Distributed optimization for cognitive radio networks using game theory

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Distributed Optimization for Cognitive Radio Networks Using Game Theory

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Abstract

The focus of this thesis is to establish a game theoretic framework for cognitive radio (CR) wireless networks in which each user could adapt the parameters and operations according to the dynamic environment and its past experience to achieve optimal performance of itself as well as the equilibrium point of the whole network. By combining recent results in information theory, wireless network, control theory, and artificial intelligence, we establish a series of distributed learning algorithms for each user to improve its performance without causing adverse effects on others.

We first introduce a simple distributed power control algorithm for the spatial spectrum sharing based multi-user CR network. In this network, maximizing the performance of the unlicensed users, called secondary users (SU), without violating the power constraints imposed by the licensed users, called primary users (PU), is a very important task. It is proved that, by using the proposed algorithm, each SU could iteratively improve its performance to approach a Nash equilibrium (NE). In addition, our setting does not require each SU to know the global channel state information as well as communicate with other SUs. It is proved that the proposed algorithm approaches to a ϵ -NE at a rate of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. To further improve the convergence speed, we also propose a sub-optimal algorithm which could approach a neighborhood of a NE at a speed of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$.

We next investigate the performance of a general multi-user CR network when

SUs are allowed to cooperate with each other. This work is motivated by the recent study which shows that the user cooperative network has a great potential to improve the performance of wireless networks. We derive the optimal power allocation methods for SUs under different assumptions and pricing functions. The conditions under which the optimal NE is obtained when all SUs use multi-hop relaying are discussed. Our results are extended into large multi-user CR networks with K source-to-destination pairs. Two distributed algorithms are proposed. The first one is a sub-gradient based power allocation algorithm in which SUs can iteratively adjust their transmit powers to approach the payoff of a NE. The other one is a reinforcement learning based relay selection algorithm which enables each SU to iteratively search for a NE-achieving relaying scheme.

We conclude by modeling the OFDMA based cooperative CR network as a hierarchical Stackelberg game in which multiple secondary users (SU) compete for the available sub-bands in the spectrum of multiple primary users (PU). In this model, PUs set the price for accessing the spectrum, and SUs optimize their transmit powers and the sub-band allocation schemes to maximize the benefits of the given price. We focus on maximizing the payoff of both SUs and PUs by jointly optimizing the three important parameters: transmit powers of SUs, sub-band allocations of SUs, and the prices of PUs. To further improve the performance of SUs, we allow SUs who share the same sub-band to cooperate with each other to send and receive signals. We first formulate the coalition formation game to study the sub-band allocation problem of SUs and introduce a simple distributed algorithm for SUs to search for the optimal sub-bands. We then establish a Stackelberg game-based hierarchical framework. We prove that the transmit powers and sub-band allocation of SUs and the prices of PUs are connected by the pricing function of PUs. This makes the joint optimization possible. We prove that if the price coefficients of PUs have the linear relationship, the sub-band allocation of SUs will be stable and

the Stackelberg equilibrium of the hierarchical game framework will be unique and optimal. A simple distributed algorithm has been proposed to achieve the NEs of all the above three games. More importantly, we prove that by allowing PUs to linearly decrease their prices, the NEs of the above three problems as well as the Stackelberg equilibrium of the hierarchical game framework can be obtained. Our algorithm does not require SUs to know the interference temperature limit of each PU, and has low communication overheads between SUs and PUs.

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List of Symbols

S_k	The k th secondary source
D_k	The k th secondary destination
P_j	The j th primary user
$\mathcal{J}$	the set of all PUs
$\mathcal{K}$	the set of all secondary source-to-destination pair / grand coalition of PUs
K	Number of SUs
J	Number of PUs
w_k or w_{ij}	Transmit power of S_k or Transmit power used by S_i to send/relay the signal of S_j
$\mathbf{w}$	Transmit power vector of SUs
h_{S_i,D_k}	Channel gain between S_i and D_k
h_{S_i,P_j}	Channel gain between S_i and P_j
$\mathbf{H}$	Channel gain matrix

q_j or $\bar{q}_j$	Interference temperature limit of P_j
$\underline{q}_j$	SNR wall of P_j
r	revenue/benefit function
c	cost function
π	payoff function
$\boldsymbol{\pi}$	payoff vector
$E(\cdot)$	Expected value
$\boldsymbol{l}$	Sub-band allocation scheme of SUs
$\mathcal{C}$	coalition
R	Channel capacity
l_{S_k}	Sub-band of S_k
β	pricing coefficient of PUs

List of Abbreviations

Cognitive radio (CR)

Channel State Information (CSI)

Cooperative Spectrum Sensing (CSS)

Frequency-division (FD)

Nash equilibrium (NE)

Orthogonal frequency-division multiple access (OFDMA)

Primary user (PU)

Proportional fairness (PF)

Quality of service (QoS)

Secondary user (SU)

Signal to noise ratio (SNR)

Chapter 1

Introduction

1.1 Multi-user Wireless Networks

Future generations of wireless systems not only have to meet the increasing demand of high-speed data services, such as multimedia transmission, video conference, etc., but also need to support various types of communication services with different qualities [1]. However, improving the performance of wireless networks in today's resource limited environments is a challenging task for the following reasons:

- (a) Traditional framework for the infrastructure based wireless networks mainly focuses on improving the performance of one source-to-destination pair without considering the interaction among mobile terminals. Such a framework cannot be directly applied into multi-user wireless networks. For example, increasing the transmit power of one user not only improves its transmission rate but also raises the interference noise levels of others. Hence, traditional methods cannot be directly applied to find the optimal transmit power of each user in a general multi-user network,

- (b) Wireless environment is time variant. Although many adaptive schemes have been proposed for optimizing the transmit parameters of wireless networks, they still lack a simple and effective algorithm to achieve the optimal performance of each user as well as that of the entire network,
- (c) Most of the existing study on the optimization problem of wireless networks assumes that each user can know the global information, i.e., strategies and states of other users, or that a central controller is installed to manage the operations of all users in the network. However, exchanging information among users requires huge communication overheads and may cause intolerable delay, especially for large networks which may not be suitable for many practical scenarios.

In a multi-user wireless network, mobile users compete or cooperate for the limited resource in the wireless environment. This makes game theory a desirable tool to investigate problem (a). One of the main advantages of game theory is its capability to study the situations in which decision makers, i.e., users, interact. Extending game theory into the wireless network could help us understand the mobile systems from a more “networked” perspective in which the performance of each user is the result of interactions among all users.

For problem (b), recent observations suggest that, although many new methods are introduced to further improve the performance of wireless networks, no single method can provide the best performance in every condition. This motivates our work to study an “intelligent” network in which the mobile users can have the ability to learn from the environment and their past experience to distributively choose different optimal strategies under different conditions. For example, a report published by the FCC [2] showed that, on one hand, a large amount of spectrum has been allocated to support the increasing demand of wireless services, and on the

other hand, however, most of the allocated spectrum is under-utilized for most of the time. Hence, a new technology, called cognitive radio (CR), has been introduced to improve the utilization efficiency of the spectrum resources in which users should first sense the spectrum of the environment and then send signals only on the under utilized frequency bands. Another example is the user cooperation network [3–7] in which users cooperate with each other to send and receive signals. It is observed that the user cooperation method provides both flexibility and complexity to the network implementation, and it cannot provide any performance gain without proper settings and operations [8–12]. The above two examples show that a self-learning “intelligent” network could revolutionize the traditional wireless network framework and has great potential in the future generation of mobile systems.

Although the intelligent network is a promising technology for the future wireless system, at present, many challenges need to be solved before its practical implementation. In this thesis, we mainly focus on the CR network. In this network, the performance of each SU (or PU) is not just affected by other SUs (or PUs), but is the result of SUs and PUs in the entire CR network. In other words, the interaction between SUs and PUs differentiate the CR network from the traditional network and hence make the traditional framework impossible to be applied to this network. In this thesis, we will take advantage of the recent advances in game theory, machine learning and control theory to establish a hierarchical game theoretic framework to study the interaction between PUs and SUs in a wireless network supporting dynamic spectrum sharing.

Figure 1.1: Network Model for Multi-user Cooperative Cognitive Radio Networks: A cell phone network shares the spectrum with a Wi-Fi network.

1.2 Game Theory and Its Application in Cognitive Radio Networks

Game theory is a branch of social science which focuses on investigating the situations where the decision makers (or players) interact [13–16]. This makes game theory a useful tool to solve many multi-user network related problems, like channel selection [17–20], power control [21–25], user cooperation analysis [26–32], bandwidth allocation [33] and coding optimization [34, 35] etc. One of the main targets of game theory is to find a balancing point of a game, called the Nash equilibrium (NE). However, it was shown that finding a NE is generally intractable even for two-player cases, i.e., there may exist more than one NE for multi-user networks and, currently, there is still a lack of effective algorithms to find the optimal NE for general games. Furthermore, the NE is generally not optimal (i.e., Pareto optimal)

for many systems. All these problems make establishing appropriate game theoretic frameworks for different types of wireless networks a very challenging task.

This thesis focuses on using game theory to solve problems in a general multi-user CR network. In this network, multiple SUs compete for limited resources owned by the PUs. A general setting is to assume two types of networks, i.e., the unlicensed network and the licensed one, trying to access the same spectrum. Users in the licensed networks (PU) are the owners and hence have the priority of sending signals in the spectrum. Users in the unlicensed network (SU), however, can only send signals on the under-utilized portion of the licensed spectrum. A simple example is shown in Figure 1.1 in which cellular service subscribers try to access the spectrum of Wi-Fi networks if this spectrum is not fully utilized by Wi-Fi devices. System modeling and optimization are more complex in CR networks than in the traditional multi-user networks because, in a CR network, both the interaction among SUs or PUs and that between PUs and SUs could greatly affect the overall system performance. In this thesis, we will establish a hierarchical game theoretic framework to study the interaction between PUs and SUs in a wireless network supporting dynamic spectrum sharing. In this framework, SUs or PUs are selfish users who try to adjust their parameters to maximize the performance, and hence the competition or cooperation among SUs or PUs is also closely related to finally achieved performance.

Two main spectrum sharing techniques were introduced in CR networks. The first technique, called temporal spectrum sharing (TSS) (or spectrum underlay [36], dynamic/opportunistic spectrum access [18, 25, 37–40] etc.), allows SUs to exploit the temporal vacancy of PU networks. The main problem for TSS based CR networks is that SUs need to continuously sense the availability of PUs even when no spectrum can be exploited. Most of the previous research focuses on simplifying the

sensing tasks of SUs by assuming that some information on the spectrum utilization information of PUs is known. For example, in cooperative spectrum sensing (CSS) methods, [41] [10] [42], the spectrum width of each SU is assumed to be much narrower than that of each PU and hence multiple SUs who occupy different sub-bands of one PU can cooperatively detect the occupancy of the licensed spectrum. The main challenge for CSS is to minimize the communication overhead among SUs. If the spectrum width of each SU is equal to or larger than that of each PU [43], each sub-band of the licensed spectrum can have more than one PU and hence SUs will compete for empty or highly under-utilized sub-bands. In this system, finding the best sub-band allocation scheme is very important. Traditional study in sub-band allocation requires communication (i.e., competitions or cooperations) among SUs [37] which is not possible for many practical systems.

In the second technique, called spatial spectrum sharing (SSS) [44] [45] [46] (or spectrum overlay [36], dynamic spectrum sharing [47], etc.), SUs and PUs can transmit signals at the same time over the same spectrum. Our approach is to take advantage of the spatial vacancy of PUs and variations of channel fading coefficients [48]. More specifically, in SSS based CR networks, the PU network can tolerate a small increase of interference level without causing much performance degradation and SUs need to control their transmit powers to maintain the total interference power of each PU to be lower than an acceptable level, called the interference temperature limit [49]. In other words, optimal power control is one of the central problems in a SSS system. Existing studies [44] [50] require PUs to inform SUs of their highest tolerable interference powers and a central controller has to be built to manage the operations of all SUs. A distributed power optimization method was proposed for SUs in [51] by allowing PUs to use a price function to manage the operations of SUs.

1.3 Main Contributions

The main contributions of this Thesis are summarized below.

- 1) In Chapter 3, we focus on the power control problem for SSS based CR networks with multiple secondary source-to-destination (SD) pairs. A simple distributed algorithm is proposed for the SUs to iteratively adjust the transmit powers to improve their performance. The proposed algorithm does not require each SU (or PU) to negotiate with other SUs (or PUs) during the communication. It is proved that the proposed algorithm can obtain a time average performance as good as that achieved when the NE is chosen in hindsight. More specifically, the average performance of CR networks will converge to an ϵ -NE at a rate of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. A sub-optimal algorithm is also introduced to converge to the neighborhood of a NE at a rate of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$. Numerical results are presented to show the performance of the proposed algorithms under different settings.
- 2) In Chapter 4, we consider SSS based multi-user CR networks with multi-hop relaying. Although recent results have shown that multi-hop relaying has great potential to improve the performance of CR networks, finding effective methods to control and manage SUs to achieve the optimal performance is still a challenging problem. In this Chapter, we model the CR network as a non-cooperative game in which each SU obtains benefits through both spectrum sharing by paying prices to PUs and multi-hop relaying by paying prices to nearby SUs. Optimal power allocation methods for SUs are investigated under different assumptions and pricing functions. The conditions under which the optimal NE is obtained when all SUs use multi-hop relaying are discussed. Our results are extended into large multi-user CR networks with K source-to-

destination pairs. Two distributed algorithms are proposed. The first one is a sub-gradient based power allocation algorithm in which SUs can iteratively adjust their transmit powers to approach the payoff of a NE. The other one is a Q-learning based relay selection algorithm which enables each SU to iteratively search for a NE-achieving relaying scheme.

- 3) In Chapter 5, we consider the OFDMA-based cooperative cognitive radio (CR) networks where multiple secondary users (SU) compete for the available sub-bands in the spectrum of multiple primary users (PU). We focus on maximizing the payoff of both SUs and PUs by jointly optimizing the three important parameters: transmit powers of SUs, sub-band allocations of SUs, and the prices of PUs. To further improve the performance of SUs, we allow SUs who share the same sub-band to cooperate with each other to send and receive signals. To help us understand the interactions among SUs and PUs, we study the proposed network model from a game theoretic perspective. More specifically, we first formulate the coalition formation game to study the sub-band allocation problem of SUs and introduce a simple distributed algorithm for SUs to search for the optimal sub-bands. We then establish a Stackelberg game-based hierarchical framework. We prove that the transmit powers and sub-band allocation of SUs and the prices of PUs are connected by the pricing function of PUs. This makes the joint optimization possible. More importantly, we prove that if the price coefficients of PUs have the linear relationship, the sub-band allocation of SUs will be stable and the Stackelberg equilibrium of the hierarchical game framework will be unique and optimal. A simple distributed algorithm has been proposed to achieve the NEs of all the above three games. More importantly, we prove that by allowing PUs to linearly decrease their prices, the NEs of the above three problems as well as the Stackelberg equilibrium of the hierarchical game framework can be obtained. Our algorithm

does not require SUs to know the interference temperature limit of each PU, and has low communication overheads between SUs and PUs.

Chapter 2

Background on Game Theory

2.1 Basic Elements in a Game

The main objective of the game theory is to study the behavior of a group of individuals in which the decision one makes depends on those of others. Generally speaking, a game should contain following elements,

- 1) A set of decision makers, called *players*, labeled as $i \in \{1, 2, \dots, K\}$,
- 2) A set of *strategies*, denoted as s_i for player i ,
- 3) *Information* which is known by each player to be used to make the decision,
- 4) *Revenue*, denoted as r_i , for the i th player is the benefit earned by choosing certain strategies,
- 5) *Cost*, denoted as c_i , paid by player i is the price paid to using certain services,
- 6) *Payoff*, denoted as π_i , of each player is the difference between its revenue and price,

7) *States* are the states of each player under certain decisions.

The names or meaning of the above elements may be different in different systems. For example, let us consider the power control game in a wireless network. In this case, we can define the players to be the mobile devices who need to control their transmit power, and the information is the statistic or instantaneous information about the channel gains. The revenue could be defined as the final performance improvement achieved by using the power control schemes. The cost can be the money charged by the telecommunication operator. And the state could be the current transmit power of each device. In addition, some systems may not have all the elements described above. For example, in the jamming game, the malicious players may like to prevent the transmission of the normal players. In this game, we can eliminate the cost and each player can only get benefits when its target is reached.

2.2 Static Games

A static game (also called one-stage game) is a simple game theoretic model in which each player can only pick up one move/decision. we also define the set of all the strategies for one player as the *pure strategy space*. Each element in the pure strategy space is called a *pure strategy*. Each player can also choose *mixed strategies* in which different strategies can be chosen by different probabilities. If one strategy (i.e., s_i) can provide a higher payoff than another (i.e., s'_i), then strategy s'_i is said to be *dominated* by s_i .

2.3 Dynamic Games

In a dynamic game, each player makes decisions in a sequential way. In other words, the process of a game can be divided into many stages. In each stage, a player chooses its next move by using the information obtained in the current and previous moves. If we assume a player knows all the previous moves, the player is said to have the *perfect information* about the game. If the play can only have limited access to the history of the game, the player is said to have the *imperfect information* about the game.

2.4 Nash Equilibrium

For most of the games, the main objective is to find a steady state in which each player cannot achieve a higher payoff by choosing a different strategy, given the strategies of other players. We call this state as *Nash equilibrium* (NE) which is defined formally below,

Definition 2.4.1. [52, Definition 23.1] *A strategy profile s^* is a NE if, for every player i and every strategy s_i for s^* , $s_i \in \mathcal{S}$ and $i \in [1, K]$, s^* is at least as good as the strategy profile (s_i, s_j^*) in which player i chooses s_i while every other player chooses s^* , i.e., for player S_i , $\pi_i(s_i^*, s_{-i}^*) \geq \pi_i(s_i, s_{-i}^*)$, where subscript $-i$ denotes all players except S_i .*

NE considers a balanced strategy profile in which no player will unilaterally change its strategy. However, for a multi-player game, more than one such balanced points can always be found. In this case, how to characterize the performance of different NEs is an very important problem. If one NE, denoted as $\mathbf{s}$, will result in better or equal payoffs for all players (and at least one player achieving a strictly

better payoff) than in another NE, denoted as $\mathbf{s}'$, then $\mathbf{s}$ is said to be *Pareto superior* than $\mathbf{s}'$, where $\mathbf{s} = [s_1, s_2, \dots, s_K]$. If there exists no other strategies that are Pareto superior to $\mathbf{s}^*$, we call $\mathbf{s}^*$ to be the *Pareto optimal* strategy of the game.

In the rest of this Thesis, we will try to apply the game theoretic concepts and models discussed above into CR networks.

Chapter 3

A Simple Distributed Power Control Algorithm for Cognitive Radio Networks

3.1 Introduction

In this chapter, we focus on SSS based CR networks. It was observed in [50] that, if SUs employ the power control methods, i.e., to decrease (or increase) their transmit powers when the conditions of SU-to-PU channels are “good” (or “poor”), the spectrum utilization and system performance can be greatly improved. However, most of the previously reported work neglects the interference among SUs or PUs and assumes each SU can have the global information, i.e., the states of other SUs and the gains of all channels in the network. This requires each SU to handle complex computation and unrealistic channel estimation, which are impossible for many practical systems. More specifically, the optimal power control methods were studied in [50] [44] for the case of one secondary source-to-destination (SD) pair sharing

the spectrum with one PU. The optimal transmit powers of SUs were derived by assuming the transmitters knowing the instantaneous channel gains of both SU-to-SU and SU-to-PU channels. The results were extended to CR networks with multiple SD pairs [45] and multi-hop relaying SUs [53] based on the similar assumptions.

Motivated by these limitations, a game theoretic framework is established in this chapter to solve the power control problem of distributed CR networks with multiple secondary SD pairs and PUs. A simple distributed algorithm is proposed to achieve the following advantages: 1) the operations of each SU are simple and fully distributed, i.e., both SUs and PUs do not know the global information, and there is no central controller to manage the network. In addition, each PU in our setting broadcasts the same information to all SUs and each SU cannot communicate with others or obtain specific instructions from PUs, 2) the computational complexity of each SU is low and does not depend on the number of SUs, 3) the proposed algorithm can be directly applied to other resource control problems, e.g., spectrum allocation, time scheduling, etc.

It is proved that the CR network with this simple algorithm can achieve a time average performance as good as that achieved when the NE is chosen in hindsight. More specifically, we show that by using the proposed algorithm, the average performance of each SU converges to an ϵ -NE at a rate of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. This is a surprising result because finding the NE for a multi-user network is generally a challenging task even when the number of users in the network is small [54, Chapter 2], and most of the previously reported NE-approaching algorithms only focus on the instantaneous performance and require more stringent conditions than ours. To further improve the convergence rate, a sub-optimal algorithm is also proposed to approach a neighborhood of the NE at a rate of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$. We discuss some potential extensions of our work and present numerical results to verify the

Figure 3.1: Network model for CR networks with multiple SD pairs.

performance of our algorithms for different applications.

The rest of this chapter is organized as follows. The network model and problem setup are discussed in section 5.2. The proposed algorithm and its convergence result are presented in Sections 3.3. Applications and numerical results are given in Section 3.4 and the chapter is concluded in Section 5.6.

3.2 Network Model and Game Formulation

Consider a CR network in which K secondary SD pairs, labeled as S_1 to D_1 , S_2 to D_2 , ..., S_K to D_K , simultaneously transmit signals in the same spectrum as that allocated to M PUs, labeled as P_1 , P_2 , ..., P_M as shown in Figure 5.1. Let the channel gain between S_i and D_j be h_{S_i, D_j} for $i, j \in \{1, 2, \dots, K\}$, and that between S_i and P_k be h_{S_i, P_k} for $i \in \{1, 2, \dots, K\}, k \in \{1, 2, \dots, M\}$.

In a practical system, P_k for $k \in \{1, 2, \dots, M\}$ can only maintain a certain level of QoS if its received interference power is lower than the interference temperature

limit, denoted by q_k . Let the transmit power of S_i be w_i . We can define the power constraints for SUs as follows:

$$\mathbf{H}_{PU}\mathbf{w}^\dagger \leq \mathbf{q}^\dagger, \quad (3.2.1)$$

where $\dagger$ denotes the transpose of a matrix, $\mathbf{w} = [w_1, w_2, \dots, w_K]$, $\mathbf{q} = [q_1, q_2, \dots, q_M]$, and $\mathbf{H}_{PU} \in \mathbb{R}^{M \times K}$ is given by

$$\mathbf{H}_{PU} = \begin{bmatrix} h_{S_1, P_1} & h_{S_2, P_1} & \cdots & h_{S_K, P_1} \\ h_{S_1, P_2} & h_{S_2, P_2} & \cdots & h_{S_K, P_2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{S_1, P_M} & h_{S_2, P_M} & \cdots & h_{S_K, P_M} \end{bmatrix}. \quad (3.2.2)$$

Based on the above notations, the signal to noise ratio (SNR) received by D_i can be written as

$$SNR_i = \frac{h_{S_i, D_i} w_i}{\sum_{\substack{j \in \{1, 2, \dots, K\} \\ j \neq i}} h_{S_j, D_i} w_j + \sigma_i}. \quad (3.2.3)$$

where if PUs keep sending signals during the SUs' communication, σ_i should contain both the transmit signals of PUs and the additive noise received by S_i . However, if PUs are absent when SUs transmit, i.e., as in the TSS based CR networks [37], σ_i should only contain the additive noise of S_i .

In this chapter, we use the game theoretic method to investigate the power control problem for CR networks. In our game, *players* are SUs who share the spectrum with PUs. We define the *revenue* of the i th SD pair, denoted by r_i , to be the benefit obtained by S_i and D_i from using the PUs' spectrum. The *price* paid by S_i , denoted by c_i , is defined as the price charged by PUs for using the licensed

spectrum. We also define the *payoff* of the i th SD pair, denoted by π_i , to be the difference between the revenue and price, i.e., $\pi_i = r_i - c_i$. In this chapter, we assume the revenue and payoff of S_i to be the function of SNR_i defined in (3.2.3).

In this chapter, a finitely repeated game model [55] is applied in which each SU plays the game repeatedly. By using subscript $[t]$ to denote the parameters and operations in the t th period, the main objective of the i th SD pair is to maximize its time average payoff $\frac{1}{T} \sum_{t=1}^T \pi_i(\mathbf{w}_{[t]})$ over T periods of transmission without causing the adverse effects on other SD pairs. This is different from most of the previously reported work in repeated game where each player only cares about the payoff of the last iteration, i.e., payoff in the T th time slot. Our setting has more practical meaning for CR networks because most mobile devices can tolerate a few short periods of “poor” performance if the average payoff is good.

It is assumed that each player cares the performance of the future as same as that of the present and hence the time discount factor [55, Definition 6.1.2] is 1. The result with other choice of the time discount factor can be similarly obtained. The main objective is to find a balance point of the entire network (i.e., NE) in which each SD pair cannot further improve its payoff by choosing a different transmit power, given the transmit powers of other SUs. More specifically, in this chapter, we try to find a strategy profile $\mathbf{w}^*$ such that, for every player i and every strategy w_i , w_i^* is at least as good as the strategy profile (w_i, w_{-i}^*) in which the player i chooses w_i while other players choose w_{-i}^* , i.e., for every player i ,

$$\pi_i(w_i^*, w_{-i}^*) \geq \pi_i(w_i, w_{-i}^*), \quad (3.2.4)$$

where subscript $-i$ means all the players except player i . We also define a strategy profile to be ϵ -Nash equilibrium (ϵ -NE), if this strategy profile is within the distance

of ϵ to the payoff achieved by a NE, i.e., the following condition is to be satisfied,

$$\pi_i(w_i^*, w_{-i}^*) - \pi_i(w_i, w_{-i}^*) \leq \epsilon \quad \text{for } \epsilon > 0. \quad (3.2.5)$$

Finding the NE of the multi-user network is difficult and intractable [54, Chapter 2]. In this chapter, we try to develop an algorithm that, during T time slots of repeated power control games, can achieve the average performance nearly as good as the system with the NE decision being chosen in hindsight, i.e., we have

$$\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \min_{l \in \{1, \dots, t\}} \|\pi(\mathbf{w}_{[l]}) - \pi(\mathbf{w}^*)\|_2^2 = 0 \quad (3.2.6)$$

where $\pi(\mathbf{w}_{[l]}) = [\pi_1(\mathbf{w}_{[l]}), \pi_2(\mathbf{w}_{[l]}), \dots, \pi_K(\mathbf{w}_{[l]})]$.

3.3 Main Results

Algorithm 1 is illustrated below.

Algorithm 1

- 1) *Initialization*: Let the transmit signal of S_i be x_i . Consider the transmission of the i th SD pair. During the time slots 1 and 2,
 - a) S_i sends signals $x_{i[1]}$ and $x_{i[2]}$ with powers $w_{i[1]}$ and $w_{i[2]}$, respectively,
 - b) After receiving the signal sent by the source S_i , the destination D_i observes its received SNR and feedbacks the revenue functions $r_{i[1]}$ and $r_{i[2]}$ in the first and second time slots, respectively. Each PU P_k also sends the prices $c_{k[1]}$ and $c_{k[2]}$ to S_i during the first two time slots.
- 2) *Iteration*: For $t = 3 : T$,

- a) In time slot t , P_k monitors its interference power $\sum_{i=1}^K h_{S_i, P_k} w_{i[t]}$. If the interference powers of all PUs satisfy (1), P_k only sends price $c_{i[t]}$ to S_i , and then S_i updates its transmit power as follows,

$$w_{i[t]} = (w_{i[t-1]} - \delta_{i[t-1]} g_{i[t]})^+, \quad (3.3.1)$$

where $(\cdot)^+ = \max\{0, \cdot\}$, $g_{i[t]}$ is the subgradient [56] of $\pi_{i[t]}$ which is a function satisfying

$$\begin{aligned} \pi_{i[t]}(\mathbf{w}_{[t]}) &\geq \pi_{i[t-1]}(\mathbf{w}_{[t-1]}) \\ &\quad + g_{i[t]}(\mathbf{w}_{[t]} - \mathbf{w}_{[t-1]}), \end{aligned} \quad (3.3.2)$$

and $\delta_{i[t-1]}$ is the step size of the t th iteration, defined as $\delta_{i[t-1]} = \frac{u_i}{t-1}$ where u_i is the step size coefficient which is a constant controlling the iteration speed,

- b) However, if the interference powers of some PUs exceed the interference temperature limit, each of these PUs should transmit more information to help SUs to adjust the transmit powers during the following iterations. More specifically, assuming a PU P_k observes a high interference power, i.e., $\sum_{i=1}^K h_{S_i, P_k} w_{i[t]} > q_k$, the operations of P_k , except for sending the price to SUs, are described as follows:

- i) P_k broadcasts a message to inform all SUs that its interference power exceeds the limit. S_i can decode this message and use it to obtain

$$h_{S_i, P_k},$$

- ii) Because we assume each PU monitors its interference power in each

iteration, P_k can calculate the following two constants in time slot t ,

$$I_{k[t]} = \sum_{i=1}^K h_{S_i, P_k} w_{i[t]} - q_k, \quad (3.3.3)$$

and

$$J_{k[t]} = \sum_{i=1}^K h_{S_i, P_k} w_{i[t]} - \sum_{i=1}^K h_{S_i, P_k} w_{i[t-1]}, \quad (3.3.4)$$

- iii) By assuming the channel gains are known by the receivers (i.e., assume a training code is involved in the transmit signals of SUs), P_k can exploit its received signal to calculate a constant

$$\Phi_{k[t]} = \sum_{i=1}^K h_{S_i, P_k}^2. \quad (3.3.5)$$

Each PU P_k broadcasts the constants $I_{k[t]}$, $J_{k[t]}$ and $\Phi_{k[t]}$ to all SUs,

- iv) S_i , knowing h_{S_i, P_k} (obtained in Step b-i)), updates its transmit power by

$$\begin{aligned} w_{i[t]} = & \left[w_{i[t-1]} - \delta_{i[t-1]} g_{i[t]} \right. \\ & \left. - h_{S_i, P_k} \Phi_{k[t-1]}^{-1} (I_{k[t-1]} - J_{k[t-1]}) \right]^+. \end{aligned} \quad (3.3.6)$$

If more than one PU detect higher-than-tolerate interference power, they should sequentially repeat the operations from Steps b-i) to b-iv) until the transmit powers of all SUs satisfy (1).

- 3) *Termination*: The above process continues until the obtained payoff is close to the optimal value within an acceptable range, i.e., $\|\pi_{i[t]}(\mathbf{w}_{[t]}) - \pi_{i[t]}(\mathbf{w}^*)\|_2^2 \leq \epsilon$, or the number of time slots reaches T , i.e., $t = T$.
-

The main idea of Algorithm 1 is that, if the transmit powers of SUs satisfy (3.2.1), each SU maximizes its payoff by using the subgradient method, as shown in Step 2-a). If any PUs detect the higher-than-tolerable interference, they will broadcast several constants, i.e., a PU P_k broadcasts Φ_k , I_k and J_k defined in Step 2) in Algorithm 1 to all SUs, and SUs use these constants to project their transmit powers into the convex hull defined in (3.2.1). Note that simply projecting the transmit powers of all SUs to (3.2.1) requires each PU to know the information of all other PUs, which does not meet the requirement that PUs cannot communicate with each other. Hence, in Algorithm 1, if more than one PU detect higher-than-tolerable interference power, they will sequentially inform SUs to project their powers to the corresponding linear equation in the convex hull defined in (3.2.1). This setting will not result in much adverse effects on the convergence performance of Algorithm 1 because if the transmit powers of SUs increase gradually and $T \gg K$, only a few PUs will first detect high-than-tolerable interference powers in each time slot. Therefore, the number of iterations used on the projection operations is negligible as the total number of iterations becomes large.

Theorem 3.3.1. *If the following assumptions*

- A1)** *The iteration step sizes are bounded, i.e., $\|g_{i[t]}\|_2^2 \leq g_i^+$, where g_i^+ is a constant,*
- A2)** *The power value changes between two iterations are bounded, i.e., $\|w_{i[t]} - w_{i[s]}\|_2^2 \leq w_i^+$ for $t \neq s$ and $t, s \in \{1, \dots, T\}$, where w_i^+ is a constant,*
- A3)** *The payoff for the i th SD pair is a continuous function of w_i ,*

are satisfied, then Algorithm 1 has the following properties:

- P1)** *It converges to a stationary point if T is large enough,*

P2) *If all SD pairs use Algorithm 1, the time average payoff of S_i converges to an ϵ -NE at a rate of $O\left(\exp\left(\frac{1}{\epsilon}\right)\right)^i$, i.e.,*

$$\left\| \frac{1}{T_\epsilon} \sum_{t=1}^{T_\epsilon} \boldsymbol{\pi}(\mathbf{w}_{[t]}) - \boldsymbol{\pi}(\mathbf{w}^*) \right\|_2^2 \leq \epsilon. \quad (3.3.7)$$

Proof 1. *See Appendix 3.6.1.*

Note that the convergence performance of Algorithm 1 is closely related to the combination of two important parameters: the initial value $w_{i[1]}$ of the transmit power and the iteration step size u_i . If $w_{i[1]}$ and u_i satisfies $|w_{i[1]} - w_i^*| \approx u_i$, Algorithm 1 could approach to the neighborhood of a NE within a few iterations. If $w_{i[1]}$ and u_i are improperly chosen, however, the convergence rate of Algorithm 1 will be very slow because we require w_i to reach w_i^* exactly without assuming any information about the systems to be known by SUs. The slow convergence performance of Algorithm 1 can be improved by assuming more information to be observed by SUs. For example, if we assume that S_i knows the payoff functions, or both the revenue and pricing functions, the optimization problem can be converted into the Lagrangian dual problem [57] which will be discussed in the next section.

Another way to improve the convergence rate of Algorithm 1 is to decrease the required accuracy of the results. In a practical digital system, mobile devices cannot adjust their parameters with an infinite accuracy, i.e., the value of w_i can only be chosen from a finite discrete set. In the rest of this section, we consider a simple case in which the transmit power of each SU is linearly quantized into L levels and w_i can only be chosen from this finite set, i.e., $w_i \in \mathcal{W}_i$ where $\mathcal{W}_i = \{0, \frac{w_i^+}{L}, \frac{2w_i^+}{L}, \dots, w_i^+\}$. Let us describe Algorithm 2 as follows.

ⁱIn this chapter, we follows Bachmann-Landau notations: $f = O(g)$ if $\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} < +\infty$.

Algorithm 2: Each SU adjusts its transmit power by using the exact same procedures as Algorithm 1. The only difference is that, in the t th iteration, the transmit power of S_i is $w_{i[t]} = l\hat{\delta}_i$ if $l\hat{\delta}_i - \frac{\hat{\delta}_i}{2} < w_{i[t]} < l\hat{\delta}_i + \frac{\hat{\delta}_i}{2}$ where $l \in \{1, 2, \dots, L\}$, $\hat{\delta}_i = \frac{w_i^+}{L}$ and $w_{i[t]}$ is calculated by using either (3.3.1) (if the power constraint in (3.2.1) is satisfied) or (3.3.6) (if (3.2.1) is not satisfied) in Algorithm 1.

The NE is a balance point where each player achieves a locally optimal solution and hence has no incentive to deviate from this point. In other words, we can always find a *neighborhood*, denoted as $\chi = [\chi_1, \chi_2, \dots, \chi_K]$, for the NE that all the elements in this neighborhood cannot achieve higher performance than that of the NE, i.e., $\pi_i(w_i^*, w_{-i}^*) \geq \pi_i(w_i^* \pm \Delta w_i, w_{-i}^*)$ where $0 < \Delta w_i < \chi_i$ for $i \in \{1, 2, \dots, K\}$. It is easy to observe that if $\mathbf{w}_{[t]} - \mathbf{w}_{[t+1]} > \chi$, Algorithm 2 may not converge to a NE. Let us summarize our observations and present the main result for Algorithm 2 as follows.

Theorem 3.3.2. *If $w_i \in \mathcal{W}_i$, $\chi_i \gg \frac{\hat{\delta}_i g_i^+}{2}$, $\forall i \in \{1, 2, \dots, K\}$ and Assumptions A1)-A3) in Theorem 3.3.1 are satisfied, Algorithm 2 approaches a $\chi = \left[\frac{\hat{\delta}_1 g_1^+}{2}, \frac{\hat{\delta}_2 g_2^+}{2}, \dots, \frac{\hat{\delta}_K g_K^+}{2}\right]$ neighborhood of a NE, i.e., $\left\| \frac{1}{T_{\epsilon'}} \sum_{t=1}^{T_{\epsilon'}} \pi \left(w_{i[t]}^* \pm \frac{\hat{\delta}_i g_i^+}{2}, w_{-i[t]}^* \right) - \pi(\mathbf{w}^*) \right\|_2^2 \leq \epsilon'$, at a rate of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$, where $\epsilon' = |\epsilon - \frac{\hat{\delta}_i g_i^+}{2}|$ and $\hat{\delta}_i = \frac{w_i^+}{L}$.*

Proof 2. See Appendix 3.6.2.

3.4 Applications and Numerical Results

In this section, the proposed algorithm is applied to the network utility maximization problem to evaluate its performance. We assume that each SU has an objective to maximize its achievable rate and define the utility function of S_i as $r_i(w_i) = \alpha_i \log(1 + \text{SNR}_i)$ where α_i is a constant. Since Algorithm 2 can be regarded as a special case of Algorithm 1, we only present the numerical results for

Algorithm 1.

Consider a special case where SUs are far from each other and assume $SNR_i \approx \frac{h_{S_i,D_i} w_i}{\sigma_i}$ and that PUs do not charge any prices for SUs if the transmit powers of SUs satisfy (3.2.1). It is observed that, in this case, the payoff function $\pi_i(\mathbf{w})$ of S_i has the following features: 1) it is continuous in $\mathbf{w}$, 2) it is concave in w_i , 3) it has a continuous first derivative with respect to w_i , and 4) there exists $r = \{r_1, r_2, \dots, r_K\} > 0$ such that $\sum_{i=1}^K r_i \pi_i(\mathbf{w})$ is diagonally and strictly concave. Following the same methods as in [58], we can claim that, for this CR network, the NE is unique and corresponds to the global utility maximization point [57]. In other words, if all SUs use Algorithm 1 to adjust their transmit powers, the average payoff of the network will converge to the global utility maximization point, defined as $\max_{\mathbf{w}} \frac{1}{T} \sum_{t=1}^T \pi_{[t]}(\mathbf{w}_{[t]})$ for $\pi_{[t]} = \frac{1}{K} \sum_{i=1}^K \pi_{i[t]}$ under the power constraint defined in (3.2.1).

To demonstrate the convergence performance of Algorithm 1, the results of the Lagrangian dual methods discussed in [57] are also provided. In this method, S_i iteratively chooses the optimal Lagrangian coefficients to maximize its payoff (see [57] for the detailed description of the dual method). In Figure 3.2, we compare the convergence rates of Algorithm 1 and the dual method. It is observed that, in general, the dual method converges to the optimal value at a faster rate than Algorithm 1. However, each SU in the dual method needs to know the payoff functions of all SUs and has to search for the optimal power to minimize the objective function in each iteration, which incurs much more computational complexity and communication overhead than those of our proposed algorithm. In addition, as is observed in Figure 3.2, if the initial values and step size are chosen optimally, the convergence rate of Algorithm 1 could outperform that of the dual methods.

Let us consider a more general case as follows. Define the revenue of S_i as $r_i = \alpha_i \log(1 + SNR_i)$ where SNR_i is given in (3.2.3). The pricing function charged

Figure 3.2: Comparison of different algorithms, where $M = 1, K = 10, q_k = 2$, for $i \in \{1, 2, \dots, K\}, k \in \{1, 2, \dots, M\}$.

Figure 3.3: Convergence rate of Algorithm 1 with different step sizes, where $w_{i[1]}$ is the least squares solution of $\mathbf{H}_{PU}\mathbf{w}^\dagger = \mathbf{q}^\dagger$, $\alpha_i = 10$, $\beta_i = 0.1$, $M = 5$, $K = 10$, $q_k = 5$, for $i \in \{1, 2, \dots, K\}$, $k \in \{1, 2, \dots, M\}$.

Figure 3.4: Convergence rate of Algorithm 1 with different initial values, where $w_{i[1]} = h_{S_i, P_k} \setminus q_k$ means $w_{i[1]}$ is the least squares solution of $\mathbf{H}_{PU} \mathbf{w}^\dagger = \mathbf{q}$, $\alpha_i = 10$, $\beta_i = 0.1$, $M = 5$, $K = 10$, $q_k = 5$, $u_i = \frac{1}{\|g_{i[t]}\|_2^2}$, for $i \in \{1, 2, \dots, K\}$, $k \in \{1, 2, \dots, M\}$.

Figure 3.5: Payoffs of power control achieved by Algorithm 1 and fixed transmit power ($w_{i[1]} = h_{S_i, P_k} \backslash q_k$) under different d_{S_i, D_i} , where $\alpha_i = 10, \beta_i = 0.1, M = 5, K = 10, q_k = 2, u_i = \frac{1}{\|g_{i[t]}\|_2^2}$, for $i \in \{1, 2, \dots, K\}, k \in \{1, 2, \dots, M\}$.

by all PUs from S_i is given by $c_i = \beta_i \sum_{k=1}^M h_{S_i, P_k} w_i$ where β_i is a constant. Note that this problem is not convex and is, in general, impossible to efficiently find the global optimal solution [59]. In this case, Algorithm 1 may not converge to the global utility maximization point, but still can converge to a NE. The convergence results of Algorithm 1 with different parameters in this case is presented in Figures 3.3 and 3.4. As observed in Section 3.3, the convergence rate of Algorithm 1 depends on the initial transmit powers of SUs and the iteration step size. It is observed that, if $w_{i[1]}$ has a distance of u_i from a NE, only a few number of iterations are required to approach the performance of this NE. However, if the initial transmit powers are not properly chosen, the adjustment on the transmit powers of SUs in Algorithm 1 may require a large number of iterations. Similarly, an optimal value of u_i should be close to the distance between $w_{i[1]}$ and w_i^* . However, if no information about initial transmit power or the optimal power is known, a large step size, i.e., $u_i \gg |w_{i[1]} - w_i^*|$, may lead to high fluctuations during the first few iterations, and a small step size, i.e., $u_i \ll |w_{i[1]} - w_i^*|$, will cause slow convergence rate. The above observation shows the importance of choosing proper combinations of initial powers and step sizes. In a practical system, SUs can first send the training code to let PUs to decide proper initial values of SUs' transmit powers. Also, the step size of each iteration, chosen by each SD pair, can be adjusted if more information can be observed or more feedback messages can be received during the communication, i.e., the dual method. Finding the distributed algorithms to allow iterative improvement of the step size in Algorithm 1 will be our future work.

Let us consider the numerical results of CR networks with Algorithm 1. Denote the distance between two users p and q as $d_{p,q}$ for $p, q \in [S_1, S_2, \dots, S_K] \cup [D_1, D_2, \dots, D_K] \cup [P_1, P_2, \dots, P_M]$ and consider the following channel models, $h_{ij} = \tilde{h}_{ij}/d_{S_i, D_j}^\eta$ and $h_{ik} = \tilde{h}_{ik}/d_{S_i, P_k}^\eta$, where $\tilde{h}_{ij}$ and $\tilde{h}_{ik}$ are average channel fading coef-

ficients unrelated to the distance of the transmission and η is the channel attenuation exponent. Assume that SUs are located on a regular planar network, i.e., $d_{S_i, P_k} = d_{S_j, P_l}$, $d_{S_i, D_i} = d_{S_j, D_j}$, $d_{S_i, D_j} = \sqrt{d_{S_i, S_j}^2 + d_{S_j, D_j}^2}$ and $d_{S_i, S_{i+1}} = d_{D_j, D_{j+1}}$, for $i, j \in \{1, 2, \dots, K\}$ and $k, l \in \{1, 2, \dots, M\}$. In Figure 3.5, the average payoff of the network obtained by using Algorithm 1 with different d_{S_i, D_i} is shown and compared with that of the fixed transmit power method. It is observed that the proposed algorithm greatly improves the performance of CR networks. More importantly, even with a small number of iterations, Algorithm 1 can achieve a significant payoff improvement.

3.5 Conclusion

In this chapter, the power control problem for SSS based CR networks with multiple secondary SD pairs and PUs has been investigated. A simple distributed algorithm has been proposed to iteratively improve the performance of CR networks. It has been proved that the proposed algorithm converges to an ϵ -NE at a rate of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. A sub-optimal algorithm has also been proposed to converge to a neighborhood of a NE at a rate of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$. Numerical results are also presented to show the convergence rates with different settings.

3.6 Appendix

3.6.1 Proof of Theorem 3.3.1

It is observed that for a NE, a neighborhood can always be found in which all the elements in this neighborhood has lower payoff than the NE. By assuming

all payoff functions of SUs to be continuous, the neighborhood of the NE can be regarded as a quasiconcave hull, i.e., for $\boldsymbol{\chi} = [\chi_1, \chi_2, \dots, \chi_K]$ neighborhood of a NE, we have $\pi_i(\mathbf{w}^* \pm \Delta \mathbf{w}) < \pi_i(\mathbf{w})$ where $\mathbf{w} \in [\mathbf{w}^* - \Delta \mathbf{w}, \mathbf{w}^* + \Delta \mathbf{w}]$ and $\Delta w_i \leq \chi_i$. Since Algorithm 1 uses a decreasing step size for iteration, it will finally fall into a small neighborhood of the NE. Therefore, if Algorithm 1 is proved to converge to a stationary point at which all elements in a neighborhood of that point has lower payoffs, we can claim that this point is the NE and Algorithm 1 converges. Defining $\mathbf{g}_{[t]} = [g_{1[t]}, g_{2[t]}, \dots, g_{K[t]}]^\dagger$, we can re-write (3.3.6) in Algorithm 1 into a vector form as follows,

$$\mathbf{w}_{[t+1]} = \begin{cases} \mathbf{w}_{[t]} - \delta_{[t]} \mathbf{g}_{[t]}, & \text{if } \mathbf{H}_{PU} \mathbf{w}_{[t]} \leq \mathbf{q}_j, \\ \mathbf{w}_{[t]} - \delta_{[t]} \mathbf{g}_{[t]} - \mathbf{H}_{PQ}^\dagger \left(\mathbf{H}_{PQ} \mathbf{H}_{PQ}^\dagger \right)^{-1} \left(\mathbf{H}_{PQ} \mathbf{w}_{[t]} - \mathbf{H}_{PQ} \delta_{[t]} \mathbf{g}_{[t]} - \mathbf{q} \right), & \text{otherwise,} \end{cases} \quad (3.6.1)$$

In (3.6.1), $\mathbf{H}_{PQ}$ is a sub-matrix of $\mathbf{H}_{PU}$ which only contains the channel gains connected with one PU observing high interference noises in its rows, and $\delta_{[t]} = \text{diag}[\delta_{1[t]}, \delta_{2[t]}, \dots, \delta_{K[t]}]$.

Let us first consider the case that the transmit powers of the SUs always satisfy (3.2.1). Denoting a NE achieving power control schemes for all the SUs as $\mathbf{w}^* =$

$[w_1^*, w_2^*, \dots, w_K^*]^\dagger$, we have

$$\begin{aligned}
& \frac{1}{T} \sum_{t=1}^T \|\mathbf{w}_{[t+1]} - \mathbf{w}^*\|_2^2 \stackrel{(a)}{=} \frac{1}{T} \sum_{t=1}^T \|\mathbf{w}_{[t]} - \mathbf{w}^* - \delta_{[t]} \mathbf{g}_{[t]}\|_2^2 \\
&= \frac{1}{T} \sum_{t=1}^T [\|\mathbf{w}_{[t]} - \mathbf{w}^*\|_2^2 - 2\delta_{[t]} \mathbf{g}_{[t]} (\mathbf{w}_{[t]} - \mathbf{w}^*) \\
&\quad + \delta_{[t]}^2 \|\mathbf{g}_{[t]}\|_2^2] \\
&\stackrel{(b)}{\leq} \frac{1}{T} \sum_{t=1}^T [\|\mathbf{w}_{[t]} - \mathbf{w}^*\|_2^2 - 2\delta_{[t]} (\boldsymbol{\pi}(\mathbf{w}_{[t]}) - \boldsymbol{\pi}(\mathbf{w}^*)) \\
&\quad + \delta_{[t]}^2 \|\mathbf{g}_{[t]}\|_2^2], \tag{3.6.2}
\end{aligned}$$

where (a) and (b) are obtained by using (3.3.1) and (3.3.2) in Algorithm 1, respectively. Equation (3.6.2) can be rewritten recursively as follows:

$$\begin{aligned}
& \frac{1}{T} \sum_{t=1}^T \|\mathbf{w}_{[t+1]} - \mathbf{w}^*\|_2^2 \leq \|\mathbf{w}_{[1]} - \mathbf{w}^*\|_2^2 \\
&\quad - \frac{2}{T} \sum_{t=1}^T \sum_{l=1}^t \delta_{[l]} (\boldsymbol{\pi}(\mathbf{w}_{[l]}) - \boldsymbol{\pi}(\mathbf{w}^*)) \\
&\quad + \frac{1}{T} \sum_{t=1}^T \sum_{l=1}^t \delta_{[l]}^2 \|\mathbf{g}_{[l]}\|_2^2. \tag{3.6.3}
\end{aligned}$$

Note that we have $\frac{1}{T} \sum_{t=1}^T \|\mathbf{w}_{[t+1]} - \mathbf{w}^*\|_2^2 \geq 0$, and the second term in the right-hand-side of (3.6.3) can be expressed as follows:

$$\begin{aligned}
& \frac{2}{T} \sum_{t=1}^T \sum_{l=1}^t \delta_{[l]} (\boldsymbol{\pi}(\mathbf{w}_{[l]}) - \boldsymbol{\pi}(\mathbf{w}^*)) \\
&\geq \frac{2}{T} \sum_{t=1}^T \left(\sum_{l=1}^t \delta_{[l]} \right) \min_{l \in [1, t]} [\boldsymbol{\pi}(\mathbf{w}_{[l]}) - \boldsymbol{\pi}(\mathbf{w}^*)] \\
&\stackrel{(c)}{\geq} \left(\frac{2}{T} \sum_{t=1}^T \sum_{l=1}^t \delta_{[l]} \right) \frac{1}{T} \sum_{t=1}^T \min_{l \in [1, t]} [\boldsymbol{\pi}(\mathbf{w}_{[l]}) - \boldsymbol{\pi}(\mathbf{w}^*)] \tag{3.6.4}
\end{aligned}$$

where (c) comes from the fact that $\sum_{l=1}^t \frac{1}{l} \geq \frac{1}{T} \sum_{t=1}^T \sum_{l=1}^t \frac{1}{l}$.

Substituting (3.6.4) into (3.6.3), we have

$$\begin{aligned} & \frac{1}{T} \sum_{t=1}^T \min_{l \in [1, t]} [\pi(\mathbf{w}_{[t]}) - \pi(\mathbf{w}^*)] \\ & \leq \frac{\|\mathbf{w}_{[1]} - \mathbf{w}^*\|_2^2 + \frac{1}{T} \sum_{t=1}^T \sum_{l=1}^t \delta_{[l]}^2 \|\mathbf{g}_{[l]}\|_2^2}{\frac{2}{T} \sum_{t=1}^T \sum_{l=1}^t \delta_{[l]}}. \end{aligned} \quad (3.6.5)$$

Substituting the step size function $\delta_{[l]} = \frac{u}{l}$ for $\mathbf{u} = [u_1, u_2, \dots, u_K]$ into the above equation and using the result $\sum_{l=1}^t \frac{1}{l^2} < \sum_{l=1}^{\infty} \frac{1}{l^2} = \frac{\pi}{6}$, and the properties of Harmonic number:

$$\sum_{l=1}^t \frac{1}{l} = \log t + \frac{1}{2}t^{-1} - \frac{1}{12}t^{-2} + O(t^{-4}) \approx \log t + \frac{1}{2}t^{-1} - \frac{1}{12}t^{-2} + \dot{\kappa}, \quad (3.6.6)$$

the following results can be obtained:

$$\begin{aligned} & \frac{1}{T} \sum_{t=1}^T \min_{l \in [1, t]} [\pi(\mathbf{w}_{[l]}) - \pi(\mathbf{w}^*)] \\ & \stackrel{(d)}{\leq} \frac{\mathbf{w}^+ + \frac{\mathbf{w}^+}{T} \sum_{t=1}^T \frac{\pi}{6}}{\frac{2}{T} \sum_{t=1}^T (\log t + 0.5t^{-1} + \dot{\kappa})} \\ & \approx \frac{\ddot{\kappa}}{\frac{2}{T} \sum_{t=1}^T \log t + \frac{\log T + 0.5T^{-1} + \dot{\kappa}}{T} + 2\dot{\kappa}} \end{aligned} \quad (3.6.7)$$

where (d) is obtained from Assumptions A1) and A2) in Theorem 3.3.1, and $\ddot{\kappa}$ is a constant defined as $\ddot{\kappa} = \mathbf{w}^+ + \mathbf{g}^+ \frac{\pi}{6}$. Because $\sum_{t=1}^T \log t = \Theta(T \log T)$ ⁱⁱ and hence $\lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \log t \rightarrow \infty$ and $\lim_{T \rightarrow \infty} \frac{\log T + 0.5T^{-1}}{T} \rightarrow 0$, it can be claimed that Algorithm 1 converges to a stationary point. Assume that for $T \geq T_\epsilon$ and T_ϵ is a

ⁱⁱAgain, we follow Bachmann-Landau notations: $f = \Theta(g)$ if $f = O(g)$ and $g = O(f)$

very large number, we try to achieve the accuracy of

$$\frac{1}{T} \sum_{t=1}^T \min_{l \in [1, t]} [\pi(\mathbf{w}_{[l]}) - \pi(\mathbf{w}^*)] = \epsilon. \quad (3.6.8)$$

From (3.6.7) and concavity of logarithm function, the following results can be obtained,

$$\begin{aligned} \frac{\ddot{\kappa}}{\epsilon} &\geq \lim_{T_\epsilon \rightarrow \infty} \frac{1}{T_\epsilon} \sum_{t=1}^{T_\epsilon} \log t + \frac{\log T_\epsilon + 0.5T_\epsilon^{-1}}{T_\epsilon} + 2\dot{\kappa} \\ &= \lim_{T \rightarrow \infty} \log \left(\frac{1}{T_\epsilon} \sum_{t=1}^{T_\epsilon} t \right) = \log \left(\frac{T_\epsilon + 1}{2} \right) \\ \Rightarrow T_\epsilon &= O \left(\exp \left(\frac{1}{\epsilon} \right) \right). \end{aligned}$$

Therefore, we can claim that by using Algorithm 1, the average payoff of SU networks converges to ϵ -NE at a rate of $T_\epsilon = O \left(\exp \left(\frac{1}{\epsilon} \right) \right)$.

Consider the case that at least one PU observes the higher-than-tolerable interference power. By multiplying (3.6.1) with $\mathbf{H}_{PQ}$, it can be easily shown that $\mathbf{H}_{PQ} \mathbf{w}_{[t+1]}^\dagger$ always converges to $\mathbf{q}^\dagger$. Substituting $\mathbf{H}_{PQ} \mathbf{w}_{[t+1]}^\dagger = \mathbf{q}^\dagger$ into (3.6.1), we rewrite (3.6.1) as

$$\mathbf{w}_{[t+2]} = \mathbf{w}_{[t+1]} - \boldsymbol{\delta}_{[t+1]} \tilde{\mathbf{g}}_{[t+1]} \mathbf{g}_{[t+1]}, \quad (3.6.9)$$

where $\tilde{\mathbf{g}}_{[t+1]} = \mathbf{I} + \mathbf{H}_{PQ}^\dagger \left(\mathbf{H}_{PQ} \mathbf{H}_{PQ}^\dagger \right)^{-1} \mathbf{H}_{PQ}$ and $\mathbf{I}$ is the identity matrix. If we denote $\mathbf{g}_{[t]} = \tilde{\mathbf{g}}_{[t]} \mathbf{g}_{[t]}$ and assume $\|\tilde{\mathbf{g}}_{[t]} \mathbf{g}_{[t]}\|_2^2 \leq \mathbf{g}^+$, the same method as the previous case can be used to obtain the similar convergence rate.

3.6.2 Proof of Theorem 3.3.2

Following the same steps as in Appendix 3.6.1, let us consider the case that each SU can only choose from L quantized power levels, i.e., $w_i \in \mathcal{W}_i$. In this case, if the initial point is not appropriately chosen, Algorithm 2 will be unstable for the first few iterations, i.e., fluctuating between different neighborhoods of NEs. However, when the number of iterations is large, the transmit powers of SUs will approach to the neighborhood of a NE. Therefore, for a large number of iterations, Algorithm 2 can be regarded as Algorithm 1 with a constant step size $\delta_{[l]} = \hat{\delta}$ where $\hat{\delta} = \frac{\mathbf{w}^+}{L}$ is a constant. Let us substitute this step size in (3.6.5), the following results can be obtained,

$$\begin{aligned}
& \frac{1}{T} \sum_{t=1}^T \min_{l \in [1, t]} [\pi(\mathbf{w}_{[l]}) - \pi(\mathbf{w}^*)] \\
& \leq \frac{1}{T} \sum_{t=1}^T \frac{\mathbf{w}^+ + (\hat{\delta}^2 \mathbf{g}^+ / t) \sum_{l=1}^t l}{(2\hat{\delta}/t) \sum_{l=1}^t l} \\
& = \frac{1}{T} \sum_{t=1}^T \frac{\mathbf{w}^+ + \hat{\delta}^2 \mathbf{g}^+ (t+1)}{2\hat{\delta} (t+1)}. \tag{3.6.10}
\end{aligned}$$

By using the same method as in Appendix 3.6.1, we can claim that if the step size is fixed, the number of iterations for Algorithm 2 is given by $\frac{T'_\epsilon}{\log T'_\epsilon} = O\left(\frac{1}{\epsilon'}\right)$ where $\epsilon' = \epsilon - \frac{\hat{\delta} \mathbf{g}^+}{2}$ and $\hat{\delta} = \frac{\mathbf{w}^+}{L}$.

Chapter 4

Game Theoretic Analysis for Spectrum Sharing with Multi-hop Relaying

4.1 Introduction

In this chapter, we also focus on the SSS based CR networks and assume PUs can maintain their QoS as long as their interference powers are lower than the interference-temperature limit. Recently, it was shown that the performance of CR networks can be further improved if SUs could help each other during the transmission [60] [61] [3]. However, it was also observed that multi-hop relaying introduces both flexibility and complexity to the network operation, and if parameters are not properly selected, it cannot provide any improvement over the direct transmission. Currently, finding effective ways to control and manage SSS based CR networks with multi-hop relaying, especially in networks with multiple secondary source-to-destination pairs, is still an open problem.

This chapter focuses on optimizing parameters and operations of multi-user CR networks. This issue becomes more complex when multi-hop relaying is allowed because if SUs are selfish, they tend to keep forcing others to serve as relays without returning the favor. In this chapter, we try to find a balanced point for the above network by using a game theoretic model in which each SU obtains benefits through both spectrum sharing by paying prices to PUs and multi-hop relaying by paying prices to SUs who serve as its relays. One advantage of our work over the previously reported results [44] [50] is that we not only consider the performance of one specific source-to-destination pair, but also investigate the interaction among SUs and that between SUs and PUs. More specifically, we define two types of pricing functions: one is charged by SUs for selling their relaying service, and the other is charged by PUs for selling the licensed spectrum. The objective for each SU is to choose proper parameters and strategies to improve its revenue and simultaneously minimize the prices paid to PUs and relays. The optimal power allocation methods for SUs are derived under three different assumptions. Except for extending two traditional assumptions [50] into our network model, i.e., transmitters know full CSI or only know average channel gains, we also introduce a new power allocation method which could optimize the performance of SUs even if transmitters cannot do any predictions but only know the current channel gains. The NE of the game is discussed, and the conditions under which all SUs choosing multi-hop relaying achieves the optimal NE are studied.

To simplify our illustration, we first consider a simple network model with two source-to-destination pairs and then extend our results to the K source-to-destination pair case. In addition, we investigate two new questions for large multi-user CR networks: how to distributively control and optimize the transmit powers of SUs and how to choose the “right” relaying schemes for each SU in terms of the number and quality of relays. To answer these questions, two distributed optimiza-

tion algorithms are proposed. The first one is a sub-gradient based power allocation algorithm which enables SUs to iteratively adjust their powers to approach the performance of a NE. The other one is a Q-learning based relay selection algorithm for each SU to search for a NE-achieving relaying scheme. We discuss possible extensions of our work and present numerical results to demonstrate the performance improvement brought by using the multi-hop relaying in CR networks.

The rest of this chapter is organized as follows. The network model and game formulation are presented in Section 5.2. The game theoretic analysis for CR networks with two secondary source-to-destination pairs is presented in Section 4.3. The results are extended into large multi-user CR networks in Section 4.4. The numerical results are presented in Section 5.5 and the chapter is concluded in Section 5.6.

4.2 Network Model and Game Theoretic Setup

4.2.1 Network Model

Consider a CR network model in which K secondary source-to-destination pairs, S_1 to D_1 , S_2 to D_2 , ..., S_K to D_K , share the licensed spectrum with M PUs, $P_1, P_2, \dots, P_M$. To simplify our illustration, in this chapter, we only focus on the transmitter cooperation in which S_i can only serve as the relay for other secondary sources, i.e., S_j for $j \neq i, j \in [1, K]$. The results of the receiver cooperation can be similarly obtained. Let the transmit power of S_i used to send/forward signals for S_j be $w_{i,j}$ for $i, j \in [1, K]$. Let the channel gain between user l and m be $h_{l,m}$ where $l, m \in \{S_1, S_2, \dots, S_K\} \cup \{D_1, D_2, \dots, D_K\} \cup \{P_1, P_2, \dots, P_K\}$

In our model, SUs can either directly send their signals or cooperate with each

other by using the decode-and-forward multi-hop relaying. Assume that SUs use time division half duplex transmission mode to void self interference and each secondary source has been allocated the same amount of time for its own transmission. For example, consider a U time duration of transmission. If S_i chooses the direct transmission, it will directly send its own information to D_i in $\frac{U}{K}$ amount of time. However, if S_i chooses to use L_i -hop relaying to send its signals for $L_i \geq 2$, it only spends $\frac{U}{KL_i}$ in sending its source information to the closest relay and leaves all the rest $\frac{U(L_i-1)}{KL_i}$ amount of time for the relaying transmission (each relay will spend $\frac{U}{KL_i}$ in forwarding the signal of S_i). Note that, in our settings, the relaying scheme chosen by each secondary source does not affect the transmission of other source-to-destination pairs. To simplify our discussion, we define a time scheduler (TS) for the multi-user CR network as follows.

Assumption 4.2.1. *We assume that there is a TS in the SU network. Two main objectives for the TS are: 1) to ensure that there are no collisions (more than one SU uses the licensed spectrum at the same time); 2) to arrange the relays and time for the transmission of each secondary source-to-destination pair. More specifically, if S_i chooses L_i -hop relaying, it will first send the request to the TS to schedule $(L_i - 1)$ intermediate SUs (relays) and the respective time segments for the relaying transmission. Note that the TS only provides the time scheduling information and each SU cannot obtain any other information from the TS. We assume the delay for each SU to wait for the availability of relays and the time scheduling information is negligible.*

Note that a TS may not necessarily be a central controller. For example, SUs can follow a pre-defined protocol to schedule their transmissions, or they can compete/cooperate with each other for the limited transmission opportunities [38].

To simplify our problem, in this chapter, we assume there is no pricing compe-

tition [62] and all PUs treat each SU equally. Hence, we can combine the effects of M PUs as one, labeled as PU 0. The interference power increment of PU 0 caused by S_i is given by $I_{S_i} = w_{i,j}h_{S_i,0}$ where $h_{S_i,0} = \sum_{k=1}^M h_{S_i,P_k}$.

4.2.2 Game Theoretic Setup

Define the following elements in our model: 1) *Players* are SUs who share the licensed spectrum, 2) *Strategies* of S_i , denoted by $s_i \in \mathcal{S}$ where $\mathcal{S}$ is the set of all strategies for SUs, are the options of actions. For example, in relay selection games, s_i can be either direct transmission or multi-hop relaying. In power allocation games, s_i can be different transmit powers of S_i , 3) *Revenue* of S_i , denoted by $r_{i,j}$, is the benefit earned by S_i for using PU 0's spectrum ($r_{i,i}$) and/or selling relaying services to S_j ($r_{i,j}$ for $j \neq i$ and $i, j \in \{1, 2, \dots, K\}$), 4) *Pricing function* paid by S_i is the price charged by PU 0, denoted by $c_{0,i}$, and/or S_j , denoted by $c_{i,j}$, for selling the spectrum and/or providing relaying services, 5) *Payoff* of S_i , denoted by π_i , is the difference between its revenue and price, i.e., $\pi_i = \sum_{j=1}^K (r_{i,j} - c_{i,j} - c_{0,i})$.

Our model can be regarded as a special case of spectrum leasing in which PUs are regarded as the spectral resource owners and could lease a part of the resource to SUs in exchange for remuneration [63] [64] [65]. The price charged by PUs and SUs can either be used to exchange services with each other [63] or be the profits of the network operators [64].

The main objective for each SU is to choose proper parameters and strategies to maximize its “long term” payoff. Let us consider a repeated game with T time slots of transmission and use subscript $[t]$ to denote parameters during the t th time slot. In every time slot, each source-to-destination pair can send a certain amount of information. We assume the transmissions in different time slots are independent

and each player cares for the performance of the future as same as that of the present i.e., the time discount factor [55, Definition 6.1.2] is 1. In this chapter, transmit powers of SUs are limited in every time slot, and hence the power constraint is defined as follows:

$$E \left(\sum_{j \in [1, K]} \sum_{i \in [1, K]} h_{S_i, 0[t]} w_{i, j[t]} \right) \leq q_{0[t]}, \quad (4.2.1)$$

where $w_{j, i[t]} = 0$ if S_i does not require S_j to serve as its relay for $i \neq j, i, j \in [1, K]$, and $q_{0[t]}$ is the interference temperature limit of PU 0 in time slot t .

Since the main effect of allowing spectrum sharing is to increase the interference level of PU 0, it is reasonable to define the price charged by PU 0 from S_i to be a function of its resulting interference power increment caused by S_i , i.e., we have

$$\hat{c}_{0, i} = \sum_{t=1}^T c_{0, i[t]} = \sum_{t=1}^T \sum_{j=1}^K \tilde{\alpha}_{0, j[t]} h_{S_j, 0[t]} w_{j, i[t]}, \quad (4.2.2)$$

where $\tilde{\alpha}_{0, i[t]}$ is the pricing coefficient of PU 0 in time slot t defined as,

$$\tilde{\alpha}_{0, i[t]} = (b_{0, i[t]} w_{i, j[t]})^\eta + a_{0, i[t]}. \quad (4.2.3)$$

and $\eta, a_{0, i[t]}$ and $b_{0, i[t]}$ are constants in time slot t . Note that $\tilde{\alpha}_{0, i[t]}$ characterizes the interaction between SUs and PU 0. More specifically, a small $\tilde{\alpha}_{0, i[t]}$ indicates that a slight increment of interference power will not cause much QoS degrading for PU 0. However, if $\tilde{\alpha}_{0, i[t]}$ is large, PU 0 is less likely to share the spectrum with SUs and each SU needs to pay a relatively high price for accessing the licensed spectrum. In this chapter, we mainly focus on two special types of $\tilde{\alpha}_{0, i[t]}$: $\tilde{\alpha}_{0, i[t]}$ is a constant ($c_{0, i[t]}$ is a linear pricing function of $w_{i, j[t]}$), i.e., $\eta = 0$, and $\tilde{\alpha}_{0, i[t]}$ is a linear function of the transmit power of S_i ($c_{0, i[t]}$ is a quadratic pricing function of $w_{i, j[t]}$), i.e., $\eta = 1$, which are considered in Subsections 4.3.2 and 4.3.3, respectively.

Assuming the decoding and re-encoding delay is negligible and the transmit power is the main resource used by each relay, we define the price charged by S_i for serving as the relay for the j th source-to-destination pair to be proportional to $w_{i,j}$, i.e., $\hat{c}_{i,j} = \sum_{t=1}^T c_{i,j[t]} = \sum_{t=1}^T \beta_{i,j} w_{i,j[t]}$, where $\beta_{i,j}$ is a constant. Assume each SU tries to maximize its data rate, and therefore the revenue of S_i should be proportional to its achievable rate $\mathbf{R}_i$, i.e., $\hat{r}_{i,i} = \sum_{t=1}^T r_{i,i[t]} = \sum_{t=1}^T \gamma_i \mathbf{R}_{i[t]}$, where $i, j \in \{1, 2, \dots, K\}$, and γ_i is a constant which should be large if spectrum sharing is the only way to maintain a certain level of achievable rate, or small if there exist some alternative ways/resources for S_i to transmit important data. Revenues obtained by S_i to serve as the relay for S_j is defined as $\hat{r}_{i,j} = \sum_{t=1}^T r_{i,j[t]} = \sum_{t=1}^T \tilde{\beta}_{i,j} w_{i,j[t]}$, for $i \neq j, i, j \in \{1, 2, \dots, K\}$ where $\tilde{\beta}_{i,j}$ is a constant.

Assumption 4.2.2. *We assume only transmitters, i.e., S_1 and S_2 , can decide which operation should be chosen (direct transmission or multi-hop relaying). Therefore, no SUs can obtain revenues by forcing others to relay information. Except from that, the revenue earned by S_i by serving as the relay of S_j can only be used for exchanging services with other SUs, i.e., S_k for $k \neq i, k, i, j \in [1, K]$. We assume, during T time slots of transmission, each SU maintains a balance between the overall price paid to buy relaying services from others and the revenue obtained for serving as relays, i.e., we have*

$$\begin{aligned} \hat{r}_{i,-i} &= \sum_{t \in [1, T]} \sum_{\substack{j \in [1, K] \\ j \neq i}} r_{i,j[t]} \\ &= \hat{c}_{-i,i} = \sum_{t \in [1, T]} \sum_{\substack{j \in [1, K] \\ j \neq i}} c_{j,i[t]}. \end{aligned} \quad (4.2.4)$$

The above assumption can be satisfied for a relatively long period of transmission in which all SUs are selfish and always tend to spend all relaying revenues on improving their payoffs. In addition, we also assume no SUs can refuse the relaying request of others, i.e., a TS is built to ensure the transmission and relaying requests

of SUs to be satisfied. These assumptions avoid the case that selfish SUs in the networks keep using multi-hop relaying and refuse to help others (free-riders) without being punished, or some SUs always forward signals for others without getting any benefits.

In the network with multiple secondary source-to-destination pairs, one of the main objectives is to find the NE. In other words, if the network operates at a NE, each SU cannot achieve a higher payoff by choosing a different strategy, given the strategies of other SUs.

4.3 Game Theoretic Analysis for Two Source-to-Destination Pair Case

Let us consider the CR network model with two secondary source-to-destination pairs, i.e., S_1 to D_1 and S_2 to D_2 .

4.3.1 Game Theoretic Analysis

Following the game theoretic model established in Subsection 4.2.2, in the t th time slot, four strategy pairs $(s_{1[t]}, s_{2[t]})$ can be selected by S_1 and S_2 as discussed below.

Both S_1 and S_2 Using Direct Transmissions

In this case, the price paid by S_i for $i \in \{1, 2\}$ to PU 0 is given by $c_{0,i[t]}^{DT} = \tilde{\alpha}_{0,i[t]} h_{S_i,0[t]} w_{i,i[t]}$. S_i obtains the following revenue which is proportional to its achiev-

able rate: $r_{i,i[t]}^{DT} = \gamma_i \log(1 + h_{S_i,D_i[t]} w_{i,i[t]})$. Therefore, the payoff of S_i is given by

$$\pi_{i[t]}^{DT} = r_{i,i[t]}^{DT} - c_{0,i[t]}^{DT}. \quad (4.3.1)$$

Both S_1 and S_2 Using Multi-hop Relaying

In this case, the cost of S_i for $i \in \{1, 2\}$ contains two parts: one is charged by PU 0 from S_i (for sending the source information) and S_{-i} (for forwarding signals of S_i), denoted by $c_{0,i[t]}^{CT} = \tilde{c}_{0,i[t]}^{CT} + c'_{0,-i[t]}^{CT} = \tilde{\alpha}_{0,i[t]} h_{S_i,0[t]} w_{i,i[t]} + \tilde{\alpha}_{0,-i[t]} h_{S_{-i},0[t]} w_{-i,i[t]}$, and the other is charged by the relay (i.e., S_{-i}), denoted by $c_{i,-i[t]}^{CT} = \beta_{-i,i} w_{-i,i[t]}$. The revenue of S_i also contains two parts: one is proportional to its achievable rate $\mathbf{R}_{i[t]}$, denoted as

$$r_{i,i[t]}^{CT} = \frac{\gamma_i}{2} \min \left\{ E \log(1 + h_{S_i,S_{-i}[t]} w_{i,i[t]}), \right. \\ \left. E \log(1 + h_{S_{-i},D_i[t]} w_{-i,i[t]}) \right\}, \quad (4.3.2)$$

and the other is earned by serving as the relay for S_{-i} , denoted as $r_{i,-i[t]}^{CT} = \tilde{\beta}_{i,-i} w_{i,-i[t]}$.

The payoff of S_i using multi-hop relaying is given by

$$\pi_{i[t]}^{CT} = r_{i,i[t]}^{CT} + r_{i,-i[t]}^{CT} - c_{0,i[t]}^{CT} - c_{i,-i[t]}^{CT}. \quad (4.3.3)$$

One SU Using Multi-hop Relaying and the Other SU Using Direct Transmission

Assume only S_i for $i \in \{1, 2\}$ uses multi-hop relaying. In this case, S_{-i} not only transmits its own information to D_{-i} but also serves as the relay for S_i . Following

the same line as the previous cases, the payoff functions for S_i and S_{-i} are given by

$$\pi_{i[t]}^{CTD} = r_{i,i[t]}^{CT} - c_{0,i[t]}^{CT} - c_{i,-i[t]}^{CT}, \quad (4.3.4)$$

$$\pi_{-i[t]}^{DTC} = r_{-i,-i[t]}^{DT} + r_{-i,i[t]}^{CT} - c_{0,-i[t]}^{DT}. \quad (4.3.5)$$

From (4.3.4) and (4.3.5), we have the following results,

$$\begin{aligned} \pi_{i[t]}^{CTD} &= \pi_{i[t]}^{CT} - r_{i,-i[t]}^{CT}, \\ \pi_{-i[t]}^{DTC} &= \pi_{-i[t]}^{DT} + r_{-i,i[t]}^{CT}. \end{aligned} \quad (4.3.6)$$

The revenue earned by each SU for serving as the relay should have different meanings for different systems. In a large network, for example, each SU can use this revenue to buy the relaying service from others [63]. In some other systems, the price charged by relays can be simply used for avoiding free-riders [66].

4.3.2 Optimal Power Allocation Methods for CR Networks with Linear Pricing Function

As observed in the previous section, the revenue of each SU is closely related to its transmit power. Hence, finding the optimal transmit power for each SU is very important for the interference limited SSS systems. Let us introduce the following assumption.

Assumption 4.3.1. *It is observed that the achievable rate of the multi-hop relay channel is always limited by the hop with the lowest capacity and hence, if the transmit powers of the source and relays are equal, there is always a part of the powers being wasted. This problem becomes more serious in our proposed model because a*

high transmit power also means a high price charged by relays and PU 0. One way to solve this problem is to adapt the forwarding powers of the relay to the CSIs of its connected channels. More specifically, if S_i chooses the multi-hop relaying, $w_{-i,i}$ should be given by

$$w_{-i,i[t]} = f_i w_{i,i[t]}, \quad (4.3.7)$$

where $f_i = \frac{h_{S_i, S_{-i}[t]}}{h_{S_{-i}, D_i[t]}}$ for $i \in \{1, 2\}$.

In this way, the overall achievable rate of multi-hop relaying channels should always be equal to that of the first hop and no powers will be wasted during the relaying transmission, i.e.,

$$\mathbf{R}_{i[t]}^{\text{CT}} = \frac{1}{2} \mathbf{E} \log \left(\mathbf{1} + \mathbf{h}_{S_i, S_{-i}[t]} \mathbf{w}_{i,i[t]} \right). \quad (4.3.8)$$

Following Assumption 4.2.2, each SU (i.e., S_i) should balance the accumulated price paid to relaying SUs and the accumulated revenue obtained by serving as relays for others. In other words, we can actually neglect the effect of $c_{-i,i[t]}$ and $r_{i,-i[t]}$ in the payoff optimization problem because these two values will be eventually canceled during T time slots of transmission. Let us consider the linear pricing function case and assume $\tilde{\alpha}_{0,j[t]}$ of PU 0 in (4.2.3) to be a constant in time slot t , i.e., $\tilde{\alpha}_{0,j[t]} = \tilde{\alpha}_{0[t]}$. Since the transmissions in different time slots are independent with each other, in the rest of this section, we focus on the transmission in the t th time slot and drop the subscript $[t]$ to simplify the notation.

Only Average Channel Gains are Known by Transmitters

In this case, $w_{i,i}$ for $i \in \{1, 2\}$ should be a constant. If both S_1 and S_2 directly transmit their signals to the corresponding destinations, $w_{i,i}$ should satisfy,

$$\begin{aligned} E(h_{S_1,0}w_{1,1} + h_{S_2,0}w_{2,2}) \\ = w_{1,1}E(h_{S_1,0}) + w_{2,2}E(h_{S_2,0}) \leq q_0, \end{aligned} \quad (4.3.9)$$

where if we assume the statistics of $h_{S_1,0}$ and $h_{S_2,0}$ are fixed during each time slot, $E(h_{S_1,0})$ and $E(h_{S_2,0})$ are two constants which can be obtained through feedbacks of PU 0.

If the SNR is low, the optimal transmit power of S_i for $i \in \{1, 2\}$ is given by

$$w_{i,i} = \left(\frac{\gamma_i}{\lambda E(h_{S_i,0})} - \frac{1}{E(h_{S_i,D_i})} \right)^+, \quad (4.3.10)$$

where λ needs to satisfy

$$\lambda \geq \frac{\gamma_1 + \gamma_2}{q + \frac{E(h_{S_1,0})}{E(h_{S_1,D_1})} + \frac{E(h_{S_2,0})}{E(h_{S_2,D_2})}} \quad (4.3.11)$$

if $w_{1,1}, w_{2,2} > 0$.

Consider the case that both S_1 and S_2 use multi-hop relaying. Following Assumption 4.3.1, if the SNR is low, the optimal transmit power of S_i for $i \in \{1, 2\}$ is given by

$$w_{i,i} = \left(\frac{\gamma_i}{\lambda [E(h_{S_i,0}) + E(h_{S_{-i},0}f_i)]} - \frac{1}{E(h_{S_i,S_{-i}})} \right)^+, \quad (4.3.12)$$

where λ needs to satisfy

$$\lambda \geq \frac{\gamma_1 + \gamma_2}{2q + \frac{E(h_{S_1,0}) + E(h_{S_2,0}f_1)}{E(h_{S_1,S_2})} + \frac{E(h_{S_2,0}) + E(h_{S_1,0}f_2)}{E(h_{S_2,S_1})}} \quad (4.3.13)$$

if both $w_{1,1}$ and $w_{2,2}$ are positive. The detailed proofs are given in Appendices 4.7.1 and 4.7.2. By using (4.3.6) and the same methods in Appendices 4.7.1 and 4.7.2, we can directly obtain the optimal power allocation method for the case with one SU choosing direct transmission and the other one choosing multi-hop relaying.

Note that our results in (4.3.10) and (4.3.12) are different from the single source-to-destination pair case in [50] in the sense that $w_{i,i}$ is limited by both SU-to-PU and SU-to-SU channels.

Full CSIs are Known by Transmitters

In this case, $w_{i,i}$ can be adapted to instantaneous values of the channel gains to further increase the revenue of S_i . If both S_1 and S_2 use direct transmission, we can obtain the following optimal power allocation methods by maximizing $\pi_1^{DT} + \pi_2^{DT}$ over $w_{1,1}$ and $w_{2,2}$, respectively. The optimal transmit power of S_i for $i \in \{1, 2\}$ is given by

$$w_{i,i} = \left(\frac{\gamma_i}{\lambda h_{S_i,0}} - \frac{1}{h_{S_i,D_i}} \right)^+ \quad (4.3.14)$$

where λ needs to satisfy

$$\lambda \geq \frac{\gamma_1 + \gamma_2}{q + \frac{h_{S_1,0}}{h_{S_1,D_1}} + \frac{h_{S_2,0}}{h_{S_2,D_2}}} \quad (4.3.15)$$

if $w_{1,1}, w_{2,2} > 0$. Note that in (4.3.14), $\frac{\gamma_1}{\lambda h_{S_1,0}}$ and $\frac{\gamma_2}{\lambda h_{S_2,0}}$ are called “water-levels” of $w_{1,1}$ and $w_{2,2}$, respectively. In other words, S_i should not transmit any signals if

h_{S_i, D_i} is lower than $\frac{\lambda h_{S_i, 0}}{\gamma_i}$.

Similarly, if both S_1 and S_2 use multi-hop relaying, the optimal transmit power of S_i can be obtained by maximizing $\pi_{1[t]}^{CT} + \pi_{2[t]}^{CT}$ over $w_{i,i}$, which is given by

$$w_{i,i} = \left(\frac{\gamma_i}{\lambda (h_{S_i, 0} + h_{S_{-i}, 0} f_i)} - \frac{1}{h_{S_1, S_2}} \right)^+, \quad (4.3.16)$$

where λ needs to satisfy

$$\lambda \geq \frac{\gamma_1 + \gamma_2}{2q + \frac{(h_{S_1, 0} + h_{S_2, 0} f_1)}{h_{S_1, S_2}} + \frac{(h_{S_2, 0} + h_{S_1, 0} f_2)}{h_{S_2, S_1}}} \quad (4.3.17)$$

if both $w_{1,1}$ and $w_{2,2}$ are positive.

Only Current Channel Gains are Known by Transmitters

In previous subsections, we study two extreme cases: 1) transmitters do not know any CSIs except for the average channel gains, 2) full CSIs can be accurately predicted by transmitters. These assumptions may not always be applied. For example, in many practical systems, transmitters cannot predict any information, i.e., future CSIs or average channel gains. In this subsection, we propose a new power allocation method to solve the above problem. In our method, transmitters only observe current channel gains.

Following the network model in Section 5.2, we assume that each time slot can be further divided into B short segments during each of which the channel fading coefficients can be regarded as constants. We use l to denote the index of segment for $l \in [1, B]$. Since SUs cannot do any predictions about future channel gains, the power constraint in (4.2.1) should be applied in every segment, i.e., we have the

following power constraint,

$$E \left(\sum_{j=1}^2 \sum_{i=1}^2 h_{S_i,0}(l) w_{i,j}(l) \right) \leq q_0. \quad (4.3.18)$$

Due to the space limit, in the rest of this subsection, we only present results for the case that both S_1 and S_2 use multi-hop relaying to transmit signals. The direct transmission case can be obtained by using the similar method. For the l th segment of transmission, $\pi_{i,i}^{CT}(l)$ can be re-written as

$$\begin{aligned} \pi_i^{CT}(l) &= \frac{\gamma_i}{2} \log \left(1 + h_{S_i,S_{-i}}(l) w_{i,i}(l) \right) \\ &\quad - \tilde{\alpha}_0 h_{S_i,0}(l) w_{i,i}(l) - \tilde{\alpha}_0 h_{S_{-i},0}(l) w_{-i,i}(l). \end{aligned} \quad (4.3.19)$$

Since $\pi_i^{CT}(l)$ is a quasi-concave function of $w_{i,i}$, the optimal transmit power of S_i can be easily obtained by maximizing (4.3.19) over $w_{i,i}$ if both $w_{1,1}$ and $w_{2,2}$ satisfy (4.3.18). However, if (4.3.18) cannot be satisfied, we need to find the optimal values of $w_{i,i}$ and $w_{-i,-i}$ in a linear function of $\frac{1}{2} \sum_{i=1}^2 (h_{S_i,0}(l) + h_{S_{-i},0}(l) f_i(l)) w_{i,i} = q_0$. Therefore, the optimal transmit power for S_i in segment l is given by

$$w_{i,i}(l) = \left(\frac{\gamma_i}{\lambda^*(l) (h_{S_i,0}(l) + h_{S_{-i},0}(l) f_i(l))} - \frac{1}{h_{S_i,S_{-i}}(l)} \right)^+, \quad (4.3.20)$$

where λ^* is given by,

$$\lambda^*(l) = \begin{cases} \frac{\gamma_i}{2q_0 + \nu_{S_i}(l)}, & \text{if (4.3.18) is not satisfied} \\ & \text{and } w_{i,i} > 0, w_{-i,-i} = 0, \\ \frac{\gamma_1 + \gamma_2}{2q_0 + \sum_{j=1}^2 \nu_{S_j}(l)}, & \text{if (4.3.18) is not satisfied} \\ & \text{and } w_{1,1}, w_{2,2} > 0, \\ 2\tilde{\alpha}_0, & \text{Otherwise,} \end{cases} \quad (4.3.21)$$

and

$$\nu_{S_i}(l) = \frac{h_{S_i,0}(l) + h_{S_{-i},0}(l)f_i(l)}{h_{S_i,S_{-i}}(l)}. \quad (4.3.22)$$

From (4.3.20), it is observed that knowing $\tilde{\alpha}_0$ from PU 0 only improves the payoffs of SUs when the received power constraint defined in (4.2.1) is not tight, i.e., q_0 being large enough.

4.3.3 Optimal Power Allocation Methods for CR Networks with Quadratic Pricing Function

In Subsection 4.3.2, we studied a CR network model in which the price charged by PU 0 is a linear function of its interference power, i.e., $\tilde{\alpha}_{0,i[t]}$ is a constant. However, the pricing function for a practical system may be more complex. For example, in an interference sensitive network, even a slight increment of the interference power could cause a huge QoS degrading for PU 0. In this case, setting $\tilde{\alpha}_{0,i[t]}$ to be a high order polynomial function of the transmit power of S_i , i.e., $\eta > 1$ in (4.2.2), could accelerate the power decreasing process of SUs to avoid a long period of high interference to PU 0. In this subsection, we consider the case that the pricing function of PU 0 is a quadratic function of the transmit power of S_i , i.e., $\tilde{\alpha}_{0,i[t]}$ is given by

$$\tilde{\alpha}_{0,i[t]} = b_{0[t]}w_{i,j[t]} + a_{0[t]}. \quad (4.3.23)$$

Following the same line as Subsection 4.3.2, we assume $\hat{c}_{i,-i} = \hat{r}_{-i,i}$ and focus on one time slot of transmission to neglect the subscript $[t]$. Let us first consider the case that both S_1 and S_2 directly transmit signals to D_1 and D_2 , respectively.

Following the same setting as Section 4.3, the payoff function of S_i can be written as follows:

$$\pi_i^{DT} = \frac{\gamma_i}{2} E \log (1 + h_{S_i, D_i} w_{i,i}) - (b_0 w_{i,i} + a_0) h_{S_i, 0} w_{i,i}.$$

By maximizing π_i^{DT} over $w_{i,i}$ under the constraint in (4.2.1), we can derive the optimal transmit power of S_i for $i \in \{1, 2\}$ as follows:

$$w_{i,i} = \left(\sqrt{\frac{1}{4h_{S_i, D_i}^2} - \frac{a_0 + \lambda}{4b_0 h_{S_i, D_i}} + \frac{(a_0 + \lambda)^2}{16b_0^2} + \frac{\gamma_i}{2b_0 h_{S_i, 0}}} - \frac{1}{2h_{S_i, D_i}} - \frac{a_0 + \lambda}{4b_0} \right)^+, \quad (4.3.24)$$

where λ is a constant to ensure the power constraint in (4.2.1) being satisfied.

Let us consider the case that both S_1 and S_2 use multi-hop relaying to transmit signals. In this setting, π_i^{CT} for $i \in \{1, 2\}$ can be re-written as follows:

$$\begin{aligned} \pi_i^{CT} = & \frac{\gamma_i}{2} E \log (1 + h_{S_i, S_{-i}} w_{i,i}) - (b_0 w_{i,i} + a_0) h_{S_i, 0} w_{i,i} \\ & - (b_0 w_{-i,i} + a_0) h_{S_{-i}, 0} w_{-i,i}. \end{aligned} \quad (4.3.25)$$

By maximizing (4.3.25) over $w_{i,i}$ and using Assumption 4.3.1, the optimal transmit power for S_i with full CSIs known by transmitters is given by

$$w_{i,i} = \left(\sqrt{\frac{1}{4h_{S_i, S_{-i}}^2} - \frac{D_i}{4A_i h_{S_i, S_{-i}}} + \frac{D_i^2}{16A_i^2} + \frac{\gamma_i}{4A_i}} - \frac{1}{2h_{S_i, S_{-i}}} - \frac{D_i}{4A_i} \right)^+, \quad (4.3.26)$$

where A_i and D_i are given by

$$A_i = b_0 \left(h_{S_i,0} + h_{S_{-i},0} \frac{h_{S_i,S_{-i}}^2}{h_{S_i,D_i}^2} \right), \quad (4.3.27)$$

$$D_i = (a_0 + \lambda) \left(h_{S_i,0} + h_{S_{-i},0} \frac{h_{S_i,S_{-i}}}{h_{S_i,D_i}} \right), \quad (4.3.28)$$

and λ is a constant to ensure that $w_{i,j}$ for $i, j \in \{1, 2\}$ satisfies the received power constraint in (4.2.1). Note that $w_{i,i}$ in (4.3.26) is different from the result in (4.3.16) and is always larger than zero if $\gamma_i > \frac{2D_i}{h_{S_i,S_{-i}}}$. By using the same method discussed in Section 4.3.2, we can calculate results for the cases that only average or current CSIs are known by transmitters.

4.3.4 Cooperative Nash Equilibrium

Based on the above discussion, there are four strategy pairs in each time slot of the considered game: $\mathcal{S} = \{(\pi_{1[t]}^{DT}, \pi_{2[t]}^{DT}), (\pi_{1[t]}^{CTD}, \pi_{2[t]}^{DTC}), (\pi_{1[t]}^{DTC}, \pi_{2[t]}^{CTD}), (\pi_{1[t]}^{CT}, \pi_{2[t]}^{CT})\}$. Assume that each SU fixes its transmission scheme during T time slots of communication. To simplify our discussion, we only focus on the symmetric network and assume $\hat{\pi}_1^{CT} = \hat{\pi}_2^{CT}$, $\hat{\pi}_1^{DT} = \hat{\pi}_2^{DT}$, $\hat{\pi}_1^{CTD} = \hat{\pi}_2^{CTD}$, and $\hat{\pi}_1^{DTC} = \hat{\pi}_2^{DTC}$ in this subsection. We have the following results about the NE supported by the 2 source-to-destination pair game,

- 1) If $\hat{\pi}_i^{DT} > \hat{\pi}_i^{CT}$ for $i \in \{1, 2\}$, then $(\hat{\pi}_1^{DT}, \hat{\pi}_2^{DT})$ is the optimal NE of the game,
- 2) If $\hat{\pi}_i^{CT} \geq \hat{\pi}_i^{DT}$ for $i \in \{1, 2\}$, then $(\hat{\pi}_1^{CT}, \hat{\pi}_2^{CT})$ is a NE. Furthermore, if we assume $\hat{\pi}_i^{CT} \geq \hat{\pi}_i^{DTC}$ and $\hat{\pi}_i^{CTD} \geq \hat{\pi}_i^{DT}$, then $(\hat{\pi}_1^{CT}, \hat{\pi}_2^{CT})$ is the only NE of the game.

Result 1) defines the conditions under which both S_1 and S_2 directly transmitting signals to the corresponding destinations achieve the optimal NE of the game. More

specifically, in our game theoretic model, S_1 and S_2 should not use multi-hop relaying to transmit signals if the extra revenue brought by using multi-hop relaying is smaller than the increased price charged by PU 0 from the relay, i.e., we have

$$\gamma_i \Delta \hat{\mathbf{R}}_i^{\mathbf{CT}-\mathbf{DT}} = \sum_{t=1}^T \gamma_i \mathbf{R}_{i[t]}^{\mathbf{CT}} - \sum_{t=1}^T \gamma_i \mathbf{R}_{i[t]}^{\mathbf{DT}} \leq \sum_{t=1}^T \mathbf{c}_{0,-i[t]}^{\mathbf{CT}}. \quad (4.3.29)$$

This setting makes each SU to cautiously decide on whether or not to use multi-hop relaying and hence avoid the scenarios that some SUs impulsively request the relaying service of others simply because they want to spend the revenues earned by selling their relaying services during previous time slots.

As observed in [52], a NE is generally not the optimal solution. However, it can be shown that, if the condition in result 2) is satisfied, applying the multi-hop relaying to both S_1 and S_2 will be the unique NE as well as the payoff maximization solution of our game. Let us summarize this observation in the following Lemma.

Lemma 4.3.1. *Assume each SU fixes its transmission scheme during T time slots of transmission. Using multi-hop relaying for both S_1 and S_2 is the optimal NE of symmetric SSS based CR networks if the improvement of the weighed achievable rate $\gamma_i \Delta \hat{\mathbf{R}}_{i[t]}^{\mathbf{CT}-\mathbf{DT}}$ for S_i is larger than the sum of the price paid to S_{-i} and the price charged by PU 0 from the relay (S_{-i}) for $i \in [1, 2]$, i.e.,*

$$\gamma_i \Delta \hat{\mathbf{R}}_i^{\mathbf{CT}-\mathbf{DT}} > \hat{\mathbf{c}}_{\mathbf{X},i}, \quad (4.3.30)$$

where $\hat{\mathbf{c}}_{X,i} = \sum_{t=1}^T \left(c_{0,-i[t]}^{\mathbf{CT}} + c_{-i,i[t]}^{\mathbf{CT}} \right)$.

4.4 Extending to Large CR Networks

In this section, we extend the results in Section 4.3 into a general CR network model in which K sources transmit signals to the corresponding destinations over the licensed spectrum. To simplify our illustration, we mainly focus on the operations of one source-to-destination pair, labeled by S_1 to D_1 , and assume there are $L - 1$ intermediate SUs, labeled as $S_2, S_3, \dots, S_L$ for $L \leq K$, available to decode-and-forward signals for S_1 . Note that these intermediate SUs are also secondary sources which have their own information to transmit. Therefore, if S_1 wants to exploit intermediate SUs to relay its signals, it needs to pay prices to each of these SUs and returns the favor when necessary. Let the set of all SUs who require the relaying service of S_1 in time slot t be $\tilde{\mathcal{L}}_{1[t]}$ for $|\tilde{\mathcal{L}}_{1[t]}| \leq K - 1$. Following the network model and game theoretic notations introduced in Section 5.2, we define the revenue of S_1 to be

$$\hat{r}_{1,1}^{CT} = \sum_{t=1}^T r_{1,1[t]}^{CT} = \sum_{t=1}^T \gamma_1 \mathbf{R}_{1[t]}, \quad (4.4.1)$$

where γ_1 is a constant. Due to space limitation, let us only provide the results for the linear pricing function case as discussed in Subsection 4.3.2. Define the revenue of S_1 , earned by being relays for other SUs, as

$$\hat{r}_{1,\tilde{\mathcal{L}}_1}^{CT} = \sum_{t=1}^T r_{1,\tilde{\mathcal{L}}_1[t]}^{CT} = \sum_{t=1}^T \sum_{i \in \tilde{\mathcal{L}}_{1[t]}} \tilde{\beta}_{1,i} w_{1,i[t]}, \quad (4.4.2)$$

where $\tilde{\beta}_{1,i}$ is a positive constant. The prices charged by intermediate SUs from S_1 are

$$\hat{c}_{L,1}^{CT} = \sum_{t=1}^T c_{L,1[t]}^{CT} = \sum_{t=1}^T \sum_{i=1}^L \beta_{i,1} w_{i,1[t]}, \quad (4.4.3)$$

where $\bar{L} = [2, 3, \dots, L]$. If S_1 uses multi-hop relaying, it also needs to pay price to PU 0 for the relaying transmission of $S_2, S_3, \dots, S_L$. Define the price charged by PU 0 to be

$$\hat{c}_{0,1}^{CT} = \sum_{t=1}^T c_{0,1[t]}^{CT} = \sum_{t=1}^T \sum_{i=2}^L \tilde{\alpha}_{0,i} h_{S_i,0[t]} w_{i,1[t]}. \quad (4.4.4)$$

Following Assumption 4.2.2, we have $\hat{r}_{1,\tilde{\mathcal{L}}_1}^{CT} = \hat{c}_{\bar{L},1}^{CT}$. Using this setting, the payoff function of S_1 can be written as,

$$\begin{aligned} \pi_1^{CT} = & \sum_{t=1}^T \left[\gamma_1 \mathbf{R}_1[t] + \sum_{j \in \tilde{\mathcal{L}}_1[t]} \tilde{\beta}_{1,j} w_{1,j[t]} \right. \\ & \left. - \sum_{k=2}^L \beta_{k,1} w_{k,1[t]} - \sum_{l=1}^L \tilde{\alpha}_{0,l} h_{S_l,0[t]} w_{l,1[t]} \right]. \end{aligned} \quad (4.4.5)$$

In a CR network with K source-to-destination pairs, the main objective is to solve the following optimization problem:

$$\begin{aligned} \max_{w_{i,j[t]}} & \sum_{t=1}^T \sum_{i=1}^K \pi_i[t] \\ \text{s.t.} & \sum_{j=1}^K \sum_{i=1}^K E(h_{S_i,0[t]} w_{i,j[t]}) \leq q_0[t]. \end{aligned} \quad (4.4.6)$$

All results presented in the previous section can be directly applied into the above model. For example, let us consider the transmission from S_1 to D_1 with $(L-1)$ -hop relaying. By assuming the transmission of SUs in each time slot to be independent and transmit powers of SUs to satisfy $w_{i,1[t]} = f_{i-1[t]} w_{i-1,1[t]}$ and $w_{L,1[t]} = \frac{h_{S_{L-1},S_L[t]}}{h_{S_L,D_1[t]}} w_{L-1,1[t]}$ where $f_{i-1[t]} = \frac{h_{S_{i-1},S_i[t]}}{h_{S_i,S_{i+1}[t]}}$ for $i \in [2, L-1]$, the optimal transmit power for S_1 with full CSIs known by transmitters can be obtained as follows:

$$w_{1,1[t]} = \left(\frac{\gamma_1}{\lambda(h_{S_1,0[t]} + \sum_{i=2}^{L-1} h_{S_i,0[t]} f_{i-1})} - \frac{1}{h_{S_1,S_2[t]}} \right)^+, \quad (4.4.7)$$

where λ is a constant to ensure transmit powers of all SUs satisfying the subjective function in (4.4.6). Similarly, the optimal power allocation methods under other operations and assumptions discussed in Subsections 4.3.2 and 4.3.3 can be directly obtained. We omit the detailed results due to the limit of space.

It is observed that in large multi-user networks, the network structures and user interactions greatly increase the complexity of the optimization problem. More specifically, two new questions that are naturally raised for this network are:

Q1) How to manage and optimize the transmit powers for CR networks in a distributed fashion?

Q2) How to choose “proper” relays (in terms of the number and quality) for each secondary source-to-destination pair to maximize the payoff?

In the following subsections, we propose two distributed algorithms to solve the above problems. The first one is a distributed power allocation method and the other is a distributed relay selection algorithm.

4.4.1 A Distributed Power Allocation Algorithm

As discussed in Section 4.3.2, achieving the optimal payoff without allowing the transmitters to know the CSIs is a very challenging task. In this subsection, we assume the channel gains can be regarded as constants during each time slot and the transmission scheme selected by each SU is fixed. A distributed power allocation algorithm, denoted as Algorithm 2, is presented in Figure 4.1. The proposed algorithm does not require the CSIs to be known by transmitters or SUs to communicate with each other. Following the same methods in [56], it can be proved that, by using Algorithm 2, the payoffs of SU networks approaches to a NE if the

Consider the transmission in time slot t with B time segments,

1) Initialization Step:

S_i chooses the initial powers $w_{i,i[t]}(1)$ and $w_{i,i[t]}(2)$ to transmit signals $x_{i,i[t]}(1)$ and $x_{i,i[t]}(2)$ during the first two time segments. After receiving signals $x_{i,i[t]}(1)$ and $x_{i,i[t]}(2)$, D_i feedbacks the revenue $r_{i,i[t]}(1)$ and $r_{i,i[t]}(2)$ to S_i . PU 0 and relaying SUs also send the prices $c_{0,i[t]}(1)$, $c_{0,i[t]}(2)$, $c_{k,i[t]}(1)$ and $c_{k,i[t]}(2)$ to S_i for $k \neq i$, $i, k \in [1, K]$. At the end of the second segment, S_i calculates payoffs $\pi_{i[t]}(1)$ and $\pi_{i[t]}(2)$.

2) Iteration Step: For $l = 3: B$,

- If the power constraint satisfies (1), S_i sends signal $x_{i,i[t]}(l)$ using power $w_{i,i[t]}(l) = (w_{i,i[t]}(l-1) + v(l-1)g_{i,i[t]}(l-1))^+$ where $(a)^+ = \max\{0, a\}$, $g_{i,i[t]}(l)$ is the sub-gradient of $\pi_{i[t]}$ which can be calculated by $\pi_{i[t]}(w_{i,i[t]}(l)) \geq \pi_{i[t]}(w_{i,i[t]}(l-1)) + g_{i,i[t]}(l)(w_{i,i[t]}(l) - w_{i,i[t]}(l-1))$, and $v(l) = \frac{u}{\sqrt{l}}$ where u is the iteration step size.
- If PU 0 detects higher than tolerable interference power (Power constraint in (1) is not satisfied), it sends the necessary information for each SU to calculate the projection of its transmit power $w_{i,i[t]}(l-1)$ on the power constraint in (4.2.1), i.e., $w_{i,i[t]}(l)$ is given by

$$w_{i,i[t]}(l) = \mathcal{P}(\mathbf{w}_{[t]}(l-1))^+$$

where $\mathbf{w}_{[t]}(l-1) = [w_{1,1[t]}(l-1), w_{2,2[t]}(l-1), \dots, w_{K,K[t]}(l-1)]^T$ and $\mathcal{P}(\mathbf{a})$ denotes the projection function of $\mathbf{a}$ on (4.2.1) defined by

$$\mathcal{P}(\mathbf{a}) = \mathbf{a} - \mathbf{h}^T(\mathbf{h}\mathbf{h}^T)^{-1}(\mathbf{h}\mathbf{a} - Q_0)$$

where $\mathbf{h} = [h_{S_1,0}, h_{S_2,0}, \dots, h_{S_K,0}]$.

End

3) Termination Step: The above process continues until $l = B$.

Figure 4.1: Illustration of Algorithm 2.

Figure 4.2: Convergence rate of Algorithm 2 with different step sizes.

number of iterations is large enough. In Figure 4.2, we show the time average payoff of S_1 under different numbers of iterations. It is observed that the convergence performance of Algorithm 2 is closely related to the iteration step size u . Generally speaking, if the initial powers are far from the optimal values, the larger the value of u , the faster rate of convergence to the payoff of a NE. However, if u is too large, the average payoff will be fluctuated during the first few iterations.

4.4.2 A Distributed Relay Selection Algorithm

Following the same setting as Algorithm 2, channel gains are assumed to be constants and the number of SUs who have positive powers is fixed in each time slot. Let us divide the time duration of each time slot into B small segments. Define

Consider the transmission in time slot t with B time segments,

- 1) Initialization Step: During the first segment, each SU (i.e., S_i) chooses a strategy (transmission scheme) $s_{i[t]}(1) = j$ to send signals. D_i feedbacks a successful-decoding message to S_i after the decoding process finished. Assume this feedback message can be eavesdropped by the TS to make sure all signals sent by secondary sources to be successfully decoded by their destinations in each time slot. S_i , after receiving the successful-decoding message from D_i and the pricing functions from relaying SUs and PU 0, calculates $\pi_{i[t]}(s_{i[t]}(1) = j)$ for its strategy j . We use $\mathcal{S}$ to denote the set of all the possible strategies that can be chosen by S_i . Assume S_i chooses the initial value of $\mathbf{Q}$ -function as $\mathbf{Q}_{S_i[t]}(s_{i[t]}(1) = j) = 0$.
- 2) Iteration Step: For $l = 2: B$,
 - S_i updates the value of $\mathbf{Q}$ -function as follows,

$$\mathbf{Q}_{S_i[t]}(s_{i[t]}(l) = j) = \mathbf{Q}_{S_i[t]}(s_{i[t]}(l-1) = k) + \Delta_{[t]}(l)(\pi_{i[t]}(s_{i[t]}(l-1) = k) \Pr(s_{i[t]}(l-1) = k))$$
 where $\Delta_{[t]}(l)$ is the iteration step size defined by $\Delta_{[t]}(l) = \frac{\Delta_{0[t]}}{l}$ and $\Delta_{0[t]}$ is a constant in time slot t .
 - S_i randomly chooses its strategies $j \in \mathcal{S}$ according to the Boltzmann distribution,

$$\Pr(s_{i[t]}(l) = j) = \frac{\exp(\mathbf{Q}_{S_i[t]}(s_{i[t]}(l) = j)/\rho)}{\sum_{k \in \mathcal{S}} \exp(\mathbf{Q}_{S_i[t]}(s_{i[t]}(l) = k)/\rho)}$$
 where $\Pr(s_{i[t]}(l) = j)$ is the probability of S_i choosing strategy j in segment l .
- 3) Termination Step: The above process stops if $l = B$.

Figure 4.3: Illustration of Algorithm 3.

the $\mathbf{Q}$ -function for S_1 in segment l of time slot t as $\mathbf{Q}_{S_1[t]}(s_1(l)) = E[\pi_{1[t]}(s_1(l))]$, where s_1 denotes the transmission scheme selected by S_1 , i.e., which and how many nearby SUs should be chosen as relays of S_1 . Assume there are L intermediate SUs located between S_1 and D_1 , and hence there are $2^{(L-1)} - 1$ strategies can be chosen. The operations of Algorithm 3 are described in Figure 4.3. Note that this algorithm is similar to Robbins-Monro algorithm [67], and hence $\mathbf{Q}_{S_1[t]}$ will converge to the average payoff of strategy s_1 . Each SU will be more likely to choose the scheme with the highest $\mathbf{Q}$ value which will eventually approach to a NE-achieving relaying scheme [68]. The convergence proof is similar to that of Robbins-Monro algorithm and hence is omitted here. The convergence rate of Algorithm 3 is closely related to the value of ρ . In Figure 4.4, we consider the 2 source-to-destination pair case and present the convergence performance of the probability of S_1 choosing different schemes (either direct transmission or multi-hop relaying). Note that in the proposed Algorithm, the value of ρ controls the frequency of the exploration, i.e., the smaller value of ρ , the higher probability for S_i to choose the high $\mathbf{Q}$ value strategy. This explains the observation in Figure 4.4 that the convergence rate of Algorithm 3 is decreased with ρ .

4.5 Numerical Results

As discussed in Section 4.4, a large multi-user CR network can be regarded as an extended version of the network with 2 source-to-destination pairs. Therefore, in this section, we will first present numerical results for the CR network model discussed in Section 4.3 and leave the results of K source-to-destination pair case to the end of this section.

Denote the distance between two users as $d_{i,j}$ for $i, j \in \{S_1, S_2, D_1, D_2, 0\}$. Let

Figure 4.4: Convergence rate of Algorithm 3 with different values of ρ .

Figure 4.5: Numerical results for the payoff of S_1 in SSS based CR networks with multi-hop relaying

Figure 4.6: Numerical results for the payoff sum of S_1 and S_2 in SSS based CR networks with multi-hop relaying

Figure 4.7: Numerical results for the payoff of S_1 in SSS based CR networks with multi-hop relaying

Figure 4.8: Numerical results for SSS based CR networks with multi-hop relaying

Figure 4.9: Numerical results for the payoff sum of S_1 and S_2 in SSS based CR networks with multi-hop relaying

Figure 4.10: Numerical results for SSS based CR networks with L -hop relaying

us consider the following channel models,

$$h_{i,j} = \frac{\tilde{h}_{i,j}}{d_{i,j}^\delta}, \quad (4.5.1)$$

where $\tilde{h}_{i,j}$ is a Rayleigh distribution random variable unrelated to the distance of transmission and δ is the channel attenuation exponent. To simplify our illustration, we focus on two types of networks: 1) *symmetric networks* in which the locations of two secondary source-to-destination pairs are geometrically symmetric, i.e., $d_{S_1,D_1} = d_{S_2,D_2} = d_{S_1,S_2} + d_{S_2,D_1} = d_{S_2,S_1} + d_{S_1,D_2}$, and hence we fix d_{S_1,D_1} to study the payoff of S_1 under different d_{S_1,S_2} , 2) *asymmetric networks* in which d_{S_1,D_2} and d_{S_1,D_1} are fixed (i.e., $d_{S_1,D_2} = \frac{d_{S_1,D_1}}{2}$) and the payoff sum of S_1 and S_2 under different d_{S_1,S_2} are considered. Based on this setting, the difference between $E(h_{S_1,S_2})$ and $E(h_{S_2,D_1})$ (or between $E(h_{S_1,S_2})$ and $E(h_{S_1,D_2})$) will become small if d_{S_1,S_2} approaches to $\frac{d_{S_1,D_1}}{2}$, or large if d_{S_1,S_2} approaches to 0 or d_{S_1,D_1} .

Consider symmetric networks with a linear pricing function first. It is observed in Figure 4.5 that the payoff of S_1 using multi-hop relaying is closely related to the location of the relay. More specifically, if S_2 is located near the middle point between S_1 and D_1 , multi-hop relaying could greatly improve the performance of CR networks. However, the improvement decreases when d_{S_1,S_2} approaches to 0 or d_{S_1,D_1} . This is because, by following Assumption 4.3.1, if $E(h_{S_1,S_2})$ is much larger than $E(h_{S_2,D_1})$, S_2 will raise its forwarding power to ensure all received signals to be successfully decoded by D_1 , which also increases the price charged by PU 0. Note that the optimal location of S_2 which maximizes the payoff of S_1 can be obtained by substituting (4.5.1) into π_1^{CT} and maximizing the π_1^{CT} over d_{S_1,S_2} . The detailed derivation is omitted due to space limit. Another interesting result in Figure 4.5 is that if S_2 is located in the middle point between S_1 and D_1 , multi-hop relaying with fixed transmit powers can provide a higher payoff than the direct transmission with

optimal power allocation methods. In other words, multi-hop relaying can be a more useful tool than optimal power allocation methods to improve the performance of SSS based CR networks under certain conditions. Numerical results for asymmetric networks with linear pricing function are presented in Figure 4.6. It is observed that, the payoff sum of S_1 and S_2 is decreased with d_{S_1, S_2} if both secondary sources use direct transmission. However, when using multi-hop relaying, the payoff sum can be greatly improved, especially when d_{S_1, S_2} approaches to $\frac{d_{S_1, D_1}}{2}$.

Note that the value of payoff in our setting can be negative. For some systems, this could mean the price is too high for SUs to use the licensed spectrum or some specific strategies. In this case, SUs should either use alternative strategies and optimization methods, such as multi-hop relaying, optimal power allocation methods, etc., to increase their payoffs, or stop sending signals over the licensed spectrum.

As discussed in Lemma 4.3.1, using multi-hop relaying for both secondary sources achieves the performance of the optimal NE if the weighed achievable rate gain $\gamma_i \Delta \mathbf{R}_i^{\text{CT-DT}}$ for S_i is larger than the price $\hat{c}_{X,i}$ charged by PU 0 and relays for $i \in [1, 2]$. Therefore, we present numerical results for the symmetric CR network in Figure 4.7 in which $\gamma_1 \Delta \mathbf{R}_1^{\text{CT-DT}}$ defined in the left-hand-side of (4.3.30) and $\hat{c}_{X,1}$ defined in the right-hand-side of (4.3.30) are compared. It is observed that multi-hop relaying can always be a NE-achieving scheme if the location of relay is close to the middle point between source and destination, i.e., $d_{S_1, S_2} \approx \frac{d_{S_1, D_1}}{2}$. However, $\gamma_1 \Delta \mathbf{R}_1^{\text{CT-DT}}$ will be exceeded by $\hat{c}_{X,1}$ if the relay is located closely to either source or destination.

Let us consider CR networks with quadratic pricing functions. Figure 4.8 shows the payoff of S_1 when the network is symmetric. It is observed that with the increasing of d_{S_1, S_2} , the payoff of S_1 with multi-hop relaying changes in a much faster speed than that in the linear pricing function case. This is because a part of the

price charged by PU 0 from the relay is a second degree polynomial function of the forwarding power which, as mentioned in Assumption 2, should be proportional to $\frac{h_{S_1,S_2}[t]}{h_{S_2,D_1}[t]}$. In addition, similar to the linear pricing function case, multi-hop relaying cannot provide a higher payoff than that of direct transmission when the relay is close to either the source or destination. For the asymmetric network, it is observed in Figure 4.9 that the payoff sum is decreased with d_{S_1,S_2} if both S_1 and S_2 use direct transmission because the revenue of S_2 is decreased with $d_{S_2,D_2} = d_{S_1,D_2} + d_{S_1,S_2}$. However, multi-hop relaying is an effective way to alleviate the payoff decreasing process caused by the increasing of d_{S_1,D_1} especially when the relay approaches to the middle point between the source and destination.

In Figure 4.10, we consider CR networks with multiple source-to-destination pairs and assume there are $L - 1$ intermediate SUs located between S_1 and D_1 . Similar to the 2 source-to-destination pair case, it is shown that $(L - 1)$ -hop relaying provides a great performance improvement for CR networks. It is also observed that there is a tradeoff between the number of relays and payoff of S_1 . More specifically, with the increasing of L , the distance of each hop is reduced, which increases the revenue (or achievable rate). On the other hand, the time duration used for S_1 to transmit the source information will be shorten, which decreases the revenue.

4.6 Conclusion

This chapter has exploited the game theoretic model to study the optimization problem for SSS based multi-user CR networks with multi-hop relaying. In the considered system, each secondary source can obtain benefits through both spectrum sharing by paying prices to PUs and multi-hop relaying by paying prices to the relaying SUs. The payoff of secondary users under different settings and different

types of pricing functions of primary users have been studied. Furthermore, we have proposed three power allocation methods corresponding to the assumptions that transmitters have average knowledge, full knowledge and current knowledge of channel gains. The NE for the proposed game under different settings have been discussed and the conditions for which applying multi-hop relaying to all secondary users achieves the optimal NE have been considered. Our results are extended into the multi-user CR network model with K secondary source-to-destination pairs. Two distributed algorithms have been proposed for each secondary source to search for the NE-achieving power allocation method and relaying scheme, respectively. Numerical results are presented to show the performance of CR networks with multi-hop relaying in both symmetric and non-symmetric network cases.

4.7 Appendix

4.7.1 Optimal Power Allocation Methods for Direct Transmission

Consider the network model in Section 5.2. Assuming the transmissions for each secondary source during different time slots are independent with each other. Our objective is to maximize the payoff sum of SUs during the t th time slot. More specifically, we need to find the solution of the following optimization problem:

$$\begin{aligned} \max_{w_{1,1[t]}, w_{2,2[t]}} \quad & \pi_{1[t]}^{DT} + \pi_{2[t]}^{DT} \\ \text{s.t.} \quad & w_{1,1[t]} E(h_{S_1,0[t]}) + w_{2,2[t]} E(h_{S_2,0[t]}) \leq q_{0[t]}. \end{aligned} \tag{4.7.1}$$

If the SNR is low, we have $E(\log(1 + SNR)) \approx \log(1 + E(SNR))$. Therefore,

the Lagrange multiplier of (4.7.1) is given by

$$\begin{aligned}
LM^{DT} = & \gamma_1 \log(1 + E(h_{S_1, D_1[t]})w_{1,1[t]}) \\
& + \gamma_2 \log(1 + E(h_{S_2, D_2[t]})w_{2,2[t]}) - \tilde{\alpha}_{0[t]}E(h_{S_1, 0[t]})w_{1,1[t]} \\
& - \tilde{\alpha}_{0[t]}E(h_{S_2, 0[t]})w_{2,2[t]} + \lambda'_t (q_{0[t]} - w_{1,1[t]}E(h_{S_1, 0[t]}) \\
& + w_{2,2[t]}E(h_{S_2, 0[t]})) .
\end{aligned} \tag{4.7.2}$$

We have KKT conditions of the above equation as follows,

$$\frac{\partial LM^{DT}}{\partial w_{1,1[t]}} = 0 \Rightarrow w_{1,1[t]} = \left(\frac{\gamma_1}{\lambda E(h_{S_1, 0[t]})} - \frac{1}{E(h_{S_1, D_1[t]})} \right)^+, \tag{4.7.3}$$

$$\frac{\partial LM^{DT}}{\partial w_{2,2[t]}} = 0 \Rightarrow w_{2,2[t]} = \left(\frac{\gamma_2}{\lambda E(h_{S_2, 0[t]})} - \frac{1}{E(h_{S_2, D_2[t]})} \right)^+. \tag{4.7.4}$$

where $\lambda_{[t]} = \alpha_{0[t]} + \lambda'_{[t]}$.

By substituting (4.7.3) and (4.7.4) into the subjective function in (4.7.1), we can show that λ needs to satisfy

$$\lambda_{[t]} \geq \frac{\gamma_1 + \gamma_2}{q_{0[t]} + \frac{E(h_{S_1, 0[t]})}{E(h_{S_1, D_1[t]})} + \frac{E(h_{S_2, 0[t]})}{E(h_{S_2, D_2[t]})}} \tag{4.7.5}$$

if the transmit powers of both S_1 and S_2 are positive.

The optimal power allocation methods for the case that transmitters have full CSIs can be obtained by replacing the average values with instantaneous value of CSIs.

4.7.2 Optimal Power Allocation Method for Both S_1 and S_2 Using Multi-hop Relaying

Following the same line as Appendix 4.7.1, let us consider the case that both S_1 and S_2 use multi-hop relaying. Using Assumption 4.3.1, we can write the optimization problem as follows:

$$\begin{aligned} & \max_{w_{1,1[t]}, w_{2,2[t]}} \pi_{1[t]}^{CT} + \pi_{2[t]}^{CT} \\ & \text{s.t. } \frac{1}{2} [E(h_{S_1,0[t]}w_{1,1[t]}) + E(h_{S_2,0[t]}w_{2,2[t]}) \\ & \quad + E(h_{S_1,0[t]}f_{2[t]}w_{2,2[t]}) + E(h_{S_2,0[t]}f_{1[t]}w_{1,1[t]})] \leq q_{0[t]} \end{aligned} \quad (4.7.6)$$

Similar to Appendix 4.7.1, we only consider the low SNR case and re-write the objective function of (4.7.6) as follows:

$$\begin{aligned} \pi_{1[t]}^{CT} + \pi_{2[t]}^{CT} &= \frac{\gamma_1}{2} \log(1 + E(h_{S_1,S_2[t]})w_{1,1[t]}) \\ &+ \frac{\gamma_2}{2} \log(1 + E(h_{S_2,S_1[t]})w_{2,2[t]}) - \tilde{\alpha}_{0[t]}E(h_{S_1,0[t]})w_{1,1[t]} \\ &- \tilde{\alpha}_{0[t]}E(h_{S_2,0[t]})w_{2,1[t]} - \tilde{\alpha}_{0[t]}E(h_{S_2,0[t]})w_{2,2[t]} \\ &- \tilde{\alpha}_{0[t]}E(h_{S_1,0[t]})w_{1,2[t]}. \end{aligned} \quad (4.7.7)$$

Solving the Lagrange multiplier problem, we have the following KKT conditions:

$$\begin{aligned} w_{1,1[t]} &= \left(\frac{\gamma_1}{\lambda [E(h_{S_1,0[t]}) + E(h_{S_2,0[t]}f_{1[t]})]} - \frac{1}{E(h_{S_1,S_2[t]})} \right)^+, \\ w_{2,2[t]} &= \left(\frac{\gamma_2}{\lambda [E(h_{S_2,0[t]}) + E(h_{S_1,0[t]}f_{2[t]})]} - \frac{1}{E(h_{S_2,S_1[t]})} \right)^+, \end{aligned}$$

Substituting the above equations into the subjective function in (4.7.6), we can have

the condition of λ as

$$\lambda \geq \frac{\gamma_1 + \gamma_2}{2q + \frac{E(h_{S_1,0[t]}) + E(h_{S_2,0[t]} f_{1[t]})}{E(h_{S_1,S_2[t]})} + \frac{E(h_{S_2,0[t]}) + E(h_{S_1,0[t]} f_{2[t]})}{E(h_{S_2,S_1[t]})}} \quad (4.7.8)$$

if both $w_{1,1[t]}$ and $w_{2,2[t]}$ are positive.

The optimal power allocation methods for the case that transmitters have full CSIs can be similarly obtained by replacing the average values with instantaneous value of CSIs.

Chapter 5

Distributed Optimization for OFDMA-based Cognitive Radio Networks

5.1 Introduction

As mentioned in previous chapters, two main techniques, TSS and SSS, have been introduced for CR networks to solve the spectrum under-utilization problems. In TSS based CR networks, how to optimally allocate the available sub-bands to all SUs is a very important problem. Typically, sub-band allocation requires communication (i.e., competitions or cooperations) among SUs [37, 63], which is not possible for many practical systems. In SSS based CR networks, as shown in the previous chapters of this thesis, the performance of SUs can be greatly improved by allowing the transmit powers of SUs to be adapted to the CSIs. In this chapter, we establish an OFDMA-based CR network model in which the SUs distributedly compete with each other for the available sub-bands in the spectrum of multiple PUs. To further

improve the efficiency of the SU network, we allow multiple SUs to share the same sub-band and cooperate with each other to send and receive signals. In our setting, the PUs do not need to know any information about the SU network (e.g., the number of SUs and how the spectrum is divided among them), or send specific information to any of the SUs.

The main focus of this chapter is to study the optimization problem for an OFDMA-based cooperative CR network. More specifically, we study how to maximize the performance of both SU and PU networks by jointly optimizing three parameters: the transmit powers of SUs, sub-band allocations of SUs and the price charged by PUs. Simultaneously optimizing these three parameters is in general complex, and hence most previous work in the literature only focuses on the optimization of one or two of these parameters. In this chapter, we show that it is actually possible to simultaneously optimize all these parameters. More specifically, we establish a coalition formation game [69] to study the possible cooperation/competition among SUs in the sub-bands. We show that the core of the grand coalition in our coalition formation game is always empty and hence how to let the SUs to distributedly form different coalitions in each sub-band is a challenging problem. We then fit the coalition formation game into a Stackelberg game-based hierarchical framework to jointly optimize the transmit powers and sub-bands selected by SUs and the pricing functions of PUs. In our framework, SUs optimize their transmit powers and compete for the sub-bands to maximize their data rate and simultaneously minimize the price paid to PUs for using the licensed spectrum, while each PU adjusts the pricing function to optimize its revenue and simultaneously limit the interference caused by SUs. We derive the Nash equilibrium (NE) for the power control game of SUs and the price adjustment game of PUs. We also present the Stackelberg equilibria (SE) of the hierarchical game. We observe that the SE changes with the sub-band allocation scheme of SUs and hence focus on the

sub-band allocation problem for SUs. We propose a simple distributed algorithm for SUs to search for the NE-achieving sub-band allocation scheme. More importantly, we prove that if the pricing coefficients of all PUs satisfy a fixed linear relationship, the optimal sub-band allocation scheme for SUs is unique and stable and a unique SE can always be found for the hierarchical game. A simple distributed algorithm has been proposed to achieve the SE. Comparing to the previously reported results, our proposed algorithm does not require SUs to know the interference temperature limit of each PU and the number of iterations is unrelated to the number of SUs.

The rest of this chapter is organized as follows. The network model and problem formulation are discussed in Section 5.2. Game theoretic analysis of the proposed model is given in Section 5.3. The joint optimization algorithm is presented in Section 5.4. Numerical results are presented in Section 5.5. We summarize the presented work and offer concluding remarks in Section 5.6.

5.2 Network Model and Problem Formulation

In this section, we first describe the network model and then formulate the game theoretic model for the OFDMA-based CR network. Finally, we provide formal mathematical descriptions of three main problems and related games studied in this chapter. We will present the solution for each of these problems in the next section.

5.2.1 Network Model

Consider a CR network model (see Figure 5.1) in which there are J PUs, $P_1, P_2, \dots, P_J$, and K secondary source-to-destination pairs, S_1 to D_1 , S_2 to D_2 , $\dots$, S_K to D_K . We assume that PUs and SUs do not have any a priori knowledge

Figure 5.1: Network model for OFDMA-based CR networks.

of each other's spectrum utilization. SUs divide the licensed spectrum into M sub-bands and each SU can only access one sub-band. The sub-band division scheme is decided by the SU network and is unknown to PUs.

Let $l_{S_k} \in \{1, 2, \dots, M\}$ denote the sub-band allocated to S_k . A sub-band allocation scheme for SUs is denoted as $\mathbf{l}_S = [l_{S_1}, l_{S_2}, \dots, l_{S_K}]$. In our setting, multiple SUs can simultaneously access one sub-band. Let the set of SUs who share the same sub-band m be $\mathcal{L}_m$, i.e., $\mathcal{L}_m = \{S_k : l_{S_k} = m, \forall k = 1, 2, \dots, K\}$. Therefore, $\mathcal{L}_m = \emptyset$ if no SUs use sub-band m , $\mathcal{L}_m = \{S_k\}$ if S_k is the only SU to access sub-band m , and $|\mathcal{L}_m| \geq 2$ if two or more SUs share the same sub-band m . Let the channel gain from S_k to P_j in the sub-band l be $h_{jk}[l]$. If it is clear from the context that a specific sub-band allocation scheme has been fixed, we drop the subscript l and use the notation h_{jk} to denote the channel gain between S_k and P_j .

In this chapter, we assume that the transmission of each PU spans the entire licensed spectrum and hence each PU is affected by the transmission of all K SUs. This model subsumes some of the previously studied scenarios in the literature. For example, if we assume $h_{jk} = 0$ for all $k \in \{2, 3, \dots, K\}$ and $j \in \{1, 2, \dots, J\}$, the system model under our consideration is equivalent to the case of one PU (i.e., PU P_1) with equal utilization efficiency in every sub-band of the licensed spectrum [41]. If we assume $J = K$ and $h_{jk} = 0$ for $j \neq k$, the system model under our consideration becomes the sub-band competition problem discussed in [43].

Let the transmit power of S_k be w_{S_k} , and the transmit power vector of K SUs be $\mathbf{w}_S = [w_{S_1}, w_{S_2}, \dots, w_{S_K}]^T$. The channel gains between SUs and PUs are expressed

in the matrix form, i.e.,

$$\mathbf{H} = \begin{bmatrix} h_{11}[l_{S_1}] & h_{12}[l_{S_2}] & \dots & h_{1K}[l_{S_K}] \\ h_{21}[l_{S_1}] & h_{22}[l_{S_2}] & \dots & h_{2K}[l_{S_K}] \\ \vdots & \vdots & \ddots & \vdots \\ h_{J1}[l_{S_1}] & h_{J2}[l_{S_2}] & \dots & h_{JK}[l_{S_K}] \end{bmatrix}. \quad (5.2.1)$$

Let the maximum interference power that can be tolerated by P_j be q_{P_j} for $j \in \{1, 2, \dots, J\}$. We require that the following power constraint

$$\mathbf{H}\mathbf{w}_S \leq \mathbf{q}_P \quad (5.2.2)$$

holds for $\mathbf{q}_P = [q_{P_1}, q_{P_2}, \dots, q_{P_J}]^T$.

In our model, the secondary sources and destinations sharing the same sub-band are allowed to cooperate with each other to send and receive signals. We assume that channel gains among the sources or among the destinations are much higher than those between the sources and destinations and hence the transmission of multiple SUs in one sub-band m can be modeled as a virtual $|\mathcal{L}_m|$ -input $|\mathcal{L}_m|$ -output MIMO channel [69]. In addition, the SUs can use optimal power control methods to maximize their benefits and simultaneously minimize the total price charged by PUs.

5.2.2 Hierarchical Game Setup

Let us consider a hierarchical game in which players of the game are the PUs (leader) who have priority in using the spectrum, and the SUs (follower) who can access the licensed spectrum by paying a certain “price”. Price is introduced and used by the PUs to regulate, in a distributed manner, the transmit powers and

the sub-band allocation of the SUs, so as to achieve an optimal trade-off between spectrum utilization and the interference at the PUs.

We follow the standard setup [69–74] to define the *benefit* of each SU S_k to be equal to its data rate. In particular, for a given sub-band allocation scheme $\mathbf{l}_S$, the benefit to the SU S_k , $k = 1, \dots, K$, is given by $r_{S_k}[l_{S_k}] = R_{S_k}(\mathbf{l}_S, \mathcal{F}_{S_k})$ where R_{S_k} is the data rate of S_k , and $\mathcal{F}_{S_k}$ is the fairness criterion agreed to by S_k when accessing sub-band l_{S_k} . If S_k is the only SU to use the sub-band l_{S_k} , we let $\mathcal{F}_{S_k} = \emptyset$, and $R_{S_k}(\mathbf{l}_S, \mathcal{F}_{S_k}) = R_{S_k}(l_{S_k}) = \log(1 + g_{kk}[l_{S_k}]w_{S_k})$ is the channel capacity between the k th source-to-destination pair, where $g_{kk} = g_{kk}[l_{S_k}]$ is the ratio of the channel gain between the k th secondary source-to-destination pair to the interference power received by S_k in the sub-band l_{S_k} . If more than two SUs use the sub-band m , $R_{S_k}(\mathbf{l}_S, \mathcal{F}_m)$ is the transmission rate being allocated to the k th source-to-destination pair for $S_k \in \mathcal{L}_m$. To capture the difference of the contributions to the payoff sum brought by different SUs in a coalition, in this chapter, we consider the proportional fairness (PF) criterion [75], i.e., $\mathcal{F}_m = \{\text{PF}\}$. Another reason for considering this fairness criterion is that, as observed in [70, 75, 76], PF corresponds to the Nash bargaining solution [77, Chapter 8] if the minimum payoff required for each SU equals zero.

Since the main effect of SUs on the PU network is to increase its interference level, let us define the price charged by each PU P_j to the SUs to be proportional to the resulting interference observed by P_j , i.e., the *cost* of SU S_k is given by $c_{S_k} = \boldsymbol{\beta}_P^T \mathbf{h}_{\bullet k} w_{S_k}$ where $\boldsymbol{\beta}_P = [\beta_{P_1}, \beta_{P_2}, \dots, \beta_{P_J}]^T$ and β_{P_j} is the pricing coefficient of P_j , which will be discussed in the next subsection, and $\mathbf{h}_{\bullet k} = [h_{1k}, h_{2k}, \dots, h_{Jk}]^T$ is the k th column of $\mathbf{H}$. We define the *payoff* of S_k to be the difference between its benefit and cost, which is given by

$$\pi_{S_k}(w_{S_k}, \boldsymbol{\beta}_P, \mathbf{l}_S, \mathcal{F}_{S_k}) = r_{S_k} - c_{S_k}. \quad (5.2.3)$$

Note that each SU S_k needs to know $\mathbf{h}_{\bullet k}[l_{S_k}]$, and β_P . Each SU needs to decode the pricing function allocated by PUs and hence can use this information to estimate $h_{jk}[l_{S_k}]$ for $j = 1, 2, \dots, J$. To simplify our description, we assume each SU can always reliably decode the price of PUs. Note that SUs do not need to communicate with each other or know the interference temperature limit q_{P_k} of each P_k , $k = 1, \dots, J$.

Let us define the *payoff* of each PU as the sum of all price paid by the SUs and hence we can express the payoff of P_j , $j = 1, \dots, J$, as [51, 78]

$$\pi_{P_j}(\mathbf{w}_S, \beta_{P_j}, \mathbf{l}_S) = \beta_{P_j} \mathbf{h}_{j\bullet} \mathbf{w}_S, \quad (5.2.4)$$

where $\mathbf{h}_{j\bullet} = [h_{j1}, \dots, h_{jK}]$ is the j th row of the channel gain matrix $\mathbf{H}$ in (5.2.1). Note that the pricing coefficient β_{P_j} of P_j quantifies the willingness of P_j to sell its licensed spectrum. For example, if P_j can tolerate relatively high interference, β_{P_j} should be set to a low value to make the licensed spectrum “affordable” to more SUs. However, if P_j is more sensitive to interference, P_j should set a relatively high β_{P_j} . To calculate its payoff function, each PU only needs to measure its overall received interference power. Note that P_j does not need to know if the SU is in a coalition or using a single sub-band, because it applies the same pricing coefficient for all SUs.

To simplify notations, in the rest of this chapter, we will drop the term $\mathbf{l}_S$ in the payoff functions, and write $\pi_{S_k}(w_{S_k}, \beta_P, \mathcal{F}_{S_k})$ and $\pi_{P_j}(\mathbf{w}_S, \beta_{P_j})$ instead when it is clear from the context that the sub-band allocation is pre-defined.

In this chapter, we seek a balanced point for the SU networks, called Nash Equilibrium (NE) [52, Definition 23.1]. In Section 5.3, we will show that the optimization problems for both PU and SU networks are linked to one another via the pricing function of PUs. Each PU selfishly attempts to maximize its own payoff, resulting

in competition among the PUs. Also, SUs try to optimize their transmission parameters for the given price of PUs. In this chapter, we also seek an equilibrium point of this hierarchical game, from which every player has no incentive to deviate. This equilibrium point is known as the Stackelberg equilibrium (SE) [79, Definition 3.27], whose formal definition is given as follows.

Definition 5.2.1. [79, Definition 3.27] For a fixed $\mathbf{l}_S$, the pricing coefficients $\boldsymbol{\beta}_P^* = [\beta_{P_1}^*, \dots, \beta_{P_J}^*]^T$, and the transmit powers $w_{S_k}^* = w_{S_k}^*(\boldsymbol{\beta}_P^*)$, $k = 1, \dots, K$, form an SE if the interference power limit (5.2.2) is satisfied, and for each $j \in \{1, \dots, J\}$, we have

$$\beta_{P_j}^* = \arg \max_{\beta_{P_j} \geq 0} \pi_{P_j} \left(\mathbf{w}_S^*(\beta_{P_j}, \boldsymbol{\beta}_{-P_j}^*), \beta_{P_j}, \boldsymbol{\beta}_{-P_j}^*, \mathbf{l}_S \right), \quad (5.2.5)$$

where for any given $\boldsymbol{\beta}_P$,

$$w_{S_k}^*(\boldsymbol{\beta}_P) = \arg \max_{w_{S_k} \geq 0} \pi_{S_k} (w_{S_k}, \boldsymbol{\beta}_P, \mathbf{l}_S, \mathcal{F}_{S_k}). \quad (5.2.6)$$

5.2.3 Problem Formulation

This chapter considers the following problems.

Sub-band Allocation/Coalition Formation Problem for SUs

We assume each SU can only choose one sub-band. Hence, each SU can either exclusively occupy a sub-band or share the same sub-band with other SUs. The main objective of each SU is to maximize its payoff by solving the following optimization problem,

$$\max_{l_{S_k} \in \{1, 2, \dots, M\}} \pi_{S_k} (w_{S_k}, \boldsymbol{\beta}_P, l_{S_k}, \mathcal{F}_{S_k} | \mathbf{l}_{-S_k}). \quad (5.2.7)$$

In this case, the sub-band allocation problem can be modeled as a coalition formation game in which more than one SU form a coalition to share a sub-band if this could maximize their payoff. Let us give a formal definition as follows.

Definition 5.2.2. [77, Chapter 9] A coalition $\mathcal{C}$ is a non-empty sub-set of the total set of players $\mathcal{K}$, i.e., $\mathcal{C} \subseteq \mathcal{K}$. We refer to the coalition of all the players as the grand coalition $\mathcal{K}$. A coalitional game is defined by the pair $(\mathcal{K}, \nu)$ where ν is called the characteristic function, which assigns a number $\nu(\mathcal{C})$ to every coalition $\mathcal{C}$ and $\nu(\emptyset) = 0$. Here $\nu(\mathcal{C})$ quantifies the worth of a coalition $\mathcal{C}$. A coalitional game is said to be super-additive if for any two disjoint coalitions $\mathcal{C}^1$ and $\mathcal{C}^2$, $\mathcal{C}^1 \cap \mathcal{C}^2 = \emptyset$ and $\mathcal{C}^1, \mathcal{C}^2 \subset \mathcal{K}$, we have

$$\nu(\mathcal{C}^1 \cup \mathcal{C}^2) \geq \nu(\mathcal{C}^1) + \nu(\mathcal{C}^2). \quad (5.2.8)$$

In our setting, if the SU S_k obtains its maximized payoff by exclusively occupying a sub-band m , i.e., S_k chooses sub-band m if $\pi_{S_k}(w_{S_k}, \beta_P, l_{S_k} = m, \mathcal{F}_m = \emptyset | \mathbf{l}_{-S_k}) > \pi_{S_k}(w_{S_k}, \beta_P, l_{S_k} = n, \mathcal{F}_n | \mathbf{l}_{-S_k})$ for $m \neq n, m, n \in \{1, 2, \dots, M\}$. However, if an SU S_k can obtain the highest payoff by sharing a sub-band with other SUs according to the PF, the SU will negotiate with other SUs in the sub-band l_{S_k} to obtain its payoff, i.e., S_k chooses sub-band m if $\pi_{S_k}(w_{S_k}, \beta_P, l_{S_k} = m, \mathcal{F}_m = \{PF\} | \mathbf{l}_{-S_k}) > \pi_{S_k}(w_{S_k}, \beta_P, l_{S_k} = n, \mathcal{F}_n | \mathbf{l}_{-S_k})$. We will give more detailed discussion on the coalition formation process and the fairness criteria in the next section.

We have the following definitions about the stability of a coalition.

Definition 5.2.3. Let us define a payoff vector for SUs to be any vector $\boldsymbol{\pi} = (\pi_{S_k})_{S_k \in \mathcal{K}}$ in $\mathbb{R}^K$ to divide the value $\nu(\mathcal{K})$. $\boldsymbol{\pi}$ is said to be group rational or efficient if $\sum_{k=1}^K \pi_{S_k} = \nu(\mathcal{K})$ and is said to be individually rational if $\pi_{S_k} \geq \nu(\{S_k\}), \forall S_k \in \mathcal{K}$. An imputation is a payoff vector satisfying both group and individual rationality.

Definition 5.2.4. An imputation $\boldsymbol{\pi}$ is said to be unstable through a coalition $\mathcal{C}$ if $\nu(\mathcal{C}) > \sum_{S_k \in \mathcal{C}} \pi_{S_k}$. The core of $\nu(\mathcal{K})$ is defined as the set of stable imputations. That is, $\boldsymbol{\pi}$ is in the core if and only if

$$\sum_{S_k \in \mathcal{K}} \pi_{S_k} = \nu(\mathcal{K}) \text{ and } \sum_{S_k \in \mathcal{C}} \pi_{S_k} \geq \nu(\mathcal{C}), \forall \mathcal{C} \subseteq \mathcal{K}. \quad (5.2.9)$$

It is observed that the core of the grand coalition of the sub-band allocation game is empty if at least one SU can improve its payoff by choosing a different sub-band, i.e., $\exists n \in \{1, 2, \dots, M\}$ such that

$$\pi_{S_k}(w_{S_k}, \boldsymbol{\beta}_P, l_{S_k} = n, \mathcal{F}_n | \mathbf{l}_{-S_k}) > \pi_{S_k}(w_{S_k}, \boldsymbol{\beta}_P, l_{S_k} = m, \mathcal{F}_m | \mathbf{l}_{-S_k}). \quad (5.2.10)$$

Power Control Problem for SUs

In our hierarchical game framework, the PUs have priority in using the spectrum and setting the price and the SUs optimize their transmit powers based on the price set by the PUs. For the given price and fixed sub-band allocation scheme, let us model the power control game for SUs. In this game, for a given price $\boldsymbol{\beta}_P$, each SU searches for an NE-achieving transmit power scheme in which this SU cannot further improve its payoff by choosing a different transmit power, given the transmit powers of other SUs, i.e., S_k solves the following optimization problem

$$\arg \max_{w_{S_k} \geq 0} \pi_{S_k}(w_{S_k}, \boldsymbol{\beta}_P, \mathcal{F}_{S_k} | \mathbf{l}_S). \quad (5.2.11)$$

Note that the optimal transmit power of each SU is closely related to the sub-band allocation scheme. We will give a more detailed discussion in the next section.

Price Adjustment Problem for PUs

We model the price adjustment problem as a game among PUs. The NE in this game can be obtained by allowing each PU P_j to solve the following optimization problem given the price of other PUs,

$$\begin{aligned} \arg \max_{\beta_{P_j} \geq 0} \quad & \pi_{P_j} \left(\mathbf{w}_S, \beta_{P_j}, \boldsymbol{\beta}_{-P_j}^* | \mathbf{l}_S \right), \\ \text{s.t.} \quad & \mathbf{H} \mathbf{w}_S \leq \mathbf{q}_P. \end{aligned} \quad (5.2.12)$$

Note that the payoff of each PU depends on the transmit powers and the sub-band allocation of SUs and the pricing coefficient of PUs. To study the joint optimization of the transmit powers of SUs and the pricing function of other PUs, we set up a Stackelberg game in which SUs are regarded as *followers* who maximize their payoffs by choosing the optimal transmission parameters, and PUs are *leaders* who can adjust the price of the licensed spectrum to maximize their payoffs. To study the sub-band allocation scheme, we formulate a coalition formation game for SUs. Our main objective is to find distributed methods to simultaneously approach the NEs of the games in problems 1) - 3) and achieve the SE of the hierarchical framework.

5.3 Game Theoretic Analysis

In this section, we give the solutions of the problems in Section 5.2.3. It is observed that these problems can be connected by the pricing function of PUs. We present the main result in Theorem 5.3.1 at the end of this section, which shows that if the pricing coefficients of PUs satisfy a fixed linear relationship, the sub-band allocation of SUs is stable and unique and the SE is also unique and optimal.

5.3.1 Distributed Sub-band Allocation Solution for SUs

In this section, we consider the sub-band allocation problem for SUs. Since we assume that the SUs in sub-band m can be regarded as an $|\mathcal{L}_m|$ -input $|\mathcal{L}_m|$ -output MIMO channel, following the same line as [80] and using X and Y to denote the input and output of the channels, respectively, we can write the capacity sum of all SUs in sub-band m as follows:

$$\sum_{S_k \in \mathcal{L}_m} R_{S_k} = I(\{X_{S_k}\}_{S_k \in \mathcal{L}_m}; \{Y_{D_k}\}_{D_k \in \mathcal{L}_m}) = \sum_{S_k \in \mathcal{L}_m} \log(1 + \lambda_{S_k}[m]w_{S_k}) \quad (5.3.1)$$

where $I(\cdot, \cdot)$ is the mutual information, and $\lambda_{S_k}[m]$ is the nonzero eigenvalue of a matrix $\mathbf{G}_{\{S_k \in \mathcal{L}_m\}} \mathbf{G}_{\{S_k \in \mathcal{L}_m\}}^T$ corresponding to $g_{kk}[m]$ and $\mathbf{G}_{\{S_k \in \mathcal{L}_m\}}$ is a sub-matrix of the channel gain matrix $\mathbf{G}$ of the SU network, defined as

$$\mathbf{G} = \begin{bmatrix} g_{11}[l_{S_1}] & g_{12}[l_{S_2}] & \cdots & g_{1K}[l_{S_K}] \\ g_{21}[l_{S_1}] & g_{22}[l_{S_2}] & \cdots & g_{2K}[l_{S_K}] \\ \vdots & \vdots & \ddots & \vdots \\ g_{K1}[l_{S_1}] & g_{K2}[l_{S_2}] & \cdots & g_{KK}[l_{S_K}] \end{bmatrix}. \quad (5.3.2)$$

Here, $g_{jk}[l_{S_k}]$ is the ratio of the channel gain between source S_j and destination D_k to the interference power at D_k in sub-band l_{S_k} . In this chapter, we assume the transmit signals of the SUs are independent of each other and the PUs cannot know the coalitions formed by the SUs but can only charge each SU based on its resulting interference. The payoff allocated to each member equals the allocated benefit minus the cost paid to the PUs. We assume each member SU in a coalition can use optimal power control method to maximize its allocated payoff. For example, if we consider the benefit allocated to the member SU S_k in a coalition $\mathcal{L}_m$ to be equal to the rate

contributed to the overall capacity sum, i.e.,

$$R_{S_k} = \log(1 + \lambda_{S_k}[m]w_{S_k}), \quad (5.3.3)$$

the optimal power of S_k is given by $w_{S_k}^* = \left(\frac{1}{\beta_P^T \mathbf{h}_{\bullet k}} - \frac{1}{\lambda_{S_k}[m]}\right)$. We will discuss the optimal power control method of SUs in Section 5.3.2. Note that in $w_{S_k}^*$, $\beta_P^T \mathbf{h}_{\bullet k}$ is defined by the price of PUs and the channel gains between SUs and PUs which cannot be modified by SUs. Therefore, the SUs in a coalition can only divide the payoff sum by adjusting the value of $\lambda_{S_k}[m]$. From the concavity and the monotonicity of the log function, *the payoff division among SUs in the coalition $\mathcal{L}_m$ is equivalent to the channel gain allocation among SUs, i.e., if we assume the power constraint in (5.2.2) is always satisfied, finding the optimal w_{S_k} to maximize $\sum_{S_k \in \mathcal{L}_m} R_{S_k}$ is equivalent to deciding the optimal $\lambda_{S_k}[m]$ to maximize $\prod_{S_k \in \mathcal{L}_m} \frac{\lambda_{S_k}[m]}{\beta_P^T \mathbf{h}_{\bullet k}}$ for all $S_k \in \mathcal{L}_m$. We refer this as channel gain division scheme.*

Let us now prove that the channel gain division scheme in (5.3.3) achieves the PF fairness criteria.

Proposition 5.3.1. *For a given coalition $\mathcal{L}_m$, the following channel gain division scheme satisfies the PF fairness criterion. The payoff of each member SU S_k in the coalition $\mathcal{L}_m$ is given by*

$$\pi_{S_k}[m] = r_{S_k}[m] - c_{S_k}[m] = \log(1 + \lambda_{S_k}[m]w_{S_k}) - \beta_P^T \mathbf{h}_{\bullet k} w_{S_k} \quad \forall S_k \in \mathcal{L}_m \quad (5.3.4)$$

Proof 3. *See Appendix 5.7.1.*

Note that, in a virtual MIMO system, the mutual information among the sources or among the destinations are always much higher than that between the sources and destinations, i.e., $I(X_{S_j}; Y_{S_k}) \geq \min\{I(X_{S_j}, X_{S_k}; Y_{S_k}), I(X_{S_j}, X_{S_k}; Y_{S_j})\}$ and $I(X_{D_j}; Y_{D_k}) \geq \min\{I(X_{S_j}; Y_{S_j}, Y_{S_k}), I(X_{S_k}; Y_{S_k}; Y_{S_j})\} \forall j = 1, 2, \dots, J$ and $k = 1, 2, \dots, K$.

Figure 5.2: Timing structure of the Algorithm.

In other words, there are high chances that two sources S_j and S_k are relatively close to each other compared to their distances to all the destinations. More specifically, if we assume the channel gains between these two sources and all the destination are similar to each other, i.e., $g_{kk}[m] \approx g_{ki}[m]$ and $g_{jj}[m] \approx g_{ji}[m]$ for $i \in \{1, 2, \dots, K\}$ and $m \in \{1, \dots, M\}$, the eigenvalue corresponds to the linear dependent entries in GG^T will approach zero. This will cause the low payoff SU to leave the coalition and search for other sub-bands. In addition, as observed in [73], in a user cooperation system, the relay among SUs may induce cooperation cost which will further discourage the SUs from joining a coalition. To summarize the above observation, we can claim that *the core of the grand coalition of the sub-band allocation game is usually empty*.

In the following, let us propose a simple sub-band coalition formation algorithm. Before we present the detailed description, let us briefly discuss the timing structure of the spectrum sensing process of our algorithm. We assume that the entire transmission process of the SUs has been divided into three periods: sensing period, negotiating period, and data transmission period. In the sensing period, each SU determines its payoff in all the sub-bands as if it is the only SU to use these sub-bands. If there are more than one SU choosing the same sub-band to maximize their payoff, they will negotiate the possible payoff division during the negotiating period. The detailed timing structure is presented in Figure 5.2. Note that the length of the negotiation period depends on the number of SUs in the coalition. Assume β_P is fixed and the detailed algorithm is presented in Figure 4.1.

Algorithm 1: Sub-band Coalition Formation Algorithm

1) *Sensing*:

- a) The SUs, after receiving the prices from PUs, sequentially send a short training message to estimate their payoff in all the sub-bands, i.e., S_k knows $\pi_{S_k}[1], \pi_{S_k}[2], \dots, \pi_{S_k}[M]$ where $\pi_{S_k}[m]$ is the payoff of S_k when sub-band m is exclusively used by S_k ,
- b) Each S_k broadcasts the sub-band $l_{S_k}^*$ that could maximize its payoff, i.e.,

$$l_{S_k}^* = \arg \max_{l \in \{1, \dots, M\}} \pi_{S_k}[l] \quad (5.3.5)$$

2) *Negotiating*:

- a) If more than one SU choose the same sub-band m to maximize their payoff, they start to negotiate with each other. After the negotiating process, each SU can obtain the payoff division $\pi_{S_k}(l_{S_k}^* | \mathcal{F}_m)$,
- b) S_k compares $\pi_{S_k}(l_{S_k}^* | \mathcal{F}_m)$ with its payoffs in other sub-bands. If $\pi_{S_k}(l_{S_k}^* | \mathcal{F}_m) < \max_{l_{S_k} \in \{1, \dots, M\} \setminus \{l_{S_k}^*\}} \pi_{S_k}[l_{S_k}]$, S_k updates its optimal sub-band as the sub-band $l_{S_k}^*$ that could provide the second highest payoff, i.e.,

$$l_{S_k}^* = \arg \max_{l_{S_k} \in \{1, \dots, M\} \setminus \{l_{S_k}^*\}} \pi_{S_k}[l_{S_k}]. \quad (5.3.6)$$

If $l_{S_k}^*$ contains other SUs, go to Step 2-A). Step 2) is repeated until no SUs want to change their sub-bands.

Let us prove in the following proposition that no SUs or partitions of SUs have the incentive to deviate from the current partition when Algorithm 1 stops and hence the partition achieved by Algorithm 1 is stable.

Proposition 5.3.2. *For the given pricing function β_P and fairness criterion, the resulting partition of Algorithm 1 is stable.*

Proof 4. *See Appendix 5.7.2.*

5.3.2 Distributed Optimal Power Control Solutions for SUs

Consider the optimization problem for the power control problem defined in (5.2.11). For now let us assume that the sub-band allocation scheme has been fixed and the price β_P are known to all SUs.

If S_k is the only SU who uses the sub-band m , the following optimal transmit power for S_k can be obtained by solving (5.2.11)

$$w_{S_k}^*(\beta_P) = \left(\frac{1}{u_{S_k}(\beta_P)} - \frac{1}{v_{S_k}} \right)^+, \quad (5.3.7)$$

where $(x)^+ = \max\{x, 0\}$, $u_{S_k}(\beta_P) = \beta_P^T \mathbf{h}_{\bullet k}$ and $v_{S_k} = v_{S_k}[l_{S_k}] = g_{kk}[l_{S_k}]$. The quantity $1/u_{S_k}(\beta_P)$ is typically called the “water-level” of w_{S_k} and S_k should not transmit if $1/v_{S_k}$ is higher than this level.

If multiple SUs share the sub-band m , they will form a coalition to send and receive signals. As mentioned in Section 5.2, in our model, sources and destinations within the sub-band m cooperate with each other by using space-time coding. Our results can be regarded as the upper bound for either the source cooperation or destination cooperation case.

Assume that every member SU in coalition $\mathcal{L}_m$ tries to maximize the rate sum and simultaneously minimize its allocated cost. Using the same method as [50], we

can derive the optimal transmit power for S_k as follows:

$$w_{S_k}^*(\boldsymbol{\beta}_P|\mathcal{F}_m) = \left(\frac{1}{u_{S_k}(\boldsymbol{\beta}_P)} - \frac{1}{\lambda_{S_k}[m]} \right)^+. \quad (5.3.8)$$

To simplify our description, let us write the $w_{S_k}^*$ in the same form as (5.3.7), with

$$v_{S_k}[m] = \begin{cases} g_{kk}[m], & \text{if } \mathcal{L}_m = \{l_{S_k}\}, \\ \lambda_{S_k}[m], & \text{if } |\mathcal{L}_m| \geq 2. \end{cases} \quad (5.3.9)$$

Since $w_{S_k}^*(\boldsymbol{\beta}_P)$ maximizes the payoff of S_k for the given $\mathbf{l}_S$ and $\boldsymbol{\beta}_P$, we can claim that $w_{S_k}^*(\boldsymbol{\beta}_P)$ is an NE for the power control game of SUs. It is observed in (5.3.7) that not all SUs can use the licensed spectrum because the optimal transmit powers of some SUs may be equal to zero.

The main difference between (5.3.7) and the results in [50] is that we do not include the power constraint (5.2.2) in the SU optimization. Instead, we let the PUs set the price and influence the transmit powers of SUs so as to ensure the interference power constraint in (5.2.2) is satisfied (cf. Section 5.3.3). In other words, each SU in [50] is required to know the interference temperature limit of each PU as well as the transmit powers of other SUs to calculate the water-level. However, (5.3.7) and (5.3.8) do not require such information. Therefore, our formulation reduces the information requirements of the SUs.

Note that SU S_k cannot transmit if the licensed spectrum is “pricier” than what it can afford. This could be due to the close location of S_k to some PUs, which prevents the SUs from using licensed spectrum. We say that S_k is *active* if $w_{S_k}^*(\boldsymbol{\beta}_P) > 0$. We have the following result.

Proposition 5.3.3. *For each $\boldsymbol{\beta}_P$, the SU S_k is active if and only if $u_{S_k}(\boldsymbol{\beta}_P) < v_{S_k}$. Furthermore, we have $\pi_{S_k}(w_{S_k}^*, \boldsymbol{\beta}_P) \geq 0$ for all $\boldsymbol{\beta}_P$.*

Proof 5. See Appendix 5.7.3.

5.3.3 Distributed Price Adjustment Solutions for PUs

In the previous section, we showed that for any given β_P and sub-band allocation scheme, there exists a unique optimal transmit power for each SU. Assuming that SUs are rational and use the optimal powers given in (5.3.7) and (5.3.8), the PUs compete with each other to maximize their payoffs without violating the power constraint in (5.2.2). We have the following result about the SE of the hierarchical game.

Proposition 5.3.4. *For a given set of active SUs $\mathcal{L} \subset \{1, 2, \dots, K\}$ and fixed sub-band allocation scheme $\mathbf{l}_S$, either*

1) *there does not exist a β_P for all $j \in \{1, 2, \dots, J\}$ such that the interference power constraint (5.2.2) is satisfied,*

or

2) *there exists a β_P^* satisfying the following equation,*

$$\max_{j \in \{1, 2, \dots, J\}} \left\{ \sum_{k=1}^K \frac{h_{jk}}{q_{P_j}} \left(\frac{1}{u_{S_k}(\beta_P^*)} - \frac{1}{v_{S_k}} \right)^+ \right\} = 1 \quad (5.3.10)$$

and $(\mathbf{w}_S^, \beta_P^*)$ is a pure strategy SE for the constrained game, where $\mathbf{w}_S^* = [w_{S_1}^*(\beta_P^*), w_{S_2}^*(\beta_P^*), \dots, w_{S_K}^*(\beta_P^*)]$ and $\beta_P^* = [\beta_{P_1}^*, \beta_{P_2}^*, \dots, \beta_{P_J}^*]$.*

Proof 6. See Appendix 5.7.4.

It can be observed that the SE of the hierarchical game is not unique but in general there may exist infinitely many SEs, i.e., there are infinity number of β_P satisfying 5.3.10. To simplify our problem, in this chapter, we focus on a linear

pricing function of PUs. More specifically, we assume $\beta_P = \gamma \mathbf{c}$ where $\mathbf{c}$ is a pre-defined vector which decides how PUs will change the price. The value of $\mathbf{c}$ can be regarded as a competition result among PUs. As we will prove later, for each given $\mathbf{c}$, the resulting SE is unique. Furthermore, the global optimal SE of our hierarchical game can be achieved by searching for the optimal $\mathbf{c}$. We have the following observations about the linear pricing of PUs.

Proposition 5.3.5. *For a given β_P , suppose that S_k maximizes its payoff over a given set $\mathcal{R}$ of sub-bands with the PF fairness criterion, and that $\beta_P = \gamma \mathbf{c}$ where $\mathbf{c}$ is a vector of fixed constants, and γ is a positive real number. Then the sub-band allocation scheme in $\mathcal{R}$ that maximizes π_{S_k} is unique and independent of γ .*

Proof 7. See Appendix 5.7.5.

Proposition 5.3.5 shows that if the changes of pricing coefficients β_P are in the form of $\gamma \mathbf{c}$ (i.e., β_{P_j} increases or decreases at a constant rate), then the NE of the sub-band allocation scheme is unique and does not depend on the value of γ . This allows us to decouple the sub-band allocation problem from the price and power control game. In other words, for a fixed $\mathbf{c}$, the SE is unique.

Let us present the main result of this chapter in the following theorem,

Theorem 5.3.1. *For a given $\mathbf{c}$, the optimal sub-band allocation scheme of SUs is unique, stable and can be obtained by Algorithm 1. The SE of the hierarchical game given by $(\mathbf{w}_S^*(\beta_P^*), \beta_P^*)$ is unique and optimal, where $\beta_P^* = \gamma^* \mathbf{c}$ and γ^* is given by*

$$\gamma^* = \frac{\max_{j \in \{1, \dots, J\}} \left\{ \sum_{k=1}^K \frac{h_{jk}}{q_{P_j} c^T \mathbf{h}_{\bullet k}} \right\}}{1 + \max_{j \in \{1, \dots, J\}} \left\{ \sum_{k=1}^K \frac{h_{jk}}{q_{P_j} v_{S_k}} \right\}}. \quad (5.3.11)$$

Proof 8. The first part of the theorem directly comes from Proposition 5.3.5, and

the second part comes from Proposition 5.3.4. We omit the detailed discussion for limit of space.

The above theorem states that the joint optimization problems of CR networks such as those relying on OFDMA can be converted into the optimization problem of the linear pricing coefficients $\mathbf{c}$, which can be easily solved by using the traditional convex optimization methods [56].

5.4 Joint Price Adjustment, Power Control, and Sub-band Allocation Algorithm

In this section, we propose a distributed algorithm that could approach the NEs of all three problems in Subsection 5.2.3 as well as a SE of the hierarchical game described in section 5.3.3.

Before presenting the algorithm, let us derive a feasible region of the pricing coefficient β_P as follows. In this chapter, PUs do not have a priori knowledge of the exact channel gains h_{jk} and v_{S_k} . However, we assume PUs can estimate an approximate range of h_{jk} and v_{S_k} for $j \in \{1, 2, \dots, J\}$ and $k \in \{1, 2, \dots, K\}$. This can be done by exploiting the common knowledge about the SU network (i.e., the maximum allowable transmit powers and the general information about the channel gains of SU networks) and detecting the highest and lowest received noise levels in the licensed spectrum.

Proposition 5.4.1. *Suppose $h_{jk} \geq \underline{h} > 0$ and $v_{S_k} \leq \bar{v}$ for all $j \in \{1, 2, \dots, J\}, k \in \{1, 2, \dots, K\}$. Then each PU P_j only needs to adjust its pricing coefficient β_{P_j} within the range of $0 \leq \beta_{P_j} \leq \bar{\beta}$, where $\bar{\beta} = \frac{\bar{v}}{\underline{h}}$.*

Proof 9. *See Appendix 5.7.6.*

The detailed description of the joint optimization algorithm is given in Figure 4.3.

Algorithm 2: Distributed Algorithm for FD-SSS based CR Networks

Definitions: At iteration t , let

- $\beta_P(t) = \gamma(t)\mathbf{c}$ be the price vector,
- $\mathcal{R}(t)$ be the set of the unallocated sub-bands,
- $\mathcal{L}(t)$ be the set of active SUs,
- $\mathcal{A}(t)$ be the SUs that joins the licensed spectrum in iteration t , and
- $\mathcal{B}(t)$ be the set of SUs that are not allocated any sub-band, but can afford $\beta_P(t)$ in some sub-band in $\mathcal{R}(t)$.

Note that the sets $\mathcal{L}(t)$ and $\mathcal{B}(t)$ are not actually maintained by any player, but is included here to facilitate the description of the algorithm. Each SU that has not been allocated a sub-band keeps track of $\mathcal{R}(t)$.

1) *Initialization:*

- Set $\mathbf{c} = [c_1, c_2, \dots, c_J]^T$ with $c_j \geq \bar{\beta}_P$, for all $j \in \{1, 2, \dots, J\}$.
- Set $\gamma(0) = 1$ and $\epsilon > 0$ be a small positive constant.
- Set $\mathcal{R}(0) = \{1, 2, \dots, M\}$, $\mathcal{L}(0) = \emptyset$ and $\mathcal{B}(0) = \emptyset$.
- Set θ to be a small positive number that is known to all SUs.

2) *Price Adjustment:*

- a) At iteration t , PUs update $\gamma(t) = (1 - \epsilon)\gamma(t - 1)$, $\beta_P(t) = \gamma(t)\mathbf{c}$, and

$$\begin{aligned} \mathcal{B}(t) = \{ & S_k \notin \mathcal{L}(t) : \beta_P(t)^T \mathbf{h}_{\bullet k}[l] < v_{S_k}[l] \\ & \text{for some } l \in \mathcal{R}(t) \}. \end{aligned} \quad (5.4.1)$$

Each PU P_j broadcasts β_{P_j} ,

- b) Each $S_k \in \mathcal{B}(t)$ uses the sensing and negotiating process in Steps 1) - 2) in Algorithm 1 to choose its sub-band $l_{S_k}^*$,
- c) If $S_k \in \mathcal{B}(t)$ joins a sub-band m that is already occupied by at least one SU, then all the SUs in this sub-band should repeat Step 2-b) to update their optimal sub-bands,
- c) If $N_j > q_{P_j}$, P_j broadcasts a “stop” message to all PUs. Go to Step 3).
- d) If $N_j \leq q_{P_j} \forall j \in \{1, 2, \dots, J\}$, i.e., no message has been broadcast by any PUs,
- e) Update $\mathcal{L}(t+1) = \mathcal{L}(t) \cup \mathcal{A}$, $\mathcal{R}(t+1) = \mathcal{R}(t) \setminus \{l_{S_k}^* : S_k \in \mathcal{A}\}$, and $t = t+1$. Go to Step 2a).

- 3) *Termination*: The algorithm ends with solution $\gamma^* = \gamma(t-1)$, $\beta_P^* = \beta_P(t-1)$, $\mathcal{L}^* = \mathcal{L}(t-1)$ and $\mathbf{w}_S^* = [w_{S_1}^*(\beta_P^*), w_{S_2}^*(\beta_P^*), \dots, w_{S_K}^*(\beta_P^*)]$ where

$$w_{S_k}^*(\beta_P^*) = \begin{cases} \alpha_{S_k} \left(\frac{1}{u_{S_k}(\beta_P^*)} - \frac{1}{v_{S_k}} \right) > 0, & \text{if } S_k \in \mathcal{L}^* \\ 0 & \text{otherwise} \end{cases} \quad (5.4.2)$$

and sub-band allocation $\mathbf{l}_S^* = [l_{S_1}^*, l_{S_2}^*, \dots, l_{S_K}^*]$ where, for $S_k \in \mathcal{L}^*$, $l_{S_k}^*$ is determined in Step 2-b).

The main idea of Algorithm 1 is to allow the PUs to gradually decrease their price and hence the high payoff SUs will always be the first to join licensed spectrum. In this way, the active SUs as well as the possible negotiation among SUs will be limited in each step. The following Theorem shows the equilibrium that can be achieved by Algorithm 1.

Theorem 5.4.1. *Suppose that Algorithm 1 terminates with γ^* given in (5.3.11). Then, we have the following equilibrium,*

- E1) The resulting $\mathbf{w}_S^*$ is the NE of the power control game for SUs with the given β_P^* and sub-band allocation scheme $\mathbf{l}_S$,*
- E2) The resulting $\mathbf{l}_S^*$ is the stable for the sub-band coalition formation game,*
- E3) The resulting β_P^* is the NE for the price adjustment game for PUs,*
- E4) The resulting $(\mathbf{w}_S^*, \mathbf{l}_S^*)$ is the SE of the power and pricing game with the given $\mathcal{L}^*$,*

Furthermore, if Algorithm 1 does not terminate with (5.3.11), the solution $\mathbf{w}_S^$, $\mathbf{l}_S^*$, β_P^* and $(\mathbf{w}_S^*, \mathbf{l}_S^*)$ are within a multiple of $(1-\epsilon)$ of the equilibrium points described in E1) - E4) above.*

Proof 10. *See Appendix 5.7.7.*

Since we assume PUs have no knowledge of the channel conditions, i.e., channel gains between SUs and PUs as well as that among PUs, the price adjustment scheme which depends on $\mathbf{c}$ should be pre-defined. It is observed that different price decreasing schemes of β_P (different value of $\mathbf{c}$) may result in different sub-band allocations, as well as $\mathbf{l}_S^*$. For example, in Figure 5.3, we show three price adjustment schemes for a CR network with two PUs and four SUs that result in three corresponding sub-band allocation schemes. More specifically, the adjustment scheme (i) in Figure 5.3 results in S_4 and S_1 being the first and last SUs, respectively, to join the licensed spectrum. On the other hand, in the adjustment scheme (iii), S_2 and S_3 are the first and last SUs to join the licensed spectrum.

A smaller value of ϵ results in a tighter bound on the possible deviation from the SE solution. However, the number of iterations increases as ϵ decreases. The

Figure 5.3: Illustration of price decreasing process in Algorithm 2: we consider the case with two PU (P_1 and P_2) and four SUs (S_1, S_2, S_3 and S_4). Dash lines are three price decreasing schemes of Algorithm 2 which result in three different sub-band allocation schemes. Note that if $l_{S_k} \neq l_{S_j}$ for $l_{S_k}, l_{S_j} \in \{1, \dots, 4\}$ there is no cooperation among SUs. If $l_{S_k} = l_{S_j}$, S_k and S_j will start a negotiation process as described in Step 2) in Algorithm 1.

following results gives a range for the number of iterations required for Algorithm 2.

Proposition 5.4.2. *Suppose $\underline{h} \leq h_{jk} \leq \bar{h}$, $\alpha_{S_k} \leq \bar{\alpha}$ and $\underline{v} \leq v_{S_k}[l] \leq \bar{v}$ for all $j \in \{1, 2, \dots, J\}, k \in \{1, 2, \dots, K\}$, and $l \in \{1, 2, \dots, M\}$. Let $\underline{q} = \min_{j \in \{1, 2, \dots, J\}} \{q_{P_j}\}$ and n be the number of iterations required by Algorithm 1. Then,*

$$\frac{\log(\underline{h}\mathbf{c}^T\mathbf{1}) - \log(\bar{v})}{\log(1 - \epsilon)^{-1}} < n \leq \frac{\log(\bar{h}\mathbf{c}^T\mathbf{1}) + \log\left(\frac{\bar{q}}{K\bar{\alpha}\bar{h}} + \frac{1}{\bar{v}}\right)}{\log(1 - \epsilon)^{-1}} \quad (5.4.3)$$

where $\mathbf{1}$ is a length J vector whose components are all 1's.

Proof 11. See Appendix 5.7.8.

5.5 Numerical Results

In this section, simulation results are presented to verify the performance of our proposed algorithm. Let us first consider the case where two SUs S_1 and S_2 share the spectrum of one PU P_1 . Let us fix the price of P_1 and assume that the channel gain h_{j1} is given by $h_{j1} = \frac{\hat{h}_{j1}}{d_{j1}^\xi}$ for $j \in \{1, 2\}$ where $\hat{h}_{j1}$ is the average channel fading coefficient, and d_{11} and d_{21} are the distances between P_1 and S_1 and P_1 and S_2 , respectively. ξ is the fading exponent. Assume that both SUs are located in a linear network and the PU P_1 moves from the left to the right as shown at the top of Figure 5.4. The payoffs of both SUs under different PU locations are shown in the middle of Figure 5.4. It is observed that the payoff of each SU is minimized when the PU is closest to it. This verifies our results in Proposition 5.3.4 that the transmit power of each SU is decreased when the channel gain between this SU and PU becomes higher. Similarly, it is observed that the payoff of the PU P_1 is also minimized when the PU approaches either SU because the payoff of the PU decreases with

Figure 5.4: Payoffs of two SUs and one PU with different distances between SUs and PUs.

Figure 5.5: Number of coalitions with different number of active SUs: $M = K = 1, 2, \dots, 150$.

the transmit powers of SUs. At the bottom of Figure 5.4, we present the payoffs of PU and SUs for a fixed interference temperature limit at P_1 . Note that the pricing coefficient of the PU also slightly decreases when P_1 approaches either SU. This is because the optimal transmit control method forces the SUs to greatly decrease their transmit powers when the channel gain between SU and PU increases and PU P_1 has to decrease its price to alleviate the payoff reduction caused by the transmit power decreasing of SUs.

Let us consider the sub-band coalition formation game presented in Section 5.2.3. As observed in Section 5.3.1, the core of the grand coalition of all SUs is in general empty and if the number of SUs and the number of sub-bands are large, only a limited number of coalitions will be formed. In Figure 5.5, we present the number of coalitions formed under different numbers of active SUs for a CR network with

Figure 5.6: Payoffs of PUs under different γ . Figure 5.7: The interference of P_1 under different γ .

Figure 5.8: Payoffs of SUs under different γ .

Figure 5.9: Coalition formation of SUs under different γ .

equal number of available sub-bands and active SUs, i.e., $M = K$. Our simulation result shows that even when the number of active SUs in the licensed spectrum is large, for most of the cases, only 1 or 2 sub-bands will be occupied by more than one SU. This verifies our observation in Section 5.3.1 that if the channel gains among SUs are similar, the SUs prefer to exclusively occupy the empty sub-bands instead of forming coalitions with others. This also verifies our motivation for Algorithm 2 where we let the PUs to choose the high initial price at the beginning and then gradually decrease their price to allow the high payoff SUs to join the licensed spectrum first. In other words, the low initial price of PUs may cause many low payoff SUs to compete and negotiate with the high payoff ones which may introduce more communication overheads.

In Figures 5.6 – 5.9, we consider a system with one PU and 10 SUs and study the effects of the pricing coefficient decreasing process of PUs on the performance of SU and PU networks. More specifically, in Figures 5.6 and 5.7, we show the payoff and the interference power of the PU and the number of active SUs under different pricing coefficients. It is observed that the value of γ controls the payoff and the interference level of P_1 as well as the number of active SUs. Similarly, we can observe from Figure 5.8 that the payoff sum of SUs also decreases with the price of PUs. Note that slight payoff jumps can be observed in some point in Figures 5.6 and 5.8. This is because when the value of γ decreases, some new SUs will be activated to join the competition with the existing SUs in some coalitions or sub-bands. This will eventually make some SUs to change their original sub-bands. When the sub-band allocation scheme changes, the SE of the hierarchical game will also change which will cause the changes on the transmit powers of these sub-band changing SUs as well as the payoffs (or payoff sum) of PU (or SUs). However, these SE changes during the pricing decreasing process in Algorithm 2 do not violate our conclusion in Theorem 5.4.1 because we always force the price of PUs to decrease

to the interference temperature limit in (5.2.2) which as proved in Theorem 5.3.1 is always a SE for the resulting sub-band allocation scheme. In addition, as observed in Figure 5.9, the number of coalitions formed by SUs is usually small and hence the SE changes are also limited during the price decreasing process of Algorithm 2.

5.6 Conclusion

We have considered OFDMA-based cooperative CR networks in which the licensed spectrum has been divided into many sub-bands, and each SU can access at most one sub-band. We have first addressed the problem of distributed allocation of sub-bands to SUs by using the coalition formation game-based framework. Then we have considered the joint optimization of both SU and PU networks by introducing a Stackelberg game-based hierarchical formulation. In this game, PUs control the price at which SUs can access a sub-band so that the total interference at each PU is below a predefined threshold. We have proved that if the pricing coefficients of PUs follow a fixed linear relationship, the SE of the hierarchical game framework will be unique and optimal and the sub-band allocation of SUs will be stable.

5.7 Appendix

5.7.1 Proof of Proposition 5.3.1

The optimal transmit power of S_k in the coalition $\mathcal{L}_m$ is obtained by maximizing $\pi_{S_k}[m]$ over $w_{S_k}[m]$ in (5.3.4). Following the same line as [80], we can prove that the resulting $w_{S_k}^*[m]$ (given in (5.3.8)) is the optimal transmit power for a $|\mathcal{L}_m|$ -input $|\mathcal{L}_m|$ -output MIMO channel with the power constraint in (5.2.2) which maximizes

the payoff sum of the coalition $\mathcal{L}_m$. Because the power constraint in (5.2.2) is always satisfied, we can neglect the cost function and focus on the benefit maximization problem as follows.

By substituting the optimal transmit power $w_{S_k}^*[m]$ into the capacity sum, we have

$$\sum_{S_k \in \mathcal{L}_m} R_{S_k}[m] = \sum_{S_k \in \mathcal{L}_m} \log \left(\frac{v_{S_k}[m]}{u_{S_k}} \right) \quad (5.7.1)$$

Because of the concavity and the monotonicity of the log function, the capacity sum maximization problem is equivalent to the following problem,

$$\max_{v_{S_k}[m] \forall S_k \in \mathcal{L}_m} \prod_{S_k \in \mathcal{L}_m} \frac{v_{S_k}[m]}{u_{S_k}} \quad (5.7.2)$$

Using the fact that $v_{S_k}[m] = \lambda_{S_k}[m]$ is the solution of the problem $\max_{w_{S_k}} \sum_{S_k \in \mathcal{L}_m} R_{S_k}[m]$, for any other point $v'_{S_k}[m]$, the following condition is satisfied,

$$\sum_{S_k \in \mathcal{L}_m} \left. \frac{\partial R_{S_k}[m]}{\partial v_{S_k}[m]} \right|_{v_{S_k}[m] = \lambda_{S_k}[m]} (v'_{S_k}[m] - \lambda_{S_k}[m]) = \sum_{S_k \in \mathcal{L}_m} \frac{v'_{S_k}[m] - \lambda_{S_k}[m]}{\lambda_{S_k}[m]} \leq 0.$$

which directly leads to the proportional fairness criteria in [75, Equation (1)] being satisfied.

5.7.2 Proof of Proposition 5.3.2

Suppose that the coalition in Algorithm 1 is not stable, i.e., the SU S_k can find another sub-band $l'_{S_k} \neq l^*_{S_k} \in \mathcal{L}$ or coalition which provide a high payoff than

$\pi_{S_k}(l_{S_k}^*)$, i.e.,

$$\pi_{S_k}[l_{S_k}^*] < \pi_{S_k}[l'_{S_k}] \quad (5.7.3)$$

where $l_{S_k}^*$ is the sub-band allocated by Algorithm 1. This implies that S_k is also active in sub-band l'_{S_k} (otherwise $\pi_{S_k}[l'_{S_k}] = 0$). From (1)–(2) in Algorithm 1, we have $\pi_{S_k}[l_{S_k}^*] \geq \pi_{S_i}[l'_{S_k}]$, which contradicts our assumption. Therefore, for every SU S_k , it has no incentive to deviate from its sub-band $l_{S_k}^*$ or join any other coalition. Therefore, $\mathbf{l}_S^*$ is a stable allocation.

5.7.3 Proof of Proposition 5.3.3

It is clear from (5.3.7) that $w_{S_k}^* > 0$ if and only if $u_{S_k}(\boldsymbol{\beta}_P) < v_{S_k}$. If $w_{S_k}^* = 0$, then $\pi_{S_k}(w_{S_k}^*, \boldsymbol{\beta}_P) = 0$. Suppose that $u_{S_k}(\boldsymbol{\beta}_P) < v_{S_k}$. By substituting the optimal transmit power $w_{S_k}^*$ in (5.3.7) into $\pi_{S_k}(w_{S_k}^*, \boldsymbol{\beta}_P)$ in (5.2.3), we have

$$\pi_{S_k}(w_{S_k}^*, \boldsymbol{\beta}_P) = \log \left(\frac{v_{S_k}}{u_{S_k}(\boldsymbol{\beta}_P)} \right) - 1 + \frac{u_{S_k}(\boldsymbol{\beta}_P)}{v_{S_k}} \geq 0, \quad (5.7.4)$$

where the inequality comes from the fact that $1 - x \leq \exp(-x)$ for all x .

5.7.4 Proof of Proposition 5.3.4

Let us first prove that, given a sub-band allocation, there exists a mixed strategy SE for the price and power control game. It can be shown that β_{P_j} takes values in a non-empty compact set. In addition, the payoff function $\pi_{P_j}(w_S^*(\boldsymbol{\beta}_P), \beta_{P_j})$ is continuous in this set. Therefore, there exists a mixed strategy SE [81].

Let us now focus on proving the existence of a pure strategy SE for the hierarchical game. Let $A_k = \{\boldsymbol{\beta}_P : \boldsymbol{\beta}_P^T \mathbf{h}_{\bullet,k} < v_{S_k} \ \forall S_k \in \mathcal{L}\}$ where the set $\mathcal{L}$ of active SUs

is fixed. Hence $\beta_P \in \bigcap_{k \in \mathcal{L}} A_k$ takes values in a convex set. Suppose there exists a β_P satisfying the condition in (5.3.10). Let us now show that $\pi_{P_j}(w_{S_k}^*(\beta_P), \beta_{P_j})$ is a quasi-concave function of β_{P_j} . Substituting (5.3.7) into (5.2.4), we have

$$\begin{aligned}
& \pi_{P_j}(\mathbf{w}_S^*, \beta_{P_j}) \\
&= \sum_{S_k \in \mathcal{L}} h_{jk} \left(\frac{\beta_{P_j}}{\beta_{P_j} h_{jk} + \sum_{\substack{i=1 \\ i \neq j}}^J \beta_{P_i} h_{ik}} - \frac{\beta_{P_j}}{v_{S_k}} \right) \\
&= \sum_{S_k \in \mathcal{L}} h_{jk} \left(\frac{1}{h_{jk}} - \frac{\sum_{\substack{i=1 \\ i \neq j}}^J \beta_{P_i} h_{ik}}{\beta_{P_j} h_{jk} + \sum_{\substack{i=1 \\ i \neq j}}^J \beta_{P_i} h_{ik}} - \frac{\beta_{P_j}}{v_{S_k}} \right)^+,
\end{aligned} \tag{5.7.5}$$

Evaluating the second derivative of $\pi_{P_j}(\mathbf{w}_S^*, \beta_{P_j})$, we obtain

$$\frac{\partial^2 \pi_{P_j}(\mathbf{w}_S^*, \beta_{P_j})}{\partial \beta_{P_j}^2} = - \sum_{S_k \in \mathcal{L}} h_{jk} \left(\frac{2h_{jk}^2 \sum_{\substack{i=1 \\ i \neq j}}^J \beta_{P_i} h_{ik}}{\left(\sum_{i=1}^J \beta_{P_i} h_{ik} \right)^3} \right) < 0. \tag{5.7.6}$$

Therefore, $\pi_{P_j}(\mathbf{w}_S^*, \beta_{P_j})$ is a concave function of β_{P_j} . In other words, we can always find a $\beta_{P_j}^*$ to maximize the value of $\pi_{P_j}(\mathbf{w}_S^*, \beta_{P_j})$ with the given β_{-P_j} . This concludes the proof.

5.7.5 Proof of Proposition 5.3.5

Let $x_k[l] = \frac{v_{S_k}[l]}{\beta_P^T \mathbf{h}_{\bullet k}[l]}$. We can re-write π_{S_k} in (5.2.3) as follows,

$$\begin{aligned}
 \pi_{S_k}(w_{S_k}^*(\beta_P), \beta_P; l_S) &= \log(1 + g_{S_k}[l] w_{S_k}^*(\beta_P)) - \beta_P^T \mathbf{h}_{\bullet k}[l] w_{S_k}^*(\beta_P) \\
 &= \log \left(1 + v_{S_k}[l] \left(\frac{1}{\beta_P^T \mathbf{h}_{\bullet k}[l]} - \frac{1}{v_{S_k}[l]} \right)^+ \right) \\
 &\quad - \beta_P^T \mathbf{h}_{\bullet k}[l] \left(\frac{1}{\beta_P^T \mathbf{h}_{\bullet k}[l]} - \frac{1}{v_{S_k}[l]} \right)^+ \\
 &= \left(\log(1 + (x_k[l] - 1)^+) - \left(1 - \frac{1}{x_k[l]} \right)^+ \right).
 \end{aligned} \tag{5.7.7}$$

If $x_k[l] \leq 1$, $\pi_{S_k} = 0$. If $x_k[l] > 1$, we have

$$\frac{\partial \pi_{S_k}}{\partial x_k[l]} = \left(\frac{1}{x_k[l]} - \frac{1}{x_k^2[l]} \right) > 0, \tag{5.7.8}$$

and therefore π_{S_k} is an increasing function of $x_k[l]$. To maximize π_{S_k} , the SU S_k will search for the sub-band $l_{S_k}^*$ that could maximize the value of $x_k[l]$. For the linear price adjustment, we have $\beta_P = \gamma \mathbf{c}$. We can re-write the optimal sub-band chosen by S_k as follows:

$$l_{S_k}^* = \arg \max_{l \in \{1, \dots, M\}} x_k[l] = \frac{1}{\gamma} \arg \max_{l \in \{1, \dots, M\}} \frac{v_{S_k}[l]}{\mathbf{c}^T \mathbf{h}_{\bullet k}[l]}. \tag{5.7.9}$$

Let us denote $y_k = \arg \max_{l \in \{1, \dots, M\}} \frac{v_{S_k}[l]}{\mathbf{c}^T \mathbf{h}_{\bullet k}[l]}$, (5.7.9) is equal to $l_{S_k}^* = \frac{y_k}{\gamma}$. It is observed that the value of y_k is fixed for each SU S_k . In other words, the change of γ does not change the sequence of the payoffs for the SU S_k in all the sub-bands. We hence can claim that decreasing the value of γ cannot change the sub-band allocation scheme of all SUs if the set of active SUs and the fairness criteria are fixed. This concludes the proof.

5.7.6 Proof of Proposition 5.4.1

Let us consider the upper bound of β_{P_j} . Suppose that for some j , $\beta_{P_j} > \bar{\beta} \geq \frac{v_{S_k}}{h_{jk}}$ for all $k \in \{1, 2, \dots, K\}$. We then have $\sum_{j=1}^J \beta_{P_j} h_{jk} > v_{S_k}$, which implies that $u_{S_k}(\boldsymbol{\beta}_P) > v_{S_k}$. From Proposition 5.3.3, we have $w_{S_k}^*(\boldsymbol{\beta}_P) = 0$ for all k and hence no SUs are active in the licensed spectrum. Therefore, we have $0 \leq \beta_{P_j} \leq \bar{\beta}$ for all j .

5.7.7 Proof of Theorem 5.4.1

The claim E1) in Theorem 5.4.1 comes from the fact that $w_{S_k}^*(\boldsymbol{\beta}_P^*)$ is the optimal transmit power for S_k with the given $l_{S_k}^*$ for all $k \in \{1, 2, \dots, K\}$. The claim E2) is a direct result in Proposition 5.3.2. Claim E3) is proved in Proposition 5.3.4. Let us consider the claim E4) as follows. By substituting $\boldsymbol{\beta} = \gamma \mathbf{c}$ into (5.2.4), we can show that

$$\pi_{P_j}(\mathbf{w}_S^*(\boldsymbol{\beta}_P), \beta_{P_j}) = \beta_{P_j} \sum_{k=1}^K h_{jk} \left(\frac{1}{\boldsymbol{\beta}_P^T \mathbf{h}_{\bullet k}} - \frac{1}{v_{S_k}} \right)^+ = c_j \sum_{k=1}^K h_{jk} \left(\frac{1}{\mathbf{c}^T \mathbf{h}_{\bullet K}} - \frac{\gamma}{v_{S_k}} \right)^+$$

is increasing as γ decreases. Following the same line as Proposition 5.3.4, we can claim that $(\gamma^*, \mathbf{w}_S^*)$ is a SE. The proof is now complete.

5.7.8 Proof of Proposition 5.4.2

Let us first consider the lower bound of the number of iterations. From Proposition 5.3.3, for a SU S_k to transmit in the sub-band l with positive power, we require that

$$(1 - \epsilon)^n \underline{h} \mathbf{c}^T \mathbf{1} \leq (1 - \epsilon)^n \mathbf{c}^T \mathbf{h}_{\bullet k}[l] < v_{S_k}[l] \leq \bar{v}. \quad (5.7.10)$$

Therefore,

$$n \log(1 - \epsilon) \leq \log(\bar{v}) - \log(\underline{h}\mathbf{c}^T\mathbf{1}) \quad \text{and} \quad n \geq \frac{\log(\underline{h}\mathbf{c}^T\mathbf{1}) - \log(\bar{v})}{\log(1 - \epsilon)^{-1}}. \quad (5.7.11)$$

Let us consider the upper bound of the number of iterations. We can rewrite the power constraint in (5.2.2) as follows,

$$\frac{1}{\gamma} \sum_{k=1}^K \frac{h_{jk}\alpha_{S_k}}{\mathbf{c}^T \mathbf{h}_{\bullet k}} \leq q_{P_j} + \sum_{k=1}^K \frac{h_{jk}\alpha_{S_k}}{v_{S_k}} \Rightarrow \frac{1}{\gamma \bar{h}} \frac{\sum_{k=1}^K h_{jk}\alpha_{S_k}}{\mathbf{c}^T \mathbf{1}} \leq \bar{q} + \frac{\sum_{k=1}^K h_{jk}\alpha_{S_k}}{\underline{v}} \quad (5.7.12)$$

$$\Rightarrow \gamma \geq \frac{1}{\bar{h}\mathbf{c}^T\mathbf{1}} \left(\frac{\bar{q}}{\sum_{k=1}^K h_{jk}\alpha_{S_k}} + \frac{1}{\underline{v}} \right)^{-1} \geq \frac{1}{\bar{h}\mathbf{c}^T\mathbf{1}} \left(\frac{\bar{q}}{K\underline{h}\underline{\alpha}} + \frac{1}{\underline{v}} \right)^{-1} \quad (5.7.13)$$

We hence have $\gamma = (1 - \epsilon)^n \geq \frac{1}{\bar{h}\mathbf{c}^T\mathbf{1}} \left(\frac{\bar{q}}{K\underline{h}\underline{\alpha}} + \frac{1}{\underline{v}} \right)^{-1}$ which leads to the following result,

$$n \leq \frac{\log(\bar{h}\mathbf{c}^T\mathbf{1}) + \log\left(\frac{\bar{q}}{K\underline{h}\underline{\alpha}} + \frac{1}{\underline{v}}\right)}{\log(1 - \epsilon)^{-1}} \quad (5.7.14)$$

Chapter 6

Conclusion and Future Work

6.1 Conclusion

This thesis considers the CR network with an emphasis on the interactions between PUs and SUs. We believe it is this interaction that differentiates the CR network from the traditional frameworks and make it possible to solve the spectrum under-utilization problem for future generations of mobile systems. Generally speaking, the interaction between PUs and SUs in CR networks is complex. The main focus of this thesis is to use the recent advances of game theory to study the cooperation or competition scenarios among autonomous decision makers (called players) in multi-user CR networks.

We start by considering the SSS based multi-user CR networks in which the SUs can send signals over the licensed spectrum as long as the interference power levels of the PUs are lower than the interference temperature limit. In this network, how to control transmit powers of SUs to satisfy the power constraints of PUs is a very important problem. Most of the previously reported results assume SUs can have global information, i.e., CSI of all channels, interference temperature limits

of all PUs, etc., and each PU can provide specific instructions to each SU. In this thesis, we proposed a distributed algorithm for the SUs to iteratively adjust their transmit powers to improve the network performance. The proposed algorithm does not require each SU (or PU) to negotiate with other SUs (or PUs) during the communication. It is proved that, if all SD pairs use the proposed algorithm, the performance of CR networks will converge to an ϵ -NE at a rate of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. To overcome the low convergence rate of the proposed algorithm, a sub-optimal algorithm is also introduced which could approach a neighborhood of the NE at a rate of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$. Numerical results show that the proposed algorithm greatly improves the performance of CR networks.

We then study the possible performance improvement brought by another technique, called multi-hop relaying, to CR networks. We proposed a non-cooperative dynamic game framework in which each SU obtains benefits through both spectrum sharing by paying prices to PUs and multi-hop relaying by paying prices to nearby SUs. The payoff of secondary users under different settings and different types of pricing functions of PUs has been studied. Furthermore, we have proposed three power allocation methods corresponding to the assumptions that transmitters have average knowledge, full knowledge and current knowledge of channel state information. The NE for the proposed game under different settings has been discussed and the conditions for which applying multi-hop relaying to all secondary users achieves the optimal NE of networks have been considered. Our results are extended into large multi-user CR networks with K source-to-destination pairs. Two distributed algorithms are proposed. The first one is a sub-gradient based power allocation algorithm in which SUs can iteratively adjust their transmit powers to approach the payoff of a NE. The other one is a Q-learning based relay selection algorithm which enables each SU to iteratively search for a NE-achieving relaying scheme.

Finally, we introduce a hybrid framework of SSS and TSS [37], in which PUs cannot tolerate any interference power increment caused by SU networks and hence each SU needs to detect the temporal vacancy (spectrum hole) of PUs before its transmission. A general assumption applied in this system is that SUs can divide the licensed spectrum into many sub-bands each of which can only be used by one or multiple SUs [82]. In this case, how to optimally allocate the sub-bands to all SUs is very important problem for TSS based CR networks. If we further assume each sub-band of SUs is much narrower than the transmission spectrum of each PU, the CSS [42] is proposed to be an effective method to improve sensing accuracy of SUs. In CSS method, SUs are required to communicate with each other which may not be possible for some practical mobile systems. In addition, how to fairly allocate the detection complexity to each SU is still an open problem. In this thesis, we formulate a Stackelberg game in which PUs set the price for accessing the spectrum, and SUs optimize their transmit power and the sub-band allocation scheme to maximize the benefits for the given prices. We derive NEs of the price adjustment game for PUs, the power control game and the sub-band allocation game for SUs. In addition, we prove that a SE can always be found for a given sub-band allocation scheme. Surprisingly, we prove that by allowing PUs to gradually decrease their prices, we can simultaneously achieve all the equilibrium points for both SU and PU networks. Our setting does not require SUs to communicate with each other and the proposed algorithm has a fast convergence rate with low communication overhead between SUs and PUs.

6.2 Future Work

As discussed above, game theory is a powerful tool to study multi-user CR networks. Let us summarize our future work as follows:

- *Improving the performance of the distributed power allocation algorithm*

As discussed in chapter 3, the convergence performance of the proposed Algorithm is closely related to two important parameters: initial transmit power value and the iteration step size. More specifically, the closer the initial values are to a NE, the fewer iterations that are required to reach the performance of this NE. However, if the initial transmit powers are not properly chosen, the adjustment of Algorithm 1 on the transmit powers of SUs may cause large fluctuations. Generally speaking, the larger value of the iteration step size coefficient u_i , the faster the rate of convergence. However, large coefficients of step size may lead to fast changes of the transmit powers of SUs. Consequently, the payoff may deviate from the optimal values at the beginning of iterations. Hence, how to choose proper initial powers and step size coefficients is very important for our Algorithm. Our future work in this direction is to investigate novel methods to decide the optimal initial values or step sizes for our algorithm by allowing each SU to know more information. In this work, how to decide the tradeoff between the required information and the improvement on the convergence speed is a key problem.

Another future direction is to apply other convex optimization algorithms into our game theoretic model. As noticed in chapter 3, the main reason for the slow convergence performance of our algorithm is that we do not assume SUs can have any channel information or communicate with each other. Recently, some power allocation algorithms are proposed by allowing SUs to communicate with each other [83–85]. In this case, how to find the tradeoff between the communication overhead among SUs and the converge performance increment is still an open problem.

- *Improving the performance of cooperative CR networks*

In chapter 4, we propose a general game theoretic framework for CR networks with multi-hop relaying by assuming each SU cares about performance of the future the same as that at present. However, there may be cases that the payoffs of the future (or current) are more important than that of the current (or future). For example, in the delay limited applications, i.e., video and audio transmissions, it is important for SUs to maintain the current level of quality of service (QoS). Hence, the time discount factor should be decreased with time. Therefore, studying the performance of the cooperative CR network with different time discount factors could improve our understanding and highlight the optimal design principles for the time related multi-user CR networks [86–90].

- *Extending the hierarchical framework into more general CR networks models*

In chapter 5, we propose a hierarchical framework for optimizing the CR networks. However, our model has some shortcomings when being applied into some practical systems. More specifically, in chapter 5, we assume that each SU can only access the sub-bands by PF fairness. In practical systems, different SUs may have different requirement on the fairness criteria [91–98], and hence a negotiation process may be more complex and how to maximize the overall system utility with these fairness criteria is important. In addition, the traffic of PUs is ignored in our settings [99]. In a practical system, the transmission of PUs on some specific sub-bands requires more protection or should charge higher prices than other sub-bands [100]. Our future is to investigate other possible forms of pricing functions of PUs to extend our results obtained in chapter 5 into more general models.

Author's Publications

1. Yong Xiao, Guoan Bi, Dusit Niyato, and Luiz A. DaSilva "Distributed Optimization for Cooperative OFDMA-based Cognitive Radio Networks", submitted to *IEEE J. Sel. Areas Commun.: Cognitive Radio Series*, 2011.
2. Yong Xiao, Guoan Bi and Dusit Niyato, "A Simple Distributed Power Control Algorithm for Cognitive Radio networks", *IEEE Trans. Wireless Commun.*, vol. 10, pp. 3594-3600, Nov. 2011.
3. Yong Xiao, Guoan Bi and Dusit Niyato, "Game Theoretic Analysis for Spectrum Sharing with Multi-hop Relaying", *IEEE Trans. Wireless Commun.*, vol. 10, no. 5, pp. 1527 -1537, May 2011.
4. Yong Xiao and Guoan Bi, "Spectrum Sharing with Multi-hop Relaying", AP-SIPA Annual Summit and Conference, 15-17 Dec., 2010, Singapore.
5. Yong Xiao, Guoan Bi and Dusit Niyato, "Distributed optimization for cognitive radio networks using Stackelberg game", IEEE International Conference on Communication Systems (ICCS), 17-19 Nov., 2010, Singapore.
6. Yong Xiao and Guoan Bi, "An Improved Amplify-and-Forward Strategy for User Cooperation Channels", ISCIT 2009, 10-13 Oct., 2009, Korea.
7. Yong Xiao and Guoan Bi, "On the Capacity of Frequency Division Gaussian MIMO Relay Channels", ICICS 2007, 10-13 Dec., 2007, Singapore.

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