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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

DEVELOPMENT OF A REVOLVING VANE EXPANDER

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2012

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Nomenclature

Dimensional parameters

C	clearance (m)
c_p	specific heat at constant pressure (J/kg·K)
c_v	specific heat at constant volume (J/kg·K)
D_h	hydraulic diameter (m)
d	diameter (m)
E	total energy (J)
e	eccentricity (m)
F	force (N)
g	gravitational acceleration (m/s ²)
h	specific enthalpy (J/kg); height (m); heat transfer coefficient (W/m ² ·K)
I	inertia (kg·m ²)
k	thermal conductivity (W/m·K)
l	length (m)
M	bending moment at shaft (N·m)
m	mass (kg)
P	power (W)
p	pressure (Pa)
Q	heat transfer (J)
q	volume flow rate (m ³ /s)
r	radius (m)
R_g	gas constant (J/kg·K)
S	section modulus (m ³)

s_n	estimated endurance strength (Pa)
s'_n	modified endurance strength (Pa)
s_y	yield strength (Pa)
T	temperature (K); torque (N·m); torsion experienced by the shaft (N·m)
t	time (s); thickness (m)
U	velocity (m/s)
u	specific internal energy (J/kg)
V	volume (m ³); velocity (m/s); shear force experienced by shaft (N)
v	velocity (m/s)
W	work done (J), bearing load (N)
w	width (m)
z	height (m)

Dimensionless parameters

C_d	coefficient of discharge
C_R	design reliability factor
C_s	design size factor
K_t	stress concentration factor
M	Mach number = $\frac{v_f}{v_{sound}}$
N	design safety factor
Nu	Nusselt number = $\frac{hL}{k_f}$
Pr	Prandtl number = $\frac{C_p \mu}{k}$
R	radius ratio = $\frac{r_r}{r_c}$

$$Re \quad \text{Reynolds number} = \frac{\rho v L}{\mu}$$

Greek letters

α	angular acceleration (rad/s ²)
δ	gap (m)
ε	eccentricity ratio (-)
ϕ	attitude angle of the bearing (rad)
γ	contact angle (rad); adiabatic index (-)
η	coefficient of friction (-); efficiency (-)
φ	operating angle (rad)
κ	heat capacity ratio (-) = $\frac{c_p}{c_v}$
λ	friction factor (-)
μ	dynamic viscosity (Pa·s)
θ	angle (rad)
ρ	density (kg/m ³)
σ	stress (Pa)
τ	shear stress (N/m ²)
ω	angular velocity (rad/s)

Subscripts

*	critical
I	RV-I
II	RV-II
Ia	RV-Ia
IIa	RV-IIa

b	bearing
c	cylinder
cc	cylinder cover
ctfg	centrifugal
cv	control volume
dccv	discharge control volume
disc	discharge condition
disc_open	discharge opening
disc_port	discharge port
endf	endface
exp	expander
f	friction; fluid
h	housing
hc	housing cover
ht	heat transfer
i	inlet, inner
isen	isentropic
j	journal
lubr	lubricant
mech	mechanical
n	normal
o	outlet, outer
orif	orifice
p	pressure
r	rotor

rad	radial clearance
rs	rotor shaft
s	isentropic
scv	suction control volume
shaft	shaft
start_exp	start of expansion
suct	suction condition
sp	suction port
suct_open	suction opening
suct_valve	suction valve
vane_side	vane side
t	tip
v	vane
vc	<i>vena contracta</i>
ve	exposed vane
vh	vane head
vol	volumetric
vs	vane slot
x	<i>x</i> -direction
y	<i>y</i> -direction

Caps

·	differentiated with time
–	average
^	vector notation

Abstract

The Revolving Vane (RV) compressor mechanism was introduced in 2006. Since then, the mechanism has been studied, developed and improved. However, it has never been investigated for expander applications. It is the main objective of this project to develop an expander with the mechanism.

In order to achieve the aforementioned objective, the RV expander was first analysed by developing a comprehensive mathematical model. The model consists of two main parts, namely the working fluid processes and the solid components dynamics. The first part of the model includes the thermodynamics, the fluid dynamics and the heat transfer of the working fluid. The internal leakage models were also included. The second part includes the geometrical, the kinematics and the dynamics models of the main components. The endface effects and the journal dynamics models were also integrated. Computer simulation codes were then developed to numerically simulate the working principles of the expander. It was found that, among others, the output torque of the expander is a function of the inertia of the driven component, the pressure differences across the vane and the frictional losses. In the range of operating conditions tested, the friction losses were mainly dominated by the losses at the vane side (70%) and the shaft bearings (21%). It was also found that the expander internal leakage is mainly through the radial clearance (79%).

The RV mechanism has at least five design variants. Mathematical models for the other four variants have also been developed in this project. Comparisons between the variants suggest that the design where the vane is attached to the rotor and the rotor is used as the driving component (the RV-I design) is the most desirable design. It has the highest mechanical efficiency of up to 96.5% and the output torque profile is the most

stable. Based on this finding, an RV-I prototype was then designed and manufactured. This is believed to be the first RV-I expander ever built in the world. The suction valve of the prototype was designed to function with air at a suction pressure of 3 barg and a discharge pressure of 0 barg. Experimental studies were then carried out at suction pressures of up to 6 barg and operational speeds of up to 600 rpm. The prototype was operated without any significant problem. It was also found that the isentropic efficiency is up to 32%. The volumetric efficiency is only up to 26% when operated in the design condition due to the larger than expected manufacturing clearances. Verifications of the theoretical models show that the theoretical model can simulate the average torque data with a standard deviation of mostly within 8%.

Another prototype was also developed based on the RV-0 design, where the vane was allowed to swivel in the cylinder and to slide relative to the rotor. This is believed to be the first RV-0 expander prototype in the world. It was found that it can successfully operate without any significant operational problem in the range of the operational pressures (suction pressures of up to 5 barg and a fixed discharge pressure of 0 barg) and speeds (up to 900 rev/min) tested. The expander exhibited a volumetric efficiency of up to 77%. From the static leakage test, the main leakage paths were found to be through the housing and the radial clearances, accounting for around 50% and 40% of the total leakage, respectively. The test also showed that the leakage through the radial clearance accounted for 67% to 93% of the total internal leakage of the expander, depending on the operational pressure. The isentropic efficiency of the prototype was found to be up to 17.5%. Comparisons between the theoretical predictions and the experimental data show that the mathematical models can predict the average torque and flow rate within average standard deviations of 20% and 11%, respectively.

Finally, an improved dynamics model was proposed. This model is based on the dynamic interactions of the forces and the torques, allowing the modelling of the dynamics and impacts between the vane and the vane slot as the expander operates. It allows the observation of previously unobservable phenomena. These include, among others, the interaction between the vane and the vane slot and its effect on the thermodynamic processes of the working fluids and to the cylinder dynamics. It was also found that the selection of the impact time is crucial to the accuracy of the model.

INTRODUCTION

1.1. Background

At the 15th session of the Conference of Parties (COP 15) to the United Nations Framework Convention on Climate Change (UNFCCC) in Copenhagen, Denmark, in 2009, climate change has been underlined as one of the greatest challenges of this time. Deep cuts in global emissions and low-emission development strategy are recognised as crucial to combat the issue [1]. These points have been reemphasised in the continuing session one year later in Cancun, Mexico.

These calls to curb greenhouse gases emissions are continuations of the 1997 Kyoto Protocol [2] which calls for the reduction of the emissions of, among others, carbon dioxide and two groups of refrigerants, i.e. hydrofluorocarbons and perfluorocarbons. The production and emission of the more environmentally damaging substance, chlorofluorocarbons (CFCs), have been regulated even earlier in 1987 under the Montreal Protocol [3]. In response, more environment friendly refrigeration systems have been investigated in recent years. Two aspects are of particular concerns, namely the use of environmentally friendly refrigerants and the energy consumption issue.

With the phasing out of the use of CFCs under the Montreal Protocol [3], chemical substances like the hydrochlorofluorocarbons (HCFCs) and the hydrofluorocarbons (HFCs), have been proposed and used as temporary alternatives. However these compounds have their own problems, such as being toxic and are considered to be greenhouse gases [4-6]. In addition, these gases are non-natural and are considered to be pollutants when released into the atmosphere. The carbon dioxide (CO₂)-based refrigeration system was proposed as a response to these concerns [7, 8]. The concept

has been used for commercial heat pumps in Japan under the general name of “Eco Cute” [9-11] since 2001.

While it is true that CO₂ is a greenhouse gas, it is also a natural by-product of many human activities. Therefore it will always exist in the atmosphere and be recycled by trees through photosynthesis. The use of it for refrigeration is just a postponement of its release into the atmosphere. Unfortunately, a CO₂-based refrigeration system needs to run at a high pressure condition. Therefore the energy losses, specifically those related to the throttling process, are comparatively larger as compared to the conventional refrigeration systems. Hence, without a significant improvement to its efficiency, the energy consumption aspect of such refrigeration system is still not yet addressed.

Energy consumption itself is actually a natural process. However, high energy consumption is damaging to the environment mainly because of our current over-dependence on energy produced by conventional fossil fuel burning processes. These processes produce greenhouse gases (especially CO₂) which in turn causes climate change problems when released into the atmosphere. Until we have finally successfully substituted this fuel burning process with sustainable energy production mechanisms, it is crucial to save whatever amount of energy we can. This is also the message that was conveyed by Mr. Lee Hsien Loong, Singapore’s Prime Minister, at the 3rd Singapore Energy Week in 2010 [12].

Meanwhile, it is reported that 40% of the world annual energy consumption is used for the provision of building-related facilities such as lighting, heating and air conditioning [13]. In the UK, for example, according to a report [14] published in 2009, buildings are responsible for around 50% of all carbon emissions in the country. It is therefore very important not only to have a refrigeration system using an environment friendly refrigerant, but also to have one with good energy efficiency. That is why it is

absolutely necessary to fix the low-efficiency issue of the CO₂ refrigeration system before the system can be used widely and commercially. It has been reported by Robinson and Groll [15] that this can be done by replacing the commonly used expansion valve with a work producing device such as an expander. It has also been reported by Goncalves and Parise [16] that an expander can also improve the coefficient of performance (COP) of a conventional R-134a refrigeration system by up to 12%. In terms of economical benefits, Henderson *et al.* [17] showed that an expander can cut the payback time of the replacement of gas boilers with heat pumps by about 20%.

Simply speaking, an expander is a compressor working in the reversed cycle direction. If a compressor absorbs work to increase the fluid pressure, an expander produces work by decreasing the fluid pressure. In other words, it works like an engine. Therefore, like compressors and engines, good expander design with high energy efficiency is desirable for optimum performance.

Recently, a new type of rotary mechanism, called the revolving vane (RV) mechanism, was introduced by Ooi and Teh [18]. This mechanism has been employed in a newly invented compressor and has been shown to eliminate most of the frictional losses normally associated with the conventional rotary compressors [19, 20]. An improved version of the mechanism, called the RV-i compressor [21], has shown an even better performance as compared to the original RV compressor [22]. Since this mechanism exhibits superiority in terms of geometrical construction and significantly low frictional losses, it is believed that it will also be a superior mechanism for an expander in particular and rotary engine applications in general. Hence, in this project, an expander will be developed using this RV-i mechanism. For simplicity, the machine will be called the Revolving Vane (RV) expander from here onwards.

1.2. Objectives and scope of the project

The main objective of this project is to develop a Revolving Vane (RV) expander. This includes mathematically modelling, designing, fabricating and testing a prototype of the Revolving Vane (RV) expander. To achieve the above objective, the following work scope is necessary:

- To conceptually design an RV expander using the RV-i mechanism.
- To develop a mathematical model of the RV expander and to numerically test it to establish its operational characteristics.
- To carry out detailed industrial design and to build a RV expander prototype.
- To instrument and to measure the performance of the RV expander prototype.
- To verify and to further improve the mathematical model.

1.3. Organisation of the thesis

The thesis consists of 12 chapters and appendices. Chapter 1 opens up the thesis with the introduction, consisting of mainly background information and the motivation of the project. It is then followed by Chapter 2 on a summary of the related literature about environmental effects of refrigeration systems, proposed methods to improve the environmental friendliness of refrigeration systems, expanders in general, the revolving vane (RV) mechanism and previous studies on the various aspects related to the expander. The detailed discussions of the RV expander mechanism are presented in Chapter 3.

The utilisation of engineering theories to formulate mathematical models of the RV expander, with the assumption that the dynamics of the components are geometrically constrained, is presented in Chapters 4 and 5. The models include the thermodynamics, the geometries, the kinematics and the dynamics aspects of the expander. Simulation

results obtained from the formulated mathematical models are presented and discussed in Chapter 6. Parametric studies are also presented in the chapter to better understand the behaviours of the expander design mechanism.

The RV expander mechanism has four alternative design configurations. These configurations and their respective mathematical models are presented in Chapter 7. The performances of all the possible RV expander design configurations are then compared to find the most desirable configuration. The finding is used as the basis to design the RV expander prototype as explained in Chapter 8. Experimental investigations are then carried out with the prototype. The procedure and the results are presented and discussed in Chapter 9. The comparison between the theoretical predictions and the experimental results are also presented in this chapter.

Another RV expander prototype was also built and tested. It uses the original RV mechanism (called the RV-0 mechanism) invented by Teh and Ooi in 2006 [23]. These are presented in Chapter 10. An improved dynamics model that includes the vane thickness and the dynamic interaction between the force and the torques of the components is proposed in Chapter 11. Finally, to close the thesis, a list of the significant findings of the project, the conclusions and recommendations for future work are presented in Chapter 12.

LITERATURE REVIEW

In this chapter, articles in the open literature which are relevant to the development of the revolving vane expander are reviewed to show the state of the art of the research in this area. The publications reviewed cover the following topics: (1) the impact of refrigeration systems on the environment; (2) the carbon dioxide refrigeration systems, where expanders are first proposed to be used; (3) existing expander designs and analysis; (4) the revolving vane mechanism; and (5) other previous works that are related to the development of the revolving vane expander.

2.1. Refrigeration Systems and the Environment

The world energy consumption had grown steadily in the past decades. In 2010, the overall consumption grew by 5.5% [24] while electricity consumption, which accounts for about 40% of the total energy consumption [25], increases by 6.4% [24]. This growth is driven mainly by the growing demand of the developing countries, particularly China (the world's largest energy consumer in 2010) and India (the third largest, below the US). In Singapore, the electricity consumption increases by 9% in 2010 [26] with about 20% of the electricity being consumed by households [27].

In 2009, about half of the household electricity consumption in Singapore is for air conditioning and refrigerators (AC&R) [28, 29]. In Guangzhou, China, AC&R contributes 30% of the total household electricity consumption in 2003 [30]. In the US, cooling and refrigeration contributed about 24% of the total electricity consumption of commercial buildings, while air conditioning consumed more than US\$25 billion or about 20% of the total household electricity consumption in 2005 [31]. In the UK, the

electricity consumption of AC&R applications contributed about 8% of the total greenhouse gases emissions in 2008 [32]. Globally, AC&R applications account for about 15% of the world electricity use [33].

Most of the electrical energy used to operate AC&R systems is, and will still be for the foreseeable future, produced from fossil fuels which produce greenhouse gases, mainly CO₂. This accounts for about 80% of the total greenhouse gases emissions of AC&R while the rest is from the release of refrigerants into the atmosphere [34]. In accordance to the call for the curbing of these gases by the Kyoto Protocol [2], and subsequently by the Copenhagen Accord [1], it is therefore important to increase the efficiencies of the AC&R systems. One of the ways is to recover the power loss during expansion by replacing the conventional expansion valve with an expander, the idea that was initially proposed to increase the efficiency of the CO₂ refrigeration system [7]. This idea will be discussed further in the subsequent sections.

Synthetic refrigerants used in most AC&R systems also damage the environment when discharged into the atmosphere. History shows that until around the 1940s, the three main refrigerants used are all natural gases, namely ammonia (NH₃), sulphur dioxide (SO₂) and carbon dioxide (CO₂) [7]. These systems functioned satisfactorily and the engineers of that time were not aware of any particular problem. However, with the introduction of R-12, the use of these refrigerants died out. Unfortunately, not long after that, in the 1970s, it was found that R-12 is destructive to the ozone layer [35, 36]. Around 10 years later, in 1987, the Montreal Protocol [3] was signed to ban the use of the substance. Alternative substances which fall under the categories of hydrochlorofluorocarbons (HCFCs), like R-22 and hydrofluorocarbons (HFCs), such as R-134a, have since been used as replacements for CFCs due to their less harmful environmental impact but appropriate thermophysical properties with CFCs. However,

since HCFCs still contain chlorine, and therefore can still damage the ozone layer, the Montreal Protocol had also called for the phasing out of the substance by the year 2020. Non-chlorine substances like HFCs and hydrofluoroethers (HFEs) have been found to have their own environmental impacts too [37, 38]. For example, although R-134a and R-410A, which are HFC refrigerants, have almost zero ozone depletion potential (ODP), they have around 1300 times and 1989 times, respectively, the global warming potential (GWP) of CO₂ [4, 37]. They have to be phased out in accordance with the spirit of the Copenhagen Accord.

Even more recently, a new synthetic refrigerant, called HFO1234yf, was introduced [39]. It is reported to be very environmentally friendly (low GWP, zero ozone depletion and excellent Life Cycle Climate Performance) and to have thermophysical properties that are very similar to R-134a which allows the refrigerant to be readily used in existing systems [40]. Preliminary tests with beverage coolers and room air conditioner units have shown that the performance of the refrigerant is comparable to R-134a [41] and R-404A [42]. On the down side, this substance is found to be mildly flammable and is potentially dangerous for humans due to its production of fluoric acid when ignited [43]. It is also suspected to damage the environment with the trifluoroacetic acid that is produced when the refrigerant is decomposed in the atmosphere [44]. In addition, learning from the experience with CFCs, there is a growing scepticism in the long term and still unknown impacts of HFO1234yf in particular and synthetic refrigerants in general [45].

2.2. Carbon-Dioxide Refrigeration System and its Energy Efficiency

In the light of the growing scepticism and doubts about synthetic refrigerants, natural refrigerants are promoted to be used again. It was Lorentzen who in the 1990s

called for the reassessment of the natural refrigerants that were used before the invention of R-12 [46]. In particular, he believed that CO₂ is the most promising refrigerant, calling it a near ideal refrigerant. His arguments include, among others, that CO₂ is natural, safe and economical. It also has appropriate thermodynamics properties, especially when used in a transcritical cycle, resulting in a “gliding” temperature profile as shown in Figure 2.1 which is desirable in many applications [7].

Figure 2.1 Temperature-entropy chart of a transcritical CO₂ heat pump [7]

Lorentzen and Pettersen [8] built a car air conditioning system based on the transcritical CO₂ refrigeration cycle. The system had to operate in a transcritical cycle due to the high ambient temperatures usually encountered in a car air conditioning system. The test results indicated that the transcritical CO₂ refrigeration system performed at least as well as the standard R-12 system. It was found that the CO₂ system had better compressor efficiency, due to the low pressure ratios of the system, but greater heat rejection and throttling losses. It was also found that if the CO₂ system

were forced to run with the common refrigeration cycle (instead of the transcritical cycle), its COP dropped to less than half of the standard R-12 refrigeration system. It was noted that effective sealing is very important due to the large pressure differentials in the CO₂ systems.

Neksa [47] compared a CO₂ unit with a conventional R-22 unit. He found that the COPs of the CO₂ and the R-22 systems are comparable. The difference is mainly attributed to the heat exchanger design issues. Brown *et al.* [48] evaluated the performances of CO₂ and R-134a automotive air conditioning systems using semi-theoretical cycle models. Their results show that the CO₂ system had an inferior COP as compared to the R-134a system. The COP disparity depends on the compressor speed (system capacity) and the ambient temperature. The higher the speed and ambient temperature are, the greater the COP difference is. Rigola *et al.* [49] compared the performance of a transcritical CO₂ system with a conventional R-134a system experimentally. They found that the COP of the CO₂ system was about 10% lower than that of the R-134a system. Girotto *et al.* [50] analysed the refrigeration system of an “all-CO₂” supermarket in the north of Italy. A comparison with a R-404A system was done. It was found that the CO₂ system had lower energy consumption than the R-404A system if it was used in colder ambient temperatures. However, the CO₂ system was less efficient in warmer climate regions. In terms of cost, the all-CO₂ system is currently about 20% more expensive than a R-404A system. This is mainly because of the higher costs of the condensing unit and the evaporator which must be specially designed to withstand the high pressures of a CO₂ system. It is interesting to note, however, that the system costs less in terms of piping and refrigerant.

From this brief review of the open literature, it can be seen that one of the main challenges facing the CO₂ refrigeration system is to achieve a competitive efficiency.

This improvement in efficiency is crucial for CO₂ refrigeration systems to break into the market, to balance the increase in the capital cost. This is especially true for the heat pump market where gas boilers, the more established heater system, have much lower capital and fuel costs [17]. It is interesting to note that even with the current design, Neksa *et al.* [51] showed that although a heat pump is more expensive than a gas boiler, by employing a CO₂ heat pump system, the energy consumption can be reduced by 75% as compared to the electrical or gas-fired systems. Commercial CO₂ heat pumps have also been produced in Japan under the brand name “Eco Cute” [10, 11].

In order to achieve the competitive efficiency, studies have been conducted to better understand the CO₂ refrigeration system. Suess and Kruse [52] studied the performances of an open-type reciprocating compressor and a wobble plate compressor when used with CO₂. Among all the findings, it was found that the heat transfer between the gas and the cylinder had only negligible influence on the system performance while leakage affected the efficiencies significantly. The high significance of leakage was also observed by Ooi [53]. It was found that leakage can reduce the COP by as high as 16%. Liu *et al.* [54] discuss how different parameters affect the performance of a CO₂ automotive air conditioner experimentally. Among others, they found that the COP was affected by the type of lubricant used, CO₂ charge, discharge pressure, evaporator outlet pressure, compressor speed and ambient air conditions.

There have been attempts to improve the performance of the CO₂ system by optimising the heat rejection pressure [55, 56] and two-stage compression with intermediate cooling [57]. However, since throttling loss is one of the most significant losses in a transcritical CO₂ system, it is crucial to reduce this particular loss. Casson *et al.* [58] tried to achieve this by controlling the gas cooler pressure by using two expansion valves. However the controlling mechanism is complicated and the

performance is difficult to predict. Ejectors have also been proposed, increasing the COP of the system by up to 22% [59, 60].

Another way to minimise the throttling loss is to replace the expansion valve with an expander [7]. An expander improves the COP of the system in two ways: 1) by increasing the cooling capacity through performing a near-isentropic expansion, hence reducing the enthalpy of the refrigerant at the evaporator inlet, and 2) by recovering the expansion energy, hence reducing the power requirement of the compressor [61]. Robinson and Groll [15] are among the first who investigated the use of expanders to improve the efficiency of a transcritical CO_2 refrigeration system. The effect of the expander to the system can be seen in the temperature-entropy diagram shown in Figure 2.2. It was found that the installation of an expander with a 60% isentropic efficiency improves the COP of the system by about 20%. The COP of this system is between 87%-146% of the COP of a conventional R-22 system, depending on the evaporation temperature. Interestingly, it was found that a CO_2 system with an internal heat exchanger and an expander had around 8% lower COP than a system with an expander alone. Salim [62] also found that the transcritical CO_2 system performed better with an expander only (without an internal heat exchanger).

Figure 2.2 Temperature-entropy diagram of a transcritical CO_2 system [15]

2.3. Existing Expander Designs and Analysis

An expander can be seen as a compressor operating in reverse. Therefore, any compressor can theoretically be used as an expander as can be seen from the literature that will be reviewed in this section. However this conversion is usually not so straightforward because of some important issues that must be addressed when designing an expander. One of them is the two-phase expansion process. Unlike compressors which usually avoid wet processes, expanders must deal with it and hence must be handled differently. Fukuta *et al.* [63, 64] investigated this phenomenon and found that the process happened under a thermodynamically equilibrium condition. This finding simplified the modelling of the expansion process significantly. The two-phase nature of the working fluid also calls for the internal leakage model of an expander to incorporate the effects [65].

Another issue about expanders that must be specifically addressed is the suction and discharge control. Many compressors utilise pressure difference to control the opening and closing of the suction and discharge valves. However these mechanisms cannot work for expanders since the pressure in the working chamber must be controlled by the suction and discharge valves, not the other way around. The simplest method may be to use externally controlled valves such as the solenoid or the pneumatically activated valves. However, the time delays inherent to these types of valves may cause problems [16, 66, 67]. Therefore, it is common to find different and novel mechanisms to control the suction and discharge processes as will be discussed in the literature reviewed subsequently. Other issues in the expander that need special attention include the sealing, cavitations, liquid slugging, lubrication, vibration and noise control problems [68].

Matsui *et al.* [69, 70] manufactured a two-stage rotary expander. The arrangement eliminated the need of a suction valve, while no discharge valve was needed in a rotary expander. The optimised design showed an expander efficiency of 60% and the addition of this expander improved the COP of the system by 6%. Yang *et al.* [71, 72] also investigated a two-stage rolling piston expander and showed that the system can function properly. Li *et al.* [73] tested a rolling piston expander with a mechanically controlled valve. It was found that the expander can increase the efficiency of the system by 19%. It is also noted that the heat caused by the friction of the expander may affect the cooling capacity of the system. Kohsokabe *et al.* [74] investigated how their expander-compressor unit (ECU) affected a CO₂ transcritical refrigeration system. The compressor is of the rolling piston type while the expander is of the scroll type. This compressor is used as a secondary compressor. They found that by installing their ECU, a 30% improvement in the COP can be obtained. Hua *et al.* [75] built a rolling piston expander and obtained a 30% increase in the COP of the CO₂ refrigeration system.

Yang *et al.* [76, 77] developed a rotary vane expander. The design characteristics of the rotary vane machine removed the requirement for inlet and outlet valves. However, as is the common problem of a rotary engine, leakage was a major issue. This problem is more serious in a transcritical CO₂ system since the operating pressure difference is large. It was found that the main contributor was the leakage through the sealing arc. They tried to overcome this problem by using different types of vane. It was found that normal shape steel vanes perform better than normal shape self-lubricating material vanes and sharp tip steel vanes. They also found that the internal leakage decreases when the rotating speed is higher. However, friction is higher with a higher speed. In another paper, Yang *et al.* [78] found that other than the sealing arc, the leakage through the end gap is also significant, while the friction losses due to the vanes are the main

source of the power loss. The theoretical studies [79] found that both the volumetric and the isentropic efficiencies of the expander can be improved by minimising the end and the sealing arc gaps. They also proposed to install springs to ensure the vane tip always touches the cylinder inner wall [80]. However, this method created a large frictional loss. Therefore, in another paper, Jia *et al.* [81] proposed to introduce high-pressure gas into the vane slots instead of the springs. This method was found to successfully minimise the leakage and at the same time, increasing the isentropic efficiency of the expander. The expander was also reported to increase the COP of their CO₂ refrigeration system by up to 27%. In yet another paper, Fukuta *et al.* [82] built a vane-type compressor-expander unit which improved the COP of the CO₂ system by 50%.

Kim *et al.* [83], developed a scroll expander-compressor unit. The expander had a total efficiency of 54.4% and the unit increased the COP of the transcritical CO₂ system by 23.5%. Westphalen and Dieckmann [84] also investigated the application of a scroll expander in a CO₂ system. The expander improved the system efficiency by 20% or more and was crucial to make the CO₂ system power competitive with a HFC-based system. They also found that a large portion of the expander work is associated with intake flow work. In another paper, Westphalen and Dieckmann [85] reported that the leakage associated with the expander was not as large as predicted. This was thought to be caused by the presence of undissolved oil which helped to seal the gaps. This oil also helped to reduce friction. Hiwata *et al.* [86] investigated the use of overexpansion and oil-film pressure to reduce the scroll expander leakages in the axial and radial directions. The methods resulted in 96% volumetric and 62% isentropic efficiencies. A back-to-back scroll compressor-expander unit has also been developed [87-89]. The compressor was used as a second stage compressor in the system.

Kovacevic *et al.* [90] investigated the screw compressor-expander unit. It was found that by combining the compressor and expander, hence reducing the number of bearing pairs from 4 to 2, the bearing mechanical losses were reduced. This was important especially with high pressure applications for instance in the transcritical CO₂ systems. Huff and Radermacher [91] compared the performances of the reciprocating, rolling piston, screw and scroll expanders. They concluded that the screw expander was the most desirable because it was efficient and required only simple control of the inlet and outlet ports.

Baek *et al.* [66, 67, 92] built a piston-cylinder expansion device using a modified four-cycle, two-piston gasoline engine that was commercially available. Fast-acting solenoid valves were used to control the inlet and outlet of the working fluid. They reported that the time delay associated with the valves made the inlet and outlet flow control difficult. The measured isentropic efficiency was found to be about 11% while the improvement in the COP was as high as 10.5% when the recovered work from the expander was used to drive the compressor in addition to the near-isentropic expansion the expander had provided. The low efficiency was thought to be caused by internal leakages. Their simulation results also show that if the expander runs at too high speeds (i.e. at more than 360 rev/min), there is not enough time to fill the expansion chamber and hence the performance will be compromised. Nickl *et al.* [61] developed a three-stage piston-type expander. The design removed the need of sophisticated control of the suction valve. This expander was used to drive the second compressor in a two-stage CO₂ compression system with intercooling. It was found that the expander isentropic efficiency can reach up to 70% and the improvement in the COP of the system when compared to the same system without an expander was more than 40%.

A free piston expander has also been built for the transcritical CO₂ refrigeration system [93-96]. The suction and discharge processes were controlled by a specially constructed mechanism. It used a slider-based control scheme, utilising the geometrical characteristics of the expander mechanism. The theoretical model indicated that the expander had a 69.2% indicated efficiency and a 51.8% adiabatic efficiency. The model also shows that losses due to flow, clearance volume, leakage and heat transfer were the key factors influencing the efficiency of the expander. Later, the model was modified to a double-acting expander [96]. Experimental tests were done with air as the working fluid. It was found that the concept works stably in a wide range of pressure difference. It was also found that when operating at higher frequencies (more than 22.6 Hz), the expander performance dropped. This was mainly due to the insufficient gas filling at the suction stage. Their tests also indicated that the expander can obtain an isentropic efficiency of 62%. The power losses due to the pressure drop in the discharge process and due to under expansion were the dominant losses, which accounted to 8.4% and 10.3% respectively. The power loss due to suction process was only around 3.1%.

Guan *et al.* [68] investigated the performance of a swing piston expander. A mechanical actuation system was used to control the inlet valve by controlling the stroke of the cam. The expander was directly connected to a motor-generator unit. The efficiency of the expander was found to be 28% to 44%.

Turbines have also been proposed to improve the performance of the CO₂ refrigeration system. Tondell [97] studied the use of the impulse turbine for CO₂ systems, while He *et al.* [98] studied the Pelton-type expander. The improvement of the COP caused by the Pelton turbine was 5.4%.

Other than the design, the way an expander is placed inside a refrigeration cycle also affects the effectiveness of the expander. According to Yang *et al.* [76], a two-stage

compression with the expander driving the main (high pressure) compressor was the most effective configuration as compared to the other arrangements they investigated which include: single-stage compression without expander, single-stage compression with an expander, two-stage compression at optimal intermediate pressure (the expander is connected the low pressure compressor using a transmission mechanism) and two-stage compression with expander driving the auxiliary (low pressure) compressor arrangements.

Although expanders are mainly proposed to be used for CO₂ refrigeration systems, they can also be used for conventional systems. Badr *et al.* [99, 100] investigated the sliding vane expander to be used in a R-113 refrigeration system. Robinson and Groll [15] reported a 15% increase in the efficiency of a conventional HCFC-12 system with the installation of an expander. Goncalves and Parise [16] showed that an expander can improve the COP of a R-134a refrigeration system by around 12%. Tamura *et al.* [101] showed that a screw expander can improve the COP of a heat-pump system up to almost 25%. Soo *et al.* [67] showed that replacing the expansion valve with a turbo expander can improve the R-134a refrigeration system performance by increasing the cooling capacity. He *et al.* [98] found that their Pelton-type expander can increase the COP of the R-410A domestic refrigeration system by 5%. Cho *et al.* [102] also proposed the use of a turbo expander to improve the performance of a R-134a refrigeration system.

Other than for refrigeration applications, expanders are also commonly used in engine and other energy generation applications. The most commonly used expander design in engine applications is obviously the reciprocating piston-type expander. It is used in most commercial engine vehicles [103]. The Wankel engine has also been proposed as an alternative in recent years [104]. Shen and Hwang [105] investigated the use of a rotary vane expander for a compressed air vehicle, which emits zero emission

[106]. Expanders are also used for air motors. Zhang and Nishi [107] studied the use of a centrifugal expander while Hwang *et al.* [108] employed the rotary vane mechanism for such an application.

To conclude, there are numerous expander designs that are currently being researched. However, there is still a lot of room for improvements, especially in the efficiencies and suction flow control of the expanders. The potential applications of the expander are not limited to the transcritical CO₂ refrigeration system either. Instead, they also include the conventional refrigeration systems, engines, energy generating devices and air motors, among others.

2.4. Revolving Vane Machine Design

After reviewing numerous studies on expander designs in the previous section, a review of the literature on the Revolving Vane (RV) machine is presented in this section. The RV mechanism design was first introduced by Teh and Ooi [23] in 2006. The design is protected under the international patent application [18]. This design promises to keep the advantages of rotary compressors, namely the compact nature and the good vibration characteristics, while reducing the frictional losses caused by the rubbing process of the components by reducing the relative velocities of the rubbing components. In its basic form, this compressor comprises a rotor, a cylinder and a vane. The vane is assembled with the rotor such that it is allowed to slide within a slot. During operation, the rotation of the rotor revolves the vane which in turn rotates the cylinder. The motion causes the volumes trapped within the rotor, vane and cylinder to vary, resulting in the suction, compression and discharge of the working fluid [19]. Some illustrations of the design are shown in Figure 2.3.

Figure 2.3 The RV compressor prototype by Teh and Ooi [19] and the dimensions:

(a) cutaway, (b) front and (c) side sectional views

In the subsequent development, many aspects of the mechanism have been studied and analysed. The frictions of the compressor have been analysed [19]. The focus was on the friction losses at the vane sides, the journal bearings, the vane tip and the cylinder endfaces. It was found that the RV compressor generated 20% less frictional losses as compared to a rolling piston compressor. It was also found that the vane side friction and the bearing friction were the significant frictional losses in a RV compressor, accounting to about 66% and 30% of the total losses, respectively.

The RV compressor has a discharge valve and no suction valve. A study on the discharge valve dynamics was necessary because unlike most rotary compressors, the valve used in the RV compressor is not stationary. Instead, it rotates together with the

cylinder. Teh *et al.* [20] found that the centrifugal force affected the valve more significantly as compared to the Coriolis force. It was also found that the centrifugal force improved the performance of the valve compared to the commonly encountered non-rotating valves. However, it is important to design the valve carefully to avoid centrifugal overloading and resonance.

From the study on the leakage loss of the RV compressor [109], it was found that the compressor exhibited a 40% reduction in leakage loss as compared to that of the rolling piston compressor. The parametric studies also indicate that a fat and short compressor has better volumetric efficiency as compared to a slim and long one. This is simply because a short compressor means a short radial clearance, hence a smaller radial clearance area reduces leakage through this area. However, this is in contrast with the mechanical efficiency behaviour found in the study of RV compressor friction losses [19] which shows that a slim and long compressor has a better mechanical efficiency compared to that of a fat and short one. Therefore, there is an optimum configuration which optimises both efficiencies.

A prototype has also been built and studied by Teh and Ooi [110, 111]. The prototype was operated up to a speed of 1200 rev/min and a discharge pressure of 8 bar without any problem. The tests confirmed that the machine design can function successfully. The theoretical models have been validated with the experimental results and were found to be acceptably accurate. The heat transfer [112] and the lubrication system [113] had also been studied. Zhou *et al.* [114] investigated a very similar mechanism called the synchronal rotary compressor. They found that the relative velocities of the mechanism are reduced to less than 20% of the conventional rotary machines. They also warned that the stress on the vane is large.

About two years after its introduction, an improved RV design mechanism, called the RV-i mechanism, was proposed by Ooi and Teh [21]. In this improved design, the vane is rigidly attached to the rotor. The vane slot is now located at the cylinder and its shape is designed such that the vane can move in the slot freely. Some illustrations of the RV-i compressor design are shown in Figure 2.4. With this design, the vane side friction was reduced significantly [22]. The overall reduction in the frictional loss as compared to the RV-compressor was about 30%. A study on the leakage losses of the RV-i compressor had also been done [115]. In agreement with the RV compressor, the RV-i compressor exhibited smaller radial clearances as compared to a rolling piston compressor. This resulted in a 40% reduction of the leakage loss. It was also found that the variation amplitude of the radial clearance in the RV-i compressor was smaller than that of the rolling piston. Therefore, it is possible to reduce the leakage loss of the RV-i compressor even further by reducing the assembly radial clearance.

A more comprehensive discussion on the RV-i mechanism can be found in Chapter 3 of this thesis. To facilitate subsequent discussions, the original RV machine design will be called the RV-0 design in this thesis to distinguish it from the RV-i design.

Figure 2.4 Sectional front and side views of the RV-i compressor [21]

2.5. Other Studies Related to RV Expander Development

The RV expander being developed in this project has many aspects, such as the dynamics, internal leakage, heat transfer, shaft bearings and lubrication. Numerous relevant studies are available to help in the studies of these aspects. They are reviewed in this section.

The dynamics of the RV expander can be analysed following the literature on the RV mechanism reviewed in Section 2.4. Other literature that may be helpful include the dynamics analysis of the rolling piston compressor by Yanagisawa and Shimizu [116], Matsuzaka and Nagatomo [117], Ooi and Wong [21] and Ooi [53, 118]. It is interesting to note that in all of these studies, the vane thickness is usually assumed to be negligible. The vibration analysis of the rolling piston compressor by Yanagisawa *et al.* [119] can also be adopted to study the vibration of the RV expander.

The RV expander will have internal leakages. These can be studied following the works of Yanagisawa and Shimizu [120, 121] who investigated the internal leakages of the rolling piston compressor. They focused on the leakages through the radial clearance and the lubricant paths. The leakage through the radial clearance was modelled by integrating the convergent nozzle and Fanno flows. The works of Pandeya and Soedel [122], Mahmoud *et al.* [65] are also useful.

The heat transfer between the working fluid and the expander walls can be analysed following the methods proposed by, among others, Tan and Ooi [112], Ooi [123], Ooi and Zhu [124] and Padhy and Dwivedi [125]. One of the useful assumptions to simplify the analysis is that the walls have high thermal masses and hence they are less susceptible to temperature fluctuation as compared to the working fluid. When two-phase fluids are involved, depending on the type of fluid used, the heat transfer correlations proposed by Agrawal and Bhattacharyya [126], Dan and Hihara [127], Lee

and Lee [128], Kandlikar [129] and Jung *et al.* [130] may be useful. The two-phase fluid flow and viscosity can also be modelled according to the methods proposed by Wong and Ooi [131, 132].

The RV expander has two rotating components, namely the rotor and the cylinder. The shafts of both components must be supported by bearings to function properly. There are many types of bearings available. However, journal bearings are commonly used. This type of bearing consists of only a journal (shaft), a bearing sleeve and the lubricant is used to lubricate the gaps between these two components. They can usually be self-made and therefore, is usually lower in cost as compared to other types of bearings such as roller or needle bearings. They are also known to have low wear, low noise and good damping characteristics. To ensure a smooth operation, the bearing must be designed properly. Hirani *et al.* [133] proposed a step-by-step procedure to design a journal bearing for static operating conditions. The method is based on solving the Reynolds equation (Equation (2.1)) using both the infinitely short and infinitely long bearings assumptions. The lubricant film pressure is then estimated based on a harmonic average of the pressures from the infinitely short and infinitely long bearings assumptions proposed by Reason and Narang [134]. The procedures employed in the prediction for the pressure rise at the lubricated gap of the bearings requires the oil feed pressure information which is approximated using the method proposed by Martin and Lee [135]. The friction loss is assumed to be a function of the shear of the oil film and the hydrodynamic film pressure. This procedure is able to predict the effect of lubricant temperature to the bearing performance. However, the main limitation of the procedure is its static operating conditions assumption because most machines usually have dynamic (or non-static) loading and operating conditions.

$$\begin{aligned}
& \frac{\partial}{\partial \theta} \left((1 + \varepsilon \cos \theta)^3 \frac{\partial p}{\partial \theta} \right) + r_j^2 \frac{\partial}{\partial z} \left((1 + \varepsilon \cos \theta)^3 \frac{\partial p}{\partial z} \right) \\
& = 12 \mu_{lubr} \left(\frac{r_j}{C_b} \right)^2 \left(\dot{\varepsilon} \cos \theta + \varepsilon \left(\dot{\phi} - \frac{\omega}{2} \right) \sin \theta \right)
\end{aligned} \tag{2.1}$$

To overcome this limitation, Hirani *et al.* [136] proposed a procedure to design a dynamically loaded journal bearing. The procedure is iterative in nature and requires the complete history of the load and the operating conditions to be known beforehand. The Newton-Rhapson method [137] is used to ensure that the iteration process is convergent. At the end of the procedure, the profiles of the eccentricity ratio and the attitude angle of the bearing are obtained. This procedure was shown to be relatively accurate and was faster as compared to the more established finite element method (FEM). In one of the cases tested, the minimum film thickness obtained by the proposed method differed from the one obtained with FEM by 7% while the computational time was shortened from 4090 to 19 s. However, the need to have the complete history of the load and the operating conditions beforehand is a major limitation of this procedure. It prevents the prediction of a “real-time” interaction between the bearing and the operating conditions. The procedure was then developed further by replacing the Newton-Rhapson method with a method developed by Hirani *et al.* [138] which links the eccentricity ratio and attitude angle directly to the lubricant pressure. Later, the effects of temperature were also included by Hirani *et al.* [139].

The RV expander will need a lubricating fluid to lubricate the rubbing components, to seal the clearance gaps and to provide cooling mechanism for the components. The lubrication system of the RV expander can be designed according to the method developed by Kim *et al.* [83, 140, 141]. They modelled the lubrication with an equivalent electrical circuit diagram. The electrical current represents the lubricant flow rate, the electrical resistance represents the flow friction and the voltage source

represents the lubrication pump. The choice of the lubricant type will depend on the working fluid used. The works of Dang *et al.* [142] and Shen and Groll [143, 144] may be useful to advise on this issue.

REVOLVING VANE EXPANDER DESIGN CONCEPT

Following the review of the open literature about expanders and a brief introduction about the Revolving Vane mechanism design, a detailed discussion of the Revolving Vane expander design concept is presented in this chapter. The chronological development of the concept is first presented. The working principles are then explained, followed by the discussions of the unique suction valve design.

3.1. The Historical Development of the Revolving Vane Mechanism

3.1.1. The Rolling Piston mechanism

The Revolving Vane (RV) mechanism design was initially developed by Ooi and Teh [18] with the intention to eliminate the major contributors to the frictional losses in the rolling piston (RP) mechanism. An illustration of a rolling piston machine is shown in Figure 3.1.

Figure 3.1 The rolling piston mechanism [118]

A RP machine has two working chambers, the high pressure and the low pressure chambers. Each is bounded by the inner wall of the cylinder, the outer wall of the roller (rotor) and the vane. The vane and the line contact between the rotor and the cylinder separate the two working chambers. When used as a compressor, the roller (rotor) is driven by a driving mechanism to roll inside the cylinder. This causes the low pressure chamber (the suction chamber) to increase in volume and, at the same time, decreases the volume of the high pressure chamber (the compressor chamber). Low pressure fluid from outside the compressor will then be induced into the low pressure chamber, while the working fluid inside the high pressure chamber will be compressed and eventually discharged out of the compressor when the pressure inside the chamber is high enough to overcome the discharge valve's stiffness and the forces, if any, acting on the outside surface of the valve reed.

When used as an expander, the high pressure fluid inside the high pressure chamber (the suction chamber) drives the rotor in the direction which will cause the volume of the chamber to increase and the volume of the low pressure chamber (the discharge chamber) to decrease. This causes the high pressure fluid from outside the expander to be induced into the expander and eventually expanded. At the same time, the volume of the low pressure chamber is decreased, causing the low pressure working fluid to be discharged out of the expander. The expansion of the working fluid starts only when the suction valve is shut, which happens when the volume of the high pressure chamber has increased to a designed value. The expansion of the working fluid causes its pressure and temperature to decrease. This process happens until the discharge port is exposed to the high pressure chamber. After which the working fluid will be discharged out through the discharge port until the working fluid is completely discharged. The process is then repeated.

Frictional losses occur due to the rubbing of components, which comes about as a result of different moving velocities of the rubbing components. Table 3.1 shows the various contributions of these frictional losses of a RP compressor. The values are based on the results from Yanagisawa and Shimizu [116], which carried out the study with a compressor with a swept volume of 10.5 cm^3 operating at a speed of 3600 rev/min. The working fluid was R-22 with suction and discharge pressures of 5.83 and 20.3 bar, respectively. To overcome these frictional losses, there must not be any relative velocity at the rubbing surfaces. The subsequent sections show how the various frictional losses in the RP machine can be eliminated, which will eventually lead to the invention of the RV mechanism.

Table 3.1 A RP compressor frictional losses and their relative contributions

Location of Friction	Contribution
Vane sides	~30%
Vane tip	~30%
Endfaces	(negligible)
Bearings	~40%

3.1.2. Elimination of vane sides friction

The frictions at the vane sides exist due to the rubbing between the vane and its slot. The magnitude of the power loss is determined by the contact force between the surfaces and the rubbing velocity between the stationary slot and the vane. Therefore, to reduce the friction, we can either reduce the contact force or the relative velocity. However, the velocity of the vane is dictated by the operating speed of the machine and therefore, is not an option to be modified. The magnitude of the force itself is

determined by the pressure difference across the vane that presses it on the slot and therefore, is not an option to be modified either if the design remains the same. However, the vane contact force can be almost eliminated by fixing the vane onto the driving component of the mechanism and not to let it be freely pressed onto the slot by the pressure force, as shown in Figure 3.2. This has a significant effect on the frictional force and hence reduces the frictional loss at the vane side. This modification, however, causes the vane to turn together with the rotor, forcing the cylinder to also rotate together with the rotor. This modification will be addressed in the subsequent sections.

It is useful to note that although the vane and the piston are shown as one piece in Figure 3.2, they can be manufactured separately and then fixed together in reality.

Figure 3.2 The new rotor-vane design

3.1.3. Elimination of vane tip friction

The vane tip friction force exists because of the rubbing between the tip of the vane and the piston. This force can be minimised by lubricating the contact or by improving the surface finishing. However, the best way is to remove the contact completely by allowing the vane to move in the slot without touching the vane tip at all. A modified vane slot shape is needed and one example is shown in Figure 3.3.

Figure 3.3 The new vane slot design

3.1.4. Reduction of endfaces friction

The endfaces friction exists due to the rubbing of the surfaces of the endfaces of the piston, the vane and the cylinder. Ooi [118] found that this loss can contribute more than 25% of the total rolling piston friction loss. Lubrication is usually introduced to reduce this friction. However, the most effective way is to reduce the relative velocities of the rubbing components. This can be done by letting the cylinder to rotate together with the piston. Coincidentally, this rotating cylinder is also needed to accommodate the new rotor-vane design as described in Section 3.1.2. Combining the new rotor-vane design, the new vane slot and the rotating cylinder concepts discussed previously produces a new mechanism design as shown in Figure 3.4. It is worthy to note that since the cylinder is eccentrically positioned to the rotor, the rotational speed of the cylinder will still be slightly different from that of the rotor. This small speed difference between the cylinder and the rotor causes frictional loss at the endfaces. However, this loss is now much smaller as compared to the loss in the rolling piston design due to the much smaller relative velocity between the two sliding components.

The arrangement shown in Figure 3.4 also effectively eliminates the friction between the eccentric and the piston, which according to Ooi [118], can contribute up to 45% of the total RP machine frictional loss. This is because the new arrangement eliminates the need of the eccentric component and arranges the rotor shaft to be concentrically situated on the rotor.

Figure 3.4 The new mechanism design concept

3.1.5. Reduction in bearing friction

The frictional loss at the shaft bearing is dependent on the type of bearing used. One of the most commonly used bearing types is the journal bearing due to its simplicity and low cost. In this type of bearing, the frictional loss is due to oil viscosity and the load. One simple way to minimise this loss is to reduce the shaft load. Fortunately, with the new design, the shaft load is reduced as the cylinder can be made shorter and fatter and hence this loss is also reduced. The reduction in the shaft load is also due to the elimination of the vane tip contact force.

3.1.6. The Revolving Vane (RV) mechanism

In order to make the new mechanism illustrated in Figure 3.4 to function, suction and discharge ports must be added. In addition, because the cylinder rotates, the suction port revolves and the suction tube cannot be connected directly to the suction port. Instead, the expander should be put inside a stationary casing which acts as a suction reservoir and the suction tube is connected to the casing according to Figure 3.5. The discharge port runs through the rotor shaft and hence it rotates but not revolves. Therefore, it can be connected to a discharge reservoir first or coupled directly to the

discharge tube. Finally, putting all the components together, the Revolving Vane (RV) mechanism was born. Illustrations of the mechanism when used as an expander are shown in Figures 3.5 and 3.6.

Figure 3.5 Front sectional view of the Revolving Vane expander design concept

Figure 3.6 Side sectional view of the Revolving Vane expander design concept

3.1.7. RV mechanism design configurations

In the previous sections, the vane has always been shown to be fixed onto the rotor and the rotor has been shown to be the driving component as shown in Figure 3.7(a). However, this is not the only RV mechanism design configuration available. There is also the original RV mechanism design proposed the first time by Teh and Ooi [23]

where the vane is not rigidly fixed onto any of the rotor and the cylinder. Moreover, when the vane is rigidly fixed to either the rotor or the cylinder, there are at least three more RV mechanism design variants. They are:

- The vane is fixed onto the rotor, the cylinder is the driver (Figure 3.7(b))
- The vane is fixed onto the cylinder, the cylinder is the driver (Figure 3.7(c))
- The vane is fixed onto the cylinder, the rotor is the driver (Figure 3.7(d))

A comparison of the four design variants when used as expanders is presented in Chapter 7. It is found that the design configuration where the vane is fixed onto the rotor and the rotor is the driving component is the most desirable RV expander design configuration from the performance point of view. Therefore, this is the RV expander design configuration that is used in the subsequent discussions in this report.

Figure 3.7 The four RV mechanism alternative design configurations

3.2. The Working Principles of the Revolving Vane Expander

The operating process of a Revolving Vane expander is illustrated in Figure 3.8. Figure 3.8(a) shows the end of expansion and thus the beginning of suction and discharge. At this moment, the pressure inside the suction chamber is high and thus the pressure difference across the vane pushes the vane (and the rotor) to move in the direction of reducing the volume of the discharge chamber. This movement causes high-pressure fluid to flow into the suction chamber and, at the same time, low-pressure working fluid to be discharged, as shown by Figure 3.8(b).

At a designated point, the suction valve is then closed as shown in Figure 3.8(c). This marks the end of the suction process and the beginning of the expansion process. The selection of this point and the response time of the valve are very important to ensure that there is no over-expansion or under-expansion. Another way is to use two expanders in series instead of a suction valve [72]. After the closing of the suction port, the working fluid inside the suction chamber is now being expanded, as shown by Figure 3.8(d). The cycle is then repeated.

Figure 3.8 The operating process of the Revolving Vane expander

3.3. Suction Valve

As has been mentioned in the previous sections, the design of the suction valve is very important because the expansion process of the expander is dependent on it. If the valve is not designed properly, over-expansion or under-expansion may happen, thus reducing the performance of the expander. However, it is also challenging to design the valve properly. This is because the valve cannot be operated based on pressure difference, unlike that in a compressor. Instead, it must be externally controlled.

The valve to be designed must open during suction and close during expansion. Suction should ideally start at the start of the cycle (Figure 3.8(a)). However, it can also start later to accommodate the suction port, but it cannot start earlier or else the high pressure working fluid will flow into the discharge chamber instead of the suction chamber and hence, preventing the expander from working the way it should. The valve should then be left open (Figure 3.8(b)) until the designated point at which the expansion should commence (Figure 3.8(c)). Then the valve should be closed (Figure 3.8(d)) until the end of the expansion process where the suction process (and the discharge process) is again started and the whole cycle is then repeated.

Meanwhile, the discharge port does not need a valve since the pressure inside the discharge chamber is always at the discharge pressure and the discharge flow occurs only due to the reduction of the discharge chamber volume as the expander rotates.

A solenoid valve can be used but the reaction time may cause some problems [66] and the rotational movement of the cylinder and the suction port may cause the wire to suffer from wear and tear. The two-stage mechanism with the smaller expander used for suction and the larger expander for expansion as shown in Figure 3.9 can also be used. This arrangement removes the need of a suction valve and at the same time, increases the volume flow rate since the full volume of the expander is now used. However, the

capital cost of this arrangement is expensive because two expanders are needed, instead of one. Another way is to design a mechanically controlled suction valve which will solve the problems encountered by the solenoid valve and the two-stage arrangement. One of the design concepts of such suction valves will be presented in the following paragraphs. The other design concepts can be found in Appendix A.

Figure 3.9 The two-stage Revolving Vane expander arrangement

The design concept of the RV expander suction valve presented here uses the relative movement between the expander and the stationary casing. This is done by having stationary blockers attached to the stationary casing of the expander. As the expander turns, the relative position of a point at the expander with respect to the stationary casing keeps changing. The blocker is designed such that as the expander turns, when the suction port is supposed to be closed, it is blocked by the blocker and when it is supposed to be open, the suction port is exposed (Figure 3.10). Ideally, the blocker should not be in direct contact with the expander to minimise friction. However, the separation gap should also be kept to be as small as possible to minimise leakage. The gap should also be sealed with lubrication and therefore the blockers can also be used as bearings if wished.

Figure 3.10 Working cycle of the RV expander suction valve concept:

- (a) suction begins, suction port is almost open,
- (b) suction continues, suction port is fully open,
- (c) expansion starts, suction port is almost closed,
- (d) expansion continues, suction port is fully closed

The shape of the blocker can be of various shapes, such as a sectioned cylinder (Figure 3.11(a)) or a cylinder with a slot (Figure 3.11(b)).

Figure 3.11 Alternative blocker shapes of the suction valve

In addition, the suction port can be moved to other locations such as at the rotor shaft or the endfaces. Therefore, the location of the blockers depends on the location of the suction port too. Some possibilities are shown in Figure 3.12.

Figure 3.12 Alternative blocker locations of the suction valve concept:

(a) at cylinder, (b) at rotor shaft, (c) at cylinder endface

In order to minimise the contact area (in other words, to reduce the frictional loss), the suction port can be modified to be protruding out from the expander as shown in Figure 3.13. With this arrangement, the contact area is now reduced to only the suction port and its wall instead of the outer wall of the cylinder as what is shown in Figure 3.10. However, the main disadvantage of this arrangement is that if the thickness of the suction port protrusion wall is not long enough, leakage path will be very short and hence, leakage may become a major issue.

Figure 3.13 Protruding suction port of the RV expander suction valve design concept

The protruding part of the suction port can also be made to bend upwards or downwards instead of straight like in Figure 3.14. In this case, the blocker needs to be modified to close the port from the top or bottom, acting like a ‘roof’.

Figure 3.14 Modified suction port of the suction valve concept

The main advantage of this suction valve design concept is its simplicity and reliability. It will always open and close at the right positions. The weaknesses of this design include friction and possibility of leakage. The rubbing between the stationary blocker and the moving expander component will cause undesirable friction loss. If not manufactured properly, the gap between the suction port and the blocker may also cause leakage and compromise the reliability of the valve.

THEORETICAL MODEL: WORKING FLUID PROCESSES

Following the presentation of the RV expander design concept and working principles in Chapter 3, the theoretical model of the RV expander will now be presented. The model consists of the simulation models of the working fluid processes, which are presented in this chapter, and the expander components dynamics processes, which are presented in the subsequent chapters.

During operation, the working fluid undergoes thermodynamics, fluid mechanics and heat transfer processes. These processes are all simulated accordingly. The discussion will therefore include, among other things, the pressure, temperature and density changes of the working fluid inside the expander. The suction and the discharge flows, the suction valve, the internal leakage models and heat transfer between the fluid and the RV expander are also discussed in this chapter.

4.1. Thermodynamics Model

The RV expander has two working chambers, namely the suction/expansion and the discharge chambers. These chambers are bounded by the inner wall of the cylinder, the outer wall of the rotor, the cylinder endfaces, the vane and the line contact between the cylinder and the rotor. However, in modelling the thermodynamics processes of the expander, the vane slot should also be considered as a separate chamber. This is because it has distinct thermodynamics behaviours from the two working chambers due to it communicating with either one or both of the working chambers during operation, depending on the location of the vane contact. Therefore, three chambers are involved in the building of the model here. For simplicity, the suction/expansion chamber will

simply be called the suction chamber from this point onward. In equations, suction, discharge and vane slot chambers parameters are given subscripts scv, dcv and vs respectively. An illustration of the chambers is shown in Figure 4.1. Points C and R are the cylinder and rotor centres, respectively.

Figure 4.1 (a) Vane slot, discharge and suction chambers; (b) Relevant dimensions

As the expander operates, the volume of the suction chamber increases while that of the discharge chamber decreases. The vane slot volume also changes as the vane slides in and out of it. These volumes can be estimated with Equations (4.1) - (4.3). The definitions of the symbols used are shown in Figure 4.1(b). For illustration, the

expander prototype developed in this project and explained in Chapter 8 has a swept volume of 30 cm³, which is shared by both the suction and discharge chambers, while the vane slot volume is about 1.8 cm³.

$$V_{scv} = V_{dead,scv} + l_{exp} \left(\frac{\varphi_c}{2} r_c^2 - \frac{er_c \sin \varphi_c}{2} - \frac{\varphi_r}{2} r_r^2 - \frac{w_v(l_{ve1} + l_{ve2})}{4} \right) \quad (4.1)$$

$$V_{dcv} = V_{dead,dcv} + l_{exp} \left(\pi(r_c^2 - r_r^2) - \frac{\varphi_c}{2} r_c^2 + \frac{er_c \sin \varphi_c}{2} + \frac{\varphi_r}{2} r_r^2 - \frac{w_v(l_{ve1} + l_{ve2})}{4} \right) \quad (4.2)$$

$$V_{vs} = V_{vs0} - l_{exp} \left(w_v l_v - \frac{w_v(l_{ve1} + l_{ve2})}{4} \right) \quad (4.3)$$

The changes in the volumes of the chambers cause pressure, temperature and mass to change inside each chamber. The increase in the suction chamber volume expands the working fluid and causes fluid from outside to be induced in through the suction port if the suction valve is open. If the suction valve is closed, the increase in volume expands the fluid only. The reduction of the discharge chamber volume compresses the fluid inside and causes it to be discharged through the discharge port. In addition, fluids from both the discharge and suction chambers communicate with the fluid in the vane slot at some point during operation, causing changes to the fluids inside all the chambers. Other factors that influence the thermodynamics of the fluids include the suction valve dynamics, internal leakage and heat transfer between the fluids and the chamber wall.

The thermodynamics processes inside each chamber is time-dependent and can be analysed using the first law of thermodynamics for a control volume [145] as shown in Equation (4.4). The meanings of the symbols are illustrated in Figure 4.2.

$$\begin{aligned} & m_{cv} \frac{d \left(u_{cv} + \frac{v_{cv}^2}{2} + gz_{cv} \right)}{dt} + \left(u_{cv} + \frac{v_{cv}^2}{2} + gz_{cv} \right) \frac{dm_{cv}}{dt} \\ &= \frac{dQ_{cv}}{dt} - \frac{dW_{cv}}{dt} + \sum_i \left(h_i + \frac{v_i^2}{2} + gz_i \right) \frac{dm_i}{dt} - \sum_o \left(h_o + \frac{v_o^2}{2} + gz_o \right) \frac{dm_o}{dt} \end{aligned} \quad (4.4)$$

Figure 4.2 The meanings of symbols of the first law of thermodynamics

Since the changes in the heights of the control volumes are relatively small, we can assume that the potential energy terms are negligible. We can also assume that the kinetic energy of the control volume is negligible. Therefore, Equation (4.4) can be simplified into Equation (4.5).

$$m_{cv} \frac{du_{cv}}{dt} + u_{cv} \frac{dm_{cv}}{dt} = \frac{dQ_{cv}}{dt} - \frac{dW_{cv}}{dt} + \sum_i \left(h_i + \frac{v_i^2}{2} \right) \frac{dm_i}{dt} - \sum_o \left(h_o + \frac{v_o^2}{2} \right) \frac{dm_o}{dt} \quad (4.5)$$

Because there are three chambers involved in this analysis, Equation (4.5) must be applied to each of the three control volumes. Each control volume has different inlets and outlets. The suction chamber has the suction port and the internal leakage paths. The discharge chamber has the discharge port and the internal leakage paths. The vane slot has the vane gaps which may link to both the discharge and suction chambers. The relationship between the mass of the control volume and the inlet and outlet masses is shown in Equation (4.6).

$$\frac{dm_{cv}}{dt} = \sum_i \frac{dm_i}{dt} - \sum_o \frac{dm_o}{dt} \quad (4.6)$$

By definition, the rate of work done, internal energy and fluid density are as expressed in Equations (4.7) - (4.9).

$$\frac{dW_{cv}}{dt} = p_{cv} \frac{dV_{cv}}{dt} \quad (4.7)$$

$$u = h - p \left(\frac{V}{m} \right) \quad (4.8)$$

$$\frac{d\rho}{dt} = -\frac{m}{V^2} \frac{dV}{dt} + \frac{1}{V} \frac{dm}{dt} \quad (4.9)$$

Enthalpy is a function of both temperature and density, $h = f(T, \rho)$. Therefore, its rate of change is as expressed in Equation (4.10).

$$\frac{dh}{dt} = \left(\frac{\partial h}{\partial T} \right)_\rho \frac{dT}{dt} + \left(\frac{\partial h}{\partial \rho} \right)_T \left(-\frac{m}{V^2} \frac{dV}{dt} + \frac{1}{V} \frac{dm}{dt} \right) \quad (4.10)$$

Differentiating Equation (4.8) with time and using Equations (4.9) and (4.10) give the rate of change of the internal energy as expressed in Equation (4.11).

$$\frac{du}{dt} = \left(\frac{\partial h}{\partial T} \right)_\rho \frac{dT}{dt} + \left(\frac{\partial h}{\partial \rho} \right)_T \left(-\frac{m}{V^2} \frac{dV}{dt} + \frac{1}{V} \frac{dm}{dt} \right) - p \left(\frac{1}{m} \frac{dV}{dt} - \frac{V}{m^2} \frac{dm}{dt} \right) - \left(\frac{V}{m} \right) \frac{dp}{dt} \quad (4.11)$$

Because pressure is a function of temperature and density, the pressure derivative can also be expressed as shown in Equation (4.12).

$$\frac{dp}{dt} = \left(\frac{\partial p}{\partial T} \right)_\rho \frac{dT}{dt} + \left(\frac{\partial p}{\partial \rho} \right)_T \frac{d\rho}{dt} \quad (4.12)$$

By substituting Equations (4.6) - (4.12) into Equation (4.5), the rate of change of the working fluid temperature can be obtained according to Equation (4.13).

$$\frac{dT_{cv}}{dt} = \frac{\left(\frac{dQ_{cv}}{dt} + \rho_{cv} \frac{dV_{cv}}{dt} \left(\rho_{cv} \left(\frac{\partial h_{cv}}{\partial \rho_{cv}} \right)_T - \left(\frac{\partial p_{cv}}{\partial \rho_{cv}} \right)_T \right) + \sum_i \left(h_i + \frac{v_i^2}{2} - h_{cv} - \rho_{cv} \left(\frac{\partial h_{cv}}{\partial \rho_{cv}} \right)_T + \left(\frac{\partial p_{cv}}{\partial \rho_{cv}} \right)_T \right) \frac{dm_i}{dt} - \sum_o \left(h_o + \frac{v_o^2}{2} - h_{cv} - \rho_{cv} \left(\frac{\partial h_{cv}}{\partial \rho_{cv}} \right)_T + \left(\frac{\partial p_{cv}}{\partial \rho_{cv}} \right)_T \right) \frac{dm_o}{dt} \right)}{m_{cv} \left(\frac{\partial h_{cv}}{\partial T_{cv}} \right)_\rho - V_{cv} \left(\frac{\partial p_{cv}}{\partial T_{cv}} \right)_\rho} \quad (4.13)$$

With the density and the temperature of the working fluids known, the rest of the fluid thermodynamics properties, i.e. the enthalpy, internal energy and pressure, can be

calculated by integrating the respective equations with time. Another method is to directly solve Equations (4.5) and (4.9) for the internal energy and the density. This is simpler but requires the correlations between internal energy and the other fluid parameters. In this case, computer routines like REFPROP [146] can be employed to help in the calculation of the fluid properties. REFPROP is a program that uses equations for the thermodynamic and transport properties to calculate the state points of the fluid [147]. The equations are the most accurate equations currently available. They are generally valid over the entire vapour and liquid regions of the fluid, including supercritical states.

4.2. Suction Flow, Discharge Flow and Suction Valve Models

The inlet and outlet mass flow rates through the suction and discharge ports can be estimated by using the energy equation of the flow through a thick orifice model [148] as illustrated in Figure 4.3.

Figure 4.3 The model of a flow through a thick orifice

Assuming steady, isentropic flow across the orifice, negligible potential energy, heat, work and velocity at the suction reservoir, the energy equation can be used to estimate the flow velocity as expressed in Equation (4.14). The velocity obtained with

this equation must then be compared with the speed of sound. If it is higher than the speed of sound, the flow is choked and the flow velocity is set to be equal to the speed of sound. Else, the velocity obtained by Equation (4.14) is valid. The ideal mass flow rate can then be calculated using Equation (4.15).

$$v_{1s} = \sqrt{2(h_1 - h_{1s})} \quad (4.14)$$

$$\frac{dm_s}{dt} = \rho_1 A_{orif} v_{1s} \quad (4.15)$$

In order to obtain a more realistic model, Equation (4.15) needs to be modified to account for the existence of the *vena contracta* (the real minimum area of the flow, see Figure 4.3) and other effects like friction and turbulence which cause the flow to be non-isentropic. This can be done by introducing a factor called the coefficient of discharge, C_d . The value of C_d for a thick orifice can be approximated to be around 0.7-0.85 [149]. Therefore, Equation (4.15) becomes Equation (4.16).

$$\frac{dm}{dt} = \rho_1 C_d A_{orif} v_{1s} \quad (4.16)$$

Chambers 1 and 2 are the discharge working chamber and discharge reservoir, respectively, when simulating the discharge flow. They are the suction reservoir and the suction working chamber when the model is used to simulate the suction flow. When simulating the discharge flow, the orifice area, A_{orif} , can be either the discharge port area, A_{disc_port} , or the discharge opening area, A_{disc_open} , whichever is smaller. The discharge opening area is derived according to Figure 4.4 and is found to be as expressed in Equation (4.17). For a first estimate, d_{disc_open} can be approximated to be equal to the vane exposed length, l_{ve} .

$$A_{disc\_open} = d_{disc\_open} \times l_{exp} \quad (4.17)$$

Figure 4.4 The discharge and suction port and opening areas

When simulating the suction flow, the orifice area, A_{orif} , can be either the suction port area, A_{suct_port} , the suction opening area, A_{suct_open} , or the suction valve opening, A_{suct_valve} , whichever is smaller. The suction opening area is estimated according to Figure 4.4 and is expressed in Equation (4.18). For a first estimate, d_{suct_open} can be approximated to be equal to the vane exposed length, l_{ve} .

$$A_{suct\_open} = d_{suct\_open} \times l_{exp} \quad (4.18)$$

The suction valve area, A_{suct_valve} , depends on the suction valve design (see Chapter 3 for more discussions about the suction valve design). With this design, the suction valve opening area can be modelled according to Figure 4.5. The ellipsoid area at the left-hand side of the figure is the opening slot of the valve while the circle at the right-hand side is the suction port. The radii of the suction port and the valve slot are set equal. This figure also assumes that the suction port is moving from right to left, coming towards the valve slot. The intersection between them, shown as the shaded area in Figure 4.5, is the suction valve opening area.

Figure 4.5 The suction valve geometrical model

If the suction port opening is at the side of the rotor shaft with the left edge of the port hole is right at the vane midline (the line which indicates the rotor angle) as illustrated in Figure 4.5, from the geometry of the RV expander, the distance between the suction port and the valve slot centres, d , can be approximated according to Equation (4.19). Figure 4.5 also gives the geometrical relationships expressed in Equations (4.20) - (4.21).

$$d = 2r_{sp} - \varphi_r r_{r,shaft} \quad \text{if } 0 \leq \varphi_r \leq \frac{2r_{sp}}{r_{r,shaft}}$$

$$d = 2r_{sp} - \left(\theta_{sv} - \left(\varphi_r - \frac{2r_{sp}}{r_{r,shaft}} \right) \right) r_{r,shaft} \quad \text{if } \theta_{sv} \leq \varphi_r \leq \theta_{sv} + \frac{2r_{sp}}{r_{r,shaft}} \quad (4.19)$$

$$2d_{sp} = d \quad (4.20)$$

$$\cos \theta_{sp} = \frac{d_{sp}}{r_{sp}} \quad (4.21)$$

Finally, the suction valve opening area can be calculated by Equation (4.22).

$$\begin{aligned}
 A_{suct\_valve} &= 2\theta_{sp} r_{sp}^2 - 2d_{sp} \sqrt{r_{sp}^2 - d_{sp}^2} & \text{if } 0 \leq \varphi_r \leq \frac{2r_{sp}}{r_{r,shaft}} \\
 A_{suct\_valve} &= \pi r_{sp}^2 & \text{if } \frac{2r_{sp}}{r_{r,shaft}} < \varphi_r < \theta_{sv} \\
 A_{suct\_valve} &= 2\theta_{sp} r_{sp}^2 - 2d_{sp} \sqrt{r_{sp}^2 - d_{sp}^2} & \text{if } \theta_{sv} \leq \varphi_r \leq \theta_{sv} + \frac{2r_{sp}}{r_{r,shaft}} \\
 A_{suct\_valve} &= 0 & \text{if } \varphi_r > \theta_{sv} + \frac{2r_{sp}}{r_{r,shaft}}
 \end{aligned} \tag{4.22}$$

4.3. Internal Leakage Flows Model

As has been mentioned in Section 4.1, there are internal leakage paths affecting the three chambers. The main ones include the radial clearance, vane endface and vane side gaps, as illustrated in Figure 4.6. The theoretical models of the leakage flows through these paths are presented in this section. Other paths, such as the rotor endface gaps and the lubrication paths, are expected to be relatively less significant as compared to the main paths as they are usually long and well sealed.

Figure 4.6 Main internal leakages in an RV expander

A parameter called the volumetric efficiency is commonly used to measure the effects of internal leakage. It is defined as the ratio between the flow rates of a perfectly sealed and a real expanders. Mathematically, this is expressed in Equation (4.23).

$$\eta_v = \frac{\dot{m}_{ideal}}{\dot{m}_{real}} \quad (4.23)$$

4.3.1. Leakage through the radial clearance gap

The leakage from the suction chamber to the discharge chamber through the radial clearance gap is modelled based on the geometrical model in Figure 4.7. Points P and Q lie at the rotor wall and the cylinder inner wall, respectively.

Figure 4.7 Geometrical model of the radial clearance gap

To find the radial clearance gap, δ_{rad} , the distance between points P and Q is needed. This is done by obtaining the distance between points C and P from the triangle CPR.

$$l_{CP} = -(e - \delta_{rad}) \cos \varphi \pm \sqrt{r_r^2 - (e - \delta_{rad})^2 \sin^2 \varphi}$$

When $\varphi = \pi$, then $l_{CP} = r_r + (e - \delta_{rad})$. Therefore, only the positive (+) sign is correct and the distance between points C and P is found to be as shown in Equation (4.24).

$$l_{CP} = -(e - \delta_{rad}) \cos \varphi + \sqrt{r_r^2 - (e - \delta_{rad})^2 \sin^2 \varphi} \quad (4.24)$$

The radial clearance profile can then be found to be as expressed in Equation (4.25).

$$h_{rad} = r_c + (e - \delta_{rad}) \cos \varphi - \sqrt{r_r^2 - (e - \delta_{rad})^2 \sin^2 \varphi} \quad (4.25)$$

Because the rotor radius term, r_r , in the square root term of Equation (4.25) is the most dominant, Equation (4.25) can be simplified into Equation (4.25a).

$$h_{rad} \approx e + (e - \delta_{rad}) \cos \varphi \quad (4.25a)$$

The leakage flow rate should be modelled with the compressibility of the fluid considered. This is because the geometry of the radial clearance gap is convergent-divergent in nature, which may affect the density of the fluid significantly, especially if the fluid is in the gaseous phase. Following the method proposed by Yanagisawa and Shimizu [121], the flow is modelled as an isentropic compressible flow through a converging nozzle followed by a Fanno flow through a channel shown in Figure 4.8.

Figure 4.8 (a) Radial clearance gap profile,

(b) Geometrical model of the radial clearance gap used in the simulation

The effective Fanno length is defined according to Equation (4.26).

$$\bar{\lambda} \frac{l_f}{2\delta_{rad}} = \int_{s_1}^{s_2} \lambda \frac{ds}{2h_{rad}} \quad (4.26)$$

where:

$$s = r_c \varphi$$

$$\lambda = 96/\text{Re} \text{ if } \text{Re} \leq 3560$$

$$\text{or } \lambda = 0.3164/\text{Re}^{0.25} \text{ if } \text{Re} > 3560$$

Assuming that $\bar{\lambda} \cong \lambda$, Equation (4.26) can be solved, resulting in Equation (4.27).

$$l_f = \frac{r_c \delta_{rad}}{e \sqrt{1 - \left(1 - \frac{\delta_{rad}}{e}\right)^2}} \left(\sin^{-1} \left(\frac{\sin \varphi_2 \sqrt{1 - \left(1 - \frac{\delta_{rad}}{e}\right)^2}}{1 + \left(1 - \frac{\delta_{rad}}{e}\right) \cos \varphi_2} \right) - \sin^{-1} \left(\frac{\sin \varphi_1 \sqrt{1 - \left(1 - \frac{\delta_{rad}}{e}\right)^2}}{1 + \left(1 - \frac{\delta_{rad}}{e}\right) \cos \varphi_1} \right) \right) \quad (4.27)$$

Yanagisawa and Shimizu [121] showed that l_f increases then saturates immediately with increasing $(\varphi_2 - \varphi_1)$. Therefore, in practice we can represent l_f with the value at $(\varphi_2 - \varphi_1) = 2\pi$, without paying attention to the selection of the frictional range. With this, Equation (4.27) can be approximated to Equation (4.28).

$$l_f = \frac{2\pi \delta_{rad} r_c}{e \sqrt{1 - \left(1 - \frac{\delta_{rad}}{e}\right)^2}} \quad (4.28)$$

To calculate the leakage flow rate, the flow at the exit (e) in Figure 4.11 is initially assumed sonic. Therefore, Fanno flow conditions give Equations (4.29) and (4.30).

$$\bar{\lambda} \frac{l_f}{2\delta_{rad}} = \frac{1 - M_t^2}{\kappa M_t^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{(\kappa + 1) M_t^2}{2 + (\kappa - 1) M_t^2} \right) \quad (4.29)$$

$$\frac{p_t}{p_e} = \frac{1}{M_t} \left(\frac{\kappa + 1}{2 + (\kappa - 1) M_t^2} \right)^{0.5} \quad (4.30)$$

Assuming that p_t is the stagnation pressure, the convergent nozzle conditions give Equation (4.31).

$$\frac{p_1}{p_t} = \left(1 + \frac{\kappa - 1}{2} M_t^2\right)^{\frac{\kappa}{\kappa - 1}} \quad (4.31)$$

Next, the total pressure ratio $\frac{p_1}{p_e} \equiv \left(\frac{p_1}{p_t}\right)\left(\frac{p_t}{p_e}\right)$ should be compared with $\frac{p_1}{p_2}$. If the former is less than the latter, the flow is choked. Therefore, leakage flow conditions can be calculated according to Equations (4.32) - (4.35).

$$p_e = \frac{p_1}{\left(\frac{p_1}{p_t}\right)\left(\frac{p_t}{p_e}\right)} \quad (4.32)$$

$$T_e = \frac{T_1}{\left(1 + \frac{(\kappa - 1)}{2} M_e^2\right)} \quad (4.33)$$

$$V_e = M_e (\kappa R_g T_e)^{0.5} \quad (4.34)$$

$$q = \left(\frac{p_e}{R_g T_e}\right) (l_{\text{exp}} \delta_{\text{rad}}) V_e \quad (4.35)$$

If the flow is not choked, iterations have to be carried out. This is initialised by guessing a value for M_t . This value is then used to get the critical channel length, l_{f*} , and other critical properties from Equations (4.36) - (4.38).

$$\bar{\lambda} \frac{l_{f*}}{2\delta_{\text{rad}}} = \frac{1 - M_t^2}{\kappa M_t^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{(\kappa + 1) M_t^2}{2 + (\kappa - 1) M_t^2} \right) \quad (4.36)$$

$$\frac{p_t}{p_*} = \frac{1}{M_t} \left(\frac{\kappa + 1}{2 + (\kappa - 1) M_t^2} \right)^{0.5} \quad (4.37)$$

$$\frac{p_1}{p_t} = \left(1 + \frac{\kappa - 1}{2} M_t^2\right)^{\frac{\kappa}{\kappa - 1}} \quad (4.38)$$

The Mach number at the exit (e), M_e , can be calculated by solving Equation (4.39).

$$\bar{\lambda} \frac{l_{f*} - l_f}{2\delta_{rad}} = \frac{1 - M_e^2}{\kappa M_e^2} + \frac{\kappa + 1}{2\kappa} \ln \left(\frac{(\kappa + 1)M_e^2}{2 + (\kappa - 1)M_e^2} \right) \quad (4.39)$$

With M_e found, the critical pressure ratio at (e) can also be found according to Equation (4.40).

$$\frac{p_e}{p_*} = \frac{1}{M_e} \left(\frac{\kappa + 1}{2 + (\kappa - 1)M_e^2} \right)^{0.5} \quad (4.40)$$

Finally, the total pressure ratio can be calculated according to Equation (4.41).

$$\frac{p_1}{p_e} = \left(\frac{p_1}{p_t} \right) \left(\frac{p_t}{p_*} \right) / \left(\frac{p_e}{p_*} \right) \quad (4.41)$$

If it is found that $\frac{p_1}{p_e} = \frac{p_1}{p_2}$, then the values of M_t and M_e are already correct.

Therefore, the leakage flow properties can be calculated using Equations (4.32) - (4.35).

If $\frac{p_1}{p_e} \neq \frac{p_1}{p_2}$, then the iteration must be repeated until $\frac{p_1}{p_e} = \frac{p_1}{p_2}$.

4.3.2. Leakage through the vane endface gaps

The leakage through the vane endface gaps can be approximated as a flow from a reservoir (the suction chamber) to another reservoir (the discharge chamber) through a narrow channel as illustrated in Figure 4.9. To accommodate the compressibility and the friction of the fluid, this flow can be modelled as a Fanno flow. The channel width, δ_{v_endf} , is a geometrical property of the RV expander assembly. If the vane is a rectangular vane, l_f is equal to the vane width, w_v . It is important to remember that there are two vane endface gaps in a RV expander. Therefore, the total leakage flow rate through the vane endface gaps is found from the summation of the flow rates through each vane endface gap.

Figure 4.9 Geometrical model of the vane endface gap

The simulation procedure is similar to the one presented in Section 4.3.1, except that the nozzle flow is now ignored. Therefore, Equation (4.38) is ignored while the other variables are adjusted accordingly.

4.3.3. Leakage through the vane side gaps

The leakage through the vane side gaps consists of two flows, between the suction chamber and the vane slot, and between the vane slot and the discharge chamber. The flow from the suction chamber to the vane slot is illustrated in Figure 4.10. The flow is similar to a flow from a reservoir to another reservoir through a thin orifice. Hence, the thin orifice flow model is used. This model is also employed to model the flow between the vane slot and the discharge chamber through the vane side gap.

Figure 4.10 (a) Flow from suction chamber to vane slot through the vane side gap,

(b) theoretical model of the flow through vane side gap

The vane side area is dictated by the vane side gap width, d_v , and is calculated according to Equation (4.43). The expression for the vane side gap width depends on whether the flow is between the suction chamber and the vane slot or between the discharge chamber and the vane slot.

$$A_{vane\_side} = d_v \times l_{exp} \quad (4.43)$$

Following the derivation for the flow through an orifice in Section 4.2, the mass flow rate of the leakage through the vane side gaps can be calculated using Equation (4.44). The value of the coefficient of discharge, C_d , can be taken as 0.61 [149].

$$\frac{dm}{dt} = \rho_1 C_d A_{vane\_side} \sqrt{2(h_1 - h_{1s})} \quad (4.44)$$

4.4. Heat Transfer Models

In each of the three chambers, i.e. the suction, discharge and vane slot chambers, heat transfer may occur between the fluid and the chamber walls when they are of different temperatures. This heat transfer can be modelled according to the method proposed by Tan and Ooi [112]. The heat transfer is approximated according to Newton's law of cooling expressed in Equation (4.45). The heat transfer coefficient, h , is calculated from the Nusselt number correlation proposed by Liu and Zhou [150], expressed in Equation (4.46). Since the expander solid components have a significantly larger thermal mass as compared to the working fluids, the wall temperature can be estimated to be constant and uniform at the average temperature between the suction and discharge reservoirs temperatures (Equation (4.47)).

$$\frac{dQ}{dt} = h A_{ht} (T_{wall} - T_f) \quad (4.45)$$

$$Nu = 0.75 Re^{0.8} Pr^{0.6} \quad (4.46)$$

where

$$T_{wall} = \frac{T_{suct} + T_{disc}}{2} \quad (4.47)$$

$$Nu = \frac{hD_h}{k_f} \quad (4.48)$$

$$Re = \frac{\rho_f U D_h}{\mu_f} \quad (4.49)$$

$$Pr = \frac{c_{p_f} \mu_f}{k_f} \quad (4.50)$$

When modelling the heat transfer of the fluids in the suction and discharge chambers, the characteristic velocities of the working fluid and the hydraulic diameter are approximated by Equations (4.51) and (4.52), while the ones of the vane slot are in Equations (4.53) and (4.54). These definitions are derived based on the assumption that the flow of the fluid inside the chamber is mainly due to the vane movement.

$$U_{ht,scv/discv} = \frac{\omega_r}{2} (l_{ve} + 2r_r) \quad (4.51)$$

$$D_{h,ht,scv/discv} = \frac{2l_{exp} l_{ve}}{l_{ve} + l_{exp}} \quad (4.52)$$

$$U_{ht,vs} = \frac{dl_{ve}}{dt} \quad (4.53)$$

$$D_{h,ht,vs} = \frac{2w_{vs} l_{exp}}{w_{vs} + l_{exp}} \quad (4.54)$$

THEORETICAL MODELS: COMPONENTS DYNAMICS

After developing the mathematical models of the working fluid processes of the Revolving Vane expander in Chapter 4, the theoretical models of the dynamics of the expander components are presented in this chapter. The models assume that the dynamics of the components are geometrically constrained. The models consist of the geometrical, kinematics and dynamics of the cylinder and the rotor.

5.1. Geometrical and Kinematics Models

To simplify the construction of the geometrical and the kinematics models of the expander, the thin vane assumption is made and therefore, the vane thickness and the vane slot volume are assumed to be negligible as compared to the other main dimensions. This is a common practice found in the open literature as discussed in Chapter 2.

The model is developed with the geometrical model shown in Figure 5.1. Points C and R are the cylinder and the rotor rotation centres, respectively. With the thin vane assumption, the contact between the vane and the vane slot can be assumed to be always at point A, which is the intersection between the cylinder inner wall and the vane midline. This point is, therefore, always of a distance of the cylinder radius, r_c , from point C. The angle between the vertical line passing through R and C and the line connecting points R and A is defined as the rotor angle, φ_r , while the angle between the vertical line and the line connecting points C and A is defined as the cylinder angle, φ_c .

Figure 5.1 Geometrical model of the RV expander and the triangle ACR

From the triangle ACR, the geometrical relationships expressed in Equations (5.1) and (5.2) are obtained.

$$\cos \varphi_c = \frac{e^2 + r_c^2 - (r_r + l_{ve})^2}{2r_c e} \quad (5.1)$$

$$l_{ve} = -e \cos \varphi_r - r_r + \sqrt{r_c^2 - e^2 \sin^2 \varphi_r} \quad (5.2)$$

The first and second derivatives of Equation (5.2) with respect to time are the sliding velocity and acceleration of the vane, respectively, which are expressed by Equations (5.3) and (5.4).

$$\frac{dl_{ve}}{dt} = \omega_r e \sin \varphi_r \left(1 - \frac{e \cos \varphi_r}{\sqrt{r_c^2 - e^2 \sin^2 \varphi_r}} \right) \quad (5.3)$$

$$\begin{aligned} \frac{d^2 l_{ve}}{dt^2} = & \alpha_r e \sin \varphi_r \left(1 - \frac{e \cos \varphi_r}{\sqrt{r_c^2 - e^2 \sin^2 \varphi_r}} \right) \\ & + \omega_r^2 e^2 \left(\frac{\cos \varphi_r}{e} - \frac{\cos^2 \varphi_r}{\sqrt{r_c^2 - e^2 \sin^2 \varphi_r}} - \frac{(e^2 - r_c^2) \sin^2 \varphi_r}{(r_c^2 - e^2 \sin^2 \varphi_r)^{1.5}} \right) \end{aligned} \quad (5.4)$$

Another geometrical relationship that can be derived from the triangle ACR is the relationship between the angle γ , which is the angle between the vane midline and the line connecting A and C, as expressed in Equation (5.5a).

$$\gamma = \cos^{-1} \left(\frac{r_c^2 + (r_r + l_{ve})^2 - e^2}{2r_c(r_r + l_{ve})} \right) \quad (5.5a)$$

The convention used in this report is that angle γ always starts from the line AC. Therefore, to be consistent with the convention used here that the clockwise direction is positive, Equation (5.5a) is applicable only for $0 \leq \varphi_r \leq \pi$. Otherwise it must be modified to Equation (5.5b).

$$\gamma = -\cos^{-1} \left(\frac{r_c^2 + (r_r + l_{ve})^2 - e^2}{2r_c(r_r + l_{ve})} \right) \quad (5.5b)$$

The tangential velocities at point A in Figure 5.1 have the relations expressed in Equations (5.6) - (5.8).

$$v_{Ac} = r_c \omega_c \quad (5.6)$$

$$v_{Ar} = (r_r + l_{ve}) \omega_r \quad (5.7)$$

$$v_{Ar} = v_{Ac} \cos \gamma \quad (5.8)$$

Combining Equations (5.5a), (5.6) - (5.8) gives Equation (5.9), which relates the angular velocity of the cylinder, ω_c , to the geometry, the angle of the rotor (through the vane exposed length, l_{ve}) and the angular velocity of the rotor, ω_r .

$$\omega_c = \omega_r \left(\frac{2(r_r + l_{ve})^2}{r_c^2 + (r_r + l_{ve})^2 - e^2} \right) \quad (5.9)$$

Taking the derivative of Equation (5.9) with time gives the angular acceleration of the cylinder as expressed in Equation (5.10), which shows that even in the case where the rotor is running at a constant angular velocity (zero rotor angular acceleration), there will still be an angular acceleration for the cylinder. This is because both the rotor and

the cylinder rotate at their respective centres and there exists an eccentricity between these centres.

$$\alpha_c = \alpha_r \left(\frac{2(r_r + l_{ve})^2}{r_c^2 + (r_r + l_{ve})^2 - e^2} \right) + \omega_r \frac{dl_{ve}}{dt} \left(\frac{4(r_r + l_{ve})(r_c^2 - e^2)}{(r_c^2 + (r_r + l_{ve})^2 - e^2)^2} \right) \quad (5.10)$$

The radial velocity of the cylinder at point A, which is the velocity with a direction that is parallel to the line AC, is actually the sliding velocity of the vane. This can be shown as follows. From the triangle ACR, Equation (5.11) can be obtained.

$$\sin \gamma = \frac{e}{r_c} \sin \varphi_r \quad (5.11)$$

Using Equations (5.2), (5.6), (5.9) and (5.11), the radial velocity of the cylinder at point A can be written as Equation (5.12) which is identical to Equation (5.3), the expression of the sliding velocity of the vane.

$$v_{Ac} \sin \gamma = \omega_r e \sin \varphi_r \left(1 - \frac{e \cos \varphi_r}{\sqrt{r_c^2 - e^2 \sin^2 \varphi_r}} \right) \quad (5.12)$$

5.2. Dynamics Model of the Cylinder

After formulating the geometrical and the kinematics models, the dynamics of the components, can now be analysed. The discussion starts with the cylinder and then continues with the rotor in the next section. It is realised that during operation, the vane may contact the slot at either side. Therefore, the analysis must include both scenarios. The vane and vane slot thicknesses are taken into consideration here, although it was previously ignored in the geometrical and kinematics models. This allows the friction at the vane side, caused by the rubbing between the vane side and the vane slot wall, to be considered more accurately. The vane is assumed to be of rectangular geometry.

The free-body diagram of the cylinder is shown in Figure 5.2. The centre of mass of the cylinder has been assumed to be at the centre of rotation of the cylinder, point C. There may be a significant contact force between the rotor and the cylinder at the line contact if the cylinder and the rotor are in direct contact. However, this situation is undesirable and can be avoided by selecting appropriate bearings to maintain a radial clearance gap between the cylinder and the rotor. In the following analysis, the latter situation is assumed and hence this force can be neglected. With this assumption, the friction at the line contact caused by the relative velocities of the cylinder and the rotor is also reduced to only the viscous force of the fluid in the radial clearance gap. Therefore, this force is also negligible. The summation of the torques acting at the cylinder, ΣT_c , includes the torques caused by the shaft bearings and the endface frictions, which will be discussed in the subsequent sections in this chapter. It is important to be consistent with the signs of the torques. A torque loss should be given a negative sign, while a torque gain should have a positive sign.

Figure 5.2 Free-body diagram of the cylinder (positive $F_{v,n}$)

By definition, the relationships in Equations (5.13) to (5.15) are known. Equation (5.14) neglects the thickness of the vane slot.

$$F_{v,f} = |F_{v,n}| \eta_v \quad (5.13)$$

$$F_{p,c} = (P_{scv} - P_{dcv}) 2r_c \sin\left(\frac{\varphi_c}{2}\right) l_{\text{exp}} \quad (5.14)$$

$$F_{p,vs} = p_{vs} w_v l_{\text{exp}} \quad (5.15)$$

To simplify the analysis, the normal force acting on the vane, $F_{v,n}$, is approximated to be at point A, instead of B, as illustrated in Figure 5.3. The accuracy of this assumption depends on the distance between the real contact point, B, and point A. However, since this distance is usually much smaller as compared to the main dimensions of the expander, it is expected that this assumption will not compromise the accuracy of the results obtained.

Figure 5.3 (a) The real and (b) approximated locations of $F_{v,n}$,
(c) the triangle ACR with the forces shown (positive $F_{v,n}$)

The force balance equation of the cylinder in the x and y directions and the torque balance equation are expressed as Equations (5.16) - (5.18a), respectively.

$$F_{cx} - F_{v,n} \cos \varphi_r + F_{v,f} \sin \varphi_r - F_{p,c} \sin \frac{\varphi_c}{2} - F_{p,vs} \sin \varphi_c + \sum_i \vec{F}_i \cdot \hat{e}_x = 0 \quad (5.16)$$

$$F_{cy} + F_{v,n} \sin \varphi_r + F_{v,f} \cos \varphi_r - F_{p,c} \cos \frac{\varphi_c}{2} - F_{p,vs} \cos \varphi_c + \sum_i \vec{F}_i \cdot \hat{e}_y = 0 \quad (5.17)$$

$$I_c \alpha_c = F_{v,n} r_c \cos \gamma - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \left(r_c - \frac{w_v}{2 \sin \gamma} \right) \sin \gamma + \sum T_c \quad (5.18a)$$

Substituting Equation (5.13) into Equation (5.18a), dropping the absolute sign and rearranging gives Equation (5.19a).

$$F_{v,n} = \frac{I_c \alpha_c - \sum T_c}{r_c \cos \gamma - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \left(r_c \sin \gamma - \frac{w_v}{2} \right)} \quad (5.19a)$$

In the situations where the normal force acting on the vane, $F_{v,n}$, is negative, the vane contact point B moves to the other side of the vane. For simplicity the force, $F_{v,n}$, is again approximated to be at point A, but the direction of $F_{v,n}$ is flipped and the location of $F_{v,f}$ is moved to the other side of the vane as illustrated in Figure 5.4.

Figure 5.4 The triangle ACR with the forces shown (negative $F_{v,n}$)

The force balance equations in the cases when $F_{v,n}$ is negative are identical to Equations (5.16) and (5.17). However, the torque balance equation becomes as expressed in Equation (5.18b).

$$I_c \alpha_c = F_{v,n} r_c \cos \gamma + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \left(r_c + \frac{w_v}{2 \sin \gamma} \right) \sin \gamma + \sum T_c \quad (5.18b)$$

Substituting Equation (5.13) into Equation (5.18b), dropping the absolute sign and rearranging gives the closed form expression of $F_{v,n}$ when it is negative.

$$F_{v,n} = \frac{I_c \alpha_c - \sum T_c}{r_c \cos \gamma + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \left(r_c \sin \gamma + \frac{w_v}{2} \right)} \quad (5.19b)$$

Equations (5.19a) and (5.19b) can be generalised into Equation (5.19), which is the general equation for $F_{v,n}$.

$$F_{v,n} = \frac{I_c \alpha_c - \sum T_c}{r_c \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \left(r_c \sin \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{w_v}{2} \right)} \quad (5.19)$$

It is interesting to observe that Equation (5.19) indicates that the vane contact force is only a function of the cylinder inertia and the summation of the other torques acting at the cylinder (mainly friction losses), but is not directly affected by the pressure difference between the suction and discharge chambers. This is a huge advantage of the RV mechanism as compared to the more common rotary machines because the vane side friction loss is now somewhat independent of the pressure conditions. In addition, the vane is exposed to less wear and tear.

5.3. Dynamics Model of the Rotor

After the dynamics analysis of the cylinder is presented, the analysis of the rotor is presented in this section. The free-body diagram of the rotor is shown in Figure 5.5. The

centre of mass of the rotor has been assumed to lie at the centre of rotation of the rotor, point R. The normal force on the vane has been approximated at point A as was done in the cylinder dynamics analysis discussed in the previous section. T_{gen} represents the expander generated output torque, while the summation of the torques acting at the rotor, ΣT_r , may include the torques caused by the endface and the bearing frictions.

Figure 5.5 Free-body diagram of the rotor (positive $F_{v,n}$)

Equations (5.20) and (5.21) are known by definition. The vane thickness has been assumed to be insignificant as compared to the rotor radius in Equation (5.21).

$$F_{v,p} = (p_{scv} - p_{dcv}) l_{ve} l_{exp} \quad (5.20)$$

$$F_{p,r} = (p_{scv} - p_{dcv}) 2r_r \sin\left(\frac{\varphi_r}{2}\right) l_{exp} \quad (5.21)$$

Based on Figure 5.5, the force balance of the rotor in the x and y directions and the torque balance equation are expressed in Equations (5.22) - (5.24).

$$F_{rx} - F_{v,p} \cos \varphi_r + F_{v,n} \cos \varphi_r + (F_{p,vs} - F_{v,f}) \sin \varphi_r + F_{p,r} \sin \frac{\varphi_r}{2} + \sum_j \vec{F}_j \cdot \hat{e}_x = 0 \quad (5.22)$$

$$F_{ry} + F_{v,p} \sin \varphi_r - F_{v,n} \sin \varphi_r + (F_{p,vs} - F_{v,f}) \cos \varphi_r + F_{p,r} \cos \frac{\varphi_r}{2} + \sum_j F_j \cdot \hat{e}_y = 0 \quad (5.23)$$

$$I_r \alpha_r = F_{v,p} \left(r_r + \frac{l_{ve}}{2} \right) - F_{v,n} (r_r + l_{ve}) - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \frac{w_v}{2} + \sum T_r - T_{gen} \quad (5.24)$$

Substituting Equation (5.13) into Equation (5.24) produces Equation (5.25).

$$I_r \alpha_r = F_{v,p} \left(r_r + \frac{l_{ve}}{2} \right) - F_{v,n} (r_r + l_{ve}) - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} |F_{v,n}| \eta_v \frac{w_v}{2} + \sum T_r - T_{gen} \quad (5.25)$$

Dropping the absolute sign from Equation (5.25) gives Equation (5.26).

$$I_r \alpha_r = F_{v,p} \left(r_r + \frac{l_{ve}}{2} \right) - F_{v,n} \left(r_r + l_{ve} + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \frac{w_v}{2} \right) + \sum T_r - T_{gen} \quad (5.26)$$

Equation (5.26) is applicable not only when the vane side contact force acts on the right hand side (positive $F_{v,n}$), but also when it is at the left hand (negative $F_{v,n}$) side of the vane. Equation (5.26) also shows the relationship between the rotor acceleration with all the forces and torques acting on the rotor. The cylinder inertia and other torques acting at the cylinder are implicitly present through the vane side contact force.

It is possible to explicitly show the relationship between the dynamics of the components and the geometries and the rotor angle by substituting Equations (5.10) and (5.19) into Equation (5.26). Then, by introducing the new parameters shown in Equations (5.27) and (5.28), Equation (5.29) is obtained. This equation explicitly shows the relationship between the rotor acceleration and the expander geometries.

$$f_1 = \frac{2(r_r + l_{ve})^2}{r_c^2 + (r_r + l_{ve})^2 - e^2} \quad (5.27)$$

$$f_2 = \frac{dl_{ve}}{dt} \left(\frac{4(r_r + l_{ve})(r_c^2 - e^2)}{(r_c^2 + (r_r + l_{ve})^2 - e^2)^2} \right) \quad (5.28)$$

$$\alpha_r = \frac{F_{v,p} \left(r_r + \frac{l_{ve}}{2} \right) + \frac{\left(\sum T_c - I_c \omega_r f_2 \right) \left(r_r + l_{ve} + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \frac{w_v}{2} \right)}{r_c \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \left(r_c \sin \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{w_v}{2} \right)} + \sum T_r - T_{gen}}{I_r + \frac{I_c f_1 \left(r_r + l_{ve} + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \frac{w_v}{2} \right)}{r_c \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \left(r_c \sin \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{w_v}{2} \right)}} \quad (5.29)$$

In many cases, the parameter of interest is the expander output torque, instead of the rotor acceleration. For such cases, Equation (5.19) can be substituted into Equation (5.26) and rearranged, resulting in Equation (5.30). This equation explicitly shows the contributions of the pressure difference, the rotor inertia, the cylinder inertia and the other torques to the expander output torque. It is useful to note that if the thin vane assumption is adopted in the dynamics analysis, the vane side friction power loss is obtained by multiplying the vane side friction force, $F_{v,f}$, with the vane sliding velocity. Naturally, the result will be slightly different from the one predicted by Equation (5.30).

$$T_{gen} = F_{v,p} \left(r_r + \frac{l_{ve}}{2} \right) - I_r \alpha_r - \left(\frac{r_r + l_{ve}}{r_c \cos \gamma} \right) I_c \alpha_c + \left(\frac{r_r + l_{ve}}{r_c \cos \gamma} \right) \sum T_c + \sum T_r - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \left(\left(r_c \sin \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{w_v}{2} \right) \left(\frac{r_r + l_{ve}}{r_c \cos \gamma} \right) + \frac{w_v}{2} \right) \quad (5.30)$$

The output torque can be converted to output power by multiplying it with the angular velocity of the rotor as expressed in Equation (5.31).

$$P_{gen} = T_{gen} \omega_r \quad (5.31)$$

It is common to measure the mechanical performance of the expander by introducing a parameter called the mechanical efficiency, which is defined as the ratio

between the real expander output power and the ideal output power if no friction is involved. Mathematically it can be defined as in Equation (5.32).

$$\eta_{mech} = \frac{P_{real}}{P_{ideal}} \quad (5.32)$$

5.4. Endface Model

A revolving vane machine has two endface gaps. These are the gaps between the end covers of the cylinder and the endfaces of the rotor (see Figure 5.6). Lubricants fill and seal these gaps [140, 141]. They also flow in these gaps, caused by the movement of the endface walls and also by the pressure difference between the suction and discharge chambers. However, since the gaps are usually very narrow, the shear flow can be expected to dominate the flow in most situations. Since the rotor and the cylinder are eccentrically located to each other, there is a relative velocity between their endfaces. This results in the fluid friction at the gaps at the endfaces which, in turn, contributes to the overall friction loss of the RV mechanism. The theoretical model of this effect has been analysed [151] and will be presented here.

Figure 5.6 Side sectional views of RV expander with different shaft bearing arrangements

In this analysis, since the mechanical losses occurring at the vane endfaces area (due to vane thickness) are small when compared to that at the rotor-cylinder endfaces, it has been neglected to simplify the model. Therefore, depending on the bearing arrangement, whether it is of the cantilever (Figure 5.6(a)) or the simply-supported arrangement (Figure 5.6(b)), the endface area of interest will be one of the two shaded areas shown in Figure 5.7.

Figure 5.7 The RV mechanism endface areas of interest:

(a) Configuration 1, (b) Configuration 2

In Figure 5.7, “C” refers to the cylinder centre while “R” the rotor centre. The circle “a” is the endface area of the cylinder while the circle “b” is the endface area of the rotor. The circle “c” in Figure 5.7(b) represents the cylinder shaft hole, while the circle “d” represents the rotor shaft. The crescent shaped area created by the difference in the area of circles “a” and “b” in Figure 5.7 is the part of the cylinder end cover that is exposed to the working chamber. The area of interest, represented by the shaded area – which is circle “a” in Figure 5.7(a) or the area of circle “a” minus the area of circle “c”

in Figure 5.7(b) - is the part of the rotor endface which interacts with the cylinder endface. In this region, shear flows occur, creating the endface friction losses.

The area of interest in Figure 5.7(a), which is circle “a”, is found only at the side of endface without the rotor shaft when the bearings are arranged in the cantilever arrangement. This is at the right-hand side of the machine in Figure 5.6(a). Meanwhile, the area of interest in Figure 5.7(b), which is the area of circle “a” minus the area of circle “c”, is found at both sides of the RV mechanism endfaces when the bearings are arranged in the simply-supported arrangement (Figure 5.6(b)). It is also encountered at the side of the RV mechanism endface with the rotor shaft, which is at the left-hand side of the RV machine in Figure 5.6(b). To ease the further presentation, the geometrical configuration of Figure 5.7(a), which is circle “a”, will now be called “Configuration 1” while the geometrical configuration of Figure 5.7(b), which is made up of the area of circle “a” minus that of circle “c” will now be called “Configuration 2”.

In this model, the rotor is the driving component which rotates at a constant velocity, ω . It is assumed that the lubricant in the gaps has a constant viscosity, μ . The endface gaps, δ , are assumed to be uniform and constant throughout. It is also assumed that the gaps are small enough such that pressure effect to the fluid flow is insignificant. Instead, the fluid only flows due to shear according to the Couette flow model expressed by Equation (5.33). The resultant force and torque produced by the fluid friction in the endface gaps can be derived using Equations (5.34) and (5.35), respectively.

$$\vec{\tau} = \frac{\mu}{\delta} \Delta \vec{V} \quad (5.33)$$

$$\vec{F} = \frac{\mu}{\delta} \int_A \Delta \vec{V} dA \quad (5.34)$$

$$\vec{T} = \frac{\mu}{\delta} \int_A (\Delta \vec{V} \times \vec{r}) dA \quad (5.35)$$

The definition of the relative velocities used in Equations (5.33) - (5.35) depends on which endface wall is under examination. When analysing the rotor endface wall, the relative velocity should be that of the cylinder minus the rotor velocities, while it is the opposite when analysing the cylinder endface wall. It is also important to note that the moment arm, r , in Equation (5.35) is referenced to point R when analysing the rotor endface and is referenced to point C when analysing the cylinder endface.

5.4.1. Endface Configuration 1

The analysis is carried out by considering an arbitrary point A in Configuration 1, shown in Figure 5.8(a).

Figure 5.8 Configuration 1 and the triangle ACR

Referring to Figure 5.8(b), the two velocity vectors of the cylinder and the rotor endfaces acting at point A are given by Equations (5.36) and (5.37) respectively.

$$\vec{V}_c = -(\omega_c x_c \cos \theta_c) \hat{x} + (\omega_c x_c \sin \theta_c) \hat{y} \quad (5.36)$$

$$\vec{V}_r = -(\omega_r x_r \cos \theta_r) \hat{x} + (\omega_r x_r \sin \theta_r) \hat{y} \quad (5.37)$$

From the triangle ACR, the vector representing the relative velocity at point A at the rotor endface is found to be expressed by Equation (5.38). A similar expression but with an opposite sign should be used when dealing with the cylinder endface.

$$\vec{V}_c - \vec{V}_r = (-\omega_c e - (\omega_c - \omega_r)x_r \cos \theta_r)\hat{x} + ((\omega_c - \omega_r)x_r \sin \theta_r)\hat{y} \quad (5.38)$$

Substituting Equation (5.38) into Equations (5.34) and (5.35) gives the resultant force and the torque produced by the fluid friction at the rotor endface wall as expressed by Equations (5.39) and (5.40), respectively.

$$\vec{F}_{r1} = \left(-\frac{\mu}{\delta} \pi \omega_c e r_r^2 \right) \hat{x} + 0 \hat{y} \quad (5.39)$$

$$\vec{T}_{r1} = \left(\frac{\mu \pi}{\delta} (\omega_c - \omega_r) r_r^4 \right) \hat{z} \quad (5.40)$$

It is interesting to observe that the resultant force acting on the rotor endface is simply a force vector that always points to the negative x -direction. The magnitude is also interestingly proportional to the area of the rotor endface multiplied by the tangential velocity of the cylinder acting at the rotor axis. It is independent of the velocity difference. As for the torque, the value can be either positive or negative depending on the difference between the cylinder and rotor angular velocities. When the cylinder angular velocity is higher than the rotor, the net effect is that the cylinder “pulls” the rotor to move along thus producing an assisting torque on the rotor. On the other hand, when the rotor is turning faster than the cylinder, the rotor is “resisted” by the cylinder indicating a torque loss at the rotor. However, the average torque is zero since the average value of the relative velocity of the cylinder and the rotor is also zero.

Performing a similar procedure to the cylinder endface wall gives the resultant force and torque as expressed by Equations (5.41) and (5.42), respectively.

$$\vec{F}_{c1} = \left(\frac{\mu}{\delta} \pi \omega_c e r_r^2 \right) \hat{x} + 0 \hat{y} \quad (5.41)$$

$$\bar{T}_{cl} = -\frac{\mu}{\delta} \left(\frac{\pi}{2} (\omega_c - \omega_r) r_r^4 + \pi \omega_c e^2 r_r^2 \right) \hat{z} \quad (5.42)$$

Equation (5.41) shows that the resultant force acting on the cylinder endface wall is the opposite of that acting at the rotor. This is expected since both the rotor and the cylinder rotate relative to each other. Meanwhile, Equation (5.42) shows that the torque on the cylinder endface wall is equal to the negative of the torque acting on the rotor endface, which is purely a result of the relative velocity between the rotor and the cylinder, plus a term due to the eccentricity, e . The first term has an average value of zero while the second is not. If the rotor and the cylinder centres are located at the same location, the term due to eccentricity in Equation (5.42) disappears. This latter term is equal to the magnitude of the resultant force multiplied by the eccentricity, e .

Combining Equations (5.40) and (5.42) with their respective angular velocities, gives the total power loss of the RV mechanism caused by the fluid friction at the endface wall. This is given in Equation (5.43).

$$P_1 = -\frac{\mu}{\delta} \left(\pi \omega_c^2 e^2 r_r^2 + \frac{\pi}{2} (\omega_c - \omega_r)^2 r_r^4 \right) \quad (5.43)$$

The negative sign in Equation (5.43) indicates that it is a power loss. Since there are two endfaces, one on each side of the rotor, the total power loss is the summation of the losses from both sides. Similar to the torque in Equation (5.42), Equation (5.43) shows that the power loss also has two terms: due to the eccentric and the velocity difference. The average power loss is given by Equation (5.44).

$$\bar{P}_1 = \frac{1}{2} \frac{\mu}{\delta} r_r^2 \left(\pi \omega_r^2 \omega_c^2 - \left(e^2 + \frac{1}{2} r_r^2 \right) \int_0^{2\pi} \omega_c^2 d\varphi_r \right) \quad (5.44)$$

The second term of Equation (5.44) which contains an integral term is difficult to solve analytically. The angular velocity, ω_c , itself is expressed by Equation (5.45) as has been derived in Section 5.1. Numerical integration may be performed to solve this

integral term. Another method is to approximate the term with a simpler expression that can be solved analytically. The latter method is chosen here because by observing the variations of the cylinder angular velocity of the RV machine, it can be seen that the velocity can be approximated with a cosine function expressed in Equation (5.46).

$$\omega_c = \omega_r \left(\frac{2(r_r + l_v)^2}{r_c^2 + (r_r + l_v)^2 - e^2} \right) \quad (5.45)$$

$$\omega_c \approx \omega_r - (\omega_r - \omega_{c0}) \cos \varphi_r \quad (5.46)$$

where ω_{c0} is the cylinder angular velocity when the operating angle $\varphi_r = 0$.

Comparing the cylinder velocities of the RV machine model investigated by Teh and Ooi [19] shows that the root mean square difference (RMSD) is only around 1.4%. Using Equation (5.46), the average power loss per cycle can now be solved analytically and is expressed by Equation (5.47).

$$\bar{P}_1 = -\frac{\pi}{2} \frac{\mu}{\delta} \left(\frac{r_r}{r_c} \right)^2 e^2 \omega_r^2 (3r_c^2 - 2r_c r_r + 1.5r_r^2) \quad (5.47)$$

5.4.2. Endface Configuration 2

The endface effects when the area of interest is in the shape of Configuration 2 can be obtained by combining the results gathered from the analysis of Configuration 1 and the effects due to the cylinder shaft hole area (circle “c” in Figure 5.7(b)) which will be investigated in this section.

The analysis begins by taking an arbitrary point B inside the cylinder shaft hole area as illustrated in Figure 5.9. A triangle made of point B, the cylinder centre, C and the rotor centre, R, can then be obtained.

Figure 5.9 Configuration 2 and the triangle BCR

Triangle BCR gives the expression for the relative velocity vector at the rotor endface as expressed by Equation (5.48).

$$\vec{V}_{ch} - \vec{V}_{rh} = (-\omega_r e - (\omega_c - \omega_r)x_{ch} \cos \theta_{ch})\hat{x} + ((\omega_c - \omega_r)x_{ch} \sin \theta_{ch})\hat{y} \quad (5.48)$$

The resultant force and torque produced by the relative velocity of Equation (5.48) acting on the rotor endface wall are shown in Equations (5.49) and (5.50), while those acting on the cylinder wall are shown in Equations (5.51) and (5.52), respectively.

$$\vec{F}_{rh} = -\left(\frac{\mu}{\delta} \pi \omega_c e r_h^2\right)\hat{x} + 0\hat{y} \quad (5.49)$$

$$\vec{T}_{rh} = \frac{\mu}{\delta} \left(\frac{\pi}{2} (\omega_c - \omega_r) r_h^4 - \pi \omega_r e^2 r_h^2 \right) \hat{z} \quad (5.50)$$

$$\vec{F}_{ch} = \left(\frac{\mu}{\delta} \pi \omega_c e r_h^2 \right) \hat{x} + 0\hat{y} \quad (5.51)$$

$$\vec{T}_{ch} = -\left(\frac{\mu}{\delta} \frac{\pi}{2} (\omega_c - \omega_r) r_h^4 \right) \hat{z} \quad (5.52)$$

Consequently, the power loss caused by the endface friction in the cylinder shaft hole area can be found as shown in Equation (5.53).

$$P_h = -\frac{\mu}{\delta} \left(\pi \omega_r^2 e^2 r_h^2 + \frac{\pi}{2} (\omega_c - \omega_r)^2 r_h^4 \right) \quad (5.53)$$

To get the endface effects of Configuration 2, we subtract the endface effects of the cylinder shaft hole area from the results of Configuration 1. Therefore, the resultant forces, the torque at the rotor and at the cylinder, and the power loss caused by the endface friction of Configuration 2 are as shown in Equations (5.54) - (5.58).

$$\bar{F}_{r2} = -\left(\frac{\mu}{\delta}\pi\omega_c e(r_r^2 - r_h^2)\right)\hat{x} + 0\hat{y} \quad (5.54)$$

$$\bar{F}_{c2} = \left(\frac{\mu}{\delta}\pi\omega_c e(r_r^2 - r_h^2)\right)\hat{x} + 0\hat{y} \quad (5.55)$$

$$\bar{T}_{r2} = \left(\frac{\mu}{\delta}\frac{\pi}{2}(\omega_c - \omega_r)(r_r^4 - r_h^4) + \frac{\mu}{\delta}\pi\omega_r e^2 r_h^2\right)\hat{z} \quad (5.56)$$

$$\bar{T}_{c2} = -\left(\frac{\mu}{\delta}\frac{\pi}{2}(\omega_c - \omega_r)(r_r^4 - r_h^4) + \frac{\mu}{\delta}\pi\omega_c e^2 r_r^2\right)\hat{z} \quad (5.57)$$

$$P_2 = -\frac{\mu}{\delta}\left(\pi e^2(\omega_c^2 r_r^2 - \omega_r^2 r_h^2) + \frac{\pi}{2}(\omega_c - \omega_r)^2(r_r^4 - r_h^4)\right) \quad (5.58)$$

Using the approximated cylinder angular speed expressed in Equation (5.46), the average power loss caused by the endface friction of Configuration 2 can be found and is given by Equation (5.59).

$$\bar{P}_2 = -\frac{\pi}{2}\frac{\mu}{\delta}\frac{e^2}{r_c^2}\omega_r^2\left(2r_c^2(r_r^2 - r_h^2) + e^2 r_r^2 + \frac{r_r^4 - r_h^4}{2}\right) \quad (5.59)$$

To close the discussion, it is appropriate to discuss the constraints that must be complied when dealing with Configuration 2. These include the requirement that the cylinder shaft hole must be small enough so that it is fully encapsulated by the rotor endface, or $e + r_h \leq r_r$, but it should be large enough to cover the rotor shaft, $r_h \geq e + r_{rs}$. The first constraint is not a necessary condition for the RV machine to function, but since the model developed here assumes that the cylinder shaft hole area is within the rotor endface area, this constraint should be conformed to. On the other hand, the second constraint is a necessary condition for the RV machine to function.

5.5. Shaft Bearing Model

The discussion in Chapter 3 has indicated that other than the vane side and the endface frictions, the other friction loss that is expected to be significant in an RV machine is the bearing loss. This loss is unavoidable since bearings are always needed to support the rotating shafts. Generally, if the friction coefficients are known, the bearing friction force can be approximated using Equation (5.60).

$$F_{b,f} = W_b \eta_{b,f} \quad (5.60)$$

There are many different types of bearings available. One of the most commonly used is the journal bearing. It consists of only a journal (shaft), a bearing sleeve and the lubricant. It can usually be self-made and therefore, is usually lower in cost as compared to other types of bearings. The schematic diagram is shown in Figure 5.10.

Journal bearings utilise the lubricant hydrodynamics to support the load of the shaft. The lubricant is guided to the gaps between the journal and the bearing sleeve. As the journal rotates with a load force acting on it, a hydrodynamic pressure is generated by the lubricant to support the journal.

Figure 5.10 Schematic diagram of a journal bearing

The Reynolds equation as expressed by Hirani *et al.* [133] and shown as Equation (5.61), which governs the dynamics of a thin film of lubricant, is used as the basic

equation for the model. The equation itself is derived under the assumptions that the lubricant is Newtonian, the inertia and the body force terms are negligible, the pressure variation across the film in the axial direction is negligible, the flow is laminar and the film thickness is much smaller as compared to the width and length of the bearing [152].

$$\begin{aligned} & \frac{\partial}{\partial \theta} \left((1 + \varepsilon \cos \theta)^3 \frac{\partial p}{\partial \theta} \right) + r_j^2 \frac{\partial}{\partial z} \left((1 + \varepsilon \cos \theta)^3 \frac{\partial p}{\partial z} \right) \\ &= 12\mu_{lubr} \left(\frac{r_j}{C_b} \right)^2 \left(\dot{\varepsilon} \cos \theta + \varepsilon \left(\dot{\phi} - \frac{\omega}{2} \right) \sin \theta \right) \end{aligned} \quad (5.61)$$

where

$$C_b = r_b - r_j$$

$$\varepsilon = \frac{e_b}{C_b}$$

The closed form solutions of Equation (5.61) can be derived for two limiting cases, the infinitely short (when slenderness ratio, $L_b/(2r_{shaft})$, is more than 2, Ocvirk's solution, p_o) and the infinitely short bearings (when slenderness ratio is less than 0.25, Sommerfield solution, p_s), as expressed in Equations (5.62) and (5.63), respectively.

$$p_o = \frac{3\mu_{lubr} U_j l_b^2}{r_j C_b} \left(0.25 - \left(\frac{z}{l_b} \right)^2 \right) \frac{\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta)^3} \quad (5.62)$$

$$p_s = \frac{6\mu_{lubr} U_j r_j}{C_b^2 (2 + \varepsilon^2)} \left(\frac{\varepsilon \sin \theta (2 + \varepsilon \cos \theta)}{(1 + \varepsilon \cos \theta)^2} \right) \quad (5.63)$$

Since most practical bearing applications use a slenderness ratio of between 0.25 to 1.5, Hirani *et al.* [133] proposed an approximation as expressed in Equation (5.64).

$$\frac{1}{p} = \frac{1 + \varepsilon \left(\frac{l_b}{2r_j} \right)^{1.2} (e^{\varepsilon^5} - 1)}{p_o} + \frac{e^{(1-\varepsilon)^3}}{p_s} \quad (5.64)$$

With the bearing pressure known, the total load supported by the bearing can be obtained by integrating the pressure components over the whole region of film where

the pressure is positive. Equations (5.62) and (5.63) indicate that this extends from 0 to π . Therefore, the load components with respect to the bearing attitude angle, ϕ , can be expressed as in Equations (5.65) and (5.66).

$$W \cos \phi = - \int_0^\pi \int_{-L/2}^{L/2} r_j p \cos \theta dz d\theta \quad (5.65)$$

$$W \sin \phi = \int_0^\pi \int_{-L/2}^{L/2} r_j p \sin \theta dz d\theta \quad (5.66)$$

where

$$\phi = \tan^{-1} \left(\frac{W \sin \phi}{W \cos \phi} \right) \quad (5.67)$$

The bearing friction force can be calculated by integrating the shear stress over the journal surface. Various theories are available to get the analytical solution of the integration. However, Hirani *et al.* [133] showed that the expression shown in Equation (5.68) is the most accurate. It can be seen that the friction force is a function of two terms, the shear of lubricant film and the hydrodynamic film pressure.

$$F_{b,f} = \frac{\mu_{lubr} U_j L_b r_j \pi}{C_b (1 - \varepsilon^2)^{0.5}} \left(\frac{2 + \varepsilon}{1 + \varepsilon} \right) + \frac{C_b \varepsilon}{2 r_j} W \sin \phi \quad (5.68)$$

where

$$h_b = C_b (1 + \varepsilon \cos \theta) \quad (5.69)$$

During operation, usually the bearing temperature increases due to friction. This reduces the lubricant viscosity, μ_{lubr} . The change in the lubricant temperature can be approximated according to the method proposed by Hirani *et al.* [133, 139] shown in Equation (5.70) while the change in the lubricant viscosity is a property of the lubricant.

$$T_{lubr} = T_{in} + \frac{\sigma F_{b,f} r_j \omega_{b,f}}{\rho_{lubr} C_{lubr} Q_{lubr}} \quad (5.70)$$

where T_{in} is the temperature of the lubricant entering the bearing and σ is the fraction of bearing friction power loss that is absorbed by the lubricant to increase its temperature.

Another important parameter is the minimum lubricant film thickness, h_{min} . This is obtained from Equation (5.69) by setting the angle θ to 0. This value should not be smaller than the roughness of the surface multiplied by some safety factor (usually around 10-100) to avoid solid to solid contact.

In most cases, the bearing load, W , is the known parameter, instead of the bearing pressure, p . Therefore, Equations (5.65) and (5.66) usually need to be solved using some iterative methods. Two such methods are discussed here. The first one requires less iteration but is only applicable for steadily loaded bearings. The second method can be used for dynamically loaded bearings but involves more iteration.

5.5.1. Steadily loaded journal bearings model [133]

In this method, the procedure begins by guessing a certain value for the eccentricity ratio, ε . It is important to note that the range of values of the eccentricity ratio is only between 0 and 1. With the guessed eccentricity ratio value, the guessed bearing load can be calculated according to Equation (5.71).

$$W_{guessed} = \sqrt{\left(\int_0^\pi Ar_j \cos \theta d\theta\right)^2 + \left(\int_0^\pi Ar_j \sin \theta d\theta\right)^2} \quad (5.71)$$

where

$$A = \frac{6U_j l_b r_f}{C_b^2 e^{(1-\varepsilon)^3}} \left(\frac{\varepsilon \sin \theta (1 + h_b)}{(2 + \varepsilon^2) h_b^2} \right) \left(1 - \left(\frac{B}{\sqrt{B + 0.25}} \log \left(\frac{0.5 + \sqrt{B + 0.25}}{-0.5 + \sqrt{B + 0.25}} \right) \right) \right)$$

$$B = \left(\frac{1 + \varepsilon \left(\frac{l_b}{2r_j} \right)^{1.2} (e^{\varepsilon^5} - 1)}{2e^{(1-\varepsilon)^3} \left(\frac{l_b}{2r_j} \right)^2 (2 + \varepsilon^2)} \right) h_b (1 + h_b)$$

The calculated guessed bearing load, $W_{guessed}$, is then compared with the real bearing load, W . If they are acceptably close, the guessed eccentricity ratio, ε , is already acceptable. However, if they are not, the guessed eccentricity ratio value must be adjusted to obtain a better guessed bearing load until the guessed bearing load is acceptably close to the real bearing load. With the real eccentricity ratio obtained, all the necessary bearing parameters such as the attitude angle, the bearing friction force and the minimum oil film thickness can now be calculated.

5.5.2. Dynamically loaded journal bearings model [136, 138, 139]

In this method, the complete history of the bearing load for one operational cycle must be available beforehand. The procedure itself is started by guessing the values of the eccentricity ratio, ε , and its rate of change, $\dot{\varepsilon}$, and the attitude angle, ϕ , and its rate of change, $\dot{\phi}$. With these guessed values, the range of angles where the bearing pressure is positive can be calculated according to Equations (5.72) and (5.73).

$$\theta_1 = \tan^{-1} \left(\frac{\dot{\varepsilon}}{\varepsilon(0.5(\omega_j + \omega_b) - \dot{\phi})} \right) \quad (5.72)$$

$$\theta_2 = \theta_1 + \left(\frac{0.5(\omega_j + \omega_b) - \dot{\phi}}{0.5(\omega_j + \omega_b) - \dot{\phi}} \right) \pi \quad (5.73)$$

Next, the corrected rates of change of the eccentricity ratio and the attitude angle can be calculated from Equations (5.74) and (5.75).

$$\dot{\varepsilon} = \frac{WC_b^2}{6\mu l_b r_j^3} \left(\frac{A_3 \cos \phi + A_2 \sin \phi}{A_1 A_3 - A_2^2} \right) \quad (5.74)$$

$$\dot{\phi} = \frac{WC_b^2}{6\mu l_b r_j^3} \left(\frac{A_1 \sin \phi + A_2 \cos \phi}{A_2^2 - A_1 A_3} \right) + 0.5(\omega_j + \omega_b) \quad (5.75)$$

where

$$A_1 = \int_{\theta_1}^{\theta_2} B \cos^2 \theta d\theta$$

$$A_2 = \int_{\theta_1}^{\theta_2} B \cos \theta \sin \theta d\theta$$

$$A_3 = \int_{\theta_1}^{\theta_2} B \sin^2 \theta d\theta$$

$$B = \left(1 - \frac{h_b(1+h_b)r_j^2/l_b^2}{\sqrt{h_b(1+h_b)r_j^2/l_b^2 + 0.25}} \log \left(\frac{0.5 + \sqrt{h_b(1+h_b)r_j^2/l_b^2 + 0.25}}{-0.5 + \sqrt{h_b(1+h_b)r_j^2/l_b^2 + 0.25}} \right) \right) \left(\frac{1+h_b}{h_b^2} \right)$$

With the rates of change known, the eccentricity ratio and the attitude angle at the next time step can be calculated according to Equations (5.76) and (5.77).

$$\varepsilon_{t+1} = \varepsilon_t + \dot{\varepsilon} \Delta t \quad (5.76)$$

$$\phi_{t+1} = \phi_t + \dot{\phi} \Delta t \quad (5.77)$$

The procedure is repeated until the last bearing load data. Because the bearing load is cyclical, the eccentricity ratio, ε_{last} , and the attitude angle, ϕ_{last} , of the last data must be the same as the ones of the first data, ε_I and ϕ_I . This requirement governs the convergence criteria. If they are not acceptably close, the iteration must be continued with ε_{last} and ϕ_{last} used as the new guessed values of ε_I and ϕ_I . The procedure is continued until the convergence criteria are satisfied.

With the histories of the eccentricity ratio and the attitude angle known, the other bearing parameters like the friction force and the minimum film thickness can be computed for the whole cycle by applying the necessary equations for every data point.

SIMULATION RESULTS

With all the necessary theoretical models of the RV expander set up in Chapters 4 and 5, the RV expander simulation can now be carried out to observe its theoretical behaviours. The simulation procedure is firstly discussed. Two simulation models are presented in this chapter. Model 1 is an idealised RV expander which operates adiabatically with no internal leakage and no frictional losses. Model 2 is a more comprehensive model with the heat transfer between the fluid and the working chambers, the internal leakage and the frictional losses considered. Both models assume that the vane is thin. Finally, parametric studies are carried out to better understand how various design parameters affect the RV expander performance.

6.1. Simulation Procedure

Based on the mathematical models developed in the preceding chapters, a computer code was developed in FORTRAN programming language. The REFPROP routines [146] were incorporated into the code to provide the thermophysical properties of the working fluid. The simulation code consists of four main blocks:

- (1) Geometries and kinematics
- (2) Thermodynamics and heat transfer
- (3) Dynamics
- (4) Internal leakage

Block (1) simulates the geometries and kinematics of the RV expander components discussed in Section 5.1. Block (2) simulates the thermodynamics processes of the working fluid (Section 4.1) including the suction flow, the discharge flow (Section 4.2)

and the heat transfer between the working fluid and the working chamber walls (Section 4.4). The thermodynamics processes are solved based on the calculation of the internal energy and the density of the working fluid. Block (3) simulates the dynamics processes of the expander components (Sections 5.2 and 5.3), including the endface (Section 5.4) and the shaft bearings (Section 5.5) effects. The dynamically loaded bearing model (Section 5.5.2) is used in this simulation. Finally, block (4) simulates the expander internal leakages (Section 4.3). There are internal leakages through the radial clearance gap (Section 4.3.1) and the vane side endfaces gaps (Section 4.3.2), but there is no internal leakage through the vane sides gaps (Section 4.3.3) due to the thin vane assumption. The 4th order Runge-Kutta method and the trapezoidal rule [137] are employed whenever initial value problems and numerical integration are involved, respectively. The Golden Section method with a convergence criterion of 0.01% is employed to solve the equations that need to be solved numerically.

To start the simulation, the required parameters and dimensions must first be initialised. Then, the four main blocks must be called in sequence because data from block (1) are needed by block (2), while the data from both blocks (1) and (2) are needed by block (3), and so on. At the end of block (4), the mass flow rates of the consecutive iterations are compared for convergence check. Convergence is said to have been achieved when they differ by less than 1% and hence, the computation is terminated. Otherwise, the iteration is repeated by recalling blocks (2) - (4) with the newly obtained internal leakage and shaft bearings data until convergence is achieved. The mass flow rate of the expander is taken as the parameter for convergence check because the internal leakage is the main cause of the fluctuations in the simulation data.

The algorithm of the code is shown in Figure 6.1.

Figure 6.1 The algorithm of the RV expander simulation computer code

6.2. Simulation Setup and Initial Conditions

In this simulation, all the assumptions mentioned in Chapters 4 to 5 are adopted. Air was used as the working fluid throughout the simulation because the experiments carried out in this project (presented in Chapters 9 and 10) used air as the working fluid. The type of working fluid used affects the expander mainly through the pressure-volume (PV) diagrams. Three different PV diagrams obtained from the same expander model which used (i) air as working fluid with a suction pressure of 8 bar and a discharge pressure of 1 bar, (ii) R-22 in a conventional refrigeration system and (iii) CO₂ in a transcritical refrigeration system, are shown in Figure 6.2. Air is in gaseous form throughout the processes while R-22 is a two-phase mixture. CO₂ undergoes changes from the supercritical gas to the two-phase mixture and therefore, an inflection point is observed in the PV diagram during expansion.

Figure 6.2 also shows that an expander that is well suited for an air expansion from 8 bar to 1 bar operating conditions may not function optimally for different applications. In the case when R-22 is used, the prediction shows that the suction volume is too small, while in the case of CO₂, its suction and discharge ports are too small.

Figure 6.2 Pressure-volume diagrams of an air expansion, an R-22 refrigeration system and a transcritical CO₂ refrigeration system

Although in actual applications, the speed of the expander may vary slightly over a complete expansion cycle as the torque varies, the present study assumed that the expander operates at a constant rotor angular velocity. The simulated expander dimensions and operational conditions are listed in Table 6.1. The inertias of the components are obtained from the SolidWorks 2010, a computer aided design software package [153]. These values have also been verified through manual calculations.

Table 6.1 Expander dimensions and operational parameters

Operating conditions	
Angular velocity of rotor	3000 rev/min
Suction reservoir temperature	25 °C
Suction reservoir pressure	800000 Pa
Discharge reservoir temperature	25 °C
Discharge reservoir pressure	101325 Pa
Rotor dimensions and parameters	
Radius of rotor	0.029 m
Length of rotor (or length of the expander)	0.025 m
Rotating inertia of rotor	0.00021 kg·m ²
Cylinder dimensions and parameters	
Radius of cylinder	0.035 m
Rotating inertia of cylinder	0.00281 kg·m ²
Non-loaded radial clearance gap between the rotor and the cylinder	15 ×10 ⁻⁶ m
Vane dimensions and parameters	
Length of vane	0.016 m

Width of vane	0.004 m
Friction coefficient between vane and vane slot	0.15 [116]
Suction ports dimensions and parameters	
Radius of suction ports	0.0032 m
Depth of suction ports	0.029 m
Number of suction ports	3
Opening angle of suction valve	84°
Coefficient of discharge of suction ports	0.7
Discharge ports dimensions and parameters	
Radius of discharge ports	0.0023 m
Depth of discharge ports	0.029 m
Number of discharge ports	3
Coefficient of discharge of discharge ports	0.7
Endfaces dimensions and parameters	
Viscosity of lubricant	0.0034 Pa·s
Width of endface gap	10×10^{-6} m
Rotor shaft bearings dimensions and parameters	
Radius of rotor shaft bearings	0.0125 m
Length of rotor upper shaft bearing	0.04 m
Length of rotor lower shaft bearing	0.02 m
Radial clearance of rotor shaft bearings	15×10^{-6} m
Distance between upper rotor shaft bearing to rotor	0.0325 m
Distance between lower rotor shaft bearing to rotor	0.0225 m
Viscosity of lubricant	0.0034 Pa·s

Cylinder shaft bearings dimensions and parameters	
Radius of cylinder shaft bearings	0.0246 m
Length of cylinder upper shaft bearing	0.01 m
Length of cylinder lower shaft bearing	0.01 m
Radial clearance of cylinder shaft bearings	12.5×10^{-6} m
Distance between upper cylinder shaft bearing to cylinder	0.0175 m
Distance between lower cylinder shaft bearing to cylinder	0.0175 m
Viscosity of lubricant	0.0034 Pa·s

The simulation begins at 0° rotor angle, which is the position where the vane is fully retracted and is completely inside the vane slot. At this point, the volume of the discharge chamber is at its maximum while the suction chamber is at its minimum (the minimum volume equals to the dead volume). The total volume of the suction ports, which is around 2.8 cm³, has been set as the dead volume of the expander. The fluids inside both chambers are initially assumed to be at the discharge reservoir conditions.

For convenience, rotor angle step, instead of time step, is used in the simulation. In order to choose the appropriate angle step, a step size independence test is conducted with the expander output torque used as the gauging parameter. For simplicity, the test is conducted under the no leakage (perfect sealing) assumption. The result of the test is shown in Figure 6.3. The result obtained with the angle step of 0.0005 rad (around 0.029°) differs by only 0.013% from that of the 0.00001 rad step size. Therefore, this angle step is selected for the simulations in this chapter.

Figure 6.3 Step size independence test result

6.3. Expander Model 1

The first set of simulation is obtained under the assumptions that the expander operates adiabatically with perfect sealing (no leakage) and with no frictional loss. The suction ports are assumed to uncover to its full area instantaneously and to close instantaneously when suction and expansion begin. The expansion is set to start at a rotor angle of 115.48° which is obtained by assuming that the fluid will expand isentropically to the maximum volume of the expander. It is necessary to note, however, that the expansion process itself is simulated with the procedure described in Chapter 4. The discharge ports are also assumed to fully uncover and to close instantaneously at the start and the end of every operational cycle.

The exposed vane length and vane sliding velocity profiles are plotted in Figure 6.4. The exposed vane length follows the variation of the cross-sectional area of the working chamber. Therefore, it is zero at 0° rotor angle, which means that the vane is fully retracted and stays in the vane slot. The exposed vane length increases as the rotor angle becomes larger until it reaches its maximum length at a rotor angle of 180° . This maximum vane exposed length is equal to twice the eccentricity of the expander, which is 12 mm. The exposed vane length then decreases until it becomes zero at the rotor angle of 360° . Accordingly, the vane sliding velocity reflects the rate of change of the

exposed vane length. It is positive during the first half of the cycle and is negative in the second half. Physically, a positive vane sliding velocity means that it is extending out of the vane slot while a negative one indicates that the vane is retracting into the vane slot.

Figure 6.4 Variations of: (a) exposed vane length of expander Model 1,
(b) vane sliding velocity of expander Model 1

The variations of the cylinder angular velocity and the acceleration are shown in Figure 6.5. The velocity variation is similar to a negative cosine function with an average of 100π rad/s (3000 rev/min), which is equal to the rotor angular velocity. It is at its minimum at 0° rotor angle, the location where the distance between the vane-cylinder contact point and the rotor centre is minimal. It increases as the rotor angle increases until it reaches its maximum at 180° rotor angle. This is the location when the distance between the vane-cylinder contact point and the rotor centre is maximal. The velocity then drops until it reaches its minimum value again at the end of the cycle. The variation of the cylinder angular acceleration is similar to a sine function with a zero average value, reflecting the rate of change of the cylinder angular velocity profile.

Figure 6.5 Variations of: (a) cylinder angular velocity of expander Model 1,
(b) cylinder angular acceleration of expander Model 1

The variations of the suction and the discharge chambers volumes and the pressure-volume diagram of the model are shown in Figure 6.6. During the initial and end stages of the cycle, the change of the rotor angle does not affect the suction and discharge volumes because of the vane thickness. The pressure increases very rapidly at the beginning of the suction process, when the volume is minimal. This is because of the assumption that the suction ports are uncovered instantaneously. In practice, the suction ports are progressively uncovered and hence, the pressure increase will be gradual. The sizes of the suction and discharge ports are large enough, indicated by the fairly constant fluid pressures during suction and discharge processes. The area encapsulated by the pressure-volume lines indicates the amount of energy produced by the fluid.

Figure 6.6 (a) Variations of working chambers volumes of expander Model 1,
(b) PV diagram of expander Model 1

The variations of the masses of the working fluids inside the two working chambers are shown in Figure 6.7. The mass inside the suction chamber increases rapidly initially due to a large pressure difference between the suction chamber and the suction reservoir. As the value of the pressure in the suction chamber becomes closer to that of the suction reservoir, the mass increases more steadily until the rotor angle is 115.48°, the point at which the suction process stops and the expansion starts. From then on, the mass of the fluid in suction chamber remains constant, since no internal leakage is considered in this case. The mass of the fluid in the discharge chamber decreases from the onset of the discharge process until the end of the cycle.

Figure 6.7 Working fluid masses in working chambers of expander Model 1

The variation of the output torque of expander Model 1 is shown in Figure 6.8. The torque variation is a strong function of the cylinder inertia and a very weak function of the pressure difference across the vane. In fact, the pressure component is very small such that the output torque trend is practically identical to the inertia component. However, while the average of the inertia component is zero, the average output torque value is 1.16 N·m. The inertia component is the product between the cylinder inertia with the negative of the angular acceleration, whose average is zero. The pressure component is a function of the pressure difference between the two working chambers and the exposed vane length. Therefore, it is maximal at the end of the suction process where the pressure difference between the suction and the discharge chambers is largest and the exposed vane length is long. This pressure component has a positive average value which results in the positive average value of the torque.

Figure 6.8 Output torque of expander Model 1 and its components

Figure 6.9 shows the variation of the output torque with operating angle at 3 different operating pressure conditions. Three cases are shown: (1) 1 bar discharge, 8 bar suction of air, a difference of 7 bar; (2) 6 bar discharge, 21 bar suction, a difference of 15 bar, which is the typical operating condition of a conventional R-22 refrigeration

system; and (3) 42 bar discharge, 110 bar suction, a difference of 68 bar, which is the typical operational pressure range of a transcritical carbon dioxide refrigeration system.

Figure 6.9 Trends of output torque of expander Model 1 at various operating pressure conditions

It can be seen from Figure 6.9 that the trend of the output torque of case (3) is no longer sinusoidal. It has a sharp peak near the 180° mark, making it distinct from the other two cases simulated. This is because the torque of case (3) is significantly affected by the pressure component. The pressure component is comparable to the inertia component, caused by the large pressure difference involved. Therefore, the output torque trend is a combination of the two components. The sharp peak near the 180° mark is related to the peak of the pressure component of case (3). In cases (1) and (2), the pressure components are significantly smaller than the inertia components. Therefore, the output torque trends practically only follow the inertia components.

It is interesting to observe that the pressure ratio of case (3), which is 2.6, is much smaller than that of case (1), which is 8. This shows that the pressure difference, not the pressure ratio, is the important parameter in the RV expander. This is because the force acting at the vane is a function of the pressure difference, not the pressure ratio.

6.4. Expander Model 2

In this section, the simulation results of expander Model 2, where the heat transfer between the working fluids and the working chamber walls, the internal leakages, the ports dynamics and the friction losses are taken into consideration, are discussed. As a consequence of the assumptions adopted to formulate the model in Chapter 5, the kinematics of the expander is wholly geometrically dependent, the kinematics and the working chambers volumes behaviours of the expander considered in this section are identical to that considered in Section 6.3 and will not be discussed again. The dimensions and operating conditions are also following those tabulated in Table 1.

The pressure-volume (PV) diagram and the variations of the fluid pressure of the expander Model 2 are shown in Figure 6.10. The PV diagram of Model 1 is also included in the figure for comparison. It can be seen that in the expander Model 2, due to the dynamics of the suction valve, the pressure drops faster and more smoothly at the end of the suction process. The fluid in Model 2 is also found to be under-expanded. It ends with the fluid pressure that is higher than the discharge reservoir pressure. This is caused mainly by the heat transfer from the chamber walls to the working fluid. More detailed discussions about this will be presented in the subsequent paragraphs. As a result, the fluid experiences a rapid pressure drop at the beginning of the discharge process before eventually discharges out from the expander in the expander Model 2.

Figure 6.10 also shows that the area encapsulated by the PV line of Model 2 is larger than that of Model 1 by about 14%. This indicates that the expander Model 2 produces more energy. However, this does not mean that Model 2 is more efficient than Model 1 because Model 2 has a higher mass flow rate as compared to Model 1 caused by the internal leakages and longer suction process due to the suction valve dynamics.

Figure 6.10 (a) PV diagrams of expander Models 1 and 2,
(b) variations of fluid pressure of expander Model 2

The variation of the suction valve opening area is shown in Figure 6.11. It can be seen that the closing (and opening) process of the suction valve is gradual. This causes the suction process to also end gradually, instead of abruptly like that in Model 1.

Figure 6.11 Variation of suction valve opening area of expander Model 2

The variations of the suction chamber fluids temperature of expander Models 1 and 2 are shown in Figure 6.12. At the beginning of the suction process, the temperature of the fluid in the suction chamber increases rapidly due to the sudden increase of its pressure during the initial stage of the suction process. As the pressure in the suction chamber settles down, the temperature decreases due to the interaction between the fluid

inside the suction chamber and that in the suction reservoir which is kept constant at 25°C. When heat transfer is introduced into the model as is the case with Model 2, the temperature quickly drops to the wall temperature (25°C) as the suction chamber pressure settles. This makes the high temperature condition in Model 2 very short lived. As the fluid expands, its temperature drops rapidly. However, because of the heat transfer with the expander wall, the temperature of the fluid in the suction chamber of Model 2 does not decrease continuously, unlike that in Model 1. Instead, it inflects at the rotor angle of around 180°, when the temperature is -15.7°C, way below the assumed wall temperature of 25°C. This increase in temperature maintains the fluid pressure even though its volume increases during the expansion process. This causes the under-expansion of the gas as shown and discussed in Figure 6.10. It is worthy to note that Figure 6.12 shows that Model 1 is unrealistic because its fluid temperature drops to a very low value (-100°C). In reality, this is impossible because of the heat transfer between the working fluid and the chamber walls.

Figure 6.12 Variations of suction chamber fluids temperature of Models 1 and 2

The variations of the fluid masses of Models 1 and 2 in the working chambers are shown in Figure 6.13. Unlike Model 1, the fluid mass in the suction chamber does not increase rapidly at the beginning of the cycle in Model 2 because the suction ports open

gradually. The mass increases steadily until the suction valve is fully closed. The mass profile inflects at a rotor angle of around 30° because the suction opening area switches from that expressed by Equation (4.18) to Equation (4.22). Unlike that in Model 1, during the expansion process, the fluid mass decreases slowly because the high pressure fluid in the suction chamber continuously leaks into the discharge chamber through the radial clearance and the vane endfaces gaps. As for the discharge chamber, the fluid mass decreases rapidly at the beginning due to a large pressure difference across the discharge ports. After that, the fluid mass of Model 2 decreases more gradually as compared to that of the Model 1 because of the internal leakage from the suction chamber into the discharge chamber.

Figure 6.13 Variations of working fluid masses of expander Models 1 and 2:

(a) suction chamber, (b) discharge chamber

Figure 6.13(a) also shows that although Models 1 and 2 begin with the same fluid masses in the suction chambers, the expander Model 2 ends the cycle with more mass in its suction chamber as compared to Model 1, despite of the internal leakages in Model 2. This is because the suction process of Model 2 is longer than that of Model 1 due to

suction valve dynamics. The mass flow rate of Model 2 is 65.8 mg/rev (3.3 g/s) and that of the Model 1 is 59.2 mg/rev (3.0 g/s).

The variations of the internal leakage mass flow rates of the expander Model 2 with operating angle are plotted in Figure 6.14(a). It is assumed that the total internal leakage rate is the summation of only two components, i.e. through the radial clearance and the vane endfaces gaps. It can be seen that the leakage through the radial clearance dominates the total internal leakage of the expander. The leakage flow rate through the radial clearance gap varies with time, or more precisely, with the operating angle, because both the instantaneous pressure difference between the suction and discharge chambers and the radial clearance gap size also vary with time. The radial clearance gap varies with time (shown in Figure 6.14(b)) because it depends on the complex interaction between the shafts and their bearings, which are time-dependent.

The mass flow rate through the radial clearance gap increases steadily from 0° until it reaches its maximum at about 120° . This corresponds to the suction process where the pressure difference increases and then remains large (see Figure 6.10(b)). At about 120° , the expansion process starts and after which the pressure difference between the suction and discharge chambers begins to decrease. Therefore, from about 120° onward, the mass flow rate decreases until the end of the cycle.

From 120° to 180° , the flow rate of the leakage through the radial clearance gap drops rapidly. Interestingly, a detailed observation of the region between rotor angles of between 180° to 240° shows that the gradient is somewhat inconsistent with the ongoing trend as a small hump is observed. This corresponds to the hump of the radial clearance gap in the same rotor angle region shown in Figure 6.14(b). This region is where the shaft bearings loads caused by the unbalanced pressure forces acting on the surfaces of the rotor and the inner wall of the cylinder are high and the directions are not towards

the rotor-cylinder line contact. Therefore, the rotor is pushed away from the rotor-cylinder line contact, resulting in a large radial clearance gap.

Comparing the indicated mass flow rate with the total internal leakage of the expander shows that the expander has a volumetric efficiency of 69.8%. The leakages through the radial clearance gap and the vane endfaces contribute to 78.9% and 21.1% of the total leakages, respectively.

Figure 6.14 Variations of: (a) internal leakage mass flow rates of expander Model 2, (b) radial clearance gap of expander Model 2

The expander output torque and its components are shown in Figure 6.15. The average total output torque value is 0.82 N·m, lower than that of Model 1, which is 1.16 N·m. The mechanical efficiency of Model 2 is 56.5%, which is a relatively low value, caused mainly by the low operating pressure condition used (pressure difference of 7 bar) and high vane side friction loss due to the large cylinder rotating inertia. More discussions about the frictional losses are presented in the subsequent paragraphs. This low efficiency value also suggests that design optimisations are possible. Parametric studies to direct such optimisations are presented in Section 6.5.

Similar to Model 1, the inertia component dominates the output torque. However, there is a loss component in the case of Model 2 that does not exist in Model 1. This loss component has a negative average value and consists of the friction losses at the vane side, the shaft bearings and the rotor endfaces. The contributions of each of these losses are plotted in Figure 6.15(b). The vane side friction loss is the main contributor of the friction losses of the simulated expander. It accounts for around 70.5% of the losses while the cylinder bearings, the rotor bearings and the rotor endfaces friction losses contribute to 20.9%, 5.9% and 2.7%, respectively. The vane side friction loss is high because of the large cylinder inertia used in this simulation, resulting in high vane side contact forces. It is therefore important to minimise the cylinder inertia as far as possible to reduce this loss and improves the mechanical efficiency.

Figure 6.15 Variations of: (a) output torque and its components of expander Model 2, (b) friction power loss components of expander Model 2

The variations of the vane contact forces of the expander Models 1 and 2 are shown in Figure 6.16. The important numbers of the figure are tabulated in Table 6.2. It can be seen that they are almost identical, although closer inspection shows that the amplitude of the contact force of Model 1 is slightly smaller than that of the Model 2 due to the

exclusion of losses in Model 1. The variation of the vane contact force also follows that of the cylinder acceleration very closely, indicating the domination of the inertia term over the other terms. Interestingly, the force flips its sign from positive to negative at a rotor angle of around 180° and from negative to positive at a rotor angle of 0° and 360° . During the first half of the cycle, the rotor pushes the cylinder, while at the other half, the cylinder pushes the rotor. This flipping of vane contact force direction is instantaneous because of the thin vane assumption. However, in reality, the flipping process takes some time and this is one of the limitations of the thin vane assumption adopted in this simulation model.

Figure 6.16 Variations of vane contact force of expander Models 1 and 2

Table 6.2 Important numbers of vane contact force of expander Models 1 and 2

	Average (N)	Maximum (N)	Minimum (N)
Model 1	0.0	1399.51	-1399.51
Model 2	5.98	1430.53	-1418.30

6.5. Parametric Studies

After discussing the behaviours of the RV expander Models 1 and 2, parametric studies of the RV expander are presented in this section. The design parameters include the cylinder rotating inertia, the operating pressure conditions and the cylinder and the rotor radii. The effect of the rotor rotating inertia is not investigated because it is not expected to affect the expander performance since the rotor is assumed to rotate at a constant angular velocity in these simulations. The performance of the expander is characterised by the mechanical and volumetric efficiencies.

6.5.1. Cylinder rotating inertia

The effects of the rotating inertia of the cylinder (subsequently referred to as cylinder inertia or just inertia) on the expander performance are discussed in this section. The study is carried out by varying the cylinder rotating inertia while keeping the rest of the dimensions and parameters listed in Section 6.2 unchanged. The cylinder inertia itself in practice can be reduced by using lighter material for the cylinder or by removing the materials to reduce weight. For example, by using aluminium (density of 2700 kg/m^3) instead of steel, the cylinder inertia of the expander in Table 6.1 will be reduced to 0.34 of the original inertia.

The effects of cylinder inertia to the efficiencies of the expander are shown in Figure 6.17. The inertia has been normalised by dividing it with the cylinder inertia listed in Table 6.1 (which is $0.00281 \text{ kg}\cdot\text{m}^2$). Therefore, a normalised cylinder inertia of 1 refers to a cylinder inertia of $0.00281 \text{ kg}\cdot\text{m}^2$ and so on. It can be seen that the mechanical efficiency increases as the inertia decreases. This is because with a smaller cylinder inertia, the vane contact force is smaller and so does the vane side friction loss. However, the mechanical efficiency value can only increase up to 80%, even in the

hypothetical case when the cylinder inertia is assumed zero. This limitation is due to the existence of the other losses, i.e. shaft bearings and endfaces losses, and the relatively low operating pressure conditions (a pressure difference of 7 bar) used here. With a better shaft bearings design and higher operating pressure conditions, the mechanical efficiency will be higher. On the other hand, if the dimensionless cylinder inertia is increased to more than four, the mechanical efficiency drops to negative. This situation means that the expander does not produce positive output power anymore. Instead, it withdraws power to operate.

Figure 6.17 Effects of cylinder inertia to expander efficiencies

The effect of cylinder inertia to volumetric efficiency is found to be the inverse of mechanical efficiency. As the cylinder inertia increases, the volumetric efficiency also increases. This is because as the cylinder inertia increases, the vane contact force also increases, pushing the rotor to be closer to the cylinder. This, in turn, reduces the radial clearance gap (see Figure 6.18 for comparison of the radial clearance gaps of expanders with dimensionless cylinder inertias of 0.01, 1 and 5) and the leakage. The leakage through the vane endfaces gaps are not affected by the inertia since the gap sizes are assumed to be constant throughout the simulation.

Figure 6.18 Effects of cylinder rotating inertia to radial clearance gap

6.5.2. Operating pressure conditions

The effects of operating pressure conditions are discussed in this section. The discharge reservoir pressure is kept constant at 1 atm throughout, while the suction reservoir pressure is varied. The suction valve opening angle is also varied accordingly and is listed in Table 6.3. These angle values are set according to the values that will result in the high pressure fluid to be expanded as close as possible to the discharge reservoir pressure when the process is adiabatic and without any leakage involved. The other dimensions and parameters listed in Table 6.1 are left unchanged.

Table 6.3 Suction valve opening angles

Suction Pressure (bar)	2	4	6	8	10	12
Suction Valve Opening Angle	164°	116°	94°	84°	67°	58°

The effects of the operating pressure conditions to the expander efficiencies are shown in Figure 6.19. It can be seen that with a larger operating pressure difference, mechanical efficiency increases while volumetric efficiency decreases. The mechanical efficiency increases because pressure is the component that produces a positive average output torque value. With a larger operating pressure difference, this pressure

component becomes more significant, increasing the average output torque and the mechanical efficiency. It is noted that the friction losses component of the torque also increases with a higher operating pressure difference. However, the increase is less significant as compared to the pressure component. It is interesting to observe that if the pressure difference is too small, the mechanical efficiency of the expander will become negative. This is when the expander stops producing a net positive output power. Instead, it becomes a power consuming device.

Figure 6.19 Effects of operating pressure conditions to expander efficiencies

The rate of increase of the mechanical efficiency is steep when the operating pressure difference is small. This is because as the pressure difference increases, the area encapsulated by the pressure-volume (PV) diagram, which is equal to the useful fluid energy, increases as its height (pressure) increases. However, at the same time, the suction process also becomes shorter, reflected by the smaller suction valve opening angle. Therefore, as the operating pressure difference increases further, the increase in the PV diagram area becomes less significant and eventually no further pressure ratio increase is possible with the current expander design. The volumetric efficiency decreases with the operating pressure. This is because a higher operating pressure

difference causes a higher internal leakage through all the leakage paths, resulting in a lower volumetric efficiency.

6.5.3. Cylinder and rotor radii

As listed in Table 6.1, the RV expander studied in this chapter has a rotor radius of 29 mm, a cylinder radius of 35 mm and an expander length of 25 mm. This results in an expander with a capacity of around 30 cm³. However, there are infinite number of possible design configurations that will result in a RV expander with the same capacity. It is the purpose of this section to study how different design configurations affect the expander performance. To generalise the conclusions, a dimensionless parameter called the radius ratio, R , which is defined as the ratio between the rotor and the cylinder radii, will be used to make the radii dimensionless.

As the dimensions of the expander are varied, the inertias of the components also vary. To simplify the study, the inertias of the cylinder and the rotor are approximated by assuming that the cylinder and the rotor are made up of simpler parts as shown in Figures 6.20 and 6.21, respectively. The suction and the discharge ports have been represented by a hole at the centres of the rotor body and the rotor shaft.

Figure 6.20 Exploded view of the simplified cylinder components

In Figure 6.20, component C_1 is the upper end cover of the cylinder with a hole to accommodate the rotor shaft. For simplicity, the radius of this hole is assumed to be the same as the rotor shaft radius, which is equal to 12.5 mm. The thickness of component C_1 is set to be 5 mm while the outer radius is set to be equal to the outer radius of component C_2 . Component C_2 is the cylinder body complete with the vane slot. The length of this component is equal to the expander length. The inner radius is equal to the cylinder radius while the wall thickness is set to be 18 mm to accommodate the vane slot. The vane slot itself is assumed to have a rectangular cross-sectional area with a width of 6.5 mm and a depth of 13 mm. Components C_3 and C_4 are the lower end cover and the shaft of the cylinder, respectively. The thickness of component C_3 is 5 mm while its outer radius is equal to the outer radius of component C_2 . Component C_4 is 50 mm long with a radius of 24.6 mm. The total inertia of the cylinder can then be estimated by summing the rotating inertias of all the cylinder components. Since the cylinder movement is a rotation about the cylinder shaft axis, the point of reference of the rotating inertia is at the cylinder shaft axis. Assuming that all the components are made of the same material with a density of ρ_c , the rotating inertias of the components can be calculated according to the established expressions [154]. The inertia of component C_2 in Figure 6.20 requires the use of the Parallel Axis Theorem [155] due to the vane slot. By assuming that the cylinder components are made of steel with a density of 7800 kg/m^3 , the cylinder rotating inertia is found to be around $0.0028 \text{ kg}\cdot\text{m}^2$.

Figure 6.21 Exploded view of the simplified rotor components

In Figure 6.21, component R_1 is the end cover of the rotor with a thickness of 1 mm and outside radius that is equal to the rotor radius. Component R_2 is the rotor body including the vane while R_3 is the rotor shaft. The radius of the rotor body is equal to the rotor radius while its length is equal to the expander length. The rotor body has a vane that is assumed to have a rectangular cross-section with a width of 4 mm and a length of 16 mm. The body also has a 5 mm radius hole at its centre that goes through all the way to the rotor shaft. The shaft is 50 mm long with a radius of 12.5 mm.

The rotor rotates about its shaft axis. Therefore, this axis should be used as the point of reference for the rotating inertia calculation. Parallel Axis Theorem [155] needs to be used to calculate the vane rotating inertia. If the rotor is made of steel with a density of 7800 kg/m^3 , the rotor rotating inertia is found to be around $0.00026 \text{ kg}\cdot\text{m}^2$.

The effects of different cylinder and rotor radii to an RV expander with a capacity of 30 cm^3 are shown in Figure 6.22. It is important to note that there is a limit to the range of the practical radius ratio as discussed in Chapter 6. Therefore, the cylinder radius is only varied from 30 mm to 38 mm in this study, which corresponds to a radius ratio of 0.76 to 0.97. The rotor radius is only varied from 27 mm to 33 mm, which corresponds to radius ratios of 0.77 to 0.94.

Figure 6.22 Effects of radius ratio to expander efficiencies:

(a) rotor radius fixed at 29 mm, (b) cylinder radius fixed at 35 mm

It can be seen that in general, as the radius ratio increases, the mechanical efficiency increases while the volumetric efficiency decreases. The figure also shows that the rate of decrease of the volumetric efficiency tends to increase as the radius ratio is higher. This is because as the radius ratio increases, the expander becomes longer. This makes the widths of the internal leakage paths at the radial clearance to also increase, resulting in more internal leakage. As the radius ratio increases further, the increase in the expander length tends to be more significant. Therefore, the rate of decrease in the volumetric efficiency increases as the radius ratio is higher.

As for the mechanical efficiency in Figure 6.22(a), as the radius ratio increases, the cylinder radius decreases but the expander length increases. Due to the relationship between the radii and the expander length to the expander capacity, where the radii has a power factor of two while the length is not, the cylinder radius decreases slower than the increase of the expander length as the radius ratio increases. This results in the vane exposed area, which dictates the pressure component of the output torque, to increase faster than the increase in the cylinder rotating inertia, which affects the total friction loss of the expander. At the same time, as the radius ratio increases, the shaft bearings

loads also increase due to the increase in the expander length, resulting in more shaft bearings friction losses. However, the rates of increase of these losses are also slower than the increase of the pressure component of the output torque.

The reason behind the mechanical efficiency trend in Figure 6.22(b) is slightly different. This is because now, unlike the case in Figure 6.22(a), as the radius ratio increases, the cylinder radius remains unchanged. Therefore, the increase in the cylinder inertia is solely due to the increase in the expander length. As the radius ratio increases, the expander length also increases. This causes the vane exposed area to increase which results in the increase of the pressure component of the output torque. At the same time, the cylinder rotating inertia and the shaft bearings loads also increase, resulting in higher friction loss. However, the increase in the friction loss is slower as compared to the increase in the pressure component of the output torque.

The contradicting behaviours of the mechanical and volumetric efficiencies create an optimum radius ratio value at where the expander will have the highest possible summation of efficiencies. This point is at the intersection of the two efficiency curves and is found to be about 0.87 for both Figures 6.22(a) and 6.22(b).

COMPARISON OF RV EXPANDER DESIGN CONFIGURATIONS

As was mentioned in Chapter 3, the Revolving Vane mechanism has at least four possible design configurations. They are as listed below and illustrated in Figure 7.1.

- RV-I: The vane is fixed to the rotor and the rotor is the driving component. This is the configuration that has been used in the previous chapters.
- RV-Ia: The vane is fixed to the rotor and the cylinder is the driving component.
- RV-II: The vane is fixed to the cylinder and the cylinder is the driving component.
- RV-IIa: The vane is fixed to the cylinder and the rotor is the driving component.

Figure 7.1 The four RV expander alternative design configurations:

(a) RV-I; (b) RV-Ia; (c) RV-II; (d) RV-IIa

In this chapter, these four alternative design configurations are analysed and compared theoretically. The theoretical models of the RV-I configuration have been formulated in Chapters 4 and 5. The theoretical models for the other three configurations are presented in the first section of this chapter. Once all the theoretical models are set up, the comparisons are carried out and discussed in the subsequent sections. Because the operating pressure range is expected to be one of the major factors affecting the performance of the machine as suggested by the results discussed in Chapter 6, the low-operational pressure range (a pressure difference between suction and discharge of less than 10 bar) and a high pressure range (a pressure difference between suction and discharge above 50 bar, typical of a transcritical CO₂ refrigeration system) applications are treated separately when evaluating the expander performance.

7.1. Theoretical Models

In this section, the theoretical models for RV-Ia, RV-II and RV-IIa will be formulated. Many of the steps and procedures used previously in deriving the theoretical models for RV-I are still applicable. Therefore, only the steps and equations that are peculiar to individual RV configuration will be shown. To differentiate the models for RV-I, RV-Ia, RV-II and RV-IIa, subscripts “I”, “Ia”, “II” and “IIa” are assigned to the respective models. However, to simplify the notations, during the derivations, these subscripts will only appear at the final forms of the equations. To simplify the study, the vane and the vane slot are assumed thin and thus, their respective thicknesses play no role in kinematics models. These thickness, however, are accounted for when formulating the thermodynamics and dynamics models.

7.1.1. RV-Ia

Geometrically, the RV-Ia design configuration is identical to the RV-I configuration. Therefore, the volumes of the working chambers can still be calculated according to the expressions used in the RV-I, shown in Equations (7.1a) and (7.2a). In these equations, the difference between the lengths of the vane exposed to the discharge and the suction chambers, l_{ve1} and l_{ve2} , has been assumed to be negligible and hence, the average vane exposed length, l_{ve} , is used. The rest of the thermodynamics models of RV-Ia are also the same as RV-I.

$$V_{scv,Ia} = V_{dead,scv,Ia} + \frac{l_{exp}}{2} (\varphi_c r_c^2 - e r_c \sin \varphi_c - \varphi_r r_r^2 - l_{ve} w_v) \quad (7.1a)$$

$$V_{dev,Ia} = l_{exp} \pi (r_c^2 - r_r^2) - V_{scv,Ia} \quad (7.2a)$$

The main difference between the RV-I and the RV-Ia design configurations are in the driving components. RV-Ia uses the cylinder while RV-I uses the rotor. Therefore, the operational angle of RV-Ia is the cylinder angle, instead of the rotor angle like that in the RV-I, causing the kinematics and the dynamics models of the two configurations to be different. To formulate the RV-Ia kinematics models, the triangle ACR used previously in Chapter 5 is shown in Figure 7.2.

Figure 7.2 The triangle ACR of RV-Ia

From the triangle ACR, the cosine rule gives Equation (7.3a) which relates the exposed vane length with the cylinder angle. Since the model requires that at $\varphi_c = 0$, $l_{ve} = 0$, only the positive result of the square term in Equation (7.3a) is considered.

$$l_{ve, Ia} = -r_r + \sqrt{r_r^2 + 2er_c(1 - \cos \varphi_c)} \quad (7.3a)$$

Differentiating Equation (7.3a) with time gives the sliding velocity of the vane as expressed in Equation (7.4a).

$$\frac{dl_{ve, Ia}}{dt} = \omega_c \left(\frac{er_c \sin \varphi_c}{r_r + l_{ve, Ia}} \right) \quad (7.4a)$$

By applying the cosine rule again to the triangle ACR, the relationship between the rotor and the cylinder angles (through l_{ve}) can be expressed as Equation (7.5a).

$$\cos \varphi_{r, Ia} = \frac{r_c^2 - e^2 - (r_r + l_{ve})^2}{2e(r_r + l_{ve})} \quad (7.5a)$$

Again, from the triangle ACR, the angle γ can be derived and is found to be as expressed in Equation (7.6a). The convention used here is that angle γ always starts from the line that connects points A and. Therefore, to be consistent with the convention used in this report that clockwise direction is positive, Equation (7.6a) is applicable only for $0 \leq \varphi_c \leq \pi$. Otherwise it must carry a negative sign.

$$\gamma_{Ia} = \cos^{-1} \left(\frac{r_c^2 + (r_r + l_{ve})^2 - e^2}{2r_c(r_r + l_{ve})} \right) \quad (7.6a)$$

The velocities at point A have the relations expressed in Equations (7.7a) - (7.9a).

$$v_{Ac} = r_c \omega_c \quad (7.7a)$$

$$v_{Ar} = (r_r + l_{ve}) \omega_r \quad (7.8a)$$

$$v_{Ar} = v_{Ac} \cos \gamma \quad (7.9a)$$

Combining Equations (7.6a) - (7.9a) gives the expression for the rotor angular velocity which is expressed in Equation (7.10a).

$$\omega_{r,la} = \omega_c \left(\frac{r_c^2 + (r_r + l_{ve})^2 - e^2}{2(r_r + l_{ve})^2} \right) \quad (7.10a)$$

Differentiating Equation (7.10a) with time gives the angular acceleration of the rotor and is expressed in Equation (7.11a).

$$\alpha_{r,la} = \alpha_c \left(\frac{r_c^2 + (r_r + l_{ve})^2 - e^2}{2(r_r + l_{ve})^2} \right) + \omega_c \frac{dl_{ve}}{dt} \left(\frac{e^2 - r_c^2}{(r_r + l_{ve})^3} \right) \quad (7.11a)$$

To formulate the dynamics models for RV-Ia, the free body diagrams of the rotor and the cylinder as shown in Figure 7.3 are referred to.

Figure 7.3 Free-body diagrams of RV-Ia components: (a) rotor (b) cylinder

From the torque balance of the rotor based on the free body diagram in Figure 7.3, the vane contact force is found to be as expressed in Equation (7.12a).

$$\begin{aligned}
I_r \alpha_r &= F_{v,p} \left(r_r + \frac{l_{ve}}{2} \right) - F_{v,n} (r_r + l_{ve}) - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \frac{w_v}{2} + \sum T_r \\
\rightarrow F_{v,n,la} &= \frac{F_{v,p} \left(r_r + \frac{l_{ve}}{2} \right) + \sum T_r - I_r \alpha_r}{\left(r_r + l_{ve} + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta \frac{w_v}{2} \right)} \quad (7.12a)
\end{aligned}$$

It can be seen that the vane contact force of RV-Ia is also a function of the pressure difference across the vane, unlike in the case of RV-I. This means that if the operating pressure conditions are high, the vane contact force and the vane side friction loss will also be high. This is obviously undesirable. However, it is interesting to observe that the pressure and the inertia terms have different signs. This opens a possibility that these terms may cancel each other and hence, reduce the vane contact force. This is discussed in Section 7.2.

From the torque balance of the cylinder, the RV-Ia expander output torque is found to be as expressed in Equation (7.13a). The output torque of the RV-Ia expander is affected implicitly by the pressure difference across the vane through the vane contact force, unlike in the case of RV-I where the relationship is explicit.

$$\begin{aligned}
I_c \alpha_c &= F_{v,n} r_c \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \left(r_c \sin \gamma - \frac{w_v}{2} \right) + \sum T_c - T_{gen} \\
\rightarrow T_{gen,la} &= F_{v,n} \left(r_c \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta \left(r_c \sin \gamma - \frac{w_v}{2} \right) \right) + \sum T_c - I_c \alpha_c \quad (7.13a)
\end{aligned}$$

7.1.2. RV-II

The two main differences between RV-II and RV-I are the geometry and the driving component. The vane of RV-II is fixed at the cylinder and hence, points towards the centre of the cylinder, instead of the rotor like in RV-I. The vane slot is at the rotor

and hence, the contact point between the vane and the vane slot should be approximated to locate at the rotor wall. The driving component of RV-II is the cylinder. Therefore, the operating angle is the cylinder angle. With these differences, some of the geometrical relationships and most of the kinematics and the dynamics models of RV-II will be different from RV-I. These are presented in this section.

The volumes of the working chambers of RV-II are as shown in Equations (7.1b) and (7.2b). The thermodynamics processes can be modelled according the RV-I models.

$$V_{scv,II} = V_{dead,scv,II} + \frac{l_{exp}}{2} (\varphi_c r_c^2 - e(r_c - l_{ve}) \sin \varphi_c - \varphi_r r_r^2 - l_{ve} w_v) \quad (7.1b)$$

$$V_{dev,II} = l_{exp} \pi (r_c^2 - r_r^2) - V_{scv,II} \quad (7.2b)$$

To formulate the kinematics models of RV-II, the triangle ACR that is suited for RV-II is shown in Figure 7.4. Point A is now located at the rotor wall. Therefore, it is always of a distance of the rotor radius, r_r , from the rotor centre, R.

Figure 7.4 The triangle ACR of RV-II

From the triangle ACR in Figure 7.3, the exposed vane length can be obtained. Considering that the system requires that at $\varphi_c = 0$, $l_{ve} = 0$. Therefore only the positive value is taken as the appropriate expression as shown in Equation (7.3b).

$$l_{ve,II} = r_c - \left(e \cos \varphi_c + \sqrt{r_r^2 - e^2 \sin^2 \varphi_c} \right) \quad (7.3b)$$

Differentiating Equation (7.3b) with time gives the sliding velocity of the vane as expressed in Equation (7.4b).

$$\frac{dl_{ve,II}}{dt} = \omega_c e \sin \varphi_r \left(\frac{r_c - l_{ve,II}}{r_c - l_{ve,II} - e \cos \varphi_c} \right) \quad (7.4b)$$

The relationships between the rotor angle, the angle γ and the cylinder angles (through l_{ve}) are also found from the triangle ACR. They are expressed in Equations (7.5b) and (7.6b). Similar to RV-I and RV-Ia, the convention is that angle γ always starts from the line that connects points A and C in Figure 7.4. Therefore, Equation (7.6b) is applicable only for $0 \leq \varphi_c \leq \pi$. Otherwise, it must carry a negative sign.

$$\cos \varphi_{r,II} = \frac{(r_c - l_{ve})^2 - e^2 - r_r^2}{2er_r} \quad (7.5b)$$

$$\gamma_{II} = \cos^{-1} \left(\frac{r_r^2 + (r_c - l_{ve})^2 - e^2}{2r_r(r_c - l_{ve})} \right) \quad (7.6b)$$

The vectors of the tangential velocities at point A in the triangle ACR in Figure 7.4 have the relationships listed in Equations (7.7b) - (7.9b).

$$v_{Ac} = (r_c - l_{ve}) \omega_c \quad (7.7b)$$

$$v_{Ar} = r_r \omega_r \quad (7.8b)$$

$$v_{Ac} = v_{Ar} \cos \gamma \quad (7.9b)$$

By combining Equations (7.6b) - (7.9b), the rotor angular velocity can be obtained as expressed in Equation (7.10b).

$$\omega_{r,II} = \omega_c \left(\frac{2(r_c - l_{ve})^2}{r_r^2 + (r_c - l_{ve})^2 - e^2} \right) \quad (7.10b)$$

Differentiating Equation (7.10b) with time gives the angular acceleration of the rotor as expressed in Equation (7.11b).

$$\alpha_{r,II} = \alpha_c \left(\frac{2(r_c - l_{ve})^2}{r_r^2 + (r_c - l_{ve})^2 - e^2} \right) + 4\omega_c \frac{dl_{ve}}{dt} \left(\frac{(r_c - l_{ve})(e^2 - r_r^2)}{(r_r^2 + (r_c - l_{ve})^2 - e^2)^2} \right) \quad (7.11b)$$

In order to formulate the dynamics models of RV-II, the free body diagrams of the rotor and the cylinder are needed and are shown in Figure 7.5.

Figure 7.5 Free-body diagrams of RV-II components: (a) rotor (b) cylinder

To obtain the vane contact force, the torque balance of the rotor is analysed. The result is expressed in Equation (7.12b). It can be seen that the vane contact force of RV-II is only a function of the rotor inertia and the other torques (such as the frictional losses) acting at the rotor, but is not a function of the pressure difference across the vane. This is similar to RV-I, but different from that of RV-Ia.

$$I_r \alpha_r = F_{v,n} r_r \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \left(r_r \sin \gamma + \frac{w_v}{2} \right) + \sum T_r$$

$$\rightarrow F_{v,n,II} = \frac{I_r \alpha_r - \sum T_r}{r_r \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta \left(r_r \sin \gamma + \frac{w_v}{2} \right)} \quad (7.12b)$$

The torque balance of the cylinder gives the output torque of the RV-II. The expression is shown in Equation (7.13b). The pressure difference across the vane affects the output torque of the RV-II expander explicitly, similar to that of the RV-I but unlike that of the RV-Ia.

$$I_c \alpha_c = F_{v,p} \left(r_c - \frac{l_{ve}}{2} \right) - F_{v,n} (r_c - l_{ve}) + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \frac{w_v}{2} + \sum T_c - T_{gen}$$

$$\rightarrow T_{gen,II} = F_{v,p} \left(r_c - \frac{l_{ve}}{2} \right) - F_{v,n} \left(r_c - l_{ve} - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta \frac{w_v}{2} \right) + \sum T_c - I_c \alpha_c \quad (7.13b)$$

7.1.3. RV-IIa

The RV-IIa configuration is geometrically identical to RV-II. The volumes of the working chambers can therefore be calculated according to Equations (7.1c) and (7.2c). However, the operational angle of RV-IIa is the rotor angle and hence, the kinematics and the dynamics models of RV-IIa are different from RV-II.

$$V_{scv,IIa} = V_{dead,scv,IIa} + \frac{l_{exp}}{2} \left(\varphi_c r_c^2 - e(r_c - l_{ve}) \sin \varphi_c - \varphi_r r_r^2 - l_{ve} w_v \right) \quad (7.1c)$$

$$V_{dcv,IIa} = l_e \pi (r_c^2 - r_r^2) - V_{scv,IIa} \quad (7.2c)$$

To develop the kinematics models, the triangle ACR of RV-IIa is used. It is the same as the one of RV-II in Figure 7.4. From the triangle ACR, the vane exposed length is found according to Equation (7.3c). Differentiating Equation (7.3c) with time gives the sliding velocity of the vane and is expressed in Equation (7.4c).

$$l_{ve,IIa} = r_c - \sqrt{r_c^2 + 2er_r (\cos \varphi_r - 1)} \quad (7.3c)$$

$$\frac{dl_{ve,IIa}}{dt} = \omega_r \left(\frac{er_r \sin \varphi_r}{r_c - l_{ve,IIa}} \right) \quad (7.4c)$$

The relationships between the cylinder angle, the angle γ and the rotor angle (through l_{ve}) are expressed in Equations (7.5c) and (7.6c). As for the other configurations, the convention used here is that angle γ always starts from the line that connects points A and C in Figure 7.4. Therefore, Equation (7.6c) is applicable only for $0 \leq \varphi_r \leq \pi$. Otherwise it must carry a negative sign.

$$\cos \varphi_{c,IIa} = \frac{e^2 + (r_c - l_{ve})^2 - r_r^2}{2e(r_c - l_{ve})} \quad (7.5c)$$

$$\gamma_{IIa} = \cos^{-1} \left(\frac{r_r^2 + (r_c - l_{ve})^2 - e^2}{2r_r(r_c - l_{ve})} \right) \quad (7.6c)$$

The vectors of the tangential velocities at point A have the relations shown in Equations (7.7c) - (7.9c).

$$v_{Ac} = (r_c - l_v) \omega_c \quad (7.7c)$$

$$v_{Ar} = r_r \omega_r \quad (7.8c)$$

$$v_{Ac} = v_{Ar} \cos \gamma \quad (7.9c)$$

The cylinder angular velocity is obtained by combining Equations (7.7c) - (7.9c) which results in Equation (7.10c).

$$\omega_{c,IIa} = \omega_r \left(\frac{r_r^2 + (r_c - l_{ve})^2 - e^2}{2(r_c - l_{ve})^2} \right) \quad (7.10c)$$

Differentiating Equation (7.10c) with time gives the angular acceleration of the cylinder as expressed in Equation (7.11c).

$$\alpha_{c,IIa} = \alpha_r \left(\frac{r_r^2 + (r_c - l_{ve})^2 - e^2}{2(r_c - l_{ve})^2} \right) + \omega_r \frac{dl_{ve}}{dt} \left(\frac{r_r^2 - e^2}{(r_c - l_{ve})^3} \right) \quad (7.11c)$$

To formulate the dynamics models of the RV-IIa expander design configuration, the free body diagrams of the rotor and the cylinder are shown in Figure 7.6.

Figure 7.6 Free-body diagrams of RV-IIa components: (a) rotor (b) cylinder

The vane contact force of RV-IIa is expressed in Equation (7.12c). By observing the vane contact force terms of all the RV expander configurations, it can be concluded that only when the vane is fixed to the driving component, such as those in RV-I and RV-II, the vane contact force is independent of the pressure difference across the vane.

$$\begin{aligned}
 I_c \alpha_c &= F_{v,p} \left(r_c - \frac{l_{ve}}{2} \right) - F_{v,n} (r_c - l_{ve}) + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \frac{w_v}{2} + \sum T_c \\
 \rightarrow F_{v,n,IIa} &= \frac{F_{v,p} \left(r_c - \frac{l_{ve}}{2} \right) + \sum T_c - I_c \alpha_c}{\left(r_c - l_{ve} - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta \frac{w_v}{2} \right)} \quad (7.12c)
 \end{aligned}$$

The output torque is obtained by analysing the torque balance on the rotor shaft and is found to be as expressed in Equation (7.13c).

$$\begin{aligned}
 I_r \alpha_r &= F_{v,n} r_r \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} F_{v,f} \left(r_r \sin \gamma + \frac{w_v}{2} \right) + \sum T_r - T_{gen} \\
 \rightarrow T_{gen,IIa} &= F_{v,n} \left(r_r \cos \gamma - \frac{F_{v,n}}{|F_{v,n}|} \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta \left(r_r \sin \gamma + \frac{w_v}{2} \right) \right) + \sum T_r - I_r \alpha_r \quad (7.13c)
 \end{aligned}$$

7.2. Low Pressure Applications

As discussed in Section 7.1, the operating pressure is expected to be a main factor that affects the performance of the machine. The study of low pressure applications, where the pressure difference is less than 10 bar will first be carried out and is presented in this section. The high pressure applications are presented in the subsequent section.

The studies will be focused on the comparison of the inherent mechanical behaviours of each design configuration because mechanical performance is the main advantage of the RV mechanism design as compared to the more established rotary mechanisms. Therefore, the studies will include, but not limited to, the effects of the vane contact force, the vane side friction loss and the endface friction loss to the expander performance.

In order to focus on the effects of different designs, all the expander dimensions, the operating conditions and the parameters are set to be consistent for all the four variants. The rotating inertias of the rotor and the cylinder are also assumed to be of the same values. In practical cases, the cylinder rotating inertia tends to be larger than that of the rotor due to its larger radius. However, it is also possible to have them the same values by removing excessive material from the cylinder like that shown in Figure 7.1.

7.2.1. Simulation procedures

To carry out the studies, computer codes for each of the RV design configurations, similar to the one used for RV-I, described in Chapter 6, have been developed. For simplicity and since the effects are independent of the type of fluid used, air is chosen as the working fluid. The air properties are obtained using the REFPROP routine [146]. The discharge pressure is fixed at 1 bar while three different suction pressure conditions are considered, i.e. 2 bar, 6 bar and 10 bar. Therefore, these correspond to three cases

with operational pressure difference of 1, 5 and 9 bar. The models are assumed to be adiabatic and are perfectly sealed (no internal leakage). The suction valves are assumed to uncover and close instantaneously when suction and expansion occur, respectively. The driving shafts are assumed to rotate at constant speeds. The main dimensions and operating conditions parameters of the expander models are shown in Table 7.1.

Table 7.1 Models dimensions and operating parameters
of the low-pressure applications comparison study

Item	Value
Suction temperature	25 °C
Discharge pressure	1 bar
Discharge temperature	25 °C
Motor speed	3000 rev/min
Rotor radius	0.018 m
Cylinder radius	0.025 m
Expander length	0.032 m
Rotor rotating inertia	0.0002 kg.m ²
Cylinder rotating inertia	0.0002 kg.m ²
Friction coefficient	0.15
Endface gap	7.5 x 10 ⁻⁶ m
Viscosity of lubricant	0.34 Pa.s

7.2.2. Results and discussions

The variations of the vane contact forces of all the four RV expanders for operational pressure difference of 1 bar, 5 bar and 9 bar are shown in Figure 7.7.

Figure 7.7 Vane contact forces of RV expander variants at pressure differences of:
(a) 1 bar, (b) 5 bar, (c) 9 bar

Figure 7.7 shows that the vane contact forces are sinusoidal, following the variation of the angular acceleration (Figure 7.8) very closely. This indicates that the forces are mainly affected by the inertia, not the pressure differences. Only at relatively high pressures, such as that in the case of 9 bar pressure difference, will the effects of pressure be more obvious, although the general trends are still unchanged.

Figure 7.7 also shows that the vane contact force of RV-Ia is observably increased with the pressure while the RV-IIa vane contact force is observably decreased with the pressure for the first half of the operation (between 0° to 180°). As for the other half of the cycle (from 180° to 360°), the pressure effects are not significant because the high pressure gas has been expanded and hence the pressure across the vane is no longer as

high as the first half. This trend is observed because the angular accelerations of the driven components behave in a sinusoidal manner, unlike that of the pressure component, which is always positive. Therefore, although the pressure and inertia components of the vane contact forces in Equations (7.12a) and (7.12c) have opposite values, they can only cancel each other for half of the cycle. They will then magnify the vane contact forces together for the other half of the cycle.

Figure 7.8 Angular accelerations of various RV design configurations

From Figure 7.7, it is also observed that in agreement with the variation of the acceleration shown in Figure 7.8, the amplitude of the vane contact force of the RV-I is consistently the smallest while the amplitude of RV-IIa is consistently the largest. The amplitude of the vane contact force of RV-Ia is smaller than RV-II when the pressure difference is 1 bar or 5 bar, but is larger when the pressure difference is 9 bar. This is because of the relationship between the pressures and the inertial components mentioned before.

The locations of the peaks of the vane contact forces are different too. Those RV-Ia are most desirable because they lie near the beginning and the end of the operating cycle, when the vane exposed length is shortest. The locations of the peak contact forces for RV-IIa are the most undesirable. They lie near the mid-point of the cycle where the

vane exposed length is the longest. This indicates that the vane of the RV-IIa design will experience the highest bending stress as compared to the other designs.

The vane side friction power losses of the four RVs are shown in Figure 7.9.

Figure 7.9 Vane side friction losses of RV expander variants at pressure differences of:

(a) 1 bar, (b) 5 bar, (c) 9 bar

From Figure 7.9, it can be seen that the RV-I always gives the lowest average vane friction power loss while RV-IIa gives consistently the largest, irrespective of the suction pressure value. Consistent with the vane contact forces behaviours, the amplitude of the vane side loss of RV-Ia is smaller than RV-II when the operational pressure difference is 1 bar or 5 bar but larger when the pressure difference is 9 bar. The observable differences in the first half and the second half peaks of the RV-Ia and the RV-IIa vane friction power losses are also due to the pressure effect. The pressure

difference across the vane during the first half of the cycle (between 0° to 180°) is larger than the second half (between 180° to 360°).

It is also useful to observe the behaviours of the vane sliding velocities as shown in Figure 7.10 to better understand the behaviours of the vane side friction power losses.

Figure 7.10 Vane sliding velocities comparison of the four RV expander configurations

Figure 7.10 shows that the peaks of the vane sliding velocities of the RV-I and, most importantly, the RV-IIa designs are closer to the mid-point of the operating cycle than the other two designs. This means that for RV-IIa, the peaks of the angular acceleration of the driven component, which are also the peaks of the vane contact forces, and the peaks of the vane sliding velocities coincide with each other. This is not the case in the other three designs. Coupling this and the fact that RV-IIa has the largest vane contact force amplitude results in RV-IIa having the largest amplitude of the vane friction power loss among the four designs, as shown in Figure 7.9.

The variations of the instantaneous rotor endface friction power loss for the four RVs are shown in Figure 7.11. The average power loss of RV-I is the lowest at 2.62 W, followed by RV-Ia at 2.64W, then RV-II at 2.81 W and the largest is RV-IIa at 2.89 W. Therefore, from the endface friction power loss point of view, RV-I is the superior design, while RV-IIa is the worst.

Figure 7.11 Rotor endface friction losses comparison of the four RV expander designs
(values shown are for one side of the endfaces only)

7.2.3. Conclusions of the low pressure applications comparison

Comparisons of the four possible RV expander designs with operational pressure difference of 1, 5 and 9 bar have been carried out. The followings are obtained:

- The vane contact forces are mostly influenced by the rotating inertia. They are not significantly affected by the operational pressures, even for RV-Ia and RV-IIa.
- RV-I has the smallest vane contact force amplitude while RV-IIa has the largest. These trends are irrespective of the operating pressures.
- The locations of the RV-Ia peak vane contact forces are most desirable because they lie when the vane exposed length is shortest, while those of RV-IIa are the most undesirable. These trends are consistent throughout the pressure range studied.
- RV-I has the lowest average vane friction power loss while RV-IIa has the largest, irrespective of the operating pressure conditions.
- RV-I has the least endface friction power loss, while RV-IIa has the most.

It can, therefore, be concluded that from the point of view of the vane contact force and the mechanical efficiency, in relatively low pressure applications the RV-I design configuration is the most superior while RV-IIa is the least desirable design.

7.3. High Pressure Applications

After comparing the performances of the four RV expander design configurations in relatively low-pressure applications, we proceed with the comparison in high-pressure applications, where the operational pressure difference is 68 bar.

7.3.1. Simulation procedures

To carry out the study, a transcritical CO₂ refrigeration system where the suction pressure is 110 bar and the discharge pressure is around 42 bar is adopted. All the dimensions, operational conditions and properties are set to be consistent for the four variants and are listed in Table 7.2. The properties of CO₂ are taken from the NIST database using REFPROP 8.0 [156]. The theoretical models are solved using the computer codes mentioned in Section 7.2.1.

Table 7.2 Main dimensions, operating conditions and parameters
of the high pressure applications comparison

Item	Value
Suction pressure	110 bar
Suction temperature	46.1 °C
Discharge temperature	7.2 °C
Cooling capacity	5.2 kW
Motor speed	3000 rev/min
Rotor radius	0.03 m
Cylinder radius	0.035 m
Expander length	0.0037 m
Rotor rotating inertia	$3.4 \times 10^{-5} \text{ kg}\cdot\text{m}^2$

Cylinder rotating inertia	$3.4 \times 10^{-5} \text{ kg}\cdot\text{m}^2$
Suction port radius	0.0035 m
Discharge port radius	0.0035 m
Coefficient of discharge of ports	0.61
Friction coefficient	0.15
Endfaces clearance	$10 \times 10^{-6} \text{ m}$
Journal bearing radius	0.01 m
Journal bearing length	0.015 m
Journal bearing clearance	$15 \times 10^{-6} \text{ m}$
Lubricant viscosity	0.0034 Pa·s

From the theoretical models derived in Section 7.1, it was found that the vane contact forces of the RV-Ia and the RV-IIa configurations are affected by the pressure difference across the vane. This means that as the operating pressures are higher, the vane contact forces will also increase, as have been confirmed by the results in Section 7.2. At exceptionally high pressure conditions, like in a transcritical CO₂ system that will be simulated here, the vane contact forces will be undesirably high. This will result in a high stress at the vane and a large vane side friction loss. Therefore, RV-Ia and RV-IIa are excluded from the comparison in high pressure applications. Instead, the study will be focused on RV-I and RV-II. The comparisons will include the thermodynamics and the dynamics behaviours of the two designs.

As in the case of the low pressure studies, the expanders are assumed to experience adiabatic processes with no leakage. The ideal suction valve response is also assumed such that the valve opens and closes instantaneously at the desired moments. The driving shafts are assumed to operate at a constant angular velocity.

7.3.2. Thermodynamics behaviours comparison

To begin the comparison of the thermodynamics behaviours of RV-I and RV-II, we refer to the pressure-volume diagrams shown in Figure 7.12. Process **e-a** represents the suction process. The state of the working fluid before the entry at **e** is supercritical gas. This process goes on until the suction valve is closed and the expansion process begins. The pressure then drops rather steeply during the beginning of the expansion process **a-b** when a supercritical gas and subsequently sub-cooled liquid is expanded. Once the pressure reaches that of the saturated condition, the fluid becomes a two-phase mixture of gas and liquid and, the pressure drop becomes much gradual, as shown by process **b-c**. The expansion process ends when the expander reaches its maximum volume at point **c**. Discharge process then follows and is represented by process **c-d**.

Figure 7.12 Pressure-volume diagrams and suction and discharge paths of RV-I and RV-II in high pressure application

The pressure-volume diagrams of RV-I and RV-II look almost identical. Since the area encapsulated by a pressure-volume diagram indicates the energy that is produced by an expander, this indicates that both designs produce practically an equal amount of energy. However, a detailed examination shows that there are small differences at the suction and the discharge processes as can be seen from the enlarged portions in Figure

7.12. The suction process **e-a** of RV-I is initially more desirable than that of RV-II but it becomes less desirable for the rest of the operation. Meanwhile, the discharge process of RV-I is not too obviously different from that of RV-II at the small regions near the start and the end of the process, but it is less desirable for the rest of the process. This causes the total area encapsulated by the pressure-volume diagram of RV-I to be slightly smaller than that of RV-II and causing RV-I design to actually have a slightly lower energy produced. One way to improve the suction and discharge processes of RV-I is by using larger ports than the ones used in RV-II.

The differences in the suction and discharge processes are caused by the different rates of change of volume of the two variants as plotted in Figure 7.13. The figure shows that the rate of increase of RV-II volume is initially larger than that of RV-I. This is the reason why the suction process of RV-I to be initially more desirable than RV-II. However, as the expander rotates further, the rate of change of RV-I volume becomes steeper than that of RV-II and causes the suction process to be less desirable than RV-II for the rest of the operation. This reconfirms that RV-I will need a larger suction port and the situation is more crucial when the speed of operation is higher. As for the region near the start and the end of the discharge process, the rates of change of volumes are small. Therefore, the discharge processes of the two designs do not differ noticeably.

Figure 7.13 Rate of change of volumes of RV-I and RV-II

7.3.3. Power production behaviours

Power is produced at the shaft of the RV expander's driving component, which is at the rotor shaft for RV-I and at the cylinder shaft for RV-II. The magnitude of the power produced depends on three parameters, i.e. the pressure difference across the vane, the rotating inertia (together with the angular acceleration) of the driven component (in the case of RV-I, it is the cylinder, and for RV-II, it is the rotor) and the friction losses. Figure 7.14 shows the variation of the instantaneous power produced over the entire expander cycle and its components. It is reminded that the scale of the vertical axis of Figure 7.14(c) is different from those of Figures 7.14(b) and (c).

Figure 7.14 Variations of power productions of RV-I and RV-II and their components

Figure 7.14 shows that the maximum power is produced at the end of the suction process before it drops sharply. This is caused by the sharp pressure drop during the beginning of the expansion process when the fluid is initially a supercritical gas and

subsequently becomes a sub-cooled liquid. The peak power of RV-I is higher than that of RV-II because RV-I has the rotor centre as the axis of the driving component and hence, it has a longer moment arm at the maximum power production location. This moment arm is the length from the centre of the driving component to the centre of the area of the protruded portion of the vane which is exposed to the pressure force as illustrated in Figure 7.15. A higher peak power production is generally desirable for the expander to better overcome the compressor peak power requirement.

Figure 7.15 Maximum moment arms of the pressure force of: (a) RV-I and (b) RV-II

Figure 7.14 also shows that the power variation of RV-I is always positive except for a small region around the 0° angular position. This consistent positive power is desirable, because it means that the expander always produces power during the whole cycle of operation. On the contrary, RV-II shows that there is a negative power production region. This is because the inertia effect of RV-I is negative when the pressure component is positive. The variation of the inertia effect itself is dictated by the variation of the angular acceleration of the driven component shown in Figure 7.16(b). Since the rotating inertia is related to the power production with a negative sign, the variation of the inertia component is thus the reverse of the angular acceleration.

The variations of the velocities of the driven components of RV-I and RV-II are shown in Figure 7.16(a). The driven component velocity of RV-I increases initially before it decreases after the π -angular position, while it is the opposite for RV-II. The reason for this behaviour is because of the different vane contact points of the two designs. The vane contact point of RV-I is at the cylinder inner wall while that of RV-II is at the outer wall of the rotor. Referring to the variations of angular velocities at the 0 and π angles will help to understand this more clearly:

$$\text{At } 0 \text{ angle: } \omega_{c,I,0} = \frac{r_r}{r_c} \omega_{r,I,0} \text{ and } \omega_{r,II,0} = \frac{r_c}{r_r} \omega_{c,II,0}.$$

$$\text{At } \pi \text{ angle: } \omega_{c,I,\pi} = \left(2 - \frac{r_r}{r_c}\right) \omega_{r,I,\pi} \text{ and } \omega_{r,II,\pi} = \left(2 - \frac{r_c}{r_r}\right) \omega_{c,II,\pi}.$$

Because the driving shaft turns at equal speed for the two expanders and the rotor radius is smaller than the cylinder radius, therefore, $\omega_{c,I,0} < \omega_{r,II,0}$ and $\omega_{c,I,\pi} > \omega_{r,I,\pi}$.

Figure 7.16 Driven component (a) angular velocities and (b) angular accelerations of RV-I and RV-II

The amplitudes of the angular velocity and acceleration of RV-II are larger than that of RV-I. This is explained as follows. Considering the driving shafts of the two expanders to rotate at an equal speed, ω , and introducing a parameter, D , defined by

Equation (7.14) to measure the difference in the amplitudes of the angular velocities of the driven components for RV-I and RV-II, Equation (7.14a) is obtained.

$$D = |\omega_{c,I} - \omega| - |\omega_{r,II} - \omega| \quad (7.14)$$

$$\rightarrow \frac{D}{\omega} = \left| \frac{r_r - r_c}{r_c} \right| - \left| \frac{r_c - r_r}{r_r} \right| \quad (7.14a)$$

Since the cylinder radius is always larger than the rotor radius, the relationship in Equation (7.14b) is obtained. This shows that the amplitudes of the angular velocity and acceleration of the driven component of RV-II are always larger than RV-I.

$$D < 0, \text{ or: } |\omega_{c,I} - \omega| < |\omega_{r,II} - \omega| \quad (7.14b)$$

7.3.4. Frictional losses

The mechanical efficiency of the expander is affected by the frictional losses. There are three major sources of frictional losses, namely at the vane side, the endfaces and the shaft bearings. The frictional power loss at the vane side is the product of the vane contact force, friction coefficient and the vane sliding velocity. In most applications, the effects due to the rotating inertia dominate and therefore, the vane contact force variation also follows the variation of the angular acceleration, as shown in Figure 7.17.

Figure 7.17 Vane contact forces of RV-I and RV-II

Figure 7.17 also shows that the amplitude of the vane contact force of RV-II vane is larger than that of RV-I, caused by the larger acceleration amplitude of RV-II. This results in a higher vane side friction loss for RV-II. The figure also shows that RV-II vane contact force at 0° is higher than that of RV-I. This is due to the different effects of friction at the endfaces of the driven component explained in the subsequent paragraphs.

The effects of the fluid in the gaps between the endfaces of the rotor and the cylinder have been discussed in Chapter 5. To ease the subsequent discussions, the equations to estimate the friction torque and power losses are re-shown in Equations (7.15a), (7.15b) and (7.16). The equations for RV-I and RV-II are the same.

$$T_{endf,r} = \frac{\mu_{lubr}}{\delta_{endf}} \pi \frac{r_r^4}{2} (\omega_c - \omega_r) \quad (7.15a)$$

$$T_{endf,c} = -\frac{\mu_{lubr}}{\delta_{endf}} \pi \left(r_r^2 e^2 \omega_c + \frac{r_r^4}{2} (\omega_c - \omega_r) \right) \quad (7.15b)$$

$$P_{endf,loss} = \frac{\mu_{lubr}}{\delta_{endf}} \pi \left(r_r^2 e^2 \omega_c^2 + \frac{r_r^4}{2} (\omega_c - \omega_r)^2 \right) \quad (7.16)$$

Equations (7.15a) and (7.15b) help to explain the variation of the vane contact force shown in Figure 7.17. The rotation of the driven component (rotor, in this case) of RV-II is initially larger than that of its cylinder, hence according to Equation (7.15b), the torque is negative. For RV-I, since the rotation of the driven component (cylinder) is less than that of the rotor, according to Equation (7.15a) the torque is positive. As the bearing friction loss torque is always negative, the summation of the bearing torque and the endface torque for both expanders will result in different net torques. For RV-I, the two torques reduce each other's effect. For RV-II, they result in a large resultant torque. This explains the non-zero torque at 0° angular position.

As for the power loss due to the friction at the endface gaps, Equation (7.16) shows that it is dependent on the eccentricity and the relative velocity of the components. The first term is larger for RV-I since the cylinder angular velocity is not a constant. The second term is larger for RV-II because the amplitude of the relative velocity of RV-II components is larger than that of RV-I. However, since the rotor radius is significantly larger than the eccentricity, the second term is usually more significant than the first one. Therefore, RV-II endfaces power loss is usually larger than that of RV-I.

7.3.5. Mechanical efficiency

Table 7.3 tabulates the comparison of the average output power production, frictional power losses and mechanical efficiency. The vane side and the endfaces friction losses of RV-II are found to be larger than that of RV-I as expected while the bearings friction losses are comparable. This results in RV-I having a larger mechanical efficiency than RV-II. The ideal power produced by RV-II is larger than that of RV-I since the PV diagrams are slightly different due to RV-II having a lower suction and discharge losses.

Table 7.3 Output powers, mechanical losses and efficiencies of RV-I and RV-II

		RV-I	RV-II
Average power production (W)		650.3	650.7
Friction loss (W)	vane side	1.7	5.8
	endfaces	7.0	8.3
	rotor bearing	7.2	7.6
	cylinder bearing	7.6	7.7
Mechanical efficiency (%)		96.5%	95.7%

As has been mentioned before, the rotating inertia of the cylinder is usually larger than that of the rotor due to its larger radius. Since the mechanical efficiency of RV-I is expected to reduce with a bigger rotating inertia of the cylinder, it is necessary to study the upper limit of the rotating inertia of the cylinder that still makes RV-I more efficient and produces a higher peak power than RV-II. This analysis is carried out by introducing three parameters, namely the inertia ratio, the peak power ratio and the efficiency ratio. The inertia ratio is defined as the ratio between the rotating inertia of the RV-I cylinder and its rotor (where the rotating inertia of the rotor is fixed at a value throughout this section). The peak power ratio is defined as the peak power of RV-I to that of RV-II (where its value is taken as constant value at 2316 W). The efficiency ratio is defined as the mechanical efficiency of RV-I with that of RV-II (where the value for RV-II is taken as a constant value at 95.7%, as presented in Section 7.3.5). The results are shown in Figure 7.18.

Figure 7.18 The effect of inertia on peak power produced and mechanical efficiency

Figure 7.18 shows that the RV-I's efficiency is still higher than RV-II even when its cylinder's inertia is four times larger than that of the rotor. When the inertia ratio is around 4.4, the mechanical efficiency of RV-I is equal to that of RV-II and if the inertia

ratio increases further, RV-I becomes less efficient. For the peak power produced, the inertia ratio limit is slightly higher, i.e. at around 4.8.

7.3.6. Conclusions of the high pressure applications comparison

A comparative study on the performances of the RV-I and the RV-II expander configurations in high pressure applications have been carried out. The study found that:

1. RV-I expander design needs larger suction and discharge ports than RV-II.
2. The amounts of energy produced by RV-I and RV-II expanders are comparable.
3. The peak power produced by RV-I is larger than RV-II.
4. The power variation of RV-I is more consistently positive than that of RV-II.
5. RV-I has higher mechanical efficiency than RV-II
6. Even when the inertia ratio is four, RV-I expander still outperforms RV-II.

From the findings, it can be concluded that RV-I design configuration is superior to RV-II for expander applications in high-pressure applications. Combining this finding with the finding in low-pressure applications, it can be concluded that RV-I is the most preferred design configuration of the four RV expander designs, regardless of the pressure range.

PROTOTYPE DESIGN

In this chapter, the detailed design of the RV expander prototype is presented. It is made up of the rotor, the vane, the cylinder, the bearings and the housing. The design considerations of each of these components are discussed in this chapter. Lubrication analysis of the prototype will also be discussed. Unless otherwise stated, all design equations used in this chapter are taken from design textbooks [157]. SolidWorks SimulationXpress 2010, which is a commercially available computer aided design and analysis software [153], was employed for the stress and deformation analysis of the components.

8.1. Major Dimensions of the Prototype

The RV expander prototype is designed with a working chamber volume of 30 cm^3 . The rotor radius is 29 mm, the cylinder radius is 35 mm and the expander length of 25 mm. The RV-I design, where the vane is fixed at the rotor and the rotor is the driving component, is chosen for it is shown to be the most desirable design in Chapter 7.

It is necessary to note that when designing a RV machine, there is a feasible design range for the eccentricity. As the minimum limit, it must be more than zero, because a zero eccentricity results in a non-functional machine. As for the maximum limit, it exists because the working chamber is sealed, among others, by the intersection between the endfaces of the rotor and the cylinder. A geometrical model is shown in Figure 8.1 to illustrate this more clearly, where d_b is the minimum thickness of the rotor shaft bearing. In theory, this thickness can be set to zero. However, in practice, a certain amount of thickness is needed to avoid sharp corners.

Figure 8.1 Working chamber is properly sealed by the endface intersection

Figure 8.1 shows that the working chamber is separated from the cylinder shaft hole by this endface intersection. This is a necessary condition for the RV compressor design. Else, the working fluid will interact with the outside fluid as illustrated in Figure 8.2 through the shaded area to the cylinder shaft hole. Mathematically, this design limitation can be expressed by Equation (8.1).

$$r_{ch} + e < r_r \quad (8.1)$$

From the relationship between the radius of the cylinder shaft hole, the radius of the rotor shaft and the bearing minimum thickness (Equation (8.2)), the maximum allowable eccentricity is found to be as expressed in Equation (8.3).

$$r_{ch} = e + r_{rs} + d_b \quad (8.2)$$

$$e < \frac{r_r - r_{rs} - d_b}{2} \quad (8.3)$$

By setting the value of d_b to be at least 2 mm, the rotor shaft radius, r_{rs} , is found to be maximum at 15 mm. This value must be noted when deciding the rotor shaft size of the prototype later.

Figure 8.2 Working chamber is not properly sealed by the endface intersection

In cases that the eccentricity limitation has to be broken, a design modification can be made by employing a big stationary journal bearing that is as big as the cylinder shaft hole. Therefore, the hole is now blocked by the bearing, not allowing working fluid to flow through the hole. However, this is not desirable because the friction between the stationary bearing and the endface of the rotor causes undesirable friction. In addition, the sealing gap between the stationary bearing and the rotating cylinder shaft inner wall may not be sufficient to prevent leakage of the working fluid from the working chamber.

8.2. Prototype Material and Design Conditions

The prototype will be designed to work with up to 20 bar of suction pressure and 1 bar of discharge pressure at the rotor speed of 3000 rev/min. The RV expander components will be made of AISI 4140 steel. Some of the properties of the material are listed in Table 8.1 and Figure 8.3.

Table 8.1 Mechanical properties of AISI 4140 steel [158]

Properties		Conditions	
		T (°C)	Treatment
Density ($\times 1000 \text{ kg/m}^3$)	7.7-8.03	25	
Poisson's Ratio	0.27-0.30	25	
Elastic Modulus (GPa)	190-210	25	
Tensile Strength (Mpa)	655.0	25	annealed at 815°C more
Yield Strength (Mpa)	417.1		
Elongation (%)	25.7		
Reduction in Area (%)	56.9		
Hardness (HB)	197	25	annealed at 815°C more
Impact Strength (J) (Izod)	54.5	25	annealed at 815°C more

Figure 8.3 Endurance strength versus tensile strength for wrought steel [157]

8.3. Rotor Design

Although the vane has always been shown to be rigidly fixed to the rotor in the previous chapters, they do not have to be machined as one piece. The latter approach is adopted for the prototype to ease the manufacturing process. Therefore, the rotor will only consist of the shaft and the rotor body (with a slot for the vane to be fixed onto). The expander will adopt the simply-supported shaft bearings arrangement. Therefore,

the rotor has a shaft at each end. The minimum diameter of the rotor shaft can be calculated with Equations (8.4) and (8.5).

$$d = \sqrt[3]{\frac{2.94 K_t (V) N}{s'_n}} \quad (8.4)$$

$$d = \left(\frac{32N}{\pi} \sqrt[3]{\left(\frac{K_t M}{s'_n} \right)^2 + \frac{3}{4} \left(\frac{T}{s_y} \right)^2} \right)^{1/3} \quad (8.5)$$

where the modified endurance strength, s'_n , is calculated according to Equation (8.6).

$$s'_n = s_n C_s C_R \quad (8.6)$$

A schematic diagram of the rotor shaft and its loads is shown in Figure 8.4. As mentioned before, the radius of the rotor body is 29 mm while its length is 25 mm. The lengths of the rotor lower and upper shafts are set to be 20 mm and 113 mm, respectively. The lengths of the rotor lower and upper bearings are set to be 20 mm and 40 mm, respectively.

Figure 8.4 Illustration and schematic diagram of rotor and rotor shaft

Using the simulation parameters in Table 6.1 and the computer code discussed in Chapter 6, the maximum torque acting on the rotor shaft, T_r , is 50.9 Nm when the operating suction and discharge pressures are 20 bar and 1 bar, respectively, and the rotor speed is constant at 3000 rev/min.

The rotor load is represented by a resultant force, F_r , which acts at the midpoint of the rotor body, point B. It is caused mainly by the pressure difference between the working chambers. The highest load occurs when the rotor angle is 180°. By assuming that the pressure difference is 19 bar, the largest rotor load can be calculated as follows:

$$F_r = (19 \times 10^5) \times (0.025 \times 0.058) = 2755 \text{ N}$$

Assuming that the torque load component at the end of the upper rotor shaft does not exert any force to the rotor shaft, the rotor shaft bearing loads, $F_{b1,r}$ acting at point C and $F_{b2,r}$ acting at point A, which hold the rotor shafts at their respective places can be calculated as follows:

$$F_{b1,r} = \frac{22.5}{22.5 + 32.5} F_r = 1127.05 \text{ N}$$

$$F_{b2,r} = \frac{32.5}{22.5 + 32.5} F_r = 1627.95 \text{ N}$$

The shear force and the bending moment diagrams of the rotor and its shaft are as shown in Figure 8.5.

Figure 8.5 Shear force and bending moment diagrams of the rotor shaft

To calculate the minimum shaft diameter at point A, a sharp fillet is assumed at the edge of the rotor body and the lower rotor shaft. Therefore, the stress concentration factor, K_t , is 2.5 [157]. Figure 8.3 suggests that the estimated endurance strength, s_n , is 240 MPa. The design size factor, C_s , is first arbitrarily assumed from Figure 8.6 to be 0.8 because the final shaft diameter is not yet known. The design reliability factor, C_R , is set to be 0.81 due to a required reliability of 0.99 while the design safety factor, N , is set to be 3 as the RV expander will be operating under a typical industrial condition [157]. Therefore, the modified endurance strength, s'_n , is calculated as follows:

$$s'_n = 240 \times 0.8 \times 0.81 = 155.52 \text{ MPa}$$

Figure 8.6 Size factor for the various shaft diameter [157]

Substituting all the values into Equation (8.4) gives the first estimation of the minimum shaft diameter at Point A, $d_{rs,A}$.

$$d_{rs,A} = \sqrt{\frac{2.94 \times 2.5 \times 1627.95 \times 3}{155.52 \times 10^6}} = 15.2 \text{ mm}$$

The above calculation has to be reiterated because C_s was earlier estimated according to a guessed shaft diameter of 75 mm. The new C_s value from Figure 8.6 is now around 0.96. Therefore, the updated s'_n and $d_{rs,A}$ values are as follow:

$$s'_n = 240 \times 0.96 \times 0.81 = 186.62 \text{ MPa}$$

$$d_{rs,A} = \sqrt{\frac{2.94 \times 2.5 \times 1627.95 \times 3}{186.62 \times 10^6}} = 13.9 \text{ mm}$$

There is neither bending moment nor torsion at point A. Therefore, Equation (8.5) is not applicable and therefore, the minimum rotor shaft diameter at point A is 13.9 mm.

At point B, the shear force is equal to point A. However, the bending moment is 36.3 Nm and the torsion is 50.9 Nm. The yield strength, s_y , is 417.1 MPa (see Table 8.1). Therefore, the minimum shaft diameter is as follows:

$$d_{rs,B} = \left(\frac{32 \times 3}{\pi} \sqrt{\left(\frac{2.5 \times 36.3}{186.62 \times 10^6} \right)^2 + \frac{3}{4} \left(\frac{50.9}{417.1 \times 10^6} \right)^2} \right)^{1/3} = 24.8 \text{ mm}$$

However, since point B is located at the rotor body, its diameter follows that of the rotor diameter, which is 58 mm and is larger than the requirement.

The shear force at point C is less than that at point A. Therefore, the minimum rotor diameter at point C required by shear stress is smaller than 13.9 mm. There is zero bending moment acting at point C, but there is a maximum torsion load of 50.9 Nm. Therefore, Equation (8.5) should be used.

$$d_{rs,C} = \left(\frac{32 \times 3}{\pi} \sqrt{\left(\frac{2.5 \times 0}{186.62 \times 10^6} \right)^2 + \frac{3}{4} \left(\frac{50.9}{417.1 \times 10^6} \right)^2} \right)^{1/3} = 14.8 \text{ mm}$$

At point D, the shaft only experiences torsion. Therefore, the minimum shaft diameters are 14.8 mm, same as that at point C.

It is required to channel the suction and the discharge ports to the respective reservoirs through channels in the rotor upper shaft as illustrated in Figure 8.7. The discharge channel connects the discharge working chamber to the outside of the housing, while the suction channel connects the suction working chamber to the suction reservoir, which is the inside of the housing, through the suction valve. The suction

valve hole is located at the side of the rotor shaft. The areas of these channels must be at least equal to the combined areas of the ports connected to the working chambers to avoid flow losses, which are set to consist of 3 suction ports with radii of 3.2 mm and 3 discharge ports with radii of 2.3 mm.

$$r_{suct\_channel} = \sqrt{3 \times 3.2^2} = 5.5 \text{ mm}$$

$$r_{disc\_channel} = \sqrt{3 \times 2.3^2} = 4.0 \text{ mm}$$

Figure 8.7 Cross-section of rotor upper shaft with suction and discharge channels

Therefore, a suction channel with a diameter 11 mm and a discharge channel with a diameter of 8 mm are designed. By designing the minimum thicknesses between the channels and between the channels with the shaft outer wall to be 2 mm, the rotor upper shaft diameter is finally designed to be 25 mm. This is less than the maximum limit imposed by Equation (8.3), which is a rotor shaft diameter of 30 mm. For convenience, the rotor lower shaft is designed to have the same diameter with the upper shaft. The suction valve opening hole located at the side of the upper rotor shaft is designed to have a diameter of 11 mm, following the diameter of the suction channel. Lastly, to minimise centrifugal force, balancing holes are added to the rotor body. Pictures of the rotor prototype (with the vane) are shown in Figure 8.8.

Figure 8.8 Pictures of the rotor prototype (with the vane)

8.4. Vane Design

The vane is fabricated separately from the rotor and is attached together by press-fitting the vane into the available slot at the rotor. The free body diagram of the vane is shown in Figure 8.9.

Figure 8.9 Free body diagram of the vane

The most severe condition is when the vane is fully extended, which is when the rotor angle is 180° , with the vane exposed length, l_{ve} , of 12 mm. The pressure force acting on the vane, $F_{v,p}$, is calculated as follows:

$$F_{v,p} = (19 \times 10^5) \times (0.025 \times 0.012) = 570 \text{ N}$$

Since the pressure force, $F_{v,p}$, acts at the middle of the vane exposed length, the vane reaction forces, F_{v1} and F_{v2} , are equal to 285 N each. Therefore, the bending moment diagram of the vane is as shown in Figure 8.10.

Figure 8.10 Bending moment diagram of the vane

To calculate the minimum thickness of the vane, t_v , the elastic flexure formula for a beam [159] as expressed in Equation (8.7) can be used. The elastic section modulus of the vane can be calculated by approximating the vane to have a rectangular cross section which is expressed in Equation (8.8).

$$\sigma_{\max} = \frac{M_{\max}}{S_v} \quad (8.7)$$

$$S_v = \frac{t_v l_{ve}^2}{6} \quad (8.8)$$

By substituting Equation (8.8) into Equation (8.7), rearranging and introducing a safety factor, N , and a reliability factor, C_R , the minimum vane width can be calculated according to Equation (8.9).

$$w_v = \frac{6M_{\max}}{\sigma_{\max} l_{ve}^2} \times \frac{N}{C_R} \quad (8.9)$$

Figure 8.9 shows that the maximum bending moment experienced by the vane, $M_{\max}$, is 1.68 Nm. The maximum stress that can be withstood by the vane is equal to the yield strength of the material, i.e. $\sigma_{\max} = 417.1$ MPa (Table 8.1). The safety factor, N , under a typical industrial condition is 3, while with a desired reliability of 0.99, the reliability factor, C_R , should be set at 0.81 [157]. Substituting the values into Equation (8.9), the minimum vane thickness is obtained.

$$t_v = \frac{6 \times 1.68}{(417.1 \times 10^6) \times 0.012^2} \times \frac{3}{0.81} = 0.6 \text{ mm}$$

For easier handling and to minimise deflection, the thickness of the vane for the RV expander prototype design is set to be 4 mm. To allow a press-fit condition between the vane and the rotor slot, the vane thickness is designed with a tolerance of 4.000 to 4.005 mm while the rotor slot width is between 3.990 and 3.995 mm. To further secure its position in the slot, the vane is also designed to have a circular “head” with a radius of 2.5 mm. The tolerance of this “head” is from 2.500 to 2.505 mm while the slot radius is from 2.496 to 2.500 mm. The depth of the rotor slot is designed to be 12 mm. The total length of the vane is set to be 28 mm. Therefore, the length of the vane protruding out from the rotor body is 16 mm.

8.5. Cylinder Design

In this prototype, the cylinder is made up of three parts: a cylinder body and two cylinder covers to be attached onto both sides of the cylinder body. This is necessary for easier machining. This also helps to ensure that the corners at the endfaces are sharp to minimise internal leakage between the working chambers. Various design calculations necessary for a reliable cylinder prototype design are discussed in this section. These include the calculations of the minimum cylinder wall thickness, the minimum cylinder shaft size and the minimum number of fasteners required to secure the cylinder cover onto the cylinder body. The geometry of the cylinder body and other design considerations that may compromise the reliability of the prototype are also addressed.

8.5.1. Cylinder wall thickness

The minimum thickness of the cylinder walls can be calculated using the thin wall cylindrical pressure vessel method [159] because the cylinder radius is usually

significantly larger than its wall thickness. The method consists of two equations shown as Equations (8.10) and (8.11) which are related to the longitudinal stress, σ_l , and the hoop stress, σ_h , of the pressure vessel, respectively. The longitudinal stress is caused by the pressure acting on the end surfaces of the cylinder while the hoop stress is caused by the pressure acting on the circular wall of the cylinder.

$$\sigma_l = \frac{P \times r}{2t} \quad (8.10)$$

$$\sigma_h = \frac{P \times r}{t} \quad (8.11)$$

where

P is the pressure difference across the pressure vessel wall (Pa)

r is the inner radius of the pressure vessel (m)

t is the wall thickness of the pressure vessel (m)

Equations (8.10) and (8.11) show that the minimum wall thickness requirement of hoop stress is more stringent than the one by longitudinal stress. The wall thickness of the cylinder is set to be uniform throughout. Therefore, using only Equation (8.11) is sufficient for the calculation. By including a safety factor, N , and rearranging Equation (8.11), the minimum wall thickness can be calculated according to Equation (8.12).

$$t = \frac{N \times P \times r}{\sigma_h} \quad (8.12)$$

The maximum pressure difference across the cylinder walls is 19 bar. The cylinder material yield strength is 417.1 MPa and a safety factor of 3 is used. Substituting all the values into Equation (8.12) gives the minimum cylinder wall thickness.

$$t_c = \frac{3 \times (19 \times 10^5) \times 0.035}{417.1 \times 10^6} = 0.5 \text{ mm}$$

This minimum wall thickness is 70 times smaller than the radius of the cylinder. Therefore, the thin-walled cylindrical pressure vessel assumption is acceptable.

During operation, high pressures may cause the cylinder cover to deflect towards the inside of the cylinder body, reducing the gap between the rotor and the cylinder endfaces. If the deflection is too large, the cylinder cover may be in contact with the rotor, creating a large friction loss or even system failures. Therefore, the cylinder body must be designed to be longer than the rotor length to allow sufficient gaps between the rotor and the cylinder endfaces. The finite element analysis shows that the maximum deflection is around 7 micron.

8.5.2. Cylinder shaft size

The method to determine the minimum diameter of the cylinder shaft is similar to that of the rotor shaft. However, the shaft bearings are now placed inside the cylinder shaft and supporting the cylinder from within. Therefore, the shafts are hollow and hence, Equations (8.4) and (8.5) must be modified to Equations (8.14) and (8.15). These are obtained by using the section modulus of a pipe expressed in Equation (8.13).

$$S_{cs} = \frac{\pi(d_o^4 - d_i^4)}{32d_o} \quad (8.13)$$

$$d_o^2 - d_i^2 = \frac{2.94K_t(V)N}{s'_n} \rightarrow d_i = \frac{2.94K_t(V)N}{s'_n} - t_{cs}^2 \quad (8.14)$$

$$\frac{d_o^4 - d_i^4}{d_o} = \left(\frac{32N}{\pi} \sqrt{\left(\frac{K_t M}{s'_n} \right)^2 + \frac{3}{4} \left(\frac{T}{s_y} \right)^2} \right) \quad (8.15)$$

where

d_i and d_o are the inner and outer diameters of the cylinder shaft, respectively (m)

t_{cs} is the wall thickness of the cylinder shaft (m)

The diagram illustrates a two-cylinder engine mechanism. The top part shows a 3D perspective view of the cylinders and their connecting rods. The bottom part shows a 2D cross-sectional view of the crankshaft with dimensions and forces.

3D View Labels:

- cylinder lower shaft
- cylinder body
- cylinder upper shaft
- cylinder lower bearing
- cylinder upper bearing

3D View Forces:

- $F_{b2,c}$ (downward force on lower shaft)
- $F_{b1,c}$ (downward force on upper shaft)
- F_c (upward force on cylinder body)

2D View Labels:

- O (crankshaft center)
- A (lower shaft center)
- B (cylinder body center)
- C (upper shaft center)
- D (crankshaft center)

2D View Dimensions:

- 5 mm (distance O to A)
- 17.5 mm (distance A to B)
- 17.5 mm (distance B to C)
- 5 mm (distance C to D)

2D View Forces:

- $F_{b2,c}$ (downward force at A)
- F_c (upward force at B)
- $F_{b1,c}$ (downward force at C)

The cylinder load, F_c , is assumed to act at the midpoint of the cylinder body. The value is calculated according to the most severe condition, which is when the rotor angle is 180° and the pressure difference between the working chambers is 19 bar.

The shaft bearing forces are assumed to act at the midpoints of the shafts. Since the lengths of the upper and the lower shafts are the same, these bearing forces are also the same and are equal to half of the cylinder load. Therefore, $F_{b1,c}$ and $F_{b2,c}$ are equal to 1662.5 N. With these data, the shear force and the bending moment diagrams of the cylinder and its shaft are as shown in Figure 8.12.

Figure 8.12 Shear force diagram of the cylinder shaft

There is only shear force at point A. Therefore, Equation (8.14) is used to determine the minimum diameter of the rotor shaft at this location. K_t is set as 2.5 due to sharp fillet between the rotor and the rotor shaft [157], s_n is 240 MPa, C_s is first guessed to be 0.8 from Figure 8.6, C_R is 0.81 due to a required reliability of 0.99 and N is 3. With these values, the modified endurance strength, s'_n , is found as follows:

$$s'_n = 240 \times 0.8 \times 0.81 = 155.52 \text{ MPa}$$

Assuming that the cylinder shaft wall thickness is 4 mm, a value that is larger than the minimum cylinder wall thickness of 0.5 mm obtained previously, the minimum inner shaft diameter at Point A, $d_{i,cs,A}$, is calculated using Equation (8.14).

$$d_{i,cs,A} = \frac{(2.94 \times 2.5 \times 1662.5 \times 3) / (155.52 \times 10^6)^{-0.004^2}}{2 \times 0.004} = 27.5 \text{ mm}$$

With the newly obtained estimated diameter, a new estimation for C_s is given as 0.92. Therefore, the revised s'_n and $d_{i,cs,A}$ are as follow.

$$s'_n = 240 \times 0.92 \times 0.81 = 178.85 \text{ MPa}$$

$$d_{i,cs,A} = \frac{(2.94 \times 2.5 \times 1662.5 \times 3) / (178.85 \times 10^6)^{-0.004^2}}{2 \times 0.004} = 23.6 \text{ mm}$$

Point B lies at the cylinder body. Therefore, the inner diameter is already set at 70 mm, following the inner diameter of the cylinder body. There are shear force and

bending moment acting at point B. The value of the shear force is equal to the one at point A. Therefore, the minimum inner diameter due to shear force at point B is equal to point A, which is 23.6 mm. This is smaller than the inner cylinder diameter. The required minimum inner diameter at point B due to bending moment is calculated according to Equation (8.15).

$$\frac{(d_{i,cs,B} + t_{cs})^4 - d_{i,cs,B}^4}{d_{i,cs,B} + t_{cs}} = \left(\frac{32 \times 3}{\pi} \sqrt{\left(\frac{2.5 \times 29.09}{178.85 \times 10^6} \right)^2 + \frac{3}{4} \left(\frac{0}{417.1 \times 10^6} \right)^2} \right) = 1.24 \times 10^{-5} \text{ m}^3$$

Assuming that the wall thickness is also 4 mm at point B and substituting the inner diameter of the cylinder body of 70 mm to the left hand side of Equation (8.15) gives a value of $8.08 \times 10^{-5} \text{ m}^3$, which is larger than the required value of $1.24 \times 10^{-5} \text{ m}^3$. This means that the cylinder body dimensions are sufficient to withstand the loads at point B.

At point C, the loading condition is the same as at Point A. Therefore, the minimum inner shaft diameter at Point C, $d_{cs,C}$, is also 23.6 mm.

For the final design of the prototype, the cylinder shaft diameter is set to be 48.2 mm, according to the requirements of the shaft bearings used. This is explained in Section 8.6.

8.5.3. Minimum number of fasteners

To secure the cylinder covers onto the cylinder body, fasteners are used. These fasteners are subjected to loadings due to pressure difference across the cylinder cover. One side of the cylinder cover, called the outer side, is always exposed to the suction reservoir with a pressure of 20 bar. The other side, called the inner side, is exposed to the cylinder endface and the working chambers. The worst condition is when the fluid in the working chamber is at its lowest at 1 bar. This happens at the end of the expansion process. The force is found according to the calculation as follows:

$$F_{cc} = (19 \times 10^5) \times (\pi(0.035^2 - 0.029^2)) = 2292.1 \text{ N}$$

The allowable stress of each fastener is expressed in Equation (8.16) [157].

$$\sigma_{fastn} = \frac{N \times F}{n \times A_{fastn}} \quad (8.16)$$

where

$$\sigma_{fastn} = 0.75 \times \sigma_{proof}$$

σ_{fastn} is the allowable stress of the individual fastener (Pa)

A_{fastn} is the tensile stress area of the fastener (m²)

n is the number of fasteners

M5 fine thread steel screws will be used to fasten the cylinder cover onto the cylinder. This type of fastener has a proof strength of 225 MPa and a tensile stress area of 16.1 mm² [157]. Substituting these values and the fastener load of 2292.1 N with a safety factor of 3 into Equation (8.16) gives the minimum number of fastener required.

$$n = \frac{3 \times 2292.1}{0.75 \times (225 \times 10^6) \times (16.1 \times 10^{-6})} = 2.5$$

To minimise deflections of the parts in between the screws, 16 screws were used in the prototype. To further prevent leakage through the gaps between the cylinder cover and the cylinder body, 3.9 mm wide O-rings were installed in the gaps. To accommodate the thread holes and the O-rings, the cylinder body wall thickness is set to be 14 mm, much larger than the requirement of 0.5 mm obtained from the pressure vessel calculation previously.

8.5.4. Geometry of cylinder

A vane slot must be fabricated at the cylinder body. The depth and width of the vane slot must be sufficient to allow the vane to move freely. From Section 8.4, the

vane length protrusion is designed to be 16 mm. Therefore, the vane slot is designed to be 18 mm deep. From the simulation, it is found that the maximum swivelling angle of the vane is around 10° . Therefore, the vane slot is designed to have an opening angle of 13° . The vane is 4 mm wide. Therefore, the vane slot entrance is designed to have a width of 4.1 mm.

To accommodate the vane slot, the cylinder is designed with a diamond shape, as shown in Figure 8.13. A balancing hole at the opposite side of the vane slot is created to minimise vibration during operation.

Figure 8.13 The top view of the cylinder prototype

During operation, because the pressure outside the cylinder may be higher than the one inside, the vane slot entrance width may be reduced. It is important to ensure that this width is always large enough for the vane to move freely. The finite element analysis shows that during the worst possible condition, when the outside pressure is 20 bar and the inside pressure is 1 bar, the entrance width only changes by less than $1\text{ }\mu\text{m}$. Therefore, the design is reliable.

Pictures of the cylinder body and the cylinder cover are shown in Figure 8.14.

Figure 8.14 Pictures of the cylinder cover and the cylinder body

8.6. Shaft Bearings Design

The rotor and the cylinder shafts of the prototype are to be supported by shaft bearings. It has been mentioned that the rotor shafts are supported from the outside while the cylinder shafts are supported from within. Therefore, it is possible to design a bearing that will serve both of these functions at the same time. This idea is adopted here. With this arrangement, only two bearings are needed. One is to support the upper shafts and the other one is to support the lower shafts. Each bearing is fixed to the housing to prevent it from moving.

The bearings are designed according to the journal bearing design procedure discussed in Chapter 5. The rotor upper shaft bearing is designed to be 40 mm long with a diameter of 25 mm, while the lower one is 20 mm long with a diameter of 25 mm. The radial clearances of the rotor bearings are set at 15 μm . The cylinder shaft bearings are 10 mm long with diameters of 48.2 mm each. The radial clearances of the cylinder bearings are set at 12.5 μm . The minimum oil film thicknesses of each shaft bearings obtained from numerical simulation are shown in Figure 8.15. It can be seen that the minimum oil film thicknesses of all the bearings are at least 1 μm throughout the cycle, indicating that the shafts are never in contact with the bearings and that the bearings are

able to perform even up to the operating conditions of 19 bar pressure difference and 3000 rev/min operational speed.

Figure 8.15 Minimum oil film thickness of the shaft bearings

To increase the reliability of the bearings, sleeve bearings are added to complement the journal bearing mechanism. The list of the sleeve bearings used is shown in Table 8.2. All of the bearings are of the TEF-MET [160] type produced by Singa Bearings Solutions Pte. Ltd.

Table 8.2 List of sleeve bearings used in the RV expander prototype

Location	Type	Bearing Diam. (mm)	Shaft Diam. (mm)
Rotor upper bearing	TF2530	27.979 - 28.000	24.959 - 24.980
Rotor lower bearing	TFZ2520	27.979 - 28.000	24.959 - 24.980
Cylinder upper bearing	TFZ444925-S	44.400 – 44.425	48.210 – 48.235
Cylinder lower bearing	TFZ444925-S	44.400 – 44.425	48.210 – 48.235

The upper bearing is also used as the suction valve in this prototype, following the concept illustrated in Figure 3.12(b) in Chapter 3. Therefore, suction valve grooves have to be fabricated to allow suction pressure to flow into the suction chamber through the

suction channel and the suction ports. Since the suction channel has a diameter of 11 mm, the suction valve grooves are also designed with a width of 11 mm. The opening angle of the groove is set to be 137° , based on the assumption of isentropic expansion of 4 bar air to 1 bar condition. Balancing grooves are also fabricated to balance the pressure force acting on the rotor shaft.

Pictures of the bearings are shown in Figure 8.16.

Figure 8.16 Pictures of the: (a) upper bearing with the suction valve opening groove shown, (b) upper bearing with the balancing grooves shown, (c) lower bearing

8.7. Housing Design

In this prototype, a housing is needed to house all the RV expander components. Hence, the inner radius of the housing must be larger than the maximum length of the cylinder body, which is 144 mm. The inner diameter of the housing is eventually set to be 146 mm. The housing also acts as the suction reservoir. Therefore, during operation, the pressure inside the housing is 20 bar while the outside pressure is 1 bar. The minimum wall thickness can be calculated according to the thin wall cylindrical pressure vessel method [159], using Equation (8.12). The housing material yield strength is 417.1 MPa and a safety factor of 3 is used. Substituting all the values into Equation (8.12) gives the minimum wall thickness of the housing.

$$t_h = \frac{3 \times (19 \times 10^5) \times 0.073}{417.1 \times 10^6} = 1.0 \text{ mm}$$

For easier fabrication, better reliability and to minimise distortion, a wall thickness of 4 mm is chosen.

The housing consists of the body and the cover. M5 fine thread steel bolts will be used to secure the cover onto the body. These fasteners should be properly designed to withstand the load caused by the pressure difference across the housing cover that will push the cover to move away from the body. This force is calculated as follows.

$$F_{hc} = (19 \times 10^5) \times \pi(0.073^2 - 0.0375^2) = 23415.0 \text{ N}$$

M5 steel bolts can be assumed to have a proof strength of 225 MPa and a tensile stress area of 16.1 mm² [157]. Substituting these values and the fastener load with a safety factor of 2, which is lower than the other components because the housing is stationary, into Equation (8.16) gives the minimum number of fastener required.

$$n = \frac{2 \times 23415.0}{0.75 \times (225 \times 10^6) \times (16.1 \times 10^{-6})} = 17.2$$

Hence, 18 M5 steel bolts will be used to secure the housing cover onto the housing. During operation, it is possible to have leakage from the inside of the housing to the outside through the gaps created by the deflections of the parts of the housing cover that are in-between the bolts. Fortunately, finite element analysis shows that the maximum deflection is only around 1 µm. In addition, to further prevent the leakage, an O-ring is inserted in between the housing cover and the housing body.

Pictures of the housing and the housing cover are shown in Figure 8.17.

Figure 8.17 Pictures of the: (a) housing body, (b) housing cover,
(c) assembled housing body and cover

8.8. Lubrication System Design

A lubrication system is needed to provide lubrication for the journal bearings and the rubbing components. Lubrication is also needed to minimise internal leakage by sealing the gaps and to provide cooling mechanism for the components during operation. In this prototype, no electrical lubrication pump is used. Instead, mechanical mechanisms are used. The lubrication system can therefore be analysed according to the method proposed by Kim *et al.* [83, 140, 141], resulting in the model shown in Figure 8.18. Point 0 refers to the oil level at the oil reservoir at the bottom of the housing. The lubricant is assumed to flow in the paths according to the arrows shown in the figure.

Figure 8.18 The RV expander prototype lubrication model

The lubrication system in Figure 8.18 can be represented with a representative electrical circuit as shown in Figure 8.19. Electrical currents represent oil flow rates, voltage sources represent lubrication pumps while electrical resistances represent oil flow resistances. The circuit has 18 unknown currents: q_{12} , q_{24} , q_{23A} , q_{23B} , q_{13C} , q_{13G} , q_{03C} , q_{03G} , q_{36S} , q_{36D} , q_{46S} , q_{46D} , q_{45C} , q_{45G} , q_{04C} , q_{04G} , q_{05B} and q_{05V} . The current q_{12} represents the oil flow from point 1 to point 2, q_{03C} represents the flow from point 0 to point 3 through path C, and so on.

Figure 8.19 Electrical circuit representation of the prototype lubrication system

From Kirchoff's Current Law, the following four equations are obtained:

1. Node 2: $q_{12} = q_{24} + q_{23A} + q_{23B}$
2. Node 3: $q_{23A} + q_{23B} = q_{13C} + q_{13G} + q_{03C} + q_{03G} + q_{36S} + q_{36D}$
3. Node 4: $q_{24} = q_{46S} + q_{46D} + q_{45C} + q_{45G} + q_{04C} + q_{04G}$
4. Node 5: $q_{45C} + q_{45G} = q_{05B} + q_{05V}$

From Kirchoff's Voltage Law, the following 14 equations are obtained:

1. $q_{23A} R_{23A} - q_{23B} R_{23B} = V_{23A} - V_{23B}$
2. $q_{12} R_{12} + q_{23B} R_{23B} + q_{13C} R_{13C} = -V_{12} + V_{23B} + V_{13}$
3. $q_{13G} R_{13G} - q_{13C} R_{13C} = V_{13G}$
4. $q_{03C} R_{03C} - q_{13G} R_{13G} = V_{01} - V_{13G} - V_{13} + V_{03}$
5. $q_{03G} R_{03G} - q_{03C} R_{03C} = V_{03G}$
6. $q_{36S} R_{36S} - q_{36D} R_{36D} = V_{36S} - V_{06S} + V_{06D} - V_{36D}$

7. $q_{36D}R_{36D} - q_{03G}R_{03G} = V_{36D} - V_{06D} - V_{03G} - V_{03}$
8. $q_{46S}R_{46S} - q_{46D}R_{46D} = V_{46S} - V_{06S} + V_{06D} - V_{46D}$
9. $q_{46D}R_{46D} - q_{36S}R_{36S} - q_{23A}R_{23A} + q_{24}R_{24} = V_{46D} - V_{06D} + V_{06S} - V_{36S} - V_{23A} - V_{24H} + V_{24R}$
10. $q_{45C}R_{45C} - q_{45G}R_{45G} = -V_{45G}$
11. $q_{45G}Q_{45G} - q_{04C}R_{04C} = V_{45G} - V_{45} + V_{04}$
12. $q_{04G}R_{04G} - q_{46S}R_{46S} = V_{04G} - V_{04} + V_{06S} - V_{46S}$
13. $q_{05V}R_{05V} - q_{05B}R_{05B} = -V_{05V}$
14. $q_{12}R_{12} + q_{24}R_{24} + q_{45C}R_{45C} + q_{05V}R_{05V} = -V_{12} - V_{24H} + V_{24R} - V_{45} - V_{05V} + V_{01}$

There are 18 unknowns and 18 equations. The Gaussian Elimination Method [137] was used to solve the equations. It was found that 3 mm wide, 0.4 mm deep and 5° angle spiral grooves machined at the shafts and 4 mm diameter radial holes are able to give a reliable lubrication system.

RV-I EXPERIMENTAL STUDY

Experimental investigations have been carried out with the prototype discussed in Chapter 8. The procedures and the results are presented in this chapter. The accuracy of the theoretical models developed in Chapters 4 and 5 are then verified by comparing the theoretical predictions with the experimental results.

9.1. Experimental Setup

The experiments were designed to test the characteristics of the expander, specifically the average output torque, average flow rate, instantaneous output torque and instantaneous working chambers pressures, when operating at various pressure and operational speed conditions. Air is used as the working fluid in the tests for simplicity. In order to control the pressure conditions, the discharge pressure is fixed at the atmospheric pressure while the suction pressure is controlled by a pressure regulator that is located between the expander and an air tank. The operational speed is maintained by connecting the expander shaft to a motor. This is necessary because as the expander operates, its instantaneous output torque fluctuates. When it is positive, the expander accelerates and vice versa. The motor ensures that the speed is constant by supplying torque when the expander output torque is negative. On the other hand, it acts as a generator (absorbs torque) when the expander produces positive output torque.

The schematic diagram of the main experimental setup is shown in Figure 9.1 while the picture is shown in Figure 9.2. The motor is connected to a frequency controller to allow the adjustment of speed. The flow meter is of the Aalborg 044-40 variable area type with a tantalum float which can measure up to 70 l/min of air. It is installed at the

suction line, because the discharge gas is discharged through the discharge holes on the rotating driving shaft into the atmosphere. The torque and the rotational speed are measured using the Torquemaster TM107 transducer linked to the DUD282 display system. This sensor is able to measure torques from -10 to +10 N·m. The suction gas pressure and temperature are measured with an analogue pressure transducer and a type-K thermocouple, respectively. When measuring dynamic pressures, WIKA S-10 pressure transducers which are able to measure up to 10 bar of pressures are used. The accuracies of the measuring devices are listed in Table 9.1. TOTAL Dacnis VS 46 compressor oil was used for lubrication.

Figure 9.1 Schematic diagram of the experimental setup

Table 9.1 Accuracies of the measuring devices

Thermocouple	$\pm(0.3\% \text{ of reading} + 1 \text{ }^{\circ}\text{C})$
Pressure transducer	$\pm 0.1 \text{ bar}$
Flow meter	$\pm 23 \text{ cm}^3/\text{s}$
Speed sensor	$\pm 5 \text{ rev/min}$
Torque transducer	$\pm 0.02 \text{ N}\cdot\text{m}$

Figure 9.2 The experimental setup

Before being used, all the measuring devices are calibrated and tested for accuracy. The calibration method and data of the pressure transducers are presented here. The tests for the rest of the devices are attached in Appendix B.

The pressure transducers were calibrated using the FLOTECH deadweight hydraulic calibrator as shown in Figure 9.3. The data is captured using the NI cDAQ-9174 data acquisition system with the LabView software platform. A digital multimeter was also used to capture the output signals for comparison. One of the test data (obtained with the WIKA pressure transducer, S/N:260H66S) is plotted in Figure 9.4.

Figure 9.3 Calibration of the pressure transducer with a dead-weight calibrator

Figure 9.4 Calibration data of the pressure transducer (S/N:260H66S)

9.2. Experimental Results and Discussions

With the experimental rig and measuring devices properly set up, the experiments were then carried out. They are discussed in the following sections. It is worthy to remember that the designated suction and discharge pressures of the expander are 3 barg and 0 barg, respectively, because the suction valve is designed for such pressure conditions.

9.2.1. Preliminary tests

Before any readings were taken, preliminary experiments were conducted. This is carried out by disconnecting the expander from the motor and then channelling the compressed air gas (up to 6 barg) into the stationary expander at various rotor angle positions. It was found that as long as the expander is started with a rotor angle that is not near 0° , the expander will immediately turn. This is because at 0° (and 360°), the vane is concealed by the vane slot and therefore, there is not enough vane surface area that is exposed to the high-pressure air. As the rotor angle moves closer to 180° , the vane exposed area increases. Hence, sufficient torque is produced to move the expander. However, the expander will then stop turning when the rotor angle is near 360° . This indicates that although the high-pressure gas is able to turn the expander, but the

momentum generated during the start-up is not sufficient to overcome the inertia and the friction of the rotating components of the expander.

In the second set of preliminary tests, the motor is used to start-up the expander. After the desired speed (set at 4 rev/min) is reached, the motor is switched off while the high-pressure air is still pumped into the expander. It was found that if the lip seal is installed, after the motor is off, even with 6 barg of air, the expander will stop to turn once the rotor angle is near 360° . However, if no lip seal is installed, the expander will sustain the rotation and even accelerate with suction pressure of 3 barg, which is the designed suction pressure value. This shows that if no lip seal is installed, the expander is able to not only overcome its own friction and inertia, but also the inertia of the motor and the torque sensor. However, the drawback is that significant gas and oil leakage is observed through the gap between the shaft and the housing if no lip seal is installed.

9.2.2. Average torque with different pressure and speed conditions

Based on the findings of the preliminary tests, experiments were set-up to observe how different suction pressure and operating speed conditions affect the average output torque. Up to four readings were taken for each operating condition. The tests were carried out in the operational speed range of 120 to 600 rev/min. No test was conducted at higher speeds because of the mechanical performance deterioration at higher speeds. This will be discussed later. The suction pressures are varied in the range of ± 2 bar of the designated suction pressure value (3 barg). The results are plotted in Figure 9.5.

Figure 9.5 The variations of average output torque at various suction pressure and operational speed conditions

It can be seen that as the suction pressure increases, the average output torque also increases, as expected. This is because with a higher suction pressure, more force is applied on the vane exposed area to turn the expander. Conversely, as the speed increases, the output torque decreases. This is because the frictions of the shaft bearings and at the vane side increase with speed. According to the simulations in Chapter 6, the suction and discharge flows are able to cope up with the operating speeds and hence, we they do not affect the torques significantly. Interestingly, the average output torques at the first two lowest operational speeds (around 120 and 240 rev/min) are sometimes contrary to this trend. This is thought to be caused by the lubricating oil failure to lubricate some of the bearing surfaces properly when the expander is operated at a very low speed. This causes more friction produced at the bearings.

The figure also shows that even with a suction pressure of 5 barg, the expander only produces positive output torque at a very low operational speed. The reasons are thought to be the significant friction produced by the lip seal and the internal leakage. In order to test this hypothesis, the lip seal is removed. This will result in more leakage from the housing to the surrounding, but will reduce the friction experienced by the expander shaft. The results are plotted in Figure 9.6.

Figure 9.6 The variations of average output torque of the expander without lip seal with various suction pressure and operational speed conditions

Figure 9.6 shows that without lip seal, the expander is now able to consistently produce positive average output torque with the designed suction pressure value (3 barg) throughout the operational speed range tested. The isentropic efficiency, as defined by Equation (9.1), is up to 32% with an uncertainty of around $\pm 10\%$. The uncertainty is computed according to Equation (9.3). Even when the pressure is lowered all the way to 1 barg, the expander still produces positive torque at a low speed. This confirms that the lip seal contributes significantly to the expander friction loss. Comparison of the data in Figures 9.5 and 9.6 shows that the lip seal contributes to frictional torque in the range of around 0.2 to 0.3 N·m.

$$\eta_i = \frac{P_{real}}{\dot{m}_{ideal} \Delta h_{isentropic}} \quad (9.1)$$

$$\dot{m}_{ideal} = \rho_{suct} \left(\frac{\omega_{real}}{2\pi} \right) V_{start\_exp} \quad (9.2)$$

$$\delta q = \sqrt{\sum_i \left(\frac{\partial q}{\partial x_i} \delta x_i \right)^2} \quad (9.3)$$

9.2.3. Average flow rate at various pressure and speed conditions

The average flow rates of the expander prototype at various suction pressure and operational speed conditions are plotted in Figure 9.7(a). It is valuable to note that they were measured at the suction line as shown in Figure 9.1. Therefore, the values were obtained at the corresponding suction pressure conditions. The data are obtained with the lip seal installed.

Figure 9.7 (a) The average flow rates (at the suction line) and (b) volumetric efficiency of the expander at various suction pressure and operational speed conditions

It can be seen that as either the suction pressure or the speed increases, the flow rate also increases. The flow rate increase with speed is linear because the internal leakage flow rate is almost independent of the operational speed. Therefore, as the speed increases, only the expander capacity increases. The flow rate also increases with the suction pressure. However, as the suction pressure increases more, the volume flow rate increase is less significant. This is thought to be due to the choked flow limitation.

Figure 9.7(b) shows that the volumetric efficiency of the expander (calculated according to Equation (9.4)) is up to 57% at suction pressure of 1 barg with average uncertainty of 8%. However, at the designed suction pressure (3 barg), the efficiency is only up to 26%, indicating a significant internal leakage. Further observations of the

expander suggests that this may be due to the larger than expected vane endface clearance gap (around 0.1 mm at each side). Moreover, the endface seal length discussed in Section 8.1 is possibly too short as it is only 0.8 mm long at a certain section of the expander assembly. The relatively low viscosity lubricant used, which provides weaker sealing as compared to the more viscous ones, is also thought to contribute to the large internal leakage observed.

$$\eta_v = \frac{\dot{m}_{ideal}}{\dot{m}_{real}} \quad (9.4)$$

The leakage of the gas from the housing to the surrounding also contributes to the total flow rate. To confirm this contribution, a test was carried out by conducting a static leakage test (measuring the flow rate with the expander being kept stationary) with all the expander discharge holes blocked. This prevents the air to flow through the expander, only through the housing gaps to the surrounding. It was found that this leakage contributes only up to 20% to the overall expander flow rate.

9.2.4. Instantaneous torque

The variations of the expander instantaneous output torque if the pressures of both working chambers are kept at the atmospheric pressure (0 barg) are shown in Figure 9.8. This study allows the observation of the torque components excluding that of the pressure. This was carried out by running the expander with the upper cylinder cover removed and without high-pressure air pumped into the housing. This arrangement allows air at atmospheric pressure to flow freely into both working chambers. The data are obtained without a lip seal installed. The data plotted in the figure has been arranged such that at the beginning of the graph (time equals to 0), the rotor angle is 0°. The rotor angle is tracked by setting the magnetic pick-up sensor to give signal only when the rotor angle is at 0°.

Figure 9.8 The variations of the instantaneous output torque of the expander if both working chambers are kept at atmospheric pressure (0 barg)

The average torque of the one at 238 rev/min is around -0.09 N·m while the one at 474 rev/min is around -0.14 N·m. This is in accordance with the trend observed in section 9.2.2. Figure 9.8 also shows that when both working chambers are kept at 0 barg, the expander torque exhibits a sinusoidal behaviour. During the first half of the cycle, the torque is below the average value and is above during the rest of the cycle. These are in accordance with the theoretical predictions presented in Chapter 6. The amplitude increases linearly with the operational speed. As the speed is doubled, the amplitude is also doubled. This is surprising because the theoretical model predicts a quadratic relationship instead of a linear one. These are thought to be caused by the fact that the motor shaft not really turning at a constant speed. This is because as the expander operates, it accelerates and decelerates. In order to keep the rotor shaft speed constant, the motor must try to balance by providing counter accelerations and decelerations. This hypothesis will be tested in the next section.

Another unexpected finding is the torque fluctuations observed. They are more obvious if the expander operates at a lower operational speed. It is thought that these are caused also by the variations of the motor speed. Another reason may be the impacts

between the vane and the vane slot wall as the expander operates which is not considered in the theoretical models developed in Chapter 5.

The variations of the instantaneous output torque with time at a suction pressure of 3 barg at two different operational speeds are shown in Figure 9.9. It can be seen that the torques are still sinusoidal but the amplitudes are no longer symmetrical. The amplitude during the second half of the cycle is now larger than the first one. The figure also shows that as the operational speed increases, the torque shifts to become more negative. The average torque at 240 rev/min is 0.267 N·m while at 478 rev/min is lower at 0.147 N·m, caused by the larger friction losses. By comparing Figures 9.8 and 9.9, it can be observed that the maximum output torques with a high pressure gas is higher than those without, as expected. For comparison, when the expander operates at 240 rev/min, the maximum torque obtained without high pressure gas is around 0.1 N·m while the one with 3 barg of air is around 0.7 N·m.

Figure 9.9 The variations of the instantaneous output torque of the expander at the suction pressure of 3 barg

9.3. Verification of Theoretical Models

In this section, the theoretical models developed in Chapters 4 and 5 are verified with the experimental data. The simulations are carried out with the expander dimensions and parameters listed in Table 9.2. It is worthy to mention here that the vane endface gap width of the manufactured prototype is 180 μm , larger than expected, because the manufactured vane length is shorter than the designed value. The discharge coefficient of ports is set at 0.2 because comparison of the theoretical and experimental data indicates that this gives the best fit. This value is low and may be due to the existence of lubricants in the ports that block the air flow.

Table 9.2 Main dimensions and parameters of the RV-I prototype

Radius of rotor body	29 mm
Inner radius of cylinder body	35 mm
Length of expander	25 mm
Diameter of rotor shaft bearing	25 mm
Length of rotor upper shaft bearing	30 mm
Length of rotor lower shaft bearing	20 mm
Radial clearance of rotor upper shaft bearing	20 μm
Radial clearance of rotor lower shaft bearing	15 μm
Diameter of cylinder shaft bearing	44.4 mm
Length of cylinder upper shaft bearing	20 mm
Length of cylinder lower shaft bearing	20 mm
Radial clearance of cylinder upper shaft bearing	40 μm
Radial clearance of cylinder lower shaft bearing	25 μm

Rotating inertia of rotor	0.21 g.m ²
Rotating inertia of cylinder	2.81 g.m ²
Rotating inertia of motor	2.4 g.m ²
Friction coefficient of vane	0.15
Discharge coefficient of ports	0.2
Lubricant viscosity	0.065 Pa.s
Diameter of suction ports (3 numbers)	6.4 mm
Diameter of discharge ports (3 numbers)	4.6 mm
Assembly radial clearance	20 µm
Vane endface total gap width	180 µm
Cylinder endface total gap width	40 µm

The comparison between the theoretical predictions and the experimental data of the average output torque at various operating conditions is shown in Figure 9.10. The experimental data used for comparisons are taken from those obtained without the lip seal installed. It can be seen that in general, the theoretical predictions are in relative agreement with the experimental data. The agreement tends to be better as the suction pressure and the operational speed increases. For most of the points, the deviations are less than 8%. Only at those points where the torque values are close to zero, the average deviations are larger. In total, if the outliers, which are the points at the lowest operational speeds, are ignored, the average standard deviation (defined according to Equation (9.5)) is found to be 22%.

$$SD = \sqrt{\frac{\sum (x_{\text{theory}} - x_{\text{experiment}})^2}{\sum x_{\text{experiment}}^2}} \quad (9.5)$$

At low pressure conditions and low operational speeds, which equate to smaller bearing loads, the theoretical model tends to over predict the output torque. This may be caused by the lubricant being more viscous than predicted, because a small bearing load generates less loss to heat up the oil. This results in more frictional loss.

Figure 9.10 Comparison of the theoretical predictions and the experimental data of the average output torque at various operating conditions

The comparison between the variations of the expander instantaneous output torque predicted by the theoretical models and the obtained from the experiments is shown in Figure 9.11. The theoretical predictions consist of those with and without rotor acceleration. It can be observed that all the profiles are sinusoidal in behaviour. However, the amplitudes predicted by the theoretical models without the rotor acceleration are around 2 to 3 times larger than those of the experimental data. Once the rotor acceleration effect is included, the theoretical prediction matches the experimental data better. This confirms the hypothesis in Section 9.2.4 about the rotor acceleration being the reason behind the unexpected instantaneous torque amplitude data.

Figure 9.11 Comparisons of the theoretical predictions and experimental data of the instantaneous output torque

The comparison of the average volume flow rate obtained from the theoretical predictions and the experimental data at various operating conditions is shown in Figure 9.14. It can be seen that the theoretical model consistently over predicts the flow rate. This is because the actual leakage clearance gaps were partially blocked by the lubricant and therefore, were smaller than the values in Table 9.2. To accommodate this effect, a blocking coefficient can be introduced. Obviously, the coefficient depends on the operating conditions. For illustration, a blocking coefficient of 0.5 was arbitrarily introduced in this study. Figure 9.15 shows that the accuracy of the theoretical prediction immediately improves with the introduction of this coefficient.

Figure 9.14 Comparison of the theoretical predictions with and without blocking coefficient and the experimental data of the average volume flow rate

9.4. Concluding Remarks

Experimental studies have been carried out with the expander prototype built according to the design discussed in Chapter 8. This is believed to be the first machine ever built based on the RV-I mechanism design. The following conclusions are drawn:

1. The RV-I expander prototype design has been shown to be able to function without any operational problem in the range of the pressure (up to 6 barg) and operational speed (up to 600 rev/min) studied.
2. The lip seal was found to contribute significantly to the expander friction loss. The expander only produces positive output torque at very low speed (120 rev/min) and pressure of 5 barg when the lip seal is installed. Without the lip seal, an isentropic efficiency of up to 32% at the designed suction pressure (3 barg) is achieved.
3. The expander exhibits a volumetric efficiency of up to 57% when operated at low suction pressure (1 barg). However, at the designed suction pressure value (3 barg), the efficiency is only up to 26%. This is due the large clearances, especially at the vane endface and the endface sealing. The leakage from the housing to the surrounding contributes to up to 20% of the overall expander flow rate.

4. The instantaneous expander torque is sinusoidal in nature. It is below the average value during the first half of the cycle and is above during the rest of time. When operated without any pressure difference, the amplitude is symmetrical with respect to the average value. The amplitude increases linearly with a higher operational speed. Fluctuations are observed and are thought to be caused by the impacts between the vane and the vane slot wall and the variations of the motor speed.
5. The comparison between the average torques obtained from the theoretical predictions and the experiments shows that the theoretical model is able to simulate the data with an average standard deviation of 22%. The theoretical models are also able to predict the variations of the instantaneous output torque with time.
6. The theoretical models tend to over predict the expander flow rates. This is because the lubricant blocks the clearance gaps during operation, resulting in smaller leakage. If a coefficient of blocking is introduced to account for this effect, the accuracy of the predictions improves.

RV-0 EXPANDER ANALYSIS

To complete the study of the development of the Revolving Vane expander, it is necessary to discuss the original RV design which is the first RV mechanism design introduced by Teh and Ooi in 2006 [110, 111]. This study is presented in this chapter. It includes the theoretical and experimental analyses. For simplicity, the design shall be called the RV-0 design from here onwards to distinguish it from the other RV mechanism designs.

10.1. Theoretical Model

Unlike the RV-I or RV-II designs, in the RV-0 design, the vane is not rigidly fixed onto either the rotor or the cylinder. Instead, it is allowed to slide in a slot at the rotor and to swivel in another slot at the cylinder. The working cycle of the RV-0 expander is shown in Figure 10.1.

Figure 10.1 The working cycle of the RV-0 expander

The thermodynamics, geometrical and kinematics models of the RV-I mechanism described in Chapter 4 and Section 5.1 are still applicable for the RV-0 mechanism. However, the dynamics model needs to be modified because the vane is no longer rigidly attached to the rotor. To start with, the free body and geometrical diagram of the cylinder is shown in Figure 10.2.

Figure 10.2 Free body and geometrical diagram of the RV-0 cylinder

The force and torque balance equations of the cylinder in Figure 10.2 are shown in Equations (10.1) - (10.3).

$$F_{cx} - F_{v,n} \cos \varphi_r - F_{v,t} \sin \varphi_r - F_{p,c} \sin \frac{\varphi_c}{2} + F_{ctfg,c,x} = 0 \quad (10.1)$$

$$F_{cy} + F_{v,n} \sin \varphi_r - F_{v,t} \cos \varphi_r - F_{p,c} \cos \frac{\varphi_c}{2} + F_{ctfg,c,y} = 0 \quad (10.2)$$

$$r_c F_{v,n} \cos \gamma + r_c F_{v,t} \sin \gamma + \sum T_c = I_c \alpha_c \quad (10.3)$$

where

$$F_{ctfg,c} = \frac{m_c \omega_c^2 r_{ctfg,c}^2}{2}$$

$$\sum T_c = -T_{c,bearing} - T_{c,endfaces} - \frac{\mu}{\omega_c} \sqrt{F_{v,n}^2 + F_{v,t}^2} \left(\frac{d_{vh}}{2} \left| \frac{d\gamma}{dt} \right| \right)$$

$$\frac{d\gamma}{dt} = \frac{d}{dt} \cos^{-1} \left(\frac{r_c^2 + (r_r + l_{ve})^2 - e^2}{2r_c(r_r + l_{ve})} \right) = - \frac{\frac{dl_{ve}}{dt} (r_c^2 - (r_r + l_{ve})^2 - e^2)}{2r_c(r_r + l_{ve})^2 \sqrt{1 - \left(\frac{r_c^2 + (r_r + l_{ve})^2 - e^2}{2r_c(r_r + l_{ve})} \right)^2}}$$

The free body and geometrical diagram of the vane is shown in Figure 10.3.

Figure 10.3 Free body and geometrical diagram of the RV-0 vane

The force and torque balance equations of the vane are shown in Equations (10.4) - (10.6). The angular velocity and acceleration of the vane are equal to that of the rotor because they rotate together. L_v is the total physical length of the vane excluding the vane head. For simplicity, the vane head is assumed to be a cylinder with a diameter of d_{vh} . Therefore, the total physical length of the vane including the vane head is $L_v + d_{vh}$. The centre of mass of the vane is assumed to be at the midpoint of the vane.

$$-F_{v,n} + F_{p,v} + F_{N1} + F_{p,vs,n} - F_{N2} = m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \quad (10.4)$$

$$F_{v,t} + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu (F_{N1} + F_{N2}) - F_{p,vs,t} = m_v \frac{d^2 l_{ve}}{dt^2} \quad (10.5)$$

$$\begin{aligned} & -F_{v,n} \left(\frac{L_v - d_{vh}}{2} \right) + F_{p,v} \left(\frac{L_v - d_{vh} - l_{ve}}{2} \right) - F_{N1} \left(l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \\ & - F_{p,vs,n} \left(\frac{l_{ve} + d_{vh}}{2} \right) + F_{N2} \left(\frac{L_v + d_{vh}}{2} \right) + \sum T_v = I_v \alpha_r \end{aligned} \quad (10.6)$$

where

$$F_{p,vs,t} = p_{vs} l_{\text{exp}} w_v$$

$$\sum T_v = -\frac{\mu}{\omega_r} \left(|F_{N1}| + |F_{N2}| \right) \left| \frac{dl_{ve}}{dt} \right|$$

$$F_{p,vs,n} = \begin{cases} (p_{vs} - p_{dcv}) l_{\text{exp}} (L_v - l_{ve}) & \text{if } F_{N1} > 0 \text{ and } F_{N2} > 0 \\ (p_{scv} - p_{vs}) l_{\text{exp}} (L_v - l_{ve}) & \text{if } F_{N1} < 0 \text{ and } F_{N2} < 0 \\ p_{dcv} l_{\text{exp}} (L_v - l_{ve}) & \text{if } F_{N1} > 0 \text{ and } F_{N2} < 0 \\ p_{scv} l_{\text{exp}} (L_v - l_{ve}) & \text{if } F_{N1} < 0 \text{ and } F_{N2} > 0 \end{cases}$$

During operation, the pressure of the fluid inside the vane slot, p_{vs} , varies. The value is usually in between the instantaneous pressures of the working chambers.

The free body diagram of the rotor is shown in Figure 10.4.

Figure 10.4 Free body diagram of the RV-0 rotor

Equations (10.7) - (10.9) show the force and torque balance equations of the rotor.

$$F_{rx} + (F_{N1} - F_{N2} + F_{p,vs,n}) \cos \varphi_r - \left(\frac{dl_{ve}/dt}{|dl_{ve}/dt|} (F_{N1,f} + F_{N2,f}) - F_{p,vs,t} \right) \sin \varphi_r \quad (10.7)$$

$$+ F_{p,r} \sin \frac{\varphi_r}{2} + F_{ctfg,r,x} = 0$$

$$F_{ry} - (F_{N1} - F_{N2} + F_{p,vs,n}) \sin \varphi_r - \left(\frac{dl_{ve}/dt}{|dl_{ve}/dt|} (F_{N1,f} + F_{N2,f}) - F_{p,vs,t} \right) \cos \varphi_r \quad (10.8)$$

$$+ F_{p,r} \cos \frac{\varphi_r}{2} + F_{ctfg,r,y} = 0$$

$$-r_r F_{N1} + (r_r + l_{ve} - L_v) F_{N2} - \left(r_r + \frac{l_{ve} - L_v}{2} \right) F_{p,vs,n} + \sum T_r - T_{gen} = I_r \alpha_r \quad (10.9)$$

where

$$F_{ctfg,r} = \frac{m_r \omega_r^2 r_{ctfg,r}^2}{2}$$

$$\sum T_r = -T_{r,bearing} - T_{r,endfaces} - \frac{\mu}{\omega_r} \left(|F_{N1}| + |F_{N2}| \right) \left| \frac{dl_{ve}}{dt} \right| - T_{lip\_seal}$$

$$T_{lip\_seal} = (2\pi r_{rs}) r_{rs} \bar{\varphi}$$

$\bar{\varphi} \approx 100 + 20P$ for a PTFE lip seal if the oil pressure is less than 8 bar [111, 161]

Equations (10.3) and (10.4) can be rearranged into Equations (10.10) and (10.11), respectively.

$$F_{v,t} = \frac{I_c \alpha_c - \sum T_c}{r_c \sin \gamma} - \frac{F_{v,n}}{\tan \gamma} \quad (10.10)$$

$$F_{N1} = F_{v,n} - F_{p,v} - F_{p,vs,n} + F_{N2} + m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \quad (10.11)$$

Substituting Equation (10.11) into Equation (10.6) gives

$$F_{v,n} = \frac{\left(F_{p,v} \left(\frac{l_{ve}}{2} \right) - F_{p,vs,n} \left(\frac{L_v - l_{ve}}{2} \right) + F_{N2} (L_v - l_{ve}) \right.}{l_{ve}} \left. - m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \left(l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) + \sum T_v - I_v \alpha_r \right) \quad (10.12)$$

Substituting Equation (10.12) back into Equation (10.11) gives

$$F_{N1} = \frac{\left(-F_{p,v} \left(\frac{l_{ve}}{2} \right) - F_{p,vs,n} \left(\frac{L_v + l_{ve}}{2} \right) + F_{N2} (L_v) \right.}{l_{ve}} \left. + m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \left(\frac{L_v - d_{vh}}{2} \right) + \sum T_v - I_v \alpha_r \right) \quad (10.13)$$

Combining Equations (10.11) and (10.13), substituting into Equation (10.10) gives

$$F_{v,t} = \frac{I_c \alpha_c - \sum T_c}{r_c \sin \gamma} - \frac{\left(F_{p,v} \left(\frac{l_{ve}}{2} \right) - F_{p,vs,n} \left(\frac{L_v - l_{ve}}{2} \right) + F_{N2} (L_v - l_{ve}) \right.}{l_{ve} \tan \gamma} \\ \left. + m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \left(\frac{L_v - d_{vh}}{2} - l_{ve} \right) + \sum T_v - I_v \alpha_r \right) \quad (10.14)$$

Substituting Equations (10.13) and (10.14) into Equation (10.5) produces the closed form solution of F_{N2} as expressed in Equation (10.15).

$$F_{N2} = \frac{\left(- \left(\sum T_v - I_v \alpha_r \right) \left(1 - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right) - F_{p,v} \left(\frac{l_{ve}}{2} \right) \left(1 + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right) \right.}{\left(L_v - l_{ve} - (L_v + l_{ve}) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right)} \\ - F_{p,vs,t} l_{ve} \tan \gamma - F_{p,vs,n} \left(\left(\frac{L_v + l_{ve}}{2} \right) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma - \left(\frac{L_v - l_{ve}}{2} \right) \right) \\ + m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \left(\left(\frac{L_v - d_{vh}}{2} \right) \left(\frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma - 1 \right) + l_{ve} \right) \\ \left. + \left(\frac{I_c \alpha_c - \sum T_c}{r_c \sin \gamma} - m_v \frac{d^2 l_{ve}}{dt^2} \right) l_{ve} \tan \gamma \right) \quad (10.15)$$

Substituting Equation (10.15) into Equation (10.12) gives the closed-form solution of $F_{v,n}$ as expressed in Equation (10.16).

$$F_{v,n} = \frac{\left(- F_{p,v} \left(L_v \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right) - F_{p,vs,t} \tan \gamma (L_v - l_{ve}) \right.}{\left(L_v - l_{ve} - (L_v + l_{ve}) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right)} \\ + m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r (d_{vh} + l_{ve}) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \\ \left. + 2(I_v \alpha_r - \sum T_v) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma + \left(\frac{I_c \alpha_c - \sum T_c}{r_c \sin \gamma} - m_v \frac{d^2 l_{ve}}{dt^2} \right) \tan \gamma (L_v - l_{ve}) \right) \quad (10.16)$$

Substituting Equations (10.15) and (10.16) into Equation (10.11) gives the expression for F_{NI} as shown in Equation (10.17).

$$F_{N1} = \frac{\left(\begin{aligned} & -F_{p,v} \left(L_v - \frac{l_{ve}}{2} - \left(\frac{l_{ve}}{2} \right) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right) - F_{p,vs,t} L_v \tan \gamma \\ & - F_{p,vs,n} \left(- \left(\frac{L_v + l_{ve}}{2} \right) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma + \left(\frac{L_v - l_{ve}}{2} \right) \right) \\ & - m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \left(\left(\frac{L_v - d_{vh}}{2} \right) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma - \left(\frac{L_v + d_{vh}}{2} \right) \right) \\ & + \left(I_v \alpha_r - \sum T_v \right) \left(\frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma + 1 \right) + \left(\frac{I_c \alpha_c - \sum T_c}{r_c \sin \gamma} - m_v \frac{d^2 l_{ve}}{dt^2} \right) L_v \tan \gamma \end{aligned} \right)}{\left(L_v - l_{ve} - (L_v + l_{ve}) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right)} \quad (10.17)$$

Lastly, substituting Equation (10.15) into Equation (10.14) gives the expression for $F_{v,t}$ as shown in Equation (10.18).

$$F_{v,t} = \frac{\left(\begin{aligned} & F_{p,v} L_v \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu + F_{p,vs,t} (L_v - l_{ve}) \\ & - m_v \left(r_r + l_{ve} - \left(\frac{L_v - d_{vh}}{2} \right) \right) \alpha_r \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu (-L_v + 2d_{vh} + l_{ve}) \\ & - \left(\frac{I_c \alpha_c - \sum T_c}{r_c \sin \gamma} \right) \left((L_v + l_{ve}) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right) \\ & + \left(m_v \frac{d^2 l_{ve}}{dt^2} \right) (L_v - l_{ve}) + 2 \left(\sum T_v - I_v \alpha_r \right) \left(\frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \right) \end{aligned} \right)}{\left(L_v - l_{ve} - (L_v + l_{ve}) \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \mu \tan \gamma \right)} \quad (10.18)$$

The rest of the parameters, such as the cylinder shaft forces, the rotor shaft forces and the rotor acceleration can be calculated by substituting Equations (10.15) - (10.18) into the respective equations.

10.2. RV-0 Expander Prototype and Experimental Setup

The RV-0 expander prototype is built by modifying the RV-0 compressor built by Teh and Ooi [110, 111]. The schematic drawing of the prototype is shown in Figure

10.5. The dimensions of the prototype are listed in Table 10.1. The original suction and discharge holes are now blocked. New discharge and suction holes are constructed at the rotor body and at the cylinder and the rotor endfaces, respectively, as shown in Figures 10.5 and 10.6. Due to the eccentric nature of the RV-0 mechanism, when the suction hole at the cylinder begins to uncover at the beginning of a cycle, high pressure gas flows in through the hole, into the suction port at the rotor and eventually into the suction chamber. As the expander rotates further, the high pressure gas flows into the working chamber directly through the cylinder suction hole, while the rotor suction port is now covered by the cylinder inner endface wall.

Figure 10.5 A schematic drawing of the RV-0 expander prototype

Figure 10.6 The prototype: (a) the rotor, (b) the cylinder with the rotor inside it

The control of the inlet flow is carried out with the suction valve design presented earlier in Figure 3.12(c). It uses a stationary blocker that will cover the suction port, which is located at the endface of the cylinder, during a predetermined range of the operational angles. This prevents high pressure gas to flow into the working chamber, causing the gas inside to expand. As the expander rotates further, the suction port is exposed and high pressure gas flows into the expander, resulting in the suction process. The design concept and the suction hole blockers are illustrated in Figure 10.7.

Figure 10.7 The suction valve and its arrangement: (a) the inlet control concept with the cylinder suction port exposed, (b) the suction hole blockers

Four suction hole blockers were designed with opening angles of 156.7° , 183.7° , 198.8° and 209.1° for suction pressures of 2 to 5 bar (abs.), respectively. These blocker sizes were obtained by assuming that the gas will expand isentropically in the expander. The valves are made of PTFE material to minimise friction. They were secured to the upper bearing with two pins. Compression springs were installed between the upper bearing and the blocker surfaces to press the blockers towards the cylinder endface to ensure proper covering of the suction holes during the expansion process (see Figure 10.8).

Figure 10.8 The prototype: (a) the suction hole blocker and the upper bearing, (b) the blocker, the spring and the cylinder with its suction port exposed

Similar to the RV-I prototype discussed in Chapter 8, in this prototype design, the expander was also inserted into a housing which also acts as a suction reservoir. High pressure gas flows into the housing first before finally entering the expander. The housing was made up of the body and the cap. The cap was secured unto the body with screws. An O-ring is inserted in between the body and the cap to prevent leakage. The rotor shaft protruded out from the housing through a hole at the housing cap. Two oil seals were installed to minimise oil and gas leakage through the clearance between the shaft and this hole. The upper bearing was suspended on the housing cap with screws. O-rings were also inserted below these screw heads to seal.

Table 10.1 Major dimensions of the RV-0 expander prototype

Radius of rotor body, r_r	30.0 mm
Length of expander, l_{exp}	49.5 mm
Diameter of rotor shaft	22.0 mm
Length of rotor upper shaft	122.0 mm

Length of rotor lower shaft	53.5 mm
Inner radius of cylinder body, r_c	33.3 mm
Outer radius of cylinder body	40.3 mm
Diameter of cylinder bearing hole	38.0 mm
Length of vane including head	22.0 mm
Diameter of vane head	6.0 mm
Thickness of vane	5.0 mm
Length of vane slot	19.0 mm
Width of vane slot	5.0 mm
Diameter of suction port	3.8 mm
Diameter of discharge ports	5.0 mm
Assembly radial clearance	15 μm
Total endface gap width	30 μm
Length of rotor bearing	68.1 mm
Length of cylinder bearing	44.0 mm

The schematic diagram of the experimental setup is shown in Figure 10.9. The motor is connected to a frequency controller to allow adjustments of the speed. The flow meter is of the Aalborg 044-40 variable area type which can measure up to 70 l/min of air. It is installed at the suction line, because the discharge process is through the rotating driving shaft into the atmosphere. The torque and the rotational speed are measured using the Torquemaster TM107 transducer linked to the DUD282 display system. The suction gas pressure and temperature are measured with an analogue pressure transducer and a type-K thermocouple, respectively. The accuracies of the measuring devices are listed in Table 10.2. Shell Corena P lubricant oil with a viscosity

grade of 100, which was used for previous experiments with the prototype for compressor applications [111], was used for lubrication.

Figure 10.9 A schematic diagram of the experimental setup

Table 10.2 Accuracies of the measuring devices

Thermocouple	$\pm(0.3\% \text{ of reading} + 1\text{ }^{\circ}\text{C})$
Pressure transducer	$\pm 10\text{ kPa}$
Flow meter	$\pm 23\text{ cm}^3/\text{s}$
Speed sensor	$\pm 5\text{ rev/min}$
Torque sensor	$\pm 0.02\text{ Nm}$

Our subsequent preliminary tests also indicate that the natural frequency of the rig is around 300-360 rev/min. Therefore, no test was carried out in this speed range. In addition, because of mechanical performance deterioration, the operational speed tested was limited to 900 rev/min. Measurements are recorded when the readings indicated by the sensors have stabilised.

10.3. Experimental Results and Discussions

With the prototype and the experimental setup properly installed, the experiments were carried out. The results are presented in this section. They include the study of the flow rate, the leakage and the dynamics of the RV-0 expander.

10.3.1. Preliminary tests

Before data collection was started, two sets of preliminary tests were carried out to understand the general behaviour of the expander. In the first set, the expander was disconnected from the motor and high pressure air (up to 7 barg) was pumped into the housing. This is to test if the expander can start with high-pressure gas alone, without the motor. It was found that the result depends on the start-up position of the vane. If the expander was started with a rotor angle that is near 0° , which is when the vane is concealed by the vane slot, the expander could not turn because there was not enough exposed vane surface area. However, when there was enough exposed vane surface area, the expander immediately turned and stopped when the rotor angle was near 360° , when the vane was again concealed by the vane slot. This indicates that although the high-pressure gas was able to turn the expander, but the momentum generated during the start-up was not sufficient to overcome the inertia and the friction of rotating components of the expander.

In the second set, the expander was connected to the motor, which was used as a start-up motor. Once the desired speed was achieved, the motor was switched off but the high-pressure gas supply was maintained. In one of the tests, the motor was used to start-up the expander to a speed of around 200 rev/min and 3 barg air was pumped into the expander housing. It was found that even after the motor is switched off, the high-pressure air was able to sustain the rotation of the expander. This shows that the

prototype is able to function properly as an expander, even with the inertia of the motor and the torque sensor to add as the load.

10.3.2. Flow rate analysis

The variations of the expander volume flow rate and volumetric efficiency with operational speed and suction pressure are shown in Figure 10.10. The definition of volumetric efficiency is the ratio between the ideal and the real mass flow rates as expressed in Equation (10.19).

$$\eta_v = \frac{\dot{m}_{ideal}}{\dot{m}_{real}} \quad (10.19)$$

$$\dot{m}_{ideal} = \rho_{suct} \left(\frac{\omega_{real}}{2\pi} \right) V_{start\_exp} \quad (10.20)$$

Figure 10.10 shows that the flow rate increases linearly with speed like what was observed in Chapter 9, indicating that the leakage flow rates are unaffected by the operational speed. This is because leakage volume flow rate is affected by the pressure difference and the size of gaps, but is relatively independent of the operational speed. However, the relative contribution of the leakage to the total flow rate decreases as the operational speed increases. This is reflected clearly by the volumetric efficiency behaviours. When the operational speed is low, the expander flow rate is small. Hence, the leakage is significant and the volumetric efficiency is low. As the speed (and the expander flow rate) increases, the leakage becomes less and less significant.

Figure 10.10(b) shows that the volumetric efficiency of the prototype is up to 77%. It decreases as the suction pressure increases. This is due to the increase in the leakage flow rate. However, due to choked flow limitations, the decrease of the volumetric efficiency becomes less significant as the pressure becomes higher.

Figure 10.10 Variations of (a) mass flow rate and (b) volumetric efficiency with suction pressure and operational speed

The uncertainty of the results was estimated with Equation (10.21). The uncertainty of the volumetric efficiency comes from the pressure transducer, the flow meter and the speed sensor. The average uncertainty of the volumetric efficiencies is 13%.

$$\delta q = \sqrt{\sum_i \left(\frac{\partial q}{\partial x_i} \delta x_i \right)^2} \quad (10.21)$$

10.3.3. Static leakage analysis

In order to observe the leakage distribution of the RV-0 expander, a static leakage test was carried out. This test will provide rough estimation of the prototype leakage components distribution. The expander shaft is prevented from turning while high pressure gas is pumped into the housing. The pressure is varied from 2 to 6 bar absolute. The volume flow rate is then recorded, indicating the leakage flow rate of the system. It is important to note that since the components do not move, the lubrication system, which usually helps to seal the gaps and clearances, is not active. Hence, the leakage flow rates obtained from the test are expected to be larger than in the actual cases.

The total leakage of the prototype consists of two components: the housing and the expander internal leakages. The housing leakage includes all the gas leakages from

inside the housing to the surrounding. These are mainly through the screw and transducer threads clearances and lip seal clearance. The expander internal leakage includes all the gas leakages from the expander high pressure to the low pressure chambers. These are mainly through the radial clearance and the vane endface gaps.

In the first set of tests, all leakage paths are left open. Therefore, the overall leakage of the prototype, including the leakage of the housing and the internal leakage of the expander is obtained. The data is plotted in Figure 10.11 with a rectangular symbol. In the second set of tests, the discharge outlets of the expander are blocked, preventing any expander internal leakage from occurring. This set of tests produce the leakage of the housing only, plotted with diamond shaped marks in Figure 10.11. The expander internal leakage flow rates can then be calculated by taking the difference between the two sets of data, plotted with a triangular symbol in Figure 10.11. All the leakage flow rates increase linearly with higher suction pressures.

Interestingly, it can be observed that almost 50% of the total leakages of the prototype are caused by the housing leakage. Further observations show that this is mainly due to the leakage at the clearance between the rotor shaft and the shaft hole at the housing cap. This finding also indicates that the volumetric efficiency plotted in Figure 10.10(b) will be improved significantly if the housing's sealing is improved.

Figure 10.11 Leakage flow rates of the prototype, the housing and the expander

The expander internal leakage consists of three components: the leakages through the radial clearance (the gap between the rotor and the cylinder at the line contact), the vane endface gaps and through the other internal leakage paths. In order to investigate the contribution of the components of the expander internal leakage, the radial clearance was first blocked with aluminium foil. The flow rates obtained from this set of tests are the leakage of the housing and the internal leakage of the expander through all paths other than the radial clearance. In the second set of tests, both the radial clearance and the vane endface gaps were blocked with aluminium foil as shown in Figure 10.12. This set gives the flow rate of the housing and the expander internal leakage excluding the leakages through the radial clearance and the vane endface gaps. With all these data, the leakages through the radial clearance, the vane endface gap and the other expander internal leakage paths can be calculated. They are plotted in Figure 10.13.

Figure 10.12 Radial clearance and vane endface gap blockages

Figure 10.13 Flow rates of the expander internal leakage and its components

It can be seen that the leakage through the radial clearance dominates the expander internal leakage, contributing to about 67% at 2 bar of suction pressure to about 93% at 6 bar of suction pressure. This is thought to be caused by the relatively large cross-sectional opening area of the radial clearance, which is over the whole length of the expander working chamber, and the short leakage path as compared to the other leakage paths. For comparison, the radial clearance assembly opening area is around 0.74 mm^2 (based on gap width and length of $15 \text{ }\mu\text{m}$ and 49.5 mm , respectively), while the total vane endface gap maximum opening area is only around 0.20 mm^2 (based on total width and maximum length are $30 \text{ }\mu\text{m}$ and 6.6 mm , respectively). This finding is also consistent with what was reported previously by Yanagisawa and Shimizu [121] in similar rotary machines like the rolling piston mechanism.

The leakage through the vane endface is found to be significant only in the low pressure regions. It accounts for about 20% of the expander internal leakage at 2 bar suction pressure. As the pressure increases, the effect becomes less significant. In this study, the flow rate dropped unexpectedly as the pressure was increased. This was thought to be caused by the squeezing of the aluminium foil as the pressure across the vane increases, sealing the endface gaps more tightly.

10.3.4. Dynamics analysis

The variations of the output torque of the RV-0 expander prototype with suction pressure and operation speed are shown in Figure 10.14. The torque decreases with increasing operational speed, indicating higher friction losses. As mentioned before, the RV-0 mechanism has two main sources of friction: the vane and the bearings. The vane side force increases quadratically with the operational speed [19], while the journal bearing friction force is proportional to the speed [133]. These increases in the forces

cause large friction torque losses at high operational speeds. Another factor that may cause the behaviour of the output torque is the fact that the expander body is cooler at low speeds as compared to at high speeds. The Corena P 100 lubricant oil used in this experiment exhibits exponentially increasing viscosity in low temperature (below 60 °C, which is the range which the prototype is found to operate in) regions, which is the range of temperature corresponding to the expander experiments, hence the friction loss at the bearings also increases exponentially in the low speed region.

Figure 10.14(a) also shows that at lower speeds, the output torque decreases more rapidly as compared to at higher speeds. This is thought to be caused mainly by leakage. At low speeds, leakage was significant and hence, the pressure difference across the vane was significantly compromised, reducing the output torque of the expander. To confirm this hypothesis, a test was conducted with the suction tube exposed to atmosphere. With this arrangement, no internal leakage will occur and hence, the output torque will be independent of the effect of the internal leakage. The results are plotted in Figure 10.14(b). It can be seen that the output torque drops fairly linearly with increasing operational speeds. This confirms the hypothesis that leakage causes the exponential decrease of the output torque in the low operational speed region.

Figure 10.14 Variation of output torque: (a) With various suction pressure and operational speed conditions; (b) Suction tube exposed to atmosphere

Figure 10.14 also shows that the expander did not produce enough torque to overcome the friction loss in many of the measurement points, indicating large friction losses throughout the operational range studied. The main reasons are thought to be the relatively low pressure conditions tested (up to only 5 bar of suction pressure), which resulted in the small working fluid energy available, the large inertia of the cylinder (around 1.63 g.m^2) and the highly viscous lubricant oil used (viscosity grade 100, kinematic viscosity of $100 \text{ mm}^2/\text{s}$ at 40°C). In addition, the compression spring at the suction blocker and the lip seal also caused significant additional friction to the system.

To confirm the small fluid energy hypothesis, a set of tests without a suction valve was carried out. The fluid energy was maximised because the pressure in the suction chamber was always maintained at the suction reservoir pressure. It must be remembered, however, that the gas is now expanded instantaneously and the ratio between the energy produced and the flow rate is now very low as compared to the ratio of the expander with a suction blocker. The tests were conducted with suction pressures of 5 and 6 bar absolute and rotational speeds of 540 to 900 rev/min, where the internal leakage is not so significant. The results of the tests are shown in Figure 10.15. The figure shows that the expander produces a net output torque at 6 bar of suction pressure at all the rotational speeds tested and at 5 bar of suction pressure at the lower speeds. This shows that if the fluid energy is increased, the expander will produce net output torque. This also confirms that the low fluid energy hypothesis is valid.

Figure 10.15 Output torque with no suction valve

As for the high oil viscosity argument, expanders operate at lower temperatures as compared to compressors because as the fluid expands, its temperature decreases. This causes the oil temperature to also be at a value which is relatively low as compared to that in the compressor applications and hence, the viscosity oil lubricant is relatively high. To test the validity of this high oil viscosity argument, two sets of tests were carried out with grade 46 lubricant (with kinematic viscosity of 46 mm²/s at 40°C). Both sets were conducted with a suction pressure of 5 bar absolute. The first set was done with the suction blocker while the second one was without. The results are compared with those obtained when grade 100 lubricant was used and are shown in Figure 10.16. It can be seen that with grade 46 lubricant, the output torques improved significantly and all the output torques are now positive. The isentropic efficiency (defined according to [162] and expressed in Equation (10.22)) of Figure 10.16(a) varies from 3.5% to 17.5%. These finding confirms the validity of the high viscosity lubricant argument.

$$\eta_i = \frac{P_{real}}{\dot{m}_{th} \Delta h_{isentropic}} \quad (10.22)$$

Figure 10.16 Output torque with oil grade 46 and suction pressure of 5 bar:

(a) with suction valve, (b) without suction valve

10.4. Verification of Theoretical Models

To verify the accuracy of the theoretical models formulated in Section 10.1, the predicted results obtained by the models are compared with the experimental results in Section 10.3. The comparisons will be focused on the output torques and the flow rates.

10.4.1. Output torque

The value of output torque predicted by Equation (10.9) depends on the friction loss which includes the bearing losses. Journal bearings are used in this prototype. From the discussion in Section 5.5, the friction loss of such bearing is dependent on the viscosity of the lubricant. Table 10.3 lists down some physical characteristics of the Shell Corena P 100 lubricant that is used in the experiments. It can be seen that the viscosity changes exponentially with temperature, especially at lower temperatures. For example, an increase of 10 °C of the Corena P 100 oil from 40 °C to 50 °C decreases its viscosity by about 50%. Thus, it can be predicted that the predicted bearing friction loss is very sensitive to temperature change too and therefore, lubricant temperature must be modelled accordingly to give good predictions of the expander output torque.

Table 10.3 Physical characteristics of Shell Corena P 100 lubricant [111]

Corena P			100
ISO Viscosity grade		ISO 3448	100
Performance Standard		DIN 51508	VDL 100
Kinematic viscosity		ASTM D445	
at 40°C	mm ² /s		100
at 50°C	mm ² /s		57.7
at 60°C	mm ² /s		35.9
at 70°C	mm ² /s		23.8
at 80°C	mm ² /s		16.6
at 90°C	mm ² /s		12.2
at 100°C	mm ² /s		9.2
Density at 15°C	kg/m ³	ASTM D1298	899
Flash point (COC)	°C	ASTM D92	240
Pour point	°C	ASTM D97	-33

Comparisons between the RV-0 expander output torques operating at various suction pressure and operational speed conditions obtained from measurements and theoretical predictions are shown in Figure 10.17. The pressure inside the vane slot has been assumed to be equal to the mean value of the pressures of the working chambers. The theoretical predictions include these lubricant temperature models: (1) constant oil temperature at 40 °C and (2) constant lubricant temperature at the gas suction temperature. It can be seen that oil temperature significantly affects the output torque. The torque predictions when the oil temperature is fixed at the suction gas temperature (model (2)) are below the predictions of when the oil temperature is fixed at 40 °C (model (1)). This is because at lower temperature, the oil is more viscous, resulting in more bearing friction losses. The figure also shows that as the operational speed increases, the experimental data move away from the predictions of model (2) and become closer to the predictions of model (1). This is because with a higher operational speed, there are higher friction losses, which result in higher oil temperature. The results also indicate the need to include the oil temperature variation model, mentioned in Chapter 5, to get a better theoretical model.

Figure 10.17 Comparison of output torques from experiment and theoretical models

To model the variation of the oil temperature, Equation (5.70) is used with the assumption that 50% of the bearing friction power loss is absorbed by the lubricant to increase its temperature [133]. The oil flow is assumed to be mainly due to the flows through the bearing clearances [139] and through the spiral grooves [141]. The comparison between the predicted and the measured data are shown in Figure 10.18.

Figure 10.18 Comparison between the output torques from various theoretical models and the measured data

It can be seen that the theoretical predictions with varying bearing temperature match the measured results better as compared to the models with constant bearing temperatures. The average deviation is 25%, contributed mostly by the deviations at lower speeds. The predictions also tend to be better at higher suction pressure and operational speed conditions. At lower speed conditions, the models tend to over-predict, indicating that the lubricant is actually cooler in the actual operational conditions. This is because of the assumption that the fraction of friction bearing loss absorbed by the lubricant is constant at 50% while it should decrease as the bearing loads decrease [133]. If all the data from operational speeds that are lower than 600 rev/min are arbitrarily assigned a fraction of 40%, instead of 50%, the average deviation reduces to 20%.

10.4.2. Flow rates

The comparisons between the expander mass flow rates obtained from measurement and predictions are shown in Figure 10.19. It can be seen that they are very close with an average standard deviation of 10%. No coefficient of blocking is needed here, unlike in Chapter 9, because the clearance gaps are relatively smaller. Hence, the effect of the leakage to the overall flow rate is also less significant.

Figure 10.19 Comparison between the expander mass flow rates from various theoretical models and the experimental data

Detailed analysis of the mass flow rate data shows that the internal leakage through the radial clearance gap dominates the internal leakage, consistent with the results obtained from the static leakage test. The proportions of the average volume flow rates through the radial clearance and the vane endfaces gaps vary from about 60% and 40%

to 52% and 48%, respectively, as shown in Figure 10.20(a). However, the proportions tend to be closer to each other as the suction pressure increases, which are different from the trend suggested by the static leakage test. This difference is because unlike in the static leakage test, the width of the radial clearance gaps varies in the real operations, as shown in Figure 10.20(b). The higher the suction pressure is, the smaller the gap is due to the larger shaft bearing load. On the other hand, the widths of the endface gaps are unaffected by the suction pressure. As a result, as the suction pressure increases, the relative proportion of the leakage through radial clearance decreases while that through the vane endface gaps increases.

Figure 10.20 (a) Proportions of the flow rates of the internal leakage components;
(b) Variation of the average predicted radial clearance gap width with suction pressure

10.5. Concluding Remarks

Theoretical and experimental investigations of the RV-0 expander have been carried out. Because the mechanism is different from the RV-I expander, the kinematics and dynamics theoretical models of the RV-0 have been derived separately in this chapter. One of the main differences between the RV-0 and the RV-I models is that the vane side contact forces of the RV-0 are dependent on the pressure difference across the vane while that of RV-I are not. This causes the RV-I design to be more desirable than

the RV-0. Experimental investigations have also been carried out with an RV-0 expander prototype. The following observations are noted:

1. The RV-0 expander prototype was successfully operated without any significant operational problem in the range of the operational pressure (from 2 to 5 bar abs.) and speed (up to 900 rev/min) tested.
2. The expander exhibits a volumetric efficiency of up to 77%, depending on the suction pressure and the operational speed used. The efficiency increases with increasing speed, but decreases with increasing pressure.
3. From the static leakage test, the main leakage paths are found to be through housing and the radial clearances, accounting for around 50% and 40% of the total leakage, respectively. The test also shows that the leakage through the radial clearance accounts for 67% to 93% of the total internal leakage of the expander, depending on the operational pressure. The leakage through the vane endface gaps accounts for around 20% of the total expander internal leakage in some of the cases studied. However, as the suction pressure increases, the contribution of this leakage becomes less significant.
4. The output torque produced by the expander is very sensitive to the viscosity of the lubricant oil used. Almost no net positive output torque is obtained with oil grade 100 and suction pressures of up to 5 bar abs. However, when oil grade 46 is used, net positive output torque readings are obtained with isentropic efficiencies of 3.5% to 17.5%.
5. The theoretical prediction of the output torque is very sensitive to the lubricant temperature. By accounting for the different fractions of bearing friction losses are absorbed by the lubricant during operation, the comparisons between the theoretical and experimental data of the output torques show an agreement with an average deviation of around 20%.
6. The comparisons between the theoretical and experimental data of the flow rates show a good agreement with an average deviation of around 10%.
7. The proportions between the internal leakage flow rates through the radial clearance and the vane endfaces gaps vary from 60% and 40% to 52% and 48%, respectively. As the suction pressure increases, the proportions tend to get closer because the average width of the radial clearance gap decreases while the width of the vane endface gap remains the same.

IMPROVED DYNAMICS MODEL

From the simulation results presented in Chapter 6, it is realised that the theoretical models developed in Chapters 4 and 5 have some limitations that are caused by the assumptions adopted, namely that the vane and the vane slot geometries are negligible in the geometrical and the kinematics models and that the movements of the components are wholly geometrically constrained. The limitations include the assumption that vane contact side flipping process is instantaneous and the inability of the models to accurately simulate: (1) the leakages through the vane side gaps, (2) the relationship between the force, the torque and the movements of the components, and (3) the effects of impact between the vane and the vane slot. Therefore, there is a need for a more developed geometrical, kinematics and dynamics models which include the vane geometries and are preferably based on the force-torque interactions, not on the geometries alone. In addition, the experimental data discussed in Chapter 9 suggests that this interaction may also be important in the dynamics of the RV-I expander. Two methods to develop the new models are presented in this chapter. The first method is a modification of the methods developed in Chapter 5 to include the vane geometries. However, force-torque interactions are still not included because the foundations of the models are still geometrically constrained as were developed in Chapter 5. The second method is novel in that it includes both the vane geometries and the force-torque interactions. The method is fundamentally different from that presented in Chapter 5. Only the important steps of the model formulations are shown here. The more detailed derivations can be found in Appendix E. Simulation results and discussions of the models are also presented in this chapter.

11.1. Geometrically Constrained Model with Inclusion of Vane Geometry

The derivations of the geometrical, kinematics and dynamics models of the RV expander discussed in Chapter 5 are based on the assumption that the vane has a negligible thickness. However, it is possible to modify the models to include the vane geometries. The method is presented in Section 11.1.1. Results and discussions of the method are presented in Section 11.1.2.

11.1.1. Formulation of the models

To start with, the definitions of the cylinder angle and the length of the exposed vane are left unchanged. Therefore, Equations (5.1) - (5.5b) are still applicable. However, the contact point is now moved from point A to point V_1 as illustrated in Figure 11.1. Point V_1 is a point where one of the sides of the vane contacts the vane slot opening. This point is located at half of the vane width, w_v , away from point A and is defined according to the triangle ARV_1 . During operation, the contact point may switch to a point at the other side of the vane called point V_2 . This will happen when the sign of the vane contact force, $F_{v,n}$, becomes negative. The definition of this point is identical to V_1 except that it is at the other side of the vane.

Figure 11.1 (a) Geometrical model of the RV expander with the vane thickness considered, (b) triangle ARV_1 and (c) triangle CRV_1

After some derivations (the steps are shown in Appendix E), the cylinder angular speed is found to be as expressed in Equation (11.1).

$$\omega_c = \omega_r \left(\frac{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2}{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right)} \right) \quad (11.1)$$

Differentiating Equation (11.1) with time gives the angular acceleration of the cylinder as expressed in Equation (11.2).

$$\begin{aligned} \alpha_c = \alpha_r & \left(\frac{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2}{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right)} \right) \\ & \left(\frac{2(r_r + l_{ve}) \frac{dl_{ve}}{dt} \left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right) \right)}{2(r_r + l_{ve}) \frac{dl_{ve}}{dt} \left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right) \right)} \right) \\ & + \frac{\left(- \left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 \right) \frac{2(r_r + l_{ve}) \frac{dl_{ve}}{dt}}{2(r_r + l_{ve}) \frac{dl_{ve}}{dt} \left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right) \right)} + e \left(\frac{dl_{ve}}{dt} \cos \varphi_r - \omega_r (r_r + l_{ve}) \sin \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \omega_r \left(\frac{w_v}{2}\right) \cos \varphi_r \right) \right)}{\left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right) \right)^2} \end{aligned} \quad (11.2)$$

As for the force and torque balance relationships of the components, because the derivations in Sections 5.2 and 5.3 have included the vane geometries, they are still applicable. Equations (5.13) - (5.27) do not need to be modified.

11.1.2. Simulation results and discussions

The variations of the vane contact force obtained from the models developed in Section 11.1.1 are shown in Figure 11.2. It can be seen that the general trend is still

sinusoidal like the one shown in Figure 6.16. Two vane contact sides flipping processes, indicated by the change in the sign of the vane contact force, are also observed. One is at the middle of the cycle, where the vane flips its contact sides from the right hand side of the vane, point V_1 , to the left hand side, point V_2 , and the other is at the end of the cycle, where the vane flips its contact sides back from point V_2 to point V_1 .

Figure 11.2 Variations of vane contact force of the geometrically constrained model with consideration of vane geometry

The vane side flipping process at the middle of the cycle is marked by the zero vane contact force region near the 180° rotor angle, corresponding to the period when the vane is not in contact with the vane slot. The vane side flipping process at the end of the cycle is marked by discontinuities of the vane contact force. It is negative at the 360° rotor angle, but instantaneously becomes positive at the 0° rotor angle. This is unrealistic and shows that even the model presented in Section 11.1.1 is not capable of simulating the vane sides flipping process adequately. In reality, the flipping process takes some time to complete, like the flipping process at the middle of the cycle. The length of time depends on the vane thickness and the size of the vane opening. Therefore, there will be some periods of time when the vane is not in contact with the vane slot, indicating that the movements of the components are not purely geometrically

constrained. Instead, it is actually a function of the force-torque interactions of the components. Unfortunately, because the models developed in Chapter 5 and in Section 11.1.1 are derived based on the assumption that the movements of the components are geometrically constrained, they are not able to simulate this flipping process adequately. In addition, the fact that the flipping process takes some time to complete indicates that there will be impact between the vane and the vane slot at the end of the flipping process. This impact cannot be modelled by the models derived in this chapter. Therefore, the vane contact force predicted by the models may be significantly lower than the real one because the effect of the impact is not accounted for.

These limitations show that in order to investigate the flipping process and to obtain a more accurate dynamics analysis, there is a need for more developed kinematics and dynamics models which are dictated wholly by the force-torque interactions, not by the geometries only. This method is presented later in the next section.

11.2. Theoretical Model with Vane Geometries and Force-Torque Interactions

To formulate the new theoretical model, the geometrical model shown in Figure 11.3 is employed. The cross-section of the vane is not rectangular as before to allow the model to be applicable for different vane geometries. Points C and R are the cylinder and rotor centres, respectively. Point V_1 is a point at the right hand side of the vane that is of a distance of the cylinder radius, r_c , from point C. This is also the vane contact point when the vane is in contact with the vane slot at its right hand side. The distance of this point from R can, therefore, vary accordingly. Point S_1 is the tip of the right hand side of the vane slot opening. The distance between this point to point C is always equal to the radius of cylinder, r_c . In the figure, points V_1 and S_1 are at the same position,

indicating that the contact point of the vane and the vane slot is currently at the right hand side of the vane.

Figure 11.3 Geometrical models of (a) the RV expander and (b) the rotor

The angle of the rotor, ϕ_r , is defined as the angle between a vertical line, which passes through both points C and R, and the midline of the vane, which passes through the rotor centre, R, and point V_0 . Point V_0 itself is a point that lies at the midline of the vane and is of a distance of the cylinder radius, r_c , from the cylinder centre, C. Point V_2 is a point at the left hand side of the vane that is of a distance of the cylinder radius, r_c , from the cylinder centre, C. This is the contact point of the vane when the vane contact is at the left hand side of the vane. Points V_{b1} and V_{b2} are points at the right hand side and left hand side of the base of the vane. They are of distances of the rotor radius, r_r , from the rotor centre, R.

The angle of the cylinder, ϕ_c , is defined as the angle between the vertical line and a line which passes through the cylinder centre, C, and a line that connects point C and the midpoint of the vane slot opening, S_0 (see Figure 11.4). Point S_2 is the tip of the left

hand side of the vane slot opening. Point R_1 is the projection of point S_1 at the midline of the vane, while point R_2 is the projection of point S_2 at the midline of the vane. When the vane is in contact with the vane slot at its right hand side, the distance between points R_1 and S_1 is equal to half of the vane width, w_v . Point C_1 is a point at the line that connects point C with the midpoint of the vane slot opening, S_0 . The distance between point C_1 and point S_1 is equal to half of the width of the vane slot opening, w_{vs} . There is also point C_2 (not shown in the figures) that is of the same distance from point S_2 . However, because this point always coincides with point C_1 , point C_2 will be merged with point C_1 in the subsequent discussions. Figure 11.4 also shows that points V_1 , V_0 , V_2 , S_1 , S_0 and S_2 are all of the distances of the cylinder radius, r_c , from C.

Figure 11.4 Geometrical model #1 (vane is in contact with vane slot at V_1)

11.2.1. The angle of the cylinder

After some derivations (see Appendix E for the steps), the angle of the cylinder can be computed using Equation (11.3a) if the vane contact point is at V_1 or with Equation (11.3b) if the point is at V_2 . The only unknown in these equations is the angle of cylinder, φ_c . Therefore by solving these equations, the angle of cylinder when the vane is in contact with the vane slot can be calculated.

$$r_c \sin(\varphi_c + \theta_{S_0CS_1}) = \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \cos \varphi_r$$

$$+ \left(\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v + \sqrt{\frac{e^2 + r_c^2 - 2er_c \cos(\varphi_c + \theta_{S_0CS_1})}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} \right) \cos^2 \theta_v (\sin \varphi_r - \tan \theta_v \cos \varphi_r)$$
(11.3a)

$$r_c \sin(\varphi_c - \theta_{S_0CS_2}) = l_{RV_2} (\sin \varphi_r \cos \theta_{V_0RV_2} - \cos \varphi_r \sin \theta_{V_0RV_2})$$

$$\rightarrow r_c \sin(\varphi_c - \theta_{S_0CS_2}) = - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \cos \varphi_r$$

$$+ \left(\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v + \sqrt{\frac{e^2 + r_c^2 - 2er_c \cos(\varphi_c - \theta_{S_0CS_2})}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} \right) \cos^2 \theta_v (\sin \varphi_r + \tan \theta_v \cos \varphi_r)$$
(11.3b)

11.2.2. The exposed length of the vane

To find the exposed vane length at both sides of the vane, Equations (11.4a) and (11.4b) can be used.

$$l_{ve1}^2 = r_r^2 + l_{RV_1}^2 - 2r_r l_{RV_1} \cos(\theta_{V_0RV_{b1}} - \theta_{V_0RV_1})$$
(11.4a)

$$l_{ve2}^2 = r_r^2 + l_{RV_2}^2 - 2r_r l_{RV_2} \cos(\theta_{V_0RV_{b2}} - \theta_{V_0RV_2})$$
(11.4b)

Differentiating Equation (11.4a) or (11.4b) with time gives the sliding velocity of the vane, which is important for the calculation of the friction at the vane side in the dynamics model. The velocity is expressed in Equations (11.5a) and (11.5b).

$$\frac{dl_{ve1}}{dt} = \frac{\omega_c e r_c \sin(\varphi_c + \theta_{S_0CS_1})}{l_{ve1}} \left(1 - \frac{(l_{RV_{b0}} - l_{V_{b0}V_{b1}} \tan \theta_v)}{\sqrt{\frac{l_{RV_1}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2}} \right) \quad (11.5a)$$

$$\frac{dl_{ve2}}{dt} = \frac{\omega_c e r_c \sin(\varphi_c - \theta_{S_0CS_2})}{l_{ve2}} \left(1 - \frac{(l_{RV_{b0}} - l_{V_{b0}V_{b2}} \tan \theta_v)}{\sqrt{\frac{l_{RV_2}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2}} \right) \quad (11.5b)$$

11.2.3. The gap between the vane sides and the vane slot opening tips

It is necessary to check the gap between the sides of the vane to the tips of the vane slot opening to see if the vane is in contact with the vane slot opening. These gaps may also cause working fluid to leak from the high pressure chamber to that of the low pressure. To help in the understanding of the procedure, a geometrical model when the vane is not in contact with the vane slot is shown in Figure 11.5.

Figure 11.5 (a) Geometrical model #2 (vane is not in contact with the vane slot), (b) triangle \$CRS_1\$ and (c) triangle \$RR_1S_1\$

The vane is not in contact with the vane slot when neither the distance between R_1 and S_1 nor between R_2 and S_2 is equal to half of the vane width. Mathematically, it means that the vane is not in contact with the vane slot when both $l_{R_1S_1} - \left(\frac{w_v}{2} + (d_v - (l_{RR_1} - l_{RV_{b0}})) \tan \theta_v \right)$ and $l_{R_2S_2} - \left(\frac{w_v}{2} + (d_v - (l_{RR_2} - l_{RV_{b0}})) \tan \theta_v \right)$ are more than 0. After some derivations (see Appendix E for the steps), the gaps between the side of the vane and the tip of the vane slot opening is found as expressed in Equation (11.6a) and (11.6b).

$$d_{v1} = l_{R_1S_1} - \left(\frac{w_v}{2} + (d_v - (l_{RR_1} - l_{RV_{b0}})) \tan \theta_v \right) \quad (11.6a)$$

$$d_{v2} = l_{R_2S_2} - \left(\frac{w_v}{2} + (d_v - (l_{RR_2} - l_{RV_{b0}})) \tan \theta_v \right) \quad (11.6b)$$

11.2.4. The dynamics of the cylinder and the rotor

To model the dynamics of the RV expander, the cylinder and the rotor must be analysed separately. The forces and torques at the cylinder when the vane and the vane slot are in contact at V_1 are shown in Figure 11.6.

Figure 11.6 The forces and torques at the cylinder (vane is in contact at V_1)

After some derivations, the vane contact force is found to be as expressed in Equation (11.7). The ΣT_c term consists of the torques acting on the cylinder which are not caused by the vane contact force nor the vane side friction. These may include, among others, the torques caused by the endface and the bearing frictions.

$$F_{v,n} = \frac{I_c \alpha_c - \Sigma T_c}{\left(\left(\cos \theta_v + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \sin \theta_v \right) r_c \cos \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0Rcont} \right) + \frac{F_{v,n}}{|F_{v,n}|} \left(\sin \theta_v - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \cos \theta_v \right) r_c \sin \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0Rcont} \right) \right)} \quad (11.7)$$

where if the vane contact point is at V_1 , then $\frac{F_{v,n}}{|F_{v,n}|} = 1$. If the vane contact point is at V_2 ,

then $\frac{F_{v,n}}{|F_{v,n}|} = -1$, and if the vane is not in contact with the vane slot, there is no contact

force and therefore $\frac{F_{v,n}}{|F_{v,n}|} = 0$

The forces and torques acting on the rotor are shown in Figure 11.7.

Figure 11.7 The forces and torques at the rotor (vane is in contact at V_1)

After some derivations, the vane contact force is found to follow Equation (11.8).

The ΣT_r term consists of all the torques acting on the rotor which are not caused by the vane contact force, the vane side friction, the fluid pressures acting on the vane or the load. Points R_1 and R_2 have been generalised as R_{cont} in this equation.

$$F_{v,n} = \frac{-I_r \alpha_r - F_{P1} \left(l_{RV_{b0}} + \frac{l_{ve1}}{2} \right) + F_{P2} \left(l_{RV_{b0}} + \frac{l_{ve2}}{2} \right) + \sum T_r - T_{gen}}{\left(\cos \theta_v l_{RR_{cont}} - \sin \theta_v l_{R_{cont}V_{cont}} \right) + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \left(\sin \theta_v l_{RR_{cont}} + \cos \theta_v l_{R_{cont}V_{cont}} \right)} \quad (11.8)$$

To simplify the subsequent discussions, let us introduce the new parameters shown in Equations (11.9) - (11.12).

$$R_{v,n,c} = \left(\cos \theta_v + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \sin \theta_v \right) r_c \cos \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0Rcont} \right) + \frac{F_{v,n}}{|F_{v,n}|} \left(\sin \theta_v - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \cos \theta_v \right) r_c \sin \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0Rcont} \right) \quad (11.9)$$

$$T_c = \sum T_c \quad (11.10)$$

$$R_{v,n,r} = \left(\cos \theta_v l_{RR_{cont}} - \sin \theta_v l_{R_{cont}V_{cont}} \right) + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \left(\sin \theta_v l_{RR_{cont}} + \cos \theta_v l_{R_{cont}V_{cont}} \right) \quad (11.11)$$

$$T_r = -F_{P1} \left(l_{RV_{b0}} + \frac{l_{ve1}}{2} \right) + F_{P2} \left(l_{RV_{b0}} + \frac{l_{ve2}}{2} \right) + \sum T_r - T_{gen} \quad (11.12)$$

Equations (11.7) and (11.8) calculate the contact force at the cylinder and the rotor, respectively. They are action and reaction forces. These forces are zero when the vane is not in contact with the vane slot. Combining this with the conservation of energy equation before and after the vane hits the vane slot gives the accelerations of the rotor and the cylinder as expressed in Equations (11.13) and (11.14). These equations should be used only when the vane is in contact with the vane slot. Else, the accelerations should be calculated using Equations (11.7) and (11.8) with the vane contact force, $F_{v,n}$, is set to zero. It can be seen that unlike in the geometrically constrained model in

Section 11.1, the accelerations of the expander components are now functions of forces and torques.

$$\alpha_r = \frac{-2 \left(\omega_{r1} - \frac{R_{v,n,c}}{R_{v,n,r}} \omega_{c1} \right) + \frac{t_{imp}}{I_c} \frac{R_{v,n,c}}{R_{v,n,r}} \left(T_r \frac{R_{v,n,c}}{R_{v,n,r}} + T_c \right)}{t_{imp} \left(1 + \frac{I_r}{I_c} \frac{R_{v,n,c}^2}{R_{v,n,r}^2} \right)} \quad (11.13)$$

$$\alpha_c = -\frac{I_r}{I_c} \alpha_r \frac{R_{v,n,c}}{R_{v,n,r}} + \frac{T_r}{I_c} \frac{R_{v,n,c}}{R_{v,n,r}} + \frac{T_c}{I_c} \quad (11.14)$$

11.2.5. Preliminary simulation results and discussions

After the models are formulated, computational simulations are now ready to be carried out. This is discussed in this section. The procedure is discussed first, followed by the discussion of the results. To simplify the discussion, the theoretical models developed in Chapter 5 will be called the “original model” while the model developed in Section 11.2 will be referred to as the “new model”.

The simulation procedure presented in Chapter 6 can no longer be used since the new model requires solving the simultaneous equations. Therefore, a different computer code has to be written. The 4th order Runge-Kutta method and the trapezoidal rule are still employed to carry out numerical integrations. The REFPROP routines [146] are also employed for the calculations of the fluid properties.

The simulation begins with the initialisation of the necessary parameters. These include the angles of the rotor and the cylinder, the angular velocities of the rotor and the cylinder and the thermodynamic properties of the working fluids in all the three chambers. Then, at each time step, the angular accelerations of the rotor and the cylinder and the rate of change of the internal energy of the working fluids in all the three chambers must be computed. To find the appropriate angular accelerations, a

check of whether the vane is in contact with the vane must first be performed at each time step. This is done by calculating the vane gaps using Equations (11.6a) and (11.6b). If the vane is found to be in contact with the vane slot, the angular accelerations must be computed according to Equations (11.13) and (11.14). In this case, the angle of the cylinder should follow either Equation (11.3a) or (11.3b). The Golden Section method was employed to solve these equations. Else, if the vane is not in contact with the vane slot, the angular accelerations should be computed using Equations (11.7) and (11.8) with the vane contact force set to zero. The rates of change of the internal energy of the fluids are computed according to Equation (4.5). Simultaneously, other necessary equations should be solved to supply the necessary data. These include, among others, the vane exposed lengths, the sliding velocity of the vane contact point, the endface friction torque, the bearing friction torque, the volumes of the chambers, the flow rates through the ports and the vane gaps. The angular accelerations will then be used to calculate the angular velocities and the angles of the rotor and the cylinder of the next time step, while the rate of change of the internal energy is used to calculate the internal energy of the working fluids of the next time step. By combining the internal energy with the mass and the volume of the working fluid, the other thermodynamics properties of the fluid, like the pressure and temperature, can be obtained.

To simplify the analysis, some assumptions were made. These include that all the potential energy and the kinetic energy of the control volume terms are negligible. Only the flows through the suction port, the discharge port and the gaps between the vane and the vane slot were considered. Flows and leakages through other paths are neglected. Therefore, the working fluid in the suction chamber can only communicate with the suction reservoir through the suction port or with the vane slot through the vane gap, d_{VI} , calculated by Equation (11.6a). The working fluid in the discharge chamber can

only communicate with the discharge tank through the discharge port or with the vane slot through the vane gap, d_{v2} , calculated by Equation (11.6b). The working fluid in the vane slot can only flow in or out through the vane gaps to both the suction and the discharge chambers.

One of the most crucial parts of the theoretical model in this theoretical model is the selection of the appropriate impact time. To the author's knowledge, there has been no reported study about the impact time of a vane and a vane slot. However, because the vane and the vane slot are made of steel, it is believed that the impact time should be in the order of 0.1 ms, which is the order of the impact time of two steel balls [163, 164].

The dimensions of the RV compressor and the parameters listed in Table 6.1 are used. The vane slot is set to be 0.1 mm wider than the vane width, which is 4 mm. The general angle step is 0.0001 rad, but it is reduced by a factor of 4 when the vane is in contact with the vane slot. The simulation begins with both the rotor and the cylinder angles at zero degree, where the vane exposed length is shortest. Hence, the vane is currently not in contact with the vane slot. The cylinder speed is assumed to be equal to the ratio between the cylinder and the rotor radii multiplied by the rotor speed. The simulation is then carried out until the convergence criterion, that the cylinder speeds at the beginning and the end of a cycle are less than 0.01%, is fulfilled. For simplicity, the shaft bearings are assumed to be of the sleeve type (with PTFE coating at the rubbing surfaces) with a friction coefficient of 0.04, which is typical from rubbing between PTFE and steel [165]. With the simulation procedures all set up, the results will be presented next.

Figure 11.8 shows the comparison of the variations of the cylinder angular speeds with the rotor angle obtained from the new and the original models. It can be seen that the profile and the amplitude are very similar. However, the new model shows some

features which are not previously observable with the original model. These are the near-vertical and near-horizontal lines which represent sudden changes of the cylinder speed and constant speed situations, respectively. The sudden change of the cylinder speed is caused by the impact of the vane with the vane slot, which causes significant acceleration or deceleration to the cylinder. The constant speed lines represent conditions when the vane is not in contact with the vane slot. In these situations, the cylinder only experiences deceleration caused by the frictions at the shaft bearings and the endfaces, which are usually relatively small.

Figure 11.8 Cylinder angular speeds of new and original models

Figure 11.9(a) shows the variation of the fluid mass in the suction chamber with the rotor angle as obtained by the new model. The effects of the vane slot chamber are observable here, especially during the expansion process. At the beginning of the expansion, since the pressure inside the vane slot is lower than that in the suction chamber (SCV) (see Figure 11.9(b)), fluid flows from the suction chamber into the vane slot, hence reducing the mass in the suction chamber. It is reminded that other leakage paths were not considered in this simulation. As the expansion process progresses, the pressure difference becomes less significant with the vane slot pressure slightly higher

than that in the suction chamber. This causes the mass in the suction chamber to increase slowly until the end of the process.

Figure 11.9 Variations of (a) suction chamber fluid mass and (b) chambers pressures with rotor angle obtained with the new model

The comparison between the vane contact force predicted by the new and the original models are shown in Figure 11.10. It shows that the general profiles of the vane contact forces obtained from both models are similar, following the profile of a sine function. This indicates that during the operation, the vane only flips its contact sides twice per cycle, as predicted by the original model. These happen somewhere near the mid-point and at the end of the cycle. The figure also shows that the new model does not give a smooth profile. Instead, it fluctuates between zero and a value throughout the cycle, indicating that the vane and the vane slot are only intermittently in contact with each other. The amplitude obtained from the new model is around 10 times larger than that from the original model. This large vane contact force amplitude causes large friction losses at the vane side and the shaft bearings. This causes the average output torque of the new model (around -1.23 Nm) to be significantly lower than the one predicted by the original model (around 0.82 N·m). The difference in the amplitudes of the vane contact forces itself is because the new model includes the impact force

between the vane and the vane slot, while the original model assumes that the vane and the vane slot are always in contact.

Figure 11.10 Vane contact forces of (a) new and (b) original models

It is also important to remember that as discussed in Section 11.2.4, this impact force depends on the impact time. The smaller the impact time is, the larger the amplitude is. This can be seen from the vane contact force data obtained with impact time of 1 ms shown in Figure 11.11. The amplitude of the force is around 10 times smaller than that with an impact time of 0.1 ms shown in Figure 11.10(a), much closer to the amplitude predicted by the original model shown in Figure 11.10(b). It can also be observed that with a larger impact time, the vane and the vane slot are also in contact for longer periods of time although the general trend remains the same, which is positive throughout the first half of a cycle and negative during the other half. More in-depth studies are required in the future to find better impact time estimations for the development of the model.

Figure 11.11 Vane contact forces of the new model with an impact time of 1 ms

11.2.6. Concluding remarks

Two new theoretical models have been introduced in this chapter which include the dynamics interaction between the vane and the vane slot. The first model is a modification of the earlier models developed in Chapters 4 and 5. While some new features like the effects of the vane geometries can be observed with this new model, many limitations inherent to the fundamentals of the model, which assumes that the movements of the components are geometrically dependent, restrict the use of the model. Therefore, a new theoretical model that is fundamentally different from the previously built model was developed.

In the second model, the dynamics of the components are purely dictated by the interactions between the forces and torques acting on the components. This allows the observations of, among others, the interactions between the vane and the vane slot and the effects of the vane slot chamber. One main parameter in this model that will affect the accuracy of the model is the impact time between the vane and the vane slot.

A preliminary simulation has been carried out with the second model. The impact time is assumed to be equal to the impact time between two steel balls. It was found that the general trends of the expander behaviours predicted by the new model are similar to

those by the original model. However, unique features previously not observable with the original models are now observable with the new model. These include:

- (1) The sudden changes and the constant cylinder speed periods due to the interactions between the vane and the vane slot.
- (2) The effects of the vane slot chamber to the thermodynamics processes in the suction and discharge chambers.
- (3) The detailed interactions between the vane and the vane slot. It was found that the vane and the vane slot are only intermittently in contact with each other. However, the vane itself only flips its contact side twice per cycle, as predicted by the original model.
- (4) The effect of the impact force between the vane and the vane slot. This causes the amplitude of the vane contact force to be larger than as predicted by the original model.
- (5) It is found that the impact time between the vane and the vane slot significantly impacts the vane contact force and, consequently, the expander output torque. The appropriate impact time should be investigated further in the future to be used with the new model and to be compared with the experimental data.

CONCLUSIONS AND RECOMMENDATIONS

FOR FUTURE WORK

A thorough study of the development of Revolving Vane expanders has been carried out and presented in this thesis. To wrap it up, the technical findings obtained from this project are presented in this chapter. Some concluding remarks are then presented to close the thesis.

12.1. Conclusion

From the discussions in the previous chapters, significant technical findings have been gathered throughout the project. They are listed below in this section.

1. The recently invented revolving vane (RV) mechanism has been employed for an expander application for the first time. The mechanism is different from the more commonly used rotary mechanisms because the cylinder is allowed to rotate together with the rotor. This feature reduces the relative velocities between the rubbing surfaces and hence, lower frictional losses are produced. The mechanism has five design variants and these are RV-0, RV-I, RV-Ia, RV-II and RV-IIa. All these variants, corresponding to various vane and component designs, are discussed and considered for expander applications in this project.
2. When designing an expander, special attention must be given to the suction valve design since its opening and closing have to be controlled externally in accordance with the designed pressure level to match the operating pressures condition. Four

mechanically controlled suction valve design concepts have been proposed in this project (one in Chapter 3, three in Appendix A).

3. Comprehensive mathematical models which include the thermodynamics, heat transfer, valve dynamics, suction and discharge flows, internal leakage, geometrical, kinematics, dynamics, endface and journal bearings of the RV expander have been formulated. The models are developed with the assumption that the dynamics of the expander components are wholly dictated by its geometry. The thin vane assumption is adopted to formulate the kinematics model since the vane thickness is usually significantly smaller than the dimensions of other major components. This latter assumption simplifies the formulation of the models significantly. However, the consequences are that the flipping process of the vane side contact must be assumed to be instantaneous and the effects of the vane slot volume on the expander performance are unobservable.
4. Theoretical models have been developed for all of the five RV expander design variants discussed in this project. It was found that when the vane is fixed to the driving component (the RV-I and RV-II design concepts), the vane side contact force is dependent only on the inertia of the driving component and the mechanical losses, but is independent of the pressure difference across the vane. This offers a superior performance, especially in high operating pressure applications. On the other hand, when the vane is attached to the driven component and in the case of the original RV mechanism (the RV-0 concept), the vane side contact force is dependent on the pressure difference across the vane.

5. Numerical simulations have been carried out to investigate the expander behaviours. It was found, among others, that the expander output torque is a function of the pressure difference across the vane, the inertia of the driven component and the friction losses. In the design configuration examined, the inertia component dominates other components. The friction losses consist of friction at the vane side, the bearings and the endfaces. In the design configuration tested, the losses at the vane side (70%) and the cylinder bearing (21%) dominate. The leakage study shows that the expander internal leakage increases its flow rate, unlike that in a compressor. It was also found that the expander internal leakage through the radial clearance (79%) dominates that through the vane endface gaps (21%).
6. The parametric studies show that to maximise the expander mechanical efficiency, it is preferable: (i) to have a driven component (which is the cylinder in the RV-I mechanism) with a small rotating inertia, (ii) to operate at high operating pressure conditions and (iii) to employ a long and slim expander, rather than a short and thick one. Unfortunately, to maximise the volumetric efficiency of the expander, the opposites are preferred. Therefore, in most cases, a compromise must be reached. In the expander model studied, it was found that a radius ratio between the rotor and the cylinder of about 0.87 gives the optimum overall expander efficiency.
7. Theoretical comparisons of the performance of the four RV expander variants have been carried out with the following pressure conditions: (i) suction pressures of up to 9 barg and discharge pressure of 0 barg, and (ii) at the CO₂ transcritical refrigeration system conditions. It was found that the RV-I design, where the vane is attached to the rotor and the rotor is used as the driving component, is the most

desirable: it has the highest mechanical efficiency, it produces the highest peak power with the power variation more consistently positive, it also has the smallest vane contact force amplitude, and hence, the lowest stress at the vane. It was also found that even when the rotating inertia of the RV-I expander's driven component (the cylinder) is four times larger than that of the RV-II (the rotor, where the vane is attached to the cylinder and the cylinder is the driving component), the RV-I expander still outperforms RV-II.

8. An RV-I expander prototype has been designed and fabricated. This is believed to be the first RV-I machine ever produced in the world. Some design considerations that must be considered include the minimum endface sealing gap, the strengths of the components and the clearances.
9. Experiments have been carried out with the RV-I prototype under the suction pressures of up to 6 barg and operational speeds of up to 600 rev/min. It was found that the RV-I mechanism design is able to function without any problem in the range of operating conditions tested. The isentropic efficiency is up to 32%. Unfortunately, the prototype is found to have larger than expected manufacturing clearances, resulting in a large internal leakage. The volumetric efficiency is found to be up to 57% when operated at 1 barg of suction pressure, but is only up to 26% when operated at the design condition (3 barg of suction pressure).
10. The comparison between the average torques obtained from the theoretical predictions and the experiments shows that the theoretical model is able to simulate the data with a standard deviation of mostly within 8% and occasionally larger

when the output torque value is near zero. The total average standard deviation is 22%. The theoretical models are also able to predict the variations of the instantaneous output torque with time. The theoretical models are found to over predict the average flow rates. This is thought to be caused by the fact that the leakage clearance gaps are actually partially blocked by lubricant. To accommodate this effect, a blocking coefficient was introduced to improve the accuracy of the prediction.

11. Another expander prototype, fabricated with the RV-0 mechanism, has also been tested. It was found that it can successfully operate without any significant operational problem in the range of the operational pressures (suction pressures of up to 4 barg and a fixed discharge pressure of 0 barg) and speeds (up to 900 rev/min) tested. The expander exhibits a volumetric efficiency of up to 77%. This efficiency increases with increasing speed, but decreases with increasing pressure. From the static leakage test, the main leakage paths are found to be through the housing and the radial clearances, accounting for around 50% and 40% of the total leakage, respectively. The test also shows that the leakage through the radial clearance accounts for 67% to 93% of the total internal leakage of the expander, depending on the operational pressure. The leakage through the vane endface gaps accounts for around 20% of the total expander internal leakage in some of the cases studied. However, as the suction pressure increases, the contribution of this leakage becomes less significant.

13. It was found that the output torque produced by the RV-0 expander prototype is very sensitive to the viscosity of the lubricant oil used. Almost no net positive output

torque is obtained with oil grade 100 and suction pressures of up to 5 barg. However, when oil grade 46 is used, net positive output torque readings are obtained with isentropic efficiencies of 3.5% to 17.5%.

14. The theoretical prediction of the output torque is very sensitive to the lubricant temperature. By accounting for the different fractions of bearing friction losses to be absorbed by the lubricant during operation, the comparisons between the theoretical and experimental data of the output torques show an agreement with an average deviation of around 20%. The comparisons between the theoretical and experimental flow rates data show a good agreement with an average deviation of 10%. The proportions between the internal leakage flow rates through the radial clearance and the vane endfaces gaps vary from 60% and 40% to 52% and 48%, respectively. As the suction pressure increases, the proportions tend to get closer because the average width of the radial clearance gap decreases while the width of the vane endface gap remains the same.

15. Based on the realisations about the limitations of the current theoretical models, an improved dynamics model has been proposed. This model is developed based on the dynamics interaction between the forces and the torques of the components. It includes the impact between the vane and the vane slot. However, the selection of the collision time is crucial and still needs further refining. It is proposed that this model should be further developed in future studies of the Revolving Vane expanders.

12.2. Concluding Remarks and Future Developments of the RV Expander

Since its inception in the year 2006, the Revolving Vane (RV) mechanism has been studied and developed by our research group. I am privileged to be given the chance to join in the project. Since the mechanism was first proposed by our group and there was nobody who has explored the exploitation of the model for expander application, I had the opportunity to be the first (of many in time to come, I believe) researcher working on this. It has definitely been a challenging but also a fulfilling journey.

Many issues related to the RV expander in particular, and the RV mechanism in general, have been resolved, developed, invented and studied in this project. However, many more aspects are still waiting to be discovered and studied. To name a few, related to experiments, the testing of the RV expander in operating conditions beyond what have been tested here is necessary. This will require a better prototype design and a compressor that is able to provide higher gas pressures than the ones tested here. The use of expander with refrigerants and the integration of the expander to a complete refrigeration system also need to be carried out. In such studies, the expander prototype must be designed and fabricated properly according to the compressor capacity and the type of refrigerant used. In my opinion, it is good to start with conventional refrigeration systems first as the pressure conditions are lower as compared to the transcritical CO₂ system. The handling of the liquid nature of the refrigerant and the two-phase expansion will also be challenging and must be properly studied.

On the theoretical aspects, an improved dynamics model that is based on the force-torque interaction between the components has been proposed in Chapter 11. However, the model is still in its infancy and needs to be developed further. It is also interesting to note that, to the author's knowledge, there is currently no specific heat transfer correlation for heat exchange between the working fluid and the compressor wall of a

rotary compressor or expander. The heat transfer correlation mentioned in Chapter 4 is originally for a reciprocating compressor. An in-depth study of the heat transfer of the RV expander, in particular, and rotary mechanisms, in general, should be explored in the future. A second law analysis of the RV expander and the refrigeration systems with and without the RV expander should also be carried out in the future. The work by Yang *et al.* [166] is a good reference for such works.

From aspects related to design improvements, a combined compressor-expander unit is desirable. One of the obstacles to the commercialisation of expanders is the extra manufacturing cost incurred considering that the conventional expansion valve is significantly cheaper than an expander. If the expander and the compressor can be designed as a single unit, the manufacturing cost will be reduced and together with the significantly improved cycle performance, the success of commercialisation of this new product may not be too far away. From my experience, some challenges to such integration include the synchronization of the expander and the compressor torques, the design of the suction and discharge ports and the lubrication system. These are especially challenging for the RV mechanism where the cylinder rotates with the rotor. One possible way to simplify the design is to combine a more conventional mechanism, such as the rolling piston or the sliding vane mechanisms, with the RV mechanism.

Finally, my wish is that this thesis will be a valuable resource for anyone who reads it, especially those who follow my footpath. I also hope that through this project, I have contributed something valuable to the furtherment of knowledge and to the betterment of the society as a whole.

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RV EXPANDER SUCTION VALVE DESIGN CONCEPTS

The need of a specially designed RV expander suction valve has been discussed in Chapter 3. A suction valve design concept has also been presented in the chapter. Three more alternative suction valve design concepts are presented here.

A.1. Concept 1

A simple spring-based valve design as shown in Figure A.1 is proposed here. It has only two main components: (a) a plate, and (b) a spring. One end of the spring is attached to the plate while the other is attached to the RV expander rotor. The suction port consists of two concentric holes, the suction port itself and the valve plate bed. The suction port is connected directly to the suction chamber and is smaller than the valve plate bed. The valve plate bed is designed to be the place for the valve plate to rest. Therefore, the radius of the suction port should be smaller than the valve plate, while the radius of the valve plate bed should be slightly larger than the valve plate.

Figure A.1 The RV expander suction valve design concept 1

To ensure that the valve is only open after the suction process starts, the spring of the valve is positioned at an offset position from the centreline of the suction port as shown in Figure A.2.

Figure A.2 The positioning of the spring of suction valve concept 1

The working principle of the valve is as follows and illustrated in Figure A.3. The spring is designed such that at the start of the suction process (Figure A.3(a)), it is fully compressed and its length is slightly longer than the total length of the suction port and the valve plate bed. As the gap between the rotor and the cylinder inner walls is virtually zero at the start of suction, the fully compressed spring forces the valve to be slightly open. This opening will then let the high pressure fluid flows into the suction chamber and eliminates the pressure difference across the valve plate. The only force acting on the valve plate is now only the pushing force of the spring and hence the valve is opened further. The spring then reaches its neutral condition and the valve is fully opened (Figure A.3(b)).

The spring must also be designed such that its neutral condition length is equal to the summation of the rotor-cylinder gap, the suction port length and the valve plate seat length at the start of the expansion process. Therefore, right at that point, the valve is closed (Figure A.3(c)). This condition causes a pressure difference across the valve plate which will keep the valve closed. Meanwhile, the spring will be stretched further

until its maximum at the half cycle mark. Therefore, the spring will also help the valve to be kept closed during this period and until the spring reaches its neutral condition again. The spring will then be in compressed condition and tries to push the valve open (Figure A.3(d)). It is important to design the spring such that this pushing force is still smaller as compared to the force produced by the pressure difference across the plate. This condition continues until the end of the expansion process at which the spring will be fully compressed forces the valve to open and suction process begins again. The cycle is therefore restarted and repeated.

Figure A.3 Working cycle of the RV expander suction valve concept 1:

- (a) suction begins, spring is fully compressed, valve is slightly open;
- (b) suction continues, spring is at neutral condition, valve open;
- (c) expansion starts, spring is at neutral condition, valve closed;
- (d) expansion continues, spring is compressed, valve closed.

The valve plate can also be made of the reed type or using a rigid plate with a pin joint as shown in Figures A.4(a) and A.4(b), respectively. With proper modifications to the shape of the valve plate bed, other types of blockers such as ball, cones, etc. can also be used instead of a plate. Additional springs and/or dampers can also be installed for better control.

Figure A.4 Using (a) a reed or (b) a rigid plate and a joint as the valve plate of suction valve concept 1

The main strength of this design concept is its simplicity, because the valve has only two main components, namely the valve plate and the spring. The spring stiffness and the fully compressed length of the spring are the only factors in the opening and closing control of the valve. It is also expected to be economical to produce. The suction port needs to be designed to have a valve plate bed as the place for the valve plate to rest on. This is done by creating a concentric hole and can be manufactured easily by casting and finished with boring process. The main weakness of this design concept is the difficulty in the precise control of the spring behaviour which may render the valve to be unreliable.

A.2. Concept 2

Design concept 2 of the RV expander suction valve uses a vane-like device to control the opening of the suction port as shown in Figure A.5. This design concept has been patented [1]. It consists of an arm, a slider and a blocker. One end of the arm is coupled to the rotor shaft to allow the mechanism to rotate with respect to the rotor shaft axis. The other end of the arm is holding the slider. The slider holds the blocker and provides it with some degree of freedom for it to be able to move together with the cylinder. The blocker itself is in contact with the cylinder and is the one that opens or closes the port. During operation, it will slide in and out with respect to the cylinder. It will also slide right and left in the slider. In this instance, the blocker is shown to be made of a rod with a hole connecting the chamber inside the cylinder with the outside allowing high pressure fluid to flow into the suction chamber of the expander.

Figure A.5 The RV expander suction valve concept 2

Instead of a straight port used in the previous concept, a tunnelled suction port is used here (see Figures A.5 and A.6). This tunnel causes an offset between the end of the suction port connected to the expander chamber and the other end connected to the

outside. This allows the geometry of the RV expander chamber to be utilized to open and close the suction port.

As mentioned above, the blocker slides in and out during operation with respect to the cylinder. This causes the hole at the blocker to be exposed to the outside for some period and sealed inside the cylinder for the rest of the cycle. When the hole is outside the cylinder (Figures A.6(a) and A.6(b)), the high pressure fluid flows into the chamber inside the cylinder (which is connected with a tunnel to the suction chamber of the expander). As the expander turns further, the rod will slide into the cylinder until the hole is covered and the suction stops (Figures A.6(c) and A.6(d)).

Figure A.6 Working cycle of the RV expander suction valve concept 2:

- (a) suction starts, valve is almost open,
- (b) suction continues, valve is fully open,
- (c) suction stops, expansion happens, valve is fully closed,
- (d) expansion continues, valve is still closed

Different types of blockers such as a solid rod, a ball, a cone, a plate, a spring-plate, etc. can also be used. The shape of the tunnel is also not crucial as long as it can provide

the offset between the expander chamber and the outside. By modifying the arm design, the hole can also be located at the cylinder endface instead of its outer wall.

The advantages of this valve include its reliability and its minimal negative effect to the performance of the expander. The main disadvantage is the number of components, which is more than the valve design concept 1 and the one presented in Chapter 3.

A.3. Concept 3

The third design concept of the RV expander suction valve uses a movable blocker that moves together with the suction port and a stationary opener of the blocker. When the port is supposed to be closed, this blocker stays at the port closing it. However, when the port is supposed to be open, the blocker is opened by the opener. A possible mechanism is illustrated in Figure A.7 with a capped plate as the blocker, a spring as the anchor which keeps the blocker to move with the expander all the time and a stationary track attached to the stationary casing as the opener. The plate is to block the suction port while the cap is to hang and guide the blocker to move along the track during opening. One end of the spring is attached to the blocker while the other is attached to the expander.

Figure A.7 RV expander suction valve concept 3:

(a) in assembly, (b) the stationary track, (c) the blocker and the spring

The working cycle of the fourth concept of the RV expander suction valve is shown in Figure A.8. The valve is normally closed in this concept. As the expander turns, the blocker will then touch the track (Figure A.8(a)) and is guided to move away from the suction port, and hence, the suction port is open. As the blocker is attached to the expander by the spring, the blocker still follows the expander even when the blocker is separated from the expander as it is being pulled away by the track. The suction port remains open as long as the blocker is still at the track (Figure A.8(b)). Once the blocker has reached the end of the track, it again rests on the suction port and closing it (Figure A.8(c)). It remains closed (Figure A.8(d)) until the end of the cycle, at which the cycle then continues.

Figure A.8 Working cycle of the RV expander suction valve concept 3: (a) suction starts, valve is almost open, (b) suction continues, valve is fully open, (c) suction stops, expansion starts, valve is almost closed, (d) expansion continues, valve is still closed

The blocker is shown to be made of a plate with a cap in this article. However, various shapes can also be used, such as a ball, a cone, etc. The anchor need not to be of a spring and can be of any other mechanism as long as it serves the purpose of keeping the blocker to move together with the expander all the time. The opener is shown to be

of a stationary track in this article. The shape of the track depends on the requirement of the expander. Different types of opening mechanism can also be used, such as a magnet.

The main advantage of this valve design is its reliability. It will always open and close at the desired moments. The disadvantages include the friction loss due to rubbing of the blocker and the stationary track during suction and the number of components involved, which is more than the other design concepts.

References

- [1] A. Subianto, K. S. Yap and K. T. Ooi. (2010). *Revolving Vane Expander*. PCT/SG2010/000044.

CALIBRATION OF THE MEASURING DEVICES

Various devices were used for the experiments presented in this thesis. They include the pressure transducers, the torque sensor, the flowmeter, the thermocouple and the speed sensor. Before being used, they were calibrated and checked for the accuracies. The calibration test of the pressure transducer has been presented in Chapter 9. The tests of the rest of the devices are discussed here.

To test the torque transducer, a static weight test was carried out as shown in Figure B.1. In this test, a known weight was hung at a certain arm length and the measured torque is recorded. It was found that the transducer behaves according to the manufacturer's specification (attached in Appendix C), where a torque of x Nm is displayed as $(x \pm 0.02)$ Nm with a $(0.5x \pm 0.1\%)$ V of output signal.

Figure B.1 Static weight test for the torque sensor

The flowmeter used in this study is of the rotameter type. Its functionality was tested by using it to measure the flow rate of an air blower and to later compare the reading with a pre-installed venturi flow meter. The picture of the test setup is shown in

Figure B.2. It was found that the manufacturer's specification (attached in Appendix C) is acceptable. It is important to remember that if the rotameter is to be used for air at different pressure and temperature conditions, a correction factor as expressed in Equation (B.1) must be used to relate the indicated and the actual flow rates.

$$Q_{actual} = Q_{indicated} \sqrt{\frac{\rho_{calibration}}{\rho_{actual}}} \quad (B.1)$$

Figure B.2 Calibration test of the rotameter

The reliability of the thermocouple is tested by using it to measure the temperature of melting and boiling de-ionized (DI) water, as shown in Figure B.3. It is also tested by comparing the readings with a thermocouple that has been calibrated with a ThermoCal Calibrator Model 18B. The result of the test is shown in Figure B.4.

Figure B.3 Reliability test of the thermocouple

Figure B.4 Test data of the thermocouple

The speed sensor is integrated with the torque sensor. It is tested by comparing the reading with a magnetic pick-up sensor. An iron screw is attached to the expander shaft and the magnetic sensor is placed such that as the expander shaft turns and the iron screw passes by the sensor, an impulse signal is sent to the DAQ system. The time between two impulse signals indicate the period of the shaft. This value is then compared with the reading of the speed sensor. The assembly of the magnetic pick-up sensor, the iron screw and the expander shaft is shown in Figure B.5.

Figure B.5 The expander and the magnetic pick-up sensor

SPECIFICATIONS OF THE MEASURING DEVICES

The relevant manufacturer's specifications of the measuring devices used in this project are attached here. They include those of the thermocouple, the torque and speed sensors and the flowmeter shown in Figures C.1-C.5.

Figure C.1 Specifications of the thermocouple (PATOS DE-305)

7.2 Torque Measuring System

The following specifications are valid for all ratings of transducers in the series TM 104 - TM 113. They are quoted for the transducer and its associated UAE 757 / UAE 763 signal conditioning module.

Accuracy class (1)	: 0.1
Rated output (RO) of TM ... unit	: ± 5 V DC (depending on direction of the applied torque)
Temperature influence on the signal	
- Specification range 1	: +10 °C to +60 °C
On zero	: $< \pm 0.1$ % of RO / 10K
On sensitivity (2)	: $< \pm 0.1$ % of RO / 10K
- Specification range 2	: -25 °C to +80 °C
On zero	: $< \pm 0.2$ % of RO / 10K
On sensitivity (2)	: $< \pm 0.2$ % of RO / 10K
Speed influence on the signal (including wobble)	
- TM 104 to TM 108	: < 0.005 % of RO / 1000 rpm
- TM 109 to TM 113	: < 0.01 % of RO / 1000 rpm
Power supply	
- AC component (20 kHz, sine wave)	: 40 mA RMS
- DC component (pre-amplifier powering)	: 24 V DC
Combined error (linearity + hysteresis)	: ≤ 0.1 % of RO
Repeatability	: < 0.1 % of RO
Long-term stability of zero signal	: $< \pm 0.05$ % of RO / year
Long-term stability of rated output	: $< \pm 0.05$ % of RO / year

Notes

- (1) Greatest individual error where temperature influence is based on ΔT of 10 K.
- (2) This value defines the influence on the whole analog measuring chain (TM ... transducer and UAE... module). Note that for this test the UAE... module was kept at ambient temperature and the temperature of the TM transducer was varied.

Figure C.2 Specifications of the torque transducer (Torquemaster TM107)

7.3 Speed Measuring System

Minimum speed	: 10 rpm
Pulse signal	
- Output voltage	: 5 V peak-peak
- Measurement range	: 10 Hz to 50 kHz
- Pulses per revolution (z)	: 30 for TM 104 to TM 108 60 for TM 109 to TM 113
Analog speed signal	
- Output voltage	: 0 to +10 V (= 0 to 100 % of selected speed range)
DC component (for temperature compensation)	: 6000 mV $\pm$ 10 mV / K at 23 °C

Figure C.3 Specifications of the speed sensor (Torquemaster TM 107)

9**SPECIFICATIONS****Torque measurement and display**

Measurement and display resolution : 1/10000 of rated torque
Display range : 5 digits with floating decimal point
Measurement accuracy : < 0.2 % of full-scale reading
Temp influence : 0.02 %/K

Speed measurement and display

Measurement and display resolution : 1 Hz with 60 pulses/rev. corresponding to 1 rpm
Display range : 5 digits
Accuracy : 0.1% of full-scale
Max. input frequency : 40 kHz

Figure C.4 Specifications of the torque and speed sensor measurement system (Vibrometer DUD 282)

044-40

FLOW DATA

DOCUMENT NO. 587

Tube Number: 044-40(GL,SA,ST,CA,TA)
Units : std ml/min

Metering Temperature: 70 deg.F
Metering Pressure : 14.70 psia

Std. Conditions: 1 atmos @ 70 deg.F
Tolerance : ±2% F.S.

SCALE READINGS AT CENTER OF FLOAT

Float Material:	Glass (GL)	Sapphire (SA)	Stainless Steel (ST)	Carboloy (CA)	Tantalum (TA)
Float Density :	2.53 g/ml	3.98 g/ml	8.04 g/ml	14.98 g/ml	16.58 g/ml

A I R

Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]
150 ---	23742	150 ---	30711	150 ---	43227	150 ---	66346
140 ---	22126	140 ---	28568	140 ---	41968	140 ---	61524
130 ---	20385	130 ---	26223	130 ---	38315	130 ---	55622
120 ---	18651	120 ---	23998	120 ---	35172	120 ---	50253
110 ---	16985	110 ---	21745	110 ---	31655	110 ---	44997
100 ---	15076	100 ---	19322	100 ---	28245	100 ---	39909
90 ---	13542	90 ---	17294	90 ---	25161	90 ---	35286
80 ---	11961	80 ---	15262	80 ---	22199	80 ---	30720
70 ---	10252	70 ---	13051	70 ---	19057	70 ---	26268
60 ---	8637	60 ---	11007	60 ---	16007	60 ---	21976
50 ---	6785	50 ---	8769	50 ---	12988	50 ---	17771
40 ---	5102	40 ---	6693	40 ---	10040	40 ---	15043
30 ---	3701	30 ---	4826	30 ---	7256	30 ---	10218
20 ---	2248	20 ---	2998	20 ---	4668	20 ---	5181
10 ---	791	10 ---	1208	10 ---	2182	10 ---	1842

W A T E R

Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]	Scale Readings	Flow [ml/min]
150 ---	541	150 ---	806	150 ---	1288	150 ---	1881
140 ---	508	140 ---	759	140 ---	1215	140 ---	1778
130 ---	470	130 ---	704	130 ---	1140	130 ---	1652
120 ---	431	120 ---	650	120 ---	1053	120 ---	1528
110 ---	395	110 ---	593	110 ---	965	110 ---	1392
100 ---	355	100 ---	530	100 ---	868	100 ---	1252
90 ---	313	90 ---	470	90 ---	771	90 ---	1117
80 ---	274	80 ---	411	80 ---	678	80 ---	981
70 ---	231	70 ---	355	70 ---	578	70 ---	848
60 ---	193	60 ---	296	60 ---	488	60 ---	714
50 ---	156	50 ---	240	50 ---	397	50 ---	536
40 ---	120	40 ---	185	40 ---	315	40 ---	457
30 ---	84	30 ---	130	30 ---	222	30 ---	336
20 ---	45	20 ---	80	20 ---	141	20 ---	212
10 ---	15	10 ---	30	10 ---	57	10 ---	96

Figure C.5 Specifications of the flowmeter (Aalborg 044-40 with the TA float)

MEASUREMENTS WITH A BRAKE MECHANISM

The experimental setups discussed in Chapters 9 and 10 show that the motor is connected directly to the expander to control the expander speed. However, a brake mechanism was initially installed in between the torque sensor and the expander as shown in Figure D.1. This was to ensure that the motor load is always larger than the expander output torque. The brake mechanism was built from bicycle brakes. The tests were carried out in two phases. The first phase was without the expander, to obtain the torque load of the brake. The second phase was with both the brake and the expander. The reading from the second phase is expected to be lower than the first one due to the expander output torque assisting the motor to overcome the torque load of the brake. The expander output torque was then obtained by taking the difference of the two readings.

Figure D.1 The experimental setup and the brake mechanism

However, the brake mechanism was found to limit the testing range of the setup since it creates too much noise at high speeds. In addition, it was suspected that the brake was not needed after all. Therefore, a preliminary test was carried out to compare the data from both the initial plan and the setup without the brake mechanism, which is the one shown in Chapters 9 and 10. The results are shown in Table D.1 and confirm that the brake is not needed since the readings obtained from the two setups are acceptably close.

Table D.1 Preliminary results to test the brake mechanism

Speed (rpm)	Brake Torq. (Nm)	Suct. P (bar g)	Brake + Expander Torq. from orig. setup (Nm)	Brake + Expander Torq. from setup w/o brake (Nm)	Difference (%)
36	-1.096	1	-0.983	-1.041	5.9
		1.5	-0.823	-0.976	18.6
54	-1.116	1	-1.004	-1.058	5.4
		1.5	-0.841	-0.968	15.1

DETAILED DERIVATIONS OF MODELS IN CHAPTER 11

In Chapter 11, two improved dynamics models are presented. The more detailed derivations of the models are presented here.

E.1. Geometrically Constrained Model with Inclusion of Vane Geometry Models

The models are derived based on Figure E.1. All the relevant information about the figure has been presented in Chapter 11.

Figure E.1 (a) Geometrical model of the RV expander with the vane thickness considered, (b) triangle ARV₁ and (c) triangle CRV₁

From the triangle ARV₁, the relationships expressed in Equations (E.1a) and (E.2a) can be obtained.

$$l_{RV_1} = \sqrt{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2} \quad (\text{E.1a})$$

$$\cos \theta_{ARV_1} = \frac{r_r + l_{ve}}{l_{RV_1}} \quad (\text{E.2a})$$

When the contact point is at V_2 , Equations (E.1a) and (E.2a) should be replaced by Equations (E.1b) and (E.2b).

$$l_{RV_2} = \sqrt{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2} \quad (\text{E.1b})$$

$$\cos \theta_{ARV_2} = \frac{r_r + l_{ve}}{l_{RV_2}} \quad (\text{E.2b})$$

From the triangle CRV_1 , Equations (E.3a)-(E.5a) are obtained.

$$\theta_{CRV_1} = \pi - \theta_{ARV_1} - \varphi_r \quad (\text{E.3a})$$

$$l_{CV_1}^2 = e^2 + l_{RV_1}^2 - 2el_{RV_1} \cos \theta_{CRV_1} \quad (\text{E.4a})$$

$$\cos \theta_{CV_1R} = \frac{l_{CV_1}^2 + l_{RV_1}^2 - e^2}{2l_{CV_1}l_{RV_1}} \quad (\text{E.5a})$$

If the contact point is at point V_2 , Equations (E.3b)-(E.5b) are used.

$$\theta_{CRV_2} = \pi + \theta_{ARV_2} - \varphi_r \quad (\text{E.3b})$$

$$l_{CV_2}^2 = e^2 + l_{RV_2}^2 - 2el_{RV_2} \cos \theta_{CRV_2} \quad (\text{E.4b})$$

$$\cos \theta_{CV_2R} = \frac{l_{CV_2}^2 + l_{RV_2}^2 - e^2}{2l_{CV_2}l_{RV_2}} \quad (\text{E.5b})$$

The tangential velocities at point V_1 have the relations expressed in Equations (E.6a)-(E.8a).

$$v_{V_1c} = l_{CV_1} \omega_c \quad (\text{E.6a})$$

$$v_{V_1r} = l_{RV_1} \omega_r \quad (\text{E.7a})$$

$$v_r = v_c \cos \theta_{CV_1R} \quad (\text{E.8a})$$

Combining (E.5a)-(E.8a) gives Equation (E.9a).

$$\omega_c = \omega_r \left(\frac{2l_{RV_1}^2}{l_{CV_1}^2 + l_{RV_1}^2 - e^2} \right) \quad (\text{E.9a})$$

Equation (E.9a) relates the angular velocity of the cylinder to the geometry, the position of the rotor and the angular velocity of the rotor. Substituting Equations (E.1a)-(E.5a) into Equation (E.9a) results in Equation (E.10a).

$$\omega_c = \omega_r \left(\frac{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2}{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \left(\frac{w_v}{2}\right) \sin \varphi_r \right)} \right) \quad (\text{E.10a})$$

If the contact point is at V₂, Equation (E.10a) is replaced by Equation (E.10b).

$$\omega_c = \omega_r \left(\frac{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2}{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r + \left(\frac{w_v}{2}\right) \sin \varphi_r \right)} \right) \quad (\text{E.10b})$$

Equations (E.10a) and (E.10b) can be generalized into Equation (E.10).

$$\omega_c = \omega_r \left(\frac{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2}{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right)} \right) \quad (\text{E.10})$$

Differentiating Equation (E.10) with time gives the angular acceleration of the cylinder as expressed in Equation (E.11).

$$\begin{aligned}
\alpha_c = \alpha_r & \left[\frac{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2}{(r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right)} \right] \\
& \left[\frac{\omega_r \left(2(r_r + l_{ve}) \frac{dl_{ve}}{dt} \left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right) \right) \right. \right. \\
& \left. \left. - \left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 \right) \left(2(r_r + l_{ve}) \frac{dl_{ve}}{dt} + e \left(\frac{dl_{ve}}{dt} \cos \varphi_r - \omega_r (r_r + l_{ve}) \sin \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \omega_r \left(\frac{w_v}{2}\right) \cos \varphi_r \right) \right) \right)}{\left((r_r + l_{ve})^2 + \left(\frac{w_v}{2}\right)^2 + e \left((r_r + l_{ve}) \cos \varphi_r - \frac{F_{v,n}}{|F_{v,n}|} \left(\frac{w_v}{2}\right) \sin \varphi_r \right) \right)^2} \right]
\end{aligned} \tag{E.11}$$

E.2. Theoretical Model with Vane Geometries and Force-Torque Interactions

To formulate the new model, the geometrical model shown in Figures E.2 and E.3 are employed. Information about the figures has been presented in Chapter 11.

Figure E.2 Geometrical models of (a) the RV expander and (b) the rotor

$$l_{RV_1}^2 = e^2 + r_c^2 - 2er_c \cos(\varphi_c + \theta_{S_0CS_1}) \quad (E.13a)$$

$$\cos \theta_{CV_1R} = \frac{l_{RV_1}^2 + r_c^2 - e^2}{2l_{RV_1}r_c} \quad (E.14a)$$

$$\begin{aligned} r_c \sin(\varphi_c + \theta_{S_0CS_1}) &= l_{RV_1} \sin(\varphi_r + \theta_{V_0RV_1}) \\ \rightarrow r_c \sin(\varphi_c + \theta_{S_0CS_1}) &= l_{RV_1} (\sin \varphi_r \cos \theta_{V_0RV_1} + \cos \varphi_r \sin \theta_{V_0RV_1}) \end{aligned} \quad (E.15a)$$

If the contact point is at the left hand side of the vane, Equations (E.13b)-(E.15b) should be used instead.

$$l_{RV_2}^2 = e^2 + r_c^2 - 2er_c \cos(\varphi_c - \theta_{S_0CS_2}) \quad (E.13b)$$

$$\cos \theta_{CV_2R} = \frac{l_{RV_2}^2 + r_c^2 - e^2}{2l_{RV_2}r_c} \quad (E.14b)$$

$$r_c \sin(\varphi_c - \theta_{S_0CS_2}) = l_{RV_2} (\sin \varphi_r \cos \theta_{V_0RV_2} - \cos \varphi_r \sin \theta_{V_0RV_2}) \quad (E.15b)$$

The unknown terms in Equation (E.15a) can be obtained from the triangle RV_0V_1 (Figure E.4(c)) and expressed in Equations (E.16a) and (E.17a).

$$\sin \theta_{V_0RV_1} = \frac{l_{R_1V_1}}{l_{RV_1}} \quad (E.16a)$$

$$\cos \theta_{V_0RV_1} = \frac{l_{RR_1}}{l_{RV_1}} \quad (E.17a)$$

$$l_{RV_1}^2 = l_{R_1V_1}^2 + l_{RR_1}^2 \quad (E.18a)$$

If the contact point is at the left hand side of the vane, Equations (E.16a)-(E.18a) are replaced by Equations (E.16b)-(E.18b).

$$\cos \theta_{V_0RV_2} = \frac{l_{RR_2}}{l_{RV_2}} \quad (E.16b)$$

$$\cos \theta_{V_0RV_2} = \frac{l_{RR_2}}{l_{RV_2}} \quad (E.17b)$$

$$l_{RV_2}^2 = l_{R_2V_2}^2 + l_{RR_2}^2 \quad (E.18b)$$

Assuming the total length of the vane from the base to the tip is d_v , the width of the vane tip is w_v and the tapered angle is θ_v , the geometrical properties expressed in Equations (E.19), (E.20a) and (E.20b) are found.

$$l_{RV_{b0}} = \sqrt{r_r^2 - \left(\frac{w_v}{2} + d_v \tan \theta_v \right)^2} \quad (E.19)$$

$$l_{R_1V_1} = \frac{w_v}{2} + (d_v - (l_{RR_1} - l_{RV_{b0}})) \tan \theta_v \quad (E.20a)$$

$$l_{R_2V_2} = \frac{w_v}{2} + (d_v - (l_{RR_2} - l_{RV_{b0}})) \tan \theta_v \quad (E.20b)$$

Combining (E.18a) and (E.20a) gives Equation (E.21a).

$$\begin{aligned} l_{RV_1}^2 &= \left(\frac{w_v}{2} + (d_v - (l_{RR_1} - l_{RV_{b0}})) \tan \theta_v \right)^2 + l_{RR_1}^2 \\ \rightarrow l_{RV_1}^2 &= \left(\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2 - 2 \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) l_{RR_1} \tan \theta_v + l_{RR_1}^2 \tan^2 \theta_v \right) + l_{RR_1}^2 \end{aligned} \quad (E.21a)$$

The solution of Equation (E.21a) is expressed in Equation (E.22a).

$$l_{RR_1} = \frac{2 \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v \pm \sqrt{4 \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2 \tan^2 \theta_v - 4 (1 + \tan^2 \theta_v) \left(\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2 - l_{RV_1}^2 \right)}}{2(1 + \tan^2 \theta_v)}$$

$$\rightarrow l_{RR_1} = \left(\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v \pm \sqrt{\frac{l_{RV_1}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} \right) \cos^2 \theta_v$$

If $\theta_v = 0$, $l_{RR_1} = \sqrt{l_{RV_1}^2 - \frac{w_v^2}{4}}$, therefore only the positive value is valid.

$$l_{RR_1} = \left(\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v + \sqrt{\frac{l_{RV_1}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} \right) \cos^2 \theta_v \quad (\text{E.22a})$$

If the contact point is at V_2 , Equation (E.22a) is replaced by Equation (E.22b).

$$l_{RR_2} = \left(\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v + \sqrt{\frac{l_{RV_2}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} \right) \cos^2 \theta_v \quad (\text{E.22b})$$

Substituting Equations (E.13a), (E.16a), (E.17a), (E.20a) and (E.22a) into Equation (E.15a) produces Equation (E.23a).

$$\begin{aligned} r_c \sin(\varphi_c + \theta_{S_0CS_1}) &= l_{RV_1} \left(\sin \varphi_r \frac{l_{RR_1}}{l_{RV_1}} + \cos \varphi_r \frac{l_{R_1V_1}}{l_{RV_1}} \right) \\ &\rightarrow r_c \sin(\varphi_c + \theta_{S_0CS_1}) = \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \cos \varphi_r \\ &\quad + \left(\frac{\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v}{\sqrt{\frac{e^2 + r_c^2 - 2er_c \cos(\varphi_c + \theta_{S_0CS_1})}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2}} \right) \cos^2 \theta_v (\sin \varphi_r - \tan \theta_v \cos \varphi_r) \end{aligned} \quad (\text{E.23a})$$

If the vane contact is at V_2 , Equation (E.23a) is replaced by Equation (E.23b).

$$\begin{aligned} r_c \sin(\varphi_c - \theta_{S_0CS_2}) &= l_{RV_2} (\sin \varphi_r \cos \theta_{V_0RV_2} - \cos \varphi_r \sin \theta_{V_0RV_2}) \\ &\rightarrow r_c \sin(\varphi_c - \theta_{S_0CS_2}) = - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \cos \varphi_r \\ &\quad + \left(\frac{\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) \tan \theta_v}{\sqrt{\frac{e^2 + r_c^2 - 2er_c \cos(\varphi_c - \theta_{S_0CS_2})}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2}} \right) \cos^2 \theta_v (\sin \varphi_r + \tan \theta_v \cos \varphi_r) \end{aligned} \quad (\text{E.23b})$$

The only unknown in Equations (E.23a) and (E.23b) is the angle of cylinder, φ_c . Therefore by solving these equations, the angle of cylinder when the vane is in contact with the vane slot can be calculated.

E.2.2. Exposed length of the vane

To find the exposed vane length at both sides of the vane, Equations (E.24) and (E.25) are needed.

$$l_{V_{b0}V_{b1}} = l_{V_{b0}V_{b2}} = \frac{w_v}{2} + d_v \tan \theta_v \quad (\text{E.24})$$

$$\sin \theta_{V_0RV_{b1}} = \sin \theta_{V_0RV_{b2}} = \frac{0.5w_v + d_v \tan \theta_v}{r_r} \quad (\text{E.25})$$

Figure E.5 The triangle RV_1V_{b1}

From the triangle RV_1V_{b1} (Figure E.5), the expression for the length of the exposed vane at its right hand side is obtained as expressed in Equation (E.25a).

$$l_{ve1}^2 = r_r^2 + l_{RV_1}^2 - 2r_r l_{RV_1} \cos(\theta_{V_0RV_{b1}} - \theta_{V_0RV_1})$$

$$\rightarrow l_{ve1}^2 = r_r^2 + l_{RV_1}^2 - 2 \left[\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) (\sin \theta_v \cos \theta_v l_{RV_{b0}} - l_{V_{b0}V_{b1}} \sin^2 \theta_v) \right. \\ \left. + \frac{w_v}{2} l_{V_{b0}V_{b1}} + (d_v + l_{RV_{b0}}) \tan \theta_v l_{V_{b0}V_{b1}} \right. \\ \left. + \sqrt{\frac{l_{RV_1}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} (l_{RV_{b0}} \cos^2 \theta_v - l_{V_{b0}V_{b1}} \sin \theta_v \cos \theta_v) \right] \quad (E.25a)$$

The length of the exposed vane at its left hand sight can be similarly derived and found to be as expressed in Equation (E.25b).

$$l_{ve2}^2 = r_r^2 + l_{RV_2}^2 - 2r_r l_{RV_2} \cos(\theta_{V_0RV_{b2}} - \theta_{V_0RV_2}) \\ \rightarrow l_{ve2}^2 = r_r^2 + l_{RV_2}^2 - 2 \left[\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) (\sin \theta_v \cos \theta_v l_{RV_{b0}} - l_{V_{b0}V_{b2}} \sin^2 \theta_v) \right. \\ \left. + \frac{w_v}{2} l_{V_{b0}V_{b2}} + (d_v + l_{RV_{b0}}) \tan \theta_v l_{V_{b0}V_{b2}} \right. \\ \left. + \sqrt{\frac{l_{RV_2}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} (l_{RV_{b0}} \cos^2 \theta_v - l_{V_{b0}V_{b2}} \sin \theta_v \cos \theta_v) \right] \quad (E.25b)$$

Differentiating Equation (E.25a) or (E.25b) with time gives the sliding velocity of the vane contact point, expressed in Equations (E.26a) and (E.26b).

$$\frac{dl_{ve1}}{dt} = \frac{2l_{RV_1} \frac{dl_{RV_1}}{dt} - \frac{(l_{RV_{b0}} \cos^2 \theta_v - l_{V_{b0}V_{b1}} \sin \theta_v \cos \theta_v)}{\sqrt{\frac{l_{RV_1}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2}} \frac{2l_{RV_1}}{\cos^2 \theta_v} \frac{dl_{RV_1}}{dt}}{2 \left[r_r^2 + l_{RV_1}^2 - 2 \left[\left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right) (\sin \theta_v \cos \theta_v l_{RV_{b0}} - l_{V_{b0}V_{b1}} \sin^2 \theta_v) \right. \right. \\ \left. \left. + \frac{w_v}{2} l_{V_{b0}V_{b1}} + (d_v + l_{RV_{b0}}) \tan \theta_v l_{V_{b0}V_{b1}} \right. \right. \\ \left. \left. + \sqrt{\frac{l_{RV_1}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2} (l_{RV_{b0}} \cos^2 \theta_v - l_{V_{b0}V_{b1}} \sin \theta_v \cos \theta_v) \right] \right]}$$

$$\rightarrow \frac{dl_{ve1}}{dt} = \frac{\omega_c e r_c \sin(\varphi_c + \theta_{S_0CS_1})}{l_{ve1}} \left(1 - \frac{(l_{RV_{b0}} - l_{V_{b0}V_{b1}} \tan \theta_v)}{\sqrt{\frac{l_{RV_1}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2}} \right) \quad (E.26a)$$

$$\frac{dl_{ve2}}{dt} = \frac{\omega_c e r_c \sin(\varphi_c - \theta_{S_0CS_2})}{l_{ve2}} \left(1 - \frac{(l_{RV_{b0}} - l_{V_{b0}V_{b2}} \tan \theta_v)}{\sqrt{\frac{l_{RV_2}^2}{\cos^2 \theta_v} - \left(\frac{w_v}{2} + (d_v + l_{RV_{b0}}) \tan \theta_v \right)^2}} \right) \quad (E.26b)$$

E.2.3. The gap between the vane sides and the vane slot opening tips

To help to formulate the calculations of the gap between the vane sides and the vane slot opening tips, a geometrical model when the vane is not in contact with the vane slot is shown in Figure E.6.

Figure E.6 (a) Geometrical model #2 (vane is not in contact with the vane slot), (b) triangle CRS_1 and (c) triangle RR_1S_1

To start the procedure, we first observe the triangle CRS_1 (Figure E.6). Applying the Cosine Rule to the triangle CRS_1 gives Equations (E.27a) and (E.28a).

$$l_{RS_1}^2 = e^2 + r_c^2 - 2er_c \cos(\varphi_c + \theta_{S_0CS_1}) \quad (\text{E.27a})$$

$$\cos(\varphi_r + \theta_{S_1RV_0}) = \frac{r_c^2 - e^2 - l_{RS_1}^2}{2el_{RS_1}} \quad (\text{E.28a})$$

Next, from the triangle R_1RS_1 , Equation (E.29a) can be obtained.

$$l_{R_1S_1} = l_{RS_1} \sin \theta_{S_1RV_0} \quad (\text{E.29a})$$

The gap between the right hand side of the vane and the right hand tip of the vane slot opening is finally expressed in Equation (E.30a).

$$d_{v1} = l_{R_1S_1} - \left(\frac{w_v}{2} + (d_v - (l_{RR_1} - l_{RV_{b0}})) \tan \theta_v \right) \quad (\text{E.30a})$$

A similar approach can be performed to find the gap between the left hand side of the vane and the left hand tip of the vane slot opening, resulting in Equations (E.27b)-(E.30b).

$$l_{RS_2}^2 = e^2 + r_c^2 - 2er_c \cos(\varphi_c - \theta_{S_0CS_2}) \quad (\text{E.27b})$$

$$\cos(\varphi_r - \theta_{S_2RV_0}) = \frac{r_c^2 - e^2 - l_{RS_2}^2}{2el_{RS_2}} \quad (\text{E.28b})$$

$$l_{R_2S_2} = l_{RS_2} \sin \theta_{S_2RV_0} \quad (\text{E.29b})$$

$$d_{v2} = l_{R_2S_2} - \left(\frac{w_v}{2} + (d_v - (l_{RR_2} - l_{RV_{b0}})) \tan \theta_v \right) \quad (\text{E.30b})$$

E.2.4. The dynamics of the cylinder and the rotor

To model the dynamics of the RV expander, the cylinder and the rotor must be analyzed separately. The forces and torques at the cylinder when the vane and the vane slot are in contact at V_1 are shown in Figure E.7.

Figure E.7 The forces and torques at the cylinder (vane is in contact at V_1)

The torque balance equation of the cylinder is expressed in Equation (E.31a).

$$I_c \alpha_c = F_{v,n} \cos \theta_v r_c \cos(\theta_{CV_1R} - \theta_{V_0RV_1}) + \frac{dl_{ve1}/dt}{|dl_{ve1}/dt|} F_{v,f} \sin \theta_v r_c \cos(\theta_{CV_1R} - \theta_{V_0RV_1}) + F_{v,n} \sin \theta_v r_c \sin(\theta_{CV_1R} - \theta_{V_0RV_1}) - \frac{dl_{ve1}/dt}{|dl_{ve1}/dt|} F_{v,f} \cos \theta_v r_c \sin(\theta_{CV_1R} - \theta_{V_0RV_1}) + \sum T_c \quad (E.31a)$$

where:

$$F_{v,f} = |F_{v,n}| \eta_v \quad (E.32)$$

If the contact point is at V_2 , Equation (E.31b) should be used instead.

$$I_c \alpha_c = F_{v,n} \cos \theta_v r_c \cos(\theta_{CV_2R} + \theta_{V_0RV_2}) - \frac{dl_{ve2}/dt}{|dl_{ve2}/dt|} F_{v,f} \sin \theta_v r_c \cos(\theta_{CV_2R} + \theta_{V_0RV_2}) - F_{v,n} \sin \theta_v r_c \sin(\theta_{CV_2R} + \theta_{V_0RV_2}) - \frac{dl_{ve2}/dt}{|dl_{ve2}/dt|} F_{v,f} \cos \theta_v r_c \sin(\theta_{CV_2R} + \theta_{V_0RV_2}) + \sum T_c \quad (E.31b)$$

The ΣT_c term consists of the torques acting on the cylinder which are not caused by the vane contact force nor the vane side friction. This may include, among others, the torques caused by the endface and the bearing frictions. Rearranging and generalizing Equation (E.31a) to be also applicable when the vane is in contact with the vane slot at V_2 and using Equation (E.32) result in Equation (E.33). In this equation, the contact points V_1 and V_2 have been replaced by a general name “cont”.

$$F_{v,n} = \frac{I_c \alpha_c - \sum T_c}{\left(\left(\cos \theta_v + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \sin \theta_v \right) r_c \cos \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0 Rcont} \right) + \frac{F_{v,n}}{|F_{v,n}|} \left(\sin \theta_v - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \cos \theta_v \right) r_c \sin \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0 Rcont} \right) \right)} \quad (E.33)$$

where: if the vane contact point is at V_1 , then $\frac{F_{v,n}}{|F_{v,n}|} = 1$. If the vane contact point is at

V_2 , then $\frac{F_{v,n}}{|F_{v,n}|} = -1$, and if the vane is not in contact with the vane slot, there is no

contact force and therefore $\frac{F_{v,n}}{|F_{v,n}|} = 0$

The forces and torques acting on the rotor are shown in Figure E.8.

Figure E.8 The forces and torques at the rotor (vane is in contact at V_1)

Based on Figure E.8, the torque balance equation of the rotor when the vane is in contact with the vane slot at V_1 can be expressed as shown in Equation (E.34a).

$$\begin{aligned}
I_r \alpha_r = & F_{P2} \left(l_{RV_{b0}} + \frac{l_{ve2}}{2} \right) - F_{P1} \left(l_{RV_{b0}} + \frac{l_{ve1}}{2} \right) - F_{v,n} \cos \theta_v l_{RR_1} + F_{v,n} \sin \theta_v l_{R_1 V_1} \\
& - \frac{dl_{ve1}/dt}{|dl_{ve1}/dt|} F_{v,f} \cos \theta_v l_{R_1 V_1} - \frac{dl_{ve1}/dt}{|dl_{ve1}/dt|} F_{v,f} \sin \theta_v l_{RR_1} + \sum T_r - T_{gen}
\end{aligned} \tag{E.34a}$$

where:

$$F_P = P l_{ve} l_{cham} \tag{E.35}$$

The ΣT_r term consists of all the torques acting on the rotor which are not caused by the vane contact force, the vane side friction, the fluid pressures acting on the vane or the load. This may include, among others, the endface and the bearing frictions. Rearranging and generalizing Equation (E.34a) to be also applicable when the vane contact is at V_2 and using Equation (E.35) give Equation (E.36). Points R_1 and R_2 have been generalized as R_{cont} in this equation.

$$F_{v,n} = \frac{-I_r \alpha_r - F_{P1} \left(l_{RV_{b0}} + \frac{l_{ve1}}{2} \right) + F_{P2} \left(l_{RV_{b0}} + \frac{l_{ve2}}{2} \right) + \sum T_r - T_{gen}}{(\cos \theta_v l_{RR_{cont}} - \sin \theta_v l_{R_{cont} V_{cont}}) + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v (\sin \theta_v l_{RR_{cont}} + \cos \theta_v l_{R_{cont} V_{cont}})} \tag{E.36}$$

To simplify the subsequent discussions, let us introduce the new parameters shown in Equations (E.37)-(E.40).

$$\begin{aligned}
R_{v,n,c} = & \left(\cos \theta_v + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \sin \theta_v \right) r_c \cos \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0 Rcont} \right) \\
& + \frac{F_{v,n}}{|F_{v,n}|} \left(\sin \theta_v - \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v \cos \theta_v \right) r_c \sin \left(\theta_{CcontR} - \frac{F_{v,n}}{|F_{v,n}|} \theta_{V_0 Rcont} \right)
\end{aligned} \tag{E.37}$$

$$T_c = \sum T_c \tag{E.38}$$

$$R_{v,n,r} = (\cos \theta_v l_{RR_{cont}} - \sin \theta_v l_{R_{cont} V_{cont}}) + \frac{dl_{ve}/dt}{|dl_{ve}/dt|} \eta_v (\sin \theta_v l_{RR_{cont}} + \cos \theta_v l_{R_{cont} V_{cont}}) \tag{E.39}$$

$$T_r = -F_{P1} \left(l_{RV_{b0}} + \frac{l_{ve1}}{2} \right) + F_{P2} \left(l_{RV_{b0}} + \frac{l_{ve2}}{2} \right) + \sum T_r - T_{gen} \tag{E.40}$$

Equations (E.33) and (E.36) calculate the contact force at the cylinder and the rotor, respectively. They are action and reaction forces. These forces are zero when the vane is not in contact with the vane slot. Substituting Equations (E.37)-(E.40) into Equations (E.33) and (E.36) and equating them give Equation (E.41).

$$\frac{I_c \alpha_c - T_c}{R_{v,n,c}} = \frac{T_r - I_r \alpha_r}{R_{v,n,r}} \quad (\text{E.41})$$

The conservation of energy equation before and after the vane hits the vane slot is expressed in Equation (E.42) with t_{imp} is the impact time between the vane and the vane slot. This parameter should be obtained experimentally. If necessary, coefficient of restitution can be introduced in Equation (E.42) by adding a term into T_r and T_c in Equations (E.38) and (E.40).

$$\begin{aligned} & \frac{1}{2} I_r \omega_{r1}^2 + \frac{1}{2} I_c \omega_{c1}^2 + \frac{1}{2} (\omega_{r1} + \omega_{r2}) t_{imp} T_r + \frac{1}{2} (\omega_{c1} + \omega_{c2}) t_{imp} T_c \\ &= \frac{1}{2} I_r \omega_{r2}^2 + \frac{1}{2} I_c \omega_{c2}^2 \end{aligned} \quad (\text{E.42})$$

where: $\omega_{r2} = \omega_{r1} + \alpha_r t_{imp}$

Finally, combining Equations (E.41) and (E.42) give the accelerations of the rotor and the cylinder as expressed in Equations (E.43) and (E.44). These equations should be used only when the vane is in contact with the vane slot. Else, the accelerations should be calculated using Equations (E.33) and (E.36) with the vane contact force, $F_{v,n}$, is set to zero.

$$\alpha_r = \frac{-2 \left(\omega_{r1} - \frac{R_{v,n,c}}{R_{v,n,r}} \omega_{c1} \right) + \frac{t_{imp}}{I_c} \frac{R_{v,n,c}}{R_{v,n,r}} \left(T_r \frac{R_{v,n,c}}{R_{v,n,r}} + T_c \right)}{t_{imp} \left(1 + \frac{I_r}{I_c} \frac{R_{v,n,c}^2}{R_{v,n,r}^2} \right)} \quad (\text{E.43})$$

$$\alpha_c = -\frac{I_r}{I_c} \alpha_r \frac{R_{v,n,c}}{R_{v,n,r}} + \frac{T_r}{I_c} \frac{R_{v,n,c}}{R_{v,n,r}} + \frac{T_c}{I_c} \quad (\text{E.44})$$