Modeling of holey fiber using boundary element method

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MODELING OF HOLEY FIBER USING BOUNDARY ELEMENT METHOD

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SCHOOL OF ELECTRICAL & ELECTRONIC ENGINEERING

NANYANG TECHNOLOGICAL UNIVERSITY

2005

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Summary

In this thesis, we present the boundary element method for modeling of holey fiber.

The nature of complexity of the holey fiber requires a general and accurate modeling method. Boundary element method is studied and proved to be suitable for modeling holey fiber. Its advantage lies in its simple mesh along the dielectric interface only since holey fiber is a waveguide structure with piecewise homogeneous regions. Additionally, its ability to model open domain problem greatly simplifies the modeling task.

The developed boundary element method algorithm is found to be efficient and accurate. It has been used to study various aspects of holey fiber, such as its fundamental space filling mode, effective core radius, symmetry and degeneracy, confinement loss, effective mode area and birefringence of holey fiber. Study of the fundamental space filling mode not only facilitates our understanding of holey fiber but allows the application of conventional theories of fiber propagation and dispersion. Analogy between HF and step index fibers is examined by studying the effective core radius of holey fiber. Symmetry analysis enables one to classify the possible modes into mode classes, predict the mode degeneracy between mode classes and specify minimum waveguides sectors for each mode class, which completely determine the modes of that mode class. It is found that holey fiber with hexagonally arranged hole is not birefringent. The structural dependence of confinement loss, effective mode area and birefringence is also studied in detail.

Chapter 1 Introduction

1.1 Motivation

Holey fiber with multiple inclusions arranged about core has attracted considerable attention from researchers around the world. Because the light confinement is due to many inclusions in a silica matrix rather than to a single doped region, the degrees of freedom, such as inclusion placement and size, is larger than that in conventional fibers. The stack-and-draw procedure for holey fiber fabrication has been proven to be very versatile, allowing very complex lattices to be assembled from individual stackable unit of the correct size and shape. The unique structure and manufacturing technique of holey fiber offer a great flexibility in the fiber design. Various designs have been reported in literature, which are not readily obtainable using conventional fiber technology.

Holey fiber has many remarkable properties such as large and adjustable dispersion, nonlinearity and birefringence, and single-mode operation over a wide range of wavelengths. Some of these properties are not readily attainable in conventional fiber. They have inspired many researches to find applications ranging from normal transmission system, WDM system to fiber devices.

To exploit the potential of holey fiber, accurate modeling tools are necessary. The nature of the complexity in the transverse geometry of holey fiber makes the modeling a challenging task. Finite difference approaches as well as finite element methods are generally accurate and can be easily applied to graded index profiles. But

both algorithms discretize the refractive index throughout the entire calculation window and are therefore somewhat inefficient when applied to waveguide profiles composed of a small number of homogeneous regions. Additionally, artificial boundary condition must be imposed for open domain problem.

We introduce here an alternative, boundary-element method (BEM) approach that is particularly applicable to waveguide structures with piecewise homogeneous regions (like holey fiber). And it can handle open domain problems as easily as close domain problems. In this method, only the waveguide boundaries are discretized, yielding a considerable reduction in matrix size compared to finite difference or finite element methods.

1.2 Objectives

The objective of this thesis is to develop a general numerical simulation method for modeling holey fiber. The proposed method should have the following characteristics:

- 1) it is general, i.e., it should enable easy implementation for different structures of holey fiber,
- 2) it has simple meshing procedure,
- 3) it should provide modeling of open domain problem without imposing any artificial boundary conditions.

1.3 Major contributions of the Thesis

In this thesis, we describe a novel boundary element method and demonstrate its application to model the holey fiber.

Boundary element method is well suited for modeling holey fiber since it reduces the dimensions of the problem by one, hence it produces a smaller number of algebraic equations, and it is able to solve open domain optical structures without imposing any artificial boundary conditions.

We have developed efficient algorithm to study various aspects of holey fiber, such as its fundamental space filling mode, analogy between HF and step index fibers, symmetry and degeneracy, confinement loss, effective mode area and birefringence of holey fiber. Study of the fundamental space filling mode not only facilitates our understanding of holey fiber but also allows the application of conventional theories of light propagation and dispersion to the study of holey fiber. Analogy between HF and step index fibers is established by studying the effective core radius of a holey fiber. Symmetry analysis enables one to classify the possible modes into mode classes, predict the mode degeneracy between mode classes and specify minimum waveguides sectors for each mode class, which completely determine the modes of that mode class. Both theoretically and numerically, we prove holey fiber with hexagonally arranged hole is not birefringent. Wavelength and structural dependence of confinement loss and effective mode area are also studied in detail. Finally, a dual core birefringent HF is studied in detail, such as its modal property, single mode operation, maximum birefringence, polarization dependent confinement loss, effective mode area, mode spot size and half divergence angle.

1.4 Organization of the Thesis

The rest of thesis is organized as follows.

Chapter 2 gives a brief review of the history of photonic crystal and holey fiber and their properties. Different guiding mechanisms and manufacturing techniques are introduced. Then properties of holey fiber and air guiding bandgap fiber are presented. However focus is on holey fiber as this thesis is devoted mainly to modeling holey fiber.

In Chapter 3, various modeling methods are reviewed and compared with each other, namely effective index method, localized function method, multipole method, finite difference and finite element method.

In Chapter 4 we describe the details of the boundary element formulation. First boundary element method is compared to finite element method and the distinct advantages are highlighted. After that, fundamental solution and Green's function are derived. Boundary integral equation and its discretized version are then presented before they are generalized to multi-domain problem.

Simulation and discussion are presented in Chapter 5. Accuracy of the BEM is discussed before it is implemented to study fundamental space filling mode, effective core radius, symmetry and degeneracy, confinement loss, effective mode area and birefringence of holey fiber.

Conclusions and recommendations for future research are presented in Chapter 6.

Chapter 2 Holey fiber

2.1 Background

Russell and co-workers have pioneered this field by the realization of microstructured silica fiber, comprising fine silica fibers with an array of air holes running down their length [1]. Cross section of such a fiber is shown in Figure 2.1. The microstructured cladding consists of air holes of diameter d arranged in with hole-to-hole distance pitch Λ . The silica core guiding the light is formed by the missing central air hole. The initial motivation was to realize fiber that would be able to guide light by photonic bandgap (PBG) effect. However the guiding mechanism of this fiber is not the photonic band gap but the total internal reflection. Despite of the resemblance with conventional optical fiber, this kind of fiber was found to have important differences with conventional fiber in terms of dispersion, nonlinearity etc. and has attracted great research interest in recent years

Figure 2.1 Cross section of an all-silica photonic crystal fiber.

2.1.1 Guiding mechanism

As well known, light guidance in such fibers may be explained by two different physical processes: total internal reflection and photonic band gap (PBG) effect.

Conventional fiber's operation usually relies on light being guided by the physical mechanism known as total internal reflection (TIR), or index guiding. In order to achieve TIR, a higher refractive index of the core compared to the surrounding media is required. Basically, the air hole pattern simply yields a lower effective refractive index, compared to the physical index of the central pure silica zone. The guidance mechanism is then essentially the same as that in conventional fibers. And such fibers are widely known as holey fibers (HFs). Analogy between a HF and conventional step index fiber is drawn in Figure 2.2. The HF can be modeled as step index fiber by knowing two basic parameters: the average index of the HF cladding n_2 , and core diameter.

Figure 2.2 Analogy between HF and step index fiber.

HFs are usually designed and fabricated with a hexagonal air hole lattice. But this periodic geometry is not strictly necessary: any arrangement of air holes in a silica

matrix would yield a reduced effective index. It was shown that many of the features attributed to periodic holey fibers, in particular, single-mode guidance at all wavelengths, can also be obtained with holey fibers with random air hole distribution [2].

In optical domain, a crystal made of dielectric media with different dielectric constants is called photonic crystal. Figure 2.3 shows 1-D and 2-D photonic crystal [3]. ϵ_1 and ϵ_2 represent different dielectric medias ($\epsilon_1 > \epsilon_2$).

Figure 2.3 Simple examples of (a) one-, (b) two-dimensional photonic crystal and (c) cross section of PBG fiber.

The simplest possible photonic crystal consists of alternating layers of material with different dielectric constants. 1-D photonic crystals have been exploited for decades, e.g., high reflection mirror and fiber Bragg grating. Suppose light wave incident in $-y$ direction to 1-D photonic crystal shown in Figure 2.3.a, each of the layers causes a fraction of the wave to be reflected due to the index difference. For certain frequency, the reflected wave interferes with incident wave constructively. Hence, the 1-D photonic crystal may exhibit high reflection with these frequency intervals.

Provided an infinite version of above 1-D photonic crystal, we can find the frequency intervals in which no mode solutions are found. Hence, no modes at such frequency can propagate throughout the structure. Such a frequency intervals are called photonic band gap.

The above idea of photonic band gap can be generalized to 2-D version. A simple 2-D photonic crystal is shown in Figure 2.3.b. The photonic crystals become more valuable by introducing spatial defects within them. The cross section of a photonic band gap fiber is shown in Figure 2.3.c. The central lower index region ($\epsilon_1 > \epsilon_2$) with larger diameter breaks the periodicity of the photonic crystal. Hereby, the photonic crystals surrounding the central defect act as reflecting boundaries and trap light at the defect region, provided that the frequency is within the photonic band gap of the photonic crystal. Such a defect can act as the core, and guide light within well-defined frequency windows [4]. In this case, light confinement does not require a core with a higher refractive index. These fibers are normally named as PCF.

Bandgap guidance can coexist alongside index guidance in fibers with high-index cores, but index guidance is so dominant that bandgap guidance would hardly be noticed. Therefore, fiber must have a low-index core in order to observe bandgap guidance. Total internal reflection then becomes impossible, and such a fiber can only support bandgap guidance. In this thesis, we only focus on modeling of the holey fiber, which rely on index guiding.

2.1.2 Fabrication

Majority of silica holey fibers have been fabricated using stack-and-draw technique [1]. Capillary tubes are stacked in a particular configuration. This perform is then reduced in scale on a fiber drawing tower. This stack-and-draw procedure has been proved very versatile, allowing very complex lattices to be assembled from individual stackable unit of the correct size and shape. Solid rod, hollow tube and doped glass rods could be easily incorporated in the process as shown in Figure 2.4.

Another promising technique is extrusion. The preform can be extruded directly from glass with low softening temperature [5]. Extrusion offers the prospect of fiber designs that could not be realized with stacking technique. Extrusion allows fiber to be drawn directly from bulk glass, and almost any structure (crystalline or amorphous) can be produced. It works for many materials, including polymers [6], and compound glasses [5] [7]. However selective doping of specified regions to introduce rare-earth ions or render the glass photosensitive is much more difficult than stacking technique.

Figure 2.4 Fabrication technique: Stack and draw.

2.2 Properties of HF

In HF, parameters such as diameter of holes, hole-to-hole distance (pitch), and arrangement of holes offer great flexibility in designing the fiber. HF possesses numerous unusual properties such as wide single-mode wavelength range, extremely high nonlinearity, large effective-core-area in the single-mode region, flexibly engineered group-velocity dispersion at various wavelengths, extremely high birefringence, wide range of device applications, etc. The applications of air-guiding PCF are also briefly introduced.

2.2.1 Endlessly single mode HF

HF has a very unique property: broadband single-mode guidance. This is not readily attainable in conventional fibers. The first HF manufactured in 1996 was single mode over remarkably wide wavelength range, from 458 to 1550 nm at least [1]. This wide single-mode range of HF is explained by considering the effective refractive index of the cladding, loosely understood as the average index of the cladding [8]. HF can be modeled as step index fiber. Assuming the core radius of HF to be a , the value of the normalized frequency V is given by

$$V = \frac{2\pi a}{\lambda} \sqrt{(n_1^2 - n_2^2)} \quad (2.1),$$

where, n_1 and n_2 are core index and average cladding index.

In practical designs of single mode fibers, the core cladding index difference varies between 0.2% and 1.0%. The core diameter of the fiber should be just below the

cutoff of the second order modes; that is, for V slightly less than 2.405. For conventional single mode fiber, the cladding index n_2 varies with wavelength through empirical equation called Sellmeier equation. But for HF, the average cladding index n_2 also depends on the unique cladding structure. At shorter wavelengths the field becomes more concentrated in the silica regions and avoids the holes, thus raising the effective cladding index. This results in a small V value for broad wavelength range for HF. This dispersion counteracts the explicit dependence of normalized frequency, V , on wavelength λ and so extends the single-mode range. Theoretically, HF can be single mode at all wavelengths. But the useful wavelength range of the fiber within the transparency window of silica is ultimately limited by bend loss.

2.2.2 Highly nonlinear HF

Fiber devices based on nonlinear optical effects are very promising applications of holey fiber technology [9]. Nonlinear effects can be used for a wide range of optical processing applications including optical data regeneration [10], wavelength conversion [11], optical demultiplexing [12][13], and amplification [14][15]. Dispersion property (magnitude, sign and slope) of HF can be easily tailored by varying the fiber design. Zero group velocity dispersion (GVD) point in HFs can be engineered over a very wide wavelength range extending from 550nm to 1500nm [16]. Such a fiber can find applications in nonlinear applications such as soliton lasers, ultra broadband continuum generation [17][18][19][20], soliton generation and propagation [21], and ultrashort pulse compression.

The refractive index n of optical material has a weak dependence on optical intensity given by:

$$n = n_0 + n_2 P / A_{eff} \quad (2.2)$$

where n_0 is the ordinary refractive index of the material and n_2 is the nonlinear index coefficient. The nonlinearity in the refractive index is known as Kerr nonlinearity. The fiber nonlinearity can be measured by Kerr nonlinearity [22]:

$$\gamma = 2\pi n_2 / (\lambda A_{eff}) \quad (2.3)$$

where λ is the wavelength and, A_{eff} is the effective mode area. Silica is not a particularly nonlinear material ($n_2 = 2.16 \times 10^{-20} \text{ m}^2/\text{W}$). Conventional Corning SMF28 fiber has a small value of $\gamma = 1 \text{ W}^{-1} \cdot \text{km}^{-1}$ at 1550 nm.

To design a fiber with high values of effective nonlinearity per unit length, we must use material with high value of n_2 and make the value of A_{eff} small. Holey fibers can have a significantly larger NA than conventional fibers because the cladding can be mostly comprised of air when the air-filling fraction is large (i.e., large d / Λ), so light can be confined more tightly within the small core. Effective mode area A_{eff} of $\sim 1.5 \mu\text{m}^2$ has been achieved in silica holey fiber [11]. The small effective area corresponds to a nonlinearity coefficient $\gamma = 70 \text{ W}^{-1} \cdot \text{km}^{-1}$, which is ~ 70 times higher than that of conventional fiber. Another pure silica holey fiber has effective fiber nonlinearities of more than $\gamma = 35 \text{ W}^{-1} \cdot \text{km}^{-1}$ at 1550 nm (with effective area of $\sim 2.8 \mu\text{m}^2$ [10]).

By using material with very high nonlinearity, it is possible to achieve significantly higher fiber nonlinearity. The chalcogenide glass As_2S_3 ($n = 2.4$ at 1550 nm, $n_2 \approx 2 \times 10^{-18} \text{ m}^2/\text{W}$) is 100 times more nonlinear than silica [23]. Chalcogenide glass holey fiber has been reported in [7]. Schott lead-glass SF57 ($n = 1.8$ at 1550 nm, $n_2 \approx 4 \times 10^{-19} \text{ m}^2/\text{W}$) is 20 times more nonlinear than silica [24]. HF manufactured using such glass has effective nonlinearity $\gamma = 550 \text{ W}^{-1} \cdot \text{km}^{-1}$ at 1550 nm [5].

The combination of HF technology and compound glass is attractive for a number of reasons: 1) Core-cladding interface problem is avoided in single-material fiber. So it allows fibers to be drawn from a wide range of glasses. 2) HF technology offers possibility of high NA with tight mode confinement. Together with compound glass having high nonlinearity, high fiber nonlinearity is easily realized. 3) Compound glasses generally have significantly lower softening temperatures than silica. A broader range of HF preform fabrication techniques are available, for example, extrusion. It is possible to increase the nonlinearity still further through the use of glass with higher nonlinearity, and to design fibers with still smaller cores. Such nonlinear fibers will lead to nonlinear fiber devices with unprecedentedly low operating powers and remarkably short device lengths.

2.2.3 Large mode area HF

Holey fiber technology is attractive for large mode area (LMA) fiber for a number of reasons. Firstly, as introduced in section 2.2.1, LMA HF can be single-mode at wide range of wavelength [25]. Such a holey fiber can be achieved by using a large hole-to-

hole spacing ($\Lambda > 5 \mu\text{m}$) and/or a small air-filling fraction ($d/\Lambda < 0.3$). Single mode HFs with mode areas as large as $680 \mu\text{m}^2$ at 1550 nm have been produced [26].

LMA HFs also have different bend-loss properties compared to conventional LMA fibers [8] and it was shown that the bend loss in an large mode area HFs is at least as bend resistant as conventional fiber types with the same mode area at a given wavelength [26]. Like conventional fibers, HF exhibits a bend-loss edge at long wavelengths, as the modes evanescent tail must travel a longer path length around the bend. This part of the energy gets radiated. HFs possesses an additional bend-loss edge at short wavelengths as a direct consequence of the novel cladding structure. If the hole-to-hole spacing is large compared with the wavelength, when the fiber is bent, the guided mode can escape via the solid silica bridges between neighboring holes. By exploiting the intrinsic flexibility of these unusual structures, it may be possible to design LMA HF structures with substantially improved bend loss properties.

Finally, HF technology provides an alternative, and potentially more accurate route, to control the index difference between core and cladding. Making large mode area single mode fiber using conventional technology becomes increasingly difficult for large cores because of the stringent requirements of the uniformity of the doping level as the refractive index difference between the core and the cladding decreases.

LMA holey fiber is important in delivering high power optical beams [27][28]. A highly efficient cladding pumped single transverse mode holey fiber laser has been demonstrated using ytterbium doped all-glass double-clad large mode area HF [29].

2.2.4 Flexibly engineered dispersion property

The group velocity dispersion can be calculated using the following equation:

$$GVD = -\frac{\lambda}{c} \frac{d^2 n_{eff}}{d\lambda^2} \quad (2.4)$$

where $n_{eff} = \beta / k_0$. The fiber dispersion consists of material dispersion and waveguide dispersion. The material dispersion can also be calculated separately by

$$GVD = -\frac{\lambda}{c} \frac{d^2 n(\lambda)}{d\lambda^2} \quad (2.5)$$

where $n(\lambda)$ is given by Sellmeier equation.

The basic material dispersion is hard to alter significantly. But it is possible to modify the waveguide dispersion by changing from a simple step index profile to more complicated index profile. The novel cladding structure can be utilized to accomplish this. The novel cladding structure consisting of air holes allows for a very flexible tailoring of the dispersion curves. It offers the possibilities of obtaining normal, anomalous, large, and zero dispersion curves.

Dependence of the fiber geometrical dispersion on the structural parameters of a HF have been studied in [30][31][32][33]. Results of nearly zero ultra-flattened dispersion curves are obtained in 1424-1627nm wavelength range. HFs with very low and flat dispersion can also be realized by varying the hole-size radially, going from small holes around the core to large holes in the last ring. However, this approach poses significant fabrication challenges and such designs have only been treated theoretically [34].

Dispersion compensation using HF is also attractive. Dispersion as large as $-2000 \text{ ps/nm}\cdot\text{km}$ is obtained from a silica HF [35]. Zero-GVD point in HFs can be engineered over a very wide wavelength range extending from 550nm to 1500nm [18]. However, the high dispersion slope limits the useful spectral bandwidth of this kind of fiber. A new design of hybrid-core HF features full control of both dispersion level and dispersion slope while maintaining low loss [36].

2.2.5 Birefringent HF

In conventional single mode fiber, there are actually two independent, degenerate propagation modes. These two modes (horizontal mode and vertical mode) are very similar but their polarization planes are orthogonal, as shown in Figure 2.5 below.

Figure 2.5 Two polarizations of the fundamental mode in a single mode fiber.

In ideal fibers with perfect rotational symmetry, the two modes are degenerate with equal propagation constant. Any polarization state launched into the fiber will propagate unchanged. High birefringence can be created by asymmetrical lateral stresses, noncircular cores, and variations in refractive index profiles. These imperfections will break the circular symmetry of the ideal fiber and lift the degeneracy of the two modes. These two modes will propagate with different phase

velocity. The difference between their effective refractive indices is called the fiber birefringence:

$$B_f = |n_y - n_x| \quad (2.6)$$

Using HF technology, highly birefringent fibers can be easily realized, because of the higher index contrast than conventional fibers and the unique fabrication process, which allows easy formation of the required asymmetric microstructure near the fiber core. It is possible to create modal birefringence of at least one order of magnitude larger than that of conventional high birefringent fibers. High birefringence in HF can be created by introducing air holes of different sizes [37][38][39], dual-core [40] or elliptical hole in cladding [41][42]. Air guiding photonic crystal fiber can also exhibit high birefringence [43]. Polarization maintaining fibers (PMFs) are essential for coherent optical communication systems and fiber sensor systems. Applications in splitter [44] and symmetrical supercontinuum generation [45] have been reported. However, the polarization-maintaining ability of these fibers is influenced by polarization cross talk and polarization mode dispersion. The remedy is the single-polarization single-mode (SPSM) fiber [46], which propagates one and only one polarization state of the fundamental mode. HF technology offers the possibilities of realizing better SPSM fibers than conventional SPSM fibers [47][48].

2.2.6 Tunable device applications

HF provides a platform for new class of optical devices [49]. In particular, the air holes allow for infusion of active materials yielding novel hybrid all-fiber optical devices that exhibit desirable properties, such as enhanced tunability, compactness, and intrinsic low insertion loss. These devices have exploited index tunable materials

to manipulate and switch mode propagation. Several examples of tunable devices have been reported: tunable optical attenuators by tapering HF where light is modulated by a polymer incorporated in capillary voids [50][51], a highly tunable birefringent HF by a tapered HF with infusion of polymer asymmetrically [52][53][54], a dynamically adjustable narrow and broadband fiber filter using electrowetting actuation in recirculating fluidic channels [55], periodic refractive index grating in air hole realized by microfluidic plugs that are infused into air holes of HF periodically [56].

2.2.7 Air-guiding photonic crystal fiber

The first bandgap-guiding fiber was devised by Broeng and co-workers [57][58]. The photonic crystal had a “honeycomb” array, and the core was an additional small air hole in the array. The fiber has a complex six-lobed modal pattern. Russell and co-workers also presented a new design: a very large hollow core was formed by completely omitting the central seven tubes from the hexagonally arranged stack of tubes [59]. It is easy to couple light into the core of the fiber, because of the Gaussian-like modal profile.

Photonic bandgap fiber guides the light by photonic bandgap but not total internal reflection. It has many unique properties, which cannot be obtained using conventional fiber. As explained in Section 2.1.1, with careful design it is possible to produce a fiber which confines light by a two-dimensional photonic bandgap. In such a PCF, light is unable to penetrate the cladding material owing to suppression of all optical modes within a range of wavelength. Light is then guided in the spatial defect,

which is normally air hole. Higher power can be carried without degradation by absorptive or nonlinear processes. And it will lead to greatly increased threshold powers for stimulated Raman and Brillouin effects.

On the other hand, if the hollow core is deliberately filled with a gas, vapor, or relatively low-index liquid, very strong interactions are possible. This may prove useful for gas sensing and monitoring, for the generation of multiple optical wavelengths by nonlinear processes, and more generally in enhanced nonlinear optics.

Fresnel reflections are eliminated in hollow core fiber because the refractive index discontinuity between the outside world and the fiber mode can be zero. Another remarkable property of the fiber is that there is also very little dispersion, since the light propagates almost entirely in air. The ability to guide light in a tube of vacuum offers possibilities in the fields of micro object transportation [60].

Chapter 3 Modeling method

3.1 Introduction

The nature of the complexity in the transverse geometry of HF makes the modeling a challenging task. Scalar approach and weakly guiding approximation cannot accurately estimate the performance because of the sharp index discontinuity of HF. Analytical methods are yet to find its place suitable in this context because geometry might become more complex for various designs aimed for different applications. Our aim is to look for a general and accurate approach for modeling the modal property of holey fibers. Several numerical methods are already in use for waveguide modeling. For example, FEM, FDM methods are well known and popular to researchers for quite some times. The choice of a method depends on the problem in particular. There is in general a need to achieve a good tradeoff between accuracy and speed. In this chapter, we will present commonly used methods for modeling holey fiber.

Modeling techniques for optical guided-wave propagation can be divided into two main classes: Some photonic devices, in which propagation is nonuniform, require techniques including the beam propagation method (BPM) and the finite-difference time-domain method (FDTD). Certainly they can be applied to nonuniform as well as uniform waveguide to investigate the propagation in these structures. Both Scalar and full vector BPM [61][62][63] has been applied to study various HFs. Imaginary-distance BPM based on finite element and finite difference [64][65] have been

studied. FDTD have been applied to study nonuniform structures (splice loss between HF and SMF [66] and tapered holey fiber [67]) and normal uniform HF [68].

Most optical waveguide structures, like HF, are uniform in propagation direction. In this context, the most fundamental question to ask is what are the optical modes that can propagate in a given cross section of a wave guiding structure. We will focus on mode-solving techniques addressing this question in this chapter. General mode-solving techniques include two categories: First category is based on indirect, modal decomposition approach using sinusoidal functions (plane wave expansion method) [69][70], Hermite-Gaussian functions (localized function method) [71][72][73][74][75], or cylindrical functions (multipole method) [76][77][78][79][80][81]. Second category utilizes other direct, space-domain-division approaches such as the finite element method (FEM) [82][83][84][85], finite difference method (FDM) [86], and boundary element method (BEM).

For bandgap photonic crystal fiber, plane wave method [3] [87][88][89] and FDTD [90] are the generally adopted method. Plane wave method is based on a plane-wave expansion of the field and expansion of the position dependent dielectric constant. This method solves the full-vector wave equation for the magnetic field. It allows one to calculate the possible existence, width, and positioning of PBG, i.e. photonic band diagram.

Since our focus is on 2D simulation on index guiding holey fiber, we briefly review several modeling methods: effective index method, localized function method, multipole method and some numerical techniques (FEM and FDM).

3.2 Effective index method

Birks *et al* proposed a simple effective index method [8] [91]. It can provide qualitative mode-propagation properties of the holey fiber with periodic air-hole arrangement. Because a HF and conventional fiber both rely on the mechanism of total internal reflection, it is believed that a HF can be modeled as a simple conventional fiber. The basic idea of this work is to first evaluate the periodically repeated hole-in-silica structure of the cladding and then replace the cladding by a properly chosen effective index. Approximate scalar method [8] and vectorial approaches [92][93][94] have been reported to calculate cladding effective index. After obtaining effective cladding index. Then the fiber is treated the same way as we treat conventional fibers.

Firstly, an infinite cladding with self-similar hexagonal lattice is assumed, as illustrated in Figure 3.1. A circular shape (dashed line) approximates the hexagonal shape (solid line), so a general circular symmetric mode solution is possible [93]. By solving either scalar-wave equation or analytical vector equation, we can determine the cladding mode field within a simple cell centered on one of the holes. The diameter of these cells is determined assuming air filling fraction is unchanged. By reflection symmetry, the boundary condition at the cell edge is $d\Psi/dr = 0$ in scalar formulation, where r is the radial direction.

Figure 3.1 Holey fiber cladding with hexagonal lattice arrangement.

After the fundamental space-filling mode (i.e., the fundamental cladding mode) is solved, we obtain the propagation constant β_{FSM} . And it is used to define the cladding effective index as $n_{eff} = \beta_{FSM} / k_0$, where k_0 is the free space wave number.

Assumptions made in this simple method are: 1) cladding structure is infinite, 2) hexagonal cell can be approximated as circular one, 3) weakly guiding is assumed, although the index difference between the central hole and the surrounding silica is actually considerable. However inaccuracy introduced by these assumptions is considered to be insignificant as this method is generally used to produce qualitative result. Effective-Index method ignores the complex refractive index profiles within HF. Hence, although it can provide some insight into HF operation, this reduced model is unable to accurately predict modal properties such as the dispersion or birefringence, which depend critically on the arrangement and size of the air holes.

3.3 Localized function method

3.3.1 Localized function method for infinite cladding

The localized function method [71][72][73] is based on the direct solution of Maxwell's equations. In this method, the refractive index and the field distributions are represented as sums of localized basis functions. It assumes that the guided modes of the HF are localized in the close vicinity of the defect forming the core. Therefore, the guided modes can be modeled by representing the fields as sums of functions localized in the vicinity of the core. For uniform HF along the z -direction, the electric field can be rewritten as

$$E_j(x, y, z) = \left(e_j^t(x, y) + e_j^z(x, y)\hat{z} \right) \exp(i\beta_j z) \quad (3.1)$$

where β_j is the propagation constant of the j^{th} mode, and $e_j^t = e_x \bar{x} + e_y \bar{y}$ and e_j^z are the transverse and longitudinal components of the modal electric field respectively.

To solve the wave equation efficiently for HF, both the transverse refractive index profile and the modal electric field are decomposed. For this modal decomposition to be efficient and accurate, it must provide accurate representations of the relevant physical quantities without requiring the use of too many terms. The modal electric field is expanded as

$$e^t(x, y) = \sum_{a,b=0}^{F-1} \left(\varepsilon_{ab}^x \psi_a^m(x) \psi_b^m(y) \bar{x} + \varepsilon_{ab}^y \psi_a^m(x) \psi_b^m(y) \bar{y} \right) \quad (3.2)$$

where F is the number of terms retained in this expansion. The ψ_a^m are elements of the following orthogonal set of Hermite-Gaussian basis functions:

$$\psi_a^m(x) = \frac{2^{-a} \pi^{-1/4}}{\sqrt{(2a)! w_m}} \exp\left(\frac{-x^2}{2w_m^2}\right) H_{2a}\left(\frac{x}{w_m}\right) \quad (3.3)$$

H_i is the i^{th} -order Hermite polynomial. The characteristic width of this basis set is w_m , which is taken to be $\Lambda/2$. Using this Hermite-Gaussian basis set, which is localized function, only a few terms are needed to reconstruct the modal profiles. The advantage of using this set of basis function is that the overlap integrals can be done analytically, which make this technique efficient.

Transverse refractive index is decomposed into two parts: periodic functions for the periodic lattice and localized Hermite-Gaussian functions for central index defect.

$$\begin{aligned} n^2(x, y) &= n_{\text{defect}}^2(x, y) + n_{\text{periodic}}^2(x, y) \\ &= \sum_{a,b=0}^{C-1} c_{ab} \psi_a^d(x) \psi_b^d(y) + \sum_{a,b=0}^{P-1} p_{ab} \cos\left(\frac{2a\pi x}{l}\right) \cos\left(\frac{2b\pi y}{l}\right) \end{aligned} \quad (3.4)$$

The schematic diagram of this decomposition is shown below.

Figure 3.2 Decomposition of the transverse refractive index profile.

Finally, eigenvalue function is obtained by inserting electrical field and index profile expansion functions into the wave equation.

The advantage of this approach is that for an appropriately chosen set of basis functions, only a modest number of functions are required to accurately describe the bound modes, thereby significantly lowering the demand for computational resources. Limitation of this method is that it assumes the cladding is infinite. And different arrangement of air holes requires different formulation. Also it requires knowledge of the characteristic width of basis function.

3.3.2 Localized function method for finite cladding

Previous work assumes an unlimited cladding structure. A variant localized function method [74] was reported to model an exact HF with limited rings of holes. It models the index profile by expanding each air hole locally into a series of Hermite-Gaussian functions. The refractive index profile is expanded as:

$$n^2(x, y) = n_{bg}^2 + \sum_j (n_j^2 - n_{bg}^2) \times \sum_{a,b}^D c_{f,g}^j \psi_{f,g}^{w_d^j}(x - r_x^j, y - r_y^j) \quad (3.5)$$

where n_{bg} is background refractive index, n_j , $c_{f,g}^j$, w_d^j and (r_x^j, r_y^j) are j^{th} hole refractive index, expansion coefficient, normalized characteristic width and Cartesian coordinates.

By doing this, the method can facilitate comparatively fast and accurate modeling of fiber structures based solely on localized functions. It provides a direct way of modeling finite structures. Each air-hole is expanded independently, which may be useful when considering gas- or polymer loaded fibers, since a change in one air-hole does not affect any of the others. It also provides possibilities to reduce the resolution

for holes far away from the core, to ease computational demands without distorting the defect mode significantly.

3.3.3 Supercell lattice method

In supercell lattice method [75], the electric field is again decomposed by use of Hermite-Gaussian functions. The novelty of this method is in how it splits the HF into two virtual perfect periodic photonic crystal lattices. A practical HF missing a central air hole has a limited number of air hole rings. A supercell lattice is constructed using the practical HF as a supercell, as shown in Figure 3.3 (a).

Figure 3.3 Construction of transverse dielectric structure.

The structural parameters of the fiber are the pitch Λ , the hole diameter d , and the supercell lattice period $N\Lambda$. To describe the dielectric structure of Figure 3.3 (a), two 2-D perfect photonic crystals (PC1 and PC2) are introduced, as shown in Figure 3.3 (b) and (c). And it can be written as:

$$n^2(x, y) = n^2|_{PC1} + n^2|_{PC2}, \ln(n^2) = \ln(n^2)|_{PC1} + \ln(n^2)|_{PC2} \quad (3.6)$$

The dielectric structure of every perfect 2-D photonic crystal, such as in the plane wave method model, can be decomposed by use of periodic function (cosines). So the terms in Equation (3.6) can be expressed as:

$$n^2|_{PC1} = \sum_{a,b=0}^{P_1} P_{1ab} \cos(2\pi ax / \Lambda) \cos(2\pi by / \sqrt{3}\Lambda) \quad (3.7)$$

$$n^2|_{PC2} = \sum_{a,b=0}^{P_2} P_{2ab} \cos(2\pi ax / N\Lambda) \cos(2\pi by / \sqrt{3}N\Lambda) \quad (3.8)$$

$$\ln(n^2)|_{PC1} = \sum_{a,b=0}^{P_1} P_{1ab}^{\ln} \cos(2\pi ax / \Lambda) \cos(2\pi by / \sqrt{3}\Lambda) \quad (3.9)$$

$$\ln(n^2)|_{PC2} = \sum_{a,b=0}^{P_2} P_{2ab}^{\ln} \cos(2\pi ax / N\Lambda) \cos(2\pi by / \sqrt{3}N\Lambda) \quad (3.10)$$

where $(P_1 + 1)$ and $(P_2 + 1)$ are the number of decomposition terms of PC1 and PC2, and P_{1ab} , P_{2ab} , $P_{1ab}^{\ln}$, $P_{2ab}^{\ln}$ are coefficients that can be obtained through the Fourier transform. Since all the decomposition coefficients and the overlap integrals are evaluated analytically, the supercell lattice method is efficient. The formulation assumes the HF has perfect periodic cladding.

3.4 Multipole method

T.P. White et al proposed Multipole method for calculating the modes of HF [79][80]. The key aspect of this method is to make use of the circularity of the air holes. So field can be expressed in terms of Hankel and Bessel functions (a natural basis for harmonic functions in cylindrical coordinates) centered at each hole. All the field expansion is then transformed into one common coordinate and boundary condition is enforced accurately.

As usual, E_z and H_z satisfy the Helmholtz equation. In the vicinity of the l^{th} hole, electrical field can be expressed in terms of Fourier-Bessel series as following:

$$E_z = \sum_m \left[A_m^{El} J_m(k_{\perp}^e r_l) + B_m^{El} H_m^{(1)}(k_{\perp}^e r_l) \right] e^{im\theta_l} \quad (3.11)$$

where $k_{\perp}^e = \sqrt{k^2 n_e^2 - \beta^2}$, J_m terms represent the regular incident part of the field E_z for cylinder l since it is finite everywhere, H_m terms represent the outgoing wave part of the field associated with a source at the center of the cylinder l . This expression is only valid in annulus extending from the hole to the nearest hole or source.

Another description of the fields is based on the theory that the field can also be written as superposition of outgoing waves from all the source bodies in the region:

$$E_z = \sum_{l=1}^{N_c} \sum_m B_m^{El} H_m^{(1)}(k_{\perp}^e |r_l|) e^{im \arg(r - c_l)} + \sum_m A_m^{E0} J_m(k_{\perp}^e r) e^{im\theta} \quad (3.12)$$

where each term of the m series is an outgoing wave field with a source at cylinder l , the final term, indexed by 0, is the regular field originating at the fiber jacket boundary.

Equating these two expressions will give the system equation. But evaluating this system equation is difficult because each term of Equation (3.11) and (3.12) is referring to its own unique coordinate system. Complicated coordinate conversion from cylinder to cylinder is applied to solve this issue. An eigenvalue equation is finally obtained. This method can be used to simulate both HF and photonic bandgap fiber. But this method is difficult to be applied for structures with non-circular holes.

3.5 Finite difference method

The FD technique represents a widely used technique in integrated optics. The basic technique has been outlined here. Figure 3.4 gives the discretization scheme.

Figure 3.4 2-D mesh for finite difference method.

Assuming the field has dependence of $\exp[i(\beta z - \omega t)]$, following equations are deduced from Maxwell's curl equation $\nabla \times E = -\partial B / \partial t$,

$$ik_0 H_x = \partial E_z / \partial y - i\beta E_y; ik_0 H_y = i\beta E_x - \partial E_z / \partial x; ik_0 H_z = \partial E_y / \partial x - \partial E_x / \partial y \quad (3.13)$$

Discretizing above equations yields:

$$ik_0 H_x(j,i) = [E_z(j,i+1) - E_z(j,i)] / \Delta y - i\beta E_y(j,i) \quad (3.14)$$

$$ik_0 H_y(j,i) = i\beta E_x(j,i) - [E_z(j+1,i) - E_z(j,i)] / \Delta x \quad (3.15)$$

$$ik_0 H_z(j,i) = [E_y(j+1,i) - E_y(j,i)] / \Delta x - [E_x(j,i+1) - E_x(j,i)] / \Delta y \quad (3.16)$$

They can be written in matrix form as

$$ik_0 \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} 0 & -i\beta I & U_y \\ i\beta I & 0 & -U_x \\ -U_y & U_x & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (3.17)$$

Similarly from the curl equation $\nabla \times H = \partial D / \partial t$, another equation is obtained:

$$-ik_0 \begin{bmatrix} \varepsilon_{rx} & 0 & 0 \\ 0 & \varepsilon_{ry} & 0 \\ 0 & 0 & \varepsilon_{rz} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} 0 & -i\beta I & U_y \\ i\beta I & 0 & -U_x \\ -U_y & U_x & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \quad (3.18)$$

where I is square identity matrix, ε_{rx} , ε_{ry} , ε_{rz} are diagonal matrices whose value are obtained by averaging the refractive indices of adjacent cells, U , V are square matrices depending on the boundary conditions of rectangular computation window.

From Equation (3.17) and (3.18), after some algebra, we obtain an eigenvalue equation in terms of transverse electric or magnetic fields in the form of:

$$[M] \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} = \beta^2 \begin{bmatrix} \varphi_x \\ \varphi_y \end{bmatrix} \quad (3.19)$$

FDM is very good at modeling complex inhomogeneous configurations. It requires the entire domain to be meshed. Since HF is open domain problem, boundary condition must be applied in simulation, hence approximation is introduced. In the finite difference analysis of HF's with curved interfaces, the staircase approximation has to be used in the rectangular mesh.

3.6 Finite element method

FEM is one of the most successful computational methods for electromagnetic simulation. The basic idea of FEM is to divide the solution domain into a number of small interconnected sub regions, called 'element'. The shape and the size of elements are arbitrary, so it is flexible to fit the different shape of the boundary. The density of

the elements may easily be adjusted according to the problem. The behavior of the unknown function within the element is approximated by interpolation function.

A formulation using three components of the electric or magnetic field is presented for illustration. The functional of wave equation is given by

$$F = \iint_{\Omega} [(\nabla \times \phi)^* \cdot ([p] \nabla \times \phi) - k_0^2 [q] \phi^* \cdot \phi] dx dy \quad (3.20)$$

where $p=1, q=n^2$ for $\phi = E$; $p=1/n^2, q=1$ for $\phi = H$. Ω is the waveguide cross section and the asterisk represents complex conjugate.

In order to eliminate spurious solutions, edge elements are introduced. The waveguide cross section is then discretized into triangular edge element as shown below.

Figure 3.5 Triangular edge element for finite element method.

The six nodes described in the triangular edge element consist of the three corner and three side points as shown above. The transverse components $[\phi] = [\phi_x \phi_y \phi_z]^T$ is approximated using

$$[\phi] = [N]^T \times [\phi]_e \quad (3.21)$$

where $[\phi]_e = [\phi_t \phi_z]_e \cdot [\phi_z]_e$ and $[\phi_t]_e$ are the nodal axial field vector and edge variables in the transversal plane for each element respectively. $[N]$ is constructed using the ordinary shape function vector for the linear triangular element.

Inserting (3.21) into (3.20), from the variational principle we obtain the following eigenvalue equation:

$$[K]\{\phi_t\} - \beta^2 [M]\{\phi_t\} = \{0\} \quad (3.22)$$

An eigenvalue equation derived here involves only edge variables in the transversal plane and can provide a direct solution for the propagation constant and the corresponding field distribution.

The remarkable advantage of FEM is that it combines geometrical adaptability and material generality for modeling arbitrary geometries and materials of any composition. It is a very versatile and general method well suited to problems with complicated geometries and complicated material distribution. FEM resolves the primary difficulty with finite differences method, i.e., it allows changing mesh shape and sizes especially in areas of rapid dependent variable change. The disadvantage of FEM lies in its complicated meshing procedure and its disability to model open domain problem directly. The requirement of absorbing boundary condition or perfectly matched layer for an open domain problem (like HF) adds more complexity.

In summary, the methods presented in this chapter have their own unique advantages, but they all have limitation in some aspects. The effective index method ignores the complex refractive index profiles within HF. Hence it is unable to accurately predict modal properties such as the dispersion or birefringence, which depend critically on

the arrangement and size of the air holes. And different arrangement of air holes requires different formulation. Localized function method requires assumption of the characteristic width of basis function. Also it assumes a periodic cladding structure. Multipole method is difficult to be applied for structures with non-circular holes. The disadvantage of FDM and FEM is their disability to model open domain problem directly. The requirement of absorbing boundary condition or perfectly matched layer for an open domain problem (like HF) adds more complexity. FEM also requires a complicated meshing procedure.

BEM, with its own unique property and implementation, will be introduced in next chapter.

Chapter 4 Boundary Element Method

4.1 Introduction

Over recent decades, the boundary element method (BEM) has attracted considerable attention from researchers and has become an important technique in the computational solution of a number of physical problems. In common with the better-known finite element method (FEM) and finite difference method (FDM), the boundary element method is essentially a method for solving partial differential equations (PDEs). The boundary element method has found application in such diverse topics as stress analysis, potential flow, fracture mechanics and electromagnetics.

BEM is based on the boundary integral equation and the principle of weighted residuals, where the fundamental solution is chosen as the weighting function. So it is also named as boundary integration equation method (BIEM). The value of the function and its normal derivative along the boundary are assumed to be the unknowns. By using discretization, the boundary integral equation is transformed into a set of algebraic equations. Then the value of the function and its normal derivative are obtained simultaneously by solving the matrix equation [95][96].

4.2 Comparison with FEM

People used to compare the two numerical methods: FEM and BEM. Even though, FEM is much more popular over BEM, BEM has its own advantage in some areas. Some of the major differences between the two methods are given below. Depending on the application, some of these differences can either be considered as advantageous or disadvantageous to a particular scheme.

- **Mesh construction**

An entire domain mesh is required for FEM. But BEM only needs a mesh on the boundary. Because of the reduction in size of the mesh, one can say that the problem size has been reduced by one dimension. This is one of the major advantages of the BEM. Construction of meshes for a complicated object, like holey fiber, is a very time consuming exercise for FEM. But it is easily accomplished using BEM.

- **Procedures of calculation**

Entire domain solution is calculated as part of the solution if FEM is used. But for BEM, solution on the boundary is calculated first, and then the solution at domain points (if required) are found as a separate step. There are many problems where the details of interest occur on the boundary, or are localized to a particular part of the domain, and hence an entire domain solution is not required.

- **Approximation**

Differential Equation is being approximated for FEM. But for BEM, only boundary conditions are being approximated. The use of the Green-Gauss theorem and a fundamental solution in the formulation means that the BEM involves no approximations of the differential Equation in the domain - only in its approximations of the boundary conditions. Another problem with FEM is that it cannot deal with open problem easily. It must use approximation like absorbing boundary condition or perfectly matched layer. But BEM can handle open domain problem as easily as bounded ones.

- **Matrix**

Normally, sparse symmetric matrix is generated for FEM and fully populated non-symmetric matrices for BEM. But complicated formulation in FEM will destroy the sparsity of the matrix. The matrices are generally of different sizes due to the differences in size of the domain mesh compared to the surface mesh. There are problems where either method can give rise to the smaller system and quickest solution - it depends partly on the volume to surface ratio. For problems involving infinite or semi-infinite domains, BEM is to be favored.

- **Applicability**

FEM is widely applicable and it can handle nonlinear problems well. But BEM cannot be applied to all the problems as in the case for FEM. BEM cannot directly handle nonlinear problem without modification. A fundamental solution must be found (or at least an approximate one) before the BEM can be applied. There are many linear problems (*e.g.*, virtually any non-homogeneous equation) for which fundamental

solutions are not known. There are certain areas in which BEM is clearly superior, but it can be rather restrictive in its applicability.

- **Implementation**

FEM is relatively easy to implement compared to BEM. The need to evaluate integrals involving singular integrands makes the BEM more difficult to implement than a corresponding finite element procedure.

In conclusion, easy meshing procedure and its ability to handle open domain problem, make BEM a suitable choice for modeling holey fiber.

4.3 BEM formulation

4.3.1 Green's function

BEM is a particular version of the method of weighted residuals [95]. BEM equations can be obtained by weighted residuals and integrating twice by parts. Such an approach is presented in [96] [97]. The so-called fundamental solution and its derivative along the outward normal to the boundary can be chosen as a weighted function. The fundamental solution is usually interpreted as a potential field within an infinite region resulting from the unit point source.

BEM equation can also be derived by making use of the reciprocity theorem, known also as a symmetrical Green's identity [98]. This identity relates any two physical fields to each other. From the mathematical point of view, it sets up the relationship

between domain and boundary integrals and as such, is used as a formula for integration by parts. We start the formulation with derivation of Green's function. Field $u(\bar{r})$ satisfies the following linear nonhomogeneous equation:

$$Lu(\bar{r}) = f(\bar{r}) \quad (4.1)$$

where L is linear operator, $\bar{r}$ is field point. If right hand side of the equation is replaced by Dirac delta function $\delta(\bar{r})$, its solution is named as fundamental solution of equation (4.1), i.e.

$$L\tilde{u}(\bar{r}) = \delta(\bar{r}) \quad (4.2)$$

where $\tilde{u}(\bar{r})$ is the fundamental solution of equation (4.1).

Knowing the fundamental solution $\tilde{u}(\bar{r})$, solution of equation (4.1) can be readily obtained by integration. Let $\bar{r}_s$ to be source point. Then the nonhomogeneous term $f(\bar{r})$ can be represented by

$$f(\bar{r}) = \iiint_{V(\infty)} f(\bar{r}_s) \delta(\bar{r} - \bar{r}_s) dv_s \quad (\bar{r}_s \in V) \quad (4.3)$$

The solution of equation (4.1) in integration form is

$$u(\bar{r}) = \iiint_V f(\bar{r}_s) \tilde{u}(\bar{r} - \bar{r}_s) dv_s \quad (\bar{r}_s \in V) \quad (4.4)$$

From Equation (4.2), (4.3) and (4.4), we know the integration core $\tilde{u}(\bar{r} - \bar{r}_s)$ is response of the excitation $\delta(\bar{r} - \bar{r}_s)$. $\tilde{u}(\bar{r} - \bar{r}_s)$ is particularly named as Green's function, and denoted as $G(\bar{r} | \bar{r}_s)$. Green's function represents the field at $\bar{r}$ generated by the source at $\bar{r}_s$. So equation (4.2) and (4.4) can be rewritten as

$$LG(\bar{r} | \bar{r}_s) = \delta(\bar{r} - \bar{r}_s) \quad (4.5)$$

$$u(\bar{r}) = \iiint_V f(\bar{r}_s) G(\bar{r} | \bar{r}_s) dv_s \quad (\bar{r}_s \in V) \quad (4.6)$$

4.3.2 Green's function of 2D Helmholtz equation

To derive the Green's function of 2D Helmholtz equation, we assume its fundamental solution $\tilde{u}(\vec{r})$. By definition:

$$-(\nabla_{\perp}^2 + k^2)\tilde{u}(\vec{r}) = \delta(\vec{r}) \quad (4.7)$$

where $\nabla_{\perp}^2$ is the Laplacian operator in transverse plane.

$$k = \begin{cases} \sqrt{k_0^2 n^2 - \beta^2} & k_0^2 n^2 > \beta^2 \\ -j\sqrt{\beta^2 - k_0^2 n^2} & \beta^2 > k_0^2 n^2 \end{cases}$$

k_0, β are free space wave number and the propagation constant, respectively.

To solve for $\tilde{u}(\vec{r})$, the problem is formulated as following three equations:

$$(\nabla_{\perp}^2 + k^2)\tilde{u}(\vec{r}) = 0 \quad \frac{\partial}{\partial \phi} = \frac{\partial}{\partial z} = 0 \quad (r > 0) \quad (4.8)$$

$$\lim_{a \rightarrow 0} \left[-(\nabla_{\perp}^2 + k^2)\tilde{u}(\vec{r}) \right]_{r \leq a} = \lim_{a \rightarrow 0} [\delta(\vec{r})]_{r \leq a} \quad (4.9)$$

$$\lim_{r \rightarrow \infty} \sqrt{r} \left(\frac{\partial}{\partial r} + jk \right) \tilde{u}(\vec{r}) = 0 \quad (4.10)$$

Equation (4.10) means field at infinite must decay to zero.

The solution should be in the form of:

$$\tilde{u}(\vec{r}) = AH_0^{(1)}(kr) + BH_0^{(2)}(kr) \quad (r > 0) \quad (4.11)$$

Inserting Equation (4.11) into equation (4.10), using Hankel function's asymptotic approximations property, the first term in the solution must be zero, i.e.

$$\tilde{u}(\vec{r}) = BH_0^{(2)}(kr) \quad (r > 0) \quad (4.12)$$

From Equation (4.12) and Equation (4.9), domain integration is calculated as:

$$\lim_{a \rightarrow 0} \iint_{r \leq a} -(\nabla_{\perp}^2 + k^2)BH_0^{(2)}(kr)ds = \lim_{a \rightarrow 0} \iint_{r \leq a} \delta(\vec{r})ds = 1 \quad (4.13)$$

In above equation, property of Dirac delta function is used. $\delta(\vec{r} - \vec{r}_s)$ represents a Dirac delta function which goes to infinity at the point $\vec{r}_s$ and is equal to zero elsewhere. The integral of Dirac delta function over the domain, however, is equal to one. The use of the Dirac delta function is an elegant way of representing unit concentrated sources and forces when dealing with differential equations. The integral of a Dirac delta function multiplied by and another function is equal to the value of the latter at the point.

Left hand side of Equation (4.13) can be transformed into line integral and simplified again by Hankel function's asymptotic approximations property:

$$\begin{aligned}
 LHS &= B \lim_{a \rightarrow 0} \left[\oint_{r=a} -\nabla_{\perp} H_0^{(2)}(kr) \cdot \hat{r} dl + \int_0^{2\pi} \int_0^a -k^2 r H_0^{(2)}(kr) dr d\phi \right] \\
 &= B \lim_{a \rightarrow 0} \left[\int_0^{2\pi} -\frac{\partial}{\partial r} H_0^{(2)}(kr) \Big|_{r=a} a d\phi - 2\pi \int_0^a (kr) H_0^{(2)}(kr) d(kr) \right] \\
 &= B \lim_{a \rightarrow 0} \left\{ 2\pi ka H_1^{(2)}(ka) - 2\pi \left[(kr) H_1^{(2)}(kr) \right]_{r=0}^{r=a} \right\} \quad (4.14) \\
 &= 2\pi B \lim_{a \rightarrow 0} \left[ka H_1^{(2)}(ka) \right] \\
 &= 4jB
 \end{aligned}$$

Hankel function's asymptotic approximations property states that:

$$\begin{aligned}
 \lim_{x \rightarrow 0} H_1^{(2)}(x) &= \lim_{x \rightarrow 0} [J_1(x) - jY_1(x)] \\
 &= \lim_{x \rightarrow 0} \left\{ \frac{1}{2^1 1!} x^1 - j \left[-\frac{2^1 (1-1)!}{\pi} x^{-1} \right] \right\} \quad (4.15) \\
 &= \lim_{x \rightarrow 0} \frac{2j}{\pi x}
 \end{aligned}$$

Solving Equation (4.13) and (4.14), the fundamental solution and Green's function are:

$$\tilde{u}(\vec{r}) = \frac{-j}{4} H_0^2(kr) \quad (r > 0) \quad (4.16)$$

$$G(\vec{r} | \vec{r}_s) = \frac{-j}{4} H_0^2(k|\vec{r} - \vec{r}_s|) \quad (\vec{r} \neq \vec{r}_s) \quad (4.17)$$

The fundamental solution or Green's function of the Helmholtz equation can be usually interpreted as a potential field generated by concentrated unit source acting at a point $\vec{r}_s$. The effect of this source is propagated from point $\vec{r}_s$ to infinity without any consideration of boundary conditions.

4.3.3 Derivation of boundary integral equation

The derivation starts with the second Green's identity [99]. Green's identities are a set of three vector derivative/integral identities, which can be derived starting with the vector derivative identities:

$$\nabla \cdot (\psi \nabla \phi) = \psi \nabla^2 \phi + (\nabla \psi) \cdot (\nabla \phi) \quad (4.18)$$

$$\nabla \cdot (\phi \nabla \psi) = \phi \nabla^2 \psi + (\nabla \phi) \cdot (\nabla \psi) \quad (4.19)$$

From the divergence theorem, we have

$$\int_V (\nabla \cdot F) dV = \int_S F \cdot da \quad (4.20)$$

Subtracting (4.19) from (4.18),

$$\nabla \cdot (\phi \nabla \psi - \psi \nabla \phi) = \phi \nabla^2 \psi - \psi \nabla^2 \phi \quad (4.21)$$

Using (4.21), Equation (4.20) becomes

$$\int_V [\phi \nabla^2 \psi - \psi \nabla^2 \phi] dV = \int_S (\phi \nabla \psi - \psi \nabla \phi) \cdot da \quad (4.22)$$

Equation (3.22) is the Green's second identity. For 2D problem, using the trial solution $u(\bar{r})$ and the weighting function $W(\bar{r})$, equation (4.22) becomes

$$\int_{\Omega} (W \nabla^2 u - u \nabla^2 W) d\Omega = \oint_{\Gamma} \left(W \frac{\partial u}{\partial n} - u \frac{\partial W}{\partial n} \right) dl \quad (4.23)$$

Equation (4.23) sets up the relationship between domain and boundary integrals.

Applying the 2D Helmholtz equation, we have

$$-\int_{\Omega} u (\nabla_{\perp}^2 + k^2) W d\Omega = \oint_{\Gamma} \left(W \frac{\partial u}{\partial n} - u \frac{\partial W}{\partial n} \right) dl \quad (4.24)$$

The aim is now to transform the domain integration on left hand side of the equation (4.24) into a boundary integral equation. This can be done by using a special type of weighting function: the Green's function of 2D Helmholtz equation.

By replacing the W in Equation (4.24) with the Green's function, we obtain

$$-\int_{\Omega} u (\nabla_{\perp}^2 + k) G d\Omega = \oint_{\Gamma} \left(G \frac{\partial u}{\partial n} - u \frac{\partial G}{\partial n} \right) dl \quad (4.25)$$

The left hand side of above equation can be simplified as

$$\begin{aligned} & -\int_{\Omega} u (\nabla_{\perp}^2 + k) G d\Omega \\ &= \int_{\Omega} u(\bar{r}) \delta(\bar{r} - \bar{r}_s) d\Omega \\ &= u(\bar{r}_s) \end{aligned} \quad (4.26)$$

Finally the boundary integral equation is written as

$$u(\bar{r}_s) = \oint_{\Gamma} \left(G \frac{\partial u(\bar{r})}{\partial n} - u(\bar{r}) \frac{\partial G}{\partial n} \right) dl \quad (4.27)$$

It needs to be remembered that above equation applies for the case of a concentrated source at $\bar{r}_s$ and consequently the values of $u(\bar{r})$, and $\frac{\partial u(\bar{r})}{\partial n}$ are those corresponding to that particular position of the source. For each other position of source position, one will obtain a new integral equation. The integral equation is valid for any point within the domain Ω . In BEM it is usually preferable for computational reasons to apply the source point on the boundary Γ and hence one needs to find out what happens when the source points are on boundary Γ .

Putting the source point on boundary will create singular integration problem. A simple way to solve this is to consider that the source point $\bar{r}_s$ is on the boundary but the domain itself is augmented by a hemisphere of radius ε (for 3-D) or semicircle (for 2-D) as shown below,

Figure 4.1 Boundary augmented by a small semicircle.

The source point $\bar{r}_s$ is considered to be at the center of the semicircle. When the radius ε tends to zero, the semicircle becomes a boundary point. So if we adopt the integration path going round the source point $\bar{r}_s$ as shown above, the integral equation (4.27) becomes

$$\begin{aligned} & u(\bar{r}_s) + \lim_{\varepsilon \rightarrow 0} \int_{\Gamma} u(\bar{r}) \frac{dG}{dn} dl + \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} u(\bar{r}) \frac{dG}{dn} dl \\ &= \lim_{\varepsilon \rightarrow 0} \int_{\Gamma} G \frac{du(\bar{r})}{dn} dl + \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} G \frac{du(\bar{r})}{dn} dl \end{aligned} \quad (4.28)$$

The terms involve integration along the boundary Γ_ε are simplified as:

$$\begin{aligned} & \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} u(\bar{r}) \frac{dG}{dn} dl \\ &= \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} u(\bar{r}) \frac{j}{4} H_1^{(2)}(k\varepsilon) k\varepsilon d\theta \\ &= \frac{j}{4} k\theta \lim_{\varepsilon \rightarrow 0} u(\bar{r}) [J_1(k\varepsilon) - jY_1(k\varepsilon)] \\ &= \frac{j}{4} k\theta \lim_{\varepsilon \rightarrow 0} u(\bar{r}) \left[\frac{k\varepsilon}{2} - j \frac{-2}{\pi k\varepsilon} \right] \varepsilon \\ &= -\frac{\theta}{2\pi} u(\bar{r}_s) \end{aligned} \quad (4.29)$$

$$\begin{aligned} & \lim_{\varepsilon \rightarrow 0} \int_{\Gamma_\varepsilon} G \frac{du(\bar{r})}{dn} dl = \lim_{\varepsilon \rightarrow 0} \frac{du(\bar{r})}{dn} \left(-\frac{j}{4} \right) H_0^{(2)}(k\varepsilon) \varepsilon \theta \\ &= \theta \left(-\frac{j}{4} \right) \lim_{\varepsilon \rightarrow 0} \left\{ \left[-j \frac{2}{\pi} \frac{\ln(k\varepsilon)}{1/\varepsilon} \right] \frac{du(\bar{r})}{dn} \right\} \\ &= \theta \left(-\frac{j}{4} \right) \lim_{\varepsilon \rightarrow 0} \left\{ \left[-j \frac{2}{\pi} \frac{1/\varepsilon}{-1/\varepsilon^2} \right] \frac{du(\bar{r})}{dn} \right\} \\ &= \theta \left(-\frac{j}{4} \right) \lim_{\varepsilon \rightarrow 0} \left\{ \left[j \frac{2}{\pi} \varepsilon \right] \frac{du(\bar{r})}{dn} \right\} \\ &= 0 \end{aligned} \quad (4.30)$$

where θ is the angle of boundary shown in Figure 4.1. For smooth boundary, θ is π .

With results from Equation (4.29) and (4.30), boundary integral equation (4.28) can be written as (For simplicity source point $\bar{r}_s$ is denoted as i , c_i is boundary dependent coefficient.):

$$c_i u_i + \lim_{\varepsilon \rightarrow 0} \int_{\Gamma} u \frac{dG}{dn} d\Gamma = \lim_{\varepsilon \rightarrow 0} \int_{\Gamma} G \frac{du}{dn} d\Gamma \quad (4.31)$$

where

$$\begin{cases} c_i = 1, & \text{source point in the domain} \\ c_i = 0.5, & \text{source point on smooth boundary} \\ c_i = 0, & \text{source point outside of domain} \\ c_i = 1 - \theta / 2\pi, & \text{source point on any boundary} \end{cases}$$

4.3.4 Boundary discretization

Let us now consider how Equation (4.31) can be discretized to find the system of equations, from which the boundary values can be found. The points where the unknown values are considered are called “nodes”. Position of nodes for different type of element are shown below:

Figure 4.2 Boundary element of BEM.

Nodes are taken to be in the middle of the element for constant element, at two ends of element for linear one, and a further mid-element node is required for quadratic element.

The boundary is assumed to be divided into N elements. Boundary integral equation (4.31) can be discretized for a given source point i before applying any boundary conditions as follows:

$$c_i u_i + \sum_{j=1}^N \left(\int_{\Gamma_j} u \frac{dG}{dn} dl \right) = \sum_{j=1}^N \left(\int_{\Gamma_j} \frac{du}{dn} G dl \right) \quad (4.32)$$

where Γ_j is boundary of element j of the boundary.

The task now is to evaluate the integration for each element. First of all, the field and its normal derivative must be expressed in terms of nodal value of element. And this is different for different element type because of different interpolation function used. The field u or normal derivative du/dn (denoted by q) along an element are interpolated using its nodal values as:

$$\begin{aligned} u(\xi) &= \{\phi(\xi)\}^T \{u\}_e \\ q(\xi) &= \{\phi(\xi)\}^T \{q\}_e \end{aligned} \quad (4.33)$$

ξ is dimensionless coordinate varying from -1 to $+1$. $\{u\}_e$ and $\{q\}_e$ are nodal values of an element, whose size is the number of nodes of an element z . $\phi(\xi)$ is the interpolation functions.

For constant element:

$$z = 1; \phi_1 = 1$$

For linear element:

$$z = 2; \phi_1 = \frac{1}{2}(1 - \xi); \phi_2 = \frac{1}{2}(1 + \xi)$$

For quadratic element

$$z = 3; \phi_1 = \frac{1}{2}\xi(\xi - 1); \phi_2 = (1 - \xi)(1 + \xi); \phi_3 = \frac{1}{2}\xi(1 + \xi)$$

Now the integration in Equation (4.32) can be evaluated as:

$$\int_{\Gamma_j} u \frac{dG}{dn} dl = \int_{\Gamma_j} [\phi_1 \phi_2 \dots \phi_z]_j \frac{dG}{dn} dl [u_1 u_2 \dots u_z]_j^T = [\bar{h}_1^{ij} \bar{h}_2^{ij} \dots \bar{h}_z^{ij}] [u_1 u_2 \dots u_z]_j^T = \bar{h}^{ij} u^j \quad (4.34)$$

$$\int_{\Gamma_j} \frac{du}{dn} G dl = \int_{\Gamma_j} [\phi_1 \phi_2 \dots \phi_z]_j G dl \left[\left(\frac{du}{dn} \right)_1^j \left(\frac{du}{dn} \right)_2^j \dots \left(\frac{du}{dn} \right)_z^j \right]^T = g^{ij} \frac{du^j}{dn} \quad (4.35)$$

These two integrals relate the source i where the fundamental solution is acting on to any other element j . Because of this their resulting values $\bar{h}^{ij}$ and g^{ij} are sometimes called influence coefficients. u^j and $\frac{du^j}{dn}$ represent column vectors of the nodal values of element j . Using above integration results, Equation (4.32) becomes

$$c_i u_i + \sum_{j=1}^N (\bar{h}^{ij} u^j) = \sum_{j=1}^N (g^{ij} \frac{du^j}{dn}) \quad (4.36)$$

To further simplify above equation, we assume

$$h^{ij} = \bar{h}^{ij} + c_i \delta^{ij} \quad (\delta = 1, \text{ if and only if } i = j) \quad (4.37)$$

Equation (4.36) can be written as

$$\sum_{j=1}^N (h^{ij} u^j) = \sum_{j=1}^N (g^{ij} \frac{du^j}{dn}) \quad (4.38)$$

If we now apply the source at each node successively one obtains a system of equations resulting from applying (4.38) to each node in turn:

$$HU = GQ \quad (4.39)$$

where H and G are matrices, U and Q are boundary nodal variables (column vectors).

4.3.5 Formulation for holey fiber

The cross section of holey fiber can be modeled by more generalized problem as shown below. An open domain D_0 has M inclusions labeled as $D_1 D_2 \dots D_M$, with dielectric constant $\epsilon_1 \epsilon_2 \dots \epsilon_M$. The outward normal direction and tangential direction for each domain are shown also.

Figure 4.3 Cross section of general multi-domain problem.

Boundaries $\Gamma_1 \Gamma_2 \dots \Gamma_M$ are discretized into $N_1 N_2 \dots N_M$ ($N_1 + N_2 + \dots + N_M = N_t$) elements respectively. For each inclusion D_k ($k \geq 1$) we can write an equation for each application of source point i to a node (refer to Equation (4.38)):

$$\sum_{j=1}^{N_k} (h^{ijk} u^{jk}) = \sum_{j=1}^{N_k} (g^{ijk} \frac{du^{jk}}{dn}) \quad (4.40)$$

The source point i is assumed to be inside the domain Γ_k but infinitely close to the boundary Γ_k .

The boundary of domain D_0 , is the sum of boundaries of all the inclusions, i.e.,

$\Gamma_0 = \sum_{k=1}^M \Gamma_k$. The discretized boundary integral equation for a source point i is:

$$\sum_{j=1}^{N_i} (h^{ij0} u^{j0}) = \sum_{j=1}^{N_i} (g^{ij0} \frac{du^{j0}}{dn}) \quad (4.41)$$

Various formulations based on different components of electromagnetic wave have been used in modeling different waveguide [100][101][102][103][104][105][106][107][108]. Holey fiber has not been studied until very recently [109][110]. The formulation presented here use the transverse magnetic fields H_x and H_y [111] [112]. It can suppress non physical (spurious) solutions [113][114][115]. Direct formulation based on E_z and H_z will give spurious solution. When dealing with nonmagnetic materials, continuity of the magnetic fields across interfaces eliminates the complication of solving for discontinuous transverse electric field components. These reasons motivate the use of the transverse magnetic fields H_x and H_y rather than others.

So equation (4.40) and (4.41) should be written for both H_x and H_y . And the relation between u^{jk} in Equation (4.40) and u^{j0} in Equation (4.41) is obtained through boundary condition: H_x and H_y , as well as axial field E_z and H_z must be continuous.

From Maxwell, axial fields E_z and H_z can be expressed in terms of transverse fields:

$$j\omega\epsilon_0\epsilon_r E_z = \frac{\partial H_l}{\partial n} - \frac{\partial H_n}{\partial l} \quad (4.42)$$

$$j\beta H_z = \frac{\partial H_n}{\partial n} + \frac{\partial H_l}{\partial l} \quad (4.43)$$

where $\frac{\partial H_n}{\partial l} \approx \frac{1}{r} \frac{\Delta H_n}{\Delta \theta} - \frac{H_l}{r}$, H_l and H_n denote tangential and normal magnetic field

respectively. They are constructed using H_x and H_y as:

$$H_n = H_x \cos \theta + H_y \sin \theta \quad (4.44)$$

$$H_l = -H_x \sin \theta + H_y \cos \theta \quad (4.45)$$

where θ is defined in Figure 4.3.

Continuity of H_z results in (for boundary (Γ_k)):

$$\left[\frac{dH_x}{dn} \cos \theta + \frac{dH_y}{dn} \sin \theta \right]_{D_0} = \left[-\left(\frac{dH_x}{dn} \cos \theta + \frac{dH_y}{dn} \sin \theta \right) \right]_{D_k} \quad (4.46)$$

Continuity of E_z results in (for boundary (Γ_k)):

$$\left[-\frac{dH_x^e}{dn} \sin \theta + \frac{dH_y^e}{dn} \cos \theta \right]_{D_0} = \left[\frac{\epsilon_{r0}}{\epsilon_{rk}} \left(\frac{dH_x}{dn} \sin \theta - \frac{dH_y}{dn} \cos \theta \right) - \left(1 - \frac{\epsilon_{r0}}{\epsilon_{rk}} \right) \frac{\partial H_n}{\partial l} \right]_{D_k} \quad (4.47)$$

where the term $\frac{\partial H_n}{\partial l}$ can be expressed in terms of H_x and H_y by using Equation (4.44)

and (4.45).

Continuity of H_x and H_y results in (for boundary (Γ_k)):

$$[H_y]_{D_0} = [H_y]_{D_k}; [H_y]_{D_0} = [H_y]_{D_k} \quad (4.48)$$

Equations (4.40), (4.41), (4.46), (4.47) and (4.48) form the complete description of the whole guiding structure in terms of H_x and H_y and their normal derivatives along the boundaries. The final equation will be in the form of

$$[M] \left\{ \begin{matrix} \{u\} \\ \{\partial u / \partial n\} \end{matrix} \right\} = 0 \quad (4.49)$$

where u is H_x and H_y .

Propagation constant is obtained by searching for zero points of the determinant of $[M]$. Since the matrix $[M]$ is complex-valued and involves propagation constant β implicitly, equation (4.49) is non-standard eigenvalue problem. It is difficult to obtain the solutions analytically and we are obliged to use direct search method. Knowing the propagation constant, boundary value of $H_x, H_y, \partial H_x / \partial n, \partial H_y / \partial n$ can be solved from equation (4.49). Then field at any point of any domain can be calculated from equation (4.36) by boundary integration.

4.4 Summary

To summarize, the whole process of solving boundary problem by boundary element method is shown in Figure 4.4.

Figure 4.4 Successive stages of BEM.

The method contains the following steps. According to reciprocity theorem, the fundamental solution is chosen as the weighting function to form the integral equation. The boundary Γ is then discretized into a number of elements over which the unknown function and its normal derivative are assumed by the interpolation functions. After the integrals over each element are evaluated analytically or numerically, the coefficients of the matrix equation are evaluated. The solutions of these equations result in the boundary value of the potential and its normal derivatives. Hence the field strength of most interest on the boundary is computed directly from the matrix equation. The value of the function at any point can be calculated once all the function values and their normal derivatives on the boundary are known through boundary integral equation.

The main characteristic of BEM is that it reduces the dimensions of the problem by one. For a 2D problem, only the boundary of the domain needs to be discretized hence

it produces a much smaller number of algebraic equations. It is especially attractive that the data preparation is simple because the tedious domain discretization is avoided. It is particularly applicable to piecewise homogenous optical structures and open domain problem because it uses Green's function as weighting function, hence no absorbing boundary condition or perfectly matched layer is required.

Chapter 5

Simulation results using BEM

5.1 Validity of BEM

Before applying BEM to complex structure of holey fiber, simulation is done on simple step index fiber to verify the accuracy of the method.

Figure 5.1 Simple step index fiber.

Figure 5.1 shows a simple step index fiber, with core index $n_1=1.452$, cladding index $n_2=1.447$, and radius $a=4.6\mu\text{m}$. Under weakly guiding assumption, the propagation constant β can be obtained by

$$\frac{J'_v(ua)}{uJ_v(ua)} + \frac{K'_v(wa)}{wK_v(wa)} = \pm \frac{v}{a} \left(\frac{1}{u^2} + \frac{1}{w^2} \right) \quad (5.1)$$

where $u^2 = k_1^2 - \beta^2$, $k_1 = 2\pi n_1 / \lambda$, $w^2 = \beta^2 - k_2^2$, $k_2 = 2\pi n_2 / \lambda$.

Using BEM, the core and cladding interface is discretized by constant elements. The integration is obtained by Gaussian quadrature. But there are few exceptions to be noted, i.e., when the integration is calculated on an element where the source is

($i = j$) (see Equation (4.36)). The term $\bar{h}^{ii}$ is always zero. The integrand of g^{ii} becomes infinite at source point. The Cauchy principal value is then constructed by Hankel function's expansion [107]:

$$g^{ii} = \int_{\Gamma_i} G(\bar{r} | \bar{r}_s) dl = \int_{\Gamma_i} \frac{1}{4j} H_0^{(2)} dl (k | (\bar{r} - \bar{r}_s) | dl$$

$$= \frac{1}{2jk} \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n} (2n+1)(n!)^2} \left(\frac{kl_i}{2} \right)^{2n+1} \times \left[1 - i \frac{2}{\pi} \left(\ln \frac{kl_i}{4} + \gamma - \frac{1}{2k+1} - \sum_{m=1}^n \frac{1}{m} \right) \right] \quad (5.2)$$

where $\gamma = 0.5772157...$ is the Euler constant.

The most important parameter of an optical waveguide is its propagation constant or effective index of the propagation mode. Simulation results based on different mesh densities are compared with the results obtained by analytical method, as shown in Figure 5.2.

Figure 5.2 Comparison between BEM results and analytical results.

It shows that BEM results converge to the exact ones when higher mesh density is used. The computed results are always slightly below the result obtained by analytical method. This is due to the fact that meshed structure has a slightly smaller core than ideal model. There is always a tradeoff between accuracy and computation time. To know what is the mesh density required for certain level of accuracy, the influence of the ratio between wavelength and element length must be studied. Figure 5.3 shows the relative error (in percentage) and normalized computing time in terms of the ratio between element length (assuming all elements are of the same length) and wavelength (fixed at 1.55 μm). As expected, shorter element length gives better accuracy but take much longer time.

Figure 5.3 Accuracy and computing time for different mesh density.

In order to check the accuracy of the BEM for holey fiber, we consider an index-guiding HF with a single ring of six air holes (hexagonal arrangement) with hole

diameter $d = 5\mu m$, hole-to-hole spacing $\Lambda = 6.75\mu m$. The background silica refractive index is 1.45, and the wavelength $\lambda = 1.45\mu m$. This set of parameters is chosen to be same as that in [79] for comparison. In Figure 5.4 we show a determinant surface in the neighborhood of a well-defined minimum, corresponding to the fundamental mode of above described HF. The computed effective index of the fundamental mode: real part is 1.4453954, imaginary part is 3.10×10^{-8} . Imaginary part corresponds to loss of 1.17dB. The subsequent effective indices of higher modes are 1.43858616, 1.4384488, 1.4383682, 1.43017648, 1.42997014, 1.42925988 respectively. These results are in good agreement with that of the full-vector multipole method [79] and FEM [34].

Figure 5.4 Logarithm of the magnitude of the determinant of matrix versus the real and the imaginary parts of the complex refractive index.

5.2 Fundamental space filling mode

In the analysis of the HF, the effective cladding index is a very important parameter. It determines if the HF is single-mode or multi-mode. It not only facilitates our understanding of holey fiber but also allows the application of conventional theories of fiber propagation and dispersion without necessarily resorting to numerical models. It is obtained by investigating the propagation characteristic of the fundamental cladding mode (also named as fundamental space filling mode (FSM)) [93]. The investigation of space filling mode has not been carried out using BEM.

Figure 5.5 Elementary piece of cladding.

Assuming the cladding is infinite. The effective cladding index n_{FSM} can be obtained by studying only the elementary piece as shown in Figure 5.5. The boundary conditions to compute the n_{FSM} are: perfect magnetic conductor (PMC) for Γ_1 and Γ_3 , perfect electric conductor (PEC) for Γ_2 and Γ_4 if the mode is x-polarized; PEC for Γ_1 and Γ_3 , PMC for Γ_2 and Γ_4 if the mode is y-polarized.

H_x and H_y are chosen to be the variables of the formulation in Chapter 4 because it can suppress spurious mode. However, in this formulation, E_z and H_z are chosen to be the variables because of the unique boundary condition. Consequently, it results in spurious modes. The reason behind the occurrence of spurious modes is the lack of continuity of tangential derivatives of the axial fields [105]. To suppress the spurious mode, tangential derivatives of the axial fields $\partial E_z / \partial t$ and $\partial H_z / \partial t$ (t is the tangential direction in transverse plane) are chosen to be the unknowns instead of E_z and H_z . $\partial E_z / \partial t$ and $\partial H_z / \partial t$ are given by:

$$E_z(\tau) = \int_0^\tau \frac{\partial E_z}{\partial t} dt + E_z(0); H_z(\tau) = \int_0^\tau \frac{\partial H_z}{\partial t} dt + H_z(0) \quad (5.3)$$

where $E_z(0)$ and $H_z(0)$ are the reference value arbitrarily taken along the integration path, and the integral is made from the reference point to point τ . Solving the above equation, finally we have new set of unknowns along the boundary:

$$\begin{aligned} & [E_z(0) \frac{\partial E_{z1}}{\partial t} \frac{\partial E_{z2}}{\partial t} \dots \frac{\partial E_{zN-1}}{\partial t}]^T, [\frac{\partial E_{z1}}{\partial n} \frac{\partial E_{z2}}{\partial n} \dots \frac{\partial E_{zN}}{\partial n}]^T \\ & [H_z(0) \frac{\partial H_{z1}}{\partial t} \frac{\partial H_{z2}}{\partial t} \dots \frac{\partial H_{zN-1}}{\partial t}]^T, [\frac{\partial H_{z1}}{\partial n} \frac{\partial H_{z2}}{\partial n} \dots \frac{\partial H_{zN}}{\partial n}]^T \end{aligned} \quad (5.4)$$

Because the outer boundary is straight line, the boundary conditions are: PEC implies

$$e_z = 0 \quad (\frac{\partial E_z}{\partial t} = 0) \quad \text{and} \quad \frac{dh_z}{dn} = 0; \quad \text{PMC implies} \quad h_z = 0 \quad (\frac{\partial H_z}{\partial t} = 0) \quad \text{and} \quad \frac{de_z}{dn} = 0.$$

For boundary Γ_a and Γ_b , the continuity of tangential derivatives of axial fields H_z and E_z are enforced. For example, at Γ_a (boundary between D_1 and region D_2), following boundary conditions are applied:

$$\begin{aligned} E_z^1(0) &= E_z^2(0), H_z^1(0) = H_z^2(0), \\ \frac{\partial E_z^1}{\partial t} &= -\frac{\partial E_z^2}{\partial t}, \frac{\partial H_z^1}{\partial t} = -\frac{\partial H_z^2}{\partial t} \end{aligned} \quad (5.5)$$

Following the stages outlined in Chapter 4, system of equations can be obtained using above boundary conditions. The final equations are similar to (4.39), but the matrix H is redefined as:

$$H_{i1} = \sum_{j=1}^N H_{ij} \quad (5.6)$$

$$H_{ij} = \left(\sum_{k=j-1}^N H_{ik} l_{j-1} \right) - H_{iN} \frac{l_N}{2} - H_{i(j-1)} \frac{l_{j-1}}{2} \quad (j > 1) \quad (5.7)$$

To show the effectiveness of spurious mode suppression, formulations with and without spurious mode suppression are implemented. Figure 5.6 shows the root searching for both implementations. As expected, spurious modes are effectively suppressed.

Figure 5.6 Spurious mode suppression.

With the propagation constant obtained, values of $E_z(0)$, $H_z(0)$, $\partial E_z / \partial t$, $\partial H_z / \partial t$, $\partial E_z / \partial n$, $\partial H_z / \partial n$ along the boundary can be calculated. Then the full axial field profile is calculated using integral equation (4.36). The conventional way of generating the transverse electromagnetic field is to use following equations:

$$\begin{aligned} \begin{bmatrix} E_x \\ H_y \end{bmatrix} &= \frac{-1}{n_j^2 - n_e^2} \begin{bmatrix} j\beta, & j\omega\mu \\ j\omega\varepsilon, & j\beta \end{bmatrix} \begin{bmatrix} \partial E_z / \partial x \\ \partial H_z / \partial y \end{bmatrix} \\ \begin{bmatrix} E_y \\ H_x \end{bmatrix} &= \frac{1}{n_j^2 - n_e^2} \begin{bmatrix} -j\beta, & j\omega\mu \\ j\omega\varepsilon, & -j\beta \end{bmatrix} \begin{bmatrix} \partial E_z / \partial y \\ \partial H_z / \partial x \end{bmatrix} \end{aligned} \quad (5.8)$$

in which, $\partial E_z / \partial x, \partial E_z / \partial y, \partial H_z / \partial x, \partial H_z / \partial y$ are evaluated using the finite difference scheme. However, due to the discontinuity of the transverse derivative of axial field across dielectric interface, the computed transverse field is not accurate when it is close to the dielectric interface.

To solve the problem, E_n, E_t, H_n and H_t are calculated using Equation (5.8) with $(\vec{x}, \vec{y})$ replaced by $(\vec{n}, \vec{t})$. Because $\partial E_z / \partial t, \partial H_z / \partial t, \partial E_z / \partial n$ and $\partial H_z / \partial n$ are known on the boundary, the finite difference scheme is not used. Then the transverse field components are obtained by applying coordinate transformation:

$$\begin{pmatrix} \vec{x} \\ \vec{y} \end{pmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{pmatrix} \vec{n} \\ \vec{t} \end{pmatrix} \quad (5.9)$$

in which, θ is the angle between $\vec{n}$ and $\vec{x}$ as shown in Figure 4.3.

As Figure 5.7 shows, the conventional finite difference scheme generates inaccurate results (Figure 5.7 a) along the boundary (light color represents high density). And the proposed method solves the problem (Figure 5.7 b). As expected, the field is strongly concentrated in the high index region (silica).

Figure 5.7 Transverse magnetic field density of fundamental space filling mode.

The fundamental cladding mode (space filling mode) is defined to be the mode with the highest effective index. Figure 5.8 shows the transverse electric field for these two degenerate modes at $\lambda=1.0\mu\text{m}$. We can see that they are essentially linearly polarized.

Figure 5.8 Transverse electric field vector of fundamental space filling mode.

Now, the dispersion of the fundamental space filling mode with different air-filling ratios can be studied. Following figure shows results obtained using scalar approach and BEM simulation.

Figure 5.9 Effective cladding index obtained by BEM (solid line) and the scalar method (dashed line) for claddings of different air-hole size.

It can be seen from Figure 5.9 that scalar method is very good approximation at short wavelength and low air-filling ratio because the modal fields tend to concentrate more in the silica region; the interaction of the modal field with the boundary is minimum. The difference is larger at longer wavelength with larger air-filling ratio.

We compute the cladding effective index for two different HFs. The first HF has $d/\Lambda = 0.6/2.3$. It remains single-moded over a wide range of wavelength [8] (Figure 5.10 a). The second HF has $d/\Lambda = 1.4/2.3$, which allows at least propagation of second order modes (Figure 5.10 b). The results agree well with [83][92].

(a) Single mode holey fiber

(b) Multi mode holey fiber

Figure 5.10 Effective index of guided mode and cladding effective index.

In conclusion, we have developed a technique of using BEM for analysis of space filling mode of HF. The method is accurate, efficient and simple. It is an important component in studying HF using BEM. The formulation is capable of suppressing spurious mode by enforcing the continuity of tangential derivatives of the axial fields. Besides this, a novel approach of generating transverse field is implemented to avoid inaccurate results along the boundary.

5.3 Analogy between HF and step index fiber

Because both HF and conventional step index fiber rely on the mechanism of total internal reflection, it is believed that a HF can be modeled as a simple conventional step index fiber (SIF). After knowing the effective cladding index, holey fiber can be treated as a conventional fiber. Assuming the core radius of HF to be pitch Λ , the value of the normalized frequency V is given by

$$V = k_0 \Lambda \sqrt{(n_{core}^2 - n_{FSM}^2)} \quad (5.10)$$

With above assumption, we have calculated effective index of HF ($d / \Lambda = 0.69 / 2.3$) and its step index model with the same NA and V ($NA = \sqrt{(n_{core}^2 - n_{FSM}^2)}$ and $V = k_0 \Lambda \sqrt{(n_{core}^2 - n_{FSM}^2)}$). The fundamental mode $U = k_0 \Lambda \sqrt{(n_{core}^2 - n_{neff}^2)}$ for both HF and SIF are plotted in Figure 5.11.

Figure 5.11 Fundamental U versus V for HF and SIF with the same V and NA .

The two sets of values are quite different, showing that direct application of SIF's V formulae to HF gives wrong answers for U (hence the propagation constant, chromatic dispersion and so on). Hence, it is necessary to redefine the core radius of HF.

Brechet et al. has proposed a reasonable definition for effective core radius [82]. It calculates the temporary effective V value with core radius Λ , V_Λ . In addition, the value of V for HF is calculated using the actual effective index given by FEM. Then the ratio V/V_Λ is approximately established to be 0.64. So the effective core radius can be approximated by 0.64Λ . Birks et al. [116] proposed another approach for determining the value of effective core radius. The approach is similar to [82] but the normalized transverse phase constant in the core region $U = k_0 \rho \sqrt{n_{core}^2 - n_{eff}^2}$ is used instead of V . The estimated effective core radius is 0.625Λ .

Our approach is outlined here: for a HF (with d and Λ), effective index n_{eff} of the guided fundamental mode and effective cladding index n_{FSM} are calculated. Then the effective index n_{eff} , n_{core} and $n_{cladding} = n_{FSM}$ are substituted into the dispersion equation of step index fiber. Finally, the core radius of the step index fiber is solved directly from the dispersion equation. And this radius is the effective core radius.

Figure 5.12 plots the effective core radius (normalized by $\Lambda = 2.3\mu m$) for the fundamental mode for HF with various hole sizes. It appears that the ratio is quite constant when the wavelength is close to the hole diameter. And the ratio is found to be approximately 0.62 for $d=0.6\mu m$. Nevertheless, for small values of d/λ , the core

radius cannot be considered constant. The effective core radius is only used when HF is modeled as step index fiber. It is the core radius of the step index fiber, which has the same propagation constant as the HF considered. Effective core radius is calculated with no consideration of mode field diameter. And it should not be used as indication of the mode field diameter. Mode field diameter will be further discussed in Section 5.6. The simulation results also do not show any correlation between these two parameters.

Figure 5.12 Effective core radius versus d/λ ratio for various air hole sizes.

In a standard step-index fiber with core radius a , core index n_{core} and cladding index $n_{cladding}$, the number of guided modes is determined by normalized frequency V (with Λ replaced with a in Equation (5.10)). The fiber is single mode if V is less than

2.405. By analogy with SIF, the effective V value for HF can be defined by Equation (5.10) with Λ replaced with a_{eff} .

Calculation of effective cladding index n_{FSM} is described in Section 5.2. Taking effective core a_{eff} to be 0.62Λ , the normalized frequency for various air-filling ratio is plotted in Figure 5.13.

Figure 5.13 Effective normalized frequency as a function of Λ/λ for different air hole size.

It is observed that HF can be single mode over a large wavelength range, or even “endlessly” single mode.

5.4 Symmetry and degeneracy in HF

Symmetry analysis is one analytical technique, which can provide basic information concerning the modal characteristics and suggest possible strategies to optimize the computation. The symmetry of a waveguide controls several mode characteristics of the waveguide. First of all, it enables one to classify the possible modes into mode classes, predict the mode degeneracy between mode classes, and determine the azimuthal symmetries of the modal electromagnetic fields in each mode class. Secondly, one can specify minimum waveguides sectors for each mode class, which completely determine the modes of that mode class. These two tasks can be accomplished from the waveguide symmetry properties without solving the boundary value problem.

For a two dimensional figure, only two types of spatial symmetry operations can exist: rotation about a symmetry axis oriented normal to the plane concerned and reflections in planes oriented normal to the plane of the figure. A pattern possesses the symmetry group C_n of order n if it possessed only rotational symmetry ($2\pi/n$ is the smallest angle associated with a symmetry operation). If a plane possesses not only C_n symmetry, but also possesses at least one plane of reflection symmetry, then it possesses precisely n planes of reflection symmetry. The symmetry group for such a figure is designated as C_{nv} (of order $2n$) [117].

A HF with hexagonally arranged holes possess symmetry group C_{6v} . Numerical methods (plane wave method [118] and FEM [119]) have been applied to show that such a fiber is not birefringent and the fundamental mode is degenerate. The effective

indices n_{eff}^x and n_{eff}^y for the fundamental modes HE_{11}^x and HE_{11}^y are calculated. The difference of these two indices converges to machine precision limit (order of 10^{-8}), showing that the fiber is not birefringent. In FEM, the difference in these two indices is due to the fact that meshed structure no longer possess perfect symmetry property. The plane wave method using a rectangular grid, also introduces numerical birefringence, which splits the HE states. But for both methods, the difference decreases with the increasing number of variables (or basis size) used.

For BEM, however, only the boundary is meshed, it is possible for the meshed structure to remain perfect symmetry as original model. We consider a fiber with a ring of six air holes symmetrically arranged around the core as shown in Figure 5.12.

($\Lambda = 6.75\mu m$, $d = 5\mu m$, $n = 1.45$, $\lambda = 1.55\mu m$)

Figure 5.14 HF with a ring of six air holes.

The angle in Figure 5.14 indicates the position of the first element along each air hole boundary. The meshed structure will still perfectly possess symmetry group C_{6v} . The calculated effective indices at different mesh density are shown below.

Figure 5.15 Effective index of fundamental modes versus mesh density.

The effective indices n_{eff}^x and n_{eff}^y for the fundamental HE_{11}^x and HE_{11}^y are found to be identical to the order of 10^{-9} . It is observed that the meshing scheme does not introduce any numerical birefringence because it still preserves the symmetry group C_{6v} . And splitting of the fundamental states is absent, showing the HF is not birefringent. The transverse electric field vectors are shown in Figure 5.16 for x-polarized and y-polarized modes of the fundamental mode.

Figure 5.16 Transverse electric field vectors for fundamental modes.

Uniform waveguide has a mode spectrum containing an infinite number of modes. However, the number of distinct azimuthal symmetries of modal electromagnetic field patterns for a structure of a given symmetry type, C_n or C_{nv} is of order n . So each mode can be assigned to a mode class according to the azimuthal symmetry of modal field patterns. These classes are called mode classes, which contain an infinite number of modes. As mentioned before, HF with hexagonally arranged holes possesses C_{6v} symmetry. It has (6+2) distinct mode classes, of which, four mode classes are degenerate. For open boundary problem like HF, modes can be labeled as E_{pq} (p designates mode class, q designates mode in that mode class, usually limited to few possible value). All the modes in a mode class are non-degenerate or all the modes are degenerate. And mode degeneracy can occur only in pairs. The mode classes and the corresponding minimum sector are summarized in Figure 5.17.

Figure 5.17 Minimum sectors for waveguide with C_{6v} symmetry.

p is mode class number. Solid lines indicate Dirichlet boundary conditions for the electric field; dashed lines indicate Neumann boundary conditions.

Mode classes $p=1, 2, 7, 8$ are non-degenerate, exhibiting the full symmetry of HF. Mode classes $p=3, 4, 5, 6$ are twofold degenerate exhibiting the full symmetry in an

appropriate linear combination. The use of minimum waveguide sector allows faster computation because only a quarter or one sixth of the waveguide needed for computation. [79][82][83][86].

We have calculated the effective index for a HF with $d = 5\mu m$, $\Lambda = 6.75\mu m$, $n = 1.45$, $\lambda = 1.45\mu m$ [79]. The fundamental mode is twofold degenerate, with effective index equal to 1.4453959. The axial field $|E_z|$, $|H_z|$ and modal intensity profile are plotted.

Figure 5.18 Modal property for degenerate fundamental mode (mode class p=3).

Figure 5.19 Modal property for degenerate fundamental mode (mode class p=4).

With reference to Figure 5.17, it is clearly understood that mode shown in Figure 5.18 belongs to class $p=3$, and mode shown in Figure 5.19 belongs to class $p=4$. Modal property of such a mode class can be completely determined by $\pi / 2$ sector.

In conclusion, Symmetry analysis enables one to classify the possible modes into mode classes, predict the mode degeneracy between mode classes and specify minimum waveguides sectors for each mode class, which completely determine the modes of that mode class.

5.5 Confinement loss of HF

Optical losses of HF have been relatively high compared with those of conventional single-mode fiber, which exhibit optical loss of 0.2 dB/Km at $1.55\mu\text{m}$ wavelength. High optical loss of HF is mainly due to following factors: imperfection loss induced by variation in hole positions and hole size along the fiber, scattering loss caused by the surface roughness of the holes, absorption losses caused by contamination in the holes, confinement loss and the material's intrinsic losses. Here, only the confinement loss is studied.

Unlike the truly bound modes of conventional fibers, the modes of HF are leaky due finite number of holes. Physically, the light is trapped by enclosure of air holes. The confinement loss arises from energy leakage between and through the holes. Computationally, confinement loss is modeled by finding the leaky mode solution corresponding to a complex effective index.

$$\text{Confinement loss} = 8.686 \times \text{Im}(n_{\text{eff}}) \times 2\pi \times 10^6 / \lambda \text{ dB/m} \quad (5.11)$$

where λ is in μm .

Multipole method [120] has been successfully used to accurately calculate the confinement loss, but it is restricted to structures with circular holes only. BEM based on Green's function is able to calculate confinement loss accurately without any artificial absorbing boundary, which attempts to minimize any back reflection and make the boundary appear as transparent as possible.

We consider HFs with two rings of 18 air holes and three rings of 36 air holes in a hexagonal lattice, where $\Lambda = 2.3\mu m$, $n = 1.45$. The wavelength dependence of real part of effective index for different air hole size is shown in Figure 5.20.

(a) Two rings of air holes

(b) Three rings of air holes

Figure 5.20 Wavelength dependence of the real part of effective index of the fundamental mode in HFs.

It is observed that the real part of the effective index is almost independent of the number of arrays of air holes if the ratio d/Λ is fixed. For example, at $\lambda = 0.8\mu\text{m}$, the difference of effective index between HFs ($d/\Lambda = 0.5$) with one and two rings of air holes is $4.28\text{e-}006$, and the difference is only $1.04\text{e-}009$ between HF with two and three rings of air holes.

The corresponding confinement loss of HF as described above is shown in Figure 5.21.

(a) Two rings of air holes

(b) Three rings of air holes

Figure 5.21 Wavelength dependence of confinement loss of HFIs.

We conclude that confinement losses have been drastically reduced by introducing additional rings of air holes or by enlarging the air hole size. This is easy to understand because it is the low index holes that provide the confinement mechanism. It is also evident that the loss increases smoothly with wavelength, with no evidence of any critical physical dimensions changing the physics of the energy leakage. This wavelength dependence of the loss phenomenon is consistent with the decrease of the real part of the effective index with wavelength (Figure 5.20), indicating that light is less confined.

5.6 Effective mode area

Effective area is a very important design parameter. It is used as a measure of the nonlinearity of an optical fiber. A small effective area gives high power density needed for nonlinear effects. The effective area is also important in analysis of confinement loss, bending loss, splicing loss and numerical aperture. Effective area [22] is defined as

$$A_{eff} = \frac{(\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy)^2}{\int \int_{-\infty}^{\infty} |F(x, y)|^4 dx dy} \quad (5.12)$$

where $F(x, y)$ is modal field distribution.

As well known, the modal profile of fundamental mode in step index fiber can be well approximated by Gaussian shape, i.e., $F(r) \propto e^{-(r/w)^2}$. For such a mode profile, the effective area is $A_{ff} = \pi w^2$. However, the mode profile in HF is considerably deformed by the existence of air holes as shown in Figure 5.22, where the HF has $d/\Lambda = 0.4$, $\Lambda = 2.3\mu m$, $n = 1.45$. The field penetrates more into cladding at longer wavelength.

Figure 5.22 Intensity profile of fundamental mode in a HF at different wavelength.

So it is necessary to adapt the second moment of the optical intensity distribution for the definition of modal spot size for HF [121][122][123]. The modal spot size in x and y direction are given by:

$$w_x^2 = \frac{4 \int \int_{-\infty}^{\infty} (x - x_c)^2 |F(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy}; w_y^2 = \frac{4 \int \int_{-\infty}^{\infty} (y - y_c)^2 |F(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy} \quad (5.13)$$

where the center of the field profile (x_c, y_c) is given by:

$$x_c = \frac{\int \int_{-\infty}^{\infty} x |F(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy}; y_c = \frac{\int \int_{-\infty}^{\infty} y |F(x, y)|^2 dx dy}{\int \int_{-\infty}^{\infty} |F(x, y)|^2 dx dy} \quad (5.14)$$

We have simulated the effective area (normalized by Λ^2) for different air hole size, as shown in Figure 5.23. Since effective area is almost independent of the number of air hole rings [124], the simulation is based on HF with three air hole rings. As expected, the mode becomes more confined, i.e., lower effective area, for increasing air hole size and reducing wavelength.

Figure 5.23 Normalized effective area of HF with different air hole size.

The modal spot size in x and y direction for HF mentioned above is also plotted in Figure 5.24.

Figure 5.24 Normalized modal spot size in x and y directions

The modal spot size in the x and y directions, w_x and w_y , are in general, different from each other. w_y is generally greater than w_x . This can be seen from the modal intensity plot in Figure 5.22. The difference is smaller at short wavelength with large air holes. It is because the field is more confined in the central core at short wavelength or with large air holes.

5.7 Birefringent HF

Holey fiber technology offers new possibilities of realizing highly birefringent fibers due to intrinsic large index contrast compared to conventional fibers and unique stack-and-draw process. It is possible to create modal birefringence of at least one order of magnitude larger than that for conventional high birefringent fibers. Here, we present theoretical analysis on birefringence of a simple HF design: dual core HF.

The schematic of a dual core birefringent HF is shown in Figure 5.25, where the dots represent the nodes of element. For the subsequent analysis, the hole-to-hole distance Λ is fixed at $2.3\mu\text{m}$, silica index is also fixed at $n = 1.45$.

Figure 5.25 Meshed cross-section of dual core birefringent holey fiber.

The asymmetrical core is formed by two missing air holes. This is easily achievable using stack-and-draw technique. Such a structure no longer possesses the C_{6v} symmetry, and the fundamental modes are no longer degenerate. And the

birefringence, which is defined as $|n_x - n_y|$, could be much larger than conventional birefringent fiber. Current polarization maintaining fiber (PMF), such as PANDA or Bow-tie fibers, achieve birefringence by applying stress to the core region of a standard fiber, thereby creating a modal birefringence up to 5×10^{-4} [125][126][127]. Geometrical birefringence of HF could be much larger. For example, for HF with $d/\Lambda = 0.6$, $\lambda = 2.0 \mu m$, the x-polarized mode HE_{11}^x has an effective index of 1.415824, while the y-polarized mode HE_{11}^y has an effective index of 1.414313. The calculated birefringence is 1.511×10^{-3} . It is possible to increase the birefringence still further through creating more asymmetrical core.

The transverse electrical field vector and the contour plot of modal field (for HE_{11}^x) are given in Figure 5.26. Modal field for HE_{11}^y is similar to that for HE_{11}^x . The field penetrates more into the hole closest to the center of the fiber.

(a) Electrical field vector for HE_{11}^x

(b) Electrical field vector for HE_{11}^y

(c) Contour plot of modal field with 1dB spacing

Figure 5.26 Modal property of dual-core birefringent HF.

We have simulated guided modes for HF with $d/\Lambda = 0.5$ and the effective cladding index (Figure 5.27 (a)). It is observed that the fiber supports at least second order mode at short wavelength with a cut off at $\lambda/\Lambda = 0.887$. The number of modes that can exist in a waveguide may be conveniently represented in terms of a normalized propagation constant b [128] defined by $b = (n_{neff}^2 - n_{FSM}^2)/(n_{silica}^2 - n_{FSM}^2)$ (see Figure 5.27 (b)).

(a) Modal index

(b) Normalized propagation constant b

Figure 5.27 Illustration of the cut off wavelength of the second order mode for birefringent HF.

Here, the lifted degeneracy of the two polarization states is apparent. Wavelength dependence of birefringence of HF with different air hole size is show in Figure 5.28.

Figure 5.28 Wavelength dependence of birefringence.

The second mode cut off is also marked in Figure 5.28 with a circle. It is observed that the maximum birefringence occurs at single mode operation region. At longer wavelength, the field extends far into the cladding. In this case, the asymmetric core has a vanishing influence on the polarization splitting, making the birefringence small. On the other hand, birefringence becomes vanishing at shorter wavelength. Hence, there is an optimum wavelength window for the design of high birefringent fibers. The above simulation is performed on HF with 2 rings of air holes. The HF with $d/\Lambda = 0.6$ exhibits maximum modal birefringence $\Delta n = 3.731 \times 10^{-3}$ at a normalized wavelength $\lambda/\Lambda = 1.607$. The information about the optimum normalized frequency is an important design parameter, which provides information on the exact fiber dimensions of optimized high birefringent fibers.

To study the influence of number of air hole rings on the birefringence, we have simulated birefringence of HFs with two and three air hole rings.

Figure 5.29 Birefringence of HFs with different air hole rings.

As it can be seen from Figure 5.29, the difference of the birefringence is almost negligible at short wavelength. But HF with more air hole rings has larger maximum birefringence at longer wavelength. So the inner two rings of air holes have more impact on the birefringence at short wavelength. However at longer wavelength, as the field penetrates more into cladding, the outer rings of air holes will enhance the birefringence.

Confinement loss has been studied in Section 5.5 for the HF with hexagonally arranged air hole. The fundamental modes of such HF are degenerate. However, the fundamental modes of birefringent fiber has two states, namely, HE_{11}^x and HE_{11}^y . The confinement losses of these two states are studied here.

For reference, simulation also includes normal HF with single missing air holes.

Figure 5.30 shows both the real part of effective index and confinement loss.

(a) Real part of effective index

(b) Confinement loss.

Figure 5.30 Polarization dependent confinement loss of birefringent fiber.

The real part of the effective index of the slow-axis mode is slightly larger than that of the fast-axis mode, as shown in Figure 5.30 (a). The normal HF with the same structure parameters has much smaller effective index than that of birefringent fiber due to a smaller core. In the normal HF, the two fundamental modes are degenerate, and so the confinement losses are independent of the polarization states. For birefringent fiber, on the other hand, it is confirmed that the confinement losses depend on the polarization state and that the fast-axis mode has larger loss. The loss of normal HF is larger as the field is less confined (a smaller effective index).

The mode spot size in x and y direction as well as effective mode area are also studied for the dual core birefringent fiber. The corresponding half divergence angle θ can be estimated as $\theta \cong \tan^{-1}(\lambda / \pi w)$ [129].

Figure 5.31 Normalized effective mode area of dual core birefringent HF.

Figure 5.32 Normalized mode spot size of dual core birefringent HF.

Figure 5.33 Half divergence angle of dual core birefringent HF.

Similar to normal HF, the mode area is larger with small air hole at longer wavelength as shown in Figure 5.31. The spot sizes at x and y directions are considerably

different from each other as expected. The effects of varying air holes size on the mode spot size and divergence angle along the y direction (i.e., w_y and θ_y) are slightly larger than that along the x direction (w_x and θ_x), as showing in Figure 5.32 and 5.33.

In conclusion, we have studied the characteristics of a dual core birefringent fiber, such as its modal property, single mode operation, wavelength dependent birefringence, polarization dependent confinement loss, effective mode area, mode spot size and half divergence angle.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

In this thesis, we present a novel boundary element method and demonstrate its applications in modeling holey fiber.

Boundary element method is a direct, space-domain-division approach similar to finite element and finite difference methods. However, it needs only to discretize the boundary, hence reducing the dimension of the problem by one and producing a much smaller number of algebraic equations. Additionally, it is able to solve open domain optical structures without imposing any artificial boundary conditions. As a result, boundary element method is well suited for modeling holey fiber, which is an open waveguide domain composed of a small number of homogeneous regions.

We have developed an efficient algorithm to study various aspects of a holey fiber, such as its fundamental space filling mode, the analogy between HF and step index fibers, symmetry and degeneracy, confinement loss, effective mode area and birefringence of holey fiber.

Study of the fundamental space filling mode not only facilitates our understanding of the holey fiber but also allows the application of conventional theories of fiber

propagation and dispersion to the study of holey fiber. The knowledge of the effective cladding index is proven to be crucial in studying HF, especially in mode cut off.

Analogy between HF and step index fibers is examined by studying the effective core radius of a holey fiber.

Symmetry analysis enables one to classify the possible modes into mode classes, predict the mode degeneracy between mode classes and specify minimum waveguides sectors for each mode class, which completely determine the modes of that mode class. Theoretically, HF with hexagonally arranged air holes possesses symmetry group C_{6v} , hence its fundamental modes are degenerate. It is also numerically proven that holey fiber is not birefringent.

Wavelength and structural dependence of confinement loss is also studied. It is found that additional air hole rings does not change the real part of effective index very much, but does reduce the confinement loss drastically, especially for air holes close to the core.

The mode profile in HF is considerably deformed by the existence of air holes. Structural and wavelength dependence of effective mode area is studied.

Finally, the structural dependence of the birefringence of a dual core HF is studied. Additionally, its modal property, single mode operation, wavelength dependent birefringence, polarization dependent confinement loss, effective mode area, mode spot size and half divergence angle are also studied.

Boundary element method with its unique advantage is found to be accurate and efficient in modeling holey fiber.

6.2 Recommendations

We propose a few aspects for future research:

1. To estimate the effect of using higher order element on computation cost and improvement of accuracy.
2. To apply BEM in studying high birefringence HF with different structures.
3. To apply BEM in three-dimensional optical problem.
4. To investigate the impact of using hybrid BEM-FEM scheme in holey fiber modeling.

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