Buckling and energy absorption behavior of shell structure with a compliant core

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**NANYANG
TECHNOLOGICAL
UNIVERSITY**

**BUCKLING AND ENERGY ABSORPTION
BEHAVIOR OF SHELL STRUCTURE
WITH A COMPLIANT CORE**

YE LEI

SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

2012

**BUCKLING AND ENERGY ABSORPTION BEHAVIOR
OF SHELL STRUCTURE WITH A COMPLIANT CORE**

YE LEI

School of Mechanical and Aerospace Engineering

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ABSTRACT

This dissertation presents the study on buckling and energy absorption behavior of a thin-walled cylindrical shell with compliant core under axial compression. Previous studies related to this subject focused on its behavior with a fully or fixed amount of infill. The principle aim of current work is to study the effect of partially filled foam core on its behavior in initial buckling, post-buckling and energy absorption.

An energy-based technique is applied in initial buckling to obtain the theoretical solution of critical buckling load, for an axisymmetric mode with partially infill. This solution has been verified by a subsequent finite element analysis. A comprehensive parametric study indicates that the filler's inner radius should not be more than 10% of the shell radius for practical design purpose.

After that, a finite element analysis is performed for post-buckling. With an initial geometric imperfection added on the shell, the large discrepancy between theory and experiment is quantitatively resolved. It reveals that the actual critical buckling load is closely located in the numerical post-buckling region. Furthermore, the plateau load of such a structure with this specific imperfection is formulated. The result shows for a shell with thickness h , that the post-buckling dimple requires a compressive force proportional to $h^{2.5}$ to hold it in place.

For energy absorption, a theoretical model is proposed to predict its axisymmetric crushing behavior with partially infill by the energy balance method.

This solution has been verified by a subsequent finite element analysis and experiments. After that, axial crushing tests and corresponding finite element simulations are extended to study its non-axisymmetric behavior. Interaction effect between the tube and foam core is formulated by counting the shortening of the wavelength (H) in the axisymmetric crushing model, which is much simpler than previous results. The study indicated that the filler's inner radius should not be more than 50% of the shell radius in the design. Furthermore, a cylindrical shell with a diameter-to-thickness (D/h) ratio up to 660 is investigated in the experimental study for non-axisymmetric behavior.

The current study is valuable in assessing and understanding the deformation or failure mechanism of the buckling and energy absorption behavior. In addition, it can help engineers engaged in the design of such structures find optimal arrangements in terms of strength, energy absorption, weight and cost.

Keywords: Thin-walled tube; cylindrical shell; compliant core; elastic buckling; non-linear buckling; post-buckling behavior; energy absorption.

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NOTATION

CHAPTER 1

D	diameter
h	thickness
R	radius
σ_f	plateau stress of the foam core

CHAPTER 2

b	outer diameter of the circular tube; side length of the square tube
C	a constant in Eq. (2.19)
C_{avg}	a constant in Eq. (2.18)
D	diameter; flexural rigidity
E	Young's modulus of the shell
E_c	Young's modulus of the core
H	half plastic wavelength
h	thickness
k_e	spring constant
L	length
M_0	full plastic bending moment per unit length
m	specified axial half-wave number; eccentricity factor
n	specified circumferential full-wave number
P	axial load
P_c	critical axial load
P_m	mean crushing load
P_{mt}	contribution of the mean crushing load from the non-filled aluminium extrusion
q	load displacement
R	radius

δ	out-of-plane displacement
δ_e	effective crushing distance
ε	strain
λ	buckling wavelength parameter
λ_{cr}	buckling half wavelength divided by π
ν	Poisson's ratio of the shell material
ν_c	Poisson's ratio of the core
σ	stress
σ_0	critical buckling stress for an empty cylinder; flow stress of the tube
σ_{cr}	critical buckling stress for a cylinder with infill
σ_y	yield stress

CHAPTER 3

A	cross-sectional area
C	coefficient in Eq. (3.47)
D	flexural rigidity
E_f	Young's modulus of the foam core
E_s	Young's modulus of the shell
h	thickness of the shell
k_e	stiffness
L	length of one local buckling cell
L_s	length of the shell
N_x	edge force intensities in x -direction
N_y	edge force intensities in y -direction
N_{xy}	shear components of the edge force intensity
p_o	pressure
q	amplitude in Rayleigh-Ritz's trial solution
R	radius of the mid-plane; polar coordinate in radial direction

r	distance from the applied stress to the centre in radial direction
R_f	inner radius of the foam core
R_s	radius of the shell
T	polar coordinate in tangential direction
U	total potential energy
U_b	strain energy stored due to bending
U_r	strain energy stored in the elastic restraining infill
U_s	membrane strain energy due to stretching
u	displacement in axial direction
V	work done by compression
v	displacement in circumferential direction
w	deformation of local buckling cell in z -direction; displacement in the direction of the inward normal to the middle surface
X	coordinate in circumferential direction
x	coordinate in axial direction
Y	coordinate in the direction of the inward normal to the middle surface
y	coordinate in circumferential direction
Z	coordinate in axial direction
z	coordinate in the direction of the inward normal to the middle surface
α	elastic restraint existing portion
γ_{xy}	shear components of the mid-surface strain
ε_0	axial strain at critical buckling stress
ε_1	mid-surface strains in x -direction
ε_2	mid-surface strains in y -direction
ς	coefficient in Eq. (3.48)
η	percentage of the infill
ν_f	Poisson's ratio of the foam core

ν_s	Poisson's ratio of the shell
σ_0	critical buckling stress for a hollow cylinder; flow stress of the tube
σ_{cr}	critical buckling stress for a cylinder with infill
σ_x	stress in x -direction
σ_y	stress in y -direction
σ_θ	stress component in polar coordinate of tangential direction
Ψ	coefficient in Eq. (3.48)

CHAPTER 4

a	radius of the spherical shell
C_s	coefficient in Eq. (4.12)
D	diameter
D_1	diameter of the shell 1
D_2	diameter of the shell 2
d	axial displacement
E_f	Young's modulus of the foam core
E_s	Young's modulus of the shell
F	axial load
$F_{cr,exp}$	experimental buckling load
$F_{p,num}$	numerical post-buckling load
f	fraction of height where dimple occurs
h	thickness
K_p	buckling load prediction factor
k_1	parameter used to relate w_{C1} and w_{C2}
L_s	length of shell
n_1	number of completed cosine wave in circumferential direction
n_2	number of completed cosine wave in longitudinal direction
P	inward-directed radial force

P_c	critical axial load
P_f	interfacial force
P_m	mean crushing load
p_o	pressure
R	shell radius; polar coordinate in radial direction
r	dimple radius
s_1	amplitude of arc length in circumferential direction
s_2	amplitude of arc length in longitudinal direction
U	total potential of the system
U_{bs}	strain energy due to bending and stretching
U_r	strain energy stored in the foam core
u	shortening of the axial length
V	work done by applied force
w_0	dimple inwards deflection
$w_{c1,2}$	radial displacement in decayed cosine wave
w_i	imperfection introduced to the shell
x	coordinate in circumferential direction
y	coordinate in radial direction
z	(polar) coordinate in axial direction
β_1	a scalar and selected by the user (in circumferential direction)
β_2	a scalar and selected by the user (in longitudinal direction)
η	percentage of the infill
θ	polar coordinate in tangential direction
θ_1	circumferential angle
ν_f	Poisson's ratio of the foam core

ν_s	Poisson's ratio of the shell
ψ	dimple subtend angle

CHAPTER 5

a	arc segment portion of half plastic wavelength
b	coefficient in Eq. (5.23)
C_{1-5}	coefficients in Eq. (5.21 & 5.22)
d	axial displacement
E	Young's modulus; energy absorption
E_b	plastic energy stored due to bending
$\dot{E}_b$	rate of bending energy
E_f	energy store in the infill due to its volume reduction
$\dot{E}_f$	energy absorption rate in the infill due to its volume reduction
E_m	membrane strain energy due to stretching
$\dot{E}_m$	membrane strain energy rate due to stretching
F	axial load
F_0	average force of the transverse test result
F_{ave}	average axial load
F_{max}	maximum axial load
H	half plastic wavelength
h	shell thickness
ID	inner diameter
L	length of the shell
M_0	full plastic bending moment per unit length
m	eccentricity factor
m_f	mass of the foam core per unit length
m_s	mass of the shell per unit length

N_0	fully plastic membrane force per unit length
OD	outer diameter
P_m	mean crushing load
R	shell radius
R_f	inner radius of the foam core
r	instantaneous radius of the arc segment in half plastic wavelength
r_1	instantaneous radius of the arc segment in half plastic wavelength - element 1
r_2	instantaneous radius of the arc segment in half plastic wavelength - element 2
s	length variable
V_0	initial volume
V_{final}	final volume
$\dot{w}$	radial velocity of the tube wall
α	instantaneous centre angle of the arc segment in half plastic wavelength - element 1
$\dot{\alpha}$	instantaneous centre angle rate of the arc segment in half plastic wavelength - element 1
α_0	initial centre angle of the arc segment in half plastic wavelength – element 1
α_f	final centre angle of the arc segment in half plastic wavelength – element 1
β	instantaneous centre angle of the arc segment in half plastic wavelength - element 2
$\dot{\beta}$	instantaneous centre angle rate of the arc segment in half plastic wavelength - element 2

β_0	initial centre angle of the arc segment in half plastic wavelength – element 2
β_f	final centre angle of the arc segment in half plastic wavelength – element 2
ΔV	volume reduction
δ	crush distance
δ_{eff}	effective crush distance
ε_f	strain of the foam core
$\dot{\varepsilon}$	strain rate
$\dot{\varepsilon}_i$	principal strain rate
η	percentage of the infill; crushing efficiency
k	curvature
$\dot{k}$	rate of curvature
ν	Poisson's ratio
ρ	density
ρ_f	density of the foam core
ρ_{f0}	density of the foam base material
σ_0	flow stress of the tube
σ_f	plateau stress of the foam core
σ_y	yield stress
ϕ	angular variable

CHAPTER 6

a	test repetition – 1 st time
b	test repetition – 2 nd time
D	diameter of the shell

D_1	diameter of the shell 1
D_2	diameter of the shell 2
d	actual deformation
d_{max}	maximum deformation
E	energy absorption; Young's modulus
E_t	total energy absorption
F	crushing force
H	vickers pyramid number; half plastic wavelength
h	thickness of the shell
L	length of the shell
l	original length of the specimen
m_f	mass of the foam core
m_s	mass of the shell
S_e	stroke efficiency
x	crushing distance
z	coordinate in axial direction
η	percentage of the infill
θ	deformation angle
ν	Poisson's ratio
ρ	density
ρ_A	density of foam A
ρ_B	density of foam B
σ_u	ultimate stress
σ_y	yield stress

CHAPTER 7

D	diameter of the shell
-----	-----------------------

E_f	Young's modulus of the foam core
H	half plastic wavelength
h	thickness of the shell
η	percentage of the infill

CHAPTER 1

INTRODUCTION

“Human subtlety will never devise an invention more beautiful, more simple, or more direct than does Nature, because in her inventions, nothing is lacking and nothing is superfluous.”

Leonardo da Vinci (ca. 1500)

1.1 Background

Da Vinci is correct. Scientists take inspirations from the nature and emulate its models and systems to solve real time problems. People gave this kind of investigation a name - ‘Biomimics’. Over a long history, researches perform with this approach and solve lots of problems by learning from nature.

As far as the main content of this thesis is concerned, thin-walled tube with foam core is a perfect example of Biomimics. This structure has found wide practical applications over a long history, such as components in the rockets, aircrafts, submarines, silos and pressure vessels. They are the dominant structures in engineering due to light-weight and high strength. Moreover, such structures have good energy absorbing capacity. It is usually applied as an energy absorber to improve the structural crashworthiness and passenger safety of high-volume industrial products such as cars, trains and ships.

With the advancement of technology, engineers are increasingly interested in structures which are even more light-weighted with better performance (*e.g.*, higher strength and better energy absorption). A honeycomb- or foam-like cellular core is filled to support the thin-walled cylindrical shell, where the infill is usually termed as a compliant core (Dawson and Gibson, 2007). This idea is originated from nature where thin-walled cylindrical structures are often supported by a low density cellular core, such as in plant stems, porcupine quills, or hedgehog spines. The micrographs of such structures can be shown in Fig. 1.1. Scientists found this soft core can significantly increase its resistance to localized loading stress and meanwhile reduce the structure weight as compared to a hollow one (Karam and Gibson, 1995).

Fig. 1.1 Nature thin-walled cylindrical structures are often supported by a cellular core, such as in plant stems, porcupine quills, or hedgehog spines. (a) Micrographs of cross-section view of grass stem with a foam-like core, and (b) Zoom-in view of one portion (Karam and Gibson, 1995).

This principle was then adapted by engineering thin-walled tubes with better properties. A soft foam core is filled in the thin-walled tube to increase the structure strength and energy absorption capacity without too much increase in its total weight. Such composite structures are referred to as “shells with compliant cores”.

1.2 Objective

Previous studies on the effect of foam core in a thin-walled cylindrical shell have merely focused on its initial buckling and energy absorption with either a fully filled foam core or a fixed amount. This may not be the optimum arrangement as the core material around the central region plays a less role in resisting lateral movement of a shell during axial compression. The principal aim of this project is to study the effect of partially filled foam core in the area of initial buckling, post-buckling and energy absorption.

More specifically, previous studies found there is a large discrepancy of the initial buckling load between the theory and experiment, but few presents quantitative results. Besides, it is well known that the interaction between the tube and foam core plays an important role in its energy absorption performance. However, its quantitative estimation presented in the past is too complex. Furthermore, previous experimental studies of thin-walled tubes are limited with a small range of diameter-to-thickness ratio (D/h) of 10-450. It is not enough for many modern

applications as a thinner shell wall is receiving more attention. Therefore, the main contributions of current study intend to solve these stated issues.

1.3 Scope

This project was designed to understand the behavior of buckling and energy absorption of thin-walled cylindrical shell with foam core. Therefore, for the study of initial buckling, an energy-based technique was applied to obtain the theoretical solution for the critical buckling load in an axisymmetric mode. This solution has been verified by a subsequent finite element analysis. Parametric studies including core thickness, ratio of Young's modulus of the core and shell materials were also performed.

The minimum post-buckling load for this structure was also obtained from finite element analysis. By adding an initial geometrical imperfection in the shell, the finite element (F.E.) results were compared with test data. In addition, the F.E. results demonstrated the effect of foam core on the post-buckling behavior. Furthermore, the plateau load of the structure with this specific imperfection was formulated by energy method and solved under the assumption of *inextentional* deformation.

For the study in energy absorption, a theoretical model was proposed to predict the

axisymmetric crushing behavior of such structures with a partial infill. Using a modified superfolding model for shell and considering the volume reduction model for the foam core, the mean crushing force was predicted by the energy balance. This solution has been verified by a subsequent finite element analysis as well. A parametric study was carried out to examine the contribution of foam core plateau stress (σ_f), amount of filling and shell's radius-to-thickness ratio (R/h) on the axial crushing behavior of the structure. In addition, experimental study of the crushing behavior with axisymmetric mode was performed with aluminium 6061 tube filled with expanded and extruded polystyrene core, respectively.

Besides, quasi-static axial crushing tests were carried out on a thin-walled ($D/h > 600$) cylindrical aluminium alloy 3104-H19 shell filled with a hollow aluminium foam core to study the crushing behavior of non-axisymmetric mode. Percentage of infill, foam density and the shell diameter were the main parameters investigated. Based on the experiments, corresponding finite element simulations were performed as well for verification and further understanding the behind mechanics.

1.4 Outline

The structure of this dissertation is as follows. Chapter 2 reviews the relevant previous works on buckling and energy absorption behaviour of a thin-walled cylindrical shell with compliant core. After that, the following chapters are

structured according to the three stages in the load-displacement curve: initial buckling, post-buckling and energy absorption.

Study in initial buckling is presented in Chapter 3. The problem is formulated using the energy method, based on which the critical buckling stress is solved using the Rayleigh-Ritz approximation. A simplified formula based on numerical results is presented. After that, finite element analysis is performed to verify the derived formula.

Chapter 4 investigates the post-buckling behaviour in order to quantitatively resolve the large discrepancy between the buckling load in theory and experiment. Based on the test data, finite element analyses are carried out with an initial geometric imperfection on the shell. In addition, the plateau load is obtained with the energy method under the assumption of *inextentional* deformation.

The energy absorption behavior for an axisymmetric mode is presented in Chapter 5. A theoretical model is proposed to predict the crushing behavior of such a structure but with partial infill. Meanwhile, a finite element simulation using axisymmetric elements is presented to validate the analytical model.

Chapter 6 presents the study on energy absorption behavior for non-axisymmetric

crushed mode. Experimental study and corresponding finite element analysis are performed to investigate the axial crushing of partially filled thin-walled cylindrical shell, with D/h ratio in a wider range than those reported, up to 660.

Finally, a conclusion which sums up the work done in the current investigation is presented in Chapter 7 and recommendations are proposed for further work.

CHAPTER 2

LITERATURE REVIEW

*“Science is the study of what Is, Engineering builds what Will Be....
The scientist merely explores that which exists, while the engineer creates
what has never existed before.”*

Theodore VonKármán (c.a. 1957)

2.1 Introduction

This chapter reviews works in the fields of initial buckling, post-buckling and energy absorption. These three areas are divided according to the load-displacement curve shape of the axial crushed behaviour of a circular tube as shown in Fig. 2.1. Studies on both empty cylindrical shell and cylindrical shell with a compliant core are reviewed. For empty cylindrical shells, only post-buckling is reviewed. To the author's best knowledge, there has been no research in post-buckling of cylindrical shell with a compliant core so far, which will be one of the present research areas. Meanwhile, issues on material properties of compliant core are reviewed as well.

The objective of this chapter is to capture and summarize past and recent investigations that are related to buckling and energy absorption behavior on a thin-walled cylindrical shell with compliant core. Previous works related to the thin-walled cylindrical shell with and without the compliant core are reported as well. Comments are given on where the interesting points, questions and inconsistencies are.

Fig. 2.1 Load-displacement curve for the axial crushed behaviour of a circular tube (Lu and Yu, 2003). Three areas can be divided, *i.e.*, initial buckling, post-buckling and energy absorption.

2.2 Initial Buckling

Usually, *buckle* is defined as a structural response subjected to compression with visibly large displacement transverse to the load. For small load, this displacement recovers once the load is removed and this buckling process is elastic. Buckling happens in two manners. One is *stable* which means that the structure can sustain the increased load and maintain the original shape. The other one is *unstable*, in which the deformations increase instantaneously with catastrophic collapse.

The pioneer work of buckling study was conducted by Euler (1744) on the elastic stability of pin-ended slender columns in the 18th century. This is an ideal study, in

which the load is coaxially applied to a straight column. By using the load-displacement curve which is also referred to as the *equilibrium path*, the buckling behavior of the structure is studied. In the equilibrium path, the out-of-plane displacement, δ , is used as it is more descriptive of buckling than the load displacement, q .

Initially, there is no visible out-of-plane displacement when the applied load increases from zero, as shown in Fig. 2.2 (a). That indicates that the column is stable and there is no buckling. The P - δ equilibrium path is characterized by a vertical segment and named as *primary path*, as illustrated in Fig. 2.3 (a). This lasts until the load reaches the critical load when buckling suddenly takes place and the out-of-plane displacement grows instantaneously as shown in Fig. 2.2 (b).

Fig. 2.2 (a) Initial shape of column. There is no visible out-of-plane displacement, δ , when the applied load increases from zero to the critical point, P_c . (b) Buckling shape of column. The buckling suddenly takes place and the out-of-plane displacement, δ , grows instantaneously in either direction.

This critical load is named as *Euler load*, and the P - δ equilibrium path bifurcates into two symmetric *secondary paths* in Fig. 2.3 (a). The corresponding P - q load-displacement curve for these two paths is plotted in Fig. 2.3 (b).

Fig. 2.3 (a) P - δ equilibrium path. Initially, there is no visible out-of-plane displacement, δ , when the applied load increases from zero. Therefore, the P - δ equilibrium path is characterized by a vertical segment and named as *primary path*. This lasts until the load reaches the critical load, the buckling suddenly takes place and δ grows instantaneously in either direction. Thus, the P - δ equilibrium path bifurcates into two symmetric *secondary paths*. (b) The corresponding P - q load-displacement curve for these two paths (q is the load displacement as shown in Fig. 2.2b).

For buckling of a thin plate, it can usually withstand load greater than the critical load. Therefore, the post-buckling path is stable as illustrated in the P - δ equilibrium path of Fig. 2.4 (a). As shown in Fig. 2.4 (b) in the P - q curve, its stiffness is greatly reduced after buckling and this system is elastic.

Fig. 2.4 (a) P - δ equilibrium path of thin plate. For buckling of a thin plate, it usually can withstand load greater than the critical load. Therefore, the post-buckling path is stable. (b) The corresponding P - q load-displacement curve of thin plate. Its stiffness is greatly reduced after buckling. Therefore, this system is totally elastic (δ is the out-of-plane displacement, q is the load displacement as shown in Fig. 2.2).

For buckling of a shell, it behaves in a *snap-buckling mode* as the displacement is uncontrollable. This snap-buckling mode is illustrated in Fig. 2.5 (a). As the increasing load reaches the bifurcation point, there is an instantaneous snap increase in deflection to point 1 in order to accommodate the increasing load. Subsequently, there is an instantaneous snap decrease in deflection until point 2 to accommodate the subsequent decreasing in load. Then, it backs off to point 3 which is on the primary path. Due to the sudden reduction of load capacity, the post-buckling path is unstable and the stiffness is negative as shown in Fig. 2.5 (b).

Fig. 2.5 (a) P - δ equilibrium path of shell. For buckling of a shell, it behaves in a *snap-buckling mode* as the displacement is uncontrollable. Due to the sudden reduction of load capacity, the postbuckling path is unstable. (b) The corresponding P - q load-displacement curve of shell. Its stiffness is negative after buckling (δ is the out-of-plane displacement, q is the load displacement as shown in Fig. 2.2).

2.2.1 Empty cylindrical shell

Linear buckling analysis

One of the major buckling research areas of interest is buckling of a cylindrical shell. The buckling behavior of axially compressed cylindrical members was initiated by Lorenz (1908). In the study, the proposed model was a perfect elastic cylinder that was simply supported and a membrane pre-buckling stress uniform

distributed. He gave the solution to this problem in his later publication (Lorenz, 1911).

At that time, other forms of loading were studied as well. Southwell (1913) and von Mises (1914) solved the case under uniform lateral pressure. Flügge (1932) considered this under combined loading and bending. Donnell (1933) studied its buckling under torsional loading and established *Donnell's nonlinear theory* for circular cylindrical shell.

Not long afterwards, Timoshenko and Gere (1961) developed the classical elastic buckling theory. This classical theory is used to determine the critical buckling load under uniform axial compression. By linearizing the governing equation and making the problem as an eigenvalue problem, of which eigenvalues are the buckling loads and the eigenvectors are the corresponding modes, this theory gives,

$$\sigma_0 = \frac{E}{\sqrt{3(1-\nu^2)}} \left(\frac{h}{R} \right) \quad (2.1)$$

where E is the Young's modulus of the shell material, ν is the Poisson's ratio of the shell material, h is the uniform shell thickness and R is the radius of the shell. This buckling stress is adaptable for any assumed initial buckling mode.

Recently, Mandal and Calladine (2000) proposed a simplified form for the classical theory. By assuming a constant Poisson's ratio and replacing the well-known term $(1-\nu^2)$ by replaced by 1, the solution was simplified as,

$$\sigma_0 = 0.6E \left(\frac{h}{R} \right) \quad (2.2)$$

Buckling configuration

According to Bulson and Allen (1980), the initial buckling of a perfect cylindrical shell under uniform axial compression in small deflection range has three possible configurations as shown in Fig. 2.6.

Fig. 2.6 Three possible configurations of the initial buckling of a perfect cylindrical shell under uniform axial compression in small deflection range. (a) Ring (or axis symmetrical) buckling, (b) Chessboard (or checkerboard) buckling and (c) Diamond buckling (Bulson and Allen, 1980).

The first one is axi-symmetric or ‘ring’ buckling. Radial displacements of any cross-section are constant around the perimeter and move in the form of waves along the shell length.

The second one is ‘chessboard’ or ‘checkerboard’ buckling, in which the wave is moving along both the longitudinal and transverse direction of the shell. Therefore, the shell surface has the pattern of rectangular depressions and bulges.

The third one is ‘diamond’ buckling. This is because the shell cannot have infinitesimal deflections. After initial buckling, the cylindrical shell snaps into another equilibrium state and the buckle pattern becomes like ‘diamond’ shape.

Non-linear buckling analysis

Besides theoretical studies, massive experiments were also performed on cylindrical shell and some early works include Robertson (1928), Flügge (1932), Wilson and Newmark (1933), Lundquist (1933) and Donnell (1934). By comparing the observations with those theoretical results, they realized that the experiment buckling loads are always much lower than the theoretical ones, usually only 20-50% of their values (Thielemann, 1962; Babcock and Sechler, 1963; Tennyson, 1963; Almroth *et al.*, 1964; Horton and Durham, 1965). Fig. 2.7 shows the discrepancy between the prediction from classic theory and the experimental data from Brush and Almroth (1975) for cylinders with closed ends subjected to axial compression over a wide range of R/h ratio.

The much higher theoretical prediction of the buckling strength implies that the linear stability analyses were inadequate and large deflections should be taken into account. Flügge (1932) and Donnell (1934) proposed a preliminary theory of large deflection in buckling. Not long afterwards, the works carried out by von Kármán *et al.* (1940) and von Kármán and Tsien (1941) firstly made a breakthrough in this field. In their study, an accurate solution for geometrically non-linear analysis was given. Moreover, they used a column which was restrained by non-linear spring in

Fig. 2.7 The discrepancy between the prediction from classic theory and the experimental data for cylinders with closed ends subjected to axial compression over a wide range of R/h ratio provided from Brush and Almroth (1975). It revealed that the experiment buckling loads are always much lower than the theoretical ones, usually only 20-50% (Mandal and Calladine, 2000).

lateral direction to simulate a cylindrical shell under uniform compression involving strongly non-linear features.

Besides, several attempts were made to explain the discrepancy between the theoretical prediction and experimental results as well. In 1945, Koiter developed a general non-linear elastic stability theory stating that the behavior of a cylindrical shell under uniform compression was essentially non-linear, and there was an unstable post-buckling equilibrium path with a minimum post-buckling load after the initial critical buckling load.

Hoff (1966) summarized all the case studies with various boundary conditions. It was found that those without constraints in tangential displacement for the boundary conditions had a buckling strength of only 38% from the classical prediction, which was also the lowest value in all cases. Besides, those with radial displacement free for the boundary conditions were around 50% of the classical prediction.

In engineering design, Kollár and Dulácska (1984) and Kenny (1984) recommended a value of 30-40% of theoretical values. This is usually known as safety factor, or “knock down” factor. This knock-down factor is usually used to estimate the actual capacity.

Recently, Mandal and Calladine (2000) carried out experimental investigations on self-weight buckling of open-topped cylindrical shells. Their results as well as previous test data revealed that the buckling load was proportional to $(h/R)^{1.5}$ instead of $(h/R)^{1.0}$ as predicated from the classical theory when other conditions are equal.

Studies on imperfection sensitivity

In practice, initial imperfections produced during the manufacturing process are unavoidable and would influence the buckling load. The imperfection affects the actual buckling load as compared with theoretical value and therefore its effect should be clarified.

Koiter (1945) found that the critical buckling load was reduced with the presence of initial geometric imperfections and used ‘imperfection sensitivity’ to describe this phenomenon. He found that even for a very small amount of geometric imperfection on the shell surface, such as 20% of the wall thickness from the ideal, the peak buckling load can be reduced by 50%. This was also confirmed by Donnell and Wan (1950).

With the presence of initial geometric imperfections, theoretical study was usually performed by two approaches. The first approach was deterministic. By using the Donnell-type theory together with Galerkin approximations of the solution, an asymptotically accurate estimate of the buckling load was obtained (Koiter, 1963; Arbocz and Babcock, 1969; Hoff and Soong, 1969).

The other was probabilistic approach. By performing series expansion of the experimental measurements, it treated the Fourier coefficients as random variables, after which the buckling load was obtained based on a Cholesky decomposition of the covariance matrix of the Fourier coefficients (Elishakoff, 1985; Elishakoff and Arbocz, 1985).

Moreover, research has shown that the difference in buckling load between the experimental and the theoretical one was not only due to the imperfections of geometry, but also due to imperfections of material, boundary conditions as well as variation of thickness and applied axial load. (Arbocz and Babcock, 1969;

Elishakoff, 2000; Arbocz, 2000; Arbocz and Starnes Jr., 2002; Elishakoff *et al.*, 2001).

However, as stated by Schenk and Schueller (2003), these analyses can only provide limited capability in modeling more complex shell structures, while a realistic description of more general and combined imperfection patterns in a rational probabilistic context was elusive. Therefore, the design process was still under case by case situation nowadays. This is the exact the reason why the European standard on shell buckling (ENV 1993-1-6, 1999) requires that when a geometrically nonlinear shell analysis with explicit representation of imperfections is used for the design, a range of potentially damaging imperfection forms should be explored.

Recently Khamlichi *et al.* (2004) studied shell surface with the presence of a localized axisymmetric imperfection. The experiments showed that critical buckling load was much less than that with distributed defects. Meanwhile, they also proposed an analytic model and suggested that the imperfection to be introduced into the model be obtained from the measurement.

2.2.2 Cylindrical shell with compliant core

Buckling behavior

In nature, thin-walled cylindrical structures are often supported by a low-density cellular core, such as in plant stems, porcupine quills or hedgehog spines. This cellular core can significantly reduce the weight and increases the resistance of

localized loading stress as compared to a hollowed one. Such structures are usually referred to as “core-rind” structures by biologists (Karam and Gibson, 1995; Niklas, 1992).

With an increased interest in light-weight and high strength structures, the idea from nature was adapted to engineering in which a compliant core was filled into the thin-walled cylindrical shell. The ranges of the radius to thickness ratio, R/h , for both natural and engineering structures are shown in Fig. 2.8 (Karam and Gibson, 1995).

Fig. 2.8 Ranges of the radius to thickness ratio, R/h , for both natural and engineering structures from Karam and Gibson (1995).

Pioneering investigations of thin-walled members filled with compliant core started in the mid-20th century, mostly inspired by the progress of solid-propellant rocket motor (Obrecht *et al.*, 2006). However, there were few explorations on the effect of

core material as well as amount of infill. At that time, researches mainly focused on the stabilizing effect of the low modulus core in an isotropic shell.

Under uniform axial compression, Seide (1962) used shell equilibrium equations and treated the compliant core as a spring constant stiffness to study the thin-walled cylindrical shell with a core modulus lower than that of the shell, while Yao (1962) used stress functions in the same problem.

In parallel to the theoretical studies, experiments were conducted by Goree and Nash (1962). Three kinds of foams were filled into the thin-walled stainless steel shell to investigate buckling behaviors under both axial compression and uniform radial-band loading. The results revealed that the buckling strength was increased with infill and this increase was more significant with a larger R/h ratio.

Besides uniform axial compression, Seide (1962) studied the same structure under both uniform radial pressure and axial loading. Yao (1965) gave a solution when the structure was under the pressure in circumferential direction as well. Brush and Almroth (1962) considered this under different situations, *i.e.* axial loading plus uniform radial pressure, axial loading plus axially varying radial pressure and axial loading plus circumferential pressure.

More recently, studies of buckling of a thin-walled cylindrical shell with foam core have led to some fruitful results. Karam and Gibson (1995) found that this structure can have a higher buckling load as compared to a hollowed one, at no extra weight

and radius. Their analysis was based on the classical buckling theory of a hollow cylindrical shell given by Timoshenko and Gere (1961). By treating the compliant core as a two-dimensional elastic foundation in the structure, they derived a theoretical solution for axisymmetric elastic buckling as,

$$\sigma_{cr} = \sqrt{3(1-\nu^2)}\sigma_0 f_1 \quad (2.3)$$

where, σ_0 is the buckling stress for an empty cylinder as given in Eq. (2.1) and

$$f_1 = \frac{1}{12(1-\nu^2)} \frac{R/h}{(\lambda_{cr}/h)^2} + \frac{(\lambda_{cr}/h)^2}{R/h} + \frac{2(E_c/E)}{(3-\nu_c)(1+\nu_c)} (\lambda_{cr}/h)(R/h) \quad (2.4)$$

in which E_c is the Young's modulus of the core, ν_c is the Poisson's ratio of the core and λ_{cr} is the buckling half wavelength divided by π with the expression of,

$$\lambda_{cr} = \left[\frac{(3-\nu_c)(1+\nu_c)}{12(1-\nu^2)} \right]^{1/3} \left[\frac{E}{E_c} \right]^{1/3} h \quad (2.5)$$

Good agreements were shown between previous experiments and this theoretical analysis.

Ghorbanpour *et al.* (2007) investigated the elastic stability of cylindrical shell with an elastic core under axial compression. They found that such a structure can significantly improve the elastic stability and reduce the weight, compared with an empty cylindrical shell of the same weight and radius. By using the energy method, they derived the critical axial load P_c as,

$$P_c = 2\pi R \cdot \left[\frac{D}{\bar{m}^2} \left(\bar{m}^2 + \frac{n^2}{R^2} \right)^2 + \frac{(1-\nu^2)}{R^2} C \frac{\bar{m}^2}{\left(\bar{m}^2 + \frac{n^2}{R^2} \right)^2} + \frac{k_e}{\bar{m}^2} \right] \quad (2.6)$$

where D is the flexural rigidity of the shell, $D = Eh^3/12(1-\nu^2)$, m and n are the specified axial half-wave number and the circumferential full-wave number, respectively; $\bar{m} = m\pi R/L$, $C = Eh/(1-\nu^2)$ and k_e is the spring constant which is given as follows,

$$k_e = \frac{2E_c}{(3-\nu_c)(1+\nu_c)} \cdot \frac{1}{\lambda} \quad (2.7)$$

where λ is the buckling wavelength parameter ($\lambda = 1/m\pi$). Compared with the results of Karam and Gibson (1995), reasonable agreement was achieved.

In 2000, Loh *et al.* studied the features of concrete as an infill in a steel hollow circular column, which was used as a structural member in high-rise buildings. There are two purposes for the concrete in such a structure: one is to resist the failure by the loading while the other is to prevent the inward buckling of the steel tube; thus the infill was treated as “rigid” infill in this situation.

Soon afterwards, Bradford *et al.* (2006) presented a solution to the buckling of a thin-walled elastic circular tube containing a “rigid” infill by using an energy technique,

$$\sigma_{cr} = \frac{E}{\sqrt{1-\nu^2}} \left(\frac{h}{R} \right) \quad (2.8)$$

Comparing this with Eq. (2.1), the rigid in-fill enhances the buckling stress of the tube by a factor of $\sqrt{3}$.

Core materials and their properties

Cellular material is used as the compliant core due to its high specific stiffness, specific strength and good thermal insulation properties compared with other materials. It is all around the nature, such as wood, sponge, sea foams and bones (Fig. 2.9). Such structures have high strength but low density due to the pores and are popular as a structural element.

However these pores could possibly reduce the strength as imperfections where the crack grows. Therefore the material property should be carefully studied in the design. Porosity or void fraction is used to measure the void spaces in the material, which is a fraction of the volume of voids over the total volume, between 0–1, or as a percentage between 0–100%.

Fig. 2.9 Cellular material is spread all around the nature, such as (a) softwood, (b) sponge and (c) bones. There are a lot of pores in the structures. It is found that these structures have high strength but low density due to the pores and are often used as a structural element.

Gibson and Ashby (1997) investigated a wide range of cellular material's structures and properties. Later, Lu and Yu (2003) further studied three common types of cellular material which are honeycomb, foam and wood. The corresponding cell

structure, relative density, stress-strain relation and plateau stress were presented in detail.

HONEYCOMB is a common type of cellular material. It is a two-dimensional repetitive structure as shown in Fig. 2.10. The analysis is therefore relatively easier compared with other three-dimensional structures, *e.g.* foam. The shape of honeycomb cell is usually hexagonal (Fig. 2.10) which could also be triangular, square, rhombic or circular as described in Chung and Waas (2002). The materials of the cell can be polymer, metal, ceramic or paper.

Fig. 2.10 Honeycomb structure in (a) Diagrammatic sketch and (b) Actual aluminium honeycomb panel. It is a two-dimensional repetitive structure. The shape of honeycomb cell is usually hexagonal which can also be triangular, square, rhombic or circular as described in Chung and Waas (2002).

FOAM is another typical type of cellular material. It is a three-dimensional and usually can be divided into two kinds based on the microstructure (Gibson and Ashby, 1997). The one with an interconnected network of microstructure is named open-cell foam while the one consists of plates is called closed-cell foam. In open-cell foam, the cell walls are broken, as shown in Fig. 2.11 (a). The strength is not

high. While for the closed-cell foam in Fig. 2.11 (b), the majority of the cells are not broken and they join together firmly to increase the strength.

Fig. 2.11 Foam usually can be divided into two kinds based on its microstructure (Gibson and Ashby, 1997). (a) The one with an interconnected network of microstructure is named as open-cell foam. (b) Microstructure consists of plates is called closed-cell foam.

The investigation on foam started around 1980s, with the introduction of new techniques for making artificial foam (Hilyard, 1982). After that, massive studies of the foam were conducted. Recently work includes Gibson and Ashby (1997) in which a systematic study on the mechanical behavior of various foams was performed. A good summary of the manufacturing techniques of the foam is also reported by Banhart (2001).

A new class of foams, metal foam, has gained popularity in engineering applications. It has the advantages of high strength, high energy absorption capacity and lightweight. Aluminium and nickel are commonly used as the material. A summary of metal foams were provided by Ashby *et al.* (2000).

A typical stress-strain curve for metal foam under axial compression is shown in Fig. 2.12. There are three stages: Initially, it is linear elastic when the strain is small. After yielding, the foam begins to compress at a fairly constant stress till 50-70% of strain, which is the region of plateau. Finally, the voids in metal foam are compacted, which results in the sharp increase in the stress.

It should be noted that the material can continuously absorb a large amount of energy in the plateau stage and is quite suitable as an energy absorber. However, its strength is usually small and should cooperate with other components for more practical applications. In other words, it has an efficient mechanism as a filler but needs support in gaining strength. Therefore, the foam is often chosen as the core material which biomimicks the “core-rind” structure in the nature.

Fig. 2.12 A typical stress-strain curve for metal foam under axial compression.

WOOD is a cellular material in nature (Fig. 2.13a with its microstructure shown in Fig. 2.9a). According to Gibson and Ashby (1997), a polar coordinate system can be used as shown in Fig. 2.13 (b). It can be noticed that the microstructure in the axial direction has a cross-section shape of hexagon and the cells are stretched as compared with the other two planes. Therefore, the compressive strength in radial direction is usually 40% higher than the other directions.

Fig. 2.13 (a) A tree trunk and (b) its schematic diagram with a polar coordinate system (Lu and Yu, 2003). It can be noticed that the array of cell in radial direction is smaller and more rectangular than other directions. Therefore, the compressive strength in radial direction is usually 40% higher than the other directions.

2.3 Post-buckling

2.3.1 Empty cylindrical shell

In the practical use of light-weight structure, safety is an important design consideration. Examination on initial buckling is not enough and post-buckling behavior is equally important. Post-buckling is a phenomenon when structures continue to carry loads after buckling. This post-buckling load may be higher, lower or unchanged as compared with the critical buckling load. Thus, a post-buckling analysis actually is a continuation of a nonlinear buckling analysis.

Pioneer work in this area goes back to 1940s with the nonlinear post-buckling theory introduced by von Kármán *et al.* (1940) and von Kármán and Tsien (1941). By applying Ritz method to Donnell non-linear theory on axial compression of thin cylindrical shell, they argued that the behavior of a cylindrical shell under axial compression is essentially nonlinear, and there is an unstable post-buckling equilibrium path with a minimum post-buckling load after the initial critical buckling load.

In order to determine the smallest post-buckling load, Koiter (1945) and Budiansky and Hutchinson (1966) extended the pioneer work to include the initial post-buckling behavior. The investigations tend to question the minimum post-buckling load as the measure of load carrying capacity. However, Hoff *et al.* (1966) found that it approaches to zero as the number of unknown parameters in Ritz method tends to infinity. Thus, the previous thoughts were untenable.

A more accurate solution was provided by Thielemann and Esslinger (1967, 1967) and Esslinger and Geier (1972) by applying Galerkin method to the Donnell non-linear theory for the post-buckling calculation. Their results matched the experiment well. Based on their method, Yamaki (1984) studied the clamped cylindrical shell under combined loading.

Besides evaluation of the ultimate strength in the post-buckling, effects of imperfection on the buckling behavior were studied. Sheinman and Simites (1983) studied the buckling and post-buckling behavior of a geometrically imperfect thin-

walled cylindrical shell under uniform axial compression. Their analysis investigated the behavior of all the pre-limit, limit and post-limit point. It revealed that the imperfection sensitivity decreases with increasing radius to thickness ratio and length to radius ratio.

By using an improved simulation technique with high computational efficiency, Li *et al.* (1995) investigated the buckling problem of a column resting on an elastic foundation with random initial geometric imperfection under a random axial compression. In this way, the probability of the structure failure was obtained and the result can be used directly in the design.

Recently, self-weight buckling study by Mandal and Calladine (2000) introduced a specific initial imperfection which is a localized initial inward displacement into the perfect shell. A post-buckling dimple (a large inward displacement) was found both in their experiment as well as in finite element analysis. Thus they assumed that there was a post-buckling load plateau and stated that the inward displacement curve governed the experimental behavior.

A new effective analytical approach was advanced by Shen and Chen (1988, 1990). A boundary layer theory was proposed to study the buckling of thin shell which includes initial geometric imperfection. The idea originated from Reissner (1912), who pointed out the existence of a boundary layer in the bending of thin shells. Using this creative method, the nonlinear pre-buckling deformation, the nonlinear large deflection in the post-buckling range and initial geometric imperfection could

be considered simultaneously. Based on this boundary layer theory, Shen and Chen (1990) analyzed the post-buckling behavior of a circular cylindrical shell under combined external pressure and axial compression using a singular perturbation technique.

The approaches for structural buckling studies which use column, plate, shell or higher order theories are not enough due to the complexity of the structure construction. By modeling the structure as a three-dimensional elastic body, the accuracy of these approaches can be assessed. The finite element analysis has been performed by many researchers to simulate the analysis, understand the mechanism of the buckling phenomena and verify the analytical results.

Endou *et al.* (1976) implemented fundamental study of initial post-buckling in finite element analyses of axisymmetric shells. Zhu *et al.* (2002) conducted several specific shells by means of ABAQUS. The results showed the existence of a post-buckling “plateau” but it is not precisely constant. It decreases a little as the displacement increases.

2.4 Energy Absorption

2.4.1 Empty cylindrical shell

Nowadays, various transportation means become the major tool for human’s travel. There is an increasing interest in improving the passenger safety of high-volume industrial products such as cars, trains and ships as well as the structural crashworthiness. Furthermore, it is desirable to employ light-weight and high

energy absorbing capacity vehicle from the point view of economy (Lu and Yu, 2003). It was found that thin-walled tubes fulfill all the requirements and have been widely applied as energy absorbers to improve the passenger safety of high-volume industrial products and structural crashworthiness. Thus, numerous theoretical and experimental studies on this topic were carried out over the last several decades.

Buckling mode

Experiments by Timoshenko and Gere (1961) and Kollár and Dulácska (1984) observed that thick, low modulus cylindrical shells under axial compression buckle axisymmetrically while thin, high modulus cylindrical shells under axial compression buckle non-axisymmetrically.

In fact, according to Lu and Yu (2003), there are five buckling modes when a cylindrical shell is under axial compression. The exact type of mode is dependent on the ratio of the diameter over thickness (D/h) and the ratio of the length over thickness (L/h).

The first mode is called '*Axisymmetric*' mode. It is known as 'ring' mode or concertina mode. It occurs when D/h is less than 50 and L/h is less than 2.

The second mode is called '*Non-symmetric*' mode. It is known as 'diamond' mode and characterized by the number of lobes. Usually, the number of lobes is two to five in practical cases. It occurs when D/h is greater than 80.

The third mode is called '*Mixed*' mode. That means the cylindrical shell starts to collapse with 'ring' mode and then switches to 'diamond' mode. It occurs when D/h is less than 50 and L/h is larger than 2.

The fourth mode is '*Euler*' mode. It occurs for the long cylindrical shell and the last mode is '*Other*' mode. It is the mode not included in the above.

The classification of these five modes is based on a large number of experiments with various dimension of cylindrical shell. It also depends on the material of cylindrical shell. Fig. 2.14 shows the mode classification of aluminium cylindrical shell by Guillow *et al.* (2001).

Fig. 2.14 Mode classification chart for of aluminium cylindrical shell by Guillow *et al.* (2001). There are five buckling modes when a cylindrical shell under axial compression and the type of mode is dependent on the ratio of the diameter and thickness (D/h) and the ratio of the length and thickness (L/h).

Studies

Pioneer study by Alexander (1960) found that axial crushing of thin-walled circular mild steel tubes was an excellent mechanism for energy absorption. In his experimental study, the metal tubes has the diameter to thickness ratio of $D/h = 29-89$. In addition, he was also the first to provide an analytical model to derive the mean crushing load (P_m) with the axisymmetric collapse mode.

Fig. 2.15 Theoretical model for axisymmetric collapse by Alexander (1960) (Lu and Yu, 2003). An axisymmetric fold pattern was adopted where the tube walls were considered as straight-line elements which folded either outward or inward at stationary circumferential hinges.

In his theoretical model, an axisymmetric fold pattern (refer to Fig. 2.15) was adopted where the tube walls were considered as straight-line elements which folded either outward or inward at stationary circumferential hinges and one fold forms with three circumferential plastic hinges. The mean crushing force was obtained as,

$$\frac{P_m}{M_0} = 20.73 \left(\frac{D}{h} \right)^{0.5} + 6.283 \quad (2.9)$$

where $M_0 = \sigma_y h^2 / 4$ is the full plastic bending moment per unit length, σ_0 is the flow stress of the tube, h is the tube wall thickness and D is the tube outer diameter. In addition, the half plastic wavelength, H (refer to Fig. 2.15) was determined as,

$$H = 0.95 \sqrt{Dh} \quad (2.10)$$

This simple formulation predicted the main features of ring mode and achieved good agreements with his experimental results. Many subsequent researches were based on this model.

The first empirical solution for the non-symmetric folding mode was presented by Pugsley and Macaulay (1960) which considered the diamond mode,

$$\frac{P_m}{\sigma_y \pi Dh} = 10 \frac{D}{h} + 0.13 \quad (2.11)$$

where σ_y is the yield stress. Johnson *et al.* (1977) developed an equation to predict average axial crush force based on the actual geometry of diamond mode folding. However, their models do not have good agreement with test results. Soon after, Johnson and Reid (1978) studied axially crushed thin-walled circular metal tubes with $D/h < 100$ using various end fixtures. It revealed that the tubes can be forced to buckle progressively, invert externally or internally, and split into curls or flat strips.

Not long afterwards, Abramowicz and Jones (1984b, 1986) conducted axial compression tests on a range of thin-walled circular and square steel tubes with $D/h = 9-65$. In addition, theoretical study was performed as well for both axisymmetric

and non-symmetric modes. An important concept of effective crushing distance, δ_e , was introduced and an improved model was proposed with a fold consisted of two oppositely curved arcs joining together with equal radii segments of length H . They found that the deforming tube wall also bends in the meridian direction instead of being a straight line and presented the mean crushing force for the axisymmetric mode as,

$$\frac{P_m}{M_0} = 20.79 \left(\frac{D}{h} \right)^{0.5} + 11.90 \quad (2.12)$$

Reasonable agreement was obtained between this prediction and their experimental results for steel tubes. For the non-symmetric mode, by taking into account effective crushing distance and material strain rate, the average crushing force was developed as,

$$\frac{P_m}{M_0} = 86.14 \left(\frac{D}{h} \right)^{0.33} \quad (2.13)$$

Further modifications were performed by Grzebieta (1990, 1990 and 1996) for both axisymmetric and non-symmetric modes. In the study of axisymmetric mode, a fold model consisted of three equal length segments with two equal radius curves connected by a straight line was adapted. Based on the equilibrium of the external work done with the internal energy from horizontal, inclined and travelling plastic hinges as well as stretching of the metal, the detailed force-displacement curve can be worked out, instead of only the average force. However, the expression for the average crush force was complicated. For the non-symmetric mode, the folds were considered as a half-diamond mechanism. Meanwhile, static and dynamic tests were carried out on steel tubes with $D/h=30-300$.

The aforementioned models of axisymmetric mode assumed the wall folded either outwardly or inwardly. In fact, experimental observations for the axisymmetric collapse mode showed that it deforms both inwardly and outwardly. To account for this, Wierzbicki *et al.* (1992) introduced a parameter m , known as the eccentricity factor, to define the outward portion over the whole wavelength using a superfolding element method. By considering energy rate equations, not only the mean crushing force can be predicted, but also the force-displacement history. In addition, the occurrence of two peak forces during a single fold formation can be explained and predicted as well. For the stationary hinge model,

$$\frac{P_m}{M_0} = 22.27 \left(\frac{D}{h} \right)^{0.5} \quad (2.14)$$

and for the moving hinge model,

$$\frac{P_m}{M_0} = 31.74 \left(\frac{D}{h} \right)^{0.5} \quad (2.15)$$

This work has been improved by Singace *et al.* (1995) with a global energy balance considered for the axisymmetric mode. With a constant value of m ($m=0.65$), the mean crushing force is expressed as,

$$\frac{P_m}{M_0} = 22.27 \left(\frac{D}{h} \right)^{0.5} + 5.632 \quad (2.16)$$

This result showed good agreement with reported experimental data. After that, Singace and Elsobky (1996) also investigated the non-symmetric mode case. With a small range of tests on circular metal tubes, the factor m was approximately equal to 0.65 as well.

Not long afterwards, by Guillow *et al.* (2001) conducted a number of experiments on axial crushing of aluminium tubes which covered a large range of $D/h=10-450$. The empirical formula showed that the normalized mean crushing load, P_m/M_0 , is proportional to $(D/h)^{0.33}$ instead of previous predictions of $(D/h)^{0.5}$ in a larger range of D/h . It was found that test results for both axi-symmetric and non-symmetric modes lie on a single curve of,

$$\frac{P_m}{M_0} = 72.3 \left(\frac{D}{h} \right)^{0.33} \quad (2.17)$$

They provided an analytical explanation subsequently to explain what might be overlooked in previous studies (Huang and Lu, 2003). In their more realistic theoretical model for axisymmetric mode, the effective plastic hinge length was introduced which is proportional to tube thickness. The tube wall was assumed to bend into two arcs linked with a straight line and the length of the arcs was assumed proportional to the thickness as well. The theoretical solution for crushing force history, mean crushing force and plastic half-wavelength showed better agreement with test data than previous solutions.

For the tube with an even smaller wall thickness ($D/h > 600$), it was observed that the deformation mode is non-axisymmetric and usually a large dent was formed by several circumferential buckles merging together. Usually, the buckling pattern was Euler buckling or skewed deformations and the axis of the tube becomes zig-zagged (Pugsley and Macaulay, 1960; Abramowicz and Jones, 1984b).

The axial crushing behavior of a tube does not only depend on its geometrical parameters but also relates to its material properties. Aluminium has received more attention as it can save up to 25% weight compared with steel with similar energy-absorbing capability. As a structure body in vehicle it can reduce the fuel consumption and consequently lower carbon dioxide (CO₂) emissions. In addition, it has good corrosion resistance and can be recycled at only 5% of the energy needed to produce primary aluminium, as reported by Langseth *et al.* (1998).

Based on test results, it was found the formula for mean crushing force proposed by Abramowicz and Jones (1986) has good prediction when the material is aluminium. The only modification was to use a different flow stress which is the mean value of the 0.2%-proof stress and the ultimate stress. Besides, it was found the aluminium alloy type and the temper selections were the key factors in design. This is because aluminium has a lower ductility than steel. For instance, although the 0.2%-proof stress of a temper T6 alloy is preferable regarding energy absorption, it has a lower ductility. Therefore, over-aged tempers should be adapted to increase ductility by eliminating the localized strains.

2.4.2 Cylindrical shell with compliant core

Besides aforementioned studies on the tube, another efficient approach to increase the energy absorption capacity is to add a filler. Such structure is more appealing because filling light-weight foam in a thin-walled tube is not only improving the stability of the structure, but also increasing the energy absorbing capacity.

Earlier work was focused on steel tubes filled with rigid polyurethane foam. Thornton (1980) found that the stability and energy absorption capacity of the steel tube were improved by adding a rigid polyurethane foam filler. Various cross-sectional shapes of foam-filled tubular structures under axial compression were studied. It was found that the mean crushing load of the foam-filled tube was higher than the sum of the individual tube and foam due to the interactive effect between the tube wall and foam core. Meanwhile, an empirical expression for the crushing load was derived. Besides, it revealed that this foam filled structure is not weight effective.

The behavior of polyurethane foam filler with various cross-sections was investigated by Lampinen and Jeryan (1982). A regression model for the mean crushing load was derived. It concluded that a structure with a higher foam density forms Euler buckling mode while others are in progressive buckling mode.

Reid *et al.* (1986) conducted experimental investigations on axial crushing of thin-walled square and rectangular metal tubes filled with polyurethane foam under quasi-static and dynamic loading. It was shown that the crushing folds are more contiguous, closely spaced and compacted by the foam filling and the plastic fold length was less affected by the foam-filler. The effect was more pronounced in structure with a thinner tube. The study revealed that the infill can enhance the mean crushing loads not only by its own crushing strength but also through interaction with the outer tube.

Besides, they tested the foam and conclude that the mechanical properties (Young's modulus, yield stress, etc.) of the foam mainly depend on its density, and the foam behavior after yielding is like a rigid, perfectly plastic material. After that, simple theoretical models are proposed to explain and quantify the interaction between the foam and the sheet metal tubes.

Not long after this, Abramowicz and Wierzbicki (1988) developed a new theoretical method by analyzing the progressive folding of foam filled prismatic columns and taking account of the interaction effect. The analytical results showed good agreement with experimental results obtained by Reid *et al.* (1986).

For the tube with small wall thickness, Wirsching and Slater (1973) carried out experiments on beer cans and found that even air can stabilize the crushing behavior and increase the energy absorbing capacity.

Reddy and Wall (1988) investigated the axial crushing behavior with low density rigid polyurethane foam under quasi-static and dynamic loading conditions. Axisymmetric deformation mode was observed instead of irregular diamond crumpling deformation mode in non-filled case. Theoretical analysis was performed using axisymmetric crushing model for the shell and volume reduction model for the infill from Ashby (1983). Good agreement was obtained between the theoretical predictions and experimental data. Besides, an optimum foam density of such structure could be found in terms of maximum specific energy absorption.

Recently, aluminium foam filler has received much attention due to its high strength to weight ratio as well as cost-effective manufacturing methods. Seitzberger *et al.* (1997) carried out a limited experimental study of mild steel tubes filled with aluminium foam with both square and circular cross sections axially crushed at low loading velocities. It concluded that aluminium foam was a suitable material for filling thin-walled tubular steel structures with a potential of enhancing the energy absorption capacity.

Extensive experimental tests on aluminium extrusions with aluminium foam filler were investigated due to its better performance. Hanssen *et al.* (1999, 2000) conducted a series of experiments on the axial crushing of aluminium extrusions with aluminium foam filler. Their study showed that there are more lobes formed and greater energy absorbed during the progressive buckling with the metallic foam-filler. This supported the finding by Reid *et al.* (1986), as described before.

Meanwhile, an empirical formula for the mean crushing load P_m was proposed. In the formula, the mean crushing load consisted of three parts: mean crushing load from the non-filled aluminium extrusion, uniaxial resistance from the aluminium foam core and the interactive effect by foam filling. For a circular extrusion, the expression is,

$$P_m = P_{mt} + \frac{\pi}{4} \sigma_f b_i^2 + C_{avg} \sqrt{\sigma_f \sigma_0} b_m h \quad (2.18)$$

where $b_i = b - 2h$, $b_m = b - h$, b is the outer diameter and h is wall thickness of the extrusion. P_{mt} is the contribution from the non-filled aluminium extrusion, σ_f is the plateau stress of the foam and σ_0 is the flow stress of the tube material, which is the

average between the initial yield stress and the ultimate stress. C_{avg} is a dimensionless constant obtained by fitting the test data. For example $C_{avg}= 2.74$ when the relative deformation (the actual deformation divided by the original length of the crushed component) is 50% in their study.

Santosa *et al.* (1998, 2000) proposed another empirical formula for the mean crushing load of foam filled square tube with side length b , based on numerical and experimental investigations. In their study, the interactive effect was regarded similar to the direct compressive resistance of aluminium foam, and thus there are only two terms in the formula,

$$P_m = P_{mt} + C\sigma_f b^2 \quad (2.19)$$

where C is a constant obtained by curve fitting the test results. Without the presence of adhesives C is 1.8 and it increases to 2.8 with adhesives.

Furthermore, the diamond collapse mode tends to be axisymmetric with foam filled in. This is due to the thickening effect formed with the foam-filler. A number of papers with different structures reported this including: filled aluminum tube by Reid *et al.* (1986), aluminum foam-filled steel tube by Seitzberger *et al.* (2000), wood sawdust-filled plastic tube by Singace (2000), and polystyrene foam-filled aluminum tubes by Toksoy and Güden (2005).

In addition, Zhang and Yu (2009) conducted an experimental study on axial crushing of thin-walled circular tubes with high pressure inside. It revealed that the energy absorption capacity can be enhanced by applying the internal pressure. The

enhancement is due to the direct effect of the internal pressure as well as an indirect effect from the interaction between the pressure and tube wall.

2.5 Concluding Remarks

The literature review provides a holistic view of researches that have been carried out on buckling and energy absorption behavior of thin-walled cylindrical shell with and without a foam core. It can be observed that the studies so far have mostly focused on an empty cylindrical shell. Previous studies on the effect of foam core have just started in initial buckling and energy absorption areas while they did not touch upon the post-buckling area.

In addition, existing studies have been performed only with a *fully* filled foam core or simply used a fixed thickness of foam core. However, this may not be the optimum arrangement in terms of design. In terms of industrial design, this requires a careful selection of infill thickness for optimal design in terms of weight and cost. Thus, there is a need to investigate the effect of a partially filled (or hollow) elastic core to obtain an optimum amount of the filler.

Moreover, although previous studies tried to resolve the large discrepancy of the critical buckling load between theory and experiment, there is no conclusion quantitatively reached yet. It is necessary to further investigate the post-buckling behavior of a thin-walled cylindrical shell with foam core and provide a quantitatively solution rather than qualitatively.

Besides, the interaction between the tube and foam core plays an important role in its energy absorption performance. However, its quantitative estimation presented in the past is too complex. Thus, there is a need to provide a much simpler quantitatively solution.

Furthermore, previous studies in energy absorption area are limited in terms of the ranges of D/h . It is not enough for many modern applications as a thinner shell wall is receiving more attention.

CHAPTER 3

BUCKLING UNDER AXIAL COMPRESSION

“...I wish to discuss the strength of hollow solids, which are employed in art - and still oftener in nature - in a thousand operations for the purpose of greatly increasing strength without adding to weight; examples of these are seen in the bones of birds and in many kinds of reeds which are light and highly resistant both to bending and breaking...”

Galileo Galilei (1638)

3.1 Introduction

Previous studies merely focused on a fully or fixed amount of filled foam core. However, this may not be the optimum arrangement in terms of design as an excessive amount of infill may not be necessary or comes only with diminishing returns. In order to maximize the strength of the structure without an unnecessary increase in the weight and cost, this chapter investigates the effect of partially filled (or hollow) foam core on the behavior of buckling in a thin-walled cylindrical shell. To perform the further investigation, a theoretical analysis is carried out using Rayleigh-Ritz approximation and a new formula is proposed to predict the critical buckling stress of an infill ranging from 0% up to 100%-rigid¹. Meanwhile, a simplified formula is provided to the practicing engineer. Furthermore, corresponding finite element simulations have been undertaken using commercial

¹ 100% refers to a fully filled foam core and rigid refers to the material's property related to Young's modulus. 0% to 100%-rigid means the range we wish to cover in the formulation.

software ABAQUS, in order to verify this formula and further understand the mechanics of the buckling phenomena of such structure.

3.2 Theoretical Analysis

3.2.1 Formulation of the governing equation by energy method

Fig. 3.1 shows a cylindrical shell with foam core. The shell has a length L_s , a radius R_s , a constant thickness h , Young's modulus E_s and Poisson's ratio ν_s . The foam core has an inner radius R_f , Young's modulus E_f and Poisson's ratio ν_f . A coordinate system (x, y, z) is used with the origin located at the end of the tube on the middle plane². In Figure 3.1, x is in the direction that is axially along the cylinder, y is in the circumferential direction and z is normal to the middle surface.

Fig. 3.1 Geometry and coordinate system of a cylindrical shell with foam core. The shell has a length L_s , a radius R_s , a constant thickness h , Young's modulus E_s and Poisson's ratio ν_s . The foam core has an inner radius R_f , Young's modulus E_f and Poisson's ratio ν_f .

² Middle plane refers to the middle of shell.

The corresponding displacements are designated by u , v and w .

When the tube is subjected to uniform axial compression, the case of axisymmetric (ring) buckling is considered here (Fig. 3.2). Here, it is assumed that there is no prevention of lateral expansion at both ends, which means the edges of this shell-foam structure are free to move after loading. As it is simply supported, there is only movement in the axial direction and rotation relative to the tangential direction.

Fig. 3.2 A cylindrical shell with foam core subjected to uniform axial compression in the case of axisymmetric (ring) buckling is considered here. It is assumed that there is no prevention of lateral expansion at both ends, which means the edges of this shell-foam structure are free to move after loading.

In order to obtain the critical load in this case, a direct and simple method - energy method was used. Due to the repeatable structure of the ring buckle “cell”, only one local buckling “cell” is studied here, as in Fig. 3.3. It has a length L , and a deformation w in z -direction.

In the case of buckling, the total potential energy of the shell increases due to four factors: the strain energy stored due to bending (U_b), the membrane strain energy due to stretching (U_s), the work done by compression (V) and the strain energy stored in the elastic restraining infill (U_r). Analysis of the first three follows the convenient method which provided by Bulson and Allen (1980), and the last one is obtained from the solution of Timoshenko (1951).

Fig. 3.3 Configuration of one buckling “cell” with length L and deformation w in z -direction. It contracts around the depression in the circumferential direction at the range of $0 < x < \alpha L$ and swells around the bulge for $\alpha L < x < L$.

Strain energy stored due to bending (U_b)

Similar to the bending of flat plate (Timoshenko and Gere, 1961), the strain energy

stored in a shell element is,

$$U_b = \frac{1}{2} D \iint \left[\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1 - \nu_s) \left(\frac{\partial^2 w}{\partial x^2} \times \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right) \right] dA \quad (3.1)$$

in which $\partial^2 w / \partial x^2$ is the change of curvatures in x -direction, $\partial^2 w / \partial y^2$ is the change of curvatures in y -direction, $\partial^2 w / \partial x \partial y$ is the twist of the element during bending, A is the cross-sectional area and D is the flexural rigidity,

$$D = \frac{E_s h^3}{12(1 - \nu_s^2)} \quad (3.2)$$

As this is an axisymmetric buckling mode, there is no change of curvatures in y -direction and no twist of the element during bending. Thus,

$$\frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial x \partial y} = 0 \quad (3.3)$$

Expression (3.1) can be reduced as,

$$U_b = \frac{1}{2} D \iint \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dA = \frac{1}{2} D \cdot (2\pi R) \int_0^L \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx = \pi R D \int_0^L \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx \quad (3.4)$$

where R is the radius of the mid-plane.

Membrane strain energy due to stretching (U_s)

This strain energy is generated due to deformation that occurs within the middle surface swelled around the bulges and contracted around the depressions in the circumferential direction. Due to stretching of the middle surface, this strain energy stored is expressed as,

$$U_s = \frac{1}{2} \iint (N_x \varepsilon_1 + N_y \varepsilon_2 + N_{xy} \gamma_{xy}) dA \quad (3.5)$$

in which N_x and N_y are the edge force intensities in x and y - direction, ε_1 and ε_2 are the mid-surface strains in x and y -direction and N_{xy} and γ_{xy} are the shear components of the edge force intensity and mid-surface strain.

Before buckling, the axial strain ε_0 at critical stress is expressed as,

$$\varepsilon_0 = \frac{\sigma_0}{E_s} \quad (3.6)$$

As in this bifurcation buckle, the axial force remains constant. The load per unit length in x and y direction, N_x and N_y can be presented as,

$$N_x = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_x dz = E_s \varepsilon_0 h \quad (3.7)$$

$$N_y = \int_{-\frac{h}{2}}^{\frac{h}{2}} \sigma_y dz = E_s \left(\frac{w}{R} \right) h \quad (3.8)$$

The circumferential strain at any section is due to two factors. One is radial deflection of w , and the other is Poisson's ratio,

$$\varepsilon_2 = \frac{w}{R} - \nu_s \varepsilon_0 \quad (3.9)$$

Similarly,

$$\varepsilon_1 = \varepsilon_0 - \nu_s \frac{w}{R} \quad (3.10)$$

As this is an axisymmetric buckling mode, there is no shear strain,

$$\gamma_{xy} = 0 \quad (3.11)$$

Therefore, substitute Eqns. (3.7), (3.8), (3.9), (3.10), (3.11) into Eq. (3.5), the membrane strain energy due to stretching of the middle surface is obtained as,

$$\begin{aligned}
U_s &= \frac{1}{2} \iint \left\{ (E_s \varepsilon_0 h) \left(\varepsilon_0 - \nu_s \frac{w}{R} \right) + \left[E_s \left(\frac{w}{R} \right) h \right] \left(\frac{w}{R} - \nu_s \varepsilon_0 \right) \right\} dA \\
&= \frac{1}{2} (2\pi R) \int_0^L \left\{ (E_s \varepsilon_0 h) \left(\varepsilon_0 - \nu_s \frac{w}{R} \right) + \left[E_s \left(\frac{w}{R} \right) h \right] \left(\frac{w}{R} - \nu_s \varepsilon_0 \right) \right\} dx \\
&= \pi R \int_0^L \left\{ (E_s \varepsilon_0 h) \left(\varepsilon_0 - \nu_s \frac{w}{R} \right) + \left[E_s \left(\frac{w}{R} \right) h \right] \left(\frac{w}{R} - \nu_s \varepsilon_0 \right) \right\} dx \\
&= \pi R (E_s h) \int_0^L \left\{ \varepsilon_0^2 + \left(\frac{w}{R} \right)^2 - 2\nu_s \varepsilon_0 \frac{w}{R} \right\} dx \tag{3.12}
\end{aligned}$$

Work done by compression (V)

The total work done by compression during buckling is equal to the end-load on the cylinder multiplied by the shortening of the axial length. The load on an element is $\sigma_0(2\pi R h)$, substituting Eq. (3.6), the load becomes $E_s \varepsilon_0(2\pi R h)$. The shortening is due to two factors, one is bending, and the other is the change in axial strain.

As the neutral plane is being considered non-extendable, the axial length shortening of an element due to bending is equal to the difference in length of the deflection curve and the length of the chord connecting the load edge. The curved length is $dx \sqrt{1 + (\partial w / \partial x)^2}$, and the chord length is dx . So that the displacement is $dx \sqrt{1 + (\partial w / \partial x)^2} - dx$, neglecting powers of $(\partial w / \partial x)$ greater than 2, we have,

$$dx \left[1 + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] - dx = \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 dx \tag{3.13}$$

Length shortening due to axial strain is $\int_0^L (\varepsilon_1 - \varepsilon_0) dx$. From Eq. (3.10), this shortening can be expressed as $\int_0^L (-\nu_s \cdot w / R) dx$. Thus the total work done is given

by,

$$\begin{aligned}
 V &= -[E_s \varepsilon_0 (2\pi R h)] \cdot \left[\frac{1}{2} \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx + \int_0^L \left(-\nu_s \frac{w}{R} \right) dx \right] \\
 &= -E_s \varepsilon_0 \pi R h \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx + 2\pi h E_s \varepsilon_0 \nu_s \int_0^L w dx
 \end{aligned} \tag{3.14}$$

Strain energy stored in the elastic restraining infill (U_r)

Fig. 3.4 shows a local buckling “cell” after buckling. The buckle penetrates the medium within the range $0 < x < \alpha L$. The elastic deformation of the medium can be simplified as a radial spring having a stiffness k_e . The parameter $0 < \alpha < 1$ is presented as the unilateral buckle is not known a priori. On the other hand, no elastic restraint exists for the range of $(1 - \alpha)L < x < L$. So the strain energy stored in this elastic restraining infill can be presented as,

$$U_r = \frac{1}{2} \int_0^{\alpha L} k_e w^2 dx \tag{3.15}$$

Fig. 3.4 A local buckling “cell” after buckling. The buckle penetrates the medium within the range $0 < x < \alpha L$. The elastic deformation of the medium can be simplified as a radial spring having a stiffness k_e . The parameter $0 < \alpha < 1$ is presented as the unilateral buckle is not known a priori. On the other hand, no elastic restraint exists for the range of

$$(1 - \alpha)L < x < L.$$

As the shell is thin, R_s is assumed the same as foam outer radius. The expression of k_e can be approximately derived from the stress distribution (Timosheko, 1951). For Fig. 3.5, the stress component σ_θ in polar coordinate is expressed as,

$$\sigma_\theta = -\frac{R_f^2 R_s^2 p_o}{R_s^2 - R_f^2} \frac{1}{r^2} - \frac{p_o R_s^2}{R_s^2 - R_f^2} \quad (3.16)$$

Fig. 3.5 Stress distribution in cross-sectional view in order to derive the expression of k_e as the shell is thin (Timosheko, 1951).

At the interfacial of shell and foam core, the pressure p_o can be directly derived from the previous equation as $r = R_s$,

$$p_o = \sigma_\theta \frac{1 - \eta^2}{1 + \eta^2} \quad (3.17)$$

where $\eta = R_f / R_s$.

As the axial force remains constant during buckling, so that $\sigma_\theta = E\varepsilon_\theta$. As this is plane strain, so $E = E_f / (1 - \nu_f^2)$. Since $\varepsilon_\theta = w/R_s$, the previous equation can be expressed as,

$$p_o = \frac{E_f}{1 - \nu_f^2} \frac{w}{R_s} \frac{1 - \eta^2}{1 + \eta^2} \quad (3.18)$$

Therefore, the interfacial force can be obtained as,

$$\begin{aligned} F_o &= p_o (2\pi R_s) \\ &= 2\pi \frac{1 - \eta^2}{1 + \eta^2} \frac{E_f}{1 - \nu_f^2} w \end{aligned} \quad (3.19)$$

For a linear elastic system,

$$F_o = k_e w \quad (3.20)$$

The stiffness can be obtained as,

$$k_e = 2\pi \frac{1 - \eta^2}{1 + \eta^2} \frac{E_f}{1 - \nu_f^2} \quad (3.21)$$

which has a unit of force per unit area.

The sum of the above four energies are the total potential energy (U) during buckling,

$$U = U_b + U_s + V + U_r \quad (3.22)$$

Thus,

$$U = \pi R D \int_0^L \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx + \pi R (E_s h) \int_0^L \left\{ \varepsilon_0^2 + \left(\frac{w}{R} \right)^2 \right\} dx - E_s \varepsilon_0 \pi R h \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx + \frac{1}{2} \int_0^{aL} k_e w^2 dx \quad (3.23)$$

3.2.2 Solution using Rayleigh-Ritz approximation method

In order to find the critical stress of buckling, Rayleigh-Ritz approximation method is applied. Let the tube be constrained to deform only in the form,

$$w = q \cdot \varphi(x) = q \cdot \sin[\pi(\alpha - \xi)] \sin[\pi(\xi - 1)] \quad (3.24)$$

in which q is an amplitude which is to be determined, $\xi = x/L$, the shape of $\varphi(x)$ is chosen to satisfy the boundary conditions,

(1) At $x = 0, x = \alpha L, x = L$: $w = 0$

(2) The function w should be periodical with the wavelength L of the cylinder:

$$\frac{\partial w}{\partial x} = \left(\frac{\pi q}{L} \right) \sin[\pi(\alpha + 1 - 2\xi)] \quad (3.25)$$

(3) Along the x -direction, the buckling mode is anti-symmetric in $0 < x < L$:

$$\left(\frac{\partial w}{\partial x} \right)_{x=0} = \begin{cases} 0 & (\alpha = 0) \\ -\frac{\pi q}{L} & (\alpha = 0.5) \end{cases} \quad \& \quad \left(\frac{\partial w}{\partial x} \right)_{x=L} = \begin{cases} 0 & (\alpha = 0) \\ -\frac{\pi q}{L} & (\alpha = 0.5) \end{cases} \quad (3.26)$$

The total potential energy is obtained in Eq. (3.23). Here, the term $\pi R(E_s h) \int_0^L \varepsilon_0^2 dx$ is ignored as it is not affected by the flexure of the member,

$$\begin{aligned} U &= \pi R D \int_0^L \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx + \pi R (E_s h) \int_0^L \left(\frac{w}{R} \right)^2 dx + \frac{1}{2} \int_0^{\alpha L} k_e w^2 dx - E_s \varepsilon_0 \pi R h \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx \\ &= \pi R D \int_0^L \left(\frac{\partial^2 w}{\partial x^2} \right)^2 dx + \frac{\pi E_s h}{R} \int_0^L w^2 dx + \frac{1}{2} k_e \int_0^{\alpha L} w^2 dx - \sigma_0 \pi R h \int_0^L \left(\frac{\partial w}{\partial x} \right)^2 dx \end{aligned} \quad (3.27)$$

Substitute the trial solution of Eq. (3.24), w , into the functional, U , which is the total

potential energy of Eq. (3.27). For the stationary point of U with respect to q from the trial solution,

$$\frac{\partial U}{\partial q} = 0 \quad (3.28)$$

Therefore, the stress can be obtained from this nontrivial solution as,

$$\sigma_0 = \frac{\pi^2 E_s h^2}{3L^2(1-\nu_s^2)} + \frac{E_s L^2(3-2\sin^2 \beta)}{4\pi^2 R_s^2} + \frac{k_e L^2 \left[3\beta - 2\beta \sin^2 \beta - \frac{3}{2} \sin(2\beta) \right]}{8\pi^4 R_s h} \quad (3.29)$$

where, $\beta = \alpha\pi$.

In order to find the critical value of this stress, Ritz-method is applied again. The previous expression of stress, Eq. (3.29), can be written as,

$$\sigma_0 = c \cdot \frac{E_s}{\sqrt{1-\nu_s^2}} \frac{h}{R_s} \quad (3.30)$$

in which, the local buckling coefficient,

$$c = \frac{\pi^2}{3\phi\sqrt{1-\nu_s^2}} + (a+b)\phi \quad (3.31)$$

with

$$a = \left(\frac{\sqrt{1-\nu_s^2}}{4\pi^2} \right) (3-2\sin^2 \beta) \quad (3.32)$$

and

$$b = \left(\frac{\psi\sqrt{1-\nu_s^2}}{8\pi^4} \right) \left[3\beta - 2\beta \sin^2 \beta - \frac{3}{2} \sin(2\beta) \right] \quad (3.33)$$

where, $\phi = L^2/(R_s h)$ is the dimensionless buckling wavelength and $\psi = (k_e/E_s) \cdot (R_s/h)$ is the dimensionless restraint stiffness.

In this case, the functional is c and the trial solution is ϕ . For the stationary point of c with respect to ϕ , the wavelength is assumed to be a constant value and we have

$$\frac{\partial c}{\partial \phi} = 0 \quad (3.34)$$

Substituting Eq. (3.31) into (3.34), the critical value of dimensionless buckling wavelength is obtained as,

$$\phi_{crit} = \frac{\pi}{\sqrt{3(a+b)}} (1 - \nu_s^2)^{\frac{1}{4}} \quad (3.35)$$

Substituting this equation back into Eq. (3.31), the local buckling coefficient is,

$$c = \frac{2\pi}{\sqrt{3}} \sqrt{a+b} (1 - \nu_s^2)^{\frac{1}{4}} \quad (3.36)$$

There is still one more variable β contained in this coefficient to be determined.

Using Ritz-method one more time to obtain the critical local buckling coefficient, now, the functional is still c and the trial solution is β . For the stationary point of c with respect to β , requiring that,

$$\frac{\partial c}{\partial \beta} = 0 \quad (3.37)$$

By substituting Eq. (3.36), this leads to,

$$\frac{\partial a}{\partial \beta} + \frac{\partial b}{\partial \beta} = 0 \quad (3.38)$$

Using Eqns. (3.32), (3.33) and (3.38), the location of the buckled region

$0 < x < \alpha L$ can be defined as,

$$\tan \beta - \beta = \frac{2\pi^2}{\psi} \quad (3.39)$$

in which $0 < \beta < \pi/2$ for the permissible solution.

Substitute Eq. (3.39) back to Eq. (3.33), the term b becomes,

$$b = \frac{\sqrt{1 - \nu_s^2} \left[3\beta - 2\beta \sin^2 \beta - \frac{3}{2} \sin(2\beta) \right]}{4\pi^2 (\tan \beta - \beta)} \quad (3.40)$$

Therefore, the critical local buckling coefficient can be obtained by substituting

Eqns. (3.32), (3.40) into Eq. (3.36),

$$c_{crit} = \sin \beta \sqrt{\frac{\tan \beta}{3(\tan \beta - \beta)}} \quad (3.41)$$

Eq. (3.39) may be expanded using Taylor series, which leads to an approximate

expansion for β . By substituting this expression of β and k_e of Eq. (3.21) into

(3.41), the critical buckling stress of Eq. (3.30) is finally obtained as,

$$\sigma_{cr} = \left[\sin A \sqrt{\frac{6A - A^3}{9B(2 - A^2)}} \right] \cdot \frac{E_s}{\sqrt{1 - \nu_s^2}} \frac{h}{R_s} \quad (3.42)$$

where,

$$A = \frac{\Phi}{2} + \frac{B^2}{2\Phi} - \frac{B}{2} \quad (3.43)$$

and

$$B = \pi(1 - \nu_f)^2 \frac{E_s}{E_f} \frac{h}{R_s} \frac{1 + \eta^2}{1 - \eta^2} \quad (3.44)$$

and

$$\Phi = \left(12B - B^3 + 2 \cdot \sqrt{36B^2 - 6B^4} \right)^{\frac{1}{3}} \quad (3.45)$$

Two extreme cases are obtained here. When $E_f \rightarrow \infty$, which indicates a tube with a rigid infill, Eq (3.42) reduces exactly to the form obtained by Bradford *et al.* (2006) (Eq. 2.8). When $\eta = 1$, σ_{cr} is for an empty tube,

$$\sigma_{cr} = 0.57 \cdot \frac{E_s}{\sqrt{1 - \nu_s^2}} \frac{h}{R_s} \quad (3.46)$$

which is close to the result from Timoshenko and Gere (1961).

3.2.3 Comparison with experiments

Limited experimental data are available, with tubes completely filled with Lockfoam P-502. Goree and Nash (1962) conducted several experiments with mild steel tubes with diameter of 152.4mm and thickness 0.1524mm. Fig. 3.6 compares the results from the present study and their tests. The solid line represents our normalized buckling stress. Similar trend is observed for the effect of shell thickness on critical buckling stress between our prediction and the experimental results produced by Goree and Nash (1962), although their test data are lower than our theoretical ones. The discrepancy in value could be due to the inherent initial imperfection on the thinner shell walls resulting from manufacturing and preparation process of the specimens.

Fig. 3.6 Plot of normalized buckling stress versus the radius/thickness ratio of the shell.

$L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$, $\eta = 0$, $E_f/E_s = 7 \times 10^{-6}$ and $\nu_f = 0.1$.

3.2.4 Discussion and design consideration

In practice engineers need an intuition into the distinct regimes of the structure behavior as well as a quick estimate to start out the design. The present formula developed seems a little complicated and a simplified one would be useful in the initial design of structures.

Parametric studies

Parametric studies are presented on the effect of key parameters on the buckling stress. A case study is carried out by further investigating the experiments by Goree

and Nash (1962) with shell dimensions of $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$ and $h = 0.1524\text{mm}$. The shell has Young's modulus $E_s = 200\text{GPa}$ and Poisson's ratio $\nu_s = 0.3$. As the foam core is filled in the shell, it has adaptive dimensions. The foam core's central bore hole radius R_f and Young's modulus E_f are changed in order to explore the effect of foam core (correspondingly, η varies from 0 to 1, and E_f/E_s varies from 1×10^{-6} to $1 \times 10^{+4}$). The Poisson's ratio of foam core is fixed at $\nu_f = 0.1$.

Fig. 3.7 Plot of critical buckling stress normalized with respect to that of an unfilled cylindrical shell against the percentage of infill, for different values of E_f/E_s . $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $h = 0.1524\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$ and $\nu_f = 0.1$.

Effect of E_f

The buckling resistance is enhanced with increasing infill modulus E_f (Fig. 3.7). In

Fig 3.7, the critical buckling stress is normalized with that of an unfilled tube. The plot shows that for the tube with a very soft foam core (*e.g.* $E_f / E_s = 1 \times 10^{-6}$), the infill has no effect on the buckling resistance, even if it is completely filled ($\eta = 0$). Conversely for a very hard foam core ($E_f / E_s = 1 \times 10^{+4}$), the infill can be regarded as rigid with σ_{cr} / σ_0 equal to 1.73 (Bradford *et al.*, 2006).

Effect of ν_f

The effect of Poisson's ratio of the foam core on the critical buckling stress is plotted in Fig. 3.8. As expected, a higher value of Poisson's ratio leads to an increase in the buckling resistance. The maximum effect occurs for the fully filled tube and the increase is about 7% when the value of ν_f changes from 0 to 0.3. In practice, the value of ν_f is small and hence its effect can be ignored in the calculations of buckling stress of the shells.

Effect of R_s/h and η

It is known that the buckling resistance is smaller for those with thinner shell wall, or large value of R_s/h . See Eq. (2.1) for empty tubes, or Eq. (3.42) for foam-filled tubes. Also, for a given value of E_f/E_s , the amount of filler, characterized by η , leads to a different enhanced buckling stress. Fig 3.9(a) plots the buckling stress versus R_s/h with different η when $E_f/E_s = 10^{-1}$.

Fig. 3.8 Plot of critical buckling stress normalized with respect to that of an unfilled cylindrical shell against the percentage of infill, for four different values of ν_f . $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $h = 0.1524\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$ and $E_f/E_s = 10^{-2}$.

Fig. 3.9 (a) Double logarithmic plots of critical buckling stress versus R_s/h , for six different values of η . (b) Plots of critical buckling stress normalized with respect to that of an unfilled cylindrical shell versus R_s/h , for fifteen different values of η . $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$, $E_f/E_s = 10^{-1}$ and $\nu_f = 0.1$.

It reveals that the buckling stress decreases with a larger R_s/h , as expected. In addition, the difference of the buckling stress between the empty tube and the foam-filled tube get larger with increasing R_s/h . This can be found in Fig 3.9(b), which plots the buckling stress normalized with respect to the corresponding buckling stress for their empty tubes. Hence when $\eta=1$, the ratio is always equal to unity. The strengthening effect of the core is more significant for those with thinner shell wall, or larger values of R_s/h . Note that when $\eta=0.9999$, the thickness of the filler is $0.0001R_s$, which is equal to $0.1h$ or h when $R_s/h = 10^3$ or 10^4 , respectively.

Effect of ν_s and η

Fig. 3.10 indicates the influence of Poisson's ratio of shell on the critical buckling stress. The effect is most significant for the fully filled tube and the increase is about 4% when the value of ν_s changes from 0.1 to 0.3. Since the paper investigates the effect of foam core on buckling behavior, the well-known term $(1-\nu_s^2)$ may be reduced to 1 (Mandal and Calladine, 2000). The plot shows again the effect of η on the buckling stress.

Fig. 3.10 Plot of normalized critical buckling stress against η for three different values of ν_s .
 $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $h = 0.1524\text{mm}$, $E_s = 200\text{GPa}$, $E_f/E_s = 10^{-2}$ and $\nu_f = 0.1$.

Simplified formulae

The discussion on the above four parameters reveals that Poisson's ratio of both the foam core and shell plays an insignificant role in the critical buckling stress. Therefore a simplified version of Eq (3.42) may be obtained by neglecting these two factors.

Fig. 3.11 Double logarithmic plot of normalized buckling stress versus the radius/thickness ratio of the shell for eight different values of E_f/E_s , $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$, $\eta = 0.8$ and $\nu_f = 0.1$.

The critical buckling stress of the cylindrical shell with a 20% infill is plotted against the ratio of shell's radius to thickness, on double-logarithmic scales, in Fig. 3.11. This figure shows that the buckling stress decreases significantly with increasing R_s/h , as expected. The highlighted line corresponds to the case for empty shells.

In Fig. 3.11, all the lines for different values of E_f/E_s fall within a region bounded by two lines, one corresponding to rigid infill and the other, empty shell. The dimensionless critical buckling stress may be presented in the form of,

$$\frac{\sigma_{cr}}{E_s} = C \cdot \left(\frac{h}{R_s} \right) \quad (3.47)$$

where C is the coefficient whose value is related to E_f/E_s .

Values of the coefficient C against E_f/E_s are plotted in Fig. 3.12. As the percentage of infill increases, C may be divided into three regions. When E_f/E_s is less than 10^{-5} , it is approximately constant with a value of 0.6. It then increases rapidly. Finally, when E_f/E_s approaches 1, C becomes constant again at the value of 1.05.

For the central region, C may be related to E_f/E_s as,

$$C = \Psi \cdot \left(\frac{E_f}{E_s} \right)^\zeta \quad (3.48)$$

where both the coefficients Ψ and ζ are functions of η . It was found that both

Ψ and ς have distinct trends. Therefore expressions of Ψ and ς can be obtained by linear fitting separately for $0 < \eta < 0.8$ and $0.8 < \eta < 1$. Thus, the final simplified critical buckling stress can be expressed as,

Fig. 3.12 Plot of C against E_f/E_s for ten different values of η . $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $h = 0.1524\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$ and $\nu_f = 0.1$.

$$\text{For } \frac{E_f}{E_s} \leq 10^{-4}, \quad \sigma_{cr} = 0.6 \cdot E_s \left(\frac{h}{R_s} \right) \quad (3.49a)$$

$$\text{For } \frac{E_f}{E_s} \geq 1, \quad \sigma_{cr} = 1.05 \cdot E_s \left(\frac{h}{R_s} \right) \quad (3.49b)$$

$$\text{For } 10^{-4} < \frac{E_f}{E_s} < 1,$$

$$\sigma_{cr} = 1.5 \cdot E_f^{0.1} \cdot E_s^{0.9} \cdot \left(\frac{h}{R_s} \right) \quad \text{when } 0 \leq \eta < 0.8 \quad (3.49c)$$

and

$$\sigma_{cr} = (8 - 7\eta) \cdot E_f^{1-\eta} \cdot E_s^\eta \cdot \left(\frac{h}{R_s} \right) \quad \text{when } 0.8 \leq \eta \leq 1 \quad (3.49d)$$

where $\eta = R_f / R_s$.

Design consideration

In practical design, it is paramount to minimize the cost and maximize the efficiency. In the present study, the critical buckling load ($\sigma_{cr} R_s h$) is firstly normalized by Young's modulus of the shell (E_s), and then divided by $(R_s^2 - R_f^2)$, which is related to the total cross-sectional area.

Variation of specific strength with different percentage of infill is shown in Fig. 3.13. It can be noted that, as the percentage of infill increases (η decreases), the specific strength decreases sharply for infill less than 10% and then it reaches constant thereafter. This indicates that for practical design, because of the diminishing weight, the infill should not be increased beyond 10% of the out radius of the shell. Any infill above 10% would seem uneconomical. This point may also be observed in Figs 3.7 and 3.8, though less obviously.

Fig. 3.13 Dimensionless plot of the critical buckling stress against η for six different values of E_f/E_s . $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $h = 0.1524\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$ and $\nu_f = 0.1$.

3.3 Finite Element Analysis

In order to verify this formula and further understand the mechanics of the buckling phenomena of the cylindrical shell with a foam core, finite element analysis has been conducted. Commercial software ABAQUS is used for this finite element analysis. One of the three core products of it, ABAQUS/Standard, is chosen to estimate the eigenvalue of bifurcation loads and modes as it is a general-purpose solver using a traditional implicit integration scheme to solve finite element analysis.

3.3.1 F.E. modeling

Modeling geometry and material properties

As one purpose of this finite element analysis is to verify the previous theoretical solution, the same geometric dimensions in previous parametric studies are used here. The shell has a length of $L_s = 100\text{mm}$, a radius of $R_s = 76.2\text{mm}$ and a thickness of $h = 0.1524\text{mm}$. The foam would fill in the shell and has adaptive dimensions. For instance, the ratio of R_f/R_s equal to 0.9 is studied in detail as shown in Fig. 3.14 with the shell colored in blue and foam core colored in yellow.

For the convenience of drawing in ABAQUS, the coordinate system switches to a right-hand set of axes (X, Y, Z) which Z and X are in the axial and circumferential directions of the tube, Y is in the direction of the inward normal to the middle surface. Meanwhile, polar coordinate system (R, T, Z) is inserted in order to set the boundary condition in the later stage.

The Young's modulus of the shell is taken as $E_s = 200\text{GPa}$, and Poisson's ratio is $\nu_s = 0.3$. The Young's modulus of the foam core E_f is varied to explore its effect on buckling behavior. The Poisson's ratio of the foam core is taken $\nu_f = 0.1$, a very low value. In this section, the ratio of E_f/E_s equal to 1×10^{-6} will be studied in detail as an example, that means $E_f = 200 \times 10^{+3} \text{ Pa}$. As $E_s = 200 \times 10^{+9} \text{ Pa}$, the shell is much stiffer than the foam core, and so the foam core receives force action through the

shell during compression. Therefore, shell is the “master” and foam core is the “slave”.

Fig. 3.14 Geometry of the cylindrical shell with foam core. The ratio of R_f/R_s equal to 0.9 is studied in detail with the shell colored in blue and foam core colored in yellow.

Element and mesh design

In this study, a three dimensional cylindrical shell with foam core was used. Therefore, it has two components: shell and foam core. For the shell, the element shape was chosen *Quad*. Because the shell is very thin, it can be treated as a two-dimensional region. Numerical integration is used to calculate the stresses and strains independently at each section point, or called integration point, through the thickness of the shell, thus allowing for the nonlinear material behavior. Therefore, the type of shell element was chosen as *S4R*, a 4-node doubly curved thin or thick shell, reduced integration, hourglass control, finite membrane strains element. For

the foam core, the element shape was chosen as *Hex*. As the foam core is solid, the type *C3D8R* was chosen for the element which is an 8-node three-dimensional solid continuum brick element with reduced integration and hourglass control.

Fig. 3.15 The rule of master and slave surface. Master surface can penetrate into the slave surface, while slave surface cannot penetrate into the master surface.

To ensure convergence of the eigenvalues, an adequately fine mesh is needed. This is the key aspect of the eigenvalue analysis. Mesh convergence is defined with the criteria of results variation less than 5%. Meanwhile, as the slave nodes cannot penetrate the master segments, shown in Fig. 3.15, the approximate global mesh size of shell (master) is chosen as $0.7 \times 0.7 \text{ mm}^2$ and for the foam core (slave) as $3 \times 3 \text{ mm}^2$. The mesh of the shell component contains 133266 S4R elements and 134064 nodal points. For the foam components, the mesh contains 14949 C3D8R elements and 20536 nodal points.

Analysis steps

The material of both the shell and foam core is taken as linear-elastic. For the eigenvalue analysis, Lanczos eigenvalue extraction method is used for the buckling modes estimation. It has the advantages of superior accuracy and fast convergence rate. The first 20 buckling modes are studied here as Lanczos eigenvalue extraction method is faster with a large number of eigenmodes (≥ 20).

Boundary and loading conditions

The purpose of defining boundary condition is to constrain the movement or rotation of the selected degrees of freedom. In this case, the cylindrical shell with foam core under uniform compression is simply supported, *i.e.* it only has movement in the axial direction and rotation relative to the tangential direction. As the slave elements follow the master's, boundary condition is only needed to apply on the shell. Thus, the top and bottom edges of the shell are fixed in both R and T directions and the rotation around both the R and Z axes, while movement in Z direction and rotation around T axis are free. Additionally, one of the bottom points is fixed in all the directions in order to make sure that the cylindrical shell with foam core is not floating during compression.

Eigenvalue buckling analysis is a linear perturbation procedure. Thus, any load applied during the eigenvalue buckling analysis is called a live load. This

incremental load describes the load pattern for which buckling sensitivity is being investigated and its magnitude is less important. In this study, a load of 10000 N is applied.

Contact between the shell and foam core

Constraint is used to define constrain in the analysis degrees of freedom between regions of a model. In this study, “tie” constraint is used to tie the two separate surfaces (shell inner surface and foam core outer surface) together, so that the slave node which is on the outer surface of the foam core and the surrounding nodes on the master surface of the shell are tied together. This is to make sure that there is no relative motion between them during buckling. Although the meshes created on these two surfaces are different, “tie” constraint can fuse these two regions. And this “tie” only constrains translational degrees of freedom of the structure.

3.3.2 F.E. result

The first mode of the buckling simulation is shown in Fig. 3.16. As introduced in the previous chapter, this is a ‘ring’ shaped buckle. The first 8 buckling modes are presented in Fig. 3.17.

Normally, the first mode has the lowest value and is of interest in critical buckling calculation. As indicated in Fig. 3.16, this eigenvalue has the value of 1.7703. As

Fig. 3.16 The first buckling mode of cylindrical shell with foam core. It has ‘ring’ shaped buckle and its value is the critical buckling load.

Fig. 3.17 The first 8 mode shapes of the buckling result. Normally, the first mode has the lowest value and it is the interest one in critical buckling calculation.

eigenvalue buckling analysis is a linear perturbation procedure, the critical load can

be obtained by the product of the live load and this lowest eigenvalue as the relation of these three is presented as,

$$Eigenvalue = \frac{CriticalLoad}{LiveLoad} \quad (3.50)$$

The live load is the initial load we apply on the cylinder in F.E., which is taken as 10000 N. Therefore, the value of critical load is,

$$CriticalLoad = 1.7703 \times 10000 = 17703 \text{ N} \quad (3.51)$$

As the cross-section area can be calculated as,

$$A = (2\pi R_s) \times h \quad (3.52)$$

Thus,

$$A = (2\pi \times 76.2 \times 10^{-3}) \times (0.1524 \times 10^{-3}) = 7.30 \times 10^{-5} \text{ m}^2 \quad (3.53)$$

Therefore, the critical buckling stress can be obtained as,

$$\sigma_{cr} = \frac{CriticalLoad}{A} \quad (3.54)$$

Hence,

$$\sigma_{cr} = \frac{17703}{7.30 \times 10^{-5}} = 2.43 \times 10^8 \text{ Pa} \quad (3.55)$$

Moreover, finite element analysis is performed for infill from 0% to 100% and E_f/E_s from 1×10^{-6} to 1×10^{-2} (Fig. 3.18). Corresponding theoretical predictions in Fig. 3.7 are re-plotted in this figure in order to verify the derived formula in the previous section. This plot indicates a good agreement between F.E. results and theoretical

predictions. The slight discrepancy between the F.E. analysis and theory is possibly due to several factors. Assumptions are normally made in the theory where as a F.E. analysis is a more comprehensive simulation. Besides, the mesh size and software setup can also slightly affect the results. But all in all, good agreement has achieved.

Fig. 3.18 Comparison of the F.E. result and theoretical prediction of critical buckling stress normalized with respect to that of an unfilled cylindrical shell against the percentage of infill, for different values of E_f/E_s . $L_s = 100\text{mm}$, $R_s = 76.2\text{mm}$, $h = 0.1524\text{mm}$, $E_s = 200\text{GPa}$, $\nu_s = 0.3$ and $\nu_f = 0.1$.

3.4 Summary

In this chapter, an analytical formula has been established for the critical buckling stress of a thin-walled cylindrical shell partially filled with a foam core. The study reveals that filling a foam core in a thin-walled cylindrical shell can enhance its resistance to buckling failure, as expected. This effect is more pronounced for those with either a thicker or a harder foam core, and the maximum enhancement factor is 1.73. The value of the Poisson's ratio of the shell or foam core has little impact in calculating the critical buckling stress and it may thus be neglected without introducing much error. It has been also found that for practical purpose the in-filler inner radius should not be more than 10% of the shell radius; any extra filler would be uneconomical. After the parametric studies of the formula, a simplified formula has been proposed, which was based on the distinct regimes of the structural behavior. Besides, finite element analysis has been conducted in order to verify the derived theoretical formula and further investigate the mechanism of buckling phenomena. Good agreement is achieved between F.E. results and theoretical prediction. This investigation can help engineers engaged in the design of such structures seek for the optimal arrangements in strength, weight and cost.

CHAPTER 4

POST-BUCKLING BEHAVIOR UNDER AXIAL COMPRESSION

“We lift ourselves by our thought, we climb upon our vision of ourselves. If you want to enlarge your life, you must first enlarge your thought of it and of yourself. Hold the ideal of yourself as you long to be, always, everywhere - your ideal of what you long to attain - the ideal of health, efficiency, success.”

Orison Swett Marden (1850 - 1924)

4.1 Introduction

When a thin-walled cylindrical shell with foam core is subject to axial compression, its strength and the effect of foam core on buckling behavior are the critical design considerations. Previous studies found there is a large discrepancy between the theoretical and experimental buckling loads, but results were rarely given. This chapter concerns this issue and starts with a preliminary experimental investigation on the response of a thin-walled cylindrical shell with foam core under axial compression. Based on the experiments, corresponding finite element simulations have been undertaken using the ABAQUS software with an initial geometric imperfection present in the shell. Furthermore, a parametric study was carried out to examine the contribution of foam core stiffness and shell's R/h ratio on the post-buckling behavior. Thereafter, the plateau load for the structure with this specific imperfection was formulated by energy method and its expression obtained under the assumption of *inextentional* deformation.

4.2 Preliminary Experimental Study

A total of 24 quasi-static crushing tests were performed on thin-walled cylindrical shell with foam core. Tests were performed, respectively, for the empty shell, the foam only and then the shell with foam core. The shell diameter had two values and two different types of foams were used.

4.2.1 Specimens and apparatus

The shell specimens were cut from two kinds of commercial beverage cans which are made of aluminium alloy 3104-H19. Two different values of the diameters, $D_1=52\text{mm}$ and $D_2=66\text{mm}$, with an average thickness $h\sim 0.10\text{mm}$ ($\pm 5\%$ variation), are used and denoted as S1 (for Shell 1) and S2 (for Shell 2) (Fig. 4.1). The two ends of the beverage can were cut off using the Electrical Discharge Machining (EDM) technique, with the final height equal to that of the infill which is $L_s=85\text{mm}$.

S1

S2

Fig. 4.1 The two types of beverage cans, both are made of aluminium alloy 3104-H19. S1 has a diameter of $D_1=52\text{mm}$ and S2 has a diameter of $D_2=66\text{mm}$, both with thickness $h\sim 0.10\text{mm}$. The two ends of the beverage can were cut off using the Electrical Discharge Machining (EDM) technique, with the final height equal to that of the infill which is $L_s=85\text{mm}$.

Two types of foam core are used, denoted as FA and FB in Fig. 4.2, which are aluminium foams with a density of 0.125g/cm^3 and 0.472g/cm^3 respectively. All the foams were cut into cylinders using EDM technique to fit into its shells with a gap of 0.05mm.

Fig. 4.2 The two types of foams. Both are aluminium foams, with density of 0.125g/cm^3 (FA) and 0.472g/cm^3 (FB), respectively.

In order to obtain the mechanical properties, tensile tests were performed on the beverage can coupons. Thus, the shell specimens were found to have a Young's modulus $E_s=70.0\text{GPa}$ and a Poisson's ratio $\nu_s=0.3$. The tests were performed with an Instron 5500 universal machine with accuracy within $\pm 5\%$ of indicated load. The load was applied at a crosshead speed of 5mm/min . Traces of compressive load against displacement were recorded automatically.

4.2.2 Test results

The test results are grouped into 4 sets of data in Table 4.1 for the comparison of P_c , which is the first peak load and P_m , which is the average force (in Fig. 2.1). Within

Table 4.1
Experimental results

		D (mm)	h (mm)	D/h	ρ_f (g/cm ³)	F_{max} (N)	F_{ave} (N)
Set 1A	S1a	52.2	0.10	522	N.A.	866.19	232.53
	S1b	52.4	0.09	582	N.A.	832.04	297.32
	FAa	N.A.	N.A.	N.A.	0.125	307.27	328.22
	FAb	N.A.	N.A.	N.A.	0.124	231.79	253.89
	S1FAa	52.1	0.11	474	0.127	1287.27	835.43
	S1FAb	52.2	0.10	522	0.131	1376.99	923.07
Set 1B	S1a	52.0	0.10	520	N.A.	869.07	328.42
	S1b	52.2	0.11	475	N.A.	897.71	150.24
	FBa	N.A.	N.A.	N.A.	0.473	5929.25	8511.11
	FBb	N.A.	N.A.	N.A.	0.475	6188.22	8512.22
	S1FBa	52.4	0.09	582	0.477	6786.18	8755.74
	S1FBb	52.0	0.11	473	0.471	5481.80	8855.89
Set 2A	S2a	66.1	0.11	601	N.A.	661.02	211.97
	S2b	65.8	0.11	598	N.A.	668.74	237.22
	FAa	N.A.	N.A.	N.A.	0.130	412.06	322.15
	FAb	N.A.	N.A.	N.A.	0.133	551.97	213.75
	S2FAa	65.9	0.10	659	0.137	1574.95	911.81
	S2FAb	66.0	0.11	600	0.132	1478.99	923.07
Set 2B	S2a	66.3	0.10	663	N.A.	570.16	162.38
	S2b	65.8	0.10	658	N.A.	581.89	170.37
	FBa	N.A.	N.A.	N.A.	0.477	8167.97	8367.23
	FBb	N.A.	N.A.	N.A.	0.475	7476.85	8231.71
	S2FBa	66.4	0.11	604	0.474	7603.21	8576.19
	S2FBb	66.0	0.10	660	0.478	8529.91	8649.68

S1

FA

S1FA

Fig. 4.3 Set 1A samples after quasi-static axial crushing. The individual shell S1, individual foam FA and the combined case S1FA (Shell 1 with infill of Foam A).

each set, the buckling load of individual shell, individual foam and shell with foam core are listed. For instance, as shown in Fig. 4.3, Set 1A includes the results of individual shell S1 (Shell 1), individual foam FA (Foam A), and the combined case S1FA (Shell 1 with infill of Foam A). In addition, each test was performed twice and denoted by a and b for 1st and 2nd test.

Deformation mode

In order to describe the deformation mode, S2FA and S2FB are presented as an example in Fig. 4.4. When it is under axial compression, a non-axisymmetric buckling mode is observed in both of them, resulting in several axial and circumferential buckles. The buckles are randomly distributed on the specimens. For the one filled with lower density foam A, the buckles are more inward as compared with the one filled with higher density foam B. This is because the lower density foam has a weaker resistance to the shell inward movement. In addition, the

number of buckles is less for the one with higher density infill. The reason is also due to it has a higher resistance of the shell inward movement. As the compression progresses, S2FA has a dent deformed in the middle position while S2FB split up at the bottom edge.

Effect of Young's modulus of foam core (E_f) on buckling behavior

Referring to Table 4.2, it shows that sample with infill foam A has a significantly higher buckling load compared to the sum of individual shell and foam. This is because the in-filled foam helps to prevent the shell from buckling by maintaining its original structure. Distinct phenomenon was observed for foam B. The load P_c for the combined case had a negative percentage increase which means that individual components actually performed better. It should be noticed that sample with foam B fractured before the first peak load (Fig. 4.5). This means the specimen with foam B filling buckles before the first peak load reached.

Table 4.2

Experimental results for the effect of Young's modulus of foam core on buckling behavior

Series no.	E_f (MPa)	P_c (N)				% increase
		Shell _{Avg}	Foam _{Avg}	Shell + Foam _{Avg}	Shell with Foam _{Avg}	
Set 1A	20	849	270	1119	1373	22.7
Set 1B	200	883	6059	6942	6134	- 11.6

S2FA

$d=1mm$

S2FB

$d=1mm$

$d=3mm$

$d=3mm$

$d=5mm$

$d=5mm$

$d=10mm$

$d=10mm$

Fig. 4.4 The deformation mode of S2FA and S2FB.

S1

FB

S1FB

Fig. 4.5 Set 1B samples after quasi-static axial crushing. The individual shell S1, individual foam FB and the combined case S1FB (Shell **1** with infill of Foam **B**). It should be noticed that sample S1FB split up before the first peak load.

From Table 4.2, it can be found that sample filled with foam B can withstand a higher load. This is because foam B has higher Young's modulus than foam A. Thus, it can be concluded that the buckling resistance is enhanced by increasing the infill Young's modulus E_f .

Effect of shell's diameter-to-thickness (D/h) ratio on buckling behavior

To the study of effect of shell's D/h ratio on buckling behaviour, foam A is used as it is the most effective infill. The shell thickness h is approximately 0.10 mm for both samples, while diameter D is varied. Table 4.3 indicates that S2FA (Fig. 4.6) had a higher buckling resistance due to a larger shell's diameter to thickness ratio (S1FA: $D/h = 520$, S2FA: $D/h = 660$).

Table 4.3

Experimental results for the effect of shell's D/h ratio on buckling behavior

Series no.	D/h	P_c (N)				
		Shell _{Avg}	Foam _{Avg}	Shell + Foam _{Avg}	Shell with Foam _{Avg}	% increase
Set 1A	520	849	270	1119	1373	22.7
Set 2A	660	665	482	1147	1476	28.7

Fig. 4.6 Set 2A samples after quasi-static axial crushing. The individual shell S2, individual foam FA and the combined case S2FA (Shell 2 with infill of Foam A).

4.3 Finite Element Analysis

4.3.1 F.E. modeling

Modeling geometry and material properties

The structure investigated in this study is based on the preliminary tests which is a beverage can with an aluminium foam core. The general geometry of the structure is defined in Fig. 3.1. A coordinate system (x, y, z) is used with the origin located at the end of the tube on the middle plane. The coordinates x , y and z are in the directions of radial, circumferential and axial.

Following the geometrical dimensions in the tests, the shell has a length $L_s=85\text{mm}$,

a radius $R = 26\text{mm}$ and various wall thicknesses (h) from 0.26 to 0.029mm to give the radius/thickness ratios from 100 to 900. The shell is filled with foam core. The foam core thickness is fixed at a value of $R_f/R = 0.9$ for illustration, as different percentage of infill only changes the magnitude of post-buckling load while the shape remains the same.

The Young's modulus and Poisson's ratio of the shell are assumed to be 70GPa and 0.3, respectively. The Young's modulus of the foam core is varied (E_f/E_s from 10^{-5} to 10^{-3}) in order to study its effect on the post-buckling behavior. The Poisson's ratio of the foam core is 0.1 in each case.

Element

The shell was modeled using the 4-node doubly curved thin shell element, S4R, with reduced integration and hourglass control. The element shape chosen was *Quad* as the shell is very thin, and thus it can be treated as a two-dimensional region. Element type C3D8R was chosen as the foam core, which is an 8-node linear brick, reduced integration and hourglass control element. The element shape chosen was *Hex* as the foam core is solid.

Analytical steps

In the present study, nonlinear analyses were conducted using the modified Riks

method. This method is particularly suitable for obtaining nonlinear static equilibrium solutions for unstable problems, in which the load and/or the displacement may decrease as the solution evolves. Here, only the geometric non-linearity is considered while the material properties of the shell and foam core are taken as linear-elastic.

Boundary and loading conditions

A uniform axial compression loading was applied to the nodes of both the ends of the cylindrical shell. The loaded ends of the cylindrical shell were kept flat so that the axial component of the displacement was uniform. With reference to the coordinates in Fig. 3.1, both end edges were restricted in translational x - and y -directions, while the rotational DOF in x - and z -directions were fixed to be zero.

In Riks method, the loading condition is similar as in eigenvalue buckling analysis. The magnitude of load and the displacement is unknown and the load is governed by a single scalar parameter (LPF , load proportionality factor). A live load of 10000 N is applied on the shell in the simulation.

Contact between the shell and foam core

In order to make sure that there is no relative motion between the shell and foam core during buckling, the inner surface of the shell and the outer surface of the foam

core were assumed to tie together. This type of contact only constrains translational degrees of freedom of the structural elements involved. Although the meshes created on these two surfaces are different, “tie” constraint can fuse these two regions.

Imperfection

Due to the bifurcation at the point of buckling, ABAQUS cannot directly analyze the post-buckling problem. Therefore, a geometric imperfection should be introduced into the perfect model to turn such a discontinuous case into a continuous problem.

Previous study by Seide (1962) found that the buckling loads of axially compressed thin cylindrical shells are sensitive to the presence of initial geometric imperfections. The initial geometric imperfections are subject to fabrication process variation. Therefore, the choice of imperfection is a critical task for better accuracy.

As described in the literature review, all of such load-deflection post-buckling analyses only show the worst imperfection form depending on the structure geometry and loading. General conclusions could not be obtained from their analyses. Therefore, the imperfection used in Mandal and Calladine (2000) was modified to have distinctive features along both the circumference and radius with

the localized buckle in this study.

In order to achieve the distinctive features, a function $e^{-k_i\theta_i^2}$ is added to the localized inward initial displacement. Therefore, the imperfection introduced to the shell can be expressed as,

$$w_i = w_0 \cdot \cos(n_1\theta_1)e^{-k_1\theta_1^2} \cdot \cos(n_2\theta_2)e^{-k_2\theta_2^2} \quad (4.1)$$

where, w_0 is the maximum amplitude of imperfection, pointing inwards and expressed as one quarter of the shell thickness,

$$w_0 = 0.25h \quad (4.2)$$

the terms of $\cos(n_1\theta_1)e^{-k_1\theta_1^2}$ and $\cos(n_2\theta_2)e^{-k_2\theta_2^2}$ are the imperfection components in circumferential and longitudinal direction and the detailed descriptions of them are provided below. It should be noticed that the subscript 1 stands for the variables in circumferential direction while 2 for longitudinal direction.

Imperfection in circumferential direction

In order to have distinctive feature around the circumference with the localized buckle, a function $e^{-k_1\theta_1^2}$ is added to the localized inward initial displacement. This is strongly decaying in the circumferential direction which is shown as Fig. 4.7. Therefore, the imperfection shape in this direction can be expressed as:

$$w_c = A \cdot \cos(n_1\theta_1)e^{-k_1\theta_1^2} \quad (4.3)$$

where, A is the amplitude of imperfection, pointing inwards. n_1 has various

amplitudes and is given by classical analysis of buckling under radial-inward pressure. As shown in Fig. 4.8, this parameter, n_1 , is the number of completed cosine wave between $[0, 2\pi]$.

Fig. 4.7 Shell imperfection in circumferential direction.

Fig. 4.8 Decayed cosine wave in the circumferential direction. The parameter k_1 is used to relate the radial displacement w_{c1} and w_{c2} .

Therefore, the expression of r in the above two figures can be presented as:

$$r = \frac{2\pi}{4n_1} \quad (4.4)$$

θ_1 is a circumferential angle as shown in Fig. 4.6,

$$\frac{\theta_1}{2} = r \quad (4.5)$$

Substitute equation (4.4) into (4.5) and rearrange,

$$\theta_1 = \frac{\pi}{n_1} \quad (4.6)$$

With the approximation of the amplitude of arc length s_1 pointed to θ_1 , we can get:

$$s_1 = 2r \quad (4.7)$$

Due to,

$$s_1 = R\theta_1 \quad (4.8)$$

where R is the shell radius. Substitute equation (4.6) into (4.8) and rearrange, therefore the expression of n_1 is:

$$n_1 = \frac{R\pi}{s_1} \quad (4.9)$$

k_1 is a constant and was selected to fulfill the relationship of the radial displacement w_{c1} and w_{c2} as shown in Fig. 4.8:

$$\beta_1 \cdot w_{c1} = w_{c2} \quad (4.10)$$

here, β_1 is a scalar and selected by the user. By using equation (4.3) into (4.10) with different circumferential angle at w_{c1} and w_{c2} :

$$\beta_1 \cdot A \cdot \cos(n_1 \cdot 0) e^{-k_1 \cdot 0^2} = A \cdot \cos(n_1 \theta_1) e^{-k_1 \theta_1^2} \quad (4.11)$$

$$k_1 = \frac{\log[\cos(n_1 \theta_1)] - \log(\beta_1)}{(\theta_1)^2} \quad (4.12)$$

Imperfection in longitudinal direction

Similar procedures are repeated in longitudinal direction. By referring to Fig. 4.9, the imperfection shape in this direction can be expressed as:

$$w_l = B \cdot \cos(n_2 \theta_2) e^{-k_2 \theta_2^2} \quad (4.13)$$

where all the parameters hold similar meaning with previous part.

Fig. 4.9 Shell imperfection in longitudinal direction.

In this case, assume the length of the shell is the circumference of a circle. Thus,

$$L_s = 2\pi \cdot r_2 \quad (4.14)$$

Therefore,

$$r_2 = \frac{L_s}{2\pi} \quad (4.15)$$

As cosine is an even function, only half wave of the cosine wave under $n_2 = 1$ is considered as shown in Fig. 4.10. In this case, the dimple occurs at the centre of the height.

Fig. 4.10 Cosine imperfection wave

From the figure, the relation between z and θ_2 can be obtained as,

$$\theta_2 = \left(\frac{fL_s - z}{fL_s} \right) \cdot \pi \quad (4.16)$$

where f is the fraction of height where dimple occurs.

The k_2 can be obtained by,

$$\beta_2 \cdot w_{l1} = w_{l2} \quad (4.17)$$

Similar with previous section, the expression of k_2 is,

$$k_2 = \frac{\log[\cos(n_2\theta_2)] - \log(\beta_2)}{(\theta_2)^2} \quad (4.18)$$

Fig. 4.11 The configuration of shell with initial cosine wave geometric imperfection. Using MATLAB, all the perfect shell nodes in polar coordinate is adding with the imperfections and become the one as shown above.

With $r = 15\text{mm}$, $f = 0.5$, $\beta_2 = \beta_1 = 1/200$, all the perfect shell nodes in polar coordinate (R, θ, z) and the imperfections in coordinate $(R + w_i, \theta, z)$ are plotted in Matlab, as shown in Fig. 4.11.

4.3.2 F.E. result

Load vs. deformation

A nonlinear analysis was performed for shells with or without foam core. Fig. 4.12 shows a typical load-end shortening response of the same shell with different Young's modulus of infill. It is evident that the structure behavior was linear elastic up to the point of maximum load. After the end-shortening curve reaches its first peak, the axial load begins to drop and finally maintains constant. This drop slows down with the foam core introduced. With an increasing Young's modulus till $E_f/E_s = 10^{-2}$, this drop is getting less significant while load keeps almost constant near the peak value.

There is an interesting phenomenon in the load-displacement curve. From the load-displacement curve of empty shell, as in Fig.4.13, the load remains virtually constant while the inward displacement increases quickly to several times of the shell thickness. This flat-plateau is also reported in Mandal and Calladine (2000). The constant load is referred to as 'the critical post-buckling load' in the following context.

Fig. 4.12 A typical load-end shortening response of the same shell with five different Young's modulus of infill. $L_s = 85\text{mm}$, $R = 26\text{mm}$, $h = 0.1\text{mm}$, $E_s = 70\text{GPa}$, $\nu_s = 0.3$, $R_f/R = 0.9$ and $\nu_f = 0.1$.

Fig. 4.13 Load against end shortening plot for an empty shell with $L_s = 85\text{mm}$, $R = 26\text{mm}$, $h = 0.1\text{mm}$, $E_s = 70\text{GPa}$ and $\nu_s = 0.3$.

It should be noticed that foam core only increases the initial buckling load slightly while it improves the critical post-buckling load considerably (Fig. 4.12). As the observed stabilizing effect of the foam core in experiment is significant, it seems that the critical post-buckling load dominates the experimental behavior.

Fig. 4.14 shows the structure deformation. An isolated dimple occurs immediately after the maximum load and it grows in size as the load falls from its peak. Since the tube with imperfection is symmetric about x - y and x - z planes, only one quarter of the structure was modeled to simplify the analysis, as the highlighted part in this figure. One should note that all conditions, such as shape, boundary conditions and loading are symmetrical in the simulation. In reality it is hard to satisfy these conditions. The use of one-quarter part of the specimen might miss some possible deformation modes¹.

Effect of R/h and E_f on post-buckling behavior

The load–displacement curves obtained by axial compression of the tubes are shown in three diagrams (Fig. 4.15) with different Young’s modulus of foam core (E_f). Within each figure, it was found that the shell radius-to-thickness ratio, R/h , has a significant influence on the critical buckling and post-buckling load. The structure buckles earlier for thinner shell wall (those with a higher R/h ratio as R is

¹ A complete model instead of one-quarter cut-out could be investigated in future studies, if stability might be a concern.

Fig. 4.14 Structure deformation with a large inward displacement after the maximum load. One quarter of the symmetric part is highlighted.

constant at 26mm). In addition, the critical buckling and post-buckling load decreases as the R/h ratio increases. The decrease is more pronounced when the R/h ratio increases from 100 to 200.

As for Young's modulus of foam core (E_f), it was found that the strengthening effect is more significant for a harder foam core (a higher value of E_f/E_s). The structure buckles earlier for a softer foam core (a lower value of E_f/E_s). In addition, the plateau load is flatted for a lower value of E_f/E_s .

Fig. 4.15 Load–displacement curves for the axial compression of the tubes depicted by Young’s modulus of foam core as (a) $E_f/E_s=10^{-5}$, (b) $E_f/E_s=10^{-4}$ and (c) $E_f/E_s=10^{-3}$. $L_s = 85\text{mm}$, $R = 26\text{mm}$, $E_s = 70\text{GPa}$, $\nu_s = 0.3$, $R_f/R = 0.9$ and $\nu_f = 0.1$.

Distributions of stress and strain energy

Figs. 4.16 and 4.17 show the load-end shortening curves, isometric view of the tubes, von Mises stress and strain energy contours for empty shell and shell with foam core. Same dimensions of the shell are used for these two cases with length $L_s = 85\text{mm}$ and $R/h = 260$. The infill has a Young’s modulus ratio $E_f/E_s = 10^{-3}$.

With a deformation scale factor of two, it shows that the deformations occur near the imperfection region soon after the peak load is reached in Figs. 4.16 and 4.17.

(a)

(b)

a

b

c

d

e

f

(c)

a

b

c

d

Fig. 4.16 (a) Plots of load vs. end shortening, (b) the tube deformations, (c) the von Mises stress states, and (d) plots of the strain energy states of the empty shell at various loading stages with $L_s = 85\text{mm}$, $R = 26\text{mm}$, $h = 0.1\text{mm}$, $E_s = 70\text{GPa}$ and $\nu_s = 0.3$.

As the load drops to a certain level, this region buckles with a significant inward deformation and the displacement near the imperfection gradually grows comparable to the radius of the shell. This deformation near the imperfection region is delayed with foam core added in.

The stress and strain energy contours are uniformly distributed in all areas of the shell, and increase with increasing load before the load reaches a critical value. In the specimen with such an imperfection, stress and strain energy contours are totally symmetrical. The stress and strain energy are concentrated in the imperfection region with an increasing load, until it reaches to a constant value.

Comparing Figs. 4.16 and 4.17, it shows that with an increasing Young's modulus of foam core, stress and energy distributions become more uniform before and after buckling. It approaches uniform stress and energy distributions in the shell, except

(a)

(b)

a

b

c

d

e

f

(c)

a

b

c

d

Fig. 4.17 (a) Plots of load vs. end shortening, (b) the tube deformations, (c) the von Mises stress states, and (d) plots of the strain energy states of shell with foam-core at various loading stages with $L_s = 85\text{mm}$, $R = 26\text{mm}$, $h = 0.1\text{mm}$, $E_s = 70\text{GPa}$, $\nu_s = 0.3$, $R_f/R = 0.9$, $E_f/E_s = 10^{-3}$ and $\nu_f = 0.1$.

Fig. 4.18 Comparison of the experimental and numerical results for (a) empty shell, (b) Shell 1 with Foam A, and (c) Shell 1 with Foam B. The experimental critical buckling load is close to numerical critical post-buckling load.

Shell 1 with Foam A

Shell 1 with Foam B

Fig. 4.18 Continued

for regions around the imperfection. This explains why the buckling load increases with the presence of infill and the increase is more significant with a harder infill.

4.3.3 Empirical equation

Simulation results are compared with experimental results in Fig. 4.18. It is evident that the experimental buckle happens near the numerical post-buckling region, with a value close to numerical post-buckling load. For the ease of computation, a buckling load prediction factor K_p is defined for a thin-walled cylindrical shell with foam core. It is the ratio of the numerical post-buckling load ($F_{p,num}$) with an initial geometric imperfection, to the experimental buckling load ($F_{cr,exp}$) as,

$$K_p = \frac{F_{p,num}}{F_{cr,exp}} = \begin{cases} 1.3 & (\text{shell with foam core}) \\ 0.53 & (\text{empty shell}) \end{cases} \quad (4.19)$$

Note that due to the heavy computation, $F_{p,num}$ is the one with 10% of infill. Extracting the post-buckling value in each line of Fig. 4.15 with the above equation and using logarithmic plot of normalized post-buckling stress versus radius/thickness ratio of shell, it was found the buckling stress follows a similar trend to Mandal and Calladine (2000)'s result for empty shell in the range of $R/h = 100-300$. Then the change slows down approaching a constant value when $R/h > 300$ (Fig. 4.19). This means the real buckling stress with foam core follows the power -1.5-line when $R/h < 300$, beyond which the foam core becomes the dominant body to sustain the stress. For $E_f/E_s = 10^{-3}$, it was found there is no shell component of the

buckling stress beyond $R/h=300$ as the shell and foam core split before it buckles.

The imperfection sensitivity of the buckling can be studied by altering the amplitude of the imperfection, w_0 . In addition to $w_0 = 0.25h$, the study is performed with $w_0 = 0.5h$ and $w_0 = h$, which are twice and four times of the previous value. The investigation revealed that the amplitude of the initial imperfection has little effect on either the initial buckling load or the "plateau" load of the post-buckling (Mandal and Calladine, 2000).

4.4 Theoretical Analysis

As described in the previous section, the buckling behavior of the structure is linear elastic up to the point of maximum load. After the end-shortening curve reaches its first peak, the axial load begins to drop and finally maintains constant. This constant load is referred to as 'critical post-buckling load'. In addition, the buckling behavior is very sensitive to the initial imperfection. In this section, the plateau load for the structure with a specific imperfection is formulated by energy method and solved under the assumption of *inextentional* deformation.

4.4.1 Formulation of the plateau load

In the structure deformation, an isolated dimple was found immediately after the peak-buckling load and it grows in size at the more-or-less constant load. This

Fig. 4.19 Logarithmic plot of normalized postbuckling stress versus radius/thickness ratio of shell, for three different normalized Young's modulus of foam core. $L_s = 85\text{mm}$, $R = 26\text{mm}$, $E_s = 70\text{GPa}$, $\nu_s = 0.3$, $R_f/R = 0.9$ and $\nu_f = 0.1$.

“plateau” load was found to be proportional to $h^{2.5}$ for the empty shell (Mandal and Calladine, 2000). Researches by Hotala (1996) and Guggenberger *et al.* (2000) also confirmed this approximate 2.5-power law for the plateau load in the empty shell.

Mandal and Calladine (2000) used a simpler analogue of the *inextensionally* dimpled cylindrical shell in the plateau load study for empty shell. This idea was

originated with a thin-walled elastic spherical shell which is inverted axisymmetrically by an inward-directed radial force P (Fig. 4.20). This figure shows the cross-sectional view of the spherical shell with radius a and thickness h , and the dimple has a radius r and subtending an angle ψ . The centre of the dimple has been pushed inwards which could be envisaged as the top portion of the spherical shell that was cut out, flipped and united. The most important point is that the inverted surface is isometric with the original spherical shell and the deformation is *inextensional*.

Fig. 4.20 Cross-section of an elastic thin-walled spherical shell that is being inverted by a central force P . The shell has a radius a , a thickness h and a central deflection w_0 . At a stage when there was a knuckle, it has a radius r and subtending an angle ψ .

The inward-directed radial force P required to hold the dimple in place in the spherical shell was reported in Holst and Calladine (1994). Mandal and Calladine

(2000) and Guggenberger *et al.* (2000) found this force P is insensitive to the dimple width and expressed as the function of inwards deflection w_0 ,

$$P = 1.7 E_s h^{2.5} w_0^{0.5} / a \quad (4.20)$$

Based on this result, Mandal and Calladine (2000) evaluated the total elastic strain energy in a cylindrical shell and derived the expression for the axial force F which keeps the dimple in place as,

$$F = C_s E_s h^{2.5} / R^{0.5} \quad (4.21)$$

Obviously, it is a direct consequence of the 2.5-power law for the plateau load F in cylindrical shell.

Inspired with similar idea for the empty cylindrical shell, the plateau load for the shell with foam core is derived in this study. In the general case of buckling, an element of the shell is subject simultaneously to the deformation of bending and stretching, compression by the applied force and the restraint by the infill. By assuming the foam only exerts a compressive pressure on the shell, the total potential of the system, U , is given by,

$$U = U_{bs} + V + U_r \quad (4.22)$$

where, U_{bs} is the strain energy due to bending and stretching, V is the work done by applied force and U_r is the strain energy stored in the foam core.

The strain energy due to bending and stretching can be obtained by evaluating $\int Pdw$ for the shell,

$$U_{bs} = \int Pdw = \int \left(C_s E_s h^{2.5} w^{0.5} / R \right) dw = C_s E_s h^{2.5} w^{3/2} / R \quad (4.23)$$

One should note that the stretching deformation is insignificant here. The reason is because, if the width of the dimple is smaller (Fig.4.20), the strain energy of meridional bending would be larger and the strain energy of the circumferential stretching would be smaller (Calladine, 2001).

The work done by the applied force during buckling is equal to the end-load on the cylinder, F , multiplied by the shortening of the axial length, u ,

$$V = -F \times u \quad (4.24)$$

The strain energy stored in the foam core,

$$U_r = \int P_f dw \quad (4.25)$$

where, P_f is the interfacial force and can be obtained by the pressure p_o on the foam core outer interface due to shell buckling. Based on the assumption of a linear, homogeneous and isotropic foam core, Timoshenko (1951) expressed the pressure p_o as,

$$p_o = \frac{E_f}{1 - \nu_f^2} \frac{w}{R} \frac{1 - \eta^2}{1 + \eta^2} \quad (4.26)$$

where, $\eta = R_f / R_s$.

Thus,

$$P_f = p_o(\pi r^2) = \frac{E_f}{1-\nu_f^2} \frac{w}{R} \frac{1-\eta^2}{1+\eta^2} (\pi r^2) = C_f \frac{r^2}{R} w \quad (4.27)$$

where, $C_f = \pi \frac{1-\eta^2}{1+\eta^2} \frac{E_f}{1-\nu_f^2}$.

Then, the total potential energy stored can be expressed as,

$$U = C_s E_s h^{2.5} w^{3/2} / R - Fu + C_f r^2 w / R \quad (4.28)$$

This assumes U as the dominant elastic energy stored with the elastic strain energy in the *inextensionally* deformed portions of the shell ignored. Thus, the axial force F which let the dimple hold in position can be obtained by,

$$\frac{dU}{du} = 0 \quad (4.29)$$

With the simple geometry, Mandal and Calladine (2000) expressed the axial displacement u of the top of the dimple using a Taylor expansion to first-order as,

$$u = 2 \left[r - R \sin\left(\frac{r}{R}\right) \right] \approx \frac{r^3}{3R^2} \quad (4.30)$$

By using the small angle approximation,

$$wa \approx r^2 \quad (4.31)$$

Here we made the assumption that $R \approx a$, such that²

$$wR \approx r^2 \quad (4.32)$$

² This empirical approximation is validated, as compared with our experimental results later.

Therefore,

$$F = \left[10 + \frac{1-\eta^2}{1+\eta^2} \frac{E_f}{E_s} \left(\frac{R}{h} \right)^{2.5} \right] E_s h^{2.5} / R^{0.5} \quad (4.33)$$

where $C_s = 10$, $C_f = \frac{1-\eta^2}{1+\eta^2} E_f$ are obtained from the numerical result and

$(1-\nu_f^2)$ is simplified to be 1.

It should be noticed that the two parts in Eq. (4.33) represent forces on the shell and foam core respectively. The equation can therefore predict the individual force borne by the shell and foam core. For example, when the coefficient of foam core equals the term of the shell,

$$\frac{1-\eta^2}{1+\eta^2} \frac{E_f}{E_s} \left(\frac{R}{h} \right)^{2.5} = 10 \quad (4.34)$$

As the specimens used in the test are fully filled ($\eta=0$), we have

$$\frac{E_f}{E_s} \left(\frac{R}{h} \right)^{2.5} = 10 \quad (4.35)$$

When $R/h=100$,

$$\frac{E_f}{E_s} = \frac{10}{(R/h)^{2.5}} = 10^{-4} \quad (4.36)$$

Fig. 4.21 plots the total axial force F , which holds the dimple in place, as well as forces on the shell and foam core. In this plot, the shell has a radius $R=26\text{mm}$, a Young's modulus $E_s=70\text{GPa}$ and its R/h ratio is varied from 100 to 900. For the foam core, it is fully filled with a Young's modulus ratio $E_f/E_s=10^{-5}$.

Fig. 4.21 Plot of the total axial force F as well as its two force components in shell and foam core. $R=26\text{mm}$, $E_s=70\text{GPa}$, $\eta=0$ and $E_f/E_s=10^{-5}$.

4.4.2 Comparison with FE. analysis and experiments

Fig. 4.22 compares the theoretical prediction with the results from the numerical study and the tests for different foam density. In each plot, the colored lines indicate the peak buckling force and plateau force in numerical analysis and the solid line represents the theoretical prediction. Similar trend is observed for the effect of shell thickness on buckling load between the theoretical prediction and the numerical analysis. As shown in Fig 4.23, the buckling of shell with foam core takes place around displacement of 2mm. At this amount of displacement, the force on foam alone is 11.9% of shell with foam core and will be ignored in the theoretical study. The plots show good correlation between the theoretical prediction, the numerical

Fig. 4.22 Comparison among the numerical study, theoretical prediction and the test results for (a) Shell with Foam A and (b) Shell with Foam B. The plots show that the theoretical prediction for plateau load agrees well with the numerical analysis as well as the test data.

Fig. 4.23 Force-displacement curves of Set 1A. It reveals that the buckling of shell with foam core takes place around displacement of 2mm. At this amount of displacement, the force on foam alone is 11.9% of shell with foam core and will be ignored in the theoretical study.

analysis as well as the measurement.

4.5 Summary

In the present study, preliminary experimental investigations have been carried out to test the response of a thin-walled cylindrical shell with and without foam core subjected to axial compression. Based on the findings from the experiments, corresponding numerical simulation analysis was performed using ABAQUS software. With a geometric imperfection added in, the simulations provide valuable insights into the behavior of the tubes.

The post-buckling dimple is identified which governs the post-buckling behavior. Structures with various R/h ratios and E_f are investigated. It shows that the Young's modulus of foam core and shell thickness have significant influence on the buckling capacity and performance of the tube. The buckling resistance is higher with stiffer infill and/or thicker shell wall. In addition, the structure buckles earlier for that with thinner shell wall as well as the one with softer foam core (smaller E_f). The plateau load is more flat with softer infill.

The numerical analyses also indicate that the presence of imperfection could significantly reduce the buckling capacity. Compared with experimental findings, it reveals that the actual critical buckling load is closely located in the numerical post-buckling region. Thus, it can be deducted that the critical post-buckling load dominates the experimental behavior. Meanwhile, a buckling load prediction factor is introduced to correct the discrepancy generally encountered between the experiment buckling load and numerical analysis result.

Finally, an analytical formula has been established to find the plateau load under the assumption of *inextensional* dimple deformation for the thin-walled cylindrical shell with foam core. The result shows that the post-buckling dimple requires a compressive force proportional to $h^{2.5}$ to hold it in place. The theoretical prediction for plateau load agrees well with the numerical analysis as well as the test data.

Empowered with this knowledge, the buckling load of such structures could be predicted more accurately.

CHAPTER 5

ENERGY ABSORPTION IN AXIAL CRUSHING - AXISYMMETRIC MODE

“Mechanics is the paradise of mathematical science because here we come to the fruits of mathematics.”

Leonardo da Vinci (ca. 1500)

5.1 Introduction

A thin-walled tube filled with light-weighted foam has wide engineering applications because of its excellent energy absorption capacity. When the structure is axially crushed, the interaction between the tube and foam core plays an important role in its energy absorption performance. Previous theoretical studies so far have largely been concerned with *fully* in-filled tubes. In addition, the approach to quantitatively estimating the interactive effect is complex.

To fill these gaps, in this chapter a theoretical model is proposed to predict the axisymmetric crushing behavior of such structures but with a partial infill. The division between axisymmetric and non-axisymmetric behavior primarily depends on the D/h , according to Lu and Yu (2003). In this chapter, axisymmetric crushing behavior is investigated, which refers to tubes with $D/h < 50$. Using a modified model for shell and considering the volume reduction for the foam core, the mean crushing force is predicted by the energy balance. A parametric study is carried out

to examine the contribution of foam core plateau stress (σ_f), amount of filling and shell's radius-to-thickness ratio (R/h) on the axial crushing behavior of the structure. Furthermore, a finite element simulation using axisymmetric elements is presented to provide some validations to the analytical model.

5.2 Theoretical Analysis

5.2.1 Analytical model for shell

The present analytical model is largely based on that developed by Huang and Lu (2003). The axisymmetric crushing model for a non-filled cylindrical shell with an initial radius R and a thickness h can be represented in Fig. 5.1. This is one complete folding cycle, which repeats through the whole progressive collapse. Within each fold, there are two stages. One is for segment 1 to complete, as in Fig. 5.1(a) to (c). The other is for the completion of segment 2, which is depicted from Fig. 5.1(c) to (e). These two-stage folding cycle leads to the periodic variation in a

Fig. 5.1 Successive deformation stages in one cycle. Within each crushing cycle, there are two stages. First stage: (a)-(c), the process for segment 1 to complete. Second stage: (c)-(e), the process for the completion of segment 2.

typical load-displacement curve for the axisymmetric collapse of a metal tube, apart from the initial peak, which is governed by local buckling.

Fig. 5.2 Instantaneous profile of the transition zone which is a zoomed-in view of Fig. 5.1(b). There are three segments consisted within each element: two equal radius arcs connected with a straight line which is tangent to the arcs.

The detailed shape of the deformation zone is shown in Fig. 5.2. Three segments are included within each element: two equal radius arcs connected with a straight line which is tangent to the arcs. The length of all the four arcs is equal to the effective hinge length, aH by definition. Note that H is the half plastic wavelength which will be determined in the analysis. For simplicity, no variation of H is assumed during the collapse. The parameter a takes a value between 0 and 1 (Wierzbicki *et al.*,

1992).

At a given axial crushing distance, the shape and instantaneous position of the deformation zone is defined by either the instantaneous radii r_1 , r_2 , or the two angles, α and β , and they are related as,

$$r_1 = \frac{aH}{2\alpha}, \quad r_2 = \frac{aH}{2\beta} \quad (5.1)$$

During the two stages of the collapse, both α and β increase, corresponding to axial shortening. Furthermore, α and β can be related by adopting the eccentricity factor m , which is the ratio of the outward fold length to the total fold length. The outward fold length is $A'D - D'G$, as shown in Fig. 5.2. The length of $A'D$ is related to the angle α and can be found easily as

$$A'D = H \left[2r_1(1 - \cos \alpha) + (1 - a)\sin \alpha \right] = H \left[\frac{a}{\alpha} (1 - \cos \alpha) + (1 - a)\sin \alpha \right] \quad (5.2a)$$

Similarly we can express $D'G$ as

$$D'G = H \left[2r_2(1 - \cos \beta) + (1 - a)\sin \beta \right] = H \left[\frac{a}{\beta} (1 - \cos \beta) + (1 - a)\sin \beta \right] \quad (5.2b)$$

The final angle α_f and β_f is equal to each other. Thus the total fold length is given by $\frac{a}{\alpha_f} (1 - \cos \alpha_f) + (1 - a)\sin \alpha_f$.

With (5.2a) - (5.2b), for the first stage of each fold,

$$\left[\frac{a}{\alpha} (1 - \cos \alpha) + (1 - a)\sin \alpha \right] - \left[\frac{a}{\beta} (1 - \cos \beta) + (1 - a)\sin \beta \right] = m \left[\frac{a}{\alpha_f} (1 - \cos \alpha_f) + (1 - a)\sin \alpha_f \right] \quad (5.2c)$$

Similarly for the second stage of each fold,

$$\left[\frac{a}{\beta} (1 - \cos \beta) + (1 - a) \sin \beta \right] - \left[\frac{a}{\alpha} (1 - \cos \alpha) + (1 - a) \sin \alpha \right] = (1 - m) \left[\frac{a}{\alpha_f} (1 - \cos \alpha_f) + (1 - a) \sin \alpha_f \right] \quad (5.2d)$$

By taking the derivative of (5.2c), the angular rates $\dot{\alpha}$ and $\dot{\beta}$ for the complete load cycle can be related by adopting the eccentricity factor m as well,

$$\left[\frac{a}{\alpha^2} (\alpha \sin \alpha - 1 + \cos \alpha) + (1 - a) \cos \alpha \right] \dot{\alpha} = \left[\frac{a}{\beta^2} (\beta \sin \beta - 1 + \cos \beta) + (1 - a) \cos \beta \right] \dot{\beta} \quad (5.2e)$$

In this way, the current model can be solved through either α or β with a given value of m as a one-dimensional system.

It is convenient to define an angle α_0 in the first element as a reference configuration when the second element is vertical. The limits for α and β in the first stage are $\alpha_0 \leq \alpha \leq \alpha_f$ and $0 \leq \beta \leq \beta_0$, whereas $0 \leq \alpha \leq \alpha_0$ and $\beta_0 \leq \beta \leq \beta_f$ for the second stage. The values of the initial angle α_0 and β_0 may not be the same while the final angle α_f and β_f are equal to each other. The initial angle (α_0 and β_0) can be determined from the model continuity requirement during crush as,

$$\frac{a}{\alpha_0} (1 - \cos \alpha_0) + (1 - a) \sin \alpha_0 = m \left[\frac{a}{\alpha_f} (1 - \cos \alpha_f) + (1 - a) \sin \alpha_f \right] \quad (5.3a)$$

$$\frac{a}{\beta_0} (1 - \cos \beta_0) + (1 - a) \sin \beta_0 = (1 - m) \left[\frac{a}{\beta_f} (1 - \cos \beta_f) + (1 - a) \sin \beta_f \right] \quad (5.3b)$$

In addition, the final angle can be obtained from the geometry of the fully collapsed model as,

$$\cos \alpha_f = \frac{a}{2(1-a)} \frac{1 - 2 \sin \alpha_f}{\alpha_f} \quad (5.4)$$

When a is zero, α_f is equal to 180° .

Finally, the shortening of the column in a complete crushing cycle at any value of the process parameter, α , is expressed as,

$$\delta = \left[2 + \frac{a}{\alpha_0} \sin \alpha_0 + (1-a) \cos \alpha_0 \right] H - \left[\frac{a}{\alpha} \sin \alpha + (1-a) \cos \alpha + \frac{a}{\beta} \sin \beta + (1-a) \cos \beta \right] H \quad (5.5)$$

The effective crush distance is defined as $\delta_{eff} = \delta_{final} - \delta_{initial}$ over a complete cycle, thus

$$\delta_{eff} = \Delta(a)H \quad (5.6)$$

where, $\Delta(a) = 2 - a/\alpha_f$. When a is zero, δ_{eff} is $2H$.

Plastic energy stored due to bending (E_b)

During one complete fold cycle, the external work done is dissipated by the plastic energy of bending and stretching. Bending energy only occurs in the arc sections with a curvature $k = 1/r$ during continuous bending. The rate of bending energy is,

$$\dot{E}_b = \int_{ABCDEFG} |M_0 \dot{k}| 2\pi R_0 ds \quad (5.7)$$

where $M_0 = \sigma_0 h^2 / 4$ and the rates of curvature in the first stage of collapse are,

$$\dot{k} = -\frac{\dot{r}_1}{r_1^2} = \frac{2\dot{\alpha}}{aH} \text{ (Sections AB and CD)} \quad (5.8a)$$

$$\dot{k} = -\frac{\dot{r}_2}{r_2^2} = \frac{2\dot{\beta}}{aH} \text{ (Sections DE and FG)} \quad (5.8b)$$

Thus, the rate of bending energy dissipation can be obtained as,

$$\dot{E}_b = 4\pi R_0 M_0 \left(|\dot{\alpha}| + |\dot{\beta}| \right) \quad (5.9)$$

In the second stage of collapse, the expression for bending energy rate is the same as the above equation. Integrating Eq. (5.9), the bending energy dissipation in a complete crushing cycle can be obtained as,

$$E_b = 8\pi R_0 M_0 A_1 \quad (5.10)$$

where $A_1 = \alpha_f$.

Membrane strain energy due to stretching (E_m)

The membrane energy from the circumferential strain is,

$$\dot{E}_m = 2\pi R_0 N_0 \int_s \left| \frac{\dot{w}}{R_0} \right| ds \quad (5.11)$$

where $N_0 = \sigma_0 h$ is the fully plastic membrane force per unit length. Note that no interaction is assumed between bending moment and membrane force. s is the coordinate measured along the deformation zones (see Fig. 5.2). $\dot{w}$ denotes the radial velocity of the tube wall and has different expressions for different element components, i.e., in the first stage of collapse,

On section AB,

$$\dot{w} = \frac{aH}{2\alpha^2} (\phi \sin \phi - 1 + \cos \phi) \dot{\alpha} \quad (5.12a)$$

On section BC,

$$\dot{w} = \left[\frac{aH}{2\alpha^2} (1 - \cos \alpha) - \frac{aH}{2\alpha} \sin \alpha - s \cos \alpha \right] \dot{\alpha} \quad (5.12b)$$

On section CD,

$$\dot{w} = \left[\frac{aH}{2\alpha^2} (1 - 2 \cos \alpha + \cos \phi + \phi \sin \phi - 2\alpha \sin \alpha) - (1 - a)H \cos \alpha \right] \dot{\alpha} \quad (5.12c)$$

Here, the angular variable ϕ and length variable s are $0 \leq \phi \leq \alpha$ and $0 \leq s \leq (1 - a)H$ in the above three equations. The expressions of $\dot{w}$ in the second stage of collapse for sections DE, EF and FG are the same as those for CD, BC and AB, respectively, except that parameter α is replaced with β . Integrating Eq. (5.11), the membrane energy dissipation in a complete crushing cycle can be obtained as,

$$E_m = 4\pi N_0 H^2 A_2 \quad (5.13)$$

where $A_2 = \int_0^{\alpha_f} f(\alpha) d\alpha$ and the function $f(\alpha)$ is,

$$f(\alpha) = \frac{a^2 \sin \alpha}{\alpha^3} + \frac{1}{2} \frac{a(1 - \cos \alpha) - a^2(1 + \cos \alpha)}{\alpha^2} - \frac{1}{2} \frac{a \sin \alpha}{\alpha} - \frac{1 - a}{2} \cos \alpha \quad (5.14)$$

Note that neither total bending energy nor membrane energy depends on the eccentricity parameter m , although eccentricity is allowed to exist in the fold formation analysis (Wierzbicki *et al.*, 1992).

5.2.2 Analytical model for foam core

Energy stored in the infill due to its volume reduction (E_f)

The crushing model for the foam core refers to the aluminium foam, as in the previous studies (Seitzberger *et al.*, 1997, 2000; Reyes *et al.*, 2004; Hanssen *et al.*, 1999, 2000; Santosa *et al.*, 1998, 2000). Under uniaxial crushing, it can be treated as a perfectly compressible material and thus its energy absorption rate can be expressed as,

$$\dot{E}_f = \int_{v_0} \sigma_f \dot{\varepsilon} dV \quad (5.15)$$

Supposing that Eq. (5.15) is valid for all combinations of principal strain rates $\dot{\varepsilon}_i$ ($i=1, 2, 3$), the equivalent strain can be written in the function of volumetric relation,

$$\varepsilon_f = \frac{V_0 - V_{final}}{V_0} = \frac{\Delta V}{V_0} \quad (5.16)$$

where V_0 , V_{final} and ΔV are the initial volume, the final volume and volume reduction of the foam column, respectively.

Assume that an original cylindrical foam of length $2H$ and inner radius R_f deforms into one with outer diameter $R-2r$ but the same inner radius, as shown in Fig. 5.3. In addition, no local indentation is assumed as well. The initial volume of foam column can be expressed as,

$$V_0 = 2H \cdot \pi (R^2 - R_f^2) \quad (5.17)$$

while the final volume of foam column is,

Fig. 5.3 Successive deformation shape of a thin-walled cylindrical shell with hollow foam core (axial section).

$$V_{final} = (2H - \delta) \cdot \pi \left[(R - 2r)^2 - R_f^2 \right] \quad (5.18)$$

where $r=r_1=r_2$ in the final volume.

Integrating Eq. (5.15), the energy absorption for uniform volume distribution of foam core is,

$$E_f = \sigma_f \Delta V \quad (5.19)$$

It should be noticed that independent interaction yield surface is assumed for the use of Eq.(5.19) when calculating the energy stored in the infill.

5.2.3 Solution by energy method

Assume that the foam-filled tube collapses in the same mode as an empty one, but with the different wavelength and effective crushing distance due to the presence of the foam. For a complete crushing cycle, the total work done by the mean external crushing load is equal to the energy dissipation,

$$P_m \delta = E_b + E_m + E_f \quad (5.20)$$

Therefore, the mean crushing force can be obtained as,

$$P_m = \left\{ M_0 \left(C_1 \frac{H}{h} + C_2 \frac{R}{H} \right) + \sigma_f \left[C_3 H^2 + C_4 RH + C_5 (1 - \eta^2) R^2 \right] \right\} \frac{2H}{\delta} \quad (5.21)$$

where $\eta = R_f / R$ and the corresponding coefficients are,

$$C_1 = 8\pi A_2(a), C_2 = 4\pi A_1(a), C_3 = \pi(\Delta(a) - 2)^3 / 2, C_4 = \pi(\Delta(a) - 2)^2 \text{ and } C_5 = \pi\Delta(a)/2. \text{ Note that } A_1, A_2 \text{ and } \Delta(a) \text{ are as given earlier.}$$

Eq. (5.21) could be minimized with respect to H to predict the mean crushing force, which is,

$$2C_3\sigma_f H + C_4\sigma_f R + C_1M_0h^{-1} - C_2M_0RH^{-2} = 0 \quad (5.22)$$

The half wavelength H and corresponding mean crushing load P_m could be easily solved using numerical iteration method by substituting geometrical parameters a, R, h and η , material parameters σ_0 and σ_f , coefficients C_1, C_2, C_3, C_4 and C_5 into Eq. (5.21) and (5.22).

It should be noticed here that the parameter a is the only unknown and can be determined by the effective hinge length. From Huang and Lu (2003), the effective hinge length can be expressed as,

$$aH = bh \quad (5.23)$$

where the value of constant b was between 2 to 4, based on their experimental observation. In the following analysis, an average value of $b=3$ was selected.

5.2.4 Comparison with experiments

Previous experimental data for axial crushing of thin-walled cylindrical shell with fully filled foam core are available by Reddy and Wall (1988). In their study, beer can was completely filled with rigid polyurethane foam. The seamless aluminium alloy can had a length of 148mm, a diameter of 65mm and a thickness 0.115mm. Fig. 5.4 compares the results from predictions of our theoretical model and their tests. A good agreement is observed for the effect of foam core density on mean crushing force between our prediction and their experimental results. The minor discrepancy is possibly due to the effect of the relatively thick top and bottom rings in the specimen on the deformation process. In addition, the results from the empirical prediction by Hanssen *et al.* (2000) are plotted in Fig. 5.4. Good agreements are observed between our model and their findings.

Fig. 5.4 Mean crushing force versus the density of the foam core. $L=148\text{mm}$, $D=65\text{mm}$, $h=0.115\text{mm}$, $\sigma_0=550\text{MPa}$ and $\eta=0$.

5.2.5 Discussion and design consideration

Parametric studies

Parametric studies are performed for the effects of key parameters on the crushing behavior of the structure. Case studies are carried out with shell dimensions of $L=85\text{mm}$, $R=26\text{mm}$ and $h=0.52\text{mm}$. A flow stress σ_0 of 295MPa is used. The foam core's central bore hole radius R_f and plateau stress σ_f are varied to explore the effect of foam core (η from 0 to 1 and σ_f/σ_0 from 1×10^{-4} to 7.8×10^{-2}).

Effect of η and σ_f on mean crushing force (P_m)

The mean crushing force increases with an increasing percentage of infill (i.e., decreasing value of η) as well as the infill plateau stress (σ_f), as expected (Fig. 5.5). The plot shows that for the tube with a very soft foam core (e.g. $\sigma_f/\sigma_0 = 1 \times 10^{-4}$), the amount of infill has little effect on the mean crushing force. As the foam core becomes stronger, the mean crushing force becomes higher. For a given value of σ_f , a smaller η leads to a higher mean crushing force.

Fig. 5.5 Plot of mean crushing force against the percentage of infill, for different values of σ_f/σ_0 . $L = 85\text{mm}$, $R = 26\text{mm}$, $h = 0.52\text{mm}$ and $\sigma_0 = 295\text{MPa}$.

Effect of shell's R/h ratio and σ_f on the half wavelength (H)

From Eq. (5.22), the influence of shell's radius-to-thickness ratio (R/h) as well as foam core plateau stress (σ_f) on the half wavelength, H , is plotted with different percentages of infill in Figs. 5.6 (a) and (b) in a double-logarithmic scale. One should note the half wavelength H here is normalized by the square root of (Rh), as R/h is non-dimensional. In addition, the analytical prediction of empty shell by Alexander (1960) is plotted as well for comparison.

Fig. 5.6(a) Double logarithmic plot of normalized half wavelength versus R/h , for different values of σ_f/σ_0 . $L = 85\text{mm}$, $R = 26\text{mm}$, $\sigma_0 = 295\text{MPa}$ and $\eta = 0.5$ (50% infill).

Fig.5.6(b) Double logarithmic plot of normalized half wavelength versus R/h , for different values of σ_f/σ_0 . $L=85\text{mm}$, $R=26\text{mm}$, $\sigma_0=295\text{MPa}$ and $\eta=0$ (100% infill).

For a fixed radius R and a given value of σ_f/σ_0 , $H/\sqrt{Rh}$ decreases with a larger R/h . For a given value of R/h , the strength of filler σ_f leads to a different $H/\sqrt{Rh}$. It shows that $H/\sqrt{Rh}$ is smaller for the tubes with infill and the amount of shortening increases with a larger value of σ_f/σ_0 . In addition, the amount of shortening also increases as the percentage of infill increases (η decreases), as shown by comparing Fig. 5.6(a) (50% infill) and (b) (100% infill).

Design consideration

In practical design, it is paramount to minimize the cost and maximize the efficiency. In the present study, the mean crushing force P_m obtained in Fig. 5.5 is normalized with respect to (m_f+m_s) , which is the mass of the foam core and shell per unit length in the axial direction. This is the same as the specific energy absorption, which is energy absorbed per unit mass of the structure. In the calculation for foam plateau stress σ_f , the foam core density ρ_f is related to it by a power law provided by Hanssen *et al.*(2000) as,

$$\sigma_f = 3640 \left[\frac{\rho_f}{\rho_{f0}} \right]^{3.0} \quad (5.24)$$

where $\rho_{f0} = 2.7 \text{ g/cm}^3$ is the foam base material density.

Variation of the specific mean crushing force per unit length with different percentages of infill is shown in Fig. 5.7. It can be noted that, as the percentage of infill increases (η decreases) till 50%, the specific mean crushing force per unit length reaches constant thereafter. This indicates that for practical design, the infill should be at least 50% from the outer radius, for improved efficiency in energy absorption. However, filling more than 50% does not further enhance the efficiency considerably.

Fig. 5.7 Plot of specific mean crushing force per unit length against the percentage of infill, for six different values of σ_f/σ_0 . $R = 26\text{mm}$, $h = 0.52\text{mm}$ and $\sigma_0 = 295\text{MPa}$.

5.3 Finite Element Analysis

In order to verify the analytical prediction and to further understand the mechanics of the crushing behaviors of foam-filled columns, a finite element analysis was conducted using ABAQUS. One of its three core products, ABAQUS/Explicit, is chosen since it uses explicit integration assignment to solve highly nonlinear transient dynamic and quasi-static analyses.

5.3.1 F.E. modeling

Modeling geometry and material properties

The analysis was carried out by axial crushing of a foam-filled deformable tube between two rigid end blocks. An axisymmetric F.E. model is used as shown in Fig. 5.8. This is to force the tube to deform in the axisymmetric mode, the same as that in the analytical model. The tube has a length $L=85\text{mm}$, an initial mid-surface radius $R=26\text{mm}$ and wall thickness $h=1.0\text{mm}$. The foam core is filled in the shell with an inner radius $R_f=0.3R$.

All the material models were time-independent, finitely deforming and elastic-plastic solid which hardens isotropically. The shell has a flow stress σ_0 of 295MPa, a Young's modulus E of 70GPa and a Poisson's ratio ν of 0.3. The plastic stress-strain behavior for shell component follows the tensile test data of an aluminium beverage can coupon (Fig. 5.8).

The isotropic hardening model was used for the crushable foam. This phenomenological constitutive model was originally developed by Deshpande and Fleck (2000). It assumes symmetric behavior in tension and compression and the evolution of the yield surface is governed by an equivalent plastic strain from both the volumetric and deviatoric plastic strain. In this case, the foam model is using the

Fig. 5.8 The uniaxial tensile stress-strain curve of profile shell material.

Fig. 5.9 The stress-strain curve of aluminum foam.

aluminium foam with a density of 0.125g/cm^3 . Fig. 5.9 shows the engineering stress-strain curve of the foam in axial compression. The end planes use steel blocks with a density of 7.8 g/cm^3 , a Young's modulus of 210GPa and a Poisson's ratio of 0.3 .

Element

The axisymmetric solid element type CAX4R was selected as the element type of entire domain for this axisymmetric collapse analysis. It is a four-nodded, bilinear axisymmetric quadrilateral and reduced integration element with hourglass control. As relatively high density of elements are in the axial direction of shell component, convergence studies were performed on the determination of element numbers through the shell thickness with the square shape in order to ensure that it is sufficient for the forming of final tightly spaced folds curvatures (see Fig. 5.1). In this case, the mesh contains 1308 elements and 1640 nodal points for the shell component, 51 elements and 72 nodal points for the foam core component, and 250 elements and 306 nodal points for the end plane components.

Analysis steps

In the present study, the quasi-static axial crushing analysis was conducted using the explicit method. This method is suitable for quasi-static analysis with complicated contact conditions. In this method, a large number of small time increments were

performed. The analysis of each increment is based on the previous step's result. Although it is more time consuming than the direct-integration dynamic analysis procedure in ABAQUS/Standard, it is more effective in non-linear analysis.

Boundary and loading conditions

A uniform axial compression was applied to the top plane while the bottom was totally fixed. The ends of the cylindrical shell were kept flat so that to make sure the axial displacement was uniform. The top edge was applied with a vertical displacement at a rate of 5mm/min which is followed the actual quasi-static crushing speed (refer to the next chapter).

Contact between the shell and foam core

In this analysis, "tie" constraint was used as well for the inner surface of the shell and the outer surface of the foam core. This is to make sure that there is no relative motion between them during axial crushing. Contact between the top platen and the tube was treated by defining two surface-to-surface contact pairs. The shell is treated via self-contact option for the contact in the folds forming process.

5.3.2 F.E. result

The analysis is performed for both an empty shell and the same shell with foam core. Fig. 5.10(b) shows the final deformed configurations. Their major deformation

characteristics appear similar to those in the analytical model. In addition, it reveals that the half plastic wavelength is less when the foam core is introduced. The values of their mean crushing forces are listed in Table 5.1. It shows that the mean crushing force is higher with foam core, as expected.

The load-displacement curve for an empty shell is shown in Fig 5.11 (a) and a set of successive deformed configurations for shell are presented in Fig. 5.11 (b). Initially, the response is elastic. The tube develops a local buckle close to the edge (b) that corresponds to the first peak load. After that, collapse continues [(c)] with the formation of an inward fold till (d). The load then increases to a new peak (e). With a new outward fold starts to form, there is a temporary decrease in the force corresponding to (f). This is a short-lived event as, after (g), the second fold continues to form [(h)]. The second folding is complete by (i) when the outer knee formed comes into contact with the first fold. The load increases once more until at (j) when a new inward fold starts to form again. These stages repeat in the whole progressively crushed tube process.

A comparison of the F.E. and theoretical results, with or without infill, is listed in Table 5.1. Good agreements are observed in the Table. Thus the analytical model gives a good prediction in term of mean crushing force as well as the half plastic wavelength with respect to the F.E. model.

Fig 5.10 (a) Initial geometries of axisymmetric models (axial section) for both empty shell and shell with 70% filled foam core. (b) Corresponding final deformed configurations. It is evident that their major characteristics are similar as in theoretical analysis. In addition, it reveals that the half plastic wavelength was shorter with the foam core added in.

Table 5.1

Comparison of F.E. and theoretical results for both empty shell and foam-filled structure

	F.E. results	Theoretical results
Empty shell		
H	7.1 mm	7.3 mm
P_m	20.1kN	19.6kN
Shell with 70% foam core		
H	6.6 mm	6.8 mm
P_m	40.8 kN	42.7kN

5.4 Experimental Study

5.4.1 Specimen and apparatus

The tube specimens were made of aluminium 6061. Three different dimensions were used with different thickness and diameter. For the purpose of easy reference, the tubes were named in alphabetical order with Tube A having the largest diameter and thickness while Tube C is the one with the smallest diameter (see Fig. 5.12).

The aluminium tubes were cut to a length of around 100mm by a radial arm saw (Fig. 5.13a). It ensures that the aluminium tubes were cut 90° perpendicular to the tubes. However, this method of cutting makes the edges very rough with slight variance in length. In order to ensure that the end faces finishing was fairly accurate and perpendicular to the specimen's axis, the tubes were further processed on a lathe machine (Fig. 5.13b).

(a)

(b)

Fig. 5.11(a) Calculated force-displacement responses for empty shell. (b) Sequence of calculated deformed configurations showing the progressive crushing of empty shell (correspond to numbered points in (a)). Its major characteristics are similar as used in theoretical analysis.

Fig. 5.12 The tube specimens in this study. They were made by aluminium 6061 and there are three different sizes of the tube. For the purpose of easy reference, the tubes were named in alphabetical order with Tube A having the largest diameter and thickness while Tube C is the one with the smallest diameter.

Fig. 5.13 (a) A radial arm saw which used to cut the aluminium tubes to a length of around 100mm; (b) The tubes were further turned on a lathe machine, in order to ensure that the end faces were perpendicular to the specimen's axis.

The yield strength of the aluminium tubes was obtained through the transverse loading experiments by the formula,

$$\sigma_y = \frac{F_0 R}{h^2 L} \quad (5.25)$$

where F_0 is the average force of the transverse test result. The dimensions and material property of the aluminium tubes are listed in Table 5.2. The corresponding curves under transverse loading are shown in Fig 5.14.

Table 5.2

The dimensions and material property of the aluminium tubes

	OD (mm)	ID (mm)	h (mm)	L (mm)	D/h	F_0 (N)	σ_y (MPa)
Tube A	50.7	48.4	1.2	96	42	924	186.1
Tube B	37.9	34.9	1.5	99	25	2259	188.7
Tube C	31.8	29.7	1.1	99	29	1434	209.9

(b)

(c)

Fig. 5.14 The transverse loading test curves of (a) Tube A, (b) Tube B, and (c) Tube C.

Two different types of core material are investigated to study the effect on axial crushing behaviour. The core materials to be used are extruded polystyrene ($\rho=0.017\text{g/cm}^3$) and expanded polystyrene ($\rho=0.036\text{g/cm}^3$) as shown in Fig. 5.15. All the foams were cut into cylinders to fit into the respective shells using specially sharpened aluminium tubes, shown in Fig. 5.16. This ensures a tight fit when the core material is placed in the aluminium tubes (see Fig. 5.17).

Fig. 5.15 Two different types of core material are investigated to study the effect on axial crushing behaviour. (a) Extruded polystyrene ($\rho=0.017\text{g/cm}^3$) and (b) expanded polystyrene ($\rho=0.036\text{g/cm}^3$).

Fig. 5.16 (a) The foams were cut into cylinders to fit into the respective shells using specially sharpened aluminium tubes; (b) Foam cutting tool for aluminium tube A and aluminium tube B&C.

Fig. 5.17 Tube specimens and corresponding infilled cases with extruded polystyrene and expanded polystyrene.

The axial crushing experiments were conducted on the Instron 5500r machine operating at a crosshead speed of 5mm/min. This allowed the tests to be conducted by a quasi-static load instead of a constant loading force. Traces of compressive load against displacement were recorded automatically using the software of Bluehill.

5.4.2 Test result

The test results with different sizes of aluminium tube as summarized in Table 5.3. All the results are taken at a crushing distance of 50mm. The corresponding

force-displacement curves are shown in Fig. 5.18. In addition, the uniaxial compression test results for the polystyrene foams are shown in Fig.5.19.

Table 5.3

The experimental results at 50mm crushing distance

	Tube A ($OD=50.7\text{mm}$, $D/h=42$)		
	Empty tube	With expanded polystyrene	With extruded polystyrene
F_{max} (N)	35193	36725	35170
F_{ave} (N)	12468	14014	12517
E (J)	608	701	623
$\eta = \frac{F_{ave}}{F_{max}}$	0.35	0.38	0.36
	Tube B ($OD=37.9\text{mm}$, $D/h=25$)		
	Empty tube	With expanded polystyrene	With extruded polystyrene
F_{max} (N)	34355	34461	33410
F_{ave} (N)	19313	19508	18433
E (J)	969	979	925
$\eta = \frac{F_{ave}}{F_{max}}$	0.56	0.57	0.55
	Tube C ($OD=31.8\text{mm}$, $D/h=29$)		
	Empty tube	With expanded polystyrene	With extruded polystyrene
F_{max} (N)	20141	22045	21898
F_{ave} (N)	9538	11240	11182
E (J)	477	563	560
$\eta = \frac{F_{ave}}{F_{max}}$	0.47	0.51	0.51

(a)

(b)

(c)

Fig. 5.18 Force-displacement curves categorized by the different sizes of aluminium tube.

(a) Tube A, (b) Tube B and (c) Tube C.

(a)

Fig. 5.19 The uniaxial quasi-static compression test results of (a) expanded polystyrene; and (b) extruded polystyrene.

By referring to the deformation of the tubes (Fig. 5.20) at the same crushing distance, axisymmetric axial crushing mode can be observed. This is the same as defined in the theoretical study. Table 5.3 shows that Tube B has the highest energy absorbed, F_{ave} and crushing efficiency, η . The empirical predictions by Guillow *et al.* (2001) also confirmed this. It reveals that the mean crushing force is related to diameter to thickness ratio (D/h), shell wall thickness (h) and yield strength (σ_y).

For the formed folding, Tube A has 5 folds while Tube B has 4 and Tube C has 6. At the same crushing distance, it indicates that the half wave-length (H) of Tube B is the largest, followed by Tube A and then Tube C. The thickness of the tube seems to

be the most important factor as it restrains the folds from forming. In other words, a thinner wall is more prone to bending and folding than a thicker wall.

Fig. 5.20 Axisymmetric axial crushing deformation mode of aluminium tube at the same crushing distance of 50mm. (a) Tube A, (b) Tube B, and (c) Tube C.

When the tubes are filled with foam core, there is an increase in the mean crushing force as well as the energy absorption capacity. The results in Table 5.3 and Fig.5.18 are higher for expanded polystyrene than extruded polystyrene. This is because expanded polystyrene has a higher density. For the same type of infill, the results show that Tube B has the highest energy absorbed, F_{ave} and crushing efficiency η .

Fig 5.21 presents the deformation mode of Tube A filled with extruded polystyrene core. It shows the start and completion of a fold, which is similar to theoretical description (Fig 5.1). Its mean crushing force (P_m) and half plastic wavelength (H) are compared with theoretical predictions and has good agreement (see Table 5.4).

Fig. 5.21 The deformation mode of Tube A filled with extruded polystyrene core. (a) A fold starts to form, and (b) A fold almost completed.

Table 5.4

Comparison of test and theoretical results for Tube A filled with extruded polystyrene core

	Experimental data _{ave}	Theoretical result
P_m	12.5 kN	12.0 kN
H	5.3 mm	5.1 mm

5.5 Summary

In the present study, an analytical model is established to predict the mean crushing force of a thin-walled cylindrical shell with partially filled foam core. Using the modified superfolding element model for the shell and the volumetric reduction model for foam core, the mean crushing force is predicted through energy balance method. As this formula relates the mean crushing force to the half plastic wavelength (H), the interactive effect can be easily evaluated from the change of H . The result reveals that filling a thin-walled cylindrical shell using a foam core can increase its mean crushing force as well as energy absorption. With foam core filled,

the half plastic wavelength (H) shortens. The amount of shortening increases when the plateau stress of foam core or shell wall thickness increases, as well as when the percentage of infill increases (η decreases). It has been also found that for practical purpose the in-filler inner radius should be at least 50% of the shell radius.

The proposed theoretical model was validated by an axisymmetric finite element model in ABAQUS as well. Furthermore, experimental study was carried out on aluminium 6061 tubes filled with expanded and extruded polystyrene core. The test results confirmed our theory. This study can give valuable design guidelines in the use of a thin-walled structure as an energy absorber.

CHAPTER 6

ENERGY ABSORPTION IN AXIAL CRUSHING - NON-AXISYMMETRIC MODE

“...those experiments be not only esteemed which have an immediate and present use, but those principally which are of most universal consequence for invention of other experiments, and those which give more light to the invention of causes; for the invention of the mariner's needle, which give the direction, is of no less benefit for navigation than the invention of the sails, which give the motion.”

Sir Francis Bacon (1852)

6.1 Introduction

A thin-walled tube filled with light-weighted foam has wide engineering applications because of its excellent energy absorption capacity. Previous studies so far have largely been concerned with *fully* in-filled tubes. Therefore, current investigation is focused on a partially in-filled tube. Axisymmetric crushing mode has been studied in Chapter 5. In this chapter, a study is carried out in non-axisymmetric crushing mode which is associated with tubes of $D/h > 80$.

A total of 48 quasi-static axial crushing tests were conducted on circular aluminium alloy 3104-H19 tubes filled with hollow aluminium foam. The D/h ratio studied here is up to 660. Six different percentages of aluminium foam (0%, 20%, 40%, 60%, 80% and 100%) were studied to investigate the axial crushing behavior. In addition to the infill amount, the effect of foam density and shell diameter was studied in the experiment. Based on the test results, corresponding finite element

simulations were performed to simulate the crushing behavior of the tested specimens for further investigation.

6.2 Experimental Study

6.2.1 Specimens and apparatus

The tests were performed with an Instron 5500 universal machine with accuracy within $\pm 5\%$ of indicated load. The load was applied at a cross-head speed of 5mm/min. Traces of compressive load against displacement were recorded automatically using the software of Bluehill. Six types of tube structure were investigated in this experiment, i.e. 0% in-filled tube (empty shell), 20% in-filled tube, 40% in-filled tube, 60% in-filled tube, 80% in-filled tube and 100% in-filled tube, as shown in Fig. 6.1.

Fig. 6.1 Six types of tube structure were investigated in this experiment. (a)empty shell, (b)20% in-filled tube, (c)40% in-filled tube, (d)60% in-filled tube, (e)80% in-filled tube and (f)100% in-filled tube.

The shell specimens were cut from commercial beverage cans, which are made of aluminium alloy 3104-H19. Two different values of the diameters, $D_1=52\text{mm}$ and

$D_2=66\text{mm}$, are used and denoted as S1 and S2 (Fig. 6.2). The two ends of the beverage can were cut off using Electrical Discharge Machining (EDM) technique, with the final height equals the infill of 56mm.

Fig. 6.2 The two types of shell specimens which were cut from commercial beverage cans, made of aluminium alloy 3104-H19, with two different values of the average diameters, being $D_1=52\text{mm}$ and $D_2=66\text{mm}$ and denoted as (a)S1 and (b)S2. The two ends of the beverage can were cut off using the Electrical Discharge Machining (EDM) technique, with the final height equals the infill of 56mm.

Tensile tests were performed on the coupons with a parallel section of 10mm wide and 45mm long, which were cut from the each kind of beverage cans by EDM in order to obtain their mechanical properties. The coupon was then fitted with an extensometer and tested in Instron 5500 universal machine. The engineering stress-strain curves for each shell are shown in Fig. 6.3.

Fig. 6.3 The uniaxial tensile stress-strain curves of profile material. The tensile tests were performed on the coupons with a parallel section of 10mm wide and 45mm long which were cut from the each kind of beverage cans by EDM in order to obtain their mechanical properties.

In addition, Vickers hardness tests were performed to check the material uniformity of the shell specimens. For each kind of shell specimen, S1 and S2, ten tests with 1kg load were conducted and the results of Vickers Pyramid Number (HV) are listed in Table 6.1. It was found that the hardness variation is within $\pm 5\%$. According to Tabor (1951), these results can be used to measure the flow stress of the shell specimens by

$$\sigma_y = H/3 \quad (6.1)$$

It was found the mean values of $H/3$ obtained here (S1: $\sigma_y = 239.0$ MPa, S2: $\sigma_y = 234.3$ MPa) have good agreement with the initial yield stresses obtained in tensile tests (Fig. 6.3).

Table 6.1

Vickers hardness test results

Test	Hardness Value (HV)*										Ave.
	1	2	3	4	5	6	7	8	9	10	
S1	72.3	75.1	72.8	70.5	73.3	73.6	73.1	74.9	71.2	74.5	73.1
S2	71.3	68.5	71.5	71.8	72.1	73.2	69.9	71.1	70.8	72.4	71.7

*Vickers hardness value normally is expressed as a number only (without the unit of kgf/mm^2).
To convert HV to MPa: multiply by 9.807.

Two types of aluminium foam core, denoted as FA and FB in Fig. 6.4, are used with a density of 0.125g/cm^3 and 0.472g/cm^3 respectively. All the foams were cut into cylinders by EDM to fit into the respective shells, with a gap of $+0.05\text{mm}$. The uniaxial quasi-static compression test results of the foams are shown in Fig. 6.5.

Fig. 6.4 Two types of aluminium foam core used in this study, denoted as (a) FA and (b) FB with a density of 0.125g/cm^3 and 0.472g/cm^3 respectively. All the foams were cut into cylinders by EDM to fit into the respective shells, with a gap of $+0.05\text{mm}$.

In this study, the main parameter is the percentage of foam filled. Meanwhile, it is also very important to investigate the interactions between this parameter and other model parameters such as shell diameter and foam density. Therefore, an experiment program was adopted here for convenience of analysis.

(a)

(b)

Fig. 6.5 The uniaxial quasi-static compression test results of (a) foam A with a density of 0.125g/cm^3 , (b) foam B with a density of 0.472g/cm^3 .

As seen in Table 6.2, five different percentages of foam infill (20%, 40%, 60%, 80% and 100%) were studied in addition to the non-filled shell. Thus, all the

characteristic tests were $6 \times 2 \times 2 = 24$ and each test was performed twice (a and b), resulting in a total of 48 tests. Based on this program, a test numbering system has been applied for the whole study, e.g. S1FA20a corresponds to S1: shell diameter 1, FA: foam density ρ_A , 20: foam filled percentage 20%, and a : repetition a .

Table 6.2

Experimental test program

	Parameters			Repetition
	Shell diameter	Foam density	Foam filled percentage	
	D_1	ρ_A	0	a
	D_2	ρ_B	20	b
			40	
			60	
			80	
			100	
No. of levels	2	2	6	2

The two values of shell diameter are $D_1=52\text{mm}$ and $D_2=66\text{mm}$: see Fig. 6.2, and the two types of aluminium foam core has the density of $\rho_A=0.125\text{g/cm}^3$ and $\rho_B=0.472\text{g/cm}^3$ respectively: see Fig. 6.4. Based on this program, a test numbering system has been applied for the whole study, e.g. S1FA20a corresponds to S1: shell diameter 1, FA: foam density ρ_A , 20: foam filled percentage 20%, and a : repetition a .

6.2.2 Test results

Deformation mode

The deformation mode of non-filled shell is presented in Fig. 6.6a. When it is axially crushed, a non-axisymmetric buckling mode is observed in the elastic range, resulting in several axial and circumferential buckles. As compression progresses,

several adjoining circumferential buckles merged to produce a large dent on one side around the 2/3-region of the specimens with the other large dent near 1/3-region of the other side of the specimen. The deformation forms an angle $\theta = 37^\circ$ from the horizontal axis and a fold length $2H=37\text{mm}$ as indicated in the figure. Meanwhile, the top and bottom surfaces were kept flat but not co-axial.

Fig. 6.6(a) The deformation mode of non-filled shell. When it is axially crushed, a non-axisymmetric buckling mode is observed in the elastic range, resulting in several axial and circumferential buckles. As compression progresses, several adjoining circumferential buckles merged to produce a large dent on one side around the 2/3-region of the specimens with the other large dent near 1/3-region of the other side of the specimen. The deformation forms an angle $\theta = 37^\circ$ from the horizontal axis and a fold length $2H=37\text{mm}$ as indicated. Meanwhile, the top and bottom surfaces were kept flat but not co-axial.

For the filled specimens, it was found that the structures were less compressible as compared with the non-filled specimen. The deformation for lower density foam infill usually started at the lower end of the specimen. As the crushing continues, a

large dent was formed as well, but with a smaller formed angle and fold length than the empty tube case. For instance, the deformation angle in S1FA40a is $\theta = 25^\circ$ and the fold length $2H=27\text{mm}$. The angle and fold length was further reduced with more infill (Fig. 6.6b). This is because the infill resists the inner movement as well as the shear force of the outer shell. The level of the resistance mostly depends on the foam core property. Tubes having higher density foams were deformed with two or three folds either in the top or the bottom. The outer shell was split up from the last fold and the foam became the sole energy absorber in Fig. 6.6c. It should also be noted that more infill could extend the displacement before the outer shell splits.

Fig. 6.6(b) The deformation mode of shell filled with lower density foam at $z=10\text{mm}$. The deformation from lower density foam usually started at the lower end of the specimen. As the crushing continues, a large dent was formed as well, but with a smaller formed angle and fold length than the empty tube case. The angle and fold length was further reduced slightly with more infill

Crushing force and energy absorption

Typical compressive force-displacement curves for a thin-walled cylindrical shell with foam core (S1FA100b) are shown in Fig. 6.7. It shows that the compressive force of shell with infill exceeds the sum of individual shell (S1b) and foam (FA100b). Referring to the force-displacement plot, enhancement of mean crushing force and energy absorbing capacity is not only related to the crush strength of the foam, but also subject to its interaction with the container. This interactive effect between the shell and foam core is the main factor that contributes to the enhancement.

Fig. 6.6(c) The deformation mode of shell filled with higher density foam at $z=10\text{mm}$. Two or three folds either in the top or the bottom were formed. The outer shell was split up from the last fold and the foam became the sole energy absorber. It should also be noted that more infill could extend the time before the outer shell splits.

The force-displacement curve of the shell specimen with or without infill shows that the axial force reaches an initial peak, followed by a drop and then crushing progresses at an almost constant or mildly increasing load with little fluctuation. After a certain level of compression, the load starts to increase rapidly, possibly indicating the starting of foam densification.

Fig. 6.7 Typical compressive force-displacement curves for a thin-walled cylindrical shell with foam core (S1FA100b). It shows that the compressive force of shell with infill exceeds the sum of individual shell (S1b) and foam (FA100b). Referring to the force-displacement plot, enhancement of mean crushing force and energy absorbing capacity is not only related to the crush strength of the foam, but also subject to its interaction with the container.

Fig. 6.8 compares the force-displacement curves of different percentages of foam infill with different foam densities as well as different shell's D/h ratios (h is keeping constant with $\sim 0.10\text{mm}$) at the same crushing distance of 25mm. Due to the good reproducibility of experiments, only one of the curves in the two repeated tests

is given. It was found that the crush force is much higher for the filled specimen as compared with the non-filled one. Thus, its stroke efficiency is smaller than for the non-filled case. In addition, the crushing load increases further with a higher amount or density of infill or a larger shell's D/h ratio.

(c)

(d)

Fig. 6.8 Force-displacement curves of different percentages of foam infill with different foam density as well as different shell's D/h ratio (h is keeping constant with $\sim 0.10\text{mm}$) at the same crushing distance of 25mm . (a)S1FA, (b)S1FB, (c)S2FA, and (d)S2FB. Due to the good reproducibility of experiments, only one of the curves is given.

In terms of energy absorption, due to the higher crushing resistance of the filled specimens, the structure with infill can absorb more energy than the empty shell

(Fig. 6.9). The presence of foam acts as a foundation and resists the inward movement of the tube wall despite having little crush resistance in itself. Thus, a higher amount or density of the infill or a larger shell's D/h ratio improves energy absorption.

(a)

(b)

(c)

(d)

Fig. 6.9 Energy absorption curves of different percentages of foam infill with different foam density as well as different shell's D/h ratio (h is keeping constant with $\sim 0.10\text{mm}$) at the same crushing distance of 25mm. (a)S1FA, (b)S1FB, (c)S2FA, and (d)S2FB.

Fig. 6.10 shows that the specific energy absorption (S.E.A.) of specimens filled with foam core is lower than the corresponding empty shell, which agrees well with

the previous conclusion (Thornton, 1980). For the specimens filled with a higher density foam core (Fig. 6.10b and d), this phenomenon only exists in the initial 5mm crushing displacement. This is because the outer shell split here and left only the foam core to absorb the energy. Among the filled specimens, it reveals that

(c)

(d)

Fig. 6.10 Specific energy absorption (S.E.A., the ratio of total energy absorption vs. total mass of the specimen) curves of different percentages of foam infill with different foam density as well as different shell's D/h ratio (h is keeping constant with $\sim 0.1\text{mm}$) at the same crushing distance of 25mm. (a)S1FA, (b)S1FB, (c)S2FA, and (d)S2FB.

S.E.A. is higher with a larger amount, higher density of the infill or a lower value of the shell's D/h ratio.

The variation of the energy absorption as well as S.E.A. with different percentages of infill is summarized in Fig. 6.11. It can be noted that, as the percentage of infill increases (η decreases), the energy absorption and S.E.A. increase as well. For the same amount of infill, that with higher density of infill has a better energy absorption and S.E.A. The plot reveals that although the shell with smaller D/h ratio has a worse energy absorption capacity, it can have a better S.E.A.

Effect of infill amount on axial crushing behavoir

Increased infill leads to a larger crush load, energy absorption and S.E.A. as shown in Figs. 6.8-6.10. However, the stroke efficiency decreases. This is because the

(a)

(b)

Fig. 6.11 The summarized variations of (a) the energy absorption as well as (b) S.E.A. with different percentages of infill. It can be noted that, as the percentage of infill increases (η decreases), the energy absorption and S.E.A. increase as well, where η is the percentage of infill. For the same amount of infill, the one with higher density of infill has a better energy absorption and S.E.A. The plot reveals that although the shell with smaller D/h ratio has a worse energy absorption capacity, it can achieve a better S.E.A.

resistance from the foam core added either by itself or by the interaction with the shell wall. Thus, the more the infill, the higher crushing force is and the more energy is absorbed.

A case study for the investigation of this effect on axial crushing behavior was performed by S1 filled with different amounts of foam A. Fig. 6.12 shows the top, bottom, front and back side views at a crushing displacement $\sim 40\text{mm}$ as well as the

deformation modes at several progressive crushing stages of the tested specimens. It reveals that the foam core resists the otherwise almost free inward movement of the shell wall, thus leading to the plastic deformation with a number of folds formed before the large dent forms. The inclined dent angle and fold length decreases with the foam core added and they are decreased slightly further as the amount of infill increase. This is because the infill resists the shear force of the outer shell. It should be noticed that there are one or two cracks appear in the outer shell for the cases with 40% infill and above. This is because the infill resists the inwards movement of the shell yet the shell is very thin, and it would split when there is not enough space for its inwards movement.

S1b S1FA20a S1FA40a S1FA60a S1FA80b S1FA100b
(a)

S1b S1FA20a S1FA40a S1FA60a S1FA80b S1FA100b
(b)

S1b S1FA20a S1FA40a S1FA60a S1FA80b S1FA100b
(c)

Fig. 6.12 Case study for the investigation of the effect of infill amount on axial crushing behavior was performed by S1 filled with different amount of foam A. Deformation modes are shown in the view of (a) top, (b)bottom, (c)front side and (d)back side views at the crushing displacement $\sim 40\text{mm}$, and (e)several progressive crushing stages.

Effect of foam density on axial crushing behaviour

Referring to Figs. 6.8-6.10, it shows that the shell specimen filled with higher density foam (FB) can withstand a larger crush load and has a better energy absorption capability as well as S.E.A. However, their stroke efficiency decreased as compared with foam A as the filler. This is because the resistance from the foam core gets stronger when the foam density increases. With a thin shell wall, cracks would appear when there is not enough space for the shell's inwards movement. This phenomenon is more pronounced for the specimens filled with higher density foam. Usually, their outer shell is totally split up during the crushing (Fig. 6.13).

For instance, the deformation modes of specimens filled with different foam densities, S2FA and S2FB, are compared in Fig. 6.13 with views in the top, bottom, front and back side. For specimens filled with lower density foam (S2FA), the crushing displacement is $\sim 40\text{mm}$ and the outer shells are still around the foam core with only one or two cracks. While for specimens filled with higher density foam

(e)

Fig. 6.12 Continued

S2FA20a

S2FA40a

S2FA60a

S2FA80a

S2FA100b

S2FB20a

S2FB40a

S2FB60a

S2FB80b

S2FB100b

(a)

S2FA20a

S2FA40a

S2FA60a

S2FA80a

S2FA100b

S2FB20a

S2FB40a

S2FB60a

S2FB80b

S2FB100b

(b)

S2FA20a

S2FA40a

S2FA60a

S2FA80a

S2FA100b

S2FB20a

S2FB40a

S2FB60a

S2FB80b

S2FB100b

(c)

Fig. 6.13 Deformation modes to show the effect of foam density on axial crushing behaviour.

The deformation modes of the specimens filled with different foam density, S2FA and S2FB, are compared in with the views in the (a) top, (b) bottom, (c) front side and (d) back side. For specimens filled with lower density foam (S2FA), the crushing displacement is ~ 40mm and the outer shells are still around the foam core with only one or two cracks. For specimens filled with higher density foam (S2FB), the outer shells are split up already at a crushing displacement ~ 25mm.

(S2FB), the outer shells are split up already at a crushing displacement ~ 25mm.

The deformation modes at several progressive crushing stages of the tested specimens S2FA60a and S2FB60a are presented in Fig. 6.14. It shows that there are a number of axial and circumferential buckles in the early stage. The one with higher density foam buckles earlier. This is possibly because of the weaker resistance of the foam initially provided to the tube wall. As compression progresses, densification of the foam occurs, resulting in an increase in foam's foundation stiffness due to the non-adhesion between the shell and foam core. After that an inclined large dent due to several adjoining circumferential buckles merged

Fig. 6.14 The deformation modes at several progressive crushing stages of the tested specimens S2FA60a and S2FB60a. It shows that the one with higher density foam buckles earlier. In addition, the outer shell started to split up at only 5mm crushing distance with a higher density infill (S2FB60a) and it was fully split up at 10mm. For the one with lower density infill, S2FA60a, the outer shell was still around during the crushing process.

is formed with a smaller angle measured from horizontal axis than the empty case. The fold length is reduced slightly as well. It reveals the fact that the higher the filler's density, the more reduction in formed angle and fold length. In addition, the outer shell started to split up at only 5mm crushing distance with a higher density infill (S2FB60a) and it is fully split up at 10mm. It is because the infill has a strong resistance yet the shell is very thin, therefore the outer shell would split when there is not enough space for its inwards movement. For the one with lower density infill, S2FA60a, the outer shell is still around during the crushing process.

The sectioned views of specimens' deformation are presented in Fig. 6.15. At a crushing displacement of ~15mm, foam A is trapped in the S2 diamond lobed folds and the foam inside forms the shear band at the location where outer shell is buckled inwards, as highlighted in the figure. With the percentage of infill increases, the resistance to the shell's inward movement increases as well. Besides, the foam is crushing toward the central axis. This is because the inner hole of the foam provides space for the inward movement as well. It should be noted that the outer shell split already for the higher density foam filler, foam B.

Fig. 6.15 The sectioned views of specimens' deformation with (a) S2FA, (b) S2FB. At a crushing displacement of $\sim 15\text{mm}$, foam A is trapped in the S2 diamond lobed folds and the foam inside forms the shear band at the location where outer shell is buckled inwards, as highlighted in the figure. With the percentage of infill increases, the resistance to the shell's inward movement increases as well. It should be noted that the outer shell split already for the higher density foam filler, foam B.

Effect of shell's D/h ratio on axial crushing behaviour

The shell thickness h in this study is approximately 0.10 mm for all the samples, while diameter D is varied. Figs. 6.8-6.10 indicate a higher crushing force and energy absorption capacity for a larger shell's diameter to thickness ratio (S1: $D/h = 520$, S2: $D/h = 660$). Earlier studies by Reddy and Wall (1988) and Guillow *et al.* (2001) confirmed this finding as well. However it has a lower stroke efficiency and S.E.A. for a large D/h ratio. As Thornton (1980) pointed out, it is possible to obtain a larger crushing load by increasing the thickness of the tube, which means a smaller D/h ratio, or by using stronger material. He argued that in either case similar or better S.E.A. can be achieved.

The deformation modes at several progressive crushing stages of the tested specimens S1FA60a and S2FA60a are presented in Fig. 6.16a. It shows that S1FA60a, with a smaller D/h ratio, has a larger angle formed by the dent and fold length. In addition, the deformation zone of S1FA60a is near the lower end of the specimen as compared with S2FA60a. That means a larger D/h ratio increases the resistance against collapse and results in less deformation. Therefore, it can withstand a higher crushing load resistance and has a better energy capacity.

For the specimen filled with higher density foam, FB, (Fig. 6.16b) deformation starts from the top side of S1FB20a, the one with a smaller D/h ratio, and its outer shell splits up soon after one axisymmetric fold ($2H=2\text{mm}$) is formed. However for S2FB20a which has a larger D/h ratio, the deformation zone is near the lower end of the specimen with a large dent formed before the outer shell fully splits up. It should be noticed that the outer shell can withstand longer crushing with a larger D/h ratio, which indicates a higher crushing load resistance. Fig. 6.16c shows various views of the higher density foam, FB, filled in different shells at the crushing displacement $\sim 25\text{mm}$. It confirms the facts as described above. Meanwhile, the sectional view in Fig. 6.16c iv as well as Fig. 6.15b reveal that there are less shear zones, as highlighted in the figures, for a smaller D/h ratio. That indicates a weaker resistance against the crushing load.

Fig. 6.16(a) The deformation modes at several progressive crushing stages of the tested specimens S1FA60a and S2FA60a. It shows that S1FA60a, with a smaller D/h ratio, has a larger angle formed by the dent and fold length. In addition, the deformation zone of S1FA60a is near the lower end of the specimen as compared with S2FA60a. That means a larger D/h ratio increases the resistance against collapse and results in less deformation. Therefore, it can withstand a higher crushing load resistance and has a better energy capacity.

(b)

Fig. 6.16(b) The deformation modes at several progressive crushing stages of the tested specimens S1FB20a and S2FB20a. For the specimen filled with higher density foam, FB, deformation starts from the top side of S1FB20a, the one with a smaller D/h ratio, and its outer shell splits up soon after one axisymmetric fold ($2H=2\text{mm}$) is formed. However for S2FB20a which has a larger D/h ratio, the deformation zone is near the lower end of the specimen with a large dent formed before the outer shell fully splits up.

S1FB40b S2FB40a S1FB80a S2FB80b S1FB100a S2FB100b

(i) Top view

S1FB40b S2FB40a S1FB80a S2FB80b S1FB100a S2FB100b

(ii) Front side view

S1FB40b S2FB40a S1FB80a S2FB80b S1FB100a S2FB100b

(iii) Back side view

S1FB40b

S1FB80a

S1FB100a

S1FB40b

S1FB80a

S1FB100a

(iv) Sectional view

(c)

Fig. 6.16(c) Various views of the higher density foam, FB, filled in different shells at the crushing displacement $\sim 25\text{mm}$. It confirmed the facts as described above. Meanwhile, the sectional view in (iv) as well as Fig. 15(b) revealed that there are less shear zones, as highlighted in the figures, for a smaller D/h ratio.

6.3 Finite Element Analysis

Numerical analysis was performed with the models corresponding to the real tests to have a better understand of the crushing behavior of foam-filled tubes. The numerical simulations were carried out using the general finite element program ABAQUS 6.9-1.

6.3.1 F.E. modeling

Modeling geometry and material properties

Finite element analysis resembling the experimental setup was conducted. Two different diameters, $D_1=52\text{mm}$ and $D_2=66\text{mm}$, were used for the shell with a thickness of $h=0.10\text{mm}$ and a height of $L=56\text{mm}$. Five different percentages of foam core (20%, 40%, 60%, 80% and 100%) were filled to investigate its effect on the axial crushing behavior.

The material properties for shell component were obtained from tensile tests of each beverage can coupons as mentioned in the experimental study. For instance, S1 shell specimen has a yield stress σ_y of 250MPa, an ultimate stress σ_u of 340MPa, a Young's modulus E of 70GPa and a Poisson's ratio ν of 0.3.

The isotropic hardening model was used for the crushable foam. This phenomenological constitutive model was originally developed by Deshpande and

Fleck (2000). It assumes symmetric behavior in tension and compression, and the evolution of the yield surface is governed by an equivalent plastic strain from both the volumetric and deviatoric plastic strain.

To define the shape of the yield surface, the yield stress ratio, k , is assumed equal to unity (Mirfendereski *et al.*, 2008). The other yield stress ratio, k_t , is 0.07 which is based on the ABAQUS user's manual. That sets the strength of the material in hydrostatic tension, p_t , equal to 7% of the initial yield stress in hydrostatic compression, p_c^0 , as $k_t = p_t/p_c^0$. The material properties for foam cores were obtained from axial crushing tests in the experimental study.

Element

The shell was modeled using the 4-node doubly curved thin or thick shell element, S4R, with reduced integration and hourglass control. The element shape was chosen *Quad* as the shell is very thin, and thus it can be treated as a two-dimensional region. Element type C3D8R was chosen as the foam core, which is an 8-node linear brick, reduced integration and hourglass control element. The element shape was chosen *Hex* as the foam core is solid.

A mesh sensitivity study was performed by using every new mesh size be the half of the previous one. The results revealed that using coarse meshes underestimates

the mean force. In addition, the resulting force converges to a unique value as the number of element increases. An adequate accuracy was achieved in this study using $3\text{mm} \times 3\text{mm}$ mesh size for shell and $1\text{mm} \times 1\text{mm}$ mesh size for foam core.

Analysis steps

The dynamic explicit method was used to study the axial crushing behavior of such structures. This method is suitable for quasi-static analysis with complicated contact conditions. In this method, a large number of small time increments were performed. It is therefore very effective since each increment analysis is based on previous result.

Boundary and loading conditions

In order to resemble the test, a uniform axial compression was applied to the top plane while the bottom plane was totally kept fixed. A vertical displacement rate of 5mm/min was applied for the top plane which simulates the loading cell crushing speed as in the physical test. In this way, both ends of the cylindrical shell were kept flat due to the end plates, so that the axial displacement was uniform.

Contact between the shell and foam core

Contact between the inner surface of the shell and the outer surface of the foam core

was modeled by using 'tie' in order to make sure that there is no relative motion between them during crushing. Although the meshes created on these two surfaces are different, 'tie' constraint can fuse these two regions. Besides, other contacts in the crushing process were defined as self-contact.

6.3.2 F.E. result

As shown in Fig. 6.17, the F.E. force-displacement history in a complete crushing process is compared with experimental tests. When an axial compression is applied on the specimen, S1FA100 for example, several axial and circumferential buckles firstly occur due to the crushing. As compression progresses, several adjoining circumferential buckles are merged to produce a large dent, which is shown at the crushing distance (d) of 5mm in Fig. 6.17. As the crushing continues, the formed dents are closer as shown in $d=10\text{mm}$, and more inward due to the crushing. Finally, the dent edges touched till a crushing distance of 20mm.

Meanwhile, the collapse modes at certain loading stages are compared as well for S1FA100 and S2FA100. The simulation and actual collapse modes show good agreement. It shows that S1FA100, the one with a smaller D/h ratio, has a slightly larger dent angle than S2FA100.

$z = 0$ mm

$z = 5$ mm

$z = 10$ mm

$z = 20$ mm

$z = 0$ mm

$z = 5$ mm

$z = 10$ mm

$z = 20$ mm

(a)

Fig. 6.17 F.E. force-displacement history in a complete crushing process compared with experimental tests. Meanwhile, the collapse modes at certain loading stages were compared as well for (a) S1FA100 and (b) S2FA100. The simulation and actual collapse modes show good agreement. It shows that S1FA100, the one with a smaller D/h ratio, has a slightly larger dent angle than S2FA100. Moreover, the deformation zone of S1FA100 is near the lower end of the specimen as compared with S2FA100.

Moreover, the deformation zone of S1FA100 is near the lower end of the specimen as compared with S2FA100. That means a higher D/h ratio results in a stronger resistance against collapse thus less deformation, or better energy absorption.

As the experiments reveal, it demonstrates that the infill resists the otherwise almost free inward movement of the shell wall. It reveals that increase percentage of the infill leads to a stronger resistance to the shell inwards movement (Fig. 6.18a, S2FA at crushing distance of 15mm). Meanwhile, the inclined large dent angle measured from horizontal line and the fold length decrease with the foam core added.

Fig. 6.18(a) Effect of different amount of infill on the crushing behavior in finite element simulations was compared with experimental tests with the instance of S2FA at crushing distance of 15mm.

Fig. 6.18b compares the force-displacement history as well as deformation mode at different collapse stages of S2FA40. Good agreements observed between the F.E.

and experiments. The force-displacement history of each component is plotted in the same graph. It shows that initially the main body withstanding the resistance is the shell component. The foam core is linearly increasing against the crushing load. This is because the density of the foam A is very low compared with the shell.

For S2FB40, the one with higher density infill, Fig. 6.18c compares the force-displacement history as well as deformation mode at different collapse stages. Since no failure criterion of the shell parts is considered in this simulation, the results differ from the experimental results. Since the density of the foam is very high compared with the outer shell, the foam core is the major load resistant component. The deformation mode at crushing distance of 15mm shows that the outer shell fractured already; this phenomenon is presented in F.E. as well with the high stress concentration as highlighted in the figure.

Fig. 6.18d shows the comparison of higher density foam, FB, filled in shells with different D/h ratio at the crushing displacement $\sim 25\text{mm}$. It reveals that the outer shell split up at a larger displacement in the specimen with a bigger D/h ratio, which indicates a higher crushing load resistance. Since no failure criterion of the shell parts is considered in this simulation, the split up zones in real test are presented by

Fig. 6.18(b) Comparison of force-displacement history as well as deformation mode at different collapse stage of S2FA40. Good agreement was observed between the F.E. and experiments. The force-displacement history of each component is plotted in the same graph. It shows that initially the main body withstanding the resistance is the shell component. The foam core is linear increasing against the crushing load. This is because the density of the foam A is very low compared with the shell.

Fig. 6.18(c) Comparison of force-displacement history as well as deformation mode at different collapse stage of S2FB40. Since no failure criterion of the shell parts was considered in this simulation, the results differ from the experimental results. The deformation mode at crushing distance of 15mm shows that the outer shell fractured already; this phenomenon is presented in F.E. as well with the high stress concentrated which highlighted in the figure.

the high stress concentrated zone highlighted in the figure. It is noted that the comparison of the split patterns and zone have good agreement with each other which reveals that the one with smaller D/h ratio has weaker resistance against the crushing load resistance, thus it has less energy absorption capacity.

Fig. 6.18(d) Comparison of higher density foam, FB, filled in shells with different D/h ratio at the crushing displacement $\sim 25\text{mm}$. Since no failure criterion of the shell parts is considered in this simulation, the split up zones in the physical test are presented by the high stress concentrated zone highlighted in the figure.

6.4 Summary

In the present study, experimental study of the axial crushing behavior of a thin-walled cylindrical shell with a hollow foam core was carried out. The study intends to figure out how the infill would influence the deformation modes as well as its force-displacement behavior.

For the deformation modes, the tube filled with lower density foam usually deforms from the lower end. As the crushing continues, a large dent is formed, but usually with a smaller angle than if an empty tube is used. The angle and the fold length would reduce slightly with more infill, as well as a bigger D/h ratio. For tubes

having higher density foams, they deform with two or three folds either in the top or the bottom. The outer shell split from the last fold and the foam became the sole energy absorber. Either more infill or a larger D/h ratio could extend the displacement before the outer shell splits.

From the force displacement behavior, it is found that the crushing load and energy absorption capacity increases with increased amount of infill, a higher density of infill or a bigger shell's D/h ratio. The added foam core is found to increase the crushing resistance beyond the sum of shell and foam acting alone due to the interactive effect. Besides, it is found that S.E.A. is higher with a higher amount of infill, a higher density of infill or a smaller shell's D/h ratio.

Based on the experiments, corresponding finite element simulations are performed for a better understanding of behavior. A good correlation is achieved between experimental and numerical simulation results. The proposed F.E. model can be considered as a valuable tool in assessing and understanding the deformation/failure mechanism and predicting the response of the axial crushing of a thin-walled cylindrical shell with a hollow foam core.

CHAPTER 7

CONCLUSION AND RECOMMENDATION

“There is nothing like looking, if you want to find something. You certainly usually find something, if you look, but it is not always quite the something you were after.”

J.R.R. Tolkien (1892-1973)

7.1 Conclusion

Buckling and energy absorption behavior of a thin-walled cylindrical shell with foam core is investigated in this research. Studies are carried out in the sequence of initial buckling, post-buckling and energy absorption. These three areas are divided according to the force-displacement curve and the outcome of this research is summarized in this chapter.

7.1.1 Initial buckling behavior of a thin-walled cylindrical shell with foam core

A direct and simple energy method was firstly used to formulate the critical buckling stress of such structure. After that, finite element analysis using ABAQUS software was carried out to verify this formulation and further explore the mechanism of its buckling behavior.

The theoretical study reveals that filling a foam core in a thin-walled cylindrical shell can enhance its resistance to buckling failure, as expected. This effect is more pronounced for those with either a thicker or a harder foam core, and the maximum

enhancement factor is 1.73. Poisson's ratio of the shell or foam core has little impact in calculating the critical buckling stress and thus it may be neglected without introducing much error.

It has also been found that for practical purpose the in-filler inner radius does not need to be more than 10% of the shell radius as any extra amount of filler would be uneconomical. After the parametric studies, a simplified formula was proposed, based on the distinct regimes of the structural behavior.

Besides, a finite element analysis was performed for the eigenvalue buckling analysis. Good agreement was achieved between F.E. results and theoretical prediction. This investigation can help engineers optimize the design in terms of strength, weight and cost.

7.1.2 Post-buckling behavior of a thin-walled cylindrical shell with foam core

The study in this area starts with an experimental investigation. Based on the experiments, corresponding finite element simulations were undertaken with an initial imperfection presented in the shell. Furthermore, the plateau load of the structure with this specific imperfection was formulated by energy method and solved under the assumption of *inextensional* deformation.

The numerical analysis indicates that the presence of imperfection could significantly reduce the buckling capacity. It shows that both the Young's modulus

of foam core (E_f) and shell thickness have a significant influence on the buckling capacity and performance of the tube. The buckling resistance is higher with stiffer infill and/or thicker shell wall. In addition, the structure buckles earlier for the one with thinner shell wall or softer foam core (smaller E_f). The plateau load tends to be flatter in the load displacement curve than with softer infill.

Compared with preliminary experimental findings, it reveals that the actual critical buckling load is closely located in the numerical post-buckling region. Thus, it can be deducted that the critical post-buckling load dominates the experimental behavior. Meanwhile, a buckling load prediction factor is introduced to correct the discrepancy generally encountered between the experiment buckling load and numerical analysis result.

The analytical result shows that the post-buckling dimple requires a compressive force proportional to $h^{2.5}$ to hold it in place. The theoretical prediction for plateau load agrees well with numerical analysis and test data. Empowered with this knowledge, the buckling load of such structures could be predicted more accurately.

7.1.3 Energy absorption behavior of a thin-walled cylindrical shell with foam core

FOR THE AXISYMMETRIC CRUSHING MODE, an analytical model was established to predict the mean crushing force of such structure. Using the modified superfolding element model for the shell and the volumetric reduction model for foam core, the mean crushing force was predicted through energy balance method. The proposed

theoretical model was validated with an axisymmetric finite element model in ABAQUS as well as the experimental test.

The result reveals that filling a thin-walled cylindrical shell using a foam core can increase its mean crushing force as well as energy absorption. With foam core filled, the half plastic wavelength (H) shortens. The amount of shortening increases when the plateau stress of foam core or shell wall thickness increases, as well as when the percentage of infill increases (η decreases).

In addition, the interactive effect can be predicted accurately through the change in the half plastic wavelength of the supporting tube. Besides, it is found that for practical purpose the in-filler inner radius should be at least 50% of the shell radius. Therefore, this study can give valuable design guidelines in the use of a thin-walled structure as an energy absorber.

FOR THE NON-AXISYMMETRIC CRUSHING MODE, experimental study of the axial crushing behavior of a thin-walled cylindrical shell with a hollow foam core was carried out. Based on the experiments, corresponding finite element simulations were performed to simulate the crushing of tested specimens for a better understanding of its behavior.

It is found that the infill has significant effects on both the deformation modes and force-displacement behavior. For the deformation modes, the one filled with lower density foam usually deforms from the lower end. As the crushing continues, a

large dent is formed, but usually with a smaller angle than if an empty tube is used. The angle and the fold length would reduce slightly with more infill, as well as a bigger D/h ratio. While for tubes having higher density foams, they deform with two or three folds either in the top or the bottom. The outer shell fractured from the last fold and the foam became the sole energy absorber. Either more infill or a larger D/h ratio could extend the displacement before the outer shell splits.

From the force displacement behavior, it is found that the crushing load and energy absorption capacity increases with amount of infill, a higher density of infill or a bigger shell's D/h ratio. The added foam core is found to increase the crushing resistance beyond the sum of shell and foam acting alone due to the interactive effect. Besides, it is found that S.E.A. is higher with a higher amount of infill, a higher density of infill or a smaller shell's D/h ratio.

A good correlation is achieved between the experimental and numerical simulation results. Therefore, the proposed F.E. model can be considered as a valuable tool in assessing and understanding the deformation/failure mechanism and predicting the response of the axial crushing of a thin-walled cylindrical shell with a hollow foam core.

7.2 Summary of Contributions

This study is concerned with the thin-walled cylindrical shell with foam core. The major contributions as a result of this research are summarized here:

- Previous studies merely focused on a fully or fixed amount of filled foam core. In order to maximize the strength of the structure at minimal weight and cost, the effect of partially filled (or hollow) foam core on the behavior of buckling in a thin-walled cylindrical shell is investigated. The author proposed an analytical formula for the critical buckling stress of a thin-walled cylindrical shell with partially filled foam core.
- Previous studies found a large discrepancy in the buckling load between the theory and experiment. The author introduced an initial geometric imperfection in the shell and conducted finite element analyses in ABAQUS. A post-buckling dimple is identified, which governs the post-buckling behavior. The author compared the experimental and numerical results and reveals that the actual critical buckling load is closely located in the numerical post-buckling region. A buckling load prediction factor is introduced to correct the discrepancy generally encountered between the experiment buckling load and numerical analysis result.
- A theoretical model is proposed to predict the axisymmetric crushing of a thin-walled tube filled with light-weighted foam. Previous studies focus on fully infilled tubes and the modeling of interaction between the shell and the foam is not straightforward. The author's investigations include variable amount of infill. The interactive effect can be easily evaluated from the change of the half plastic wavelength.

- Experiments are conducted to study the energy absorption of non-axisymmetric crushing mode. Compared to previous studies, two additions are made to the literature: (1) The D/h ratio studied here is up to 660. (2) Partial infill from 0% to 100% is experimented. The study gives a clue on how the infill would influence the deformation modes as well as its energy absorption behavior.

7.3 Recommendation for Future Work

Future study can be performed to address the following issues which arise from performing the current work.

- Dynamic analysis of such structures is still in its early stage. This is because a dynamic analysis is more complex and costly than a quasi-static analysis. However, it has wide engineering applications, especially in structural impact, vehicle crash *etc.*. Current work on quasi-static condition can serve as a foundation to future work in dynamic analysis.
- Current work reveals that the outer shell restrict the outer movement of the inner foam core under axial compression. This restriction can enhance its loading resistance. Therefore an inner shell can be added to have a double cylindrical shells structure as compared to the conventional single shell.
- The current investigation is focused on the thin-walled structure with foam core of a circular cross-section. Various cross-section shape of the shell can be studied for performance optimization.

- With an aging population globally, health care is a primary concern in the future. Methodology in this thesis could be applied similarly in biomedical application as well. For instance, investigations can be performed on human bone in order to enhance safety precautions in sports or to provide prospects on low cost high strength implant design.

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LIST OF PUBLICATIONS

1. Ye, L., Lu, G. and Ong L.S. (2011) Buckling of a thin-walled cylindrical shell with foam core under axial compression. *Thin-Walled Structures* **49**, 106-11.
2. Ye, L. and Lu, G. (2011) Axial crushing of thin walled cylindrical shell with foam core. *Materials Research Innovations* **16**, 3-6.
3. Ye, L., Lu, G. and Yang J.L. (2011) An analytical model for axial crushing of a thin-walled cylindrical shell with a hollow foam core. *Thin-Walled Structures* **49**, 1460-7.
4. Ye, L. and Lu, G. Post-buckling behavior of a thin-walled cylindrical shell with foam core under axial compression. *International Journal of Stability and Dynamics*. (Submitted)
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