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**NANYANG
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**FATIGUE LIFE ESTIMATION OF 1-D ALUMINUM BEAM
USING ELECTROMECHANICAL IMPEDANCE
TECHNIQUE**

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SCHOOL OF CIVIL AND ENVIRONMENTAL ENGINEERING

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FATIGUE LIFE ESTIMATION OF 1-D ALUMINUM BEAM USING ELECTROMECHANICAL IMPEDANCE TECHNIQUE

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SUMMARY

Ageing of structures over the years are creating maintenance problems, ushering the development of automated, real-time and online structural health monitoring (SHM) and nondestructive evaluation (NDE) systems to provide cost-effective alternative to the traditional SHM techniques.

A comprehensive SHM shall be able to detect damage, characterize the severity of damage, as well as to predict the remaining useful life of the structure before maintenance is required. Damage prognosis (DP) is increasingly attracting attention, which could not only widen the safety margin but also lead to huge cost savings.

The recent advent of smart materials applicable in SHM alleviates the shortcomings of the conventional techniques. Autonomous, real-time, remote monitoring becomes possible with the use of smart piezo-impedance transducers and Fiber Bragg grating (FBG) based strain sensors. The electro-mechanical impedance (EMI) technique, employing the piezo-impedance transducers as collocated actuators and sensors, is known for its ability in damage detection and characterization.

Up to date, researchers in the field of EMI technique have conducted many studies, proving the robustness of the technique in damage monitoring. However, no study has been conducted to develop a more comprehensive damage prognosis model.

Thus, this PhD study attempts to push forward this area of research, by developing a damage prognosis model, capable of estimating the remaining useful life of structure subjected to fatigue loading using the EMI technique as SHM tool. This proof-of-concept semi-analytical damage model utilizes the ability of EMI technique in detecting and characterizing crack (which can be simulated using finite element method) as well as the theory of linear elastic fracture mechanics (LEFM) in predicting the remaining life of fatigue loaded structure. The model presented in this study involves simple beam structure with single propagating crack subjected to mode I fatigue loading. This proof-of-concept model has been verified experimentally, which opens up a path for the development of more rigorous and complicated models

for the evaluation of structural life under fatigue loading. Finite element (FE) model is adopted to enable more general application. The theory of LEFM is incorporated to provide greater insight into the underlying concepts involving damage mechanism and fatigue life estimation.

A wide range of structural resonance frequency can be selected as damage quantifier. The reduction in resonance frequency (leftward shift of peak) of the real-part of admittance signatures turns out to be a useful index for characterizing crack length. The reduction in resonance frequency simulated in FE model agrees well with the experimental counterparts, making it possible to replace experimental test with FE model.

Statistical damage index, namely the cross correlation deviation mean (CCDM), known for its sensitivity towards change in resonance frequency has also shown its ability in characterizing fatigue damage.

On one hand, the EMI technique demonstrates its robustness in characterizing macroscopic crack as well as in detecting microscopic crack. On the other hand, qualitative identification of critical crack is also possible by observing the changes in frequency spectrum.

The effects of axial loading and boundary conditions on the admittance signatures acquired by the EMI technique are also studied through rigorous modeling and experimentations. 1-D analytical model considering the interaction between surface-bonded PZT transducer and axially loaded uniform beam is developed. The model shows that the presence of axial load affects the transverse vibration but not the extensional vibration. The only parameter affected is the natural frequency (transverse mode) of the beam. The theoretical model is verified by the FE model for the cases of tensile and compressive stress. In the case of experimental test, the stiffening effect in the tensile test can be observed in the experiment but not the softening effect in the case of compression. It is found that by using the resonance peak induced by extensional vibration, the effect of stiffening can be compensated, rendering a closer match between the models and the experiments.

The damage detection capability of EMI technique on host structure under static axial load and constant boundary conditions is finally investigated through FE simulation and experimental test. The FE model is capable of simulating the propagation of crack in the presence of axial load, without being affected by the boundary condition. Experimental test was conducted in the presence of both factors. Both results show that the damage detection capability of EMI technique is unaffected as long as the baseline signatures are recorded at different loading.

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LIST OF SYMBOLS

$()'$	Spatial differential operator
$()^{\bullet}$	Temporal differential operator
a	Crack length
a_0	Initial crack length
a_c	Critical crack length
$a_{c, LEFM}$	Critical crack length predicted by LEFM
$a_{c, S1}$	Critical crack length of specimen S1
$a_{c, S2}$	Critical crack length of specimen S2
$a_{c, S3}$	Critical crack length of specimen S3
a_f	Final crack length
a_{small}	Threshold of crack length for small crack growth behavior
$A_{0.95}$	Cross sectional area of non-circular section subjected to 95% of maximum stress
A_1	Constant of partial differential equation
A_2	Constant of partial differential equation
A_a	Cross sectional area of actuator
A_s	Cross sectional area of beam structure
α	Constant mass matrix multiplier
b	Slope of S-N curve
b_a	Width of PZT patch / actuator
b_{notch}	Modified slope of S-N curve
b_s	Width of beam
β	Constant stiffness matrix multiplier
β_a	Constant stiffness matrix multiplier of PZT patch
β_b	Constant stiffness multiplier of bonding layer
β_s	Constant stiffness multiplier of host structure

β_j	Constant stiffness multiplier for material, j
$[C]$	Damping matrix
c_a	Wave speed in actuator
c_s	Wave speed in beam
C	Intercept of Paris equation (normalized K-gradient)
C_a	Zero load capacitance of PZT patch
C_k	Aging rate of relative dielectric constant
C_ε	Aging rate of coupling factor
C_{str}	Dynamic receptance
$C_{str,e}$	Dynamic receptance of extensional vibration
$C_{str,t}$	Dynamic receptance of transverse vibration
$[d_d]$	Piezoelectric strain coefficient (direct effect)
$[d_c]$	Piezoelectric strain coefficient (converse effect)
$[D]$	Electric displacement (charge density) vector
d	Diameter
d_{31}	Piezoelectric strain coefficients in direction 1 (voltage in direction 3)
d_{32}	Piezoelectric strain coefficients in direction 2 (voltage in direction 3)
d_{33}	Piezoelectric strain coefficients in direction 3 (voltage in direction 3)
d_{15}	Piezoelectric strain coefficients for shear strain in plane 1-3 (voltage in direction 1)
d_{24}	Piezoelectric strain coefficients for shear strain in plane 2-3 (voltage in direction 2)
da/dN	Fatigue crack growth rate
D_3	Electric displacement in direction 3
δ	Dielectric loss factor
$\delta(x)$	Dirac Delta distribution function
Δf	Reduction in resonance frequency
ΔK	Stress intensity factor range
ΔK_{th}	Threshold stress intensity factor range (Fatigue crack growth threshold)

ΔK_I	Stress intensity factor range under mode I loading
$\Delta\Omega$	Change in natural frequency
$\Delta\sigma$	Range of stress
$\Delta\sigma_f$	Fatigue limit range (twice fatigue limit)
$[E]$	External electric field vector
E_3	Externally applied electric field along direction 3
$\hat{E}_3$	Magnitude of externally applied electric field along direction 3
$[\overline{\varepsilon^T}]$	Second order complex dielectric permittivity tensor under constant stress
ε_{11}^T	Dielectric Permittivity in direction 1
ε_{22}^T	Dielectric Permittivity in direction 2
ε_{33}^T	Dielectric Permittivity in direction 3
$\overline{\varepsilon_{33}^T}$	Complex dielectric permittivity in direction 3
η	Mechanical loss factor of PZT patch
$\{F\}$	Applied harmonic loading
$f(a/w)$	Stress intensity factor function
$f_m(x,t)$	Distributed loading function of transverse vibration (function of x and t)
$\hat{f}_m(x)$	Distributed loading function of transverse vibration (function of x)
$f_n(x,t)$	Distributed loading function of extensional vibration (function of x and t)
$\hat{f}_n(x)$	Distributed loading function of extensional vibration (function of x)
$F(a)$	Function of crack length
F_0	Magnitude of force
F_m	Modal participation coefficient of transverse vibration
F_n	Modal participation coefficient of extensional vibration
F_{PZT}	Force induced by PZT at both ends (function of x and t)
$\hat{F}_{PZT}$	Force induced by PZT at both ends (function of x)

$\overline{G_d}$	Averaged conductance signatures in damaged condition
$\overline{G_h}$	Averaged conductance signatures in healthy condition
$G_{i,d}$	Conductance signatures for host structure in damaged condition at frequency i
$G_{i,h}$	Conductance signatures for host structure in healthy condition at frequency i
γ	Wave number
h	Thickness
h_a	Thickness of PZT patch / actuator
h_b	Thickness of bonding layer
h_s	Thickness of beam structure
I	Electric current
$\bar{I}$	Electric current (complex notation)
I_s	Second moment of inertia of beam structure
j	Imaginary number
$[K]$	Stiffness matrix
$[K_{11}]$	Random matrix
$[K_{12}]$	Random off-diagonal sub-matrices
$[K_{21}]$	Random off-diagonal sub-matrices
$[K_{22}]$	Random matrix
$[K^d]$	Dielectric conductivity
$[K^z]$	Piezoelectric coupling matrix
k	Dimensionless coefficient (Euler buckling equation)
k_f	Fatigue notch factor
k_L	Load Factor
k_{size}	Size factor
k_t	Stress concentration factor
k_{SF}	Surface finish factor
K	Stress intensity factor

K_j	Portion of structure stiffness matrix based on material, j
K_{min}	Minimum stress intensity factor
K_{max}	Maximum stress intensity factor
K_{str}	Dynamic stiffness of structure
K_C	Corrected critical fracture toughness (6mm)
K_{IC}	Critical fracture toughness (plane strain)
K_{PZT}	Dynamic stiffness of PZT
κ	Wave number
κ_{31}	Electro-mechanical cross coupling coefficient of PZT patch
$\{L\}$	Vector of nodal, surface and body charges
l_a	Length of PZT patch / actuator
l_s	Length of beam structure
L	Unsupported length (Euler buckling equation)
λ	Material constant (Walker equation)
$[M]$	Structural mass matrix
m	Gradient of Paris equation
m	Mode numbers of transverse vibration
m_0	Lower mode number of transverse vibration
m_1	Upper mode number of transverse vibration
$M_s(x, t)$	Distributed forcing moment (function of x and t)
$\hat{M}_s$	Distributed forcing moment (function of x)
μ	Mechanical loss factor of beam structure
n	Factor of safety
n	Mode numbers of extensional vibration
n_0	Lower mode number of extensional vibration
n_1	Upper mode number of extensional vibration
N	Total number of frequency points selected
N	Number of loading cycles
$N(\%)$	Number of loading cycles (%)

N_f	Number of cycles to fracture
$N_s(x, t)$	Distributed loading function (function of x and t)
$\hat{N}_s$	Distributed loading function (function of x)
N_x	Magnitude of static axial load on beam
ω	Angular frequency
Ω_0	m^{th} natural frequency of beam (transverse vibration) without axial load
Ω_1	m^{th} natural frequency of beam (transverse vibration) in the presence of axial load, N_I
Ω_m	m^{th} natural frequency of beam in transverse vibration
P_{cr}	Euler buckling load
ϕ	Displacement phase shift (phase angle)
$\varphi_m(x)$	Mode shape of transverse vibration
$\phi_n(x)$	Mode shape of extensional vibration
Φ_n	n^{th} natural frequency of the beam in axial vibration
r	Radial coordinate
r_y'	Radius of plastic zone
R	Mean stress ratio
R	Stiffness ratio
R^2	Coefficient of determination
ρ_a	Density of PZT patch
ρ_b	Density of bonding layer
ρ_s	Density of beam structure
$[S]$	Second order strain tensor
$[\overline{s^E}]$	Fourth order complex elastic compliance tensor (constant electric field)
s_{11}	Elastic compliance in direction 1
s_{22}	Elastic compliance in direction 2
s_{33}	Elastic compliance in direction 3
s_{12}	Elastic compliance in direction 1 (Load in direction 2)

s_{21}	Elastic compliance in direction 2 (Load in direction 1)
s_{13}	Elastic compliance in direction 1 (Load in direction 3)
s_{31}	Elastic compliance in direction 3 (Load in direction 1)
s_{23}	Elastic compliance in direction 2 (Load in direction 3)
s_{32}	Elastic compliance in direction 3 (Load in direction 2)
s_{44}	Elastic (shear) compliance in plane 2-3
s_{55}	Elastic (shear) compliance in plane 1-3
s_{66}	Elastic shear compliance in plane 1-2
S_I	Mechanical strain in direction 1
S_a	Alternating stress/ stress amplitude
S_{eq}	Equivalent completely reversed stress
S_f'	Intercept of S-N curve
S_{mean}	Mean stress
S_u	Ultimate stress
S_{FL}	Fatigue limit
$S_{FL,component}$	Fatigue limit of the component
σ	Stress applied perpendicular to direction of crack
σ_I	Mechanical stress in direction 1
σ_{max}	Maximum stress
σ_{min}	Minimum stress
σ_{ult}	Ultimate strength
σ_y	Yield stress
σ_{yy}	Stress in y-direction
σ_{xx}	Stress in x-direction
τ_{xy}	Shear in plane x-y
$[T]$	Stress tensor
t	Time
θ	Angular coordinate
$\{u\}$	Nodal displacement vector

$\{\dot{u}\}$	Nodal velocity vector
$\{\ddot{u}\}$	Nodal acceleration vector
$\{u_1\}$	Real component of displacement vector
$\{u_2\}$	Imaginary component of displacement vector
$u(x,t)$	Displacement of beam in x -direction (function of x and t)
$u_a(x,t)$	Displacement of actuator in x -direction (function of x and t)
u_{PZT}	Displacement of PZT in x -direction
$U(x)$	Displacement of beam in x -direction (function of x)
$U_a(x)$	Displacement of actuator in x -direction (function of x)
$\{V\}$	Vector of nodal electric potential
ν_a	Poisson ratio of actuator / PZT patch
ν_b	Poisson ratio of bonding layer
ν_s	Poisson ratio of beam
V	Voltage
$\bar{V}$	Voltage (in complex notation)
w	Width
w_a	Width of actuator / PZT patch
$w(x,t)$	Displacement of beam in z -direction (function of x and t)
$W(x)$	Displacement of beam in z -direction (function of x)
x_1	Location of left end of PZT patch along x -axis
x_2	Location of right end of PZT patch along x -axis
X_1	Random degree of freedom 1
X_2	Random degree of freedom 2
Y	Dimensionless shape factor
$\bar{Y}$	Complex electrical admittance
Y_{11}^E	Young's modulus of PZT patch along direction 1 at zero electric field
$\bar{Y}_{11}^E$	Complex Young's modulus of PZT patch along direction 1 at zero electric field

Y_b	Young's modulus of bonding layer
Y_s	Young's modulus of beam structure
$\bar{Y}_s$	Complex Young's modulus of beam structure
Z	1-D mechanical impedance of structure
Z_a	1-D mechanical impedance of actuator
$Z_{a,eff}$	2-D effective actuator's impedance
Z_{xx}	2-D direct impedance in x -direction
Z_{yy}	2-D direct impedance in y -direction
Z_{yx}	2-D cross impedance in x -direction
Z_{xy}	2-D cross impedance in y -direction
Z_{axx}	2-D mechanical impedance of PZT patch in x -directions
Z_{ayy}	2-D mechanical impedance of PZT patch in y -directions
$Z_{s,eff}$	2-D effective structural impedance

LIST OF ACRONYMS

ACPD	Alternating current potential drop
ANN	Artificial neural networks
ASTM	American Society for Testing and Materials
ATM	Adaptive template machining
CCDM	Cross correlation deviation mean
CTOD	Crack tip opening displacement
DP	Damage prognosis
EDM	Electric discharge machining
EMI	Electro-mechanical impedance
EPFM	Elastic plastic fracture mechanics
ER	Electrorheological
FBG	Fiber Bragg grating
FE	Finite element
FEA	Finite element analysis
FEM	Finite element method
FRF	Frequency response functions
GDP	Gross domestic product
KPCA	Kernel principal component analysis
LEFM	Linear elastic fracture mechanics
LPCA	Linear principal component analysis
MEMS	Micro-electromechanical system
MR	Magnetorheological
NDE	Nondestructive evaluation
NITINOL	Alloy of Nickel, Titanium developed by Naval Ordnance Laboratory
PNN	Probabilistic neural network
PVDF	Polyvinylidene Fluoride
PZT	Lead Zirconate Titanate

RF	Radio Frequency
RMSD	Root mean square deviation
SAC	Signature assurance criteria
SEF	Static equivalent-force
SHM	Structural health monitoring
SMA	Shape memory alloys
UTM	Universal Test Machine
WCC	Waveform chain code

CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

Since the existence of human being, structures have become an essential part of human life. Structures are assemblies of load carrying members capable of transferring the superimposed loads to the foundations. They are either constructed (e.g. buildings, bridges, dams, transmission towers, etc.) or manufactured (e.g. machines, trains, ships, aircraft, etc.) to serve specific functions during their design lives. Each structure forms an integral component of civil, mechanical or aerospace systems. The structures are expected to satisfy both strength and serviceability criteria throughout their stipulated design lives.

Historically, safety of the structures emerges to be one of the main concerns during their construction and service for accommodating humans and their properties. In the olden days, design of structures depended highly on experiences as well as trial and error. Thus, the designs could be uneconomical due to excessive use of materials but on the other hand, may not be structurally safe.

Throughout the years of research and development, the rapid advancement in structural design theory and the advancements in engineering materials have resulted in more efficient structures. However, the safety of real life structures remains to be a major challenge for engineers when loadings are accompanied by surrounding factors, such as environmental or human factors which deteriorate the structures, and unpredicted events such as earthquake and blast loading. With current technology, even the best designed structures, constructed from advanced high strength materials, could not be thoroughly freed from damage.

Yao (1985) defined ‘damage’ as a deficiency or deterioration in the strength of a structure, caused by external loads, environmental conditions, or human errors. Physically, a damage may be a crack, delamination, debonding, reduction in

thickness/ cross-section, or exfoliation. The term ‘damage’ differs itself from the term ‘failure’ in which ‘failure’ generally refers to any action leading to inability of a part structure or machine to function in intended manner (Ugural and Fenster, 1995). Fracture, permanent deformation, buckling and even excessive linear elastic deformation may be regarded as modes of failure. Failure results when a particular type of damage exceeds its threshold value, thereby significantly impairing the safety and the function of the structure (Bhalla, 2004).

1.2 STRUCTURAL FAILURES

The German Intercityexpress, or ICE provides comfortable train service for travelling passengers up to a maximum speed of 280 km/h. On 5 June 1998, ICE number 884, serving the Munich-Hamburg route crashed near the village of Eschede at a speed of 250 km/h. The accident resulted in 100 deaths, 100 injuries, destruction of a bridge, the track, the train and interruption of train service (Zehnder, 2007).

Investigation revealed that a tire was detached from the wheel, which dragged along and jammed under the floor of the carriage, subsequently stuck in the tongue of a switch. By this, the switch was toggled to the neighboring track and the hind part of the train redirected there. This led to derailment and collision of the derailed train part with the pylon of a road bridge leading over the tracks. The collapsing bridge buried a part of the train.

The cause of the tire detachment was a fatigue crack, as depicted in Figure 1.1, that grew from the inner rim of the tire. The crack grew slowly by fatigue to about 80% of the cross sectional area of the tire before the final, rapid fracture.

Figure 1.1: Fracture surface of broken ICE train wheel tire (Zehnder, 2007).

On 28 April 1988, the passengers on board a Boeing 737 of Aloha Airlines were horrified when they discovered that the fuselage panels of the forward cabin were ripped apart from the main body, as shown in Figure 1.2. Fortunately, the passengers remained held against air pressure by their safety belts. In the incidence, one flight attendant was killed and many passengers were injured. The underlying cause of this accident was later found to be the appearance of multi-site cracks in the skin joints, as a result of accumulated loading cycles, corrosion and maintenance problem. These cracks were undetected during the routine pre-flight inspections. The accident challenged the notion that fracture was well understood and under control in modern structures.

Figure 1.2: Accident involving Aloha Airlines (LAMSS, 2003).

On 12 November 2001, the mid air crashing of the American Airlines Airbus A300-600 (Flight 587) appeared to be one of the deadliest accidents in the American aviation history. The cause of failure was discovered to be the breaking off of the tail (vertical stabilizer), right from the root of the connection to the main body during take off. The investigators found that the existence of an undetected damage in the tail, caused by previous mid air events involving severe loading had weakened the composite joint. Conventional NDE techniques, including visual inspections failed to detect the presence of the previous damages. This incipient damage was further aggravated by the aerodynamic loads during subsequent flight and until a critical stage where the tail finally broke apart, as shown in Figure 1.3.

Figure 1.3: Close-up view of breakaway composite joint in the accident involving American Airlines Airbus A300-600 (LAMSS, 2003).

In addition to the abovementioned failures, numerous civil-structural failures have also occurred in the history of human being. For instance, the June 1983 Mianus River Bridge collapse in Greenwich, which resulted from a hanger pin connection failure due to excessive corrosion accumulation (USDT, 2003).

On 20 April 2004, the Nicoll Highway collapse (Levey, 2008) in Singapore was considered as one of the largest failures of a civil engineering project under construction in the city state. A tunnel being constructed for future use by trains collapsed when the temporary supporting structure for the deep excavation work failed, resulting in a 30-metre deep cave-in damaging six lanes of the adjacent Nicoll Highway (Figure 1.4). Four people were killed and three were injured. Two construction cranes were swallowed by the cave-in.

The cause of the collapse lies in the stress imposed on the temporary retaining wall exceeded its capacity, forcing it to give way. In fact, large deflections occurred during the excavation but the strut load measured appeared to be smaller than expected, giving false impression to the engineer in charge.

All these failures emphasize that reliable inspection techniques are essential for structures in service, since visual inspection is likely to miss out certain critical damages. To date, many buildings and bridges constructed during the economical boom of the eighties are now showing signs of ageing, for which the maintenance engineers are not logistically prepared.

Figure 1.4: The scene following April, 2004 collapse of a section of tunnel being built for the Circle Line in Singapore (Image courtesy / Committee of Inquiry, 2005).

1.3 STRUCTURAL HEALTH MONITORING

Continuous SHM is essential to ensure timely repair and maintenance, in order to preserve the integrity and serviceability of the structure at minimum cost. In the case of potential structural failure, early warning can minimize human and property losses. A comprehensive SHM shall be able to detect damage, characterize its severity, as well as to predict the remaining useful life (damage prognosis) of the structure (Giurgiutiu, 2007) before maintenance is required. This requires technology from multi disciplines including material science, engineering, statistical science, etc.

The United States spends more than USD200 billion each year on the maintenance of plants, equipments and facilities. The increasing age of existing structures demands careful monitoring and costly maintenance. Systematic and reliable SHM may alleviate the labor intensive and financial intensive maintenance, such as replacing scheduled maintenance by condition-based (as-needed) maintenance.

The conventional non-destructive SHM techniques are broadly classified into two categories, namely global and local interrogation techniques, based on the way of interrogating the host structure. Each of these techniques is found to be restricted by

some inherent limitations. For instance, the global interrogation techniques require large force of actuation which is uneconomical and almost impossible for large structures. Moreover, the outcome only reflects global defects, and is unable to detect incipient but potentially hazardous damages. The modeling and analysis involved in the process can be excruciatingly laborious, if not impossible for complex structures.

On the other hand, the application of local interrogation techniques first require experienced inspectors to conduct periodic visual inspection to search for possible damaged locations. The local interrogation techniques are then applied to pin-point the exact spot. In short, local techniques are highly inefficient as they require *a priori* knowledge of the approximate damaged location, which relies heavily on the experience of inspectors.

The recent advent of smart materials applicable in SHM could alleviate the shortcomings of the conventional techniques. Autonomous, real-time, remote monitoring could become possible with the use of smart piezo-impedance transducers and fiber Bragg grating (FBG) based strain sensors (Lim, 2006). These materials possess distinct advantages such as non-intrusive to host structure, highly sensitive to incipient damage or strain, capable of localizing damage and potentially low-cost with mass production. Their potentials in substituting the currently available NDE techniques are intensively explored.

In this research, application of piezo-impedance transducer in SHM is the main focus of study. Generically, two SHM techniques employing piezo-impedance transducer are available, namely, the electromechanical impedance (EMI) technique and the wave propagation technique.

The EMI technique, employing piezoceramics (PZT) materials as collocated actuators and sensors is proven to be effective in damage detection and characterization (Sun et al., 1995; Ayres et al., 1998; Soh et al., 2000; Park, 2000; Bhalla, 2001). The technique is found to be effective in detecting incipient crack, such as fatigue crack (Lim and Soh, 2010).

The wave propagation technique (Ihn and Chang, 2004a, b) adopts similar idea as the conventional ultrasonic technique which is proven to be robust in localizing

damage as well as determining the progress and direction of crack propagation. Pitch-catch method and pulse-echo method are commonly used in the wave propagation techniques (Giurgiutiu, 2007).

1.4 DAMAGE PROGNOSIS

In recent years, the concept of Damage Prognosis (DP) has been introduced to extend SHM from damage detection and characterization to estimation of the remaining useful life of structure (Inman, 2005). Continuous monitoring of the remaining useful life of structure ensures timely repair which not only widens the safety margin but also leads to huge cost savings. There are generally three categories of damage prognosis problems namely, gradual wear, predictable discrete events and unpredictable discrete events.

Conceptually, a comprehensive DP model requires at least three sub-models including physics based model, state awareness model and future loading model. The physics based model would be useful in defining the sensor properties such as amount of sensors required, locations, bandwidth, etc. State awareness model involves SHM techniques for the assessment of current structural state, and the future loading model requires operational and environmental measurements data from usage monitoring for the development of future system loading.

The estimation shall be achieved with a reliability-based predictive tool, which is expected to be able to estimate in terms of probability, the remaining useful life of a system or components or the probability of completing certain mission or job. In short, the development of DP capability requires multidisciplinary approaches and is labeled as of “grand challenge” in nature. At current stage, research and development of a comprehensive DP is still in its embryonic stage.

In this thesis, focus is placed on the problems of gradual wear, which involve the slow accumulation of damage at the material and component level of microscopic scale such as fatigue induced and corrosion induced cracking.

1.5 RESEARCH OBJECTIVES

Up to date, researchers in the field of EMI technique have conducted many studies, proving the robustness of the technique in monitoring damage. SHM on civil, mechanical and aerospace structures have been extensively studied. However, as yet no study has been conducted to develop a more comprehensive damage prognosis model.

Thus, the main objective of this PhD research is to investigate the feasibility of performing structural damage prognosis using the EMI technique. This is approached by developing a proof-of-concept semi-analytical damage model capable of monitoring structural health and predicting the remaining useful life of structure under fatigue loading. The model utilizes the ability of the EMI technique in detecting and characterizing crack as well as the theory of linear elastic fracture mechanics (LEFM) in predicting the remaining life of fatigue loaded structure. The proof-of-concept model developed in this study involved lab-sized aluminum beam structure with single propagating crack under mode I fatigue loading. This model is anticipated to serve as a stepping stone, opening a path for future development of more rigorous and comprehensive models for the evaluation of remaining life of real-life structures. The finite element method (FEM) is adopted, enabling the model to stand independent from experiment. The theory of LEFM is also incorporated to provide greater insight into the underlying concepts involving damage mechanism and fatigue life estimation. Furthermore, the ability of the EMI technique in monitoring incipient crack, such as microscopic fatigue crack is studied.

Research works are also conducted to investigate the effect of static axial load and boundary condition on the admittance signatures acquired by the EMI technique. Both theoretical model and FE model for simple beam structure are developed for this purpose. Experimental verification is performed. The damage detection capability of the EMI technique under varying load is also studied.

1.6 RESEARCH ORIGINALITY AND CONTRIBUTIONS

The research reported in this thesis aims to expand the present capabilities of the EMI technique for SHM and DP. This research attempts to balance theoretical developments through analytical and numerical modeling as well as practical applications by experimental tests in order to maximize the benefits of the EMI technique. The original contributions of this research can be summarized as follows:

- (a) A novel semi-analytical damage model useful for estimation of remaining fatigue life of structure with single propagating crack is developed. The model incorporates the FEM in simulating structures monitored by the EMI technique and the theory of LEFM for the prediction of remaining useful life. Proof-of-concept application is presented through simple beam structure, which is experimentally verified. The robustness of the semi-analytical model lies in the fact that it can be independent of experimental test. Generically, the damage prognosis model can be developed through semi-analytical, empirical or numerical modeling, depending on the availability of resources.
- (b) Ability of the EMI technique in detecting microscopic crack not visible to the naked eyes using the high frequency range of excitation is presented. Method of identifying critical crack condition of a structure is also proposed.
- (c) The effect of axial loading under fixed boundary conditions on the admittance signatures acquired by the EMI technique is studied through rigorous modeling and experimentation. The stiffening effect induced by the boundary condition is found to be more dominant than the effect of axial stress. A novel approach, using the extensional mode of vibration to investigate the stiffening effect is proposed, explaining the variations between experiments and models, as observed by previous researchers.

- (d) The damage detection capability of EMI technique on host structure under static axial load with fixed boundary condition is studied and confirmed.

The original findings of the research work have been submitted to or published in international journals and conferences, whose details are attached at the end of this thesis.

1.7 THESIS ORGANIZATION

This thesis consists of a total of eight chapters including this introductory chapter. Chapter 2 presents a detailed literature review on state-of-the art in SHM, concepts and solutions of DP, fatigue, the concept and applications of smart systems and materials as well as descriptions on the EMI technique and other relevant issues.

Chapter 3 studies the damage detection and characterization capability of the EMI technique through experimental tests on lab-sized aluminum beam structures. Beam specimens of different sizes and configurations were cyclically loaded till failure with surface-bonded PZT patch affixed to monitor the propagation of crack. The reduction in resonance frequency of structural peaks in the admittance signatures spectrum is adopted as the damage quantifier. Other non-parametric damage indices such as root mean square deviation (RMSD) and cross correlation deviation mean (CCDM) are also utilized for damage quantification. Pros and cons of each approach are discussed. Ability of the EMI technique in detecting micro-crack invisible to the naked eyes as well as its ability to identify critical crack is investigated.

Chapter 4 outlines the feasibility of using the FEM in modeling the EMI technique for monitoring of fatigue crack. The chapter first reviews the previous works performed on the numerical modeling of PZT-structure interaction. Theories underlying the FE simulation of PZT-structure interaction are presented in brief. Investigations started from the FE modeling of PZT patch under free vibration, comparing it with the analytical and experimental counterparts. Next, the PZT-structure interaction in the EMI technique is studied by modeling selected

specimens used in the experiment in Chapter 3. Finally, the possibility of using the EMI technique for monitoring the propagation of crack is investigated by simulating crack of varying length in the model. All results are compared with the experimental reported in Chapter 3.

In Chapter 5, the theory of LEFM and stress life are first introduced. The process of incorporating the LEFM theory into the FE model developed in Chapter 4 in forming a semi-analytical damage model for fatigue life estimation is narrated in details. An empirical model serving similar function is also developed based on the experimental data reported in Chapter 3. The possibility of developing pure numerical model is finally discussed.

Chapter 6 presents a step-by-step development of the theoretical model describing the dynamic interaction between PZT transducer and 1-D continuous systems, specifically beam structure. The model shows its robustness in evaluating the admittance signatures based on the extensional and transverse vibration of the beam. Chapter 7 extends the theoretical model developed in Chapter 6 to include the effect of axial loading. A 3-D FE model is also developed with experimental test conducted to verify the analytical model. The stiffening effect induced by the boundary conditions is examined. Ability of the EMI technique in detecting and characterizing damage under varying static axial loading is then presented.

Finally, conclusions and recommendations are presented in Chapter 8, which are followed by a list of the author's publications, a comprehensive list of references, and appendices.

CHAPTER 2

LITERATURE REVIEW ON STRUCTURAL HEALTH MONITORING AND DAMAGE PROGNOSIS

2.1 OVERVIEW OF STRUCTURAL HEALTH MONITORING

Aging and deterioration of structures are inevitable as a result of repeated or excessive usage, overloading, corrosion due to climatic conditions, lack of maintenance or inefficient monitoring and other unforeseen factors. All these factors contribute to the obsolescence of the structures. If maintenance could not be performed in time, the structures could be rendered unserviceable and even failure, causing loss of lives and properties.

The US Federal Reserve Board has reported that the failure of civil infrastructure systems to perform at the expected level may cause a reduction in the national gross domestic product (GDP) by 1% (Aktan et al., 1998).

Thus, periodical monitoring and maintenance of structures, to ensure their serviceability and safety, are essential especially for the civil structures which are expected to accommodate large numbers of humans for years. Early detection of degradation such as cracks, overstress or overstrain, decay of cement matrix, concrete spalls and corrosion losses is desirable to reduce cost of maintenance and to mitigate potential hazards.

These concerns have subsequently triggered research and development in the area of SHM. SHM assesses the state of structural health and through appropriate data processing and interpretation, may predict the remaining life of the structure. (Giurgiutiu, 2007).

The United States spends more than USD200 billion each year on the maintenance of plant, equipment and facilities. However in real life, many civil,

mechanical and aerospace structures are able to perform longer than their design life, which is, they are able to remain in service despite their original design life has been reached or exceeded due to inherent factor of safety incorporated during the design. On the other hand, structures or structural parts with tolerable degree of damages may still be able to perform for certain period of time before repair is needed.

SHM could therefore maximize the usage of structures that would lead to great reduction in maintenance and replacement costs. Conventional schedule-driven monitoring can also be replaced by more efficient condition based monitoring. A highly reliable SHM system with damage prognosis capability is compulsory to achieve the abovementioned goal.

Broadly speaking, conventional structural defect detection and analysis techniques can be divided into two categories, namely destructive and non-destructive evaluation (NDE). Destructive test, such as the pull out test on concrete is generally undesirable as the test itself causes damage on the structure.

NDE is widely used in the engineering community as it is more efficient, cost effective, and easier to apply. Based on the way of interrogation with the host structure, non-destructive health monitoring techniques can be classified into global and local interrogation techniques. In a broad sense, both techniques utilize the response of the structure under investigation as an indication of the structural health condition under external excitation, which can be intentionally or naturally induced.

The following sections present a series of review on SHM, initiating with an introduction to various currently available NDE techniques followed by more advanced techniques employing smart materials. In depth review on piezoelectric materials including its properties, basic principals, applications and EMI technique, etc. will then be presented. The concept of DP and its role in forming a comprehensive SHM system is discussed. The chapter ends with an overview on fatigue damage and relevant issues.

2.2 GLOBAL SHM TECHNIQUES

In the global interrogation techniques, the structure under investigation will be subjected to an externally applied global disturbance/excitation (static force or vibration) in its healthy condition and the corresponding responses (displacement, mode shape, modal frequency, etc.) measured will serve as a baseline. According to structural theory, any subsequent global damages incurred will change the structural parameters (such as stiffness) of the structure. Thus, when the same disturbance is inflicted onto the structure, deviation from the baseline will serve as an indication of damage. Depending on the nature of the disturbance, global interrogation technique can be classified into static and dynamic global interrogations.

2.2.1 Global Static Response Based Techniques

A technique based on static displacement response was formulated by Banan and Hjelmstad (1994). The technique involves applying static forces to a structure at certain nodal points and measuring the corresponding displacements. With this, a set of member constitutive properties or structural parameters can then be derived. Any change in the parameters from the baseline healthy state is an indicator of damage. Major drawbacks of this technique arise from its practical implementation, such as difficulty in establishing a frame of reference and tremendous effort involved in the application of loads, large enough to cause measurable deflections. Besides, laborious computations are involved to evaluate the useful information.

Another similar technique based on static strain measurements was proposed by Sanayei and Saletnik (1996). The advantage of this technique over the displacement approach is that strain measurements can be made with a higher degree of precision. However, the application on real-life structure remains as tedious. In short, applications of these techniques are too tiresome, expensive and impractical.

2.2.2 Low Frequency Global Dynamic Techniques

The principle of application of global dynamic techniques lies in the structural

dynamic theory, following the fundamental concept that any damage induced to a structure will change the modal parameters (Zimmerman and Kaouk, 1994) and mode shapes (Pandey and Biswas, 1994) of the structure. When the entire structure under investigation is excited by a low frequency dynamic actuation (harmonic or impulsive), the resulting vibrational responses (displacements, velocities or accelerations) at some discrete points along the structure will enable the derivation of the first few mode shapes and their corresponding natural frequencies. Subsequent damage in the structure will alter its modal parameters and structural parameters, namely the stiffness matrix and the damping matrix. Theoretically, information on damage severity and damage location can also be derived from the structural response. These techniques are relatively easy to implement.

However, these techniques are essentially based on the low frequency dynamic response of structures involving only the first few modes of vibration. Therefore, only a limited number of modal frequencies and their corresponding mode shape vectors can be extracted. As the stiffness change has different sensitivity to each mode, it may not significantly affect some of the mode shapes resulting in certain damages unnoticed.

Moreover, these techniques rely heavily on the measurement of global properties to identify the localized changes. At low frequency, small cracks or incipient damages are unable to significantly affect the global modal parameters to permit effective damage detection. Therefore, the low frequency techniques are not discernible for the detection of relatively small sized cracks. In physical terms, the reason for this is attributed to the fact that the long wavelength stress wave of the low frequency modes is unable to detect small local crack.

In addition, the procedures involved are also very time-consuming and likely to be contaminated by noise associated with the ambient vibrations (typically less than 100Hz) thereby adding an element of inaccuracy. Reliability and health assessment of these global techniques also requires accurate modeling of the damaged structure, restricting its application to simple geometries. Expensive hardware and sensors such as shaker and accelerometer further restricts its real life application.

2.3 LOCAL SHM TECHNIQUES

Similar to the global techniques, the local techniques make use of the host structural response as a measure of the degree of damage. However, the local SHM techniques rely on local interrogation of the structure. In other words, instead of investigating the structure as a whole, local SHM techniques detect damages only at specific portion of the structure. Some of these techniques are briefly described in the following paragraphs (Pook, 2007):

(1) Visual inspection

Visual inspection is the simplest technique for detecting surface cracks. Generally, a surface crack of 25mm can be comfortably detected using naked eye. Detection of 3mm crack may be possible if the surface condition is good. The advantage of visual inspection is that it is relatively inexpensive and does not require bulky instruments. However, routine visual inspection is tedious and some fatigue cracks could be omitted. In practice, visual inspection is used to detect surface breaking cracks. It is also not amenable to automation. The sizing of crack could later be performed using ultrasonic or alternating current potential drop.

(2) Magnetic particle inspection (MPI)

MPI is an established technique in detecting surface breaking crack. It is limited to the use of ferromagnetic materials that can be strongly magnetized. The application of MPI in crack detection requires magnetic poles to be placed in between a suspected crack and in contact with the material. The magnetic flux generated by the poles will be distorted when it encounters a transverse surface breaking crack. As a result, some of the flux passes through the crack while some passes around the crack tip. Flux will be leaking around the crack. Ferromagnetic materials would be attracted by the leaked flux, and the crack could be visible as the magnetic particles concentrate. The magnetic particles, such as black iron oxide can be dyed to improve visibility.

MPI is easy to apply with portable equipment but requires expertise for

satisfactory results. Disadvantages include only surface crack can be detected and low accuracy in crack size.

(3) Dye penetrant

Dye penetrant is useful in detecting surface breaking cracks. Generally, the crack that is found by the dye penetrant could visually be seen under good condition. The application of dye penetrant makes them easier to be detected.

The principle of application is by applying dye penetrant on the targeted surface so that it wets the material and fills the crack by capillary action. A thin layer of porous developer is then applied with the excess penetrant at the surface removed. The penetrant in the crack is finally drawn out, which would leave the crack visible.

The dye penetrant is widely applied in aluminium alloys and other metallic materials which cannot be magnetized, refraining the usage of MPI. The application of dye penetrant is relatively simple but is limited to the surface length.

(4) Radiography

As opposed to some of the previously described technique, radiography is useful in detecting voluminous defects such as internal cavity, porosity and inclusion. The application of radiography requires the emission of X-rays which would travel through the structure of interest in straight line. The X-ray penetrates through the object (though partly absorbed) and be reflected in a photographic film placed on the other side. The resulting radiograph would reflect the 2-D shape and location of any internal cavity since less absorption is expected when the X-rays passes through the void. The cavity could even be outlined in 3-D form if the radiograph is obtained from different directions.

Radiography is one of the oldest NDE techniques which is proven to be quite versatile and relatively easy to apply. The application, however, requires strict safety precautions as X-ray poses a health hazard to the inspector. Furthermore, it could hardly detect and size tiny cracks whose opposite surfaces are close together or touching.

(5) Ultrasonic

As its name implies, ultrasonic utilizes the propagating sound waves with frequency above the audible range for sizing of crack. With a wave frequency in the MHz range, the resulting wavelength is of the order of millimeter. Two main types of wave used in ultrasonics are longitudinal (compressive) waves and transverse (shear) waves. Waves generated are in the form of pulses. Short electric pulses are triggered to activate the piezoelectric disc (known as probe) designed to resonate at particular frequency. The waves generated are then received by a suitably positioned probe to give useful information about any defects within the specimen.

There are two approaches commonly used in the application of ultrasonic wave for crack detection, namely the impact-echo and the time of flight diffraction. In impulse echo, an ultrasonic wave pulse is triggered and when it hits a crack, it will be reflected. The echoes are then displayed on an oscilloscope. When comparing the reflected wave with the base pulse, the positions of the echoes on the time axis provide information on the crack location. However in practice, dispersion effects within the specimen which lead to subsidiary echoes can complicate interpretation of the display.

On the other hand, the time of flight diffraction technique requires separate transmitter and receiver probes. The principle of application lies in the fact that when a compressive ultrasonic wave encounters a crack tip, some of the energy is diffracted. The diffracted waves would be distributed over a large angular range, which could be detected by a receiver probe. Estimation of the position of crack tip and crack depth is possible with symmetrically placed transmitter and receiver. For the detection of deeper crack, shear waves are sometimes preferred.

Major drawbacks of this technique include high cost of application and unsuitability for small, thin specimens. Application of ultrasonic techniques also requires large and bulky piezo-actuator to generate ultrasonic waves. For good coupling during the test, the structure will be rendered unavailable throughout the process. Complex data processing forms another shortcoming of this technique.

(6) Electromagnetic fields

Two commonly used techniques based on the principle of electromagnetic field are eddy current and alternating current potential drop (ACPD). As the name implies, these techniques make use of uniform alternating current field being injected into the specimen for crack detection and for crack sizing.

For actual application, eddy current technique is more suitable for material that has low magnetic permeability (non-magnetic material) whereas the ACPD is preferable in material with high magnetic permeability (magnetic material). For both techniques, the effect of skin depth shall be taken into consideration. Satisfactory skin depth must be small in comparison to the crack depth.

The working principle of eddy current lies in the use of a probe with a coil carrying alternating current, which produces alternating magnetic flux. Eddy current, on the other hand, is induced in the host structure and produces opposing magnetic flux to the current carrying coil which changes the coil's impedance. With the probe passing over a crack, eddy currents are distorted and the presence of crack could be detected.

Eddy current is very effective in detecting crack depth as small as 10 micrometer. Frequency of application is usually kept high at around 1 to 10 MHz to limit the skin depth. It is also suitable for automatic collection of fatigue crack propagation data during structural fatigue tests. Application of ACPD is free from issue of safety due to its low voltage involved. Another advantage is its unnecessary contact with the structural surface. However, the technique is costly and sometimes poses difficulty in interpretation of results.

Drawback of this technique is shown when it is measuring an oblique crack, where the crack length is measured instead of the depth of the crack tip below the surface. As a result, the projected crack length is overestimated.

Other common drawbacks of the conventional local techniques include its dependency on the known vicinity of the damage. Essentially, this requires knowledge of the approximate location of the damage *a priori*. Thus, visual inspection for gross assessment is required prior to the application of the local techniques to pinpoint the

damage. These techniques require experienced inspectors to evaluate the external signs of structural damage such as corrosion, wear and visible deterioration.

However, in large scale structure, critical parts of a structure may not be readily accessible. This situation is further aggravated by the requirement of bulky probes or equipments to be carried for inspection. Inspectors are also often exposed to dangerous conditions or uncomfortable working environments. High dependency on experience of inspector, high cost and unsystematic procedures render the technique uneconomical and unreliable.

In general, most of these local techniques work best for assessing the condition of portions of a structure or individual members at selected time but are quite impractical for comprehensive monitoring of large structures with complex geometry. Therefore, an autonomous, real-time, reliable and cost efficient monitoring technique, capable of replacing the conventional ones, is constantly being pursued by SHM researchers (Chang, 1997).

2.4 ADVENT OF SMART MATERIALS, STRUCTURES AND SYSTEMS IN SHM

2.4.1 Background

Technological advancement of mankind is closely related to the development of materials. In the olden days, before the emergence of systematic structural design methodology, civil structures were built based on experience or trial and error, which often, resulted in structures been over-designed but may not be structurally sound.

Subsequent development in structural design and materials aimed at achieving a set of intended functions under pre-selected loads or a selection of the dimensions of load-bearing components of the structure. Further research in materials led to the advent of man made materials such as plastics and composites. Successive revolutions then led to the ideas of embedding sensors to monitor complex strain fields of the structures. In recent decades, research and development in materials and structures have witnessed the emergence of a new concept – “smartness” in materials, structures

and systems (Rogers et al., 1988; Takagi, 1996), in which the structure is expected to actively adapt itself to the external or internal changes (such as damage) and remains functional.

2.4.2 Smart Materials, Structures and Systems

The concept of smart materials arises from observations on some recently discovered and developed materials which possess unique capability of changing their physical properties in response to certain external stimuli such as temperature, poling direction, electric field, magnetic field and strain. The change in properties can be strain, shape, viscosity, stiffness and damping. When they are properly utilized, their behavior resembles or exhibits certain smartness. Some of them have the ability to convert energy, acting as sensor or actuator. Suitably incorporating the smart materials in structures and systems will lead to smart structures and smart systems.

Rogers et al. (1988) defined a “smart”, “intelligent” or “adaptive” material as one which possesses extraordinary and useful properties that can be designed and developed into a smart structure. Ideally, smart structure should have the capability to sense, measure, process, and diagnose any change in selected variables at critical locations, and to command appropriate action in a predetermined manner within a certain time frame and ensure the performance of the intended function. An engineered smart structure must meet the following six criteria: functionality, reliability, durability, affordability, safety and cost effectiveness (Srinivasan and McRarland, 2001).

Some of the notable developed smart materials are piezoelectric materials, shape memory alloys (SMA), electrostrictive and magnetostrictive materials, electrorheological (ER) and magnetorheological (MR) fluids, and fiber optics. The following sections briefly describe the properties, applications and limitations of the commonly available smart materials.

2.4.2.1 Piezoelectric materials

When an electric field is applied across the pre-poled direction of a piezoelectric material, deformation occurs in the other two directions. Conversely, it produces a dielectric polarization when subjected to mechanical strain. These unique properties, commonly known as direct and converse effects, enable piezoelectric materials to be utilized as actuators and sensors.

In comparison to other smart materials which possess non-linear characteristics, such as electrostrictive materials, the piezoelectric effect exhibits linearity within certain range. Piezoelectric materials are also less noisy and in general, more efficient. The frequency bandwidth is usually orders of magnitude larger than that of the conventional modal analysis equipment. For these reasons, they are found to be highly desirable for smart system applications. Piezoelectric materials are commercially available in two principal forms, namely, piezoceramics and piezopolymers.

Piezoceramics exhibits most of the characteristics of ceramics, including high elastic modulus, high rigidity, brittleness and low tensile strength. Efficient conversion of electrical energy to mechanical energy renders piezoceramics suitable for actuation. The most commonly available type of piezoceramics is the Lead Zirconate Titanate (PZT).

On the other hand, piezopolymers are characterized by low charge characteristics. They are electro-mechanically weak when compared to piezoceramics. However, piezopolymers are more sensitive to mechanical loads over a larger range, which renders them to be better sensors. The most commonly available form of piezopolymers is the Polyvinylidene Fluoride (PVDF).

Piezoelectric materials have been successfully implemented in various applications such as vibration control and distributed structural excitations (Zhou et al., 1996; Fairweather, 1998), structural health monitoring (Sun et al., 1995; Ayres et al., 1998; Soh et al., 2000; Naidu and Soh, 2004; Ihn and Chang, 2004a, b), and systems identification (Liang et al., 1994; Giurgiutiu and Rogers, 1998; Giurgiutiu and Zagari, 2000). Other applications include ultrasonic sensors, piezoceramic stacks, bimorphs, micro-positioning actuators, and controls in robotic systems.

2.4.2.2 *Electrostrictive materials*

Electrostrictive materials are characterized by the phenomenon known as electrostriction, a phenomenon similar to piezoelectricity in which mechanical deformation occurs upon application of electric field. However, they are isotropic and possess no net polarization. The electromechanical coupling is also non-linear; and the strain generated is approximately proportional to the square of induced polarization at low electric field. Any increase in electric field will lead to asymptotically constant value. Another distinct difference from piezoelectric materials is that it does not generate strain upon the reversal of electric field. Thus, this unidirectional effect limits the application of electrostrictive material for bidirectional actuation.

2.4.2.3 *Magnetostrictive materials*

Similar to electrostrictive materials, magnetostrictive materials are solids that develop considerable deformation when subjected to magnetic field, and conversely produce magnetic field when mechanically deformed. Magnetostrictive materials can act as actuator as well as sensor. When compared to the piezoelectric materials, magnetostrictive materials can produce considerably larger strain. However, the relationship between magnetic field and resulting strain are highly non-linear. They also suffer from problems such as high hysteresis loss, bulkiness and large in size.

2.4.2.4 *Shape memory alloy (SMA)*

As implied by its name, SMA is essentially a kind of metal alloy, which possesses the inherent ability to remember a specific shape. When plastically deformed at low temperature, it can regain its original “memorized” shape when temperature is elevated above certain threshold, called the characteristic transition temperature. One of the most commonly used SMA is NITINOL (an alloy of Nickel, Titanium developed by the Naval Ordnance Laboratory).

Restoration of shape can be easily achieved by heating the SMA. Up to four times increase in Young’s modulus and ten times increase in yield strength during the

heating process generate large recovery force if they are restrained. Restraining stress of magnitude 100,000 psi allows the SMA to be utilized as actuators.

Active vibration control, active buckling control, shape/motion control, artificial muscle are amongst the applications of SMA. Biocompatibility of SMA enables applications in the biomedical field. Limitations of the materials lie in the difficulty in the control of heating and cooling speed, problem of hysteresis and dissipation of extreme heat within short period.

2.4.2.5 Electrorheological (ER) fluids

ER fluid could be viewed as a kind of smart fluid capable of changing its viscosity upon application of external electric field. In the absence of an electric field, the ER fluid behaves like a Newtonian fluid.

Application of electric field forces the particles to form into chains that resist the flow or shear movement, thus changing its flow characteristic. This in turn causes the fluid to undergo significantly large and instantaneous reversible changes in their mechanical properties, such as mass distribution and energy dissipation characteristics. Upon removal of the electric field, the ER fluid flows smoothly like water. Its reaction time is relatively short, in the order of milli seconds. The main disadvantage of this smart material is that very high voltage is required for effective actuation.

2.4.2.6 Optical fiber

Originally developed for telecommunication purposes, optical fibers (Gandhi and Thompson, 1992) have recently found their roles as smart sensors. Consisting of glass and silica, optical fibers are very thin fibers which utilize fiber properties for generation of optoelectronic signals, serving as an indication of the external parameters to be measured. Laser diode or a light emitting diode is commonly used to generate light signal. At the other end of the fiber, an optical receiver system containing a photodiode will receive the conducted light signal. Any modulation of the light signal (frequency, intensity, wavelength, amplitude, phase, color, modal distribution or polarization characteristics) indicates some disturbances caused by

external stimuli (such as strain, pressure, temperature etc.) along the optical fiber. Careful calibration allows the monitoring of changes in the external parameters. Differing from the active smart materials such as piezoelectric materials and SMA, pure sensing based optical fiber is considered as a passive smart material.

Many novel sensors have been developed using optical fibers, such as velocity sensors, pressure sensors, strain gauges (Rao, 1997), temperature sensor (Baran, 1996) and displacement sensors, which find themselves numerous applications in SHM. Distinct advantages of the optical fibers include light weight, non-conductive, rugged and can be easily embedded in composite structures due to their shape adaptability. Moreover, unlike piezoelectric materials, they do not require electrical isolation as they are immune to electromagnetic interference. Ability of multiplexing numerous sensors into a single fiber renders the fiber optics to be space efficient, easy to handle and cost efficient. Among the various types of fiber based technologies, the Fiber Bragg Grating (FBG) is proven to be one of the most versatile (Dewter-Marty et al., 1998; Rao, 1999; Tjin et al., 2002; Lin et al., 2004).

However, multiplexing of a number of sensors into a single fiber suffers from the risk of whole sensor system dysfunctional even when only single damage is incurred at any where along the optical fiber. Being fragile, proper protection is very important to ensure its use under harsh environment.

2.4.3 Application of Smart Materials in SHM

Ideally, an intelligent structure is one which is capable of monitoring its own health and preferably performing self-maintenance to retain its functionality upon detection of malfunctions. It is expected that the intelligent structure would possess an integrated package with a monitoring system consisting of sensors, data acquisition, control and communications hardware, and relevant software to perform the intended functions. Successful application of such system is conceived to lead to significant cost savings and enhancement in safety with the autonomous and real time operation.

SHM techniques employing smart materials such as piezoelectric materials and optical fiber are proven to be robust in damage detection and strain measurement.

They are also found to be able to overcome most of the shortages present in the conventional techniques. Detailed descriptions of the piezoelectric material, which is the main focus of this study, are presented in the following sections.

2.5 PIEZOELECTRICITY AND PIEZOELECTRIC MATERIALS

2.5.1 Background

The discovery of piezoelectricity (Schwartz, 2005) can be traced back to 1880 when Pierre and Jacques Curie were studying the effects of pressure on the generation of electrical charge on a group of crystals such as quartz, tourmaline, and Rochelle salt. They found that these crystals developed electrical charges on their surface when mechanically deformed in the orthogonal direction. Converse effect was also found to present where deformation occurred when a potential difference was applied across certain direction. W. Hankel suggested the term “piezoelectricity” in describing this behavior in 1881, where *piezo* means “pressure” in Greek. At that time, limited performance of these materials inhibited their commercialization. This situation persisted until a major breakthrough came with the discovery of piezoceramics, namely Barium Titanate and Lead Zirconate Titanate (PZT) in the 1940s and 1950s respectively. These families of materials exhibit much higher dielectric and piezoelectric properties. In 1960s, success of the Japanese companies in developing new processes and applications of piezoelectric devices officially opened their commercial market.

2.5.2 Fundamentals of Piezoelectric Materials

Piezoelectric effect occurs in certain anisotropic crystal, where the crystal lattice does not have any center of symmetry. In this non-centro symmetric crystal, net dipole moment will be induced upon deformation of the crystal. On the other hand, converse effect where electric field induces mechanical strain is similarly present.

When piezoelectric material is first manufactured, it is isotropic without the piezoelectric properties, as their electric dipoles are arranged in random directions.

For the material to exhibit piezoelectricity, a process known as poling is required to permanently align all the dipoles in certain direction. To achieve this, the piezoelectric material is first heated to a transition temperature known as Curie temperature. Above this temperature, the dipoles in the solid phase material are free to be re-orientated. Maintaining this temperature, an intense electric field (> 2000 V/mm) is applied across the desired direction (polarization direction) to align the dipoles. With the electric field maintained, the temperature is reduced below Curie temperature. As a result, all dipoles are permanently fixed despite the removal of electric field.

Subsequently, application of a small electric field across the poling direction will cause a collective response of all the dipoles and lead to macroscopic expansion along the poling axis and contraction in the perpendicular axis (or vice versa depending on the direction of applied electric field). This is generally known as converse piezoelectric effect. On the other hand, direct piezoelectric effect exists where induction of mechanical strain generates potential difference across the poling direction. From molecular point of view, piezoelectricity arises from the displacement of ions in the crystal lattice when the polarized material is deformed mechanically.

2.5.3 Piezoelectric Constitutive Relations

The direct and converse piezoelectric effects involve cross coupling interaction between mechanical and electrical behavior of the piezoelectric material. They can be modeled accordingly by linear constitutive equations involving two mechanical variables and two electrical variables (IEEE standard, 1987):

$$D_i = \overline{\varepsilon}_{ij}^T E_j + d_{im}^d T_m \quad (2.1)$$

$$S_k = d_{jk}^c E_j + \overline{s}_{km}^E T_m \quad (2.2)$$

Equations (2.1) and (2.2) can be expressed in generic terms as a compressed matrix notation:

$$\begin{bmatrix} D \\ S \end{bmatrix} = \begin{bmatrix} \overline{\varepsilon}^T & d_d \\ d_c & \overline{s}^E \end{bmatrix} \begin{bmatrix} E \\ T \end{bmatrix} \quad (2.3)$$

where $[D]$ (C/m²) is the electric displacement (charge density) vector, $[S]$ is the second order strain tensor, $[E]$ (V/m) is the applied external electric field vector and $[T]$ (N/m²) is the stress tensor. $[\overline{\varepsilon^T}]$ (F/m) is the second order complex dielectric permittivity tensor under constant stress, $[d_d]$ (C/N) and $[d_c]$ (m/V) are the third order piezoelectric strain coefficient tensors (the superscripts d and c indicate the direct and converse effect respectively), and $[\overline{s^E}]$ (m²/N) is the fourth order complex elastic compliance tensor under constant electric field. Note that the bar above the variables indicates complex number.

It is clear that two basic equations could readily be refined from the constitutive equations. When no stress is applied, Equation (2.1) is interpreted as an electrical expression indicating a material subjected to an electrical field having electric displacement. Equation (2.2) can be reduced to a material's stress-strain relationship at zero field strength. Thus, the piezoelectric effect is governed by the piezoelectric strain coefficient, d , which appears in both equations. $[d_c]$ defines the strain per unit field at constant stress, while $[d_d]$ defines the electric displacement per unit stress at constant electric field.

Mechanical and electrical behavior of the piezoelectric material can be expressed using a well accepted set of conventions indicating various directions. A commonly used rectangular piezoelectric plate as shown in Figure 2.1 is taken as an example for description.

Figure 2.1: A piezoelectric element with conventional labels of axes.

The poling direction is normally along the thickness direction, denoted as axis-3. Therefore axis-1 and axis-2 indicate the plane of the piezoelectric sheet. Taking advantage of symmetry of the stress and strain tensor, the original matrix can be reduced to a more concise (6x1) vector form: $[T] = [T_{11}, T_{22}, T_{33}, T_{23}, T_{31}, T_{12}]^T$ and $[S] = [S_{11}, S_{22}, S_{33}, S_{23}, S_{31}, S_{12}]^T$ respectively. The piezoelectric strain coefficients can accordingly be reduced to second order tensors, as $[d_d]$ (3x6) and $[d_c]$ (6x3) with $[d_d] = [d_c]^T$. Piezoelectric strain coefficient varies with different types of crystal structures. The matrix $[d_c]$ of the commonly used Lead Zirconate Titanate, PZT is:

$$d_c = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{24} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (2.4)$$

The first subscript indicates the direction of externally applied field whereas the second subscript denotes the plane of resulting strain with 4 and 5 indicating the shear strain in plane 2-3 and plane 1-3 respectively. Shear in the plane 1-2 will not generate any electrical response and is thus excluded in the equation. This also implies that application of electric field in any direction will not cause any shear strain in plane 1-2. For good sensing capability, the algebraic sum of d_{31} and d_{33} should be maximized, whereas ε_{33} and the mechanical loss factor should be minimized (Kumar, 1991).

The compliance (inverse of Young's modulus or shear modulus) matrix could be expressed as:

$$\overline{s^E} = \begin{bmatrix} \overline{s_{11}} & \overline{s_{12}} & \overline{s_{13}} & 0 & 0 & 0 \\ \overline{s_{21}} & \overline{s_{22}} & \overline{s_{23}} & 0 & 0 & 0 \\ \overline{s_{31}} & \overline{s_{32}} & \overline{s_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{s_{44}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \overline{s_{55}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \overline{s_{66}} \end{bmatrix} \quad (2.5)$$

The electric permittivity matrix (for PZT) can be written as:

$$\overline{\varepsilon^T} = \begin{bmatrix} \overline{\varepsilon_{11}^T} & 0 & 0 \\ 0 & \overline{\varepsilon_{22}^T} & 0 \\ 0 & 0 & \overline{\varepsilon_{33}^T} \end{bmatrix} \quad (2.6)$$

The electric displacement vector and electric field vector remain the same:

$$D = \begin{bmatrix} D_{11} \\ D_{22} \\ D_{33} \end{bmatrix} \quad (2.7)$$

$$E = \begin{bmatrix} E_{11} \\ E_{22} \\ E_{33} \end{bmatrix} \quad (2.8)$$

2.5.4 Classifications of Piezoelectric Materials

Nowadays, piezoelectric materials are commercially employed in various applications such as medicine, military, communication and automobile. As a transducer, its electro-mechanical property continues to attract considerable attention from a wide range of research communities. Two commercially available piezoelectric materials, piezoceramics and piezopolymer are described in the following sections.

2.5.4.1 Piezoceramics

The commercially available piezoceramics possess a perovskite structure as shown in Figure 2.2. This simple octahedral arrangement consists of 8 corners, sharing oxygen octahedral and forming a cube. A small cation (Ti, Zr) occupies its centre with larger cations (Pb, Ba) filling the interstices between the octahedral.

Figure 2.2: Perovskite structure of PZT.

The most commonly seen piezoceramics, namely lead zirconate titanate (PZT) comprises a solid solution of PbZrO_3 (53%) and PbTiO_3 (46%). When dealing with piezoelectricity, piezoceramics are transversely isotropic in the plane normal to the poling direction, implying $d_{31} = d_{32} \neq d_{33}$ and $d_{15} = d_{24}$. Mechanically, they are isotropic (Sirohi and Chopra, 2000a, b).

Being chemically inert, PZT demonstrates competitive characteristics such as light weight, low-cost, small size and good dynamic performance. Besides, it exhibits large range of linearity, fast response, long term stability and high energy conversion efficiency. The PZT patch can be manufactured in any desired shape, size and thickness at relatively low-cost. High elastic modulus (comparable to that of aluminium) renders PZT a good actuator. On the other hand, high strain coefficients also ensure good sensing capability. PZT is thus an ideal material to be used as collocated actuator and sensor.

However, inherent deficiencies such as being fragile, brittle and having low tensile strength sometimes limit its application. Ensuing problems caused by the abovementioned deficiencies include inability to withstand bending and poor conformity to curved surface. Considerable fluctuations of electrical properties with temperature also pose difficulties when it is used under environment with unstable ambient temperature.

2.5.4.2 Piezopolymers

Polyvinylidene Fluoride (PVDF) is the most frequently used piezopolymer. As the name implies, it is a polymer made up of long chains of repeating monomer ($-\text{CH}_2-\text{CF}_2-$). High dipole moment of this polymer results in stronger piezoelectric effect than the other organic materials.

PVDF film is manufactured by solidification from its molten phase, which is then stretched in a particular direction and poled. PVDF is characterized by its lightness and transparency. Ease of shaping and formation into thin sheets as well as flexibility allow the PVDF films to be customized to fit the intended application such as adhering to curved surfaces. Low stiffness (Young's modulus is 1/12th that of

aluminium) of PVDF ensures negligible stiffening effect when attached to the host structure. PVDF is more suitable for sensor applications when compared to piezoceramics. Table 2.1 lists the typical properties of piezoceramics and piezopolymers (Ghandi and Thompson, 1992).

Table 2.1: Typical properties of PVDF and PZT (Ghandi and Thompson, 1992).

Properties	PVDF	PZT	Units
Density	1.78	5.7	(10 ³) kg/m ³
Relative permittivity	12	1700	--
Piezoelectric constant	23	78	(10 ⁻¹²) m/V
Piezoelectric voltage constant	216	5	(10 ⁻³) Vm/N
Coupling factor	12	21	% @ 1 kHz

A broad range of properties, sizes and shapes are readily available in the industry (PI Ceramic, 2010) for both types of piezoelectric materials. When employed as smart materials in SHM, thin sheet of piezoelectric material is preferred to prevent intervention to the integrity of the host structure.

2.5.5 Secondary Effect

The linear constitutive equations (Equation 2.1 and Equation 2.2) are generally valid under low electric fields. Under high externally applied electric field, non-linearity comes into play due to the effect of electrostriction, which can be accounted for with an additional term in the equation.

In the case of piezoceramics, the non-linearity effect may be complicated by the aging effect as the properties of the ceramics decay logarithmically with time. Therefore, the validity of the piezoelectric strain coefficient should be investigated periodically, especially when the materials have been exposed to elevated temperatures for long period of time.

2.5.6 Practical Considerations

2.5.6.1 Depoling

Throughout the service life of the piezoelectric materials, the working temperature should be well below Curie temperature, typically varies from 150°C to 350°C. On the other hand, an excessively strong field (greater than 80% of the rated coercive field, $> 12 \text{ kV/cm}$), opposite to the poling direction should be prevented as it will cause the dipoles to be shifted away from the original direction.

2.5.6.2 Hysteresis

Magnitude of hysteresis is closely related to the applied electric field (Littlefield, 2000). The presence of hysteresis may be significant in some area of applications such as the active control schemes for smart structures and micro-positioning. These devices rely heavily on the direct strain induced by the piezoceramic actuators.

When employed as piezo-impedance transducers in SHM and system identification using the EMI technique, the effect of hysteresis is usually negligible as the applied electric field is small.

2.6 APPLICATION OF PIEZOELECTRIC MATERIALS IN SHM

The transduction capability of piezoelectric material has been adopted for applications in various fields. Piezoceramics such as PZT possesses good actuating and sensing capabilities. This special feature of PZT transducer acting as a collocated actuator and sensor has recently been employed for NDE in the field of SHM. Piezoelectric transducer based SHM technique has the potential to bring about a revolution in SHM, damage detection and NDE just as significant as ultrasonic inspection did 50 years ago.

The EMI technique, which emerged a decade ago, employs PZT transducers or piezo-impedance transducers (normally PZT patches) to dynamically actuate the host structure and simultaneously sense its responses. Damage in the structure will be reflected from its change in structural response. Changes in the structural vibrational

response will in turn be sensed by the piezo-impedance transducer. The underlying physical principles of the EMI technique are discussed in the ensuing section.

Another novel technique, commonly known as wave propagation technique, also employs PZT patches in damage detection (Ihn and Chang, 2004a, b; Ihn and Chang, 2008). In this technique, built-in PZT patches are used as transmitters and receivers to generate diagnostic stress waves along the host structures. Structural damage can be detected from changes in the received signal. This technique is proven to be effective in detecting debonding or delamination in composites and fatigue crack growth.

In short, the EMI change is a high frequency standing waves effect while the Lamb-wave transmission change is due to waves being reflected and diffracted by the crack. In this study, focus is placed on investigating the applications of EMI technique employing surface-bonded PZT patches.

2.6.1 Physical Principles of EMI Technique

The EMI technique shares similar working principles as the conventional global dynamic response techniques. However, the frequency range employed in the EMI technique ($30 - 1000 \text{ kHz}$) is much higher. The method of interrogation also differs from each other.

For the EMI technique, a mechanically attached (surface-bonded or embedded) PZT patch is dynamically excited by an alternating (sinusoidal) voltage, sourced by an impedance analyzer, uniformly across the patch. The vibrational force generated by the PZT patch can then be transferred to the host structure. The corresponding structural response at different excitation frequency will modulate the electric current across the PZT patch.

The modulated current, in terms of complex electrical admittance (conductance and susceptance signatures) is also measured and recorded by the impedance analyzer at predetermined frequency interval. This frequency response function can be graphically plotted, which yields a spectrum that serves as an indication of the structural response as exemplified in Figure 2.3 using an aluminium beam (331mm x 31mm x 6mm) with surface-bonded PZT patch located at the centre.

Figure 2.3: Typical plot of admittance signatures versus frequency acquired from PZT patch surface-bonded on aluminium beam (331mm x 31mm x 6mm).

- (a) Conductance (real component) signatures (0 ~ 1000 kHz)
- (b) Susceptance (imaginary component) signatures (0 ~ 1000 kHz)
- (c) Conductance (real component) signatures (0 ~ 200 kHz)

The frequency response functions generally consist of large amount of resonance peaks (conductance) and valleys (susceptance). These resonance peaks represent various modal frequencies of the host structure as well as the PZT itself. For this particular beam specimen, densely spaced resonance peaks below 200 kHz (Figure 2.3c) are the host structural resonances, induced by different modes of vibration such as axial, transverse, torsion etc. Three dominant peaks (see dotted circles in Figures 2.3a & b) are the lateral modes of resonance of the PZT patch.

Any subsequent damage or interference on the structure, which causes a change in structural response, can be reflected qualitatively from the alteration in the

spectrum as exemplified in Figure 2.4. This figure zoomed into one of the structural resonance peak occurring at 58.1 kHz at healthy stage. Comparing the different damaged (induced by drilling 5mm holes) stages of the plot, it is apparent that the progressive leftward shift in resonance peak (reduction in resonance frequency) could serve as an indication of the existence of damage. This agrees well with the structural dynamic theory stating that occurrence of damage would reduce the overall structural stiffness which is reflected through the reduction in modal frequency.

Figure 2.4: Typical plot of admittance signatures versus frequency (57 ~ 59 kHz) acquired from PZT patch surface-bonded on aluminium beam under different health conditions.

2.6.2 Modes of Wave Propagation

In an unbounded 3-D elastic solid, two basic wave types exist: dilatational and rotational. Dilatational (longitudinal) waves are those in which the particle motions are parallel to the direction of wave propagation. They are also known as P-waves, or ‘Principal’ or ‘Pressure’ waves. The rotational (transverse) waves, on the other hand, correspond to incompressible distortion of solids, like shear. The particles move perpendicularly to the direction of wave propagation. They are often referred to as S-waves or ‘Secondary’ or ‘Shear’ waves.

When the solid medium is not infinite, two additional aspects need to be considered (i) wave reflection and refraction on account of boundary, and (ii)

existence of additional wave types closely related to the boundary effects. When a pure P-wave or S-wave traveling at an oblique angle hits a boundary, both pressure and shear waves are generated in the reflection process. A free boundary, on the other hand, gives rise to two new wave types – Rayleigh waves and Lamb waves.

The actuation of a surface-bonded PZT patch generates propagating waves, mainly surface acoustic waves such as Rayleigh waves (a.k.a. surface guided waves), as shown in Figure 2.5. Rayleigh waves present in solids containing free surface. Its amplitude decreases rapidly with depth, and becomes almost zero at a depth of approximately 1.6 times of its wavelength. The waves travel radially outwards on the plane at which the PZT patch is attached.

The wave propagation dynamics (reflection, refraction and transmission) determines the mechanical impedance of the. They play crucial role in detecting any defects which tend to obstruct their path. Due to the nature of the wave, damage detection capability of surface-bonded PZT patch is usually stronger in the parallel plane than the thickness direction.

Lamb waves are confined to a superficial layer existing on the top of a homogeneous solid. At high frequency of excitation (> 5 kHz), the particle motions of both symmetric and anti-symmetric modes of Lamb waves would be restricted to the proximity of free-surfaces, resembling Rayleigh waves.

Figure 2.5: Modes of wave propagation associated with PZT patch.

- (a) PZT patch affixed to host structure
- (b) Surface waves generated by vibrating PZT patch

2.6.3 Application of EMI Technique

2.6.3.1 SHM

Application of the EMI technique employing piezoceramics in the field of SHM was proposed by Chaudhry et al. (1994). The initial attempt was focused on the use of raw electrical impedance/admittance signatures. The application of the EMI technique for SHM of a lab-sized truss structure was first reported by Sun et al. (1995). Ayres et al. (1998) studied qualitatively the feasibility of employing the EMI technique on a quarter-scale deck truss bridge joint. Park et al. (2000) reported some proof of concept applications of the EMI technique on civil-structural components including composite reinforced masonry walls, steel bridge joints and pipe joints. The technique is proven to be tolerant to mechanical noise.

Soh et al. (2000) studied the damage detection and localization ability of the PZT transducers for reinforced concrete structures, by monitoring a 5m span RC bridge during a loading testing. They provided recommendations for issues related to sensor positioning, damage localization and sensor validation. Inman et al. (2005) proposed a novel technique to utilize a single PZT transducer to simultaneously monitor the structural health as well as to control its vibration. Giurgiutiu et al. (2004) combined the EMI technique with wave propagation technique for detecting crack in aircraft components. It was found that both techniques could complement each other where the EMI technique was employed for near field damage detection and the guided ultrasonic wave propagation technique (pulse echo) was used for far field damage detection.

Bhalla and Soh (2003) reported the feasibility of diagnosing blast/seismic induced damages using the EMI technique. Naidu and Bhalla (2002) showed the robustness of the EMI technique in characterizing damages induced in concrete structure. Giurgiutiu (2007) performed experimental test to monitor the debonding of FRP composite reinforcing strips surface mounted on concrete beam during a fatigue test. The PZT patch's resonance was selected as indication of damage, whereby the peak's amplitude gradually increased upon debonding.

Conventionally, damage detection and characterization are achieved through

statistical quantifiers. These statistical approaches are non-parametric (Winston et al., 2001) in nature, as quantification of damages is based on the measured electrical admittance instead of the physical parameters related to the structural response. Root mean square deviation (RMSD), signature assurance criteria (SAC), waveform chain code (WCC), adaptive template machining (ATM) and cross correlation deviation mean (CCDM) are among the non-parametric quantification approaches reported in the literature. RMSD appears to be one of the most commonly used approaches (Giurgiutiu and Rogers 1998). However, Yang et al. (2008a) reported that the RMSD, which measures essentially variation in magnitude, is more susceptible to the effect of temperature and bonding.

Lopes et al. (2000) employed multiple sets of artificial neural networks (ANN) to quantitatively assess the state of a massive quarter scale model of a steel bridge section and a space truss structure. The measured electrical impedance signals were used as input patterns. By incorporating neural network features, the EMI technique has shown its robustness in detecting damage in its early stage as well as in estimating the nature of damage without prior knowledge of the model of structures. Similar neural network was also presented by Giurgiutiu and Zagari (2005). They developed an EMI-based analytical model for 2D thin-wall structures to access local dynamics of structure and adopted probabilistic neural network (PNN) to classify the EMI data and identify damage severity.

Park et al. (2003a) presented an outlier analysis framework for EMI technique to quantify damage. A modified auto-regressive model with exogenous inputs in the frequency domain was developed. The method of outlier analysis was then adopted to determine the damage state of a structure. Furthermore, they used extreme value statistics to establish proper confidence limits. It has been found that the proposed algorithm could assess the condition of a structure in a more quantifiable manner over the traditional impedance approaches.

Park et al. (2008a) adopted the PCA-data compression technique as a pre-processing module to reduce the data dimensionality and eliminate the unwanted noises. The proposed PCA-data compression approach is especially useful for

EMI-based wireless SHM. Experimental study inspecting loose bolts in a bolt-jointed aluminium structure was conducted. The damage detection capability is significantly enhanced in comparison to the traditional RMSD approach.

Generically, pros and cons exist in these non-parametric approaches. Advantages can be seen from the fact that no information regarding the structure is required in advance. The EMI technique does not require a detailed knowledge of the failure modes and can deal with unpredicted failure patterns.

However, physical changes in the host structure remain unknown and its corresponding structural parameters can hardly be derived. This causes extreme difficulty for the damage mode identification and damage severity quantification.

Bhalla (2004) and Lim et al. (2006) attempted some parametric based damage detection using equivalent structural parameters in characterizing the severity of damage in a structure. The approach shows considerable advantages over the conventional non-parametric approaches. Xu et al. (2004) successfully attempted damage quantification (limited to one hole) using the EMI technique with evolutionary programming.

Annamdas and Soh (2010) concluded that the present form of EMI technology is still relatively new. Various practical problems such as wiring, instrumentation and systematic frequency spectrum analysis have yet to be fully addressed. Detailed reviews considering various issues on the applications of EMI technique in SHM can be obtained in publications by Park et al. (2003b), Park et al. (2008b) and Annamdas and Soh (2010).

2.6.3.2 Other applications of EMI technique

The EMI technique also shows its ability in monitoring the hydration process of concrete. For instance, Shin et al. (2008) showed that the EMI signatures obtained from PZT patch surface bonded on concrete gradually shift to the right and subside with curing time. They concluded that these behaviors are attributed to stiffening action caused by strength gain of concrete. They also proposed to use both RMSD and the resonant frequency shift index to monitor the strength gain because RMSD alone

cannot distinguish if the strength increases or decreases. On the other hand, using embedded PZT transducer as smart aggregate for concrete strength monitoring is also reported in the literature (Song et al., 2008, Qin and Li, 2008).

Yang et al. (2010) proposed a reusable PZT transducer setup for simultaneous monitoring of concrete hydration as well as structural health. A piece of PZT is bonded to an enclosure with two bolts tightened inside the holes drilled in the enclosure. RMSD statistical index was used to monitor the increase in concrete strength upon curing with changes in the admittance signatures. The results indicated that the setup is effective in monitoring the initial hydration of concrete. The setup can also be detached from the concrete for future application.

Through the intrinsic electromechanical coupling characteristic, the PZT transducer can simultaneously act as actuator and sensor permitting effective modal identification of host structure (Liang et al., 1994). Wide frequency band width of the EMI technique in comparison to conventional shaker allows for modal extraction at very high frequency.

Some more advanced developments are also pursued such as the evolvement of new piezoelectric materials and the application of EMI technique to medical field (Dugnani and Chang, 2008), and biological and green materials like bamboo (Gaza et al., 2006).

2.7 MODELING OF EMI TECHNIQUE

2.7.1 Analytical models

A number of analytical models have been proposed to simulate the interaction between the piezo-impedance transducer and the host structure. The static equivalent-force (SEF) modeling approach, first proposed by Crawley and de Luis (1987) assumes a frequency independent PZT-structure interaction. The actuation force from the PZT patch is determined through static equilibrium and strain compatibility. The host structure is simplified into a static stiffness. Despite the convenient simplification in the modeling equation brought about by the omission of

the complex terms in the piezoelectric constitutive equations, considerable loss in accuracy is inevitable. This is a result of ignorance on the effect of inertia and damping. In fact, this modeling approach is highly impractical for the EMI technique which adopts high frequency (in the order of tens to hundreds of kHz) of excitation. In depth discussions on SEF model can be found in Lalande (1995), Fairweather (1998) and Littlefield (2000).

Some other modeling approaches are also available in the literature. The energy-based modeling technique proposed by Pan et al. (1991) investigated the development of a dynamic beam model based on Euler-Bernoulli beam theory. The model prediction shows closer agreement with experimental measurement when compared to those from the SEF model. However, it has been reported that the energy-based technique can be extremely sophisticated when applied to structure such as plates (Fairweather, 1998).

A brilliant impedance based modeling approach was proposed by Liang et al. (1993). This approach simplifies the host structure into a skeletal structure (Figure 2.6). The PZT patch is assumed to be a vibrating thin bar undergoing axial vibration with its interaction with the structure confined at the end points (Figure 2.6a). Therefore, the entire structure can be represented by its drive point mechanical impedance, Z , resembling a black box (Figure 2.6b). Incorporating the dynamic force equilibrium and PZT constitutive equations, Liang et al. (1994) derived the following expression for the 1-D electro-mechanical admittance.

$$\bar{Y} = 2\omega j \frac{w_a l_a}{h_a} \left[\bar{\epsilon}_{33}^T + \left(\frac{Z_a}{Z + Z_a} \right) d_{31}^2 \bar{Y}_{11}^E \left(\frac{\tan \kappa l_a}{\kappa l_a} \right) - d_{31}^2 \bar{Y}_{11}^E \right] \quad (2.9)$$

where j is the imaginary number, ω is the angular frequency of the driving voltage, w_a , l_a and h_a are the width, half length and thickness of the PZT patch, respectively, and κ is the wave number. $\bar{Y}_{11}^E$ and $\bar{\epsilon}_{33}^T$ are the complex Young's modulus and complex dielectric permittivity in x -direction and z -direction respectively. Z_a and Z are the mechanical impedance of the actuator and the structure respectively. According to the impedance based modeling approach, the actuator impedance Z_a is defined as:

$$Z_a = \frac{\kappa \omega_a h_a \bar{Y}_{11}^E}{(j\omega) \tan(\kappa l_a)} \quad (2.10)$$

Figure 2.6 (a): Idealized interactions between a PZT patch and host structure.

Figure 2.6 (b): A generic single degree of freedom electro-mechanical interaction where host structure is represented by drive point mechanical impedance.

Structural stiffness, actuator stiffness and displacement in static based model are replaced by the structural impedance, actuator impedance and velocity respectively in the impedance model. Integration of the dynamic PZT patch's impedance and the structural impedance renders the 1-D electromechanical coupling equation more comprehensive than the static based equation. Lalande (1995) studied the impedance model as applied to ring and shell structures, which included the effect of transverse shear. Detailed description of this method is provided by Cheng and Wang (2001).

Giurgiutiu and Zagrai (2000) proposed a PZT-structure interaction model for transducer dynamics by an elastically constrained (EC) boundary condition represented by a pair of springs (dynamic stiffness). In their study which focuses on a thin aluminium beam, reasonable accuracy has been achieved between the analytical result and the experimental outcome.

The abovementioned 1-D model is considered oversimplified for the modeling of some more complex structures. Zhou et al. (1995) extended the impedance based 1-D (skeletal) model to a more generic 2-D (planar) model. Direct impedances Z_{xx} and Z_{yy} , and the cross impedances Z_{xy} and Z_{yx} are incorporated in the equation:

$$\bar{Y} = 4j\omega \frac{w_a l_a}{h_a} \left[\frac{\bar{\epsilon}_{33}^T}{\epsilon_{33}^T} - \frac{2d_{31}^2 \bar{Y}^E}{(1-\nu_a)} + \frac{d_{31}^2 \bar{Y}^E}{(1-\nu_a)} \left\{ \frac{\sin \kappa l_a}{l_a} \quad \frac{\sin \kappa w_a}{w_a} \right\} N^{-1} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \right] \quad (2.11)$$

where ν_a is the Poisson ratio, w_a in this case is half width of the PZT patch,

$\kappa = \omega \sqrt{\frac{\rho_a (1-\nu^2)}{Y^E}}$ indicating the 2-D wave number (ρ_a is the density of the PZT

patch) and N is a 2 x 2 matrix, given by

$$N = \begin{bmatrix} \kappa \cos(\kappa l_a) \left\{ 1 - \nu_a \frac{w_a}{l_a} \frac{Z_{xy}}{Z_{axx}} + \frac{Z_{xx}}{Z_{axx}} \right\} & \kappa \cos(\kappa w_a) \left\{ \frac{l_a}{w_a} \frac{Z_{yx}}{Z_{ayy}} - \nu_a \frac{Z_{yy}}{Z_{ayy}} \right\} \\ \kappa \cos(\kappa l_a) \left\{ \frac{w_a}{l_a} \frac{Z_{xy}}{Z_{axx}} - \nu_a \frac{Z_{xx}}{Z_{axx}} \right\} & \kappa \cos(\kappa w_a) \left\{ 1 - \nu_a \frac{l_a}{w_a} \frac{Z_{yx}}{Z_{pyy}} + \frac{Z_{yy}}{Z_{ayy}} \right\} \end{bmatrix} \quad (2.12)$$

Z_{axx} and Z_{ayy} are the two components of the mechanical impedance of the PZT patch in the two principal directions, derived in a similar manner as for the 1-D impedance based model.

However, this model possesses 4 impedance parameters which have altogether 8 unknowns but only 2 equations. The large number of unknowns renders the system of equations highly indeterminate and thus inapplicable.

On the other hand, it should be noted that the additional factors of 2 and 4 are introduced respectively to Equations 2.18 and 2.20 when compared to the original derivations of Liang et al. (1994) and Zhou et al. (1995) due to the fact that only one-half and one-quarter of the patch are modeled in these initial derivations.

The shortcoming of the above 2-D equation is later overcome by Bhalla and Soh (2004a, b) who introduced the concept of “effective impedance”. It was suggested that the actual interaction between the patch and the structure is not restricted at the end points but extended all over the finite size of the PZT patch. They introduced the concept of effective velocity rather than drive point velocity, thus ensuring the force transmission between the PZT patch and the structure occurs along the entire boundary of the patch. The effective displacement is also redefined as change in area divided by undeformed perimeter length. This approach overcomes the problem of unsolvable unknowns as there is only one complex term, effective host structural impedance $Z_{s,eff}$ involved in the equation:

$$\bar{Y} = 4\omega j \frac{l_a^2}{h_a} \left[\frac{\bar{\epsilon}_{33}^T}{\epsilon_{33}^T} - \frac{2d_{31}^2 \bar{Y}^E}{(1-\nu_a)} + \frac{2d_{31}^2 \bar{Y}^E}{(1-\nu_a)} \left(\frac{Z_{a,eff}}{Z_{s,eff} + Z_{a,eff}} \right) \left(\frac{\tan \kappa l_a}{\kappa l_a} \right) \right] \quad (2.13)$$

where $Z_{a,eff}$ is the effective actuator's impedance. Bhalla (2004) showed that the effective impedance based model produces higher accuracy than the 1-D model.

Zagrai and Giurgiutiu (2001) developed an analytical model for 2-D structural thin plate. Good matching was found between the axial and flexural components when compared with the experimental results. However, they found that the model is only applicable to the axis-symmetric mode. Slight misalignment from the centre will generate additional local peaks.

Annamdas and Soh (2007) presented a semi-analytical 3-D PZT–structure interaction model incorporating the adhesive. In their model, the effect of the bonding layer is incorporated collectively in the impedance of the structure.

2.7.2 Numerical Models

Despite the ability of analytical models to simulate the PZT-structure interaction, the solution is limited to simple structures with simple geometries and boundary conditions. When the structure to be studied is relatively complex or with complicated boundary conditions, or when the targeted model involves a system of structures interacting with each other, analytical modeling is usually impossible.

Numerical models turn up to be a viable option, which often provide close enough approximation to the exact solution, satisfactory for engineering applications. Recent developments of various finite element method (FEM) based software package and advancement in computer hardware render the numerical modeling technique more attractive than ever. Lalande (1995) summarized three approaches of FEM based modeling of the dynamic PZT-structure interaction, namely direct formulation of element for specific applications, thermo-elastic analogy and use of commercial FEM codes. All these models incorporate the piezo-transducer in the simulation.

Fairweather (1998) proposed a semi-analytical modeling approach incorporating the FE model and the impedance based model, commonly known as finite element analysis (FEA)-based impedance model. This model enables the modeling of PZT-structure interaction without the presence of PZT patch in the model, as it has been simplified and represented by a force or moment. This approach retains the simplicity of the impedance based model while utilizes the strength of FEM including the ability in modeling generic distributed structures possessing anisotropic material, mass loading and non-uniform boundary conditions.

Bhalla (2001) and Lim (2004) extended the FEA-based impedance model to the frequency range commonly used in the EMI technique. Bhalla (2001) attempted the use of FEM to simulate the damage in concrete by assuming the experimental stress-strain curve to follow the Drucker-Prager plasticity model.

Despite the advantages discussed above, the FEM possesses some inherent shortcomings such as its solution removes the physical insight that can be obtained with analytical solution process. Besides, a new solution is always required for each adjustment made, causes the process of analysis time consuming and expensive.

Yang et al. (2008b) studied the ability of coupled-field FE model in simulating the PZT-structure interaction. PZT patch surface-bonded onto aluminium structures of various shapes were simulated and compared with the experimental counterparts. The overall outcome of the numerical simulation showed excellent agreement with the experimental tests up to a frequency as high as 1000 kHz.

Liu and Giurgiutiu (2007) compared the real part of the impedance from both the FEA-based impedance model (non-coupled) and coupled field model of a 1-D narrow beam structure to those of the experimental tests. The coupled field model exhibited closer agreement to the experimental results.

Advantages of FE simulation using coupled elements include: results in terms of electrical admittance can be readily obtained and compared with the experimental counterparts, higher accuracy can be achieved as the entire PZT patch can be simulated instead of being replaced by a force or moment, local modes omitted in the analytical model can be excited in the coupled field model, and the bonding layer and shear lag effect can be physically simulated.

2.8 PRACTICAL CONSIDERATIONS RELATED TO APPLICATION OF EMI TECHNIQUE IN SHM

Despite plenty of research works conducted on realizing the application of EMI technique, real-life and large scale commercial application remain at its embryonic stage. Various practical issues and constraints have yet to be fully overcome. For instance, wiring problem, energy requirement for wireless application, appropriate protection for transducer, reliability of damage identification and quantification, installation procedures, placement of transducers, economy of application, monitoring, analysis and maintenance guidelines, instrumentations, etc. are pending more studies and standardization. Some of them are discussed in the following sections.

2.8.1 Frequency Range Selection

One of the unique characteristics of the EMI technique is its high frequency of

excitation which ensures high sensitivity in sensing. This enables the detection of incipient crack invisible to the naked eyes (Bhalla, 2001, Lim and Soh, 2010).

According to the recommendation by Sun et al. (1995), in the application of the EMI technique, major vibration modes of structure should be included and the frequency range with high mode density (large number of resonance peaks) is preferred. Park et al. (2003b) additionally recommended that the frequency range should be kept within 30 kHz to 400 kHz to ensure high sensitivity to incipient damage. They further recommended that frequency larger than 500 kHz is unfavorable because the sensing region becomes extremely localized and the lateral modes of PZT patch shows adverse sensitivity to their bonding conditions. At extremely high frequency, the sensing range becomes very limited and the resonance of the PZT patch dominates that of the structure, thus rendering the outcome of sensing ambiguous and inefficient.

The PZT peaks below 300 kHz are mainly anti-resonances of in plane modes (Giurgiutiu, 2007) with the fundamental modes being the strongest. The peaks diminish with increase in frequency due to higher amount of energy required for excitation. The fundamental resonance of the thickness mode occurs at about 11 MHz .

Some researchers (Park et al., 2005) presented successful SHM application by adopting the thickness modes of PZT transducer in the MHz range. They showed that for frequency range higher than 1 MHz , the thickness mode-vibration of PZT patch is sensitive (in terms of resonant frequency shift) in identifying localities of incipient and small damages. They found that the thickness mode can work as a supplement to conventional lateral modes ($> 20\text{ kHz}$) for damage localization.

Yang et al. (2008a) discovered that frequency above 200 kHz is prone to contamination by changes in ambient temperature due to softening of the bonding layer especially when the bonding layer is thick, caused by leftward shift of dominating PZT patch's resonance. Peairs et al. (2007) presented similar results, concluding that PZT patch's resonances will diminish the damage detection capability of EMI technique. They recommended that the optimal frequency should be those slightly higher than the unbonded PZT resonances.

Baptista and Filho (2010) proposed a formal procedure to determine the damage sensitive frequency ranges through a modified equivalent electromechanical circuit. They recommended that minimizing the ratio of mechanical impedance of host structure to mechanical impedance of PZT transducer would ensure good sensitivity of the transducer. The metric indices are higher at frequency around point of maximum sensitivity of the transducer, which is preferred for damage detection.

2.8.2 Excitation Voltage and Signature Acquisition

The PZT patch is normally excited by an impedance analyzer with alternating voltage signal of 1 volt r.m.s. (root mean square) over the user specified frequency range. The magnitude and phase of the corresponding steady state, modulated current are directly recorded in the form of complex admittance (conductance and susceptance) signatures in the frequency domain, thereby eliminating the requirements of intensive domain transforms (Bhalla, 2004).

Sun et al. (1995) reported that the conductance signatures remained practically constant when the excitation voltage was increased from 0.5V to 15V, and concluded that the excitation level does not affect the signature. They further suggested that higher excitation voltage could improve the signal to noise ratio, which could correspondingly improve the ability to identify weak modes.

2.8.3 Sensing Range and Optimal Placement of PZT Patch

High frequency of excitation induced by the PZT patch renders the actuation and sensing zone on the host structure to be localized. Esteban (1996) carried out extensive numerical modeling based on wave propagation theory, as well as conducted comprehensive parametric studies to identify the sensing zone of the piezo-impedance transducers. He found that at such high frequencies, exact quantification of energy dissipation was very difficult and hence the sensing zone could not be exactly identified. However, it was found that this zone depends on the material of the host structure, its geometry, the frequency of excitation and the

presence of structural discontinuities. The structural discontinuities acting as the sources of multiple reflections cause maximum attenuation to the propagating waves.

Based on the experimental data from a large number of case studies, Park et al. (2000) claimed that the sensing radius of a typical PZT patch might vary anywhere from 0.4m on composite reinforced structures to about 2m on simple metal beams. The region is also affected by the structure's geometry, the frequency of excitation and the presence of structural discontinuities.

Hu and Yang (2007) investigated the PZT sensing region based on the elasticity solution of PZT generated wave propagation and PZT–structure interaction effect. The material and structural damping were considered. They found that with an excitation voltage of 1V applied in the typical excitation frequency range of 100–200 kHz for the EMI technique, the valid sensing region of PZT sensors is about 2–2.5 m, subjected to a sensing limit of PZT transducer, i.e. 0.01 V. The sensing zone could be extended with the use of higher excitation voltage.

In actual application, spacing between the PZT patches should be reasonably chosen for optimal sensing. Soh et al. (2000) suggested that the PZT patches should be placed at critical location such as those susceptible to shear crack and bending failure. The number of PZT patches required in monitoring the entire structure can be optimized if they are located wisely.

2.8.4 Long Term Performance of PZT Patch

Consistency and reliability of the EMI technique under various environmental conditions are two critical issues in actual application. Repeatability of signatures acquired from the PZT patch acquired from an intact host should be reasonably high throughout the period of monitoring to prevent false alarm. The technique should also be functional and sustainable under various environmental conditions.

This is especially important in the field of civil engineering where the patches (surface-bonded on structure) are exposed to dirt, fluctuations of humidity and temperature, frost, and other severe conditions. Durability of the patch in terms of perseverance of the piezoelectric properties and resistance to wear and tear should be

satisfactory. Investigation into appropriate protection for surface-bonded PZT patch is also crucial. Embedding the PZT patch (Annamdas, 2007) may be a viable alternative to protect the patch. However, the vulnerable ceramic based PZT patch could easily fail during the process of concrete casting. Embedment also renders replacement and maintenance of the transducer impossible. Embedded PZT transducer must be properly isolated, using inert materials, to make it chemically stable (Paget et al., 2002). They should also be able to withstand curing pressures and temperatures of the host material.

Giurgiutiu (2007) reported that the resonance frequencies of admittance signatures acquired from 25 identical PWAS specimens (7mm square, 0.2mm thick) were relatively consistent with less than 1.5% variations. The amplitude, however showed wider dispersion of 21%.

Giurgiutiu et al. (2004) investigated the effect of cyclic temperature change (oven test), climatic factors (outdoors tests) and operational fluids (immersion tests) on the durability of EMI technique. No significant changes were reported on the impedance spectrum. Yang et al. (2008a) studied the long term repeatability of admittance signatures from PZT patches exposed to different environmental conditions. Repeatability of admittance signatures is high up to a period of one and a half year for specimen placed in room condition. A layer of silicone gel applied over the surface-bonded PZT patch could serve as an excellent protection for both the wire connection and the patch itself.

2.8.5 Piezoelectric Sensor Diagnosis

Park et al. (2006a, b) introduced a piezoelectric sensor diagnostic procedure by tracking the imaginary parts of the measured electrical admittance. They confirmed that the bonding layer between the PZT sensor and the host structure significantly influences the measured electrical admittance. Park et al. (2007) continued to develop a novel electro-mechanical impedance model that can be utilized for both functions of SHM and sensor diagnostics. The authors established a rigorous impedance model incorporating the sensor quality index of PZT and coupling degradation effects

between PZT and bonding layer/host structure. It was confirmed that the degradation of the sensor quality and/or bonding layer can be identified by monitoring the slope of the imaginary part of the electrical admittance. The degradation of sensor quality would cause a downward shift in the slope of the imaginary part of the electrical admittance. Contrary to the sensor degradation, the bonding defects would cause an upward shift in the slope of the imaginary part of the admittance.

2.8.6 Effect of Temperature

The electrical admittance signatures acquired from the impedance analyzer are temperature sensitive (Sun et al., 1995; Park et al., 1999). The undesirable effects caused by temperature on the acquired signatures are the horizontal shift, due to changes in the host material's stiffness, and the vertical shift due to variations in the parameters ϵ_{33} and d_{31} of the PZT patch (Bhalla, 2004).

Bhalla (2001) conducted several numerical simulations using FEM to study the effect of temperature on the admittance signature caused by changes in the PZT patch's or host structural properties. Influence of temperature on bonding layer, actuation capability and physical properties of PZT patch were investigated by Nguyen et al. (2004) and Schulz et al. (2003). All researchers concluded that the overall shift in the frequency spectrum varies linearly with temperature over narrow frequency bands. The most critical vertical shift was caused by the changes of ϵ_{33} .

In real situation, it is essential to differentiate the effects between damage and temperature, as the actual working condition is generally subjected to considerable temperature fluctuations. This necessitates the development of a compensation algorithm to decouple the two. Sun et al. (1995) observed that the overall effect of temperature has distinct difference from the abrupt and localized variation in signature caused by damages. They also suggested a compensation technique based on cross-correlation coefficient to compensate for temperature induced horizontal shift. Park et al. (1999) later proposed an empirical based method for temperature compensation, proving the damage detection capability of the EMI technique under fluctuating temperature.

2.8.7 Effect of Bonding Film

For a surface-bonded PZT patch, efficiency of strain transfer between the PZT patch and the host structure is closely related to the adhesive layer, mechanically joining the two. The performance and behavior of the bonding layer is crucial for an effective strain transfer and thus efficient actuation and sensing.

Bhalla (2004) suggested that the shear lag effect caused by the adhesive on the PZT patch is negligible if the bonding layer is less than one third of the patch's thickness. Impedance based modeling of the EMI technique proposed by Liang et al. (1994) neglected the bonding layer. Xu and Liu (2003) extended the abovementioned model by assuming the bonding layer as a single degree of freedom (SDOF) spring-mass-damper system placed in between the PZT patch and the structure.

In actual application, the effect of bonding layer turns up to be more complicated when external factors such as temperature changes are involved. Nguyen et al. (2004) studied the actuation efficiency of the PZT patch under varying environmental temperature and adhesive layers. They found that the quality of bonding is affected by the effective working condition such as the ambient temperature and type of bonding. Alfredo (2003) investigated the impact of finite stiffness bonding on the sensing effectiveness of piezoelectric patch. He suggested that the shear lag effect must be carefully assessed in order to enhance the quality of the sensing output. Ong (2003) studied the effects of adhesive on the electro-mechanical response of a piezoceramic transducer coupled smart system.

Yang et al. (2008a and 2008b) studied the effect of bonding with varying temperature on the admittance signatures through experiment and FE simulation. They discovered that the detrimental effect of bonding thickness is less significant for frequency range below 200 kHz. Most of the frequency of structural resonance peaks remained unaltered with thicker bonding, with slight reduction in amplitude. This implies that the structural modes could still be excited with relatively thick bonding but at lower efficiency due to shear lag effect. Above 200 kHz, the PZT resonance will dominate the structural resonance if the bonding thickness exceed one-third of the PZT patch's thickness. They recommended the frequency range to be limited within

100 kHz at elevated temperature because the rise in temperature would soften the bonding layer, magnifying the contamination caused by the PZT's resonance.

2.8.8 Effect of Loading and Boundary Condition

In practice, structures in service are constantly subjected to loading. Even with the structure in standby mode, static load caused by self weight or dead load would normally exist. Annamdas et al. (2007) showed that the electrical admittance signatures, especially its imaginary component is susceptible to variation in the applied load on the host structure.

Abe et al. (2000) first proposed the use of the EMI technique for the identification of in-situ stress in thin structural members. They showed that the tensile stress increased the resonance frequency in the electrical impedance spectrum. Generally, tensile stress tends to stiffen a structure whereas compressive stress would soften it.

Ong et al. (2002) investigated the effects of stress on the frequency response functions of beam and plate structures by applying pure bending actuation through a pair of symmetrically located PZT transducers, activated in out-of phase mode.

Esteban (1996) reported that the advantage of having localized sensing region is that the transducer is less sensitive to boundary condition changes, which usually affect lower order global modes.

Annamdas and Soh (2010) reported that if the host structure dimensions are smaller than the sensing range of the PZT transducer bonded on it, the boundary conditions can influence the PZT-based signatures. The boundary conditions in real life structures are however, extremely hard to characterize analytically (David, 2006).

2.8.9 Instrumentation, Wireless Power, Energy Harvesting and Other Considerations

One existing shortcoming of the EMI technique in real-life application is the use of bulky and heavy impedance analyzer. This reduces the agility and mobility of the entire monitoring system. Besides being not portable, the impedance analyzer is quite costly as they possess extra functions that are redundant in the application of EMI

technique. Peairs et al. (2004a) developed a miniaturized, portable, operational amplifier-based turnkey device capable of measuring electrical impedance of PZT patch with accuracy comparable to the conventional impedance analyzer.

For real life monitoring, a large number of PZT transducers are expected to be placed at various major structural elements. Corresponding large numbers of wires could pose inconveniences, for instance, unduly long monitoring time, enormous number of data for analysis, difficulty in handling the wires during installations, etc. Multiplexing of the PZT patches could be a solution to the problem. Some successful applications on multiplexing had been attempted by Bhalla (2004) and Madhav and Soh (2008). However, the technique is relatively immature when compared to the well established multiplexing of fiber optic strain sensor.

Another way of overcoming the hassle of wires is to employ the wireless technology. However, active sensing device such as piezo-transducer would normally require relatively high electrical power. Battery replacement of remotely placed sensors could be extremely tedious if not impossible. A sustainable power supply is thus anticipated. Researchers in the field of SHM and sensing network have shown interest in power harvesting as an alternative power sources, which include sunlight, thermal gradient, human motion, wind and vibration (Yun and Min, 2010).

In lieu of energy harvesting, some researchers attempted to minimize the battery power consumption for wireless smart sensors. Rice et al. (2010) proposed a flexible framework for autonomous full-scale SHM of large civil structures to minimize the power consumption in a large-scale wireless sensor network. All sensors are set in the sleep mode by default and activated periodically for data acquisition.

The data processing and compression has been studied to reduce the power consumption caused by wireless transmission. Park et al. (2008a) applied the PCA-data compression technique as a pre-processing module to reduce the data dimensionality and to eliminate the unwanted noises. Only the most significant principal components were obtained from the raw impedances. The effectiveness of the compression technique for EMI based wireless SHM was verified through an experimental study inspecting loose bolts aluminium structure. They concluded that

the PCA-data compression provides noise elimination effects and enhances the damage detection capability of the traditional approach such as the RMSD.

Alternative power supply is another possibility such as through Radio Frequency (RF) microwave transmission technology to wirelessly transmit the power to an active sensor node. The RF microwaves are transmitted through atmosphere to a receiver to be converted into DC power (Mascarenas et al. 2007).

A self-contained wireless sensor incorporating various functions including on-board actuating/sensing, power generation, on-board data processing/damage diagnostic and RF module is continuously being pursued by researchers such as Inman and Grisso (2006), Mascarenas et al. (2006) and Park et al. (2006c). Overly et al. (2008) developed a compact, self-contained wireless active-sensor node (WID2.0) for various SHM applications. The low-cost, low power consumption sensor node integrates several components, including local computing, telemetry for wirelessly transmitting data, multiplexing, energy harvesting and storage mediums. The inclusion of a wireless telemetry solution allows the sensor node to be placed in remote and inaccessible locations. They compared the measurements from both conventional impedance analyzer and WID2.0 in monitoring corrosion and bolt loosening. Both devices produced the same shape impedance response, with only slight difference in magnitude, as a result of differences in A/D conversion.

In terms of software, one which is able to control the impedance analyzer for actuating the PZT transducers and acquiring data by the personal computer is essential. The basic software should be written in a way that enables real-time, autonomous data acquisition. Further, advanced software should include autonomous temperature and humidity compensation algorithm and be able to alert the inspector when significant deviation from the healthy state signatures is detected.

Another limitation of the EMI technique is its inability to identify the failure modes or nature of damage occurring in the host structure. Also, the exact quantification of damage is very difficult unless appropriate damage model is constructed. The extent of damage can only be qualitatively recognized through changes in the admittance signatures' spectrum.

2.9 INTEGRATED USE OF SMART MATERIALS IN SHM

Smart materials often possess unique characteristics or behavior. However, they carry inherent limitations, which are sometimes insurmountable. For instance, optical fiber can only act as a passive sensor, i.e. it cannot produce actuation. SMA on the other hand, can only actuate and piezoceramics essentially provides non-parametric measurements. These limitations could hinder the development of an efficient smart structures and systems. Theoretically, if the strengths of each smart material are utilized to complement the shortcomings of the others, a more robust intelligent system can be developed

In the field of SHM, Peairs et al. (2004b) investigated the feasibility of a self-sensing and self-repairing bolted joints. PZT transducer was used for sensing of loosened bolt while SMA was used as actuator to restore the tension of the loose bolt.

Various studies have proven the ability of the FBG sensor in measuring strain of attached structure. On the other hand, piezo-transducer is also found to be useful in damage detection. Incorporation of both smart materials is expected to complement the shortcomings of each other and thus forming a more comprehensive structural health monitoring system. Lim (2006) and Yang et al. (2008c) investigated the feasibility of damage detection and quantification in rock specimens using both FBG based strain sensor and EMI technique. Both techniques were found to be well correlated and the corresponding results also complemented each others, providing more information on the nature of damage and its severity. Yang et al. (2009) presented an experimental investigation on the sensing capability of FBG and PZT transducer for load monitoring. The PZT transducer was able to monitor load increment through RMSD index as well as reflecting the damage on host structure.

2.10 DAMAGE PROGNOSIS

2.10.1 Concept

A SHM technology without any comprehensive process of damage prognosis (DP) shall not be considered as a complete SHM system. Conventional concept of SHM

involves the process of detecting and characterizing damage induced in a structure or structural component through some NDE techniques. These techniques however, generally give no indication on the remaining useful life of the structure under current health condition. In other words, with current monitoring technique, it is usually unsure about the necessity of repair or replacement of the monitored structure. This often results in unnecessarily early maintenance or components replacement, which is deemed uneconomical as the structure concerned may still be able to perform for a certain period of time. The potentially huge cost savings of DP has ushered the exploration and development of this area.

The development of DP capability requires multidisciplinary approaches and is therefore labeled as of “grand challenge” in nature (Farrar et al., 2005). At current stage, research and development of a comprehensive DP is still in its embryonic stage. Before venturing deeper into the concept and application of DP, it is essential to understand several terms related to this technology.

2.10.2 Definition of Damage

Damage can be viewed as unacceptable and irreversible changes that occur in a material or a structure under loading. In structural or mechanical system, damage can be defined as a change in material or geometrical properties of the system, including changes in boundary conditions and system connectivity which could adversely affect the current or future performance of that system (Farrar et al., 2005).

Damage occurs in virtually all length scales. In terms of length scale, it usually initiates at material level and propagates to component and system level. In terms of time scale, damage may accumulate over long periods of time such as fatigue and corrosion induced damage or at much shorter time such as those attributed to aircraft landings and accidental overloading. Evolution of damage is strongly dependent on the imposed loading as well as the material micro-structure.

Signs of damage can typically be reflected in certain quantifiable form such as reduction in strength, stiffness or toughness. Damage results in nonlinear, irreversible or dissipative deformation, which is history-dependent.

2.10.3 Usage Monitoring and Structural Health Monitoring in DP

Usage monitoring and SHM are also pre-requisite to a successful application of DP. As described in the previous section, SHM is a damage detection or diagnosis process. Usage monitoring is a process of monitoring the structural input (loading) and environmental (operational) condition as well as its response during service.

DP is defined as the estimate of an engineered system's remaining useful life (Farrar and Lieven, 2007). The estimation is based on the output of a predictive model, which couples the information from usage monitoring, SHM, past, current and anticipated future environmental and operational conditions, the original design assumptions regarding loading and operational environments, and previous component and system level testing (Farrar et al., 2005). "Softer" information such as the "feeling" of user towards the system's response should also be considered and utilized when developing the DP solutions. In simple terms, DP attempts to forecast the system's performance by measuring the current state, estimating the future loading environment as well as predicting through simulation and past experience.

Three main technology components for DP application include instrumentation, data processing hardware and comprehensive predictive modeling. Each of them is problem specific. Disciplines required to address DP include, engineering mechanics, reliability engineering, electrical engineering, computer science, information science, material science, statistics and mathematics. Solving of this problem will lead to significant economical and social impact.

2.10.4 Damage Prognosis Solution Process

It is important to classify the damage prognosis problem before arriving at different damage prognosis solutions. A summary of general DP solution procedures is summarized and depicted in Figure 2.7.

We shall first understand the sources or causes of damage concerned and their corresponding potential failure modes. There are generally three categories namely, gradual wear, predictable discrete events and unpredictable discrete events.

Figure 2.7: General procedures for damage prognosis solution (Farrar et al., 2005).

Gradual wear is the slow accumulation of damage at the material and component level of microscopic scale such as fatigue induced and corrosion induced cracking. For predictable discrete events, damage initiates at microscopic scale, but accumulates at a much faster rate caused by predictable, sudden and discrete event. Examples include aircraft landings and explosions in confinement vessels. Unpredictable discrete events are usually caused by unknown, discrete and usually severe loading at unpredictable times such as foreign object induced fan blade-off in turbine engine, earthquake induced damage in buildings and battle damage in military hardware.

The next step would be to determine the techniques to be used in damage assessment. Depending on the nature of damage and importance of structure, the

assessment may be done online, near real-time or offline at discrete intervals. Various SHM techniques outlined in previous sections could be used.

Then, one is required to collect as much initial information as possible for the development of physics based model, definition of state awareness assessment, establishment of sensors for operational and environmental conditions monitoring. The physics based model can be useful in defining the sensors properties such as number of sensors required, locations, bandwidth, etc. Data collected from the sensors together with those from physics based model can be used to assess the current state of the structure. Updating of the physics based model is also possible with the availability of data with time. Data from operational and environmental measurements are also essential to predict the future system loading and the environmental effect.

With the output from physics based model, state awareness model and future loading model, estimation of remaining useful life of system (prognosis model) is possible. A reliability-based predictive tool can also be incorporated to estimate in terms of probability the remaining useful life or the probability of completing certain mission or job. The DP process is an iterative process, which is improvable with experience and accumulation of data.

For DP using PZT transducer as diagnostic sensors, plenty of works are yet to be conducted. Chang et al. (2007) developed a piezo based SHM system to monitor the damage initiation based on acoustic signals and to detect its growth based on ultrasonic waves propagation generated by the piezoelectric sensors. In their study, they found that it is possible to integrate the built-in diagnostic system with a progressive failure modeling for monitoring and prediction of composite structures in service, from damage initiation to final failure.

In this research, the LEFM theory and FE simulation are used as the physics based model, the EMI technique serves as the state awareness measurement system. The loading, on the other hand, is a controlled variable. All these are combined to form a semi-analytical damage prognosis model for estimation of remaining life. Data from experimental test is used to strengthen the physics based model.

2.10.5 Material Modeling

Traditionally, a structure is considered failed or become useless when any macroscopic evidence of damage has been detected. The concept of “damage tolerant” structure has brought structure to another level. A damage tolerant structure is one which can survive the imposed service environment in the presence of evolving or even extensive damage without failure. More than one type of damage is usually needed to be considered. Extensive material modeling is necessary regardless of the method of manufacturing, for the design of a damage tolerant structure.

Material modeling generally requires certain constitutive models or failure criteria. A material model shall be able to establish a tractable mathematical version of the actual material behavior which could relate the applied loading state (stress/strain, temperature, rate of application, etc.) to the bulk response of the material by considering the subsequent deformation and damage mechanism over the relevant length scale.

If the dominant mechanism or interaction governing material response is not known *a priori*, the modeling process shall include the determination of the relevant mechanism. Be it qualitative or quantitative, an appropriate model is required for predicting material behavior and structural response, up to failure in order to develop an appropriate damage prognosis solution.

Separation of length scales ought to be considered in the development of material models. Typical length scales include: micro length scale, material length scale and structural length scale. A material model needs to provide at least a bridge between material and structural length scale. Feasibility of incorporating a model into relevant structural analysis shall always be investigated.

An accurate material model with strong physical basis can serve as a tool for studying the behavior of material beyond the experimental data. It can potentially be used for DP as it is able to predict the remaining life of structure through monitoring the initiation and extent of damage. Uncertainties and factor of safety can be reduced with reliable prediction on damage evolution, implying huge economic impact.

A material model can be classified as either based on a continuum or

micromechanical approach. Continuum theories are more classic approach based on the analysis of material behavior at the bulk or macroscopic length scale. This approach is often computationally efficient and is useful for examining the trends in material and structural behavior under various loading conditions. Some of the typical examples are the LEFM, Weibull model and phenomenological failure theories

As for the micromechanical (homogenization) theory, it attempts to predict bulk material behavior based on direct analysis of the material micro-structure, response of individual phases and interfacial properties. Micro-level damage initiation and evolution are directly modeled, which is often useful for heterogeneous materials. An appropriate averaging scheme is necessary to predict the bulk material response. Insight into the mechanisms governing the local and global behavior requires micromechanical theory. This theory is also less demanding on the experimental data when there are changes in material micro-structure or processing history. However, computational requirements can be much higher than the continuum theories.

It is important to determine the dominant damage mechanism as parts of the modeling process as some damage mechanism need not be included in the failure model. Various forms of damages may exist in a material simultaneously. Thus, some failure modes can be interacting in a highly complex manner.

2.10.6 Damage and Failure Modes

Structural failures often involve complicated interactions of various factors. The interaction between load, time and environment along with particular material, geometry, processing and residual stresses generates a wide range of failure possibilities in various fields of engineering. It is important to understand the potential failure modes in a structure for successful SHM and DP. A list of possible failure modes in metals as well as their brief descriptions is tabulated in Table 2.2.

Fuchs and Stephens (1980) reported that 50 to 90% of structural failures are caused by fatigue. It is thus essential to establish a systematic procedure for fatigue design, fatigue crack monitoring and fatigue life estimation. In this study, focus will be placed on monitoring of fatigue crack using the EMI technique.

Table 2.2: Mechanical failure modes in metals.

Failures Modes	Descriptions
Excess deformation	Can be elastic, yielding and onset of plasticity
Ductile fracture	Substantial plasticity and high energy absorption
Brittle fracture	Little plasticity, low energy absorption and high crack growth velocity during fracture
Impact/dynamic loading	High strain rate, excess deformation or fracture
Creep	Predominant at elevated temperature, excess permanent deformation or fracture
Relaxation	Predominant at elevated temperature, loss of residual stress or external loading
Wear	Crack nucleation and crack growth with plenty of possible failure mechanism
Buckling	Induced by external loading or thermal condition, may involve elastic or plastic instability
Corrosion	Pitting and crack nucleation. Stress corrosion cracking or environmental assisted cracking
Fatigue	Cyclic loading. Contributes to at least 50% of mechanical failures.

2.11 FATIGUE

2.11.1 Background

A comprehensive study by Stephens et al. (2001) indicated that the cost of fracture failure in United States is about USD119 billion in 1978 or about 4% of the gross national product. The report emphasized that the cost could be significantly reduced if proper fatigue design with state-of-the-art technology imposed. Fatigue design can sometimes be extremely complicated and expensive but could still benefit in terms of financial saving in the long run. Proper fatigue design involved not only simple model and testing but requires synthesis, analysis and testing which should

include service loading condition, ambient temperature, corrosion, residual stress and variable amplitude loading. An appropriate safety factor is always in need to ensure safety while maintaining its market's competitiveness. However, the factor of safety shall not be viewed as a replacement of proper fatigue design.

Fatigue failures, whether thermal or mechanical occurs in almost every field of engineering, from electric circuit board in electrical engineering, bridges in civil engineering, automobiles in engineering, farm tractor in agricultural engineering, aircraft in aeronautical engineering, heart valve implants in biomedical engineering, pressure vessel in chemical engineering to nuclear piping in nuclear engineering.

2.11.2 Historical Overview

The term fatigue was first discovered in print in 1854, used by Braithwaite (Pook, 2007). A general opinion had been developed that material will become “tired” of carrying the load.

August Wohler conducted many laboratory fatigue tests during 1850s and 1860s concerning railway axle failures in Germany, which were considered the first systematic investigation of failure. He introduced the concept of S-N curve, fatigue limit and importance of stress range over maximum stress. In 1870s, Gerber investigated the influence of mean stress while Goodman later proposed a simplified theory regarding mean stress.

In early 1900s, optical microscope was used to study the micro mechanics of fatigue by Ewing and Humfrey where localized slip lines and slip bands leading to the formation of micro-cracks were observed. Basquin in 1910 established the log-log linear relationship of alternating stress against number of cycles in finite life region of the S-N curve.

In 1920, Griffith pioneered the concept of fracture mechanics by showing that the strength of glass was dependent on the size of crack. In 1945, Miner formulated a linear cumulative fatigue damage criterion and was later termed the Palmgren-Miner linear damage rule. The American Society for Testing and Materials (ASTM) on fatigue was formed in 1946, thus standardizing tests and design for fatigue.

The advent of electron microscope in the 1950s opened up the path to better understanding of basic fatigue mechanism. Irwin introduced the concept of stress intensity factor which is the basis of LEFM for fatigue crack growth.

In early 1960s, low-cycle strain controlled fatigue behavior was studied by Manson and Coffin, which established the Manson-Coffin relationship between plastic strain amplitude and fatigue life. Paris at the same period of time developed the relationship between fatigue crack growth rate and stress intensity factor. He also showed in 1970 that a threshold stress intensity factor range existed, below which fatigue crack will not be initiated. Elber demonstrated the importance of crack closure in fatigue crack growth and developed the concept of effective stress intensity factor.

In 1980s and 1990s, researchers investigated more complex multiaxial fatigue cases. The advent of computer technology had also brought significant changes in fatigue design which includes various fatigue life models and ability to simulate real loadings under variable amplitude loadings.

2.11.3 Fundamentals of Fatigue Crack

Structures in service are often subjected to repeated fluctuating stress of magnitude lower than its strength limit. However, after enduring a large number of loading cycles, the structures or some components would start to develop progressive localized damage. This phenomenon is often known as fatigue crack and the cyclic load applied is known as fatigue load.

The number of loading cycles a structural member or component could sustain before fracture occurs is referred to as its fatigue life. The exact number of cycles, in turns, is dependent on several factors including the nature of load, the load displacement curve, the frequency of repetition, load history, the size of member, the flaw that initially present, operation temperature and environmental conditions. Therefore in practice, it is difficult to accurately estimate the fatigue life of a member. Designer usually relies on either full scale test or more often, laboratory test.

The total fatigue life of a member is usually consisted of three phases (Boresi and Schmidt, 2003) namely, initial fatigue damage that produces crack initiation,

propagation of crack that lead to partial separation of a cross section and final fracture of member.

Crack initiation involves dislocation movements, formation of slip bands, extrusion and intrusion of slip bands, micro-crack nucleation, growth and coalescence as well as the initiation of macroscopic crack (Sevostianov et al., 2010). Macroscopic crack will propagate steadily under same cyclic load until a critical crack length is reached. Upon reaching the critical crack length, the crack becomes unstable (propagation rate is high) and failure will be fast approaching.

Fracture resulting from fatigue load on ductile materials can be ductile or brittle, depending on the stress and number of cycles applied. Low cycle fatigue life usually presents in those members subjected to high stress in which large plastic strain occurs. This type of fatigue failure is labeled as failure in ductile state. Brittle state failure, on the other hand, occurs in fatigue failure with small plastic strain (small stress) but very high cycles, above 10^6 cycles.

In general only one crack develops to any considerable depth, and this crack propagates across the material until the section is so reduced that static failure takes place (Pook, 2007). Up to now, fatigue propagation path is not completely understood and is difficult to be predicted which often requires structural test.

Further descriptions on approaches for fatigue life estimation namely the stress life, strain life and LEFM is presented in Chapter 5.

CHAPTER 3

DETECTION AND CHARACTERIZATION OF FATIGUE INDUCED DAMAGE USING EMI TECHNIQUE

3.1 INTRODUCTION

Fatigue induced damage is often progressive and gradual in nature. Structures subjected to large number of fatigue load cycles will progressively encounter the process of crack initiation, crack propagation and finally fracture. Fatigue load are generally lower than the static strength of the structure.

In the conventional structural design, conservative estimation of fatigue life is adopted to ensure safety especially in the aerospace and aircraft industry. However, it is often noticed that the structure or structural member remains functional despite the initial design life been reached.

In practice, it is essential to develop a technique capable of detecting crack at the earliest possible stage. In terms of metal fatigue, the NDE is expected to detect the presence of crack as well as to measure its size. Some of the commonly used NDE techniques, and their pros and cons have been described in Chapter 2. Their major drawbacks include location of crack has to be known *in priori*, impossible to be automated, cost and labor intensive.

The emergence of smart material based sensing such as the EMI technique can possibly provide an alternative, overcoming some of current NDEs' drawbacks. The EMI technique, employing piezo impedance transducer as collocated actuator and sensor is potentially applicable in this aspect with its widely known capability to detect and characterize damage. It offers advantages such as autonomous, real-time and remote monitoring. The installation of piezo-impedance transducers is non-intrusive to the structure, and the transducers are capable of detecting various

damages, including incipient fatigue cracks (Lim and Soh 2010).

The following sections start with presenting some literature review on topics related to fatigue crack monitoring using piezo-impedance transducer. Experimental studies on fatigue crack detection and characterization using the EMI technique are then presented. Several damage indices applied for damage characterization are also.

3.2 SURVIVABILITY OF PZT TRANSDUCER UNDER CYCLIC LOADING

Structures subjected to fatigue loading could be stressed up to tens of millions of cycles before failure is reached. To monitor fatigue cracks using piezo-impedance transducers affixed on the structure, the PZT transducer/patch shall possess sufficient durability and survivability under the cyclic stress to avoid premature failure.

Giurgiutiu et al. (2004) investigated the effect of cyclic temperature change (oven test), climatic factors (outdoor tests) and operational fluids (immersion tests) on the durability of the EMI technique. No significant changes were reported on the impedance spectrum. They also studied the effect of strain on impedance signatures acquired from PZT patch. For a PZT patch surface bonded on aluminium-beam, which failed at about 7200 micro-strain, the impedance signatures were minimally affected when the strain does not exceed 3000 micro-strain. Above 6000 micro-strain, the effect of strain on the signatures was significant. They also studied the durability of surface-bonded PZT patch when the structure was subjected to fatigue loading of varying magnitude. It was found that the PZT patch survived and remained functional after all tests, despite the failure of the specimen itself (Giurgiutiu, 2007).

Thielicke et al. (2003) studied the reliability of piezoceramic sensors under cyclic mechanical loading. Two types of PZT sensors, namely piezoceramic foils and fibers were bonded on carbon fiber reinforced plastic, which were subjected to four point bending cyclic loading with different strain levels. All specimens tested were found to be able to survive 10^7 cycles with 0.12% strain without remarkable loss of performance.

Hoon et al. (2004) concluded that the performance of piezoelectric elements was not affected by the fatigue loading imposed throughout a fatigue test, until the failure of host structure. The patch was not de-bonded at the end of the test.

Under normal working condition, the consistency of piezoelectric properties is relatively high. Aging rates of some typical piezoceramics are summarized in Table 3.1. It is apparent that the aging rate of the key piezoceramics properties are generally lower than 5% per decade (PI Ceramic, 2010). It is also reported that the lifespan of a piezoelectric material is not limited by wear and tear as they are specifically designed for high duty cycle applications. Endurance test conducted on piezoelectric actuator showed that the material performs consistently even after several billion cycles. Therefore, it could be inferred that the deterioration of PZT patch itself is not a major concern in monitoring fatigue damage in host structure.

Table 3.1: Aging stability (relative change of the parameters in % per decade) of typical piezoceramics (PI Ceramic, 2010).

Types of PZT	PIC 110	PIC 140	PIC 181	PIC 151	PIC 155	PIC 255
Aging rate of relative dielectric constant, C_k	-8.5	-1.0	-1.0	-2.0	-2.0	-1.0
Aging rate of coupling factor, C_e	-5.0	-4.0	-0.25	-4.0	-3.0	-2.5

Yang et al. (2008a) studied the repeatability of electrical admittance signatures acquired from PZT patches surface-bonded on aluminum structures under various environmental conditions. They reported that the repeatability was excellent up to a period of one and a half years.

3.3 FATIGUE CRACK MONITORING USING PIEZO-IMPEDANCE TRANSDUCERS

Giurgiutiu et al. (1999) performed incremental damage tests of spot-welded lap-joint shear specimen under fatigue loading. The RMSD index of impedance

signatures in the frequency range of 200-1100 kHz was correlated with the stiffness loss, which in turns quantify the structural damage. The damage index values increased in conjunction to increase in crack length. The stiffness-damage correlation principle was used to quantify progression of damage and estimation of fatigue life.

Giurgiutiu et al. (2006) showed that the EMI technique and the Lamb-wave technique applied with piezoelectric wafer active sensor are both able to detect the presence and propagation of a crack under mixed-mode fatigue loading. They observed that some PZT patches disbonded from the specimen after the test, which could be caused by plasticity effect near the crack tip. Thus, they suggested that the placing of transducers should avoid the highly strained areas.

Sovostianov et al. (2010) conducted experimental investigation on the relationship between strength reduction caused by accumulated damage in elastic electrically conductive material, its corresponding electrical resistance across the damaged specimen and the EMI response. The structural impedance peak was found to decrease steadily in peak frequency over the number of loading cycles. After about 40,000 cycles of loading, the impedance peaks between 52 to 55 kHz were found to shift progressively to the left in comparison to its healthy state. They also presented, in brief, the possibility of using the EMI technique to detect initiating crack.

Kim (2006) demonstrated the effectiveness of EMI technique in monitoring welded structural members while in service. He also adopted one PZT transducer as sensor and another as actuator. He found that the measurement that crosses the crack path produces increase in damage metric. FBG sensor was also attached for stress measurement and this integrated technique was found to provide ideal characteristics for detecting damage on structure in service.

Ihn and Chang (2004a) employed the wave propagation technique using built-in PZT patches for damage detection. Structural damage was detected from changes in the received signal. They proved that the technique is effective in detecting debonding or delamination as well as in monitoring fatigue crack growth (Ihn and Chang 2004b).

Soh and Lim (2009) investigated the feasibility of fatigue induced damage detection and characterization using the EMI technique. Experimental study on an

aluminum beam with a pre-induced circular notch indicated that the EMI technique is excellent in detecting incipient crack through changes in admittance signature's frequency even at its crack initiation stage. Lim and Soh (2010) studied the feasibility of fatigue life estimation of an aluminum beam structure using the EMI technique incorporating the LEFM theory.

3.3.1 Fatigue Crack Monitoring of Beam Specimen with Single Edge Notch

3.3.1.1 Experimental setup and procedures

In this section, feasibility of fatigue damage detection and characterization using the EMI technique is experimentally investigated. Two sets of aluminium beams (T6061-T6) were prepared. Each sets of the specimens consisted of three aluminium beams of identical dimensions as shown in Table 3.2(a). The physical and fatigue related properties of the aluminium beams are tabulated in Table 3.2(b)

Table 3.2(a): Dimensions of aluminium beam specimens.

Label	Size
S1, S2 & S3	300mm x 50mm x 6mm
B1, B2 & B3	300mm x 75mm x 6mm

Table 3.2(b): Physical and fatigue related properties of aluminium (T6061-T6).

Parameters	Symbols	Values	Unit
Ultimate strength	σ_{ult}	352	MPa
Yield stress	σ_y	299	MPa
Critical fracture toughness (plane strain)	K_{IC}	29	MPa(m) ^{1/2}
Corrected critical fracture toughness (6mm)	K_C	61.11	MPa(m) ^{1/2}
Gradient (Paris equation)	m	4.5	--
Intercept (Paris equation)	$\log C$	-11.5	--

One piece of PZT patch dimensioned 10mm x 10mm x 0.3mm, manufactured by PI Ceramic (PI Ceramic, 2010), was surface-bonded on each of the specimens (located at one-quarter of the specimen's length from one end) using two parts high strength epoxy (RS component, 2006) as shown in Figure 3.1(a). The PZT patches used in this study were PIC 151, a modified lead zirconate titanate ceramic with high permittivity, high coupling factor and high charge constant. Figure 3.1(b) shows the configuration of a typical PZT patch used in this study. The characteristic feature of the patch is that the electrode from the bottom edge is wrapped around the thickness. As a result, both electrodes are available on the top face. For each patch, a pair of wires, each of them 1m length were soldered to each electrode and connected to the impedance analyzer. The bottom side could be bonded to the host structure.

The adhesive was allowed to cure for at least 24 hours to attain maximum bonding strength. A layer of silicone rubber (Hi-Bond, 2006) was applied above the PZT patch to protect it against wear and tear during specimen preparation and testing (Yang et al., 2008a).

For each of the specimens, an initial single edge crack (measured 4.75 mm) was created at the center of the specimen using electric discharge machining (EDM) (Figure 3.2) to serve as stress concentration. The edge notch was prepared according to British Standard (BS ISO 12108, 2002). The detailed configuration is shown in Figure 3.3. Fatigue crack is expected to initiate at the tip of the edge notch and propagate perpendicularly to the direction of loading. All specimens were successfully tested to failure, except specimen B3. Specimen B3 was abandoned because the wire soldered on one of the PZT patch's electrode jerked off, spoiling the patch. Since the results obtained from all five specimens were generally consistent and repeatable, the repair of specimen B3 was deemed unnecessary.

Figure 3.1(a): Aluminum beam specimen with single edge notch and a piece of PZT patch surface-bonded at one-quarter length from one end.

Figure 3.1(b): Schematic plot of PZT patch (PIC 151).

Figure 3.2: Electric discharge machining (EDM) in operation for creating single edge notch. Water jet applied continuously for cooling.

Figure 3.3: Schematic plot of a single edge notch located at center of aluminium beam specimen.

Uniaxial cyclic tensile load was applied to the specimen using a 25 ton dynamic testing machine (Figure 3.4a) at predetermined interval to induce mode I (opening mode) constant stress amplitude fatigue load. Nominal stress applied was controlled between 40 ~ 50% of the yield stress of the aluminium beam, which asserted a mean stress of 134.6MPa and an alternating stress of 15.0MPa. In this case, the maximum strain asserted on the structure was approximately 2200 micron, much lesser than the limiting value (3000 micron) suggest by Giurgiutiu (2007).

Frequency of cyclic loading was fixed at 30Hz at the initial stage. When a crack can be clearly identified through naked eyes, the rate was reduced to 15Hz. According to the data published by Kapp and Duquette (1986), the rate of cyclic load application between 5 ~ 30 Hz will not affect the crack growth rate of aluminum 6061-T6.

Wayne Kerr precision impedance analyzer 6420 (Figure 3.4b) was used to supply the alternating voltage and to measure the corresponding current in the application of the EMI technique. A notebook computer (Figure 3.4c) with customized software was used to record the admittance signatures. A calibration process known as zero

correction was performed on the impedance analyzer using a pair of wires in short and open circuit conditions before each test. This zero correction could eliminate the interference of test wires into the measured admittance signatures, for a more precise measurement.

A crack detector of 0.02mm resolution was used for measuring crack length as shown in Figure 3.4(d). A built in light bulb was useful in brightening up the area surrounding the crack thus assisting the detection and sizing of crack. The crack size on both the top and bottom surfaces of the specimen was recorded. Crack length was evaluated by averaging the two.

Figure 3.4: (a) 25 ton dynamic testing machine (b) Precision impedance analyzer
(c) Note book computer (d) Crack detector

Baseline admittance signatures of the PZT patch surface-bonded on the beam specimen was first recorded at healthy stage (before application of load) in the frequency range of 0 ~ 200 kHz. Cyclic tensile load was then applied at stages with predetermined number of cycles (Figure 3.5). The specimen was removed from the

machine after each stage for the acquisition of admittance signatures in free-ended condition to prevent contamination caused by the effect of clamping and loading. Initiation and propagation of crack from the edge notch was closely monitored using the crack detector.

The specimen was loaded in stages up to failure. Other specimens were similarly tested. The number of cycles at crack initiation, critical crack length and failure were recorded. Specimens S1, S2 and S3 failed at 240,000, 225,000 and 220,000 cycles respectively. The corresponding critical crack length were approximately 18mm, 17mm and 17mm for specimens S1, S2 and S3 respectively including the machined notch (Figure 3.6). All PZT patches remained functional after failure of the host structures. In this chapter, the results and discussions presented will be mainly for specimen S1, S2 and S3. Some repetitive results from the S series are attached in Appendix A. Experimental outcomes from the B series are very similar and are thus omitted to avoid a too lengthy thesis. However, some of the results from the B series are attached in Appendix B for reference.

Figure 3.5: Pictorial illustration of aluminium beam specimen cyclically loaded on dynamic testing machine.

Figure 3.6: Critical crack length (18mm) in aluminium beam specimen S1 after being subjected to 240'000 cycles of uniaxial tension (mean stress = 134.6MPa, alternating stress = 15.0MPa).

3.4 ADMITTANCE SIGNATURES OF EMI TECHNIQUE FOR DAMAGE DETECTION AND CHARACTERIZATION

In this study, conductance (real component of admittance) signatures are used as damage quantifier due to its higher sensitivity towards damage than its imaginary counterpart (Sun et al., 1995). Leftward horizontal movement of the structural resonance peak (reduction in resonance frequency) serves as a useful guideline for fatigue crack characterization (Lim and Soh, 2010). Other damage indices such as RMSD and CCDM are also adopted for damage quantification.

3.4.1 Repeatability of Admittance Signatures from EMI Technique

In order to successfully generate a health monitoring and damage prognosis solution, consistency of the admittance signatures acquired from EMI technique is essential. In this section, the repeatability of admittance signatures acquired from different specimens of identical configurations under the same health and environmental conditions is plotted in Figure 3.7.

The figure shows that the signatures exhibit reasonably high repeatability in terms of resonance frequencies. This implies that, for the specimens of same geometrical shape and boundary condition, the modal frequencies are the same, which can be reflected by the electromechanical resonances.

Figure 3.7: Comparison of conductance signatures versus frequency (40 ~ 50 kHz) acquired from PZT patch surface-bonded on free-ended beam specimens S1, S2 and S3 at their healthy state (without pre-induced notch).

However, the amplitudes of signatures vary slightly among each others. A number of factors may contribute to this variation. Firstly, the bonding thicknesses among different specimens are not the same. Although it is known that a sufficiently thin bonding layer (less than 0.05mm) can usually be achieved with careful preparation (Yang et al. 2008a), slight variations are inevitable which will lead to difference in magnitude of strain transferred. Thicker bonding implies higher shear lag, lower strain and thus lower amplitude. Secondly, the admittance signatures were acquired in discrete manner. To obtain an exact value for the amplitude can be extremely time consuming, if not impossible, due to the fact that very narrow frequency step is required. In this study, the resonance frequency is of our interest, which is less sensitive to bonding condition but more sensitive to damage (Soh and Lim, 2010).

3.4.2 Selection of Resonance Peak

For metallic structures, closely spaced host structural peaks can be observed throughout the frequency spectrum, from 0 to the range of Mega Hertz. The peaks appear more densely spaced at higher frequency range due to the excitations of various local modes. At higher frequency range such as above 200 kHz, the

occurrence of PZT patch's resonances (which dominate the structural peaks) could be detrimental to the damage detection ability of the EMI technique. Yang et al. (2008a, b) presented the advantages of adopting frequency below 200 kHz to reduce contamination by the effect of bonding and temperature. Thus in this study, the frequency range selected for damage detection is limited to 200 kHz unless otherwise stated. Peairs et al. (2007) similarly showed that the PZT patch's resonance will diminish the damage detection ability of EMI technique.

The conductance signatures are plotted against frequency in the range of 30 ~ 50 kHz, as shown in Figure 3.8(a). Various structural peaks appear within the range. The legend "Baseline" indicates healthy state of the host structure (with single edge notch). After different number of loading cycles, structural peaks shifted to the left from their original (healthy/baseline) positions indicating reductions in resonance frequency. This observation is inline with the structural dynamic theory in which the occurrence of damage reduces the stiffness of the structure and thus causes a reduction in resonance frequency.

As shown in Figure 3.8(a), one can easily identify the structural resonances at different damaged state with respect to the healthy state, as circled in red. This observation is valid throughout the entire spectrum thus allowing almost all structural peaks to be selected as damage detector and quantifier. However at higher frequency range such as those presented in Figure 3.8(b), peaks are closely spaced such that overlapping could easily occur especially when damage became severe. Therefore, the use of frequency range below 100 kHz is recommended for characterizing crack throughout the entire crack propagation process.

It should however be noted that higher frequency range possesses inherent advantage in detecting very small crack due to its high frequency (short wavelength) of excitation which is suitable for detecting incipient crack. This is presented in the ensuing section.

In this study, peaks at moderate range, occurring between 40 to 50 kHz are chosen. In this range, the sensitivity to crack is sufficiently high and the peaks are relatively loosely spaced, allowing rapid and accurate identification.

(a)

(b)

Figure 3.8: Comparison of conductance signatures versus frequency acquired from PZT patch surface-bonded on specimen S2 from 0 cycles (baseline) to 210,000 cycles (before critical crack).
 (a) 40 ~ 50 kHz (b) 100 ~ 120 kHz

3.4.3 Fatigue Crack Characterization

With the experimental data (admittance signatures) collected at different health conditions for all specimens, the results can be analyzed. As described previously, the increase in number of cycles will alter the admittance signatures measured. To further understand its effect, the movement of the structural peaks is investigated.

Figure 3.9 shows a plot of the selected structural resonance peaks at different health conditions (different number of loading cycles) for specimen S1. The corresponding crack length and shift in frequency at different loading cycles for all three specimens are summarized in Table 3.3(a). The peak of baseline signature occurred at 41.4 kHz. Progressive horizontal leftward movements of the resonance peak at different loading stages could be observed. This observation agrees well with the structural dynamics theory, in which damage would cause a reduction in structural stiffness. In this case, reduction in structural stiffness of aluminium beam specimen is reflected from the reduction in resonance frequency (horizontal leftward movements) of peaks in the admittance signatures, measured through the electromechanical interaction between the PZT patch and the specimen. Similar observations could be found from specimen S2 and S3 (Appendix A) as well as B1 and B2 (Appendix B).

Figure 3.9: Comparison of conductance signatures versus frequency (39.8 ~ 41.5 kHz) acquired from PZT patch surface-bonded on specimen S1 after different number of loading cycles.

Table 3.3(a): Relationship between number of cycles, crack length and frequency reduction (healthy peak at 41.4 kHz) for specimens S1, S2 and S3.

	No. of cycles (1000)	Cumulative life cycles (%)	Crack length (mm)	Peak's frequency (kHz)	Frequency reduction (kHz)
S1	0	0.0	0	41.4	0
	35	14.6	0	41.4	0
	55	22.9	0	41.4	0
	85	35.4	0	41.4	0
	120	50.0	0.2	41.3	0.1
	160	66.7	1.3	41.2	0.2
	190	79.2	3	41.1	0.3
	205	85.4	3.6	41	0.4
	225	93.8	6	40.8	0.6
	235	97.9	7	40.4	1
	240	100.0	14	39.9	1.5
S2	0	0.0	0	41.45	0
	35	15.6	0	41.45	0
	55	24.4	0	41.45	0
	85	37.8	0	41.45	0
	120	53.3	0.2	41.4	0.05
	160	71.1	1.1	41.27	0.2
	180	80.0	4.2	41.15	0.3
	210	93.3	8.3	40.8	0.65
	225	100.0	13	39.9	1.55
S3	0	0.0	0	41.45	0
	35	15.9	0	41.45	0
	55	25.0	0	41.45	0
	85	38.6	0	41.45	0
	120	54.5	0.3	41.35	0.1
	160	72.7	1.7	41.32	0.15
	180	81.8	4.2	41.2	0.25
	210	95.5	9	40.75	0.7
	220	100.0	13	40	1.45

Further scrutinizing the movement reflects that the shift in resonance peak (Figure 3.9) could also reflect the severity of damage (in terms of crack length) encountered by the specimen. Observing the first 3 peaks (baseline, 120,000 cycles and 160,000 cycles), leftward movements was minimal indicating only mild structural damage was inflicted. Referring to the physical condition of the specimen, 1.3mm

crack was observed after 160,000 cycles. If this 1.3mm crack is defined as the first crack, this implies that below 160,000 cycles, the crack remained predominantly in its microscopic level. One could therefore infer that up to this stage, the specimen is undergoing the process of crack initiation, which consists of up to 70% of its total life.

Table 3.3(b): Relationship between number of cycles, crack length, frequency reduction (peak at 41.4 kHz) and stages of fatigue for specimen S1.

No. of cycles (1,000)	Cumulative life cycles (%)	Crack length (mm)	Frequency reduction (kHz)	Physical descriptions	Stages of fatigue
0	0.0	0.0	0.00	Intact	Healthy
35	14.6	0.0	0.00	Micro-crack	Micro-crack initiation
55	22.9	0.0	0.00		
85	35.4	0.0	0.00		
120	50.0	0.2	0.10		
160	66.7	1.3	0.20	First crack	
190	79.2	3.0	0.30	Crack length increasing steadily	Macro-crack propagation
205	85.4	3.6	0.40		
225	93.8	6.0	0.60		
235	97.9	8.0	1.00		
240	100.0	14.0	1.50	Unstable crack propagation	Critical crack length

Above 160,000 cycles, the rate of peak's movement increases. One could easily identify that from the healthy state to 160,000 cycles (consisting of 66.7% of total fatigue life), the peak's movement is only 0.2 kHz. Comparatively, from 225,000 to 235,000 (4.1% of total fatigue life), the peak's frequency reduces by 0.4 kHz. Physically, the increment in crack length from healthy to 160,000 cycles is 1.3mm while from 225,000 to 235,000 is 2.0mm.

The status of fatigue crack with physical descriptions for specimen S1 is tabulated in Table 3.3(b). The behaviors of other specimens are similar, and can be found in Appendix A.

Upon loading from 235,000 cycles to 240,000 cycles, which indicates the occurrence of critical crack, an abrupt 0.5 kHz reduction in resonance frequency can be observed, implying that very serious damage have been inflicted after this 5,000 cycles of loading. Upon reaching critical crack, the specimen failed shortly in less than a thousand cycles.

One could therefore infer that the reduction in resonance frequency or peak's movement is effective in characterizing the fatigue crack and indirectly reflecting its corresponding number of loading cycles. In other words, the EMI technique is useful in reflecting the severity of fatigue induced damage (fatigue crack). It can also be very efficient in providing early warning. Whenever there is a rapid shift in resonance frequency, say caused by overloading, one should be alerted that failure may be fast approaching and maintenance should be carried out accordingly.

3.4.4 Damage Quantification

3.4.4.1 Reduction in resonance frequency

The reduction in resonance frequency is found to be an effective damage quantifier as shown in the previous section. This can be further exemplified through Figure 3.10. When the crack length and the reduction in resonance frequency are plotted against the life cycles respectively, they exhibit similar trend. The slopes of the curves from 0 to 70% of life cycles are mild, indicating that crack increment is at its initiation stage. Exceeding 70% of life cycles, the slopes suddenly increase at an exponential rate up to failure.

Moving a step further, the relationship between frequency reduction and crack length can be established, as shown in Figure 3.11(a). They exhibit an approximately linear relationship. Therefore, one could possibly use frequency reduction to characterize the crack length for SHM. It is also worth mentioning that another advantage shown by the EMI technique is the repeatability of signatures among the

three specimens at different crack lengths. Note that the “crack length” indicated in Figure 3.10(b) and 3.11(a) excludes the pre-induced edge crack. Combining the data from all three specimens, a linear relationship could be established through simple curve fitting as shown in Figure 3.11(b) with coefficient of determination, $R^2 = 0.927$. These observations are numerically verified using FEM in Chapter 4.

Figure 3.10: Schematic plots comparing reduction in resonance frequency (peak at 41.4 kHz) and crack length versus % life cycles for specimens S1, S2 and S3.

- (a) Frequency reduction versus % life cycles
(b) Crack length versus % life cycles

Figure 3.11: Reduction in resonance frequency (peak at 41.4 kHz) versus crack length for specimens S1, S2 and S3.

(a) S1, S2 and S3 plotted individually

(b) Linear curve fitting (data from S1, S2 and S3)

3.4.4.2 Statistical damage index

In order to fully automate the process of SHM especially in damage characterization, the use of appropriate statistical damage index is desirable. In this section, two commonly used indices, the RMSD and CCDM indices are adopted.

(i) Root Mean Square Deviation (RMSD)

The RMSD can be expressed as:

$$RMSD(\%) = \sqrt{\frac{\sum_{i=1}^N (G_{i,d} - G_{i,h})^2}{\sum_{i=1}^N (G_{i,h})^2}} \times 100\% \quad (1)$$

where $G_{i,d}$ and $G_{i,h}$ are the conductance signatures at damaged and healthy state respectively, measured at frequency i . N is the total number of frequency points selected. The RMSD of conductance signatures acquired from PZT patch bonded on specimen S1 in the frequency range of 20 ~ 30 kHz and 40 ~ 50 kHz at different number of cycles were evaluated and plotted in Figures 3.12(a) and (b).

Figure 3.12: RMSD index against number of loading cycles for specimen S1.
(a) 20 ~ 30 kHz (b) 40 ~ 50 kHz

Apparently, the RMSD values fluctuate with the number of cycles showing no regular pattern. This is expected because RMSD index is more sensitive to variations in amplitude (vertical shift). However, as shown in the previous section, leftward horizontal shift or reduction in resonance frequency is a better representation of damage severity whereas amplitude (vertical value) exhibits less information about damage in this test. Therefore, the RMSD index could not reflect crack propagation satisfactorily.

(ii) Correlation Coefficient Deviation Mean (CCDM)

Another damage index, known as the CCDM is attempted. According to Marqui et al. (2008), CCDM is more sensitive to changes in the shape between the impedance signatures, such as frequency shifts. It CCDM index can be expressed as:

$$CCDM = 1 - \left| \frac{\sum_{i=1}^N (G_{i,h} - \overline{G}_h)(G_{i,d} - \overline{G}_d)}{\sqrt{\sum_{i=1}^N (G_{i,h} - \overline{G}_h)^2} \sqrt{\sum_{i=1}^N (G_{i,d} - \overline{G}_d)^2}} \right| \quad (2)$$

where $\overline{G}_h$ and $\overline{G}_d$ are the averaged signatures in healthy and damaged condition, respectively.

The CCDM index acquired from specimen S1 in the frequency range of 20 ~ 30 kHz and 40 ~ 50 kHz at different number of loading cycles are plotted in Figure 3.13 (a) and (b), respectively. As shown in the figures, this index is a much better representation of fatigue crack propagation whereby a progressive increase in damage index can be observed. Between 120,000 and 160,000 cycles, a sudden increase in CCDM value indicates the occurrence of first crack. This is a transition point where the crack enters the macroscopic propagation zone, and a higher rate of crack growth is expected thereafter.

Comparing all three damage quantifiers, the reduction in resonance frequency is the most effective and accurate quantifier for characterizing fatigue crack propagation. It is also repeatable among different specimens. There are many peaks available for selection throughout the frequency spectrum. It also enables the inspector to “see” and “feel” the changes in signatures directly, giving more physical meaning. However, it requires experiences to analyze the results.

The CCDM index is a good representation of crack propagation. Automated monitoring can be easily achieved with the use of CCDM. However, the results are less repeatable among different specimens of identical configurations. Selection of frequency range for different sensitivity and repeatability also require further study. The RMSD index failed to yield reasonable results and is thus not recommended.

Figure 3.13: CCDM index against number of loading cycles for specimen S1.
(a) 20 ~ 30 kHz (b) 40 ~ 50 kHz

3.5 MICRO-CRACK DETECTION

In this section, the ability of the EMI technique to detect microscopic crack is investigated. The conductance signatures after different number of loading cycles (below 120,000 cycles) are plotted in Figure 3.14. One can easily identify that the lower frequency range (Figure 3.14a) is less sensitive to microscopic crack development, as reflected by minimal peak movement in comparison to the higher frequency range (Figure 3.14b), showing clear movements of peak.

It should be noted that at 120,000 cycles, the crack length is 0.2 mm (invisible to the naked eyes). At 100,000 and 55,000 cycles, the crack is even smaller in size, which can hardly be detected using the crack detector. However, using the frequency range of 106 ~ 107 kHz, the development of micro crack could be clearly

characterized through the peak movements.

This can be explained by the fact that at higher frequency range, the wavelength of waves actuated by the PZT patch would be shorter, enabling it to interact with crack of smaller size. Therefore, it is recommended to use the higher frequency range, typically larger than 100 kHz for the detection of micro crack or when the crack is in its initiation stage. However, selecting resonance peaks in this densely packed range shall be carefully performed to prevent confusion with adjacent peaks. Moreover, the peaks induced by different modes of vibration will have different sensitivity towards a crack, which deserve further study.

Figure 3.14: Plot of conductance signatures versus frequency acquired from PZT patch surface-bonded on specimen S2 after different number of loading cycles.

(a) 32.5 ~ 33.5 kHz

(b) 106.2 ~ 107.2 kHz

3.6 IDENTIFICATION OF CRITICAL CRACK

Monitoring fatigue crack in a host structure as it attains critical crack length can be qualitatively achieved by observing the conductance signatures versus frequency spectrum for a wider range, say from 60 ~ 80 kHz. Observing Figure 3.15(a), when the damage is relatively “minor” (in the propagation stage), most structural peaks moved only slightly to the left. No new peaks emerged or existing peaks disappeared at these states. The outlooks of the entire spectrum at these damaged states are generally the same as the healthy state.

Upon reaching critical crack length, the damage is considered “severe” and is reflected by random peaks movements in comparison to the healthy state, as reflected in Figure 3.15(b). At this stage, identification of original peaks is difficult as some existing peaks disappeared and some new peaks emerged thus changing the outlook of the entire spectrum. A quick glance at the frequency spectrum thus provides a quick way of identifying the status of crack.

This phenomenon can be physically explained by the fact that the resonance frequencies in the admittance signature spectrum represent the modes of vibration of the host structure. The presence of a relatively small crack would slightly reduce the stiffness of the structure but may not be significant enough to alter its vibrational behavior. As the crack length increases, its effect on the modes of vibration of the beam becomes more impactful, to a point where it alters some of the modes, which can be reflected by the emergence of new resonance peaks or disappearance of the existing resonance peaks.

Figure 3.15: Plot of conductance signatures versus frequency (60 ~ 80 kHz) acquired from PZT patch surface-bonded on specimen S2 after different number of loading cycles.

(a) Between 0 (baseline), 120,000 and 160,000 cycles (first crack)

(b) Between 0 (baseline) and 225,000 cycles (critical crack)

3.7 FATIGUE CRACK MONITORING OF BEAM SPECIMEN WITH CIRCULAR NOTCH

In this section, an aluminum beam specimen (300mm x 50mm x 6 mm) (T6061-T6) with a 20mm circular symmetric notch at the center was prepared for constant stress amplitude fatigue test. A PZT patch (10mm x 10mm x 0.3mm) was surface-bonded at 75mm from one end of the specimen as shown in Figure 3.16.

Figure 3.16: Pictorial illustration of aluminium beam specimen with surface-bonded PZT patch (fatigue failure occurred).

The experimental procedures outlined in section 3.3.1.1 were similarly carried out. The mean load and alternative load were set at 24 kN and 12 kN, corresponding to mean stress and nominal alternative stress of 80 MPa and 40 MPa, respectively.

During the fatigue test, slip lines could be observed around the circular notch at about 80,000 cycles. Its density increased with the number of loading cycles. At 104,000 cycles, a 5 mm crack can be clearly seen. The direction of propagation is perpendicular to the direction of loading. The crack first occurred at one side of the notch, which is the point of highest local stress. The specimen failed and broke into two parts (Figure 3.16) after 105,200 cycles. The final crack length measured is 10 mm. Again, the PZT patch remained functional despite the failure of host structure.

The conductance signatures recorded after various loading cycles are presented in Figure 3.17. The number of loading cycles 104,000, 104,800 and 105,200 correspond to 5mm, 8mm and 10mm crack respectively. The resonance peak at 45.4 kHz shifts progressively to the left with increasing number of loading cycles (increase in crack

length), having similar behavior as the previously presented single edge crack. The movement of peak again correlates closely with the crack length. Peak movement is minimal when the crack length is relatively small. After the formation of macro-crack, the peak moves at higher rate during the propagation stage.

Figure 3.17: Plot of conductance signatures versus frequency (44.2 ~ 45.6 kHz) acquired from PZT patch surface-bonded on aluminium beam specimen with circular symmetric notch after different number of loading cycles.

The ability of the EMI technique in characterizing micro-crack not visible by naked eyes as described in the previous section is also observed in this case, as presented in Figure 3.18. Using the higher frequency range (> 100 kHz), the peak shift is more sensitive to loading cycles in comparison to the lower frequency range. No crack can be seen by naked eyes at 70,000 cycles but a reduction in frequency of 0.2 kHz is observed, indicating that micro-crack is initiating.

Finally, the conductance signatures acquired before failure (10mm crack) and at healthy state are presented in Figure 3.19. If 10mm crack is defined as the critical crack length (which is reasonable as failure occurred shortly), one could again deduce that upon reaching the critical crack length, the specimen is seriously damaged and the outlook of the entire spectrum is altered.

Figure 3.18: Plot of conductance signatures versus frequency (107 ~ 108 kHz) acquired from PZT patch surface-bonded on aluminium beam specimen with circular symmetric notch after different number of loading cycles.

Figure 3.19: Plot of conductance signatures versus frequency (30 ~ 70 kHz) acquired from PZT patch surface-bonded on aluminium beam specimen with circular symmetric notch at healthy and critical (10mm) condition.

3.8 CONCLUDING REMARKS

A series of experimental tests have been conducted in this chapter. The EMI technique is found to be useful for detecting and characterizing fatigue crack, propagating from either an intentionally induced single edge crack or a circular notch. The study shows that a wide range of structural resonance frequency can be selected as damage quantifier. The reduction in resonance frequency (leftward shift of peak) of the conductance signatures turns out to be a useful index for characterizing crack length. CCDM index is preferred over RMSD index for statistical damage quantification. The repeatability of conductance signatures at various damaged states is also reasonably high among different specimens of identical geometrical shapes and boundary conditions.

Higher frequency range (> 100 kHz) of the conductance spectrum is found to be more sensitive to incipient crack. Identification of critical crack is made possible by observing the changes in frequency spectrum. Further study into the micro-crack detection is recommended to obtain a better picture on damage sensitivity and frequency range.

It is also worth mentioning that the EMI technique for fatigue crack monitoring is proven workable for lab-sized structures, but extension to real-life structures or structural components requires further study. In practice, the structure being monitored is expected to be much larger with complicated geometrical configurations and boundary conditions, accompanied by external static and dynamic loading. Environmental effects also come in picture, which shall be compensated for effective monitoring (Lim, 2006). Wave propagation technique may need to be employed to locate the crack if its location is not known *a priori*. The effect of size, sensing range, optimal placement of PZT transducer and sensitivity versus frequency range ought to be carefully studied.

In this chapter, the admittance signatures were acquired from the host structure in stress free and free-ended boundary condition. The presence of axial stress and complicated boundary condition could affect the admittance signatures acquired in

one way or another, contaminating the signatures and causing confusion during the process of analysis. Consequently, erroneous conclusions may be drawn.

These variables are purposely set aside in the experiment conducted in this chapter to reduce complication. However, they should not be ignored in actual application. Separate studies on the effects of boundary condition and axial stress will be presented in Chapter 7.

In Chapter 4, a FE model will be developed to simulate the experimental tests conducted in this chapter. A semi-analytical solution for fatigue life estimation is then proposed in Chapter 5 by incorporating the theory of LEFM into the FE model.

CHAPTER 4

FINITE ELEMENT MODELING OF EMI TECHNIQUE IN FATIGUE CRACK MONITORING

4.1 BACKGROUND

In engineering design, modeling of engineering system is often vital to have a better understanding of the problem domain as well as to achieve an efficient design. In the field of EMI technique, various analytical models simulating the PZT–structure interactions have been developed (Liang et al., 1994; Yang et al., 2005; Giurgiutiu and Zagrai, 2000) and applied to the SHM of different engineering systems.

Despite its advantage of providing better insight into the mechanism and corresponding theoretical background, analytical models are often limited in actual application because they are only applicable to simple structures such as beams, plates and shells with easy to simulate boundary conditions. Subsequent developments in various numerical methods, such as the FEM are found to be an ideal alternative.

Historically, the FEM was first developed for solving problems involving stress analysis. It was subsequently extended to other engineering problems including thermal analysis, fluid flow analysis, piezoelectric analysis and electromagnetic analysis under steady, transient or harmonic states.

In FE modeling, the targeted system (such as a structure) is first modeled using numerous discrete elements (finite element) with each of them ‘tied’ together at their perimeters nodes through various physical parameters, depending on the requirements. This process of dividing into discrete elements, commonly known as meshing is extremely useful for solving real-life engineering problems which are often complex in terms of shapes, materials, boundary conditions and loadings.

Existing literatures reveal that a number of approaches for establishing the FE

problems are readily available, such as direct formulation, minimum total potential energy formulation and weighted residual formulations. Solving of the systems of equations often employs the direct method and the iterative method (Moaveni, 2003).

The outcome of the analysis, which usually involves vast amount of data, can be conveniently analyzed and presented with today's computer. Rapid advancement of the computer's technology, especially the processing speed and graphics display capability renders the FEM an increasingly robust and indispensable tool in today's engineering problem solving.

4.2 REVIEW ON FE MODELING OF PZT-STRUCTURE INTERACTION

Lalande (1995) provided an excellent, insight review into the FE modeling approaches for the simulation of PZT-structure interaction. They could be broadly classified into 3 categories, namely direct formulation of elements for specific application, utilization of thermoelastic analogy, and the use of commercially available FEA codes incorporated with piezoelectric element formulation. He attempted the dynamic FEA of ring and shell structures using commercially available software ANSYS 5.0. Good correlation was found between the results obtained from FEA and the impedance based model.

Fairweather (1998) developed a FEA-based impedance model for the prediction of structural response to induced-strain actuation. The model utilized the FEM to determine the host structural impedance. He computed the frequency response of a structure based on eigenvalues and mass normalized eigenvectors. The simplicity of this model was reflected from the fact that modeling of the actuator (PZT patch) was not required as it was represented by a force or moment. The driving point mechanical impedance could be derived by evaluating the ratio of force to velocity. For EMI technique applications, the mechanical impedance obtained could be used to calculate the PZT's admittance signature, through the impedance based electromechanical coupling equation (Liang et al, 1994, Bhalla, 2004).

Initial applications of the abovementioned models were mainly focusing on relatively low frequency of excitation, typically lower than 1 kHz. The FEA-based impedance model was later applied to the EMI technique, which involved much higher frequency of excitation, in the order of tens to hundreds of kHz.

Bhalla (2001) simulated a concrete FE model with damages incorporating 1-D FEA-based impedance model. Lim (2004) showed reasonably good comparison of mechanical impedance between experiment and 1-D FEA-based impedance model for aluminum beam, truss and concrete cube. Bhalla (2004) improved the model by incorporating 2-D effective impedance. In fact, the FEA-based impedance model is a semi-analytical model, incorporating the impedance based analytical model and the FE model. This model uses the robustness of the FEM in modeling complex system while retaining the simplicity of impedance based analytical model.

At low frequency of excitation, simplification of the PZT patch into a force or moment is normally acceptable. However, at high frequency of excitation, such simplification can lead to considerable loss in accuracy (Yang et al., 2008b). For instance, accurate prediction of modal frequencies of the host structure is essential in the EMI technique. Therefore, such losses in accuracy can be intolerable.

Liu and Giurgiutiu (2007) compared the real part of the impedance from both the FEA based impedance model (non-coupled) and coupled field FE model of a 1-D narrow beam structure to experiment. The coupled field FE model exhibits closer agreement to the experimental results.

Yang et al. (2008b) presented various FE simulations on the interaction between piezo-impedance transducer and lab-sized structures with varying bonding thickness and temperature. Simulation of the PZT–structure interaction at high frequency range (up to 1000 kHz) using the commercially available FEM software, ANSYS version 8.1, was successfully performed using coupled field element. Promising results were found when compared to the experimental results.

In comparison with other models in the field of EMI technique, the coupled field FE model exhibits exceptional robustness. For instance, most of the previously studied models, either purely analytical through impedance-based modeling or semi-analytical

through FEA-based impedance modeling, were often unable to model the minor peaks (Zagrai and Giurgiutiu, 2001), exhibited large variation in magnitudes (Lim, 2004, Giurgiutiu and Zagrai, 2000) and showed low accuracy when frequency range exceeded 60 kHz (Ong, 2003).

These shortcomings are attributed to a number of reasons (Yang et al., 2008b). Firstly, at high frequency of excitation, modes of the vibrating structure are numerous and complicated. Under such context, the FEA-based impedance model which converts the actuator (PZT patch) into a force or moment could be oversimplified. As a result, some of the vibrational modes are not excited. On the other hand, the full FE model incorporating the PZT patch simulates the entire finite area beneath the patch, instead of simplified end points interaction in the FEA-based analytical model.

Moreover, the basic assumption which neglects the effect of bonding (ideal bonding) is not realistic at high frequency due to highly localized actuation. The effect of shear lag is usually not negligible unless the bonding film is sufficiently thin. Yang et al. (2008a, b) suggested the frequency range to be kept within 200 kHz for thick bonding and further limited to 100 kHz for varying temperature to prevent any contamination by the PZT patch's resonance.

Direct acquisition of electrical admittance/impedance is also simple in the coupled-field analysis. This saves all the hassle of converting the mechanical impedance into electrical admittance through the impedance based electromechanical coupling equation as required in the FEA-based impedance model. A study conducted by Makkonen (2001) showed that fairly accurate results could be obtained from harmonic analysis using FEM, up to frequency of GHz range.

In this chapter, the feasibility of simulating EMI technique for fatigue crack monitoring using commercial FE code, ANSYS 12.1 (ANSYS, 2010) is presented. The effect of damping ratio on the amplitude of admittance signatures is also briefly studied using free-suspended PZT patch. The outcomes are compared with the experimental counterparts from Chapter 3. Accuracy, and pros and cons of FEM in modeling the EMI technique are discussed. The results are then carried forward to Chapter 5 for the development of semi-analytical damage model for fatigue life estimation.

4.3 FE MODELING – THEORY AND APPLICATIONS

With the help of powerful commercial FE software such as ANSYS, ABAQUS and ATILA, the FE modeling and analysis process are becoming increasingly efficient. However, it would be useful for the analysts to have certain level of understanding on the theoretical background of the method. This section briefly reviews some basic theories and concepts related to the simulation of the PZT – structure interaction based on the commercial FE software, ANSYS 12.1.

General equation of motion for a forced structural system using the Galerkin FE discretization (Moveni, 2003) can be expressed by the following differential equation:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \quad (4.1)$$

where $[M]$, $[C]$ and $[K]$ are the mass matrix, damping matrix and stiffness matrix respectively. $\{F\}$ denotes the applied harmonic loading, and $\{\ddot{u}\}$, $\{\dot{u}\}$ and $\{u\}$ indicate the nodal acceleration vector, velocity vector and displacement vector, respectively.

4.3.1 FEA-Based Impedance Model

In FEA-based impedance model, the PZT patch is simplified into some arbitrary steady state harmonic forces applied at the ends of the patch.

$$\{F\} = \{F_0\}e^{j\omega t} \quad (4.2)$$

where F_0 denotes the magnitude of the force, j is the imaginary number, ω is the angular frequency and t is time.

All the frequency dependent parameters will follow the applied frequency, though not necessarily in phase due to the presence of damping. The resulting displacement could then be defined as:

$$\{u\} = \{u_0 e^{j\phi}\}e^{j\omega t} \quad (4.3)$$

with ϕ representing the displacement phase shift (phase angle). The displacement could be expressed in a more convenient form, which is the usual output of commercial FE software:

$$\{u\} = \{\{u_1\} + j\{u_2\}\}e^{j\omega t} \quad (4.4)$$

where $\{u_1\}$ and $\{u_2\}$ denote the real and imaginary displacement vector respectively.

For instance, in a 2-D analysis, two equal and opposite forces are normally applied at both ends of the patch. Performing the numerical analysis (say using ANSYS), the resulting displacement, u can be readily evaluated at both loading points. Thus the drive point mechanical impedance can be conveniently evaluated as:

$$Z = \frac{F}{\dot{u}} \quad (4.5)$$

where the velocity, $\dot{u}$ can be derived by differentiating the displacement.

The mechanical impedance obtained could be converted into admittance signature through the impedance based electromechanical coupling equation.

4.3.2 Coupled-field FE Model

In ANSYS version 12.1, piezoelectric analysis comes under the category of coupled field analysis. Coupled-field analysis considers the interaction or coupling between two or more disciplines of engineering. Piezoelectric analysis caters for the interaction between structural and electric fields.

Coupled-field analysis derives solutions to problems not possible with the usual FEM, by simplifying the modeling of coupled-field problems. Piezoelectric analysis makes use of direct coupling method, which involves only one analysis with the use of one coupled-field element containing all necessary degrees of freedom. The FE formulation used for developing the matrix equations is the strong coupling method (ANSYS, 2010):

$$\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \quad (4.6)$$

where X_1 and X_2 are two different types of degrees of freedom. The coupled effect is taken into account by the off-diagonal sub-matrices $[K_{12}]$ and $[K_{21}]$.

Substituting the linear electromechanical constitutive equations (Equation 2.3) into Equation 4.1, the FE discretization can be performed by establishing nodal solution variables and element shape functions over an element domain. With the application of

variational principle and FE discretization, the coupled FE matrix for one element model can be expressed as (ANSYS, 2010):

$$\begin{bmatrix} [M] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{u}\} \\ \{\ddot{V}\} \end{Bmatrix} + \begin{bmatrix} [C] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{u}\} \\ \{\dot{V}\} \end{Bmatrix} + \begin{bmatrix} [K] & [K^z] \\ [K^z]^T & [K^d] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{V\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{L\} \end{Bmatrix} \quad (4.7)$$

in which $\{V\}$ is the vector of nodal electric potential where the dot above variables denotes time derivative, $\{L\}$ is the vector of nodal, surface and body charges, $[K^z]$ is the piezoelectric coupling matrix and $[K^d]$ is the dielectric conductivity matrix.

This formulation is very convenient for evaluating the admittance signatures used in the EMI technique. The complex electrical admittance signature, which is the ratio of electric current to voltage, can be expressed as:

$$\bar{Y} = \frac{\bar{I}}{\bar{V}} \quad (4.8)$$

where $\bar{V}$ is the sinusoidal voltage applied and $\bar{I}$ is the modulated current, with the bars above indicating complex terms.

4.3.3 Modeling of Structural Damping

The effect of damping is inevitable in structure under vibration. For harmonic analysis, the structural damping matrix could be expressed in terms of the stiffness and the mass matrix as (ANSYS, 2010):

$$[C] = \alpha[M] + \beta[K] + \sum_{p=1}^{NMAT} \beta_p [K_p] \quad (4.9)$$

where α is the constant mass matrix multiplier, β is the constant stiffness matrix multiplier, NMAT indicates the number of materials with damping input, β_p is the constant stiffness multiplier for material p and K_p is the portion of structural stiffness matrix of material p . This type of damping is in general known as Rayleigh damping and Equation 4.9 is often known as Rayleigh equation.

Both α and β are dependent on energy dissipation of structure, which is in turn, frequency dependent. Due to the nature of damping which is uncertain, predicting the various parameters listed above is not straightforward and depends heavily on

experience as well as trial and error. In this study, $\alpha = 0$ is adopted (Bhalla, 2004). β can be approximated by $\beta = \eta / \omega$, where η is the mechanical loss factor and ω is the angular frequency. It should be mentioned that, the β value adopted in this study is an average as it often needs to be adjusted through trial and error to fit the experimental results. Yang et al. (2008b) reported that the variation of β value in metallic material such as aluminium should not be too large ($1 \times 10^{-9} < \beta < 6 \times 10^{-9}$).

4.3.4 Modeling of Crack

Analysis of crack propagation or crack growth is an important topic in fracture mechanics. The FEM is one of the most popular techniques for analyzing fracture problem in composite materials. However, the FEA for crack growth can sometimes be tedious and time consuming as remeshing is required for each crack length, and the interior points of domain are also restricted to the nodal points or ought to be interpolated from nodal values (Inman et al. 2005). It is sometimes more convenient to apply the boundary element method in analyzing fracture mechanics problem due to its ability to describe both stress and displacements fields accurately, even near to the crack tip. In this study, the crack is simulated using FEM by simplifying it into a discontinuity, and is presented in section 4.5.

4.4 COUPLED FIELD FE MODELING AND ANALYSIS

Generically, the entire process of modeling and analysis comprises of three stages, namely the pre-processing phase, solution phase and post-processing phase. In the pre-processing phase, the element types are selected and the material properties are input, after which the geometrical shapes of the model is built up. The model is then discretized (meshed) into a number of FE with element's attributes and material properties assigned accordingly. At this stage, loadings and boundary conditions are then applied on the elements, nodes, surface area or volume accordingly. The final step is the selection of the type of analysis. At the solution phase, sets of linear or non-linear algebraic equations are solved simultaneously to obtain the nodal solutions,

such as displacements and electric current. Post-processing is a phase where the analyst acquires the desired information in data or graphical form.

4.4.1 FE Modeling of Freely Suspended PZT Patch

In order to gain an understanding of the piezoelectric analysis, modeling of the PZT patch was first performed without the presence of the host structure. In this section, 3-D modeling was performed with using both Solid 5 and Solid 226 elements. Solid 5 and Solid 226 are coupled field elements with 8 and 20 nodes respectively.

A freely suspended PZT patch dimensioned 10mm x 10mm x 0.3mm was modeled in ANSYS 12.1 workspace as depicted schematically in Figure 4.1. The material properties, in accordance to PIC 151 (PI Ceramic, 2010) were assigned to the PZT patch, as tabulated in Table 4.1.

An alternating voltage of 1V was applied across the patch for excitation along the z -direction. Due to the symmetry, only one-quarter of the patch is modeled. The interfacial nodes along the x -plane were restrained in the x -direction and those along the y -plane were restrained in the y -direction as shown in Figure 4.1. One node at the center of the patch was also restrained in the z -direction to maintain stability.

Figure 4.1: Isometric view of one-quarter of PZT patch modeled in ANSYS 12.1 workspace.

Table 4.1: Piezoelectric properties of PIC 151 (PI Ceramics, 2010).

Parameters	Symbols	Values	Units
Density	ρ_a	7800	kg/m^3
Dielectric loss factor	$\tan \delta$	0.02	--
Compliance	$s_{11} = s_{22}$	16.1	$\times 10^{-12} m^2/N$
	s_{33}	20.4	
	$s_{12} = s_{21}$	-4.84	
	$s_{13} = s_{31} = s_{23} = s_{32}$	-6.13	
	$s_{44} = s_{55}$	41.9	
	s_{66}	53.1	
Electric Permittivity	$\epsilon_{11}^T = \epsilon_{22}^T$	1.75	$\times 10^{-8} F/m$
	ϵ_{33}^T	2.12	
Piezoelectric Strain	$d_{31} = d_{32}$	-2.10	$\times 10^{-10} m/V$
Coefficients	d_{33}	5.00	
Constant stiffness multiplier	β_a	3	$\times 10^{-9}$

4.4.1.1 Convergence of solution

Convergence of the solutions was tested in two ways. Firstly, according to the recommendation by Makkonen (2001), to ensure sufficient accuracy of the solution, the size of the element should typically lies between two to three elements per half wavelength for harmonic analysis. Bhalla (2004) conducted modal analysis using elements of different sizes on a lab-sized aluminium beam structure. He concluded that good convergence can be achieved for 200 kHz at an element size of 1mm.

In this case, the wavelength estimated through the 1-D wave equation, assuming the highest frequency to be 1000 kHz, was approximately 5mm. The smallest element achievable (considering the time consumption and processing capability of the computer) in this analysis is 0.2mm, equivalent to 25 elements per wavelength. The calculations are attached in Appendix C.

On the other hand, convergence was also tested by performing several analyses,

each time reducing the elements' size. Convergence for Solid 5 element and Solid 226 element was achieved with element sizes of 0.2mm and 0.5mm respectively, as illustrated by the twin resonance peak near 800 kHz in Figure 4.2

In actual application, element size as small as 0.2mm is not recommended as it requires intensive computation which can be extremely time consuming. With a personal computer of model Pentium 4 (3.0GHz processing speed), the computational time of the model using Solid 226 of 0.5mm in size is approximately half an hour.

Figure 4.2: Plot of admittance signatures versus frequency (0 ~ 1000 kHz) acquired from numerically simulated free-ended PZT patch (10mm x 10mm x 0.3mm) with different element sizes.

(a) Conductance signatures (b) Susceptance signatures

4.4.2 Comparison with Existing Impedance-Based Analytical Model and Experiment

In this section, outcome from the analytical model and experimental test are compared with the numerical results.

4.4.2.1 Analytical modeling of free-ended PZT patch

Analytical model of the free-ended PZT patch can be conveniently derived by setting the mechanical impedance of the host structure, Z to zero in the impedance based electromechanical coupling equations. For the 1-D free PZT patch model, Equation 2.9 can be reduced to:

$$\bar{Y} = \left\{ -4\pi f \frac{w_a l_a}{h_a} \left[d_{31}^2 Y_{11}^E (t + \eta(r-1) - \delta \varepsilon_{33}^T) \right] \right\} + j \left\{ 4\pi f \frac{w_a l_a}{h_a} \left[\varepsilon_{33}^T + d_{31}^2 Y_{11}^E (r - \eta t - 1) \right] \right\} \quad (4.10)$$

where $r + tj = \frac{\tan \kappa l_a}{\kappa l_a}$, $\bar{Y}_{11}^E = Y_{11}^E (1 + \eta j)$, $\bar{\varepsilon}_{33}^T = \varepsilon_{33}^T (1 - \delta j)$ with δ and η indicating

the electrical loss factor and mechanical loss factor respectively. A simple program code written in Matlab 7.1 (Palm, 2001) for the evaluation of electrical admittance is attached in Appendix D.

In the case of 2-D model based on cross impedance using the equation proposed by Zhou et al. (1995), free PZT vibration could be modeled by setting all 4 terms related to the structural mechanical impedance to zero. Equation 2.11 and Equation 2.12 can thus be reduced to:

$$\bar{Y} = \frac{8\pi f w_a l_a}{h_a} \left\{ \left[\varepsilon_{33}^T \delta - \frac{d_{31}^2 Y^E}{(1-\nu)} (t + T + \eta(r + R - 2)) \right] + j \left[\varepsilon_{33}^T + \frac{d_{31}^2 Y^E}{(1-\nu)} [r + R - \eta(t + T) - 2] \right] \right\} \quad (4.11)$$

where $R + Tj = \frac{\tan \kappa w_a}{\kappa w_a}$.

Similarly, setting the effective structural impedance to zero in the 2-D effective impedance equation (Equation 2.13) yields:

$$\bar{Y} = 8\pi f \frac{l_a^2}{h_a} \left\{ \left[\delta \varepsilon_{33}^T - N(t + \eta r - \eta) \right] + j \left[\varepsilon_{33}^T + N(r - \eta t - 1) \right] \right\} \quad (4.12)$$

where $N = \frac{2d_{31}^2 Y^E}{(1-\nu)}$. Appendix D lists the program codes written in Matlab 7.1 for

the derivation of admittance signatures for Equation 4.11 and Equation 4.12.

4.4.2.2 Experimental test on free-ended PZT patch

The admittance signatures obtained analytically, numerically and experimentally from a free-ended PZT patch (10mm x 10mm x 0.3mm) are plotted in Figure 4.3.

Figure 4.3: Plot of conductance signatures versus frequency acquired from free-ended PZT patch (10mm x 10mm x 0.3mm).

(a) Experimental vs. numerical (100 ~ 900 kHz)

(b) Experimental vs. analytical (100 ~ 900 kHz)

The numerical results are obtained from the analysis using Solid 226 element with a mesh size of 0.5mm. On the other hand, it is found that the analytical outcome of the 2-D model based on cross impedance and effective impedance yield exactly the same results and thus only one of them is plotted, representing both.

A glance at Figure 4.3(a) and 4.3(b) suggests that both the numerical and analytical model provide reasonably good predictions on the actual vibrational behavior (experiment) of the PZT patch as the major resonance peaks were accurately predicted. The susceptance plot exhibits similar outcome, as attached in Appendix E.

However, a closer view into frequency range between 100 to 600 kHz suggests that the analytical models were unable to predict two resonance peaks and a twin peak as indicated in Figure 4.3(b). On the other hand, these peaks could be predicted by the numerical model (Figure 4.3a). Both twin peaks and two smaller resonance peaks are successfully matched in the numerical model. This outcome reflects the inherent shortcoming of the analytical models, which were oversimplified. At high frequency of excitation, both the 1-D and 2-D simplifications omitted some of the resonance modes that can only be generated using the 3-D FE model.

This outcome proves the robustness and capability of ANSYS 12.1 in simulating dynamic motion of PZT patch under high frequency of excitation.

4.4.3 Effect of Damping Ratio

Despite having higher accuracy in predicting the resonance peaks, one limitation existed in the FE model, where the predicted peak magnitudes differ from the experimental counterparts. Adjustment of peak height is very cumbersome as it depends heavily on trial and error of different parameters, especially the damping ratio. Excessive trials and errors with very small frequency step can be excruciatingly laborious. A balance between accuracy and time consumed is essential.

In this study, an attempt was made to predict a set of suitable damping ratios for each frequency ranges by matching the resonance peaks from the FE simulation with the experiment. Figure 4.4 shows the outcome of the adjustment.

Analyzing the results, the stiffness-damping factor, β generally decreases with frequency, agreeing intuitively with Equation 4.9. However, a perfect match between the conductance signature of the experiment and simulation throughout the wide frequency range cannot be achieved without extensive trial and errors. The situation can be more complicated when the host structure and bonding layer are considered.

Figure 4.4: Plot of conductance signatures versus frequency (0 ~ 1000 kHz) for numerically simulated free-ended PZT patch (10mm x 10mm x 0.3mm) best matched with the experimental counterparts.

In the following sections involving the host structure, the frequency range of simulation was limited to 100 kHz in accordance to the experimentation in Chapter 3. For the sake of convenience, within such range, only one β value was assumed. The determination of β value and other potential reasons leading to variation in peak amplitudes require further study to achieve FE model of higher accuracy.

4.5 FE MODELING OF EMI TECHNIQUE IN FATIGUE CRACK MONITORING

In this section, the simulation is extended to include the PZT-structure interaction. The beam specimen with surface-bonded PZT patch was modeled and a crack of

varying length representing the crack propagation process was simulated. The outcome was then compared with the experimental counterparts given in Chapter 3.

All PZT patches were modeled with Solid 5 elements instead of Solid 226. Solid 5 element possesses less nodes which are more convenient for multiple structures interactions. In addition, ANSYS (2010) recommends smaller size element with less nodes than larger size element with more nodes.

4.5.1 Modeling of PZT - Structure Interaction

A 3-D model of the aluminium beam dimensioned 300mm x 50mm x 6mm was simulated using 8 nodes, Solid 45 brick element in ANSYS workspace as illustrated in Figure 4.5. A PZT patch (10mm x 10mm x 0.3mm) modeled using 8 nodes, Solid 5 coupled field element was “surface-bonded” onto the structure. This configuration models the S series aluminium beam specimens used in the experiment.

Elements of size 1mm were adopted for both the structure and the PZT patch. The wavelength at 200 kHz calculated using 1-D wave equation (Appendix C) is 14.6mm. The corresponding number of elements available for each wavelength is about 14, fulfilling the requirements by Makkonen (2001).

Convergence was also tested by conducting modal analysis on the aluminium beam. The mesh sizes used for modal analysis are 2mm, 1.5mm, 1mm, 0.8mm and 0.5mm. A total of 73 modes were extracted for a frequency range of 20 to 120 kHz. Using 0.5mm as benchmark, satisfactory convergence was attained at 1mm mesh, whereby the percentage error (in terms of modal frequency), was generally less than 0.5%. Table 4.2 tabulates a selected frequency range (40 to 50 kHz), commonly used in this study. A longer list of all other modes (20 to 120 kHz) can be found in Appendix C. Further reducing the element size to 0.8mm will not lead to significant increase in accuracy, but will demand great computational capacity. Thus the mesh size was set at 1mm.

The material properties of the aluminum beam are listed in Table 4.3. In simulation, the patch was ‘bonded’ to the aluminum beam by merging the interfacial nodes between the patch and the host structure. Merging of the nodes ensured that the interfacial nodes carry the same nodal displacements.

Table 4.2: Modal frequencies and percentage error (relative to 0.5mm) evaluated at different element sizes for aluminium beam specimen (300mm x 50mm x 6mm).

Mode no.	Element sizes								
	2mm		1.5mm		1mm		0.8mm		0.5mm
	Modal F (Hz)	Err (%)	Modal F (Hz)	Err (%)	Modal F (Hz)	Err (%)	Modal F (Hz)	Err (%)	Modal F (Hz)
21	40,432	2.32	40,016	1.26	39,708	0.25	39,610	0.24	39,517
22	42,675	0.12	42,652	0.06	42,635	0.01	42,630	0.01	42,625
23	45,194	0.12	45,170	0.06	45,153	0.01	45,147	0.01	45,142
24	46,483	2.06	46,085	1.18	45,771	0.25	45,659	0.25	45,547
25	48,722	2.57	48,169	1.41	47,758	0.28	47,626	0.27	47,499
26	49,268	0.11	49,243	0.06	49,224	0.01	49,218	0.01	49,212
27	50,550	0.15	50,515	0.08	50,489	0.02	50,481	0.02	50,473

Figure 4.5: Isometric view of aluminum beam (300mm x 50mm x 6mm) with surface-bonded PZT patch (10mm x 10mm x 0.3mm) modeled in ANSYS 12.1 workspace.

According to the study by Yang et al (2008b), when the frequency range does not exceed 200 kHz, the effect of bonding layer (shear lag) on the resonance frequency of structural peaks is negligible. Satisfying this requirement, the bonding layer was not simulated in this study.

Table 4.3: Material properties of aluminum beam.

Parameters	Symbols	Values	Units
Density	ρ_s	2715	kg/m ³
Poisson ratio	ν_s	0.3	--
Young's modulus (Isotropic)	Y_s	65 x 10 ⁹	N/ m ²
Constant stiffness multiplier	β_s	1 x 10 ⁻⁹	--

A free-ended boundary condition was modeled to simulate the signature acquisition process during the experiment. The displacement of a selected node (directly below the center of the PZT patch) was fixed (Figure 4.5) in all 3 directions to ensure stability of the entire structure during analysis.

The numerical outcome was compared with the experimental results, as plotted in Figure 4.6 and Figure 4.7, for two different frequency ranges. From the figures, one can conclude that the simulation is successful as apparent from the good matching of the slopes and resonance frequencies, even up to frequency as high as 120 kHz for both real (conductance) and imaginary (susceptance) components. Better agreement is achieved in the range of 40 to 50 kHz (Figure 4.6) in which almost all of the simulated structural resonances match their experimental counterparts.

In the range of 100 to 120 kHz (Figure 4.7), the densely spaced peaks are less accurately predicted. Some of the resonances occurring in the experiment are not simulated while some are simulated, but not at the exact frequency. At higher frequency range, the resonances are highly sensitive to various local imperfections. Slight variations in boundary conditions, location of PZT patch, geometrical shapes or the presence of roughness of edges, etc., in the experiment could lead to the excitations of

various local modes. These local modes cannot be predicted in the FE model, which is only an ideal representation of the experimental tests.

Figure 4.6: Comparison of admittance signatures versus frequency between experiment and FE model acquired from PZT patch surface-bonded on aluminum beam specimen (300mm x 50mm x 6mm).
 (a) Conductance signatures (30 ~ 50 kHz)
 (b) Susceptance signatures (50 ~ 50 kHz)

Figure 4.7: Comparison of admittance signatures versus frequency between experiment and FE model acquired from PZT patch surface-bonded on aluminum beam specimen (300mm x 50mm x 6mm).

(a) Conductance signatures (100 ~ 120 kHz)

(b) Susceptance signatures (100 ~ 120 kHz)

The amplitudes of the signatures for both the real and imaginary parts exhibit some variations. These were also observed by Zagrai and Giurgiutiu (2001) and Yang et al (2008b). One of the main reasons is the difficulty involved in determining the damping ratio, as discussed in section 4.4.3. Ignoring the bonding layer also contributes to this variation. Since the amplitude is not a major concern of this study, in depth investigations into this issue is omitted.

4.5.2 Simulation of Crack Propagation

Moving a step further, the propagating crack under fatigue loading was simulated. For the sake of simplicity, the crack was assumed to be through-the-thickness and propagated perpendicularly to the direction of loading. Nodal displacements along the crack were uncoupled so that the interfacial nodes could move freely relative to each others. Similar models with different crack length were rebuilt after each successful analysis. Admittance signatures were acquired for each crack length.

Figure 4.8 depicts an exaggerated view of the mode shape of aluminium beam under the excitation of PZT patch at 42.2 kHz. A discontinuity caused by the through-the-thickness crack can be clearly seen at the center of the beam (circled in red).

Figure 4.8: Exaggerated view of aluminum beam (300mm x 50mm x 6mm) actuated by surface-bonded PZT patch (10mm x 10mm x 0.3mm) at 42.2 kHz modeled in ANSYS 12.1 workspace.

The resonance peak at 42.2 kHz (healthy state) was selected as the baseline, as shown in Figure 4.9. As the crack length increases, the resonance peak moves progressively to the leftward, indicating a reduction in resonance frequency. This observation is inline with the experimental counterparts, in which increase in crack length progressively reduces the structural stiffness causing leftward shift of peaks.

It is worth mentioning that the crack length labeled in Figure 4.9 is the crack length beyond the pre-induced edge crack. This finding proved, from numerical point of view, that the reduction in structural resonance frequency is a useful damage quantifier for characterizing fatigue crack propagation.

Figure 4.9: Plot of conductance signatures versus frequency (41.6 ~ 42.4 kHz) acquired from PZT patch surface-bonded on aluminium beam at different crack length (beyond pre-induced edge crack) simulated in ANSYS 12.1.

Hence, the reductions in resonance frequency at different crack lengths (beyond the pre-induced edge crack) acquired from both FE simulation and experimental test (Chapter 3) are plotted in Figure 4.10. The outcome from FE simulation exhibits an approximately linear relationship which agrees closely with the experimental results. The ability of EMI technique in characterizing fatigue crack as well as its repeatability among identical specimens is further proven using FEM.

Figure 4.10: Comparison of reduction in resonance frequency (peak at 42.2 kHz) versus crack length between FE model and experiment (specimens S1, S2 and S3).

In this section, a different peak frequency (42.2 kHz) has purposely been selected as compared to the peak used in Chapter 3 (41.4 kHz). This is to illustrate that various structural resonance peaks in the spectrum are available for damage detection and characterization. Different peaks represent different modes of vibration which have different sensitivity to damage of different forms, locations and sizes. One could select more than one peak in actual application, which would render higher confidence to the EMI technique as a SHM tool.

In this simulation, the cyclic loading, stress intensity factor and crack propagation were not included because incorporating these factors would render the problem too complex, considering the already complicated coupled field analysis. The capacity of the computational system available would become a limiting factor in such complicated analysis. This is justifiable because the main focus of this study is to investigate the ability of FEM in simulating the admittance signatures at various crack length instead of focusing on the details of crack propagation. The focal point of this simulation is to investigate the effect of crack on the wave excited by the PZT patch and thus the corresponding electrical admittance measured.

Damage model can be incorporated into the results of this FEA for evaluating the remaining life of structure under fatigue load, which is presented in next chapter.

4.5.3 Identification of Critical Crack

To verify the experimental results presented in section 3.6, the critical crack length (18mm inclusive of pre-induced edge notch) was simulated in the FE model. The conductance signatures obtained at both the healthy and critical stage are depicted in Figure 4.11. Similar to the experimental findings, some new peaks emerged and others disappeared from the spectrum, indicating a significant change in vibrational behavior of the host structure and thus implying serious damage had been inflicted.

Figure 4.11: Plot of conductance signatures versus frequency (40 ~ 50 kHz) acquired from PZT patch surface-bonded on aluminium beam at healthy and critical state (18mm) simulated in ANSYS 12.1.

4.6 CONCLUDING REMARKS

This chapter investigates the ability of the EMI technique in detecting and characterizing fatigue crack propagation through FE simulation using the commercial FEA software ANSYS version 12.1. Investigation commences with the modeling of free-ended PZT patch. Both linear and quadratic elements were adopted, and their accuracy and convergence discussed. The numerical outcomes were compared with the various existing impedance-based analytical models as well as with the

experimental tests. Closed agreement between the two is observed up to 1000 kHz. The effect of damping ratio on the peak amplitudes for different frequency ranges is briefly studied.

Next, the dynamic PZT-structure interaction was simulated. The peaks' resonance frequencies agree well with the experimental counterparts. The FEM is proven to be a robust tool in the modeling of dynamic interactions between the structure and the PZT patch at various frequency ranges.

Finally, FE simulation on the aluminum specimen with varying crack size, resembling the experimental fatigue test in Chapter 3 was performed. The effectiveness of adopting the reduction in resonance frequency for fatigue damage quantification is proven numerically through FEM.

In the following chapter, the outcome of FE simulation on the beam specimen generated in this chapter is incorporated into the LEFM theory for the development of a proof-of-concept semi-analytical damage model capable of predicting the remaining fatigue life of the specimen.

CHAPTER 5

FATIGUE LIFE ESTIMATION USING EMI TECHNIQUE INCORPORATING LEFM

5.1 INTRODUCTION

Fatigue damages are likely to occur in structures subjected to cyclic load, unless properly designed. Theoretical and practical understandings of fatigue and fracture mechanics are essential for the proper design of structures subjected to fatigue loading. The effective use of fatigue technology and analysis is an essential part of assuring the fatigue resistance and durability of all mechanical components (eFatigue, 2011).

Some important issues in fatigue design of structures or components involve the understanding of any pre-existing crack or potentially developing crack, its size and configuration, its rate of propagation under current service load, the tolerable crack size before failure and the remaining time for failure to occur. It is almost impossible to construct a structure or even an aircraft that is completely free from crack or flaw. One has to assume crack exist and design accordingly.

Conventional fatigue test often utilizes specimens with smooth surfaces under conditions of rotating-bending or uniaxial tension-compression cyclic loading. The allowable numbers of cycles to failure is related to the alternating stress applied.

Several methods are available to describe the relationships between critical parameters in structure under fatigue load, such as stress, strain, crack length, number of cycles, etc. The common one, including stress-life, strain life and fracture mechanics, will be reviewed in the following sections.

In this chapter, stress-life and linear elastic fracture mechanics (LEFM) are adopted for estimating the fatigue life of the aluminium beam specimens tested in Chapter 3. LEFM is incorporated into the FE model presented in Chapter 4 to form a proof-of-concept semi-analytical damage model for fatigue life prediction.

5.2 STRESS LIFE

5.2.1 Constant Amplitude Stress Life Method

The stress life method is the classical method for long life fatigue analysis of metals. It provides a simple and quick estimate of the lifetime of a structure under long life (high cycles), constant cyclic stress. It is originated by Wöhler in 1850.

In the stress life, plastic strain at short life is ignored where the stress applied is assumed sufficiently small. Stresses in the structure or component are compared to the fatigue limit of the material. The basis of the method is the materials S-N curve (Figure 5.1), describing the relationship between alternating stress applied and number of cycles to failure, which is obtained by testing small laboratory specimens until failure.

Wide ranges of data are available at various surface finish, load configuration and environment. The prediction is however highly empirical and sometimes provides limited insight. For instance, there is no differentiation between initiation and propagation of crack.

Figure 5.1: Common S-N curve depicting allowable alternating stress versus number of loading cycles to failure.

The number of cycles to failure reduces with increase in magnitude of the alternating stress as shown in Figure 5.1. Some materials such as steel exhibit a

behavior known as endurance limit in which a material can undergo repeated cycling of stress almost indefinitely when the stress applied is below certain threshold, known as fatigue limit, S_{FL} . However, fracture may still occur if the repetition of load is continued for a sufficiently large number of cycles, say, longer than 10^6 or 10^7 cycles.

The stress applied at the indicated cycles which falls below the line simply means that fatigue failure will not happen. Conventionally, this kind of S-N curve is applicable for endurances less than 10^8 cycles, often called high cycle fatigue. On the other hand, fatigue failure which takes place in less than about 10^4 cycles is called low cycle fatigue. The relationship between stress amplitude (alternating stress), S_a and fatigue life cycles, N_f can be expressed as:

$$S_a = S_f' (N_f)^b \quad (5.1)$$

where S_f' is the intercept and b is the slope, which depend on the material, size, surface and loading condition as well as the presence of stress concentration.

Most other materials such as aluminum do not exhibit an obvious endurance limit but the S-N curve continues to lower down with increasing number of cycles. The bend in the S-N curve would not exist, and the slope of the line is relatively gentle. For these materials, the fatigue limit can be defined as the stress at predetermined cycles of loading such as 10^6 , 10^7 , or 10^8 , depending on the type of application.

5.2.2 Modification Factors

The S-N relationship presented above may not be directly applicable in actual application due to the difference between real-life and test condition. For instance, the presence of mean stress and notch in the structure could significantly alter its fatigue life, ignoring them could render the structure significantly under-designed. Modification factors must be included to account for such discrepancies. Some of the factors relevant to this study are described below:

5.2.2.1 Surface finish factor, k_{SF}

Fatigue cracks usually nucleate on the surface. The surface condition plays a

major role in the fatigue resistance of a component. Rough surface could significantly reduce the fatigue life of a structure. Test specimens are often polished to reduce the effects of surface finish. Generally, higher strength materials are more susceptible to surface damage. A perfectly polished surface would have its surface finish factor equals to 1. Typical values of k_{SF} for steel are listed in Table 5.1 (Hertzberg, 1996).

Table 5.1: Surface finish factor for steel of strength 570MPa (Hertzberg, 1996).

Conditions	Fatigue limit (MPa)	Surface factor
Polished. 0000 emery	± 280	1
Buffed with red lead	± 276	0.99
Polished, 0 emery	± 272	0.97
Ground, 120 wheel (fine)	± 268	0.96
Ground, 46 wheel (medium)	± 258	0.92
Ground, 30 wheel (coarse)	± 232	0.83

5.2.2.2 Load Factor, k_L

Historically, fatigue limits have been determined from simple bending tests where there is a stress gradient in the test specimen. A specimen loaded axially will have a relatively low fatigue limit. An empirical correction factor, known as the load factor, k_L is introduced to account for various loading conditions as presented in Table 5.2 (eFatigue, 2011).

Table 5.2: Loading Factor, k_L (eFatigue, 2011).

Types of loading	k_L
Tension, Ultimate stress ≤ 1500 MPa	0.92
Tension, Ultimate stress > 1500 MPa	1.0
Bending	1.0
Torsion	0.58

5.2.2.3 Size factor, k_{size}

Smaller components normally exhibit higher fatigue limits in comparison to the larger ones. This is accommodated in the analysis by a size factor, k_{size} . One of the most widely used corrections is based on the diameter of a bar with diameter, d (eFatigue, 2011):

$$k_{size} = \left(\frac{d}{7.62}\right)^{-0.1133} \quad 3mm \leq d \leq 50mm \quad (5.2)$$

For a section that is not round, one could estimate its equivalent diameter through equating the cross sectional area of the non-circular section subjected to 95% of the maximum stress, $A_{0.95}$ to the area of an equivalent round bar, with the effective diameter given by:

$$d = \sqrt{\left(\frac{A_{0.95}}{0.077}\right)} \quad (5.3)$$

With three of the abovementioned factors determined, the fatigue limit of the component, $S_{FL,component}$ can be evaluated from the fatigue limit of the standard test specimen, S_{FL} :

$$S_{FL,component} = S_{FL} \times k_{SF} \times k_L \times k_{size} \quad (5.4)$$

5.2.2.4 Stress concentration factor, k_t and fatigue notch factor, k_f

Stress concentration factor is considered as one of the most important factors affecting the fatigue life of a component or structure. These stress concentrations may be intentionally introduced in the design or unintentionally created such as processing related flaws. Visually, stress concentration induces a downward shift and change in slope on the S-N curve.

Stress concentration factor for simple structures of different notches are readily available in the literature (Boresi and Schmidt, 2003). For a plate with a circular symmetric hole, the k_t values against ratio of hole's diameter to specimen's width is shown in Figure 5.2. The presence of notch reduces the allowable stress amplitude by a factor, k_t . However, k_t itself often over-estimates the actual stress concentration factor, due to variation in the size of notch and strength of material. Thus, the

effective stress concentration factor, also known as the fatigue notch factor, k_f is introduced (for aluminium):

$$k_f = 1 + \frac{k_t - 1}{1 + \frac{0.5}{r}} \quad (5.5)$$

where r is the notch radius in mm.

Figure 5.2: Relationship between stress concentration factor, k_t and hole diameter for plate with circular symmetric hole (eFatigue, 2011).

The fatigue notch factor will affect the slope of the material's S-N curve, but not the intercept. In the presence of a notch, a new slope, b_{notch} can subsequently be defined for the S-N curve:

$$b_{notch} = b - \frac{\log\left(\frac{k_f}{k_{SF} \cdot k_L \cdot k_{size}}\right)}{\log(N_f)} \quad (5.6)$$

5.2.2.5 Mean Stress Effect

It should be noted that the S-N curve described above presents the fatigue strength of members subjected to reversed cyclic load (loading with zero mean stress). In real life application, the presence of mean stress, S_{mean} either tensile or compressive

is unavoidable. Tensile mean stress would cause a reduction in fatigue strength while compressive mean stress imposes an opposite effect. For loading of non-zero mean stress, several relationships have been proposed, such as the Soderberg relation, Gerber relation and Goodman relation. Goodman relation, with stress concentration and modification factors included for 1-D testing is herein presented:

$$\frac{k_f \times S_a}{S_{FL,component}} + \frac{k_f \times S_{mean}}{S_u} = \frac{1}{n} \quad (5.7)$$

where S_u denotes the ultimate stress of the material and n is the factor of safety.

The Goodman relationship is represented schematically in Figure 5.3. It is clearly shown that for a given design life (number of cycles), the allowable alternating stress reduces with increase in mean stress. Fatigue lives are assumed to be “infinite” in the safe region ($n < 1$), defined by the triangle bound by the y -axis, x -axis and Goodman curve with “ $n = 1$ ”. For a combination of S_a and S_{mean} that falls outside the safe region ($n > 1$), fatigue failure is expected to occur within a finite life. It should be noted that the definition of “infinite” is generally application and material dependent. One can define 10^7 or 10^6 or 10^5 cycles as “infinity” as shown in Figure 5.3. Goodman relation generally yields a more conservative estimate.

Figure 5.3: Variation of alternating stress to mean stress according to Goodman relationship ($n = 1$).

In the presence of mean stress, an equivalent completely reversed stress, S_{eq} can be estimated and applied with the standard S-N curve:

$$S_{eq} = \frac{S_a}{1 - \frac{k_f \times S_{mean}}{S_u}} \quad (5.8)$$

It is worth mentioning that k_f is used to modify the mean stress but not the stress amplitude because stress concentration effects are already included in the component S-N curve. Finally the finite fatigue life, considering all external effects can be evaluated as:

$$S_{eq} = S'_f (N_f)^{b_{notch}} \quad (5.9)$$

5.2.3 Fatigue Life Estimation of Aluminium Beam Specimen with Circular Symmetric Hole

The stress life approach outlined above is adopted to evaluate the fatigue life of the aluminium beam specimen (300mm x 50mm x 6mm) with a circular symmetric hole (diameter = 20mm) as tested in section 3.4. The parameters used for the calculation is listed in Table 5.3. It should be noted that one of the most uncertain parameter is the surface finish factor whereby the machining of notch was roughly ground during preparation, rendering the surface finish factor to be ranging from approximately 0.8 to 0.95 (Table 5.1). In this case, the surface finish factor was evaluated as 0.837 by trial and error, the finite fatigue life was estimated to be 105,294 cycles in comparison to 105,200 as recorded experimentally.

Pros and cons exist when using the stress-life approach for fatigue life estimation. This highly empirical technique possesses an abundant amount of experimental data collected for various materials and configurations accumulated throughout the years. Its application is also relatively direct and simple as standard formulae and charts are readily available. Fatigue life can be acquired without considering the details of crack initiation and propagation. This approach however provides less insight into the process of crack propagation due to its empirical nature. Estimation of remaining fatigue life at various crack length is also impossible unless sufficient number of

experimental tests are conducted to calibrate an empirical model. The fracture mechanics approach, described in section 5.4, provides an excellent alternative to the stress life approach.

Table 5.3: List of parameters for fatigue life estimation of aluminium beam specimen (T6061-T6) with a circular symmetric hole.

Parameters	Labels	Values	Units
Ultimate stress	S_u	340	MPa
Mean Stress	S_{mean}	80	MPa
Stress amplitude	S_a	40	MPa
Equivalent stress amplitude	S_{eq}	82.16	MPa
Intercept (S-N)	S_f'	603	MPa
Fatigue limit (material)	S_{FL}	158	MPa
Fatigue limit (component)	$S_{FL, component}$	95.27	MPa
Slope (S-N)	b	-0.097	
Modified slope (S-N)	b_{notch}	-0.173	
Surface finish factor	k_{SF}	0.83	
Loading factor	k_L	0.92	
Size factor	k_{size}	1	
Stress concentration factor	k_t	2.24	
Fatigue notch factor	k_f	2.181	
Factor of safety	n	0.7	
Finite fatigue life	N_f	105,294	Cycles

5.3 STRAIN LIFE APPROACH

The stress life approach is suitable for high cycles (low stress) fatigue analysis. However, for highly stressed structure, an important aspect of fatigue cracking is the nucleation of micro-cracks from localized plastic straining. Cyclic strain controlled

tests often better characterize the high stress fatigue behavior of structures. This strain controlled approach, often known as strain life approach, can be applied to structure subjected to low cycle fatigue loading when plastic strain is significant. In this method, high or low plastic strain leading to crack initiation is considered.

The strain life method had its major development during the 1960's. As the name implied, it is a fatigue test in which the strain amplitude is fixed and of constant strain rate. Triangular waveform or sinusoidal waveform is commonly used as input signal. Similar to stress life approach, some of the parameters are highly empirical. It is based on the premise that the local stresses and strains around a stress concentration control the fatigue life. The strain life is relatively more complicated in terms of testing machine and control than the stress life.

The strain life approach has an advantage of being able to model residual stress from sequence loading, more realistic than stress life which considers cumulative damage. It is sometimes used together with the LEFM, capable of predicting crack propagation, for total fatigue life estimation.

5.4 LINEAR ELASTIC FRACTURE MECHANICS (LEFM)

5.4.1 Background

Nowadays, fracture mechanics is widely used in the aerospace, nuclear, ship industries and ground vehicle industry. In comparison to the stress life and strain life approaches, classical fracture mechanics provides better insight into actual mechanism of fatigue especially propagation of crack. Fracture mechanics can be approached from different aspects, including energy to cause failure, stress analysis, micro-mechanisms of fracture, applications of fracture, computational approaches and so on. This study focuses predominantly on the use of stress intensity factor for fatigue crack growth prediction.

The theory of fracture mechanics, which adopts the stress intensity factor as an important indication to initiation of crack is incorporated in the LEFM for prediction of critical crack length and fatigue life.

The LEFM is able to characterize final failure due to fracture based on existing cracked section. An initial crack needs to be assumed in the application of fracture mechanics. Propagation or growth of crack can be caused by fatigue, corrosion, creeping, etc. If the crack growth rate, stress state and crack length can be determined, the remaining safe life of the cracked component can be predicted. The accuracy can be very high for simple structure as closed form solution is readily available

This field of research was first introduced by Griffith in 1920 when he recognized that material could fail at stresses far lower than the theoretical strength. On the other hand, he found that the local stress field around the crack can be significantly amplified in comparison to the far field stress state. He considered a single through-the-thickness crack in an infinite body subjected to axial load perpendicular the crack direction. This provides strain energy required to propagate a crack. The location and size of crack is usually assumed *a priori*.

However, LEFM is unable to consider the process of crack initiation due to difficulty in estimating the initial crack size. It can also be incompetent in complicated structure as a result of difficulty in finding its stress intensity factor.

5.4.2 Loading Modes in Fracture Mechanics

Fracture mechanics is solid mechanics of cracked bodies. The deformation or failure of crack can be characterized by the relative movement between the upper and lower crack surfaces. Three basic modes of deformation are depicted in Figure 5.4.

Mode I is the opening or tensile mode in which the crack surfaces move directly apart. This mode is the most commonly encountered mode in fatigue, primarily because cracks tend to grow on plane of maximum tensile stress.

Mode II is known as the sliding or in-plane shear mode where the crack surfaces slide over one another in a direction perpendicular to the leading edge of the crack. Mode III is the tearing or out of plane shear mode where the crack surfaces move relative to one another and parallel to the leading edge of the crack. Mixed mode crack extension can also occur. This study will be focusing on mode I loading, which appears to be the predominant mode of macroscopic fatigue crack growth.

Figure 5.4: Three basic modes of loading causing crack extension.

5.4.3 Stress Intensity Factor

Stress intensity factor is the most important parameter in the application of LEFM for quantifying fatigue damage. It defines the intensity of stress and strain in the vicinity of a crack that includes the geometrical parameter according to various modes of loading. Stress intensity is directly proportional to the applied load.

Consider a through-the-thickness sharp crack in a linear elastic isotropic body subjected to mode I loading. The stresses in an arbitrary stress element at polar coordinates r and θ relative to the crack tip, as shown schematically in Figure 5.5, can be expressed using mathematical theory of linear elasticity and Westergaard stress function in complex form (Stephen et al. 2001):

$$\sigma_{xx} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 - \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \quad (5.10)$$

$$\sigma_{yy} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \left[1 + \sin \frac{\theta}{2} \sin \frac{3\theta}{2} \right] \quad (5.11)$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \cos \frac{\theta}{2} \sin \frac{\theta}{2} \cos \frac{3\theta}{2} \quad (5.12)$$

where σ_{xx} , σ_{yy} and τ_{xy} are the normal stress in x -direction, normal stress in y -direction and shear stress on x - y plane respectively.

Figure 5.5: Elastic stresses near the crack tip with $r/a < 1$.

To ensure that the relationship remains valid despite stress singularity at $r = 0$, the plastic zone at the crack tip must be small relative to the local geometry. In general, the stress intensity factor, K is dependent on externally applied stress, geometry, crack size and shape. It can be expressed as:

$$K = \sigma \sqrt{\pi a} Y \quad (5.13)$$

where a denotes half length of crack size, σ is the stress applied, Y is the dimensionless shape factor, which depends on the specimen geometry, and the crack length. For simple geometry, Y can be obtained from the standard handbook.

In one of its simplest form, the stress intensity factor for a through crack of length $2a$ in an infinite plane subjected to a uniform stress field, σ can be expressed as (Figure 5.6):

$$K = \sigma \sqrt{\pi a} \quad (5.14)$$

Figure 5.6: A through-the-thickness crack of length $2a$ subjected to tensile stress, σ in an infinite sheet.

The expressions of K for some simple configurations can be calculated using the theory of elasticity involving analytical and computational approaches as well as experimental methods (Stephen et al. 2001). Stress intensity factor for complicated shapes or boundary conditions can also be obtained through numerical analysis using FE or boundary element modeling. In this study, a single edge crack in tension is considered, in conjunction with the experimental test given in Chapter 3.

5.4.4 Validity of LEFM

As its name implied, LEFM is only applicable when the stress applied does not exceed the linearity of the material. In fact, the material surrounding the crack is considered to be homogeneous and linear elastic except for a small non-linear zone at the vicinity of the crack tip. This non-linear region shall be small relative to the crack

length as well as the geometrical dimensions of the specimen. Generally, one needs to ensure that the nominal net section stress is smaller than 80% of the yield stress.

Figure 5.7: The size of plastic zone at the tip of through-the-thickness crack for plane strain and plane stress conditions.

Specifically, to ensure the validity of LEFM condition, the radius of plastic zone, r_y' shall not exceed $a/4$ for the case of cyclic loading. Irwin (1957) described the radius of plastic zone for the case of cyclic loading as:

Plane stress:

$$2r_y' = \frac{1}{4\pi} \left(\frac{\Delta K}{\sigma_y} \right)^2 \quad (5.15)$$

Plane strain:

$$2r_y' = \frac{1}{12\pi} \left(\frac{\Delta K}{\sigma_y} \right)^2 \quad (5.16)$$

where ΔK is the stress intensity factor range and σ_y is the yield stress of the material.

The plastic zone in plane stress is three times larger than that of the plane strain, as illustrated in Figure 5.7. If the stress state leads to non-linear response i.e. when the surrounding of crack tip present large plasticity, non-linear approach or the elastic plastic fracture mechanics (EPFM) shall be adopted. Two commonly used EPFM theories are (1) the J-integral developed on the basis of deformation theory of plasticity and (2) the experimentally motivated British crack tip opening displacement (CTOD).

5.4.5 Fracture Toughness and Critical Crack Length

Critical value of stress intensity factor refers to the condition in which a crack will extend rapidly or unstably without any increase in loading. Critical value of K is commonly known as “fracture toughness”, and denoted with a subscript c :

$$K_c = \sigma \sqrt{\pi a_c} f\left(\frac{a_c}{w}\right) \quad (5.17)$$

where a_c is the crack length at instability (critical crack length) and w is the width of specimen. The fracture toughness, often determined experimentally, is highly dependent on the material, strain rate, temperature, thickness and sometimes crack length. In other words, K_c represents the critical value of stress intensity factor for a given combination of load, crack length and geometry that fracture would occur.

The values of K_c given by the standard handbook are often assumed to be under plane strain condition, i.e. the specimen is sufficiently thick. Plane strain fracture toughness is often denoted as: K_{IC} . According to ASTM E674 (ASTM, 2000), plane strain condition holds if the crack length, a and the thickness, h satisfy the following relationship:

$$a, h \geq 2.5 \left(\frac{K_{IC}}{Y} \right)^2 \quad (5.18)$$

On the other hand, a correction factor for specimen of thickness lesser than that required for plane strain condition is suggested by the British Standard, BS7910 (2005) in semi-empirical form:

$$K_c = K_{IC} \sqrt{1 + \frac{1.4}{h^2} \left(\frac{K_{IC}}{\sigma_y} \right)^4} \quad (5.19)$$

5.4.6 Fatigue Crack Growth Rate

The presence of crack in a structure or component could significantly reduces its strength but not necessarily critical enough to cause catastrophic failure. Subcritical crack could grow until a critical size is reached. One of the common causes of subcritical crack growth is fatigue. For specimens containing the same initial crack length, the crack growth rate increases with the applied cyclic stress. The relationship

between fatigue crack growth rate, da/dN is conveniently related to the applied stress intensity factor range, as shown in Figure 5.8 in log scale, which can generally be divided into 3 regions. Many models attempting to describe this relationship have been proposed based on this basic form:

$$da / dN = f(\Delta\sigma, a, Y)^m = f(\Delta K) \quad (5.20)$$

Region I is the threshold region as indicated by a threshold value, ΔK_{th} , below which fatigue crack is characterized as non-propagating. Typical threshold value is lesser than 1×10^{-10} m/cycle as defined in the ASTM standard E647 (ASTM, 2000). Microstructure, mean stress, frequency and environment are important factors in controlling the crack growth of region I.

Figure 5.8: Schematic representation of sigmoidal behavior of typical fatigue crack growth rate versus stress intensity factor range of metal.

Region II depicts an essentially linear relationship between crack growth rate, da/dN and stress intensity factor range, ΔK in logarithmic scale. In typical fatigue design, region II is often adopted as it covers the largest range of intensity. This region is often known as the Paris region, named after Paris, Gomez and Anderson. The

equation is correspondingly known as Paris' equation (Paris et al 1961):

$$da / dN = C(\Delta K)^m \quad (5.21)$$

where C and m are material dependent constants. The Paris region represents the fatigue crack growth corresponds to stable macroscopic crack growth that is typically controlled by the environment.

In region III, the fatigue crack growth rate is very high resulting in unstable propagation. Although plastic zone size is often big in this region, it is normally negligible because very little fatigue life is involved in this region as fracture is fast approaching. In real-life design, extrapolation of region II into both region I and III are often acceptable as it provides conservative fatigue life prediction.

5.4.7 Other Considerations: Mean Stress Effect and Small Fatigue Crack

The mean stress effect and small crack growth, where applicable, should be considered in the fatigue crack growth relationship.

Loss of LEFM similitude may occur when the crack is small in comparison to the plastic zone or micro-structural dimensions. It is therefore important to ensure the crack length exceeds the limiting value for LEFM application. The crack length, a_{small} below which small crack growth behavior is expected can be evaluated by:

$$a_{small} \approx \frac{1}{\pi} \left(\frac{\Delta K_{th}}{\Delta \sigma_f} \right)^2 \quad (5.22)$$

where $\Delta \sigma_f$ is the fatigue limit range, which equals to twice the fatigue limit.

On the other hand, the mean stress applied would have certain influence on the fatigue crack behavior. An increase in the mean stress ratio, R would increase the crack growth rates in all three regions of the sigmoidal crack growth curve. The R value is highly material dependent. Two commonly used equations to incorporate the effect of mean stress into the Paris' equation are the Forman equation and Walker equation. In this study, the Walker equation with empirical correction factor for mean stress, $R \neq 0$ is adopted (Stephens et al., 2001):

$$da / dN = C'(\Delta K)^m \quad (5.23)$$

where $C' = \frac{C}{(1-R)^{m(1-\lambda)}}$ and $R = K_{min}/K_{max}$. K_{min} and K_{max} are the minimum and maximum stress intensity factor applied. λ is a material constant which ranges from 0.3 ~ 1 (with a typical value of 0.5) for metals. Lower value of λ implies stronger influence of R on the fatigue crack growth behavior.

5.4.8 Estimation of Fatigue Life Using LEFM

In actual application, if the stress intensity factor range is known, the fatigue life of the component can be obtained by integrating the sigmoidal curve between the limits of the initial crack size and final crack size. For instance within the Paris region, the number of cycles to failure, N can be expressed as:

$$N = \frac{1}{C'(\Delta\sigma)^m} \int_{a_0}^{a_c} \frac{da}{(\sqrt{\pi a} f(a/w))^m} \quad (5.24)$$

in which a_0 is the initial crack size and a_c is the critical crack length, and $f(a/w)$ is the stress intensity factor function.

In this study, a rectangular specimen with single edge crack being loaded in tension was used in the experiment. Mathematically, for single edge crack subjected to tension in an infinitely wide plate ($w \gg a$), the stress intensity factor range under mode I loading, ΔK_I can be expressed as:

$$\Delta K_I = F(a) \Delta\sigma \sqrt{\pi a} \quad (5.25)$$

where $F(a) = 1.1215$, ΔK_I is the stress intensity factor range and $\Delta\sigma$ is the stress range.

On the other hand, if the single edge crack appears in a plate of finite width ($0 < a/w < 0.95$) as illustrated in Figure 5.9, the above expression shall be corrected for the effect of free edge (Stephen et al., 2001):

$$\Delta K_I = \Delta\sigma \sqrt{a} \left[1.99 - 0.41\left(\frac{a}{w}\right) + 18.7\left(\frac{a}{w}\right)^2 - 38.48\left(\frac{a}{w}\right)^3 + 53.85\left(\frac{a}{w}\right)^4 \right] \quad (5.26)$$

More specifically,

$$\Delta K_I = \Delta\sigma \left[1.99a^{1/2} - 0.41\frac{a^{3/2}}{w} + 18.7\frac{a^{5/2}}{w^2} - 38.48\frac{a^{7/2}}{w^3} + 53.85\frac{a^{9/2}}{w^4} \right] \quad (5.27)$$

Figure 5.9: A plate with finite width containing a single edge crack in tension.

Substituting Equation (5.27) into Equation (5.23) and upon integration yields the predicted number of cycles for a crack to propagate from a_0 to a_f (final crack length):

$$N = \frac{1}{C'(\Delta\sigma)^m} \int_{a_0}^{a_f} \left[1.99a^{1/2} - 0.41 \frac{a^{3/2}}{w} + 18.7 \frac{a^{5/2}}{w^2} - 38.48 \frac{a^{7/2}}{w^3} + 53.85 \frac{a^{9/2}}{w^4} \right]^{-m} da \quad (5.28)$$

Due to its complexity, numerical integration is necessary to solve the integral. In this study, the program written in Matlab 7.1 workspace for solving the integration is attached in Appendix F.

5.5 EMI TECHNIQUE BASED SEMI-ANALYTICAL DAMAGE MODEL FOR FATIGUE LIFE PREDICTION

The effectiveness of the EMI technique employing smart piezoceramics

transducer in monitoring crack growth in structure has been proven in a wide range of research. It is well known that the EMI technique can provide real time, remote and autonomous monitoring. In this study, its effectiveness in characterizing fatigue crack length as well as its consistency among identical specimens have been proven experimentally and numerically in Chapter 3 and Chapter 4, respectively.

Lim and Soh (2010) derived a semi-empirical solution by incorporating the LEFM theory into the experimental outcome from EMI technique for fatigue life prediction. This chapter describes further investigation into the possibility of performing damage prognosis on simple structure subjected to gradual wear (fatigue loading in this case) using a semi-analytical damage model developed through incorporating FE model of the EMI technique (from Chapter 4) and LEFM theory through proof-of-concept application. The semi-analytical damage model described in this study supersedes the semi-empirical model developed by Lim and Soh (2010) whereby this model need not rely on any experimental data. The experimental data from Chapter 3 is only used to verify the validity of this damage model.

A similar empirical model for fatigue life prediction is also presented at the end of the chapter. The possibility of developing a pure numerical model for the same purpose is finally discussed.

5.5.1 Incorporating LEFM into EMI Technique for Fatigue Life Estimation

As discussed in the previous chapters, the conductance signatures acquired from the EMI technique (both experimentally and numerically) could be used to quantify the crack length of aluminum beam specimens. On the other hand, if the crack length in the specimen can be monitored at any instance during the monitoring process, the remaining life could be predicted using Equation 5.28 (for single edge crack).

Fatigue related parameters for aluminum beam specimen (T6061-T6) dimensioned 300mm x 50mm x 6mm required for solving Equation 5.28 are tabulated in Table 5.4. Values of m and C in the Paris equation are adopted from a study conducted by Kapp and Duquette (1986).

Table 5.4: Physical and fatigue properties of aluminum (Al6061-T6) beam specimen.

Parameters	Symbols	Values	Units
Ultimate strength	σ_u	352	MPa
Yield stress	σ_y	299	MPa
Critical fracture toughness (plane strain)	K_{IC}	29	MPa(m) ^{1/2}
Corrected critical fracture toughness ($w = 6mm$)	K_C	61.11	MPa(m) ^{1/2}
Gradient of Paris equation (log scale)	m	4.5	
Intercept of Paris equation (log scale)	$\log C$	-11.5	
Maximum stress	σ_{max}	149.5	MPa
Minimum stress	σ_{min}	119.6	MPa
Range of stress	$\Delta\sigma$	29.9	MPa
Stress ratio	R	0.8	
Material constant	λ	0.805	
Initial crack length	a_0	4.75	mm

The critical crack length, a_c can be evaluated using Equation 5.26, by substituting $K = K_c$ and $\sigma = \sigma_{max}$

$$K_C = \sigma_{max} \sqrt{a_c} \left[1.99 - 0.41 \frac{a_c}{w} + 18.7 \frac{a_c^2}{w^2} - 38.48 \frac{a_c^3}{w^3} + 53.85 \frac{a_c^4}{w^4} \right] \quad (5.29)$$

The above equation is solved using the in mathematical software Matlab 7.1 and presented in Appendix F. The computed critical crack length is found to be in closed agreement with the values measured from experimental tests, as listed in Table 5.5.

Table 5.5: Comparison of critical crack length between analytical prediction and experimental measurements.

Specimens	Label	Critical crack length
LEFM	$a_{c, LEFM}$	16.7 mm
S1	$a_{c, S1}$	18 mm
S2	$a_{c, S2}$	17 mm
S3	$a_{c, S3}$	17 mm

With all the relevant parameters determined, the number of loading cycles, N at various crack length, a (with $a_0 = 4.75\text{mm}$) can be calculated using Equation 5.28 as tabulated in Table 5.6. For illustration, the relationship is also plotted in Figure 5.10 for comparison with the experimental results. Note that the number of cycles, N to failure is normalized in terms of life cycles (%), in which 100% denotes failure.

Table 5.6: Crack length versus number of cycles predicted analytically using Equation 5.28.

$a \text{ (mm)}$	N	$N \text{ (%)}$
4.75	0	0
6	98,081	43
7.8	166,036	73
9.2	192,203	84
11	210,284	92
16.7	227,490	100

Figure 5.10: Schematic representation of crack length versus life cycles (%) estimated by LEFM theory and measured experimentally (S1, S2 and S3).

It could be deduced from Figure 5.10 that the LEFM theory yields a fairly accurate prediction of the number of loading cycles encountered at different crack length as reflected from its closed agreement with the experimental counterparts.

Finally, the relationship between reductions in resonance frequency against crack length as simulated by the FEM (Figure 4.11) is incorporated into the relationship of crack length against number of cycles from Equation 5.28 to form a semi-analytical solution which is presented graphically in Figure 5.11. In this figure, the reduction in frequency based on resonance peak at 42.4 kHz and loading cycles acquired from the semi-analytical damage model and the experiment are compared against each other. Relatively close agreement between the semi-analytical prediction and experimental results is achieved.

The proof-of-concept semi-analytical damage model presented above can be useful for fatigue crack monitoring as well as for the estimation of remaining life. It can serve as an alternative to experimentation to acquire the baseline data for damage detection and damage prognosis. With the use of LEFM, a deeper insight into the fatigue crack propagation, rate of propagation, stages of fatigue crack at various instance, critical crack length, etc. could be attained.

Figure 5.11: Graphical plot depicting relationship between reductions in resonance frequency (peak at 42.4 kHz) versus life cycles (%) obtained experimentally and semi-analytically.

5.6 EMI TECHNIQUE BASED EMPIRICAL DAMAGE MODEL FOR FATIGUE LIFE PREDICTION

Pure empirical relationship can also be conveniently established if sufficient experiments are conducted. A curve fitting can be performed using exponential function, $y = Ae^{kx}$ for establishing the relationship between reduction in resonance frequency against life cycles. The exponential function is selected because it yields the smallest R^2 , implying a best fit. Experimental data measured from all test specimens at 42.2 kHz are used for curve fitting as shown in Figure 5.12.

Figure 5.12: Empirical relationship established through curve fitting using exponential function, $y = Ae^{kx}$ adopting data from specimens S1, S2 and S3.

- (a) Crack length (beyond edge notch) versus life cycles (%)
- (b) Reduction in frequency (peak at 42.2 kHz) versus life cycles (%)

The relationship between crack length (beyond single edge notch) and life cycles (%) can be expressed as:

$$a = 0.002e^{0.0896N} \quad (5.30)$$

and the relationship between frequency reduction and life cycles (%) as:

$$\Delta f = 0.0024e^{0.052N} \quad (5.31)$$

Similarly, the relationship between reduction in frequency and crack length (beyond single edge notch) can be established. In this case, a power function is adopted so that when plotted in logarithmic scale, they exhibit a linear relationship (Figure 5.13):

$$\Delta f = 0.0945a^{0.4674} \quad (5.32)$$

One could infer that the empirical correlation acquired from experimental tests can be presented in a more concise form and applied to actual health monitoring and fatigue life prediction. However, the cost incurred in performing sufficient number of successful experimental tests would render the semi-analytical solution an excellent alternative.

Figure 5.13: Empirical relationship established between reduction in resonance frequency and crack length through curve fitting using power function, $y = Ax^a$ adopting data from specimens S1, S2 and S3.

5.7 EMI TECHNIQUE BASED NUMERICAL DAMAGE MODEL FOR FATIGUE LIFE PREDICTION

In this study, the FE simulation described in the previous chapter focuses on investigating the effect of varying crack length on the admittance signatures measured from the PZT patch. Similar to the experimental counterparts, the reduction in frequency versus crack length (beyond single edge notch) also exhibits a linear relationship when plotted in logarithmic scale as shown in Figure 5.14. It could similarly be fitted using the power function, $y = Ax^k$:

$$\Delta f = 0.0415a^{1.1859} \quad (5.33)$$

Figure 5.14: Linear relationship in logarithmic scale between reduction in resonance frequency and crack length through curve fitting using power function, $y = Ax^k$ based on data obtained from FE simulation.

In fact, the FE software allows the user to model the crack propagation induced by cyclic loading. The stress intensity factor, critical crack length and number of loading cycles at various crack length can also be evaluated.

In other words, pure numerical solution could also be obtained for fatigue life estimation using the EMI technique. However, this is not performed in this study due to the complexity involved in modeling of crack propagation, which requires fine mesh around the crack tip. When added to the computationally intensive coupled field

analysis, the limited computational resources prohibited the analysis.

However, the FEM can be very useful due to its ability to simulate complicated structural and crack configurations which is not possible through analytical modeling.

5.8 POTENTIAL REAL-LIFE APPLICATIONS

For real-life applications, the actual number of loading cycles encountered by a structure or structural component under service is usually not known. Furthermore, the actual crack length is also unknown unless noticed and physically measured. Thus, it is possible to use the proposed damage model for crack monitoring and estimation of the number of cycles experienced, as well as predicting the remaining life based on pre-calibrated data from experimental test, numerical model or semi-analytical model.

It is worth mentioning that one could also detect accidental overloading using the EMI technique through observing any abrupt change in the resonance peaks or the shape of conductance plot. However, one obvious shortcoming of this damage model is the necessity of rebuilding new models for different applications. Further study to investigate into the repeatability of this approach, the effect of environment and loading, applicability on more complex and larger scale structures shall be carefully studied before real-life application is possible.

5.9 CONCLUDING REMARKS

This chapter investigates the feasibility of employing the EMI technique for estimating the remaining fatigue life of lab-sized aluminum beam structure through the development of a proof-of-concept semi-analytical damage model consisting of the FEM and the LEFM theory. The model's prediction agrees well with the experimental counterparts. It can be developed independent of experimentation.

An empirical model serving the same purpose was also developed using the experimental results from Chapter 3. The possibilities of constructing pure FE model as well as the potential real-life application are also discussed.

CHAPTER 6

THEORETICAL MODELING OF 1-D PZT–BEAM INTERACTION

6.1 INTRODUCTION

1-D continuous systems, such as uniform beams usually form the basic of investigation for many engineering problems. Their simplicity in terms of geometry and loading conditions allow the development of theoretical model. Generally, experimental verifications may be performed with ease. Fundamental analysis can be carried out on the core mechanisms, where exact solutions can be sought, providing insight into the underlying mechanisms. Extension into more realistic engineering problems is thus possible with approximate solutions such as through FEA.

In this chapter, fundamental theories related to the modeling of EMI technique are reviewed. It should be noted that the modeling has been conducted by researchers such as Giurgiutiu (2007) and Ong (2003). Particular interest is placed on the electro-dynamics of 1-D continuous system, such as simple beam structure, actuated by surface-bonded PZT transducer. The validity of the model is examined through comparison with 3-D FE model and experiment. The ability of EMI technique in structural identification is also presented. The model built in this chapter is then extended to investigate the axially loaded beams in Chapter 7.

6.2 1-D ELECTRO-DYNAMICS OF PZT TRANSDUCER

Consider the dynamic behavior of a PZT transducer surface-bonded on a uniform beam of arbitrary boundary condition as shown in Figure 6.1. As discussed in Chapter 2, the PZT transducer bonded on a surface of the continuous structure is relatively small (non-intrusive) in comparison to the host structure. The mass and stiffness imposed by the transducer on the system can thus be ignored.

Figure 6.1(a): Pictorial illustration of PZT patch surface-bonded on beam structure.

Figure 6.1(b): PZT-structure interaction simplified into equivalent force and moment.

As shown in Figure 6.1, the interaction between the PZT patch and the host structure is linked through the bonding film sandwiched in between them. Extensional vibration of the patch induces shear stresses at the bottom of the patch, which are transmitted to the host structure through the adhesive interface.

Liang and Rogers (1989) showed that the shear stresses concentrate at the end points of the transducer. The force transfer mechanism can be simplified into a pair of opposing forces at two discrete points on the surface of the beam. Based on force equilibrium and compatibility, the unsymmetrical configuration is equivalent to a pair of axial forces and bending moments acting at the center of the beam (Figure 6.1b).

Giurgiutiu and Zagrai (2000) proposed a model for transducer dynamics by an elastically constrained (EC) boundary condition represented by a pair of springs having dynamic stiffness, $2K_{str}$ at both ends of the patch as shown in Figure 6.2. The drive point dynamic stiffness represents the host structure's interaction with the PZT patch at its end points. This approach of modeling allows for a complete range of transducer boundary condition to be investigated.

Figure 6.2: Elastically constrained PZT-structure interaction model (Giurgiutiu and Zagrai, 2000).

Consider a PZT patch (length, l_a , width, b_a and thickness, h_a) being excited by a sinusoidal voltage, the patch resembles a thin bar undergoing axial vibration which can be mathematically expressed using the equation of motion as:

$$c_a^2 u_a'' - \ddot{u}_a = 0 \quad (6.1)$$

where $c_a = \sqrt{\frac{\bar{Y}_{11}^E}{\rho_a}}$ is the wave speed in the PZT patch, $\bar{Y}_{11}^E$ is the complex Young's modulus and ρ_a is the density of the patch. u_a is the displacement in the x -direction with $()'$ and $()^\bullet$ denote spatial and temporal differential operator, respectively.

Assuming a variable separable solution for u_a , in the form of $u_a(x_a, t) = U_a(x_a)e^{j\omega t}$, Equation 6.1 can be rewritten as:

$$c_a^2 U_a'' + \omega^2 U_a = 0 \quad (6.2)$$

The general eigenfunction for 1-D axial vibration is:

$$U_a = A_1 \sin(\gamma x_a) + A_2 \cos(\gamma x_a) \quad (6.3)$$

in which $\gamma = \frac{\omega}{c_a}$ is the wave number, and A_1 and A_2 are constants to be determined.

Note that the angular frequency, $\omega = 2\pi f$, where f is the frequency.

The first boundary condition can be easily identified as $U_a = 0$ when $x_a = 0$. Substituting into Equation 6.3 gives $A_2 = 0$:

$$U_a = A_1 \sin(\gamma x_a) \quad (6.3a)$$

Next, consider the 1-D linear constitutive piezoelectric relationships:

$$S_1 = \frac{\sigma_1}{Y_{11}^E} + d_{31} E_3 \quad (6.4a)$$

$$D_3 = d_{31} \sigma_1 + \overline{\epsilon}_{33}^T E_3 \quad (6.4b)$$

where S_1 and σ_1 represent the mechanical strain and stress respectively, D_3 is the electric displacement, d_{31} is the piezoelectric constant, E_3 is the externally applied electric field and $\overline{\epsilon}_{33}^T$ is the complex electrical permittivity.

One can then utilize the force-displacement relations of the PZT patch at both ends as two of the boundary conditions:

$$A_a \overline{Y}_{11}^E u_a'(-0.5l_a, t) = 2K_{str} u_a(-0.5l_a, t) \quad (6.5a)$$

$$A_a \overline{Y}_{11}^E u_a'(0.5l_a, t) = -2K_{str} u_a(0.5l_a, t) \quad (6.5b)$$

Substituting the above equations into Equation 6.4a and upon rearranging:

$$d_{31}\hat{E}_3 = U_a'(-0.5l_a) - 2\frac{R}{l_a}U_a(-0.5l_a) \quad (6.6a)$$

$$d_{31}\hat{E}_3 = U_a'(0.5l_a) + 2\frac{R}{l_a}U_a(0.5l_a) \quad (6.6b)$$

where $R = K_{str}/K_{PZT}$ is the stiffness ratio between host structure and PZT patch, $K_{PZT} = A_a \overline{Y}_{11}^E / l_a$ and $A_a = b_a h_a$ are the quasi-static stiffness and the cross-sectional area of the patch, respectively. Note that $E_3 = \hat{E}_3 e^{j\omega t}$.

Substituting either one of the above equation into Equation 6.3(a) solves the coefficient of the general eigenfunction:

$$A_1 = \frac{d_{31}\hat{E}_3}{\gamma \cos(\frac{\mathcal{N}_a}{2}) + 2\frac{R}{l_a} \sin(\frac{\mathcal{N}_a}{2})} \quad (6.7)$$

The displacement function of the elastically constrained PZT patch is:

$$u_a = \frac{d_{31}\hat{E}_3}{\gamma \cos(\frac{\mathcal{N}_a}{2}) + 2\frac{R}{l_a} \sin(\frac{\mathcal{N}_a}{2})} \sin(\gamma x_a) e^{j\omega t} \quad (6.8)$$

Consequently, the electric current passing through the PZT patch can be obtained by integrating the derivative of dielectric displacement, $\dot{D}_3$ over the area:

$$I = \int_0^{b_a} \int_0^{l_a} \dot{D}_3 dx_a dy_a \quad (6.9)$$

The dielectric displacement can be obtained by substituting Equations 6.4(a) and 6.8 into Equation 6.4(b), and expressed concisely as:

$$D_3 = \overline{Y}_{11}^E d_{31} (u_a' - d_{31} E_3) + \overline{\epsilon}_{33}^T E_3 \quad (6.10)$$

The electrical admittance $\bar{Y}$ can finally be evaluated by finding the ratio of current, I to applied voltage, $V = E_3 h_a$:

$$\bar{Y} = j\omega C_a \left[1 - \kappa_{31}^2 \left(1 - \frac{1}{\theta \cot(\theta) + R} \right) \right] \quad (6.11)$$

where $\theta = \frac{\mathcal{N}_a}{2}$, $C_a = l_a b_a \overline{\varepsilon_{33}^T} h_a^{-1}$ is the zero load capacitance and $\kappa_{31} = \sqrt{\frac{d_{31}^2 \overline{Y_{11}^E}}{\overline{\varepsilon_{33}^T}}}$ is

the electro-mechanical cross coupling coefficient of the PZT patch.

Equation 6.11 describes the electro-mechanical interaction between the PZT transducer and the host structure, defined by the drive-point dynamic stiffness. A vanishing R term ($R = 0$) implies a free vibration of the patch.

It is worth mentioning that both mechanical and electrical damping effect of the PZT patch are implicitly considered in the Young's modulus and electrical permittivity respectively by the addition of an imaginary term into each parameters:

$$\overline{Y_{11}^E} = Y_{11}^E (1 + j\eta) \quad (6.12)$$

$$\overline{\varepsilon_{33}^T} = \varepsilon_{33}^E (1 - j\delta) \quad (6.13)$$

in which η and μ are the mechanical loss factor and dielectric loss factor, respectively.

6.3 1-D DYNAMICS OF BEAM MODEL

In this section, the structural dynamics of a uniform Euler-Bernoulli beam is expounded and derived from first principle. Both axial and transverse vibrations are considered. The process of extraction of drive point dynamic stiffness and receptance is outlined, and subsequently utilized to estimate the electrical admittance.

Firstly, consider the PZT-beam interaction model as depicted in Figure 6.1(b). The application of a high frequency sinusoidal voltage will excite the PZT patch, and thus actuate the beam structure through the end points of the patch. Harmonic axial forces, N_s and bending moments, M_s will be generated due to the unsymmetrical configuration of the system. This activates both extensional and transverse vibration in the beam. Harmonic elastic waves will be propagating into the structure and upon stabilization, the structure will vibrate at the same frequency as the PZT transducer. Rao (1999) suggests that both motions can be considered uncoupled if the amplitude of vibration is small. Separate solutions for each mode can thus be derived and superimposed to obtain the complete solution.

6.3.1 Extensional Vibration of Beam

Consider a thin beam (Figure 6.1a) of length, l_s width, b_s and thickness, h_s being subjected to extensional forced vibration. Its dynamic response resembles a rod vibrating axially and can be expressed as (Rao, 1999):

$$\bar{Y}_s A_s u'' - \rho_s A_s \ddot{u} = f_n(x, t) \quad (6.14)$$

where u is the displacement in x -direction, $\bar{Y}_s$, A_s and ρ_s are the complex Young's modulus, cross sectional area and material density of the beam, respectively. $f_n(x, t) = \hat{f}_n(x) e^{j\omega t}$ is the distributed loading function.

Basic assumptions of an Euler-Bernoulli beam are adopted including the material is elastic, homogeneous and isotropic, cross sectional plane remains plane after loading and the normal stress in the axial direction is the only stress.

Assuming a variable separable solution of complex harmonic form, $f(x, t) = \hat{f}(x, t) e^{j\omega t}$, Equation 6.14 can be rewritten as:

$$\bar{Y}_s A_s U'' + \rho_s A_s \omega^2 U = \hat{f}_n(x, t) \quad (6.15)$$

Rao (1999) recommended that the spatial component of the distributed loading function can be expressed in terms of the mode shape, $\phi_n(x)$ as:

$$\hat{f}_n(x) = \sum_{n=n_0}^{n_1} F_n \phi_n(x) \quad (6.16)$$

in which F_n is the modal participation coefficient, $n = 1, 2, 3, 4, \dots$ are the mode numbers, n_0 and n_1 are the lower and upper mode numbers encompassing the frequency interval under investigation.

Substituting Equation 6.16 into Equation 6.15, and considering only the n^{th} term:

$$\bar{Y}_s A_s U_n'' + \omega^2 \rho_s A_s U_n = F_n \phi_n(x) \quad (6.17)$$

To solve for the above equation, the free vibration of beam shall first be considered by omitting the forcing function:

$$c_s^2 U_n'' + \Phi_n^2 U_n = 0 \quad (6.18)$$

where $c_s = \sqrt{\frac{\bar{Y}_s}{\rho_s}}$ is the wave speed and Φ_n is the n^{th} natural frequency of the beam in axial vibration.

The solution of Equation 6.18 yields the general eigenfunction:

$$U_n = A_1 \sin\left(\frac{\Phi_n x}{c_s}\right) + A_2 \cos\left(\frac{\Phi_n x}{c_s}\right) \quad (6.19)$$

To evaluate A_1 and A_2 , one needs to impose the boundary conditions. Solutions of various boundary conditions are readily available in vibration textbook such as Inman (2001). In this case, a simply supported boundary condition was considered. For extensional vibration, the simple support is equivalent to free-ended.

Substituting the strain at the left end of the beam, $u'(0, t) = 0$ into Equation 6.19 yields $A_1 = 0$:

$$U_n = A_2 \cos\left(\frac{\Phi_n x}{c_s}\right) \quad (6.20)$$

Similarly, inserting the strain into the right end of the beam, $u'(l_s, t) = 0$, gives:

$$-\frac{\Phi_n}{c_s} A_2 \sin\left(\frac{\Phi_n l_s}{c_s}\right) = 0 \quad (6.21)$$

A_2 shall not be zero for a non-trivial solution, which yields the n^{th} natural frequency:

$$\Phi_n = \frac{n\pi}{l_s} c_s \quad (6.22)$$

The general eigenfunction from Equation 6.20 can thus be rewritten as:

$$U_n = A_2 \phi_n(x) \quad (6.23)$$

where $\phi_n(x) = \cos\left(\frac{n\pi x}{l_s}\right)$ is the n^{th} mode shape of the beam.

Next, the general solution is substituted into Equation 6.17 to search for a particular solution. A_2 may be correspondingly derived as:

$$A_2 = \frac{F_n}{\rho_s A_s (\Phi_n^2 - \omega^2)} \quad (6.24)$$

The total forced response of the system is thus given by:

$$u(x, t) = \sum_{n=n_0}^{n_1} \frac{F_n \phi_n(x)}{\rho_s A_s (\Phi_n^2 - \omega^2)} e^{j\omega t} \quad (6.25)$$

It is worth mentioning that Equation 6.25 is applicable to other boundary conditions subjected to differences in mode shape and natural frequency.

The harmonic actuation from the PZT patch can be characterized as point loads concentrating at both ends of the patch acting on the beam's surface. A space-wise distribution function can be adopted to represent the forcing function:

$$N_s(x, t) = \hat{N}_s [\delta(x - x_1) - \delta(x - x_2)] e^{j\omega t} \quad (6.26)$$

where x_1 and x_2 represents the location of the ends of PZT patch as shown in Figure 6.1, $N_s(x, t)$ is the distributed loading function with $\hat{N}_s$ denotes its amplitude and $\delta(x)$ is the Dirac Delta distribution function.

Force equilibrium between PZT patch and beam gives $\hat{N}_s = \hat{F}_{PZT}$. Consider the axial force distribution on an infinitesimally small beam element, $f_n(x, t) = -N_s(x, t)$. Substituting into Equation 6.16 yields the modal participation factor:

$$\sum_{n=n_0}^{n_1} F_n \phi_n(x) = \hat{F}_{PZT} [\delta(x - x_2) - \delta(x - x_1)] \quad (6.27)$$

One could then orthogonalize the above equation with $\phi_{n'}(x)$, for the n^{th} mode:

$$\hat{F}_{PZT} \int_0^{l_s} [\delta(x - x_2) - \delta(x - x_1)] \phi_{n'}(x) dx = F_n \int_0^{l_s} \phi_n(x) \phi_{n'}(x) dx \quad (6.28)$$

The mode shape shall satisfy the orthogonal property.

If $n \neq n'$

$$\int_0^{l_s} \phi_n(x) \phi_{n'}(x) dx = 0 \quad (6.29a)$$

If $n = n'$

$$\int_0^{l_s} \phi_n(x) \phi_{n'}(x) dx = Q_u \quad (6.29b)$$

The n^{th} modal participation factor can thus be expressed as:

$$F_n = Q_u^{-1} \hat{F}_{PZT} [\phi_n(x_2) - \phi_n(x_1)] \quad (6.30)$$

$Q_u = 0.5l_s$ for simply supported, fixed-ended and cantilevered beam.

6.3.2 Transverse Vibration of Beam

The equation of motion for transverse vibration of an Euler-bernoulli thin beam can be derived from the dynamic equilibrium as given by Rao (1999):

$$\bar{Y}_s I_s w^{iv} + \rho_s A_s \ddot{w} = f_m(x, t) \quad (6.31)$$

where $w(x, t)$ is the displacement in z -direction (Figure 6.1a) and $I_s = h_s^3 b_s / 12$ is the second moment of inertia of the beam. w^{iv} is the forth order partial differential of transverse displacement with respect to x . Assumptions for Equation 6.31 include negligible deformation caused by transverse shear, negligible rotational inertia and no net longitudinal force.

Assume that the solution is variable separable, in which $w(x, t) = W(x)e^{j\omega t}$, Equation 6.31 becomes:

$$\bar{Y}_s I_s W^{iv} - \rho_s A_s \omega^2 W = \hat{f}_m(x) \quad (6.32)$$

where $\hat{f}_m(x) = \sum_{m=m_0}^{m_l} F_m \varphi_m(x)$, F_m and $\varphi_m(x)$ is the modal participation coefficient and the mode shape for transverse vibration, $m = 1, 2, 3, 4 \dots$ are the mode numbers, and m_0 and m_l are the lower and upper mode numbers encompassing the frequency under investigation.

To seek for a general solution, consider the free transverse vibration by dropping the forcing function:

$$\bar{Y}_s I_s W_m^{iv} - \rho_s A_s \Omega_m^2 W_m = 0 \quad (6.33)$$

Ω_m is the m^{th} natural frequency of the beam.

For the transverse vibration of 1-D continuous systems, Warburton (1976) suggested a general solution for the eigenfunction equation as:

$$W_m = A_1 \cosh(\lambda x) + A_2 \sinh(\lambda x) + A_3 \cos(\lambda x) + A_4 \sin(\lambda x) \quad (6.34)$$

where A_1, A_2, A_3 and A_4 are constants and $\lambda = \sqrt[4]{\frac{\rho_s A_s \Omega_m^2}{\bar{Y}_s I_s}}$.

Again, by considering simply supported boundary conditions, $w(0, t) = 0$ and

$w''(0, t) = 0$ yields $A_1 = A_3 = 0$. Equation 6.34 becomes:

$$W_m = A_2 \sinh(\lambda x) + A_4 \sin(\lambda x) \quad (6.35)$$

Then, by considering $w(l_s, t) = 0$ and $w''(l_s, t) = 0$ yields:

$$A_2 \sinh(\lambda l_s) + A_4 \sin(\lambda l_s) = 0 \quad (6.36a)$$

$$A_2 \sinh(\lambda l_s) - A_4 \sin(\lambda l_s) = 0 \quad (6.36b)$$

To achieve a non-trivial solution, $A_2 = 0$ and $\lambda = m\pi / l_s$. The m^{th} natural frequency can be written as:

$$\Omega_m = \sqrt{\frac{Y_s I_s}{\rho_s A_s} \left(\frac{m\pi}{l_s} \right)^4} \quad (6.37)$$

The general eigenfunction of free vibration can be reduced to:

$$W_m = A_4 \varphi_m(x) \quad (6.38)$$

where $\varphi_m(x) = \sin\left(\frac{m\pi x}{l_s}\right)$ is the m^{th} mode shape.

To evaluate A_4 , Equation 6.38 is substituted back to the equation of motion (Equation 6.32) incorporating the forcing function. Upon solving:

$$A_4 = \frac{F_m}{\rho_s A_s (\Omega_m^2 - \omega^2)} \quad (6.39)$$

The corresponding total response of the forced vibration can be derived:

$$w(x, t) = \sum_{m=m_0}^{m_1} \frac{F_m}{\rho_s A_s (\Omega_m^2 - \omega^2)} \varphi_m(x) e^{j\omega t} \quad (6.40)$$

Similar to the extensional vibration, the general expression from Equation 6.40 is applicable to other boundary conditions with differences in natural frequency and mode shape. For free-ended boundary condition, the mode shape and natural frequency are expressed as:

$$\varphi_m(x) = \left[-\frac{\cosh(\lambda l_s) - \cos(\lambda l_s)}{\sinh(\lambda l_s) - \sin(\lambda l_s)} (\sinh(\lambda x) + \sin(\lambda x)) + (\cosh(\lambda x) + \cos(\lambda x)) \right] \quad (6.41)$$

$$\Omega_m = \sqrt{\frac{Y_s I_s}{\rho_s A_s} \left(\frac{(2m+1)\pi}{2l_s} \right)^4} \quad (6.42)$$

To evaluate the modal participation factor, the loading mechanism with a set of space-wise distribution function was first expressed:

$$M_s(x, t) = \hat{M}_s [\delta(x - x_1) - \delta(x - x_2)] e^{j\omega t} \quad (6.43)$$

where $M_s(x, t)$ is the distributed forcing moment (Figure 6.1) with its amplitude equals to $\hat{M}_s$. Based on the moment equilibrium between the PZT patch and the beam, $\hat{M}_s = 0.5(h_s + h_a)\hat{F}_{PZT}$. Again, consider an infinitesimally small beam element:

$$f(x, t) = M_s'(x, t) \quad (6.44)$$

Incorporating the term containing the modal participation factor, Equation 6.43 can be rewritten as:

$$\sum_{m=m_0}^{m_l} F_m \varphi_m(x) = 0.5(h_s + h_a)\hat{F}_{PZT} [\delta'(x - x_1) - \delta'(x - x_2)] \quad (6.45)$$

Orthogonalize Equation 6.45 with $\varphi_{m'}(x)$:

$$\int_0^{l_s} F_m \varphi_m(x) \varphi_{m'}(x) dx = \frac{(h_s + h_a)}{2} \hat{F}_{PZT} \int_0^{l_s} [\delta'(x - x_1) - \delta'(x - x_2)] \varphi_{m'}(x) dx \quad (6.46)$$

Similar to the extensional vibration, considering the orthogonal property:

If $m \neq m'$

$$\int_0^{l_s} \varphi_m(x) \varphi_{m'}(x) dx = 0 \quad (6.47a)$$

If $m = m'$

$$\int_0^{l_s} \varphi_m(x) \varphi_{m'}(x) dx = Q_w \quad (6.47b)$$

Finally, the modal participation factor is expressed in terms of the actuation force by the PZT patch:

$$F_m = Q_w^{-1} \frac{(h_s + h_a)}{2} \hat{F}_{PZT} [\varphi_m'(x_1) - \varphi_m'(x_2)] \quad (6.48)$$

where $Q_w = 0.5l_s$ for simply supported beam.

6.3.3 Drive Point Dynamic Stiffness and Receptance

To derive the electrical admittance, the drive point dynamic stiffness of the host structure is required. With the displacement function for both extensional and transverse vibration under forced vibration by PZT patch derived, one could move a step further to evaluate the drive point dynamic stiffness and receptance of the structure. In continuous linear elastic system, the dynamic stiffness, K_{str} is defined as the ratio between the excitation force to the displacement at that point:

$$K_{str} = \frac{F_{PZT}}{u_{PZT}} \quad (6.49)$$

Since the PZT patch is significantly smaller than the host structure, the end points of the PZT patch are selected as drive points, implying that K_{str} is the point wise dynamic stiffness (Giurgiutiu and Rogers, 1999).

Based on small displacement assumption, displacement in the x -direction of a generic point on the surface of the beam can be derived by superimposing the point-wise extensional and transverse displacement:

$$u_s(x, t) = u(x, t) - \frac{h_s}{2} w'(x, t) \quad (6.50)$$

From response compatibility, the total induced displacement of the PZT transducer is evaluated by finding the difference in displacement between two discrete points (x_1 and x_2) on the surface of the beam, coincident to the ends of the transducer:

$$u_{PZT} = [u(x_2, t) - u(x_1, t)] + \left[\frac{h_s}{2} w'(x_1, t) - \frac{h_s}{2} w'(x_2, t) \right] \quad (6.51)$$

Consequently the dynamic stiffness, K_{str} can be derived by substituting Equations 6.40, 6.25 and 6.51 into Equation 6.49:

$$K_{str} = \rho_s A_s \left[\sum_{n=n_0}^{n_1} \frac{k_u (\phi_n(x_2) - \phi_n(x_1))}{(\Phi_n^2 - \omega^2)} + \left(\frac{h_s}{2} \sum_{m=m_0}^{m_1} \frac{k_w (\phi_m'(x_1) - \phi_m'(x_2))}{(\Omega_m^2 - \omega^2)} \right) \right]^{-1} \quad (6.52)$$

where $k_u = \frac{F_n}{\hat{F}_{PZT}}$ and $k_w = \frac{F_m}{\hat{F}_{PZT}}$ respectively.

Similarly the inverse of dynamic stiffness, which is often known as the dynamic receptance, C_{str} is given as:

$$C_{str} = \frac{1}{\rho_s A_s} \left[\sum_{n=n_0}^{n_1} \frac{k_u (\phi_n(x_2) - \phi_n(x_1))}{(\Phi_n^2 - \omega^2)} + \left(\frac{h_s}{2} \sum_{m=m_0}^{m_1} \frac{k_w (\phi_m'(x_1) - \phi_m'(x_2))}{(\Omega_m^2 - \omega^2)} \right) \right] \quad (6.53)$$

As the total induced displacement of the PZT transducer is obtained by linearly combining the displacement from extensional vibration and transverse vibration, the dynamic receptance can be expediently separated into two components according to the modes of vibration.

Dynamic stiffness of extensional vibration:

$$C_{str,e} = \frac{1}{\rho_s A_s} \left[\sum_{n=n_0}^{n_1} \frac{k_u (\phi_n(x_2) - \phi_n(x_1))}{(\Phi_n^2 - \omega^2)} \right] \quad (6.53a)$$

Dynamic stiffness of transverse vibration:

$$C_{str,t} = \frac{1}{\rho_s A_s} \left[\frac{h_s}{2} \sum_{m=m_0}^{m_1} \frac{k_w (\phi_m'(x_1) - \phi_m'(x_2))}{(\Omega_m^2 - \omega^2)} \right] \quad (6.53b)$$

6.4 SIMULATION OF 1-D THIN BEAM

Consider a free-ended aluminium beam (T6061-T6) of rectangular cross section dimensioned 236mm x 26mm x 4mm with a PZT patch (10mm x 10mm x 0.3mm) surface-bonded at the center. Basic parameters of the beam and the PZT patch, required for the simulation of the analytical model, are tabulated in Table 6.1. A Matlab-coded program is written in Matlab 7.1 workspace to solve for the desired parameters, and is attached in Appendix G.

Table 6.1(a): Geometrical and material properties of aluminium beam.

Parameters	Symbols	Values	Units
Area	A_s	0.000104	m ²
Moment of Inertia	I_s	1.39 x 10 ⁻¹⁰	m ⁴
Density	ρ_s	2715	kg/m ³
Young's modulus	Y_s	68 x 10 ⁹	N/m ²
Mechanical loss factor	μ	0.005	

Table 6.1(b): Material properties of PZT patch.

Parameters	Symbols	Values	Units
Piezoelectric constant	d_{31}	-2.10×10^{-10}	m/V
Electric Permittivity	ϵ_{33}^T	2.12×10^{-8}	F/m
Density	ρ_a	7800	kg/m ³
Young's modulus	Y_{11}^E	62.1×10^9	N/m ²
Mechanical loss factor	η	0.003	
Dielectric loss factor	δ	0.02	

6.4.1 Natural Frequencies of Beam

The natural frequencies of the beam at free-ended boundary condition are acquired to understand its behavior under free vibration. Using Equation 6.22 and 6.42, the natural frequencies between 10 kHz to 100 kHz are calculated (Table 6.2).

Table 6.2: Natural frequencies of free-ended beam.

Extensional vibration		Transverse vibration	
Mode numbers	Natural frequency	Mode numbers	Natural frequency
n	Φ_n	m	Ω_m
1	10.7	8	11.9
2	21.4	9	14.8
3	32.0	10	18.1
4	42.7	11	21.7
5	53.4	12	25.6
6	64.1	13	29.9
7	74.7	14	34.5
8	85.4	15	39.4
9	96.1	16	44.7
		17	50.3
		18	56.2
		19	62.4
		20	69.0
		21	75.9
		22	83.1
		23	90.6
		24	98.5

6.4.2 Theoretical Prediction of Point-Wise Structural Response

The point-wise dynamic receptance of Equation 6.53 are evaluated throughout the frequency range of interest, and illustrated in Figure 6.3. The corresponding admittance signatures are plotted in Figure 6.4. The dynamic stiffness behaves in a similar manner as the dynamic receptance. Since the dynamic receptance better correlates with the admittance signatures, only the dynamic receptance is presented.

Figure 6.3: Plot of dynamic receptance (analytical) versus frequency (10 ~ 100 kHz) of free-ended beam specimen (236mm x 26mm x 4mm).

(a) Real component (b) Imaginary component

Figure 6.4: Plot of admittance signatures (analytical) versus frequency (10 ~ 100 kHz) of free-ended beam specimen (236mm x 26mm x 4mm).
 (a) Conductance signatures (b) Susceptance signatures

Close examination of the resonance frequencies of both dynamic receptance and electrical admittance suggests that the point-wise structural resonance and the electromechanical resonances match seamlessly with each other. Furthermore, the amplitude of the structural resonance, represented by the dynamic receptance, is also closely related to the amplitude of the electromechanical resonances measured by the PZT patch. This proves that the EMI technique is effective in reflecting the point-wise structural vibrational behavior.

6.4.3 Extensional Vibration vs Transverse Vibration

As elucidated in the previous section, the point-wise dynamic receptance consists of two components (Equation 6.53). It is possible to separately evaluate the dynamic receptance, for each mode of vibrations as tabulated in Table 6.3.

Figure 6.5(a) and 6.5(b) depict the dynamic receptance calculated based on the transverse vibration and extensional vibration, respectively, which are plotted on the same scale. There are altogether five and eight resonances being excited for each mode. Further analysis shows that the transverse mode of vibration is more dominant in terms of magnitude than the longitudinal counterparts.

This observation can similarly be observed in the conductance signature plot of the same frequency range as shown in Figure 6.5(c). The magnitudes of peaks from the extensional modes are obviously smaller than those induced by the transverse modes. The resonance frequencies of peaks in the conductance signatures match seamlessly with peaks in the dynamic receptance. Understanding the sources of resonance peaks can be very useful as they have different sensitivity towards different conditions, which is further discussed in the next chapter.

Table 6.3: Comparison of resonance frequencies between dynamic receptance and electrical conductance for free-ended beam derived analytically.

Dynamic receptance		Electrical conductance (kHz)
Extensional (kHz)	Transverse (kHz)	
10.7		10.7
	14.9	14.9
	21.8	21.8
	29.9	29.9
32.05		32.05
	39.45	39.45
	50.25	50.25
53.4		53.4
	62.4	62.4
74.75		74.75
	75.85	75.85
	90.65	90.65
96.1		96.1

Figure 6.5: Plot of resonance peaks induced by extensional and transverse modes of vibration in the frequency range of 10 ~ 100 kHz of beam specimen (236mm x 26mm x 4mm) derived analytically.

- (a) Point-wise dynamic receptance (extensional vibration)
- (b) Point-wise dynamic receptance (transverse vibration)
- (c) Electrical conductance signatures

6.5 VERIFICATION OF ANALYTICAL MODEL

The theoretical model developed in the previous sections has shown its robustness in providing insight into the fundamental mechanism of the PZT-structure interaction. However, the theoretical model is derived based on numerous simplifications and assumptions. Verification of the model is essential to confirm its validity before utilizing it for further study.

In this section, experimental test and 3-D numerical simulation using the FEM are performed to compare against the analytical model. Experimental outcome is most realistic, serving as a control. 3-D FE model is more comprehensive because the entire PZT-structure interaction system including the PZT patch can be simulated. Free-ended boundary condition is applied for all three cases.

For the purpose of comparison, the conductance signatures acquired from the analytical model (Equation 6.11 & 6.52), experiment and FEA are plotted in two different frequency ranges as presented in Figures 6.6 and 6.7 respectively. Higher agreement with the experiment, in terms of resonance frequency implies more accurate model, or a better representation of the PZT-structure interaction system.

A quick examination of Figure 6.6 suggests that both the FE model and analytical model are successful in simulating the vibrational behavior of the system in the frequency range of 10 to 30 kHz as reflected in the reasonably accurate matching of resonance frequencies. However, in the higher frequency range of 32 to 50 kHz (Figure 6.7), the weakness of the analytical model is exposed. Only one resonance peak presents in the analytical model (Figure 6.7a) in comparison to a total of six resonance peaks in the experiment (Figure 6.7b) and the FE model (Figure 6.7c).

The lack of accuracy in the analytical model can be easily explained by looking at its limitations in comparison to the FE model. Firstly, the analytical model is one dimensional in nature, considering only the longitudinal and transverse vibration. The other modes of vibration, including the thickness, width and torsion are ignored. This simplification can be oversimplified in the EMI technique, involving high frequency range of vibration.

Figure 6.6: Plot of conductance signatures versus frequency (10 ~ 30 kHz) acquired from PZT patch surface-bonded on beam specimen (236mm x 26mm x 4mm).

(a) Analytical (b) Experimental (c) Numerical

Figure 6.7: Plot of conductance signatures versus frequency (32 ~ 50 kHz) acquired from PZT patch surface-bonded on beam specimen (236mm x 26mm x 4mm).

(a) Analytical (b) Experimental (c) Numerical

Another obvious shortcoming is the simplification of PZT patch into a pair of forces. In reality, the dynamic interaction between PZT patch and host structure is through the transfer of shear stress over the finite area between the bottom surface of the PZT patch and the top surface of the beam. Simplification of the PZT patch into forces may render certain local modes of vibration being omitted. This shortcoming is overcome by the 3-D FE model, where the simulation incorporates the PZT patch.

However, it should be noted that the 3-D FE model will also lose its accuracy with further increase in frequency. At very high frequency (say larger than 100 kHz), vast amount of local modes may present in the experiment attributed to deficiency in surface finish, edge roughness and imperfect geometrical shape in the host structure or varying bonding thickness and misalignment of the PZT patch, etc., which are hardly, if not impossibly, simulated.

Despite possessing some shortcomings, the analytical model remains useful in providing insight into the PZT-structure interaction system. Moreover, some of the resonance frequencies could still be accurately simulated. In the following chapter, the analytical model will consider the presence of axial load, and its effect on the admittance signatures.

6.6 IDENTIFICATION OF BEAM'S NATURAL FREQUENCIES USING EMI TECHNIQUE

Another interesting observation is made by comparing the resonance frequencies of both the natural vibration and forced vibration of the beam at the same boundary condition. Consider a PZT patch being surface-bonded on the beam at three different locations: at the center, at one-quarter of the beam length and at one-eighth of the beam length. Table 6.4 tabulates the resonance frequencies excited by the PZT patch at the different locations using the natural frequencies as a yardstick.

Figures 6.8(a) and 6.8(b) depict the conductance signatures versus frequency acquired from the theoretical beam model (with PZT patch located at the center and at 1/4 length) in two different frequency ranges. Natural frequencies of the beam for

both extensional and transverse modes of vibrations are marked as discrete points separated by different labels in the same figures.

Table 6.4 summarizes the occurrence of forced resonance frequencies as acquired from the conductance signatures spectrum for PZT patch located at $1/8$, $1/4$ and $1/2$ length of the beam. If the forced resonance frequency is excited, as well as matching the natural frequency, a “Yes” is labeled.

A few useful observations are herein summarized:

1. All of the resonance peaks from the conductance signatures match the natural frequencies. Comparing the numerical values, the resonance frequencies of peaks and the natural frequencies are identical.
2. By varying the PZT patch’s location along the beam, different number of resonance modes can be excited. With the patch located at the center, only half of the natural frequencies are matched. This is conceivable due to geometrical symmetry. At $1/4$ length, all natural frequencies for the transverse modes are matched. Two natural frequencies from the extensional modes are missing. Shifting to $1/8$ length, only one extensional mode is missing.
3. Despite its advantage of being able to excite almost all resonances, the magnitudes are generally weaker in the case of PZT patch at $1/8$ length.
4. The forced resonance peaks for the transverse modes are generally, though not necessarily, stronger than the extensional modes.

Figure 6.8: Plot of conductance signatures versus frequency (natural frequencies of beam shown by discrete markings) acquired from PZT patch surface-bonded on beam specimen derived analytically.

(a) 10 ~ 40 kHz (b) 40 ~ 80 kHz

Table 6.4: Comparison of resonance peaks with natural frequency acquired from PZT patch surface-bonded on beam at varying locations derived analytically.

Extensional vibration				Transverse vibration			
Free vib.	Forced (PZT) vibration			Free vib.	Forced (PZT) vibration		
Nat. freq.	Location of PZT patch			Nat. freq.	Location of PZT patch		
Φ_n	$1/2 l_s$	$1/4 l_s$	$1/8 l_s$	Ω_m	$1/2 l_s$	$1/4 l_s$	$1/8 l_s$
10.7	Yes	Yes	Yes	11.9	--	Yes	Yes
21.4	--	Yes	Yes	14.8	Yes	Yes	Yes
32.0	Yes	Yes	Yes	18.1	--	Yes	Yes
42.7	--	--	Yes	21.7	Yes	Yes	Yes
53.4	Yes	Yes	Yes	25.6	--	Yes	Yes
64.1	--	Yes	Yes	29.9	Yes	Yes	Yes
74.7	Yes	Yes	Yes	34.5	--	Yes	Yes
85.4	--	--	--	39.4	Yes	Yes	Yes
96.1	Yes	Yes	Yes	44.7	--	Yes	Yes
				50.3	Yes	Yes	Yes
				56.2	--	Yes	Yes
				62.4	Yes	Yes	Yes
				69.0	--	Yes	Yes
				75.9	Yes	Yes	Yes
				83.1	--	Yes	Yes
				90.6	Yes	Yes	Yes
				98.5	--	Yes	Yes

To verify the analytical results, a simple experimental test was carried out on the beam specimen having the same dimension but with the PZT patch bonded at one-quarter length of the beam. The outcome is illustrated in Figure 6.9. The forced structural resonances calculated from the theoretical model match reasonably well with the experimental counterparts up to a frequency range of 30 kHz. Similar to the previous cases, many peaks excited by modes other than the extensional and transverse modes beyond 30 kHz shown in the experiment cannot be simulated by the theoretical model.

This finding provides further evidence on the robustness of the admittance signatures of the EMI technique in representing the structural dynamical behavior, or in other words, structural identification. The natural frequencies acquired from the EMI technique are more realistic for the structure concern in comparison to the theoretical prediction. This is because physical measurement can account for all practical conditions including complicated geometrical shape, sophisticated boundary conditions, and the presence of imperfections in the actual structure.

Figure 6.9: Plot of conductance signatures versus frequency (10 ~ 40 kHz) acquired analytically and experimentally from PZT patch surface-bonded at 1/4 length of beam specimen.

6.7 CONCLUDING REMARKS

This chapter presents the step-by-step development of a theoretical model to describe the dynamic interaction between PZT transducer and 1-D beam structure. The PZT patch is modeled as a thin bar with axial vibration, equivalent to a pair of forces and moments. Its interaction with the beam is limited to the patch's end points where the beam is represented by drive point dynamic stiffness. The electrical admittance is found to be representative of the beam dynamic receptance. It is also useful for identification of the beam's natural frequencies. The model shows its robustness in evaluating the admittance signatures based on the extensional and transverse vibration of the beam. Comparison with experiment and FEA suggests that the model can accurately predict the beam resonances up to 30 kHz. Above the range, other modes of vibration emerged which cannot be reflected from the model considering only axial and transverse vibration. The 1-D model is useful in providing insight into the electromechanical interaction between the PZT transducer and the beam. The model is extended to investigate the effect of static axial load in Chapter 7.

CHAPTER 7

EFFECT OF AXIAL STRESS ON ADMITTANCE SIGNATURES UNDER FIXED BOUNDARY CONDITION

7.1 INTRODUCTION

The EMI technique utilizes high frequency of excitations induced by surface-bonded piezo-impedance transducer to actuate the host structure and sense its corresponding vibrational response through electromechanical coupling ability of the piezoelectric material. Changes in vibrational behavior of host structure, including damage, can be indirectly measured from the alterations in electrical impedance/admittance signatures. In the previous chapters, the admittance signatures were acquired under stress free and free-ended boundary conditions, which may not be that realistic in actual application.

In practice, structures in service are often subjected to constant loading, be it static or dynamic (Yang and Miao, 2010). Even with the structure in standby mode, static load caused by self weight, dead load or dynamic load caused by low level of vibration would normally exist. Since the flexural wave propagation properties in structure can be altered by applied stress, the effect of loading should not be ignored.

Abe et al. (2000) first proposed the use of EMI technique for the identification of in-situ stress in thin structural members. Both 1-D and 2-D EMI model based on wave propagation theory for beam and plate structures were developed, incorporating the effect of axial stress. They reported that the wave number and wave frequencies can be directly related to the electrical admittance through the electromechanical coupling equation suggested by Liang et al. (1994). They showed that tensile stress will increase the resonance frequency in the electrical impedance spectrum. Generally, tensile stress tends to stiffen a structure whereas compressive stress would soften a structure. However, they found that considerable discrepancy exist between the

measured and predicted stress. No in depth reasoning is provided.

Ong et al. (2002) investigated the effects of in-situ stress on the frequency response functions of beam structure by applying pure bending actuation through a pair of symmetrically located surface-bonded PZT transducers, activated in out-of phase mode. They simulated the axially loaded beam structure with PZT patch using 1-D EMI model considering an elastically constrained transducer developed by Giurgiutiu and Zagrai (2000). Effect of bonding was neglected. Back calculation of the in-situ stress from the electrical frequency response function (FRF) of the transducer was presented. No experimental results were presented. Both of the abovementioned models concentrate only on the flexural vibration. Compression test was also not conducted.

Annamdas et al. (2007) showed that the electrical admittance signatures, especially its imaginary component, adopted in the EMI technique for SHM is susceptible to variation upon application of static load on the host structure. Annamdas and Soh (2010) reported in a review paper that if the host structure dimensions are smaller than the sensing range of the PZT transducer bonded on it, the boundary conditions can also influence the PZT-based signatures. The boundary conditions in real life structures are however, extremely hard to characterize analytically, and tend to exhibit poor repeatability between structures (David, 2006).

Esteban (1996) concluded that structural discontinuities (at boundaries) acting as the sources of multiple reflections cause maximum attenuation to the propagating waves. Hu and Yang (2007) investigated the PZT sensing region based on the elasticity solution of PZT generated wave propagation and PZT–structure interaction effect. They found that with an excitation voltage of 1V, applied in the typical excitation frequency range of 100–200 kHz for the EMI technique, the valid sensing region of PZT sensors is about 2–2.5 m, subjected to a sensing limit of PZT transducer, i.e. 0.01 V. The sensing zone could be extended with the use of higher excitation voltage. The effect of boundary condition, especially for metallic structure shall be carefully considered for effective SHM.

In a nutshell, there are only a few studies (Ong et al., 2002; Abe et al., 2000)

conducted to investigate the ability of employing PZT transducer in monitoring static load. Annamdas et al. (2007) presented briefly the effect of different boundary conditions on admittance signatures without the presence of loading. However, the effect of boundary condition with varying loading has yet to be investigated.

This chapter aims at bridging this gap, by presenting a series of experimental, analytical and numerical studies to investigate the effect of axial stress on the admittance signatures acquired through the EMI technique in the presence of fixed boundary condition. The problem is approached by using the 1-D analytical model involving axially loaded simple beam with surface-bonded PZT patch. Both axial and flexural vibrations are considered. Experimental test and FE simulation are carried out to verify the model. The effect of loading and boundary condition are discussed.

In the following section, the model developed by Ong (2003) involving an axially loaded Euler-Bernoulli beam is revisited. The EMI model based on elastically constrained transducer conditions (Giurgiutiu and Zagari, 2000) is adopted to evaluate the electrical admittance from the PZT transducer. In this study, the extensional vibration is considered. Experimental tests and FE simulation are presented. Repeatability of admittance signatures and damage detection capability of EMI technique under varying load with fixed boundary condition are also investigated.

7.2 DYNAMICS OF AXIALLY LOADED BEAM

Consider an axially loaded uniform thin beam with a surface-bonded PZT patch as shown in Figure 7.1(a). To investigate the effect of static axial force on the admittance signatures acquired from the EMI technique, the simplified dynamic PZT-structure interaction model is once again considered as depicted in Figure 7.1(b). Similar to the beam model without axial load, the PZT patch's actuation can be modeled by a pair of equivalent axial forces and bending moments which can be separately considered for extensional and transverse vibrations, respectively.

Since the static axial force is acting in the same direction as the extensional vibration, it will not alter the extensional dynamic presented in Equation 6.14.

Therefore, the derivation of the corresponding forced displacement function for axially loaded beam is exactly the same as the one without axial loading as presented in section 6.3.1, and is herein omitted.

However, the effect of axial force is substantial in transverse vibration, which can be incorporated into the dynamic equation for transverse vibration (Equation 6.31) by inserting a second order differential term as suggested by Rao (1986):

$$\bar{Y}_s I_s w^{iv}(x,t) - N_x w''(x,t) + \rho_s A_s \ddot{w}(x,t) = f_m(x,t) \quad (7.1)$$

where N_x is the magnitude of the axial load with positive value denoting tension.

Figure 7.1(a): Schematic representation of PZT patch surface-bonded on axially loaded beam structure.

Figure 7.1(b): Simplified PZT-beam interaction in the presence of axial load.

Again, assuming that the general solution is variable separable in the form of $w(x,t) = W(x)e^{j\omega t}$. Equation 7.1 gives:

$$\bar{Y}_s I_s W^{iv}(x) - N_x W''(x) - \rho_s A_s \omega^2 W(x) = \hat{f}_m(x) \quad (7.2)$$

where $\hat{f}_m(x) = \sum_{m=m_0}^{m_1} F_m \varphi_m(x)$, and F_m and $\varphi_m(x)$ are the modal participation coefficient and the mode shape respectively, for transverse vibration.

Once again, considering free vibration, through dropping the distributed loading function:

$$\bar{Y}_s I_s W_m^{iv}(x) - N_x W_m''(x) - \rho_s A_s \omega^2 W_m(x) = 0 \quad (7.3)$$

The general solution can once again be expressed as:

$$W_m = A_1 \cosh(\lambda x) + A_2 \sinh(\lambda x) + A_3 \cos(\lambda x) + A_4 \sin(\lambda x) \quad (7.4)$$

Consider simply supported boundary conditions, $w(0,t) = 0$, $w'(0,t) = 0$, $w(l_s,t) = 0$ and $w'(l_s,t) = 0$, yields $A_1 = A_2 = A_3 = 0$. The general eigenfunction can thus be reduced to:

$$W_m = A_m \sin\left(\frac{m\pi}{l_s} x\right) \quad (7.5)$$

The eigenfunction derived is identical to the one obtained from the transverse vibration without axial loading (Equation 6.34). The mode shape is again, $\varphi_m(x) = \sin\left(\frac{m\pi}{l_s} x\right)$. The m^{th} natural frequency can then be derived by substituting Equation 7.5 into Equation 7.3:

$$\Omega_m = \sqrt{\frac{1}{\rho_s A_s} \left[\bar{Y}_s I_s \left(\frac{m\pi}{l_s} \right)^4 + N_x \left(\frac{m\pi}{l_s} \right)^2 \right]} \quad (7.6)$$

The above equation is then substituted into Equation 7.2 to seek for particular solution, thereby yields the constant, A_m :

$$A_m = \frac{F_m}{\rho_s A_s (\Omega_m^2 - \omega^2)} \quad (7.7)$$

The forced transverse vibration response of axially loaded beam can thus be found:

$$w(x, t) = \sum_{m=m_0}^{m_1} \frac{F_m}{\rho_s A_s (\Omega_m^2 - \omega^2)} \sin\left(\frac{m\pi}{l_s} x\right) e^{j\omega t} \quad (7.8)$$

Equation 7.8 is identical to Equation 6.40, derived based on the forced transverse vibration of beam without axial loading.

On the other hand, the modal participation factor, the total induced displacement of the PZT transducer and the point-wise dynamic stiffness are also identical to those derived from beam without axial loading, as presented in Chapter 6. The final outcome of the modal participation factor and the dynamic stiffness are herein repeated for the sake of completeness:

$$F_m = Q_w^{-1} \frac{(h_s + h_a)}{2} \hat{F}_{PZT} [\varphi_m'(x_1) - \varphi_m'(x_2)] \quad (7.9)$$

$$C_{str} = \frac{1}{\rho_s A_s} \left[\sum_{n=n_0}^{n_1} \frac{k_u (\phi_n(x_2) - \phi_n(x_1))}{(\omega^2 - \Phi_n^2)} + \left(\frac{h_s}{2} \sum_{m=m_0}^{m_1} \frac{k_w (\varphi_m'(x_1) - \varphi_m'(x_2))}{(\Omega_m^2 - \omega^2)} \right) \right] \quad (7.10)$$

$$K_{str} = \rho_s A_s \left[\sum_{n=n_0}^{n_1} \frac{k_u (\phi_n(x_2) - \phi_n(x_1))}{(\Phi_n^2 - \omega^2)} + \left(\frac{h_s}{2} \sum_{m=m_0}^{m_1} \frac{k_w (\varphi_m'(x_1) - \varphi_m'(x_2))}{(\Omega_m^2 - \omega^2)} \right) \right]^{-1} \quad (7.11)$$

Equations 7.1 to 7.11 elucidate a more generalized interaction between PZT transducer and beam substrate whereby the effect of axial loading on the beam is incorporated. The difference between the load free beam and axially loaded beam is highlighted in the natural frequency, as shown in Equation 7.6. Natural frequency of the beam is directly correlated to the magnitude of axial force. If the axial force is equal to zero, Equation 7.6 reduces to Equation 6.37, representing the natural frequency of beam in transverse vibration without axial loading.

7.2.1 Effect of Axial Force on Natural Frequency

The application of axial force would not affect the extensional vibration but only the transverse vibration, in terms of natural frequency. Thus, a relationship between the natural frequency of a beam in transverse vibration and the magnitude of axial force can be established. Firstly, assuming no axial force, the m^{th} natural frequency of the beam is:

$$\Omega_0 = \sqrt{\frac{\bar{Y}_s I_s}{\rho_s A_s} \left(\frac{m\pi}{l_s} \right)^4} \quad (7.12)$$

Suppose a force of magnitude, N_I is applied. The natural frequency is changed to:

$$\Omega_1 = \sqrt{\frac{1}{\rho_s A_s} \left[\bar{Y}_s I_s \left(\frac{m\pi}{l_s} \right)^4 + N_I \left(\frac{m\pi}{l_s} \right)^2 \right]} \quad (7.13)$$

Rearranging:

$$N_I = \rho_s A_s (\Omega_1^2 - \Omega_0^2) \times \left(\frac{l_s}{m\pi} \right)^2 \quad (7.14)$$

Suppose when the axial force is increased from 0 to N_I , a change in natural frequency, $\Delta\Omega = \Omega_1 - \Omega_0$ occurs:

$$N_I = \rho_s A_s (\Delta\Omega)(2\Omega_0 + \Delta\Omega) \times \left(\frac{l_s}{m\pi} \right)^2 \quad (7.15)$$

Since $\Delta\Omega \ll \Omega_0$, $\Delta\Omega$ can be omitted in the summation term. Substituting Equation 7.12 into Equation 7.15 gives:

$$N_I = \sqrt{4\bar{Y}_s I_s \rho_s A_s} \Delta\Omega \quad (7.16)$$

Equation 7.16 illustrates a linear relationship between the axial force and in natural frequency. Generalizing their relationship:

$$\Delta N \propto \Delta\Omega \quad (7.17)$$

In fact, the outcome suggests that a tensile stress will cause an increase in natural frequency while a compressive stress will lead to a reduction. The variation in natural frequency is directly proportional to the variation in axial force, regardless of the mode number and range of frequency. Thus, it is conceivable that the effect of axial force will be relatively insignificant in the higher frequency range. Equation 7.17 is similar to the outcome presented by Abe et al. (2000).

Using the same aluminium beam (236mm x 26mm x 4mm) presented in Chapter 6, the shifts in natural frequency against axial force (Equation 7.16) are calculated and plotted in Figure 7.2(a). The figure shows a linear reduction in resonance frequency when the force is compressive, and vice versa. The rate of shift (slope) for this particular case is about 0.048 kHz / kN.

Figure 7.2: Variations in natural frequency under different axial load.

(a) Shift in natural frequency versus force

(b) Shift in natural frequency versus stress with varying beam thickness

The value of stress instead of force is plotted on the x -axis as shown in Figure 7.2(b). The figure shows an interesting observation whereby the shift in natural frequency is not only affected by the magnitude of stress but also the cross-sectional area of the beam. Doubling or halving the thickness results in halving or doubling the amount of shift in natural frequency, respectively. Annamdas et al. (2007) presented

similar findings in his experimental study. This observation gives meaningful implication for real-life applications of the EMI technique. In practice, the cross-sectional area of a structure can be much larger than the beam adopted in this study, thus minimizing the contamination of axial load on the admittance signatures.

Knowing that the natural frequencies can be directly reflected by the resonance peaks in the admittance signatures, the relationship between force and frequency shift derived in Equation 7.6 shall be equally applicable to the peak shift in admittance signatures, as presented in the ensuing section.

7.2.2 Effect of Axial Force on Admittance Signatures

To investigate the effect of axial force on admittance signatures, the theoretical electrical admittance signatures of PZT transducer surface-bonded (at centre) on axially loaded aluminum beam (236mm x 26mm x 4mm) is first derived. Figure 7.3 depicts the admittance signatures evaluated at varying tensile forces, up to 10kN, in the frequency range of 10 to 50 kHz. In general, the slope and magnitudes of the signatures are unaffected by the static axial load.

Figure 7.3: Plot of conductance signatures versus frequency (10 ~ 50 kHz) derived theoretically from PZT patch surface-bonded on aluminium beam (236mm x 26mm x 4mm) at varying tensile forces.

Five out of seven of the structural resonance peaks shift progressively to the right with increase in tensile force. These peaks are induced by transverse modes of vibration. Two of the resonance peaks (dotted circle) with relatively low amplitudes, excited by the extensional modes, are unaffected.

7.2.2.1 Resonance peaks induced by transverse mode of vibration

Zooming into 46.5 to 48.5 kHz of Figure 7.3, one could observe a progressive rightward shift in peaks against increasing tension as shown in Figure 7.4(a). In contrary, leftward shift is observed with increasing compression (Figure 7.4b).

Figure 7.4: Plot of resonance peaks (transverse mode) versus frequency (46.5 ~ 48.5 kHz) acquired from beam specimen (236mm x 26mm x 4mm) at varying axial load.

(a) Tensile force (b) Compressive force

To verify the theoretical model, experimental test and FE simulation were performed. In the experiment, tensile force was applied using a Universal Test Machine (UTM) as shown in Figure 7.5. Axial force was applied progressively at 1kN steps up to a maximum of 10kN, equivalent to a maximum stress of 96.2MPa (32% of yield stress). Admittance signatures were acquired after each load step. The load was allowed to stabilize before initiating the signature acquisition process. The baseline signatures (0N) were taken after the specimen was clamped on the UTM.

Two pieces of strain gauges (3mm in length), type FLA-3-11-5LT manufactured by TML, were installed at the center of the beam (opposite to the face with PZT patch) and 5cm from the center of the beam respectively, serving as controllers for load monitoring. The values of strain measured were consistent with the load cell's reading from the UTM. The values of strain recorded near the gripping jaw (located at 5cm from center) are consistently larger than those recorded by the strain gauge at the center. This is caused by the effect of stress concentration near the clamping end.

Giurgiutiu (2007) reported that for a PZT patch surface bonded on aluminium beam which failed at about 7200 micro-strain, the impedance signatures were minimally affected when the strain does not exceed 3000 micro-strain. In this study, the maximum strain is 1360 micro (at 10kN), well below the suggested threshold. The PZT patch remained functional at the end of the test.

Figure 7.5: Pictorial illustration of aluminum beam monitored by PZT transducer and strain gauges tested on UTM.

On the other hand, a 3-D FE model was also developed for the same purpose. In this case, the FEA was performed in two stages. Firstly, a static analysis was performed in the presence of axial load, exerted by applying “structural pressure” at both ends of the beam. The outcome of this static analysis was then brought forward to perform harmonic analysis under the excitation of PZT transducer.

The admittance signatures acquired from the experiment and numerical analysis are plotted in Figures 7.6 (a) and (b) respectively. The frequencies of resonance peak at zero load are 44.55 kHz and 43.55 kHz respectively.

Figure 7.6: Plot of resonance peaks(transverse mode) versus frequency (43 ~ 46 kHz) acquired from PZT patch surface-bonded on aluminium beam (236mm x 26mm x 4mm) at varying tensile load.

(a) Experimental (b) Numerical

Both cases behave in the same manner as the theoretical model where the peaks shift progressively to the right with increase in tensile force. Note that the resonance peaks selected for all three cases were excited by transverse mode of vibration. Similar behavior can be observed in other peaks actuated by this mode of vibration.

It is worth mentioning that the variations between the magnitudes of peaks from experimental, numerical and analytical results are attributed to the uncertainties involved in determining the damping ratios, which are highly depend on trial and error. In this case, the damping ratios are obtained from previous study without performing rigorous trial and error. This is because the magnitude of peak does not affect the resonance frequency, which is the major parameter used in the EMI technique (Yang et al. 2008).

For ease of comparisons, the increase in resonance frequencies obtained from the numerical simulation, the analytical model and the experimental test as well as the theoretical increase in natural frequency are plotted against the axial load as shown in Figure 7.7. Two resonance peaks induced by transverse vibration in different frequency ranges are selected. The peak frequencies without loading obtained from the experiment are 44.6 kHz and 61.9 kHz, respectively.

The results presented herein demonstrate the capability of the numerical and the analytical models in describing the interaction between PZT patch and beam substrate under varying axial load. All of them exhibit a linear relationship as predicted by Equation 7.16. The analytical and numerical results match seamlessly. The experimental results, however, are consistently larger than those obtained by the models. The amount of shift at 10kN is equal to 0.8 kHz for both resonance frequencies in the experiment and 0.5 kHz in both the models. These variations are mainly attributed to the difficulties involved in simulating the stiffening effect caused by the boundary conditions which present only in the experiment. This issue will be further discussed in the ensuing section.

Figure 7.7: Plot of frequency increment versus axial load (0 ~ 10kN) obtained from numerical model, analytical model and experiment.

(a) Frequency of peak at zero load (experiment) = 44.4 kHz

(b) Frequency of peak at zero load (experiment) = 61.9 kHz

7.2.2.2 Resonance peak induced by extensional mode of vibration

Theoretical derivation of the extensional mode of vibration indicates that the corresponding resonance peaks are unaffected by externally applied axial load, as exemplified in Figure 7.8(a) (zoom into 30 to 35 kHz of Figure 7.3). One can easily identify that the admittance signatures for all 11 cases (from 0kN to 10kN) are exactly

the same, overlapping each other and resembling only one curve.

Similar to the previous section, experimental test and FE simulation were carried out with their outcome presented in Figures 7.8(b) and (c) respectively. In the FE model, three scenarios were simulated: without loading, with 5kN and with 10kN. The peaks for all three cases turn out to be at the same frequency despite some variations in the magnitude, thus verifying the analytical counterpart. The other signatures ranging in between 10kN and 0kN can therefore be omitted.

In the experiment, a relatively small amount of shift from 0 to 10kN is recorded as 0.35 kHz. This value is less than half of the transverse mode induced peak shift, which is 0.8 kHz.

Revisiting both the theoretical and FE models, the beams were assumed to be simply supported with forces acting at both ends, without considering any contact with the loading mechanism. In reality, this can never be achieved because the application of axial load mandates certain interaction with the loading instrument. Therefore the peak shift (extensional) observed in the experiment, absent from the theoretical and numerical analysis is most likely to be caused by interaction at the boundary condition.

Figure 7.9 illustrates the actual boundary conditions during the tensile test. The specimen was tightly held by the upper and lower jaws throughout the test.

Despite maintaining the clamping force, the increase in tensile load increases the shear stress required to hold the beam specimen in place, as depicted in the figure. The tensile load was transferred to the beam via the shear stress developed at the interface between the specimen holding jaws and the beam.

The increase in surface shear stress thus stiffened the portion of beam around the clamping zone, resulting in a 0.35 kHz increase in resonance frequency upon loading to 10kN. This observation could provide an explanation to Figure 7.7, in which the shift in resonance frequency for the peak induced by transverse vibration is also about 0.35 kHz more than the theoretical and numerical predictions.

Figure 7.8: Plot of resonance peak (extensional vibration) versus frequency acquired from PZT patch surface-bonded on beam specimen (236mm x 26mm x 4mm) at varying tensile load.

(a) Analytical (b) Numerical (c) Experimental

Figure 7.9: Pictorial illustration of shear stress built up at upper and lower jaws of aluminium beam specimen under tensile test.

Therefore, upon compensating the stiffening effect caused by the boundary condition, Figure 7.7 can be reproduced, as presented in Figure 7.10. A much closer agreement between the experimental test and the models is achieved. The discrepancy between experiment and both models at 10kN is reduced to approximately 0.05 kHz.

This result suggests that the effect of boundary condition must be considered along with the axial load in practical application. Large discrepancy between the theoretical value and experimental measurement as reported by Abe et al. (2000) was likely to be caused by the ignorant of boundary stiffening. Similar compensation can be achieved by using other peak induced by extensional vibration.

It is worth mentioning that the peak acquired from experiment and FE model match each other closer than from the analytical model. This is mainly attributed to the fact that the theoretical model is 1-D in nature, being less accurate than the 3-D FE model. Exact matching is however very difficult due to the difficulty in realistically simulate the boundary condition used in the experiment.

Figure 7.10: Plot of frequency increment versus axial load (0 ~ 10kN) obtained from numerical model, analytical model and experiment, after compensating for stiffening effect from boundary condition.

(a) Frequency of peak at zero load (experiment) = 44.6 kHz

(b) Frequency of peak at zero load (experiment) = 61.9 kHz

7.2.3 Effect of Adhesive Layer on Admittance Signatures under Axial Loading

Both the numerical and analytical models presented in the previous sections ignored the presence of bonding film between the beam and the PZT patch. According to study by Yang et al. (2008a, b), the effect of bonding layer on the resonance

frequencies is negligible when the frequency of excitation is kept within 200 kHz. In this case, the effect of bonding thickness is revisited in the presence of axial load. Since it is impossible to perform experimental study without the presence of axial load, FEM can be used for this purpose.

Following similar procedures outlined in Chapter 4, a FE model was developed with a 0.05mm thick adhesive sandwiched between the PZT patch and the beam. The bonding layer was modeled using Solid 45 element. The properties of the adhesive are outlined in Table 7.1 according to Lim (2006).

The admittance signatures obtained from the FE models with or without loading are plotted against the frequency. One of the resonance peaks at 43.5 kHz is presented in Figure 7.11 for illustration. Both signatures acquired with and without bonding at zero load (dark blue) occurred at 43.5 kHz. On the other hand, both signatures acquired with and without bonding at 5kN (pink) shifted to the right, to 43.7 kHz. The results clearly indicate that the bonding layer did not affect the structural resonance frequency. The amplitudes of the peaks in the presence of bonding layer are slightly smaller due to effect of shear lag.

This observation justifies the assumption made to omit the bonding layer, used in the previous theoretical and analytical models. However, this may not be true at higher frequency range (above 200 kHz) when the PZT patch's resonances come into picture.

Table 7.1: Material properties of bonding film for FE simulation.

Parameters	Symbols	Values	Unit
Density	ρ_b	1000	kg/m^3
Poisson ratio	ν_b	0.4	--
Young's modulus (Isotropic)	Y_b	5.1	$\times 10^9 N/m^2$
Constant stiffness multiplier	β_b	6×10^{-9}	--

Figure 7.11: Plot of conductance signatures versus frequency (42.5 ~ 44.5 kHz) acquired from PZT patch surface-bonded (with and without bonding) on aluminium beam (236mm x 26mm x 4mm) simulated by FEM.

7.3 STIFFENING EFFECT BY BOUNDARY CONDITION FOR BEAM STRUCTURE UNDER COMPRESSION

In this section, two aluminium beams were prepared to investigate the effect of compression on the admittance signatures under fixed boundary condition. A short beam (reusing specimen of the previous tensile test, 236mm x 26mm x 4mm) and a long beam (786mm x 26mm x 4mm) having identical cross sectional area were loaded in compression using a strut rig (TQ SM 105 Mk II). One piece of PZT patch is surface bonded at the center of each specimens.

Maximum load exerted on the short beam was 600N. Both ends of the beam were unconstrained. In this study, a certain amount of compressive load was initially applied to secure the beam on the strut rig, which is treated as the baseline. This is because when the load is below 100N, the boundary condition is very unstable, resulting in instability in the admittance signatures acquired.

The experimental setup for compression test is illustrated in Figure 7.12. Both ends of the beam were left unconstrained. The elastic column buckling load was evaluated using the Euler buckling equation:

$$P_{cr} = \frac{\pi^2 Y_s I_s}{(kL)^2} \quad (7.18)$$

where P_{cr} is the critical buckling load, L is the unsupported length, and k is the dimensionless coefficient. The corresponding critical buckling load calculated for pin-pin boundary condition is 2650N and 240N for short and long beams, respectively.

Figure 7.12: Pictorial representation of compression test where an aluminium beam specimen is loaded on strut rig.

7.3.1 Compression Test on Short Beam

The conductance signatures acquired at various compressive loads for the short beam are plotted in Figure 7.13. Two of the resonance peaks occurring at 32.1 kHz and 45 kHz are selected for illustrations. Note that they are not exactly the same as for the previous tensile test because the initial boundary conditions were different.

Observing the figure, both resonance peaks move to the right, contradicting the theoretical outcome from Equation 7.16 and Figure 7.4, in which compression will soften the host structure. Analyzing the peak movements, one will find that both peaks shifted about 0.3 kHz when loaded to 600N (400N increase in compressive load). The corresponding rates of shift averaged to 0.75 kHz/kN in comparison to -0.048 kHz/kN as calculated theoretically from Equation 7.16. The large variation in magnitude (15

times difference) and opposite in direction suggest that the rightward shift of peak must be induced by some other factors instead of the compressive load. Once again, the stiffening effect from boundary condition is the main factor as proven by another experiment in the ensuing section. Note that the rightward shift of peak happens in almost all peaks throughout the frequency spectrum.

Figure 7.13: Plot of conductance signatures versus frequency acquired experimentally from PZT patch surface-bonded on short aluminum beam (236mm x 26mm x 4mm) at varying compressive load.
 (a) 31.6 ~ 32.6 kHz
 (b) 44.5 ~ 45.5 kHz

7.3.2 Compression Test on Long Beam

The conductance signatures acquired at varying compressive load for the long beam are plotted in Figure 7.14. The peak once again moves to the right when loaded to 200N and 300N. However, upon loading to 400N, the peak becomes stagnant. At 500N, the peak shifted back to the left. Observing the specimen, obvious curvature presents in the beam caused by the P-delta effect at 500N. In other words, the beam failed in buckling at 500N.

This phenomenon leads to two important observations. Firstly, the effect of local stiffening remains dominant (though slightly less than the short beam) over the effect

of compressive stress in spite of the length of long beam (more than three times of the short beam). The PZT patch remains sensitive to the end conditions. Secondly, buckling of beam (a severe damage / failure) induced a leftward shift indicating a stiffness reduction.

Figure 7.14: Plot of conductance signatures versus frequency (49.8 ~ 50.5) acquired experimentally from PZT patch surface-bonded on long aluminium beam (786mm x 25mm x 4mm) at varying compressive load.

7.3.3 Local Stiffening Caused by Boundary Condition

A simple test was conducted to separately investigate the stiffening effect caused by the boundary condition. The same aluminum specimen (short beam) was clamped on a bench vise at one end as shown in Figure 7.15. An attempt was made to minimize the area being clamped to simulate the boundary condition in the compression test.

Clamping force was increased progressively by tightening the screw. Admittance signatures were acquired after each stage. The degree of clamping force applied was labeled as “Wrench” and classified according to the amount of rotation on the screw as tabulated in Table 7.2. Note that no axial force was applied.

Figure 7.16(a) plots the conductance signatures versus frequency with different levels of clamping forces applied. Using resonance peak at 32.2 kHz as baseline, the

peak shifted progressively to the right, similar to previous observation (compression test). The increase in resonance frequency is plotted against relative rotations as presented in Figure 7.16(b).

Table 7.2: Different levels of clamping forces applied on aluminium beam specimen (236mm x 26mm x 4mm) with surface-bonded PZT patch.

Labels	Relative rotations ($^{\circ}$)	Descriptions	Increase in peak's resonance frequency (kHz)
Wrench 0	0	Very loose (Touching)	0
Wrench 1	10	Loose	0.05
Wrench 2	20	Medium	0.1
Wrench 3	40	Tight	0.15
Wrench 4	60	Very tight	0.18
Wrench 5	64	Maximum tightness (by hands)	0.2

Figure 7.15: Pictorial illustration of aluminium beam clamped on bench vise.

Figure 7.16: Effect of local stress on conductance signatures acquired from PZT patch surface-bonded on aluminum beam (236mm x 26mm x 4mm) with one end clamped on bench vice.

(a) Conductance signatures versus frequency (31.7 ~ 32.7 kHz)

(b) Increase in resonance frequency versus rotation

This result clearly shows that the increase in clamping force caused a rightward shift in resonance peaks. Local compressive stress was built up in the vicinity of clamp resulting in local stiffening, inducing a rightward shift in resonance peaks. This observation qualitatively proves the statement suggested in the previous compression test, where the leftward shift of peak caused by compressive force was overshadowed

by the local stiffening at the boundary condition.

It is worth mentioning that the boundary condition in the compression test differed from the tensile test. In the compression test, the surface compressive pressures were increasingly built up around the ends by direct compression, instead of increased in surface shear stress without increase in surface compressive pressure in tensile test. 10kN of shear force in the tensile test and 400N of surface compressive force both induced an approximately 0.3 kHz of stiffening. Obviously, the effect of surface compressive pressure is more dominant than the surface shear stress as well as the axial stress itself.

From three of the tests presented in this section, it can be concluded that the EMI technique is very sensitive to local stress induced by the boundary condition, even if it is fixed. Throughout the experiments, it is found that the admittance signatures acquired from the specimens above are repeatable for each load step upon loading and unloading as long as the initial boundary condition is not disturbed. The effect may also diminish with the increase in length and size of structure, which deserves further investigations.

This suggests that SHM could still be performed by acquiring the admittance signatures using different loads as baseline. Certain compensation algorithm could also be used to prevent false alarm, which demands further study. On the other hand, this area can be further explored to monitor the integrity of structural connections.

7.4 DAMAGE MONITORING USING EMI TECHNIQUE UNDER VARYING LOAD WITH FIXED BOUNDARY CONDITIONS

The previous sections presented studies on the effect of axial loading and boundary conditions on the admittance signatures of the EMI technique. An ensuing question would be the effectiveness of EMI technique in damage monitoring in the presence of axial load and boundary conditions.

Firstly, ensure that the admittance signatures measured remain intact even after the specimen has undergone repetitive tensile and compressive axial load as well as

numerous gripping, without any actual damage. This can be easily achieved by comparing the admittance signatures spectra of the aluminum beam specimen (236mm x 26mm x 4mm) at free-ended boundary conditions before and after the tests, as shown in Figure 7.17.

Obviously, the beam specimen remains intact in spite of all the tests, as reflected from the exact match of resonance peaks' frequencies in the range of 30 to 70 kHz, indicating no change in the vibrational behavior of the beam specimen. Note that all tests conducted did not exceed the linear elastic range.

Figure 7.17: Plot of conductance signatures versus frequency (30 ~ 70) acquired experimentally from PZT patch surface-bonded on aluminum beam (236mm x 26mm x 4mm) at healthy state and after undergoing various tests.

7.4.1 Characterization of Crack Propagation under Varying Compressive Load

7.4.1.1 Considering only axial load using FEM

Knowing the fact that the effect of boundary condition cannot be completely isolated when axial load is applied in experiment, the FEM is used as an alternative tool for investigation. In the FE simulation, axial load can be applied without disturbing the boundary condition.

In the simulation, the beam specimen was modeled with a crack of varying length (along the width direction) at 5cm from the center of the beam. For each crack length, admittance signatures were evaluated with three load cases, namely -10kN, 0kN and 10kN respectively. A resonance peak occurring at 43.5 kHz at healthy state (without loading) was selected as the baseline. A crack with varying length of 3mm, 5mm and 7mm was simulated with 10kN of tensile or compressive load applied.

The admittance signature plot for 5mm crack is shown in Figure 7.18. Consider the resonance peak without loading, the peak shifts to the left relative to its healthy counterpart upon creation of a 5mm crack. Similar observations can be made for the case of tensile or compressive load of 10kN. The amount of shift upon cracking is the same for all three cases.

This can be further exemplified using Figure 7.19, showing the frequencies of the resonance peaks at different loading under different health conditions. The lines at each health condition are parallel to each other (Figure 7.19a), indicating that the amounts of reduction in resonance frequencies are the same in spite of different axial load. The exact amount of shift is reflected in Figure 7.19(b).

Figure 7.18: Plot of conductance signatures versus frequency (42.3 ~ 44.3 kHz) acquired numerically from PZT patch surface-bonded on aluminum beam (236mm x 26mm x 4mm) at healthy state and with 5mm crack in the presence of different axial load.

Figure 7.19: Plot of resonance frequency (peak at 43.5 kHz) at different crack length under varying load (Numerical simulation).

(a) Frequency of peaks versus load

(b) Reduction in resonance frequency versus load

7.4.1.2 Considering axial load and boundary condition by experimentation

(I) Crack propagation

The above numerical study proves that the EMI technique remains effective for damage detection under varying load. Moving a step further, a crack of varying length was induced in the presence of compressive axial load with fixed boundary condition.

Three baseline signatures at 900N, 1000N and 1100N were taken.

In this case, an aluminum beam having the same size as in the previous test (236mm x 26mm x 4mm) was used but the PZT patch was bonded at one-quarter length of the beam. A progressively severing crack of 1mm, 3mm, 5mm, 7mm and 10mm was induced by a handsaw, as shown in Figure 7.20. The thickness of the saw tooth is approximately 0.5mm. For each crack length, admittance signatures were acquired under three different compressive loads, namely 900N, 1000N and 1100N.

Figure 7.20: Aluminium beam specimen (236mm x 26mm x 4mm) with a propagating edge crack monitored by PZT patch surface-bonded at one-quarter length.

Figure 7.21: Plot of conductance signatures versus frequency (45.5 ~ 46.5 kHz) acquired from PZT patch surface-bonded on aluminum beam (236mm x 26mm x 4mm) at different crack length with 900N compressive load.

A resonance peak (46 kHz) at healthy state with 900N compressive load is adopted for characterizing the crack. Admittance versus frequency spectrum for 900N

is selected for illustration as shown in Figure 7.21. The plots for 1000N and 1100N are very similar and can be found in Appendix I. Similar to those without loading, increase in crack length reduces the peak's resonance frequency.

Figure 7.22: Characterization of crack on aluminum beam specimen (236mm x 26mm x 4mm) using PZT patch surface-bonded.
 (a) Reduction in resonance frequency versus crack length
 (b) CCDM index (40 ~ 80 kHz) versus crack length

For the sake of comparison, the frequency of peaks at various crack length under different compressive load are plotted in Figure 7.22(a). The corresponding CCDM

index evaluated in the frequency range of 40 to 80 kHz is presented in Figure 7.22(b).

The figures indicate that the EMI technique remains effective for damage detection in the presence of load with fixed boundary condition. However, the CCDM index suggests that the sensitivity to damage would be slightly lower under larger applied load. The monitoring process is also more cumbersome as acquisition of various baselines at different loading is required.

In the presence of axial load, characterization of crack as small as 1mm remains possible, as exemplified in Figure 7.23. In the presence of compressive load (900N), the EMI technique can characterize cracks of 1mm, 2mm and 3mm in length using the peak at 93.85 kHz.

Figure 7.23: Plot of conductance signatures versus frequency (93.3 ~ 94.3 kHz) acquired from PZT patch surface-bonded on aluminium beam (236mm x 26mm x 4mm) at different crack length with 900N compressive load.

(II) Bolt loosening

Besides cracking, many forms of damage can occur in a structure. Another experimental test was conducted to monitor the loosening of bolts under different compressive axial load using the EMI technique. Three holes of 5mm diameter were created in an aluminum beam specimen (again 236mm x 26mm x 4mm). Three set of identical bolts (5mm diameter) and nuts were tightened by hand through the holes as

shown in Figure 7.24. Damage was simulated by loosening the nuts (360° turn), one at a time starting from the bolt furthest away from the PZT patch, labeled as bolt C. After acquiring the signatures, bolt B was loosened. Finally, bolt A was loosened. For the sake of presentation, each stage of damage was classified into damage levels as tabulated in Table 7.3.

Figure 7.25 illustrates the conductance signatures with different damage levels in the frequency range of 40 to 80 kHz under 900N of compression. In this case, the changes in admittance signatures are more erratic. Identification of peak movement is impossible because many peaks disappeared and new peaks emerged after each stage of damage. Figures for 800N and 1000N show similar results, as attached in Appendix J. This phenomenon is conceivable because loosening one bolt is equivalent to creating a 5mm hole, which can be considered as a severe damage, in view of the size of the beam.

Figure 7.24: Aluminum beam specimen (236mm x 26mm x 4mm) with three bolts and a piece of PZT patch surface-bonded at centre of beam.

Table 7.3: Different levels of damages induced on aluminum beam specimen (236mm x 26mm x 4mm) through loosening of bolts.

Damage levels	No. of bolts loosened	Descriptions
0	0	Healthy (All bolts tightened)
1	1	Bolt C loosened
2	2	Bolt B & C loosened
3	3	Bolt A, B & C loosened
4	--	All bolts removed

Figure 7.25: Plot of conductance signatures versus frequency (40 ~ 80 kHz) acquired experimentally from PZT patch surface-bonded on aluminum beam (236mm x 26mm x 4mm) with 3 bolts loosened progressively with 900N compressive load.

Figure 7.26: CCDM index evaluated from conductance signatures (40 ~ 80 kHz) acquired from PZT patch surface-bonded at center of aluminum beam specimen (236mm x 26mm x 4mm) with different number of bolts loosened.

Thus, the CCDM damage index was used to quantify the severity of damage, as shown in Figure 7.26. In this case, the frequency range from 40 ~ 80 kHz is adopted. The CCDM damage increases progressively as the number of loosened bolts increases.

The CCDM index acquired from damage level 3 and level 4 are identical. This implies that upon loosening all the bolts, the extent of damage is equivalent to no bolts. Referring back to the conductance plot in Figure 7.25, damage level 3 and level 4 are basically overlapping each other, confirming the outcome from CCDM. In this case, the sensitivity to damage detection is much consistent, regardless of the axial load.

Both of the studies presented above concentrate on the damage detection and characterization of the EMI technique under varying static axial load in the presence of fixed boundary conditions. Both sets of results indicate that the EMI technique remains efficient in damage monitoring. However, different baseline signatures are required for each conceivable load. In real-life application, it is recommended to use certain data normalization technique to compensate these effects, for a more efficient SHM. Some of the commonly used data normalization techniques, such as effective frequency shift based on cross-correlation coefficient, artificial neural network (ANN), linear principal component analysis (LPCA), Kernel principal component analysis (KPCA), etc., can be adopted for detecting damage while compensating the effects of loading as well as environmental conditions.

7.5 CONCLUDING REMARKS

This chapter investigates the effect of axial loading and boundary condition on the admittance signatures acquired from the PZT patch. After a brief literature review, a theoretical model was constructed to simulate the interaction between PZT patch and aluminum beam structure in the presence of axial load. Both extensional and transverse modes of vibrations were considered. Axial load is influential to the transverse vibration but not the extensional vibration. Tensile stress will induce a stiffening effect, resulting in an increase in natural frequency as well as the resonance frequency of peaks in the admittance signature spectrum. Theoretically, opposite effect can be found in compression. All results obtained from the theoretical model were verified by the FE simulation. The effect of stiffening by boundary condition

was investigated using extensional mode of vibration, explaining the variations between experiment and models.

Compression test was also conducted but the expected softening effect derived theoretically cannot be observed because it was totally overshadowed by the local stiffening around the boundary condition. The effect of boundary condition is also found to be more dominant than the effect of axial stress.

Damage detection and characterization using the EMI technique remains effective in the presence of load and boundary conditions. Baseline signatures are required for each load. Two types of damages, crack propagation and bolts loosening were simulated. The CCDM index is found to be efficient in characterizing the damages in the presence of axial loading.

CHAPTER 8

CONCLUSIONS AND RECOMMENDATIONS

8.1 INTRODUCTION

This thesis embodies findings from the research carried out on the EMI technique employing surface-bonded piezo-impedance transducers in DP and SHM. This is approached by developing a proof-of-concept semi-analytical damage model capable of monitoring the host structural health and predicting its remaining useful life under fatigue loading.

The proof-of-concept model developed in this study involves simple aluminum beam structures with single propagating edge crack under mode I fatigue loading. The model utilizes the ability of the EMI technique in detecting and characterizing crack as well as the theory of LEFM in predicting the remaining life of fatigue loaded structure. FEM is adopted for more general application. The theory of LEFM is incorporated to provide greater insight into the underlying damage mechanism and structural life estimation. This proof-of-concept model serves as a stepping stone, opening up a path for the development of more rigorous models for fatigue life prediction of real-life structures.

The following sections outline the major contributions, conclusions and recommendations stemming out from this research.

8.2 RESEARCH CONCLUSIONS AND CONTRIBUTIONS

This research attempts to balance between theoretical developments, through analytical and numerical modeling, and experiment, through experimental tests in order to maximize the benefits of the EMI technique. The original contributions of this research can be summarized as follows.

(a) Fatigue Crack Detection and Characterization Using EMI Technique

The conductance (real part of admittance) signatures measured from the PZT transducer are found to be an effective parameter for the detection and characterization of fatigue crack. The repeatability of the signatures at various damaged states is also reasonably high among identical test specimens.

(b) Damage Quantification

A wide range of structural resonance frequency can be selected as damage quantifier, and the reduction in resonance frequency (leftward shift of peak) of the conductance signatures turns out to be a useful index for characterizing fatigue crack. CCDM index, which is sensitive to changes in resonance frequency, can be used as a statistical damage quantifier, enabling automated monitoring. On the other hand, the commonly used RMSD index is found to be less effective in characterizing crack propagation due to its sensitivity towards variations in amplitudes rather than changes in resonance frequency.

(c) Detection of Microscopic Crack

The EMI technique is well-known for its robustness in detecting crack. In this study, its ability to detect incipient crack is extended. The higher frequency range (> 100 kHz) of the admittance signatures is found to be very sensitive to fatigue induced microscopic crack, as small as 0.2mm, which is invisible to naked eyes.

(d) Identification of Critical Crack

Qualitative identification of critical crack is made possible by observing the changes in the frequency spectrum. Whenever new peaks emerge and existing peaks disappear, the host structure is expected to have encountered severe damage (critical crack is reached). As a result, the vibrational behavior and the frequency spectrum would be significantly changed.

(e) FE Simulation of PZT-Structure Interaction in EMI Technique

3-D FE simulation of the PZT-structure interaction in the frequency range normally employed in the EMI technique was conducted in this study. In the 3-D modeling of freely suspended PZT patch, reasonable accuracy was achieved up to 1000 kHz. Simulation of PZT-structure interaction using a lab-sized aluminum beam is also successful. Closed agreement between the numerical and experimental results in terms of structural resonance frequencies was achieved up to *120 kHz*. Damping ratio was identified as the main factor affecting the amplitude of the signatures. Trial and error is required to achieve an optimum value to match the experimental data.

(f) FE Modeling of EMI Technique in Fatigue Crack Monitoring

Through-the-thickness fatigue crack was modeled using FEM by uncoupling the displacements of nodes along the crack. The reduction in resonance frequency simulated in the FE model matches well with the experimental counterparts. It is thus possible to replace experiment with FE model in the application of EMI technique for fatigue crack monitoring.

(g) Semi-Analytical Damage Model for Fatigue Life Estimation

A novel semi-analytical damage model was developed for the estimation of remaining fatigue life of structure with single propagating crack. The model incorporates the outcome from the FE simulation of EMI based fatigue crack monitoring and the theory of LEFM, for the prediction of remaining life. Proof-of-concept application is presented using simple beam structure, which is also experimentally verified. The model is robust because it is standalone from the experiment. Generically, the damage prognosis model can be developed through semi-analytical and numerical modeling, or purely empirical by experimentation.

(h) Theoretical Modeling of 1-D PZT-Beam Structure Interaction

1-D analytical model considering the interaction between surface-bonded PZT transducer and uniform beam was studied. The PZT patch was modeled as a thin bar in axial vibration with an elastically constrained boundary condition. Both extensional and transverse vibrations of beam were modeled. Electrical admittance can be derived by substituting the point-wise dynamic structural stiffness into the electromechanical coupling equation. The corresponding resonance peaks are found to match seamlessly with the natural frequencies of the beam, which strengthens the fact that the electrical admittance is an excellent representation of the structural vibrational behavior. In comparison to the experimental counterparts, matching of resonance peaks are generally accurate up to about 40 kHz, above which some of the peaks measured in the experiment are found to be absent from the analytical model. These peaks, however can be simulated using the 3-D FE model, implying that those resonance peaks are excited by other modes of vibrations, such as width and thickness modes, omitted by the simplified 1-D analytical model.

(i) Theoretical 1-D Interaction Model between PZT Transducer and Axially Loaded Beam

An analytical model describing the interaction between surface-bonded PZT transducer and axially loaded uniform beam is developed. Theoretically, the presence of static axial load only affects the transverse vibration but not the extensional vibration. The only parameter affected is the natural frequency (induced by transverse vibration) of the beam. Application of tensile load will stiffen the structure, inducing an increase in natural frequency. On the other hand, opposite effect is induced by the compressive load. The theoretical model is successfully verified by 3-D FE simulation.

Outcome from the experiment indicates that the actual electrical admittance measured not only contains the effect of axial load, but also the effect of boundary condition. Consequently, the movement of peak induced by the extensional mode of vibration can be used to filter the stiffening effect caused by the boundary condition.

Similar stiffening effect can also be observed in the compression test, resulting in a rightward shift in peak. The theoretical leftward shift induced by compressive force is overshadowed by the stiffening effect from boundary condition.

Conductance signatures acquired from the beam specimen after undergoing various tests show no difference from the baseline (before any test), implying that no damage has been inflicted. The repeatability of admittance signatures and reliability of EMI technique in SHM is further proven.

(j) Damage Detection Capability of EMI Technique under Varying Axial Load with Fixed Boundary Condition

The damage detection capability of EMI technique on host structure under static axial load and constant boundary conditions is investigated through FE simulation and experimental test. The FE model is capable of simulating the propagation of crack in the presence of axial load, without being affected by the boundary condition. Both results show that the damage detection capability is unaffected as long as the baseline signatures are recorded for each conceivable load.

8.3 RECOMMENDATIONS FOR FUTURE WORK

Based on the experiences accumulated throughout this research study, the author believes that the research in the field of EMI technique can be further extended as below:

(a) Fatigue Crack Detection and Characterization using EMI Technique

Further study into crack detection shall be conducted to obtain a better picture on damage sensitivity and optimal frequency range for EMI application. It is worth mentioning that, the EMI technique for fatigue crack monitoring is proven to be effective on lab-sized structures but extension to real-life structures or structural components requires further study. The sensitivity of the technique on a crack may differ between PZT patches and host structures of different sizes.

(b) Parameters for Damage Quantification

In this study, the parameter adopted for damage quantification is the reduction in structural resonance frequency. Although most of the resonance peaks can be used to detect fatigue crack, each of them may possess different sensitivities. This is due to the fact that these resonance frequencies are excited by different modes of vibration. One of the modes may have advantages over the others for a particular type or orientation of damage, and thus deserves further study.

On the other hand, the identification of resonance frequency is carried out physically, which can be cumbersome for real-life application. It is thus recommended to conduct further study to develop an algorithm capable of automatically identifying the peak movements. The use of damage index can be an alternative but systematic application also requires further investigations.

(c) Detection of Microscopic Crack and Identification of Critical Crack

The EMI technique is found to be very sensitive to microscopic crack especially when the higher frequency range is adopted. However, the physical relationship between crack length and frequency has not been established. In this study, we found that the occurrence of critical crack could significantly alter the vibrational behavior of the beam. This is a physical explanation based on observation without in depth understanding and systematic quantification. Further study may focus on theoretical modeling or analysis to establish certain relationship between critical crack length and vibrational behavior.

(d) FE Simulation of EMI Technique in Fatigue Crack Monitoring

In this study, the fatigue crack is simulated by separating the interfacial nodal displacements along the crack. The FE model thus only provides information regarding the admittance signatures acquired from beam structure with varying crack length. The actual crack propagation mechanism is not physically modeled. Future study may focus on developing a more complete numerical model, incorporating the details of crack propagation, such as the stress intensity factor, so that the model can

be more robust, and capable of estimating fatigue life of complicated structures.

(e) Damage Model for Fatigue Life Estimation

The semi-analytical damage model proposed in this thesis is useful for estimating the fatigue life of small-scale structure with single propagating crack. The model may be extended to include structure of more complicated shape, boundary condition and larger size. This would require detail investigation especially for larger size structure because its sensitivity towards crack may be different. Moreover, when more than one crack exists, more than one sensor may be required. The wave propagation technique may also need to be employed for crack localization.

(f) Theoretical Modeling of PZT-Beam Interaction

Theoretical model is very useful in understanding the interaction between the PZT transducer and the host structure. It is found that different modes of vibration possess different sensitivity towards different forms of damages, with varying locations and orientations. Understanding this could be very useful in damage identification and characterization, leading to a more efficient SHM.

(g) Damage Detection Capability of EMI Technique under Varying Axial Load with Fixed Boundary Condition

In real-life applications, varying environmental conditions and fluctuating static loads during operation are inevitable. It is thus essential to establish a robust compensation technique, capable of filtering these adverse effects so as to maintain the effectiveness of EMI technique in damage monitoring. In addition to understanding the physical interaction as presented in this research, it is recommended to study the feasibility of adopting certain data normalization technique such as ANN, LPCA, KPCA, etc., for detecting damage while compensating the effect of loading and boundary condition.

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APPENDIX A

RESULTS & ANALYSIS OF ADMITTANCE SIGNATURES OF BEAM SPECIMENS S2 & S3

Table A1: Relationship between number of cycles, crack length, frequency reduction (peak at 41.4 kHz) and stages of fatigue for specimen S2.

No. of cycles (1000)	Cumulative life cycles (%)	Crack length (mm)	Freq reduction (kHz)	Physical descriptions	Stages of fatigue
0	0.0	0.0	0.00	Intact	Healthy
35	15.6	0.0	0.00	Micro-crack	Crack initiation
55	24.4	0.0	0.00		
100	44.4	0.0	0.03		
120	53.3	0.2	0.05		
160	71.1	1.1	0.18	First crack	Crack propagation
180	80.0	4.2	0.30	Crack increasing steadily	
190	84.4	5.3	0.35		
210	93.3	8.3	0.65		
225	100.0	13.0	1.55	Unstable	Critical crack

Table A2: Relationship between number of cycles, crack length, frequency reduction (peak at 41.4 kHz) and stages of fatigue for specimen S3.

No. of cycles (1000)	Cumulative life cycles (%)	Crack length (mm)	Freq reduction (kHz)	Physical descriptions	Stages of fatigue
0	0.0	0.0	0.00	Intact	Healthy
35	15.9	0.0	0.00	Micro-crack	Crack initiation
55	25.0	0.0	0.00		
100	45.5	0.2	0.05		
120	54.5	0.3	0.10		
160	72.7	1.7	0.13	First crack	Crack propagation
180	81.8	4.2	0.25	Crack increasing steadily	
210	95.5	9.0	0.70		
220	100.0	13.0	1.45	Unstable

Figure A.1: Statistical damage index plotted against number of loading cycles for specimen S2 in the frequency range of 20 ~ 30 kHz

(a) RMSD

(b) CCDM

APPENDIX B

RESULTS & ANALYSIS OF ADMITTANCE SIGNATURES OF BEAM SPECIMENS B1 & B2

Figure B.1: Plot of conductance signatures versus frequency (76.9 ~ 77.8 kHz) acquired from PZT patch surface-bonded on aluminium beam specimen after different number of loading cycles.

(a) B1 (b) B2

Figure B.2: Plot of conductance signatures versus frequency (70 ~ 80 kHz) acquired from PZT patch surface-bonded on aluminium beam specimen B1 between healthy and critical crack (217,000 cycles).

Figure B.3: Statistical damage index plotted against number of loading cycles for specimen B2 in the frequency range of (20 ~ 30 kHz)
(a) RMSD (b) CCDM

APPENDIX C

CONVERGENCE TEST FOR FINITE ELEMENT ANALYSIS

C.1 Wave Speed and Wavelength Based on 1-D Wave Equation

According to Makkonen (2001), to ensure sufficient accuracy of the solution, the size of element should lie between three to five nodal points (two to three elements) per half wavelength for harmonic analysis.

1-D Wave speed:
$$c = \sqrt{\frac{Y}{\rho}}$$

(I) PZT patch

Wave speed = 2923.6m/s

$f_{max} = 1000 \text{ kHz}$, $\lambda \approx 5.0\text{mm}$

- a) No. of elements (0.2mm) per wavelength = 25
- b) No. of elements (0.5mm) per wavelength = 10

(II) Aluminum beam

Wave speed = 5039.4m/s

$f_{max} = 200 \text{ kHz}$, $\lambda \approx 14.6\text{mm}$

- a) No. of elements (2mm) per wavelength = 7
- b) No. of elements (1mm) per wavelength = 14

C.2 Modal Analysis

Modal analysis was performed on aluminum beam (300mm x 50mm x 6mm) using ANSYS 12.1. Percentage errors was evaluated w.r.t 0.5mm.

Table C.1: Modal frequencies and percentage error (relative to 0.5mm) evaluated at different element sizes for aluminium beam specimen (300mm x 50mm x 6mm).

Mode no.	Element sizes									
	2mm		1.5mm		1mm		0.8mm		0.5mm	
	Modal F (Hz)	Err %	Modal F (Hz)	Err %	Modal F (Hz)	Err %	Modal F (Hz)	Err %	Modal F (Hz)	
12	19,822	4.59	19,527	3.03	19,233	0.65	19,108	0.82	18,952	
13	23,197	2.35	23,000	1.48	22,821	0.32	22,749	0.37	22,665	
14	23,928	0.02	23,926	0.01	23,924	0.00	23,923	0.00	23,923	
15	25,599	2.41	25,358	1.45	25,156	0.30	25,081	0.34	24,996	
16	28,150	2.25	27,906	1.36	27,698	0.29	27,619	0.32	27,531	
17	32,646	2.15	32,359	1.25	32,129	0.25	32,048	0.28	31,960	
18	33,686	2.08	33,400	1.22	33,169	0.25	33,085	0.26	32,998	
19	37,673	0.06	37,662	0.03	37,654	0.01	37,651	0.01	37,649	
20	39,785	2.00	39,470	1.20	39,208	0.25	39,109	0.27	39,003	
21	40,432	2.32	40,016	1.26	39,708	0.25	39,610	0.24	39,517	
22	42,675	0.12	42,652	0.06	42,635	0.01	42,630	0.01	42,625	
23	45,194	0.12	45,170	0.06	45,153	0.01	45,147	0.01	45,142	
24	46,483	2.06	46,085	1.18	45,771	0.25	45,659	0.25	45,547	
25	48,722	2.57	48,169	1.41	47,758	0.28	47,626	0.27	47,499	
26	49,268	0.11	49,243	0.06	49,224	0.01	49,218	0.01	49,212	
27	50,550	0.15	50,515	0.08	50,489	0.02	50,481	0.02	50,473	
28	50,738	0.09	50,717	0.05	50,701	0.01	50,695	0.01	50,690	
29	51,984	0.11	51,958	0.06	51,938	0.01	51,932	0.01	51,926	
30	53,586	2.13	53,097	1.20	52,721	0.25	52,591	0.24	52,467	
31	55,592	2.25	55,055	1.26	54,643	0.26	54,500	0.24	54,371	
32	56,204	2.35	55,647	1.33	55,212	0.28	55,057	0.26	54,914	
33	57,175	2.17	56,889	1.66	56,332	0.33	56,145	0.33	55,960	
34	57,618	2.02	57,105	1.12	56,784	0.29	56,622	0.26	56,475	
35	57,857	1.46	57,250	0.40	57,054	0.03	57,039	0.03	57,023	
36	59,927	2.31	59,339	1.30	58,883	0.27	58,722	0.25	58,576	
37	61,602	2.42	61,056	1.51	60,514	0.31	60,327	0.30	60,145	
38	61,762	0.38	61,571	0.07	61,544	0.01	61,536	0.01	61,528	
39	62,772	1.72	62,505	1.29	62,027	0.27	61,860	0.24	61,711	
40	63,127	0.85	62,694	0.15	62,635	0.03	62,616	0.03	62,598	
41	65,230	0.83	65,142	0.70	65,077	0.25	64,914	0.34	64,692	
42	66,838	2.77	65,866	1.27	65,146	0.14	65,058	0.03	65,038	

43	67,133	2.39	66,447	1.34	65,919	0.28	65,734	0.25	65,567
44	69,721	2.65	68,912	1.46	68,306	0.29	68,106	0.27	67,920
45	69,898	0.34	69,790	0.19	69,710	0.04	69,685	0.04	69,660
46	71,792	2.48	71,034	1.40	70,448	0.29	70,241	0.27	70,054
47	73,802	0.39	73,670	0.21	73,573	0.04	73,543	0.04	73,513
48	76,259	3.49	75,107	1.93	74,228	0.38	73,949	0.36	73,685
49	76,683	2.23	76,008	1.33	75,399	0.28	75,192	0.24	75,012
50	76,868	0.89	76,617	0.56	76,567	0.16	76,447	0.33	76,193
51	78,065	2.00	77,526	1.29	76,717	0.21	76,554	0.02	76,538
52	78,607	1.12	77,917	0.23	77,808	0.04	77,774	0.04	77,740
53	82,521	2.37	81,677	1.32	81,030	0.28	80,805	0.24	80,611
54	82,933	0.58	82,713	0.32	82,552	0.06	82,504	0.06	82,453
55	86,332	4.10	84,766	2.21	83,627	0.43	83,271	0.41	82,931
56	86,721	2.58	86,148	1.90	85,164	0.38	84,844	0.36	84,543
57	87,481	1.30	86,556	0.23	86,435	0.04	86,397	0.04	86,360
58	88,808	2.47	87,860	1.37	87,137	0.29	86,887	0.25	86,671
59	91,558	0.75	91,256	0.42	91,020	0.08	90,947	0.08	90,872
60	92,174	0.42	91,983	0.21	91,861	0.04	91,826	0.04	91,791
61	95,493	3.48	94,371	2.26	93,129	0.47	92,696	0.45	92,282
62	95,836	3.10	94,573	1.74	93,558	0.33	93,246	0.31	92,954
63	96,451	3.31	95,185	1.95	94,030	0.38	93,675	0.34	93,361
64	96,818	1.40	95,674	0.20	95,555	0.04	95,516	0.04	95,479
65	99,906	1.12	99,393	0.61	99,021	0.11	98,910	0.12	98,795
66	102,620	2.80	101,380	1.56	100,430	0.32	100,110	0.28	99,827
67	105,070	3.20	104,370	2.51	102,830	0.52	102,300	0.48	101,810
68	105,750	3.59	104,540	2.40	102,970	0.45	102,510	0.41	102,090
69	106,330	1.71	104,830	0.28	104,650	0.06	104,590	0.05	104,540
70	106,880	1.50	105,550	0.24	105,390	0.04	105,350	0.05	105,300
71	107,830	1.21	107,230	0.65	106,800	0.12	106,670	0.12	106,540
72	110,080	2.99	108,650	1.66	107,570	0.34	107,210	0.31	106,880
73	114,290	4.51	112,090	2.50	110,420	0.52	109,850	0.45	109,360
74	114,340	3.66	113,060	2.50	111,390	0.52	110,810	0.46	110,300
75	115,250	3.82	113,700	2.42	112,050	0.49	111,500	0.44	111,010
76	115,900	4.15	113,730	2.20	112,380	0.51	111,810	0.48	111,280
77	115,910	3.71	114,140	2.13	112,940	0.54	112,330	0.51	111,760
78	116,470	3.28	114,870	1.86	113,270	0.12	113,130	0.32	112,770
79	116,790	3.37	115,520	2.25	113,840	0.50	113,270	0.26	112,980
80	117,460	2.78	115,550	1.11	115,090	0.38	114,660	0.33	114,280
81	117,720	2.39	116,380	1.23	115,290	0.07	115,210	0.21	114,970
82	118,210	2.68	116,460	1.16	116,050	0.49	115,480	0.30	115,130
83	119,980	3.36	117,750	1.44	116,220	0.06	116,150	0.06	116,080
84	120,480	2.51	119,970	2.08	118,600	0.48	118,030	0.43	117,530

APPENDIX D

ALGORITHMS FOR DERIVATION OF ADMITTANCE SIGNATURES OF FREELY SUSPENDED PZT PATCH IN MATLAB 7.1 WORKSPACE

D.1 1-D Impedance Based Electromechanical Coupling Equation (Equation 4.11)

$$\bar{Y} = \left\{ -4\pi f \frac{w_a l_a}{h_a} \left[d_{31}^2 Y_{11}^E (t + \eta(r-1) - \delta \varepsilon_{33}^T) \right] \right\} + j \left\{ 4\pi f \frac{w_a l_a}{h_a} \left[\varepsilon_{33}^T + d_{31}^2 Y_{11}^E (r - \eta t - 1) \right] \right\}$$

% This program calculates the analytical value of electrical admittance
 % Obtained from free PZT through 1-D impedance-based electromechanical coupling
 equation (Liang et al. 1994)

% Z = 0

M = dlmread('changing frequency_s.txt','t');
 L = 0.005; H = 0.0003; RHO = 7800; D31 = -0.00000000021;

%In this case, L is half length but w is full width
 Y11E= 66700000000; E33T=0.00000002124; ETA= 0.035; DELTA= 0.015;
 B = D31*D31*Y11E;

f = M(:,1); %frequency in Hz
 N = size(f);

```
for l = 1:N,
    OMEGA(l) = 2*pi*f(l);
    A(l) = 2*OMEGA(l)*2*L*H;

    %Calculation of wave number
    cons = (RHO / (Y11E * (1 + ETA * ETA)))^0.5;
    rl(l) = L * cons * OMEGA(l);
    im(l) = L * cons * OMEGA(l) * (-0.5 * ETA);

    a(l) = (exp(-im(l)) + exp(im(l))) * sin(rl(l));
    b(l) = (exp(-im(l)) - exp(im(l))) * cos(rl(l));
```

```

c(l) = (exp(-im(l)) + exp(im(l))) * cos(rl(l));
d(l) = (exp(-im(l)) - exp(im(l))) * sin(rl(l));
u(l) = c(l) * rl(l) - d(l) * im(l);
v(l) = d(l) * rl(l) + c(l) * im(l);
h(l) = u(l)^2 + v(l)^2;

r(l) = (a(l) * u(l) - b(l) * v(l)) / h(l);
t(l) = (-1.0) * (a(l) * v(l) + b(l) * u(l)) / h(l);

G1(l) = -A(l) * (B * (t(l) + ETA*(-1 + r(l))) - DELTA * E33T);
B1(l) = A(l) * (E33T + B * (-1 + r(l) - ETA * t(l)));

End

Y(:,1) = f/1000;
Y(:,2) = transpose(G1);
Y(:,3) = transpose(B1);

dlmwrite('Y for free PZT_Liang.xls', Y, '\t');

```

D.2 2-D Impedance Based Electromechanical Coupling Equation (Equation 4.13)

$$\bar{Y} = 8\pi f \frac{w_a l_a}{h_a} \left\{ \left[\varepsilon_{33}^T \delta - \frac{d_{31}^2 Y^E}{(1-\nu)} (t + T + \eta(r + R - 2)) \right] + j \left[\varepsilon_{33}^T + \frac{d_{31}^2 Y^E}{(1-\nu)} [r + R - \eta(t + T) - 2] \right] \right\}$$

% This program calculates the analytical value of electrical admittance
 % Obtained from free PZT through 2-D impedance-based electromechanical coupling
 equation (Zhou et al. 1995)

% Z = 0

```

M = dlmread('changing frequency.txt', '\t');
L = 0.005; H = 0.0003; RHO = 7800; D31 = -0.00000000021;

%In this case, l and w are half length and half width
Y11E= 66700000000; E33T=0.00000002124; ETA= 0.035; DELTA= 0.015; mu = 0.3; D =
D31*D31*Y11E/(1-mu);

f = M(:,1); %frequency in Hz
N = size(f);

for l = 1:N,
    OMEGA(l) = 2*pi*f(l);

```



```

C(l) = OMEGA(l)*L*L/H;

%Calculation of wave number
cons = ( (RHO * (1 - mu^2)) / (Y11E * (1 + ETA * ETA)) )^0.5;
rl(l) = L * cons * OMEGA(l);
im(l) = L * cons * OMEGA(l) * (-0.5 * ETA);

a(l) = (exp(-im(l)) + exp(im(l))) * sin(rl(l));
b(l) = (exp(-im(l)) - exp(im(l))) * cos(rl(l));
c(l) = (exp(-im(l)) + exp(im(l))) * cos(rl(l));
d(l) = (exp(-im(l)) - exp(im(l))) * sin(rl(l));
u(l) = c(l) * rl(l) - d(l) * im(l);
v(l) = d(l) * rl(l) + c(l) * im(l);
h(l) = u(l)^2 + v(l)^2;

r(l) = (a(l) * u(l) - b(l) * v(l)) / h(l);
t(l) = (-1.0) * (a(l) * v(l) + b(l) * u(l)) / h(l);
E(l) = E33T + D * (-2 + 2*r(l) - ETA * (2 * t(l)));
F(l) = -E33T * DELTA + D * (-2 * ETA + 2 * t(l) + ETA * 2 * r(l));

G(l) = 4 * (-C(l) * F(l));
B(l) = 4 * (C(l) * E(l));

End

Y(:,1) = f/1000;
Y(:,2) = transpose(G);
Y(:,3) = transpose(B);

dlmwrite('Y for free PZT_Zhou.xls',Y,'\t');

```

D.3 2-D Effective Impedance Based Electromechanical Coupling Equation (Equation 4.15)

$$\bar{Y} = 8\pi f \frac{l_a^2}{h_a} \left\{ \left[\delta \varepsilon_{33}^T - N(t + \eta r - \eta) \right] + j \left[\varepsilon_{33}^T + N(r - \eta t - 1) \right] \right\}$$

% This program calculate the analytical value of electrical admittance
 % Obtained from free PZT through 2-D effective impedance-based electromechanical coupling
 equation (Bhalla et al. 2004)

% Z = 0

```

X = dlmread('changing frequency.txt','t');
L = 0.005; H = 0.0003; RHO = 7800; D31 = -0.00000000021;

%In this case, l and w are half length and half width
Y11E= 66700000000; E33T=0.00000002124; ETA= 0.035; DELTA= 0.015; mu = 0.3;

N = 2*D31*D31*Y11E/(1-mu);
f = X(:,1); %frequency in Hz
U = size(f);

for l = 1:U,
    OMEGA(l) = 2*pi*f(l);
    M(l) = 4*OMEGA(l)*L*L/H;

    %Calculation of wave number
    cons = ( (RHO * (1 - mu^2)) / (Y11E * (1 + ETA * ETA)) )^0.5;
    rl(l) = L * cons * OMEGA(l);
    im(l) = L * cons * OMEGA(l) * (-0.5 * ETA);

    a(l) = (exp(-im(l)) + exp(im(l))) * sin(rl(l));
    b(l) = (exp(-im(l)) - exp(im(l))) * cos(rl(l));
    c(l) = (exp(-im(l)) + exp(im(l))) * cos(rl(l));
    d(l) = (exp(-im(l)) - exp(im(l))) * sin(rl(l));
    u(l) = c(l) * rl(l) - d(l) * im(l);
    v(l) = d(l) * rl(l) + c(l) * im(l);
    h(l) = u(l)^2 + v(l)^2;

    r(l) = (a(l) * u(l) - b(l) * v(l)) / h(l);
    t(l) = (-1.0) * (a(l) * v(l) + b(l) * u(l)) / h(l);

    O(l) = E33T + N * (r(l) - 1 - ETA * t(l)) ;
    P(l) = -DELTA * E33T + N * (t(l) + ETA * r(l) - ETA);
    G(l) = -P(l) * M(l);
    B(l) = O(l) * M(l);
End

y(:,1) = f/1000;
y(:,2) = transpose(G);
y(:,3) = transpose(B);

dlmwrite('Y for free PZT_Effective.xls',y,'t');

```

APPENDIX E

SUSCEPTANCE SIGNATURES FOR FREE-ENDED PZT PATCH OF EXPERIMENT, ANALYTICAL & NUMERICAL MODEL

Figure E.1: Plot of susceptance signatures versus frequency acquired from free-ended PZT patch (10mm x 10mm x 0.3mm).

(a) Experimental vs. numerical (0 ~ 900 kHz)

(b) Experimental vs. analytical (0 ~ 900 kHz)

APPENDIX F

ALGORITHM FOR NUMERICAL INTEGRATION OF FATIGUE LIFE & EVALUATION OF CRITICAL CRACK LENGTH IN MATLAB 7.1 WORKSPACE

F.1 Numerical Integration of Integral (Equation 5.28)

$$N = \frac{1}{C'(\Delta\sigma)^m} \int_{a_0}^{a_f} \left[1.99a^{1/2} - 0.41\frac{a^{3/2}}{w} + 18.7\frac{a^{5/2}}{w^2} - 38.48\frac{a^{7/2}}{w^3} + 53.85\frac{a^{9/2}}{w^4} \right]^{-m} da$$

%call function for integration: quad('Integration_Fa', a₀, a_f)

function poly = Integration_Fa(a)

w = 0.05;

poly = (1.9878*a.^(0.5) - 0.41*a.^(3/2)/w + 18.7*a.^(5/2)/(w^2) - 38.48*a.^(7/2)/(w^3) + 53.85*a.^(9/2)/(w^4)).^(-4.5);

F.2 Evaluation of Critical Crack Length (Equation 5.29)

$$K_C = \sigma_{\max} \sqrt{a_c} \left[1.99 - 0.41\frac{a_c}{w} + 18.7\frac{a_c^2}{w^2} - 38.48\frac{a_c^3}{w^3} + 53.85\frac{a_c^4}{w^4} \right]$$

%AC=solve(ac,'a')

function poly = ac

w = 0.05;

poly = '1.9878*a^(1/2) - 0.41*a^(3/2)/w + 18.7*a^(5/2)/(w^2) - 38.48*a^(7/2)/(w^3) + 53.85*a^(9/2)/(w^4)-0.4088=0';

%To solve for poly: acrit = solve(poly);

APPENDIX G

ALGORITHM FOR EVALUATION OF DYNAMIC STIFFNESS, DYNAMIC RECEPTANCE & ELECTRICAL ADMITTANCE OF 1-D FREE-ENDED BEAM IN MATLAB 7.1 WORKSPACE

% This algorithm is written for free ended 1-D beam actuated by single surface-bonded PZT patch

% Input properties of beam

```
pi = 3.1415926535898;
ls = 0.236; bs = 0.026; hs = 0.004; ha = 0.0003;
```

```
fs = 10000;
fe = 100000;
step = (fe - fs) / 50;
x2 = 0.123;
x1 = 0.113;
```

```
rhos = 2715;
As = hs * bs;
ls = bs * hs^3 / 12;
mu = 0.005;
Ys = 68.95e9 * (1 + mu * j);
cs = (Ys / rhos)^0.5;
```

% Input PZT properties

```
la = 0.01; ba = 0.01;
Aa = ba * ha;
rhoa = 7800; d31 = -2.1e-10; eta = 0.001; delta = 0.02;
E33T = 2.124e-8 * (1 - j * delta);
Y11E = 6.21e10 * (1 + j * eta);
k31 = (d31^2 * Y11E / E33T)^0.5;
Ca = la * ba * E33T / ha;
ca = (Y11E / rhoa) ^ 0.5;
KPZT = Aa * Y11E / la;
```

```

for l = 1 : step
    f(l) = fs + l * 50;
    omega(l) = f(l) * 2 * pi;

    CSTR_Temp_E = 0;
    CSTR_Temp_T = 0;

    %Extensional vibration
    for n = 1 : 100

        OMEGA_E(n) = n * pi * cs / ls;
        phi1_E(n) = cos(n * pi * x1 / ls);
        phi2_E(n) = cos(n * pi * x2 / ls);
        F_E(n) = 1 / (0.5 * ls) * (phi2_E(n)-phi1_E(n));

        Cstr_E(n) = 1 / (rhos * As) * F_E(n) * (phi2_E(n) - phi1_E(n)) / (OMEGA_E(n)^2 -
        omega(l)^2 );

        CSTR_Temp_E = CSTR_Temp_E + Cstr_E(n);

    end

    %Transverse vibration
    for m = 1 : 100
        lambda(m) = (2 * m +1) * pi / (2 * ls);

        OMEGA_T(m) = ( (Ys * ls / (rhos * As) * lambda(m)^4 ) )^0.5;
        beta(m) = - ( (cosh(lambda(m)*ls) - cos(lambda(m)*ls) ) / (sinh(lambda(m)*ls) -
        sin(lambda(m)*ls)) );

        phip1_T(m) = lambda(m) * (beta(m) * (cosh(lambda(m)*x1) + cos(lambda(m)*x1)) +
        sinh(lambda(m)*x1) - sin(lambda(m)*x1) );
        phip2_T(m) = lambda(m) * (beta(m) * (cosh(lambda(m)*x2) + cos(lambda(m)*x2)) +
        sinh(lambda(m)*x2) - sin(lambda(m)*x2) );
        F_T(m) = 1 / ls * (hs+ha)*(phip1_T(m)-phip2_T(m));

        Cstr_T(m) = hs / (2 * rhos * As) * F_T(m) * (phip1_T(m) - phip2_T(m)) /
        (OMEGA_T(m)^2 - omega(l)^2);

        CSTR_Temp_T = CSTR_Temp_T + Cstr_T(m);

    end

    % Combine both extensional and transverse

```

```

CSTR(l) = CSTR_Temp_T + CSTR_Temp_E;
CSTR_E(l) = CSTR_Temp_E;
CSTR_T(l) = CSTR_Temp_T;

% Evaluate KSTR
KSTR(l) = 1 / CSTR(l);

% Derivation of electrical admittance Y
gamma(l) = omega(l) / ca;
theta(l) = gamma(l) * la / 2;

R(l) = KSTR(l) / KPZT;
Y(l) = j * omega(l) * Ca * (1 - k31^2 * (1 - (theta(l) * cot(theta(l)) + R(l))^-1));

End

C(:,1) = f'/1000;
C(:,2) = real(CSTR)';
C(:,3) = imag(CSTR)';

C(:,5) = real(KSTR)';
C(:,6) = imag(KSTR)';

C(:,8) = real(CSTR_E)';
C(:,9) = imag(CSTR_E)';

C(:,11) = real(CSTR_T)';
C(:,12) = imag(CSTR_T)';

C(:,14) = real(Y)';
C(:,15) = imag(Y)';

dlmwrite('C_Y_FreeEnd.xls',C,'t');

```

APPENDIX H

ALGORITHM FOR EVALUATION OF DYNAMIC STIFFNESS, DYNAMIC RECEPTANCE & ELECTRICAL ADMITTANCE OF 1-D AXIALLY LOADED BEAM IN MATLAB 7.1 WORKSPACE

% This algorithm is written for simply supported 1-D beam actuated by single surface-bonded PZT patch in the presence of axial load

% Input properties of beam

```
pi = 3.1415926535898
ls = 0.236; bs = 0.026; hs = 0.004; ha = 0.0003;
```

```
fs = 10000;
fe = 100000;
step = (fe - fs) / 50;
```

```
x2 = 0.123;
x1 = 0.113;
```

```
rhos = 2715;
As = hs * bs;
Is = bs * hs^3 / 12;
mu = 0.005;
Ys = 68.95e9 * (1 + mu * j);
cs = (Ys / rhos)^0.5;
```

% Input PZT properties

```
la = 0.01; ba = 0.01;
Aa = ba * ha;
rhoa = 7800; d31 = -2.1e-10; eta = 0.001; delta = 0.02;
E33T = 2.124e-8 * (1 - j * delta);
Y11E = 6.21e10 * (1 + j * eta);
k31 = (d31^2 * Y11E / E33T)^0.5;
Ca = la * ba * E33T / ha;
ca = (Y11E / rhoa)^0.5;
KPZT = Aa * Y11E / la;
```



```

for K = 0:10
    Nx = 1000*K;

for l = 1 : step
    f(l) = fs + l * 50;
    omega(l) = f(l) * 2 * pi;

    CSTR_Temp_E = 0;
    CSTR_Temp_T = 0;

    %Extensional vibration
    for n = 1 : 100

        OMEGA_E(n) = n * pi * cs /ls;
        phi1_E(n) = cos(n * pi * x1 /ls);
        phi2_E(n) = cos(n * pi * x2 /ls);
        F_E(n) = 1 / (0.5 * ls) * (phi2_E(n)-phi1_E(n));

        Cstr_E(n) = 1 / (rhos * As) * F_E(n) * (phi2_E(n) - phi1_E(n)) / (OMEGA_E(n)^2 -
            omega(l)^2 );

        CSTR_Temp_E = CSTR_Temp_E + Cstr_E(n);

    end

    %Transverse vibration
    for m = 1 : 100

        OMEGA_T(m) = ( (Ys * ls *(m*pi/ls)^4 + Nx * (m*pi/ls)^2) / (rhos * As) )^0.5;
        phip1_T(m) = (m * pi /ls) * cos(m * pi * x1 /ls);
        phip2_T(m) = (m * pi /ls) * cos(m * pi * x2 /ls);
        F_T(m) = 1 / ls * (hs+ha)*(phip1_T(m)-phip2_T(m));

        Cstr_T(m) = hs / (2 * rhos * As) * F_T(m) * (phip1_T(m) - phip2_T(m)) /
            (OMEGA_T(m)^2 - omega(l)^2);

        CSTR_Temp_T = CSTR_Temp_T + Cstr_T(m);

    end

    % Combine both extensional and transverse

    CSTR(l) = CSTR_Temp_T + CSTR_Temp_E;
    CSTR_E(l) = CSTR_Temp_E;

```

```

CSTR_T(l) = CSTR_Temp_T;

% Evaluate KSTR
KSTR(l) = 1 / CSTR(l);

% Derivation of electrical admittance Y
gamma(l) = omega(l) / ca;
theta(l) = gamma(l) * la / 2;

R(l) = KSTR(l) / KPZT;
Y(l) = j * omega(l) * Ca * (1 - k31^2 * (1 - (theta(l) * cot(theta(l)) + R(l))^-1));

end

C(:,1) = f/1000;
C(:,2) = real(CSTR)';
C(:,3) = imag(CSTR)';
C(:,5) = real(KSTR)';
C(:,6) = imag(KSTR)';

C(:,8) = real(CSTR_E)';
C(:,9) = imag(CSTR_E)';

C(:,11) = real(CSTR_T)';
C(:,12) = imag(CSTR_T)';

C(:,14) = real(Y)';
C(:,15) = imag(Y)';

if (K == 0)
    dlmwrite('C_Y_0N.xls',C,'\t');
elseif (K == 1)
    dlmwrite('C_Y_1000N.xls',C,'\t');
elseif (K == 2)
    dlmwrite('C_Y_2000N.xls',C,'\t');
elseif (K == 3)
    dlmwrite('C_Y_3000N.xls',C,'\t');
elseif (K == 4)
    dlmwrite('C_Y_4000N.xls',C,'\t');
elseif (K == 5)
    dlmwrite('C_Y_5000N.xls',C,'\t');
elseif (K == 6)
    dlmwrite('C_Y_6000N.xls',C,'\t');
elseif (K == 7)
    dlmwrite('C_Y_7000N.xls',C,'\t');

```

```
elseif (K == 8)
    dlmwrite('C_Y_8000N.xls',C,'\t');
elseif (K == 9)
    dlmwrite('C_Y_9000N.xls',C,'\t');
elseif (K == 10)
    dlmwrite('C_Y_10000N.xls',C,'\t');

end

end
```

APPENDIX I

CONDUCTANCE SIGNATURES OF CRACKED ALUMINIUM BEAM UNDER COMPRESSION

Figure I.1: Plot of conductance signatures versus frequency (45.5 ~ 46.5 kHz) acquired from PZT patch surface-bonded on aluminum beam (236mm x 26mm x 4mm) at different crack length.

(a) Compression = 1000N (b) Compression = 1100N

APPENDIX J

CONDUCTANCE SIGNATURES OF ALUMINUM BEAM WITH BOLTS LOOSENED UNDER COMPRESSION

Figure J.1: Plot of conductance signatures versus frequency (40 ~ 80 kHz) acquired experimentally from PZT patch surface-bonded on aluminum beam (236mm x 26mm x 4mm) with 3 bolts loosened progressively.

(a) Compression = 800N (b) Compression = 1000N