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DOUBLE PERIODIC COMPOSITE RIGHT/LEFT HANDED METAMATERIALS

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Dedicated to my parents

Ruishan JIN and Shuying DONG

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Summary

Electromagnetic metamaterials (MTMs) with simultaneous negative permittivity and permeability, or left-handed materials (LHMs) are a very interesting and eye opening research topic in recent times. The concept was first proposed theoretically by Veselago in 1968. The first experimental realization and verification of LHMs was successfully achieved by incorporating the well-known split-ring resonators (SRRs) and metallic thin-wires. However, this kind of MTMs are bulky and highly resonant and they are more suitable for specific narrow band applications.

Recently, an innovative concept of composite right/left handed (CRLH) transmission lines (TLs) is becoming an interesting and promising topic of research. These are well suited for the realization of MTMs offering advantages of non-resonant, more compact, less lossy and wider bandwidth. These CRLH TLs are used in practice to design compact devices, like small leaky-wave antennas (LWAs), compact filters, and many other applications. Most of the CRLH TLs known till now are single periodically loaded with uniform inductance and capacitance, and thus are termed as single periodic composite right/left handed (SP-CRLH) TLs in this thesis.

A new transmission line, which is double periodic in nature, is loaded with a pair of alternate capacitances and/or inductances has been proposed and termed as double periodic composite right/left handed (DP-CRLH) TL in this thesis. In comparison to the SP-CRLH TL, the number of degrees of freedom offered by the DP-CRLH TL is doubled. As a result, a new branch can be observed in the dispersion diagram.

Apart from the fact that DP-CRLH TL can radiate in the left-handed (LH) region and forward in the right-handed (RH) region (similar to SP-CRLH TLs), DP-CRLH TL promises a new forward leaky-wave (LW) radiation at low frequency below the LH passband with narrow bandwidth. This is an attractive and promising method to design compact LWAs. The new radiation frequency is significantly lower than the LH passband of the SP-CRLH TL with the corresponding similar period. Hence, it is possible to design LWAs which are more compact than the one based on SP-CRLH TLs.

The proposed DP-CRLH TLs support three band LW radiations: First one being the new-found LW radiation at very low frequency. Second and third ones are the LH and the conventional RH radiations at higher frequencies. By controlling the values of the loaded components and their period, it is possible to obtain a desirable compact LW radiation behavior. With a suitable selection of the substrate material and geometrical dimensions, it is possible to design super-compact LWAs working at a lower frequency, unbalanced triple-band or balanced dual-band LWAs with frequency-scanning property.

The DP-CRLH TL has been analyzed using TL theory. It aims at determining the propagation characteristics obtained from the dispersion behavior using the complex propagation constant (phase and attenuation constants). The new LW radiation is observed at very low frequency below the LH passband. It is noted that TLs double periodically loaded with either alternate capacitances or alternate inductances will contribute to the new LW behavior, although the other components are single periodically loaded. In order to analyze further the radiation characteristics of the proposed DP-CRLH TLs, especially for the microstrip line implementation of DP-CRLH TLs, singular perturbation procedure has been used. Dispersion characteristics are investigated, and the radiation efficiency and radiation angle for the optimum radiation efficiency are numerically estimated.

Both the propagation and radiation characteristics of the proposed DP-CRLH TLs have been studied using a number of parametric studies, which include variation of geometrical dimensions, the loaded capacitances and inductances, and the period of the unit cell. The expressions for the cut-off frequencies have been derived for the infinite periodic structures. Also, a qualitative estimation of the cut-off frequencies for the periodic structure has been provided (for three cells). Based on these parametric studies, and suitable modeling of the proposed DP-CRLH TLs, a list of design guidelines has been developed. A few design examples of DP-CRLH TLs are presented. The proposed structures have also been fabricated and measured results are found to be in agreement with the expected results.

The thesis concludes by pointing out a few limitations of the proposed DP-CRLH TLs. Multiple unit cells have to be cascaded to get better radiation efficiency, but this increases the geometrical dimensions of the antenna. Another limitation is that it is very difficult to design a wideband LWA at low frequency based on the proposed DP-CRLH TLs, because the boundary of the fast wave region is extremely close to the vertical axis of the dispersion diagram and the fast wave region at low frequency is too narrow. Since the bandwidth of the new LW radiation of the proposed DP-CRLH TLs is narrow, it is hard to design LWAs with frequency-scanning property. Although the bandwidth of the new LW radiation is narrow at very low frequency, the new method to design compact LWAs at a fixed low frequency based on DP-CRLH TLs is still attractive and promising. With further research on the proposed DP-CRLH TLs, it is expected that present limitations can be overcome, and this structure might find applications in more areas of microwave engineering.

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List of Acronyms and Symbols

ACRONYMS	FULL EXPRESSIONS
1-D	One-dimensional
2-D	Two-dimensional
CPW	Coplanar waveguide
CRLH	Composite right/left handed
CST	Computer simulation technology
DNG	Double negative
DP-CRLH	Double periodic composite right/left handed
DPS	Double positive
EBG	Electromagnetic bandgap
ENG	Epsilon-negative
HFSS	High frequency structure simulator
HMSIW	Half mode substrate integrated waveguide
LH	Left-handed
LHM	Left-handed material
LPF	Lowpass filter
LTCC	Low-temperature co-fired ceramic
LW	Leaky-wave
LWA	Leaky-wave antenna

MNG	μ -negative
MTM	Metamaterial
NRI	Negative refractive index
RH	Right-handed
RHM	Right-handed material
RPA	Relative power absorbed
SIW	Substrate integrated waveguide
SP-CRLH	Single periodic composite right/left handed
SRR	Split ring resonator
TE	Transverse electric
TEM	Transverse electric-magnetic
TL	Transmission line
TM	Transverse magnetic
WLAN	Wireless local area network

SYMBOLS

FULL EXPRESSIONS

a_g	Principal guided wave
a_i	Incident wave
b_i	Radiated wave
f_L	Cutoff frequency of the lower edge of the LH branch
f_{nR_H}	Cutoff frequency of the upper edge of the new RH branch
f_{nR_L}	Cutoff frequency of the lower edge of the new RH branch
f_{nRLW}	Cutoff frequency of the upper edge of the new LW branch
$f_{\Gamma 1}$	Cutoff frequency of the upper edge of the LH branch
$f_{\Gamma 2}$	Cutoff frequency of the lower edge of the RH branch

g	Geometric factor
k	Wave number in a medium
k_0	Free space wave number
s	Substrate thickness of the microstrip line
v_g	Group velocity
v_p	Phase velocity
$\vec{B}$	Magnetic flux density
C_{gg}	Extinction coefficient
C_L	Uniform series capacitance
C_{L1}, C_{L2}	Alternate series capacitances
C_{rg}	Coupling coefficient
C_{rr}	Reflection coefficient
C_R	Parasitic shunt capacitance
$\vec{D}$	Electric flux density
$\vec{E}$	Electric field intensity
$\vec{H}$	Magnetic field intensity
H_y	Magnetic field in y -direction
H_{ya}	Magnetic field in y -direction in the freespace
H_{ys}	Magnetic field in y -direction in the medium
I	Current
$\vec{J}$	Electric current density
L_g	The length of the investigated transmission line
L_L	Uniform shunt inductance
L_{L1}, L_{L2}	Alternate shunt inductances
L_R	Parasitic series inductance
P_x	Poynting vector in x -direction

Q_0	Radiation efficiency
S_{11}	Reflection coefficient
S_{21}	Transmission coefficient
V	Voltage
W	Conductor width of the microstrip line
Y_{C_R}	Per-unit admittance of the equivalent per-unit series inductance
Y_{L_L}	Admittance of the periodic loading uniform series inductance
Z_B	Bloch impedance
Z_{C_L}	Impedance of the periodic loading uniform series capacitance
Z_{L_R}	Per-unit impedance of the equivalent per-unit shunt capacitance
Z_0	Characteristic impedance
α	Attenuation constant
β	Phase constant
γ	Complex propagation constant
δ	Perturbation order or smallness parameter
ε	Permittivity of the medium
ε_0	Permittivity of free-space
ε_{eff}	Effective permittivity of the medium
ε_r	Relative permittivity of the medium
η_1	Amplitude of the fundamental (first harmonic) component
η_2	Amplitude of the second harmonic
θ_r	Radiation angle of main lobe measured in broadside direction
λ_g	Guided wavelength
Λ	Period of the unit cell
Λ_D	Period of the unit cell of DP-CRLH TLs
Λ_S	Period of the unit cell of SP-CRLH TLs

μ	Permeability of the medium
μ_0	Permeability of the free-space
μ_{eff}	Effective permeability of the mudium
μ_r	Relative permeability of the medium
ρ	Electric charge density
τ_g	Group delay of the signal
τ_ϕ	Phase delay of the signal
ϕ	Instaneous phase
ω	Angular frequency
ω_L	Angular cutoff frequency of the lower edge of the LH branch
ω_{nR_H}	Angular cutoff frequency of the upper edge of the new RH branch
ω_{nR_L}	Angular cutoff frequency of the lower edge of the new RH branch
$\omega_{nR_{LW}}$	Angular cutoff frequency of the upper edge of the new LW branch
$\omega_{\Gamma 1}$	Angular cutoff frequency of the upper edge of the LH branch
$\omega_{\Gamma 2}$	Angular cutoff frequency of the lower edge of the RH branch

Chapter 1

INTRODUCTION

Recently composite right/left handed (CRLH) transmission lines (TLs) realized by periodic loading of series capacitance and shunt inductance in a host medium are becoming an interesting and promising topic of research. Since the resonant frequency of CRLH TLs is independent of the physical length [1, 2], the TLs are used to design many compact devices for communication system applications, like small leaky-wave antennas (LWAs) and compact filters. Another advantage of CRLH TLs is the unusual propagation property of backfire-to-endfire (including broadside) radiation, and many frequency-scanned antenna applications have been designed to radiate backward in the left-handed (LH) region and forward in the right-handed (RH) region. Most of the well-known CRLH TLs are single periodically loaded with uniform inductance and capacitance until now, and may be named as single periodic composite right/left handed (SP-CRLH) TLs in this thesis.

A novel transmission line double periodically loaded with a pair of alternate capacitances and/or inductances is presented in this thesis. This new TL is named as double periodic composite right/left handed (DP-CRLH) TL to differentiate it from the SP-CRLH TL. It is evident that the number of degrees of freedom of the structure is doubled for the double periodic structures and therefore there is a new branch in

the dispersion diagram comparing with the one for the single periodic structures [3,4]. In addition to the CRLH property as exhibited by SP-CRLH TLs, the DP-CRLH TL is promising a new forward leaky-wave (LW) radiation at low frequency below the LH passband with narrow bandwidth. It is possible to design compact LWAs exciting a radiation at low frequency, which may also lead to multi-band radiations including the new-found RH LW radiation, the LH radiation and the conventional RH radiation. This novelty of the proposed DP-CRLH TLs raises intriguing questions on the excitation and physical meaning of spatial harmonics in such structures. The new method to design compact LWAs based on the proposed DP-CRLH TLs is attractive and promising since its resonant frequency is independent of the physical length and is even below the LH region.

1.1 Motivation

1.1.1 Basics of Electromagnetic Metamaterials

Electromagnetic metamaterials (MTMs) are defined as artificial effectively homogeneous electromagnetic structures with unusual properties not readily available in nature [2], and the term is most often applied to structures with simultaneously equivalent negative permittivity (ϵ) and permeability (μ), more commonly referred to as left-handed materials (LHMs) [1,2,5], which exhibit specific electromagnetic properties not commonly found in nature. Science magazine named LHMs as one of the top ten scientific breakthroughs of 2003 [6]. The theoretically anticipated [5] and experimentally demonstrated unique electromagnetic properties of LHM include the inverse Snell's Law (negative refraction) [7–9], the inverse Doppler Effect [10], the reverse Vavilov-Cerenkov effect [11] (radiation produced by a fast-moving particle as it travels through a medium), the reverse Goos-Hanchen Shift [12,13] and the reverse

Fermat Principle [14]. Based on these extraordinary electromagnetic properties that are rarely found in the natural materials, LHMs provides a novel method and concept to develop various kinds of microwave circuits and applications.

A periodic array of conducting elements can behave as an effective medium for electromagnetic scattering when the wavelength is much longer than both the element dimension and lattice spacing. What makes the resulting media special is that the equivalent permittivity and permeability can have values not observed in ordinary materials. In natural life, photonic crystals are one kind of the materials that possess negative permittivity [2, 15]. Although in the crystal structure, the wavelength is comparable with the unit cell, which is different from the so called MTM, the internal periodic structure of crystal presents a practical proposal for designing LHMs. The first LHM was realized as a composite of conducting wires and split ring resonators (SRRs) [14, 16]. The Rotman's rodged artificial dielectric was unearthed and recast working as a microwave plasma with $\varepsilon < 0$, and the SRR [17] exhibits $\mu < 0$ at microwave frequencies. However, these structures are resonant, and consequently exhibit high loss and narrow bandwidth [2, 18]. A structure made of resonating elements generally does not constitute a good transmission medium for a modulated signal because of the quality factor intrinsically associated with each resonator [19].

1.1.2 Unique Property of Composite Right/Left Handed Metamaterials

Due to the weaknesses of resonant-type LHMs, alternate architectures, named as CRLH TLs recognizing the analogy between LH waves and conventional backward waves, were introduced by three groups Eleftheriades et. al [1, 20], Oliner [21] and Caloz et al as shown in Fig. 1.1(b). [2, 22–24] in 2002. Because of unavoidable parasitic series inductance (L_R), shunt capacitance (C_R) effects and the spacing between the

Figure 1.1: Equivalent circuit of the structures. (a) Pure RH TL. (b) SP-CRLH TL. (c) Line implementation in the form of a periodic network.

conductors and the metallic enclosure, a purely LHM structure does not exist, even in a restricted frequency range [2]. The CRLH TL is necessary to be analyzed and designed in practical applications. Most of the CRLH TLs are periodically loaded with uniform shunt inductance (L_L) and series capacitance (C_L) until now, and may be named as SP-CRLH TLs in this thesis to differentiate it from the proposed DP-CRLH TLs.

The SP-CRLH TLs have many unique properties in microwave circuits:

- SP-CRLH TLs can support electromagnetic waves with group and phase velocities antiparallel in the LH region [24]. While energy travels away from the source, wavefronts travel backward toward the source. When a SP-CRLH TL is open to free space and supports a fast-wave mode, a LW radiation will be obtained and the SP-CRLH TL can be used as an LWA. Unique radiation effects can be obtained with SP-CRLH TLs due to the unusual propagation property of CRLH

LW backfire-to-endfire (including broadside) radiation with a frequency-scanned antenna application. They can be used to design broadband LWA which can radiate backward in the LH region and forward in the RH region.

- SP-CRLH TLs are operating in the fundamental mode (effectively homogeneous), where structural average cell size Λ is much smaller than the guided wavelength λ_g ($\Lambda < \lambda_g/4$) and the structures behave as an effective homogeneous medium for electromagnetic scattering [2]. Many guided-wave applications can be devised on the foundation of the SP-CRLH TL, where electromagnetic energy remains confined in the metal and dielectric media constituting the components. The advantages of the SP-CRLH TLs are

- Size reduction. Ref [2] points out that the SP-CRLH TL can be considered to be constituted by essentially nondispersive (independent on frequency) $L - C$ components over the frequency range of interest although a SP-CRLH TL is naturally dispersive. Compact devices for communication system applications, like small LWAs [2, 25–30] and compact filters [31–34] are realized based on SP-CRLH TLs since the resonant frequency is independent of the physical length [1, 2].
- Bandwidth enhancement [30]. In the balanced-resonance case of the SP-CRLH TLs, the real CRLH dispersion curve is, according to Ref [2], precisely the superposition (sum) of the LH hyperbolic dispersion function (dominant at lower frequencies) and of the RH linear dispersion function (dominant at higher frequencies). When the hyperbolic-linear dispersion characteristic of the SP-CRLH TL is judiciously combined with the linear dispersion characteristic of the conventional pure RH TL section, it is possible to maximize the bandwidth of the component. This is also referred as dispersion engineering.

- Dual-band operation. Because unbalanced SP-CRLH TLs exhibit a dispersion relation satisfying the double resonance condition where the two frequencies and the two propagation constants are arbitrary, this kind of SP-CRLH TLs with the dual-band property can be used to transform virtually any microwave TL-based component into a dual-band component [35].

1.1.3 Proposed Double Periodic Composite Right/Left Handed Transmission Lines

Most of the CRLH TLs are periodically loaded with uniform inductance and capacitance until now, a novel TL, double periodically loaded by a pair of alternate capacitances and/or inductances as shown in Fig. 1.2, is proposed in this thesis. The new TLs is named as DP-CRLH TLs to differentiate it from SP-CRLH TLs. The series capacitances as well as the shunt inductances are modulated within the double periodic unit cell to obtain DP-CRLH TLs.

In addition to the CRLH property as exhibited by SP-CRLH TLs, the behavior of DP-CRLH TLs, analyzed by perturbation method in [29, 36], TL theory in [37, 38] and verified by two kinds of designed LWAs [36–38], is promising a new forward LW radiation at extremely low frequency below the LH passband with narrow bandwidth. Although the bandwidth of the new leaky-wave radiation is narrow at very low frequency, the new method to design compact LWAs at a fixed low frequency based on DP-CRLH TLs is still attractive and promising since its resonant frequency is independent of the physical length and is even below the LH region.

Because the number of degrees of freedom of DP-CRLH TLs is doubled, a new branch is obtained in the dispersion curve compared with the one for the SP-CRLH TL. Resultantly, a multi-band radiation can be excited including a new LW radiation at very low frequency and the CRLH LW backfire-to-endfire radiation.

Figure 1.2: Equivalent circuit of the proposed DP-CRLH TLs. (a) The unit cell. (b) Line implementation in the form of a periodic network.

The advantages of the proposed DP-CRLH TLs may be:

1. Triple-band LWAs design [36].

When the dispersion characteristic of DP-CRLH TL is in the fast wave region ($\beta < k_0$), the structure is able to radiate, which can be used for potential LWAs design [1, 2, 28, 32], where the β and k_0 are the phase constant of the structure and the free space wave number, respectively. The dispersion behavior shows three bands are observed in the fast wave region which indicates that DP-CRLH TLs are potentially devised as triple-band LWAs.

2. Super compact LWAs design [37].

As discussed above, SP-CRLH TLs are good structures to design compact devices for communication system applications, because the SP-CRLH TL is constituted by essentially nondispersive (independent on frequency) $L - C$ compo-

Figure 1.3: Prototypes of the proposed DP-CRLH TLs. (a) Microstrip line implementation of the DP-CRLH TL. (b) SIW implementation of the DP-CRLH TL.

nents over the frequency range of interest. For the proposed DP-CRLH TLs, the new radiation frequency is significantly lower than the LH passband of the SP-CRLH TL with the corresponding similar period. Hence, it is possible to design LWAs which is more compact than the one based on SP-CRLH TLs.

3. CRLH property [36–38].

It is noted that the pursuit of the new LW radiation or the compact antennas is not at the expense of losing this unique CRLH property as is the case for SP-CRLH TLs. By comparing with the corresponding SP-CRLH TLs and DP-CRLH TLs, the scattering parameters and the radiation characteristics is almost similar in the LH region and the conventional RH region. Hence, DP-CRLH TLs modified from SP-CRLH TLs also hold the unique properties as it is done on

SP-CRLH TLs as discussed above.

It is noted that TLs double periodically loaded with either alternate capacitances or alternate inductances will contribute the new LW behavior, although the other components are single periodically loaded. This feature gives a great opportunity for easy design and fabrication of a SP-CRLH TL. For this reason, a substrate integrated waveguide (SIW) implementation of a DP-CRLH TL is designed and illustrated with simulated and experimentally demonstrated results.

It should be pointed out that the bandwidth of the new LW region is narrow. When the dispersion characteristics are in the fast wave region ($\beta < k_0$), the structure is able to radiate, which can be used for potential LWAs design. Therefore, the target to widen the bandwidth is to make the dispersion curve located in the fast wave region wider. This can be obtained by tuning the loading capacitances and inductances. However, it is very difficult to design a broadband LWA at low frequency, because the boundary of the fast wave region or air line ($\beta = k_0$) is extremely close to the vertical axis ($\beta = 0$) and the fast wave region is too narrow at low frequency.

Nevertheless, there are three LW regions: the new RH LW region, the LH LW region, and the conventional RH LW region for the DP-CRLH TLs. It is possible to design broadband and/or multi-band applications by considering all these LW radiations. The greatest advantage of the DP-CRLH TL is for designing compact antennas at very low frequency. It has been demonstrated that the SP-CRLH TLs are a good structure to design compact antennas since its resonant frequency is independent of the physical length. It is proposed in this thesis that the resonant frequency of DP-CRLH TLs is much lower than the SP-CRLH TLs. That means this new structure can be used to design even more compact antennas compared to the SP-CRLH TLs. Although the bandwidth of the new LW radiation is narrow, the new method to design compact LWAs at a fixed low frequency is still attractive and promising.

1.2 Objectives

The main objective in this work, is to propose the new concept of DP-CRLH TLs. As discussed in the previous section that apart from the CRLH characteristics as SP-CRLH TLs present, the proposed DP-CRLH TLs exhibit a new LW radiation below the LH passband with narrow bandwidth. The principal part of the research aims at completely investigating the novel DP-CRLH TL. The qualitative propagation characteristics, including the complex propagation constant (phase and attenuation constants), are studied based on TL theory. The expressions of the cutoff frequencies of each band edge are derived. The radiation behaviour, such as low frequency LW radiation, triple bands LW radiation and RH property of the new LW radiation, are obtained from TL theory. Additionally, dispersion characteristics and the radiation efficiency are investigated with the help of a singular perturbation procedure, and the radiation efficiency is numerically estimated as a function of the antenna length. Also, design guidelines and demonstration of the related DP-CRLH TLs are part of the objectives.

It includes the following goals:

1. A comprehensive review of the literature on MTMs is the first step for the proposed research. The primary target of this step is to understand the fundamental operating principle of the SP-CRLH TLs and the recent designs, especially LWAs. This review also aims to investigate the advantages of the SP-CRLH TLs comparing with SRR and other approaches, the analytical methods for improvements, and the future direction of research. Finally, the motivation for the proposed research may be established more clearly.
2. As the next objective, propagation characteristics of SP-CRLH TLs need to be investigated using TL theory. Understanding these propagation characteristics is important to formulate useful guidelines for the design of SP-CRLH TLs

or compact LWAs. Because the balanced or unbalanced SP-CRLH TLs are dependent on the cutoff frequencies of the lower edge of the RH branch and the upper edge of the LH branch of the dispersion curve, which are the series and shunt resonance frequencies for the equivalent circuit, it is important to analyze the effect of the cutoff frequencies to design balanced or unbalanced SP-CRLH LWAs.

3. Based on the understanding of the principle of SP-CRLH TLs, a novel DP-CRLH TL, modified from the SP-CRLH TLs, is proposed. As is the case for SP-CRLH TLs, TL theory is also used to analyze DP-CRLH TLs aiming at determining the propagation characteristics through the dispersion behavior based on the complex propagation constant (phase and attenuation constants). In addition to the CRLH properties as SP-CRLH TLs excited, a new LW is observed at very low frequency below the LH passband. The expressions of the cutoff frequencies are derived for the infinitely periodic structures and provide an qualitative estimation of the cutoff frequencies for the periodic structure including over three cells.
4. In order to further analyze the radiation characteristics of the proposed DP-CRLH TLs, especially for the microstrip line implementation of the DP-CRLH TLs, the singular perturbation procedure based on multiple space scales is used. Dispersion characteristics and the radiation efficiency are investigated with the help of the singular perturbation procedure, and the radiation efficiency is numerically estimated as a function of the antenna length. The analysis based on TL theory, the Floquet theorem and the equivalent circuit approach are static methods or quasi-static methods. The nature of the mode of the propagation is considered to be a pure transverse electric-magnetic (TEM) mode and the TL characteristics are calculated from the electrostatic capacitance and inductance

of the TLs. However, the singular perturbation procedure takes into account of the hybrid nature of the mode of the propagation. The singular perturbation procedure, by starting from the exact solution of Maxwell's equations, comprises mathematical methods to find an approximate solution about the radiation and propagation characteristics of the structure. This method is much more accurate, especially for TLs where transverse electric (TE), transverse magnetic (TM) or quasi TEM modes are supported.

The dispersion properties of waves in the double periodic inductance loaded transverse magnetic waveguide are rigorously established with the help of a singular perturbation method and the behaviors of the dispersion diagram are analyzed to illustrate the guiding characteristics of the double periodic structure. Based on the exact formulation, numerical analysis of specific structures may be carried out by truncating the Floquet modes, and the first order Floquet mode is considered along with perturbation orders of δ^1 and δ^2 in the first and second order analysis. This would cause some degree of approximation, but the accuracy may be systematically improved by increasing the number of Floquet components to higher orders. It has been earlier reported that this approximation is valid until the periodic perturbation is within 30% [39].

The purposes of such a rigorous analysis are threefold:

- To provide understanding of radiation characteristics of DP-CRLH TLs in more detail.
- To investigate the radiation efficiency and the radiation angle of the LWAs.
- To give accurate numerical data as bench-mark results against approximate ones from TL theory.

5. Based on the above studies about DP-CRLH TLs, it is aimed to outline use-

ful guidelines and procedures to design working DP-CRLH TLs and compact LWAs. To achieve this, circuit models of the proposed DP-CRLH TLs need to be considered and analyzed.

6. For a demonstration of the proposed concept of DP-CRLH TLs, it is targeted to fabricate a number of prototype structures, and measurement results shall be compared with the predicted performance of those prototypes. These exercises are also expected to show whether the response of a finite-sized practical structure is close to the theoretical results obtained above whose modeling is primarily based on the assumption of an infinite array as required by TL theory, the Floquet theorem and the perturbation procedure.

1.3 Major Contributions of the Thesis

This thesis introduces a novel DP-CRLH TL with LW radiating performance at very low frequency based on understanding of the basic principle of the SP-CRLH TLs. Both of them are based on periodic structures, working as artificial effectively homogeneous electromagnetic structures when the period is much smaller than the guided wavelength ($\Lambda \ll \lambda_g$). The analysis and design procedures are presented in this work. Major contributions of the thesis may be identified as the following:

1. Theoretical analysis of the proposed DP-CRLH TLs.
 - Firstly, the TL theory is used to analyze the proposed DP-CRLH TLs [37]. The model of $L - C$ loaded TL is adopted as the analyzed equivalent circuit. The basic information about LW radiation characteristics of the DP-CRLH TLs is obtained through the dispersion behavior of the DP-CRLH TLs based on the complex propagation constant (phase and attenuation constants). In addition to the CRLH property as SP-CRLH TLs excite, a

new LW is observed at low frequency below the LH passband at first time and the new LW is demonstrated to work as RH characteristic.

- Next, to extract the expressions of the cutoff frequencies about the edges of each branch in the dispersion curve, the ideal $L - C$ network is used as an equivalent circuit model of the analyzed DP-CRLH TLs [38]. The cutoff frequencies of the upper edge of the LH branch and the lower edge of conventional RH branch at the spectral axis are obtained by the series and shunt resonance frequencies. When the phase constant of the DP-CRLH TL approaches one of the dispersion curve of the corresponding SP-CRLH TLs at $\beta = \pi/2\Lambda$, the cutoff frequencies of lower edge of LH branch and upper edge of the new RH branch occur. The cutoff frequency of the lower edge of the new RH branch is located between that of corresponding SP-CRLH TLs.
- Finally, in order to further analyze the radiation characteristics of the DP-CRLH TLs, especially for the microstrip line implementation of the DP-CRLH TLs, the singular perturbation procedure is used [36]. The dispersion characteristics are obtained and also illustrate the new RH LW property of the proposed DP-CRLH TLs. Besides the propagation characteristics, the radiation efficiency is numerically estimated around the new RH LW region and the LH LW region as a function of the TL length. It is noted that the radiation efficiency in the new RH LW region is smaller than the radiation behavior in the LH LW region for the same length of TLs. Longer TLs are needed to get more radiation efficiency. The radiation angle of the new LW is also estimated for the length fixed DP-CRLH TL with the particular amplitude of the fundamental component of the periodically loaded inductance.

2. Experimental demonstration of the new concept of DP-CRLH TLs.

The thesis introduces the design procedure and a number of prototype structures are designed, fabricated and measured. The measurement results compared with the predicted performance of those prototypes demonstrate the proposed concept.

- The first experiment is performed using microstrip lines with double periodically loaded shunt short-circuited stub inductances and series interdigital capacitances. The compact LWA [37] based on the proposed DP-CRLH TL displays the expected new-found forward LW radiation in the direction of $\theta_r = +3^\circ$ and with gain of 4.12 dBi at 1.6 GHz, the LH backward radiation in the direction of $\theta_r = -19^\circ$ and with gain of 8.62 dBi at 3.3 GHz, and the conventional RH forward radiation in the direction of $\theta_r = +12^\circ$ and with gain of 8.78 dBi at 3.9 GHz.
- Another kind of compact LWAs is designed using longer microstrip lines double/single periodically loaded with series interdigital capacitances and/or single-side shunt short-circuited stub inductances [36]. The radiation characteristics of the optimized antenna are measured at $f = 1.023$ GHz (new LW, forward radiation), $f = 2.467$ GHz (LH, backward radiation) and $f = 4.567$ GHz (conventional RH, forward radiation). The radiation efficiency and gain of these longer LWAs are improved compared to the last experiment. As the analysis estimates, the LWA radiates at $f = 1.023$ GHz with gain of 5.1 dBi and radiation angle $\theta_r = 18^\circ$ toward the direction of propagation, at $f = 2.467$ GHz with gain of 7.9 dBi and radiation angle $\theta_r = -16^\circ$, and at $f = 4.567$ GHz with gain of 9.3 dBi and $\theta_r = 21^\circ$.
- It is noted that the new LW radiation is obtained through the TLs double periodically loaded with either alternate series capacitances or alternate

shunt inductances, although the other components are single periodically loaded with uniform value [38]. Following the analysis, an SIW double periodically loaded with alternate shunt inductances and single periodically loaded with uniform series capacitance is designed and realized as a DP-CRLH TL. Full wave simulations and experiments have been done to support the analysis. The frequency responses and radiation characteristics are measured. The dispersion characteristics extracted from the measured results are matched well with the theory and the numerical simulation results. The measured radiation characteristics of the DP-CRLH SIW LWA display the new forward LW radiation from 5.2 GHz to 5.8 GHz in the direction of $\theta_r = +33^\circ$ and with gain of 8.62 dBi, and broadband frequency-scanned LW radiation from -42° at 11.5 GHz in the LH region to 40° at 18 GHz in the conventional RH region through the broadside radiation at 14 GHz. The new LW radiation band is tuned to cover the IEEE 802.11a band [40] for wireless local area network (WLAN) applications.

1.4 Organization of the Report

The thesis has been organized into six chapters. The first chapter primarily explains the motivation to propose the new concept of DP-CRLH TLs by identifying unique properties following the theory of previous MEMs and SP-CRLH TLs. It establishes that in addition to the CRLH property as excited by the SP-CRLH TLs, the proposed DP-CRLH TLs, in principle, can radiate in a low frequency region below the LH passband.

Chapter 2 presents a comprehensive literature review of MTMs and SP-CRLH TLs. Recent designs, especially the SP-CRLH TLs, have been considered. Operating principle and the main design techniques are discussed in sufficient details. Some

design examples based on SP-CRLH TLs have also been presented. The previous study of double-stub structures has been reviewed and the basic principle of singular perturbation method based on multiple space scales is also introduced.

Chapter 3 proposes the new concept of the DP-CRLH TLs based on the understanding of SP-CRLH TLs. The TL theory and Floquet theorem are used to analyze the fundamental propagation characteristics of the proposed DP-CRLH TLs. The new forward LW radiation is observed at low frequency below the LH passband with narrow bandwidth. A number of parametric studies have been presented to understand the effect of geometrical dimensions and excitation on the propagation characteristics of the proposed DP-CRLH TLs.

Chapter 4 formulates the LW problem where an air-to-medium discontinuity has been analyzed using a singular perturbation method. The modes in the air and substrate regions are expressed in terms of TM mode, while the mode in the substrate region has been considered as TEM mode or quasi-TEM mode in Chapter 3. A number of parametric studies have been presented in more detail to develop a better understanding and physical significance of the proposed DP-CRLH TLs. Effect of structural dimensions, radiation efficiency and angle of incidence have been studied in sufficient details.

Based on the above two major studies (Chapter 3 and Chapter 4), Chapter 5 demonstrates a number of applications of the proposed DP-CRLH TLs. The TL approach of double periodic structures, based on DP-CRLH TLs concept, represents the foundation for most of the applications presented in this work. Circuit model and design guidelines of the proposed compact DP-CRLH LWAs have been presented to more clearly explain the operating principle, and lay down a faster design procedure. Two kinds of compact LWAs are implemented by microstrip lines modified from the previous SP-CRLH TLs and SIW modified from a proposed SP-CRLH SIW. A number of prototype structures have been fabricated and the measurement results validate the

design procedures presented in this chapter.

Chapter 6 concludes this thesis by summarizing the findings and results of this research. It also points out certain limitations of the proposed structure. Some suggestions for future explorations leading to possible improvements/extensions of the proposed structure have also been listed in this last chapter.

Chapter 2

LITERATURE REVIEW

2.1 Introduction

Electromagnetic metamaterials (MTMs) with simultaneous negative permittivity and permeability, or left-handed materials (LHMs) are a very interesting research topic in recent times, and a large number of geometries and techniques have been proposed for over past ten years. The concept was first investigated theoretically by Veselago in 1968 [5]. Negative permittivity materials [41], negative permeability ring resonators [17] and composite medium [7,14,16,42–45] were practically realized after two decades.

For practical applications, it is desirable to realize MTMs with high selectivity performance, low loss and wide bandwidth. However, all these features are difficult to obtain. A new technique, named as CRLH MTMs, was introduced by three groups Eleftheriades et. al [1,20], Oliner [21] and Caloz et al [2,22–24] in 2002. These new methods and designs have been reviewed in this chapter. These groups authored a comprehensive account of design techniques of CRLH MTMs. The subject of CRLH MTMs is treated as single periodic composite right/left handed (SP-CRLH) transmission lines (TLs) in this thesis since these TLs are single periodically loaded with uniform capacitance and inductance. The understanding and principles of SP-CRLH

TLs presented in this chapter are mainly based on the work of [1, 2].

The last section of this chapter explores a double-stub structure and singular perturbation procedure. Based on this convention, the proposed double periodic composite right/left handed (DP-CRLH) TLs have been extended to realize compact leaky-wave antennas (LWAs). Therefore, a comprehensive review of previously known MTMs becomes relevant to more clearly distinguish this research from the previous studies on SP-CRLH TLs.

2.2 Fundamental Description of Metamaterials

It is well known that the response of a system to the presence of an electromagnetic field is determined to a large extent by the properties of the materials involved. These properties are described by defining macroscopic parameters of the materials, permittivity (ε) and permeability (μ), based on Maxwell's Equations [19, 46, 47]. This leads to classification of a medium as follows,

- A medium with both positive permittivity and permeability ($\mu > 0, \varepsilon > 0$) will be designated a double positive (DPS) medium. Most naturally occurring media fall under this designation.
- A medium with negative permittivity, but positive permeability ($\mu > 0, \varepsilon < 0$) will be designated an epsilon-negative (ENG) medium. In certain frequency regimes materials with plasmas exhibit this characteristic.
- A medium with negative permeability, but positive permittivity ($\mu < 0, \varepsilon > 0$) will be designated a mu-negative (MNG) medium. In certain frequencies some gyrotropic materials exhibit this characteristic.
- Finally the medium with both negative real permittivity and permeability ($\mu <$

Figure 2.1: All possible properties of isotropic materials in the $\varepsilon \sim \mu$ domain.

$0, \varepsilon < 0$) will be termed as double negative (DNG) medium or left-handed (LH) medium.

The medium classification can be graphically illustrated as shown in Fig. 2.1 [45]. Most natural materials occur at points on the line $\mu = \mu_0$ and $\varepsilon \geq \varepsilon_0$. Few natural electric plasma and magnetic plasma occur in very small parts in the second and fourth quadrants. The best-known examples of media with negative permittivity are low-loss plasmas, metals and semiconductors at optical and infrared frequencies (sometimes called solid-state plasmas) [48]. Ferrites magnetized to saturation present a negative magnetic permeability near the ferrimagnetic resonance (which usually occurs at microwave frequencies) [49], although media with negative permeability are less common comparing to the ENG in nature due to the weak magnetic interactions in most solid-state materials [50]. In the $\varepsilon - \mu$ domain, some particular lines and points

in Fig. 2.1 indicating special material properties, such as the point $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$ representing an anti-air in the LHM region, which will be used to design perfect lens; the point $\mu = 0$ and $\varepsilon = 0$ showing a nihility, which can support a perfect tunneling effect; the line $\varepsilon = \mu$ in both DPS and DNG regions denote impedance-matching materials, which have perfect impedance matching with air. Also, the vicinity of $\mu = 0$ is called as μ -near zero (MNZ) material, and the vicinity of $\varepsilon = 0$ is called as ε -near zero (ENZ) material [45].

Much of the recent research has been focused on MTMs that exhibit simultaneous negative permittivity (ε) and permeability (μ), which are usually termed as DNG material [1, 2, 43–45]. Although the media of negative permeability or negative permittivity have been found in nature, the simultaneous negative permittivity and permeability is a strange phenomenon which is not found in nature until now. However, Veselago theoretically investigated plane-wave propagation in a material whose permittivity and permeability are assumed to be simultaneously negative at the end 1960s [5]. His theoretical studies showed that for a monochromatic uniform plane wave in such a medium the direction of a Poynting vector is antiparallel to the direction of the phase velocity, contrary to the case of plane wave propagation in conventional media.

Several fundamental phenomena occurring in or in association with LHM were theoretically predicted by Veselago [5] and demonstrated by several groups worldwide (this is summarized in [2]):

1. Necessary frequency dispersion of the constitutive parameters.
2. Reversal of Doppler effect [10].
3. Reversal of Vavilov-Cerenkov radiation [11].
4. Reversal of the boundary conditions relating the normal components of the elec-

tric and magnetic fields at the interface between a conventional right-handed (RH) medium and a LH medium.

5. Reversal of Snells law [7, 8].
6. Subsequent negative refraction at the interface between a RHM and a LHM [9].
7. Transformation of a point source into a point image by a LH slab.
8. Interchange of convergence and divergence effects in convex and concave lenses, respectively, when the lens is made LH.
9. Plasmonic expressions of the constitutive parameters in resonant-type LHM.

2.3 Demonstration of First Left-Handed Medium

The systematic analysis of electromagnetic behaviors in LHM is a significant progress [5]. However, due to the non-existence in nature and lack of experimental verification, the realization of LHM did not occur until three decades later. The first revolution was proposed by Pendry et al. that the artificially electric plasma with negative permittivity is realized using the thin-wire medium as shown in Fig. 2.2(a) in 1996 [41]. Then in 1999, the well-known split-ring resonators (SRRs) were proposed also by Pendry to realize the artificially magnetic plasma with negative permeability as shown in Fig. 2.2(b) [17]. The first experimental verification of LHMs was successfully achieved by Smith et al., who incorporated the copper SRRs and metallic thin-wires to develop a novel medium with simultaneously negative permeability and permittivity in microwave frequency region as shown in Fig. 2.2(c) [7, 14, 16, 51].

The SRRs provide negative permeability while the metallic thin-wire provides negative permittivity, respectively. The structure demonstrated the concept of a negative

Figure 2.2: Revolutions on the first experimentally realization of MTMs. (a) Thin-wire structure exhibiting negative- ϵ . (b) Split-ring resonator structure exhibiting negative- μ . (c) Bidimensionally LH structure exhibiting negative- ϵ and μ .

index of refraction and it is confirmed by a group at Massachusetts Institute of Technology (MIT) more completely [52]. Motivated by the experimental realization, LHM has attracted growing attention in both theoretical exploration [8, 11–14, 18, 42, 52–54] and experimental study [33, 51, 55–63]. Features of MTMs beyond the negative refraction are studied and the gradient refractive index medium was realized to bend electromagnetic waves in 2005 [64], including the exciting discovery of three-dimensional optical MTM with a negative refractive index (NRI) [9] and the optical transformation making invisible cloaks [65–68].

2.4 Study of SP-CRLH TLs

Despite the good properties, the MTMs discussed in previous section is bulky and highly resonant. They are difficult to realize microwave applications because of the high quality-factor resonance. Despite the LHM based on the thin-wire and SRR structure demonstrates the realization of MTMs, the LHM suffers from the large loss and narrow bandwidth [2]. An alternate representation of MTMs, named as CRLH MTMs (SP-CRLH TLs in this thesis) with several advantages of non-resonant, more compact, less lossy and wider bandwidth, are developed using the TL approach recognizing the analogy between LH waves and conventional backward waves by three groups Eleftheriades et. al [1, 20], Oliner [21] and Caloz et al. [2, 22–24] in 2002.

It is well known that a conventional TL is composed of distributed series inductance (L_R) and shunt capacitance (C_R), which can be shown equivalent to a one-dimensional (1D) RHM as shown in Fig. 2.3. As a dual model of the conventional TL, a new TL consisting of series capacitance (C_L) and shunt inductance (L_L) (Fig. 2.4(a)) will support backward waves and hence can represent LHM [2, 24, 69]. In the realization of LH TL, the distributed components have been used. Since the series capacitance is always accompanied by an inductance and the shunt inductance is accompanied by a capacitance, a general SP-CRLH TL model has been proposed to represent RHM and LHM as shown in Fig. 2.4(b) [2].

The presented SP-CRLH TLs are periodic structures by nature. The unit cells are arranged periodically into a one-/two- dimensional array or three-dimensional lattice, and the array is excited at guide wavelengths longer than the cell dimension to enforce the homogeneity condition, so that the resulting periodic structures can be regarded as distributed. As a result, wave propagations within periodic structures are firstly introduced with a review of the Floquet theorem.

Figure 2.3: Equivalent circuit of the conventional RHM. (a) Unit cell. (b) Line implementation in the form of a periodic network.

Figure 2.4: Equivalent circuit of the unit cell of (a) Pure LH TL, (b) SP-CRLH TL.

2.4.1 Periodic Structure

Periodic structures can be designed by repeating a special kind of unit cell [47, 70]. They are classified into two basic types, structures with continuous but periodically varying electrical properties and structures with periodic boundary conditions as shown in Fig. 2.5 [71]. There are two extraordinary properties made periodic structures essential,

- (1) In the periodic structures there are an infinite number of space-harmonics with phase velocities varying from zero to infinity;
- (2) They can support waves with well-specified propagation and stop bands.

The first property is important because that it expresses a separate set of passbands differentiated by stopbands. The second property is attractive since it provides an application which can support waves with phase velocities much less than the velocity of light. The metallic diaphragm in a hollow metallic wave guide behaves as a lumped capacitance and inductance added at interval, which forms the series capacitance and shunt inductance per unit length. So the negative permittivity and permeability will be realized at certain frequency of interest [2].

As early as 1887, Lord Rayleigh proposed a new concept about the propagation of waves in periodically stratified media [72]. Since then, Kelvin and Vincent built a theory for analyzing the dispersion properties of lattices which were made up of a series of identical particles and an electric filter was designed by the same group during 19th century [73]. In 1928, the most important theory about periodic structures was found by Floquet [74]. After that, the interest in periodic structures increased. The applications including the relatively slow electron beam and antennas were designed [75].

Recently, experiments focusing on designing MTMs based on periodic structures with artificially fabricating diaphragm embedded in host media or connected to host

Figure 2.5: Periodic structures. (a) The structure with continuous but periodically varying electrical properties. (b) The structure with periodic boundary conditions.

surfaces will realize the uniform properties. The SP-CRLH TLs [1, 2, 20–24] have been realized based on the periodic structure approach. Due to many unique properties such as supporting a fundamental backward wave and zero propagation constant with zero or non-zero group velocity at a discrete frequency, these periodic structures have been used to realize novel, compact antennas [2, 25–30].

2.4.2 Floquet Theorem

Floquet theorem [76] (or Bloch’s theorem [77]) is a basic theorem underlying the theory of wave propagation in periodic structures. It gives a method to determine the solution of the wave equation in a periodic structure. The Brillouin zones (or $\omega - \beta$ diagram) are usually used in expressing the properties of propagation of electromagnetic waves in periodic structures. For simplicity the following discussion of the Floquet theorem will focus on structures periodic in one direction [19].

In the electromagnetic field, the periodic structure satisfies the Maxwell's equations, which can be considered as a kind of linear differential equations. It is convenient to formulate the problem in terms of Helmholtz equations [47]

$$\begin{cases} \nabla^2 E + \omega^2 \mu \varepsilon E = 0, \\ \nabla^2 H + \omega^2 \mu \varepsilon H = 0. \end{cases} \quad (2.1)$$

The Floquet theorem states that the field distribution $\varphi(x, y, z)$ along a periodic structure changes by only a multiplicative complex constant for translations of one period along the structure [19]. For a structure that is axially periodic along the z direction, this means

$$\varphi(x, y, z) = e^{-\gamma \Lambda} \phi(x, y, z), \quad (2.2)$$

where $\gamma = \alpha + j\beta$ is effective complex propagation constant of the periodic structure and Λ is the length of the unit cell. Therefore, an electromagnetic field in any point of the unit cell takes the same value of the field in similar point at the adjacent cell except for complex constant $e^{-\gamma \Lambda}$ which introduce an attenuation constant (α) and phase constant (β).

In a periodic medium, the propagation constant ($\gamma = j\omega\sqrt{\varepsilon\mu}$) in the wave equation is periodic along z direction because of the underlying periodic boundary conditions. The wave equation in a periodic structure along z direction is:

$$\frac{d^2 \varphi(x, y, z)}{dz^2} + k(z)^2 \varphi(x, y, z) = 0, \quad (2.3)$$

where $k(z)$ is periodic with period of Λ . In addition, based on the Floquet theorem, the wave equation for the periodic structure satisfies

$$\frac{d^2 \varphi(x, y, z + \Lambda)}{d(z + \Lambda)^2} + k(z + \Lambda)^2 \varphi(x, y, z + \Lambda) = 0. \quad (2.4)$$

Figure 2.6: A two-port network of one unit cell of the periodic structure.

Because

$$\begin{cases} \frac{d^2 \varphi(x, y, z + \Lambda)}{dz^2} = \frac{d^2 \varphi(x, y, z + \Lambda)}{dz^2}, \\ k(z + \Lambda) = k(z), \end{cases} \quad (2.5)$$

the wave equation is derived as

$$\frac{d^2 \varphi(x, y, z + \Lambda)}{dz^2} + k(z)^2 \varphi(x, y, z + \Lambda) = 0. \quad (2.6)$$

Hence, the solution $\varphi(x, y, z + \Lambda)$ is also a solution of the wave equation for the periodic structure with period of Λ .

The ABCD matrices is defined for a two-port network in terms of the total voltages $V(z)$ and currents $I(z)$ as shown in Fig. 2.6 [47]. The two linearly independent solutions $V(z)$ and $I(z)$ are following:

$$\begin{bmatrix} V_1(z) \\ I_1(z) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2(z) \\ I_2(z) \end{bmatrix}. \quad (2.7)$$

Based on the Floquet theorem, if $V(z)$ and $I(z)$ are two linearly independent solutions, $V(z + \Lambda)$ and $v(z + \Lambda)$ are also solutions for the periodic structure with period of Λ .

Therefore, $V(z + \Lambda)$ and $I(z + \Lambda)$ are linear combinations of $V(z)$ and $I(z)$, i.e.

$$\begin{bmatrix} V(z + \Lambda) \\ I(z + \Lambda) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(z) \\ I(z) \end{bmatrix}. \quad (2.8)$$

In the periodic structure, the output current and voltage variables are related to the input current and voltage by the propagation term $e^{-\gamma\Lambda}$ as

$$\begin{bmatrix} V(z + \Lambda) \\ I(z + \Lambda) \end{bmatrix} = e^{-\gamma\Lambda} \begin{bmatrix} V(z) \\ I(z) \end{bmatrix}. \quad (2.9)$$

The statement of the Floquet theorem shows that solutions in one unit cell differ from the solutions in an adjacent unit cell by only a complex propagation constant. Substituting (2.9) into (2.11) gives

$$e^{-\gamma\Lambda} \begin{bmatrix} V(z) \\ I(z) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V(z) \\ I(z) \end{bmatrix}. \quad (2.10)$$

The corresponding determinant is set to zero to obtain non-trivial solutions for the propagation constant, γ :

$$\begin{vmatrix} A - e^{-\gamma\Lambda} & B \\ C & D - e^{-\gamma\Lambda} \end{vmatrix} = 0. \quad (2.11)$$

So that,

$$e^{\gamma\Lambda} + (AD - BC)e^{-\gamma\Lambda} - (A + D) = 0, \quad (2.12)$$

yielding:

$$e^{-\gamma\Lambda} = \frac{(A + D) + \sqrt{(A + D)^2 - 4(AD - BC)}}{2}. \quad (2.13)$$

Since $AD - BC = 1$ due to reciprocity, then:

$$\cosh(\gamma\Lambda) = \frac{A + D}{2}. \quad (2.14)$$

This is the key equation determining the dispersion characteristics for analyzing periodic structure.

2.4.3 Theoretical Treatment on SP-CRLH TLs.

The equivalent circuit of conventional RH TLs is low pass as shown in Fig. 2.3. By exchanging the capacitance and the inductance and relocating the series/ parallel reactance, the LH TL is obtained as shown in Fig. 2.4(a), which is the duality of conventional TL [24]. Combining the series RH inductance and LH capacitance with the shunt RH capacitance and LH inductance, the homogeneous model is obtained for the SP-CRLH TLs as shown in Fig. 2.7.

The ABCD matrices for the two-port-network unit cell of the SP-CRLH TLs shown in Fig. 2.7 is obtained by multiplying the individual element matrices [2, 24, 47], i.e.,

$$\begin{aligned} \begin{bmatrix} A & B \\ C & D \end{bmatrix} &= \begin{bmatrix} 1 & Z_{LR} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_{CL} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{CR} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{LL} & 1 \end{bmatrix} \\ &= \begin{bmatrix} (Z_{CL} + Z_{LR})(Y_{CR} + Y_{LL}) + 1 & Z_{CL} + Z_{LR} \\ Y_{CR} + Y_{LL} & 1 \end{bmatrix}, \end{aligned} \quad (2.15)$$

where $Z_{CL} = 1/j\omega C_L$, $Y_{LL} = 1/j\omega L_L$ are the impedance and admittance of the periodic loading elements and $Z_{LR} = j\omega L_R$, $Y_{CR} = j\omega C_R$ are the per-unit impedance and admittance contributed by the equivalent per-unit shunt capacitance and series inductance of the host medium.

The dispersion relation of an infinite SP-CRLH TL is derived by applying the

Figure 2.7: The homogeneous model of the SP-CRLH unit cell obtained from combination of PRH and PLH structures.

Floquet theorem to ABCD matrices of the unit cell (2.4.2) and is obtained as

$$\cosh(\gamma\Lambda) = \frac{A + D}{2} = \frac{(Z_{C_L} + Z_{L_R})(Y_{C_R} + Y_{L_L}) + 2}{2}, \quad (2.16)$$

where

$$\begin{cases} Z_{L_R} = j\omega L_R & Y_{C_R} = j\omega C_R, \\ Z_{C_L} = 1/j\omega C_L & Y_{L_L} = 1/j\omega L_L. \end{cases} \quad (2.17)$$

Substituting (2.17) into (2.4.3) gives

$$\begin{aligned} \cosh(\gamma\Lambda) &= \frac{(\frac{1}{j\omega C_L} + j\omega L_R)(j\omega C_R + \frac{1}{j\omega L_L}) + 2}{2} \\ &= \frac{(\frac{C_R}{C_L} + \frac{L_R}{L_L}) - (\frac{1}{\omega^2 C_L L_L} + \omega^2 C_R L_R) + 2}{2}. \end{aligned} \quad (2.18)$$

The dispersion characteristics of the SP-CRLH TL using the average phase shift incurred per unit cell ($\beta\Lambda$) with period of Λ are obtained as shown in Fig. 2.8. The

Figure 2.8: Dispersion characteristics of the unbalanced SP-CRLH TL. $C_L = 0.5$ pF, $C_R = 0.3$ pF, $L_L = 0.5$ nH, $L_R = 0.4$ nH, $\Lambda = 5$ mm.

cutoff frequency of the lower edge of the RH branch ($f_{\Gamma 2}$) and the upper edge of the LH branch ($f_{\Gamma 1}$) at the spectral axis are obtained when the attenuation constant $\alpha = 0$ and the phase constant $\beta = 0$, so that the propagation constant $\gamma = \alpha + j\beta = 0$ as shown in Fig. 2.8. At the cutoff frequencies, the dispersion relation satisfies

$$\cosh(\gamma\Lambda) = \cosh(0) = 1. \quad (2.19)$$

Hence,

$$\frac{(\frac{C_R}{C_L} + \frac{L_R}{L_L}) - (\frac{1}{\omega^2 C_L L_L} + \omega^2 C_R L_R) + 2}{2} = 1. \quad (2.20)$$

So that,

$$\omega^4 C_L L_L C_R L_R - \omega^2 C_L L_L (\frac{C_R}{C_L} + \frac{L_R}{L_L}) + 1 = 0. \quad (2.21)$$

Therefore, the cutoff frequencies of the upper edge of the LH branch ($\omega_{\Gamma_1} = 2\pi f_{\Gamma_1}$) and the lower edge of the RH branch ($\omega_{\Gamma_2} = 2\pi f_{\Gamma_2}$) at the spectral axis are obtained by

$$\begin{cases} \omega_{\Gamma_1} = \min(\omega_{se}, \omega_{sh}), \\ \omega_{\Gamma_2} = \max(\omega_{se}, \omega_{sh}), \end{cases} \quad (2.22)$$

where

$$\begin{cases} \omega_{se} = \frac{1}{\sqrt{C_L L_R}}, \\ \omega_{sh} = \frac{1}{\sqrt{C_R L_L}}. \end{cases} \quad (2.23)$$

The dispersion characteristics show the typical LH passband ($f_L \sim f_{\Gamma_1}$) and conventional RH passband ($f > f_{\Gamma_2}$) for the SP-CRLH TL. The phase velocity $v_p = \omega/\beta$, and the group velocity $v_g = (d\omega)/(d\beta)$, where $\omega = 2\pi f$ is the angular frequency. The typical dispersion behavior of left-handedness exhibiting antiparallel phase and group velocities is obtained from f_L to f_{Γ_1} . The conventional right-handedness exhibiting parallel phase and group velocities is obtained when $f > f_{\Gamma_2}$. When the dispersion curve is in the fast wave region ($\beta < k_0$), the structure is able to radiate and can be used for LWAs which radiate backward at $f_{L-LW} < f < f_{\Gamma_1}$ and forward at $f > f_{\Gamma_2}$, where $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ is the free space wave number [1, 2, 28, 32].

Generally the series and shunt resonance frequencies in SP-CRLH TLs are not equal ($L_R C_L \neq C_R L_L$), which leads to term this class of SP-CRLH medium as unbalanced SP-CRLH TLs. However, when the series and shunt resonance frequencies are equal ($L_R C_L = C_R L_L$), the LH and RH contributions exactly balance each other in the same frequency at

$$\omega_0 = \frac{1}{\sqrt[4]{L_R C_R L_L C_L}}. \quad (2.24)$$

There is a seamless transition from left-handedness to right-handedness for the balanced SP-CRLH TLs, because γ is always purely imaginary, unlike the unbalanced case. As a result, the dispersion curve of the balanced SP-CRLH TL does not have a

Figure 2.9: Dispersion characteristics of the balanced SP-CRLH TL. $C_L = 0.5$ pF, $C_R = 0.4$ pF, $L_L = 0.5$ nH, $L_R = 0.4$ nH, $\Lambda = 5$ mm.

stop-band. Although β is zero at ω_0 , which corresponds to an infinite guided wavelength $\lambda_g = 2\pi/|\beta|$, the wave propagation still occurs since group velocity (v_g) is nonzero at ω_0 . In addition, the phase shift for a TL of length Λ is zero $\phi = -\beta\Lambda = 0$) at ω_0 . The phase advance ($\phi > 0$) occurs in the LH frequency range ($\omega < \omega_0$), and phase delay ($\phi < 0$) occurs in the RH frequency range ($\omega > \omega_0$) [25].

SP-CRLH TLs are highly dispersive as frequency increases, since the phase velocity ($v_p = \omega/\beta$) becomes increasingly dependent on frequency, which also illustrates the dual nature of the SP-CRLH TLs; at low frequencies the SP-CRLH TLs are dominantly left-handedness, while at high frequencies the SP-CRLH TLs are dominantly right-handedness. The dispersion diagrams of balanced and unbalanced SP-CRLH TLs as shown in Fig. 2.9 and Fig. 2.8 indicate that the LH to RH transitions occur

at:

$$\omega = \begin{cases} \omega_0 = \frac{1}{\sqrt[4]{L_R C_R L_L C_L}} & (balanced), \\ \begin{cases} \omega_{\Gamma 1} = \min(\omega_{se} = \frac{1}{\sqrt{C_L L_R}}, \omega_{sh} = \frac{1}{\sqrt{C_R L_L}}) \\ \omega_{\Gamma 2} = \max(\omega_{se} = \frac{1}{\sqrt{C_L L_R}}, \omega_{sh} = \frac{1}{\sqrt{C_R L_L}}) \end{cases} & (unbalanced), \end{cases} \quad (2.25)$$

where ω is referred as the transition angular frequency. On the other hand, the lower edge of the LH branch is estimated by

$$\omega_L = \frac{1}{2\sqrt{C_L L_L}}. \quad (2.26)$$

The dispersion diagram of SP-CRLH TLs demonstrates the existence of both LH region ($v_p v_g < 0$) and RH region ($v_p v_g > 0$) region, which means that the structures exhibit both equivalent positive permittivity and permeability and equivalent negative permittivity and permeability depending on the frequency.

2.4.4 Physical Implementation Approach to SP-CRLH TLs

The $L - C$ network of SP-CRLH TLs as shown in Fig. 2.4(b) can be realized with physical components that can generate the required capacitances and inductances. Distributed components such as microstrip, substrate integrated waveguide (SIW), stripline, coplanar waveguide, and surface-mount technology (SMT) chip components are used to generate the $L - C$ network.

Microstrip Implementation

Some examples of distributed components are based on one-dimensional (1-D) SP-CRLH TLs. The first distributed SP-CRLH TL is implemented on microstrip line

Figure 2.10: Microstrip SP-CRLH TL using series interdigital capacitors and short-circuited stub inductors.

with series interdigital capacitors and shunt short-circuited stub inductors connected to the ground plane as shown in Fig. 2.10 [24]. Comparing with $L - C$ network relationship of (2.18), the contributions C_L and L_L are provided by the series interdigital capacitors and shunt short-circuited stub inductors, respectively. The RH capacitance C_R is attributed to the capacitance between the trace and ground plane, and the RH inductance L_R is caused by the magnetic flux generated by the current flow in the digits of the interdigital capacitor [24].

Various applications are designed based on this kind of microstrip line implementation of SP-CRLH TLs.

- Leaky-wave antennas

The leaky-wave phenomenon of microstrip lines was first reported by W. Menzel in [78]. A number of LWAs have been designed because of the advantages of simple construction, low-profiles, easy to match, narrow beam-width, frequency-scanning ability and inherently more broadband than resonant type antennas [79, 80]. The SP-CRLH TLs realized by introducing the series capacitances and shunt inductances in host mediums [2] can be used to design compact broadband LWAs which can radiate backward in the LH region and forward in the RH region [2].

Figure 2.11: Examples of the microstrip line implementations of SP-CRLH TLs on antennas. (a) A CPW backward-wave antenna based on negative refractive index L-C networks. (b) Electronically controlled SP-CRLH TL as a LWA with tunable radiation angle and beamwidth. (c) The microstrip interdigital-C/stub-L SP-CRLH LW antenna. (d) Vialess microstrip implementation of the SP-CRLH zeroth resonator.

A. Grbic and G. V. Eleftheriades first proposed Coplanar Waveguide (CPW) MTM LWA as shown in Fig. 2.11(a) in 2002 [81]. Figure 2.11(c) shows a microstrip interdigital-C/stub-L SP-CRLH TL [82] providing a unique property of backfire-to-endfire frequency-scanning capability. A microstrip line alternate to the structure of Fig. 2.11(c) is the via-less configuration as shown Fig. 2.11(d) [83]. Such a via-less configuration may be advantageous in processes where implementation of vias is problematic. A MTM-based electronically controlled TL as shown in Fig. 2.11(b) was presented as a LWA with tunable radiation angle and beamwidth functionalities [84]. The structure is a SP-CRLH microstrip line incorporating varactor diodes for fixed-frequency voltage-controlled operation.

Angle scanning at a fixed frequency is achieved by modulating the capacitances of the structure by adjusting the (uniform) bias voltage applied to the varactors.

- Guided wave applications

In the development of the communication and electronic systems, small size and high performance guided wave applications, such as filters and couplers, are needed to reduce the cost and improve the system performance. This brings increased opportunities for designing compact applications for many modern microwave systems, such as automotive radar and broadband wireless communication. A series microwave applications such as dual-band branch-line coupler [85], asymmetric backward-wave directional coupler [86], zeroth order resonator [87] have been proposed.

A number of compact filters have been developed before, such as, using meander spurline and capacitively loaded stubs [24], sierpinski-based resonator [88], defected ground structure combined with lumped capacitor [89], multilayered TL [31], MCM-D technology [90], and superconductor [91]. Unfortunately, these kinds of compact filters have few disadvantages: the size of filters in [88, 92] are 23mm length resonating at 5.6GHz and 40mm length resonating at 2.2 GHz respectively, the filters in [31, 89, 90] are difficult to fabricate because they are based on MCM-D technology or lumped capacitor. The superconductor [91] filter can only work at very low temperature and it is too expensive for these filters to be widely used.

Many compact guided wave applications have also been developed before. Fig. 2.12(a) shows a nine-cell SP-CRLH microstrip coupler prototype [93], which can achieve any arbitrary level of coupling. Physical length of the coupler depends only on the reactive loadings of the line, which is shorter than $\lambda/4$ in general case. The even/odd-mode characteristic impedances and of the SP-

Figure 2.12: Examples of the microstrip line implementations of SP-CRLH TLs on guided wave applications. (a) A nine-cell SP-CRLH microstrip coupler. (b) Phase coupling edge-coupled directional coupler. (c) Coupled-line MTM coupler having co-directional phase but contra-directional power flow.

CRLH coupler are purely imaginary in the coupling range. Another backward directional coupler, composed of a conventional microstrip line and SP-CRLH line, was proposed in [25] as shown in Fig. 2.12(b). This coupler is functionally backward but it is based on a forward-type coupling phenomenon. The coupler is shown to exhibit broad bandwidth and tight coupling characteristics.

A coupled-line MTM coupler having co-directional phase but contra-directional power flow was proposed in 2004 [94]. Comprising a microstrip line, edge-coupled to a negative-refractive-index line, the coupler exhibits co-directional phase but contra-directional Poynting vectors on the lines, and this leads to backward coupling. Hence, the coupler is proposed to support complex coupled modes and the resulting exponential field decay enables enhanced coupling with moderate line lengths and spacing.

Some other passive components were designed based on SP-CRLT TLs with dual-band and compact properties. For example, a dual-band quadrature hybrid was designed by replacing the -90° RH branches by SP-CRLH branches exhibiting -90° and -270° phase shifts at two frequencies [95]. A dual-band rat-race coupler, or ring hybrid, based on the same principle as this SP-CRLH dual-band quadrature hybrid was also proposed in [95]. A dual-band Wilkinson power divider was introduced by replacing the -90° branches of the conventional Wilkinson power divider by SP-CRLH branches inducing a phase of $+90^\circ$ (LH range, phase advance) at the one frequency and the phase shift of -90° (RH range, phase delay) at the other frequency [95].

Substrate Integrated Waveguide (SIW) Implementation

As the deployment of microwave technologies, broadband and high resolution microwave applications are critical for the evolution of wireless systems as they are naturally supported by the use of microwaves. The availability of technology with cost-effective, suitable for the mass-production of components and systems is expected for the developing wireless communication systems. Hence, substrate-integrated waveguide (SIW) is proposed as high-density integration technique with a low-cost fabrication process [96–98].

Figure 2.13: Examples of the SIW implementations of SP-CRLH TLs. (a) A SP-CRLH SIW with transverse slots. (b) SP-CRLH SIWs with interdigital slots. (c) SP-CRLH HMSIW with interdigital slots.

SIWs are integrated waveguide-like structures, which make the non-planar rectangular waveguide into a planar form and are compatible with the other planar structures, such as standard printed circuit board (PCB) or low-temperature co-fired ceramic (LTCC). The propagation characteristics exhibited by SIW structures are similar to the ones of rectangular waveguides, including the most advantages such as high quality-factor and high power-handling capability, and self-consistent electrical shielding [99]. The SIW technology has been used to many microwave components, such as

post and cavity filters [97], directional couplers [100], circulators [101] and slot array and leaky antennas [102].

Few SP-CRLH SIWs have been designed, such as the antenna with transverse slots etched on the waveguide surface which are bridged by metallic strips located in a plane slightly below the surface as shown in Fig. 2.13(a) [103], the antenna with interdigital capacitors etched on the waveguide surface as shown in Fig. 2.13(b) [104]. Fig. 2.13(c) shows the two SP-CRLH half mode substrate integrated waveguide (HMSIW) LWAs [104].

Their propagation properties and radiation characteristics are studied extensively to demonstrate the backfire-to-endfire beam-steering capabilities through frequency scanning. The radiating structures are realized by etching periodically transverse slots in the upper side of the waveguide and the ground. The slot contributes the series capacitor as well as a radiator leading to a CRLH LW application. Under the configuration of HMSIW, wave can propagate and radiate both below and above the cutoff frequency of HMSIW while still keeping the half mode field distribution. The antennas behave the advantages of low profile, low cost, and low weight, while they also have unique features, such as high directivity, quasi-omnidirectional radiation, miniaturized size, continuous beam-steering capabilities covering both the backward and forward quadrants, etc., providing much design flexibility for the real applications [104].

Surface-Mount Technology Chip Components Implementation

Many lumped implementations are also possible for SP-CRLH TLs. For instance, Fig 2.14(a) shows the photograph of a SP-CRLH TL implemented in the form of the series connection of an SMT-chips LH section and of a conventional RH microstrip line [95]. Figs. 2.14(b) and 2.14(c) show a dual-band quarter-wavelength short-circuited stub and a dual-band quarter-wavelength open-circuited stub, respectively [95]. Fig 2.14(d)

Figure 2.14: Examples of the SMT chip components implementations of SP-CRLH TLs. (a) A SP-CRLH TL implemented in the form of the series connection of an SMT-chips LH. (b) A dual-band quarter-wavelength short-circuited stub. (c) A dual-band quarter-wavelength open-circuited stub. (d) a dual-band LH-SMT/RH-microstrip quadrature hybrid.

shows a dual-band LH-SMT/RH-microstrip quadrature hybrid operating at the two frequencies ($f_1 = 0.93$ GHz and $f_2 = 1.78$ GHz) [95].

2.5 Double-Stub Structures

Back in 1950, the periodic guide with systematic modifications of resonators, such as double-stub structures and symmetrical triple-stub structures as shown in Fig. 2.15 were analyzed [105].

Figure 2.15: Periodic strip TLs. (a) A parallel-plate line loaded with uniform stubs. (b) Inductive double-stub structure. (c) Triple-stub multiply-periodic structure.

The qualitative analysis of such structures had been carried out using TL theory [105]. Fig. 2.16 shows the dispersion characteristics of the periodic strip TLs [70]. It was presented that the useful properties of the double-stub periodic structure occur when the ratio l_l/l_s is not too larger [70]. Fig. 2.16(a) shows that cutoff frequencies of ω at $\beta = \pi/2p$ for the double-stub structure occur at all points where the ordinate $\pi/2p$ cuts the curves of the two uniformly-loaded lines. The cutoff frequencies depend upon the alternate loaded components and can be derived when adjacent resonators are oscillating in antiphase [105]. The dispersion characteristic of the double-stub

(a)

(b)

Figure 2.16: Dispersion characteristics of the periodic strip TLs. (a) Inductive double-stub structure. (b) Triple-stub multiply-periodic structure.

periodic structure is similar to that of the simple structure until the phase constant approaches the values at which the cutoffs occur. It was proposed that when stubs were double periodically inserted in a finite waveguide, it can lead simultaneously to transmittance resonances in the gaps and antiresonances in the bands that widen into new bands and gaps [106, 107].

It is noted that the number of degrees of freedom of the system is doubled for the double periodic structure and therefore there are twice the number of branches in the frequency characteristics of single periodic structure [70]. For periodic structures with more than two lengths of stubs, the number of branches in the dispersion characteristic will equal the number of different lengths of resonators; i.e., the number of frequencies corresponding to a given phase constant is equal to the number of degrees of freedom associated with each section of the line. Fig. 2.16(b) shows one of many possible dispersion characteristics for a structure in which there are three stubs per section and every third is modified as shown in Fig. 2.15(c).

Although the reasoning deals with the modification of every N th resonator, the results may be generalized to a periodic guide consisting of like groups of N resonators [105]. Similar dispersion characteristics can be obtained by keeping all resonators the same length and modifying their separation in a periodic manner. However, this type of modification is less effective when the dispersion characteristics show low values of phase velocity at $\lambda_g = 2p$ compared with the free-space velocity. Although qualitative analysis of the structures are carried out using TL theory, the authors of [105] mentioned that further discussion does not seem justified since no radically new features are introduced and the effects produced are small. Hence, not too many works have been done on the traditional RH double or triple periodic structures. Nevertheless, some new and interesting unique properties have been found in the DP-CRLH TLs and will be introduced in this thesis.

2.6 Singular Perturbation Method based on Multiple Space Scales

The analysis for determining the propagation constant (phase constant and attenuation constant) and radiation characteristics of the TLs is necessary to get more physical insights. Previously, many researchers have investigated on the SP-CRLH TLs based on the TL theory, the Floquet Theorem [24] and the equivalent circuit approach [108]. In these works, the analysis used is static methods or quasi-static methods. The nature of the mode of propagation is considered to be a pure transverse electric-magnetic (TEM) mode and the TL characteristics are calculated from the electrostatic capacitance and inductance of the TLs, but the hybrid nature of the mode of the propagation should be taken into account which is more appropriate [47]. Especially for microstrip lines, they are not capable of supporting a pure TEM mode wave, and small longitudinal components of both the electric and magnetic fields need to be considered to satisfy the boundary conditions at the dielectric-air interface. These hybrid modes cannot be fully described in terms of static capacitances and inductances only. Because the power density distributions of the symmetric E component, which is very close and similar to the transverse magnetic (TM) mode comparing with other higher order modes [109], the dispersion characteristics of the TM mode are investigated to determine the propagation constant.

The singular perturbation procedure [39], by starting from the exact solution of a related un-perturbed problem, comprises mathematical methods to find an approximate solution to a problem which cannot be solved exactly by full wave analysis. When an approximate solution to a problem is needed which cannot be solved exactly and if the problem can be formulated by adding a "smallness" term to the mathematical description of the exactly solvable problem, the perturbation theory may be used. Perturbation theory leads to an expression for the desired solution in terms

Figure 2.17: Analytical model of periodic structures.

of a formal power series in some "smallness" parameter that quantifies the deviation from the exactly solvable problem. When a series smallness parameter is referred as δ , for the structure has a periodical perturbation of material constants that occurs along z -axis as shown in Fig. 2.17, the approximation to the full solution " H_y " may be given as

$$H_y = \delta^0 H_{y0} + \delta^1 H_{y1} + \delta^2 H_{y2} + \cdots + \delta^n H_{yn}, \quad (2.27)$$

where H_{y0} would be the known solution to the exactly solvable initial problem and $H_{y1}, H_{y2}, \cdots, H_{yn}$ represent the higher-order terms which may be found iteratively by some systematic procedure [39].

For a thin homogeneous isotropic medium, because the wave variation is weak in x -direction as shown in Fig. 2.17, a TM mode wave is considered here with revernishing field components H_y, E_x and E_z , having no variation in the y -direction [20]. Based on the Maxwell's equations, the differential equations satisfying the fields for each order is derived as [110]

$$\frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial z^2} + \omega^2 \mu \varepsilon H_y = \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial z} \frac{\partial H_y}{\partial z}, \quad (2.28)$$

where $\varepsilon = f(z)$, which is a function of z , is the equivalent electric permittivity of the investigated structure. Second order space scale is enough for analyzing the LWA based on the periodic structure [110]. All of the necessary information, like dispersion

characteristics, radiation efficiency and radiation angle, can be calculated. Therefore, space scales is introduced until second order:

$$z_0 = z, \quad z_1 = \delta z, \quad z_2 = \delta^2 z. \quad (2.29)$$

The original space derivative transforms is,

$$\frac{\partial}{\partial z} = \frac{\partial z_0}{\partial z} \frac{\partial}{\partial z_0} + \frac{\partial z_1}{\partial z} \frac{\partial}{\partial z_1} + \frac{\partial z_2}{\partial z} \frac{\partial}{\partial z_2} = \frac{\partial}{\partial z_0} + \delta \frac{\partial}{\partial z_1} + \delta^2 \frac{\partial}{\partial z_2}. \quad (2.30)$$

Because $\frac{\partial}{\partial z_2} \gg \frac{\partial}{\partial z_1}$ [39], to simplify the analysis, it is usually assumed that $\frac{\partial}{\partial z_1} = 0$ [110]. Therefore,

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial z_0} + \delta^2 \frac{\partial}{\partial z_2}. \quad (2.31)$$

and

$$\frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z_0^2} + 2\delta^2 \frac{\partial^2}{\partial z_0 \partial z_2}. \quad (2.32)$$

Substituting (2.31) and (2.32) into the differential equations (2.28), and the solutions will be obtained as the form composited by a power series expansion:

$$H_y = \delta^0 H_{y0} + \delta^1 H_{y1} + \delta^2 H_{y2}. \quad (2.33)$$

It is noted that in addition to the dispersion characteristic derived from the zeroth-order problem, the perturbation method also gives the information about the radiation efficiency and the radiation angle for the optimum radiation efficiency, which are derived from the first-order and second-order problem [110].

2.7 Conclusion

MTMs and SP-CRLH TLs have been comprehensively reviewed with a focus on the design principles and recent trends. It has been noted that there exists a clear direction of future explorations on SP-CRLH based microwave applications as small LWAs with backfire-to-endfire frequency-scanning capability and compact guided wave applications. There is a room for further significant research on DP-CRLH TLs in order to examine different double periodic unit-cell geometries for their effect on the new leaky-wave response. Based on that trend, this thesis marks a significant contribution, since it presents a complete study of a new DP-CRLH TL.

Following the review of SP-CRLH TLs, it is seen that the recent trend of analysis has been based on TL theory and the Floquet Theorem. However, the nature of the mode of the propagation is considered to be a pure TEM mode and the TL characteristics are calculated from the electrostatic capacitance and inductance of the TLs, but the hybrid nature of the mode of the propagation should be taken into account. Based on the review of open literature, in addition to the TL theory, the singular perturbation procedure is used to generate the dispersion characteristics of the TM mode of the DP-CRLH TLs to determine the propagation constant and radiation characteristics. In addition to the dispersion characteristic derived from the zeroth-order problem, the perturbation method also gives the information about the radiation efficiency and the radiation angle for the optimum radiation efficiency, which are derived from the first-order and second-order problem.

The theoretical work in [29] shows that DP-CRLH TLs is a superior choice for designing compact LWAs. With that motivation, this thesis proposes that TLs double periodically loaded with alternate capacitances and inductances are suitable structures to implement DP-CRLH TLs. Although the structure proposed in this thesis carries certain limitations, the new method to design compact LWAs based on DP-CRLH TLs

is still attractive and promising, and it may lead to a preferred direction for future explorations whereby further improvements may overcome the limitations, and may render an enhanced performance.

Chapter 3

PROPAGATION CHARACTERISTICS OF DP-CRLH TLs

3.1 Introduction

This chapter presents the fundamental transmission line (TL) theory of double periodic composite right/left handed (DP-CRLH) TLs. Understanding the propagation characteristics of DP-CRLH TLs is important for designing the applications as leaky-wave antennas (LWAs). The TL analysis gives the qualitative information about the propagation characteristics of DP-CRLH TLs. The period of the unit cell is much smaller than the guide wavelength ($\Lambda \ll \lambda_g$) for electromagnetic homogeneity. When this condition is satisfied for the analyzed DP-CRLH TLs, the TL can be considered as an effective transmission medium [2]. Because DP-CRLH TLs are effectively homogeneous structures, they can be essentially modeled by TLs, whose propagation direction represents any direction in the material.

Previously, many researchers have made investigations on single periodic composite

right/left handed (SP-CRLH) TLs based on the TL theory, Floquet theory [1, 2] and equivalent circuit theory [108]. TL theory provides a powerful tool for the analysis and design of SP-CRLH TLs. In the same way, the analysis for DP-CRLH TLs is aimed at determining the propagation characteristics through the dispersion behavior of TLs based on the complex propagation constant (phase and attenuation constants). Based on the Floquet theory and the TL theory, dispersion characteristic of DP-CRLH TLs is analyzed, and compared with SP-CRLH TLs. From the dispersion behavior, three bands, the new-found leaky-wave (LW) region, the left-handed (LH) region and the conventional right-handed (RH) region, are observed in the fast wave region which indicates that DP-CRLH TLs are able to radiate in these bands. It has been also demonstrated that the group velocity and phase velocity are parallel in the new-found LW region. The expected radiation regions of DP-CRLH TLs are the new forward LW radiation at a low frequency, backward direction radiation in the LH region and the forward direction radiation again in the conventional RH region.

The TL theory of DP-CRLH TLs is presented in two progressive steps. Starting with the model of the host TLs loaded with L and C (Section 3.2), the qualitative information about the propagation characteristics of DP-CRLH TLs is obtained. Besides the CRLH property, a new LW radiation is observed at a lower frequency with narrow bandwidth for the TL double periodically loaded with both alternate capacitances and inductances. Next, based on the analysis of the equivalent circuit model of ideal $L - C$ network of DP-CRLH TLs, the expressions for the cutoff frequencies delimiting the operational radiation frequencies will be derived in Section 3.3. In addition, it is found that TLs double periodically loaded with either alternate capacitances or alternate inductances also contribute the new LW behavior.

Figure 3.1: Equivalent circuit of the structures. (a) Unit cell of L-C network featuring the host TL medium (Z_0, k) of SP-CRLH TLs. (b) Unit cell for the L-C network featuring the host TL medium (Z_0, k) of DP-CRLH TLs. (c) Line implementation in the form of a periodic network.

3.2 Model of Host TLs Loaded with L and C

A TL periodically loaded with capacitances and inductances can be considered as an effective transmission medium provided that the dimensions of the unit cell are small compared to the operating wavelength [20]. The model of SP-CRLH TLs consists of uniform shunt inductances L_L and uniform series capacitances C_L featuring the host medium with period Λ_S as shown in Fig. 3.1(a) [20, 24]. The model represents the general form of a structure with SP-CRLH attributes. Many applications including filters and LWAs have been designed based on SP-CRLH TLs [2, 33, 34, 84].

The model of proposed DP-CRLH TLs consists of double periodically loaded shunt inductances L_{Li} and double periodically loaded series capacitances C_{Li} ($i=1,2$) featuring the host medium with period Λ_D as shown in Fig. 3.1(b). The realization of the DP-CRLH TLs requires the continuous medium with small physical dimensions provided by a host TL medium (Z_0, k) double periodically loaded with alternate inductances (L_{L1}, L_{L2}) and capacitances (C_{L1}, C_{L2}) that may be realized using either lumped printed or discrete (chip) inductors and capacitors. Z_0 and k refer to the characteristic impedance and wave number of the TL used. Many cells are arranged periodically into a one-dimensional (1-D) array, and the array is excited at guide wavelengths longer than the cell dimension to enforce the homogeneity condition, so that the resulting periodic structure can be regarded as distributed.

The period Λ_D and Λ_S are normally small compared to wavelength. The loaded inductances L_{L1} , L_{L2} and capacitances C_{L1} , C_{L2} are different within one unit cell, and the unit cell of DP-CRLH TLs can be seen as combination of two unit cells of different SP-CRLH TLs. Therefore the proposed structure is named as DP-CRLH TL. The period of the analyzed DP-CRLH TL is chosen as double the period of the SP-CRLH TL ($\Lambda_D = 2\Lambda_S$) for simple analysis and without loss of generality, so that the total length of the DP-CRLH TL and the SP-CRLH TL are comparable when the same amount of capacitances and inductances are loaded in the host medium.

The transmission matrix [1, 2, 47] of the two-port-network unit cell of DP-CRLH TLs can be written as

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = U_1 * U_2 * \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}, \quad (3.1)$$

where

$$U_i = \begin{bmatrix} \cos(k\frac{\Lambda_D}{2}) & jZ_0\sin(k\frac{\Lambda_D}{2}) \\ j\frac{1}{Z_0}\sin(k\frac{\Lambda_D}{2}) & \cos(k\frac{\Lambda_D}{2}) \end{bmatrix} \begin{bmatrix} 1 & Z_{C_L i} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{L_L i} & 1 \end{bmatrix} \quad (3.2)$$

($i = 1, 2$) are the ABCD matrices of one part of unit cell of DP-CRLH TLs, which is the same as the transmission matrix of unit cell of SP-CRLH TLs with period $\Lambda_S = \Lambda_D/2$ as shown in [20]. $Z_{C_L i} = 1/(j\omega C_{L i})$ and $Y_{L_L i} = 1/(j\omega L_{L i})$ ($i = 1, 2$) represent the impedances of double periodically loaded series capacitances $C_{L i}$ and admittances of double periodically loaded shunt inductances $L_{L i}$, where $\omega = 2\pi f$ is the angular frequency. The series capacitances and shunt inductances are repeated with period of Λ_D , which is explicitly provided by the TL segments. Z_0 and k refer to the characteristic impedance and wave number of the TL used. The parameters A and D are given by

$$\left\{ \begin{array}{l} A = \cos^2(k\frac{\Lambda_D}{2})[(1 + Z_{C_L 1}Y_{L_L 1})(1 + Z_{C_L 2}Y_{L_L 2})] \\ \quad - \sin^2(k\frac{\Lambda_D}{2})[1 + Z_{C_L 2}Y_{L_L 2} + Z_0^2Y_{L_L 1}Y_{L_L 2}] \\ \quad + j\cos(k\frac{\Lambda_D}{2})\sin(k\frac{\Lambda_D}{2})[(2 + Z_{C_L 1}Y_{L_L 1})Z_0Y_{L_L 2} \\ \quad \quad + (1 + Z_{C_L 2}Y_{L_L 2} + \frac{Z_{C_L 1}}{Z_0})Z_0Y_{L_L 1}] \\ D = \cos^2(k\frac{\Lambda_D}{2})(1 + Z_{C_L 2}Y_{L_L 1}) - \sin^2(k\frac{\Lambda_D}{2})[1 + Z_{C_L 1}Y_{L_L 1} - \frac{1}{Z_0^2}Z_{C_L 1}Z_{C_L 2}] \\ \quad + j\cos(k\frac{\Lambda_D}{2})\sin(k\frac{\Lambda_D}{2})[\frac{Z_{C_L 2}(2 + Z_{C_L 1}Y_{L_L 1})}{Z_0} + Z_0Y_{L_L 1} + \frac{Z_{C_L 1}}{Z_0}] \end{array} \right. \quad (3.3)$$

Based on the Floquet theorem, if V_n is a fundamental matrix solution of the periodic TL with period Λ_D , then

$$\left\{ \begin{array}{l} V_{n+1} = V_n e^{-\gamma\Lambda_D} \\ I_{n+1} = I_n e^{-\gamma\Lambda_D}, \end{array} \right. \quad (3.4)$$

Figure 3.2: The flow chart of dispersion characteristics calculation procedure based on the periodic structure model of host TLs loaded with L and C.

so that, the complex propagation constant ($\gamma = \alpha + j\beta$) is derived as [47]

$$\cosh\gamma\Lambda_D = \frac{A + D}{2} = \frac{P\cos^2(k\frac{\Lambda_D}{2}) + Q\sin^2(k\frac{\Lambda_D}{2}) + R\sin(k\frac{\Lambda_D}{2})\cos(k\frac{\Lambda_D}{2})}{2}, \quad (3.5)$$

where

$$\begin{cases} P = 2 + Z_{CL1}Y_{LL1} + Z_{CL2}Y_{LL2} + Z_{CL1}Y_{LL2} + Z_{CL2}Y_{LL1} + Z_{CL1}Y_{LL1}Z_{CL2}Y_{LL2} \\ Q = -2 - Z_{CL1}Y_{LL1} - Z_{CL2}Y_{LL2} - Z_0^2Y_{LL1}Y_{LL2} - \frac{Z_{CL1}Z_{CL2}}{Z_0^2} \\ R = j[(Y_{LL1} + Y_{LL2})(2Z_0 + \frac{Z_{CL1}Z_{CL2}}{Z_0}) + (Z_{CL1} + Z_{CL2})(\frac{2}{Z_0} + Z_0Y_{LL1}Y_{LL2})]. \end{cases} \quad (3.6)$$

Here α and β are the attenuation and phase constants of the DP-CRLH TL, respectively.

The flow chart of the dispersion characteristics calculation procedure based on the periodic structure model of host TLs loaded with L and C is shown in Fig. 3.2. Fig. 3.3 and Fig. 3.4 shows the dispersion behavior of the proposed DP-CRLH TL using the average phase shift incurred per unit cell ($\beta\Lambda_D$) with period of $\Lambda_D = 10$ mm (being small compared to wavelength at 1.6 GHz) (red solid line) and the dispersion behavior ($\beta\Lambda_S$) of the SP-CRLH TL with period of $\Lambda_S = 5$ mm (blue dashed line). It shows the typical LH passband ($f_L \sim f_{\Gamma1}$) and conventional RH passband ($f > f_{\Gamma2}$) for both the DP-CRLH TL and the SP-CRLH TL. When the dispersion characteristic of DP-CRLH TL is in the fast wave region ($\beta < k_0$), the structure is able to radiate, which can be used for potential LWAs design [1,2,28,32], where the k_0 is the free space wave number. Frequencies, f_{nR-L} , $f_{\Gamma1}$ and $f_{\Gamma2}$ in Fig. 3.3 mark the limits of various passbands so leaky-mode radiation occurs just above f_{nR-L} , just below $f_{\Gamma1}$ and above $f_{\Gamma2}$. It is noted that the slope of k_0 line for DP-CRLH TL shown in Fig. 3.3 is twice that of the SP-CRLH TL, since length of per unit cell incurring the average phase shift of the DP-CRLH TL ($\Lambda_D = 10$ mm) is twice as much as the period of SP-CRLH

(a)

(b)

Figure 3.3: Dispersion characteristics of the proposed structures. (a) Over the frequency range 0 to 10 GHz, and (b) in the vicinity of the new LW region. DP-CRLH TL (red solid line): $C_{L1}=0.5$ pF, $C_{L2} = 1$ pF, $L_{L1} = 3.3$ nH, $L_{L2} = 6.8$ nH, $Z_0 = 50\Omega$, $\Lambda_D = 10$ mm. SP-CRLH TL (blue dashed line): $C_L = 0.5$ pF, $L_L = 3.3$ nH, $Z_0 = 50\Omega$, $\Lambda_S = 5$ mm.

Figure 3.4: Dispersion characteristics of the proposed DP-CLRHL structure compared with the dispersion curve of SP-CRLH TL.

TL ($\Lambda_S = 5$ mm). Fig. 3.3(a) shows the DP-CRLH TL radiates LWs in LH mode ($f = f_{\Gamma 1}$) and RH mode ($f = f_{\Gamma 2}$) like the SP-CRLH TL.

In addition to the CRLH property as SP-CRLH TLs, the DP-CRLH TL provides a new RH passband ($f_{nR_L} \sim f_{nR_H}$) below the LH region as shown in Fig. 3.3. The enlarged dispersion characteristic in the new leak-wave region due to the double periodically loaded capacitances and inductances is shown in Fig. 3.3(b). The new LW occurs at very low frequency ($f_{nR_L} \sim f_{nR_LW}$) with narrow bandwidth. The DP-CRLH TL is expected to radiate forward LW in this region.

The cutoff frequencies between the new LW region, the typical LH region and the conventional RH region are affected by the difference between the loaded series capacitances (C_{L1}/C_{L2}) and the difference between the loaded shunt inductances (L_{L1}/L_{L2}). Fig. 3.5 shows the effect on dispersion behavior by changing the ratio of capacitances (C_{L1}/C_{L2}) while the loaded inductances are equal ($L_{L1} = L_{L2}$). The calculation follows the procedure as the flow chart shown in Fig. 3.2. It is noted that TLs with

Figure 3.5: Dispersion characteristics of DP-CRLH TLs for different ratio of capacitances (C_{L1}/C_{L2}) while keeping the loaded inductances equal, ($C_{L1} = 0.5$ pF, $L_{L1} = L_{L2} = 3.3$ nH). (a) Dispersion curves. (b) Cutoff frequencies.

(a)

(b)

Figure 3.6: Dispersion characteristics of DP-CRLH TLs for different ratio of inductances (L_{L1}/L_{L2}) while keeping the loaded capacitances equal, ($L_{L1} = 3.3$ nH, $C_{L1} = C_{L2} = 0.5$ pF). (a) Dispersion curves. (b) Cutoff frequencies.

double periodically loaded different capacitances ($C_{L1} \neq C_{L2}$) and uniform inductances ($L_{L1} = L_{L2}$) work as a DP-CRLH TL with period of $\Lambda = \Lambda_D$ and can provide a new LW radiation at the lower frequency. When $C_{L1} = C_{L2}$, the structure works as a SP-CRLH TL with period of $\Lambda = \Lambda_S = \Lambda_D/2$, and no passband and LW characteristic occur below the LH passband as shown in Fig. 3.5 (bold blue dashed line). Fig. 3.5(b) explains the changes of the cutoff frequencies depending on the ratio of the loaded capacitances. As the ratio increases, the lower cutoff frequency (f_{nR-L}) of the new LW decreases (bold red solid line) while the upper cutoff frequency (f_{nR-H}) is unchanged when $C_{L1}/C_{L2} < 1$. It then decreases slightly when $C_{L1}/C_{L2} > 1$ (blue dashed line). The lower cutoff frequency of the LH passband (f_L) decreases when $C_{L1}/C_{L2} < 1$, and then remains unchanged when $C_{L1}/C_{L2} > 1$ (thin black solid line) while the upper cutoff frequency ($f_{\Gamma 1}$) is unchanged and independent of the ratio (C_{L1}/C_{L2}) (thin green dotted line). The lower cutoff frequency of conventional RH passband ($f_{\Gamma 2}$) decreases distinctly (bold pink dotted line). It is noted that when $C_{L1}/C_{L2} = 1$, no LW occurs below the LH passband and the structure works as SP-CRLH TLs. The cutoff frequencies f_{nR-H} and f_L drop to zero when $C_{L1}/C_{L2} = 1$ as shown in the Fig. 3.5.

Figure 3.6 shows the effect on the dispersion behavior of DP-CRLH TLs by changing the ratio of loaded inductances (L_{L1}/L_{L2}) while keeping the loaded capacitances equal ($C_{L1} = C_{L2}$). The effect is almost similar with the variation by the difference of capacitances except that the upper-cutoff frequency of the LH passband ($f_{\Gamma 1}$) decreases with an increase in the ratio L_{L1}/L_{L2} as shown in Fig. 3.6(b). It is noted that the TLs with double periodically loaded inductances ($L_{L1} \neq L_{L2}$) and uniform capacitances ($C_{L1} = C_{L2}$) also provides the new LW radiation. It is a good way to get a better performance of the DP-CRLH TLs by combining a pair of different series capacitances and a pair of different shunt inductances together double periodically loaded in the TL.

3.3 Equivalent Circuit Model of Ideal $L - C$ Network of DP-CRLH TLs

In order to analyze more in detail about the proposed DP-CRLH TLs, especially on the relationships about the cutoff frequencies, the equivalent circuit of the ideal pure RH material (Fig. 3.7(a)), the ideal SP-CRLH TLs (Fig. 3.7(b)) and the ideal DP-CRLH TLs (Fig. 3.7(c)) are used here. The period of the unit cell is also considered to be much smaller than the guide wavelength ($\Lambda \ll \lambda_g$) for electromagnetic homogeneity. When this condition is satisfied, the TL can be considered as an effective transmission medium [2]. The unit cell of SP-CRLH TL model shown in [2, 20, 24] consists of a series inductance L_R cascading with a capacitance C_L and a shunt capacitance C_R paralleling with an inductance L_L , which represents the most general form of a structure with CRLH attributes with period Λ_S . The unit cell of DP-CRLH-TL consists of double periodically loaded alternate shunt inductances (L_{L1} , L_{L2}) and alternate series capacitances (C_{L1} , C_{L2}) featuring the host medium with period $\Lambda_D = 2\Lambda_S$ as shown in Fig. 3.7(c). The period of DP-CRLH TL is also chosen as double the period of SP-CRLH TL for simple analysis and without loss of generality.

The general transmission matrix for the two-port-network unit cell of the DP-CRLH TL shown in Fig. 3.7(c) is obtained by multiplying the individual element matrices [2, 24, 47], i.e.,

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = U_1 * U_2 * \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_{n+1} \\ I_{n+1} \end{bmatrix}, \quad (3.7)$$

where

$$U_i = \begin{bmatrix} 1 & Z_{LR} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & Z_{CLi} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{CR} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y_{LLi} & 1 \end{bmatrix}, \quad (3.8)$$

Figure 3.7: Equivalent circuit of the structures. (a) Pure RH TL. (b) SP-CRLH TL. (c) DP-CRLH TL. (d) Line implementation in the form of a periodic network.

($i = 1, 2$) are the ABCD matrices of one part of the double unit cell, which is the same as the transmission matrix of SP-CRLH TLs [2]. $Z_{C_{Li}} = 1/j\omega C_{Li}$, $Y_{L_{Li}} = 1/j\omega L_{Li}$ ($i = 1, 2$) are the impedances and admittances of the double periodic loaded elements and $Z_{L_R} = j\omega L_R$, $Y_{C_R} = j\omega C_R$ are the per-unit impedance and admittance contributed by the equivalent per-unit shunt capacitance and series inductance of the

host medium and they are having uniform values. The parameters A and D can be obtained as

$$\begin{cases} A = \prod_{i=1}^2 [(Z_{L_R} + Z_{C_L i})(Y_{C_R} + Y_{L_L i}) + 1] + (Z_{L_R} + Z_{C_L 1})(Y_{C_R} + Y_{L_L 2}), \\ D = (Z_{L_R} + Z_{C_L 2})(Y_{C_R} + Y_{L_L 1}) + 1. \end{cases} \quad (3.9)$$

Based on the Floquet theorem, if V_n and I_n are the fundamental solutions of the periodic TL with period Λ , then

$$\begin{cases} V_{n+1} = V_n e^{-\gamma \Lambda_D}, \\ I_{n+1} = I_n e^{-\gamma \Lambda_D}, \end{cases} \quad (3.10)$$

so that, the dispersion relation of an infinitely DP-CRLH TL is derived by applying Floquet theorem to ABCD matrices of the unit cell (3.9) [24, 47]

$$\begin{aligned} \cosh(\gamma \Lambda_D) = & \frac{1}{2} \left\{ \left[\left(\frac{C_R}{C_{L1}} + \frac{L_R}{L_{L1}} \right) - \left(\frac{1}{\omega^2 C_{L1} L_{L1}} + \omega^2 C_R L_R \right) + 1 \right] \right. \\ & \left[\left(\frac{C_R}{C_{L2}} + \frac{L_R}{L_{L2}} \right) - \left(\frac{1}{\omega^2 C_{L2} L_{L2}} + \omega^2 C_R L_R \right) + 1 \right] \\ & + \left[\left(\frac{L_{L2}}{C_{L1}} + \frac{L_R}{C_R} \right) - \left(\frac{1}{\omega^2 C_{L1} L_R} + \omega^2 L_R L_{L2} \right) \right] \\ & \left. + \left[\left(\frac{C_R}{C_{L2}} + \frac{L_R}{L_{L2}} \right) - \left(\frac{1}{\omega^2 L_{L1} C_{L2}} + \omega^2 C_R L_R \right) + 1 \right] \right\}. \end{aligned} \quad (3.11)$$

Here ω and $\Lambda_D = 2\Lambda_S$ are the angular frequency and period of the loaded DP-CRLH TL, respectively. $\gamma = \alpha + j\beta$ is the complex propagation constant, and $\beta\Lambda_D$ is the phase change across one complete section of the unit cell with period of Λ_D .

The dispersion characteristics of the DP-CRLH TL with alternate loaded shunt inductances (Fig. 3.8(a), red solid line) and alternate loaded series capacitances (Fig. 3.8(b) red solid line) using the average phase shift incurred per unit cell ($\beta\Lambda_D$) with period of $\Lambda_D = 2\Lambda_S$ are obtained. The phase velocity $v_p = \omega/\beta$, and the group velocity

(a)

(b)

Figure 3.8: Dispersion characteristics of the proposed DP-CRLH TLs. (a) DP-CRLH TLs depending on loaded alternate shunt inductors and same capacitors, $C_{L1} = C_{L2} = 0.5$ pF. (b) DP-CRLH TLs depending on loaded alternate series capacitors and same shunt inductors, $L_{L1} = L_{L2} = 0.5$ nH. For all the analyzed structures: $C_R = 0.3$ pF, $L_R = 0.4$ nH, $\Lambda_S = 5$ mm.

$v_g = (d\omega)/(d\beta)$, where $\omega = 2\pi f$ is the angular frequency. The typical dispersion behavior of left-handedness exhibiting antiparallel phase and group velocities is obtained from f_L to $f_{\Gamma 1}$. The conventional right-handedness exhibiting parallel phase and group velocities is obtained when $f > f_{\Gamma 2}$. When the dispersion curve is in the fast wave region ($\beta < \omega\sqrt{\varepsilon_0\mu_0}$), the structure is able to radiate and can be used for LWAs which radiate backward at $f_{L-LW} < f < f_{\Gamma 1}$ and forward at $f > f_{\Gamma 2}$ [1, 2, 28, 32]. In addition to the CRLH property, Fig. 3.8 shows that below the LH passband ($f < f_L$), a new RH passband ($f_{nR-L} \sim f_{nR-H}$) is obtained in the DP-CRLH TL, and the LW occurs at very low frequency ($f_{nR-L} \sim f_{nR-LW}$) with narrow bandwidth due to double periodically loaded alternate shunt inductances (Fig. 3.8(a)) or alternate series capacitances (Fig. 3.8(b)) when the frequency is located in the fast wave region ($\beta < \omega\sqrt{\mu_0\varepsilon_0}$).

The dispersion characteristics ($\beta\Lambda_S$) of two SP-CRLH TLs loaded with different shunt inductances ($L_{L1} = 0.5$ nH, $L_{L2} = 0.7$ nH) are also shown in Fig. 3.8(a) comparing with the DP-CRLH TL loaded with corresponding alternate shunt inductances. The figure shows that the frequency characteristics in the conventional RH branch and the LH branch of DP-CRLH TLs are similar to that of the SP-CRLH TL and between the dispersion curves of the SP-CRLH TLs. Here, (3.3) is rewritten as

$$\begin{aligned} \cosh\gamma\Lambda_D &= \frac{A + D}{2} \\ &= \frac{1}{2} \left\{ \prod_{i=1}^2 \left[\frac{(\omega^2 C_R L_{Li} - 1)(1 - \omega^2 L_R C_{Li})}{\omega^2 C_{Li} L_{Li}} + 1 \right] + \left[\frac{(\omega^2 C_R L_{L2} - 1)(1 - \omega^2 L_R C_{L1})}{\omega^2 C_R C_{L1}} \right] \right. \\ &\quad \left. + \left[\frac{(\omega^2 C_R L_{L1} - 1)(1 - \omega^2 L_R C_{L2})}{\omega^2 C_{L2} L_{L1}} + 1 \right] \right\}. \end{aligned} \quad (3.12)$$

The dispersion relation of DP-CRLH TL is much more complex than the one of SP-CRLH TLs (2.8) as shown in Chapter 2.

- The cutoff frequencies of the lower edge of the RH branch ($f_{\Gamma 2}$) and the upper edge of the LH branch ($f_{\Gamma 1}$) at the spectral axis are obtained when the atten-

uation constant $\alpha = 0$ and the phase constant $\beta = 0$, so that the propagation constant $\gamma = \alpha + j\beta = 0$ as shown in Fig. 3.8 (blue dashed line and green dotted line). When $\gamma = \alpha + j\beta = 0$,

$$\cosh(\gamma\Lambda_D) = \cosh(0) = 1. \quad (3.13)$$

So that,

$$\begin{aligned} \frac{1}{2} \left\{ \prod_{i=1}^2 \left[\frac{(\omega^2 C_R L_{Li} - 1)(1 - \omega^2 L_R C_{Li})}{\omega^2 C_{Li} L_{Li}} + 1 \right] + \left[\frac{(\omega^2 C_R L_{L2} - 1)(1 - \omega^2 L_R C_{L1})}{\omega^2 C_R C_{L1}} \right] \right. \\ \left. + \left[\frac{(\omega^2 C_R L_{L1} - 1)(1 - \omega^2 L_R C_{L2})}{\omega^2 C_{L2} L_{L1}} + 1 \right] \right\} = 1, \end{aligned} \quad (3.14)$$

and then,

$$\begin{aligned} & \frac{(\omega^2 C_R L_{L1} - 1)(1 - \omega^2 L_R C_{L1})}{\omega^2 C_{L1} L_{L1}} \frac{(\omega^2 C_R L_{L2} - 1)(1 - \omega^2 L_R C_{L2})}{\omega^2 C_{L2} L_{L2}} \\ & + \frac{(\omega^2 C_R L_{L1} - 1)(1 - \omega^2 L_R C_{L1})}{\omega^2 C_{L1} L_{L1}} + \frac{(\omega^2 C_R L_{L2} - 1)(1 - \omega^2 L_R C_{L2})}{\omega^2 C_{L2} L_{L2}} \\ & + \frac{(\omega^2 C_R L_{L2} - 1)(1 - \omega^2 L_R C_{L1})}{\omega^2 C_R C_{L1}} + \frac{(\omega^2 C_R L_{L1} - 1)(1 - \omega^2 L_R C_{L2})}{\omega^2 C_{L2} L_{L1}} \} = 0. \end{aligned} \quad (3.15)$$

Two of the possible solutions are the series and shunt resonance frequencies:

$$\begin{cases} \omega_{se} = \sqrt{\frac{1/C_{L1} + 1/C_{L2}}{2L_R}}, \\ \omega_{sh} = \sqrt{\frac{1/L_{L1} + 1/L_{L2}}{2C_R}}, \end{cases} \quad (3.16)$$

respectively.

The cutoff frequencies of the upper edge of the LH branch ($\omega_{\Gamma 1}$) and the lower

edge of conventional RH branch ($\omega_{\Gamma 2}$) at the spectral axis are obtained by

$$\begin{cases} \omega_{\Gamma 1} = \min(\omega_{se}, \omega_{sh}), \\ \omega_{\Gamma 2} = \max(\omega_{se}, \omega_{sh}), \end{cases} \quad (3.17)$$

where $\omega = 2\pi f$ is the angular frequency.

When $L_{L1} = L_{L2}$ and $C_{L1} = C_{L2}$, the structure works as SP-CRLH TL with period of Λ_S . The cutoff frequencies become

$$\begin{cases} \omega_{\Gamma 1} = \min\left(\frac{1}{\sqrt{C_R L_L}}, \frac{1}{\sqrt{C_L L_R}}\right), \\ \omega_{\Gamma 2} = \max\left(\frac{1}{\sqrt{C_R L_L}}, \frac{1}{\sqrt{C_L L_R}}\right), \end{cases} \quad (3.18)$$

where $L_L = L_{L1} = L_{L2}$, $C_L = C_{L1} = C_{L2}$ as shown in [24].

- It is noted from Fig. 3.8 that the cutoff frequency of the lower edge of the new RH branch (f_{nR-L}) is located between that of corresponding SP-CRLH TLs

$$1/(2\sqrt{C_{L1}L_{L1}}) < f_{nR-L} < 1/(2\sqrt{C_{L2}L_{L2}}). \quad (3.19)$$

The numerical result can be calculated based on the dispersion relationship as shown in (3.3).

- Because of double periodically loaded alternate shunt inductances, Fig. 3.8 shows that the dispersion curve of DP-CRLH TL is similar to that of corresponding SP-CRLH TLs until the phase constant approaches one of the dispersion curves of SP-CRLH TLs, at which the cutoff frequencies of lower edge of LH branch (f_L) and upper edge of the new RH branch (f_{nR-H}) occur. At these cutoff frequencies, standing wave comes out with group velocity $v_g = 0$ and phase velocity $v_p = \pi$. When the attenuation constant $\alpha = 0$ and the phase change

across one complete section of the unit cell with period of Λ_D is π ($\gamma\Lambda_D = j\pi$), the cut off frequency of the lower edge of the LH branch f_L and the upper edge of the new RH branch $f_{n_{RH}}$ should be derived as shown in Fig. 3.3 (red solid line). However, it is much more difficult to calculate the concise expression as discussion above.

Observed from the dispersion characteristics of the DP-CRLH TL and SP-CRLH TLs loaded with the corresponding capacitances and inductances as shown in Fig. 3.3, it is noted that the cutoff frequencies of the upper edge of the new RH branch and the lower edge of the LH branch are matched with the dispersion curve of the single periodic structures when $\alpha\Lambda_S = 0$, $\beta\Lambda_S = \pi/2$. It seems that these cutoff frequencies can be derived from the dispersion relation of the SP-CRLH TL when $\gamma\Lambda_S = j\pi/2$ as

$$\cosh(\gamma\Lambda_S) = \cosh(j\pi/2) = 0. \quad (3.20)$$

Hence, from the dispersion relationship of SP-CRLH TLs as shown in (2.8),

$$\cosh(\gamma\Lambda_S) = \frac{(\frac{C_R}{C_L} + \frac{L_R}{L_L}) - (\frac{1}{\omega^2 C_L L_L} + \omega^2 C_R L_R) + 2}{2} = 0. \quad (3.21)$$

So that,

$$\omega^4 C_L L_L C_R L_R - \omega^2 C_L L_L (\frac{C_R}{C_L} + \frac{L_R}{L_L} + 2) + 1 = 0. \quad (3.22)$$

The cutoff frequency of the lower edge of the LH branch f_L and the upper edge of the new RH branch $f_{n_{RH}}$ can be calculated based on this equation. However, it is also difficult to get a formula explicitly with the clear indication of series/shunt resonant frequencies. Fortunately, the numerical solution of these cutoff frequencies can be obtained based on the dispersion relationship. Because the cutoff frequencies of the upper edge of the new RH branch and the lower

edge of the LH branch for the DP-CRLH TL are matched with the dispersion curve of the single periodic structure when $\alpha\Lambda_S = 0$ and $\beta\Lambda_S = \pi/2$, this also provides a method to estimate the cutoff frequencies of the DP-CRLH TL from the property of the corresponding SP-CRLH TLs.

3.4 Conclusion

Propagation characteristics of a 1-D periodic array of DP-CRLH TLs have been investigated based on two models. By invoking Floquet theorem, a unit-cell of the array has been analyzed using TL theory. In addition to the CRLH property, a new LW radiation is observed at a lower frequency with narrow bandwidth for the TL double periodically loaded with alternate capacitances and/or inductances. The expressions of the cutoff frequencies of each band edge are derived. In addition, it is found that TLs double periodically loaded with either alternate capacitances or alternate inductances will contribute the new LW behavior.

It has been noted that the proposed DP-CRLH TL supports triple bands LW radiations. The expected radiation regions are the new-found forward LW radiation at a low frequency, backward direction radiation in the LH region and the forward direction radiation again in the conventional RH region. These triple-band LW radiations, especially the new-found LW region at very low frequency, are actually the prime motivation behind the research presented in this thesis. Based on their importance in the design of compact LWAs and compact triple-band LWAs with backfire-to-endfire frequency-scanning capability, propagation characteristics of DP-CRLH TLs have been investigated in sufficient detail based on the TL theory discussed in this chapter and with the help of a singular perturbation procedure, which is discussed in the next chapter.

At last, it is pointed out that the LH TL is necessarily frequency dispersive [2].

A frequency-dispersive medium is a medium in which the propagation constant (β) is a nonlinear function of frequency, which results in frequency-dependent group delay. Hence, it is possible to demonstrate the LH property of the TL based on the frequency-dependent group delay, which is shown in Chapter 5. Another point is that the lossless DP-CRLH TLs considered here is only with lumped inductive/capacitive loading. The design of DP-CLRH structures which include frequency-dispersive nature of the material parameters $\mu(\omega)$ and $\varepsilon(\omega)$ is discussed in the singular perturbation procedure in Chapter 4.

Chapter 4

RADIATION CHARACTERISTICS OF DP-CRLH TLs

4.1 Introduction

A novel transmission line (TL) where capacitances and/or inductances are double periodically loaded has been proposed and analyzed based on TL theory in Chapter 3. In order to further analyze the radiation characteristics of the double periodic composite right/left handed (DP-CRLH) TLs, especially for the microstrip line implementation of the DP-CRLH TLs, a singular perturbation procedure is used in this chapter. The leaky-wave (LW) characteristics from the fast wave region in the DP-CRLH TLs implemented by the microstrip lines double periodically loaded with interdigital capacitors and shunt short-circuited stub inductances, as shown in Fig. 4.1, are analyzed with the help of the singular perturbation procedure. The new LW region is also found below the left-handed (LH) passband for the DP-CRLH TLs based on this analysis method. The analysis shows that the wave is forward radiated and the TLs work as

a right-handed material (RHM) in the new LW region. Dispersion characteristics and the radiation efficiency are investigated with the help of the singular perturbation procedure. The radiation efficiency is numerically estimated as a function of the antenna length, and the radiation angle is estimated in the LW region. Validating experiments are given later in Chapter 5.

The analysis for determining the radiation characteristics of the DP-CRLH TLs is necessary to get more physical insights and generate the guidelines for designing compact leaky-wave antennas (LWAs). Previously, the DP-CRLH TLs have been investigated based on the TL theory, the Floquet theorem and the equivalent circuit approach. In these works, the analysis used is static methods or quasi-static methods. The nature of the mode of the propagation is considered to be a pure transverse electric-magnetic (TEM) mode and the TL characteristics are calculated from the electrostatic capacitance and inductance of the TLs. However, the hybrid nature of the mode of the propagation should be taken into account [47]. Microstrip lines in particular are not capable of supporting a pure TEM mode wave, and small longitudinal components of both the electric and magnetic fields need to be considered to satisfy the boundary conditions at the dielectric-air interface. These hybrid modes cannot be fully described in terms of static capacitances and inductances only. The power density distributions of the symmetric E -component are almost similar to the transverse magnetic (TM) mode [109], and hence the TM mode dispersion analysis is considered here to determine the propagation characteristics. Especially for the microstrip line as shown in Fig. 4.1, the fields in x -direction are weakly varying because of the thin homogenous isotropic medium; thus consideration of TM-like mode wave is justified [20].

The singular perturbation procedure, by starting from the exact solution of a related problem, comprises mathematical methods to find an approximate solution to a problem which cannot be solved exactly by full wave analysis. Perturbation theory

Figure 4.1: The periodic TLs. (a) Equivalent circuit of unit cell of double periodic TLs. (b) Layout of DP-CRLH TL with different capacitances and inductances loaded. (c) Layout of SP-CRLH TL. (d) Layout of DP-CRLH TL with different inductances loaded. (e) Layout of DP-CRLH TL with different capacitances loaded.

leads to an expression for the desired solution in terms of a formal power series in some "smallness" parameter that quantifies the deviation from the exactly solvable problem. When series in the small parameter is referred as δ , for the structure has a periodical component that occurs along z -axis as shown in Fig. 4.1, the approximation to the full solution " H_y " may be given as

$$H_y = \delta^0 H_{y0} + \delta^1 H_{y1} + \delta^2 H_{y2} + \dots + \delta^n H_{yn}, \quad (4.1)$$

Figure 4.2: The flow chart of the singular perturbation procedure.

where H_{y0} represents the known solution to the exactly solvable initial problem and $H_{y1}, H_{y2}, \dots, H_{yn}$ represent the higher-order terms which may be found iteratively by some systematic procedure [39]. The flow chart of the singular perturbation procedure for the analysis of the periodic structure is shown in Fig. 4.2. It is noted that in addition to the dispersion characteristic derived from the zeroth-order problem, the perturbation method also gives information about the radiation efficiency and the radiation angle for the optimum radiation efficiency, which are derived from the first-order and second-order problem [110].

The purpose of this chapter is to investigate the double periodic TLs with the help of the singular perturbation procedure, and study in sufficient detail on the new-found right-handed (RH) LW radiation with narrow bandwidth below the LH passband. The structures are performed using microstrip lines with shunt short-circuited stub inductances and/or series interdigital capacitances double periodically loaded to design compact LWAs. Radiation characteristics are analyzed for the investigated DP-CRLH TLs, and the results display the expected new LW behavior. The results of the analysis verify the new LW radiation property of the DP-CRLH TLs proposed by the TL theory.

4.2 Formulation of Singular Perturbation Analysis

The layouts of the DP-CRLH TLs investigated in this study are shown in Fig. 4.1. The dispersion characteristics and the radiation efficiency of the compact LWAs are analyzed with the help of the singular perturbation procedure, and they are numerically estimated as a function of the frequency and the antenna length, respectively. The radiation angle is estimated in the novel LW region.

Figure 4.3: Processing of the layout of the DP-CRLH TL equivalent to the analytical model of double periodic dielectric material. (a) Layout of the unit cell of the DP-CRLH TL. (b) Equivalent circuit of the L-C network featuring the host TL medium (Z_0, k) of DP-CRLH TLs. (c) Layout of the DP-CRLH TL. (d) Analytical model of double periodic dielectric material on a ground plane.

4.2.1 Statement of the problem and parameters extraction

It is noted that the wire/SRR structures [17] and the single periodic composite right/left handed (SP-CRLH) TLs [24] possess a direct L-C equivalent circuit. As similar to the case for the SP-CRLH TL, the equivalent permeability (μ) and permittivity (ε) in DP-CRLH TLs are represented by the series inductance $L_R = \mu\Lambda_D$ and the shunt capacitance $C_R = \varepsilon\Lambda_D$, where $\Lambda_D = 10mm$ is the period of the unit cell as shown in Fig. 4.1(a). The cell dimension ($\Delta z = \Lambda_D$) is normally small compared to the guided wavelength (λ_g). The series capacitances C_{Li} and the shunt inductances L_{Li} ($i = 1, 2$) are equivalent models for the effect of the LH property. The approach is known to possess simultaneously negative equivalent permittivity and negative equivalent permeability, resulting in left-handedness behavior. In this case, the free-space permittivity and permeability can be directly related to the per-unit-length capacitance and inductance [20].

The DP-CRLH TLs can be constructed using the geometry in Fig. 4.3. It is assumed that the wave is propagating along the z -direction, and independent of the y -direction ($\partial/\partial y = 0$). In a thin homogeneous isotropic medium, such as microstrip lines, a TM mode wave is considered because of a weak x -variation [20]. The TM mode is considered here with non-vanishing field components H_y , E_x and E_z , having no variation in the y -direction. The material is assumed to be lossless.

Arising from the series capacitances (C_L) and shunt inductances (L_L), the dielectric and magnetic properties consist of a positive contribution due to the host medium and a negative dispersive contribution due to the loaded shunt inductances and series capacitances (corresponding to negative refractive index (NRI) materials). The frequency-dispersive nature of the material parameters equivalent permittivity $\varepsilon(\omega)$

and permeability $\mu(\omega)$ of the investigated TL are given by [20]

$$\varepsilon(\omega) = \varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_L \Lambda_D}, \quad \mu(\omega) = \mu_0 - \frac{1}{\omega^2 C_L \Lambda_D}, \quad (4.2)$$

where L_L and C_L are the double periodically loaded inductances and capacitances, respectively. ε_{eff} is the effective permittivity of the unperturbed host medium. For example, ε_{eff} of the microstrip lines is

$$\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \sqrt{\frac{1}{1 + 12s/W}}, \quad (4.3)$$

where s/W is the ratio of substrate thickness to the conductor width of the microstrip line and ε_r is the relative permittivity of the substrate. The Bloch impedance is $Z_B = g\sqrt{\mu/\varepsilon}$, where g is the geometric factor, and it can be determined through the width and substrate height of the microstrip line. From the expressions of (4.2), it is noted that the relative permittivity of the host medium is not doubled compared with the expressions in [20], in which an additional capacitive contribution to the unit cell is considered because each of the two transverse TL segments become open-circuited at the center for the two-dimensional (2-D) version of the medium. However, in this work one-dimensional (1-D) structure is considered and shunt short-circuited stubs are used.

These are the key parameters in the analysis of the DP-CRLH TLs. The investigation of the material parameters requires that the series and shunt branches are dominated by C_{Li} and L_{Li} ($i=1, 2$). The analytical model of the TLs investigated is shown in Fig. 4.3. The equivalent material permittivity and permeability of the guide are expressed as a function of frequency, periodically loaded capacitances and inductances as shown in (4.2).

In DP-CRLH TLs, alternate inductances L_{Li} and alternate capacitances C_{Li} are

loaded with period Λ_D , as shown in Fig. 4.1(a). It is found that TLs double periodically loaded with either alternate capacitances or alternate inductances will contribute the new LW behavior based on the TL theory in Chapter 3. For simplicity, the TL with the double periodically loaded alternate inductances is considered (Fig. 5.12(c)), and it is easy to extend to the TL with double periodically loaded alternate capacitances (Fig. 5.12(d)) and the TL with double periodically loaded both alternate inductances and alternate capacitances (Fig. 5.12(a)). For the TL with double periodically loaded alternate inductances, the inductances are expanded by

$$L_L = L_{L0}[1 + \delta\eta_1 \cos(Kz + \theta_1) + \delta^2\eta_2 \cos(2Kz + \theta_2) + \dots], \quad (4.4)$$

where L_{L0} is the average inductance, η_1 is the amplitude of the fundamental (first harmonic) component of the periodically loaded inductance while η_2 is the amplitude of the second harmonic, δ is a smallness parameter, and K is satisfied by the Bragg condition $K = 2\pi/\Lambda_D$. θ_1 is the phase of the first harmonic component and θ_2 is the phase of the second harmonic. The material is assumed to be lossless. The TM mode is considered here with nonvanishing field components H_y , E_x and E_z having no variation in the y -direction. By substituting (4.4) into (4.2), a periodically modulated permittivity profile in z -direction can be expressed as

$$\begin{cases} \varepsilon = \varepsilon(z) = \varepsilon_0\varepsilon_{eff} - \frac{1}{\omega^2 L_{L0}\Lambda_D} + \frac{\delta\eta_1 \cos(Kz + \theta_1)}{\omega^2 L_{L0}\Lambda_D} + \frac{\delta^2\eta_2 \cos(2Kz + \theta_2)}{\omega^2 L_{L0}\Lambda_D} + \dots, \\ \frac{\partial \varepsilon}{\partial z} = \frac{1}{\omega^2 L_{L0}\Lambda_D} (-\delta\eta_1 K \sin(Kz + \theta_1) - 2\delta^2\eta_2 K \sin(2Kz + \theta_2) - \dots), \end{cases} \quad (4.5)$$

where

$$\varepsilon_0\varepsilon_{eff} - \frac{1}{\omega^2 L_{L0}\Lambda_D} = \varepsilon_{s0}, \quad (4.6)$$

which is the equivalent dielectric constant without any perturbation [1, 20].

4.2.2 Derivation of the Helmholtz equation for the TM mode field

The magnetic field H_y satisfies the differential equation from Maxwell's equations. Maxwell's equations describe the electromagnetic phenomena as [47]:

$$\begin{cases} \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \\ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \\ \nabla \cdot \vec{D} = \rho, \\ \nabla \cdot \vec{B} = 0. \end{cases} \quad (4.7)$$

In these equations, $\vec{E}$ is the electric field intensity. $\vec{H}$ is the magnetic field intensity. $\vec{B} = \mu \vec{H}$ is the magnetic flux density, where $\mu = \mu_r \mu_0$ is the permeability of the medium, μ_r is the relative permeability of the medium and μ_0 is permeability of free-space. $\vec{J}$ is the electric current density, and $\vec{J} = 0$ in source-free regions of space. $\vec{D} = \varepsilon \vec{E}$ is the electric flux density, where $\varepsilon = \varepsilon_r \varepsilon_0$ is the permittivity of the medium, ε_r is the relative permittivity of the medium and ε_0 is the permittivity of free-space. ρ is the electric charge density, and $\rho = 0$ in source-free regions of space. Taking the curl of $\nabla \times \vec{H}$ and using $\nabla \times \vec{E}$ gives

$$\nabla \times \nabla \times \vec{H} = \nabla \times \left(\frac{\partial \vec{D}}{\partial t} \right) = \nabla \times \left(\frac{\partial \varepsilon \vec{E}}{\partial t} \right) = \frac{\partial}{\partial t} \nabla \times (\varepsilon \vec{E}). \quad (4.8)$$

Here, the TM mode is considered with field components $\vec{H} = \vec{e}_y H_y$ and $\vec{E} = \vec{e}_x E_x + \vec{e}_z E_z$. Hence,

$$\frac{\partial}{\partial t} \nabla \times (\varepsilon \vec{E}) = \frac{\partial}{\partial t} \nabla \times (\varepsilon (\vec{e}_x E_x + \vec{e}_z E_z)). \quad (4.9)$$

The equivalent permittivity of the medium is modulated in z -direction as (4.5), and then,

$$\begin{aligned}
& \frac{\partial}{\partial t} \nabla \times (\varepsilon(\vec{e}_x E_x + \vec{e}_z E_z)) \\
&= \frac{\partial}{\partial t} [\vec{e}_x \frac{\partial \varepsilon E_z}{\partial y} + \vec{e}_y (\frac{\partial \varepsilon E_x}{\partial z} - \frac{\partial \varepsilon E_z}{\partial x}) + \vec{e}_z (-\frac{\partial \varepsilon E_x}{\partial y})] \\
&= \frac{\partial}{\partial t} [\vec{e}_x \varepsilon \frac{\partial E_z}{\partial y} + \vec{e}_y (E_x \frac{\partial \varepsilon}{\partial z} + \varepsilon \frac{\partial E_x}{\partial z} - \varepsilon \frac{\partial E_z}{\partial x}) + \vec{e}_z (-\varepsilon \frac{\partial E_x}{\partial y})] \\
&= \frac{\partial}{\partial t} \{ \varepsilon [\vec{e}_x \frac{\partial E_z}{\partial y} + \vec{e}_y (\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}) + \vec{e}_z (-\frac{\partial E_x}{\partial y})] + \vec{e}_y E_x \frac{\partial \varepsilon}{\partial z} \} \\
&= \frac{\partial}{\partial t} [\varepsilon \nabla \times \vec{E} + \vec{e}_y E_x \frac{\partial \varepsilon}{\partial z}] \\
&= \frac{\partial}{\partial t} \varepsilon \nabla \times \vec{E} + \vec{e}_y \frac{\partial}{\partial t} (E_x \frac{\partial \varepsilon}{\partial z}).
\end{aligned} \tag{4.10}$$

Substituting $\nabla \times \vec{E} = -\frac{\partial \mu \vec{H}}{\partial t}$ and $E_x = -\frac{1}{j\omega \varepsilon} \frac{\partial H_y}{\partial x}$ into the last equation, it is obtained that

$$\begin{aligned}
\frac{\partial}{\partial t} \varepsilon \nabla \times \vec{E} + \vec{e}_y \frac{\partial}{\partial t} (E_x \frac{\partial \varepsilon}{\partial z}) &= \frac{\partial}{\partial t} \varepsilon (-\frac{\partial \mu \vec{H}}{\partial t}) + \vec{e}_y \frac{\partial}{\partial t} [\frac{\partial \varepsilon}{\partial z} (-\frac{1}{j\omega \varepsilon} \frac{\partial H_y}{\partial z})] \\
&= -\varepsilon \frac{\partial^2 \mu \vec{H}}{\partial t^2} - \vec{e}_y [\frac{\partial \varepsilon}{\varepsilon \partial z} (\frac{1}{j\omega} \frac{\partial H_y}{\partial z})].
\end{aligned} \tag{4.11}$$

Hence,

$$\nabla \times \nabla \times \vec{H} = -\varepsilon \frac{\partial^2 \mu \vec{H}}{\partial t^2} - \vec{e}_y [\frac{\partial \varepsilon}{\varepsilon \partial z} (\frac{1}{j\omega} \frac{\partial H_y}{\partial z})]. \tag{4.12}$$

This equation in phasor form can be obtained through replacing the time derivatives by $j\omega$, and

$$\nabla \times \nabla \times \vec{H} = \omega^2 \mu \varepsilon \vec{H} - \vec{e}_y \frac{\partial \varepsilon}{\varepsilon \partial z} \frac{\partial H_y}{\partial z}. \tag{4.13}$$

The Helmholtz equation are given from by

$$\nabla^2 \vec{H} = \nabla(\nabla \times \vec{H}) - \nabla \times \nabla \times \vec{H} = -\nabla \times \nabla \times \vec{H}. \tag{4.14}$$

The magnetic field equation in y -direction (H_y) of the TM mode satisfies the Helmholtz equation as shown

$$\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \omega^2 \mu \varepsilon\right\} H_y = \frac{1}{\varepsilon} \frac{\partial \varepsilon}{\partial z} \frac{\partial H_y}{\partial z} \quad (4.15)$$

and

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial x}, \quad E_x = -\frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial z}. \quad (4.16)$$

4.2.3 Perturbation expansions

When the singular perturbation method is used to analyze the periodic structure, the asymptotic expansions are extracted to match the corresponding parameters, which are permittivity (ε), permeability (μ), the electric field (E_x and E_z) and the magnetic field (H_y). These parameters are considered in different orders that are then combined together to form a composite expansion. The structure has a periodical component that occurs along the z -axis and has a periodic variation in the parameters (permittivity and permeability) and the solution as

$$\begin{aligned} L_L(z) &= \delta^0 L_{L0}(z_0, z_1, z_2, \dots) + \delta^1 L_{L1}(z_0, z_1, z_2, \dots) + \delta^2 L_{L2}(z_0, z_1, z_2, \dots) + \dots \\ &= L_{L0}[1 + \delta\eta_1 \cos(Kz + \theta_1) + \delta^2 \eta_2 \cos(2Kz + \theta_2) + \dots] \\ \varepsilon(z) &= \delta^0 \varepsilon_0(z_0, z_1, z_2, \dots) + \delta^1 \varepsilon_1(z_0, z_1, z_2, \dots) + \delta^2 \varepsilon_2(z_0, z_1, z_2, \dots) + \dots \\ &= \varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D} + \frac{\delta\eta_1 \cos(Kz + \theta_1)}{\omega^2 L_{L0} \Lambda_D} + \frac{\delta^2 \eta_2 \cos(2Kz + \theta_2)}{\omega^2 L_{L0} \Lambda_D} + \dots \end{aligned} \quad (4.17)$$

and

$$\begin{aligned} H_y(x, z) &= \delta^0 H_{y0}(x, z_0, z_1, z_2, \dots) + \delta^1 H_{y1}(x, z_0, z_1, z_2, \dots) \\ &\quad + \delta^2 H_{y2}(x, z_0, z_1, z_2, \dots) + \dots \end{aligned} \quad (4.18)$$

The perturbation is carried up by introducing space scales in the z -direction as

$$z_0 = z, \quad z_1 = \delta z \quad z_2 = \delta^2 z, \dots, z_n = \delta^n z. \quad (4.19)$$

Based on the analysis in [110], second order space scale is enough for analyzing the periodic structures working as LWAs. All of the necessary information, like dispersion characteristics, radiation efficiency and radiation angle, can be calculated. Therefore, space scales is introduced until second order:

$$z_0 = z, \quad z_1 = \delta z \quad z_2 = \delta^2 z. \quad (4.20)$$

The original space derivative transforms is

$$\frac{\partial}{\partial z} = \frac{\partial z_0}{\partial z} \frac{\partial}{\partial z_0} + \frac{\partial z_1}{\partial z} \frac{\partial}{\partial z_1} + \frac{\partial z_2}{\partial z} \frac{\partial}{\partial z_2} = \frac{\partial}{\partial z_0} + \delta \frac{\partial}{\partial z_1} + \delta^2 \frac{\partial}{\partial z_2}. \quad (4.21)$$

Because it is assumed that $\partial/\partial z_1 \ll \partial/\partial z_2$, hence $\partial/\partial z_1 \simeq 0$ [110]

$$\begin{cases} \frac{\partial}{\partial z} = \frac{\partial}{\partial z_0} + \delta^2 \frac{\partial}{\partial z_2}, \\ \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial z_0^2} + 2\delta^2 \frac{\partial^2}{\partial z_0 \partial z_2}, \end{cases} \quad (4.22)$$

and the expression for H_y is given by

$$\begin{cases} H_y(x, z) = H_{y0}(x, z_0, z_2) + \delta H_{y1}(x, z_0, z_2) + \delta^2 H_{y2}(x, z_0, z_2), \\ \frac{\partial H_y}{\partial z} = \frac{\partial H_{y0}}{\partial z_0} + \delta \frac{\partial H_{y1}}{\partial z_0} + \delta^2 \left(\frac{\partial H_{y0}}{\partial z_2} + \frac{\partial H_{y2}}{\partial z_0} \right). \end{cases} \quad (4.23)$$

Substituting (4.5) and (4.23) into (4.16), the differential equation is obtained as

$$\begin{aligned}
& \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_0^2} + 2\delta^2 \frac{\partial^2}{\partial z_0 \partial z_2} + \omega^2 \mu \left[\varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D} \right] + \delta \eta_1 \frac{\mu \cos(Kz_0 + \theta_1)}{L_{L0} \Lambda_D} \right. \\
& \quad \left. + \delta^2 \eta_2 \frac{\mu \cos(2Kz_0 + \theta_2)}{L_{L0} \Lambda_D} \right\} [H_{y0} + \delta H_{y1} + \delta^2 H_{y2}] \\
& = - \frac{\delta \eta_1 \frac{K \sin(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D} + \delta^2 \eta_2 \frac{2K \sin(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D}}{(\varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D}) + \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D} + \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D}} \\
& \quad \left[\frac{\partial H_{y0}}{\partial z_0} + \delta \frac{\partial H_{y1}}{\partial z_0} + \delta^2 \left(\frac{\partial H_{y0}}{\partial z_2} + \frac{\partial H_{y2}}{\partial z_0} \right) \right], \tag{4.24}
\end{aligned}$$

where H_y satisfies the equations (4.23). Because δ is a series smallness parameter for the singular perturbation procedure, then

$$\begin{aligned}
& \frac{\delta \eta_1 \frac{K \sin(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D} + \delta^2 \eta_2 \frac{2K \sin(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D}}{(\varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D}) + \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D} + \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D}} \\
& = \frac{\delta \eta_1 K \sin(Kz_0 + \theta_1) + \delta^2 \eta_2 2K \sin(2Kz_0 + \theta_2)}{(\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1) \left[1 + \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} + \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} \right]} \\
& \approx \frac{\delta \eta_1 K \sin(Kz_0 + \theta_1) + \delta^2 \eta_2 2K \sin(2Kz_0 + \theta_2)}{(\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1)} \\
& \quad \left[1 - \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} - \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} \right]. \tag{4.25}
\end{aligned}$$

Therefore,

$$\begin{aligned}
& \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_0^2} + 2\delta^2 \frac{\partial^2}{\partial z_0 \partial z_2} + \omega^2 \mu \left[\varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D} \right] + \delta \eta_1 \frac{\mu \cos(Kz_0 + \theta_1)}{L_{L0} \Lambda_D} \right. \\
& \quad \left. + \delta^2 \eta_2 \frac{\mu \cos(2Kz_0 + \theta_2)}{L_{L0} \Lambda_D} \right\} [H_{y0} + \delta H_{y1} + \delta^2 H_{y2}] \\
& = - \frac{\delta \eta_1 K \sin(Kz_0 + \theta_1) + \delta^2 \eta_2 2K \sin(2Kz_0 + \theta_2)}{(\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1)} \\
& \quad \left[1 - \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} - \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} \right] \\
& \quad \left[\frac{\partial H_{y0}}{\partial z_0} + \delta \frac{\partial H_{y1}}{\partial z_0} + \delta^2 \left(\frac{\partial H_{y0}}{\partial z_2} + \frac{\partial H_{y2}}{\partial z_0} \right) \right]. \tag{4.26}
\end{aligned}$$

- The differential equations for different orders are obtained as

Zeroth order $O(\delta^0)$:

$$\left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_0^2} + \omega^2 \mu \varepsilon_{s0} \right\} H_{y0} = 0, \quad (4.27)$$

First order $O(\delta^1)$:

$$\begin{aligned} & \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_0^2} + \omega^2 \mu \varepsilon_{s0} \right\} H_{y1} \\ &= -\frac{\mu \eta_1 \cos(Kz_0 + \theta_1)}{L_{L0} \Lambda_D} H_{y0} - \frac{\eta_1 K \sin(Kz_0 + \theta_1)}{\varepsilon_0 \varepsilon_{eff} \omega^2 L_{L0} \Lambda_D - 1} \frac{\partial H_{y0}}{\partial z_0}, \end{aligned} \quad (4.28)$$

Second order $O(\delta^2)$:

$$\begin{aligned} & \left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_0^2} + \omega^2 \mu \varepsilon_{s0} \right\} H_{y2} \\ &= \frac{\eta_1^2 K \sin(2Kz_0 + 2\theta_1)}{2(\varepsilon_0 \varepsilon_{eff} \omega^2 L_{L0} \Lambda_D - 1)^2} \frac{\partial H_{y0}}{\partial z_0} - 2 \frac{\partial^2 H_{y0}}{\partial z_0 \partial z_2} - \frac{\mu \eta_1 \cos(Kz_0 + \theta_1)}{L_{L0} \Lambda_D} H_{y1} \\ & \quad - \frac{\mu \eta_2 \cos(2Kz_0 + \theta_2)}{L_{L0} \Lambda_D} H_{y0} - \frac{\eta_1 K \sin(Kz_0 + \theta_1)}{\varepsilon_0 \varepsilon_{eff} \omega^2 L_{L0} \Lambda_D - 1} \frac{\partial H_{y1}}{\partial z_0} \\ & \quad - \frac{2\eta_2 K \sin(2Kz_0 + \theta_2)}{\varepsilon_0 \varepsilon_{eff} \omega^2 L_{L0} \Lambda_D - 1} \frac{\partial H_{y0}}{\partial z_0}, \end{aligned} \quad (4.29)$$

where $\varepsilon_{s0} = \varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D}$, which is the equivalent dielectric constant without any perturbation.

For each order δ^n , the field H_y and E_z are continuous at $x = s$, and the field $E_z = 0$ at $x = 0$. Substituting (4.5) and (4.23) into the electric fields as shown in (4.16), the electric field E_z is derived as

$$\begin{aligned} E_z &= \frac{1}{j\omega \varepsilon} \frac{\partial H_y}{\partial x} \\ &= \frac{1}{j\omega} \frac{1}{\varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D} + \frac{\delta \eta_1 \cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D} + \frac{\delta^2 \eta_2 \cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D}} \frac{\partial}{\partial x} (H_{y0} + \delta H_{y1} + \delta^2 H_{y2}). \end{aligned} \quad (4.30)$$

It is also because δ is a series smallness parameter for the singular perturbation pro-

cedure, then

$$\begin{aligned}
& \frac{1}{\varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_{L0} \Lambda_D} + \frac{\delta \eta_1 \cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D} + \frac{\delta^2 \eta_2 \cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D}} \\
&= \frac{1}{\varepsilon_0 \varepsilon_{eff} - \omega^2 L_{L0} \Lambda_D} \frac{1}{1 + \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} + \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1}} \\
&\approx \frac{1}{\varepsilon_0 \varepsilon_{eff} - \omega^2 L_{L0} \Lambda_D} \left[1 - \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} - \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} \right],
\end{aligned} \tag{4.31}$$

and then,

$$\begin{aligned}
E_z = \frac{1}{j\omega} \frac{1}{\varepsilon_0 \varepsilon_{eff} - \omega^2 L_{L0} \Lambda_D} & \left[1 - \delta \eta_1 \frac{\cos(Kz_0 + \theta_1)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} - \delta^2 \eta_2 \frac{\cos(2Kz_0 + \theta_2)}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} \right] \\
& \frac{\partial}{\partial x} (H_{y0} + \delta H_{y1} + \delta^2 H_{y2}).
\end{aligned} \tag{4.32}$$

- The boundary conditions for each order are obtained as

Zeroth order $O(\delta^0)$:

$$\begin{cases} \frac{\partial H_{y0s}}{\partial x} = 0 & x = 0, \\ H_{y0s} = H_{y0a} \Rightarrow \frac{1}{\varepsilon_{s0}} \frac{\partial}{\partial x} H_{y0s} = \frac{1}{\varepsilon_0} \frac{\partial}{\partial x} H_{y0a} & x = s, \end{cases} \tag{4.33}$$

First order $O(\delta^1)$:

$$\begin{cases} \frac{\partial H_{y1s}}{\partial x} = 0 & (x = 0), \\ H_{y1s} = H_{y1a} \Rightarrow \frac{1}{\varepsilon_{s0}} \frac{\partial}{\partial x} H_{y1s} - \frac{\omega^2 L_0 \Lambda \eta_1 \cos(Kz_0 + \theta_1)}{(2\varepsilon_0 \varepsilon_{eff} \omega^2 L_0 \Lambda - 1)^2} \frac{\partial}{\partial x} H_{y0s} = \frac{1}{\varepsilon_0} \frac{\partial}{\partial x} H_{y1a} & (x = s), \end{cases} \tag{4.34}$$

Second order $O(\delta^2)$:

$$\left\{ \begin{array}{l} \frac{\partial H_{y2s}}{\partial x} = 0 \\ H_{y2s} = H_{y2a} \Rightarrow \\ \frac{1}{\varepsilon_{s0}} \frac{\partial}{\partial x} H_{y2s} - \frac{\omega^2 L_0 \Lambda \eta_1 \cos(Kz_0 + \theta_1)}{(2\varepsilon_0 \varepsilon_{eff} \omega^2 L_0 \Lambda - 1)^2} \frac{\partial}{\partial x} H_{y1s} = \frac{1}{\varepsilon_0} \frac{\partial}{\partial x} H_{y2a} \end{array} \right. \quad \begin{array}{l} (x = 0), \\ \\ (x = s), \end{array} \quad (4.35)$$

where H_{y0s} and H_{y0a} are the fields in the medium and freespace, respectively.

4.3 Solutions of each order problem

4.3.1 Zeroth-order problem

The zeroth-order problem will generate the dispersion relationship of the investigated DP-CRLH TLs.

The zeroth-order component H_{y0} inside the substrate satisfies the following differential equation (4.27). The fields H_{y0} and E_{z0} are continuous at the air-medium interface ($x = s$), and the field $E_{z0} = 0$ at the ground plane ($x = 0$). The boundary conditions are obtained as (4.33), where H_{y0s} and H_{y0a} are the fields in the medium and freespace, respectively.

If a product solution is assumed as $H_{y0} = f(x)g(z_0)$, then substituting H_{y0} into (4.27), it is obtained as

$$\frac{f''(x)}{f(x)} + \frac{g''(z_0)}{g(z_0)} + \omega^2 \mu \varepsilon_{s0} = 0. \quad (4.36)$$

After substituting $f(x)$ and $g(z_0)$ for H_{y0} and dividing the equations by $f(x)$ and $g(z_0)$, the term $\frac{f''(x)}{f(x)}$ is a function of x only and the term $\frac{g''(z_0)}{g(z_0)}$ is a function of z_0 only.

Figure 4.4: Wave number supportable by a planer DP-CRLH TL.

- In the substrate, it is assumed that

$$\begin{cases} \frac{f''(x)}{f(x)} = -k_{xs}^2, \\ \frac{g''(z_0)}{g(z_0)} = -k_z^2, \end{cases} \quad (4.37)$$

Solutions for $f(x)$ and $g(z_0)$ in the substrate are supposed to be

$$\begin{cases} f_s(x) = A\cos(k_{xs}x) + B\sin(k_{xs}x), \\ g(z_0) = e^{-jk_z z_0}. \end{cases} \quad (4.38)$$

- In air, it is assumed that

$$\begin{cases} \frac{f''(x)}{f(x)} = -k_{xa}^2, \\ \frac{g''(z_0)}{g(z_0)} = -k_z^2. \end{cases} \quad (4.39)$$

Solutions for $f(x)$ and $g(z_0)$ in air are supposed to be

$$\begin{cases} f_a(x) = C\cos(k_{xa}x) + D\sin(k_{xa}x), \\ g(z_0) = e^{-jk_z z_0}. \end{cases} \quad (4.40)$$

The k_{xs} and k_{xa} are the complex wave numbers along x -direction in substrate and air, respectively, and k_z is the complex wave number along the z -direction as shown in Fig. 4.4. These wave numbers satisfy

$$\begin{cases} k_{xa}^2 + k_z^2 = \omega^2 \mu_0 \varepsilon_0 & x > s \\ k_{xs}^2 + k_z^2 = \omega^2 \mu \varepsilon_{s0} & 0 < x < s \end{cases} \quad (4.41)$$

and they are complex numbers as

$$\begin{cases} k_{xa} = k'_{xa} + jk''_{xa} = \beta_{xa} - j\alpha_{xa}, \\ k_{xs} = k'_{xs} + jk''_{xs} = \beta_{xs} - j\alpha_{xs}, \\ k_z = k'_z + jk''_z = \beta_z - j\alpha_z, \end{cases} \quad (4.42)$$

where β_{xa} , β_{xs} and β_z are the corresponding phase constants and α_{xa} , α_{xs} and α_z are the corresponding attenuation constants.

- Based on the boundary conditions as shown in (4.33), at $x = 0$, $E_{z0s} = 0$, and then $\frac{\partial H_{y0s}}{\partial x} = 0$, hence

$$\begin{aligned} \frac{\partial H_{y0s}}{\partial x} &= \frac{\partial f_s(x)g(z_0)}{\partial x} \Big|_{x=0} \\ &= \frac{\partial}{\partial x} ([A \cos(k_{xs}x) + B \sin(k_{xs}x)]e^{-jk_z z_0}) \Big|_{x=0} = Bk_{xs}e^{-jk_z z_0} = 0. \end{aligned} \quad (4.43)$$

Therefore, $B = 0$.

- At $x = \infty$, $f_a(x)$ should have limit value as $f_a(x)|_{x=\infty} < \infty$. Therefore

$$\begin{aligned} f_a(x)|_{x=\infty} &= C \cos(k_{xa}x)|_{x=\infty} + D \sin(k_{xa}x)|_{x=\infty} \\ &= C \frac{e^{jk_{xa}x} + e^{-jk_{xa}x}}{2} \Big|_{x=\infty} + D \frac{e^{jk_{xa}x} - e^{-jk_{xa}x}}{2} \Big|_{x=\infty}. \end{aligned} \quad (4.44)$$

Because $\alpha_{xa} > 0$, hence

$$\begin{cases} e^{jk_{xa}x}|_{x=\infty} = e^{(\alpha_{xa}+j\beta_{xa})x}|_{x=\infty} = \infty, \\ e^{-jk_{xa}x}|_{x=\infty} = e^{(-\alpha_{xa}-j\beta_{xa})x}|_{x=\infty} < \infty. \end{cases} \quad (4.45)$$

Therefore, based on the boundary condition, (4.44) is assumed as

$$f_a(x) = C \frac{e^{-jk_{xa}x}}{2} - D \frac{e^{-jk_{xa}x}}{2} \doteq E e^{-jk_{xa}x}. \quad (4.46)$$

- At $x = s$, $H_{y0s} = H_{y0a}$, so that,

$$A \cos(k_{xs}x) e^{-jk_z z_0}|_{x=s} = E e^{-jk_{xa}x} e^{-jk_z z_0}|_{x=s}. \quad (4.47)$$

Hence, the parameters A and E are

$$\begin{cases} A = \frac{1}{\cos(k_{xs}s)}, \\ E = e^{jk_{xa}s}. \end{cases} \quad (4.48)$$

Based on the differential equations (4.27) and the boundary conditions (4.33), the fields is satisfied as

$$\begin{cases} H_{y0a} = N_g a_g e^{-jk_{xa}(x-s)} e^{-jk_z z_0} & x > s, \\ H_{y0s} = N_g a_g \frac{\cos(k_{xs}x)}{\cos(k_{xs}s)} e^{-jk_z z_0} & 0 < x < s. \end{cases} \quad (4.49)$$

The K_{xa} , k_{xs} and k_z are defined from (4.41) and (4.42).

By applying the boundary condition that the electric field (E_z) are continuous on the air-medium interface $x = s$ (4.33) to the fields in the medium and air with (4.49),

the solution satisfies the relationship as

$$\begin{cases} \frac{\partial}{\partial x} H_{y0s}|_{x=s} = -N_g a_g k_{xs} \tan(k_{xs} s) e^{-jk_z z_0}, \\ \frac{\partial}{\partial x} H_{y0a}|_{x=s} = -j N_g a_g k_{xa} e^{-jk_z z_0}. \end{cases} \quad (4.50)$$

Hence, it is obtained that

$$N_g a_g k_{xs} \tan(k_{xs} s) e^{-jk_z z_0} = j N_g a_g k_{xa} e^{-jk_z z_0}. \quad (4.51)$$

The dispersion relation can be obtained from (4.50) and satisfies

$$k_{xs} \tan(k_{xs} s) = \frac{\varepsilon_{s0}}{\varepsilon_0} j k_{xa}. \quad (4.52)$$

The field is a wave of varying amplitude a_g and the unknown normalization constant N_g in (4.49) is chosen by considering the power carrier by the guided wave in the z -direction as $|a_g|^2$ over a unit width in the y -direction, where a_g is a guided power along the TL. In the passband, the attenuation constant of the fundamental harmonic in the z -direction (α_z) is zero. The Poynting vector of $|a_g|^2$ toward the z -direction is

$$\begin{aligned} P_z &= \frac{1}{2} \text{Re} \iint (E \times H^*) \cdot e_z dx dy = \frac{1}{2} \text{Re} \iint (E_x H_y^*) dx dy \\ &= \frac{1}{2} \text{Re} \iint \left(-\frac{1}{j\omega\varepsilon} \frac{\partial H_y}{\partial z} H_y^* \right) dx dy \\ &= \frac{1}{2} \text{Re} \int_{y=0}^{y=1} \int_{x=0}^{x=s} \left(-\frac{1}{j\omega\varepsilon} \frac{\partial H_{y0s}}{\partial z} H_{y0s}^* \right) dx dy \\ &\quad + \frac{1}{2} \text{Re} \int_{y=0}^{y=1} \int_{x=s}^{x=\infty} \left(-\frac{1}{j\omega\varepsilon} \frac{\partial H_{y0a}}{\partial z} H_{y0a}^* \right) dx dy. \end{aligned} \quad (4.53)$$

Substituting (4.49) and (4.42) into (4.53), the Poynting vector is obtained as

$$P_z = \frac{\beta_z N_g^2 \alpha_g^2}{4\omega\varepsilon_{s0}\alpha_{xa}\beta_{xs}^2} \left[\frac{\varepsilon_{s0}}{\varepsilon_0} (\alpha_{xa}^2 + \beta_{xs}^2) + s\alpha_{xa} (\beta_{xs}^2 + \alpha_{xa}^2 \frac{\varepsilon_{s0}^2}{\varepsilon_0^2}) \right]. \quad (4.54)$$

Hence, the normalization constant N_g in the passband is obtained as

$$|N_g|^2 = \frac{4\omega\varepsilon_{s0}\alpha_{xa}\beta_{xs}^2}{\beta_z[\frac{\varepsilon_{s0}}{\varepsilon_0}(\alpha_{xa}^2 + \beta_{xs}^2) + s\alpha_{xa}(\beta_{xs}^2 + \alpha_{xa}^2\frac{\varepsilon_{s0}}{\varepsilon_0^2})]}. \quad (4.55)$$

4.3.2 First-order problem

The harmonic variation of the permittivity generates many higher order Floquet modes. For simplicity, first order Floquet mode is considered along with perturbation orders of δ^1 and δ^2 in first and second order analyses [110]. The corresponding differential equations and boundary conditions are given in (4.28) and (4.34), respectively.

Based on the differential equations and boundary conditions, the solutions (H_{y1}) of (4.28) for the first-order are assumed as

$$\begin{cases} H_{y1a} = F_{1a}e^{-\alpha_{1a}(x-s)}e^{-j(k_z+K)z_0} + N_re^{j\theta_1}(a_ie^{jk_{-1a}(x-s)} + b_ie^{-jk_{-1a}(x-s)})e^{-j(k_z-K)z_0}, \\ H_{y1s} = \sum_{i=\pm 1} e^{-j(k_z\pm K)z_0}[\frac{F_{\pm 1s}\cos(k_{\pm 1s}x)}{\cos(k_{\pm 1s}s)} + \frac{G_{\pm 1s}\cos(k_{xs}x)}{\cos(k_{xs}s)}], \end{cases} \quad (4.56)$$

where

$$G_{\pm 1s} = \frac{\eta_1 N_g a_g e^{j\theta_1}}{2(2Kk_z \pm K^2)}(\frac{k_z K}{\varepsilon_0 \varepsilon_{eff} \omega^2 L_{L0} \Lambda_D - 1} \pm \frac{\mu}{L_{L0} \Lambda_D}), \quad (4.57)$$

and

In air :

$$\begin{cases} k_{-1a}^2 = \omega^2 \mu_0 \varepsilon_0 - (k_z - K)^2, \\ \alpha_{1a}^2 = -\omega^2 \mu_0 \varepsilon_0 + (k_z + K)^2. \end{cases} \quad (4.58)$$

In the substrate :

$$\begin{cases} k_{-1s}^2 = \omega^2 \mu \varepsilon_{s0} - (k_z - K)^2, \\ k_{1s}^2 = \omega^2 \mu \varepsilon_{s0} - (k_z + K)^2. \end{cases}$$

The first-order solutions contain $(k_z - K)$ and $(k_z + K)$ terms, which are the wave

numbers in the z -direction, at least one of the scattered Floquet modes must be a fast wave and then the LW phenomenon occurs. This can be accomplished by choosing k such that only $(k_z - K)$ lies in the fast wave region.

The power flow in the $-x$ and $+x$ -directions is given by the incident power $|a_i|^2$ and radiated power $|b_i|^2$, respectively. The relation between the radiated wave b_i , incident wave a_i and guided wave a_g (as shown in Fig. 4.5) with in-phase condition $e^{-j(k_z - K)z_0}$ of $\cos(Kx)$ can also be derived as [110]

$$b_i = C_{rg}a_g + C_{rr}a_i, \quad (4.59)$$

where C_{rg} is the coupling coefficient and C_{rr} is the reflection coefficient.

The normalization factor N_r is evaluated by considering the power flow in the $-x$ and $+x$ direction as incident $|a_i|^2$ and radiated $|b_i|^2$ powers. The Poynting vector in x -direction is

$$\begin{aligned} P_x &= \frac{1}{2} \text{Re} \int_{z=0}^{z=1} \int_{y=0}^{y=1} (E \times H^*) \cdot e_x dy dz = \frac{1}{2} \text{Re} \int_{z=0}^{z=1} \int_{y=0}^{y=1} (E_z H_y^*) dy dz \\ &= \frac{1}{2} \text{Re} \int_{z=0}^{z=1} \int_{y=0}^{y=1} \left(\frac{1}{j\omega\epsilon_0} \frac{\partial H_y}{\partial x} H_y^* \right) dy dz = \frac{1}{j2\omega\epsilon_0} \frac{\partial H_y}{\partial x} H_y^*. \end{aligned} \quad (4.60)$$

Substituting (4.56) and (4.58) into (4.60), the Poynting vector is obtained as

$$P_x = \frac{k_{-1a}}{2\omega\epsilon_0} N_r^2 (|a_i|^2 - |b_i|^2). \quad (4.61)$$

By normalizing the incident wave and radiated wave, the normalization factor N_r is evaluated as

$$N_r = \sqrt{\frac{2\omega\epsilon_0}{k_{-1a}}}. \quad (4.62)$$

The electric fields E_z and E_x can be obtained from Maxwell's equations as (4.16). Substituting (4.56) into the boundary conditions of the first-order, F_{-1s} , F_{1s} are ob-

Figure 4.5: Interaction of the coupling: a_i . incident wave, b_i . radiated wave, a_g . principal guided wave.

tained and the coupling coefficient C_{rg} , the reflection coefficient C_{rr} are also obtained as

$$\left\{ \begin{array}{l} C_{rr} = -\frac{\varepsilon_0 k_{-1s} \tan(k_{-1s}s) + j k_{-1a} \varepsilon_{s0}}{\varepsilon_0 k_{-1s} \tan(k_{-1s}s) - j k_{-1a} \varepsilon_{s0}}, \\ C_{rg} = \frac{\eta_1 N_g}{2 N_r e^{j\theta_1}} \frac{\varepsilon_0}{\varepsilon_0 k_{-1s} \tan(k_{-1s}s) - j \varepsilon_{s0} k_{-1a}} \\ \quad \left\{ \left(\frac{2 k_z K}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} - \frac{\mu}{L_{L0} \Lambda_D} \right) k_{xs} \tan(k_{xs}s) \right. \\ \quad \left. - \frac{K [k_{xs} \tan(k_{xs}s) (K - k_z) - k_z k_{-1s} \tan(k_{-1s}s)]}{\omega^2 L_{L0} \Lambda_D \varepsilon_0 \varepsilon_{eff} - 1} - \frac{\mu k_{-1s} \tan(k_{-1s}s)}{L_{L0} \Lambda_D} \right\}. \end{array} \right. \quad (4.63)$$

4.3.3 Second-order problem

The treatment of second-order analysis is necessary for the determination of the guide-mode amplitude and the phase change of the periodic structure. The inhomogeneous second-order problem has a solution if a certain solvability condition is satisfied. The solvability condition is the amplitude transport equation governing the nature of interactions between the zero, first, and second-order fields.

The corresponding differential equations and boundary conditions are given in

(4.29) and (4.35), respectively. Based on the differential equations and boundary conditions, the solutions (H_{y2}) of (4.29) for the second-order are sought in the form

$$\begin{cases} H_{y2a} = \phi_a(x)e^{-jk_z z_0} & x > s, \\ H_{y2s} = \phi_s(x)e^{-jk_z z_0} & 0 < x < s. \end{cases} \quad (4.64)$$

Substituting (4.64) into (4.29), and solving the differential equations for $\phi_a(x)$ and $\phi_s(x)$, the solutions are obtained as

$$\begin{cases} \phi_a(x) = A_1 e^{-jk_{xa}(x-s)} - \frac{jk_z}{jk_{xa}} N_g \frac{da_g}{dz_2} x e^{-jk_{xa}(x-s)}, \\ \phi_s(x) = B_1 \frac{\cos(k_{xs}x)}{\cos(k_{xs}s)} + C_1 \frac{\sin(k_{xs}x)}{\cos(k_{xs}s)} + \frac{jk_z N_g}{k_{xs} \tan(k_{xs}s)} \frac{da_g}{dz_2} x \sin(k_{xs}x). \end{cases} \quad (4.65)$$

Substituting (4.64) into the boundary conditions for the second-order at $x = 0$, yields $C_1 = 0$. At $x = s$, followed by the elimination of the arbitrary constants, the amplitude transport equation is obtained as

$$\frac{da_g}{dz_2} = C_{gg}a_g + C_{gr}a_i, \quad (4.66)$$

where

$$\begin{cases} C_{gr} = \frac{\eta k_{-1s} N_r (1 + C_{rr}) \tan(k_{-1s}s)}{j2k_z N_g \omega^2 \varepsilon_{s0}^2 L_{L0} \Lambda_D T}, \\ C_{gg} = \frac{\eta k_{-1s} N_r C_{rg} \tan(k_{-1s}s)}{j2k_z N_g \omega^2 \varepsilon_{s0}^2 L_{L0} \Lambda_D T}, \end{cases} \quad (4.67)$$

and

$$T = \frac{1}{jk_{xa}\varepsilon_0} + \frac{[\sin(k_{xs}x) + sk_{xs}\cos(k_{xs}x)]/\varepsilon_{s0} + jk_{xa}s\sin(k_{xs}x)/\varepsilon_0}{k_{xs}\cos(k_{xs}s)}. \quad (4.68)$$

Equation (4.67) constitutes a pair of canonical equations relating the guided wave, the incident wave, and the radiated wave. $C_{gg} = C_{ggr} + jC_{ggi}$ is the extinction coefficient. Its real part C_{ggr} gives the leakage coefficient and the imaginary part C_{ggi} determines

the exact radiation angle for the optimum radiation efficiency.

In the LWA problem, the incident wave a_i is not considered, since it is based on the scattering of guided waves into a radiated wave due to the presence of a nonuniformity along the waveguide and hence the relations reduce to

$$b_i = C_{rg}a_g \quad \text{and} \quad \frac{da_g}{dz_2} = C_{gg}a_g. \quad (4.69)$$

In a finite length L_g of the permittivity modulated TL, the radiation efficiency Q_0 and radiation angle θ_r of the LW radiating from the TL with double periodically loaded inductances and capacitances can be obtained from

$$\begin{cases} Q_0 = \frac{\int_0^{L_g} |b_i|^2 dz_2}{|a_g|_{z_2=0}^2} = 1 - e^{2C_{gg}L_g}, \\ \theta_r = \tan^{-1}\left(\frac{k_{-1a}}{\beta - K - C_{ggi}}\right), \end{cases} \quad (4.70)$$

where L_g is the total length of the periodic TL. The radiation efficiency is defined as the ratio of the total power radiated from the modulated region to the guided wave power incident at $z_2 = 0$ [110].

4.4 Numerical results of the problem

Figure 4.6 shows a dispersion diagram of the fundamental TM mode supported by the unperturbed microstrip lines double periodically loaded with alternate inductances. It is assumed that the thickness of the TL $s = 1.6$ mm, the average loaded inductance $L_{L0} = 5.05$ nH, the loaded capacitance $C = 0.5$ pF, the period of the loaded elements is $\Lambda_D = 10$ mm, the relative permittivity of the substrate $\varepsilon_r = 4.5$, and it is considered to be non-magnetic. The phase constant of the fundamental space harmonic can be calculated when the periodic permittivity modulation applied is assumed to be

the same as the un-perturbed phase constant. The TM mode solutions need to be considered in this case for nonzero k_{xs} .

The dispersion characteristics are calculated based on the dispersion relation as predicted by (4.52), and illustrates the new RH passband property ($f_{nR\text{-}L} \sim f_{nR\text{-}H}$), the new LW region ($f_{leaky} = f_{nR\text{-}L}$) and the LH passband ($f_L \sim f_{\Gamma 1}$) that exhibits a distinct backward wave characteristic as shown in Fig. 4.6. A stopband ($f_{\Gamma 1} \sim f_{\Gamma 2}$) and a higher-order passband ($f > f_{\Gamma 2}$) possessing a RH characteristic appear even in the SP-CRLH TL proposed in [2]. The curves at $f_{nR\text{-}L}$, $f_{\Gamma 1}$ and $f_{\Gamma 2}$ are inside the fast wave region within the boundary $\beta < \omega\sqrt{\mu_0\epsilon_0}$. It indicates that besides the curves at $f_{\Gamma 1}$ and $f_{\Gamma 2}$, where the TL can radiate in the backward and forward directions, respectively, the proposed DP-CRLH TL can radiate a forward wave at $f_{leaky} = f_{nR\text{-}L}$, which is below the LH passband with narrow bandwidth.

With the effect of double periodically loaded inductances, the upper cutoff frequency ($f_{\Gamma 2}$) of the TL is similar for both double and single periodic TLs, but the lower cutoff frequency ($f_{\Gamma 1}$) of the DP-CRLH TL is lower than that of the SP-CRLH TL, which also gives that the bandwidth of the stopband is wider than SP-CRLH TL. In both the RH and LH passband, the phase constant of the DP-CRLH TL approaches that of the SP-CRLH TL. In the stopband ($f_{\Gamma 1} \sim f_{\Gamma 2}$), the phase constant is equal to that in air ($\beta = \omega\sqrt{\mu_0\epsilon_0}$), and then with increasing frequency it changes from a surface wave to a guided wave.

The dispersion behavior of the DP-CRLH TL is also derived from the TL theory on TEM mode. Fig. 3.3 and Fig. 3.8 show the dispersion diagram of SP-CRLH TL and DP-CRLH TLs double periodically loaded with alternate inductances and/or alternate capacitances. It is noted that the new LW region is also found in the DP-CRLH TL on the TEM mode. This analysis is carried out to show the quasi TEM mode and the considered TM mode has almost similar dispersion characteristics.

The radiation efficiency Q_0 of (4.70) is numerically estimated around the new RH

Figure 4.6: Dispersion characteristics of the DP-CRLH TL based on singular perturbation procedure, TM mode.

LW region ($f_{leaky} = f_{nR_L}$) and the LH LW region ($f = f_{\Gamma1}$) as a function of TL length L_g as shown in Fig. 4.7. It can be seen from Fig. 4.7 that the radiation efficiency for the new LW RH region is less than the radiation behavior in the LH region and longer TLs is needed to get more radiation efficiency. This comparison is made for $\eta_1 = 0.1$. It may be developed for large η_1 upto 0.3 beyond which it exceeds the limit of the singular perturbation procedure [39]. If the length of the DP-CRLH TL $L_g = 8$ cm and $\eta_1 = 0.1$, the radiation angle of the new LW is estimated to be around 23° at $f_{leaky} = f_{nR_L}$.

Figure 4.7: Radiation efficiency as a function of antenna length for three modulation indices.

4.5 Conclusion

A singular perturbation procedure has been formulated to study the radiation characteristics of the microstrip lines with double periodically loaded with alternate shunt short-circuited stub inductances. The dispersion characteristics in the array have been calculated on TM mode, and comparing with the one derived from TL theory based on TEM mode earlier in Chapter 3. Based on those analyses, the detailed steps of the formulation have been explained in this chapter. In addition to the LH characteristic of the structure, the new LW radiation is also obtained below the LH passband with narrow bandwidth, and this point is focused on the LW phenomenon, which is also obtained from the TL theory.

The proposed DP-CRLH TL is a radiation structure whose radiation efficiency is

a function of the length of the investigated TL. Based on the singular perturbation theory, the information derived from the first-order and second-order problem shows that the radiation efficiency around the new RH LW region is less than that around the LH LW region. It is noted that longer TLs is needed to get more radiation efficiency and larger amplitude of the fundamental component of the periodically loaded inductances is also important for increasing the radiation efficiency.

Chapter 5

LEAKY-WAVE ANTENNAS IMPLEMENTATION BASED ON DP-CRLH TLs

The primary attractive feature of the proposed double periodic composite right/left handed (DP-CRLH) transmission lines (TLs) is that they can support three leaky-wave (LW) radiation bands, including the new-found LW radiation at very low frequency, left-handed (LH) and conventional right-handed (RH) LW radiations at higher frequencies. This unique property leads to design super-compact leaky-wave antennas (LWAs) working at a low frequency, unbalanced triple-band or balanced dual-band LWAs with frequency-scanning property. This chapter considers the applications of the proposed DP-CRLH TL approaches based on the microstrip lines and substrate integrated waveguide (SIW), and lays down useful design guidelines. Three LWAs are designed for the purpose of demonstrating the unique properties of the proposed DP-CRLH TLs. All of the designed examples of LWAs have been fabricated, and the measurement results are in agreement with the expected performance, which validate the guidelines and formulae presented in the Chapter 3 and Chapter 4.

5.1 Double Periodic Composite Right/Left Handed Transmission Lines

The proposed DP-CRLH TLs may be treated as transmission lines double periodically loaded with alternate capacitances and/or inductances. The capacitances and the inductances are controlled to realize a desirable radiation response. In principle, it is possible to obtain a required new LW radiation at desired very low frequency locations by choosing appropriate dimensions of the unit-cell. The operating principle of the DP-CRLH TLs based on this array is comprehensively explained through its circuit model. Based on the understanding developed in the previous chapters, useful design guidelines are listed. Examples of compact unbalanced triple-band and balanced dual-band LWAs have been designed and the measurement results are also presented in this chapter.

5.1.1 Circuit Model

It has been already shown in Chapter 3 and Chapter 4 that the proposed DP-CRLH TLs support a new LW radiation because when capacitances and/or inductances are double periodically inserted in a transmission line, it can lead simultaneously to transmittance resonances in the gaps and anti-resonances in the bands that widen into new bands and gaps, including a new LW region occurs. The number of the degrees of the system is doubled for the DP-CRLH TLs comparing with the single periodic composite right/left handed (SP-CRLH) TLs and therefore there is twice the number of branches in the frequency characteristics of SP-CRLH TLs. It is also observed that one of these loaded components, series capacitances or shunt inductances, are double periodically loaded in the TL will contribute this unique LW radiation, although the other one is single periodically loaded.

Figure 5.1: Equivalent circuit model of the proposed DP-CRLH TLs.

The loaded inductances and/or inductances are alternate within one unit cell and the unit cell of DP-CRLH TLs can be seen as combination of two unit cells of different SP-CRLH TLs. A cascaded junction of two such unit cells of SP-CRLH TLs then leads to the circuit model of one unit cell of the proposed DP-CRLH TLs as shown in Fig. 5.1, where the two unit cells (U_1 and U_2) are formed by the transmission lines consisting of series capacitances (C_{L1} and C_{L2}) and shunt inductances (L_{L1} and L_{L2}). In the actual case of the proposed DP-CRLH TLs, the host mediums are approached with different kinds of structures, such as microstrip line implementation and SIW implementation, while the loaded components are also implemented based on the corresponding host mediums, such as interdigital capacitors and short-circuited stubs for the microstrip lines and the etched transverse slots and metallic multiple posts in the SIW representing the series capacitances (C_{L1} and C_{L2}) and shunt inductances (L_{L1} and L_{L2}).

5.1.2 Design Guidelines

The frequency responses of the proposed DP-CRLH TLs are mainly controlled by these design parameters:

1. The host transmission line medium.
 - Implementation of the structure (Microstrip lines, waveguide, SIW, split ring resonator (SRR), etc).
 - Dielectric constant ε_r ,
 - Characteristic impedance Z_0 ,
2. The values of the loaded series capacitors (C_{L1} and C_{L2}),
3. The values of the loaded shunt inductors (L_{L1} and L_{L2}),
4. Period Λ_D .

Effect of these parameters on scattering parameters of the DP-CRLH TLs is related to the propagation characteristics of the proposed array. Based on that, certain useful guidelines may be deduced for designing a high-performance DP-CRLH TL. These guidelines are intuitive, and lead to a good estimate of the required DP-CRLH TL dimensions. Those dimensions may then be verified and fine-tuned using the efficient a suitable commercial full-wave software package, such as CST Microwave Studio [111]. Fig. 5.2 shows the DP-CRLH LWA design procedure.

For designing a compact LWA based on the proposed DP-CRLH TLs, the following steps may be considered.

1. Based on the requirement, one has to determine the implementation structure for the host transmission line medium. For low-profile cases, microstrip line or SIW implementation may be considered. For high Q-factor and high power handling capability, waveguide or SIW implementation may be required.

Figure 5.2: The DP-CRLH LWA design procedure.

2. The approaches of the loaded capacitances and inductances are also required to be considered to match with different host transmission line medium. If a microstrip line is adopted as the host medium, the interdigital capacitors and short-circuited stub inductors may be used as the loaded series capacitances and shunt inductances, respectively. If a SIW is chosen, periodic transverse slots etched on the upper side of the waveguide and the metallic inductive vias shorted to the ground in the rectangular SIW may contribute the series capacitances and shunt inductances, respectively.
3. In order to design a LWA, a lower value of relative permittivity (ε_r) may be selected. For compact guided wave applications, a higher ε_r is more suitable, because a higher value of ε_r reduces the thickness and the length of the structure.
4. Before designing DP-CRLH TLs, a corresponding SP-CRLH TL is design first based on the requirement of the higher cutoff frequency of the backward radiation ($f_{\Gamma 1}$) and the lower cutoff frequency of the conventional RH radiation ($f_{\Gamma 2}$). The SP-CRLH TL will work as a basic model and give instructions for designing DP-CRLH TL.

- The series capacitance (C_L) and the shunt inductance (L_L) may be chosen based on

$$\begin{cases} \omega_{\Gamma 1} = \min(\frac{1}{\sqrt{C_R L_L}}, \frac{1}{\sqrt{C_L L_R}}), \\ \omega_{\Gamma 2} = \max(\frac{1}{\sqrt{C_R L_L}}, \frac{1}{\sqrt{C_L L_R}}), \end{cases} \quad (5.1)$$

where $\omega = 2\pi f$ is the angular frequency, and C_R , L_R are equivalent series capacitance and shunt inductance of the host transmission line medium, respectively. When designing a unbalanced dual-band SP-CRLH LWA, $f_{\Gamma 1} \neq f_{\Gamma 2}$, while $f_{\Gamma 1} = f_{\Gamma 2} = f_{\Gamma}$ for the balanced continuous SP-CRLH LWA. It is better to choose lager values of the series capacitance and shunt

inductance, because larger amplitude of the periodically loaded components can support higher radiation efficiency as predict in Chapter 4.

- Period of Λ_S is also needed to modified, and it qualitatively follows that the center frequencies increases as the period decreasing.
- The number of unit cells is requested to be decided. In order to obtain a high performance of LWAs, multiple unit cells have to be cascaded. However, this increases the geometrical dimensions of the antenna.

Until now, the SP-CRLH TL has been designed to radiate at $f_{\Gamma 1}$ in the LH region and $f_{\Gamma 2}$ in the conventional RH region.

5. In order to obtain the DP-CRLH TL, two unit cells of designed different SP-CRLH TLs are cascaded together.
 - Based on the lower center frequency of the new-found LW radiation ($f_{nR,L}$), the series capacitances (C_{L1} and C_{L2}) and shunt inductances (L_{L1} and L_{L2}) may be tuned as

$$1/(2\sqrt{C_{L1}L_{L1}}) < f_{nR,L} < 1/(2\sqrt{C_{L2}L_{L2}}). \quad (5.2)$$

At least one of the values of series capacitances (C_{L1} and C_{L2}) and shunt inductances (L_{L1} and L_{L2}) are alternate different.

- More tuning is needed to match the center frequencies of the LH radiation and the conventional RH radiation, because after tuning the C_{L1} , C_{L2} and L_{L1} , L_{L2} they shift as

$$\begin{cases} \omega_{\Gamma 1} = \min(\sqrt{\frac{1/C_{L1} + 1/C_{L2}}{2L_R}}, \sqrt{\frac{1/L_{L1} + 1/L_{L2}}{2C_R}}), \\ \omega_{\Gamma 2} = \max(\sqrt{\frac{1/C_{L1} + 1/C_{L2}}{2L_R}}, \sqrt{\frac{1/L_{L1} + 1/L_{L2}}{2C_R}}). \end{cases} \quad (5.3)$$

- To obtain higher radiation efficiency in the new LW region, more unit cells are required to be cascaded together to get longer aperture length, which leads to both a high radiation efficiency and small sidelobes. It is derived that the radiation efficiency for the new LW region is less than the radiation behavior in the LH region and conventional RH region.
6. The scanning angle of main lobe measured in broadside direction can be calculated from the beam-scanning expression as

$$\theta_r = \sin^{-1}\left[\frac{\beta}{k_0}\right], \quad (5.4)$$

where k_0 is the free-space wavenumber and β is the phase constant of the waveguide.

The design guidelines have not been described and several approaches of DP-CRLH LWAs, such as microstrip lines implementation and SIW implementation will be introduced in the following paragraphs based on this design guidelines.

5.2 Microstrip Line Implementation

The first approach of the proposed DP-CRLH TLs is based on the microstrip line double periodically loaded with alternate series interdigital capacitors and alternate shunt stub inductors shorted to the ground plane by a via. The similar structure, implemented by microstrip lines constituted by uniform interdigital capacitors and shunt short-circuited stub inductors shorted to the ground plane by via-holes, was first introduced to design SP-CRLH TLs [24] and subsequently used in various applications [69]. The unit cell of the microstrip line implementation of the DP-CRLH TLs, centered in the plane defined by the axis of the stub, represents the equivalent transmission line network as shown in Fig. 5.1. The unit cell is cascaded of two cells

of different SP-CRLH TLs, which are constituted by impedance branch with capacitances C_{Li} and by an admittance branch with inductances L_{Ri} ($i = 1, 2$). A compact LWA is designed by cascading 4 unit cells of the DP-CRLH TL implemented by the proposed microstrip line. The antenna displays the expected new forward LW radiation at 1.6 GHz, the backward radiation at 3.3 GHz in the LH region and forward again radiation at 3.9 GHz in the conventional RH region.

It is observed that when one of the loaded series capacitances or shunt inductances are alternate changed in the host transmission line medium, the structure will contribute this unique LW radiation of the DP-CRLH TLs, although the other component is single periodically loaded. Therefore, another four different structures are designed to demonstrate this observation. One is the microstrip line periodically loaded with uniform series capacitance and uniform shunt inductance. The second is the microstrip line double periodically loaded with alternate series capacitances and alternate shunt inductances. The third one is the microstrip line double periodically loaded with uniform series capacitance and alternate shunt inductances. The last one is the microstrip line double periodically loaded with alternate series capacitances and uniform shunt inductance. In order to get high gain and high radiation efficiency, 8 unit cells are cascaded together in this experiment.

5.2.1 Compact Leaky-Wave Antenna With Both Sides Of Shunt Inductances

Configurations

Figure 5.3 shows the configurations of the proposed DP-CRLH TL and SP-CRLH TL using microstrip line with double/single periodically loaded series interdigital capacitors and shunt short-circuited stub inductors according to the equivalent circuits shown in Fig. 3.1. The interdigital capacitors implement the series capacitances while

Figure 5.3: Layouts of the unit cell of the periodic structures. (a) DP-CRLH TL. (b) SP-CRLH TL. Substrate: $\epsilon_r=4.5$, loss tangent=0.016, thickness=1.6 mm. $l_1=4.35$, $l_2=3.7$, $l_3^*=4.5$, $l_3^{\#}=7.9$, $l_4=1$, $l_5=1$, $l_6=0.5$, $S_1=0.15$, $S_2=0.15$, $w=0.2$, $\Lambda_D=10$, $\Lambda_S=5$. All are in mm.

stubs are shorted to the ground to obtain the shunt inductances as proposed in [24]. When all the interdigital capacitors and short-circuited stubs are uniform as shown in Fig. 5.3(b), the structure works as a SP-CRLH TL with period of Λ_S , and it has been experimentally demonstrated for composite right/left handed (CRLH) property [1,2,24,25,93]. In the proposed DP-CRLH TL, one of the interdigitated structures is constituted by 11 fingers while the other is realized by 7 fingers to obtain unequal capacitances ($C_{L1} \neq C_{L2}$). The length of short-circuited stubs is also kept unequable to obtain the different inductances ($L_{L1} \neq L_{L2}$). The DP-CRLH TL consists of 4 unit cells with period of $\Lambda_D = 10$ mm. The SP-CRLH TL is printed with 8 unit cells and a period of $\Lambda_S = 5$ mm. The overall length of both the DP-CRLH TL and SP-CRLH TL is 4 cm. The output ports of the TLs are terminated with 50Ω . The CST Microwave Studio [111] was employed to optimize the dimensions of the TLs and they are listed

in Fig. 5.3. The double and single periodic microstrip lines were fabricated on an inexpensive FR-4 substrate with relative dielectric constant $\varepsilon_r = 4.5$, loss tangent of 0.016, and thickness of 1.6 mm. Although FR4 is known to be highly lossy, and may lead to high insertion loss in the passband of the structure, the new LW radiation is still observed with considerable gain for the proposed DP-CRLH TL fabricated on this substrate.

Simulation Results

Numerical simulations have been carried out on both the DP-CRLH TL and the SP-CRLH TL. Fig. 5.4 shows the simulated scattering parameters of the SP-CRLH TL, which present the frequency response of the CRLH features as predicted by [1, 2, 24, 25, 93]. The typical LH passband is obtained from f_L to $f_{\Gamma 1}$, followed by a stopband ($f_{\Gamma 1} \sim f_{\Gamma 2}$), and RH passband ($f_{\Gamma 2} \sim f_{RH}$), which are qualitatively matched with the dispersion characteristic analysis as shown in Fig. 3.3 (blue dashed line). Fig. 5.5 shows the simulated scattering parameters of DP-CRLH TL. Besides the CRLH characteristic like SP-CRLH TL, the new LW radiation is obtained just above f_{nRL} below the LH passband ($f_L \sim f_{\Gamma 1}$) as shown in Fig. 5.5(b).

It has been pointed out that the LH TL is frequency dispersive, in which the propagation constant (β) is a nonlinear function of frequency, which results in frequency-dependent group delay. Hence, it is possible to demonstrate the LH property of the TL based on the frequency-dependent group delay. Fig. 5.4 and Fig. 5.5 also show the frequency-dependent group delay for the wave propagating along the TLs. The group delay τ_g is defined as the negative rate of change of the insertion phase $\angle S_{21}(\omega)$ with frequency $\tau_g = -\partial \angle S_{21}(\omega) / \partial \omega$. Phase delay $\tau_p(\omega)$ is the time delay of a signal at a specific frequency, and is related to the phase of the system by $\tau_p(\omega) = -\angle S_{21}(\omega) / \omega$ [112]. The group velocity and group delay have the same sign since $\tau_g = L_g / v_g$, where L_g is the length of system. For the case of left-handed materials (LHMs), the reversal of

Figure 5.4: Simulated frequency response and group delay of the SP-CRLH TL.

phase velocity can be used as an indication of LH behavior: where the group velocity v_g is positive, the phase velocity v_p becomes negative [2, 113, 114]. This means that the group delay τ_g decreases with increasing frequency [29, 115–117]. It is convenient to get the group delay directly from the CST Microwave Studio and compare with the measurement results obtained directly from the Vector Network Analyzer. The backward wave characteristics can be seen in the frequency region from f_L to $f_{\Gamma 1}$ as shown in Fig. 5.4 and Fig. 5.5. Because the accuracy of the group delay is dependent on the data being free of noise and sufficient number of data points, the group delay prediction below the narrow band LW region may not be accurate. However confirmation of RH LW region at this band is done through dispersion analysis and measurements.

Figure 5.5: (a) Simulated frequency response and group delay of the DP-CRLH TL. (b) Enlarged version in the new LW region.

(a)

(b)

Figure 5.6: Prototype, measured frequency response and group delay of the SP-CRLH TL. (a) Prototype. (b) Frequency response and group delay.

Figure 5.7: Prototype of the proposed DP-CRLH TL.

Measurement Results

To confirm the characteristics of the new LW radiation of the DP-CRLH TL, the LWAs based on both the DP-CRLH TL and the SP-CRLH TL are fabricated and their frequency responses, group delay, dispersion behavior and radiation characteristics are measured. The fabricated prototypes are shown in Fig. 5.6(a) and Fig. 5.7, corresponding to the SP-CRLH TL and DP-CRLH TL, respectively. The measured scattering parameters using the Agilent N5230A PNA-L network analyzer are shown in Fig. 5.6(b) and Fig. 5.8. The LH passband, stopband and RH passband are found at $f_L \sim f_{\Gamma 1}$, $f_{\Gamma 1} \sim f_{\Gamma 2}$ and $f_{\Gamma 2} \sim f_{RH}$, respectively in the measurement results. The novel LW radiation is found just above f_{nRL} , which is located below the LH passband in the DP-CRLH TL. It is noted that the satisfactory return loss better than 10 dB is achieved in the new LW region while the insertion loss is around and below 20 dB as shown in Fig. 5.8(b). Small difference between the measured and simulated results may be attributed by the effects of manufacturing tolerances and the SMA-connectors mismatch with the circuit. The fingers of the fabricated interdigital structures are thinner, which decreases the capacitance. Due to soldering via-holes, some spurious

Figure 5.8: Measured frequency response and group delay of the proposed DP-CRLH TL. (a) Frequency response and group delay. (b) Frequency response and group delay in the new LW region.

resonances have also been observed, which may be improved by better fabrication approaches.

In order to validate the CRLH characteristics of TLs, Fig. 5.9 shows dispersion characteristics of the proposed DP-CRLH TL (red solid line) and the SP-CRLH TL (blue dashed line) extracted from the measured complex scattering parameters using the average phase shift incurred per unit cell ($\beta\Lambda$) derived by

$$\gamma = \frac{1}{\Lambda} \cosh^{-1} \frac{[(1 + S_{11})(1 - S_{22}) + S_{12}S_{21}] + \frac{Z_{01}}{Z_{02}}[(1 - S_{11})(1 + S_{22}) + S_{12}S_{21}]}{4S_{21}}, \quad (5.5)$$

where the complex propagation constant $\gamma = \alpha + j\beta$ and α, β are the attenuation and phase constant, respectively [19,118]. Z_{01} and Z_{02} are the impedance of ports 1 and 2, respectively, which are 50Ω in this study. $\Lambda = \Lambda_S$ is the period of the SP-CRLH SIW. $\Lambda = \Lambda_S = 5$ mm for the SP-CRLH TL and $\Lambda = \Lambda_D = 10$ mm for the DP-CRLH TL. The new RH LW characteristic is expected to appear in the region of $f_{nR,L} \sim f_{nR,LW}$ around 1.6 GHz that supports a forward LW radiation. The LH LW is expected to be obtained at $f_{\Gamma 1}$ around 3.3 GHz that supports the backward LW. The conventional RH LW is expected to be at $f_{\Gamma 2}$ around 3.9 GHz. The regions $f_{nR,H} \sim f_L$ and $f_{\Gamma 1} \sim f_{\Gamma 2}$ correspond to two stopbands. A comparison with the dispersion characteristics of SP-CRLH TL is also shown in Fig. 5.9 (blue dashed line) with period of $\Lambda_S = 5$ mm. The LH LW radiation and the conventional RH LW radiation are also observed in the SP-CRLH TL. However, it is noted that no LW performance occurs at the frequency band below the LH LW region.

It is pointed out that the measured scattering parameters are de-embedded at the two specified reference planes [119–121]. When a model is used for the test fixture, in the measurement system, it is necessary to perform the de-embedding process. The considered de-embedding is port extensions, which mathematically extends the measurement plane towards the device under test. It is assumed the fixture has no

Figure 5.9: Dispersion characteristics (extracted from measured scattering parameters) of the DP-CRLH TL and the SP-CRLH TL.

Figure 5.10: Normalized measured radiation patterns at three different LW frequencies.

loss, a linear phase response, and constant impedance. Port extension only adds or subtracts phase length from the measured scattering parameters. Well calibration of the vector network analyzer is required with Open-Short-Load-Through standard to get the accurate measurement results. This calibration process creates a reference plane at the point where the standards are connected.

The measured far-field radiation characteristics of the proposed DP-CRLH TL at 1.6 GHz, 3.3 GHz, and 3.9 GHz are shown in Fig. 5.10. As expected, the antenna radiates forward in the direction of $\theta_r = +3^\circ$ at $f = 1.6$ GHz in the new LW region just above f_{nRL} (red solid line), backward in the direction of $\theta_r = -19^\circ$ at $f = 3.3$ GHz around the LH LW frequency f_{r1} (blue dashed line), and forward again in the direction of $\theta_r = +12^\circ$ at $f = 3.9$ GHz around the conventional RH LW frequency f_{r2} (purple dotted line), where θ_r is the angle of main lobe measured in broadside direction. The gain is 4.12 dBi at the new LW frequency, 8.62 dBi at the LH LW frequency and 8.78 dBi at the conventional RH LW frequency.

The radiation efficiency can be estimated as $\eta = (1 - |s_{11}|^2 - |s_{21}|^2)$, based upon the relative power absorbed (RPA) of the element [122–125] as shown in Fig. 5.11. It is mentioned that the value of the RPA represents the total power losses of the electromagnetic device under investigation. The losses may include radiation loss, metal loss, dielectric loss, and surface-wave loss. Notably, neither the conductor loss nor the dielectric loss is separated from the leakage power in the RPA calculation. These non-separated losses are the main reason causing the difference between the simulated and the measured results. However, the conductor loss and the dielectric loss are much smaller than the leakage power of the experimental antenna in this work, because the RPA calculation considers the proposed experiment transmission line with only around one guided wavelengthes at the corresponding radiation frequency region (The overall length of the LWA, excluding the ports, is 40 mm, which is $0.45\lambda_g$ at 1.6 GHz, $0.93\lambda_g$ at 3.3 GHz, and only $1.1\lambda_g$ at 3.9 GHz.) although the loss tangent

Figure 5.11: RPA value of the proposed DP-CRLH TL extracted from the measured scattering parameters.

of the material is not very small ($\tan\delta = 0.016, \varepsilon_r = 4.5$). Hence, the RPA value is adopted as an approximate measure of the antenna radiation efficiency and provides an additional viewpoint to investigate the properties of the antenna. It is noted that the proposed DP-CRLH TL radiates at the new LW region, the LH LW region and the conventional RH region. It shows that the RPA is up to 80% in the new LW region, which confirms that the significant radiation of the new LW is obtained for the DP-CRLH TL. The RPA is around 85% in the LH region, and it is a little higher than the one in the new LW region. This property validates the analysis results based on the perturbation procedure method, that the new LW RH region is less than the radiation behavior in the LH region, as shown in Fig. 4.7.

Figure 5.12: Layouts of the periodic TLs. (a) DP-CRLH TL with different capacitances and inductances loaded. (b) SP-CRLH TL. (c) DP-CRLH TL with different inductances loaded. (d) DP-CRLH TL with different capacitances loaded. Substrate: $\epsilon_r=4.5$, thickness=1.6 mm. $l_1=4.35$, $l_2=3.7$, $l_3^*=4.5$, $l_3^\#=7.9$, $l_4=1$, $l_5=1$, $l_6=0.5$, $S_1=0.15$, $S_2=0.15$, $w=0.2$, $\Lambda_D=10$. All are in mm.

5.2.2 Longer Microstrip Line Approach

Configurations

Some other experiments have been carried out using both double and single periodic inductively and/or capacitively loaded microstrip lines which are equivalent to the TLs as shown in Fig. 5.12. It is noted that because the microstrip line can be seen as a thin

Figure 5.13: Prototypes of the proposed TLs. (a) SP-CRLH TL. (b) DP-CRLH TL with different both series interdigital capacitances and shunt inductances loaded. (c) DP-CRLH TL with different shunt inductances loaded. (d) DP-CRLH TL with different series interdigital capacitances loaded.

Figure 5.14: Simulated and measured results. (a) Frequency responses and measured group delay of SP-CRLH TL. (b) RPA value extracted from the simulated and measured scattering parameters.

homogenous isotropic medium, it is reasonable to follow the perturbation analysis on the transverse magnetic (TM)-like mode (i.e. the quasi-static TM mode) [20]. The period Λ_D is 10 mm with 8 unit cells for the DP-CRLH TLs and the period is 5 mm with 16 unit cells for the SP-CRLH TL. Therefore, the total length for both the SP-CRLH and DP-CRLH TLs is the same. Here, all the dimensions of the TLs follow the singular perturbation analysis and they are given in Fig. 5.12. The microstrip line has a width of 3.7 mm and thickness of 1.6 mm on FR-4 substrate with relative dielectric constant $\epsilon_r = 4.5$.

The fabricated prototypes are shown in Fig. 5.13, corresponding to the DP-CRLH TLs and SP-CRLH TL. The full wave simulated and measured scattering parameters of the TLs are shown in Fig. 5.14(a), Fig. 5.15(a) and Fig. 5.2.2, which present the frequency responses of the periodic TLs as predicted by (3.3), (3.5) and (4.52) and illustrate the new LW region ($f_{leaky} = f_{nR_L}$) followed by the LH passband ($f_L \sim f_{\Gamma 1}$) exhibiting a distinct backward wave characteristic, the stopband ($f_{\Gamma 1} \sim f_{\Gamma 2}$), and the RH passband and higher order mode band ($f > f_{\Gamma 2}$), which are corresponding with the dispersion characteristics analyzed in Fig. 3.3, Fig. 3.8 and Fig. 4.6.

It is noted that when the inductances or capacitances are double periodically loaded in the TL as shown in Fig. 5.13, the new LW phenomenon is obtained as shown in Fig. 5.2.2, although the other component is single periodically loaded. To get better performance, the TL is double periodically loaded with both alternate capacitances and alternate inductances, whose prototype and frequency responses are shown in Fig. 5.13(b) and Fig. 5.15(a), respectively. The frequencies $f_{nR_L} = 1.023$ GHz, $f_{\Gamma 1} = 2.467$ GHz and $f_{\Gamma 2} = 4.567$ GHz shown in Fig. 5.15(a) correspond to f_{nR_L} , $f_{\Gamma 1}$ and $f_{\Gamma 2}$ depicted in Fig. 3.3, Fig. 3.8 and Fig. 4.6. Below the LH region, the point ($f_{leaky} = f_{nR_L}$) exhibits in the LW phenomenon. Thus the new LW radiation occurs at very low frequency with narrow bandwidth. However, no LW property below the LH passband can be found in the SP-CRLH TL, as shown in Fig. 5.14(a).

(a)

(b)

Figure 5.15: Simulated and measured frequency responses and RPA value of DP-CRLH TL with different both series interdigital capacitances and shunt inductances loaded. (a) Frequency responses. (b) RPA value extracted from the simulated and measured scattering parameters.

Figure 5.16: Simulated and measured frequency responses of DP-CRLH TL with different shunt inductances loaded (up) and different series interdigital capacitances loaded (down).

Figure 5.14(b) and Fig. 5.15(b) show the RPA of SP-CRLH LWA and DP-CRLH LWA with different both series interdigital capacitances and shunt inductances loaded. They are also estimated as $\eta = (1 - |s_{11}|^2 - |s_{21}|^2)$ to indicate the radiation efficiency of the LWAs, based upon the RPA of the element [122–125]. It shows that the RPA is up to 98% in the new LW region, which confirms that the significant radiation of the new LW is obtained for the DP-CRLH LWA.

Figure 5.14(a) and Fig. 5.15(a) shows the group delays of the DP-CRLH TLs. The phase velocity v_p , group velocity v_g and group delay τ_g are

$$\begin{cases} v_p = \frac{\omega}{\beta} = \frac{L_g}{\tau_\phi} = -\frac{L_g}{\phi/\omega} \Rightarrow \text{Sign}(v_p) = \text{Sign}(\tau_\phi) = \text{Sign}(-\frac{\phi}{\omega}), \\ v_g = \frac{d\omega}{d\beta} = \frac{L_g}{\tau_g} = -\frac{L_g}{d\phi/d\omega} \Rightarrow \text{Sign}(v_g) = \text{Sign}(\tau_g) = \text{Sign}(-\frac{d\phi}{d\omega}), \\ \tau_g = -\frac{d\phi}{d\omega} = \frac{d\beta L_g}{d\omega} \Rightarrow \beta = \frac{1}{L_g} \int_0^{\omega_0} \tau_g d\omega, \end{cases} \quad (5.6)$$

where ϕ is the phase of the transfer function of the periodic structure, and L_g is the physical length of the transmission line. If the interface effect is negligible for a matched TL, the phase of the transfer function is related to the propagation constant by $\phi = -\beta L_g$. In the case of the LH TL, the phase velocity and group velocity are antiparallel, which means the sign of $-\phi/\omega$ and $-d\phi/d\omega$ is reversed.

In the lossless case, phase constant (β) exhibits the dispersion characteristic as $\beta = -1/\omega\sqrt{LC}$ for the pure LH TLs [24], while it is $\beta = \omega\sqrt{LC}$ for the RH TLs [47]. Hence, the phase and group velocities in the LH TLs are antiparallel hyperbolic as shown

$$v_p = -\omega^2\sqrt{LC} \quad \text{and} \quad v_g = +\omega^2\sqrt{LC}. \quad (5.7)$$

Then, the group delay τ_g is

$$\tau_g = -\frac{d\phi}{d\omega} = +\frac{d(\beta L_g)}{d\omega} = \frac{L_g}{\sqrt{LC}} * \frac{1}{\omega^2} \propto \omega^{-2}. \quad (5.8)$$

Therefore, the group delay is dependent on the term $1/\omega^2$, and the dispersion decreases as the frequency increases indicating left-handedness [24]. The reduction of τ_g reduces with increasing frequency until τ_g remains unchanged or shows little increase. At this point, the TL is going into the RH region, where the phase velocity and group velocity are parallel. In Fig. 5.14(a) and Fig. 5.15(a), the LH characteristic is found between f_L and $f_{\Gamma 1}$, and then the stop band and the RH region ($f_{\Gamma 2} \sim f_{RH}$) occur as the frequency increases.

The far-field radiation characteristics of the proposed antenna designed using microstrip line double periodically loaded with both series interdigital capacitances and shunt inductances are measured at $f_{n_{RL}} = 1.023$ GHz (new LW, forward radiation), $f_{\Gamma 1} = 2.467$ GHz (LH, backward radiation) and $f_{\Gamma 2} = 4.567$ GHz (conventional RH, forward radiation) as shown in Fig. 5.17. As the singular perturbation analysis

Figure 5.17: Measurement E -field radiation patterns in decibel scale at different frequencies. (a) $f_{nRL} = 1.023$ GHz (new RH LW, forward radiation), (b) $f_{\Gamma 1} = 2.467$ GHz (LH radiation, backward radiation), (c) $f_{\Gamma 2} = 4.567$ GHz (conventional RH radiation, forward radiation)

estimates, the antenna radiates at $f_{nRL} = 1.023$ GHz with a radiation angle $\theta_r = 18^\circ$ toward the direction of propagation. The radiation patterns for the LH and the conventional RH LW at $f = 2.467$ GHz and $f = 4.567$ GHz with radiation angle $\theta_r = -16^\circ$ and $\theta_r = 21^\circ$, respectively are shown in Fig. 5.17. The gain at the new LW frequency is 4 dB less than the gain at the LH radiation frequency, which is also predicted in the singular perturbation analysis, as shown in Fig. 4.7.

5.3 Substrate Integrated Waveguide Implementation

Some SP-CRLH TLs have been designed based on the SIW because the SIW takes advantage of the characteristics of conventional rectangular waveguides, such as high Q-factor, high power handling capacity, and low loss. The transmission line analysis gives the qualitative information about DP-CRLH TLs. It is noted that transmission lines double periodically loaded with either alternate capacitances or alternate inductances will contribute the new LW behavior, although the other component is single periodically loaded. Following the analysis of the proposed DP-CRLH TLs, SIW with double periodically loaded shunt alternate inductances and single periodically loaded uniform series capacitance is designed and realized as a DP-CRLH SIW. A compact LWA is designed based on the proposed DP-CRLH SIW, simulated using the numerical full-wave simulation tool and the performance is measured. The measurement results show that the designed LWA exhibit forward radiation at the new LW region from 5.2 GHz to 5.8 GHz (covering IEEE 802.11a band) and frequency-scanning from backward to forward directions in the LH and conventional RH region from 12.6 GHz to 17.8 GHz.

5.3.1 SP-CRLH SIW Implementation

At first, a SP-CRLH SIW implementation is proposed based on the general SP-CRLH TL model as shown in Fig. 2.4(b) [2]. The equivalent circuit to a one-dimensional (1D) SP-CRLH TL is composed of distributed series inductance (L_R) and capacitance (C_L) with shunt capacitance (C_R) and inductance (L_L) [2, 24, 69].

Description of the Structure

Figure 5.18 shows the model of the proposed SP-CRLH SIW, which consists of the host medium (C_R, L_R) periodically loaded with shunt inductance L_L and series capacitances C_L with period Λ_S . The period Λ_S is normally small compared to the guided wavelength λ_g . The etched periodically transverse slots in the upper side of the waveguide contribute the series capacitance and metallic inductive multiple posts shorted to the ground generate shunt inductance. Periodic loading of the series capacitances and shunt inductances are the key elements to obtain SP-CRLH TLs [20, 24]. The configuration of the unit cell of the SP-CRLH TL is shown in Fig. 5.19(a).

Figure 5.19 shows the detailed configuration and equivalent circuits of the proposed SP-CRLH SIW which consists of TLs with periodically loaded series capacitances (C_L) and shunt inductances (L_L). By etching periodically transverse slots in the upper side of the waveguide and metallic inductive multiple posts shorted to the ground in the rectangular SIW result the series capacitances (C_L) and shunt inductances (L_L), respectively. Because it is difficult to extract the equivalent capacitances and inductances for the proposed complicated SIW, the CST Microwave Studio simulation tool is used to analyze the structure.

Feeding technique is an important design part, which influences the input impedance and characteristics of the antenna. The bandwidth can be widened by the inset microstrip feed [126]. The inset microstrip feed is used in this paper to widen the band-

Figure 5.18: Equivalent circuit and layout of the proposed SP-CRLH SIW.

Figure 5.19: Layout of the proposed structure. (a) CRLH SIW unit cell. (b) Layout of the impedance matching network of the proposed structure. Substrate is Rogers RT5880: $\epsilon_r = 2.2$, $\tan\delta = 0.0011$, thickness = 0.787 mm. $W = 14.5$, $g = 0.143$, $\phi = 0.5$, $R_1 = 2$, $R_2 = 0.5$, $R_3 = 1.02$. $P_1 = 9.5$, $P_2 = 2.6$, $P_3 = 0.127$, $P_4 = 1.6$, $P_5 = 9.0$, $P_6 = 0.5$, $P_7 = 0.68$, $P_8 = 0.5$, $P_9 = 0.8$. $\Lambda_S = 4.143$. All are in mm.

width of the passband of the transmission line to match the need of the wide band LW radiation. A small shunt tuning stub is also required to tune the frequency to match to the LW region based on the concept of the single-stub impedance-matching technique. To accommodate the stub, the inset feed is modified on one side only. Fig. 5.19(b) shows the modified inset-feed and tuning stub combination impedance matching network.

Figure 5.18 shows the structure of the proposed SP-CRLH SIW which is constituted by cascading 13 unit cells with the same dimensions as shown in Fig. 5.19. The SP-CRLH SIW is designed on Rogers RT5880 substrate with the thickness of 0.787 mm and dielectric constant $\epsilon_r = 2.2$. The output port of the SP-CRLH SIW is terminated to 50Ω by introducing the modified inset-feed and tuning stub combination impedance matching network as shown in Fig. 5.19(b) to suppress the reflection inducing spurious beams on the other side of the normal to the SIW.

Results Discussion

Figure 5.20 shows the dispersion diagram calculated from the simulated complex scattering parameters using the average phase shift incurred per unit cell ($\beta\Lambda_S$) derived by (5.5). $\Lambda = \Lambda_S$ is the period of the SP-CRLH SIW. Typical LH characteristic is found below $f_{\Gamma 1}$, where the phase constant decreases with increasing frequency while the group velocity and phase velocity are antiparallel. At higher frequency beyond $f_{\Gamma 2}$, the structure works as RHM, where the group velocity and phase velocity are parallel in this region. When the dispersion characteristic of SP-CRLH SIW is in the fast wave region ($\beta < k_0$), the structure is able to radiate, which can be used for potential LWAs design [1,2], where the k_0 is the free space wave number. Fig. 5.20 shows that a region of LH propagation extending below $f_{\Gamma 1}$ supports a backward wave, while the conventional RH leaky-wave (LW) is at a frequency band higher than $f_{\Gamma 2}$.

The radiation property of the proposed SP-CRLH SIW can be seen from the

Figure 5.20: Dispersion characteristics (extracted from measured complex scattering parameters) comparing with dispersion characteristics obtained from transmission line theory. $C_R = 0.32$ pF, $L_R = 0.68$ nH, $C_L = 0.2$ pF, $L_L = 0.4$ nH, $\Lambda_S = 4.143$ mm.

frequency responses as shown in Fig. 5.21. The satisfactory return loss below -10 dB is achieved from f_{L_LW} to $f_{\Gamma 1}$ and from $f_{\Gamma 2}$ to f_{R_LW} while the insertion loss is almost below -15 dB. It is noted that the return loss is slightly up and a stop band occurs from $f_{\Gamma 1}$ to $f_{\Gamma 2}$.

5.3.2 DP-CRLH SIW Implementation

As discussed in Chapter 3 and Chapter 4, in addition to the CRLH property as exhibited by SP-CRLH TLs, a new forward LW radiation below the LH passband with narrow bandwidth will be obtained for the DP-CRLH TL double periodically loaded by a pair alternate series capacitances and/or shunt inductances. It is also proposed that the new LW radiation can be excited by the TLs double periodically loaded with

Figure 5.21: Simulated frequency responses of the SP-CRLH SIW.

either alternate series capacitances or alternate shunt inductances, although the other components are single periodically loaded with uniform value. Hence, an SIW double periodically loaded with alternate shunt inductances and single periodically loaded uniform series capacitances is designed based on the proposed SP-CRLH SIW.

The new structure is expected to contribute to design of compact LWAs for short range dedicated communication applications. Firstly, an implementation of the DP-CRLH TL is performed using SIW with double periodic loaded alternate shunt inductances and periodic loaded uniform series capacitances. The full wave animated field maps of the electric field distribution are extracted from CST Microwave Studio. Next, the dispersion characteristics of the new structure extracted from the simulated and experimentally measured complex scattering parameters are compared with the one calculated from the transmission line theory discussed in Chapter 3. Finally, the

frequency responses and radiation characteristics are simulated and measured for the proposed DP-CRLH SIW LWA with the new LW radiation covering the IEEE 802.11a WLAN band. They display the expected new RH LW radiation below LH region with narrow bandwidth. Besides the LH LW radiation from 12.6 to 14 GHz, the typical new RH LW is obtained at 5.6 GHz with gain of 6.5 dBi and main lobe radiating at direction of $+33^\circ$ to broadside direction in the DP-CRLH SIW.

Description of the Structure

Figure 5.22 shows the model of DP-CRLH TLs, which consists of the host medium (C_R, L_R) double periodically loaded with alternative shunt inductances L_{L1}, L_{L2} and alternative series capacitances C_{L1}, C_{L2} with period Λ_D . The period Λ_D and $\Lambda_S = \Lambda_D/2$ are normally small compared to wavelength. The shunt inductances L_{L1}, L_{L2} and/or series capacitances C_{L1}, C_{L2} are alternative within one unit cell to achieve the DP-CRLH TL as proposed in Chapter 3 and Chapter 4.

Figure 5.23 shows the unit cell of the DP-CRLH SIW based on the transmission line theory, respectively. To get the series capacitances (C_L) in the equivalent circuit of SP-CRLH TLs as shown in Fig. 3.1(a) and Fig. 3.7(b), periodic transverse slots are etched on the upper side of the waveguide [30,127,128]. To get the shunt inductances (L_L), metallic inductive via-holes are introduced and shorted to the ground in the rectangular SIW [30,128,129]. As it is proposed that transmission lines with the double periodically loaded alternate inductances and single periodically loaded with uniform capacitance (Fig. 3.8(a)) will contribute a new RH LW radiation, the SIW with double periodically loaded alternate shunt inductances ($L_{L1} \neq L_{L2}$) is designed by tuning the number of metallic inductive multiple posts shorted to the ground in the rectangular SIW while keeping the loaded series capacitance uniform ($C_{L1} = C_{L2}$) as shown in Fig. 5.22.

Figure 5.24 shows the fabricated LWA based on DP-CRLH SIW with the same

Figure 5.22: Equivalent circuit and layout of the proposed DP-CRLH SIW. The dimensions are in mm.

dimensions as shown in Fig. 5.23. The DP-CRLH SIW LWA is fabricated on Rogers RT5880 substrate with the thickness of 0.787 mm and dielectric constant $\epsilon_r = 2.2$ and $\tan\delta = 0.0011$. The overall length of the LWA, excluding the ports, is 53.86 mm.

Results Discussion

Numerical full wave simulations have been carried out on both double and single periodic SIW structures with the same dimensions as shown in Fig. 5.23 using the commercial software CST Microwave Studio. The period is 8.286 mm with 6.5 unit cells for the DP-CRLH SIW and 4.143 mm with 13 unit cells for SP-CRLH SIW. A half unit cell of the DP-CRLH SIW is used to make the structure symmetrical and the port matching to the modified inset-feed and tuning stub combination impedance matching network. For LWAs, a substrate with a low dielectric constant ($\epsilon_r = 2.2$)

Figure 5.23: Unit cell of the DP-CRLH SIW. Substrate is Rogers RT5880: $\epsilon_r = 2.2$, $\tan\delta = 0.0011$, thickness $h = 0.787$ mm. $L = 4$, $W = 14.5$, $g = 0.143$, $\phi = 0.5$, $R_1 = 2$, $R_2 = 0.5$, $R_3 = 7.25$, $R_4 = 0.7$, $\Lambda_D = 8.286$. All are in mm.

Figure 5.24: Prototypes of the fabricated DP-CRLH SIW LWA.

and a low loss tangent ($\tan\delta = 0.0011$) is preferred to enhance the fringing fields and hence get higher efficiency of the radiation. The output port of the LWA is terminated to 50Ω to suppress the reflection inducing spurious beams on the other side of the normal to the SIW.

The simulated and measured scattering parameters are shown in Fig. 5.25, which

(a)

(b)

Figure 5.25: Simulated and measured frequency responses of the DP-CRLH SIW. (a) S_{11} . (b) S_{21} .

present the frequency responses of the DP-CRLH SIW as predicted by (3.5), (4.52) and (3.3). The typical new LW radiation occurs from $f_{nR-L} = 5.9$ GHz to $f_{nR-LW} = 6.2$ GHz, the LH LW radiation is obtained from $f_{L-LW} = 11.9$ GHz to $f_{\Gamma_1} = 13.9$ GHz and the conventional RH LW radiation is found from $f_{\Gamma_2} = 14.5$ GHz to $f_{R-LW} = 17.9$ GHz for the DP-CRLH SIW as shown in Fig. 5.25. The measured resonant frequency has been slightly shifted down due to the effects of manufacturing tolerances, like nonuniform series capacitances, and the simulated resonant frequency is tuned to be a little higher than the expected frequency band to counteract the effects of manufacturing tolerances. However, the new LW is not found in the SP-CRLH SIW as shown in Fig. 5.21, while the LH LW ($f_{L-LW} \sim f_{\Gamma_1}$) and conventional RH LW ($f_{\Gamma_2} \sim f_{LW}$) are obtained. The LH passband is obtained around f_L as shown in Fig. 5.25 corresponding to the dispersion characteristics as shown in Fig. 3.8(a).

Figure 5.25 also shows the measured transmission characteristics of the DP-CRLH SIW LWA using the Agilent N5230A PNA-L network analyzer. The new RH LW is obtained from $f_{nR-L} = 5.2$ GHz in the DP-CRLH SIW LWA covering IEEE 802.11a band. The resonant frequency is slightly shifted down compared to the simulation result ($5.9 \sim 6.2$ GHz) due to the effects of manufacturing tolerances. The LH and conventional RH LWs are obtained from $f_{L-LW} = 12.6$ GHz to $f_{\Gamma_1} = 14$ GHz and from $f_{\Gamma_2} = 14.6$ GHz to $f_{R-LW} = 17.8$ GHz. Because of the multiple reflections and mutual couplings from other adjacent cells, the transmission and reflection characteristics are slightly deviated in the band between LH and conventional RH regions from 14 to 14.6 GHz. The cutoff frequencies depicted in Fig. 5.25 correspond to that shown in Fig. 3.3, Fig. 3.8 and Fig. 4.6.

Figure 5.26 shows the dispersion characteristics extracted from the simulated and experimentally measured complex scattering parameters using the average phase shift incurred per unit cell ($\beta\Lambda_D$) derived by Eq. 5.5 [19, 118], where $\Lambda = \Lambda_D$ is the period of the DP-CRLH SIW. This dispersion relation reveals the new RH LW radiation

Figure 5.26: Dispersion characteristics (extracted from measured complex scattering parameters) comparing with dispersion characteristics obtained from transmission line theory. $C_R = 0.32$ pF, $L_R = 0.68$ nH, $C_{L1} = C_{L2} = 0.2$ pF, $L_{L1} = 0.3$ nH, $L_{L2} = 0.5$ nH, $\Lambda_D = 8.286$ mm.

near 5.6 GHz from f_{nR_L} to f_{nR_LW} that supports a forward wave. A region of LH propagation extending near 13.5 GHz from f_{L_LW} to $f_{\Gamma 1}$ supports a backward wave. The conventional RH LW is at a frequency band higher than $f_{\Gamma 2}$. A comparison with the DP-CRLH TL dispersion characteristic derived from the transmission line theory shows a qualitative correspondence. The simulation results and experimentally observed results have good agreement with the analyzed dispersion characteristic, which further validates the new RH property and the LH property of the DP-CRLH TL.

The forward and backward wave propagation phenomena in the new RH region and the typical LH region can be clearly viewed from the movement of concentrated field

Figure 5.27: The electric distribution (x-y plane) of the transmission line in z direction at 6 GHz for different instants. (a) Phase = 0° ($t = 0$), (b) Phase = 90° ($t = T/4$), (c) Phase = 180° ($t = T/2$), (d) Phase = 270° ($t = 3T/4$), (e) Phase = 360° ($t = T$).

vector of the electric field distribution, where the phase constant (β) and wave vector ($\hat{k}$) are antiparallel. The electric field distribution in z -direction maps extracted using CST Microwave Studio at 6 GHz (new RH propagation) and 13 GHz (LH propagation) are shown in Fig. 5.27 and Fig. 5.28 in one period T (phase = $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$ corresponding to $t = 0, T/4, T/2, 3T/4, T$), respectively. The polarized incident waves propagate along x -direction with electric field in the z -direction and magnetic field in the $-y$ -direction. The frames at the different instants in Fig. 5.27 show clearly the forward wave propagation which verifies the RH property of the DP-CRLH TL at lower frequency, and the frames in Fig. 5.28 show the backward wave propagation which demonstrates the LH nature of the DP-CRLH TL. Fig. 5.29 shows the electric field distribution in z -direction at 6 GHz, 13 GHz and 16.5 GHz.

The radiation characteristics are simulated and measured to confirm the new RH

Figure 5.28: The electric distribution (x-y plane) of the transmission line in z direction at 13 GHz for different instants. (a) Phase = 0° ($t = 0$), (b) Phase = 90° ($t = T/4$), (c) Phase = 180° ($t = T/2$), (d) Phase = 270° ($t = 3T/4$), (e) Phase = 360° ($t = T$).

Figure 5.29: The electric field distribution (x-y plane) of the transmission line in x direction at different frequencies. (a) $f = 6$ GHz (new RH), (b) $f = 13$ GHz (LH), (c) $f = 16.5$ GHz (conventional RH).

Figure 5.30: Measured and simulated antenna radiation patterns, E-plane (y-z plane). Solid line: measurement results. Dotted line: simulation results. (a) the new RH LW radiation pattern, (b),(c) the LH LW radiation pattern, (d)-(f) the conventional RH LW radiation pattern.

and LH phenomena. The actual radiation spectrum in the new RH region (5.2 ~ 5.8 GHz) lowered in comparison with the simulation (5.9 ~ 6.2 GHz) as seen from scattering parameters shown in Fig. 5.25. The radiation characteristics are measured in the reasonable LW region from 5.2 to 5.8 GHz and the simulated result is given from 5.9 to 6.2 GHz. Fig. 5.30 shows the normalized far-field radiation patterns of the LWA based on the proposed DP-CRLH SIW at different frequencies. The measurement results show that the antenna radiates forward in the direction of $+33^\circ$ at 5.6 GHz (Fig. 5.30(a)), backward with angle of -22° at 12.5 GHz and -11° at 13.5 GHz (Fig. 5.30(b) and Fig. 5.30(c)) and forward again with angle of $+8^\circ$, $+23^\circ$ and $+34^\circ$ at 14.5, 15.5 and 16.5 GHz, respectively (Fig. 5.30 (d-f)), corresponding with the new RH LW radiation, LH LW radiation and conventional RH LW radiation.

The leaky-wave antennas radiate continuously within the fast-wave region to a beam direction related to the phase constant in both the LH and the conventional RH regions and can be calculated from the beam-scanning expression [79, 130]

$$\theta = \sin^{-1}\left[\frac{\beta}{k_0}\right], \quad (5.9)$$

where θ is the scanning angle of main lobe measured in broadside direction (z -direction), β is propagation constant of the waveguide, k_0 is the free-space wavenumber. From (5.9), it is noted that the radiation angle of the LWA is determined by the phase constant (β) which is dependent on shunt inductances and series capacitances of the periodic structure. The radiation angle of the LWA is also an evidence to demonstrate the sign of phase velocity of the transmission line.

The frequency dependence of the simulated and measured beam directions are shown in Fig. 5.31. In the new right-handed region, the beam directions are measured in the reasonable leaky-wave region from 5.2 to 5.8 GHz and the simulated result is given from 5.9 to 6.2 GHz. It is noted that the dispersion curve slightly increases

Figure 5.31: Measured and simulated beam direction against the radiation frequency.

with frequency increasing as shown in Fig. 5.26. That means the scanning angle should also scan from broadside to the endfire direction with frequency increasing. A small frequency scanning can be observed from $+33^\circ$ to $+37^\circ$ in the simulation results as shown in Fig. 5.31 (solid blue line from 5.9 to 6.2 GHz). The frequency scanning is shown in the measurement results too. Because the measurement system is not sensitive enough and because of the fabrication tolerance of the structure, a little angle shift occurs, especially at $f = 5.4$ GHz. However, since the leaky-wave region is too narrow, the scanning angle is not wide even in the simulation results. The beam directions are measured around $+33^\circ$ to the broadside direction in the new leaky-wave region and this property that the main lobe steers to forward direction demonstrates the right-handed characteristic occurs in the DP-CRLH TL below the left-handed passband at very low frequency.

In the LH and conventional right-handed region, the main lobe steers to end fire in $x - z$ plane with the increase frequency and it scans from -42° at 11.5 GHz to 40° at 18 GHz. The total beam scanning angle is up to 82° in measurement as shown in Fig. 5.31 (red stars). The beam directions are also observed in the radiation patterns as shown in Fig. 5.30.

Figure 5.32 shows the frequency dependence of the simulated and measured peak gain and simulated directivity. The measured gain is around 6.5 dBi in the new LW region, 7.3 dBi in the LH region, and 8.2 dBi in the conventional RH LW region. The radiation gain is reduced at 14 GHz because of the resonance point [131] which could also be observed in Fig. 5.25 and Fig. 5.26. The gain of the proposed antenna is simulated and measured only in the LW region, because most of the input power is reflected and the gain should be low and do not convey useful information in the bandgap region. Fig. 5.32 shows the simulated directivity increases with frequency. In the new LW radiation region, the simulated gain is about 2.5 dBi lower than the simulated directivity. The difference between the simulated gain and directivity increases with frequency increasing.

The radiation efficiency of the proposed DP-CRLH SIW LWA is also estimated based on $\eta = (1 - |s_{11}|^2 - |s_{21}|^2)$, based upon the RPA of the element [122–125] as shown in Fig. 5.33. It is also mentioned that the value of the RPA represents the total power losses, including radiation loss, metal loss, dielectric loss, and surface-wave loss. However, the conductor loss and the dielectric loss are much smaller than the leakage power of the experimental antenna, because the RPA calculation considers the proposed experiment transmission line with only around one or two guided wavelengths at the corresponding radiation frequency region (The overall length of the LWA, excluding the ports, is 53.86 mm, which is $2.87\lambda_g$ at 16 GHz, $2.33\lambda_g$ at 13 GHz, and only $1.07\lambda_g$ at 6 GHz.) and the loss tangent of the material is very small ($\tan\delta = 0.0011, \varepsilon_r = 2.2$). Hence, the RPA value is adopted as an approximate

Figure 5.32: Simulated and measured gain and simulated directivity of the DP-CRLH SIW LWA against the radiation frequency.

measure of the antenna radiation efficiency and provides an additional viewpoint to investigate the properties of the antenna. In addition, to make up the limitation of the indirection method of calculating the radiation efficiency based on the RPA of the element, the radiation efficiency extracted from the full wave simulation tool of CST Microwave Studio is given in Fig. 5.33 (red star points) to compare with the radiation efficiency estimated from the measured scattering parameters. It is noted that the simulated radiation efficiency is slightly lower than the RPA extracted from the measured scattering parameter based on the RPA of the element. However, both of the results show that the proposed DP-CRLH SIW radiates at the new LW region, the LH LW region and the conventional RH region. It shows that the radiation efficiency is up to 60% in the new LW region, which confirms that the significant radiation of the

Figure 5.33: RPA value extracted from the measured scattering parameters and simulated radiation efficiency of the proposed DP-CRLH SIW LWA as a function of frequency.

new LW is obtained for the DP-CRLH SIW LWA. The radiation efficiency is around 80% in the LH region as shown in Fig. 5.33, and it is a little higher than the one in the new LW region. This property validates the analysis results based on the perturbation procedure method, that the new LW RH region is less than the radiation behavior in the LH region, as shown in Fig. 4.7.

5.4 Conclusion

A DP-CRLH TL is proposed and analyzed based on transmission line theory in Chapter 3 and singular perturbation procedure in Chapter 4. Besides the CRLH property, a new LW radiation is observed at a lower frequency with narrow bandwidth. The

new structure is expected to contribute toward the design of compact LWAs for communication system. The one-dimensional implementations by microstrip lines and SIW, presented in this thesis, are useful for designing compact LWAs, which has been demonstrated in this chapter. Design guidelines and the procedures have been explained through a number of examples, which have also been fabricated, and the measurement results clearly validate the methods presented here.

Firstly, experiments are performed using microstrip lines with double periodically loaded shunt short-circuited stub inductors and series interdigital capacitors. Frequency response, dispersion behavior and radiation characteristics are measured from the prototype structure and they display the expected novel forward LW radiation in the direction of $\theta_r = +3^\circ$ and gain of 4.12 dBi at 1.6 GHz, the LH backward radiation in the direction of $\theta_r = -19^\circ$ and gain of 8.62 dBi at 3.3 GHz, and the conventional RH forward radiation in the direction of $\theta_r = +12^\circ$ and gain of 8.78 dBi at 3.9 GHz.

Next, It is noted that transmission lines double periodically loaded with either alternate capacitances or alternate inductances will contribute the new LW behavior. Another series of experiments on both DP-CRLH TLs and the SP-CRLH TL are performed using microstrip lines with double/single periodically loaded series interdigital capacitances and/or shunt inductances. In order to get high gain and high radiation efficiency, 8 unit cells of the DP-CRLH TLs are cascaded together in this experiment. Frequency response and radiation characteristics are measured for the proposed DP-CRLH LWA, and they display the new forward LW radiation below the LH LW region with narrow bandwidth in the DP-CRLH TL. The new RH LW is obtained at 1.023 GHz with gain of 5.1 dBi and main lobe radiating at $\theta_r = +18^\circ$ to broadside direction.

Finally, a DP-CRLH TL is proposed based on SIW, which takes advantage of the characteristics of conventional rectangular waveguides, such as high Q-factor, high power handling capacity, and low loss. It is also because of the observation that transmission lines double periodically loaded with either alternate capacitances or alternate

inductances will contribute the new LW behavior. SIW with double periodically loaded shunt alternate inductances and periodic loaded uniform series capacitance is designed and realized as a DP-CRLH SIW. The SIW implementation of a DP-CRLH transmission line is designed and illustrated with simulated and experimentally demonstrated results. The electric field distribution plots are extracted from full wave simulation tool which exhibit the forward and backward wave propagation in the new LW region and the typical LH region, respectively. The dispersion characteristics are extracted from the simulated and measured complex scattering parameters. They display the expected new RH LW radiation below LH region with narrow bandwidth. A compact LWA based on the DP-CRLH SIW is designed and the radiation characteristics are tested and demonstrated to exhibit forward radiation at the new LW region from 5.2 GHz to 5.8 GHz (covering IEEE 802.11a band) and frequency-scanning from backward to forward directions in the LH and conventional RH region from 12.6 GHz to 17.8 GHz.

Parameters	LWA based on Microstrip line	LWA based on SIW
Bandwidth	narrow	wide
Gain	low	high
Radiation efficiency	low	high
Directivity	normal	normal
Frequency scanning realization	difficult	easy
Matching network design	easy	difficult
fabrication	easy	difficult

Table 5.1: Comparison between microstrip line and SIW implementations of DP-CRLH LWAs.

The applications of the proposed DP-CRLH TL approaches have been designed based on the microstrip lines and SIW as discussed in this chapter. Three LWAs are designed and the designed examples have been fabricated. The measurement results are in agreement with the expected performance, which validate the unique properties of the proposed DP-CRLH TLs. The comparison between microstrip line and SIW implementations of DP-CRLH LWAs are listed in Table 5.1. Because the SIW

takes advantage of the characteristics of conventional rectangular waveguides, such as high Q-factor, high power handling capacity, and low loss, the performance of the designed DP-CRLH SIW LWA is better than the one approached by the microstrip line. The gain and radiation efficiency of SIW LWA is higher. The bandwidth is wider than the one based on microstrip line. Furthermore, it is easy to realize continuous frequency scanning property and design balanced CRLH TL based on the SIW structure. However, since there is no viahole in the microstrip line approaches, it is easy to be fabricated. The matching network design is also easier for the microstrip line than the SIW.

Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

A new class of double periodic composite right/left handed (DP-CRLH) transmission lines (TLs) has been presented. It promises a new forward leaky-wave (LW) radiation at a lower microwave band below the left-handed (LH) passband, in addition to the composite right/left handed (CRLH) properties as exhibited by single periodic composite right/left handed (SP-CRLH) TLs. The proposed DP-CRLH TLs provide an attractive and promising method to design leaky-wave antennas (LWAs) which is more compact than the one based on SP-CRLH TLs.

Before analyzing the proposed DP-CRLH TLs, a comprehensive literature review on metamaterials (MTMs) has been done as the first step for the proposed research. The primary target of this step is to understand the fundamental operating principle of the SP-CRLH TLs and the recent designs, especially LWAs.

Based on the understanding the principle of SP-CRLH TLs, a novel DP-CRLH TL, by modifying the SP-CRLH TLs, has been proposed. TL theory was used to

analyze DP-CRLH TLs to determine the LW radiation characteristics through the dispersion behavior based on the complex propagation constant (phase and attenuation constants). A singular perturbation procedure was used to further analyze the radiation characteristics, especially for the microstrip line implementation of the DP-CRLH TLs. Apart from the fact that DP-CRLH TL can radiate in the left-handed (LH) region and forward in the right-handed (RH) region (similar to SP-CRLH TLs), DP-CRLH TL promises a new forward leaky-wave (LW) radiation at low frequency below the LH passband with narrow bandwidth. This is an attractive and promising method to design compact LWAs. The new radiation frequency is significantly lower than the LH passband of the SP-CRLH TL with the corresponding similar period. Hence, it is possible to design LWAs which are more compact than the one based on SP-CRLH TLs.

A number of parameters including variation of geometrical dimensions, the loaded capacitances and inductances, and the period of the unit cell have been studied. The expressions for the cut-off frequencies have been derived for the infinite periodic structures. Also, a qualitative estimation of the cut-off frequencies for the periodic structure has been provided (for three cells). Based on these parametric studies, and suitable modeling of the proposed DP-CRLH TLs, a list of design guidelines has been developed. A few design examples of DP-CRLH TLs are presented. The proposed structures have also been fabricated and measured results are found to be in agreement with the expected results. Dispersion characteristics have been investigated with the help of the singular perturbation procedure. This method also gives information about the radiation efficiency and the radiation angle for the optimum radiation efficiency.

- The first experiment was performed using microstrip lines with double periodically loaded shunt short-circuited stub inductances and series interdigital capacitances. The designed compact LWA displays the expected new-found forward

LW radiation in the direction of $\theta_r = +3^\circ$ and with gain of 4.12 dBi at 1.6 GHz, the LH backward radiation in the direction of $\theta_r = -19^\circ$ and with gain of 8.62 dBi at 3.3 GHz, and the conventional RH forward radiation in the direction of $\theta_r = +12^\circ$ and with gain of 8.78 dBi at 3.9 GHz.

- Another kind of compact LWAs have been designed using longer microstrip lines double/single periodically loaded with series interdigital capacitances and/or single-side shunt short-circuited stub inductances. As the analysis estimates, the LWA radiates at $f = 1.023$ GHz with gain of 5.1 dBi and radiation angle $\theta_r = 18^\circ$ toward the direction of propagation, at $f = 2.467$ GHz with gain of 7.9 dBi and radiation angle $\theta_r = -16^\circ$, and at $f = 4.567$ GHz with gain of 9.3 dBi and $\theta_r = 21^\circ$.
- It is noted that the new LW radiation is obtained through the TLs double periodically loaded with either alternate series capacitances or alternate shunt inductances, although the other components are single periodically loaded. A substrate integrated waveguide (SIW) double periodically loaded with alternate shunt alternate inductances and single periodically loaded with uniform series capacitance has been designed and realized as a DP-CRLH TL. The measured radiation characteristics of the DP-CRLH SIW LWA display the new forward LW radiation from 5.2 GHz to 5.8 GHz in the direction of $\theta_r = +33^\circ$ and with gain of 8.62 dBi, and broadband frequency-scanned LW radiation from -42° at 11.5 GHz in the LH region to 40° at 18 GHz in the conventional RH region through the broadside radiation at 14 GHz. The new LW radiation band is tuned to cover the IEEE 802.11a band for wireless local area network (WLAN) applications.

It should also be mentioned that the proposed DP-CRLH TLs suffer from a few limitations, and hence it may not be a suitable selection for every application. One lim-

itation is that multiple unit cells have to be cascaded to get more radiation efficiency, but this increases the geometrical dimensions of the antenna. Another limitation is that it will be very difficult to design a wideband LWA at low frequency based on the proposed DP-CRLH TLs, because the boundary of the fast wave region ($\beta = k_0$) is extremely close to the vertical axis of the dispersion diagram ($\beta = 0$) and the fast wave region at low frequency is too narrow.

6.2 Recommendations for Future Work

This thesis has presented design of novel DP-CRLH TLs. Complete analysis and guidelines of a primitive form of this concept have been presented. However, like any useful new idea, it offers opportunities for many future investigations on the same or similar structures. Some of these possible studies are suggested as follows:

- 1 Propagation analysis based on the perturbation method for the DP-CRLH TLs.
 - Based on the exact formulation, numerical analysis of specific structures may be carried out by truncating the Fourier components to a finite number. This would cause some degree of approximation, but the accuracy may be systematically improved by increasing the number of Fourier components. Until now, the analysis of the DP-CRLH TLs is based on the singular sinusoidal form of the singular perturbation procedure. In the future, working on the double sinusoidal form of the perturbation procedure may be considered, which would be more accurate for analysis of the periodic structure.
 - In order to get more accurate analysis, the periodic structure from double perturbation procedure may also be analyzed. That means both the permeability and permittivity will have perturbed change because of the periodically loaded

shunt inductances and series capacitances.

$$\varepsilon = \varepsilon_0 \varepsilon_{eff} - \frac{1}{\omega^2 L_L \Lambda_D}, \quad \mu = \mu_0 - \frac{1}{\omega^2 C_L \Lambda_D}. \quad (6.1)$$

Where both the inductance and the capacitance are periodically modulated as follow:

$$\begin{cases} L_L = L_{L0}[1 + \delta\eta_1^L \cos(Kz + \theta_1^L) + \delta^2\eta_2^L \cos(2Kz + \theta_2^L) + \dots], \\ C_L = C_{L0}[1 + \delta\eta_1^C \cos(Kz + \theta_1^C) + \delta^2\eta_2^C \cos(2Kz + \theta_2^C) + \dots]. \end{cases} \quad (6.2)$$

That means both the periodically modulated permittivity profile and the periodically modulated permeability are considered. Because in practice, the capacitance is periodically loaded on the structure, this processing will complement the propagation analysis system for the double periodic structure.

2 Small and efficient filters design.

Based on the analysis of the DP-CRLH TLs based on the TL theory and the singular perturbation procedure, it is observed that in addition to the CRLH property as exhibited by SP-CRLH TLs, the DP-CRLH TL should also promise a new passband at low frequency below the LH passband with narrow bandwidth. This unique property may provide a new method to design super-compact filters working at a low frequency, unbalanced triple-band or balanced dual-band filters. To reduce the dimensions, compact filters may be considered to be achieved by vertically stacking.

3 Wide bandwidth of the new leaky-wave region.

It is possible to widen the bandwidth of the LW region. When the dispersion characteristics is in the fast wave region ($\beta < k_0$), the structure is able to radiate, which can be used for potential LWAs. Therefore, the target to widen the bandwidth

is to make the dispersion curve located in the fast wave region wider. This may be obtained by tuning the loading capacitances and inductances. However, these designed values should be possible to realize and implement in the TL configuration. It will be very difficult to design a wide band LWA at low frequency, because the boundary of the fast wave region ($\beta = k_0$) is extremely close to the vertical axis ($\beta = 0$) and the fast wave region at low frequency is too narrow. Another way to widen the bandwidth may be to design the antenna at higher frequency, where the fast wave region is wide.

4 Increase the gain of leaky-wave antennas.

Although LWAs, including SP-/DP-CRLH LWAs, have many advantages such as compact, low-profile, beam scanning property, they have the major disadvantage of low gain. In order to increase the gain of the LWAs, there are several methods that can be used.

- Placing a dielectric superstrate, or cover layer, over the original dielectric layer will enhance significantly the gain of LWAs [132, 133]. A large gain can be obtained for radiation at any desired angle by choosing the layer thicknesses and the dielectric constant values appropriately. Another way is to replace the single superstrate by a periodic array of such superstrates [134] to increase the reflection from the partially reflective surface.
- Another method is using a photonic-band-gap (PBG) structure, which can be approached to a periodic array of low/high permittivity superstrate layers and serves as the cover for the bottom dielectric layer [135]. An extra mode inside the frequency gap has been opened by inserting a defect into the dielectric medium. The directive antenna uses the defect mode of a photonic crystal to achieve electromagnetic radiation. The PBG material is used as a cover to enhance the gain of a usual patch antenna.

- The third method is that amplifiers can be interconnected in the passive leaky-wave sections of LWAs, which regenerate the power progressively leaked out of the structure in the radiation process and increase the effective aperture of the antenna and thereby its gain [136].

5 Increase the radiation efficiency of leaky-wave antennas.

Figure 4.7 shows the radiation efficiency, numerically estimated around the new RH LW region and the LH LW region as a function of TL length. It can be seen that the radiation efficiency for the new LW RH region is less than the radiation behavior in the LH region and longer TLs is needed to get more radiation efficiency. Increasing radiation efficiency is also one of the major problems of LWAs, especially for the new proposed RH LW radiation. In the future, enhancing the radiation efficiency is also one of important issues for the proposed LWAs.

- The comparison shown in Fig. 4.7 is made for modulation index of $\eta_1 = 0.1, 0.2$ and 0.3 . It is found that it may be developed for large η_1 upto 0.3 to increase the radiation efficiency [39]. However, it requires more rigorous analysis as it includes higher order perturbation terms.
- A typical leaky-wave antenna might be about 20 wavelengths long in order to design a leaky-wave antenna which allows about 90 percent of the power in the guide to be leaked away (radiated) by the time the wave reaches the end of the antenna aperture [137]. The remaining power is absorbed by a matched load placed at the end of the waveguide. In order to obtain higher radiation efficiency, long transmission line is required. Power recycling schemes may be used for enhancing the radiation efficiency of LWAs. One of the schemes enhance the radiation efficiency of a single LWA by feeding the energy remaining at the output of the antenna back into its input terminal via a rat-race hybrid coupler is

proposed in [138]. In this self-recycling scheme, the power associated with each information symbol loops several times along the structure until all its energy has been fully radiated. Another scheme, which is a cross-recycling scheme, enhances the radiation efficiency of a LWA by re-injecting the energy remaining at the output of the excited element into the adjacent elements of the array via a series feeding network, and possibly including phase-shifters for full-space pencil-beam scanning [138].

Author's Publications

Journal Papers

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