Optimal price and delivery time quotation with production scheduling for make-to-order manufacturing

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**Optimal Price and Delivery Time Quotation with Production
Scheduling for Make-to-Order Manufacturing**

A thesis submitted in fulfillment of
the requirements for the degree of
Doctor of Philosophy

by

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Abstract

Customers have varying requirements on price and delivery time in make-to-order manufacturing environment. Due to limited availability of the production capacity, the firm can increase revenue if it allocates priority to time-sensitive customers, charging them a higher price for the same product. The remaining capacity can then be allocated amongst the price-sensitive customers with a relatively lower price. This strategy is aimed at maximizing the overall revenue for the firm. Moreover, the quoted price and delivery time have effects on the demand in the market. In literature, the demand function is considered as a linear decreasing function with respect to price and delivery time, respectively. The challenge is to coordinate the price quotation and the delivery time quotation for customers so that it results in maximizing the net revenue for a manufacturer.

There are two types of capacity settings to differentiate products by price and delivery time. These are dedicated capacity and shared capacity. In the dedicated capacity, customers are categorized into several groups. Different customer groups are served in different production facilities. Customers from one customer group share a common price quotation and a common delivery time quotation. The common price and delivery time quotation problem is formulated for a single customer group. The optimality of this problem is studied and applied to develop an algorithm to find the optimal solution in polynomial time. Numerical examples show that the common price and delivery time quotation strategy is applicable when customers are more price-sensitive rather than time-sensitive.

In the situation of shared capacity, all customers are served in the same production facility. The production priority is given to the customers who are more

time-sensitive. Orders from the customers who are more price-sensitive are scheduled to the rear of the production sequence. Each customer will have a unique quotation of price and a unique quotation of delivery time. The problem is formulated and proven to be NP-complete even when prices are pre-determined parameters. A branch-and-bound (B&B) algorithm is developed to obtain optimal solutions for the moderate-sized problem. Moreover, a heuristic algorithm is proposed to obtain the near-optimal solution in a short time.

Numerical experiments indicate that B&B algorithm is able to find the optimal solution in a reasonable time when order size is less than 20. The heuristic algorithm can achieve the optimal solutions in most of cases in a short time. The heuristic algorithm is also compared with the methodology from literature and practice. Results show that the heuristic is significantly superior to other methods especially when customers are time-sensitive.

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Dedicated to
My Parents

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List of Abbreviations

ATO	Assemble-to-Order
B&B	Branch-and-Bound
CNL	Child Node List
CV	Coefficient of Variation
EDD	Earliest Due Date
EOQ	Economic Order Quantity
FCFS	First Come First Serve
FIFO	First In First Out
HAC	Heuristic Algorithm for $C_{\max}$
HAD	Heuristic Algorithm for Distinct prices and delivery time quotations problem
HAP	Heuristic Algorithm regarding Price
KS	Known Sequence
LB	Lower Bound
MIP	Mixed Integer Programming
MTO	Make-to-Order
OPS	Optimal Price Searching
PNL	Parent Node List
PS	Pending Sequence
PTA	Price-Time problem Algorithm
PTS	Price and Time Sensitive
SLA	Sequence Listing Algorithm
SPT	Shortest Processing Time
Stdev	Standard deviation
TPS	Time and Price Sensitive
UP	Upper Bound

List of Variables

c	: a constant value in common price and delivery time quotation problem
c	: production rate in distinct prices and delivery time quotations problem
f	: the total net profit
i	: order index
j	: order index
m	: the number of on-time orders for a specific t in common price and delivery time quotation problem
m_0^t	: the number of on-time orders when $p=0$ at a given t
n	: the total number of orders
p	: quoted price for all orders in common price and delivery time quotation problem
$[p_c,$	: the boundary of a specific price zone c
$p_{c+1}]$	
p_o^t	: the quoted price when revenue achieves maximum at a given t
p_n^t	: the quoted price when all order are on time at a given t
pt_i	: processing time of order i in common price and delivery time quotation problem
p_i	: quoted price for order i
$p_{(i)}$	: the i_{th} p when the set of p_j are arranged in ascending order
$p_{[i]}$	: the i_{th} p in an arbitrary sequence
s	: the number of edges in a specific price-time zone in common price and delivery time quotation problem
t	: quoted delivery time for all orders in common price and delivery time quotation problem
$[t_c, t_{c+1}]$	: the boundary of a specific time zone c
t_i	: quoted delivery time for order i
v_i	: the value of $\theta_i/(c + \theta_i)$
$v_{(i)}$	: the i_{th} v when the set of v_j are arranged in ascending order
$v_{[i]}$	: the i_{th} v in an arbitrary sequence
w	: common weight of tardiness

C_i	: completion time of order i
Q_i	: order quantity of order i
R	: the total revenue by sales
S_i	: the process starting time of order i
T	: the total tardiness penalty
T_i	: tardiness penalty of order i
X_i	: the value of $(\alpha_i - \beta_i p_i)/(c + \theta_i)$
$X_{(i)}$	: the i_{th} X when the set of X_j are arranged in ascending order
$X_{[i]}$	: the i_{th} X in an arbitrary sequence
Y_i	: processing time of order i
α_i	: potential market size of order i
β_i	: price sensitivity of order i
θ_i	: time sensitivity of order i
δ_{ij}	: a binary variable
π	: a production sequence

Chapter 1

Introduction

1.1 Overview

In the present scenario, the customer's awareness and concerns on choosing a proper product or service is not only based on characteristics like product's price, performance and quality but also on the availability of the product's waiting time. Hence, price and lead time (or delivery time) for product/service are two important factors in determining the competitive advantage.

The recent literature on time-based competition presents ample evidence to show that firms can make use of response time as a strategic weapon to gain competitive advantage (Stalk and Hout 1990, Hum and Sim 1996, Suri 1998). These strategies have a strong demand-enhancing effect in industries where response time is a key concern. Therefore it is essential to maintain a high degree of responsiveness while sustaining a competitive cost structure (Hum and Sim 1996). Examples of industries in which firms establish different price and delivery time (or lead time) for different customers include:

- **Transportation and logistics industry.** A logistics service provider often has a quotation table, from which customers can select their preferred delivery time

and price quotation. For example, FedEx have different services, named as Customer Critical, SameDay Freight and Express Freight. When the delivery time is shorter, the price is naturally higher (Zhao et al. 2006).

- **Communications and information services.** The computing service providers offer multiple grades of service because users' quality-of-service (QoS) requirements and willingness to pay are different. The first service is delivered subject to a "guaranteed" processing rate, and the second is a "best-effort" type service in which the residual capacity is used. The prices charged are different for the two kinds of service (Maglaras and Zeevi 2005).
- **Production and manufacturing.** Just-in-time firms such as printing and packaging companies negotiate with customers on the delivery time of their orders. They can leave the production priority to the most valuable customers, who can afford a high price in order to get products immediately. On the other hand, customers can enjoy a budget price if they can wait for a long time.

Although the price and time differentiation has attracted much attention from industrial practitioners, the concept of coordinating pricing and delivery time quotation on an operational level is still relatively new. In majority of the literature, it is assumed that the price quotation is decided on a strategic level by marketing department of the company, while the delivery time quotation is processed on an operational level by the manufacturing department.

The relationship among price, delivery time, scheduling and profit for the manufacturing department facing time-sensitive customers is shown in Figure 1.1.

When the market is price-sensitive and time-sensitive, both quoted price and quoted delivery time can affect demand. Demand (or order quantity) is an input of the production sequence. The delivery time quotation is dependent on the production sequence. Meanwhile, customers' demand is affected by the delivery time. If the delivery time is too long, the demand is decreased because customers are not satisfied. Finally, the profit is based on price, demand and the earliness or tardiness penalties incurred by the difference of the completion time and delivery time quotation. These factors are closely related to each other. Therefore the coordination problem of price quotation, delivery time quotation and the production sequence is challenging and could result in a considerable amount of increased profits for a make-to-order manufacturer.

Figure 1.1: Relationship among price, delivery time, scheduling and profit

1.2 Motivation

The present research on coordination of price and delivery time quotation is motivated primarily to deal with price quotation, delivery time quotation and

production scheduling in a make-to-order environment. These are described in detail in the following subsections.

Need to integrate price with delivery time quotation

A major portion of the past scheduling research deals with the due-date quotation (or delivery time quotation) without price consideration (Kaminsky and Hochbaum 2004; Hopp and Sturgis 2000; Keskinocak et al. 2001). While some work has been done with the assumption that demand is sensitive to the due-date quotation or other factors (Duenyas and Hopp 1995; Chatterjee et al. 2002; Keskinocak et al. 2001), few researchers have simultaneously considered price and delivery time quotation (for example: Plambeck (2004) and Celik and Maglaras (2008)). However, these researches all used queueing models to estimate the delivery time, which is not suitable for day-to-day operations. It is important to integrate price quotation into due-date problems, because both due date and price affect the demand significantly (Keskinocak and Tayur 2003). Therefore, the primary focus of this research is to develop a scheduling methodology to deal with the delivery time quotation with price consideration.

Need to consider diversity of customers' requirements on time and price

For a traditional manufacturer, valuation of a product is mainly based on material, labor cost, the competitive market, and other factors related to cost. That is, the price for a product is same for all customers in a period of time. However, customers' valuation of a product can differ tremendously, depending on the customers' preference and ability to pay (Charnsirisakskul et al. 2006). Hence, to maximize the profit, it is essential that the firm's pricing policy effectively reflects customers' willingness to pay according to customers' preferences. In the make-to-order or make-

to-assemble supply chain, customers' preferences are mainly represented by the delivery time of products (Lederder and Li 1997; Plambeck 2004; Maglaras and Zeevi 2005). The printing or packaging service industry is a good potential application area. For example, the small printing company has demand from different kinds of customers. The company's capacity is limited. Some customers may be very eager to get the product (such as printing ID photos) and do not care about paying the higher price. On the other hand, other customers are willing to wait for their personal photos to be printed later at a lower price. It is a challenge to study the product differentiation problem based on delivery time and price in make-to-order (make-to-assemble) supply chain, because of diversity of customers' requirements.

Need to study integration of price and delivery time quotation on an operational level

Investigating delivery time quotation on an operational level can provide accurate estimation of delivery time. Time related penalty, such as earliness penalty and tardiness penalty, can be taken into account to estimate the revenue. It can guide the daily production operations to maximize the revenue for the manufacturer. However, most of the literature which study delivery time and price quotation formulates the production stage as a stationary queueing model (Palaka et al. 1998; So and Song 1998; Boyaci and Ray 2006; Pekgün et al. 2008). All these existing models consider price quotations and delivery time quotations from a tactical or strategic perspective of production. That is, the detailed scheduling of each individual order is not considered (Chen and Hall 2010). The costs considered in these models are setup, production, and finished product inventory holding costs. However, the consideration of detailed order-by-order scheduling decisions makes the costs more relevant to many practical

make-to-order or assemble-to-order environments (Li et al. 2005, Stecke and Zhao 2007). Therefore, this motivates the study of coordination of price and delivery time quotation on an operational level.

1.3 Coordination of price and delivery time quotation

According to the motivation of this research, the product differentiation is based on the delivery time quotation and price quotation. There are two common model families to cater for customers' distinct preferences on delivery time and price.

The first one is a dedicated capacity model, which is shown in Figure 1.2. This model categorizes customers into different groups based on their sensitivities to price and delivery time, so that customers are served by groups. Within one group, customers have comparable price sensitivities and time sensitivities and will share the same price quotation and delivery time quotation. Since different customer groups have different requirements on price and delivery time, the price quotations and delivery time quotations are different among different groups. Orders from different customer groups are processed in different production facilities. The objective is to find the price quotation and delivery time quotation within each customer group. Relevant studies based on this model are found in Boyaci and Ray (2006), Zhao et al. (2006) and Pangburn and Stavroulakis (2008) among others.

The advantage of the dedicated capacity model is that it can provide better service level to different customer groups with relatively simple scheduling rules. However, there are drawbacks in this model. The resources may not be fully utilized and may require more cost for facility configuration.

Figure 1.2: Dedicated capacity model

Shared capacity model for time/price differentiation has been addressed by many researchers (Ledera and Li 1997; So and Song 1998; Plambeck 2004; Ray and Jewkes 2004; Rao et al. 2005; Celik and Maglaras 2008). The model is as shown in Figure 1.3. This setting allows sharing of resources and facilities among all customers. The production priority is allocated to the customers who are more sensitive to delivery time, while orders from the customers who are more sensitive to price are scheduled to the rear of the production sequence. In this case, each customer will have a unique quotation of price and a unique quotation of delivery time.

The advantage of this model is that the customer demand is pooled across groups, making it easier to fully utilize capacity. Since the delivery time quotation only depends on the production sequence, this model requires impeccable scheduling techniques to optimize the performance.

Figure 1.3: Shared capacity model

The coordination of price and delivery time quotation problem is investigated under these two different scenarios, i.e. dedicated capacity model and shared capacity model. In the dedicated capacity model, a single customer group with a dedicated production facility is considered. In this model, it is investigated how to find an optimal common price quotation and an optimal delivery time quotation for the customer group. It is also analyzed which customers should be categorized into the same group. In the shared capacity model, the focus is on finding the optimal delivery time quotation for each order with price consideration. Based on these two models, two solution methodologies are developed in this research.

1.4 Scope of Thesis

In this study, the coordination of price and delivery time quotation with detailed scheduling decisions is considered in a make-to-order environment. A deterministic customer's demand function with respect to price and delivery time quotation is assumed throughout this thesis. This deterministic demand function is adapted from the commonly linear stochastic demand function, which is (e.g. Palaka et al. 1998; Webster 2002; Liu et al. 2007 and Pekgün et al. 2008),

$$\lambda_i(p, t) = \alpha_i - \beta_i p - \theta_i t$$

where:

p = price quotation for customer i ,

t = quoted delivery time for customer i ,

$\lambda_i(p, t)$ = mean order arrival rate for customer i ,

α_i = total market potential for customer i ,

β_i = price sensitivity of demand,

θ_i = time sensitivity of demand.

The non-linear demand model can also be applied to study this problem. For example, So and Song (1998) adapted a log-linear (Cobb-Douglas) demand function to study the price and delivery time guarantee problem. However, the linear demand function is the basic assumption for the research in the thesis. The results achieved in this thesis can be applied to study the non-linear demand models in the future research.

Since detailed scheduling decisions typically involve a short planning horizon, the deterministic demand function in this planning horizon is originally represented by,

$$D_i(p, t, \varepsilon) = \alpha_i - \beta_i p - \theta_i t + \varepsilon_i$$

where $D_i(p, t, \varepsilon)$ represents the order quantity from customer i in the planning horizon; ε_i represents the marginal error. However, the deterministic portion of $D_i(p, t, \varepsilon)$ is applied as the demand function,

$$D_i(p, t) = \alpha_i - \beta_i p - \theta_i t$$

There are three justifications for this approach. Firstly, it is often difficult in practice to accurately estimate the error term in a planning period. A deterministic portion is used to estimate expected demand for planning in most decision support systems. Secondly, the error term is generally replaced by a random variable for a customer (Ray 2005). Since the research is the first to study the coordination problem in deterministic model, the situation when ε_i is negligible is only considered in this thesis. Thirdly, a deterministic portion is oftentimes reasonably accurate (Chen and Hall 2010) and is a first step toward understanding the more realistic and complicated environment with stochastic demand.

It is assumed that the demand from a customer realizes immediately after a price quotation and a delivery time quotation are given at the beginning of the planning horizon. The finished order is delivered immediately upon its completion.

This is common in make-to-order situations involving customized consumer products such as packing, printing, electronics and fashion items.

1.5 Research Objective

The main objective of this thesis is to devise a price and delivery time quotation methodology to maximize the profit of a manufacturer. The issue is examined on the operational planning level. The higher level of production planning is not a factor considered in this research. Based on the outline of the research problem presented in section 1.3, the coordination problem of price and delivery time quotation will be investigated in the situation of dedicated capacity model and shared capacity model, respectively. The specific objectives are as follows:

- To develop an operational approach to coordinate the price quotation and delivery time quotation for a manufacturer.
- To define and formulate the coordination problem of price and delivery time quotation in the dedicated capacity model.
- To propose a methodology to solve the coordination problem in dedicated capacity model optimally and to analyze the applicability of this model with respect to the characteristics of the market.
- To define and formulate the coordination problem of price and delivery time quotation in the shared capacity model.
- To study the complexity of the coordination problem in shared capacity model and to identify approaches to solve the problem.

In summary, the primary research objective is to propose an operational product differentiation approach, encompassing both price and delivery time quotation and production scheduling, for dedicated and shared capacity models.

1.6 Contributions of Thesis

The main contributions of this research are listed below in detail.

1. This thesis presents an operational methodology for product differentiation regarding price and delivery time in the dedicated capacity model. The joint optimization of pricing, delivery time quotation and production sequencing is firstly studied and solved.
2. This thesis provides managerial insights of common price and delivery time quotation in dedicated capacity. For example, the experimental results show that this approach is more applicable in the market where customers are more sensitive to price compared to delivery time.
3. This thesis identifies that the coordination problem of price and delivery time quotation in the shared capacity model is NP-complete. A MIP formulation is developed to find the solution in small sized problem. A branch-and-bound algorithm is presented to find the solutions in moderate sized problem.
4. This thesis presents a heuristic algorithm for the above NP-complete problem. A heuristic algorithm is developed to find the near-optimal solution. It is proven that the heuristic algorithm performs well regarding to computational time and accuracy.
5. The thesis also includes a detailed review of existing literature on integration of price and production.

1.7 Organization of Thesis

This thesis consists of seven chapters.

Chapter 2 reviews the published literature addressing the problem of integrated price and production. This review also covers the research that deals with the lead time quotation (or due date quotation). In addition, the literature addressing scheduling problem with time-dependent processing times is reviewed.

Chapter 3 proposes the coordination problem of price and delivery time quotation in the dedicated capacity model. Two special problems are studied under the specific constraints. Based on the results obtained from the special problems, an optimal solution is developed.

In Chapter 4, the characteristics of common price and delivery time quotation model is studied through numerical experiments and the mathematical analysis.

Chapter 5 formulates the coordination problem of price and delivery time quotation in the shared capacity model. It is proven that the computational complexity of this problem is NP-complete. Two solution methods, MIP formulation and B&B algorithm are presented to find the optimal solutions. In addition, a heuristic algorithm is developed to solve this problem in a short time.

In Chapter 6, computational results present the performance of the methods. The applicability of the heuristic algorithm is also tested in the numerical experiments.

Finally, the conclusions of this research are drawn in the last chapter. The directions for further work are also summarized.

Chapter 2

Literature Review

The literature related to problems in joint price-production, due date assignment and single machine scheduling with time-dependent processing times is reviewed in this chapter.

The price/time related production problem is classified as shown in Figure 2.1. Firstly, the related literature can be divided into three categories. In the first category, it is the joint price-production problem where lead time is not considered. This category can be further divided into price and capacity management, pricing and inventory management, and pricing and supply chain management. The due date assignment category focus on lead time/due date quotation. Price is not a concern in this part. The due date assignment can be divided into static model and dynamic model. In the third category, joint price/lead-time/production problem considers both price and lead time in the production. In this category, most of literature is applying queueing models to estimate the lead time. The decisions are on the tactical level which cannot be applied in day-to-day operations. This research integrates due date assignment problem into the joint price/lead time/production problem. It can obtain the accurate job lead time/due date based on the information from production. Thus this research models the problem on the operational level.

Figure 2.1: Structure of price/time related production problem

2.1 Joint Price-Production Problem

In the last decade, the relationship between demand and price is usually ignored in the research on pricing and production problem. The relationship between price and production is not significant. Thomas (1970) reviewed a variant of this problem which simultaneously makes price and production decisions under deterministic demand function. Since then, more and more literature addressed the dynamic demand model in which the order arriving rate is affected by prices. Ample literature has addressed the requirement for improved communication and cooperation between the production and marketing functions of the manufacturing firms (Karmarkar and Lele, 2004; Karmarkar 1996; Montergeomery and Hausman, 1986). There are a plenty of papers studied in joint price-production problem. Only the most relevant papers are reviewed in this section. The review can be divided into three categories. In the first category, papers considering price and capacity management in assemble-to-order manufacturing are discussed. The other two categories which are focused on pricing

and inventory management and pricing and supply chain management are less related to this research.

2.1.1 Price and Capacity Management Problem

Since the customer demand is sensitive to the price of the product, studies on price and capacity management which involve choosing product prices and managing capacity have emerged as a popular research area in operations research in recent years. These problems are largely motivated by the adoption of the assemble-to-order strategy incorporating component production in today's manufacturing industry. The related literature aims to satisfy orders at a high service level and reduce inventory costs for components through adjusting the price.

With respect to types of customers (or types of final products) in the market, the research in this area can be classified into two categories.

2.1.1.1 Single Product/Single Customer

There is limited literature addressing the system that produces a single product and sells at variable prices. Feng et al. (2008) study this problem with multiple make-to-stock components. In their research, the model of two components and two price levels is presented. Some properties of the optimal control policy have been characterized and summarized for the general problem with multiple components and multiple prices.

2.1.1.2 Multiple Products/Multiple Customers

The assemble-to-order manufacturers always face the customers who are heterogeneous and demand a variety of products. In order to maximize the profit, the primary challenge is to choose product prices to determine revenue and the demand for each component. Production capacity for each component is also a decision variable in this paper (Plambeck and Ward 2005).

Maglaras and Zeevi (2005) consider a model of a service system that delivers two non-substitutable services to heterogeneous users. In this model, the first service is delivered subject to a “guaranteed” (G) processing rate. The second is a “best-effort” (BE) type service in which residual capacity not allocated to G is shared among BE users. Users are sensitive to both price and congestion-related effects. The objective is to optimally design the system so as to extract maximum revenues. The decision variables consist of a pair of static prices for the two services, a policy that controls admission of G users, and the mechanism estimating the state of congestion for users.

Plambeck and Ward (2005) consider an ATO system in which J different components are assembled into K different finished products. In the paper, a discrete-review assembly policy is proposed to minimize instantaneous financial holding costs. An asymptotic analysis has been undertaken when the customer arrival rate increases. The asymptotical behavior is incorporated into the optimal policy. Plambeck and Ward (2008) further study a similar ATO system that considers delay cost and leadtime quotation. The prices, maximum leadtimes, and nominal component production rates can be updated in response to periodic, random shifts in demand and supply conditions.

Benjaafa and ElHafsi (2006) study a similar problem as Feng et al. (2008) with the consideration of multiple customer classes. However, pricing for customer classes has not been discussed.

Dai and Jiang (2007) examine an assemble-to-order system with batch arrival components. Their work refines the solution of some static planning problem and a discrete review policy to batch arrival environment. An asymptotically optimal policy is developed for the system operating under heavy traffic to maximize expected infinite-horizon discounted profit.

Revenue Management

Revenue management is commonly applied in airline and service industry. Related revenue management problems contain the following common characteristics: capacity is perishable and limited, demand is stochastic, and there are different customer classes (Bitran and Caldentey, 2003; Modarres and Sharifyazdi, 2009). Since some MTO/ATO manufacturing systems have the common characteristics, the concept and techniques of revenue management is also applied to capacity allocation problems in a manufacturing system. A handful of literature is available on studying pricing and capacity allocation decisions in revenue management models.

Feng and Xiao (2006) present a continuous-time model, in which the supplier sells the same products to different customer classes at distinct prices. With the objective of maximizing the expected revenues under time and capacity constraints, a threshold control policy is developed to decide which customer class should be served at what price at a given time and inventory level.

Modarres and Sharifyazdi (2009) study a manufacturer serving two different customer classes with stochastic capacity. In their model, prices are not decision variables. The optimal policy is to find the maximum capacity which can be assigned to each customer class. It is proved that the objective function is a unimodal function. By taking the advantage of this property, the optimal solution is determined.

2.1.2 Pricing and Inventory Management

It has been nearly 50 years since researchers began to develop analytical models to aid simultaneous decisions in terms of pricing and inventory stocking decisions. A recent review on dynamic pricing with the consideration of inventory has been presented by Elmaghraby and Keskinocak (2003). Literature is reviewed in two market settings, one where there is no inventory replenishment and independent

demand over time; the other one concern inventory replenishment, independent demand and myopic customers. The paper considers the characteristics of E-Commerce market and points out that the dynamical pricing with inventory considerations will be a promising future direction for research.

2.1.2.1 Dynamic Inventory Model

A dynamic inventory model is firstly investigated by Thomas (1970), in which the demand function is deterministic and non-increasing in price. The paper considers the problem of a monopolist who must make price decisions in each of T period planning horizons. An efficient forward algorithm is developed to find decisions in each period.

Yano and Gilbert (2005) made a comprehensive review of analytical models on coordination of pricing and production. The paper examines the trade-offs between the holding inventory cost and setup cost with the consideration of demand smoothing and demand uncertainty. In this paper, the economic order quantity (EOQ) models, in which external demand is price-sensitive, are studied. It is proposed that capacity constraints and heterogeneity of customers' preference are the future research directions.

Deng and Yano (2006) address an extended work of Thomas (1970) to consider capacity constraints. It is assumed that the manufacturer faces a known demand curve, which is continuous differentiable and strictly decreasing with respect to price, in each period. A joint-pricing and lot-sizing problem is solved by applying the characteristics of the optimal policy.

Chan et al. (2006) discuss the joint price and production problem in a strategic level. The authors analyze and compare two delayed strategies, one where pricing is fixed at the beginning of the horizon while the production decision is made at the

beginning of each period; the other one where production decision is determined at the beginning of the horizon while the pricing is made at the beginning of each period.

2.1.2.2 Newsvendor Model

The newsvendor model is adopted to study the integration problem of operations and marketing by Ray (2005) and Rao et al. (2005). In these two papers, customer demand is sensitive to the price as well as the non-price factors.

Ray (2005) studies a firm which stocks single product and sells them to end customers. The task is to determine the optimal pricing, stocking level and attribute level, in which all three factors can affect the demand. The research also shows how attribute-sensitivity and randomness of demand affect the firm's optimal decisions.

The impact of lead time guarantee on customer demand is considered in Rao et al. (2005). In their model, price is specified exogenously. This research focuses on integrating demand and production decisions to optimize expected profits by quoting a uniform guaranteed maximum lead time to all customers.

2.1.2.3 Competition Model

When considering that services are provided by different firms in a supply chain, Chen and Wan (2003) study the (non-cooperative) competition between two make-to-order firms. In this model, customers are sensitive to capacity allocated to them or service delay. The firms have different values of service and different service capacities, as well as firm-dependent unit cost of waiting. The authors have found sufficient conditions for the existence of Nash equilibrium. The equilibrium is analytically or numerically characterized in some specific cases. The results indicate that whenever the equilibrium exists and is unique, the firm with the higher capacity can usually charge a premium price and take a larger market share.

Liu et al. (2007a) discussed a competition model between two firms in which customers are sensitive to the product availability. This work is an extension of the research on a dynamic game model (Hall and Porteus, 2000) by incorporating a general demand model. It is proven that there exists a stationary equilibrium policy. The dynamic equilibrium policy always converges to this stationary equilibrium policy.

2.1.3 Pricing and Supply Chain Management

Pricing and production problems on supply chain management are common concerns for outsourcing and assembly-related industries. It involves several firms negotiating prices and coordinating the production/inventory in each firm. There is relatively little research addressing pricing and production on supply chain management (Weng 1995; Chen et al. 2001; Zhao and Wang 2002). Weng (1995) considers a single manufacturer with multiple identical retailers. The channel coordination can be achieved by using a quantity discount policy. Chen et al. (2001) extend Weng's Model to non-identical retailers. Both papers consider static models (i.e., stationary demand) with concave (EOQ) cost functions. Zhao and Wang (2002) present dynamic models with convex cost function. The authors model a decentralized supply chain, where a manufacturer outsources her product distribution/retail function to an independent distributor/retailer. An incentive scheme is developed for the manufacturer to achieve channel coordination.

2.2 Joint Price/Lead-time/Production Problem

Most of the traditional economics and marketing literature models the demand of a product or service as a function of price. However in the last decade, Just-In-Time (JIT) has highlighted the importance of shorter lead-times. Additionally, in today's world of globalized finance and manufacturing, firms are increasingly depending on

fast response times as an important source of sustainable competitive advantage. The fact is that customers' perception of value and their purchasing decisions are not influenced exclusively by an item's selling price, but also by other non-price attributes, such as lead time, delivery reliability and product quality (Baker and Bertrand 1981; Stalk and Hout 1990; Palaka et al. 1998). This is not considered in the section 2.1. Upasani and Uzsoy (2008) reviewed the existing literature on integrated production-marketing function focusing on models that consider lead times and capacity.

The literature in this section is most related to this research. Papers jointly considering price, lead time and production are examined in detail in terms of model formulation and decision variables. Hence the following part will categorize and review the related literature according to two approaches. Firstly, the papers are categorized with respect to the modeling assumptions. Then, the related research is reviewed based on types of decision level, for example tactical level or operational level.

2.2.1 Models according to Assumptions

The literature reviewed in this section can be classified into several categories according to the modeling assumptions, such as types of demand function, types of capacity setting and types of lead time quotation.

2.2.1.1 Linear Demand Function

Since customers are delay sensitive, the expected demand is a function of quoted lead-time as well as price. To simplify the demand function, some researchers assume that the demand is downward-sloping in both price and quoted lead-time. Furthermore, they assume the demand to be a linear function of price and quoted lead time as shown in the equation (2.1) (Palaka et al. 1998; Pekkn et al. 2008; Lederer and Li 1997; Plambeck 2004).

$$\Lambda(p,l) = a - b_1p - b_2l \quad (2.1)$$

where:

p = price of the good/service set by the firm,

l = quoted lead time,

$\Lambda(p,l)$ = expected demand for the good/service at price p and quoted lead time l ,

a = the potential market size which is a constant,

b_1 = price sensitivity of demand,

b_2 = lead time sensitivity of demand.

It may be noted that the equation (2.1) has several desirable properties. Firstly, the price elasticity of demand given by $(-b_1p/(a - b_1p - b_2l))$, is increasing in both price and quoted lead-time. Subsequently, the lead time elasticity $(-b_2l/(a - b_1p - b_2l))$, has the similar properties. These properties are desirable since intuitively, customers would be more sensitive to long lead times when they are paying more for the goods or service. Similarly, customers would be more sensitive to high prices when they have longer waiting times. Another desirable property of the linear demand function is separability of price and quoted lead time. This separability is desirable because an underlying premise of time-based competition is that customers perceive time and money as substitutes (i.e., time is money), which would imply that the demand function should be separable in price and quoted lead time (Lederer and Li 1997).

Palaka (1998) considered congestion costs caused by stochasticity in order arrivals and production environments. The primary objective of this research is to develop a model that captures the impact of stochasticity in demand arrivals and the production process on the profit. Customers are delay-sensitive. The firm incurs

congestion costs as well as tardiness penalties. M/M/1 queuing system is applied to represent the production process.

Considering the coordination between marketing and production departments in a firm, Pekgün et al. (2008) study the impact of the decentralization of the marketing and production departments in a Make-To-Order (MTO) firm, where pricing decisions are made by the marketing department and lead time decisions by the production department. Moreover, a transfer price contract with bonus payments is shown to generate higher profits with an efficient output.

Generally, customers may be willing to pay a price premium for shorter delivery times. Hence there is an inverse relationship between price and lead time. Ray and Jewkes (2004) extend the previous research by explicitly modeling such a relationship between price and delivery times. The paper includes economies of scale by modeling the unit operating cost as a decreasing function of the mean demand rate. In this model, price is determined by the length of lead time, which is subordinate to a linear form. Ray and Jewkes suggest that modeling the non-linear relationship (So and Song, 1998) between delivery time and price as a possible extension to their work.

Lederer and Li (1997) study competition between firms that produce goods or services for customers who are sensitive to delay time. Firms compete by setting prices and production rates for each type of customer by choosing scheduling policies. In their work, customers are differentiated by demand function and delay sensitivity. The demand rate of customers of type z is given by a differentiation of a strictly decreasing function $d(z, p + c(z)W)$, where p is the price, $c(z)$ is the delay cost per unit delay time and W is the expected delay time. A full price in the market is given as $P(z) = p(z) + c(z)W(z, \lambda, f)$ with λ representing the production rate and f representing scheduling policy (such as FIFO). Plambeck (2004) quotes dynamic lead

times to potential customers and decides the orders from which customers are processed. In the model, impatient customers pay a premium for immediate delivery and receive priority in scheduling, whereas patient customers are quoted a lead time proportional to the current queue length. Queue length and lead time can be closely approximated by a reflected Ornstein-Uhlenbeck diffusion process.

Boyaci and Ray (2006) develop an analytical framework for shedding light on relationship between the capacity costs and customer preferences towards delivery times, reliabilities, prices. The authors consider a demand model with substitution with the following assumptions: (i) each product's mean demand is decreasing in its own delivery time and price, and increasing in the reliability level offered, and (ii) each product's increase in delivery time and/or price can only increase the other product's mean demand, and each product's increase in delivery reliability can only decrease the other product's mean demand. These are similar to what is reported by Tsay and Agrawal (2000).

To the best of our knowledge, most papers study the relationship between price and demand as a linear decreasing function. Only few of papers study it under a non-linear demand function. In the following section, the literature with non-linear demand function is presented.

2.2.1.2 Non-linear Demand Function

To reflect customer sensitivity in price and delivery time, So and Song (1998) assume the following log-linear (Cobb-Douglas) demand function $D(p, x) = \lambda p^{-a} x^{-b}$ where λ , a and b are positive constants with a representing the price elasticity and b representing the delivery time guarantee elasticity. In general, since the demand also depends on the delivery reliability, they consider the reliability level and investment in capacity expansion which is plausible in order to maintain a high probability of on-

time delivery. The objective is to find joint optimal decisions in pricing, delivery time quotation and capacity expansion level to maximize the overall profit of the firm while maintaining a predetermined level of delivery reliability.

Despite the influence of lead time on demand rate, Celik and Maglaras (2008) assume that there exists an inverse demand function $p(\lambda; d)$. A one-dimensional non-homogeneous Poisson process is assumed that maps an achievable vector of demand rates λ into a corresponding vector of prices $p(\lambda; d)$. The objective is to maximize the profit for a make-to-order manufacturer that offers multiple products to price and delay sensitive users. An approximating diffusion control problem is formulated to derive near-optimal dynamic pricing, lead-time quotation, sequencing and expediting policies. It also provides structural insights which lead to practically implementable recommendations.

As discussed in section 1.3, there are two kinds of configurations for production capacity; one is shared capacity for all demands from different customers, the other one is dedicated capacity for each customer cluster. In the following two sections, literature in these two categories is reviewed, respectively.

2.2.1.3 Shared Capacity

Pangburn and Stavrulaki (2008) studied both shared capacity and separated capacity for different customer clusters. Consumer clusters differ with respect to their reservation prices and time sensitivities. The firm serves them using a process that is modeled as a G/G/1 queuing system. The authors find that a hybrid strategy based on a prioritized queuing discipline, that combines elements of segmentation (by offering different waiting times) and pooling (by sharing capacity across consumer segments), can outperform both the pure segmentation and pooling strategies.

A large number of consider that a manufacturer produces identical products and sells to price and time sensitive customers. They model the firm's operations as a G/G/1 queue system (Ray and Jewkes, 2004; Palaka et al. 1998; Pekgün et al. 2008; So and Song, 1998; Pangburn and Stavroulakis, 2008; So 2000; Zhao et al. 2006).

In many practical situations, different product types are produced in the single production line especially in consumer electronics assembly facility. Hence, several researchers have modeled production facility as a multi-product (or multi-class) single-server queue (Celik and Maglaras, 2008; Plambeck 2004). Diffusion models are commonly applied to get the near-optimal solutions.

2.2.1.4 Dedicated Capacity

Recent research in the lead-time/price differentiation has studied dedicated capacity for each customer type (Boyaci and Ray, 2003; Zhao et al. 2006; Elhafsi and Rolland, 1999; Elhafsi 2000).

Boyaci and Ray (2003) provide optimal lead times and prices for a firm maintaining separate queues. Each queue has different service rates. The firm produces two substitutable products/services which only differ in price and lead time. Three special cases are studied, the first one ignores the effect of reliability, the second one treats prices as exogenous variables, the third one considers features for an express variant while a regular product already exists in the market. Based on the results from the special cases, the integrative framework of time-and-price-based differentiation for both products is addressed.

Considering switchover among different capacity queues, Zhao et al. (2006) studied a similar problem and modeled customer preferences by customer utility functions. The demand rate is endogenous, which is dependent on the number of customers placing orders. The authors illustrate the production system in a firm as two

M/M/1 queues, while the results are also approximately valid for a general G/G/s queues.

The above literature dealt with lead-time and pricing decisions at a strategic level focuses solely on simple models (e.g., M/M/1 and M/G/1 systems). Only a single product and a single stage production system is considered. However, Elhafsi and Rolland (1999) studied the day to day operational problem on a make-to-order manufacturing system consisting of several processing centers that are subject to failures and repairs. The authors propose a normative model that can be applied as a tool for negotiating the delivery date and the price of a certain upcoming order. The efficiency of the solution method for the model allows real-time decision-making while negotiating the price and delivery date of the order to be placed. Since the decisions are made based on a snapshot of the congestion level at the shop floor, usage of this model will reduce the conflict between the marketing and the production activities in manufacturing organizations. Elhafsi (2000) illustrates the similar problem without considering the price and non-price effects on demand.

Recently, innovation-oriented firms have been competing along dimensions other than price. Lead time has been one such dimension. Increasingly, customers are favoring lead time guarantees as a method to hedge against supply chain risks. Researchers have investigated the Price/Lead Time/Production problems under a common lead time assumption.

2.2.1.5 Common Lead Time

Rao et al. (2005) explicitly modeled the impact of common lead time guarantee on customer demands and production planning. The analysis highlights the increasing importance of lead time for customers as well as the tradeoffs in achieving a proper

balance between revenue and costs associated with lead time guarantees. A closed-form solution with a newsvendor-like structure is shown to get the optimal lead time.

A common price and delivery time quotation assumption has also been made for a group of related problems (e.g., So 2000; So and Song, 1998; Ray and Jewkes, 2004; Pekgün et al. 2008). Although there are different prices and time quotations for different customer groups discussed in the literature, a common price and time quotation is still proposed within a customer group (e.g., Boyaci and Ray, 2003; Boyaci and Ray, 2006; Pangburn and Stavroulakis, 2008; Zhao et al. 2006; Lederer and Li, 1997; Palaka et al. 1998).

2.2.1.6 Distinct Lead Times

In ATO/MTO environment, customers in the market also have large diversity on delivery time and price preference. Some customers may prefer to obtain products immediately and do not care how much the products cost, some customers are willing to wait and get a discount on the price. Consequently, several researchers assume distinct lead time quotations for different customer groups dependent on the congestion level and marketing (Boyaci and Ray 2003; Boyaci and Ray 2006; Zhao et al. 2006; Pangburn and Stavroulakis 2008; Lederer and Li 1997). Boyaci and Ray (2003) discuss the interaction between product differentiation and capacity cost. Pangburn and Stavroulakis (2008) consider joint pricing and capacity decisions for dispersed, time-sensitive customer segments.

When orders have large diversity on the quantity and require much different processing times, it is not appropriate to estimate a common lead time for all customers in a make-to-order environment especially when the congestion levels and scheduling policies of the facility varies. Dynamic pricing and lead-time policies are

applicable to these systems (e.g., Webster 2002; Celik and Maglaras 2008; Plambeck 2004; Duenyas 1995).

Webster (2002) examine policies for adjusting price and capacity in response to periodic and unpredictable shift in a MTO firm. It is suggested that maintaining a fixed capacity while using lead-time and/or price to absorb changes in the market will be most attractive when throughput is stable. From a pure profit maximization perspective, it is best to strive for short and consistent lead-times by adjusting both capacity and price in response to market changes.

Both Plambeck (2004) and Celik and Maglaras (2008) apply diffusion models to study the production queue. Plambeck (2004) presents an optimal lead-time differentiation for the same product, while Celik and Maglaras (2008) model a multiclass queue for multiple products.

Duenyas (1995) consider a dynamic due date setting problem with multiple customer classes. A due date is quoted to each customer arriving to a production system, which is modeled as a single server queue. Different customer classes have different preferences for price and lead time. Common price is applied within one customer class. The results suggest that policies that consider customer price and due date preferences in scheduling significantly outperform due date setting policies that do not.

2.2.2 Models according to Decision Level

In this section, the research related on joint price/lead-time/production problem is reviewed with regard to the planning level where the decision making process happens. To the best of our knowledge, majority of the relevant literature address the problem in a tactical level or strategic level. Only a few papers study the coordination problem in an operational level. Since the lead-time is studied as a critical factor in the related

problems, it is important to study the joint price/lead-time/production in the operational level. The reason is that the estimation of lead time is more accurate and reliable in the operational level. It is meaningful to study the joint problem with reliable lead times and production operations which can be applied in real practice.

2.2.2.1 Tactical Level

Most of the related literature deals with lead-time and pricing decisions at a tactical level or strategic level and focuses on simple models (e.g., M/M/1 and M/G/1 systems). The studies in this area provide insight into the nature of the pricing, due date setting, capacity selection problem.

Most papers illustrate the production process by a queueing model. Among them, some papers investigate the policy of delivery time guarantees and capacity selection (So and Song 1998; Palaka et al. 1998; Ray and Jewkes, 2004). Product differentiation problems based on price and time are examined by Boyaci and Ray (2003), Boyaci and Ray (2006), Zhao et al. (2006), Pangburn and Stavroulakis (2008) and Plambeck (2004). Dynamic lead-time quotation problems are studied by Ata and Olsen (2009) and Celik and Maglaras (2008).

Lederer and Li (1997) and So (2000) address the competition between firms. So (2000) investigates the marketplace in which many firms use delivery time guarantees to compete for customers. In the paper, the optimization problem for individual firms are analyzed and applied to study the equilibrium solution in a multiple-firm competition. The paper analyzes how the different firm and market characteristics would affect the price and delivery time competition in the market. Lederer and Li (1997) study the time-based competition through an analytical model which captures the effect of time performance on prices, demands, and profit.

Bidding models in MTO are presented by Easton and Moodie (1999) and Watanapa and Techanitisawad (2005). In these two models, MTO firms estimate the resource requirements, and generate bids to meet a customer's requirements. Then, the customer decides which bid to accept or whether to initiate negotiations for more favorable terms. Easton and Moodie (1999) consider the situation in which the MTO firm bid for multiple projects that require the same capacity. If two or more projects contend for the same resource, the risk of penalties for late deliveries is inevitable. A technique is developed to optimize pricing and lead time decisions simultaneously for MTO firms with contingent orders. Watanapa and Techanitisawad (2005) extend the model proposed by Easton and Moodie (1999) and incorporates market segmentability. Customers are classified into multiple segments based on willingness to pay, sensitivity to short delivery time, quality level requirement, and intensity of competition. Two sequencing rules, namely the early-due-date (EDD) and first-come-first-serve (FCFS) are applied to different customer segments. A partial search algorithm is proposed to find an optimal bid price and due date.

Supply chain management problems with pricing and lead-time decisions are studied by Liu et al. (2007b) and Pekkün et al. (2008). Liu et al. (2007b) investigate a decentralized supply chain consisting of a supplier and a retailer facing price and lead-time sensitive demands. A Stackelberg game is formulated to describe the decentralized decision problem. It is shown that decentralized decisions in general are inefficient and lead to inferior performance compared to centralized systems, due to the double marginalization effect. Pekkün et al. (2008) examine a firm which serves customers who are sensitive to quoted price and lead time. The pricing and lead time decisions are made by the marketing and production departments, respectively. Both a centralized model and a decentralized model are studied and compared. It is shown

that the coordination of marketing and production can be achieved by using a transfer price contract with bonus payments.

In addition, Webster (2002) uses system dynamics to study the dynamic pricing and lead-time policies for MTO firms.

2.2.2.2 Operational Level

Although majority of literature examines the integration problem of pricing, lead time and production at a tactical level, it cannot be used to make day-to-day operational decisions. Such decisions are crucial to the survival of firms operating in highly competitive markets (Elhafsi 2000).

Elhafsi (2000) develops a model of a manufacturing system consisting of several processing centers to achieve realistic lead-time due date and price quoted to the customer. The order can be split among several processing centers to meet the constraint imposed on its delivery time. Two cases are studied in the paper: the case of rushed order (more sensitive to time) and the case of regular order (more sensitive to price). Two options are considered in each case: partial deliveries allowed and partial delivery is not allowed. Heuristic algorithms are developed for each situation.

Charnsirisakskul et al. (2006) investigate a problem where a manufacturer has the ability to set prices to influence demand, reject orders, and set lead-times or due dates for accepted orders. Two scenarios are studied, where the manufacturer charges the same price or different prices to different customers. Heuristics are developed to obtain the initial solutions, which is within 87% of the optimal solution. The initial solution can be improved through solving a Mixed Integer Programming (MIP) problem.

Chen and Hall (2010) address the scheduling problem in which the demand is a deterministic non-increasing function of price. Prices are chosen from a determined

discrete set. Three alternative measures of scheduling cost are considered: total work-in-process inventory cost of orders, total penalty for orders delivered late to customers, and total capacity usage. The objective is to maximize the net profit, which is revenue minus scheduling cost. The value of coordinating pricing and production scheduling decisions is estimated by comparing solutions from an uncoordinated approach, a partially coordinated approach, and the coordinated approach. It is proven that there is a significant benefit even if pricing and scheduling are only partially coordinated. The effect of lead time on demand is not considered in their paper.

2.3 Due Date Assignment

This section attempts to present a review of a particular segment of scheduling research in which the due date assignment (or known as lead time quotation) decision is of primary interest. The exogenous case, when the due dates are set by some independent external agency and are announced before arrival of the job, is not reviewed.

In general, there are four types of due date setting rules which are commonly used to estimate due dates (Kaminsky and Hochbaum 2004; Cheng and Gupta 1989):

1. CON: Jobs are given constant lead times, so that for job i , $d_i = r_i + k$.
2. SLK: Jobs are given lead times that reflect equal slacks, so that for job i ,
$$d_i = r_i + p_i + k.$$
3. TWK: Jobs are given lead times proportional to their work content, so that for job i , $d_i = r_i + kp_i$.
4. NOP: Jobs are assigned lead times on the basis of number of operations to be performed on them, so that for job i , $d_i = r_i + kn_i$, where n_i is number of operations of job i .

Since the due-date quotation literature has a long history, there have been multiple review papers, which have addressed the problems in different perspectives (e.g., Cheng and Gupta 1989; Gordon et al. 2002; Kaminsky and Hochbaum 2004; Keskinocak and Tayur 2003). Some papers which are closely related to this research are presented in this section. The following literature review is classified into static models and dynamic models. In static models, release times for jobs are 0 at the start of the scheduling horizon. In dynamic models some jobs are not available to be processed at the start of the scheduling horizon.

2.3.1 Static Models

Panwalkar et al. (1982) and Seidmann et al. (1981) propose a pioneering research in the area of due date assignment in scheduling. Panwalkar et al. (1982) study the constrained version where the scheduler must decide a common due date for all jobs, while Seidmann et al. (1981) dealt with the unrestricted case where each job can have a unique due date. These two papers were followed by extensive research in the area of due date assignment, with most papers focusing on the static models (e.g., Bagchi et al. 1986; Bagchi et al. 1987; Birman and Mosheiov 2004; Cheng and Kovalyov 1996; Mosheiov 2001).

2.3.1.1 Common Due Date

A typical instance of a common due date model corresponds to an assembly system in which the components of the product should be ready at the same time, or corresponds to a shop where several jobs constitute a single customer's order. In the related problems, the objective is to find an optimal common due date and the corresponding optimal schedule for a given criterion based on the due date and the completion times of jobs.

A recent survey on common due date assignment problems is given by Gordon et al. (2002). This review paper provides a unified framework of the common due date assignment and scheduling problems in the deterministic case. The models involving single machine and parallel machines are reviewed. Problems with different objectives and constraints are summarized regarding to the algorithms and computational complexity.

Most of the literature published assumes that release times are identical for all jobs. The review is divided into two categories, single machine scheduling and parallel machine scheduling (Gordon et al. 2002). In the single machine problem, literature which aims at minimizing the mean absolute deviation of completion times can be found in Panwalkar et al. (1982), Cheng and Kovalyov (1996), Biskup and Jahnke (2001) and Bagchi et al. (1986). With the objective to minimize the weighted sum of absolute deviation of the completion times, Panwalkar and Rajagapalan (1992) and Liman et al. (1998) studied due date assignment problem. Articles focusing on minimizing total weighted earliness and tardiness are found in Cheng (1990), Quaddus (1987), Dileepan (1993), De et al. (1994) and Gupta et al. (1990).

In the parallel machine problem, there are fewer publications compared to the single machine problem. Emmons (1987) considered the parallel machine scheduling problems with due date assignment. Related literature can be found in Alidaee and Panwalkar (1993), Alidaee and Ahmadian (1993), Cheng and Chen (1994).

2.3.1.2 Distinct Due Dates

To the best of our knowledge, few researchers have addressed scheduling problems with distinct lead times. The representative one is authored by Seidmann et al. (1981). The paper presents a polynomial-time optimization algorithm for the single machine scheduling problem, which determines the set of due dates $d = (d_1, d_2, \dots, d_n)$

and job sequence π minimizing the following objective function as shown in the equation (2.2).

$$Z(\pi, d) = \sum_{i=1}^n (\alpha \max(0, d_i - A) + \beta E_i + \gamma T_i) \quad (2.2)$$

where C_i is the completion time of job i in the sequence π , $E_i = \max(0, d_i - C_i)$ is the earliness of job i , $T_i = \max(0, C_i - d_i)$ is the tardiness of job i , A represents the lead time that customers consider to be acceptable, and α , β and γ are non-negative parameters representing the per unit lead-time, earliness and tardiness penalties, respectively. There is no lead time cost if the due date is set to be less than or equal to A .

Shabtay and Steiner (2006) extend the work of Seidmann et al. (1981) to the individually weighted case where each job can have different lead time, earliness and tardiness penalties. The authors consider different acceptable lead times for the jobs to be scheduled. The objective function is as shown in the equation (2.3).

$$Z(\pi, d) = \sum_{i=1}^n (\alpha_i \max(0, d_i - A) + \beta_i E_i + \gamma_i T_i) \quad (2.3)$$

This problem is referred to total weighted earliness and tardiness with due date assignment. The authors also consider situations where earliness penalties are not considered.

Qi and Tu (1998) consider the problem in which the due dates are determined by the SLK method. The objective of the scheduling problem is to minimize the penalty of earliness subject to non-tardy jobs. Two types of penalty are investigated, i.e. equally weighted penalty and weighted linear penalty. Two algorithms are proposed to find the solutions in polynomial time. It is also proven that the problem becomes NP-complete when due-dates are arbitrary.

The scheduling problems with random processing times are addressed by several papers (e.g., Cheng 1984; Cheng 1988; Cheng 1991).

2.3.2 Dynamic Models

In many real world problems, some of jobs are not available to be processed at the start time of the planning horizon. In this section, the literature concerning dynamic models, in which jobs have associated release times, and cannot be processed until these times, is reviewed. The section is further divided into offline and online models. Offline algorithms can use information about jobs which will be released in the future to make sequencing and due date quotation decisions. In contrast, online algorithms sequence jobs at any time using only information pertaining to jobs which have been released by that time (Kaminsky and Hochbaum, 2004).

2.3.2.1 Offline Models

Baker and Bertrand (1981) study the due date assignment rules for single machine models with preemption under the objective of minimizing the average due date subject to a 100% service rate. The authors consider three simple due date setting rules, CON, SLK, and TWK. For each rule, an algorithm is proposed to sequence jobs optimally and determine the lead times.

Gordon (1993) presents the SLK rule for a single machine with job preemption under the objective of minimizing the maximum tardiness plus a penalty associated with the slack.

Charnsirisakskul et al. (2004) consider a profit-maximization problem with order acceptance, scheduling, and due-date setting decisions. The objective is to decide how much of each order to produce in each period to maximize profit, which is revenue minus holding cost, tardiness penalty and production cost. The problem is formulated as a mixed integer linear program. Different manufacturing environments are

investigated based on several factors, such as demand load, order size and seasonality of the demand. The benefits of lead time and partial fulfillment flexibility are discussed via a numerical study.

2.3.2.2 Online Models

There are a plenty of papers that discussed the online models (e.g., Wein, 1991; Duenyas 1995; Hopp and Sturgis, 2000; Kapuscinski and Tayur, 2007; Ata and Olsen, 2009). Queueing theory has been utilized by various researchers to study the single machine dynamic due date quotation problem. (e.g., Duenyas and Hopp, 1995; Duenyas, 1995; Ata and Olsen, 2009; Wein, 1991).

Due date quotation when demand is sensitive to time quotation or other factors has been addressed by a number of researchers (see, e.g., Duenyas and Hopp, 1995; Chatterjee et al. 2002; Ata and Olsen, 2009).

Since customers may not be homogeneous in the real world, some literature considers multiple customer groups in the queueing models (e.g., Wein, 1991; Duenyas, 1995; Keskinocak et al., 2001; Kapuscinski and Tayur, 2007).

In addition to queueing theory, Kaminsky and Kaya (2008) introduced on-line scheduling algorithms to quote due-dates for a two-stage supply chain. In this model, jobs or orders arrive at the manufacturer over time. Due dates must be quoted immediately upon job arrival. Order rejection is not considered in this model. In order to complete the production, the manufacturer needs to obtain a customized component from a supplier. Each order takes a different amount of time to process at the manufacturer, and at the supplier. The objective is to determine a schedule and quote due dates in order to minimize a function of quoted lead time and lateness. An on-line due date quotation algorithm is proposed with several variations. The paper

characterizes the asymptotic performance of this algorithm and then analyzes probabilistic bounds on its performance.

2.4 Scheduling with time-dependent processing times

In make-to-order environment, customers may be sensitive to the lead times of their orders. The demand of a customer depends on the lead time quoted to it. Taking this into account, the general scheduling problem can be formulated as a scheduling problem with time dependent processing times (for details of formulation, see section 5.2). There are two types of time dependent processing time. One is that the processing time of a job is increasing when the job starting time is increased. The other one is that the processing time of a job is decreasing when the job starting time is increased. Examples of increasing processing times occur in many industrial applications, such as scheduling maintenance or cleaning tasks, where any delay in job starting increases the duration of the job. There are also some examples of decreasing processing times in industrial applications, such as learning, where learning time is decreasing while starting time to learn becomes late.

Related models of time-dependent processing time have been extensively studied from a variety of perspectives in recent two decades. Full reviews on this area have been done by Alidaee and Womer (1999), and Cheng et al. (2005). In this section, some related research is highlighted, focusing on single machine scheduling problem.

2.4.1 Increasing start time dependent processing times

In the literature considering time dependent processing times, most of the publications deal with non-decreasing start time dependent job processing times

(namely, deteriorating jobs). It is common to use a linear increasing function to describe the processing time. For example, the processing time p_i of job i is given as a linear increasing function of its starting time s_i :

$$p_i(s_i) = a_i + b_i s_i$$

where $a_i > 0$ and b_i denote the normal processing time and the deteriorate rate of job i , respectively.

Browne and Yechiali (1990) firstly examine the single machine scheduling problem with linear increasing processing time of start time. The optimal sequence is presented with the objective of minimizing (maximizing) the makespan based on the theory proved in Rau (1971). Gupta and Gupta (1988) conduct a pioneer research on the similar problem through a combinatorial approach. Despite linear function, non-linear processing time function is also considered.

Properties of the optimal schedule in some special cases are studied by Mosheiov (1991), Mosheiov (1994), and Mosheiov (1996). Mosheiov (1991) studies the special model in which a_i are identical for all jobs. The objective is to minimize follow time. It is shown that the optimal sequence is V-shaped: jobs are arranged in descending order of b_i if they are placed before the minimal b_i , and in ascending order if placed after it. When $a_i = 0$, Mosheiov (1994) investigates the optimal schedule of various objectives, such as, makespan, total flow time, sum of weighted completion times, total lateness, and number of tardy jobs. When b_i are identical for all jobs, Mosheiov (1996) discusses this special case with the objective of minimizing the sum of weighted completion times. The optimal schedule is proved to be Λ -shaped, i.e., jobs are arranged in ascending order of a_i if they are placed before the job with largest a_i , but in descending order if placed after it.

Bachman et al. (2002b) deal with a single machine scheduling problem to minimize the total weighted completion time. In their paper, this problem is proved to be NP-complete. Wu et al. (2007) develop a branch-and-bound algorithm to solve the total weighted completion time minimization problem. Several heuristic algorithms are developed to find near optimal solutions.

Wang (2007) consider both learning effect (decreasing) and deterioration effect (increasing) of job processing times. The linear processing time of job i in position r is

$$p_{i,r}(t) = p_i(\alpha(t) + \beta r^a)$$

where p_i is the basic processing time; $\alpha(t)$ is the increasing function; $a \leq 0$ is the learning index; $t \geq 0$ is the starting time for job i . It is proved that makespan and sum of completion times minimization problems are polynomially solvable.

Lee et al. (2010) introduce the time-dependent scheduling problem with the consideration of machine availability. In their model, the machine is not available in a specific time interval due to machine maintenance. It is also assume that $a_i = 0$. A branch-and-bound algorithm and a heuristic are developed to obtain the optimal and near optimal solution of minimizing the total completion time, respectively.

2.4.2 Decreasing start time dependent processing times

Literature related to single machine scheduling problem with decreasing start time dependent processing times are highlighted. It is common that the processing time p_i of job i is given as a linear decreasing function of its starting time s_i :

$$p_i(s_i) = a_i - b_i s_i$$

where $a_i > 0$ and b_i denote the normal processing time and the decreasing rate of job i , respectively.

Chen (1995) considers the number of tardy jobs minimization problem in which the due dates for jobs are identical. A dynamic programming algorithm is developed to

solve this problem in $O(n^2)$ time. The total weighted completion time minimization problem is studied by Bachman et al. (2002a). In the paper, the authors prove that the problem is NP-complete, and present some special cases which are polynomially solvable.

Cheng et al. (2005) address a special single machine scheduling problem, in which b_i are identical for all jobs. A common due date is quoted for all jobs. The objective is to minimize the sum of due date, earliness and tardiness penalties. A algorithm is developed to solve this problem in $O(n \log n)$ time.

Job precedence constraints are examined in Wang (2009). In this model, two precedence constraints, the parallel chains and series-parallel graph, are investigated with the objective of minimizing the makespan. These constrained problems are proven to be polynomially solvable. As an extent to Wang (2009), Gao et al. (2010) consider a special case, where the processing time function is, $p_i = a_i(1 - bs_i)$. It is shown that there exist polynomial algorithms for the problems of minimization of total weighted completion time.

Voutsinas and Pappis (2010) present a value maximization problem for remanufacturing of PCs. It is assume that the value of components, V_i are decreasing over time t , that is

$$V_i = K_i t^{a_i}$$

where K_i is the value of one unit of component i at time zero; a_i is the value deterioration rate for component i . Since this problem is proven to be NP-hard, a branch and bound algorithm is developed to find the optimal sequence in disassembly.

In addition, other related single machine scheduling problems are proven to be NP-complete in Cheng and Ding (1998) and Cheng and Ding (2003).

2.5 Discussion

The related literature on jointly Price/Lead-time/Production problem and single machine scheduling problem with time-dependent processing times are summarized and evaluated in this section. A total of 20 evaluation studies on jointly Price/Lead-time/Production problem are summarized in Table 2.1 and Table 2.2. A total of 16 evaluation studies on single machine scheduling problem with time-dependent processing times are summarized in Table 2.3 and classified into two categories, increasing processing times and decreasing processing times.

From the above review of the evaluation works, the following important observations, which would highlight the significance of the research problem addressed in this thesis, can be obtained:

- The research considering price/lead-time/production problem mainly investigate system congestion using queueing models linking lead time dynamics and service levels (Table 2.1 and 2.2). These models cannot give accurate lead time quotation on each job. Hence it can be only applied to make tactical decisions.
- For pricing and production problem, most of the research is focusing on the tactical level. There is lack of literature addressing the models that can be used to make day to day operational decisions (Table 2.1 and 2.2).
- The due date assignment problems are well studied. However, there are only a handful of published results considering price- and time-dependent demand (Ata and Olsen (2009) in Table 2.1). Price and time are two important factors which cannot be ignored when customers are heterogeneous with price-sensitivity and time-sensitivity.
- Although the scheduling problems with time-dependent processing times have been fully discussed, to the best of our knowledge, the minimization of total

weighted completion times for decreasing processing times remains an unsolved problem (Table 2.3).

From the view of the industrial applications, there are several important factors that justify the significance of the research problem studied in this thesis:

- In the make-to-order manufacturing, it is often the case that customers have heterogeneous preferences on the product price and delivery time. It is valuable to study the delivery time quotation with consideration of different price and time sensitivities.
- The estimation of lead time is more accurate and reliable in the operational level. It is meaningful to study the joint problem with reliable lead times and production operations which can be applied in real practice.
- Dedicated prices and delivery time quotations for different customers can utilize the capacity efficiently by prioritizing and producing orders from different customers to maximize the profit for the manufacturing/service company.

This thesis presents an operational model of the coordination problem for price and delivery time quotation, which is also considered as a joint Price/Lead-time/Production problem. Compared to the evaluation studies in Table 2.1 and 2.2, there are only 3 papers located at the operational level involving factors such as quoted lead time, capacity model, scheduling priority and demand function. To the best of our knowledge, the problem of jointly considering dynamic sequencing priority problem in an operational level with price and lead time quotation is absent in previously published literature. By integrating the analysis of sequencing with price and lead time quotation, this research gives a full and clear understanding on how price/time production differentiation performs in the day-to-day operations.

Table 2.1: Summary of Evaluation Studies (I)

Study	Year	Decision Level	Methodology	Capacity model	Customer group	Demand function	Quoted lead time type	Notes
So and Song	1998	Tactical	Queueing	Shared	Single	Log-linear	Common	
Palaka	1998	Tactical	Queueing	Shared	Single	Linear	Common	
Ray and Jewkes	2004	Tactical	Queueing	Shared	Single	Linear	Common	
Boyaci and Ray	2003	Tactical	Queueing	Dedicate	Multiple	Linear	Common	Switchover between customer groups
Boyaci and Ray	2006	Tactical	Queueing	Dedicate	Multiple	Linear	Common	Switchover between customer groups
Zhao et al.	2006	Tactical	Queueing	Dedicate	Multiple	Linear	Common	
Pangburn and Stavoulaki	2008	Tactical	Queueing	Shared/dedicate	Single	Linear	Common	
Plambeck	2004	Tactical	Diffusion approximation	Shared	Two	Linear	Distinct	Dynamic Sequence
Ata and Olsen	2009	Tactical	Queueing	Shared	Single	Renewal process	Distinct	No price
Celik and Maglaras	2008	Tactical	Diffusion approximation	Shared	Multiple	Mapping	Single/Two	Multiple products

Table 2.2: Summary of Evaluation Studies (II)

Study	Year	Decision Level	Methodology	Capacity model	Customer group	Demand function	Quoted lead time type	Notes
Lederer and Li	1997	Tactical	Game theory	Shared	Single/Multiple	D. D. F.	Common	Competitive firms
So	2000	Tactical	Game theory	Shared	Single	MCI model	Common	Competitive firms
Easton and Moodie	1999	Tactical	Bidding model	Shared	Single	N.A.	Distinct	
Watanapa et al.	2005	Tactical	Bidding model	Shared	Multiple	N.A.	Distinct	Dynamic sequencing
Liu et al.	2007b	Strategic	Stackelberg game	Shared	Single	Linear	Common	Supply chain management
Pekun et al.	2008	Strategic	Stackelberg game	Shared	Single	Linear	Common	Supply chain management
Webster	2002	Strategic	System dynamics	Shared	Single	Linear	N.A.	
Elhafsi	2000	Operational	Probability	Shared	Single	N.A.	Distinct	FCFS sequence
Charnisrisakskul et al.	2006	Operational	MIP	Shared	Single	Mapping	N.A.	
Chen and Hall	2008	Operational	Dynamic Programming	Shared	Single	Mapping	N.A.	Dynamic pricing and sequencing
This research		Operational		Shared/Dedicated	Single/N.A.	Linear	Common/Distinct	Dynamic sequencing

D.D.F.: Differentiable decreasing function; MCI: Multiplicative competitive interaction; N.A.: Not applicable, means the factor is not considered

Table 2.3: Summary of Evaluation Studies (III)

Study	Year	Problem	Complexity	Solution
Gupta and Gupta	1988	$1 p_i = a_i + b_i s_i C_{max}$	$O(n \log n)$	A combinatorial approach
Mosheiov	1991	$1 p_i = 1 + b_i s_i \sum C_i$	$O(n \log n)$	V-shaped property
Mosheiov	1994	$1 p_i = b_i s_i C_{max}, \sum C_i, etc$	Polynomial solvable	Simply policy and algorithms
Mosheiov	1996	$1 p_i = a_i + b_i s_i \sum w_i C_i$	$O(n \log n)$	$\wedge$ -shaped property
Bachman et al.	2002b	$1 p_i = a_i + b_i s_i \sum w_i C_i$	NP complete	Open question
Wu et al.	2007	$1 p_i = a_i + b_i s_i \sum w_i C_i$	NP complete	B&B algorithm near optimal heuristic
Wang	2007	$1 p_{i,r}(t) = p_i(\alpha(t) + \beta r^a C_{max}, \sum C_i$	Polynomial solvable	Simple rules
Lee et al.	2010	$1 p_i = b_i s_i \sum C_i$ machine availability constraint	Open question	B&B algorithm near optimal heuristic
Chen	1995	$1 p_i = a_i - b_i s_i, d_i = D \sum U_i$	Polynomial solvable	An $O(n^2)$ -time dynamic programming algorithm
Cheng and Ding	1998	$1 p_i = a_i - b_i s_i, d_i \in \{D_1, D_2\} C_{max}$	NP complete	Open question
Bachman et al.	2002a	$1 p_i = a_i - b_i s_i \sum w_i C_i$	NP complete	Open question
Cheng and Ding	2003	$1 p_i = 1 - b_i s_i, d_i C_{max}$	NP complete	Open question
Cheng et al.	2005	$1 p_i = a_i - b_i s_i, d_i = D \sum(\alpha E_i + \beta T_i + \gamma d)$	Polynomial solvable	An $O(n \log n)$ time algorithm
Wang	2009	$1 p_i = a_i - b_i s_i C_{max}$ precedence constraints	Polynomial solvable	Polynomial time algorithms
Gao	2010	$1 p_i = a_i(1 - b_i s_i) w_i C_i$ precedence constraints	Polynomial solvable	Polynomial time algorithms
This research		$1 p_i = a_i - b_i s_i \sum w_i p_i$	NP complete	MIP formulation, B&B, heuristic

In Table 2.3, the total weighted completion time problem is proven to be NP complete. To the best of our knowledge, no research has developed algorithms to find optimal or near-optimal solution for this problem. It is shown that this problem is also related to the coordination problem (see in Section 5.2), and will be examined in this thesis.

These observations give the inspiration to develop an operational model of the coordination of price and delivery time quotation.

2.6 Summary

This chapter presented the literature review pertaining to the coordination of price and delivery time quotation production with a focus on tactical and operational level decisions. To achieve a clear understanding of the integration of price, lead time and production, three typical research categories, price/production, price/lead-time/production, and due-date assignment, are investigated. Related single machine scheduling problems with time-dependent processing times are also reviewed.

The gaps identified in the research literature provide the motivation for the issues addressed in this research work. In the next chapter, the mathematical model for the coordination of price quotation and lead time quotation is presented in detail.

Chapter 3

Common Price and Delivery Time Quotation Problem

In this chapter, a supply chain which consists of a firm and a set of customers is studied. The firm quotes one price and one delivery time for all customers. The demand from each customer depends on the quoted price and delivery time. The firm's objective is to maximize the revenue by quoting the optimal price and delivery time. A mathematical model is developed. Also, the optimal price and delivery time quotation policy, as well as the production scheduling algorithm, are identified.

3.1 Introduction

In a make-to-order manufacturing environment, delivery time (or lead time) guarantee has been applied as a strategic weapon to compete with other firms. Moreover, customers are heterogeneous with their time sensitivity and price sensitivity in the market. Some customers may prefer to pay more to get a shorter delivery time, while some customers would like to wait longer to get a price discount. These customers' characteristics are depicted through their demand (or order quantity) in this

model. The demand from a customer depends on the delivery time guarantee and quoted price.

A new strategy is emerging to divide customers into different groups regarding to their sensitivities to price and time. Within one group, customers share one combination of delivery time guarantee and price quotation. Different quotations of price and delivery time are offered to different customer groups, which is most evident in industries such as printing and packaging. This problem is known as customer segmentation and pooling problems in literature (Pangburn and Stavroulakis 2008, Plambeck 2004). The following example of printing.com applies the segmentation and pooling strategy. Printing.com is a printing service provider. A menu of delivery times associated with different prices is provided for the customer to choose from. For shorter delivery, the price is naturally higher. Figure 3.1 (obtained from <http://www.printing.com/services/guarantee>) gives an example of the delivery time and price menu from Printing.com.

our normal turnaround	choose your turnaround	choose your price
9 working days	add 20%
	8 working days	add 35%
	7 working days	add 50%
	5/6 working days	add 100%

Figure 3.1: Delivery time and price menu from printing.com

Generally, there are two kinds of capacity setting in this strategy cases, i.e., shared capacity among all customer groups or dedicated capacity for each niche. Dedicated capacity is advocated by two reasons. First, it is compatible to use dedicated capacities to provide each customer group with different lead time guarantees (Boyaci and Ray 2003). The other one is that there is less interference in production between customers in different groups (Pangburn and Stavroulakis 2008).

Based on the dedicated capacity setting, the implementation of segmentation and pooling strategy can be considered as multiple single-customer-group problems. In addition, the capacity for each group and the range of customer sensitivities to price and time for each group need to be identified. Therefore, the preliminary work of this strategy is to quote a common optimal price and a common delivery time for all customers in one customer group. Motivated by this, the coordination of pricing and delivery time quotation is considered with detailed production scheduling decisions in the make-to-order environment. It is assumed that a common price and a common delivery time quotation will be given to all customers at the beginning of the planning period. Then, customer determines his/her order quantity. Similar assumptions are also made in the existing literature (Charnsirisakskul et al. 2006, Chen and Hall 2010). It is also assumed that each order cannot be split in production and delivery. Furthermore, each order is delivered immediately upon its completion in production. This is common in make-to-order situations such as consumer electronics and fashion items.

One of the technical contributions is a solution procedure for the coordination problem of pricing and scheduling faced by a manufacturer. This research shows the properties of the optimal delivery time quotation, and develops a forward scheduling algorithm to solve the problem optimally. This research extends the work of Chen and Hall (2010) on pricing and scheduling coordination problem by consideration of joint time and price quotations.

3.2 Problem Formulation

There is a set of customers, indexed by $i = 1, \dots, n$, placing orders to a single firm for an identical product. The manufacturer will quote a common price and delivery time for all the customers. Each customer will then place a single order with the

amount of the product. The firm will deliver each order in full to its customer at the time when the order is completed in production. The transportation time is not taken into account in this model. It is also assumed that the facility has a stable production rate. In other words, the processing time of per unit product is fixed in the model. The capacity cost is also fixed and known.

To reflect customer sensitivity to price (p) and delivery time quotation (t), the demand per customer is assumed to be the following linear function, which is represented by the quantity of a customer order

$$Q_i(p, t) = \alpha_i - \beta_i \cdot p - \theta_i \cdot t \quad (3.1)$$

where α_i , β_i and θ_i are positive constants. α_i is potential market size for one customer. β_i represents the price sensitivity, and θ_i represents the delivery time sensitivity. The reason of using a linear demand from literature has been discussed in Chapter 1.4. Besides that the linear demand function helps to obtain quantitative insights without much analytical complexity. It also has the desirable properties on the price and delivery time elasticity of demand (refer to Palaka et al., 1998, Pekgün et al. 2008). To differentiate the customers with respect to their sensitivities of price and delivery time quotation, combinations of (β_i, θ_i) are used to describe the attribute of customers.

In real practice, there are also some examples in which the demand is linearly decreasing as the time increases. For example, a third-party company provides cleaning service to warehouses owned by other companies. The amount of cleaning work of one warehouse depends on the amount of its inventory. Since there is a constant usage rate of the inventory in all warehouses, the amount of cleaning work decreases as time passed. It is true that the cleaning service has a positive impact on the value of the stocks in all warehouses. If the cleaning price for per unit inventory

goes high, the warehouse will only clean part of the inventory which will stay there for a relative long period. In other words, the amount of inventory to be cleaned is decreasing as the price goes up. Each warehouse has a different price expectation and time expectation on the cleaning service because of their different inventory. Moreover, as the relationship between price and demand is hard to describe in a single form, it is general to use a linear function to model it in many cases. Since it is important to gain the competence by give a time guarantee for its customers, the cleaning service company is looking forward to quoting a common price and a common finished time for all warehouses and maximizes its revenue.

A customer's demand quantity is dependent on the quoted price and quoted delivery time. If a customer's demand quantity turns to 0 or negative, the manufacturer rejects the order from this customer. It is assumed that an order with positive order quantity must be produced and delivered without splitting. This restriction is reasonable when customers prefer to receive an order all at one time, especially in a delivery time guarantee situation.

The firm is assumed to be a single machine. This assumption is also made in several papers including So and Song (1998), Zhao et al. (2006) and Celik and Maglaras (2008). Order i represents the order from Customer i . Processing time per unit product is denoted by u which is identical and predetermined. Hence the processing time of order i can be described as $pt_i = u \cdot Q_i$. It is assumed that $u = 1$ (per time)/(per unit product) to generalize the formulation. Thus, the equation $pt_i = Q_i$ is valid. Since the price and delivery time quotation are set at the beginning of the planning horizon, the processing times of all orders are determined. The release time of each order is 0.

Orders which are completed later than the delivery time quotation will incur tardiness penalties. Since all customers are from the same customer group, it is reasonable to apply a common fixed tardiness weight for all customers. The tardiness cost function is $T_i(p, t) = w \cdot \max(0, C_i - t)$. C_i represents the completion time of order i . w represents the common weight of tardiness for all customers.

The objective is to maximize the expected net profit. The profit function does not include other fixed facility costs, because they will not affect the optimal decisions of price and time quotations. The problem can be formulated by the following non-linear optimization model:

$$\text{Maximize} \quad f(p, t, \pi) = p \sum_{i=1}^n \max(Q_i(p, t), 0) - \sum_{i=1}^n (g_i \cdot T_i(p, t)) \quad (3.2)$$

$$\text{subject to} \quad g_i = \begin{cases} 0, & Q_i(p, t) \leq 0 \\ 1, & Q_i(p, t) > 0 \end{cases}$$

$$Q_i(p, t) = \alpha_i - \beta_i \cdot p - \theta_i \cdot t$$

$$T_i(p, t) = w \cdot \max(0, C_i - t)$$

$$C_i(\pi) = \sum_{s_1}^{s_i} g_i Q_i$$

$$s_i = j \quad \forall i, j \in \{1, 2, \dots, n\}$$

$$s_i \neq s_j \quad \forall i \neq j \text{ and } i, j \in \{1, 2, \dots, n\}$$

$$p, t \geq 0$$

Let p^* and t^* denote the optimal values of p and t , respectively. s_i denotes the index of the i^{th} order in a production sequence π . The first term in (3.2) represents the revenue by selling all orders. The second term represents the tardiness penalty for all orders.

3.3 Derivation of Optimal Algorithm

According to the objective function (3.2), the issue of production schedule is involved in the model. In section 3.3.1, the optimal production sequence for this

problem is examined. Then, two special cases of the general model are studied. Section 3.3.2 considers one sub-model, namely Fixed Price Problem, in which quoted price is fixed. Section 3.3.3 discusses the other sub-model, namely Fixed Time Problem, in which the quoted delivery time is fixed. Based on the results of sub-problems, the general model is analyzed in Section 3.3.4.

3.3.1 Decision on Sequence

The objective function (3.2) shows that the optimal net profit depends not only on the quoted delivery time and price, but also depends on the production sequence of orders.

Proposition 3.1 *Shortest Processing Time (SPT) is the optimal sequence rule in production to achieve the best service rate and maximum profit, simultaneously.*

Proof Proposition 3.1 is intuitive because of two reasons. Firstly, it is well known that the total lateness, $L = \sum L_i$, is minimized by sequencing orders in non-decreasing order of order processing times (Emmons 1987). When due dates are identical and determined, tardiness is equivalent to lateness. Secondly, the revenue achieved by sales,

$$R(p, t) = p \sum_{i=1}^n (\alpha_i - p \cdot \beta_i - t \cdot \theta_i)$$

$R(p, t)$ is not affected by sequencing. Hence SPT should be applied for production to maximize the net profit, which is sum of tardiness penalty ($-\sum T_i$) and revenue. Moreover, considering the objective of completing as many jobs as possible before a pre-specified deadline, no matter what deadline is specified, SPT sequencing (orders are arranged in non-decreasing order of its processing time) maximizes the number of orders completed by that time. Hence, it is also proven that SPT is optimal to achieve best service rate when the delivery time quotation is fixed. $\square$

Therefore, SPT rule is applied in the production scheduling throughout this problem. The following discussions illustrate how to incorporate the sequence constraint in the profit maximization problems.

3.3.1.1 Sequence Decision in Fixed Price Problem

In Fixed Price Problem, the price is assumed to be fixed. The order processing time is a linear function of quoted delivery time t , $pt_i = (\alpha_i - \beta_i \cdot p) - \theta_i \cdot t$. Figure 3.2 shows the processing times of three customer orders in respect to delivery time quotation. The functions of processing times are represented by three straight lines 1, 2 and 3. In Figure 3.2, there are 3 intersections, C_{12} , C_{13} and C_{23} , formed by these three lines. One intersection denotes where two order processing times are equal. For example, C_{12} is the point when $pt_1 = pt_2$ in Figure 3.2. As shown in the figure, the t -coordinate of lines' t -intercepts and intersections are numbered by t_1 to t_6 . It is clear that the SPT sequence for three orders is unique and determined within any interval of $[0, t_1]$, $[t_1, t_2]$, ..., and $[t_5, t_6]$. Table 3.1 lists all SPT production sequences for three orders within each time interval, namely time zone.

Figure 3.2: Order processing times with respect to time

Table 3.1: SPT sequence in different time zones

Time zone	$0-t_1$	t_1-t_2	t_2-t_3	t_3-t_4	t_4-t_5	t_5-t_6
Sequence	3-2-1	3-1-2	1-3-2	3-2	2-3	3

When t is increased from 0 to ∞ , the SPT sequence changes at two situations, 1) two order processing times are equal, which is represented by an intersection (e.g. C_{12}) as shown in Figure 3.2; 2) an order processing time is equal to 0, which is represented by the t -intercept of a line (e.g. t_I) as shown in Figure 3.2.

If C_{ij} is the intersection, the equation, $pt_i = pt_j$ is satisfied. When (3.1) is substituted into this equation, the t -coordinate of C_{ij} can be derived as

$$t_{ij} = \frac{\alpha_i - \alpha_j + p(\beta_j - \beta_i)}{\theta_i - \theta_j} \quad (3.3)$$

The t -coordinate of the intercept point I_i can also be derived from $pt_i = 0$. That is

$$t_i = \frac{\alpha_i - p\beta_i}{\theta_i} \quad (3.4)$$

As a result, all boundaries of time zones except the original point 0 can be calculated through (3.3) or (3.4). Then, they are sequenced in ascending order. In Fixed Price Problem, the time constraint, $t_c \leq t < t_{c+1}$ formulates one time zone in general, where t_c and t_{c+1} represent any two adjacent boundaries of time zones. Within this time constraint, the optimal sequence is unique and determined.

Figure 3.3: Order processing times with respect to price

3.3.1.2 Sequence Decision in Fixed Time Problem

Similar results can be achieved in Fixed Time Problem, in which delivery time quotation is fixed. In Fixed Time Problem, the price constraint $p_c \leq p < p_{c+1}$

formulates one price zone, where p_c and p_{c+1} represent any two adjacent boundaries of price zones. As shown in Figure 3.3, the p -coordinate of C_{ij} is,

$$p_{ij} = \frac{\alpha_j - \alpha_i + t(\theta_i - \theta_j)}{\beta_j - \beta_i} \quad (3.5)$$

The p -coordinate of I_i is,

$$p_i = \frac{\alpha_i - t\theta_i}{\beta_i} \quad (3.6)$$

The boundaries of price zones can be calculated by (3.5) or (3.6). Within one price zone, the optimal production sequence is unique and determined.

3.3.1.3 Sequence Decision in General Problem

In the general problem, the price quotation and delivery time quotation are both decision variables. For example, there are three orders with different price and time sensitivities. The order processing time is a linear function of both t and p , $pt_i = \alpha_i - \beta_i \cdot p - \theta_i \cdot t$. Figure 3.4 illustrates the processing times of three orders in a 3-dimensional coordinate system.

Figure 3.4: Order processing time function in general problem

There are 4 intersection points on (pt, t) , (pt, p) and (p, t) plane formed by these 3 orders, C_1 , C_2 , C_3 and C_4 as shown in Figure 3.4. The coordinate of each projection point (p_{Ci}, t_{Ci}) of C_i on (p, t) plane can be obtained by simultaneously solving the two equations below,

$$\alpha_i - \beta_i \cdot p - \theta_i \cdot t = \alpha_j - \beta_j \cdot p - \theta_j \cdot t \quad (3.7)$$

$$p = 0, t = 0 \text{ or } pt = 0 \quad (3.8)$$

As shown in Figure 3.4, points E, F, G are the projections of points C_1, C_2 and C_3 on (p, t) plane, respectively. The projection of C_4 on (p, t) plane is itself. It is known that the straight line segment C_1C_2 is comprised of the points satisfying $pt_2 = pt_3$. Since EF is the projection of C_1C_2 , a point (p, t) on EF will also satisfies $pt_2 = pt_3$.

Figure 3.5: Price-time zones on the p - t plane

Figure 3.5 illustrates the (p, t) plane of Figure 3.4 in a two dimensional coordinate system. In Figure 3.5, the straight line T_iP_i is the intersection of plane i and the p - t plane, which is comprised of the set of points satisfying $pt_i = 0$. That is, points (p, t) on T_iP_i satisfy the equation,

$$\alpha_i - \beta_i \cdot p - \theta_i \cdot t = 0 \quad (3.9)$$

Similar to what is observed in the Fixed Price Problem and Fixed Time Problem, the optimal production sequence in the general problem changes only at two cases: (1) two order processing times are equal; (2) one order processing time is equal to 0. This means that the optimal production sequence changes on the straight lines P_iT_i , the straight lines EF and GC_4 in Figure 3.5.

Hence, the optimal production sequence is unique in a convex region which is separated by EF , GC_4 , P_iT_i , and t/p -axis in Figure 3.5. This convex region is defined as the price-time zone. In Figure 3.5, regions numbered by ① to ⑦ are all price-time zones in this example. Decision variables (p, t) which are in one price-time zone will have one unique optimal production sequence. Table 3.2 presents the optimal sequences in each price-time zone for this example.

Table 3.2: SPT sequence in all price-time zones

Zone Index	1	2	3	4	5	6	7
Sequence	2-3-1	3-2-1	2-1	1	1-2	2	n.a.

The key to generate all possible SPT sequence is finding the boundaries of each price-time zone in Figure 3.5. An algorithm is developed for creating detailed price-time zone list. The main idea of this algorithm is as follows: Firstly, Quadrant I in Figure 3.5 is considered as a single price-time zone and divided by each of the segment lines P_iT_i to obtain a series of price-time zones. Secondly, each of the price-time zones is checked to see whether it is further divided by the projections of intersection lines C_iC_j on the p - t plane. Finally, all price-time zones are formulated. The algorithm SLA is listed below.

Algorithm SLA (Sequence Listing Algorithm)

Input

Given $\alpha_i, \beta_i, \theta_i, i = 1, \dots, n$;

Initialization

Create Order List $L = \{O_1, O_2, \dots, O_n\} = \{(\alpha_1, \beta_1, \theta_1), (\alpha_2, \beta_2, \theta_2), \dots, (\alpha_n, \beta_n, \theta_n)\}$;

Formulate Quadrant I as the initial price-time zone, Z_I with edges, $p = 0$ and $t = 0$;

Create Price-time Zone List $ZL = \{Z_1\}$;

Price-time Zone index $s = 1$.

Loop 1:

1. Check Order List L : If L is empty, End Loop 1 and go to Step 5. Else select one order O_i in L and remove it from L ;

Loop 2:

2. Check the size of ZL , nz ; Select one Price-time Zone Z_j (initially, $j = 0$) from ZL ;
3. Cut Z_j by the segment line (3.9) from O_i : If there are 2 intersection points on the edges of Z_j , create a new Price-time Zone Z_{s+1} and attach it to ZL , revise Z_j (An example of Step 3 is given in at the end of this algorithm); $s = s+1$;
4. Increase j by 1: If $j = nz + 1$, End Loop 2 and go to Step 1; Else, go to Step 2;
5. Find the intersection lines of two orders, L_{ij} through (3.7) and (3.8). Find the projection of L_{ij} on the p - t plane, E_{ij} . Create Line Segment List, $SL = \{E_{ij}\}$. If there is no intersection line, SL is empty.

Loop 3:

6. Check Line Segment List SL : If SL is empty, End Loop 3 and go to Step 10. Else select one line segment E_{ij} in SL and remove it from SL ;

Loop 4:

7. Check the size of ZL , nz ; Select one Price-time Zone Z_j from ZL ;
8. Cut Z_j by the segment line E_{ij} : If there are 2 intersection points on the edges of Z_j , create a new Price-time Zone Z_{s+1} and attach it to ZL , revise Z_j (similar to Step 3); $s = s+1$;

9. Increase j by 1: If $j = nz+1$, End Loop 4 and go to Step 6; Else, go to Step 7;

10. Stop

Computational complexity analysis:

Algorithm SLA is of polynomial time. It is shown that there are 2 loops in series, Loop 1 and Loop 2. The number of iterations in Loop 1 depends on n and the size of ZL for a given n . It is proven that the maximum number of ZL is $((n^2+n+2)/2)$ by Buck (1943). Thus, the computational complexity of Loop 1 is $O(n^3)$. The estimation of the number of iterations in Loop 2 is similar to Loop 1. The computational complexity of Loop 2 is $O(n^6)$. As a result, the computational complexity of algorithm SLA is $O(n^6)$.

A simple example: splitting a price-time zone by a segment line

In Figure 3.6, a price-time zone ABC is defined by its edges $[A, B)$, $[B, C)$ and $[C, A)$ according the clockwise direction with the start point A . For example, Edge $[A, B)$ denotes that the end point A is on the edge, while the end point B is not on the edge. Based on this definition, the relationship between the segment line GH and the price-time zone ABC is discussed. Firstly, every edge of the price-time zone ABC is checked to find that whether it has an intersection point with the segment line GH . Secondly, the number of all intersection points is counted. If the number is 0 or 1, the price-time zone ABC is not intersected by the segment line GH . If the number of intersection points is 2, the price-time zone ABC is intersected by the segment line GH . As shown in Figure 3.4, there are two intersection points D and E on Edge $[A, B)$ and Edge $[C, A)$ respectively. The following steps show the detail of how to split a price-time zone ABC with the segment line GH .

Figure 3.6: Splitting a price-time zone ABC with a segment line GH

Initial:

Price-time Zone ABC: $[A, B)$, $[B, C)$ and $[C, A)$

Segment Line GH: $[G, H]$

Intersection Point D on $[A, B)$, Intersection Point E on $[C, A)$

Step 1: Create the revised price-time zone from ABC.

1.1 From the start point A, find the corners and edges along the edges of ABC until there is an intersection point on the edge. Create a new corner with this intersection point D. After this step, the revised price-time zone is partially formed by $[A, D)$.

1.2 From the corner D, create a new edge $[D, E)$ with the other end point, which is the other intersection point E. After this step, the revised price-time zone is partially formed by $[A, D)$ and $[D, E)$.

1.3 From the corner E, find other corners and edges along the previous edges of ABC until the last corner is same as the start corner. After this step, the revised price-time zone is fully formed by $[A, D)$, $[D, E)$ and $[E, A)$ and named by ADE.

Step 2: Create a new price-time zone.

2.1 From the intersection point D which is obtained in Step 1.1, find the corner and edges along the edges of ABC until there is the other intersection point on the edge. Create new a corner with this intersection point E . After this step, the new price-time zone is partially formed by $[D, B)$, $[B, C)$ and $[C, E)$ and named by $DBCE$.

2.2 From the other intersection point E , create a new edge with the other end point which is the start point in Step 2.1. After this step, the new price-time zone is fully form by $[D, B)$, $[B, C)$, $[C, E)$ and $[E, D)$.

These steps are also applicable when the intersection point coincides with one corner of the original price-time zone. $\square$

Once a price-time zone is selected in Table 3.2, there is a unique production sequence according to SPT rule. All information of edges on each price-time zone is given by equations (3.7), (3.8) or (3.9). Therefore any price-time zone can be formulated by a series of linear constraints according to the above three equations. A series of linear constraints for a price-time zone can be generalized in the format of (3.10),

$$\begin{aligned}
 c_{11} \cdot p + c_{12} \cdot t &\leq c_{13}; \\
 c_{21} \cdot p + c_{22} \cdot t &\leq c_{23}; \\
 &\dots \\
 c_{s1} \cdot p + c_{s2} \cdot t &\leq c_{s3}; \\
 p &\geq 0; t \geq 0
 \end{aligned} \tag{3.10}$$

where c are constant values which can be derived from (3.7), (3.8) and (3.9); s denotes the number of edges in the price-time zone.

3.3.2 Decisions of Price and Delivery time

In previous section, the sequence decision has been discussed. Based on the sequence decision, the optimal price quotation and delivery time quotation is discussed in this section. Before the general problem is discussed, two special problems, Fixed Price Problem and Fixed Time Problem are examined to find the properties of the optimal solution. The optimal properties found in these special problems will be applied to solve the general problem.

3.3.2.1 Decision of Delivery Time Quotation in Fixed Price Problem

As stated in section 3.3.1.1, the SPT sequence is fixed for each time zone, (t_c, t_{c+1}) . If Fixed Price Problem is investigated within a specific time zone, the scheduling decision is trivial. Thus, the overall problem can be decomposed into several simplified sub-problems according to the number of time zones. The objective function (3.2) is decomposed into two parts for investigation. The first part is the revenue by sales. That is

$$R(t) = p \sum_{i=1}^n (\alpha_i - p \cdot \beta_i - t \cdot \theta_i)$$

The other part is the tardiness penalty,

$$T(t) = w \sum_{i=m+1}^n \sum_{j=1}^i (\alpha_i - p \cdot \beta_i - t \cdot \theta_i) - w(n - m)t$$

where m is the number of on-time orders at a given value of t .

If m is independent of t , the gradient of the profit with respect to t is,

$$\frac{\partial f}{\partial t} = \frac{\partial R}{\partial t} - \frac{\partial T}{\partial t} = -p \sum_{i=1}^n \theta_i - w \sum_{i=m+1}^n \sum_{j=1}^i (-\theta_i) + w(n - m) \quad (3.11)$$

Actually m is not independent to t . The relationship between m and t is presented in detail in the proof of Proposition 3.2.

When m is constant, the gradient of the revenue function and the tardiness function respect to t are

$$\frac{\partial R}{\partial t} = -p \sum_{i=1}^n \theta_i \quad (3.12)$$

$$\frac{\partial T}{\partial t} = -w \sum_{i=m+1}^n \sum_{j=1}^i \theta_i - w(n-m) \quad (3.13)$$

Proposition 3.2 *The profit function is a concave function with respect to t as shown in Figure 3.7. The function has following properties: (i) it is a piece wise linear function; (ii) the gradient of each piece is decreasing with respect to t ; (iii) The gradient of the first piece is positive, while the gradient of the last piece is negative; (iv) A maximum point is at a conjunction point between two pieces.*

Figure 3.7: The profit function respect to delivery time quotation

Proof Through (3.12), it is proven that the revenue function is a decreasing straight line. In the function (3.13), the variable m is not independent to t actually. The relationship between t and m is presented in Figure 3.8. When t is increased to t' , the processing times of all orders are decreased, as order quantities are reduced. As a result, the distance between C_{m+1} and t is reduced. Although m is not changed when t increases to t' in the figure, it is intuitive that m can be increased by 1 when t is increased to a certain value when the distance between C_{m+1} and t is reduced to 0. The relationship between t and m is that, m is increased by 1 intermittently when t is continuously increased to a series of certain values. The value of m is an integer between 0 to n . After m is increased to n , it will not be increased any more when t is increasing.

 Figure 3.8: The relationship between m and t

From (3.13) it is found that 1) the gradient of the tardiness is decreasing when m is increasing; 2) the tardiness function is a straight line when m is constant. In the following step, it is proven that the tardiness function is continuous.

In Figure 3.8 the tardiness function is

$$T = w \cdot \sum_{i=1}^n \max(C_i - t) = w \cdot (C_{m+1} + C_{m+2} + \dots + C_n - (n - m)t)$$

When t is increased from C_m to C_{m+1}' , it is known that $C_{m+2}, C_{m+3}, \dots, C_n$ are all continuous function with respect to t . If the term C_{m+1} in the tardiness function is proven to be continuous functions with respect to t , the full tardiness function is proven to be continuous. An example with a single order is listed below.

When $t = 0$, $C_1 = \alpha_1 - p\beta_1$. The tardiness penalty is $T = w(\alpha - p\beta_1)$. The gradient of tardiness penalty is $\frac{\partial T}{\partial t} = -w(\theta_1 + 1)$. When $t' = C_1$, the tardiness penalty is $T' = 0$. At this time, the equation $t' = \alpha_1 - p\beta_1 - t'\theta_1$ is satisfied. Hence $t' = \frac{\alpha_1 - p\beta_1}{1 + \theta_1}$, and the equation, $T + \frac{\partial T}{\partial t}(t' - t) = T' = 0$ exists.

Since $T + \frac{\partial T}{\partial t}(t' - t) = T'$, it is proven that the tardiness function is continuous in this example, which is a general case.

Based on the results, it concludes that the tardiness function is a piece-wise line respect to t with the gradient of each piece is decreasing. Since the revenue function is

a straight line with a negative gradient, the profit function is a concave function which is shown as Figure 3.7.

According to practical experience, the profit cannot be optimal when t is set at 0 or ∞ . If the tardiness weight is close to 0, the optimal t is set at 0. If the tardiness weight is ∞ , the optimal t will always at the maximum completion time. It can be proven that the gradient of the first piece is positive as well as the gradient of the last piece is negative. Finally, it is concluded that a maximum point is always at a conjunction point between two pieces. $\square$

Lemma 3.1: *The optimal delivery time quotation in Fixed Price Problem must satisfy one of the following two conditions:*

1. *The optimal delivery time quotation coincides with an order completion time;*
2. *The optimal delivery time quotation is at either boundaries of the time zone.*

Proof. Figure 3.7 shows the profit function with respect to t . In the figure, the number on each segment line denotes the number of on-time orders in that status. For example, m^* is shown above the segment line EF. It indicates that there are m^* on-time orders when t locates within that segment line EF. As t is increased, m is increased by 1 intermittently. The t -coordinate of a conjunction point between two segments represents the time when t is equal to an order completion time. The maximum profit point is a conjunction point between two segments. The gradient of the one segment is positive, while the gradient of the other one is non-positive. According to this, the following two inequalities are satisfied,

$$\frac{\partial f}{\partial t} | (m = m^*) > 0$$

$$\frac{\partial f}{\partial t} | (m = m^* + 1) \leq 0$$

$m = m^*$ and $m = (m^*+1)$ are substituted into (3.11) to derive (3.14) and (3.15) according to above two inequalities, respectively. By solving (3.14) and (3.15) simultaneously, the value of m^* can be obtained.

$$w(n - m^*) + w \sum_{i=m^*+1}^n \sum_{j=1}^i \theta_i \leq p \sum_{i=1}^n \theta_i \quad (3.14)$$

$$w(n - m^* + 1) + w \sum_{i=m^*}^n \sum_{j=1}^i \theta_i > p \sum_{i=1}^n \theta_i \quad (3.15)$$

As shown in Figure 3.7, when $t = t^*$, the profit reaches the maximum point. t^* is the completion time of the $(m^*)_{th}$ order in the sequence. Hence the optimal delivery time quotation will satisfy the function,

$$t^* = \sum_{i=1}^{m^*} (\alpha_i - p\beta_i - t^* \theta_i)$$

Thus, t^* can be calculated by,

$$t^* = \frac{\sum_{i=1}^{m^*} \alpha_i - p \sum_{i=1}^{m^*} \beta_i}{1 + \sum_{i=1}^{m^*} \theta_i}$$

The solution for Fixed Price Problem is based on one specific time zone ($t_c \leq t \leq t_{c+1}$) with a corresponding optimal production sequence. Thus, it is doubtful that whether t^* is located in the corresponding time zone or not. As a result, the value of t^* is compared with the boundaries of the time zone. If the value of t^* locates in the time zone, the optimal delivery time quotation is equal to t^* . If the value of t^* locates outside of the time zone ($t^* < t_c$), the optimal delivery time quotation is either t_c or t_{c+1} . The optimal delivery time quotation of Fixed Price Problem is listed below,

$$t^* = \begin{cases} t_{c+1}, & t_{c+1} < \frac{\sum_{i=1}^{m^*} \alpha_i - p \sum_{i=1}^{m^*} \beta_i}{1 + \sum_{i=1}^{m^*} \theta_i} \\ \frac{\sum_{i=1}^{m^*} \alpha_i - p \sum_{i=1}^{m^*} \beta_i}{1 + \sum_{i=1}^{m^*} \theta_i}, & t_c \leq \frac{\sum_{i=1}^{m^*} \alpha_i - p \sum_{i=1}^{m^*} \beta_i}{1 + \sum_{i=1}^{m^*} \theta_i} \leq t_{c+1} \\ t_c, & t_c > \frac{\sum_{i=1}^{m^*} \alpha_i - p \sum_{i=1}^{m^*} \beta_i}{1 + \sum_{i=1}^{m^*} \theta_i} \end{cases} \quad (3.16)$$

As shown in (3.16), the optimal delivery time quotation can be equal to t_c and t_{c+1} .

These two values are the boundaries of a specific time zone. When $t^* = \frac{\sum_{i=1}^{m^*} \alpha_i - p \sum_{i=1}^{m^*} \beta_i}{1 + \sum_{i=1}^{m^*} \theta_i}$,

it is proven that t^* is the completion time of the $(m^*)_{\text{th}}$ order in the sequence. As a result, Equation (3.16) illustrates the same result as listed in Lemma 3.1. $\square$

Till now, the optimal delivery time quotation within one specific time zone is analytically solved in Fixed Price Problem. The optimal solution can be obtained by iterating all time zones. However, it is not shown in the thesis. The purpose of studying this special case is to find the properties of the optimal solution for the general problem. In the following section, the optimal price quotation is examined in Fixed Time Problem.

3.3.2.2 Decision of Price Quotation in Fixed Time Problem

In this section, the impact of adjusting price quotation on the optimal net profit is explored when delivery time quotation is given. According to the methodology to find the optimal SPT sequence in each zone, the price horizon can be divided into price zones as in Table 3.1. Then, a specific optimal order sequence can be found within each price zone, $p_c \leq p < p_{c+1}$. If Fixed Time Problem is investigated within a specific price zone, the scheduling decision is trivial. Thus, Fixed Time Problem can be decomposed into several simplified sub-problems according to the number of price zones.

Similar to previous section, the objective function is analyzed in terms of the revenue function and tardiness penalty function. The gradient formulation of revenue is given below,

$$\frac{\partial R}{\partial p} = -2p \sum_{i=1}^n \beta_i + \sum_{i=1}^n (\alpha_i - t\theta_i) \quad (3.17)$$

When m is assumed to be independent to p , the gradient of tardiness respect to price can be represented as

$$\frac{\partial T}{\partial p} = \begin{cases} -w \sum_{i=m+1}^n \sum_{j=1}^i \beta_i, & m = [m_0^t, n-1] \\ 0, & m = n \end{cases} \quad (3.18)$$

Actually, m is not independent to p . The relationship between m and p is discussed in the proof of Proposition 3.3.

Proposition 3.3 *Negative tardiness penalty ($-T$) is a piecewise linear function with respect to p . The function has following properties: (i) it is continuous and non-decreasing with respect to p ; (ii) the absolute gradient of each piece is decreasing with respect to p ; (iii) the gradient is 0 when p is larger than a certain value.*

Proof As illustrated in (3.18), the gradient of tardiness is formulated with one variable, m ,

$$\frac{\partial T}{\partial p} = \begin{cases} -w \sum_{i=m+1}^n \sum_{j=1}^i \beta_j, & m = [m_0^t, n-1] \\ 0, & m = n \end{cases}$$

when m is assumed to be independent to p .

The relationship between p and m is presented in Figure 3.9. When p is increased to p' , the processing times of all orders are decreased. As a result, the distance between C_{m+1} and t is reduced when t remains unchanged. Although m is not changed when p increases to p' in this example, it is intuitive that m can be increased by 1 when p is increased to a certain value that the distance between C_{m+1} and t is reduced to 0. At that time, the distance between C_{m+1} and t is reduced to 0. The relationship between p and m is that, m is increased by 1 intermittently when p is continuously increased. The m is an integer between m_0^t to n . After m is increased to n , it remains unchanged when p is increasing.

 Figure 3.9: The relationship between m and p

From (3.18) it is shown that 1) the gradient of the tardiness function is decreasing when m is increasing; 2) the tardiness function is a straight line when m is constant. Similar to Proof of Proposition 3.2, the tardiness function can be proven to be continuous.

Based on the results, it is concluded that the tardiness function is a piece-wise line with respect to p with the gradient of each piece decreasing. When the value of p is increased to a certain value, p_n^t , $m = n$. Then, the value of the gradient will remain 0 even when p is increased. $\square$

Since the profit is the summation of R and $-T$, it is also a concave function with only one optimal solution which is shown in Figure 3.10.

Figure 3.10: The net profit function

Figure 3.11: The revenue function and the tardiness function

Lemma 3.2: *The optimal price must satisfy one of the following 3 conditions in Fixed Time Problem:*

- (1) *The optimal price makes an order completion time coincides with the delivery time quotation;*
- (2) *The optimal price locates at either boundary of a price zone;*
- (3) *The optimal price makes the gradient of revenue, (3.17) equal to the gradient of tardiness penalty, (3.18).*

Proof In Figure 3.11, the negative tardiness function ($-T$) is below the p -axis. The number above each segment of $-T$ denotes the number of on-time orders in that status of price quotation. When $p = 0$, the number of on-time orders is assumed to be m_0^t at a given value of t . Similar to Fixed Price Problem, each conjunction point between two segments on $-T$ represents the situation when t is equal to an order completion time. When there are m orders on time, the p -coordinate of the conjunction point satisfies the equation, $t = \sum_{i=1}^m (\alpha_i - p\beta_i - t\theta_i)$. This equation can be derived into (3.19).

$$p = \frac{m\alpha_i - (1 + \sum_{i=1}^m \theta_i)t}{\sum_{i=1}^m \beta_i} \quad (3.19)$$

As shown in Figure 3.11, when p is greater than p_n^t , each order processing time is decreased. All orders are still on time. In other words, the tardiness penalty is 0 when $p \geq p_n^t$.

The function of revenue by sales can be expressed as a concave quadratic function according to (3.17), which is shown above p -axis in Figure 3.9. When $\partial R / \partial p = 0$, it achieves the maximum value. The corresponding value of p is,

$$p_0^t = \frac{\sum_{i=1}^n \alpha_i - t \sum_{i=1}^n \theta_i}{2 \sum_{i=1}^n \beta_i}$$

If $p_n^t \leq p_0^t$, the optimal point is at $p^* = p_0^t$, because the tardiness penalty is always at 0 when $p \geq p_n^t$. The cases of $p_n^t > p_0^t$ is more complex, which will be discussed in detail.

In Figure 3.11, when $p = p_0^t$, the gradient of the revenue function R is 0. Meanwhile, the gradient of $-T$ is positive. When $p \geq p_0^t$ and p is increasing, the gradient of R is decreasing from 0 to $-\infty$ continuously. At the same time, the gradient of $-T$ is decreasing from a positive value to 0 discretely. Hence, the optimal solution can be achieved, when the sum of the gradient of $-T$ and the gradient of R , $\partial f / \partial p$ reaches its least non-negative value. If $\partial f / \partial p$ can be equal to 0, the optimal price quotation is given by,

$$p = \frac{\sum_{i=1}^n (\alpha_i - t \theta_i) + w \sum_{i=m+1}^n \sum_{j=1}^i \beta_j}{2 \sum_{i=1}^n \beta_i} \quad (3.20)$$

where m is the number of on-time order is at this point.

If f' cannot be equal to 0, function (3.20) is not applicable to find the optimal solution. A search algorithm is developed for finding optimal price quotation based on Figure 3.11. The main idea of this algorithm is to find the value of p when $(\frac{\partial R}{\partial p} - \frac{\partial T}{\partial p})$ reaches the least non-negative value. $\square$

Algorithm OPS (Optimal Price Search)

1. Find the conjunction point X on $-T$ which is the first one with p -coordinate larger than p_o^t through (3.19). As shown in Figure 3.11, the p -coordinate of X is p_x , and the number of on time orders is x .
2. Calculate the gradient of R at p_x , $R_{x'}$ through (3.17); and obtain two gradients of segments on $-T$ at X , $Tl_{x'}$ and $Tr_{x'}$ through (3.18). $Tl_{x'}$ is the gradient of the left segment to X . $Tr_{x'}$ is the gradient of the right segment to X .
3. If $|Tr_{x'}| > |R_{x'}|$, $x = x + 1$ and go to Step 2;
 If $|Tl_{x'}| \geq |R_{x'}| \geq |Tr_{x'}|$, the optimal point is X . Go to Step 4.
 If $|R_{x'}| > |Tl_{x'}|$, the optimal point is at the point where $|\partial R / \partial p| = |Tl_{x'}|$.
 The p coordinate of this point can be obtained through (3.20). Go to Step 4.
4. Stop.

Computational complexity analysis

Since the number of iterations of the algorithm OPS only depends on the size of x , the computational complexity of this algorithm is $O(n)$.

An example of algorithm OPS

In this example, it is assumed that 4 orders in a sequence to be produced. It is assumed that the revenue function is $R = -(p - 1)^2 + 4$ after initial calculation. The tardiness penalty function ($-T$) is listed as shown in Table 3.3.

Table 3.3: An example of tardiness penalty function

No. of on-time orders	0	1	2	3	4
Constraint	$0 \leq p \leq 0.2$	$0.2 \leq p \leq 1.3$	$1.3 \leq p \leq 2.8$	$2.8 \leq p \leq 15.05$	$p \geq 15.05$
Function	$T = 7p - 9$	$T = 4p - 8.4$	$T = 0.5p - 3.85$	$T = 0.2p - 3.01$	$T = 0$

In the first step of the algorithm, the conjunction point set, X is $\{0, 0.2, 1.3, 2.8, 15.05\}$; p_0^t is equal to 1. $p = 1.3$ is selected because it is the first one larger than p_0^t . The number of on time orders, x is equal to 1.

In Step 2, $R_{-x'}$ is equal to -0.6 when $p = 1.3$. The two gradients of piece-wise lines are $Tl_{-x'} = 4$ and $Tr_{-x'} = 0.5$.

In Step 3, it is true that $|Tl_{-x'}| \geq |R_{-x'}| \geq |Tr_{-x'}|$. The optimal point is at $p = 1.3$ with 2 on time orders. The algorithm ends.

So far, the solution for Fixed Time Problem is based on the overall price domain. However, the overall price domain is divided into price zones. Assuming that there is a price zone, $p_c \leq p^* \leq p_{c+1}$, the value of p^* is compared with the boundaries of the price zone. If the value of p^* is located in the price zone, the optimal price quotation is p^* . If the value of p^* is located on the left side of the price zone ($p^* < p_c$), the optimal delivery time quotation is p_c , because the profit function is monotonically decreasing when $p > p^*$ as shown in Figure 3.10. If the value of p^* is located on the right side of the price zone ($p^* > p_{c+1}$), the optimal price quotation is p_{c+1} , because the profit function is monotonically increasing when $p < p^*$. As a result, the optimal delivery time quotation of this problem can be three options listed below,

$$p = \begin{cases} p_{c+1}, & p_{c+1} < p^* \\ p^*, & p_c \leq p^* \leq p_{c+1} \\ p_c, & p_c > p^* \end{cases} \quad (3.21)$$

$$\text{where } p^* = \begin{cases} \frac{\sum_{i=1}^m \alpha_i - t \sum_{i=1}^m \theta_i}{2 \sum_{i=1}^m \beta_i}, & \frac{\partial R}{\partial p} \neq \frac{\partial T}{\partial p} \\ \frac{\sum_{i=1}^n \alpha_i - t + w \sum_{i=m+1}^n \sum_{j=1}^i \beta_j}{2 \sum_{i=1}^n \beta_i}, & \frac{\partial R}{\partial p} = \frac{\partial T}{\partial p} \end{cases} \quad \square$$

Till now, the optimal delivery time quotation within one specific price zone is analytically solved in Fixed Time Problem. The optimal solution can be obtained by iterating all price zones. However, it is not shown in the thesis. The purpose of studying this special case is to find the properties of the optimal solution for the

general problem. In the following section, the general problem when price and delivery time are both decision variables is discussed.

3.3.2.3 Decisions of Price and Delivery Time Quotation in General Problem

In Section 3.3.1.3, it is known that for every price-time zone of (p, t) there is a fixed SPT production sequence. All the price-time zones with corresponding production sequences are obtained by the algorithm SLA. The solution for the general problem can be investigated in each price-time zone, so that the scheduling decision is trivial. Therefore, the whole problem can be decomposed into simplified problems according to a number of price-time zones.

According to the algorithm SLA, the price-time zone is a convex region whose edges are formed by two types of straight lines, such as EF and P_1T_1 illustrated in Figure 3.5. The mathematical formulation of the price-time zone has been formulated in a general form in (3.10). When the general problem is investigated in a specific price-time zone, constraints in (3.10) must be satisfied.

Proposition 3.4 *The optimal solution must satisfy one of the following two conditions:*

1) the delivery time quotation coincides with an order completion time; 2) the combination, (p^, t^*) is at the sequence changing point, which means that (p^*, t^*) makes either processing times of two orders equal or one order processing time is reduced to 0.*

Proof As it has been proven that the profit function $f(p, t)$ is concave with respect to each variable p or t (Lemma 3.1 and Lemma 3.2). Hence it is true that the optimal solution for the general case must satisfy Lemma 3.1 and Lemma 3.2 simultaneously. There are two common conditions which are shared between Lemma 3.1 and Lemma 3.2. They are, 1) the delivery time quotation coincides with an order completion time; 2) the optimal solution makes either processing times of two orders equal, or one order

processing time is just equal to 0. As a result, one of these two conditions must be satisfied when the profit achieves maximum. $\square$

It is now ready to discuss how to find the optimal solution under the two situations outlined in Proposition 3.4. Firstly, it is assumed that the profit is maximized when the delivery time quotation coincides with an order completion time, which is the first condition of Proposition 3.4. The decision variables (p, t) satisfy the equation (3.19) where m is an integer in $[1, 2, \dots, n]$. Each possible value for m is enumerated and substituted in (3.19) to form a straight line $L_m: p = \frac{\sum_{i=1}^m \alpha_i - (1 + \sum_{i=1}^m \theta_i)t}{\sum_{i=1}^m \beta_i}$ in p - t coordinate system. If there are 2 intersection points between L_m and edges of the price-time zone, the current value of m is a valid candidate and saved in the set M ,

$$M = [m, m+1, \dots, m+c]$$

where c is an integer value.

Then, each element in M is enumerated to find the optimal profit. When (3.19) is substituted into (3.10), all the constraints of the price-time zone can be simplified with only one decision variable t , such that, all the inequalities in (3.10) can be integrated into one inequality with the format of $t_{l1} \leq t \leq t_{r1}$, where t_{l1} and t_{r1} are determined during the procedure of the inequality integration. After (3.19) is substituted into the objective function (3.2), the objective function is changed to a quadratic function with only one variable t , that is,

$$f(t) = \frac{\sum_{i=1}^m \alpha_i - (1 + \sum_{i=1}^m \theta_i)t}{\sum_{i=1}^m \beta_i} \sum_{i=1}^n Q_i(t) - w \sum_{i=m+1}^n (C_i(t) - t) \quad (3.22)$$

This problem can be directly solved by selecting the maximum profit among $f(t_{l1})$, $f(t_{r1})$ and $f\left(t \left| \frac{\partial f}{\partial t} = 0 \right. \right)$. Solutions are obtained from all candidates in M . Then, the optimal solution for this price-time zone is the one with the maximum profit among all solutions.

The following study finds the optimal solution when (p^*, t^*) is at the sequence changing point according to the second condition in Proposition 3.4. As discussed in Section 3.3.1, the sequence changing point of a price-time zone is the points on edges of this zone in p - t coordinate system. Each edge is represented by one equation in (3.10). In general, an edge of a specific price-time zone can be represented by the equation, $c_{j1} \cdot p + c_{j2} \cdot t = c_{j3}$, where $j = [1, 2, \dots, s]$. Hence, the decision variable p is a function of the decision variable t , that is

$$p = (c_{j3} - c_{j2}t)/c_{j1} \quad (3.23)$$

The equation (3.23) is substituted into the objective function (3.2). The objective function is formulated by a quadratic function in terms of the delivery time quotation, t , only.

$$f(t) = \left(\frac{c_{j3} - c_{j2}t}{c_{j1}} \right) \sum_{i=1}^n (\alpha_i - (c_{j3} - c_{j2}t)\beta_i/c_{j1} - t \cdot \theta_i) - w \sum_{i=1}^n \max(0, C_i - t) \quad (3.24)$$

When (3.23) is substituted into (3.10), all the constraints of the price-time zone can be simplified with only one decision variable t , such that, all the inequalities in (3.10) can be integrated into one inequality with the format of $t_{l2} \leq t \leq t_{r2}$, where t_{l2} and t_{r2} are determined during the procedure of the inequality integration. As a result, this problem can be directly solved by selecting the maximum profit among $f(t_{l2})$, $f(t_{r2})$ and $f\left(t \left| \frac{\partial f}{\partial t} = 0 \right. \right)$. Solutions are found on all edges of the price-time zone. Then, the optimal solution for this price-time zone is the one with the maximum profit among all solutions.

Both optimization problems (3.22) and (3.24) are analytically solvable in one specific price-time zone. The optimal solution with larger profit for one specific price-time zone is selected between solutions of (3.22) and (3.24). An algorithm PTA is developed for finding optimal delivery time quotation in the general problem as shown

below. The main idea of this algorithm is as follows: firstly, the all price-time zones is formed with the corresponding SPT sequences; secondly, the two optimal solutions are obtained in each price-time zone through (3.22) and (3.24); finally, the optimal solution is the one which has the maximum profit among solutions obtained in all price-time zones.

Algorithm PTA (Price-Time problem Algorithm)

Input

Given $\alpha_i, \beta_i, \theta_i, p$, Price-time Zone List $ZL=\{Z_j\}$, Sequence List $SL=\{\sigma_j\}$, $i = 1, \dots, n; j = 0, \dots, n_1$.

Initialization

Set the optimal delivery time quotation $t = 0$, the optimal price quotation $p = 0$ and the optimal profit $f = 0$; Set initial $j = 0$.

1. Select Price-time zone Z_j from ZL and corresponding optimal production sequence σ_j from SL .
2. Generate the set M . For each element in M , form the optimization problem according to (3.22) and obtain results, t_m, p_m and f_m .
3. $f_1 = \max(f_m), p_1 = p_m | \max(f_m), t_1 = t_m | \max(f_m)$.
4. If $f_1 > f, f = f_1, p = p_1$ and $t = t_1$.
5. For each edge of Z_j , form the optimization problem according to (3.24) and obtain the results, t_c, p_c and f_c .
6. $f_2 = \max(f_c), p_2 = p_c | \max(f_c), t_2 = t_c | \max(f_c)$.
7. If $f_2 > f, f = f_2, p = p_2$ and $t = t_2$.
8. $j = j+1$; if $j > n_1$, go to step 9; else, go to step 1.
9. Stop.

A simple example of Algorithm PTA

In this example, it is assumed that a price-time zone Z is selected from ZL . Z is shown in Figure 3.12. The information of four edges of Z , l_1 , l_2 , l_3 and l_4 are known. It is assumed that there are 4 orders with a given sequence (1->2->3->4).

Figure 3.12: An example of algorithm PTA

In Step 2, the set M is generated, which is initially empty. The number of on time orders, m is iterated from 1 to 4. When $m = 1, 2, 3$ or 4, it is find out that whether Z is intersected by the straight line $t = \sum_{i=1}^m \alpha_i - \sum_{i=1}^m \theta_i t - p \sum_{i=1}^m \beta_i$. If there is intersection, the current m is saved in the set of M .

After M has been generated, the profit as well as the solution are calculated through (3.22) for each element of M .

In Step 3 and 4, the optimal solution is selected through all the solutions obtained in Step 2.

In Step 5, the profit as well as the solution are calculated through (3.24) when the point (p^*, t^*) is on l_1 , l_2 , l_3 or l_4 . Thus, 4 solutions have been obtained.

In Step 6 and 7, the maximum value among the solutions obtained in Step 3 and Step 4 is selected as the optimal solution in Z .

In Step 8, the algorithm will start to find the optimal solution in another price-time zone.

After all the price-time zones are visited, the algorithm stops. $\square$

Proposition 3.5 *Algorithm PTA finds an optimal solution (p^*, t^*) for the general problem in $O(n^3)$ time.*

Proof In the Initialization step, all price-time zones are generated. The amount of price-time zones depends on the number of customers, n . The optimality of Algorithm PTA is obtained by comparing the net profits among all price-time zones. Hence, the number of possible values of the net profit f_j is the same as the price-time zones. According to Figure 3.5, the amount of price-time zones is represented by the amount of simple polygon in Quadrant I. It is well known that N straight lines divide the plane into at most $\frac{N^2+N+2}{2}$ simple polygons (Buck 1943). From Figure 3.5, there are 3 lines (equal to the number of customers) and another 2 lines (equal to the number of intersections between any two customers). There are at most $(n*(n-1))/2$ intersections between any two customers. As a result, there are at most $(n^2+n)/2$ straight lines in this problem. The plane is divided into at most $(n^4+2n^3+3n^2+2n+8)/8$ simple polygons.

In each price-time zone, the algorithm iterates at Step 2 then Step 5, respectively. The number of iterations at Step 2 depends on the size of the set M . It is known that M records the number of on-time orders. Thus, the size of M is at most n . The number of iterations at Step 5 depends on the number of edges of a simple polygon, which is also at most n . Thus, in each price-time zone the computation time is $O(n)$. Therefore, the overall computation time of Algorithm PTA is $O(n^5)$. $\square$

3.4 Summary

In this chapter, a profit-maximizing firm selling a product to customers with a common price and delivery time quotation was studied. The primary objective was to explore the optimal solution of simultaneous price and delivery time quotation. Firstly the optimal production sequence for this problem was examined. It was shown that

SPT rule can generate the optimal production sequence. The production sequence was taken into account through the constraints of quoted price and quoted delivery time. Two sub-problems were investigated while fixing one decision variable, price or delivery time. The properties of the optimal solution for each sub-problem were developed and presented. From the results achieved in these two sub-problems, an algorithm was developed to find exact optimal solution for the general problem. The computational complexity of the algorithm was discussed at the end of the chapter.

Chapter 4

Numerical Study of Common Price and Delivery Time Model

This chapter presents a numerical example of the model in Chapter 3. It illustrates how the algorithm can be used to analyze and make decisions optimally on quoted price and delivery time.

4.1 Preliminaries

The following parameters are chosen for design of experiment: (1) number of customers (n), (2) potential market size of each customer (α_i), and (3) price sensitivity (β_i) and time sensitivity (θ_i). The values of these parameters are based on Boyaci and Ray (2006). The settings of parameters are as follows:

1. The number of customers $n \in \{4, 8, 12, 16\}$;
2. The potential market size α_i for each customer is generated from $U[500, 700]$ or $U[800, 1000]$;
3. Price sensitivity β_i and time sensitivity θ_i are generated as $\beta_i \sim U[1, 3]$ and $\theta_i \sim U[0.3, 0.5]$; or as $\beta_i \sim U[3, 5]$ and $\theta_i \sim U[0.1, 0.3]$;
4. The weight of tardiness penalty, $w = 100$ for all customer orders, which is constant.

Table 4.1: Comparison of optimal solutions

n	α_i	β_i	θ_i	p^*		t^*		Profit ($\times 10^4$)		No. of Orders Accepted	
				Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev
4	[500,700]	[1,3]	[0.3,0.5]	195.34	60.63	353.89	111.84	7.99	1.96	3.36	0.85
		[3,5]	[0.1,0.3]	79.30	10.27	665.04	74.20	5.25	0.73	3.94	0.24
	[800-1000]	[1,3]	[0.3,0.5]	317.49	83.38	324.35	152.97	19.66	4.80	3.40	0.70
		[3,5]	[0.1,0.3]	116.52	13.50	978.39	136.11	11.23	1.44	3.90	0.30
8	[500,700]	[1,3]	[0.3,0.5]	217.06	44.68	363.84	77.47	10.32	1.84	4.94	1.25
		[3,5]	[0.1,0.3]	76.71	6.03	898.26	62.40	6.89	0.72	7.26	0.78
	[800-1000]	[1,3]	[0.3,0.5]	348.51	69.41	358.54	168.26	25.39	4.66	5.30	1.25
		[3,5]	[0.1,0.3]	115.02	7.09	1356.48	79.46	15.61	1.38	7.42	0.67
12	[500,700]	[1,3]	[0.3,0.5]	226.78	39.93	407.01	77.72	11.81	1.80	6.04	1.67
		[3,5]	[0.1,0.3]	78.07	3.97	1063.68	72.40	8.30	0.68	9.76	1.27
	[800-1000]	[1,3]	[0.3,0.5]	399.72	60.69	326.06	172.41	30.66	4.70	5.94	1.48
		[3,5]	[0.1,0.3]	116.61	6.13	1571.35	105.65	18.30	1.25	10.06	1.11
16	[500,700]	[1,3]	[0.3,0.5]	239.24	37.50	420.57	69.74	12.88	1.90	6.54	1.68
		[3,5]	[0.1,0.3]	79.00	4.71	1131.06	85.09	8.93	0.86	11.62	1.35
	[800-1000]	[1,3]	[0.3,0.5]	412.99	80.66	370.92	214.92	33.45	5.84	6.96	1.48
		[3,5]	[0.1,0.3]	119.04	5.98	1745.31	127.26	20.77	1.74	11.82	1.41

4.2 Results Analysis with the identical weight

Table 4.1 shows computational results of optimal solutions by PTA algorithm. Each row shows the results over 50 randomly generated instances. The columns “Mean” and “Stdev” represent the mean and standard deviation of the results, respectively. Instances with $\beta_i \sim U[1, 3]$ and $\theta_i \sim U[0.3, 0.5]$ is defined as “time and price sensitive” (TPS) market, where the market is more sensitive to delivery time. For instances with $\beta_i \sim U[3, 5]$ and $\theta_i \sim U[0.1, 0.3]$, the market is more sensitive to price, which will be referred as “price and time sensitive” (PTS) market. For an instance when $n = 16$, the PTA algorithm takes about 20 CPU seconds to get the optimal solution on a 3.00 GHz personal computer. When $n = 8$ or 4, the PTA algorithm costs less than 1 CPU second to find the optimal solution.

4.2.1 Comparison of PTS market and TPS market

In this section, the profit obtained in PTS market and TPS market is compared at the same settings to discover the applicability of this solution in different market scenarios. The impacts of n and α_i on the profit are discussed, respectively.

In Figure 4.1 and 4.2, it is observed that TPS market always contributes more profit compared to PTS market when n and α_i are at the same settings. This is because customers in TPS market would like to pay more to compete for shorter completion time, when capacity is constrained. The standard deviation of profit is much higher in TPS market than that in PTS market. For example, in Figure 4.2, when $n = 8$, the standard deviation of the profit is 4.66 in TPS, while it is 1.38 in PTS. This indicates that the PTA algorithm is more consistent in PTS market than in TPS market. As a result, it is indicated that the common price and delivery strategy is not suitable when demand is more sensitive to time. There may be two reasons. First, a common delivery

time is not suitable for time sensitive customers while there is no earliness penalty incurred. Second, tardiness weight is identical for different customers. It is possible that the manufacturer uses an unreliable quoted delivery time to attract more demand from time sensitive customer. This phenomenon will be discussed in detail in Chapter 4.3.

Figure 4.1: Comparison of Profits when $\alpha_i \sim U[500, 700]$

Figure 4.2: Comparison of Profits when $\alpha_i \sim U[800, 1000]$

Figure 4.3: Comparison of Mean Profits among TPS/PTS and α_i

In Figure 4.3, it is observed that the profit is significantly increased as the potential market size (α_i) is increased, while the profit is slowly increased as the number of customers (n) in the market is increased. As a result, the effect of n on promoting profit is not as significant as the potential market size (α_i). For example, the average profit is increased from 5.25 to 8.93 when n is increased from 4 to 16 on PTS-Low alpha line, as the profit is increased from 5.25 to 11.23 when α_i is increased from [500, 700] to [800, 1000] in the case ($n = 4$, PTS). This may be because when orders are merged, potential market size is increased. Meanwhile, the price sensitivities are pooled. These two factors give the opportunity of price increment. When the number of customers increases, the capacity is unchanged. The manufacturer will select which customer to be served. As customers are from the same group and have similar sensitivities to price and time, it is hard to force some customers to drop out (order quantity decreases to 0) simply by increasing price. The order quantity from each customer is decreased. The profit earned in this situation is almost similar as when the number of customers is not changed. Therefore, in order to increase the demand, it is

suggested that the manufacturer to focus on how to increase the potential market size rather than the number of customers.

4.2.2 Impact on price and delivery time quotation

In this section, the effect of market characteristics on price quotation and delivery time quotation is discussed. The purpose is to see how the solution is affected by the market characteristics.

In Figure 4.4, it is observed that the coefficient of variation (CV) of price quotation are significant larger in TPS market than that in PTS market. In Figure 4.5, it is shown that the CV of delivery time quotation are significantly larger in TPS market than that in PTS market. These two facts are consistent with the conclusion that the PTA algorithm results are more consistent in meeting the quoted delivery time in PTS market than in TPS market. It is because small change in TPS market may result in a dramatic change in optimal price quotation and optimal time quotation. Thus the common price and delivery time quotation is not suitable under TPS market, especially when the market is not stable.

Figure 4.4: CV of quoted prices among different settings

Figure 4.5: CV of quoted delivery times among different settings

In Figure 4.6, it is shown that the average delivery time quotation in PTS market is significantly larger than that in TPS market. In Figure 4.7, the average price quotation in PTS market is significantly less than that in TPS market. This is consistent with real practice.

Figure 4.6: Comparison of quoted price among different settings

Figure 4.7: Comparison of quoted delivery times among different settings

In Figure 4.7, it is found that the average delivery time quotation is increasing in PTS market when n is increased. However, in TPS market the delivery time quotation is non-increasing when n is increased. The reason is that, the time sensitivity in TPS market is larger than that in PTS market. The weight of the tardiness penalty is identical. It is possible that the manufacturer can earn profit by quoting unreliable but short delivery time. In this situation, time quotation is not as reliable as that in PTS market. Hence it is suggested that it is not appropriate to apply common delivery time and price quotations in TPS market when the number of customers is relatively large. Meanwhile, the weight of the tardiness should be re-estimated and may be consistent to the time sensitivity.

4.2.3 Impact on order acceptance ratio

In this section, the number of accepted orders is examined in the experiment. The acceptance ratio is defined by the number of accepted orders divided by the total number of orders in the system. It helps to find out what proportion of the customers is accepted with the constraint capacity and whether it is affected by market characteristics.

Figure 4.8 and Figure 4.9 shows Mean and Stdev of acceptance ratio in two settings of α_i . It is shown that 84% to 97% orders have been accepted when the total number of customer in the market (n) is 4. The percentage of the accepted orders is reduced to 40% to 74% when n is 16. This is because that the capacity is assumed constant, regardless of customer quantity in all the test instances. The number of accepted orders in PTS market is always larger than that in TPS market according to Figure 4.8 and Figure 4.9. This means that more orders can be accepted because the customers have more tolerance on the delivery time quotation. The difference of deviations between the PTS market and the TPS market is not significant as that in profit, price quotation and delivery time quotation. This means that the proportion of the most valuable customers to be processed is stable.

Figure 4.8: Comparison of acceptance ratios ($\alpha_i \sim U[500, 700]$)

Figure 4.9: Comparison of acceptance ratios ($\alpha_i \sim U[800, 1000]$)

4.3 Results Analysis under the different tardiness weights

In this section, experiments are conducted at different values of tardiness weight to validate the results which are concluded in section 4.2 when the tardiness weight has been changed. The following results are examined:

1. Common price and delivery time quotation is more applicable (consistent) in the PTS market regardless of the tardiness weight;
2. Profit fluctuation is more significant in TPS market compared to that in PTS market regardless of the tardiness weight;
3. The delivery time quotation in TPS market is not as reliable as that in the PTS market when the same tardiness weight is given;
4. The proportion of the accepted customers is stable regardless of the tardiness weight.

In the following experiments, the instances with different tardiness weights will be studied under the identical number of customers and the identical range for potential market size. Thus, the number of customers, n is fixed at 12. The potential market size for each customer, α_i is generated from $U[500, 700]$. The tardiness weight, w is set at three different values [50, 100, 150]. Similar to the experiments in section 4.2, $\beta_i \sim U[1, 3]$ and $\theta_i \sim U[0.3, 0.5]$ is defined as “time and price sensitive” (TPS) market, where the market is more sensitive to delivery time. $\beta_i \sim U[3, 5]$ and $\theta_i \sim U[0.1, 0.3]$ is referred as “price and time sensitive” (PTS) market, where the market is more sensitive to price.

Totally, there are 6 settings of parameters as shown in Table 4.2. In this table, it shows computational results of optimal solutions at different settings. Each row shows

the results over 50 randomly generated instances. The columns “Mean” and “Stdev” represent the mean and standard deviation of the results, respectively.

Table 4.2: Optimal solutions at different weights

w	β_i	θ_i	p^*		t^*		Profit ($\times 10^4$)		No. of Orders		Accepted	
			Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean	Stdev	Mean
50	[1,3]	[0.3,0.5]	41.24	267.88	118.29	112.38	3.15	15.59	1.64	2.30	1.05	6.86
	[3,5]	[0.1,0.3]	6.41	83.67	127.52	928.84	0.75	8.41	0.94	9.04	0.96	9.82
100	[1,3]	[0.3,0.5]	39.93	226.78	77.72	407.01	1.80	11.81	1.73	4.94	1.67	6.04
	[3,5]	[0.1,0.3]	3.97	78.08	72.40	1063.68	0.68	8.30	1.27	9.76	1.27	9.76
200	[1,3]	[0.3,0.5]	44.52	220.20	88.00	488.65	1.83	11.50	1.58	5.62	1.47	6.12
	[3,5]	[0.1,0.3]	3.85	79.24	86.46	1069.39	0.80	8.47	1.27	9.48	1.27	9.48

Figure 4.10 shows the coefficient of variation (CV) of both the quoted delivery time and profit at different settings of the tardiness weight. It is observed that CV of the quoted delivery time in TPS is significantly larger than that in PTS at all setting of w . This is because the common price and delivery time quotation approach is more suitable or consistent when customers are more sensitive to price and less sensitive to delivery time. The CV of the obtained profit has the similar characteristics. It indicates that profit fluctuation in TPS is more significant compared to that in PTS regardless of the tardiness weight.

Figure 4.10: CV of quoted delivery time and profit among different weights

In Figure 4.11, the number of accepted customers is shown among different values of the tardiness weight. It is observed that the mean value and the standard deviation of the number of accepted customers are almost stable as the tardiness weight varies. In other words, the acceptance ratio is stable regardless of the tardiness weight, while its standard deviation is small and almost unchanged. It is also shown that the acceptance ratio in PTS is larger than that TPS, which is consistent with the result in section 4.2.3.

Figure 4.11: No. of accepted customers among different weights

The relationship between the number of on-time orders and the tardiness weight is shown in Figure 4.12. Through Figure 4.11 and Figure 4.12, it can be observed that almost all orders are on time in PTS when the weight is larger than 100. Almost all orders are on time in TPS when the weight is at 150. This is because the company can afford higher tardiness penalty in TPS than in PTS. The number of on-time orders is very less in TPS when w is at 50. The reason is w is under-weighted in this circumstance. The company can earn profit by quoting an unreliable delivery time and incurring tardiness penalty. This circumstance is more significant in TPS than in PTS because the company can afford higher tardiness penalty in TPS.

Figure 4.12: No. of on-time orders among different weights

4.4 Managerial Implications

The main contribution of the chapter is to the derivation of managerial insights through numerical studies. The main managerial insights are given as follows.

1. Common price and delivery time quotation is more applicable in the PTS market.
2. Although higher profit can be achieved in the TPS market, there is a high risk of profit fluctuation. The delivery time quotation in this market is not as reliable as that in the PTS market.
3. If the manager wants to increase the demand, it is more profitable to increase the potential market size than to increase the number of customers in the market. In the other words, it is profitable if customer orders can be merged into single customer order.
4. Under the same capacity, the number of accepted orders in the PTS market is larger than that in the TPS market.
5. If customers are too time-sensitive and less price-sensitive, there may be a chance that the manufacturer earns profit by quoting unreliable delivery time quotation.

Chapter 5

Distinct Prices and Delivery Time Quotations Problem

In the previous chapter, the common price and delivery time quotation problem is studied where customers share the same price quotation and delivery time quotation. In this chapter, the distinct prices and delivery time quotations problem where each customer has a unique price quotation and delivery time quotation is analyzed. It is assumed that the price quotations are predetermined. The firm's objective is to maximize the profit by quoting different delivery times for all customers. A mathematical model is developed. In this section, the problem is proven to be NP-complete. Both a MIP formulation and the branch-and-bound algorithm are built to find the optimal solution. Also, a heuristic algorithm is developed to obtain the near-optimal solution within a fraction of the time required to obtain the optimal solution.

5.1 Introduction

In the previous chapter, the coordination problem is investigated in the dedicated capacity model with common price and delivery time quotation. In that model,

customers are categorized into different customer groups. Each customer group is processed in a dedicated production facility.

In this chapter, the coordination problem of price and delivery time is studied in the shared capacity model. The slack capacity in dedicated facilities is minimized in the shared capacity model, because all orders are processed in a single production facility without being categorized and separated into groups. Each order from a customer will be given a specific delivery time quotation as well as a price quotation. It is often the case in many industrial applications, that the firm offers the same product or service, differentiated only in terms of the delivery times offered and the prices charged (Boyaci and Ray 2003). This strategy is an effective way to maintain customer responsiveness and to enhance demand. Examples of such product/service differentiation strategy are abundant. Printing companies (as shown in Figure 3.1) offer 5 days express service and a cheap 10-day regular service among a lot of options. In the transportation sector, transportation companies create a quotation table with different combinations of time and price even within a particular delivery mode (ground-delivery, sea-delivery or air-delivery). For example, there are different “time-definite” fast delivery options (1-day delivery, 2-day delivery, 3-day delivery) regarding to customers’ preference.

The delivery time quotation for an order is reliable, meaning that the quoted delivery time will be not less than the completion time of this order. This means that the tardiness weight is large enough that the manufacturer cannot make money by cheating customers with an unreal short delivery time quotation.

With the objective of maximizing the profit, a price and delivery time quotation problem is studied in a single machine system to find the optimal production sequence in this chapter. In this problem, it is assumed that an order’s delivery time quotation is

set just at its production completion time. Hence there is no earliness or tardiness penalty in this problem. The delivery time quotation is decided by the production sequence directly. The customer's demand (or order size) is a linear function with respect to its price and delivery time quotation.

Without the consideration of price, this problem is related to the due-date quotation literature. Researchers have introduced a variety of models in an attempt to investigate this subject (Kaminsky and Hochbaum 2004; Hopp and Sturgis 2000; Gordon et al. 2002). These models assume that demand is independent of lead time. The problem, when demand is sensitive to time quotation or other factors, has been addressed by some other researchers (Duenyas and Hopp 1995; Chatterjee et al. 2002; Keskinocak et al. 2001). To the best of our knowledge, this research is the first one to study due-date quotation problem in which demand is dependent on both price and due date quoted.

Since the firm is formulated as a single machine with a constant production rate, this problem is similar to the single machine scheduling problem with time-dependent processing times (e.g., Alidaee and Womer 1999, Cheng et al. 2005). The details on how to formulate this problem is illustrated in Section 5.2. This research is the first one to study optimal solutions for the total weighted completion time problem with learning effect.

Motivated by the above discussions, the delivery time quotation with price consideration is presented in a make-to-order environment. It is assumed that different price quotations for all customers are known at the beginning of each planning period. Similar assumptions are also made in existing papers (Charnsirisakskul et al. 2006, Chen and Hall 2010). Then, customer orders are scheduled for production. The delivery time quotations are decided by the production schedule. It is also assumed

that each order cannot be split in production and delivery. Furthermore, each order is delivered immediately upon completion of production. This is common in make-to-order situations such as consumer electronics and fashion items.

5.2 Problem formulation

There is a set of customers, indexed by $i = 1, \dots, n$, placing orders to a single firm for an identical product. The manufacturer will quote distinct prices and distinct delivery times for different customers. Each customer will place a single order with an order quantity. The firm will deliver each order to its customer when the order is completed in production. The transportation time is not taken into account in this model. It is assumed that the processing time of per unit product is fixed and known in the model. The capacity cost is also fixed and known.

To reflect customer sensitivity to price (p_i) and delivery time quotation (t_i), the demand is assumed as the following linear function, which represents the quantity of a customer order

$$Q_i = \alpha_i - \beta_i \cdot p_i - \theta_i \cdot t_i \quad (5.1)$$

where α_i , β_i and θ_i are positive constants. α_i represents the potential market size, β_i represents the price sensitivity and θ_i represents the delivery time sensitivity. To differentiate the customers, combinations of (β_i, θ_i) are used to describe the sensitivities of customers. It is assumed that an order must be produced and delivered without splitting. This restriction is reasonable when customers prefer to receive an order all at one time, especially since delivery times are guaranteed. The linear demand function helps us to obtain quantitative insights without much analytical complexity. It also has the desirable properties of price and delivery time elasticity of demand.

The firm is assumed to be a single machine (an assumption similar to that made by So and Song 1998; Zhao et al. 2006; Celik and Maglaras 2008). Production rate is denoted by c product units / unit time, which is fixed. Hence the processing time of order i can be described as $Y_i = Q_i/c$. The release time of all orders is 0. The single machine can only produce one order at a time. The process starting time for order i is represented by S_i . It is assumed that, there is no earliness and tardiness allowed in this problem. In the other words, the delivery time guarantee for order i is equivalent to its completion time. Hence,

$$S_i + Y_i = t_i \quad (5.2)$$

The processing time for order i can be derived from (5.1), (5.2) and $Y_i = Q_i/c$,

$$Y_i = \frac{\alpha_i - \beta_i p_i}{c + \theta_i} - \frac{\theta_i}{c + \theta_i} S_i \quad (5.3)$$

Assume that, $X_i = \frac{\alpha_i - \beta_i p_i}{c + \theta_i}$ and $v_i = \frac{\theta_i}{c + \theta_i}$, (5.3) can be simplified as follows,

$$Y_i = X_i - v_i S_i$$

In this problem, it is assumed that no order will be rejected by the firm. In other words, the processing time of order i is positive in any production sequence π . Hence it is assumed that $\frac{\theta_i}{c + \theta_i} \left(\sum_{j=1}^n \frac{\alpha_j - \beta_j p_j}{c + \theta_j} - \frac{\alpha_i - \beta_i p_i}{c + \theta_i} \right) < \frac{\alpha_i - \beta_i p_i}{c + \theta_i}$ (see also Ho et al. (1993) for detailed explanation).

The objective is to find a production sequence π to maximize the profit for the firm achieved by selling all orders, which is illustrated as follows,

$$\max f = \sum_{i=1}^n p_i Q_i = c \sum_{i=1}^n p_i Y_i$$

Proposition 5.1 *This problem is a NP-complete problem even when p_i are fixed parameters.*

Proof It is assumed that price for each order i is known. The objective is to find a schedule π for which the value of the total weighted processing time is maximized,

$$TWPT = \sum_{i=1}^n p_i Y_i = \sum_{i=1}^n p_i (X_i - v_i S_i) \quad (5.4)$$

Finally, this problem is derived to a single machine scheduling problem, in which the processing time for each order is linearly decreasing with respect to its starting time of processing. The objective of this problem is to maximize the total weighted processing time.

$$\max \sum_{i=1}^n p_i Y_i = \max \sum_{i=1}^n p_i (X_i - v_i S_i) = \max \{ \sum_{i=1}^n p_i X_i - \sum_{i=1}^n p_i v_i S_i \}$$

It is known that $S_i = C_{i-1}$, where C_{i-1} represents the completion time of order $i-1$, and $\sum_{i=1}^n p_i X_i$ is a constant. The objective function is derived to $\min \sum_{i=1}^n p_i v_i C_{i-1}$, where $C_0 = 0$.

Since p_i, v_i are both a series of independent parameters, a new variable, w_i , is constructed, which satisfies $w_i = p_i v_i$. Finally, the objective function is formulated into $\min \sum_{i=1}^n w_i C_i$. It is equivalent to the problem to minimize the total weighted completion time. It is proven that $1|p_i(S_i) = a_i - b_i S_i| \min \sum w_i C_i$ is NP-complete (Bachman et al. 2002). The problem $1|Y_i(S_i) = X_i - v_i S_i| \max \sum_{i=1}^n p_i Y_i$ is NP-complete. $\square$

Thus this problem is NP-complete even when prices are given parameters. Regarding the complexity of the general problem where both prices and delivery time quotations are decision variables, a common way to solve the general problem is firstly to study the situation when price decisions are given; secondly to search in the feasible price domain with results obtained in the first step. Therefore, it is essential to investigate the special case where price quotations are given, which can be further applied as a preliminary methodology to solve the general problem considering prices as the input factor. The results can be also commonly applied by others in this research field.

The following research in this chapter attempts to solve the problem with given price quotations is solved. Thus, $(\alpha_i - \beta_i p_i)$ is assumed to be predetermined.

5.3 Heuristic solution approach

In this section, a heuristic algorithm is developed for this problem. Firstly, the special case, in which all the prices are same, is studied. The objective function is equivalent to maximizing the total order quantity produced (or the total completion time) in the system. Then, a pair-wise exchange between any two consecutive orders is examined in a production sequence. The heuristic algorithm is developed based on these results obtained.

5.3.1 Properties and analysis

Proposition 5.2 *The production sequence π^* , in which orders are sorted in non-decreasing ratio of $\frac{X_i}{v_i}$, is optimal for the problem when all p_i are same, even without the constraint, $v_i(\sum_{j=1}^n X_j - X_i) < X_i$.*

Proof which is different from Browne and Yechiali (1990) is found in Appendix at the end of this chapter. $\square$

The methodology developed to prove Proposition 2 is able to model the problem in which orders can be rejected (or order quantity can decrease to 0 in a production sequence). The mathematical form of this constraint is

$$v_i \left(\sum_{j=1}^n X_j - X_i \right) < X_i$$

which is relaxed in Proposition 5.2.

Since the scheduling problem is a NP-complete problem, it is hard to derive a direct solution to form an optimal production sequence in this case. The sequence of any two consecutive orders is investigated firstly.

Proposition 5.3 *For two consecutive orders $(j, j+1)$, it is optimal to schedule order $j+1$ ahead of order j when both constraints are satisfied,*

$$\frac{X_j}{v_j} < \frac{X_{j+1}}{v_{j+1}} \text{ and } p_{j+1} \left(C_{j-1} - \frac{X_j}{v_j} \right) < p_j \left(C_{j-1} - \frac{X_{j+1}}{v_{j+1}} \right)$$

Proof In Proposition 5.2, it is proven that the maximum completion time C_n is maximized (minimized) when orders are sequenced by increasing (decreasing) values of $\frac{X_i}{v_i}$.

Assume that there is a permutation,

$$\pi_0 = (1, 2, \dots, j-1, j, j+1, \dots, n)$$

If the sequence of any two consecutive orders is changed in π_0 , it is obtained

$$\pi_1 = (1, 2, \dots, j-1, j+1, j, \dots, n)$$

The sequence of the first $i-1$ orders is unchanged. Let $C_j(\pi_0)$ and $Y_j(\pi_0)$ denote the completion time and the processing time of order j in π_0 , respectively.

$$Y_j(\pi_0) = X_j - v_j C_{j-1}$$

$$C_j(\pi_0) = C_{j-1} + Y_j(\pi_0) = X_j + (1 - v_j) C_{j-1}$$

$$Y_{j+1}(\pi_0) = X_{j+1} - v_{j+1} X_j - v_{j+1} (1 - v_j) C_{j-1}$$

$$C_{j+1}(\pi_0) = (1 - v_{j+1}) X_j + X_{j+1} + (1 - v_j) (1 - v_{j+1}) C_{j-1}$$

Similarly, in π_1

$$Y_j(\pi_1) = X_{j+1} - v_{j+1} C_{j-1}$$

$$C_j(\pi_1) = X_{j+1} + (1 - v_{j+1}) C_{j-1}$$

$$Y_{j+1}(\pi_1) = X_j - v_j X_{j+1} - v_j (1 - v_{j+1}) C_{j-1}$$

$$C_{j+1}(\pi_1) = (1 - v_j) X_{j+1} + X_j + (1 - v_j) (1 - v_{j+1}) C_{j-1}$$

When $\frac{x_j}{v_j} \leq \frac{x_{j+1}}{v_{j+1}}$, it is proven that $C_{j+1}(\pi_0) \geq C_{j+1}(\pi_1)$ through Proposition 5.2.

The processing times of orders $(j+2, j+3, \dots, n)$ are all increased in π_1 . The total profit of orders $(j+2, j+3, \dots, n)$ are increased in π_1 .

The difference between the values of $(p_j Y_j + p_{j+1} Y_{j+1})$ in these two permutations is

$$\begin{aligned} & (p_j Y_j + p_{j+1} Y_{j+1})_{\pi_1} - (p_j Y_j + p_{j+1} Y_{j+1})_{\pi_0} \\ &= p_j v_j (v_{j+1} C_{j-1} - X_{j+1}) - p_{j+1} v_{j+1} (v_j C_{j-1} - X_j) \end{aligned}$$

Substitute $X_i = \frac{\alpha_i - \beta_i p_i}{c + \theta_i}$ and $v_i = \frac{\theta_i}{c + \theta_i}$ into the above function

$$\begin{aligned} & (p_j Y_j + p_{j+1} Y_{j+1})_{\pi_1} - (p_j Y_j + p_{j+1} Y_{j+1})_{\pi_0} = \\ & \frac{\theta_j \theta_{j+1}}{(c + \theta_j)(c + \theta_{j+1})} \left[p_j \left(C_{j-1} - \frac{\alpha_{j+1} - \beta_{j+1} p_{j+1}}{\theta_{j+1}} \right) - p_{j+1} \left(C_{j-1} - \frac{\alpha_j - \beta_j p_j}{\theta_j} \right) \right] \end{aligned} \quad (5.5)$$

Therefore, if $p_j \left(C_{j-1} - \frac{x_{j+1}}{v_{j+1}} \right) > p_{j+1} \left(C_{j-1} - \frac{x_j}{v_j} \right)$, the total profit of π_1 is greater than the profit of π_0 . $\square$

The criterion in Proposition 5.3 is only a sufficient condition. It cannot guarantee to get the optimal sequence of any two consecutive orders. A more general rule for finding the optimal sequence of two consecutive orders is required.

Proposition 5.4 *There are two consecutive orders (i, j) with their positions $(s_{th}, (s+1)_{th})$ in a production sequence π_1 . By exchanging positions of (i, j) , a new production sequence π_2 is formed, where orders (j, i) are at positions $(s_{th}, (s+1)_{th})$. Let C_1 represents the completion time of $(s+1)_{th}$ order in π_1 ; C_2 represents the completion time of $(s+1)_{th}$ order in π_2 . The difference of C_1 and C_2 is Δt which is independent of s ,*

$$\Delta t = \frac{x_j v_i}{c + \theta_j} - \frac{x_i v_j}{c + \theta_i}$$

Proof It is assumed that the completion times of $(s+1)_{th}$ orders in π_1 and π_2 are both C_{s+1} . In production sequence π_1 , the following expressions can be derived,

$$C_s|\pi_1 = x_i + (1 - v_i)C_{s-1}$$

$$C_{s+1}|\pi_1 = x_j + (1 - v_j)(x_i + (1 - v_i)C_{s-1})$$

$$C_s|\pi_2 = x_j + (1 - v_j)C_{s-1}$$

$$C_{s+1}|\pi_2 = x_i + (1 - v_i)(x_j + (1 - v_j)C_{s-1})$$

After substituting $X_i = \frac{\alpha_i - \beta_i p_i}{c + \theta_i}$ and $v_i = \frac{\theta_i}{c + \theta_i}$, the difference of completion times

$$\text{of } (s+1)_{th} \text{ order in } \pi_1 \text{ and } \pi_2 \text{ is } \Delta t = C_{s+1}|\pi_1 - C_{s+1}|\pi_2 = \frac{x_j v_i}{c + \theta_j} - \frac{x_i v_j}{c + \theta_i} \quad \square$$

From Proposition 5.4, when the last two orders in a sequence exchange their positions, the difference of the total completion time of this sequence can be calculated. Then, a given partial production sequence is studied when the starting time of the first order in this partial sequence is changed. Assume that the part of the production sequence π is $\pi_e = (i, i+1, i+2, \dots, n)$. The starting time of order i is S_0 . The processing times for each order in π_e is

$$Y_i = X_i - v_i S_0$$

$$Y_{i+1} = X_{i+1} - v_{i+1} X_i - v_{i+1} (1 - v_i) S_0$$

...

$$\begin{aligned} Y_n &= X_n - v_n X_{n-1} - v_n (1 - v_{n-1}) X_{n-2} - \dots - v_n (1 - v_{n-1}) \dots (1 - v_{i+1}) X_i \\ &\quad - v_n (1 - v_{n-1}) \dots (1 - v_i) S_0 \end{aligned}$$

If S_0 is decreased by Δt , the profit of π_e is increased by

$$\Delta f_{\pi_e} = \Delta t (p_i v_i + \sum_{j=i+1}^n p_j v_j \prod_{k=i}^{j-1} (1 - v_k)) \quad (5.6)$$

Figure 5.1: Illustration of exchanging two consecutive orders in a sequence

If there is a production sequence $\pi = (1, 2, \dots, j-1, j, j+1, \dots, n)$, a new production sequence $\pi' = (1, 2, \dots, j, j-1, j+1, \dots, n)$ is obtained by exchanging order $(j-1)$ and j as shown in Figure 5.1. Both production sequences can be divided into three parts for analysis, which is $(1, 2, \dots, j-2)$, $[j-1, j]$ and $(j+1, \dots, n)$. It is known that Part I of the sequence, $(1, 2, \dots, j-2)$, is not changed. The profit contributed by this part is unchanged. The profit increment of Part II, order $(j-1)$ and j , which may be negative, is obtained by (5.5). Meanwhile, the difference of the starting time of $(j+1)_{\text{th}}$ order, Δt as shown in Figure 5.1 is calculated by Proposition (5.4). The profit increment of Part III of sequence, $(j+1, j+2, \dots, n)$, which may be negative, can be calculated by (5.6), taking Δt as the input. If the summation of these two profit increments is positive, the pair-wise exchange of $(j-1)$ and j makes an improvement. Otherwise, it is better to keep their positions unchanged. Following the above procedures, it helps to validate that whether a pair-wise exchange of two consecutive orders in a given sequence is profitable or not.

The proposed methodology provides an efficient algorithm for evaluating the change in profit due to pairwise exchange of two consecutive orders for any given sequence. The following proposition identifies a sufficient condition for when it is profitable to perform a pairwise exchange of any two orders in a given sequence. If two orders satisfy specific constraints, exchanging their positions in the sequence will improve the profit.

Proposition 5.5 *There are two orders (i, j) in a production sequence π , which satisfy*

$$\alpha_j - p_j\beta_j > \alpha_i - p_i\beta_i \text{ and } \frac{\alpha_j - p_j\beta_j}{\theta_j} < \frac{\alpha_i - p_i\beta_i}{\theta_i}$$

If the starting time of order j is less than t_0 and the starting time of order i is greater than t_0 , the total profit (except order i and j) is increased when order i and j exchange their positions in this sequence π , where t_0 satisfies $Y_i(t_0) = Y_j(t_0)$. It is shown in Figure 5.2.

Figure 5.2: Illustration of Proposition 5.5

Proof It is assumed that $\pi_1 = (\dots, j-1, j, j+1, \dots, i-1, i, i+1, \dots)$. After the positions of order i and j are exchanged in π_1 , a new production sequence is obtained as $\pi_2 = (\dots, j-1, i, j+1, \dots, i-1, j, i+1, \dots)$. It is known that the starting time of order j in π_1 is equal to the starting time of order i in π_2 . That is $C_{j-1}|\pi_1 = C_{j-1}|\pi_2$.

Since $C_{j-1}|\pi_1 < t_0$ and $\alpha_j - p_j\beta_j > \alpha_i - p_i\beta_i$, it is known that $Y_i|\pi_2 < Y_j|\pi_1$.

As a result, $C_i|\pi_2 < C_j|\pi_1$.

For order $j+1$,

$$Y_{j+1}|\pi_1 = X_{j+1} - v_{j+1}C_j|\pi_1$$

$$C_{j+1}|\pi_1 = X_{j+1} + (1 - v_{j+1})C_j|\pi_1$$

$$Y_{j+1}|\pi_2 = X_{j+1} - v_{j+1}C_i|\pi_2$$

$$C_{j+1}|\pi_2 = X_{j+1} + (1 - v_{j+1})C_i|\pi_2$$

Therefore, $Y_{j+1}|\pi_1 < Y_{j+1}|\pi_2$ and $C_{j+1}|\pi_2 < C_{j+1}|\pi_1$. Iterating these steps, the same conclusion is drawn for orders $(j+1, \dots, i-1)$, i.e., the processing time and completion time for each order in π_1 is less than that in π_2 .

It is known that $C_{i-1}|\pi_2 < C_{i-1}|\pi_1$. According the assumption that the starting time of order i is larger than t_0 in π_1 , that is $t_0 < C_{i-1}|\pi_1$.

If $C_{i-1}|\pi_2 = C_{i-1}|\pi_1$, $C_i|\pi_1 > C_j|\pi_2$. When $C_{i-1}|\pi_2$ is decreasing, $C_j|\pi_2$ is decreasing. Therefore, $C_i|\pi_1 > C_j|\pi_2$.

For order $i+1$, the similar results can be obtained that, $Y_{i+1}|\pi_1 < Y_{i+1}|\pi_2$ and $C_{i+1}|\pi_2 < C_{i+1}|\pi_1$.

Finally, a conclusion is made that the processing time of orders $(j+1, \dots, i-1)$ and orders $(i+1, \dots, n)$ all are increased in π_2 compared that in π_1 . $\square$

A sufficient condition to find when it is profitable to perform a pairwise exchange of any two orders has been proposed. An efficient methodology has been proposed to evaluate the change in profit due to exchange of two consecutive orders for any given sequence. In the following section, the above results will be applied to derive a heuristic algorithm to find the near-optimal solution for this problem.

5.3.2 A heuristic algorithm

Since this problem is proven to be NP-complete, conventional search and optimization methods generally require intensive computations. In order to construct

accurate and easily implemented algorithms, the heuristic algorithm is presented in this section. The algorithm is developed from three phases; the first phase involves generating an initial solution by a simple procedure, the second phase further improves the quality of the solution by a neighborhood search, and the third phase exchanges the positions of any two orders which satisfy the constraints in Proposition 5. Once special cases are found, a neighborhood search is invoked again to refine the result.

It is known that the processing time is decreasing with respect to the increasing start time for each order. If the decreasing rates among orders are similar to each other, it is profitable to place the order with higher price at the beginning of the sequence. Therefore, it is better to generate the initial sequence by sorting the orders according to the value of p_i . Furthermore, as shown in Proposition 5.2, if orders are sorted in non-decreasing order of the ratio $\frac{x_i}{v_i}$, the total processing time is maximized. This sequence is optimal when the prices for all orders are the same as illustrated in Proposition 5.2. The principle of the algorithm is to take these two characteristics into consideration. To do so, orders are arranged in non-increasing order of the ratio $\frac{p_i v_i}{x_i}$. The details of the algorithm are given as follows.

Step 1

Arrange jobs in non-increasing order of the ratio $\frac{p_i v_i}{x_i}$ to obtain an initial solution

Step 2

Exchange any two orders if they satisfy the constraints in Proposition 5.5.

Step 3

Make pair-wise exchanges between any two consecutive orders to improve the solution (applied Proposition 5.4 and expression (5.11)) until no improvement can be found.

Step 4

If improvement of the sequence has been found in *Step 3*, go to *Step 2*; otherwise, *END*.

5.4 MIP formulation

In this section, a mixed integer programming (MIP) is applied to formulate this single machine scheduling problem. The purpose of MIP formulation is to verify the performance of the heuristic algorithm proposed in Section 5.3. Since this problem is proven to be NP-complete, MIP formulation is strict to find optimal solutions for the small sized problem.

Let i and j be any two orders selected from a production sequence π . The processing start times for the two orders are S_i and S_j . Hence the processing times for these two orders are $(X_i - v_i S_i)$ and $(X_j - v_j S_j)$, respectively. A binary variable δ_{ij} is introduced, which is equal to 0 when order i is processed before order j and equal to 1 otherwise.

The objective function is

$$\max \left\{ \sum_{i=1}^n p_i X_i - \sum_{i=1}^n p_i v_i S_i \right\}$$

subject to

$$C_i - S_j \leq \delta_{ij} \cdot R \quad \forall i, j \in N \text{ and } i < j \quad (5.7)$$

$$C_j - S_i \leq (1 - \delta_{ij}) \cdot R \quad \forall i, j \in N \text{ and } i < j \quad (5.8)$$

$$C_i = X_i + (1 - v_i) S_i \quad \forall i, j \in N \text{ and } i < j \quad (5.9)$$

$$S_i \geq 0 \quad \forall i \in N \quad (5.10)$$

$$\delta_{ij} \in \{0,1\} \quad \forall i, j \in N \text{ and } i < j \quad (5.11)$$

Constraint set (5.7) and (5.8) are disjunctive constraints which enforce that either order i is processed before order j or order j is processed before order i . Constraint set (5.9) illustrates that the completion time of one order is equal to the summation of its processing start time and its processing time. Further, constraint sets (5.10) and (5.11)

are the non-negativity and integrality constraints. In this formulation, the value of R is big positive constant, for example $R = 100,000$ through this problem.

The Proposition (5.3) in section 5.3 can be formulated as the MIP constraints as follows,

$$\frac{x_i}{v_i} - \frac{x_j}{v_j} \leq (1 - a_{ij})R \quad \forall i, j \in N \text{ and } i \neq j \quad (5.12)$$

$$\frac{x_i}{v_i} - \frac{x_j}{v_j} \geq -a_{ij}R \quad \forall i, j \in N \text{ and } i \neq j \quad (5.13)$$

$$p_j \left(S_i - \frac{x_i}{v_i} \right) - p_i \left(S_i - \frac{x_j}{v_j} \right) \leq (1 - b_{ij})R \quad \forall i, j \in N \text{ and } i \neq j \quad (5.14)$$

$$p_j \left(S_i - \frac{x_i}{v_i} \right) - p_i \left(S_i - \frac{x_j}{v_j} \right) \geq -b_{ij}R \quad \forall i, j \in N \text{ and } i \neq j \quad (5.15)$$

$$|X_i + (1 - v_i)S_i - S_j + 2 - a_{ij} - b_{ij}| > 0 \quad (5.16)$$

$$a_{ij}, b_{ij} \in \{0,1\} \quad \forall i, j \in N \text{ and } i \neq j \quad (5.17)$$

From these constraints, it is seen that if $a_{ij} = b_{ij} = 1$, $\frac{x_i}{v_i} \leq \frac{x_j}{v_j}$ (derived from (5.12) and (5.13)) and $p_j \left(S_i - \frac{x_i}{v_i} \right) \leq p_i \left(S_i - \frac{x_j}{v_j} \right)$ (derived from (5.14) and (5.15)). In this situation, order j cannot be assigned directly after order i , which is concluded in Proposition 5.3. Thus, when $a_{ij} = b_{ij} = 1$, $|X_i + (1 - v_i)S_i - S_j| > 0$ (derived from (5.16)) which is equivalent to $C_i \neq S_j$.

It is observed that the number of integer has been increased to 3 times when Proposition 5.3 is not included in the MIP problem. It is also known that the computational time of solving a MIP problem through a commercial solver (such as LINGO, CPLEX) is essentially decided by the number of integer variables. Although the constraints (5.12) to (5.17) are built to limit the search domain, the computational time is significantly increased when these constraints are included in the formulation. This is observed through the initial numerical test. As a result, only the basic

constraints, (5.7) to (5.11) are programmed into the MIP solver. The constraints for Proposition 5.5 are not further studied.

This MIP formulation is modeled and solved in LINGO. The experimental results are discussed in chapter 6.

5.5 B&B algorithm

A Branch-and-bound method is introduced to reduce the search time and achieve the optimal solution. In this section, two upper bounds are developed for B&B algorithm for this maximization problem. It is assumed that a production sequence consists of two parts, $\pi = (\text{KS}, \text{PS})$. KS denotes the partial sequence which consists of the first m jobs. The completion time of KS is known as $C_{[m]}$. PS denotes the remaining part of the sequence. It consists of order $m+1, \dots, m+r$ which are waiting to be scheduled.

For each order i , there are three corresponding parameters X_i , v_i and p_i . These three parameters are arranged in ascending order, separately, so that $X_{(m+1)} \leq X_{(m+2)} \leq \dots \leq X_{(m+r)}$, $v_{(m+1)} \leq v_{(m+2)} \leq \dots \leq v_{(m+r)}$ and $p_{(m+1)} \leq p_{(m+2)} \leq \dots \leq p_{(m+r)}$. It is noted that $X_{(m+k)}$, $v_{(m+k)}$ and $p_{(m+k)}$ do not necessarily belong to the same order.

The following steps are trying to find the least possible value of the completion time for each order in PS.

Firstly, the process completion time for the $(m+1)_{\text{th}}$ order in PS is

$$C_{[m+1]} = X_{[m+1]} + (1 - v_{[m+1]})C_{[m]}$$

Hence, $C_{[m+1]}$ satisfies the following inequalities,

$$C_{[m+1]} \geq X_{(m+1)} + (1 - v_{(m+r)})C_{[m]}$$

Similarly, the completion time of the $(m+k)$ th order in PS is

$$C_{[m+k]} = X_{[m+1]} + \sum_{i=1}^{k-1} X_{[m+k-i]} \prod_{j=k+1-i}^k (1 - v_{[m+j]}) + C_{[m]} \prod_{i=0}^{k-1} (1 - v_{[m+r-i]}) \quad (5.18)$$

Substituted $\sum_{i=1}^{k-1} X_{[m+k-i]} \prod_{j=k+1-i}^k (1 - v_{[m+j]}) = X_{[m+k-1]}(1 - v_{[m+k]}) + X_{[m+k-2]}(1 - v_{[m+k]})(1 - v_{[m+k-1]}) + \dots + X_{[m+1]}(1 - v_{[m+k]})(1 - v_{[m+k-1]}) \dots (1 - v_{[m+2]})$ into (5.18),

the following inequality is derived,

$$\begin{aligned} \sum_{i=1}^{k-1} X_{[m+k-i]} \prod_{j=k+1-i}^k (1 - v_{[m+j]}) &\geq X_{[m+k-1]}(1 - v_{(m+r)}) + X_{[m+k-2]}(1 - v_{(m+r)})(1 - v_{(m+r-1)}) \\ &\quad + \dots + X_{[m+1]}(1 - v_{(m+r)})(1 - v_{(m+r-1)}) \dots (1 - v_{(m+r-k+2)}) \end{aligned} \quad (5.19)$$

Since rearrangement inequality (as shown in Lemma 5.1) is applied to obtain the minimum value of completion time for each order, this well-known result is stated below for reference (see Wayne 1946).

Lemma 5.1 *The following inequality*

$$x_n y_1 + \dots + x_1 y_n \leq x_{\sigma(1)} y_1 + \dots + x_{\sigma(n)} y_n \leq x_1 y_1 + \dots + x_n y_n$$

for every choice of real numbers $x_1 \leq \dots \leq x_n$ and $y_1 \leq \dots \leq y_n$ for every permutation $x_{\sigma(1)}, \dots, x_{\sigma(n)}$ of $x_1, \dots, x_n$.

In the expression (5.19), it is known that $(1 - v_{(m+r)})$, $(1 - v_{(m+r)})(1 - v_{(m+r-1)})$, ..., $(1 - v_{(m+r)})(1 - v_{(m+r-1)}) \dots (1 - v_{(m+r-k+2)})$ are in descending order. According to rearrangement inequality, $X_{[m+k-1]}$, $X_{[m+k-2]}$, ..., $X_{[m+1]}$ should be in ascending order to make sure that the left side of (5.19) is minimum. Since

$X_{[m+k-1]}, X_{[m+k-2]}, \dots, X_{[m+1]}$ are selected from $X_{(m+i)}$, the following inequality exists,

$$\begin{aligned} \sum_{i=1}^{k-1} X_{[m+k-i]} \prod_{j=k+1-i}^k (1 - v_{[m+j]}) &\geq X_{(m+1)}(1 - v_{(m+r)}) + \\ X_{(m+2)}(1 - v_{(m+r)})(1 - v_{(m+r-1)}) &+ \dots + X_{(m+k-1)}(1 - v_{(m+r)}) \\ (1 - v_{(m+r-1)}) \dots (1 - v_{(m+r-k+2)}) & \end{aligned} \quad (5.20)$$

The term in (5.18), $C_{[m]} \prod_{i=0}^{k-1} (1 - v_{[m+r-i]})$ satisfies that,

$$C_{[m]} \prod_{i=0}^{k-1} (1 - v_{[m+r-i]}) \geq C_{[m]} \prod_{i=0}^{k-1} (1 - v_{(m+r-i)}) \quad (5.21)$$

Hence, through (5.20) and (5.21), the completion time of the $(m+k)_{\text{th}}$ order in PS satisfies,

$$C_{[m+k]} \geq X_{(m+1)} + \sum_{i=1}^{k-1} X_{(m+i)} \prod_{j=1}^i (1 - v_{(m+r+1-j)}) + C_{[m]} \prod_{i=0}^{k-1} (1 - v_{(m+r-i)}) \quad (5.22)$$

In this manner, the least possible value of the completion time for the $(m+k)_{\text{th}}$ order in PS is obtained. Then, this value is substituted into the objective function to obtain the upper bound. As it is proven in Proposition 5.1, the objective of this problem can be converted into $\max\{\sum_{i=1}^n p_i X_i - \sum_{i=1}^n p_i v_i S_i\}$. The objective of PS is equal to

$$f_{PS} = \sum_{i=m+1}^{m+r} p_i X_i - \sum_{i=m+1}^{m+r} p_{[i]} v_{[i]} C_{[i-1]} \quad (5.23)$$

Since it is assumed that p_i and v_i are fixed parameters, a new parameter w_i is incorporated, which satisfies $w_i = p_i v_i$. w_i is arranged in ascending order and represented by $w_{(m+1)} \leq w_{(m+2)} \leq \dots \leq w_{(m+r)}$. Therefore, (5.23) can be derived as,

$$f_{PS} = \sum_{i=m+1}^{m+r} p_{(i)} X_{(i)} - \sum_{i=m+1}^{m+r} w_{[i]} C_{[i-1]}$$

It is known that $C_{[m]}, C_{[m+1]}, \dots, C_{[m+r-1]}$ are in ascending order. Based on Lemma 5.1, the following inequality exists,

$$\sum_{i=m+1}^{m+r} w_{[i]} C_{[i-1]} \geq \sum_{i=m+1}^{m+r} w_{(2m+r+1-i)} C_{[i-1]}$$

When (5.22) is substituted into above inequality, it is

$$\sum_{i=m+1}^{m+r} w_{[i]} C_{[i-1]} \geq w_{(m+r)} C_{[m]} + \sum_{k=1}^{r-1} w_{(m+r-k)} (X_{(m+1)} + \sum_{i=1}^{k-1} X_{(m+i)} \prod_{j=1}^i (1 - v_{(m+r+1-j)})) + C_{[m]} \prod_{i=0}^{k-1} (1 - v_{(m+r-i)}))$$

Finally, the objective function of PS satisfies,

$$f_{PS} \leq \sum_{i=m+1}^{m+r} p_{(i)} X_{(i)} - w_{(m+r)} C_{[m]} - \sum_{k=1}^{r-1} w_{(m+r-k)} (X_{(m+1)} + \sum_{i=1}^{k-1} X_{(m+i)} \prod_{j=1}^i (1 - v_{(m+r+1-j)})) + C_{[m]} \prod_{i=0}^{k-1} (1 - v_{(m+r-i)}))$$

Through the following expression (5.24), one upper bound of the total weighted processing time of partial sequence PS is obtained as follows,

$$UB1 = \sum_{i=m+1}^{m+r} p_{(i)} X_{(i)} - w_{(m+r)} C_{[m]} - \sum_{k=1}^{r-1} w_{(m+r-k)} (X_{(m+1)} + \sum_{i=1}^{k-1} X_{(m+i)} \prod_{j=1}^i (1 - v_{(m+r+1-j)})) + C_{[m]} \prod_{i=0}^{k-1} (1 - v_{(m+r-i)})) \quad (5.24)$$

However, if the number of orders in PS is small and $C_{[m]}$ is large, this estimation of the least possible starting times may not be tight. To relax this constraint, it is necessary to reevaluate the least possible starting time of each order in a different way. In the new method, there are two main steps. Firstly, $C_{[m]}$ is the starting time of the $(m+1)_{th}$ order. The completion time is calculated, as well as the processing time for each order in PS through (5.4), and select the minimum completion time and maximum processing time for the $(m+1)_{th}$ order, as shown below,

$$C_{[m+1]min}(C_{[m]}) = \min C_{[m+1]}(C_{[m]}) \text{ and } Y_{[m+1]max}(C_{[m]}) = \max Y_{[m+1]}(C_{[m]})$$

Secondly, $C_{[m+1]min}$ is compared with $C_{(m+1)}$ through (5.22), the larger one is selected as the revaluated least possible starting time,

$$C'_{(m+1)} = \max(C_{[m+1]min}, C_{(m+1)})$$

The reevaluated least possible starting times and largest possible processing times are generated through the following expression,

$$C'_{(m+k+1)} = \max\{C_{[m+k]\min}(C'_{(m+k)}), C_{(m+k+1)}\}$$

$$Y'_{(m+k+1)} = \max Y_{[m+1]}(C'_{(m+k)})$$

Therefore, the total weighted processing time of orders in PS satisfies the condition that

$$f_{PS} \leq \sum_{k=1}^r p_{(m+r-k+1)} Y'_{(m+k)}$$

Thus, the second of the total upper bound of the total weighted processing time for PS is obtained as

$$UB_2 = \sum_{k=1}^r p_{(m+r-k+1)} Y'_{m+k} \quad (5.25)$$

In order to make the upper bound tighter, the maximum value from (5.24) and (5.25) is chosen, that is

$$UB^* = \min\{UB_1, UB_2\} \quad (5.26)$$

Besides a tight upper bound, a tight lower bound is also required to make the branch-and-bound algorithm performs well. In this case, a tight lower bound is obtained from a feasible solution. As shown in Proposition 5.2, if orders are sorted in non-decreasing order of the ratio $\frac{X_i}{v_i}$, the total processing time is maximized. It is intuitive that it is profitable to place the order with a high price at the beginning. Therefore, if orders are arranged in non-decreasing order of the ratio $\frac{p_i v_i}{X_i}$, it can give a relative good solution. Thus, the lower bound, LB , is obtained by calculating the profit of PS , in which orders are arranged in non-decreasing order of the ratio $\frac{p_i v_i}{X_i}$.

Procedure (B&B algorithm)

- Step 1.* Form two sequences KS, PS (order positions are fixed in KS, orders are arranged in non-decreasing order of the ratio $\frac{p_i v_i}{X_i}$ in PS). Initially, KS is empty; PS has all orders in it. Calculate the profit of KS, f_{KS} , and the completion time of KS, C_{KS} . Calculate the profit of PS, f_{LB} , and the upper bound of PS, f_{UB} , through (5.26).
- Step 2.* Build a node, which consists of the sequence of PS and KS, the lower bound, $LB = (f_{KS} + f_{LB})$ and the upper bound, $UB = (f_{KS} + f_{UB})$. Attach the node to the parent node list, PNL (initially, PNL is empty).
- Step 3.* Select and remove node j from PNL (initially, $j = 1$).
- Step 4.* Branching node j .
- Step 4.1.* Select and remove i_{th} (initially, $i = 1$) order in PS of node j , and attach it to KS of node j . Calculate the profit of KS, f_{KS} , and the completion time of KS, C_{KS} . Calculate the profit of PS, f_{LB} , and the upper bound of PS, f_{UB} , through (5.26).
- Step 4.2.* If $f_{LB} + f_{KS} > LB^*$, $LB^* = f_{LB} + f_{KS}$ (initially, $LB^* = 0$).
- Step 4.3.* If $f_{UB} + f_{KS} > LB^*$, build a node from changes in *Step 4.1*. Attach the node to the child node list, CNL (CNL is initialize to empty for every j).
- Step 4.4.* $i = i + 1$. If all orders in PS are iterated, go to *Step 5*; else go to *Step 4.1*.
- Step 5.* If PNL is empty, $PNL = CNL$; else $j = j + 1$, and go to *Step 3*.
- Step 6.* Prune nodes from PNL: iterate each node in PNL, if $LB^* > UB$, delete this node from PNL then.
- Step 7.* If there is only one node in PNL, it is the optimal solution and *END*; else go to *Step 3*.

A simple example of the B&B algorithm

In this example, it is assumed that there are 4 orders. It is also assumed that the sequence is (1, 2, 3, 4) when orders are arranged in non-decreasing order of the ratio $\frac{p_i v_i}{X_i}$.

In Step 1, KS is () which is empty; PS is (1,2,3,4). $f_{KS} = 0$ and $C_{KS} = 0$. For the purpose of simple illustration, the calculation value is given directly through this example. $f_{KS} + f_{LB} = 100$ and $f_{KS} + f_{UB} = 120$.

In Step 2, a node, N_1 is built as $N_1 = \{KS=(); PS=(1,2,3,4); LB=100; UB=120\}$. The set PNL is $PNL = \{N_1\}$.

In Step 3, iterate each node in PNL. In this situation, N_1 is selected and removed from PNL. $PNL = \{\}$.

In Step 4, branching node N_1 . Each order in PS of N_1 is iterated. For example, the first order in PS is processed. In Step 4.1, KS is changed to (1), and PS is changed to (2,3,4). $f_{KS} + f_{LB} = 100$. $f_{KS} + f_{UB} = 180$. In Step 4.2, $LB^* = 100$. In Step 4.3, $CNL = \{N_2\}$ where $N_2 = \{(1); (2,3,4); 100; 120\}$.

After the iterations in Step 4, $CNL = \{(1); (2,3,4); 100; 120\}; \{(2); (1,3,4); 85; 135\}; \{(3); (1,2,4); 90; 137\}; \{(4); (1,2,3); 74; 93\}$.

In Step 5, since PNL is empty, $PNL = \{(1); (2,3,4); 100; 120\}; \{(2); (1,3,4); 85; 135\}; \{(3); (1,2,4); 90; 137\}; \{(4); (1,2,3); 74; 93\}$.

In Step 6, the last node in PNL can be deleted because $LB^* = 100 > 94$.

In Step 7, PNL is because there is more than 1 node in PNL. The algorithm will go to Step 3 again with the current value of PNL as the input.

The algorithm will end when there is only one node in PNL. □

5.6 Summary

In this chapter, a production scheduling problem is studied assuming that customers are sensitive to delivery time quotations and price. The objective of the problem is to maximize the profit under the constraint that orders are completed on its delivery time quotation. This problem is proven to be NP-complete even when prices are determined parameters. The optimization problem in which prices are input parameters is studied. A MIP formulation is proposed to find an optimal solution for the small sized problem. A B&B algorithm is developed to find an optimal solution for the moderate sized problem. In the B&B algorithm, two upper bounds and one lower bound are developed. A heuristic is also developed to find a near-optimal solution within a short time. The performance of the heuristic and the B&B algorithm is presented in the following chapter.

In many real circumstances, however, the prices of customers are also decision variables. It may be necessary to study a production scheduling problem in which both prices and delivery time quotations are decision variables.

Appendix: Proof of Proposition 5.2

A method other than Browne and Yechiali (1990) is shown below, which can omit the constraint that any order's order quantity is large than 0 in any sequence.

It is assumed that order 1 and 2 are two consecutive orders in any production sequence. Each demand function of two orders is a linear decreasing function respect to time, which is represented by the line l_1 , l_2 respectively in Figure 5.1A. l_0 represents the production capacity function respect to time, since the production facility is modeled by a single machine with a constant production rate.

$$\begin{cases} l_0: & y = ct \\ l_1: & y = b_1 - a_1t \\ l_2: & y = b_2 - a_2t \end{cases}$$

It is assumed that the t -intercepts of l_1 , l_2 always satisfy the following constraint,

$$\frac{b_1}{a_1} < \frac{b_2}{a_2}$$

Hence there are only three different possible cases considering the segments of l_1 and l_2 below l_0 , which are shown in Figure 5.1A,

Figure 5.1A: All possible cases for two orders

In the following part, each possible case is discussed. It is needed to be proven that the optimal order sequence is order 1 followed by order 2 in all cases. Before go

through each case, an example is given to illustrate how to represent scheduling in a figure.

Figure 5.2A: Two orders in two different sequences

As shown in Figure 5.2A, point A and B are the intersections by l_1 , l_2 with l_0 respectively. Since it is assumed that no earliness or tardiness in the system, point A (B) is the delivery time quotation for order 1 (2) when it is firstly produced. Point C, which is the projection of A, is the starting point to produce order 2 (I). If the demand of the next order is decreased below 0 when t is at C, this order is not produced because its order quantity is 0. Otherwise, the new line CD , which is parallel to l_0 , is the new production capacity function. Point D is the intersection by l_1 or l_2 with CD . Point E, which is the projection of D, is the completion time of order 1(2). It is intuitive that the total order quantity of these two orders is increased when the t -coordinate of E increases.

Case 1:

As shown in Figure 5.3A, the intersection of l_0 and l_1 , A is on the left side of the intersection of l_0 and l_2 , B, that is

$$\frac{b_1}{c + a_1} < \frac{b_2}{c + a_2}$$

And it is known that $a_1 > a_2$, $\frac{b_1}{a_1} < \frac{b_2}{a_2}$. If order 1 is first produced, followed by order 2, as shown in Figure 5.3A-(a). The t -coordinate of C is $\frac{b_1}{c+a_1}$. The function of CD is that,

$$y = c(t - \frac{b_1}{c+a_1})$$

Figure 5.3A: Case 1 in different sequences

Therefore, the t -coordinate of E can be achieved by solving the two functions below,

$$\begin{cases} y = c(t - \frac{b_1}{c+a_1}) \\ y = b_2 - a_2 t \end{cases}$$

The t -coordinate of E is

$$E_{-t_1} = \frac{cb_1 + cb_2 + a_1b_2}{(c+a_1)(c+a_2)}$$

If order 2 is first produced, followed by order 1, there are two possible conditions, one is that the order quantity of order 1 has been diminished to 0 when Order 2 is completed (Figure 5.3A-(b)); the other one is that there is still some demand from Order 1 when Order 2 is completed (Figure 5.3A-(c)).

In the first condition (Figure 5.3A-(b)), $\frac{b_2}{c+a_2} \geq \frac{b_1}{a_1}$, the t -coordinate of E is

$$E_{-t_2} = \frac{b_2}{c+a_2}$$

It is easy to find that $Ex_2 < Ex_1$.

In the second condition (Figure 5.3A-(c)), $\frac{b_2}{c+a_2} < \frac{b_1}{a_1}$, the t -coordinate of E is

$$Ex_3 = \frac{cb_1 + cb_2 + a_2b_1}{(c+a_1)(c+a_2)}$$

Since $\frac{b_1}{a_1} < \frac{b_2}{a_2}$, $Ex_3 < Ex_1$.

Hence, it is concluded that to produce Order 1 first followed by Order 2 can yield a larger quantity of orders produced than the other production sequence in Case 1.

Case 2:

Similar to Case 1, there are three assumptions initially,

$$\frac{b_1}{c+a_1} < \frac{b_2}{c+a_2}; \frac{b_1}{a_1} < \frac{b_2}{a_2}; a_1 > a_2$$

Figure 5.4A: Case 2 in different sequences

If order 1 is first produced, followed by order 2, as shown in Figure 5.4A-(a), the t -coordinate of E is

$$E_{-t_1} = \frac{cb_1 + cb_2 + a_1b_2}{(c+a_1)(c+a_2)}$$

If order 2 is first produced, followed order 1, as shown in Figure 5.4A-(b), the t -coordinate of E is

$$E_{-t_2} = \frac{cb_1 + cb_2 + a_2b_1}{(c+a_1)(c+a_2)}$$

Since $\frac{b_1}{a_1} < \frac{b_2}{a_2}$, $E_{t_1} > E_{t_2}$.

Hence, it is concluded that to produce Order 1 first followed by Order 2 can yield a larger quantity of orders produced than the other production sequence in Case 2.

Case 3:

Similar to Case 1, there are three assumptions initially,

$$\frac{b_1}{c+a_1} < \frac{b_2}{c+a_2}; \quad \frac{b_1}{a_1} < \frac{b_2}{a_2}; \quad a_1 < a_2$$

Figure 5.5A: Case 3 in different sequences

If order 1 is first produced, followed by order 2, as shown in Figure 5.5A-(a), the t -coordinate of E is

$$E_{t_1} = \frac{cb_1 + cb_2 + a_1b_2}{(c + a_1)(c + a_2)}$$

If order 2 is first produced, followed by order 1, there are two conditions.

In the first condition, as shown in Figure 5.5A-(b), the demand of order 1 is diminished to 0, that is $\frac{b_2}{c+a_2} \geq \frac{b_1}{a_1}$. The t -coordinate of E is

$$E_{t_2} = \frac{b_2}{c + a_2}$$

In the second condition, as shown in Figure 5.5A-(c), the demand of order 1 exists, that is $\frac{b_2}{c+a_2} < \frac{b_1}{a_1}$. The t -coordinate of E is

$$E_{-t_3} = \frac{cb_1 + cb_2 + a_2b_1}{(c + a_1)(c + a_2)}$$

It is easy to find that $E_{-t_1} > E_{-t_2}$. Since $\frac{b_1}{a_1} < \frac{b_2}{a_2}$, $E_{-t_1} > E_{-t_3}$.

Hence, it is concluded that to produce Order 1 first followed by Order 2 can yield a larger quantity of orders than the other production sequence in Case 3.

Finally, it is concluded that for any two consecutive orders, to process the order with a small value of $\frac{b}{a}$ first can produce more total quantity of orders. Since $a = \alpha - p\beta$ and $b = \theta$, hence the production sequence, in which orders are sorted in ascending order of the ratio, $\frac{\alpha_i - p_i\beta_i}{\theta_i} = \frac{x_i}{v_i}$, can accept and produce the maximum total order quantity. $\square$

Chapter 6

Numerical Study of Distinct Prices and Delivery Time Quotations Model

In chapter 5, the model is extended to the scenario, where each order in the system will be quoted a unique price and corresponding reliable delivery time guarantee. A branch-and-bound algorithm and a heuristic algorithm were proposed to find the solutions to maximize the profit. In this chapter, the performance of the branch-and-bound algorithm and the heuristic algorithm is tested. The comparison of the heuristic algorithm with other heuristics is also presented through a numerical experiment.

6.1 Preliminaries

In the distinct prices and delivery time quotations model, the production facility is modeled as a single machine. There are n customers in the system. Each of them makes a single order for the same product to the manufacturer. The order is characterized by its size, which is the quantity of the product in the order. The processing time of each order is dependent on its order quantity.

In the beginning, the price for each customer is known. The manufacturer will quote each customer order delivery time guarantee. In this problem, it is assumed that the quoted delivery time for each order is reliable and equals to its completion time. Thus, there is no earliness and tardiness in the system. It is assumed that customers are sensitive to the quoted price and quoted delivery time. In other words, the order quantity is a linear function of the quoted price and quoted delivery time, $Q_i = \alpha_i - p_i\beta_i - t_i\theta_i$. The objective is to maximize the profit, $f = \sum p_i Q_i$.

In this model, it is assumed that quoted prices are fixed. The decision variables are the production sequence, as well as the quoted delivery times.

6.2 Performance of B & B and HAD Algorithms

In chapter 5, three methods are developed to find solutions for production sequence and quoted delivery times in the distinct prices and delivery time quotations model. Among them, a Mixed Integer Programming formulation (namely, MIP) and a branch-and-bound algorithm (namely, B&B algorithm) can find optimal solutions for this problem. The heuristic algorithm (namely, HAD) is proposed to find the near-optimal solution within a short time. In the first design of experiment, the objective is to estimate the performance of B&B and HAD algorithm in terms of two aspects, computational time and accuracy.

6.2.1 Design of Experiment

In this problem, it is assumed that p_i is fixed as an input parameter. Since the problem is proven to be NP-complete, an optimal algorithm solving the problem in polynomial-time is highly unlikely. The MIP formulation is modeled using LINGO 12.0 with default settings to solve the generated problem instances. At the same time, the branch-and-bound algorithm is also applied to achieve the optimal solution. The

results from MIP formulation are compared with the results achieved by the branch-and-bound algorithm to evaluate the effectiveness and efficiency of UB and LB of the branch-and-bound algorithm. Furthermore, heuristics are compared with the optimal solutions to validate its performance. The branch-and-bound algorithm and the heuristic algorithm are coded in C++. All programs run on a personal computer with a Core 2 Duo processor at 3.00 GHz and 3 GB memory.

$[\alpha_i - p_i\beta_i]$ is treated as a single parameter, φ_i , which is randomly generated from three uniform distributions, $[300, 350]$, $[300, 400]$ and $[300, 500]$. For all tests, the price, p_i and the time sensitivity, θ_i are both randomly generated from the uniform distribution, $[1, 10]$. All parameter settings are based on the work of Boyaci and Ray (2006) and Bachman et al. (2002a). Since it is assumed that the processing time (or order quantity) of order i is positive in any production sequence π , the following constraint must be satisfied,

$$\frac{\theta_i}{c+\theta_i} \left(\sum_{j=1}^n \frac{\varphi_j}{c+\theta_j} - \frac{\varphi_i}{c+\theta_i} \right) < \frac{\varphi_i}{c+\theta_i}$$

which is illustrated in section 5.2. Since $\sum_{j=1}^n \frac{\varphi_j}{c+\theta_j} < n \frac{\varphi_{max}}{c+\theta_{min}}$, if the production rate, c satisfies the following inequality in each test, the above constraint can be satisfied,

$$n\theta_{max}\varphi_{max}(c + \theta_{max}) < \varphi_{min}(c + \theta_{min})^2 + \theta_{max}\varphi_{min}(c + \theta_{min})$$

Therefore, the production rate, c , is selected at $[140, 170, 200, 270, 340]$ when order size is set at $[8, 10, 12, 16, 20]$, respectively. MIP formulation is only tested when order size is at 8 and 10, because its computation time is exponentially increased when order size is increased. For each test setting for the three heuristic simultaneously, 20 replications are randomly generated, and a total of 300 problems are tested.

All the parameter in this experiment settings are listed in the following table.

Table 6.1: Parameters setting for numerical experiments

Parameter	Number of Choices	Possible Values
$(\alpha_i - p_i\beta_i)$	3	{U(300, 350), U(300, 400), U(300, 500)}
θ_i	1	{U(1, 10)}
n	6	{ 8, 10, 12, 16, 20}

6.2.2 Results Analysis

Table 6.2 and Table 6.3 show the results of the tests when the number of orders is at [8, 10] and [12, 16, 20] respectively. Error percentage is calculated as $100 \times (z^* - z')/z^*$, where z^* is the optimal profit achieved by B&B algorithm and z' is the profit achieved by the heuristic algorithm. Number of nodes represents the maximum number of nodes expanded in the B&B algorithm. It can be seen that the number of orders has more impact on the difficulty (CPU time) of the problem than the range of $(\alpha - p\beta)$ from both tables. This is because the difficulty is affected by the number of orders to be scheduled.

In table 6.2, it is noted that B&B algorithm is able to find the optimal solution very fast, and the number of nodes in B&B algorithm is small. This implies that the upper bound is very tight when the number of orders is no larger than 10. From the results achieved by HAD, it is observed that HAD can find the optimal solution in most of the cases. Although there are two instances which cannot be obtained optimally through HAD, the error percentage is less than 0.1%. It is clear that the computational time in MIP is larger than B&B significantly. CPU time in MIP is increasing exponentially when the number of orders is increased. When the number of

Table 6.2: Test results for $n \leq 10$

No	$(\alpha - p\beta)$	MIP		B&B algorithm				HAD algorithm		Optimal solutions count
		CPU time (s)		CPU time (s)		No. of nodes		Error percentage		
		Mean	Max	Mean	Max	Mean	Max	Mean	Max	
8	(300,350)	5.3	29	0.02	0.03	4	12	0.0	0.0	20
	(300,400)	3.2	10	0.02	0.04	6	21	0.0	0.0	20
	(300,500)	4.6	8	0.03	0.03	9	25	6.9e-4	6.9e-4	19
10	(300,350)	276	1,195	0.02	0.04	26	158	0.0	0.0	20
	(300,400)	156	472	0.02	0.05	26	89	4e-5	4e-5	19
	(300,500)	141	453	0.03	0.09	59	641	0.0	0.0	20

Table 6.3: Test results for $10 < n \leq 20$

No	$(\alpha - p\beta)$	B&B algorithm				HAD algorithm		Optimal solutions count
		CPU time (s)		No. of nodes		Error percentage		
		Mean	Max	Mean	Max	Mean	Max	
12	(300,350)	0.03	0.09	55	260	0.0	0.0	20
	(300,400)	0.04	0.12	73	373	0.0	0.0	20
	(300,500)	0.09	0.22	293	800	0.0	0.0	20
16	(300,350)	0.69	4.84	1,365	10,987	5e-6	7e-6	18
	(300,400)	1.57	8.93	2,838	17,414	0.0	0.0	20
	(300,500)	3.31	8.78	5,858	16,834	0.0	0.0	20
20	(300,350)	2,041	40,049	56,208	627,376	0.0	0.0	20
	(300,400)	2,868	21,618	129,401	435,216	0.0	0.0	16/(16)*
	(300,500)	5,168	23,056	174,392	442,432	0.0	0.0	15/(15)*

* represents not all 20 instances can be found solutions through B&B. For example, 16/(16) denotes 16 instances can be obtained optimal solutions through HAD algorithm, while (16) instances have been obtained solutions through B&B algorithm.

orders is greater than 10, it is hard to find the optimal solution by MIP within a reasonable time (6 hours). Hence MIP is not used to obtain results in Table 6.2.

Through Table 6.2, a conclusion can be made that the B&B algorithm is the best choice when the number of orders is less than or equal to 10, which guarantees an optimal solution.

In Table 6.3, it is observed that CPU times for the B&B algorithm increase as the range of $(\alpha - p\beta)$ increases. This is because the upper bound is getting loose when $(\alpha - p\beta)$ increases, which can be observed from the formulation of the upper bound. It is noticed that the number of nodes is increasing dramatically when the number of orders is increased in B&B algorithm. CPU times increases more intensively than the number of orders when the number of orders increases. The reason for this is that it takes much longer time to compute an upper bound for each node when the number of orders is increased.

When $n = 20$, most of the optimal solutions can be obtained by B&B. However there are few instances where optimal solutions cannot be obtained by B&B in with 12 hours. 16 out of 20 optimal solutions have been found by B&B when $(\alpha - p\beta)$ is at (300, 400), while 15 out of 20 instances have been found the solutions when $(\alpha - p\beta)$ is at (300, 500). The worst case among the tests was solved within 11.1 hours. Although the CPU times for the worst case is very long, the mean of the CPU times is close to 1 hour. This means that the upper bound is effective in most of the cases even when $n = 20$. Since the computational complexity of HAD algorithm in worst case is $O(n^2)$, it takes only a few seconds for the heuristic algorithm to obtain results when $n = 20$.

It is also observed that only 2 among 171 instances cannot be obtained optimally by HAD algorithm. The error percentage is very small, less than 10^{-5} . Therefore, it is

suggested that the HAD algorithm can be used to find near-optimal solutions for problems of $n = 20$ or above.

6.3 Applicability of HAD: A Numerical Example

In order to validate the applicability of HAD algorithm, it is compared against a method adopted from literature. When quoted prices are identical, the profit maximization problem is proven to be equivalent to a single machine scheduling problem to minimize the total completion times in chapter 5. Browne and Yechiali (1990) studied this special problem, $1|a_i - b_i S_i|C_{\max}$. The optimal sequence is that the jobs are scheduled in non-decreasing order of the ratio a_i/b_i , which is equivalent to X_i/v_i in this problem. This method from the special case is applied and named ‘HAC’ in the following discussion.

In addition, it is often the case in the industrial practice that the production priority is given to an order with higher quoted price. Thus, the scheduling sequence, in which orders are arranged in non-increasing sequence of the quoted price p_i , is used to compare with HAD. It is named ‘HAP’ in the following discussion.

6.3.1 Design of Experiment

The model setting described in chapter 5 involves the following parameters: n , α_i , β_i , θ_i , p_i and c . In this example, it is considered that there are 20 orders ($n = 20$) in the system. The production rate c is set as 340. The quoted prices are input parameters, which are shown in Table 6.4.

Other parameters in the experiments are setting as shown in Table 6.5. Totally, there are 8 combinations for the parameter settings.

Table 6.4: Quoted Price List

Order Index	1	2	3	4	5	6	7	8	9	10
Quoted Price	83.32	91.52	21.42	42.20	66.91	18.77	35.06	59.21	26.17	96.84
	11	12	13	14	15	16	17	18	19	20
	24.18	97.35	56.14	53.68	82.02	22.76	47.95	92.14	51.29	36.35

Table 6.5: Parameters setting for numerical experiments

Parameter	Number of Choices	Possible Values
α_i	2	{U(400, 500), U(400, 700)}
β_i	2	{U(0.1, 0.5), U(0.1, 1.0),}
θ_i	2	{U(1, 5), U(1, 10)}

6.3.2 Result Analysis

Three algorithms, HAD, HAC and HAP are tested in 12 situations, which are based on the combinations of the parameters in Table 6.5. For each test, 30 randomly generated instances are compared. Totally, 240 instances are tested in the numerical study. All the parameters are randomly generated according to the uniform distribution in Table 6.5.

Table 6.6: Results of the paired t -tests of (HAD-HAP) for $\theta_i \in (1,5)$

		$\alpha_i \in (400,500)$	$\alpha_i \in (400,700)$
$\beta_i \in (0.1,0.5)$	Mean	7.929	11.032
	Stdev	2.820	3.768
	t -value	15.400	16.036
$\beta_i \in (0.1,1.0)$	Mean	7.464	10.572
	Stdev	2.749	3.582
	t -value	14.871	16.166

Table 6.7: Results of the paired t -tests of (HAD-HAC) for $\theta_i \in (1,5)$

		$\alpha_i \in (400,500)$	$\alpha_i \in (400,700)$
$\beta_i \in (0.1,0.5)$	Mean	14.444	18.666
	Stdev	4.436	5.257
	t -value	17.834	19.448
$\beta_i \in (0.1,1.0)$	Mean	13.650	17.421
	Stdev	4.264	5.175
	t -value	17.534	18.438

Table 6.8: Results of the paired t -tests of (HAP-HAC) for $\theta_i \in (1,5)$

		$\alpha_i \in (400,500)$	$\alpha_i \in (400,700)$
$\beta_i \in (0.1,0.5)$	Mean	6.515	7.634
	Stdev	5.740	6.812
	t -value	6.217	6.138
$\beta_i \in (0.1,1.0)$	Mean	6.186	6.848
	Stdev	5.649	6.488
	t -value	5.998	5.781

* The benchmark is 1.699 for confidence level 95% and D.O.F 29 in Table 6.6-6.10.

Table 6.9: Results of the paired t -tests of (HAD-HAP) for $\theta_i \in (1,10)$

		$\alpha_i \in (400,500)$	$\alpha_i \in (400,700)$
$\beta_i \in (0.1,0.5)$	Mean	18.177	25.656
	Stdev	5.146	7.786
	t -value	19.347	18.048
$\beta_i \in (0.1,1.0)$	Mean	17.554	26.293
	Stdev	4.961	7.850
	t -value	19.381	18.346

Table 6.10: Results of the paired t -tests of (HAD-HAC) for $\theta_i \in (1,10)$

		$\alpha_i \in (400,500)$	$\alpha_i \in (400,700)$
$\beta_i \in (0.1,0.5)$	Mean	21.199	25.919
	Stdev	7.162	7.164
	t -value	16.212	19.816
$\beta_i \in (0.1,1.0)$	Mean	19.668	26.019
	Stdev	6.017	6.679
	t -value	17.903	21.337

Table 6.11: Results of the paired t -tests of (HAP-HAC) for $\theta_i \in (1,10)$

		$\alpha_i \in (400,500)$	$\alpha_i \in (400,700)$
$\beta_i \in (0.1,0.5)$	Mean	3.022	0.263
	Stdev	9.171	11.708
	t -value	1.805	0.123
$\beta_i \in (0.1,1.0)$	Mean	2.114	-0.274
	Stdev	8.006	11.737
	t -value	1.446	-0.128

* The benchmark is 2.045 for confidence level 95% and D.O.F 29 in Table 6.11.

The results when the time sensitive, θ_i is set at (1, 5) are presented in Table 6.6, Table 6.7 and Table 6.8. Paired t -tests are applied to compare the performance between HAD and HAP (HAC). The hypothesis test is built with H_0 : $HAD - HAP = 0$ and H_1 : $HAD - HAP > 0$. The results of the t -tests of (HAD-HAP) are shown in Table 6.6. The t -values are all larger than the benchmark 1.699. It is indicated that HAD significantly outperforms HAP. The similar results of the t -tests of (HAD-HAC) are shown in Table 6.7. It is also concluded that HAD outperforms HAC significantly. In Table 6.8, it is shown that HAP outperforms HAC. This is because order position has little effect on order quantity when the time sensitivity is small and with little variety. In this case, it is better to give production priority to the order with a high quoted price.

The results when the time sensitive, θ_i is set at (1, 10) are presented in Table 6.9, Table 6.10 and Table 6.11. Similar results can be obtained from Table 6.9 and Table 6.10 compared to cases when $\theta_i \in (1,10)$. That is the performance of HAD is better than either HAC or HAP. In Table 6.11, it is shown that the t -values are less than the benchmark 2.045. It denotes that the performance of HAP is comparable to HAC.

It can be concluded that both HAC and HAP algorithms have limitations when the market condition changes. When customers' time sensitivities are negligible, HAP can be applied to get the near-optimal solution. When the quoted prices are similar, HAC can be applied to obtain the near-optimal solution. HAD is able to find a near-optimal solution in all tested scenarios.

6.4 Summary

This chapter firstly discusses the performance the B&B algorithm and the HAD algorithm. Results of tests on the performance show that the B&B algorithm can find

optimal solutions of moderate-sized problems in a reasonable time. When the number of orders is at 20, it takes a long time to find the optimal solutions in the B&B algorithm. Moreover, in a few of cases it is hard to obtain the optimal results within 12 hours. The results from the HAD algorithm suggest that it is able to find the optimal solutions in a very short time in most of the cases, and the error percentage is very small.

The applicability of the HAD algorithm is examined in the second numerical experiment. It is compared with two algorithms, which are from the literature and the practice, respectively. The comparison results indicate that the HAD algorithm outperforms two algorithms in all situations. If the customers are more sensitive to price and time, the advantage of applying HAD algorithm is more significant.

Chapter 7

Conclusions

This chapter summarizes the conclusions, contributions, and limitations of this thesis. The possible extensions of this research are proposed for the future work.

7.1 Summary of the Research

In this research, an operational decision model for price quotation and delivery time quotation is investigated for a manufacturer, serving customers which are sensitive to price and time. The overall problem is studied in two scenarios. In the first scenario, it is assumed that all customers share the same price quotation and same delivery time quotation. This phenomenon is consistent with the dedicated capacity model, in which the manufacturer provides service to different customer groups through different production facilities. An optimal methodology is developed to solve this problem. In the other scenario, it is assumed that different customers have different price quotations as well as different delivery time quotations. This phenomenon is consistent with the shared capacity model, in which different customers are served in a single production facility. This problem is considered to be similar to the single machine scheduling problem with start-time dependent processing times. The problem is proven to be NP-complete even when quoted prices are

predetermined variables. A MIP formulation is proposed to find the optimal solution for small-sized problems. A branch-and-bound algorithm is developed to find the optimal solution for moderate-sized problems. A heuristic algorithm is also proposed; the experimental results show that it can obtain the near-optimal solutions in a fraction of the time required for the branch-and-bound algorithm.

Based on this exhaustive literature review, previous researchers have focused on the decision making for lead time quotation and price quotation for queueing models. To the best of our knowledge, most of planning models are proposed on either tactical decision level or strategic decision level. Queueing models that jointly quote price and lead-time are studied under steady state conditions. Furthermore, the due date quotation problems ignore the effects of both price and time on demand. These limitations have been partially addressed with the work presented in this thesis.

An operational decision model to coordinate price and delivery time is discussed in this thesis. It is suggested to employ a linear decreasing function to describe the relationship between demand and price/delivery time. This feature is consistent with the characteristics in the price time sensitive market, and it is commonly applied in the literature. The delivery time quotation is also accurate and obtained from the information of production.

In chapter 3, the coordination problem of price and delivery time is formulated in the situation, when different customers share a common price quotation and a common delivery time quotation. Orders completed after the delivery time quotation will incur tardiness penalty. The objective is to find the price quotation and delivery time quotation as well as the production sequence to maximize the profit, which is the revenue from sales minus the tardiness penalty. Two special problems are studied to find the properties for optimal solutions of the general problem. In the first special

problem, it is assumed that the price is fixed. In the second problem, it is assumed that the quoted delivery time is fixed. Through the results obtained from these two special problems, an algorithm is developed to find the optimal solution for the general problem in polynomial time.

Numerical studies for the common price and delivery time problem is presented in chapter 4. The experiments indicate that the common price and delivery time quotation strategy is more applicable in the market where customers are more time sensitive compared to price sensitive. When customers are more time sensitive, the average profit achieved is higher. However, there are some drawbacks. The profit variance is higher, and the delivery time quotation is less reliable. If the manager wants to increase the demand, it is more profitable to increase the potential market size than to increase the number of customers in the market. In other words, it is profitable if some customer orders can be merged into a single customer order.

In chapter 5, the coordination problem of price and delivery time is formulated under the assumption that different customers receive different price quotations and different delivery time quotations. It is assumed that an order's delivery time quotation is equal to its completion time in production. Thus, all the delivery time quotations are reliable. There is no tardiness penalty in this problem. The objective is to maximize the profit by sales. This problem is proven to be NP-complete even when price quotations are fixed variables. A MIP formulation is proposed to find optimal solution for small-sized problems. A heuristic algorithm is developed to find a near-optimal solution in a short time. A branch-and-bound algorithm is built for moderate-sized problems.

Numerical studies for the three algorithms proposed in chapter 5 are presented in chapter 6. Firstly, the performances of the three algorithms are compared through

experiment results. The computational time of MIP formulation is increasing exponentially as the problem size increased. The branch-and-bound algorithm is able to find optimal solutions when $n < 20$ within about one hour. The heuristic algorithm can obtain the near-optimal solutions very fast and with little error. Secondly, a numerical example of the heuristic is given and compared with the algorithms from literature and practice. It is shown that the heuristic developed in chapter 5 is capable to maximize the profit for the manufacturer especially when customers are both price sensitive and time sensitive.

In summary, the contributions of this thesis are listed as follows:

- This thesis presents a new decision model to investigate the coordination of price quotation and delivery time quotation from a make-to-order manufacturer's perspective.
- The coordination problem is studied in common price and delivery time quotation model. An algorithm is developed to find the optimal solution for the general problem in polynomial time.
- The coordination problem is studied in distinct prices and delivery time quotations model. This problem is proven to be NP-complete. An efficient branch-and-bound algorithm is proposed to solve moderate-sized problems. A heuristic algorithm is developed to find near-optimal solutions in a short time.

7.2 Limitations

The linear demand function respect to price and time, $Q_i = \alpha_i - \beta_i p_i - \theta_i t_i$ is a fundamental assumption for describing customers who are sensitive to price and time. Different customers have different parameters (α_i , β_i and θ_i). There are errors omitted through the transformation from stochastic demand to deterministic demand. Hence

using deterministic demand cannot fully describe the relationship between demand, price and delivery time quotation.

The common price and delivery time quotation model (chapter 3) is built for preliminary work on the dedicated capacity model. There is only one customer group with its dedicated capacity is studied. In the real world, there are multiple production facilities serving multiple customer groups. Customers are able to select which facility to get service. In addition, the allocation of capacity among different customer groups is also important for further research.

In the distinct prices and delivery time quotations model, all the orders are assigned to the production schedule. In real practice, some orders may be rejected to reserve the capacity for the high profitable orders. Further analysis should be carried out to consider the order acceptance.

In chapter 5, the tardiness weight for each order is assumed to be very large. In this situation, the quoted delivery time is reliable for each order. Tardiness penalty can be taken into consideration in the distinct prices and delivery time quotations model. It is also assumed that customers do not know other customers' price sensitivity, time sensitivity and the potential market size. Thus there is no strategic competition among customers.

An order's delivery time is equal to its completion time plus the transportation time. In this thesis, the transportation time is not considered. In the real world, there are also some constraints in the transportation part. The delivery time should be reevaluated on account of the uncertainty of transportation.

In this research, all orders are assumed to be immediately available at the beginning of the planning horizon. Actually, orders arrive throughout the planning horizon. The release times could be taken into account in the problem modeling.

7.3 Future Work

In general, this research can be extended in the following directions:

- In the dedicated capacity model, only a single customer group with the limited capacity is studied. In future, multiple customers groups with different capacity can be modeled to fulfill the segment and pooling strategy. The total capacity is fixed. The focus is on the capacity allocation among different customer groups, as well as selecting appropriate price quotation and delivery time quotation for each customer group. In addition, if a customer can decide which customer group to join, the conditions for incentive compatibility should be studied in this problem.
- In chapter 5, the solution is developed for the case with fixed prices. The general problem where both prices and delivery time quotations are decision variables can be further explored based on the results obtained in the fixed price problem.
- Further work needs to be conducted considering order acceptance for the distinct prices and delivery time quotations problem. In reality, it is common that a manufacturer can decide whether an incoming order can be accepted or rejected. In this sense, the problem becomes more complex. Since the inherent problem is NP-complete, the heuristic algorithm developed in chapter 5 can be applied after orders are selected to be processed. Furthermore, the rejection of orders may be examined through two criteria. Firstly, an order is rejected when the order quantity is decreased to 0. The other one is the delivery time quotation; for example, an order is rejected when the order's delivery time quotation exceeds to customer's expected time.

- All order release times are considered to be 0 in the two models, common price and delivery time quotation model and distinct prices and delivery time quotations model. In reality, orders are arriving during a time period. Thus, the release times of orders are different. If the release times are known at the beginning of the planning horizon, the production problem can be modeled as an off-line scheduling problem. Otherwise, it is modeled as an on-line scheduling problem. The scheduling problem can be extended to consider the release time in both models, as illustrated above.
- It is assumed that transportation time is 0 through the thesis. Future research can be conducted addressing constraints in transportation. For example, the transportation vehicles have their capacity. According to the locations of the customers, the transportation time is also dependent on the delivery route.
- Last but not least, the production facility is simplified to single machine in this research. The production problem can be extended to consider the batch processor, which is more similar to make-to-assemble facility.

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