cos^2 \alpha_w \\ C_{Do} + 0.8 \sin^2 \alpha_w \\ -0.19 \sin \alpha_w \end{bmatrix} \quad (5.36)$$

Using the same trigonometric relationships, the aerodynamic coefficients can be expressed in terms of horizontal and vertical velocities in body axes system.

$$\begin{bmatrix} C_L \\ C_D \\ C_M \end{bmatrix} = \begin{bmatrix} 1.75 \frac{u_w^2}{V^3} \\ C_{Do} + 0.8 \frac{w^2}{V^2} \\ -0.19 \frac{w}{V} \end{bmatrix} \quad (5.37)$$

Now the contraction formulation is invoked on the Equation 5.35 in terms of virtual dynamics relationships i.e. $\delta \dot{\mathbf{z}} = \Theta \delta \dot{\mathbf{v}}$. If $\Theta = I_4$ then the equations of longitudinal dynamics can be written as

$$\begin{bmatrix} \delta \dot{u} \\ \delta \dot{w} \\ \delta \dot{\theta} \\ \delta \dot{q} \end{bmatrix} = \begin{bmatrix} \frac{\partial \delta \dot{u}}{\partial \delta u} & \frac{\partial \delta \dot{u}}{\partial \delta w} & \frac{\partial \delta \dot{u}}{\partial \delta \theta} & \frac{\partial \delta \dot{u}}{\partial \delta q} \\ \frac{\partial \delta \dot{w}}{\partial \delta u} & \frac{\partial \delta \dot{w}}{\partial \delta w} & \frac{\partial \delta \dot{w}}{\partial \delta \theta} & \frac{\partial \delta \dot{w}}{\partial \delta q} \\ \frac{\partial \delta \dot{\theta}}{\partial \delta u} & \frac{\partial \delta \dot{\theta}}{\partial \delta w} & \frac{\partial \delta \dot{\theta}}{\partial \delta \theta} & \frac{\partial \delta \dot{\theta}}{\partial \delta q} \\ \frac{\partial \delta \dot{q}}{\partial \delta u} & \frac{\partial \delta \dot{q}}{\partial \delta w} & \frac{\partial \delta \dot{q}}{\partial \delta \theta} & \frac{\partial \delta \dot{q}}{\partial \delta q} \end{bmatrix} \begin{bmatrix} \delta u \\ \delta w \\ \delta \theta \\ \delta q \end{bmatrix} = J \begin{bmatrix} \delta u \\ \delta w \\ \delta \theta \\ \delta q \end{bmatrix} \quad (5.38)$$

where J is the Jacobian of the system which can be expressed in more detailed as

$$J = \begin{bmatrix}
kC_{L_{\delta u}} \sqrt{\delta u^2 \delta w^2 + \delta w^4} & kC_{L_{\delta w}} \sqrt{\delta u^2 \delta w^2 + \delta w^4} & -g \cos \delta \theta & -\delta w \\
-kC_{D_{\delta u}} \sqrt{\delta u^4 + \delta u^2 \delta w^2} & -kC_{D_{\delta w}} \sqrt{\delta u^4 + \delta u^2 \delta w^2} & & \\
+\frac{kC_L \delta u \delta w}{\delta V} - \frac{kC_D (2\delta u^2 + \delta w^2)}{\delta V} & +\frac{kC_L (\delta u^2 + 2\delta w^2)}{\delta V} - \frac{kC_D \delta u \delta w}{\delta V} - \delta q & & \\
-kC_{L_{\delta u}} \sqrt{\delta u^4 + \delta u^2 \delta w^2} & -kC_{L_{\delta w}} \sqrt{\delta u^4 + \delta u^2 \delta w^2} & -g \sin \delta \theta & \delta u \\
-kC_{D_{\delta u}} \sqrt{\delta u^2 \delta w^2 + \delta w^4} & -kC_{D_{\delta w}} \sqrt{\delta u^2 \delta w^2 + \delta w^4} & & \\
-\frac{kC_L (2\delta u^2 + \delta w^2)}{\delta V} - \frac{kC_D \delta u \delta w}{\delta V} + \delta q & +\frac{kC_L \delta u \delta w}{\delta V} - \frac{kC_D (\delta u^2 + 2\delta w^2)}{\delta V} & & \\
0 & 0 & 0 & 1 \\
pC_{M_{\delta u}} & pC_{M_{\delta w}} & 0 & 0
\end{bmatrix}$$

(5.39)

Now the final symmetric real Jacobian is evaluated as of Equation 5.27. The typical stability characteristics of the optimal solutions can be examined by applying the contraction theory above to the sample optimal hover-to-cruise transition maneuver in Section 5.4, where the vehicle transitions from hover to cruise in 2 s with the terminal velocity is set at 15 m/s. The eigenvalues of the symmetric Jacobian from contraction analysis (Equation 5.39) during the maneuver are plotted in Figure 5-19.

Figure 5-19 Eigenvalues of the symmetric Jacobian for hover-to-cruise transition

It can be observed in Figure 5-19 that the eigenvalues associated with the translational dynamics suggest exponentially stable response across the whole

maneuver. The stability of translational dynamics increases as the aircraft picks up speed towards the end of the transition maneuver. However, it can be seen that the rotational dynamics is unstable across the complete transition maneuver. This instability worsens as the aircraft picks up the speed.

Similarly the eigenvalue pattern for cruise-to-hover transition case, as discussed in Section 5.4.8.2, is evaluated. The vehicle transitions from cruise to near-hover situation in 2 s (Figure 5-16). The eigenvalues from the symmetric Jacobian (Equation 5.39) for cruise-to-hover case are plotted in Figure 5-20.

Figure 5-20 Eigenvalues of the symmetric Jacobian for cruise-to-hover transition

As per expectation from previous cases, the eigenvalues associated with the translational dynamics show exponentially stable response across the whole maneuver. In this case, the aircraft is decelerating and results in the reduction of the absolute magnitude of translational eigenvalues. Similar to the hover-to-cruise transitions, it can be seen that the rotational dynamics is unstable across the complete transition maneuver. The above stability results suggest that a controller to stabilize the rotational dynamics of the aircraft is necessary to follow the optimized results.

5.6.2.1 Control-input Perturbations

The stability results obtained previously will be confirmed through a series of simulations of the aircraft responses due to perturbations. Since we are dealing with stability of the solutions, the perturbation can be inserted at the control-input or the system states. In Figure 5-21, the optimal elevator command is plotted along with four perturbation cases across the transition time.

Figure 5-21 Perturbation in elevator optimal response

Figure 5-22 Perturbation in optimal trajectory in inertial space

The optimal trajectories, plotted in inertial space (Figure 5-22), are corresponding to the optimal and perturbed elevator command histories. It can be observed that due to the perturbation, the aircraft fails to track the reference

trajectory and either will take negative or positive flight path angle based on perturbation.

The velocity response for the optimal and perturbed cases is also plotted in Figure 5-23. It can be observed that the perturbation causes the system to deviate from the optimal condition.

Figure 5-23 Velocity response based on perturbation in elevator command

Figure 5-24 Horizontal and vertical velocity response based on perturbation in elevator command

It can further be observed from Figure 5-24 and Figure 5-25 that once perturbed by the elevator command the states deviate and do not return to the nominal/optimal values. In some cases, the aircraft develops a high pitch rate at the end of the transition maneuver and thereby result in the divergent response.

Figure 5-25 Pitch angle and pitch rate response based on pertrubation in elevator command

5.6.2.2 State Perturbations

In this section the states are perturbed around the nominal/optimal solution and corresponding perturbed responses are observed. A step disturbance in pitch angle is introduced at $t = 1$ s and the corresponding response is shown in Figure 5-26.

Figure 5-26 Perturbation in aircraft attitude state

It can be observed from the figure above that the aircraft fails to follow the nominal pitch angle response after the disturbance and plunge nose down. The corresponding trajectory is plotted in Figure 5-27. As can be seen, the aircraft looses altitude significantly due to this nose-down plunge.

Figure 5-27 Trajectory response from state perturbation

The deviation in the velocity response is also shown in Figure 5-28.

Figure 5-28 Velocity response from state perturbation

The corresponding horizontal and vertical velocities are shown in Figure 5-29. The aircraft vertical velocity deviates significantly as a consequence of perturbation.

Figure 5-29 Horizontal and vertical velocity response in inertial frame from state perturbation

The pitch rate of the aircraft also departs its nominal history as a result of perturbation as plotted in Figure 5-30.

Figure 5-30 Pitch rate response from state perturbation

5.6.3 Concluding Remarks

The advantages associated with the aerodynamic vectoring are formulated through optimization of hover-to-cruise transition maneuver for small agile UAV. As compared to fixed-wing scheme, the results with aerodynamic vectoring indicate improvement in the transition performance over a variety of parameters like $(T/W)_{max}$, allocated transition time and specified terminal velocity. Thus the aerodynamic vectoring phenomena can be harnessed to achieve advantages during hover-to-cruise transition maneuvers under spatial and time constraints ensuring the agility of the flying platforms. For cruise-to-hover transitions, the aerodynamic-vectoring is less advantageous and no significant improvement in transition trajectories has been found.

The contraction analysis provides us the essence of the stability in terms of the solution of whole trajectory instead of a single point as mentioned earlier. Based on the eigenvalue analysis and trajectory response subjected to perturbation in elevator command history (control variable) and pitch angle

(state variable), it can be seen that the controller is required for two purposes. First, the rotational motion needs to be stabilized in order to achieve a certain desired attitude and pitch rate at every instant of the transition maneuver. Secondly, feedback controller will help us to track the desired translational motion.

CHAPTER 6

On Closing the Loop of the Transition Maneuver

6.1 Background

Up to this point, the dynamic analysis has been carried out for the steady and unsteady transition maneuvers. From the steady transitions, we have observed the possibility of the occurrence of limit-cycle oscillations in certain situations for the aerodynamic-vectoring case. From the unsteady transitions, contraction analyses of the optimal solutions reveal tracking and regulation issues associated with such agile maneuvers. All these point to the need of a closed loop control system to overcome any particular stability issues and enhance the performance of such agile transition maneuvers.

This chapter deals with the closed-loop control design approach to delineate the advantages of aerodynamic vectoring over conventional scheme. Based on the open-loop analysis from the last chapter, it was observed that the rotational dynamics needs to be stabilized to get a stabilized overall motion. For this reason, a single degree of freedom simple rotational control is first demonstrated for providing the stability to the whole system. Then later, for better performance and tracking, nonlinear control synthesis is carried out using feedback linearization for two configurations – conventional UAV and ‘aerodynamic vectored’ UAV. In any case, the stability of its internal dynamics is ensured using contraction analysis. The prospects and limitations of each control scheme are discussed explicitly. Subsequently, the closed-loop transition

maneuver response simulations with and without ‘aerodynamic vectoring’ are carried out. Avoidance of singularities and improvement in performance in terms of agility from the proposed aerodynamic vectoring during the transition maneuver will be discussed and highlighted.

6.2 Dynamics Modeling

For the aerodynamic data, higher-order polynomial fits are generated using the curve-fitting toolbox of MATLAB®. As an example, the trend-lines are shown in Figure 6-1 and Figure 6-2 for the aerodynamic data of the wing submerged under the free-stream. The coefficients associated with the wings can then be interpolated using the higher-order polynomial relations (Equation 6.1) for the angle of attack ranging from 0° to 90°.

$$C_{L_w} = C_{L_{w5}} \alpha_w^5 + C_{L_{w4}} \alpha_w^4 + C_{L_{w3}} \alpha_w^3 + C_{L_{w2}} \alpha_w^2 + C_{L_{w1}} \alpha_w + C_{L_{w0}} \quad (6.1a)$$

$$C_{D_w} = C_{D_{w3}} \alpha_w^3 + C_{D_{w2}} \alpha_w^2 + C_{D_{w1}} \alpha_w + C_{D_{w0}} \quad (6.1b)$$

$$C_{M_w} = C_{M_{w4}} \alpha_w^4 + C_{M_{w3}} \alpha_w^3 + C_{M_{w2}} \alpha_w^2 + C_{M_{w1}} \alpha_w + C_{M_{w0}} \quad (6.1c)$$

where $C_{L_{wi}}$, $C_{D_{wi}}$ and $C_{M_{wi}}$ are constant coefficients.

Figure 6-1 Polynomial Curve-Fitting: Coefficient of lift (left) and coefficient of drag (right)

Figure 6-2 Polynomial Curve-Fitting for coefficient of pitching moment

The slipstream bending effects under cross-wind conditions are neglected. This mild assumption simplifies the model such that its contribution to the aerodynamic forces can also be neglected. The influence of drag posed by the elevator deflection is taken under consideration for initial aerodynamic modeling. The vehicle is supposed to draw its control authority in pitch near hover from the elevator area submerged in the slipstream.

$$C_{D_f} = C_{D_f}^1 \delta_e \quad (6.2a)$$

$$C_{M_f} = C_{M_f}^1 \delta_e \quad (6.2b)$$

where $C_{D_f}^1$ and $C_{M_f}^1$ are constant coefficients. The part of the elevator in free stream primarily affects the rotational moment and can be represented by the relationship as shown in Equation 6.3.

$$C_{M_{fe}} = C_{M_{fe}}^1 (\alpha + \delta_e) + C_{M_{fe}}^0 \quad (6.3)$$

where $C_{M_{fe}}^1$ and $C_{M_{fe}}^0$ are constant coefficients. The free-stream and slipstream aerodynamic forces and moment effects can then be presented separately. The coefficient of lift relative to different flow regimes can be presented as in Equation 6.4.

$$C_{LF} = c_o\alpha_w^5 + c_1\alpha_w^4 + c_2\alpha_w^3 + c_3\alpha_w^2 + c_4\alpha_w + c_5 \quad (6.4a)$$

$$C_{LS} = -d_1 \sin \alpha \delta_e \quad (6.4b)$$

where c_i and d_1 are constant coefficients. Similarly, the aerodynamic drag coefficient can be written explicitly with respect to separate flow regimes as follows.

$$C_{DF} = c_6\alpha_w^3 + c_7\alpha_w^2 + c_8\alpha_w + c_9 \quad (6.5a)$$

$$C_{DS} = d_2 \cos \alpha \delta_e \quad (6.5b)$$

where c_i and d_2 are constant coefficients. The coefficient of pitching moment is expressed in Equation 6.6 as

$$C_{MF} = C_{MF1} + c_{15}\alpha + d_3\delta_e \quad (6.6a)$$

$$C_{MS} = d_4\delta_e \quad (6.6b)$$

where $C_{MF1} = c_{10}\alpha_w^4 + c_{11}\alpha_w^3 + c_{12}\alpha_w^2 + c_{13}\alpha_w + c_{14}$, c_i and d_i are constant coefficients. The slip-stream modeling has already been described earlier and the equations are then plugged into the relevant aerodynamic relationships. The overall aerodynamic forces divided by the mass of vehicle and pitching moment divided by the moment of inertia of the vehicle can be expressed as follows:

$$\frac{L}{m} = k_2 V^2 C_{LF} - k_2 d_1 V^2 \cos \theta \sin \alpha \delta_e - k_1 k_2 d_1 \sin \alpha T \delta_e \quad (6.7a)$$

$$\frac{D}{m} = k_2 V^2 C_{DF} + k_2 d_2 V^2 \cos \theta \cos \alpha \delta_e - k_1 k_2 d_2 \sin \alpha T \delta_e \quad (6.7b)$$

$$\frac{M}{I_{yy}} = k_3 V^2 C_{MF1} + k_3 d_3 V^2 \delta_e + k_3 d_4 V^2 \cos^2 \alpha \delta_e + k_1 k_3 d_4 T \delta_e \quad (6.7c)$$

where $k_2 = \frac{\rho S_{ref}}{2m}$ and $k_3 = \frac{\rho S_{ref} \bar{c}}{2I_{yy}}$. By inspection of Equations 6.7a and 6.7b, the

order of magnitude of the coefficients d_1 and d_2 is near to zero. In order to simplify the relationships yet still maintaining the dominant flow characteristics in the governing equations, the terms multiplied by d_1 and d_2 are neglected. Physically, this means that the lift and drag contributions due to the slip-stream are neglected as their order of magnitude is less than the free-stream ones. The pitching moment effects within the slip-stream are conserved as they play a vital role in the aircraft controllability at very low speeds.

In this section, the longitudinal equations of motion in earth-fixed frame of reference are shown in Equation 6.8 (the axes notation can be seen in Figure 4-2).

$$\ddot{x}_F = \frac{T}{m} \cos \theta + \frac{L}{m} \sin(\alpha - \theta) - \frac{D}{m} \cos(\alpha - \theta) - q\dot{z}_F \quad (6.8a)$$

$$\ddot{z}_F = -\frac{T}{m} \sin \theta + g - \frac{L}{m} \cos(\alpha - \theta) - \frac{D}{m} \sin(\alpha - \theta) + q\dot{x}_F \quad (6.8b)$$

$$\ddot{\theta} = \frac{M}{I_{yy}} \quad (6.8c)$$

The dynamic equations in Equation 6.8 can be rewritten in state variable form $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ where $\mathbf{x} \in \mathfrak{R}^5$ is the state vector $\mathbf{x}^T = (\dot{x}, \dot{z}, \dot{z}, \theta, \dot{\theta})$. It is notable here that the reference variable from the horizontal equation is the desired velocity vector and not the position vector. Also the control vector space is referred as $\mathbf{u} \in \mathfrak{R}^2$ such that $\mathbf{u}^T = (T, \delta_e)$ for fixed- wing case. The control vector space for variable-incidence wing case is referred as $\mathbf{u} \in \mathfrak{R}^3$ such that $\mathbf{u}^T = (T, \delta_e, \alpha_w)$. The state space form for the variable-incidence wing case will

be presented later. The following state space form is obtained for the fixed-wing case only.

$$\dot{x}_1 = \frac{L}{m} \sin(\alpha - x_4) - \frac{D}{m} \cos(\alpha - x_4) - x_3 x_5 + \frac{u_1 \cos x_4}{m} \quad (6.9a)$$

$$\dot{x}_2 = x_3 \quad (6.9b)$$

$$\dot{x}_3 = g - \frac{L}{m} \cos(\alpha - x_4) - \frac{D}{m} \sin(\alpha - x_4) + x_1 x_5 - \frac{u_1 \sin x_4}{m} \quad (6.9c)$$

$$\dot{x}_4 = x_5 \quad (6.9d)$$

$$\dot{x}_5 = \frac{M}{I_{yy}} \quad (6.9e)$$

The angle of attack of the aircraft can be computed from the state variables in Equation 6.9 as

$$\alpha = \tan^{-1} \left(\frac{x_1 \sin x_4 + x_3 \cos x_4}{x_1 \cos x_4 - x_3 \sin x_4} \right) \quad (6.10)$$

6.3 Simple Closed Loop Control

In order to support the results obtained from contraction analysis in the last chapter, we first consider a simple rotational control law to demonstrate that the system stability can be achieved by controlling the rotational motion only. The rotational dynamics (Equation 6.8c) can be further represented as:

$$\ddot{\theta} = M_1 + M_{\delta_e} \delta_e \quad (6.11)$$

where $M_1 = k_3 V^2 C_{MF1}$ and $M_{\delta_e} = k_3 d_3 V^2 + k_3 d_4 V^2 \cos^2 \alpha + k_1 k_3 d_4 T$. A simple control law that can stabilize the rotational dynamics is given below:

$$\delta_e = \frac{-M_1 + K_1(\theta_d(t) - \theta(t)) + K_2(\dot{\theta}_d(t) - \dot{\theta}) + \ddot{\theta}_d}{M_{\delta_e}} \quad (6.12)$$

where $(\cdot)_d$ indicates the desired values to achieve; K_1 and K_2 are the controller gains. The desired pitch motion used in the simulation is the same as the optimal maneuver discussed in the last chapter (Figure 5-22), where the vehicle executes the transition maneuver from hover to forward flight in 2 s. The final velocity of the transition maneuver is set at 15 m/s. The aircraft is supposed to track the pre-described attitude and pitch rate along the transition maneuver. The responses of the system with the control law above are shown in Figure 6-3.

Several observations can be made from Figure 6-3. The optimal solution in the last chapter exits the hover to cruise transition maneuver in accelerated mode. It can be observed that because of this tracking of desired attitude, the aircraft accelerates with the pre-defined thrust history and then later settles down. The aircraft tracks the pre-defined thrust history obtained from the optimization results up to 2 s and then uses the cruise thrust to fly at 15 m/s. Moreover, there can be observed a significant altitude variation due to lack of control. With only elevator feedback, there is only limited performance that can be achieved. Nevertheless, the simulation here demonstrates that stabilizing the rotational motion only is sufficient to achieve a stable system. After this demonstration, a more sophisticated approach based on feedback linearization is further examined to achieve better transition performance.

Figure 6-3 State history of the transition maneuver with one degree of freedom control ($K_1=9$; $K_2=13$)

6.4 Feedback Linearization Control

The basic idea in feedback linearization control approach is to transform a nonlinear system into a (fully or partially) linear system, and then use the well-known and powerful linear design techniques to complete the control design. An alternative name for feedback linearization is dynamic-inversion. A brief review of the concept is given below and readers are referred to Ref. [65, 66] for detailed discussion.

Consider a standard smooth non-linear control system affine in the input variables

$$\begin{aligned}
 \dot{\mathbf{x}} &= f(\mathbf{x}) + g(\mathbf{x})\mathbf{u} = f(\mathbf{x}) + \sum_{i=1}^m g_i(\mathbf{x})\mathbf{u}_i \\
 \mathbf{y}_1 &= h_1(\mathbf{x}) \\
 &\dots \\
 &\dots \\
 \mathbf{y}_m &= h_m(\mathbf{x})
 \end{aligned} \tag{6.13}$$

where $\mathbf{x} \in \mathfrak{R}^n$ is the state vector, $\mathbf{u}_i \in \mathfrak{R}^m$ is the input vector and $\mathbf{y}_j \in \mathfrak{R}^m$ is the output vector. It is assumed that the system has a well-defined vector relative degree $r = (r_1, r_2, \dots, r_m)$ at the origin, which means for all $1 \leq j \leq m, 1 \leq i \leq m, 0 \leq k < r_i - 1$ and for all $\mathbf{x}$ in a neighborhood of the origin $L_{g_j} L_f^k h_i(\mathbf{x}) = 0$.

where L is the Lie derivative. The input-output linearization is carried out by differentiating the outputs y_j until at least one input appears. Let the differentiated outputs be expressed in compact form as

$$\begin{bmatrix} y_1^{(r_1)} \\ \dots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(\mathbf{x}) \\ \dots \\ L_f^{r_m} h_m(\mathbf{x}) \end{bmatrix} + E(\mathbf{x}) \begin{bmatrix} u_1 \\ \dots \\ u_m \end{bmatrix} \quad (6.14)$$

where $E(\mathbf{x})$ is called the decoupling matrix, defined as

$$E(\mathbf{x}) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1 & \dots & L_{g_m} L_f^{r_1-1} h_1 \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{r_m-1} h_m & \dots & L_{g_m} L_f^{r_m-1} h_m \end{bmatrix} \quad (6.15)$$

If the decoupling matrix is non-singular then a static control law can be implemented such that

$$\begin{bmatrix} u_1 \\ \dots \\ u_m \end{bmatrix} = -E^{-1} \begin{bmatrix} L_f^{r_1} h_1(\mathbf{x}) \\ \dots \\ L_f^{r_m} h_m(\mathbf{x}) \end{bmatrix} + E^{-1} \begin{bmatrix} v_1 \\ \dots \\ v_m \end{bmatrix} \quad (6.16)$$

The control law is named static-feedback linearizable control law and the resulting system can be expressed as

$$\begin{bmatrix} y_1^{(r_1)} \\ \dots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} v_1 \\ \dots \\ v_m \end{bmatrix} \quad (6.17)$$

In addition of being linearized, the input-output equations of the above system also become decoupled. As a result of decoupling, one can use SISO design techniques on each input-output channel in the above decoupled dynamics to construct tracking or stabilization controllers. The new inputs can be easily designed to regulate y and z , provided that the associated internal dynamics is stable. It is assumed that the distribution spanned by $\{g_1(\mathbf{x}), \dots, g_m(\mathbf{x})\}$ is involutive and the internal dynamics will not depend explicitly on the inputs.

The feedback linearization control technique, apparently attractive in its nature, has some limitations as well:

- It cannot be used for all nonlinear systems
- The input to the system is the state. Therefore the full state has to be measured.
- No robustness is guaranteed in the presence of parameter uncertainty or unmodeled dynamics.

However, in the present work, the scheme is used to analyze the qualitative behavior/advantage of aerodynamic-vectoring over fixed-wing counterpart.

6.5 Control Synthesis

In this section, control synthesis exercise for the conventional and aerodynamic vectoring aircraft cases is carried out. The framework for the problem formulation is laid out beforehand. It should be noted that the control

design is restricted to the longitudinal plane only and lateral-directional coupling is neglected.

6.5.1 Conventional UAV Control Synthesis

6.5.1.1 Mathematical Formulation

Now the control design for the aircraft dynamic system without aerodynamic vectoring is carried out. Two outputs of interest selected are vertical height in the earth fixed frame of reference and pitch angle.

$$\begin{aligned} y_1 &= x_2 \\ y_2 &= x_4 \end{aligned} \quad (6.18)$$

By differentiating the output function y_i twice so that it is explicitly related to the input, we get,

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} g - k_2(x_1^2 + x_3^2)(C_{LF} \cos(\alpha - x_4) + C_{DF} \sin(\alpha - x_4)) + x_1 x_5 \\ k_3(x_1^2 + x_3^2)(C_{MF1} + c_{15}\alpha) \end{bmatrix} + E \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (6.19)$$

where the decoupling matrix is

$$E = \begin{bmatrix} -\frac{\sin x_4}{m} & 0 \\ 0 & k_3(x_1^2 + x_3^2)(d_3 + d_4 \cos^2 \alpha) + k_1 k_3 d_4 u_1 \end{bmatrix} \quad (6.20)$$

The invertibility of the decoupling matrix depends on the singularities of Equation 6.20. It can be observed that one singularity is encountered when the aircraft pitch angle is equal to zero, corresponding to the situation where there is no lift produced. Another singularity is associated with the zero velocity, i.e. hover condition when thrust is also zero at the same time. This is, however, an unlikely flying condition and therefore it is of no interest. The control input from Equation 6.4 can be expressed as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -E^{-1} \begin{bmatrix} g - k_2(x_1^2 + x_3^2)(C_{LF} \cos(\alpha - x_4) + C_{DF} \sin(\alpha - x_4)) + x_1 x_5 \\ k_3(x_1^2 + x_3^2)(C_{MF1} + c_{15}\alpha) \end{bmatrix} + E^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (6.21)$$

which can also be written as

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \frac{mg - k_2m(x_1^2 + x_3^2)(C_{LF} \cos(\alpha - x_4) + C_{DF} \sin(\alpha - x_4)) + mx_1 x_5 - mv_1}{\sin x_4} \\ \frac{-k_3(x_1^2 + x_3^2)(C_{MF1} + c_{15}\alpha) + v_2}{k_3(x_1^2 + x_3^2)(d_3 + d_4 \cos^2 \alpha) + k_1 k_3 d_4 u_1} \end{bmatrix} \quad (6.22)$$

Because of the singularity at $x_4=0$, in order to attain the altitude hold position, the thrust needed tends to be very high. The resulting system can be written in compact state space form as

$$\begin{aligned} \dot{x}_1 &= k_2(x_1^2 + x_3^2)(C_{LF} \sin(\alpha - x_4) - C_{DF} \cos(\alpha - x_4)) - x_3 x_5 \\ &+ \frac{g - k_2(x_1^2 + x_3^2)(C_{LF} \cos(\alpha - x_4) + C_{DF} \sin(\alpha - x_4)) + x_1 x_5 - v_1}{\tan x_4} \end{aligned} \quad (6.23a)$$

$$\dot{x}_2 = x_3 \quad (6.23b)$$

$$\dot{x}_3 = v_1 \quad (6.23c)$$

$$\dot{x}_4 = x_5 \quad (6.23d)$$

$$\dot{x}_5 = v_2 \quad (6.23e)$$

The following control law, obtained from feedback linearization approach, can result in convergent tracking errors

$$v_1 = \ddot{y}_1 + K_{p11}k_2V^2(y_{1_{des}} - y_1) + K_{d12}k_2V^2(\dot{y}_{1_{des}} - \dot{y}_1) \quad (6.24a)$$

$$v_2 = \ddot{y}_2 + K_{p21}k_2V^2(y_{2_{des}} - y_2) + K_{d22}k_2V^2(\dot{y}_{2_{des}} - \dot{y}_2) \quad (6.24b)$$

It should be noted that the gains are scaled by using the term k_2V^2 . By doing this scaling, the control actuations can be easily kept under saturation

limits and significant overshoots can be avoided. It must be noted that gains K_{d12} , K_{p11} , K_{d22} and K_{p21} are taken as positive constants. The stability of the system using this control law can be easily determined because of its double-integrator relationship. Let the tracking error e be defined as $e = y_{des} - y$, then the resulting error dynamics of the system can be expressed as

$$\ddot{e}_1 + K_{d12}k_2V^2\dot{e}_1 + K_{p11}k_2V^2e_1 = 0 \quad (6.25a)$$

$$\ddot{e}_2 + K_{d22}k_2V^2\dot{e}_2 + K_{p21}k_2V^2e_2 = 0 \quad (6.25b)$$

which represents an exponentially stable error dynamics.

The internal dynamics of the above system is associated with horizontal velocity x_1 . The stability of the internal dynamics can be determined by evaluating the eigenvalues of the symmetric Jacobian of Equation 6.23a as follows

$$\lambda_1 = \left(\begin{array}{c} -\frac{2k_2x_1(C_{LF}\cos\alpha + C_{DF}\sin\alpha)}{\sin x_4} - \frac{2K_{p11}k_2x_1(y_{1des} - y_1)}{\tan x_4} \\ -\frac{2K_{d12}k_2x_1(\dot{y}_{1des} - \dot{y}_1)}{\tan x_4} + \frac{x_5}{\tan x_4} \end{array} \right) \quad (6.26)$$

The gains K_{p11} and K_{d12} in Equation 6.26 need to be selected so that the system response is over damped and such that the aircraft should not cross or approach the singularity condition. The singularity condition in this case is the attitude of the aircraft. It should be noted that the singularity in this approach may be avoided by using another singularity suppression technique such as quaternion. The thesis emphasizes the control advantage using aerodynamic vectoring and the conventional fixed-wing case is developed just as a benchmark.

6.5.1.2 Fixed-Wing Response Simulation

The conventional UAV controller simulation is carried out and discrepancies are highlighted with reference to the state convergence and control inputs. The initial conditions for near-hover flight regime and are kept the same for both control schemes. The relationship between the desired aircraft attitude θ_d , aircraft velocity V and wing angle of attack is computed in Figure 6-4. For the fixed-wing UAV, $\theta = \alpha_w$ and the desired attitude can be calculated to achieve certain cruise velocity.

Figure 6-4 3-D plot of relationship between aircraft velocity, θ and α_w

A simulation of the controlled response for near-hover to cruise transition for the conventional UAV is carried out. The optimal maneuver of achieving 15 m/s in 2 s time as discussed in Figure 5-22 is used as the reference maneuver to track. The cruise attitude of the aircraft can be computed from Figure 6-4. A particular set of gains is selected so that the transition is completed within 2 s. The resultant state histories are plotted in Figure 6-5. It can be seen that the desired states of aircraft attitude, pitch rate, vertical velocity and altitude converge within the specified time. The horizontal velocity is a part of internal dynamics and therefore cannot be controlled explicitly. It can be seen that the

desired horizontal velocity has some overshoot because of the lack of control in that particular degree of freedom.

Figure 6-5 State histories for conventional UAV from near-hover to cruise ($K_{p11}=0.8$; $K_{d12}=1$; $K_{p21}=2.5$; $K_{d22}=2$)

The associated control history to the simulated transition maneuver above is plotted in Figure 6-6. For the T/W control input, the aircraft starts its maneuver from hover thrust and eventually settles down to cruise value. In the transition regime, its value goes as high as 1.48. The elevator has a significant design control authority and its deflection history is well within saturation limits.

Figure 6-6 Control histories for conventional UAV from near-hover to cruise

6.5.2 Aerodynamic Vectoring UAV Control Synthesis

6.5.2.1 Mathematical Formulation

The dynamic equations in Equation 6.9 are written in state variable form $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$ where $\mathbf{x} \in \mathbb{R}^5$ is the state vector $\mathbf{x}^T = (\dot{x}, z, \dot{z}, \theta, \dot{\theta})$. For the variable-incidence wing case, the control vector space is referred as $\mathbf{u} \in \mathbb{R}^3$ where $\mathbf{u}^T = (T, \delta_e, \alpha_w)$. Here we can take three outputs of interest and can model the system such that there is no internal dynamics. Three outputs of interest selected are horizontal velocity, vertical height and pitch angle in the fixed frame of reference.

$$\begin{aligned} y_1 &= x_1 \\ y_2 &= x_2 \\ y_3 &= x_4 \end{aligned} \tag{6.27}$$

Equation 6.27 is differentiated (repeatedly as necessary) such that at least one input appears. The derived model is long enough to compute and is written in symbolic form in Equation 6.28.

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \end{bmatrix} = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} + \begin{bmatrix} a & b & c \\ d & e & f \\ p & h & i \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \quad (6.28)$$

where vector H represents,

$$\begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} = \begin{bmatrix} -x_3x_5 \\ p + x_1x_5 \\ k_3(x_1^2 + x_3^2)c_{15}\alpha \end{bmatrix} \quad (6.29)$$

And the decoupling matrix can be written as

$$E = \begin{bmatrix} \frac{\cos x_4}{m} & 0 & k_2(x_1^2 + x_3^2) \begin{bmatrix} C_{LF} \sin(\alpha - x_4) \\ -C_{DF} \cos(\alpha - x_4) \\ u_3 \end{bmatrix} \\ \frac{-\sin x_4}{m} & 0 & -k_2(x_1^2 + x_3^2) \begin{bmatrix} C_{LF} \cos(\alpha - x_4) \\ +C_{DF} \cos(\alpha - x_4) \\ u_3 \end{bmatrix} \\ 0 & k_3(x_1^2 + x_3^2)(d_3 + d_4 \cos^2 \alpha) + k_1k_3d_4u_1 & k_3(x_1^2 + x_3^2) \frac{C_{MF1}}{u_3} \end{bmatrix} \quad (6.30)$$

which corresponds to the matrix multiplied to the control vector in Equation 6.28. The invertibility of the decoupling matrix (Equation 6.30) is dependent on the singularity of Equation 6.21. Since the zero velocity (perfect hover) condition is not the point of interest, therefore the discussion pertaining to zero velocity is out of the current scope. Next, it is evident that the singularity of pitch angle at zero degrees is removed therefore we can now fly the aerodynamic vectored UAV at zero pitch angle subject to the condition that the wing angle of attack is

not zero – a control variable. Therefore, it can be said that Equation 6.30 is non-singular in our prescribed envelope of interest. Therefore, the inverse of the decoupling matrix exists during the transition maneuver. Taking the symbolic representation of decoupling matrix and H vector from Equations 6.28, 6.29 and 6.30, the actual control input vector u can be calculated from this equation.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{fH_1 - cH_2 - fv_1 + cv_2}{cd - af} \\ \frac{diH_1 - aiH_2 - div_1 + aiv_2}{h(cd - af)} + \frac{v_3 - H_3}{h} \\ \frac{aH_2 - dH_1 + dv_1 - av_2}{cd - af} \end{bmatrix} \quad (6.31)$$

The resultant feedback-linearized state-space model can be written as

$$\dot{x}_1 = v_1 \quad (6.32a)$$

$$\dot{x}_2 = x_3 \quad (6.32b)$$

$$\dot{x}_3 = v_2 \quad (6.32c)$$

$$\dot{x}_4 = x_5 \quad (6.32d)$$

$$\dot{x}_5 = v_3 \quad (6.32e)$$

It can be seen that with the inclusion of additional control variable, the relative degree r of a system is equal to n , the number of states, and thus input-output linearization leads to input-state linearization. There is no internal dynamics left on the system and the longitudinal motion becomes completely controllable. The control law for the aerodynamic vectored UAV is formulated similar to Equation 6.25 with scaling effects incorporated as well as the exponential convergence guaranteed.

$$v_1 = \dot{y}_{1_{des}} + K_{p11}k_2V^2(y_{1_{des}} - y_1) \quad (6.33a)$$

$$\ddot{y}_2 = \ddot{y}_{2_{des}} + K_{p21}k_2V^2(y_{2_{des}} - y_2) + K_{d22}k_2V^2(\dot{y}_{2_{des}} - \dot{y}_2) \quad (6.33b)$$

$$\ddot{y}_3 = \ddot{y}_{3_{des}} + K_{p31}k_2V^2(y_{3_{des}} - y_3) + K_{d32}k_2V^2(\dot{y}_{3_{des}} - \dot{y}_3) \quad (6.33c)$$

6.5.2.2 Aerodynamic-Vectored Controlled Response Simulation

6.5.2.2.1 Unsteady Transition Control

In this control scheme, the desired cruise velocity can be declared explicitly. The transition responses to the cruise value of 15 m/s are shown in Figure 6-7. The transition trajectory tracks the optimal trajectory of 15 m/s to be achieved in 2 s. For this purpose, the gains are tuned accordingly to achieve the desired trajectory. The response of the horizontal velocity clearly shows that the aircraft dynamics is more under control with the aerodynamic-vectored versatile architecture.

Figure 6-7 State histories for aerodynamic-vectored UAV from near-hover to cruise (Gains: $K_{p11}=0.7$; $K_{p21}=1$; $K_{d22}=1$; $K_{p31}=1$; $K_{d32}=1$)

The associated control history of the aerodynamic-vector UAV with the simulated transition maneuver is plotted in Figure 6-8. The wing angle of attack starts from approximately zero lift angle and gradually settles down to the cruise value. Since the wing angle of attack primarily remains in the pre-stall regime, the elevator of the aircraft has to deflect slightly more in order to achieve the desired pitching moment. During the transient response, the T/W reduces more rapidly than its conventional counterpart because the drag posed to the dynamics of the aircraft is significantly less. It can also be observed that the altitude variation is smaller for the variable-incidence wing case as the controller is more efficient in tracking the optimal altitude scenario.

Figure 6-8 Control histories for aerodynamic-vector UAV from near-hover to cruise

6.5.2.2.2 Steady Transition Control

As discussed in Chapter 4, the limit-cycle appears across certain trim conditions for the variable-incidence wing. In the following set of simulation, the aircraft is perturbed for the same trim state as in Figure 4-24 and the controlled

response is captured in Figure 6-9. It can be observed that the proposed controller (6.33) is able to achieve asymptotically stable response of the system for the trim point that generates limit-cycle in the open-loop situation.

Figure 6-9 State histories for aerodynamic-vector UAV for limit-cycle case ($K_{p11}=0.48$; $K_{p21}=0.9$; $K_{d22}=0.9$; $K_{p31}=1$; $K_{d32}=1$)

The control histories associated with the following trim state are recorded and shown in Figure 6-10. It can be seen that there is no unrealistically high peak control requirements to reject any perturbation/disturbance from the system dynamics.

Figure 6-10 Control histories of aerodynamic vectored UAV for limit cycle case

6.5.3 Concluding Remarks

A feedback linearization algorithm is used to design the nonlinear controller for conventional as well as proposed aerodynamic vectored feature. The proposed scheme show advantages over conventional scheme in terms of shaping state convergence criteria and avoidance of singularities at high speeds. The internal dynamics disappears for the aerodynamic vectoring case and therefore, the velocity response can be tailored to specific requirements. The uncontrollable response from velocity is observed because of internal dynamics and high drag situation for fixed-wing case.

CHAPTER 7

Conclusions & Future Work

7.1 Conclusions

The conclusions drawn from the work presented in the previous chapters can be divided into three major groups. The grouping is based on whether the conclusions are related to steady transition maneuvers, unsteady transition maneuvers and the associated control of transition maneuvers.

7.1.1 Conclusions Related to Steady Transition Dynamics

- The trim analysis shows significant decrease in the thrust requirement over the whole velocity range (Figure 4-3) for aircraft equipped with variable-incidence wing as compared to fixed-wing. The primary cause of the thrust reduction can be attributed to the wing positioning in the pre-stall regime thereby posing significantly less drag.
- The trim analysis also shows that the favorable region for the variable-incidence wing to trim across the complete velocity envelope is always in the pre-stall regime (Figure 4-5).
- Since the variable-incidence wing always trims in the pre-stall regime, the pitching moment generated from the wing will be smaller than in the post-stall regime (Figure 3-25). The associated elevator control effort required to trim the whole aircraft is therefore significantly reduced when compared to the fixed-wing configuration. Therefore, the additional available elevator control authority can be used to better reject the disturbances.

- The linear longitudinal dynamic analysis shows that the fixed-wing aircraft short-period motion is stable across the complete velocity envelope. However, this is not the case for the variable-incidence wing case, as eigenvalue migration to the right-side of the imaginary axis in the complex plane is observed in certain region of flight. The numerical simulations as well as multiple degree-of-freedom MTS analysis concurs the existence of limit cycle in this region. The important parameters and their effects on transition characteristics, such as amplitude and frequency can be easily seen in explicit functional relationships from the approximate solutions obtained using MTS approach.
- It is shown that the combination of the significant effect of cubic nonlinearity associated with lift and the use of low thrust during the maneuver (because of aerodynamic vectoring) give birth to the occurrence of limit cycles in the post-stall regime.

7.1.2 Conclusions Related to Unsteady Transition Maneuvers

- From the optimization analysis, the improvement in the transition performance over a variety of parameters with the use of aerodynamic vectoring is observed. The requirement of $(T/W)_{max}$ has a direct relation with payload capacity on-board. The advantage of variable-incidence wing is more pronounced at high payload configurations. Moreover, the $(T/W)_{max}$ has an inverse relationship with the transition time.
- For a same $(T/W)_{max}$ for both configurations, altitude loss is more pronounced and increases significantly with the increase in payload capacity.

- It has been observed that the control variations for the aerodynamic vectoring case are smaller (thrust) than the fixed wing configuration. This is primarily due to the fact that the wing is always in the pre-stall regime.
- It is observed from the optimization study that unsteady aerodynamic effects play a significant role in fast transitions. However, as the time allocated to the transition maneuvers is increased, the unsteady aerodynamic effects diminish. Moreover, the unsteady aerodynamic effect is more pronounced on the conventional fixed-wing configuration because the wing undergoes large pitching motion from the post-stall to the pre-stall flow regime.
- The effect of elevator effectiveness is the key parameter in transition performance of the aircraft. With the decrease in the elevator control authority, the advantage of aerodynamic vectoring reduces because of the lack of generation of high pitching moments. Generally, high pitching moments are required in the initial phase of the maneuver to bring the nose of the aircraft down.
- For cruise-to-hover optimal transition maneuvers, the advantage of aerodynamic vectoring is less obvious and no significant improvement in transition maneuvers over the fixed-wing case has been found.
- The open-loop stability analysis of the longitudinal dynamics using contraction theory highlights the stability characteristics of the translational and rotational dynamics explicitly. The translational dynamics appears to be stable across the whole maneuver range whereas, the rotational dynamics is unstable. The instability of the rotational dynamics contributes to the overall instability of the transition maneuvers. A simple closed loop control design around rotational

dynamics will stabilize the overall transition dynamics. However, for improved performance and agility, a better control approach would be preferred.

7.1.3 Conclusions Related to Control of Transition Maneuver

- Based on the contraction analysis of the transition maneuver, a simple closed loop control on the rotational dynamics is sufficient to address the stability issues. However this simple controller cannot achieve good tracking performance and significant deviations from the optimal path are observed. Moreover, the settling time of the response using this simple controller is long.
- The comparison of feedback linearization control architectures for the fixed and variable-incidence wing configurations show that the internal dynamics disappears with the inclusion of additional control variable in the form of the wing-incidence angle, leading to better performance in the variable-incidence wing case. Moreover, the decoupling matrix for this case is non-singular in the operational envelope of the aircraft, which is not true for the fixed-wing configuration.
- The enhancement for the transition maneuver control using aerodynamic vectoring in terms of less T/W requirement and better tracking performance as compared to the fixed-wing configuration is also observed.

7.2 Future Work

Some recommendations for future research based on the work in this dissertation are as follows:

- The ultimate validation of the analytical, numerical and experimental research presented in the thesis is the flight demonstration of the

aerodynamic-vectoring UAV. Flight tests of the UAV equipped with aerodynamic-vectoring feature are planned as part of the future work. Specific emphasis will be made on the actuation mechanism of the variable-incidence wing. Moreover, flight data will be collected and analyzed for comparison with comparison with the current results and for further parametric and performance studies.

- The variable-incidence wing actuation mechanism development has already been in progress. A prototype of actuation mechanism is shown in Figure 7-1. The actuation mechanism is controlled by a commercially available servo-motor. Both wings are controlled by a single servo-motor mounted on top of the fuselage. The push-pull rod is linked to the rotatable rod passing through the fuselage and is inserted into both wings spanwisely at quarter-chord location. Future emphasis will be made on the design refinement as well as alternative power efficient design.

Figure 7-1 Prototype Actuation Mechanism for Aerodynamic-Vectoring

- The present work is restricted to longitudinal plane as the aerodynamic vectoring will have pronounced effect in this plane. As a future guideline, six degree-of-freedom dynamics may be incorporated in the study to examine any anomalous lateral-directional phenomena during the aerodynamic-vectoring applications. Moreover, because of large attitude changes, singularity avoidance issue will also be investigated.
- During the change of angle of incidence of the outboard wing, significant contribution from actuation power is anticipated. The actuation power may be evaluated as function of aspect ratio in the future study.
- The concept of differential variable-incidence wings may be explored to generate desired roll moments. The concept is somewhat similar to ‘all-moving-elevators’ of advanced fighter aircrafts. By having ‘All-Moving-Ailerons’, a large roll moment can be generated. However this feature may

also induce a strong coupling effect to the longitudinal plane forces. This can become a nice area of research in the future.

- The response to gust is not quantified in this study. As a future work, estimation of the gust tolerance for the variable-incidence wing may be examined as well.

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Appendix A

Stability Derivatives at Various Velocities

Velocity	Cl_u	Cd_u	Cm_u	CL_α	CD_α	CM_α	ϵ_α
15	0.0012	-0.0003	-0.00002	1.6617	0.14898	-0.28077	0.17763
14	0.0011	-0.0003	-0.00004	1.66743	0.17763	-0.28077	0.17763
13	0.0021	-0.0003	-0.0001	1.6617	0.21774	-0.28077	0.17763
12	0.0047	0.0009	-0.0002	1.62159	0.25212	-0.28077	0.1719
11	0.0068	0.0012	-0.0003	1.56429	0.29223	-0.28077	0.1719
10	0.0085	0.0018	-0.0005	1.51272	0.3438	-0.28077	0.16617
9	0.012	0.0046	-0.0009	1.29498	0.39537	-0.2865	0.14325
8	0.0387	-0.0026	-0.0024	-0.64749	0.52716	-0.30369	-0.07449
7	0.0222	-0.0149	-0.0053	-0.7449	0.5157	-0.16044	-0.08022
6	0.0259	-0.0182	-0.0085	-0.19482	0.56154	-0.12606	-0.02292
5	0.0126	-0.0142	-0.0149	0.1719	0.56727	-0.12033	0.02292
4	-0.0381	0.0014	-0.0298	0.1719	0.50997	-0.13179	0.02292
3	-0.0137	0.0074	-0.0322	-0.14898	0.39537	-0.1719	-0.01719
2	-0.2198	0.0334	-0.0723	-1.7763	0.25785	-0.20628	-0.21774
1	-0.4168	0.0193	-0.0665	-2.82489	0.72771	-0.10314	-0.33807
Velocity	X_u	Z_u	M_u	X_w	Z_w	M_w	Z_q
15	-0.09272	-0.6646	-1.01417E-05	0.013471	-3.58859	-0.14237	-0.10643
14	-0.09688	-0.70683	-1.89311E-05	-0.00105	-3.36593	-0.13288	-0.10987
13	-0.10437	-0.75019	-4.39473E-05	-0.02908	-3.12212	-0.12339	-0.10202
12	-0.11681	-0.80537	-8.11334E-05	-0.03124	-2.82277	-0.1139	-0.0774
11	-0.12911	-0.87252	-0.000111558	-0.02584	-2.50876	-0.10441	-0.04849
10	-0.14579	-0.94511	-0.000169028	-0.02203	-2.22119	-0.09492	-0.02043
9	-0.16344	-1.02107	-0.000273825	-0.00278	-1.73493	-0.08717	-0.02225
8	-0.24691	-1.08098	-0.000649067	-0.08078	0.611132	-0.08213	0.00344
7	-0.20799	-0.73469	-0.001254187	-0.15667	0.629546	-0.03797	0.016556
6	-0.19729	-0.58804	-0.001724085	-0.19579	0.059699	-0.02557	0.010966
5	-0.19368	-0.4388	-0.002518516	-0.18812	-0.22402	-0.02034	0.011288
4	-0.16711	-0.30142	-0.004029626	-0.12833	-0.18086	-0.01782	0.009031
3	-0.12849	-0.27339	-0.00326562	-0.02893	0.000844	-0.01743	0.00774
2	-0.08043	-0.20934	-0.004888288	0.062623	0.469356	-0.01395	0.005043
1	-0.03585	-0.22921	-0.002248072	0.04081	0.384862	-0.00349	0.002688

Velocity	M_q	Cz_{αdot}	Z_{wdot}	Z_{αdot}	Cm_{αdot}	M_{wdot}	M_{αdot}
15	-0.05854	-0.07457	-0.00126037	-0.01891	-0.17239	-0.000693	-0.0104
14	-0.06043	-0.08247	-0.00139404	-0.01952	-0.19067	-0.0007667	-0.01073
13	-0.05611	-0.08247	-0.00139404	-0.01812	-0.19067	-0.0007667	-0.00997
12	-0.04257	-0.0656	-0.00110883	-0.01331	-0.15166	-0.0006098	-0.00732
11	-0.02667	-0.04483	-0.0007577	-0.00833	-0.10363	-0.0004167	-0.00458
10	-0.01123	-0.02008	-0.00033942	-0.00339	-0.04643	-0.0001866	-0.00187
9	-0.01224	-0.02096	-0.00035421	-0.00319	-0.04845	-0.0001948	-0.00175
8	0.001892	-0.0019	-3.2033E-05	-0.00026	-0.00438	-1.761E-05	-0.00014
7	0.009106	-0.01122	-0.00018973	-0.00133	-0.02595	-0.0001043	-0.00073
6	0.006031	-0.00248	-4.1889E-05	-0.00025	-0.00573	-2.303E-05	-0.00014
5	0.006209	0.003061	5.17452E-05	0.000259	0.007077	2.8459E-05	0.000142
4	0.004967	0.003061	5.17452E-05	0.000207	0.007077	2.8459E-05	0.000114
3	0.004257	-0.00262	-4.4353E-05	-0.00013	-0.00607	-2.439E-05	-7.3E-05
2	0.002773	-0.03248	-0.00054898	-0.0011	-0.07509	-0.0003019	-0.0006
1	0.001478	-0.05376	-0.00090862	-0.00091	-0.12428	-0.0004997	-0.0005