Study of low frequency FOPEN imaging

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Study of Low Frequency FOPEN Imaging

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School of Electrical & Electronic Engineering

A thesis submitted to the Nanyang Technological University
in fulfillment of the requirement for the degree of
Master of Engineering

2005

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Statement of Originality

I hereby certify that the work embodied in the thesis is the result of original research and has not been submitted for a higher degree to any other University or Institution.

27/08/2005

Date

A handwritten signature in black ink, appearing to be 'Na Yibo', written over a horizontal line.

Na Yibo

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Summary

FOLIage PENetration (FOPEN) imaging is the Synthetic Aperture Radar (SAR) imaging for foliage penetration application to image the targets concealed by foliage areas, which is of special interests for both military and civilian applications [1]. Usually FOPEN SAR is in spotlight mode, wherein radar beam is continually steered to image an area. Spotlight SAR has an advantage over the strip-map SAR in terms of its resolution. To provide higher resolution, UWB waveform and large integration angle are required in FOPEN radar. However, they will induce very large range migration and bring new complexities and challenges to the traditional SAR research. So some hypotheses are not valid and some classical techniques cannot be applied at all. In this thesis, two relatively novel SAR imaging algorithms, namely the back-projection algorithm and range migration algorithm, are studied. As well, a new approach of range migration algorithm is developed. In their application to FOPEN SAR imaging, performance comparison is made by theoretical analysis, simulation and experimental results. It will be shown that both of these two algorithms are suitable for FOPEN radar imaging where theoretical imaging results can be achieved. The thesis also presents some aspects of image formation in both image (spatial time) domain and phase history (spatial frequency) domain. These two domains are Fourier pair and go through the basis of image reconstruction of FOPEN SAR. In addition, some enhancement techniques for improving the quality of the reconstructed SAR image are introduced and a system model for understanding the SAR imaging process is given in the thesis. Finally, the thesis discusses some possible future research work on FOPEN SAR imaging.

Contents

Acknowledgements i

Summary..... ii

List of Figures..... vii

List of Tables xi

1 Introduction..... 1

1.1 A Brief Introduction to the FOPEN SAR Imaging 1

1.2 Motivation..... 2

1.3 Objectives 3

1.4 Major Contribution of the Thesis..... 3

1.5 Organization of the Thesis 5

2 Literature Review 6

2.1 Radar 6

2.2 Synthetic Aperture Radar..... 7

2.3 UWB Radar..... 13

2.4 FOPEN SAR 15

2.5 Conclusion of the Chapter 16

3 SAR Resolution 18

3.1 The Azimuth Resolution and Spotlight SAR..... 18

3.1.1 Real Aperture Azimuth Resolution.....	18
3.1.2 Concept of Synthetic Aperture.....	20
3.1.3 Spotlight SAR over Strip-map SAR in Azimuth Resolution.....	21
3.2 The Range Resolution and Waveform.....	24
3.2.1 Slant Range Concept.....	24
3.2.2 Range Resolving Techniques Using CW Pulses	25
3.2.3 Range Resolving Techniques Using Chirp Waveform.....	32
3.2.4 Range Resolving Techniques Using Step-Frequency Waveform.....	35
3.2.5 Comparison in Range Resolution for Different Waveforms	36
3.3 Conclusion of the Chapter	38
4 Aspects of SAR Image Formation	39
4.1 Fast Fourier Transform and Image Properties	39
4.2 Basic Algorithm for SAR Imaging	42
4.3 Phase History Data and Resolution.....	45
4.4 Enhancement Techniques for Image Formation	48
4.4.1 Multi-Look Concept.....	48
4.4.2 Over-Sampling.....	49
4.4.3 Aperture Weighting	50
4.4.4 Spreading Correction	51
4.4.5 Phase Gradient Autofocus.....	53
4.5 Conclusion of the Chapter	61

5 SAR Model.....	62
5.1 SAR System Model.....	62
5.1.1 General Waveform.....	64
5.1.2 Chirp Waveform	69
5.2 SAR Simulation Model.....	71
5.3 Conclusion of the Chapter	77
 6 Imaging Algorithms for FOPEN SAR	 78
6.1 Back-Projection Algorithm.....	78
6.1.1 Introduction of Back-Projection Algorithm.....	78
6.1.2 Procedures of Back-Projection Algorithm.....	79
6.2 Range Migration Algorithm.....	82
6.2.1 Introduction of Range Migration Algorithm.....	82
6.2.2 Procedures of Conventional RMA.....	83
6.2.3 New Approach of RMA and Simulation	87
6.2.4 Doppler Bandwidth Analysis.....	91
6.2.5 Performance Evaluation in Sampling Requirement.....	99
6.3 Conclusion of the Chapter	105
 7 Comparison of Imaging Algorithms.....	 106
7.1.2 Data Sampling Error	110
7.1.3 Radar Trajectory Error	111
7.2 Experimental Results Analysis	113

7.2.1 Microwave Anechoic Chamber Measurements 113

7.2.2 Foliage Penetration Boom-SAR Measurement..... 117

7.3 Comparison of Implementation Issues..... 122

7.3.1 Data Collection 122

7.3.2 Computational Complexity..... 123

7.3.3 Memory Requirement 126

7.3.4 Over-sampling..... 127

7.4 Conclusion of the Chapter 131

8 Conclusion and Recommendations 132

8.1 Conclusion 132

8.2 Recommendations for Future Research 134

Author’s Publications 136

References..... 137

Appendix..... 144

A. Principle of Stationary Phase 144

B. Background of Back-Projection Algorithm 144

List of Figures

Figure 2.1: A typical airborne SAR geometry	9
Figure 2.2: The block diagram of SAR process.....	10
Figure 2.3: SAR modes in side view	11
Figure 2.4: SAR modes in top view.....	12
Figure 2.5: The tree pattern for different frequency bands.....	16
Figure 3.1: Real aperture beam pattern.....	19
Figure 3.2: Collection scenario of strip-map mode	21
Figure 3.3: Collection scenario of spotlight mode.....	22
Figure 3.4: Slant range and range direction	24
Figure 3.5: Continuous wave pulses with envelope $b(t)$	25
Figure 3.6: Collection geometry in range direction	26
Figure 3.7: Raised cosine pulse envelope with effective duration and effective bandwidth	29
Figure 3.8: Offset spatial frequencies of processed signal	32
Figure 3.9: A step-frequency waveform	35
Figure 4.1: Data collection model after demodulation	43
Figure 4.2: Geometry of Polar to Cartesian transform	45
Figure 4.3: Phase history properties in rectangular grid	46
Figure 4.4: Sub-apertures selected to form images.....	49
Figure 4.5: Over-sampling effect on one target image	50

List of Figures

Figure 4.6: Aperture weighting.....	51
Figure 4.7: Original image of PGA.....	59
Figure 4.8: Defocused image of PGA.....	60
Figure 4.9: Final image of PGA.....	60
Figure 5.1: Spotlight SAR imaging geometry	63
Figure 5.2: The rectangular-formatted demodulated data model.....	67
Figure 5.3: The polar-formatted demodulated data model	68
Figure 5.4: Spotlight SAR imaging system geometry in simulation	72
Figure 5.5: Range-compressed image with motion compensation to a line	75
Figure 5.6: Range-compressed image with motion compensation to a point	76
Figure 5.7: One target imaging. (a) With removal of RVP. (b) Without removal of RVP	77
Figure 6.1: 2-D spatial frequency image.....	84
Figure 6.2: 2-D spatial frequency image after Stolt interpolation	86
Figure 6.3: Final SAR image Using RMA	87
Figure 6.4: 2-D spatial frequency image.....	89
Figure 6.5: 2-D spatial frequency image without Doppler bandwidth widening	89
Figure 6.6: Final SAR image with new approach of RMA	90
Figure 6.7: SAR Geometry 1 for Doppler bandwidth analysis.....	93
Figure 6.8: Spatial frequency image from MCTL for SAR Geometry 1	94
Figure 6.9: Spatial frequency image from MCTP for SAR Geometry 1	94
Figure 6.10: SAR Geometry 2 for Doppler bandwidth analysis.....	95
Figure 6.11: Spatial frequency image from MCTL for SAR Geometry 2.....	96

List of Figures

Figure 6.12: Spatial frequency image from MCTP for SAR Geometry 2	96
Figure 6.13: SAR Geometry 3 for Doppler bandwidth analysis.....	97
Figure 6.14: Spatial frequency image from MCTL for SAR Geometry 3	98
Figure 6.15: Spatial frequency image from MCTP for SAR Geometry 3	98
Figure 6.16: SAR image with 0.53m sampling distance for both approaches of RMA .	100
Figure 6.17: SAR image using conventional approach of RMA with 1.1m sampling distance	100
Figure 6.18: SAR image using new approach of RMA with 1.1m sampling distance ...	101
Figure 6.19: Corner reflector image with 0.2m sampling distance.....	102
Figure 6.20: Corner reflector image using conventional RMA with 3m sampling distance	102
Figure 6.21: Corner reflector image using new approach of RMA with 3m sampling distance	103
Figure 6.22: Corner reflector image using conventional RMA with 3.4m sampling distance	104
Figure 6.23: Corner reflector image using new approach of RMA with 3.4m sampling distance	104
Figure 7.1: Simulation result by using back-projection algorithm	108
Figure 7.2: Simulation result by using range migration algorithm.....	108
Figure 7.3: Cuts of reconstructed images around the scatter (200,0) along range and azimuth direction	109
Figure 7.4: Cuts of scatterer response in range and azimuth directions	110
Figure 7.5: Radar trajectory with a sine shape.....	112

List of Figures

Figure 7.6: Cuts of scatterer response in range and azimuth directions	112
Figure 7.7: Radar trajectory with random disturbance	112
Figure 7.8: Cuts of scatterer response in range and azimuth directions	113
Figure 7.9: The hardware configuration of the VNA based SAR system	114
Figure 7.10: Photo of measurement in microwave anechoic chamber	114
Figure 7.11: Geometrical configuration of the measurement	115
Figure 7.12: Experimental result by using back-projection algorithm	116
Figure 7.13: Experimental result by using range migration algorithm	116
Figure 7.14: Cuts of cylinder response in range and azimuth direction	117
Figure 7.15: The photo of experimental Boom-SAR system	118
Figure 7.16: The photo of field trial site	119
Figure 7.17: Experimental result by using back-projection algorithm	120
Figure 7.18: Experimental result by using range migration algorithm	120
Figure 7.19: Cuts of corner reflector response in range and azimuth direction.....	121
Figure 7.20: Cuts of trunk response in range and azimuth direction	121
Figure 7.21: Simulation result by using back-projection algorithm	128
Figure 7.22: Simulation result by using back-projection algorithm	129
Figure 7.23: Simulation result by using back-projection algorithm	129
Figure 7.24: Simulation result by using back-projection algorithm	130
Figure 7.25: Cuts of simulated image around the center scatter	131
Figure B.1: A parallel-beam X-ray CAT system	147

List of Tables

Table 1: Comparison in range resolution for different waveforms..... 37

Chapter 1

Introduction

1.1 A Brief Introduction to the FOPEN SAR Imaging

National security and safety issues created the needs for developing FOliage PENetrating (FOPEN) SAR systems that utilize ultra wideband (UWB) UHF/VHF radars for detection of concealed targets [2]. The SAR digital signal processing issues associated with these FOPEN SAR systems bring new complexities and misunderstandings for those familiar with the traditional SAR systems, it is mainly due to the wide bandwidth and wide integration angle of these SAR systems. The wide coherent angular interval associated with fine resolution UWB imaging has a significant impact on the choice of image formation algorithm [3]. The line-of-sight range to a scatter varying hyperbolically over a synthetic aperture is called range curvature. The difficulty for fine resolution imaging, in general, is this range curvature itself in the imaged scene. While it is a straightforward matter to compensate range curvature for a given range bin, it is difficult to compensate range curvature for all range bins simultaneously. For UWB imaging, this differential range curvature across the imaged swath can be large. Hence when designing a UWB SAR system, it is important to select image formation algorithm that compensates differential range curvature effectively and efficiently. A number of algorithms are available for fine resolution SAR image formation. However, the majority of these

algorithms use approximations in compensating differential range curvature. These approximations result in residual differential range curvature, and lead to space variant phase errors that cause defocusing and geometric distortion. Thus imaging with FOPEN SAR data requires the use of new SAR imaging methods. Since 1980s, two novel SAR imaging algorithms, namely back-projection algorithm and range migration algorithm, have been developed. These two algorithms have capability to compensate the differential range curvature effectively and efficiently, thus they are applicable to the ultra-wideband imaging scenario with a large integration angle.

1.2 Motivation

Foliage PENetration (FOPEN) imaging is the Synthetic Aperture Radar (SAR) imaging for foliage penetration application to image the targets concealed by foliage areas. For the study of FOPEN propagation and SAR imaging in military application, a joint group was formed by NTU (Nanyang Technological University, Singapore) and ONERA (Office National d'Etudes et de Recherches Aerospatiales, France). A boom SAR system was developed and a series of field experiments were conducted. In the experiment, the raw data was collected by radar. Hence, to convert the raw data into final image becomes an attractive topic in research.

In the research of FOPEN SAR imaging, the large bandwidth waveform (several hundreds MHz) is used to get high range resolution ($\leq 1\text{m}$), the fractional bandwidth of

transmitting waveform may be as high as 0.5m or more, i.e., the Ultra-Wide Bandwidth (UWB). On the other hand, to get high resolution in the azimuth, as well as range, the integration angle of FOPEN imaging may be more than 60 degrees, which induces very large range migration. Therefore, the use of UWB waveform and large integration angle brings new complexities and challenges for the traditional SAR imaging techniques.

1.3 Objectives

The basic goal of FOPEN SAR imaging is to generate a two-dimensional reflectivity function of the illuminated foliage area and to detect targets concealed by foliage. To achieve this objective, study and simulation of SAR image formation algorithms are conducted. Also, FOPEN SAR image processing codes are developed for the raw data collected by real FOPEN SAR experiments. In this thesis, two relatively novel imaging algorithms for FOPEN SAR are discussed. They are the back-projection algorithm and the range migration algorithm.

1.4 Major Contribution of the Thesis

In this thesis, two relatively novel imaging algorithms are applied to FOPEN SAR Imaging, there are back-projection algorithm (BPA) and range migration algorithm (RMA). The performances of these two algorithms are compared in detail through

simulation and experimental results [4]. It will be shown that these two algorithms are suitable for FOPEN SAR imaging where theoretical image results can be achieved. Furthermore, some issues of implementation and performance difference about these two algorithms are discussed [5].

Usually the range migration algorithm is formulated from the received signal after motion compensation to a line. Motion compensation to a line is one type of demodulation model. In the thesis, a new approach of range migration algorithm is developed to process the raw data, which can be modeled as the received data followed by motion compensation to a point. This compensation strategy has more advantages over motion compensation to a line strategy in sampling requirement.

A convenient system model is developed in the thesis. It is useful to understand and study the image formation algorithms. Most importantly, it is the link that connects the SAR imaging algorithms and the raw data received from FOPEN radar system.

In addition, some enhancement techniques are discussed to improve the image quality. In particular, a phase estimation technique called phase gradient autofocus (PGA) is applied to restore the defocused image due to the unexpected phase error. And the simulated results of PGA are described and analyzed in the thesis.

1.5 Organization of the Thesis

The thesis is organized as follows: The literature review is presented in Chapter 2. It gives some basic concepts about radar, synthetic aperture radar, UWB radar and FOPEN radar. In Chapter 3, the fundamentals of spotlight SAR in image resolution are introduced. Chapter 4 describes some aspects of SAR imaging, where some enhancement techniques about SAR imaging are presented and analyzed. In Chapter 5, a SAR system model is provided. It is designed to simulate the radar system process and generate the model of raw data. Two relatively novel imaging algorithms for FOPEN SAR, back-projection algorithm and range migration algorithm, are presented in Chapter 6. In Chapter 7, simulation and experimental results of these two algorithms are given and performance comparison is made. Finally, Chapter 8 concludes the thesis and gives recommendations for the possible future work.

Chapter 2

Literature Review

Some brief introductions about radar, synthetic aperture radar (SAR), ultra-wide bandwidth (UWB) radar and FOPEN SAR are given in this chapter. These topics will be useful for the understanding of FOPEN SAR imaging.

2.1 Radar

The word radar is an acronym based on the phrase “RADio Detection And Ranging”. Radar is an important sensor technology because it provides an all-weather, day/night capability to not only detect and locate objects, but also to generate a spatial, visual representation (that is, an image) of the radar reflectivity of an illuminated scene. Radar has day/night capability because it supplies its own illumination; and has all-weather capability because radio waves propagate through clouds and rain with only limited attenuation [6]. Of particular interest in my research is the extension of this capability to the imaging of terrain and cultural features through foliage.

In reality, a radar system transmits a sequence of bursts, or pulses, or radio waves at regularly spaced intervals in time. The illuminated object will absorb a fraction of the

incident energy and scatter the remainder. The transmitted pulses propagate outward to illuminate the area within the antenna beam. The part of the scattered energy reflected back to the radar antenna is the received signal. Generally radio wave propagates at the speed of light. The time delay between transmission of a pulse and the return of a reflection from an object provides a measure of the two-way distance, or twice of the range between the radar and the object, and the two-way distance divided by the speed of light is the two-way time delay. Accuracy in the measurement of this two-way time delay directly impacts on the ability to determine an object's range direction. Therefore, it can be concluded that radar is a radio detection system and that its basic application of radar is ranging.

However, the capabilities of modern radar systems had been widely extended besides the target detection and ranging. Many more exterior and concealed characteristics of targets can be obtained and revealed with advanced radar technologies. Modern imaging radars, known as synthetic aperture radars (SAR), are capable of producing high quality images of target or ground terrain of interest [7].

2.2 Synthetic Aperture Radar

The synthetic aperture radar (SAR) is the radar for imaging. Historically, image formation has been accomplished by analog means. However, with the availability of high-speed digital computers, a new field called computed imaging or digital imaging has

been created. Computed imaging refers to the synthesis or computation of imagery using data collected from an object, a material or a scene. This computer makes the computation of the plenty of collected data an easy process. Imaging modalities include computer-aided tomography (CAT), magnetic resonance imaging (MRI), ultra-sound and acoustic imaging, x-ray crystallography, radio astronomy, and synthetic aperture radar [8]. Some of these computed imaging systems utilizes similar image signal processing principle. There are mainly five imaging algorithms used for SAR imaging, namely range-Doppler algorithm, polar format algorithm, chirp-scaling algorithm, back-projection algorithm and range migration algorithm. My research involves both back-projection algorithm and range migration algorithm, because of their ability to deal with the ultra-wideband waveform that is indispensable in FOPEN SAR application. Back-projection algorithm is also the imaging principle behind CAT, and range migration algorithm is identical to the seismic migration techniques [9]. Usually these two algorithms are used for high-resolution imaging, while the other algorithms are only applied for ranging, detection and low-resolution imaging.

A SAR system consists of a transmitter, a receiver, an antenna (including a pointing or steering mechanism), image processor and display unit. A typical geometry where an airborne SAR imager illuminates a patch of ground is shown in Figure 2.1. The beam of the radar looks out to the side of the aircraft, pointing nominally in the direction orthogonal to the flight path. This direction of radiation propagation is referred to as the range direction and the one nominally parallel to the flight path is called cross-range direction or azimuth direction. These two directions form the slant plane that is shown in

the Figure 2.1. In this thesis, all the two-dimensional reconstruction images are based on this slant plane.

Figure 2.1: A typical airborne SAR geometry

SAR process is conducted when radar moves along a sequence of locations in the azimuth direction. At every location, the radar periodically transmits and receives signals. These locations are the elements of the synthetic aperture and the synthesis involves the proper summation of the signals received at these locations. After receiving the signal, some pre-processes such as demodulation, are performed. The collected and pre-processed data are referred to as the raw data. Then the raw data will flow into the imaging processing as input. The reconstruction of the electromagnetic reflectivity of the illuminated ground is then produced. The reflectivity is treated as two-dimensional imaging, the range and cross-range dimensions. My research is focused on the image process that converts raw data into high quality image depending on imaging algorithms and some enhancement techniques, i.e., some automatic phase error detection and correction techniques. The block diagram of the SAR process is shown in Figure 2.2.

Figure 2.2: The block diagram of SAR process

An imaging radar system must distinguish between single and multiple scatters located in close proximity. Resolution is the minimum distance needed between adjacent scatters to separate them in the imaging map. Fine resolution provides the capability to image a complex object or scene as a number of separate scattering centers. The bulk of the development effort in radar imaging is at improving the radar resolution. Generally, the range resolution is inversely proportional to the bandwidth of the transmitted signal. A wide bandwidth means finer range resolution. In conventional radar, resolution in the azimuth direction improves as the antenna beamwidth becomes smaller. Antenna beamwidth becomes smaller as antenna aperture size or radar frequency increases. Hence practical constraints such as antenna size and transmit frequency will limit the azimuth resolution of the conventional aperture radar. The synthetic aperture concept offers improvement to the azimuth resolution. The synthetic aperture is formed when radar moves along a sequence of locations. These locations are the elements of synthetic aperture and they serve to enlarge the effective aperture size so as to improve the azimuth

resolution. A practical means for moving through these locations is to mount a SAR on a moving platform, such as an aircraft (Figure 2.1).

SAR has evolved to satisfy a variety of applications for both civilian and military. Military SAR applications include intelligence gathering, battlefield reconnaissance, and weapons guidance. Civilian applications include topographic mapping, oil spill monitoring, sea ice characterization and tracking, agricultural classification and assessment, land use monitoring and planetary or celestial investigations [6].

Figure 2.3 and Figure 2.4 show the side view and top view of the three common SAR imaging modes, respectively, namely the spotlight, strip-map and scan mode.

Figure 2.3: SAR modes in side view

Figure 2.4: SAR modes in top view

During a spotlight mode data collection, the sensor steers its antenna beam to continuously illuminate the scene being imaged. While in the strip-map mode, antenna pointing is fixed relative to the line of flight. The result is a moving antenna footprint that sweeps along a strip of scene parallel to the path of motion. In the scan mode, the sensor steers the antenna beam to illuminate a strip of terrain at any angle to the path of motion. The scan mode is a versatile operating mode that encompasses both the spotlight and strip-map modes at special case. Because scan mode involves additional operational and processing complexity, spotlight and strip-map modes are the more common SAR modes. The strip-map mode is most efficient when used for coarse resolution mapping over large regions. The spotlight mode is a practical choice when the objective is to collect fine resolution data from one or more localized areas. Furthermore, spotlight imagery provides the possibility of imaging a scene at multiple viewing angles during a single pass. My research is mainly focused on the spotlight mode.

2.3 UWB Radar

The majority of traditional radio systems use a narrow band of signal frequencies. Narrow band means that radio systems have a band of frequencies that is much lower than their carrier signal. Narrowband signals limit the information capability of radio systems, because the amount of the information transmitted in a unit of time is proportional to its bandwidth. Increasing the system's information capacity requires expanding its band of frequencies. The only alternative is to increase the information transmitting time. This information problem is especially important for radiolocation system, where the surveillance time of the target is limited. Past radars have used a band of frequency that does not exceed 10 percent of the carrier frequency. Therefore, they have practically exhausted the information opportunities in terms of range resolution and target characteristics. A new radar development is the transition to signals with a wide and ultra-wide bandwidth (UWB).

For designing UWB radars, as with any other equipment, we must understand the required theory that will allow us to correctly design and specify their characteristics. The theory is also necessary for defining the requirements of radars and for developing the equipment needed to create, radiate, receive and process UWB signals. In spite of recent developments and experimental work, there is no satisfactory and systematized theory of UWB radars available. The reason is that the process of radar tracking and surveillance with UWB signals differs considerably from similar processes when using traditional

narrowband signal. Hence, it brings some difficulties in the study of UWB radar, and new methods and algorithms need to be developed.

The informational content of the UWB radar increases because of the smaller pulse volume of signal. We also can say that the radar instrument probing the surveillance space becomes finer and more sensitive. The UWB radar's reduced signal length can

1. Improve detected target and range measurement accuracy.
2. Identify target classes and types.
3. Reduce the radar effects of passive interference.
4. Improve stability when observing targets at low elevation.
5. Increase the probability of target detection and improve stability while observing a target.
6. Provide a narrow antenna pattern by changing the radiated signal characteristics.
7. Improve the radar's immunity to external narrowband electromagnetic radiation effects and noise.
8. Decrease the radar "dead zone".
9. Increase the radar's secretiveness by using a signal that will be hard to detect.

In my research, UWB signal are applied to SAR system. The reason is that, it can greatly improve the radar resolution in imaging. Some discussions about signal processing of UWB waveform are presented in the following chapters.

2.4 FOPEN SAR

FOPEN SAR is the synthetic aperture radar for foliage penetration application, it is used to image targets concealed by foliage areas. For achieving the higher range resolution, UWB waveform is utilized into FOPEN SAR. To further achieve high resolution in the azimuth as well as range, long synthetic apertures or large integration angles are required. However, these large integration angles lead to severe range migration, or motion through resolution cells (MTRC). Scatterers at different locations in an imaged scene experience different levels of MTRC. The variation in MTRC makes selection of proper image formation algorithms critical. Moreover, the large integration angle, together with UWB waveform, brings about new complexities and challenges to the traditional SAR imaging techniques. In this thesis, two relatively novel Synthetic Aperture Radar (SAR) imaging algorithms, namely the back-projection algorithm and the range migration algorithm, are selected. They can compensate the range migration effectively and efficiently.

In foliage penetration, most of the radio energy attenuation is due to the leaves and braches. To reduce this attenuation induced by the leaves and branches, FOPEN SAR always operates in the VHF/UHF frequency bands. Figure 2.5 shows the image of tree with crown canopy when using different frequency bands. It can be found that only low frequency at VHF/UHF bands can penetrate the canopy of tree and detect the target under the foliage, where the image is formed by tree trunks and ground as shown in Figure 2.5. However, at such low frequency bands, the FOPEN data will contain radio frequency interference (RFI) due to transmission by local television stations [10], FM radio stations,

etc. Hence some additional techniques are required to remove the RFI during the pre-processing. These topics will not be discussed in this thesis, as it is not in my scope of study.

Figure 2.5: The tree pattern for different frequency bands

CARABAS system and P-3 radar system are typical low frequency SAR systems. They can be used for FOPEN application. CARABAS system [40][41] is a VHF-band radar system and is used for stem volume retrieval. It has been developed by Swedish Defense Research Establishment (FOA). P-3 system [2][42] is a UHF-band radar system, which was built by the Environmental Research Institute of Michigan (ERIM) and the Naval Air Warfare Center (NAWC) for the Advanced Research Projects Agency (ARPA). And the final processing of P-3 data was done by a group at MIT Lincoln Laboratory (MIT-LL).

2.5 Conclusion of the Chapter

Synthetic aperture radar is the radar for imaging. FOPEN SAR Imaging is the synthetic aperture radar for foliage penetration application. Due to the high resolution required in

this application, UWB waveform is used in FOPEN SAR. Therefore, some background and introduction about radar, synthetic aperture radar, UWB SAR and FOPEN SAR are presented in this chapter.

Chapter 3

SAR Resolution

The term resolution is defined as the minimum distance between adjacent scatterers, such that they can be separately identified in imaging map. This is a crucial factor in determining the image quality. This chapter details SAR resolution in both range and azimuth directions respectively.

3.1 The Azimuth Resolution and Spotlight SAR

Due to the limitation of conventional real aperture radar, the concept of synthetic aperture is used to improve the azimuth resolution. This section presents the difference between real aperture and synthetic aperture in term of azimuth resolution. Two data collection modes of synthetic aperture are also introduced and an example is given to show the advantages of spotlight SAR.

3.1.1 Real Aperture Azimuth Resolution

Figure 3.1 shows the top view of a real aperture beam pattern of radar, where β is the angular beam-width (measured in radians) of radar illumination and R is the distance

between the center of the illuminated area and radar.

Figure 3.1: Real aperture beam pattern

For the real aperture radar system, the nominal angular width of a beam produced by a radiation aperture is given by

$$\beta = \frac{K_a \lambda}{D} \quad (3.1)$$

where D is the width of the antenna aperture, λ is wavelength of the center frequency, and K_a represents a generalized main-lobe broadening factor and is approximately equal to one by simplicity. The azimuth width of the beam on the ground, called azimuth resolution, is given by

$$\rho_y = R\beta = R \frac{K_a \lambda}{D} \quad (3.2)$$

where R is the range from the ground location to the radar platform (see Figure 3.1). Depends on this equation, the real aperture imaging radar has azimuth resolution that are range-dependent, and narrower beamwidth is required for higher azimuth resolution.

Real aperture azimuth resolution can be explained in a hypothetical case, in which we desire $p_y = 1\text{m}$, and assume that $\lambda = 0.03\text{m}$ and $R = 50\text{km}$. Then result of the required width of the physical antenna aperture onboard the aircraft D is calculated as 1500 meters. Such a physical structure is clearly impractical, therefore in this case real aperture radar is incapable of achieving the desired azimuth resolution of one meter. On the other hand, if a realistic physical antenna size of 10 meters is used, then the radar will only have a lower azimuth resolution of 150 meters. Hence, the way to improve the azimuth resolution for real aperture radar is to either reduce the platform standoff range R or to reduce the angular beamwidth β by reducing the center wavelength. However, both of these methods are undesirable as reducing the standoff range is disadvantages for many applications, and reducing the wavelength is limited due to the electromagnetic propagation effects (atmospheric absorption).

3.1.2 Concept of Synthetic Aperture

Since both the reduction in platform standoff range and wavelength are undesirable, in order to improve the real aperture radar resolution in the azimuth direction, the concept of synthetic aperture can be used. The synthetic aperture is formed when a radar moves along a sequence of positions. At each position, the transmission and reception is conducted in coherence. These positions are the elements of synthetic aperture and the total motion distance is the synthetic aperture value. Hence creating an effective synthetic aperture which is significantly larger than the actual physical size of the antenna aperture, thereby improving the azimuth resolution as the length of the synthetic aperture is

effectively increased. Aperture synthesis is accomplished by the coherent integration of a series of radar returns transmitted from interval positions along a flight path.

3.1.3 Spotlight SAR over Strip-map SAR in Azimuth Resolution

As we have mentioned, there are two basic modes for SAR, the spotlight mode and the strip-map mode. In my study, for both experiment and simulation, spotlight SAR is applied. This is because spotlight SAR is generally advantageous where high resolution imagery is desired over small patch areas. In the following, an example is given to verify how spotlight SAR is better than strip-map one. Now, assuming that the patch to be imaged is 1km^2 , the desired azimuth resolution is 1 foot (0.33m), the center wavelength is 3cm and the range to scene center is 70km.

Figure 3.2: Collection scenario of strip-map mode

For the strip-map mode SAR, the achievable azimuth resolution according to [11] is equal to one-half of the width (along-track dimension) of the antenna. An important condition that accompanies this resolution limit is that pulses must be transmitted with equal spacing along the flight path equal to the resolution (one-half of antenna width) which will be explained by Equation (4.15). Figure 3.2 shows the collection scenario of strip-map SAR, the area of deep gray color is the imaged area and desired resolution requires an antenna width of 0.66m. With the wavelength at 0.03m, it produces a 2.6 degrees angular beam. As mentioned above, the pulses must be transmitted with spacing of 0.33m along the synthetic aperture to avoid spatial aliasing. For the total length of flight path, or called synthetic aperture length, is equal to 4km (3km + 1km), total number of transmitted pulses is about 12,121.

Figure 3.3: Collection scenario of spotlight mode

Figure 3.3 shows the collection scenario of spotlight mode SAR. For the spotlight mode SAR, the radar beam is now continually steered, so that the narrower angular beam with

only 0.82 degrees is required. Although the width of antenna is now 2.1m larger than that of the strip-map one, it is still a practical value. In order to achieve the same azimuth resolution, the integration angle that is defined in Figure 3.3 should be the same with the angular beam of strip-map mode (2.6 degrees). With a range distance of 70km, the total length of flight path can be calculated as 3.18km. For spotlight mode, it is required that the spacing of transmitted pulses is equal to one-half antenna width (1.05m). Hence only 3,029 pulses are required, which is much less than that of the numbers of strip-map SAR. The details about exact azimuth equation and information of integration angle will be discussed in Chapter 4.

From this example, it is clearly observed that spotlight mode SAR is better than strip-map mode for achieving the same resolution. In contrast, the required spacing between transmitted pulses for spotlight mode is larger by the same proportion (1m versus 0.33m), also only 3,029 pulses are required by spotlight radar system, instead of 12,121 pulses needed for the strip-map mode. In addition, the spotlight mode collection offers an advantageous power situation, the power demands is less for obtaining the same SNR. It is because the illuminated beam of spotlight mode SAR is narrower.

In conclusion, three attributes can distinguish spotlight and strip-map modes. First, spotlight mode offers finer azimuth resolution than that achievable in strip-map mode using the same physical antenna. Second, spotlight imagery provides the possibility of imaging a scene at multiple viewing angles during a single pass. Third, spotlight mode

allows efficient imaging of multiple smaller scenes whereas strip-map mode naturally images a long strip of terrain.

3.2 The Range Resolution and Waveform

This section presents the range resolution problem associated with the types of waveform. Three main types of waveform are discussed. Nominally they are continuous wave pulse, chirp waveform and step-frequency waveform. Chirp and step-frequency waveforms belong to ultra wideband signals and they are widely used in FOPEN SAR imaging for achieving higher range resolution.

3.2.1 Slant Range Concept

As we have demonstrated before, the range direction is the beam-illuminated direction and is perpendicular to the flight path. But this direction is not the exact range direction and is nominally called slant range direction. The exact range direction is on the ground and orthogonal to the flight path of aircraft, it is shown in Figure 3.4.

Figure 3.4: Slant range and range direction

In this thesis, for the sake of simplicity all the imaging problems are based on the slant plane and not on the ground plane. Hence the slant range direction is heavily used, instead of the range direction. For easy understanding, the slant range direction will be known as the range direction.

3.2.2 Range Resolving Techniques Using CW Pulses

The continuous wave (CW) pulses are discussed with respect to the range direction. Suppose that the radar launches a CW burst of microwave energy that is described by

$$s(t) = b(t) \cos(w_0 t) \quad (3.3)$$

where $b(t)$ is an envelope function of duration τ_b seconds and amplitude-modulates the carrier wave of frequency, w_0 . That is, $b(t) = 0$ outside the interval of $-\tau_b/2 \leq t \leq \tau_b/2$.

In this case, the envelope function is generated as a raised cosine pulse, which can be expressed as

$$b(t) = \frac{1}{2} \left[1 + \cos\left(\frac{2\pi}{\tau_b} t\right) \right] \quad (3.4)$$

Figure 3.5: Continuous wave pulses with envelope $b(t)$

A typical burst waveform is shown in Figure 3.5. Now consider the CW pulses is illuminated by the radar antenna, some scatters in the illuminated path will reflect some of the incident energy back toward the radar. Figure 3.6 shows the collection geometry to describe this situation, where the x dimension stands for the range direction that exactly is the slant range, y dimension depicts the azimuth direction and the illuminated patch is projected onto slant plane.

Figure 3.6: Collection geometry in range direction

In the figure, the center of the illuminated path is located at $x = 0$ and is offset from the platform's x position by the amount of x_0 . It also shows the radius of illuminated patch is x_1 . Then the signal received by radar from a single scatter can be expressed as

$$r(t) = A|g(x)|\cos(w_0(t - \tau_0 - \tau(x)) + \angle g(x))b(t - \tau_0 - \tau(x)) \quad (3.5)$$

where $g(x)$ is the complex microwave reflectivity of the scatter located at range (slant range) position x , and A is a scale factor that accounts for propagation attenuation and other effects. The magnitude of complex reflectivity determines the amount of the incident energy that is reflected back as the return signal. The value of $\angle g(x)$ is determined by the electrical properties of target material (at center frequency), as well as

other factors, such as target shape. The signal delay is simply the propagation time to and from the target (round trip) and is calculated according to the collection geometry as shown in Figure 3.6 and represented as $\tau_0 + \tau(x)$ in Equation (3.5), where

$$\tau(x) = \frac{2x}{c} \quad (3.6)$$

and

$$\tau_0 = \frac{2x_0}{c} \quad (3.7)$$

Equation (3.5) can also be represented in a complex form as:

$$r(t) = A \operatorname{Re}\{g(x) \exp[jw_0(t - \tau_0 - \tau(x))]b(t - \tau_0 - \tau(x))\} \quad (3.8)$$

A more realistic target structure consists of a continuum of scatters placed along the range direction, so the total returned radar waveform from all the scatters within the illuminated patch is given by the integral expression as

$$r(t) = A \operatorname{Re}\left\{\int_{-x1}^{x1} g(x) \exp[jw_0(t - \tau_0 - \tau(x))]b(t - \tau_0 - \tau(x))dx\right\} \quad (3.9)$$

We next define the patch propagation time τ_p as the difference in two-way propagation delay between a target at the near-edge and a target at the far-edge of the illuminated patch. It is given by

$$\tau_p = 2\tau(x1) = 2\left(\frac{2x1}{c}\right) \quad (3.10)$$

Thus, the key idea behind resolving targets in range when using this type of CW pulses is to keep the pulse as narrow as possible, so that close targets can be separated in range. In such a case, the duration of the received signal is approximately equal to τ_p , because τ_b as the duration of pulse is chosen to be much smaller than τ_p .

The resolution to which $g(x)$ can be reconstructed from the returned pulses depends on the duration of the transmitted CW burst waveform. One useful measure of effective duration that is appropriated for a pulse envelope waveform $f(t)$, is given by

$$\tau_e = \frac{\int_{-\infty}^{\infty} f(t)dt}{f(0)} \quad (3.11)$$

A corresponding measure of effective bandwidth in hertz is given by a similar expression, using the Fourier transform of the pulse, $F(w)$

$$B_e = \frac{1}{2\pi} \frac{\int_{-\infty}^{\infty} F(w)dw}{F(0)} \quad (3.12)$$

While a rectangular envelope function would have an effective duration equal to τ_b , shaped pulses in general have smaller effective durations, and correspondingly larger effective bandwidths. From the defining equation for the Fourier transform, the above measures can be shown to have a product, which is constant [12]

$$\tau_e B_e = 1 \quad (3.13)$$

The time-bandwidth product of pulse waveform is always equal to unity. For the particular raised cosine pulse envelope in Equation (3.3), the effective duration is equal to $\tau_b / 2$ by the calculation of Equation (3.11). The corresponding effective bandwidth in radians is $4\pi / \tau_b$. Figure 3.7 shows the raised cosine burst and its Fourier transform, and indicates the effective duration and bandwidth.

From the above discussion and observation in Figure 3.7, it can be reasoned that targets having return signal that are separated in time by τ_b seconds or more should be separable.

Figure 3.7: Raised cosine pulse envelope with effective duration and effective bandwidth

For analyzing the range resolution problem, pre-processing for CW pulses is required. Pre-processing basically is the process to convert the returned signal into the processed signal. It is conducted to remove the effect of carrier frequency. Pre-process mainly consists of two stages for CW pulses. The first stage is the in-phase and quadrature demodulation and the second stage is the low-pass filtering. Demodulation is conducted by mixing the returned signal with a reference signal, which is given by

$$r_{ref} = \text{Re}\{\exp[jw_0(t - \tau_0)]\} \quad (3.14)$$

where τ_0 is the round trip time between radar platform and calibrator or reference target in the image area. The calibrator or reference target is usually located at the center of illuminated patch. Then the result with the in-phase term can be easily expressed as

$$\begin{aligned} r_I(t) = & \frac{A}{2} \text{Re} \left\{ \int_{-x_1}^1 g(x) \exp[jw_0(2t - \tau(x) - 2\tau_0)] b(t - \tau(x) - \tau_0) dx \right\} \\ & + \frac{A}{2} \text{Re} \left\{ \int_{-x_1}^1 g(x) \exp[-jw_0\tau(x)] b(t - \tau(x) - \tau_0) dx \right\} \end{aligned} \quad (3.15)$$

After the low-pass filtering, only the second term is kept. Meanwhile, the quadrature term is in a similar process. Therefore, finally the complex expression for the processed signal becomes

$$r_p(t) = a \int_{-\tau_p/2}^{\tau_p/2} g\left(\frac{c\tau}{2}\right) \exp(-jw_0\tau) b(t - \tau_0 - \tau) d\tau \quad (3.16)$$

where a is the constant. The new limit of integration using a time term is $\pm \tau_p/2$, instead of range term $\pm x_l$. Especially, $r_p(t)$ can be viewed as the convolution of the pulse envelope with the function $\bar{g}(\tau)$, which can be defined as

$$\bar{g}(\tau) = g\left(\frac{c\tau}{2}\right) \exp(-jw_0\tau) \quad (3.17)$$

That is

$$r_p(t) = a\bar{g}(t) \otimes b(t) \quad (3.18)$$

where the symbol $\otimes$ represents convolution and $r_p(t)$ can be considered as smoothing function of $\bar{g}(t)$. Hence the resolution of processed signal in range is clearly related to $b(t)$, it can be expressed as

$$\rho_x = \frac{c\tau_e}{2} \quad (3.19)$$

Also we can re-write the resolution equation equivalently as

$$\rho_x = \frac{c}{2B_e} \quad (3.20)$$

Then the Fourier transform of $r_p(t)$, $R_p(w)$, can be expressed as

$$R_p(w) = a_1 G\left[\frac{2}{c}(w + w_0)\right] B(w) \quad (3.21)$$

where a_1 is the constant, and $B(w)$ is the Fourier transform of the envelope pulse. From this equation, it indicates that bandwidth of processed signal can be taken as the limited bandwidth of reflectivity function by the effective bandwidth of envelope. Suppose that $G(X)$ is the Fourier transform of $g(x)$, where X is the spatial-frequency variable. The spatial frequency variable is related with transmitted frequency as

$$X = \frac{2w}{c} \quad (3.22)$$

And the range of offset spatial frequencies of $R_p(X)$ is given by

$$\frac{2}{c}(w_0 - \pi B_e) \leq X \leq \frac{2}{c}(w_0 + \pi B_e) \quad (3.23)$$

Figure 3.8 shows the interesting relation in frequency domain between $R_p(X)$ and $G(X)$, where $R_p(X)$ is the narrowband of $G(X)$ around X_0 . Also as shown in the figure, the spatial bandwidth ΔX of this reconstruction $R_p(X)$ is proportional to the effective bandwidth of the transmitted pulse B_e , and the center spatial frequency X_0 is proportional to the radar center frequency w_0 .

From the above discussion about range problem of CW pulses, it can be concluded that the resolution in the range dimension is inversely proportional to the radar bandwidth. Therefore, in order to achieve high resolution, a short duration CW pulses is required. However, this short duration causes the low energy per pulse. Therefore, we can conclude that CW pulse is impractical.

Figure 3.8: Offset spatial frequencies of processed signal

3.2.3 Range Resolving Techniques Using Chirp Waveform

The chirp signals are used to achieve high resolution in range due to its wide-band property, but that do not suffer the same consequence of low average power levels that short CW bursts do. The chirp waveform is described by $\text{Re}\{s(t)\}$, with

$$s(t) = \exp[j(\omega_0 t + 0.5\alpha t^2)] \quad (3.24)$$

where ω_0 is the center frequency and the quantity α is the constant, which is usually called chirp rate. Generally the frequency information can be interpreted as the first derivative of phase in time domain, hence the chirp signal in frequency domain can be expressed as $\omega_0 + \alpha t$. It can be observed that frequency is linearly increased with time t . So chirp signal is also called linear FM chirp. Suppose the chirp duration is τ_c , so that the bandwidth of signal in hertz is given by

$$B_c = \frac{\alpha \tau_c}{2\pi} \quad (3.25)$$

The linear FM chirp is the most utilized of large time-bandwidth waveform for imaging [13]. Because longer duration chirp signal can generate the same effective bandwidth with CW pulse. Hence chirp signal can achieve the same resolution, but with much greater average energy. Although the chirp in fact has a duration equal to τ_c , it can behave like a pulse with duration equivalent to the inverse of its bandwidth, $\tau_e = 1/B_c$. The signal processing that allows this to happen is known as pulse compression. Hence chirp signal is also called pulse compression waveform. And the amount of this compression is given by

$$\frac{\tau_c}{\tau_e} = \frac{\alpha \tau_c^2}{2\pi} \quad (3.26)$$

which is the time-bandwidth of the waveform. Therefore, it can be concluded that chirp signal is a long duration and wide bandwidth waveform.

Similar with the case of the CW pulse, the returned signal can be expressed as:

$$r(t) = A \operatorname{Re} \left\{ \int_{-x_1}^{x_1} g(x) \exp[j\omega_0(t - \tau_0 - \tau(x)) + 0.5\alpha(t - \tau_0 - \tau(x))^2] dx \right\} \quad (3.27)$$

After we obtained the returned signal, the processing is followed by pre-process. The pre-process is discussed to understand the range resolution problem. For chirp signal, it is also referred as de-chirp processing and accomplished by three steps. The first two steps are similar with CW pulses quadrature demodulation. They are mixing with in-phase and quadrature versions of the chirp and low-pass filtering. And the last additional step is the Fourier transforming in range direction. De-chirp process also represents one particular form of the generic procedure called pulse compression. Basically the mixing term of de-chirp is given by

$$r_m(t) = \exp[j(\omega_0(t - \tau_0) + 0.5\alpha(t - \tau_0)^2)] \quad (3.28)$$

Then the complex expression for the processed signal after low-pass filtering becomes

$$r_p(t) = \frac{A}{2} \int_{-x_1}^{x_1} g(x) \exp[j[0.5\alpha\tau^2(x) - \tau(x)(w_0 + \alpha(t - \tau_0))]] dx \quad (3.29)$$

For the sake of simplicity, we can ignore the quadrature phase term, with

$$\Phi = 0.5\alpha\tau^2(x) = 0.5\alpha\left(\frac{2x}{c}\right)^2 = \frac{2\alpha x^2}{c^2} \quad (3.30)$$

If the effect of this quadrature phase term is too large to ignore, some special techniques, e.g. range de-skew [6], can be used to remove it in pre-process. Now we can re-write Equation 3.29 as

$$r_p(t) = \frac{A}{2} \int_{-x_1}^{x_1} g(x) \exp[j[-\frac{2x}{c}(w_0 + \alpha(t - \tau_0))]] dx \quad (3.31)$$

Notice that the integrand of Equation (3.31) involves the general form of the Fourier transform kernel, $\exp(-jxX)$, when we let

$$X = \frac{2w}{c} = \frac{2}{c}(w_0 + \alpha(t - \tau_0)) \quad (3.32)$$

where the capital X is the spatial frequency variable and it can be considered as the Fourier pair of spatial time variable x . Now the processed signal $r_p(t)$ can be referred as the Fourier transform of the reflectivity function, and evaluated on a specific limited spatial frequency X . Therefore, if the inverse Fourier transform (the final step of pre-process) is conducted, the reflectivity with relative to a function of illuminated range is obtained, which is also called range-compressed signal.

From Equation (3.32), the spatial frequency X is determined by the duration of chirp, and it is also relative to the frequency interval of chirp. Then the range of spatial frequencies is given by

$$\frac{2}{c}(w_0 - \pi B_c) \leq X \leq \frac{2}{c}(w_0 + \pi B_c) \quad (3.33)$$

As the bandwidth of spatial frequency is equal to

$$\Delta X = \frac{2}{c}(2\pi B_c) \quad (3.34)$$

which is the same with the results of CW pulses. Hence the expression of range resolution for the chirp waveform must be same with CW pulses, which is shown as

$$\rho_x = \frac{c}{2B_c} \quad (3.35)$$

3.2.4 Range Resolving Techniques Using Step-Frequency Waveform

The step-frequency waveform [14][15] is the third category for achieving high range resolution. Radars employing a step-frequency waveform increase the frequency of successive pulses linearly in discrete steps. A step-frequency can be shown as

Figure 3.9: A step-frequency waveform

As shown in the figure, the bandwidth of this step-frequency waveform is $N\Delta f$ over the duration of N pulses. A step-frequency waveform has a narrow instantaneous bandwidth (corresponding to individual pulse) and attains a large effective bandwidth (corresponding to frequency spread of pulses within a burst). Therefore, step-frequency

can achieve as high range resolution as chirp waveform due to its effective wide bandwidth. The range resolution of step-frequency radar is given by

$$\rho_x = \frac{c}{2B_{eff}} = \frac{c}{2N\Delta f} \quad (3.36)$$

In addition, a step-frequency waveform can be viewed as discrete implementation of chirp waveform. Hence step-frequency waveform possesses the same pre-process modeling as chirp waveform. But the real pre-processing with step-frequency waveform has a little more complex than that with chirp waveform.

Due to narrower instantaneous bandwidth as compared with UWB waveforms, the hardware requirements of step-frequency become less stringent. The step-frequency waveform requires slower A/D converter, lower peak power sources, and slower computers to process smaller sets of data. Also, the receiver bandwidth would be smaller, resulting in lower noise bandwidth and a higher signal-to-noise ratio. In short, step-frequency waveform provides the range resolution of wideband systems with the advantages of narrowband systems.

3.2.5 Comparison in Range Resolution for Different Waveforms

Three types of waveforms, CW pulses, chirp waveform and step-frequency waveform, associated with their range resolution problems are discussed in the above parts. Now the performance comparison in range resolution for these three types of waveforms are given in Table 1.

	Duration	Bandwidth	Power Level	Practicality
CW Pulses	Short	Wide	High	Impractical
Chirp Waveform	Long	Wide	Middle	Practical
Step Frequency Waveform	Long	Effective Wide	Low	Most Practical

Table 1: Comparison in range resolution for different waveforms

As we have verified, the range resolution is inversely proportional to the bandwidth of the waveform. This statement is suitable for all types of waveforms. For achieving high range resolution, the wide bandwidth CW pulses require the short duration, but short duration is impractical. Hence two long duration and wide or effective wide bandwidth waveforms, chirp waveform and step-frequency waveform, are induced. As compared with CW pulse and chirp waveform, step-frequency waveform is most practical. It is because the hardware requirements for step-frequency are less stringent.

To study the FOPEN SAR imaging, the step-frequency waveform is primarily applied in experiments. But due to its discrete property, it is not easy to simulate the signals. Therefore, the chirp waveform is widely utilized in simulation.

3.3 Conclusion of the Chapter

The SAR images are reconstructed based on range and azimuth direction. The concept of synthetic aperture is involved to realize the imaging application of radar by improving the azimuth resolution. It is shown that Spotlight SAR has more advantages over strip-map SAR, especially in term of azimuth resolution. The only factor that decides the range resolution is the bandwidth of the waveform, and higher range resolution needs wider bandwidth. Finally the performance of different types of waveform are compared and discussed. It indicates that the step-frequency waveform is the most practical one used in real experiment.

Chapter 4

Aspects of SAR Image Formation

This chapter presents some aspects of SAR image formation. Most importantly, the Fourier transform between phase history (spatial frequency) domain and image (spatial time) domain will go through the basis of image construction of FOPEN SAR. Moreover, the resolutions of image are analyzed from phase history domain data due to the property of Fourier based imaging algorithms. In addition, some enhancement techniques are introduced to make the final image more visualized and focused.

4.1 Fast Fourier Transform and Image Properties

As we have discussed in the previous chapter, the processed signal after quadrature demodulation is the spatial frequency domain signal in the range direction. The spatial frequency domain is also called the phase history domain. Basically the SAR imagery is obtained when we take Fourier transform to the phase history domain data. The phase history domain and image domain are Fourier pairs to go through the basis of SAR imaging. In digital signal processing, the processed data are discrete. The discrete Fourier transform (DFT) and its efficient implementation as a fast Fourier transform (FFT) are the basic principles for SAR imaging.

The one-dimensional (continuous) Fourier transform is defined by the following relationships:

$$H(f) = \int_{-\infty}^{\infty} h(t) \exp(-j2\pi ft) dt \quad (4.1)$$

$$h(t) = \int_{-\infty}^{\infty} H(f) \exp(j2\pi ft) df \quad (4.2)$$

where t typically represents time and f denotes frequency (in Hertz). Mathematically, those two transform expressions are identical except for the signs of the exponential phase terms. Nonetheless, it is customary and instructive to make a distinction between them. A common way of differentiating between them is to call the first one the forward transform of $h(t)$ and the second the inverse transform of $H(f)$. A more descriptive distinction is to refer to Equation (4.1) as the analysis transform and to refer to Equation (4.2) as the synthesis transform. We think of the analysis transform decomposing the function $h(t)$ into its frequency components, or spectrum, expressed by $H(f)$, while the synthesis transform reconstructs $h(t)$ from its spectrum $H(f)$. In the context of SAR image formation, we often think of synthesizing the image in space from its spatial frequency components. Now t , f represent the spatial time and spatial frequency variables respectively.

In practice, we have only a certain set of discrete samples that cover a limited time interval to represent one of these continuous signals. It follows that we should be talking about a discrete version of the Fourier transform. The discrete Fourier transform is represented by

$$H_k = \sum_{n=0}^{N-1} h_n \exp(-j2\pi nk / N) , k = 0,1,\dots,N-1 \quad (4.3)$$

$$h_n = \sum_{k=0}^{N-1} H_k \exp(j2\pi nk / N) , n = 0,1,\dots,N-1 \quad (4.4)$$

where the quantities n and k here denote discrete sample indices in the time and frequency domains, respectively. And N denotes the number of samples in both domains. We will concentrate on Equation (4.3), which represents the forward or analysis transform analogous to Equation (4.1). The quantity h_n will be thought of a sample of the continuous function $h(t)$ at time $t = n\delta t$, where δt is sample interval in time. And H_k will be interpreted as an approximation of the continuous spectrum $H(f)$ at the frequency $f_k = k/(N\delta t)$. Therefore, the following relationship can be obtained as

$$\delta f = \frac{1}{N\delta t} = \frac{1}{\Delta t} \quad (4.5)$$

where δf is the frequency interval and Δt is the time duration. Also, with respect to synthesis transform in Equation (4.4), the relationship is given by

$$\delta t = \frac{1}{N\delta f} = \frac{1}{\Delta f} \quad (4.6)$$

where δf is the sample interval in frequency and Δf is the bandwidth of spectrum. In short, the discrete Fourier transform in Equation (4.3) evaluates N frequency domain samples from N time domain samples. The corresponding inverse discrete Fourier transform in Equation (4.4) reconstructs the original time series from the frequency samples.

In above, the discrete Fourier transform problems are discussed in time-frequency domains. However, these relationships are also suitable for any Fourier pairs including phase history (spatial frequency) and image (spatial time) domains in context of SAR imaging. Therefore, when we let t and f represent the spatial time and spatial frequency variables respectively in the above equations, it can be found the image resolution (spatial time interval) and image area coverage (total length of spatial time data) are determined by the content of phase history domain data. Based on Equation (4.6), the image resolution is limited to the reciprocal of the signal extent in phase history domain. Then if parts of phase history data are used to reconstruct the image, the resolution in image will become lower. Furthermore, if we down-sample the spatial frequency data, then the image area coverage will be reduced based on Equation (4.5). However, the down-sampling will not affect the image resolution.

4.2 Basic Algorithm for SAR Imaging

In the above section, Fourier transform is presented, which is the basis for SAR imaging. Moreover, the phase history domain can be thought as the analyzed domain. Hence we need primarily transfer the processed data into phase history domain in image processing. Then in turn, the final image can be achieved. As we know, the processed signal after quadrature demodulation for UWB waveforms is the spatial frequency domain signal in range. During collection for spotlight SAR, the radar beam is always focused on image scene and moves along a sequence of positions. At all positions, the aspect angle of radar

for image area is variant. Hence radar illuminating the beam is not really in the range direction, but in radial range direction. And the data collection after quadrature demodulation can be modeled as Figure 4.1, where the bold line in azimuth direction denotes the radar path and θ represents the aspect angle.

Figure 4.1: Data collection model after demodulation

Since we have known the content of phase history data, the image is easily obtained by the Fourier transform. As shown in Figure 4.1, the collected data in phase history domain is polar-formatted. The expression for the inverse Fourier transform of the spatial frequency domain data in polar coordinates is given as

$$g(\rho \cos \phi, \rho \sin \phi) = \frac{1}{4\pi^2} \int_{-\pi/2}^{\pi/2} d\theta \int_{-\infty}^{\infty} G(r \cos \theta, r \sin \theta) |r| \exp(jr\rho \cos(\phi - \theta)) dr \quad (4.7)$$

where G is the processed data in phase history (spatial frequency) domain, and g is the reconstructed image of the complex reflectivity that is also in the polar coordinates as (ρ, ϕ) . This expression is the basis of one SAR imaging algorithm called the back-projection algorithm. In Chapter 6, some details about this radar imaging algorithm are given.

Sometimes, the phase history domain data could be rectangular-formatted as $G(X,Y)$. Then the reconstructed image is obtained by taking two-dimensional inverse Fourier transform to the phase history data in Cartesian coordinates. The expression for this process can be written as

$$g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(X,Y) \exp(jxX + jyY) dXdY \quad (4.8)$$

where $G(X,Y)$ stands for the phase history data and $g(x,y)$ depicts the reconstructed SAR image, they both are in Cartesian coordinates. On the condition of fractional bandwidth of transmitted signal much less than one ($\Delta f/f_c \ll 1$) and integration angle much less than one ($\theta \ll 1$), the collected data in polar coordinates, which is shown in Figure 4.1, can be approximately in Cartesian coordinates. Therefore, the SAR image can be achievable when direct taking two-dimensional inverse Fourier transform to the phase history data. This simple SAR imaging algorithm is called range-Doppler algorithm, it is applied to the conventional SAR imaging. However, when fractional bandwidth is larger than 0.25 (UWB) and integration angle is larger than 10 degrees, the direct approximation will cause large errors. So the range-Doppler algorithm is not applicable. To solve this problem, one additional approximate process is involved, which is called polar-to-Cartesian re-sampling. It converts the data from polar coordinates to Cartesian coordinates, which is shown in Figure 4.2.

Now if the two-dimensional Fourier transform is conducted, then image can be reconstructed with fewer errors. This SAR imaging algorithm is called polar format algorithm. It is just mentioned and not applied to FOPEN SAR imaging in the thesis, since it is also an approximate algorithm and will degrade the quality of image. High

quality is required for FOPEN application. Therefore, a more precise and newer algorithm for UWB imaging is introduced, which is called range migration algorithm. It uses an accurate re-sampling method to convert the data from polar coordinates to Cartesian coordinates. And the details about range migration algorithm are discussed in Chapter 6.

Figure 4.2: Geometry of Polar to Cartesian transform

4.3 Phase History Data and Resolution

SAR imagery is achieved by taking inverse Fourier transform from the phase history data. Hence the image properties are determined by the phase history content. Figure 4.3 gives some phase history information in Cartesian coordinates.

As we have discussed in chapter 3, the spatial frequency bandwidth in the range direction with relative to the bandwidth of transmitted waveform is expressed by

$$\Delta X = \frac{4\pi}{c} B_c \quad (4.9)$$

Figure 4.3: Phase history properties in rectangular grid

From Figure 4.3, we can easily obtain the spatial frequency bandwidth in azimuth (ΔY), when the spatial frequency bandwidth in range (ΔX) and integration angle (θ) is known. It can be expressed as

$$\Delta Y = 2R_1 \sin \frac{\Delta \theta}{2} \quad (4.10)$$

where

$$R_1 = R_0 - \frac{\Delta X}{2} = \frac{2}{c}(w_0 - \pi B_c) \quad (4.11)$$

where w_0 is the center frequency of transmitted waveform. If we suppose that SAR signal is in narrow band and the integration angle is very small, then Equation (4.10) can be approximately written as

$$\Delta Y = \frac{4\pi}{\lambda} \Delta \theta \quad (4.12)$$

Therefore, the corresponding interval expression is similar as

$$\delta\theta = \frac{\delta Y}{(4\pi / \lambda)} \quad (4.13)$$

If the image scene is a $L * L$ square area, then required sampling interval in azimuth δY should be equal or less than $2\pi/L$. Depends on Equation (4.13), the required interval value in integration angle can be rewritten as

$$\delta\theta = \frac{\lambda}{2L} \quad (4.14)$$

Therefore, the required along-track sampling interval for radar platform is

$$\delta y = r\delta\theta = \frac{r\lambda}{2L} = \frac{D}{2} \quad (4.15)$$

where D is the physical antenna length, and

$$D = \frac{r\lambda}{L} \quad (4.16)$$

The result of Equation (4.15) proves that radar platform motion interval is one-half of the antenna size, which has been mentioned in Chapter 3. In Figure 3.3, it shows that beam-width required for spotlight SAR is approximately equal to L/r . It can be verified by Equation (4.16), because the beam-width is determined by the ratio of center wavelength λ to the physical antenna length D . However, these statements are only applicable for narrow band radar.

In fact, Equation (4.09) is not only suitable for the UWB system, but also for the narrow band condition. When we have fully known the information of phase history domain data in two dimensions, then it is easy to estimate image resolution depends on imaging properties that we have discussed before. Thus the resolution in range, as well as in azimuth, are given respectively as

$$\rho_x = \frac{2\pi}{\Delta X} = \frac{c}{2B_c} \quad (4.17)$$

$$\rho_y = \frac{2\pi}{\Delta Y} = \frac{\pi c}{2(w_0 - \pi B_c) \sin(\Delta\theta/2)} \quad (4.18)$$

For the visualization of SAR image, we need to equalize the range resolution with azimuth resolution. Then from the above two equations, it can be found that the integration angle is proportional to the bandwidth of transmitted signal, when ρ_x is equal to ρ_y . It means that SAR illuminating a wider bandwidth signal must move along a wider integration angle.

4.4 Enhancement Techniques for Image Formation

4.4.1 Multi-Look Concept

The spotlight SAR phase history can be considered as a hologram. Hence the SAR image will show that the brightness distribution is not smooth and continuous but is instead composed of a complicated granular pattern of bright and dark spots called speckle. Speckle pattern occur in any form of coherent imaging, for example, optical holography and SAR. So multi-look processing is involved to produce smoother and more interpretable image [16]. Multi-look processing is accomplished by non-coherent summing the image intensities formed from some sub-apertures. Non-coherent summing refers to summing the image intensities (amplitudes) from sub-apertures rather than summing the complex reflectivity. In Figure 4.4, it is shown that there are two sub-

apertures. Then the final image is resulted from summing the images reconstructed by every sub-aperture. As we discussed in the above section, the less size of the phase history data, the lower the resolution of the corresponding reconstructed imagery. Therefore, the essential feature of multi-look concept is that losing resolution makes the image more visible.

Figure 4.4: Sub-apertures selected to form images

4.4.2 Over-Sampling

Over-sampling means that samples are evaluated more densely than the Nyquist criterion. The reason for over-sampling an image is to make it more pleasing visually and to facilitate certain image processing procedures that may be applied to the image. Over-sampling is accomplished by zero padding the processed data prior to Fourier transform. Figure 4.5(a) shows the one target image without over-sampling, while image is focused with over-sampling by factor four in Figure 4.5(b). Over-sampling by factor four means that the number of zeros padded is three times of original sampling number. And these zeros are equally dispatched to the both sides of the original data. Form the results of one

target image shown in Figure 4.5, it can be proved that the over-sampling highly impact on the quality of image. Sometimes, this technique is very important, especially when back-projection algorithm is applied.

Figure 4.5: Over-sampling effect on one target image. (a) Without over-sampling.

(b) With over-sampling by a factor of four

4.4.3 Aperture Weighting

Aperture Weighting, as its name implies, involves multiplying the time-domain sampled data sequence by an appropriate function, such as Hamming Window, Hann Window and some other window functions. Aperture weighting is a technique used to suppress side-lobe levels and thereby reduce interference of large amplitude signals with neighboring low-level components. Figure 4.6(c) shows the spectrum obtained from the same set of data when weighting has been applied. It can be found that the reduction in side-lobe

level in this spectrum has allowed the low-amplitude components in the original signal to be clearly visible, while it is mixed with low-amplitude noise in Figure 4.6(b). In addition, two closely spaced frequency components that are distinct in the un-weighted spectrum and are fused together in the weighted spectrum. So aperture weighting suffer the loss of resolution. Therefore, it is essential that the interference effect of side-lobes associated with large targets be minimized even if resolution must be sacrificed in the process.

Figure 4.6: Aperture weighting. (a) A hypothetical high-resolution image (b) Imaging without aperture weighting (c) Imaging with aperture weighting

4.4.4 Spreading Correction

Spreading correction is a technique using a function with respect to range to strengthen the back-scattered energy from far range target.

Consider the radar with an omni direction antenna (one that radiates energy equally in all directions). Since these kinds of antennas have a spherical radiation pattern, the power density P_D at range R away from the radar (assuming a lossless propagation medium) is

$$P_D = \frac{P_t}{4\pi R^2} \quad (4.19)$$

where P_t is the peak transmitted power and $4\pi R^2$ is the surface area of a sphere of radius R . The equation shows the one-way propagation, wherein the magnitude of wave field is inversely proportional to R . While in real case of SAR imaging, it is inversely proportional to R^2 due to the two-way propagation path. Therefore, generally the backscattered wave field including phase term is proportional to

$$\frac{\exp(-j4\pi fR/c)}{R^2} \quad (4.20)$$

where R is the radar to target distance and f is the radiation frequency. However, in practice, radar systems always utilize the directional antennas in order to increase the power density in a certain direction. Then the power factor of range R will become smaller in the denominator of Equation (4.20).

As a result, radar signals are dispersed in range, which leads to loss of range resolution, range accuracy and signal to inference ratio. Hence spreading correction is required to undo these negative effects and restore the full potential of the waveform in image processing. And spreading correction is conducted by digital multiplying R^α to compensate the power loss due to the propagation. The α value is usually chosen to be equal or less than 2, it depends on the property of real antenna.

4.4.5 Phase Gradient Autofocus

4.4.5.1 Introduction of phase errors

We have introduced some aspects on quadrature demodulation. When SAR is moving at each transmission position, we use the reference signal for demodulation. The reference signal is two-way time delay signal. Two-way time refers to the time that radar pulses travel from the SAR platform to the reference target in image area and return to the radar, which is also called demodulation time. The demodulation time τ_0 can be defined by the equation

$$\tau_0 = \frac{2R_0}{c} \quad (4.21)$$

where R_0 is the distance between the SAR platform and the reference target and c is the propagation velocity of the electromagnetic wave. But in practice, the demodulation time is not accurate when radar moves with unexpected condition. When the demodulation time is not known precisely, it will cause demodulation phase error and finally degrade the reconstructed image quality. The standard approach for estimating R_0 for each pulse is to employ electronic navigation systems, which use inertial measurement units (IMUs) placed onboard the collection platform. Modern IMU systems incorporate ring laser gyro technology and also include Global Positioning System (GPS) updates to increase the system accuracy. So increasing the accuracy of the IMU system is required for high-resolution imagery. But the option of improving the accuracy of IMU systems does not help other situations such as when the demodulation errors are not directly due to platform position uncertainty. For example, propagating radar energy through

atmospheric turbulence can cause random delays in the signal. At such condition, image restoration techniques known as autofocus, offers an attractive alternative. Autofocus techniques are automatic phase error detection and correction methods. Using these techniques, we can remove the effects of demodulation errors independent of the error source.

4.4.5.2 Model for demodulation phase errors

Depending on Equation (3.31), the processed signal after quadrature demodulation can be described by

$$F(R) = \int f(r) \exp(-jrR) dr \quad (4.22)$$

where $f(r)$ stands for reflectivity function of target at position r , and R is defined by

$$R = \frac{2}{c} (w_0 + 2\alpha(t - \tau_0)) \quad (4.23)$$

Now we assume that a time demodulation error is ε , then the corresponding in-phase and quadrature terms become

$$C_I(t) = \cos(w_0(t - \tau_0 + \varepsilon) + \alpha(t - \tau_0 + \varepsilon)^2) \quad (4.24)$$

and

$$C_Q(t) = -\sin(w_0(t - \tau_0 + \varepsilon) + \alpha(t - \tau_0 + \varepsilon)^2) \quad (4.25)$$

Hence the output of the quadrature demodulation after we ignore the residual video phase can be expressed as

$$F\varepsilon(R) = \exp(-j\frac{\varepsilon \cdot c}{2} R) \int f(r) \exp(-jrR) dr = \exp(-j\frac{\varepsilon \cdot c}{2} R) F(R) \quad (4.26)$$

Also, we know that the data after demodulation in spatial frequency R domain are band-limited and centered at $R_0 (2w_0 / c)$, where w_0 is the center frequency of the transmitted signal. Then the base-band data for the case without demodulation errors can thereby be represented by

$$F_b(R) = F(R + R_0)W(R) \quad (4.27)$$

where $W(R)$ is a windowing function that imposes the restricted range of R . Therefore, base-band version of $F\varepsilon$ can be obtained as

$$\begin{aligned} F_b\varepsilon(R) &= F\varepsilon(R + R_0)W(R) = \exp\left[-j\frac{\varepsilon \cdot c}{2}(R + R_0)\right]F(R + R_0)W(R) \\ &= \exp(-j\varepsilon w_0)\exp\left(j\frac{\varepsilon \cdot c}{2}R\right)F_b(R) \end{aligned} \quad (4.28)$$

We ultimately concern about how the phase error terms affect the range-compressed data. The range-compressed data can be achieved by taking the one-dimensional inverse Fourier transform on both sides of Equation (4.28), then the relationship between range-compressed data without phase error and range-compressed data with phase error becomes

$$f\varepsilon(r) = \exp(-j\varepsilon w_0)f\left(r + \frac{\varepsilon \cdot c}{2}\right) \quad (4.29)$$

Thus, the corrupted range-compressed signal $f\varepsilon$ has been altered by a constant phase and also shifted by an amount that is equal to $(\varepsilon \cdot c / 2)$. But the shift is small enough to be ignored. It then remains for us to analyze the effects of the phase error term on the reconstructed image. Accordingly, we ignore the shift and model the corrupted compressed signal simply as

$$f\varepsilon(r) = \exp(-j\varepsilon w_0)f(r) \quad (4.30)$$

But this equation is obtained when radar moves at certain position. For the whole radar path, the model in two dimensions can be expressed as

$$ff\epsilon(r, y) = \exp(-j\epsilon w_0) ff(r, y) \quad (4.31)$$

where y indicates the radar positions. Now the signal model is just range-compressed, but is not azimuth-compressed. Hence we need to take the Fourier transform in the azimuth direction and estimate the phase error in the azimuth phase history domain. Finally the phase error equation is described as

$$fF\epsilon(r, Y) = \exp(j\phi(Y)) fF(r, Y) \quad (4.32)$$

where the phase error is

$$\phi(Y) = -\epsilon(Y)w_0 \quad (4.33)$$

and Y is the phase history variable in the azimuth direction and the Fourier pair of y .

4.4.5.3 Phase Gradient Autofocus and simulation

(1) Phase Gradient Autofocus algorithm

Phase gradient autofocus (PGA) is one of the autofocus techniques. The PGA algorithm was developed to make a robust estimation of the derivative (gradient) of the phase error using only the defocused complex SAR image. The estimation process exploits the redundancy of the phase-error information contained in the degraded image, independent of the underlying scene content [17]. There are four crucial processing steps required to allow robust phase error estimation and image restoration. These four processing steps are referred to as center (circular) shifting, windowing, phase estimation and iterative correction.

Step 1: Center shifting

The first step in PGA is to select the strongest scatter for each range bin and shift it to the center of the data in the azimuth. This process relates to signal $ff\epsilon(r, y)$ and each selection is at one r value.

Step 2: Windowing

This step is windowing the circularly shifted imagery. Windowing has the effect of preserving the width of the dominant blur for each range bin while discarding the data that cannot contribute to the phase error estimation to proceed using input data having the highest signal-to-noise ratio.

Step 3: Phase Estimation

The center shifting and windowing operations isolate single defocused targets and position them at the center of each range bin. Then these targets are decompressed using a one-dimensional Fourier transform in the azimuth dimension for each range line. Now the signal is the same with the corrupted one in Equation (4.33). Suppose the data set of $fF\epsilon(r, Y)$ is composed of N range lines and M azimuth columns. A direct method to the phase estimation problem is to use all $N * M$ data points to derive a maximum-likelihood (ML) estimate of $\varphi(Y)$. A simpler approach for ML estimation, and the one that has become the most widely used in real SAR systems, is to use data on two adjacent pulses at a time to estimate the phase difference between them. These differences may then be integrated to obtain an estimate for the entire $\varphi(Y)$. This approach can be shown to be a special case of the more general procedure that uses all the data simultaneously.

For more details on the more general approach, reference [18] can be consulted. Here, the simpler approach is discussed. Firstly the phase difference

$$\Delta\phi(Y_M) = \phi(Y_M) - \phi(Y_{M-1}) \quad (4.34)$$

which is estimated for each value of Y_M . Then the expression for the ML estimator of this adjacent-pulse phase difference derived in Appendix F of reference [8] is found to be

$$\Delta\hat{\phi}(Y_M) = \angle \sum_{K=1}^N \{fF\varepsilon(r_K, Y_{M-1}) * fF\varepsilon(r_K, Y_M)\} \quad (4.35)$$

where the superscript (*) indicates the complex conjugate operation, and $\Delta\hat{\phi}(Y_M)$ denotes the ML estimator of $\Delta\phi(Y_M)$. Once the estimate for $\Delta\phi(Y_M)$ is obtained for all Y_M , the entire aperture phase error is estimated by integrating the $\Delta\hat{\phi}(Y_M)$ values. Mathematically, the estimate of phase error is given by

$$\begin{aligned} \hat{\phi}(Y_M) &= \sum_{K=2}^M \Delta\hat{\phi}(Y_K) \\ \hat{\phi}(Y_1) &= 0 \end{aligned} \quad (4.36)$$

Thus, the phase estimation portion of algorithm consists of estimating the phase differences for all adjacent pulses, followed by integrating the result using Equation 4.36.

Step 4: Iterative correction

The phase error estimates produced by the last step can be used to correct the degraded image. The phase correction process is identical for all phase correction algorithms and simply involves multiplying the degraded range-compressed data by the complex conjugate of our phase error estimate. For the reason that attempting to converge on the correct center shifting and target isolation, it is necessary to execute the PGA algorithm in an iterative manner. Therefore, after the image is corrected using the initial ML

estimate of the phase error function, the entire process is repeated on this refocused image. As more iteration are conducted, phase estimation becomes more accurate, the targets become more compact, and it allows more accurate center shifting and smaller windowing to be used.

(2) Phase Gradient Autofocus Simulation

In the simulation, the original image is an airplane, which is shown in Figure 4.7.

Figure 4.7: Original image of PGA

Then a random phase error with uniform distribution in between -0.7π and 0.7π radians is generated. Accordingly, the defocused image is shown in Figure 4.8, where we cannot identify the image. A program based on PGA is written in Matlab program, which is used to process the defocused image. Then the final result is shown as Figure 4.9. It indicates that the final image is perfectly focused and is similar with the original one. Also, it verifies that the PGA is an efficient technique to restore the image with phase error.

Figure 4.8: Defocused image of PGA

Figure 4.9: Final image of PGA

4.5 Conclusion of the Chapter

Synthetic aperture radar is a Fourier transform based imaging system. Hence the basic imaging algorithms are discussed from the characteristics of Fourier transform. Furthermore, the SAR resolution as synthesis domain information is analyzed from the phase history data (analysis domain data). In addition, some enhancement techniques for image formation are presented. They are necessary, so that good quality in SAR image can be achieved.

Chapter 5

SAR Model

This chapter presents the model of synthetic aperture radar. SAR system model concerns some topics about how to model the SAR operation, how to model the receiver process of SAR into mathematic expression and so on. All these pre-process models are essential for the success of the SAR image processing. Moreover, a simulation model for SAR system is given, which is used to simulate the SAR process in computer. At last, all the models are based on the UWB waveform due to the requirement of FOPEN application.

5.1 SAR System Model

SAR system model is generated for simulation analysis and mathematic derivation of imaging algorithms. A convenient imaging geometry is set up and shown in Figure 5.1, where radar moves along a sequence of positions located by $(0, y)$. These positions form the synthetic aperture and the mid-point of synthetic aperture is considered as the origin of radar coordinates. At every position, the radar will transmit and receive signal. In this system model, the “start-stop” approximation is adopted, i.e., no radar movement is assumed between transmission and reception. The direction of radar movement, as well the direction of synthetic aperture, is called azimuth direction. For spotlight SAR, radar

always points to the image area. The pointing direction of the antenna is called radial range direction denoted as r , which continuously changes with the motion of radar. The location of image center or reference target in image area is (x_c, y_c) , which is considered as the origin of image Coordinates

Figure 5.1: Spotlight SAR imaging geometry

In Figure 5.1, the imaging geometry presents the general case of spotlight SAR, which is called the squint mode. However, when y_c is equal to zero, it means the center of synthetic aperture and the center of image area are at the same azimuth value, then the spotlight SAR becomes broadside mode. This specialized and simple mode will be applied to simulation. Furthermore, the range and azimuth directions are depicted by x and y respectively on the radar Coordinates. Whereas on image Coordinates, variables x_n and y_n are instead used. As shown, the shadow area describes the imaging area and image center is the origin of image coordinates. Thus an arbitrary scatter in image coordinates can be expressed by (x_n, y_n) . When based on the radar Coordinates, the

position of scatter can be expressed as $(x_n + x_c, y_n + y_c)$. Through the thesis, vectors (x, y) or (x_n, y_n) and (X, Y) or (X_n, Y_n) identify, respectively, the spatial time and spatial frequency domain variables [16]. In the following chapters, they are widely used in mathematic development of FOPEN SAR imaging algorithms. In addition, the SAR imaging is based on the slant plane and the height of radar is not discussed in the thesis.

5.1.1 General Waveform

The general expression for a UWB waveform can be given by

$$S_T(t) = \exp(j\omega t) \quad (5.1)$$

where ω stands for a wide band of frequencies. Then the received signal from a single point scatter can be described as

$$S_R(t) = \exp[j\omega(t - \frac{2r_n}{c})] \quad (5.2)$$

where r_n is the distance between radar and the point scatter and $2r_n/c$ depicts the round-trip time delay. Firstly, we assume that the radar's radiation pattern is spherical. For the purpose of simplicity, Equation (5.2) neglects antenna gain, amplitude effects of propagation on the signal and any additional time delays due to atmospheric effects. Also, assumes that backscattered amplitude and phase characteristics of the point scatter do not vary with frequency or aspect angle and the value of backscattered voltage level is equal to one. Also, the losses due to the free-space propagation are not involved into this equation.

After receiving, a demodulation process in radar receiver is conducted. The demodulation process can be modeled as a real time motion compensation operation by mixing the received signal with a reference function to produce the demodulated signal. The reference signal is

$$S_{ref}(t) = \exp[j\omega(t - \frac{2r_{ref}}{c})] \quad (5.3)$$

where r_{ref} is the reference range and $2r_{ref}/c$ is the reference delay, they may be fixed or varied that relies on the motion compensation strategy of SAR system. Then the resultant demodulated signal can be expressed as

$$S_D(R) = \exp[-j\frac{2\omega}{c}(r_n - r_{ref})] = \exp[-jR(r_n - r_{ref})] \quad (5.4)$$

where

$$R = \frac{2\omega}{c} \quad (5.5)$$

is twice the wave-number value (ω/c).

Demodulation process is also called motion compensation process. Depends on different situations, SAR systems will choose different motion compensation strategies. Motion compensation to a moving point or a line requires the fixed reference range or reference delay. And the value of reference range can be the distance between reference target and the center of synthetic aperture. Hence, depending on the figure of imaging geometry, the reference range can be written as

$$r_{ref} = r_{ML} = \sqrt{x_c^2 + y_c^2} \quad (5.6)$$

In common, conventional strip-map SAR employs this motion compensation strategy. Another strategy is motion compensation to a fixed point that spotlight SAR usually

employs, where reference range varies with radar positions. In general, the reference range is the distance between reference target and radar. And it can be written with respect to radar position $(0, y)$ as

$$r_{ref} = r_{MP} = \sqrt{x_c^2 + (y_c - y)^2} \quad (5.7)$$

In essence, motion compensation or demodulation operation aims at stabilizing the signal received from reference target. Therefore, based on motion compensation to a point, the response of reference target is always at the center of range-compressed signal data for all the pulses. The range-compressed signal is the demodulated signal in range spatial time domain. However, based on motion compensation to a line, the response of reference target is at the center only when pulse or waveform is transmitted at the center of synthetic aperture. And the response from reference target will form a hyperbolic curve in the range-compressed image.

Assume that r_0 is the radius of image area and $f(x_n, y_n)$ stands for the reflectivity coefficient of an arbitrary scatter (x_n, y_n) , then the demodulated baseband signal from all the scatters in the image area can be expressed as

$$S_D(R) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-jR(r_n - r_{ref})] dx_n dy_n \quad (5.8)$$

where r_n is the distance between radar and certain scatter. Based on the imaging geometry of Figure 5.1, it can be written as

$$r_n = \sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} \quad (5.9)$$

Hence, the demodulated signal with relative to radar position y after motion compensation to a line can be expressed as

$$S_{DL}(R, y) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-jR(\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + y_c^2})] dx_n dy_n \quad (5.10)$$

And after motion compensation to a point, it can be expressed as

$$S_{DP}(R, y) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-jR(\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + (y_c - y)^2})] dx_n dy_n \quad (5.11)$$

Therefore, the model of signal after reception and demodulation, namely demodulated signal, is obtained. In real case, the demodulated signal is called as raw data, which is the input of image processing. Furthermore, it can be observed that the demodulated data is a two-dimensional data rectangular-formatted on (R, y) Coordinates. Also, it can be considered as a column of spatial frequency data with respect to azimuth y , which is shown in Figure 5.2.

Figure 5.2: The rectangular-formatted demodulated data model

In addition, Equation (5.8) can be considered as the process of one-dimensional Fourier transform when R is thought of as the Fourier pair of radial range r , where r is expressed as

$$r = r_n - r_{ref} \quad (5.12)$$

Thus, now if taking Fourier transform to the demodulated data for every azimuth y value, a column of spatial time data is obtained, which is called range-compressed data. Basically the rectangular format model is widely used in range migration algorithm. On the other hand, the demodulated data can be interpreted as the SAR signal in the spatial frequency domain and expressed as polar-formatted data on (R, θ) Coordinates, where θ is the direction of radar illumination at every collected position. This polar format model is shown in Figure 5.3, and it is mainly utilized in back-projection algorithm that is another FOPEN SAR imaging algorithm and will be discussed in the next chapter.

Figure 5.3: The polar-formatted demodulated data model

5.1.2 Chirp Waveform

The chirp waveform is extensively used in computer simulation. The chirp waveform as a transmitted signal can be expressed by

$$S_T(t) = \exp[jw_0t + j0.5\alpha t^2] \quad (5.13)$$

where w_0 is the carrier frequency and α is the chirp rate. Also, the step-frequency waveform can be modeled by this equation. By taking the first derivative of the phase term with relative to time t , the instantaneous frequency of the transmitted signal is obtained as

$$w = w_0 + \alpha t \quad (5.14)$$

All the assumptions that were mentioned in the general waveform are still applied to the chirp waveform. So the received signal from a single point scatter can be described as

$$S_R(t) = \exp\left[jw_0\left(t - \frac{2r_n}{c}\right) + j0.5\alpha\left(t - \frac{2r_n}{c}\right)^2\right] \quad (5.15)$$

where r_n is the distance between radar and the point scatter and $2r_n/c$ depicts the round-trip time delay. Similar with the general waveform analysis, the reference signal is required in demodulation, which is

$$S_{ref}(t) = \exp\left[jw_0\left(t - \frac{2r_{ref}}{c}\right) + j0.5\alpha\left(t - \frac{2r_{ref}}{c}\right)^2\right] \quad (5.16)$$

with the instantaneous frequency

$$w = w_0 + \alpha\left(t - \frac{2r_{ref}}{c}\right) \quad (5.17)$$

where r_{ref} is reference range and $2r_{ref}/c$ is the reference time delay. After mixing the received signal with the reference signal, then the resultant demodulated signal can be expressed as

$$\begin{aligned}
 S_D(t) &= \exp\left[-j\frac{2(w_0 + \alpha)}{c}(r_n - r_{ref}) + j\frac{2\alpha}{c^2}(r_n^2 - r_{ref}^2)\right] \\
 &= \exp\left[-j\frac{2[w_0 + \alpha(t - 2r_{ref}/c)]}{c}(r_n - r_{ref}) + j\frac{2\alpha}{c^2}(r_n - r_{ref})^2\right] \\
 &= \exp\left[-j\frac{2w}{c}(r_n - r_{ref}) + j\frac{2\alpha}{c^2}(r_n - r_{ref})^2\right] \\
 &= \exp\left[-jR(r_n - r_{ref}) + j\frac{2\alpha}{c^2}(r_n - r_{ref})^2\right]
 \end{aligned} \tag{5.18}$$

where R is twice the wave-number (w/c). The second phase term is called residual video phase (RVP). SAR system may ignore this term during image formation, then the demodulated signal becomes

$$S_D(R) = \exp[-jR(r_n - r_{ref})] \tag{5.19}$$

It can be observed that this equation is the same as Equation (5.4), which is from the general waveform. Then the demodulated signal from all the scatterers in the image area can be expressed as

$$S_D(R) = \int_{r_0}^0 \int_{r_0}^0 f(x_n, y_n) \cdot \exp[-jR(r_n - r_{ref})] dx_n dy_n \tag{5.20}$$

where r_0 is the radius of image area and $f(x_n, y_n)$ depicts the reflectivity function of any scatter located at (x_n, y_n) . Then the demodulated signal after motion compensation to a line can be expressed as

$$S_{DL}(R, y) = \int_{r_0}^0 \int_{r_0}^0 f(x_n, y_n) \cdot \exp[-jR(\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + y_c^2})] dx_n dy_n \tag{5.21}$$

And after motion compensation to a point, it can be expressed as

$$S_{DP}(R, y) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-jR(\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + (y_c - y)^2})] dx_n dy_n \quad (5.22)$$

The above two equations for expressing the demodulated signal are obtained when the RVP term is ignored. If the RVP term is taken into account, it will cause the degradation of image formation. In general, the RVP term is larger with motion compensation to a line than with motion compensation to a point. It can be explained by Equation (5.18), RVP is proportional to the square of the quantity $(r_n - r_{ref})$. Typically, the value of $(r_n - r_{ref})$ in the case of motion compensation to a line is substantially larger.

5.2 SAR Simulation Model

This section presents the model used in simulation. The simple SAR image geometry for simulation is shown in Figure 5.4. Based on the figure, the demodulated signal, or called raw data, can be generated in computer simulation. In the next chapter, the simulated raw data will be used for image processing, and finally the SAR image is reconstructed. Accordingly, the similarity between the SAR image and the presumed imaging geometry will verify the validity of the image algorithms.

As shown in Figure 5.4, five point scatters including reference target are located in the imaging area. The reference target locates at 200 meters away from the center of radar

path (synthetic aperture) in range, but at the same level in azimuth. So its location can be expressed as $(200,0)$. Hence the image area is at the case of broadside mode. In the simulation, the transmitting signal is chirp waveform $p(t)$ with a center frequency of 225MHz and bandwidth of 150MHz. It is also a UWB waveform with 0.67 of fractional bandwidth. From Figure 5.4, radar moves along -50 to 50 meters in the azimuth direction. At every 0.5 meters, the radar will be stationary and will transmit and receive signal. Hence a total of 200 demodulated signals are obtained. All parameters above are selected approximately the same as the parameters of the real field experiment. Furthermore, all the assumptions discussed before will still be valid for this simulation model.

Figure 5.4: Spotlight SAR imaging system geometry in simulation

Based on the image geometry shown in Figure 5.4, the corresponding received signal can easily be generated in simulation, since we know fully the information of the point scatters. Then the process of converting the received signal into the demodulated signal is

required. This pre-processing can be modeled by using several steps. For the sake of simplicity, all these pre-processing steps will be explained using a simple case where only one point scatter is located in the image area.

Step1: Generate the received signal based on the location of the point scatter in above image geometry. It can be expressed as

$$S_R(t) = p(t - \frac{2r_n}{c}) = \exp\left[jw_0(t - \frac{2r_n}{c}) + j0.5\alpha(t - \frac{2r_n}{c})^2\right] \quad (5.23)$$

where w_0 is the center frequency and equal to 225MHz and r_n is the distance between the radar and point scatter. For the value of chirp rate α , it is selected with relative to the time duration of the chirp waveform. Usually the time duration is much longer than the transmission time of radiated signals.

Step 2: Multiply a phase term $\exp(-jw_0t)$ to the received signal. This step serves as moving the signal into base-band.

Step 3: Take the Fourier transform of the processed signal. The signal becomes

$$S_R(w) = P(w + w_0) \exp[j(w + w_0)\frac{2r_n}{c}] \quad (5.24)$$

where w stands for a base-band of frequencies and is centered at 0Hz.

Step 4: Generate the reference signal. It can be expressed as

$$S_{ref}(t) = p(t - \frac{2r_{ref}}{c}) \quad (5.25)$$

where the value of r_{ref} is relied on the motion compensation strategy. Conventionally, it is equal to 200m that is shown in Figure 5.4, when the motion compensation to a line is applied. On the other hand, when using motion compensation to a point, the reference distance r_{ref} is given by

$$r_{ref} = \sqrt{200^2 + y^2} \quad (5.26)$$

which is relative to the radar position $(0, y)$.

Step 5: Multiple the same phase term and take the Fourier transform of the reference signal. This is analogous to the process for the received signal. Now the reference signal is brought down to base-band in the frequency domain, and can be described as

$$S_{ref}(w) = P(w + w_0) \exp[j(w + w_0) \frac{2r_{ref}}{c}] \quad (5.27)$$

Step 6: Mix the processed received signal $S_R(w)$ with the conjugate term of $S_{ref}(w)$.

Then the demodulated signal becomes

$$\begin{aligned} R(w) &= R(w) [\text{conj}(R_m(w))] = |P(w + w_0)|^2 \exp[-j \frac{2(r_n - r_{ref})}{c} (w + w_0)] \\ &= \exp[-j(r_n - r_{ref})R] \end{aligned} \quad (5.28)$$

where

$$R = \frac{2(w + w_0)}{c} \quad (5.29)$$

It can be observed that the expression of the demodulated signal is exactly the same as the results obtained from the system model.

Step 7: Take the inverse Fourier transform of the demodulated signal. Then the signal becomes range-compressed. The range-compressed image is used to verify the validity of the modeling process. Based on the simulation model mentioned above, the range-compressed image with motion compensation to a line and with motion compensation to a point are shown in Figure 5.5 and Figure 5.6, respectively.

Figure 5.5: Range-compressed image with motion compensation to a line

Figure 5.6: Range-compressed image with motion compensation to a point

It shows the range-compressed images describing the distance between the radar and the scatters, relative to the reference distance when the radar is moved along a sequence of locations. For different motion compensation strategies, the reference distance are different. Hence the different range-compressed images are resulted in. Specifically, the range-compressed response from the reference scatter (deep color line in Figure 5.5 and Figure 5.6) is curved in Figure 5.5 when motion compensation to a line is applied. It becomes a straight-line after motion compensation to a point is applied, which is shown in Figure 5.6.

In addition, the most important advantage of this simulation model is the removal of the residual video phase (RVP) term. Hence the quality of simulation result is improved and therefore it is good for the future study on SAR image algorithms. Figure 5.7 shows the

one target SAR imaging with the removal of RVP and without the removal of RVP. It indicates that the one with the removal of RVP can achieve better image quality.

Figure 5.7: One target imaging. (a) With removal of RVP. (b) Without removal of RVP

5.3 Conclusion of the Chapter

In this chapter, a SAR system model is developed to understand and simulate SAR imaging algorithms. It is the link to connect the imaging algorithms with the raw data from the real experiment. Some assumptions about this system model are made. In simulation model, the chirp waveform with UWB is used and a simple image geometry is assumed.

Chapter 6

Imaging Algorithms for FOPEN SAR

In this chapter, two relatively novel imaging algorithms applied for FOPEN SAR, namely back-projection algorithm and range migration algorithm, are introduced and analyzed. These two imaging algorithms have capability to compensate differential range curvature effectively and efficiently, thus they are applicable to near-field scenario. Originally, the back-projection algorithm was applied in the medical image reconstruction known as computer-aided tomography technique [20]. On the other hand, the range migration algorithm is derived from the seismic migration techniques [9].

6.1 Back-Projection Algorithm

6.1.1 Introduction of Back-Projection Algorithm

Back-Projection algorithm is the most popular technique to construct the SAR image. It was derived from the medical image reconstruction technique known as Computer-Aided Tomography (CAT). Several years ago, it was shown that the mathematical structure of SAR image reconstruction is similar with the image reconstruction problem encountered in CAT [22]. The similarity in these two different imaging systems suggested that the

reconstruction algorithm used in CAT, i.e., back-projection algorithm, could also be used in SAR, and vice versa. More details about the background of back-projection algorithms are presented in Appendix B.

6.1.2 Procedures of Back-Projection Algorithm

The back-projection algorithm (BPA) is a popular and simple approach for reconstructing the accurate SAR images, though it needs to carry relatively heavy computational burden. It can be easily derived by starting with the two-dimensional Fourier transform pair. Consider the complex reflectivity function $f(x, y)$ with its two-dimensional Fourier transform $F(X, Y)$, we have

$$f(x, y) = \frac{1}{4\pi^2} \iint_L F(X, Y) e^{j(xX+yY)} dXdY \quad (6.1)$$

where L is the support of the function $f(x, y)$. Let (R, θ) represent the polar Coordinates in the (X, Y) plane. Therefore, $F(R, \theta)$ denotes the values of $F(X, Y)$ along a line at an angle θ with the X axis, which can be interpreted as the demodulated data $S_D(R, \theta)$ in the Section 5.1.1. The model of $S_D(R, \theta)$ is shown in Figure 5.3. The cell $\Delta X \cdot \Delta Y$ in the (X, Y) plane can be expressed as $|R| \cdot \Delta R \cdot \Delta \theta$ in the polar Coordinates. Therefore, expressing $f(x, y)$ in polar Coordinates with $|R|$ being the Jacobian, then the Equation (6.1) can be rewritten as

$$f(x, y) = \frac{1}{4\pi^2} \int_{-(\theta_m/2)}^{\theta_m/2} \int_{\omega_1/c}^{\omega_2/c} S_D(R, \theta) e^{jR(x \cos \theta + y \sin \theta)} |R| dR d\theta \quad (6.2)$$

where θ_m is the integration angle and UWB waveform is range at $[w_1, w_2]$. And it is accomplished by taking

$$\begin{aligned} X &= R \cdot \cos \theta \\ Y &= R \cdot \sin \theta \end{aligned} \quad (6.3)$$

into Equation (6.2). Additionally, the resulted image $f(x, y)$ should also be in the polar Coordinates. Therefore, the SAR image reconstruction is reduced to solve the following integral

$$f(r, \phi) = \frac{1}{4\pi^2} \int_{-(\theta_m/2)}^{\theta_m/2} \int_{\omega_1/c}^{\omega_2/c} S_D(R, \theta) e^{jRr \cos(\theta - \phi)} |R| dR d\theta \quad (6.4)$$

where (r, ϕ) represents the polar Coordinates in the (x, y) plane with the following relationships

$$\begin{aligned} x &= r \cos \phi \\ y &= r \sin \phi \end{aligned} \quad (6.5)$$

Now the main process of back-projection algorithm is obtained. The processing in Equation (6.4) can be separated into two steps:

$$Q(r') = \frac{1}{4\pi^2} \int_{\omega_1/c}^{\omega_2/c} S_D(R, \theta) |R| e^{jRr'} dR \quad (6.6a)$$

$$\tilde{f}(r, \phi) = \int_{-(\theta_m/2)}^{\theta_m/2} Q(r \cos(\theta - \phi)) d\theta \quad (6.6b)$$

where $r' = r \cos(\theta - \phi)$. The first step, as shown in Equation (6.6a), can be regarded as a convolution processing between $p_\theta(r)$, the projection of $g(x, y)$ at a look angle θ , and a filter whose spatial frequency response is $|R|$. Therefore, the back-projection algorithm is

also called the “convolution back-projection” or “filtered back-projection” algorithm. As well, it can be considered as inverse Fourier transform process by the demodulated data multiplying the Jacobian function. Subsequently, the final image $f(r, \phi)$ can be obtained by integrating the function Q over the look angle θ .

It should be noted that in the back-projection step of Equation (6.6b), a one-dimensional interpolation is involved for each point (x, y) . This interpolation processing is the major contribution to the computational cost of back-projection algorithm. Moreover, the focusing performance of back-projection algorithm mainly depends on the interpolation precision. In addition, note that in SAR process, the radar signal is modulated by a carrier frequency w_c , so the demodulated data $S_D(R, \theta)$ in Equation (6.6a) need to be shifted to the base-band prior to taking the one dimensional inverse Fourier transform in the radial range R direction. Let center spatial frequency R_c be equal to the value $(2w_c / c)$. Then the Equation (6.6a) can be re-expressed in base-band as

$$\begin{aligned} Q(r') &= \frac{1}{4\pi^2} \int_{2(\omega_1 - w_c)/c}^{2(\omega_2 - w_c)/c} S_D(R + R_c, \theta) |R + R_c| \exp[j(R + R_c)r'] dR \\ &= \left\{ \frac{1}{4\pi^2} \int_{2(\omega_1 - w_c)/c}^{2(\omega_2 - w_c)/c} S_D(R + R_c, \theta) |R + R_c| \exp(jRr') dR \right\} * \exp(jR_c r') \end{aligned} \quad (6.7)$$

Therefore, if we directly take the inverse Fourier transform to the demodulated data, then after that an additional exponential term $\exp(jR_c r')$ needs to be multiplied.

6.2 Range Migration Algorithm

6.2.1 Introduction of Range Migration Algorithm

Range Migration algorithm (RMA) [6],[9],[26]-[28] is also known as wave-number (w-k) processing technique, and it originates from seismic engineering and geophysics. The algorithm can completely compensate the curvature of the wavefront and is computationally efficient, requiring a one-dimensional interpolation known as Stolt interpolation and a single matched filter. The RMA was firstly introduced to focus two-dimensional SAR data acquired from a space-borne platform in the strip-map mode. Later it was adapted in the spotlight mode. Unlike other approximate SAR imaging algorithms, the reconstructed image quality from range migration algorithm does not suffer from the space variant defocusing and geometric distortion induced by range curvature. These characteristics make the RMA particularly attractive for imaging situations that involve several differential range curvatures over the scene of interest. Such situations include imaging large scenes at fine resolution, imaging at short ranges, and imaging with a low carrier frequency, such as FOPEN imaging. In convention, RMA works with motion compensation to a moving point, or a line. The motion compensation to a line (MCTL) produces a signal history that is chirped in azimuth and de-chirped in range. It causes one drawback of RMA processing that higher sampling rate in azimuth direction is required. Hence methods to alleviate the RMA sampling requirement in azimuth direction are an active area of research. In this section, a new approach of RMA

is discussed. With operation on the signal data after motion compensation to a fixed point (MCTP), it can further reduce the sampling rate in the azimuth direction.

6.2.2 Procedures of Conventional RMA

Conventionally, range migration algorithm is developed with the signal data after motion compensation to a line. As mentioned in the Section 5.1.2, the RVP term is larger for the strategy of motion compensation to a line. Hence, an additional pre-processing is required before the operations of RMA, called range de-skew [6]. This process is necessary so that the RVP effect can be safely ignored. Besides the pre-process, the range migration algorithm mainly consists of four processing steps: (1) conversion of the processed data into two dimensional spatial frequency domains; (2) two-dimensional phase compensation; (3) Stolt interpolation; and (4) two-dimensional inverse Fourier transform.

(1) Conversion of the processed data into 2-D spatial frequency domain

The first step of range migration algorithm is to apply a one-dimensional azimuth Fourier transform to the demodulated data $S_D(R, y)$ that is modeled in Figure 5.2 or a two dimensional Fourier transform to the range-compressed data [27]. After this step, the processed data is converted into two dimensional spatial frequency domain (R, Y) , where R, Y are the Fourier pairs of radial range r and azimuth y variables, respectively. As we have demonstrated, the demodulated data working with motion compensation to a straight line can be expressed in Equation (5.10). Based on the principle of stationary phase explained in Appendix A, we can easily Fourier-transfer the complex equation and

obtain the expression of the processed data in two-dimensional spatial frequency domain as

$$P_{C1}(R, Y) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-j\sqrt{R^2 - Y^2}(x_c + x_n) - jY(y_c + y_n) + jRr_c] dx_n dy_n \quad (6.8)$$

where amplitude functions are suppressed. It is because that we always emphasize on the phase term rather than the magnitude term in the coherent radar problem. Also, all the variables in this equation are associated with the system model in Chapter 5, where (x_c, y_c) shows the position of reference scatter and (x_n, y_n) depicts any target location in image area with relative to reference scatter. Using the simulation model in Section 5.2, after two-dimensional Fourier transform to the range-compressed data shown in Figure 5.5 or one-dimensional Azimuth Fourier transform to the demodulated data, the simulation results can be shown in Figure 6.1.

Figure 6.1: 2-D spatial frequency image

(2) Two-dimension phase compensation

The second step of RMA is the two-dimensional phase compensation, which is a matched filtering process and is made to partially remove the residual range curvature of scatters.

It is accomplished by multiplying an additional phase term, which is expressed as

$$P_{CM}(R, Y) = \exp[j\sqrt{R^2 - Y^2} \cdot x_c + jYy_c - jRr_c] \quad (6.9)$$

Then, the processed data becomes

$$P_{C2}(R, Y) = P_{C1}(R, Y) * P_{CM}(R, Y) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-j\sqrt{R^2 - Y^2} \cdot x_n - jYy_n] dx_n dy_n \quad (6.10)$$

where (x_n, y_n) is the coordinate of pixel to be imaged.

Apparently, this compensation term can remove the residual phase term and make the reference scatter or reference point at image center. In essence, this matched filter under-compensates the curvature of targets at ranges nearer than reference scatter and over-compensates the curvature of targets at ranges farther than reference scatter. Thus, this operation perfectly corrects the range curvature of all targets at the same range as reference scatter and it only partially compensates targets at other ranges. Hence the job of the next operation is to remove the residual range curvature of all targets.

(3) Stolt interpolation

The third step performed by the RMA is known as Stolt interpolation. It is used to fully compensate the range curvature of all targets by an appropriate warping of the processed data. The warping is operated by a one-dimensional variable changing, where the new variable X_n is given by

$$X_n = \sqrt{R^2 - Y^2} \quad (6.11)$$

Now, we let $Y_n = Y$, it means the sampling criterion for both azimuth variable on radar Coordinate and azimuth variable on image Coordinate are the same. Then the corresponding data becomes

$$P_{C3}(X_n, Y_n) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-jX_n x_n - jY_n y_n] dx_n dy_n \quad (6.12)$$

Then the signal from a single target located anywhere in the imaging area now has linear phase in Y and X domains and nominally constant magnitude over the two-dimensional support. Hence the processed data can be viewed as the Fourier transform of the reflectivity functions image. In the simulation program, the sinc interpolating function is applied for Stolt interpolation. Also, to reduce the computational cost, the interpolating function is in the windowed version. Then the simulation results of two-dimensional spatial frequency image after Stolt interpolation can be shown in Figure 6.2.

Figure 6.2: 2-D spatial frequency image after Stolt interpolation

(4) Two-dimensional inverse Fourier Transform

Since the processed signal from all targets is two-dimensional linear phase gratings, a two-dimensional inverse Fourier Transform can be involved to fully compress the processed data in both range and azimuth. This process is shown as

$$I(x_n, y_n) = P_{C4}(x_n, y_n) = IFFT[P_{C3}]_{2D} \quad (6.13)$$

Then the simulation result of final SAR image is shown in Figure 6.3 and it could be found that there are exactly five targets in the image.

Figure 6.3: Final SAR image Using RMA

6.2.3 New Approach of RMA and Simulation

The new approach of range migration algorithm is developed with the signal data after motion compensation to a point. And this compensation strategy can eliminate azimuth chirp. It causes the lower sampling rate in the azimuth direction. However, azimuth de-

chirping causes a narrower spatial bandwidth in the azimuth direction, which is also called Doppler bandwidth. In general, narrower Doppler bandwidth results in worse azimuth resolution in image formation. Hence, a necessary step is required to reinstate the azimuth chirp of the processed data for the new approach of RMA. However, this strategy can safely ignore RVP without additional operation. It is because the RVP from motion compensation to point is much smaller, which is explained in Section 5.1. RVP term will only become significant in situations when a UWB waveform and when imaging over a large area is done. As demonstrated before, the processed data after motion compensation to a point can be expressed as Equation (5.12). Different with the conventional approach, the new approach of RMA is beginning at the digital reinstating the azimuth chirp for widening the Doppler bandwidth. It is conducted by multiplying a phase term, which is written as

$$P_a = \exp[-jR\sqrt{x_c^2 + (y_c - y)^2}] \quad (6.14)$$

Then the processed data becomes

$$P_{N1}(R, y) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-jR\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2}] dx_n dy_n \quad (6.15)$$

Next, the processed data is converted into 2-D spatial frequency domain, which is same as the first step of conventional RMA, since RMA mainly operates the data in spatial frequency domain for both range and azimuth directions. Based on the principle of stationary phase in Appendix A, the processed data now can be written as

$$P_{N2}(R, Y) = \int_{-r_0}^0 \int_{-r_0}^0 f(x_n, y_n) \cdot \exp[-j\sqrt{R^2 - Y^2}(x_c + x_n) - jY(y_c + y_n)] dx_n dy_n \quad (6.16)$$

Using the simulation model in Section 5.2, the 2-D spatial frequency image can be obtained as Figure 6.4. It is found that the Doppler bandwidth is about 12 rad/m, which is

similar to the result obtained from the conventional approach of RMA in Figure 6.1. If the simulation mode is processed without the first step of widening the Doppler bandwidth, then the 2-D spatial frequency image is shown in Figure 6.5, where the Doppler bandwidth is about 4 rad/m. Since the value of Doppler bandwidth is about one-third of the value from conventional approach, the lower sampling rate is required for the new MCTP-based RMA.

Figure 6.4: 2-D spatial frequency image

Figure 6.5: 2-D spatial frequency image without Doppler bandwidth widening

Similar to the conventional approach, the next step is 2-D phase compensation. The compensated phase term is

$$P_{NM}(R, Y) = \exp[j\sqrt{R^2 - Y^2} \cdot x_c + jYy_c] \quad (6.17)$$

Compared with the conventional one, only the last phase term is lost. Then the processed data is expressed as

$$P_{N3}(R, Y) = \int_{r_0}^0 \int_{r_0}^0 f(x_n, y_n) \cdot \exp[-j\sqrt{R^2 - Y^2} \cdot x_n - jYy_n] dx_n dy_n \quad (6.18)$$

After phase compensation, the expression of processed data becomes the same as the one from conventional RMA. Similarly, the processed data then goes through Stolt interpolation and 2-D inverse Fourier transform. They can be expressed as

$$P_{N4}(X_n, Y_n) = \int_{r_0}^0 \int_{r_0}^0 f(x_n, y_n) \cdot \exp[-jX_n x_n - jY_n y_n] dx_n dy_n$$

$$I(x_n, y_n) = P_{N5}(x_n, y_n) = 2DFFT[P_{N4}] \quad (6.19)$$

Finally the SAR image is simulated as

Figure 6.6: Final SAR image with new approach of RMA

From the figure, it can be found that the imaging result is successfully reconstructed, which is in consistency with the simulation model.

6.2.4 Doppler Bandwidth Analysis

As mentioned before, the new approach of RMA with operation on the signal data after motion compensation to a fixed point (MCTP) can further reduce the sampling rate. It means fewer collected points are used during data collection. Basically sampling requirement in azimuth direction is decided by the Doppler (azimuth spatial frequency domain) bandwidth. In the conventional approach of RMA, image formation is based on motion compensation to a line (MPTL) that produces the signal data with wider Doppler bandwidth. Hence, higher azimuth sampling rate is required. Undoubtedly, it will bring complexities in real radar system. The new approach of RMA is developed to overcome this problem. In the following, Doppler bandwidth from different compensation strategies is analyzed by some simulations.

Based on Equation 5.10, the phase term after MCTL can be modeled as

$$\Phi_{ML} = -R(\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + y_c^2}) \quad (6.20)$$

When taking the first derivative of this phase term, the instantaneous spatial frequency with relative to radar position domain y is

$$Y(y) = \frac{\partial}{\partial y} [-R(\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + y_c^2})]$$

$$\begin{aligned}
&= \frac{R(y_c + y_n - y)}{\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2}} \\
&= R \sin \theta_n(y)
\end{aligned} \tag{6.21}$$

where

$$\theta_n(y) = \arctan\left(\frac{y_c + y_n - y}{x_c + x_n}\right) \tag{6.22}$$

is the aspect angle of the radar for scatter $(x_c + x_n, y_c + y_n)$ when radar is located at $(0, y)$. Hence, Doppler bandwidth can be obtained as

$$DB_{ML} = 2R_{\max} \sin \theta_n(y)_{\max} \tag{6.23}$$

However, for the strategy of MCTP, the phase term in Equation (5.11) can be written as

$$\Phi_{MP} = -R\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + (y_c - y)^2} \tag{6.24}$$

And the corresponding instantaneous spatial frequency in the azimuth direction is

$$\begin{aligned}
Y(y) &= \frac{\partial}{\partial y} [-R\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2} - \sqrt{x_c^2 + (y_c - y)^2}] \\
&= \frac{R(y_c + y_n - y)}{\sqrt{(x_c + x_n)^2 + (y_c + y_n - y)^2}} - \frac{R(y_c - y)}{\sqrt{x_c^2 + (y_c - y)^2}} \\
&= R \sin \theta_n(y) - R \sin \theta_c(y)
\end{aligned} \tag{6.25}$$

where

$$\theta_c(y) = \arctan\left(\frac{y_c - y}{x_c}\right) \tag{6.26}$$

is the aspect angle of radar for the image center. Hence, Doppler bandwidth becomes

$$DB_{MP} = R_{\max} \cdot \text{MAX}[\sin \theta_n(y) - \sin \theta_c(y)] \tag{6.27}$$

For further analyzing the Doppler bandwidth, three simple SAR geometries at broadside mode ($y_c = 0$) are set up. Assume radar moves from -50m to 50m in the azimuth direction, then $y \in [-50, 50]$. And the transmitting frequency is ranged from 150MHz to 300MHz .

(1) Some scatters in imaged area are regularly placed and shown in Figure 6.7. The image center is at $(200, 0)$.

Figure 6.7: SAR Geometry 1 for Doppler bandwidth analysis

Based on Equation (6.23) and (6.26), the theoretic values of Doppler bandwidth from these two compensation strategies can be calculated. They are

$$DB_{ML} = 2R_{\max} \sin \theta_n (-50)_{\max} = (2 * 5.9136) = 11.8272 \text{ rad / m} \quad (6.28)$$

$$DB_{MP} = 2R_{\max} \cdot \text{MAX}[\sin \theta_n (-50) - \sin \theta_c (-50)] = (2 * 2.8658) = 5.7316 \text{ rad / m} \quad (6.29)$$

Then the ratio of DB_{ML} to DB_{MP} is obtained as

$$r = \frac{DB_{ML}}{DB_{MP}} = 2.064 \quad (6.30)$$

In these simulations, spatial frequency image from these two compensation strategies are reconstructed, where the Doppler bandwidth can be observed as $2Y_{\max}$. The simulation images are shown in Figure 6.8 and Figure 6.9 respectively.

Figure 6.8: Spatial frequency image from MCTL for SAR Geometry 1

Figure 6.9: Spatial frequency image from MCTP for SAR Geometry 1

From the simulated figures, it can be found that the observed Doppler bandwidth values are quite matched with the theoretical ones we have calculated.

(2) The image center is changed to (400, 0) and the image area is shown in Figure 6.10.

Figure 6.10: SAR Geometry 2 for Doppler bandwidth analysis

Based on this SAR geometry, the theoretical Doppler bandwidth values can be obtained as

$$DB_{ML} = (2 * 2.800) = 5.600 \text{ rad / m}$$

$$DB_{MP} = (2 * 1.241) = 2.482 \text{ rad / m} \quad (6.31)$$

And the ratio of these two Doppler bandwidth values becomes

$$r = \frac{DB_{ML}}{DB_{MP}} = 2.256 \quad (6.32)$$

In simulation, the images in two dimensional spatial frequency domains from MCTL and MCTP are shown in Figure 6.11 and 6.12 respectively. It can be found that the Doppler bandwidth values are approximately the same with the theoretic ones

Figure 6.11: Spatial frequency image from MCTL for SAR Geometry 2

Figure 6.12: Spatial frequency image from MCTP for SAR Geometry 2

(3) The reduced image area is shown in Figure 6.13, where the image center is at (200, 0).

Figure 6.13: SAR Geometry 3 for Doppler bandwidth analysis

Then the theoretical Doppler bandwidth values and the ratio value can be calculated as

$$DB_{ML} = (2 * 5.3141) = 10.6282 \text{ rad / m}$$

$$DB_{MP} = (2 * 2.2663) = 4.5326 \text{ rad / m} \quad (6.33)$$

$$r = \frac{DB_{ML}}{DB_{MP}} = 2.3448 \quad (6.34)$$

The simulated images in 2-D spatial frequency domain from MCTL and MCTP are shown in Figure 6.14 and Figure 6.15 respectively.

From the simulation results, we can find that the observed Doppler bandwidth values are approximately equal to theoretical ones.

Through the analysis from these three cases, it can be concluded that when imaging a smaller area and imaging at larger range, the larger ratio value is obtained. Therefore,

under such conditions, the MCTP strategy has more advantage over MCTL in azimuth sampling requirement.

Figure 6.14: Spatial frequency image from MCTL for SAR Geometry 3

Figure 6.15: Spatial frequency image from MCTP for SAR Geometry 3

6.2.5 Performance Evaluation in Sampling Requirement

As demonstrated above, when sufficient sampling points are selected during radar collection, the reconstructed images are the same for both approaches of range migration algorithm. However, analysis shows lower sampling rate is required for the new approach of MCTP-based range migration algorithm. Thus compared with conventional approach of MCTL-based RMA, better quality images can be achieved when low sampling rate is applied to both approaches. This advantage of MCTP based RMA will be shown using both simulation and experimental results.

6.2.5.1 Simulation Analysis

Figure 6.7 is used as simulation geometry. Then the theoretical Doppler bandwidth values from both approaches based on Equation (6.28) and (6.29) are 11.8272 rad/m and 5.7316 rad/m respectively. Hence the sampling distances are 0.53m and 1.1m respectively. Assume that radar will collect the data at every 0.53m in the synthetic aperture. Then the total number of sampling points becomes 189. At such condition, the sampling status satisfies both compensation strategies. So the simulation results from the two approaches are the same. It is shown in Figure 6.16. Now, if we assume that radar will collect the data at every 1.1m, it means the total number of sampling points during collection is about 91. Thus the sampling status is sufficient for the strategy of motion compensation to a point, but it is not enough for the strategy of motion compensation to a line. Then the simulated image based on MCTL is shown in Figure 6.17, while the simulated result based on MCTP is shown in Figure 6.18. Comparing these two images, it is clear that

image from the MCTP-based new approach of RMA results in better quality. Therefore the advantage of this new approach range migration algorithm is verified in term of sampling requirement.

Figure 6.16: SAR image with 0.53m sampling distance for both approaches of RMA

Figure 6.17: SAR image using conventional approach of RMA with 1.1m sampling distance

Figure 6.18: SAR image using new approach of RMA with 1.1m sampling distance

6.2.5.2. Experimental Results Analysis

The processed results are reconstructed from the raw data of FOPEN SAR experiment. The details about this experiment will be described in Section 7.2.2. A single corner reflector is placed in the imaging area and it is concealed under the foliage areas. When the data is collected at every 0.2m in the path of radar movement. At such high sampling rate, the experimental results are the same for two motion compensation strategies and can be seen in Figure 6.19.

If the signal data is collected at every 3m, the experiment results from MCTL-based and MCTP-based RMA are shown in Figure 6.20 and 6.21 respectively. It is found that the corner reflector response from the MCTL-based RMA results in many surrounding clutters and it has a bad resolution in the azimuth direction.

Figure 6.19: Corner reflector image with 0.2m sampling distance for both approaches of RMA

Figure 6.20: Corner reflector image using conventional RMA with 3m sampling distance

Figure 6.21: Corner reflector image using new approach of RMA with 3m sampling distance

However, when the sampling distance increase more, the sampling rate during the data collection becomes lower, so the reconstructed image of the corner reflector using the MCTL-based RMA will result in worse quality. Figure 6.22 shows the experimental result from MCTL-based RMA. It can be found that the corner reflector response becomes unfocused, while the result in Figure 6.23 from MCTP-based RMA is still perfectly focused and approximately the same with the previous image shown in Figure 6.19.

Therefore, from the comparison of experimental results using these two approaches of RMA, we can further verify the advantage of the MCTP-based RMA in sampling requirement.

Figure 6.22: Corner reflector image using conventional RMA with 3.4m sampling distance

Figure 6.23: Corner reflector image using new approach of RMA with 3.4m sampling distance

6.3 Conclusion of the Chapter

The background and procedures of two imaging algorithms for FOPEN SAR, back-projection algorithm and range migration algorithm, are presented in detail. A new approach of RMA is developed in this chapter. Based on numerical simulation and experiment results, it is found that the new approach of RMA has more advantages in term of sampling requirement than the conventional one.

Chapter 7

Comparison of Imaging Algorithms

Although both back-projection and range migration algorithms had been applied to SAR imaging for more than one decade, no performance comparison was reported. In this chapter, performance of two algorithms, back-projection algorithm and range migration algorithm, is evaluated by using simulated and experimental data and comparisons are made. Based on numerical simulation, performance is illustrated under the ideal condition, data sampling error, and radar trajectory error respectively. Experimental results are presented from the experiments conducted in microwave anechoic chamber and foliage penetration Boom-SAR field trial. Finally the issues of the implementation of these two algorithms are compared and analyzed.

7.1 Simulation Results Analysis

7.1.1 Ideal Condition

To verify the performance of the above mentioned two algorithms, numerical simulations are applied to the SAR imaging system geometry shown in Figure 5.4, where five ideal scatterers are located in the imaging area. As stated in SAR simulation model (Section 5.2), the transmitting signal is a chirp waveform with a center frequency of the 225MHz

and bandwidth of 150 MHz. Therefore, the range resolution based on Equation (4.17) can be calculated as

$$\rho_x = \frac{c}{2B} = 1m \quad (7.1)$$

where B is the bandwidth of the transmitted signal and c is the speed of light. In the model, suppose the radar moves along from $-50m$ to $50m$ in the azimuth direction. Based on Equation (4.18), the azimuth resolution with relative to the largest wavelength (at 150MHz) can be obtained as

$$\rho_y = \frac{\lambda_L}{4 \sin(\vartheta/2)} = 2.1m \quad (7.2)$$

where ϑ is the integration angle and equals $2 \arctan(50/200)$.

Sufficient sampling points are chosen in the simulation, so that both approaches of RMA will achieve the same result. For BPA, an additional process, over-sampling by a factor of 6 in the spatial frequency domain, is used to improve the image quality. Figure 7.1 and Figure 7.2 show the imaging results by using the back-projection and range migration algorithm respectively. We can see that both algorithms are able to successfully reconstruct the image of scatters and the simulated image is in good agreement with the expected ones, where five scatters are clearly seen and are accurately located. An interesting thing to note is that the reflectivity peak of the scatter closer to the radar, which is located at $(100,0)$ in the simulated image area, is narrower than other scatters for both imaging techniques. This is because the effective synthetic aperture is larger in the near range.

In order to depict the resolution performance, Figure 7.3 shows the cuts of image reconstructed by the two algorithms around the center scatter $(200,0)$ along the range and

azimuth direction. It is found that both the back-projection and range migration algorithms can achieve the theoretical resolution performance in both range and azimuth directions.

Figure 7.1: Simulation result by using back-projection algorithm

Figure 7.2: Simulation result by using range migration algorithm

Figure 7.3: Cuts of reconstructed images around the scatter (200,0) along range and azimuth direction

(-●- : back-projection algorithm; -○- : range migration algorithm)

From the above simulation results, it is shown that benefiting from the full compensation of the differential range curvature, both algorithms are suitable for ultra-wideband SAR imaging and the theoretical resolution can be achieved. As demonstrated in the previous chapter, the full compensation processing includes two steps for back-projection algorithm instead of four steps for range migration algorithm. Thus the back-projection algorithm is a simpler approach with fewer code lines, but it needs longer operational time. More detailed analysis on the computational complexities of two algorithms will be discussed in Section 7.3.

Note that the above simulation results are based on ideal conditions. To verify the robustness of back-projection and range migration algorithms, in the following parts, some practical implementation aspects, including the effects of receiver sampling error and radar trajectory error on both algorithms, will be discussed and compared.

7.1.2 Data Sampling Error

In practical applications, the sampling error is often induced by the data acquisition system in the radar receiver. The digitized receiving signal inevitably includes amplitude and phase errors, which will degrade the focusing of target scatters, particularly in the azimuth direction. Considering the same configuration as illustrated in Figure 5.4 but with only one scatter at (200,0), the normally distributed amplitude error with a variance of 0.5, and the uniformly distributed phase error at $(-0.4\pi, 0.4\pi)$, are induced to the scattering response at each observation. Figure 7.4 shows the cuts of target images reconstructed by back-projection and range migration algorithms in range and azimuth directions respectively. It can be found that the effect of sampling error is much more serious in azimuth direction for both algorithms. The focusing performances of different algorithms are similar.

Figure 7.4: Cuts of scatterer response in range and azimuth directions
 (-●- : back-projection algorithm; -○- : range migration algorithm)

7.1.3 Radar Trajectory Error

First, it should be noted that in the range migration algorithm, the data collection positions are required to be evenly spaced in azimuth. Therefore, the radar trajectory is often supposed to be a straight line. This is because the azimuth Fourier transform is applied in range migration algorithm and the errors due to improper uneven sampling along the azimuth will induce significant defocusing in the reconstructed image. However, the back-projection algorithm has no assumption on the azimuth sampling spacing and does not require uniform angular increments. When the radar trajectory is not a straight line but already known, the back-projection algorithm can be directly applied. However, more complicated azimuth interpolation is necessary before applying the range migration algorithm. In this section, we will verify and compare the performance robustness of both algorithms with data collection trajectory error that is not known, i.e., regardless any motion compensation. Taking the same configuration as illustrated in Figure 5.4, also with only one scatterer at (200,0), we consider two types of radar trajectory errors. The first one is “slow” variance with respect to data collection time. As an example, the radar trajectory has a sine shape instead of a straight line, as illustrated in Figure 7.5. The range and azimuth cuts of scatterer response reconstructed by back-projection and range migration algorithms are shown in Figure 7.6. Another type of radar trajectory error is relatively “fast” with respect to data collection time. Figure 7.7 illustrates an example of radar trajectory with random disturbance and the corresponding results are shown in Figure 7.8. It can be found that the performances of different algorithms are similar in the case of radar trajectory with a sine shape. When taking the random disturbance, the

azimuth resolution of back-projection algorithm is slightly better than that of range migration algorithm.

Figure 7.5: Radar trajectory with a sine shape

Figure 7.6: Cuts of scatterer response in range and azimuth directions
(-●- : back-projection algorithm; -○- : range migration algorithm)

Figure 7.7: Radar trajectory with random disturbance

Figure 7.8: Cuts of scatterer response in range and azimuth directions
 (-●- : back-projection algorithm; -○- : range migration algorithm)

7.2 Experimental Results Analysis

7.2.1 Microwave Anechoic Chamber Measurements

Based on the previous theoretical analysis and numerical simulations, to verify the performance of back-projection and range migration algorithms in application of near-field synthetic aperture radar (SAR) imaging, a simple Vector Network Analyzer (VNA) based experimental SAR system was set up and some measurements were conducted in the microwave anechoic chamber. The hardware configuration of the VNA based SAR system is shown in Figure 7.9. In this experimental system, HP-8753D Vector Network Analyzer (VNA) with frequency range from 30KHz to 6GHz acts as a SAR transceiver,

including the function of step-frequency CW waveform generation, target echo signal reception, mixing and data acquisition. It reduces the hardware design to the consideration of only the radar system. A pair of WJ-48430 Horn Antennas with operating frequency range from 3GHz to 18GHz are selected for this near-field SAR experiment, that can provide high-gain directional patterns over octave bandwidths. Figure 7.10 shows the photo of SAR imaging measurement in microwave anechoic chamber.

Figure 7.9: The hardware configuration of the VNA based SAR system

Figure 7.10: Photo of measurement in microwave anechoic chamber

Figure 7.11 shows the geometrical configuration of the measurement. The antennas are placed on the same level as the target and raw data are collected at 33 equally spaced positions with a total synthetic aperture of 1.6m. Therefore, the antennas are shifted by an interval of 0.05m after data has been collected. As a result, 33 data files are collected for one set of SAR measurement. Finally 33 data files are combined and sent to MATLAB for off-line processing.

Figure 7.11: Geometrical configuration of the measurement

In this experiment, a steel cylinder with a diameter of 20 cm and a height of 35 cm was selected as the target. The step-frequency CW waveform from 3 to 6 GHz is transmitted by the VNA with 201 steps. Therefore, the theoretical range resolution is 5 cm. On the other hand, for the geometry configuration illustrated in Figure 7.11, the azimuth resolution relative to Equation (4.18) can be calculated to be around 3.4 cm.

Figure 7.12 and Figure 7.13 show the final two-dimensional radar image of the target reconstructed by back-projection and range migration algorithms respectively. It can be

observed that both target images are well focused and exactly located at the center of the azimuth and around 0.75m away from the antenna, which is consistent with the location of the target in the physical set up.

Figure 7.12: Experimental result by using back-projection algorithm

Figure 7.13: Experimental result by using range migration algorithm

To analyze image resolution in range and azimuth directions, Figure 7.14 shows the cuts of target response in range and azimuth directions respectively when using the back-projection and range migration algorithm. It also indicates that the actual physical size of steel cylinder was successfully measured by using both algorithms.

Figure 7.14: Cuts of cylinder response in range and azimuth direction
 (-●- : back-projection algorithm; -○- : range migration algorithm)

7.2.2 Foliage Penetration Boom-SAR Measurement

For the study of foliage penetration propagation and radar imaging, an experimental ultra-wideband SAR system was developed by a joint group of NTU (Nanyang Technological University, Singapore) and ONERA (Office National d'Etudes et de Recherches Aérospatiales, France). A series of field experiments had been conducted

[29][30]. This system is based on a SAR device embarked in the basket of a boom lift whose motion can produce the synthetic aperture effect. Such system is also called Boom-SAR. Operating in low frequency bands (VHF/UHF), the experimental system can achieve the FOPEN imaging capability with a resolution of 0.5×0.5 m. Figure 7.15 shows the photo of experimental radar system and Figure 7.16 shows an example of the imaged area including many regularly planted tropical trees, that is similar to the trial site, except that the crown of trees at trial site is much more dense and compact.

Figure 7.15: The photo of experimental Boom-SAR system

Figure 7.16: The photo of field trial site

The results presented in this section are obtained for VHF band (150-300 MHz). The crane altitude is 35 m. In this trial, a 4×4 m trihedral corner reflector was set up under the foliage as a reference target. Figure 7.17 and Figure 7.18 show the reconstructed SAR images of a local region of trial site by using the back-projection and range migration algorithms respectively. For back-projection algorithm, over-sampling by a factor of six is used. In both images, we can find a strong scatter corresponding to the corner reflector locating under the foliage and many other uniformly distributed points corresponding to the trunks.

To precisely analyze the range and azimuth resolution achieved, Figure 7.19 shows the cuts of corner reflector response in range and azimuth directions respectively when using the back-projection and range migration algorithms. Figure 7.20 shows the corresponding results for an arbitrarily selected trunk response. It can be found that for both the corner

reflector and trunk, the responses in range and azimuth directions coincide and the theoretical resolution can also be achieved when using different algorithms. These results verify both the success of experimental Boom-SAR system and the validity of back-projection and range migration algorithms in the application to FOPEN SAR imaging. However, the notable disadvantage of BPA is that more processing time is required.

Figure 7.17: Experimental result by using back-projection algorithm

Figure 7.18: Experimental result by using range migration algorithm

Figure 7.19: Cuts of corner reflector response in range and azimuth direction

(-●- : back-projection algorithm; -○- : range migration algorithm)

Figure 7.20: Cuts of trunk response in range and azimuth direction

(-●- : back-projection algorithm; -○- : range migration algorithm)

7.3 Comparison of Implementation Issues

From the above simulated and experimental results, both back-projection algorithm (BPA) and range migration algorithm (RMA) can achieve image with high resolution. It is due to the fact that these two algorithms can fully compensate the differential range curvature. This no phase aberration property results in the high resolution. As demonstrated in the last chapter, the full compensation process includes two steps for BPA instead of four steps for RMA. Thus BPA is a simpler approach with fewer code lines, but it needs longer operational time. The most important step in RMA for curvature compensation is Stolt interpolation, which complicates the implementation and induces the interpolation errors. However, BPA mainly suffers from artifacts due to spatial time domain interpolation in its send step. In contrast, RMA starts with the rectangular-formatted data, while BPA begins at the polar-formatted data. In the following parts, the implementation aspects including data collection, computational complexity, memory requirement and over-sampling approach for both BPA and RMA will be compared and discussed.

7.3.1 Data Collection

For data collection, RMA needs that the collection points, where the signals transmitted and received are evenly spaced. And the radar trajectory must be in a straight line. This is because azimuth Fourier transform is used in the processing of RMA and errors due to improper sampling along the azimuth will lead to very significant artifacts in the imagery.

However, BPA is applicable for any shape trajectory, because Equation (6.14b) in back-projection algorithm can be modified into

$$\tilde{f}(r, \phi) = \sum_{k=0}^{N-1} Q(r \cos(\theta_k - \phi)) \Delta \theta \quad (7.3)$$

where k is point number of the collection and N is the total number of the collection points. Hence BPA does not need uniform angular increments and only the difference in the successive angles $\Delta \theta$ is required for computation. It makes BPA implement more easily.

7.3.2 Computational Complexity

In general, the computational complexity of the BPA is $O(N^3)$ for a $N \times N$ pixels image with N collection pulses. And the complexity for the Fourier-based imaging algorithm (RMA), is $O(N^2 \log_2 N)$ [31]. Therefore, the BPA will suffer more computational complexity. In the following, we will present a quantitative analysis on the computational complexities of the two algorithms in detail.

Many possible approaches are applied to evaluate the computational complexity of the imaging algorithm. This part presents an approach based on estimating the number of real operations when a $N \times N$ pixels image is reconstructed with N collection points, in which the following assumptions are taken:

- (1) Both back-projection and range migration algorithms start from the range compressed SAR data.
- (2) The same data interpolation technique is applied to both algorithms.

- (3) The real multiplication and real addition are with equal computational cost.
- (4) The radix-2 based approach is adopted to implement the computation of fast Fourier transform (FFT).

We start from the computation cost of FFT. It is well known that when using the radix-2 based approach, the number of complex multiplications and complex additions required for one-dimensional FFT are $(N/2)\log_2 N$ and $N\log_2 N$, respectively. For two-dimensional FFT operation, $N^2\log_2 N$ complex multiplications and $2N^2\log_2 N$ complex additions are required. On the other hand, each complex multiplication consists of 4 real multiplications and 2 real additions, and each complex addition consists of 2 real additions. Therefore, the computation of N points one-dimensional FFT need $2N\log_2 N$ real multiplications and $3N\log_2 N$ real additions, totally $5N\log_2 N$ real operations. Similarly, the computation of N -by- N points two-dimensional FFT need $4N^2\log_2 N$ real multiplications and $6N^2\log_2 N$ real additions, totally $10N^2\log_2 N$ real operations.

The first step of back-projection algorithm, as expressed in Equation (6.14a), is a convolution processing, which can be implemented with the FFT-multiplication-IFFT technique. Thus, the number of real operations involved in the convolution processing is $5N^2\log_2 N + 2N + 5N^2\log_2 N$. Next, for the back-projection of each pixel, N complex additions are needed. Therefore, to reconstruct an image with $N \times N$ pixels, the number of real operations required is $2N^3$. In addition, to get good back-projection precision of each image pixel, data interpolation should be applied to the N range profiles before the summation. The data interpolation can be implemented by many techniques with different computational cost. Supposing the number of real operations required to

interpolate one data sample is M_{interp} , the number of real operations required in the second step of back-projection algorithm becomes $2N^3 + N^3 M_{\text{interp}}$. Hence, the total number of real operations required for back-projection algorithm is

$$10N^2 \log_2 N + 2N^2 + 2N^3 + N^3 M_{\text{interp}} \quad (7.4)$$

In the range migration algorithm, the first step is one-dimensional azimuth Fourier transform applying to N range cells. So the number of real operation required is $5N^2 \log_2 N$. Next, the step of phase compensation possesses N^2 complex multiplications, i.e., $6N^2$ real operations. In the step of Stolt interpolation, the nonuniform-to-uniform data interpolation is involved in the variable changing. When using the same interpolation technique as that is used in back-projection algorithm, the number of real operations in the Stolt interpolation is $N^2 M_{\text{interp}}$. At last, as above discussed, the processing of two-dimensional IFFT consists of $10N^2 \log_2 N$ real operations. Therefore, the total number of real operations for range migration algorithm is

$$5N^2 \log_2 N + 6N^2 + N^2 M_{\text{interp}} + 10N^2 \log_2 N \quad (7.5)$$

From the above quantitative analysis, we can conclude that:

- (1) In the back-projection algorithm, the main contribution to the computational cost is the data interpolation.
- (2) In the range migration algorithm, the computational complexity mainly depends on one and two-dimensional FFT operations as well as the data interpolation.
- (3) Relatively, the computational complexity of back-projection algorithm is much higher than that of range-migration algorithm.

However, the computational complexity of back-projection algorithm may not be serious for the following two reasons: First, some fast computation techniques [31]-[36] have been developed to mitigate the computational burden of standard back-projection algorithm. Second, the processing scheme of back-projection algorithm can be interpreted as an evolution process, i.e., a point-by-point reconstruction, that greatly benefits its practical applications. Using the back-projection algorithm, one can choose the points to be reconstructed and then compute the intensity values for these points only.

7.3.3 Memory Requirement

As we have discussed above, although BPA has a larger computational burden, it benefits from parallel processing. The whole image can be segmented into some subarrays, the back-projection of pixels in each subarray can be implemented in a separate processor. Thus, the parallel processing can be easily implemented in real-time. This type of architecture also provides a degree of fault tolerance. If a processor fails, a subarray will be lost, but the other processors will continue to produce the rest of image correctly. In addition, the data from different collection points can be separately processed, it allows image reconstruction simultaneously with data collection. So we would not need to wait for all the measurements to be collected before a partial image could be produced. Therefore, BPA can save memory and is more applicable for real-time processing. However, RMA needs to synthesize all the collected data and process together.

7.3.4 Over-sampling

Over-sampling is conducted just by zero-padding the data in its Fourier pair domain. When using BPA, the over-sampling in spatial frequency domain is necessary. It will reduce the process error due to the data sampling near the Nyquist rate and improve the image quality. The more we over-sample, the more accurate the results will be. In practice, over-sampling factors between 2 and 8 offer a reasonable combination of speed and accuracy. The simulated images with different over-sampling factors are shown in Figure 7.21-7.24. It can be found that using less over-sampling will obtain poorer image quality. Figure 7.25 shows the cuts of simulated image around center scatter from the different over-sampling factors along both range and azimuth direction. It indicates that the result with over-sampling by a factor of 2 gets worst resolution, especially in the azimuth direction. However, using RMA without over-sampling, it still can achieve high-quality image that is shown in Figure 7.2. Hence over-sampling does not have much effect on the reconstruction in RMA except when the synthetic aperture is shorter than the imaged distance in the azimuth direction. In addition, over-sampling for BPA will increase the computational load.

Figure 7.21: Simulation result by using back-projection algorithm
(over-sampling by a factor of 2)

Figure 7.22: Simulation result by using back-projection algorithm
(over-sampling by a factor of 4)

Figure 7.23: Simulation result by using back-projection algorithm
(over-sampling by a factor of 6)

*Figure 7.24: Simulation result by using back-projection algorithm
(over-sampling by a factor of 8)*

Figure 7.25: Cuts of simulated image around the center scatterer along range and azimuth direction
 (over-sampling factor: -o- => 2; -* => 4; -+ => 6; -• => 8)

7.4 Conclusion of the Chapter

In this chapter, performance of back-projection algorithm and range migration is analyzed by using simulation and experimental results. It proves that both algorithms can be applied in FOPEN radar imaging where theoretical resolution can be achieved. In addition, more theoretic analysis and comparison of these two algorithms are made. The computational cost of back-projection algorithm is relatively higher than that of the range migration algorithm and over-sampling is required for back-projection algorithm. However, the back-projection is more suitable for hardware parallel implementation, with fewer software code lines and a memory saving algorithm.

Chapter 8

Conclusion and Recommendations

8.1 Conclusion

FOLIage PENetration (FOPEN) imaging is the Synthetic Aperture Radar (SAR) imaging for foliage penetration application, which is of special interests for both military and civilian applications. To reduce the radio energy attenuation induced by the leaves and branches and achieve foliage penetration, the VHF/UHF bands are often selected as the operation frequency of FOPEN radar systems. The Ultra-WideBand (UWB) waveform is used to get high range resolution. On the other hand, to get high resolution in azimuth as well as range, the integration angle of FOPEN imaging will be very large, which will induce very large range migration. Therefore, using the UWB waveform and large integration angle bring new complexities and challenges to traditional SAR imaging techniques. Some hypotheses used in traditional techniques are therefore not valid and some classical techniques cannot even be applied at all. In my research, two relatively novel SAR imaging algorithms, namely back-projection algorithm and range migration algorithm, are studied.

Some topics about SAR fundamentals are firstly presented, which are useful to understand FOPEN SAR imaging. It shows that resolution at two dimensions, range and

azimuth, are relative to UWB waveform and integration angle respectively. Spotlight SAR has more advantages over strip-map SAR and performance for different types of waveform are compared and discussed. Furthermore, some aspects of SAR image formation including enhancement techniques to improve the image quality are introduced.

A SAR system model is developed in the thesis. It is convenient to understand and simulate SAR imaging algorithms and it is the link to connect the imaging algorithms with the real raw data. In simulation model, the chirp waveform with UWB is used to study the FOPEN SAR image. It is due to its easy expression in software development. However, in experiment, a more practical UWB waveform called step-frequency is applied.

Back-projection algorithm and range migration algorithms are discussed in details in the thesis. A new approach of RMA is developed. It is found from numerical simulation and experimental results that the new approach has more advantages in sampling requirement than the conventional one. The simulation results and experimental results from the foliage penetration Boom-SAR measurements and microwave anechoic chamber measurements also prove that both the back-projection and range migration algorithms can be applied in FOPEN SAR imaging when using UWB waveform and large integration angle, and the theoretical resolution can be achieved. In addition, more theoretic analysis and comparison of these two algorithms are made in the thesis. Although the computational cost of back-projection algorithm is relatively higher than that of the range migration algorithm, the back-projection algorithm is more suitable for

hardware parallel implementation and with fewer software code lines. In the cases with data sampling error and radar trajectory error, the performance of two algorithms are similar.

8.2 Recommendations for Future Research

In the thesis, radars have demonstrated by operating at VHF and UHF frequency bands and using either chirp waveform or step-frequency waveform. There is also another type of radar, the UWB random noise radar, which has obtained more attention in recent years [37]. When applied in foliage penetration applications, UWB random noise radar has its special merits. Because of the randomness and the ultra-wide bandwidth of the transmit waveform, such a radar has potential for covert detection and identification, and is relatively immune from radio frequency interference (RFI) and hostile detection and jamming while preserving very high resolution. In the future research, the FOPEN SAR imaging processes need to be developed when the UWB random noise waveform is used.

The chirp scaling algorithm (CSA) [38] is another conventional synthetic aperture radar imaging algorithm. A new approach of chirp scaling algorithm associated with UWB waveform is developed in reference [6]. The new approach makes the CSA become one of wave-number domain algorithm, which range migration algorithm also belongs to. The chirp scaling algorithm is a computationally efficient variation to the range migration algorithm for SAR image formation. It is because it requires only FFTs and complex

multiplies and it requires no interpolation. These attributes lead to efficient implementations. The chirp scaling algorithm achieves its unique properties by approximating the Stolt transformation of the range migration algorithm. With these approximations, the chirp scaling algorithm does not totally accomplish the phase compensation. In consequence, it causes the images resulted from chirp scaling algorithm are not as accurate as the results from range migration algorithm. However, interest in and use of the chirp scaling algorithm is widespread within the SAR community because of algorithm's ease of implementation and reasonable compensation. Therefore, the works, including the simulating the new approach of chirp scaling algorithm, processing the FOPEN data and comparing with other imaging algorithms, can be future-researched.

For the sake of simplicity, my research is based on FOPEN SAR imaging with stationary targets. If there are moving targets in the imaging scene, SAR cannot simultaneously produce clear image of both these stationary targets and moving targets [39]. Usually moving targets appear as defocused and spatially displaced objects superimposed on the SAR map. Therefore, how to detect and clearly image moving targets becomes an important issue. Future research can embark on SAR imaging of moving targets with three basic issues: (1) how to detect moving targets in the background of stationary objects such as trunks called clutter; (2) how to focus images of moving targets; and (3) how to place the detected moving targets into their true location in the SAR scene.

Author's Publications

1. Y. Na, H. Sun, Y.H. Lee, L.C. Tai and H.L. Chan, "Performance Evaluation of Back-Projection and Range Migration Algorithms in Foliage Penetration Radar Imaging," *IEEE International Conference on Image Processing (ICIP2004)*, 24-27 October 2004, Singapore
2. H. Sun, Y. Na, Y.H. Lee, Y. Lu, "Spotlight SAR Focusing with a Modified Range Migration Algorithm," submitted to *IEEE Trans. On Image Processing*

References

- [1] M.E.Davis, P.G.Tomlinson, R.P.Maloney, "Technical challenges in ultra-wideband radar development for target detection and terrain mapping", *1999 IEEE National Radar Conference*, Boston, MA, pp.1-6, April 1999

- [2] Mehrdad Soumekh, David A. Nobles, Michael C. Wicks, and Gerard J. Genello, "Signal processing of wide bandwidth and wide beamwidth P-3 SAR data", *IEEE Trans. Aerosp. Electron. Syst.*, vol.37, pp.1122-1141, 2001

- [3] R.Goodman, S.Tummala, W.Carrara, "Issues in ultra-wideband, widebeam SAR image formation", In *Proceedings of the IEEE International Radar Conference*, pp.479-485, May 1995

- [4] Y. Na, H. Sun, Y.H. Lee, L.C. Tai and H.L. Chan, "Performance Evaluation of Back-Projection and Range Migration Algorithms in Foliage Penetration Radar Imaging," *IEEE International Conference on Image Processing (ICIP2004)*, 24-27 October 2004, Singapore

- [5] Y. Na, H. Sun, Y.H. Lee, Y. Lu, "A Comparison of Back-Projection Algorithm and Range Migration Algorithm," submitted to *EURASIP Journal on Applied Signal Processing*

References

- [6] W.G.Carrara, R.S.Goodman, and R.M.Majewski, *Spotlight Synthetic Aperture Radar Signal Processing Algorithms*. Boston, MA: Artech House, 1995
- [7] D.A.Ausherman, A. Kozma, J. L. Walker, H. M. Jones, E. C. Poggio, "Developments in radar imaging", *IEEE Transaction on Aerospace and Electronic Systems*, Vol.20, No.4, pp.363-400, July 1984
- [8] Charles V. Jakovatz, Jr., Daniel E. Wahl, Paul H. Eichel, Dennis C. Ghiglia, and Paul A. Thompson, *Spotlight-Mode Synthetic Aperture Radar: A Signal Processing Approach*. Kluwer Academic Publishers, 1994
- [9] C.Cafforio, C.Prati, and F.Rocca, "SAR data focusing using seismic migration techniques," *IEEE Trans. Aerosp. Electron. Syst.*, vol.27, pp.194-207, 1991
- [10] Soumekh, M., *Synthetic Aperture Radar Signal Processing with Matlab Algorithms*. New York: Wiley, 1999
- [11] L.J.Cutrona, W.E.Vivian, E.N.Leith, and G.O.Hall, "A high resolution radar combat surveillance system," *IRE Trans. Milirary Elect.*, MIL-5, pp.127-131,1961
- [12] G.R.Cooper and C.D.McGillem, *Methods of Signal and System Analysis*. Holt, Rinehart, and Winston, Inc., New York, 1967

References

- [13] A.W.Rihaczek, *Principles of High-Resolution Radar*. Peninsula Publishing, Los Altos, CA, 1985.
- [14] J.D.Taylor, *Ultra-wideband Radar Technology*, CRC Press, Boca Raton, FL, 2001
- [15] G.S.Gill, "Step frequency waveform design and processing for detection of moving targets in clutter," *IEEE International Radar Conference*, pp.573-578, Alexandria, VA USA, May 1995
- [16] F.K.Li, C.Croft, and D.H.Held, "Comparison of several techniques to obtain multiple-look SRA imagery," *IEEE Transaction on Geoscience and Remote Sensing*, vol.GE-2, No.3, pp.370-375, July 1984
- [17] D.E.Wahl, P.H.Eichei, D.C.Ghiglia and C.V.Jakowatz,JR., "Phase gradient autofocus-a robust tool for high resolution SAR phase correction," *IEEE Trans. Aerosp. Electron. Syst.*, vol.30, pp.827-835,July 1994
- [18] C.V.Jakowatz, Jr., and D.E.Wahl, " An eigenvector method for maximum likelihood estimation for phase errors in SAR imagery," *J. Opt. Soc. Am. A*, vol.10, No.12, pp.2539-2546, December,1993
- [19] M.Soumekh, "A system model and inversion for synthetic aperture radar imaging", *IEEE Trans. On Image Processing*, pp.64-76, Jan. 1992

- [20] M.D.Desai, W.K.Jenkins, "Convolution back-projection image reconstruction for spotlight mode synthetic aperture radar", *IEEE Transaction on Image Processing*, vol.1, No.4, pp.505-516, October 1992
- [21] D. C. Munson Jr., J. D. O'Brien, W. K. Jenkins, "A tomographic formulation of spotlight mode synthetic aperture radar", *Proceedings of the IEEE*, Vol.72, No.8, pp.917-925, August 1983
- [22] D.C.Munson Jr., Jorge L.C. Sanz., W.K.Jenkins, G.kakazu, B.Mather, " A comparison of algorithms for polar-to-cartesian interpolation in spotlight mode SAR," *Proc.Int.Conf. on Acoustic Speech and Signal Processing*, Tampa, FL, Mar. 1985
- [23] H.J.Scudder, "Introduction to computer aided tomography", *Proc.IEEE*, vol.66, pp.628-637, June 1978
- [24] G.T.herman, Ed., *Image Reconstruction from Projections*. New York: Springer, 1979
- [25] R.M.Mersereau, "Recovering multidimensional signals from their projections," *Computer Graphics and Image Processing*, vol.1, pp179-195, October 1973

[26] Juan M. Lopez Sanchez, and Joaquim Fortuny Guasch, "3-D radar imaging using range migration techniques," *IEEE Trans. On Antennas and Propagat.*, vol.48, No.5, May 2000

[27] Richard Bamler, "A comparison of range-doppler and wavenumber domain SAR focusing algorithms," *IEEE Trans. On Geoscience and Remote Sensing*, vol.30, No.4, July 1992

[28] H.Jiang, C.H.Hou, "Spaceborne SAR Imaging in Wavenumber Domain", 1995 *International Conference on Acoustics, Speech, and Signal Processing*, pp.2141-2144, May 1995

[29] Y.Lu, Y.Cheng, W.Liu, H.W.Seah, H.L.Chan, L.C.Tai, M.Lesturgie, P.Borderies, R.Guern, "Low frequency radar phenomenology study in equatorial vegetation - preliminary results", 2001 *IEE International Radar Conference*, London, UK, pp.70-74, October 2001

[30] R. Guern, M. Lesturgie, J. Derosch, P. Borderies, L. C. Tai, H. L. Chan, H. W. Seah, Y. Lu, W. Liu, "Benefits of ultra wide band for radar imaging through the forest: examples from a BoomSAR experimentation", *International Conference on Radar Systems*, October 2004, 11A-UWB-3, Toulouse, France

References

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- [31] Lance M. Kaplan, James H. McClellan and Seung-Mok Oh, “ Prescreening during image formation for ultrawideband radar”, *IEEE Trans. Aerosp. Electron. Syst.*, vol.38, NO.1, January 2001
- [32] S.Basu, Y.Bresler, “ $O(N^2 \log_2 N)$ Filtered backprojection reconstruction algorithm for Tomography”, *IEEE Trans. On Image Processing*, vol.9, No.10, pp.1760-1773, October 2000
- [33] Seung-mok Oh, J.H.McClellan, “ Multiresolution imaging with quadtree backprojection”, *the Thirty-Fifth Asilomar Conference on Signals, Systems and Computers*, vol.1, pp.105-109, Pacific Grove, CA USA, Nov. 2001
- [34] Shu Xiao, D.C.Munson,Jr., S.Basu, Y.Bresler, “An $N^2 \log N$ back-projection algorithm for SAR image formation”, *the Thirty-Fifth Asilomar Conference on Signals, Systems and Computers*, vol.1, pp.3-7, Pacific Grove, CA USA, Nov. 2000
- [35] A.F.Yegulalp, “Fast backprojection algorithm for synthetic aperture radar”, *1999 IEEE Radar Conference*, pp.60-65, Waltham, MA USA, April 1999
- [36] L. M. H. Ulander, H. Hellsten, G. Stenstrom, “Synthetic-aperture radar processing using fast factorized back-projection” *IEEE Transactions on Aerospace and Electronic Systems*, Vol.39, No.3, July 2003, pp.760-776

[37] X.J.Xu, R.M.NARAYANAN, "FOPEN SAR imaging using UWB step-frequency and random noise waveforms", *IEEE Trans. Aerosp. Electron. Syst.*, vol.37, NO.4, October 2001

[38] R.K.Raney, H.Runge, R.Bamler, I.G. Cumming and F.H.Wong, " Precision SAR Processing Using Chirp Scaling", *IEEE Trans. On Geoscience and Remote Sensing*, vol.32, No.4, pp.786-799, July 1994

[39] Victor C.Chen, Hao Ling, *Time-Frequency Transforms for Radar Imaging and Signal Analysis*. Boston, MA: Artech House, 2002

[40] Gustavsson A., P.O.Frolind, H. Hellsten, T.Jonsson, B.Larsson, and G.Stenstrom, "The Airborne VHF SAR System CARABAS", *IGARSS'93*, Tokyo, Japan, 1993, pp.558-562

[41] H.Israelsson, L.M.H. Ulander, J.I.Askne, J.E.S. Fransson, P.O.Frolind, A.Gustavsson, and H.Hellsten, "Retrieval of Forest Stem Volume Using VHF SAR", *IEEE Trans. On Geoscience and Remote Sensing*, vol.35, pp.36-40, Jan. 1997

[42] Sheen,D.R., C.M.Strawitch, and T.B.Lewis, "UHF Wideband SAR Design and Preliminary Results", *Proceedings of the 1994 IEEE International Geoscience and Remote Sensing Symposium (IGARSS)*, Pasadena, CA, August 1994

Appendix

A. Principle of Stationary Phase

The principle of stationary phase (PSP) is introduced as an integration technique. This technique is widely used to evaluate the complex azimuth Fourier transform in the process of range migration algorithm. Usually this integration method applies to integrals of the form as

$$S(X) = \int_a^b s(x) \exp[j\varphi(x)] dx \quad (\text{A.1})$$

where $s(x)$ is a slowly varying function while $\varphi(x)$ changes by many cycles over the interval of integration. Under these conditions, contributions to the above integral over most of the x interval tend to cancel each other, and thus the little to the overall values of the integral. An exception occurs for contributions at the stationary points of the phase $\varphi(x)$, defined as those values of x for which

$$\frac{\partial}{\partial x} \varphi(x) = 0 \quad (\text{A.2})$$

These points in the integration interval may contribute substantially to the integral in Equation (A.1). The PSP states that the integral of Equation (A.1) has its greatest contributions from those values of x that are stationary points of $\varphi(x)$ on the interval $[a, b]$. Evaluation of the integral involves two steps:

- (1) Determine the location of the stationary point of $\varphi(x)$;
- (2) Evaluate the integrand of Equation A.01 at the stationary point.

If the (x^*) is the only stationary point on $[a, b]$, then an approximation for the integral can be evaluated as

$$\int_a^b s(x) \exp[j\phi(x)] dx = \left[\frac{-\pi}{2\phi''(x^*)} \exp(-j\pi/2) \right]^{\frac{1}{2}} s(x^*) \exp[j\phi(x^*)] \quad (\text{A.3})$$

For the radar signal processing of interest here, the factor in square brackets in Equation (A.3), including magnitude and phase, is essentially constant and we need not consider them in the following analysis. Also in coherent radar problems, we emphasize phase rather than magnitude.

Appendix

B. Background of Back-Projection Algorithm

Back-Projection algorithm was derived from the medical image reconstruction technique known as Computer-Aided Tomography (CAT). It was shown that the mathematical structure of SAR image reconstruction is similar with the image reconstruction problem encountered in CAT. The similarity in these two different imaging systems suggested that the reconstruction algorithm (back-projection algorithm) used in CAT could also be used in SAR, and vice versa. Thus the spotlight SAR can be interpreted as a tomographic reconstruction problem, and the signal processing theory can be characterized in terms of the projection-slice theorem [21]. The signal recorded at each SAR transmission point is modeled as a portion of the Fourier transform of a central projection of the imaged ground area.

B.1 Computer-Aided Tomography

(1) Introduction to Medical X-Ray CAT

The computer-aided tomography (CAT) scan is an X-ray technique that enables the imaging of two-dimensional cross sections of objects [23][24]. In particular, tomography is used extensively for noninvasive medical examination of internal organs and in nondestructive testing of manufactured item. The basic geometry for collecting the X-ray data is shown in Figure B.1.

Figure B.1: A parallel-beam X-ray CAT system

A highly collimated beam of X-ray is directed at the target so that only a thin slice of the object is irradiated. In fact, tomography is derived from the Greek, meaning section or slice. The beam is then translated and rotated, collecting data that can be used to reconstruct an image of that thin cross section. As the source and detector are translated in synchrony, the received X-ray intensity after transmission through the target is measured at a series of positions. The result of the source and detector translation is to sweep out a single projection function, $p_\theta(u)$. In Figure B.1, (u, v) Coordinate system is relative to (x, y) Coordinate by counterclockwise rotation from the x axis through angle θ , which is similar with spotlight mode SAR. The orthogonal linear transformation between these two coordinates system is

$$\begin{aligned} x &= u \cos \theta - v \sin \theta \\ y &= u \sin \theta + v \cos \theta \end{aligned} \tag{B.1}$$

The corresponding inverse transformation is

$$\begin{aligned} u &= x \cos \theta + y \sin \theta \\ v &= -x \sin \theta + y \cos \theta \end{aligned} \quad (\text{B.2})$$

Each value of the projection function is the result of integrating the two dimensional X-ray attenuation coefficient profile $g(x, y)$ for the given slice along a single ray. That is, for a given transmitted X-ray intensity I_0 , the received X-ray intensity I_r measured by the detector, and the attenuation coefficient function $g(x, y)$ are related by the exponential relationship

$$I_r = I_0 \exp\left\{-\int g(x(u, v), y(u, v)) dv\right\} \quad (\text{B.3})$$

Rewriting Equation (B.3) with respect to Equation (B.1) gives

$$\ln\left[\frac{I_0}{I_r}\right] = p_\theta(u) = \int g(u \cos \theta - v \sin \theta, u \sin \theta + v \cos \theta) dv \quad (\text{B.4})$$

The X-ray source and the detector are rotated about center of the target. At each of a series of angular orientation θ , spanning 180 degrees, a projection function is obtained. The set of projection functions are then processed by computer according to a tomographic reconstruction algorithm to produce an estimate of the attenuation profile for the particular slice. The digital display of this computed estimate becomes the tomographic image.

(2) Projection Slice Theorem and Tomographic Reconstruction

The principle underlying the theory of CAT is the projection-slice theorem [25]. This theory states that one dimensional Fourier transform of any projection function $p_\theta(u)$ is equal to the two dimensional Fourier transform $G(X, Y)$ of the image to be reconstructed,

and is evaluated along a line in the Fourier plane that lies at the same angle θ measured from the X axis. The foundational relationship is given by

$$G(U \cos \theta, U \sin \theta) = P_{\theta}(U) \quad (\text{B.5})$$

where

$$P_{\theta}(U) = \int_{-\infty}^{\infty} p_{\theta}(u) \exp(-juU) du \quad (\text{B.6})$$

and

$$G(U \cos \theta, U \sin \theta) = G(X, Y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) \exp[-j(xX + yY)] dx dy \quad (\text{B.7})$$

where $G(U \cos \theta, U \sin \theta)$ stands for the spatial frequency domain data in polar Coordinates (U, θ) , while $G(X, Y)$ describes the data in Cartesian Coordinates (X, Y) so that

$$g(x, y) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} G(X, Y) \exp[j(xX + yY)] dXdY \quad (\text{B.8})$$

This theorem is the basis of an early algorithm used for tomographic image reconstruction from the projection data in medical X-ray CAT systems, called polar format algorithm. The first process relative to Equation (B.6) is the Fourier transform of projection function. Depends on Equation (B.5), we now obtain the spatial frequency domain data in polar Coordinates. Hence before two-dimensional inverse Fourier transform into the final image, a polar to Cartesian Coordinates transform is required. The purpose of this process is to make $G(U \cos \theta, U \sin \theta)$ equal to $G(X, Y)$ in discrete sense of digital processing. Since the Coordinates transform is an approximate process, modern medical CAT systems uses accurate back-projection algorithm for imaging. The key principle of back-projection algorithm is that direct processing the Fourier transform

of the spatial frequency domain data in polar Coordinates, then CAT imaging in polar coordinates is achievable.

B.2 X-ray CAT and Spotlight SAR

When the image algorithms are applied to X-ray CAT and spotlight mode SAR, there are several obvious differences between them. Firstly, medical X-ray CAT uses data from a transmission mode, while SAR employs reflected signals. And the scene reflectivity function and its associated projection functions in SAR are complex-valued, while the analogous quantities in medical CAT are real-values. In addition, for spotlight mode SAR, the platform path appears to span a rather limited set of viewing angel, while the medical CAT scanner obtains projection that completely encircle the target, for example, span the full 180 degrees of viewing angles. Furthermore, there exists one more important difference between these two imaging modalities at the view of data collection, in medical X-ray CAT the projection data are transduced in the image domain, presented by $p_{\theta}(u)$. However, as we have discussed, spotlight mode SAR employs UWB waveform and the associated quadrature demodulation processing, the processed signal (prior to range compression) yields samples of phase history domain data in polar Coordinates and can be presented by $P_{\theta}(U)$. In spite of these and several other more subtle differences, reconstruction algorithm used in the X-ray CAT is similar with the algorithm used in spotlight mode SAR.