Cooperative spectrum sensing in cognitive radio networks

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**COOPERATIVE SPECTRUM SENSING IN COGNITIVE
RADIO NETWORKS**

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SCHOOL OF ELECTRICAL AND ELECTRONIC
ENGINEERING
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Abstract

The demand for radio spectrum has dramatically increased over the past decade due to the growing proliferation of wireless services and applications. However, under the current static spectrum allocation policy, almost all the spectrum has been allocated and that there is a shortage of spectrum for new wireless services. On the other hand, actual measurements of the spectrum usage have shown that most of the licensed spectrum is largely under-utilized. Cognitive radio (CR) is proposed as a promising technology to increase the efficiency of spectrum usage by introducing secondary (unlicensed) users to opportunistically or concurrently access the spectrum allocated to primary (licensed) users. To enable the secondary users (SUs) to utilize the under-utilized spectrum, they need to obtain the necessary observations about their surrounding radio environment. Therefore, spectrum sensing is required by the SUs to learn about the activities of the primary users (PUs). However, the performance of spectrum sensing is limited by multipath fading and shadowing which are the fundamental characteristics of wireless channels. To overcome these challenges, cooperation among SUs to perform spectrum sensing has been proposed in the literature.

This thesis studies the designs of cooperative spectrum sensing schemes and exploits the spectrum sensing information to improve the performance of the cognitive radio networks (CRNs) in various scenarios. First, in a small area CRN, where the SUs are close to one another such that the detection signal-to-noise ratios (SNRs) at the various SUs do not vary much, a suitable cooperative sensing scheme

is the k -out-of- N fusion rule. In this scenario, the fusion rule threshold, common energy detector threshold, and the sensing time are proposed to be optimized in order to maximize the throughput of the CRN. In this thesis, a closed-form equation for the optimal common energy detector threshold is derived and an iterative algorithm to jointly obtain the optimal fusion rule threshold and the sensing time is proposed.

In a large area CRN, where the detection SNRs at various SUs can differ greatly, weightings should be given to the decisions of different SUs. Therefore, weightings of the SUs' decisions have to be designed as well under large area CRNs. In this thesis, the optimal decision fusion rule is derived and is shown to be a weighted sum of the decisions based on the likelihood-ratio test. An iterative algorithm is also proposed to compute the individual SUs' energy thresholds and the fusion rule threshold.

Performing spectrum sensing and transmission of sensing data require energy. This can be detrimental to the CRN if the SUs are battery-powered wireless devices where their energies are limited. In this scenario, maximizing the energy efficiency of the CRN is important. Therefore, cooperative spectrum sensing parameters are designed to maximize the energy efficiency of the CRN. An iterative algorithm is proposed to find the optimal fusion rule threshold and the expression at which the energy threshold is optimal is derived. The impacts of the number of cooperating SUs and the length of the sensing time to the energy efficiency of the CRN are also studied in this thesis.

Finally, instead of using the spectrum sensing information just for detecting the PUs, it is used to design power allocation strategies as well. Optimal power allocation strategies under both cooperative and non-cooperative spectrum sensing schemes to maximize the average data rate and to minimize the outage probability of the SU are derived in this thesis.

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List of Abbreviations

AWGN	Additive White Gaussian Noise
BS	Base Station
CAF	Cyclic Autocorrelation Function
CLPC	Close-Loop Power Control
CR	Cognitive Radio
CRN	Cognitive Radio Network
CSCG	Circularly Symmetric Complex Gaussian
CSI	Channel State Information
DSA	Dynamic Spectrum Access
FC	Fusion Center
FCC	Federal Communications Commission
FFT	Fast Fourier Transform
i.i.d.	Independent and Identically Distributed
INR	Interference-to-Noise Ratio
KKT	Karush-Kuhn-Tucker
LLR	Log-Likelihood Ratio
LO	Local Oscillator
OSA	Opportunistic Spectrum Access
PDF	Probability Density Function
PSD	Power Spectral Density

PU	Primary User
PU-Tx	Primary User Transmitter
PU-Rx	Primary User Receiver
QoS	Quality-of-Service
SCD	Spectral Correlation Density
SINR	Signal-to-Interference-Plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SSI	Spectrum Sensing Information
SU	Secondary User
SU-Tx	Secondary User Transmitter
SU-Rx	Secondary User Receiver
s.t.	subject to

List of Notations

$\mathbf{E}[\cdot]$	statistical expectation operator
$ \cdot $	absolute function
$\forall$	for all
$\mathcal{Q}(\cdot)$	right-tail probability of a normalized Gaussian distribution
$\binom{n}{k}$	binomial coefficient
$\triangleq$	defined as
$\nabla f(\cdot)$	differentiation of a function with respect to its argument
$\exp(\cdot)$	exponential function
$\log_2(\cdot)$	binary logarithm
$\ln(\cdot)$	natural logarithm
$\in$	in the set
$\lceil \cdot \rceil$	ceiling operator
$\max(x, y)$	maximum value of the argument(s).
$\min(x, y)$	minimum value of the argument(s).
$[\cdot]^+$	$\max(0, \cdot)$

Chapter 1

Introduction

1.1 Motivation

The current static spectrum allocation policy, which allocates radio spectrum exclusively to specific wireless services, has been working very well in mitigating interferences among the wireless services. However, as the demand for radio spectrum dramatically increases due to the growing proliferation of wireless services and applications, this poses a big challenge to the current static spectrum allocation policy. From the United States frequency allocation chart [1], it can be seen that almost all the spectrum has been allocated and it seems that there is a serious shortage of spectrum for new wireless services or expanding existing ones.

However, actual spectrum usage measurements have shown a different story. In the Federal Communications Commission (FCC) spectrum policy task force report [2], it is shown that the allocated spectrum experiences low utilization rate when measurements of the spectrum usage were taken at various time and locations. Measurements taken in Singapore have also shown similar results [3]. These findings indicate that spectrum scarcity is resulted from the spectrum management policy rather than the lack of physical usable spectrum. Although allocating spectrum bands exclusively to specific wireless services prevents harmful interferences

among the wireless services, it causes low spectrum utilization efficiency as the licensed wireless services may not utilize the spectrum all the time or at all locations. To cater to the increasing demand of radio spectrum for wireless services, the spectrum utilization efficiency must be increased [4].

This motivates the introduction of dynamic spectrum access (DSA), which allows some wireless services or devices to dynamically access or share the radio spectrum with the licensed wireless services as long as the disruptions to the communication of the licensed wireless services are kept to the minimum [5]. Hence, DSA offers the potential to dramatically increase the efficiency of spectrum usage and meet the growing demand of wireless communications. Cognitive radio (CR), which is defined as an intelligent wireless communication system that is aware of its surrounding environment through learning and changes its operating parameters according to the environment [6], is the enabling technology for DSA. Cognitive radio technology enables the unlicensed wireless devices to use the under-utilized spectrum by obtaining the necessary observations about their surrounding radio environment [7, 8]. The licensed users, referred to as the primary users (PUs), are in general not equipped with the CR functions since changing legacy systems may incur huge cost. The unlicensed users, referred to as the secondary users (SUs), equipped with the CR functions are responsible for opportunistically utilizing or sharing the spectrum with the PUs without affecting the PUs' quality-of-service (QoS).

In December 2003, FCC has identified CR as the candidate to implement spectrum sharing on a negotiated or an opportunistic basis to increase the efficiency of spectrum usage [9]. A flurry of research on CR and its applications have since begun such as the DARPA XG [10], CORVUS [11], KNOWS [12], and etc. [13, 14]. The development of CR requires multidisciplinary fields such as signal processing techniques, machine learning, stochastic games, optimization, and etc. [6]. Some

Figure 1.1: Spectrum sharing model: The SU coexists with the PU provided that its interference to the PU-Rx is constrained.

of the research challenges include spectrum sensing, spectrum management, spectrum mobility, spectrum sharing, routing, and cross-layer design in cognitive radio networks (CRNs) [5, 15]. Wireless services based on CR technology to reuse under-utilized frequency bands have also been proposed in various standardizations such as IEEE 802 [16–19], IEEE Standard Coordination Committee (SCC) 41 [20, 21], European Telecommunications Standards Institute (ETSI) [22], European Association for Standardizing Information and Communication Systems (ECMA) [23], and etc. [24, 25].

The intelligence of the SU to learn its surrounding radio environment will determine how the SU is able to utilize the spectrum. If the cognitive level of the SU enables it to predict its power level received at the PU receiver (PU-Rx), the SU is then able to transmit simultaneously with the PU within the same frequency band, as long as the SU transmitter (SU-Tx) limits its transmit power such that the interference it causes to the PU-Rx is below a tolerable threshold [26–28]. This CR model is referred to as the spectrum sharing model or spectrum underlay model in [29]. The spectrum sharing model is illustrated in Fig. 1.1. In this model, SUs are required to estimate either the channel state information (CSI) between the

Figure 1.2: Opportunistic spectrum access (OSA) model: The SU is able to opportunistically utilize the spectrum whenever the PU is inactive.

SU-Tx and the PU-RX or the interference power the SU-Tx caused to the PU-Rx. A blind-estimation approach is proposed in [30] to estimate this partial CSI. Upon knowing the CSI or the interference power, the SU is required to dynamically adjust its transmit parameters, such as transmit power, bit-rate, bandwidth, and antenna beam for its own communication subject to the interference power constraint [31–35].

However, if the cognitive level of the SU only enables it to know the ON/OFF status of the PUs, it can utilize the spectrum only when the PUs are inactive. This CR model is referred to as the Opportunistic Spectrum Access (OSA) model. This model is also referred to as the spectrum overlay model [15] or the interweave paradigm [29]. The OSA model is illustrated in Fig. 1.2. In this model, SUs have to conduct spectrum sensing to detect the presence of the PUs. Upon detecting that the PUs are inactive, the SUs can then operate in the unused spectrum. The SUs are required to frequently sense the spectrum and monitor the status of the PUs so as to vacate the frequency band whenever the PUs become active.

Spectrum sensing to detect the presence of the PU is by far the most important component for CR in the OSA model. The accuracy of the sensing results will determine the spectrum reuse efficiency by the SUs and also, the probability that

the signals of the PUs will be interfered by the SUs. The two key performance metrics associated with spectrum sensing are the probabilities of detection and false alarm. Probability of detection is defined as the probability of detecting the presence of the PU correctly when the PU is active; while the probability of false alarm is defined as the probability of erroneously declaring the presence of the PU when the PU is in fact inactive. A lower probability of false alarm increases the spectrum reuse efficiency by the SUs, which in turn, increases their throughput. A higher probability of detection will result in the PU being less likely to be interfered by the SUs. Hence, there is vast research interest in spectrum sensing algorithms in recent years to improve the detection performance using less time and complexity [36–41].

The performance of spectrum sensing is limited by multipath fading and shadowing which are the fundamental characteristics of wireless channels [42]. To overcome these challenges, cooperation among SUs to perform spectrum sensing has been proposed in the literature [43, 44]. By exploiting the advantages of spatial and multiuser diversities, cooperative spectrum sensing has been shown to improve the reliability of spectrum sensing. When there is noise uncertainty and the received PU's signal-to-noise ratio (SNR) at the SU is very low, it has been shown in [45, 46] that there exists a SNR wall at which reliable spectrum detection is impossible even with a long sensing time. However, it has been proven in [47] that cooperative spectrum sensing is able to alleviate the noise uncertainty problem. In addition, cooperative spectrum sensing is able to decrease the required amount of sensing time [48, 49]. Therefore, cooperative spectrum sensing will have an important role in the implementation of CRNs under the OSA model.

Although cooperative sensing has been well studied in wireless sensor networks and radar detection, cooperative spectrum sensing in CRN introduces new and different objectives and constraints. The design objective of CRN is to enable its

SUs to use the unoccupied spectrum as efficiently as possible while minimizing the interference to the PUs. Hence, the objectives in designing the fusion rule of the cooperative spectrum sensing scheme are the throughput of SUs and the protection of PUs. In CRN, the fusion center (FC), where the sensing data are combined, is likely to have communications with the sensing nodes. For example, the secondary base station acts as the FC and the SUs are the sensing nodes, hence it is possible for the FC to control the sensing nodes' sensing parameters adaptively. In spectrum sensing, it is possible to obtain a high probability of detection with low probability of false alarm given a long sensing time. However, a longer sensing time means lesser time for transmission which affects the SUs' throughput. Therefore, sensing time is an important design parameter in CRN. In conclusion, the design parameters for cooperative spectrum sensing in CRN consist of the fusion rule threshold, sensing nodes' thresholds, and the length of sensing time where the SUs' throughput and the protection of the PU are the objective and constraint, respectively.

Secondary users can be battery-powered devices where energy is precious to them. In the literature, there are studies on minimizing the energy consumed in cooperative spectrum sensing. This includes sending their sensing data to the FC through forms of relaying [50–52] or censoring techniques to reduce the amount of sensing data being transmitted [52–56]. However, the impacts of the energy reduction in cooperative spectrum sensing to the throughput of the SUs are not discussed in these studies. A better performance metric is the energy efficiency where the throughput is also taken into consideration. Cooperative spectrum sensing parameters will affect the amount of sensing and transmission energies used by the CRN and also, the probability of the SUs transmitting when the PU is inactive. Therefore, cooperative spectrum sensing parameters can be designed to improve the energy-efficiency of the CRN when the SUs are energy constrained.

In cooperative spectrum sensing, SUs are required to transmit their respective sensing information to a FC to combine their sensing information and make a final decision on the presence of the PU. However, other than using the sensing information for detecting the presence of the PU, is there any other use for the sensing information at the FC? The sensing information is an additional information in CRN which conventional wireless systems do not have and hence, it is intriguing to explore if this information can be exploited in other ways to improve the system performance other than for spectrum detection. This motivates the idea of using the sensing information for power control in the OSA model.

1.2 Objectives

The objectives of the research in this thesis are to investigate the design of cooperative spectrum sensing schemes and exploit the sensing information to improve the performance of the CRNs in various scenarios.

In a small area CRN, where the SUs are close to one another such that the PU's SNRs at the various SUs do not vary much, a suitable cooperative sensing scheme is the k -out-of- N fusion rule. The k -out-of- N fusion rule requires limited bandwidth and gives equal weightings to the SUs' decisions. The objective in this scenario is to jointly find the optimal SUs' sensing thresholds, fusion rule threshold, and the sensing time to maximize the throughput of the SUs and at the same time, achieve adequate protection to the PU.

In a large area CRN, where the PU's SNRs at various SUs differ greatly, weightings should be given to the decisions from different SUs so as to distinguish between a SU with high PU's SNR and a SU with low PU's SNR. Therefore, in addition to the previous mentioned parameters, the weightings for the decisions are required to be optimized as well for a large area CRN.

In CRN, the sensing performance will affect the average throughput and the

average transmission energy of the CRNs since the transmission opportunities of the CRN depend on the sensing performance. The sensing performance depends on the fusion rule threshold, SUs' sensing thresholds, the sensing time, and the number of cooperating SUs. Longer sensing time and larger number of cooperating SUs will improve the sensing performance but they require more sensing energy. This thesis studies the impacts of these four parameters to the energy efficiency of the CRN and attempts to maximize the energy efficiency of the CRN which is important when the SUs are battery-powered.

The previous objectives are on designing the cooperative spectrum sensing schemes for different CR systems with the aim of either maximizing the throughput or the energy efficiency of the SUs. The last objective is to use the sensing data from the SUs to design power allocation strategies instead of just using the sensing data for detection. Sensing data provides information about the channel condition of the SU's link, i.e., if the PU is active, the channel condition of the SU's link becomes worse due to the interference from the PU. Conventional power control schemes, which do not consider sensing data, may not be optimal in OSA model because when the SU miss-detects the PU's presence, the interference from the PU will cause a lower data rate or a higher outage probability for the SU. Therefore, this thesis attempts to use the sensing data for power control of the SU to either maximize its average throughput or minimize its outage probability.

1.3 Contributions and Thesis Organization

The main contributions of the thesis are as follows.

Optimizing cooperative spectrum sensing schemes to maximize the throughput of the CRN. Several different CR systems are considered in this thesis. In a small area CRN, the parameters to be optimized are the threshold of the k -out-of- N fusion rule, a common energy detector threshold for the SUs,

and the sensing time. In this thesis, a closed-form solution to obtain the optimal common energy detector threshold is derived. The SUs' throughput as a function of the sensing time when k -out-of- N fusion rule is used, is proven to be a unimodal function which allows the optimal sensing time to be solved efficiently. An iterative algorithm to jointly obtain the optimal fusion rule threshold and the sensing time is proposed.

For a large area CRN, the optimal decision fusion rule is derived and is shown to be a weighted sum of the decisions based on the likelihood-ratio test. An iterative algorithm is also proposed for the FC to compute the individual SUs' energy detector thresholds and the fusion rule threshold. It is shown that when the SUs' energy detector thresholds, fusion rule threshold, and the sensing time are jointly optimized, the throughput of the SUs increases significantly.

Optimizing cooperative spectrum sensing schemes to maximize the energy efficiency of the CRN. To maximize the energy efficiency of the CRN, an iterative algorithm is proposed to achieve the optimal fusion rule threshold and the equation at which the common SUs' energy detector threshold is optimal is also derived. It is shown that optimizing the fusion rule threshold can achieve significant gain in the energy efficiency of the CRN. However, if the fusion rule threshold is at optimal, optimizing the SUs' energy detector threshold does not achieve much gain.

Power allocation strategies based on the spectrum sensing information. In non-cooperative spectrum sensing, only the SU-Tx does spectrum sensing and therefore, it has only its spectrum sensing information (SSI). In cooperative spectrum sensing, both the SU-Tx and the SU-Rx do spectrum sensing. The SU-Rx transmits its SSI to the SU-Tx, which acts as the FC. Hence, the SU-Tx will have both SSI for power control when cooperative spectrum sensing is used. Optimal power allocation strategies, under both cooperative and non-cooperative

spectrum sensing scenarios, to maximize the average data rate and to minimize the outage probability of the SU are derived. Not surprisingly, it is shown that in the non-cooperative spectrum sensing, the optimal power controls are functions of the SU-Tx's SSI. However, in the cooperative spectrum schemes, it is shown that the optimal power controls are only functions of the SU-Rx's SSI instead of both SSI.

The remainder of this thesis is organized as follows. Chapter 2 provides a brief background of the CRN under the OSA model and the reviews of the various spectrum sensing algorithms and cooperative spectrum sensing schemes found in the literature. In Chapter 3, optimization of cooperative spectrum sensing scheme to maximize the throughput of a small area CRN is studied. In Chapter 4, a large area CRN is considered and optimization of cooperative spectrum sensing scheme with different number of SUs are investigated. Optimization of cooperative spectrum sensing scheme such that the energy efficiency of the CRN is maximized is proposed in Chapter 5. In Chapter 6, power allocation strategies for OSA CRN under cooperative and non-cooperative spectrum sensing are presented. Chapter 7 concludes this dissertation and discusses the future work.

Chapter 2

Background

2.1 Cognitive Radio Processes under OSA model

This thesis focuses on CRN under the OSA model and therefore, a brief background of the CRN under the OSA model is given in this section. The basic processes of the CRN under the OSA model are shown in Fig. 2.1. The processes include channel sensing selection, spectrum sensing, channel access policy, and transmission process. We will review some existing works that have been done in each of the process. The survey of the existing works in each process is by no means exhaustive and is provided to let readers have a more complete understanding of the CRN under the OSA model.

The first process of the CRN is channel sensing selection. Before the SUs perform spectrum sensing, they must first decide on which frequency channels to sense. Some of the considerations in this process include whether the hardware of the SU allows it to sense multiple channels at a time. If the SU can only sense one channel at a time, how to choose the channel to be sensed and what is their sensing order? In [57–61], the channel sensing order is designed to maximize the discoverable spectrum opportunities, transmission rewards or the energy efficiency of the CRN. In [62, 63], the channels are selected to sense and transmit such that

Figure 2.1: Processes of CRN under OSA model.

the throughput of the CRN is maximized. Joint optimization of channel selection and power control is considered in [64]. When the SUs are able to sense multiple channels, the sensing thresholds and the cooperative sensing scheme within each channel are optimized to maximize the aggregate throughput of the CRN in [65]. An extension of [65] to include sensing time as an optimization parameter is proposed in [66].

After the channel to be sensed is decided, the next process is the spectrum sensing. Some of the issues in this process include the length of the sensing time, the sensing algorithm, and the cooperative sensing scheme to be used. Sensing time to maximize the throughput of CRN in multi-channels scenario is designed in [67–71], while in [72–74], the sensing time is minimized with a targeted detection performance. There are vast amount of spectrum sensing algorithms being researched and which spectrum sensing algorithm to use normally depends on the amount of knowledge that the SU has about the PU’s signal and its noise power [38]. As the main topic of this thesis is on cooperative spectrum sensing, we will provide reviews of the various spectrum sensing algorithms and cooperative spectrum sensing schemes in the next two sections. The contributions of this thesis on cooperative sensing sensing will be further discussed in Chapters 3, 4, and 5.

When there is one or more channels being sensed to be idle, the next process

is to decide the channel access strategy [75, 76]. Protocols are needed to coordinate the channel selection within a communication group or between different communication groups [77]. Rules are implemented in [78] such that channel assignments are according to proportional fairness among the SUs. Channel access schemes have been proposed to maximize the CRN's throughput [79, 80] and for delay constraint applications [81]. A framework for competition among SUs to access the spectrum is proposed in [82]. In [83–87], spectrum sensing and access strategies to maximize the total throughput based on the framework of Markov decision process are proposed. Spectrum sensing and access strategies with energy constraint are further considered in [88].

After a SU is being assigned a channel for transmission, it has to decide its transmission parameters, multicarrier methods [89], and modulation techniques [90]. One of the transmission parameters is the transmission length. The PU may reclaim the channel anytime, hence it is important that the SU's transmission is not too long before the next sensing phase starts so that if the PU re-appears, the SU can detect it in time and switch to other channel. Hence, in the literature, the transmission duration or the next sensing period is designed based on the transition rate of the PU's ON/OFF states for interference avoidance, spectrum discovery efficiency or throughput maximization [91–93]. Another important transmission parameter is the power allocation, which will be further discussed in Chapter 6 of this thesis.

2.2 Spectrum Sensing Techniques

In this section, the concept of spectrum sensing in CRN is discussed and a review of spectrum sensing algorithms is provided. The aim of spectrum sensing is to detect spectrum holes. Spectrum hole is defined as spectrum that is usable by the SU without interfering the PU's communication [94]. The concept of using

Figure 2.2: Concept of spectrum sensing in CRN.

spectrum sensing to detect the spectrum hole will be discussed by using a simple system setup shown in Fig. 2.2. A PU-Tx is transmitting data to its receivers, either PU-Rx1 or PU-Rx2 or both, in a particular licensed band and a secondary communication system, formed by a pair of SU-Tx and SU-Rx, intends to access the same frequency band. It is obvious that if PU-Rx1 is not active, a spectrum hole existed as SU-Tx can transmit to SU-Rx without interfering the PU system. Hence, as long as the SU-Tx detects that there is no active PU-Rxs within its coverage range d_{SU} , it is safe to utilize the spectrum. Conducting spectrum sensing to detect PU-Rx within the SU-Tx's coverage range is referred to as direct spectrum sensing. However, direct spectrum sensing is a challenging task especially if the PU-Rx does not transmit any signal.

Most of the spectrum sensing schemes in the literature aim at detecting the PU-Tx instead. In Fig. 2.2, d_{PU} is denoted as the transmission range of the PU-Tx. If the SU-Tx is farther than $d_{PU} + d_{SU}$ from the PU-Tx, then surely none of the PU-Rx will be within the SU-Tx's coverage range and therefore a spectrum hole exists. The task of the SU-Tx is then to detect any PU-Tx within the range $d_{PU} + d_{SU}$ which is referred to as indirect spectrum sensing. If no PU-Tx is detected to be within $d_{PU} + d_{SU}$ of the SU-Tx, the SU-Tx can access the spectrum. However,

indirect spectrum sensing can be a challenging task as the detection distance is far and requires detection of very weak signals. A disadvantage of indirect spectrum sensing is that even if a PU-Tx is detected to be within $d_{PU} + d_{SU}$ of the SU-Tx, it does not necessarily mean that there is no spectrum hole. For example, in Fig. 2.2, if PU-Tx is transmitting data to PU-Rx2 and not to PU-Rx1, even if SU-Tx transmits concurrently in the same channel, it will not interfere with the PU communication. In this case, although the SU-Tx detects a PU-Tx to be within $d_{PU} + d_{SU}$, there exists a spectrum hole. If the locations of the PU-Rxs are unknown to the SU-Tx, the SU-Tx has to assume the worst case scenario that a PU-Rx is located within the SU-Tx's coverage range and therefore, it is not allowed to utilize the spectrum. The amount of opportunity to utilize spectrum hole will be lower for indirect spectrum sensing. Next, an overview of some well-known spectrum sensing algorithms found in the literature for both direct and indirect spectrum sensing are provided.

2.2.1 Direct Spectrum Sensing

1) *Local Oscillator Leakage Detector*

In most wireless communication systems, the receiver will down convert its received signal from the radio frequency band to the intermediate frequency band. In the process of down conversion, there will be a local oscillator (LO) leakage power emitted by the radio frequency front-end of the receiver. Therefore, it is proposed in [95] to detect the PU-Rx's presence by using this LO leakage signal. However, the LO leakage signal is typically weak which makes it difficult to detect the PU-Rx at larger distance within a short sensing time. It is shown in [95] that it would take in the order of seconds to detect a PU-Rx at a distance of 20m with a high probability. Hence, for this scheme to work, low cost sensor nodes have to be placed near the PU-Rxs.

2) *Proactive Sensing Detector*

Close-loop power control (CLPC) is usually applied in wireless communication systems to maintain communication links with certain transmission QoS guarantee. The CLPC will adjust the transmitter's signal power to compensate the power loss caused by the characteristics of wireless channels such as path-loss and shadowing to meet the required QoS. A creative scheme to make use of the CLPC of the primary communication system to detect PU-Rx within the transmission range of the SU-Tx is proposed in [96, 97]. The SU-Tx transmits a sounding signal first before sensing. If a PU-Rx is nearby, the sounding signal will change the interference level experienced by the PU-Rx. The PU-Rx will therefore feedback the interference change caused by the sounding signal to the PU-Tx for it to adjust its transmission signal to adapt the new interference. Hence, during the sensing period, the SU-Tx will sense whether the PU-Tx's signal did respond to the sounding signal. If the PU-Tx is detected to change its transmission signal in response to the sounding signal, then the SU-Tx can conclude that a PU-Rx is within its transmission range. The design of the sounding signal and the detector to detect the PU-Tx's response to the sounding signal are proposed in [97]. Cooperation of multiple SUs for this proactive spectrum sensing scheme is proposed in [98].

2.2.2 Indirect Spectrum Sensing

1) *Energy Detector*

When there is no prior knowledge of the PU's signal, energy detector can be used by comparing the received signal's energy with a threshold determined by the noise level [99]. The signal's energy can be measured in both the time and frequency domains. In the time domain, the signal's energy in a particular frequency band is measured by applying a bandpass filter to the received signal

and summing the energy of the filtered signal samples. In the frequency domain, the time domain signal is transformed to the frequency domain using fast Fourier transform (FFT) and the signal's energy over all the frequency bins in the targeted frequency band are summed and measured [100]. Closed-form expressions for the energy detector's probability of detection under additive white Gaussian noise (AWGN) and fading channels are derived in [101]. It is shown that the energy detector is optimal for detecting any unknown zero-mean constellation signals [102]. Other advantages of the energy detector include its low computational complexity and easy implementation which enable large number of frequency bands to be sensed at the same time.

However, the energy detector cannot differentiate the PU's signal from other unknown signal sources which may cause the detector to wrongly detect the presence of the PU if other signal sources exist. Energy detector is also susceptible to noise uncertainty where in low SNR regimes, reliable detection of the PU is not possible even with a large amount of sensing time [46]. To mitigate the noise uncertainty problem, the noise level is estimated adaptively by decoupling the noise and the signal subspaces using the multiple signal classification algorithm in [103]. A forward method is also proposed in [104] to adaptively estimate the noise level.

2) Match Filter and Coherent Detector

When prior knowledge of the PU's signal is known to the SU, the optimal detection method is the match filter [105]. By correlating the known SU's signal with the received signal, it maximizes the SNR if the received signal contains the PU's signal. An advantage of match filter is that it requires less time to achieve a certain detection performance [102]. However, match filter requires perfect knowledge of the PU's signal which is hardly available to the SU. Hence, partial information of the PU's signal such as known patterns of pilot symbols, preamble, spreading sequences, and etc., can be used instead for coherent detection in CRN [106].

Detection can be performed by correlating the received signal with the known pattern. If the known pattern is longer, the sensing performance will be better [100]. Coherent detection can also be performed in the frequency domain [107]. It is shown to be more robust to noise uncertainty with a lower SNR wall if the number of coherent samples is large [108], however, the complexity for synchronization could be an issue [109]. Another disadvantage of coherent detection is that to detect different types of PU's signals, the received signal will have to be tested for correlation with many different known signal patterns which will increase the implementation complexity and power consumption.

3) Cyclostationary Detector

Wireless communication signals, either analog [110] or digital [111], are usually cyclostationary. Cyclostationary process is a random process with statistical parameters that vary periodically in time [110]. These cyclostationary features in wireless signals are due to the periodicity of their sine wave carriers, pulse trains, repeating spreading codes, etc., while noise is generally wide-sense stationary with no correlation and has no features at non-zero cycle frequencies [112]. Hence, spectrum sensing can be done to detect these cyclostationary features in the PUs' signals at non-zero cycle frequencies. These cyclostationary features can be extracted by either the cyclic autocorrelation function (CAF) or the Fourier transform of it, which is referred to as spectral correlation density function (SCD). The CAF is reduced to the well known autocorrelation function and SCD is reduced to power spectral density when the cyclic frequency is at zero. For more information about the mathematical interpretation of CAF and SCD, readers can refer to [110–113].

Since the spectral correlation properties of different signals are usually unique, cyclostationary detectors allow the SUs to differentiate the PU's signal from interference signals and noise [114]. Cyclostationary features can also be intentionally

embedded into the SU's signals to enable SU to differentiate other SUs' signals from the PU's [115]. It is also shown that the cyclostationary detector is robust to noise uncertainty but will still encounter a SNR wall because of the fading process [45]. Furthermore, cyclostationary detector requires high sampling rate and computational complexity especially if the cyclic frequency of the PU's signal is unknown. The cyclostationary detectors can use generalized likelihood-ratio test [116] for detection or other detectors proposed in [117, 118]. Cooperative spectrum sensing schemes using cyclostationary detectors are proposed in [116, 119, 120]. Censoring technique for cooperative cyclostationary detectors to reduce the amount of sensing data being transmitted is also proposed in [120]. Hardware implementation of cyclostationary detector is presented in [121].

4) *Covariance-based Detector*

When a PU's signal exists, the received PU's signal samples at the SU are usually correlated because of the time-dispersive nature of the propagation channel, oversampling or utilization of multiple receive antennas [122] while noise samples are usually independent and identically distributed (i.i.d.) and are not correlated. The statistical covariance matrix of the noise is a diagonal matrix with the diagonal elements equal to the noise power. The statistical covariance matrix of the received PU's signal is usually not diagonal. Therefore, the covariance matrices of the received PU's signal and the noise are generally different and can be used to differentiate them.

Various covariance-based detectors are proposed in the literature. In [122, 123], the off-diagonal elements are compared with the diagonal elements of the covariance matrix to detect the existence of PU's signal. The ratio between the maximum and minimum eigenvalues of the covariance matrix is proposed to detect the PU's signal [124] and the threshold for this detector can be found using the limiting eigenvalue ratio distribution in Wishart matrices derived in [125, 126].

In [127], the ratio between the largest eigenvalue and the normalized trace of the covariance matrix is proposed instead. The covariance-based detectors do not require a priori knowledge of the PU's signal, channel, and noise power. No synchronization is required for the covariance-based detectors and they are shown to be robust to noise uncertainty [122]. If the noise variance is known, then the maximum eigenvalue detector can be used [128].

5) Other Detectors

Wavelet detector is proposed in [129] to detect the changes in the power spectral density (PSD) of the received sensing signal. In a wideband spectrum, there will be abrupt changes in the PSD between a frequency band that is occupied and its neighbor frequency bands that are free. The edge locations of the frequency bands can then be found using wavelet detector and the power in between the edges can be used to decide whether PU exists in that band. A multitaper spectral estimator, which is a nonparametric method for spectrum sensing, has been proposed in [6]. The multitaper spectral estimator is capable of high spectral resolution in both average power and frequency. A tutorial of the multitaper spectral estimator is given in [130].

The overview on the various common spectrum sensing detectors is provided where the advantages and disadvantages of the detectors have been discussed. A summary of the pros and cons of the detectors is given in Table 2.1. From the table, it can be seen that different detectors are applicable to different scenarios or CR systems. When the SUs have low computation power and have totally no prior information about the PUs, energy detector may be preferred in this scenario. Energy detector may also be useful in wide-band spectrum sensing to obtain rough sensing information due to its ease of implementation before zooming into individual bands for more accurate sensing. If the PU's signal is correlated, the SUs can use covariance-based detectors to achieve better sensing performance. Covariance-

based detectors are also useful when the system has high noise uncertainty. For systems that are required to differentiate the PU's signal from the interference, cyclostationary-based detectors can be used, especially if the cyclic frequencies of the PU's signal are known to the SUs. When whole or part of the PU signal's information such as the waveform patterns or the pilot symbols, are known to the SUs, the match filter or coherent detector can be used to improve the sensing performance. For more information and in-depth discussions on spectrum sensing detectors, readers can refer to [36–41].

Table 2.1: Advantages and disadvantages of various indirect spectrum sensing techniques.

Detector Type	Advantages	Disadvantages
Energy Detector	<ul style="list-style-type: none"> • Do not require prior knowledge of PU's signal • Low computational complexity and easy implementation 	<ul style="list-style-type: none"> • Unable to differentiate PU's signal from interference • Susceptible to noise uncertainty
Match Filter / Coherent Detector	<ul style="list-style-type: none"> • Robust to noise uncertainty • Require less signal samples to achieve a good detection performance 	<ul style="list-style-type: none"> • Require prior knowledge of PU's signal • Require to correlate with multiple known signal patterns if there are multiple types of PU's signals • Difficult to synchronize the signal especially at very low SNRs
Cyclostationary Detector	<ul style="list-style-type: none"> • Able to differentiate PU's signal from interference • Relatively robust to noise uncertainty 	<ul style="list-style-type: none"> • High computational complexity if the cyclic frequency of the PU's signal is unknown • Sampling time error and frequency offset can affect the cyclic frequencies
Covariance-based Detector	<ul style="list-style-type: none"> • Do not require prior knowledge of PU's signal • Robust to noise uncertainty 	<ul style="list-style-type: none"> • Unable to differentiate PU's signal from interference

2.3 Cooperative Spectrum Sensing

As mentioned in Chapter 1, cooperative spectrum sensing is able to mitigate the impact of multipath fading, shadowing, and noise uncertainty. Cooperative spectrum sensing is also able to reduce the sensing time for a given sensing performance. In this subsection, we will give a background and a review of cooperative spectrum sensing schemes in CRNs in the literature.

The structures of cooperative spectrum sensing in CRNs can be classified into centralized and distributed cooperative spectrum sensing. In centralized cooperative spectrum sensing, there is a FC, e.g., a secondary base station or access point, which is able to communicate or receive sensing information from multiple spectrum sensors, e.g., the SUs. The SUs send their sensing information to the FC and the FC will combine the sensing information received and then makes a decision on the presence of the PU for the whole network. Since the FC is able to communicate with the SUs, it is able to coordinate them such that the CRN obtains an optimal performance. In distributed cooperative spectrum sensing, the SUs exchange their sensing information among themselves and make their own individual decision on the presence of the PU. To ensure consensus on the global decision, multiple rounds of message passing among the SUs may be required. The advantage of distributed sensing is that there is no need for a backbone infrastructure. Cooperative spectrum sensing schemes with relays using amplify-and-forward protocol have been investigated in [131, 132], however, we will not delve into relay systems in this thesis.

When the SUs send their sensing information either to the FC or among themselves, fusion schemes are required to combine the sensing information. Fusion schemes are classified as data fusion and decision fusion depending on the type of sensing information being sent by the SUs. In data fusion, the SUs send their observed data or processed data out for combining and decision making. In de-

Figure 2.3: Centralized cooperative spectrum sensing.

cision fusion, individual SU will make a decision based on its own observed data first and sends out its decision for combining and final decision making. Generally, data fusion scheme has better sensing performance but requires a much larger bandwidth for transmission of sensing information [133]. The transmission of the sensing information can be done using a dedicated band, an unlicensed band such as ISM/UNII, or ultra wideband [134]. To reduce the error of transmitting the sensing information, space-time block code has been proposed in [135].

2.3.1 Centralized Cooperative Spectrum Sensing

An illustration of centralized cooperative spectrum sensing scheme is shown in Fig. 2.3. We will first review some of the existing works under this structure using data fusion. When the SUs measure their received energy level and send it to the FC for combining, the optimal fusion scheme to maximize the detection probability for a given false alarm probability is the log-likelihood ratio (LLR) test [136]. However, because of its quadratic form, there is no closed-form solution for the fusion rule threshold and therefore, linear weighted combination of the energy levels measured at the SUs is proposed in [137]. The optimal weighting for combining the energy

levels is found in [137] and is later being solved using more efficient algorithms, such as semidefinite programming in [138], and using direct analytic method in [139]. When Rician fading channel is considered, the exact detection and false alarm probabilities of the linear combination of the energy levels are given in [140]. The problem has been extended to multiband scenario where the weights and the fusion rule threshold of each individual channel are jointly optimized to maximize the aggregated throughput subject to interference constraints [65, 141]. The multiband problem can also be solved using genetic algorithms [142].

Data fusion requires more bandwidth and there are various proposed techniques to reduce the amount of sensing information being transmitted from the SUs to the FC. In [143], each SU sends a linear combination of sensing information of multiple channels instead of all the sensing information of all channels. Matrix completion and joint sparsity recovery techniques are then used at the FC to reconstruct all the channel sensing information. In [144], quantizer is proposed for the LLRs to reduce the amount of data to be sent by the SUs to the FC. Since having more SUs in cooperative spectrum sensing consumes more bandwidth and resources, the optimal number of cooperative SUs is found by trading the detection performance with resource consumption in [145]. In [146], SUs are selected for cooperative spectrum sensing based on their locations. The SUs are chosen such that they are sufficiently spatially separated from each other so that their data are less correlated. This is because it has been shown in [147] that the detection performance of cooperative spectrum sensing will be degraded if the data are correlated. In [148], the FC uses the energy levels sent by the SUs to construct channel gain map and finds the locations of the PUs and their transmit powers.

We will next discuss some of the existing works using decision fusion. The most common decision fusion scheme is the k -out-of- N fusion rule where if k or more decisions out of the total N decisions received by the FC decide that the

PU is present, then the FC will decide that the PU is present [149]. When $k = 1$, the fusion rule becomes OR fusion rule and when $k = N$, it becomes AND fusion rule. In [150], the k -out-of- N fusion rule is optimized such that the total error probability, which is the sum of the missed detection probability and the false alarm probability, is minimized. The k -out-of- N fusion rule is also being optimized such that the sum of PU's throughput and SU's throughput is maximized in [151] and the detection probability is maximized for a given false alarm probability in [152].

In [153], it is shown that cooperating all the SUs in OR and AND fusion rules do not achieve the maximum detection probability for a given false alarm probability. Instead, it is shown that selecting a subset of SUs with the highest detection SNRs for cooperation will achieve a better detection performance. To minimize the amount of decisions to be transmitted from the SUs, it is also proposed in [154] that a subset of SUs with the highest detection performance to transmit their decisions instead of all the SUs. Another technique to minimize the amount of decisions to be transmitted is proposed in [54]. Two thresholds are proposed in [54] such that decisions will only be made when the energy level is very large or small. The SUs which have energy levels that fall in between the two thresholds will not make any decisions and will also not send any signal to the FC. Therefore, only decisions with high reliability will be transmitted to the FC.

In [155], three thresholds are used to classified the energy level into four regions. Therefore, the SUs will transmit two bits of sensing information to the FC instead of one bit. At the FC, weightings will be given to the sensing information based on the region it is in and the FC will compute the sum of the weightings as its test statistic. It is shown in [155] that with an extra bit, the fusion scheme's detection performance is comparable to equal gain combining of the SUs' energy levels.

A linear test statistic that can robustly combine the sensing information, both

Figure 2.4: Distributed cooperative spectrum sensing.

data and decisions, when the statistics of the PU's signal and the PU-to-SU CSI are unknown is proposed in [156]. The total error probability is minimized for both data fusion and decision fusion in [157] by optimizing the energy detectors' thresholds and the fusion rule threshold. In [158], the SUs are assigned for cooperative spectrum sensing in various channels depending on the channel gains from the PU in each channel to the SUs. The energy detector thresholds of the SUs are assumed to be the same and are optimized to maximize the sum throughput and also, to maximize the minimum throughput across all the channels. In [159], a linear-quadratic fusion scheme is proposed when the decisions of the SUs are correlated.

2.3.2 Distributed Cooperative Spectrum Sensing

In a distributed cooperative spectrum sensing scheme, it is usually assumed that either the SUs are fully connected with each other such that the exchange of sensing information can be completed in one transmission hop or the SUs are sparsely connected as shown in Fig. 2.4, where the exchange of sensing information requires a few transmission hops. In a sparsely connected network, iterative algorithms are

required to ensure that all the SUs in the network will reach the same final decision. In [160], an average-consensus algorithm is proposed where the sensing information are exchanged iteratively and all the SUs will eventually reach the same consensus value, which is the average of all the local decisions, and therefore, make the same decision. Distributed cooperative compressive sensing schemes are proposed in [161, 162] for wideband spectrum sensing where each SU performs compressive sensing and makes local estimates or decisions on the spectrum occupancy. Iterative consensus algorithms are then used to fuse the sensing information that are being distributively exchanged among the SUs.

In a distributed CRN, some selfish SUs may decide not to participate in cooperative spectrum sensing and wait for other SUs to transmit their sensing information to them in order to increase its own transmission time or reduce its energy consumption. In [163], an evolutionary game is proposed for the distributed SUs to determine how they should cooperate in order to increase their own throughput, since if none of them wants to cooperate, all the SUs will have low throughput due to a high false alarm probability. When the SUs are distributed, it is important that the SUs are able to self-organize and form coalitions for cooperative spectrum sensing. In [164], a merge-and-split algorithm is proposed for the SUs to form coalition distributively that maximizes the detection performance.

Chapter 3

Optimization of Cooperative Spectrum Sensing in a Small Area Cognitive Radio Network

In this chapter, we consider a CRN where there are N SUs and one secondary base station. The CRN is located in a small area, meaning that the SUs and their base station are relatively close to each other. Before the SUs can utilize a spectrum, they need to ensure that the PU is not occupying the channel. Therefore, spectrum sensing is required to be performed before any transmission as in any OSA CR system. A longer sensing time will improve the sensing performance; however, with a fixed frame size, the longer sensing time will shorten the allowable data transmission time of the SUs. Hence, a sensing-throughput tradeoff problem was formulated in [165] to find the optimal sensing time that maximizes the SUs' throughput while providing adequate protection to the PU.

Another technique to improve the spectrum sensing performance is cooperative spectrum sensing. In this chapter, hard decision fusion scheme, such as k -out-of- N fusion rule [149], is considered as it requires the least communication overhead and is easy to implement. The SUs make their own decision on the presence of the

PU and then transmit their one bit decision to the secondary base station. The secondary base station will make a final decision on the PU's activity based on the received decisions.

Both the sensing time and the cooperative sensing scheme affect the spectrum sensing performance, such as the probabilities of detection and false alarm. These probabilities affect the throughput of the SUs since they determine the re-usability of the frequency bands. Therefore, in this chapter, we propose joint optimization of the sensing time and the parameters of the cooperative sensing scheme to maximize the throughput of the SUs. In the literature, there are some studies on optimizing the k value of the k -out-of- N fusion rule [149, 166] that minimize the total decision error probability instead of maximizing the SUs' throughput. Furthermore, sensing time was not considered in their optimization formulations. There are also studies on optimizing the total frame time of the SUs if the PUs' traffic statistics are known to the SUs [57, 91]. However, in this chapter, the PUs' traffic statistics are assumed unknown and therefore the total frame time is fixed.

The main contributions in this chapter are as follows. First, using the k -out-of- N fusion rule as the basis, an optimization problem using the sensing time and the fusion rule threshold k as the optimization variables to jointly maximize the throughput of the SUs while giving adequate protection to the PU is formulated. Secondly, an iterative algorithm to obtain both the optimal sensing time and the k value for the optimization problem is proposed. In this chapter, we prove the unimodal characteristics of the SUs' throughput as a function of the sensing time when k -out-of- N fusion rule is used. Lastly, using computer simulations, it is shown that optimizing both the sensing time and the fusion scheme together increases the throughput of the SUs significantly.

The remainder of this chapter is organized as follows. The system model is introduced in Section 3.1. In Section 3.2, the mathematical formulation of sensing-

throughput tradeoff problem with cooperative spectrum sensing in a small area CRN is presented. An iterative algorithm to find both the optimal sensing time and the fusion parameter k is presented in Section 3.3. Computer simulations are provided in Section 3.4 to show the performances of the proposed algorithm. We draw a summary of this chapter in Section 3.5. Appendices of some mathematics proofs required in this chapter are given in Section 3.6.

3.1 System Model

In a CRN, the SUs cannot use a channel continuously without first checking the availability of the channel. Therefore, sensing periods have to be allocated periodically in between transmissions of SUs to verify the continual absence of the PU before the CRN can continue to utilize the licensed channel [165]. To accommodate both data transmission and channel sensing, the basic frame structure of a CRN consists of, at least, a sensing slot and a transmission slot as shown in Fig. 3.1, where the sensing time and the total frame time are denoted as τ and T respectively.

Figure 3.1: Frame structure of CR.

Spectrum sensing is conducted during the sensing period. Consider that the PU is either idle or active throughout the sensing period, the spectrum sensing is essentially a binary hypothesis testing problem. Denote $\mathcal{H}_0$ and $\mathcal{H}_1$ as the hypotheses of the absence and presence of the PU respectively, the sampled signals

that are received at the i th SU during the sensing period are given as

$$\mathcal{H}_0 : \quad x_i(n) = u_i(n), \quad (3.1)$$

$$\mathcal{H}_1 : \quad x_i(n) = h_i(n)s(n) + u_i(n), \quad (3.2)$$

where $s(n)$ denotes the signal from the PU where each sample is assumed to be an i.i.d. random process with zero mean and variance $\mathbf{E}[|s(n)|^2] = \sigma_s^2$. We denote $\mathbf{E}[\cdot]$ as the statistical expectation operator and $|\cdot|$ as the absolute function. Similar to [166], in this chapter, we assume a small area CRN where the distances between any SUs are small compared to the distance from any SU to the PU-Tx. Therefore, it is assumed that each channel gain $|h_i(n)|$ is Rayleigh distributed with same variance $\mathbf{E}[|h_i(n)|^2] = \sigma_h^2$. The noise $u_i(n)$ is assumed to be i.i.d. circularly symmetric complex Gaussian (CSCG) with zero mean and variance $\mathbf{E}[|u_i(n)|^2] = \sigma_u^2$. Assume $s(n)$, $h_i(n)$ and $u_i(n)$ are independent of each other and the average received SNR at each SU is given as $\gamma = \frac{\sigma_h^2 \sigma_s^2}{\sigma_u^2}$.

Consider that the SUs employ energy detector and measure their received powers during the sensing period, their measured received powers are given as

$$y_i = \frac{1}{M} \sum_{n=1}^M |x_i(n)|^2, \quad \forall i = 1, \dots, N. \quad (3.3)$$

Denote M as the number of signal samples collected at each SU during the sensing period, which is the product of the sensing time τ and the sampling frequency f_s . Denote ε_i as the threshold of the energy detector at the i th SU where when $y_i \geq \varepsilon_i$, the energy detector declares that the PU is present; otherwise, it decides that the channel is unoccupied. When the PU's signal is a complex-valued PSK signal, the energy detector's probabilities of detection and false alarm at each SU

are approximated respectively as [165]

$$P_{di}(\tau, \varepsilon_i) = \mathcal{Q} \left(\left(\frac{\varepsilon_i}{\sigma_u^2(\gamma + 1)} - 1 \right) \sqrt{\tau f_s} \right), \quad i = 1, \dots, N, \quad (3.4)$$

$$P_{fi}(\tau, \varepsilon_i) = \mathcal{Q} \left(\left(\frac{\varepsilon_i}{\sigma_u^2} - 1 \right) \sqrt{\tau f_s} \right), \quad i = 1, \dots, N, \quad (3.5)$$

where $\mathcal{Q}(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$ denotes the right-tail probability of a normalized Gaussian distribution.

After every SU makes its individual decision d_i , where $d_i = 1$ denotes PU is being detected and $d_i = 0$ denotes otherwise, their decisions are transmitted to the FC. The FC combines the decisions it received and makes a final decision d_{FC} on the presence of the PU for the CRN. Suppose the k -out-of- N fusion rule is adopted as the decision fusion scheme, the FC makes the final decision by the following test:

$$d_{FC} = \begin{cases} 1, & \text{if } \sum_{i=1}^N d_i \geq k \\ 0, & \text{otherwise} \end{cases} \quad (3.6)$$

where k is the fusion rule threshold and is an integer between 1 and N . When $k = 1$, the fusion rule becomes an OR-fusion rule and if $k = N$, the fusion rule becomes an AND-fusion rule.

By setting a common threshold, $\varepsilon_i = \varepsilon$, for the energy detectors at the SUs, the overall probabilities of detection and false alarm of the CRN when the k -out-of- N fusion rule is applied, are given as, respectively,

$$\mathbb{P}_d(\tau, k, \varepsilon) = \sum_{i=k}^N \binom{N}{i} P_d(\tau, \varepsilon)^i (1 - P_d(\tau, \varepsilon))^{N-i}, \quad (3.7)$$

$$\mathbb{P}_f(\tau, k, \varepsilon) = \sum_{i=k}^N \binom{N}{i} P_f(\tau, \varepsilon)^i (1 - P_f(\tau, \varepsilon))^{N-i} \quad (3.8)$$

where $\binom{N}{i}$ is the binomial coefficient. The system model considered in this chapter

Figure 3.2: System model of a small area cognitive radio network where k -out-of- N fusion rule is used.

is shown in Fig. 3.2, where the SUs use the same ε and the FC uses the k -out-of- N fusion rule. A centralized cooperative spectrum sensing system is considered where the FC optimizes τ and ε and feeds them back to the SUs.

3.2 Sensing-throughput Tradeoff Problem with Cooperative Spectrum Sensing

There are two possible scenarios for the SUs to utilize the channel. The first scenario occurs when the CRN manages to detect the PU's absence. The probability of this scenario occurring is $Pr(\mathcal{H}_0)(1 - \mathbb{P}_f(\tau, k, \varepsilon))$, where $Pr(\mathcal{H}_0)$ is the probability that the PU is absent. The second scenario is when the CRN fails to detect the presence of the PU. The probability of this scenario occurring is $Pr(\mathcal{H}_1)(1 - \mathbb{P}_d(\tau, k, \varepsilon))$, where $Pr(\mathcal{H}_1)$ is the probability of the PU being active in the channel. We denote C_0 and C_1 as the throughputs of the SUs if they are allowed to operate continuously in the absence and presence of the PU, respectively. Since a length of τ period out of the total frame time T is used for sensing, the average

achievable throughput of the SUs under these scenarios are given respectively as

$$R_0(\tau, k, \varepsilon) = C_0 Pr(\mathcal{H}_0) \left(1 - \frac{\tau}{T}\right) (1 - \mathbb{P}_f(\tau, k, \varepsilon)), \quad (3.9)$$

$$R_1(\tau, k, \varepsilon) = C_1 Pr(\mathcal{H}_1) \left(1 - \frac{\tau}{T}\right) (1 - \mathbb{P}_d(\tau, k, \varepsilon)). \quad (3.10)$$

The average achievable throughput of the SUs is given as

$$R(\tau, k, \varepsilon) = R_0(\tau, k, \varepsilon) + R_1(\tau, k, \varepsilon). \quad (3.11)$$

From (3.11), the average achievable throughput of the CRN depends on the sensing time τ , the threshold ε , and k . Hence, the question is, “What are the values of k , τ and ε that achieve the maximum average throughput while giving PU adequate protection?”.

The sensing-throughput tradeoff problem with cooperative spectrum sensing in a small area CRN is formulated as shown below

$$\begin{aligned} \max_{\tau, k, \varepsilon} \quad & R(\tau, k, \varepsilon) \\ \text{s.t.} \quad & \mathbb{P}_d(\tau, k, \varepsilon) \geq \bar{\mathbb{P}}_d \\ & 0 \leq \tau \leq T \\ & 1 \leq k \leq N \end{aligned} \quad (3.12)$$

where $\bar{\mathbb{P}}_d$ is the targeted probability of detection that the FC needs to achieve in order to protect the PU. In the scenario when the CRN miss-detects the PU and transmits, its transmission signal experiences interference from the PU and therefore $C_0 > C_1$. From (3.4) and (3.5), we conclude that $P_f(\tau, \varepsilon) < P_d(\tau, \varepsilon)$ and therefore $(1 - \mathbb{P}_f(\tau, k, \varepsilon)) > (1 - \mathbb{P}_d(\tau, k, \varepsilon))$. Furthermore in CRN, we are interested in frequency bands that are under-utilized, such as frequency bands with $Pr(\mathcal{H}_0) > Pr(\mathcal{H}_1)$, then from these results, we have $R_0(\tau, k, \varepsilon) \gg R_1(\tau, k, \varepsilon)$.

Hence, we replace the objective function $R(\tau, k, \varepsilon)$ with $R_0(\tau, k, \varepsilon)$ to simplify the problem.

Lemma 3.1. *The optimal solution occurs when the constraint $\mathbb{P}_d(\tau, k, \varepsilon) \geq \bar{\mathbb{P}}_d$ is at equality.*

Proof: For any given τ and k , we may choose a threshold ε_0 such that $\mathbb{P}_d(\tau, k, \varepsilon_0) = \bar{\mathbb{P}}_d$. Any other threshold that is smaller than ε_0 , will also be able to satisfy the constraint for the same τ and k . This is because when k -out-of- N fusion rule is used, from (3.4), (3.5), (3.7) and (3.8), we can observe that $\mathbb{P}_d(\tau, k, \varepsilon)$ and $\mathbb{P}_f(\tau, k, \varepsilon)$ is monotonically decreasing in ε for a fixed τ and k . This means if $\varepsilon_1 < \varepsilon_0$, then $\mathbb{P}_d(\tau, k, \varepsilon_1) > \mathbb{P}_d(\tau, k, \varepsilon_0)$ and $\mathbb{P}_f(\tau, k, \varepsilon_1) > \mathbb{P}_f(\tau, k, \varepsilon_0)$. These lead to both $R_1(\tau, k, \varepsilon_1) < R_1(\tau, k, \varepsilon_0)$ and $R_0(\tau, k, \varepsilon_1) < R_0(\tau, k, \varepsilon_0)$ and therefore, $R(\tau, k, \varepsilon_1) < R(\tau, k, \varepsilon_0)$. Choosing a smaller threshold can satisfy the constraint $\mathbb{P}_d(\tau, k, \varepsilon) \geq \bar{\mathbb{P}}_d$ but always resulted in a lower average throughput for the SUs. Therefore, the optimal solution must occur when the threshold is chosen such that $\mathbb{P}_d(\tau, k, \varepsilon_0) = \bar{\mathbb{P}}_d$. ■

We further delve into the constraint when it is at equality. The detailed expression is given by

$$\sum_{i=k}^N \binom{N}{i} P_d(\tau, \varepsilon)^i (1 - P_d(\tau, \varepsilon))^{N-i} = \bar{\mathbb{P}}_d. \quad (3.13)$$

Define $\bar{P}_d(k)$ as the targeted probability of detection that each SU needs to achieve so that $\mathbb{P}_d(\tau, k, \varepsilon) = \bar{\mathbb{P}}_d$ at a given k . Then, for any given k , the equality constraint is satisfied when the SUs set $P_d(\tau, \varepsilon) = \bar{P}_d(k)$ where

$$\sum_{i=k}^N \binom{N}{i} \bar{P}_d(k)^i (1 - \bar{P}_d(k))^{N-i} = \bar{\mathbb{P}}_d. \quad (3.14)$$

For any given pair of τ and k , we are able to determine a threshold from (3.4) that is able to satisfy the equality constraint and is also the optimal threshold for the

problem (3.12) given by

$$\varepsilon(\tau, k) = \sigma_u^2 \left(\sqrt{\frac{2\gamma + 1}{\tau f_s}} \mathcal{Q}^{-1}(\bar{P}_d(k)) + \gamma + 1 \right). \quad (3.15)$$

Since the optimal threshold is given by (3.15) for any given τ and k , the optimization problem (3.12) is reduced to

$$\begin{aligned} \max_{\tau, k} \quad & \hat{R}_0(\tau, k) \\ \text{s.t.} \quad & 0 \leq \tau \leq T \\ & 1 \leq k \leq N \end{aligned} \quad (3.16)$$

where $\hat{R}_0(\tau, k)$ denotes the value of $R_0(\tau, k, \varepsilon)$ with the threshold ε chosen by (3.15). The probabilities of false alarm at each SU and the FC with the threshold chosen by (3.15) are given respectively as

$$\hat{P}_f(\tau, k) = \mathcal{Q}(\alpha + \beta\sqrt{\tau}) \quad (3.17)$$

and

$$\hat{\mathbb{P}}_f(\tau, k) = \sum_{i=k}^N \binom{N}{i} \hat{P}_f(\tau, k)^i \left(1 - \hat{P}_f(\tau, k)\right)^{N-i} \quad (3.18)$$

where $\alpha = (\gamma + 1)\mathcal{Q}^{-1}(\bar{P}_d(k))$ and $\beta = \gamma\sqrt{f_s}$. In the next section, we propose an iterative algorithm to find the solutions of τ and k for the optimization problem (3.16).

3.3 Proposed Iterative Algorithm for Small Area CRN

Instead of solving the two-variable optimization problem (3.16) directly, we propose an algorithm that solves (3.16) by decoupling it into two single-variable sub-optimization problems. The main idea of the proposed algorithm is to maximize the objective function (3.16) over k with τ fixed and then over τ with k fixed, iteratively. Denote j as the number of iterations, the proposed algorithm iterates until $k^{(j)} = k^{(j-1)}$ and $\tau^{(j)} = \tau^{(j-1)}$.

3.3.1 Sub-optimization Problem One

The first sub-optimization problem is that for a given $k = \tilde{k}$, we find the optimal value of τ that maximizes the throughput of the CRN subject to the probability of detection of the FC meeting the required target probability of detection $\bar{\mathbb{P}}_d$. In this sub-optimization problem, k is treated as a constant and the threshold ε depends only on τ to meet the probability of detection constraint. The throughput of the CRN is just a function of τ and the optimization problem (3.16) reduces to

$$\begin{aligned} \max_{\tau} \quad & \tilde{R}_0(\tau) \triangleq \hat{R}_0(\tau, k)|_{k=\tilde{k}} = C_0 Pr(\mathcal{H}_0) \left(1 - \frac{\tau}{T}\right) (1 - \tilde{\mathbb{P}}_f(\tau)) \quad (3.19) \\ \text{s.t.} \quad & 0 \leq \tau \leq T \end{aligned}$$

where $\tilde{\mathbb{P}}_f(\tau)$ is $\hat{\mathbb{P}}_f(\tau, k)$ for a given $k = \tilde{k}$. There is no closed-form solution of τ for any of the possible $\tilde{k}$ values. Hence, a search for the optimal τ over $0 \leq \tau \leq T$ is required for any given $\tilde{k}$. However, instead of an exhaustive search for τ , efficient search algorithms can be used if $\tilde{R}_0(\tau)$ is a unimodal function in the range $0 \leq \tau \leq T$ for any given $\tilde{k}$. Denote τ^* as the optimal sensing time for any given $\tilde{k}$. If $\tilde{R}_0(\tau)$ is a unimodal function, then $\tilde{R}_0(\tau)$ is monotonically increasing from $0 \leq \tau < \tau^*$

and is monotonically decreasing from $\tau^* < \tau \leq T$. Hence, $\tilde{R}_0(\tau^*)$ is the only local maximum in the entire range of $0 \leq \tau \leq T$. Next, we will prove that $\tilde{R}_0(\tau)$ is a unimodal function in the range $0 \leq \tau \leq T$ for any given $\tilde{k}$.

Proposition 3.1: If $\tilde{R}_0(\tau)$ satisfies the three conditions listed below, then $\tilde{R}_0(\tau)$ must be a unimodal function in the range of $0 \leq \tau \leq T$ for any $\tilde{k}$ value between 1 and N .

1. $\nabla \tilde{R}_0(0) > 0, \quad \forall \tilde{k} = 1, \dots, N.$
2. $\nabla \tilde{R}_0(T) < 0, \quad \forall \tilde{k} = 1, \dots, N.$
3. There is a unique τ^* where $0 < \tau^* < T$ such that $\nabla \tilde{R}_0(\tau^*) = 0, \quad \forall \tilde{k} = 1, \dots, N.$

where ∇ denotes the differentiation of the function with respect to its argument.

Proof: The first condition means that $\tilde{R}_0(\tau)$ is an increasing function at the point $\tau = 0$ and the second condition means that the function $\tilde{R}_0(\tau)$ is decreasing at the point $\tau = T$. Hence, these two conditions imply that there must be at least a point in $0 < \tau < T$ that maximizes $\tilde{R}_0(\tau)$. If the third condition is true, then together with the first condition, $\nabla \tilde{R}_0(\tau)$ must be positive in the range of $0 \leq \tau < \tau^*$ which infers that $\tilde{R}_0(\tau)$ is strictly increasing in this region for any given $\tilde{k}$. The second and third conditions together implies that $\nabla \tilde{R}_0(\tau)$ must be negative in the range of $\tau^* < \tau \leq T$ which infers that $\tilde{R}_0(\tau)$ is strictly decreasing in this region for any given $\tilde{k}$. The three conditions jointly imply that $\tilde{R}_0(\tau^*)$ must be the only local maximum in the entire range of $0 \leq \tau \leq T$ and thus $\tilde{R}_0(\tau)$ is a unimodal function in the range $0 \leq \tau \leq T$. ■

To show that the function $\tilde{R}_0(\tau)$ does satisfy all the three conditions for it to be a unimodal function, we first derive the derivative of the function $\tilde{R}_0(\tau)$ which is given as

$$\nabla \tilde{R}_0(\tau) = -C_T \left(1 - \tilde{\mathbb{P}}_f(\tau) + (T - \tau) \nabla \tilde{\mathbb{P}}_f(\tau) \right) \quad (3.20)$$

where $C_T = \frac{1}{T}C_0P(\mathcal{H}_0)$ is a positive constant, and

$$\nabla \tilde{\mathbb{P}}_f(\tau) = \tilde{k} \binom{N}{\tilde{k}} \tilde{P}_f(\tau)^{\tilde{k}-1} (1 - \tilde{P}_f(\tau))^{N-\tilde{k}} \nabla \tilde{P}_f(\tau) \quad (3.21)$$

where $\tilde{P}_f(\tau)$ is $\hat{P}_f(\tau, \tilde{k})$, and

$$\nabla \tilde{P}_f(\tau) = -\frac{\beta}{\sqrt{8\pi\tau}} \exp\left(-\frac{(\alpha + \beta\sqrt{\tau})^2}{2}\right). \quad (3.22)$$

At $\tau = 0$, from (3.22), we have $\nabla \tilde{P}_f(0) = -\infty$, and hence, $\nabla \tilde{\mathbb{P}}_f(0) = -\infty$ for all possible $\tilde{k}$ values in (3.21). Therefore, $\nabla \tilde{R}_0(0) = \infty$, and thus the first condition is satisfied. At $\tau = T$, from (3.20), we obtain

$$\begin{aligned} \nabla \tilde{R}_0(T) &= -C_T \left(1 - \tilde{\mathbb{P}}_f(T)\right) \\ &= -C_T \left(1 - \sum_{i=\tilde{k}}^N \binom{N}{i} \tilde{P}_f(T)^i \left(1 - \tilde{P}_f(T)\right)^{N-i}\right). \end{aligned} \quad (3.23)$$

Since $0 \leq \tilde{P}_f(T) \leq 1$, we have

$$\sum_{i=\tilde{k}}^N \binom{N}{i} \tilde{P}_f(T)^i \left(1 - \tilde{P}_f(T)\right)^{N-i} < 1, \quad \forall \tilde{k} = 1, \dots, N. \quad (3.24)$$

Therefore, $\nabla \tilde{R}_0(T)$ must be a negative value for all the possible $\tilde{k}$ values, and hence the second condition is satisfied.

To prove that $\tilde{R}_0(\tau)$ satisfies the third condition, let $\nabla \tilde{R}_0(\tau) = 0$ to obtain $g(\tau) = h(\tau, \tilde{k})$ where

$$g(\tau) = (\alpha + \beta\sqrt{\tau})^2, \quad (3.25)$$

and

$$h(\tau, \tilde{k}) = -2 \ln \left[\left(\frac{\sqrt{8\pi\tau}}{\beta \tilde{k} \binom{N}{\tilde{k}} (T - \tau)} \right) \sum_{i=0}^{\tilde{k}-1} \binom{N}{i} \tilde{P}_f(\tau)^{i-\tilde{k}+1} \left(1 - \tilde{P}_f(\tau)\right)^{\tilde{k}-i} \right]. \quad (3.26)$$

The third condition is satisfied if the two functions $g(\tau)$ and $h(\tau, \tilde{k})$ intersect each other only once in the entire range of $0 \leq \tau \leq T$ for all $\tilde{k} = 1, \dots, N$. The τ that corresponds to the intersection point is the unique τ^* which gives $\nabla \tilde{R}_0(\tau^*) = 0$ and is also the optimal sensing time that maximizes the function (3.19). First, we partition the region of τ into two regions, where the lower and higher regions of τ are defined respectively as

$$\mathbb{R}_L = \{ \tau \mid (\alpha + \beta\sqrt{\tau}) \leq 0, 0 \leq \tau \leq T \}, \quad (3.27)$$

and

$$\mathbb{R}_H = \{ \tau \mid (\alpha + \beta\sqrt{\tau}) > 0, 0 \leq \tau \leq T \}. \quad (3.28)$$

At the lower region of τ , since $\beta > 0$, from (3.25), the function $g(\tau)$ is monotonically decreasing in τ . However, at the higher region, $g(\tau)$ is monotonically increasing in τ . For the function $h(\tau, \tilde{k})$, it is always monotonically decreasing in τ for all $\tilde{k} = 1, \dots, N$, as shown in Appendix 3.6.1, where we have proved that

$$\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < 0, \quad \forall 0 \leq \tau \leq T, \tilde{k} = 1, \dots, N. \quad (3.29)$$

There are only two possible scenarios on the manner in which these two functions intersect. In the first scenario, if $g(\tau)$ and $h(\tau, \tilde{k})$ intersect each other in the lower region, it is impossible for them to intersect more than once because $h(\tau, \tilde{k})$ is always decreasing at a faster rate than $g(\tau)$ at the lower region $\mathbb{R}_L$. The proof of the property,

$$\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < \frac{\partial}{\partial \tau} g(\tau), \quad \forall \tau \in \mathbb{R}_L, \tilde{k} = 1, \dots, N$$

is given in Appendix 3.6.2. If $g(\tau)$ and $h(\tau, \tilde{k})$ have intersected in the lower region, then $g(\tau)$ must be greater than $h(\tau, \tilde{k})$ at the higher region $\mathbb{R}_H$. Since in $\mathbb{R}_H$, $g(\tau)$

is a monotonically increasing function while $h(\tau, \tilde{k})$ is a monotonically decreasing function, it is impossible for them to intersect in the higher region. Hence, in this scenario, only one intersection occurs in the range of $0 \leq \tau \leq T$ and it occurs at the lower region.

In the second scenario, when the two functions do not intersect in the lower region, then they must intersect at the higher region. This is true as the first two conditions in proposition 3.1 infer that there must be at least one intersection in the entire range $0 \leq \tau \leq T$. In the region $\mathbb{R}_H$, $g(\tau)$ is monotonically increasing, and $h(\tau, \tilde{k})$ is monotonically decreasing; therefore, they can only intersect at most once. In either scenario, there is only one intersection between the two functions in the entire range of τ , and therefore, $\tilde{R}_0(\tau)$ satisfies the third condition. ■

We have proven that $\tilde{R}_0(\tau)$ satisfies all the three conditions and is a unimodal function. Therefore, the optimization problem (3.19) can be solved by algorithms such as Bisection search method, Golden section method, Newton's method, etc., [167] for any given $\tilde{k}$. An example of using Golden section method to obtain the optimal τ in this sub-optimization problem is provided in Table 3.1.

3.3.2 Sub-optimization Problem Two

The second sub-optimization problem is that for a given value of $\tau = \check{\tau}$, find the optimal value of k that maximizes the problem (3.16). Since τ is treated as a constant in this sub-problem, the optimization problem reduces to

$$\begin{aligned} \max_k \quad & \tilde{R}_0(k) \triangleq \hat{R}_0(\tau, k) |_{\tau=\check{\tau}} = C_0 Pr(\mathcal{H}_0) \left(1 - \frac{\check{\tau}}{T}\right) (1 - \check{\mathbb{P}}_f(k)) \quad (3.30) \\ \text{s.t.} \quad & 0 \leq k \leq N. \end{aligned}$$

Table 3.1: Golden section method

Algorithm 3.1: Solving the optimal τ that maximize $\tilde{R}_0(\tau)$ for a given k .
Input: k, T, μ $\{\mu$ is the tolerant of the accuracy of $\tau^*\}$
Initialization : $\tau_{min} \leftarrow 0 ; \tau_{max} \leftarrow T$
$n = (\sqrt{5} - 1) / 2$
$\tau_1 = \tau_{min} + (1 - n)(\tau_{max} - \tau_{min})$
$f_1 = \tilde{R}_0(\tau_1)$
$\tau_2 = \tau_{min} + n(\tau_{max} - \tau_{min})$
$f_2 = \tilde{R}_0(\tau_2)$
while $(\tau_{max} - \tau_{min}) > \mu$ do
if $f_1 > f_2$ then
$\tau_{min} \leftarrow \tau_1$
$\tau_1 \leftarrow \tau_2$
$f_1 \leftarrow f_2$
$\tau_2 = \tau_{min} + n(\tau_{max} - \tau_{min})$
$f_2 = \tilde{R}_0(\tau_2)$
else
$\tau_{max} \leftarrow \tau_2$
$\tau_2 \leftarrow \tau_1$
$f_2 \leftarrow f_1$
$\tau_1 = \tau_{min} + (1 - n)(\tau_{max} - \tau_{min})$
$f_1 = \tilde{R}_0(\tau_1)$
end if
end while
Output: $\tau^* = \tau_{min} + (\tau_{max} - \tau_{min})/2$

Removing the constant terms in (3.30), the second sub-optimization problem is equivalent to

$$\begin{aligned} \min_k \quad & \check{P}_f(k) \triangleq \hat{P}_f(\tau, k) |_{\tau=\check{\tau}} = \sum_{i=k}^N \binom{N}{i} \check{P}_f(k)^i (1 - \check{P}_f(k))^{N-i} \\ \text{s.t.} \quad & 0 \leq k \leq N \end{aligned} \quad (3.31)$$

where $\check{P}_f(k)$ is $\hat{P}_f(\tau, k)$ for a given $\tau = \check{\tau}$. No closed-form solution for k is available for this sub-problem, and a search over all possible k values is required. However, since k is an integer and range from 1 to N , it is not computationally expensive to search for the optimal k^* .

Combining the steps for solving the two sub-optimization problems, the proposed iterative algorithm is summarized in Table 3.2. Next we analyze the properties of the proposed iterative algorithm. The objective function of the optimization problem $\hat{R}_0(\tau, k)$, is non-decreasing at every iteration, as one can easily conclude from Algorithm 3.2 that

$$\hat{R}_0(\tau^{(j)}, k^{(j)}) \leq \hat{R}_0(\tau^{(j)}, k^{(j+1)}) \leq \hat{R}_0(\tau^{(j+1)}, k^{(j+1)}), \quad \forall j. \quad (3.32)$$

The proposed iterative algorithm iterates until $\hat{R}_0(\tau, k)$ converges to a maximum point in the two-dimensional space since one can conclude that, at the converged solutions τ^* and k^* , $\hat{R}_0(\tau^*, k^*) \geq \hat{R}_0(\tau^*, k)$ for all $k = 1, \dots, N$ and $\hat{R}_0(\tau^*, k^*) \geq \hat{R}_0(\tau, k^*)$ for $0 \leq \tau \leq T$. At τ^* , $\hat{R}_0(\tau^*, k^*)$ is the largest across the k dimension and at k^* , $\hat{R}_0(\tau^*, k^*)$ is the largest across the τ dimension. Although the proposed iterative algorithm may converge to a solution of a possible local maximum point, this point is actually the global maximum point as we have found from the simulations described in the Section 3.4. This is true for any initial value of $\tau^{(0)}$ that is chosen in between zero and T , which suggests that the proposed iterative algorithm is not sensitive to the initial value of $\tau^{(0)}$ and always

Table 3.2: Proposed iterative algorithm for small area CRN

Algorithm 3.2: Find the τ and k -out-of- N fusion rule that maximize $\hat{R}_0(\tau)$.
Input: $\tau^{(0)}$ {Any value of τ in between 0 and T }
Initialization: $j \leftarrow 0$; $k^{(0)} \leftarrow 1$
repeat
1) Given $\tau^{(j)}$, find the k^* that solves (3.31) by computing $\check{\mathbb{P}}_f(k)$ from $k = 1$ to N .
2) $k^{(j+1)} \leftarrow k^*$
3) Given $k^{(j+1)}$, find τ^* using Algorithm 3.1.
4) $\tau^{(j+1)} \leftarrow \tau^*$
5) $j \leftarrow (j + 1)$
until $\tau^{(j)} == \tau^{(j-1)}$ and $k^{(j)} == k^{(j-1)}$
Output: $\tau^{(j)}$; $k^{(j)}$

converges to the same maximum point.

As the unimodal characteristic of SUs' throughput as a function of τ regardless of the k value is proven, the sub-optimization problem one can be solved by Bisection search, Golden section method, etc.. If Bisection search is used in the proposed iterative algorithm, the number of iterations needed to solve for τ at each k is $\lceil \log_2 \left(\frac{T}{\mu} \right) \rceil$ where μ is the maximum allowable error in the optimal sensing time. In the simulations that are provided in the next section, μ is set at $0.1/f_s = 1.67 * 10^{-5}$ ms, hence 21 iterations, are needed. However, if the unimodal characteristic is not proven, $\tilde{R}_0(\tau)$ may have multiple local maximum points and an exhaustive search is needed to obtain the maximum $\tilde{R}_0(\tau)$. An exhaustive search with the same μ will need $\frac{T}{\mu} = 1.2$ million points. In the exhaustive search, all the k values are needed to be searched for. However, from the simulations, the average number of iterations needed for the proposed algorithm to search for k when $N = 25$ is 4. The average total iterations needed for the proposed algorithm is $4 \times 21 = 84$ iterations while an exhaustive search needs $25 \times 1.2\text{M} = 30$ million iterations.

3.4 Simulation Results and Discussion

In this section, computer simulation results are presented to evaluate the sensing-throughput tradeoff with k -out-of- N fusion rule where each SU is assumed to use the energy detector. Simulations are carried out to find the maximum $R_0(\tau, k, \varepsilon)$ by optimizing ε , k , and τ while providing sufficient protection to the PU.

In the following simulations, we set the frame duration to be $T = 20\text{ms}$. The sampling frequency of the received signal is assumed to be 6MHz, and $\bar{\mathbb{P}}_d$ is set at 99.99%. The SNR γ of the PU's signal that is received at the SUs and the number of SUs N are varied in the simulations.

Figures 3.3 and 3.4 show the optimal k at different γ and N values, respectively. First, a time consuming two-dimensional exhaustive search over k and τ is used to find the global maximum of the throughput $\hat{R}_0(\tau, k)$. Next, the optimization problem is solved using the proposed iterative algorithm, and the obtained solutions are compared with those determined by the two-dimensional exhaustive search method. It can be seen from Figs. 3.3 and 3.4 that the proposed iterative algorithm does obtain the global optimal values for k at different γ and N values, respectively. It is also observed that the optimal k value in the k -out-of- N fusion rule varies across different γ and N values. This shows that there is no single k value that is optimal for all cases, and hence, there is a need to optimize the k value to enhance the throughput of the CRN.

Figures 3.5 and 3.6 show the optimal $\frac{\tau}{T}$ at different γ and N values, respectively. From both figures, it can be seen that the proposed iterative algorithm does obtain the global optimal values for τ at different γ and N values. Figure 3.5 shows an interesting result of the optimal sensing time with different SNRs values at $N = 25$. When the SNR decreases from 0dB to -27dB , the optimal fraction of frame time used for sensing increases. Since the throughput of the CRN is proportional to the length of the transmission $(T - \tau)$, and is inversely proportional to the probability

Figure 3.3: Optimal k -out-of- N fusion rule that maximizes the throughput of a small area CRN when $N = 25$.

Figure 3.4: Optimal k -out-of- N fusion rule that maximizes the throughput of a small area CRN at different N values.

Figure 3.5: Optimal sensing time that maximizes the throughput of a small area CRN when $N = 25$.

Figure 3.6: Optimal sensing time that maximizes the throughput of a small area CRN at different N values.

Figure 3.7: Maximum achievable throughput of CRN by various k -out-of- N fusion rules at different SNR values.

of false alarm $\hat{\mathbb{P}}_f(\tau, k)$, the simulation results suggest that, at low SNRs, i.e., 0dB to -27 dB, allocating a larger portion of the total frame time for sensing to lower $\hat{\mathbb{P}}_f(\tau, k)$ and spending less time for transmission, in fact, increases the throughput of the CRN. However, at extremely low SNR, i.e., below -27 dB, the optimal fraction of frame time used for sensing starts to decrease. At this SNR region, $\hat{\mathbb{P}}_f(\tau, k)$ is very close to 1; although a large amount of the total frame time has been allocated to sensing, as from Fig. 3.5, nearly 69% of the frame time is spent on sensing at -27 dB. This suggests that since $\hat{\mathbb{P}}_f(\tau, k)$ does not improve much with more sensing time at extremely low SNRs, it is wiser to allocate more time for transmission to improve the throughput of the CRN. When the number of SUs increases, the sensing performance improves. Hence, with greater number of SUs, the targeted probability of detection can be achieved with less sensing time and therefore, in Fig. 3.6, as N increases, the optimal fraction of frame time used for sensing decreases.

Figures 3.7 and 3.8 compare the maximum normalized throughput when k is

Figure 3.8: Maximum achievable throughput of CRN by various k -out-of- N fusion rules at different N values.

optimized to cases where k is fixed to 1 (OR fusion rule), and k is fixed to N (AND fusion rule) when the γ and N values are varied, respectively. The normalized throughput is defined as $(1 - \frac{\tau}{T}) (1 - \hat{\mathbb{P}}_f(\tau, k))$. Optimal sensing time is used in each of the fusion rules, and it is seen that AND rule always performs the worst for all γ and N values. In particular, from Fig. 3.7, at SNR range of -25dB to -15dB , when k is optimized, its throughput is much higher than that obtained by using the OR and AND fusion rules. From Fig. 3.8, at $\gamma = -20\text{dB}$, when N is large, the throughput with an optimal k is about 4 times higher than that obtained by using AND fusion rule and 1.37 times higher than that obtained by using OR fusion rule.

Figures 3.9 and 3.10 compare the maximum normalized throughput when τ is optimized, as opposed to those with τ fixed at 5%, 10%, and 20% of the total frame time when the γ and N values are varied, respectively. Optimal k values are used for each fixed τ . From Fig. 3.9, it is clear that the disadvantage with a fixed sensing time is that, at high SNR levels where the PU can be easily detected, the

Figure 3.9: Maximum achievable throughput of CRN at various sensing time and at different SNR values.

Figure 3.10: Maximum achievable throughput of CRN at various sensing time and at different N values.

Figure 3.11: Sensing-throughput tradeoff with various k -out-of- N fusion rules.

throughput of the CRN is bounded by the percentage of the total frame time that is spent for sensing. From Fig 3.6, we have concluded that at small N values, it is better to increase the sensing time. Therefore, in Fig. 3.10, it can be observed that if τ is fixed at a larger percentage of the frame time, the throughput is better at small N values. However, when N becomes large, the throughput will be bounded by the ratio of the frame time used for sensing. Hence, optimizing the sensing time is important at different N values.

In Fig. 3.11, the effect of the fusion rule threshold k on the sensing-throughput tradeoff performance is illustrated. The normalized throughput curves are plotted at SNR $\gamma = -20\text{dB}$. It is clear that optimizing k essentially maximizes the achievable throughput for a given sensing time. With the above simulation results, the inter-relationships between the throughput, the sensing time, and the k value are presented, and we have shown the significant performance gains by jointly optimizing ε , k , and sensing time, as proposed.

3.5 Summary

In this chapter, we have proposed to optimize the energy threshold, fusion rule threshold and the sensing time to improve the achievable throughput of a small area CRN. By using the k -out-of- N fusion rule as the basic fusion scheme, we have proposed an iterative algorithm to obtain the sensing time and the k value that maximizes the throughput of the CRN, subject to adequate protection to the PU, in the form of a minimum target probability of detection. The results of the proposed iterative algorithm have been verified to be optimum by comparing them with exhaustive search results. We have shown that significant improvement in the throughput of the CRN has been achieved when both the parameters for the fusion scheme and sensing time are jointly optimized.

3.6 Appendices

3.6.1 Proof of $h(\tau, \tilde{k})$ is monotonically decreasing in τ .

We prove that $\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < 0$ for the range $0 \leq \tau \leq T$ and for all $\tilde{k} = 1, \dots, N$. From (3.26),

$$\begin{aligned} h(\tau, \tilde{k}) &= -2 \ln \left[\left(\frac{\sqrt{8\pi\tau}}{\beta \tilde{k} \binom{N}{\tilde{k}} (T - \tau)} \right) \sum_{i=0}^{\tilde{k}-1} \binom{N}{i} \tilde{P}_f(\tau)^{i-\tilde{k}+1} \left(1 - \tilde{P}_f(\tau) \right)^{\tilde{k}-i} \right] \\ &= -2 \ln \left(\frac{\sqrt{8\pi}}{\beta \tilde{k} \binom{N}{\tilde{k}}} \right) - \ln(\tau) + 2 \ln(T - \tau) \\ &\quad - 2 \ln \left[\sum_{i=0}^{\tilde{k}-1} \binom{N}{i} \tilde{P}_f(\tau)^{i-\tilde{k}+1} \left(1 - \tilde{P}_f(\tau) \right)^{\tilde{k}-i} \right]. \end{aligned}$$

Taking its derivative,

$$\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) = -\frac{1}{\tau} - \frac{2}{T - \tau} + \frac{2(A + B + C)}{\sum_{i=0}^{\tilde{k}-1} \binom{N}{i} \tilde{P}_f(\tau)^{i-\tilde{k}+1} (1 - \tilde{P}_f(\tau))^{\tilde{k}-i}} \nabla \tilde{P}_f(\tau) \quad (3.33)$$

where

$$A = \sum_{i=0}^{\tilde{k}-1} \binom{N}{i} \tilde{P}_f(\tau)^{i-\tilde{k}+1} (1 - \tilde{P}_f(\tau))^{\tilde{k}-i-1}, \quad (3.34)$$

$$B = \sum_{i=1}^{\tilde{k}-1} \left[(i-1) \binom{N}{\tilde{k}-i} + i \binom{N}{\tilde{k}-i-1} \right] \left(\frac{1 - \tilde{P}_f(\tau)}{\tilde{P}_f(\tau)} \right)^i, \quad (3.35)$$

and

$$C = (\tilde{k} - 1) \left(\frac{1 - \tilde{P}_f(\tau)}{\tilde{P}_f(\tau)} \right)^{\tilde{k}}. \quad (3.36)$$

From (3.22), we have $\nabla \tilde{P}_f(\tau) < 0$ for $0 \leq \tau \leq T$, and from (3.34)-(3.36), we have A, B , and $C \geq 0$ for all $\tilde{k} = 1, \dots, N$. Hence, from (3.33), we obtain $\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < 0$ for $0 \leq \tau \leq T$ and all possible $\tilde{k}$.

3.6.2 Proof of $\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < \frac{\partial}{\partial \tau} g(\tau)$.

We first prove for the case when $\tilde{k} = 1$ that $\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < \frac{\partial}{\partial \tau} g(\tau)$, in the lower region of τ , i.e., $\mathbb{R}_L$. From (3.34)-(3.36), when $\tilde{k} = 1$, we have $A = 1$, B and $C = 0$. Therefore, from (3.33), we have

$$\begin{aligned} \frac{\partial}{\partial \tau} h(\tau, 1) &= -\frac{1}{\tau} - \frac{2}{T - \tau} + \frac{2}{1 - \tilde{P}_f(\tau)} \nabla \tilde{P}_f(\tau) \\ &= -\frac{1}{\tau} - \frac{2}{T - \tau} \\ &\quad - \frac{\beta}{\sqrt{2\pi\tau} (1 - \mathcal{Q}(\alpha + \beta\sqrt{\tau}))} \exp\left(-\frac{(\alpha + \beta\sqrt{\tau})^2}{2}\right). \end{aligned} \quad (3.37)$$

The derivative of the function $g(\tau)$ is given as

$$\frac{\partial}{\partial \tau} g(\tau) = \frac{\beta}{\sqrt{\tau}} (\alpha + \beta\sqrt{\tau}). \quad (3.38)$$

From (3.37) and (3.38), the detailed expression of $\frac{\partial}{\partial \tau} h(\tau, 1) < \frac{\partial}{\partial \tau} g(\tau)$ is given as

$$-\frac{1}{\tau} - \frac{2}{T - \tau} - \frac{\beta}{\sqrt{2\pi\tau}(1 - \mathcal{Q}(\alpha + \beta\sqrt{\tau}))} \exp\left(-\frac{(\alpha + \beta\sqrt{\tau})^2}{2}\right) < \frac{\beta}{\sqrt{\tau}} (\alpha + \beta\sqrt{\tau}). \quad (3.39)$$

Let $x = -(\alpha + \beta\sqrt{\tau})$, and since in the region $\mathbb{R}_L$, we have $(\alpha + \beta\sqrt{\tau}) \leq 0$, therefore, $x \geq 0$. Substitute x into (3.39), we obtain

$$-\frac{1}{\beta\sqrt{\tau}} - \frac{2\sqrt{\tau}}{\beta(T - \tau)} - \frac{1}{\sqrt{2\pi}\mathcal{Q}(x)} \exp\left(-\frac{x^2}{2}\right) < -x, \quad \forall x \geq 0. \quad (3.40)$$

From [168], it is shown that

$$\frac{1}{\sqrt{2\pi}\mathcal{Q}(x)} \exp\left(-\frac{x^2}{2}\right) > x, \quad \forall x \geq 0.$$

Therefore, the inequality at (3.40) is verified, and we have proven $\frac{\partial}{\partial \tau} h(\tau, 1) < \frac{\partial}{\partial \tau} g(\tau)$ at the lower region of τ . Next we will prove $\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < \frac{\partial}{\partial \tau} h(\tau, 1)$ for $\tilde{k} = 2, \dots, N$. From (3.33), and since $\nabla \tilde{P}_f(\tau) < 0$ for $0 \leq \tau \leq T$, we have

$$\frac{2(A + B + C)}{\sum_{i=0}^{\tilde{k}-1} \binom{N}{i} \tilde{P}_f(\tau)^{i-\tilde{k}+1} (1 - \tilde{P}_f(\tau))^{\tilde{k}-i}} \nabla \tilde{P}_f(\tau) < \frac{2}{1 - \tilde{P}_f(\tau)} \nabla \tilde{P}_f(\tau), \quad \forall \tilde{k} = 2, \dots, N.$$

Since $\nabla \tilde{P}_f(\tau) < 0$ for $0 \leq \tau \leq T$,

$$\frac{2(A + B + C)}{\sum_{i=0}^{\tilde{k}-1} \binom{N}{i} \tilde{P}_f(\tau)^{i-\tilde{k}+1} (1 - \tilde{P}_f(\tau))^{\tilde{k}-i}} > \frac{2}{1 - \tilde{P}_f(\tau)}, \quad \forall \tilde{k} = 2, \dots, N,$$

and after some algebraic manipulations, we have

$$1 + \frac{B + C}{A} > 1, \quad \forall \tilde{k} = 2, \dots, N. \quad (3.41)$$

Since $A, B, C > 0$ for all $\tilde{k} = 2, \dots, N$, the inequality at (3.41) is verified. This completes the proof that

$$\frac{\partial}{\partial \tau} h(\tau, \tilde{k}) < \frac{\partial}{\partial \tau} h(\tau, 1) < \frac{\partial}{\partial \tau} g(\tau), \quad \forall \tau \in \mathbb{R}_L, \quad \tilde{k} = 2, \dots, N. \quad (3.42)$$

Chapter 4

Optimization of Cooperative Spectrum Sensing in a Large Area Cognitive Radio Network

In Chapter 3, it is shown that both the sensing time and the cooperative fusion scheme affect the throughput of the CRN, and hence, a joint optimization of the sensing time and the cooperative fusion scheme is performed. A small area CRN is considered in Chapter 3 and in this chapter, we will consider a large area CRN such that the SUs, in general, have different detection SNRs. In a large area CRN, it is not efficient to use the k -out-of- N fusion rule, as the fusion rule makes no distinction between a decision from a SU with high detection SNR and a decision from a SU with low detection SNR. Decisions made by SUs with high detection SNRs should be more accurate and reliable. Therefore, an efficient decision fusion rule should take into account the differences in the reliability of the decisions made by different SUs. In this chapter, the optimal decision fusion rule for large area CRN will be derived during the optimization process of the sensing-throughput tradeoff problem with cooperative spectrum sensing. It will be shown that the optimal decision fusion rule is a weighted one based on the likelihood-ratio test.

Figure 4.1: System model of a large area cognitive radio network where weighted decision fusion rule is used.

The differences in the reliability of the decisions made by different SUs will be reflected in the weightings of the decisions in the fusion rule. The system model considered in this chapter is shown in Fig. 4.1, where the SUs can have different threshold values and weightings are given to the decisions of the SUs at the FC.

In this chapter, we further consider three scenarios. In the first scenario, we consider a CRN with a small number of SUs. The FC will optimize the sensing-throughput tradeoff problem with the SUs' thresholds, the fusion rule threshold, and the sensing time as the optimization variables. Since the number of SUs is small, the FC optimizes the individual thresholds of the SUs and feeds them back to the SUs as shown in Fig. 4.1. An algorithm is proposed for the FC to compute the thresholds of the individual SUs and the fusion rule jointly. When the SUs' thresholds are optimized, the weightings for the SUs' decisions are optimized as well since the weightings are functions of the thresholds.

In the second scenario, we consider large number of SUs. When the number of SUs is large, computing the optimal thresholds of individual SUs can be time consuming. Therefore, in this scenario, the SUs' thresholds are constrained to be the same and the FC needs to compute only one optimal threshold that applies to

all the SUs. This reduces the complexity of the optimization problem and shortens the computational time compared to optimizing the individual SUs' thresholds. To further reduce the computational time, the upper and lower bounds of the threshold are derived and a one-dimensional numerical search between the bounds can be conducted to obtain the optimal value of the threshold.

In the first two scenarios, the FC acts as a centralized processor and computes the thresholds for the SUs. However, if the channel conditions between the SUs and the PU change frequently or there is no feedback channel available, computing the SUs' thresholds at the FC may not be practical. In the third scenario, we consider that the SUs optimize their own thresholds and the FC optimizes only the fusion rule threshold. If the sensing time is pre-fixed, then the feedback channel shown in Fig. 4.1 is not required in this scenario. When the SUs have no knowledge of each other's channel conditions, it is impossible for the SUs to optimize their thresholds such that the system-wide objective function is optimized at the FC. Hence, in this chapter, we propose two heuristic methods for the SUs to optimize their own thresholds under this scenario.

The remainder of this chapter is organized as follows. The system model is introduced in Section 4.1. In Section 4.2, the problem formulation of optimizing the sensing-throughput tradeoff problem in a large area CRN with small number of SUs is presented. The proposed algorithm to compute the thresholds of the individual SUs and the fusion rule jointly is also presented in Section 4.2. The solution for large number of SUs is presented in Section 4.3. In Section 4.4, the proposed methods to optimize the SUs' thresholds in a decentralized manner are introduced. Computer simulations are provided in Section 4.5. Finally, we draw a summary of this chapter in Section 4.6.

4.1 System Model

Similar to Chapter 3, the CRN consists of N SUs which cooperatively detect the presence of the PU by sending their sensing decisions to the secondary base station. We consider the channel coefficient from the PU to the i th SU, denoted as h_i , undergo flat fading and is constant during the sensing period. Therefore, the average detection SNR at the i th SU is given as $\gamma_i = \frac{|h_i|^2 \sigma_s^2}{\sigma_u^2}$. In IEEE 802.22, the TV station (PU-Tx) and the consumer premise equipments (SUs) are usually geographically fixed. It is reasonable to assume that the channels are relative constant throughout the period of interest and the average detection SNR can be measured a prior during a period when the TV station is known for sure to be broadcasting.

If the h_i is constant during the sensing period instead of fast fading as assumed in Chapter 3, the energy detector's probabilities of detection and false alarm at the i th SU are, respectively, approximated as [165]

$$P_{di}(\tau, \varepsilon_i) = \mathcal{Q} \left(\left(\frac{\varepsilon_i}{\sigma_u^2} - \gamma_i - 1 \right) \sqrt{\frac{\tau f_s}{2\gamma_i + 1}} \right), \quad (4.1)$$

$$P_{fi}(\tau, \varepsilon_i) = \mathcal{Q} \left(\left(\frac{\varepsilon_i}{\sigma_u^2} - 1 \right) \sqrt{\tau f_s} \right). \quad (4.2)$$

In this chapter, since a large area CRN is assumed, the h_i for every SU is, in general, different. Therefore, even if a common energy threshold is used, the probabilities of detection of the SUs will be different.

After every SU makes its own decision d_i and transmits to the secondary base station, we denote $\mathbf{d} = [d_1 \ d_2 \ \cdots \ d_N]$ as the received decisions at the FC and there will be 2^N possible outcomes of $\mathbf{d}$. The FC will make a final decision d_{FC} on the presence of the PU based on $\mathbf{d}$. Denote $\mathbf{D}^+$ as the set of $\mathbf{d}$ values that the FC will decide that the PU is present, while its complementary set $\mathbf{D}^-$ contains the set of $\mathbf{d}$ values that the FC will decide that the PU is absent. The FC has to determine

which $\mathbf{d}$ values should be included into $\mathbf{D}^+$ so that an optimal decision fusion rule is achieved. For example, if k -out-of- N fusion rule is used, all $\mathbf{d}$ values which have k or more number of $d_i = 1$ in it will be included into $\mathbf{D}^+$. The general equations for the probabilities of detection and false alarm at the FC using decision fusion rule are, respectively, given as

$$\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \sum_{\mathbf{d} \in \mathbf{D}^+} Pr(\mathbf{d} | \mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma}), \quad (4.3)$$

$$\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \sum_{\mathbf{d} \in \mathbf{D}^+} Pr(\mathbf{d} | \mathcal{H}_0, \tau, \boldsymbol{\varepsilon}) \quad (4.4)$$

where $\boldsymbol{\varepsilon} = [\varepsilon_1 \ \varepsilon_2 \ \cdots \ \varepsilon_N]$ and $\boldsymbol{\gamma} = [\gamma_1 \ \gamma_2 \ \cdots \ \gamma_N]$. Assume independence of the SUs' decisions, $Pr(\mathbf{d} | \mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma})$ and $Pr(\mathbf{d} | \mathcal{H}_0, \tau, \boldsymbol{\varepsilon})$ are, respectively, expressed as

$$Pr(\mathbf{d} | \mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma}) = \prod_{i=1}^N (1 - P_{di}(\tau, \varepsilon_i))^{(1-d_i)} (P_{di}(\tau, \varepsilon_i))^{d_i}, \quad (4.5)$$

$$Pr(\mathbf{d} | \mathcal{H}_0, \tau, \boldsymbol{\varepsilon}) = \prod_{i=1}^N (1 - P_{fi}(\tau, \varepsilon_i))^{(1-d_i)} (P_{fi}(\tau, \varepsilon_i))^{d_i}. \quad (4.6)$$

The rest of the system parameters are assumed to be the same as in Chapter 3, and hence, will not be repeated here.

4.2 Large Area CRN with Small Number of SUs

The average achievable throughput of the CRN is given as

$$\begin{aligned} R(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) &= \left(1 - \frac{\tau}{T}\right) C_0 Pr(\mathcal{H}_0) (1 - \mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)) \\ &\quad + \left(1 - \frac{\tau}{T}\right) C_1 Pr(\mathcal{H}_1) (1 - \mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)), \end{aligned} \quad (4.7)$$

and the objective in this section is to maximize the throughput of the CRN by finding the optimal decision fusion rule $\mathbf{D}^+$, sensing time τ , and the individual

energy detector thresholds ε_i of the SUs subject to the constraint that the PU is sufficiently protected. The problem is formulated as

$$\begin{aligned} \max_{\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+} \quad & R(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) \\ \text{s.t.} \quad & \mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) \geq \bar{\mathbb{P}}_d. \end{aligned} \quad (4.8)$$

Generally, the sensing-throughput tradeoff problem is not a convex problem and in order to solve (4.8), we need to transform the problem into subproblems with lower complexity.

First, we solve (4.8) for a specific τ value. The optimal solution occurs when the constraint is at equality. The proof is similar to that in Chapter 3 except that in this section, instead of a common SUs' threshold, the thresholds of SUs are allowed to be different. Since, the proof is similar, we will only provide a brief explanation here. For a specific τ and $\mathbf{D}^+$, the values of $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ and $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ are inversely proportional to the individual threshold ε_i . Increasing any of the energy detector thresholds ε_i decreases both values of $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ and $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$. From (4.7) and (4.8), it can be seen that the objective function is maximized when $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ and $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ are minimized. Hence, the thresholds ε_i should always be chosen to meet the minimum requirement of $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \bar{\mathbb{P}}_d$ so that $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ will also be at the minimum for a specific τ and $\mathbf{D}^+$.

When $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ is constrained to be equal to $\bar{\mathbb{P}}_d$, the second part of the average throughput in (4.7) becomes a constant for a specific τ value. Hence, the problem is simplified to

$$\begin{aligned} \min_{\boldsymbol{\varepsilon}, \mathbf{D}^+} \quad & \mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \sum_{\mathbf{d} \in \mathbf{D}^+} Pr(\mathbf{d} | \mathcal{H}_0, \tau, \boldsymbol{\varepsilon}) \\ \text{s.t.} \quad & \mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \sum_{\mathbf{d} \in \mathbf{D}^+} Pr(\mathbf{d} | \mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma}) = \bar{\mathbb{P}}_d. \end{aligned} \quad (4.9)$$

The above problem (4.9) has the form of the Neyman-Pearson problem. Therefore, the solution of (4.9) has to satisfy the likelihood-ratio test [169],

$$LR(\mathbf{d}) = \frac{Pr(\mathbf{d}|\mathcal{H}_1, \tau, \varepsilon, \gamma)}{Pr(\mathbf{d}|\mathcal{H}_0, \tau, \varepsilon)} \geq k, \quad (4.10)$$

and

$$\sum_{\mathbf{d}: LR(\mathbf{d}) \geq k} Pr(\mathbf{d}|\mathcal{H}_1, \tau, \varepsilon, \gamma) = \bar{\mathbb{P}}_d. \quad (4.11)$$

Note that since this is a likelihood-ratio test, the fusion rule's threshold k does not necessary be an integer as in Chapter 3 which k -out-of- N fusion rule is used. The optimal fusion rule $\mathbf{D}^+$ will consist of the $\mathbf{d}$ values that are able to satisfy both (4.10) and (4.11) simultaneously. From (4.5) and (4.6), (4.10) can be further expressed as

$$\prod_{i=1}^N \left[\frac{(1 - P_{di}(\tau, \varepsilon_i))}{(1 - P_{fi}(\tau, \varepsilon_i))} \right]^{(1-d_i)} \left[\frac{P_{di}(\tau, \varepsilon_i)}{P_{fi}(\tau, \varepsilon_i)} \right]^{d_i} \geq k \quad (4.12)$$

and by taking logarithm on both sides,

$$LLR(\mathbf{d}) = \sum_{i=1}^N [(1 - d_i)w_{0i}(\tau, \varepsilon_i) + d_iw_{1i}(\tau, \varepsilon_i)] \geq \log k \quad (4.13)$$

where

$$w_{0i}(\tau, \varepsilon_i) = \log \frac{1 - P_{di}(\tau, \varepsilon_i)}{1 - P_{fi}(\tau, \varepsilon_i)} \quad (4.14)$$

and

$$w_{1i}(\tau, \varepsilon_i) = \log \frac{P_{di}(\tau, \varepsilon_i)}{P_{fi}(\tau, \varepsilon_i)}. \quad (4.15)$$

Equation (4.13) corresponds to the optimal decision fusion rule in [170], where the fusion rule puts a weighting of $w_{1i}(\tau, \varepsilon_i)$ onto those SUs' decisions that detect the PU and a weighting of $w_{0i}(\tau, \varepsilon_i)$ onto those decisions that do not detect the PU, before summing up all the weighted decisions. If the summation of the terms is greater than the threshold $\log k$, the FC will decide that the PU is present.

There are infinite number of solutions of k and $\boldsymbol{\varepsilon}$ that can satisfy both (4.11) and (4.13) simultaneously, and the difficulty is to determine the optimal pair of k and $\boldsymbol{\varepsilon}$ that satisfies both (4.11) and (4.13) while minimizes $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$. In a Bayesian problem, e.g., minimizing the objective function of (4.9) without the constraint, a global optimum solution for k and $\boldsymbol{\varepsilon}$ can be found using person-by-person optimization methodology [149], where each threshold is optimized individually while all the other thresholds are assumed to be fixed at particular values. However, this approach cannot be applied to a Neyman-Person problem, i.e., problem (4.9), because optimizing ε_i while keeping all the other thresholds fixed cannot lead to a new value for ε_i since the constraint $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \bar{\mathbb{P}}_d$ would be violated [171].

In this section, we propose a heuristic algorithm to find the SUs' thresholds $\boldsymbol{\varepsilon}$ that will minimize $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ for a given $\mathbf{D}^+$. First, we define a term β_i for all the N SUs as follows:

$$\beta_i = \left[\frac{\partial \mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)}{\partial \varepsilon_i} \right] \left[\frac{\partial \mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)}{\partial \varepsilon_i} \right]^{-1} \quad \forall i = 1, \dots, N. \quad (4.16)$$

The term β_i is to measure the impact of the i th SU's threshold ε_i on the overall probabilities of detection and false alarm at the FC. A small variation in ε_i with β_i value greater than one will cause a bigger change in $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ than in $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ while a small variation in ε_i with a β_i value less than one will cause a bigger change in $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ than in $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$. For a given $\mathbf{D}^+$, other than (4.3) and (4.4), $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ and $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ can be expressed into the following forms:

$$\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = a_i P_{di}(\tau, \varepsilon_i) + b_i, \quad (4.17)$$

$$\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = c_i P_{fi}(\tau, \varepsilon_i) + e_i \quad (4.18)$$

where

$$\begin{aligned} a_i &= f_{i1}(P_{d1}, \dots, P_{di-1}, P_{di+1}, \dots, P_{dN} | \mathbf{D}^+), \\ b_i &= f_{i2}(P_{d1}, \dots, P_{di-1}, P_{di+1}, \dots, P_{dN} | \mathbf{D}^+), \\ c_i &= f_{i1}(P_{f1}, \dots, P_{fi-1}, P_{fi+1}, \dots, P_{fN} | \mathbf{D}^+), \\ e_i &= f_{i2}(P_{f1}, \dots, P_{fi-1}, P_{fi+1}, \dots, P_{fN} | \mathbf{D}^+), \end{aligned}$$

and the functions f_{i1} and f_{i2} are obtainable from (4.3)-(4.6). Therefore,

$$\frac{\partial \mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)}{\partial \varepsilon_i} = -\frac{a_i}{\sigma_u^2} \sqrt{\frac{\tau f_s}{2\pi(2\gamma_i + 1)}} \exp \left(-\frac{\left[\left(\frac{\varepsilon_i}{\sigma_u^2} - \gamma_i - 1 \right) \sqrt{\frac{\tau f_s}{2\gamma_i + 1}} \right]^2}{2} \right), \quad (4.19)$$

and

$$\frac{\partial \mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)}{\partial \varepsilon_i} = -\frac{c_i}{\sigma_u^2} \sqrt{\frac{\tau f_s}{2\pi}} \exp \left(-\frac{\left[\left(\frac{\varepsilon_i}{\sigma_u^2} - 1 \right) \sqrt{\tau f_s} \right]^2}{2} \right). \quad (4.20)$$

The proposed heuristic algorithm works as follows. Suppose for a given $\mathbf{D}^+$, there is a set of non-optimal thresholds $\boldsymbol{\varepsilon}^{(0)} = [\varepsilon_1^{(0)}, \varepsilon_2^{(0)}, \dots, \varepsilon_N^{(0)}]$ which satisfies the constraint (4.11). Since it is not possible to optimize a SU's threshold ε_i while keeping all the other thresholds fixed due to the constraint, the proposed algorithm optimizes two SUs' thresholds at a time while keeping the rest of the thresholds fixed so that the constraint (4.11) can be maintained during the iteration process. When one of the thresholds changes such that the value of $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ increases, the other threshold should change accordingly to maintain the value of $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ at $\bar{\mathbb{P}}_d$ while minimizing the value of $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$. When a threshold with small β_i value is used to decrease the value of $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$, the value of $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ will be slightly decreased. On the other hand, using a threshold with large β_i value to increase the value of $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$, the value of $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ will only be slightly increased. Hence, choosing the thresholds with the largest and the smallest β_i

values to minimize the value of $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ while maintaining $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \bar{\mathbb{P}}_d$ will provide the best tradeoff.

At the start of each iteration in the proposed algorithm, the values of β_i for all the thresholds are computed for a particular $\mathbf{D}^+$. The threshold ε_i with the smallest β_i value is increased to decrease the value of $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ while the threshold ε_i with the largest β_i value is decreased to maintain $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \bar{\mathbb{P}}_d$. The two thresholds values are updated and the values of β_i are recomputed for all the thresholds for the next iteration. The iteration stops when the difference between the largest and the smallest β_i values becomes smaller than a predetermined value ρ . Since if one of the thresholds is changed to reduce $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ and as a result $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ is also reduced; the other threshold has to change to increase $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ so as to maintain the constraint. However, if they have the same β_i value, $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ will be increased back to the original value and hence, there is no gain in using two thresholds with the same β_i value. The k value is then computed using the $\boldsymbol{\varepsilon}$ and $\mathbf{D}^+$.

In general, there are 2^{2^N} possible sets of fusion rule $\mathbf{D}^+$ when there are N decisions sent to the FC. However, if the fusion rule is constrained to make a decision based on (4.13), the number of possible sets of $\mathbf{D}^+$ reduces to $2^N - 1$ for a given $\boldsymbol{\varepsilon}$. The maximum possible cardinality of $\mathbf{D}^+$ is 2^N , that is when all the 2^N possible values of $\mathbf{d}$ are elements of $\mathbf{D}^+$. However, it is obvious that $\mathbf{d} = [0, 0, \dots, 0]$ cannot be an element of $\mathbf{D}^+$ since it does not make sense for a fusion rule to decide that the PU is present when all the SUs' decisions decide otherwise. The practical maximum cardinality of $\mathbf{D}^+$ is therefore $2^N - 1$. The proposed algorithm obtains the $\boldsymbol{\varepsilon}$ values as described earlier for $2^N - 1$ possible sets of $\mathbf{D}^+$ and chooses the pair of k and $\boldsymbol{\varepsilon}$ that has the minimum $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$. The proposed heuristic algorithm is summarized in Table 4.1. Although the procedure of optimizing two thresholds at a time is not proven to achieve global optimal

Table 4.1: Proposed iterative algorithm that optimizes the individual SUs' thresholds

Algorithm 4.1: Find the optimal k and ϵ for (4.9).
Initialization:
1) Set $\mathbf{d} = [1, 1, \dots, 1]$ be the only element in $\mathbf{D}^{+(1)}$.
2) Given $\mathbf{D}^{+(1)}$, set $\epsilon^{(1)}$ to be any value that satisfy (4.11).
repeat
3) Compute β_i for all ϵ_i .
4) $\epsilon_S := \epsilon_i$ with the smallest β_i .
5) $\epsilon_L := \epsilon_i$ with the largest β_i .
repeat
6) $\epsilon_S := \epsilon_S + \Delta$.
7) Decrease the ϵ_L such that $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+) = \bar{\mathbb{P}}_d$.
8) Repeat steps (7) and (8) till $\mathbb{P}_f(\tau, \epsilon, \mathbf{D}^+)$ is minimized.
9) Do steps (3) to (5).
until
10) $ \text{largest } \beta_i - \text{smallest } \beta_i \leq \rho$.
Output: $\epsilon^{(j)}$
11) Compute $k^{(j)}$ with $\epsilon^{(j)}$ and $\mathbf{D}^{+(j)}$.
12) Add another $\mathbf{d}$ with the highest (4.10) into $\mathbf{D}^{+(j+1)}$.
13) Given $\mathbf{D}^{+(j+1)}$, set $\epsilon^{(j+1)}$ to be any value that satisfy (4.11).
until
14) $\mathbf{D}^{+(2^N-1)}$, $\epsilon^{(2^N-1)}$, and $k^{(2^N-1)}$ are reached.
Output: Every $k^{(j)}$ and $\epsilon^{(j)}$ is a pair of solutions that satisfy both (4.11) and (4.13). Choose the pair of $k^{(j)}$ and $\epsilon^{(j)}$ with the minimum $\mathbb{P}_f(\tau, \epsilon, \mathbf{D}^+)$.

solution, the proposed heuristic algorithm provides the highest throughput in all the simulations when we compare it with other scenarios shown in Section 4.5.

4.3 Large Area CRN with Large Number of SUs

In a CRN that consists of a large number of SUs, optimizing the threshold of every SU becomes time consuming even with the proposed heuristic algorithm developed in the previous section. Hence, in a CRN with large number of SUs, optimizing every threshold of the SUs at the FC may become impractical. Therefore, in this section, we propose a single threshold value ϵ for all the SUs even when the

detection SNRs of the SUs are different. Assume that the noise power at every SU is normalized to be the same and all the thresholds ε_i are constrained to be of the same value, then $P_{fi}(\tau, \varepsilon) = P_f(\tau, \varepsilon)$ for all i , and the weightings in (4.13) become

$$w_{0i}(\tau, \varepsilon) = \log \frac{1 - P_{di}(\tau, \varepsilon)}{1 - P_f(\tau, \varepsilon)}, \quad (4.21)$$

and

$$w_{1i}(\tau, \varepsilon) = \log \frac{P_{di}(\tau, \varepsilon)}{P_f(\tau, \varepsilon)}. \quad (4.22)$$

Although, with the same threshold ε , all the SUs will have the same probability of false alarm $P_f(\tau, \varepsilon)$, the SUs with higher detection SNR γ_i will have higher $P_{di}(\tau, \varepsilon)$. It becomes apparent from (4.21) and (4.22) that SUs with higher detection SNR will definitely have higher absolute weightings in both $|w_{0i}|$ and $|w_{1i}|$ in the fusion rule, compared to SUs with lower detection SNR.

In the previous section when the thresholds are not constrained to be the same, there are an infinite number of solutions of k and ε that can satisfy (4.11). However, when all the thresholds ε_i are constrained to be the same ε , there are limited pairs of k and ε values that can satisfy (4.11). With limited pairs of k and ε values that can satisfy both (4.11) and (4.13), the optimal solution for (4.9), with all the thresholds constrained to the same value ε , can be obtained much easier.

We first obtain the lower and upper bounds of ε that are able to satisfy both (4.11) and (4.13). The probability of detection at the FC can be expressed as

$$\mathbb{P}_d(\tau, \varepsilon, \mathbf{D}^+) = \sum_{\mathbf{d} \in \mathbf{D}^+} \left[\prod_{i=1}^N (1 - P_{di}(\tau, \varepsilon))^{1-d_i} P_{di}(\tau, \varepsilon)^{d_i} \right]. \quad (4.23)$$

For a given τ and ε , $\mathbb{P}_d(\tau, \varepsilon, \mathbf{D}^+)$ will be maximized when $\mathbf{D}^+$ contains all the $\mathbf{d}$ values except for $\mathbf{d} = [0, 0, \dots, 0]$, which is essentially an OR fusion rule. Denote $\mathbf{D}^{\text{OR}}$ as the OR fusion rule, this fusion rule can be obtained by setting $\log k =$

$\sum_{i=1}^N w_{0i} + \Delta$ where Δ denotes an extremely small positive value. The upper bound of ε , which is denoted as ε_U , can be obtained by solving

$$\prod_{i=1}^N (1 - P_{di}(\tau, \varepsilon_U)) = 1 - \bar{\mathbb{P}}_d. \quad (4.24)$$

Any other ε_0 that is greater than ε_U is unable to meet the constraint (4.11), as $\mathbb{P}_d(\tau, \varepsilon_0, \mathbf{D}^{\text{OR}}) < \mathbb{P}_d(\tau, \varepsilon_U, \mathbf{D}^{\text{OR}}) = \bar{\mathbb{P}}_d$. There is no other fusion rule $\mathbf{D}^+$, that can increase $\mathbb{P}_d(\tau, \varepsilon_0, \mathbf{D}^+)$ to meet $\bar{\mathbb{P}}_d$ for the given τ and ε_0 . Hence, the maximum value of ε that can meet the constraint (4.11) for a given τ is at ε_U .

Similarly, for a given τ and ε , $\mathbb{P}_d(\tau, \varepsilon, \mathbf{D}^+)$ is at minimum when $\mathbf{D}^+$ contains only one $\mathbf{d}$ value. Based on the fusion rule in (4.13), the only possible element in $\mathbf{D}^+$ is $\mathbf{d} = [1, 1, \dots, 1]$ which is obtained by setting $\log k = \sum_{i=1}^N w_{1i}$. This fusion rule is, in fact, an AND fusion rule and we denote it as $\mathbf{D}^{\text{AND}}$. The lower bound of ε , which is denoted as ε_L , is obtained by solving

$$\prod_{i=1}^N P_{di}(\tau, \varepsilon_L) = \bar{\mathbb{P}}_d. \quad (4.25)$$

Any other ε_0 that is lower than ε_L is unable to meet the constraint (4.11) as $\mathbb{P}_d(\tau, \varepsilon_0, \mathbf{D}^{\text{AND}}) > \mathbb{P}_d(\tau, \varepsilon_L, \mathbf{D}^{\text{AND}}) = \bar{\mathbb{P}}_d$. For the given τ and ε_0 , there is no other fusion rule $\mathbf{D}^+$ that can reduce $\mathbb{P}_d(\tau, \varepsilon_0, \mathbf{D}^+)$ to meet $\bar{\mathbb{P}}_d$. Hence, ε_L is the lower bound of ε for a given τ that can meet the constraint (4.11). With both the upper and lower bounds of ε known, the task of obtaining all the possible pairs of k and ε values that can satisfy both (4.11) and (4.13) is further simplified since we know that the ranges of k and ε lie between $\sum_{i=1}^N w_{0i}$ to $\sum_{i=1}^N w_{1i}$ and ε_L to ε_U , respectively.

For a given ε value, there are only $2^N - 1$ possible k values which correspond to the $2^N - 1$ possible sets of $\mathbf{D}^+$ that can be obtained from (4.13). On the other hand, the values of k and ε have to meet the constraint $\mathbb{P}_d(\tau, \varepsilon, \mathbf{D}^+) = \bar{\mathbb{P}}_d$. The

value $\mathbb{P}_d(\tau, \varepsilon, \mathbf{D}^+)$ is inversely proportional to both the value of ε for a fixed k and also the value of k for a fixed ε . By using the above mentioned properties, a program can be easily written to extract the pairs of k and ε values that will satisfy both (4.11) and (4.13) when conducting a numerical search across ε from ε_L to ε_U . The objective in (4.9) is to minimize $\mathbb{P}_f(\tau, \varepsilon, \mathbf{D}^+)$ with the constraint (4.11) and the optimal fusion rule to minimize (4.9) is proven to be a log likelihood-ratio test (4.13). Therefore, the pair of k and ε that has the minimum $\mathbb{P}_f(\tau, \varepsilon, \mathbf{D}^+)$, among all the pairs of k and ε values that satisfy both (4.11) and (4.13), is the global optimal solution.

4.4 Decentralized Optimization of SUs' Thresholds

In the previous sections, the FC computes the thresholds for the SUs and feeds back the thresholds to the SUs for them to make their decisions. This is possible if the FC has a large computation power and the channel conditions between the SUs and the PU are relatively constant such that the thresholds fed back to the SUs are still valid for the next sensing period. However, if the FC lacks the computation power or the channel conditions between the SUs and the PU change rapidly, computing the SUs' thresholds at the FC may not be possible or practical.

Under the above scenario, the SUs should optimize their own thresholds while the FC optimizes the problem (4.9) with ε_i as fixed variables instead of optimization variables. The question for the SUs is that without the knowledge of the other SUs' channel information and the fusion rule $\mathbf{D}^+$, how are the SUs going to set their own thresholds? The objective of (4.9) is to minimize the total false alarm probability $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ while maintaining the total detection probability $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ at the targeted probability $\bar{\mathbb{P}}_d$. It is obvious that if the SUs minimize their own

false alarm probabilities $P_{fi}(\tau, \varepsilon_i)$, they can help to minimize $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ at the FC. However, it is impossible for them to set an optimal targeted probability $\bar{P}_{di}$ such that $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \bar{\mathbb{P}}_d$ since they have no knowledge of each other's channel information and the fusion rule $\mathbf{D}^+$. If the SUs know their optimal values of $\bar{P}_{di}$, then setting the threshold ε_i based on minimizing $P_{fi}(\tau, \varepsilon_i)$ subject to the constraint that $P_{di}(\tau, \varepsilon_i) = \bar{P}_{di}$, will obtain the global optimal solution. However, since the optimal value of $\bar{P}_{di}$ is unknown, a heuristic approach is to set $\bar{P}_{di} = \bar{\mathbb{P}}_{di}$. Using this criteria, the threshold for the i th SU is

$$\varepsilon_i = \sigma_u^2 \left(\sqrt{\frac{2\gamma_i + 1}{\tau f_s}} \mathcal{Q}^{-1}(\bar{\mathbb{P}}_{di}) + \gamma_i + 1 \right). \quad (4.26)$$

From the constraint of (4.9), it is observed that if the SUs' detection probabilities $P_{di}(\tau, \varepsilon_i)$ are high, it will require fewer $\mathbf{d}$ elements in $\mathbf{D}^+$ for $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ to reach $\bar{\mathbb{P}}_d$. This, in turn, will reduce $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ when there are fewer $\mathbf{d}$ elements in $\mathbf{D}^+$. Hence, not only having a low $P_{fi}(\tau, \varepsilon_i)$ contributes to minimizing $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$, a high $P_{di}(\tau, \varepsilon_i)$ contributes in minimizing $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ as well, due to the constraint. Using this knowledge, another heuristic approach of setting the individual SUs' thresholds without any information on each other's channel conditions can be based on

$$\min_{\varepsilon_i} P_{fi}(\tau, \varepsilon_i) + (1 - P_{di}(\tau, \varepsilon_i)). \quad (4.27)$$

This formulation does not guarantee that the threshold ε_i will be optimal for (4.9) but it balances the needs of a low $P_{fi}(\tau, \varepsilon_i)$ and a high $P_{di}(\tau, \varepsilon_i)$. This formulation essentially selects the threshold that minimizes the total decision error probability of the individual SU and has a closed-form solution as follows:

$$\varepsilon_i = \frac{\sigma_u^2}{2} \left(1 + \sqrt{1 + 2(\gamma_i + \psi)} \right) \quad (4.28)$$

where

$$\psi = \frac{(2\gamma_i + 1) \ln(2\gamma_i + 1)}{\gamma_i \tau f_s}.$$

When the SUs decide their own thresholds and make their decisions, the FC will still need to decide the optimal fusion rule $\mathbf{D}^+$ such that $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ is minimized and $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \bar{\mathbb{P}}_d$. In this scenario, for a fixed τ value, the value of $Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma})$ in (4.11) which is made up of individual $P_{di}(\tau, \varepsilon_i)$ becomes a fixed constant at the FC in contrast to the previous scenarios when the FC has control over the thresholds ε_i . With $Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma})$ being a fixed constant, which is a probability mass function, the probability of having a k value that can exactly satisfy $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) = \bar{\mathbb{P}}_d$ based on (4.13) is almost zero. The FC can instead choose a threshold $\hat{k}$ which is equal to the maximum k value that satisfies

$$\sum_{\mathbf{d}: LR(\mathbf{d}) \geq k} Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma}) \geq \bar{\mathbb{P}}_d.$$

This ensures that the FC's probability of detection at every frame is at least greater than the targeted probability of detection $\bar{\mathbb{P}}_d$ and satisfies the requirement to protect the PU.

However, the solution based on the likelihood-ratio test is optimal only when the constraint $\sum_{\mathbf{d}: LR(\mathbf{d}) \geq k} Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma})$ is exactly equal to $\bar{\mathbb{P}}_d$. This means that adding $\mathbf{d}$ values with the highest likelihood-ratio into $\mathbf{D}^+$ until $\sum_{\mathbf{d}: LR(\mathbf{d}) \geq k} Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma})$ exactly equal to $\bar{\mathbb{P}}_d$ is the optimal fusion rule while exceeding $\bar{\mathbb{P}}_d$ may not be optimal. For example, we choose a fusion rule threshold $\check{k}$ to be the minimum k value that satisfies

$$\sum_{\mathbf{d}: LR(\mathbf{d}) \geq k} Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma}) < \bar{\mathbb{P}}_d.$$

The FC's probability of detection $\mathbb{P}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ is less than $\bar{\mathbb{P}}_d$ using $\check{k}$. By adding the next $\mathbf{d}$ value with the highest likelihood-ratio from $\mathbf{D}^-$ to $\mathbf{D}^+$ will make

$\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ greater than or equal to $\bar{\mathbb{P}}_d$ which is equivalent to using $\hat{k}$. The problem is that it is possible that the next $\mathbf{d}$ value with the highest likelihood-ratio may have a large $Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma})$ value and adding it into $\mathbf{D}^+$ may increase $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ to be much greater than $\bar{\mathbb{P}}_d$. A $\mathbf{d}$ value with a lower likelihood-ratio but has a $Pr(\mathbf{d}|\mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma})$ value that can increase $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ to exactly or slightly over $\bar{\mathbb{P}}_d$ may perform better. Hence, the method to get a better performance is to first use $\check{k}$ to include the $\mathbf{d}$ values with high likelihood-ratio into $\mathbf{D}^+$ and then select a or combination of $\mathbf{d}$ values from those remaining in $\mathbf{D}^-$ to increase $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ to or exceeds $\bar{\mathbb{P}}_d$ with the least combined $Pr(\mathbf{d}|\mathcal{H}_0, \tau, \boldsymbol{\varepsilon})$.

In the previous sections, the $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ for every sensing frame is constrained to be equal to or greater than $\bar{\mathbb{P}}_d$. However, if this constraint can be relaxed such that $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ at every sensing frame can be higher or lower than $\bar{\mathbb{P}}_d$, while the average $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ of the sensing frames over time is constrained to be equal to or greater than $\bar{\mathbb{P}}_d$, then not only can the cumbersome procedure of selecting $\mathbf{d}$ values from $\mathbf{D}^-$ be avoided, the average throughput of the SUs can also be further increased. Let the average $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ and $\mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ of the sensing frames over time be denoted as $\mathbb{P}_{d,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ and $\mathbb{P}_{f,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$, respectively. Therefore, $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) \geq \bar{\mathbb{P}}_d$ is an instantaneous detection probability constraint and $\mathbb{P}_{d,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+) \geq \bar{\mathbb{P}}_d$ is an average detection probability constraint. If the constraint is relaxed from the instantaneous detection probability constraint to the average detection probability constraint, the optimal solution is to use two fusion rule thresholds, both $\hat{k}$ and $\check{k}$. Denote α as the probability of using the threshold $\check{k}$ and $1 - \alpha$ as the probability of using the threshold $\hat{k}$. The value of α is computed from

$$\alpha = \frac{\bar{\mathbb{P}}_d - \phi(\hat{k})}{\phi(\check{k}) - \phi(\hat{k})} \quad (4.29)$$

where

$$\phi(x) = \sum_{\mathbf{d}: LR(\mathbf{d}) \geq x} Pr(\mathbf{d} | \mathcal{H}_1, \tau, \boldsymbol{\varepsilon}, \boldsymbol{\gamma}).$$

By using the threshold $\check{k}$ with probability α and $\hat{k}$ with probability $(1 - \alpha)$, the $\mathbb{P}_{d,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ is exactly equal to the targeted probability of detection $\bar{\mathbb{P}}_d$, while the average probability of false alarm $\mathbb{P}_{f,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ is being minimized.

4.5 Simulation Results and Discussion

In the computer simulations, we set the number of SUs to be $N = 5$ and the frame duration to be $T = 20\text{ms}$. The sampling frequency of the received signal is assumed to be 6MHz and $\bar{\mathbb{P}}_d$ is set at 99%. Figure 4.2 shows the maximum normalized throughput achieved under different scenarios when the sensing time is varied from 1ms to 19ms. For ease of labeling the curves in the figures, we denote Scenario I as the case when the individual thresholds of the SUs are optimized by the proposed algorithm given in Table 4.1. Scenario II is denoted as the case when all the thresholds of the SUs are constrained to be the same and Scenario III as the case when the thresholds of the SUs are decentralized optimized. The SUs' SNRs are assumed to be $\boldsymbol{\gamma} = [-20, -22, -24, -26, -28]\text{dB}$ unless otherwise stated. The normalized throughput is defined as $(1 - \frac{\tau}{T})(1 - \mathbb{P}_f(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+))$.

From Fig. 4.2, it is observed that Scenario I achieves the highest normalized throughput for all sensing time. This result is expected as the individual thresholds are optimized using a centralized processor which has the channel information of all the SUs. The performance of the maximum normalized throughput under Scenario II, which constrained all the SUs' thresholds to be the same, is only slightly lower than that under Scenario I. Therefore, constraining all the SUs' thresholds to be the same is a good option if the number of SUs is large. The throughputs of Scenario III under the average $\mathbb{P}_{d,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ constraint, where the SUs compute

Figure 4.2: Maximum normalized throughputs in different scenarios when the SUs' SNRs are 2dB apart.

their own thresholds, based on minimizing their own total decision errors and based on minimizing $P_{fi}(\tau, \varepsilon_i)$ subject to $P_{di}(\tau, \varepsilon_i) = \bar{\mathbb{P}}_d$, are much lower. The reason for the lower performances in Scenario III is because each SU computes its own threshold without the knowledge of the other SUs' channel information. The OR and the AND fusion rules are shown in Fig. 4.2 for comparison. We would like to highlight that in both OR and AND fusion rules, it is necessary for the SUs to coordinate their thresholds such that $\mathbb{P}_d(\tau, \varepsilon, \mathbf{D}^{\text{OR}})$ and $\mathbb{P}_d(\tau, \varepsilon, \mathbf{D}^{\text{AND}})$ equal to $\bar{\mathbb{P}}_d$, while in Scenario III, the SUs do not need any coordination in setting their thresholds. Nevertheless, the schemes in Scenario III still perform better than the OR and AND fusion rules in this system setup as shown in Fig. 4.2. From Fig. 4.2, it is observed that the maximum normalized throughputs for these scenarios are quasiconvex in terms of the sensing time. The problem formulation (4.8) can be solved by combining the various proposed algorithms with Golden search method [167] as an outer iteration loop to solve for the optimal τ .

Figure 4.3 shows the differences in performance between the average and the in-

Figure 4.3: Maximum normalized throughputs of decentralized optimization of SUs' thresholds under instantaneous and average total detection probability constraints when the SUs' SNRs are 1dB apart.

stantaneous detection probability constraints under Scenario III. The SUs' SNRs are assumed to be $\gamma = [-20, -21, -22, -23, -24]$ dB. The performance of Scenario I is also shown as the upper bound. The average $\mathbb{P}_{d,avg}(\tau, \epsilon, \mathbf{D}^+)$ constraint provides equal or higher average normalized throughputs as compared to the instantaneous $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ constraint. For the method based on minimizing the total decision error, the performance curve under the instantaneous $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ constraint is not smooth across the sensing time. This can be explained using Fig. 4.4. As mentioned in Section 4.4, under instantaneous $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ constraint, the value of $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ may not be exactly equal to $\bar{\mathbb{P}}_d$. Hence, we plotted out the $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ value at various sensing time in Fig. 4.4. From Fig. 4.4, it can be observed that the $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ value when minimizing the total decision error under instantaneous $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ constraint exceeds the required $\bar{\mathbb{P}}_d = 0.99$ by different amount at different sensing time. This causes the curve of its normalized throughput to be irregular across the sensing time. We comment that

Figure 4.4: Probabilities of detection at the FC in decentralized optimization of SUs' thresholds under instantaneous and average total detection probability constraints when the secondary users' SNRs are 1dB apart.

a major disadvantage of using the instantaneous $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ constraint is that its normalized throughput is not a quasiconvex function in terms of the sensing time. Hence, the optimal sensing time has to be determined through an exhaustive search. The $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ value based on setting $P_{di}(\tau, \varepsilon_i) = \bar{\mathbb{P}}_d$ under the instantaneous $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ constraint is just slightly above $\bar{\mathbb{P}}_d$ that it is not observable in Fig. 4.4. Although, its $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ under the instantaneous constraint is very close to $\bar{\mathbb{P}}_d$, its normalized throughput may still be lower than the case under the average $\mathbb{P}_{d,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ constraint as shown in Fig. 4.3. This is because under average $\mathbb{P}_{d,avg}(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ constraint, only $\mathbf{d}$ values with the highest likelihood ratio are included in $\mathbf{D}^+$, however, under instantaneous $\mathbb{P}_d(\tau, \boldsymbol{\varepsilon}, \mathbf{D}^+)$ constraint, $\mathbf{d}$ values with lower likelihood-ratio may be included in $\mathbf{D}^+$.

Figures 4.5 and 4.6 are similar to Figs. 4.3 and 4.4, except that the SUs' SNRs are set to be $\boldsymbol{\gamma} = [-20, -23, -26, -29, -32]\text{dB}$. The main purpose in showing Figs. 4.5 and 4.6, is to show that the method based on minimizing total decision error

Figure 4.5: Maximum normalized throughputs of decentralized optimization of SUs' thresholds under instantaneous and average total detection probability constraints when the SUs' SNRs are 3dB apart.

is not always better than the method based on setting $P_{di}(\tau, \varepsilon_i) = \bar{\mathbb{P}}_d$. When the differences in the SNRs of each SU are large, the performance basically depends heavily on the SU with the highest SNR. The gain from cooperative decision fusion with the other SUs having much lower SNRs is limited. Setting the threshold such that $P_{di}(\tau, \varepsilon_i) = \bar{\mathbb{P}}_d$ is, in fact, close to optimal for the SU with the highest SNR with itself being able to meet the required $P_{di}(\tau, \varepsilon_i) = \bar{\mathbb{P}}_d$. However, when using the method based on minimizing total decision error, the $P_{di}(\tau, \varepsilon_i)$ of the SU with the highest SNR may not be near the required $\bar{\mathbb{P}}_d$ and therefore, its threshold is further from the optimal value. In general, if the differences in SNRs among the SUs are small, it is better to use the method based on minimizing total decision error. However, if the differences in SNRs among the SUs are large, it is better to use the method based on setting $P_{di}(\tau, \varepsilon_i) = \bar{\mathbb{P}}_d$. Note that although the SUs are uncoordinated in the schemes presented in Scenario III, at least one of the two proposed schemes can still outperform the OR fusion rule shown in Figs. 4.3 and

Figure 4.6: Probabilities of detection at the FC in decentralized optimization of SUs' thresholds under instantaneous and average total detection probability constraints when the secondary users' SNRs are 3dB apart.

4.5.

Using the same parameters as in Fig. 4.2, we show the absolute weightings $|w_{0i}|$ and $|w_{1i}|$ of the SU with the highest SNR value ($\gamma = -20$ dB), denoted as user 1, and the SU with the least SNR value ($\gamma = -28$ dB), denoted as user 5, in Figs. 4.7 and 4.8, respectively. From Figs. 4.7 and 4.8, user 5's weightings have more fluctuations than user 1's weightings under Scenario I. For user 1, its decision is highly correlated to the final decision of the optimal fusion rule $\mathbf{D}^+$. The $\mathbf{d}$ values with element $d_1 = 1$ are more likely to be included in the set $\mathbf{D}^+$ than the $\mathbf{d}$ values with element $d_5 = 1$. User 1's weightings are not affected much by the solutions of the other SUs' weightings for all τ values. On the other hand, user 5's decision is the least correlated to the final decision and its weightings are sensitive to the changes of the other SUs' weightings. Hence, user 5's weightings tend to fluctuate more across different sensing times although its influence to the overall throughput is the least.

Figure 4.7: Weightings of the decisions of SU with the highest detection SNR (user 1) under different scenarios.

From Fig. 4.7, it is observed that the weightings of both $|w_{01}|$ and $|w_{11}|$ of user 1 under Scenario II are close to the weightings obtained under Scenario I. However, from Fig. 4.8, the weightings of both $|w_{05}|$ and $|w_{15}|$ of user 5 under Scenario II are very different from the weightings obtained under Scenario I. This shows that when all the thresholds are constrained to be the same, the optimal single threshold is highly correlated to the optimal thresholds of the SUs with high SNRs. This is not surprising as those SUs with high SNRs have more impact on $\mathbb{P}_d(\tau, \epsilon, \mathbf{D}^+)$ and $\mathbb{P}_f(\tau, \epsilon, \mathbf{D}^+)$. Although the single threshold is nowhere near optimal for the SUs with low SNRs, the loss in throughput is trivial since their contributions to the overall performance are not large.

From both Figs. 4.7 and 4.8, an interesting finding is that using the method based on minimizing the total decision error, the curves of $|w_{0i}|$ and $|w_{1i}|$ are overlapped when plotted across different sensing time for all SUs. This method seems to place equal importance on the results of $d_i = 0$ and $d_i = 1$. On the other hand, using the method based on minimizing $P_{fi}(\tau, \epsilon_i)$ subject to $P_{di}(\tau, \epsilon_i) = \bar{\mathbb{P}}_d$

Figure 4.8: Weightings of the decisions of SU with the lowest detection SNR (user 5) under different scenarios.

essentially gives a very heavy weighting to $d_i = 0$ compared to $d_i = 1$ as seen in Figs. 4.7 and 4.8. If $\bar{P}_d$ is very high and the $P_{di}(\tau, \varepsilon_i)$ is set to that level, the occurrence of $d_i = 1$ will be high. Hence, $d_i = 1$ is not as informative as $d_i = 0$, therefore, the weighting of $d_i = 0$ will be much heavier than $d_i = 1$.

Figure 4.9 shows the average weightings allocated to user 1 ($\gamma = -20\text{dB}$), user 3 ($\gamma = -24\text{dB}$), and user 5 ($\gamma = -28\text{dB}$), under the various scenarios. The average weighting is defined as $(|w_{0i}(\tau, \varepsilon_i)| + |w_{1i}(\tau, \varepsilon_i)|) / 2$, which shows the average weight each SU has on the fusion rule (4.13). Although different SUs may weigh heavier on either $d_i = 0$ or $d_i = 1$ under different scenarios at different sensing time, SUs with higher SNRs should have higher average weightings than SUs with lower SNRs under all scenarios. The higher reliability of the decisions made by SUs with higher SNRs are clearly factored into the fusion rule by their heavier average weightings, unlike the common k -out-of- N fusion rule that makes no distinction between the decisions.

A system model of a large area CRN with small number of SUs is further used

Figure 4.9: Average weighting of each SU under different scenarios.

to simulate the performance of the various scenarios. The system model is shown in Fig. 4.10, where a SU base station transmitter (BS-Tx) is located at 150Km away from a PU BS-Tx and three SUs are uniformly and randomly distributed within 50Km of the SU BS-Tx. The received power of the PU BS-Tx at the i th SU is given as $P_{SUi} = \frac{L_f P_p}{d_{SUi}^\nu}$, where P_p is the PU BS-Tx's signal power, d_{SUi} is the distance of the i th SU from the PU BS-Tx, ν is the path loss exponent factor and L_f is the loss in near field. The received SNR (in dB) at the i th SU is then given by $\gamma_i = 10 \log \frac{P_{SUi}}{\sigma_u^2} + x_i$, where σ_u^2 is the noise power and x_i is a zero-mean Gaussian distributed random variable with variances of 3dB, 5dB, and 8dB for SU1, SU2, and SU3, respectively to account for the shadowing effects. With this setup, the randomly generated distances are $d_{SU1} = 119.6\text{Km}$, $d_{SU2} = 179.6\text{Km}$, and $d_{SU3} = 176.5\text{Km}$. Assume that the received power of the PU BS-Tx at the SU BS-Tx is at -20dB and the noise power is normalized to 1, the obtained SNRs at a particular instant are $\gamma_1 = -15.1\text{dB}$, $\gamma_2 = -17.6\text{dB}$, and $\gamma_3 = -25.7\text{dB}$.

The maximum normalized throughputs of different scenarios with the γ values

Figure 4.10: A scenario of a large area CRN with small number of SUs.

Figure 4.11: Maximum normalized throughputs of different methods of setting the cooperative sensing parameters.

given above are shown in Fig. 4.11. The results concur with the previous simulation results shown. The throughput of optimizing the individual threshold of the SUs using the proposed heuristic algorithm in Table 4.1 achieves the highest throughput and the throughput by optimizing the thresholds of the SUs with all the thresholds constrained to be the same, is slightly lower. The throughput with the SUs' thresholds set as $P_{di} = \bar{\mathbb{P}}_d$ is better than the throughput with the SUs' thresholds set by minimizing their individual decision errors. This is because the differences between γ_1 and γ_3 and between γ_2 and γ_3 are large. Similar to Fig. 4.5, when the differences in γ among the SUs are large, the OR fusion rule is able to outperform the method that set the SUs' thresholds by minimizing their individual decision errors.

4.6 Summary

In this chapter, analysis and discussions on the optimal decision fusion rule for the sensing-throughput tradeoff design in a large area CRN with SUs having different SNR levels are provided. The optimal decision fusion rule is a weighted rule based on the likelihood-ratio test. Various proposed methods to compute the SUs' thresholds, their decisions' weightings and the threshold of the fusion rule under various scenarios are discussed. The throughput is the highest when the individual SUs' thresholds are optimized using a centralized processor. The throughput is lower if the SUs' thresholds are constrained to be the same, however, the loss is minimized if the threshold is set at a level such that the SUs with high SNRs operate near their optimal thresholds while letting the SUs with low SNRs lose their optimality. Two heuristic methods are proposed to set the individual SUs' thresholds in a distributed manner, without the knowledge of each other's channel information. Two different constraints are also discussed in the decentralized optimization scenario: the average constraint and the instantaneous constraint.

Chapter 5

Energy-Efficient Cooperative Spectrum Sensing in a Small Area Cognitive Radio Network

From the previous chapters, we know that cooperative spectrum sensing is able to improve the sensing performance of CRNs and also able to mitigate the shadowing and multipath effects. However, when we collect sensing information from more SUs for cooperative spectrum sensing, it consumes more energies. This is because performing spectrum sensing and transmission of sensing data require energies. This can be detrimental to the CRN if the SUs are battery-powered wireless devices where energy is limited.

In the literature, there are some studies on minimizing the energy consumed by the CRN when performing cooperative spectrum sensing. When the SUs transmit their sensing information to the FC, those SUs that are farther away from the FC require more transmit energies. Therefore, in [50–52], the SUs are grouped into clusters based on their geometric locations, and each SU will send its sensing information to an assigned cluster head, which is nearer to the FC. The cluster head may compress the collected sensing information first before sending the sensing

information to the FC. In [52–55], censoring techniques are used where only SUs with higher reliability/confidence sensing information will transmit their sensing information to the FC for combining. The motivation of the above methods is to reduce the energy used in transmitting the sensing information from the SUs to the FC either by relaying or reducing the amount of transmitting sensing information.

In the above mentioned works, the sensing energy is not considered and the impact of the energy reduction in the cooperative spectrum sensing schemes to the throughput of the CRN is not discussed. In the previous chapters, the parameters of the cooperative spectrum sensing schemes are designed to maximize the throughput of the CRN. In this chapter, we will focus on the energy efficiency of a small area CRN, which is defined as the ratio of the average successful bits transmitted by the CRN over the average energy consumed by the CRN. We will design the fusion rule threshold, energy detector thresholds, number of cooperating SUs and the sensing time to maximize the energy efficiency of the small area CRN. The fusion rule threshold at the FC and the energy detector thresholds at the SUs affect the sensing performance which in turn will affect both the average throughput and the average transmission energy. This is because the transmission opportunities of the CRN depend on the sensing performance. The number of cooperating SUs and the length of the sensing time will affect the sensing performance and also the amount of energy used for sensing. Larger number of cooperating SUs and longer sensing time will also decrease the transmission time if the frame duration is fixed, therefore, affecting the throughput of the CRN. In this chapter, we will focus on obtaining the analytical solutions of the fusion rule threshold and the energy detector threshold that will maximize the energy efficiency of the small area CRN for a given sensing time and number of cooperating SUs. We will also show how the sensing time and the number of cooperating SUs impact the energy efficiency of the CRN through simulations.

The remainder of this chapter is organized as follows. The system model is introduced in Section 5.1. The energy efficiency of a CRN is introduced in Section 5.2. The problem formulations and solutions of maximizing the energy efficiency of a CRN using the fusion rule threshold and the energy detector threshold are given in Sections 5.3 and 5.4, respectively. Numerical results of the energy efficiency of the CRN with the parameters optimized are provided in Section 5.5. Finally, we draw a summary of this chapter in Section 5.6.

5.1 System Model

In this chapter, similarly, we consider a CRN with N SUs and a secondary base station. However, during the sensing period, only L out of the N SUs will sense the presence of the PU and make their own decisions. The L SUs will then transmit their sensing decisions to the FC for it to make a final decision. In the previous chapters, we consider that the time required for the transmission of the sensing decisions from the SUs to the FC is negligible since the sensing decision is only a bit long. However, in this chapter, we will include the transmission time of the sensing decisions into the system model. The frame structure of the CRN is shown in Fig. 5.1, where r represents the time taken by each SU to report its sensing decision to the FC, while τ and T still represent the sensing time and the frame time, respectively. We will like to note that if the transmission time of the sensing decisions is included in Chapters 3 and 4, the solutions presented in the chapters will remain the same by replacing the total frame time from T to $T - Nr$.

In this chapter, we assume a small area CRN similar to Chapter 3, where the SUs experience almost similar path loss and have the same average detection SNR γ . The system model considered in this chapter is therefore similar to Fig. 3.2 of Chapter 3, where the k -out-of- N fusion rule is used. However, in this chapter, we consider that only L out of the total N SUs perform spectrum sensing and

Figure 5.1: Frame structure of cooperative spectrum sensing CRN.

the FC is able to control which of the L SUs performs the spectrum sensing. Hence, the fusion rule is k -out-of- L and the feedback information from the FC to the SUs includes the sensing ON/OFF information of the SUs. We assume the channel gains of the PU to the SUs are constant during the sensing period and the probabilities of detection and false alarm are given in (4.1) and (4.2), respectively.

5.2 Energy Efficiency of a CRN

In a CRN, there are four possible scenarios between the activities of the PU and the SUs. In scenario one, when the PU is present and the CRN detects it successfully, no data will be transmitted by the SU. Denote P_s as the power consumed by each SU to perform spectrum sensing and P_t as the transmission power of each SU, the energy used by the CRN in this scenario is given by $L(P_s\tau + P_t r)$. Similarly, if the PU is absent and the CRN detects it as present, no data will be transmitted and the energy used by the CRN is given by $L(P_s\tau + P_t r)$. When the PU is present but the CRN miss-detects it, one of the SUs in the CRN will transmit data but its signal will be interfered by the PU's signal. In this scenario, we assume that the SU-Rx will not be able to decode its data due to the PU's interference. The last scenario is when the PU is absent and the CRN detects it correctly, therefore, one of the SUs will perform data transmission. The energies consumed by the CRN in the last

Table 5.1: Energy consumption and throughput achieved by the CRN under different scenarios.

Scenarios	Energy (joules)	Throughput (bits/Hz)
1) $Pr(H_1)\mathbb{P}_d$	$L(P_s\tau + P_tr)$	0
2) $Pr(H_0)\mathbb{P}_f$	$L(P_s\tau + P_tr)$	0
3) $Pr(H_1)(1 - \mathbb{P}_d)$	$LP_s\tau + P_t(T - \tau)$	0
4) $Pr(H_0)(1 - \mathbb{P}_f)$	$LP_s\tau + P_t(T - \tau)$	$(T - \tau - Lr)C_0$

two scenarios mentioned above are both given by $LP_s + LP_tr + P_t(T - \tau - Lr) = LP_s\tau + P_t(T - \tau)$. Similar to previous chapters, we denote C_0 as the achievable data rate when a SU transmits under no interference from the PU. The energy consumption and throughput achieved by the CRN under the different scenarios are summarized in Table 5.1.

From Table 5.1, the average throughput of the CRN is given by

$$R(\varepsilon, k, L, \tau) = Pr(H_0)(1 - \mathbb{P}_f)(T - \tau - Lr)C_0, \quad (5.1)$$

and the average energy consumed by the CRN is given by

$$E_{CRN}(\varepsilon, k, L, \tau) = x + y(1 - \mathbb{P}_d) + z(1 - \mathbb{P}_f) \quad (5.2)$$

where

$$\begin{aligned} x &= L(P_s\tau + P_tr) \\ y &= Pr(H_1)P_t(T - \tau - Lr) \\ z &= Pr(H_0)P_t(T - \tau - Lr). \end{aligned}$$

In this chapter, we are interested in the energy efficiency of the CRN, which is defined as

$$\eta(\varepsilon, k, L, \tau) = \frac{R(\varepsilon, k, L, \tau)}{E_{CRN}(\varepsilon, k, L, \tau)}. \quad (5.3)$$

From (5.1), the overall probability of false alarm affects the average throughput of the CRN while, from (5.2), both the overall probabilities of false alarm and detection affect the average amount of energy consumed by the CRN. Therefore, the probabilities of the overall false alarm and detection will impact the energy efficiency of the CRN.

Intuitively from (5.1)-(5.3), for a given L and τ , when both $\mathbb{P}_f$ and $\mathbb{P}_d$ approach one, the average energy consumed by the CRN will be minimized, however, the average throughput will become zero as this practically means that the CRN always decides that the PU is active and does not transmit data. On the other hand, when both $\mathbb{P}_f$ and $\mathbb{P}_d$ approach zero, which means that the CRN always decides that the PU is inactive, the average throughput will be maximized, however, it also maximizes the energy consumption of the CRN. Hence, designing parameters such as ε and k is needed so that the $\mathbb{P}_f$ and $\mathbb{P}_d$ are at the optimal point where the energy efficiency of the CRN is optimal.

If the $\mathbb{P}_f$ is able to approach zero while the $\mathbb{P}_d$ approaches one, the energy efficiency of the CRN seems to be optimal, however, this is not possible by adjusting k and ε because $\mathbb{P}_f$ and $\mathbb{P}_d$ increase or decrease in the same direction as ε or k changes. There are two possible methods to decrease $\mathbb{P}_f$ while increasing $\mathbb{P}_d$. They are either to increase the sensing time τ or the number of cooperating SUs L . However, increasing τ and/or L will decrease the amount of transmission time available to the CRN and may not necessarily increase the average throughput even though $\mathbb{P}_f$ decreases. Increasing τ and/or L will also increase the sensing energy consumed by the CRN, although it may reduce the transmission energy wasted in the missed detection scenario (Scenario 3 in Table 5.1) because of a higher $\mathbb{P}_d$. Hence, there are tradeoffs among the sensing performance, transmission period and energy consumption when designing the τ and L .

5.3 Optimal Fusion Rule Threshold that Maximizes the Energy Efficiency of a CRN

In a CRN, it is possible that the FC does not feed back any information to the SUs to control their sensing parameters due to a lack of feedback channel. In this case, the L cooperating SUs can decide their own thresholds by a targeted P_f value or the proposed methods presented in Section 4.4. In this section, after the FC receives the L sensing decisions, it will optimize k to maximize the energy efficiency of the CRN instead of maximizing the CRN's throughput in the previous chapters. The problem formulation to be solved by the FC for a given ε , L and τ is given by

$$\max_k \eta(k) = \frac{w(1 - \mathbb{P}_f(k))}{x + y(1 - \mathbb{P}_d(k)) + z(1 - \mathbb{P}_f(k))} \quad (5.4)$$

where $w = Pr(H_0)(T - \tau - Lr)C_0$.

The methodology used in [61] to solve for the design of the energy-efficient sequential channel sensing can also be used to solve the optimization problem (5.4). In [61], a ratio-type objective function is changed to a parametric problem which treats the throughput as the monetary reward and the energy consumption as the cost. The price per unit of energy consumption is λ , where $\lambda \geq 0$, and the net reward function can be expressed as

$$f(k, \lambda) = w(1 - \mathbb{P}_f(k)) - \lambda[x + y(1 - \mathbb{P}_d(k)) + z(1 - \mathbb{P}_f(k))]. \quad (5.5)$$

The function of the maximum net reward for a given λ is expressed as

$$g(\lambda) = \max_k f(k, \lambda). \quad (5.6)$$

From Proposition 1 in [61], the optimal solution of the ratio-type objective function is obtained, if and only if, the maximum net reward function equates to zero.

Therefore, if $g(\lambda)$ is a monotonically decreasing function of λ , then the optimal solution of (5.4) occurs at $g(\lambda) = 0$.

Proof: From (5.4) and (5.5), for a given k , $f(k, \eta(k)) = 0$. Define k^* as the optimal solution of (5.4), then $\lambda = \eta(k^*)$ is the largest λ value that can satisfy $f(k, \lambda) = 0$ since $\eta(k^*)$ is the maximum energy efficiency value for any k . Next, we denote λ° as $g(\lambda^\circ) = 0$ and k° as its corresponding k value that maximizes $f(k, \lambda^\circ)$. Any other $\lambda^\dagger > \lambda^\circ$ is unable to satisfy $f(k, \lambda^\dagger) = 0$ for any k value because $g(\lambda^\dagger) = \max_k f(k, \lambda^\dagger) < 0$ due to monotonicity. Hence, λ° is the largest λ value that can satisfy $f(k, \lambda) = 0$ and therefore, λ° must be equal to $\eta(k^*)$ and $k^\circ = k^*$. ■

It is easy to show that $g(\lambda)$ is a monotonically decreasing function in terms of λ . Since for any k value,

$$x + y(1 - \mathbb{P}_d(k)) + z(1 - \mathbb{P}_f(k)) > 0$$

hence from (5.5), $f(k, \lambda)$ must decrease as λ increases for any given k . Next, we have

$$\begin{aligned} g(\lambda) &= \max_k f(k, \lambda) \\ &\geq f(k, \lambda) \quad \forall k \\ &> f(k, \lambda + \Delta) \quad \forall k \end{aligned}$$

where Δ is any positive number. Therefore,

$$g(\lambda) > \max_k f(k, \lambda + \Delta) = g(\lambda + \Delta)$$

which proves $g(\lambda)$ is a monotonically decreasing function in terms of λ . Due to the monotonicity of $g(\lambda)$, the optimal energy efficiency value $\eta(k^*)$ which occurs

at $g(\eta(k^*)) = 0$ can simply be found using bisection algorithm. Furthermore, the closed-form solution of $g(\lambda)$ can be found as shown below. First, we have

$$\begin{aligned} \frac{\partial f(k, \lambda)}{\partial k} &\approx f(k+1, \lambda) - f(k, \lambda) \\ &= (w - \lambda z) \left(\binom{L}{k} P_f^k (1 - P_f)^{L-k} \right) \\ &\quad - \lambda y \left(\binom{L}{k} P_d^k (1 - P_d)^{L-k} \right). \end{aligned} \quad (5.7)$$

The k value that maximizes $f(k, \lambda)$ is obtained when $\frac{\partial f(k, \lambda)}{\partial k} = 0$, and we derive

$$k = \left\lceil \frac{\log \frac{w - \lambda z}{\lambda y} - L \log \frac{1 - P_d}{1 - P_f}}{\log \frac{P_d}{P_f} - \log \frac{1 - P_d}{1 - P_f}} \right\rceil \quad (5.8)$$

where $\lceil \cdot \rceil$ denotes the ceiling function. Since $k \geq 1$,

$$\log \frac{w - \lambda z}{\lambda y} - L \log \frac{1 - P_d}{1 - P_f} > 0,$$

and we obtain

$$\lambda < \frac{w}{y \left(\frac{1 - P_d}{1 - P_f} \right)^L + z}. \quad (5.9)$$

Therefore, the initial maximum value of λ for the bisection algorithm can be set to $\lambda = \frac{w}{y \left(\frac{1 - P_d}{1 - P_f} \right)^L + z}$. The algorithm to solve the problem (5.4) is given in Table 5.2 where ρ is the tolerable error of λ . From the simulations, when γ is at -16 dB, -20 dB, and -22 dB, with ε fixed at $P_f = 0.1$, the number of iterations for the proposed algorithm to converge for $L = 50$ is 4 iterations and for γ at -18 dB is 5 iterations, while for exhaustive search, it needs to search through all the 50 possible k values to obtain the optimal k values.

Table 5.2: Proposed iterative algorithm that optimizes fusion rule threshold to maximize energy efficiency

Algorithm 5.1: Find the optimal k for (5.4).
Initialization:
$\lambda_{min} \leftarrow 0; \lambda_{max} \leftarrow \frac{w}{y\left(\frac{1-P_d}{1-P_f}\right)^L + z};$
$k_{\lambda_{min}} \leftarrow 1; k_{\lambda_{max}} \leftarrow L;$
while ($\lambda_{max} - \lambda_{min} > \rho$ and $k_{\lambda_{max}} \neq k_{\lambda_{min}}$ do
$\lambda \leftarrow (\lambda_{max} + \lambda_{min})/2$
$k \leftarrow \left\lceil \frac{\log \frac{w-\lambda z}{\lambda y} - L \log \frac{1-P_d}{1-P_f}}{\log \frac{P_d}{P_f} - \log \frac{1-P_d}{1-P_f}} \right\rceil$
if $g(\lambda) > 0$ then
$\lambda_{min} \leftarrow \lambda; k_{\lambda_{min}} \leftarrow k$
else
$\lambda_{max} \leftarrow \lambda; k_{\lambda_{max}} \leftarrow k$
end if
end while
Output: k

5.4 Optimal Energy Detector Threshold that Maximizes the Energy Efficiency of a CRN

When the FC has a feedback channel to control the SUs' sensing parameters, it is be able to control the SUs' energy detector threshold ε , the number of cooperating SUs L out of the N SUs and the sensing time τ in order to improve the energy efficiency of the CRN. In this section, the FC will optimize the SUs' energy detector threshold when all the other parameters are fixed and feeds back only the optimal energy detector threshold for the SUs to operate on. The problem formulation to be solved by the FC for a given k , L and τ is given by

$$\max_{\varepsilon} \quad \eta(\varepsilon) = \frac{w(1 - \mathbb{P}_f(\varepsilon))}{x + y(1 - \mathbb{P}_d(\varepsilon)) + z(1 - \mathbb{P}_f(\varepsilon))}. \quad (5.10)$$

The optimal solution occurs when $\frac{\partial \eta(\varepsilon)}{\partial \varepsilon} = 0$, which we obtain

$$1 - \mathbb{P}_d(\varepsilon) = (1 - \mathbb{P}_f(\varepsilon)) \frac{\nabla \mathbb{P}_d(\varepsilon)}{\nabla \mathbb{P}_f(\varepsilon)} - \frac{x}{y} \quad (5.11)$$

where from [150],

$$\nabla \mathbb{P}_d(\varepsilon) = \frac{\partial \mathbb{P}_d(\varepsilon)}{\partial \varepsilon} = \nabla P_d \sum_{i=k}^L \binom{L}{i} P_d^{i-1} (1 - P_d)^{L-i} \left(i - (L - i) \frac{P_d}{1 - P_d} \right),$$

$$\nabla \mathbb{P}_f(\varepsilon) = \frac{\partial \mathbb{P}_f(\varepsilon)}{\partial \varepsilon} = \nabla P_f \sum_{i=k}^L \binom{L}{i} P_f^{i-1} (1 - P_f)^{L-i} \left(i - (L - i) \frac{P_f}{1 - P_f} \right),$$

$$\nabla P_d = \frac{-1}{\sigma_u^2} \sqrt{\frac{\tau f s}{2\pi(2\gamma + 1)}} \exp \left(-\frac{\tau f s}{4\gamma + 2} \left(\frac{\varepsilon}{\sigma_u^2} - \gamma - 1 \right)^2 \right),$$

and

$$\nabla P_f = \frac{-1}{\sigma_u^2} \sqrt{\frac{\tau f s}{2\pi}} \exp \left(-\frac{\tau f s}{2} \left(\frac{\varepsilon}{\sigma_u^2} - 1 \right)^2 \right).$$

The optimal threshold ε^* can be evaluated numerically from (5.11) and by substituting (5.11) into (5.10), the energy efficiency of the CRN operating at the optimal energy detector threshold is given by

$$\eta(\varepsilon^*) = \frac{w}{y \frac{\nabla \mathbb{P}_d(\varepsilon^*)}{\nabla \mathbb{P}_f(\varepsilon^*)} + z}. \quad (5.12)$$

5.5 Simulation Results and Discussion

In this section, we present the numerical results of the energy efficiency of the CRN when the parameters of the cooperative spectrum sensing scheme are optimized. In the simulations, we assume that the probability of the PU being active $Pr(H_1)$ is equal to 0.5. The total duration of each frame T is set to 20ms. The sampling frequency f_s during the sensing period is set to 6MHz. The sensing time τ is

Figure 5.2: The optimal fusion rule threshold when the probability of false alarm at individual SU is fixed.

set to 1ms unless otherwise stated. The time r required for each SU to report its decision is assumed to be $10\mu\text{s}$. The rate achieved by the CRN whenever one of the SUs transmits data successfully without the interference from the PU is assumed to be $C_0 = \log_2(1 + \text{SNR}_s) = 6.6582\text{bits/sec/Hz}$ where the SNR for the secondary transmission is assumed to be $\text{SNR}_s = 20\text{dB}$. The transmission power P_t and sensing power P_s consumed by each SU are assumed to be 3W and 0.1W, respectively. The noise power σ_u^2 is normalized to be 1.

When there is no feedback channel for the FC to control the sensing parameters of the SUs, the FC will optimize k . In this simulation, the SUs will set their thresholds such that $P_f = 0.1$. From Fig. 5.2, the optimal k obtained by using the proposed Algorithm 5.1 is the same as using exhaustive search for different L and γ values. This shows that the proposed Algorithm 5.1 is able to obtain the optimal k value.

In Fig. 5.3, we compare the energy efficiency of the CRN when the threshold of the SUs is fixed to when it is optimized at different k values. The number of

Figure 5.3: Energy efficiency of CRN using different fusion rule thresholds.

cooperating SUs is set to $L = 25$. When ϵ is fixed at $P_f = 0.1$, it can be observed from Fig. 5.3, that optimizing k is important as the difference in energy efficiency can be more than double between different k values. For example, when $\gamma = -18\text{dB}$, using OR rule will obtain an energy efficiency of about $0.85\text{bits/Hz/joules}$ but if $k = 6$ (optimal), the energy efficiency is about $1.97\text{bits/Hz/joules}$. However, if there is a feedback channel for the FC to feed back the optimal ϵ to the SUs, it seems that optimizing k is no longer as important because the energy efficiency across different k values becomes relatively flat. In fact, if one chooses the majority fusion rule $k = \lceil L/2 \rceil$, the energy efficiency with ϵ optimized will be at or close to optimal.

From Fig. 5.3, it can also be observed that when k is at optimal value, its energy efficiency is actually not far from the energy efficiency of when k and ϵ are jointly optimized. Optimizing k does not require any feedback channel while optimizing ϵ does. Hence, is it worth to optimize ϵ and feeds back it to the SUs? In Fig. 5.4, we compare the energy efficiency when k is optimized with $P_f = 0.1$

Figure 5.4: Energy efficiency of CRN at different SNR regions.

to the energy efficiency when k and ϵ are jointly optimized. It can be observed that at low γ region, optimizing k and ϵ jointly is about 0.1bits/Hz/joule higher than just optimizing k . At high γ region, there is hardly any gain. Therefore, we can conclude that having a feedback channel to feedback the optimal ϵ may not be necessary if k is optimized, especially at high γ region.

As mention in the previous section, when the number of cooperating SUs L or sensing time τ increases, it improves the sensing performance but decreases the transmission time. This is because as L increases, the total time for all the cooperating SUs to report their decisions also increases. The amount of sensing energy will also be increased as there are more cooperating SUs or longer sensing time. However, with a better sensing performance, the “useful” transmission energy may be increased if the $\mathbb{P}_f$ is decreased because the chance of secondary transmission when the PU is inactive becomes higher. While the “wasteful” transmission energy may be decreased if the $\mathbb{P}_d$ is increased because the chance of secondary transmission colliding with PU’s signal becomes lower. Hence, L and τ affect the energy

Figure 5.5: Energy efficiency of CRN when number of cooperative users increases with sensing time fixed at 1ms.

efficiency of the CRN in numerous ways. We plot the energy efficiency of CRN when k and ε are jointly optimized with different L and τ values in Figs. 5.5 and 5.6, respectively.

From Fig. 5.5, it can be observed that when L initially increases, the energy efficiency increases. This means that the improvement in sensing performance is able to outweigh the loss in shorter transmission time and the larger energy used for sensing and reporting the sensing decisions. However, when L is increased further, the energy efficiency decreases. This means that it is no longer worth to improve the sensing performance by having more cooperating SUs. From Fig. 5.5, when the SNR is high, the optimal L is smaller as a small number of cooperating SUs is enough to achieve a good sensing performance. When a small number of SUs are needed for cooperating spectrum sensing, the SUs can take turns to perform spectrum sensing which can increase their battery life.

The curves in Fig. 5.6, where the sensing time varies instead of L , have similar trends as Fig. 5.5. This is because increasing the sensing time has similar effects

Figure 5.6: Energy efficiency of CRN when sensing time increases with N fixed at 25.

as increasing L where the sensing performance improves but the transmission time becomes shorter and the sensing energy consumed becomes higher.

5.6 Summary

In this chapter, we proposed to maximize the energy efficiency of the CRN using cooperative spectrum sensing. The parameters optimized in the design include the fusion rule threshold, energy detector threshold, number of cooperating SUs, and the length of the sensing time. An iterative algorithm is proposed to find the optimal k . The equation at which ε is optimal is derived. The impacts of L and τ on the energy efficiency are shown using simulations. From the numerical results, optimizing k is important as the gain may be more than double. However, when k is optimized, optimizing ε does not bring much gain especially in the high SNR region. We have also shown that an optimal L exists since the sensing performance gain from having more cooperating SUs is being tradeoff by more sensing energy

and less transmission time. The same conclusion can be applied to the length of the sensing time.

Chapter 6

Power Control in Cognitive Radio Networks under Cooperative and Non-Cooperative Spectrum Sensing

Power allocation strategies for CRNs have attracted a lot of attention as the transmission powers of the SUs will affect their opportunities to utilize the licensed spectrum and their interferences to the PUs [172]. A large portion of the literature on power allocation designs for CRNs is on the spectrum sharing model where the SU coexists with the PU by limiting its interference power to the PU-Rx. Some of the power allocation strategies for the spectrum sharing model are listed as follows. In [173–175], power allocation strategies have been designed to maximize the ergodic and/or outage capacities of the SU with the constraint that the interference power from the SU to the PU-Rx is limited to a certain level. In [176], in addition to the interference power constraint, each SU has to achieve a minimum QoS while maximizing the number of SUs' admission links.

In [173–176], the PU is assumed to be always in operational and hence, the SU is not required to perform spectrum sensing to detect the PU. In [177–180], the PU is not assumed to be always active and hence, spectrum sensing is carried out to determine the probability of the presence of the PU. The probability of the PU's presence is then taken into account when maximizing the ergodic capacity [177, 178], data rate [179], and outage capacity [180] of the SU, while limiting the SU's average interference power to the PU-Rx below a certain level. In [177–180], no decision is required to be made on the presence of the PU after sensing, since the PU and the SU coexist as long as the average interference power constraint is met. However, the above studies [173–180], which protect the PU by limiting the SU's interference power at the PU-Rx, require the CSI between the SU-Tx and the PU-Rx. This CSI is usually unknown to the SU-Tx unless the PU-Rx is willing to help the SU to estimate this information.

Another group of power allocation strategies for CRNs that has been investigated in the literature is to find the maximum transmit power of the SU-Tx such that even if the SU transmits data in the presence of the PU, the interference from the SU to the PU will be limited. In [181, 182], spectrum sensing information (SSI) gathered during the sensing period is used to estimate the distance between the PU and the SU. The maximum transmit power of the SU is then determined by the estimated distance. By having an upper limit on the transmit power of the SU-Tx, this ensures that the SU can coexist with the PU without fear of interfering it.

For the OSA model, an intra-frame power allocation strategy has been designed in [183], where more power is allocated to data samples located at the beginning of the data frame since the probability of the PU returning to an active state is higher at the tail end of the data frame. In OSA model, the SSI is used in making decision on the presence of the PU but is seldom used in the power allocation strategies.

When the SU miss-detects the PU and transmits data, its throughput and outage probability will be affected due to the interference from the PU. The SSI provides soft information about the PU's activities and its signal strength received at the SU. Hence, in this chapter, we proposed to utilize the SSI in the power allocation strategies under the OSA model in both cooperative and non-cooperative spectrum sensing scenarios. In non-cooperative spectrum sensing scenario, only the SU-Tx performs spectrum sensing while in the cooperative spectrum sensing scenario, both the SU-Tx and the SU-Rx perform spectrum sensing. In cooperative spectrum sensing, the SU-Rx will transmit its SSI to the SU-Tx and the SU-Tx will combine its SSI with the SU-Rx's SSI. In this chapter, we will present the difference between the power allocation strategies under the cooperative and non-cooperative spectrum sensing scenarios when we maximize the average data rate and minimize the outage probability of the SU. The transmit power of the SU is constrained to satisfy an average power constraint and the percentage of time that the PU can be interfered by the SU is also constrained below a threshold. Lastly, the proposed power allocation strategies are functions of the sensing time and therefore, we will show the performance gains of the power allocation strategies when the sensing time is optimized as well.

The remainder of this chapter is organized as follows. The system model is introduced in Section 6.1. We provide both the power allocation strategies for maximizing the average data rate and minimizing the outage probability of the SU under the non-cooperative spectrum sensing scenario in Section 6.2 and under the cooperative spectrum sensing scenario in Section 6.3. Discussions on optimizing the sensing time are provided in Section 6.4. Computer simulations of the power allocation strategies are shown and discussed in Section 6.5. Finally, we draw a summary of this chapter in Section 6.6.

Figure 6.1: Primary and secondary users' activities under cooperative and non-cooperative spectrum sensing.

6.1 System Model

Consider a CRN, formed by a pair of SU-Tx and SU-Rx, that opportunistically operates over a frequency band that has been licensed to a primary communication system. The frame structure of the SU consists of a sensing period and a data transmission period and if cooperative spectrum sensing is performed, there will be a reporting period between the sensing period and the data transmission period. The PU's and SU's activities under the cooperative and non-cooperative spectrum sensing are shown in Fig. 6.1. In Fig. 6.1, T_{ON} and T_{OFF} are denoted as the PU's average ON and OFF periods, respectively, and other symbols remain the same as previous chapters. From Fig. 6.1, when the SU miss-detects the PU and transmits data, the PU will be interfered with a time period of $(T - \tau)$ under the non-cooperative spectrum sensing scenario and $(T - \tau - r)$ under the cooperative spectrum sensing scenario, assuming that the PU is active throughout the whole frame of the SU.

From Fig. 6.1, assuming that T_{ON} and T_{OFF} are much greater than T such that the PU is active or inactive throughout most of the SU's frames, then the approximate percentage of time that the PU is interfered whenever the PU is active

under the non-cooperative spectrum sensing is given by

$$I = (1 - P_d) \left(1 - \frac{\tau}{T}\right) \times 100\% \quad (6.1)$$

where P_d is the probability of detecting the PU by the CRN. Under cooperative spectrum sensing, the term $\frac{\tau}{T}$ in (6.1) is replaced by $\frac{\tau+r}{T}$.

The non-cooperative and cooperative spectrum sensing models considered in this chapter are shown in Fig. 6.2. In the non-cooperative spectrum sensing model, only the SU-Tx senses for the presence of the PU during the spectrum sensing period. We denote $x_1(n)$ as a received sample at the SU-Tx during a sensing period and y_1 as the sum of the energy levels of all $x_1(n)$ collected during a sensing period. Energy detector is considered and therefore, if y_1 of a particular frame is below the energy detector's threshold ε_1 , the SU-Tx will transmit data to the SU-Rx, otherwise, the SU-Tx is not allowed to transmit. For every frame that has a measured energy level y_1 below the threshold ε_1 , the transmit power of the SU-Tx, denoted as $P_t(y_1)$, will be determined by the value of y_1 measured in that frame. The rationale is that when y_1 is below ε_1 and the SU decides that the PU is absent, this does not mean that the PU is definitely absent because it may be a case of miss-detection. Transmitting the data with a power that assumes the PU is absent, when in fact it is present, may result in a lower data rate or a loss of data packet for the SU due to the interference from the PU. Therefore, power control for each data frame is required to include the possibility of miss-detection in the frame even though the SU perceives that the PU is absent. The measured energy level y_1 in each of the frames is used to gauge the probability of miss-detection occurring in the frame.

In the cooperative spectrum sensing model shown in Fig. 6.2, both the SU-Tx and the SU-Rx sense for the presence of the PU where $x_2(n)$ denotes the received sample at the SU-Rx during a sensing period and y_2 denotes the total

Figure 6.2: System models of cooperative and non-cooperative spectrum sensing.

energy level of all $x_2(n)$ collected during the sensing period. After the sensing period, the SU-Rx sends y_2 to the SU-Tx through a dedicated control channel. The SU-Tx acting as the FC will combine the two SSI, y_1 and y_2 , by doing a weighted sum of them. We denote y_c as the weighted sum of y_1 and y_2 , and ε_c as the threshold of the fusion rule. Similar to the non-cooperative model, the SU-Tx will only transmit when y_c is below the threshold ε_c and the power allocated to each frame is determined by the values of y_1 and y_2 where the SU-Tx has both information. There are two advantages for cooperative spectrum sensing compared to the non-cooperative one. First, cooperative spectrum sensing improves the sensing performance, which further improves the SU's utilization of unoccupied channels, and hence, increases the average data rate of the SU. The second advantage is that the SU-Tx has the knowledge of the energy level of the noise power (or interference-plus-noise power when PU is present) at the SU-Rx, which is y_2 . Hence, the SU-Tx can allocate transmit power such that its SNR (or signal-to-interference-plus-noise ratio (SINR) when PU is present but it fails to

detect it) at the SU-Rx is above a level that the data can be decoded successfully. However, the disadvantages of the cooperative spectrum sensing model are that it needs a dedicated control channel and additional time for the SU-Rx to feedback its SSI to the SU-Tx.

The statistical assumptions of the noise remain the same as the previous chapters, therefore, the probability density functions (PDFs) of y_1 and y_2 under the hypothesis that the PU is inactive H_0 , are approximated as

$$f_{y_i|H_0}(y_i) = \sqrt{\frac{N}{2\pi\sigma_u^4}} \exp\left(-\frac{N(y_i - \sigma_u^2)^2}{2\sigma_u^4}\right), \quad \forall i = 1, 2. \quad (6.2)$$

Similar to the previous chapters, the PU's signal is assumed to be complex-valued phase-shift keying modulated and under the hypothesis that the PU is active H_1 , the PDFs of y_1 and y_2 are approximated as

$$f_{y_i|H_1}(y_i) = \sqrt{\frac{N}{2\pi(2\gamma_i + 1)\sigma_u^4}} \exp\left(-\frac{N(y_i - (\gamma_i + 1)\sigma_u^2)^2}{2(2\gamma_i + 1)\sigma_u^4}\right), \quad \forall i = 1, 2 \quad (6.3)$$

where γ_1 and γ_2 are the average SNRs of the PU's signal received at the SU-Tx and the SU-Rx, respectively. The test statistic for the FC under the cooperative spectrum sensing scenario is given as $y_c = w_1y_1 + w_2y_2$ where w_1 and w_2 are the weights given to the SSI of the SU-Tx and the SU-Rx, respectively. The PDFs of y_c under H_0 and H_1 are respectively approximated as

$$f_{y_c|H_0}(y_c) = \sqrt{\frac{N}{2\pi\sigma_u^4(w_1^2 + w_2^2)}} \exp\left(-\frac{N(y_c - \sigma_u^2(w_1 + w_2))^2}{2\sigma_u^4(w_1^2 + w_2^2)}\right), \quad (6.4)$$

$$\begin{aligned} f_{y_c|H_1}(y_c) &= \sqrt{\frac{N}{2\pi\sigma_u^4(w_1^2(2\gamma_1 + 1) + w_2^2(2\gamma_2 + 1))}} \\ &\times \exp\left(-\frac{N(y_c - \sigma_u^2(w_1(\gamma_1 + 1) + w_2(\gamma_2 + 1)))^2}{2\sigma_u^4(w_1^2(2\gamma_1 + 1) + w_2^2(2\gamma_2 + 1))}\right). \end{aligned} \quad (6.5)$$

6.2 Power Control under Non-Cooperative Spectrum Sensing

When only the SU-Tx is used to sense the channel, it is able to utilize the channel without delay when the PU is detected to be inactive for a particular frame. We consider two power allocation strategies for the transmission of the SU's data in each frame based on the SSI y_1 . The first strategy is to maximize the average data rate of the SU and the second strategy is to minimize the outage probability of the SU for a pre-determined rate $\bar{R}$. The average transmit power is restricted below a certain level $\bar{P}_t$ to satisfy the long-term power budget so as to prolong the battery life span of the SUs. The maximum percentage of time that the SU can interfere with the PU is fixed at a pre-determined level denoted as $\hat{I}$. We consider that the sensing and data transmission periods have been designed and are fixed in Sections 6.2 and 6.3.

6.2.1 Maximizing the average data rate

From (6.3), for a given y_1 , the probability that the PU is present is given by, using Bayes' theorem,

$$Pr(H_1|y_1) = \frac{Pr(H_1)}{Pr(H_1) + Pr(H_0)\sqrt{2\gamma_1 + 1} \exp\left(-\frac{N(2\gamma_1 y_1^2 - 2\gamma_1 \sigma_u^2 y_1 - \gamma_1^2 \sigma_u^4)}{2\sigma_u^4(2\gamma_1 + 1)}\right)}. \quad (6.6)$$

The probability that the PU is absent for a given y_1 is given as $Pr(H_0|y_1) = 1 - Pr(H_1|y_1)$. The average data rate of the SU for a given y_1 normalized with the time spent on sensing the channel is given by

$$\begin{aligned} R(P_t(y_1)) &= \left(1 - \frac{\tau}{T}\right) Pr(H_0|y_1) \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_u^2}\right) \\ &\quad + \left(1 - \frac{\tau}{T}\right) Pr(H_1|y_1) \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_{s,2}^2 + \sigma_u^2}\right) \end{aligned} \quad (6.7)$$

where $P_t(y_1)$ is the transmit power of the SU-Tx for a given y_1 . The channel gain between the SU-Tx and the SU-Rx is denoted as g_s . We denote $\sigma_{s,2}^2$ as the PU's average signal power received at the SU-Rx when the PU is active.

The problem formulation to maximize the average data rate of the SU with an average transmit power limit $\bar{P}_t$ while limiting the percentage of time that the PU is interfered by the SU to $\hat{I}$ is given as

$$\begin{aligned} \max_{P_t(y_1)} \quad & \int_{-\infty}^{\infty} R(P_t(y_1)) f_{y_1}(y_1) dy_1 \\ \text{s.t.} \quad & \left(1 - \frac{\tau}{T}\right) \int_{-\infty}^{\infty} P_t(y_1) f_{y_1}(y_1) dy_1 \leq \bar{P}_t, \\ & I \leq \hat{I}, \quad P_t(y_1) \geq 0, \forall y_1 \end{aligned} \quad (6.8)$$

where $f_{y_1}(y_1) = Pr(H_0)f_{y_1|H_0}(y_1) + Pr(H_1)f_{y_1|H_1}(y_1)$. From (6.1), $I \leq \hat{I}$ is equivalent to $P_d \geq 1 - (\frac{T}{T-\tau})\hat{I}$, and from the probability of detection equation given in (4.1), the threshold to achieve $P_d = 1 - (\frac{T}{T-\tau})\hat{I}$ is given by

$$\varepsilon_1 = \sigma_u^2 \left(\sqrt{\frac{2\gamma_1 + 1}{N}} \mathcal{Q}^{-1} \left(1 - \left(\frac{T}{T-\tau} \right) \hat{I} \right) + \gamma_1 + 1 \right). \quad (6.9)$$

By not transmitting when $y_1 > \varepsilon_1$, this limits the percentage of time that the PU is interfered by the SU to $\hat{I}$ whenever the PU is active. However, there is a probability that y_1 is greater than ε_1 even when the PU is inactive. In this case, the SU loses the opportunity to transmit a data frame without interference from and to the PU. This loss in data transmission opportunity is the cost of limiting the percentage of time that the PU is being interfered. If the SU-Tx knows the CSI between the SU-Tx and the PU-Rx, it is possible for the SU-Tx to transmit even if $y_1 > \varepsilon_1$, as in the spectrum sharing model [177–180]. However, in this thesis, we assume this CSI is unavailable to the SU-Tx and hence, there are these losses of data transmission opportunities for the SU in order to protect the PU.

By setting $P_t(y_1) = 0$ for all $y_1 > \varepsilon_1$, the interference time constraint in (6.8) is always satisfied with equality and the problem becomes

$$\begin{aligned} \max_{P_t(y_1)} \quad & \int_{-\infty}^{\varepsilon_1} R(P_t(y_1)) f_{y_1}(y_1) dy_1 \\ \text{s.t.} \quad & \left(1 - \frac{\tau}{T}\right) \int_{-\infty}^{\varepsilon_1} P_t(y_1) f_{y_1}(y_1) dy_1 \leq \bar{P}_t, \quad P_t(y_1) \geq 0, \forall y_1 \leq \varepsilon_1. \end{aligned} \quad (6.10)$$

Setting the threshold given by (6.9) will optimize the problem (6.8). The proof is as follows: Any other $\varepsilon^\dagger < \varepsilon_1$ will yield $I < \hat{I}$, which also satisfies the interference time constraint. When $\varepsilon^\dagger$ is chosen instead of ε_1 , the problem (6.8) will still become (6.10) with ε_1 replaced by $\varepsilon^\dagger$. Since $\varepsilon_1 > \varepsilon^\dagger$, the integration region in (6.10) with $\varepsilon^\dagger$ being chosen is a subset of the integration region when ε_1 is chosen. Therefore, the solution of (6.10) with ε_1 being chosen is always greater than or equal to the solution of (6.10) with $\varepsilon^\dagger$ being chosen. ■

The objective function in (6.10) is concave in $P_t(y_1)$ and the constraint sets are convex, therefore, the problem can be solved via the Lagrangian methods [184]. We form the partial Lagrangian

$$L(P_t(y_1), \lambda) = \int_{-\infty}^{\varepsilon_1} R(P_t(y_1)) f_{y_1}(y_1) dy_1 - \lambda \left[\int_{-\infty}^{\varepsilon_1} P_t(y_1) f_{y_1}(y_1) dy_1 - \frac{T\bar{P}_t}{T - \tau} \right] \quad (6.11)$$

where λ is the Lagrange multiplier. Taking the derivative of $L(P_t(y_1), \lambda)$ with respect to $P_t(y_1)$ and setting it to zero, we derive

$$P_t(y_1) = \frac{-b \pm \sqrt{b^2 - 4c(y_1)}}{2} \quad (6.12)$$

where

$$b = \frac{\sigma_{s,2}^2 + 2\sigma_u^2}{g_s} - \frac{1}{\lambda \ln(2)},$$

and

$$c(y_1) = \frac{\sigma_{s,2}^2 \sigma_u^2 + \sigma_u^4}{g_s^2} - \frac{\sigma_{s,2}^2 Pr(H_0|y) + \sigma_u^2}{\lambda g_s \ln(2)}.$$

Given that the constraint $P_t(y_1) \geq 0$ for all $y_1 \leq \varepsilon_1$, it can be proven (shown in Appendix 6.7.1) that

$$\frac{-b - \sqrt{b^2 - 4c(y_1)}}{2} < 0, \forall y_1 \leq \varepsilon_1,$$

therefore, the solution of the optimal power allocation is given by

$$P_t(y_1) = \left[\frac{-b + \sqrt{b^2 - 4c(y_1)}}{2} \right]^+, \forall y_1 \leq \varepsilon_1 \quad (6.13)$$

where $[x]^+ = \max(x, 0)$ and λ is chosen such that $(1 - \frac{\tau}{T}) \int_{-\infty}^{\varepsilon_1} P_t(y_1) f_{y_1}(y_1) dy_1 = \bar{P}_t$ as one of the Karush-Kuhn-Tucker (KKT) conditions [184] is

$$\lambda \left(\left(1 - \frac{\tau}{T}\right) \int_{-\infty}^{\varepsilon_1} P_t(y_1) f_{y_1}(y_1) dy_1 - \bar{P}_t \right) = 0. \quad (6.14)$$

It can be shown (in Appendix 6.7.2) that when $\lambda < \frac{g_s}{\ln(2)(\sigma_{s,2}^2 + 2\sigma_u^2)}$, which occurs when the average transmit power limit $\bar{P}_t$ is high, $P_t(y_1)$ in (6.13) is greater than zero for all y_1 . Therefore, when $\bar{P}_t$ is high, the SU-Tx should transmit with a power proportional to $Pr(H_0|y_1)$ whenever it detects that the PU is inactive. However, when $\lambda > \frac{g_s}{\ln(2)(\sigma_{s,2}^2 + 2\sigma_u^2)}$, which occurs when $\bar{P}_t$ is low, $P_t(y_1)$ is greater than zero only when

$$Pr(H_0|y_1) > \frac{\lambda \ln(2) \sigma_u^2}{g_s} \left(1 + \frac{\sigma_u^2}{\sigma_{s,2}^2} \right) - \frac{\sigma_u^2}{\sigma_{s,2}^2}. \quad (6.15)$$

This means that if the average transmit power limit is low, even if the PU is detected to be inactive, the SU-Tx should not always transmit data. The SU-Tx should transmit only when the probability of the PU being inactive for a given y_1 is higher than the threshold given in (6.15). Note that although the above result

is derived by assuming g_s is a constant, it can be generalized to fading channel. Assumed that g_s experiences block fading with the PDF of its distribution denoted as $f_{g_s}(g_s)$, we will still obtain the same power allocation solution (6.13) with λ chosen such that $(1 - \frac{\tau}{T}) \int_{-\infty}^{\varepsilon_1} \int_{-\infty}^{\infty} P_t(y_1) f_{g_s}(g_s) f_{y_1}(y_1) dg_s dy_1 = \bar{P}_t$.

6.2.2 Minimizing the outage probability

We consider that whenever a frame fails to transmit data above a pre-determined data rate $\bar{R}$, it is considered as an outage regardless of the status of the PU, similar to [185]. This means that if the SU-Tx decides not to transmit in order to protect the PU, we consider the SU is in outage regardless whether the PU is indeed present or absent. When the data rate is normalized to account for the time spent on sensing the channel, the outage probability of the SU with a targeted rate of $\bar{R}$ for a given SSI y_1 , is given by

$$P_{out}(P_t(y_1)) = Pr(H_0|y_1)Pr\left[\left(1 - \frac{\tau}{T}\right) \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_u^2}\right) < \bar{R}\right] \\ + Pr(H_1|y_1)Pr\left[\left(1 - \frac{\tau}{T}\right) \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_{s,2}^2 + \sigma_u^2}\right) < \bar{R}\right]. \quad (6.16)$$

Similar to (6.8), by setting $I = \hat{I}$ using the threshold ε_1 given by (6.9) will minimize the outage probability given the same constraints in (6.8). The proof is quite similar to that in (6.8) and hence, will not be repeated. The problem formulation to minimize the outage probability of the SU with an average transmit power limit $\bar{P}_t$ while setting $I = \hat{I}$ is given by

$$\min_{P_t(y_1)} \quad \int_{-\infty}^{\varepsilon_1} P_{out}(P_t(y_1)) f_{y_1}(y_1) dy_1 + \int_{\varepsilon_1}^{\infty} f_{y_1}(y_1) dy_1 \\ \text{s.t.} \quad \int_{-\infty}^{\varepsilon_1} P_t(y_1) f_{y_1}(y_1) dy_1 \leq \frac{T\bar{P}_t}{T - \tau}, \quad P_t(y_1) \geq 0, \forall y_1 \leq \varepsilon_1 \quad (6.17)$$

where $\int_{\varepsilon_1}^{\infty} f_{y_1}(y_1)dy_1$ is a constant term independent of $P_t(y_1)$. From (6.17), we can observe that the outage probability is lower bounded by

$$\int_{\varepsilon_1}^{\infty} f_{y_1}(y_1)dy_1 = Pr(H_0)\mathcal{Q}\left(\frac{\sqrt{N}(\varepsilon_1 - \sigma_u^2)}{\sigma_u^2}\right) + Pr(H_1)\mathcal{Q}\left(\frac{\sqrt{N}(\varepsilon_1 - (\gamma_1 + 1)\sigma_u^2)}{\sqrt{2\gamma_1 + 1}\sigma_u^2}\right)$$

because of the interference time constraint to protect the PU. Whenever the SSI y_1 is greater than ε_1 , the SU will be in outage for that data frame since the SU-Tx will not transmit any data. Note that ε_1 given by (6.9) is the largest possible threshold value that satisfies $I \leq \hat{I}$, and if any $\varepsilon^\dagger < \varepsilon_1$ is chosen instead, the lower bound of the outage probability will be increased. Hence, choosing the threshold given by (6.9) will minimize both the outage probability and the outage floor.

The optimal solution of (6.17) (The proof is given in Appendix 6.7.3) for $y_1 \leq \varepsilon_1$ is given by

$$P_t(y_1) = \begin{cases} \frac{(\sigma_{s,2}^2 + \sigma_u^2)(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s}, & \alpha < \min\{\beta, Pr(H_1|y_1)\} \\ \frac{\sigma_u^2(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s}, & Pr(H_1|y_1) < \min\{\alpha, \beta\} \\ 0, & \text{otherwise} \end{cases} \quad (6.18)$$

where

$$\alpha = \frac{\lambda\sigma_{s,2}^2(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s},$$

and

$$\beta = 1 - \frac{\lambda\sigma_u^2(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s}$$

in this chapter. The Lagrange multiplier λ in (6.18) is chosen such that (6.18) satisfies the average transmit power constraint. The solution (6.18) has two layers of truncated channel inversion compared to the conventional truncated channel inversion [186]. This is because the SU experiences two different possible channel conditions: channel with and without the presence of the PU. Therefore, we refer this power allocation strategy as two-layered truncated channel inversion power

allocation.

6.3 Power Control under Cooperative Spectrum Sensing

In cooperative spectrum sensing, the SU-Rx will send its SSI to the SU-Tx through a dedicated control channel after the sensing period. Assume that a delay of r is required for the SU-Rx to send its SSI y_2 to the SU-Tx, the data transmission period will become $(T - \tau - r)$. The SU-Tx with the knowledge of y_2 is able to combine both the SSIs to achieve a better sensing performance. When the PU is present during a particular frame, y_2 is the interference-plus-noise power experiencing at the SU-Rx. Hence, even when the SU miss-detects the PU, the SU-Tx can transmit its data successfully to the SU-Rx by making sure its SINR is above a level that the SU-Rx is able to decode its data. In the previous scenario where the SU-Tx does not have the value of y_2 , when the SU fails to detect the PU, the SU-Tx may transmit the data frame using low power assuming that there will be no interference from the PU and may cause the SU-Rx to fail to decode the data. In both cooperative and non-cooperative spectrum sensing, the SU is going to interfere with the PU during a miss-detection for a length of the data transmission time. The difference is that the SU-Tx with the knowledge of y_2 is able to use more power to ensure that its data frame is successfully received at the SU-Rx while the SU-Tx in the non-cooperative scenario may just waste some power over a failed transmission attempt.

6.3.1 Maximizing the average data rate

The average data rate of the SU for a given y_1 and y_2 normalized by the time spent on sensing and reporting the SSI y_2 is given by

$$R_c(P_t(y_1, y_2)) = \left(1 - \frac{\tau + r}{T}\right) \log_2 \left(1 + \frac{P_t(y_1, y_2)g_s}{y_2}\right). \quad (6.19)$$

Comparing (6.19) with (6.7), the probabilities of H_0 and H_1 for a given y_1 and y_2 are not required in (6.19) since if the PU is absent, the value of y_2 will just be the noise power instead of the interference-plus-noise power. The PDF of y_2 , is denoted as $f_{y_2}(y_2) = Pr(H_0)f_{y_2|H_0}(y_2) + Pr(H_1)f_{y_2|H_1}(y_2)$.

The problem formulation to maximize the average data rate of the SU based on y_1 and y_2 with an average transmit power limit $\bar{P}_t$ while limiting the percentage of time that the PU is interfered by the SU to $\hat{I}$ is given by

$$\begin{aligned} \max_{P_t(y_1, y_2)} \quad & \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_c(P_t(y_1, y_2)) f_{y_1, y_2}(y_1, y_2) dy_1 dy_2 \\ \text{s.t.} \quad & \left(1 - \frac{\tau + r}{T}\right) \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P_t(y_1, y_2) f_{y_1, y_2}(y_1, y_2) dy_1 dy_2 \leq \bar{P}_t, \\ & I \leq \hat{I}, \quad P_t(y_1, y_2) \geq 0, \forall y_1, y_2 \end{aligned} \quad (6.20)$$

where $f_{y_1, y_2}(y_1, y_2)$ is the joint PDF of y_1 and y_2 . Assuming y_1 and y_2 are independent for a given hypothesis, the joint PDF of y_1 and y_2 is given by

$$\begin{aligned} f_{y_1, y_2}(y_1, y_2) &= f_{y_1, y_2, H_0}(y_1, y_2) + f_{y_1, y_2, H_1}(y_1, y_2) \\ &= \frac{Pr(H_0)N}{2\pi\sigma_u^4} \exp\left(-\frac{N((y_1 - \sigma_u^2)^2 + (y_2 - \sigma_u^2)^2)}{2\sigma_u^4}\right) \\ &\quad + \frac{Pr(H_1)N}{2\pi\sigma_u^4 \sqrt{(2\gamma_1 + 1)(2\gamma_2 + 1)}} \\ &\quad \times \exp\left(-\frac{N(y_1 - (\gamma_1 + 1)\sigma_u^2)^2}{(4\gamma_1 + 2)\sigma_u^4} - \frac{N(y_2 - (\gamma_2 + 1)\sigma_u^2)^2}{(4\gamma_2 + 2)\sigma_u^4}\right). \end{aligned}$$

The SU-Tx uses the fusion rule, $\sum_{i=1}^2 w_i y_i \underset{H_0}{\gtrless}^{H_1} \varepsilon_c$. In [155], when Neyman-Pearson criterion is used, i.e., maximizing the detection probability for a given false alarm probability, the optimal weightings to combine y_i is given by $w_i = \frac{\gamma_i}{\gamma_i + 1}$.

Note that if the SU miss-detects the PU and transmits during a frame, the interference time experienced by the PU is $(T - \tau - r)$ since a dedicated control channel is used and therefore, the reporting transmission from the SU-Rx to the SU-Tx does not interfere with the PU. Similar to the non-cooperative sensing scenario, setting the threshold such that $I = \hat{I}$ will optimize (6.20). From (6.5), to achieve $I = \hat{I}$, the threshold is given by

$$\begin{aligned} \varepsilon_c = & \sigma_u^2 \left(\sqrt{\frac{w_1^2(2\gamma_1 + 1) + w_2^2(2\gamma_2 + 1)}{N}} \mathcal{Q}^{-1} \left(1 - \left(\frac{T}{T - \tau - r} \hat{I} \right) \right) \right) \\ & + \sigma_u^2 (w_1(\gamma_1 + 1) + w_2(\gamma_2 + 1)). \end{aligned} \quad (6.21)$$

Hence, the problem (6.20) becomes

$$\begin{aligned} \max_{P_t(y_1, y_2)} & \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\varepsilon_c - w_2 y_2}{w_1}} R_c(P_t(y_1, y_2)) f_{y_1, y_2}(y_1, y_2) dy_1 dy_2 \\ s.t. & \left(1 - \frac{\tau + r}{T} \right) \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\varepsilon_c - w_2 y_2}{w_1}} P_t(y_1, y_2) f_{y_1, y_2}(y_1, y_2) dy_1 dy_2 \leq \bar{P}_t, \\ & P_t(y_1, y_2) \geq 0, \forall y_c \leq \varepsilon_c. \end{aligned} \quad (6.22)$$

The solution of the optimal power allocation can be found by taking the derivative of the partial Lagrangian of the convex problem (6.22) with respect to $P_t(y_1, y_2)$ and equate it to zero. The solution of the optimal power allocation is then derived as

$$P_t(y_1, y_2) = \left[\frac{1}{\lambda \ln(2)} - \frac{y_2}{g_s} \right]^+, \quad \forall y_2 \leq \min \left\{ \frac{g_s}{\lambda \ln(2)}, \frac{\varepsilon_c - w_1 y_1}{w_2} \right\} \quad (6.23)$$

where λ is chosen such that (6.23) satisfies the average transmit power constraint.

It is interesting to note that the water level of the power allocation solution (6.23) is a function of only y_2 and does not include y_1 . The SSI y_1 only helps in the sensing decision which is in the term $y_2 \leq \frac{\varepsilon_c - w_1 y_1}{w_2}$ found in (6.23). However, when referring back to the optimal power allocation solution for the non-cooperative scenario (6.13), y_1 is a variable in both the water level and in the sensing decision.

The reason is that after a channel is sensed to be free and the SU is able to utilize the channel, the optimal power allocation should be similar to a conventional water filling solution which depends on the channel condition [186]. However, different from the conventional wireless networks, the channel condition of a CRN has an additional element of the presence or absence of the PU. There is an uncertainty of whether the channel contains interference from the PU, unless the sensing decision is perfect. When only the SU-Tx is used for sensing, the probability of whether the SU-Rx's interference contains just the noise or includes the PU's signal is estimated using y_1 and hence the optimal power allocation in (6.13) is a function of y_1 . However, when the SU-Rx is involved in the sensing and sends its SSI to the SU-Tx, the SU-Tx then does not need to estimate the probability that the SU-Rx's interference contains just the noise or includes the PU's signal since it has that knowledge from the value of y_2 . Hence, the water level in (6.23) is no longer a function of y_1 but just a function of y_2 . From here, we conclude that in cooperative spectrum sensing, SSI y_1 only helps to improve the sensing capability but takes no part in the power level of each data frame. We assumed that g_s is constant in the derivation of the results in this section but the results can be generalized when we consider g_s experiences block fading.

6.3.2 Minimizing the outage probability

In this subsection, we aim to find the optimal power allocation strategy to minimize the outage probability of the SU for a targeted data rate $\bar{R}$ when the SU-Tx and

the SU-Rx cooperatively sense the channel. The outage probability of the SU with a targeted rate of $\bar{R}$ for a given y_1 and y_2 after taking account of the time spent on sensing and reporting is given by

$$P_{out}(P_t(y_1, y_2)) = Pr \left[\left(1 - \frac{\tau + r}{T} \right) \log_2 \left(1 + \frac{P_t(y_1, y_2)g_s}{y_2} \right) < \bar{R} \right]. \quad (6.24)$$

The problem formulation after limiting I to $\hat{I}$ using the threshold (6.21) is given by

$$\begin{aligned} \min_{P_t(y_1, y_2)} \quad & \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\varepsilon_c - w_2 y_2}{w_1}} P_{out}(P_t(y_1, y_2)) f_{y_1, y_2}(y_1, y_2) dy_1 dy_2 \\ & + \int_{-\infty}^{\infty} \int_{\frac{\varepsilon_c - w_2 y_2}{w_1}}^{\infty} f_{y_1, y_2}(y_1, y_2) dy_1 dy_2 \\ \text{s.t.} \quad & \int_{-\infty}^{\infty} \int_{-\infty}^{\frac{\varepsilon_c - w_2 y_2}{w_1}} P_t(y_1, y_2) f_{y_1, y_2}(y_1, y_2) dy_1 dy_2 \leq \frac{T \bar{P}_t}{T - \tau - r}, \\ & P_t(y_1, y_2) \geq 0, \forall y_c \leq \varepsilon_c. \end{aligned} \quad (6.25)$$

The lower bound of the outage probability, $\int_{-\infty}^{\infty} \int_{\frac{\varepsilon_c - w_2 y_2}{w_1}}^{\infty} f_{y_1, y_2}(y_1, y_2) dy_1 dy_2$, is given by

$$\begin{aligned} & Pr(H_0) \mathcal{Q} \left(\frac{\sqrt{N}(\varepsilon_c - \sigma_u^2(w_1 + w_2))}{\sqrt{w_1^2 + w_2^2 \sigma_u^2}} \right) \\ & + Pr(H_1) \mathcal{Q} \left(\frac{\sqrt{N}(\varepsilon_c - \sigma_u^2(w_1(\gamma_1 + 1) + w_2(\gamma_2 + 1)))}{\sqrt{w_1^2(2\gamma_1 + 1) + w_2^2(2\gamma_2 + 1) \sigma_u^2}} \right). \end{aligned}$$

The optimal solution of (6.25) is given by

$$P_t(y_1, y_2) = \begin{cases} \left(2^{\frac{T \bar{R}}{T - \tau - r}} - 1 \right) \frac{y_2}{g_s}, & y_2 \leq \min \left\{ \frac{g_s}{\lambda(2^{(T \bar{R})/(T - \tau - r)} - 1)}, \frac{\varepsilon_c - w_1 y_1}{w_2} \right\} \\ 0, & \text{otherwise} \end{cases} \quad (6.26)$$

where the Lagrange multiplier is chosen such that (6.26) satisfies the average transmit power limit. The procedure to solve (6.25) is similar to (6.17) found in Ap-

pendix 6.7.3 and therefore is not shown. The solution (6.26) has only one layer of truncated channel inversion compared to (6.18) which has two layers since the SU-Tx now knows the exact interference power experiencing in each data frame at the SU-Rx. Similarly, it can be observed that y_1 does not involve in the power level of each data frame but only involve in the sensing decision under the cooperative sensing scenario.

6.4 Optimization of the Sensing Time

Other than optimizing the power allocation strategies for each data frame to maximize the average data rate or minimize the outage probability of the SU for a targeted data rate $\bar{R}$, another parameter that can be designed to further improve the performance is the sensing time τ . For a given frame time T , the length of τ will affect the CRN in several ways. First, it affects the interference time that the PU will experience whenever the SU miss-detects the PU. Longer sensing time decreases the interference time in each miss-detection frame which benefits the PU. Secondly, longer sensing time also increases the sensing performance, which could either increase the probability of detection and hence, reducing the average interference time experienced by the PU or reduce the probability of false alarm which allows the SU to utilize the spectrum opportunity more efficiently. This in turn can increase the average data rate or decrease the lower bound of the outage probability of the SU since the SU is able to transmit more often.

However, longer sensing time decreases the amount of time for data transmission for a given frame time. If the transmission power and the false alarm probability remain the same, this will decrease the average data rate or increase the outage probability of the SU since the shorter data transmission period may no longer achieve the targeted rate $\bar{R}$. Lastly, if the transmission power remains the same, each frame will consume less power when the transmission period is shorter.

Hence, longer sensing time will lower the long-term power consumption if the false alarm probability and the transmission power remain the same.

Therefore, it is important to design the sensing time together with the power allocation strategies for the data frames to obtain further gain in performances. In Section 6.2 and 6.3, the power allocation strategies are designed based on a given sensing time. However, if the sensing time is included as an optimization variable in (6.10), (6.17), (6.22) and (6.25), the problems are no longer convex problems.

Since the power allocation problems are convex for a fixed sensing time, we will evaluate the performances from $\tau = 0$ to T for Section 6.2 and $\tau = 0$ to $(T - r)$ for Section 6.3 to determine the optimal sensing time. The sensing time and the power allocation strategies are designed to meet the average transmit power limit and hence, it is an one-off design. The simulation results will be shown in the next section.

6.5 Simulation Results and Discussion

In this section, we present the numerical results of the proposed power allocation strategies based on the SSIs under cooperative and non-cooperative spectrum sensing scenarios. In the simulations, we assume that the duration of each frame is $T = 2\text{ms}$. The PU is only allowed to be interfered at most 10% of the time whenever the PU is active. The probability that the PU is active, $Pr(H_1)$ is assumed to be 0.5. We assume that the sensing time has been pre-designed and is fixed at 5% of the total frame time unless otherwise stated. The sampling frequency is assumed to be 10MHz. Under cooperative spectrum sensing, we assume the time needed for the SU-Rx to feed back y_2 to the SU-Tx takes 1% of the total frame time.

The maximum data rates of the SU achieved by the proposed power control (6.13) under non-cooperative spectrum sensing are shown in Fig. 6.3. When the

Figure 6.3: Maximum data rate of the SU by using the proposed power allocation strategy under non-cooperative spectrum sensing.

PU's SNR at the SU-Tx γ_1 increases, the maximum data rate improves because the sensing performance is improved. When the PU's SNR at the SU-Rx γ_2 increases, the maximum data rate deteriorates because of a higher interference power from the PU to the SU-Rx. An interesting observation is that whenever the PU's SNR at the SU-Tx increases by 5dB, the maximum data rate improves greatly, however, if the PU's SNR at the SU-Rx decreases by 5dB, the maximum data rate only improves slightly. It seems that the data rate of the SU is more sensitive to the sensing performance than to the interference caused by the PU.

The minimum outage probabilities of the SU achieved by the proposed power control (6.18) under non-cooperative spectrum sensing are shown in Fig. 6.4. The target rate for each frame is set at $\bar{R} = 5$ bits/sec/Hz. From Fig. 6.4, we can observe that there is a slight discontinuity in every outage probability curve. The slight discontinuity is caused by the change between the power layers in the two-layered truncated channel inversion power allocation strategy. It can be observed that when γ_2 is decreased, there is no improvement in the outage probability at

Figure 6.4: Minimum outage probabilities of the SU by using the proposed power allocation strategy under non-cooperative spectrum sensing.

the low $\bar{P}_t$ region. This is because with a low $\bar{P}_t$, the power allocation strategy allocates power such that frames are able to maintain the targeted rate $\bar{R}$ only when the PU is indeed absent. If there is a miss-detection, regardless of the value of γ_2 , the frame will fail to maintain $\bar{R}$ because of the interference from the PU. From Fig. 6.4, γ_2 has also no effect on the outage floor since it does not affect the sensing performance of the SU. One last observation from Fig. 6.4 is that when γ_1 is increased, it improves the outage probability and also reduces the outage floor. This is because the sensing performance has improved with a larger γ_1 .

The maximum data rates of the SU achieved by the proposed power control (6.23) under cooperative spectrum sensing are shown in Fig. 6.5. The weightings, $w_i = \frac{\gamma_i}{\gamma_i + 1}$ proposed in [155], are used in the simulations. The non-cooperative spectrum sensing results are also shown for comparison. From Fig. 6.5, it can be observed that cooperative spectrum sensing always achieve a higher data rate than non-cooperative spectrum sensing. The improvement is caused by two factors. Firstly, the sensing performance is improved with cooperative spectrum sensing

Figure 6.5: Maximum data rate of the SU achieved by using the proposed power allocation strategy under both cooperative and non-cooperative spectrum sensing.

and secondly, when the SU-Rx sends its sensing information to the SU-Tx, the SU-Tx can make use of this information to gauge the noise or the interference-plus-noise level at the SU-Rx and allocate transmit power more efficiently. It can be observed that when γ_2 increases, the maximum data rate improves, unlike in the non-cooperative scenario where the maximum data rate actually deteriorates. This is because, although γ_2 increases and causes more interference to the SU-Rx, it helps to increase the sensing performance of the SU since the SU-Rx now participates in spectrum sensing. From Fig. 6.5, it seems that the gain from the improved sensing performance is able to outweigh the loss from an increased interference power causing the maximum data rate to increase. The performance gap between the cooperative and non-cooperative spectrum sensing is also greater when γ_2 increases because of the better sensing performance and also the impact of the more efficient power control of cooperative spectrum sensing is larger when γ_2 is higher.

The minimum outage probabilities of the SU achieved by the proposed power

Figure 6.6: Minimum outage probabilities of the SU achieved by using the proposed power allocation strategy under both cooperative and non-cooperative spectrum sensing.

control (6.26) under cooperative spectrum sensing are shown in Fig. 6.6. The non-cooperative spectrum sensing results are also shown for comparison. From Fig. 6.6, it can be observed that cooperative spectrum sensing always achieve a lower outage probability than non-cooperative spectrum sensing. The improvement is caused by the two same factors mentioned in the maximizing data rate case. It can also be observed that at low $\bar{P}_t$, the outage probabilities of the cooperative spectrum sensing are the same even when both γ_1 and γ_2 increased by 5dB. The improved sensing performance has no effect on the outage probabilities unlike the case in non-cooperative spectrum sensing. This is because in cooperative spectrum sensing, the SU-Tx will adjust its power such that every frame that it transmits, regardless whether its sensing decision is correct, will always meet the targeted data rate $\bar{R}$ since the SU-Tx knows y_2 . Hence, the sensing performance does not affect the outage probability in the cooperative spectrum sensing scenario. However, better sensing performance is able to lower the outage floor since

Figure 6.7: Maximum data rate of the SU achieved by using the proposed power allocation strategy across different sensing time under non-cooperative spectrum sensing.

the outage floor is related to the false-alarm probability. The non-cooperative spectrum sensing curves experience a sudden drop to their outage floor at 7dB for $\gamma_1 = -20\text{dB}$ and 9dB for $\gamma_1 = -15\text{dB}$. As mentioned before in this section, the optimal power control for non-cooperative spectrum sensing is a two-layered truncated channel inversion power allocation and therefore, the simulation curves are discontinuous. Comparing Figs. 6.4 and 6.6, the discontinuities occur at the same average transmit power limit $\bar{P}_t$, however, because the γ_2 is at extremely low level in Fig. 6.6, meaning that the SU-Rx experiences very small interference from the PU if it exists, then with a small increase of $\bar{P}_t$, the SU-Tx is able to always transmit at the targeted rate even when the PU exist. Therefore, when the power level jumps to the higher layer of the power strategy, the outage probability drops to the outage floor level with a small increase of $\bar{P}_t$.

The maximum data rates of the SU achieved by the proposed power controls (6.13) and (6.23) under different sensing time are shown in Figs. 6.7 and 6.8,

Figure 6.8: Maximum data rate of the SU achieved by using the proposed power allocation strategy across different sensing time under cooperative spectrum sensing.

respectively. From both figures, it is obvious that different channel conditions will require different amount of sensing time to obtain the optimal data rate. Previously, we have assumed that the sensing time is designed and fixed at 5% of the total time frame, however, by optimizing the sensing time, the improvement can be large. For example, in non-cooperative spectrum sensing, when $\gamma_1 = -15\text{dB}$ and $\gamma_2 = -5\text{dB}$, the gain in data rate is 41% by changing the sensing time from 0.1ms to 0.5ms. Similarly, for cooperative spectrum sensing, when $\gamma_1 = \gamma_2 = -15\text{dB}$, the gain in data rate is 25% by changing the sensing time from 0.1ms to 0.35ms.

The minimum outage probabilities of the SU achieved by the proposed power controls (6.18) and (6.26) under different sensing time are shown in Figs. 6.9 and 6.10, respectively. Similarly, the minimum outage probabilities obtained by the proposed power allocation strategies can be further improved by optimizing the sensing time. From the figures, the sensing time at $\tau = 0.1\text{ms}$ which we set

Figure 6.9: Minimum outage probability of the SU achieved by using the proposed power allocation strategy across different sensing time under non-cooperative spectrum sensing.

previously, is already close to optimal in most of the simulated channel conditions, therefore, the performance gain by optimizing the sensing time for the outage probability is not as great as in the average data rate case.

6.6 Summary

In this chapter, we have designed per-frame power allocation strategies for both cooperative and non-cooperative spectrum sensing scenarios in CRNs. In the non-cooperative spectrum sensing scenario, the optimal transmit power to maximize the average data rate is proportional to the probability of the PU being absent for a given SU-Tx's SSI. The optimal transmit power scheme to minimize the outage probability turns out to be a two-layer truncated channel inversion power control scheme where the regions of the power layers also depend on the probability of the PU being absent for a given SU-Tx's SSI. In the cooperative spectrum

Figure 6.10: Minimum outage probability of the SU achieved by using the proposed power allocation strategy across different sensing time under cooperative spectrum sensing.

sensing scenario, the transmit power scheme to maximize the average data rate is no longer dependent on the SU-Tx's SSI but depends only on the SU-Rx's SSI. The SU-Tx's SSI is only used to improve the sensing performance. The transmit power scheme to minimize the outage probability becomes a single-layer truncated channel inversion power control where the power level is dependent only on the SU-Rx's SSI too. Lastly, we have shown that by optimizing the sensing time together with the power allocation strategies, the performances of the average data rate and the outage probability can be further improved.

6.7 Appendices

6.7.1 Proof of $-b - \sqrt{b^2 - 4c(y_1)} < 0$, for all $y_1 \leq \varepsilon_1$

Expand $\sqrt{b^2 - 4c(y_1)}$, we obtain

$$\sqrt{\left(\frac{1}{\lambda \ln(2)} - \frac{\sigma_{s,2}^2}{g_s}\right)^2 + \frac{4\sigma_{s,2}^2}{g_s \lambda \ln(2)} Pr(H_0|y_1)} \geq 0. \quad (6.27)$$

Since $0 \leq Pr(H_0|y_1) \leq 1$, the maximum value of the term $-b - \sqrt{b^2 - 4c(y_1)}$ is when $Pr(H_0|y_1) = 0$ which will obtain $-b - \sqrt{b^2 - 4c(y_1)} = -\frac{2\sigma_u^2}{g_s} < 0$. Hence, $\frac{-b - \sqrt{b^2 - 4c(y_1)}}{2} < 0$ for all $y_1 \leq \varepsilon_1$.

6.7.2 Proof of $P_t(y_1) > 0$ only when $Pr(H_0|y_1) > \frac{\lambda \ln(2)\sigma_u^2}{g_s} \left(1 + \frac{\sigma_u^2}{\sigma_{s,2}^2}\right) - \frac{\sigma_u^2}{\sigma_{s,2}^2}$ at small $\bar{P}_t$

When $\lambda < \frac{g_s}{\ln(2)(\sigma_{s,2}^2 + 2\sigma_u^2)}$, b is always less than zero. Hence, from (6.27), $\sqrt{b^2 - 4c(y_1)} - b > 0$ for all y_1 and hence, $P_t(y_1) > 0$ for all y_1 too. When $\lambda > \frac{g_s}{\ln(2)(\sigma_{s,2}^2 + 2\sigma_u^2)}$, leads to $b > 0$. For $P_t(y_1)$ to be greater than zero, then $\sqrt{b^2 - 4c(y_1)}$ must be greater than b which is when $Pr(H_0|y_1) > \frac{\lambda \ln(2)\sigma_u^2}{g_s} \left(1 + \frac{\sigma_u^2}{\sigma_{s,2}^2}\right) - \frac{\sigma_u^2}{\sigma_{s,2}^2}$.

6.7.3 Proof of the optimal solution of (6.17)

The partial Lagrangian of (6.17) with the constant term removed is expressed as

$$L(P_t(y_1), \lambda) = \int_{-\infty}^{\varepsilon_1} P_{out}(P_t(y_1)) f_{y_1}(y_1) dy_1 + \lambda \left[\int_{-\infty}^{\varepsilon_1} P_t(y_1) f_{y_1}(y_1) dy_1 - \frac{T\bar{P}_t}{T - \tau} \right]. \quad (6.28)$$

By dual decomposition [175], we have a series of similar sub-dual-functions, one for each SSI y_1 . For a particular SSI y_1 , the problem can be shown to be equivalent

to

$$\min_{P_t: P_t(y_1) \geq 0, \forall y_1 \leq \varepsilon_1} P_{out}(P_t(y_1))f_{y_1}(y_1) + \lambda P_t(y_1)f_{y_1}(y_1). \quad (6.29)$$

From (6.16), we derive that

$$P_{out}(P_t(y_1)) = \begin{cases} 1, & \text{if } \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_u^2} \right) < \frac{T\bar{R}}{T-\tau}, \\ Pr(H_1|y_1), & \text{if } \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_{s,2}^2 + \sigma_u^2} \right) < \frac{T\bar{R}}{T-\tau} \leq \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_u^2} \right), \\ 0, & \text{if } \log_2 \left(1 + \frac{P_t(y_1)g_s}{\sigma_{s,2}^2 + \sigma_u^2} \right) \geq \frac{T\bar{R}}{T-\tau}. \end{cases}$$

When $P_{out}(P_t(y_1)) = 1$, the objective in (6.29) is minimized when $P_t(y_1) = 0$ and the objective becomes $f_{y_1}(y_1)$. When $P_{out}(P_t(y_1)) = Pr(H_1|y_1)$, the objective in (6.29) is minimized when $P_t(y_1) = \frac{\sigma_u^2(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s}$ and the objective becomes $f_{y_1}(y_1) \left(Pr(H_1|y_1) + \frac{\lambda \sigma_u^2(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s} \right)$. When $P_{out}(P_t(y_1)) = 0$, the objective in (6.29) is minimized when $P_t(y_1) = \frac{(\sigma_{s,2}^2 + \sigma_u^2)(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s}$ and the objective becomes $f_{y_1}(y_1) \frac{\lambda(\sigma_{s,2}^2 + \sigma_u^2)(2^{(T\bar{R})/(T-\tau)} - 1)}{g_s}$. Therefore, the optimal solution of $P_t(y_1)$ is given as (6.18).

Chapter 7

Conclusions and Future Work

7.1 Conclusions

Cooperation of SUs for spectrum sensing is able to mitigate the impact of multipath fading, shadowing, and noise uncertainty on the detection performance. In addition, cooperative spectrum sensing is able to improve the detection performance and reduce the detection time, therefore, it will have an important role in the implementation of CRNs. The research studies reported in this dissertation have focused on the optimization of cooperative spectrum sensing parameters under different CRN scenarios. Furthermore, this dissertation studies the power allocation strategies under both cooperative and non-cooperative spectrum sensing settings.

The throughput of a CRN depends on the probabilities of detection and false alarm of spectrum sensing. These probabilities depend on the sensing time, detectors' thresholds, and fusion rule threshold. Therefore, in Chapter 3, these parameters are optimized to maximize the throughput of a small area CRN subject to a targeted probability of detection. The optimal energy detector's threshold is derived for a given sensing time and fusion rule threshold. The SU's throughput as a function of the sensing time when k -out-of- N fusion rule is used is proven

to be a unimodal function and the optimal sensing time can be easily solved by efficient optimization algorithms. An iterative algorithm is proposed to obtain the optimal sensing time and the fusion rule threshold. The results have shown that when the detection SNRs of the SUs are lower, more sensing time is required to improve the throughput although it will reduce the transmission time. However, at extremely low detection SNRs, it is better to reduce the sensing time to improve the throughput since the false alarm probability does not improve significantly with more sensing time. We have also shown that significant improvement in the throughput of the CRN can be achieved when all the three parameters are jointly optimized.

In a large area CRN, the SUs can have different average detection SNRs. Therefore, when combining the SUs' decisions at the FC, weightings should be given to the SUs' decisions based on their detection SNRs. In a large area CRN, other than optimizing the sensing time, energy detectors' thresholds, and fusion rule threshold, the weightings of the SUs' decisions should be optimized as well in order to maximize the throughput of the CRN. It is shown in Chapter 4 that the optimal weightings to maximize the throughput are derived from the likelihood-ratio test. We further consider three different large area CR system setup: 1) small number of SUs, 2) large number of SUs, and 3) no feedback of sensing parameters from the FC to the SUs. When the number of SUs is small, the FC is able to optimize the individual energy detector's threshold of the SUs. An algorithm is proposed to compute the individual energy detectors' thresholds and the fusion rule threshold jointly. When the number of SUs is large, computing the optimal thresholds of individual SU is time consuming, therefore, one optimal threshold to be applied to all SUs is considered. The upper and lower bounds of the threshold are derived. The results showed that the performance of constraining all the SUs' thresholds to be the same is only slightly lower than that obtained by optimizing individual SUs'

thresholds. When the FC lacks the computation power to compute the thresholds for the SUs or there is no available feedback channel for it to send the optimal thresholds to the SUs, the SUs have to compute their own thresholds and the FC will only optimize the fusion rule threshold. Two heuristic methods are proposed for the SUs to optimize their own thresholds: 1) minimizing their own decision errors and 2) minimizing the false alarm probability while fixing their detection probabilities to the targeted total detection probability. The results show that when the differences in detection SNRs among the SUs are small, it is better for the SUs to minimize their own decision errors, otherwise, it is better to minimize the false alarm probability for a given targeted detection probability.

Energy efficiency of the CRN is important when the SUs are battery-powered devices. The probabilities of detection and false alarm will affect the average transmission energy as the transmission opportunities of the CRN depend on the sensing performance. The number of cooperating SUs and the length of the sensing time will also affect the sensing performance and the amount of energy used for sensing. An iterative algorithm is proposed to find the optimal k -out-of- N fusion rule that maximizes the energy efficiency of the CRN. The equation at which the thresholds of the SUs are optimal is derived. The results show that optimizing the fusion rule threshold improves the energy efficiency significantly; however, when the fusion rule threshold is optimized, optimizing the SUs' thresholds does not improve the energy efficiency much, especially in the high detection SNR region. The impacts of the number of cooperating SUs and sensing time on the energy efficiency of the CRN are shown. It is possible to trade sensing energy and transmission time for a better sensing performance by increasing the number of cooperating SUs or the length of sensing time.

The spectrum sensing information can be used to measure the miss-detection probability at a particular frame and also, the interference power experienced at

the SU-Rx due to the PU. Therefore, power allocation strategies to achieve the maximum throughput and the minimum outage probability of the CRN based on SSI are investigated under both cooperative and non-cooperative spectrum sensing settings. In the non-cooperative spectrum sensing scenario, the optimal power allocation strategy to minimize the outage probability is derived to be a two-layered truncated channel inversion power allocation while in the cooperative spectrum sensing scenario, the optimal power allocation strategy is derived to be a single-layered truncated channel inversion power allocation. Both the power allocation strategies to maximize the throughput under cooperative and non-cooperative spectrum sensing are derived to have water-filling power allocation structures. It is shown that the power level of cooperative scenario depends on the SU-Rx's SSI while the power level of non-cooperative scenario depends on the SU-Tx's SSI. The results show that cooperative spectrum sensing always achieves a higher throughput and a lower outage probability than those obtained under the non-cooperative spectrum sensing due to an improved sensing performance and more efficient power allocation strategies.

7.2 Future Work

In this dissertation, we have addressed several issues in the optimization of cooperative spectrum sensing in CRNs. However, there are still many problems before cooperative spectrum sensing of CRNs can be implemented in practical systems. In this section, some of the issues in cooperative spectrum sensing that are of practical importance are discussed for possible future research works.

When multiple SUs are conducting spectrum sensing, it is important that they synchronize their sensing and transmission timing to prevent the transmissions of some SUs from interfering with the sensing operations of the other SUs. However, synchronizing the sensing and transmission time for the SUs is not an easy task

even for centralized cooperative spectrum sensing. For example, one particular SU transmits its signal to the secondary base station during a transmission slot and after which, it switches to sense for the PU's presence. Other SUs in the CRN may also be requested to perform spectrum sensing at the same time. However, the SUs that are located farther away from this particular SU, may still receive the transmitted signal from the particular SU due to propagation delay when they start to perform spectrum sensing. Consequently, the sensing performance of these SUs will be degraded. Hence, accurate synchronization of sensing and transmission time, which includes the signal propagation time among the SUs is important. The synchronization problem may also have an adverse impact on other SUs operating in neighboring channels as the transmission signals can leak to adjacent channels, deteriorating their sensing performance. This synchronization issue will be even more challenging in a distributed cooperative spectrum sensing system.

In a CRN, it is possible that the SUs are different devices with different computational powers and energy lifetime. SUs that have more computational powers or energy can choose to use more sophisticated sensing algorithms to increase their own performance. As mentioned in Chapter 2, there are many available sensing algorithms with their respective advantages and disadvantages. For instance, some algorithms are able to differentiate signal types, or to combat noise uncertainty while some algorithms can have better sensing performance. Therefore, it is possible that the SUs in a CRN use different sensing algorithms. Hence, one possible future research is to investigate fusion techniques that are able to combine sensing information from different types of sensing algorithms to enhance the sensing performance of the CRN while preserving the advantages of the individual sensing algorithm.

One of the main distinctions between CR systems and conventional communication systems is that CR systems perform spectrum sensing which requires extra

resources such as time and energy. However, because of spectrum sensing, CRNs gain extra knowledge about its surrounding environment, i.e., the SSI which conventional communication systems do not have. In Chapter 6, the SSI is proposed for power allocation strategies, however, there may be more usage of the SSI. It would be interesting to investigate for future research if the SSI can be further exploited to enhance the performance of CRNs. For example, the SSI may be used for other resource allocation strategies such as channel selection for the next spectrum sensing period, selection of SUs for transmission, rate allocation, and sensing time for the subsequent sensing slot. Last but not least, can the SSI also be used to improve security issues in CRNs?

Chapter 8

Author's Publications

Journal papers

1. E. C. Y. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, "Optimization of co-operative sensing in cognitive radio networks: A sensing-throughput tradeoff view," *IEEE Trans. Veh. Technol.*, vol. 58, no. 9, pp. 5294-5299, Nov. 2009.
2. E. C. Y. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, "Cooperative spectrum sensing in cognitive radio networks with weighted decision fusion scheme," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3838-3847, Dec. 2010.
3. E. C. Y. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, "Power control in cognitive radio under cooperative and non-cooperative spectrum sensing," *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4238-4248, Dec. 2011.

Conference papers

1. E. C. Y. Peh, Y.-C. Liang, and Y. L. Guan, "Power control for physical-layer network coding in fading environments," in *Proc. IEEE 19th Int. Symp. on Pers., Indoor and Mobile Radio Commun. (PIMRC)*, Sep. 2008, pp. 1-5.

2. E. C. Y. Peh, Y.-C. Liang, and Y. L. Guan, "Optimization of cooperative sensing in cognitive radio networks: A sensing-throughput tradeoff view," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2009, pp. 1-5.
3. E. C. Y. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, "Cooperative spectrum sensing in cognitive radio networks with weighted decision fusion scheme," in *Proc. IEEE 71st Veh. Technol. Conf. (VTC 2010-Spring)*, May 2010, pp. 1-5.
4. E. C. Y. Peh, Y.-C. Liang, Y. L. Guan, and Y. Zeng, "Power control in opportunistic spectrum access cognitive radio with sensing information at transmitter," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Jun. 2011, pp. 1-5.
5. E. C. Y. Peh, Y.-C. Liang, Y. L. Guan, and Y. Pei, "Energy-efficient cooperative spectrum sensing in cognitive radio networks," accepted in *Proc. IEEE Global Commun. Conf. (GLOBECOM)* 2011.

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